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# **Hybrid DS/FH-CDMA Systems Employing FSK Based Modulation Schemes**

by

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A thesis submitted to the  
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in partial fulfillment of the requirements  
for the degree of  
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Ottawa-Carleton Institute for Electrical Engineering  
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# Abstract

This thesis examines a hybrid DS/FH-CDMA system employing FSK based modulation schemes. The proposed modulation schemes are: non-coherent MFSK, a combination of MFSK and DPSK (called MFSK-DPSK) and wideband multitone (MT) FSK. In each case, the signal to be transmitted is modulated by a high rate BPSK signal (the PN sequence) and then it is hopped at a rate higher than the symbol rate into different frequency bins that are sufficiently spaced so that the fading in each channel appears to be an independent process. The main difference between each system is that a DS/FH-CDMA system employing wideband MT-FSK must employ fewer frequency bins to have the same (or comparable) bandwidth as systems employing MFSK or MFSK-DPSK. However, the advantage gained by using MT-FSK is the inherent diversity of the modulation scheme.

The bit error rate performance of each modulation scheme in a Rayleigh fading channel is found (in some cases, upper bounds are used). Both frequency-nonselective and frequency-selective fading are considered. Different error control coding schemes are also considered. A hybrid DS/FH-CDMA system is presented. The multiple access interference is modelled as additional white Gaussian noise. Spectral efficiency expressions are obtained for the system employing the different modulation and coding schemes considered. It is shown that the spectral efficiency of the system is inversely proportional to the bit energy to noise spectral density ratio required to achieve the maximum allowable

bit error rate. Furthermore, it is shown that coded MFSK and coded MFSK-DPSK can guarantee lower bit energy to noise spectral density ratio required to achieve the maximum allowable bit error rate of  $10^{-3}$  than coded MT-FSK, and thus the lower bounds on the spectral efficiency of DS/FH-CDMA systems employing coded MFSK or MFSK-DPSK are higher than DS/FH-CDMA systems employing coded MT-FSK.

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# Contents

|                                            |              |
|--------------------------------------------|--------------|
| <b>Abstract</b>                            | <b>i</b>     |
| <b>Acknowledgements</b>                    | <b>iii</b>   |
| <b>Contents</b>                            | <b>iii</b>   |
| <b>List of Figures</b>                     | <b>ix</b>    |
| <b>List of Tables</b>                      | <b>xii</b>   |
| <b>List of Acronyms</b>                    | <b>xviii</b> |
| <b>List of Symbols</b>                     | <b>xx</b>    |
| <b>1 Introduction</b>                      | <b>1</b>     |
| 1.1 Motivation . . . . .                   | 1            |
| 1.2 Contributions of This Thesis . . . . . | 5            |
| 1.3 Organization of This Thesis . . . . .  | 5            |

|          |                                                                                              |           |
|----------|----------------------------------------------------------------------------------------------|-----------|
| <b>2</b> | <b>Background and Literature Survey</b>                                                      | <b>7</b>  |
| 2.1      | Introduction . . . . .                                                                       | 7         |
| 2.2      | Code Division Multiple Access . . . . .                                                      | 8         |
| 2.2.1    | Asynchronous DS-CDMA Systems . . . . .                                                       | 8         |
| 2.2.2    | FH-CDMA Systems . . . . .                                                                    | 13        |
| 2.2.3    | Hybrid DS/FH-CDMA . . . . .                                                                  | 14        |
| 2.3      | Bandwidth Efficiency . . . . .                                                               | 18        |
| 2.3.1    | Spectral Efficiency of DS-CDMA . . . . .                                                     | 19        |
| 2.3.2    | Spectral Efficiency of FH-CDMA . . . . .                                                     | 20        |
| 2.4      | Existing CDMA Systems . . . . .                                                              | 22        |
| <b>3</b> | <b>Variants of MFSK Modulation for Rayleigh Fading Channels</b>                              | <b>24</b> |
| 3.1      | Introduction . . . . .                                                                       | 24        |
| 3.2      | Non-Coherent MFSK . . . . .                                                                  | 25        |
| 3.2.1    | Performance of MFSK in an AWGN channel . . . . .                                             | 26        |
| 3.2.2    | Performance of Uncoded MFSK in a Frequency-Nonselective<br>Rayleigh Fading Channel . . . . . | 31        |
| 3.2.3    | Performance of Uncoded MFSK in Rayleigh Fading With<br>Diversity . . . . .                   | 32        |
| 3.3      | Joint Frequency-Phase Modulation . . . . .                                                   | 36        |
| 3.3.1    | Combined MFSK-DPSK Modulation . . . . .                                                      | 36        |

|          |                                                                                    |           |
|----------|------------------------------------------------------------------------------------|-----------|
| 3.3.2    | Performance of Uncoded MFSK-DPSK Modulation in a Rayleigh Fading Channel . . . . . | 38        |
| 3.4      | Multitone FSK Modulation . . . . .                                                 | 47        |
| 3.4.1    | Permutation Modulation . . . . .                                                   | 47        |
| 3.4.2    | Design of Efficient MT-FSK Waveforms . . . . .                                     | 48        |
| 3.4.3    | Performance of (6,3) MT-FSK in an AWGN Channel . . . . .                           | 53        |
| 3.4.4    | Performance of MT-FSK in Frequency-Nonselective Rayleigh Fading . . . . .          | 60        |
| 3.4.5    | Performance of Wideband MT-FSK in Frequency-Selective Rayleigh Fading . . . . .    | 63        |
| 3.4.6    | Performance of Wideband MT-FSK in Rayleigh Fading with Channel Diversity . . . . . | 65        |
| 3.5      | Combined Wideband MT-FSK/DPSK Modulation . . . . .                                 | 69        |
| 3.5.1    | MT-FSK/DPSK Modem . . . . .                                                        | 69        |
| 3.5.2    | Performance of Combined Wideband MT-FSK/DPSK in Rayleigh Fading . . . . .          | 70        |
| 3.6      | Discussion . . . . .                                                               | 77        |
| <b>4</b> | <b>Error Control Coding</b>                                                        | <b>80</b> |
| 4.1      | Introduction . . . . .                                                             | 80        |
| 4.2      | Non-Binary BCH and RS Codes . . . . .                                              | 81        |
| 4.3      | Concatenated Coding . . . . .                                                      | 88        |
| 4.4      | Nonbinary Convolutional Codes . . . . .                                            | 94        |

|          |                                                                                              |            |
|----------|----------------------------------------------------------------------------------------------|------------|
| 4.5      | Discussion . . . . .                                                                         | 98         |
| <b>5</b> | <b>Hybrid DS/FH-CDMA System for Rayleigh Fading Channels</b>                                 | <b>100</b> |
| 5.1      | Introduction . . . . .                                                                       | 100        |
| 5.2      | Hybrid DS/FH-CDMA System Employing MFSK . . . . .                                            | 101        |
| 5.3      | Spectral Efficiency of Hybrid DS/FH-CDMA System Employing<br>MFSK . . . . .                  | 104        |
| 5.3.1    | Spectral Efficiency of DS/FH-CDMA Employing MFSK<br>Without Coding . . . . .                 | 106        |
| 5.3.2    | Spectral Efficiency of DS/FH-CDMA Employing MFSK<br>With Coding . . . . .                    | 108        |
| 5.4      | Hybrid DS/FH-CDMA System Employing JPFM . . . . .                                            | 111        |
| 5.4.1    | Spectral Efficiency of Hybrid DS/FH-CDMA Employing<br>MFSK-DPSK . . . . .                    | 113        |
| 5.4.2    | Spectral Efficiency of DS/FH-CDMA Employing MFSK-<br>DPSK Without Coding . . . . .           | 113        |
| 5.4.3    | Spectral Efficiency of DS/FH-CDMA Employing MFSK-<br>DPSK and Error Control Coding . . . . . | 116        |
| 5.5      | Hybrid DS/FH-CDMA Employing MT-FSK . . . . .                                                 | 120        |
| 5.5.1    | Performance of Hybrid DS/FH-CDMA Employing MT-FSK                                            | 122        |
| 5.5.2    | Performance of Hybrid DS/FH-CDMA System with Error<br>Control Coding . . . . .               | 126        |
| 5.6      | Discussion . . . . .                                                                         | 127        |

|          |                                                         |            |
|----------|---------------------------------------------------------|------------|
| <b>6</b> | <b>Conclusions and Suggestions for Further Research</b> | <b>129</b> |
| 6.1      | Conclusions . . . . .                                   | 129        |
| 6.2      | Suggestions for Further Research . . . . .              | 133        |
| <b>A</b> | <b>Basic Spread Spectrum Techniques and Terminology</b> | <b>135</b> |
| A.1      | Spread Spectrum Techniques . . . . .                    | 135        |
| A.1.1    | Direct-Sequence Spread Spectrum . . . . .               | 135        |
| A.1.2    | Frequency-Hopped Spread Spectrum . . . . .              | 136        |
| A.1.3    | Hybrid DS/FH-SS Systems . . . . .                       | 139        |
| A.2      | PN Sequences . . . . .                                  | 139        |
| <b>B</b> | <b>Balanced Incomplete Block Design</b>                 | <b>141</b> |
| <b>C</b> | <b>Design of MT-FSK Waveforms Using the BIB Design</b>  | <b>144</b> |
|          | <b>Bibliography</b>                                     | <b>147</b> |

# List of Figures

|     |                                                                                                                                                                                                                                  |    |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 2.1 | The sampled signal components after despreading and matched filtering. . . . .                                                                                                                                                   | 9  |
| 2.2 | The effect of the number of simultaneous users on the effective bit energy to noise spectral density ratio ( $\frac{E_b}{N_o}$ ) of a DS-CDMA system for different $\frac{E_b}{N_o}$ and bandwidth expansion factors $B_e$ . . . | 12 |
| 2.3 | The effect of multiple users on the effective bit error rate performance of a FH-CDMA system for $P_b(1) = 10^{-6}$ and different number of frequency slots $N_f$ . . . . .                                                      | 15 |
| 3.1 | Optimum receiver of non-coherent MFSK. . . . .                                                                                                                                                                                   | 27 |
| 3.2 | Bit error rate performance of noncoherent MFSK in AWGN for $M = 8, 16$ , and $32$ . . . . .                                                                                                                                      | 30 |
| 3.3 | Bit error rate performance of noncoherent MFSK in flat Rayleigh fading. . . . .                                                                                                                                                  | 33 |
| 3.4 | (a) MFSK-DPSK modulator (b) MFSK-DPSK demodulator. . .                                                                                                                                                                           | 37 |
| 3.5 | MFSK-DPSK diversity receiver ( $L = 2$ ). . . . .                                                                                                                                                                                | 39 |
| 3.6 | Markov chain to determine $P_d$ . . . . .                                                                                                                                                                                        | 42 |

|      |                                                                                                                            |     |
|------|----------------------------------------------------------------------------------------------------------------------------|-----|
| 3.7  | Bit error rate of MFSK-DBPSK in Rayleigh fading for $L = 1, 2$<br>and 3. . . . .                                           | 44  |
| 3.8  | Bit error rate of MFSK-DQPSK in Rayleigh fading for $L = 1, 2$<br>and 3. . . . .                                           | 45  |
| 3.9  | Optimum receiver for (6,3) MT-FSK. . . . .                                                                                 | 54  |
| 3.10 | Bit error rate upper bounds for MT-FSK in an AWGN channel. . . . .                                                         | 59  |
| 3.11 | Performance of MT-FSK in frequency-nonselective Rayleigh fading. . . . .                                                   | 62  |
| 3.12 | Bit error performance of wideband MT-FSK in slow Rayleigh<br>fading. . . . .                                               | 66  |
| 3.13 | Performance of wideband MT-FSK in Rayleigh fading with ad-<br>ditional channel diversity. . . . .                          | 68  |
| 3.14 | (a) MT-FSK/DPSK modulation. (b) MT-FSK/DPSK demodu-<br>lation. . . . .                                                     | 71  |
| 3.15 | Markov chain used to determine $P_d$ for MT-FSK/DPSK modu-<br>lation. . . . .                                              | 73  |
| 3.16 | Comparison of MT-FSK/DBPSK to MT-FSK modulation schemes<br>in frequency-nonselective Rayleigh fading. . . . .              | 75  |
| 3.17 | Comparison of MT-FSK/DQPSK to MT-FSK modulation schemes<br>in frequency-nonselective Rayleigh fading. . . . .              | 76  |
| 3.18 | Comparison of MT-FSK/DPSK to MFSK and MFSK/DPSK mod-<br>ulation schemes in frequency-nonselective Rayleigh fading. . . . . | 78  |
| 5.1  | Hybrid DS/FH-CDMA system in forward link. . . . .                                                                          | 102 |
| 5.2  | DS/FH-CDMA signal in time and frequency. . . . .                                                                           | 103 |

|     |                                                                                                  |     |
|-----|--------------------------------------------------------------------------------------------------|-----|
| 5.3 | DS/FH-CDMA signal in time and frequency when MFSK-DPSK<br>is selected modulation scheme. . . . . | 112 |
| 5.4 | Hybrid DS/FH-CDMA system employing MT-FSK. . . . .                                               | 121 |
| A.1 | A DS-SS communications system. . . . .                                                           | 136 |
| A.2 | Direct-sequence spread spectrum: (a) in the time domain (b) in<br>the frequency domain. . . . .  | 137 |
| A.3 | A FH-SS communication system. . . . .                                                            | 138 |

# List of Tables

|     |                                                                                                                                                        |    |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 3.1 | $\bar{\gamma}_b$ required to obtain a bit error rate of $10^{-3}$ for an $L$ th order diversity system employing MFSK in Rayleigh fading. . . . .      | 35 |
| 3.2 | $\bar{\gamma}_b$ required to obtain a bit error rate of $10^{-3}$ for uncoded MFSK-DPSK for different $L$ in Rayleigh fading. . . . .                  | 46 |
| 3.3 | (7,4) Hamming code . . . . .                                                                                                                           | 49 |
| 3.4 | Equal energy waveforms derived from (7,4) Hamming codewords of weight 3. . . . .                                                                       | 50 |
| 3.5 | (6,3) MT-FSK waveforms. . . . .                                                                                                                        | 51 |
| 3.6 | (8,3) MT-FSK waveforms. . . . .                                                                                                                        | 51 |
| 3.7 | (10,4) MT-FSK waveforms. . . . .                                                                                                                       | 52 |
| 3.8 | (12,4) MT-FSK waveforms. . . . .                                                                                                                       | 52 |
| 3.9 | $\gamma'_b$ required for a bit error rate of $10^{-3}$ for wideband MT-FSK in Rayleigh fading. . . . .                                                 | 67 |
| 4.1 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing (6,3) MT-FSK with different non-binary BCH codes. . . . . | 83 |

|      |                                                                                                                                                                       |    |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 4.2  | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing (10,4) MT-FSK with different non-binary BCH codes.                       | 84 |
| 4.3  | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing 8FSK with different non-binary BCH codes. . . . .                        | 84 |
| 4.4  | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing 4FSK-DBPSK with different non-binary BCH codes.                          | 84 |
| 4.5  | $\bar{\gamma}_b$ required for a bit error rate of $10^{-3}$ for an $L$ th order diversity system employing (8,3) MT-FSK with different non-binary BCH codes. . . . .  | 85 |
| 4.6  | $\bar{\gamma}_b$ required for a bit error rate of $10^{-3}$ for an $L$ th order diversity system employing (12,4) MT-FSK with different non-binary BCH codes. . . . . | 85 |
| 4.7  | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing 16FSK with different non-binary BCH codes. . . . .                       | 85 |
| 4.8  | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing 8FSK-DBPSK with different non-binary BCH codes.                          | 86 |
| 4.9  | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing 4FSK-DQPSK with different non-binary BCH codes.                          | 86 |
| 4.10 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing 16FSK-DBPSK with different non-binary BCH codes.                         | 86 |
| 4.11 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing 8FSK-DQPSK with different non-binary BCH codes.                          | 87 |
| 4.12 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing 16FSK-DQPSK with different non-binary BCH codes.                         | 87 |

|      |                                                                                                                                                                           |    |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 4.13 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing (6,3) MT-FSK with different concatenated codes. . .                       | 89 |
| 4.14 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing (10,4) MT-FSK with different concatenated codes. . .                      | 90 |
| 4.15 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing 8FSK with different concatenated codes. . . . .                           | 90 |
| 4.16 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing 4FSK-DBPSK with different concatenated codes. . .                         | 90 |
| 4.17 | $\bar{\gamma}_b$ required for a bit error rate of $10^{-3}$ for an $L$ th order diversity<br>system employing (8,3) MT-FSK with different concatenated codes.             | 91 |
| 4.18 | $\bar{\gamma}_b$ required for a bit error rate of $10^{-3}$ for an $L$ th order diversity<br>system employing (12,4) MT-FSK with different concatenated<br>codes. . . . . | 91 |
| 4.19 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing 16FSK with different concatenated codes. . . . .                          | 91 |
| 4.20 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing 8FSK-DBPSK with different concatenated codes. . .                         | 92 |
| 4.21 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing 4FSK-DQPSK with different concatenated codes. . .                         | 92 |
| 4.22 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing 16FSK-DBPSK with different concatenated codes. . .                        | 92 |
| 4.23 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing 8FSK-DQPSK with different concatenated codes. . .                         | 93 |
| 4.24 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system<br>employing 16FSK-DQPSK with different concatenated codes. . .                        | 93 |

|      |                                                                                                                                                                           |     |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 4.25 | $\bar{\gamma}_b$ required for $P_b < 10^{-3}$ for an $L$ th order diversity system employing MFSK or MT-FSK with dual- $k$ convolutional coding. . .                      | 97  |
| 5.1  | $\Delta f$ , $R_p$ and $\beta(M, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK without coding ( $R_b = 4.8$ kbps, $W_{ds} = 195$ kHz). . .                    | 107 |
| 5.2  | $\Delta f$ , $R_p$ and $\beta(M, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK without coding ( $R_b = 9.6$ kbps, $W_{ds} = 195$ kHz). . .                    | 107 |
| 5.3  | Spectral efficiency (in bps/Hz) of the uncoded hybrid DS/FH-CDMA system employing uncoded MFSK in Rayleigh fading. .                                                      | 107 |
| 5.4  | $\Delta f$ , $R_p$ and $\beta(M, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK and BCH coding ( $R_b = 4.8$ kbps, $W_{ds} = 195$ kHz). . .                    | 109 |
| 5.5  | $\Delta f$ , $R_p$ and $\beta(M, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK and BCH coding ( $R_b = 9.6$ kbps, $W_{ds} = 195$ kHz). . .                    | 109 |
| 5.6  | $\Delta f$ , $R_p$ and $\beta(M, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK and rate 1/2 dual- $k$ coding ( $R_b = 4.8$ kbps, $W_{ds} = 195$ kHz). . . . . | 110 |
| 5.7  | $\Delta f$ , $R_p$ and $\beta(M, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK and rate 1/2 dual- $k$ coding ( $R_b = 9.6$ kbps, $W_{ds} = 195$ kHz). . . . . | 110 |
| 5.8  | Spectral efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing MFSK and non-binary BCH coding. . . . .                                                                    | 110 |
| 5.9  | Spectral efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing MFSK and rate 1/2 dual- $k$ convolutional coding. . . . .                                                  | 111 |
| 5.10 | $\Delta f$ , $R_p$ and $\beta(M_f, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK-DPSK without coding ( $R_b = 4.8$ kbps, $W_{ds} = 195$ kHz). . . . .         | 114 |

|                                                                                                                                                                                     |     |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 5.11 $\Delta f$ , $R_p$ and $\beta(M_f, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK-DPSK without coding ( $R_b = 9.6$ kbps, $W_{ds} = 195$ kHz). . . . .              | 115 |
| 5.12 Spectral efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing uncoded MFSK-DPSK. . . . .                                                                                      | 116 |
| 5.13 $\Delta f$ , $R_p$ and $\beta(M_f, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK-DPSK with BCH coding ( $R_b = 4.8$ kbps, $W_{ds} = 195$ kHz). . . . .             | 117 |
| 5.14 $\Delta f$ , $R_p$ and $\beta(M_f, R_b, R_p, \Delta f)$ for hybrid DS/FH-CDMA employing MFSK-DPSK with BCH coding ( $R_b = 9.6$ kbps, $W_{ds} = 195$ kHz). . . . .             | 118 |
| 5.15 Spectral efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing nonbinary BCH encoded MFSK-DPSK. . . . .                                                                        | 119 |
| 5.16 Bandwidth efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing uncoded MT-FSK in Rayleigh fading. . . . .                                                                     | 125 |
| 5.17 Bandwidth efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing MT-FSK and BCH coding. . . . .                                                                                 | 126 |
| 5.18 Bandwidth efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing MT-FSK and dual- $k$ coding. . . . .                                                                           | 126 |
| 6.1 A comparison of the spectral efficiencies (in bps/Hz) of hybrid DS/FH-CDMA systems employing MFSK, MFSK-DPSK or MT-FSK without the application of error control coding. . . . . | 131 |
| 6.2 A comparison of the spectral efficiencies (in bps/Hz) of hybrid DS/FH-CDMA systems employing MFSK, MFSK-DPSK or MT-FSK with the application of error control coding. . . . .    | 132 |

|     |                                     |     |
|-----|-------------------------------------|-----|
| C.1 | BIB Design of (8,3) MT-FSK. . . . . | 145 |
|-----|-------------------------------------|-----|

# List of Acronyms

AWGN: additive white Gaussian noise.

BPSK: binary phase shift keying.

CDMA: code division multiple access.

DS-CDMA: direct sequence code division multiple access.

DS/FH-CDMA: direct sequence / frequency-hopped code division multiple access.

DS/FH-SS: direct sequence / frequency-hopped spread spectrum.

DS-SS: direct sequence spread spectrum.

DBPSK: differential binary phase shift keying.

DQPSK: differential quaternary phase shift keying.

DPSK: differential phase shift keying.

FDMA: frequency division multiple access.

FFH: fast frequency hopping.

FH-CDMA: frequency-hopped code division multiple access.

FH-SS: frequency-hopped spread spectrum.

FSK: frequency shift keying.

FSR: feedback shift register.

JFPM: joint frequency-phase modulation.

MAI: multiple access interference.

MFSK: M-ary frequency shift keying.

MFSK-DPSK: Combined M-ary frequency shift keying / differential phase shift keying modulation.

MT-FSK: multiple tone frequency shift keying.

PCN: Personal Communications Network.

PN: pseudonoise.

QPSK: quaternary phase shift keying.

SFH: slow frequency hopping.

SNR: signal to noise ratio.

SS: spread spectrum.

TDMA: time division multiple access.

# List of Symbols

$\alpha$ : channel gain.

$\alpha_{ro}$ : rolloff factor.

$\beta(M, R_s, R_p, \Delta f)$ : function used in the determination of the MAI in a DS/FH-CDMA system employing MFSK or MFSK-DPSK modulation.

$\Delta f$ : frequency separation between adjacent tones in an FSK based modulation scheme.

$(\Delta f)_c$ : coherence bandwidth of a fading channel.

$\mathcal{E}$ : energy per tone for MT-FSK modulation.

$\mathcal{N}$ : noise power.

$\mathcal{N}'$ : equivalent noise power (including effects of MAI).

$\eta$ : spectral efficiency in bps/Hz.

$\pi_i$ : steady state probability of a system being in state  $i$ .

$\rho$ : relationship between  $2R_p$  and  $\Delta f$  in a DS/FH-CDMA system employing wideband MT-FSK.

$\rho_{km}$ : cross-correlation between symbol  $k$  and symbol  $m$ .

$\sigma(v, \rho)$ : function used in the determination of the MAI in a DS/FH-CDMA

system employing wideband MT-FSK.

$\theta$ : channel phase.

$B_e$ : processing gain, bandwidth expansion factor.

$C$ : signal power.

$c_i(t)$ : PN sequence of the  $i$ th user.

$c_t$ : number of common tones contained in two symbols from an MT-FSK signalling set.

$D$ : equivalent diversity order of a MT-FSK symbol.

$d$ : distance between two codewords.

$d_{free}$ : free distance of a convolutional code.

$d_{min}$ : minimum distance between codewords for a given error control code.

$E_b$ : energy per bit.

$E_c$ : symbol energy per diversity channel.

$E_s$ : energy per symbol.

$\frac{E_b}{N_o}$ ,  $\gamma_b$ : energy per bit to noise spectral density ratio.

$\frac{E_b}{N'_o}$ ,  $\gamma'_b$ : equivalent energy per bit to noise spectral density ratio (including effects of MAI).

$\frac{E_s}{N_o}$ ,  $\gamma_s$ : energy per symbol to noise spectral density ratio.

$\frac{E_s}{N'_o}$ ,  $\gamma'_s$ : equivalent energy per symbol to noise spectral density ratio (including effects of MAI).

$\gamma_{eff}$ : effective symbol energy to noise spectral density ratio of a MT-FSK symbol.

$f_c$ : carrier frequency.

$f_m$ : frequency used to represent the  $m$  th tone of a FSK based modulation scheme.

$k$ : number of bits per symbol in an M-ary signalling set.

$k_f$ : number of bits represented by frequency sub-symbol in MFSK-DPSK modulation.

$k_p$ : number of sub-symbol bits represented by differentially encoded phase in MFSK-DPSK modulation.

$L$ : order of diversity obtained by using FFH techniques ( $L$  hops per symbol).

$M$ : number of frequency tones for MFSK modulation.

$M_f$ : number of frequency tones for MFSK-DPSK modulation.

$N_f$ : number of frequency slots in a FH-CDMA or a DS/FH-CDMA system.

$N_o$ : noise spectral density.

$N'_o$ : equivalent noise spectral density (including effect of MAI).

$P$ : transition probability matrix for a Markov chain.

$P_b$ : probability of bit error.

$P_{b_{M-D}}$ : probability of bit error of MFSK-DPSK modulation.

$P_c$ : probability of a correct decision.

$P_d$ : probability of bit error of the differentially encoded phase sub-symbol in MFSK-DPSK modulation.

$P_{DPSK}$ : probability of bit error of the differentially encoded phase subsymbol in MFSK-DPSK modulation provided that the frequency sub-symbol is correctly detected.

$P_M$ : probability of symbol error for  $M$ -ary modulation schemes.  
 $P_{Mf}$ : probability of frequency sub-symbol error for MFSK-DPSK modulation.  
 $R_b$ : bit rate.  
 $\tau_c$ : code rate.  
 $R_h$ : hop rate.  
 $R_p$ : chip rate.  
 $R_s$ : symbol rate.  
 $t$ : maximum number of correctable errors in a BCH codeword.  
 $T_b$ : bit duration.  
 $T_c$ : chip duration.  
 $T_s$ : symbol duration.  
 $U$ : number of simultaneous users employing channel.  
 $U_{max}$ : maximum number of simultaneous users that can access a channel.  
 $U_m$ : decision variable corresponding to the  $m$ th symbol.  
 $u_{ni}$ : lowpass equivalent of the  $n$ th symbol sent by user  $i$ .  
 $v$ : total number of distinct tones employed by an MT-FSK signalling set.  
 $w$ : total number of tones per symbol in MT-FSK.  
 $W$ : bandwidth of narrowband (or unspread) signal.  
 $w_d$ : number of codewords that are a distance  $d$  from the all 0 codeword.  
 $W_{ds}$ : bandwidth of signal following direct sequence spreading but prior to hopping in a DS/FH-CDMA system.

$w_{eq}$ : equivalent weight of MT-FSK decision variable obtained by eliminating common terms in other decision variable.

$W_{ss}$ : total bandwidth of spread spectrum signal.

$z_n$ : filtered Gaussian noise sample at time unit  $n$ .

# Chapter 1

## Introduction

### 1.1 Motivation

In 1991, there were roughly 5 million cellular telephone subscribers in North America [1], and the need for this service is still growing. In many cities, analog and first generation digital cellular operators are already reaching their maximum capacity [2].

Personal Communications Networks (PCNs) [3] are the logical evolution of cellular telephone networks. This technology will allow the use of a small portable communication device from virtually anywhere by combining cellular radio, satellite, and wireline communications. One will be able to receive voice as well as data (e.g. fax). It is estimated that 25 to 60 million consumers in North America will subscribe to this service [2].

One can see that a system supporting so many users using currently employed techniques would require a large amount of bandwidth, but at the present time, this bandwidth is not available in the commercially feasible low frequency bands. Therefore, most researchers are directing their efforts to finding a combination

of modulation and multiple access schemes which can better utilize the existing bandwidth.

The three basic multiple access schemes are: frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA). In FDMA, a user is assigned a specific frequency slot which makes up a fraction of the overall bandwidth; while in TDMA, a user occupies the entire bandwidth for a brief time interval. In CDMA each user is assigned a specific code which is employed by the sender and is required by the receiver to retrieve the desired signal from the common channel. These frequency, time or code allocations can be fixed, or assigned on demand. Thus we can have fixed assignment or demand assignment FDMA, TDMA or CDMA networks.

In CDMA, a code is used to spread a user's signal over a large bandwidth. This code is also called a pseudonoise (PN) sequence. All other users spread their signals over the same bandwidth using different codes. At the receiver, only the desired signal is despread using the required code while the remaining spread signals act as interference. As the number of simultaneous users increases, the bit error rate performance of each individual user degrades. Thus the capacity of the system is limited by the highest acceptable bit error rate performance of a user's signal. This type of network is said to have a soft capacity [4] as the actual capacity can be surpassed, although one would choose the proper parameters to keep the probability of network overload low.

We say that CDMA systems are interference limited. This means that the maximum number of users allowed on the channel is limited by the amount of interference that can be tolerated. In other words, one way that the capacity of the system can be increased is by finding modulation and coding schemes which are more tolerant of interference<sup>1</sup>. These modulation and coding schemes are

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<sup>1</sup>The capacity of CDMA systems can also be increased by employing interference cancellation techniques, and by improved detection and signal processing schemes.

said to be power efficient because they require less power to achieve a given bit error rate than non power efficient schemes. The search for higher capacities by using power efficient modulation and coding schemes in a Rayleigh fading environment is documented in [5]-[9].

CDMA has been compared favourably to FDMA and TDMA [10, 11]. For voice applications, it has been shown that CDMA can take better advantage of the voice activity factor than TDMA or FDMA. In other words, when an active user is not speaking, the interference to other users is reduced. Thus more users can simultaneously access the channel. It has also been shown that CDMA is more tolerant of co-channel and adjacent channel interference.

DS-CDMA systems for satellite [12, 13], land-mobile or personal communications [5, 14, 15] are being extensively researched. DS-CDMA systems can reduce the interference caused by all multipath components which are more than one chip or hop out of sync with the receiver's PN sequence. The bandwidth expansion aspect of the signal also allows for diversity reception of the signal. DS-CDMA systems can obtain path diversity by using a RAKE receiver [5, 8, 9, 14].

One problem with RAKE received DS-CDMA systems in a fading environment is the need to sound the channel [5]. Since the receiver does not know the nature of the channel, it cannot determine with a good deal of certainty which taps of the RAKE receiver contain information without obtaining information about the channel. This is accomplished by channel sounding. Channel sounding techniques are discussed in [16].

Another problem with all DS-CDMA systems is the need for power control. The capacity of a DS-CDMA system is maximized when all signals are received at the same power level. The proximity of the transmitters to the base station as well as the independent fading process encountered by each signal in a mobile environment can greatly affect the received power level of each individual signal.

Thus the need for power control.

The alternative to DS-CDMA is FH-CDMA. A FH-CDMA system can take advantage of its large bandwidth to obtain diversity as well. By hopping a symbol into many frequency slots, frequency or channel diversity is achieved. The frequency slots are sufficiently spaced so that the fading process encountered by a signal in each slot appears to be independent of the fading in the other slots. Significant improvements in bit error rate performance can be achieved in this manner [17].

The bit error rate performance of FH-CDMA systems is determined by the hit probability. A hit occurs when two or more signals are hopped into the same frequency slot. Thus to achieve an acceptable bit error rate, the probability of a hit must be kept quite low. To achieve this, the number of simultaneous users must be much lower than the number of available frequency slots [18]. Thus these systems are generally bandwidth inefficient. The advantage of FH-CDMA is that it is not affected by the near-far problem.

Hybrid DS/FH-CDMA systems are now being investigated [15], [19]-[23] as they may combine some of the advantages of both DS and FH systems while avoiding some of their disadvantages. In this thesis, a hybrid DS/FH-CDMA system is proposed for use in the base to mobile link (forward link) of a land mobile communication system. The channel is modelled by a slow Rayleigh fading. The system will employ different variants of  $M$ -ary frequency shift keying (MFSK). The performance of the system employing each variant of MFSK is analyzed and the results are compared. The figure of merit used for comparison purposes in this thesis is the spectral efficiency (or bandwidth efficiency) of the system, as system capacity is directly related to its spectral efficiency.

## 1.2 Contributions of This Thesis

The objective of this thesis is to consider a hybrid DS/FH-CDMA system in a mobile fading environment. The research presented in this thesis hopes to add the following information to the existing literature:

- (1) To provide a model for a hybrid DS/FH-CDMA system employing fast frequency hopping as opposed to slow frequency hopping as demonstrated in [19] - [22].
- (2) To introduce the use of MFSK variants in hybrid DS/FH-CDMA systems, while obtaining bit error rate performance equations for these modulation schemes with diversity.
- (3) Provide analytical evaluations of the spectral efficiency of hybrid DS/FH-CDMA employing MFSK variants for specific channel conditions.
- (4) Investigate the impact of error control coding on the spectral efficiency of these systems.

## 1.3 Organization of This Thesis

Having discussed the motivation and contributions of this thesis as above, the remainder of this thesis is organized as follows:

Chapter 2 presents an overview of basic CDMA techniques and terminology. The issue of spectral efficiency is presented. Previous research is highlighted.

In Chapter 3, the modulation schemes under consideration are discussed in detail. We begin with noncoherent MFSK. The derivation of its bit error rate performance in Rayleigh fading is shown. We then present the concept of joint frequency-phase modulation (JFPM). The symbol is divided into two

sub-symbols; one conveyed by a frequency tone, while the other is conveyed by differential phase. Multitone FSK (MT-FSK) is discussed and a number of signalling sets are provided. Finally, JFPM is extended to include MT-FSK as the frequency sub-symbol. Bit error rate performances are derived for all modulation schemes with and without diversity.

Error control coding schemes are discussed in Chapter 4 as a means of improving the bit error rate results obtained for the different modulation schemes discussed in Chapter 3. Depending on the modulation scheme employed, different types of codes as well as different code rates are considered.

Chapter 5 provides a description of the hybrid DS/FH-CDMA system as well as its performance with the different modulation schemes under certain conditions. Equations for the spectral efficiency are derived for each system, and the results are compared.

Conclusions concerning the research presented in this thesis as well as suggestions for further research in this field are given in Chapter 6.

Appendix A discusses basic spread spectrum techniques and terminology. It is intended to be a tutorial for those readers who may need a quick review of spread spectrum fundamentals.

Appendix B describes the balanced incomplete block (BIB) design, which is a method of arranging  $v$  distinct objects into  $b$  blocks. This is used in designing MT-FSK signalling sets.

An example of designing MT-FSK signalling sets using the BIB design technique is shown in Appendix C.

## Chapter 2

# Background and Literature Survey

### 2.1 Introduction

Spread spectrum (SS) communication was originally conceived for wartime communications as a strategy against intentional interference, also known as jamming. A jammer's goal is to prevent reliable communications by introducing high power carrier waves or wideband noise into the communication environment. The use of spread spectrum provides resistance to these jamming techniques by sacrificing bandwidth. It can also provide information security [24]. Research into spread spectrum's performance in jamming is still ongoing [25]-[27].

In the mid 70s, commercial use of spread spectrum became a hot topic in communication engineering. Recently, SS techniques have been considered for cellular radio [4, 10], packet radio [28, 29], mobile satellite [7, 30], and PCN [31, 32]. Although spread spectrum's original goal was to combat jamming through spectral inefficiency, it appears that its new goal is to improve spectral efficiency in

an interference limited environment.

This chapter will provide an overview of CDMA technology. The spectral efficiency of CDMA systems is discussed. Spectral efficiency, which describes how efficiently bandwidth is used to transmit information, is often used in the comparison of different multiple access schemes. Important results from previous research will also be discussed.

There are two basic types of spread spectrum systems: direct-sequence spread spectrum (DS-SS) systems and frequency-hopped spread spectrum (FH-SS) systems. A basic tutorial on spread spectrum technology is given in Appendix A. More detailed tutorials on spread spectrum techniques can be found in [33] and [34, Chapter 7].

## 2.2 Code Division Multiple Access

In time division multiple access, the channel is shared by assigning each user a specific time slot when it can access the system. In frequency division multiple access, the channel is shared by assigning each user a different carrier frequency. Similarly the term code division multiple access denotes that the channel is shared by several users by assigning each user a different PN code (or sequence), although common code SS multiple access systems also exist [35]. There are two basic types of CDMA systems. Those which employ direct sequence techniques are called DS-CDMA systems, and those which employ frequency-hopped techniques are referred to as FH-CDMA systems.

### 2.2.1 Asynchronous DS-CDMA Systems

In an asynchronous DS-CDMA system, no attempt is made to align the PN sequences of the different users. User  $i$  is assigned PN sequence  $c_i(t)$ . For user

1, the received signal is the sum of the desired signal, which is spread by  $c_1(t)$  and the  $U - 1$  other spread signals. These components may or may not be affected by fading, shadowing or other impairments. For the purpose of demonstration, let us assume that the only impairment is additive white Gaussian noise. This is shown in Figure 2.1. The sampled noise component at time  $n$  after matched filtering at receiver 1 is:

$$z'_n = z_n + \sum_{i=2}^U \int_{(n-1)T_s}^{nT_s} u_{ni}(t) u_{n1}^*(t) c_i(t + \tau_i) c_1(t) dt \quad (2.1)$$

where  $z_n$  is the output noise component, sampled at time  $n$ , and  $u_{ni}$  is the  $n$ th received symbol sent by user  $i$ . If the modulation is BPSK or QPSK,  $\text{var}[|u_{ni} u_{n1}^*|] = C$ , where  $C$  is the carrier power. Note that this statement assumes that the received power of each user is equal. It can be shown that the channel capacity of DS-CDMA is maximized when this is true [34, 36, 37]. Therefore, there is a need for power control.

