

Robust Computationally Efficient Control of Cooperative Closed-Chain Manipulators With Uncertain Dynamics

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Abstract

This article presents a decentralized control scheme for the complex problem of simultaneous position and internal force control in cooperative multiple manipulator systems. The proposed controller is composed of a sliding mode control term and a force robustifying term to simultaneously control the payload's position/orientation as well as the internal forces induced in the system. This is accomplished independently of the manipulators dynamics. Unlike most controllers that do not require prior knowledge of the manipulators dynamics, the suggested controller does not use fuzzy logic inferencing and is computationally inexpensive. Using a Lyapunov stability approach, the controller is proven to be robust in the face of varying system's dynamics. The payload's position/orientation and the internal force errors are also shown to asymptotically converge to zero under such conditions.

Key words: coordinated robots, uncertain systems, hybrid position and force control, sliding mode control, robust control.
PACS:

1 Introduction

Advanced control of cooperative multiple manipulators has been witnessing an increasing interest due to their versatility in the tasks they can perform and due to their high productivity and potential of reducing the cost in many industrial applications [1, 2]. In particular, the bulk of the research has focused on the control aspect of weakly coupled cooperative manipulators. On the other hand, relatively less work has been done in the area of the control of tightly coupled manipulators where two or more robot arms cooperate to simultaneously move a certain object along a predefined path. When multiple manipulators grasp a common object, they form a closed kinematic chain mechanism. This implicitly imposes a set of kinematic and dynamic constraints on the position and velocity of the manipulators as they have to remain in contact with the object while in motion. As a result, the degrees of freedom of the whole system decrease leading to the generation of internal forces. Such forces, which have to be controlled appropriately, stem from the direct interaction between the end-effectors and the object. As a result, controlling multiple manipulators interacting with an object, or a given environment, is usually a much more complex problem than that of a single robot control. The motion of the manipulators has to be kinematically and dynamically coordinated.

The kinematic coordination refers to the fact that all the end-effectors involved have to move synchronously to track a certain prespecified desired position and orientation of the manipulated object without losing contact of it. In the context of this paper, the dynamic coordination means that the end-effectors have to move in a certain manner as to control the internal forces induced between the payload and the end-effectors. Because of that and because of the high complexity of closed-chain robotic systems, in general, the efficient control of such systems still stands as one of the challenging problems in the field of robotics control.

Since the emergence of the first cooperative manipulator models, the control problem of such systems has drawn the attention of several researchers [3]. Tarn et al. [4] proposed a linear transformation method that transforms the nonlinear dynamic equations of the system to a linear one with decoupled matrix-form equations. A nonlinear feedback controller was proposed in [5], and a controller based on a PD plus gravity compensation was discussed in [6]. Lately, a distributed impedance control technique was proposed in [7]. Such types of controllers along with most non-adaptive control schemes for the coordinated control of multiple manipulator systems, usually assume a full knowledge of the system's dynamics [8]. Although this is true for some cases, it might not be for many oth-

ers since complex systems, in general, are usually subject to the ubiquitous presence of uncertainties [9]. These uncertainties may have a dramatic effect on the controller’s performance and may even induce instability. To deal with such uncertainties, several adaptive control schemes were proposed [10–17]. These control algorithms approximate the system’s dynamics using a continuous online estimation of a set of the plant’s physical parameters through well-defined adaptation laws. For it to provide a satisfactory performance, a typical adaptive control algorithm assumes that the dynamic model is perfectly known and free of significant external (unmodeled) disturbances. In other words, the controller is only robust to parametric, or structured, uncertainties and to minor unstructured uncertainties. In addition, the unknown physical parameters must have a constant or slowly varying nominal values. An explicit linear parameterization of the uncertain dynamics parameters has also to exist, and even if it does, it might not be trivial especially with complex dynamic systems. Although the latter condition is guaranteed for every robotic dynamic equation, it might not be the case for many other systems. It is worth mentioning that all the aforementioned conditions are necessary but may not be sufficient for a satisfactory performance and stability of a large number of adaptive controllers.

Modeling imperfection of complex systems, such as closed-chain robotic manipulators, is inevitable in most cases. This makes the development of a robust control approach for the increasingly complex cooperative manipulator systems a necessary step to keep up with the increasingly demanding design requirements of such systems. Most efforts for compensating for modeling uncertainties have focused on the use of fuzzy logic based controllers given their high credibility in controlling ill-defined systems [18]. In spite of the significant advances made in this direction [19–26], the proposed fuzzy logic controllers still suffer from their relatively high computational complexities, which makes their integrability within an embedded control system a challenge in itself.

Even with the increasing number of control systems proposed in the literature for the control of closed-chain manipulator systems, overlooking the dynamic coordination of the manipulators and failing to study the controller’s stability criteria in the existence of parametric and modeling uncertainties are still among the common deficiencies in designing these control systems. On the other hand, most of the controllers that take into account the manipulators dynamic coordination (hybrid position/force controllers), divide the robots working space into two orthogonal subspaces in which position/orientation and force are independently controlled as they happen to be in the null control space of each other. Although the subspace orthogonality condition may be satisfied in a number of applications, it is not generally the case for strongly coupled multiple manipulator systems, and hence such control techniques become

inapplicable.

In this manuscript, we extend the pioneering work previously presented in [11] to the control of closed-chain multi-manipulator systems with no a priori knowledge of the system’s dynamics. In here we propose a decentralized hybrid position/force tracking controller whose objectives are to track a predefined desired position and orientation of the payload, while controlling the internal forces of the closed-chain system and make them converge to their predefined desired values. The proposed controller is based on a decentralized adaptive control scheme forcing each robot to operate independently of the others saving the communication time overhead between the robots. This full autonomy of the controller is one of the main reasons behind its high computational efficiency, which makes its embedding and real-time execution within an integrated control system quite easy to achieve. In addition, the proposed controller does not require the orthogonality of the end-effectors position and the internal force control sub-spaces saving the need to decouple the internal force and the payload’s position feedback signals. The controller is also independent of the manipulators dynamics, which makes it suitable for a large variety of cooperative manipulator systems.¹ The controller’s flexibility extends to giving the designer the luxury of not using internal force feedback signals. In this case, mounting force sensors at the manipulators’ wrists becomes unnecessary, which makes the realization of the controller in a real-world cooperative manipulator system simpler and more economical. The rest of the paper is organized as follows. Section 2 formally defines the control problem in hand and outlines the dynamic and kinematic modeling aspects of coordinated closed-chain manipulator systems. The formulation of the proposed controller and its stability analysis are discussed in Section 3. Numerical simulations are carried out to assess the performance of the proposed controller and to illustrate its robustness and ability to meet the predefined goals independently of the manipulators dynamics. The results are illustrated and analyzed in Section 4. Eventually, we provide a brief summary of the main contributions of this work and conclude with a few concluding remarks in Section 5.

