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BEHAVIOUR OF SLENDER REINFORCED CONCRETE COLUMNS UNDER BIAXIALLY ECCENTRIC LOADING

by

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ABSTRACT

The problem of designing slender reinforced concrete columns for biaxially eccentric loads has been considered by many investigators to date, but due to the complexity of the problem, a simple accurate design method is not available.

An improved method for the design of biaxially loaded slender reinforced concrete columns is presented in this thesis. The assumptions upon which it is based are well accepted in engineering practice and the major variables involved in the problem are taken into consideration either singly or in groups. The method uses developed section property curves which were produced for square section symmetrically reinforced columns with a cover ratio of 0.75. The curves give the relationships between ultimate load, eccentricity, position of the neutral axis and maximum strain for the two extreme eccentricity angles 0° and 45°. A calibrating factor is presented to enable the use of the section property curves for other cover ratios. Rectangular section columns can be designed using the same section property curves with the aid of a second calibration factor.

A companion experimental investigation was carried out to study the effects of eccentricity, angle of eccentricity, steel ratio and length on the ultimate load and the ultimate lateral deflection. A total of 44 columns divided into four groups of 11 each depending on

length and steel reinforcement used were tested until failure in a horizontal pinned-pinned configuration. The values of ultimate loads were recorded and deflection values were measured at different load intervals in two orthogonal planes till failure.

The section property curves described above were found to adequately describe the experimental behaviour of the test columns. Furthermore, using the presented section property curves to design columns for biaxially eccentric loads gave results in close agreement with those predicted by ACI 318-71.

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NOTATIONS

Ac = area of concrete

Ast = area of steel

b = smaller dimension of columns

C_m = factor relating the actual moment diagram to an equivalent
= moment diagram as per ACI 318-71

d = effective depth of section

d' = distance from extreme compression fiber to centroid of compression reinforcement

e = eccentricity of load

e_x = eccentricity of load on the X axis

e_y = eccentricity of load on the Y axis

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of steel

F = factor to correct for cover ratio

f_c' = specified cylinder strength of concrete

f_s = allowable stress in steel

f_y = specified yield strength of reinforcement

g = cover ratio

I_g = second moment of area of gross concrete area

I = second moment of area

K = effective length factor for compression members

L = length of column

M_u' = ultimate moment

M_x = design ultimate moment about the Y axis

M_y = design ultimate moment about the X axis

Mf_x = ultimate moment about Y axis

Mf_y = ultimate moment about X axis

N = defined in Figure (2-3)

P = allowable load

P_u = design ultimate load in compression member

P_u' = ultimate load

P_x' = ultimate load at e_x with $e_y = 0$

P_y' = ultimate load at e_y with $e_x = 0$

P_o' = axial ultimate load

r = radius of gyration

t = larger dimension of columns cross-section

Z = perpendicular distance from neutral axis to point of maximum strain

β_d = ratio of maximum design dead load to maximum design total load

Δ = lateral displacement of column

Δ_V = lateral displacement on the Y axis

Δ_H = lateral displacement on the X axis

δ = moment magnifier - as per ACI 318-71

ε = strain

ε_y = yield strain

σ_{max} = maximum strain

ρ = percentage of reinforcement

ϕ = curvature

ϕ_x = curvature in the X plane

ϕ_y = curvature in the Y plane

θ = angle of eccentricity

ϕ' = capacity reduction factor

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CHAPTER I

INTRODUCTION

Column is the name given to the vertical structural element which typically supports the floors and roofs of a building. The column is basically a compression member which transfers the loads down to the foundations where they are absorbed by the substratas.

In the classic structures of Greece and Egypt, the columns were of regular section, short and completely in compression. However, the column in a modern building may be slender, subjected to uniaxial or biaxial bending, shear and perhaps even transverse forces.

Structural elements can be estimated following two different design philosophies. The first is the service load design method in which the properties of the section are calculated assuming that the stresses are proportional to the strains and ultimate material stresses are divided by a safety factor to give allowable material stresses. In the second design philosophy, namely the ultimate load method, the structural element is designed to carry a multiple of the service loads at failure, and the properties of the section are usually calculated ignoring the compatability of strains. The ultimate load method is generally preferred for the design of reinforced concrete members as the problem of compatability of strains is avoided, the actual failure load is easy to obtain by experiment and can be used to validate the prediction equation(s) and different load factors can be used for the various components of load reflecting the lack of precision in their estimations.

Material failure governs the load capacity of short columns under purely axial load. However, an alternate mode of failure can occur for slender columns which is failure by elastic, or inelastic instability. Euler in 1744 presented a theoretical means of calculating the buckling load of initially straight columns under purely axial loads. Engesser (1889) extended Euler's work to apply to non-elastic buckling by introducing the concept of a tangent modulus of elasticity.

However in buildings, columns are neither perfectly straight or perfectly axially loaded. The Building Codes specify that columns must be designed for the greater of the calculated design eccentricities or a minimum eccentricity to account for initial lack of member straightness or the load not being applied perfectly concentrically.

A column, or more correctly a beam column, can be subjected to a complete spectrum of loadings ranging from pure axial load and zero moment to zero axial load and pure moment. This problem has been resolved for short columns under uniaxially eccentric loads by interaction diagrams which give the envelope of column capacity as a continuous function of axial load and moment (eccentricity). Interaction diagrams are generally available only for uniaxially applied loads.

The complete problem of the design of slender columns under uniaxial eccentric loads has usually been treated by modifying the results of a short column to take into account slenderness effects. Typically, the recommended Building Code requirement of the ACI-318-63 allows the section to be designed as short columns but the load capacity is linearly reduced with respect to the slenderness ratio. More recently the ACI and CSA adopted an alternative approach whereby the long column moment is magnified to account for slenderness and the column is then designed as a short column with the magnified moment. Using the magnification method, no account is taken of the strain compatability of the section in calculating the behaviour of a slender column, i.e. the relationship between the curvatures necessary to develop the strength characteristics of the section and lateral deflection due to the curvature is ignored.

The problem of slender columns under biaxially eccentric loads is complicated by the lack of adequate short column axial load biaxial moment interaction curves and difficulties in determining the final eccentricities of the slender column. The 1971 version of the ACI Building Code requirements (ACI 318-71), recommends that the moments in each direction be magnified separately and ignores curvatures - lateral displacement compatability, and the fact that the maximum moments do not necessarily occur at the same location on the column.

The present work introduces a method for the design of biaxially loaded slender reinforced concrete columns. The method takes into account the compatability of strains and lateral deformations. It depends on a set of produced section property curves which give the relationships between ultimate load, eccentricity, angle of eccentricity and the position of the neutral axis for different concrete and steel strengths, percentage of reinforcement and maximum strain values. The curves provide the designer with the means to obtain a good initial estimate for the cross section needed for the column as well as controlling the trial and error procedure which is unavoidable in the checking procedure of a biaxially loaded slender column.

An experiment investigation was carried out to compare the values of ultimate loads and deflections predicted by the proposed design method with those obtained experimentally.

CHAPTER II

REVIEW OF LITERATURE

The structural behaviour of steel reinforced concrete columns under concentric, uniaxially eccentric and biaxially eccentric loads has been under intensive investigation over the past 80 years. This Chapter reviews the most significant contributions to the analysis and design of reinforced concrete columns under the subheadings of (1) short columns under concentric load; (2) short columns under uniaxially eccentric load; (3) short columns under biaxially eccentric load; (4) slender columns under concentric loads; (5) slender columns under uniaxially eccentric load; (6) slender columns under biaxially eccentric load, and (7) ACI 318-71 design method for reinforced concrete columns.

2-1 Short Columns Under Concentric Load

Considére (17) about 1900, suggested that the allowable load of a short reinforced concrete column under axial load was simply the algebraic sum of the allowable load of the reinforcement plus the allowable load of the concrete.

$$P = Ac\ f_c{'} + Ast\ f_s \qquad (2\text{-}1)$$

where

P = allowable load

f_c' = maximum allowable concrete stress

f_s = allowable stress in steel

Ac and Ast = areas of concrete and steel respectively

Considére's equation ignored completely the compatability of strain between the concrete and the steel.

Another approach taking into consideration compatability of strains in the concrete and the steel assuming that both are elastic materials was presented in 1894 by Coignet and Tedesco (14) and is commonly known as the straight line theory

$$P = Ac\ f_c' \ [1 + (n-1)\ Pst] \tag{2-2}$$

where

$$n = \frac{\text{modulus of elasticity for steel } E_s}{\text{modulus of elasticity for concrete } E_c}$$

$$Pst = Ast/Ac$$

However in 1921, McMillan (27) suggested that the steel reinforcement in reinforced concrete columns may develop stresses greater than those predicted from elastic theory due to the plastic, (creep), deformation of the concrete.

This suggestion resulted in a series of major experimental investigations sponsored by the ACI column research section. These investigations were carried out by Slater (40), Slater and Lyse (41), and Richart and Brown (36) and others in the 1930s to develop rational equations to predict the behaviour of short concentrically loaded columns. These investigations concluded that the load deflection behaviour was incalculable, at that time, but the ultimate load of such columns could be predicted by the equation

$$P_o' = 0.85\ Ac\ f_c' + As\ f_y \qquad (2\text{-}3)$$

where

P_o' = ultimate axial load

f_c' = ultimate concrete cylinder strength

f_y = yield stress of reinforcement

However the case of pure axial load is unlikely to occur in practice as columns are typically cast monolithic with beams and are subject to moments from those beams. Following the development of the Slope-Deflection and Moment-Distribution Methods from 1920 to 1930, the moments and axial loads in the columns of frames could be calculated and a knowledge of the behaviour of reinforced concrete columns under eccentric loads became important.

2-2 Short Columns Under Uniaxially Eccentric Loads

Whitney (43) in 1937 presented a semi-empirical formula to predict the failure load of rectangular section reinforced columns under combined axial load and bending when the failure was governed by compression.

$$P_u' = \frac{2\,As'\,f_y}{\frac{2e}{d'} + 1} + \frac{bt\,f_c'}{\frac{3te}{d^2} + \frac{6dt-3t^2}{2d^2}} \qquad (2\text{-}4)$$

where

As' = area of steel in compression

d' = distance from extreme compression fiber to centroid of compression reinforcement

d = effective depth of section

b = width of section

t = total depth of section

In 1938, Richart and Olsen (37) presented an empirical formula derived from test results to relate the ultimate load capacity of eccentrically loaded columns to the ultimate load of the same column section concentrically loaded.

$$\frac{P_{ecc}}{P_{axial}} = \frac{1}{1 - C\,\frac{e.c}{k^2}} \qquad (2\text{-}5)$$

where

e = eccentricity of load

c = distance from centroid to extreme fiber in compression

C = an empirical constant less than unity

k = radius of gyration of the column section

This formula was useful for compression failures but not for failure by yielding of the reinforcement or tension failure.

To predict the failure load of columns governed by the yield of the tensile reinforcement Whitney (44) in 1942 introduced the rectangular stress block with a uniform stress of 0.85 f_c' as equivalent to the non linear compressive stress distribution in the concrete. Whitney developed the theoretical relationships for combined axial load and moment using the rectangular stress block. Other descriptions of the shape of the compressive stress distribution (stress block) for the concrete have been suggested by Stüssi and others.

Hognestad (24), in 1951 introduced the dimensionless interaction diagram giving the failure surface of a given reinforced concrete section under combined axial load and uniaxially eccentric moment. Hognestad assumed the concrete to have no tension capability, perfect bond between the concrete and the steel, a linear variation of strain and that the stresses in the concrete and steel were determined from their respective stress strain characteristics. A typical interaction diagram is shown in Figure (2-1).

In the same work, Hognestad also presented a stress strain characteristic for concrete in compression which is initially parabolic and then linear. The initial tangent modulus of elasticity of Hognestad's stress strain curve is given by

$$E_c = 1,800,000 + 460\ f_c' \qquad (2\text{-}6)$$

2-3 Short Columns Under Biaxially Eccentric Loads

A three-dimensional surface is required (see Figure (2-2)) to describe the failure of a short reinforced concrete column under biaxially eccentric loads. In general, such three dimensional interactions are not available. Au (2), Chu and Pabarcius (12), and Mattock, Kriz and Hognestad (26) all presented an approximately similar trial and error design method to determine the section needed to sustain a given ultimate load at a given biaxial eccentricity. The procedure would start by assuming a section and a position of the neutral axis, then the value of the ultimate load could be calculated using Whitney's stress block for any general stress distribution, assuming a maximum strain value for the concrete. Subsequently, the assumed position of the neutral axis can be checked by the use of the equations of equilibrium. By trial and error the correct position of the neutral axis can be determined for the assumed section and eventually the ultimate load it can sustain.

Furlong (23) and Fleming and Werner (22) presented interaction diagrams for biaxially loaded columns, but due to the complexity of the problem, these diagrams were confined to certain section geometries, concrete and steel strengths and percentages of reinforcement.

In 1960, Bresler (3) suggested a very convenient method by which the failure load of a short column under biaxial eccentricities could be obtained from the failure loads of the same section under uniaxial eccentricity.

$$\frac{1}{P_u'} = \frac{1}{P_x'} + \frac{1}{P_y'} - \frac{1}{P_o'} \qquad (2\text{-}7a)$$

where

P_u' = ultimate load at eccentricities e_x and e_y simultaneously

P_x' = ultimate load at e_x with $e_y = 0$

P_y' = ultimate load at e_y with $e_x = 0$

P_o' = axial ultimate load

In the same paper Bresler also suggested a second approximation to describe the biaxial moment-axial load behaviour by an equation representing the moment contours at constant axial load

$$\left(\frac{M_x}{Mf_x}\right)^{\alpha} + \left(\frac{M_y}{Mf_y}\right)^{\beta} = 1.0 \qquad (2\text{-}7b)$$

where

M_x M_y = design moments

Mf_x Mf_y = uniaxial bending capacity

α β = empirical constants

Both of Bresler's suggested approximations were criticised by Pannel (3a) in a discussion to the paper. Pannel stated that equation (2-7a) under estimated the actual failure load by up to 20 percent and that equation (2-7b) was up to 8 percent inacurate.

Pannel (29) in 1963 proposed that the three dimensional axial load biaxial moment interaction surface could be represented by a uniaxial load moment interaction diagram plus a biaxial interaction diagram. Pannel further assumed that the biaxial moment interaction could be related to the uniaxial behaviour by the deviation between the major axis moment and the moment about the diagonal and a simple trigonometric relationship as shown in Figure (2-3).

$$Mf_y = \frac{M_y \sec \theta}{1 - N \sin^2 2\theta} \qquad (2\text{-}8)$$

where

Mf_y = ultimate moment about the Y axis

Mf_x = ultimate moment about the X axis

M_y = design ultimate moment about the Y axis

M_x = design ultimate moment about the X axis

ϕ = Mf_y/Mf_x

$\tan \theta$ = M_x/M_y

Pannel also presented a transformation factor by which the results for square section columns could be extended to apply to rectangular sections.

Meek (28) in 1963 suggested modifying the interaction contours obtained when cutting the failure surface horizontally, i.e. at constant values of failure load P_u to two straight lines joining uniaxial bending in the X and Y planes with the skew bending about the diagonal of the section.

In 1966 the Portland Cement Association (34) presented a design aid for the design of short rectangular reinforced concrete columns subjected to biaxial bending. The design aid used Bresler's (3) moment equation (equation (2-76) for obtaining the moment contours obtained when cutting the failure surface of a biaxially loaded column at constant values of the ultimate load P_u'.

Besler's equation was reworked into the following form

$$\left(\frac{M_x}{Mf_x}\right)^{\text{Log } 0.5/\text{Log } \beta} + \left(\frac{M_y}{Mf_y}\right)^{\text{Log } 0.5/\text{Log } \beta} = 1 \qquad (2\text{-}7c)$$

where

Mf_x Mf_y = uniaxial bending capacity

M_x M_y = applied moments

β = the ordinate to the contours at the point at which the relative moments are equal.

The equation is stated to describe the moment contours with a difference of less than 5 percent compared to those obtained by a conventional method using equations of equilibrium with a rectangular stress block.

To obtain numerical values of β, a computer programme was prepared and curves were produced giving the variation of β with P_u'/P_o', where P_o' is the axial column load, for different values of $\rho f_y/f_c'$. Variations in g (cover ratio) b/t and F_c' were found to have only a minor effect on the values of β. The design aid used uniaxial load intersection tables given in a preceeding design aid (33).

Ramamurthy 1966 (35) presented approximate equations to describe the biaxial moment interaction of square and rectangular section columns under biaxially eccentric loads. The equations were derived assuming that the plane of action of the load to be normal to the neutral axis. A transformation factor was provided to transform the load contour of a rectangular section column into an equivalent square section column. The results given by the approximate method were compared with a stress block type analysis and experimental results and was found to be conservative for high loads and adequate for low loads.

Weber (42) in 1966 presented axial load diagonal moment (skew bending) interaction charts for square section columns. Taken, together with the conventional axial load uniaxial moment interaction curves, the prime points on the interaction surface were defined.

Brettle and Warner (4) in 1968 presented interaction diagrams for rectangular section reinforced concrete columns which related the axial load, eccentricity, angle of eccentricity and the aspect of ratio of the cross sectional dimensions of the section. These interaction diagrams were only produced for a restricted number of aspect ratio's, one cover ratio and for sections with 16 or more bars.

The Comité Européen du Béton (15, 16), and Row and Paulay (39) have recently presented biaxial interaction diagrams for rectangular section short reinforced concrete columns.

All the biaxial interaction diagrams discussed in this section were obtained for the ultimate behaviour of the section and were calculated by either imposing a planar strain distribution with a maximum limiting strain on the section and assumed stress strain relationships or by assuming the location of the neutral axis and using a rectangular stress block.

When the effect of slenderness is introduced, a reduction in the ultimate load capacity should be considered due to the P – Δ effects, i.e. a further complication is introduced into the design procedure.

2-4 Slender Columns Under Concentric Load

In 1744 Euler presented the mathematical solution to the problem of buckling of elastic compression members namely

$$P_E = \frac{\pi^2 EI}{L^2} \qquad (2\text{-}9)$$

where

P_E = buckling load

E = modulus of elasticity

I = moment of inertia

L = length of column

The Euler buckling formula was modified to the Tangent Modulus Method by Engesser in 1889 to take into account the material stress strain characteristics.

$$P_{TM} = \frac{\pi^2 \, Et \, I}{L^2} \qquad (2\text{-}10)$$

where

Et = tangent modulus of elasticity at load P_{TM}

2-5 Slender Columns Under Uniaxially Eccentric Loads

In 1958 Broms and Viest (5, 6) presented a theoretical analysis to predict the ultimate load of pinned and restrained slender reinforced concrete columns under uniaxial eccentric loads. The procedure assumed the deflected shape as a consine wave and the relationships between load, strain and moments were derived assuming Hognestad stress strain curve for the concrete and an elastic perfectly plastic stress strain curve for the steel. Restraining moments were assumed proportional to the end rotations of the column.

In 1963 Cheng and Ferguson (9) derived relations for column slope and deflections. The solution of these relations was illustrated by a numerical method using the relations between load, strain and moments previously derived by Broms and Viest. The method does not depend on the cosine wave assumption for the deflected shape.

In 1964 Pfrang and Siess (31, 32) presented an analytical study for long restrained and unrestrained columns. The study used Hognestad's stress strain curve for the concrete and an elastic perfectly plastic stress strain curve for the steel but no assumption was made concerning the deflected shape of the column. A numerical solution using a computer to handle slenderness ratio, eccentricities, percentage of reinforcement, and degree of end restraint, was developed.

In 1965 Riad (38) introduced a strain analysis to obtain the ultimate load of a uniaxially loaded slender column. Riad's approach was based on trial and error. Riad derived curves giving the relationships between load, eccentricity and position of the neutral axis, and these curves were used to establish a curvature diagram from ~~which~~ eccentricities can be checked. Riad's curves however were confined to a particular problem ~~with~~ certain geomentry and several reiterations were needed to achieve a solution.

2-6 Slender Columns Under Biaxially Eccentric Loads

Considering slenderness effects together with biaxially eccentric loads for reinforced concrete columns include numerous variables. Either simplifying assumptions are made which reduce the accuracy or a laborious trial and error procedure is needed. In both cases, for design purposes, the first choice of section is an educated guess relying on experience.

Abolitz (1) presented equations for the working stress design of symmetrically reinforced concrete short and slender columns subjected to flexure. For the problem of combined biaxial bending and axial load weighted averages of certain parameters were used. Straight line interaction diagrams were used for symmetric columns and since these interaction curves are super ellipses in the case of rectangular columns, a correction factor was used in the design method. All equations apply to both uniaxial and biaxial flecture and were divided into sets for tension and for compression failure respectively. To account for slenderness, reduction factors for slender columns bending about the X and Y axes respectively were used as per ACI 318-63. Then, an overall reduction factor was introduced for biaxial bending using the uniaxial slenderness reduction factors and the weighted average of the eccentricities to take account of biaxial slenderness effects.

$$R = \frac{E}{E_x/R_y + E_y/R_y} \qquad (2\text{-}11)$$

where

e_x and e_y = eccentricities about the X and Y axes respectively

t_x and t_y = column depth and width respectively

E_x = e_x/t_x

E_y = e_y/t_y

E = $E_x \quad E_y$

R_x and R_y = reduction factors for slender columns bending about the X and Y axes respectively.

In 1968 Pannel and Robinson (30) suggested an approximate means of calculating the final eccentricity of slender reinforced concrete columns under biaxially eccentric loads. It was assumed that the failure of a slender column under biaxial load could be predicted in terms of an equivalent axially loaded column and the moment failure surface of a similar short column under biaxial bending. Linear relationships were assumed between the interaction curves in the X and Y planes and the plane of bending in the failure surface. The maximum eccentricities e_x and e_y were given by the following equations

$$e_x = \frac{ei_x}{1 - \dfrac{es_x}{ef_x}}$$

$$e_y = \frac{ei_y}{1 - \dfrac{es_y}{ef_y}} \tag{2-12}$$

where

ei_x and ei_y - initial eccentricities

P_{sx} and P_{sy} - axial failure loads of a similar slender column constrained to fail in the X plane and in the Y plane respectively

M_{sx} and M_{sy} - moments opposite to P_{sx} in the X plane and P_{sy} in the U plane respectively in the interaction curves of a similar short column

es_x - M_{sx}/P_{sx}

es_y - M_{sy}/P_{sy}

Pf = failure load of the biaxially loaded slender column

Mf_x and Mf_y - failure moments opposite to Pf in the X and Y planes respectively in the interaction curves of the similar short column

ef_x - Mf_x/Pf

ef_y = Mf_y/Pf

In 1969 Farah and Huggins (21) presented an intergration method to analyse reinforced concrete columns subjected to longitudinal load and biaxial bending. It lead to three non linear simultaneous equations which are solved by a procedure based on the Newton Raphson method. A linear strain distribution was defined in terms of the values of strain at the corners ε_1, ε_2, and ε_3 and the corresponding values of load $\overline{P}$, and moments $\overline{M_x}$, and $\overline{M_y}$ calculated.

Then, having

$$\begin{aligned} P &= P(\varepsilon_1, \varepsilon_2, \varepsilon_3) \\ M_x &= M_x(\varepsilon_1, \varepsilon_2, \varepsilon_3) \\ M_y &= M_y(\varepsilon_1, \varepsilon_2, \varepsilon_3) \end{aligned} \tag{2-13}$$

The expressions for $\overline{P}$, $\overline{M_x}$, and $\overline{M_y}$ were expanded by Taylor's theorem neglecting second order and higher terms to get

$$\begin{aligned} P &= \overline{P} - \frac{\partial P}{\partial \varepsilon_1} \delta \varepsilon_1 - \frac{\partial P}{\partial \varepsilon_2} \delta \varepsilon_2 - \frac{\partial P}{\partial \varepsilon_3} \delta \varepsilon_3 \\ M_x &= \overline{M_x} - \frac{\partial M_x}{\partial \varepsilon_1} \delta \varepsilon_1 - \frac{\partial M_x}{\partial \varepsilon_2} \delta \varepsilon_2 - \frac{\partial M_x}{\partial \varepsilon_3} \delta \varepsilon_3 \\ M_y &= \overline{M_y} - \frac{\partial M_y}{\partial \varepsilon_1} \delta \varepsilon_1 - \frac{\partial M_y}{\partial \varepsilon_2} \delta \varepsilon_2 - \frac{\partial M_y}{\partial \varepsilon_3} \delta \varepsilon_3 \end{aligned} \tag{2-14}$$

Then if the partial derivatives in the previous equation are replaced by the corresponding difference gradients and by incrementing each strain quantity at a time, the rates of change could be calculated, then the simultaneous equations (2-14) are solved and $\delta \varepsilon_1$, $\delta \varepsilon_2$ and $\delta \varepsilon_3$ determined. These increments if added to the initial strains would give a new

$\overline{P}$, $\overline{M_x}$, and $\overline{M_y}$. If the values of $\overline{P}$, $\overline{M_x}$, and $\overline{M_y}$ are not equal to P, M_x, and M_y the process should be repeated until convergence is obtained. For slender columns, the column is divided into segements, then eccentricities plus deflection at mid-column height are assumed, thus giving values of M_x and M_y with the given load P. Using equations (2-14), the strain distribution giving P, M_x, and M_y could be determined. Having the strain distribution, curvature at the middle segment could be calculated and subsequently the values of eccentricities at the end of the first segment. The procedure would continue until the eccentricities at the top of the column are obtained. If these calculated eccentricities are not equal to the given ones, the procedure is repeated using readjusted eccentricities (curvatures) at mid-column height until a correct answer is obtained.

In 1973 (10) and 1974 (11) Cheng and Mirza presented a study of the behaviour of reinforced concrete square section columns under biaxially eccentric loading. Their method of analysis is essentially equivalent to that of Huggins and Farah (21), but using strain and curvature instead of corner strains as the deformation variables in the Taylors expansion to satisfy equilibrium at a section.

Lateral deflection was calculated by moment area from the curvatures rather than numerically as in Huggins and Farah.

A similar method to Huggins and Farah (21) was presented by Cranston (13) in 1967 for restrained uniaxially loaded slender columns using a computer programme.

Drysdale 1967 (18) and Drysdale and Huggins 1971 (19) investigated the behaviour of slender reinforced concrete columns subjected to sustained biaxially eccentric loading.

Drysdale's theoretical investigation considered the strain on a section to be composed of three components namely elastic strain, shrinkage and creep. At given time intervals the strain distribution at any section was modified by the strain due to creep and shrinkage. As element stress strain compatability and equilibrium would no longer be satisfied, the elastic strain distribution was modified until compatability and equilibrium were again satisfied.

Drysdale also carried out an experimental investigation and concluded that the load capacity of a moderately slender square section symmetrically reinforced concrete column can decrease by 20 to 25 percent due to a sustained load of 25 years duration.

2-7 ACI Design Method for Reinforced Concrete Columns

The 1963 version of ACI 318-63 Building Code requirements (7) recommended that a slender column under uniaxially eccentric load could be designed as short columns but both the moment and axial load be increased to allow for slenderness effects. The decrease of load capability with slenderness ratio was linear of the form given below

$$R = 1.07 - 0.0075 \frac{kL}{r} \qquad (2\text{-}15)$$

The ACI 318-63 Building Code Requirements recommended that long columns under biaxially eccentric loads should be designed as short columns with the axial load and both moments increased by the inverse slenderness factor. The slenderness factor being calculated for the most severe case of the two axes.

The 1971 version of ACI 318-71 Building Code Requirements (8) recommends that a slender column be designed taking into account the influence of the axial loads, variable stiffness, the effect of lateral deflection and the duration of the loads.

Alternatively an approximate method given in the Code may be used which states that compression members shall be designed for the axial load obtained from frame analysis and a magnified moment.

The moment magnifier is given by:

$$\delta = \frac{C_m}{1 - P_u/\phi' P_c} \quad 1.0 \qquad (2\text{-}16)$$

where

P_u = axial design load

P_c = critical load

ϕ' = capacity reduction fraction

C_m = $0.6 + 0.4\, M_1/M_2$

M_1 and M_2 values of smaller and larger moments respectively on compression member calculated from conventional frame analysis.

The use of C_m is needed as in the derivation of the moment magnifier it is assumed that the maximum moment is at or near mid height. If the maximum moment occurs at one end, the design must be based on an equivalent uniform moment, $C_m M_2$.

However all the terms in the moment magnifier are not well defined or easily obtained. In the original derivation of the equation the factor ϕ' in the denominator was not included and secondly the values of EI suggested are only approximate.

The errors in the formulae for EI suggested by MacGroger, Breen, and Pfrang (25) were a factor of between 0 and 150 percent for equation (1) and up to 500 percent for equation (2). The effect of creep on EI values was taken into consideration by dividing values given by equations (1) and (2) by $1 + B_d$ where B_d is the ratio of dead load moment to total load moment.

Drysdale, Sallam, and Tan (20) have concluded that the moment magnifier method does not result in consistent safety factors.

2-8 Summary

Interaction curves are available which give the biaxial failure envelope for restricted reinforced concrete sections. These interaction curves have generally been derived assuming a rectangular stress block, and the derived interaction curves give no information concerning the curvature or-strain necessary to develop a given point on the interaction envelope.

Alternatively many complete computer programmes exist which can analyse the behaviour of any given column under any load pattern including the effects of creep and shrinkage.

Currant design methods ACI 318-71 and CSA A23.3 1974 allow the designer to detail a long column using short column interaction curves by increasing the design moment by a moment magnifier. For biaxially eccentrically loaded long columns the design moments in each orthogonal plane must be magnified separately and then combined - even if the magnified moments do not occur at the same section. The moment magnifier does not take into account the strain or curvature on the section.

More detailed interaction curves for both uniaxially and biaxially eccentric loading are needed which include information on the strain and curvature inherent in generating any point on the interaction surface. By summation of the load-moment-curvature along a column, a better estimate of the column behaviour could be made.

CHAPTER III

SCOPE OF PRESENT INVESTIGATION

If a strain distribution is imposed upon a reinforced concrete section, stresses result in the various steel and concrete elements of the section. By algebraically summing the elemental forces due to the stress on the various elements, the external applied force and moment necessary for equilibrium can be determined corresponding to the imposed strain distribution. By successively imposing a number of strain distributions onto an assumed section, the total axial load, moment, strain distribution interaction of the assumed section can be determined.

In engineering design applications, the reverse situation exists - the external force and moment are known and it is necessary to determine the section required to satisfactorily support the known loads. A further complication in design examples is that the external moment applied to a section is composed of the external applied moment plus a contribution from slenderness (PΔ) effects.

The object of this thesis was to develop section property charts giving the relationships between axial load, biaxial moment, curvature and maximum strain. Hence a design method for slender reinforced concrete columns under biaxially eccentric loads was devised using the derived charts. The $P\Delta$ moment of a slender column under a given axial load and end moments can be estimated from compatability considerations knowing the end curvatures and using an assumed deflected shape. The method facilitates first choice of section and also provides a means of control on the trial and error procedure inherent in the design. The method satisfies compatability of strain (curvature) and lateral deformation along the height of the column. However it does not consider the effect of creep and shrinkage and these must be considered in the same manner as in current design methods.

CHAPTER IV

THEORETICAL INVESTIGATION

4-1 Introduction

A method for calculating section property charts is presented in this Chapter. These charts were produced to illustrate the relationships between the different properties of a reinforced concrete section. The study is first limited to square symmetrically reinforced concrete sections, then the use of the same curves for rectangular sections will be studied. A strain analysis was used to produce these charts which can be summarized as imposing a number of strain distributions on the cross section and studying the equilibrium of internal stress and external forces to give the complete axial load, moment and strain distribution behaviour. Before calculating section properties, a series of assumptions must be made. The assumptions listed below are well accepted in engineering practice and are not peculiar to this thesis.

4-2 Assumptions

1) A planar strain distribution exists over the cross section at all load levels;

2) The stress-strain relationship for concrete in compression is assumed as Hognestad's relationship as shown in Figure (4-1);

3) No tensile stress can be taken by the concrete;

4) Maximum strain at the extreme compression fiber at ultimate strength shall not exceed 0.003;

5) Steel in both tension and compression behaves in an elastic perfectly plastic manner (see Figure (4-2));

6) The angle of eccentricity is constant along the column.

4-3 Section Property Curves

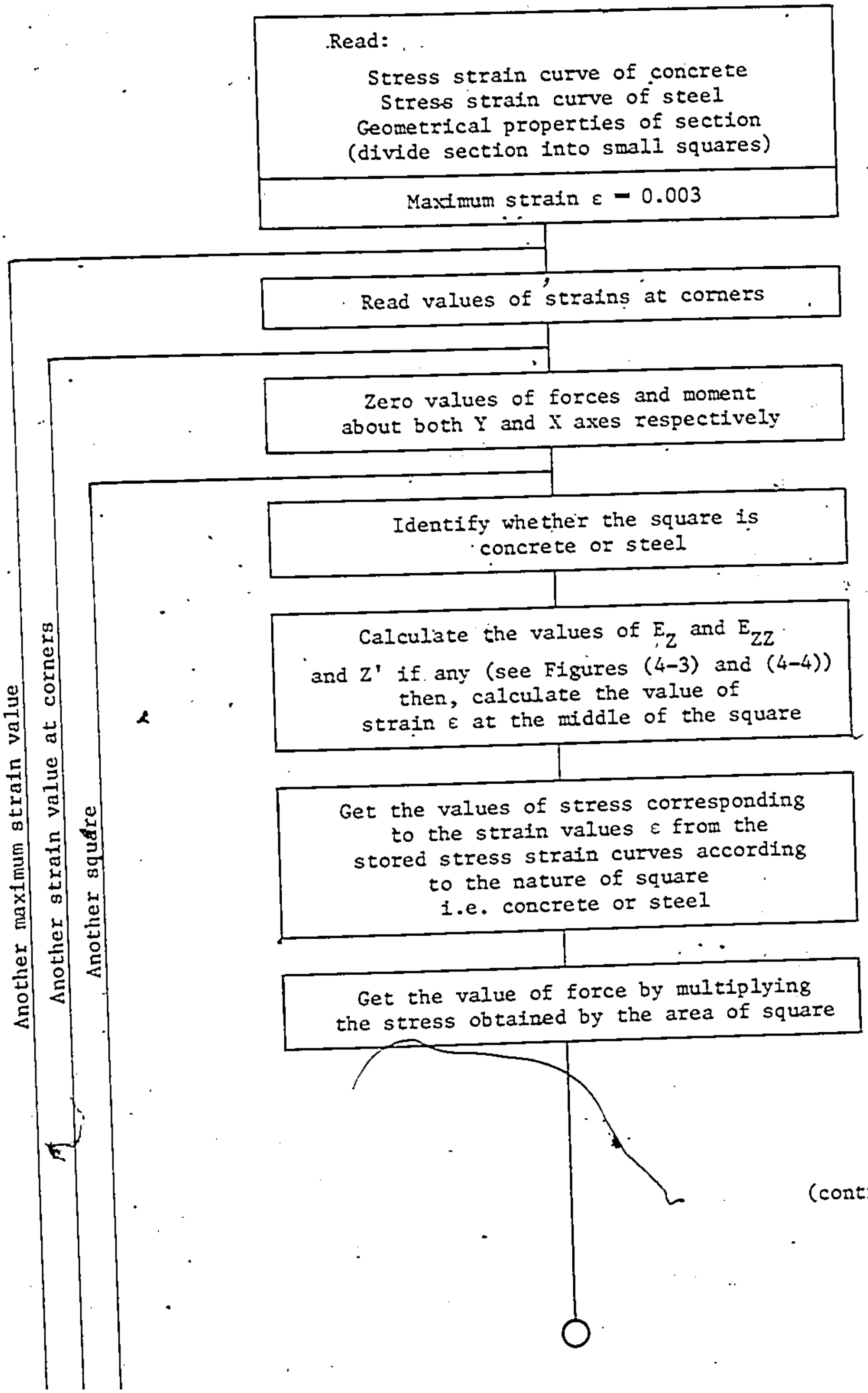
Numerous strain distributions were imposed on a nominal cross section (t.t) to give resultant forces at angles of eccentricity 0° and 45°, see Figures (4-3) and (4-4). The nominal cross section was divided into 144 elements each one square unit in area. For each strain distribution, the strain at the center of each element was determined and the elemental stress was obtained using the relevant (concrete or steel) stress-strain curves. Subsequently, the external force resultant needed to satisfy equilibrium was calculated by summing up elemental forces (product of element area with the stress of the centroid of the area). The external moments about the X and Y axes respectively were determined by summing up the moments of the external forces (product of elemental forces with the distance of squares centers from the X and Y axes respectively). To obtain the eccentricity e, the appropriate moments were divided by the total force for each strain distribution assumed to give e_x and e_y and hence e (which equals $\sqrt{e_x^2 + e_y^2}$). The value of

Z (distance from the point of maximum strain normal to the neutral axis) was determined from the geometry of each assumed strain distribution. Subsequently all values were non dimensionalized to obtain P_u / f_c' bt, e/t and Z/t, where t is the section depth and b is its width. The previous steps were executed by a computer program whose flow chart is shown in Figure (4-5) and listings for the different cases are included in the Appendix. The results of the section analyses were plotted as curves showing the relationships between P_u'/f_c' bt, e/t, and Z/t for different concrete and steel strengths (governed by the assumed stress-strain curves of concrete and steel), different steel ratios (governed by the area of reinforcement of the nominal section), different maximum strain values (governed by the maximum strain value assumed) and two angles of eccentricity 0° and 45° and a linear interpolation can be used for angles in between.

4-4 Calculating the Values of Curvature

If any cross section of a column is subjected to a linear strain distribution, the value of the maximum strain ε_{max} and the distance from the point of maximum strain normal to the neutral axis Z are known. Hence the curvature which equals ε_{max}/Z can be obtained.

However, the plane of curvature need not in general be the plane of action of the applied moment. If the line passing through the


Read:
Stress strain curve of concrete
Stress strain curve of steel
Geometrical properties of section
(divide section into small squares)
Maximum strain ε = 0.003
Read values of strains at corners
Zero values of forces and moment about both Y and X axes respectively
Identify whether the square is concrete or steel
Calculate the values of E_Z and E_{ZZ} and Z' if any (see Figures (4-3) and (4-4)) then, calculate the value of strain ε at the middle of the square
Get the values of stress corresponding to the strain values ε from the stored stress strain curves according to the nature of square i.e. concrete or steel
Get the value of force by multiplying the stress obtained by the area of square
Another maximum strain value
Another strain value at corners
Another square

(continued)

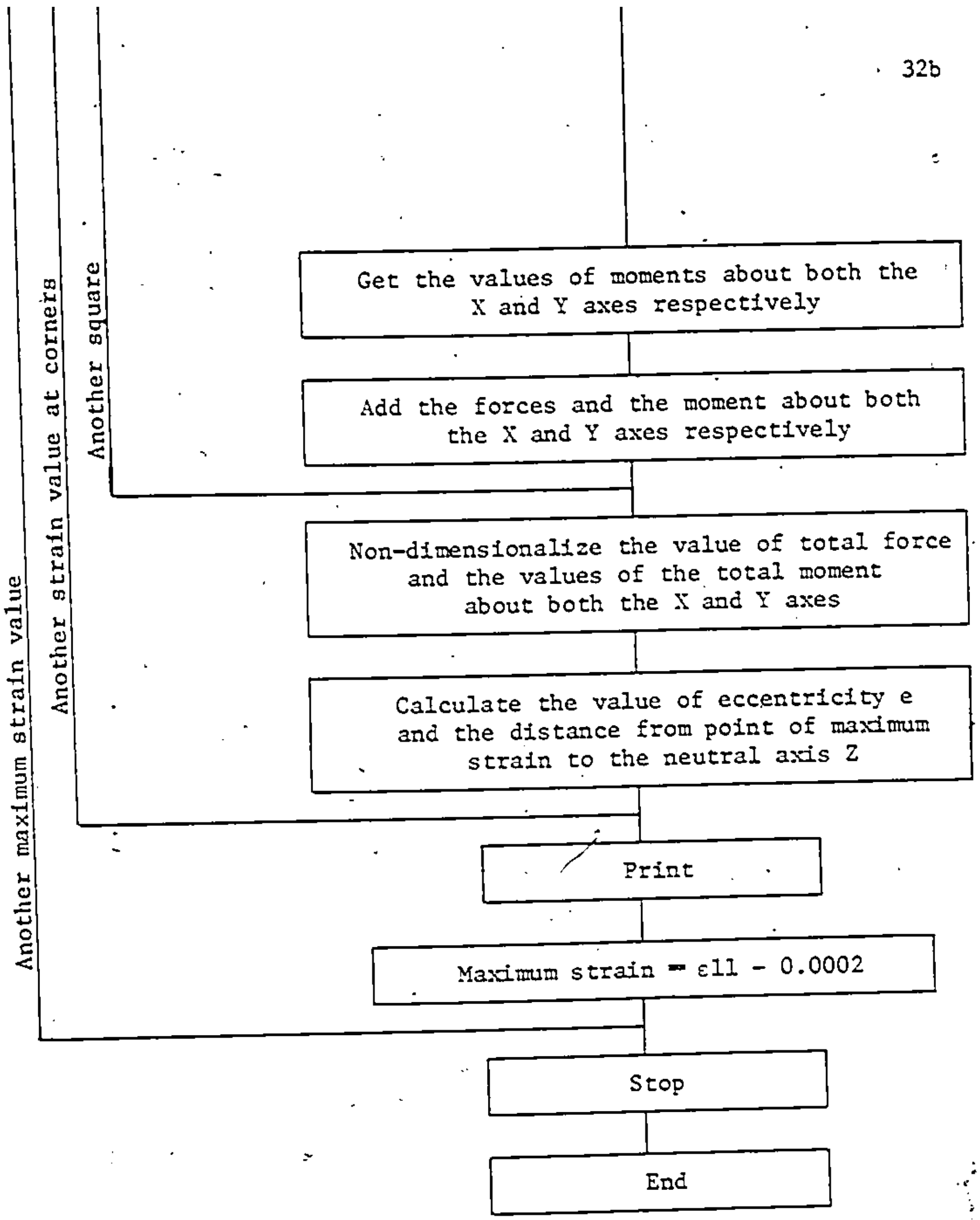


FIGURE (4-5) FLOW CHART OF PROGRAM FOR DEVELOPING INTERACTION CURVES point of application of the load and the centroid of the section intersects the neutral axis at a right angle, then the plane of curvature is parallel to the plane of action of the load. To determine if the line of curvature is parallel to the plane of action of the load two different strain distribution (shown in Figures (4-6) and (4-7)) were imposed on a symmetrically reinforced concrete section. Two strain distributions with maximum strain values of 0.002 and 0.003 were chosen to give different values of load, eccentricities and angle of eccentricity. As shown, a line drawn from the point of application of the load passing through the center of gravity of the section is normal to the neutral axis irrespective of the values of load, eccentricity, angle of eccentricity and maximum strain. From these demonstrations and others, it is concluded that the curvature in symmetric sections is in a plane parallel to the plane of action of the load. Pannel (29) and Ramamurthy (35) arrived at the same conclusion by the same reasoning.

4-5 Effect of Combining Parameters

The difficulty in analyzing the behaviour of reinforced concrete sections under biaxially eccentric loads is due to the interaction of many parameters. However, it would be very helpful if some of these parameters could be combined together without significantly effecting the results. In this work the reinforcement ratio ρf_y was identified as reasonably independent parameter and the effect of cover ratio was taken into account by using a single factor to modify results obtained for one cover ratio to another cover ratio.

4-5-1 Effect of combining $\rho f_y/f_c'$ in one factor

The parameters $\rho f_y/f_c'$ where ρ is the percentage of steel, f_y is steel yield strength and f_c' is the concrete compressive strength, has been used by numerous investigators (22), (29), and is a reasonably independent parameter of column strength irrespective of eccentricity, angle of eccentricity, maximum strain or magnitude of axial load. This parameter can be logically identified as independent for pure bending or pure moment where strain compatability effects are unimportant. In general, however, it is only possible to demonstrate that this parameter is independent. Figure (2-8) was obtained using two combinations of ρ, f_y and f_c' to give a value of $\rho f_y/f_c' = 0.6$. The two combinations were $\rho = 3\%$, $f_y = 60{,}000$ psi and $f_c' = 6{,}000$ psi for the first and $\rho = 6\%$, $f_y = 45{,}000$ psi and $f_c' = 4{,}500$ psi for the second. For each of the two combinations, the interaction of $M_u'/f_c'\, bt^2$ with $P_u'/f_c'\, bt$ curve was obtained for the extreme angles of eccentricity of 0^o and 45^o. Two values of maximum strain $\varepsilon_{max} = 0.0030$ and 0.0020 were used, thus covering the effect of combining ρ, f_y and f_c' in one factor on eccentricity and its angle, load and maximum strain. The figure shows less than 2.5% difference in case of $\varepsilon_{max} = 0.0030$ and less than 5% difference in case of $\varepsilon_{max} = 0.0020$ according to a Chi Square test. To investigate the effect of combining $\rho f_y/f_c'$ in one factor on curvature the relationships between Z/t and P_u'/f_c' bt was plotted in Figure (4-9) for the above mentioned two combinations of ρ, f_y and f_c' giving $\rho f_y/f_c' = 0.6$. Again two maximum strain values $\varepsilon_{max} = 0.0030$ and 0.0020 were used and the curves were obtained for the extreme angles of eccentricity of 0^o and 45^o. The figure shows less than 3% of difference for the different cases according to the Chi Square Test. Thus the use of $\rho f_y/f_c'$ as one factor is justified as it does not have a significant effect on results irrespective of the values of axial load, eccentricity and its angle, maximum strain and curvature.

Four sets of section property curves were produced for corner reinforced square section columns with reinforcement ratio's $\frac{\rho f_y}{f_c'} = 0.2$, 0.6, 1.0, and 1.4 respectively for a nominal cover ratio of 0.75. Each set of curves consists of ten similar graphs for various maximum strain values ranging from 0.0030 down to 0.0012 at an interval of 0.0002. The curves are presented in Figures (4-10) through (4-29).

4-6 Cover Ratio Effect

In developing section behaviour curves, the cover ratio effect cannot be ignored. However, if the effect on section properties is univalued, one set of section property curves can be developed for a certain value of cover ratio, and used with a modifying factor for other cover ratios. To study the effect of cover ratio, typical $M_u'/f_c'\ bt^2$ versus $P_u'/f_c'\ bt$ curves were plotted for two different cover ratios, 0.6, and 0.75, for angles of eccentricity of 0^o and 45^o and for two values of maximum strain ε_{max} = 0.0030 and 0.0020. The results were plotted for two different values of $\rho f_y/f_c'$, namely 0.2 and 1.0, and are given in Figures (4-30) through (4-33). To investigate the effect of cover ratio on curvature, Z/t versus P_u'/f_c' bt relationship was plotted for the same two cover ratios, 0.6 and 0.75, for the same $\rho f_y/f_c'$ = 0.2 and 1.0, two maximum strain values of ε_{max} = 0.0030 and 0.0020 and the extreme eccentricity angles of 0° and 45° in Figures (4-34) through (4-37).

It was found from Figures (4-30) through (4-33) that M_u'/f_c' bt^2 versus P_u'/f_c' bt curves would coincide with each other if one is multiplied by the ratio of the balanced moments irrespective of the values of eccentricity, angle of eccentricity load, maximum strain and $\rho f_y/f_c'$ value. In these figures, the use of the single value factor F which is the ratio of the two balanced moments for the two different cover ratios gives a difference ranging from 2.5% to 8% according to the Chi-Square Test.

The effect of cover ratio on values of Z/t as shown in Figures (4-34) through (4-37) is very small and the Chi-Square Test shows an average difference of 3% in Z/t values obtained with cover ratios equal 0.6 and 0.75. Thus, if the effect of cover ratio on the values of maximum strain is taken care of, the error in curvature will be insignificant.

In Figure (4-38) a set of curves is presented to give the value of F for different cover ratios and different values of $\rho f_y/f_c'$

relative to a cover ratio of 0.75 (the cover ratio of the nominal cross section used to develop the given set of property curves). These values were produced by a computer program whose flow chart is shown in Figure (4-39) and listing included in the Appendix.

4-7 Accuracy of the Numerical Calculation

In producing the section capacity curves, a 12 x 12 grid was used. To check the accuracy of the chosen grid, a 18 x 18 grid was tried for $\rho f_y/f_c' = 0.6$ and it was found that the arithmetic mean of error due to using a 12 x 12 grid instead of 18 x 18 grid is $^+0.89$ percent. Similar conclusions were also reached by Drysdale (18) and Brettle and Warner (4).

4-8 Slenderness

In long columns, the reduction of ultimate load due to the PΔ effect where P is the load and Δ is the lateral deflection must be considered. To calculate the PΔ effect without affecting the accuracy, compatability of strains and lateral deformation along the column height should be considered. If the values of curvature are known at the ends of the column and the deflected shape and the curvature assumed at the critical section, the lateral deflection at the critical section can be

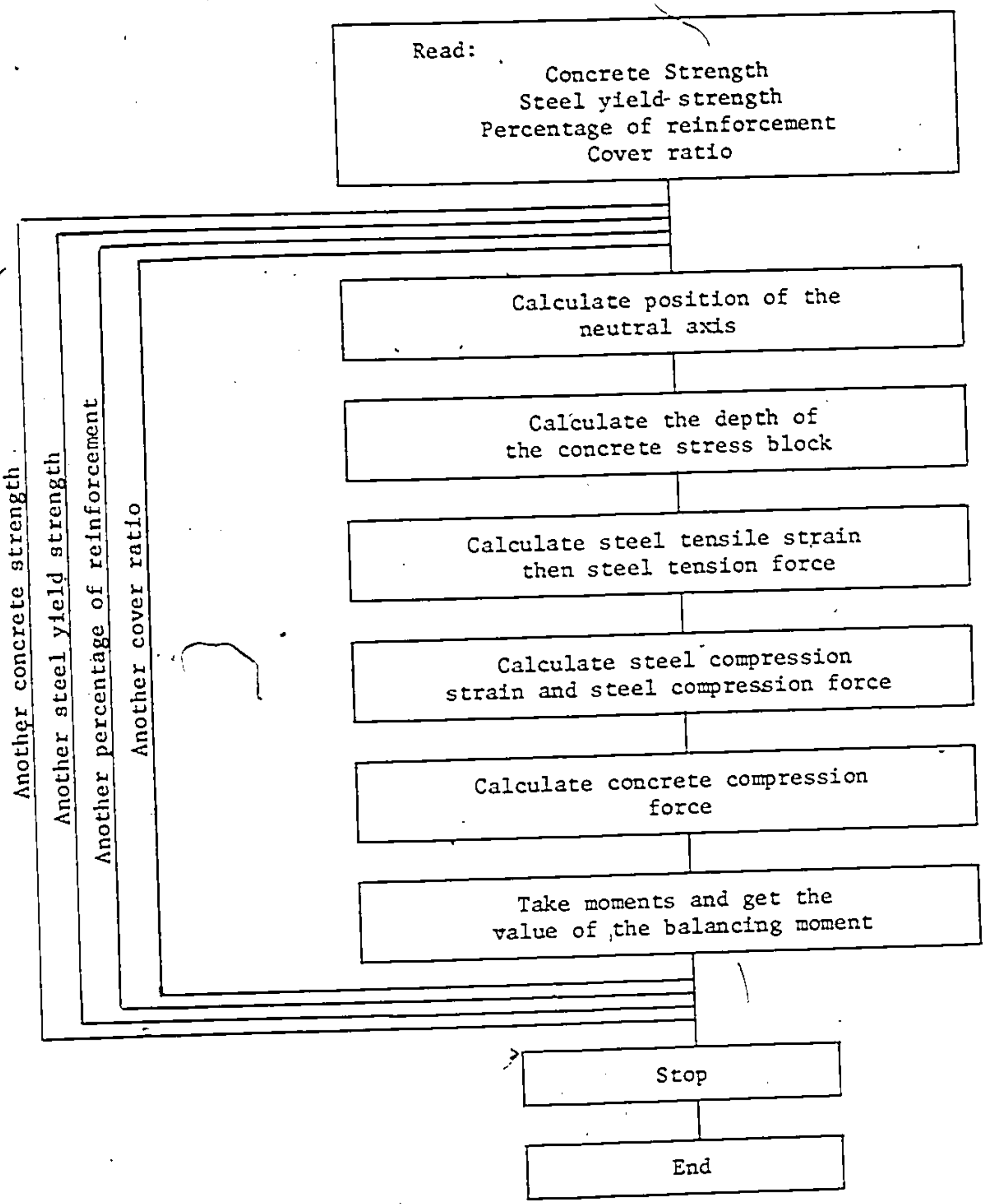


FIGURE (4-39) FLOW CHART FOR CALCULATION OF BALANCED MOMENTS FOR DIFFERENT CONCRETE AND STEEL STRENGTHS, PERCENTAGE OF REINFORCEMENT AND COVER RATIO calculated. To satisfy equilibrium at the critical section under the axial load and the calculated moment a certain value of curvature and strain are needed. If this curvature at the critical section is approximately equal to the assumed curvature then the calculated lateral deflection is exact. If the assumed and calculated curvatures are not equal, a further reiteration must be undertaken. If the strain necessary to satisfy curvature - lateral displacement compatability is less then 0.0030 then the section is adequate.

4-9 Rectangular Sections

As it was illustrated previously, the presented curves were calculated for corner reinforced square section concrete columns. However, Pannel (29) suggested that the use of a single valued factor will enable the use of the same curves for corner reinforced rectangular section columns.

If the failure surface (see Figure (4-40)) is cut horizontally, i.e. for constant values of ultimate load P_u', moment contours will be obtained. These contours are approximately of the shape of circles in square columns and approximately of the shape of elliptical arcs in rectangular columns (see Pannel (29)). However, when the horizontal coordinates of the moment contours for the rectangular column are multiplied by Mf_y/Mf_x (see Figure (4-41)), the moment contours of the analogous square column will be obtained.

Then, if a rectangular column is to be designed to carry an ultimate load P_u' with eccentricities e_x and e_y about the X and Y axes respectively, the presented curves could be used, to design the analogous square column having the same depth, if e_x is multiplied by the ratio of Mf_y/Mf_x. Then having the appropriate square section to sustain P_u' at $\phi\ e_x$ and e_y where $\phi = Mf_y/Mf_x$, the analogous rectangular column will be able to carry P_u' at eccentricities e_x and e_y.

The ratios of Mf_y/Mf_x for a rectangular column is equal to Mb_y/Mb_x where Mb_y and Mb_x are the balanced moments about the Y and X axes respectively. To justify the $Mf_y/Mf_x = Mb_y/Mb_x$, the P_u'/f_c' bt versus $M_u'/f_c\ bt^2$, curves were plotted for a rectangular column (12 x 20) about both the X and Y axes. These curves were produced for two maximum strain values $\varepsilon_{max} = 0.0030$ and 0.0016. Thus, the curves of P_u'/f_c' bt versus $Mf_y/f_c'\ bt^2$ were multiplied by $\phi = \frac{Mb_y}{Mb_x}$ to coincide with those of P_u'/f_c' bt versus $Mf_x/f_c'\ bt^2$. The Chi-Square Test gives less than 5% difference (see Figure (4-42)).

Furthermore, to check the effect of using the single value factor to convert the contours of a rectangular column to its analogous square one with the same depth, the previously mentioned curves of P_u'/f_c' bt versus $M_u/f_c'\ bt^2$ were plotted for two columns, a rectangular column (12 x 20) and a square one (20 x 20). The curves were calculated for two values of maximum strain $\varepsilon_{max} = 0.0030$ and 0.0016 and for the extreme eccentricity angles of 0° and 45°. Subsequently, the curves of the rectangular column were multiplied by ϕ to coincide with those of the analogous square column having the same depth. The Chi-Zquare Test shows a difference of 5% to 9% (see Figures (4-43) and (4.44)). It should be noted however that, in case of 45° eccentricity, a modification in the value of ϕ should be made to account for the elliptical shape contours of the rectangular column, refering to the mathematical equation of the ellipse (see Figure (4-45))

$$\phi_{45} = \frac{\sqrt{a^2 + b^2}}{a\sqrt{2}} \phi_0$$

where a and b are half the length of the major and minor axes of the ellipse respresenting Mf_y and Mf_x respectively.

To investigate the effect of using ϕ in case of rectangular columns on curvature, the values of Z/t were plotted against P_u'/f_c' bt for both a square (20 x 20) and a rectangular (12 x 20) column. Curves were plotted for $\epsilon_{max} = 0.0030$ and 0.0016 and two angles of eccentricity 0° and 45°. The Chi-Square Test shows a difference ranging from 5% to 8% (see Figures (4-46) and (4-47)).

The value of $\phi = Mb_x/Mb_y$ in rectangular columns symmetrically reinforced concrete sections is always equal to b/t.

It should be noted however that g for the rectangular column should be the same in both directions irrespective of t/b value.

CHAPTER V

EXPERIMENTAL INVESTIGATION

5-1 Introduction

An experimental investigation was carried out on four groups of columns to compare the values of ultimate loads and deflections predicted by the proposed method with those obtained experimentally. Steel ratio, eccentricity and its angle to a reference axis and length of column were the major variables taken into consideration in this program. A total of 44 columns, divided into four groups of 11 each, according to length and steel ratio were investigated. All samples were cast and tested horizontally in a pinned-pinned configuration until failure and deflections were measured in two orthogonal planes perpendicular to the faces of the column. Table (5-1) shows the combination of variables used in that test program.

5-2 Fabrication of Test Columns

5-2-1 Description of samples

All columns had a nominal cross section of 6" x 6" and enlarged ends of 12" x 12" to enable loads to be applied outside the section. The columns were 90" or 134" long to give (for a pinned-pinned configuration) slenderness ratios of 50 and approximately 75 respectively.

Steel Ratio	Cylinder Strength	Length	e_x	e_y
			0.00	2.50
ρ = 6.67%			1.25	2.50
# 7 bars		L/r = 50	2.50	2.50
f_y = 67000 psi	nominal	L = 90"	0.00	5.00
6" x 6" section.	4000 psi		1.66	5.00
			3.33	5.00
ρ = 1.22%			5.00	5.00
# 3 bars		L/r = 75	0.00	7.50
f_y = 65000 psi		L = 134"	2.50	7.50
6" x 6" section			5.00	7.50
			7.50	7.50

TABLE (5-1) Combination of Variables Considered in the Experimental Investigation

* Accuracy of eccentricities is ± 0.05 inch

5-2-2 Details of reinforcing cages

Two sizes of steel reinforcing were used, # 3 and # 7 bars giving steel ratios of 1.22% and 6.67% respectively. All columns cross sections were symmetrically reinforced with 4 bars, one at each corner giving 0.45" and 0.55" of cover for # 3 bars and # 7 bars respectively. Number 3 stirrups at 4" centers were used in columns while the spacing was reduced to 2.25" at the enlarged ends to prevent shear failure. The details of the columns, reinforcing bars and stirrups are shown in Figures (5-1) through (5-3).

