Teachers' Beliefs about Mathematics and Multilingual Students

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Abstract

I used a sociocultural perspective (Vygotsky, 1978) to examine teachers' epistemological and efficacy beliefs about the teaching and learning of mathematics with multilingual students. Specifically, I use the work of Negueruela-Azarola (2011) who suggests that teachers' beliefs are conceptualizing tools for thinking about activity. Beliefs, which are social and dynamic, arise from teachers' lived experiences. What teachers believe is relevant because beliefs inform pedagogical practices and once established are hard to change (Brownlee, Boulton-Lewis & Purdie, 2002; Cross, 2009; Pajares, 1992). Established beliefs with regards to mathematics hold that it is the easiest subject for multilingual students since there is little language involved. On the other hand, established beliefs are that increased English vocabulary is mainly what students need to be successful in mathematics. Barwell (2009) and Moschkovich (2002), using sociocultural perspectives, argue that language is important in mathematics and that multilingual students can participate in mathematical discussions when using resources such as their own mathematical knowledge, objects, and codeswitching. I interviewed five teachers who had experience teaching mathematics to students whose first language was other than English. I found that some teachers had beliefs which contrasted with the philosophies of their training institutions and with their schools. Teachers were found to hold contradictory beliefs. The study showed the importance of terminology in that how teachers referred to their multilingual students reflected their beliefs. Multilingual students were welcomed in the classroom for their contribution to cultural diversity but teachers acknowledged increased workload, and periods of frustration when supporting their multilingual students in mathematics.

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Introduction

The student gets up quickly from his mat in the corridor where he has been using the red metal circles to trace perimeters. He wants to speak with Mrs. Ray. He walks into the bright classroom and his eye is caught by the sight of a confused hamster having its hair brushed. He sees his friends Lucy and Santiago with their heads down, poring over a relief map of Italy. He looks around for Mrs. Ray but cannot see her. His eyes run over the room. The twins are having a heated argument over the definition of some word he doesn't recognize. He suddenly spots Mrs. Ray at the back of the room near the armchair. She is mopping up some spilled water. He walks over. She sees him. "Hello Sam. I was working on my painting of these lovely fruits and vegetables but accidentally knocked water all over my work!" "Never mind, Mrs. Ray," says another student, "I think your painting can be saved; I will help you". Turning to Sam, Rebecca asks, "Here Sam, wouldn't you like to paint with us, too?" Sam smiles, "No thanks. I am really excited about the perimeter stuff I am doing! Mrs. Ray! Did you know..?"

Recent research into the role of teachers' beliefs shows that beliefs play a critical role in teachers' pedagogical practices (Brownlee, Boulton-Lewis & Purdie, 2002; Chan & Elliott, 2004; Ernest, 1989; Goddard, Hoy & Hoy, 2004; Takahashi, 2011; Turner, Bogner Warzon, & Christensen, 2011; Pajares, 1992; Pettit, 2011). Looking at the above introductory vignette to consider Mrs Ray's pedagogical practices, we can make some inferences about her educational beliefs. In this classroom the students are not sitting still and listening to the teacher talking. They are moving around freely, concentrating on their own activities. From this we can infer that Mrs. Ray does not hold a transmission model of learning. This inference is supported by the observation that the students are doing a variety of activities: painting, taking care of a pet, using the dictionary, finding perimeter, and working on a relief map. As they engage in these activities

we see the children have a degree of autonomy in the classroom. We can surmise that Mrs Ray thinks it is important for the children to make decisions for themselves. From these interactions we can infer that establishing a caring and welcoming environment is a priority for Mrs Ray, both for how it allows the children to interact with each other and for how it allows Mrs Ray to interact with the children. While Mrs Ray is the person who has enabled this learning environment, she, too, is working on a project alongside her students. From this observation we can plausibly assume that for Mrs Ray everyone is a learner. In conclusion, from the description of the various activities in the classroom and from the manner in which Mrs Ray interacts with the students, we can infer that Mrs Ray believes in a collaborative and dialogic approach to engaging students. Such a belief motivates her pedagogical practices which include creating ample opportunities for the children to work together, to have meaningful discussions about their work, and to choose what they might do next.

In this qualitative study I used interviews to investigate 5 teachers' beliefs about mathematics teaching and learning with multilingual students. This study arose as a result of my experiences as a Montessori teacher and as someone who struggled with mathematics learning. I was curious to explore nature of teachers' beliefs and how they inform action. As a mathematics student I was taught using transmission models. There was limited discussion in class as the teacher expected us to work alone. As a consequence, I struggled with mathematics at school and did not enjoy the subject at all. Fortunately, at a later date while taking Montessori teacher training, I was introduced to different ways of thinking about mathematics. In contrast to transmission models, the Montessori approach to mathematics stresses problem solving and exploration. Discussions with other student teachers were considered important. Using mathematical manipulatives during my training allowed me much more time to explore concepts

and to reflect upon what I was trying to understand. I did not work from textbooks, but was encouraged to construct my own questions and find my own answers. While there was a great deal of sharing about what we were learning with classmates, I could also work on my own. I was liberated from the yoke of desperately seeking the one answer that would meet a teacher's approval and I learned that there were many ways to solve mathematical problems. My Montessori training allowed me to experience mathematics differently and this profoundly changed my beliefs about my ability to learn and do mathematics.

In addition to shedding light on teachers' beliefs about mathematics teaching and learning in general, this research explores those beliefs with regards to multilingual students in particular. This area of research is increasingly important because of the linguistically diverse nature of Ontario classrooms which research suggests creates substantial challenges for teachers as they try to support their multilingual students (Mady, 2012). Teachers are exposed to situations which may be new to them causing them to feel uncertain about the best course of action for their students in mathematics (Chitera, 2009; Clarkson, 2009; Moschkovich, 2010; Setati, 2005). In addition to attitudes about mathematics, there are numerous commonly held beliefs about language learning. These beliefs about mathematics and language come together in the teaching of mathematics to multilingual students. The setting where these beliefs come together is full of tensions (Barwell, 2009, p.161) which must be successfully navigated by the teacher. There is an increasing interest in this area of educational research (Adler, 2001; Barwell, 2009; Clarkson, 2009; Gutierrez, 2002; Moschkovich, 2010; Setati, 2005).

The focus of this research is on teachers' beliefs because of the powerful role these beliefs play in the decisions teachers make in their classrooms, decisions, which in turn, affect students' learning (Chan & Elliot, 2004; Pettit, 2011; Takahashi, 2011). I explore teachers' beliefs about

mathematics and multilingual students in the context of Ontario. This research is timely and relevant for two important reasons: First, Canadian classrooms are increasingly linguistically diverse which present substantial challenges for which teachers may not be prepared. Second, the current mathematics curriculum is based on inquiry (Holm & Kajander, 2012; Moschkovich, 2002). This curriculum places a heavy emphasis on communication. Students are expected to take part in discussions where they argue, explain, justify, and communicate their mathematical understanding to others.

Theoretical Framework

Beliefs: "A messy construct"

There is a debate within the field of educational research about the meaning and classification of beliefs. Researchers have been unsuccessful in coming to agreement (Brownlee et al, 2002; Hofer & Pintrich, 1997; Pajares, 1992; Pettit, 2011; Tschannen-Moran & Hoy, 2001). For instance, Schommer (1990) refers to core and peripheral beliefs, Pettit (2011) refers to attitude, and Nespor (1987) refers to knowledge, to name a few. Pajares (1992) considers teachers' beliefs in education research a "messy construct" because defining beliefs seems like "a game of player's choice" as "beliefs travel in disguise and under aliases" (p.309).

For my research I draw on the work of Negueruela-Azarola (2011) who, within the larger framework of sociocultural theory sees beliefs as conceptualizing activities that emerge from sense making tasks. But first I discuss broader sociocultural perspectives that allow us to enquire into the contextual nature of teachers' pedagogical practices with multilingual students. Sociocultural theories build on the work of Vygotsky (1986).

