CAHIER DE RECHERCHE #2604E
Département de science économique
Faculté des sciences sociales
Université d'Ottawa

WORKING PAPER #2604E
Department of Economics
Faculty of Social Sciences
University of Ottawa

Solving the Problem of Socially-Improving Multivariate Tax Reform with s-order Stochastic Dominance: An Application to Egyptian Consumption

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May 2026

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Abstract

This paper integrates two pans of the operations research literature, optimal tax design and stochastic dominance analysis, to characterize indirect tax reforms that improve welfare or reduce poverty. We develop a general multivariate framework based on s-order stochastic dominance that extends existing approaches beyond the traditional two-good setting and beyond second-order dominance. The proposed method identifies budget-neutral tax reforms that are robust for broad classes of social welfare functions. Tax reforms are derived from a convex optimization problem and assessed with a wild bootstrap test adapted to dominance conditions. An application to Egyptian household data illustrates the empirical relevance of the approach and underscores the importance of higher-order dominance criteria for designing socially improving tax policies.

Key words: *Optimal taxation, Stochastic dominance, Poverty, Wild bootstrap.*

JEL Classification: D12, D31, H21, H23

1 Introduction

Operational Research (OR) encompasses a broad range of economic studies that extend beyond tax emissions and optimal production to include issues of redistribution and welfare. Although many research in economics and OR are related to tax emission and optimal production Li, Wang, Cheng, and Nie [2024], Bai, Xu, Gong, and Chauhan [2022], Alegoz, Kaya, and Bayindir [2021], the literature is also aligned with taxation, redistribution and welfare concerns.

A first line of research dedicated to OR and taxation is related to the design of the tax structure within a country. For example, Morini and Pellegrino [2018] employ a genetic algorithm to find the structure of income tax in Italy, which maximizes the redistribution while preventing taxpayers being worse off than with the present tax structure. Brown, Cooper, and Pidd [2006] aim to understand the taxpayers served by the Revenue and their needs within the personal tax system. Using OR techniques, they analyze customer heterogeneity to determine what systems are required in case of heterogeneous needs.

Rather than focusing on the design of tax structures, a second line of research (grounded in stochastic dominance used in OR) examines the effects of actual tax systems on welfare and poverty. While first-order and higher-order stochastic dominance are well-established tools in OR, particularly in portfolio management [Kopa, Post, and Junová, 2026, Levy, 2022] and decision-making [Lagravinese, Liberati, and Resce, 2019, Jiang, Liang, Liang, and Yang, 2018, Montes, Miranda, and Montes, 2014], they can also be employed to assess welfare outcomes. Mussard and Pi Alperin [2021] introduce s-order stochastic dominance to find the main drivers of health inequality in Luxembourg, interpreted as a decrease of welfare in the society.

In our study, we seek to integrate the two previously discussed pans of the literature by characterizing indirect tax reforms that, for any order of stochastic dominance, result in either welfare improvements or a decline in poverty levels in society. The literature linking stochastic dominance to indirect tax reform has developed considerably, driven in large part by Yitzhaki's foundational contributions. In particular, Yitzhaki and Thirsk [1990], Yitzhaki and Slemrod [1991] propose a method for identifying welfare-improving and budget-neutral tax reforms that are valid for any social welfare function with stochastic dominance of second order, later generalized at any order for welfare and poverty by Aaberge [2000, 2009], Aaberge, Peluso, and Sigstad [2019] and by Makdissi and Wodon [2002], Duclos and Makdissi [2004], Duclos, Makdissi, and Wodon [2008]. Their approach shows how tax adjustments can balance redistributive objectives with efficiency considerations, although it is limited to a two-good framework. While Mayshar and Yitzhaki [1996] extend the analysis to a multidimensional setting (*i.e.* multiple goods) without requiring a parametric specification of the social welfare function, their framework remains restricted to second-order stochastic dominance.

We propose a general model of indirect tax reforms with multiple goods, using s-order stochastic dominance, extending existing approaches beyond the traditional two-good setting or the second-order stochastic dominance. It identifies tax reforms that are robustly welfare-improving or poverty-reducing for broad classes of social welfare functions without imposing restrictive parametric assumptions. The search for such reforms is formulated as a convex optimization problem as in Mayshar and Yitzhaki [1996] under a budget-neutrality constraint, but combined with a two-step procedure that integrates optimization and statistical testing. We also introduce a wild bootstrap test, to assess multivariate dominance conditions. By linking consumption behavior, marginal efficiency costs of public funds, and redistribution, the proposed approach bridges operational research and welfare economics. An application to Egyptian household data illustrates the implementation of higher-order dominance criteria for designing socially improving tax policies. The optimal reforms are strongly structured around a food–Energy scheme, whereby increasing subsidies on staple Cereals and financing it through higher taxation of Energy yields gains across the entire income distribution (welfare-improving reform). In addition, Meat consistently emerges as a key financing instrument at higher orders, reflecting its high expenditure elasticity and the fact that taxing it allows redistribution toward goods more intensively consumed by poorer households. These results are statistically validated with wild bootstrap tests and remain stable under increasingly inequality-averse social welfare criteria, highlighting the central role of higher-order dominance in policy design.

To summarize, our three main objectives are the following:

• To search for indirect tax schemes with convex programming that are welfare improving and/or poverty reducing.

• To use a s-order stochastic dominance condition if lower-orders stochastic dominance criteria fail to identify welfare-improving (or poverty-reducing) tax reforms.

• To employ a two-stage wild bootstrap test to assess the statistical significance of the s-order stochastic dominance test.

The paper is organized as follows. Section 2 describes the methodology linking welfare (and poverty) to indirect taxation. Section 3 presents the s-order test to identify welfare-improving and poverty-reducing indirect tax reforms. Section 4 proposes an application on Egyptian consumption. Section 5 closes the paper.

2 Measurement framework

We consider an economy in which consumers face a vector of L consumer prices, q. Working in a partial equilibrium context, we assume without loss of generality that market prices are $\mathbf{1_L}$, a vector of ones. The L goods are subject to a vector t of tax rates, such that $q = \mathbf{1_L} + t$ and $dq_\ell = dt_\ell$, where q_ℓ and t_ℓ denote the price of and the tax rate on good ℓ, respectively.

Consumers are heterogeneous in both preferences and nominal income levels. Let $F_{Y,\Theta}$ denote the joint distribution of income and preferences over the support $[0, a] \times \Psi$, with F_Y the marginal cumulative distribution of income and $F_{\Theta|Y}$ the cumulative distribution of preferences conditional on income.

2.1 Impact of indirect tax reforms on consumers

To analyze the effects of policy reforms on consumer prices, we rely on the equivalent income function $y^E(y, q, q^R, \theta)$ that gives the equivalent income under a reference price vector q^R for a consumer with nominal income y and preferences θ facing prices q. This function is implicitly defined by

$$v\left(y^E\left(y, q, q^R, \theta\right), q^R, \theta\right) = v(y, q, \theta), \tag{1}$$

where $v(\cdot)$ represents the indirect utility function.

Our aim is to identify how a consumer's equivalent income is impacted by changes in tax rates. Applying Roy's identity and using the pre-reform price vector as the reference price, we obtain

$$\frac{\partial y^E\left(y, q, q^R, \theta\right)}{\partial t_\ell} = -x_\ell\left(y, q^R, \theta\right). \tag{2}$$

Equation (2) shows that the consumption of good ℓ by a consumer with preferences θ and income y is a sufficient statistic for evaluating the impact of a marginal change in the tax on that good.

In applied welfare economics, the anonymity axiom implies that the government distinguishes among consumers only by nominal income level. From the policy maker's perspective, what matters is therefore the average impact at each income level y:

$$\begin{aligned} \frac{\partial y^E\left(y, q, q^R\right)}{\partial t_\ell} &= \mathbb{E}_\Theta\left[\frac{\partial y^E\left(y, q, q^R, \theta\right)}{\partial t_\ell}\right] \\ &= -\int_\Theta x_\ell\left(y, q^R, \theta\right) dF_{\Theta|Y}(\theta|Y=y) \\ &= -x_\ell\left(y, q^R\right). \end{aligned} \tag{3}$$

When several tax changes occur simultaneously and these changes have no impact on consumers' nominal income, the total impact is given by

$$dy^E\left(y, q, q^R\right) = -\sum_{\ell=1}^{L} x_\ell\left(y, q^R\right) dt_\ell. \tag{4}$$

Since the pre-reform price vector is always used as the reference price vector, for expositional ease, from now on use a simplified notation $x_\ell(y) = x_\ell\left(y, q^R\right)$.

2.2 Impact of indirect tax reforms on the government budget

The focus of this paper is on identifying directions for budget-neutral marginal tax reforms. The government's indirect tax revenue (budget) is:

$$R(t) = \sum_{\ell=1}^{L} t_\ell \cdot X_\ell, \tag{5}$$

where $X_\ell = \int_0^a x_\ell(y)\, dF_Y(y)$ represents aggregate demand for good ℓ at prices q. A marginal tax reform impacts tax revenue by

$$dR(t) = \sum_{\ell=1}^{L} \left[t_\ell \cdot \frac{\partial X_\ell}{\partial t_\ell} + X_\ell \right] dt_\ell = \sum_{\ell=1}^{L} MR_\ell \cdot dt_\ell. \tag{6}$$

2.3 Budget-neutral indirect tax reforms

For budget-neutral tax reforms, we denote $\delta_\ell = MR_\ell \cdot dt_\ell$ and impose the constraint $\sum_{\ell=1}^{L} \delta_\ell = 0$. Let $MECF_\ell = X_\ell / MR_\ell$ denote the marginal efficiency cost of public funds when taxing good ℓ at consumer prices $q = \mathbf{1_L} + t$. We can then rewrite equation (4) as

$$\begin{aligned} dy^E\left(y, q, q^R\right) &= -\sum_{\ell=1}^{L} MECF_\ell \cdot \delta_\ell \cdot \frac{x_\ell(y)}{X_\ell(q)} \\ &= -\sum_{\ell=1}^{L} MECF_\ell \cdot \delta_\ell \cdot CD_\ell^1(y), \end{aligned} \tag{7}$$

where $CD_\ell^1(y)$ is the first-order consumption dominance curve defined in Makdissi and Wodon [2002]. Following Makdissi and Wodon [2002], we define higher-order consumption dominance curves as $CD_\ell^2(y) = \int_0^y CD_\ell^1(u) dF_Y(u)$ and,

$$CD_\ell^s(y) = \int_0^y CD_\ell^{s-1}(u) dF_Y(u) \text{ for } s \in \{3, 4, \dots\}.$$

Using these curves, we define reform dominance curves as

$$RD^s(y, \delta) = -\sum_{\ell=1}^{L} MECF_\ell \cdot \delta_\ell \cdot CD_\ell^s(y) \quad \text{for } s \in \{1, 2, \dots\}. \tag{8}$$

Consider a government searching for an indirect tax reform that would improve social welfare, defined as,

$$W(F_Y) = \int_0^a w\left(y^E(q, y)\right) dF_Y(y), \tag{9}$$

where $w\left(y^E(q, y)\right)$ represents the social evaluation of the well-being of a consumer with equivalent income y^E. We define the following class of welfare indices:

$$\Omega^s := \left\{ W | w(\cdot) \in C^s(\infty), (-1)^{i+1} w^{(i)}(\cdot) \geq 0 \quad \text{for } i = 1, 2, \dots, s \right\}. \tag{10}$$

Theorem 1. *A necessary and sufficient condition for a marginal tax reform δ to be s-order socially-improving, that is, to increase social welfare weakly for all $W \in \Omega^s$ for a given $s \in \{1, 2, 3, \dots\}$, is that*

$$RD^s(y, \delta) \geq 0 \quad \forall y \in [0, \infty).$$

Proof. See Appendix C. □

It is helpful to provide a normative interpretation of these different classes of social welfare indices. When $s = 1$, welfare indices in Ω^1 weakly increase with individual income. These indices are thus Paretian and obey the well-known symmetry or anonymity axiom: interchanging any two individuals' incomes leaves the social welfare index unchanged. Ordering two distributions by imposing only this requirement is known as first-order dominance and is equivalent to making living standards "parades" walk simultaneously alongside each other and verifying if one weakly dominates the other (an exercise first suggested by Pen, 1971, chapter III). For simplicity, we follow Duclos et al. [2008] and refer to tax reforms that weakly increase any index obeying this minimal requirement as "Pen-improving tax reforms."