5-2-3 Details of forms

Steel forms which consist of No. 15 channel at the bottom and two No. 12 channels, 12" apart for the sides were used. One side of the form was fixed while the other side could be moved. A wood filling was prepared to give a cross section of 6" x 6" changing smoothly to 12" x 12" at the ends. The form had threaded rods with nuts at each end to be able to adjust the movable side to the right dimensions without developing difficulty when removing the columns from the forms. Three screws with nuts welded to the bottom of the form were used to provide the movable side with horizontal supports (one in the middle and two near the ends) in order to prevent it from outside bowing under the fresh weight of concrete. The details of the forms used are shown in Figure (5-4).

To prepare forms for casting, they were cleaned and the wood filling was placed in its position in the middle of the form. The required dimensions of the form were adjusted by the two end threaded rods and the three side screws. The wood filler was covered with polyethylene and the whole form oiled to prevent the concrete from sticking to it. The steel cages were centered on small precast concrete blocks to provide the specified cover.

5-2-4 Casting and curing

Ready-mix concrete of the specified compressive strength, aggregate size and slump were used to cast the columns. Columns were cast horizontally in groups of five. To determine compressive strength four 6" diameter x 12" cylinders were cast with each column, making a total of 20 for a single cast. An electrically driven vibrator was used to vibrate the concrete and a slump test was taken for each pour to check workability. After 24 hours, columns and cylinders were removed from their forms and covered with wet burlap for 28 days.

5-3 Test Apparatus

5-3-1 Details of roller bearing and end plates

Since it was desired to test columns in a pinned-pinned configuration, special pin bearings were fabricated. Basically, each bearing consisted of a 2" ball seating into two machined sockets. Sockets were cylinders 6" in diameter with a conical groove 0.625" deep to accommodate the 2.0" ball in between. To minimize the effect of friction during testing, the bearings were pre-loaded to 280 kips to plastically deform the conical shaped groove into a spherical one. The end bearings were mounted to end plates 1" thick. Each end plate had 11 machined holes at the desired locations from the column center line in order to locate the axial load accurately. Two angles were welded to the back of each of the end plates at right angles to help mount the end plates to the columns ends accurately. The details of end bearings and end plates are shown in Figures (5-5) and (5-6).

5-3-2 Details of end brackets

Since the columns were to be tested horizontally where the load was to be applied through a jack at one end and measured by a load cell at the other end, two end brackets were needed on which to mount both the jack and the load cell. The two end brackets were fabricated with steel channels and plates and fixed to the 3 feet thick strong floor of the laboratory. Each bracket was bolted to the strong floor with two 2.0" diameter high tensile steel bolts which passed through holes in the floor. To prevent twisting, each backet was braced horizontally to the strong floor at two points by means of two steel plates welded to it at one end and fixed the strong floor with bolts at the other end.

The details of the end brackets are shown in Figures (5-7) ~~and (5-8)~~.

5-4 Test Procedure

5-4-1 Testing of concrete cylinders and steel reinforcing bars

To determine the column concrete compressive strength, the four cylinders casted with it were tested to failure on the same day. To prepare the cylinders for testing, they were capped with a standard sulfur compound. A 300,000 lbs capacity Forney hydraulic concrete testing machine was used to test the cylinders until failure. The average compressive strength of the four cylinders was taken to be the compressive strength of concrete. These results are shown in Tables (5-2) through (5-5).

The steel yield strength of the reinforcement was determined from samples tested in tension until failure. Stress-strain diagrams were drawn for each sample to obtain the yield stress and again the average value of yield strength would give the yield strength of the reinforcing bars used. The stress-strain curves for both # 3 and # 7 reinforcement are shown in Figures (5-10) to (5-11).

5-4-2 Testing of columns

Columns were tested in a horizontal pinned-pinned configuration with load applied by a hydraulic jack at one end and measured

Column No. / Compressive Force	Cylinder 1 kips	Cylinder 2 kips	Cylinder 3 kips	Cylinder 4 kips	Average kips	Compressive Force of Concrete psi	Date of Cast
A_1	89	93	95	98	94	3320	6/7/73
A_2	93	119	109	103	106	3750	26/6/73
A_3	112	110	105	121	112	3960	26/6/73
A_4	123	121	118	110	118	4170	26/6/73
A_5	110	113	116	107	111.5	3940	26/6/73
A_6	90	96	96	92	94	3310	6/7/73
A_7	98	100	95	103	99	3500	6/7/73
A_8	125	135	144	150	138	4900	15/6/73
A_9	123	117	112	124	119	4210	7/6/74
A_{10}	140	133	123	133	132	4680	15/6/73
A_{11}	133	143	119	148	135	4780	15/6/73

TABLE (5-2) Concrete Compressive Strength

GROUP A

Compressive Force / Column No.	Cylinder 1 kips	Cylinder 2 kips	Cylinder 3 kips	Cylinder 4 kips	Average kips	Compressive Force of Concrete psi	Date of Cast
B_1	119	120	113	122	118	4190	7/6/73
B_2	124	111	115	109	114	4060	7/6/73
B_3	118	114	116	118	116	4140	7/6/73
B_4	111	117	125	121	118	4190	7/6/73
B_5	114	121	122	124	120	4250	4/6/73
B_6	117	125	115	120	119	4220	4/6/73
B_7	123	114	107	120	116	4100	4/6/73
B_8	106	108	109	103	106	3770	10/4/73
B_9	115	109	107	123	113	4010	4/6/73
B_{10}	96	115	102	106	105	4000	10/4/73
B_{11}	104	107	103	108	105	3730	10/4/73

TABLE (5-3) Concrete Compressive Strength

GROUP B

Compressive Force / Column No.	Cylinder 1 kips	Cylinder 2 kips	Cylinder 3 kips	Cylinder 4 kips	Average kips	Compressive Force of Concrete psi	Date of Cast
C_1	165	145	141	148	149	5300	30/1/73
C_2	140	154	150	151	148	5260	30/1/73
C_3	138	132	139	139	137	4850	9/3/73
C_4	141	150	130	133	138	4900	30/1/73
C_5	144	138	143	140	141	5000	30/1/73
C_6	112	128	124	118	120	4260	9/3/73
C_7	123	115	124	114	119	4210	6/3/73
C_8	120	119	122	123	121	4280	9/3/73
C_9	118	120	117	120	118	4200	9/3/73
C_{10}	123	126	123	120	123	4350	6/3/73
C_{11}	135	128	130	138	132	4690	9/3/73

TABLE (5-4) Concrete Compressive Strength

GROUP C

Compressive Force / Column No.	Cylinder 1 kips	Cylinder 2 kips	Cylinder 3 kips	Cylinder 4 kips	Average kips	Compressive Force of Concrete psi	Date of Cast
D_1	120	130	131	125	126	4470	9/1/73
D_2	112	129	136	118	123	4370	4/1/73
D_3	130	110	128	130	124	4400	9/1/73
D_4	130	131	130	120	127	4520	9/1/73
D_5	130	112	124	130	124	4390	4/1/73
D_6	130	120	122	124	124	4390	4/1/73
D_7	135	110	106	130	120	4250	4/1/73
D_8	126	130	130	123	127	4500	9/1/73
D_9	128	120	130	120	124	4400	9/1/73
D_{10}	117	135	120	130	125	4420	4/1/73
D_{11}	135	134	110	130	127	4500	14/12/72

TABLE (5-5) Concrete Compressive Strength

GROUP D

by a load cell at the other. The desired eccentricity was obtained by moving the ball bearings to the proper position relative to the end plates which were mounted on the specimen ends. The end plates, complete with the ball bearings fixed, were mounted on the specimens and then the specimens were adjusted so that the center line of the end bearing facing the jack coincides with its center line and also, so that the center line of the other end bearing facing the load cell coincides with the center line of the load cell. Subsequently, a small load was applied by the jack to keep the specimens in place. The Gilmore hydraulic jack used to apply the load had a 6" stroke and 100 kips capacity. The Gilmore load cell at the other end was connected to a digital millivolt indicator, Gilmore model 471. After adjusting the specimens to the desired position, three mechanical dial gauges were fixed in the two orthogonal planes along each face (see Figure (5-9)). One dial gauge was placed at the mid-length of the column and the two others near the ends. The dial gauges were adjusted to zero and then the load was applied. The load was applied in small intervals and at each interval the load was kept constant and the readings of the six dial gauges were recorded to give the deflection of the two column faces under that load. Columns were tested until failure and deflections were recorded at each load interval. The time needed for testing ranged from 20 to 30 minutes. An outline of test programs is shown in Tables (5-6) through (5-10).

5-5 Relationship Between Load, Eccentricity, Maximum Deflection and Curvature

To illustrate the effect of load and eccentricity on maximum deflection, contours were drawn to give the values of deflections at various intervals of the load for each combination of eccentricities. The curves also show the effect of e_x/e_y on $\Delta H/\Delta V$ where e_x and e_y are eccentricities on the X and Y axis respectively and ΔH and ΔV are deflections on the X and Y axis respectively. Those relationships are shown for the different groups in Figures (5-12) through (5-15).

To illustrate the effect of load and eccentricity on curvature, contours were drawn to give values of curvature at various intervals of load for each combination of eccentricities. The graphs also illustrate the effect of e_x/e_y on ϕ_x/ϕ_y where ϕ_x and ϕ_y are curvatures in the X and Y planes respectively. Those relationships are shown for the different groups in Figures (5-16) through (5-19). Pictures of samples after failure are shown in Figures (5-20) through (5-24).

5-6 Experimental Errors

The accuracy of the applied eccentricities was ±0.05 ins. The effect of such an error in eccentricity on the failure loads for the eccentricities used in this study would be insignificant as the error in the eccentricity itself would only be 2 percent in the extreme case of an end eccentricity of 2.5 inches.

Group No.	Column No.	Eccentricity e inchs		Length L inch	Reinforcement Size #	Steel Yield Strength (f_y) psi	Concrete Compressive Strength fc' psi	$\frac{\rho f_y}{fc'}$
		e_y	e_x					
A	A_1	2.50	0.00	90"	7	67000	3320	1.34
	A_2	2.50	1.25				3750	1.19
	A_3	2.50	2.50				3960	1.13
	A_4	5.00	0.00				4170	1.07
	A_5	5.00	1.66				3940	1.13
	A_6	5.00	3.33				3310	1.35
	A_7	5.00	5.00				3500	1.28
	A_8	7.50	0.00				4900	0.91
	A_9	7.50	2.50				4210	1.06
	A_{10}	7.50	5.00				4680	0.96
	A_{11}	7.50	7.50				4780	0.93

TABLE (5-6). Outline of Tests

GROUP A

* Accuracy of eccentricities is ± 0.05 inch

Group No.	Column No.	Eccentricity e inchs		Length L inch	Reinforcement Size #	Steel Yield Strength (f_y) psi	Concrete Compressive Strength fc' psi	$\frac{\rho f_y}{fc'}$
		e_y	e_x.					
B	B_1	2.50	0.00	90"	3	65000	4190	0.19
	B_2	2.50	1.25				4060	0.20
	B_3	2.50	2.50				4140	0.19
	B_4	5.00	0.00				4190	0.19
	B_5	5.00	1.66				4250	0.19
	B_6	5.00	3.33				4220	0.19
	B_7	5.00	5.00				4100	0.19
	B_8	7.50	0.00				3770	0.21
	B_9	7.50	2.50				4010	0.20
	B_{10}	7.50	5.00				4000	0.20
	B_{11}	7.50	7.50				3730	0.21

TABLE (5-7) Outline of Tests

GROUP B

* Accuracy of eccentricities is ± 0.05 inch

Group No.	Column No.	Eccentricity e inchs		Length L inch	Reinforcement Size #	Steel Yield Strength (f_y) psi	Concrete Compressive Strength fc' psi	$\frac{\rho f_y}{fc'}$
		e_y	e_x					
C	C_1	2.50	0.00	134"	7	67000	5300	0.84
	C_2	2.50	1.25				5260	0.85
	C_3	2.50	2.50				4850	0.92
	C_4	5.00	0.00				4900	0.91
	C_5	5.00	1.66				5000	0.89
	C_6	5.00	3.33				4260	1.05
	C_7	5.00	5.00				4219	1.06
	C_8	7.50	0.00				4280	1.04
	C_9	7.50	2.50				4200	1.06
	C_{10}	7.50	5.00				4350	1.03
	C_{11}	7.50	7.50				4690	0.94

TABLE (5-8) Outline of Tests

GROUP C

* Accuracy of eccentricities is ± 0.05 inch

Group No.	Column No.	Eccentricity e inchs		Length L inch	Reinforcement Size #	Steel Yield Strength (f_y) psi	Concrete Compressive Strength fc' psi	$\rho \frac{f_y}{fc'}$
		e_y	e_x					
D	D_1	2.50	0.00	134"	3	65000	44.70	0.18
	D_2	2.50	1.25				4370	0.18
	D_3	2.50	2.50				4400	0.18
	D_4	5.00	0.00				4520	0.18
	D_5	5.00	1.66				4390	0.18
	D_6	5.00	3.33				4380	0.18
	D_7	5.00	5.00				4250	0.19
	D_8	7.50	0.00				4500	0.18
	D_9	7.50	2.50				4400	0.18
	D_{10}	7.50	5.00				4440	0.18
	D_{11}	7.50	7.50				4500	0.18

TABLE (5-9) Outline of Tests

GROUP D

* Accuracy of eccentricities is ± 0.05 inch

The reinforcing steel had some strain hardening immediately after yield. Some error obviously arises in comparing the experimental results with the theoretical results using the design charts as the charts were derived assuming an elastic-perfectly plastic stress-strain characteristic for the steel. The effect would give experimental ultimate loads greater than those predicted and experimental deflections smaller than predicted and would be more noticed for the columns reinforced with # 3 bars than those reinforced by the # 7 bars.

No account was taken for creep and shrinkage effect on the results. Since the time needed for the test was approximately 20 to 30 minutes and shrinkage strain is very small, these effects are thought to be minor.

CHAPTER VI

COMPARISON OF PROPOSED METHOD WITH EXPERIMENTAL RESULTS

6-1 Summary of Results

The object of the experimental program was to provide data to determine the validity of the proposed method to predict the behaviour of slender reinforced concrete columns under biaxially eccentric loads. The experimental program covered the effect of eccentricity, angle of eccentricity, slenderness ratio and percentage of reinforcement on ultimate loads and deflections. Values of eccentricity varied from $e/t = 0.42$ to $e/t = 1.8$ at angles of eccentricity ranging from 0° to 45°. The percentages of reinforcement used were 1.2% and 6.7%. Two sets of columns were fabricated with slenderness ratios of 50 and 75 respectively.

The ultimate loads and lateral deflections were calculated by the proposed method for the 44 test samples and compared with those measured from the companion experimental program in Tables (6-1) through (6-4). However as the columns were tested horizontally, the effect of the moment due to the columns self weight had to be included.

Specimen Number	e_x in	e_y in	θ^o	P_{ex} kips	P_{ca} kips	$\frac{P_{ex}}{P_{ca}}$	Δ_{ex} in	Δ_{ca} in	$\frac{\Delta_{ex}}{\Delta_{ca}}$
A_1	0.00	2.50	0.0	62.5	65.7	0.95	0.90	0.89	1.01
A_2	1.25	2.50	26.6	56.9	57.9	0.98	0.81	0.76	1.06
A_3	2.50	2.50	45	46.2	45.2	1.02	0.83	0.71	1.17
A_4	0.00	5.00	0.0	39.2	39.4	0.99	1.10	1.26	0.87
A_5	1.66	5.00	18.45	33.8	34.3	0.99	1.00	0.94	1.06
A_6	3.33	5.00	33.7	26.7	26.9	0.99	0.96	0.86	1.12
A_7	5.00	5.00	45	22.0	21.1	1.04	0.94	0.79	1.19
A_8	0.00	7.50	0.0	29.7	29.7	1.00	1.26	1.43	0.88
A_9	2.50	7.50	18.45	23.4	22.4	1.04	1.00	1.01	0.99
A_{10}	5.00	7.50	33.7	22.6	20.6	1.10	0.87	0.89	0.98
A_{11}	7.50	7.50	45	15.8	15.2	1.04	0.87	0.88	0.99
Average						1.01			1.03

TABLE (6-1) Comparison of Predicted and Experimental Results

GROUP A

90" Long reinforced with # 7 bars

Specimen Number	e_x in	e_y in	θ^o	P_{ex} kips	P_{ca} kips	$\frac{P_{ex}}{P_{ca}}$	Δ_{ex} in	Δ_{ca} in	$\frac{\Delta_{ex}}{\Delta_{ca}}$
B_1	0.0	2.50	0.0	44.0	39.6	1.11	0.80	1.05	0.76
B_2	1.250	2.50	26.6	31.2	28.9	1.08	0.61	0.82	0.76
B_3	2.50	2.50	45.0	27.2	23.1	1.18	0.65	0.77	0.84
B_4	0.00	5.00	0.0	18.1	15.3	1.18	0.97	1.32	0.74
B_5	1.66	5.00	18.45	16.5	14.6	1.13	0.86	1.05	0.82
B_6	3.33	5.00	33.70	12.4	11.8	1.05	0.76	1.03	0.74
B_7	5.00	5.00	45.0	10.4	9.3	1.12	0.75	0.97	0.77
B_8	0.00	7.50	0.0	10.1	9.1	1.11	1.18	1.45	0.82
B_9	2.50	7.50	18.45	9.7	8.5	1.14	1.05	1.09	0.96
B_{10}	5.00	7.50	33.70	8.0	7.2	1.11	1.02	1.07	0.95
B_{11}	7.50	7.50	45.0	6.4	5.4	1.18	1.03	1.03	1.0
Average						1.13			0.83

TABLE (6-2) Comparison of Predicted and Experimental Results

GROUP B

90" Long reinforced with # 3 bars

Specimen Number	e_x in	e_y in	θ^o	P_{ex} kips	P_{ca} kips	$\frac{P_{ex}}{P_{ca}}$	Δ_{ex} in	Δ_{ca} in	$\frac{\Delta_{ex}}{\Delta_{ca}}$
C_1	0.00	2.50	0.0	48.5	58.8	0.82	2.41	2.39	1.0
C_2	1.25	2.50	26.6	39.0	48.8	0.80	1.84	1.98	0.93
C_3	2.50	2.50	45.0	36.0	37.9	0.95	1.82	1.70	1.07
C_4	0.00	5.00	0.0	31.9	33.1	0.96	3.0	3.02	0.99
C_5	1.66	5.00	18.45	28.8	29.9	0.96	2.05	2.07	0.99
C_6	3.33	5.00	33.70	20.6	21.5	0.96	1.83	1.85	0.99
C_7	5.00	5.00	45.0	17.9	18.6	0.96	1.84	1.63	1.13
C_8	0.00	7.50	0.0	22.6	23.3	0.97	2.41	3.11	0.77
C_9	2.50	7.50	18.45	19.0	18.2	1.04	1.74	1.80	0.97
C_{10}	5.00	7.50	33.70	14.9	15.4	0.96	1.76	1.93	0.91
C_{11}	7.50	7.50	45.0	13.8	13.7	1.01	1.81	1.82	0.99
Average						0.95			0.98

TABLE (6-3) Comparison of Predicted and Experimental Results

GROUP C

134" Long reinforced with # 7 bars

Specimen Number	e_x in	e_y in	θ^o	P_{ex} kips	P_{ca} kips	$\frac{P_{ex}}{P_{ca}}$	Δ_{ex} in	Δ_{ca} in	$\frac{\Delta_{ex}}{\Delta_{ca}}$
D_1	0.00	2.50	0.0	28.5	29.0	0.98	1.40	1.65	0.85
D_2	1.25	2.50	26.6	24.0	23.8	1.01	1.30	1.63	0.80
D_3	2.50	2.50	45.0	16.0	17.0	0.94	1.20	1.31	0.92
D_4	0.00	5.00	0.0	14.0	14.2	0.99	1.99	2.31	0.86
D_5	1.66	5.00	18.45	11.9	12.3	0.97	1.51	1.93	0.78
D_6	3.33	5.00	33.70	10.7	11.0	0.97	1.41	1.84	0.77
D_7	5.00	5.00	45.0	8.3	8.3	1.00	1.46	1.57	0.93
D_8	0.00	7.50	0.0	9.8	9.0	1.09	2.19	2.30	0.95
D_9	2.50	7.50	18.45	8.7	8.0	1.09	2.23	2.26	0.98
D_{10}	5.00	7.50	33.70	7.7	7.2	1.07	1.83	2.09	0.88
D_{11}	7.50	7.50	45.0	7.0	6.6	1.06	1.84	1.95	0.94
Average						1.02			0.88

TABLE (6-4) Comparison of Predicted and Experimental Results

GROUP D

134" Long reinforced with # 3 bars

The theoretical capacity of the column was first determined using the section property charts not including the self weight moment of the column. Hence the self weight moment was divided by the calculated ultimate load to obtain a correction to the eccentricity. Subsequently, the capacity of the column was again calculated using the corrected eccentricity.

Further, as the proposed method of analysis gives the strain distribution in the column at any load, the load/deflection (curvature) behaviour of a given column can be predicted. Unfortunately, the described method being a design aid rather than a method of analysis, obtaining the load curvature characteristics of a given column is rather tiresome. However, the load deflection curves of eight (8) columns were calculated and compared with the appropriate experimental curves shown in Figures (6-1) through (6-4).

6-1-1 Comparison of predicted and measured load-deflection behaviours

It can be observed from Figures (6-1) through (6-4) that the predicted load deflection curves are similar in form to those measured. However, the theoretical curves have higher values of deflections and the difference between the theoretical and experimental results is greater for the columns reinforced with # 3 bars than those reinforced with # 7 bars. This is due to the assumptions inherent in calculating the deflections of reinforced concrete members even with access to moment curvature relationships and partly due to ignoring strain hardening in the deviation of the theoretical section property curves.

6-1-2 Comparison of predicted ultimate loads with measured ultimate loads

From Tables (6-1) through (6-4), it can be seen that the proposed method closely predicts the experimental ultimate loads with mean values of P experimental/P calculated of 1.01, 1.13, 0.95, and 1.02 for the four groups of experiments respectively. The overall average for the four groups was 1.03 with a coefficient of variation of 0.072. It can be noted that the difference between the experimental and predicted results is greater in Tables (6-2) and (6-4) (# 3 bars) than that from Tables (6-1) and (6-3) (# 7 bars). This is attributed to the theoretical method ignoring strain hardening and as the # 3 bars exhibited more strain hardening than the # 7 bars, the discrepancy is greater.

6-1-3 Comparison of predicted ultimate deflections with measured ultimate deflections

Comparison between the predicted and experimental ultimate deflections in Tables (6-1) through (6-4) shows that the proposed method predicts the experimental ultimate deflections with average values of Δ experimental/Δ calculated of 1.03, 0.83, 0.98, and 0.88 for the four groups of test program respectively. The overall average was 0.93 with a coefficient of variation of 0.091.

It must be noted that it was difficult to measure experimentally the ultimate deflections of the columns at loads near the ultimate load with the present test arrangement as the load deflection curves were relatively flat topped. Secondly, the theoretical deflections were calculated assuming no strain hardening in the steel and that the deflected shape of the column was parabolic.

6-1-4 Comparison of the proposed method with available test data

A comparison of the results obtained by the proposed design method for square and rectangular columns with corner reinforcement with experimental data obtained from Drysdale (18) for square columns and Bresler (3) for rectangular columns is shown in Tables (6-5) and (6-6) for short term loading.

Column Size	Length	$\rho f_y/f_c'$	g	e	θ	P test/P cal
5"x5"	13'	0.42 0.46 0.46 0.41	0.7	1.0" 1.0" 1.0" 1.5"	0.0° 22.5° 45° 22.5°	0.92 1.03 1.02 1.06

TABLE (6-5) Comparison of Calculated Results with Drysdale's Test Data

Column Size	Length	$\rho f_y/f_c'$	g	e_x	e_y	P test/P cal
6"x8"	4'	0.37	g_x = 0.42	6	-	0.96
		0.39	g_y = 0.56	3	-	0.95
		0.37	an average	-	4	1.0
		0.30	value of	-	8	1.01
		0.43	0.49 was	3	4	0.94
		0.37	used	6	8	1.01
		0.39		6	4	0.98
		0.38		3	8	1.06

TABLE (6-6) Comparison of Calculated Results with Bresler's Test Data

6-2 Conclusions

From the comparisons between the predicted and experimental results given in sections 6-1-1, 6-1-2, 6-1-3, and 6-1-4 it can be concluded that the given section property charts adequately describe the behaviour of slender corner reinforced square section columns under biaxially eccentric loads. Hence a design method based upon these section property charts could reliably be used to design such columns.

CHAPTER VII

DESIGN METHOD

7-1 Introduction

To completely describe the behaviour of a reinforced concrete column, the load, eccentricity, angle of eccentricity, maximum strain and the position of the neutral axis (to determine curvature) need to be known. In addition, to describe the behaviour of a slender reinforced concrete column, the lengthwise lateral deflection compatability with curvature and load at any section are needed.

Section behaviour charts were prepared for square columns with cover ratio g = 0.75 giving the relationship between non-dimensional load P_u'/f_c bt, eccentricity e/t, location of the neutral axis Z/t and maximum strain for angles of eccentricity 0° and 45°. A calibration curve was derived to use the same curves for cover ratios other than 0.75 and a method to design rectangular columns using the same curves was also illustrated. By using these section behaviour curves, it is possible to determine the complete load deflection behaviour of a slender reinforced concrete column.

7-2 Calculating Lateral Deflection - Curvature Compatability

When a reinforced concrete column is designed to carry a biaxially eccentric load, a cross section with steel arrangement is assumed. Then, at the top of column, knowing the magnitude of the load and the eccentricities, there is only one linear strain distribution which would satisfy these conditions. If that strain distribution is found, the value of maximum strain ε_{max} and the normal distance from the point of maximum strain to the neutral axis Z at the column top are known and the curvature at the top $(\varepsilon_{max}/Z)_t$ can be calculated. At mid-column height, the value of load is still the same, then assuming a value for maximum strain and assuming that the angle of eccentricity is still the same, there is only one linear strain distribution which would satisfy both load and strain constraints from the linear strain distribution. The value of curvature at the middle section $(\varepsilon_{max}/Z)_M$ and eccentricity of load e_{middle} can be calculated. Since the curvature at top and middle of column are known and assuming a deflected shape for the column, the value of maximum deflection at mid-column height Δ can be calculated using the moment area method or any elementary method for calculating deflection since the curvature diagram $(\varepsilon_{max}/Z)_{diagram}$ is equivalent to the elastic weight diagram $(M/EI)_{diagram}$. To satisfy compatability of strains and lateral deformation $e_{top} + \Delta$ should equal e_{middle}, otherwise another value of maximum strain at the middle should be assumed until convergence takes place. The trial and error involved in finding the value of maximum strain at the middle is controlled since it is logical to assume larger value of ϵ_{max} at the middle if $e_{top} + \Delta > e_{middle}$ and a smaller value if $e_{top} + \Delta < e_{middle}$.

7-3 Design Method by the Use of Curves

First a concrete strength f_c' steel yield strength f_y and a percentage of reinforcement ρ are assumed and using the value of $\rho f_y/f_c'$, the proper set of curves can be selected. Then a cross section is assumed for the column which is thought to be suitable. Having values of ultimate load P_u' and eccentricity e, P_u'/f_c' bt and e/t at the top of column are defined, then by the use of curves, the value of maximum strain ϵ_{max} and also Z/t at the top can be found. From the value of ϵ_{max} and Z at the top, the curvature $(\epsilon_{max}/X)_t$ at the top can be calculated. Since P_u'/f_c' bt is constant along the column, and assuming no change in the angle of the eccentricity, a maximum strain at the middle can be assumed and again, by the use of curves, the value of e/t and Z/t at the middle can be obtained and the curvature $(\epsilon_{max}/Z)_M$ at the middle can be calculated. Then, as explained previously, having the curvature diagram and assuming a deflected shape, the value of maximum deflection Δ can be calculated. The assumed ϵ_{max} at mid-column height would be controlled by equating $e_{top} + \Delta$ to e_{middle}.

If the maximum strain at the middle of the column needed to satisfy compatability ($e_{top} + \Delta = e_{middle}$) is larger than 0.0030, the chosen section is too small and a bigger one needs to be assumed. However, if that strain is too much less than 0.0030, a smaller section may be tried because if it is found to be adequate, it would be a more economical solution.

7-4 Solved Examples

Example 1:

Determine the section and reinforcement required for a reinforced concrete column 14' long monolithically cast with slabs at both ends to carry a design ultimate load of 150 kips and design ultimate moments of 665 kip.ins. and 1000 kip.ins. respectively about the two axes. The column is assumed to be bent in single curvature about both the axis and a design under strength factor of 0.7 is to be used.

Assume a concrete strength of 4000 psi, a steel yield stress of 60000 psi, 4% reinforcement and a cover ratio of g = 0.7

$$\rho f_y / f_c' = 0.6$$

$$e = \sqrt{(4.42)^2 + (6.67)^2} = 8 \text{ ins}$$

$$\theta = \tan^{-1} \frac{665}{1000} = 33.6^\circ$$

From Figure (4-38) with $g = 0.7$ and $\rho f_y/f_c' = 0.6$

$$F = 1.08$$

$$e_{modified} = F.e = 1.08 \times 8 = 8.64 \text{ ins}$$

$$P_u' = P_u/\phi' = 214 \text{ kips}$$

Assume column size to be 11 ins x 11 ins,

$$P_u'/f_c' \, bt = \frac{214000}{4000 \times 11 \times 11} = 0.442$$

$$e/t = \frac{8.64}{11} = 0.786$$

At the ends of the column, using Figures (4-15) through (4-19) and estimating θ between $\theta = 0$ and $\theta = 45$, no point for $e/t = 0.786$ and P_u'/f_c' bt = 0.442 exists - hence, section is too small.

Assume column size to be 13 ins x 13 ins,

$$P_u'/f_c' \, bt = \frac{214000}{4000 \times 13 \times 13} = 0.317$$

$$e/t = \frac{8.64}{13.00} = 0.665$$

At the ends of the column, using Figure (4-18) and estimating θ between $\theta = 0$ and $\theta = 45^o$

$$\text{Maximum strain } \varepsilon = 0.0018$$

$Z/t = 0.72$ $\quad\therefore\; Z = 9.36 \text{ in}^2$

(Z is the perpendicular distance between the point of maximum strain and the neutral axis)

$$\text{Curvature} = \frac{0.0018}{9.36} = 0.000193 \text{ ins}^{-1}$$

At mid column height, the load is the same but the eccentricity and its angle are unknown.

Assume maximum strain = 0.0030. Again from Figure (4-15)

$e/t = 0.765$ $\qquad e = 9.95$ ins

$Z/t = 0.710$ $\qquad Z = 9.22$ ins

$$\text{Curvature} = \frac{0.0030}{9.22} = 0.000326 \text{ ins}^{-1}$$

Assuming the deflected shape, a second degree parabola and using moment area method

$R = 0.000193 \times 7 \times 12 + 0.000133 \times 7 \times 12 \times 2/3$

$= 0.0162 + 0.0075 = 0.0237$

$\Delta = 0.0237 \times 7 \times 12 - 0.0162 \times 7 \times 12/2 - .0075 \times 7 \times 12 \times 3/8$

$= 1.99 - 0.68 - 0.23 = 1.08$ ins.

Hence, $e_{actual} = e_{top} + \Delta = 8.64 + 1.08 = 9.72$ ins

$$e_{possible} = 9.95 \text{ ins}$$

$e_{possible} > e_{actual}$ $\therefore$ section is O.K.

Example 2:

If we change the design ultimate load and moments in the previous example to 200 kips, 1000 kip.ins. and 2000 kip.ins. respectively for a 20' column with g = 0.80, the solution would be:

Assume a concrete strength of 3000 psi, a steel yield stress of 60000 psi, 5% reinforcement and a cover ratio of g = 0.80

$$\rho f_y/f_c' = 1.0$$

Assume column size 15 ins x 15 ins

$$P_u' = P_u/\phi' = 286 \text{ kips}$$

$$P_u'/f_c'\,bt = \frac{286000}{3000 \times 15 \times 15} = 0.423$$

$$e = \sqrt{25 + 100} = 11.2 \text{ ins}$$

From Figure (4-38) with $g = 0.8$ and $\rho f_y/f_c' = 1.0$

$$F = 0.92$$

$$e_{modified} = F.e = 0.92 \times 11.2 = 10.3$$

$$\theta = \tan^{-1} = \frac{1000}{2000} = 26.5^{\circ}$$

$$e/t = \frac{10.3}{15} = 0.687$$

At the ends of column using Figure (4-22) and estimating θ between $\theta = 0$ and $\theta = 45^{\circ}$

Maximum strain $\varepsilon = .0020$

$Z/t = 0.69$ $\therefore Z = 10.35$

$$\text{Curvature} = \frac{0.0020}{10.35} = 0.000193 \text{ in}^{-1}$$

At mid column height the load is the same but the eccentricity and its angle are unknown.

Assume maximum strain = 0.0030. Again from Figure (4-20)

$e/t = 0.89$ $\qquad$ $e = 13.35$

$Z/t = 0.68$ $\qquad$ $Z = 10.2$

$$\text{Curvature} = \frac{0.0030}{10.2} = 0.000294$$

$R = 0.000193 \times 120 + 0.0010 \times 120 \times 2/3$

$= 0.0232 + 0.0081 = .0313$

$\Delta = 0.0313 \times 120 - 0.0232 \times 120/2 - 0.0081 \times 120 \times 3/8$

$= 3.75 - 1.39 - 0.36$

$= 3.75 - 1.75 = 2.00''$

Hence, $e_{actual} = e_{top} + \Delta = 10.3 + 2.0$

$= 12.3''$

$\therefore$ $e_{possible} > e_{actual}$ $\qquad$ $\therefore$ section is O.K.

Try Maximum strain = 0.0024

$e/t = 0.815$ $e = 12.2$

$Z/t = 0.67$ $Z = 10.0$

$$\text{Curvature} = \frac{0.0024}{10.0} = 0.00024$$

$$R = 0.000193 \times 120 + 0.000047 \times 120 \times 2/3$$
$$= 0.0232 + 0.0038 = .0270$$

$$\Delta = 0.0270 \times 120 - 0.0232 \times 120/2 - 0.0054 \times 120 \times 3/8$$
$$= 3.24 - 1.39 - 0.17 = 3.24 - 1.56 = 1.68$$

Hence, $e_{actual} = e_{top} + \Delta = 10.3 - 1.68 = 11.98''$

$e_{possible} > e_{actual}$ ∴ section is O.K.

Since $e_{possible}$ (12.3) is fairly close to e_{actual} (11.98).

∴ Maximum strain = 0.0024

It is noticed when assuming Maximum strain = 0.0030, that $e_{possible}$ (13.35 ins) is much bigger than e_{actual} (12.3 ins) while, when assuming Maximum strain = 0.0024, $e_{possible}$ (12.2 ins) is close enough to e_{actual} (11.98 ins) which means that in the above mentioned example, the maximum strain is = 0.0024.

If more economy is required a column size 14" x 14" may be assumed and used if ε_{max} does not exceed 0.0030 i.e. if $e_{possible}$ is still bigger than e_{actual} when maximum strain is equal or less than 0.0030.

Try column size 14" x 14"

$$P_u'/f_c'\ bt = 0.486$$

$$e/t = \frac{10.3}{14} = 0.735$$

At column ends using Figure (4-21) and estimating θ between $\theta = 0^\circ$ and $\theta = 45^\circ$

Maximum strain = 0.0026

$Z/t = 0.7$ $\qquad Z = 9.8$

$$\text{Curvature} = \frac{0.0026}{9.8} = 0.000265$$

At mid column height, assume maximum strain = 0.0030

$e/t = 0.77$ $\qquad e = 10.78$

$Z/t = 0.69$ $\qquad Z = 9.65$

$$\text{Curvature} = \frac{0.0030}{9.65} = 0.00031$$

$$R = .000265 \times 120 + 0.00045 \times 120 \times 2/3$$

$$= 0.0318 + 0.0036 = .0354$$

$$\Delta = 0.0354 \times 120 - 0.0318 \times 60 - 0.0036 \times 45$$

$$= 4.25 - 1.91 - 0.16 = 4.25 - 2.07$$

$$= 2.18$$

$$e_{actual} = 10.3 + 2.07 = 12.37$$

$$e_{possible} = 10.78$$

$\therefore$ $e_{actual} > e_{possible}$ $\therefore$ section is not O.K. and 15" x 15" column should be used

Example 3:

Determine the section and the reinforcement required for a reinforced concrete column 24' long monolithically cast with slabs at both ends to carry a design ultimate load of 200 kips and design ultimate moments of 2000 kip.ins. and 1000 kip.ins. respectively about the two axes. The column is assumed to be bent in single curvature about both the axes and a design under strength factor of 0.70 is to be used.

Assume

$f_c' = 4000$ psi $\qquad f_y = 60000$ psi $\qquad$ 4% reinforcement

$$\therefore \quad \rho f_y/f_c' = 0.6$$

If a rectangular section of aspect ratio $t/b \leq 2.0$ is needed, then

$$\phi = t/b = 2.0$$

$$\therefore \quad \phi\, e_x = 2.0 \times \frac{1000}{200} = 10''$$

$$e_y = \frac{2000}{200} = 10'' \qquad e = \sqrt{\phi e_x^{\,2} + e_y^{\,2}}$$

$$= 14.1''$$

$$\theta = 45^\circ$$

$$P_u' = P_u/\phi' = \frac{200}{0.7} = 286 \text{ kips}$$

Now design the analogous square column to carry P_u' at ϕe_x and e_y

Assume 17" x 17" analogous column

$$\therefore \quad P_u'/f_c'\, bt = \frac{286000}{4000 \times 17 \times 17} = 0.247$$

$$e/t = 0.830$$

At the ends of column, using Figure (4-17)

$$\varepsilon_{max} = 0.0020$$

$$Z/t = 0.73 \qquad Z = 12.4''$$

$$\text{Curvature} = \frac{0.0020}{12.4} = 0.000161 \text{ ins}^{-1}$$

At mid column height assume $\varepsilon_{max} = 0.0030$

$$e/t = 0.96 \qquad e = 16.35$$

$$Z/t = 0.7 \qquad Z = 11.9$$

$$\text{Curvature} = \frac{0.0030}{11.9} = 0.000252$$

$$R = 0.000161 \times 12 \times 12 - 0.000091 \times 12 \times 12 \times 2/3$$
$$= 0.023 - 0.0087 = .0317$$

$$\Delta = 0.0317 \times 12 \times 12 - 0.0230 \times 6 \times 12 - 0.0087 \times 12 \times 12 \times 2/3$$
$$= 4.60 - 1.65 - 0.94 = 4.60 - 2.49$$
$$= 2.11''$$

$$e_{top} + \Delta = 14.1 + 2.11 = 16.22$$

$$e_{middle} = 16.35 \qquad \text{O.K.}$$

Then the rectangular column can carry P_u' at e_x and e_y.

Use a 17" x 8.5" with $As = 17 \times 17 \times \frac{4}{100} = 11.56 \text{ in}^2$

i.e. 8%

7-5 Comparison with ACI 318-71

The design of slender columns in accordance with the approximate procedure of ACI 318-71 involves two stages - determining the moment magnifier and then using an appropriate short column biaxially eccentric load interaction curves. The comparison between the proposed method and ACI 318-71 is on the basis of moment magnifier and section required to support a stated load at defined eccentricities.

The moment magnifier term itself can be used in two versions (1) as presented in ACI 318-71 with the simple EI and ϕ' in the denominator and (2) as presented without ϕ' in the denominator as originally proposed by MacGregor, Breen, Pfrang (25).

The comparison is shown in detail for the previously solved examples # 1 and # 2 pages 57 and 60 respectively.

A more extensive comparison is summarised in Table (7-1).

For Example 1:

Exact analysis moment magnifier $\delta_{exact} = \frac{9.72}{8.64} = 1.125$

(1) Calculation as per ACI 318-71 - ϕ' included

$$C_m = 1$$

$\beta_d = 0.0$ to correspond with previous (all live load)

$$I_g = \frac{13 \times 13^3}{12} = 2380 \text{ in}^4$$

$$E_c = 57000 \sqrt{f_c'}$$

$$= 3.6 \times 10^6$$

$$EI = \frac{\frac{E_c I_g}{2.5}}{1 + \beta_d} = \frac{\frac{3.6 \times 10^6 \times 2380}{2.5}}{1 + 0.0}$$

$$= 3420 \times 10^6 \text{ lbs.ins}^2$$

$$P_c = \frac{\pi^2 EI}{(K l_u)^2} = \frac{\pi^2 \times 3420 \times 10^6}{(1 \times 14 \times 12)^2}$$

$$= 1190000 \text{ lbs}$$

$$\delta = \frac{C_m}{1 - \frac{P_u}{\phi' P_c}} = \frac{1}{1 - \frac{150 \times 1000}{0.7 \times 1190000}}$$

$$\delta = \frac{1}{1 - 0.18} = \frac{1}{0.82} = 1.22$$

Column is bent in single curvature $\delta_x = \delta_y = \delta = 1.22$

$$\delta_{ACI} = \sqrt{(\delta \ e_x)^2 + (\delta \ e_y)^2}$$

$$\frac{\delta_{ACI}}{\delta_{exact}} = \frac{1.22}{1.125} = 1.08$$

$$L/r = 43$$

Hence the section needed to sustain P_u = 150 kips at an eccentricity of 10.54 ins (1.22 x 1.08 x 8)

Try 13" x 13"

$$\frac{P_u}{\phi f_c' \, bt} = \frac{150,000}{0.7 \times 4000 \times 13 \times 13} = 0.317$$

$$e/t = \frac{10.54}{13} = .81$$

$$\Theta = 33.6^{\circ}$$

With reference to Figure (4-15) it can be shown that this section is adequate.

(2) Calculation as per ACI-318-71 - ϕ not included in denominator

$C_m = 1$ $\beta_d = 0.0$ $I_g = 2380 \text{ in}^4$.

$$E_c = 3.6 \times 10^6$$

$$EI = 3420 \times 10^6 \text{ lbs.ins}^2$$

$$P_c = 1190000 \text{ lbs}$$

$$\delta = \frac{C_m}{1 - \frac{P_u}{P_c}} = \frac{1}{1 - \frac{150 \times 1000}{1190000}}$$

$$= \frac{1}{1 - 0.126} = \frac{1}{0.874} = 1.14$$

Column is bent in single curvature $\delta_x = \delta_y = \delta = 1.14$

$$e_{ACI} = \sqrt{(\delta\ e_x)^2 + (\delta\ e_y)^2} = \delta\ e$$

$$\delta_{ACI} = 1.14$$

$$\delta_{ACI}/\delta_{exact} = \frac{1.14}{1.125} = 1.013$$

Hence determine section required.

Try section 13" x 13"

$$\frac{P_u}{\phi f_c'\ bt} = \frac{150,000}{0.7 \times 4000 \times 13 \times 13} = 0.317$$

$$e/t = \frac{8.64 \times 1.14}{13} = 0.76$$

With reference to Figure (4-15) this section can be seen to be adequate.

<u>For Example 2</u>:

Exact analysis moment magnifier $\delta_{exact} = \frac{11.98}{10.3} = 1.165$

(1) Calculation as per ACI 318-71 - ϕ' included in denominator

$$C_m = 1$$

$\beta_d = 0.0$ to correspond with previous

$$I_s = 5.625 \times 2 \times 6^2 = 405$$

$$E_c = 57000 \sqrt{f_c'}$$

$$= 3.12 \times 10^6$$

$$EI = \frac{\frac{E_c I_g}{5} + E_s I_s}{1 + \beta_d}$$

$$= \frac{\frac{3.12 \times 10^6 \times 4210}{5} + 405 \times 29 \times 10^6}{1 + 0.0}$$

$$= 2625 \times 10^6 + 11745 \times 10^6$$

$$= 14390 \times 10^6$$

$$P_c = \frac{\pi^2 EI}{(K l_u)^2} = \frac{\pi^2 \times 14370 \times 10^6}{(1 \times 20 \times 12)^2}$$

$$= 2455217$$

$$\delta = \frac{C_m}{1 - \frac{P_u}{\phi' P_c}} = \frac{1}{1 - \frac{200 \times 1000}{0.7 \times 2455217}}$$

$$= \frac{1}{1 - 0.116} = \frac{1}{0.884} = 1.13$$

Column is bent in single curvature $\delta_x = \delta_y = \delta = 1.13$

$$\delta_{ACI} = \sqrt{(\delta\ e_x)^2 + (\delta\ e_y)^2} = \delta\ e$$

$$\frac{\delta_{ACI}}{\delta_{exact}} = 1.03$$

$$L/r = 53$$

Hence determine section to sustain $P_u = 200$ kips at 15.09 ins ~~(10.30 x 1.465)~~ at 26.5^o.

Try 15" x 15"

$$\frac{P_u}{\phi f_c'\ bt} = \frac{200,000}{0.7 \times 3000 \times 15 \times 15} = 0.42$$

$$e/t = \frac{15.09}{15} = 1.0$$

With reference to Figure (4-20) it can be seen that the section is adequate.

(2) Calculated as per ACI 318-71 - ϕ' not included in denominator

$C_m = 1$ $\beta_d = 0.0$ $I_g = 4210\ in^4$

$$E_c = 3.12 \times 10^6$$

$$EI = 14370 \times 10^6$$

$$P_c = 2455217 \text{ lbs}$$

$$\delta = \frac{C_m}{1 - \frac{P_u}{P_c}} = \frac{1}{1 - \frac{200000}{2455217}}$$

$$= \frac{1}{1 - 0.08} = 1.09$$

$$e_{ACI} = \sqrt{(\delta\, e_x)^2 + (\delta\, e_y)^2} = \delta\, e$$

$$\delta_{ACI} = 1.09$$

$$\delta_{ACI}/\delta_{exact} = 0.94$$

Hence the section to sustain a load of 200 kips at 13.25" and 26.5° needs to be determined.

Assume a 15" x 15" section

$$\frac{P_u}{\phi f_c' \, bt} = \frac{200{,}000}{0.7 \times 3000 \times 15 \times 15} = 0.42$$

$$e/t = \frac{13.25}{15} = .88$$

Again, with reference to Figure (4-20) it can be shown that this section is adequate.

$\rho f_y/f_c'$	P_u Kip	M_x Kip in.	M_y Kip in.	g	e ins	L	Section ins x ins	L/r	Δ	δ_{exact}	δ_{ACI} φ' Included	Section ins x ins	δ_{ACI} φ' Excluded	Section ins x ins
0.2	100	300	500	0.7	6.3	9'	12 x 12	30	0.57	1.09	1.122	12 x 12	1.08	12 x 12
0.2	150	600	1200	0.85	8.95	20'	15 x 15	53	2.23	1.32	1.26	16 x 16	1.17	16 x 16
0.2	80	240	240	0.75	4.24	16'	11 x 11	58	1.68	1.39	1.38	11 x 11	1.24	11 x 11
0.2	100	240	400	0.80	6.1	18'	12 x 12	60	2.18	1.36	1.26	13 x 13	1.17	13 x 13
0.2	200	600	300	0.75	3.35	16'	14 x 14	46	1.17	1.35	1.30	14 x 14	1.19	14 x 14
0.6	200	1000	1000	0.75	7.06	10'	13 x 13	31	0.52	1.08	1.135	13 x 13	1.09	13 x 13
0.6	150	665	1000	0.70	8.64	24'	13 x 13	43	1.08	1.125	1.32	13 x 13	1.14	13 x 13
0.6	110	440	550	0.8	5.85	19'	11 x 11	69	2.31	1.4	1.89	12 x 12	1.49	11 x 11
0.6	160	480	480	0.75	4.24	23'	12 x 12	80	2.84	1.67	3.45	15 x 15	2.00	13 x 13
0.6	100	130	130	0.75	1.84	14'	8 x 8	70	1.34	1.73	5.90	12 x 12	2.39	10 x 10
1.0	100	400	400	0.75	5.65	12'	10 x 10	48	0.63	1.11	1.25	10 x 10	1.16	10 x 10
1.0	200	1000	2000	0.8	10.3	20'	15 x 15	53	1.68	1.165	1.47	15 x 15	1.29	15 x 15
1.0	300	1500	150	0.75	7.06	25'	16 x 16	63	2.38	1.34	2.35	17 x 17	1.67	16 x 16
1.4	100	600	600	0.75	8.47	10'	10 x 10	40	0.67	1.08	1.25	10 x 10	1.16	10 x 10
1.4	300	1500	2100	0.8	7.86	24'	15 x 15	64	2.5	1.32	3.18	17 x 17	1.92	15 x 15

TABLE (7-1) Comparison Between δ_{exact} and δ_{ACI}

* All columns are calculated for a pinned-pinned configuration and bent in single curvature

7-6 Discussion of Comparisons

Comparing the magnification factor obtained by the proposed design method with that obtained by the ACI 318-71 design method (see Table (7-1)), shows that the ACI is always on the conservative side except for $\rho f_y/f_c' = 0.2$. However, there are two versions for calculating the ACI magnification factor; the first including the ϕ' which is very conservative and the second excluding the ϕ' yielding reasonably close values to the proposed design method. However it should be noted that crude values for EI were used to obtain the ACI magnifier; had a more accurate values been available, it would yield less conservative magnification factors.

Comparing the sections obtained by the proposed design method to those needed to sustain the load with the magnified moment (moments multiplied by the ACI 318-71 magnification factor) shows a small or no change except for high values of K l/r (see Table (7-1)).

It must be concluded than in spite of the errors inherent in calculating EI for the moment magnifier method, the sections obtained are remarkably close to those given by the more exact method.

CHAPTER VIII

CONCLUSIONS

8-1 Conclusions

Section capacity curves which give the relationships between load, eccentricity, angle of eccentricity and position of the neutral axis are produced for different concrete and steel strengths, percentage of reinforcement and maximum strain values. These section capacity curves are produced for square section columns with corner reinforcement and a cover ratio of 0.75. However they can be used for the design of columns with other cover ratios and also rectangular columns.

By the use of these section capacity curves, reinforced concrete column subjected to biaxial bending can be designed considering the compatability and equilibrium across the cross-section and along the columns without that need of using sophisticated computer programs. This makes the method available for use in ordinary design offices.

From the comparisons between the predicted and experimental results described in this thesis and the comparisons between the predicted results and the test data provided by Drysdale (18) and Bresler (3), it can be concluded that the proposed method reasonably describes the behaviour of slender square section corner reinforced concrete columns under biaxially eccentric loads. Hence, a design method based upon the above method of analysis can be used to reliably design such columns. A possible criticism would be that the range of eccentricities in the companion test program was for e/t values between 0.42 and 1.8 and thus not covering the low range of e/t values. However the comparison with Drysdale's test date for e/t = 0.2 and 0.3 gave reasonable agreement.

The accuracy of the chosen grid 12 x 12 to produce the section capacity curves was proved to be adequate by comparing the results with those obtained using an 18 x 18 grid, the arithmetic mean of error being +0.89 percent.

Some of the discrepancy between the experimental and the calculated results would be due to an inaccuracy in the eccentricity applied and the strain hardening of the steel reinforcing bars immediately after yield (see Figure (5-11)).

Lateral buckling is not generally considered critical for reinforced concrete columns but circumstances can be envisaged where the lateral stiffness of the partially 'plastic' section is not adequate to prevent buckling lateral to the line of action of the load. Hence, the lateral flexural stiffness of the plastic section is required. However, as reinforced concrete columns fail by material failure, the problem of determining the lateral stiffness can be avoided by resolving the curvature onto the symmetric axes and calculating the lateral deflections about each axis separately.

The provided section capacity curves are for columns with corner reinforcement: other arrangements of reinforcement would need additional sets of section property curves. The same method used in the development of the present curves may be used to develop the new ones.

Comparison of the magnification factor obtained by the proposed design method with that of ACI 318-71 shows that the ACI magnification factor is always conservative except for $\rho f_y/f_c' = 0.2$. The comparison of the sections obtained by the proposed method with those needed to sustain the load with the magnified moment according to the ACI 318-71 shows small or no change except for high K l/r values.

