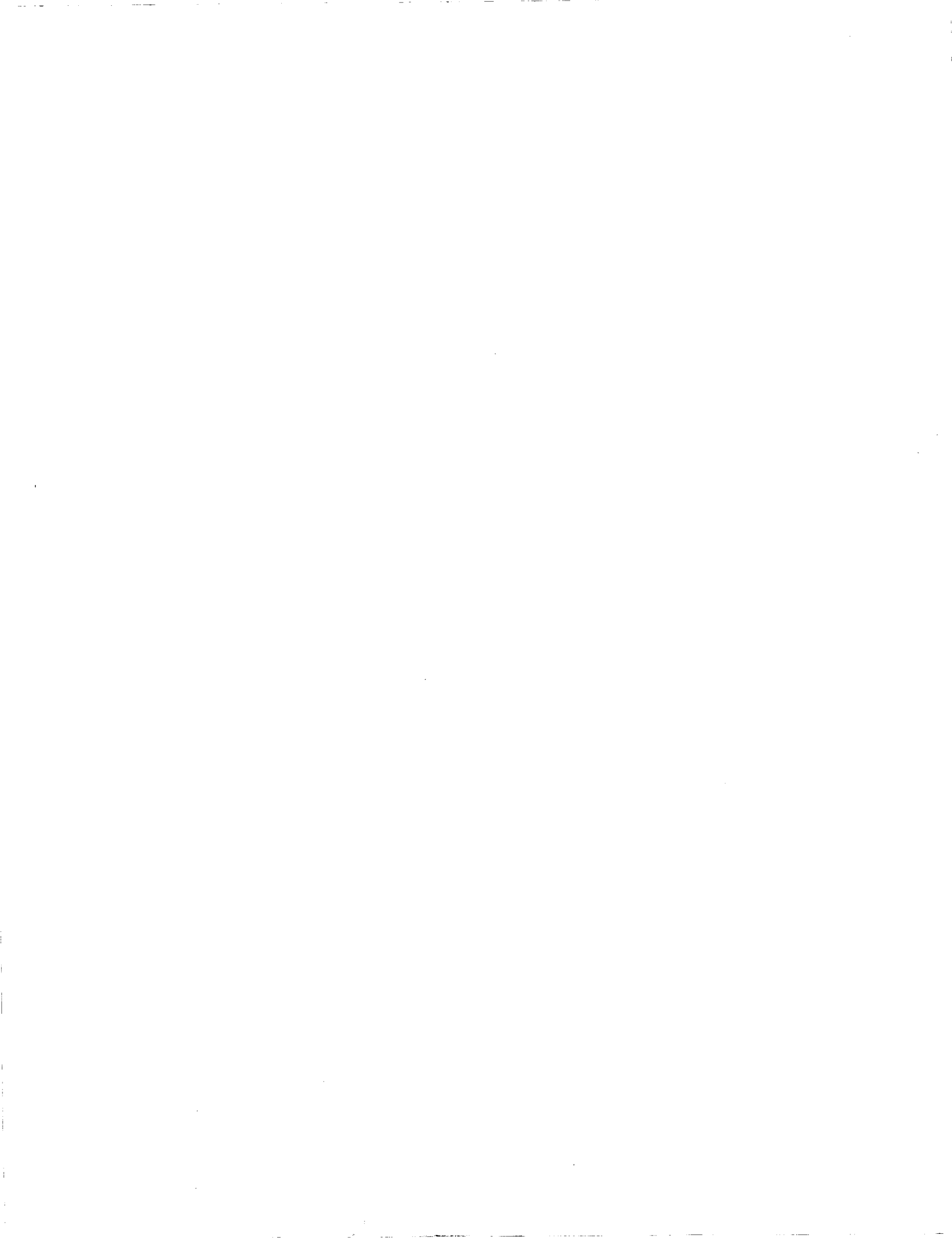


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Secondary School Mathematics Teachers' Views of Manipulative and Their use in the Classroom

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**SECONDARY SCHOOL MATHEMATICS TEACHERS' VIEWS OF
MANIPULATIVES AND THEIR USE IN THE CLASSROOM**

Adrian Jones

**A dissertation submitted in partial fulfillment of the
requirements for the degree of Master of Arts in Education**

**Faculty of Education
University of Ottawa
February 2010**

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ABSTRACT

Mathematical manipulative materials (manipulatives) invite students to explore and represent abstract mathematical concepts in varied, concrete, tactile, and visually rich ways. Considering the prominence of the use of mathematical manipulatives in current K-12 curricula, pedagogical resources and professional development, research studies show that few secondary school teachers use them. While these studies do not examine this issue from the teachers' perspective, they posit that some teachers lack the mathematical knowledge connected to manipulatives, are uncomfortable with or uncertain how to use them, or do not believe that manipulatives have value in the teaching of secondary school mathematics. As a result there is a great need for research that provides further detail as to why and how secondary school mathematics teachers use manipulatives in their classrooms. This study, guided by the research questions: 'How do secondary school teachers view the use of manipulatives in teaching mathematics?' and 'How do secondary school teachers describe their use of manipulatives in teaching mathematics?', sought to examine these issues through semi-structured interviews with six secondary school mathematics teachers. This study supports the notion that the use of manipulatives in secondary school mathematics classrooms is influenced by teachers' views and experience with manipulatives. It highlights some of the challenges that teachers face, and supports from which they gain confidence and competence in their efforts to integrate the use of mathematical manipulatives into their teaching practice.

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CHAPTER 1: INTRODUCTION

Overview

This is a study that examines secondary school mathematics teachers' views and use of mathematical manipulatives as tools to support their students' understanding of abstract concepts. Mathematical manipulatives (and, in keeping with other researchers in the field (e.g. Howard, Perry & Tracey, 1997), used synonymously with 'concrete materials') refer to "concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students" (Hynes, 1986, p. 11). Considerable research conducted over many decades supports the value of using manipulatives in the mathematics classroom as concrete models of abstract mathematical concepts. Influenced by such research and current theories of how students learn, current curricula (often referred to as reform curricula), pedagogical resources, and teachers' professional development promotes the use of manipulatives to develop and strengthen students' understanding of mathematical concepts (e.g.: NCTM, 2000a; OME, 2005a, 2005b). The National Council of Teachers of Mathematics (NCTM, 1989) challenged teachers to revamp the way in which mathematics is taught and shift their teaching role from one of dispenser of knowledge to facilitator of learning and call for increased emphasis on creating meaningful experiences for students. They specifically recommend that to provide "... students with a lasting sense of number and number relationships, learning should be grounded in experience related to aspects of everyday life or to the use of concrete materials designed to reflect underlying mathematical ideas" (p. 87) and encourage a shift in emphasis:

...from a curriculum dominated by memorization of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understandings, multiple representations and connections, mathematical modeling, and mathematical problem solving. (NCTM, 1989, p. 125)

While research demonstrates the value of using concrete materials in the teaching and learning of mathematics, several studies have noted a sharp decline in the use of manipulatives as students progress through the grades, to the point where there is little use of manipulatives in secondary school mathematics classes (e.g.: Perry & Howard, 1997; Weiss, 1994). Reasons offered for this are that some teachers lack the mathematical knowledge connected to manipulatives (Moyer, 2001), are uncomfortable with or uncertain as to how to use them (Suurtamm & Graves, 2007; Tooke, Hyatt, Leigh, Snyder, & Borda, 1992), or do not believe that manipulatives have value in the teaching of secondary school mathematics (Sherin, Mendez, & Louis, 2004). However these suggestions need further investigation and elaboration in order to develop a thorough understanding of the situation. As a result there is a great need for research that provides further detail as to the reasons why secondary school mathematics teachers choose or choose not to use manipulatives and how they are actually used in their classrooms.

Current theories of how students learn posit that mathematical understanding is enhanced by connecting and translating within and between multiple representations of mathematical concepts (Clements, 1999; Goldin, 1987, 2003; Lesh, Post, & Behr, 1987). Such mathematical representations consist of both external conventional representations (concrete models, pictures, symbols, language, and real-world situations (see Lesh, Landau, & Hamilton, 1980)), and the students' own internal mental models. The interactions with, and connections between the students' mental models, their production of external representations, and their talk during mathematical learning and problem solving, are seen as fundamental for effective teaching and learning in mathematics (Goldin & Shteingold, 2001; Lesh, et al., 1987). This suggests that a rich learning experience for students involves the exploration and investigations of mathematical concepts by using and connecting many different representational forms. Indeed, we see that theories of multiple representation, and specifically the role of manipulatives as concrete representations, and current theories of learning,

are two (of many) areas that influence current curriculum, pedagogical resources and the content of much professional development.

As concrete models, manipulatives are considered important teaching tools with which students can deepen their understanding of mathematical concepts (NCTM, 1989). While much of the early research on the use of manipulatives is somewhat inconclusive in regard to benefits to students' academic achievement (e.g. Fennema, 1972; Friedman, 1978), Suydam and Higgins (1977) suggest that early studies merely looked for the presence of manipulatives in the classroom and not how and why teachers used them in the classroom. Indeed, they found that using manipulatives produced greater academic achievement in students than not using them across every grade and ability level in elementary (K-8) school. Sowell's (1989) meta analysis of 60 prior studies on the use of manipulatives found similar improvements in academic achievement but only when mediated by teachers knowledgeable in their use. More recent studies (e.g.: Cramer, Post, & delMas, 2002; Moyer & Jones, 2004; Stein & Bovalino, 2001) focused on how teachers actually use manipulatives in the context of teaching mathematics in the classroom and how they are used in relation to students' outcomes. Moyer and Jones (2004) concluded their study by saying:

By demonstrating how to use the manipulatives as tools for better understanding, teachers open doors for many students who struggle with abstract symbols. Often this struggle can be minimized or avoided entirely by simply using different representations before using abstract symbols alone, thus giving students a firm conceptual base on which to build higher mathematical thinking. (p. 29)

These recent studies recognize the critical importance of the mediating role of the teacher and reinforce the need for teachers to introduce and use manipulatives with careful thought and with strong connections to underlying mathematical concepts so that students understand relations between the actions with the manipulatives and other forms of mathematical representations, such as symbols, graphs, and tables (Uttal, Scudder, & DeLoache, 1997). Conversely, teachers may not be effective if they lack the mathematical knowledge to use manipulatives to represent concepts as

concrete experiences for their students (Moyer, 2001). Similarly, if teachers do not help students to connect their experiences using manipulatives with the mathematical concepts and alternative forms of representation under investigation, the results may be counter-productive in that students are forced to learn two separate and disconnected systems (Boulton-Lewis, 1998).

Yet while the use of manipulatives is encouraged and strongly recommended by current curricula (NCTM, 2000a; OME, 2005a, 2005b) and supporting materials, many teachers do not use them. Studies into teachers' practice show that their actions are influenced by their views about, and knowledge of, mathematics and its teaching and learning (Ernest, 1989b; Thompson, 1984), and their understanding of students as learners (van der Sandt, 2007). As teachers translate their knowledge of mathematics and pedagogy into practice through the filter of their beliefs (Manouchehri, 1997), their views about manipulatives are also situated in these beliefs and will affect the way teachers use them (Sherin, et al., 2004).

Beyond a few quantitative studies reporting the comparatively lower use of manipulatives in secondary school mathematics (e.g.: Perry & Howard, 1997; Suurtamm & Graves, 2007; Weiss, 1994) than in elementary school, no research has been found that details secondary school teachers' views about, and use of manipulatives in their teaching. Given the prominence of mathematical manipulatives in current curricula and activities that encourage their use, it raises the question as to why few secondary school teachers use them.

Addressing the Research Gap

In order to investigate why secondary school mathematics teachers choose or choose not to use manipulatives in their classrooms, this study examines the views and use of manipulatives by secondary school teachers to understand how and why they choose to use them in their teaching. The study was guided by the following primary research questions:

How do secondary school teachers view the use of manipulatives in teaching mathematics?

How do secondary school teachers describe their use of manipulatives in teaching mathematics?

Data was gathered through semi-structured interviews with six secondary school mathematics teachers. An interview guide was developed around the key dimensions of (a) teachers' mathematics background and teaching experience; (b) teachers' views of mathematics, and mathematics teaching and learning; (c) teachers' views of the use of manipulatives for teaching mathematics; and (d) teachers' descriptions of their use of manipulatives in the classroom. The discussions that follow elaborate on the analysis of these key dimensions.

Personal Significance

I was first introduced to the use of mathematical manipulatives during my teacher education program in 2006. The value of manipulatives in developing and strengthening understanding of a wide range of mathematical concepts has been of particular interest to me ever since. The many "aha" moments that I experienced using them encouraged me to further understand their usefulness while facilitating problem-solving activities with pre-service teachers and my own children, and the overwhelmingly positive results continue to surprise. In particular, my children found great benefit in connecting the manipulation of algebra tiles to symbolic algebraic operations such as simplifying like terms, multiplying binomials and factoring quadratics. I have found that the use of manipulatives helps make concrete and meaningful connections to abstract mathematical concepts, and serve to shift the locus of understanding (and power) from the teacher and the textbook to the student as knowledge-creator. My own and my children's positive experiences with manipulatives leaves me wondering why all teachers, especially those engaged in secondary school mathematics, are not using them for their students' and their own benefit.

Thesis Overview

In order to provide a ‘road-map’ for the reader, the following structure is employed in this thesis. Chapter 2 offers an overview of the conceptual framework that situates and provides structure to consider the many influences on teachers’ views and use of manipulatives. In Chapter 3, literature related to the benefits of and challenges to the use of manipulative is reviewed. Chapter 4 outlines the methodology used in this study. Chapter 5 provides a description of each participant based on the analysis of their background and teaching experiences with mathematics in order to contextualize their views and use of manipulatives. In Chapter 6, analysis is provided across the participants to specifically address the two research questions that deal with the participants’ views and use of manipulatives in the classroom. Chapter 7 describes the analysis of two emergent themes that dealt with both the supports that assisted and challenges that hindered the participants in the integration of manipulative use into their teaching practice. The thesis concludes in Chapter 8 with implications for education, a discussion of the contributions and limitations of the study, suggestions for future research directions, and concluding comments about the study.

CHAPTER 2: CONCEPTUAL FRAMEWORK

Conceptual Framework Overview

In order to address the research questions of teachers' views of and use of manipulatives in the classroom, it is necessary to situate and contextualize them in terms of the myriad influences that act on the teacher and shape the teachers' practice. In this chapter, I describe those ideas and theories that consider teachers' views and knowledge of mathematics, and its teaching and learning as influences on the teachers' enactment of the curriculum in the classroom. I have crafted these influences into a conceptual framework, as illustrated in Figure 1, to situate my own perspectives, provide the context within which to consider teachers' views and use of manipulatives and to lay the theoretical structure that integrates the key dimensions used to address the research questions.

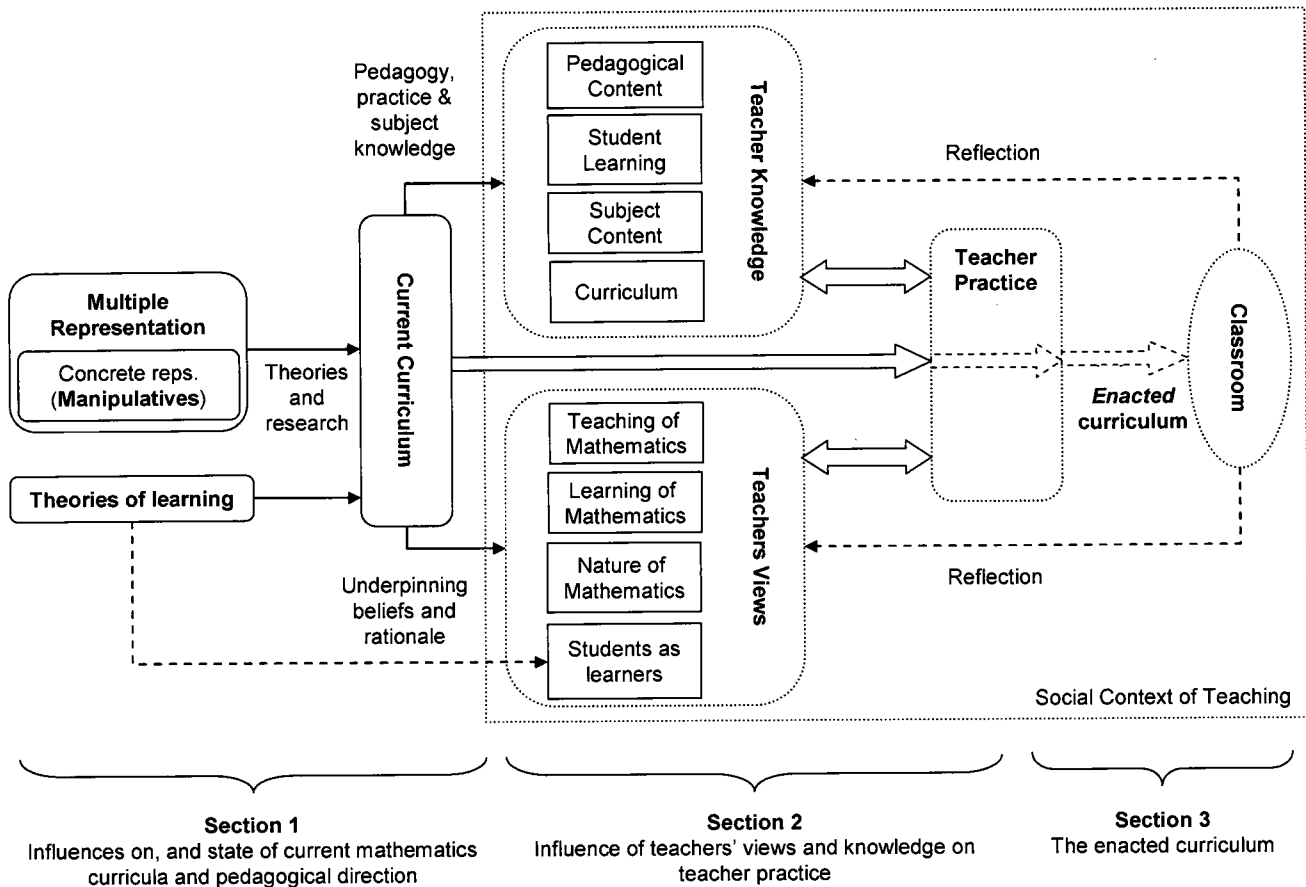
Details of the Conceptual Framework

Many conceptual frameworks have been created by researchers to ground studies that examine the influence of teachers' mathematical knowledge and views on teacher practice (e.g.: Ernest, 1989b; Koehler & Grouws, 1992; Shulman, 1986; van der Sandt, 2007). The conceptual framework developed for this study, shown in Figure 1, is adapted from a research framework proposed by van der Sandt's (2007) to examine teachers' behaviour from the perspective of teachers' knowledge and views. Van de Sandt's model is built upon the work of Koehler and Grouws (1992) who examined progress made in educational research on teaching. Koehler and Grouws presented four levels of complexity, each with a corresponding model, to reflect the changes made and progress in research on teaching, the highest level of which represented current (at that time) research that examined research questions in teaching and learning from multiple perspectives. They postulated that teachers' actions were influenced by the teacher's knowledge (the content to be taught, how learners learn and understand that content, and methods to teach that content), and teachers' views about teaching and mathematics. To Koehler and Grouws' research framework, van

der Sandt added extra elements. These additions included: teachers' knowledge of curriculum, in keeping with recommendations by Shulman (1986); teachers' views of the learning of mathematics, as posited by Ernest (1989a); and, in keeping with more current research, teachers' knowledge and views of students as learners (Leung, 1995). Consideration was also given for the social context in which teaching occurred, influenced primarily by the work of Fennema and Franke (1992). These changes, presented by van der Sandt (2007) as a more complete framework for research on teacher behaviour, is used as the conceptual framework in this research study.

The framework provides the focus of, and organizational structure for the three main sections of the literature review and the key dimensions of the data collection instrument. The following discussion is organized into three parts, each of which details the sections of the conceptual framework. These sections are (1) some influences on and state of current mathematics curricula and pedagogical direction; (2) the influence of teachers' views and knowledge on teacher practice; and (3) the enacted curriculum in the classroom.

Figure 1: Conceptual Framework



Section 1: Influences on and State of Current Mathematics Curricula and Pedagogical Direction

As shown in Figure 1 – Section 1, theories of multiple representations, and specifically the researched benefits of the use of mathematical manipulatives as concrete representation, are important influences on current curricula development. Current theories of learning (e.g.: constructivist, social constructivist, and socio-cultural) also influence the beliefs that underpin the current reform mathematics curriculum content, its pedagogical direction, and the nature and focus of classroom activities (NCTM, 2000a). This curriculum provides the teacher with an important source of pedagogical and subject content knowledge that is founded on and assumes a set of pedagogical beliefs about the nature of mathematics, its teaching and learning, and about students as learners (e.g.: NCTM, 2000a; OME 2005a, 2005b).

The following discussions consider two (of many) influences on current mathematics curricula and pedagogical direction: current theories of learning, and theories of multiple representation in mathematics, within which is detailed the specific research into the use of mathematical manipulatives as concrete representations.

Current Theories of Learning

Current (cognitive) theories of learning posit students as active participants who construct knowledge by reorganizing their current ways of knowing through personal experience and participation in specific sociocultural practices. Piaget's (1952) theory of cognitive development explains, among other things, how children progressively enrich (construct) their understanding of things by interaction with objects and materials, reflecting on their own prior experiences and previous knowledge, and organizing new and existing knowledge in increasingly complex and interconnected structures. This constructivist view posits the learning of mathematics as a complex interaction of existing knowledge, views, skills, experiences, challenges and opportunities for resolution, mediated by the teacher (Davis, Maher, & Noddings, 1990). Vygotsky's (1978) sociocultural theories of development of cognition emphasize the fundamental role of culture, cultural artifacts and social interactions in learning. Through social interaction, the child learns the habits and tools of the culture, including speech patterns, written language, and other symbolic knowledge through which (cultural mediation) the child derives meaning and constructs knowledge. Cobb (1994) convincingly argues that these two theories can be considered complementary in that "sociocultural perspective informs theories of the conditions for the possibility of learning, whereas theories developed from the constructivist perspective focus on what students learn and the processes by which they do so" (p. 13). This view of students as active knowledge-constructing participants within a classroom mathematical learning community underpins important aspects of current

mathematics curricular development, educational pedagogy and classroom practice (e.g.: NCTM, 2000a, 2000b).

Theories of Multiple Representation in Mathematics Learning

While there are many ways to describe the discipline of mathematics, Post's (1981) description of mathematics as a discipline whose practitioners represent the real world using concrete and symbolic models is useful when considering theories of representation in mathematics. Such representation can be any configuration of characters, images, concrete objects, etc., that can be made to denote, symbolize, or otherwise "represent" something else (Kaput, 1987; Goldin, 1987; Goldin & Shteingold, 2001). For instance, the fractional notation " $1/4$ " could refer to, or represent, a diagram of a circle in which one of four equal divisions is coloured. Similarly such a diagram could itself represent the fraction " $1/4$ ". Such representations belong to representational systems, defined by conventions for forming representational configurations, and structured such that different representations within a system are richly related to one another (Goldin & Shteingold, 2001). Base-ten Hindu-Arabic numeration, fractional notation, and Cartesian graphs are examples of culturally created and conventionally established mathematical representational systems (DeWindt-King & Goldin, 2003). A major goal of mathematics education is to cumulatively develop and broaden students' mathematical understanding based on the capacity to translate and make connections between ever-enriching sets of representation systems (Janvier, 1987; Kaput, Noss, & Hoyles, 2002): "When students gain access to mathematical representations and the ideas they represent, they have a set of tools that significantly expand their capacity to think mathematically" (NCTM, 2000a, p. 66).

Bruner's (1966) theory of discovery learning proposed that mathematical concepts can be represented by, and that learners need experiences in, three modes of representation: enactive (concrete action-based representation), iconic (pictorial representation), and symbolic (written symbols). Rousseau, Bates and Beattie (1985) considered a fourth mode – that of language

representations – which are the accepted oral and written vocabulary that has been given specialized meaning. To this, Lesh, Landau, and Hamilton (1980) added representations of experience-based real-world situations, and considered this expanded set of five different modes (concrete¹, pictorial, symbolic, language, and real-world) to be interconnected mesh-like in a non-hierarchical manner. These and other learning theorists have advocated for mathematical manipulatives, functioning as concrete representations, to be used as valuable tools to help students establish an understanding of mathematics (e.g.: Dienes, 1960; Bruner, 1966; Rousseau et al., 1985; Lesh, et al., 1980, 1987).

Diagrams, drawings, notes, equations, tables, Cartesian graphs, to name a few, have long been part of school mathematics formal (external) representations that cut across multiple mathematical concepts and levels of complexity. These provide powerful ways to unify students' experiences (Greeno & Hall, 1997). The NCTM's Principles and Standards for School Mathematics (2000a) emphasized the importance for students to develop their own idiosyncratic (internal) representations and connect them coherently with these conventionally established mathematical representations (Dufour-Janvier, Berdnarz, & Belanger, 1987; Goldin & Shteingold, 2001; Lesh, et al., 1987). It is within the translation process between external and internal representations, that a student's mathematical power (knowledge) arises (Goldin, 2003; Lesh, et al., 1987; Pimm, 1995; Smith, 2003). Studies by Hiebert and Carpenter (1992) showed that students who construct strong connections between mathematical representations remember new concepts better, find them easier to recall, more effectively transfer their understanding to new problems and may be more inclined to see mathematics as a cohesive body of highly interconnected knowledge.

Mathematical Manipulatives as Concrete Representations

Manipulatives can be considered a subset of myriad concrete materials that may be used to represent concepts and ideas in the teaching and learning of mathematics. Mathematical

¹ Also referred to as “manipulative models” by the authors in subsequent articles (e.g.: Lesh et al. 1987).

manipulatives may be objects from the real-world environment, such as money, blocks and solid objects, or materials designed to teach specific sets of mathematical concepts, such as algebra tiles, fraction circles, and base10 number blocks (Moyer, 2001). While such manipulatives are designed to be handled by students (Kennedy, 1986), they are not used solely as objects in their own right, but simultaneously as representations of mathematical concepts (Uttal, Scudder, & DeLoache, 1997). They operate as representations with which, rather than from which, students can learn mathematics (Pimm, 1995). As Clements (1999) notes:

Good manipulatives are those that are meaningful to the learner, provide control and flexibility to the learner, have characteristics that mirror, or are consistent with, cognitive and mathematical structures, and assist the learner in making connections between various pieces and types of knowledge (p. 50).

Current Mathematics Curricula and Pedagogical Direction

Current curriculum development, informed in part by constructivist theories of learning and theories and research in mathematical representations, represents a rethink of the teacher's role from a dispenser of knowledge to one of facilitating students' conceptual understanding, and a commensurate shift from a teaching of mathematics as facts, rote, drills, and procedures towards an emphasis on conceptual understandings, multiple representations and connections, mathematical modeling, and mathematical problem solving (NCTM, 1989). NCTM's Principles and Standards for School Mathematics (2000a) reiterated this emphasis in developing students' conceptual understanding: "Well-connected, conceptually grounded ideas are more readily accessed for use in new situations... Conceptual understanding is an essential component of the knowledge needed to deal with novel problems and settings" (p.19).

The Ontario Ministry of Education (OME) mathematics curriculum (OME, 2005a, 2005b), as an example of such reform curriculum, also places strong emphasis on the need for students to construct their own conceptual understandings of mathematical ideas, and the significant role that multiple representations play in effective learning:

When students are able to represent concepts in various ways, they develop flexibility in their thinking about those concepts. They are not inclined to perceive any single representation as “the math”; rather, they understand that it is just one of many representations that help them understand a concept (OME, 2005b, p. 16).

Manipulative use is identified as a catalyst in building representations of mathematical concepts to help students in all grades see patterns and relationships, make connections between different representations, and communicate their reasoning:

Even at the secondary level, manipulatives are necessary tools for supporting the effective learning of mathematics by all students. These concrete learning tools invite students to explore and represent abstract mathematical ideas in varied, concrete, tactile, and visually rich ways. Moreover, using a variety of manipulatives helps deepen and extend students’ understanding of mathematical concepts (OME, 2005b, p. 23).

Multiple representations and the effective use of mathematical manipulatives as concrete representations are believed to play an important role in developing students’ conceptual understanding of mathematics. Specifically speaking about the use of manipulatives, the Ontario mathematics curriculum states:

Students should be encouraged to select and use concrete learning tools to make models of mathematical ideas. Students need to understand that making their own models is a powerful means of building understanding and explaining their thinking to others. Using manipulatives to construct representations helps students to see patterns and relationships; make connections between the concrete and the abstract; test, revise, and confirm their reasoning; remember how they solved a problem; communicate their reasoning to others. (OME, 2005a, p. 15)

From such curriculum documents we gain an active² view of mathematics where students come to understand by *doing* mathematics within a collaborative mathematical learning environment that encourages discourse and the use of tools to advance and enhance their thinking through a multiplicity of rich experiences (NCTM, 2000a; OME, 2005a, 2005b).

² Verbs used in NCTM (2000a) and other reform curricular materials include *explore, communicate, construct, conjecture, investigate*.

Section 2: The Influence of Teachers' Views and Knowledge on Teacher Practice

This section (as shown in Figure 1 – Section 2) discusses research into the influence of mathematics teachers' views (consciously or unconsciously held) about what mathematics *is* and what constitutes mathematical knowledge, on their methods of instruction and delivery (Thompson, 1992a), their views about how students should learn and engage with mathematics (Ernest, 1989b), and the choice of texts and activities – including the role and use of manipulatives (Sherin et al., 2004). Research suggests that teacher practice is not solely influenced by curriculum, but by teachers' knowledge of mathematics, its teaching and learning (Ernest, 1989b; Thompson, 1984), and knowledge of how students think and learn (Ball & Cohen, 1996; NCTM, 2000b). The following discussions elaborate on these two themes of teachers' knowledge and views.

Teachers' Knowledge

Teachers need knowledge of the subject of mathematics (content knowledge) that is identified as “crucial to being inventive in creating worthwhile opportunities for learning that take learners' experiences, interests and needs into account” (Ball, 2000, p. 242). Teachers also need the special knowledge that links content and pedagogy, referred to as pedagogical content knowledge by Shulman (1986). Pedagogical content knowledge allows teachers to represent and formulate mathematical concepts to make them comprehensible to students (Fennema & Franke, 1992), create suitable opportunities for learning (Ball & Bass, 2000) and includes an understanding of students' conceptions, difficulties and common errors (Fennema & Franke, 1992) and an appreciation of the representations most useful for teaching a specific content idea (Shulman, 1986). The value of such knowledge is succinctly characterized by Ball and Bass (2000) as:

Teachers who were accustomed to viewing subject matter from the perspective of its growth and development would be prepared to notice nascent intellectual activity in learners. Such individuals would know subject matter in ways that prepared them to hear and extend students' thinking (p. 85).