2 Kinematics and Dynamics of Cooperative Manipulators

2.1 Problem Statement

Consider m cooperative manipulators handling a common object as shown in Fig. 1. The objectives of the

¹ It should be understood that although the proposed controller is independent of the manipulators dynamics, porting it to different cooperative manipulators with different dynamics might need fine tuning of the controller’s parameters to maintain a satisfactory performance.

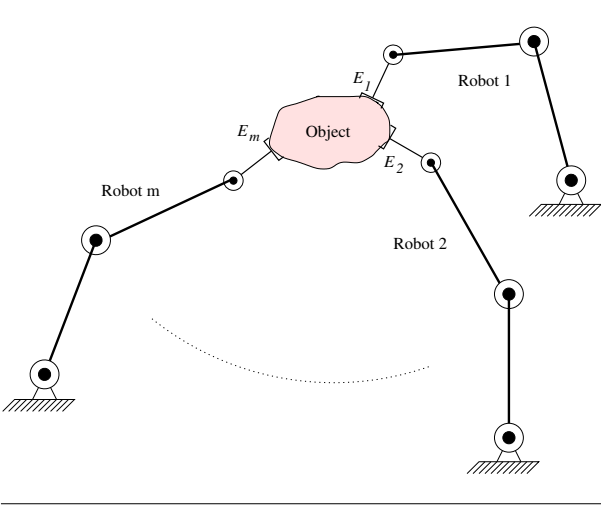


Fig. 1. Multiple Cooperative Robots Manipulating a Common Object.

coordinated robots are to simultaneously (i) move that object so that its center of mass tracks a predefined trajectory (position and orientation), and (ii) control the internal forces between the object and the end-effectors so that they converge to their predetermined desired values. These objectives are to be achieved within a decentralized control scheme and with no prior knowledge of the robots and payload's dynamics. That is, the control law τ_i for each robot i , for $i = 1, \dots, m$, has to be independent of the other robots parameters. To facilitate the formulation of the closed-chain system's dynamics, the payload is supposed to be rigidly grasped by the end-effectors. In other words, there is no relative motion between the end-effectors and the object in order not to increase the system's degrees of freedom. Also, the manipulators forward kinematics are supposed to be known. The manipulators inverse kinematics, on the other hand, are not needed as they are not used by the controller.

2.2 Kinematics

At any instant of time, the location of the manipulated object can be defined by the vector $x = [x_{(p)}^T, x_{(o)}^T]^T \in \mathbb{R}^{k_0}$, where $x_{(p)} \in \mathbb{R}^p$ and $x_{(o)} \in \mathbb{R}^o$ are vectors defining the position and orientation of the object, respectively. The superscript integers p and o belong to the set $\{0, 1, 2, 3\}$, where \mathbb{R}^0 denotes the empty vector, and k_0 ($k_0 = p + o$) is the object's total degrees of freedom. In what follows, all positions and orientations of the object and the end-effectors coordinates are expressed relative to a common reference frame unless otherwise stated. Using the manipulators forward kinematics equations, x can be rewritten as

$$x = \phi_1(q_1) = \phi_2(q_2) = \dots = \phi_m(q_m), \quad (1)$$

where $q_i \in \mathbb{R}^{k_i}$ and k_i are the respective joint coordinates and the degrees of freedom of robot i , for $i = 1, \dots, m$. Differentiating equation (1) with respect to time yields

$$\dot{x} = \dot{\phi}_i(q_i) = J_{\phi_i}(q_i) \dot{q}_i, \quad i = 1, \dots, m, \quad (2)$$

where $J_{\phi_i}(q_i) \in \mathbb{R}^{k_0 \times k_i}$ is the Jacobian matrix from the object's center of mass to q_i . Differentiating equation (2) with respect to time results in

$$\ddot{x} = \dot{J}_{\phi_i}(q_i) \dot{q}_i + J_{\phi_i}(q_i) \ddot{q}_i, \quad i = 1, \dots, m. \quad (3)$$

Equations (1), (2) and (3) represent the kinematics equations of cooperative robotic systems.

2.3 Dynamics

In a closed-chain robotic system, the dynamics of the i th manipulator in the joint space is given by

$$M_i(q_i) \ddot{q}_i + Q_i(q_i, \dot{q}_i) \dot{q}_i + W_i(q_i) - J_{e_i}^T(q_i) F_{e_i} = \tau_i, \quad (4)$$

where $\tau_i \in \mathbb{R}^{k_i}$ denotes the joint torque/force applied by the actuators on the i th manipulator, $M_i(q_i) \in \mathbb{R}^{k_i \times k_i}$ is the positive definite inertial matrix, $Q_i(q_i, \dot{q}_i) \in \mathbb{R}^{k_i \times k_i}$ is a matrix representing the Coriolis and centrifugal forces and joint friction coefficients, $W_i(q_i) \in \mathbb{R}^{k_i}$ represents the vector of gravitational forces, $J_{e_i}(q_i) \in \mathbb{R}^{k_0 \times k_i}$ is the manipulator Jacobian matrix from the end-effector E_i to q_i , and $F_{e_i} \in \mathbb{R}^{k_0}$ is the force exerted by the object on the end-effector E_i . The dynamics equation of the object in the task space is given by