Mayshar and Yitzhaki [1996] argue that since the search for Pareto-improving tax reforms is doomed to failure, one should instead focus on indices obeying inequality aversion and search for Dalton-improving tax reforms. However, by imposing only symmetry, anonymity, and monotonicity, Pen-improving tax reforms are more general in that they do not require inequality aversion. This class thus includes all indices obeying inequality aversion together with those that are not inequality averse.

When $s = 2$, $w(\cdot)$ is concave. Welfare indices in Ω^2, therefore, respect the Pigou-Dalton principle of transfers. This principle postulates that a mean-preserving transfer of income from a higher-income person to a lower-income person constitutes a social improvement. Following Mayshar and Yitzhaki [1996], we refer to reforms that are second-order welfare-improving as "Dalton-improving tax reforms", that is, taking into account inequality aversion.

Through their third-order derivative ($s = 3$), social welfare indices belonging to Ω^3 are also sensitive to diminishing transfers [Kolm, 1976]. These transfers are such that a beneficial Pigou-Dalton transfer within the lower part of the distribution, accompanied by an adverse Pigou-Dalton transfer within the upper part of the distribution, will add to social welfare provided that the variance of the distribution is not increased. Kolm [1976] was the first to introduce this condition into the inequality literature, and following Duclos et al. [2008], we refer to third-order welfare-improving tax reforms as "Kolm-improving tax reforms."

For higher orders of dominance, we can invoke the generalized transfer principles of Fishburn and Willig (1984). For instance, for $s = 4$, consider a combination of two exactly opposite and symmetric composite transfers: the first favorable and occurring within the lower part of the distribution, the second unfavorable and occurring within the higher part. Because the favorable composite transfer occurs lower down in the income distribution, indices that are members of the Ω^4 class will respond favorably to this combination. Generalized higher-order transfer principles essentially postulate that as s increases, the weight assigned to transfers occurring at the bottom of the distribution also increases. Blackorby and Donaldson [1978] describe these indices as becoming more Rawlsian.

One can also restrict focus to "poverty-reducing" reforms. As pointed out in Duclos and Makdissi

[2004], poverty indices are equivalent to "censored" social welfare indices, which we define as

$$\Omega^s(z^+) := \left\{ W \;\middle|\; \begin{array}{l} w(\cdot) \in C^s(\infty), w^{(1)}(y) = 0 \quad \forall y \in [z^+, \infty), \\ \text{and } (-1)^{i+1} w^{(i)}(\cdot) \geq 0 \quad \text{for } i = 1, 2, \ldots, s \end{array} \right\}. \tag{11}$$

Theorem 2. *A necessary and sufficient condition for a marginal tax reform δ to be s-order poverty-reducing, that is, to decrease poverty or increase social welfare weakly for all $W \in \Omega^s(z^+)$ for a given $s \in \{1, 2, 3, \ldots\}$, is that*

$$RD^s(y, \delta) \geq 0 \quad \forall y \in [0, z^+].$$

Proof. See Appendix C. □

3 Searching for socially improving tax reforms

Since no change at all (i.e., $dt_\ell = 0$ for all goods) is always a feasible solution, we must arbitrarily fix dt_ℓ for at least one good ℓ. Without loss of generality, we fix this for good 1. If the government wishes to increase the subsidy or reduce the tax on good 1, we set $\delta_1 = -1$ and search for a budget-neutral socially improving tax reform to finance this change. Alternatively, if the objective is to increase the tax on good 1 to decrease taxes or increase subsidies on other goods, we set $\delta_1 = 1$.

3.1 Estimators

To search for an s-order socially-improving or poverty-reducing tax reform, we first estimate the various mathematical objects that constitute the reform dominance curves. We estimate $CD^s_\ell(y)$ and its confidence bands on a grid of points $[0, y_1, y_2, \cdots, y_{d-1}, y^{++}]$ with d points for socially-improving tax reforms. Note that since the dominance condition is on the $[0, \infty)$ interval, one needs to set a y^{++} that is substantially higher than a. For poverty-reducing tax reforms with poverty lines up to z^+, the grid becomes $[0, y_1, y_2, \cdots, y_{d-1}, z^+]$. The estimator of the first-order consumption-dominance curves is,

$$\widehat{CD}^1_\ell(y_j) = \frac{\widehat{\mathbb{E}[X_\ell | Y = y_j]}}{\widehat{X}_\ell}, \tag{12}$$

where $\widehat{X}_\ell$ is the sample average consumption of good ℓ and the expectation $\widehat{\mathbb{E}[X_\ell | Y = y_j]}$ can be estimated using a Nadaraya-Watson estimator or any other non-parametric regression method. For $s \in \{2, 3, \ldots\}$:

$$\widehat{CD}^s_\ell(y_j) = \frac{1}{\widehat{X}_\ell \cdot (s-2)!} \sum_{i=1}^{N} \mathbb{1}\left(y_i \leq y_j\right) \cdot (y_j - y_i)^{s-2} \cdot x_{\ell,i}. \tag{13}$$

The estimator of the marginal efficiency cost of public funds is,

$$\widehat{MECF}_\ell = \frac{\widehat{X}_\ell}{\widehat{X_\ell} + \frac{1}{1+t_\ell} \times \sum_{k=1}^{L} R(t_k) \widehat{\epsilon_{\ell k}}}, \tag{14}$$

where $\widehat{\epsilon_{\ell k}}$ is estimated from a demand system QUAIDS (see Appendix D). Using these estimators, we obtain an expression for the s-th order reform dominance curve:

$$\widehat{RD}^s(y_j, \delta) = \widehat{MECF}_1 \cdot \widehat{CD}_1^s(y_j) - \sum_{\ell=2}^{L} \widehat{MECF}_\ell \cdot \delta_\ell \cdot \widehat{CD}_\ell^s(y_j). \tag{15}$$

This estimator depends on δ, the policy reform vector we seek to identify. Our empirical procedure comprises two steps: (1) finding a potential tax reform vector δ and (2) testing for dominance of the reform using this vector.

3.2 Finding a potential tax reform vector

In the first step, we numerically solve

$$\begin{aligned} V = \quad & \min_\delta \textstyle\sum_{j=1}^{d} \left(\max\left[-\widehat{RD}^s(y_j, \delta), 0\right]\right)^2 \\ & \text{subject to } \textstyle\sum_{\ell=1}^{L} \delta_\ell = 0. \end{aligned} \tag{16}$$

The solution to this optimization problem may be multiple or non-existent. If $V \neq 0$, there is no socially-improving tax reform at order s. If $V = 0$, we have identified a potentially socially-improving tax reform at order s. In this case, we define δ^s as

$$\begin{aligned} \delta^s = \quad & \arg\min_\delta \textstyle\sum_{j=1}^{d} \left(\max\left[-\widehat{RD}^s(y_j, \delta), 0\right]\right)^2 \\ & \text{subject to } \textstyle\sum_{\ell=1}^{L} \delta_\ell = 0, \end{aligned} \tag{17}$$

and proceed to the dominance test. *The interpretation of δ^s is the following:*
Each component $\delta_\ell^s = MR_\ell \cdot dt_\ell$ capture the direction and magnitude of the government's revenue generated by a tax change on good ℓ. A negative value ($\delta_\ell^s < 0$) indicates that the tax on good ℓ is reduced, equivalently that good ℓ is subsidized. A positive value ($\delta_\ell^s > 0$) indicates a tax increase that generates a positive government's revenue. The budget-neutrality constraint $\sum_\ell \delta_\ell^s = 0$ ensures that the revenue raised by taxing some goods is exactly offset by the revenue foregone on others. The normalization $\delta_1^s = \pm 1$ fixes the scale of the reform: it defines good 1 as the numeraire commodity whose tax change is set to one unit, so that all other δ_ℓ^s are expressed relative to this reference. A reform vector δ^s derived from program (17) identifies which goods should bear a greater fiscal burden and which should be reduced, in a way that is robustly welfare-improving or poverty-reducing at dominance order s.

In practice, we implement this step with the Adam optimizer, complemented by multiple random restarts. This choice is motivated by the fact that, although the criterion is convex, the estimated objective may become irregular and locally non-convex because of sampling noise, the kink introduced by the $\max[\cdot, 0]$ operator, and the dependence of $\widehat{RD}^s$ on estimated MECFs and demand elasticities. Adam provides a robust first-order method in this environment, thanks to its adaptive learning rates and its empirical performance on non-smooth, high-dimensional problems.

3.3 Stochastic dominance test

This section presents a testing procedure that builds on Schechtman, Shelef, Yitzhaki, and Zitikis [2008] and Khaled, Makdissi, and Yazbeck [2018], except that here we test whether the reform dominance curve lies everywhere above zero. Assume we have an i.i.d. sample S of size n from our population of interest. For the dominance test in Theorem 1, we formally define the null and alternative hypotheses as:

$$\begin{aligned} H_0 &: RD^s(y) \geq 0, \forall y \in [0, a] \\ H_1 &: RD^s(y) < 0, \text{ for some } y \in [0, a]. \end{aligned}$$

A similar test can be constructed for Theorem 2. Importantly, we do not attempt to establish dominance by imposing a null of non-dominance. Instead, we impose a null of dominance and test whether this null can be rejected. We adopt this testing approach, which may seem counterintuitive, for two reasons. First, in a similar context, Davidson and Duclos [2013] have shown that testing a null hypothesis of non-dominance requires strong evidence against the null, which may be difficult to obtain. Second, since the conditions in Theorems 1 and 2 require only weak dominance, we follow the standard practice from the empirical stochastic dominance literature and test both $H_0^1 : RD^s(y) \geq 0, \forall y \in [0, a]$ and $H_0^2 : RD^s(y) \leq 0, \forall y \in [0, a]$. Table 1 displays the decision rules for the dominance tests. For a significance level α, we conclude that there is strong evidence in favor of a dominant reform if the p-value of the first test is greater than or equal to α while the p-value of the second test is strictly lower than α.