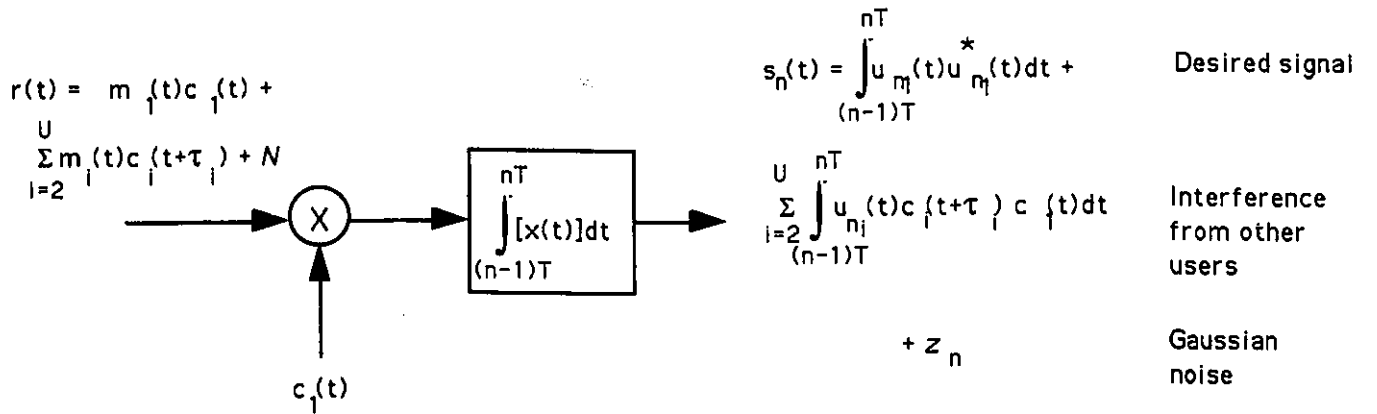


Figure 2.1 : The sampled signal components after despreading and matched filtering.

The MAI component is the sum of  $U - 1$  independent random variables, and therefore the central limit theorem states that it can be approximated by a Gaussian random variable [38]. The power of the noise plus MAI signal from which we obtain the samples  $z'_n$  is:

$$\mathcal{N}' = \mathcal{N} + C \text{var} \left[ \sum_{i=2}^U \int_{(n-1)T_s}^{nT_s} c_i(t + \tau_i) c_1(t) dt \right] \quad (2.2)$$

The effective noise spectral density,  $N'_o = \mathcal{N}'/W$  (where  $W$  is the bandwidth of the non-spread signal) is given by:

$$\begin{aligned} N'_o &= N_o + \frac{C}{W} \text{var} \left[ \sum_{i=2}^U \int_{(n-1)T_s}^{nT_s} c_i(t + \tau_i) c_1(t) dt \right] \\ &= N_o \left( 1 + \frac{C}{N_o W} \text{var} \left[ \sum_{i=2}^U \int_{(n-1)T_s}^{nT_s} c_i(t + \tau_i) c_1(t) dt \right] \right) \end{aligned} \quad (2.3)$$

Thus as  $U$  increases,  $N'_o$  also increases. This means that the performance of the individual user degrades as the number of users simultaneously accessing the channel increases. Thus the capacity of this channel depends on the maximum symbol error rate that is tolerable by all individual users (or equivalently, the maximum  $N'_o$  that the system can tolerate). Note that  $N'_o$  depends on the variance of the cross correlation between the PN sequences of each user, and this depends on the type of PN sequences employed by the system. Therefore, the additional noise produced by the MAI must be found by averaging equation (2.3) over  $\tau_i$  and  $i$ .

A generally accepted derivation for the effect of MAI in a DS-CDMA system is given in [39], where it is stated that the effective symbol energy to noise spectral density ratio,  $E_s/N'_o$  for DS-CDMA systems employing random signature PN sequences and a processing gain of  $B_c$  is given by:

$$\frac{E_s}{N'_o} = \left[ \left( \frac{E_s}{N_o} \right)^{-1} + \left( \frac{3B_e}{2(U-1)} \right)^{-1} \right]^{-1} \quad (2.4)$$

Simplifying this equation, the equivalent energy per symbol to noise spectral density ratio is

$$\gamma'_s = \frac{\gamma_s}{1 + \frac{2}{3} \frac{(U-1)}{B_e} \gamma_s} \quad (2.5)$$

where  $\gamma'_s = E_s/N'_o$  and  $\gamma_s = E_s/N_o$ . For BPSK modulation, the symbol and bit signal-to-noise ratios are the same, i.e.  $\gamma_s = \gamma_b$ . Furthermore the processing gain is given by  $B_e = R_p/R_b$ , where  $R_p$  is the symbol rate of the PN sequence, otherwise known as the *chip rate*, and  $R_b$  is the bit rate of the information signal. Typically  $R_p \gg R_b$ . Equation (2.5) can be rewritten as:

$$\gamma'_b = \frac{\gamma_b}{1 + \frac{2}{3} \frac{(U-1)R_b}{R_p} \gamma_b} \quad (2.6)$$

Figure 2.2 shows  $\gamma'_b$  as a function of the number of simultaneous users for different  $B_e$  and  $\gamma_b$ . Note that for systems employing the same  $B_e$ , increases in  $E_b/N_o$  cause negligible increase in  $E_b/N'_o$  when the number of simultaneous users is large.

In a synchronous DS-CDMA system, the PN sequences of all users are aligned, thus making  $\tau_i = 0$  for all  $i$ . It then becomes simple to choose a set of PN sequences  $c_i(t)$  which are mutually orthogonal. This can be achieved, for example, by using Hadamard codes [40]. Therefore the cross-correlation of different codes becomes 0, and the MAI is eliminated. However, in a mobile environment, maintaining synchronism and thus orthogonality for all users becomes difficult due to the motion of the mobiles and multipath. In this thesis, only asynchronous DS-CDMA systems will be considered.

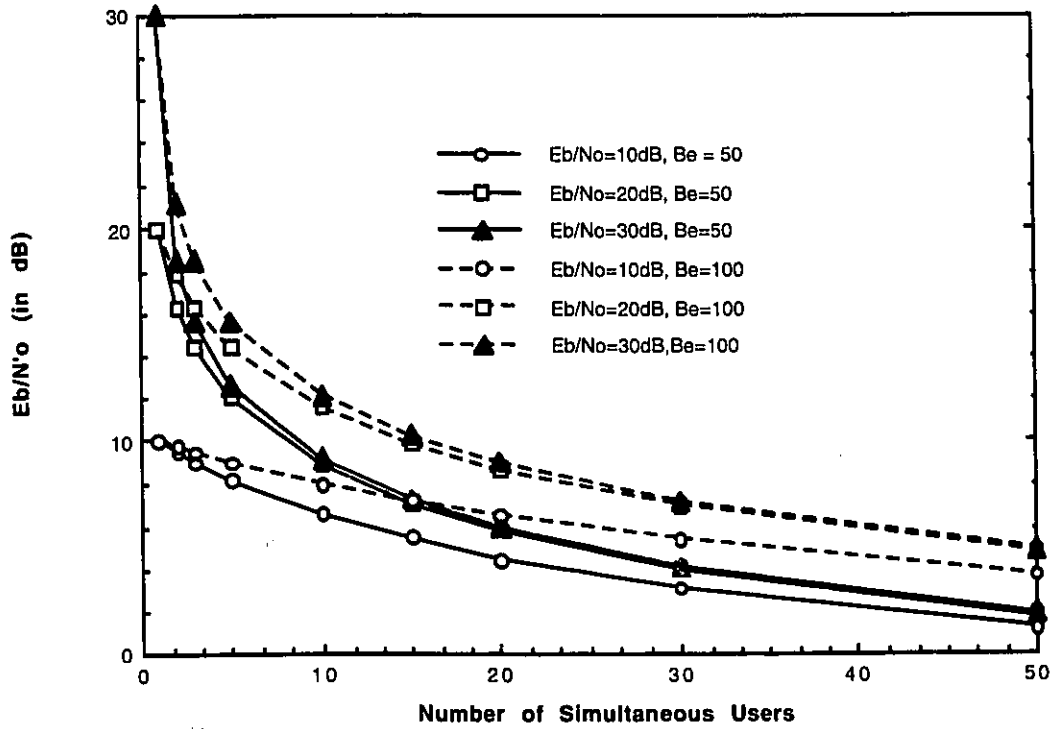


Figure 2.2 : The effect of the number of simultaneous users on the effective bit energy to noise spectral density ratio ( $\frac{E_b}{N_o'}$ ) of a DS-CDMA system for different  $\frac{E_b}{N_o}$  and bandwidth expansion factors  $B_e$ .

### 2.2.2 FH-CDMA Systems

Frequency hopping can be used in a multiple access system by assigning a distinct hop sequence to each user. If slow hopping is used, (one or more symbols per hop), each user's carrier occupies a range of frequencies, which is smaller than or equal to the bandwidth required by the signal in a non-spread system,  $W$ . The frequency slots are usually separated by a range of frequencies equal to or greater than the non-spread bandwidth, and thus the total number of frequency slots is  $N_f = W_{ss}/W$  (assuming minimum frequency separation between the frequency bins), where  $W_{ss}$  is the total spread bandwidth. For fast hopping, the minimum frequency separation between bins is  $LW$ , where  $L$  is the number of hops per symbol, and thus  $N_f = W_{ss}/LW$ .

Assuming slow random hopping for all users, the probability that one or more undesired signals are hopped into the same frequency slot as the desired signal (referred to as the probability of a hit) is given as [18]:

$$\begin{aligned} P_h(U) &= 1 - \frac{(N_f - 1)^{U-1}}{N_f^{U-1}} \\ &\approx \frac{U-1}{N_f} \quad \text{for } U \ll N_f \end{aligned} \quad (2.7)$$

A simple approach to estimating the effects of hits on the overall bit error rate is to assume that only full hits occur. This implies synchronous FH-CDMA, but in asynchronous FH-CDMA, partial hits occur as the desired signal hops into an occupied frequency slot, and when an undesired signal hops into the desired signal's frequency slot. As a result, there are 2 opportunities for each user to create partial hits, and two partial hits may resemble one full hit. Finally, one can assume that a full hit produces a 50 % bit error rate. Therefore, the bit error rate in a FH-CDMA system is approximated as [18]:

$$\begin{aligned}
P_b(U) &\approx \frac{1}{2}P_h(U) + (1 - P_h(U))P_b(1) \\
&\approx \frac{U-1}{2N_f} + \frac{N_f+1-U}{N_f}P_b(1) \quad U \ll N_f
\end{aligned} \tag{2.8}$$

where  $P_b(1)$  is the bit error rate when  $U = 1$ . From equation (2.8), one can see that when  $P_b(1) \ll (U-1)/2N_f$ , then  $P_b(U) \approx \frac{1}{2}P_h$ . Therefore, as  $P_b(1) \rightarrow 0$ , the bit error rate approaches  $\frac{U-1}{2N_f}$ . Without error control coding, an acceptable bit error rate can only be achieved when  $U-1 \ll 2N_f$ , which means that most frequency slots are empty, which in turn means that the system is bandwidth inefficient. When error control coding is used, the pre-decoding (or raw) code symbol error rate can be higher, meaning that bandwidth efficiency is improved. Figure 2.3 shows the effect of multiple users on a FH-CDMA system when  $P_b(1) = 10^{-6}$ . It can be seen that FH-CDMA systems are extremely sensitive to multiple users even when  $N_f = 1000$ .

One method of improving this is to employ fast hopping ( $L > 1$ ). When fast hopping is used, the number of frequency slots decreases from  $N_f$  to  $N'_f = N_f/L$ . Thus  $P_h$  increases as well, but the probability that all hops suffer from a hit is  $P_h^L$ . Using a good diversity combining scheme, the “hit” hops will generally contribute less to the decision variable than the “non-hit” hops, thus improving  $P_b(U)$ .

### 2.2.3 Hybrid DS/FH-CDMA

In a hybrid DS/FH-CDMA system, each user is assigned a PN sequence for direct sequence spreading and a hop pattern for frequency hopping. The channel is made up of a number of DS-SS channels that each user hops in and out of. By combining the two types of CDMA systems in this manner, a good compromise of the advantages and disadvantages of both types of systems is realized. The

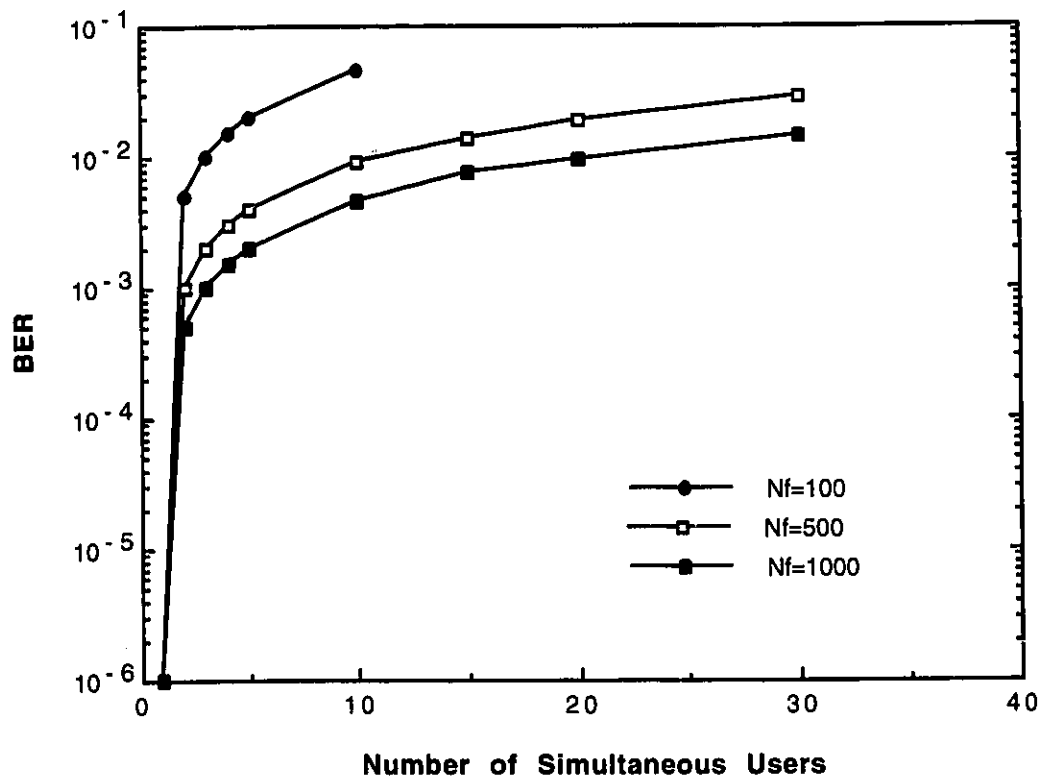


Figure 2.3 : The effect of multiple users on the effective bit error rate performance of a FH-CDMA system for  $P_b(1) = 10^{-6}$  and different number of frequency slots  $N_f$ .

advantages of employing hybrid DS/FH-CDMA are many, and the main ones are as follows.

(1) Low Probability of Detection: a direct sequence spread spectrum signal has a low probability of detection due to its low spectral density signal. Frequency-hopped spread spectrum signals are often undetected because the signal occupies a specific band of the spectrum for very short periods of time. Hybrid DS/FH-SS systems exploit both aspects and can become quite difficult to detect.

(2) Spectrum Sharing: low probability of detection is basically a concern for military communications. However, a signal that is difficult to detect will create little interference to narrowband signals that may also occupy the same frequency band. The concept of having spread spectrum systems share spectrum with other services such as point-to-point microwave tower communications is discussed in [32].

The problem created by spectrum to a DS-SS system is that each user will encounter this additional interference caused by the narrowband interference. In hybrid DS/FH-SS systems, the total bandwidth can be partitioned into channels, those containing narrowband interference and those without. Each user's signal will encounter the interference caused by spectrum sharing equally, but can also take advantage of the knowledge about which hops are corrupted by interference. If fast hopping is used, diversity can be exploited. Hop symbols corrupted by the interference can be weighted so that their influence to the final decision is less than the uncorrupted hop symbols.

(3) Power Control: To maximize system capacity, DS-CDMA systems require that each transmitted signal arrive at the receiver with equal power so that a few users do not dominate the network<sup>1</sup>. This requires power control. Although

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<sup>1</sup>In a star topology network, where all users transmit to a central receiver (e.g. cellular systems), power control is not a huge factor. Since the receiver has all the information about every signal transmitted over the channel, it can eliminate much of the MAI using a multiple

power control is needed in hybrid DS/FH-CDMA systems, the algorithm does not need to be as complex as the DS-CDMA case to achieve the same level of performance.

When one or more signals arrive at the receiver with much higher power than the majority, all other users must encounter this interference. In a hybrid DS/FH-CDMA systems, each user encounters the interfering signals only a fraction of the time. If fast frequency hopping is used, then the overall bit error rate of the desired signal will not suffer much from this interference if proper diversity combining techniques are used.

(4) Security: In DS-CDMA or FH-CDMA the spreading sequence (either the PN sequence or the hop pattern) acts much like an encryption key. The security of the system can be compromised if the key is discovered. In hybrid DS/FH-CDMA, there are now two keys, the PN sequence and the hop pattern. The PN sequence is generally shorter in a DS/FH-CDMA system compared to a DS-CDMA system (since some of the spreading is achieved by frequency hopping), but the hop sequence can be made as long as one wishes. This increases the level of complexity of the system and makes a cryptanalysis attack on the system more difficult.

(5) Synchronization delay: In CDMA systems, the PN sequence of the receiver must be synchronized with that of the transmitter. The synchronization of the PN sequences requires time during which no useful information may be sent. Synchronization delay can be made low by shortening the PN sequence which decreases information security. Let us consider a hybrid DS/FH-CDMA system with a comparable level of security as either a DS or an FH-CDMA system. Some of the spreading is accomplished by direct-sequence means while the rest is achieved by frequency hopping. Therefore, the direct sequence PN code and the hop pattern are shorter than their counterparts in non hybrid

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user maximum likelihood detection receiver [41]-[43].

systems. The synchronization of these two codes can be done jointly, reducing the synchronization delay, and thus reducing channel overhead.

(6) Spectral Efficiency: It will be shown in Chapter 5 that the spectral efficiency of a DS/FH-CDMA system is comparable to that of a DS-CDMA system. This is because a “hit” will not result in large error probabilities. When a signal is “hit” (in other words it share a frequency bin with one or more signals), the additional signals contribute to the MAI. Therefore the system is still limited by the amount of interference that a user can tolerate. Thus the spectral efficiency of hybrid DS/FH-CDMA is much like that of DS-CDMA which is generally higher than that of FH-CDMA.

## 2.3 Bandwidth Efficiency

In a CDMA system, where each simultaneous user employs the entire bandwidth, the maximum number of simultaneous users that the system can support is limited by the amount of interference that each user can tolerate. In other words, the addition of a new user to the channel degrades the bit error performance of every other user. Supposing that there exists a maximum bit error rate that the system can tolerate while maintaining an acceptable level of communication, then the maximum number of simultaneous users that the system can support is that which produces enough interference to attain this maximum bit error rate. We will denote the maximum number of users that the system can support as  $U_{max}$ .

Spectral efficiency is a measure of information transfer per unit time and unit bandwidth. For example, BPSK employing minimum bandwidth filtering requires 1 Hz of bandwidth to transmit information at a rate of 1 bit/sec. Thus the bandwidth efficiency is 1 bit/sec/Hz or 1 bps/Hz. Also, BPSK employing raised cosine filtering with an excess bandwidth factor of 0.2 requires 1.2 Hz to

transmit 1 bps. Its bandwidth efficiency is 0.833 bps/Hz. The spectral efficiency of any multiple access system is given by the maximum amount of information transmitted over the common channel per second divided by the total bandwidth of the channel. The spectral efficiency,  $\eta$ , of a CDMA system is given by:

$$\eta = \frac{U_{max} R_b}{W_{ss}} \quad (2.9)$$

where  $R_b$  is the bit rate that each user is transmitting information (this assumes that all users transmit at the same bit rate), and  $W_{ss}$  is the total spread bandwidth employed by the system.

### 2.3.1 Spectral Efficiency of DS-CDMA

The equivalent bit energy to noise spectral density ratio of a PSK modulated DS-CDMA system is given in eq. (2.6), and is repeated here for convenience.

$$\gamma'_b = \frac{\gamma_b}{1 + \frac{2}{3} \frac{(U-1)R_b}{R_p} \gamma_b} \quad (2.10)$$

The maximum number of simultaneous users is found by determining the lowest possible  $\gamma'_b$  that the system can support while maintaining acceptable communications. This is generally in the form of a maximum bit error rate (for voice,  $10^{-3}$  to  $10^{-2}$  is generally accepted while for data it is generally less than  $10^{-5}$ ). Suppose that the equivalent energy per bit to noise spectral density ratio  $\gamma'_b$  produces the highest tolerable bit error rate. The channel is occupied by the maximum number of simultaneous users that the system can support. Rearranging eq. (2.10) in terms of  $U_{max}$ , we get:

$$U_{max} = \frac{\gamma_b - \gamma'_b}{\frac{2R_b}{3R_p} \gamma_b \gamma'_b} + 1 \quad (2.11)$$

Thus, assuming  $U_{max} \gg 1$ , the spectral efficiency in bps/Hz is:

$$\eta \simeq \frac{\frac{3}{2} R_p (\gamma_b - \gamma'_b)}{\gamma_b \gamma'_b W_{ss}} \quad (2.12)$$

For DS-CDMA systems employing PSK modulation and BPSK PN sequences,  $W_{ss} = R_p$ . Therefore

$$\eta \simeq \frac{3}{2} \frac{\gamma_b - \gamma'_b}{\gamma_b \gamma'_b} \quad (2.13)$$

$$\simeq \frac{3}{2\gamma'_b} \quad \gamma_b \gg \gamma'_b \quad (2.14)$$

It is seen in equation (2.14) that the spectral efficiency (and thus the capacity) of a DS-CDMA system is inversely proportional to the bit energy to noise spectral density ratio required to give the maximum bit error rate allowed. Thus the spectral efficiency of a DS-CDMA system can be improved by lowering  $\gamma'_b$  by using power efficient modulation schemes, and/or by using error control coding.

### 2.3.2 Spectral Efficiency of FH-CDMA

As seen in subsection 2.4.2, the bit error rate of a FH-CDMA system is dependent on  $P_h$ . Therefore, for a maximum allowable bit error rate  $P_{b-max}$ , there exists a hit probability  $P_{h-max}$ . From equation (2.7),  $P_h \approx (U - 1)/N_f$ . Therefore

$$P_{h-max} \approx \frac{U_{max} - 1}{N_f} \quad (2.15)$$

In the slow-hopped system described in subsection 2.4.4,  $P_b \approx P_h/2$ . Therefore

$$U_{max} \approx 2N_f P_{b-max} + 1 \quad (2.16)$$

The spectral efficiency is:

$$\eta \approx \frac{2N_f P_{b-max} R_b}{W_{ss}} \quad (2.17)$$

$N_f = W_{ss}/W$ , therefore

$$\eta \approx \frac{2P_{b-max} R_b}{W} \quad (2.18)$$

For minimum bandwidth BPSK modulation,  $R_b = W$ . Therefore

$$\eta \approx 2P_{b-max} \quad (2.19)$$

We can see from equation (2.19) that even for voice applications (e.g.  $P_{b-max} = 10^{-3}$ ), the system is extremely bandwidth inefficient. When error control coding is employed,  $W = R_b/r_c$ , where  $r_c$  is the code rate. Therefore equation (2.19) becomes

$$\eta \approx \frac{2}{r_c} P_{b-max} \quad (2.20)$$

However, equation (2.20) is only valid when  $U \ll N_f$ . Thus the spectral efficiency cannot increase simply by choosing extremely low rate codes.

## 2.4 Existing CDMA Systems

In this chapter, basic CDMA systems have been discussed. Throughout the discussions BPSK modulation was implied. However, research has been conducted on CDMA systems employing different modulation schemes.

In [18], a DS-CDMA system employing wideband noncoherent MFSK is discussed for satellite communications. It shows how the bandwidth efficiency of this system can be higher than that of a DS-CDMA system employing BPSK modulation. The advantage of MFSK is that it is more power efficient than BPSK in an AWGN channel. Also, since the modulation scheme employs a large bandwidth, the chip rate required to produce a given spread bandwidth is smaller for a system employing MFSK than for BPSK. Therefore, the PN sequence length (which is generally one symbol duration) is smaller, which reduces PN synchronization times.

In [44], it is shown that noncoherent MFSK/DS-CDMA systems do indeed provide a higher spectral efficiency than do coherent BPSK/DS-CDMA systems, but only for uncoded systems. When convolutional coding is introduced, the spectral efficiency of BPSK/DS-CDMA systems is higher. This is because the coding gain of an error control code depends on the modulation and detection scheme employed, and the gain is higher for coherent BPSK than for noncoherent MFSK.

The research in [18] is augmented in [45]. Here, the MFSK modulation is improved by combining it with phase modulation. This system is termed MFSK-MPSK/DS-CDMA. In AWGN, with Reed Solomon (RS) coding, a bandwidth efficiency of more than 0.55 bps/Hz is obtained using MFSK-MPSK combined modulation.

Also in [45], MFSK modulation is used in a DS-CDMA system. While some of the information bits select the frequency tone to be transmitted, other infor-

mation bits are used to select the specific PN sequence that will modulate the symbol. The different PN sequences are orthogonal to each other. This system is referred to as MFSK/DS-CDMA employing orthogonal codes or MFSK-OC/CDMA. Bandwidth efficiencies up to 0.56 bps/Hz are obtained for this system in AWGN for a bit error rate of  $10^{-3}$ .

One more system is presented in [45]. This DS-CDMA system employs MT-FSK. The waveforms for MT-FSK are selected using permutation modulation theory which was devised by Slepian [46]. Spectral efficiencies of up to 0.81 bps/Hz in AWGN are obtained employing MT-FSK and convolutional coding. However, in Rayleigh fading, all the systems described in [45] have poor spectral efficiencies. Improving the spectral efficiencies in Rayleigh fading is the topic of the next two chapters.

## Chapter 3

# Variants of MFSK Modulation for Rayleigh Fading Channels

### 3.1 Introduction

The modulation schemes to be utilized in the hybrid DS/FH-CDMA system are described in this chapter. Equations describing the bit error rate performance of each modulation scheme in diversity and nondiversity systems are derived. Error control coding is also considered. The effects of binary and non-binary convolutional codes on the bit error rate performance of these modulation schemes are examined.

MFSK and its variants are considered because non-coherent MFSK has been shown to be more power efficient than DPSK for large values of  $M$  in AWGN channels [44]. Also, it does not require a carrier phase recovery circuit which makes it suitable for mobile communications. Evaluation of non-coherent MFSK bit error probabilities in AWGN [18, 40] shows that a bit error rate of  $10^{-6}$  is achieved for  $E_b/N_o = 9$  dB for  $M = 16$  and 8 dB for  $M = 32$  as opposed to 10.6 dB for coherently demodulated BPSK or QPSK. As we have shown in

Chapter 2, the bandwidth efficiency of spread spectrum systems is determined by the amount of inter-user interference. Therefore higher power efficiencies translate into higher bandwidth efficiencies.

In non-spread systems, the use of MFSK has the disadvantage of bandwidth inefficiency. However, in spread spectrum systems, where one intends to spread the bandwidth anyway, minimization of the non-spread bandwidth is not a concern.

## 3.2 Non-Coherent MFSK

In MFSK, information symbols are represented by specific frequencies,  $f_m = f_c + m\Delta f$ ,  $m = 1, 2, \dots, M$ . The symbols can be represented by [40]:

$$s_m(t) = \sqrt{\frac{2E_s}{T_s}} \cos[2\pi f_c t + 2\pi m\Delta f t] \quad (3.1)$$

$$= \operatorname{Re}\left[\sqrt{\frac{2E_s}{T_s}} e^{j2\pi m\Delta f t} e^{j2\pi f_c t}\right] \quad m = 1, 2, \dots, M, \quad 0 \leq t \leq T_s \quad (3.2)$$

The corresponding equivalent lowpass waveforms are thus (for  $f_c T \ll 1$ )

$$u_m(t) = \sqrt{\frac{2E_s}{T_s}} e^{j2\pi m\Delta f t}, \quad m = 1, 2, \dots, M; \quad 0 \leq t \leq T_s \quad (3.3)$$

The cross-correlation coefficient between the  $k$ th and  $m$ th waveforms is given by [40]:

$$\rho_{km} = \frac{\left(\frac{2E_s}{T_s}\right)}{2E_s} \int_0^{T_s} e^{j2\pi(m-k)\Delta f t} dt \quad (3.4)$$

$$= \frac{\sin \pi T_s (m-k)\Delta f}{\pi T_s (m-k)\Delta f} e^{j\pi T_s (m-k)\Delta f} \quad (3.5)$$

$|m - k| = 1$  corresponds to adjacent frequency tones. The minimum frequency separation needed for orthogonality is found when  $|\rho_{km}| = 0$ . Since  $|e^{j\pi T_s(m-k)\Delta f}| = 1$ ,  $|\rho_{km}| = 0$  when  $\Delta f$  is a multiple of  $1/T_s$ . Note that for coherent detection,  $e^{j\pi T_s(m-k)\Delta f}$  can be replaced by  $\cos \pi T_s(m - k)\Delta f$ , and  $\Delta f = 1/2T_s$  is the minimum spacing between adjacent signals for orthogonality.

### 3.2.1 Performance of MFSK in an AWGN channel

The optimum receiver for non-coherent MFSK is given in Figure 3.1. It computes the decision variables [40]:

$$U_m = \left| \int_0^{T_s} r(t) u_m^*(t) dt \right|, \quad m = 1, 2, \dots, M \quad (3.6)$$

and selects the signal corresponding to the largest  $U_m$ .

Assuming that the transmitted signal is  $u_1(t)$ , then the received lowpass signal,  $r(t) = \alpha e^{-j\phi_1} u_1(t) + z(t)$ , where  $\alpha$  is the channel gain,  $\phi$  is the channel phase, and  $z(t)$  is the equivalent lowpass Gaussian noise signal. Since the signals are detected non-coherently, the phase is irrelevant and thus  $U_m$  can be expressed by the following

$$U_1 = |2\alpha E_s + N_1| \quad (3.7)$$

$$U_m = |N_m| \quad m = 2, 3, \dots, M \quad (3.8)$$

where  $N_m = \int_0^{T_s} z(t) u_m^*(t) dt$   $m = 1, 2, \dots, M$ , and they are complex-valued independent, identically distributed zero mean Gaussian random variables with variance  $\sigma_n^2 = 2E_s N_0$ . The decision variables  $U_1, U_2, \dots, U_M$  are statistically

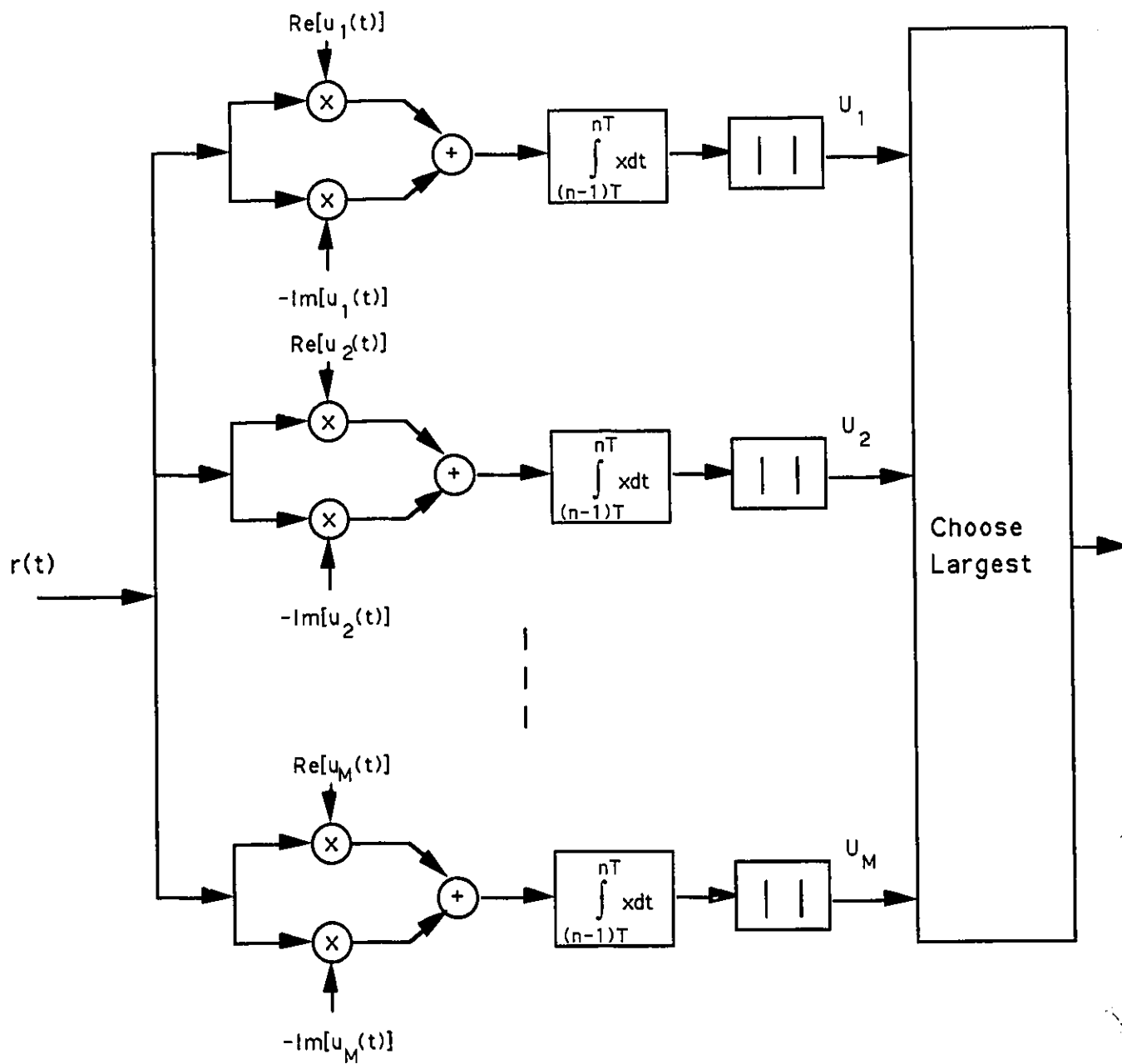


Figure 3.1 : Optimum receiver of non-coherent MFSK.

independent. Since  $U_m = |N_m|$  for  $m = 2, 3, \dots, M$ , they are statistically described by the Rayleigh probability density function.

$$p(u_m) = \frac{u_m}{2E_s N_o} e^{-u_m^2/4E_s N_o} \quad m = 2, 3, \dots, M \quad (3.9)$$

while  $U_1$  is described by the Ricean probability density function

$$p(u_1) = \frac{u_1}{2E_s N_o} e^{-(u_1^2 + 4\alpha^2 E_s^2)/4E_s N_o} I_0\left(\frac{\alpha u_1}{N_o}\right) \quad (3.10)$$

The probability of a correct decision is the probability that  $U_1$  exceeds all other decision variables. Thus

$$P_c = P(U_2 < U_1, U_3 < U_1, \dots, U_M < U_1) \quad (3.11)$$

$$= \int_0^\infty P(U_2 < u_1, U_3 < u_1, \dots, U_M < u_1 | U_1 = u_1) p(u_1) du_1 \quad (3.12)$$

Since  $U_m$  for  $m = 2, 3, \dots, M$  are independent identically distributed random variables, the joint probability conditioned on  $U_1$  factors into a product of  $M-1$  identical terms. Therefore

$$P_c = \int_0^\infty P(U_2 < u_1 | U_1 = u_1) P(U_3 < u_1 | U_1 = u_1) \dots \quad (3.13)$$

$$P(U_M < u_1 | U_1 = u_1) p(u_1) du_1 \\ = \int_0^\infty [P(U_2 < u_1 | U_1 = u_1)]^{M-1} p(u_1) du_1 \quad (3.14)$$

The decision variable  $U_2$  is a Rayleigh random variable and thus

$$P(U_2 < u_1 | U_1 = u_1) = \int_0^{u_1} p(u_2) du_2 \quad (3.15)$$

$$= 1 - e^{-u_1^2/4E_s N_o} \quad (3.16)$$

Raising this quantity to the  $(M - 1)$ th power yields

$$[P(U_2 < u_1 | U_1 = u_1)]^{M-1} = \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \frac{1}{n+1} e^{-\frac{nu_1^2}{4E_s N_o}} \quad (3.17)$$

Substitution of this result into eq (3.14) yields [40]:

$$P_c = \sum_{n=0}^{M-1} (-1)^n \binom{M-1}{n} \frac{1}{n+1} e^{-\gamma_s n/(n+1)} \quad (3.18)$$

where  $\gamma_s = \alpha^2 E_s / N_o$  is the signal to noise ratio per symbol. Then the probability of a symbol error  $P_M = 1 - P_c$  is

$$P_M = \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{n+1} e^{-nk\gamma_b/(n+1)} \quad (3.19)$$

where  $\gamma_b = \alpha^2 E_b / N_o$  and  $k = \log_2 M$ .

The probability of bit error is given by

$$P_b = \frac{2^{k-1}}{2^k - 1} P_M \quad (3.20)$$

The bit error rate performance of noncoherently demodulated MFSK in AWGN is shown in Figure 3.2. It can be seen that the performance of MFSK improves greatly as we increase  $M$ . Please note that, throughout this thesis when we computed the performance at certain values using analytical expressions, we used different symbols to differentiate the curves. Hence, the points on the curves do not indicate that they are obtained by simulations or experiments. They are merely the results of analytical evaluations at certain values.