$$M_0(x) \ddot{x} + Q_0(x, \dot{x}) \dot{x} + W_0(x) = F_0, \quad (5)$$

where $M_0(x) \in \mathbb{R}^{k_0 \times k_0}$ is the object's symmetric positive definite inertial matrix, $Q_0(x, \dot{x}) \in \mathbb{R}^{k_0 \times k_0}$ denotes the object's Coriolis and centrifugal terms and the coefficient terms due to the friction between the object and the environment. The vector $W_0(x) \in \mathbb{R}^{k_0}$ represents the gravitational force acting on the object, and $F_0 \in \mathbb{R}^{k_0}$ is the resulting force of the m manipulators acting on the object's center of mass. The force F_0 can be expressed as $F_0 = -\sum_{i=1}^m F_{ce_i}$, where $F_{ce_i} \in \mathbb{R}^{k_0}$ is the force "indirectly" applied by the object's center of mass on the i th manipulator, and is known as the interaction force. The forces F_{ce_i} and F_{e_i} are related by

$$F_{ce_i} = J_{ce_i}^T(x) F_{e_i}, \quad (6)$$

where $J_{ce_i}(x) \in \mathbb{R}^{k_0 \times k_0}$ is the Jacobian matrix from the end-effector E_i to the object's center of mass. Notice that the Jacobian matrices $J_{\phi_i}(q_i)$, $J_{e_i}(q_i)$, and $J_{ce_i}(x)$, are related by $J_{e_i}(q_i) = J_{ce_i}(x) J_{\phi_i}(q_i)$, $i = 1, \dots, m$. The force F_{ce_i} can be regarded as the sum of an internal force f_i and an external force δ_i .

$$F_{ce_i} = f_i + \delta_i. \quad (7)$$

From the property of internal forces it is known that

$$\sum_{i=1}^m f_i = 0. \quad (8)$$

The internal forces cancel each other and only external forces contribute to the motion of the object. Hence, equation (5) can now be rewritten as

$$M_0(x)\ddot{x} + Q_0(x, \dot{x})\dot{x} + W_0(x) = -\sum_{i=1}^m \delta_i. \quad (9)$$

The external force δ_i can then be formulated as

$$\delta_i = -c_i(t)(M_0(x)\ddot{x} + Q_0(x, \dot{x})\dot{x} + W_0(x)), \quad (10)$$

where $t \geq 0$ denotes the time variable, and $c_i(t) \in \mathbb{R}^{k_0 \times k_0}$ is a positive definite diagonal matrix representing the load distribution of the object onto the i th manipulator. An important physical property of load distribution matrices is that they sum up to the identity matrix $I_{k_0} \in \mathbb{R}^{k_0 \times k_0}$. That is,

$$\sum_{i=1}^m c_i(t) = I_{k_0}, \quad \text{for } t \geq 0. \quad (11)$$

Substituting equation (9) into (10) results in

$$\delta_i = c_i(t) \sum_{j=1}^m \delta_j. \quad (12)$$

Substituting equation (12) into (7) yields $f_i = F_{ce_i} - c_i(t) \sum_{j=1}^m \delta_j$. To support the following derivation of the manipulators' dynamics, we will consider $c_i(t)$ to be in the form of $c_i(t) = \varkappa_i(t)I_{k_0}$, where $\varkappa_i(t)$ is a positive real scalar. Hence, using equations (4), (6), (7), (12), and the kinematics equations (1), (2), and (3), the dynamics equation of the i th manipulator can be rewritten as

$$\tau_i = D_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) - J_{\phi_i}^T(q_i)f_i, \quad (13)$$

where

$$\begin{aligned} D_i(q_i) &= M_i(q_i) + c_i(t) M_0(q_i) \\ G_i(q_i) &= W_i(q_i) + c_i(t) W_0(q_i) \\ C_i(q_i, \dot{q}_i) &= Q_i(q_i, \dot{q}_i) + c_i(t) (\dot{M}_0(q_i) + Q_0(q_i, \dot{q}_i)) \\ M_0(q_i) &= J_{\phi_i}^T(q_i) M_0(x) J_{\phi_i}(q_i), W_0(q_i) = J_{\phi_i}^T(q_i) W_0(x) \\ Q_0(q_i, \dot{q}_i) &= J_{\phi_i}^T(q_i) Q_0(x, \dot{x}) J_{\phi_i}(q_i). \end{aligned}$$

The matrix $\{2C_i(q_i, \dot{q}_i) - (\dot{D}_i(q_i) - \dot{c}_i(t) Q_0(q_i, \dot{q}_i))\}$ is a skew symmetric matrix. In other words, $\forall r \in \mathbb{R}^{k_i}$,

$$r^T \left\{ 2C_i(q_i, \dot{q}_i) - (\dot{D}_i(q_i) - \dot{c}_i(t) Q_0(q_i, \dot{q}_i)) \right\} r = 0. \quad (14)$$

Assumption 1 The matrix $(\dot{c}_i(t) Q_0(q_i, \dot{q}_i))$ is uniformly continuous and bounded. Thus, there exists a positive constant η_i such that

$$\frac{1}{2} \|\dot{c}_i(t) Q_0(q_i, \dot{q}_i)\| \leq \eta_i, \quad \forall t \geq 0. \quad (15)$$

Since the matrices $D_i(q_i)$, $C_i(q_i, \dot{q}_i)$, and $G_i(q_i)$, are linear in terms of the manipulators physical parameters, the first terms of the dynamics equation (13) can be rewritten as

$$\begin{aligned} D_i(q_i)\ddot{q}_{r_i} + C_i(q_i, \dot{q}_i)\dot{q}_{r_i} + G_i(q_i) \\ = Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i}, c_i(t)) \mu_i, \end{aligned} \quad (16)$$

for $i = 1, \dots, m$, where $\mu_i \in \mathbb{R}^{k_{\mu_i}}$ is a vector of the physical parameters (mass, moments of inertia, friction coefficients, etc.) of the i th manipulator and the payload, k_{μ_i} is a positive integer denoting the number of such parameters, $Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i}, c_i(t)) \in \mathbb{R}^{k_i \times k_{\mu_i}}$ is the regression matrix, which is independent of the physical parameters in μ_i , and $\dot{q}_{r_i} \in \mathbb{R}^{k_i}$ is a design nominal reference signal to be determined later. Substituting (16) into (13) yields the following error dynamics equation,

$$\begin{aligned} D_i(q_i)\dot{s}_{r_i} + C_i(q_i, \dot{q}_i)s_{r_i} \\ = \tau_i - Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i}, c_i(t)) \mu_i + J_{\phi_i}^T(q_i)f_i, \end{aligned} \quad (17)$$

with $s_{r_i} \stackrel{\text{def}}{=} \dot{q} - \dot{q}_{r_i}$ ($\in \mathbb{R}^{k_i}$) being a residual error signal corresponding to robot i .