Table 1: Interpretation of dominance tests for a significance level α

p-values	Interpretation
$p_1 \geq \alpha$ and $p_2 \geq \alpha$	$RD^s(y, \delta) = 0, \forall y \in [0, a]$
$p_1 < \alpha$ and $p_2 \geq \alpha$	$RD^s(y, \delta) \leq 0, \forall y \in [0, a]$
$p_1 \geq \alpha$ and $p_2 < \alpha$	$RD^s(y, \delta) \geq 0, \forall y \in [0, a]$
$p_1 < \alpha$ and $p_2 < \alpha$	$RD^s(y, \delta)$ intersects the horizontal axis

Let $\tau = \sup_y RD^s(y, \delta)$. It is straightforward to construct Kolmogorov-Smirnov type directional test statistics that are non-parametric estimators of τ. For H_0^1, we use

$$\widehat{\tau}_1 = \sqrt{n} \sup_y \left[-\widehat{RD}^s(y, \delta) \right], \tag{18}$$

and for H_0^2, we use

$$\widehat{\tau}_2 = \sqrt{n} \sup_y \widehat{RD}^s(y, \delta). \tag{19}$$

The asymptotic distribution of $\widehat{\tau}_1$ and $\widehat{\tau}_2$ is that of a functional of a two-dimensional Gaussian process, which is very complicated to compute.[1] To overcome this computational challenge, we use a bootstrap approach. For a detailed description of the bootstrap procedure, please refer to Appendix A.

4 Application on Egyptian data

4.1 Data

To implement our methodology, we use the 2021/2022 Egyptian Household Income, Expenditure, and Consumption Survey (HIECS), conducted by the Central Agency for Public Mobilization and Statistics (CAPMAS) and accessed through the harmonized files provided by Economic Research Forum (ERF) via OAMDI.[2] This survey is nationally representative of the Egyptian household population and is composed of 12,463 observations. It provides detailed information on income and expenditures, goods and Energy consumption used for self consumption, as well as socio-economic characteristics of all household members, thereby offering a rich micro-data framework for the analysis of consumption behavior and welfare distribution.

The price data used in this study are taken from the monthly Consumer Price Index (CPI) bulletin published by CAPMAS (February 2021). CAPMAS collects data monthly from the 1st to the 28th of each month in both urban and rural areas. The index is calculated using the 2018/2019 period as the base period (100).

Regarding the poverty line, since the value for 2021 was not available, we used the 2018 threshold (857 EGP per person per month), adjusting it based on the information revealed by the Vice Minister of Planning, Ahmed Kamali, in September 2021, according to which the poverty line for 2021/2022 would have increased by about 63% to reach 1,400 EGP per person per month compared to the 2019 level. In our study, the poverty line is $z \approx 46$ EGP per person per day. As for subsidy rates, they are obtained by cross-referencing several reliable sources.

Consistent with the consumption and welfare literature, we use total household expenditure as a proxy for living standards, as it better captures permanent resources in contexts where income may be volatile or underresported. Expenditures are desaggreagated into eleven broad commodity groups, including food categories and Energy-related items such as electricity and other essential utilities.

Table 2 shows that household spending is heavily concentrated on a few essential goods. Energy and Meat alone account for nearly 40% of Egyptian households' consumer spending while grains account for 12% and Vegetables, accounting for just over 13% of total household spending, while Fruits and Fish each account for less than 6%. More than 98% of households consume nine of the eleven commodity groups, with the exception of the Fish and Seafood (consumed approximately by 76% of households) and other food products (consumed by less than 83% of households). Like

[1] Chernozhukov, Fernandez-Vál, and Melly (2013) refer to this issue as the Durbin problem.
[2] Open Access Micro Data Initiative

others, Cereals and bread are a staple food almost everywhere.

Table 2: Expenditures and Budget shares

Expenditure Item	**Init.**	**Sum**	**Share**	**% HH consuming**
Cereals and bread	CB	42.07	12.00	99.96
Meat	M	87.70	22.82	98.75
Dairy products	DP	38.49	10.5	99.75
Oil and fat	OF	26.97	7.57	99.59
Fish and Seafood	FS	21.59	5.71	75.92
Fruits	F	19.42	5.31	99.11
Vegetables	V	49.19	13.93	99.95
Sugar and confectionery	SC	14.11	3.87	99.69
Electricity, gas & fuels	EGF	51.79	14.99	99.93
Beverages	B	11.51	3.12	99.82
Other food products	OFP	2.67	0.76	82.56

Notes: $N = 12{,}463$ households. Sums are sample aggregates in billions of EGP
Total consumer spending: 365,55 billions of EGP (Egyptian pounds).
HIECS survey (2021/2022). % HH consuming weighted by `hweight`.
Sum equals total expenditures, in billions of Egyptian pounds .

4.2 MECFs computation

In order to calculate the marginal efficiency cost of public funds, we need to know how sensitive households are to price changes. To do this, we estimate a Quadratic Almost Ideal Demand System (QUAIDS) model, which will give us income elasticities, direct price elasticities, and cross-price elasticities (non-compensating). Eleven commodities groups are considered, and total consumer spending was considered a proxy for income. This measure is preferred over reported income, as the latter is more susceptible to transitory shocks and measurement errors.

As mentioned by Mayshar and Yitzhaki [1996], the marginal efficiency cost of public funds captures the loss of social welfare per unit of tax revenue collected, taking into account both behavioral responses (elasticities) and distributional weights. It measures the trade-off between an increase in government revenues and the erosion of social welfare included by changes in tax rates. Within the framework pioneered by Ramsey [1927], the optimal tax system is achieved when the social planner equalizes marginal distorsions across all taxed goods. At the second-best steady state, any reallocation of the tax burden between two different goods is neutral in term of efficiency, since the ratio of marginal cost is equal to unity. Consequently, no Pareto-improving reform can be achieved through a marginal reallocation of taxe rates alone. At the Ramsey optimum, all MECFs are equal.

We employ a demand system approach to estimate expenditure and price elasticities based on the QUAIDS model developed by Banks, Blundell, and Lewbel [1997]. It models budget shares as functions of prices, total expenditure and some demographics, while capturing nonlinear Engel effects via a quadratic term in log-expenditure. The expenditure elasticities estimated from this model (Table 3) reveal a clear partition of goods according to their nature. Meat (1.43), Fish and

Seafood (1.33), Fruits (1.14), Energy (1.07) and Dairy products (1.05) display elasticities above unity, classifying them as luxury goods whose consumption grows more than proportionally with income. Leksono, Anasiru, et al. [2024] notes that Fish is the primary source of animal protein and is highly valued by low- and middle-income households because it is readily available and affordable. However, this elasticity could be explained by the fact that Fish and Seafood were grouped together. In Egypt, Fish is an essential good with inelastic demand, and the price elasticity of demand is close to one (0.98) Afifi [2022]. This means that households maintain their Fish consumption even when prices rise, but adjust it slightly in response to changes in their income. For these goods, a 1% increase in total food spending leads to a rise of more than 1% of the consumption of these goods.

Conversely, cerals, Sugar and confectionery, Oils and fats, Vegetables an Beverages are necessities whose budget share declines as income rises. Turning to own-price elasticities, all goods exhibit negative direct elasticities consistent with the law of demand, with the notable exception of Fish and Seafood, whose own-price elasticity is positive (0.36). This provides a Giffen-good effect behavior among certain population segments, strong geographical heterogeneity in consumption patterns. This result is similar to the estimates in Bettah, Ezzrari, and Mourji [2022] for Morocco (near Egypt in terms of consumption habits). The fact that this value is positive reflects the actual situation in the country (see Table D.4).

Table 3: Uncompensated Price Elasticities and Expenditure Elasticities – QUAIDS

Good *i*	Cereals (1)	Meat (2)	Dairy (3)	Oil & Fat (4)	Fish (5)	Fruits (6)	Vegetables (7)	Sugar (8)	Beverages (9)	Elec./Gas (10)	Other (11)	Exp. elas.
(1) C	−1.766***	1.448***	−0.282***	0.543***	−1.577***	−0.412***	0.241***	0.254***	0.760***	0.022	−0.160***	0.928***
(2) M	0.667***	−1.227***	0.170***	−0.734***	−0.271***	0.160***	0.462***	−0.209***	−0.448***	−0.087***	0.092***	1.425***
(3) DP	−0.321***	0.454***	−0.509***	0.273***	−0.612***	−0.027	−0.371***	−0.014	−0.138***	0.236***	−0.025***	1.053***
(4) OF	0.836***	−2.066***	0.408***	−2.277***	0.643***	0.641***	0.382***	−0.483***	1.021***	0.094*	0.018	0.781***
(5) FS	−3.197***	−1.058***	−1.154***	0.811***	0.363	0.003	0.740***	1.118***	1.466***	−0.730***	0.312***	1.325***
(6) F	−0.910***	0.753***	−0.061	0.888***	0.013	−1.381***	−0.511***	−0.245***	0.214**	0.182***	−0.079***	1.136***
(7) V	0.228***	0.934***	−0.239***	0.217***	0.342***	−0.169***	−1.487***	−0.082***	−0.338***	−0.144***	0.078***	0.660***
(8) SC	0.751***	−1.115***	−0.023	−0.952***	1.671***	−0.323***	−0.326***	−2.432***	0.845***	0.732***	0.268***	0.905***
(9) B	0.611***	−0.505***	−0.054	0.527***	0.597***	0.102***	−0.309***	0.228***	−1.676***	0.050	−0.214***	0.643***
(10) EGF	0.065	−0.560***	0.794***	0.207	−1.323***	0.314***	−0.700***	0.904***	0.177	−0.869***	−0.075	1.065***
(11) OTF	−2.401***	2.892***	−0.319**	0.177	2.380***	−0.535***	1.405***	1.369***	−4.257***	−0.302	−1.260***	0.851***
Obs.						12,463						

Sugar and confectionary (-2.43), Oils and fats (-2.28) and Cereals (-1.77) are the most price-responsive goods, while Energy (-0.87) and Dairy products (-0.51) exhibit the most inelastic demands. This has direct implications for the computation of the MECF: a more inelastic demand translates into a lower marginal efficiency cost of public funds, making the taxation of these goods relatively less distorsionary and therefore less costly from a fiscal efficiency standpoint.

Notes: Columns (1)–(11): uncompensated (Marshallian) price elasticities. Row i, column j gives the percentage change in the budget share of good i for a 1% increase in the price of good j, evaluated at sample means. Diagonal entries are own-price elasticities. Last column (*Exp. elas.*): expenditure elasticity; values > 1 (< 1) indicate luxury (necessity) goods. EGF = Electricity, gas and other fuels. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$. Standard errors via the delta method. Model estimated with *quaids* (Poi, 2012, *Stata Journal*).

In Egypt, the standard VAT rate is 14% (Article 3 of the VAT Law 2016 enacted by the Ministry

Table 4: Effective tax rates t_ℓ with implicit subsidies

Init.	**VAT regime**	t_ℓ	**Basis and sources**
CB	Exempt (Law 67/2016)	-0.94^a / -0.60^b	Baladi bread: production cost 0.90 EGP/loaf, ration price 0.05 EGP ⇒ subsidy ≈ 94%. Mixed rate for aggregated CB category. [1,2,3]
M	Exempt	0.00	No structural subsidy identified. [4]
DP	Exempt	0.00	No subsidy identified.
OF	Exempt	−0.38	Ration card price: 25 EGP/litre (Nov. 2021); market price ≈40 EGP/litre. $(40-25)/40 \approx 0.38$. [4,5]
FS	Exempt	0.00	No subsidy identified.
F	Exempt	0.00	No subsidy identified.
V	Exempt	0.00	No subsidy identified.
SC	Exempt	−0.42	Ration card price: 10.5 EGP/kg (Jan. 2022); market price ≈18 EGP/kg. $(18-10.5)/18 \approx 0.42$. [4,6]
B	Exempt (non-alc.)	0.00	Non-alcoholic Beverages effectively exempt. [7]
EGF	Schedule tax 10% + subsidy	−0.50	Residual cross-subsidy: 5.6 bn EGP in FY2021/22. Residential tariff 0.48–1.45 EGP/kWh vs. production cost ≈2 EGP/kWh. Liberalisation postponed to FY2024/25. [8,9,10]
OFP	Exempt or 14%	0.00	Residual category; no targeted subsidy identified.