The teacher also needs a full knowledge of the curriculum (curricular knowledge) that includes the curricular texts and materials and various schemes to use them (Ernest, 1989b; Shulman, 1986). Brophy (1991) makes a strong connection between teachers' knowledge(s) and its use in classroom practice:

Where teachers' knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways, and encourage and respond fully to students' comments and questions. Where their knowledge is limited, they will tend to depend on the text for content, de-emphasize interactive discourse in favour of seatwork assignments, and in general, portray the subject as a collection of static factual knowledge (p. 352).

In the context of enacting current curriculum, teachers also require knowledge of how to use manipulatives effectively. For many teachers their mathematics knowledge consists only of the school curriculum (that generally privileged symbolic representations) that they themselves were taught (Thompson, 1992b). As many teachers teach as they were taught (Owens, 1993) – which generally did not include the use of manipulatives – the consequence is the privileged position afforded to symbolic representation of mathematical concepts and the notion of mathematics achievement as mastery of symbolic manipulation and procedural fluency. Teachers' conceptions of mathematics are grounded in their understanding of the content of mathematics, which consequently has a significant impact on the mathematical thinking that takes place in the classroom (Ma, 1999). Teachers may need help to develop greater mathematical competency (content knowledge), reflect on their own practices (Thompson, 1984), and learn how to represent the mathematical ideas they wish their students to learn. They also need to understand the strengths and weaknesses of different representations and connections among and across concepts (Ball & Cohen, 1996; Joyner, 1990; NCTM, 2000a). However, studies suggest that some teachers lack the mathematical knowledge connected to manipulatives (Moyer, 2001), are uncertain as to how to use them (Suurtamm & Graves, 2007), or do not believe that manipulatives have value in the teaching of secondary school mathematics (Sherin, et al., 2004).

Teachers' Views

Teachers' views, as defined by Kuhs and Ball (1986), and cited and used by Ernest (1989b), are conceptions, ideologies and values that shape practice and orient knowledge. Research findings suggest that teachers' views about the nature of mathematics and mathematics teaching and learning have a significant, if not always conscious, impact on teaching practice (Cooney, 1999; Ernest, 1989a, 1989b; Manouchehri, 1997; Pajares, 1992) and significantly influence their interpretation and implementation of curriculum (Thompson, 1992a). While the directionality of influence between views and instructional practices is debatable, Cobb, Wood and Yackel (1990) suggest that they are dialectically related in that "beliefs are expressed in practice, and problems or surprises encountered in practice give rise to opportunities to reorganize beliefs" (p. 145).

At the classroom level, teacher views are influenced by prior school experiences, including experiences as a mathematics student, the influence of prior teachers, teacher preparation programs, and prior teaching episodes (Raymond, 1997). As well, the body of research on teachers' beliefs show these beliefs to be highly robust (Pajares, 1992), resistant to change (Kagan, 1992), serve as filters for new knowledge (Manouchehri, 1997) and – acting as barriers to changes in teaching practices – influence the success of curricula reforms (Buzeika, 1996; Hollingworth, 1989; Pajares, 1992; Ross, McDougall, & Hogaboam-Gray, 2002).

Thompson's (1984) case studies of three junior secondary school mathematics teachers found them to possess varying levels of consistency between their professed beliefs, views and preferences about mathematics and its teaching, and their classroom practices. She identified several factors that appeared to account for these differences that included the teacher's level of awareness of the interrelationship between beliefs and practice, and the teacher's level of reflectiveness – the tendency to think critically about their actions in relation to their beliefs, their students, and the subject matter. Buzeika (1996) found that a mathematics teacher's beliefs would be more likely to change if a new

teaching practice resulted in a successful outcome and the teacher reflected upon this new practice. Teachers' critical reflection on their practice can serve to challenge personal beliefs and established practices in the face of changes to curriculum and new pedagogical directions:

A practitioner's reflection can serve as a corrective to over-learning. Through reflection, he can surface and criticize the tacit understandings that have grown up around the repetitive experiences of a specialized practice, and can make new sense of the situations of uncertainty or uniqueness which he may allow himself to experience. (Schön, 1983, p. 61)

Section 3: The Enacted Curriculum

This section, as illustrated in Figure 1 – Section 3, considers how the current mathematics curriculum is actually enacted by teachers through the mediation of the 'filter' of their knowledge and views (Manouchehri, 1997). Research shows that changes to curricular and pedagogical direction alone are unlikely to get teachers to rethink their own views and knowledge of mathematics and readily adopt new teaching practices (Andrews & Hatch, 1999; Franke, Kazemi, & Battey; 2007; Hoyles, 1992; Ross et al., 2002). Cohen and Ball (1990) stated that the adoption of curricular reforms:

...signalled the need for a revolution in most teachers' knowledge of mathematics... [and]... invited basic changes in teachers' beliefs about mathematics and in their beliefs about how students learn mathematics. Additionally, the policy [curricular reforms] called for change in how teachers thought about their role and how they conducted their classes. (p. 237)

Similarly, teachers' views about manipulatives are situated in their own assumptions and experiences with mathematics education. This affects the way they use new materials (Sherin, et al., 2004). As Lappan and Theule-Lubienski (1994) suggest, the teacher dictates and determines the curriculum enacted in the classroom:

There is no other decision that a teacher makes that has a greater impact on students' opportunity to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages the students in studying mathematics. Here the teacher is the architect, the designer of the curriculum. (p. 250)

In addition, the teachers' critical reflection on their practice is identified as a key component in harmonizing teachers' views and knowledge with their practice (Thompson, 1984).

Conceptual Framework Summary

The conceptual framework provides a structure with which to contextualize the underlying principles on which the curriculum is built, and the influence of teachers' knowledge and views on teacher practice in enacting the curriculum in the classroom. Using the framework as a guide, I have shown how the curriculum promotes the use of mathematical manipulatives as concrete representations and encourages their use as tools to help develop students' conceptual understanding of mathematics (Section 1). I have also shown how teachers' knowledge (subject content, pedagogical content, curricular content and students as learners), and teachers' views (of the teaching and learning of mathematics, the nature of mathematics and students as learners) influence teachers' practice (Section 2) that results in the *enacted* curriculum (Section 3) that takes place in the classroom.

The framework thus provides the theoretical structure to ground the key dimensions used to address the research questions and shows that in order to examine the reasons why and how teachers use manipulatives, it is necessary to examine the teachers' views and knowledge of mathematics, its teaching and learning, and their understanding of the ways in which students learn.

CHAPTER 3: REVIEW OF THE LITERATURE

Literature Review Overview

In this chapter, I first describe studies that demonstrate accrued benefits to students in the use of manipulatives as tools in the teaching and learning of mathematics. Next, I present literature that describes challenges that teachers face in implementing reform curriculum that encourages the integration of manipulatives into instruction delivery. I then discuss the findings of studies that examine the actual use of manipulatives in elementary, intermediate and secondary schools. This is followed with an in-depth discussion of four studies that have influenced and offered insight into the questions posed in this research study. I conclude the chapter by summarizing the main themes found in the related literature, and outline how my study connects to and builds upon this research literature.

Research into Manipulative Use in the Classroom

More than 100 studies have been conducted in the last 30 years on the effectiveness of manipulatives to teach a variety of mathematics topics. Research on mathematical instruction involving manipulatives prior to the mid-1980s appears to yield inconclusive results as to their benefit to the academic achievement of students (Fennema, 1972; Hiebert & Wearne, 1992; Resnick & Omanson, 1987; Suydam & Higgins, 1977). However, many researchers concur that, of the more recent studies conducted, the use of manipulatives is found to be worthwhile, and that students who use manipulatives generally outperform those who do not (e.g.: Erickson & Niess, 1996; Raphael & Wahlstrom, 1989; Sowell, 1989). Long-term use of manipulatives is related to increases in mathematics academic achievement (Cain-Caston, 1996; Cramer, Post, & delMas, 2002; Ernest, 1994), improved attitudes towards mathematics (Leinenbach & Raymond, 1996), and is of value to students in all school grades (Clements, 1999; LeNoir, 1989; Perry & Howard, 1997). However, the research also shows that these improvements require that the choice of manipulatives make sense for

the topic (Raphael & Wahlstrom, 1989), and that their use be facilitated by teachers knowledgeable in their use (Sowell, 1989), two elements potentially absent from earlier studies. Research also points to the importance for students to gain familiarity with manipulatives for them to be able to see them as viable and valuable learning tools that are well connected to the mathematical concepts under investigation. As Boulton-Lewis (1998) observed, “if children are to use structured concrete representations... effectively, without increasing their processing load, they must know the materials so well that they can use them automatically” (p. 234). As Hynes (1986) noted, teachers should use manipulatives in mathematics instruction regularly in order to give students hands-on experience to help them construct useful meanings for the mathematical ideas they are learning. Long term use of the same manipulatives to teach multiple ideas can help students identify connections between mathematical ideas and familiarity with the manipulatives may lessen the amount of time taken to introduce new concepts.

Manipulatives in Mathematics Teaching and Learning

While research shows that manipulatives have an important place in learning, their physicality does not carry the meaning of the mathematical concept (Clements, 1999), and Ball (1992) cautioned against assuming that students will automatically see what their teachers expect simply by using the manipulatives: “Although kinesthetic experience can enhance perception and thinking, understanding does not travel through the fingertips and up the arm... Mathematical ideas really do not reside in cardboard and plastic materials.” (p. 47). However, the mere presence of manipulatives is insufficient to guarantee meaningful learning (Baroody, 1989; Fennema, 1973; Thompson & Lambdin, 1994), and seeing mathematical ideas in the manipulatives may be challenging (Thompson, 1984) – while the material may be concrete, the idea or concept that the student is intended to see is not *in* the material. For students to connect their use of manipulatives to alternative representations of mathematical concepts, “mathematical relationships must be imposed

on the materials” (Moyer, 2001, p. 192). While making these connections explicit is the role of the teacher, it is the way in which the teacher understands the manipulatives and actions with them that reveals to the students the underlying mathematical concepts being explored. By acting on manipulatives, reflecting on those actions, and connecting the actions to the mathematical concepts being developed, the student develops a repertoire of images that make their mental constructs more clear (Kamii, Lewis, & Kirkland, 2001). As Moyer (2001) notes:

The understanding and value of particular manipulatives becomes known to users in the process of using them within shared environments... It is the mediation by students and teachers in shared and meaningful practices that determines the utility of the manipulatives. (Moyer, 2001, p. 177)

Teachers can also gain insight into students’ cognitive representations by observing their actions on manipulatives (NCTM, 2000a), and by using them as public objects of discussion (conjecture, explanation, evaluation, and argument) by students and the entire class (Hall, 1998; Hiebert & Carpenter, 1992; Kalathil & Sherin, 2000). An additional benefit is that when students use manipulatives, actions performed on them are easier for them to describe than similar operations on symbols (Meira, 1998).

Challenges to Manipulatives Use with Curricula Changes

While some teachers either fail to take up reforms or actively resist pedagogical innovations (Buzeika, 1996), others may make surface changes to their teaching (Cohen & Ball, 1990), or adopt more easily imported practices, such as the use of manipulatives in mathematics teaching in the elementary school grades, although while these practices may have become part of teachers’ repertoires, the full understanding of what the practices mean has not (Windschitl, 2002). As Ball (1992) claims, “Teaching with manipulatives is not just a matter of pedagogical strategy and technique” (p. 47) but requires a significant reframing of instructional practice. Moyer’s (1998) study of intermediate level mathematics teachers found that with the use of manipulatives comes the

mandate to change the delivery of instruction, which in turn requires changes to teachers' knowledge and views about manipulatives. She concluded that:

The data suggests that manipulative materials do have the potential to mediate teachers' instructional practices, encouraging paradigm shifts as teachers make accommodations in their teaching to implement manipulatives as tools for mathematical learning. Manipulatives also have the potential to change student attitudes and motivations... (p. 153)

However, Moyer also noted that teachers who view manipulatives as time wasting and secondary to learning mathematics trivialize their value, encouraging their students to also consider manipulatives for play, rather than as effective tools for 'real' mathematical learning and understanding. Moyer (2001) found that manipulative use was little more than a diversion in classrooms where teachers were unable to represent mathematics concepts themselves. In a similar manner, Stigler and Hiebert (1999) noted that if "teachers act as if student interest will be generated only by diversions outside of mathematics" (p. 89) they send a message to students that explorations and conjectures with representations are not connected to real mathematical learning. Highlighting the importance of teachers having depth, breadth and thoroughness in their understanding of mathematics, Beattie (1986) concluded that "teachers with deep, vast and thorough understanding of mathematical concepts do not invent connections between and among mathematical ideas, but reveal and represent them in terms of mathematics teaching and learning" (p. 24).

Manipulative Use in Secondary School Mathematics Classrooms

While research shows that students of all ages and all levels of mathematical achievement can benefit from experiences with manipulatives (e.g.: LeNoir, 1989; Suydam & Higgins, 1977), and current curriculum strongly encourage their use, many studies report declining manipulative use through the elementary school grades (e.g.: Gilbert & Bush, 1988; Hatfield, 1994) and into the later years of schooling (e.g.: Howard, Perry, & Tracey, 1997; Scott, 1983; Suydam, 1984). The 1993 National Survey of Science and Mathematics Education (Weiss, 1994) showed that intermediate and

secondary school mathematics teachers were less likely than elementary school teachers to believe manipulatives were important for effective mathematics education. This outcome was also reflected in class-time devoted to activities with manipulatives. Other studies have reported a similar decline in the use of manipulatives in secondary school relative to elementary school classrooms, identifying factors that include teachers' limited understanding of mathematical concepts behind the manipulatives (e.g.: Marshall & Swan, 2005; Moyer, 1998; Tooke, et al., 1992), lack of experience with manipulatives (Lesh, et al., 1987), and institutional factors such as lack of teacher support, and increased reliance on textbook-based lessons (Howard, et al., 1997). These studies report that some teachers viewed manipulatives as more useful in the elementary school grades than for more advanced grades, and were more useful with lower ability students (Fennema, 1972; Friedman, 1978). Tooke et al. (1992) reported that some teachers felt that after the fourth grade, students needed "abstract teaching for abstract concepts" (p. 61), with consequences that teachers neither value nor use manipulatives with their students. Similarly, Stewart (2003) reported that, "As mathematical ideas become more abstract in Grades 3 and 4, the mode of instruction tends to become more abstract" (pp. 20-21). However, while often cited, these views are not upheld by research (e.g.: LeNoir, 1998; Suydam & Higgins, 1977) that overwhelmingly shows the value to all students of using manipulatives. As Driscoll (1981) warned, "If there is any risk related to the use of manipulatives in these [intermediate] grades, it derives from their being ignored or abandoned too quickly" (p. 24).

With teachers' views that question the value of manipulatives for older and higher academically achieving students, it is perhaps not surprising that their use in secondary school mathematical classrooms is limited, and that research into manipulative use in secondary schools is conspicuously lacking (Kennedy, 1986; Moyer, 2001). While several studies report significant decreases in the use of manipulatives from elementary to secondary schools (e.g.: Howard, et al.,

1997; Suurtamm & Graves, 2007; Weiss, 1994), these largely quantitative studies do not provide detailed insight into how secondary school teachers view and use manipulatives.

Similar Studies

While no studies have been found that provide specific insights into secondary school mathematics teachers' views and use of manipulatives, a few studies have been identified that address elements of this inquiry. Four studies in particular offer a glimpse into the use of manipulatives in mathematics classrooms and provide some perspectives as how and why teachers use them. In the following sections, each of these studies is described in detail and their conclusions highlighted.

Mathematics and Manipulatives: Comparing Primary School and Secondary School Mathematics Teachers' Views

In their 1997 paper, Howard, Perry and Tracey (1997) detailed their ongoing investigation of teachers' views about mathematics, mathematics learning, and mathematics teaching using data collected in 1996 and 1997 from more than 900 primary³ and secondary school mathematics teachers in Australia. The data were collected in order to help answer the following key questions: (i) What manipulatives are employed in the learning and teaching of mathematics in primary schools and secondary schools?, (ii) How are these manipulatives used?; (iii) What factors influence the choice of primary school teachers and secondary school mathematics teachers to either use or not use manipulatives in their mathematics lessons?

Data were collected using a questionnaire containing both multiple choice and open-ended questions and responses were obtained from 603 primary school and 336 secondary school teachers. Analysis of the data indicated that while a significant number of both the secondary school (87%) and elementary school (97%) teachers reported feeling confident in using manipulatives, 66% of the

³ In Australia, elementary schools are referred to as *primary* schools.

secondary school teachers and 47% of the primary school teachers said that they would like more training in the use of manipulatives. The frequency of use of manipulatives in secondary schools was seen to be significantly lower than that found in primary schools, and the researchers noted that this was not surprising given that even within primary schools, a significant decrease in manipulative use could be seen in years 5 and 6 compared to years K to 4. They reported that 55% of primary school teachers use manipulatives in every lesson compared to only 4% of secondary school teachers and whereas most primary school teachers reported using manipulatives weekly, most secondary school teachers used them every two weeks or monthly. While Base10 blocks was the manipulative most frequently used by primary school teachers, pattern blocks and polydrons materials were used more often in secondary schools. While no particular manipulative was used by more than 40% of secondary school teachers, manipulatives used to teach algebra was reported by 37% of secondary school teachers. The researchers reported that secondary school teachers who used manipulatives did so because they believed that the materials benefited students' mathematical learning (84%) and that students enjoyed using them (53%), while school policies (8%) and prescribed syllabi (4%) appeared to have little influence on the use of manipulatives in their lessons. While almost all felt confident in the use of manipulatives, two-thirds of the secondary school teachers and one-half of the primary school teachers indicated that they would like more training in their use. Howard, Perry and Tracey also found that the majority of teachers used manipulatives primarily for teacher demonstration, and secondary school teachers were less willing than primary school teachers to allow students to use manipulatives as they wished, use manipulatives to check their work, or use them for remedial support. The researchers suggested several possible reasons to account for the lower use of manipulatives in secondary schools that included: the rigidity of classroom timetables and school-wide programs, logistical challenges in organizing manipulatives while teachers and students move between classes, and the challenges of integrating manipulative use into text-book dominated

lessons. In summarising their findings of declining uses of manipulatives from primary school through secondary school classes, the authors concluded that:

This study suggests a need to develop a greater awareness among secondary teachers of the ways in which manipulatives can be used to support students' mathematical learning during the transition from primary to secondary schooling, particularly as a source of remedial assistance for students and through increasing their availability for students to work with as they wish. This may require an increase in the overall flexibility of secondary mathematics teaching. (Howard, Perry, & Tracy, 1997, p. 8)

Howard, Perry and Tracy's study highlights major differences in the use of manipulatives between primary school and secondary school teachers and offers several suggestions as to the cause.

However, the study did not specifically report on teachers' views about manipulatives to help us understand why and how they use them in the classroom.

Using Mathematics Manipulatives: Control- Versus Autonomy-Orientated Middle Grades Teachers.

A second study sheds further light on the views and use of manipulatives in school by intermediate school teachers. A longitudinal study by Moyer (1998), undertaken as research for her doctoral thesis, reported findings from 10 intermediate school teachers' use of manipulatives for teaching mathematics to explore how and why the teachers used the manipulatives as they did. The study investigated the effect of the teachers' control-orientation (*control* versus *autonomy*) on their use of manipulatives during normal class activities, and also during months when students had free access to manipulatives. These 10 teachers were selected based on their identified control orientation (half were deemed to be control- and half were autonomy-oriented) from a group of 18 intermediate grade enrolled in a hands-on summer manipulatives training workshop. A Pre-Post-1/Post-2 design was used with two treatments. In treatment 1, teachers used the manipulatives for instruction using the strategies learned in the summer workshop in a more traditional classroom setting while during treatment 2, teachers provided their students free access to the manipulatives. Data for the mixed-methods study were collected over the following school year and included 40 classroom observations

by the researcher, 30 semi-structured interviews and 70 lesson plans from the teachers. Considering the two control-orientation populations, Moyer did identify many differences in teaching strategies when manipulatives were used for instruction, and noted differences in the way students considered, accessed and engaged with the materials. However, she cautioned that many of these differences could have more to do with the teachers' own experiences as students and understanding of mathematics, their views about how students learn, and their professional learning experiences with manipulatives, than specifically as a result of the teachers' control orientation. Moyer concluded that:

This data suggests that manipulative materials do have the potential to mediate teachers' instructional practices, encourage paradigm shifts as teachers make accommodations in their teaching to implement manipulatives as tools for mathematical learning. Manipulatives also have the potential to change student attitudes and motivation orientations, even in classrooms where teachers' control orientations may be an opposing influence. (p. 153)

In a paper based on this study that elaborated on the data, Moyer (2001) reported that mathematics instruction in these teachers' classrooms followed a typical traditional lesson pattern and that for some teachers, the use of manipulatives was deemed disruptive in that their use encouraged student discussion and interaction with peers and materials. She found that "although the teachers gave verbal assent to the notion that manipulatives could be used to teach mathematical concepts, the actual lessons reflected traditional teaching routines with manipulatives used primarily to supplement the lesson plan" (p. 190). Rather than being used by students to explore new mathematical ideas, manipulatives were used to reinforce previously learned content or for 'fun' activities; notions that Moyer felt limited "...the possibilities for students' explorations of meaningful mathematical content in engaging and interesting ways" (p. 191). She went on to suggest that it was unclear that the teachers in the study "were able to make connections or represent mathematics ideas in meaningful ways while using the manipulatives" (p. 192) and concluded that:

...using manipulatives was little more than a diversion in classrooms where teachers were not able to represent mathematics concepts themselves. The teachers communicated that

the manipulatives were fun, but not necessary, for teaching and learning mathematics (Moyer, 2001, p. 175)

While this study and the papers it spawned provide insight into many aspects of the intermediate teachers' understanding of manipulatives and offer rich descriptions of how they were used in the classroom, the focus of the study was to examine the teachers' classroom strategies and choices based primarily on their identified control orientation. While Moyer's study has been a huge influence on my study, its focus on intermediate mathematics teachers does not offer any specific insights into the context of teaching and learning with manipulatives in secondary school mathematics.

A profile of science and mathematics education in the United States.

Weiss (1994) reported on the 1993 National Survey of Science and Mathematics Education which involved questionnaire responses from approximately 6,000 teachers in Grades 1-12 in the US. The survey was designed to provide up-to-date information to identify trends in the areas of teacher background and experience, curriculum and instruction, and the availability and use of instructional resources. In an attempt to gauge teacher support for reform recommendations, science and mathematics teachers were provided with a list of instructional strategies and asked how important they believed each was for effective instruction. Weiss reported that support for hands-on activities is very high, "although the middle and secondary school mathematics teachers are less likely than elementary school teachers to believe that the use of manipulatives is important for effective mathematics education" (p. 7). The difference in importance attributed to manipulatives between elementary school and secondary school teachers was also identified in the time allocated to various class activities. Weiss reported that:

The typical high school⁴ mathematics class spends 48% of class time on whole group lecture/ discussion, only 14% on non-manipulative small group work, and only 7%

⁴ High school and secondary school are used synonymously in this thesis.

working with manipulatives. In contrast, the typical elementary mathematics class spends roughly 25 to 30% of class time on each of these activities. (p14)

Weiss also reported that within the elementary school grades, a decline in the use of manipulatives is seen between Grades 1-4, reporting 29% of class time spent using manipulatives, compared to only 12% in Grades 5-6; a finding similarly reported in the study by Howard, Perry, and Tracy (1997). Weiss found that “traditional” activities continued to dominate mathematics instruction with over 86% of secondary school classes doing mathematics problems from the textbooks on a daily basis. In contrast, Weiss reported that that only four out of 10 secondary school mathematics classes “engaged in making conjectures and exploring possible methods to solve mathematics problems as frequently as once a week” (p. 15). While the results confirm that both the importance attributed to, and the actual time spent using manipulatives are considerably lower with teachers in the secondary school than with teachers in the elementary school, the only reason offered for this is inadequate facilities and equipment and “lack of money to purchase consumable mathematical supplies” (p. 25).

The study, while useful in providing broad context around mathematical teaching in the secondary school, offers no particular insight into the views of secondary school mathematics teachers of manipulatives or how they are actually used in the classroom.

Curriculum Implementation of Intermediate Mathematics (CIIM) Research Report: The use of manipulatives in mathematics class.

The Curriculum Implementation in Intermediate Mathematics (CIIM) research project was initiated in January 2006 to examine how the inquiry-oriented mathematics curriculum for Grades 7-10 in Ontario is implemented and understood by the multiple partners involved (see OME, 2005a). In their research report, Suurtamm and Graves (2007) described the background and evolution of this large-scale research initiative and summarized some of the research findings from the first year of the project. The mixed-methods study gathered quantitative data using a 45-item online survey accessible to all Grade 7-10 teachers in Ontario and completed by 1,096 mathematics teachers. The

survey solicited information about mathematics teachers' teaching assignment, instruction, and assessment practices in mathematics, and professional development experiences and needs. In particular, several of the items asked the participants for their views of, comfort level with and classroom use of manipulatives. A section of the report is dedicated to reporting on these findings.

The findings show that 48% of the Grade 7 and Grade 8 teachers strongly agreed that manipulatives are necessary tools to support effective learning of mathematics for all students” compared with 16% of the Grade 9 and Grade 10 teachers. Similarly, the data show that 60% of Grade 7 and 53% of Grade 8 teachers viewed promoting the use of multiple representations as very important compared to 37% of Grade 9 Academic⁵, 50% of Grade 9 Applied⁵, 30% of Grade 10 Academic and 47% of Grade 10 Applied teachers. In addition to showing a decline in importance of promoting the use of multiple representations from Grade 7 to 10, the authors also suggest that “teachers in Grade 9 and 10 see the use of multiple representations as being more important for teaching students in Applied courses than for those in Academic courses” (Suurtamm & Graves, 2007, p. 92). The data also show a similar decline from Grade 7 to 10 in the reported comfort level of teachers with the use of concrete materials (that also encompasses mathematical manipulatives): 46% of Grade 7 and 43% of Grade 8 teachers reported feeling very comfortable with using concrete materials compared to 30% of Grade 9 Academic and 27% Grade 10 Academic teachers. Again, the data showed higher numbers reported by teachers in the Applied programs. The researchers presented data on the frequency of classes in which students work with manipulatives (such as algebra tiles, geoboards etc.) in their course. The data showed that while only 1% of Grade 7 and 4% of Grade 8 teachers never use them, 40% of Grade 9 Academic and 48% of Grade 10 Academic

⁵ In Ontario, mathematics courses in the Grade 9 and 10 curriculum are offered in two types, academic and applied, which are defined as follows: “Academic courses develop students’ knowledge and skills through the study of theory and abstract problems. These courses focus on the essential concepts of a subject and explore related concepts as well. They incorporate practical applications as appropriate. Applied courses focus on the essential concepts of a subject, and develop students’ knowledge and skills through practical applications and concrete examples. Familiar situations are used to illustrate ideas, and students are given more opportunities to experience hands-on applications of the concepts and theories they study.” (OME, 2005b, p. 6). These courses, in the context of all high-school, can be seen in Figure 2.

teachers reported never using manipulatives in their classroom. The researchers conclude that these data identify two trends:

The first is that as students move up the grades, they use manipulatives in fewer classes. The second is that students in Grade 9 and 10 Applied classes use manipulatives more frequently than their peers enrolled in Academic classes. (Suurtamm & Graves, 2007, p. 96)

The report also notes that 79% of the Grade 7 and 8 teachers felt that further professional development in the use of manipulatives would be somewhat or very helpful, compared with 57% of the Grade 9 and 10 teachers. They conclude this section of the study as follows:

One interpretation of [these findings] may be that the Grade 9 and 10 teachers do not attribute as much value to the use of manipulatives in mathematical instruction, consequently use them less frequently, and thus see less need for additional professional development in their use. (Suurtamm & Graves, 2007, p. 96)

As seen in previous studies and the research literature, Suurtamm and Graves' study also highlights the decline in use of manipulatives as students move up the grades. They suggest that reasons this include secondary school teachers' lower comfort level with the manipulatives and a lower value being attributed to the use of manipulatives in the teaching and learning of senior mathematics. The study also sheds light on considerable differences between teachers in the Applied and Academic programs that may speak to how teachers view the student groups who derive greater benefit from manipulative use. While the study provides valuable data on patterns of manipulative use, it does not offer detail of the teachers' own views about manipulatives and why and how they use them in the classroom.

Literature Review Summary and Implications for this Research.

As described in the preceding sections, many studies show that using manipulatives can be valuable in building and strengthening both the students' and teachers' mathematical understanding and current curriculum and pedagogical directions support their use. We have also seen from studies,

several of which have been described in detail, that many secondary school mathematics teachers are not using manipulatives in their classrooms for myriad conjectured reasons, including the teachers' lack of perceived value, comfort level with the manipulatives, and difficulties in representing mathematics ideas in meaningful ways with them. To better understand how teachers integrate the use of manipulatives into their teaching practice and the challenges that this presents, it is necessary to examine their views about and uses of manipulatives. As Thompson's (1984) study of intermediate-school mathematics teachers concluded:

If teachers' characteristic patterns of behaviour are indeed a function of their views, beliefs and preferences ... then any attempt to improve the quality of mathematics teaching must begin with an understanding of the conceptions held by teachers and how these relate to their instructional practice. (p. 106)

CHAPTER 4: METHODOLOGY

This chapter provides an overview of the methodology used in this study. The chapter starts with a review of the research questions that guide the study and a description of the methodology used. I then offer a rationale for the selection of semi-structured interviews as the data collection instrument and the subsequent development of the interview guide. I then describe the strategies used for participant recruitment followed by details as to how the data were collected and subsequently analysed.

Research Question

This study examines secondary school mathematics teachers' views of manipulatives to understand how and why they use them in their classrooms and is guided by the following primary research questions:

How do secondary school teachers view the use of manipulatives in teaching mathematics?

How do secondary school teachers describe their use of manipulatives in teaching mathematics?

Rationale for Methodological Framework

The focus of this study was to gain understanding and meaning from teachers' descriptions of their views of and experience with manipulatives. It was important to select a methodological framework and data collection instrument to capture the "lived experience of other people and the meaning they make of that experience" (Seidman, 2006) as teachers describe elements of their professional life and teacher practice. This led to the use of a qualitative research discipline that (Creswell, 1998) describes as one "where the researcher is an instrument of data collection who gathers words or pictures, analyzes them inductively, focuses on the meaning of participants, and describes a process that is expressive and persuasive in language" (p. 14).

In order to hear teachers describe their personally held views and experiences with mathematical manipulatives, it was important to meet with them face-to-face. To gather their personal stories, and to encourage them to speak naturally and freely, one-on-one interviews were considered to be the most appropriate format in which to gather data. A semi-structured interview was selected as the primary data collection instrument as it has been commonly used in small-scale qualitative research (Fontana & Frey, 2003; Hammersley & Atkinson, 1995). The semi-structured approach is both flexible and dynamic for the interviewer (Taylor & Bogdan, 1984), affords the opportunity for the participant to reflect, and allows the researcher to follow up on participant responses (Creswell, 2003). As further validation, Tyson's (1991) research into teacher education and lesson planning concluded that the semi-structured interview is an effective data gathering tool in assessing teachers' views and knowledge. General developmental guidelines for semi-structured interviews and the interview guide were followed (Drever, 1995) in an attempt to maximize "the match between the researcher's categories and interpretations and what is actually true" (McMillan, 2004, p. 278).