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APPENDIX 1

List of Figures

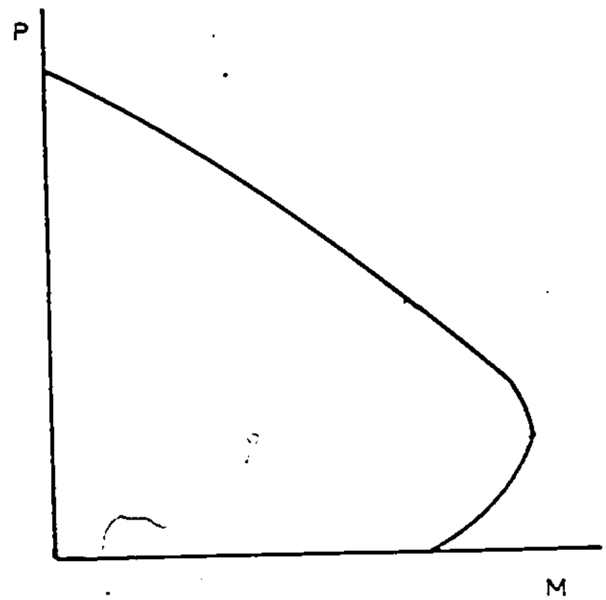


FIGURE (2-1) Load Uniaxial Moment Interaction Diagram

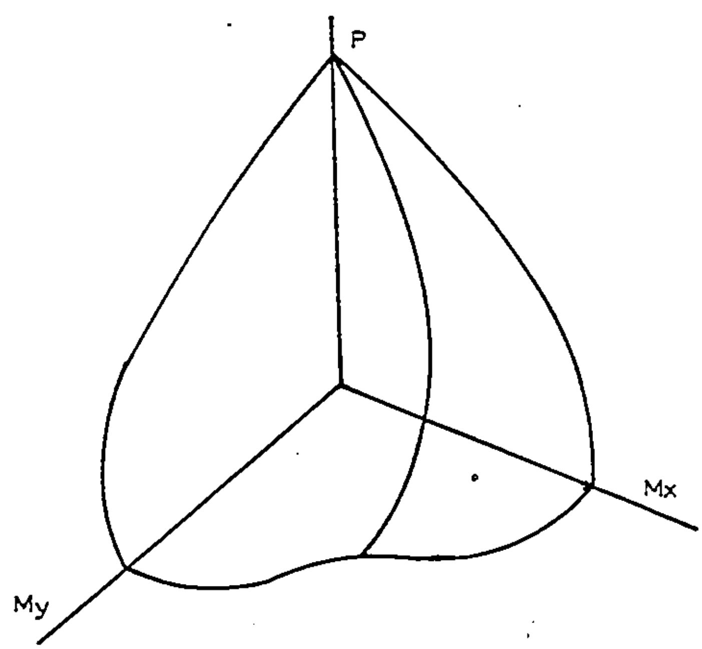


FIGURE (2-2) Load Biaxial Moment Interaction Surface

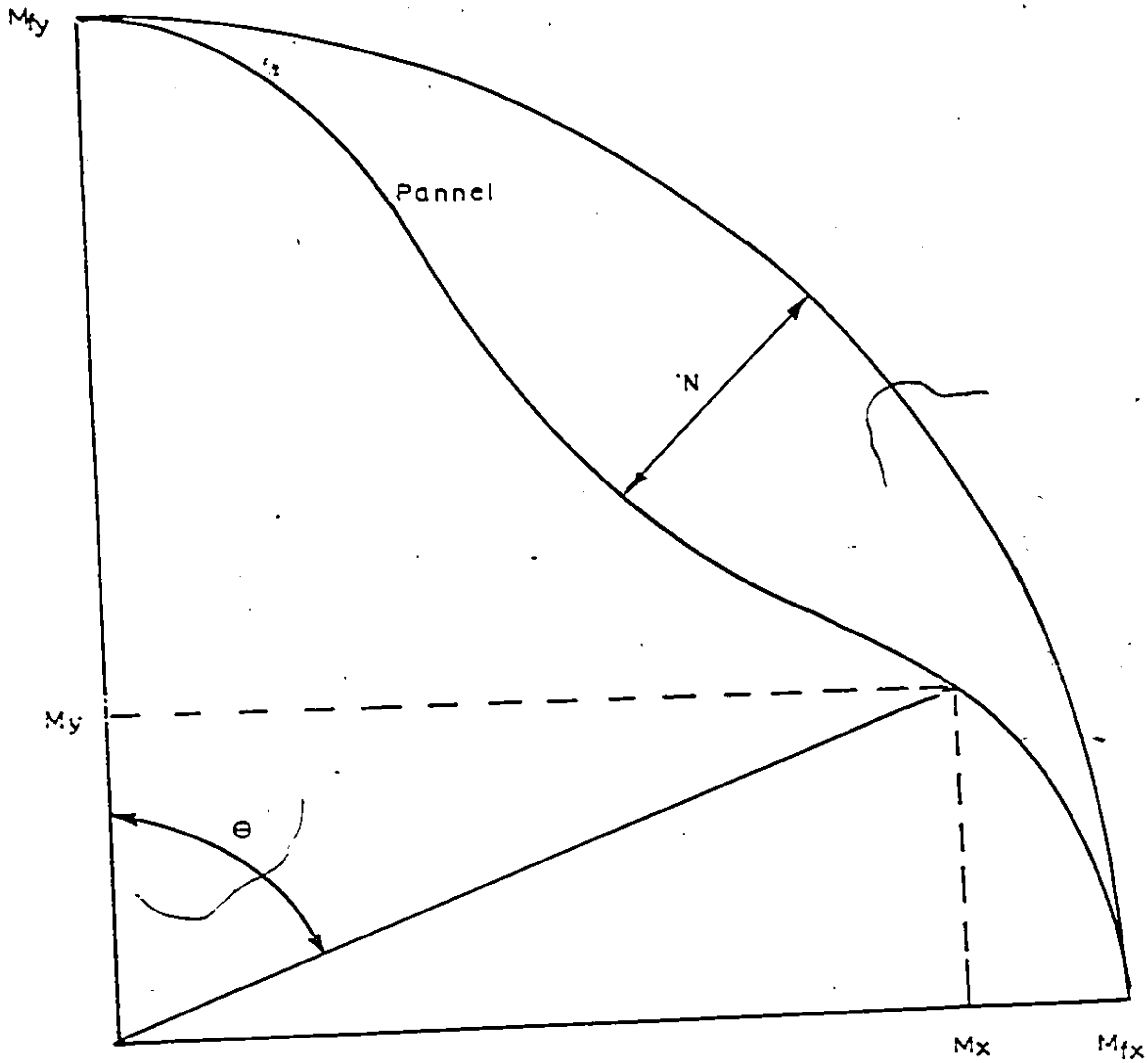
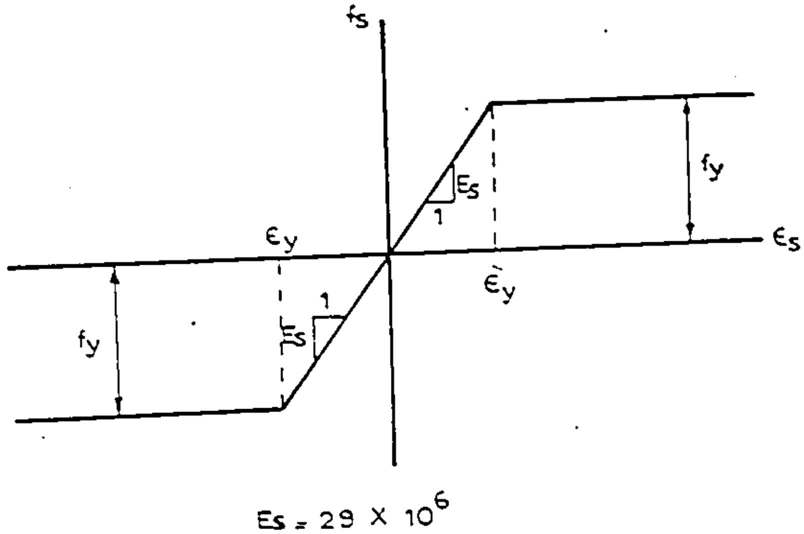
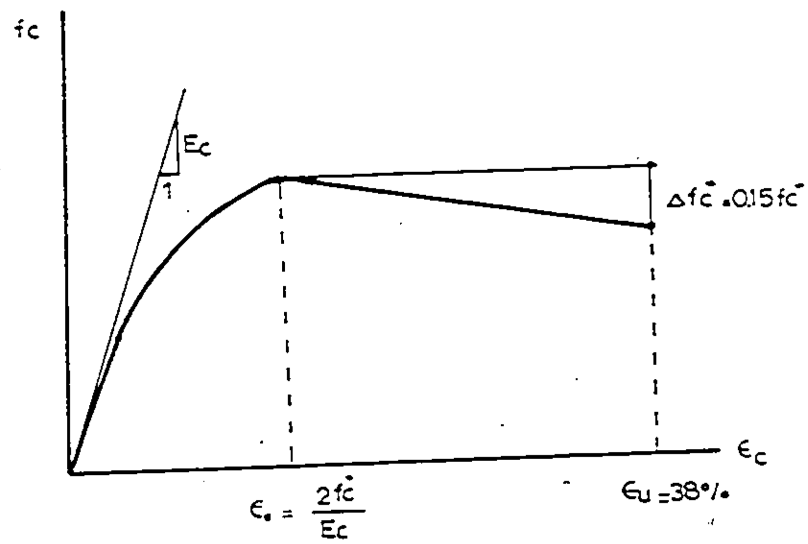


FIGURE (2-3) Horizontal Section Thru Interaction Surface



$Es = 29 \times 10^6$

FIGURE (4-2) Assumed Stress-Strain Curve for Steel



$$fc = fc' \left[\frac{2\epsilon}{\epsilon_0} - \left(\frac{\epsilon}{\epsilon_u}\right)^2 \right]$$

$Ec = 18\,000000 + 460\,fc'$

fc' = Compression Strength of 6X12 in Concrete Cylinders

FIGURE (4-1) Assumed Stress-Strain Curve for Concrete

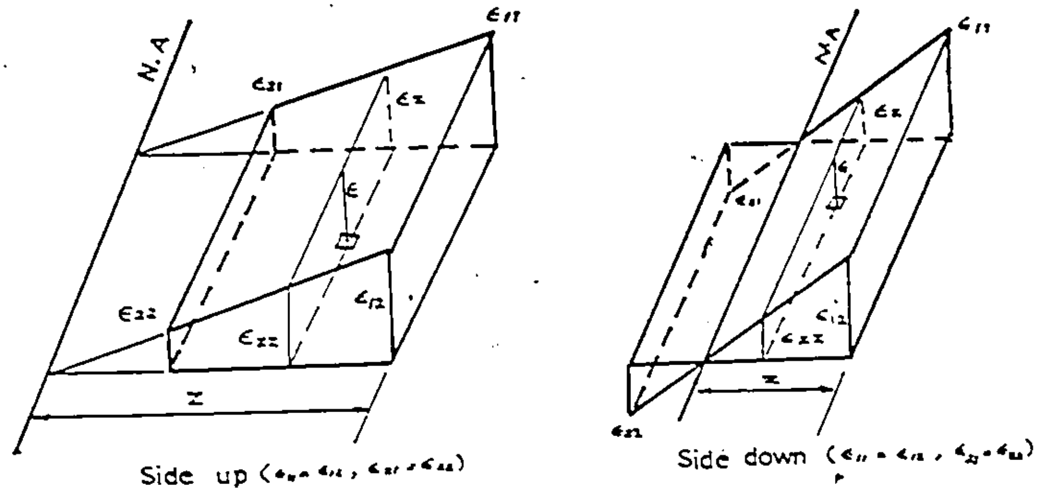


FIGURE (4-3) Planer Strain Distributions Giving $\theta = 0°$

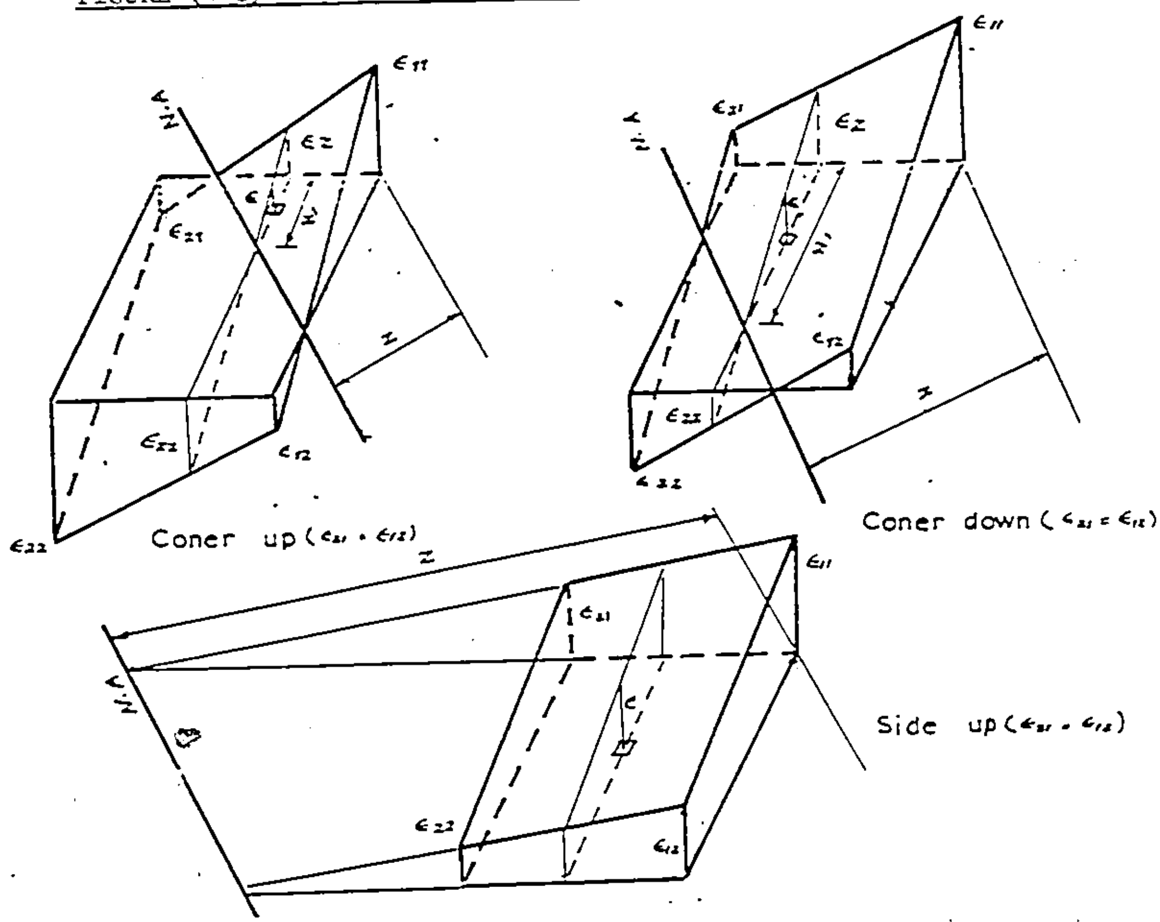


FIGURE (4-4) Planer Strain Distribution Giving $\theta = 45°$

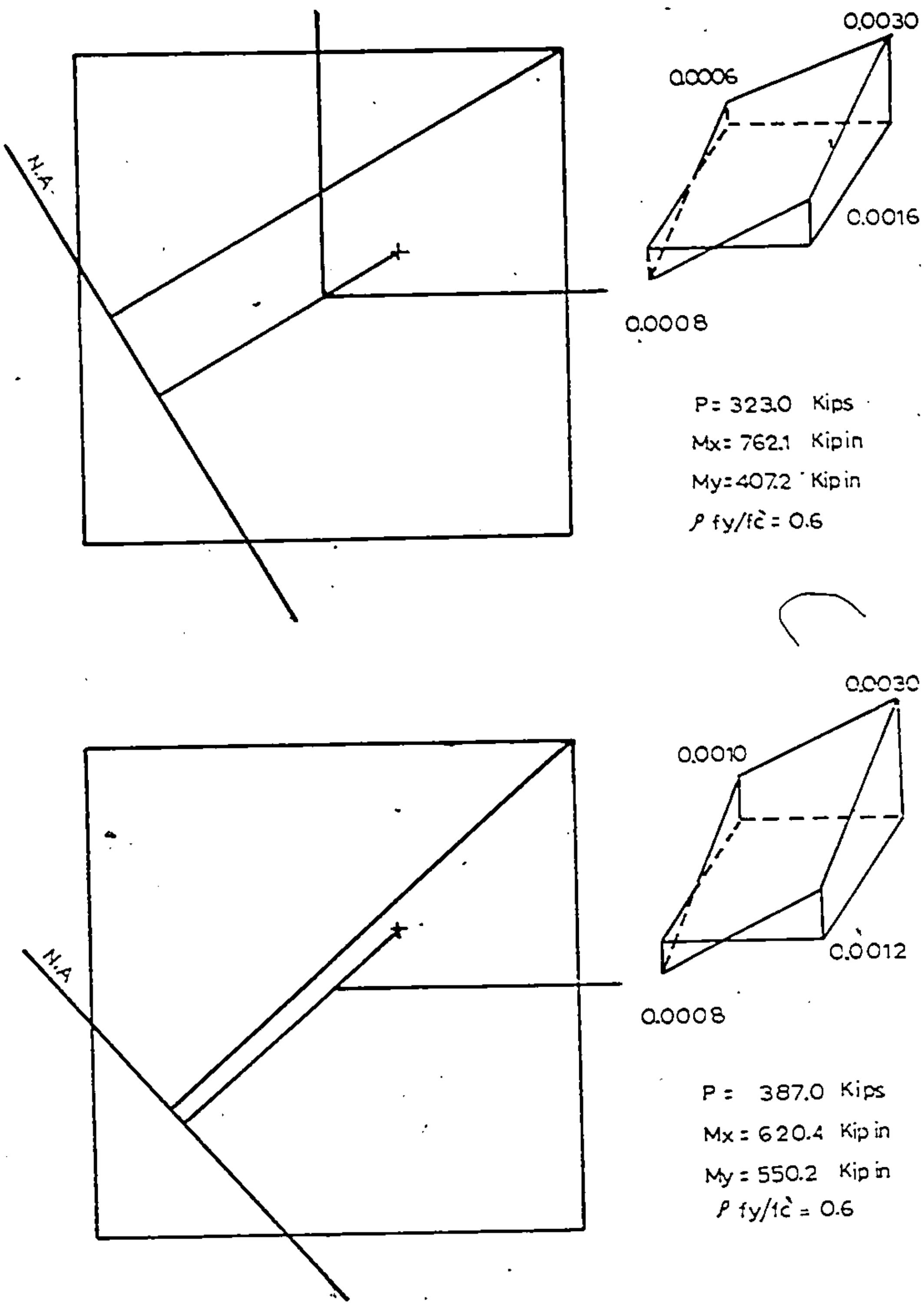


FIGURE (4-6) Justification of Curvature Values in Symmetric Sections

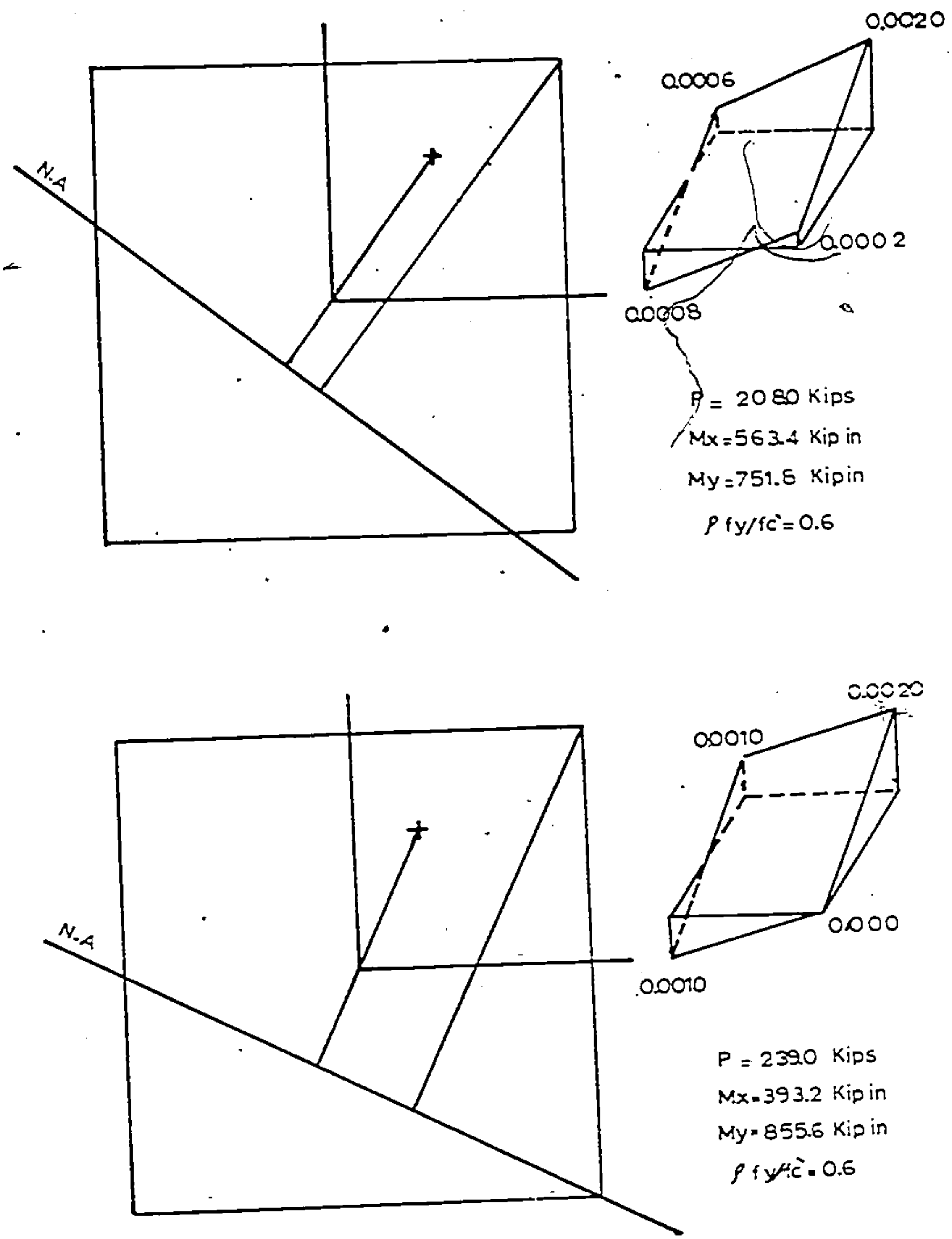
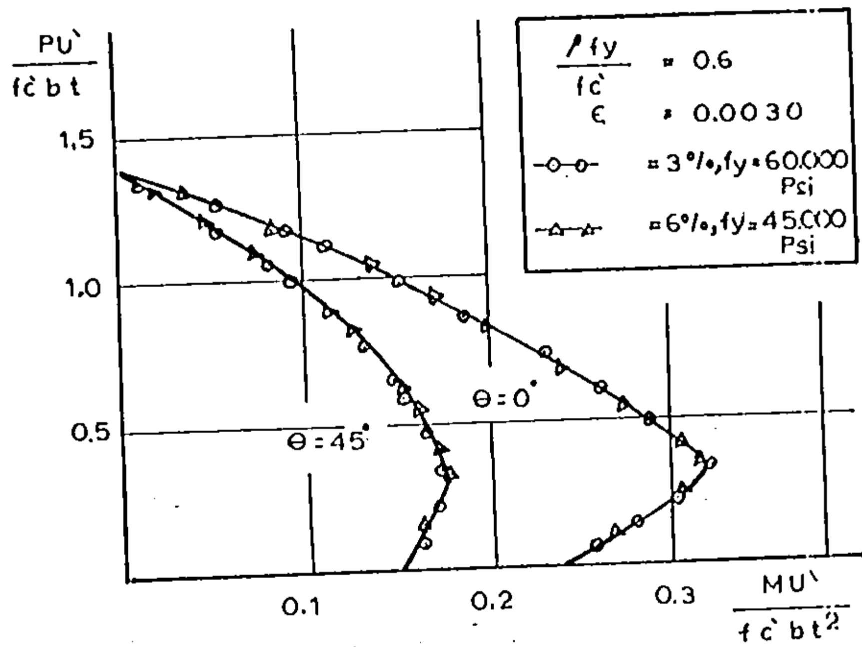


FIGURE (4-7) Justification of Curvature Values in Symmetric Sections



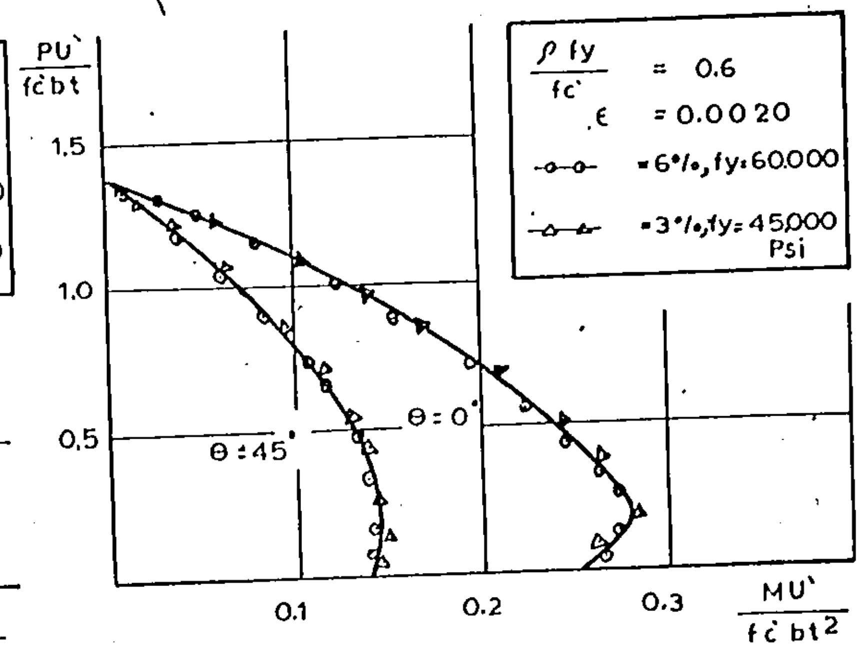


FIGURE (4-8) Justification of $\rho f_y/f_c'$ as an Independent Parameter

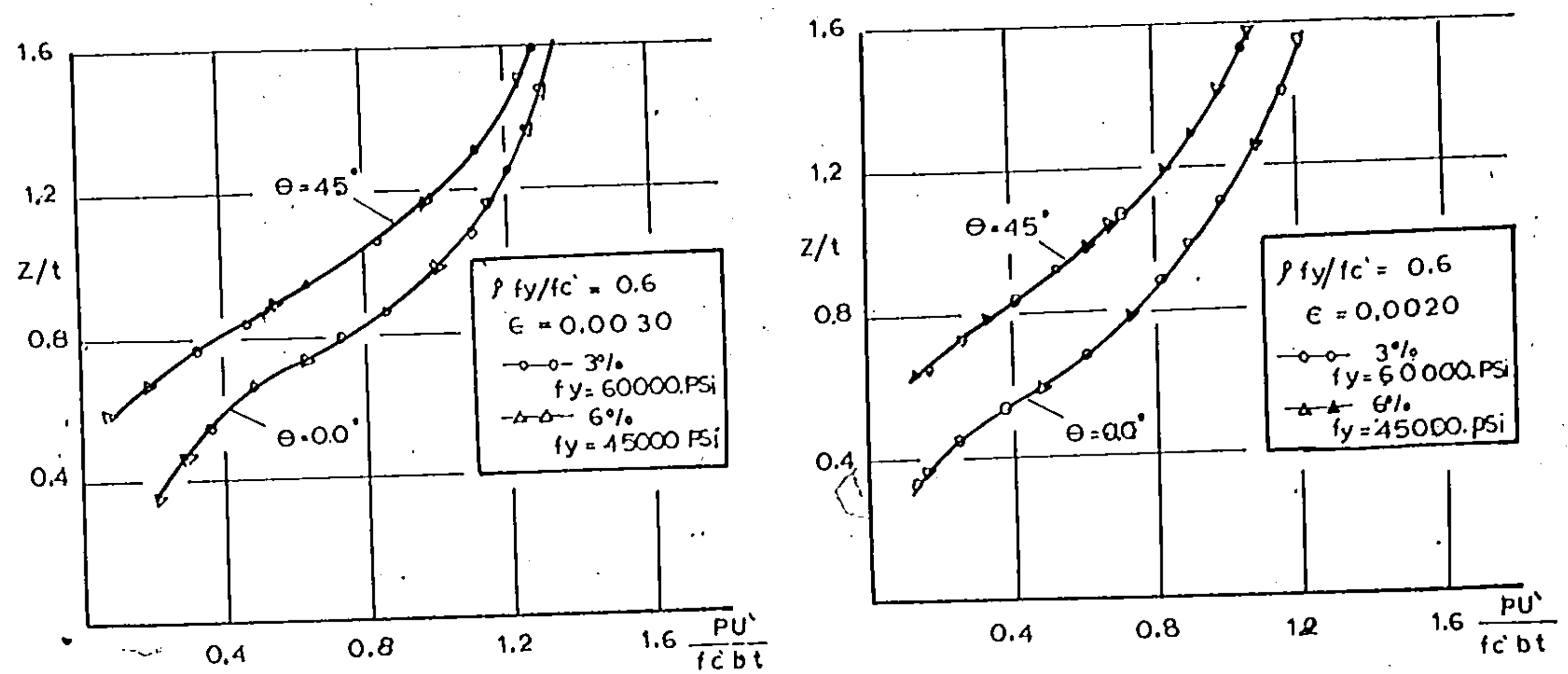


FIGURE (4-9) Justification of $\rho f_y/f_c'$ as an Independent Parameter

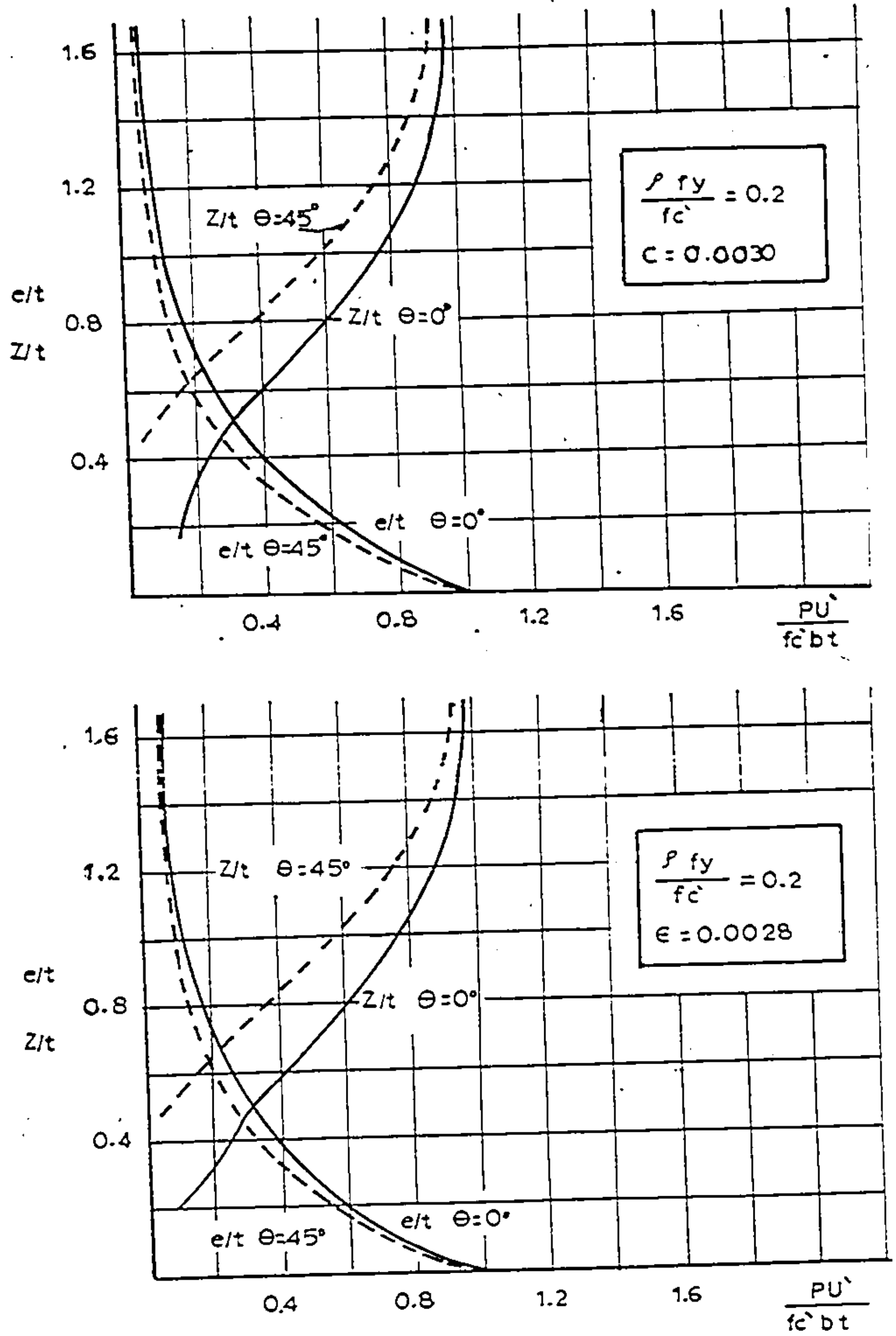
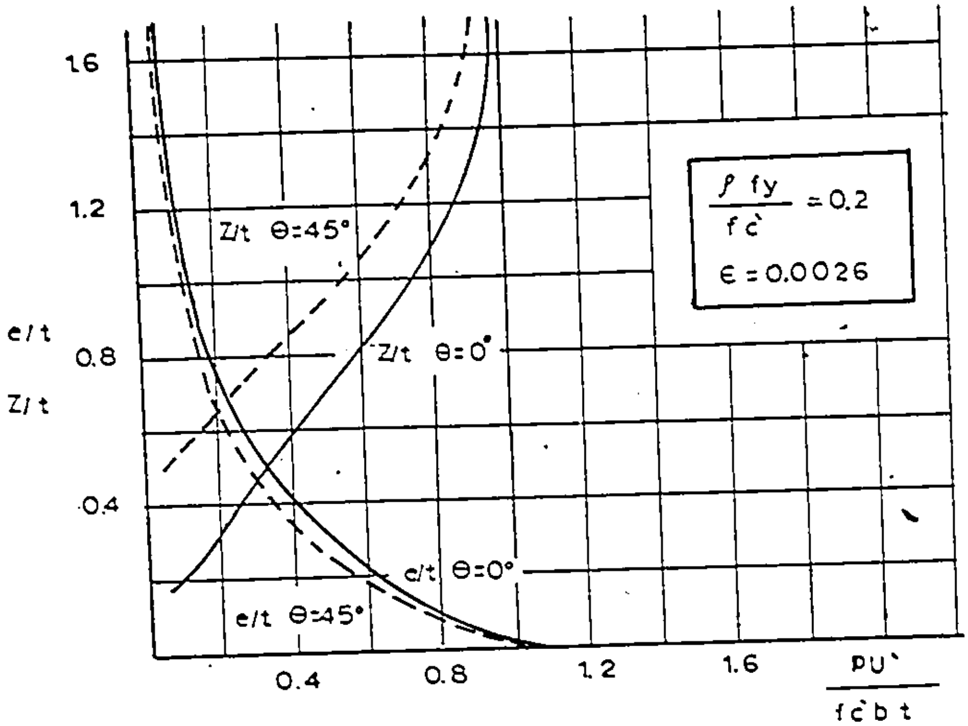
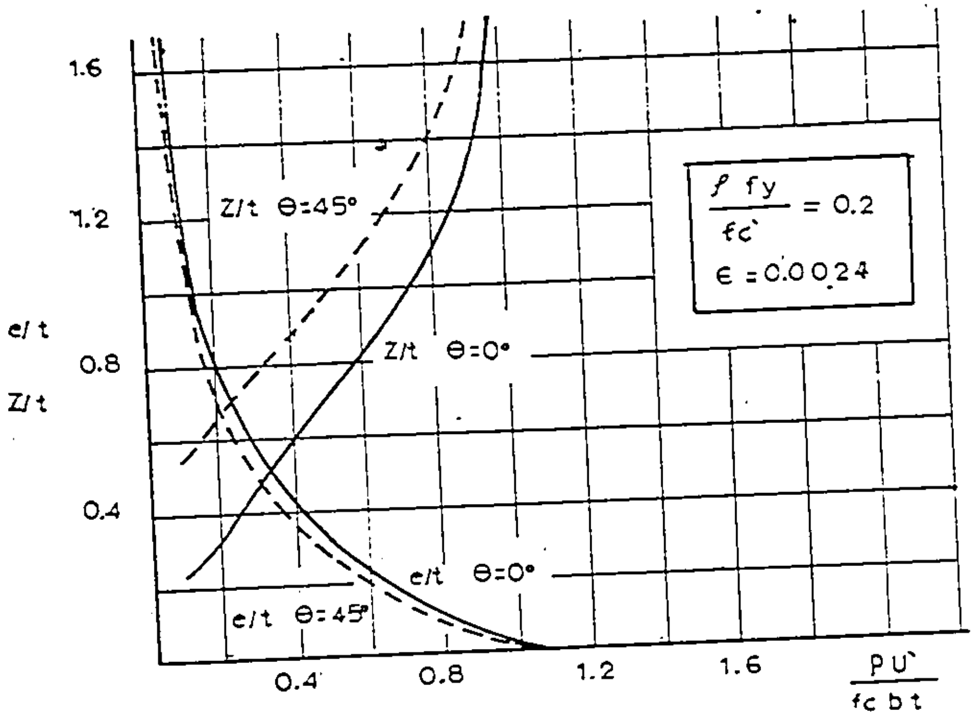


FIGURE (4-10) Interaction Curves for $\rho f_y/f_c' = 0.2$





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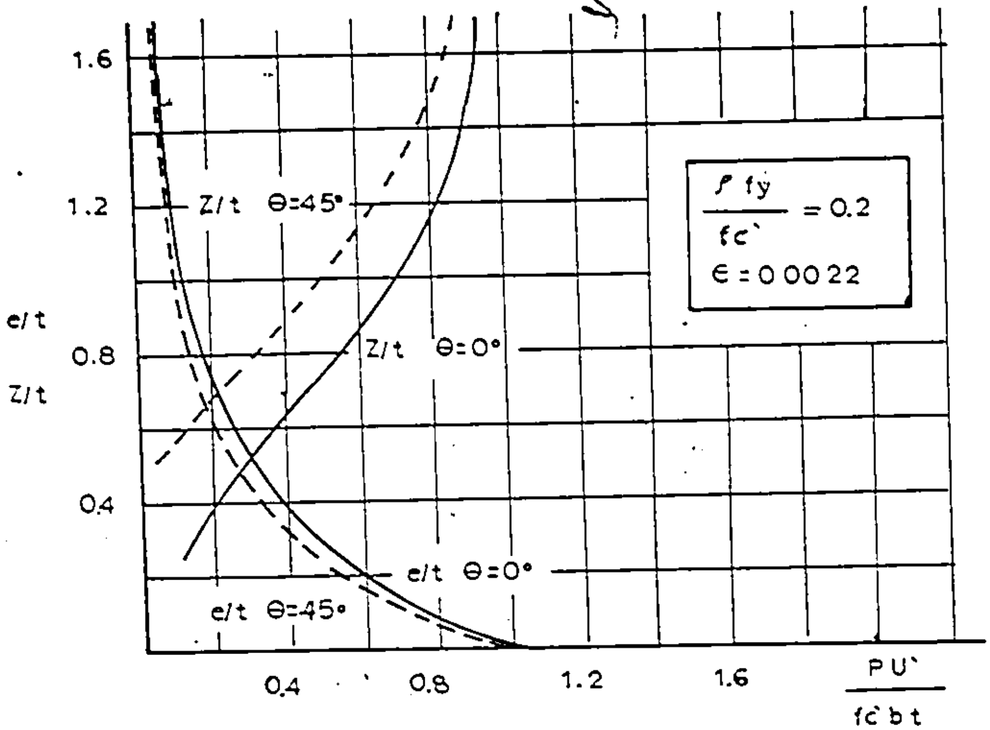

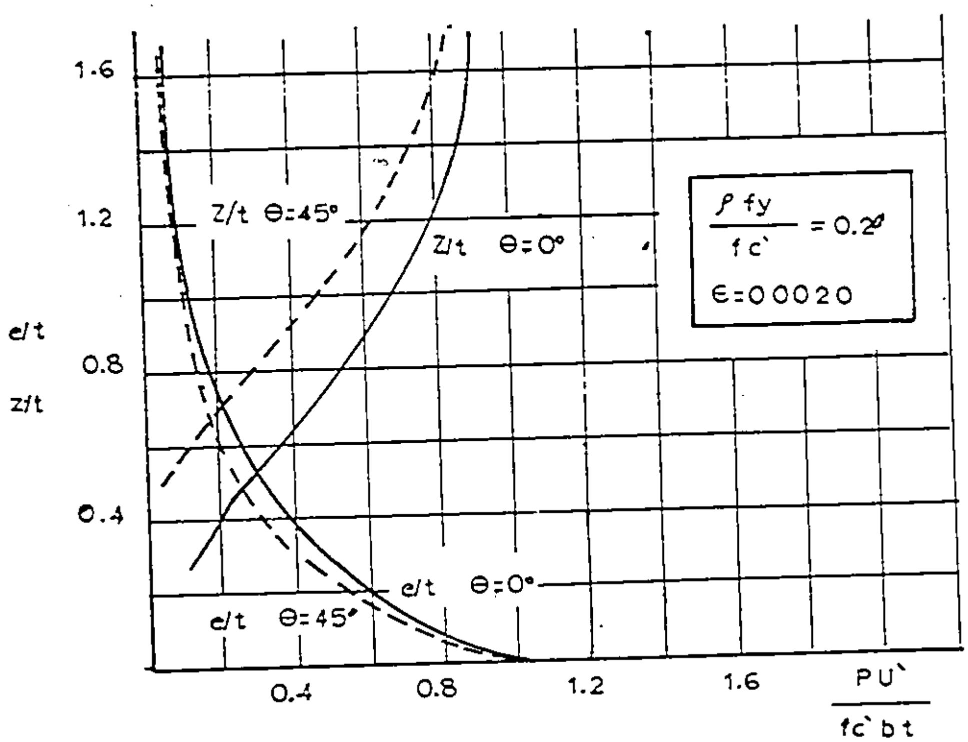

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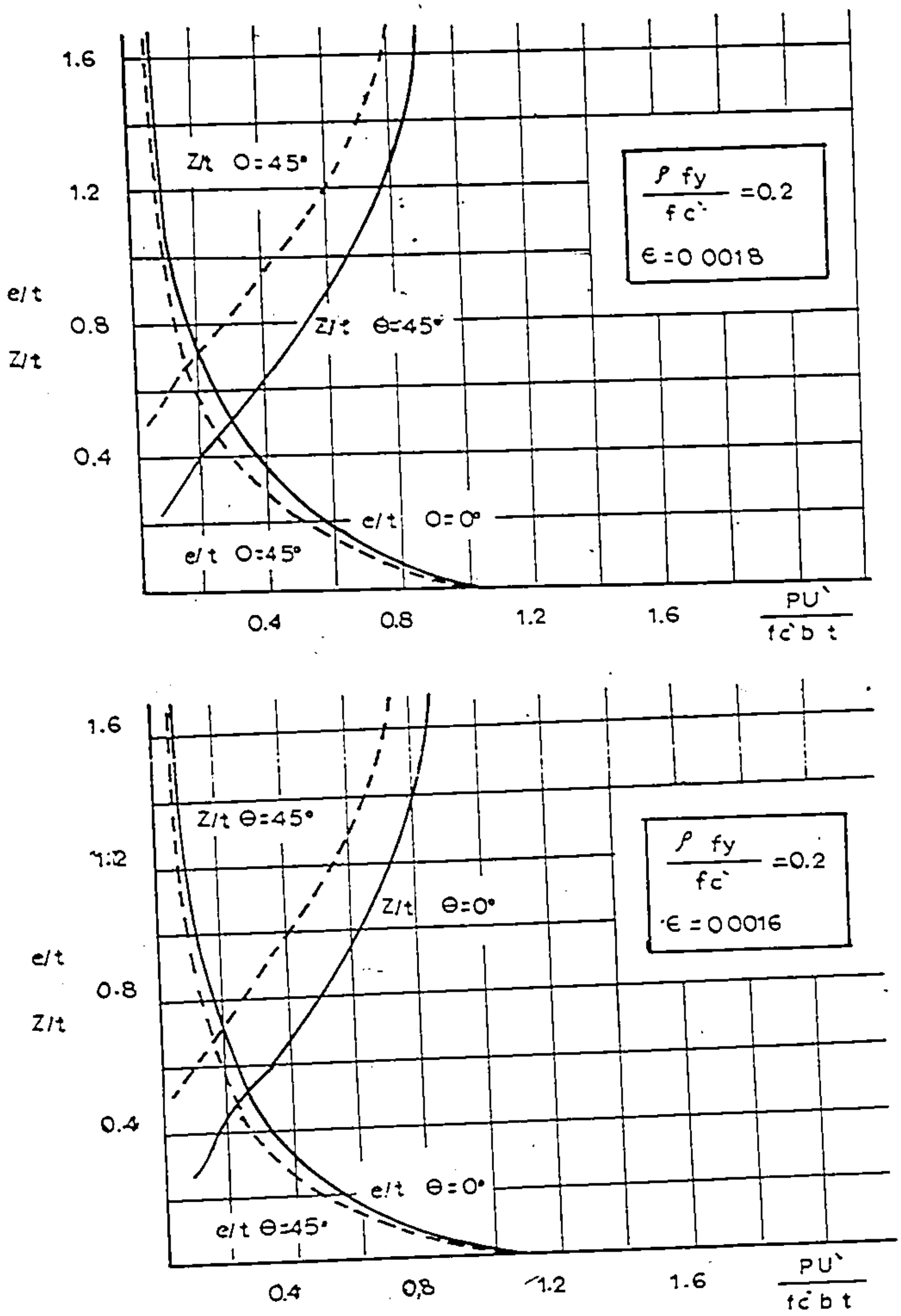

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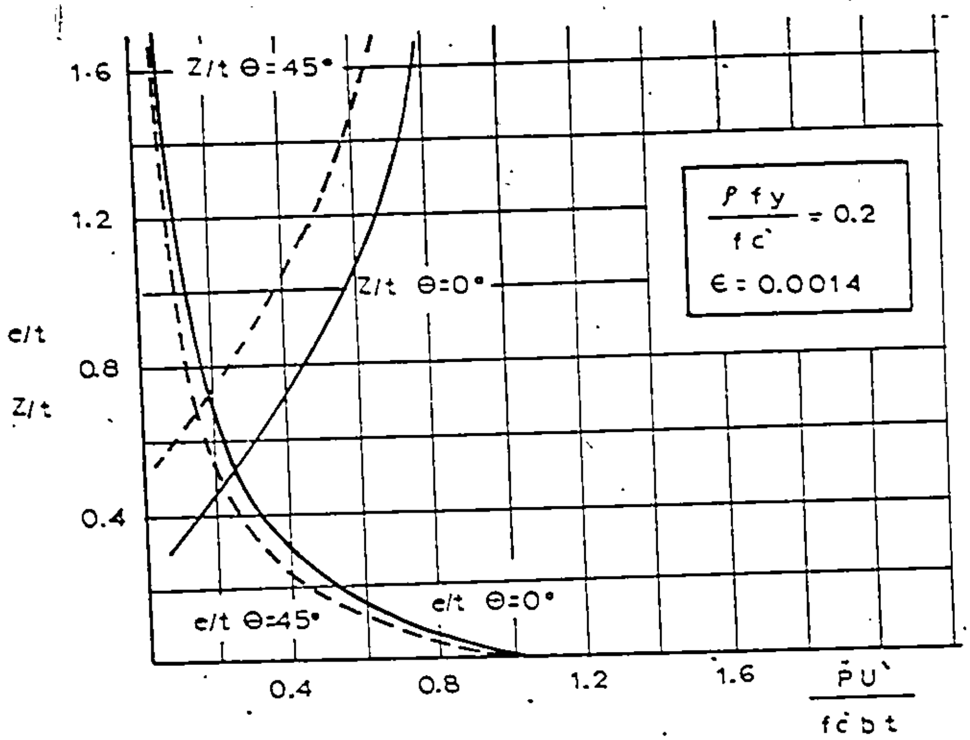

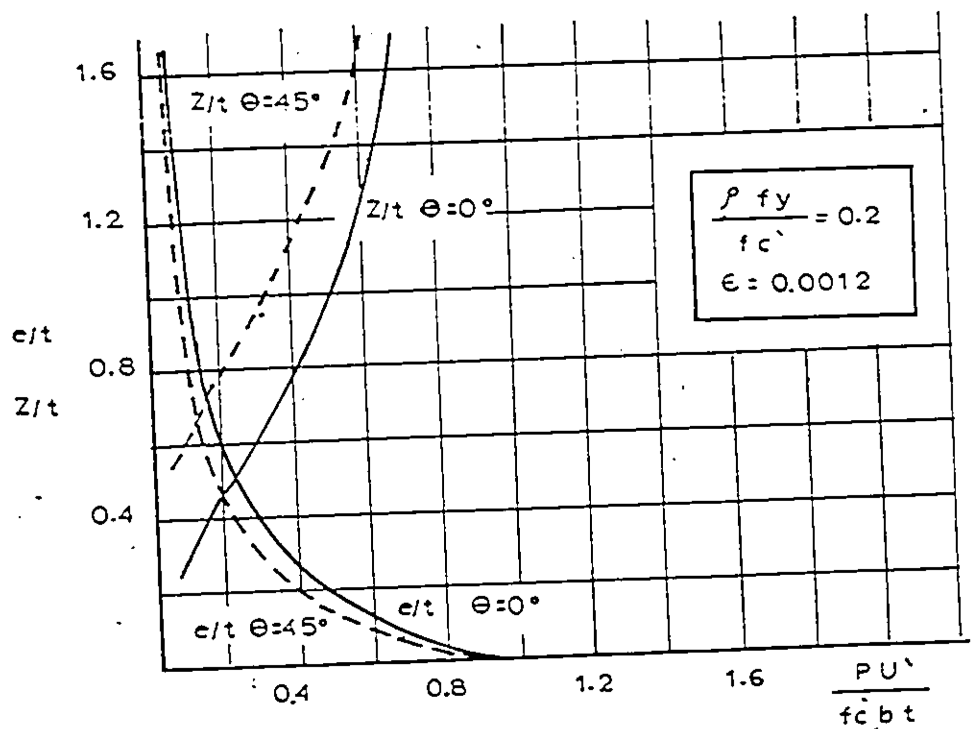

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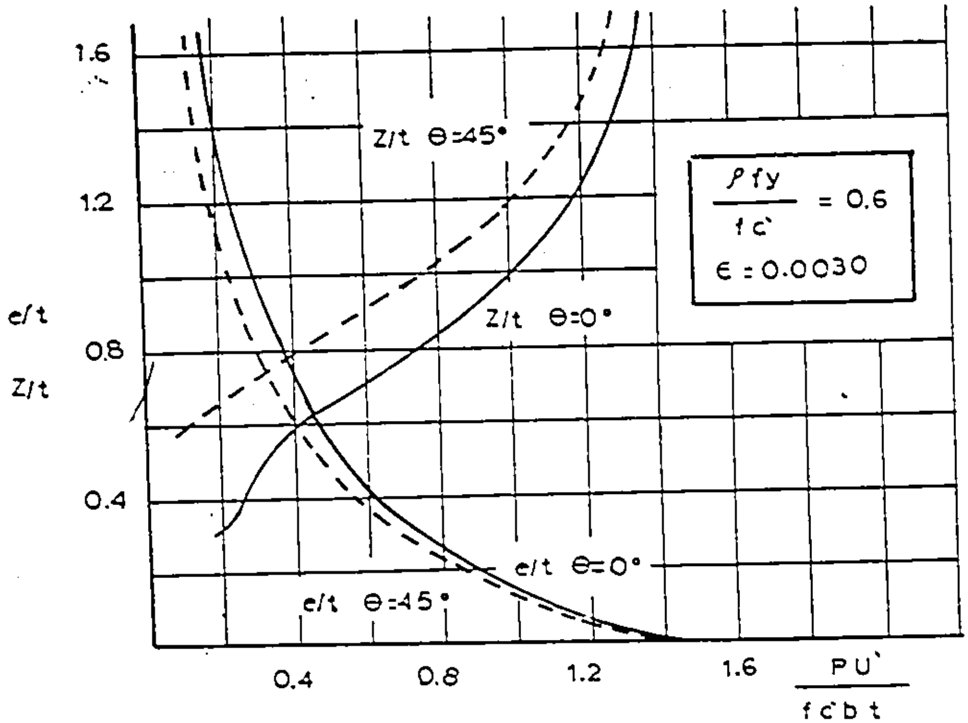


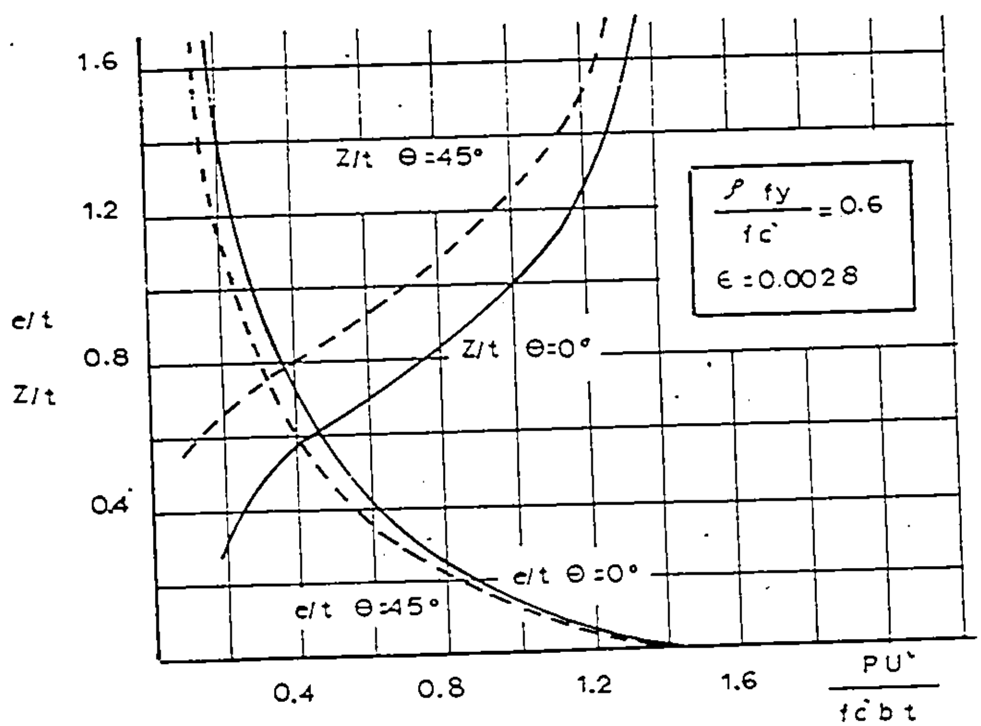
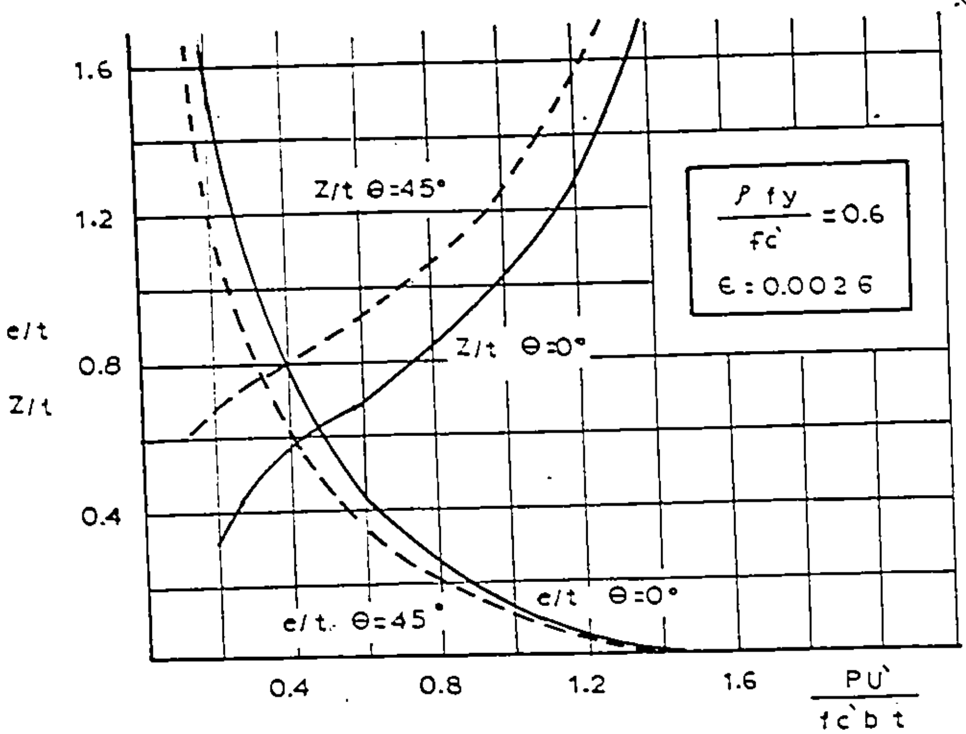
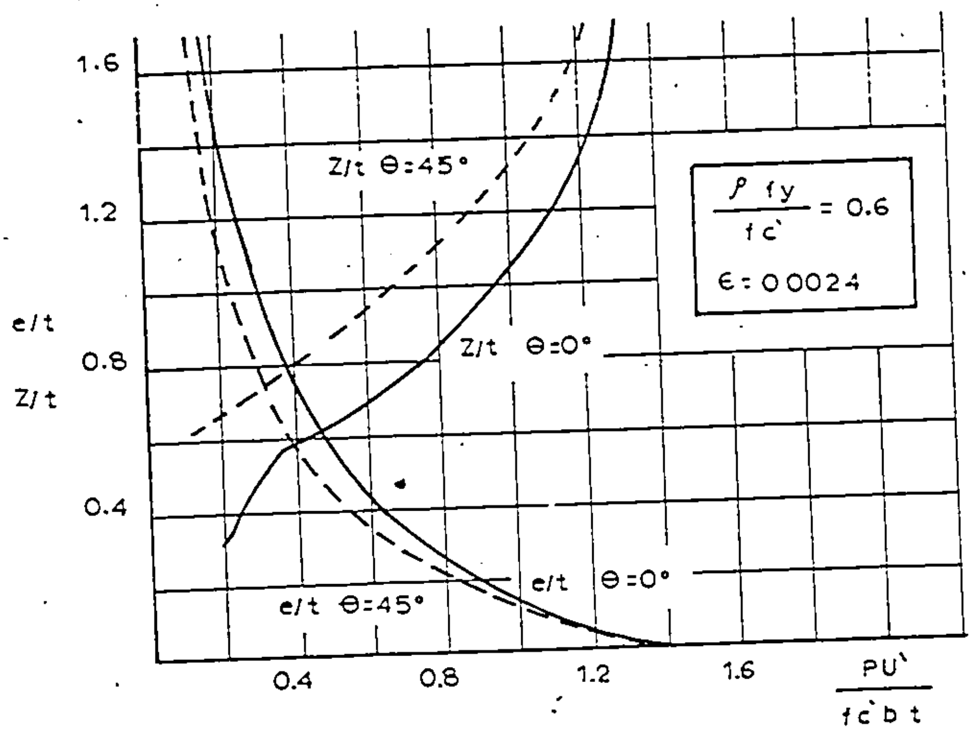


FIGURE (4-15) Interaction Curves for $\rho f_y/f_c' = 0.6$





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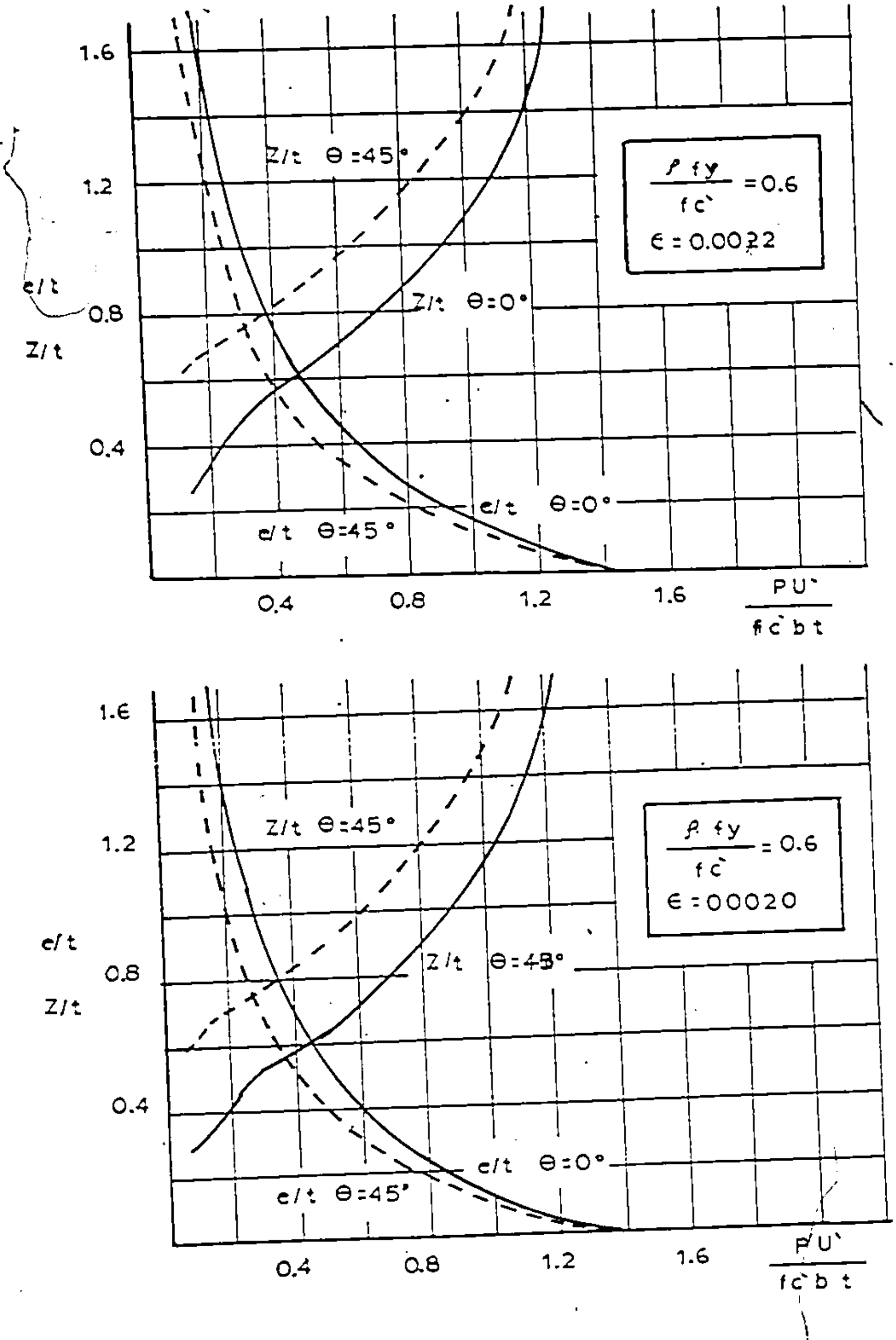
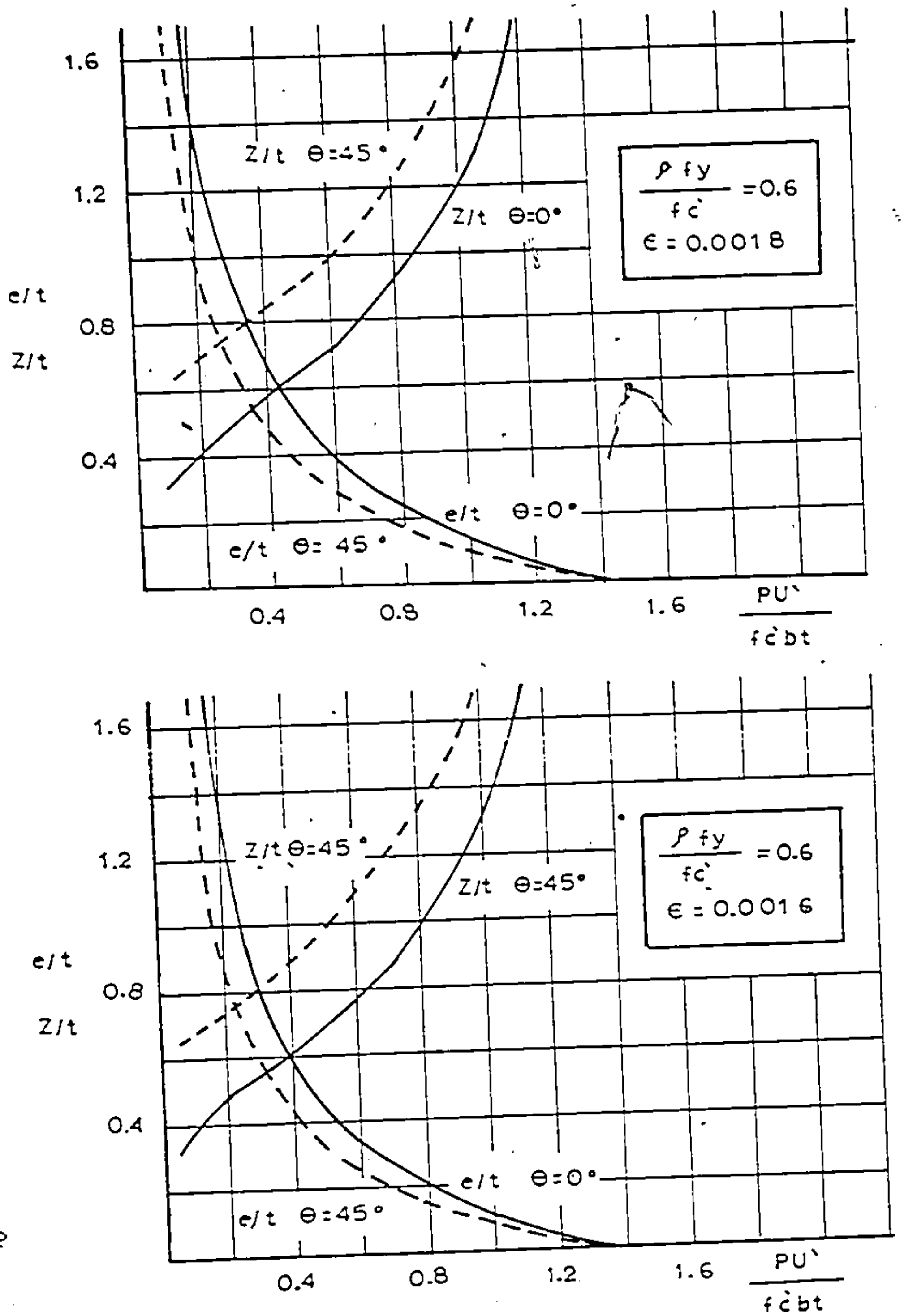