3 Controller Design

Consider the following joint reference signal of robot i ,

$$\dot{q}_{r_i} \stackrel{\text{def}}{=} J_{\phi_i}^+(q_i)(\dot{x}_d - \gamma_i \tilde{x}) + s_{d_i} - \alpha_i \sigma_i,$$

with

$$\dot{\sigma}_i = \text{sgn}(s_{q_i}). \quad (18)$$

The following residual errors corresponding to the i th robot are then defined as

$$s_i \stackrel{\text{def}}{=} J_{\phi_i}^+(q_i) (\dot{\tilde{x}} + \gamma_i \tilde{x}), \quad (19)$$

$$s_{d_i} \stackrel{\text{def}}{=} s_i(t_0) e^{-\kappa_i(t-t_0)}, \quad (20)$$

$$s_{q_i} \stackrel{\text{def}}{=} s_i - s_{d_i} = s_{r_i} - \alpha_i \sigma_i, \quad (21)$$

where $J_{\phi_i}^+(q_i) \stackrel{\text{def}}{=} J_{\phi_i}^T(q_i) (J_{\phi_i}(q_i) J_{\phi_i}^T(q_i))^{-1}$ is the pseudo-inverse of the Jacobian matrix $J_{\phi_i}(q_i)$, κ_i is a positive constant, γ_i and α_i are positive definite matrices, x_d is the desired position of the object's

center of mass, and \tilde{x} is the corresponding position error, i.e., $\tilde{x} \stackrel{\text{def}}{=} x - x_d$. Note that $J_{\phi_i}^+(q_i)$ exists only if robot i is not at a singularity position, i.e., only if $\text{rank}[J_{\phi_i}(q_i)] = k_0$.

Assumption 2 *At every instant of time,*

$$\text{rank}[J_{\phi_i}(q_i)] = k_0, \quad i = 1, \dots, m. \quad (22)$$

Consider the following control law corresponding to the i th manipulator:

$$\tau_i = -K_{d_i} s_{r_i} - J_{\phi_i}^T(q_i)(f_{d_i} - K_{f_i} \tilde{f}_i), \quad (23)$$

for $i = 1, \dots, m$, where K_{d_i} and K_{f_i} are positive definite square matrices of appropriate dimensions and $\tilde{f}_i \stackrel{\text{def}}{=} f_i - f_{d_i}$ is the internal force error at manipulator i .

Lemma 1 *If, for $i = 1, \dots, m$, s_{r_i} , \dot{q}_i , $\dot{J}_{\phi_i}(q_i)$, and $Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i$, are bounded, then \tilde{f}_i is bounded.*

PROOF. Premultiplying the residual error signal s_{r_i} by $J_{\phi_i}(q_i)$ results in

$$\begin{aligned} J_{\phi_i}(q_i) s_{r_i} &= J_{\phi_i}(q_i) s_i + J_{\phi_i}(q_i) [-s_{d_i} + \alpha_i \sigma_i] \\ &= (\dot{\tilde{x}} + \gamma_i \tilde{x}) + J_{\phi_i}(q_i) [-s_{d_i} + \alpha_i \sigma_i]. \end{aligned}$$

Taking the time-derivative of both sides leads to

$$\begin{aligned} \dot{J}_{\phi_i}(q_i) s_{r_i} + J_{\phi_i}(q_i) \dot{s}_{r_i} &= (\ddot{\tilde{x}} + \gamma_i \dot{\tilde{x}}) \\ &+ \dot{J}_{\phi_i}(q_i) [-s_{d_i} + \alpha_i \sigma_i] + J_{\phi_i}(q_i) [-\dot{s}_{d_i} + \alpha_i \dot{\sigma}_i]. \end{aligned}$$

Thus,

$$\begin{aligned} J_{\phi_i}(q_i) \dot{s}_{r_i} &= (\ddot{\tilde{x}} + \gamma_i \dot{\tilde{x}}) + \dot{J}_{\phi_i}(q_i) (-s_i) \\ &+ J_{\phi_i}(q_i) [-\dot{s}_{d_i} + \alpha_i \text{sgn}(s_{q_i})]. \end{aligned}$$

Now, premultiplying the error dynamics (17) by $J_{\phi_i}(q_i) D_i^{-1}(q_i)$ and substituting control law (23), yields

$$\begin{aligned} J_{\phi_i}(q_i) D_i^{-1}(q_i) J_{\phi_i}^T(q_i) (K_{f_i} + 1) \tilde{f}_i \\ = J_{\phi_i}(q_i) \dot{s}_{r_i} + J_{\phi_i}(q_i) D_i^{-1}(q_i) (K_{d_i} + C_i(q_i, \dot{q}_i)) s_{r_i} \\ + J_{\phi_i}(q_i) D_i^{-1}(q_i) Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i \end{aligned}$$

Letting $D_i'(q_i) = J_{\phi_i}(q_i) D_i^{-1}(q_i) J_{\phi_i}^T(q_i) (K_{f_i} + 1)$ leads to,

$$D_i'(q_i) \tilde{f}_i = (\ddot{\tilde{x}} + \gamma_i \dot{\tilde{x}}) + b_i s_i + u_i, \quad (24)$$

where

$$b_i = J_{\phi_i}(q_i) D_i^{-1}(q_i) (K_{d_i} + C_i(q_i, \dot{q}_i)) - \dot{J}_{\phi_i}(q_i)$$

$$\begin{aligned} u_i &= J_{\phi_i}(q_i) D_i^{-1}(q_i) [(K_{d_i} + C_i(q_i, \dot{q}_i)) (-s_{d_i} + \alpha_i \sigma_i) \\ &+ Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i] \\ &+ J_{\phi_i}(q_i) [-\dot{s}_{d_i} + \alpha_i \text{sgn}(s_{q_i})]. \end{aligned}$$