[a] Baladi bread only.
[b] Mixed rate for the aggregated CB category (baladi + commercial Cereals).
[1] Al-Ahram Weekly.
[2] USDA GAIN EG2022-0025.
[3] IMES GWU (2022).
[4] EgyptWatch (Jan. 2022).
[5] Middle East Eye (May 2022).
[6] Ahram Online (May 2022).
[7] PwC Egypt VAT.
[8] Daily News Egypt.
[9] ResearchGate ERA tariff.
[10] Ecofin Agency (2025).

Note: Negative rates reflect implicit subsidies (ration card price < market price). VAT regime: Law No. 67 of 2016. Tax rates for goods without subsidy set to zero pending.

of Finance and effective as of 2017/2018). However, many food products are tax-exempt. It is particularly difficult to find data on subsidies and taxes for consumer goods. Nevertheless, based on various sources cited in the footnotes of Table 4, the subsidy table below has been compiled.

Table 5 reports the estimated marginal efficiency costs of public funds for each of the eleven commodity groups. The results fall into three distinct categories.

Cereals ($MECF_C = 0.04$), Fish ($MECF_{FS} = 0.02$), and Energy ($MECF_{EGF} = 0.03$) display values close to zero and close the one other, indicating that these goods are close to their Ramsey-optimal tax rates [Ramsey, 1927]: marginal changes to their tax rates generate minimal distortions and produce little efficiency loss per unit of additional fiscal revenue.[3] These goods therefore constitute efficient fiscal instruments: any reallocation of the tax burden toward them involves a relatively low efficiency cost compared to the rest of the tax system.

Dairy products ($MECF_{DP} = 0.22$), Fruits ($MECF_F = 0.07$), and Other food products ($MECF_{OFP} = 0.69$) display positive marginal efficiency costs. Each additional unit of revenue raised by taxing these goods entails a marginal social welfare loss, reflecting the distortionary cost of taxation above the optimum. Other food products stands out with the highest MECF in the sample (0.69), making it the most costly good to tax from an efficiency standpoint.

Five goods exhibit negative MECFs: Meat (-0.11), Oil & Fat (-0.09), Vegetables (-0.65), Sugar (-0.09), and Beverages (-0.02). Negative MECFs are discussed in Mayshar [1990], Feldstein [1999], and others: they indicate that the current tax rate on these goods lies *above* the revenue-maximising level, so that reducing the subsidy (or increasing the tax) simultaneously raises fiscal revenues *and* reduces deadweight loss. In line with the predictions of Mayshar and Yitzhaki [1996], reforming the taxation of these goods would improve public finances while eliminating inefficient consumption induced by excessive subsidisation. Vegetables carries the largest negative MECF in absolute value (-0.65), implying that the existing implicit subsidy generates very large distortions relative to the fiscal revenue it protects. Meat (-0.11) presents a comparable case: with an expenditure elasticity of 1.43 (Table 3), Meat consumption grows more than proportionally with income, meaning that the current subsidy disproportionately benefits higher-income households. Reducing it would therefore increase fiscal revenues, reduce deadweight loss, and not penalise the poor disproportionately (a configuration that Yitzhaki and Thirsk [1990] identify as a Pareto-improving direction). Oil & Fat (-0.09), Sugar (-0.09), and Beverages (-0.02) present more moderate distortions but remain candidates for efficiency-improving tax increases.

Table 5: Marginal efficiency cost of public funds ($MECF_\ell$)

Good	C	M	DP	OF	FS	F	V	SC	B	EGF	OFP
$\widehat{MECF}_\ell$	0.04	-0.11	0.22	-0.09	0.02	0.07	-0.65	-0.09	-0.02	0.03	0.69

Notes: $MECF_\ell = X_\ell / MR_\ell$ where $MR_\ell = X_\ell + t_\ell \cdot \partial X_\ell / \partial t_\ell$ is the marginal revenue from taxing good ℓ, estimated using QUAIDS price elasticities (Table 3) and effective tax rates (Table 4). A positive (negative) $MECF_\ell$ indicates that taxing good ℓ generates a marginal welfare loss (gain) per unit of additional revenue. Values close to zero signal proximity to the Ramsey optimum [Ramsey, 1927]. N = 12,463 households. HIECS 2021/22.

[3] At the Ramsey optimum, all MECFs are equal; a value close to zero reflects that the good is already approximately optimally taxed relative to its demand elasticity.

4.3 Bivariate dominance tests

In the bivariate case, we apply Theorems 1 and 2 on two goods: increase the tax on good i to subsidize good j. We find either (i) welfare-improving tax reforms or (ii) poverty-reducing tax reforms (given that if (i) holds, then (ii) arises). The bootstrap test of Khaled et al. [2018] is given in Appendix A.

4.3.1 Welfare-improving tax reforms

Under the assumption of optimal taxation, we set the ratio of the Marginal Efficiency Cost of Funds of unity for all pairs of commodities. This assumption is equivalent to assuming that the government has already equalized marginal distortions across all taxed goods, then the tax system is at its Ramsey [1927]'s optimum. In this limiting case, no reallocation of the tax burden between any two goods can improve welfare based on efficiency considerations alone [Ahmad and Stern, 1984]. Consequently, any dominance result is entirely driven by the redistributive profile of consumption, independent of the distortionary nature of existing taxes. This serves as a "neutrality-efficiency benchmark" to isolate the pure redistributive effect.

Results of optimal taxation: $MECF_i/MECF_j = 1$.

Bivariate dominance results under the optimal-tax assumption are reported in Appendix E. They reveal a much sparser dominance structure: only Fish dominates several goods and Energy is the sole universally dominated good, which confirms that the Ramsey optimum is far from being reached in Egypt, leaving substantial room for welfare-improving fiscal reallocations once estimated MECFs are taken into account.

Results with estimated $MECF_\ell$ of Table 5.

Tables 6 and 7 report pairwise dominance results at orders $s = 2$ and $s = 3$ over $[0, y^{++}]$. An entry ">" in row i, column j means $RD^s(y, \delta^s) \geq 0$ for all $y \in [0, y^{++}]$ when taxing good i and subsidising good j under budget neutrality, i.e. the reform weakly increases welfare for all $W \in \Omega^s$.

The dominance hierarchy at order $s = 2$ (Table 6) is clear. Energy is dominated by every other good without exception: any budget-neutral reform shifting the tax burden toward Energy constitutes a robust second-order welfare improvement regardless of the offsetting good. This result is consistent with the empirical profile of electricity, gas, and fuels in Egypt: with an expenditure elasticity above unity ($\hat{\eta} = 1.065$, Table 3), a relatively inelastic own-price response (-0.87), and the lowest positive $MECF$ in the sample (0.03, Table 5), Energy combines the characteristics of a relative luxury and a low-distortion fiscal base. At the opposite end of the hierarchy, Cereals, Fish, and Meat each dominate eight or more goods, while Fruits and Beverages dominate seven. For Cereals and Beverages, the result reflects necessity-good profiles (expenditure elasticities of 0.93 and 0.64, respectively) whose MECF-weighted consumption-dominance curves lie uniformly above those of most other goods. For Meat and Fish, which are luxury goods ($\hat{\eta} = 1.43$ and 1.33), the result is driven by their relatively high MECFs: the existing subsidy structures create large efficiency costs, making it socially desirable to tax these goods and redistribute the revenue.

Table 6: Bivariate dominance tests for $y \in [0,\, y^{++}]$ and order $s = 2$

$i \setminus j$	*Cereals*	*Meat*	*Fish*	*Dairy*	*Oils*	*Fruits*	*Vegetables*	*Sugar*	*Beverages*	*Other*	*Energy*
Cereals		$>$	–	$>$	$>$	–	$>$	$>$	$>$	$>$	$>$
Meat			$>$	$>$	–	$>$	$>$	–	$>$	$>$	$>$
Fish				$>$	$>$	–	$>$	$>$	$>$	$>$	$>$
Dairy					$\times_{0.008}$	–	$>$	$>$	$>$	–	$>$
Oils						$>$	–	$>$	$>$	$>$	$>$
Fruits							$>$	–	$>$	$>$	$>$
Vegetables								$<$	$>$	$>$	$>$
Sugar									$<$	$<$	$>$
Beverages										$>$	$>$
Other											$>$
Energy											

Notes: $\alpha = 0.05$, $B = 500$ bootstrap (A) replications. The table is read by row: an entry “$>$” in row i, column j indicates that $RD^2(y, \delta^s) \geq 0$ for all $y \in [0, y^{++}]$, i.e. taxing good i and subsidising good j weakly increases social welfare for all $W \in \Omega^2$. “$<$”: j dominates i (reverse reform is welfare-improving). “$\times_{y^*}$”: the reform dominance curve crosses zero at y^* (expressed as a ratio y/z^+); dominance fails strictly but holds for virtually all income levels above that threshold. “–”: the bootstrap cannot reject non-dominance in either direction at the 5% significance level. The single crossing entry ($\times_{0.008}$, Dairy–Oils) corresponds to EGP per capita per day. Lower triangle is left blank by symmetry. $MECF$ estimates are taken from Table 5; consumption dominance curves $CD^2_\ell(y)$ are estimated nonparametrically on a grid of d points over $[0, y^{++}]$.

Moving to $s = 3$ (Table 7) imposes sensitivity to the Kolm transfer principle, assigning increasing weight to income transfers at the bottom of the distribution. The overall hierarchy is preserved, but four notable changes emerge.

First, Fruits strengthens considerably and now dominates all ten other goods without exception, becoming the strongest dominator at this order. This reflects the particularly pro-poor profile of fruit consumption at the very bottom of the income distribution, which becomes decisive once higher-order inequality aversion is imposed. Second, two new reversals appear that were invisible at $s = 2$. Fish dominates Cereals (entry “$<$” in row Cereals, column Fish): under stronger aversion to extreme poverty, subsidising Cereals financed by a Fish tax is welfare-improving, suggesting that the poorest households are more sensitive to (decreasing) cereal prices to Fish prices at the third order. Similarly, Other food dominates Dairy (entry “$<$” in row Dairy, column Other), a ranking that did not emerge at $s = 2$. Third, several dominances that were significant at $s = 2$ lose statistical support at $s = 3$: Meat versus Sugar and Beverages both turn “–”, as does Dairy versus Energy. These reforms are welfare-improving when only inequality aversion is imposed ($s = 2$), but are not robust to a stronger concern for the bottom of the distribution, a finding with direct policy relevance. Fourth, the Vegetables–Beverages relationship reverses: at $s = 3$, Beverages dominates

Vegetables (entry "<" in row Veg, column Bev), meaning that subsidising Beverages while taxing Vegetables is welfare-improving under Kolm-type inequality aversion.

Table 7: Bivariate dominance tests for $y \in [0,\, y^{++}]$ and order $s = 3$

$i \setminus j$	Cereals	Meat	Fish	Dairy	Oils	Fruits	Vegetables	Sugar	Beverages	Other	Energy
Cereals		>	<	>	>	–	>	>	>	–	>
Meat			>	>	–	>	>	–	–	>	>
Fish				>	>	–	>	>	>	>	>
Dairy					>	–	>	>	>	<	–
Oils						>	–	–	>	>	>
Fruits							>	>	>	>	>
Vegetables								<	<	>	>
Sugar									<	>	>
Beverages										>	>
Other											>
Energy											

Notes: $\alpha = 0.05$, $B = 500$ bootstrap (A) replications. Lower triangle left blank by symmetry. $MECF$ estimates from Table 5; $CD^3_\ell(y)$ estimated nonparametrically over $[0, y^{++}]$.