While the format and convenience of semi-structured interviews is well suited to gathering the teachers' narrative, it was acknowledged at the outset that due to time constraints⁶ the data gathered would be based upon the teacher's own descriptions of their views and actions rather than from direct observations of their teaching practice. Taylor and Bogdan (1984) recognise that the lack of first-hand knowledge of how people act can make it difficult for the researcher "to sort out the difference between purposeful distortions and gross exaggerations, on the one hand, and genuine perspectives, on the other" (p. 99). However, they suggest that by creating an interview atmosphere in which participants are likely to talk freely, by prompting participants to elaborate and hence provide in-depth descriptions, and by encouraging participants to respond with specific and concrete

⁶ Primarily those associated with the scope and timely completion of a Masters' thesis.

examples of their practice, the researcher can “usually obtain an accurate amount of past events and current activities” (p. 100). These three elements – a comfortable atmosphere, in-depth interviewing and emphasis on descriptions of real events – were central to the design and testing of the interview guide, and selection of the interview setting.

In order to allow me to concentrate on the participant’s descriptions of their views and practice during the interview, I used a small digital recorder to record the complete conversation. Taylor and Bogdan (1984) suggest that while tape recording can alter what people say in the early stages of the interview, “a tape recorder allows the interviewer to capture so much more than he or she could rely on memory” (p. 103). While I wished to keep my own note taking to a minimum so that I could concentrate on the participants’ comments, I kept a notebook on hand so that I could make simple notes – often just a word or two – of interesting ideas and comments made by the participant. As Seidman (2006) suggested, these simple and abbreviated working notes “help interviewers concentrate on what the participant is saying” (p. 79) and helped to remind me to seek clarification and additional elaboration in a natural and conversational manner.

Instrument Design

To obtain the data to address the two research questions, the dimensions of the data collection instrument needed to include the teachers’ views about and their descriptions of the use of manipulatives. As research shows, teachers’ use of manipulatives is mediated by many factors including their views of the subject of mathematics, and its teaching and learning. Consequently, the dimensions of the instrument included (a) teachers mathematics background, teaching qualifications and experience; (b) teachers’ knowledge and views about mathematics and its teaching and learning; (c) teachers’ knowledge and views of manipulatives for teaching mathematics; and (d) teachers’ discussion of their actual use of manipulatives in the classroom. The first three dimensions tie into section 2 of the conceptual framework (Figure 1) by examining the influence of teachers’ views and

knowledge of manipulatives on practice. The fourth dimension ties into section 3 by examining teachers' descriptions of how manipulatives are actually used in the classroom.

The semi-structured interview guide, detailed in Appendix A, contained an introduction that includes instructions for starting the interview and opening the dialogue with the participant, followed by some key open-ended questions arranged in four parts, described below. Each part of the interview guide is directly connected to a specific dimension of the study. The parts were as follows:

Part 1: *General teacher background information*. This contained two questions that asked the participant to describe their mathematics background and teaching qualifications and experience;

Part 2: *Teacher views about mathematics, and mathematics teaching and learning*. This contained three main questions that asked the participant to describe their teaching practices and their own understanding of the nature of mathematics;

Part 3: *Teacher knowledge and views of manipulative use in mathematics instruction*. This contained three main questions to solicit information from the teacher as to their views about manipulatives as representational forms of mathematical concepts, if they are useful for all or some mathematics concepts, how mathematical knowledge is gained by students, and their thoughts about the underlying mathematics behind manipulatives;

Part 4: *Teacher descriptions of actual use of manipulatives within the classroom*. This contained four main questions to solicit information about participants' experiences with manipulatives in the classroom, how they actually use them, for whom they appear to have value, and reflections on activities that have or have not worked and why.

The guide concluded with a wrap-up, thanking the participant for their time and effort, and provided an opportunity for the participant to offer additional comments or suggestions.

In preparation for the interviews, I conducted two pilot semi-structured interviews with university colleagues, one of whom is a secondary school mathematics teacher. For these interviews, I used the same letter of invitation to participate, interview consent form and interview guide that I subsequently used for the participant interviews. My own reflection of the pilot interviews and the feedback from the two participants allowed me to hone both the text of the questions and the order in which they were asked to establish a more natural flow to the interview.

Recruitment Strategies

In order to solicit participation in the research study, I contacted executive members of a local chapter of a provincial mathematics organization. I asked that once I had received approval from the University of Ottawa's Research Ethics Board to start recruitment, whether they would be prepared to email their membership with a letter of invitation to participate in this study. My request was warmly received and I was informed in late February 2008 that upon gaining ethics approval I should send my letter of invitation to the membership director who would then forward it to the organization's membership.

In the invitation, I introduced myself and provided a brief description of the study. I detailed that the participants would be invited to participate in a single face-to-face interview of approximately one hour's duration that would be audio taped for subsequent transcription. In order to qualify for participation in the study, I stated that the participant must be a secondary school mathematics teacher currently teaching the Ontario mathematics curriculum to at least one secondary school (Grades 9 - 12) class. I reassured the recipients of the invitation that participation was entirely voluntary and that they would be free to refuse to answer any questions or withdraw from the study at any time without penalty or prejudice. I also informed potential participants that any information collected during the interview would remain strictly confidential, their participation anonymous. In addition, I also stated that if they were quoted, pseudonyms would be used and any identifying

details removed. Finally, I asked that if any teachers were interested in learning more about my study or wished to participate, that they should contact me directly by email.

Prior to sending out the letter of invitation, I asked several colleagues to review and comment on the letter. Several minor edits were suggested to make the letter more welcoming and friendly and to improve its readability. These suggestions were incorporated into the final text of the letter of invitation. At the end of May, 2008, I received ethics board approval to start my recruitment and immediately emailed the revised letter of invitation to the membership director of the mathematics organization. Within a few days, I received emails from two teachers who subsequently became the first two participants, Alice and Barry. We used email to arrange a mutually agreeable time and date to meet for a face-to-face interview and in both cases, the interview was held at the University of Ottawa. The first interview was conducted at the end of June, 2008.

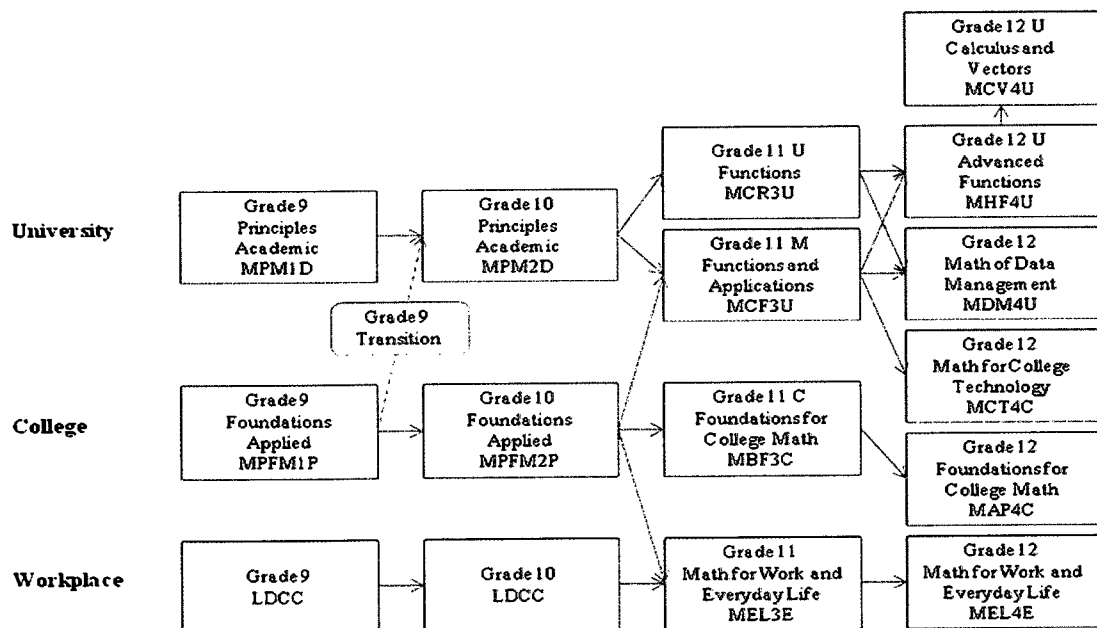
Following the initial two interviews, I received no further emails from teachers for a month and after consulting with my supervisor, requested the mathematics organization to resend the letter of invitation. This was subsequently emailed to their membership in early September, 2008. In the meantime I followed up with two mathematics teachers, met during local professional development workshops and conferences, who had expressed interest in the study. Both teachers agreed to participate and became the next two participants, Celia and Diane, interviewed in early August and early September respectively. Within a week of the second invitation, I received an email from an interested teacher who subsequently became the fifth participant, Elly, whom I interviewed during the first week of October. At around the same time I also received an email from another mathematics teacher who agreed to participate, and Frances, my final participant, was interviewed mid October. The recruitment process and all six interviews were conducted over a period of 18 weeks.

Participants

Five women and one man participated in this research and throughout the study are referred to using the following pseudonyms: Alice, Barry, Celia, Diane, Elly and Frances. At the time of the interviews, all participants were just completing or had just completed teaching at least one course of secondary school mathematics. The participants varied greatly in the length of their mathematics teaching experience. Diane and Elly had more than 15 years experience, Celia more than 10 years (although she had been a language teacher for several years), Frances nearly 9 years, Alice 3 years and Barry just one semester. All of the participants had obtained their teaching certificate within Ontario and consequently their teaching experience had been obtained using one of more revisions of the Ontario mathematics curriculum.

To provide context in the discussions of the participants' teaching experience, Figure 2 shows the Ontario secondary school mathematics courses (OME, 2005c, p. 60) with the addition of the Grade 9 and Grade 10 Locally Developed Compulsory Credit (LDCC) courses.

Figure 2: Secondary school Mathematics Courses in Ontario



A brief *cameo* of each participant is provided.

Alice

Alice was schooled abroad and obtained her teaching certificate in Ontario to teach mathematics and science in the secondary panel. Alice was in her third year of teaching and had taught the accelerated pre-International Baccalaureate (IB) mathematics curriculum⁷ to Grade 10 students, the Ontario curriculum Grade 12 Mathematics of Data Management and the two Grade 11 College courses. In the upcoming academic year she planned to move to a school to teach the Grade 9 Academic⁸ level course and some of the Grade 11 College and Grade 12 University courses.

Barry

Barry graduated with a science engineering degree and completed his teaching degree for the Primary/Junior (P/J) division. Barry went on to take an advanced basic qualification (ABQ) course in senior mathematics and at the time of the interview, Barry had just completed his first semester teaching where he was contracted to teach a Grade 10 Applied level course and two other non-mathematics courses. He was hoping for another contract as a long-term occasional (LTO) teacher in the area.

Celia

Celia obtained an undergraduate degree in chemistry with a minor in mathematics and after many years in private industry, started teaching her mother-tongue to public servants. She continued to teach English as a second language to seniors from her home country and language, history, geography and politics in a secondary school International Studies program. Celia obtained her teaching certificate in mathematics and chemistry and at the time of her interview had been teaching full-time for over 10 years, having taught almost every mathematics course - from Grade 9 to OAC⁹ – and covering the Academic, Applied and LDCC programs.

⁷ The IB program is not part of the Ontario Mathematics curriculum and thus does not appear in the Figure 2.

⁸ For simplicity, the Grade 9 and 10 mathematics courses are referred to using “Academic” and “Applied”.

⁹ Ontario Academic Credit: the fifth year of secondary school (Grade 13) for students taking pre-university courses.

Diane

Diane had been teaching mathematics for more than 15 years and, like Celia, had taught almost all of the secondary school courses from Grade 9 to 11, both Applied and Academic, and one Grade 12 course. She had been teaching at her present school for 10 years and at the time of the interview was teaching two Grade 10 Applied classes and a Grade 11 Advanced Functions course. While Diane majored in biology at university and her teaching certificate is for both science and mathematics, her teaching experience had been entirely in mathematics.

Elly

Elly had been teaching secondary school mathematics for 22 years immediately following graduation from her Teacher Education program. She had been teaching mathematics at the same school for her entire teaching career and had taught all of the mathematics courses across all of the grades and both Applied and Academic levels. The majority of her experience had been teaching the senior grades - Grade 11, 12 and OAC. At the time of the interview she was teaching two Grade 12 Advanced Functions courses and one Grade 11 Functions course. The following semester she was scheduled to teach two Grade 12 Calculus and Vectors courses and the Grade 11 Functions course again.

Frances

Frances completed an undergraduate degree majoring in chemistry with a minor in mathematics and holds a teaching certificate to teach both mathematics and chemistry in the senior panel. Prior to the interview, Frances had taught in two large urban secondary schools for 9 years and had experience teaching the Grade 9 and 10 courses in both the Applied and Academic levels, all Grade 11 College and University courses, and the Grade 12 Mathematics for Data Management course. Outside of math, she has also taught music, science, computers and “many other things”.

Along with her teaching duties, Frances was also a department head for a couple of years - a position that provided her many professional development opportunities.

Researcher's Background

In this study I found that I shared several things in common with the participants that enabled me to quickly establish a level of trust that encouraged them to share their insights and perspectives willingly and openly. My educational background was very similar to five of the participants who also had obtained an undergraduate degree in science and a teaching certificate to teach secondary school mathematics and science. I am also a similar age to three of the participants, and, like two of them, my undergraduate degree was obtained outside Canada. I had met five of the six participants in my research prior to the interview and several on multiple occasions during local professional development workshops, provincial conferences and seminars. In addition, I had met two of the participants during a mathematics summer camp¹⁰ for pre-service teachers held at the University of Ottawa. These participants and I had spent time together in professional mathematics learning environments, some of which involved discussions about and use of manipulatives in the classroom. I believe that as I was already known by these five participants, the interview atmosphere was less intimidating, more relaxed and friendly than may have been the case had we been meeting for the first time. The participants were aware that I had little classroom teaching experience and I believe that this helped to remove potential concerns that their teaching practice would be evaluated or judged during the interview.

Data Collection

All data for this study were collected from the six single semi-structured interviews conducted with the participants. The six interviews were conducted between late June and mid

¹⁰ This Math Camp, held in the summer just before the start of the university teacher training program, was offered to pre-service teachers to help them with concerns over having to teach math. It was structured around rich problem-solving with the use manipulatives within an inquiry-based learning environment.

October 2008 and as the participants responded at different times during the recruitment phase, each interview was arranged to be held as quickly as possible at a time and place that was most convenient for the participant. Five of the interviews were conducted in a quiet meeting room in the Faculty of Education building on the University of Ottawa campus. At her request, the interview with Diane was held close to her school in a large private room in a public library. Not surprisingly, the audio quality of this interview contained more echoes and other sounds than with the other five interviews.

Each of the interviews was conducted by the researcher using the interview guide and recorded using a digital recorder for subsequent manual transcription. I also had on hand a collection of different sets of manipulatives that included algebra tiles, linking cubes, fraction circles, geoboards, and pattern blocks, all of which are mentioned in the Ontario curriculum documents, and many of which are frequently the focus of professional development workshops for the senior panel. Rather than have these manipulatives out on the table and potentially leading or encouraging the participants into using them, I mentioned to each interviewee that the manipulatives were available should they wish to use them. Alice and Celia used algebra tiles and linking cubes during the interview to illustrate different discussion points.

While the interview was in progress, I decided not to make any detailed notes to ensure that the discussion progressed naturally. However, I did use the interview guide at all times to ensure that each of the main topic areas were explored and specific questions asked, although not necessarily in the order that I had originally identified; illustrating the flexibility of semi-structured interviews. I made some simple notes and jottings while the participants were speaking as reminders for me to ask additional follow up questions. For instance, during the interview with Barry, I jotted down the following phrases: “teacher support”, “comfort level”, “curriculum and its emphasis on manipulatives” and “Math Camp ah-ha”. I used these as prompts to solicit further information

through additional probing questions. In another instance, I made a simple drawing of something that Diane spoke about and she subsequently referred to this illustration during further elaborations.

Although the invitation and other email correspondence between the researcher and the participants suggested that the interview would last for one hour, the duration of the interviews ranged in length from between 57 and 97 minutes, with an average duration of 74 minutes. In each of the four interviews that lasted in excess of an hour, the discussion was happily continued by the participant. On four occasions the researcher and participant continued to discuss and exchange ideas following the taped part of the interview. Following three of the interviews, participants specifically asked the researcher to provide ideas and examples as to how manipulatives (specifically algebra tiles and linking cubes) could be used to assist the teaching of various mathematical concepts. While these discussions, two of which were more than an hour in length, followed the taped part of the interview, the researcher took brief notes and included them, and subsequent emails in notes appended to the end of the transcriptions.

Following the interview, I asked each participant if they would like to receive a copy of the completed verbatim transcription - in part because I believe, as Seidman (2006), that the participant has a basic right to the data, and also to offer them the opportunity to clarify their ideas. I reiterated this offer in an email following the interview in which I also thanked the interviewees for their participation. Two of the participants (Alice and Diane) requested a copy of their interview transcription although neither asked for any modifications to be made.

Data Sets and Data Analysis

The audio recording of each interview was transcribed by the researcher into a single data set using a combination of speech recognition and audio manipulation software. Each audio recording was played through audio processing software while the researcher repeated the interview conversation verbatim into the speech recognition program that converted the spoken words to text.

The transcription text was subsequently compared against the audio recording and edited and the process repeated until the final version was believed to be word perfect. The six interviews conducted created 102 pages of single-spaced transcript data (19, 13, 20, 20, 14, and 16 pages respectively for each of the interviews). As previously mentioned, all participants were invited to review their transcript although only two asked to do so. Neither of them asked for revisions of any sort.

The initial phase of the data analysis was conducted considering the key dimensions of the instrument. Using these categories, codes were developed for (a) mathematics background and teaching experience, (b) views of mathematics and mathematics teaching and learning, (c) views of manipulatives and (d) actual use of manipulatives. Data codes were also generated for participants' mathematics background and experiences and their teaching qualifications and experience. The second phase of the data analysis was primarily informed by McMillan (2004) and Charmaz (2005) to look for patterns and themes that emerged directly from the data. During this phase, the data was re-examined to look for additional patterns and themes emerging through successive and iterative levels of data analysis and conceptual development that allowed for the creation of new inductive coding categories. An initial codebook was then formed that included both the deductive (the four instrument dimensions) and inductive (derived from the data) code categories.

Using the code book, each data set was then coded using a combination of software (NVivo¹¹), word-processing software and hand annotations to provide an audit trail¹² of the data analysis process and progress. Starting with the first data set, the data was analysed using the deductive codes derived from the key dimensions and edited into separate summaries. The inductive

¹¹ NVivo is a qualitative data analysis (QDA) computer software package produced by QSR International. It is designed for qualitative researchers working with very rich text-based and/or multimedia information, where deep levels of analysis on small or large volumes of data are required.

¹² This audit trail (adapted from Berkowitz, 1997) shall consist of an archive containing the raw data gathered, descriptions of data analysis and reduction processes, data reconstruction and synthesis including structuring of categories and themes, process notes and instrument development information

codes were then used to analyse these summaries into smaller sub-themes and constructs. The results of the initial coding were reviewed in detail with the study supervisor and revisions made to the codebook to include additional identified themes. The process of applying the code book to each interview transcript and refining the codes continued iteratively until a final set of codes emerged that appeared to uncover all themes in the data sets. Finally, this master code set, seen in Appendix B, was reapplied to each of the data sets for consistent coding comparisons. Separate files were created for each of the interview participants containing the fully coded transcription. Additional set of files were then created for each of the major coding categories and re-analyzed using the finer-grain codes to identify coherence and differences across the sub-themes and constructs.

Summary

The methodology of the study was driven by a qualitative framework developed to gain insight from teachers' descriptions of their professional practice. The research used semi-structured interviews as the data collection instrument. An interview guide was constructed around the research questions and used to solicit information in the four dimensions of teachers' mathematics background and teaching experience; knowledge and views about mathematics, its teaching and learning; teachers' knowledge and views of the use of manipulatives for teaching mathematics; and teachers' discussion of their actual use of manipulatives in the classroom. The data from the participants was then analysed using a combination of deductive codes derived from the key dimensions of the instrument and inductive codes that emerged during multiple iterations of reading within and across data sets. The coded data was then organized by participant and again thematically across the participants for subsequent discussion.

The principal codes that emerged from the analysis were captured in individual files by participant and included (i) mathematics background, (ii) teaching qualifications and experience, (iii) view of mathematics, (iv) view of mathematics teaching and learning, (v) view of manipulatives,

(vii) use of manipulatives, (vii) challenges using manipulatives, and (viii) supports for the use of manipulatives.

Coded data in the first four of these files (mathematics background, teaching qualifications and experience, views of mathematics, and views of mathematics teaching and learning) were used to create a richly detailed context for each participant in order to frame their stories and descriptions that pertain to the two research questions. This analysis is presented by participant in Chapter 5. Coded data on each participant's views and use of manipulatives were then reanalysed into two 'thematic' files – views of manipulatives, and use of manipulatives – to directly address one of the two research questions. This cross-participant analysis is discussed in Chapter 6. Similarly, data coded on the emergent themes of supports for and challenges to the use of manipulatives, was reanalysed into separate thematic files, the results of which are presented in Chapter 7.

CHAPTER 5: ANALYSIS BY PARTICIPANT

In this chapter, I provide for each participant a rich contextual ‘wrapper’ within which to subsequently consider the teachers’ views on – and use of – manipulatives as mathematical learning tools. These discussions are grouped into the two dimensions examined in the first two parts of the interview guide (see Appendix A): Mathematics Background and Teaching Experience, and Views of Mathematics and its Teaching and Learning. Specifically, in the interview guide, Part 1: *General teacher background information* contained two questions that asked the participant to describe their mathematics background and detail their teaching experiences; while Part 2: *Teacher views about mathematics, and mathematics teaching and learning* contained three questions that asked the participant to describe the way they learned mathematics as a student, what mathematics meant to them, and their views of teaching and learning mathematics. In addition, I have included the participants’ discussion of their students as learners. I conclude this chapter by summarizing in tables the participants’ descriptions of their teaching qualifications and teaching experience, background in mathematics, experiences as students in secondary school and university, and their views on the subject and discipline of mathematics. In the following descriptions of each participant, all quotations used have been taken verbatim from their interview transcript. In the case of the one participant whose first language is not English, minor editing of the quote may have been made to aid clarity and readability.

ALICE

Mathematics Background

Alice was schooled abroad and described her early school experiences with mathematics as enjoyable but challenging. Alice stated that due in part to competition to gain entry into a good secondary school and then university, she often worked beyond the curriculum studying on her own with school textbooks, mathematics gazettes and material from mathematics clubs. While she spoke

of the pressure to perform academically, she also described a feeling of accomplishment in her mathematics skills and proficiency and her enjoyment of math, and in particular the exploration, investigation and modeling in 3D geometry. She described how the use of models in this subject “helped me very much and after that I didn't need them anymore after the first few lessons, but mathematics was already very easy because of those models”. Alice described finding challenging the content of secondary school mathematics where it was no longer the case that she could “just follow what the teacher did, understand it, and then proceed with my own problem-solving”. In her early secondary school years, she found that in order to fully understand the mathematical concepts, she needed to put in lots of additional effort by going over the class material at home to re-learn each lesson. She reported finding reviewing a mathematics concept for the second time was really helpful to her understanding and offered several examples that demonstrated the value she found in this review.

Later on when the same thing appeared, that feeling that we've seen it before and that we already know it gave us... I don't know if it was confidence or something stuck with us without realising... but I found it really helpful that only afterwards.

Alice described her overall school experience as being taught in a traditional “chalk-and-talk” manner which, with the exception of 3D geometry, did not include the use of manipulatives or models. Not surprisingly, Alice described her view of mathematics as: “everything had to be traditional math. It's abstract. You learn it for the love of it, for the sake of it... You don't need a teaspoon of sugar to take it” and stated that, to a large extent, this view of mathematics as an abstract discipline held until she started teaching.

Teaching Experience

At the time of the interview Alice was in her third year of teaching having taught the accelerated pre-International Baccalaureate (IB) mathematics curriculum to Grade 10 students and the Ontario curriculum Grade 12 Mathematics of Data Management and the Grade 11 College. For

her next school year, she had been hired by school closer to home to teach three mathematics courses: Grade 9 Academic, Grade 11 College and Grade 12 University.

Alice described her teaching experiences as moving from the “survival mode” of her first year of teaching to becoming more comfortable with her own teaching style and philosophy. She felt that she was now more reflective of her practice and better able to assess her students’ understanding and predict many of their misunderstandings. Through continual self-reflection, carefully listening to her students’ questions, and repeated experiences teaching the same class, she felt better able to scaffold and enrich her students’ learning, manage student group dynamics, and manage the classroom so that she could spend more time and effort attending to her students’ needs:

I reflected back of my first year teaching and I saw well I could use in scaffolding here. I should emphasize that, I should bring in different examples, you know, not just jumping from an easy to difficult example, that kind of refinement trying to make.

Alice described how she now paid particular attention to the affective side of her students’ learning by helping to increase their motivation, perseverance and positive attitude towards solving problems by developing their logical thinking skills. She reported that many of her students, even in senior grades, sought a rule or procedure, rather than working diligently and logically to solve a problem and when faced with difficulties, many “just give up”. She stated that her most important teaching goals were to challenge her students to think more deeply when solving problems, to help them develop the skill competence and confidence to select appropriate mathematical “tools”, and to step through problems in a logical manner. She described that the logical thinking entailed in problem-solving would be a valuable life-long skill applicable beyond the discipline of mathematics. Alice described how she used *drill-type* exercise problems to help her students develop procedural fluency, helping them to identify and explore patterns within the mathematical ideas and strengthen and deepen their conceptual understanding. Rather than reaching for a rule, she hoped that through a

deeper understanding of the concepts her students would have the confidence to tackle more challenging problems and be able to transfer their knowledge to different contexts:

In these classes, I want the students to have.... Well, there's this expression *enduring understanding*... but what I really want from them is the skills, the drive to try to solve the problems. Because I know what I teach they will forget and whatever I teach is on the web and so it's not the information that will stay with them, but the logical thinking.

View of Mathematics and its Teaching and Learning

Alice reported that her view of mathematics, formed during her school and university years followed very traditional lines where mathematics is considered completely abstract and that students learn it “for the love of it, for the sake of it”. Alice stated that her secondary school notions of mathematics as a collection of disparate and disconnected ideas had subsequently evolved into a continually expanding web of interconnected concepts and tools. She noted that as she teaches material that she learned as a student, she saw connections across and between concepts not seen before. She gave the example of being able to solve a geometry problem in so many ways:

...you can use Euclidean geometry or you could use trigonometry, which can lead you to using algebra and algebraic manipulations and all these ways of solving a problem. And how I see it is that it is a puzzle of many pieces and they're all connected.... So there is a beauty in it.... In the functions and the trig. There are phenomena... when you study the locus problems; there is no single answer, no single way to reach the answer. So I see it as an exploration.

Alice reiterated her belief that mathematics consisted of more than sets of rules or procedures when she referred to responding to her students' fixations on when to apply rules and procedures by claiming, “That's not math. That's just a set of rules.”

Alice spoke of the benefits in having her students working in groups and learning from each other. She described encouraging her students to work collaboratively together on solving problems “because there will always be someone who says ‘Hey, you can't do that. Remember?’ and they teach each other.” To facilitate effective group work, Alice described building in solo time to allow students time to work on the problem on their own to see how far they can get. She stated “I want

them all to be virtually around the same level before we move onto group work because I don't want some people still at zero and some people already at level one." As they worked on their own, Alice offered individual assistance so when students did move into groups, they would be better able to build on their own work and gain help from their colleagues:

So when we move onto more complex examples they have something to build on. And that's when some people who don't get it will be tremendously helped by those who do right away, or those who do a little...

Alice also found that group work helped students articulate their mathematical understanding and develop their self-assessment skills:

There are students who if they don't do group work by the end of the year, I haven't heard their voice. They are persons not calculators. So when they're in groups, they express their math, their feelings about math, they know how they are doing, because they have the instant feedback from their peers and they are forced to use their knowledge. So they're more engaged definitely. (Alice, p. 17)

Students as Learners

Alice expressed concern that when faced with a difficult problem or one that they had not seen before, many of her students – even her older students in the accelerated mathematics class – just give up. She described feeling that she had to “mother them, to hold their hands through multi-step problems” and wondered whether this lack of perseverance had its roots in the belief of some students’ family members that some people can’t do math. Alice reported that such students were often the hardest to motivate. Alice described her students as falling into three broad categories based on the way in which they tended to learn. She reported that one third of her students learned everything very easily and seemed to understand everything the first time it was taught; while another third were students – who she described as bright, thorough, meticulous and good problem solvers – solely focused on learning methods and procedures by rote, and who often resisted trying alternative ways of doing mathematics. The final group were students who did not quite fully understand what was taught in class but who would then work at home on their own to reach a better

understanding. This latter group, Alice reported, would often come to good understanding of both the concepts and procedures eventually, and were the ones who seemed to find using manipulatives to be most helpful in their learning. Interestingly, Alice's description of the learning style of this latter group mirrors the learning style that she described having to adopt during her own secondary school experiences where she found it necessary to relearn class work at home to understand the math.

BARRY

Mathematics Background

Barry described himself as having a strong background in mathematics and science with high grades in each subject throughout elementary school, secondary school and Quebec's CEGEP. He spoke about the strong familial support to do well in mathematics and science as a "cultural thing" and while he reflected on the pressure he was under to do well, he claimed that he "...enjoyed it and liked it, and I took along a lot of pride in learning it and doing it well...but it was really to please my parents." Barry's experience in learning mathematics was largely through being taught in a traditional way with emphasis on the "textbook, examples, blackboard and drill". He described his learning as being primarily through memorization, rote, and "learning by examples... lots of examples." While this learning style appeared to allow him to do well in secondary school, Barry described experiencing significant problems at university when he struggled with the applied mathematics courses of his engineering degree. While he passed all of his university mathematics courses, he acknowledged that he really did not understand much of the content. Barry graduated from his teaching college course with a certificate to teach at the Primary/Junior division. Both of his practicums were in elementary schools with students in Grade 3 and Grade 6 respectively. During the following summer, he took an advanced basic qualification to enable him to teach senior mathematics. While completion of this course meant that he was qualified to teach in the secondary panel, the online course did not offer any exposure or experience in teaching in the secondary school.

Teaching Experience

At the time of the interview, Barry had just completed his first semester teaching the Grade 10 Applied mathematics course and two other non-mathematics courses - Grade 10 Careers and Civics and Grade 9 Computer Art. Although Barry had been a supply-teacher at the same school and recognized many of the students when he started to teach, this was the one and only mathematics class that he had taught and almost all of the learning experiences that he described came from this one episode. Barry described this first teaching experience as very challenging, citing many reasons including having had no prior experience teaching in a secondary school, feeling unsupported by other teachers, feeling under-prepared with the course mathematics content, and having a class of academically-challenged students, many of whom had behavioural and learning difficulties. In addition, he learned of his teaching assignment just two weeks before the start of the semester and had little time to prepare for his classes:

And it wasn't such a positive experience, it's just... I learned a lot. I think I learnt a lot compressed in one semester that some new teachers say that they learned in two or three years and then they discover that they don't want to do this anymore... [laughter] So, for me I'm glad it happened very quickly all at once so that I know what I'm going to be confronted with... and I see it as a "no pain, no gain" sort of thing. So it was a struggle.