FIGURE (4-17) Interaction Curves for $\rho f_y/f_c' = 0.6$



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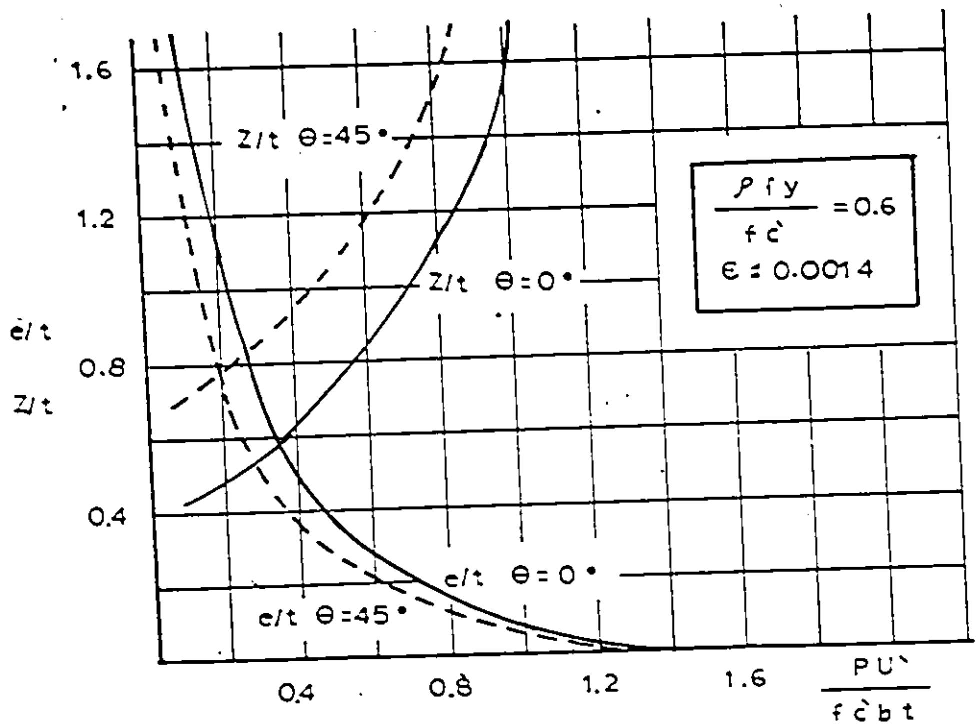

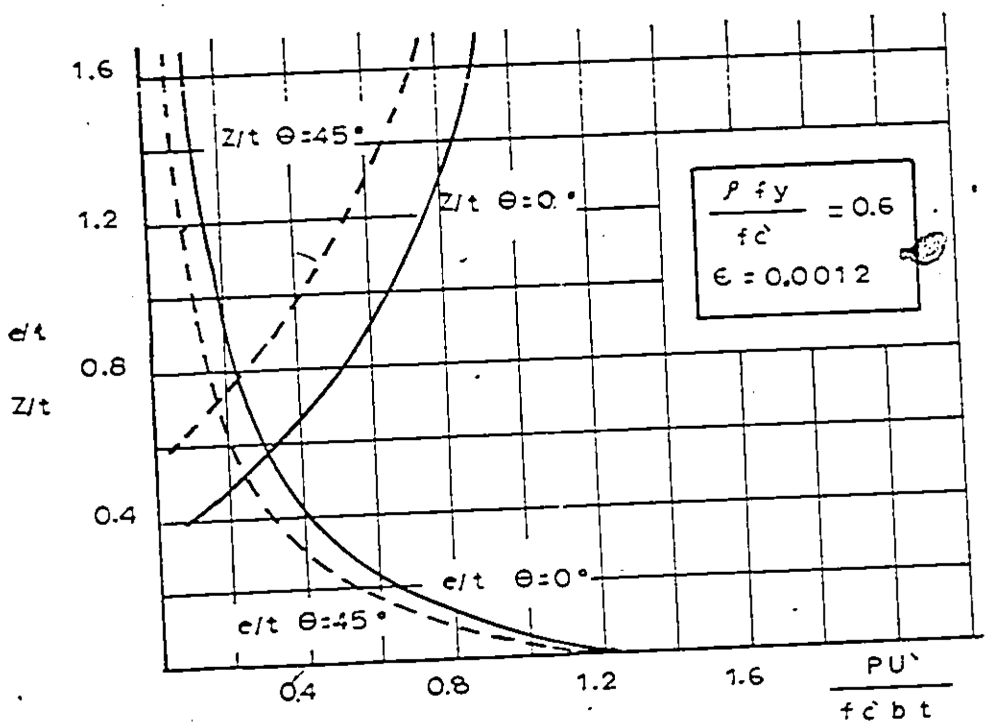

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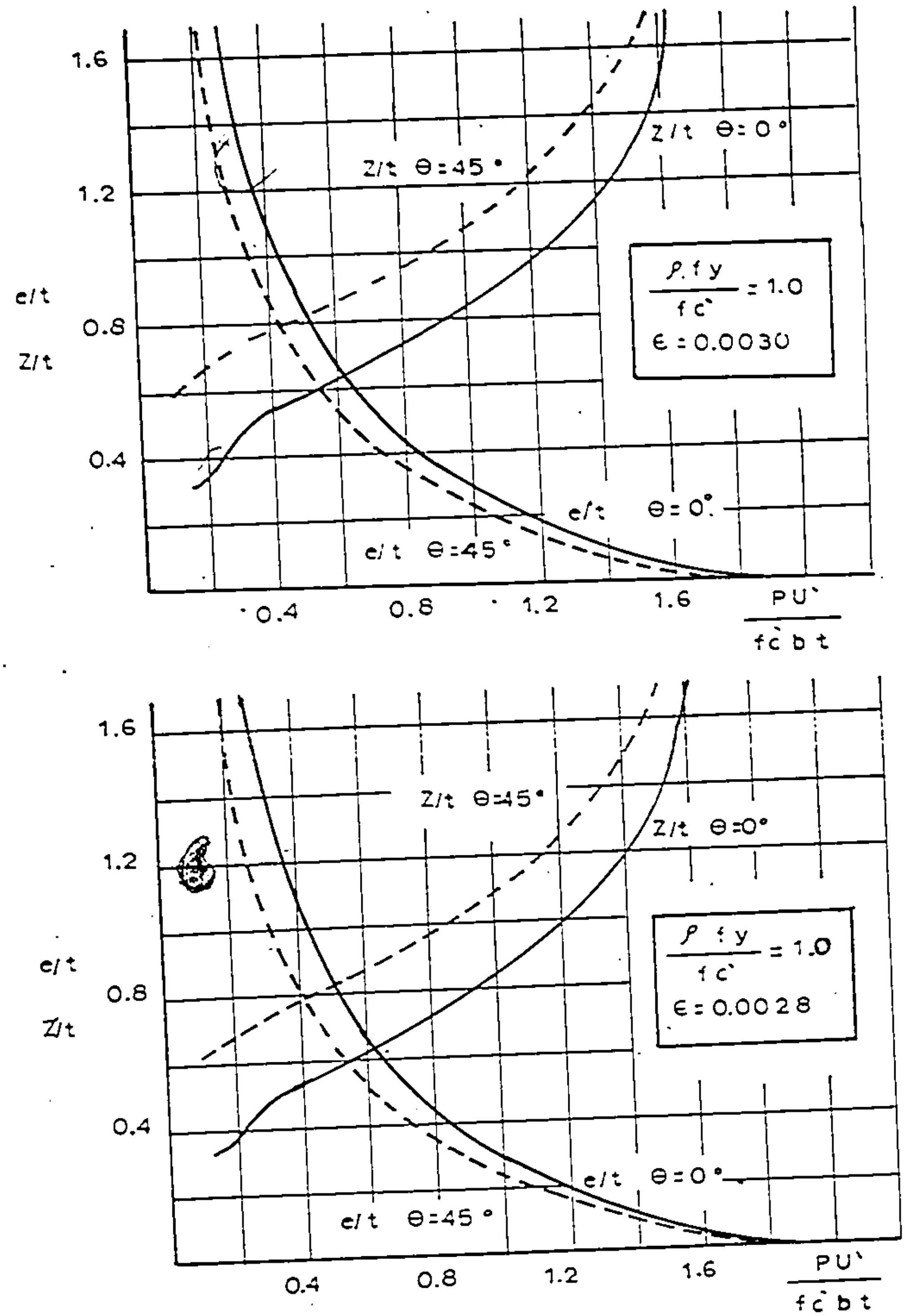
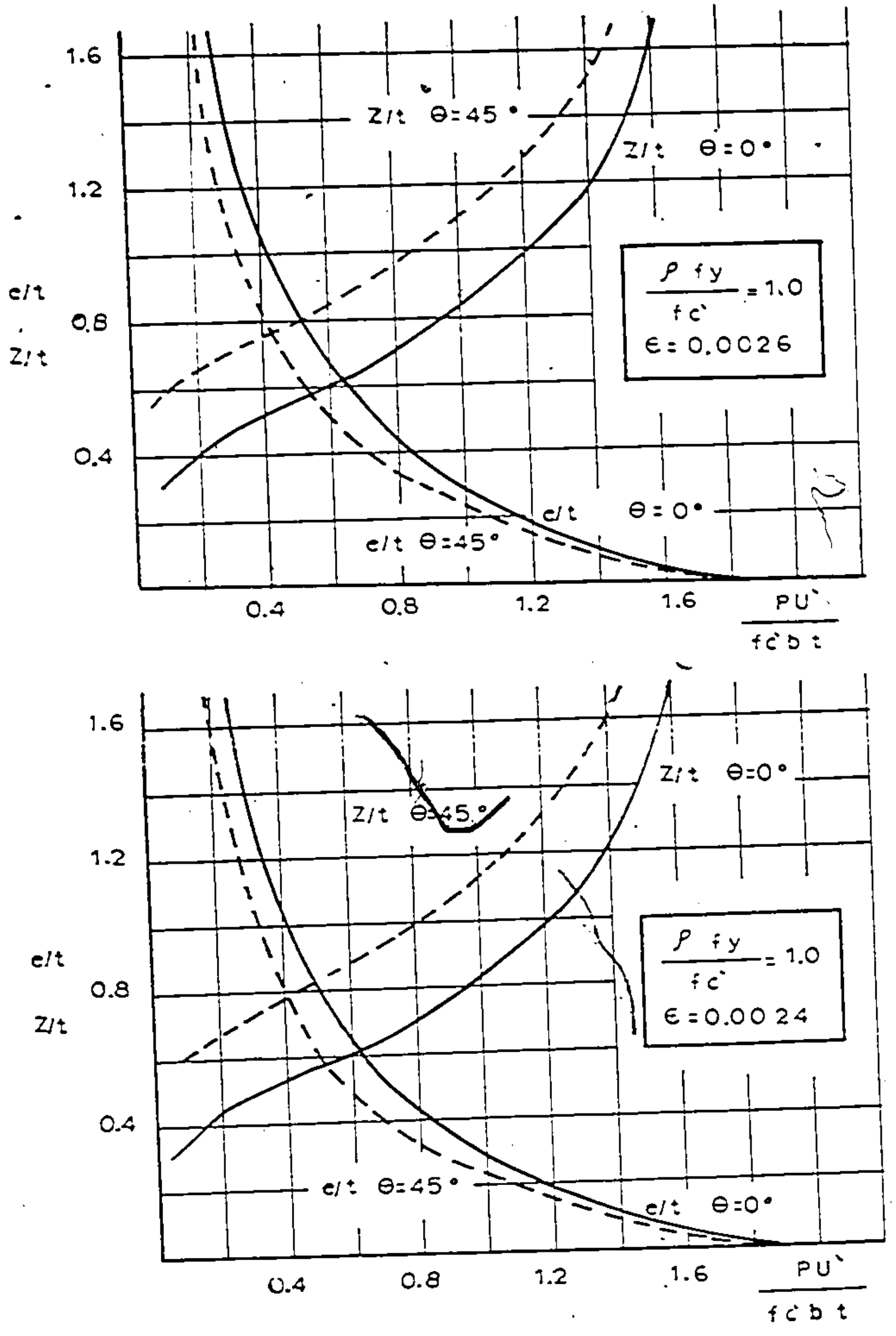


FIGURE (4-20) Interaction Curves for $\rho f_y / f_c' = 1.0$



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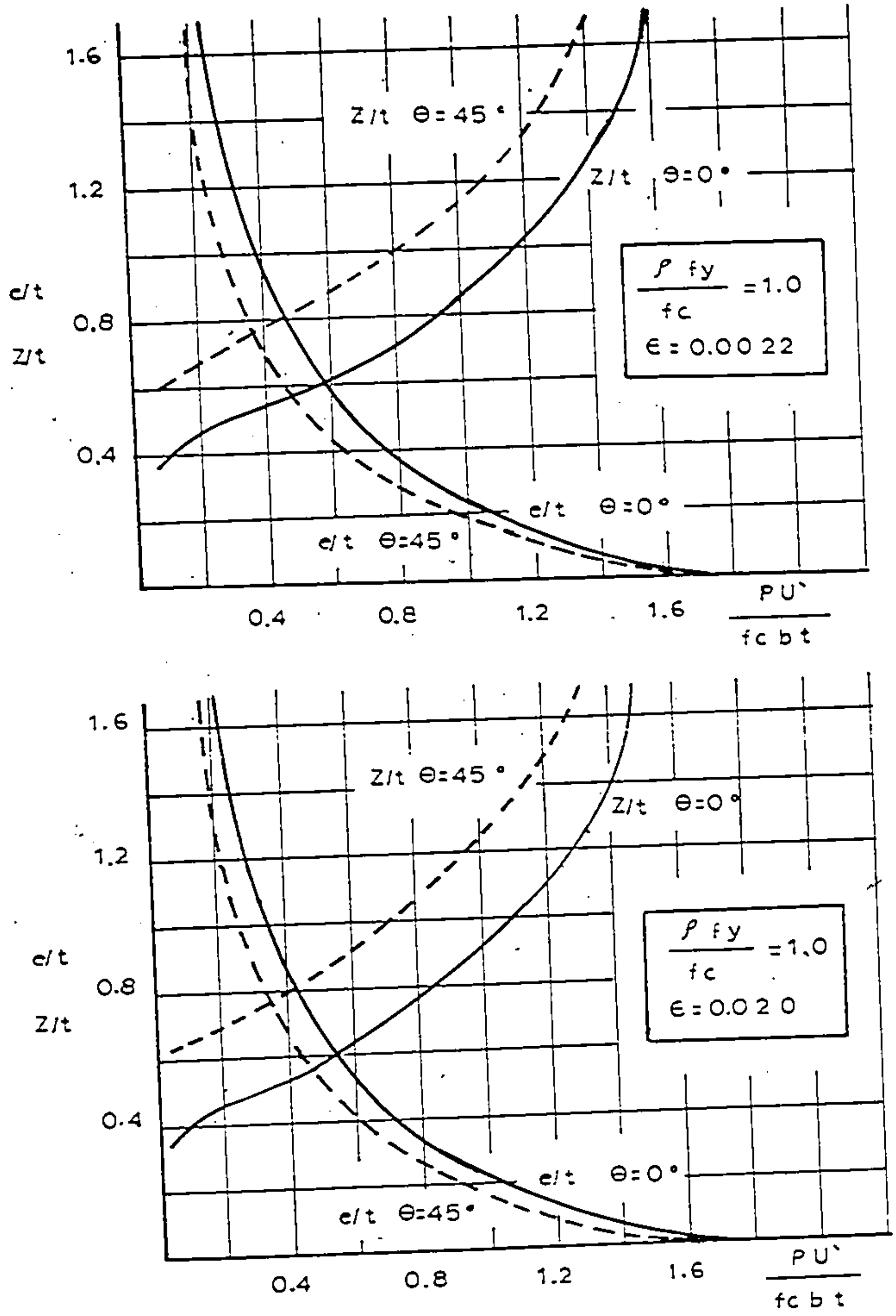

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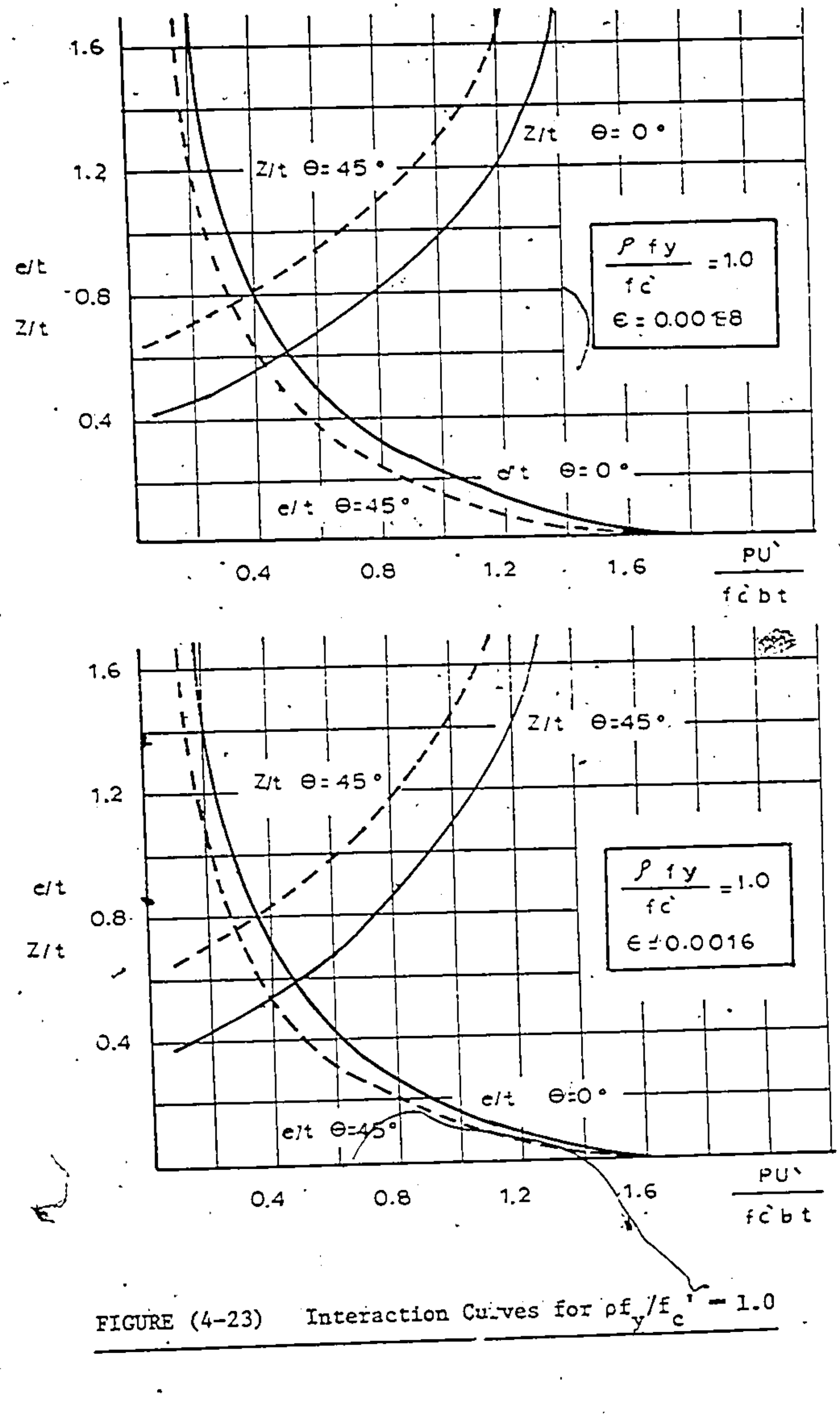

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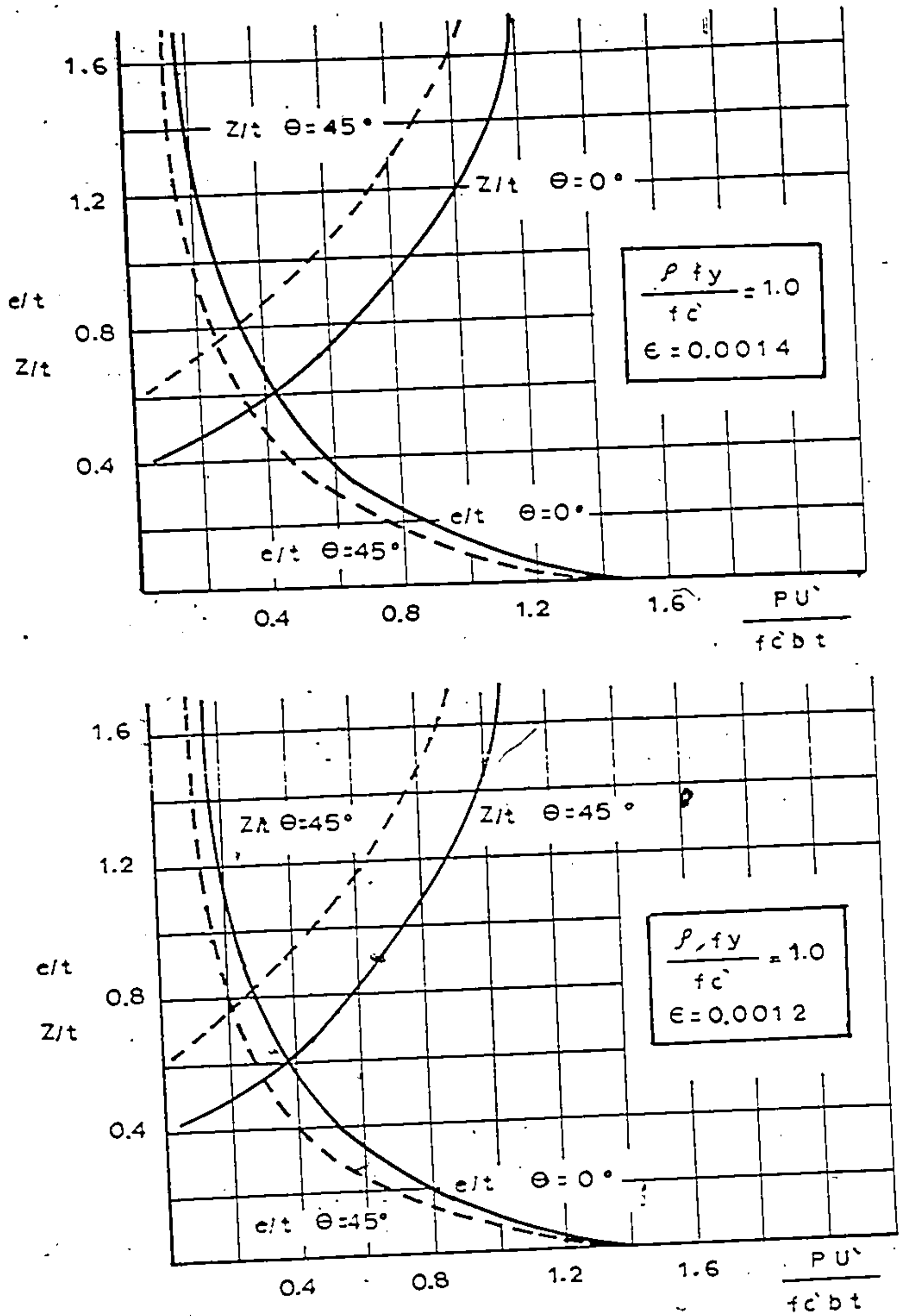

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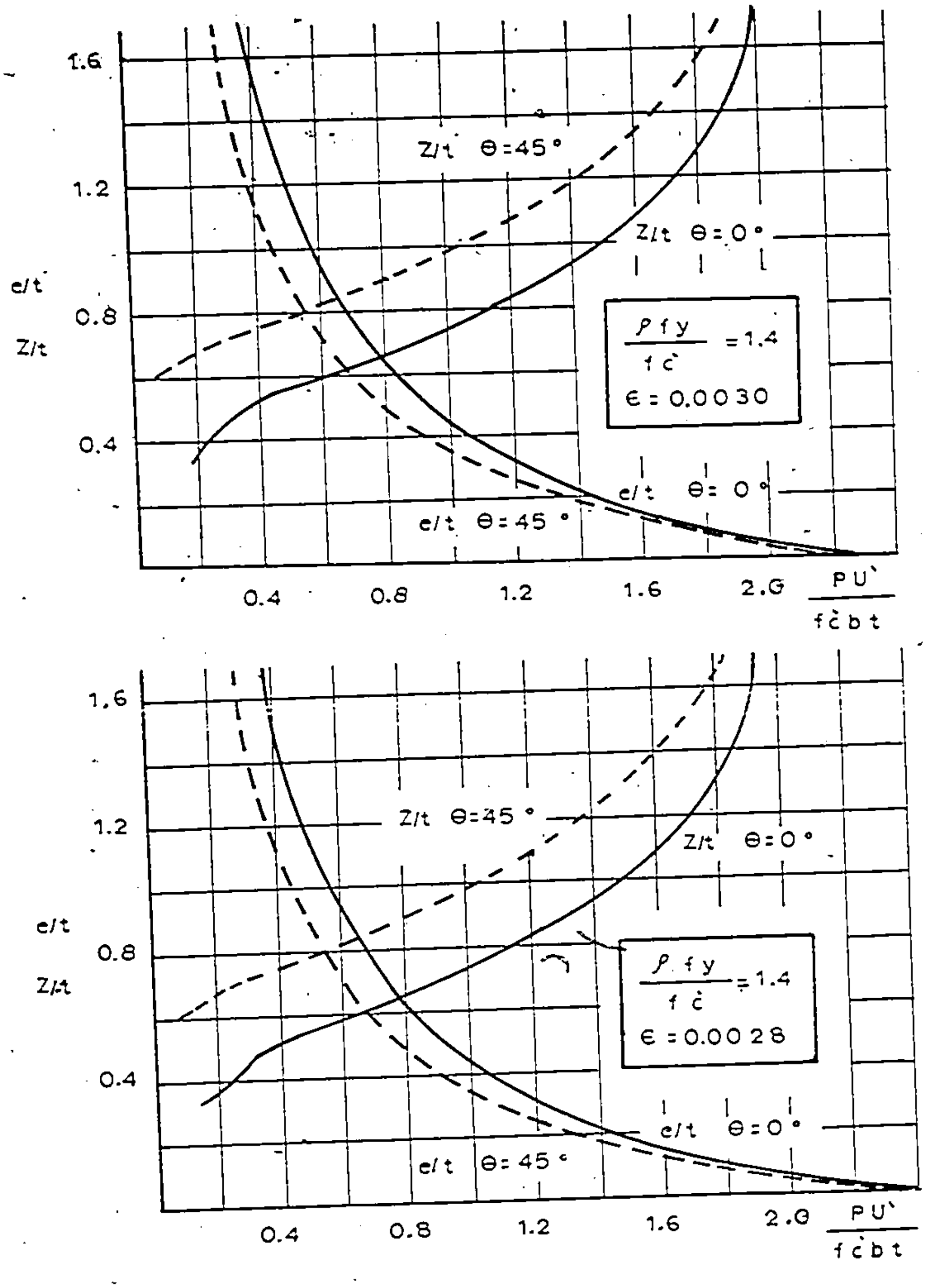
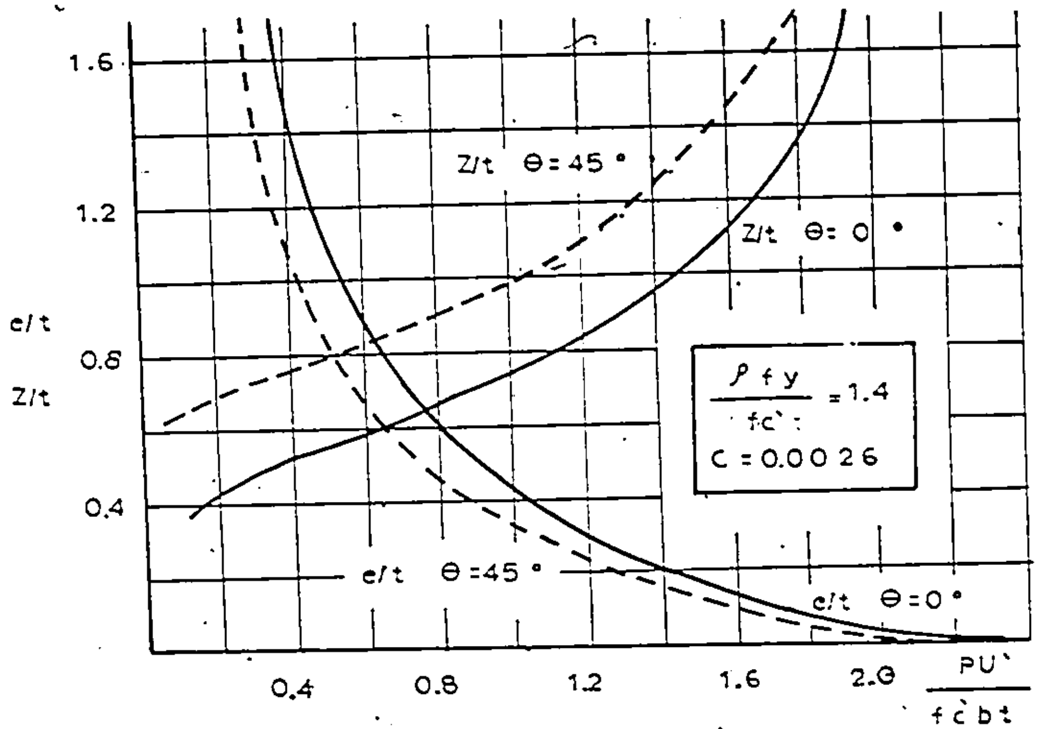


FIGURE (4-25) Interaction Curves for $\rho f_y/f_c' = 1.4$



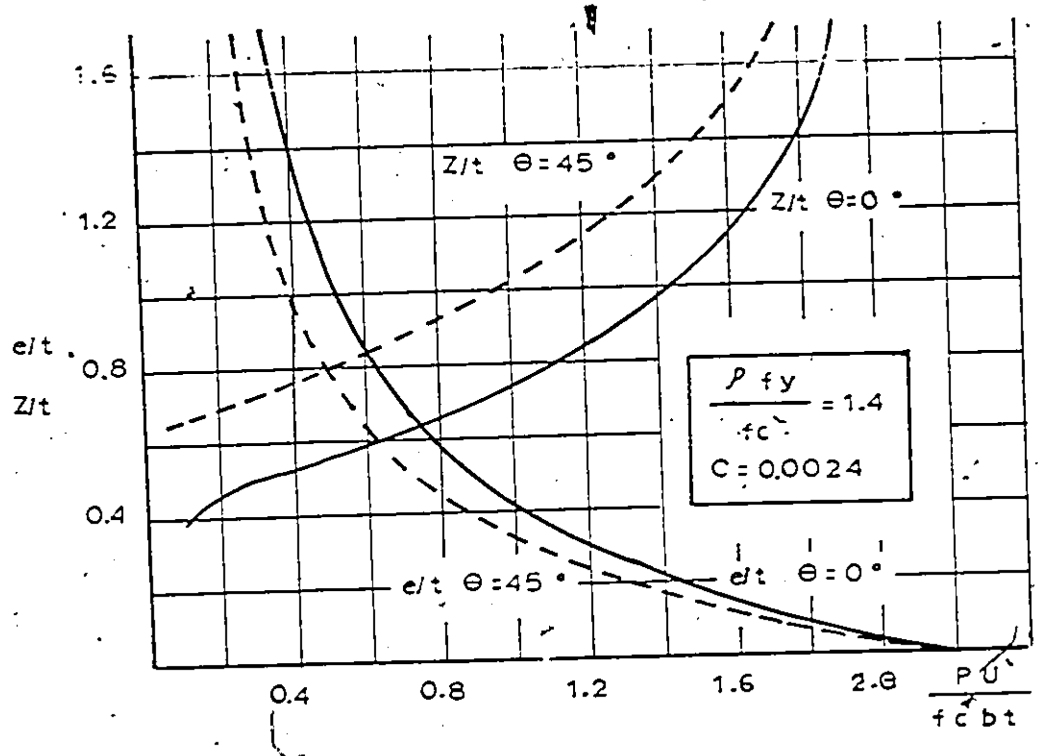
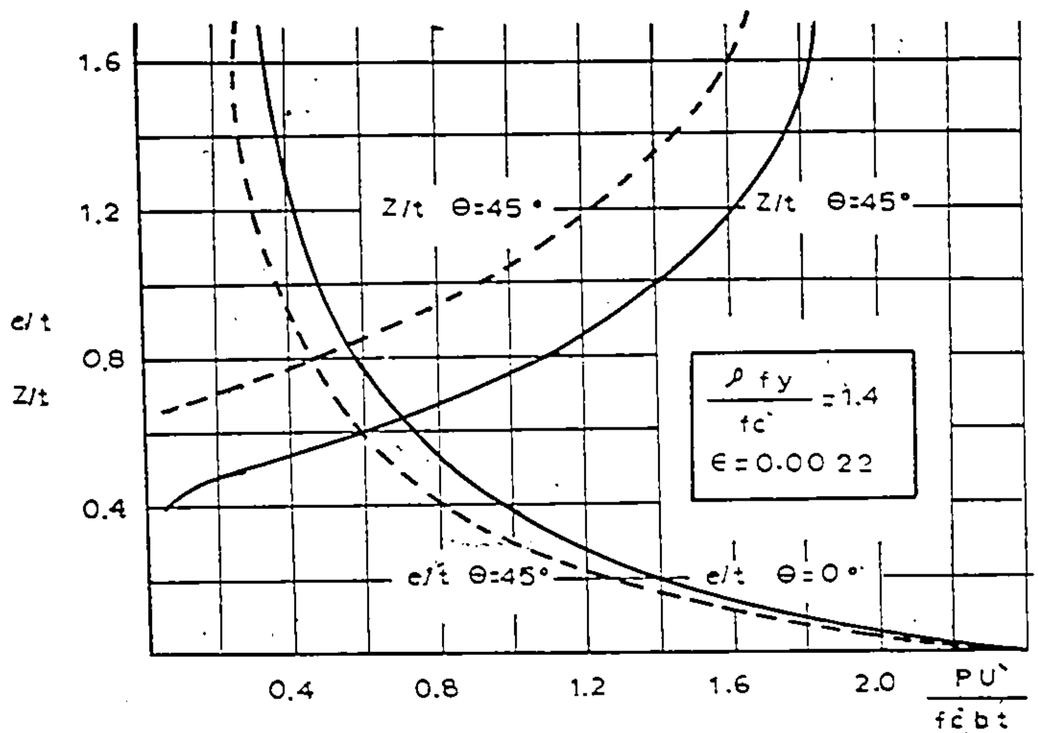


FIGURE (4-26) Interaction Curves for $\rho f_y/f_c' = 1.4$



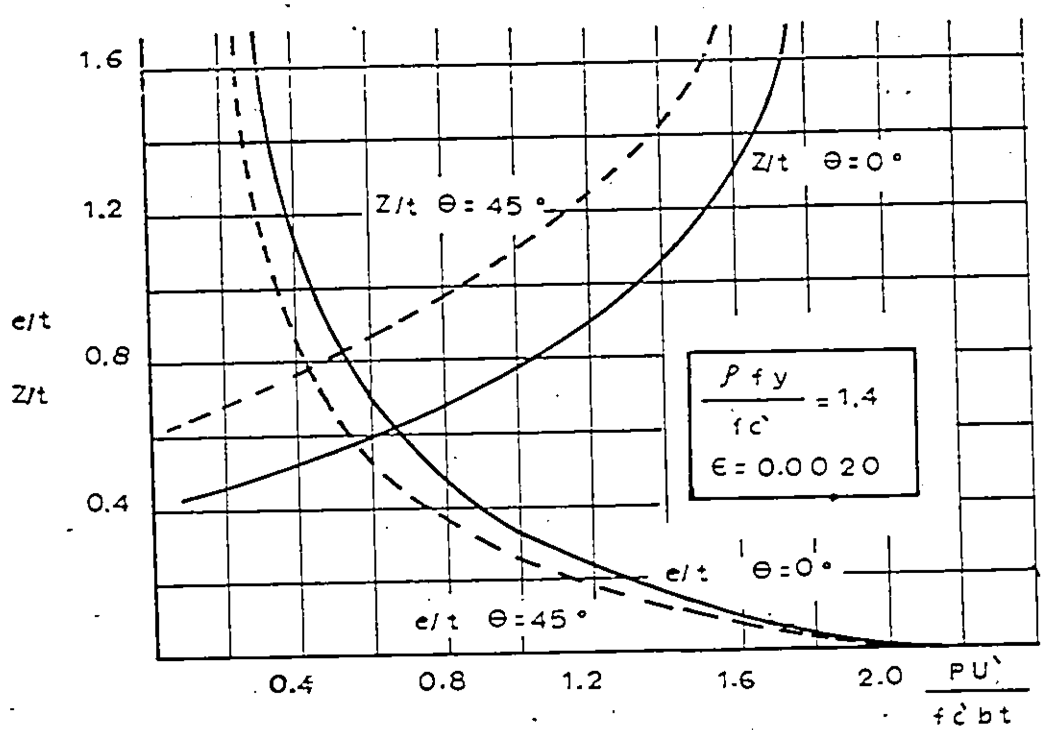
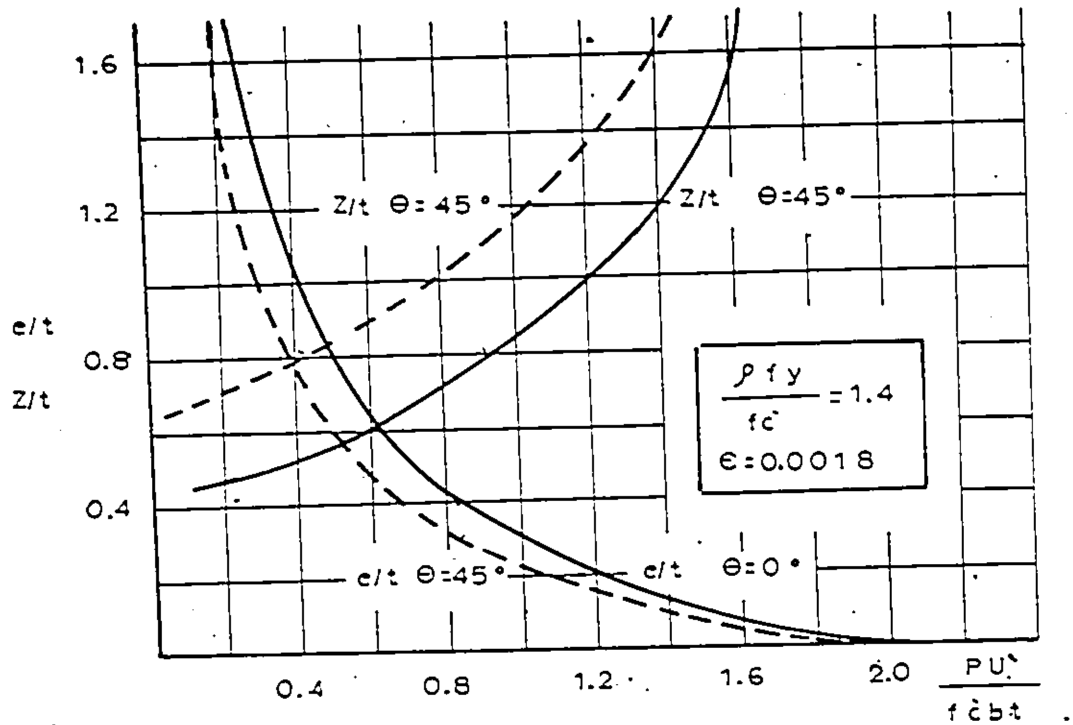
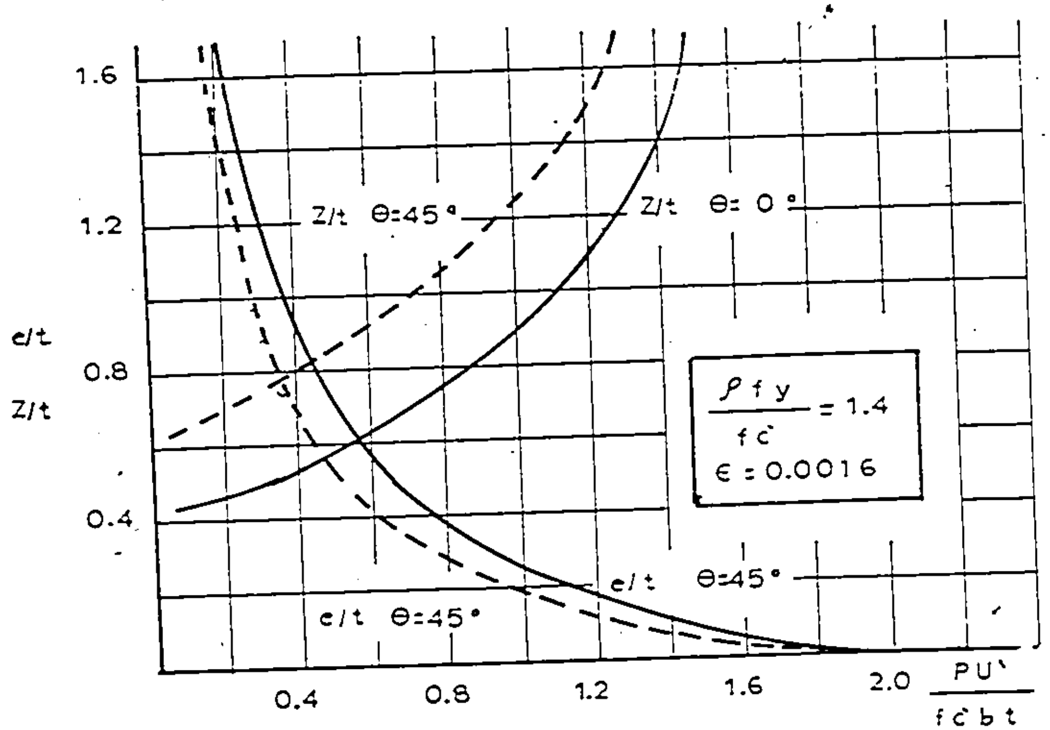


FIGURE (4-27) Interaction Curves for $\rho f_y/f_c' = 1.4$





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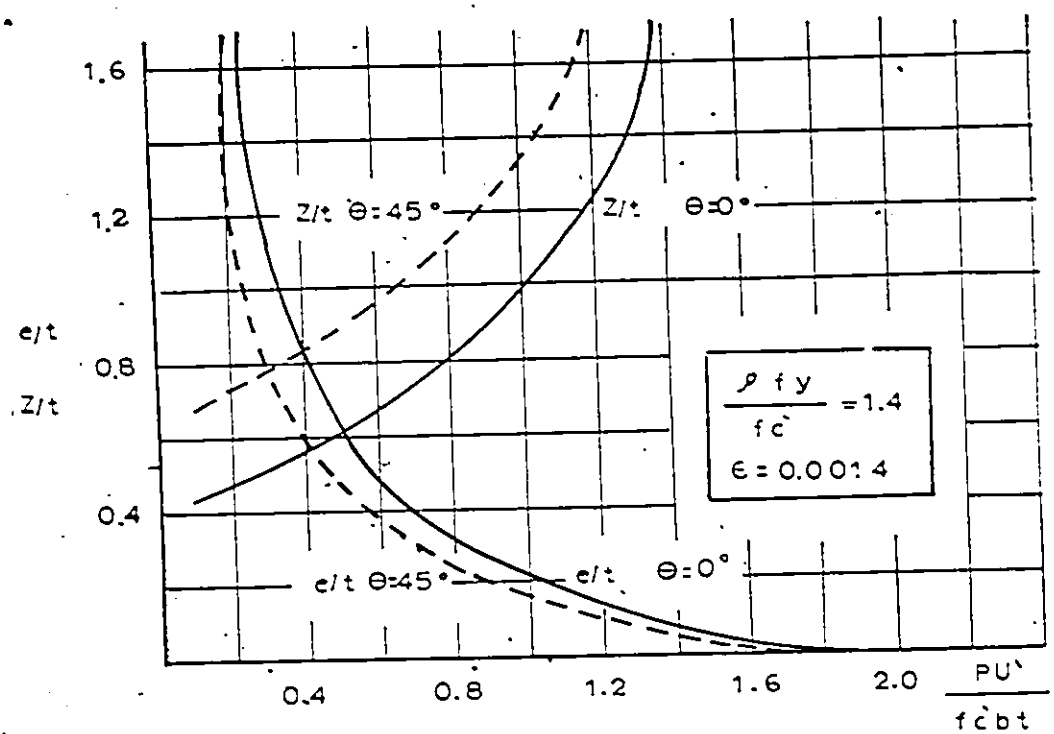

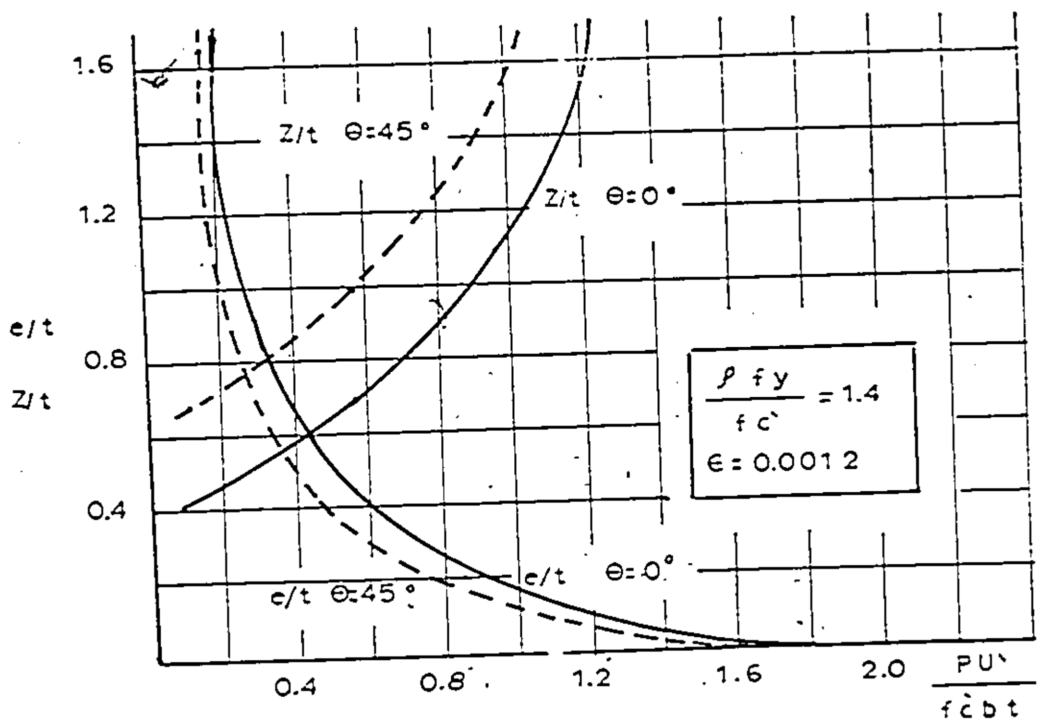

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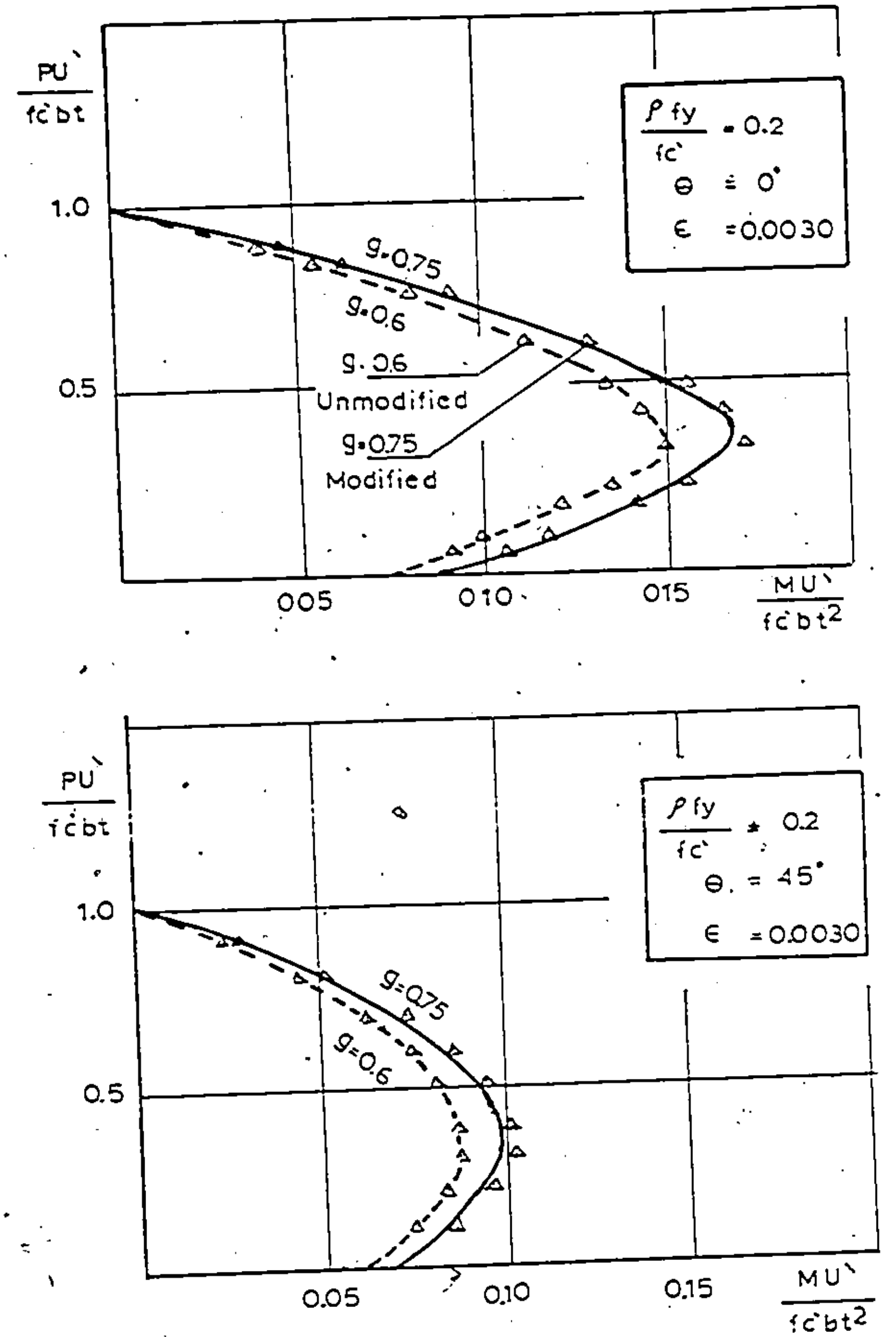
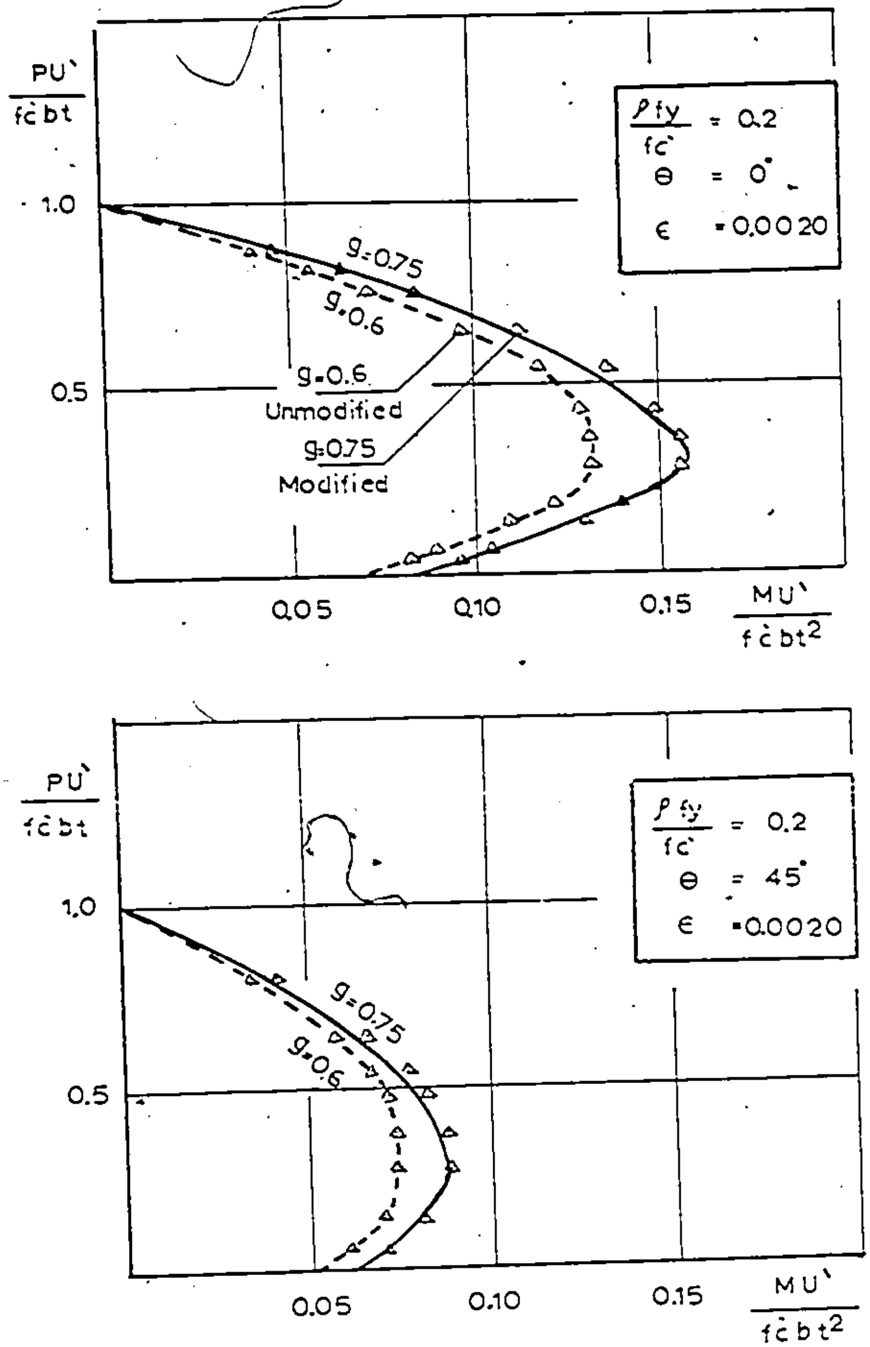