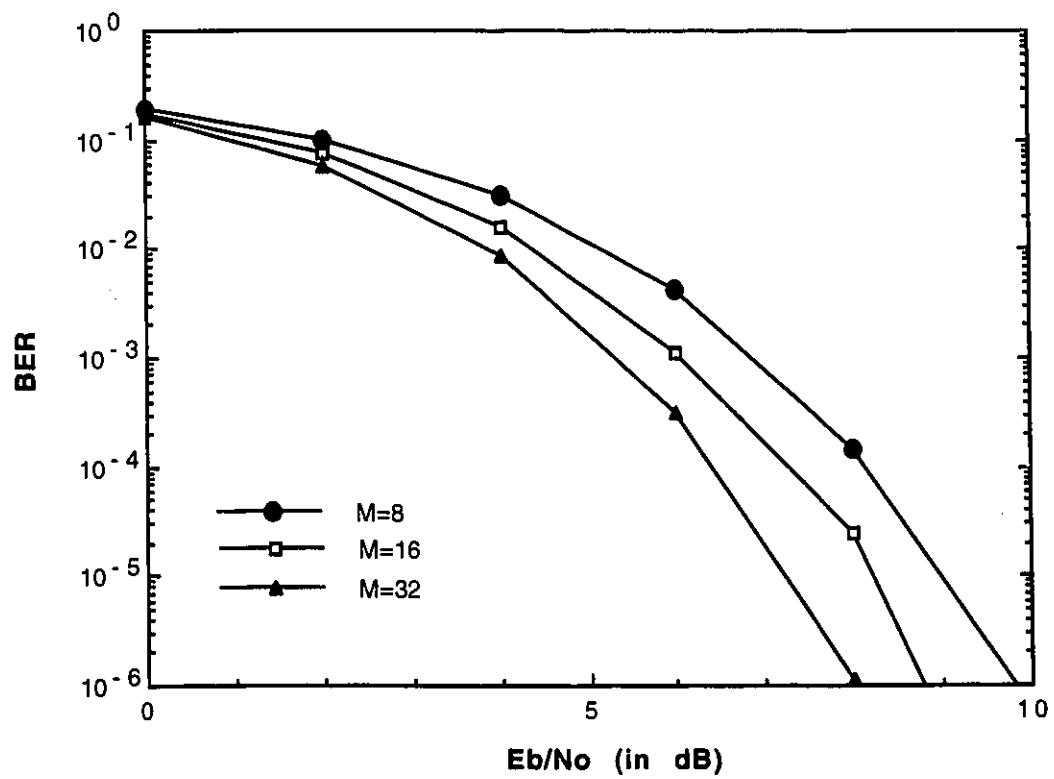


Figure 3.2 : Bit error rate performance of noncoherent MFSK in AWGN for  $M = 8, 16$ , and  $32$ .

### 3.2.2 Performance of Uncoded MFSK in a Frequency-Nonselective Rayleigh Fading Channel

The frequency-nonselective Rayleigh fading channel results in a multiplicative distortion of the transmitted signal  $u_m(t)$ . The received signal is:

$$r(t) = \alpha(t)e^{-j\phi(t)}u_m(t) + z(t), \quad 0 \leq t \leq T_s \quad (3.21)$$

In this case,  $\alpha(t)$  and  $\phi(t)$  are time variant random variables.

Therefore the symbol error probability in frequency-nonselective Rayleigh fading is obtained by averaging the symbol error probability for fixed signal to noise ratios over all possible signal to noise ratios brought on by the fading. Thus

$$P_M = \int_0^\infty P_M(\gamma_b)p(\gamma_b)d\gamma_b \quad (3.22)$$

Since  $\alpha$  is Rayleigh distributed,  $\alpha^2$  is chi-square distributed with 2 degrees of freedom [40]. Thus  $\gamma_b$  is also chi-square distributed with 2 degrees of freedom. Therefore

$$p(\gamma_b) = \frac{1}{\bar{\gamma}_b} e^{-\gamma_b/\bar{\gamma}_b} \quad (3.23)$$

where  $\bar{\gamma}_b$  is the average signal energy per bit to noise spectral density ratio, which is given by

$$\bar{\gamma}_b = \frac{E_b}{N_o} E(\alpha^2) \quad (3.24)$$

where  $E(\alpha^2)$  is the average of  $\alpha^2$ .

Combining equations (3.19), (3.22) and (3.23), the symbol error probability in Rayleigh fading can be found to be:

$$P_M = \int_0^\infty \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{n+1} e^{-nk\gamma_b/(n+1)} \frac{1}{\tilde{\gamma}_b} e^{-\gamma_b/\tilde{\gamma}_b} d\gamma_b \quad (3.25)$$

$$= \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{(n+1)\tilde{\gamma}_b} \int_0^\infty e^{-\frac{(nk\tilde{\gamma}_b+n+1)\gamma_b}{(n+1)\tilde{\gamma}_b}} d\gamma_b \quad (3.26)$$

$$= \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{nk\tilde{\gamma}_b + n + 1} \quad (3.27)$$

Again  $P_b = \frac{2^{k-1}}{2^k-1} P_M$ . The bit error rate performance of MFSK in frequency-nonselective Rayleigh fading is shown in Figure 3.3. As we increase  $M$  from 8 to 32, the performance improves marginally as opposed to in the AWGN case where increasing  $M$  results in a very significant performance improvement.

### 3.2.3 Performance of Uncoded MFSK in Rayleigh Fading With Diversity

Suppose that the same information-bearing noncoherent MFSK signal is transmitted over  $L$  frequency-nonselective slowly Rayleigh fading channels which are mutually statistically independent. This is an  $L$ th order diversity system.

Practical receivers employ square-law combining of FSK signals, thus the decision variables are given by:

$$U_1 = \sum_{k=1}^L |2E_c \alpha_k e^{-j\phi_k} + N_{k1}|^2 \quad (3.28)$$

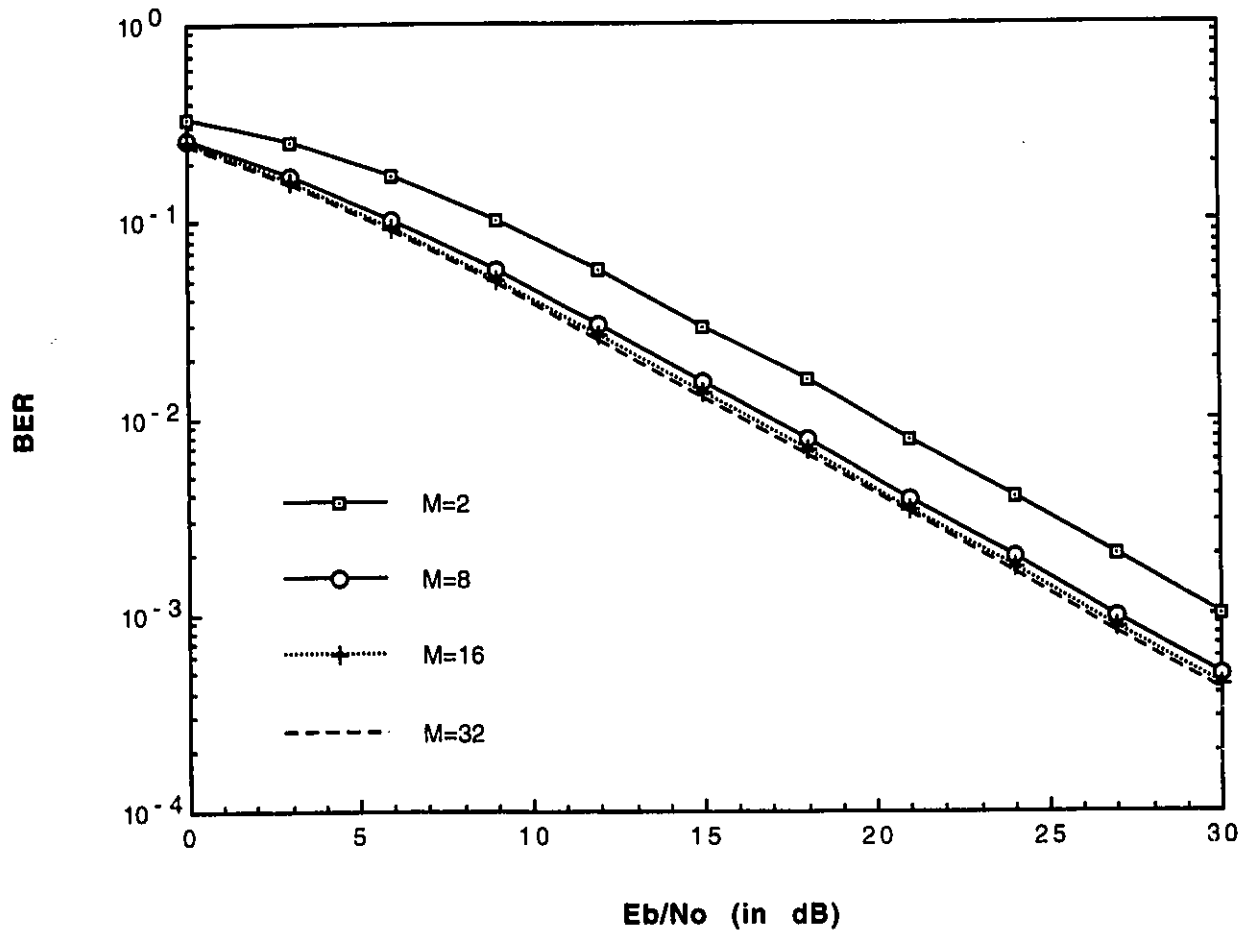


Figure 3.3 : Bit error rate performance of noncoherent MFSK in flat Rayleigh fading.

$$U_m = \sum_{k=1}^L |N_{km}|^2 \quad m = 2, 3, \dots, M \quad (3.29)$$

where  $E_c$  is the energy per symbol per channel.

The probability of symbol error is given by:

$$P_M = 1 - \int_0^\infty [P(U_2 < u_1 | U_1 = u_1)]^{M-1} p(u_1) du_1 \quad (3.30)$$

$p(u_1)$  is given by:

$$p(u_1) = \frac{1}{(2\sigma_1^2)^L (L-1)!} u_1^{L-1} e^{-u_1/2\sigma_1^2} \quad (3.31)$$

where  $\sigma_1^2 = 2E_c N_o (1 + \bar{\gamma}_c)$ , and  $\bar{\gamma}_c$  is the average symbol energy to noise spectral density ratio per channel.

$p(u_2)$  is given by:

$$p(u_2) = \frac{2}{(2\sigma_2^2)^L (L-1)!} u_2^{L-1} e^{-u_2/2\sigma_2^2} \quad (3.32)$$

where  $\sigma_2^2 = 2E_c N_o$ .

From this,  $P(U_2 < u_1 | U_1 = u_1) = \int_0^{u_1} p(u_2) du_2$ . This is given by:

$$P(U_2 < u_1 | U_1 = u_1) = 1 - e^{-u_1/2\sigma_2^2} \sum_{i=0}^{L-1} \frac{(u_1/2\sigma_2^2)^i}{i!} \quad (3.33)$$

Placing this result into eq. (3.30) yields a symbol error rate,  $P_M$ , which is given by [40]:

$$P_M = \frac{1}{(L-1)!} \sum_{m=1}^{M-1} \frac{(-1)^{m+1} \binom{M-1}{m}}{(1+m+m\bar{\gamma}_c)^L} \times \sum_{i=0}^{m(L-1)} \beta_{im} (L-1+i)! \left( \frac{1+\bar{\gamma}_c}{1+m+m\bar{\gamma}_c} \right)^i \quad (3.34)$$

where  $\beta_{im}$  is the set of coefficients in the following expansion:

$$\left( \sum_{i=0}^{L-1} \frac{u_1^i}{i!} \right)^m = \sum_{i=0}^{m(L-1)} \beta_{im} u_1^i \quad (3.35)$$

Note that equation (3.34) reduces to equation (3.27) when  $L = 1$ . Table 3.1 provides the  $\bar{\gamma}_b$  required to provide a bit error rate of  $10^{-3}$ .

*Table 3.1 :  $\bar{\gamma}_b$  required to obtain a bit error rate of  $10^{-3}$  for an  $L$ th order diversity system employing MFSK in Rayleigh fading.*

| $L$ | M=8     | M=16    | M=32    |
|-----|---------|---------|---------|
| 1   | 27.0 dB | 26.4 dB | 26.2 dB |
| 2   | 17.2 dB | 16.7 dB | 16.5 dB |
| 3   | 14.1 dB | 13.5 dB | 13.3 dB |
| 4   | 13.5 dB | 12.9 dB | 12.7 dB |

One should note that the gain obtained by employing a fourth order diversity system as opposed to a third order system is small. Thus increasing the diversity order further will result in only small increases in power efficiency.

### 3.3 Joint Frequency-Phase Modulation

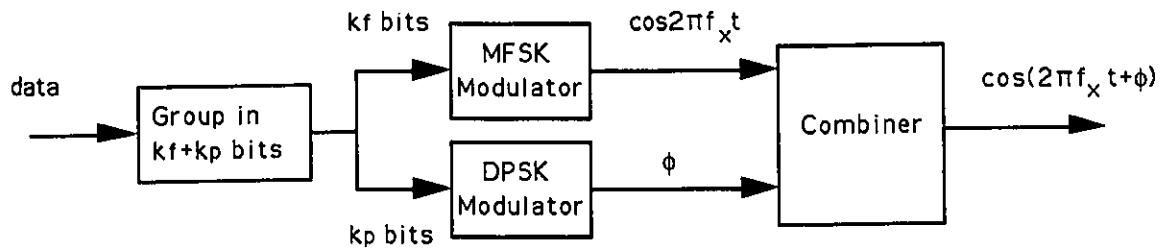
Orthogonal phase modulated signals [47, 48], or joint frequency-phase modulation (JFPM) [49] is discussed in this section. Since coherent detection is difficult in fading channels, the phase aspect of this modulation type will be differentially encoded. This modulation scheme combines MFSK and differential detection techniques, and is thus referred to as MFSK-DPSK modulation [45]. Its performance has been analysed in Rayleigh fading in [45]. Its performance in nondiversity and diversity systems is examined in this section

#### 3.3.1 Combined MFSK-DPSK Modulation

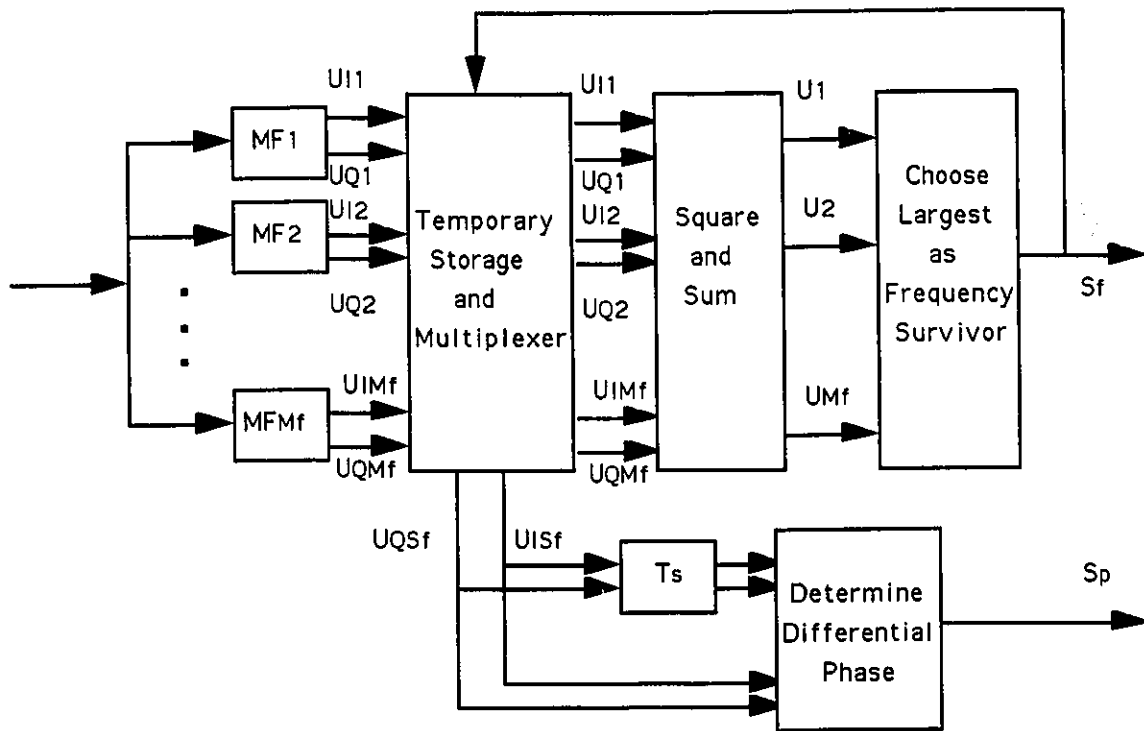
In MFSK-DPSK,  $k_f + k_p$  bits are grouped into a symbol. The first  $k_f = \log_2 M_f$  bits are represented by one of  $M_f$  possible frequencies. For orthogonality, the frequency separation is:  $\Delta f = \rho R_s = \rho \frac{R_b}{k_f + k_p}$ , where  $\rho$  is an integer equal to or greater than 1. The remaining  $k_p$  bits are represented by the differential phase of the symbol. In this thesis we will only consider two-phase and four-phase modulation schemes ( $k_p = 1$  or 2) because it is well known that PSK signals with more than 4 phases are not power efficient.

At the receiver, the transmitted frequency must first be determined, while information about the phase of each possible symbol is retained in buffers. Once the frequency is determined, the phase from the output of the matched filter of the surviving frequency is passed to a differential detector, where the differential phase sub-symbol of the symbol is determined. A MFSK-DPSK modulator is shown in Figure 3.4(a), while the demodulator is shown in Figure 3.4(b).

When diversity is employed, the receiver becomes more complicated. The MFSK part of the modulation scheme is demodulated by  $L$  MFSK demodulators, and then the decision variables are combined using a square-law combiner. The MFSK sub-symbol is then determined. This information is fed back to the



(a)



(b)

Figure 3.4 : (a) MFSK-DPSK modulator (b) MFSK-DPSK demodulator.

buffers containing the phase information. The phase information related to the selected frequency is passed to  $L$  differential detectors and the outputs are combined using equal-gain combining. The phase encoded bits of the symbol are then determined. A MFSK-DPSK diversity receiver is pictured in Figure 3.5.

A problem with this modulation scheme is that phase shifts introduced by the channel for different frequencies are different. Because of this, subsequent symbols may have different channel phases, thus making differential detection difficult. But it has been shown in [51] that we can employ a time tracking loop in the receiver, not only for symbol timing recovery, but also for estimating the frequency dependent phase shift.

### 3.3.2 Performance of Uncoded MFSK-DPSK Modulation in a Rayleigh Fading Channel

The average symbol energy to noise spectral density is  $\bar{\gamma}_s$ . Thus the average bit energy to noise spectral density is  $\bar{\gamma}_b = \bar{\gamma}_s / (k_f + k_p)$ . The bit error performance of MFSK-DPSK modulation is determined from the results of Section 3.2 and [40]. First, the MFSK sub-symbol is demodulated. The frequency modulated part of the symbol represents the first  $k_f$  bits of the symbol, yet the MFSK demodulator receives the frequency tone at a symbol energy to noise spectral density ratio of  $\bar{\gamma}_s$ . This has a symbol error rate of  $P_{Mf}$ , which, in slow Rayleigh fading, is given by:

$$P_{Mf} = \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{n\bar{\gamma}_s + n + 1} \quad (3.36)$$

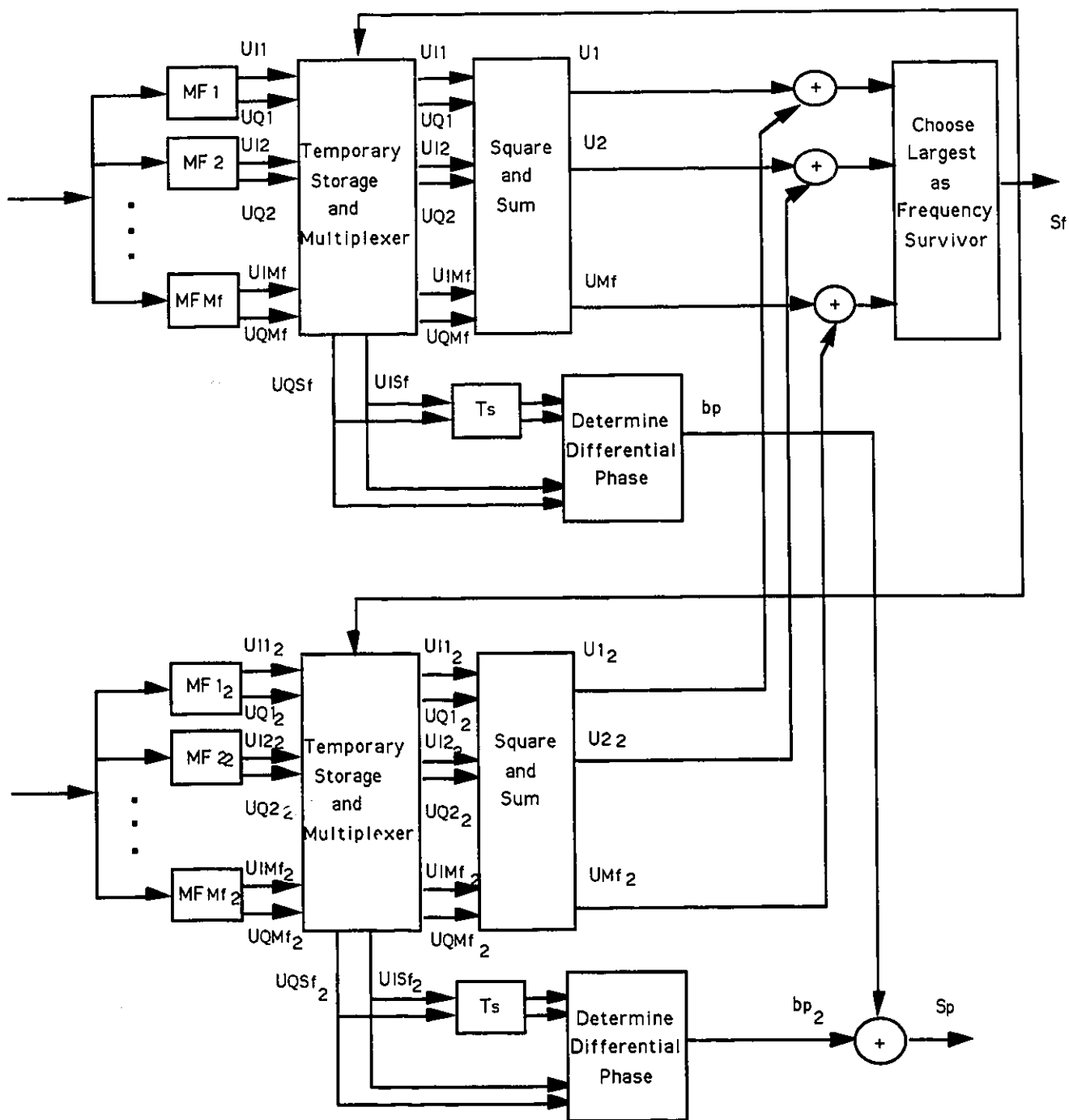


Figure 3.5 : MFSK-DPSK diversity receiver ( $L = 2$ ).

when no diversity is used and

$$P_{Mf} = \frac{1}{(L-1)!} \sum_{m=1}^{M-1} \frac{(-1)^{m+1} \binom{M-1}{m}}{(1+m+m\bar{\gamma}_s/L)^L} \times \sum_{i=0}^{m(L-1)} \beta_{in}(L-1+i)! \left( \frac{1+\bar{\gamma}_s/L}{1+m+m\bar{\gamma}_s/L} \right)^i \quad (3.37)$$

when diversity is used.

These are essentially eqns. (3.27) and (3.34) and they are defined in the previous section.

Once this symbol is determined, the received phase of this symbol is passed to the differential detector. Given that the frequency encoded symbol is correctly determined, the average bit energy to noise spectral density ratio of this phase encoded bit as seen by the differential detector is  $\bar{\gamma}_s$ . The probability of a phase encoded bit error,  $P_{DPSK}$ , for a two-phase system (MFSK-DBPSK) is given by [40]:

$$P_{DPSK} = \frac{1}{2(1+\bar{\gamma}_s)} \quad (3.38)$$

for a system without diversity and

$$P_{DPSK} = \left( \frac{1-\mu}{2} \right)^L \sum_{i=0}^{L-1} \binom{L-1+i}{i} \left( \frac{1+\mu}{2} \right)^i \quad (3.39)$$

for a diversity system, where

$$\mu = \frac{\bar{\gamma}_s/L}{1+\bar{\gamma}_s/L} \quad (3.40)$$

For a four-phase system (MFSK-DQPSK),  $P_{DPSK}$  without diversity is given by:

$$P_{DPSK} = \frac{1}{2} \left[ 1 - \frac{\mu}{\sqrt{2-\mu}} \right] \quad (3.41)$$

where  $\mu = \bar{\gamma}_s / (1 + \bar{\gamma}_s)$ . For a system with diversity, the expression becomes

$$P_{DPSK} = \frac{1}{2} \left[ 1 - \frac{\mu}{\sqrt{2-\mu}} \sum_{i=0}^{L-1} \binom{2i}{i} \left( \frac{1-\mu^2}{4-2\mu^2} \right)^i \right] \quad (3.42)$$

where  $\mu$  is given by eq. (3.40).

These equations assume that the fading rate  $B_d$  is slow compared to the symbol rate. Therefore  $B_d T_s \ll 1$ .

The probability that the phase encoded bits are incorrectly determined is dependent on the correct demodulation of the MFSK sub-symbol. If the frequency encoded sub-symbol is correctly received, the probability that the differentially decoded bits are incorrect is given by  $P_{DPSK}$ . However, when the MFSK symbol is incorrectly determined, the probability that the differentially detected bits are incorrectly detected is 1/2. Also, the next differentially detected phase will be affected since the previous phase is used in its detection. The probability of error for the differentially encoded bits,  $P_d$ , is thus found by using the Markov chain depicted in Figure 3.6. Note that after an error in detecting the MFSK symbol, it requires two consecutive correct detections before the probability of bit error in the differential detected bits to return to  $P_{DPSK}$ .

The transition probability matrix for this Markov chain is:

$$P = \begin{bmatrix} 1 - P_{Mf} & P_{Mf} & 0 \\ 0 & P_{Mf} & 1 - P_{Mf} \\ 1 - P_{Mf} & 0 & P_{Mf} \end{bmatrix} \quad (3.43)$$

The steady state probabilities,  $\pi_i$ , are satisfied by the following equations:

$$\pi = \pi P \quad (3.44)$$

$$\sum_i \pi_i = 1 \quad (3.45)$$

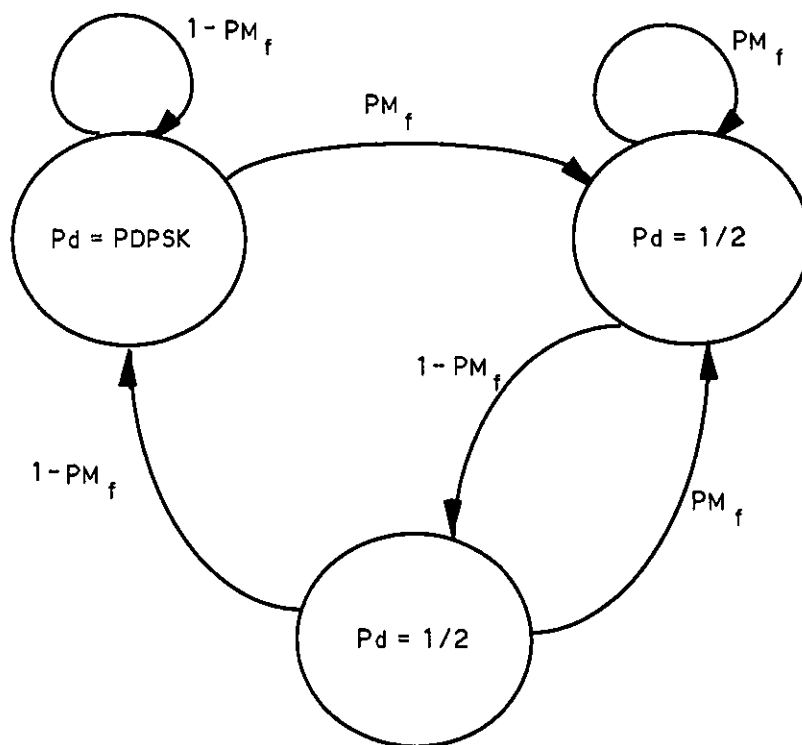


Figure 3.6 : Markov chain to determine  $P_d$ .

Solving these equations, we obtain:

$$\pi_0 = \frac{1 - P_{Mf}}{1 + P_{Mf}} \quad (3.46)$$

$$\pi_1 = \pi_2 = \frac{P_{Mf}}{1 + P_{Mf}} \quad (3.47)$$

Thus the probability of bit error in the differentially encoded bit stream is:

$$\begin{aligned} P_d &= \pi_0 P_{DPSK} + \frac{\pi_1}{2} + \frac{\pi_2}{2} \\ &= \frac{1 - P_{Mf}}{1 + P_{Mf}} P_{DPSK} + \frac{P_{Mf}}{1 + P_{Mf}} \\ &= \frac{P_{Mf}(1 - P_{DPSK}) + P_{DPSK}}{1 + P_{Mf}} \end{aligned} \quad (3.48)$$

The overall probability of bit error of MFSK-DPSK modulation is given by:

$$P_{b_{M-D}} = \frac{k_f}{k_f + k_p} \frac{2^{k_f-1}}{2^{k_f} - 1} P_{Mf} + \frac{k_p}{k_f + k_p} P_d \quad (3.49)$$

The probability of bit error of MFSK-DBPSK modulation in Rayleigh fading for different orders of diversity is shown in Figure 3.7. The probability of bit error of MFSK-DQPSK with different  $L$  is shown in Figure 3.8.

Table 3.2 shows the average bit energy to spectral noise density ratio required for these uncoded modulation schemes to yield a bit error rate performance of  $10^{-3}$ .

The symbol error rate of MFSK-DPSK is easily found. If the frequency sub-symbol is erroneously detected, the symbol is incorrect. Also, if the frequency sub-symbol is correctly detected, then the symbol will be incorrect if the phase

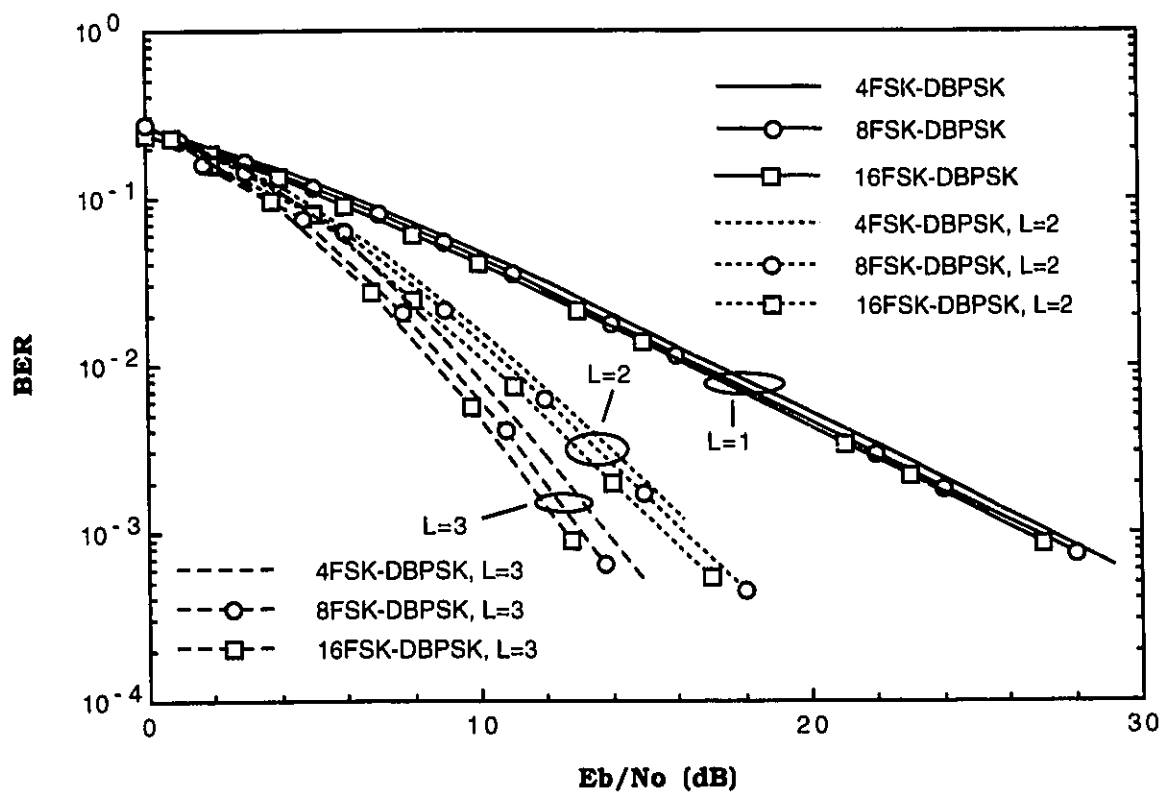


Figure 3.7 : Bit error rate of MFSK-DBPSK in Rayleigh fading for  $L = 1, 2$  and 3.

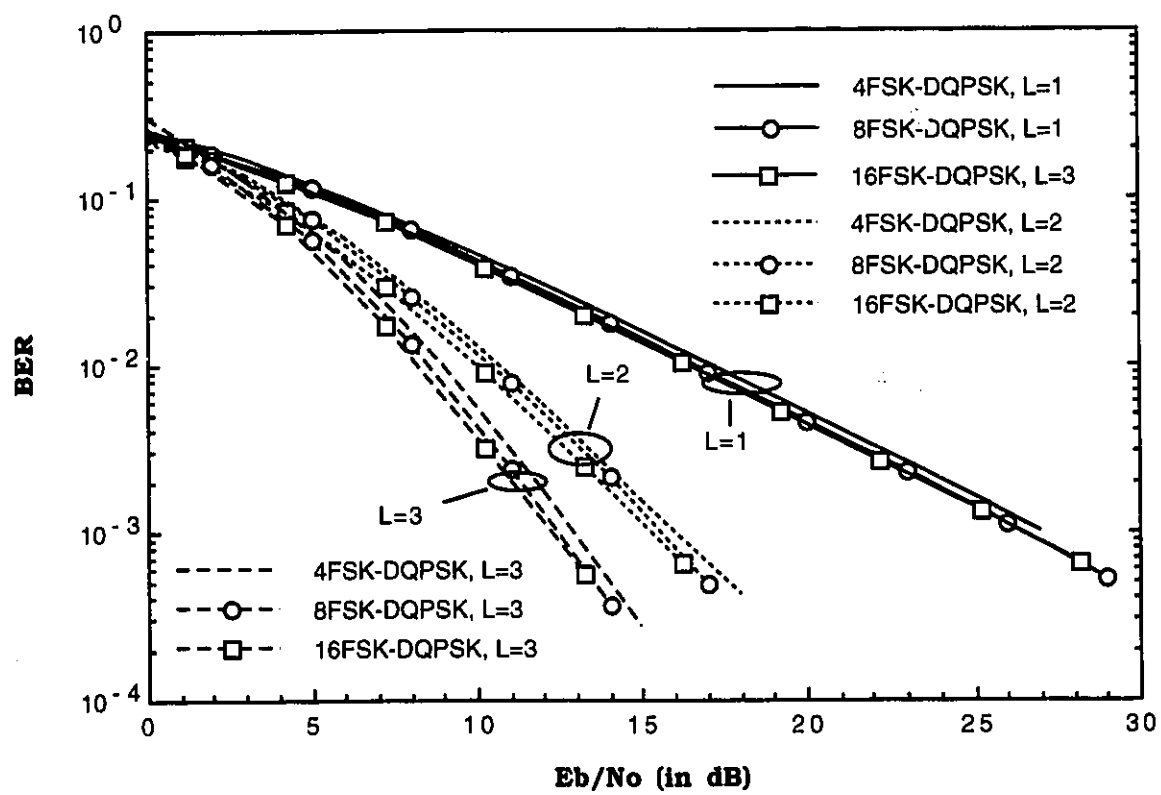


Figure 3.8 : Bit error rate of MFSK-DQPSK in Rayleigh fading for  $L = 1, 2$  and 3.

Table 3.2 :  $\bar{\gamma}_b$  required to obtain a bit error rate of  $10^{-3}$  for uncoded MFSK-DPSK for different  $L$  in Rayleigh fading.

|             | $L = 1$ | $L = 2$ | $L = 3$ |
|-------------|---------|---------|---------|
| 4FSK-DBPSK  | 27.4 dB | 17.1 dB | 14.0 dB |
| 8FSK-DBPSK  | 27.0 dB | 16.5 dB | 13.1 dB |
| 16FSK-DBPSK | 26.6 dB | 16.0 dB | 12.6 dB |
| 4FSK-DQPSK  | 27.3 dB | 16.3 dB | 13.0 dB |
| 8FSK-DQPSK  | 26.8 dB | 15.8 dB | 12.5 dB |
| 16FSK-DQPSK | 26.6 dB | 15.5 dB | 12.2 dB |

sub-symbol is incorrectly detected. Therefore, the symbol error rate is given by [50]:

$$P_M = P_{Mf} + (1 - P_{Mf})P_{Md} \quad (3.50)$$

where  $P_{Md}$  is the probability of a differential phase encoded sub-symbol is in error given that the frequency sub-symbol is correct. This is given by:

$$P_{Md} = P_{DPSK} + \frac{1}{2}P_{Mf} \quad (3.51)$$

for MFSK-DBPSK modulation schemes. the  $\frac{1}{2}P_{Mf}$  term takes into account the differential phase errors caused a frequency sub-symbol error in the previous symbol (recall that its phase is fedforward to detect the current phase). Also  $P_{DPSK}$  is given by eq. (3.38) or (3.39) depending on whether diversity is employed or not.