Barry acknowledged that during the interview he tended to concentrate on the negative aspects of his experience. However, as frustrating as he found the overall teaching experience, in his class of 23 students, he did single out a couple of students who he felt that he had helped in some small way to overcome their fear of mathematics:

...there were two girls who were afraid of math, and I checked all the grades last year... they had 50s and 60s and they were the ones who needed a lot more attention and support and a little cheerleading and keep trying, work at it and actually they succeeded so I was quite pleased with that.

Barry described at length his struggle to find a classroom management style that appeared to work with his students, many of whom had significant difficulties with mathematics having entered his class with grades around 50% – a mark he suggested that had been “a gift from the teacher the

previous year.” Barry felt that many of the behavioural problems in his class, and in particular some of the ploys to distract the class, came from the students’ fear of mathematics and of further failure. As Barry put it, for his students “the stress of ‘My god, I really don't understand’” would lead to distractions and diversions such as “Sir, that's a really nice tie!”

Barry stated that at the start of the semester he was completely out of his depth, being caught off guard by how academically weak some of his students were and how challenging he found classroom management. He spoke about how his expectations had been much higher starting the semester and how, against his nature, he found himself following the advice of other teachers who encouraged him to be more strict with the class and adopt a more traditional teaching style based on memorization and rote learning. He describes some of these comments:

They're not capable doing what you're trying to get them to do. They just need to be told. These are the rules of behaviour. These are the rules in mathematics, and give them categories, simplify as much as you can, give them a quiz in a day or two before they forget because they will forget.

Barry admitted that when he became stressed he resorted to just rote teaching, mirroring the way that he had been taught and although he acknowledged that a more traditional teaching style appeared to work with his students, he was clearly conflicted with this position. Close to the end of the interview, Barry commented:

I kick myself when I believed them... at the beginning I was fighting them in my mind that “no... These kids can think, they weren't taught properly” and I'm the new style mathematics teacher. And each time, I think I felt discouraged when I came up with new innovative ways and maybe these teachers, in retrospect looking back, you know, they're a bit older and a different generation and I think that had a big influence on me, and I felt discouraged from using [manipulatives] because ... it was a challenging.

View of Mathematics and its Teaching and Learning

Barry did not describe specifically what he felt was the nature of mathematics, but suggested that for him, mathematics was a very visual and hands-on subject. Barry spoke about his experience of learning mathematics and coming to understand that mathematics was so much more than its

content. To Barry, there was a procedural component to solving problems that included “understanding what are you looking for, what you need, and what you need to do to achieve the next level.”

Students as Learners

Barry described his class as consisting of very different groups of learners. One group consisted of students who made little effort to learn math, possibly because of past difficulties and repeated failures. Barry referred to another group of primarily identified students¹³ who were trying, yet failing and giving up. A third group, consisting of some ESL¹⁴ students – who Barry described as “fabulous to work with... They both got 80s and 90s” – appeared motivated and achieved excellent grades. Barry quoted one of his colleagues describing his Applied level class students as “...either they get it or they’re not going to get it and they don’t care about getting it.” Additionally, other teacher colleagues suggested that rather than focus on any of the ‘foundational’ pieces of mathematics that many of his students had not mastered, Barry would be better off teaching them “how to use the calculator all the time.” Another suggested that his students are:

... not capable doing what you're trying to get them to do. They just need to be told. These are the rules of behaviour. These are the rules in mathematics, and give them categories, simplify as much as you can, give them a quiz in a day or two before they forget because they will forget.

While it may be easy to consider many of Barry’s experiences as negative, it is worth reminding the reader that Barry was a novice contract teacher working on significant classroom management issues; a situation that could be typical for many beginning teachers.

¹³ This refers to students who have an Individual Education Plan (IEP) to address some special learning need.

¹⁴ ESL is the acronym for English-as-a-Second Language and refers to students for whom English is not their mother tongue.

CELIA

Mathematics background

Celia received her elementary and secondary schooling abroad and went to university in Canada. She described her mathematics (and science) experience of school as consisting primarily of rote learning, lots of memorization and huge amounts of drill through homework, which she described as “bizarre”. She noted that, “they made us memorize everything so I hated it.” Celia suggested that compared to her birth country, the Canadian education system “is a little more liberated, a little bit more student-oriented in teaching math.” She stated that the teaching style of her school days, and especially the emphasis on practice, helped her when she came to Canada as a university student as she had little understanding of English:

I didn't understand much English at that time, and yet I knew what kind of problem I was going to face - even the word problems because I had done so many. I could just look at the question and well [fingers-clicking]; I could tell what it was asking even though I didn't understand the language. So I didn't have to worry about it.... I had a huge advantage among the kids because I was always good at mathematics and physics and chemistry.

Teaching Experience

Celia holds a bachelors degree in chemistry with a minor in mathematics and worked as researched for several years. Prior to returning to university to obtain her teaching certificate, Celia taught her mother tongue to public servants, ESL to seniors and her home country's language, history, geography and politics to secondary school students in an International Studies program. Celia described that she was encouraged to obtain her B. Ed. by the school principal and obtained her teaching certificate for mathematics and chemistry. Her experience to date had been almost exclusively teaching mathematics with only a couple of science courses. She had been teaching full-time for over 10 years, teaching almost every course in mathematics from Grade 9 to OAC and covering the Academic, Applied and LDCC programs.

View of Mathematics and its Teaching and Learning

Celia spoke about mathematics as being a very logical subject that helped students develop a “problem-solving attitude”. She described seeing mathematics in terms of numbers, relations and patterns and that by doing mathematics “...we're dealing with all these relations and were looking at the patterns; we're developing the ideas” and providing multiple ways to look at and solve problems. She described seeing mathematics as all around her, pervading every facet of life, and claimed that, “we need math... whether you have no interest in math, whether you have a huge interest in math, it is with you”. She encouraged her students to identify with and embrace the mathematics that was around them:

So whether you like it or not, mathematics is all around us. People just don't recognize it. From that basic thing, we expand more and more. How far you go depends on what kind of career you might choose or how your mind works and how interested you are... don't assume you don't deal with math.

Celia felt that as students engaged in mathematical problem-solving, they developed skills of organization and logical thinking that she considered “much more important nowadays than ever before”. She encouraged her students to recognize these as transferable life skills that have value far beyond their interest in mathematics. Celia also spoke about the role of practice and memorization in learning the discipline of mathematics, and used the analogy that in order to appreciate literature and develop understanding from reading, it is necessary to master a certain amount of vocabulary and grammar. She stated that practice and memorization in mathematics is important and necessary because “in order for us to understand things there are some [fundamentals] you have to learn”. She added:

In mathematics, and somebody might argue, [in other subjects too] it's the same thing. If you cannot do basic arithmetic or algebra, even if I give them huge concepts, they really cannot attack those problems. They might know how it should be done but what can you do with that? I think that teaching theories are always changing, but there are some things we have to remember.

Celia claimed that one of her strengths as a teacher was her ability to quickly sense her students' attitude and aptitude to mathematics and to differentiate instruction to offer them what they need. She described how she responded to students' questions as to why they learn particular topics by offering them real-world problems such as making the connections between ideas in trigonometry and polynomial expressions to the trajectories of comets and the design of bridges. She spoke of her desire to help each student achieve their mathematical potential based on their own interest and career goals:

So there is mathematics in every corner of their life and now where do you want to go? How interested are you? Where is your goal? I will let you get there. I will try my best to get you there. So I start from there and ... you've got to look at the big goal and where do you want to go with mathematics.

Celia described that the content of what she had to teach as one "big circle", where key mathematical concepts are reinforced and developed with increasing sophistication, complexity and depth as students progress through secondary school. She felt that teachers need to create a consistent learning environment for students to continually build upon, reinforce and develop mathematical concepts from grade to grade, as outlined in the curriculum. She lamented that within her school "nothing like that is happening. Every teacher does everything differently." Celia went on to claim that when, as in her present situation, teachers work in such isolation, "It doesn't serve the kids well."

Students as Learners

Celia described some of the different learning styles and attitudes of her students in her mathematics classes. She identified one group of very capable and gifted students that she jokingly referred to as "six months waiting, two months learning" who spent a considerable amount of class time waiting for new and challenging mathematics concepts. She described another group of students who continuously struggled with the material yet persevered. Celia spoke of a third group of students who complain about mathematics and give up without really trying. She described these student

groupings and their attitudes to learning within the larger context of the culture of learning in the school and their own conceptions of what class routine should be and their role within it. In Celia's opinion, her school did not model a culture that promoted and encouraged learning through investigation:

I'm a strong believer that the institution [of] the school... is geared for certain kids. If the kid is reasonably submissive and reasonably structured... or have great support at home and believe in authority, they do well at school. It's an institution after all... it's a structure that we are imposing on them.

DIANE

Mathematics Background

Diane talked about her early mathematics learning as being very traditional, with strong emphasis on solo work, memorization and rote learning centred on text-book lessons with lots of drill-based questions for homework. She commented that there was little group work and no use of models – “I never got out of my seat to touch anything” – describing the entire learning experience as “brutal”:

Tools? Well we had a yardstick, we had a ruler. We had graph paper but all that graphing stuff was just point plotting. There was nothing else.... Chalk-and-talk for sure, study the book, study it again and try to predict what [the teacher] was going to put on the test.

However, Diane liked mathematics, found that she was good at it and was typically the student with the highest marks throughout her school years. Diane described in detail her senior years in secondary school and her specific challenge with learning calculus in Grade 13. It was then that she realized that rote memorization was insufficient for her to understand the fundamental concepts and described that when she came across calculus again during university mathematics classes, she found herself being “absolutely clueless”, requiring her to spend many hours in additional study. While she did pass the course, she felt that she “never understood a single thing” and it was not until she was a teacher and took lessons from a tutor that she finally understood calculus:

Everything else I understood up to calculus. No problem because I understood it. You could just say it and I would understand it. But calculus was rote... [There was] no understanding. I never knew what we were doing in calculus.

Teaching Experience

Diane majored in biology at university and went on to gain a teaching certificate in science and mathematics. At the time of the interview Diane had been teaching mathematics in the secondary school for more than 15 years and has taught all of the courses from Grade 9 to 11 in both Applied and Academic programs, and one Grade 12 course, through several revisions of the Ontario mathematics curriculum. She had been teaching at her present school for nearly 10 years and was teaching two Grade 10 Applied level classes and one Grade 11 Functions course. Although qualified to teach both science and mathematics, all of Diane's teaching experience had been in secondary school mathematics.

View of Mathematics and its Teaching and Learning

Diane described her view of mathematics as being "anything that has numbers in it... commonality of numbers" and that to do mathematics was being "able to figure out stuff that has numbers in it". She suggested that examples of mathematical proficiency would be working out your pay cheque and understanding how to determine a correct medicine dosage. Diane suggested that she saw this commonality of numbers as the connection between and across the many different strands in the secondary school curriculum.

Diane described her current school as being ethnically very diverse with many of her students coming from recent immigrant families where their first language was other than English, and whose cultural experiences were very varied. She stated that this diversity required her to present mathematical concepts and ideas in many different ways in order to connect with her students' prior experience and knowledge. She also felt this she had to be more carefully monitor her students' understanding: "I go back to school and ... realize that they don't know what I'm talking about. I have

to either redo it, re-say it, do it a different way, emphasize it, repeat it more often.” Diane also reported changes in her students’ interest in, engagement with and attitudes towards mathematics since she started her teaching career. She stated that nowadays there is greater onus placed on her to keep the students in class and motivated to be there – especially problematic as there was a chronic problem with absenteeism in the school – and to make mathematics more interesting and engaging by having the students investigate and explore through the use of models:

In the old, old days, children would go to the Academic [Grade] 10 class and it was their job to be interested. They would be interested. They would stay there and watch you and then do the math. But now they're not interested. So if I tried to do [math] the old school way they wouldn't listen. They would talk. They would fail and they would skip. They would leave, they would do anything they want... throw stuff. So I had to figure out how to make it interesting enough for them to participate....So how can I be interesting? I can't do what I used to do, I can't hold their attention. There are other things to look at besides me. So now I have to be a little more interesting.

Diane also described how her own thinking about teaching and learning mathematics and the role of the curriculum has changed over her years of teaching. She noted that at the start of her career there was enormous emphasis placed on covering the curriculum content, with little regard for how, or whether, students really understood the material. Diane stated that she now tries to take a broader view of the curriculum by covering the key ideas while being prepared to skip some of the finer expectations if her students need extra time to better understand the concepts.

I sort of skip so I do touch the main points and sometimes we just have to let something go and we miss some stuff. And according to the curriculum I don't always hit all the fine points. I have to get all the titles and I do the best I can at that. I've tried the other way... I go through the curriculum and leave them behind. They don't understand. Or [another teachers] does and I get them and they don't get it. Well, as soon as you're teaching somebody who doesn't get it, you don't have their attention and they become a problem.

Speaking primarily about the academically-challenged students in her Grade 10 Applied classes, Diane outlined some of the strategies that she used to help keep her students motivated and engaged in doing mathematics. She described breaking down larger problems and concepts into smaller steps and setting enough questions at just the right difficulty level that she knows her

students can complete. She found that once her students see that they can do some mathematics, they have the confidence to continue to try more challenging problems:

...if I go down low enough so they can do stuff, they'll keep doing it. But if I jump in too high and they can't do it they just die on me. And then you get behaviour problems. The behaviour problems are such a negative experience that I don't like to do that. I'd rather take more time to teach.

It is perhaps not surprising that when Diane spoke about the goals she has for her students – largely as they apply to her students in the Grade 10 classes – she emphasised the overwhelming need to instil and develop their self-confidence to see that they can actually do some mathematics because “when they can do the mathematics it's amazing”. A further, yet significant, goal is that her students actually learn the mathematics she teaches them. As Diane stated, she wants to teach mathematics “so that they can have a choice” to follow any career path, so long as they are prepared to make the necessary effort:

I'd like to be able to teach so that the kids can understand and if you don't want to get a high mark on your test it's because you don't want to get a high mark, or you don't want to write the test or whatever, as opposed to I couldn't communicate to you how to do Pythagoras or I couldn't show you how to do that.

Along with these motivating strategies, Diane described how she now assesses and evaluates her student's academic performance by offering more flexibility in completing in-class tests and submitting other pieces of evaluation. She noted that the more traditional methods of evaluation that involved setting a test for a particular time and date resulted in some of her students skipping school and ultimately dropping out. She reported that now she allows her students to retake tests – “like a driver's test” – during the lunch hour or after school and as many times as they wish, counting only the highest mark that they achieve. She reported that this flexibility appeared to motivate her students to learn more math, and often from each other. As she reported:

Asking [students] to be ready on this day for a test and this is your one chance I don't think works any more... So now I let them write as many tests as they want, as often as they wish...In Grade 10 Applied if you miss your test or quiz for any reason you still can

rewrite it and you can still get the mark. And I think they learn more mathematics that way... The highest mark you can give me... I don't care how many tries. Because I want you to learn the math... The catch is that you can't do it in class. You have to do it after school or at lunch.... And they do.

Along with strategies for motivating and assessing her students, Diane also spoke about trying to fill any knowledge gaps that she can identify in her students' understanding. She gave the example of how many of her students seemed unable to use a calculator and while she was initially reluctant, she reasoned that if she didn't take the time to teach them to use it, no one else would do so. As she concluded "so now I teach the calculator. Sometimes I get [students] and they don't know how to use a ruler. Any skill that I can identify that they can't do, I try to teach them."

ELLY

Mathematics Background

Elly provided a very rich description of her own mathematics learning experiences as a student throughout her school years. She said that she found mathematics quite easy throughout her elementary school years until she moved abroad and attended a local school for her Grade 9 year. In this school, Elly was placed in a class working at a more advanced level of mathematics that she was used to and reported that the combination of learning a foreign language while working with very hard mathematics was particularly challenging and frustrating. While she got a lot of help from her father (who was a mathematics professor), she recalled how difficult she found this period:

First of all it would take me about two hours to read a one-page [foreign language] story and then I'd sit down with my mathematics and I'd be crying all night because I couldn't get this... "I can't get this, I'll never get this, it's so hard"... and I think I was more interested in crying than actually figuring it out. So I understand frustration, let's put it that way.

Elly described that once the family returned to Canada, and thanks to having been pushed to work at a more advanced level of mathematics while abroad, the remainder of secondary school mathematics "was easy". It was not until she was at university that she began to find mathematics

courses to be hard, largely because the content was more theoretically-based. She described these difficulties as challenging the way in which she had learnt and come to think of mathematics up until that point:

...university was the first place where... mathematics became really hard because up to then, we would be doing all number stuff and it's easy... you just have to remember some formulas... High school mathematics was easy. University math, when you are getting into more of that theory and third-year algebra... that was horrible and scary.

While she did get through her university mathematics courses, she had to “study like crazy” yet acknowledged that she “didn’t understand some of the harder stuff”. Like Diane, Elly described in detail her challenging experiences with learning university calculus:

So like in your first year calculus they teach you the Delta Epsilon proof of the limit. So I memorized it and I didn’t have a clue what it meant. In second year calculus, of course, you do the same theorem because it is the basis of the whole calculus. So I memorized it again and I didn't make any sense to me. Finally, in third year calculus, I sat down to memorize it for the third time and I went ‘gosh, there's no need to memorize it... it makes perfect sense’.

Elly mentioned that she tells her own students this story in order to encourage them to persevere with their studies - especially when the mathematics becomes more challenging – to emphasize that it takes time, and often several times, to form a deep understanding of some mathematical concepts.

Elly described her experiences at teachers college and during the additional mathematics specialist courses as doing many “fun activities” that helped her to see beyond the content and explore many different pedagogical approaches to teaching mathematics:

At [name of university] what I learned from the mathematics section wasn't the mathematics because the mathematics you teach at the grades we teach... the difficulty isn't the math, it's how you present it ... and put together a lesson, that kind of thing, that engages the kids and gets them doing something other than just sitting there staring at you wishing you would stop.... It's just the thinking of what do I really want to get at the end of this, what is my actual objective, and then how do I get there through a motivator. And I hadn't thought of it that way really before.

Teaching Experiences

At the time of the interview, Elly had been teaching at the secondary school level for 22 years immediately following graduation from her teacher education program. She had been teaching mathematics in the same school for her entire teaching career, covering almost all courses across all of the grades, and levels (Applied, Academic and Transition). She reported that she was currently teaching two Grade 12 Advanced Functions classes and one Grade 11 Functions and Applications course. The following semester she was slated to teach two Grade 12 Calculus and Vectors courses and another Grade 11 Functions and Applications course. Elly described that she had initially been hired to teach the senior grade courses and that once she was comfortable with her assignments it had been easier for her and her schools' administration to allow her to continue teaching the same grades. She stated that her administration generally allowed her to pick the senior courses although more recently in the mathematics department, teachers were required to teach a combination of senior and junior classes. She described the younger grades and the Applied classes as being more challenging for her because, "in a way there's less marking but they're more hands-on, like they are more tiring on a daily basis." Elly claimed that initially she found Grade 9 students harder to manage, although now that her own children are of a similar age, she found that she was better able to understand them.

View of Mathematics and its Teaching and Learning

Elly described her view of mathematics as ideas that can be ordered in a logical and linear manner and said that she saw doing mathematics as "playing with ideas and thinking of some logical and linear way of getting those ideas across...and modelling situations". Similarly to Alice and Celia, she explained that the discipline of thinking logically and linearly were useful skills that her students should develop to help them way beyond the field of mathematics. In Elly's view students needed to memorize some mathematics facts and procedures because those students who don't memorize their multiplication tables, for example, are at a disadvantage. She also suggested, like Celia, that rote memorization may even be useful for some mathematics concepts, in that if they are

memorized first, and at some point later on, revisited or built upon, they may subsequently make sense. As she stated:

There is nothing fancy about it. You just memorize some stuff. And sometimes some concepts, if you memorize them first, then they'll eventually make sense to you but you would like people to actually know what the idea is behind it... what the real idea is, what is the concept... It really doesn't matter whether the answer is five or six? But do you know what it's really about?

Of interest is that Elly's views about the potential value to students of rote memorization preceding comprehension appear to mirror her university experience with calculus concepts where for the first two years of university calculus she worked hard to memorize procedures and formulae while the meaning of calculus concepts eluded her. It was only in her third year, revisiting those same concepts, did calculus concepts start to make sense.

Elly also described her efforts to teach for understanding so that her students grasped the curriculum's concepts and big ideas. However, she acknowledged that the time necessary to obtain this deeper understanding came at the expense of course coverage and the number and difficulty of practice problems her students had to solve. While she recognised the importance of curriculum coverage, she felt that once her students had truly grasped the mathematical concept, they would have the tools be able to apply that knowledge to solve more difficult and contextually different problems:

I guess for me it really comes down to... I'd rather they understood the concept of adding like terms and really got it than we did six pages of really hard ones... So I think if you know what you really want out of that unit, you can make the time because if the idea is the concept behind it, sure maybe I take some of the time that they'd spend on more complicated [problems] but I'm not sure that's better or worse. Because in the end, when they have a hard one they may be able to figure it out. So I find I can make time even though I may have to save on something else.

Elly described her goals for her senior grades students as wanting to develop their mathematical competence and confidence so that when faced with new problems, they can understand what is required to be solved, and know how to select the right tools to use. For her

younger grade students, she defined a slightly different set of goals for them – that of helping to develop their self-confidence to believe that they can actually do some math. Elly emphasised the importance of instilling confidence and building competence in these students so that when faced with a new problem, they could identify and use the right tools. Similar to Diane, as a strategy to help students Elly described breaking down larger mathematics problems into “bite-size” pieces and reinforcing the big concepts through lots of repetition. For all her students, Elly identified mathematical problem solving as the means by which her students could develop logical thinking skills.

Elly described how she attempted to build new ideas upon her students’ prior knowledge by reinforcing the connections within and between different strands of the curriculum, presenting the mathematical concepts as elements of a united whole. She gave as an example having her students identify the connection between the geometric distance between two points on a graph and the Pythagorean relationship with which they were already familiar. She wanted her students to reason that rather than having “to memorize 50 formulas, I already have a bank of five. I only need an additional three and I can bring everything together if I truly understand”, although she acknowledged that such an endeavour was “hard.”

Students as Learners

Elly described how many of her students expect a particular school routine that follows a familiar traditional script of going to class, passively listening to a lesson, writing notes and then having additional practice questions assigned for homework. Elly noted that her students reacted in two very different ways when this routine was disrupted when, for instance, she used manipulatives in the classroom. While some students would resist doing or trying anything different, other students would relish the disruption and engage thoroughly with the manipulatives. Elly stated that she wanted her students to see change as a permanent part of the class routine and as a result liked to mix

up the classroom to keep it interesting. As she concluded "...anything you do different is always good".

It's like when you put together some fun game like play MathO, which is just like bingo but with mathematics questions... And they don't think they're doing any work. Well of course they are! Or you do mathematics Jeopardy and they think they're not working that day. Or they don't want to play the game, that's the worst part, and then you're like "but you always say you don't want to do the regular work" so it's always a challenge but fun...

FRANCES

Mathematics Background

During her elementary school years, Frances was enrolled in an advanced mathematics program with about a dozen other students that compressed the Grade 6, 7 and 8 curriculum into two years – an experience that she described as “terrible. She entered Grade 9 mathematics course a year ahead of her peers feeling “horribly unprepared” and obtained the lowest mark of her entire secondary school classes. However, Frances gained considerable support from her father, who she described as very strong in math, during this difficult year:

... but I do think that having skipped some of that mathematics while my dad was able to help me in quite a bit of it, it was devastating for some of the foundations... So I had a lot of one-on-one tutoring and ended up with a B in the course, which to me was a very poor mark, but there it is.

While she enjoyed the boost in confidence in being considered one of the few students to be enrolled in this accelerated mathematics program, she acknowledged that she had some serious gaps in her knowledge that left her grappling with a lot of mathematics that she really didn't understand:

I was in tears quite frequently that year because I just felt like I didn't understand the mathematics so my confidence was quite shaken. But again at school I felt quite proud to be in the accelerated program which is a funny thing.

Throughout secondary school, Frances was enrolled in a gifted program with a cadre of around 20 other academically advanced students who came together to work and support each other. She and two other students remained a year ahead in math, yet found no support from their older

classmates who treated them as outsiders, reporting that “the three of us really struggled through that together.” She noted that all of her Grade 10 to Grade 13 mathematics teachers were excellent and described their teaching methods (and that of her father in her home tutoring) as being “very traditional”, relying heavily on textbooks and offering no experiences with concrete materials, computers or graphing calculators. Frances gave an account of the typical classroom script that consisted of “sitting in traditional rows, in a class of 30. The teacher generally started by taking up the homework questions, then there was a board lesson and then time to work.” She spoke of her calculus teacher as the only one who provided a different experience in the classroom by having his students work together in regularly rearranged groups, where each member would be given a different question to answer for homework. Frances claimed that through this group work “he really fostered a culture of relying on each other which was nice; it was quite different from anything else I had experienced.” Frances stated that overall she did very well in secondary school with marks consistently in the high 90s, although of all the courses that she took, mathematics was often her lowest mark. Frances spoke about her sense of identity as a student principally coming from her strength in arts and science courses and not in mathematics.

Frances recounted that throughout secondary school she considered herself to be a “very good book learner so I was very good at taking things from the textbook”. However, she reported that she never felt that mathematics came naturally to her and that she always felt challenged by an incomplete understanding of the concepts:

I really had to work at it, to be honest with you, and I think that’s largely why I am so interested in teaching mathematics because, while I did get quite strong marks in the subject, it was something that I didn't feel came naturally to me.... I always felt that on the mathematics test that the one question that really showed whether or not you understood... less than half the time I was really able to demonstrate a real understanding... It always felt to me that there was something missing. It always felt to me like there was a trick or something that I was missing and it's unfortunate that I seemed to think of it more as a trick than a real understanding, but that's how I felt as a high school student.

Despite her misgiving over the depth of her mathematics understanding and feeling that she missed some of the ‘fundamentals’, Frances found that being in the accelerated mathematics class and being a year ahead in mathematics to be useful in that it allowed her to spread the three OAC mathematics courses (calculus, algebra and geometry) over two years rather than having to take all three in her final year.

Frances completed an undergraduate degree majoring in chemistry with enough courses in mathematics for mathematics to be considered as a minor. Frances spoke about a particularly good calculus teacher at university who, despite lecturing to more than 400 students at a time, presented the material in an interesting and engaging manner by using multiple representations of mathematics and making connections to science, music and art. He encouraged his students to work together in groups to create and critique multiple solutions to solve problems. For Frances “...it was the first time for me that mathematics was presented in terms of something more than right and wrong. So I really enjoyed that.” Frances stated that she did “okay” in her other university mathematics courses and doing particularly well in those courses that connected with and supported the physical sciences of her major. She commented that her two highest marks in the first year at university were in calculus and algebra. She reported that as she neared the end of her degree she “felt more confident in my mathematics and I really enjoyed the courses”.

Teaching Experiences

At the time of the interview Frances had taught mathematics in two large urban secondary schools for a total of 9 years. She had taught the Grade 9 and 10 courses in Applied and Academic programs, the Grade 11 College and University courses and the Grade 12 Data Management course, claiming greater experience in teaching the Grade 10 Academic and Grade 9 Applied courses. Outside of math, she had also taught music, science, computers and “many other things”. In addition to her teaching duties, Frances had also been a department head for a couple of years – a position that

provided her “interesting professional development opportunities”. Frances indicated that her upcoming teaching assignment was in a secondary school closer to home, and she was eager to see the culture of her new school’s mathematics department regarding access to, and use of manipulatives and technology.

Frances described how when she first started teaching many of the experienced teachers preferentially chose the more senior classes, leaving her and other new teachers, with the junior courses “because those were the leftover courses in the timetable. So I got pretty comfortable with them.” Frances gave three reasons for the preference of some of her teaching colleagues for the senior classes. She claimed that several years ago, many veteran teachers resisted teaching the Grade 9 courses because of the EQAO¹⁵ test and the additional workload that accompanied it. Frances also described a more recent emphasis – primarily driven by revisions to the mathematics curriculum and commensurate changes in focus of board-wide professional development – in providing Grade 9 and 10 students a richer mathematical experience through exploration, investigation and problem solving that encouraged the use of technology and manipulatives. She suggested that some of the more experienced teachers found the consequent reframing of content, pedagogy and the teachers’ role in the classroom to be particularly challenging and where possible, avoided:.

So if you're teaching a Grade 9 course with three other teachers say and you're not necessarily comfortable with some of the manipulatives and technology you sort of get pulled into it by some of the teachers that you working with, which is a great learning opportunity. But it can be a little bit disconcerting if you've been teaching for 20 to 25 years and you're working with some, what you would consider younger or junior teachers, for them to have an expertise with the technology and the manipulatives, it does take some reconsideration of the material and quite a bit of effort to reconsider the approach to the course, the philosophy of how you're approaching things, rather than perhaps a more traditional Socratic method.

¹⁵ The Education Quality and Accountability Office (EQAO), is an agency of the Ontario government that designs and supervises tests for all students in Grades 3, 6, 9. In this discussion, EQAO specifically refers to the Grade 9 Assessment of Mathematics undertaken by students in both the Academic and Applied programs.

The third reason Frances cited had to do with what she referred to as “a little bit of elitism” that she heard from teachers who believed that the senior courses offered them the opportunity to do more interesting and difficult ‘real’ mathematics with their students:

I do see that expressed now and again, not by a majority of teachers, but I have certainly encountered a few in timetabling that have told me, and come to lobby me for senior courses saying that they really believe they have a really strong mathematics background and believe they are the best to teach that “deeper” mathematics.

Frances also noted that in her experience, some principals had been prepared to allow several of the junior grades, and in particular Grade 10 Applied classes, to be staffed with teachers unqualified in intermediate or senior mathematics. She stated that she has found that mathematics specialist teachers were preferentially selected to teach the senior grades rather than the junior grades.

... If there are holes in the timetable they tend to go in the junior courses and again, particular in the Grade 10 Applied. The reason for that... the Grade 9 Applied has the EQAO test and the principals are interested in keeping the scores high in that so it tends to be the 10 Applied course, and sometimes the workplace mathematics courses end up with some of the unqualified teachers, in my experience.

View of Mathematics and its Teaching and Learning

Frances stated that to her mathematics is a language to describe the natural world and how it works and a way to model physical phenomena. She felt that the discipline of mathematics and being engaged in doing mathematics allowed students to develop logical thinking skills and debating skills through making and defending assertions. She described how in her secondary school experience she felt that the big ideas in mathematics were largely based on procedural fluency and a “whole lot of algebraic techniques to simplify problems“. She stated that while she still believed in the importance of procedural fluency in order to be able to function and explore mathematics, she no longer sees this as the “sole idea of math“. Despite her high marks in mathematics through secondary school and university, Frances also spoke about her early perceptions of the field of mathematics as being an almost exclusively white male domain, noting that the most successful people in her university

cohort in mathematics were male. She commented that it "...seemed to me to be a bit of a deterrent. I felt that my confidence was a bit shaken."