FIGURE (4-30) Justification of the Use of a Factor to Correct for Cover Ratio



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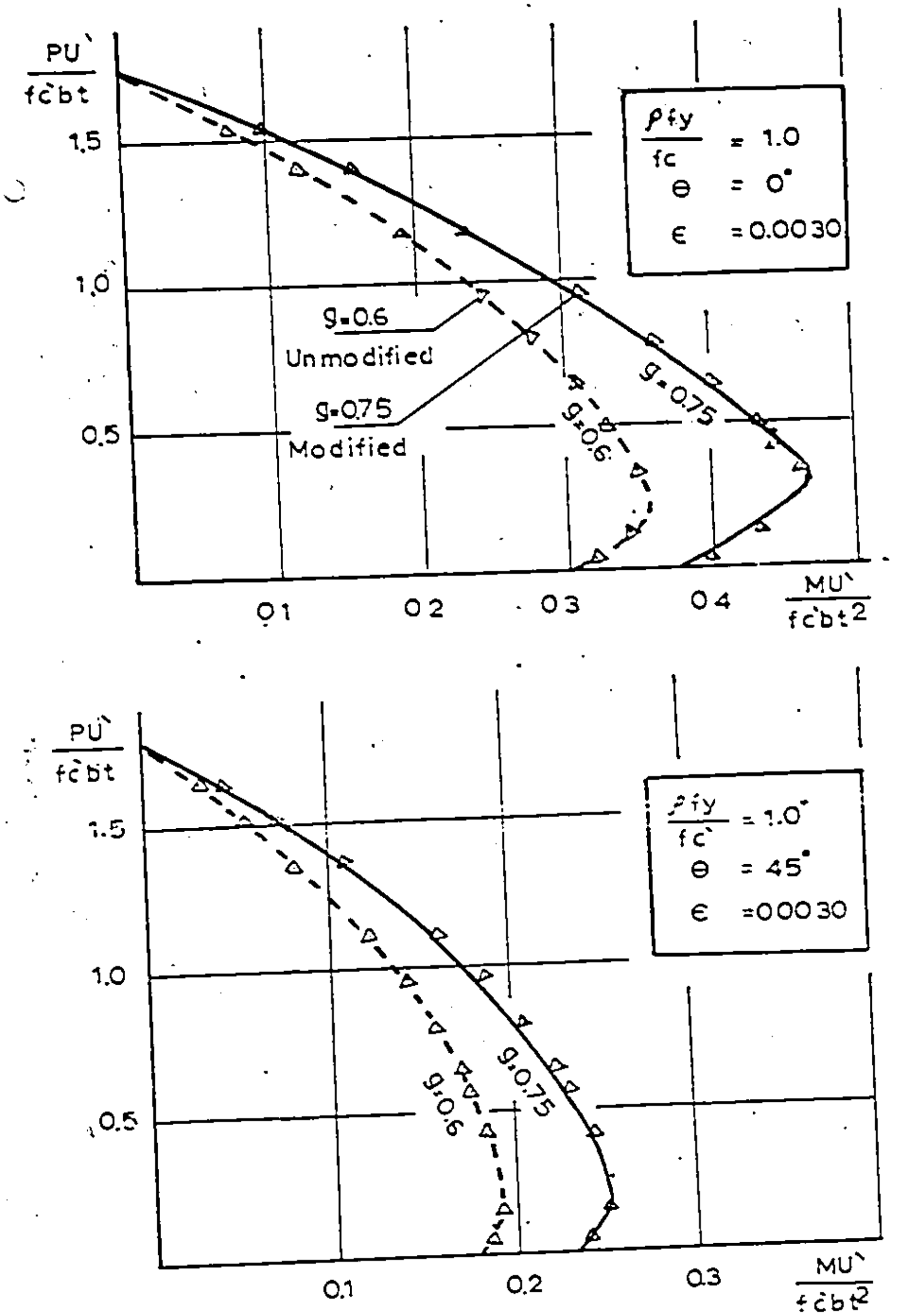

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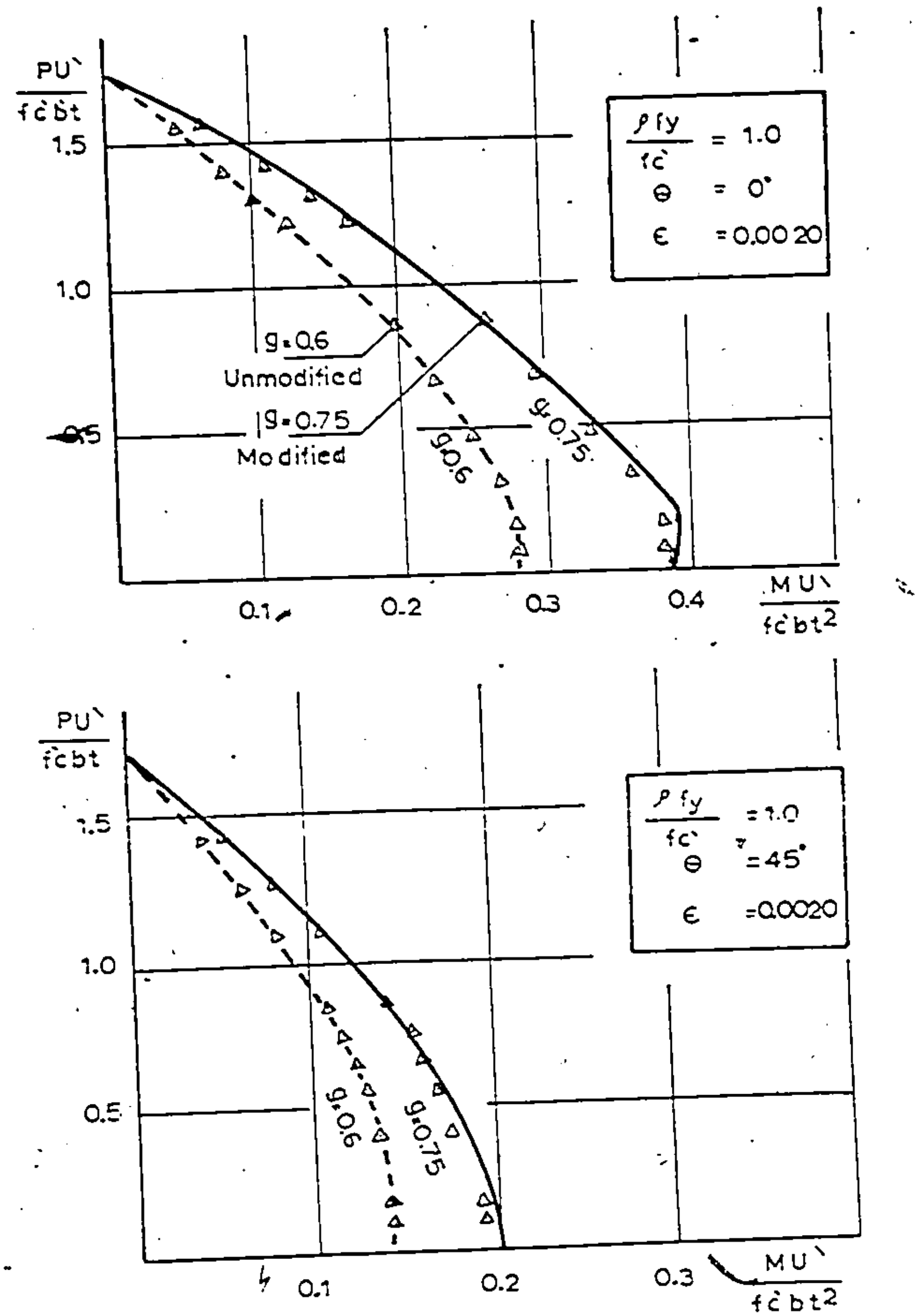

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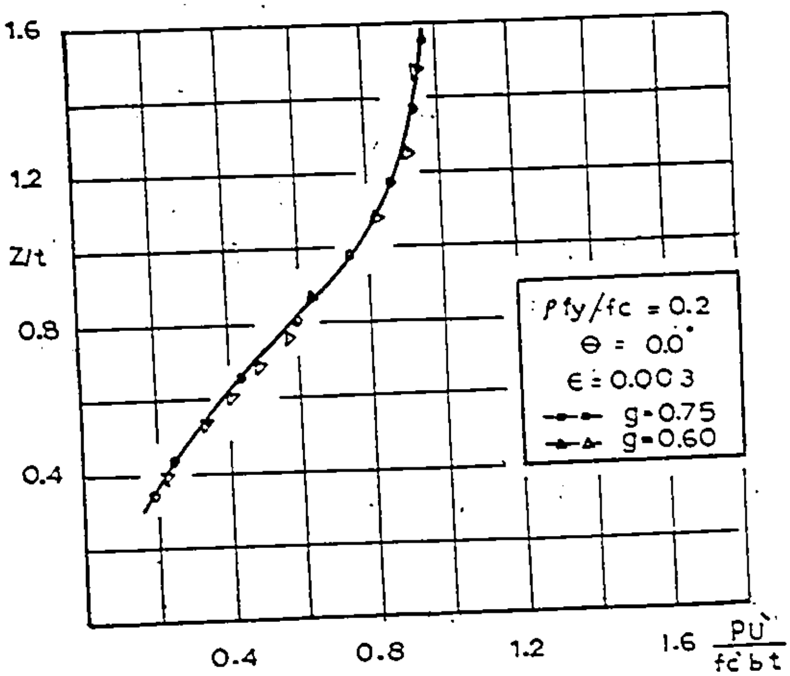


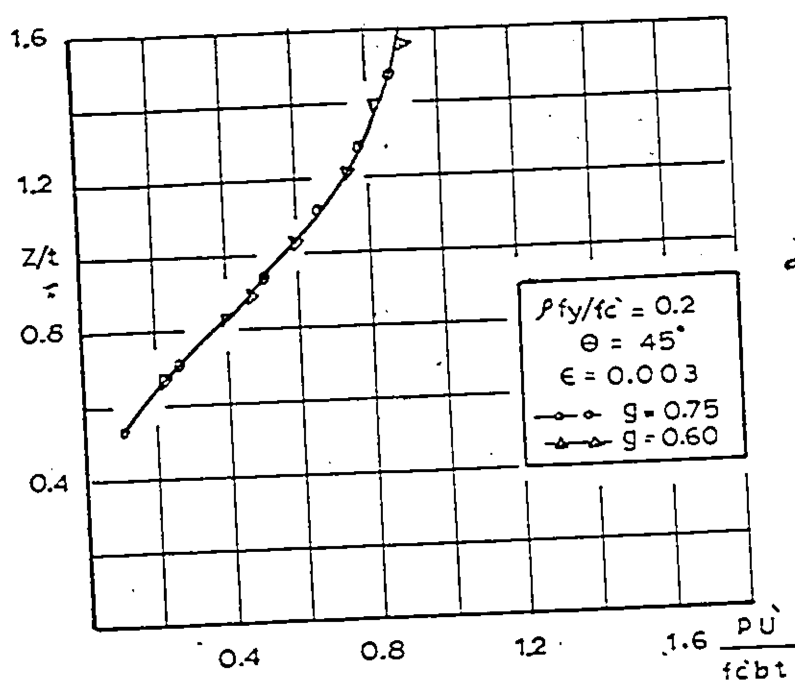
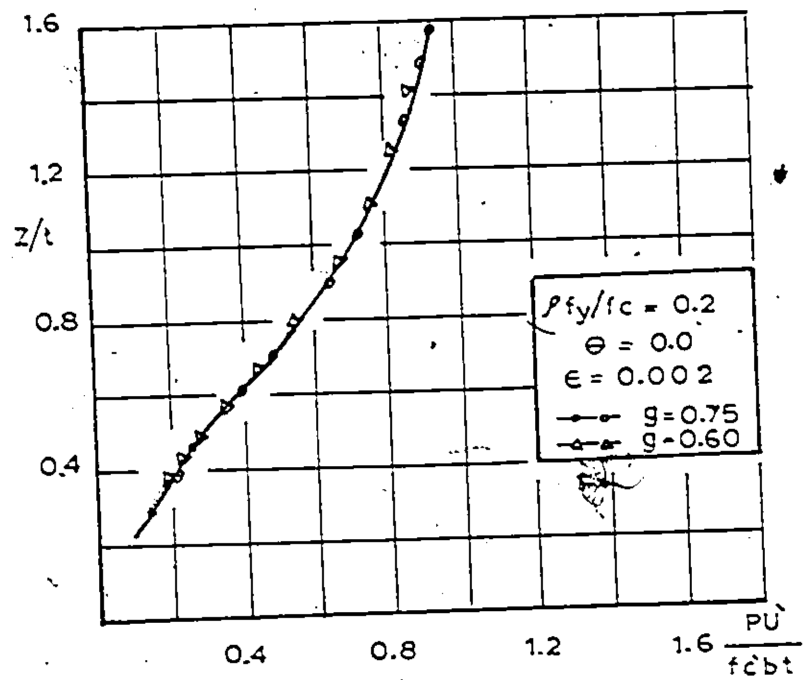
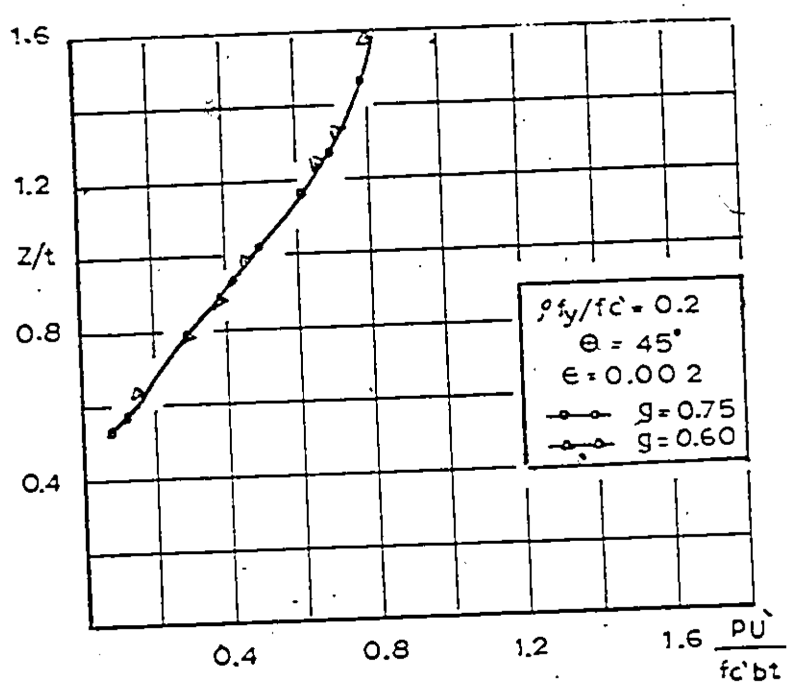


FIGURE (4-34) Effect of Cover Ratio on Z/t





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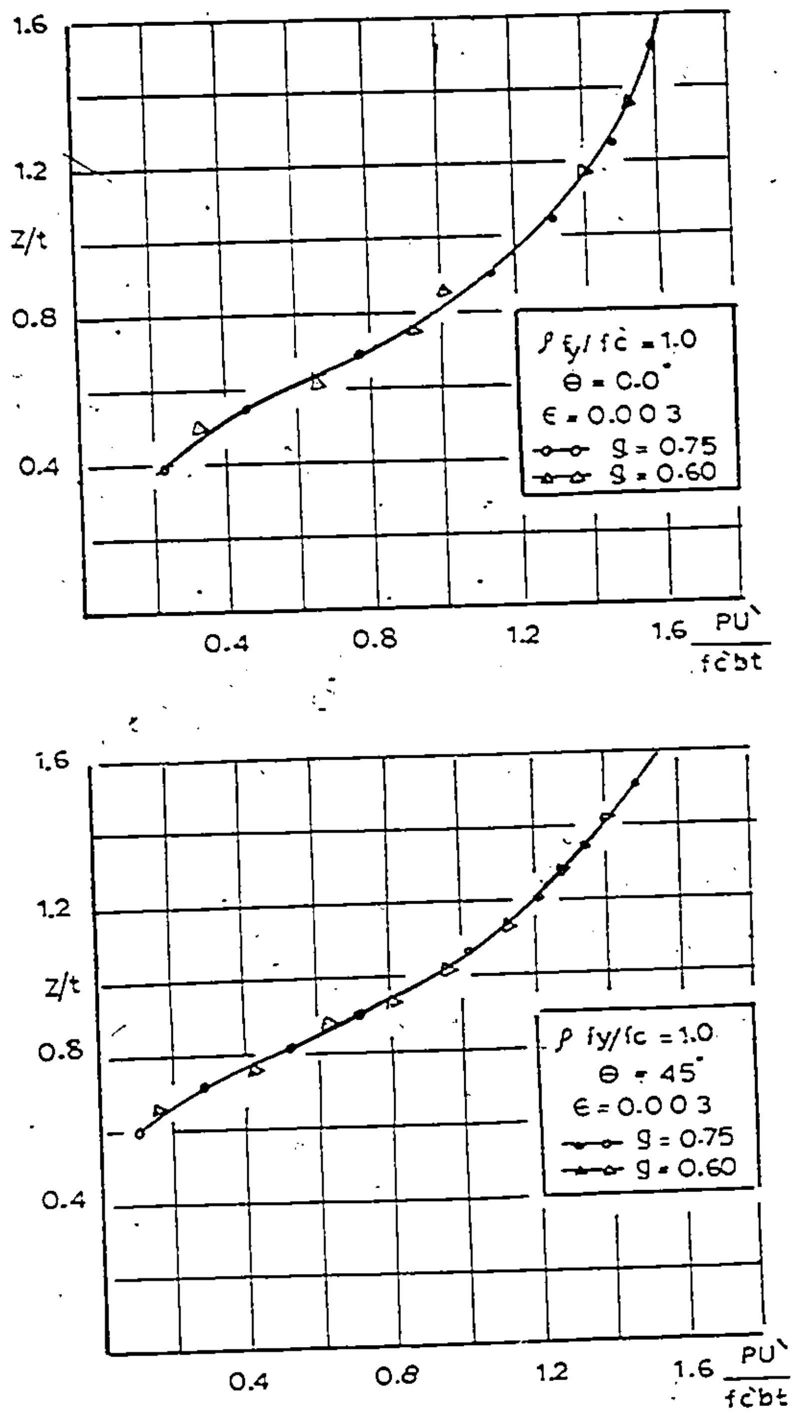

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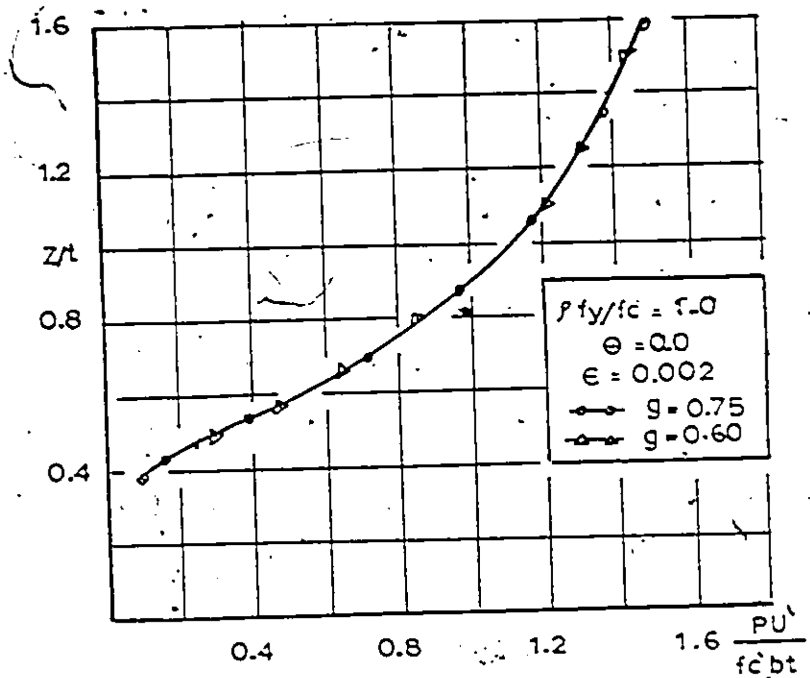

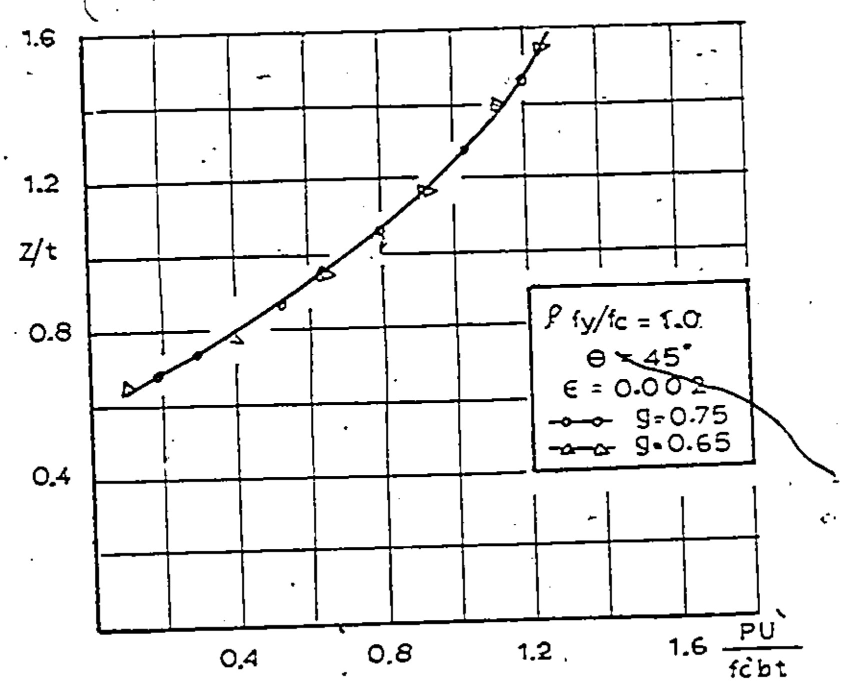

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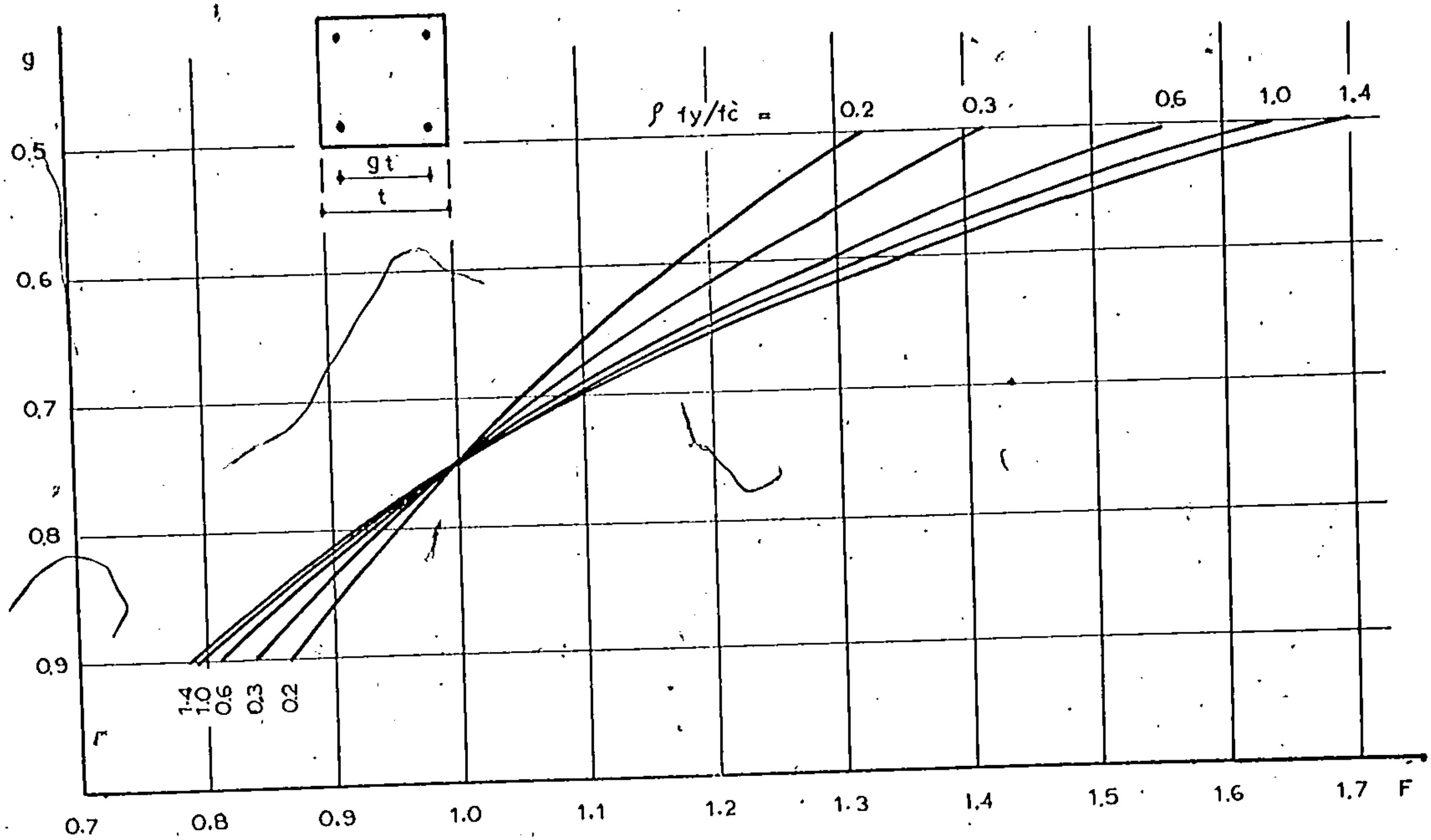

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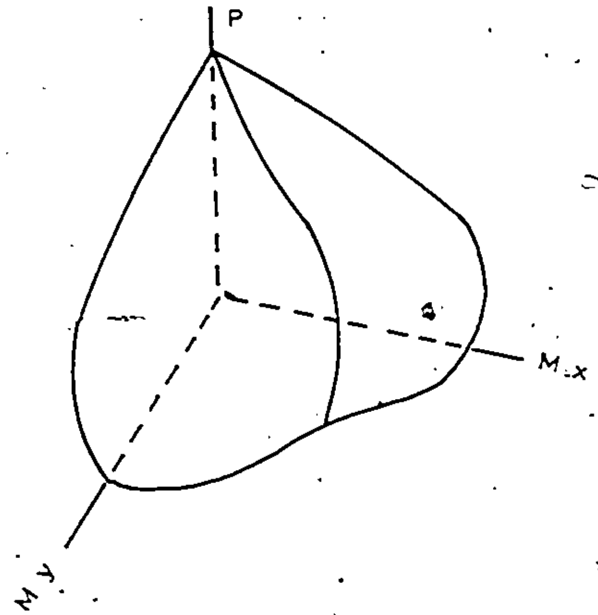


FIGURE (4-40) Failure Surface of a Biaxially Loaded Column

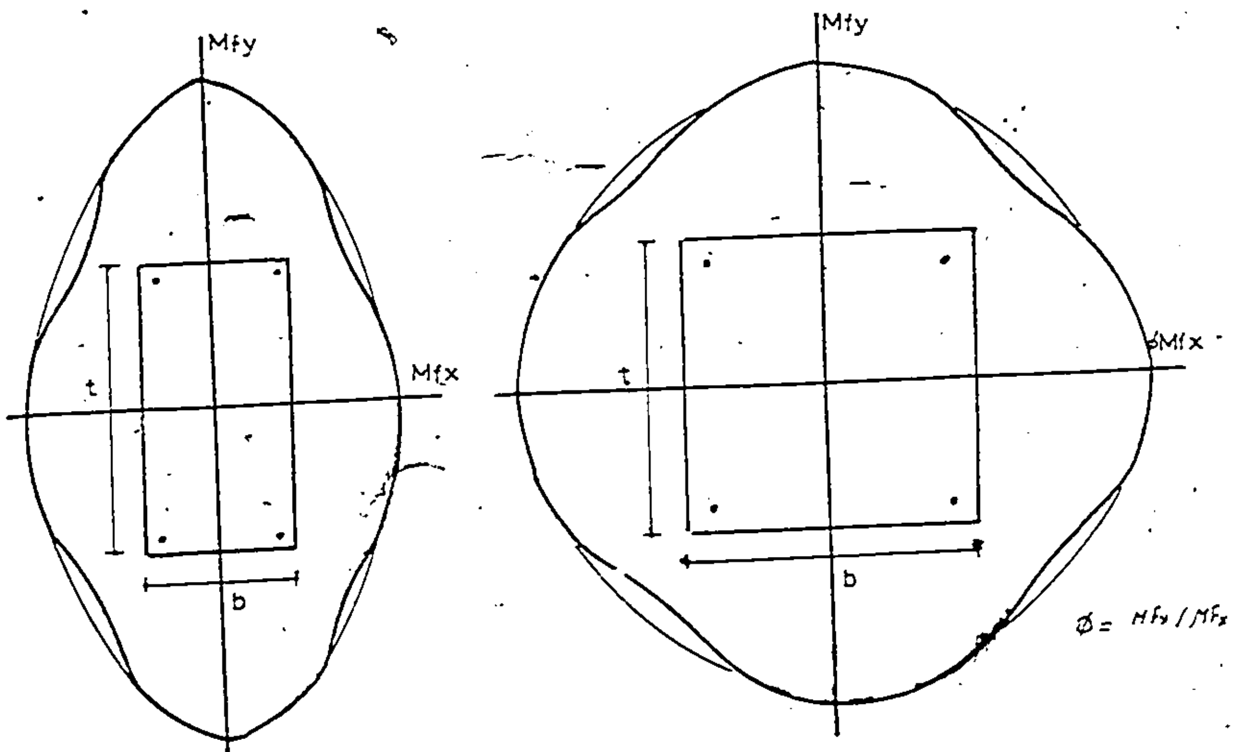


FIGURE (4-41) Interaction Contours for a Rectangular Column and it's Analogous Square Column

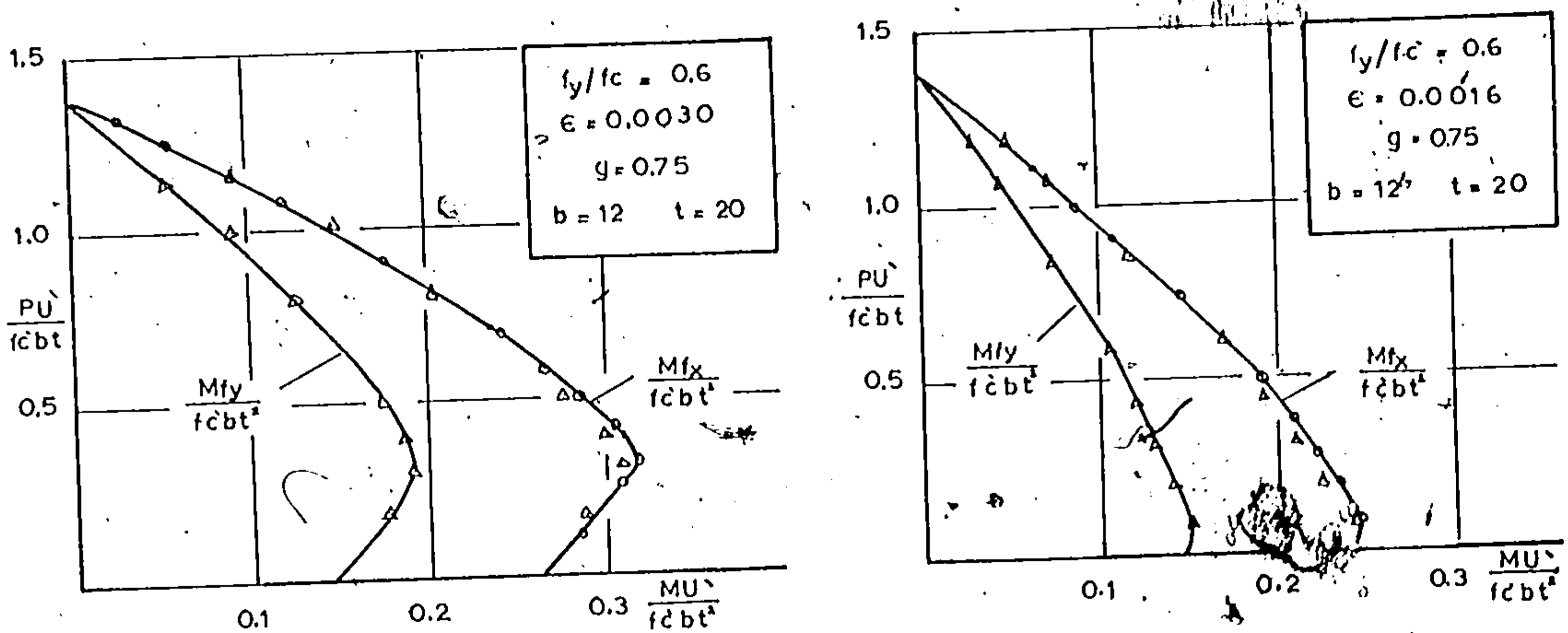


FIGURE (4-42) Use of $\phi = \frac{Mb_y}{Mb_x}$ to Transfer $Mf_y/f_c'bt^2$ Curves to it's Analogous $Mf_x/f_c'bt^2$

MARKS ON ORIGINAL

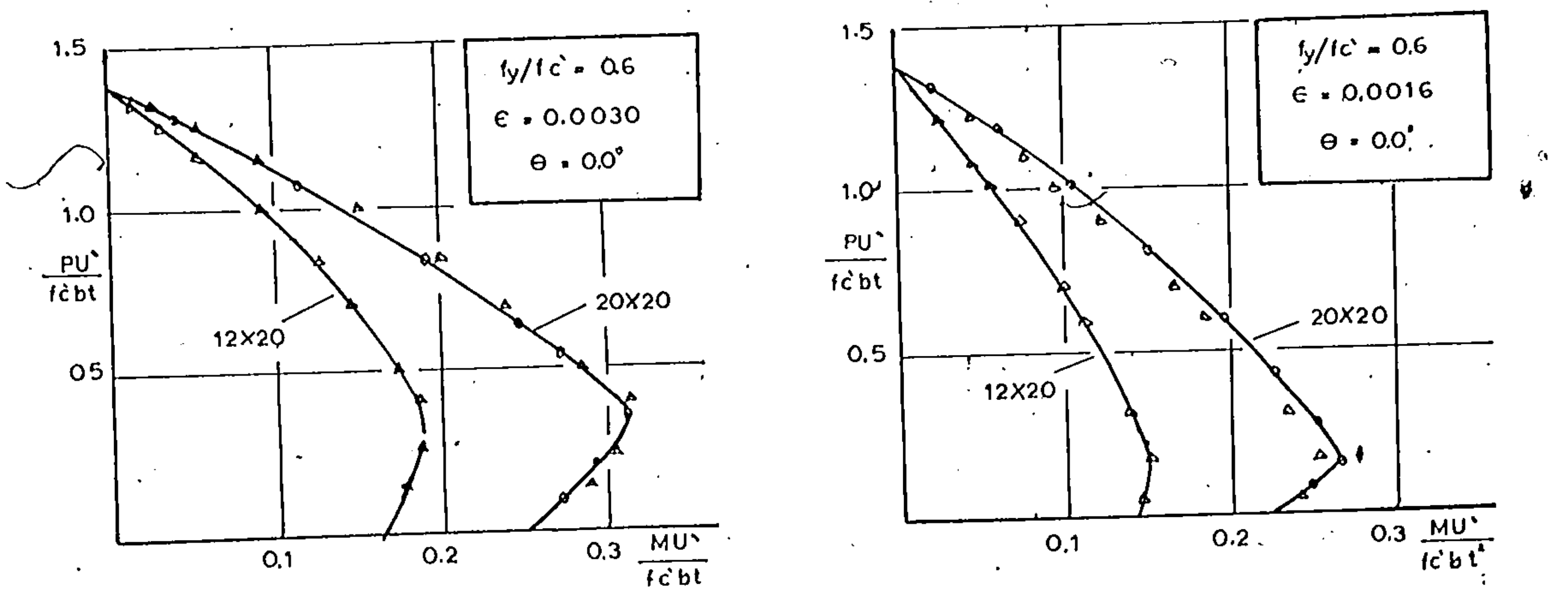


FIGURE (4-43) Use of ϕ to Transfer a Rectangular Column Interaction, Curves to it's Analogous Square Column

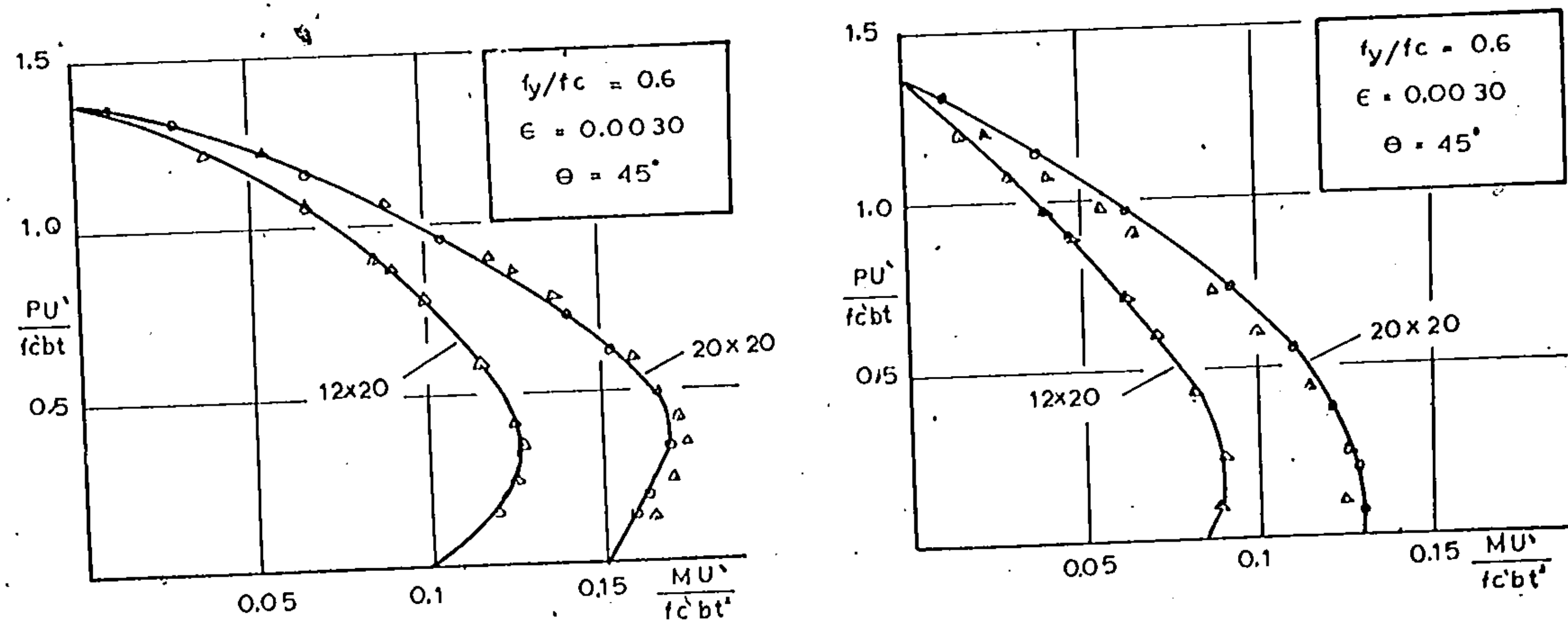
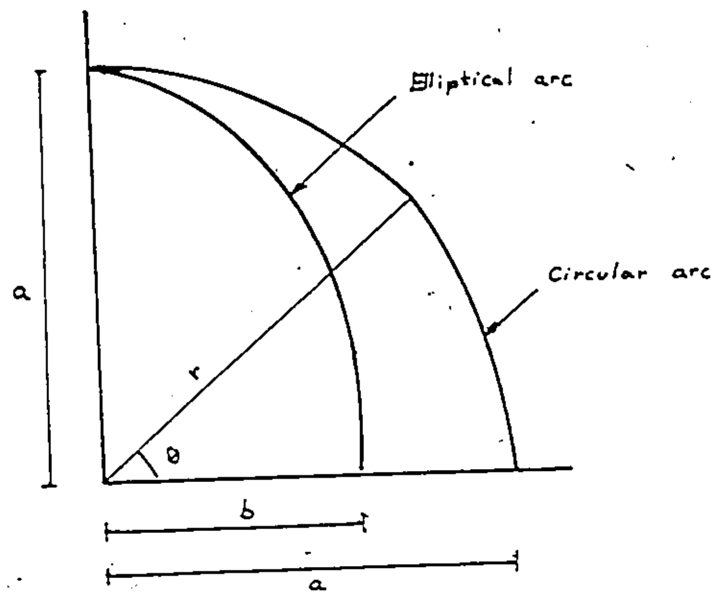


FIGURE (4-44) Use of ϕ to Transfer a Rectangular Column Interaction Curves to it's Analogous Square Column b = Mfx

a = Mfy



$$r^2 = \frac{a^2 b^2}{a^2 \sin^2\theta + b^2 \cos^2\theta}$$

at $\theta = 45°$

$$r^2 = \frac{2a^2 b^2}{a^2 + b^2}$$

$$\therefore r = \frac{\sqrt{2}\, a b}{\sqrt{a^2 + b^2}}$$

$$\phi_{00} = a / b$$

$$\phi_{45} = \frac{a}{r_{45}} = \frac{\sqrt{a^2 + b^2}}{\sqrt{2}\, b}$$

$$\therefore \frac{\phi_{45}}{\phi_{00}} = \frac{\sqrt{a^2 + b^2}}{a\sqrt{2}}$$

FIGURE (4-45) Value of ϕ_{45} Relative to ϕ_0

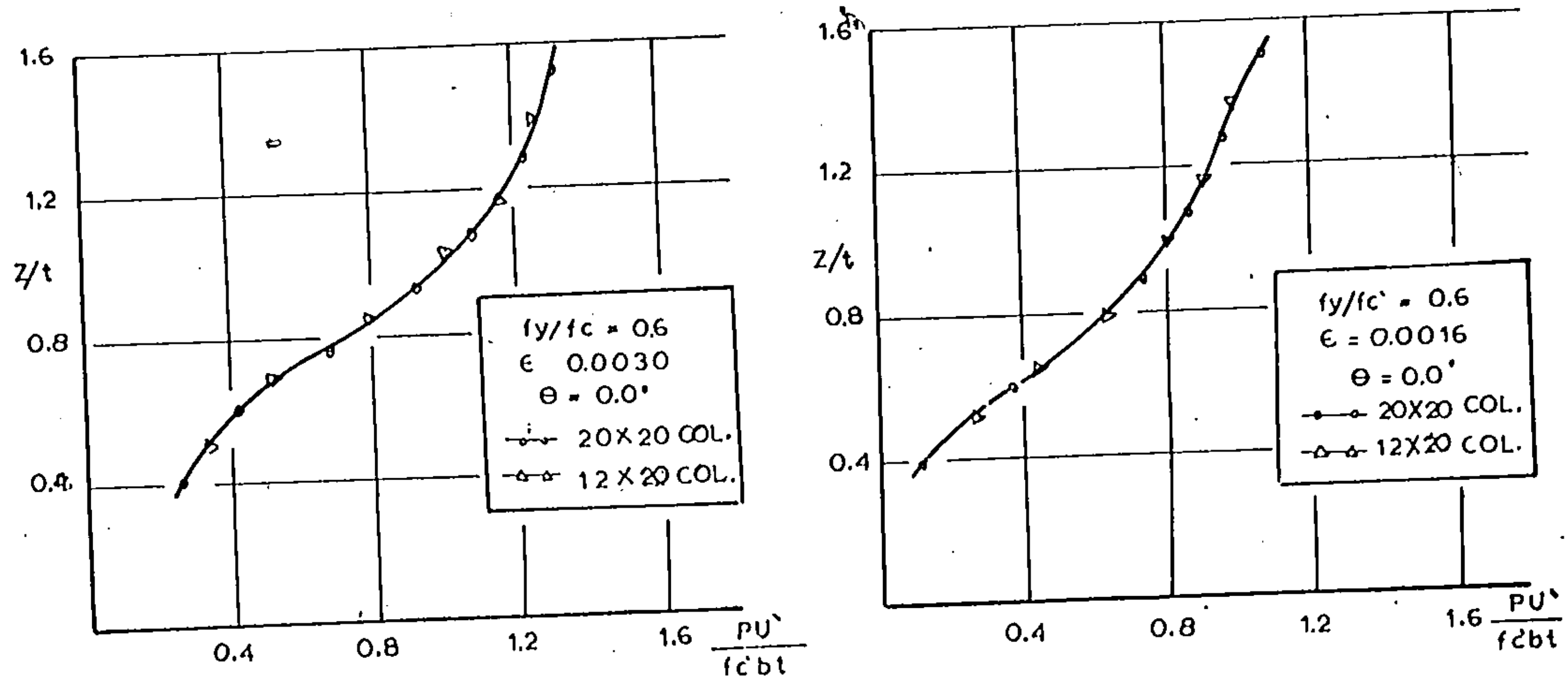


FIGURE (4-46) Values of Z/t for a Rectangular Column and its Analogous Square Column

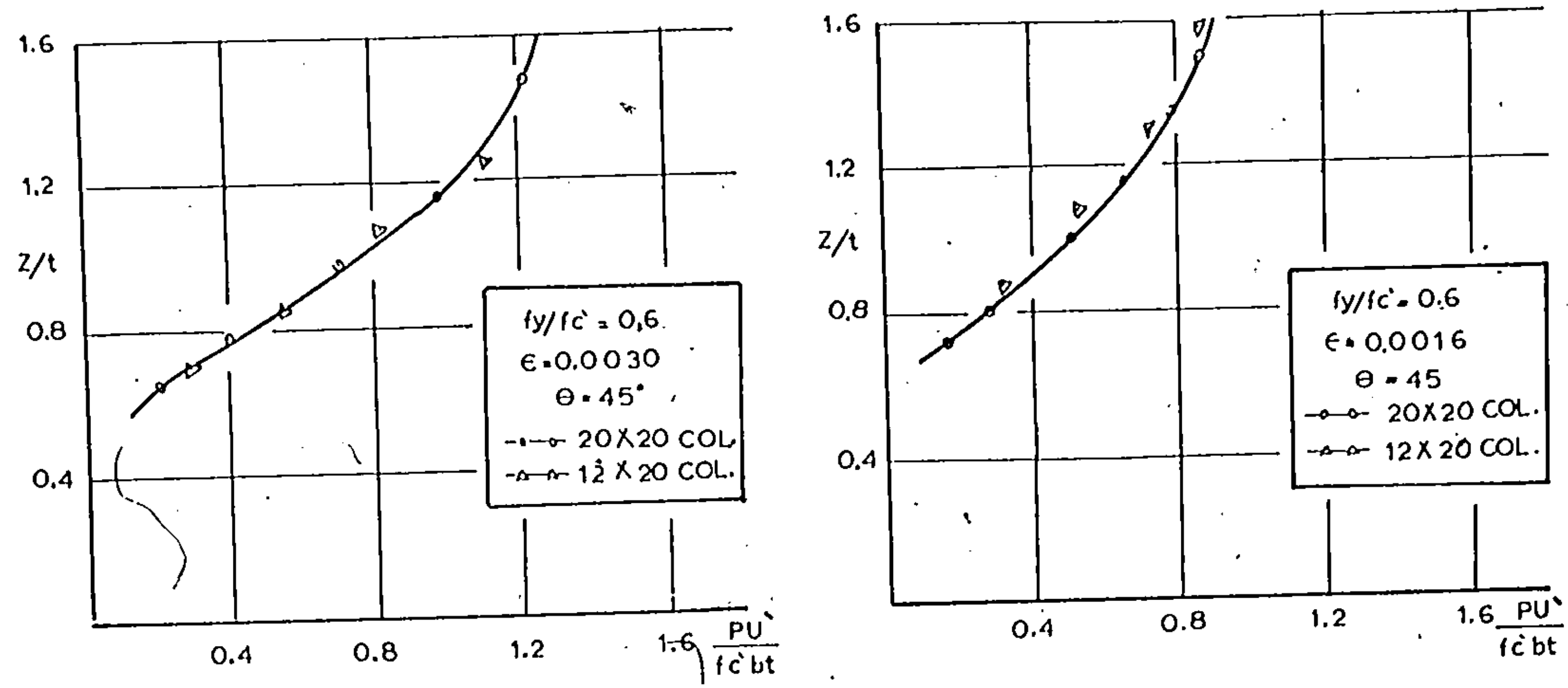
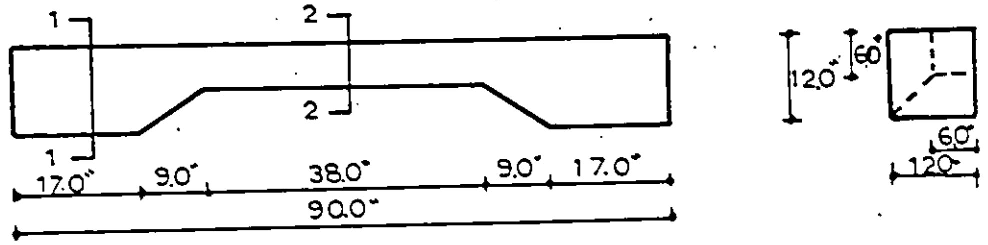
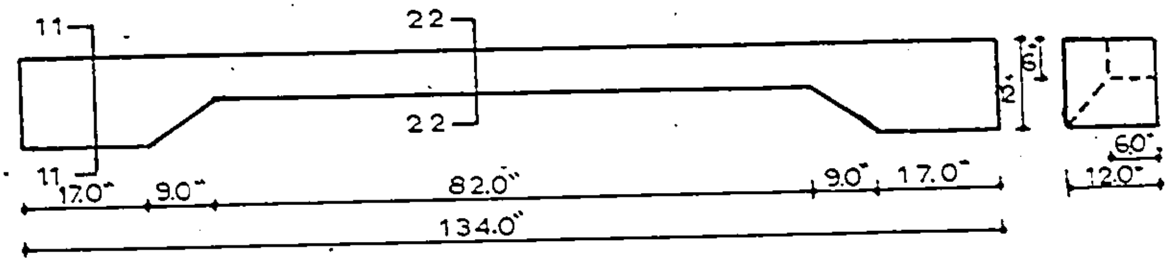


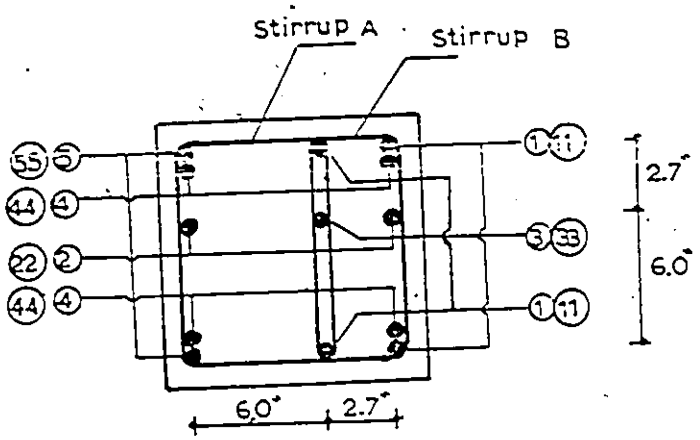
FIGURE (4-47) Values of Z/t for a Rectangular Column and its Analogous Square Columns



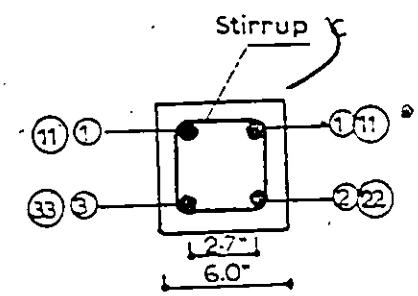
Group C and D



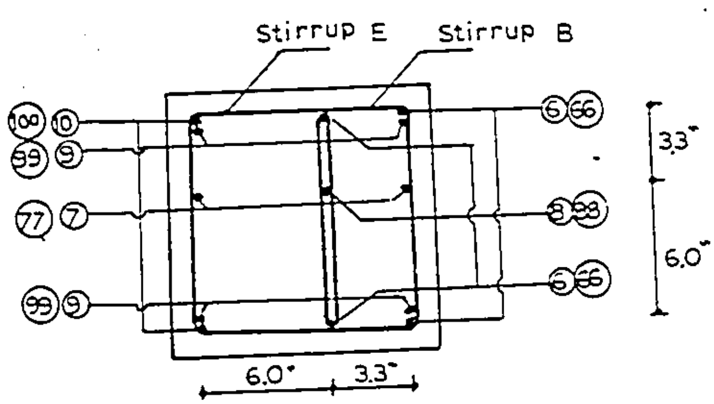
Group A and B



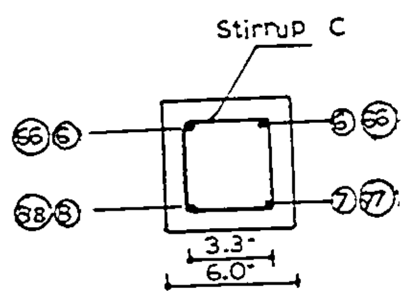
Section 1-1 and 11-11 (Group A and C)



Section 2-2 and 22-22 (Group A and C)



Section 1-1 and 11-11 (Group B and D)



Section 2-2 and 22-22 (Group B and D)

FIGURE (5-1) Details of Columns and Reinforcement

bar NO.	bar size	NO. of bars	Length	Group
66	3	30	178.2"	Group B
77	3	15	176.65"	Group B
88	3	15	131.6"	Group B
99	3	60	37.4"	Group B
100	3	30	38.2"	Group B
1	7	30	176.4"	Group A
2	7	15	172.85"	Group A
3	7	15	131.0"	Group A
4	7	60	34.9"	Group A
5	7	30	36.7"	Group A
6	3	30	134.6"	Group D
7	3	15	129.85"	Group D
8	3	15	87.6"	Group D
9	3	60	36.5"	Group D
10	3	30	37.3"	Group D
11	7	30	132.4"	Group C
22	7	15	128.85	Group C
33	7	15	87.0"	Group C
44	7	60	34.9"	Group C
55	7	30	36.7"	Group C