According to Adler, (2001), a Vygotskian framework is very useful because a teacher's understanding of her pedagogical practice is a reflection of herself, her educational and personal history, her identity as a teacher, and the context in which she works as well as the wider

communities in which she participates (p. 36). Johnson and Golombek (2011) believe a sociocultural perspective has:

> the potential to explicate the origins, mechanisms, nature, and consequences of teacher professional development at all phases of teachers' careers and in all contexts where they live, learn, and work. (p. 1)

Takahashi (2011) argues that a sociocultural view of teachers' beliefs allows one to consider how teachers may be co-constructing their efficacy with their colleagues through engagement in shared activities (p. 733).

Within mathematics research, Gutierrez, Sengupta-Irving, and Dieckmann (2010) favour a sociocultural perspective when they work from the assumption that mathematics involves a socially organized way of seeing, understanding, envisioning, and doing mathematics and in ways that are accountable to the distinct norms of the mathematical community (2010, p. 30).

Finally, an important aspect of Vygotskian theory is that it offers answers to contemporary questions that had not before been asked. Two of these questions are relevant to this study. One is the issue of multiculturalism. Historically, learning and culture have come together in the classroom in the goal to pass on culture to the next generation. In a homogenous setting, culture may seem invisible and educators may be unaware of it. It is when two or more cultures are present in a classroom that teachers would begin to notice it (Kozulin, 2003). Vygotsky answers the issue of multiculturalism by viewing learning as a sociocultural process. Vygotsky suggested that each culture has its own set of psychological tools, such as signs, symbols, and text. A multilingual classroom can be understood as containing many different systems of psychological tools. Education, then, can be seen as the challenge of integrating multiple systems of psychological tools by the students and by the teachers (Kozulin, 2003). A second issue is one of

mediation. Based on Vygotsky's theory, current pedagogies now favour models where a student's learning depends on a process of mediation whereby activities started by the adult become part of the student's own psychological functions.

I leave the broader discussion on sociocultural theory and its relevance to my research to introduce and discuss a sociocultural approach to understanding beliefs.

A sociocultural understanding of beliefs

Negueruela-Azarola (2011) suggests that Vygotskian theory "provides new understandings of beliefs" and encourages us to enquire into the contextual nature of beliefs (p. 361). From this perspective beliefs are ideas which are connected with both the context and the experiences of participants (p. 360). They may be considered as psychological tools and as conceptualizing activity of the mind. As such beliefs are "always a tool for thinking about activity and the very result of conceptualizing activity" (p. 361). Negueruela-Azarola suggests that as conceptualizing activity, beliefs "are about significant transformations for the sense-making self". He notes that a transformative approach to beliefs "takes into account that there are preferred outcomes for teaching and learning, and that these are contingent upon specific contexts and agents" (p. 362). Negueruela-Azarola refers to Kozulin (1998), who argues that the acquisition of psychological tools has to be done through intentional activity, that is, systematically organized (p. 362).

Finally, beliefs are stable historically because of their social meaning but susceptible to change because of their contextual nature. In other words they are "dynamically and personally transformed in the process of internalization" (Negueruela-Azarola, 2011, p. 360). In order for beliefs to be sustained, they must be meaningful and relevant to the self.

I understand beliefs to be the result of the lived experiences, both professional and personal, of teachers. A way to consider beliefs that I found useful from the literature is the work that

views beliefs as epistemological beliefs and efficacy beliefs.

Epistemological and efficacy beliefs

Epistemological beliefs, or beliefs about knowing, reflect an individual's views on what knowledge is, how it can be gained, its degree of certainty, and the limits and criteria for determining knowledge (Brownlee, Boulton-Lewis, & Purdie, 2002). Hofer (2004) talks about "personal epistemology" when examining what individuals believe about how knowing occurs, what counts as knowledge and where it resides, and how knowledge is constructed and evaluated (p. 1).

Efficacy beliefs, derived from the theories of Bandura (1997) have to do with individuals' beliefs about their ability to effect change. Tschannen-Moran, Woolfolk-Hoy, & Hoy (1998), referring to teachers, suggest efficacy beliefs are teachers' perceptions of their own capability to enact certain pedagogical practices and their beliefs that these actions can bring about student learning. Both epistemological beliefs and efficacy beliefs are important aspects of a teacher's pedagogical practice and will be the central focus of this research project.

Literature review

I begin with an introduction to some of the key issues in the literature with respect to research on teachers' beliefs irrespective of their subject domain. Following this, I look at the literature on teachers' beliefs about mathematics. The teaching of mathematics has changed over the past few decades and the current mathematics curriculum places a strong emphasis on communication as an essential aspect of mathematics learning and teaching. I focus on how teachers experience these changes. Finally, I look at teachers' beliefs about multilingual students in the context of learning mathematics. Since all students are now encouraged to explain their mathematical thinking as they work to solve mathematical problems it is important to understand

what teachers believe about language and mathematics in this context.

Epistemological Beliefs

In their paper on the development of epistemological theories, Hofer and Pintrich (1997) reviewed epistemological models from the time of William Perry, who has been influential in the field. In the 1960s, Perry (1968) studied the intellectual development of university undergraduates. At that time, he found students entering university believed that knowledge was simple, certain, and handed down by authority. Perry found that, over time, students developed a different view of knowledge and saw it as complex, reasoned out and tentative. Perry suggested that the students' epistemological beliefs changed by progressing through a fixed series of stages. Other researchers since Perry have also suggested models that view beliefs as progressing through stages. Schommer (1990) challenged this view.

In an early study Schommer, Crouse, and Rhodes (1992) suggested that beliefs were independent and multidimensional. Their study looked at students' epistemological beliefs and their comprehension of a mathematical text. They examined whether students' epistemological beliefs affected their learning. The researchers hypothesized that if students had strong beliefs that knowledge was merely isolated facts they would perform poorly on a test that required application and integration (p. 436). They found that the more students believed knowledge to be isolated facts, the more poorly the students comprehended the mathematical text. Schommer et al. (1992) suggested this made sense because when the students believed knowledge to be a collection of facts they studied to master lists of facts, with little effort to interrelate those facts.

Context in epistemological beliefs

In a further study, Schommer and Walker (1995) challenged the assumption that epistemological beliefs are generalizable across domains or subjects. The researchers examined

college undergraduates' epistemological beliefs with regards to two subject areas; social sciences and mathematics. The researchers gave the students a questionnaire to complete twice; once with the social sciences in mind and once with mathematics in mind. The students also had to read a passage and answer questions pertaining to that passage. The passage was either from social sciences or from mathematics. Schommer and Walker found that these students held beliefs about knowledge that were adjusted when they considered different subjects. For example, they believed knowledge was uncertain, but they also believed there are more certain facts in mathematics than in the social sciences. Based on these findings, the researchers claimed that when viewing beliefs from a multidimensional perspective it is possible that individuals may hold a variety of beliefs about knowledge and learning which are not always consistent. Students may think that knowledge is certain and unchanging in one domain yet think that knowledge is complex and evolving in another.

A study by Chan and Elliot (2004) examined student teachers' epistemological beliefs and their concepts of teaching. At a tertiary institution in Hong Kong, 385 student teachers answered two questionnaires each: the first measured epistemological beliefs and the second examined teaching concepts. Chan and Elliott found a correlation between teachers' epistemological beliefs and their teaching methods. The teachers who held that ability was innate and that knowledge was centred in authority believed in a transmission model of teaching. Those who believed knowledge was evolving and tentative and that ability was not fixed tended toward a problem-solving model of teaching. Chan and Elliott demonstrated that students' epistemological beliefs were mediated by the context of their educational environments.

Efficacy Beliefs

The concept of self-efficacy was first introduced by Bandura in 1977. Bandura asserted that self efficacy beliefs are one's own assessment about one's capabilities to attain a desired level of performance in a given endeavour. Bandura (1997) proposed that beliefs about efficacy strongly influence one's desire to act and are a more powerful force than one's actual ability for the task to be undertaken. Self efficacy beliefs influence one's persistence in putting forth effort for a desired goal and also influence how one faces setbacks while attempting to reach that goal.

The concept of efficacy beliefs in teachers developed from two strands: Firstly, and importantly, from Bandura's (1997) work on self efficacy and secondly from a study in the 1970s known as the RAND study. Researchers Armor, Conroy-Oseguera, Cox, King, McDonnell, Pascal, Pauly and Zellman (1976) looked at reading instruction among low income and minority students in an urban setting. The RAND researchers, looking for a variable to explain the effectiveness of certain teachers, examined the extent to which the teachers believed they could control student outcomes. They found that variations in reading achievement were positively related to teachers' efficacy beliefs. As a result of this study the concept of teacher efficacy was born (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998).