Frances mentioned that re-learning secondary school mathematics over again as a teacher through the process of doing investigations with her students and exploring different ways to model problems had helped to make connections to other mathematical concepts much clearer. She described how through teaching and learning using technology, concrete materials and manipulatives she had deepened and more richly connected her understanding of mathematical concepts. She stated that:

... Encountering the area model of multiplication to me was like a bolt out of the blue. To see that there could be this physical connection with algebra was something to me that was just miraculous.... Going through my first few years of teaching I found some beautiful ways of presenting material that I had not experienced as a student that really began to clarify many of the fundamental understandings that I didn't realize I didn't have until I became a teacher.

Frances noted that along with seeing mathematics as a more richly interconnected discipline, so too did she feel her own approaches to solving problems had evolved. She described her slow shift away from the more comfortable tendency to algebraic, symbolic and more traditional teaching approaches, towards an active classroom learning community that used a wide variety of tools and techniques. Frances also spoke about how teaching and learning through the investigatory process had increased her comfort level and confidence with the mathematics itself, which has also helped her become more aware of and to look for student solutions that she hadn't necessarily anticipated:

There's quite a bit of research indicating that one's own experience as a student has a very powerful model for one's own teaching. So as I become part of a community of people exploring ways to engage students with mathematics, I do feel so much more comfortable... I really have felt more confident over the last few years having students collecting data, arguing about things, investigating ideas.

Participants at a glance

In this chapter, I have provided detailed descriptions of the participants' views and experiences as both students and teachers of mathematics. These descriptions provide a broad context within which I consider the teachers' views on – and use of – manipulatives as mathematical learning tools in the following chapter. However, several key findings emerged from the data, which offer some insight into the experiences of the participants. The first point that became apparent was that the participants' experience of mathematics during school and university years was characterized by traditional teaching methods that posited a passive learner who was expected to learn by rote, memorization and drill. While each of the participants appeared to do well academically in this environment, with the exception of Celia, all found this traditional teaching approach was insufficient during the demanding university mathematics courses that required a thorough understanding of the concepts and an ability to flexibly apply this understanding to novel and different situations. As a result, albeit through very different routes, each participant came to experience a fundamental challenge to the traditional learning approach as an appropriate and effective way to learn. Secondly, and it should come as no surprise, none of the participants had gained any appreciable experience in the use of manipulatives in the teaching and learning of mathematics until attending teachers' college or, in the case of the three older teachers – Celia, Diane and Elly – until they had already begun teaching.

The following tables summarize key facets of the participants' teaching qualifications and experiences (Table 1), descriptions of their background in mathematics (Table 2), succinct quotations of their experiences as students in secondary school (Table 3) and university (Table 4), and views on the subject and discipline of mathematics (Table 5).

Table 1 details the participants' teaching qualifications, number of years teaching, number of schools where they have gained this teaching experience and the grades of the Ontario mathematics curriculum that they have taught.

Table 1: Participants' Teaching Qualifications and Experience

	<i>Qualifications</i>	<i>Teaching Experience</i>	<i>Grades Taught</i>
Alice	Chemistry and Mathematics degree; Teachers College [I/S]: Mathematics & Science	3 years teaching experience in one school	Grade 10 pre-IB, Grade 12 Data Management, Grade 11 College
Barry	Engineering degree; Teachers College [P/J]; ABQ for senior math	1 year mathematics teaching experience in one school	Grade 10 Applied
Celia	Chemistry degree; Teachers College [I/S]: Mathematics & Chemistry	>10 years mathematics teaching experience in three schools	All courses
Diane	Science degree; Teachers College [I/S]: Mathematics and Biology	15 years mathematics teaching experience in two schools	All courses
Elly	Mathematics degree; Teachers College [I/S]: Mathematics and Foreign Language	22 years mathematics teaching experience in one school	All courses
Frances	Mathematics degree (concurrent), B.Ed. [I/S] Chemistry and Mathematics	9 years mathematics teaching experience in two schools	Most Grade 9-11 courses

The table below (Table 2) presents the participants' background in mathematics and includes details of where they went to school, the teaching style they experienced when learning mathematics, and any experiences with manipulatives and models. Some quotations have been included to provide pithy insights into these areas of the participants' background.

Table 2: Participants' Background in Mathematics

	<i>Learning Environment</i>	<i>Teaching Style when Learning Mathematics</i>	<i>Experience with Manipulatives</i>
Alice	JK-13 abroad	Traditional “chalk-and-talk” delivery “Everything had to be traditional math, it’s abstract. You learn it to the love of it, for the sake of it...”	Platonic solids in spatial geometry “...everything in secondary school and ...university there was no manipulatives at all.”
Barry	JK-11, CEGEP in Quebec	Traditional learning approach: “textbook, examples, blackboard and drill” and described his learning as being through memorization, rote, and “learning by examples... lots of examples.”	No use of models or manipulatives
Celia	JK-13 abroad	Traditional learning approach: rote learning, memorization and huge amounts of drill through homework, described as “bizarre”. Aced mathematics & science.	No use of models or manipulatives
Diane	JK-13 Canada	Traditional learning approach: memorization, rote learning, text-book lessons and lots of drill. Described the learning experience as “brutal”.	No use of models – “I never got out of my seat to touch anything.”
Elly	JK-13 Canada, Grade 9 abroad	Traditional learning approach.	No use of models or manipulatives
Frances	JK-13 Canada	Traditional learning approach; gifted program & advanced math; textbook lessons,	No manipulatives or technology

Table 3 and Table 4 present the secondary school and university experiences of the participants, described in their own words through meaningful and illustrative quotes.

Table 3: Participants' Secondary School Experiences

<i>Learning Experiences in Secondary School</i>	
Alice	"I really liked the way in which we explored geometry and algebra, because there were the drill exercises that were assigned as home work and then there were the challenging problems from the mathematical."
Barry	"I enjoyed it and liked it, and I took along a lot of pride in learning it and doing it well... it was really to please my parents.... it's a [cultural] thing"
Celia	"I was always good at mathematics and [science]. I didn't study much, compared to other kids and I jumped through the whole thing. So it's a good thing, that I have that kind of brain."
Diane	"I liked mathematics and I was very successful in mathematics and very competitive so I wanted to be the top guy and I pretty well was. You could just say it and I would understand it."
Elly	"[In secondary school] we would be doing all number stuff and it's easy... you just have to remember some formulas... I don't know, high school mathematics was easy"
Frances	"[Math] was something that I didn't feel came naturally to me. I was a very good book learner so I was very good at taking things from the textbook but ... I didn't feel I had the understanding."

Table 4: Participants' University Experiences

<i>Learning Experiences in University</i>	
Alice	"..to ensure admission to university there was a lot of pressure.. [However] we enjoyed it because the more we knew it the more we liked it when we developed those skills."
Barry	"I did well until university with Fourier transforms and that's where I started to struggle. I tripped and fell at university. That was the only time that I sort of struggled with math."
Celia	"The first year I took all the mathematics required and I got perfect ... So second year... I took the exemption exam. I passed every one of those too."
Diane	"I had a very hard time in calculus... I just memorized it in Grade 13... I was absolutely clueless. I spent hours studying calculus and got an A. Never understood a single thing."
Elly	"University math... more of that theory and third-year algebra... that was horrible and scary... I just studied like crazy but I can't say I understood some of the harder stuff."
Frances	"I did well. I had an exceptionally good calculus teacher [who] had a beautiful way of showing multiple representations of mathematics and making connections to the sciences."

The final table (Table 5) presents the participants' view of the subject and discipline of mathematics. Quotes from each participant's interview have been used to illustrate their perspective.

Table 5: Participants' View of Mathematics

	<i>View of Mathematics</i>
Alice	Mathematics is a web of connections and tools spanning many different concepts “It is a puzzle of many pieces and they're all connected... There is a beauty in it.”
Barry	“For math, I think, you need to be almost visual and very hands-on.”
Celia	Mathematics is “...dealing with all these relations and we're looking at the patterns; we're developing the ideas [and a] problem-solving attitude.”
Diane	Mathematics is “anything that has numbers in it. Able to figure out stuff that has numbers in it.”
Elly	Mathematics is “playing with ideas and thinking of some logical and linear way of getting those ideas across...and modelling situations.”
Frances	Mathematics is “a language to describe the natural world and how it works” and doing mathematics provided “the language to model physical phenomena.”

CHAPTER 6: ADDRESSING THE RESEARCH QUESTIONS

In the preceding chapter, I discussed the analysis of each participant's descriptions with respect to the first two dimensions examined in the study – Mathematics Background and Teaching Experience, and Views of Mathematics and its Teaching and Learning – in order to contextualize descriptions of their views and use of manipulatives. In this chapter, I present an analysis across all six participants with respect to the two dimensions examined in the third and fourth part of the interview guide that correspond with the two research questions: Teacher Views of Manipulative Use, and Teachers Use of Manipulatives. Specifically, in the interview guide, Part 3: *Teacher views of manipulatives use in mathematics instruction* contained three questions that asked the participant to describe their views on the purpose of using manipulatives for mathematics instruction, how they felt manipulatives help to teach mathematical concepts, and whether they felt that all students benefit from the use of manipulatives and why. Part 4: *Teacher descriptions of the use of manipulatives within the classroom* contained four questions that asked the participant to describe their experiences with using manipulatives in the classroom, their comfort with using manipulatives to teach mathematics, which manipulatives they use and how are they used, and whether students found using manipulatives to be helpful to their understanding.

These two dimensions: Teachers' Views of Manipulative Use, and Teachers' Use of Manipulatives, directly address the two research questions, namely: (i) How do secondary school teachers view the use of manipulatives in teaching mathematics; and (ii) How do secondary school teachers describe their use of manipulatives in teaching mathematics? In order to clearly present the analysis of each of these key dimensions, the data is presented across the participants so that similarities and differences in the participants' descriptions can be elaborated.

Views of Manipulatives

To address the first research question of how do secondary school teachers view the use of manipulatives in teaching mathematics, I offer descriptions as to how the participants' views of manipulatives have evolved over time. These discussions are organized to include the participants' introduction to manipulatives, their initial reactions to them and the role of manipulatives in changing these views. I then discuss the participants' current views about manipulatives that include their confidence and competence with the materials, views about modeling the value of manipulatives for their students and the benefits to students' of the use of manipulatives. The section is concluded with a summary of these findings.

Introduction to Manipulatives

As previously reported, all of the participants described their school mathematics background as having been predominantly based on a traditional teaching style centered on the authority of the teacher and the textbook, and a teaching approach that preferentially favoured memorization of symbolic algebraic rules and procedures. With the exception of Alice, all of the participants reported that they had not used manipulatives during their elementary and secondary school years. Frances's description of her school experiences typify those of the other participants of a traditional 'chalk-and-talk' teaching approach, and a lesson script that did not see students engage in activities with manipulatives:

I had no experiences in high school with technology, concrete materials, manipulatives... any computers, no experiences. My experience in high school was sitting in traditional rows, in a class of 30. The teacher generally started by taking up the homework questions, then there was a board lesson and then time to work. (Frances, p. 4)¹⁶

Alice is the only participant who specifically mentioned using manipulatives during her school years; she described how helpful the use of 3-dimensional solid objects in the geometry class in her elementary school had been.

¹⁶ Citations refer to the participant and the page number of the quotation in their interview transcript.

When I was taught mathematics I only used manipulatives for 3D geometry in Grade 8 when we did spatial geometry using those models with plastic sticks and Play-Do. So we'd build the models and there was a lot of Euclidean mathematics that I did from Grade 6 to Grade 8 ... and I think we did a lot more with planes cutting through pyramids, and perpendiculars and all of that stuff, and spheres like cones inscribed in spheres, and... But we started with those 3D models with sticks and Play-Do and they helped me very much and after that I didn't need them anymore after the first few lessons, but mathematics was already very easy because of those models, but that was it. (Alice, p. 18)

However, with the exception of this one experience Alice went on to say "... everything in high school and, of course, university... there were no manipulatives at all" (Alice, p. 18).

Beyond school, none of the participants specifically speak of seeing mathematical manipulatives during their mathematics courses at university although Frances spoke of a mathematics professor who provided her with a very different view and experience of mathematics by presenting "...a graphical approach to mathematics which made a lot of sense to me" (Frances, p. 6). Similar to Frances's university experience, Diane also singled out the one mathematics professor at teachers' college who presented his mathematics classes in a non-traditional way, through hands-on activities and lots of group work. She claimed that "I had this epiphany with [professor's name]", describing him and his classes as "amazing and I just loved it... [He] was huge into ... interesting puzzle things that you could do. Just an amazing guy" (Diane, p. 5). In both of these experiences, Diane and Frances describe their professors as offering them different ways to present and think about mathematics, and consequently both found them more interesting and engaging.

Alice and Barry, the two least experienced teachers, had been introduced to manipulatives during mathematics classes in teachers' college. While Barry did not offer any specific details, Alice described seeing a variety of manipulatives in mathematics classes yet acknowledged that the time constraints of the program did not allow her sufficient time to get to use them properly. Alice also reported that she did not use them or see them being used in either practicum conducted in two different secondary schools. At this point, she described her feelings about manipulatives as a

somewhat superfluous “alternative to traditional teaching” with little, if any, connection to her own understanding of math. Barry, who was also the sole participant enrolled in the primary/junior program in teachers’ college, was the only one who used manipulatives and see them in use during his two practicums. He stated that in both practicums he felt the students enjoyed activities with the manipulatives and appeared to be learning through their play.

When I taught [Grade] six in my practicum some of the kids worked well ...a lot of them worked well. The other ones that were weak, they got it with those little blocks and stuff.... the kids enjoyed it... I think the kids really enjoyed... they learn from the fun aspect of moving things around. (Barry, p. 8)

It is interesting to note that Barry used the word *play* on several occasions in the interview when speaking about students’ use of mathematical manipulatives. As the elementary school classroom was the only place he saw manipulatives in use, and described the students as *playing* with them, it is perhaps not surprising that he felt manipulatives to have limited value in the teaching of secondary school mathematics where leaning through *play* may be considered inappropriate or unacceptable to students.

For Celia, Diane, Elly and Frances, their initial introductions to mathematical manipulatives occurred during their teaching practice and, with the exception of Frances, this occurred after they had been teaching secondary school mathematics for many years. Elly, for example, had already been teaching senior mathematics for 12 years. For these four participants, their introduction to manipulatives came about in board-wide professional development workshops aimed at introducing teachers to the pedagogical approach of presenting multiple representations of mathematics.

Alice and Barry reported having their first significant introduction to manipulatives at a week long summer Math camp. This camp was organized for pre-service teachers to engage in mathematical investigation through the use of manipulatives, prior to the start of their teacher training program. Following her graduation from teachers’ college, Alice had been recruited as one

of the Math camp facilitators and was involved in a week of planning and trialing problem-solving activities. This week allowed the facilitators to work together with manipulatives to solve problems that would subsequently be used with those enrolled in the camp. Barry, on the other hand, enrolled as a participant in the Math camp. However, for both Barry and Alice, the camp provided an opportunity to investigate and explore mathematics through problem-solving activities that used a wide variety of manipulatives within an inquiry-based mathematics environment. They both described significant and surprising engagement with the mathematics as they used manipulatives to solve problems.

Initial Reactions to Manipulatives.

Several participants described having very strong initial reactions to manipulatives, and specifically algebra tiles¹⁷, when they first encountered them, yet interestingly, these reactions ranged from strongly positive to strongly negative. Elly described her initial reaction to seeing algebra tiles for the first time as "...I thought they were the stupidest things alive" (Elly, p. 7) although she does not describe exactly why. Alice also described her initial reluctance and resistance to use manipulatives (specifically algebra tiles), primarily in terms of the challenge to her own "traditional" view of mathematics, her inability to identify any value in their use, and her view that they were unnecessary and superfluous 'toys':

The first time I saw algebra tiles I was very reluctant to use them. I thought toys were for kids and mathematics was abstract. I had all of these elitist approach for math... you do it because you like it, and you don't need toys to help you like it. So I was very much against it. (Alice, p. 6)

Diane also spoke about the challenges that she had with her initial introduction to algebra tiles "I had a real hard time learning algebra tiles. No matter who taught me, I couldn't learn it" and went on to say that "... I didn't like it and was kicking and screaming and hated it..." (Diane, p. 11).

¹⁷ Algebra tiles are manipulatives built using an area model. Sets of tiles commonly consist of three different tiles made from the combinations of two different dimensions: x and unity. There is an x^2 tile (a square of dimension x), a unit tile (a square of unity dimension), and an x tile (a rectangle of dimension x and unity). See Appendix C, Part 3 for more details.

However, not all participants shared this negative reaction to their initial encounter to with manipulatives. On the contrary, Barry and Frances described very positive initial reactions to manipulatives. Frances described that as a new teacher she attended many professional development workshops where she first encountered algebra tiles and the area model of multiplication. This encounter was “like a bolt out of the blue” (Frances, p. 11) and she went on to describe that making a physical connection to algebra through the use of algebra tiles as being “just miraculous” (Frances, p. 11). She described that through the use of manipulative and technology, her own understanding of mathematics grew deeper and more richly connected:

In our own learning of the material that our own understandings were so much more connected and so much deeper having a chance to “play” with the concrete materials, the technology, and the manipulatives (Frances, p. 12)

Barry also spoke of very meaningful encounters with manipulatives at Math camp, claiming that the approach to solving problems using manipulatives appeared to mirror and reinforce his own problem-solving strategies and techniques:

When I came and took the summer Math camp that was like a really enlightening experience for me to learn a different way of learning math... When I went through the Math camp, certainly, what I walked away from was that I discovered that “Oh yeah, this is how I do it anyway”. And I've been doing it for the last so many years but I remember saying that this is so cool that there are objects now because it was a novelty for me at Math camp that I can actually use this for school. (Barry, p. 3)

Changing Views of Manipulatives

Several of the participants spoke about how their initial views of manipulatives changed after taking the time to use them and, in some cases, learn about their use in professional development activities. Alice, for instance, who reported an initial reluctance and resistance to use manipulatives, described a significant change in her view during her experiences as a facilitator of a summer Math camp. As she stated, the experiences while using manipulatives on her own, and in helping pre-service teachers to use them while solving problems, provided her a completely different way to look at mathematics.

I only looked at them as an alternative to traditional teaching which I was totally for at the time and everything had to be traditional math, it's abstract. You learn it for the love of it, for the sake of it... And then I started to see all of these connections... between geometry, between completing the square and not just manipulation of polynomials or factoring but completing the square was very powerful. And I started to like them... With Math camp everything changed, even with fraction circles I understood fractions once again from a different way of looking at them. (Alice, p. 18)

The transformation of Alice's views towards manipulatives seems to mirror her description of her preferred learning style in secondary school where she found that reviewing mathematics concepts for the second time was really helpful to her understanding. In this case, Math camp may have provided her the environment within which to revisit the use of manipulatives in solving mathematics problems following her initial introduction to them in teachers' college. Frances, on the other hand, credited her profound experiences with manipulatives as stemming from a professional development manipulatives workshop, describing it as a catalyst that encouraged her to relearn and rethink much of her understanding of secondary school mathematics. Frances described that first professional development workshop in which teachers were shown how manipulatives could be used to teach a wide variety of topics across secondary school grades:

I went to a workshop by a colleague in the board, [name of colleague]... He did a 75 minute workshop, really quick workshop, where we all got to work with our own box of algebra tiles. He went from about Grade 8/9 material up to Grade 11/12 material and just demonstrated... he did it quickly ... but he demonstrated how a lot of ideas could be used. (Frances, p. 12)

Frances went on to describe how she took a set of algebra tiles and an instructional guide home with her and worked with her husband – also a mathematics teacher – to figure out how they could be used. While her husband seemed to take to them right away, Frances reported that she was still very focused on the symbolic-based algebraic approaches but stated “I felt like I really needed to understand it and pull it apart for myself” (Frances, p. 13). As her experience with manipulatives grew, Frances described coming to see secondary school mathematics in a new light, with myriad new connections and ways to represent abstract mathematical concepts:

Going through my first few years of teaching I found some beautiful ways of presenting material that I had not experienced as a student that really began to clarify many of the fundamental understandings that I didn't realise I didn't have until I became a teacher. (Frances, p. 6)

Elly also reported that professional development workshops helped her overcome her initially negative reactions to manipulatives and gave credit to the staff development department in her school board for their long-term commitment in helping her come to see them as viable and valuable learning tools. She playfully claimed that “[name of board consultant] was the one who actually beat it into my head” (Elly, p. 8). Similarly, Celia claimed that she also found professional development workshops, and the opportunity that these workshops provided to work with other teachers, were the best way for her to learn about using manipulatives and “what worked and how it worked” (Celia, p. 18).

However, not all participants described such professional development workshops to be as useful an introduction to manipulatives as Celia, Elly and Frances. Diane, for example, described her experiences attending manipulative workshops as failing to engage her with the manipulatives. As she stated, “I go to workshops but I find the majority of stuff doesn't suit me. I just don't like using it. It doesn't make sense, it is too hard to figure out, or it doesn't fly or it's too dry...” (Diane, p. 10). She also described attempting to learn from a companion series of video guides made by teachers for teachers in the board that illustrate using manipulatives to teach particular algebraic concepts:

I don't use [the] videos on manipulatives ever. Maybe it's a lack of time... there are 22 of them! I think I watched part of one. It was boring. You have to be really keenly interested in building a tower [referring to the use of linking cubes to model relationships] to sit through it. It didn't hold my attention so how could it hold their attention? And if I could learn it, I would be more exciting. Maybe I could hold their attention. But it has to hold my attention first before it will hold their attention. (Diane, p. 10)

While all of the participants described views of manipulatives that changed over time, to some, these changes did not occur either easily or without challenges. Five of the participants spoke about how teaching and learning with manipulatives initially challenged their more traditional view

of mathematics that had formed during their school years. They also stated that gaining the confidence and competence to use manipulatives in the classroom came neither easily nor naturally, and was still ongoing, often taking considerable time and repeated attempts to learn how to use them. However, each reported that working with manipulatives provided, if not a greater understanding of the mathematics, then at least another way to view and approach mathematical concepts. As Celia stated that, "...even as an adult, I learn with manipulatives" (Celia, p. 12).

While Barry claimed that he discovered that working with manipulatives mirrored the way that he himself did mathematics, the connection between mental and physical actions he experienced was not shared by all participants. Celia's views echoed those by Alice, Elly and Frances in suggesting that "because we're not the ones who grew with the manipulatives, it's really hard to [make connections]. It doesn't come naturally" (Celia, p. 13). Frances reported that even as profound as her experiences with manipulatives had been, she still had to work hard to incorporate models and manipulatives into her teaching practice as her learned preference was towards symbolic algebraic approaches:

...certainly four or five years ago that was something that I really had to incorporate in my lesson planning because it didn't come naturally to me to think that way. Certainly as I'm learning more about these materials and using them more and becoming more confident with them, it does start to become part of my approach to planning that I think is so important. (Frances, p. 15)

With experience and effort using manipulatives Frances described that she had now "...shifted the way that I think my own brain is put together. I used to be so strong down the algebraic path" (Frances, p. 11). Similarly, through her experiences at Math camp, Alice also described coming to see manipulatives as offering her an approach and view of mathematics that she had not seen before and that she had initially rejected as being anathema to her traditional and highly abstract view of math. She spoke about how she came to "understand fractions once again from a different way of looking at them" (Alice, p. 18) by using fraction manipulatives in problem-solving activities, and

stated that “with Math Camp everything changed” (Alice, p. 18). While Elly claimed to be unsure as to whether the use of manipulatives had, in fact, challenged or altered her views and understanding of mathematics, she did acknowledge that she valued the use of manipulatives in allowing her to show alternative representations of mathematics concepts to her students:

My understanding? I'm trying to think as to whether that really helped me understand it more.... I like the picture of it but did it help me understand it more? I guess I have to say I'm not sure. I certainly like having another way of showing it. I certainly like it, because I am very visual myself... I do like some of the visual aspects of it. Did I understand it more because of it? I understand it differently because I never thought of it as a visual thing. (Elly, p. 8)

Current Views of Manipulatives

Each of the participants described different changes to their views of manipulatives in the teaching and learning of mathematics based on their teaching experiences.

Confidence and Competence. Alice described how over the past three years she had steadily gained both competence and confidence in using manipulatives to represent mathematical concepts to the point that she readily integrates their use into lessons. Not only had this experience given her an overall comfort level with manipulatives, but she stated that she planned to preferentially introduce them “in the beginning” (Alice, p. 8) with her new class of Grade 9 students in an attempt to allay potential fears in working with fractions:

After I fully liked them I felt I was confident enough to sell them to my students so that's when I brought them to class...So, right now I would rate myself as very comfortable. I would have said the same last year, but not this time two years ago... So I find that I'm really comfortable now and I'm looking forward to using them. (Alice, p. 6)

Barry also indicated how useful manipulatives had been in developing his own understanding of mathematics and stated that “for myself, I really believe in it” (Barry, p. 4). However, Barry described feeling woefully ill-prepared to use manipulatives with his Grade 10 students, citing that he was unsure how to use them in the teaching and learning of secondary school mathematics, and in a manner that his students would find engaging; quite understandable given his lack of experience in

the secondary panel. While he acknowledged that their use required significant effort in planning, Barry stated that he would not consider introducing manipulatives into his classroom until he felt more comfortable with his own style of teaching and very familiar with the manipulatives themselves:

For my own comfort level I would actually have to design a lesson in advance anticipating using them... For the future I would go and plan a lesson in advance and get myself familiar with it before the lesson came up.... You need to be familiar with the tools before you use them. (Barry, p. 9)

Diane acknowledged that she still had difficulties using manipulatives (specifically algebra tiles and linking cubes) despite attending many workshops and accessing many Internet resources. As she stated: “to make models that work for me takes a long time. It takes years” (Diane, p. 9). However, Diane did not make it clear whether, for instance, her difficulties stemmed from having trouble learning within the format of professional development workshops, or whether they arose from challenges to her own understanding of mathematics and its teaching and learning. While it is interesting that she found it hard to learn to use those manipulatives often referred to in curricula resources (such as algebra tiles, linking cubes, etc.), Diane preferred a much broader definition of manipulatives as being “something that the kids could touch that's not flat” (Diane, p. 7) which included all manner of concrete materials (such as string, marbles and Bristol board). Of these, she stated that “I love using them... [I've] got to have them because if you don't have a manipulative you're teaching all this mathematics theoretically and that's brutal” (Diane, p. 7). She went on to describe several examples of activities that she designed for her students that used these materials.

Elly acknowledged that over the last 10 years of teaching her growth in confidence and competence in the use of manipulatives had been incremental yet slow. Rather than learning to use manipulatives proactively, she stated that she tended to learn how to use them to teach particular concepts in response to what she felt her students needed. However, Elly went on to say that as she

has gained familiarity and comfort with algebra tiles, has found more uses for them and consequently uses them very frequently:

I have yet to use it to solve equations although I know it can be done. So who knows, one year when I feel like my class needs it to learn how to do it... how to do that, I'll need to figure that one out a little more for myself. So I would say it came very slowly but I feel more and more comfortable the more I use it and I always find new things to show them, which is cool. (Elly, p. 8)

Frances reported that through experience in using manipulatives she has found her own understanding of mathematics to have become much deeper and more richly connected; attributing much of this new understanding as having come from the collaborative support of her peers. She described that she wanted a similar experience for all of her students and particularly those in the senior mathematics courses who, she felt, had likely been taught in a more traditional manner. Frances recognized that her comfort level and confidence with manipulatives was such that she tended to default to some of the pictorial representations while preparing her lessons and added:

...when I say manipulatives I'm including concrete materials and technology with that. I think anything that students can manipulate like the Geometer's Sketchpad sketch I would incorporate that into manipulatives. (Frances, p. 14)

Frances also described how her role as department head and the responsibility she had in the selection and purchase of manipulatives for the department was a further factor in her fluency with them. She described seeking assistance from other school and board colleagues and searching the Internet when she came across unfamiliar manipulatives to determine how and for which mathematics concepts they could be used. As she stated, Frances felt a responsibility to her mathematics colleagues "to learn what all of these materials were for and why should I be spending some of the school's money on some of these things" (Frances, p. 13).

Learning from Students. Diane claimed that she did not understand how to use algebra tiles to represent algebraic concepts, yet on occasions allowed her students to use them. While she acknowledged her own difficulties in using the tiles, Diane recognised that sometimes her students

benefited from their use and were able to make connections between the use of manipulatives and mathematical concepts that she was unable to see:

I used the algebra tiles using TIPS4RM on my 2Ps [Grade 10 Applied course] last year... with no hope in connecting at all, none whatsoever. And they loved it so much that they could still FOIL¹⁸ on the exam correctly using what was in their mind algebra tiles... they hadn't forgotten it and I've never had anyone on 2P FOIL on an exam correctly... using FOIL. Even though I didn't like [algebra tiles] and was kicking and screaming and hated it... I pretended to like it. They loved it... Well, they didn't love it at all but they retained it. (Diane, p. 11)

After prompting to elaborate further, Diane described herself as being open and willing to learn from her students, recognizing that sometimes they can make sense of and connections with concepts with which she struggles:

So I turn them loose and tell them that what we're trying to do is this and then somebody will be ahead of me and will think differently and it expands the class. If you learn from the kids, you learn more. If I only let them do exactly what I'm doing then I'll come away knowing exactly what I came in with. (Diane, p. 11)

It is also interesting that Celia and Frances also noted that their students have found unexpected and unanticipated ways to use manipulatives, and, as with Diane, described valuing the learning that emerged from these experiences. In this vein, Celia spoke of the need to allow students sufficient time and space with the manipulatives to encourage as much learning to take place as possible. As she stated, this learning – now largely in the hands of the student – may go well beyond what the teacher had identified and into places that students create for themselves:

Given the chance, the students might learn a lot more [than me]. They might learn to create something different and ... I'm a strong believer in letting them use them and see what happens. (Celia, p. 12)

Like Celia, Frances described manipulatives as tools that help students focus on mathematical ideas and, in a similar manner, spoke about how her students often found ways to use them in unanticipated ways:

¹⁸ FOIL is the acronym for the algorithmic method (First, Outer, Inner, Last) often taught for multiplying two binomials.