For 4-phase systems  $P_{Md}$  is:

$$P_{Md} = \frac{3}{2}(P_{DPSK} + \frac{1}{2}P_{Mf}) \quad (3.52)$$

where  $P_{DPSK}$  is given by eq. (3.41) or (3.42).

### 3.4 Multitone FSK Modulation

Multitone FSK (MT-FSK) is a multifrequency modulation scheme in which energy is transmitted simultaneously over  $w$  orthogonal frequencies out of  $v$  possible orthogonal frequencies. Thus it can convey at most  $\log_2 \binom{v}{w}$  bits of information per character. It has the potential to be more bandwidth efficient than conventional MFSK.<sup>1</sup> MT-FSK is an application of a more general modulation system described by David Slepian called permutation modulation [46].

#### 3.4.1 Permutation Modulation

Suppose we have a finite number of light emitting diodes (LEDs)  $l_1, l_2, \dots, l_v$ . We also have an alphabet  $a_1, a_2, \dots, a_z$  that we wish to represent by a combination of illuminated LEDs. If  $z = v$ , we can represent each letter in the alphabet by illuminating only one LED per symbol. For example  $a_1 = l_1$ ,  $a_2 = l_2$ , etc. If  $z > v$ , we can illuminate a pair of LEDs to represent each letter. For example  $a_1 = (l_1, l_2)$ ,  $a_2 = (l_1, l_3)$ , etc. Thus by using pairs of LEDs to represent each letter, we can represent at most  $\binom{v}{2} = v(v-1)/2$  letters using 2 illuminated LEDs. More generally,  $\binom{v}{w}$  letters can be represented by only  $v$  LEDs by representing each letter by a combination of  $w$  illuminated LEDs. The

---

<sup>1</sup>MFSK is simply a special case of MT-FSK where  $w = 1$  and  $v = M$ .

number of letters that can be represented in this manner is maximized when  $w = \lceil v/2 \rceil$ .

The preceding example was a demonstration of permutation representation. In this example, the medium was LEDs, where the information was conveyed which LEDs were switched on. Permutation modulation is generally implemented using orthogonal waveforms, mainly sinusoids. MT-FSK is a permutation modulation system which employs sinusoids.

### 3.4.2 Design of Efficient MT-FSK Waveforms

An analysis of MT-FSK using permutation modulation theory is given in [54]. In [54] the author shows that MT-FSK signalling employing permutation modulation theory may be more bandwidth efficient than 16 or 32FSK, but it is generally less power efficient for the different MT-FSK signalling sets described. An example of this is that  $\binom{16}{2}$  MT-FSK can transmit 120 possible symbols using half the number of frequency tones as 32FSK, yet 32FSK has about a 1dB power advantage. This is because in  $\binom{16}{2}$  MT-FSK, 15 symbols share the same frequency tone, therefore 1 symbol differs from 28 other symbols by 1 tone. This reduces the pairwise probability of error between adjacent symbols to that of MFSK, with only half the symbol energy (since the other half of the symbol energy is contained by the shared tone). Therefore it seems logical to suggest that MT-FSK using permutation modulation, although more bandwidth efficient, is not as power efficient as MFSK in AWGN or flat fading channels. For CDMA applications, power efficiency is more of a concern than bandwidth efficiency.

However, suppose we do not use all of the 120 possible permutations to represent our symbols. Suppose only a fraction of these are actually used to represent an alphabet. Then we can introduce a redundancy into the modulation itself, giving us larger distances between symbols, and possibly some power gain.

By employing efficient methods to design MT-FSK signalling sets, we can design MT-FSK systems which have much better bit error rate performance than seen in [54]. In effect, the use of this technique creates a form of coded modulation [52].

In this thesis, we will design a number of waveforms. The first will be obtained by expurgating a block code. In other words, we begin with a block code and select from it a subset of equal weight codewords. This method proves to be cumbersome when we wish to have a large signalling set. Therefore the remaining waveforms are designed using the Balanced Incomplete Block Design (BIB) [55]. This design method is detailed in Appendix B.

For the first waveform design, let us examine the (7,4) Hamming code [56, 57]. The (7,4) Hamming code vectors are given in Table 3.3.

Table 3.3 : (7,4) Hamming code

|               |               |
|---------------|---------------|
| 0 0 0 0 0 0 0 | 0 0 0 1 1 0 1 |
| 1 1 0 1 0 0 0 | 1 1 0 0 1 0 1 |
| 0 1 1 0 1 0 0 | 0 1 1 1 0 0 1 |
| 1 0 1 1 1 0 0 | 1 0 1 0 0 0 1 |
| 0 0 1 1 0 1 0 | 0 0 1 0 1 1 1 |
| 1 1 1 0 0 1 0 | 0 1 0 0 0 1 1 |
| 0 1 0 1 1 1 0 | 1 0 0 1 0 1 1 |
| 1 0 0 0 1 1 0 | 1 1 1 1 1 1 1 |

Each position in a codeword represents a specific frequency. For example the first bit represents  $f_1$ , the second bit represents  $f_2$  etc. Also a 0 means that the frequency is unused while a 1 indicates that the frequency is used in that specific waveform.

In order to obtain equal energy waveforms, we must use equal weight codewords.

Let us examine all codewords of weight 3. The design of the waveforms using these codewords is shown in Table 3.4.

*Table 3.4 : Equal energy waveforms derived from (7,4) Hamming codewords of weight 3.*

| Codeword      | Waveform                                              |
|---------------|-------------------------------------------------------|
| 1 1 0 1 0 0 0 | $\cos 2\pi f_1 t + \cos 2\pi f_2 t + \cos 2\pi f_4 t$ |
| 0 1 1 0 1 0 0 | $\cos 2\pi f_2 t + \cos 2\pi f_3 t + \cos 2\pi f_5 t$ |
| 0 0 1 1 0 1 0 | $\cos 2\pi f_3 t + \cos 2\pi f_4 t + \cos 2\pi f_6 t$ |
| 1 0 0 0 1 1 0 | $\cos 2\pi f_1 t + \cos 2\pi f_5 t + \cos 2\pi f_6 t$ |
| 0 0 0 1 1 0 1 | $\cos 2\pi f_4 t + \cos 2\pi f_5 t + \cos 2\pi f_7 t$ |
| 1 0 1 0 0 0 1 | $\cos 2\pi f_1 t + \cos 2\pi f_3 t + \cos 2\pi f_7 t$ |
| 0 1 0 0 0 1 1 | $\cos 2\pi f_2 t + \cos 2\pi f_6 t + \cos 2\pi f_7 t$ |

We can see from Table 3.4 that 7 waveforms are obtained by using all (7,4) Hamming codewords of weight 3. However, for binary communications it is desirable to have  $M = 2^k$  where  $M$  is the number of symbols (waveforms) in the set. Therefore, in this case, we must set  $M = 4$ . By eliminating all waveforms with  $\cos 2\pi f_7 t$ , we obtain 4 waveforms which employ 6 different frequencies. We will denote this signalling set as (6,3) MT-FSK. The waveforms are shown in Table 3.5. It is interesting to note from Table 3.5 that each symbol of (6,3) MT-FSK shares one and only one frequency with every other symbol in the set.

Using the BIB design detailed in Appendix B, three different signalling sets are designed. The first signalling set is an 8-ary modulation scheme. Each waveform is made up of three frequency tones that are selected from a set of eight possible tones. Thus it is referred to as (8,3) MT-FSK. The waveforms used in (8,3) MT-FSK are shown in Table 3.6.

The remaining two signalling sets are (10,4) and (12,4) MT-FSK. (10,4) MT-

Table 3.5 : (6,3) MT-FSK waveforms.

| Symbol | Codeword    | Waveform                                              |
|--------|-------------|-------------------------------------------------------|
| 1      | 1 1 0 1 0 0 | $\cos 2\pi f_1 t + \cos 2\pi f_2 t + \cos 2\pi f_4 t$ |
| 2      | 0 1 1 0 1 0 | $\cos 2\pi f_2 t + \cos 2\pi f_3 t + \cos 2\pi f_5 t$ |
| 3      | 0 0 1 1 0 1 | $\cos 2\pi f_3 t + \cos 2\pi f_4 t + \cos 2\pi f_6 t$ |
| 4      | 1 0 0 0 1 1 | $\cos 2\pi f_1 t + \cos 2\pi f_5 t + \cos 2\pi f_6 t$ |

Table 3.6 : (8,3) MT-FSK waveforms.

| Symbol | Codeword        | Waveform                                              |
|--------|-----------------|-------------------------------------------------------|
| 1      | 1 1 0 1 0 0 0 0 | $\cos 2\pi f_1 t + \cos 2\pi f_2 t + \cos 2\pi f_4 t$ |
| 2      | 0 1 1 0 1 0 0 0 | $\cos 2\pi f_2 t + \cos 2\pi f_3 t + \cos 2\pi f_5 t$ |
| 3      | 0 0 1 1 0 1 0 0 | $\cos 2\pi f_3 t + \cos 2\pi f_4 t + \cos 2\pi f_6 t$ |
| 4      | 0 0 0 1 1 0 1 0 | $\cos 2\pi f_4 t + \cos 2\pi f_5 t + \cos 2\pi f_7 t$ |
| 5      | 0 0 0 0 1 1 0 1 | $\cos 2\pi f_5 t + \cos 2\pi f_6 t + \cos 2\pi f_8 t$ |
| 6      | 1 0 1 0 0 0 0 1 | $\cos 2\pi f_1 t + \cos 2\pi f_3 t + \cos 2\pi f_8 t$ |
| 7      | 1 0 0 0 0 1 1 0 | $\cos 2\pi f_1 t + \cos 2\pi f_6 t + \cos 2\pi f_7 t$ |
| 8      | 0 1 0 0 0 0 1 1 | $\cos 2\pi f_2 t + \cos 2\pi f_7 t + \cos 2\pi f_8 t$ |

FSK is a 4-ary modulation scheme, where each symbol is represented by the sum of four tones which are selected from a set of ten. (12,4) MT-FSK is an 8-ary modulation scheme where the four tones which make up a signal are selected from a set of twelve possible tones. The waveforms which make up (10,4) MT-FSK are shown in Table 3.7 and the (12,4) MT-FSK waveforms are shown in Table 3.8. The design of (8,3), (10,4) and (12,4) MT-FSK waveforms is shown in Appendix C.

Table 3.7 : (10,4) MT-FSK waveforms.

| Symbol | Codeword            | Waveform                                                                   |
|--------|---------------------|----------------------------------------------------------------------------|
| 1      | 1 0 0 1 1 0 1 0 0 0 | $\cos 2\pi f_1 t + \cos 2\pi f_4 t + \cos 2\pi f_5 t + \cos 2\pi f_7 t$    |
| 2      | 0 0 1 1 0 0 0 1 0 1 | $\cos 2\pi f_3 t + \cos 2\pi f_4 t + \cos 2\pi f_8 t + \cos 2\pi f_{10} t$ |
| 3      | 0 1 1 0 1 1 0 0 0 0 | $\cos 2\pi f_2 t + \cos 2\pi f_3 t + \cos 2\pi f_5 t + \cos 2\pi f_6 t$    |
| 4      | 1 0 0 0 0 1 0 1 1 0 | $\cos 2\pi f_1 t + \cos 2\pi f_6 t + \cos 2\pi f_8 t + \cos 2\pi f_9 t$    |

Table 3.8 : (12,4) MT-FSK waveforms.

| Symbol | Codeword                | Waveform                                                                      |
|--------|-------------------------|-------------------------------------------------------------------------------|
| 1      | 1 0 1 0 0 0 1 0 1 0 0 0 | $\cos 2\pi f_1 t + \cos 2\pi f_3 t + \cos 2\pi f_7 t + \cos 2\pi f_9 t$       |
| 2      | 1 0 0 1 1 0 0 0 0 0 1 0 | $\cos 2\pi f_1 t + \cos 2\pi f_4 t + \cos 2\pi f_5 t + \cos 2\pi f_{11} t$    |
| 3      | 0 0 1 1 0 1 0 1 0 0 0 0 | $\cos 2\pi f_3 t + \cos 2\pi f_4 t + \cos 2\pi f_6 t + \cos 2\pi f_8 t$       |
| 4      | 0 1 0 0 1 1 1 0 0 0 0 0 | $\cos 2\pi f_2 t + \cos 2\pi f_5 t + \cos 2\pi f_6 t + \cos 2\pi f_7 t$       |
| 5      | 0 0 0 0 1 0 0 1 1 1 0 0 | $\cos 2\pi f_5 t + \cos 2\pi f_8 t + \cos 2\pi f_9 t + \cos 2\pi f_{10} t$    |
| 6      | 0 0 0 1 0 0 1 0 0 1 0 1 | $\cos 2\pi f_4 t + \cos 2\pi f_7 t + \cos 2\pi f_{10} t + \cos 2\pi f_{12} t$ |
| 7      | 0 1 1 0 0 0 0 0 0 1 1 0 | $\cos 2\pi f_2 t + \cos 2\pi f_3 t + \cos 2\pi f_{10} t + \cos 2\pi f_{11} t$ |
| 8      | 1 1 0 0 0 0 0 1 0 0 0 1 | $\cos 2\pi f_1 t + \cos 2\pi f_2 t + \cos 2\pi f_8 t + \cos 2\pi f_{12} t$    |

One can see from inspection that in (10,4) and (12,4) MT-FSK, each waveform

shares one and only one frequency tone with every other waveform. In the case of (8,3) MT-FSK, each waveform shares one and only one frequency tone with 6 other waveforms, and is completely orthogonal to the remaining waveform.

### 3.4.3 Performance of (6,3) MT-FSK in an AWGN Channel

The optimum receiver for (6,3) MT-FSK is shown in Figure 3.9. It consists of 6 frequency detectors whose outputs are summed together to form the four decision variables. The largest decision variable is chosen as the received symbol.

Supposing symbol 1 is transmitted. At the receiver, the decision variables are:

$$\begin{aligned}
 U_1 &= |2\alpha_1\mathcal{E} + N_1|^2 + |2\alpha_2\mathcal{E} + N_2|^2 + |2\alpha_4\mathcal{E} + N_4|^2 \\
 U_2 &= |2\alpha_2\mathcal{E} + N_2|^2 + |N_3|^2 + |N_5|^2 \\
 U_3 &= |N_3|^2 + |2\alpha_4\mathcal{E} + N_4|^2 + |N_6|^2 \\
 U_4 &= |2\alpha_1\mathcal{E} + N_1|^2 + |N_5|^2 + |N_6|^2
 \end{aligned} \tag{3.53}$$

The total energy contained in the received symbol is:

$$E_s = \sum_{i=1}^3 \alpha_i \mathcal{E} \tag{3.54}$$

Assuming that each frequency tone contains an equal amount of energy, and the channel gains are all equal (and normalized to 1), the received symbol energy is:

$$E_s = 3\mathcal{E} \tag{3.55}$$

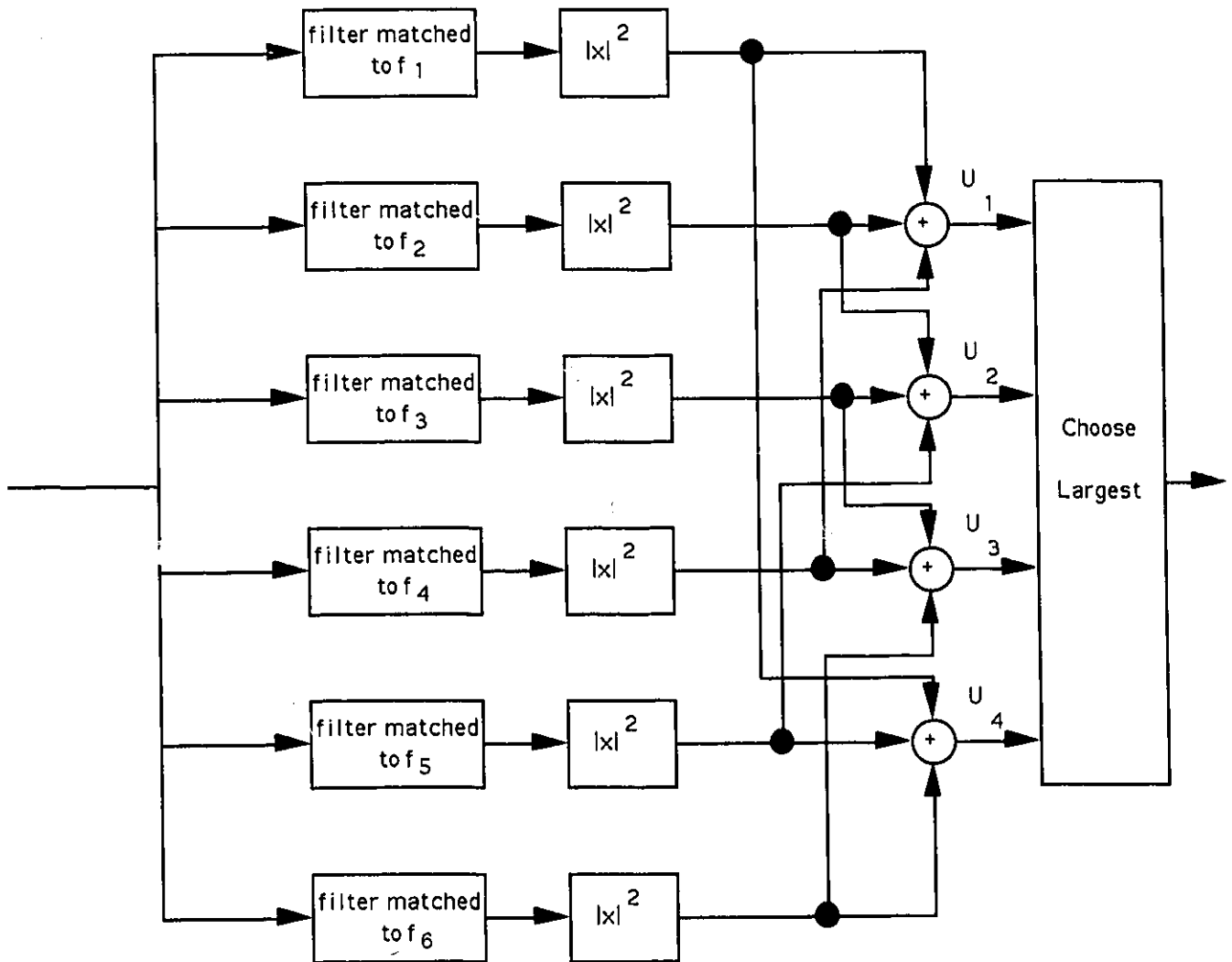


Figure 3.9 : Optimum receiver for (6,3) MT-FSK.

The probability of a symbol error is:

$$\begin{aligned} P_M &= 1 - P_c \\ &= 1 - P(U_2 < U_1, U_3 < U_1, U_4 < U_1) \end{aligned} \quad (3.56)$$

Since  $U_2$ ,  $U_3$ , and  $U_4$  are described statistically by the same distribution function, then  $P_M$  becomes:

$$P_M = 1 - [P(U_2 < U_1)]^3 \quad (3.57)$$

$P(U_2 < U_1)$  is the same as  $P(U'_2 < U'_1)$  where

$$\begin{aligned} U'_1 &= |2\alpha_1\mathcal{E} + N_1|^2 + |2\alpha_4\mathcal{E} + N_4|^2 \\ U'_2 &= |N_3|^2 + |N_5|^2 \end{aligned} \quad (3.58)$$

Thus the pdf of  $U'_1$  is found in [40] to be:

$$p(u'_1) = \frac{1}{4\mathcal{E}N_o} \left( \frac{u'_1}{s^2} \right)^{1/2} e^{-(s^2 + u'_1)/4\mathcal{E}N_o} I_1 \left( \frac{s\sqrt{u'_1}}{2\mathcal{E}N_o} \right) \quad u'_1 \geq 0 \quad (3.59)$$

where  $s^2$  is the noncentrality parameter and is given by:

$$s^2 = 4\mathcal{E}^2(\alpha_1^2 + \alpha_4^2) \quad (3.60)$$

The pdf of  $U'_2$  is:

$$p(u'_2) = \frac{1}{(4\mathcal{E}N_o)^2} u'_2 e^{-u'_2/4\mathcal{E}N_o} \quad u'_2 \geq 0 \quad (3.61)$$

Using these pdf's, we find the symbol error rate to be:

$$\begin{aligned}
P_M &= 1 - \int_0^\infty [P(U'_2 < u'_1 | U'_1 = u'_1)]^3 p(u'_1) du'_1 \\
&= 1 - \int_0^\infty [1 - e^{-\frac{u'_1}{4\mathcal{E}N_o}} \sum_{i=0}^1 \frac{1}{i!} \left(\frac{u'_1}{4\mathcal{E}N_o}\right)^i]^2 p(u'_1) du'_1 \quad (3.62)
\end{aligned}$$

This is evaluated in [40] and the following equation for the symbol error rate is found:

$$P_M = 1 - \int_0^\infty [1 - e^{-\gamma_{eff}v} \sum_{i=1}^2 \frac{1}{i!} v^i]^2 \gamma_{eff} v e^{-\gamma_{eff}(1+v)} I_1(2\gamma_{eff}\sqrt{v}) \quad (3.63)$$

where  $\gamma_{eff}$  is given by:

$$\gamma_{eff} = \frac{s^2}{4\mathcal{E}N_o} = \frac{\mathcal{E}}{N_o}(\alpha_1^2 + \alpha_4^2) = \frac{2}{3}\gamma_s \quad (3.64)$$

Equation (3.63) has no closed form expression. It can be evaluated numerically, however, due to the presence of the modified Bessel function within the integral, a great deal of accuracy is required in order to produce a reasonable approximation.

An alternative approach is to use the union bound

$$P_M < (M - 1)P_2(w_{eq}) \quad (3.65)$$

where  $P_2(w_{eq})$  is the probability of error in choosing  $U'_1$  and any one of the other  $M - 1$  decision variables  $U'_m$  (in the case of (6,3) MT-FSK,  $M=4$ ) and  $w_{eq}$  is the equivalent weight of the decision variable (for (6,3) MT-FSK,  $w_{eq}$  is 2). In [40],  $P_2(w_{eq})$  is found to be:

$$P_2(w_{eq}) = \frac{1}{2^{2w_{eq}-1}} e^{-\gamma_{eff}/2} \sum_{n=0}^{w_{eq}-1} c_n \left( \frac{\gamma_{eff}}{2} \right)^n \quad (3.66)$$

where  $c_n$  is given by

$$c_n = \frac{1}{n!} \sum_{i=0}^{w_{eq}-1-n} \binom{2w_{eq}-1}{i} \quad (3.67)$$

Note that in the case of (6,3) MT-FSK,  $\gamma_{eff} = 2\gamma_s/3$ . More generally,  $\gamma_{eff} = \frac{w-c_t}{w}\gamma_s$ , where  $c_t$  is the number of frequency tones shared by the two symbols.

For (6,3) MT-FSK, the symbol error rate is given by the following union bound:

$$P_M < \frac{3}{8} e^{-\gamma_s/3} \sum_{n=0}^1 c_n \left( \frac{\gamma_s}{3} \right)^n \quad (3.68)$$

By extension, a general expression for any MT-FSK modulation scheme in AWGN can be found to be

$$P_M < \sum_{c_t=0}^{w-1} \frac{x_{c_t}}{2^{2(w-c_t)-1}} e^{-(w-c_t)\gamma_s/2w} \sum_{n=0}^{w-c_t-1} c_n \left( \frac{w-c_t}{2w} \gamma_s \right)^n \quad (3.69)$$

where  $x_{c_t}$  is the number of symbols that share  $c_t$  terms with symbol 1. Note that  $\sum_{c_t=0}^{w-1} x_{c_t} = M - 1$ .

The bit error rate of (6,3) MT-FSK is:

$$P_b < \frac{1}{4} e^{-2\gamma_b/3} \sum_{n=0}^1 c_n \left( \frac{2\gamma_b}{3} \right)^n \quad (3.70)$$

These results can be extended to the other MT-FSK schemes. For (8,3) MT-FSK, the symbol error performance can be shown to be:

$$\begin{aligned}
P_M &< \frac{6}{2^3} e^{-\gamma_s/3} \sum_{n=0}^1 c_n \left(\frac{\gamma_s}{3}\right)^n + \frac{1}{2^5} e^{-\gamma_s/2} \sum_{x=0}^2 c_x \left(\frac{\gamma_s}{2}\right)^x \\
&< \frac{3}{4} e^{-\gamma_s/3} \sum_{n=0}^1 c_n \left(\frac{\gamma_s}{3}\right)^n + \frac{1}{32} e^{-\gamma_s/2} \sum_{x=0}^2 c_x \left(\frac{\gamma_s}{2}\right)^x
\end{aligned} \tag{3.71}$$

Since all errors are not equiprobable, the bit error rate of (8,3) MT-FSK is given by:

$$P_b < \frac{3}{8} e^{-\gamma_b} \sum_{n=0}^1 c_n (\gamma_b)^n + \frac{1}{32} e^{-3\gamma_b/2} \sum_{x=0}^2 c_x \left(\frac{3\gamma_b}{2}\right)^x \tag{3.72}$$

For (10,3) MT-FSK, the symbol error rate performance is:

$$P_M < \frac{3}{2^5} e^{-3\gamma_s/8} \sum_{n=0}^2 c_n \left(\frac{3\gamma_s}{4}\right)^n \tag{3.73}$$

The bit error probability of (10,3) MT-FSK is

$$P_b < \frac{1}{16} e^{-3\gamma_b/4} \sum_{n=0}^2 c_n \left(\frac{3\gamma_b}{4}\right)^n \tag{3.74}$$

For (12,4) MT-FSK, the bit error rate performance can be found to be:

$$P_b < \frac{1}{8} e^{-9\gamma_b/8} \sum_{n=0}^2 c_n \left(\frac{9\gamma_b}{8}\right)^n \tag{3.75}$$

The bit error rate performance of all MT-FSK schemes in an AWGN channel is shown in Figure 3.10.

From Figure 3.10, we can see that the bit error rate of the different MT-FSK schemes is more dependent on the number of bits per symbol rather than the

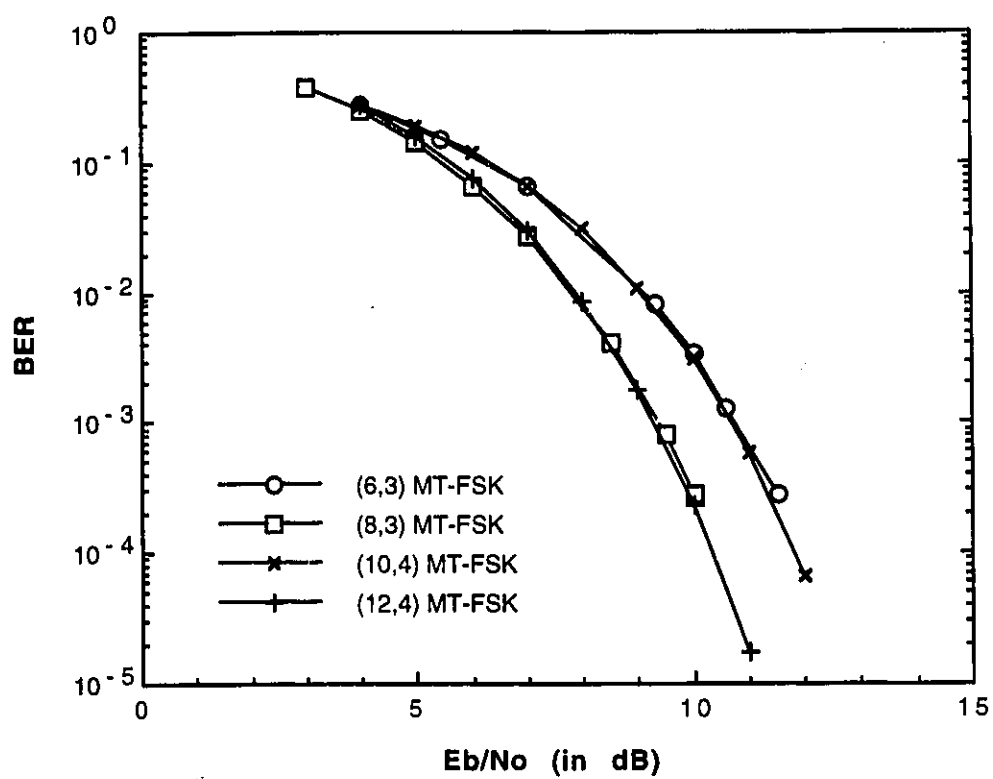


Figure 3.10 : Bit error rate upper bounds for MT-FSK in an AWGN channel.

equivalent diversity order. Modulation schemes with more bits per symbol have a better bit error performance than do those with less bits per symbol. This behaviour is also observed for MFSK modulation.

### 3.4.4 Performance of MT-FSK in Frequency-Nonselective Rayleigh Fading

As previously discussed in the case of MFSK, the energy per bit to noise spectral density ratio of a signal which encounters Rayleigh fading becomes a random variable whose statistics are described by a chi-square distribution function. For frequency non-selective fading, all frequency tones of each waveform encounter the same fading process, therefore the energy per bit to noise spectral density ratio has the following probability distribution function:

$$p(\gamma_b) = \frac{1}{\bar{\gamma}_b} e^{-\frac{\gamma_b}{\bar{\gamma}_b}} \quad (3.76)$$

The bit error rate of (6,3) MT-FSK in frequency-nonselective Rayleigh fading is

$$\begin{aligned} P_b &< \int_0^\infty \frac{1}{4} e^{-2\gamma_b/3} \sum_{n=0}^1 c_n \left( \frac{2\gamma_b}{3} \right)^n \frac{1}{\bar{\gamma}_b} e^{-\frac{\gamma_b}{\bar{\gamma}_b}} d\gamma_b \\ &< \frac{1}{4\bar{\gamma}_b} \sum_{n=0}^1 c_n \left( \frac{2}{3} \right)^n \int_0^\infty e^{-\gamma_b \frac{(2/3)\bar{\gamma}_b + 1}{\bar{\gamma}_b}} \gamma_b^n d\gamma_b \end{aligned} \quad (3.77)$$

Performing integration by parts on eqn. (3.77) yields

$$P_b < -\frac{1}{4\bar{\gamma}_b} \sum_{n=0}^1 c_n \left( \frac{2}{3} \right)^n \sum_{i=0}^n \frac{\gamma_b^{n-i}}{K^{i+1}} e^{-K\gamma_b} \left( \begin{matrix} n \\ i \end{matrix} \right) \Bigg|_0^\infty \quad (3.78)$$

where  $K = \frac{(2/3)\bar{\gamma}_b + 1}{\bar{\gamma}_b}$ . The only non-zero term in eqn. (3.78) occurs when  $n = i$ , therefore  $P_b$  becomes:

$$P_b < \frac{1}{(8/3)\bar{\gamma}_b + 4} \sum_{n=0}^1 c_n \left(\frac{2}{3}\right)^n \quad (3.79)$$

Similiarly, the expressions for the bit error rate of (8,3), (10,4) and (12,4) MT-FSK can be found. The bit error rate expression for (8,3) MT-FSK is:

$$P_b < \frac{3}{8\bar{\gamma}_b + 8} \sum_{n=0}^1 c_n + \frac{1}{48\bar{\gamma}_b + 32} \sum_{x=0}^2 c_x \left(\frac{3}{2}\right)^x \quad (3.80)$$

For (10,4) MT-FSK, the bit error rate expression is:

$$P_b < \frac{1}{12\bar{\gamma}_b + 16} \sum_{n=0}^2 c_n \left(\frac{3}{4}\right)^n \quad (3.81)$$

For (12,4) MT-FSK in frequency-nonselective Rayleigh fading, the bit error rate is:

$$P_b < \frac{1}{9\bar{\gamma}_b + 8} \sum_{n=0}^2 c_n \left(\frac{9}{8}\right)^n \quad (3.82)$$

The bit error rate of the different MT-FSK signalling sets in frequency-nonselective Rayleigh fading is shown in Figure 3.11.

It is interesting to note from Figure 3.11 that (12,4) and (8,3) MT-FSK perform worse than (6,3) and (10,4) MT-FSK in frequency-nonselective Rayleigh fading when the opposite is true in AWGN. This is because (12,4) and (8,3) MT-FSK are 8-ary modulation schemes, therefore the probability of making a

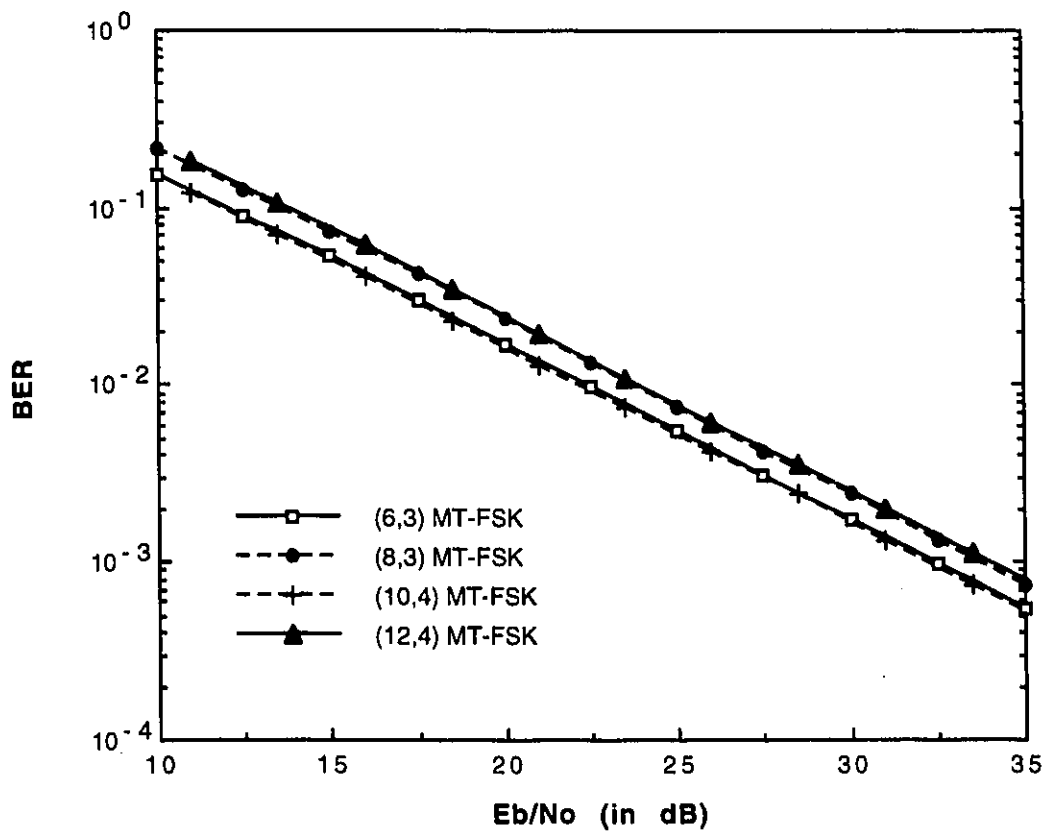


Figure 3.11 : Performance of MT-FSK in frequency-nonselective Rayleigh fading.

correct decision during a deep fade (or equivalently, when the received symbol is unrecognizable) is less because the receiver has more symbols to choose from<sup>2</sup>.

### 3.4.5 Performance of Wideband MT-FSK in Frequency-Selective Rayleigh Fading

The poor performance of MT-FSK in frequency-nonsselective Rayleigh fading is due to the improper use of its inherent diversity. In wideband MT-FSK, the frequency tone separation  $\Delta f \gg R_s$ . In this thesis, we will choose  $\Delta f$  to be greater than the coherence bandwidth of the channel  $(\Delta f)_c$ . Therefore, each frequency tone encounters independent fading processes, and frequency diversity is obtained. The decision variables  $U_1$  and  $U_2$  for (6,3) MT-FSK become:

$$\begin{aligned} U_1 &= |2\mathcal{E}\alpha_1 + N_1|^2 + |2\mathcal{E}\alpha_2 + N_2|^2 + |2\mathcal{E}\alpha_4 + N_4|^2 \\ U_2 &= |2\mathcal{E}\alpha_2 + N_2|^2 + |N_3|^2 + |N_5|^2 \end{aligned} \quad (3.83)$$

where  $\alpha_i$  is the time varying complex gain of the  $i$ th channel. The symbol error rate can be estimated by the union bound

$$P_M < (M - 1)P_2(D) \quad (3.84)$$

where  $D$  is the equivalent diversity order of the modulation scheme, and  $P_2(D)$  is given by

$$P_2(D) = P[U_2 > U_1]$$

---

<sup>2</sup>Recall that  $P_M < (M - 1)P_2(w_{eq})$ . In frequency-nonsselective Rayleigh fading,  $w_{eq}$  has little influence on the symbol error rate, therefore signalling sets with large  $M$  will perform worse than signalling sets with lower  $M$ .

$$= P[U'_2 > U'_1] \quad (3.85)$$

$U'_1$  and  $U'_2$  are formed by eliminating the common term shared by  $U_1$  and  $U_2$ .  
Therefore

$$\begin{aligned} U'_1 &= |2\mathcal{E}\alpha_1 + N_1|^2 + |2\mathcal{E}\alpha_4 + N_4|^2 \\ U'_2 &= |N_3|^2 + |N_5|^2 \end{aligned} \quad (3.86)$$

It is shown in [40] that for the preceding decision variables,  $P_2(D)$  is given by

$$P_2(D) = \left(\frac{1-\mu}{2}\right)^D \sum_{i=0}^{D-1} \binom{D-1+i}{i} \left(\frac{1+\mu}{2}\right)^i \quad (3.87)$$

where  $D = w - 1 = 2$  and  $\mu$  is given by

$$\mu = \frac{\frac{\bar{\mathcal{E}}}{N_o}}{2 + \frac{\bar{\mathcal{E}}}{N_o}} \quad (3.88)$$

where  $\frac{\bar{\mathcal{E}}}{N_o}$  is the average energy to noise spectral density ratio per frequency tone. Therefore  $\frac{\bar{\mathcal{E}}}{N_o} = \frac{\bar{E}_s}{N_o w} = \frac{k}{w} \frac{\bar{E}_b}{N_o}$ .