In their review of the literature on efficacy beliefs, Tschannen-Moran, Woolfolk Hoy, and Hoy, (1998) cited studies that showed teachers with high efficacy beliefs were more likely to have a higher level of commitment to teaching (Gibson & Dembo, 1984), were more likely to experiment with instructional methods (Allinder 1994), and were more willing to implement innovative ideas (Fuchs, Fuchs, & Bishop, 1992). In addition, teachers who viewed themselves as effective were more likely to show a willingness to work with students who were experiencing

difficulties rather than referring them to special education sessions (Soodak & Podell, 1993). Moreover, Tschannen-Moran et al. (1998) reported that studies showed the high efficacy beliefs of teachers shaped students' attitude to school (Woolfolk, Rosoff, & Hoy, 1990) and were related to student outcomes (Gibson & Dembo, 1984; Ross, 1992).

Context in efficacy beliefs

According to Tschannen-Moran, Woolfolk Hoy, and Hoy (1998) teachers' efficacy beliefs are mediated by context so that when teachers assess their teaching capabilities in a given context the assessment is made in the light of the perceived requirements of the task. Factors that mediate teachers' efficacy beliefs are collegial support, the resources available, the quality of curricula, the socio-economic status of students, and the students' perceived willingness to engage at school. Tshannen-Moran et al. (1998) found that contextual factors play more of a role in the efficacy beliefs of new teachers than teachers with longer teaching experience who have mastery experiences to draw on. According to Bandura (1997), teachers' efficacy beliefs are most in flux early on in their careers when they do not yet have mastery experience.

Klassen, Foster, Rajani, and Bowman (2009) designed a mixed methods study to examine and compare teachers' efficacy beliefs and levels of stress in the remote areas of the Yukon and Western Canada. They also wished to research how social, cultural, and geographic factors mediated teachers' beliefs in remote areas. There were two parts to their study: Study 1 and Study 2. Questionnaires were used in Study 1 to examine the efficacy beliefs and levels of stress of 221 teachers teaching in the Yukon and Western Canada. Interviews were used in Study 2 to examine how geographical, cultural, and community factors related to teachers' work beliefs. The researchers found that factors outside the classroom, such as links to the community, social interactions, and cultural differences played a crucial role in teachers' work-related beliefs.

Tschannen-Moran and Johnson (2011) reported that the notion of teachers' sense of efficacy has proved to be a powerful construct in research in the classroom. In a study of 684 teachers the researchers examined teachers' efficacy beliefs in literacy instruction and compared these results with teachers' efficacy beliefs in teaching generally. They used two surveys; the *Teachers' Survey for Efficacy Beliefs in Literacy Instruction* (TSELI), and the *Teachers' Sense of Efficacy Survey* (TSES). One finding from *The Teachers' Sense of Efficacy Survey (TSES)* on general efficacy beliefs was that the number of years a teacher had taught did not have much bearing on the teacher's sense of efficacy. This is consistent with findings of an earlier study by Tschannen-Moran and Woolfolk Hoy (2007) that teachers who began their careers with strong efficacy beliefs were likely to continue building on those positive beliefs. Tschannen-Moran and Johnson (2011) said their findings were consistent with Bandura's (1997) assertion that efficacy beliefs are established early in learning and are stable and resistant to change unless reassessment is provoked by some kind of shock or by education (Tschannen-Moran & Johnson, 2011, p. 754).

A very recent mixed methods study by Lee, Cawthon, and Dawson (2013) looked at the possibility that a high sense of efficacy would facilitate a teacher's openness to pedagogical change. This would be important where teachers' pedagogical practices needed to change in order to incorporate new evidence based practices. Lee et al. (2013) argued that an essential key to facilitating this change would be to increase teachers' sense of efficacy. Of the teachers in the study, 12 were elementary teachers and 18 were secondary teachers. The teachers were encouraged to act out new instructional strategies in the belief that these activities would create opportunities for the positive experiences Bandura (1986) believed were necessary for creating higher efficacy beliefs. Data were collected from three sources. These were the Participant Intake Form, Lesson Planning Evaluations, and the *Teacher's Sense of Efficacy Scale* (TSES) which

was used to survey the teachers. It was found that the elementary teachers experienced greater conceptual change. The secondary teachers were more reluctant than the elementary teachers to implement alternative strategies. Lee et al. (2013) reported, however, that their findings cannot support the hypothesis that efficacy beliefs contributed to the changes experienced by the elementary teachers.

The researchers also found that efficacy beliefs increased for secondary teachers over the years, but decreased for elementary teachers as their number of teaching years increased. Elementary teachers, however, began their teaching with significantly higher efficacy scores than secondary teachers. The researchers' observation for this discrepancy was that elementary teachers are expected to master new pedagogies as they are adopted by schools, while secondary teachers, who do not see as many pedagogical changes, are required to master a curriculum. This finding contrasts with findings by Tschannen-Moran and Johnson, (2011), and also with Bandura's (1997) assertion that beliefs once stable are difficult to change. In conclusion, Lee et al. (2013) observed that raising teachers' efficacy beliefs has been the focus of numerous professional development programs, but depending on the teacher's context, this may not be a productive route towards pedagogical change. The researchers remarked that further research is necessary to clarify the construct of self-efficacy and its possible relationship to pedagogical conceptual change (p. 91).

Summary of teachers' epistemological and efficacy beliefs.

Research on epistemological and efficacy beliefs helps us understand what teachers think about the nature of knowledge and learning as well as their effectiveness as teachers. The important conclusions that I can draw from the review of the literature of teachers' epistemological and efficacy beliefs are the following: How teachers view knowledge informs

their teaching and learning which can result in teaching that is not consistent with current pedagogies (Chan & Elliott, 2004). Teachers' epistemological beliefs may not generalize across domains, or in other words, they are dependent on context so that their teaching and learning in one subject may be different from their teaching and learning in another subject (Schommer & Walker, 1995). Efficacy beliefs are context-dependent as well (Klassen et al., 2009; Tschannen-Moran et al., 1998). Once beliefs are stable, or established, they are resistant to change regardless of their domain or subject area. Implementing new pedagogies, therefore, may be difficult for teachers (Lee et al., 2013). Beliefs are established early in teachers' lives (Tschannen-Moran & Johnson, 2011). Teachers' efficacy beliefs affect the amount of effort and perseverance they will put into their teaching and learning so that teachers with a high sense of efficacy are more likely to put greater effort and perseverance into their teaching than teachers with a low sense of efficacy (Tshannen-Moran & Johnson, 2011).

Teachers' Beliefs about Mathematics Teaching and Learning

Given the importance of epistemological and efficacy beliefs in pedagogical choices, what teachers believe about mathematics is relevant to how they teach and understand the subject. Ernest (1989) suggested that the successful implementation of current pedagogical practices, which sees mathematics as a dynamic, problem solving "continually expanding field of human enquiry" (p. 21), requires an understanding of teachers' mathematical beliefs**.**

A case study by Cross (2009) investigated teachers' understandings of their epistemological mathematical beliefs, the alignment of their professed beliefs with their pedagogical practices, and how these beliefs facilitated or impeded their implementation of current mathematical pedagogical practices. The study was part of a larger project focusing on the effects of mathematical argumentation and writing on the mathematical understanding and achievement of

Algebra 1 students. The participants were 5 teachers of grade 9 Algebra 1 students. The teachers received professional development to aid in the incorporation of writing and argumentation in their classrooms. Before the start of the professional intervention, each teacher was asked a series of questions that focused on epistemological beliefs about mathematics. The questions asked were:

> If you were to think of four words you thought were closely related to mathematics, what would they be?
>
> How would you describe your role in the classroom? and,
>
> How do you think students learn mathematics best? (2009, p. 330)