Certainly the more that I've experienced teaching with these tools I see students coming up with solutions that I haven't necessarily anticipated. When you've done a course seven or eight times you generally can predict how many of the students will approach a problem but when you bring out... and again because I was never taught with many of these materials... I find that I am always impressed with some of the ways that students have of representing problems particularly with linking cubes which I haven't necessarily thought of myself. (Frances, p. 12)

Toys or Tools? The participants spoke about manipulatives using very different words and metaphors to describe them, and those aspects of the manipulatives that make them beneficial to student thinking. For instance, Alice, Celia and Frances described manipulatives on several occasions generically as 'learning tools' that facilitate additional ways to think about and explore mathematics and through this exploration, students can deepen and better connect mathematical concepts:

I think they are useful tools. So the more ways we can show our students how to solve a problem, the more means for exploration the better, so definitely, I think it can only enhance their learning. So there is no reason why we shouldn't use them at all. (Alice, p. 12)

Elly, on the other hand, did not use the term 'tools' at all yet referred on several occasions to manipulatives as 'toys'. She explained that this was neither to neither diminish their value nor treat their use flippantly but to suggest that students learn from toys through play, and that manipulatives as toys "...are useful. They are effective as toys. You learn from toys anyway so I don't care if [students] think it's a toy... I don't think that's a bad thing" (Elly, p. 9). Elly, echoing similar comments from Celia, explained that when students *played* with manipulatives, they may not even be aware that they were, in fact, learning mathematics and developing understanding. She claimed that, in a similar manner, when students use mathematics computer software, such as Mathematics Trek, they often consider that they are playing and not doing 'real math':

...when they're sitting there working with paper doing the worksheet, are they actually doing more math? Are they actually understanding more than if some of them might think they are playing with the tiles? ... In the end, and same with the computers, like they work through and they do all these things on the computer and then they'll be like

‘When are we doing the unit?’ And I’m like ‘Well, you’ve just done it and you’ve demonstrated that you know what to do.’ You didn’t think you were doing anything. (Elly, p. 9)

In a similar manner, Barry described his practicum experiences with elementary school students where he suggested they were learning through play and the ‘fun’ aspect of using Tangram pieces to construct different shapes. However, when he referred to his tutoring experiences, he made a distinction between students *playing* with the manipulatives and those who used them in their sense-making. “Many kids actually appreciated learning that way of how to add and subtract positive and negative numbers and others were just playing with the chips” (Barry, p. 4). However, in the context of secondary school mathematics, Barry was so concerned about the potential negative reaction of his students to using manipulatives, citing this as one of several reasons for not using them at all. He acknowledged that he did not understand how some of the manipulatives that he had seen used in the elementary school could be used effectively in the teaching of secondary school math, claiming that some of them were “... so low level that I’m not comfortable using it at a high level because the kids would consider them as toys. So that’s what sort of holding me back” (Barry, p. 9). It is not surprising that Barry held these views given his lack of experience and encouragement to use manipulatives to investigate and explore the secondary school mathematical. While Barry found manipulatives helpful in exploring mathematical concepts during Math Camp, his views would suggest that pedagogically he viewed manipulatives as novel “toys” suitable for play activities by elementary school-aged students.

Teacher Modeling. Celia offered a different perspective by suggesting that students’ view of the value of manipulatives is strongly influenced by the attitude of the teacher towards their use. She claimed that the teacher must explicitly model for students the serious nature of using manipulatives in order for them to consider manipulatives as valuable tools for learning mathematics. As a corollary, she noted that “when the teacher does not consider [using manipulatives is] the right thing

to do, the kids just wreck the whole thing” (Celia, p. 10). Interestingly this view is consistent with Barry’s concerns that students would consider manipulatives as toys and would treat them as such, although he conceded that he was ill-prepared to model their use otherwise. Celia framed the emphasis on teacher modeling in the larger context of the learning culture of the entire school:

Learning is a cultural thing... I started teaching adding and subtracting polynomials and I made sure that when I started manipulatives I let my students know that this is a serious business because you know what your kids will say when you give them those tiles... “Miss, when are we going to learn something... we've been playing with these for a couple of days.” ... When I make it serious while I use them they seem to use these things properly. (Celia, p. 10)

Benefits to Students. All of the participants expressed the view that their students benefited from using manipulatives in their learning of mathematics, although there were a range of opinions offered as to whether certain groups of students benefited more than others. Alice, Elly and Diane expressed almost identical views in that the benefits to their students in the use of manipulatives varied greatly, ranging from significant to little or none. Alice stated that manipulatives were “more beneficial for some students and neutral to others” (Alice, p. 9) while Elly felt that “... it's either of neutral effect or of some benefit” (Elly, p. 7) Alice spoke of three groupings of students who derive different benefits from the use of manipulatives in their learning of mathematics. The group of students who had grasped the topic being taught and “have the abstract thinking already developed... wouldn’t necessarily make any additional connections because they had already developed their own understanding” (Alice, p. 9). Another group of students who had mastered rules and procedures would not think any further with the manipulatives and “wouldn’t see the connections” (Alice, p. 9). For the third group of students who paid attention but still did not fully understand the ideas, “sometimes you bring the manipulatives in, and that's their missing link” (Alice, p. 9). Diane similarly agreed that all students benefited from the use of manipulatives yet acknowledged that for some of her higher-achieving students, they “can do this all in their heads and maybe they think [the use of manipulatives] is trivial or maybe they enjoy the break” (Diane, p. 11). Elly offered very

similar views about her students as Alice, stating that different students think differently and that those students who “already understood it don't necessarily need [manipulatives], although it doesn't hurt them” (Elly, p. 6). She claimed that for some of her students using manipulatives was the catalyst to understanding the mathematical concepts. Elly suggested, and this was also mentioned by Diane, that even those students who understood the ideas from a ‘chalk-and-talk’ delivery could also benefit from having to think about alternative ways to view the math. Elly summarized her ideas about using manipulatives in the larger context of being one of many teaching strategies that she used to reach as many students in the class as possible by showing them many different ways to view the mathematics:

I think it's like any teaching strategy.... there will always be people who benefit from you doing something new. Manipulatives might be the best thing for kid A but kid B couldn't care less but they'll listen and kid C might just get confused... but isn't it the same as on the board and kid C gets it and kid B doesn't care and kid A who needed the manipulatives doesn't have a clue and gets confused. So I think you just show more things... (Elly, p. 6)

Elly stated that while she now uses them in all classes, she had started using manipulatives in her Applied level class because she felt that these students needed them more and because she felt there was “more time in applied to spend on it too. It's easier to convince yourself to use it” (Elly, p. 14). She suggested that even when using them in her Academic classes “there's always somebody who will need it” (Elly, p. 14). Celia described using manipulatives in her Applied classes more because “in the Applied classes there are fewer concepts I have to teach... Students have to see it to learn it in some ways so I try to use more manipulatives. But I think it should be done in Academic classes too” (Celia, p. 11).

Frances also stated that all students (and teachers) benefit from the use of manipulatives, even for students in the senior grades, and noted that “no matter how confident or not confident the student is, or how good or not good a student is at math, I really believe that manipulatives benefit

everyone. I've certainly experienced that myself" (Frances, p. 15). However, she understood that this view was not shared by all teachers, in part because manipulatives had initially been promoted by the school board for junior students and as remedial aids for struggling students. She explained that for her, as with many teachers, the first time she encountered manipulatives was through the TIPS¹⁹ material that was initially focused on the Grade 9 Applied course. However, she noted that more recently the focus appeared to have shifted to promoting the use of manipulatives as beneficial and helpful to all students, in both Academic and Applied classes, and across all grades:

Certainly the Grade 10 [TIPS] and the OAME [Ontario Association for Mathematics Education], OMCA [Ontario Mathematics Coordinators Association] produced materials for the Grade 11 course really began to show teachers that these materials were not merely for junior level mathematics ... by junior I mean 7 to 10... but that there are important ideas that should be incorporated into the senior mathematics courses. (Frances, p. 13)

Celia also stated that she believed manipulatives were valuable learning tools that had the potential for helping all students, in both Applied and Academic level classes, and from early grades through to Grade 12:

... if it's beneficial for slow learners, well then, it's definitely beneficial for fast learners because if that takes 10 minutes for these kids, they look at it and they'll get them in 10 seconds. So it is beneficial. It's just, you know, timing or different... degrees" (Celia, p. 11)

However, she reiterated that the benefits of manipulatives accrue from long-term and consistent use, so that students become accustomed to – and comfortable with – using them to solve mathematics problems and coming to “recognize them as a tool to figure out things” (Celia, p. 9). She suggested that teachers should start using them with their Grade 1 or 2 students for introducing integers, negative numbers and the notion of the zero sum pair “when you have the same number of a different

¹⁹ Targeted Implementation and Planning Supports (TIPS) and Targeted Implementation and Planning Supports for Revised Mathematics (TIPS4RM) refer to a comprehensive suite of grade-level support materials for Grades 7 to 10 mathematic teachers.

colour”²⁰ (Celia, p. 9). This theme of consistency of models and long-term manipulative use across grades was repeated many times throughout Celia’s interview.

Several participants spoke about the visual and tactile aspects of the manipulatives as being particularly helpful for students. Alice described how manipulating algebra tiles appeared to help students make connections with other representations, especially those students who had already identified themselves as having significant challenges with math. She added that “using concrete materials... helped their abstract thought” (Alice, p. 7). Celia echoed these ideas about the value of handling manipulatives when dealing with and making connections to very abstract concepts, such as polynomial manipulations:

So I think it's easier for them to understand and it's a little bit more concrete, so there's a huge benefit to do these things. And I think there is a big connection to the brain and hands. (Celia, p. 11)

Elly also felt that the benefit to students of the visual aspects of using manipulatives helped students develop a repertoire of mental pictures of the operations performed with the tiles that they could subsequently draw on when solving more complex problems. She gave the example that when students are collecting like terms using algebra tiles students find the operations to be more intuitive as they are dealing with different shapes; a task not as obvious when dealing with symbols. Similarly, she described the richness of being able to see the results of completing the square with algebra tiles (see Appendix C, Part 3) when students are learning about factoring. Elly concluded that:

I do think some people are more concrete and if they can actually hold and touch something they feel that it makes more sense to them plus it's good for the kids who don't think that way to have to be forced to have to think about that. (Elly, p. 5)

²⁰ Zero-sum pair refers to the algebraic action of adding an equal number of positive and negative coefficients (i.e. zero) to an equation as a step in developing a solution. Celia’s reference is to algebra tiles where positive and negative values are represented using different colours.

View of Manipulatives Summary

While each participant described a different path to their current understanding of the value and utility of manipulatives in the teaching and learning of mathematics, several common themes appeared during the analysis that provide insight on how teachers view and interact with manipulatives.

Bearing in mind that the participants in this study had little, if any, experience with manipulatives as mathematic students, many of the participants described their initial view of manipulatives as being fundamentally at odds with what up until that point had been the value of – and preference for – symbolic and abstract representations of mathematical concepts. The participants' notions of mathematics revolved around collections of largely unrelated abstract concepts where mathematics achievement was based on symbolic manipulation and procedural fluency. Some participants suggested that their initial view was not only that manipulatives were superfluous in the teaching of abstract mathematics but that their presence would merely confuse rather than enhance in students' learning. These views suggest that, initially at least, some participants may not have recognised value in considering multiple representations of mathematical ideas or failed to make connections between these concepts and the manipulatives used as concrete representations. All participants noted that the use of manipulatives in the classroom required them to reconsider their teaching practices and the delivery of instruction in the classroom. In addition, some participants found that working with manipulatives also helped to reframe their notions of what constitutes mathematics and mathematical knowledge.

Participants varied in both the degree and speed of the changes to their teaching practices encouraged by using manipulatives. All participants found that considerable time and effort were required in order to gain sufficient confidence and competence in the use of manipulatives to be comfortable using them with their students. While the time needed for participants to gain this

confidence ranged from months to many years (and was perhaps still underway with Barry and Diane), participants reported that as they became able to represent mathematics ideas in meaningful ways with manipulatives themselves, they started to use them as tools for mathematical learning with their students. Over time, as they gained more experience exploring and representing mathematical concepts with manipulatives, and witnessed the accrued benefits to their students, manipulatives as learning tools became more integrated into their lesson planning and teaching practice. All participants described this process to be slow yet valuable for their own and their students' learning.

It was also apparent that the participants, like their students, learned in different ways, such that multiple and different learning opportunities were necessary in order for them to gain an understanding of, and confidence with manipulatives. Whether this time and effort occurred in professional development workshops, in a supportive community of teachers, or as an individual effort, participants reported having to learn how to use manipulatives as their use rarely appeared to be either intuitive or obvious. Over time most of the participants had come to recognise the value of manipulatives as one of several representations that can help deepen and better connect mathematical concepts, and provide the learner with a richer repertoire of mental images. However, as they themselves had never learned with these materials, their subsequent integration into their classroom practice had come only with continuous and considerable effort. As the three most experienced participants were introduced to manipulatives long after they started teaching, and described taking a long time to become comfortable with them, this may suggest that learning to use manipulatives may be particularly problematic if this new learning requires a reconsideration of well established views and teaching practices.

A further theme revolved around the different ways in which the participants described manipulatives as either tools or toys and whether students' learned from them through play or by using them in a serious manner. While at first blush these might appear opposing views, it may be

that, in fact, the participants were agreeing that their students' benefit from playing with manipulatives when there is a serious and legitimate reason for their use. Those participants who used manipulatives in the classroom, the issue may not be about whether students viewed and used manipulatives as either *tools* or *toys* but whether the teacher modeled for their students that manipulatives were viable and valuable tools in the exploration and investigation of mathematical concepts. While acknowledging the role of the teacher to model the value of manipulatives, several participants also suggested that students gained greater benefit from long-term use of manipulatives, and were more likely to consider them useful mathematical tools, if they were consistently and repeatedly used, and preferably starting in junior grades.

An additional theme was that students found the tactile and visual aspect of manipulatives to be beneficial to their understanding of mathematical concepts. Some participants felt that the use of manipulatives helped students build a rich repertoire of mental models that could be usefully applied to new and more challenging problems. The ability to 'see' the results of actions on the manipulatives was also identified in helping students better connect and deepen their understanding of procedures such as collecting like terms, factoring quadratics and completing the square.

Finally, participants acknowledged that their early views of manipulatives, influenced by the focus of professional development activities and curricula support materials, suggested that they were mainly beneficial for younger and struggling students. The current view of those participants who use manipulatives in the classroom was that all students derive at least some benefit from their use. The participants suggested that the level of benefit to students – from little to considerable – depended on myriad influences including their confidence in and attitude towards mathematics, and their prior understanding of the concepts being explored. Some participants felt that benefit was also derived from the shift of the locus of learning from the teacher to the student when students used manipulatives, serving as the catalyst for new and unanticipated understanding.

Discussions on the use of manipulatives

This section deals with the second research question of how secondary school teachers use (or more strictly, describe using) manipulatives in the teaching and learning of mathematics with their students in the classroom. I start by describing the types of manipulatives that the participants use to explore and investigate particular mathematics concepts, and some of the activities in which they are more commonly used. This is followed by a discussion of the various teaching strategies that the participants have adopted for using manipulatives, the effects of using manipulatives on the traditional classroom ‘script’ and how manipulatives are used in students’ group work. I then discuss the participants’ views of students’ reactions to manipulatives and their efforts to increase students’ comfort level with them followed by the participants’ experiences with the availability of manipulatives and how they control students’ access to them. I detail some problems identified with particular manipulatives and conclude the section with a summary of the use of manipulatives.

Manipulatives in use: Topics and Activities

While several participants described manipulatives, in general, as being useful tools that allow students to see and explore mathematical concepts in different ways, their descriptions showed marked preferences for which manipulatives they used for particular topics. Participants offered many specific examples using certain manipulatives in the teaching and learning of particular mathematics concepts but by far, algebra tiles were discussed more frequently than all others. As a way of an explanation for the apparent preference and emphasis on the use of algebra tiles, several participants claimed that it was principally driven by the curriculum content. As an example, Elly stated that while she had used linking cubes for doing “three-dimensional things” and algebraic patterning, “it’s just that there isn’t that much of it [activities using cubes] in the curriculum” (Elly, p. 7) whereas there were many more opportunities identified to use algebra tiles.

Alice, Celia, Elly and Frances stated that they now always use algebra tiles when investigating algebraic concepts, including collecting like terms, simplifying expressions, solving linear equations, binomial multiplication and factoring quadratics. In particular, all participants spoke about the value of using algebra tiles when exploring the factoring procedure of *completing the square* (see Appendix C, Part 3). Elly described how each year her students found manipulating polynomials and particularly factoring, really hard abstract concepts to grasp and suggested that “the fact that I can use [algebra] tiles to show it and some kids might understand that... that's a bonus” (Elly, p. 7). Indeed, the connection between manipulation of the algebra tiles and the symbolic algebra was so strong as to be instrumental in changing Alice’s previously guarded reaction to manipulatives. She stated that “I started to see all of these connections... between geometry, between completing the square and not just manipulation of polynomials and factoring but completing the square was very powerful. And I started to like them” (Alice, p. 6). Elly also described that the use of algebra tiles was effective in exploring factoring and in particular, *completing the square*, because the process and result was such a powerful visual aid for students:

And you have completing the square where you see that you’ve made a square and what's left over... and [as an example] all of a sudden this becomes x plus one all squared and plus we're missing two. So the actual visualization of it... .. In the end, at least they have a mental picture of it and you can remind them ... “Remember what you were doing, you were borrowing some zeros and putting things in.” So I think it gives them another tool to visualize it rather than just symbols... They can visualize those pictures which I think is nice for some kids. (Elly, p. 6)

In a further example, Elly described using algebra tiles to help her Grade 11 students avoid making common mistakes when collecting like terms and simplifying expressions. She noted that once her students recognized the different physical representations of the algebra tiles for units, x , and x^2 terms, they could visualize them and were then less likely to make those same errors symbolically. Using the example of her students’ challenges in mixing multiplication and addition, Diane reiterated

her belief that a suitable manipulative²¹ that allowed her students to visualize the differences between the different operations would help them avoid making these mistakes over again:

I've got all kinds of kids who can't count like x plus x or whatever and x times x is whatever. Now I've got an idea about that [manipulative] but I've never got round to getting it. But I can see that they make the same mistake all the time and they need something. Whether it's sorting colour papers or sorting shapes. But they're not seeing any significance to a plus or a [multiplication]. (Diane, p. 9)

While Diane struggled to find the illusive model to help her students better understand arithmetic operations, other participants described using algebra tiles to be very effective manipulatives for students to explore these concepts.

While most of the specific activities described by the participants focussed on the use of algebra tiles, activities with other manipulatives were also mentioned. Linking cubes were identified as useful in exploring specific topics of algebraic patterning; examining the difference between linear, quadratic, cubic and exponential relationships; 3D geometry and isometrics, and (by Celia) in the senior mathematics of rates of change of area and volume. Alice also mentioned using fraction circles which she found useful in reviewing and overcoming challenges that many students faced with fractional arithmetic, especially new Grade 9 students:

These students are coming from all over the place when they start grade 9 so I don't know what mathematics skills they have ... And I should know what they know, and hopefully I can also bring them up to date by introducing fraction circles in the beginning so they see that fractions are not that scary after all, as some people may think. (Alice, p. 8)

It is interesting that Alice herself described that using fraction circles manipulatives in Math Camp had given her a completely new way to consider fractions. She stated that, “with fraction circles I understood fractions once again from a different way of looking at them” (Alice, p. 18). Her experiences with fraction circles appear to have been so significant that she saw them as key in helping to dispel her students’ fears – and identify and strengthen their understanding – of fractions.

²¹ While Diane did not know what such a manipulative may look like, she felt that if one could be devised for this purpose it would help the students.

While algebra tiles, linking cubes and fraction circles were the only manipulatives specifically identified in the activities described by the participants, Celia, Diane and Elly described using paper to construct simple 2D shapes to help explore topics such as the Pythagorean Theorem and trigonometric ratios, and more elaborate 3D models for the examination of geometric properties. Diane described several activities where her students had to construct 3D models using paper and then deconstruct them in order to examine their nets. She reported that compared to chalk-and-talk lessons, students found such physical activities to be both engaging and very helpful to their understanding of geometric properties:

...sometimes they'll pick up the model and say "Oh yeah right, look. And, let's see now, the slant height is longer than the altitude, right." Huge understanding compared to doing it on the board. (Diane, p. 8)

While Elly also described that such construction-based activities offered significant learning opportunities for her students, she added that she spent little, if any, time on them, because "it's not so much in the curriculum so I just don't tend to do it" (Elly, p. 7).

Teaching Strategies with Manipulatives

Manipulatives as Mixers: A common theme discussed by all participants was how activities with manipulatives could be used to mix-up and change the routine of a lesson; a strategy that was identified in helping to keep students' interest, attention and engagement with the math. Elly spoke about the need to keep the lessons fresh to hold the students' interest and explained that:

the difficulty isn't the math, it's how you present it and make it... put together a lesson, that kind of thing, that engages the kids and gets them doing something other than just sitting there staring at you wishing you would stop. (Elly, p. 3)

Celia stated that within her school her students expect a traditional classroom structure and routine in which their role was that of a passive learner. She described using manipulatives as a way to help change her students' attitude from passive to more active engagement with the mathematics. Alice also spoke about how the use of manipulatives helped to change the routine of the lesson by

providing her students a different stimulus, which she found especially important for those students with shorter attention spans. She described how students were more engaged with and focused on the mathematics when they were using manipulatives. Similarly, Frances described the increased engagement of students when she used manipulatives:

I find that the first thing that manipulatives do within the classroom is engage students. I find that the first thing if I bring out a box of linking cubes, the students are so engaged by what that is... they immediately are just interested. They sort of put their pencils and paper and push them aside and say “Wow, I get to play right now” and the level of engagement I see throughout the class is really high. So that is a great reason for using manipulatives. In a pencil and paper exercise with students, I don't find that the level of engagement is as high so that's one aspect of it. (Frances, p. 11)

However, Celia cautioned that the mere presence of manipulatives in the class was insufficient to maintain the students' interest and engagement once they were no longer a novelty. She claimed that ongoing engagement with the materials was only to be expected if the teacher was able to seriously model the use of manipulatives as valuable tools in the exploration and investigation of mathematical topics. As she stated, “when I make it serious, look and then do it, [the students] seem to use these things properly.... When the teacher is not serious about it... When the teacher does not consider that this is the proper [thing] to do, the kids just wreck the whole thing” (Celia, p. 10). Furthermore, she posited that the use of manipulatives and the modeling by teachers of their value must also be an inherent part of the learning culture of the entire mathematics department, and consistently applied over all grade levels so that students experience their use over the long-term:

So the cultural thing is really important. There has to be collective agreement that these are important learning tools. We play with it but we learn it. It's a process of learning. And if you don't have that culture every time you introduce those manipulatives you spend half the time persuading the kids that this is a serious business. (Celia, p. 10)

Several of the participants described strategies for introducing manipulatives to their students, many of whom had few, if any, prior experience with their use in secondary school mathematics classes. For instance, Alice described that when using algebra tiles in the classroom for the first time,

she allowed her students some free time to just play with the tiles – “to hold the squares and move them on the table” (Alice, p. 10) – before getting her students to do anything mathematical with them. She felt that this time was essential for the students to get over the initial novelty of the manipulatives and to “grow bored with them” (Alice, p. 10).

Manipulatives and Group Work: To most of the participants, the use of manipulatives and group work appeared to go hand-in-hand in that activities with manipulatives were typically undertaken in pairs or groups of 3 or 4 students. While Elly described having to pair students because she had insufficient sets of algebra tiles, she stated that even if she had a sufficient quantity, she would group students because they “have more fun because if they’re doing it together they think they’re having a good time” (Elly, p. 10). While the ‘fun’ aspect was repeated by several, participants described that generally within groups, and in particular with activities that used manipulatives, they found students more engaged with the problem, open to share their mathematics thinking and strategies, and more willing to help and learn from each other. Alice echoed many of the participants’ sentiments when she discussed the synergy that occurred in groups when students helped each other and prompted each other’s thinking and learning:

I think group work is important, and I’m encouraging them to work in groups, because there will always be someone who says ‘hey, you can’t do that remember’, and they teach each other and I think it’s very nice. (Alice, p. 14)

Frances also noted that even though her Grade 9 and 10 students were very engaged with the mathematics they were exploring with manipulatives, the class tended to be a lot more noisy and the students “a little more boisterous” (Frances, p. 3). However, Frances reported that within groups, students challenge and build upon each others’ ideas more readily and that when these ideas are expressed using manipulatives, her students talk more about their mathematical thinking. Similarly, Celia found compared to paper-and-pencil exercises, while using manipulatives in groups her

students learned from and helped each other and explained their mathematical thinking more effectively:

Like if the instructions says try these things and they give you five different things, each kid will try at least one if there are four people in the group and then they'll write it down. And there's always one or two who will do something bizarre. And then they figure out "Whoa, that didn't work" and "that does this, why is that doing that?" So I think there are kids always coming up with questions in that sense. So that's good. And another thing... when they make you work in groups in the paper-and-pencil, the kid who's smart will write the whole thing down and the other kids watch it and they don't even ask for a reason. But when you give them manipulatives they will try something and then one kid says "That doesn't work. Let's put it this way and then it will work"... they actually explain, they can actually explain better... It is easier for the kids to explain to somebody else... (Celia, p. 18)

Alice noted that her own development as a teacher had better enabled her to monitor and read group dynamics and described being able to quickly see how her students are faring by observing and hearing them express their mathematics through their use of manipulatives. She stated that being able to see her students work through the use of manipulatives enabled her to focus her assistance as and when required. It was also clear that during these group activities, Alice saw her role as being a guide to her students:

With the algebra class it's all visual [when using manipulatives] so I can simply scan who's doing the right thing and who is not and intervene right away and put them on the right track. So it's faster in that sense... I'm there to monitor the groups and make sure that every group is moving up. (Alice, p. 17)

Even with the careful orchestration of solo and group work, Celia spoke of the balancing that she faced when using manipulatives in groups, expressing (like Frances) that sometimes "there's a lot more mischievous things going on so I have to really focus [the students]..." yet "...on the other hand at least I know they are looking at things mathematically" (Celia, p. 18). She concluded that "yes there is some uncertainty about using manipulatives in class but there's also self-learning which is comforting" (Celia, p. 18). While Celia had already identified that within groups students tended to better express their mathematical understanding, like Alice, she also found that it provided her

similar access and insight into the mathematical thinking being developed. Frances further elaborated on this theme that within group work, manipulative models – far more so than pencil-and-paper work – provided a valuable focus for students to debate, conjecture and reason mathematically:

I like a noisy classroom where students work in pairs or groups of four and they ask each other how a particular representation is related to a problem. They challenge each others' notion and I find that more so than with pencil and paper where you're reading other people's messy work, that it's not always easy to challenge a mathematical idea if it is represented in the shorthand of mathematics. But if there's a picture, or a graph or a diagram or any kind of representation that can be held up for public scrutiny, for public debate, to share ideas, where students can challenge each other or support each other's ideas I find that having a manipulative or concrete material or a sketch or a Geometer's Sketchpad file allows students to talk about mathematics. (Frances, p. 12)

Like Celia, Frances acknowledged that activities with manipulatives changed the classroom norm but in ways that helped to expose and develop her students' mathematical thinking and encouraging more mathematics talk. This discourse, Frances felt, not only helped to build an effective mathematical learning community, but also served to develop her students' communication skills.

Students and Manipulatives

The participants described how their students initially reacted to, and came to view the use of manipulatives as learning tools, and perhaps not surprisingly, these descriptions mirror the participants' own initial and subsequent views of manipulatives. For instance, Alice described that like her, her students' initial reaction to manipulatives was often quite negative. She found that her students needed time to gain a comfort level with the manipulatives by using them in meaningful activities before coming to consider them as useful mathematical tools; they needed to *see* that manipulatives had value, and she had to model this value for them. She compared her students' comfort level with manipulatives to their use of graphing calculators in that “when they see them the first time they're puzzled by all those keys with different colour codes. They use them a few times, and they're fine” (Alice, p. 9). Both Celia and Elly described similar experiences with their senior students, many of whom had limited, if any, experience with manipulatives during their previous

secondary school mathematics courses. Elly described that while her students' initial reaction was to question their value, especially if they had been shown alternative methods to solve similar tasks symbolically, she found that with repeated use, most would come to see the value of manipulatives:

...after they've had time to play, their first reaction is either why [use them], or I just want to play for the ones who have already seen other ways of doing that topic. But I would say the vast majority of them will get down and understand what's going on and they will... they'll buy into it (Elly, p. 8)

Frances also found that familiarity with the use of manipulatives in meaningful activities encouraged students to see them as useful tools to investigating mathematical concepts. Without this exposure, “there's a bit of resistance sometimes in the senior grades if students haven't had a lot of experience with these materials in the junior grades, Grades 9 and 10” (Frances, p. 14).

While the participants acknowledged their own role in influencing students' views of – and attitudes towards – manipulatives, they also recognised the broader role played by the leadership of the mathematics department in creating and promoting a culture of learning across the school. While Celia and Diane bemoaned the lack of such a learning culture in their own mathematics departments, both Frances and Alice described such a learning culture that existed within their schools. Frances described how in one of her schools, a department-wide initiative was created to encourage all teachers to teach certain topics in Grade 9 and 10 with the use of manipulatives (primarily algebra tiles and linking cubes) and technology to ensure that all students received a rich experience in exploring mathematics through multiple representations. She described that the culture in the department supported teachers' continual learning through both formal professional development and through informal work with peers. In this atmosphere, Frances stated, students gained experience, confidence and competence to use technology and manipulatives as tools “to focus on the mathematics instead of taking many periods to teach students how to use algebra tiles” (Frances, p. 12). Alice, too, spoke of the effort by her mathematics department to promote and encourage the use

of manipulatives in all of the classes starting in Grade 9 and described a growing momentum and engagement by her mathematics teacher colleagues in learning to use them and taking the time to share experiences. For those participants who used manipulatives, long-term exposure to manipulatives and their use in meaningful mathematical activities was viewed as instrumental in their students coming to recognize manipulatives as legitimate and valuable mathematical tools.

Availability and access to the manipulatives

In general, participants described that there were enough manipulatives available and accessible to them, as and when they were needed. While most participants described simple procedures to schedule access to sufficient quantities of manipulatives when they were needed, Elly and Celia claimed that their easy access was largely as a result of being the only one in the school using them. While some participants had class sets of manipulatives on hand, others described having to sign them out from mathematics resource areas and on occasion, negotiate with colleagues to ensure availability ahead of time. Overall, the participants did not report any problems in access to or availability of manipulatives that they needed. For instance, Alice described that while she did not currently have her own mathematics room, she was able to bring sufficient sets of manipulatives to class as she needed. She had no issues with availability as there were sufficient sets of manipulatives for all of the teachers:

So last year I had to teach mathematics in the language class... I would simply lug my box downstairs and lug it back... We had enough sets yes, and well-organized..... We had class sets, like two large bins like this. And more than one class set in each bin so in two or three bins we'd probably had four class sets. And no, there wasn't... access was not an issue, lugging them around was not an issue. (Alice, p. 11)

While Frances spoke about the importance of ensuring her students had ready access to manipulative materials in the classroom at all times, she described challenges she faced in a former school which adopted a strict signing out policy as there were insufficient quantities of manipulatives for each class. As department head, responsible for, among other things, the selection and purchase

of manipulatives she felt a responsibility “to learn what all of these materials were for and why should I be spending some of the schools money on some of these things” (Frances, p. 14) and made sure that her teachers had both time to learn and sufficient quantities of manipulatives. However, with the exception of Alice and Frances, participants described that many of the mathematics manipulatives in the school remained largely unused. Celia described that she had ready access to algebra tiles, linking cubes and sets of pattern blocks but only “because nobody uses them” (Celia, p. 16). Elly described that while she had sufficient sets of algebra tiles in her class for each pair of students, the mathematics office had plenty of manipulatives that “just sits there collecting dust... it's a shame” (Elly, p. 12). Similarly, Barry repeated a conversation with a mathematics coach who claimed that there were lots of manipulatives ordered for them by the board, “just sitting there in the mathematics room not used or used only by me for the locally developed [class] kids” (Barry, p. 2-1). It is interesting to note that the participants who spoke about the low level of use of manipulatives in their school were also those who described the absence of a learning community in their mathematics department.