By computing $[D_1'(q_1) \tilde{f}_1 - D_i'(q_i) \tilde{f}_i]$ for $i = 2, \dots, m$, and from the internal force property (8), we obtain $R(q) \tilde{f} = Q(t)$, where $\tilde{f}^T = [\tilde{f}_1^T, \dots, \tilde{f}_m^T]$, $Q^T \stackrel{\text{def}}{=} [q_1^T, \dots, q_m^T]$, and $R(q)$ and $Q(t)$ are defined as follows

$$\begin{aligned} R(q) &= \begin{bmatrix} D_1'(q_1) & -D_2'(q_2) & 0 & \cdots & 0 \\ D_1'(q_1) & 0 & -D_3'(q_3) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ D_1'(q_1) & 0 & 0 & \cdots & -D_m'(q_m) \\ I & I & I & \cdots & I \end{bmatrix} \\ Q(t) &= \begin{pmatrix} (b_1 s_1 - b_2 s_2) + (u_1 - u_2) \\ (b_1 s_1 - b_3 s_3) + (u_2 - u_3) \\ \vdots \\ (b_1 s_1 - b_m s_m) + (u_1 - u_m) \\ 0 \end{pmatrix} \end{aligned}$$

From the structure of matrix $R(q)$ and since $D_i(q_i)$, $i = 1, \dots, m$, is positive definite, it can be deduced that $R(q)$ has full rank. The matrices $J_{\phi_i}(q_i)$ and $D_i^{-1}(q_i)$ are bounded. Thus, $R(q)$ and $R^{-1}(q)$ are also bounded. The matrix $C_i(q_i, \dot{q}_i)$ is bounded for a bounded joint velocity \dot{q}_i . Since $\dot{J}_{\phi_i}(q_i)$ is bounded, the term b_i must also be bounded. $s_{d_i} \leq s_i(t_0)$, so s_{d_i} is bounded. s_{r_i} is bounded which implies that s_i and σ_i are bounded as well. $\dot{s}_{d_i} \leq -\kappa_i s_i(t_0)$, so \dot{s}_{d_i} is bounded. Since $Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i$ is bounded, the term u_i must also be bounded. Therefore, $Q(t)$ is bounded, which implies that $\tilde{f} = R^{-1}(q) Q(t)$ is bounded. ■

Theorem 2 *Consider the robot dynamics (13) and control law (23). If, for $i = 1, \dots, m$, K_{d_i} and the eigenvalues of α_i are set to be large enough (more details in the proof), then the payload's position/orientation error \tilde{x} exponentially converges to zero.*

PROOF. Consider the Lyapunov function candidate $V = \sum_{i=1}^m V_i$, where $V_i = \frac{1}{2} s_{r_i}^T D_i(q_i) s_{r_i}$. Since $D_i(q_i)$ is a positive definite matrix, it becomes clear that V_i , and hence V , are nonnegative scalars.

$$\dot{V}_i = s_{r_i}^T D_i(q_i) \dot{s}_{r_i} + \frac{1}{2} s_{r_i}^T \dot{D}_i(q_i) s_{r_i}.$$

From error dynamics (17),

$$\dot{V}_i = s_{r_i}^T [-C_i(q_i, \dot{q}_i) s_{r_i} + \tau_i - Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i]$$

$$+ J_{\phi_i}^T(q_i) f_i] + \frac{1}{2} s_{r_i}^T \dot{D}_i(q_i) s_{r_i} \leq (1 + \bar{K}_f) \bar{\varrho}^T \sum_{i=1}^m \begin{pmatrix} \tilde{f}_i \end{pmatrix} = 0$$

Substituting (23) and using (14) leads to

$$\begin{aligned} \dot{V}_i &= s_{r_i}^T \left[-K_{d_i} s_{r_i} - J_{\phi_i}^T(q_i) (f_{d_i} - K_{f_i} \tilde{f}_i) \right. \\ &\quad \left. - Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i + J_{\phi_i}^T(q_i) f_i \right] \\ &\quad + \frac{1}{2} s_{r_i}^T \dot{c}_i(t) Q_0(q_i, \dot{q}_i) s_{r_i} \\ &= s_{r_i}^T \left[(-K_{d_i} + \frac{1}{2} \dot{c}_i(t) Q_0(q_i, \dot{q}_i)) s_{r_i} \right. \\ &\quad \left. - Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i \right] \\ &\quad + s_{r_i}^T J_{\phi_i}^T(q_i) (1 + K_{f_i}) \tilde{f}_i, \end{aligned}$$

From (21), we get

$$\begin{aligned} \dot{V}_i &= s_{r_i}^T \left[(-K_{d_i} + \frac{1}{2} \dot{c}_i(t) Q_0(q_i, \dot{q}_i)) s_{r_i} \right. \\ &\quad \left. - Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i \right] \\ &\quad + (J_{\phi_i}(q_i) s_i)^T (1 + K_{f_i}) \tilde{f}_i \\ &\quad + [J_{\phi_i}(q_i) (\alpha_i \sigma_i - s_{d_i})]^T (1 + K_{f_i}) \tilde{f}_i \\ &= \dot{V}_{i1} + \dot{V}_{i2} + \dot{V}_{i3}, \end{aligned}$$

where

$$\begin{aligned} \dot{V}_{i1} &= s_{r_i}^T \left[(-K_{d_i} + \frac{1}{2} \dot{c}_i(t) Q_0(q_i, \dot{q}_i)) s_{r_i} \right. \\ &\quad \left. - Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i \right] \\ \dot{V}_{i2} &= (J_{\phi_i}(q_i) s_i)^T (1 + K_{f_i}) \tilde{f}_i \\ \dot{V}_{i3} &= [J_{\phi_i}(q_i) (\alpha_i \sigma_i - s_{d_i})]^T (1 + K_{f_i}) \tilde{f}_i. \end{aligned}$$

Equation (19) yields $\dot{V}_{i2} = (\dot{\tilde{x}} + \gamma_i \tilde{x})^T (1 + K_{f_i}) \tilde{f}_i$. Let \bar{K}_f be an upper bound of K_{f_i} for $i = 1, \dots, m$, i.e., $\bar{K}_f \stackrel{\text{def}}{=} \max_{i=1, \dots, m} K_{f_i}$. Thus,

$$\begin{aligned} \sum_{i=1}^m \dot{V}_{i2} &= (\dot{\tilde{x}} + \gamma_i \tilde{x})^T \left(\sum_{i=1}^m (1 + K_{f_i}) \tilde{f}_i \right) \\ &\leq (\dot{\tilde{x}} + \gamma_i \tilde{x})^T (1 + \bar{K}_f) \left(\sum_{i=1}^m \tilde{f}_i \right) \end{aligned}$$

Taking into account property (8) of the system's internal forces leads to $\sum_{i=1}^m \tilde{f}_i = 0$, and so $\sum_{i=1}^m \dot{V}_{i2} \leq 0$.