Cross' findings were that the teachers held fairly strong beliefs about what constituted mathematical knowledge and their beliefs had stemmed from their own early experiences at school. These beliefs shaped how the teachers designed their instructional activities, the tasks students were given, the quality of interaction they encouraged in the classroom, and the types of evaluation methods they employed (p. 336). Three teachers believed that mathematical knowledge was an absolute and established set of concepts and their classroom practices reflected this. Cross (2009) found that for these three teachers, mathematical learning meant being competent in basic arithmetic operations, applying procedures appropriately, and performing accurate calculations (p. 336). A fourth teacher viewed mathematics as a thinking and problem solving activity, prioritizing understanding over obtaining a correct answer. Through ongoing classroom discussion with the students this teacher allowed the students to examine their own thinking and the thinking of their peers (p. 333). Cross suggested that the teacher's pedagogical practices were indicative of the teacher's belief that thinking and reasoning were fundamental to mathematics teaching and learning. The researcher found that the fifth

teacher organized her geometry and algebra classes differently. Algebra classes followed a traditional, teacher centred model, while in the geometry classes students worked in small groups. Cross found a notable difference between the types of questions the teacher asked in the two classes. The teacher asked more probing questions in geometry and persisted until the students produced a valid justification for their responses. Cross reported that the fifth teacher had conflicting beliefs about mathematics, namely that mathematics was about problem solving and critical thought, but the teacher also thought of mathematics as a huge bank of knowledge rooted in numbers (p. 338). This qualitative case study by Cross shows the relationship between epistemological beliefs and practice, and in the case of the fifth teacher, shows that beliefs may be held independently and may be specific to a domain.

Holm and Kajander (2012) conducted a qualitative study with five student teachers. This study was part of a 5 year project that examined student teachers' knowledge and beliefs about mathematics. On the premise that teachers would continue to teach the way they have been taught, the researchers attempted to break this cycle through giving the student teachers opportunities to experience mathematics using current pedagogies. The methods course focused on problem solving and inquiry methods and student teachers were encouraged to use mathematical manipulatives and models. The student teachers were also encouraged to reflect on their interpretations of the activity and the mathematics involved. At the initial interview students teachers completed a Perceptions of Mathematics (POM) survey which related to their mathematical knowledge. This survey was repeated in the last weeks of the school term. In addition, the student teachers also completed an informal beliefs survey. It was found that each of the five students had a fear and dislike of mathematics and did not feel confident in their ability to explain their mathematical thinking. They recalled their school mathematics

experiences as needing to memorize formulas and procedures. While some of the student teachers recalled that they could not remember formulae, others recalled they could not explain their mathematical thinking. When confronted with these memories they expressed resentment about the way they had been taught mathematics (p. 17). The researchers found that as the student teachers' knowledge changed through the methods course, their beliefs changed as well and the student teachers reported a desire to teach mathematics differently from how they had been taught. This finding is in keeping with similar findings that teachers may change their beliefs through being introduced to different ideas in professional development initiatives and other educational opportunities (Tschannen-Moran et al. 1998).

In contrast to Cross's (2009) small group qualitative study Wilkins (2008) conducted a larger scale quantitative study involving 481 elementary teachers. The teachers were taking part in a professional development project focussing on the implementation of inquiry based mathematics instruction. Wilkins wished to investigate the relationship between teachers' knowledge, beliefs, and attitudes towards mathematics and their practices, specifically with regard to inquiry-based teaching. The study used multiple surveys. One survey measured teachers' content knowledge, another measured teachers' attitudes towards mathematics and mathematics teaching and a third survey measured teachers' beliefs about teaching strategies and how frequently they used 29 teaching practices. Wilkins observed that teachers with high levels of mathematical knowledge tended to use fewer inquiry based methods. Wilkins pointed out that while it would seem that strong mathematical knowledge facilitated teachers' use of inquiry based pedagogies, this was not necessarily the case. In fact, teachers' high efficacy beliefs in their usual methods of teaching were likely to have prevented them from trying new ideas.

Wilkins (2008) concluded that when:

> comparing knowledge, attitudes, and beliefs, teachers' beliefs in the effectiveness of inquiry-based methods was found to have the strongest overall effect on teachers' use of inquiry-based instructional practices. (p. 156)

Summary of teachers' beliefs about mathematics teaching and learning

Most beliefs about mathematics stem from teachers' own early experiences at school (Cross, 2009; Holm & Kajander, 2012). Beliefs may change with education or with an intervention of some kind, but newly changed beliefs are the most unstable and are vulnerable to reverting back to the older, more stable beliefs (Holm & Kajander, 2012). Beliefs play a strong role in the instructional activities teachers choose to present to the students. Teachers' beliefs about mathematics inform how teachers assess their students, the quality of interactions they have with their students and the sorts of activities they give students (Cross, 2009). In teaching mathematics, teachers are often likely to hold epistemological beliefs which are "vastly different from reform-based methodologies and new pedagogies" (Holm & Kajander, 2012, p. 8). Wilkins (2008) showed that while on one hand teachers with high efficacy beliefs were more likely to engage positively with students in the classroom, on the other hand they were less likely to be open to new pedagogies.

Teachers' Beliefs about Mathematics in a Multilingual Context

I begin this section with a report on a study by Reeves (2006), which examined teachers' beliefs with regards to including multilingual students in the regular classroom. While the study does not specifically focus on mathematics, it is useful in understanding attitudes towards multilingual students. In her study of 279 high school teachers from a south-eastern city of the United States, Reeves found that although most teachers responded positively to the idea of

including multilingual students in the classroom, when it came to their own classrooms 75% did not agree. Teachers did not believe students should be included in the classroom until a certain level of English proficiency had been attained, which they believed would take two years. The survey reported that teachers believed that the English as a Second Language (ESL) teachers should be responsible for the language proficiency of multilingual students and 39% of teachers thought that students should not use their home language at school. Reeves' study showed that teachers felt they did not have enough time to deal with what they perceived as the needs of the multilingual students, and 81.7% felt inadequately trained to teach them effectively. That being said, only 53% of teachers were interested in receiving additional training.

A study by Hansen-Thomas and Cavagnetto (2010) investigated teachers' beliefs towards multilingual learners focussing particularly on beliefs regarding mathematics. Teachers of three middle schools in the United States of America took part. A questionnaire using both qualitative and quantitative measures was conducted with 118 teachers. These teachers were mathematics, social studies, and science teachers, as well as physical education teachers, special education teachers, and ESL teachers. One of the major findings pertinent to this study was that the majority of the teachers believed mathematics was the easiest subject for the multilingual learners. Teachers believed this because "numbers are universal", "numbers are a universal language" and "it's a uniform code", "numbers are the same", and "math transcends language" (p. 256). Teachers also believed that mathematics contained less reading and writing than other subjects, and that "There is hardly any English involved" (p. 257).

In a different study, Silver (2008) was interested in student teachers' beliefs about mathematics and language. As a teacher educator she observed that student teachers clearly distinguished between mathematics teaching and language teaching, and were unaware of the

role of language in mathematics. To explore this she designed a multidisciplinary project to track what she hoped was their changing understanding of the role of language in mathematics. The main challenge for Silver (2008) was to disrupt the view, voiced by a student teacher, that:

> there's really no English in Math. It's just Maths. Why don't we do a project like this with Social studies? (p. 112)

For the project Silver asked students to develop mathematics lessons and then to identify crucial language connections for those lessons. Student teachers were then required to demonstrate appropriate ways to use and teach the selected language features within the mathematics lesson. At the end of the multidisciplinary project, Silver reported that although students agreed that language did play a role in mathematics instruction they were:

> still unconvinced that language needed to be addressed in a mathematics lesson or that a mathematics class provided a context for language learning… Working on language within a mathematics lesson was still seen as largely irrelevant and unnecessary. (p. 115)

Language and mathematics

In recognizing the importance of language in mathematics with multilingual students, Khisty and Chval (2002) point out the crucial role played by the teacher. While reforms in mathematics have moved away from a teacher-centered model of instruction, the teacher remains an essential "actor" and participant in the communication that takes place (p. 154). This is particularly so in a classroom with multilingual students who are learning mathematics in their weaker academic language (p. 154). In these settings, however, a teacher's beliefs about linguistic diversity, academic ability, and second language proficiency, may lead the teacher to "emphasize remedial work instead of focusing instruction on students' thinking and experiences' (Khisty, 2002, p. 32). One of the challenges for teachers is to "understand and positively use the linguistic strengths

and experiences that children bring to school" (Khisty, 2002, p. 32). When these challenges are examined from a sociocultural perspective, the focus "moves away from the description of obstacles and deficiencies to a description of resources and competencies" and enlarges what may be considered mathematical communication (Moshkovich, 2002, p. 197).