Classroom Control of Manipulatives

The participants described many different ways in which they controlled the availability of and students' access to manipulatives in the classroom. This control ranged from making particular manipulatives available for specific activities, to allowing students unfettered access to many different manipulatives at any time. For instance, Barry brought no manipulatives into the classroom, while Diane allowed students to use some manipulatives on occasions, albeit without her guidance. On the other hand, Alice, Celia and Elly described making manipulatives available to their students for certain activities that explored specific concepts, while Frances reported making all manipulatives freely available to all students at all times.

Celia stated that she regularly used manipulatives in the class and spoke about making the manipulatives available for students to use on their tests. “Sometimes I do allow them. If it's a quiz and if the concept is in the process of soaking in, I definitely let them use it. But guess what; a lot of kids don't use [them]” (Celia, p. 17). She also commented that some of her older students make use of manipulatives because for them “...anything goes if they want to get a higher mark” (Celia, p. 17). However, she did state that in the future she would allow her students more freedom to choose which manipulatives to use and when, although she offered no further elaboration. Celia stated that while she wished to use manipulatives more frequently, she felt guilty not doing so “because there are times when I get too frustrated... because within a given time it doesn't always get them to the point where I want them to get” (Celia, p. 11). These and other comments suggest that Celia struggled to balance the benefits to students from using manipulatives with the commensurate challenges that they bring to both the management and pace of the classroom. Indeed, this might also suggest that Celia uses manipulatives in addition to, rather than as an integral part of her more traditional teaching practice.

Diane stated that she brings manipulatives into the classroom for specific activities although normally they would not be available for her students to use at any other time. However, despite her own lack of comfort with using either algebra tiles or linking cubes, she did mention making algebra tiles available for her students during exams, and being surprised that they actually used them:

They are allowed to use the algebra tiles on the test... I was astounded and they actually wanted it so I said fine. So in those cases [they're] available... it will be sitting in my room. (Diane, p. 16)

Alice described that she often created specific activities that allowed, but did not necessarily require, the use of manipulatives, which she then made available to the class. Once they were in the classroom, she would let her students access them as and when they deemed necessary. She stated that “sometimes I would casually mention that this can also be done using manipulatives [and ask]

‘Would you like them?’ And I may get a yes or no and [they would] just grab them” (Alice, p. 17). In a very similar manner, Elly described that she determined when to bring manipulatives into class, but once there, she gave her students full control over whether and when they use them. “Oh I just let them have free access to the tiles so you know if you want them I’m going in bring in a big box of them and you can have them as you wish” (Elly, p. 5). In this way, both Alice and Elly maintained a level of control over the availability of manipulatives yet allowed their students control over their using them when they were present.

These discussions seem to suggest that both Alice and Elly see algebra tiles, in particular, as having a valuable yet limited role in helping develop students’ understanding of certain algebraic concepts. Indeed, later in the interview, Elly acknowledged that she knew that there were many more concepts for which algebra tiles could be used and stated that she would take the time to learn these only if and when she felt her students needed it. This suggests that for Elly, at least, algebra tiles fulfilled a specific role in her teaching repertoire and had minimal utility in exploring other areas of mathematics. Thus, Elly and Alice make manipulatives available when they are able to identify how they could be of value; in other words, they mediate students’ access to the tools based on their understanding of their utility.

Frances on the other hand, claimed that for manipulatives to be fully integrated into mathematics teaching and be considered as serious learning tools, then they need to be readily available to students at all times, and their use not mediated by either the whim of the teacher or the nature of the activity. She cited one of the process expectations in the Ontario mathematics curriculum document that states that students be able to select tools and strategies appropriately (OME, 2005b). For that, she argued, students need unfettered access to manipulatives. To Frances, manipulatives should not just be made available, but be seen as available by students to use as and when they choose.

What I think is so important if manipulatives and concrete materials are really going to be integrated in the way we teach mathematics, students need access to them. If we're going to be “differentiating instruction”, if we're going to allow students to investigate, it shouldn't be that the investigation materials come when the teacher decides to sign them out from the book room. They should be available materials and tools. (Frances, p. 15)

Not surprisingly, Frances described that she attempted to keep full sets of manipulatives in her classroom at all times (to the chagrin of her former department head) so that her students truly had unfettered access to multiple manipulatives, and could use them as and when they saw fit. Indeed, Frances described (similar to comments by Celia) that she had witnessed many innovative and unanticipated uses for manipulatives:

Certainly the more that I've experienced teaching with these tools I see students coming up with solutions that I haven't necessarily anticipated... I find that I am always impressed with some of the ways that students have of representing problems particularly with linking cubes which I haven't necessarily thought of myself. (Frances, p. 12)

Problems with Manipulatives

Alice spoke about what she considered to be a design flaw with the particular type of algebra tiles that she had in her class. She noted that the actual length of the x dimension of the tiles was very close to five units and was a continual source of confusion for some students, especially those who had limited understanding of the notion of variables. However, she reported that she was able to address this and other misunderstandings by encouraging her students to consider *rules* for the use of algebra tiles, similar to the way in which they use rules to play board games. She described this idea in detail:

I try to address that, after those first five to 10 minutes of playing with plastic pieces I say okay, now these are fun to play with but there are some rules to this game. So now you have to understand the rules of the game so you can have fun playing it. So I may make some analogy with Monopoly... you can't just take money from the bank or you can't just put your houses anywhere. So now let's have some rules of the game. (Alice, p. 14)

Alice also described struggling to explain how to multiply two negative numbers when using algebra tiles: “Well, of course, they already know that negative times a negative equals a positive, but in an

area model it's hard. I have yet to find a satisfactory explanation for that" (Alice, p. 12). Once again, her solution was to tell her students that it was one of the given *rules* that did not necessarily have to make much sense but just had to be "obeyed for the representation to make sense" (Alice, p. 12), a situation with which she was clearly very uncomfortable:

That's one issue with students' understanding why you cannot have red and...what was it, negative, three colours together. When you have positive times positive is positive, but you can't have negative and negative and negative in the corner. ... That's one of the rules that come from above and the students have to obey it for the representation to make sense. So that's probably the only issue that I have with them. (Alice, p. 12)

Interestingly, Diane also referred to finding on the Internet a set of 10 rules associated with algebra tiles. "I thought 'Oh my God. I can't stand having ten rules' so I just gave the kids one rule: Make all the colours the same, and they got it right away" (Diane, p. 15). While Diane did not make it clear how and for what concepts she was using algebra tiles, by forcing her students to use only one colour of tile (which is synonymous to only considering positive coefficient values), Diane may have inadvertently avoided the problem that Alice discussed. Alice also described that despite explaining and reinforcing these rules, she had to watch carefully what her students were doing with them and, where necessary, intervene to address and rebuild their understanding.

Sure enough when you start talking about rules, some people automatically switch off... They are there but their minds are not. So I explain everything carefully and I get a lot of nods from the class. And when it comes to actually doing this stuff I find that some no-no's on some of the desks. So I have to watch the class a lot of make sure they follow those rules. (Alice, p. 14)

Use of Manipulatives Summary

All of the participants noted that using manipulatives in the classroom significantly disrupted the traditional classroom 'script' that encouraged students to shift from being passive learners to becoming more actively engaged with the mathematics; sometimes to the point where students did not recognise that that they were, in fact, doing mathematics. While some participants reported that students were often, initially at least, reluctant to use manipulatives, especially if they had already

seen alternative algebraic ways to tackle the same problem, on the whole, participants acknowledged that students enjoyed *playing* with manipulatives. However, they acknowledged that students were unlikely to see for themselves how manipulatives could benefit their mathematical understanding. Participants agreed that for students to consider manipulatives as serious mathematical objects, their use had to be modeled by teachers through meaningful activities that explored mathematical concepts. For some participants, the role and responsibility for modeling the serious use of manipulatives extended beyond the teacher to include the learning culture of the entire mathematics department. Those participants who did not gain support to model serious use of manipulatives from other teachers or their mathematics department reported finding this endeavour to be more challenging, problematic and time-consuming. The influence of the mathematics department culture on the teachers and their teaching practices in the classroom is explored further in subsequent sections.

Participants who used manipulatives reported that activities with them were particularly effective when students worked together in pairs or small groups. Participants claimed that when solving problems in groups, students were more engaged with the problem, more willing to demonstrate and articulate their mathematics understanding and strategies, and more willing to learn from and assist each other. While the participants did not attribute all of these benefits solely to the use of manipulatives, several described that the manipulatives served as a public focus for students to debate, conjecture and reason mathematically. Furthermore, participants noted that their own ability to assess students' understanding was enhanced as students appeared better able to describe their thinking with manipulatives compared to purely symbolic work on paper.

Of all the manipulatives available to them, algebra tiles and linking cubes were the manipulatives most discussed and used by the participants. Not surprisingly, they were used by participants to investigate and review many patterning and algebraic concepts. Activities using

linking cubes included investigation of relations and rates of change of volume and area and those using algebra tiles included collecting like terms, manipulating trinomials and factoring quadratics and the process of *completing the square*. Participants agreed that students' understanding of these concepts were strengthened by the visual nature of the manipulatives and in connecting their actions on the manipulatives with symbolic algebraic processes.

None of the teachers reported any difficulties in obtaining suitable manipulatives in sufficient quantities for their class. However, several reported that they were either the only teacher, or one of a small number of teachers, in their school who used mathematics manipulatives. Indeed, so significant was the isolation felt, that two participants, who actually teach in the same secondary school and are members of the same mathematics department, each describe themselves to be the only teacher in their school to use manipulatives.

Lastly, there appeared to be a connection between the participants' descriptions of the attributed value of manipulatives, how they are used in the classroom and how students' access to them is controlled. To illustrate this connection, in the case of algebra tiles, Barry and Diane described that they have yet to find either comfort or value in their use and describe that algebra tiles are neither integrated into their instructional practice nor routinely made available to students. Alice, Celia and Elly described that the use of algebra tiles is beneficial to their students to investigate particular algebraic concepts; and this appeared to lead to the manipulatives being used in a somewhat proscribed manner (that is, used in a specific way or for a specific purpose) and being made available to students as and when they felt that the activities warrant their use. While not stated explicitly, there is a sense from these participants' descriptions that manipulatives were often used to reinforce previously covered material while not always present in the investigation of new mathematical concepts. Frances, who considered algebra tiles (and other manipulatives) as being valuable mathematical tools with which to explore mathematics, described ensuring that a wide

variety of manipulatives were always available to her students. With her guidance, but not control, she felt that students would select the manipulative that they found to be most helpful for the activity. In this way, students were able to access manipulatives to both explore and reinforce mathematical concepts. This seems to align with findings by Moyer and Jones (2004) who concluded: "Allowing students some choice in their selection of mathematical tools is a minor step in encouraging responsibility for their own learning" (p. 29).

CHAPTER 7: EMERGENT THEMES

During the analysis phase, two significant themes emerged from the data: supports for and challenges to the use of manipulatives in the classroom. The first dealt with the teaching environment and resources from which participants drew support to help them gain confidence and competence in using manipulatives with their students. The second dealt with challenges that the participants identified that thwarted or hindered their efforts learning about and using manipulatives. While these data were identified initially during the analysis of each participant, many of the themes were common across all participants. As a result, the discussion of the data analysis is presented thematically and incorporates all of the participants' views and experience.

These two themes contribute new and important data to the educational community to support teachers to learn about and integrate the use of manipulatives into their teaching practice, and to become aware of and help address the specific challenges that teachers face in this endeavour.

Discussions on Supports for Teachers

The following section details the participants' views on the support and encouragement that they received for their use of manipulatives in the classroom. First, I describe the support received informally from within the mathematics department and the participants' mathematics teacher colleagues, followed by a description of the more formal supports offered through school board professional development workshops and mathematic consultants. I then describe the support some participants have found from curricular resources created by the Ministry of Education and provincial mathematics associations. I then conclude the section with a summary of these supports.

Support from Within

All of the participants provided a glimpse into the culture of their mathematics department and the role played by the head of the department in leading and supporting teachers. However, the

participants highlighted huge differences in the cultures of their departments, ranging from Alice and Frances' descriptions of a supportive and engaged community of mathematics teacher learners, to Elly's description of little, and Barry, Celia and Diane's description of no support and encouragement from their mathematics teacher colleagues.

Alice described the effort by the head of her mathematics department to encourage teachers to use manipulatives in all of the secondary school classes starting in Grade 9:

[Manipulatives] are used in Grade 9 whenever [the students] are introduced to polynomials and the multiplication of polynomials. It's not the sole emphasis. This is presented to the students has an alternative method. It's not THE [only] way to do things. (Alice, p. 6)

She reported feeling a growing momentum and engagement by mathematics teacher colleagues in learning how to use manipulatives. Alice spoke fondly of the supportive environment at her school and the value she gained from the informal sharing of ideas, resources and experiences, where teachers often demonstrated their use of manipulatives to each other around the lunch table. Alice also expressed her pleasure in being able to contribute her own experiences and understanding to the teaching community:

So what I find is, we're all very busy, and it's crazy at the school, but the teachers are all very supportive so all you have to do is ask a question ... I find it was a great environment in that sense.... I found even the casual question thrown at someone, if anyone else is in the office, they will add to it and say that "oh, by the way I had that happen to me once..." and they would explain what they did. So, it was casual, but it was there and it was substantial. So they would work on different examples together or... someone would put a handout together and share it with us. (Alice, p. 11)

Frances also described the support she had received in her professional development from both peers and administration, emphasising the culture of support and collaboration that the head of the mathematics department had developed. Similar to Alice, she described a department-wide initiative to ensure that all teachers teach using consistent models, manipulatives (in particular, algebra tiles and linking cubes) and technology in their lessons from Grade 9 onwards. Frances described that

both formal professional development workshops and informal lunch time conversations with her peers helped her gain the confidence and competence to fully integrate the use of manipulatives and technology into her classroom. She gave an account of working closely with a colleague with whom she developed an expanded repertoire of activities for teaching senior mathematics by sharing ideas and co-developing lesson plans. She also described learning through osmosis, by just being around other teachers engaged in investigating and exploring mathematics teaching and learning using manipulatives and technology:

You're not necessarily comfortable with some of the manipulatives and technology you sort of get pulled into it by some of the teachers that you working with, which is a great learning opportunity. (Frances, p. 3)

Like Alice, Frances also described the value of the informal support that she gained from her department colleagues around the lunch table when they would share and critique ideas, and discuss, develop and practice activities using manipulatives.

Invariably someone improved what you were doing. That they had thought about it in a slightly different way and if you were being a bit too regimented in your approach with the manipulatives because you were just learning how to use them, someone would show you something else or make a connection for you... Strengthening those connections was really important having that dialogue in the department. (Frances, p. 13)

Elly also acknowledged the strong support and encouragement to use manipulatives that she had received from her department head (yet little from other mathematics teachers), who was also one of the few teachers in the department to use manipulatives. She described that as they taught the same Grade 12 class, the two of them would often work together, stay in lock-step in their teaching and share tests, activities and resources. They had also jointly given mini-workshops for other mathematics teachers to help develop techniques and greater understanding of manipulatives. However, she noted that her department head initiated few formal math-related activities for the departments' teachers or made any effort to encourage informal discussions between teachers – identified as so useful by Alice and Frances.

Support from Without

Celia noted that professional development workshops organized by the school board were the principle way she learned about using manipulatives. As Celia claimed that she gained no support from within her own mathematics department, she credited these professional development days for not only providing training and resources but as opportunities to connect with and learn from other teachers and finding out about “what worked and how it worked” (Celia, p. 18). Celia also mentioned finding useful manipulative activities that teachers had posted on the school board’s internal communication system. Elly, too, described the value of professional development workshops offered by her school board and the efforts of board consultants who offered individual assistance in the use of manipulatives, crediting both in helping her overcome her initial dislike for manipulatives. Similarly, Frances claimed that most of what she has learned about manipulatives had come through professional development workshops and complemented the efforts of board consultants in creating comprehensive activities to explore using manipulatives for teaching mathematical concepts across all grade levels of secondary school. She lauded her school board’s willingness to pay for release time that gave all mathematics teachers the opportunity to participate in these workshops. In addition, as department head, Frances described working with other department heads at the board on a number of initiatives. One of these initiatives was the development of a workshop for heads of mathematics departments to help them see the value in teaching with manipulatives, and assist them in creating a supportive and encouraging environment for their teachers.

Curricular Supports

Alice spoke of the value of the current Ontario mathematics curriculum in its consistent promotion of the use of manipulatives in each grade but lamented that manipulatives were not a significant part of the International Baccalaureate curriculum or curricular support materials. Frances also referred to the value of the curriculum in encouraging students to construct and deepen their

understanding of mathematical concepts through problem solving, mediated through the use of manipulatives and technology. She contrasted the spiralling approach of the current curriculum - where many of the major mathematical concepts are revisited and explored in greater depth in subsequent grades - to the more traditional *linear* way of teaching mathematics. Frances believed that this linear approach (that she experienced as a secondary school student) reinforced the idea with students of “marching through a textbook and that once you've done it, it's finished and there's no connection between materials” (Frances, p. 11).

Other Supports

Other than the support received from other teachers and school-board professional development activities, only Diane and Frances referred to accessing other sources to seek inspiration and suggestions for activities that used manipulatives. Diane mentioned that she found the Internet to be a great source for fun activities for her class but did not offer further elaboration. Frances, on the other hand, went into great detail about the resources that she had found useful in planning and developing activities for using manipulatives. Frances claimed that the new Grade 10 and 11 textbooks included more activities that used manipulatives to explore mathematical concepts. She found the provincial ministry's TIPS4RM materials for the Grade 9 and Grade 10 Applied courses to be “pretty good” (Frances, p. 13), but felt that that there were many topics that would benefit from more manipulative-based explorations and investigations. Frances also referred to supplementary Grade 11 College and University course materials from provincial mathematics organizations as being particularly useful in showing teachers how manipulatives can be used with greater sophistication in the senior grades. Frances also spoke about finding “some fantastic things on the NCTM Illustrations website that has some really nice investigations that use concrete materials” (Frances, p. 13).

Summary of Supports

As seen in the participants' vivid descriptions, the level, quality and nature of the encouragement and support that they received in their growth as teachers varied considerably – ranging from almost non-existent to extensive. The participants placed significant emphasis on the important role played by the head of the mathematics department in establishing and fostering a community of learners in the department from which much of this support flowed.

At one end of this range, Alice and Frances described being immersed in a vibrant and supportive culture of learning that encouraged them to enact within their classroom a curriculum founded on conceptual understandings, multiple representations, mathematical modeling and problem solving. This support was both personal – by encouraging and motivating the teachers to try new approaches and techniques – and practical, as teaching colleagues shared assessments, lesson plans and tried-and-tested activities. At the other end of this range, Barry, Celia and Diane claim to have received little meaningful support from colleagues and administration and described feeling frustrated and isolated in their efforts to enrich the learning opportunities of students in their classroom. Not only was there little encouragement to try new approaches, the teachers had to work harder to find and develop activities that they could use. It is perhaps of no surprise that Barry, Celia and Diane described having difficulties (albeit to varying degrees) in finding value in and integrating the use of manipulatives into their teaching practice. While it is unclear whether these specific challenges would have been alleviated or least ameliorated through the support of colleagues, it is clear that both Alice and Frances *felt* encouraged and supported to try new approaches and take risks in their classrooms. In addition, Alice and Frances claimed that department initiatives helped to ensure that all students received and benefited from a consistent mathematics experience that included the use of models, manipulatives and technology.

Table 6 captures some of the salient features of the participants' descriptions of the support received in their own school and from mathematics teacher colleagues for learning about and trying out activities that used manipulatives.

Table 6: Participants' View of Support and Encouragement to use Manipulatives

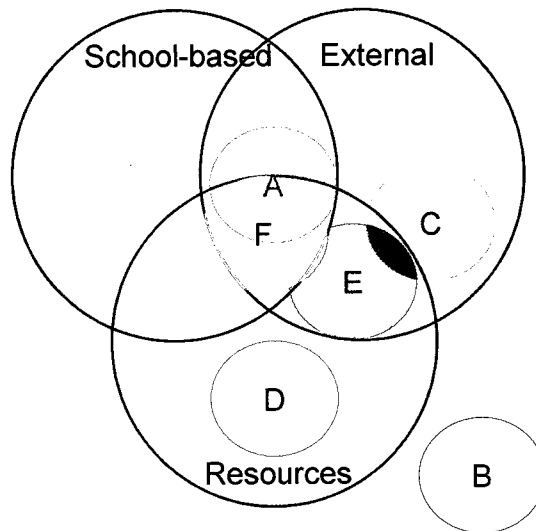
	<i>Descriptions of School-Based Supports</i>
Alice	"It's casual... the teachers are all very supportive... So I've never had a problem sharing ideas and resources and everything. So, yes, it was a great environment."
Barry	"Being a new teacher I'm not sure what... is expected of me... So there was a lot of trying to figure my way out of the dark. I didn't really feel a lot of real support. "
Celia	"So in [name of school] we need a mathematics department head. We need consistency... Every teacher does everything differently. What the hell is that? This is my rant!"
Diane	"When you have departmental head who doesn't know math, it's really hard organizing colleagues.... [Name of school] doesn't even have a course outline, it's incredible!"
Elly	"We do tend not to work too well... which is not a good thing... We do very little collaboration type stuff"
Frances	"You're not necessarily comfortable with some of the manipulatives...you get pulled into it by some of the teachers that you work with, which is a great learning opportunity."

While participants reported that the support they received within their school varied considerably, support networks for the use of manipulatives outside the school were also described. School-board professional development conferences and workshops were identified as being particularly supportive forums for gaining confidence and competence with manipulatives. All participants (except Diane) described these workshops, led by knowledgeable teachers and mathematics consultants, as being very helpful in learning about and trying activities using manipulatives. Several participants (including Celia who did not feel encouraged or supported at school) reported that professional development workshops and other mathematics forums (such as locally held mathematics association conferences) also afforded excellent opportunities to meet informally with other mathematics teachers to discuss and share activities.

Celia, for one, reported that online communities, bulletin boards and email with other mathematics teachers in the school board were also valuable ways to share and discuss activities using manipulatives. Alice, Frances and Diane also spoke about curricular resources and the Internet as providing useful ideas, inspiration and activities for using manipulatives.

The illustration in Figure 3 attempts to show the relative value of these different facets of support in each participant's endeavour to integrate manipulatives use into their teaching practice.

Figure 3: Participants' Views of Supports



Discussions on Challenges

In this section I describe some of the challenges faced by participants in their views about and use of manipulatives. I start by discussing the time and effort that the participants felt was needed to learn how to use manipulatives in the classroom. This is followed with a description of the participants' concerns over time pressure to cover the curriculum that pit depth of concept development and student exploration and investigation against each other. I then describe participants' challenges in motivating students to value manipulatives as valued learning tools. The section is concluded with a summary of these challenges.

Time and Effort to Learn how to use Manipulatives

Alice described her greatest challenge to be in finding sufficient time to learn how to use manipulatives to solve mathematical problems. She described that, similar to the learning style that she found effective during secondary school, she needed to work with manipulatives on her own before she gained sufficient comfort and confidence with them to use them in class and subsequently “sell them to my students” (Alice, p. 6). She described that finding this time was particularly challenging in light of the other responsibilities she had at the school:

But my big challenge is finding the time to engage more in problem solving besides designing the lessons and the activities and the marking and the extra help we give students before school, at lunch or after school. That time, which allows me to sit with a nice problem and find different ways of solving it.... That’s what I’d like to have more [of]. (Alice, p. 5)

Diane also spoke about how long she needed before she was confident enough to use manipulatives with her students. As she stated, “To make models that work for me takes a long time. It takes years” (Diane, p. 9). Celia claimed that as she had not been exposed to manipulatives when learning mathematics, their use did not come naturally or easily to her and admitted that she was often “guilty of lecturing her students a lot of times” (Celia, p. 8). She felt that for teachers like her to become sufficiently comfortable with manipulatives to be able to use them effectively with students demanded considerable effort and a positive attitude towards learning new things:

It's a lot more work as a teacher and because we're not the ones who grew with the manipulatives so it's really hard to do... it's not natural. And in many senses you have to like it. You have to think it's valuable. The teacher has to have the positive attitude or else it doesn't work. So the experience and attitude of the teacher is a big thing. (Celia, p. 13)

Frances also spoke about challenges to using manipulatives framed in the larger context of the current reform curriculum and its call for changes to teacher practice. She felt that for more experienced teachers, the reconsideration of mathematical content, pedagogy and the teachers’ role promoted in the current curriculum was often challenging, especially when it came to using manipulatives and technology in the classroom. Frances described that some older teachers find

Grade 9 and 10 classes particularly challenging as the curriculum strongly promotes the use of technology and manipulatives as valuable tools for mathematical exploration, investigation and problem solving. She stated that, “It does take some reconsideration of the material and quite a bit of effort to reconsider the approach to the course, the philosophy of how you're approaching things, rather than perhaps a more traditional Socratic method” (Frances, p. 3).

Elly also spoke about teachers' resistance to change their teaching practice noting that at her school few teachers use manipulatives and that “it's always hard to try to get [teachers] to do something different” (Elly, p. 7). Elly claimed that her colleagues were reluctant to use manipulatives because they represented something new and fell outside their comfort zone. She found that these teachers often justified their reluctance by claiming that manipulatives had little or no value and that their use takes up too much class time:

I'm already doing it this way. I can't put in an extra lesson to teach tiles when I've already taught it...like it's worked for me this way and it's going well. I have everything and organize things. I can teach it in one lesson here and would have to spend two lessons on it [if I used manipulatives]. Maybe not thinking it's worthwhile enough in the long run.
(p. 5)

Time Pressure and Curriculum Coverage

Although all of the participants referred to feeling under pressure to cover course content, Celia described in detail the difficulties she faced in covering the large number of concepts in the curriculum in sufficient depth within the allotted time. She found this particularly challenging when having to differentiate instruction in a class of 30 students. She stated that:

The curriculum itself is too jam-packed the way I figure it, concept wise... There are too many concepts to teach. I wish there were less concepts and [each explored] a little deeper... It's almost impossible for me to teach those concepts, those new concepts, within [the] given time. (Celia, p. 9)

She went on to say that in order to cover the material she often had to skim over important concepts “rather than go deep down so that the kids actually try it out and see ‘Ahhh...this is what it means’”

(Celia, p. 8). Celia described that she sometimes found that student investigations and explorations that used manipulatives only seemed to exacerbate this problem. On the one hand she found manipulatives to be useful learning tools that should be used more often, yet on the other she found that their use often took more time than she felt she could justify given the amount of material to cover. For Celia the time constraints continually forced her to trade off in-depth student investigations with manipulatives for content coverage:

So all these manipulatives and experiments that we do take a lot more time and sometimes it is totally necessary for me to give them time to... figure it out, some of them, right,... not all of them but some of them do take time. And if that is the case I think we really have to think about the curriculum itself. Can we teach all of those concepts within the given time? ... I wish I had more time to investigate and see how [manipulatives] can be used at a higher level but sometimes I just don't have time. (Celia, p. 9)

This became even more problematic as she was often faced with having to introduce manipulatives to her students for the first time. Elly echoed similar sentiments with being faced with a class of students who have very different prior experiences with manipulatives:

It's actually harder when half have and half haven't [used manipulatives] because then it's like how do I run this lesson? Do I make half bored out of their minds? Do I get them in groups where they help each other? But then half of them don't get it explained well because their partner doesn't happen to be good. (Elly, p. 12)

In addition to her concerns about covering the grade level curriculum content, Celia also described feeling challenged trying to address the substantial knowledge gaps of some of her students. She positioned this discussion in her view that students need a consistent mathematical learning experience throughout their secondary school years that includes repeated – and hence familiar – use of manipulatives, introduced in early grade levels. As she discussed this, she built a large but incomplete cube from linking cubes as a model of her students' mathematical understanding, referring to the missing blocks as gaps in their knowledge. Celia reported that as no such consistent

learning experience existed in her school, she had to find time to address the gaps in her students' knowledge while also completing the grade level course content:

We have to provide a consistent learning experience for students.... So that's a problem, and in mathematics unlike other subjects, you really need every block in here... And for some of these kids I know exactly what part is missing and yet I cannot do it... When they send me those kids in Grade 11 Academic class, right, and [prior learning] has not been done and the whole group doesn't know it, where the heck am I going to get the time to go through that again? (p. 14)

Motivating Students

While several of the participants spoke of their students' resistance to using manipulatives and the challenge to motivate them to consider manipulatives as useful mathematical learning tools, there were few details as to how these issues were affectively addressed. For example, Alice described problems with some of her Grade 12 class that included maintaining student engagement with the subject matter, overcoming their resistance to the use of manipulatives and dealing with significant absenteeism, yet did not offer any details as to how she coped with these problems. Barry talked about the difficulty of motivating students who did not want to work and acknowledged that for some of them, their fear of mathematics and of further failure was probably the main reason for their apparent unwillingness to try. Barry reported that for students "who don't care, who have discipline issues, the last thing they have on their mind is to learn math" and for them the use of manipulatives is almost "too late" (Barry, p. 8). Barry noted that students' attitude is a significant factor in how they viewed the use of manipulatives and his classroom experience led him to believe that were he to attempt to use manipulatives his students would regard them as a joke, and their presence would be little more than a distraction:

I said to myself, "oh this will be great to apply in an elementary Grade level 5, 6" but to bring it a little higher... to Grade 10s and trying to apply it to kids who are almost too cool to learn math... with their attitude and discipline issues it would just become like a farce. (Barry, Interview Transcript 1, p. 4)

Celia, on the other hand, acknowledged students' reluctance to use manipulatives yet suggested that these problems could be effectively addressed through a department-wide culture that promoted and encouraged student learning through investigation exploration aided by the use of technology and manipulatives. She spoke of a school with such a culture, led primarily by the efforts of the department head, where manipulatives and technology were used across Grades 9 through 12, and reported that the teachers' use of manipulatives was "much more effective" (Celia, p.10). Celia restated her view that such a culture must be present throughout the entire school and commented:

There has to be collective agreement that [manipulatives] are important learning tools. We play with them but we learn with them. It's a process of learning. And if you don't have that culture every time you introduce those manipulatives you spend half the time persuading the kids that this is a serious business... So it takes time and it takes a communal effort, not just one teacher. Because if one teacher tries to do it alone, [the students] throw them, don't do the work, and 30 kids throwing algebra tiles is ridiculous. (Celia, p. 10)

Summary of Challenges

The common challenge described by all participants involved the amount of time and effort needed to learn about and integrate the use of manipulatives into their teaching practices. However, it appears that participants may be referring to two quite separate challenges. For Alice, for instance, the challenge seems to be in actually finding the time to diligently work through problems using manipulatives – her preferred learning style – along with all of her other teaching duties. This could be illustrative of the time pressures that many teachers face in lesson preparation. For the three older and more experienced teachers, however, this challenge would appear to be less about finding sufficient preparation time and more to do with how readily they have been able to integrate the use of manipulatives into their teaching practice. Celia, Diane and Elly describe having taken a lot of time to learn about and become comfortable with using manipulatives with their students and acknowledge that this process of 'integration' has been very slow and is still in progress. It is also interesting to remember that these teachers were first introduced to manipulatives after having taught

mathematics for many years. This may be an example of research studies (e.g.: Kagan, 1992; Manouchehri, 1997; Pajares, 1992) that show teacher practice to be resilient and slow to change.