4.3.2 Poverty-reducing tax reforms

Results of optimal taxation: $MECF_i/MECF_j = 1$.

Under the $MECF_i/MECF_j = 1$ assumption and over the income support $[0, z^+]$, results are reported in Appendix D. Only Fish and Meat dominate Sugar and Energy. Most other pairs of goods remain statistically unresolved, confirming the limited discriminatory power of the optimal-tax benchmark for poverty analysis.

Results with estimated $MECF_\ell$ (Table 8).

Tables 8 and 9 restrict the dominance condition to the income range $[0, z^+]$. By Theorem 2, an entry ">" in row i, column j means that taxing good i while subsidising good j is s-order poverty-reducing for all $W \in \Omega^s(z^+)$. The poverty hierarchy at $s = 2$ (Table 8) broadly mirrors the welfare ranking, with three noteworthy departures.

First, Dairy versus Energyturns "–": over the support of poor households only, the MECF-weighted consumption-dominance curves of Dairy and Energy are statistically indistinguishable, so the welfare gain from taxing Dairy to subsidise Energy is not significant among the poor. Second, Oils loses its dominance over Vegetables, Sugar, and Beverages, and Beverages loses its dominance over Other food, when the analysis is restricted to $[0, z^+]$. This indicates that the consumption profiles of these goods converge among poor households and that the full-sample dominance rankings are driven by distributional differences above the poverty line. Third, and most strikingly, Energy remains universally dominated: every good except Dairy dominates Energy over $[0, z^+]$, confirming that reallocating fiscal revenues from Energy toward any food good is poverty-reducing, regardless of the social planner's exact normative weights within $\Omega^2(z^+)$.

Table 8: Bivariate dominance tests for $y \in [0,\ z^+]$ and order $s = 2$

$i \setminus j$	*Cereals*	*Meat*	*Fish*	*Dairy*	*Oils*	*Fruits*	*Vegetables*	*Sugar*	*Beverages*	*Other*	*Energy*
Cereals		>	–	>	>	–	>	>	>	–	>
Meat			>	>	–	>	>	–	–	>	>
Fish				>	>	–	>	>	>	>	>
Dairy					>	–	>	>	>	–	–
Oils						>	–	–	–	>	>
Fruits							>	>	>	–	>
Vegetables								<	<	>	>
Sugar									<	>	>
Beverages										–	>
Other											>
Energy											

$\alpha = 0.05$, $B = 500$. ">": i dominates j; "<": j dominates i; "–": not significant.

The poverty rankings at $s = 3$ (Table 9) are identical to the welfare rankings at $s = 3$ (Table 7).

Table 9: Bivariate dominance tests for $y \in [0,\ z^+]$ and order $s = 3$

$i \setminus j$	*Cereals*	*Meat*	*Fish*	*Dairy*	*Oils*	*Fruits*	*Vegetables*	*Sugar*	*Beverages*	*Other*	*Energy*
Cereals		>	<	>	>	–	>	>	>	–	>
Meat			>	>	–	>	>	–	–	>	>
Fish				>	>	–	>	>	>	>	>
Dairy					>	–	>	>	>	<	–
Oils						>	–	–	>	>	>
Fruits							>	>	>	>	>
Vegetables								<	<	>	>
Sugar									<	>	>
Beverages										>	>
Other											>
Energy											

$\alpha = 0.05$, $B = 500$. ">": i dominates j; "<": j dominates i; "–": not significant.

This equivalence reflects the fact that the dominance structure over $[0, z^+]$ at the third order is entirely determined by the left tail of the distribution, which coincides with the poverty support. The key results are: Fruits universally dominates all goods; Fish dominates Cereals; Other food dominates Dairy; and Energy is the sole universally dominated good. The structural stability of the $s = 3$ rankings across welfare and poverty dominance provides strong evidence of the robustness of these findings to the choice of upper bound.

The four bivariate Tables seen above yield three robust conclusions for Egypt's indirect tax system. First, taxing Energy is the most unambiguously welfare-improving and poverty-reducing reform available: Energy is dominated by every good at every order and over every support. Second, Cereals, Fish, and Fruits are the strongest dominators, so subsidising these goods financed by taxes elsewhere is broadly desirable. Third, the dominance structure changes materially between $s = 2$ and $s = 3$, and between the welfare and poverty dominance, warning against relying solely on second-order results. These findings motivate the multivariate analysis of the next section, which optimally combines all eleven goods simultaneously.

4.4 Multivariate dominance tests

Estimating the optimal reform vectors δ^s for the entire Egyptian population may reveal tax reform paths at both the second ($s = 2$) and third ($s = 3$) orders. Numerical convergence is nearly perfect, with the values of the objective function V extremely close to zero (see Tables 10 and 11).

The convergence of the optimization program may be long, therefore the boostrap test of Khaled et al. [2018] is adapted to a wild bootstrap test (see Appendix B). Statistically, wild bootstrap tests reject the non-dominance hypothesis. The test results confirm that the identified tax reforms reduce poverty (Table 11) and increase well-being (Table 10) for all utility functions that respect inequality aversion ($s = 2$) and sensitivity to transfers at the bottom of the distribution ($s = 3$).

4.4.1 Welfare-improving tax reforms

Table 10 reports the estimated reform vectors δ^s and associated wild bootstrap test statistics for social welfare improvement at dominance orders $s = 2$ and $s = 3$. In all tested cases (increasing or decreasing taxes), the algorithm finds a reform with zero loss ($V \approx 0$), and the bootstrap p-values uniformely satisfy $p_1 \geq 0.05$ and $p_2 < 0.05$), so the decision rule in Table 1 yields outcome $>$ in every column: the estimated reform dominant curve lies strictly above zero for all income levels (Theorem 2).

Scenario $\delta^s_{Cereals} = -1$ or $\delta^s_{Cereals} = -5$: Tax reduction on Cereals

The reform that reduces the cereal tax ($\delta^s_{Cereals} = \{-1, -5\}$) is financed entirely by a symmetric increase on Energy ($\delta^s_{Energy} = +1$), with all other goods assigned zero weight ($\delta^s_\ell = 0$ for $\ell \notin \{Cereals, Energy\}$). This is a pure bivariate reform: it shifts the tax burden from the most consumed staple food (Cereals represent 12% of the food budget and are purchased by 99.96% of households, see Table 2). In contrast, Energy exhibits an expenditure elasticity exceeding unity (1.065), characterizing it as a relative luxury good consumed disproportionately by higher-income households. The result holds identically at both $s = 2$ and $s = 3$, indicating that the welfare gain is robust to progressively stronger aversion to inequality among the poor. The fact that the dominance curve remains strictly positive for all income levels and across both orders simultaneously signals that this "Cereals-Energy" swap represents a Pareto-improving direction for the Egyptian fiscal system, robustly protecting the poorest deciles regardless of social planner's exact normative weights.

Scenario $\delta^s_{Cereals} = +1$ or $\delta^s_{Cereals} = +5$: Tax increase on Cereals

In contrast, a tax increase on Cereals requires a more complex, multi-sectoral compensatory mechanism to satisfy the welfare dominance condition. A price hike on Cereals requires aggressive subsidies on nutritional bridge goods to mitigate the regressive impact. At $s = 2$ $(\delta^s_{Cereals} = +1)$, the dominant reform requires significant tax reductions on Dairy ($\delta^s_{Dairy} = -1.067$) and Vegetables ($\delta^s_{Vegetables} = -1.067$). Economically, this reflects a strategy of substituting basic calories with higher-quality nutrients. These goods have a higher income elasticity (see Table 3) than bread but remain fundamental to the diet of the Egyptian lower-middle class.

Table 10: Estimated vectors δ^s and test statistics – Welfare

	s=2		s=3		s=2		s=3	
	$\delta^s = -1$	$\delta^s = +1$	$\delta^s = -1$	$\delta^s = +1$	$\delta^s = -5$	$\delta^s = +5$	$\delta^s = -5$	$\delta^s = +5$
V	0.000	0.000	0.000	0.000	0.000	1.52×10^{-9}	0.000	4.47×10^{-9}
p1	1.000	1.000	1.000	1.000	1.000	0.988	1.000	0.876
p2	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.000
Outcome	RD≥ 0		RD≥ 0		RD≥ 0		RD≥ 0	
Cereals	−1.000	+1.000	−1.000	+1.000	−5.000	+5.000	−5.000	+5.000
Meat	0.000	−0.212	0.000	+0.707	0.000	−1.386	0.000	−0.002
Fish	0.000	+0.576	0.000	−0.776	0.000	−0.423	0.000	−0.165
Dairy	0.000	−1.067	0.000	−1.570	0.000	+0.053	0.000	−1.126
Oils	0.000	+0.269	0.000	−0.717	0.000	−0.604	0.000	−0.604
Fruits	0.000	+0.545	0.000	+0.691	0.000	−0.381	0.000	−1.080
Vegetables	0.000	−1.139	0.000	+0.355	0.000	+0.431	0.000	−0.170
Sugar	0.000	−0.295	0.000	−0.719	0.000	−1.459	0.000	−0.703
Beverages	0.000	−0.386	0.000	+0.193	0.000	−0.533	0.000	−0.691
Other food	0.000	−0.816	0.000	+0.122	0.000	−0.661	0.000	−0.434
Energy	+1.000	+1.526	+1.000	+0.714	+5.000	−0.038	+5.000	−0.025

Notes: See Table 11 for definitions of V, p_1, p_2, and the test outcome. The parameter δ^s captures directional changes in consumption. RD≥ 0 indicates that the dominance condition is satisfied. See bootstrap B.
Source: Authors' calculations based on HIECS survey (2021/2022).

When the dominance order increases to $s = 3$, reflecting a stronger aversion to poverty, the compensation profile shifts further. The subsidy on Dairy intensifies to −1.570, while Meat enters the vector with a positive weight ($\delta^s = +0.707$). This reveals a "Protein-Swap": taxing the protein consumed by the wealthy (Meat) to subsidize the protein consumed by the poor (Dairy). Furthermore, for large price shocks ($\delta^s_{Cereals} = +5$), the model identifies Sugar ($\delta^s_{Sugar} = -1.459$) as a critical compensatory lever. Given its central role in the Tamween (ration card) system, Sugar acts as a high income transfer proxy.

4.4.2 Poverty-reducing tax reforms

Table 11 presents the estimated reform vectors δ^s and associated bootstrap test statistics for poverty reduction at dominance orders $s = 2$ and $s = 3$.

Scenario $\delta^s_{\textbf{Cereals}} = -1$ or $\delta^s_{\textbf{Cereals}} = -5$: Tax reduction on Cereals

Consistent with the welfare analysis, the dominant poverty-reducing reform is a pure cereal-to-Energy swap across all configurations.

Subsidizing Cereals $\delta^s_{\text{Cereals}} = -1$ (or -5) can be entirely financed by $\delta^s_{\text{Energy}} = +1$ (or $+5$), with all other goods at zero. This tax scheme emerges endogenously from the optimization program rather than being imposed as we did in the bivariate case. This result holds identically at $s = 2$ and $s = 3$, confirming that the reform satisfies simultaneously the Pigou-Dalton and Kolm transfer principles without any trade-off. The economic rationale is straightforward: Cereals are consumed by 99.96% of households with a sub-unitary expenditure elasticity ($\hat{\eta}_{\text{CB}} = 0.928$, while Energy carries an elasticity exceeding unity ($\hat{\eta}_{\text{EGF}} = 1.065$), making it disproportionately consumed by higher-income households.