A sociocultural perspective on mathematics and language

From the above perspective Moschkovich (2002) examined the role of language in the multilingual classroom. She considered how this framework allows for a more developed and nuanced investigation of multilingual students in a mathematical context. Moschkovich challenged some views that consider the acquisition of vocabulary as essential for solving word problems, and carrying out computations. Moschkovich indicated that:

> reading textbooks and solving traditional word problems are …no longer the best examples of how language and learning mathematics intersect. (p. 193)

Moschkovich said that students now are expected to:

> communicate mathematically, both orally and in writing, and participate in mathematical practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments. (p. 190)

Moschkovich suggested that by adopting a sociocultural view one can show how students use a number of resources to communicate mathematically. This view "widens what counts as competence in mathematical communication" (p. 197) by focusing on mathematical discourse practices. Moschkovich using Gee's (1996) definition of Discourse, says:

> mathematical Discourses (in Gee's sense) include not only ways of talking, acting, interacting, thinking, believing, reading, and writing but also mathematical values, beliefs, and points of view of a situation. (2002, p. 198)

In one study Moschkovich (2002) examined the mathematical thinking of a group of multilingual students as they worked together on a geometry problem regarding the area and perimeter of rectangles. One of the multilingual students struggled to find the English word for a rectangle. Despite not finding the word, the student was able to communicate her mathematical thinking to the teacher and the rest of the class by using gestures and objects in the classroom. In addition, the student shared a common language with some fellow students so she inserted a word from her first language into her English sentence. This process of codeswitching allowed her to communicate effectively with her peers. When viewed from a sociocultural stance this student successfully used her first language as a resource when she used a Spanish word in the place of an English one.

Barwell (2009) situated his research within a sociocultural framework in order to investigate how multilingual students in a primary school in England participated in mathematics classes, specifically with regard to word problems. Word problems place heavy linguistic demands on multilingual students as word problems have their own genre and are "highly complex linguistic forms" (p. 67). In the study, students working in pairs were asked to create and solve their own word problems. Barwell recorded 28 sets of interactions between students in a Year 5 class over a two year period. Additional data comprised field notes and interviews with teachers. In his analysis of the data, Barwell showed that the students, through their communication with one another as they created and solved the word problems, were able to fully participate in the task of writing word problems (p. 68). Considerable discussion took place between the students as they negotiated meanings and context. For Barwell, the value of the tasks was that they allowed the students to situate the word problems in their personal context, to pay attention to the form of the word problem while working together in a mutual activity. Like Moschkovich (2002), Barwell

concluded that teaching vocabulary for word problems is probably not sufficient support for multilingual learners to be successful at problem solving. By allowing the students to create and engage in their own problem solving the students could "deepen their familiarity with how word problems worked", "bring their own lives and worlds into the task" and promote "language learning in a mathematical context" (p. 76).

Summary of teachers' beliefs about mathematics in a multilingual context

We see from the research in the literature review that there are some strongly held beliefs that teachers have both about mathematics and about language. These established beliefs are that mathematics has little to do with language, that mathematics is a universal language, and that students need to have fluency in English before they can do mathematics. In contrast, more recent research into mathematics and multilingual students viewed from a sociocultural perspective underlines the dynamic and rich possibilities in a classroom, both with respect to language expression and with respect to mathematics. According to Moschkovich (2002) an important and practical implication of a sociocultural theoretical framework is that it emphasizes the contextual nature of language and mathematics learning which allows researchers "to more fully describe the variety of resources that students use to communicate mathematically" (p. 191). These resources include the students' own mathematical knowledge, objects in the classroom, gestures, drawing, codeswitching, and students' first language. Barwell's (2009) research shows how students use their existing resources to write word problems: they draw on their own experiences and incorporate these into their work; they build on their existing mathematical knowledge to deepen their understanding of word problems; and their work prompts language learning in the context of mathematics, as they talk and negotiate understandings with fellow students. In addition, as Barwell points out, the process of creating

their own word problems allows the students to understand the links between language, setting, and mathematics (2009, p. 77).

My research question

Long held beliefs among teachers with regards to the teaching and learning of mathematics, in conjunction with long held beliefs about second language learning, appear to contrast with the findings of current research on multilingual students and their learning of mathematics. For example, Barwell (2009) and Moschkovich (2002), using a sociocultural perspective**,** both argue that language is very important in the learning of mathematics. Furthermore, they suggest multilingual students can quite readily participate in mathematical discussions in the classroom when they use available resources such as their own mathematical knowledge, objects in the classroom, gestures, and codeswitching. I wished to explore how teachers viewed both multilingual students and mathematics in their classrooms which are linguistically diverse and where the mathematics curriculum is progressive. As I consider teachers' beliefs as having been formed through their lived experiences my research question is: "What are teachers' experiences in the teaching and learning of mathematics with multilingual students?"

Methodology

In order to address the research question: "What are teachers' experiences in the teaching and learning of mathematics with multilingual students?", I adopted a sociocultural theoretical framework as it allowed me to investigate their beliefs as having been formed in the contextual settings of their histories and their present practices (Cross, 2009; Negueruela-Azarola, 2011). For this reason the design of the study was qualitative. I anticipated that a qualitative approach using interviews would allow me to understand more fully how teachers' beliefs are mediated by the competing influences they deal with**.** I also anticipated that the use of interviews and a focus

group would foster a rich discussion as the teachers talked about their experiences, challenges, and demands of teaching mathematics to multilingual students. In speaking about their experiences they may add to what they have said, recall different incidents, and reveal feelings such as frustration or joy. Interviews offer the opportunity for a sense of "real time" and immediacy to the teachers' lived experiences. I anticipated that these discussions would be a powerful experience for them and for me.

Research participants

In my research design I planned to interview 5 or 6 teachers. I felt this was a reasonable number of research participants as I allowed for interviews of 45 minutes to an hour and intended to follow these at a later date with a focus group of an hour and a half. I believed the time allotted combined with the number of participants would allow a rich collection of data. Once I received ethics approval I began the process of recruiting the research participants. As a teacher of fifteen years experience myself, I have contact with a number of teachers. I approached several teachers and explained my research to them. I asked them to extend my recruitment text to any of their colleagues whose experiences are pertinent to the study and whom they thought might be interested in taking part (see Appendix A for recruitment text). My recruitment text explained my research and also outlined the eligibility criteria for participants. Interested teachers were asked to contact me by telephone or by email. All the research participants contacted me by email. Eventually five teachers took part in individual interviews with me over the course of the late spring and summer (see Table 1 below for background of participants). The participants were Lindy, Yasmine, Geri, Melissa, and Linda. All teachers except Yasmine taught at the elementary level. Yasmine taught grades 7 and 8. Geri and Melissa were the only two teachers who taught in public schools**.**

Table 1: Background of Research Participants

Name	Gender	Languages	School	Ages of students	Experience
Lindy	Female	English, French	Private, progressive	6 to 9	10 years
Yasmine	Female	English, Farsi	Private, for students with dyslexia	12 to 14	Unclear
Geri	Female	English	Public, alternative program	5 to 7	12 years
Melissa	Female	English	Public, alternative program	5 to 7	Supply teaching since 2008. First year with own class
Linda	Female	English	Private, Montessori	6 to 9	12 years

The research participants were female and four chose to use their own first names in the study. An outcome of this choice was that there were two participants with the first name Linda, so I changed one name to Lindy. The fifth participant chose to use the name Gerald as her pseudonym. This name is generally recognized as masculine. In order to prevent the use of a masculine noun with feminine pronouns, I shortened the name to Geri which is a gender neutral name. I anticipated this would allow for easier reading. I introduced Geri and Melissa together in my findings as they were two participants teaching at the same school. I did this rather than

discuss twice what was common in their work environments.

Interviews

I began each interview asking the participant for brief background information about themselves. I asked the participants to talk their experiences teaching mathematics to multilingual students. I used my interview guide (see Appendix B) to guide me and direct my questions. While the interviews did not last as long as I had anticipated, lasting on average 30 to 40 minutes, I believed the interviews gave me the rich data I was looking for.