The bit error rate  $P_b$  for (6,3) MT-FSK in Rayleigh fading is

$$P_b = \frac{2^{k-1}}{2^k - 1} P_M = \frac{2}{3} P_M \quad (3.89)$$

For (8,3) MT-FSK, each waveform shares 1 tone with 6 other waveforms and is completely orthogonal to the remaining waveform. Therefore, the symbol error rate performance of (8,3) MT-FSK is given by

$$P_M < 6P_2(D = w - 1) + P_2(D = w) \quad (3.90)$$

The bit error rate performance for wideband (8,3) MT-FSK in Rayleigh fading is:

$$\begin{aligned} P_b &= \frac{1}{2} \times 6P_2(D = 2) + P_2(D = 3) \\ &= 3P_2(D = 2) + P_2(D = 3) \end{aligned} \quad (3.91)$$

Similarly,

$$P_b < 2P_2(D = 3) \quad \text{for (10,4) MT - FSK} \quad (3.92)$$

$$< 4P_2(D = 3) \quad \text{for (12,4) MT - FSK} \quad (3.93)$$

The bit error rate performance of wideband MT-FSK in Rayleigh fading is shown in Figure 3.12. We can see in Figure 3.12 that (10,4) and (12,4) MT-FSK perform better than do (8,3) and (6,3) MT-FSK. This is due to the increased diversity provided by the extra frequency tone in each waveform.

### 3.4.6 Performance of Wideband MT-FSK in Rayleigh Fading with Channel Diversity

Suppose we had the available bandwidth to transmit a wideband MT-FSK signal over  $L$  different channels. Such a scheme would require a minimum bandwidth of  $vL\Delta f$ . At the receiver, the  $L$  symbols are combined using square-law combining. The equivalent decision variables become

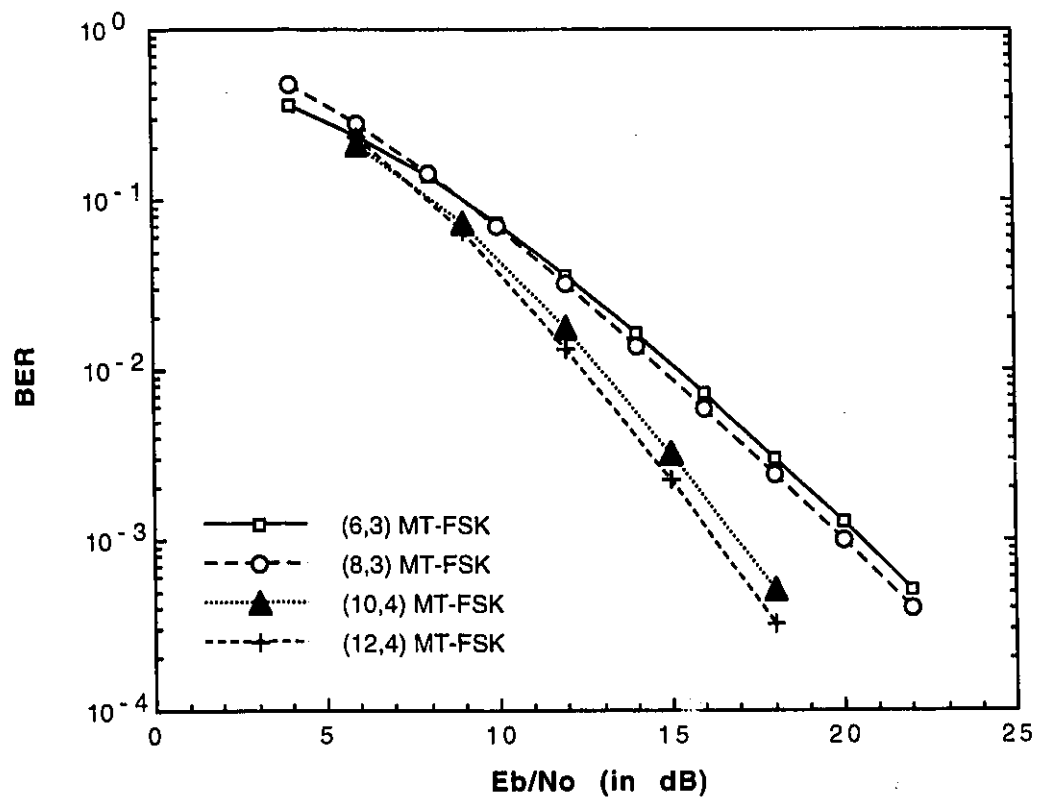


Figure 3.12 : Bit error performance of wideband MT-FSK in slow Rayleigh fading.

$$U_m^* = U_m^{(1)} + U_m^{(2)} + \dots + U_m^{(L)} \quad (3.94)$$

It is easy to show that for these decision variables,  $P_2(D)$  shown in eq.5 becomes  $P_2(LD)$  and  $\frac{E_b}{N_o}$  must be replaced by  $\frac{E_b}{N_o L}$ . In other words, we have simply increased the inherent diversity of the modulation scheme by a factor of  $L$ .

The bit error rate performance of MT-FSK is shown in Figure 3.13 for  $L = 1, 2$  and 3. Note that increasing the diversity order causes a degradation in bit error rate performance for small  $E_b/N_o$ , while the performance improves greatly for large  $E_b/N_o$ .

The values of  $\gamma'_b$  required to provide a bit error rate performance of  $10^{-3}$  are shown in Table 3.9 for different orders of diversity. We can see from the results shown in Figure 3.13 and Table 3.9 that (12,4) MT-FSK provides the target bit error rate of  $10^{-3}$  employing lower  $E_b/N_o$  for all  $L$ . It is also interesting to note that (12,4) MT-FSK with  $L = 2$  provides better results than do the other MT-FSK schemes employing a diversity order of 3.

Table 3.9 :  $\gamma'_b$  required for a bit error rate of  $10^{-3}$  for wideband MT-FSK in Rayleigh fading.

|               | $L = 1$ | $L = 2$ | $L = 3$ |
|---------------|---------|---------|---------|
| (6,3) MT-FSK  | 20.5 dB | 16.1 dB | 15.0 dB |
| (8,3) MT-FSK  | 20.0 dB | 15.1 dB | 13.8 dB |
| (10,4) MT-FSK | 17.2 dB | 14.0 dB | 13.4 dB |
| (12,4) MT-FSK | 16.4 dB | 13.0 dB | 12.4 dB |

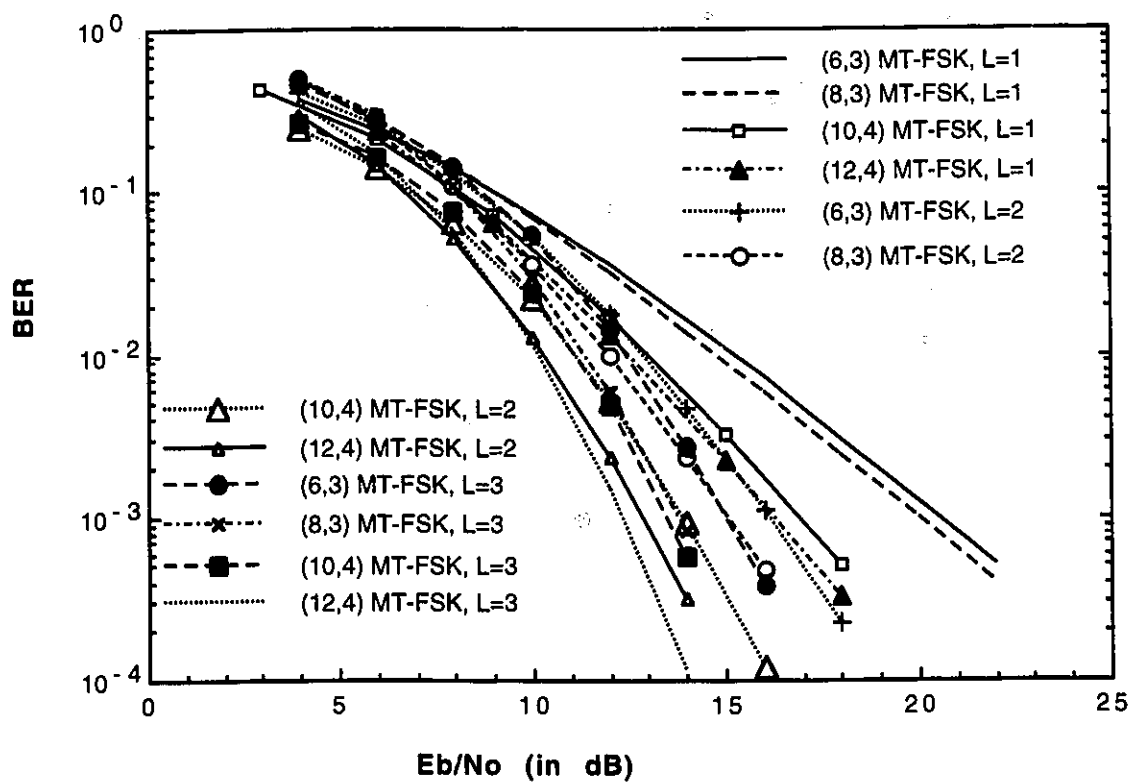


Figure 3.13 : Performance of wideband MT-FSK in Rayleigh fading with additional channel diversity.

## 3.5 Combined Wideband MT-FSK/DPSK Modulation

The concept of combining frequency and phase modulation discussed previously can be extended to MT-FSK. The resulting modulation scheme is MT-FSK/DPSK. Since MT-FSK only performs well in Rayleigh fading when  $\Delta f > (\Delta f)_c$ , then wideband MT-FSK/DPSK is discussed in this section.

### 3.5.1 MT-FSK/DPSK Modem

The phase information is differentially encoded. In this particular design, each tone of the MT-FSK sub-symbol will carry the same phase information. This is because differential phase detection can only be carried out on symbols which are affected by dependent fading (in other words, the differential phase can only be determined on symbols which are transmitted by the same carrier). Recall that a wideband MT-FSK symbol is comprised of a number of tones that are separated by a bandwidth that is greater than the coherence bandwidth of the channel. Therefore, the differential phase can only be determined by the phase information carried by the common tones of consecutive symbols.

For example, suppose (6,3) MT-FSK is used and each symbol carries phase information on each tone. Suppose symbol 1 is sent followed by symbol 2. Symbol one is made up of the sum of 3 frequency tones  $f_1$ ,  $f_2$ , and  $f_4$  while symbol 2 is composed of the sum of tones  $f_2$ ,  $f_3$ , and  $f_5$ . The differential phase can only be determined by the phase information carried by  $f_2$  from each symbol. However, suppose that symbol 1 is followed by symbol 1. The consecutive symbols are equal, thus all tones are common and all tones can be used in determining the differential phase (and diversity combining can be exploited).

It is obvious that we cannot use (8,3) MT-FSK in an MT-FSK/DPSK scheme since some waveforms are orthogonal to others, and thus they have no tones in common. It would be possible to employ some source coding algorithm which would eliminate orthogonal waveforms from following each other, but this analysis is left for future studies should MT-FSK/DPSK prove to be a power efficient modulation scheme.

The frequency sub-symbol is used to represent  $k_f$  bits while the phase sub-symbol represents  $k_p$  bits. As before, we will only consider two and four phase systems, therefore  $k_p = 1$  or 2. The phase is differentially encoded and is modulated by the frequency encoded sub-symbol. This is shown in Figure 3.14(a).

The demodulation process is the same as in the case of MFSK/DPSK. The frequency sub-symbol is determined and its appropriate phase information as well as that from the previous symbol is retrieved from memory and differentially decoded. This is shown in Figure 3.14(b).

### 3.5.2 Performance of Combined Wideband MT-FSK/DPSK in Rayleigh Fading

The probability of correctly receiving an MT-FSK/DPSK symbol is highly dependent on the correct determination of the transmitted frequency symbols. If these are correctly determined, the proper phase information will then be extracted.

The probability that the frequency encoded symbol is incorrectly received is given by eqn. (3.87) when (6,3) MT-FSK is used to transmit the frequency sub-symbol, eqn. (3.92) for (10,4) MT-FSK and eqn. (3.93) for (12,4) MT-FSK. When using these equations, one must employ  $\bar{\gamma}_s = (k_f + k_p)\bar{\gamma}_b$ .

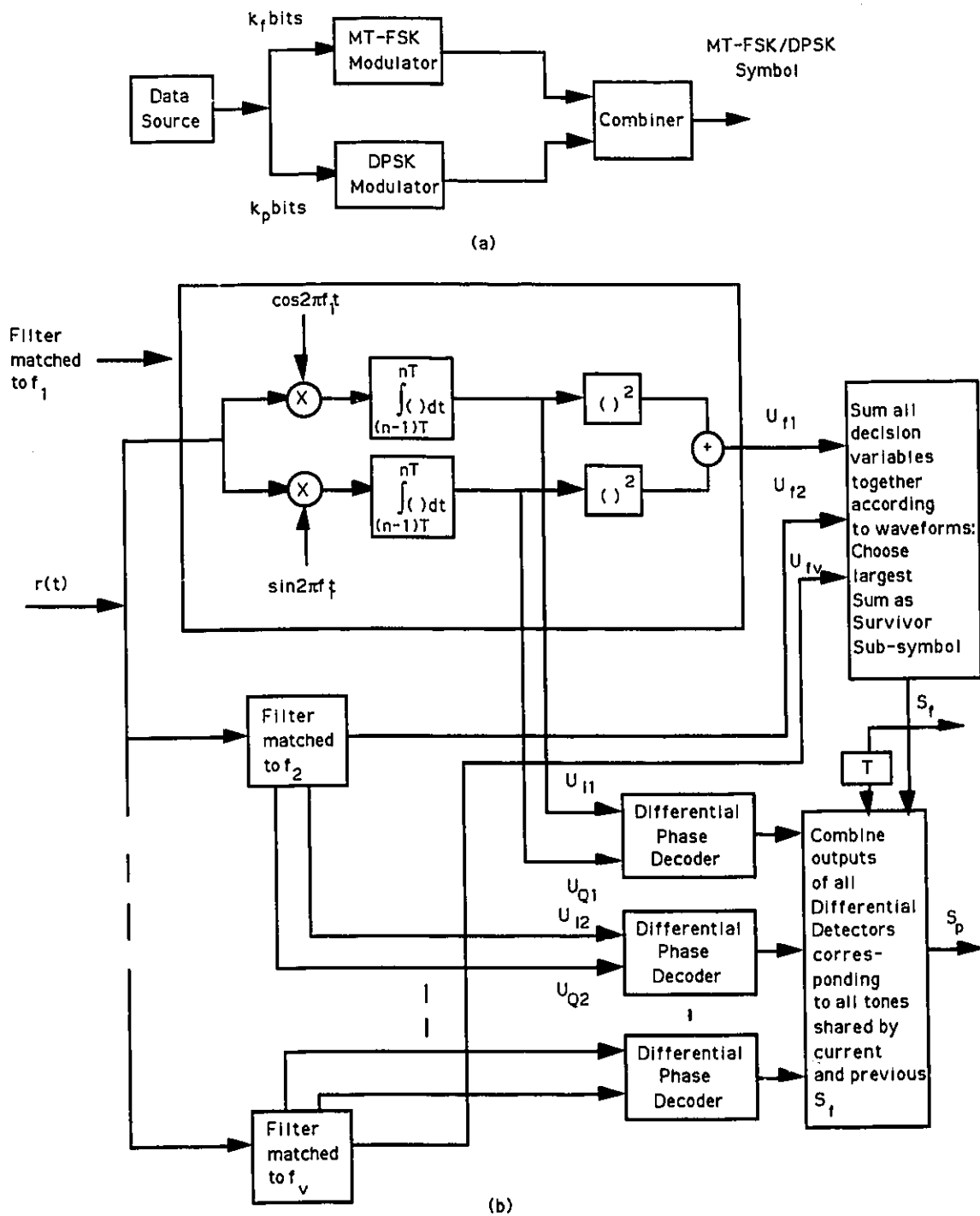


Figure 3.14 : (a) MT-FSK/DPSK modulation. (b) MT-FSK/DPSK demodulation.

If the frequency encoded sub-symbol is incorrectly received, the probability of bit error in the phase encoded symbol is 0.5. Also, since differential phase detection is used, one error in the differential phase detection affects the next symbol as well. Therefore, once an error occurs in the phase encoded symbol due to errors in the frequency sub-symbol, two successive correctly demodulated frequency sub-symbols are needed to restore the integrity of the phase encoded symbols.

If two successive frequency encoded symbols are identical, phase contained in all tones can be used to demodulate the phase encoded symbol. Therefore, the probability that this phase encoded symbol will be incorrectly demodulated diminishes. The probability that one symbol is repeated given that it was previously sent is  $1/M_f$  where  $M_f$  is the number of different frequency sub-symbols.

The probability of bit error in the phase encoded symbol,  $P_d$ , can be found from the Markov chain shown in Figure 3.15, where  $P_{DPSK}(D = 1)$  is given by eqn. (3.38) for a two phase system and eqn. (3.41) for a four phase system, and  $P_{DPSK}(D = w)$  is given by eqns. (3.39) and (3.41) for two phase and four phase systems respectively. When employing these equations, we must replace  $L$  by  $D$  (so as not to confuse tone diversity with channel diversity) and  $\bar{\gamma}_s$  by  $\bar{\gamma}_s/w$ .

As in the case of MFSK-DPSK, we must find the steady state probabilities that the instantaneous  $P_d$  equals one of the different bit error probabilities shown in Figure 3.15. To do this, we need the state transition probability matrix,  $P$ .

$$P = \begin{bmatrix} (1 - P_M)\frac{M-1}{M} & (1 - P_M)\frac{1}{M} & P_M & 0 \\ (1 - P_M)\frac{M-1}{M} & (1 - P_M)\frac{1}{M} & P_M & 0 \\ 0 & 0 & P_M & (1 - P_M) \\ (1 - P_M)\frac{M-1}{M} & (1 - P_M)\frac{1}{M} & P_M & 0 \end{bmatrix} \quad (3.95)$$

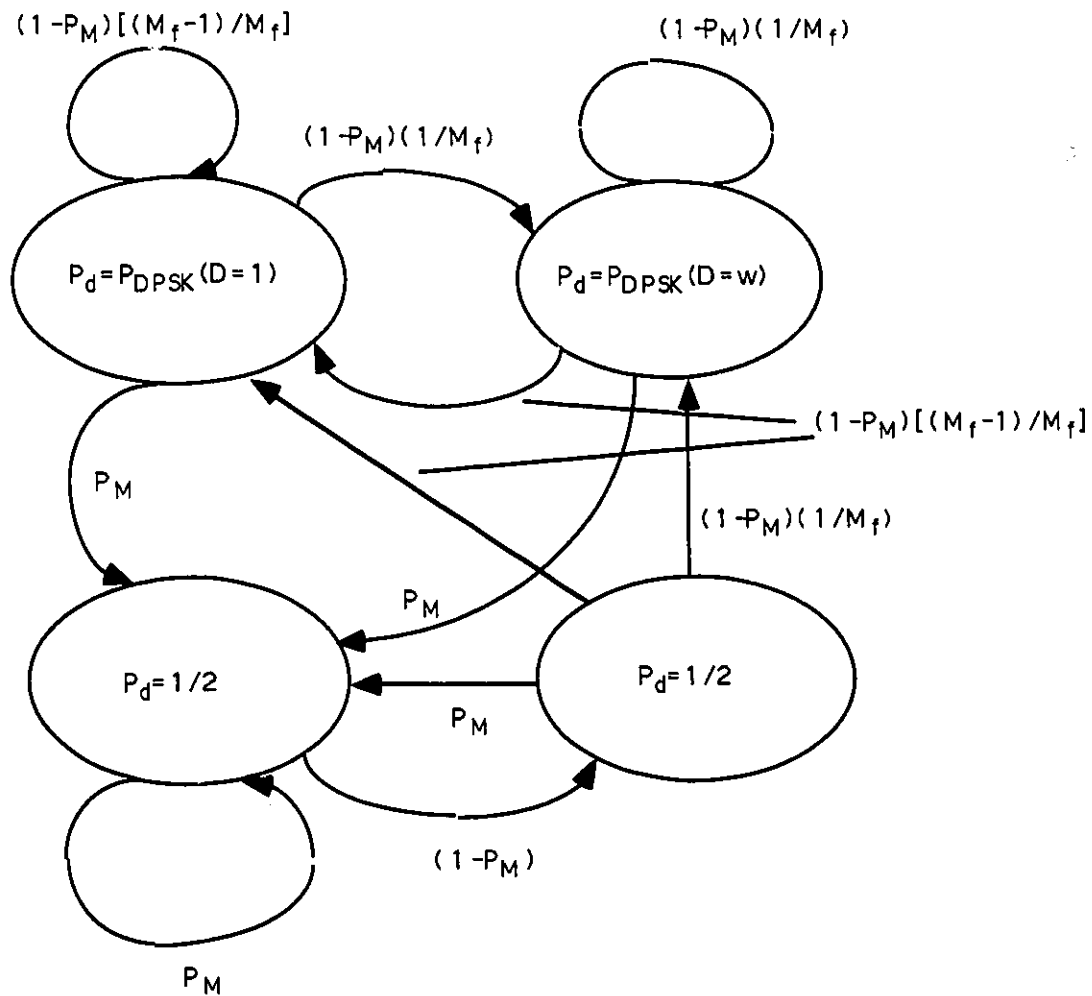


Figure 3.15 : Markov chain used to determine  $P_d$  for MT-FSK/DPSK modulation.

The steady state probabilities  $\pi_0, \pi_1, \pi_2, \pi_3$  are the probabilities that  $P_d$  is  $P_{DPSK}(D = 1), P_{DPSK}(D = w), 1/2$  and  $1/2$  respectively<sup>3</sup>. Recall that these steady state probabilities are found when

$$\pi P = \pi \quad \text{where } \pi = [\pi_0, \pi_1, \pi_2, \pi_3] \quad (3.96)$$

Also,  $\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$ . From this we find

$$\pi_0 = (1 - P_M)^2 \frac{M-1}{M} \quad (3.97)$$

$$\pi_1 = (1 - P_M)^2 \frac{1}{M} \quad (3.98)$$

$$\pi_2 = P_M \quad (3.99)$$

$$\pi_3 = (1 - P_M)P_M \quad (3.100)$$

Therefore  $P_d$  for MT-FSK/DPSK is found to be

$$\begin{aligned} P_d = & (1 - P_M)^2 \frac{M-1}{M} P_{DPSK}(D = 1) + (1 - P_M)^2 \frac{1}{M} \\ & + \frac{1}{2} P_M (2 - P_M) \end{aligned} \quad (3.101)$$

The bit error rate for MT-FSK/DPSK is given by

$$P_b = \frac{k_f}{k_p + k_f} \frac{2^{k_f-1}}{2^{k_f} - 1} P_M + \frac{k_f}{k_p + k_f} P_d \quad (3.102)$$

---

<sup>3</sup>There is a difference between  $\pi_2$  and  $\pi_3$ ;  $\pi_2$  is the probability that  $P_d = 1/2$  because the frequency sub-symbol is in error and  $\pi_3$  is the probability that  $P_d = 1/2$  because the previous frequency sub-symbol was in error.

The performance of MT-FSK/DBPSK is shown in Figure 3.16, while the bit error rate of MT-FSK/DQPSK is shown in Figure 3.17. For comparison purposes, bit error rate curves for (6,3), (10,4) and (12,4) MT-FSK are included.

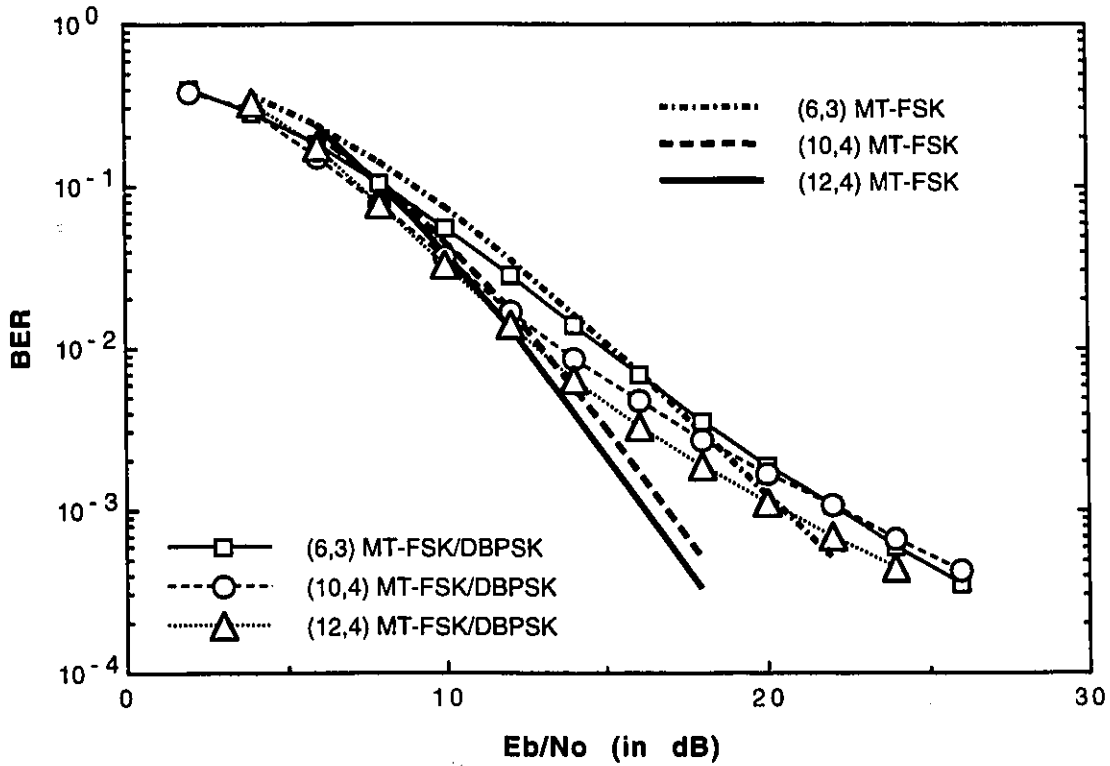


Figure 3.16 : Comparison of MT-FSK/DBPSK to MT-FSK modulation schemes in frequency-nonselective Rayleigh fading.

We can see from Figures 3.16 and 3.17 that MT-FSK/DPSK outperforms MT-FSK at low  $E_b/N_o$ . This is because the frequency and phase sub-symbols benefit greatly from the energy sharing aspect of the modulation scheme. However, at

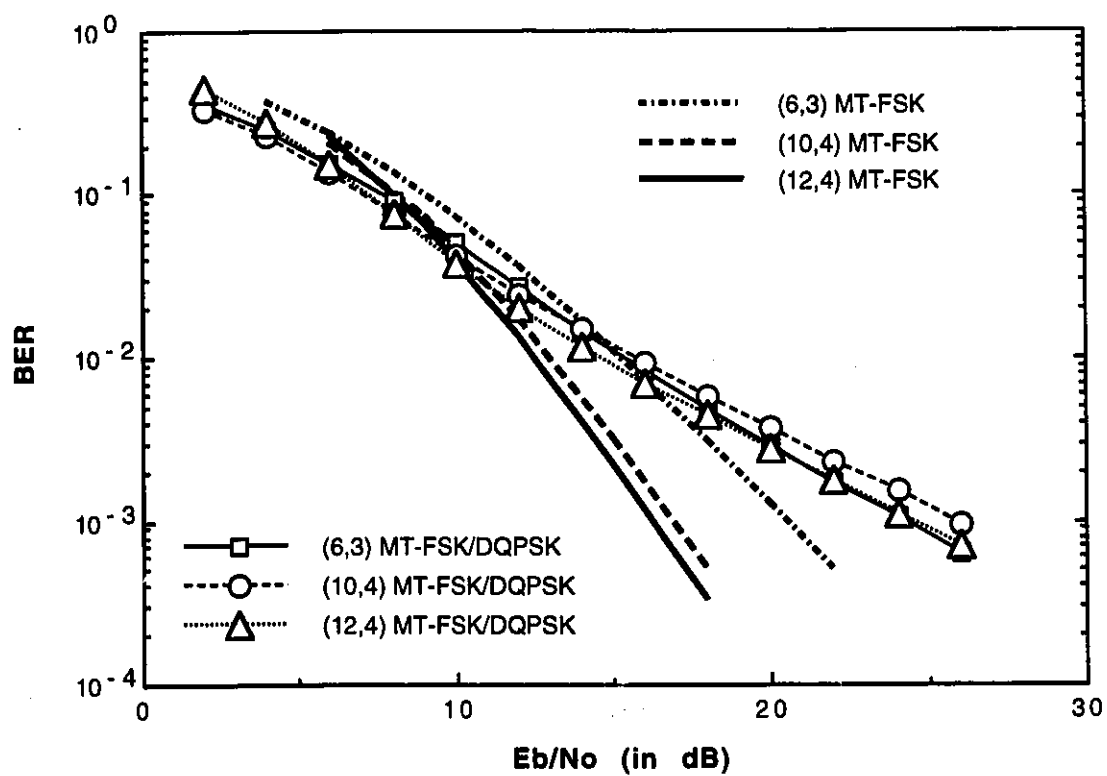


Figure 3.17: Comparison of MT-FSK/DQPSK to MT-FSK modulation schemes in frequency-nonsselective Rayleigh fading.

high  $E_b/N_o$ , the bit error rate of MT-FSK/DPSK is dominated by  $P_d$ , particularly by the term  $(1 - P_M)^2 \frac{M-1}{M} P_{DPSK}(D = 1)$ . Since the MT-FSK modulation schemes discussed in this thesis have an inherent diversity  $D = w - 1$ , it is obvious that its bit error performance will overtake a modulation scheme whose bit error rate performance is dominated by a function of  $P_{DPSK}(D = 1)$  which has no inherent diversity.

MFSK/DPSK, previously discussed in Section 3.3 outperforms MFSK for all  $E_b/N_o$ , which justifies the added complexity in implementing MFSK/DPSK. Although MT-FSK/DPSK outperforms MT-FSK at low  $E_b/N_o$ , which may be useful when error control coding is used, its performance at high  $E_b/N_o$  is cause for concern. This is especially evident when MT-FSK/DPSK is compared to MFSK and MFSK/DPSK. A comparison is made in Figure 3.18.

In Figure 3.18, we selected the best MT-FSK/DPSK bit error rate curves and compared them to some of the better performing MFSK and MFSK/DPSK schemes. We see that at low  $E_b/N_o$ , the MFSK and MFSK/DPSK curves outperform MT-FSK/DPSK. Thus we can see that at any given  $E_b/N_o$ , we can find a modulation scheme previously discussed in this thesis which outperforms MT-FSK/DPSK. Because of this, and the high complexity of MT-FSK/DPSK, we will not consider this modulation scheme any further.

## 3.6 Discussion

In this chapter, we considered a number of modulation schemes based upon MFSK signalling techniques. We first considered MFSK modulation itself, then extended the results to a combined frequency-phase modulation. The advantages of doing this are many. Firstly, we can increase the number of bits per symbol without increasing the number of frequency tones required. This results in a modulation scheme which is more bandwidth efficient. The second advan-

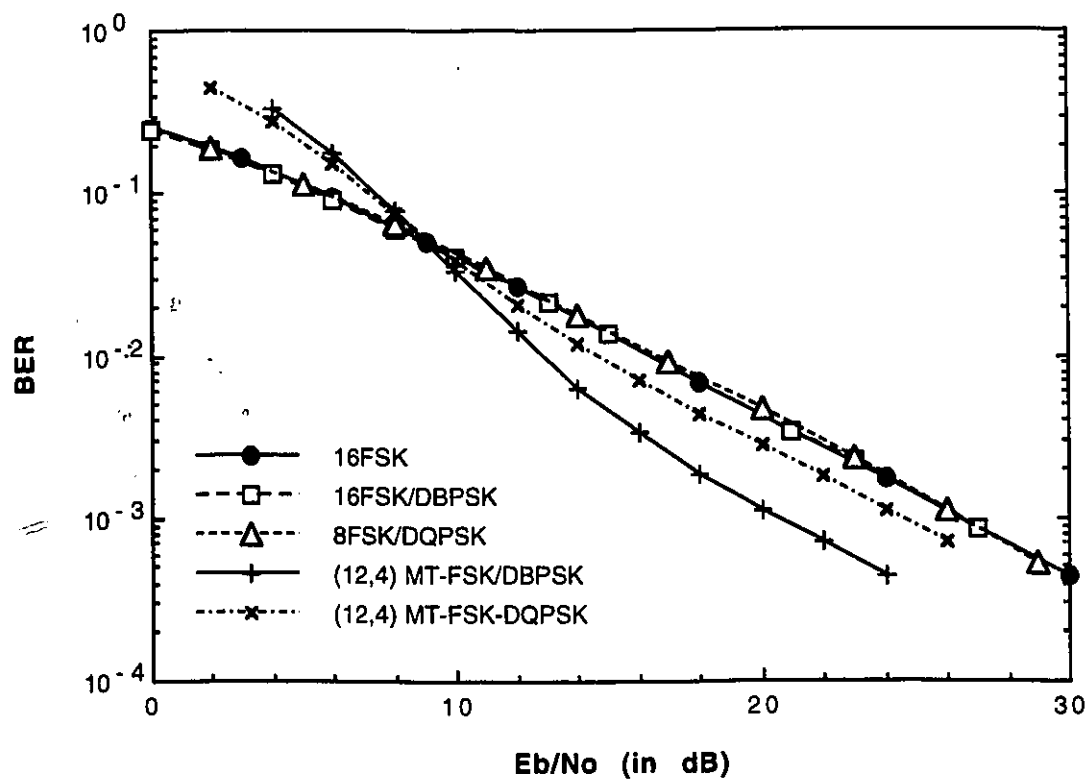


Figure 3.18 : Comparison of MT-FSK/DPSK to MFSK and MFSK/DPSK modulation schemes in frequency-nonselective Rayleigh fading.

tage obtained by having the signal phase carry information is that the symbol is now composed of two sub-symbols which share signal power, and therefore this signalling method is more power efficient than conventional MFSK. Of course the price paid by having a modulation scheme which is both more bandwidth and power efficient is an increase in complexity.

We examined MT-FSK as a method of transmitting data over a Rayleigh fading channel. The waveforms are designed so that each waveform is different from another by many tones. By employing wideband MT-FSK, the modulation scheme has an inherent frequency diversity, and thus has a relatively good bit error rate performance compared to MFSK and MFSK-DPSK at moderate to high  $E_b/N_o$  values.

The concept of introducing phase carried information to MT-FSK was also discussed. Compared to the other modulation schemes discussed in this thesis, MT-FSK/DPSK has the highest complexity. Yet, we concluded that for any  $E_b/N_o$  MT-FSK/DPSK outperformed some modulation schemes discussed previously but not all of them. Therefore at any value of  $E_b/N_o$ , some other modulation scheme previously discussed in this thesis is more cost-effective to employ. Therefore we will not consider MT-FSK/DPSK any further.

The goal of this thesis is to present a CDMA system which employs the modulation schemes discussed in this chapter. One figure of merit of a communication system is its spectral efficiency. In CDMA systems, this is inversely proportional to the value of  $E_b/N_o$  required so that the system is operating at its highest acceptable bit error rate. The results seen in this chapter can be further improved by the addition of error control coding which can decrease the required value of  $E_b/N_o$  that provides this bit error rate.

## Chapter 4

# Error Control Coding

### 4.1 Introduction

Error control coding is used to improve the power efficiency of a communications system. By introducing redundancy into the information, channel errors can be corrected. In a heavy interference environment such as a CDMA channel, coding may provide significant improvements in bit error rate which can then be traded off against spectral efficiency.

In this chapter, we will examine different error control codes as a means of improving the bit error rate performance of the modulation schemes discussed in Chapter 3. The codes discussed in this chapter have different coding gains and complexities. The goal of this chapter is not to find an optimum code, but rather to demonstrate the effect of coding on the spectral efficiency of the CDMA systems to be discussed in the next chapter.

Since the modulation schemes under consideration are non-binary, we will focus on the use of non-binary codes, namely non-binary BCH and Reed Solomon (RS) codes. We examine a number of different BCH and/or RS codes for each

modulation scheme. We also examine the concatenation of these codes with hard decision convolutional coding.

## 4.2 Non-Binary BCH and RS Codes

A popular coding scheme for nonbinary modulation schemes is RS coding. The problem with using RS codes in this case is that the length of an RS codeword is  $2^k - 1$ . For small signalling sets ( $M = 4$  or  $M = 8$ ),  $k = 2$  or  $3$ . Therefore, the use of RS codes will produce short codewords, which have little distance between them. A mismatched RS coding scheme allows one to group a number of symbols to form a larger signalling set for RS coding. This increases the codeword length, but it also greatly increases the code symbol error probability [58], which results in a poor performance.

RS codes are a subset of a set of codes called Bose-Chaudhuri-Hocquenghem (BCH) codes. RS codes are defined on  $\text{GF}(M)$  and have length equal to  $M - 1$  while primitive BCH codes are defined on  $\text{GF}(M)$  and have length equal to  $M^x - 1$ . Therefore RS codes are primitive BCH codes with  $x = 1$ . We can see that the length of a non-binary code is not limited, therefore we need not “mismatch” the code and the symbol set to obtain longer codewords.

We will limit our discussion to primitive BCH and RS codes although non-primitive BCH or RS codes are also applicable. For small alphabets ( $M = 4$  or  $8$ ) we will examine only BCH codes, while for larger alphabets, we will examine both RS and BCH codes.

Most BCH and RS decoding algorithms do not allow direct soft decision decoding [63, 59]. Some algorithms have been developed to approximate soft decision decoding<sup>1</sup>, and work has been done on new BCH soft decision algorithms [60, 61,

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<sup>1</sup>The Weldon and Chase algorithms which approximate soft decision decoding require multiple applications of the hard decision decoding algorithm which greatly increases the

Chapter 6]. However, we will examine hard decision detection of BCH and RS codes because this technique is more commonly employed in literature.

The number of parity bits in a  $(n, m)$  BCH code is  $n - m \leq 2xt$ . A BCH code can correct up to  $t$  errors where  $t \geq (n - m)/2x$  where  $x = \log_M(n + 1)$  (for RS codes,  $x = 1$ ).