A further challenge that was identified involved the apparent conflict between covering all of the curriculum concepts and the time necessary to explore them all in depth. While all participants acknowledged the challenge of getting through the course material, some described making a conscious decision to skip over certain concepts in order to allow time for more thorough student investigation of others. It would seem that the trade-off between curriculum coverage and concept depth was easier for some participants to make than others.

Finally, on the theme of motivating students to develop positive attitudes towards using manipulatives, participants seemed to agree that the culture of the mathematics department had a significant influence on their students' attitude. There was a belief, generally expressed by all participants, that a department-wide culture that encouraged student learning through investigation, aided by the use of technology and manipulatives, practiced consistently throughout the grades, encouraged students to see manipulatives as useful tools and to be used seriously.

CHAPTER 8: CONCLUSIONS

This chapter begins with a summary of the findings of the study and their connections to research literature followed by a discussion of implications of the study's findings for educational practice. I then outline contributions of this study to the discussion of manipulative use in the teaching and learning of secondary school mathematics, and detail some limitations of the study. I follow this with suggestions for future research directions and conclude with some final comments about the study.

Summary of Findings

While we see that current thinking and research in mathematics education as well as current curriculum encourage the use of concrete models in teaching and learning mathematics, this study supports the notion that the use of manipulatives in secondary mathematics classrooms is influenced by teachers' views of and experience with manipulatives. The findings support the idea, noted in literature on research into teacher behaviour (e.g.: Ernest, 1989b; Koehler & Grouws, 1992; van der Sandt, 2007), that the influences on teacher practice are both myriad and complex. Furthermore, this study shows that van der Sandt's (2007) conceptual framework for research on teacher practice, through consideration of teachers' knowledge and views, is an effective framework with which to examine teachers' views and use of manipulatives.

Overall, the findings from this study corroborate (albeit to varying degree) several of the reasons offered by research studies of elementary and middle school teachers as to why some teachers do not use manipulatives in their teaching of mathematics. These reasons include teachers' limited understanding of mathematical concepts behind the manipulatives (Moyer, 1998), lack of experience with manipulatives (Lesh, et al., 1987), lack of teacher support in using manipulatives (Howard, et al., 1997), and the belief that manipulatives are more suitable for elementary school

grades than for more advanced mathematics (Fennema, 1972; Tooke, et al., 1992). Concrete examples of each of these four reasons have been identified within this study and are detailed below.

This study shows a full spectrum of views in the participants' descriptions of their use of manipulatives to model mathematical relationships. For example, Frances described being capable of using many manipulatives to model a variety of different mathematical concepts, while Diane, on the other hand, claimed that she did not understand how to use algebra tiles to represent algebraic concepts. Despite many opportunities and much effort to try to learn how to use them, Diane did not use them with her students when investigating algebra. However, she did report making them available to her students on occasion as she recognised that her students were making connections with these manipulatives that she, herself, was unable to see. This would appear to corroborate Moyer's (1998) finding that teachers may not use manipulatives if they have limited understanding of the mathematical concepts behind them.

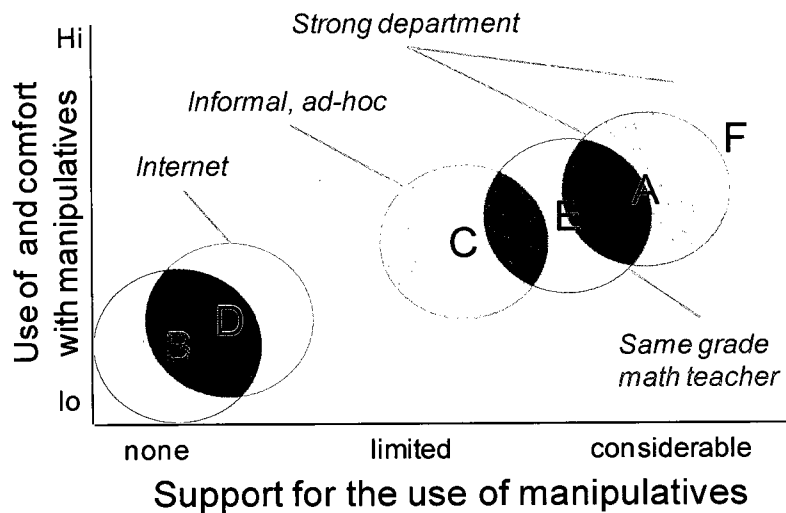
Findings from this study show that the participants needed time and guidance with using manipulatives before they were sufficiently confident to use them with their students in the classroom. Although the study recognized the dynamic nature of gaining experience, for at least two of the participants (Barry and Diane) lack of experience with manipulatives was cited as a reason why they did not use them; findings that support conclusions drawn by Lesh and his colleagues (1987). In addition, participants, like their students, did not gain their understanding of manipulatives in the same way or at the same speed. While professional development workshops were effective environments for some, not all participants benefited equally from these venues.

Findings from this study also reinforce the need for many and varied learning opportunities – including time alone, and informal time spent with other teachers – for teachers to feel sufficiently comfortable with the manipulatives before modeling their use for students in the classroom. For some teachers, the necessary changes to their teaching practice to accommodate the use of

manipulatives may require repeated learning experiences over extended periods of time. These findings align closely with Ball (1992) who noted that teaching with manipulatives requires not only an understanding of appropriate pedagogical strategies and techniques but a significant reframing of instructional practice.

While findings of this study cannot claim a direct connection between a lack of support for the teacher and their failure to use manipulatives in the classroom, as reported by Howard and his colleagues (1997), it is noteworthy that two of the six participants (Barry and Diane) described that they received no local support for integrating manipulative use into their teaching practice and were the only two who did not use manipulatives in their classroom. Conversely, the findings of this study reinforce the value of assistance and encouragement that teachers can obtain in this endeavour when immersed in a supportive environment of mathematics teacher colleagues. The findings show that this often informally created environment not only provided opportunities for teachers to contribute and exchange techniques, but were venues to discuss how and what students are thinking and learning as they engage with the manipulatives. This reflects findings by Ball (1992) who concluded that for manipulatives to play an important role in improving mathematics education, teachers require many opportunities for reflection and professional discourse. While I acknowledge that I draw from a very small sample, Figure 4 illustrates the finding of a suggested connection between the support that participants received and their comfort with and use of manipulatives. This connection suggests that those participants with higher levels of support seem to be more comfortable with and make greater use of manipulatives in the classroom.

Figure 4: Suggested Connection between School Support and Use of and Comfort with Manipulatives



This study highlights that not all teachers hold the same view about the appropriateness of manipulatives in the teaching and learning of secondary school mathematics. Indeed, several of the participants reported that their initial introduction to manipulatives was contextualized around the notion that such materials were more beneficial for younger and struggling students. While most of the participants acknowledged that through their own teaching experience and with the encouragement of curricular materials, their views have changed to consider manipulatives as valuable learning tools in secondary school mathematics. However, this view was not shared by all. Barry, for example, expressed the view that manipulatives were only appropriate for elementary school mathematics, citing this as one of the reasons why he did not use manipulatives with his students. While Barry offered several reasons as to why he did not use manipulatives, his view that they were more appropriate in the teaching of elementary school mathematics supports the findings by research by Fennema (1972) and others.

In addition, findings from this study show that the participants' views – at least initially – reinforce the idea that manipulatives themselves do not automatically carry mathematical meaning,

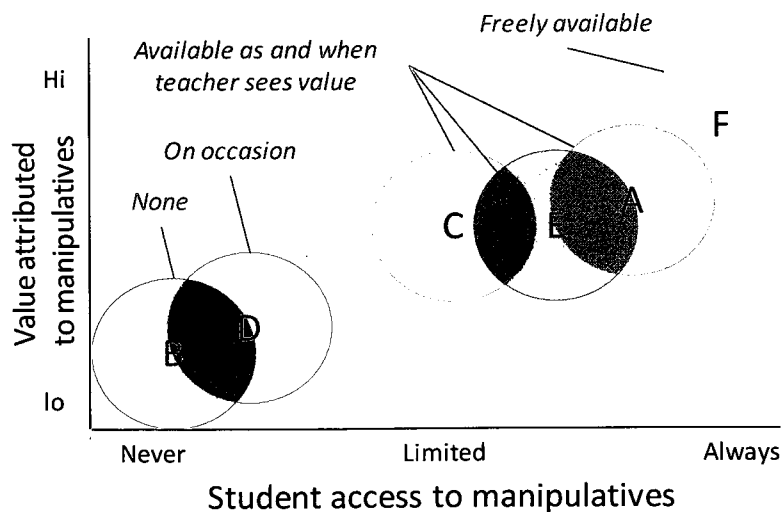
and how manipulatives can be used to model mathematical representations is not always intuitive or obvious. These findings are congruent with other researchers (e.g.: Ball, 1992) who noted that neither the physicality of manipulatives nor their mere presence is sufficient to guarantee meaningful learning for teachers and students alike. This study shows that coming to see the mathematical concepts in the manipulatives can be a challenging and lengthy process; also noted in the research literature (e.g.: Thompson, 1994). Additionally, gaining sufficient confidence to model the use of manipulatives for meaningful exploration of mathematical concepts to students often demands a significant amount of time and considerable effort on the teacher's part.

The participants in this study suggest that their students also require time to become familiar with manipulatives before they consider them as valuable learning tools; a factor noted in the literature (e.g.: Boulton-Lewis, 1998). Participants also noted that the connection between the manipulatives and the mathematical concepts under investigation may not be immediately apparent to students, closely aligning with conclusions drawn by Ball (1992) and others. Participants acknowledged the crucial role of the teacher in modelling mathematical relationships for their students and the need for them to make explicit the connections between actions on the manipulatives and other notational representations, echoing the findings of studies by Sowell (1989), Thompson (1992a) and Moyer (2001). The participants also stressed that the task of imposing mathematical relationships on the manipulatives was made more challenging when their students have limited or no prior experience with manipulatives. Conversely, they claim that students' familiarity with the manipulatives has the benefit of lessening the time necessary to introduce new concepts, further reinforcing findings by Hynes (1986) and others.

The findings suggest a connection between the participants' attributed value of manipulatives and how and when their students were able to use them. To illustrate this connection, for the three participants who described using particular manipulatives to explore specific mathematical concepts,

the manipulatives were made available to students as and when the teacher deemed them necessary or appropriate for the activity. Put another way, these participants appeared to control the availability of manipulatives based on their own understanding of the mathematics that could be explored with them. For the participant (Frances) who viewed manipulatives more generally as mathematical learning tools and described finding considerable value in a wide range of manipulatives, students were given free access to them at all times; these students, and not their teacher, selected and accessed manipulatives to mediate their own learning. This connection also seems to apply to the two participants who claimed to be unable to express mathematical concepts using manipulatives, described seeing little value in their use, and consequently did not use them or make them generally available for their students' use in their classroom. The illustration, in Figure 5, attempts to locate each participant based on the value attributed to the manipulatives, when manipulatives were made available and how students' access to manipulatives was controlled.

Figure 5: Suggested Connection between Value Attributed to Manipulatives, Availability and Control of Students' Access



This finding suggests that the participants both mediate the students' learning *and* control the availability and access to manipulatives based on the participants' own views and knowledge of the

manipulatives. It suggests that the greater the teacher values the use of manipulatives, the less they control their students' access to and use of manipulatives. Put another way, the greater the value the teacher attributes to using manipulative as mathematical learning tools, the greater the autonomy their students have in mediating their own learning with them.

Implications for Education

While the focus of this study considered teachers' views and use of manipulatives from their personal perspective, what was striking was the high value placed by the participants on being part of a supportive learning community within their mathematics department and the central role of the head of the department in fostering this environment. This professional learning community was seen as instrumental in assisting and encouraging teachers to use manipulatives for their students' investigation of mathematics, and the informal support offered by other teachers similarly involved was deemed crucial for this endeavour. Findings from this study echo conclusions from other research (e.g.: Ball, 1992) that reinforce the value to teachers of engagement in professional learning communities that, along with other benefits, serve to embed professional development into daily work. An environment that promotes long-term and consistent use of manipulatives across all grades was identified as helping students consider manipulatives as valuable learning tools, corroborating the findings of Ernest (1994) and others.

In this study, the participants acknowledged that changes to teaching practice to accommodate the use of manipulatives was often slow and incremental, especially so if their teaching practice had been well established. While professional development workshops on using manipulatives were seen as valuable, this lengthy accommodation suggests that helping teachers become knowledgeable about and comfortable with manipulatives requires multiple, varied and long-term opportunities to learn. These findings are also consistent with research on teacher practice

that shows teachers' views to be highly robust (Pajares, 1992) and slow and often resistant to change (Kagan, 1992).

Contributions and Limitations of this Study

Contributions

As previously discussed, several research studies confirm that there is a decline in the use of manipulatives in mathematics classrooms as students' progress from their early years in elementary schooling, upwards into secondary school. While these studies offer suggestions as to the causes of this decline, there is little research that attempts to understand this trend from the perspective of secondary school teachers. This research offers insight into this relatively unexplored area of the influence of secondary school teachers' knowledge and views of mathematics manipulatives on their actual use in the classroom. This study adds to the body of literature that has examined manipulative use in both elementary and middle schools. It also extends the understanding of the complex interactions between teachers' knowledge about and use of manipulatives for the teaching and learning of secondary school mathematics. In addition, this study describes, from the teacher's perspective, some of the challenges faced in enacting elements of current curriculum that promotes and encourages the use of mathematical manipulatives. It also highlights the value of formal and informal networks of support that help teachers become knowledgeable about and integrate manipulatives into their teaching practice.

Limitations

This study is limited by its small scale: six participants, all of whom taught in the same small geographic area. As such, findings from this study are not generalizable or transferable to the larger population of secondary school mathematics teachers. As these teachers volunteered to participate in this study and provide their insights freely and openly, their views may not necessarily represent those of their teaching colleagues less willing to share their experiences in this manner. As a

consequence, views from this latter group of teachers may not be voiced in this study. Another limitation stems from the fact that all data used in the study was based on the teachers' own descriptions of their views and actions rather than from direct observation of their teaching practice.

Directions for Future Research

Upon reflection, several outstanding issues and questions remain, some of which may present useful avenues for further research. Several questions involve the number and gender of participants in the study and the subsequent range of experiences represented.

For instance, all of the participants – none of who had been taught using manipulatives – cited the lack of familiarity with the materials as having hindered (albeit to varying degrees) the integration of manipulative use into their teaching practice. This raises the question whether teachers who had learned mathematics using manipulatives would be able to readily transfer this experience and knowledge into their teaching practice, and what, if any, additional challenges may emerge. Similarly, as mathematics has frequently been believed to be a 'male domain' (as identified in studies by, for example, Fennema & Sherman (1977)), would differences in findings have emerged if more, and more experienced, male teachers had contributed to the study.

Furthermore, these teachers all taught using the Ontario mathematics curriculum. While this curriculum promotes the use of manipulatives in the teaching and learning of mathematics, involving participants with experience in other curricula may offer further insight into the actual role played by curriculum in the adoption of new teacher practices. Supplementing the interview data with classroom observations may offer greater insight into how the teacher orchestrates, controls and mediates students' learning with manipulatives during mathematical problem solving.

While the social context of teaching was identified in the conceptual framework as influencing the enactment of the curriculum in the classroom, this area was only superficially

addressed and was not one of the primary dimensions that guided data collection. However, the findings that emerged from the data show the significant role played by informal support from teacher colleagues in helping and encouraging the participants as they adopted new teaching approaches that align with the pedagogy of current curriculum. Of particular value to the participants' comfort and confidence in using manipulatives, was the support derived from within their school's mathematics department and the lead role played by the head of the department in fostering a collegial learning environment. As this facet of the teachers' experience was so central to this study, the precise role played by the supportive environment of other mathematics teachers may be a fruitful area for further research. However, further research into this area may benefit from a revision to the conceptual framework upon which this study is built to foreground the social context within which teachers teach. This may include closer examination of teacher interactions with teaching colleagues, mathematics departments and other professional learning communities. Such a change to the conceptual framework would also require that teachers' social context be given equal weight to both the teachers' views and teachers' knowledge in the deductive dimensions of the data collection instrument and subsequent analysis.

Another area that warrants further study involves the possible connection between the value participants found in using specific manipulatives to teach particular mathematical concepts and the way in which they controlled students' access to them. Further exploration of teachers' perceived value and control of manipulatives could offer insight into how teachers' own understanding of the mathematics explored by the manipulatives mediates their use in the classroom.

Concluding Comments

While this study raised many questions and issues regarding teachers' views and use of mathematical manipulatives, its findings have reinforced the conclusions of prior research studies and have offered some further insights. The findings reinforce the notion that influences on teachers'

practice are myriad and complex. Specifically, teachers' views and use of manipulatives are mediated by many influences that include their mathematical background, teaching experiences and opportunities to learn. This study has highlighted the need for many and varied learning opportunities, and the importance of both formal and informal supports, in encouraging and building participants' confidence and competence to use manipulatives with their students in the classroom. The findings also tantalisingly suggest that the understanding that students obtain from using manipulatives to explore mathematical concepts is strongly mediated by their teachers' understanding of and value attributed to these materials.

While there may be many similarities to the findings of other research, this study offers a glimpse into the professional lives of these secondary school mathematics teachers and sheds light on the complex world of integrating the use of manipulatives into teaching practice.

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APPENDIX A: SEMI-STRUCTURED INTERVIEW GUIDE

Instructions: Introduction and overview of interview guidelines

Introduction

- Greet participant, make personal introduction and offer thanks for participation
- Session will be recorded and transcribed
- All reports and excerpts will respect the confidentiality and anonymity of participants
- Review, sign and date the consent form
- Let the participant know that manipulatives are available if they wish to refer to or use them

Overview of the interview guidelines

- Recording starts
- Starting with general questions, the interview will become more focussed
- The researcher may ask a participant to elaborate on an answer or comment
- Participants are not obligated to answer questions and can ask questions at any time

Part 1: General teacher background information questions

- What subjects and grade levels do you currently teach?
- How many years have you been teaching? At what grade levels and subjects?

Why: Simple ice-breakers to get participant comfortable talking about their own formal education and teaching experience and to contextualise and situate the participant.

Part 2: Teacher views about mathematics, and mathematics teaching and learning

- How would you describe the way you learned mathematics?
- What are your goals for teaching mathematics?
- How would you describe what mathematics means to you?

Why: How does the participant describe and contextualize their teaching and mathematics understanding?

Part 3: Teacher views of manipulatives use in mathematics instruction

- What do you believe is the purpose of using manipulatives for mathematics instruction?
- How do you think manipulatives help to teach mathematical concepts?
- Do you feel that all students benefit from the use of manipulatives and why? Give examples of how they were, or were not beneficial

Why: see manipulatives as representational form of concept, value for some/all students, useful for some/all concepts, knowledge from manipulatives or gained through use and reflection, underlying mathematics 'behind' manipulatives?

Part 4: Teacher descriptions of the use of manipulatives within the classroom

- What have been your overall experiences with using manipulatives?
- How comfortable are you with using manipulatives to teach mathematics?
- What manipulatives do you use, and how are they used in your classroom?
- Describe the results of using manipulatives in your classroom. Can you think of examples when you or your students found manipulatives to be helpful or unhelpful?

Why: student & teacher purpose/motivation in using manipulatives, props or valuable tool, experiences with their use, accessibility, students model teacher's attitudes to value and benefit?

Instructions: Final comments and wrap-up

- Final comments or thoughts that the participant would like to offer
- Offer sincere thanks for participation
- Recording ends

APPENDIX B: ANALYSIS MASTER CODE BOOK

The key dimensions of the study were: (a) teachers' mathematics background and teaching experience; (b) teachers' views of mathematics, and mathematics teaching and learning; (c) teachers' views of the use of manipulatives for teaching mathematics; and (d) teachers' descriptions of their use of manipulatives in the classroom. Each of these four principle dimensions was designated a single letter (a → B, T; b → V, L; c → M; d → U) while sub-themes and topics were designated using an additional suffix letter. During the analysis phase, additional inductive codes were added to include themes emerging from the data. The final codes book, shown below, was then used to annotate and collate data from the participant transcripts.

B	Math background
B.S	Learning experiences as student
B.T	Learning experiences as teacher
T	Teaching background
T.E	Teaching experience
T.Q	Teaching qualifications
V	View of mathematics
L	View of mathematics teaching and learning
L.A	Teaching approach (in general)
L.G	Goals for students
L.S	Views of students as learners
U	Actual use of manipulatives
U.T	Teaching strategies
U.A	Access and availability
M	View of manipulatives
M.O	own perspective
M.S	students perspective
E	Teaching environment
C	Challenges
C.T	to teacher
C.S	to students
S	Supports
S.P	Admin, dept., peers
S.R	Resources

APPENDIX C: COMPLETING THE SQUARE

The following examples illustrate three approaches to *completing the square*. The first is extracted from the *Mathematics Is Fun* website that offers support to students learning mathematics and represents an entirely symbolic procedure. The second is extracted verbatim from *Principles of Mathematics*, Addison Wesley (2000) Ontario Grade 10 Mathematics textbook and offers both symbolic and geometric approaches. The third illustrates a geometric approach using algebra tiles.

1. Symbolic Technique for Completing the Square

Completing the Square is where you take a Quadratic equation like this: $ax^2 + bx + c = 0$ and turn it into: $a(x + d)^2 + e = 0$. Here's how it is done:

Simplest Case

Let's first work on: $x^2 + bx = 0$

Add $(b/2)^2$ to both sides: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2$

It is now in the form $x^2 + 2dx + d^2$ where $d=b/2$, so we can rewrite it

Complete the Square: $\left(x + \left(\frac{b}{2}\right)\right)^2 = \left(\frac{b}{2}\right)^2$

See? Not hard. *Tricky* but not hard.

The Full One

OK, now for the full case. This one has a coefficient of a in front of x^2 :

Start with $ax^2 + bx + c = 0$

Divide the equation by a $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Put c/a on other side $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Add $(b/2a)^2$ to both sides $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

*Aha! we have the $x^2 + 2dx + d^2$ format that we wanted!
(if we treat " $b/2a$ " as " d ", that is)*

"Complete the Square" $\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

Now bring everything back...

... to the left side $\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2 = 0$

... to the same multiple of x^2 $a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 0$

Source: <http://www.mathsisfun.com/algebra/completing-square.html>

2. Solving Quadratic Equations by Using Square Roots

In Section 5.6, we solved quadratic equations by factoring. Quadratic equations can be solved in a different way. To see how, we will solve some equations in two ways and compare the methods.

Solving the equation $x^2 - 9 = 0$

Consider the quadratic equation $x^2 - 9 = 0$.

We can solve this equation in either of these ways:

Solution by factoring

$$x^2 - 9 = 0$$

$$(x - 3)(x + 3) = 0$$

Either $x - 3 = 0$ or $x + 3 = 0$

$$x = 3 \quad x = -3$$

Solution by using square roots

$$x^2 - 9 = 0$$

$$x^2 = 9$$

Take the square root of each side.

$$x = \pm 3$$

Either $x = 3$ or $x = -3$

The roots of $x^2 - 9 = 0$ are $x = 3$ and $x = -3$.

Notice the strategy in the solution using square roots. The equation was rearranged so each side was a perfect square, then the square root of each side was taken. If the equation had been $x^2 - 7 = 0$, we could not have solved the equation by factoring. However, we can solve it by using square roots. Compare the solution below with the solution above right.

$$x^2 - 7 = 0$$

$$x^2 = 7$$

Take the square root of each side.

$$x = \pm\sqrt{7}$$

Either $x = \sqrt{7}$ or $x = -\sqrt{7}$

In decimal form, $x = 2.646$ or $x = -2.646$

Solving the equation $x^2 + 10x + 21 = 0$

Consider the quadratic equation $x^2 + 10x + 21 = 0$.

We can solve this equation in the same two ways as the previous equation.

Solution by factoring

$$x^2 + 10x + 21 = 0$$

$$(x + 7)(x + 3) = 0$$

Either $x + 7 = 0$ or $x + 3 = 0$

$$x = -7 \quad x = -3$$

The roots of $x^2 + 10x + 21 = 0$ are $x = -7$ and $x = -3$.

Solution by using square roots

To solve $x^2 + 10x + 21 = 0$ using square roots, follow these steps.

Step 1. Rearrange the equation to isolate the constant term.

$$x^2 + 10x = -21$$

Step 2. Recall the Binomial Square Property $(x + a)^2 = x^2 + 2ax + a^2$.

The coefficient of x is $2a$, and the constant term is the square of one-half of this number.

To make the left side of the equation a binomial square, add to each side the square of one-half the coefficient of the x -term; that is,

$$\frac{1}{2}(10) = 5, \text{ and } (5)^2 = 25$$

Add 25 to each side of the equation.

$$x^2 + 10x + 25 = -21 + 25$$

$$x^2 + 10x + 25 = 4$$

Step 3. Factor the trinomial on the left side as a binomial square.

$$(x + 5)^2 = 4$$

Step 4. Take the square root of each side.

$$x + 5 = \pm 2$$

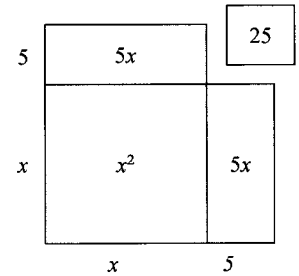
Either $x + 5 = 2$ or $x + 5 = -2$

$$x = -3 \quad x = -7$$

The roots of $x^2 + 10x + 21 = 0$ are $x = -3$ and $x = -7$.

If an equation can be solved by factoring, such as $x^2 + 10x + 21 = 0$, it is not necessary to solve it by using square roots. However, using square roots is a powerful method, because we can use it to solve quadratic equations that cannot be solved by factoring. To use the method, the equation must be converted so that the left side becomes a binomial square. This process is called *completing the square*. The left side of the equation in Step 1 was $x^2 + 10x$. When 25 was added to each side of this equation, the left side became $x^2 + 10x + 25$, which is a binomial square. Adding 25 to each side of the equation has the effect of completing the square on the left side.

This step can be illustrated geometrically. In the diagram, the shaded region represents the expression $x^2 + 10x$. To complete the square with side $(x + 5)$, a square with area 25 must be inserted in the corner. Hence, 25 must be added to the expression $x^2 + 10x$ to complete the square. The completed square represents the expression $(x + 5)^2$.



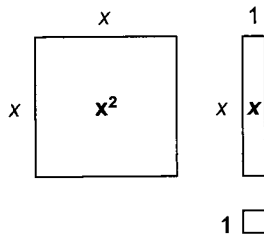
By completing the square, we converted the equation $x^2 + 10x + 21 = 0$ to the equation $(x + 5)^2 = 4$, which we solved by taking the square root of each side.

Source: Principles of Mathematics, Addison Wesley, Ontario Grade 10, p. 257.

3. Completing the Square Using Algebra Tiles

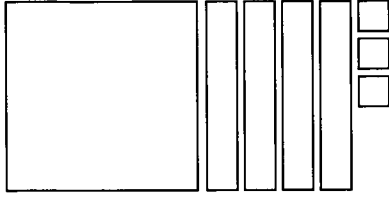
Introduction to Algebra Tiles

Algebra tiles are manipulatives constructed around an area model and generally consist of tiles with the dimensions x and unity (1). The three different tiles consist of: an x^2 tile (a square with dimension x), an x tile (a rectangle of dimensions x and 1) and a unit tile (a square of dimension 1), as shown below.



Tiles of different polarity (positive and negative) are represented by different colours. For instance, blue coloured tiles may be chosen to represent x^2 , x , 1, while red tiles may represent $-x^2$, $-x$ and -1 .

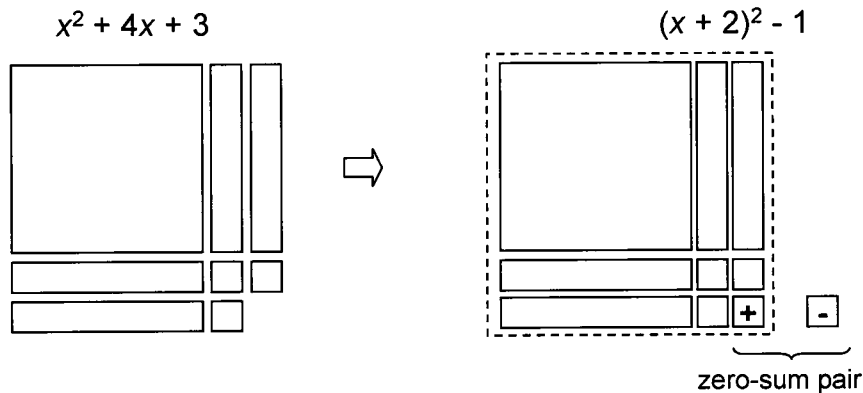
Using the area model, polynomials can be represented by the sum of the areas of their monomials. By way of an example, using the area model, the expression $x^2 + 4x + 3$ can be represented as the total area of 3 x^2 tiles, 4 x tiles and 3 unit tiles, as seen below:



This combination of tiles represents the expression $x^2 + 4x + 3$

The process of *completing the square* using algebra tiles has its analog in the symbolic manipulations (show in examples 1 and 2 above) by seeking to create a perfect square of the area representation of the entire trinomial expression. Starting with the x^2 and x tiles, an attempt is made to create a perfect square. Pairs of zero-sum unit tiles (each consisting of a negative [-] tile and a positive [+] tile) may be added to complete the square.

As an example, the area representation of the expression $x^2 + 4x + 3$, can be rearranged into a square of dimension $(x + 2)$ but only with the addition of a single (positive) unit tile. As only zero-sum pairs (i.e. zero) can be added to the expression, there is thus a remaining negative unit tile (shown as [-] below).



The total areas of each expression are identical. Thus the expression $x^2 + 4x + 3$ can be expressed as $(x + 2)^2 - 1$ by the process of *completing the square*.

The process of *completing the square* can be used as a method to convert the Standard Form of a quadratic: $ax^2 + bx + c$, into its Vertex Form: $a(x - h)^2 + k$.

Completing the square can also be used within a more general discussion of the partial factoring of trinomials, where the Standard Form of a quadratic: $ax^2 + bx + c$, may also be expressed in many different forms of: $a(x - m)(x - n) + p$.