I met the teachers in various locations: two teachers in their classrooms, two teachers at the University of Ottawa where I had booked a study room for the purpose, and one teacher in a coffee shop. I found the meeting in the coffee shop the most comfortable for me and, I think, for the teacher. It was the longest interview, lasting an hour, and I attribute this to her being relaxed in her surroundings. I had anticipated that meeting teachers in their classrooms would be a space conducive for the teachers to talk, but the two teachers I met in classrooms appeared hurried and both were interrupted by either students or other staff during our discussion. It appeared to me that the possibility of further interruptions and being in their place of work may have contributed to a lack of calmness.

Challenges

It was a challenge recruiting the teachers and arranging the meetings. The teachers spoke of being very busy and two teachers suggested it was the wrong time of year to meet as they were so busy. A sixth teacher changed her appointment with me five times until I was compelled to realize that we were not going to meet. At the interviews the teachers were more than happy to talk to me but I got the impression they were very busy and could not spare me much time. After the difficulty of recruiting the teachers for interviews and after completing the interviews I

decided to forego the focus group. I also felt that I had sufficient data from the interviews.

Data sources

My research design incorporated interviews with the 5 teachers mentioned above as participants. I had anticipated my question, "What are your experiences teaching mathematics with multilingual students?" would generate a rich description of the teachers' pedagogical practices in the classroom, the teachers' interactions with students, the teachers' observations of the multilingual students, recognition of their learning and their difficulties, thoughts about solutions, strategies, steps taken and perceived outcomes. I explained my research project to them, and also explained what I meant by the term multilingual when referring to students whose first language was other than English, whether or not they had been born in Canada. I did this because I suspected that this term was not one they would readily use when referring to their students in the classroom. I believed the terms "ESL" and "bilingual" would be more commonly used by the teachers, and this was the case. I went over the consent form underlining that the research is voluntary and assured them of their confidentiality. The participants each signed their own consent form.

Data Analysis

The interviews with teachers took place over the course of 4 months starting in May and finishing in August. I transcribed each interview once it had taken place. I began my analysis with one teacher at a time. I dealt with the analysis on an individual basis as I wanted to get a sense of each teacher practising within the context of her classroom. For my analysis I printed out the transcribed interview and read it a number of times while asking myself, "What is she saying here? What does she mean? What is important to her?" I did not start thinking about the teacher's beliefs and was not looking to uncover beliefs at this point. My goal was to get an idea

of the teacher. For the next step I took the printed text of the interview and began to insert my own notes and observations between the teacher's comments. This was to help me get a clearer idea of what the teacher thought. I came back to this work a number of times to reread what I had written and to see if it still corresponded to what I understood the teacher to mean.

Once I felt I had a sense of a teacher I read the teacher's descriptions and my notes again to look at their epistemological and efficacy beliefs. I asked myself "Why did she do what she said she did?", "What does it mean if she did that?" Again, I went through the data and inserted notes to justify the inferences I had made about the teacher's epistemological belief or an efficacy belief. I colour-coded my notes on epistemological beliefs by typing in pale blue and my notes on efficacy beliefs I typed in light brown. This helped me manage my data. I found it difficult to separate epistemological beliefs from efficacy beliefs because I believe they are connected. It was not easy to say "This is epistemological, but this is efficacy". I wanted to address them separately so when I was unsure about where to put a belief, epistemological beliefs would take precedence since I understood that efficacy beliefs were informed by epistemological beliefs.

While I initially considered the teachers one by one and did not have them all in my mind at once, as I continued working I became more experienced at understanding what they I thought they were saying and I began to be aware of the differences between the teachers. At this stage the analysis became very interesting as I was able to compare the teachers' beliefs and practices. I was able to notice differences between what the teachers said they did and what they said they believed.

I began this dissertation with a descriptive vignette of some of the activities in Mrs. Ray's classroom. From this I was able to make some inferences about her beliefs. I approached the analysis of the interview data in the same way. I had anticipated that the teachers' conversational

responses to the research question would prompt them to describe their classroom and their practices much like the description of the vignette. So a teacher, for example, may say, "*When I saw Juan struggling to read from the textbook, I decided that his vocabulary was not adequate for understanding the word problems. In order to support him I arranged for him to see the ESL teacher more frequently."* From this I could infer that the teacher believed the acquisition of vocabulary was necessary for students and that it was the responsibility of the ESL teacher to look after this. This suggests that the teacher believed supporting the student's language in mathematics was not her role. Another teacher might say, "*I put My-Linh with Sam at the table with some different material to see what they could work out together."* This allowed me to consider that the teacher believed the students would make meaning of their work together, as they used and talked about the material. A further teacher may say "*I got the feeling that Thandi understood the work but was having difficulty expressing the idea of the volume of a cylinder, so I suggested she make a cardboard model and show the other students, using her own language when stuck. That would be fun for everyone concerned!"* From this description it would appear that for this teacher, mathematical learning involved communication and shared meaning. It would seem that the teacher believed that learning was dialogic and that meaning was contextual. While not using epistemology and efficacy as mutually exclusive categories, this process of inferring beliefs from the interviews allows us information about teachers' epistemological stance and how they view their own efficacy.

Trustworthiness

While there are different understandings of trustworthiness in qualitative research, I found Guba's (1981) proposed model for assessing the trustworthiness of qualitative data useful. One of the aspects of Guba's model that I used is truth value, or credibility. Credibility may be

obtained from “the discovery of human experiences as they are lived and perceived by informants” (Krefting, 1991, p. 215). As a researcher in this study my goal was to discover teachers’ experiences teaching mathematics with multilingual students. I am a teacher myself and so I was able to use my background knowledge and my own teaching experiences to guide me in my research design. I designed interviews that I believed would be pertinent to the experiences of the research participants. I believed that interviews would allow for a relaxed and informal atmosphere where the teachers would feel at ease talking with a fellow teacher.

I was careful not to use the word beliefs during the interviews, but rather focused on discussing and understanding teachers’ experiences. I believed this distinction of terminology between beliefs and experiences to be important because my planned methodology was to infer beliefs from the teachers’ experiences and not to directly ask them about their beliefs. Had I asked about beliefs I anticipated that the credibility of the study would have been threatened by the research participants responding with what they thought was the preferred social response rather than responding with thoughts of their own personal experiences (Krefting, 1991, p. 218). During the interviews I was sensitive to the need to reframe questions should a participant be unsure of what was being asked, and I was careful to ask for clarification if I was unsure of the meaning of a participant’s comments. At all times I endeavored to give the participants the sense that they were being listened to and that they had my complete attention.

To strengthen the credibility of my research I endeavored to ensure that there were no unexplained inconsistencies between the data and my interpretation of the data. In other words I linked the data to the inferences I had made by carefully and openly explaining the reasoning for my thinking. The research participants did not review their own transcripts as the teachers’ time was limited and I thought this might be a further imposition on them.

While open to any findings my research might have revealed, I gained confidence when I observed that my findings aligned with the findings of other research on teachers' beliefs.

Findings

In this section I discuss my findings with the teachers one by one and again give some background to each of them for the purpose of refreshing the reader's memory as to who they are. My choice of discussing the participants individually is in an attempt to reveal the lived and very human experiences of the teachers' lives within their classrooms, and to reveal, as well, the very human essence of beliefs. I attempt to situate each teacher within the context of their own classrooms in order to create a picture of how their beliefs are coherent with their pedagogical practices. I share my understanding of the participants' epistemological and efficacy beliefs as I have inferred them from my discussions with the teachers. I look first at a teacher's epistemological beliefs, then at her efficacy beliefs, followed by a summary of my findings with regards to both epistemological and efficacy beliefs. I begin with Lindy.

Lindy

Background

Lindy teaches in a small private progressive school in a multi-age classroom (grades 1, 2 and 3). She was trained in the Montessori Method of teaching where learning is believed to be facilitated by a problem solving approach and by the use of manipulatives within a specially prepared classroom environment. While her current school is not a Montessori school in her classroom the children use manipulatives in their mathematical work. Although Lindy has taught for 10 years this was her first experience teaching mathematics to multilingual students.