The probability of incorrectly decoding a  $\text{BCH}(n, m)$  codeword is

$$P_E = 1 - P_c \quad (4.1)$$

where  $P_c$  is the probability of correctly decoding a codeword. To correctly decode a codeword, then the codeword must contain  $t$  or fewer symbol errors. Therefore

$$\begin{aligned} P_c &= \sum_{i=0}^t \binom{n}{i} p^i (1-p)^{n-i} \\ &= \sum_{i=0}^t P_s(i) \end{aligned} \quad (4.2)$$

where  $p$  is the code symbol error probability and is equal to  $P_M(\frac{m}{n}\gamma_b)$  and  $P_s(i)$  is the probability of having  $i$  code symbol errors in one codeword.

A codeword which has more than  $t$  errors will be decoded in error. When the decoder attempts to correct this codeword, it adds at most  $t$  more errors to the codeword [62]. Therefore a codeword that is received with  $i$  errors will exit the decoder with no more than  $t + i$  symbol errors if  $i > t$ . The symbol error rate from the decoder  $P_{M_{dec}}$  is:

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complexity of the decoder.

$$P_{M_{dec}} < \sum_{i=t+1}^n \frac{t+i}{n} P_s(i) \quad (4.3)$$

The bit error rate becomes

$$P_b < \frac{2^{k-1}}{2^k - 1} P_{M_{dec}} \quad (4.4)$$

The performance of the different modulation schemes of Chapter 3 employing different BCH codes in a Rayleigh fading environment is examined, and the values of  $\bar{\gamma}_b$  required to obtain a bit error rate less than  $10^{-3}$  are determined from eq. (4.4). Different orders of diversity are also considered. The results are tabulated in Tables 4.1-4.12 (Results appear in order of alphabet size).

*Table 4.1 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing (6,3) MT-FSK with different non-binary BCH codes.*

| code         | $L = 1$ | $L = 2$ | $L = 3$ |
|--------------|---------|---------|---------|
| BCH(63,51)   | 19.5 dB | 15.9 dB | 15.1 dB |
| BCH(63,45)   | 18.2 dB | 15.3 dB | 14.7 dB |
| BCH(63,39)   | 17.5 dB | 15.1 dB | 14.7 dB |
| BCH(255,207) | 18.0 dB | 15.0 dB | 14.3 dB |
| BCH(255,199) | 17.6 dB | 14.8 dB | 14.2 dB |

The results shown in Tables 4.1-4.12 show great improvement over the uncoded results of Chapter 3. Generally, the results for MFSK and MFSK-DPSK are better than those seen for MT-FSK, however, we must remember that bounds have been used to estimate the performance of the error control codes and furthermore bounds were used to find the uncoded symbol error rate for MT-FSK. Therefore, we can assume that the bound for this modulation scheme

Table 4.2 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing (10,4) MT-FSK with different non-binary BCH codes.

| code         | $L = 1$ | $L = 2$ | $L = 3$ |
|--------------|---------|---------|---------|
| BCH(63,51)   | 16.7 dB | 14.8 dB | 14.2 dB |
| BCH(63,45)   | 16.0 dB | 14.6 dB | 14.0 dB |
| BCH(63,39)   | 15.7 dB | 14.6 dB | 14.1 dB |
| BCH(255,207) | 15.7 dB | 14.2 dB | 13.6 dB |
| BCH(255,199) | 15.4 dB | 14.1 dB | 13.5 dB |

Table 4.3 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 8FSK with different non-binary BCH codes.

| code       | $L = 1$ | $L = 2$ | $L = 3$ |
|------------|---------|---------|---------|
| BCH(63,51) | 18.4 dB | 12.9 dB | 11.3 dB |
| BCH(63,47) | 17.2 dB | 12.4 dB | 11.0 dB |
| BCH(63,43) | 16.3 dB | 12.0 dB | 10.8 dB |
| BCH(63,39) | 15.7 dB | 11.9 dB | 10.7 dB |

Table 4.4 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 4FSK-DBPSK with different non-binary BCH codes.

| code       | $L = 1$ | $L = 2$ | $L = 3$ |
|------------|---------|---------|---------|
| BCH(63,51) | 19.3 dB | 12.8 dB | 11.2 dB |
| BCH(63,47) | 18.2 dB | 12.3 dB | 10.9 dB |
| BCH(63,43) | 17.3 dB | 12.0 dB | 10.8 dB |
| BCH(63,39) | 16.6 dB | 11.8 dB | 10.7 dB |

Table 4.5 :  $\bar{\gamma}_b$  required for a bit error rate of  $10^{-3}$  for an  $L$  th order diversity system employing (8,3) MT-FSK with different non-binary BCH codes.

| code       | $L = 1$ | $L = 2$ | $L = 3$ |
|------------|---------|---------|---------|
| BCH(63,51) | 17.4 dB | 14.0 dB | 13.1 dB |
| BCH(63,47) | 16.6 dB | 13.6 dB | 12.9 dB |
| BCH(63,43) | 16.1 dB | 13.4 dB | 12.9 dB |
| BCH(63,39) | 15.8 dB | 13.4 dB | 12.9 dB |

Table 4.6 :  $\bar{\gamma}_b$  required for a bit error rate of  $10^{-3}$  for an  $L$  th order diversity system employing (12,4) MT-FSK with different non-binary BCH codes.

| code       | $L = 1$ | $L = 2$ | $L = 3$ |
|------------|---------|---------|---------|
| BCH(63,51) | 15.0 dB | 13.1 dB | 12.5 dB |
| BCH(63,47) | 14.5 dB | 12.9 dB | 12.3 dB |
| BCH(63,43) | 14.2 dB | 12.9 dB | 12.3 dB |
| BCH(63,39) | 14.1 dB | 13.0 dB | 12.4 dB |

Table 4.7 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 16FSK with different non-binary BCH codes.

| code         | $L = 1$ | $L = 2$ | $L = 3$ |
|--------------|---------|---------|---------|
| RS(15,9)     | 14.6 dB | 10.8 dB | 9.9 dB  |
| RS(15,7)     | 13.7 dB | 10.7 dB | 10.0 dB |
| RS(15,5)     | 13.4 dB | 11.1 dB | 10.7 dB |
| BCH(255,207) | 15.8 dB | 10.9 dB | 9.7 dB  |
| BCH(255,199) | 15.1 dB | 10.6 dB | 9.5 dB  |

Table 4.8 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 8FSK-DBPSK with different non-binary BCH codes.

| code         | $L = 1$ | $L = 2$ | $L = 3$ |
|--------------|---------|---------|---------|
| RS(15,9)     | 15.8 dB | 11.2 dB | 10.0 dB |
| RS(15,7)     | 14.9 dB | 11.2 dB | 10.2 dB |
| RS(15,5)     | 14.7 dB | 11.6 dB | 10.8 dB |
| BCH(255,207) | 17.0 dB | 11.4 dB | 9.8 dB  |
| BCH(255,199) | 16.4 dB | 11.1 dB | 9.6 dB  |

Table 4.9 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 4FSK-DQPSK with different non-binary BCH codes.

| code         | $L = 1$ | $L = 2$ | $L = 3$ |
|--------------|---------|---------|---------|
| RS(15,9)     | 16.1 dB | 11.1 dB | 9.9 dB  |
| RS(15,7)     | 15.2 dB | 11.0 dB | 10.0 dB |
| RS(15,5)     | 15.0 dB | 11.4 dB | 10.7 dB |
| BCH(255,207) | 17.4 dB | 11.2 dB | 9.6 dB  |
| BCH(255,199) | 16.6 dB | 11.0 dB | 9.5 dB  |

Table 4.10 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 16FSK-DBPSK with different non-binary BCH codes.

| code      | $L = 1$ | $L = 2$ | $L = 3$ |
|-----------|---------|---------|---------|
| RS(31,25) | 17.1 dB | 10.5 dB | 9.5 dB  |
| RS(31,23) | 15.7 dB | 9.9 dB  | 9.2 dB  |
| RS(31,21) | 14.6 dB | 9.5 dB  | 8.9 dB  |
| RS(31,19) | 13.9 dB | 9.3 dB  | 8.8 dB  |

Table 4.11 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 8FSK-DQPSK with different non-binary BCH codes.

| code      | $L = 1$ | $L = 2$ | $L = 3$ |
|-----------|---------|---------|---------|
| RS(31,25) | 17.2 dB | 10.7 dB | 9.8 dB  |
| RS(31,23) | 15.8 dB | 10.1 dB | 9.4 dB  |
| RS(31,21) | 14.8 dB | 9.7 dB  | 9.1 dB  |
| RS(31,19) | 14.1 dB | 9.5 dB  | 9.0 dB  |

Table 4.12 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 16FSK-DQPSK with different non-binary BCH codes.

| code      | $L = 1$ | $L = 2$ | $L = 3$ |
|-----------|---------|---------|---------|
| RS(63,53) | 16.8 dB | 10.2 dB | 9.1 dB  |
| RS(63,51) | 15.9 dB | 9.9 dB  | 8.8 dB  |
| RS(63,49) | 15.2 dB | 9.6 dB  | 8.6 dB  |
| RS(63,47) | 14.6 dB | 9.3 dB  | 8.4 dB  |

is not very tight compared to the other two modulation techniques. For an accurate assessment, simulations are needed.

### 4.3 Concatenated Coding

A concatenated code consists of two separate codes which are combined to form a larger code. The input data stream is encoded by the outer code which has rate  $r_{c_{out}} = \frac{m_{out}}{n_{out}}$ . The coded data is then encoded by the inner code which has rate  $r_{c_{in}} = \frac{m_{in}}{n_{in}}$ . The result is a code with rate  $r_{con} = \frac{m_{out}m_{in}}{n_{out}n_{in}}$ .

In this thesis, we will consider a concatenated coding system which consists of a convolutional code with hard decision detection as the outer code, and one of the nonbinary BCH or RS code from the previous section as the inner code. Since convolutional coding works best when the errors are random, an interleaving should be used between the two codes.

A convolutional code is generated by passing an information stream through  $m$   $l$ -stage linear shift register. The output is obtained through  $n$  linear algebraic functions performed on the contents of the shift register. The code rate is  $r_c = m/n$  and the constraint length of the code is  $l + 1$ .

A convolutional code has no defined block size. Generally, the code is truncated periodically, forcing it to have a fixed length. To do this, 0's must be appended to the last information bit for the purpose of clearing the shift register of all information bits.

An upper bound of the probability of bit error for hard decision decoding of convolutional codes is given by [40]:

$$P_b < \frac{1}{m} \sum_{d=d_{free}}^{\infty} w_d [4p(1-p)]^{d/2} \quad (4.5)$$

where  $d_{free}$  is the minimum free distance of the code,  $w_d = a_d * f(a_d)$ ,  $a_d$  is the number of different paths that are a distance  $d$  from the true path, and  $f(a_d)$  is the fraction of bits that are in error when one of the incorrect paths of distance  $d$  is selected by the decoder, and  $p$  is the code bit error rate. For the concatenated code,  $p$  is the output bit error rate of the BCH code. A list of  $w_d$  for different convolutional codes is given in [63, page 402].

This bound is rather loose as shown in [40]. Therefore, the results in this section will provide pessimistic estimates of the energy per bit to noise spectral density ratios needed to guarantee a bit error rate less than  $10^{-3}$ .

In this section, we will employ some BCH codes from the previous section along with one of two convolutional codes as the outer code. These two codes are the rate 3/4 constraint length 6 convolutional code (r3/4c6) and the rate 2/3 constraint length 6 convolutional code (r2/3c6). These two codes have similar coding gain in the AWGN channel. Using eqs. (4.3) and (4.4) of the previous section and eq. (4.5), we find estimates of  $\bar{\gamma}_b$  required for each modulation scheme so that  $P_b < 10^{-3}$ . The results are tabulated in Tables 4.13-4.24 (shown in order of alphabet size).

*Table 4.13 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing (6,3) MT-FSK with different concatenated codes.*

| code              | $L = 1$ | $L = 2$ | $L = 3$ |
|-------------------|---------|---------|---------|
| BCH(63,51)-r3/4c6 | 17.5 dB | 15.1 dB | 14.7 dB |
| BCH(63,45)-r3/4c6 | 17.1 dB | 15.1 dB | 14.7 dB |
| BCH(63,51)-r2/3c6 | 17.1 dB | 15.1 dB | 14.7 dB |
| BCH(63,45)-r2/3c6 | 16.9 dB | 15.1 dB | 14.8 dB |

In most cases, the concatenated schemes discussed in this section cannot guarantee better power efficiency at a bit error rate of  $10^{-3}$  than do the BCH codes

Table 4.14 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing (10,4) MT-FSK with different concatenated codes.

| code              | $L = 1$ | $L = 2$ | $L = 3$ |
|-------------------|---------|---------|---------|
| BCH(63,51)-r3/4c6 | 15.4 dB | 14.2 dB | 14.1 dB |
| BCH(63,45)-r3/4c6 | 15.2 dB | 14.2 dB | 14.2 dB |
| BCH(63,51)-r2/3c6 | 15.2 dB | 14.2 dB | 14.2 dB |
| BCH(63,45)-r2/3c6 | 15.2 dB | 14.3 dB | 14.3 dB |

Table 4.15 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 8FSK with different concatenated codes.

| code              | $L = 1$ | $L = 2$ | $L = 3$ |
|-------------------|---------|---------|---------|
| BCH(63,51)-r3/4c6 | 18.0 dB | 13.2 dB | 11.8 dB |
| BCH(63,47)-r3/4c6 | 17.0 dB | 12.8 dB | 11.6 dB |
| BCH(63,51)-r2/3c6 | 17.1 dB | 13.0 dB | 11.7 dB |
| BCH(63,47)-r2/3c6 | 16.4 dB | 12.7 dB | 11.6 dB |

Table 4.16 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 4FSK-DBPSK with different concatenated codes.

| code              | $L = 1$ | $L = 2$ | $L = 3$ |
|-------------------|---------|---------|---------|
| BCH(63,51)-r3/4c6 | 19.0 dB | 13.6 dB | 12.7 dB |
| BCH(63,47)-r3/4c6 | 18.0 dB | 13.3 dB | 12.5 dB |
| BCH(63,51)-r2/3c6 | 18.1 dB | 13.4 dB | 12.6 dB |
| BCH(63,47)-r2/3c6 | 17.4 dB | 13.1 dB | 12.5 dB |

Table 4.17 :  $\bar{\gamma}_b$  required for a bit error rate of  $10^{-3}$  for an  $L$  th order diversity system employing (8,3) MT-FSK with different concatenated codes.

| code              | $L = 1$ | $L = 2$ | $L = 3$ |
|-------------------|---------|---------|---------|
| BCH(63,51)-r3/4c6 | 16.4 dB | 13.2 dB | 13.1 dB |
| BCH(63,47)-r3/4c6 | 16.1 dB | 13.2 dB | 13.2 dB |
| BCH(63,51)-r2/3c6 | 16.2 dB | 13.2 dB | 13.1 dB |
| BCH(63,47)-r2/3c6 | 16.0 dB | 13.2 dB | 13.1 dB |

Table 4.18 :  $\bar{\gamma}_b$  required for a bit error rate of  $10^{-3}$  for an  $L$  th order diversity system employing (12,4) MT-FSK with different concatenated codes.

| code              | $L = 1$ | $L = 2$ | $L = 3$ |
|-------------------|---------|---------|---------|
| BCH(63,51)-r3/4c6 | 14.3 dB | 12.2 dB | 12.4 dB |
| BCH(63,47)-r3/4c6 | 14.1 dB | 12.1 dB | 12.7 dB |
| BCH(63,51)-r2/3c6 | 14.2 dB | 12.3 dB | 12.5 dB |
| BCH(63,47)-r2/3c6 | 14.1 dB | 12.3 dB | 12.4 dB |

Table 4.19 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 16FSK with different concatenated codes.

| code             | $L = 1$ | $L = 2$ | $L = 3$ |
|------------------|---------|---------|---------|
| RS(15,11)-r3/4c6 | 15.7 dB | 11.6 dB | 10.6 dB |
| RS(15,9)-r3/4c6  | 14.3 dB | 11.1 dB | 10.4 dB |
| RS(15,11)-r2/3c6 | 14.7 dB | 11.2 dB | 10.4 dB |
| RS(15,9)-r2/3c6  | 13.6 dB | 10.9 dB | 10.3 dB |

Table 4.20 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 8FSK-DBPSK with different concatenated codes.

| code             | $L = 1$ | $L = 2$ | $L = 3$ |
|------------------|---------|---------|---------|
| RS(15,11)-r3/4c6 | 17.0 dB | 12.1 dB | 10.7 dB |
| RS(15,9)-r3/4c6  | 15.5 dB | 11.7 dB | 10.5 dB |
| RS(15,11)-r2/3c6 | 16.0 dB | 11.8 dB | 10.5 dB |
| RS(15,9)-r2/3c6  | 14.9 dB | 11.5 dB | 10.5 dB |

Table 4.21 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 4FSK-DQPSK with different concatenated codes.

| code             | $L = 1$ | $L = 2$ | $L = 3$ |
|------------------|---------|---------|---------|
| RS(15,11)-r3/4c6 | 17.3 dB | 11.9 dB | 10.6 dB |
| RS(15,9)-r3/4c6  | 15.8 dB | 11.5 dB | 10.4 dB |
| RS(15,11)-r2/3c6 | 16.3 dB | 11.6 dB | 10.4 dB |
| RS(15,9)-r2/3c6  | 15.2 dB | 11.3 dB | 10.4 dB |

Table 4.22 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 16FSK-DBPSK with different concatenated codes.

| code             | $L = 1$ | $L = 2$ | $L = 3$ |
|------------------|---------|---------|---------|
| RS(31,23)-r3/4c6 | 14.7 dB | 11.6 dB | 10.4 dB |
| RS(31,21)-r3/4c6 | 14.0 dB | 10.8 dB | 9.8 dB  |
| RS(31,23)-r2/3c6 | 15.0 dB | 11.3 dB | 10.2 dB |
| RS(31,21)-r2/3c6 | 14.3 dB | 10.6 dB | 9.8 dB  |

Table 4.23 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 8FSK-DQPSK with different concatenated codes.

| code             | $L = 1$ | $L = 2$ | $L = 3$ |
|------------------|---------|---------|---------|
| RS(31,23)-r3/4c6 | 17.3 dB | 11.9 dB | 10.6 dB |
| RS(31,21)-r3/4c6 | 15.4 dB | 11.0 dB | 10.0 dB |
| RS(31,23)-r2/3c6 | 16.4 dB | 11.6 dB | 10.4 dB |
| RS(31,21)-r2/3c6 | 14.8 dB | 10.8 dB | 9.9 dB  |

Table 4.24 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$  th order diversity system employing 16FSK-DQPSK with different concatenated codes.

| code             | $L = 1$ | $L = 2$ | $L = 3$ |
|------------------|---------|---------|---------|
| RS(63,51)-r3/4c6 | 16.8 dB | 10.2 dB | 9.1 dB  |
| RS(63,51)-r3/4c6 | 15.9 dB | 9.9 dB  | 8.8 dB  |
| RS(63,49)-r2/3c6 | 15.2 dB | 9.6 dB  | 8.6 dB  |
| RS(63,47)-r2/3c6 | 14.6 dB | 9.3 dB  | 8.4 dB  |

discussed in the previous section. However, the bounds used in the calculations may affect the results. Simulations may be one method of comparing the true performance of the concatenated codes in relation to the BCH codes alone.

## 4.4 Nonbinary Convolutional Codes

In the previous sections, we discussed error control codes which employed hard decision decoding. However, additional coding gain can be achieved by using soft decision decoding techniques. Although soft decision decoding of BCH or RS codes is possible, convolutional codes allow for easy soft decision decoding using the Viterbi algorithm.

Dual- $k$  codes [64] are particularly matched to noncoherently demodulated  $M$ -ary signals ( $k = \log_2 M$ ), and are easily decoded using the Viterbi algorithm [40]. A rate  $1/2^r$  dual- $k$  encoder takes a  $k$ -bit codeword and generates  $2^r$   $k$ -bit codewords. Each of these  $k$ -bit codewords are transmitted using an  $M$ -ary modulation scheme. Since convolutional coding works best for random error channels, interleaving of the symbol stream is needed.

Dual- $k$  codes have a constraint length of 2 (in  $M$ -ary units). Thus there are  $M$  states and  $M$  branches entering and leaving a state. The transfer function of a rate  $1/2^r$  dual- $k$  code is given by [40]:

$$T(D, N, J) = \frac{(2^k - 1)D^{4r}J^2N}{1 - NJ[2D^r + (2^k - 3)D^{2r}]} \quad (4.6)$$

where  $r$  is the number of times each symbol generated by the rate  $1/2$  dual- $k$  code is repeated.

By setting  $J = 1$ , and differentiating the resulting transfer function  $T(D, N)$  with respect to  $N$ , and then setting  $N$  to unity, the following is obtained:

$$\frac{dT(D, N)}{dN} \Big|_{N=1} = \frac{(2^k - 1)D^{4r}}{[1 - 2D^r - (2^k - 3)D^{2r}]^2} \quad (4.7)$$

$$= \sum_{d=4r}^{\infty} w_d D^d \quad (4.8)$$

where  $w_d$  is found by performing the division in eq. (4.7). The performance of the code is union bounded by [40]:

$$P_b < \frac{2^{k-1}}{2^k - 1} \sum_{d=4r}^{\infty} w_d P_2(d) \quad (4.9)$$

where  $P_2(d)$  is the probability that the decoder selects one incorrect path of weight  $d$  from the correct one (differs from correct path by  $d$  symbols). A union bound is obtained by determining the fraction of post-decoding symbol errors each incorrect path of weight  $d$  generates, and summing them together. This is  $w_d$ .

Let us consider an  $M$ -ary FSK signal encoded by rate  $1/2$  dual- $k$  code. The minimum free distance of this code is 4 symbols. Let us derive the probability of selecting a specific path of weight 4 over the correct path. Let us assume that the all-zero codeword has been sent (Assume that symbol 1 corresponds to the all-zero symbol). The metrics used by the decoder are the decision variables corresponding to each possible symbol. We will denote the decision variable for the correct path as  $U_1$  and for the incorrect path as  $U_x$ . The incorrect path will be selected only if  $U_1 - U_x < 0$ . Therefore

$$\begin{aligned} U_1 - U_2 &= |2E_c \alpha_1^{(a)} + N_1^{(a)}|^2 + |2E_c \alpha_1^{(b)} + N_2^{(b)}|^2 + |2E_c \alpha_1^{(c)} + N_1^{(c)}|^2 \\ &+ |2E_c \alpha_1^{(d)} + N_1^{(d)}|^2 - |N_x^{(a)}|^2 - |N_x^{(b)}|^2 - |N_x^{(c)}|^2 \\ &- |N_x^{(d)}|^2 \end{aligned} \quad (4.10)$$

where  $E_c = r_c E_s$  is the energy per code symbol,  $a, b, c, d$  are time indices and  $\alpha_1$  is the fading process encountered by symbol 1. If we assume that interleaving is used, then  $\alpha_1^{(a)}, \alpha_1^{(b)}, \alpha_1^{(c)}, \alpha_1^{(d)}$  are mutually independent. Therefore, we can show that  $P_2(d)$  can be given by eq. (3.87). Therefore

$$P_2(d) = \left(\frac{1-\mu}{2}\right)^d \sum_{i=0}^{d-1} \binom{d-1+i}{i} \left(\frac{1+\mu}{2}\right)^d \quad (4.11)$$

where  $\mu = \frac{r_c \gamma_s}{2+r_c \gamma_s}$ . We can rewrite this equation to be

$$P_2(d) = p^d \sum_{i=0}^{d-1} \binom{d-1+i}{i} (1-p)^i \quad (4.12)$$

where  $p = \frac{1}{2+r_c \gamma_s}$ .

When  $L$ th order diversity is used, the number of terms in eq. (4.10) is increased by a factor of  $L$ . Therefore  $P_2(d)$  becomes  $P_2(dL)$ , where  $\mu = \frac{\frac{r_c}{L} \gamma_s}{2+\frac{r_c}{L} \gamma_s}$  or equivalently  $p = \frac{1}{2+\frac{r_c}{L} \gamma_s}$ .

For MT-FSK, each symbol decision variable is made up of the sum of  $LD$  frequency detector outputs, and therefore, for MT-FSK  $P_2(d)$  becomes  $P_2(dD)$  where  $D$  is the inherent diversity of the modulation scheme and  $L$  is the diversity obtained by using multiple channels. In this case,  $\mu = \frac{\frac{r_c}{wL} \gamma_s}{2+\frac{r_c}{wL} \gamma_s}$  or equivalently,  $p = \frac{1}{2+\frac{r_c}{w} \gamma_s}$  where  $w$  is the number of tones per symbol. The average energy per bit to noise spectral density ratio required for a bit error rate less than  $10^{-3}$  for MFSK and MT-FSK in Rayleigh fading is shown in Table 4.25.

MFSK-DPSK modulation is not very well suited for Viterbi decoding. This is because the phase information must be retrieved from a specific frequency tone before it can be differentially detected. Therefore, a "decision" is made on the phase information. In other words, the detector may send an invalid metric

Table 4.25 :  $\bar{\gamma}_b$  required for  $P_b < 10^{-3}$  for an  $L$ th order diversity system employing MFSK or MT-FSK with dual- $k$  convolutional coding.

| Modulation/<br>code      | $L = 1$ | $L = 2$ | $L = 3$ |
|--------------------------|---------|---------|---------|
| (6,3) MT-FSK/<br>dual-2  | 12.4 dB | 12.0 dB | 12.2 dB |
| (10,4) MT-FSK/<br>dual-2 | 11.6 dB | 11.4 dB | 11.5 dB |
| 8FSK/<br>dual-3          | 11.9 dB | 9.5 dB  | 9.1 dB  |
| (8,3) MT-FSK/<br>dual-3  | 11.2 dB | 10.7 dB | 10.8 dB |
| (12,4) MT-FSK/<br>dual-3 | 10.3 dB | 10.3 dB | 10.4 dB |
| 16FSK/<br>dual-4         | 11.4 dB | 8.8 dB  | 8.3 dB  |

due to its having retrieved phase information from the wrong frequency tone. True Viterbi decoding would require that all phase information is kept until a sequence estimate on the frequency sub-symbols is made. This would require large amounts of memory, and thus is not practical.

One method that was investigated involved encoding the frequency sub-symbol stream using dual- $k$  coding and the phase sub-symbol stream employing another code of the same rate. Since soft decision is not practical on the phase sub-symbol stream, hard decision detection is used for the decoding of the phase sub-symbol stream. However, it can be shown that the bit error rate of this scheme is dominated by errors in the phase sub-symbols, and that the performance of this scheme is poor compared to the other modulation schemes. Therefore, the problem of decoding MFSK-DPSK modulation employing soft decision is left to future work.

## 4.5 Discussion

In this chapter, we have investigated a number of coding schemes to improve the bit error rate performance of the modulation schemes discussed in Chapter 3. Firstly, we considered non-binary BCH and RS codes with hard decision decoding for the different modulation schemes. We then attempted to improve these results by concatenating these non-binary codes with hard decision decoded binary convolutional codes, but in most cases, the values for  $\bar{\gamma}_b$  required to assure a bit error rate below  $10^{-3}$  was higher with concatenation than without. Since bounds are used in the bit error rate performance calculations, this result may not be true. However, this thesis is concerned with determining a level of guaranteed performance. Thus we will not consider the concatenated case any further. Another method of improving performance of BCH codes not discussed in this thesis is erasure decoding of BCH codes. By selecting symbols

in which an error is more likely and erasing it, and then finding the closest codeword in the non-erased symbols, the performance of the code can be improved. However, finding the correct erasure rate is important in improving the performance of the code, and this is left to future work.

Lastly, we investigated the use of a nonbinary class of convolutional codes known as dual- $k$  codes. These codes are easily decoded using the Viterbi algorithm and soft decision detection is possible. These codes guarantee a bit error rate less than  $10^{-3}$  at values of  $\bar{\gamma}_b$  which are lower than with BCH codes. This is due to the soft decision decoding. MFSK-DPSK modulation is not considered with these codes because the MFSK-DPSK detector is required to make a decision about which frequency tones carry the phase information. Therefore the decoder does not receive all possible phase metrics. Thus optimum soft decision decoding is not possible. Note that it is possible to modify the MFSK-DPSK detector to allow this, however, this would require the buffering of all phase information and for each possible path, the differential phase must be determined. Such an approach is impractical in terms of decoder complexity.

We have seen that the different coding schemes improve the power efficiency of the modulation schemes discussed in this thesis. In the next chapter, we will see how this improved power efficiency translates into improved spectral efficiency of a hybrid DS/FH-CDMA system.

## Chapter 5

# Hybrid DS/FH-CDMA System for Rayleigh Fading Channels

### 5.1 Introduction

In this chapter, the proposed hybrid DS/FH-CDMA system employing the different modulation schemes discussed in Chapter 3 is analyzed. The analysis focuses on the system capacity in a mobile communication link. The capacity of a CDMA system employing direct sequence techniques is maximized when all signals sharing the channel have the same received power. Thus, there is a need for power control. This is especially true in the return link, where mobiles transmit information to the base station. Not only does their distance from the base station determine the received power, but so does the fading nature of the channel. Through the use of power control, the transmitter can roughly determine the instantaneous gain of the channel. With this information, the transmitter can boost or lower the transmitted signal power so that the base station receives a signal of quasi-constant power.

In the forward link, all signals originate at the base station, and thus all signals

traverse the same channel in transit to a receiving mobile. Thus power control is not needed in this link.

In this chapter, the capacity of the hybrid DS/FH-CDMA system will be found for systems operating in the forward link. This is because all signals (desired and interfering) experience the same fading process. Thus the analysis is simpler than for the return link. However, the results found in this chapter can be applied to the return link with the assumption of perfect power control.

## 5.2 Hybrid DS/FH-CDMA System Employing MFSK

The proposed hybrid DS/FH-CDMA system in the forward link (base to mobile) is pictured in Figure 5.1. In the reverse link, the figure remains the same with the exception that each additional user's signal is passed through an independent fading channel and they are summed at the receiver.

The data bits from the source, at a bit rate of  $R_b$  bits/sec, are input into a forward error correction encoder and interleaved to protect against burst errors. If error correction is not employed, these blocks are omitted from Figure 5.1. The coded bit rate is now  $R_b/r_c$ , where  $r_c$  is the code rate.

The coded bit stream is then MFSK modulated. The symbol rate is  $R_s = R_b/(r_c \log_2 M) = R_b/(r_c k)$ . The frequency separation between adjacent symbols is given by  $\Delta f$ . The MFSK modulated data is then spread by a PSK modulated pseudo-noise (PN) sequence of rate  $R_p$  chips/sec. Generally  $R_p \gg R_s$ . The bandwidth of the signal at this point is  $W_{ds} = 2R_p + (M - 1)\Delta f$  [18].  $W_{ds}$  refers to the direct sequence spread bandwidth (the bandwidth prior to frequency hopping). This bandwidth is chosen to be smaller than the coherence bandwidth,  $(\Delta f)_c$ , of the channel.

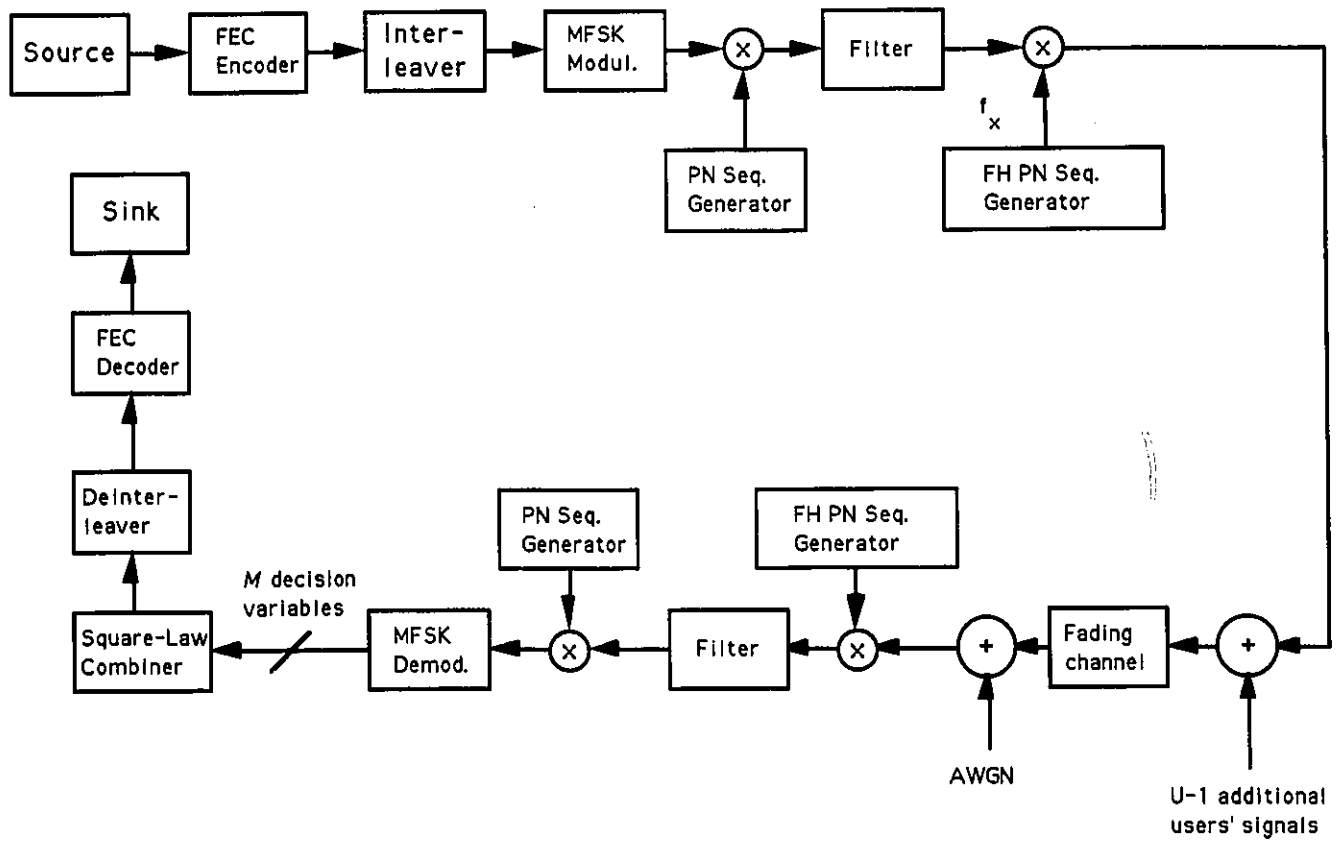


Figure 5.1 : Hybrid DS/FH-CDMA system in forward link.

The signal is then filtered (so that it will not interfere much with adjacent hop bins). The signal is then frequency-hopped at a rate  $R_h = LR_s$ . Because the filter generally has a rolloff factor  $\alpha_{ro}$ , the minimum required frequency separation between adjacent frequency slots, assuming  $R_p \gg R_h$ , is about  $W_{ds}(1 + \alpha_{ro})$ . This frequency separation should be larger than  $(\Delta f)_c$ . Thus the overall bandwidth needed is  $W_{ss} = N_f W_{ds}(1 + \alpha_{ro})$ . This is shown by means of a time/frequency diagram in Figure 5.2.

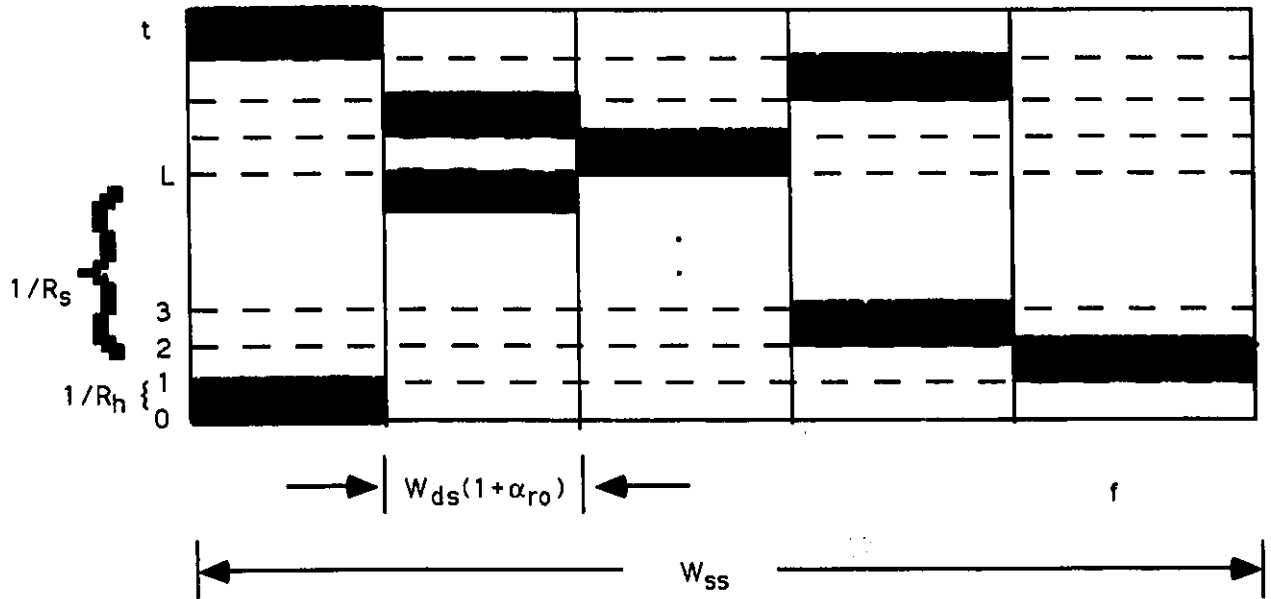


Figure 5.2 : DS/FH-CDMA signal in time and frequency.

At the receiver the signal is dehopped and despread. Each hop symbol is MFSK demodulated. Since the hop rate is  $R_h = LR_s$ , the minimum frequency separation between adjacent symbols to assure orthogonality for all symbols is  $\Delta f = LR_s$ . The decision variables are then square-law combined and a soft decision is performed. Finally the data stream is deinterleaved and FEC decoded.