In addition, $\alpha_i \sigma_i = \alpha_i \int_{t_0}^t \text{sgn}(s_{q_i}(\zeta)) d\zeta \leq \alpha_i (t - t_0)$. Similarly, (20) implies that $s_{d_i} \leq s_i(t_0)$. Therefore, there exists a state-independent constant vector $\bar{\varrho} \in \mathbb{R}^{k_i}$ such that $J_{\phi_i}(q_i) (\alpha_i \sigma_i - s_{d_i}) \leq \bar{\varrho}$. Hence,

$$\sum_{i=1}^m \dot{V}_{i3} \leq \bar{\varrho}^T \sum_{i=1}^m \left((1 + K_{f_i}) \tilde{f}_i \right)$$

It can be easily concluded following the results in [11] that for a bounded payload's desired trajectory, x_d , $Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i \leq \rho_i(t)$, where $\rho_i(t) \in \mathbb{R}^{k_i}$ is a time-variant state-dependent vector.

Let $K_i \stackrel{\text{def}}{=} K_{d_i} - \eta_i I_{k_i}$, where η_i is defined in (15). If K_{d_i} is chosen in such a way that $K_{d_i} > \eta_i I_{k_i}$, then K_i is a positive definite matrix. Hence, there exists a matrix K_{i1} such that $K_i = K_{i1}^T K_{i1}$. Thus, \dot{V}_{i1} may be rewritten as

$$\begin{aligned} \dot{V}_{i1} &= -s_{r_i}^T K_i s_{r_i} - s_{r_i}^T Y_i(q_i, \dot{q}_i, \dot{q}_{r_i}, \ddot{q}_{r_i} c_i(t)) \mu_i \\ &\leq -\|K_{i1} s_{r_i}\|^2 + \|s_{r_i}\| \rho_i(t). \end{aligned}$$

Hence, setting $K_{i1} > \|\rho_i(t)\|$ guarantees \dot{V}_{i1} , and so \dot{V} , to be non-positive. Therefore, V_{i1} and V are bounded functions. This implies that s_{r_i} is bounded. Using (21) and (22), it follows that σ_i , \tilde{x} , \tilde{x} , $\dot{\tilde{q}}_i$, and \tilde{q}_i , are bounded. Lemma 1 then implies that \tilde{f}_i is bounded for $i = 1, \dots, m$. Therefore, it can be concluded from equation (17) that \dot{s}_{r_i} is bounded, i.e., there exists $\bar{\zeta}_i \in \mathbb{R}^{k_i}$ such that $\dot{s}_{r_i} \leq \bar{\zeta}_i$, for $i = 1, \dots, m$.

So far, we have proved that the system's control signals are bounded. The rest of the proof is dedicated to proving that a sliding mode is induced on the surface $s_{q_i} = 0$ with equilibrium at $\tilde{q}_i = \dot{\tilde{q}}_i = 0$, for $i = 1, \dots, m$.

Consider the energy function corresponding to robot i , $V_{q_i} = \frac{1}{2} s_{q_i}^T s_{q_i}$. From (18) and (21) we get, $\dot{s}_{q_i} = \dot{s}_{r_i} - \alpha_i \text{sgn}(s_{q_i})$. Let α_{i_m} be the minimum eigenvalue of α_i and $\bar{\zeta}_{i_N} \stackrel{\text{def}}{=} \|\bar{\zeta}_i\|$. Hence,

$$\begin{aligned} \dot{V}_{q_i} &= s_{q_i}^T \dot{s}_{q_i} = s_{q_i}^T (\dot{s}_{r_i} - \alpha_i \text{sgn}(s_{q_i})) \\ &\leq \|s_{q_i}\| \|\dot{s}_{r_i}\| - \alpha_{i_m} \|s_{q_i}\| \\ &= (\bar{\zeta}_{i_N} - \alpha_{i_m}) \|s_{q_i}\| = -\varepsilon_i \|s_{q_i}\|, \end{aligned}$$

where $\varepsilon_i \stackrel{\text{def}}{=} \alpha_{i_m} - \bar{\zeta}_{i_N}$. Choosing α_i large enough so that $\varepsilon_i > 0$, enforces a sliding mode with sliding mode condition $s_{q_i}^T \dot{s}_{q_i} \leq -\varepsilon_i \|s_{q_i}\|$, [11]. The sliding mode is established at time $t_{s_i} = s_{q_i}(t_0)/\varepsilon_i = 0$, since $s_{q_i}(t_0) = 0$. This implies that the payload's trajectory/orientation tracking errors are always constrained within a manifold that represents an exponential solution towards trajectory/orientation, x_d . Therefore, $\lim_{t \rightarrow \infty} \tilde{x} = 0$ and $\lim_{t \rightarrow \infty} \dot{\tilde{x}} = 0$. \blacksquare

Lemma 1 and theorem 2 show that using control law (23) for the cooperative robotic system defined by (13) leads to asymptotic convergence of the payload's position and orientation to their respective pre-defined desired values. In the following, we will show that it also forces the internal force errors to asymptotically decay to zero.

Theorem 3 *If a common gain $\gamma_i = \gamma$ for all m robots, then controller (23) also leads to the asymptotic convergence of the internal force error, \tilde{f}_i , to zero, for $i = 1, \dots, m$.*

PROOF. Using (24) and since $D'_i(q_i)$ is nonsingular, the internal force error, \tilde{f}_i , can be expressed as

$$\tilde{f}_i = (D'_i(q_i))^{-1}(\ddot{x} + \gamma_i \dot{x}) + (D'_i(q_i))^{-1}b_i s_i + (D'_i(q_i))^{-1}u_i. \quad (25)$$

Using the internal force property (8), we get

$$\lim_{t \rightarrow \infty} \sum_{i=1}^m \left[(D'_i(q_i))^{-1}(\ddot{x} + \gamma_i \dot{x}) + (D'_i(q_i))^{-1}b_i s_i + (D'_i(q_i))^{-1}u_i \right] = \lim_{t \rightarrow \infty} \sum_{i=1}^m \tilde{f}_i = 0.$$