In our discussion Lindy referred to her three French speaking students when she talked about multilingual students. These French speaking girls had previously been home schooled. They

came from two different families.

Epistemological beliefs

As I talked with Lindy I saw she had a clear understanding of what she believed was required to teach mathematics to multilingual students:

> I saw the two, sort of-, the two skills that were necessary; number one, the skill of understanding the mathematical concepts and the language. I-, you know, they sort of worked together and once they-, the child mastered the language, they were able just to progress in the math or continue to progress, and I think I would do it the same. The child needs time to understand the language and really math is another language, so they're learning like two languages simultaneously.

When Lindy referred to the "skill" of understanding a mathematical concept and the "skill" of learning a language I took this to mean that multilingual students needed to master two skills in order to learn mathematics. That Lindy talked about the students having to think mathematically and then having to think about language, suggested that Lindy viewed mathematics and language as discrete entities independent of each other. Lindy explained what she thought were the challenges for the multilingual students in her classroom when they were doing their mathematical work:

> What-, at the beginning, what wasn't working, was the fact that the child didn't have the language, you know, and I found, particularly learning the math facts, you know, the multiplication tables, where they had to think mathematical and then they had to think language.

Lindy's use of the phrase "wasn't working" led me to think that Lindy saw the students' proficiency in English as a problem that needed resolving. The implication of the phrase, too,

was that students would not be able to learn unless they had more English than they currently had. Lindy referred later to the length of time it took the multilingual children to do their mathematical work and commented that it took them longer than the other students:

> They can say "45" in French or in English quicker than they could, say three months ago, so as-, as they are getting stronger with the language they are able to do the math quicker, but until they have the language you're-, you're-, it's slow going.

This suggested to me that Lindy's focus was on having the students speaking English as soon as possible so that they could understand the mathematical tasks she gave them to do and to do them with speed. She had a two pronged approach for doing this. Firstly, she focused on building up the students' vocabulary:

> I would give them vocabulary, spelling lists as well, for their math material. So there was a week they had to do all of the polygons. Those were their spelling words for the week, yeah, you know, and even their numbers were spelling words, so mixing the two – the language you know, just the vocabulary and the math together.

By using weekly spelling lists and additional vocabulary Lindy aimed at building up the students language skills. She gave them the names of polygons as well as the names of numbers as their spelling words in order to develop their vocabulary in mathematics. Another way Lindy tried to build up the French students' language skills was by insisting they only spoke English to her, although she could speak French fluently. She said "I didn't help them out with speaking French. I spoke strictly English." These comments led me to believe that Lindy might think that if she spoke French to the students it would impede their learning of English, and their learning English as quickly as possible was important to her.

The second prong in Lindy's pedagogy in teaching mathematics to the multilingual children

was to focus what she called the basics and from this I inferred she meant the students knowing their addition and multiplication numbers by rote. With regards to mathematics, Lindy commented that "in math it's so easy because the isolation is in the numbers" from which I understood Lindy to mean that she thought numbers were discrete units of knowledge readily comprehended by all students. She said with regard to the multilingual students:

> They needed the language and they needed to be solid in, you know, the basics, first, like numbers to ten, then numbers to, you know, being able to do base one to a thousand and once they had that they could progress with the math language.

Lindy referred to the importance of the students knowing their number facts when she told me about one of her multilingual students, a young girl of six. Lindy explained to me that this little girl found it very hard at school because had not made any friends. Lindy observed that the student was young and the day was long. But over the course of the year the young student learned more English and made progress. Lindy described this when she talked about her multilingual students in general:

> They don't have their math facts, their multiplication facts, for example, memorized, but they love doing it because they are much more successful now, and we do fast math, for example, and they are able to do it much faster than they were.

Lindy said that despite the student not knowing her multiplication tables by rote the student was able to work faster. Lindy talked about "fast math" and the students becoming more successful which led me again to think that speed is important for Lindy and criteria by which she judged success.

Lindy talked with me about her role as the teacher of multilingual students:

> Just constantly working with them and observing where their strengths are, where their weaknesses are and building on strengthening their weaknesses, basically.

When Lindy referred to strengths and weaknesses it was clear that she thought of learning in terms of negative and positive attributes. Another responsibility she believed she had with regards to her students was helping them obtain the correct answers to their mathematical computations. She did this by letting them know if their answers were wrong:

> With just the basic math facts, it's black and white. It is either correct or it is not correct. It if wasn't correct they could go to the math charts to check their answers on their own or I would say, "No, this is not correct. Can you go and use material to give me the correct answer?" And just by the repetition of that they became more successful in giving correct answers.

Lindy did not allow for any ambiguity in the students' answers. When she described her requirement that the students give the correct answer Lindy sees a clear divide between an answer being correct and an answer being incorrect. Lindy required an outcome and that outcome was correct answers to questions. This was important to Lindy as shown when she redirected a student to repeat work, using the help of the manipulatives if necessary, to obtain a correct answer. With repetition the students became more successful in getting their answers right.

Lindy's efficacy beliefs

In our discussion Lindy came across as a confident, competent, and cheerful teacher who enjoyed her work and took pleasure in her students' successes. She laughed a lot and gave the impression that she knew exactly what needed to be done in the classroom to ensure that her students were successful. Lindy shared her understanding of her role as a teacher with regards to

the students:

> It's the student, you know, and following the student, again, that guides you as a teacher in terms of "What is this student getting? What is the student understanding? What does the student need in order to be able to go that next step?"

I believed that Lindy showed herself as a reflective teacher when she asked herself these questions about her students. Lindy then used her perceptions of her students to guide her pedagogical choices. When I asked her how she dealt with the challenge of teaching mathematics to the multilingual students, Lindy revealed that she had a clear idea of what the multilingual students needed for their mathematics learning. First, she focused on teaching English vocabulary, second, she checked on their mathematical knowledge, and third, she relied on her own assessment of the students in deciding what they needed:

> I just think that you have to be very creative with the students. Again seeing where they are weak and bring that material to them in a different way to strengthen they knowledge of- whether it be just the vocabulary numbers or the operations of numbers.

Lindy gave me an example of her being creative with the students when she explained how she chose spelling words for one week:

> So there was a week they had to do all the polygons. Those were their spelling words for the week, yeah, you know, and even their numbers were spelling words. So mixing the two- the language you know, just the vocabulary and the math together.

When she pointed out that "even their numbers were spelling words" I inferred Lindy felt proud of bringing language into the mathematical sphere.

Lindy was not teaching in a Montessori school when I met with her, but it appeared to me that her strong sense of efficacy in herself as a teacher had to do with her relying on her previous

experiences teaching in a Montessori school, and her use of the Montessori methodology and manipulatives in her current classroom:

> I took them back to the c*asa* material, working with the number rods, right to counting spindles, recognizing numbers, and then taking them back to the bead work.

In the above, Lindy referred to specific Montessori manipulatives for mathematics for children in the kindergarten classes which are called the *casa* classrooms. Lindy appeared to believe that the Montessori manipulatives allowed the students some autonomy in checking whether their own work was correct or not:

> They would be able to go and work with the labels and the material and the control charts to make sure. "Okay, where is the hexagon? Where is the, you know, nonagon? Where is the pentagon?" you know, and by just checking their work on the charts, on the control chart, they could tell whether they were correct or not.

I asked Lindy why she thought the manipulative material was helpful for the multilingual students and her answer, referring to the manipulatives, was "because it-, it was the teacher, basically". Her comment with regards to the manipulatives appeared to underscore how Lindy viewed another of her responsibilities as teacher and that was to ensure the students got correct answers to their work. Here she implied that the control chart for the manipulatives acted like a teacher because the students could cross-check their work and see if they have the right answers.

I thought Lindy a conscientious and diligent teacher who was acutely aware of her students in terms of what they need for their mathematical learning and in terms of their strengths and weaknesses. Lindy spoke as a teacher who was confident in her ability to teach and in her ability to make pedagogical choices that support the students' learning. Lindy's strong sense of confidence came through when we discussed how she had found teaching this year considering

she had had no prior experience teaching multilingual students. She said teaching them was:

> Just sort of a natural process ... with how it unfolded this year and just following the students.