One advantage of this system over conventional spread spectrum systems is that

the spreading is achieved in two steps, thus the length of both PN sequences employed (DS and FH) are shorter than the one PN sequence employed by either a FH or DS system. This can greatly help in synchronization times. Another advantage is that the complexity of the power control algorithm in the reverse link can be relaxed. A user's signal which is received at a higher power than other users' signals interferes on average with each other signal  $1/N_f$  of the symbol duration. Thus its effect on the system performance is reduced as compared to a DS-CDMA system, where the strong signal will continually interfere with the weaker ones.

### 5.3 Spectral Efficiency of Hybrid DS/FH-CDMA System Employing MFSK

The effective noise spectral density (noise plus MAI) of a MFSK/DS-CDMA system in an AWGN channel is given in [18, 44] as:

$$N'_o = N_o + \frac{2(U-1)E_b R_b R_p}{M^2} \sum_{i=1}^M \sum_{j=1}^M \frac{1}{4R_p^2 + (\pi\Delta f)^2(i-j)^2} \quad (5.1)$$

For a MFSK DS/FH-CDMA system, equation (5.1) must be modified to include the effect of fading which all signals encounter in the forward link, and to include the fact that, on average, only  $1/N_f$  of the additional users are present in a frequency slot at a given time. Then it becomes

$$N'_o = N_o + \frac{2\frac{(U-1)}{N_f}\alpha^2 E_b R_b R_p}{M^2} \sum_{i=1}^M \sum_{j=1}^M \frac{1}{4R_p^2 + (\pi\Delta f)^2(i-j)^2} \quad (5.2)$$

The average effective spectral noise density becomes:

$$\begin{aligned}
E[N_o'] &= N_o[1 + (U - 1)E[\alpha^2] \frac{E_b}{N_o} \frac{\beta(M, R_b, R_p, \Delta f)}{N_f}] \\
&= N_o[1 + (U - 1)\bar{\gamma}_b \frac{\beta(M, R_b, R_p, \Delta f)}{N_f}]
\end{aligned} \tag{5.3}$$

where

$$\beta(M, R_b, R_p, \Delta f) = \frac{2R_b R_p}{M^2} \sum_{i=1}^M \sum_{j=1}^M \frac{1}{4R_p^2 + (\pi\Delta f)^2(i-j)^2} \tag{5.4}$$

The effective energy per bit to noise spectral density ratio,  $\bar{\gamma}_b'$ , is given by:

$$\bar{\gamma}_b' = \frac{\bar{\gamma}_b}{1 + (U - 1) \frac{\beta(M, R_b, R_p, \Delta f)}{N_f} \bar{\gamma}_b} \tag{5.5}$$

Rewriting this equation in terms of the number of simultaneous users, we obtain:

$$U = \frac{\bar{\gamma}_b - \bar{\gamma}_b'}{\bar{\gamma}_b \bar{\gamma}_b' \frac{\beta(M, R_b, R_p, \Delta f)}{N_f}} + 1 \tag{5.6}$$

The spectral efficiency of this system is given by:

$$\eta = \frac{UR_b}{W_{ss}} \tag{5.7}$$

where  $W_{ss} = N_f[2R_p + (M - 1)\Delta f](1 + \alpha_{ro})$ . Since  $W_{ss} \gg R_b$ , and if we assume that  $\bar{\gamma}_b \rightarrow \infty$ , the spectral efficiency can be expressed as follows:

$$\eta = \frac{R_b}{\bar{\gamma}_b' \frac{\beta(M, R_b, R_p, \Delta f)}{N_f} W_{ss}} \tag{5.8}$$

By replacing  $W_{ss}$  by  $N_f W_{ds}(1 + \alpha_{ro})$ , we can see that the spectral efficiency does not depend on  $N_f$ , since the  $1/N_f$  term cancels with  $N_f$  in  $W_{ss}$ . The equation can be rewritten as:

$$\eta = \frac{R_b}{\bar{\gamma}_b' \beta(M, R_b, R_p, \Delta f) W_{ds}(1 + \alpha_{ro})} \quad (5.9)$$

### 5.3.1 Spectral Efficiency of DS/FH-CDMA Employing MFSK Without Coding

In a typical land mobile channel, the delay spread of the multipath is about 5  $\mu$ seconds, and the coherence bandwidth is about 200 kHz [40]. Thus one could spread the MFSK signal to a bandwidth  $W_{ds} < 200$  kHz, and hop the spread signal into frequency slots that are separated by 200 kHz or more. In this example, we will assume that  $W_{ds} = 195$  kHz, and that the frequency slots are separated by 205 kHz. Thus  $\alpha_{ro} \approx 0.05$ , and the overall bandwidth is  $W_{ss} = N_f \times 205$  kHz.

A typical voice coder operates at bit rates between 4.8 and 10 kbps. We will investigate the performance of this system for  $R_b = 4.8$  and 9.6 kbps. We can now obtain  $\Delta f$ ,  $R_p$ , and  $\beta(M, R_b, R_p, \Delta f)$ . This is shown in Table 5.1 for  $R_b = 4.8$  kbps and Table 4.2 for  $R_b = 9.6$  kbps.

By obtaining  $\bar{\gamma}_b'$  for a worst case bit error rate of  $10^{-3}$  from Table 3.1, the spectral efficiency of the system can be calculated. This is shown in Table 5.3.

From Table 5.3, one can plainly see that the spectral efficiencies of the uncoded hybrid DS/FH-CDMA system is quite low, and therefore coding should be employed to increase the channel capacity.

Table 5.1 :  $\Delta f$ ,  $R_p$  and  $\beta(M, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK without coding ( $R_b = 4.8$  kbps,  $W_{ds} = 195$  kHz).

| Modulation | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M, R_b, R_p, \Delta f)$ |
|------------|---|-------------------------|------------------------|--------------------------|--------------------------------|
| 8FSK       | 1 | -                       | 1.6                    | 91.2                     | 0.0261                         |
|            | 2 | 3.2                     | 3.2                    | 83.2                     | 0.0278                         |
|            | 3 | 4.8                     | 4.8                    | 76.8                     | 0.0287                         |
| 16FSK      | 1 | -                       | 1.2                    | 87.6                     | 0.0269                         |
|            | 2 | 2.4                     | 2.4                    | 79.2                     | 0.0279                         |
|            | 3 | 3.6                     | 3.6                    | 68.4                     | 0.0287                         |

Table 5.2 :  $\Delta f$ ,  $R_p$  and  $\beta(M, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK without coding ( $R_b = 9.6$  kbps,  $W_{ds} = 195$  kHz).

| Modulation | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M, R_b, R_p, \Delta f)$ |
|------------|---|-------------------------|------------------------|--------------------------|--------------------------------|
| 8FSK       | 1 | -                       | 3.2                    | 83.2                     | 0.0557                         |
|            | 2 | 6.4                     | 6.4                    | 70.4                     | 0.0581                         |
|            | 3 | 9.6                     | 9.6                    | 57.6                     | 0.0577                         |
| 16FSK      | 1 | -                       | 2.4                    | 79.2                     | 0.0558                         |
|            | 2 | 4.8                     | 4.8                    | 57.6                     | 0.0576                         |
|            | 3 | 7.2                     | 7.2                    | 43.2                     | 0.0525                         |

Table 5.3 : Spectral efficiency (in bps/Hz) of the uncoded hybrid DS/FH-CDMA system employing uncoded MFSK in Rayleigh fading.

|       | $R_b = 4.8$ kbps |         |         | $R_b = 9.6$ kbps |         |         |
|-------|------------------|---------|---------|------------------|---------|---------|
|       | $L = 1$          | $L = 2$ | $L = 3$ | $L = 1$          | $L = 2$ | $L = 3$ |
| 8FSK  | 0.00179          | 0.0160  | 0.0317  | 0.00167          | 0.0154  | 0.0316  |
| 16FSK | 0.00199          | 0.0179  | 0.0364  | 0.00192          | 0.0173  | 0.0398  |

### 5.3.2 Spectral Efficiency of DS/FH-CDMA Employing MFSK With Coding

When forward error correction codes are used,  $\Delta f = LR_b/\tau_c k$ , and thus  $R_c$  must be decreased so that  $W_{ds}$  remains unchanged. For DS-CDMA systems, it has been shown that forward error correction does not alter the processing gain of the system [33], however in this system, the processing gain will be slightly changed due to coding since the double summation in  $\beta(M, R_b, R_p, \Delta f)$  depends on a nonlinear combination of  $R_p$  and  $\Delta f$ .

In this thesis, we have examined many different error control codes, however, we will consider only one BCH code and the dual- $k$  code for each modulation scheme. For 8FSK, we will examine the nonbinary BCH(63,39) code and the rate 1/2 dual-3 code. For 16FSK, we will consider the BCH(255,199) code and the rate 1/2 dual-4 code. Their selection does not imply that these codes are optimal ones for their respective modulation schemes.

The values for  $\Delta f$ ,  $R_p$ , and  $\beta(M, R_b, R_p, \Delta f)$  are given in Table 5.4 for  $R_b = 4.8$  kbps and in Table 5.5 for  $R_b = 9.6$  kbps for BCH coding. These parameters are given in Table 5.6 for  $R_b = 4.8$  kbps and Table 5.7 for  $R_b = 9.6$  kbps for rate 1/2 dual- $k$  coded FSK. When no value is given for  $R_p$  or  $\beta(M, R_b, R_p, \Delta f)$ , then  $R_p$  is not significantly greater than  $R_b$  to allow this scheme to function properly (since  $R_p$  must be an integer multiple of  $R_b$ ).

The spectral efficiency for the hybrid DS/FH-CDMA system employing MFSK with BCH coding is given in Table 5.8 and in Table 5.9 for rate 1/2 dual- $k$  coded MFSK.

We can see by comparing Tables 5.8 and 5.9 to Table 5.3 that coding improves the spectral efficiency of the system significantly. We also see that the dual- $k$  codes, by virtue of the soft decision decoder, provides larger spectral efficiencies than the hard decision decoded BCH codes.

Table 5.4 :  $\Delta f$ ,  $R_p$  and  $\beta(M, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK and BCH coding ( $R_b = 4.8$  kbps,  $W_{ds} = 195$  kHz).

| Modulation                    | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M, R_b, R_p, \Delta f)$ |
|-------------------------------|---|-------------------------|------------------------|--------------------------|--------------------------------|
| 8FSK<br>with<br>BCH(63,39)    | 1 | -                       | 2.585                  | 87.87                    | 0.0267                         |
|                               | 2 | 5.170                   | 5.170                  | 77.55                    | 0.0281                         |
|                               | 3 | 7.755                   | 7.755                  | 67.80                    | 0.0277                         |
| 16FSK<br>with<br>BCH(255,199) | 1 | -                       | 1.54                   | 84.57                    | 0.0275                         |
|                               | 2 | 3.08                    | 3.08                   | 73.8                     | 0.0283                         |
|                               | 3 | 4.61                    | 4.61                   | 59.97                    | 0.0286                         |

Table 5.5 :  $\Delta f$ ,  $R_p$  and  $\beta(M, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK and BCH coding ( $R_b = 9.6$  kbps,  $W_{ds} = 195$  kHz).

| Modulation                    | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M, R_b, R_p, \Delta f)$ |
|-------------------------------|---|-------------------------|------------------------|--------------------------|--------------------------------|
| 8FSK<br>with<br>BCH(63,39)    | 1 | -                       | 5.17                   | 77.55                    | 0.0505                         |
|                               | 2 | 10.54                   | 10.54                  | 51.70                    | 0.0541                         |
|                               | 3 | 15.51                   | 15.51                  | 31.02                    | 0.0470                         |
| 16FSK<br>with<br>BCH(255,199) | 1 | -                       | 3.08                   | 73.80                    | 0.0565                         |
|                               | 2 | 6.15                    | 6.15                   | 49.21                    | 0.0548                         |
|                               | 3 | 9.23                    | 9.23                   | 27.80                    | 0.0496                         |

Table 5.6 :  $\Delta f$ ,  $R_p$  and  $\beta(M, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK and rate 1/2 dual-k coding ( $R_b = 4.8$  kbps,  $W_{ds} = 195$  kHz).

| Modulation | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M, R_b, R_p, \Delta f)$ |
|------------|---|-------------------------|------------------------|--------------------------|--------------------------------|
| 8FSK       | 1 | -                       | 3.2                    | 83.2                     | 0.0278                         |
|            | 2 | 6.4                     | 6.4                    | 70.4                     | 0.0291                         |
|            | 3 | 9.6                     | 9.6                    | 57.6                     | 0.0289                         |
| 16FSK      | 1 | -                       | 2.4                    | 79.2                     | 0.0279                         |
|            | 2 | 4.8                     | 4.8                    | 57.6                     | 0.0285                         |
|            | 3 | 7.2                     | 7.2                    | 43.2                     | 0.0262                         |

Table 5.7 :  $\Delta f$ ,  $R_p$  and  $\beta(M, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK and rate 1/2 dual-k coding ( $R_b = 9.6$  kbps,  $W_{ds} = 195$  kHz).

| Modulation | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M, R_b, R_p, \Delta f)$ |
|------------|---|-------------------------|------------------------|--------------------------|--------------------------------|
| 8FSK       | 1 | -                       | 6.4                    | 83.2                     | 0.0581                         |
|            | 2 | 12.8                    | 12.8                   | 51.2                     | 0.0529                         |
|            | 3 | 19.2                    | 19.2                   | 19.2                     | -                              |
| 16FSK      | 1 | -                       | 4.8                    | 57.6                     | 0.0576                         |
|            | 2 | 9.6                     | 9.6                    | 19.2                     | 0.0515                         |
|            | 3 | 14.4                    | 14.4                   | -                        | -                              |

Table 5.8 : Spectral efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing MFSK and non-binary BCH coding.

|                    | $R_b = 4.8$ kbps |         |         | $R_b = 9.6$ kbps |         |         |
|--------------------|------------------|---------|---------|------------------|---------|---------|
|                    | $L = 1$          | $L = 2$ | $L = 3$ | $L = 1$          | $L = 2$ | $L = 3$ |
| 8FSK-BCH(63,39)    | 0.0236           | 0.0539  | 0.0721  | 0.0250           | 0.0512  | 0.0848  |
| 16FSK-BCH(255,199) | 0.0264           | 0.0723  | 0.0920  | 0.0256           | 0.0743  | 0.106   |

Table 5.9 : Spectral efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing MFSK and rate 1/2 dual- $k$  convolutional coding.

|       | $R_b = 4.8$ kbps |         |         | $R_b = 9.6$ kbps |         |         |
|-------|------------------|---------|---------|------------------|---------|---------|
|       | $L = 1$          | $L = 2$ | $L = 3$ | $L = 1$          | $L = 2$ | $L = 3$ |
| 8FSK  | 0.0544           | 0.0903  | 0.0997  | 0.0521           | 0.0993  | —       |
| 16FSK | 0.0608           | 0.108   | 0.132   | 0.0589           | 0.120   | —       |

## 5.4 Hybrid DS/FH-CDMA System Employing JPFM

The hybrid DS/FH-CDMA system employing JPFM is identical to the system discussed in Section 5.2 with the exception that the modulation scheme has changed, and we must place a constraint on the hopping sequence. Here, MFSK-DPSK will be used in an effort to improve the spectral efficiency of the system. Because this modulation scheme employs differential phase detection, the fading encountered by hop  $x$  of symbol  $i$  must not be independent of the fading encountered by hop  $x$  of symbol  $i + 1$ . This means that  $x$ th hop of each symbol must be sent to the same frequency bin. This is demonstrated in Figure 5.3.

The idea behind this system is to either decrease the number of frequency tones while maintaining the same symbol rate or by increasing the number of bits/symbol (and thus decreasing the symbol rate and by extension reducing  $\Delta f$ ) by including phase information while maintaining the same number of frequency tones. In each case, we can increase  $R_p$  compared to the DS/FH-CDMA system employing MFSK without increasing  $W_{ds}$ .

When employing MFSK-DPSK, the symbol rate is  $R_s = R_b/r_c(k_f + k_p)$ , where  $r_c$  is the code rate,  $k_f$  is the number of bits represented by frequency, and  $k_p$

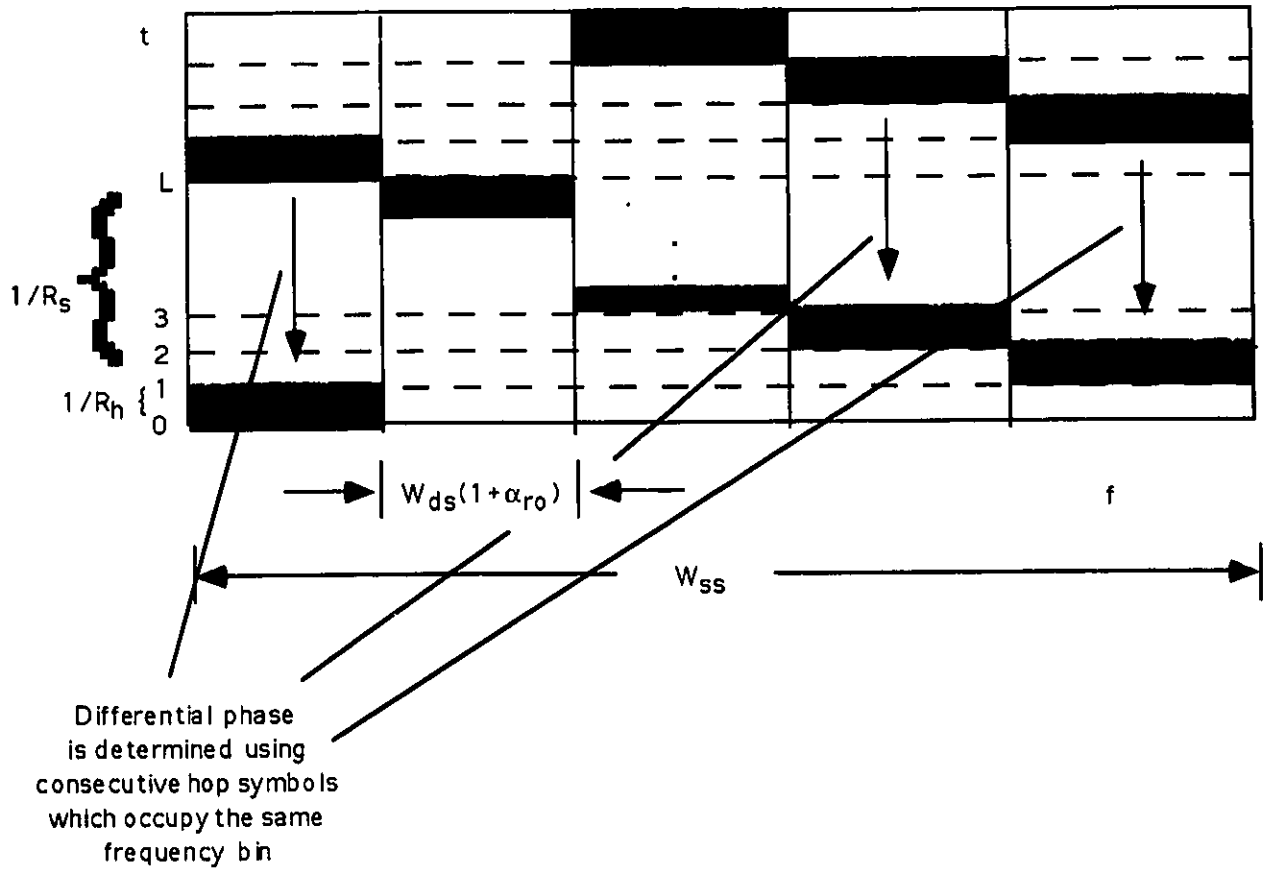


Figure 5.3 : DS/FH-CDMA signal in time and frequency when MFSK-DPSK is selected modulation scheme.

is the number of bits represented by phase. Also,  $\Delta f = L \times R_s$ . The direct sequence signal bandwidth (pre-hopped bandwidth) is  $W_{ds} = 2R_p + (M_f - 1)\Delta f$ , where  $M_f = 2^{k_f}$  is the number of frequency tones employed.

#### 5.4.1 Spectral Efficiency of Hybrid DS/FH-CDMA Employing MFSK-DPSK

The equations derived for the MAI in Section 5.3 are valid for this system except that we must replace  $M$  by  $M_f$ . Therefore

$$\beta(M_f, R_b, R_p, \Delta f) = \frac{2R_b R_p}{M_f^2} \sum_{i=1}^{M_f} \sum_{j=1}^{M_f} \frac{1}{4R_p^2 + (\pi\Delta f)^2(i-j)^2} \quad (5.10)$$

and

$$\eta = \frac{R_b}{\bar{\gamma}_b' \beta(M_f, R_b, R_p, \Delta f) W_{ds} (1 + \alpha_{ro})} \quad (5.11)$$

#### 5.4.2 Spectral Efficiency of DS/FH-CDMA Employing MFSK-DPSK Without Coding

Once again we assume that  $W_{ds} = 195$  kHz and  $\alpha_{ro} = 0.05$ . The overall bandwidth is  $W_{ss} = 195(1.05) \times N_f$  kHz =  $N_f \times 205$  kHz.

For bit rates of 4.8 and 9.6 kbps, we can find  $R_p$  and  $\beta(M_f, R_b, R_p, \Delta f)$ . These values are shown in Table 5.10 for  $R_b = 4.8$  kbps and Table 5.11 for  $R_b = 9.6$  kbps.

Using these values for  $\beta(M_f, R_b, R_p, \Delta f)$ , we can find the spectral efficiency of DS/FH-CDMA employing uncoded MFSK-DPSK. This is shown in Table 5.12.

Table 5.10 :  $\Delta f$ ,  $R_p$  and  $\beta(M_f, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK-DPSK without coding ( $R_b = 4.8$  kbps,  $W_{ds} = 195$  kHz).

| Modulation  | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M_f, R_b, R_p, \Delta f)$ |
|-------------|---|-------------------------|------------------------|--------------------------|----------------------------------|
| 4FSK-DBPSK  | 1 | -                       | 1.6                    | 94.4                     | 0.0254                           |
|             | 2 | 3.2                     | 3.2                    | 89.6                     | 0.0266                           |
|             | 3 | 4.8                     | 4.8                    | 86.4                     | 0.0273                           |
| 8FSK-DBPSK  | 1 | -                       | 1.2                    | 92.4                     | 0.0259                           |
|             | 2 | 2.4                     | 2.4                    | 88.8                     | 0.0265                           |
|             | 3 | 3.6                     | 3.6                    | 82.8                     | 0.0277                           |
| 16FSK-DBPSK | 1 | -                       | 0.96                   | 90.24                    | 0.0263                           |
|             | 2 | 1.92                    | 1.92                   | 82.56                    | 0.0276                           |
|             | 3 | 2.88                    | 2.88                   | 74.88                    | 0.0283                           |
| 4FSK-DQPSK  | 1 | -                       | 1.2                    | 94.8                     | 0.0253                           |
|             | 2 | 2.4                     | 2.4                    | 93.6                     | 0.0255                           |
|             | 3 | 3.6                     | 3.6                    | 90.0                     | 0.0264                           |
| 8FSK-DQPSK  | 1 | -                       | 0.96                   | 94.08                    | 0.0254                           |
|             | 2 | 1.92                    | 1.92                   | 90.24                    | 0.0263                           |
|             | 3 | 2.88                    | 2.88                   | 86.4                     | 0.0270                           |
| 16FSK-DQPSK | 1 | -                       | 0.8                    | 91.2                     | 0.0261                           |
|             | 2 | 1.6                     | 1.6                    | 84.8                     | 0.0273                           |
|             | 3 | 2.4                     | 2.4                    | 79.2                     | 0.0279                           |

Table 5.11 :  $\Delta f$ ,  $R_p$  and  $\beta(M_f, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK-DPSK without coding ( $R_b = 9.6$  kbps,  $W_{ds} = 195$  kHz).

| Modulation  | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M_f, R_b, R_p, \Delta f)$ |
|-------------|---|-------------------------|------------------------|--------------------------|----------------------------------|
| 4FSK-DBPSK  | 1 | -                       | 3.2                    | 89.6                     | 0.0532                           |
|             | 2 | 6.4                     | 6.4                    | 83.2                     | 0.0557                           |
|             | 3 | 9.6                     | 9.6                    | 76.8                     | 0.0575                           |
| 8FSK-DBPSK  | 1 | -                       | 2.4                    | 88.8                     | 0.0531                           |
|             | 2 | 4.8                     | 4.8                    | 76.8                     | 0.0573                           |
|             | 3 | 7.2                     | 7.2                    | 72.8                     | 0.0550                           |
| 16FSK-DBPSK | 1 | -                       | 1.92                   | 82.56                    | 0.0552                           |
|             | 2 | 3.84                    | 3.84                   | 65.28                    | 0.0581                           |
|             | 3 | 5.76                    | 5.76                   | 51.84                    | 0.0556                           |
| 4FSK-DQPSK  | 1 | -                       | 2.4                    | 93.6                     | 0.0511                           |
|             | 2 | 4.8                     | 4.8                    | 86.4                     | 0.0545                           |
|             | 3 | 7.2                     | 7.2                    | 86.4                     | 0.0534                           |
| 8FSK-DQPSK  | 1 | -                       | 1.92                   | 90.24                    | 0.0526                           |
|             | 2 | 3.84                    | 3.84                   | 80.64                    | 0.0564                           |
|             | 3 | 5.76                    | 5.76                   | 74.88                    | 0.0567                           |
| 16FSK-DQPSK | 1 | -                       | 1.6                    | 84.8                     | 0.0547                           |
|             | 2 | 3.2                     | 3.2                    | 70.4                     | 0.0581                           |
|             | 3 | 4.8                     | 4.8                    | 57.6                     | 0.0576                           |

Table 5.12 : Spectral efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing uncoded MFSK-DPSK.

|             | $R_b = 4.8$ kbps |         |         | $R_b = 9.6$ kbps |         |         |
|-------------|------------------|---------|---------|------------------|---------|---------|
|             | $L = 1$          | $L = 2$ | $L = 3$ | $L = 1$          | $L = 2$ | $L = 3$ |
| 4FSK-DBPSK  | 0.00168          | 0.0172  | 0.0341  | 0.00160          | 0.0164  | 0.0324  |
| 8FSK-DBPSK  | 0.00180          | 0.0198  | 0.0414  | 0.00176          | 0.0183  | 0.0417  |
| 16FSK-DBPSK | 0.00195          | 0.0224  | 0.0455  | 0.00186          | 0.0202  | 0.0463  |
| 4FSK-DQPSK  | 0.00172          | 0.0215  | 0.0445  | 0.00171          | 0.0201  | 0.0440  |
| 8FSK-DQPSK  | 0.00193          | 0.0234  | 0.0488  | 0.00186          | 0.0218  | 0.0464  |
| 16FSK-DQPSK | 0.00196          | 0.0242  | 0.0506  | 0.00187          | 0.0226  | 0.0489  |

We can infer from Table 5.12 that increasing the number of bits represented by the phase from 1 to 2 has a significant effect on the spectral efficiency of the system when no error control coding is used. However, the results here indicate that coding must be used in order to have a respectable spectral efficiency.

### 5.4.3 Spectral Efficiency of DS/FH-CDMA Employing MFSK-DPSK and Error Control Coding

Once again the use of coding increases the symbol rate, and therefore increases  $\Delta f$ . Because of this we must again determine  $R_p$  to give  $W_{ds} = 195$  kHz. With these new values of  $R_p$ , we can find  $\beta(M_f, R_b, R_p, \Delta f)$ . This is shown in Table 5.13 for  $R_b = 4.8$  kbps and in Table 5.14 for  $R_b = 9.6$  kbps.

With the values of  $\beta(M_f, R_b, R_p, \Delta f)$  from Tables 5.13 and 5.14, we can find the spectral efficiency of DS/FH-CDMA employing MFSK-DPSK and the two error control codes considered. The spectral efficiency of the system employing BCH coding is shown in Table 5.15.

Table 5.13:  $\Delta f$ ,  $R_p$  and  $\beta(M_f, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK-DPSK with BCH coding ( $R_b = 4.8$  kbps,  $W_{ds} = 195$  kHz).

| Modulation                         | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M_f, R_b, R_p, \Delta f)$ |
|------------------------------------|---|-------------------------|------------------------|--------------------------|----------------------------------|
| 4FSK-DBPSK<br>with<br>BCH(63,39)   | 1 | -                       | 2.59                   | 93.05                    | 0.0256                           |
|                                    | 2 | 5.17                    | 5.17                   | 87.89                    | 0.0267                           |
|                                    | 3 | 7.75                    | 7.75                   | 85.29                    | 0.0269                           |
| 8FSK-DBPSK<br>with<br>BCH(255,199) | 1 | -                       | 1.54                   | 90.72                    | 0.0263                           |
|                                    | 2 | 3.08                    | 3.08                   | 86.11                    | 0.0270                           |
|                                    | 3 | 4.61                    | 4.61                   | 78.92                    | 0.0283                           |
| 16FSK-DBPSK<br>with<br>RS(31,19)   | 1 | -                       | 1.57                   | 84.58                    | 0.0274                           |
|                                    | 2 | 3.13                    | 3.13                   | 72.05                    | 0.0287                           |
|                                    | 3 | 4.70                    | 4.70                   | 61.09                    | 0.0281                           |
| 4FSK-DQPSK<br>with<br>BCH(255,199) | 1 | -                       | 1.54                   | 93.80                    | 0.0256                           |
|                                    | 2 | 3.08                    | 3.08                   | 92.26                    | 0.0259                           |
|                                    | 3 | 4.61                    | 4.61                   | 87.65                    | 0.0273                           |
| 8FSK-DQPSK<br>with<br>RS(31,19)    | 1 | -                       | 1.57                   | 90.85                    | 0.0262                           |
|                                    | 2 | 3.13                    | 3.13                   | 84.58                    | 0.0274                           |
|                                    | 3 | 4.70                    | 4.70                   | 79.88                    | 0.0278                           |
| 16FSK-DQPSK<br>with<br>RS(63,47)   | 1 | -                       | 1.07                   | 89.00                    | 0.0266                           |
|                                    | 2 | 2.14                    | 2.14                   | 79.35                    | 0.0282                           |
|                                    | 3 | 3.22                    | 3.22                   | 70.77                    | 0.0289                           |

Table 5.14 :  $\Delta f$ ,  $R_p$  and  $\beta(M_f, R_b, R_p, \Delta f)$  for hybrid DS/FH-CDMA employing MFSK-DPSK with BCH coding ( $R_b = 9.6$  kbps,  $W_{ds} = 195$  kHz).

| Modulation                         | L | $R_h$<br>(in khops/sec) | $\Delta f$<br>(in kHz) | $R_p$<br>(in kchips/sec) | $\beta(M_f, R_b, R_p, \Delta f)$ |
|------------------------------------|---|-------------------------|------------------------|--------------------------|----------------------------------|
| 4FSK-DBPSK<br>with<br>BCH(63,39)   | 1 | -                       | 5.17                   | 87.88                    | 0.0535                           |
|                                    | 2 | 10.34                   | 10.34                  | 72.57                    | 0.0598                           |
|                                    | 3 | 15.51                   | 15.51                  | 62.03                    | 0.0605                           |
| 8FSK-DBPSK<br>with<br>BCH(255,199) | 1 | -                       | 3.08                   | 86.11                    | 0.0540                           |
|                                    | 2 | 6.15                    | 6.15                   | 73.81                    | 0.0566                           |
|                                    | 3 | 9.23                    | 9.23                   | 64.58                    | 0.0549                           |
| 16FSK-DBPSK<br>with<br>RS(31,19)   | 1 | -                       | 3.13                   | 72.05                    | 0.0573                           |
|                                    | 2 | 6.27                    | 6.27                   | 50.12                    | 0.0539                           |
|                                    | 3 | 9.40                    | 9.40                   | 18.80                    | 0.0526                           |
| 4FSK-DQPSK<br>with<br>BCH(255,199) | 1 | -                       | 3.08                   | 92.20                    | 0.0517                           |
|                                    | 2 | 6.15                    | 6.15                   | 86.11                    | 0.0542                           |
|                                    | 3 | 9.23                    | 9.23                   | 83.04                    | 0.0540                           |
| 8FSK-DQPSK<br>with<br>RS(31,19)    | 1 | -                       | 3.13                   | 84.58                    | 0.0549                           |
|                                    | 2 | 6.26                    | 6.26                   | 75.18                    | 0.0556                           |
|                                    | 3 | 9.40                    | 9.40                   | 56.34                    | 0.0590                           |
| 16FSK-DQPSK<br>with<br>RS(63,47)   | 1 | -                       | 2.14                   | 79.35                    | 0.0565                           |
|                                    | 2 | 4.29                    | 4.29                   | 69.24                    | 0.0565                           |
|                                    | 3 | 6.43                    | 6.43                   | 45.04                    | 0.0554                           |

Table 5.15 : Spectral efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing nonbinary BCH encoded MFSK-DPSK.

|                             | $R_b = 4.8$ kbps |         |         | $R_b = 9.6$ kbps |         |         |
|-----------------------------|------------------|---------|---------|------------------|---------|---------|
|                             | $L = 1$          | $L = 2$ | $L = 3$ | $L = 1$          | $L = 2$ | $L = 3$ |
| 4FSK-DBPSK/<br>BCH(63,39)   | 0.0200           | 0.0579  | 0.0743  | 0.0191           | 0.0530  | 0.0659  |
| 8FSK-DBPSK/<br>BCH(255,199) | 0.0204           | 0.0674  | 0.0908  | 0.0199           | 0.0643  | 0.0936  |
| 16FSK-DBPSK/<br>RS(31,19)   | 0.0349           | 0.0960  | 0.110   | 0.0333           | 0.102   | 0.118   |
| 4FSK-DQPSK/<br>BCH(255,199) | 0.0201           | 0.0718  | 0.0970  | 0.0199           | 0.0696  | 0.0970  |
| 8FSK-DQPSK/<br>RS(31,19)    | 0.0348           | 0.0960  | 0.106   | 0.0332           | 0.0946  | 0.100   |
| 16FSK-DQPSK/<br>RS(63,47)   | 0.0306           | 0.0777  | 0.118   | 0.0287           | 0.0974  | 0.112   |

Comparing these results to those obtained in the previous section indicate that significantly greater spectral efficiencies can be obtained by employing MFSK-DPSK as opposed to conventional MFSK employing the same type of coding. However, the complexity concerns that arise when attempting to employ a Viterbi decoder with this modulation scheme prevents us from employing the dual- $k$  code which provides higher spectral efficiencies.

## 5.5 Hybrid DS/FH-CDMA Employing MT-FSK

The hybrid DS/FH-CDMA system discussed in this paper is shown in Figure 5.4. Wideband MT-FSK is the employed modulation scheme. The constraint is that the minimum frequency tone spacing be greater than the channel coherence bandwidth ( $\Delta f > (\Delta f)_c$ ).

The output of the MT-FSK modulator is denoted by point (a) in Figure 5.2. The signal is then multiplied by a pseudonoise (PN) sequence which has a rate of  $R_p$  chips/sec. By choosing the chip rate to be a multiple of  $R_s$ , the null-to-null bandwidth of one “spread” frequency tone is  $W_{ss} = 2R_p$ . We can select a value for  $R_p$  such that this bandwidth is less than the coherence bandwidth of the channel ( $2R_p < (\Delta f)_c$ ). We have the DS spread signal at point (b) in Figure 5.4.

The signal is then hopped at a rate  $R_h$ . We wish to hop the signal  $L$  times per symbol to obtain additional diversity. Therefore  $R_h = LR_s$ , and  $R_h < R_p$ . This signal is transmitted over the channel at point (c) in Figure 5.4 along with signals from other users. Thus, assuming a large number of users, the entire available spectrum will contain components from different users.

At the receiver, the received signal is dehopped, then despread. Assuming that

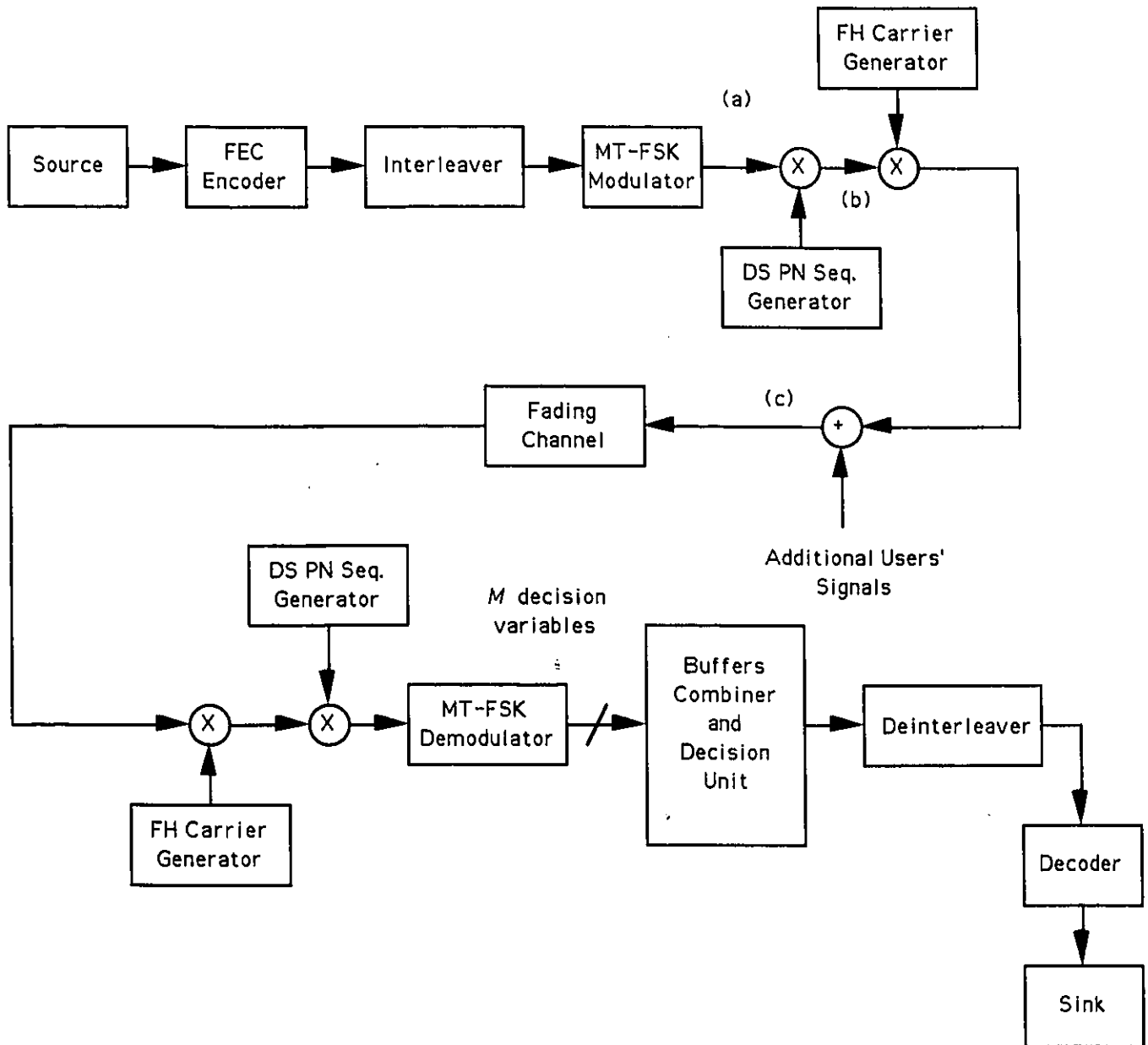


Figure 5.4 : Hybrid DS/FH-CDMA system employing MT-FSK.

the locally generated replicas of the two PN sequences are perfectly aligned with those at the transmitter, the original signal corrupted by noise and multiple access interference (MAI) is obtained. Because the signal was transmitted using  $L$  carriers, the signal can be demodulated at the hop rate. By doing this, it is as if we are using a repetition code, with the advantage that each component of the symbol has been acted upon by an independent fading process.