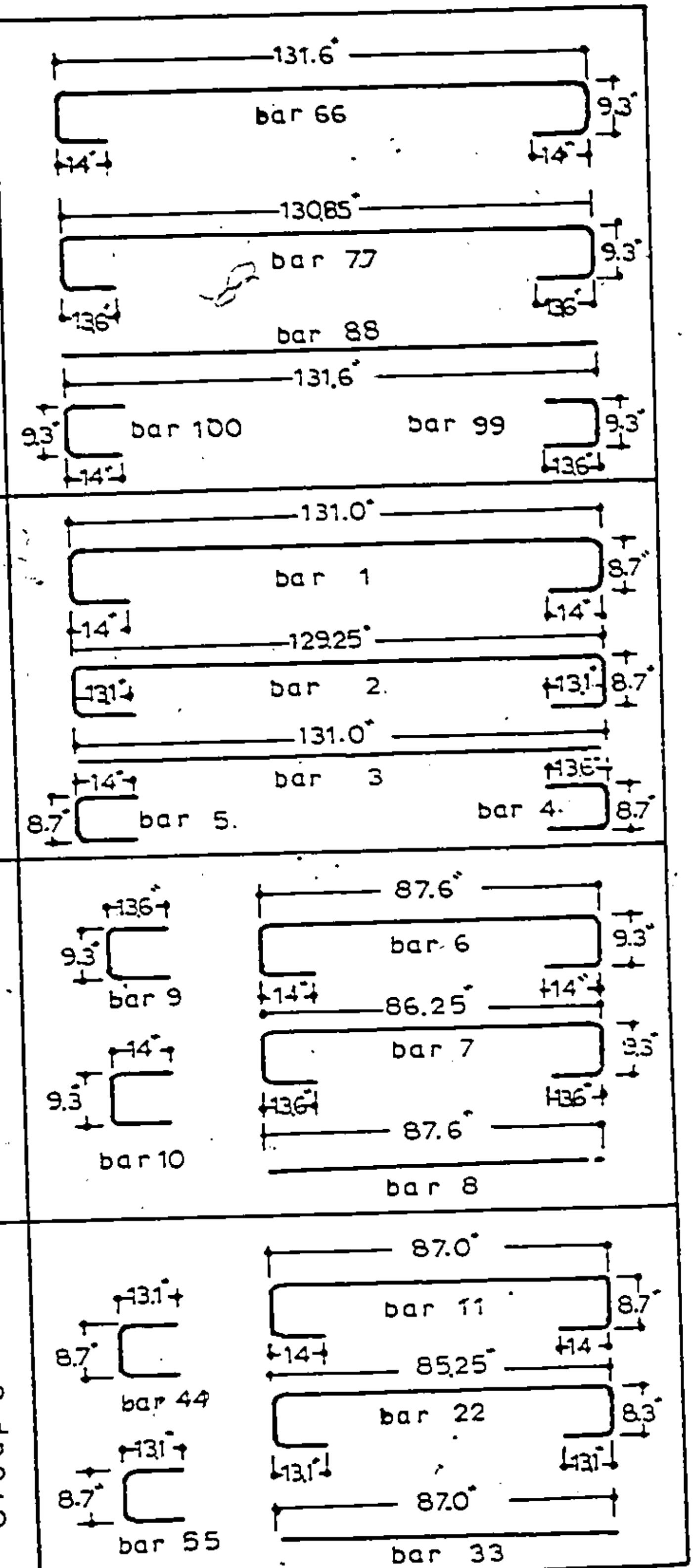


FIGURE (5-2) Details of Reinforcing Bars

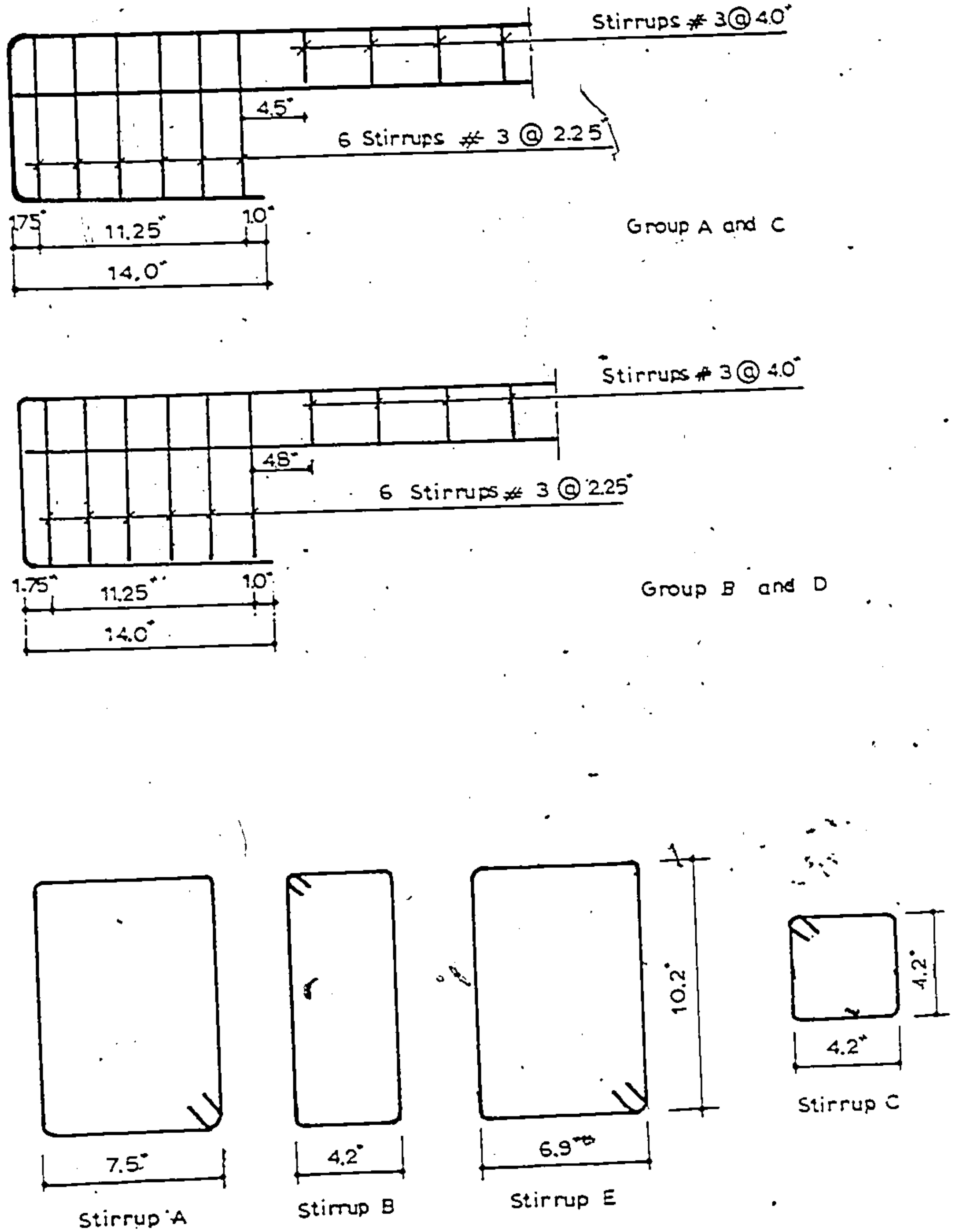


FIGURE (5-3) Details of Stirrups

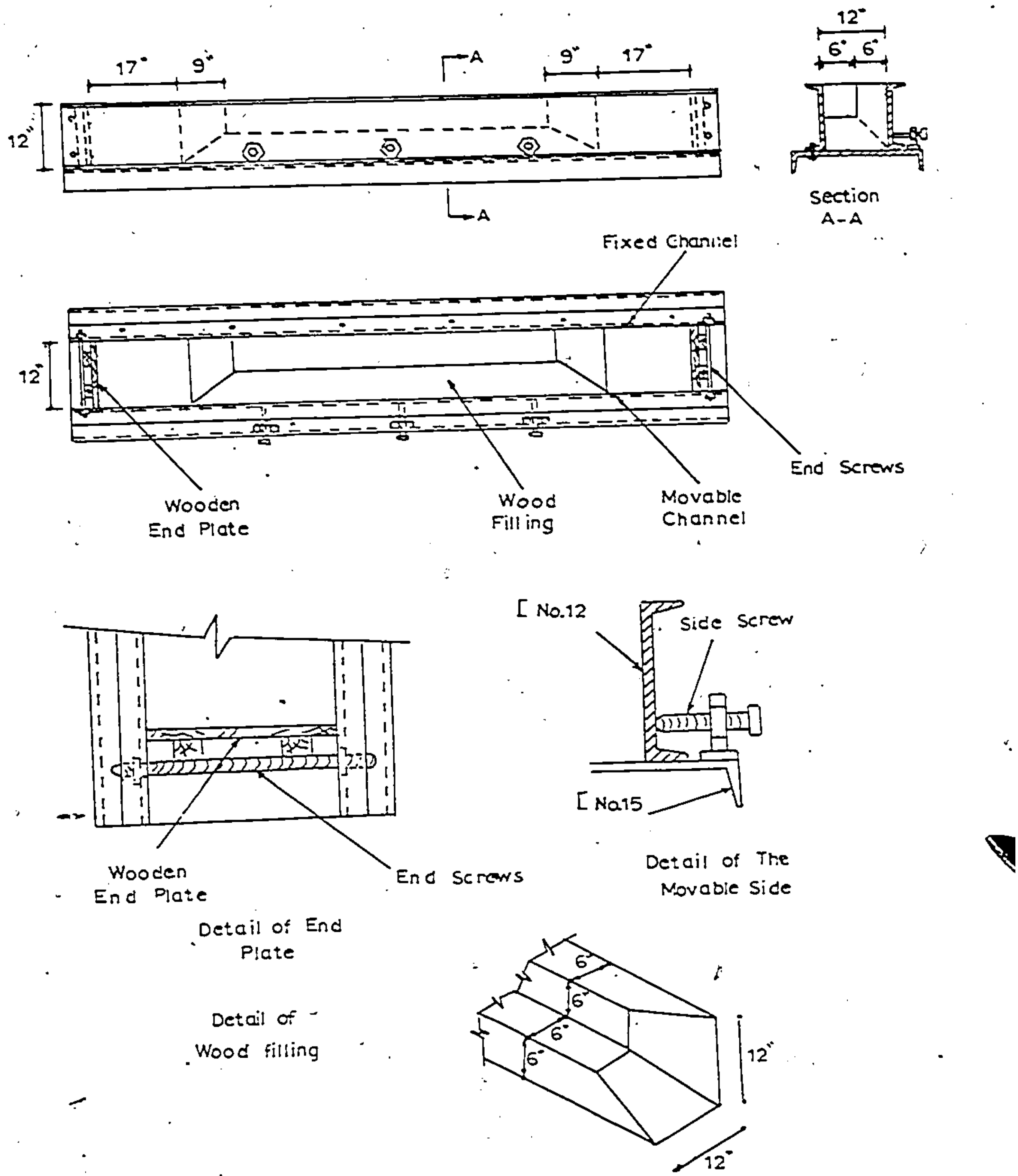


FIGURE (5-4) Details of Forms

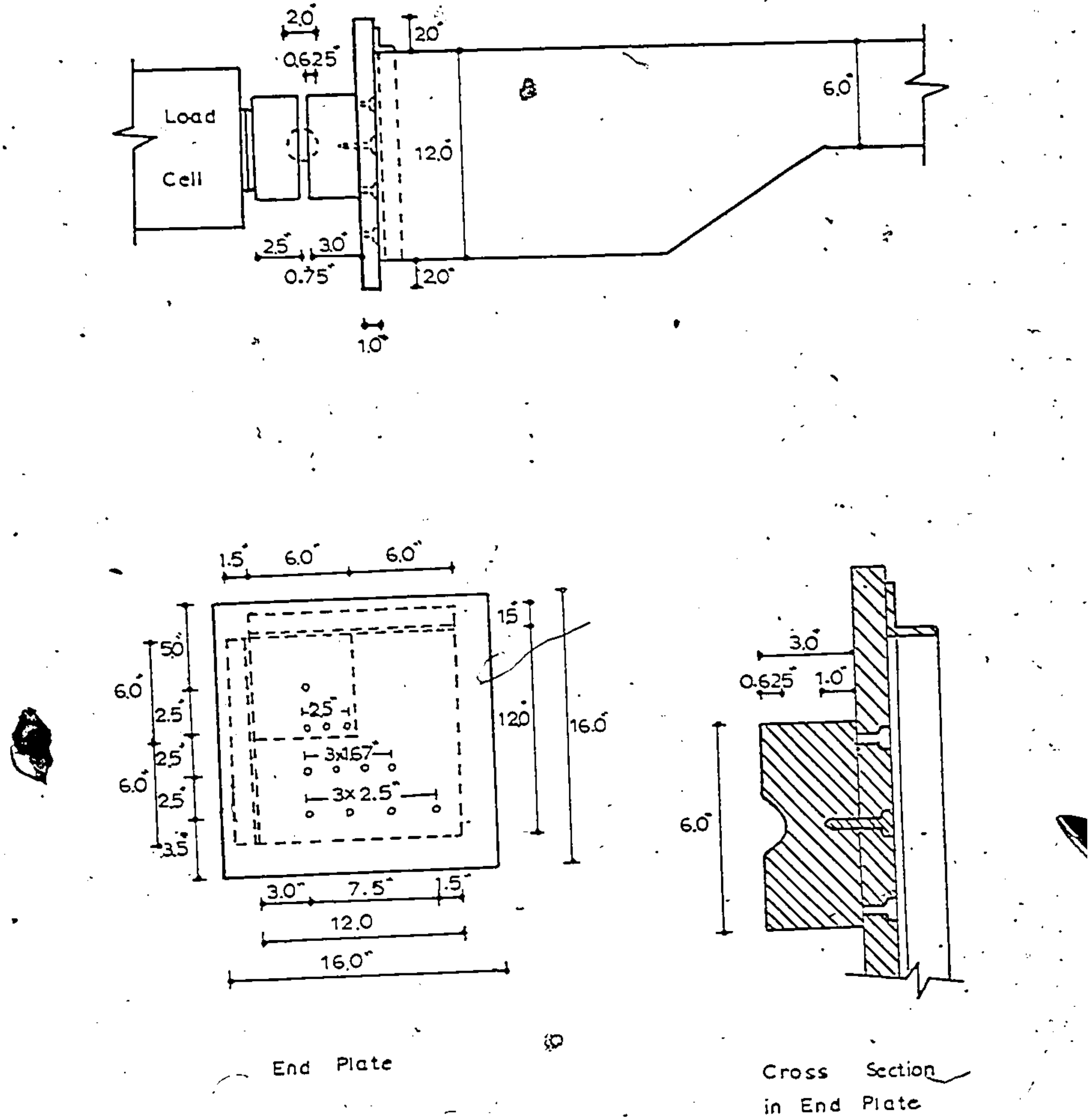


FIGURE (5-5) Details of End Plates and Ball Bearings

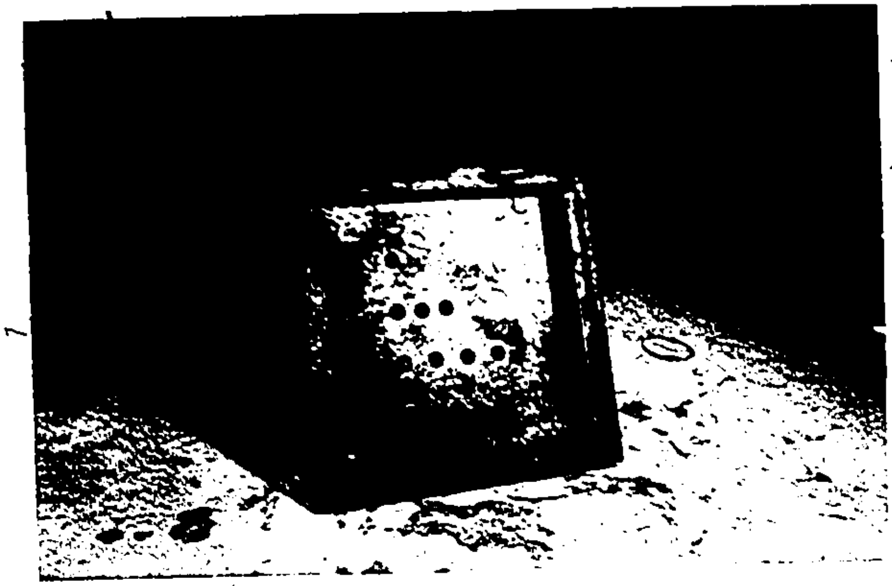
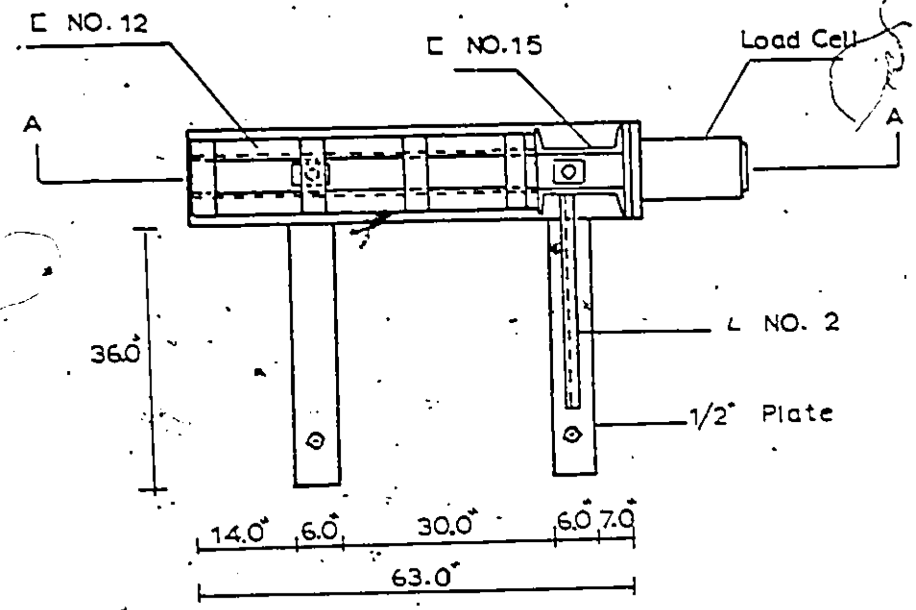
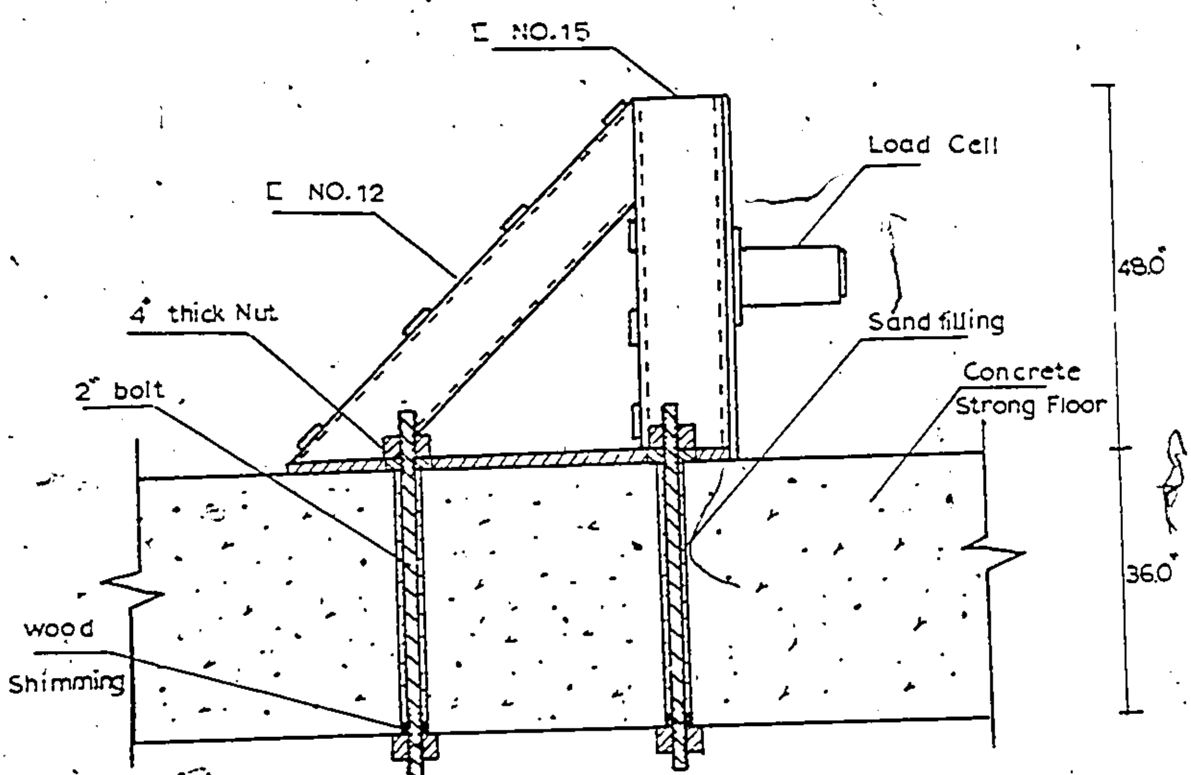


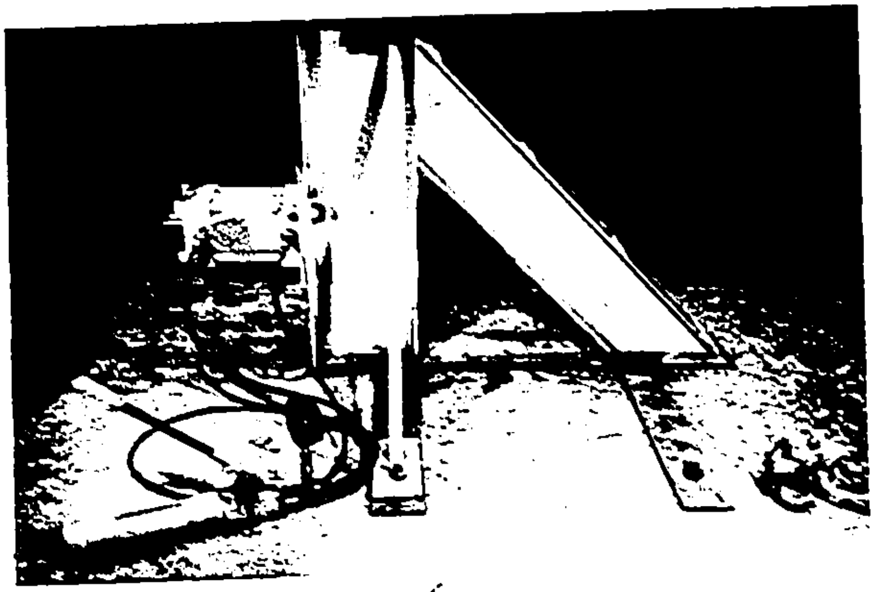
Fig.(5-6) End Plates and Ball Bearings



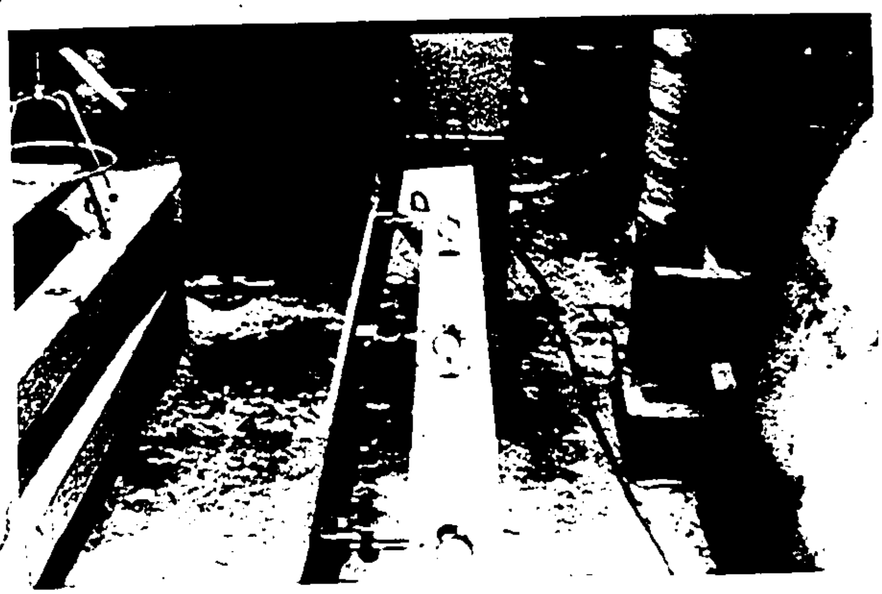


Section A-A

FIGURE (5-7) Details of End Brackets



Fig(5-8) Jack Mounted to End Bracket



Fig(5-9) Dial Gages Mounted to a Sample

FIGURE (5-10)

Stress-Strain Curves for Samples of # 3 Bars

f_y (average) = 65000 psi

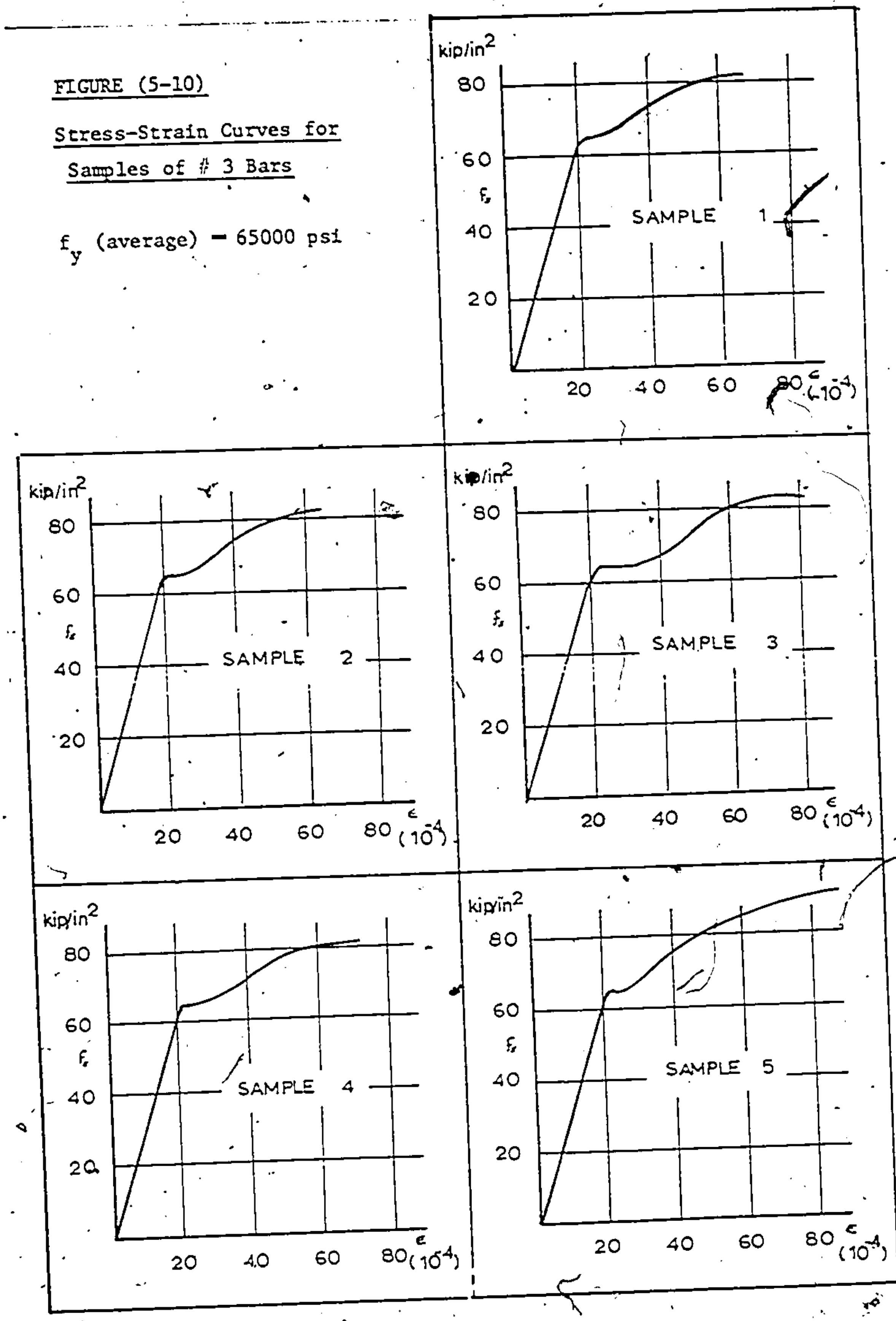


FIGURE (5-11)

Stress-Strain Curves for Samples of # 7 Bars

f_y (average) = 67000 psi

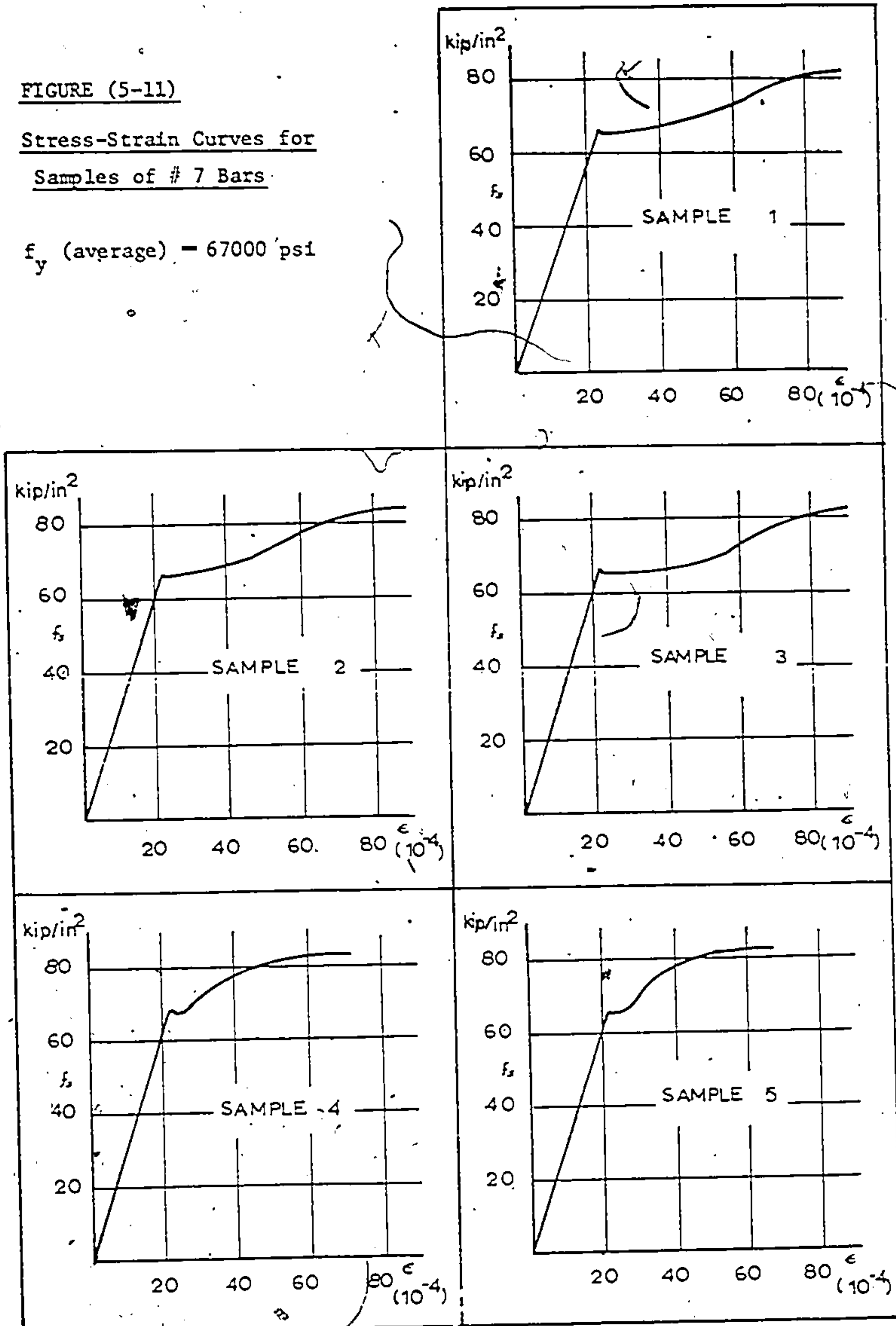
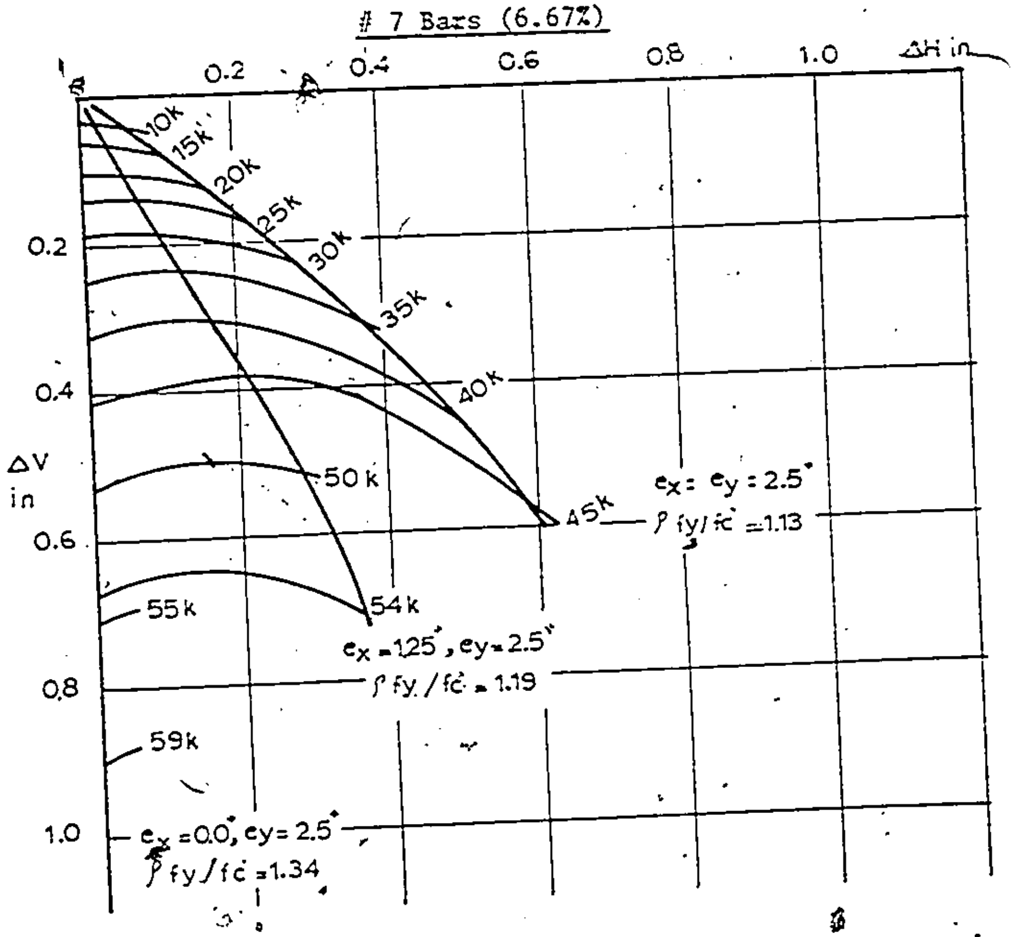
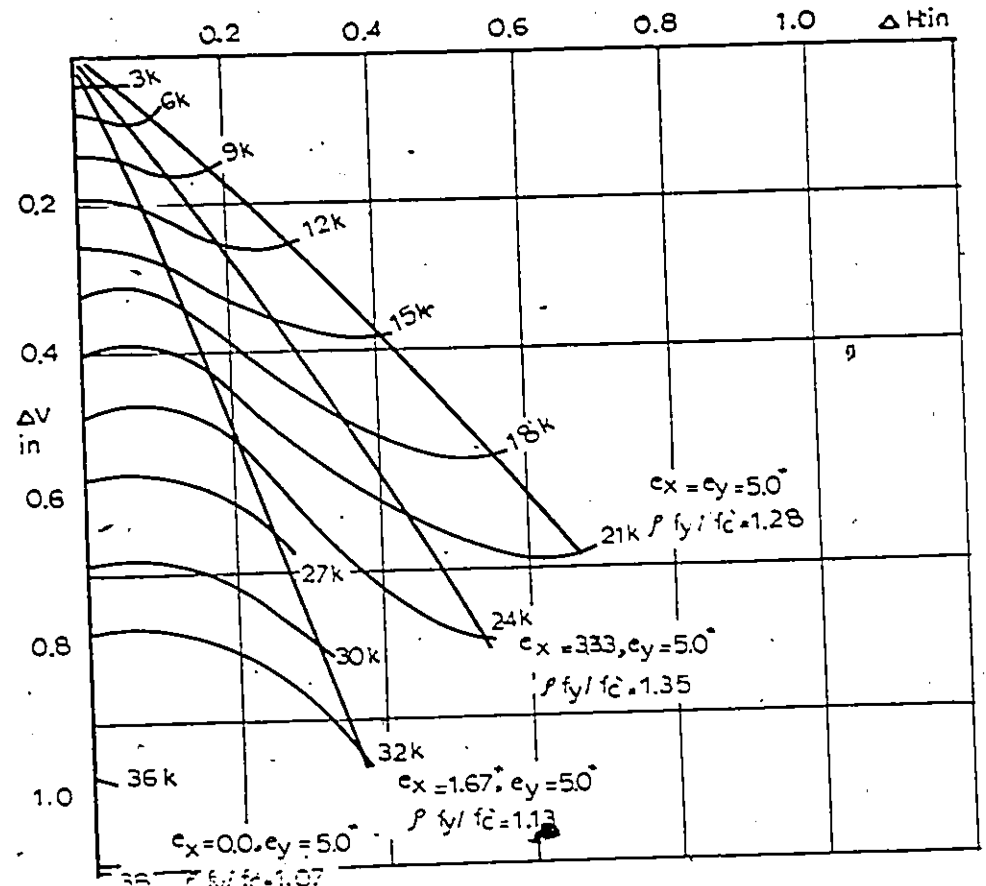


FIGURE (5-12) Mid Deflection, Eccentricity and it's Angle and Load Relationships for Group A

L = 90" (L/r = 50)

7 Bars (6.67%)




0.2
0.4
0.6
0.8
1.0
ΔH in
ΔV in
3k
6k
9k
12k
15k
18k
21k
24k
27k
30k
32k
36k
36
ex = ey = 5.0"
ρ fy / fc' = 1.28
ex = 3.33, ey = 5.0"
ρ fy / fc' = 1.35
ex = 1.67", ey = 5.0"
ρ fy / fc' = 1.13
ex = 0.0, ey = 5.0"
ρ fy / fc' = 1.07

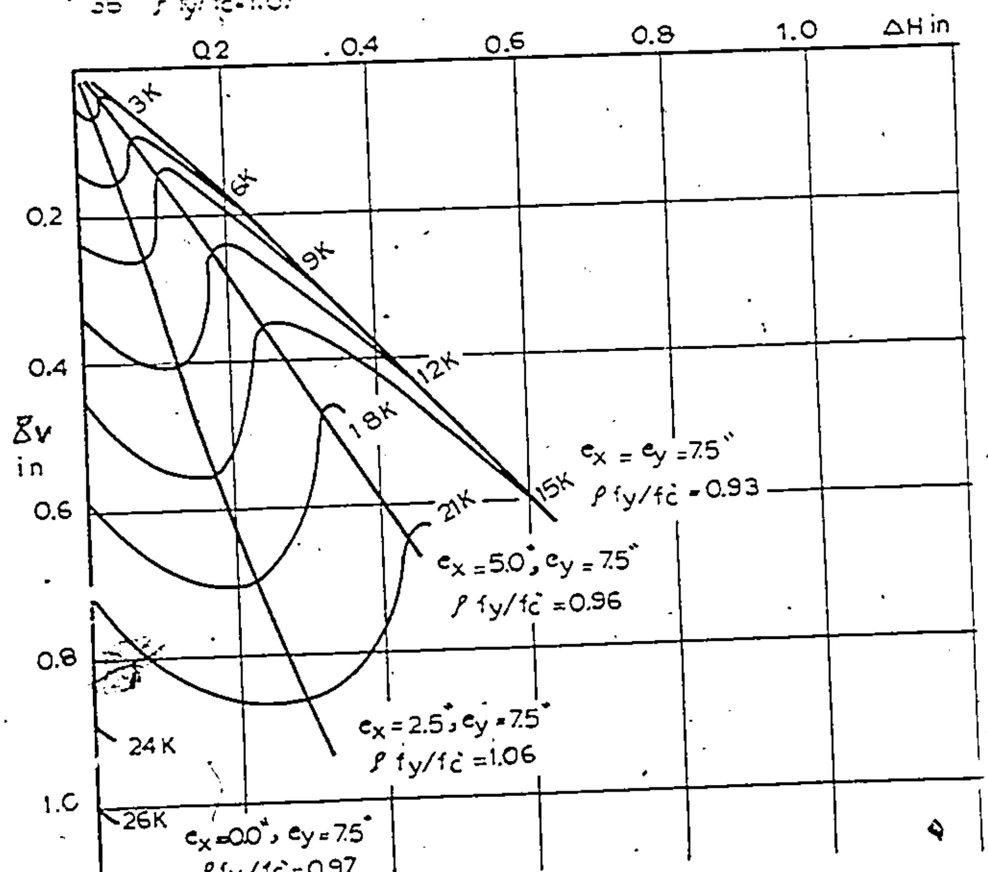
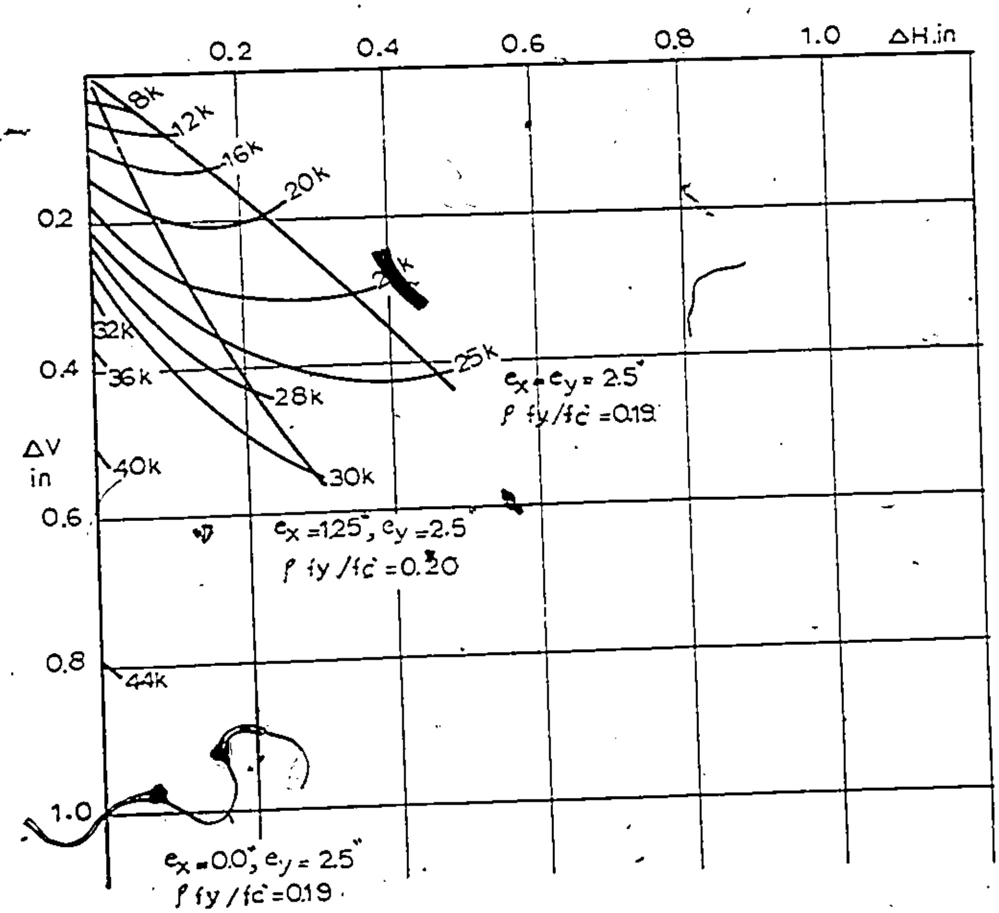
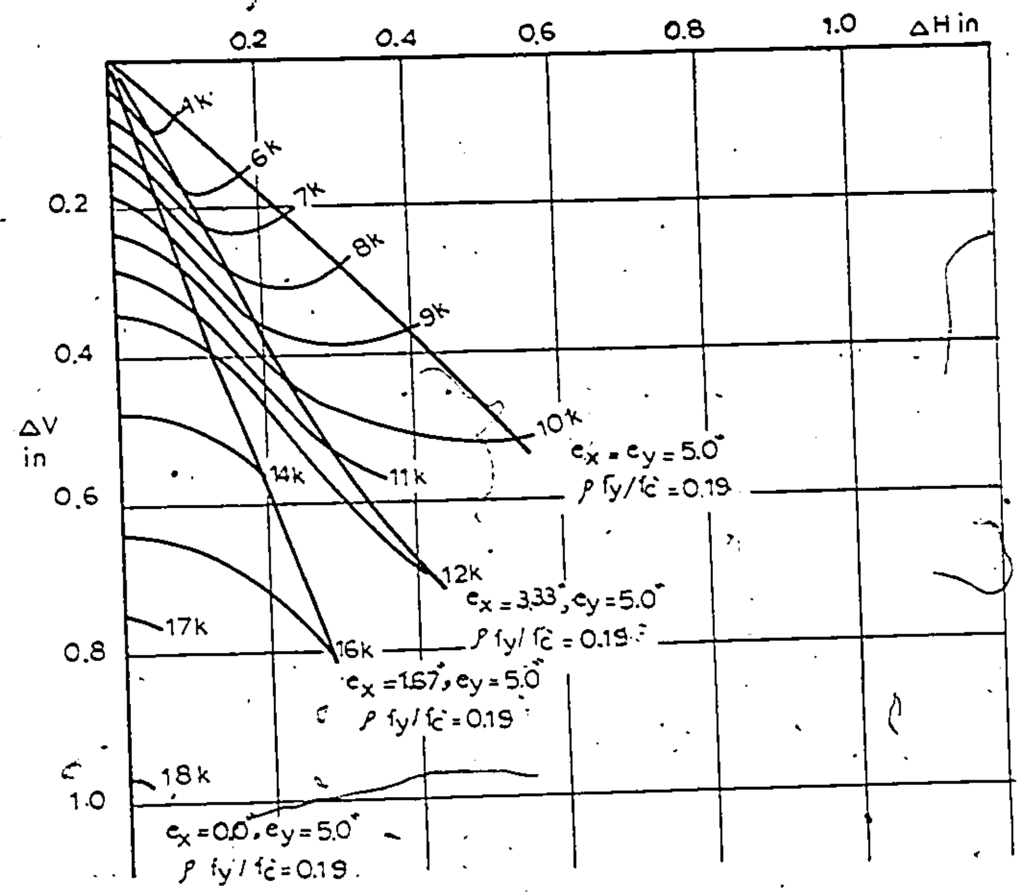

0.2
0.4
0.6
0.8
1.0
ΔH in
δv in
3K
6K
9K
12K
15K
18K
21K
24K
26K
ex = ey = 7.5"
ρ fy/fc' = 0.93
ex = 5.0", ey = 7.5"
ρ fy/fc' = 0.96
ex = 2.5", ey = 7.5"
ρ fy/fc' = 1.06
ex = 0.0", ey = 7.5"
ρ fy/fc' = 0.97

FIGURE (5-13) Mid Deflection, Eccentricity and it's Angle and Load Relationships for Group B

L = 90" (L/r = 50)

3 Bars (1.22%)




0.2
0.4
0.6
0.8
1.0
ΔH in
ΔV in
4k
6k
7k
8k
9k
10k
11k
12k
14k
16k
17k
18k
ex = ey = 5.0"
ρ fy/fc = 0.19
ex = 3.33", ey = 5.0"
ρ fy/fc = 0.19
ex = 1.67", ey = 5.0"
ρ fy/fc = 0.19
ex = 0.0", ey = 5.0"
ρ fy/fc = 0.19

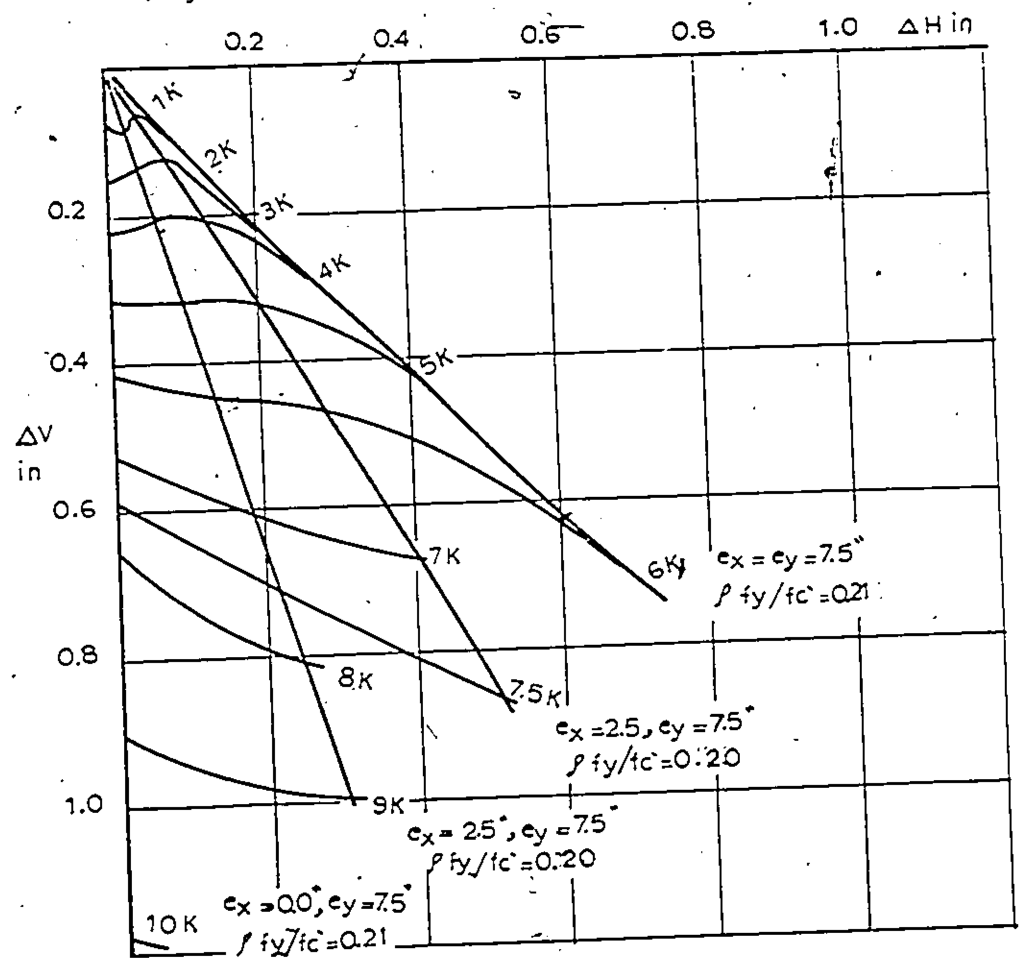
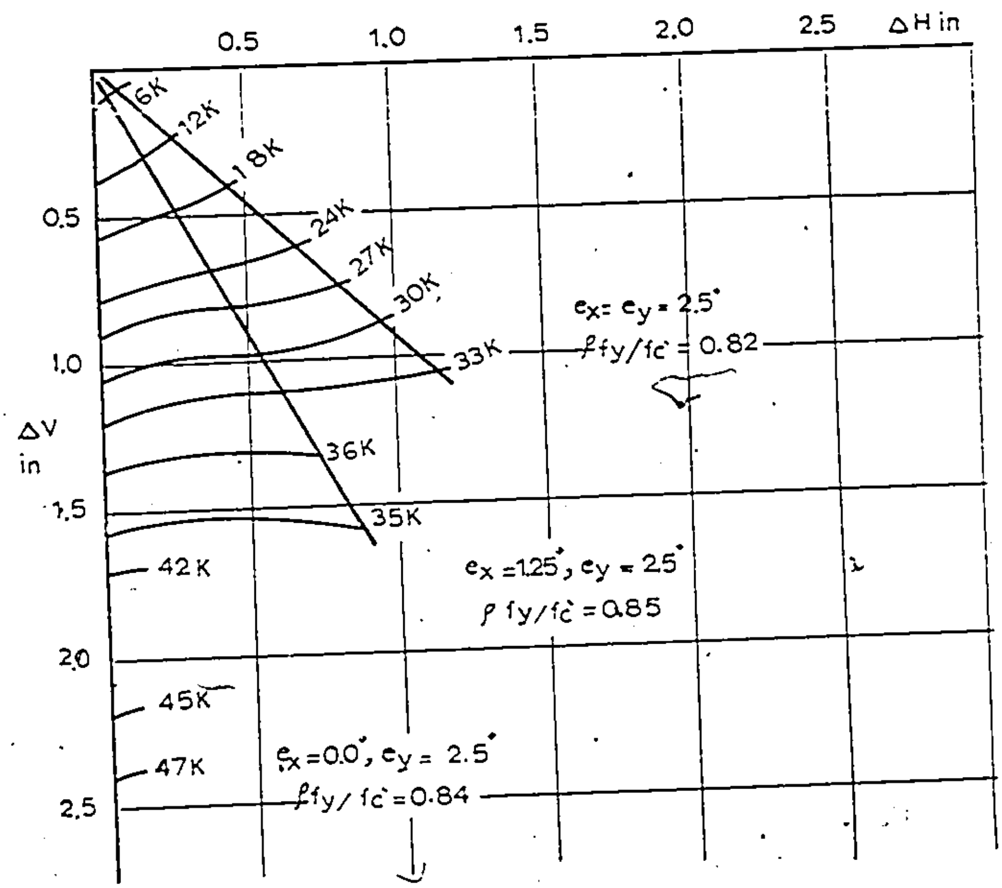
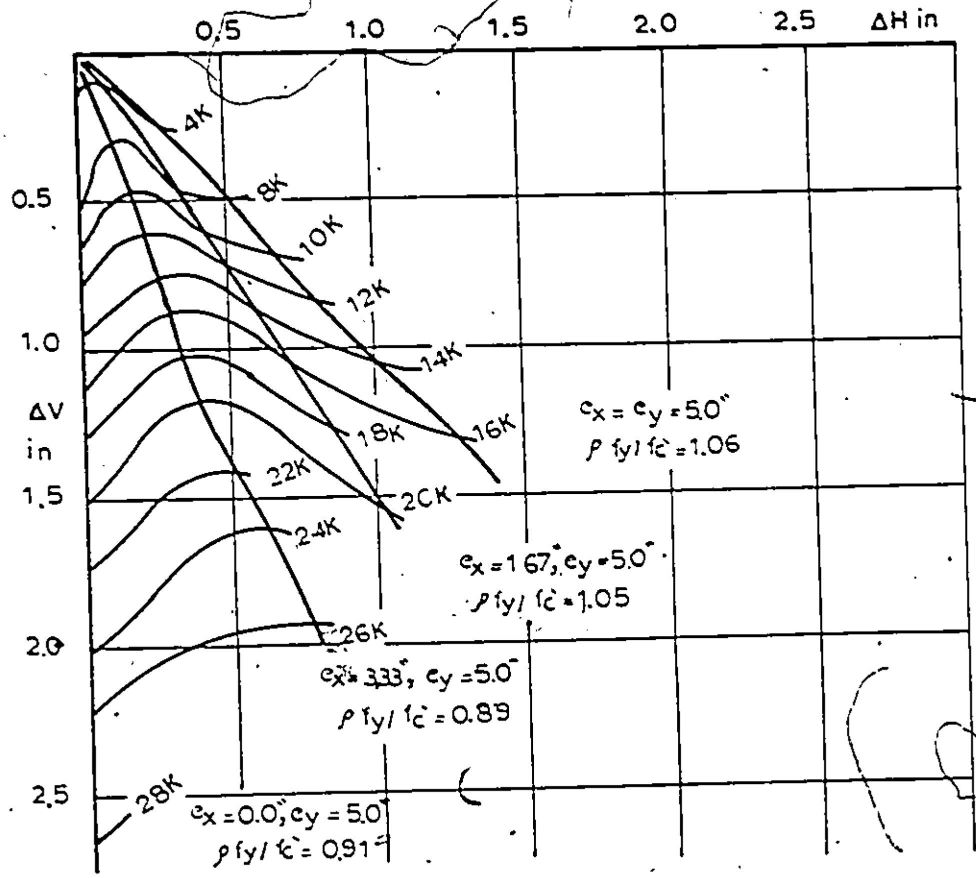

0.2
0.4
0.6
0.8
1.0
ΔH in
ΔV in
1K
2K
3K
4K
5K
6K
7K
7.5K
8K
9K
10K
ex = ey = 7.5"
ρ fy/fc = 0.21
ex = 2.5, ey = 7.5"
ρ fy/fc = 0.20
ex = 2.5", ey = 7.5"
ρ fy/fc = 0.20
ex = 0.0", ey = 7.5"
ρ fy/fc = 0.21

FIGURE (5-14) Mid Deflection, Eccentricity and it's Angle and Load Relationships for Group C

L = 134 (L/r = 75)

7 Bars (6.67%)




0.5
1.0
1.5
2.0
2.5
ΔH in
ΔV in
4K
8K
10K
12K
14K
16K
18K
20K
22K
24K
26K
28K
ex = ey = 5.0"
ρ fy/ fc' = 1.06
ex = 1.67", ey = 5.0"
ρ fy/ fc' = 1.05
ex = 3.33", ey = 5.0"
ρ fy/ fc' = 0.89
ex = 0.0", ey = 5.0"
ρ fy/ fc' = 0.91

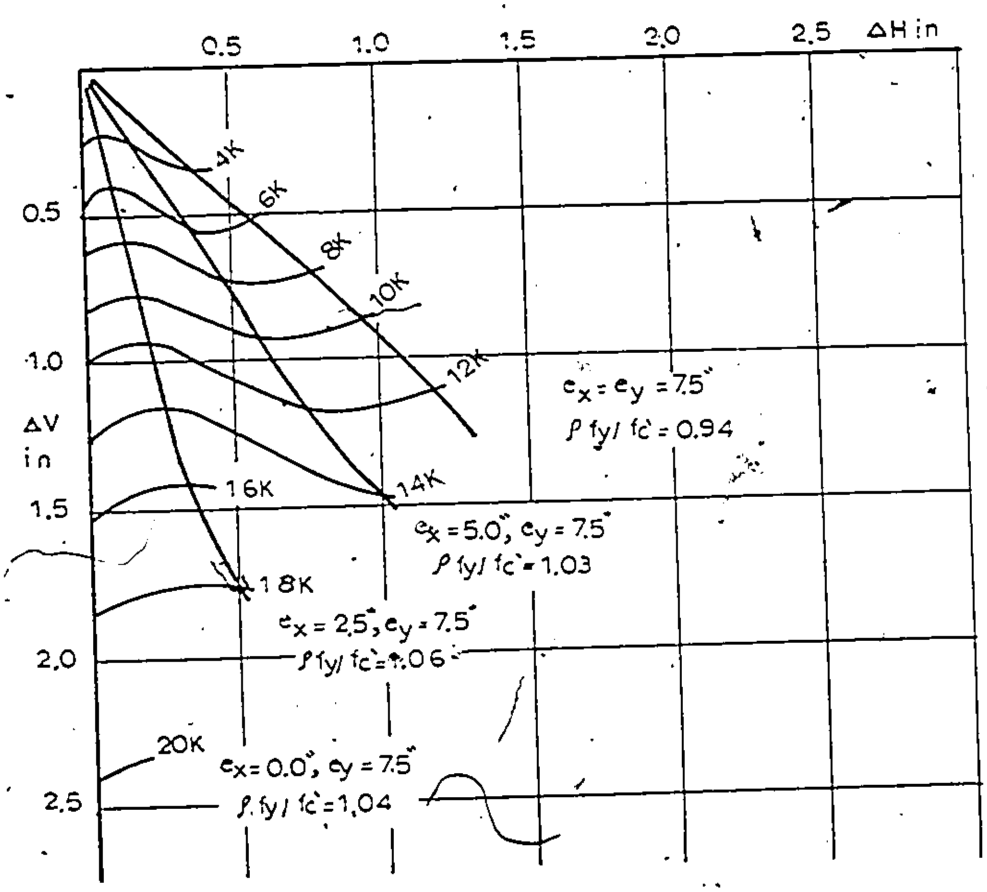
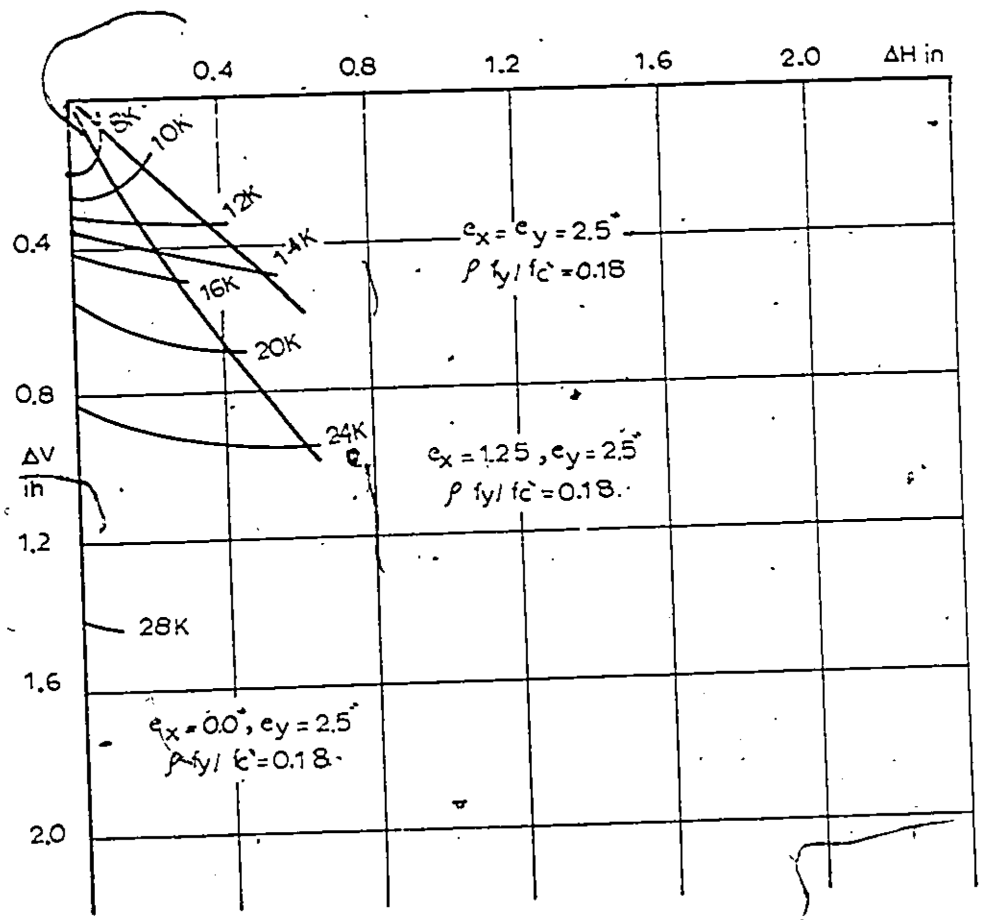
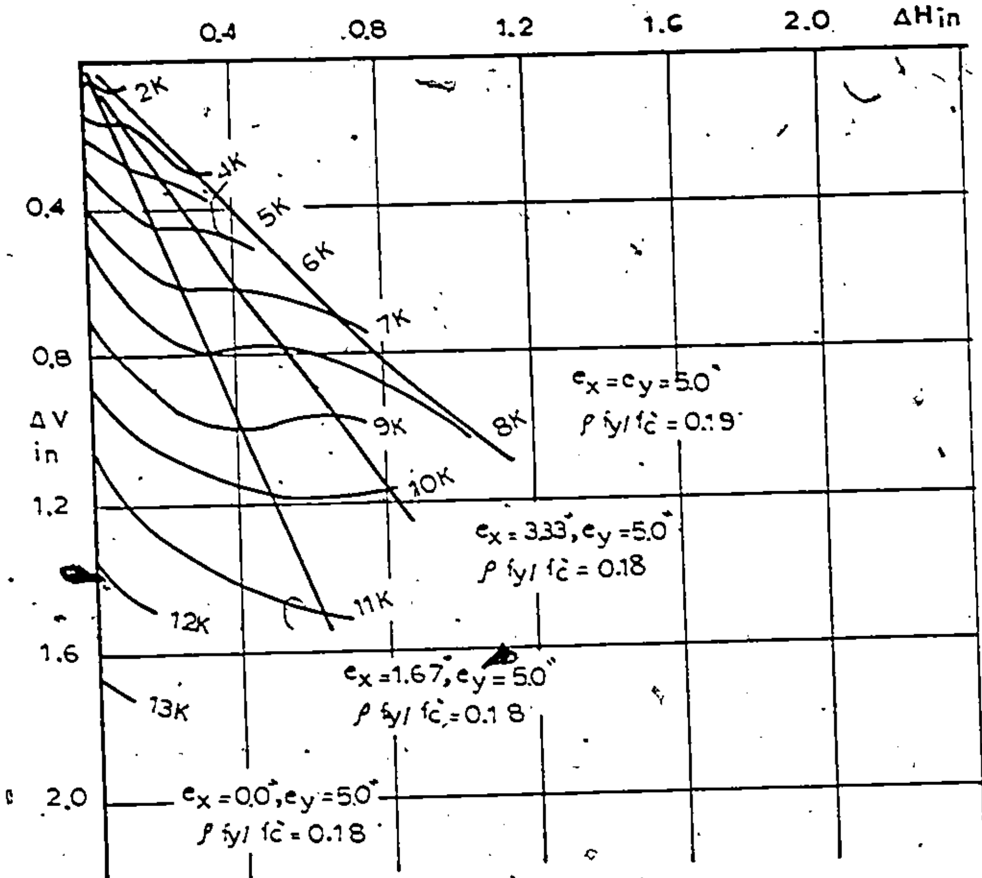