This implies that

$$\lim_{t \rightarrow \infty} \sum_{i=1}^m (D'_i(q_i))^{-1}(\ddot{x} + \gamma \dot{x}) + \lim_{t \rightarrow \infty} \sum_{i=1}^m (D'_i(q_i))^{-1}b_i s_i + \lim_{t \rightarrow \infty} \sum_{i=1}^m (D'_i(q_i))^{-1}u_i = 0.$$

It is shown in theorem 2 that $\lim_{t \rightarrow \infty} \ddot{x} = 0$ and $\lim_{t \rightarrow \infty} \dot{x} = 0$. Then, $\lim_{t \rightarrow \infty} \ddot{x} = 0$ and $\lim_{t \rightarrow \infty} s_i = 0$. Hence, the above equation implies that

$$\lim_{t \rightarrow \infty} \sum_{i=1}^m (D'_i(q_i))^{-1}u_i = 0. \quad (26)$$

The matrix $D'_i(q_i)$, and so $(D'_i(q_i))^{-1}$, is linearly independent of u_i . Hence, the joint velocity q_i , and hence the matrix $(D'_i(q_i))^{-1}$, can be continuously varying in time and so may assume an infinite number of possible values for $i = 1, \dots, m$. Since equation (26) has to hold for all those values and since, both, $(D'_i(q_i))^{-1}$ and u_i are bounded, the only solution to (26) is $\lim_{t \rightarrow \infty} u_i = 0$. Going back to (25) leads to $\lim_{t \rightarrow \infty} \tilde{f}_i = \lim_{t \rightarrow \infty} (D'_i(q_i))^{-1}(\ddot{x} + \gamma_i \dot{x}) + \lim_{t \rightarrow \infty} (D'_i(q_i))^{-1}b_i s_i + \lim_{t \rightarrow \infty} (D'_i(q_i))^{-1}u_i = 0$. ■

It is important to mention that the proposed control scheme does not explicitly compensate for external disturbances. However, it does compensate for the viscous friction in the manipulators' joints as it can be implicitly incorporated within matrix $Q_i(q_i, \dot{q}_i)$ in (4). Although this may also apply to other adaptive algorithms [10–17], the main advantage of the proposed technique lies in its

independence of a precise explicit mathematical model of the system's dynamics without the need of a computationally demanding soft computing engine.

It is also worth pointing out that it is possible to choose $K_{f_i} = 0$, for $i = 1, \dots, m$, in which case controller (23) becomes independent of f_i , and hence it would be unnecessary to measure the internal force f_i at manipulator i . This makes the use of noisy force sensors unnecessary provided there is a certain mechanism to ensure that the manipulators remain in contact with the object. Although this is possible, the rate of convergence of \tilde{f}_i to zero is proportional to $(K_{f_i} + 1)$. Hence, a larger K_{f_i} would lead to a faster convergence of \tilde{f}_i to zero.

4 Numerical Results and Discussion

4.1 Experimental Setup

To demonstrate the performance of the proposed controller (23), numerical simulations are carried out on two 3-DOF identical Adept manipulators. Both manipulators are to cooperate to handle an object whose mass $m = 11.2$ kg, length $d = 0.24$ m, moment of inertia $I_c = \frac{1}{12}md^2$, and stiffness $k_c = 300$ kN/m. It is important to bring to the reader's attention the fact that the object's stiffness is only assumed to be known to compute the contact and the internal forces during the simulation. In a real robotic system, and whenever needed, such forces are determined through means of force sensors mounted at the manipulators wrists. The manipulators physical parameters are summarized in Table 1. The bases of the two manipulators are located at $(X_{1(base)}, Y_{1(base)}) = (0, 0)$ and $(X_{2(base)}, Y_{2(base)}) = (1, 0)$, respectively. The manipulated object is set to move in a straight line in the horizontal plane following the following desired trajectory:

$$\begin{pmatrix} x_d \\ y_d \\ \psi_d \end{pmatrix} = \begin{pmatrix} 0.4 - 0.2e^{(-t/6.6)} + 0.5e^{(-t/6.8)} \\ \frac{0.4}{\sqrt{3}} - 0.2 - \frac{0.2}{\sqrt{3}}e^{(-t/6.6)} + \frac{0.5}{\sqrt{3}}e^{(-t/6.8)} \\ \pi/3 \end{pmatrix},$$

where ψ_d denotes the object's desired orientation in rad, $(x_d, y_d)^T$ is the desired position of its center of mass in meters, and t denotes the time index in seconds. The object initially starts at $(0.6, (0.6/\sqrt{3}) - 0.2, \pi/3 - 0.1)^T$, that is $(0.1 \text{ m}, 0.1 \text{ m}, 0.1 \text{ rad})^T$ off its desired initial position.

The manipulators dynamics are defined as follows.

$$M_i(q_i) = \begin{bmatrix} a & b\phi_i & 0 \\ b\phi_i & c & I_3 \\ 0 & I_3 & I_3 \end{bmatrix}, W_i(q_i) = \begin{pmatrix} VS_1 \text{sign}(\dot{q}_{i1}) \\ VS_2 \text{sign}(\dot{q}_{i2}) \\ VS_3 \text{sign}(\dot{q}_{i3}) \end{pmatrix},$$

Table 1
Values of the manipulators physical parameters.

Manipulator's parameter	Value
I_j , $j = 1, 2, 3$ - rotational inertia of link j	$I_1 = 1.1 \text{ kg}\cdot\text{m}^2$, $I_2 = 0.3 \text{ kg}\cdot\text{m}^2$, $I_3 = 0.01 \text{ kg}\cdot\text{m}^2$
m_j^* , $j = 1, 2, 3$ - mass of link j	$m_1^* = 30 \text{ kg}$, $m_2^* = 15 \text{ kg}$, $m_3^* = 5 \text{ kg}$
I_{col} - rotational inertia of the inner transmission column for joint 2	$I_{\text{col}} = 0.57 \text{ kg}\cdot\text{m}^2$
l_1 and l_2 - lengths of links 1 and 2	$l_1 = 0.425 \text{ m}$, $l_2 = 0.375 \text{ m}$
r_2 - distance of the center of mass of link 2 from the axis of joint 2	$r_2 = 0.165 \text{ m}$
VV_j , $j = 1, 2, 3$ - viscous friction coefficients	$VV_j = 1.2$
VS_j , $j = 1, 2, 3$ - Coulomb friction coefficients	$VS_j = 0.01$