Table 11: Estimated vectors δ^s and test statistics – Poverty

	s=2		s=3		s=2		s=3	
	$\delta^s = -1$	$\delta^s = +1$	$\delta^s = -1$	$\delta^s = +1$	$\delta^s = -5$	$\delta^s = +5$	$\delta^s = -5$	$\delta^s = +5$
V	0.000	0.000	0.000	0.000	0.000	1.00×10^{-8}	0.000	5.14×10^{-9}
p1	1.000	1.000	1.000	1.000	1.000	0.944	1.000	0.548
p2	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.000
Outcome	RD≥ 0		RD≥ 0		RD≥ 0		RD≥ 0	
Cereals	-1.000	$+1.000$	-1.000	$+1.000$	-5.000	$+5.000$	-5.000	$+5.000$
Meat	0.000	-0.348	0.000	$+0.182$	0.000	-0.981	0.000	-0.441
Fish	0.000	$+0.158$	0.000	-0.235	0.000	-0.128	0.000	-0.482
Dairy	0.000	-0.422	0.000	-0.767	0.000	-0.732	0.000	-0.643
Oils	0.000	-0.033	0.000	-0.597	0.000	-0.963	0.000	$+0.350$
Fruits	0.000	-0.177	0.000	$+0.465$	0.000	$+0.538$	0.000	-0.718
Vegetables	0.000	$+0.148$	0.000	-0.767	0.000	-0.145	0.000	-0.230
Sugar	0.000	-0.573	0.000	-0.570	0.000	-0.908	0.000	-0.428
Beverages	0.000	-0.335	0.000	$+0.193$	0.000	-0.757	0.000	-1.045
Other food	0.000	$+0.487$	0.000	$+0.353$	0.000	-0.766	0.000	-1.312
Energy	$+1.000$	$+0.094$	$+1.000$	$+0.744$	$+5.000$	-0.158	$+5.000$	-0.051

Notes: V is the minimized objective function value. p_1 (p_2) is the bootstrap p-value for the null that $RD^s(y) \geq 0$ (≤ 0). "→>" indicates that the test rejects the null of non-dominance in favor of $RD^s(y) \geq 0$ at the 5% level. All coefficients rounded to three decimals.

Scenario $\delta^s_{\textbf{Cereals}} = +1$ or $\delta^s_{\textbf{Cereals}} = +5$: Tax increase on Cereals

Raising the cereal tax generates a regressive burden that requires a multi-sectoral compensatory structure, whose composition varies systematically with the dominance order. At $s = 2$, the Adam optimizer allocates the government's revenue primarily to Sugar ($\delta^s_{Sugar} = -0.573$), Dairy ($\delta^s_{Dairy} = -0.422$) and Meat ($\delta^s_{Meat} = -0.348$) under $\delta^s_{\text{Cereals}} = +1$, and to Oils ($\delta^s_{Oils} = -0.963$), Meat ($\delta^s_{Meat} = -0.981$) and Sugar ($\delta^s_{Sugar} = -0.908$) under $\delta^s_{\text{Cereals}} = +5$. These goods are consumed relatively uniformly among the poor, consistent with the Pigou-Dalton requirement of redistribution. At $s = 3$, the portfolio shifts decisively toward goods concentrated at the extreme bottom of the distribution: Dairy and Vegetables ($\delta^s_{Vegetables} = -0.767$ each) under $\delta^s_{\text{Cereals}} = +1$, and other food ($\delta^s_{other} = -1.312$) and Beverages ($\delta^s_{Beverages} = -1.045$) under $\delta^s_{\text{Cereals}} = +5$. Meat reverses from beneficiary to financing source ($\delta^s_{Meat} = +0.182$ at $s = 3$ under $\delta^s_{\text{Cereals}} = +1$), reflecting its high expenditure elasticity ($\hat{\eta}_{\text{M}} = 1.425$) which disqualifies it as a redistribution instrument toward the extreme poor under the Kolm criterion. The Oils reversal between $s = 2$ ($\delta^s_{Oils} = -0.963$) and $s = 3$ ($\delta^s_{Oils} = +0.350$) under wide bounds further illustrates this normative gradient: Oils are sufficiently pro-poor under Dalton but not concentrated enough at the extreme bottom to warrant subsidization under the more demanding third-order criterion.

What happens when we tax Meat?

The analysis of the Reform Dominance curves (RD^s) reveals contrasting results regarding Meat taxation in Egypt.

Figure 1 illustrates the impact of increasing the tax on Meat, with the generated revenue redistributed toward other essential commodities. This reform produces a sharp welfare peak precisely for households earning an income equal to two poverty lines ($y/z \approx 2$) more than for households at the poverty line (red segment).

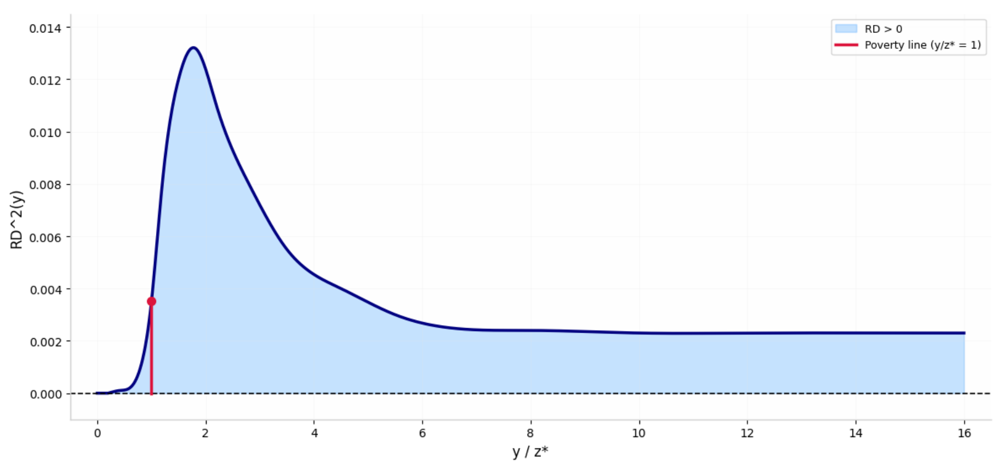

Figure 1: Increasing tax on Meat by 1 EGP

This demonstrates a high redistributive efficiency, but not targeting the poor (the pike should be before the poverty line). By extracting resources from a high-elasticity good (Meat) to finance staples, the social planner optimizes the transfer of welfare toward households with the highest marginal utility of consumption. While the curve declines after the peak, showing that the upper classes bear the fiscal weight of the reform, the fact that RD^s remains positive across the entire income distribution confirms, according to Theorem 2, that the aggregate social gain compensates for the individual welfare loss of Meat consumers.

5 Conclusion

This paper has developed a unified multivariate framework for identifying budget-neutral indirect tax reforms that unambiguously improve social welfare or reduce poverty. By integrating s-order stochastic dominance conditions with a convex optimization procedure, we have extended the existing literature, traditionally limited to two-good settings or to second-order dominance.

From a methodological standpoint, our contribution bridges the gap between theoretical welfare economics and computational operations research. The use of the Adam optimizer allowed us to navigate non-convexities in the reform space, particularly those induced by negative Marginal Efficiency Costs of Funds (MECF). Furthermore, the implementation of a two-stage wild bootstrap procedure ensures that the identified dominance is statistically significant, providing a robust decision-support system for policy-makers.

The empirical application to Egyptian household data (HIECS 2021/2022) yields critical insights for emerging economies. Our results demonstrate that a "Food-Energy Swap" strategy—specifically increasing taxes on luxury proteins (Meat) and rationalizing Energy subsidies to fund essential staples like Cereals—constitutes a robust welfare-improving path. We find that higher-order dominance ($s = 3$) confirms the stability of these reforms even under extreme redistributive preferences, such as those of a Rawlsian social planner. Finally, for policy-makers, the necessary and sufficient conditions for indirect tax reforms at the order s of stochastic dominance avoid the arbitrary selection of a specific social welfare function.

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A Bootstrap algorithm: bivariate case

The bootstrap algorithm is constructed as follows. Assume we have an i.i.d. sample of size n from the population corresponding to the theoretical policy reform curve $RD^s(y,\delta)$. Denote this sample by $\mathcal{S}$, and let $\widehat{RD}^s(y,\delta)$ be the nonparametric estimator of this curve. Define the test statistics

$$\widehat{\tau}_1 = \sqrt{n}\sup_y\left[-\widehat{RD}^s(y,\delta)\right],\ \widehat{\tau}_2 = \sqrt{n}\sup_y \widehat{RD}^s(y,\delta).$$

The bootstrap procedure proceeds as follows:

1. For each bootstrap replication $b = 1,\ldots,B$:

   (a) Draw a sample with replacement of size n from $\mathcal{S}$ and compute the nonparametric estimator $\widehat{RD}^s_b(y,\delta)$ using this bootstrap sample.

   (b) Compute the bootstrap test statistics

   $$\widehat{\tau}_{1b} = \sqrt{n}\sup_y\left[-\widehat{RD}^s_b(y,\delta)\right],\ \widehat{\tau}_{2b} = \sqrt{n}\sup_y \widehat{RD}^s_b(y,\delta).$$

2. Using the bootstrap sample $\{\widehat{\tau}_{1b}\}_{b=1}^B$ and $\{\widehat{\tau}_{2b}\}_{b=1}^B$, compute the bootstrap p-values as:

$$p_1 = \frac{1}{B}\sum_{b=1}^{B}\mathbb{1}(\widehat{\tau}_{1b} > \widehat{\tau}_1)\ \text{ and } p_2 = \frac{1}{B}\sum_{b=1}^{B}\mathbb{1}(\widehat{\tau}_{2b} > \widehat{\tau}_2).$$

B Wild bootstrap algorithm: multivariate case

In the multivariate setting, the reform dominance curve depends on a vector of parameters δ. In practice, δ is unknown and is replaced by an estimator $\widehat{\delta}$ obtained from a constrained optimization problem. Let $\widehat{RD}^s_\ell(y,\delta)$ denote the estimated reform dominance curves at the order s. The estimator $\widehat{\delta}$ is the one issued from the convex programming. The estimated reform dominance curve is therefore evaluated at the estimated parameter vector, $\widehat{RD}^s(y) = \widehat{RD}^s(y,\widehat{\delta})$. Inference is conducted using a wild multiplier bootstrap based on the estimated influence function of the estimator evaluated at $\widehat{\delta}$. Such procedures provide valid approximations for supremum-type statistics of empirical processes [Chernozhukov, Chetverikov, and Kato, 2013].

Let $\widehat{\phi}_i(y)$ denote the estimated influence function associated with observation i. The estimator admits the asymptotic linear representation

$$\sqrt{n}\left(\widehat{RD}^s(y) - RD^s(y)\right) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\widehat{\phi}_i(y) + o_p(1).$$

Define $\widehat{RD}(y) = \frac{1}{n}\sum_{i=1}^n \widehat{\phi}_i(y)$. The test statistics are: $\widehat{\tau}_1 = \sqrt{n}\sup_y[-\widehat{RD}(y)]$, $\widehat{\tau}_2 = \sqrt{n}\sup_y \widehat{RD}(y)$. The multiplier bootstrap procedure proceeds as follows.