### 5.5.1 Performance of Hybrid DS/FH-CDMA Employing MT-FSK

Let us first assume that there is no hopping. In other words,  $L = 1$ . Therefore all users share the same  $v$  frequencies. If (6,3) MT-FSK is the modulation employed by this system then the equivalent decision variables from eqn. (3.55) become

$$\begin{aligned} U'_1 &= |2\mathcal{E}\alpha_1 + N_1 + I_1|^2 + |2\mathcal{E}\alpha_4 + N_4 + I_4|^2 \\ U'_2 &= |N_3 + I_3|^2 + |N_5 + I_5|^2 \end{aligned} \quad (5.12)$$

where  $I_i$  is the effect of the MAI seen at the output of the  $i$ th filter. It has been shown in [18] that for DS-CDMA systems employing wideband FSK modulation schemes  $N_i + I_i$  can be approximated by a Gaussian random variable with variance  $N'_o W$  where

$$N'_o = N_o + \frac{2(U-1)E_b R_b R_p}{v^2} \sum_{i=1}^v \sum_{j=1}^v \frac{1}{(2R_p)^2 + (\pi\Delta f)^2(i-j)^2} \quad (5.13)$$

Previously, we stated that  $2R_p \leq (\Delta f)_c \leq \Delta f$ . Therefore  $2R_p = \rho\Delta f$ , where  $\rho \geq 1$ . Thus eq. (5.13) becomes

$$\begin{aligned}
N'_o &= N_o \left[ 1 + \frac{(U-1) \frac{E_b}{N_o} \frac{R_b}{R_p}}{2v^2} \sum_{i=1}^v \sum_{j=1}^v \frac{1}{1 + (\pi\rho)^2(i-j)^2} \right] \\
&= N_o \left[ 1 + (U-1) \frac{E_b}{N_o} \frac{R_b}{R_p} \sigma(v, \rho) \right]
\end{aligned} \tag{5.14}$$

where

$$\sigma(v, \rho) = \frac{1}{2v^2} \sum_{i=1}^v \sum_{j=1}^v \frac{1}{1 + (\pi\rho)^2(i-j)^2} \tag{5.15}$$

The average bit energy to equivalent noise spectral density ratio is

$$\frac{\bar{E}_b}{N'_o} = \frac{E_b/N_o}{1 + (U-1) \frac{E_b}{N_o} \frac{R_b}{R_p} \sigma(v, \rho)} \tag{5.16}$$

The total bandwidth of the system without hopping is  $W_{tot} = 2R_c + (v-1)\Delta f = W_{ph}$ . The bandwidth efficiency of this system is:

$$\eta = \frac{U_{max} R_b}{W_{tot}} \tag{5.17}$$

where  $U_{max}$  is the maximum number of users that the system can support. We define  $\gamma'_b$  to be the average bit energy to equivalent noise spectral density when the number of users simultaneously accessing the channel is  $U_{max}$ . Therefore

$$\gamma'_b = \frac{E_b/N_o}{1 + (U_{max}-1) \frac{E_b}{N_o} \frac{R_b}{R_c} \sigma(v, \rho)} \tag{5.18}$$

Rearranging eqn. (5.18), we find

$$\begin{aligned}
U_{max} &= \frac{\frac{E_b}{N_o} - \gamma'_b}{\frac{E_b R_b}{N_o R_c} \gamma'_b \sigma(v, \rho)} + 1 \\
&\approx \frac{R_c}{\gamma'_b R_b \sigma(v, \rho)} \quad \text{for } \frac{E_b}{N_o} \gg \gamma'_b
\end{aligned} \tag{5.19}$$

Therefore the bandwidth efficiency is

$$\begin{aligned}
\eta &\approx \frac{R_c}{\gamma'_b \sigma(v, \rho) [2R_c + (v-1)\Delta f]} \\
&\approx \frac{1}{2\gamma'_b \sigma(v, \rho) [1 + (v-1)\rho]}
\end{aligned} \tag{5.20}$$

Now let us investigate the effect of hopping on this system. Each symbol is hopped into each of  $L$  independent channels. The total bandwidth  $W_{tot} = LW_{ph} = L[2R_c + (v-1)\Delta f]$ . The equivalent noise spectral density becomes:

$$N'_o = N_o \left[ 1 + \frac{U-1}{L} \frac{E_b R_b}{N_o R_p} \sigma(v, \rho) \right] \tag{5.21}$$

Therefore we can show that  $U_{max}$  increases by a factor of  $L$  for a hopped system ( $L > 1$ ) compared to a system without hopping ( $L = 1$ ). However, the total bandwidth is also increased by a factor of  $L$ ; therefore the bandwidth efficiency for a hybrid DS/FH-CDMA system is also given by eq. 17. The only difference between the two is that as  $L$  increases,  $\gamma'_b$  decreases, thus improving the bandwidth efficiency.

In previous sections, we employed a channel model with a coherence bandwidth of  $(\Delta f)_c \approx 200$  kHz. Therefore  $2R_p \leq 200$  kchips/sec, and  $\Delta f \geq 200$  kHz. Since  $\eta$  is not dependent on  $R_b$  for this particular system, any value of  $R_b$  can be used for illustration. If we choose  $R_b = 9.6$  kbps, then a good choice for  $R_p$  is 96 kchips/sec. We also choose  $\Delta f = 211.2$  kHz. Therefore  $\rho = 1.1$ .

For (6,3) MT-FSK,  $v = 6$ , therefore  $\sigma(v, \rho) = 0.098$ . Similarly,  $\sigma(v, \rho) = 0.074$ , for (8,3) MT-FSK, 0.06 for (10,4) MT-FSK and 0.05 for (12,4) MT-FSK. The values for  $\gamma'_b$  are taken from Table 3.9. The bandwidth efficiency of the hybrid DS/FH-CDMA system employing MT-FSK is shown in Table 5.16 the MT-FSK modulation schemes and different orders of diversity.

*Table 5.16 : Bandwidth efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing uncoded MT-FSK in Rayleigh fading.*

|               | $L = 1$ | $L = 2$ | $L = 3$ |
|---------------|---------|---------|---------|
| (6,3) MT-FSK  | 0.0092  | 0.0194  | 0.0250  |
| (8,3) MT-FSK  | 0.0078  | 0.0240  | 0.0324  |
| (10,4) MT-FSK | 0.0146  | 0.0304  | 0.0349  |
| (12,4) MT-FSK | 0.0175  | 0.0383  | 0.0439  |

Compared with the results of uncoded MFSK and MFSK-DPSK, we see at  $L = 1$  or 2, the MT-FSK generally provides better results while at  $L = 3$  some of the better MFSK-DPSK schemes provide slightly higher spectral efficiencies (see Tables 5.3 and 5.12).

Of course, the spectral efficiencies of these uncoded systems are very low, therefore we must again examine the spectral efficiency of this system with error control coding. We know from the previous chapter that using error control coding with MT-FSK does not provide large power gains over the uncoded case at  $P_b = 10^{-3}$ . Therefore, we should not expect large improvements compared to the results seen in Tables 5.16.

### 5.5.2 Performance of Hybrid DS/FH-CDMA System with Error Control Coding

Using the values of  $\gamma_b$  found in Chapter 4, the bandwidth efficiency of DS/FH-CDMA employing coded MT-FSK can be found. Table 5.17 shows the spectral efficiency when BCH coding<sup>1</sup> is used. Table 5.18 gives the spectral efficiency of DS/FH-CDMA employing MT-FSK and dual- $k$  coding.

Table 5.17 : Bandwidth efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing MT-FSK and BCH coding.

|               | $L = 1$ | $L = 2$ | $L = 3$ |
|---------------|---------|---------|---------|
| (6,3) MT-FSK  | 0.0153  | 0.0272  | 0.0305  |
| (8,3) MT-FSK  | 0.0204  | 0.0389  | 0.0447  |
| (10,4) MT-FSK | 0.0236  | 0.0304  | 0.0342  |
| (12,4) MT-FSK | 0.0311  | 0.0439  | 0.0493  |

Table 5.18 : Bandwidth efficiency (in bps/Hz) of hybrid DS/FH-CDMA employing MT-FSK and dual- $k$  coding.

|               | $L = 1$ | $L = 2$ | $L = 3$ |
|---------------|---------|---------|---------|
| (6,3) MT-FSK  | 0.0452  | 0.0495  | 0.0473  |
| (8,3) MT-FSK  | 0.0589  | 0.0661  | 0.0646  |
| (10,4) MT-FSK | 0.0529  | 0.0554  | 0.0541  |
| (12,4) MT-FSK | 0.0712  | 0.0712  | 0.0696  |

The results shown in Tables 5.17 and 5.18 show improvement in the spectral efficiency of a DS/FH-CDMA system employing MT-FSK modulation compared

<sup>1</sup>BCH(255,199) is used with (6,3) and (10,4) MT-FSK while BCH(63,39) is used with (8,3) and (12,4) MT-FSK.

to the uncoded case. Higher spectral efficiencies are obtained employing the dual- $k$  codes compared to the BCH codes. Also, increasing  $L$  with the dual- $k$  code may not result in any improvement and may actually cause degradation as seen in Table 5.18. However, the level of improvement or degradation cannot actually be known since bounds are used in the performance evaluation. But these results suggest that hopping might not be beneficial to CDMA systems employing MT-FSK. The results tend to suggest that we can improve the spectral efficiency of a CDMA system employing rate 1/2 dual- $k$  encoded MT-FSK by increasing the diversity of the modulation scheme (by increasing  $v$  and  $w$ ) without the additional diversity of hopping.

## 5.6 Discussion

In this chapter, the hybrid DS/FH-CDMA system was described. Three modulation schemes were considered for this system. These modulation schemes were: non-coherent MFSK, MFSK-DPSK and MT-FSK. We compared the performance of the system with each modulation scheme by deriving expressions for the spectral efficiency of the system. In each case, the spectral efficiency was inversely proportional to the bit energy to spectral noise density ratio required by the specific modulation scheme to achieve the maximum tolerable bit error rate. For voice communications, this maximum tolerable bit error rate is  $10^{-3}$ .

We examined the spectral efficiency of the system without error control coding. Although some modulation schemes performed better than others, all results were particularly low since the bit energy to noise spectral density ratios for a bit error rate of  $10^{-3}$  for each modulation scheme are rather high. However, at  $L \leq 2$  (12,4) MT-FSK provided the highest spectral efficiencies. When  $L = 3$ , 16FSK-DQPSK provided the highest spectral efficiency. This is because when  $L = 3$ , the signal energy is divided among three channels. Therefore, the energy

per channel to noise spectral density ratio is quite low. We saw in Chapter 3 that MFSK-DPSK outperforms MT-FSK when the signal to noise ratio is low. Therefore this result seems appropriate.

When hard decision decoded BCH coding is employed, the results favor MFSK-DPSK in most cases. However, the spectral efficiency of the system when MFSK is used is quite comparable to MFSK-DPSK. As for MT-FSK, the performance bounds indicate that less coding gain is achieved at a bit error rate of  $10^{-3}$  than for MFSK or MFSK-DPSK. The poor bit error rate of MT-FSK at low  $E_b/N_o$  accounts for the poor spectral efficiency of the system when MT-FSK is used.

Dual- $k$  convolutional codes which are easily decoded using the Viterbi decoder. This allows for simpler soft decision decoding as opposed to BCH codes, which do not directly allow soft decision decoding. Implementation of a Viterbi decoder with the MFSK-DPSK modulation scheme would require the storage of large amounts of phase information for true Viterbi decoding of this modulation scheme, and thus it is not practical to use this code with this scheme. The spectral efficiencies of MFSK and MT-FSK systems are improved when this code is used. With MT-FSK,  $L > 2$  has lower spectral efficiencies than  $L \leq 2$  and as we increase the inherent diversity of the modulation scheme,  $L = 1$  may be sufficient. This suggests that dual- $k$  coded MT-FSK CDMA systems may perform well without hopping and with larger  $v$  and  $w$  than employed by the signalling sets discussed in this thesis

## Chapter 6

# Conclusions and Suggestions for Further Research

### 6.1 Conclusions

In this thesis, we have discussed different modulation and coding schemes for a hybrid DS/FH-CDMA system. Each modulation scheme was analysed, and its performance in the presence of Rayleigh fading was determined. We investigated the effect of multi-channel diversity (obtained by fast frequency hopping) and error control coding.

The spectral efficiency  $\eta$  was determined for the hybrid DS/FH-CDMA system employing each of these modulation schemes. The spectral efficiency is determined jointly by the amount of interference generated by each user, and the amount of interference that can be tolerated by each user. This means that the spectral efficiency of the system is limited by a maximum bit error rate that can be tolerated. The spectral efficiency is inversely proportional to the bit energy to noise spectral density ratio  $\gamma_b'$  that achieves this maximum bit error rate. Therefore, to increase  $\eta$ , one must decrease  $\gamma_b'$ . This can be done by employing

power efficient modulation and coding schemes.

Three hybrid DS/FH-CDMA systems were presented in this thesis. One employed MFSK modulation, the second employed MFSK-DSPK and the third employed wideband MT-FSK. The first two systems obtained diversity reception by employing fast frequency hopping (from one to three hops per symbol). The faster the hop rate, the greater the spectral efficiency of the system. The difference between the two systems is that there is a constraint on the frequency bins that can be employed when MFSK-DPSK is used. Since the phase aspect of the modulation scheme requires that the fading process be relatively unchanged on consecutive symbols, the hopping pattern must be repeated for each symbol.

The third system employed MT-FSK. Some diversity was obtained by the modulation scheme itself and more diversity was achieved through fast frequency hopping. Because the modulation scheme is wideband, the number of frequency bins is greatly reduced; perhaps only two or three frequency bins depending on the available bandwidth. Therefore, for this scheme to be practical, a user should employ all frequency bins on one signalling interval.

Table 6.1 compares some of the better systems when error control coding is not used. The bit rate of the systems shown in this Table is 9.6 kbps<sup>1</sup>. We can see that when  $L \leq 2$ , the highest spectral efficiency is obtained when (12,4) MT-FSK is employed. However, when  $L = 3$ , MFSK-DPSK modulated system overtakes the MT-FSK system. This reflects the poor performance of MT-FSK at low symbol energy to noise ratios. When  $L = 3$  the channel symbol energy to noise spectral density ratio is one-third the overall symbol energy to noise spectral density ratio. If this is too low, MFSK-DPSK outperforms MT-FSK.

When error control coding is used, the spectral efficiency of the system im-

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<sup>1</sup>The bit rate slightly effects the spectral efficiency of the system when MFSK and MFSK-DPSK is used because of the constraints on  $R_p$ . The spectral efficiency of the system employing MT-FSK is not dependent on  $R_b$ .

Table 6.1 : A comparison of the spectral efficiencies (in bps/Hz) of hybrid DS/FH-CDMA systems employing MFSK, MFSK-DPSK or MT-FSK without the application of error control coding.

|               | $L = 1$ | $L = 2$ | $L = 3$ |
|---------------|---------|---------|---------|
| 16FSK         | 0.00192 | 0.0173  | 0.0398  |
| 16FSK-DBPSK   | 0.00186 | 0.0202  | 0.0463  |
| 16FSK-DQPSK   | 0.00187 | 0.0226  | 0.0489  |
| (10,4) MT-FSK | 0.0146  | 0.0304  | 0.0349  |
| (12,4) MT-FSK | 0.0175  | 0.0383  | 0.0439  |

proves. This is quite noticeable in the case of MFSK and MFSK-DPSK, while the improvement to MT-FSK is not so great. Again this is due to the poor performance of MT-FSK at low signal to noise ratios. A comparison is made in Table 6.2(a bit rate of 9.6 kbps is used in the comparison).

These results suggest that the hybrid DS/FH-CDMA system performs best with MT-FSK modulation only when  $L \leq 2$  and no forward error correction is employed. However, the spectral efficiency of the system when this is the case is too poor for this system to be feasible (around 0.035 bps/Hz). Forward error correction is definitely needed to have acceptable spectral efficiencies.

When error control coding is used, the results shown in Table 6.2 favor MFSK modulated systems when  $L > 1$ <sup>2</sup>. The results indicate that for DS-CDMA, one may consider coded MT-FSK, however, for the hybrid DS/FH-CDMA, MFSK is a good choice. The results obtained when MFSK-DPSK is used are promising, however complexities arise when soft decision coding is desired. It may be possible to employ soft decision detection with MFSK-DPSK, but practically

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<sup>2</sup>Note that in Table 6.2, 16FSK has no entry for  $L = 3$ . This is because the unspread bandwidth is too large for these modulation schemes when  $R_b = 9.6$  kbps. However, we saw in Chapter 5 that when  $R_b = 4.8$  kbps,  $L = 3$  is possible for this modulation schemes

Table 6.2 : A comparison of the spectral efficiencies (in bps/Hz) of hybrid DS/FH-CDMA systems employing MFSK, MFSK-DPSK or MT-FSK with the application of error control coding.

|                              | $L = 1$ | $L = 2$ | $L = 3$ |
|------------------------------|---------|---------|---------|
| 16FSK/<br>dual-4             | 0.0589  | 0.120   | —       |
| 16FSK-DBPSK/<br>BCH(255,199) | 0.0333  | 0.102   | 0.118   |
| 16FSK-DQPSK/<br>RS(63,47)    | 0.0287  | 0.0974  | 0.112   |
| (10,4) MT-FSK/<br>dual-2     | 0.0529  | 0.0554  | 0.0541  |
| (12,4) MT-FSK/<br>dual-3     | 0.0712  | 0.0712  | 0.0696  |

we must compare the complexity of this with that of an MFSK system.

## 6.2 Suggestions for Further Research

The spectral efficiencies of DS/FH-CDMA systems employing different modulation and error control coding schemes were discussed in this thesis. However, not all possible modulation schemes and coding techniques could be addressed in this thesis. So inevitably, a number of questions arise, necessitating further research into this subject. Of course, the number of ideas for possible research that stems from this thesis is probably many. Listed below are but a few.

Firstly, we considered only frequency based modulation schemes in this thesis. Phase based modulation schemes may increase the spectral efficiency of this system significantly provided that phase coherence can be maintained between hops.

In [40], it is shown that BPSK and QPSK outperform DPSK in Rayleigh fading provided a phase correction circuit is possible. In [65, 66], a pseudo-coherent PSK system for Rayleigh fading channels is implemented by employing pilot symbols of known phase to track the channel fading process. It is shown in [67] that CDMA systems employing pilot symbol assisted modulation have higher spectral efficiencies compared to those employing DPSK. It would be interesting to see what gain (if any) can be realized by employing MFSK-MPSK with pilot symbols as opposed to MFSK-DPSK.

In [45], a DS-CDMA system was devised where the selection of a specific PN sequence carried some information. This can be applied to the hybrid DS/FH-CDMA system. Information can be carried by the order that the hop frequencies are used during each signalling interval. For example, if a symbol is to be hopped into channels  $x$ ,  $y$  and  $z$ , It can be hopped into channel  $z$  first followed by channel  $x$  then channel  $y$ . Or channel  $x$  can be used first, followed

by  $y$  then  $z$ . This way, information can be transmitted by the hopping sequence.

We considered the use of the hybrid DS/FH-CDMA system in the forward link (base-to-mobile). In the reverse link, where the base receives all incoming signals, it is possible for the base to estimate and decrease the MAI using cancellation or decorrelation techniques [41]-[43]. DS/FH-CDMA employing MAI cancellation should be investigated for the mobile-to-base link. One should also take into consideration the complexity of such a system, and examine possible avenues of making such a system feasible.

Finally, complexity concerns have not been a main issue in this thesis. The complexity of an MFSK-DPSK modem is greater than that of an MFSK modem. Also, a problem arises when fast hopping is introduced. Phase errors are introduced between hops, which can make the detection of the phase sub-symbol difficult. This problem must be resolved. Intuitively, DS/FH-CDMA systems employing MFSK appears less complex than those employing MFSK-DPSK.

# Appendix A

## Basic Spread Spectrum Techniques and Terminology

### A.1 Spread Spectrum Techniques

Basic spread spectrum techniques are discussed in this section. It is intended for those readers who may need a quick review of the fundamentals of spread spectrum communications.

#### A.1.1 Direct-Sequence Spread Spectrum

In a DS-SS system, a digital signal,  $m(t)$ , is modulated by a pseudonoise (PN) sequence,  $c(t)$ , which is generated by a feedback shift register (FSR). The transmitted signal is given by:

$$s(t) = m(t)c(t) \quad (\text{A.1})$$

Although  $m(t)$  can be a digital signal of any modulation scheme,  $c(t)$  is usually a BPSK signal with a rate,  $R_p$ , which is much higher than the symbol rate,

$R_s$ , of  $m(t)$ .  $R_p$  is termed the *chip rate* of the system. The resultant signal bandwidth is much higher than that of  $m(t)$ , and the signal is said to be spread.

At the receiver, a local replica of  $c(t)$  is generated and used to despread the received signal. This is accomplished by multiplying the received signal by  $c(t)$  which in turn reproduces  $m(t)$  since  $c^2(t) = 1$ . A typical DS-SS receiver-transmitter pair is shown in Figure A.1, and Figure A.2 shows direct sequence spread spectrum in both time and frequency.

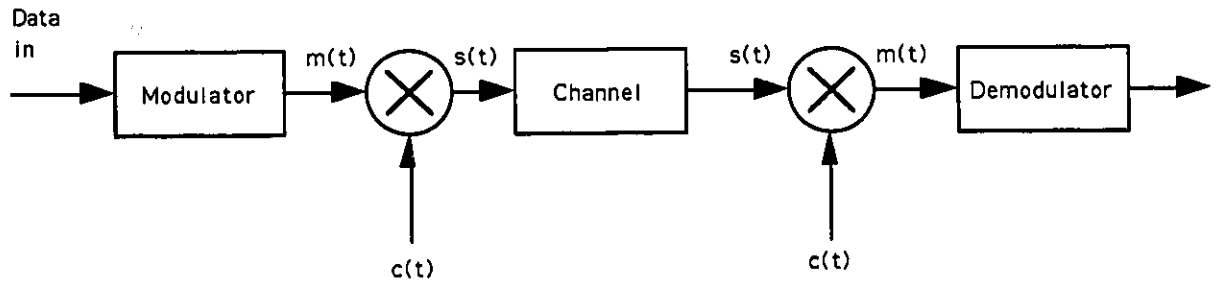


Figure A.1 : A DS-SS communications system.

The ratio of the PN sequence chip rate to the information symbol rate is defined as the processing gain. In other words, if there are  $B_c$  chips per information symbol, then the processing gain is equal to  $B_c$ . This is also called the bandwidth expansion factor.

### A.1.2 Frequency-Hopped Spread Spectrum

Frequency-hopped spread spectrum is so named because the transmitted signal appears as a data modulated carrier which is hopping from one frequency to another. This is accomplished by changing the frequency of the carrier periodically. Figure A.3 shows a typical frequency-hop transmitter-receiver pair.

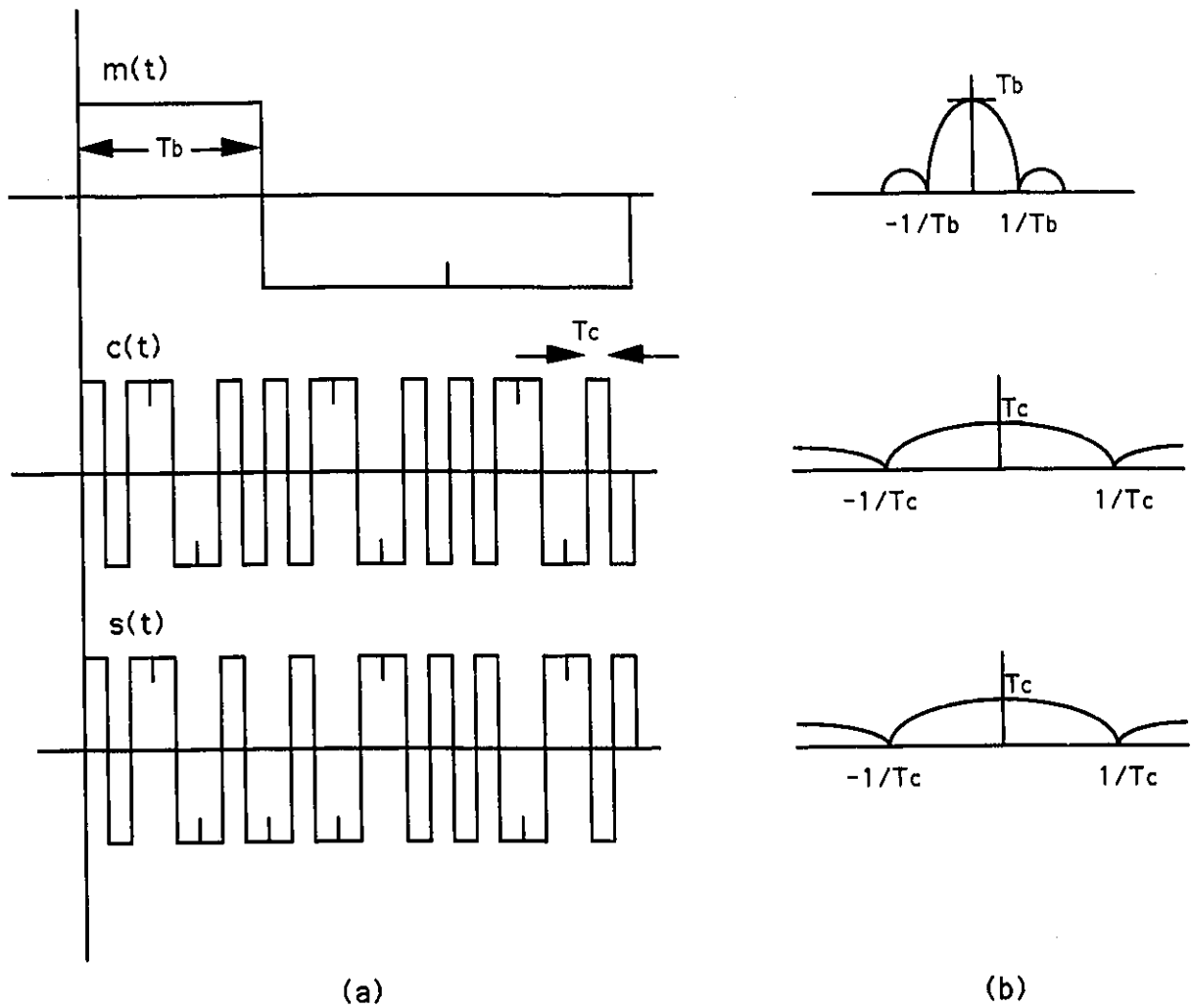


Figure A.2 : Direct-sequence spread spectrum: (a) in the time domain (b) in the frequency domain.

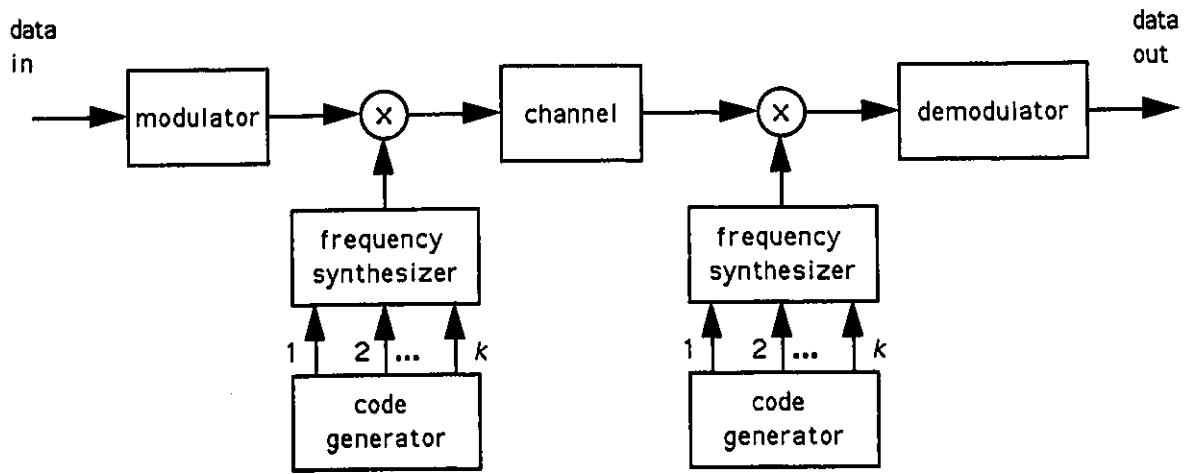


Figure A.3 : A FH-SS communication system.

From Figure A.3, it is seen that the carrier is selected by a  $k$ -bit word which is produced by a code generator (implemented by a FSR). The output of the code generator is also referred to as the frequency hopping pattern. The carrier is chosen at a rate of  $R_h$  selections per second. This is termed the *hop rate*.

When  $R_h < R_s$ , where  $R_s$  is the symbol rate, then it is a slow frequency-hopped (SFH) system. Conversely, when  $R_h > R_s$ , it is called fast frequency-hopping (FFH). The latter allows frequency diversity reception of the signal, which is desirable in Rayleigh fading channels (provided that the frequency slots are spaced such that the fading appears to be independent of the fading in the other slots).

In this system as well, the processing gain is the ratio of the spread bandwidth  $W_{ss}$  to that of the unspread bandwidth  $W$ . Therefore, the processing gain  $B_e = W_{ss}/W$ . If the frequency slots have a frequency separation equal to the unspread bandwidth of the signal, then the processing gain is equal to the number of frequency slots,  $N_f$ .

### A.1.3 Hybrid DS/FH-SS Systems

Spread spectrum systems where part of the spectral spreading is accomplished by direct sequence spreading of the signal and the remainder of the spectral spreading is achieved by hopping the spread signal over a number of frequency bins (or channels) are called hybrid DS/FH-SS systems. Hybrid techniques are widely used in military communications [34]. They are also the only practical way to achieve spreading over extremely large bandwidths.

## A.2 PN Sequences

As previously mentioned, PN sequences are generated by FSRs. FSR design is discussed in [68], which covers both linear and nonlinear FSR designs.

There are many different types of PN sequences. Some of the more common types are maximal length sequences, Gold codes and rapid acquisition codes. Maximal length PN sequences generate long sequences using the minimum amount of binary storage devices. Gold codes add the output of two or more FSRs to create a large number of possible codes for multiple access purposes.

The selection of a particular type of code depends on the most important aspect of the communication system. When long codes are desired (when large bandwidth expansion is needed, generally long codes are used), maximal-length codes can be selected. However, for multiple access purposes, there may not exist many maximal-length codes with good auto and cross correlation properties to warrant the selection of these codes, and thus Gold codes may be selected. Another consideration is security. When a high level of information security is required, maximal-length PN sequences generated by nonlinear FSRs provide more security than the other codes discussed here [24].

Finally, a concern for any code selection is code acquisition. Clearly one can see that in both DS and FH systems, the PN sequence generated at the receiver must be synchronized to that of the transmitter. Thus PN sequence acquisition and tracking algorithms must be employed by the receiver. More common schemes are serial search or matched filter acquisition and early-late gate tracking. These, along with other PN sequence synchronization schemes, are discussed in [69, 70].

## Appendix B

# Balanced Incomplete Block Design

A BIB design is defined as [55] an arrangement of  $v$  distinct objects into  $b$  blocks, where each block contains  $w$  distinct objects, each object occurs in  $r$  different blocks, and all pairs of distinct objects occur together in  $\lambda$  different blocks. From this definition of BIB design, we can derive one important relationship shown in eq. B.1.

$$bw = vr \tag{B.1}$$

This equation states that there are  $b$  blocks each containing  $w$  objects. This is the total number of elements used in the signalling set. Another way of representing this is by saying that there are  $v$  objects that are each used  $r$  times. Therefore eq. B.1 is valid.

Each block contains  $w(w - 1)$  pairs, and there are  $b$  blocks in total. There are also  $v(v - 1)$  possible pairs each occurring  $\lambda$  times. Therefore

$$bw(w-1) = \lambda v(v-1) \quad (\text{B.2})$$

Rewriting eq. B.2, we get

$$b = \lambda \frac{v(v-1)}{w(w-1)} \quad (\text{B.3})$$

Replacing  $b$  in eq. B.1 by the expression in eq. B.3, we obtain

$$\lambda(v-1) = r(w-1) \quad (\text{B.4})$$

A special case of the BIB design is the Steiner system. A Steiner system is a BIB design with  $\lambda = 1$ . This means that pairs appear in one block only. For a Steiner system,  $b$  and  $r$  become

$$b = \frac{v(v-1)}{w(w-1)} \quad (\text{B.5})$$

$$r = \frac{v-1}{w-1} \quad (\text{B.6})$$

Any object contained by a given block  $B$  occurs in  $r-1$  other blocks. For a Steiner system, the number of blocks intersecting  $B$  on a single object is given by

$$x_1 = w(r-1) = \frac{w}{w-1}(v-w) \quad (\text{B.7})$$

and the number of blocks orthogonal to  $B$  is

$$x_0 = b - x_1 - 1 \tag{B.8}$$

We will employ these principles to design MT-FSK signalling sets.

## Appendix C

# Design of MT-FSK Waveforms Using the BIB Design

Let us begin by examining the (8,3) MT-FSK design. We want to design an 8-ary ( $b \geq 8$ )<sup>1</sup> alphabet that employs 3 frequency tones ( $w = 3$ ) out of a set of 8 possible tones ( $v = 8$ ). Using eq. B.1, we have

$$3b = 8r \tag{C.1}$$

By selecting  $b = 8$  and  $r = 3$ , we can satisfy eq. B.1. However, we have made the additional constraint that no waveform will intersect another in more than 1 tone. Therefore no pair of tones may be repeated. This means that  $\lambda \leq 1$ . To verify this, we can rearrange eq. B.3 to obtain:

$$\lambda = \frac{bw(w-1)}{v(v-1)} \tag{C.2}$$

---

<sup>1</sup>We say that  $b \geq 8$  because in some cases, it is not possible to design a signalling set with  $b = M$ . Therefore, we must design an alphabet larger than the desired signalling set and then eliminate some of the waveforms to obtain an  $M$ -ary alphabet.

Solution of eq. C.3 yields  $\lambda = \frac{6}{7}$ . This means that no pair is repeated more than once throughout the signalling set (it also reveals that  $\frac{1}{7}$  of all possible pairs are not used at all).

From eq. B.7, we can find that a given symbol will intersect 6 other symbols on one tone. Eq. B.8 tells us that this same symbol will be orthogonal to the remaining symbol.

In designing the signalling set, one must select each tone  $r$  times, while taking care never to repeat a pair of tones over two symbols. The easiest method of designing such a signalling set is to employ a grid, with each row representing a symbol, and each column representing a frequency. The design of (8,3) MT-FSK is shown in Table C.1. An X indicates that the frequency tone has been selected to be used in that particular symbol.

*Table C.1 : BIB Design of (8,3) MT-FSK.*

| Symbol | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1      | X     | X     |       | X     |       |       |       |       |
| 2      |       | X     | X     |       | X     |       |       |       |
| 3      |       |       | X     | X     |       | X     |       |       |
| 4      |       |       |       | X     | X     |       | X     |       |
| 5      |       |       |       |       | X     | X     |       | X     |
| 6      | X     |       | X     |       |       |       |       | X     |
| 7      | X     |       |       |       |       | X     | X     |       |
| 8      |       | X     |       |       |       |       | X     | X     |

The same procedure is followed for (10,4) and (12,4) MT-FSK. (10,4) MT-FSK is a 4-ary signalling set where each symbol is made up of 4 out of a set of 10 possible frequencies. In designing the signalling set, it can be shown that if each tone appears twice ( $r = 2$ ), we can design a 5-ary signalling set where any

symbol intersects every other symbol on one tone. To obtain a 4-ary signalling set, we simply discard one symbol.

(12,4) MT-FSK is an 8-ary signalling set where each symbol contains 4 of 12 tones. If we reuse each tone 3 times ( $r = 3$ ), we can design a 9-ary signalling set. All symbols share one tone with the other 8. Again, we discard one symbol to obtain an 8-ary alphabet.

The (10,4) and (12,4) MT-FSK signalling sets are shown in Tables 3.7 and 3.8 in Chapter 3.

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