0.5
1.0
1.5
2.0
2.5
ΔH in
ΔV in
4K
6K
8K
10K
12K
14K
16K
18K
20K
ex = ey = 7.5"
ρ fy/ fc' = 0.94
ex = 5.0", ey = 7.5"
ρ fy/ fc' = 1.03
ex = 2.5", ey = 7.5"
ρ fy/ fc' = 1.06
ex = 0.0", ey = 7.5"
ρ fy/ fc' = 1.04

FIGURE (5-15) Mid Deflection, Eccentricity and it's Angle and Load Relationships for Groupd D

L = 134 (L/r = 75)

3 Bars (1.22%)




0.4
.08
1.2
1.6
2.0
ΔH in
2K
4K
5K
6K
7K
8K
9K
10K
11K
12K
13K
0.4
0.8
ΔV in
1.2
1.6
2.0
ex = ey = 5.0"
ρ fy/ fc = 0.19
ex = 3.33", ey = 5.0"
ρ fy/ fc = 0.18
ex = 1.67", ey = 5.0"
ρ fy/ fc = 0.18
ex = 0.0", ey = 5.0"
ρ fy/ fc = 0.18

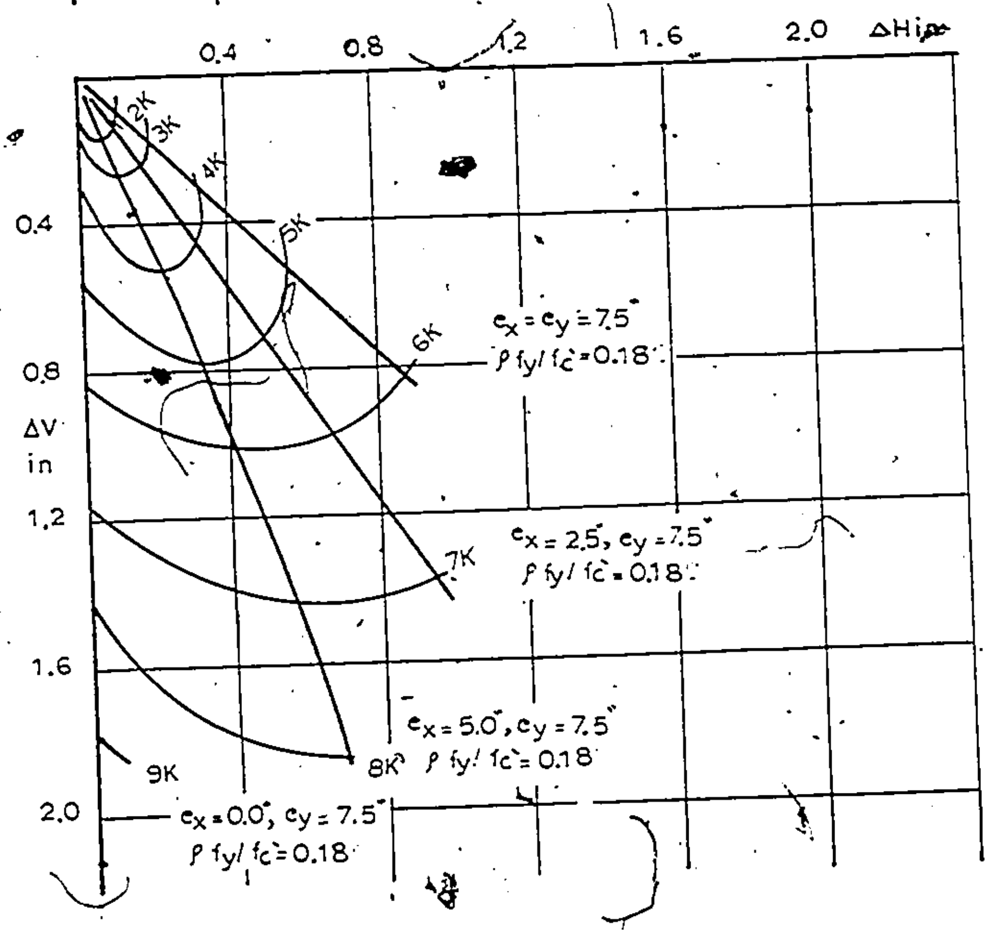
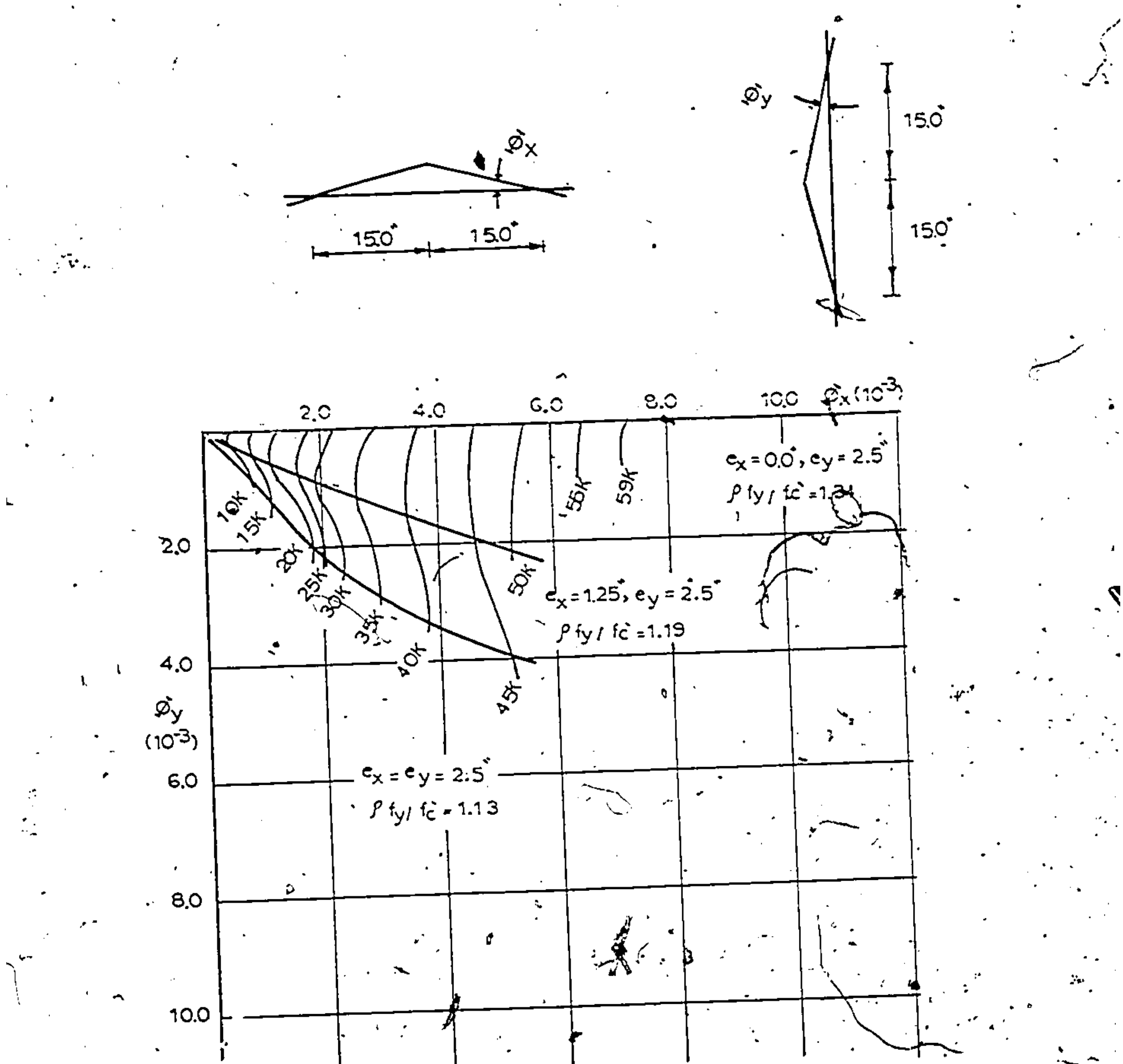
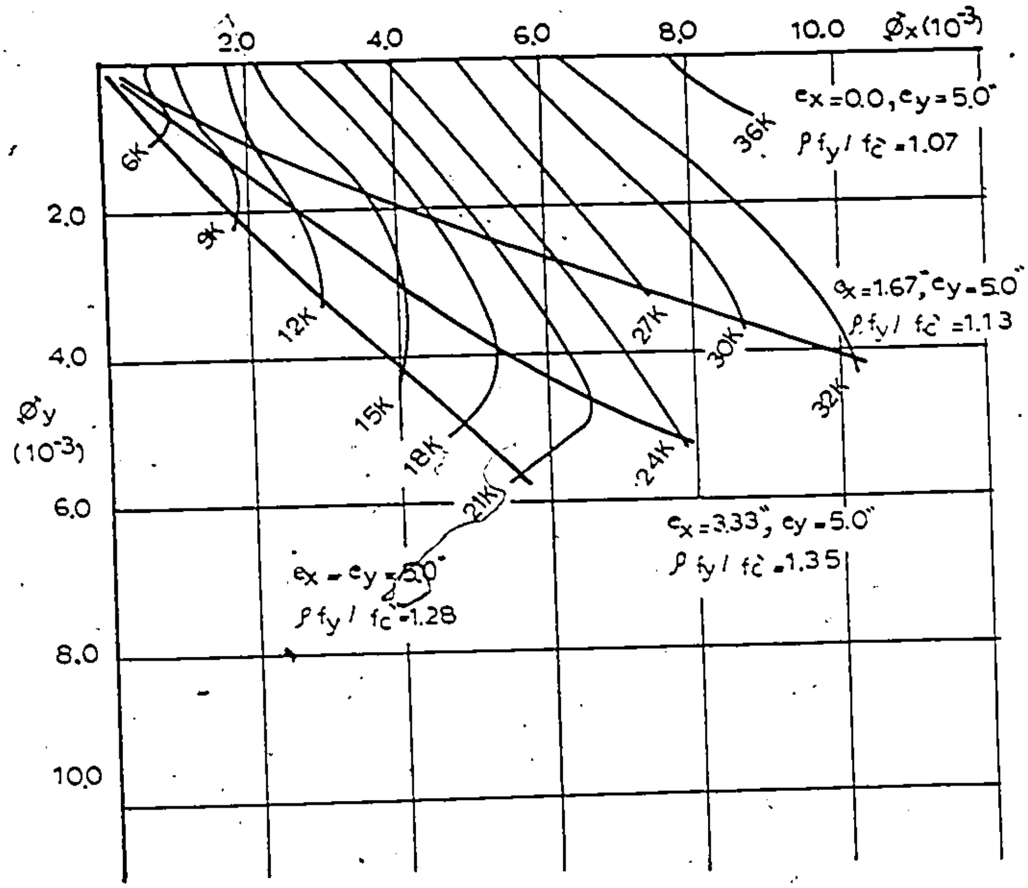

0.4
0.8
1.2
1.6
2.0
ΔH in
2K
3K
4K
5K
6K
7K
8K
9K
0.4
0.8
ΔV in
1.2
1.6
2.0
ex = ey = 7.5"
ρ fy/ fc = 0.18
ex = 2.5", ey = 7.5"
ρ fy/ fc = 0.18
ex = 5.0", ey = 7.5"
ρ fy/ fc = 0.18
ex = 0.0", ey = 7.5"
ρ fy/ fc = 0.18

FIGURE (5-16) Curvature, Eccentricity and it's Angle and Load Relationships for Group A

L = 90" (L/r = 50)

7 Bars (6.67%)




2.0
4.0
6.0
8.0
10.0
Øx(10-3)
ex=0.0, ey=5.0"
ρ fy / fc' =1.07
ex=1.67", ey=5.0"
ρ fy / fc' =1.13
ex=3.33", ey=5.0"
ρ fy / fc' =1.35
ex = ey = 5.0"
ρ fy / fc' =1.28
6K
9K
12K
15K
18K
21K
24K
27K
30K
32K
36K
Øy
(10-3)

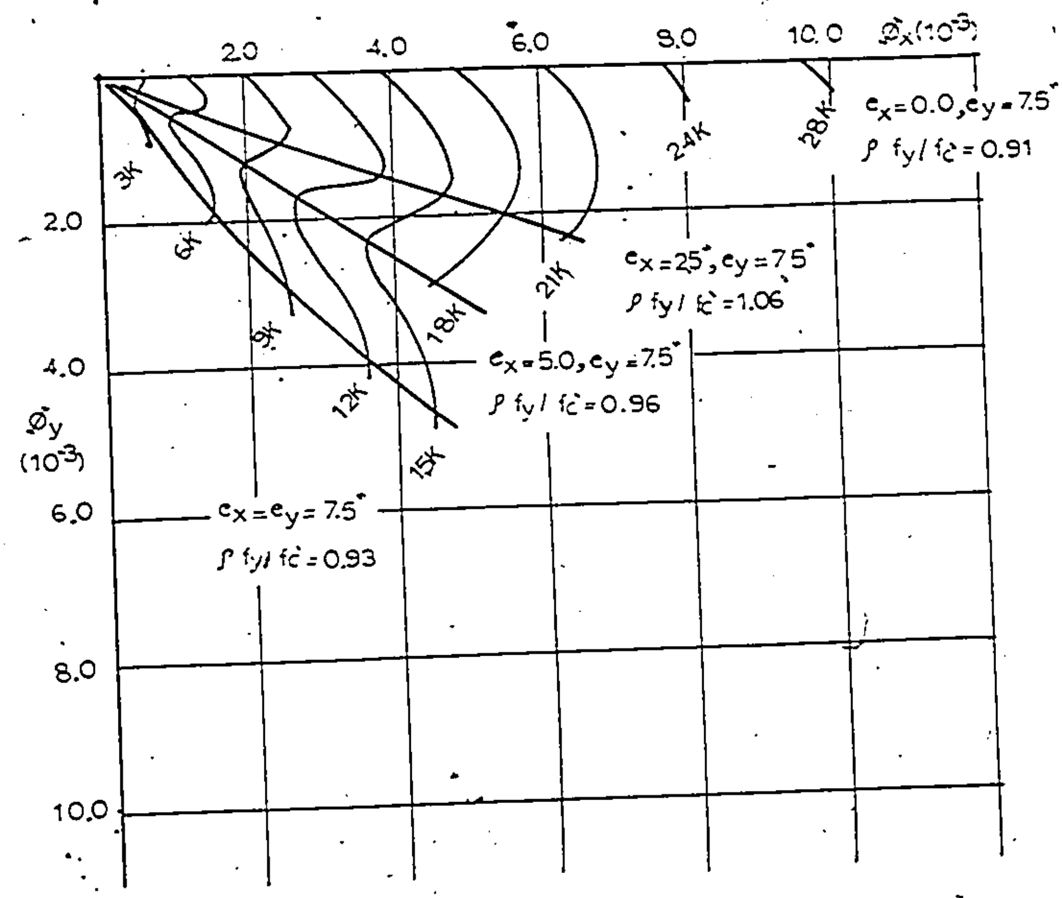
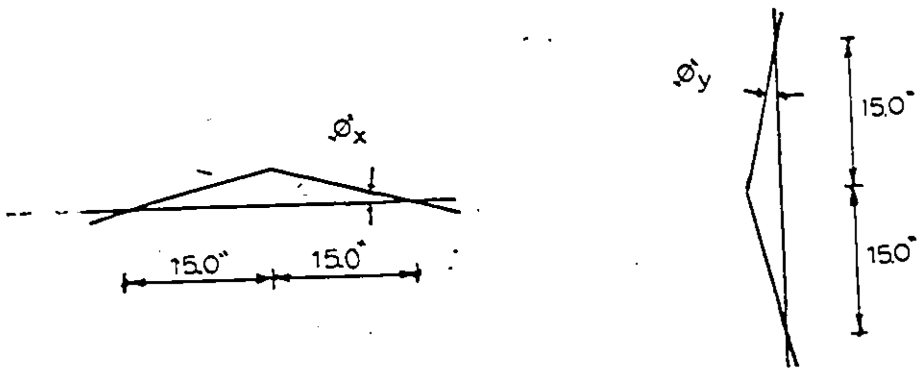
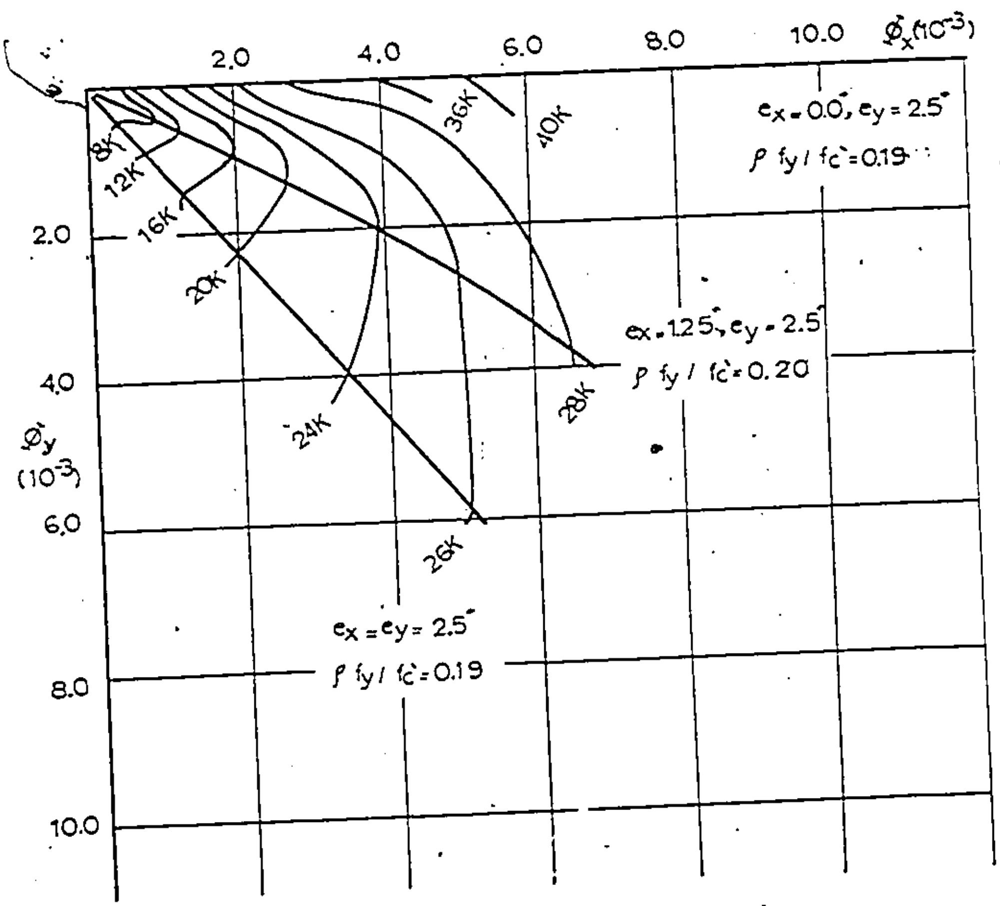

2.0
4.0
6.0
8.0
10.0
Øx(10-3)
ex=0.0, ey=7.5"
ρ fy / fc' = 0.91
ex=2.5", ey=7.5"
ρ fy / fc' =1.06
ex=5.0, ey=7.5"
ρ fy / fc' = 0.96
ex=ey= 7.5"
ρ fy / fc' = 0.93
3K
6K
9K
12K
15K
18K
21K
24K
28K
Øy
(10-3)

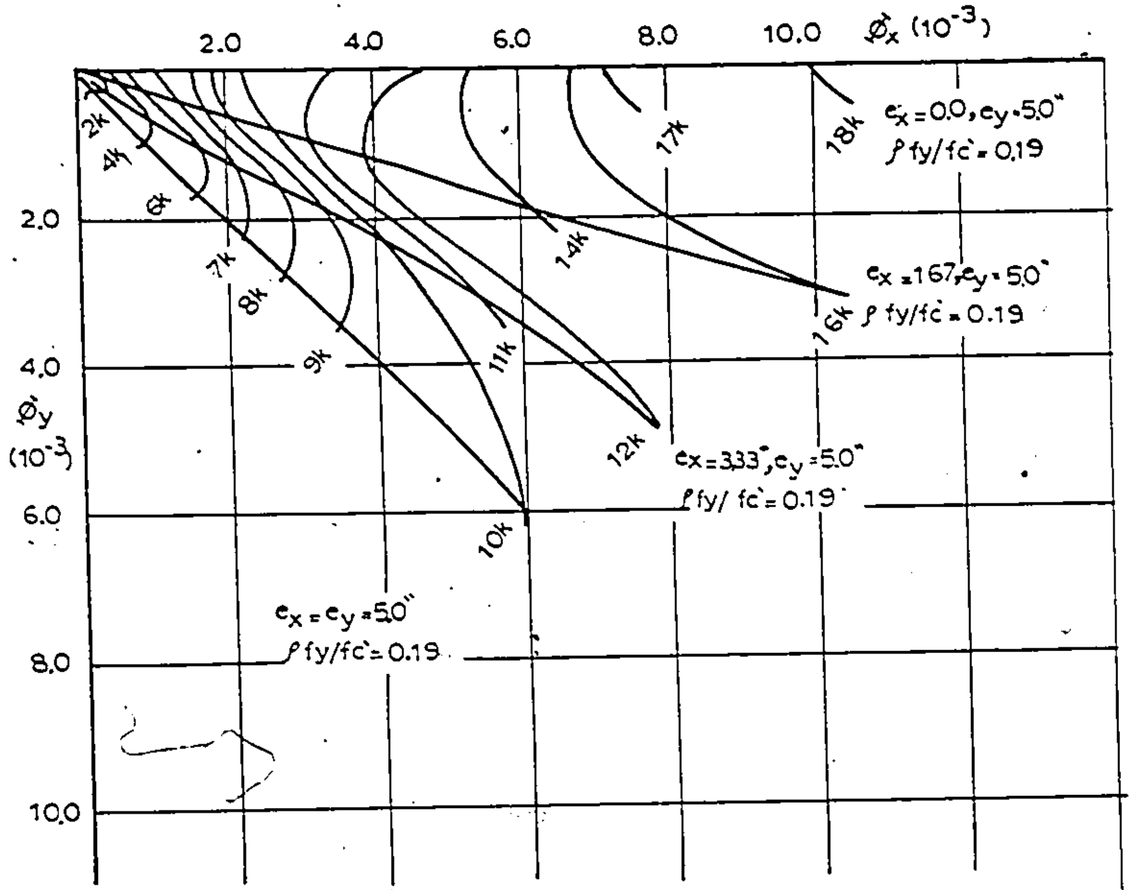
FIGURE (5-17) Curvature, Eccentricity and it's Angle and Load Relationships for Group B

L = 90" (L/r = 50)

3 Bars (1.22%)






2.0
4.0
6.0
8.0
10.0
Øx (10-3)
Øy (10-3)
ex = 0.0, ey = 5.0"
ρ fy/fc' = 0.19
ex = 1.67, ey = 5.0"
ρ fy/fc' = 0.19
ex = 3.33", ey = 5.0"
ρ fy/fc' = 0.19
ex = ey = 5.0"
ρ fy/fc' = 0.19
2k
4k
6k
7k
8k
9k
10k
11k
12k
14k
16k
17k
18k

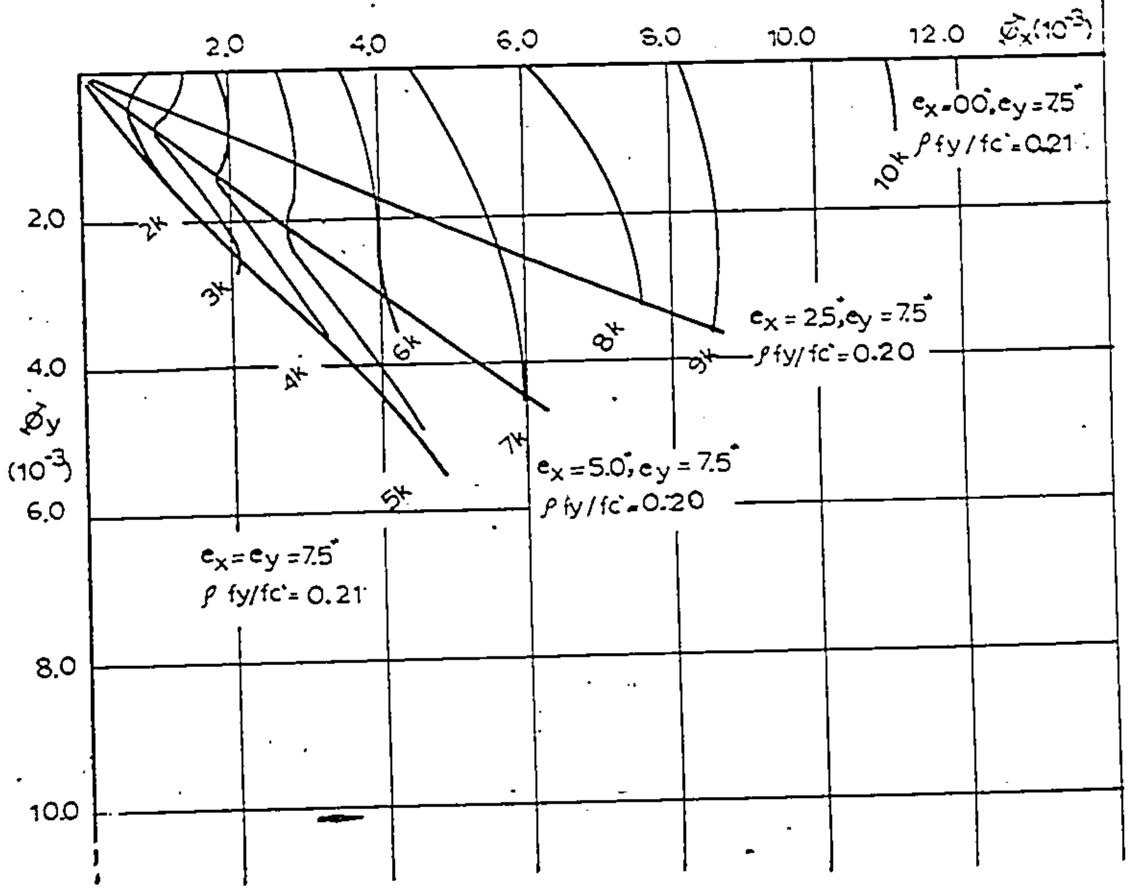
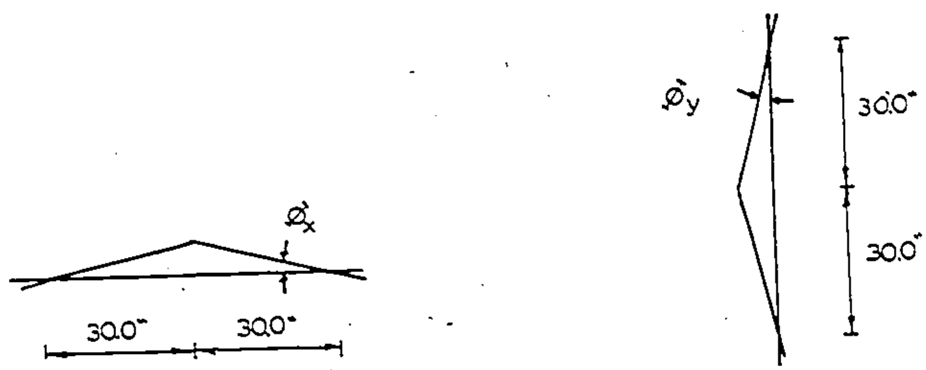
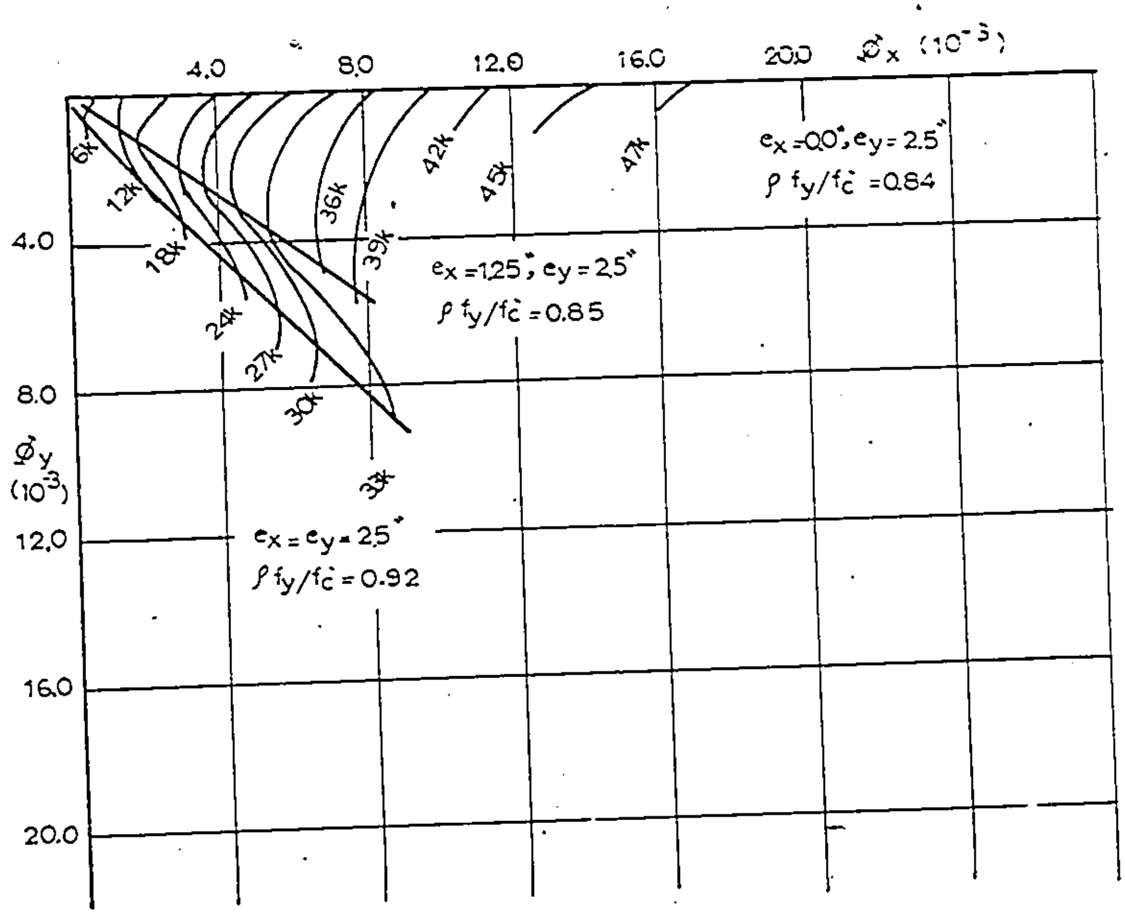

2.0
4.0
6.0
8.0
10.0
12.0
Øx (10-3)
Øy (10-3)
ex = 0.0", ey = 7.5"
ρ fy/fc' = 0.21
ex = 2.5", ey = 7.5"
ρ fy/fc' = 0.20
ex = 5.0", ey = 7.5"
ρ fy/fc' = 0.20
ex = ey = 7.5"
ρ fy/fc' = 0.21
2k
3k
4k
5k
6k
7k
8k
9k
10k

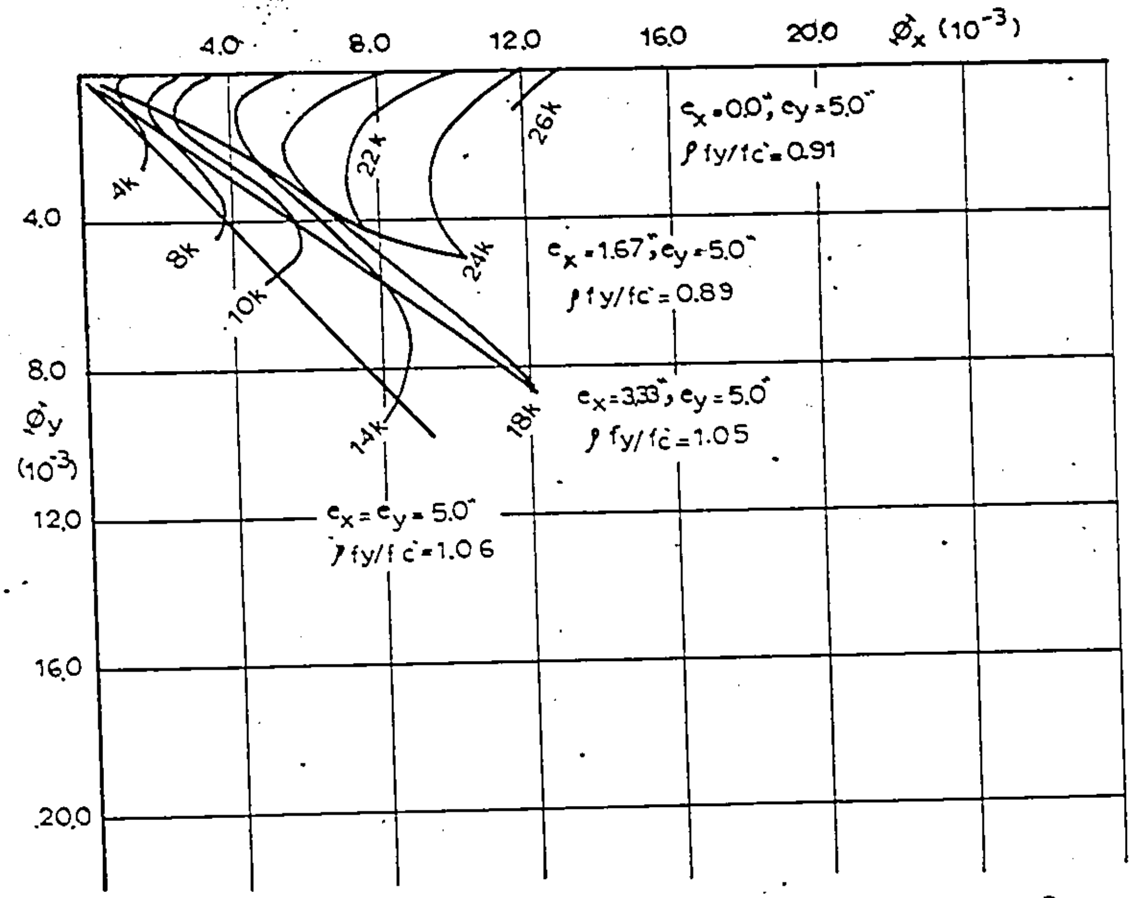
FIGURE (5-18) Curvature, Eccentricity and it's Angle and Load Relationships for Group C

L = 134" (L/r = 75)

7 Bars (6.67%)






4.0
8.0
12.0
16.0
20.0
ϕ_x (10^-3)
ϕ_y (10^-3)
e_x = 0.0", e_y = 5.0"
ρ fy/fc' = 0.91
e_x = 1.67", e_y = 5.0"
ρ fy/fc' = 0.89
e_x = 3.33", e_y = 5.0"
ρ fy/fc' = 1.05
e_x = e_y = 5.0"
ρ fy/fc' = 1.06
4k
8k
10k
14k
18k
22k
24k
26k

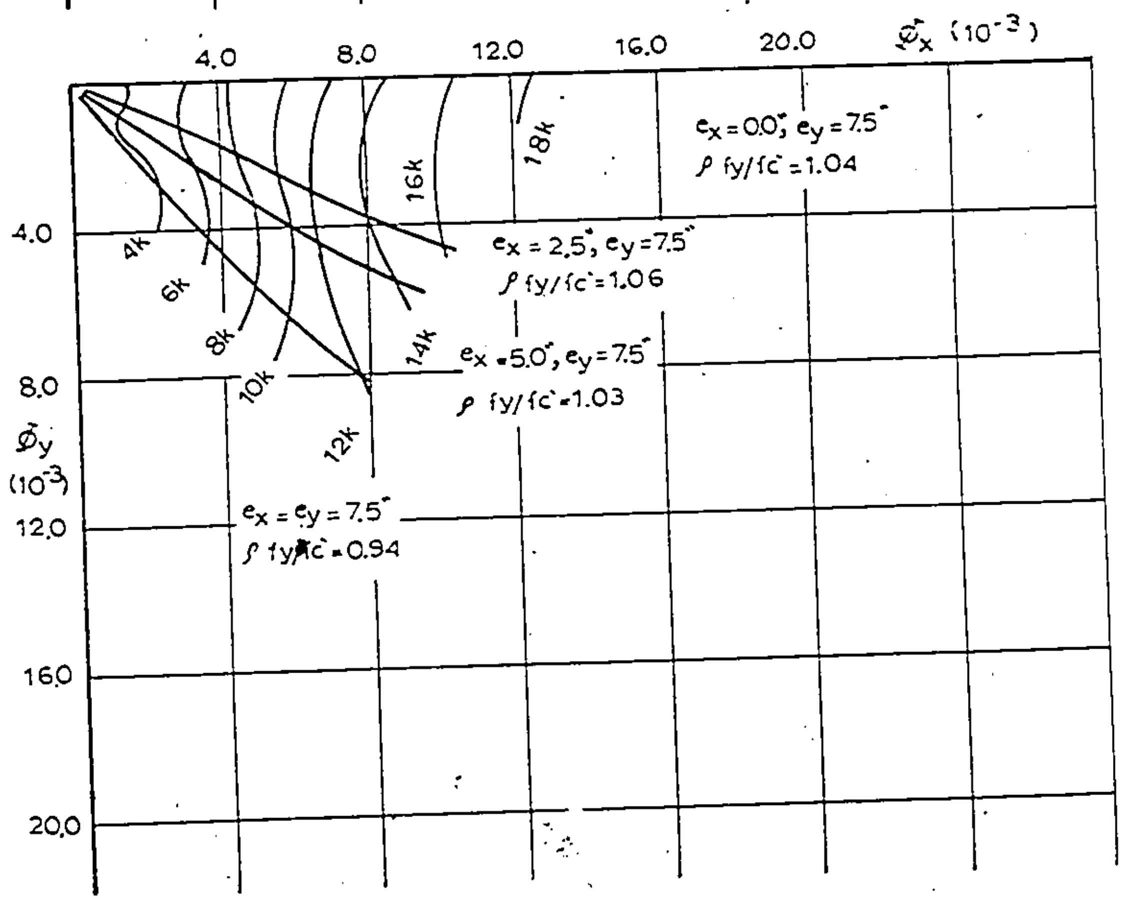
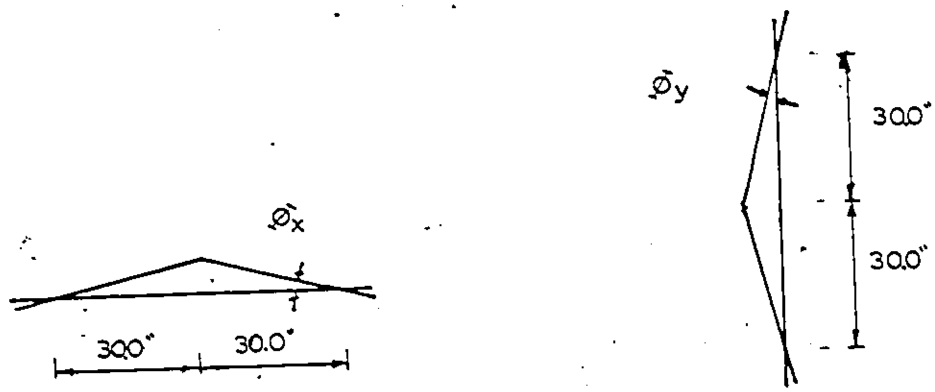
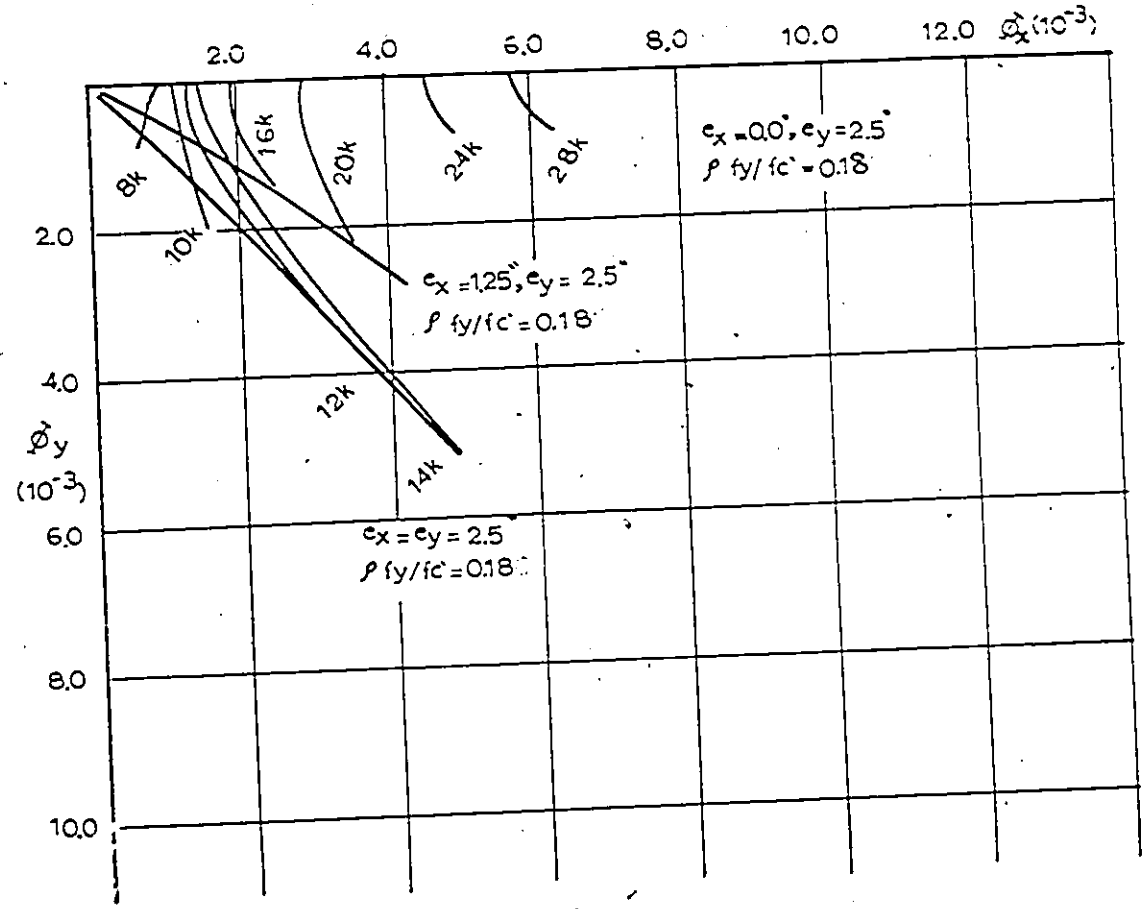

4.0
8.0
12.0
16.0
20.0
ϕ_x (10^-3)
ϕ_y (10^-3)
e_x = 0.0", e_y = 7.5"
ρ fy/fc' = 1.04
e_x = 2.5", e_y = 7.5"
ρ fy/fc' = 1.06
e_x = 5.0", e_y = 7.5"
ρ fy/fc' = 1.03
e_x = e_y = 7.5"
ρ fy/fc' = 0.94
4k
6k
8k
10k
12k
14k
16k
18k

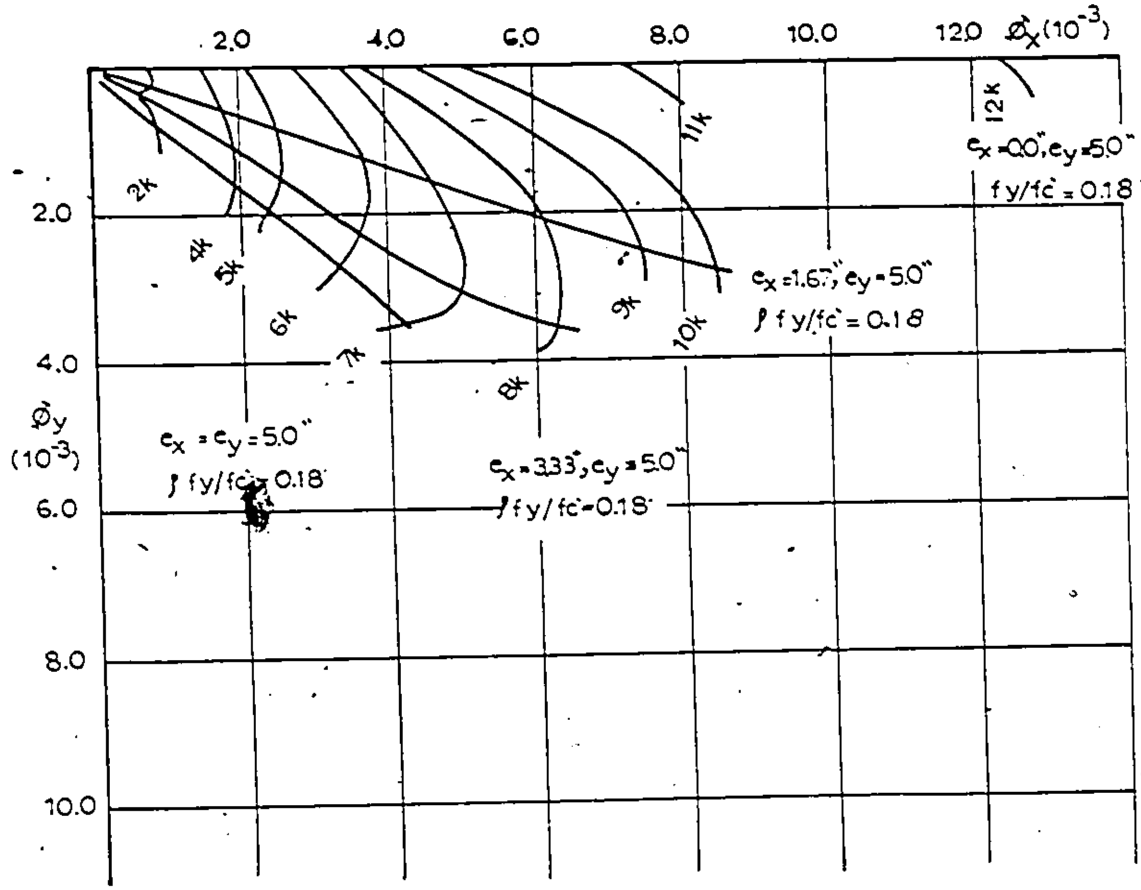
FIGURE (5-19) Curvature, Eccentricity and it's Angle and Load Relationships for Group D

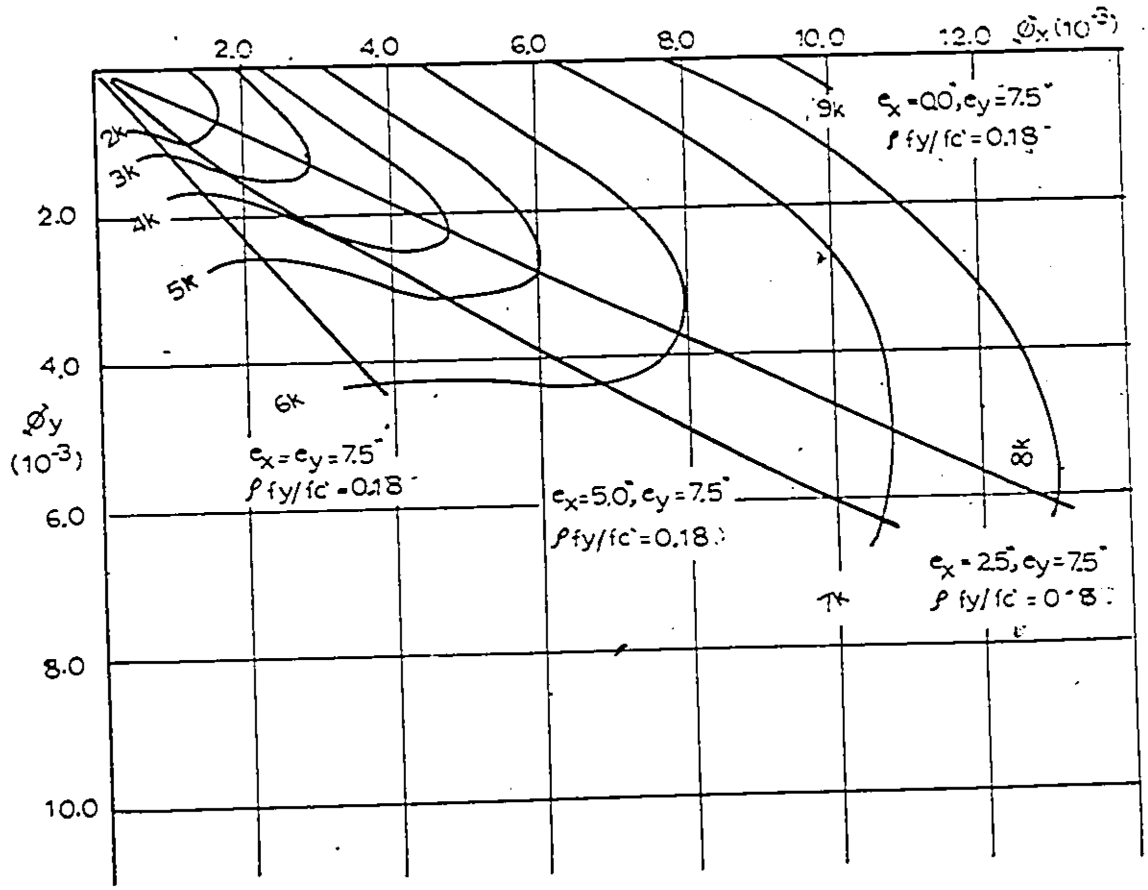
L = 134" (L/r = 75)

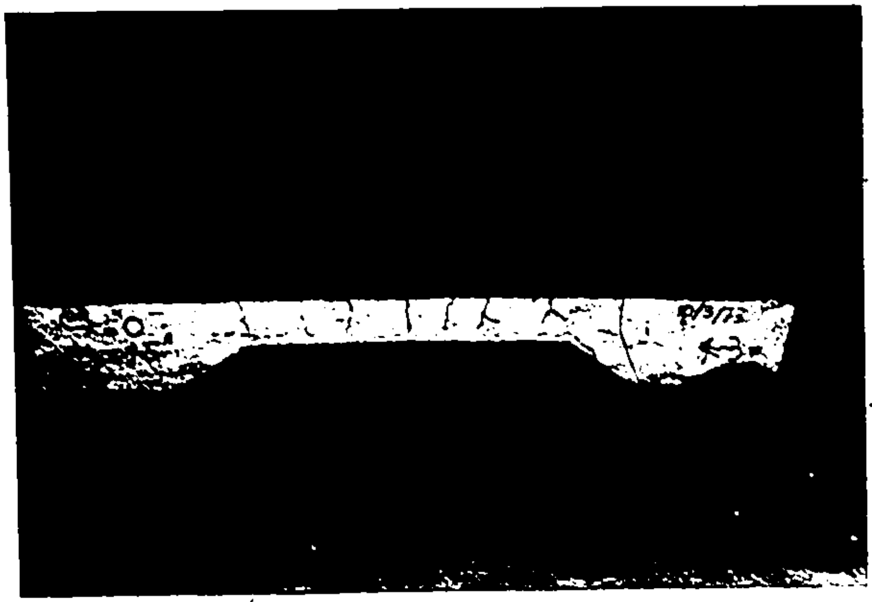
3 Bars (1.22%)



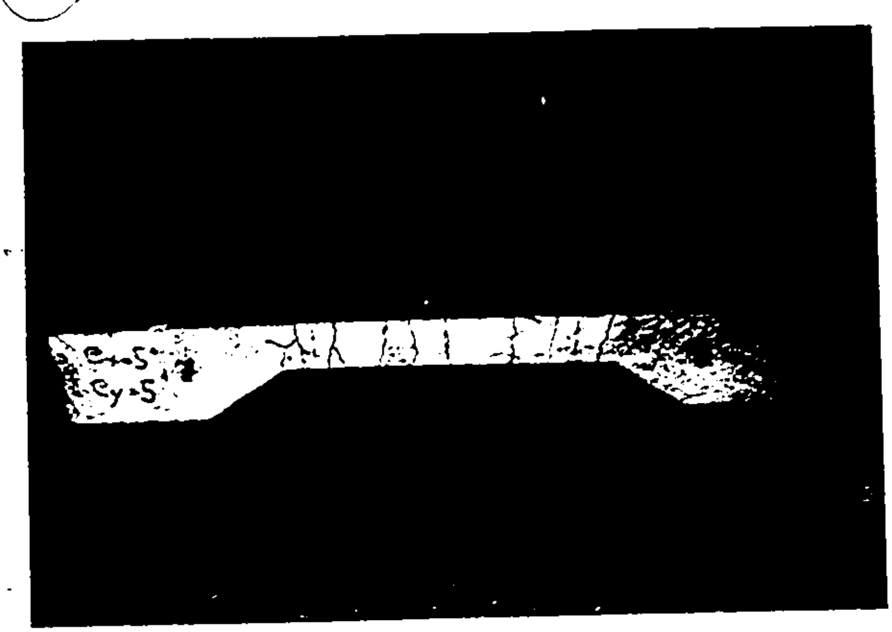



2.0
4.0
6.0
8.0
10.0
12.0
$\phi_x (10^{-3})$
2.0
4.0
6.0
8.0
10.0
ϕ_y (10^{-3})
2k
4k
5k
6k
7k
8k
9k
10k
11k
12k
$e_x = 0.0", e_y = 5.0"$
$f_y/f_c' = 0.18$
$e_x = 1.67", e_y = 5.0"$
$\rho f_y/f_c' = 0.18$
$e_x = e_y = 5.0"$
$\rho f_y/f_c' = 0.18$
$e_x = 3.33", e_y = 5.0"$
$\rho f_y/f_c' = 0.18$


2.0
4.0
6.0
8.0
10.0
12.0
$\phi_x (10^{-3})$
2.0
4.0
6.0
8.0
10.0
ϕ_y (10^{-3})
2k
3k
4k
5k
6k
7k
8k
9k
$e_x = 0.0", e_y = 7.5"$
$\rho f_y/f_c' = 0.18$
$e_x = e_y = 7.5"$
$\rho f_y/f_c' = 0.18$
$e_x = 5.0", e_y = 7.5"$
$\rho f_y/f_c' = 0.18$
$e_x = 2.5", e_y = 7.5"$



#3 bars L=90" e_x=0.0 e_y=7.5



#3 bars L=90" e_x=5.0 e_y=5.0

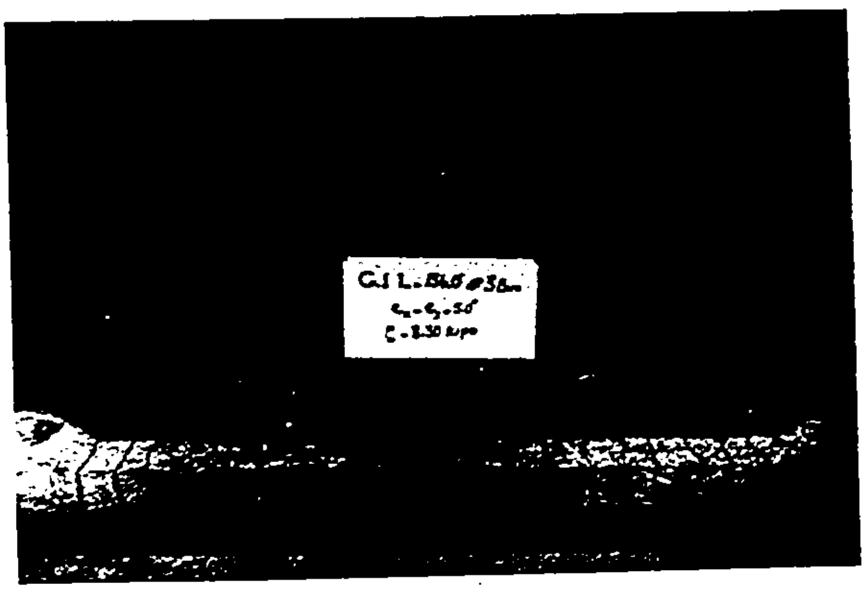
Fig (5-20) Samples after Failure

7 bars L=90° e_x=0.0 e_y=5.0

7 bars L=90° e_x = 5.0 e_y = 5.0

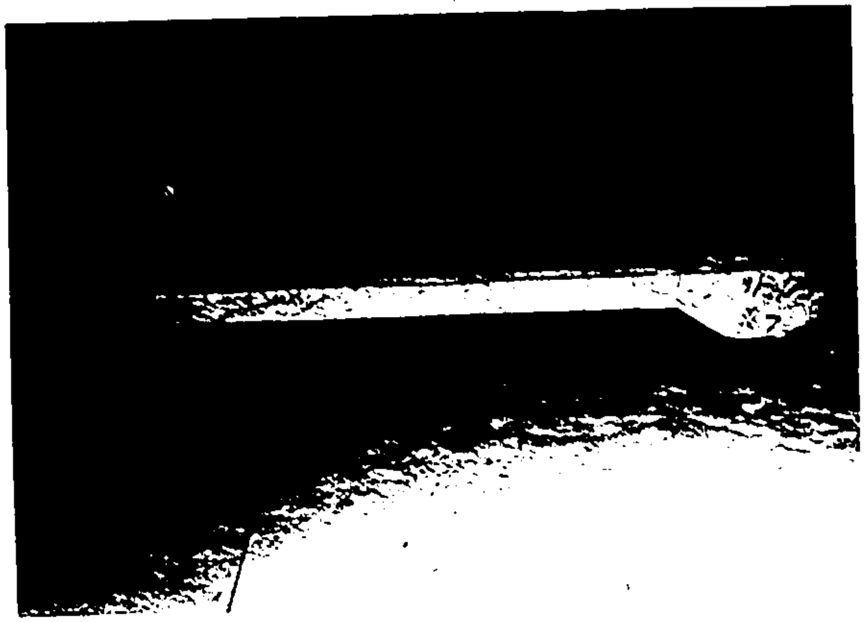
Fig (5-21) Samples after Failure

3 bars L=134" e_x=0.0 e_y=5.0



3 bars L=134" e_x=5.0 e_y=5.0

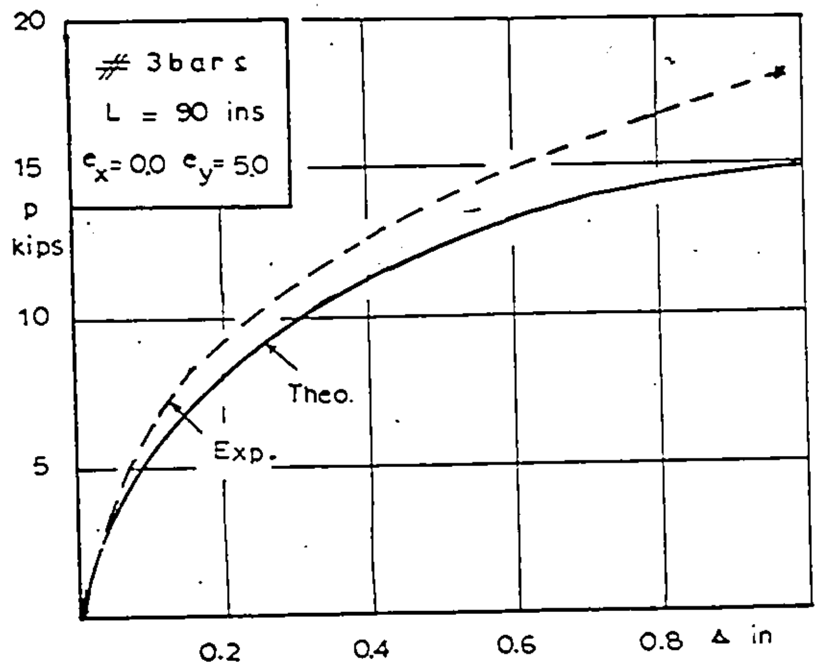
Fig (5-22) Samples after Failure



7bars L=134" e_x=0.0 e_y=7.5.