1. Compute the centered influence functions: $\tilde{\phi}_i(y) = \widehat{\phi}_i(y) - \frac{1}{n}\sum_{j=1}^{n} \widehat{\phi}_j(y)$.
2. For each bootstrap replication $b = 1, \ldots, B$:
   (a) Draw i.i.d. multipliers $\varepsilon_i^{(b)} \sim \mathcal{N}(0,1)$ for $i = 1, \ldots, n$.
   (b) Construct the bootstrap process $RD_b^*(y) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} \varepsilon_i^{(b)} \tilde{\phi}_i(y)$.
   (c) Compute the bootstrap test statistics: $\widehat{\tau}_{1b} = \sup_y[-RD_b^*(y)]$, $\widehat{\tau}_{2b} = \sup_y RD_b^*(y)$.
3. Using the bootstrap samples $\{\widehat{\tau}_{1b}\}_{b=1}^{B}$ and $\{\widehat{\tau}_{2b}\}_{b=1}^{B}$, compute the bootstrap p-values

$$p_1 = \frac{1}{B}\sum_{b=1}^{B} \mathbb{1}(\widehat{\tau}_{1b} \geq \widehat{\tau}_1), \quad p_2 = \frac{1}{B}\sum_{b=1}^{B} \mathbb{1}(\widehat{\tau}_{2b} \geq \widehat{\tau}_2).$$

C Proof of theorems 1 and 2

Proof of Theorem 1. The impact of a tax reform on a consumer is measured through the *equivalent income* $y^E(y, q, q^R, \theta)$, defined implicitly by:

$$v\big(y^E(y, q, q^R, \theta),\, q^R,\, \theta\big) = v(y, q, \theta), \tag{20}$$

where $v(\cdot)$ is the indirect utility function.

Totally differentiating (20) and since q^R is the fixed reference price vector, $dq_j^R = 0$. Differentiating (20) with respect to the tax on good ℓ:

$$v_{y^E}(y^E, q^R, \theta)\,\frac{\partial y^E}{\partial t_\ell} = v_{q_\ell}(y, q, \theta)\,\frac{\partial q_\ell}{\partial t_\ell} \implies \frac{\partial y^E}{\partial t_\ell} = \frac{v_{q_\ell}(y, q, \theta)}{v_{y^E}(y^E, q^R, \theta)} \times \frac{\partial q_\ell}{\partial t_\ell}. \tag{21}$$

Since consumer prices are $q_\ell = 1 + t_\ell$, so all price changes are driven solely by the tax variation. By Roy's identity: $x_\ell(y, q, \theta) = -\frac{v_{q_\ell}(y,q,\theta)}{v_{y^E}(y,q,\theta)}$. Because at the pre-reform stage, $q^R = q \implies v_{y^E}(y^E, q, \theta) = v_y(y, q, \theta)$, substituting into (21):

$$\frac{\partial y^E(y, q, q^R, \theta)}{\partial t_\ell} = -x_\ell(y, q^R, \theta). \tag{22}$$

By the anonymity axiom, the policy-maker distinguishes consumers only by their nominal income. Taking the expectation over unobserved heterogeneity θ conditional on income y, integrating over Θ:

$$\frac{\partial y^E(y, q, q^R)}{\partial t_\ell} = E_\Theta\left[\frac{\partial y^E}{\partial t_\ell}\right] = -\int_\Theta x_\ell(y, q^R, \theta)\, dF_{\Theta|Y}(\theta \mid y) = -x_\ell(y, q^R), \tag{23}$$

where $x_\ell(y, q^R)$ denotes mean conditional Marshallian demand at income y. When several taxes change simultaneously and do not affect consumers' nominal income, the total impact on equivalent income is:

$$dy^E(y,q,q^R) = -\sum_{\ell=1}^{L} x_\ell(y,q^R)\,dt_\ell. \tag{24}$$

Since the pre-reform price vector is always used as the reference, we write $x_\ell(y) := x_\ell(y,q^R)$ hereafter. Public revenues are $R(t) = \sum_\ell t_\ell X_\ell$, with $X_\ell = \int_0^a x_\ell(y)\,dF_Y(y)$. Total differentiation gives:

$$dR(t) = \sum_{\ell=1}^{L}\left(t_\ell \frac{\partial X_\ell}{\partial t_\ell} + X_\ell\right) dt_\ell = \sum_{\ell=1}^{L} MR_\ell\,dt_\ell, \tag{25}$$

where $MR_\ell := t_\ell\,\partial X_\ell/\partial t_\ell + X_\ell$. For budget-neutral reforms, we set $\delta_\ell = MR_\ell\,dt_\ell$ and impose $\sum_\ell \delta_\ell = 0$. The marginal efficiency cost of public funds is $MECF_\ell := X_\ell/MR_\ell$, so $dt_\ell = \delta_\ell/MR_\ell$ and (24) becomes:

$$dy^E(y) = -\sum_{\ell=1}^{L} MECF_\ell\,\delta_\ell\,\frac{x_\ell(y)}{X_\ell}. \tag{26}$$

Following Makdissi and Wodon [2002], we define the first-order consumption dominance curve,

$$CD^1_\ell(y) := \frac{x_\ell(y)}{X_\ell}\,f_Y(y), \tag{27}$$

where f_Y is the density of F_Y. Higher-order curves are defined recursively: $CD^s_\ell(y) = \int_0^y CD^{s-1}_\ell(u)\,du$ $(s \geq 2)$. Using (27), (26) may be written as: $dy^E(y)\,f_Y(y) = -\sum_{\ell=1}^{L} MECF_\ell\,\delta_\ell\,CD^1_\ell(y)$. The s-th order reform dominance curve is:

$$RD^s(y,\delta) := -\sum_{\ell=1}^{L} MECF_\ell\,\delta_\ell\,CD^s_\ell(y). \tag{28}$$

Let $W(F_Y) = \int_0^\infty w(y^E(y))\,dF_Y(y)$ with $W \in \Omega^s$. We claim that the welfare change admits:

$$dW = (-1)^{s-1}\int_0^\infty w^{(s)}(y)\,RD^s(y,\delta)\,dy. \tag{29}$$

We establish (29) by induction on s.[4]

Initialization $s = 1$. By definition of W and linearity of d, evaluating at the pre-reform point where $y^E(y) = y$: $dW = \int_0^\infty w'(y)\,dy^E(y)\,dF_Y(y) = \int_0^\infty w'(y)\,RD^1(y,\delta)\,dy$, which is (29) for $s = 1$.

Inductive step. Assume (29) holds at rank s. By the recursive definition of CD^s_ℓ:

$$d\big[RD^{s+1}(y,\delta)\big] = RD^s(y,\delta)\,dF_Y(y).$$

Substituting into the inductive hypothesis and applying integration by parts with $u = w^{(s)}(y)$ and $dv = d[RD^{s+1}(y,\delta)]$: $dW = (-1)^{s-1}\left\{\Big[w^{(s)}(y)\,RD^{s+1}(y,\delta)\Big]_0^\infty - \int_0^\infty w^{(s+1)}(y)\,RD^{s+1}(y,\delta)\,dy\right\}$.

[4] Note that the proof could be done for each order s without nested sets Ω^s following Dubois [2019].

The boundary term vanishes:

- at $y = 0$: $RD^{s+1}(0, \delta) = 0$ since $CD_\ell^{s+1}(0) = 0$ by definition;
- as $y \to \infty$: by Corollary 1 of Duclos and Makdissi [2004] (condition A40 in their Appendix), if $RD^s(y, \delta) \geq 0$ for all $y \in \mathbb{R}^+$, then the sign of $RD^s(y, \delta)$ as $y \to \infty$ is entirely determined by the smallest non-zero RD^{s-i}, for $(i \leq s-1)$ so the dominance condition over the unrestricted domain automatically implies $w^{(s)}(y)\, RD^{s+1}(y, \delta) \to 0$. Conversely, if the reform is not s-order dominant, necessity is established directly without appealing to any boundary condition at infinity (see the necessity argument below).

Therefore, $dW = (-1)^s \int_0^\infty w^{(s+1)}(y)\, RD^{s+1}(y, \delta)\, dy$, which is (29) at rank $s+1$. By induction, (29) holds for all $s \geq 1$.

Sufficiency. Suppose $RD^s(y, \delta) \geq 0$ for all $y \in [0, \infty)$. By definition of Ω^s (10): $(-1)^{s+1} w^{(s)}(y) \geq 0$, equivalently $(-1)^{s-1} w^{(s)}(y) \geq 0$. Thus the integrand in (29) is non-negative everywhere:

$$dW = (-1)^{s-1} \int_0^\infty w^{(s)}(y)\, RD^s(y, \delta)\, dy \geq 0.$$

Necessity. Suppose, by contradiction, that $RD^s(y_0, \delta) < 0$ at some $y_0 > 0$. By continuity, there exists $\varepsilon > 0$ such that $RD^s(y, \delta) < 0$ for all $y \in [y_0, y_0 + \varepsilon]$.

Consider w whose $(s-1)$-th derivative is: $w^{(s-1)}(y) = \begin{cases} (-1)^{s-1}\,\varepsilon & y \leq y_0, \\ (-1)^{s-1}(y_0 + \varepsilon - y) & y_0 < y \leq y_0 + \varepsilon, \\ 0 & y > y_0 + \varepsilon. \end{cases}$

Such w belongs to Ω^s, and its s-th derivative is:

$$w^{(s)}(y) = \begin{cases} 0 & y < y_0, \\ (-1)^s & y_0 < y < y_0 + \varepsilon, \\ 0 & y > y_0 + \varepsilon. \end{cases} \tag{30}$$

Note that $w^{(k)}(y) = 0$ for all $y > y_0 + \varepsilon$ and all $k \geq 0$, so no boundary condition at infinity is required. Substituting (30) into (29): $dW = (-1)^{s-1} \int_{y_0}^{y_0+\varepsilon} (-1)^s\, RD^s(y, \delta)\, dy = -\int_{y_0}^{y_0+\varepsilon} RD^s(y, \delta)\, dy > 0$, a contradiction. Hence the condition is necessary. □

Proof of Theorem 2. For $W \in \Omega^s(z^+)$, the censoring condition $w^{(1)}(y) = 0$ for all $y \geq z^+$ reduces the welfare change to:

$$dW = \int_0^{z^+} w'(y)\, dy^E(y)\, dF_Y(y). \tag{31}$$

We apply $s-1$ successive integrations by parts to (31), using at each step $u = w^{(k)}(y)$ and $dv = d[RD^{k+1}(y, \delta)]$. The boundary terms vanish throughout:

- at $y = z^+$: by definition of $\Omega^s(z^+)$, $w^{(i)}(z^+) = 0$ for $i = 1, \ldots, s-1$, so the upper boundary terms vanish without any condition at infinity;

- at $y = 0$: $CD_\ell^k(0) = 0$ by definition, so $RD^k(0, \delta) = 0$ for all $k \geq 1$.

After $s - 1$ iterations one obtains:

$$dW = (-1)^{s-1} \int_0^{z^+} w^{(s)}(y)\, RD^s(y, \delta)\, dy. \tag{32}$$

Sufficiency. Suppose $RD^s(y, \delta) \geq 0$ for all $y \in [0, z^+]$. By definition of $\Omega^s(z^+)$: $(-1)^{s-1} w^{(s)}(y) \geq 0$. The integrand in (32) is non-negative on $[0, z^+]$: $dW = (-1)^{s-1} \int_0^{z^+} w^{(s)}(y)\, RD^s(y, \delta)\, dy \geq 0$.