$$Q_i(q_i, \dot{q}_i) = \begin{bmatrix} VV_1 & -b\varphi_i \dot{q}_{i2} & 0 \\ b\varphi_i \dot{q}_{i1} & VV_2 & 0 \\ 0 & 0 & VV_3 \end{bmatrix},$$

where $i = 1, 2$, $\phi_i = \cos(q_{i2} - q_{i1})$, $\varphi_i = \sin(q_{i2} - q_{i1})$, $a = I_1 + m_2^* l_1^2 + m_3^* l_1^2$, $b = m_2^* l_1 r_2 + m_3^* l_1 l_2$, $c = I_{\text{col}} + I_2 + I_3 + m_2^* r_2^2 + m_3^* l_2^2$. The position and the orientation of end-effectors E_1 and E_2 are defined by the following forward kinematics equation

$$\begin{pmatrix} x_{ie} \\ y_{ie} \\ \psi_{ie} \end{pmatrix} = \begin{pmatrix} l_1 \cos q_{i1} + l_2 \cos q_{i2} + X_{i(\text{base})} \\ l_1 \sin q_{i1} + l_2 \sin q_{i2} + Y_{i(\text{base})} \\ q_{i2} + q_{i3} \end{pmatrix}.$$

The load distribution matrices are set to $c_1(t) = c_2(t) = \text{diag}(0.5, 0.5, 0.5)$. The desired internal forces are given the values of $f_{d1} = -f_{d2} = -10 \text{ N}$ acting on the line connecting the two end-effectors. The manipulators are also set to have a joint friction term in the form of $F_r(\dot{q}_i) = \text{diag}(0.15, 0.06, 0.015)\dot{q}_i$, for $i = 1, 2$. The controller is programmed to operate at 80 Hz.

4.2 Experimental Results

The performance of the proposed controller is depicted in Figs. 2 and 3. It can be seen from Fig. 2 that the controller is able to deem down the errors of the payload's position/orientation and those of the internal forces to zero within about 2 sec. To better illustrate the robustness of the controller, an unknown external disturbance is instantaneously applied to the system halfway through the payload's trajectory, i.e., at $t = 10 \text{ sec}$. At this time, the payload's mass is also dropped to half its original value to be 5.6 kg, which results in an abrupt change of

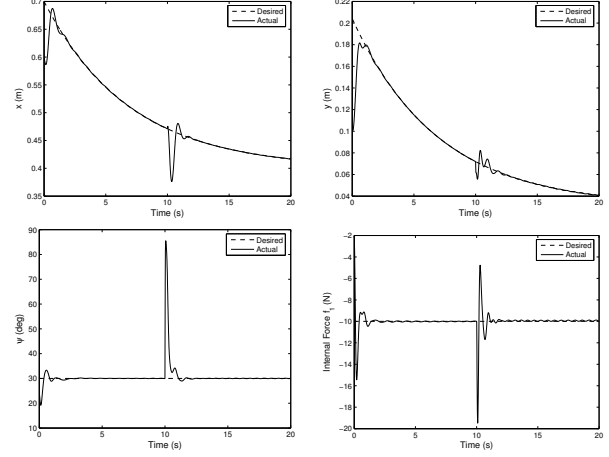


Fig. 2. Tracking errors of the payload's x - y position, orientation, and internal force.

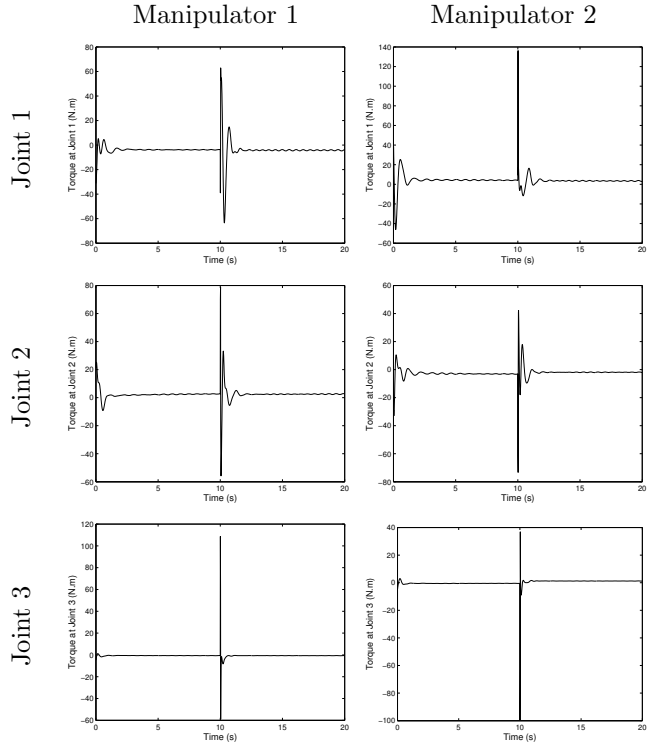


Fig. 3. Controller's output torque.

the payload's inertia as well. Once again, and as Fig. 2 shows, it took about 2 sec. for the controller to bring all control signals back to their desired values.

5 Conclusion

In this article, a decentralized hybrid position/force control scheme is proposed for the control of multiple tightly coupled manipulators handling a common object. The controller is proven to track, both, the payload's position/orientation and the system's internal forces to

their respective desired values even when they share the same lines of action. This is accomplished without a prior knowledge of the system's dynamics, which makes the controller highly modular and portable to various types of cooperative manipulators with different dynamics. Thanks to its decentralized architecture and its independence of any fuzzy logic inferencing mechanism, unlike many other controllers suggested in the literature for similar control objectives, the proposed controller is computationally inexpensive. This drastically eases its embedding and enhance its real-time execution performance within an integrated control system. In addition, it offers the flexibility of not using the internal force feedback signals, which makes its real-world realization simpler and more economical, since it saves the need for force sensors in this case. Finally, numerical simulations are carried out to illustrate the validity of the proposed controller and to confirm its proven anonymous position/force tracking ability under the aforementioned operating conditions. A potential future research avenue to extend this work is to consider multi-fingered manipulators where fingertips just contact the object instead of rigidly grasping it.

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