7bars L=134" e_x=5.0 e_y=5.0

Fig(5-23) Samples after Failure



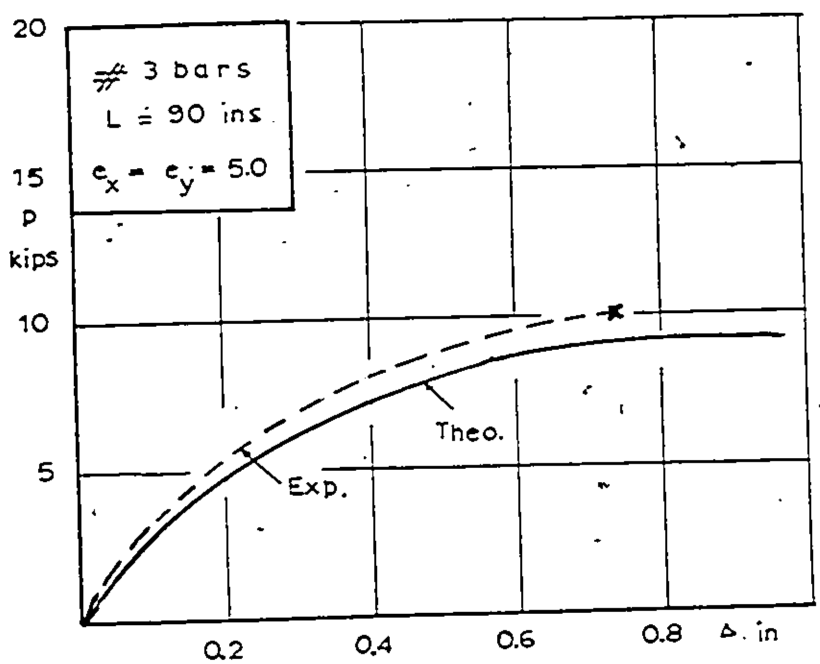
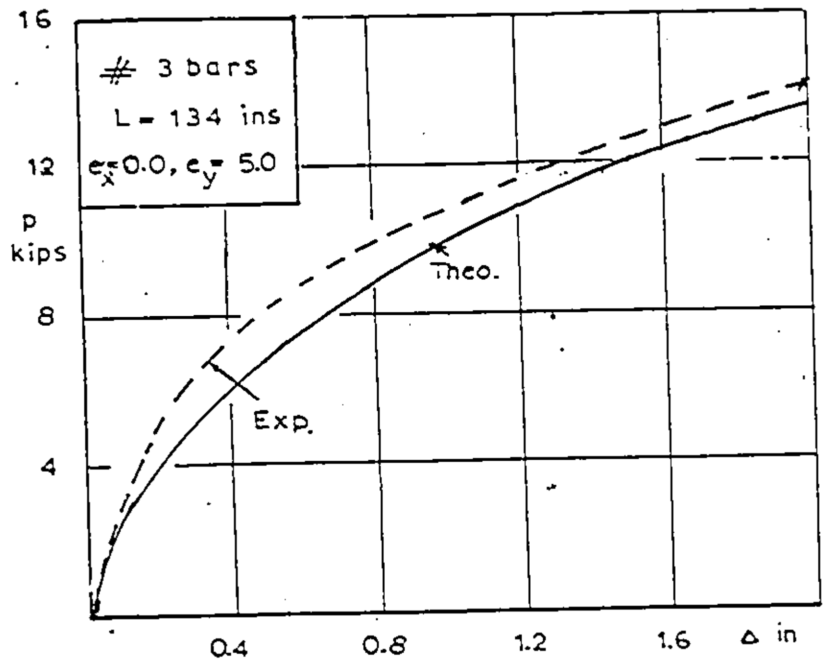


FIGURE (6-1) Comparison Between Theoretical and Experimental Load-Deflection Curves at Mid Column Height



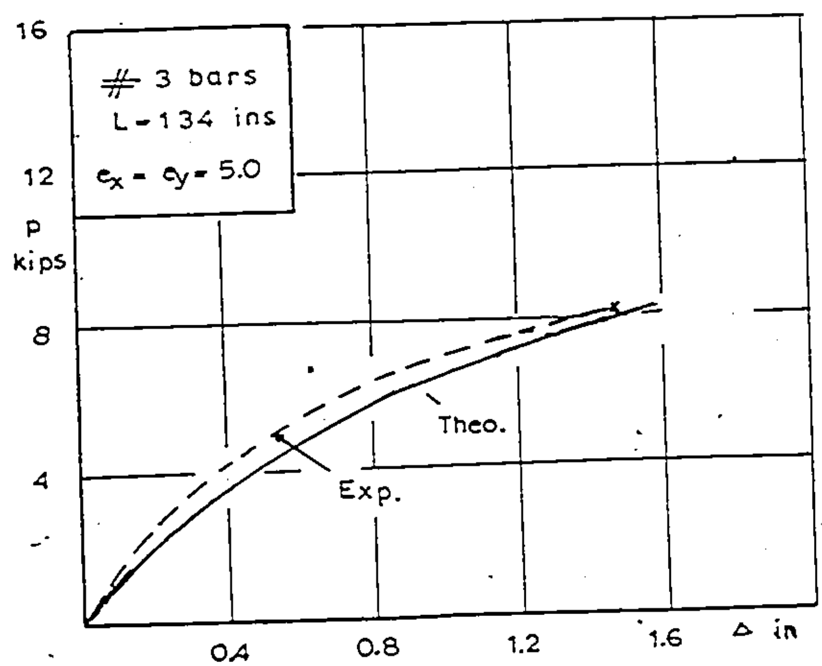
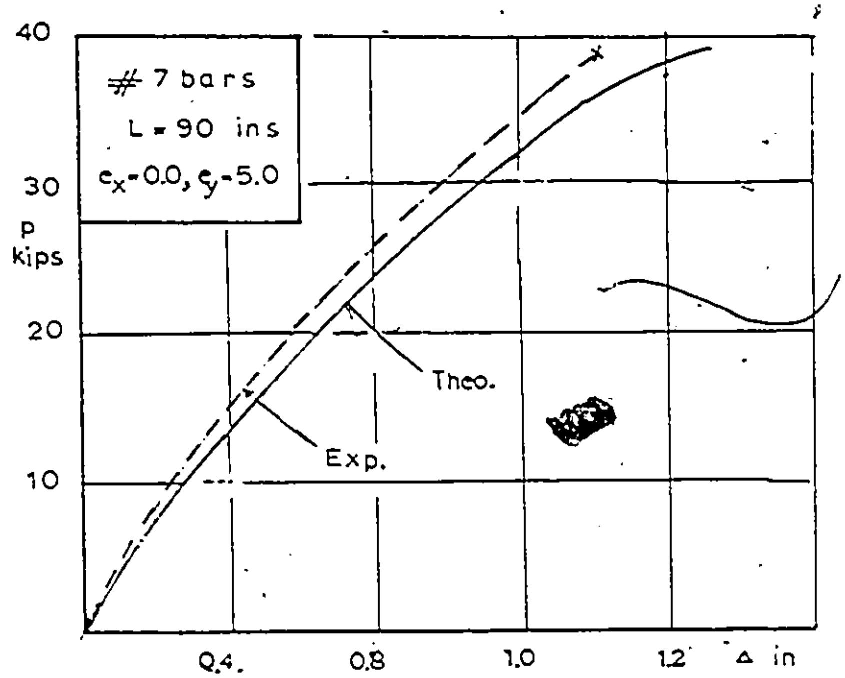


FIGURE (6-2) Comparison Between Theoretical and Experimental Load-Deflection Curves at Mid Column Height



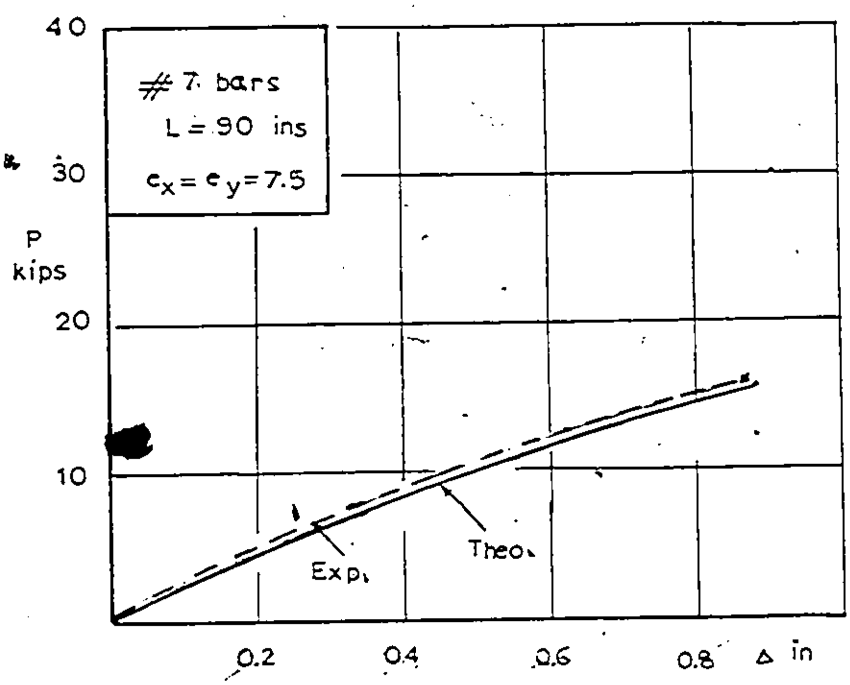
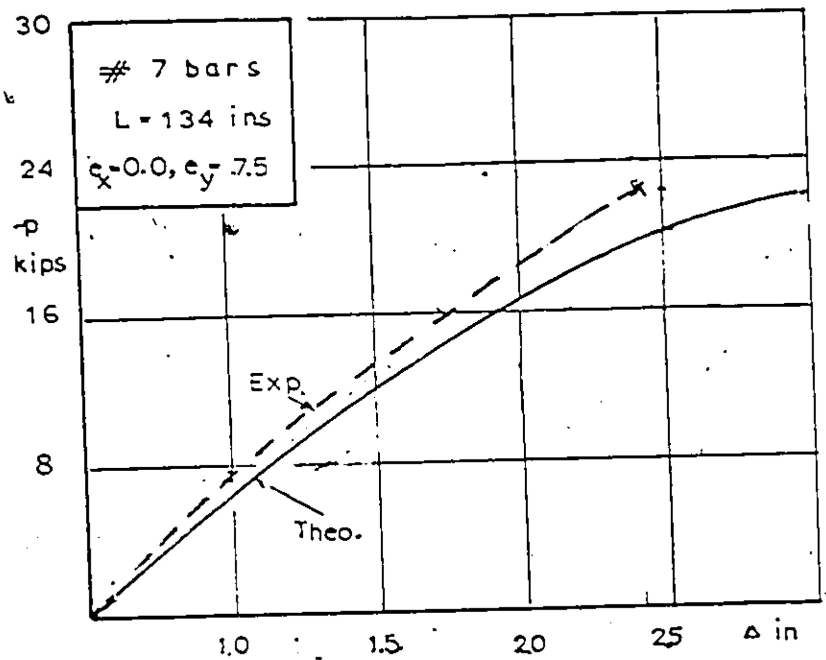


FIGURE (6-3) Comparison Between Theoretical and Experimental Load-Deflection Curves at Mid Column Height



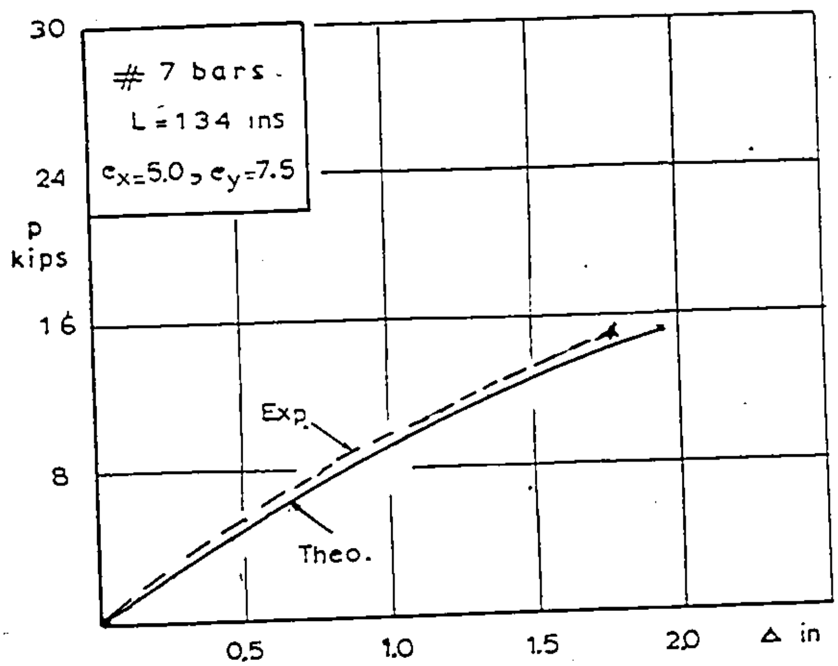


FIGURE (6-4) Comparison Between Theoretical and Experimental Load-Deflection Curves at Mid Column Height

APPENDIX 2

Computer Program

```
C
C
C             CALCULATING THE RELATIONS BETWEEN FORCES,MOMENTS,STRAINS,
C
C               ECCENTRICITIES AND THE LOCATION OF THE NEUTRAL AXIS
C
C
C
C                                 SIDE UP
C
C
C
C                          FC=3000 PSI,FS=60000 PSI,P=3%
C
C
C
C                 ES11=MAXIMAM STRAIN VALUE
C                 ES21,ES22,ES12=SRAIN VALUES AT CORNERS OF A CONCRETE SQUARE
C                 EESC=STRAIN VALUE AT THE CENTER OF A STEEL SQUARE
C                 EESS=STRAIN VALUE AT THE CENTER OF A STEEL SQUARE
C                 PPSC=STRESS VALUE AT THE CENTER OF A CONCRETE SQUARE
C                 PPSS=STRESS VALUE AT THE CENTER OF A STEEL SQUARE
C                 RMXT=TOTAL MOMENT ABOUT THE X AXIS
C                 RMYT=TOTAL MOMENT ABOUT THE Y AXIS
C                 FT=TOTAL FORCE
C                 E=ECCENTRICITY
C                 Z=LOCATION OF THE NEUTRAL AXIS
C
C
      DIMENSION ESC(150),ESS(150),PSC(150),PSS(150),ES21(150),ES22(150),
     1AC(15,15),AS(12,15),FC(15,15),FS(15,15),RMXC(15,15),RMYC(15,15),
     2RMXS(15,15),RMYS(15,15),FT(15,15),RMXT(15,15),RMYT(15,15),X(150),
     3Y(150)
C
C
C         READING STRESS STRAIN RELATIONSHIP FOR CONCRETE
C
C
  180 READ (1,18)  K,ESC(K),PSC(K),NEXT
   18 FORMAT ( I5,2F10.5, I6)
      IF (NEXT.EQ.0) GO TO 180
      NESC=K
C
C
C         READING STRESS STRAIN RELATIONSHIP FOR STEEL
C
C
  190 READ (1,33) K,ESS(K),PSS(K),NEXT
   33 FORMAT  (I5,2F12.6,I6)
      IF(NEXT.EQ.0) GO TO 190
      NESS=K
C
C
C         GEOMETERICAL PROPERTIES OF SECTION
C
C
      READ (1,15) (X(I), I=1,12),(Y(J), J=1,12)
   15 FORMAT (8F8.2)
C
C
```

```
      C     MAXIMAM STRAIN
      C
      C
            ES11=0.0030
      C
      C
      C     READ VALUES UF STRAINS AT THE CORNERS
      C
      C
      C
   66 DO  500 II=1.10
      READ (1.16) (II.(ES21(L).L=1.6).(ES22(M).M=1.6).PHI)
   16 FORMAT (I4.12F6.4.I4)
      WRITE  (3.38)( ES21(L). L=1.6).(ES22(M).  M=1.6)
   38 FORMAT  (//1H1.3X.12F9.5)
      C
      C
      C     ZERO VALUES OF FORCES AND MOMENTS
      C
      C
      C
            DO 1000   L=1.6
            DO  1000     M=1.6
            SUM1=0.0
            SUM2=0.0
            SUM3=0.0
            SUM4=0.0
            SUM5=0.0
            SUM6=0.0
            SUM7=0.0
            SUM8=0.0
            SUM9=0.0
            DO 2000   I=1.12
            DO 2000   J=1.12
      C
      C
      C     IDENTIFY THE LOCATION OF REINFORCEMENT
      C
      C
      C
            AC(I.J)=1.0
            AS(I.J)=0.0
            IF(X(I).EQ.-4.5.AND.Y(J).EQ.-4.5)GO TO 17
            GO TO 27
   17       AC(I.J)=0.0
            AS(I.J)=1.0
   27       IF(X(I).EQ.-4.5.AND.Y(J).EQ. 4.5)   GO TO 37
            GO TO 47
   37       AC(I.J)=0.0
            AS(I.J)=1.0
   47       IF(X(I).EQ. 4.5.AND.Y(J).EQ. 4.5)GO TO 57
            GO TO 67
   57       AC(I.J)=0.0
            AS(I.J)= 1.0
   67       IF(X(I).EQ. 4.5.AND.Y(J).EQ.-4.5)   GO TO 77
            GO TO 87
   77       AC(I.J)=0.0
            AS(I.J)= 1.0
   87       CONTINUE
      C
      C
```

```
C      CLCULATION OF STRAINS
C
C
      ES12=ES11-ES21(L)+ES22(M)
      IF    (ES12.GT.0.003)    GO   TO   81
      GO  TO  91
   81 ES12=0.003
   91 IF (ES11.GT.ES12)        GO    TO 13
      GO TO 14
   13 ESZ=(ES12       )+(ES11-ES12)*(6.0+Y(J))/12.0
      ESZZ=ES22(M)+(ES21(L)-ES22(M))*(6.0+Y(J))/12.0
      IF   (ESZ.GT.ESZZ)   GO TO 10
      GO TO 21
   10 EESC=ESZZ+(ESZ-ESZZ)*(6+X(I))/12.0
      EESS=ESZZ+(ESZ-ESZZ)*(6+X(I))/12.0
      GO TO 5
   21 EESC=ESZ  +(ESZZ-ESZ)*(6-X(I))/12.0
      EESS=ESZ  +(ESZZ-ESZ)*(6-X(I))/12.0
      GO TO 5
   14 ESZ=ES11+(ES12-ES11)*(6.0-Y(J))/12.0
      ESZZ=ES21(L)+(ES22(M)-ES21(L))*(5.0-Y(J))/12.0
      IF   (ESZ.GT.ESZZ)   GO TO 101
      GO TO 201
  101 EESC=ESZZ+(ESZ-ESZZ)*(6.0+X(I))/12.0
      EESS=ESZZ+(ESZ-ESZZ)*(6.0+X(I))/12.0
      GO TO 5
  201 EESC=ESZ  +(ESZZ-ESZ)*(6.0-X(I))/12.0
      EESS=ESZ  +(ESZZ-ESZ)*(6.0-X(I))/12.0
    5 CONTINUE
C
C
C      CALCULATION OF STRESSES
C
C
      IF   (AC(I,J).EQ.1.0)   GO   TO   6
      GO TO 25
    6 S =ABS(EESC   )
      K=1
   11 IF(ESC(K)-S)1,2,2
    1 K=K+1
      GO TO 11
    2 PPSC=PSC(K-1)+(S-ESC(K-1))*(PSC(K)-PSC(K-1))/(ESC(K)-ESC(K-1))
  133 PPSS=0.0
      GO   TO   109
   25 SS=ABS(EESS   )
      IF   (SS.GE.0.00200) GO  TO  99
      K=1
  110 IF (ESS(K)-SS)19,20,20
   19 K=K+1
      GO TO 110
   20 PPSS=PSS(K-1)+(SS-ESS(K-1))*(PSS(K)-PSS(K-1))/(ESS(K)-ESS(K-1))
      GO   TO   134
   99 PPSS=65000.0
  134 PPSC=0.0
  109 FC(I,J)=PPSC   *AC(I,J)
C
C
C      SUMMING UP FORCES AND MOMENTS
```

```
      C
      C
            SUM1=SUM1+FC(I,J)
            FS(I,J)=PPSS  *AS(I,J)
            SUM2=SUM2+FS(I,J)
            RMXC(I,J)=FC(I,J)*Y(J)
            SUM3=SUM3+RMXC(I,J)
            RMYC(I,J)=FC(I,J)*X(I)
            SUM4=SUM4+RMYC(I,J)
            RMXS(I,J)=FS(I,J)*Y(J)
            SUM5=SUM5+RMXS(I,J)
            RMYS(I,J)=FS(I,J)*X(I)
            SUM6=SUM6+RMYS(I,J)
            FT(I,J)=FC(I,J)+FS(I,J)
            SUM7=SUM7+FT(I,J)
            RMXT(I,J)=RMXC(I,J)+RMXS(I,J)
            SUM8=SUM8+RMXT(I,J)
            RMYT(I,J)=RMYC(I,J)+RMYS(I,J)
            SUM9=SUM9+RMYT(I,J)
 2000       CONTINUE
      C
      C
      C     NONDIMENTIONING
      C
      C
            SUM10=SUM7/423.0
            SUM11=SUM8/5180.0
            SUM12=SUM9/5180.0
            SUM13=SUM11/SUM12
            SUM14=SUM12/SUM11
      C
      C
      C     CALCULATE THE ECCENTRICITY AND LOCATE THE NEUTRAL AXIS
      C
      C
            Z1=ES11*12.0/(ES11-ES21(L))
            Z2=Z1/1.414
            SUM15=Z2/12.0
            E=SUM9/SUM7
            F1=SQRT(2.0*E**2)
            SUM16=F1/12.0
            IF ((SUM9/SUM8).GT.(0.97).AND.(SUM9/SUM8).LT.(1.03)) GO TO 7
            GO TO 1000
    7       WRITE (3,250) ES11
  250       FORMAT (F6.4)
            WRITE(3,150) SUM1,SUM2,SUM3,SUM4,SUM5,SUM6,SUM7,SUM8,SUM9
  150       FORMAT (//2X,9(F10.2,3X))
            WRITE(3,151)  SUM10,SUM11,SUM12,SUM13,SUM14,SUM15,SUM16
  151       FORMAT  (//2X,7(F14.6,2X))
 1000       CONTINUE
      C
      C     CHANGE THE VALUE OF MAXIMAM STRAIN
      C
            ES11=ES11-0.0002
            IF (PHI.EQ.0) GO TO 66
  500       CONTINUE
            STOP
            END
```

```
      C
      C
      C              CALCULATING THE RELATIONS BETWEEN FORCES,MOMENTS,STRAINS,
      C                ECCENTRICITIES AND THE LOCATION OF THE NEUTRAL AXIS
      C
      C
      C                             SIDE DOWN
      C
      C
      C
      C                   FC=3CCO PSI,FS=60000 PSI,P=7%
      C
      C
      C
      C
      C                 ES11=MAXIMAM STRAIN VALUE
      C                 ES21,ES22,ES12=SRAIN VALUES AT CORNERS
      C                 EESC=STRAIN VALUE AT THE CENTER OF A CONCRETE SQUARE
      C                 EESS=STRAIN VALUE AT THE CENTER OF A STEEL SQUARE
      C                 PPSC=STRESS VALUE AT THE CENTER OF A CONCRETE SQUARE
      C                 PPSS=STRESS VALUE AT THE CENTER OF A STEEL SQUARE
      C                 RMXT=TOTAL MOMENT ABOUT THE X AXIS
      C                 RMYT=TOTAL MOMENT ABOUT THE Y AXIS
      C                 FT=TOTAL FORCE
      C                 E=ECCENTRICITY
      C                 Z=LOCATION OF THE NEUTRAL AXIS
      C
      C
      C
            DIMENSION ESC(150),ESS(150),PSC(150),PSS(150),ES21(150),ES22(150),
           1AC(15,15),AS(12,15),FC(15,15),FS(15,15),RMXC(15,15),RMYC(15,15),
           2RMXS(15,15),RMYS(15,15),FT(15,15),RMXT(15,15),RMYT(15,15),X(150),
           3Y(150)
            READING STRESS STRAIN RELATIONSHIP FOR CONCRETE
      C
      C
      C
        180 READ (1,18)  K,ESC(K),PSC(K),NEXT
      C
      C
         18 FORMAT ( I5,2F10.5, I6)
            IF (NEXT.EQ.0) GO TO 180
            NESC=K
      C
      C
      C     READING STRESS STRAIN RELATIONSHIP FOR STEEL
      C
      C
      C
        190 READ (1,33) K,ESS(K),PSS(K),NEXT
         33 FORMAT  (I5,2F12.6,I6)
            IF(NEXT.EQ.0) GO TO 190
            NESS=K
      C
      C
      C     GEOMETERICAL PROPERTIES OF SECTION
      C
      C
      C
            READ (1,15) (X(I), I=1,12),(Y(J), J=1,12)
         15 FORMAT  (8F8.2)
      C
      C
```

```
C
C     MAXIMAM STRAIN
C
      ES11=0.003
C
C
C     READ VALUES OF STRAINS AT THE CORNERS
C
C
C
   66 DO  500  II=1.10
      READ (1,16) (II.(ES21(L).L=1.6).(ES22(M).M=1.6).PHI)
   16 FORMAT (I4.12F6.4.I4)
      WRITE  (3.38)( ES21(L). L=1.6).(ES22(M).  M=1.6)
   38 FORMAT  (//1H1.3X.12F8.5)
C
C
C     ZERO VALUES OF FORCES AND MOMENTS
C
C
C
      DO 1000   L=1.6
      DO  1000     M=1.6
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      SUM4=0.0
      SUM5=0.0
      SUM6=0.0
      SUM7=0.0
      SUM8=0.0
      SUM9=0.0
      DO 2000   I=1.12
      DO   2000   J=1.12
C
C
C     IDENTIFY THE LOCATION OF REINFORCEMENT
C
C
C
      AC(I.J)=1.0
      AS(I.J)=0.0
      IF(X(I).EQ.-4.5.AND.Y(J).EQ.-4.5)GO TO 17
      GO TO 27
   17 AC(I.J)=0.0
      AS(I.J)=-1.0
   27 IF(X(I).EQ.-4.5.AND.Y(J).EQ. 4.5)   GO TO 37
      GO TO 47
   37 AC(I.J)=0.0
      AS(I.J)=-1.0
   47 IF(X(I).EQ. 4.5.AND.Y(J).EQ. 4.5)GO TO 57
      GO TO 67
   57 AC(I.J)=0.0
      AS(I.J)= 1.0
   67 IF(X(I).EQ. 4.5.AND.Y(J).EQ.-4.5)   GO TO 77
      GO TO 87
   77 AC(I.J)=0.0
      AS(I.J)= 1.0
   87 CONTINUE
C
C
```

```
C       CLCULATION OF STRAINS
C
C
        ES12=ES11+ES21(L)-ES22(M)
        IF    (ES12.GT.C.CC3)     GO   TO   81
        GC   TO   91
     81 ES12=0.0C3
     91 IF  (ES11.GT.ES12)      GO    TO 13
        GO TO 14
     13 ESZ=(ES12          )+(ES11-ES12)*(6.C+Y(J))/12.0
        ESZZ=(ES21(L)          )+(ES22(M)-ES21(L))*(6.0-Y(J))/12.0
        Z=12.C*ESZ/(ESZ+ESZZ)
        IF ((6.C+X(I)).LT.(12.0-Z))   GO TO 10
        EESC   =ESZ*(6.C+X(I)-(12.C-Z))/Z
        EESS   =ESZ*(6.C+X(I)-(12.0-Z))/Z
        GO TO 5
     1C EESC   =C.C
        EESS   =ESZZ*((12.C-Z)-(6.0+X(I)))/(12.0-Z)
        GO TO 5
     14 ESZ=(ES11          )+(ES12-ES11)*(6.0-Y(J))/12.0
        ESZZ=(ES22(M)          )+(ES21(L)-ES22(M))*(6.C+Y(J))/12.0
        Z=12.C*ESZ/(ESZ+ESZZ)
        IF  ((6.C+X(I)).LT.(12.0-Z))  GO TO 9
        EESC   =ESZ*(6.C+X(I)-(12.0-Z))/Z
        EESS   =ESZ*(6.C+X(I)-(12.C-Z))/Z
        GO TO 5
      9 EESC   =0.0
        EESS   =ESZZ*((12.0-Z)-(6.0+X(I)))/(12.0-Z)
      5 CONTINUE
C
C       CALCULATION OF STRESSES
C
C
        IF    (AC(I,J).EQ.1.C)    GO    TO    6
        GO   TO   25
      6 S =ABS(EESC    )
        IF  (S.EQ.C.C)   GO TO 89
        K=1
     11 IF(ESC(K)-S)1,2,2
      1 K=K+1
        GO TO 11
      2 PPSC=PSC(K-1)+(S-ESC(K-1))*(PSC(K)-PSC(K-1))/(ESC(K)-ESC(K-1))
        GO   TO   133
     89 PPSC    =C.0
    133 PPSS=0.0
        GO    TO    1C9
     25 SS=ABS(EESS    )
        IF     (SS.GE.C.CC2CC) GO    TO    99
        K=1
    110 IF  (ESS(K)-SS)19,20,20
     19 K=K+1
        GO TO 110
     20 PPSS=RSS(K-1)+(SS-ESS(K-1))*(PSS(K)-PSS(K-1))/(ESS(K)-ESS(K-1))
        GO    TO    134
     99 PPSS=152CCC.0
    134 PPSC=C.C
    109 FC(I,J)=PPSC    *AC(I,J)
C
```

```
C
C      SUMMING UP FORCES AND MOMENTS
C
C
       SUM1=SUM1+FC(I,J)
       FS(I,J)=PPSS   *AS(I,J)
       SUM2=SUM2+FS(I,J)
       RMXC(I,J)=FC(I,J)*Y(J)
       SUM3=SUM3+RMXC(I,J)
       RMYC(I,J)=FC(I,J)*X(I)
       SUM4=SUM4+RMYC(I,J)
       RMXS(I,J)=FS(I,J)*Y(J)
       SUM5=SUM5+RMXS(I,J)
       RMYS(I,J)=FS(I,J)*X(I)
       SUM6=SUM6+RMYS(I,J)
       FT(I,J)=FC(I,J)+FS(I,J)
       SUM7=SUM7+FT(I,J)
       RMXT(I,J)=RMXC(I,J)+RMXS(I,J)
       SUM8=SUM8+RMXT(I,J)
       RMYT(I,J)=RMYC(I,J)+RMYS(I,J)
       SUM9=SUM9+RMYT(I,J)
 2000  CONTINUE
C
C
C      NONDIMENTIONING
C
C
       SUM10=SUM7/433.0
       SUM11=SUM8/5180.0
       SUM12=SUM9/5180.0
       SUM13=SUM11/SUM12
       SUM14=SUM12/SUM11
C
C
C      CALCULATE THE ECCENTRICITY AND LOCATE THE NEUTRAL AXIS
C
C
       Z=ES11*12.0/(ES11+ES21(L))
       SUM15=Z/12.0
       E=SUM9/SUM7
       SUM16=E/12.0
       IF  ((SUM8).GT.(-5.0).AND.(SUM8).LT.(5.0))   GO  TO  7
       GO  TO  1000
    7  WRITE  (3,250)ES11
  250  FORMAT  (F6.4)
       WRITE(3,150)  SUM1,SUM2,SUM3,SUM4,SUM5,SUM6,SUM7,SUM8,SUM9
  150  FORMAT  (//2X,9(F10.2,3X))
       WRITE(3,151)   SUM10,SUM11,SUM12,SUM13,SUM14,SUM15,SUM16
  151  FORMAT   (//2X,7(F14.6,2X))
 1000  CONTINUE
C
C      CHANGE THE VALUE OF MAXIMAM STRAIN
C
       ES11=ES11-0.0002
       IF  (PHI.EQ.0)   GO  TO  66
  500  CONTINUE
       STOP
       END
```

```
C
C     CALCULATING THE RELATIONS BETWEEN FORCES.MOMENTS.STRAINS.
C
C     ECCENTRICITIES AND THE LOCATION OF THE NEUTRAL AXIS
C
C
C                         CORNER DOWN
C
C
C                  FC=3000 PSI.FS=60000 PSI.P=5%
C
C
C
C            ES11=MAXIMAM STRAIN VALUE
C            ES21.ES22.ES12=SRAIN VALUES AT CORNERS OF A CONCRETE SQUARE
C            EFSC=STRAIN VALUE AT THE CENTER OF A STEEL SQUARE
C            EESS=STRAIN VALUE AT THE CENTER OF A CONCRETE SQUARE
C            PPSC=STRESS VALUE AT THE CENTER OF A STEEL SQUARE
C            PPSS=STRESS VALUE AT THE CENTER OF A STEEL SQUARE
C            RMXT=TOTAL MOMENT ABOUT THE X AXIS
C            RMYT=TOTAL MOMENT ABOUT THE Y AXIS
C            FT=TOTAL FORCE
C            E=ECCENTRICITY
C            Z=LOCATION OF THE NEUTRAL AXIS
C
C
      DIMENSION ESC(150).ESS(150).PSC(150).PSS(150).ES12(150).ES22(150).
     1AC(15.15).AS(15.15).FC(15.15).FS(15.15).RMXC(15.15).RMYC(15.15).
     2RMXS(15.15).RMYS(15.15).FT(15.15).RMXT(15.15).RMYT(15.15).X(150).
     3Y(150)
C
C     READING STRESS STRAIN RELATIONSHIP FOR CONCRETE
C
C
  180 READ (1.18)  K.ESC(K).FSC(K).NEXT
   18 FORMAT ( I5.2F10.5. I6)
      IF (NEXT.EQ.0) GO TO 180
      NESC=K
C
C     READING STRESS STRAIN RELATIONSHIP FOR STEEL
C
C
  190 READ (1.34) K.ESS(K).PSS(K).NEXT
   34 FORMAT ( I5.2F12.6. I6)
      IF(NEXT.EQ.0) GO TO 190
      NESS=K
C
C     GEOMETERICAL PROPERTIES OF SECTION
C
C
      READ (1.15) (X(I). I=1.12).(Y(J). J=1.12)
   15 FORMAT ( 8F8.2)
C
C
```

```
C     MAXIMAM STRAIN
C
C
      ES11=C.0030
C
C
C     READ VALUES OF STRAINS AT THE CORNERS
C
C
C
   66 DO  500 II=1,10
      READ (1,16) (II,(ES12(L),L=1,6),(ES22(M),M=1,6),PHI)
   16 FORMAT (I4,12F6.5,I4)
      WRITE  (3,38)( ES12(L), L=1,6),(ES22(M),  M=1,6)
   38 FORMAT  (//1H1,3X,12F9.5)
C
C
C     ZERO VALUES OF FORCES AND MOMENTS.
C
C
      DO 1C00   L=1,6
      DO  1000    M=1,6
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      SUM4=0.0
      SUM5=0.0
      SUM6=0.0
      SUM7=0.0
      SUM8=0.0
      SUM9=0.0
      DO 2C00   J=1,12
      DO 2000   I=1,12
C
C
C     IDENTIFY THE LOCATION OF REINFORCEMENT
C
C
      AC(I,J)=1.0
      AS(I,J)=0.0
      IF(X(I).EQ.-4.5.AND.Y(J).EQ.-4.5)GO TO 17
      GO TO 27
   17 AC(I,J)=0.0
      AS(I,J)=-1.0
   27 IF(X(I).EQ.-4.5.AND.Y(J).EQ. 4.5)  GO TO 37
      GO TO 47
   37 AC(I,J)=0.0
      AS(I,J)=1.0
   47 IF(X(I).EQ. 4.5.AND.Y(J).EQ. 4.5)GO TO 57
      GO TO 67
   57 AC(I,J)=0.0
      AS(I,J)= 1.0
   67 IF(X(I).EQ. 4.5.AND.Y(J).EQ.-4.5)  GO TO 77
      GO TO 87
   77 AC(I,J)=0.0
      AS(I,J)= 1.0
   87 CONTINUE
C
C
```

```
C
C     CLCULATION OF STRAINS
C
      ES21=ES11-ES12(L)-ES22(M)
      ESZ=ES21+(ES11-ES21)*(6.0+X(I))/12.0
      XX=(ES12(L)*12.0)/(ES12(L)+ES22(M))
      IF ((12.0-XX).LT.(6.0+X(I)))  GO  TO  3
      GO  TO  31
    3 ESZZ=ES12(L)*((6.0+X(I))-(12.0-XX))/XX
      EESC=ESZZ+(ESZ-ESZZ)*(6.0+Y(J))/12.0
      EESS=ESZZ+(ESZ-ESZZ)*(6.0+Y(J))/12.0
      GO  TO  33
   31 ESZZ=ES22(M)*((12.0-XX)-(6.0+X(I)))/(12.0-XX)
      Z=(12.0*ESZ)/(ESZ+ESZZ)
      IF ((12.0-Z).LT.(6.0+Y(J)))  GO  TO  97
      GO  TO  98
   97 EESC=ESZ*((6.0+Y(J))-(12.0-Z))/Z
      EESS=ESZ*((6.0+Y(J))-(12.0-Z))/Z
      GO  TO  33
   98 EESC=0.0
      EESS=ESZZ*((12.0-Z)-(6.0+Y(J)))/(12.0-Z)
   33 CONTINUE
C
C
C     CALCULATION OF STRESSES
C
C
      IF  (AC(I,J).EQ.1.0)  GO  TO  5
      GO  TO  25
    5 S =ABS(EESC   )
      IF (S.EQ.0.0)  GO TO 89
      K=1
   11 IF(ESC(K)-S)1,2,2
    1 K=K+1
      GO TO 11
    2 PPSC=PSC(K-1)+(S-ESC(K-1))*(PSC(K)-PSC(K-1))/(ESC(K)-ESC(K-1))
      GO  TO  133
   89 PPSC  =0.0
  133 PPSS=0.0
      GO  TO  109
   25 SS=ABS(EESS   )
      IF  (SS.GE.0.00200)  GO  TO  99
      K=1
  110 IF  (ESS(K)-SS)19,20,20
   19 K=K+1
      GO TO 110
   20 PPSS=PSS(K-1)+(SS-ESS(K-1))*(PSS(K)-PSS(K-1))/(ESS(K)-ESS(K-1))
      GO  TO  134
   99 PPSS=108000.0
  134 PPSC=0.0
  109 FC(I,J)=PPSC   *AC(I,J)
C
C
C     SUMMING UP FORCES AND MOMENTS
C
C
      SUM1=SUM1+FC(I,J)
      FS(I,J)=PPSS   *AS(I,J)
```

```
      SUM2=SUM2+FS(I,J)
      RMXC(I,J)=FC(I,J)*Y(J)
      SUM3=SUM3+RMXC(I,J)
      RMYC(I,J)=FC(I,J)*X(I)
      SUM4=SUM4+RMYC(I,J)
      RMXS(I,J)=FS(I,J)*Y(J)
      SUM5=SUM5+RMXS(I,J)
      RMYS(I,J)=FS(I,J)*X(I)
      SUM6=SUM6+RMYS(I,J)
      FT(I,J)=FC(I,J)+FS(I,J)
      SUM7=SUM7+FT(I,J)
      RMXT(I,J)=RMXC(I,J)+RMXS(I,J)
      SUM8=SUM8+RMXT(I,J)
      RMYT(I,J)=RMYC(I,J)+RMYS(I,J)
      SUM9=SUM9+RMYT(I,J)
 2000 CONTINUE
C
C
C     NONDIMENTIONING
C
C
C
      SUM10=SUM7/433.0
      SUM11=SUM8/5180.0
      SUM12=SUM9/5180.0
      SUM13=SUM11/SUM12
      SUM14=SUM12/SUM11
C
C
C     CALCULATE THE ECCENTRICITY AND LOCATE THE NEUTRAL AXIS
C
C
      Z1=ES22(M)*12.0/(ES12(L)+ES22(M))
      Z2=Z1/1.414
      SUM15=(17.0-Z2)/12.0
      E=SUM9/SUM7
      E1=SQRT(2.0*E**2)
      SUM16=E1/12.0
    7 WRITE (3,250) ES11
  250 FORMAT (F6.4)
      WRITE(3,150) SUM1,SUM2,SUM3,SUM4,SUM5,SUM6,SUM7,SUM8,SUM9
  150 FORMAT (//2X,9(F10.2,3X))
      WRITE (3,151) SUM10,SUM11,SUM12,SUM13,SUM14,SUM15,SUM16
  151 FORMAT (//2X,7(F14.6,2X))
 1000 CONTINUE
C
C
C     CHANGE THE VALUE OF MAXIMAM STRAIN
C
C
C
      ES11=ES11-0.0002
      IF (PHI.EQ.0) GO TO 66
  500 CONTINUE
      STOP
      END
```

```
C
C
C         CALCULATING THE RELATIONS BETWEEN FORCES.MOMENTS.STRAINS.
C           ECCENTRICITIES AND THE LOCATION OF THE NEUTRAL AXIS
C
C
C
C                          CORNER UP
C
C
C
C                   FC=4500 PSI.FS=45000 PSI.P=2X
C
C
C
C               ES11=MAXIMAM STRAIN VALUE
C               ES21.ES22.ES12=SRAIN VALUES AT CORNERS
C               EFSC=STRAIN VALUE AT THE CENTER OF A CONCRETE SQUARE
C               EESS=STRAIN VALUE AT THE CENTER OF A STEEL SQUARE
C               PPSC=STRESS VALUE AT THE CENTER OF A CONCRETE SQUARE
C               PPSS=STRESS VALUE AT THE CENTER OF A STEEL SQUARE
C               RMXT=TOTAL MOMENT ABOUT THE X AXIS
C               RMYT=TOTAL MOMENT ABOUT THE Y AXIS
C               FT=TOTAL FORCE
C               E=ECCENTRICITY
C               Z=LOCATION OF THE NEUTRAL AXIS
C
C
      DIMENSION ESC(150).ESS(150).PSC(150).PSS(150).ES21(150).ES22(150)
     1AC(15.15).AS(12.15).FC(15,15).FS(15.15).RMXC(15,15).RMYC(15.15).
     2RMXS(15.15).RMYS(15.15).FT(15.15).RMXT(15.15).RMYT(15.15).X(150).
     3Y(150)
C
C
C     READING STRESS STRAIN RELATIONSHIP FOR CONCRETE
C
C
  180 READ (1.18)  K.ESC(K).PSC(K).NEXT
   18 FORMAT ( I5.2F10.5. I6)
      IF (NEXT.EQ.0) GO TO 180
      NESC=K
C
C
C     READING STRESS STRAIN RELATIONSHIP FOR STEEL
C
C
  190 READ (1.33) K.ESS(K).PSS(K).NEXT
   33 FORMAT  (I5.2F12.6.I6)
      IF(NEXT.EQ.0) GO TO 190
      NESS=K
C
C
C     GEOMETERICAL PROPERTIES OF SECTION
C
C
      READ (1.15) (X(I). I=1.12).(Y(J). J=1.12)
   15 FORMAT  (8F8.2)
C
```

```
C
C      MAXIMAM STRAIN
C
C
       ES11=0.0030
C
C
C      READ VALUES OF STRAINS AT THE CORNERS
C
C
C
   66  DO  500  II=1.10
       READ (1.16)  (II.(ES21(L).L=1.6).(ES22(M).M=1.6).PHI)
   16  FORMAT (I5.12F6.5.I3)
       WRITE  (3.38)( ES21(L). L=1.6).(ES22(M).  M=1.6)
   38  FORMAT  (//1H1.3X.12F8.5)
C
C
C      ZERO VALUES OF FORCES AND MOMENTS
C
C
C
       DO 1000    L=1.6
       DO  1000     M=1.6
       SUM1=0.0
       SUM2=0.0
       SUM3=0.0
       SUM4=0.0
       SUM5=0.0
       SUM6=0.0
       SUM7=0.0
       SUM8=0.0
       SUM9=0.0
       DO 2000   I=1.12
       DO  2000   J=1.12
C
C
C      IDENTIFY THE LOCATION OF REINFORCEMENT
C
C
C
       AC(I.J)=1.0
       AS(I.J)=0.0
       IF(X(I).EQ.-4.5.AND.Y(J).EQ.-4.5)GO TO 17
       GO TO 27
   17  AC(I.J)=0.0
       AS(I.J)=-1.0
   27  IF(X(I).EQ.-4.5.AND.Y(J).EQ. 4.5)  GO TO 37
       GO TO 47
   37  AC(I.J)=0.0
       AS(I.J)=-1.0
   47  IF(X(I).EQ. 4.5.AND.Y(J).EQ. 4.5)GO TO 57
       GO TO 67
   57  AC(I.J)=0.0
       AS(I.J)= 1.0
   67  IF(X(I).EQ. 4.5.AND.Y(J).EQ.-4.5)  GO TO 77
       GO TO 87
   77  AC(I.J)=0.0
       AS(I.J)=-1.0
   87  CONTINUE
C
```

```
C
C     CLCULATION OF STRAINS
C
C
      ES12=FS22(M)-ES21(L)-ES11
      ESZ=ES12+(ES22(M)-ES12)*(6.0-X(I))/12.0
      XX=ES11*12.0/(ES11+ES21(L))
      IF   ((12.0-XX).LT.(6.0+X(I)))   GO  TO  3
      GO  TO  31
    3 ESZZ=ES11*((6.0+X(I))-(12.0-XX))/XX
      Z=ESZZ*12.0/(ESZ+ESZZ)
      IF   ((12.0-Z).LT.(6.0+Y(J)))   GO  TO  97
      GO  TO  98
   97 EESC=ESZZ*((6.0+Y(J))-(12.0-Z))/Z
      EESS=ESZZ*((6.0+Y(J))-(12.0-Z))/Z
      GO TO 5
   98 EESC=0.0
      EESS=((12.0-Z)-(6.0+Y(J)))*ESZ/(12.0-Z)
      GO TO 5
   31 ESZZ=ES21(L)*((12.0-XX)-(6.0+X(I)))/(12.0-XX)
      EESC=0.0
      EESS=ESZZ+(ESZ-ESZZ)*(6.0-Y(J))/12.0
    5 CONTINUE
C
C
C     CALCULATION OF STRESSES
C
C
      IF   (AC(I,J).EQ.1.0)   GO   TO   6
      GO   TO   25
    6 S =ABS(EESC   )
      IF  (S.EQ.0.0)   GO TO 89
      K=1
   11 IF(ESC(K)-S)1,2,2
    1 K=K+1
      GO  TO 11
    2 PPSC=PSC(K-1)+(S-ESC(K-1))*(PSC(K)-PSC(K-1))/(ESC(K)-ESC(K-1))
      GO   TO   133
   89 PPSC   =0.0
  133 PPSS=0.0
      GO   TO   109
   25 SS=ABS(EESS   )
      IF   (SS.GE.0.00150)   GO  TO   99
      K=1
  110 IF (ESS(K)-SS)19,20,20
   19 K=K+1
      GO TO 110
   20 PPSS=PSS(K-1)+(SS-ESS(K-1))*(PSS(K)-PSS(K-1))/(ESS(K)-ESS(K-1))
      GO   TO   134
   99 PPSS=32400.0
  134 PPSC=0.0
  109 FC(I,J)=PPSC   *AC(I,J)
C
C
C     SUMMING UP FORCES AND MOMENTS
C
C
      SUM1=SUM1+FC(I,J)
```

```
      FS(I,J)=PPSS   *AS(I,J)
      SUM2=SUM2+FS(I,J)
      RMXC(I,J)=FC(I,J)*Y(J)
      SUM3=SUM3+RMXC(I,J)
      RMYC(I,J)=FC(I,J)*X(I)
      SUM4=SUM4+RMYC(I,J)
      RMXS(I,J)=FS(I,J)*Y(J)
      SUM5=SUM5+RMXS(I,J)
      RMYS(I,J)=FS(I,J)*X(I)
      SUM6=SUM6+RMYS(I,J)
      FT(I,J)=FC(I,J)+FS(I,J)
      SUM7=SUM7+FT(I,J)
      RMXT(I,J)=RMXC(I,J)+RMXS(I,J)
      SUM8=SUM8+RMXT(I,J)
      RMYT(I,J)=RMYC(I,J)+RMYS(I,J)
      SUM9=SUM9+RMYT(I,J)
 2000 CONTINUE
C
C
C     NONDIMENTIONING
C
C
      SUM10=SUM7/650.0
      SUM11=SUM8/7800.0
      SUM12=SUM9/7800.0
      SUM13=SUM11/SUM12
      SUM14=SUM12/SUM11
C
C
C     CALCULATE THE ECCENTRICITY AND LOCATE THE NEUTRAL AXIS
C
C
      Z1=ES11*12.0/(ES11+ES21(L))
      SUM15=Z1/(1.414*12.0)
      E=SUM9/SUM7
      E1=SQRT(2.0*E**2)
      SUM16=E1/12.0
      IF  ((SUM9/SUM8).GT.(0.97).AND.(SUM9/SUM8).LT.(1.03))  GO  TO  7
      GO  TO  1000
    7 WRITE (3,250) ES11
  250 FORMAT  (F6.4)
      WRITE(3,150)  SUM1,SUM2,SUM3,SUM4,SUM5,SUM6,SUM7,SUM8,SUM9
  150 FORMAT  (//2X,9(F10.2,3X))
      WRITE(3,151)   SUM10,SUM11,SUM12,SUM13,SUM14,SUM15,SUM16
  151 FORMAT   (//2X,7(F14.6,2X))
 1000 CONTINUE
C
C
C     CHANGE THE VALUE OF MAXIMAM STRAIN
C
C
      ES11=ES11-0.0002
      IF (PHI.EQ.0)   GO   TO  66
  500 CONTINUE
      STOP
      END
```

```
C
C
C
C      CALCULATING THE BALANCING MOMENTS FOR DIFFERENT CONCRETE STRENGTH.
C
C      STEEL YEILD STRENGTH,PERCENTAGE OF REINFORCEMENT AND COVER RATIOS
C
C
C
C              CC=POSSION OF THE NEUTRAL AXIS
C              A=DEPTH OF THE CONCRETE BLOCK
C              T=STEEL TENSILT FORCE
C              C1=STEEL COMPRESSION FORCE
C              C=CONCRETE COMPRESSION FORCE
C              FMB=BALANCING MOMENTS
C
C
C
C
       DIMENSION FC(100),FY(100),AS(100),G(100)
C
C      READ CONCRETE STRENGTH
C
       READ (1,10) (FC(I),I=1,3)
    10 FORMAT (3F7.1)
C
C      READ STEEL YEILD STRENGTH
C
       READ (1,20) (FY(J),J=1,4)
    20 FORMAT (4F6.1)
C
C      READ PERCENTAGE OF REINFORCEMENT
C
       READ (1,30) (AS(K),K=1,8)
    30 FORMAT (8F6.3)
C
C      READ COVER RATIO
C
       READ (1,40) (G(L),L=1,7)
    40 FORMAT (7F6.3)
C
       DO 15 I=1,3
       DO 16 J=1,4
       DO 17 K=1,8
       WRITE (3,99) FC(I),FY(J),AS(K)
    99 FORMAT (1H0,3F14.6)
       DO 18 L=1,7
C
C      CALCULATE THE POSSION OF THE NEUTRAL AXIS
C
       CC=(87000.0/(87000.0+FY(J)))*(10.0-((10.0-G(L)*10.0)/2.0))
C
C      CALCULATE THE DEPTH OF THE CONCRETE BLOCK
C
       IF   (FC(I).GT.4000.0)   GO  TO  22
       A=0.85*CC
       GO  TO  23
    22 A=0.80*CC
C
```

```
C      CALCULATE STEEL TENSILE STRAIN AND STEEL TENSILE FORCE
C
   23 H=10.0-CC
      HH=(0.003*H)/CC
      Z=H-((10.0-G(L)*10.0)/2.0)
      ZZ=HH*Z/H
      IF(ZZ.LT.FY(J)/29*10**6) GO TO 8
      GO TO 9
    8 FYT=ZZ*29*10**6
      T=2.0*AS(K)*FYT
      GO TO 11
    9 T=2.0*AS(K)*FY(J)
C
C      CALCULATE STEEL COMPRESSION STRAIN AND STEEL COMPRESSION FORCE
C
   11 W=CC-((10.0-G(L)*10.0)/2.0)
      WW=(0.003*W)/CC
      IF (WW.LT.FY(J)/29*10**6) GO TO 12
      GO TO 13
   12 FYC=WW*29*10**6
      C1=2.0*AS(K)*FYC
      GO TO 14
   13 C1=2.0*AS(K)*FY(J)
C
C      CALCULATE CONCRETE COMPRESSION FORCE
C
   14 C=A*0.85*10.0*FC(I)
C
C      CALCULATE THE BALANCING MOMENTS
C
      FMB=C*(5.0-(A /2.0))+(C1+T)*G(L)*5.0
      FMB=FMB/1000.0
      WRITE (3,909)  FMB
  909 FORMAT (1H0, F15.2)
   18 CONTINUE
   17 CONTINUE
   16 CONTINUE
   15 CONTINUE
      STOP
      END
```