Necessity. Suppose $RD^s(y_0, \delta) < 0$ at some $y_0 \in (0, z^+)$. By continuity, $\exists\, \varepsilon > 0$ with $[y_0, y_0 + \varepsilon] \subset (0, z^+)$ and $RD^s(y, \delta) < 0$ on $[y_0, y_0 + \varepsilon]$. Consider w whose $(s-1)$-th derivative is:

$$w^{(s-1)}(y) = \begin{cases} (-1)^{s-1}\, \varepsilon & y \leq y_0, \\ (-1)^{s-1}(y_0 + \varepsilon - y) & y_0 < y \leq y_0 + \varepsilon, \\ 0 & y > y_0 + \varepsilon. \end{cases}$$

Since $y_0 + \varepsilon < z^+$, we have $w^{(i)}(z^+) = 0$ for all i, so this w belongs to $\Omega^s(z^+)$. Substituting its s-th derivative into (32): $dW = (-1)^{s-1} \int_{y_0}^{y_0+\varepsilon} (-1)^s\, RD^s(y, \delta)\, dy = -\int_{y_0}^{y_0+\varepsilon} RD^s(y, \delta)\, dy > 0$, a contradiction. Hence the condition is necessary. □

D QUAIDS Results

Table D.1: QUAIDS Estimation Results — Intercept ($\hat{\alpha}$), Expenditure ($\hat{\beta}$) and Quadratic Expenditure ($\hat{\lambda}$) Parameters

	$\hat{\alpha}_i$	$\hat{\beta}_i$	$\hat{\lambda}_i$
Good	(1)	(2)	(3)
Cereals	0.053	-0.013	-0.001
	(0.033)	(0.015)	(0.002)
Meat	0.121^{*}	-0.186^{***}	-0.032^{***}
	(0.067)	(0.030)	(0.003)
Dairy	0.179^{***}	0.026^{*}	0.002
	(0.034)	(0.015)	(0.002)
Oil & Fat	-0.102^{***}	-0.048^{***}	-0.003^{***}
	(0.026)	(0.011)	(0.001)
Fish	0.024	-0.025	-0.005^{***}
	(0.037)	(0.016)	(0.002)
Fruits	0.099^{***}	0.014	0.001
	(0.020)	(0.009)	(0.001)
Vegetables	-0.349^{***}	-0.168^{***}	-0.014^{***}
	(0.035)	(0.015)	(0.002)
Sugar	0.092^{***}	0.038^{***}	0.005^{***}
	(0.018)	(0.008)	(0.001)
Beverages	0.705^{***}	0.304^{***}	0.041^{***}
	(0.052)	(0.024)	(0.003)
Elec./Gas	0.172^{***}	0.063^{***}	0.007^{***}
	(0.016)	(0.007)	(0.001)
Other	0.006	-0.005	-0.000
	(0.008)	(0.004)	(0.000)
Observations		12,463	

Notes: Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.
The 11^{th} equation (Other food) is excluded to satisfy the adding-up restriction.

Table D.2: QUAIDS Estimation Results — Price Parameters $\hat{\gamma}_{ij}$ (Upper Triangle, Symmetry Imposed)

	Cer.	Meat	Dairy	Oil	Fish	Fruit	Veg.	Sugar	Bev.	E/G	Other
Good	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Cereals	-0.087^{***}	0.170^{***}	-0.034^{***}	0.064^{***}	-0.180^{***}	-0.048^{***}	0.035^{***}	0.027^{***}	0.072^{***}	-0.001	-0.018^{***}
	(0.005)	(0.011)	(0.003)	(0.004)	(0.005)	(0.003)	(0.008)	(0.003)	(0.014)	(0.003)	(0.001)
Meat		0.081^{**}	0.038^{***}	-0.142^{***}	-0.039^{**}	0.036^{***}	0.188^{***}	-0.062^{***}	-0.247^{***}	-0.046^{***}	0.024^{***}
		(0.041)	(0.011)	(0.012)	(0.016)	(0.008)	(0.017)	(0.010)	(0.036)	(0.009)	(0.004)
Dairy			0.054^{***}	0.026^{***}	-0.065^{***}	-0.001	-0.050^{***}	0.001	0.007	0.029^{***}	-0.003^{***}
			(0.005)	(0.004)	(0.005)	(0.002)	(0.008)	(0.003)	(0.013)	(0.003)	(0.001)
Oil & Fat				-0.091^{***}	0.050^{***}	0.045^{***}	0.049^{***}	-0.042^{***}	0.040^{***}	-0.001	0.002
				(0.008)	(0.008)	(0.003)	(0.007)	(0.005)	(0.014)	(0.005)	(0.002)
Fish					0.082^{***}	0.000	0.054^{***}	0.062^{***}	0.063^{***}	-0.045^{***}	0.018^{***}
					(0.013)	(0.004)	(0.009)	(0.006)	(0.018)	(0.006)	(0.003)
Fruits						-0.019^{***}	-0.034^{***}	-0.011^{***}	0.024^{***}	0.013^{***}	-0.004^{***}
						(0.003)	(0.005)	(0.003)	(0.009)	(0.003)	(0.001)
Vegetables							0.005	-0.029^{***}	-0.182^{***}	-0.049^{***}	0.013^{***}
							(0.015)	(0.005)	(0.020)	(0.005)	(0.002)
Sugar								-0.052^{***}	0.062^{***}	0.034^{***}	0.010^{***}
								(0.007)	(0.011)	(0.005)	(0.002)
Beverages									0.141^{***}	0.055^{***}	-0.036^{***}
									(0.046)	(0.010)	(0.005)
Elec./Gas										0.014^{**}	-0.003
										(0.006)	(0.002)
Other											-0.002
											(0.001)
Observations						12,463					

Notes: Standard errors in parentheses. Symmetry restriction $\gamma_{ij} = \gamma_{ji}$ imposed; only the upper triangle is reported. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

Table D.3: QUAIDS Estimation Results — Demographic Scaling Parameters $\hat{\eta}_{ik}$ and $\hat{\rho}_k$

Good i	$\hat{\eta}_{ik}$ Household size ($hnum$)	Rural dummy ($rururb$)
Cereals	-0.001^{***}	-0.000
	(0.000)	(0.000)
Meat	-0.000^{***}	0.004^{***}
	(0.000)	(0.001)
Dairy	0.001^{***}	-0.003^{***}
	(0.000)	(0.000)
Oil & Fat	-0.000^{**}	0.001^{***}
	(0.000)	(0.000)
Fish	-0.000	-0.003^{***}
	(0.000)	(0.000)
Fruits	0.001^{***}	0.000^{***}
	(0.000)	(0.000)
Vegetables	-0.001^{***}	-0.000
	(0.000)	(0.000)
Sugar	-0.000^{***}	0.001^{***}
	(0.000)	(0.000)
Beverages	0.001^{***}	0.001^{**}
	(0.000)	(0.000)
Elec./Gas	0.000^{***}	-0.000^{***}
	(0.000)	(0.000)
Other	0.000^{***}	-0.000^{***}
	(0.000)	(0.000)
Translation parameters $\hat{\rho}_k$:		
Household size ($hnum$)	0.142^{***}	(0.014)
Rural dummy ($rururb$)	-0.291^{***}	(0.034)
Observations	12,463	

Notes: Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$. $\hat{\eta}_{ik}$ measures the effect of demographic variable k on the budget share of good i. $\hat{\rho}_k$ is the corresponding translation parameter in the demographic scaling function $\ln m(\mathbf{z}) = \sum_k \rho_k \ln z_k$.

Table D.4: Own-Price Elasticities by Subgroup

Good	Full	Rural	Urban	Expenditure quintile			HH head gender	
				Q1	Q3	Q5	Male	Female
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cereals	−1.766	−1.787	−1.749	−1.823	−1.739	−1.757	−1.747	−1.854
Meat	−1.227	−1.179	−1.268	−1.415	−1.225	−1.111	−1.220	−1.255
Dairy	−0.509	−0.550	−0.467	−0.513	−0.499	−0.522	−0.506	−0.518
Oil & Fat	−2.277	−2.363	−2.212	−2.191	−2.264	−2.413	−2.292	−2.218
Fish	0.363	0.247	0.481	0.567	0.381	0.218	0.335	0.492
Fruits	−1.381	−1.377	−1.384	−1.365	−1.386	−1.374	−1.386	−1.360
Vegetables	−1.487	−1.501	−1.477	−1.468	−1.483	−1.528	−1.487	−1.487
Sugar	−2.432	−2.528	−2.359	−2.410	−2.446	−2.433	−2.425	−2.458
Beverages	−1.676	−1.645	−1.698	−1.660	−1.694	−1.670	−1.694	−1.606
Elec./Gas	−0.869	−0.872	−0.866	−0.890	−0.867	−0.859	−0.869	−0.869
Other	−1.260	−1.251	−1.269	−1.221	−1.272	−1.281	−1.270	−1.227
Obs.	12,463							

Notes: Own-price elasticities evaluated at subgroup means using the QUAIDS model. Q1 = poorest quintile; Q3 = middle quintile; Q5 = richest quintile. HH = household. Elec./Gas = Electricity, gas and other fuels. Fish is the only good with a positive own-price elasticity, suggesting Giffen-like behaviour in this sample.

E Results of Optimal Taxation

Table E.1: Bivariate dominance tests for $y \in [0, y^{++}]$ and order $s = 3$ (optimal taxation).

	Cereals	*Meat*	*Fish*	*Dairy*	*Oils*	*Fruits*	*Vegetables*	*Sugar*	*Beverages*	*Other*	*Energy*
Cereals		–	–	–	–	–	–	–	–	–	–
Meat			–	–	–	–	–	>	–	>	>
Fish				>	>	–	–	>	>	>	>
Dairy					–	–	–	–	–	–	–
Oils						–	–	–	–	–	>
Fruits							–	–	–	–	–
Vegetables								–	–	–	–
Sugar									–	–	–
Beverages										–	–
Other											–
Energy											

Table E.2: Bivariate dominance test for $y \in [0, z^{+}]$ and order $s = 2$ (optimal taxation).

$\alpha = 0.10$, $B = 500$, ">": i dominates j; "<": j dominates i; "–": not significant.

	Cereals	*Meat*	*Fish*	*Dairy*	*Oils*	*Fruits*	*Vegetables*	*Sugar*	*Beverages*	*Other*	*Energy*
Cereals		–	–	–	–	–	–	–	–	–	–
Meat			–	–	–	–	–	>	–	>	>
Fish				>	–	–	–	–	>	>	>
Dairy					–	–	–	–	–	–	–
Oils						–	–	–	–	–	–
Fruits							–	–	–	–	–
Vegetables								–	–	–	–
Sugar									–	–	–
Beverages										–	–
Other											–
Energy											–

Table E.3: Bivariate dominance tests $MECF_i \cdot \widehat{CD}_i^3(y)$ vs $MECF_j \cdot \widehat{CD}_j^3(y)$ with $\frac{MECF_i}{MECF_j} = 1$ and $y \in [0, z^+]$, order $s = 3$ (optimal taxation).

$\alpha = 0.10$, $B = 500$, "$>$": i dominates j; "$<$": j dominates i; "–": not significant.

	Cereals	Meat	Fish	Dairy	Oils	Fruits	Vegetables	Sugar	Beverages	Other	Energy
Cereals		–	–	–	–	–	–	–	–	–	–
Meat			–	–	–	–	–	>	–	>	>
Fish				>	>	–	–	>	>	>	>
Dairy					–	–	–	–	–	–	–
Oils						–	–	–	–	–	>
Fruits							–	–	–	–	–
Vegetables								–	–	–	–
Sugar									–	–	–
Beverages										–	–
Other											–
Energy											