She said that as a teacher she did not really experience any difficulties teaching her French speaking students. She considered it a very successful year and that she would do it the same way again. Lindy appeared to be looking back on her year with satisfaction.

Summary of Lindy's epistemological and efficacy beliefs

I saw Lindy's epistemological beliefs as typical of those belonging to a traditional teacher where learning is considered to be the accumulation of knowledge and the mastery of skills. Lindy specifically called mathematical understanding a skill. This view of knowledge sees learning as a skill which is measurable, cumulative, and achieved through the repetition of activities and the memorization of facts on the part of the student. Lindy's energies as a teacher were directed towards supporting the students in their learning. Therefore, as she saw learning as an accumulation of knowledge and development of skills, she focused on the students' English vocabulary so that they would have a better chance of understanding the mathematical work, obtaining a correct answer and working faster. She did not speak French to the students because her goal was to improve their English. It seems possible that Lindy did not view the French the students knew as a help in their learning of mathematics. As she pointed out earlier, their lack of English was holding them back in their learning. She also focused on giving the students lots of opportunities for repetition with their calculations so they would have their multiplication and addition facts memorized, which would allow them to work faster in their computations. The proof of this learning for Lindy would be the students completing their mathematical work with speed, ease, and obtaining correct answers. While Lindy's pedagogical practices were consistent

with her epistemological beliefs these beliefs were not consistent with her Montessori training where the focus for students in the classroom is on enquiry and not only on obtaining correct answers. Lindy appeared to have a clear idea in her mind of what constitutes learning and the steps that she needed to take in order to ensure that this learning was accomplished by the students. This clarity informed Lindy's pedagogical choices and also appeared to be expressed in her strong sense of efficacy as a teacher.

Yasmine

Background

Yasmine teaches mathematics to students in Grade 7 and 8 in a private school for children with dyslexia. This is her second year teaching at the school. In her class Yasmine had three multilingual students, two whose first language is French and one whose first language is Arabic. The French students spoke very little English, while the Arabic student could. Yasmine spoke English with the Arabic student. Yasmine herself could speak English and Farsi.

Epistemological beliefs

During our meeting Yasmine spoke often about the students' understanding and her own understanding and she regularly mentioned that this understanding was the focus of her pedagogical practice. When I asked Yasmine if there were any surprises for her as a teacher of multilingual students, she replied:

> There were times that it would take us longer to just get to a mutual understanding of what I say and I-, not what I say, but me understanding that I understood that they understood the concept, you know what I mean, because it takes a level of communication to get my point across, but then they need to show me that they understood what I said.

Her answer clearly indicated that understanding was a key element for Yasmine, both her understanding and the students' understanding and she began that response by speaking of mutual understanding. Yasmine referred to this mutual understanding taking longer. This understanding did not happen immediately as she taught but rather was a process that took varying lengths of time. Yasmine related this longer time to her own experience translating from Farsi to English. She gave the example of an occasion when she was doing simple addition on the chalk board in front of the class. Her students were calling out the answers to her. She asked them to be quiet because their calling out disturbed her thinking. She told me she needed them to be quiet so she could translate the question to Farsi, do the calculation then translate the answer back to English again. She noticed that this took time. Yasmine reflected on her need for time to do the translations and her reflections helped her with her understanding of what the French speaking students may be experiencing as they do their mathematical calculations. Her reflections led her to say with regards to the French speaking students in the classroom that:

> Maybe I know how you are thinking, so maybe you think the same way as I think and let's just tackle it that way so you have a longer time to think about these simple things because you need to do the translation. It is absolutely fine.

When Yasmine talked about the possibility of knowing how the students thought because she had reflected on how she thought, it seemed that for her there may be something common to their experiences when learning in a second language. She used this understanding of their common experiences to adjust her teaching for these students in such a way that it may help the students with their learning by allowing them extra time.

Yasmine went on to describe a situation when she found it difficult to come to an understanding with the students. She had been teaching a unit on fractions and defined the term

common denominator for them, but she did not feel this was really enough for the students to comprehend what she was trying to convey. Yasmine repeated to me what she was thinking at time she was teaching fractions:

> How am I saying it that they don't understand it? Like-, what is it that we are not in the same plane now? We don't have the same understanding of things-, and then the next lesson-, oh, that went well, but still I am not feeling very comfortable.

In the above Yasmine expressed her discomfort at the lack of understanding between herself and her French speaking students about the concepts she was trying to teach. This was clear when she said that she and they do not share the same understanding and for Yasmine then this would imply that the students were not learning. In addition to the terminology *shared meaning*, and *same understanding* Yasmine also uses *similar understanding* and these expressions led me to think that for her learning was a shared activity that came about when meaning was made and understood collectively between those taking part in the activity. Yasmine saw herself as party to the process of meaning making and perhaps even considered herself as a co-learner with the students.

If Yasmine saw learning as meaning made through a shared activity, then it is likely that her pedagogical practices would complement this epistemological belief. This would imply that Yasmine, in her role as teacher, used her understanding of the students' understanding to decide how best to facilitate their learning. For instance, as mentioned earlier, Yasmine recognized that the French speaking students need more time to do their work and she allowed this. Although the language of instruction for mathematics was English, Yasmine was quite comfortable allowing the children to do their mathematical work in French:

> I was okay with them talking in French because I thought they are explaining things- probably better than-, like they are understanding but they are doing French. Why not?

Clearly here, for Yasmine, the goal was the students' understanding because by saying that she was okay with the students speaking French, Yasmine implied that the students were more likely to come to an understanding by discussing the work between themselves in French than by her trying to communicate with them in English. In this way she acknowledged and respected the resources the children may have brought to their learning. Also by allowing the students to communicate in French, Yasmine showed that she did not need to control the conversation or to know what was being discussed but rather that she could relinquish that authority to them. The students were able to work with this degree of autonomy because the focus for Yasmine, implied in her discussions, was the students' understanding of their work. This also indicated that Yasmine may not have thought she was essential to the learning process. As Yasmine described her pedagogical practices in the classroom she illustrated how she made her choices based on what she believed the students need in order to learn:

> So if we need to do a project-, if we need to make a model to understand it, I wouldn't stick to the text book. We would make a model because we need to make a model to understand it. Or if we need to do a drawing on the floor then we do a drawing on the floor because we need to do that to understand. Like, it depends on what the kids need.

What Yasmine appeared to be illustrating here was that she and the students would use a variety of resources in a shared activity in order for them to come to an understanding of what was being taught. Yasmine used the word "we" and this may show that she identified herself as a learner with the students. If Yasmine's epistemological beliefs were enacted through her pedagogical choices then she demonstrated that she believed learning took place in a specific context

therefore she adapted her teaching to the needs of the students. Yasmine was continuously reflecting on her teaching and trying to adapt to the situation. This can be seen when she described her approach to teaching a unit on measurement when her efforts on teaching a unit on fractions had been frustrated. The unit on fractions had a lot of nomenclature that Yasmine herself found difficult to explain and difficult to understand in a second language. She said that "After that I decided we will do measurement with a totally different approach" and she changed how she taught this to the students. With the unit on measurement she did not initially give the students any terminology as she had done in the fraction unit, but rather created a lesson that would use the students' own experiences and knowledge as a resource. Yasmine called this going backwards because the students would play first with manipulatives and materials, then be introduced to the terminology:

> We should come up with a drawing, or we should come up with other means of-, or let's make a poster, or let's make-, let's play with clay. Something else that we could just communicate, not just through words.

Yasmine decided to use other resources to facilitate the students' learning. She was open to using multiple resources including drawings, clay, a poster, but as she clearly implied, whatever it took to help the students, she was willing to go beyond words. Yasmine perceived learning as more than communicating with only words. For the measurement unit, she decided they would make a house and each team would get one room to measure and then to make a model of it. Yasmine chose this approach because for the students:

> The context was familiar. It was a home, there was a chair-, like you know the chair is bigger than a table, like you know all-, the sofa is bigger than a table-, all those kinds of things. You have something in your home which they can-, were able to connect it to.

Yasmine explained how a home setting would be familiar to all the students and this familiarity would enable them to connect with the task they were given to do on measurement. This discussion suggested that she believed that a familiar or known context would support the students in their learning. By this comment Yasmine showed she believed that it is more helpful for the students to understand a concept through manipulation of materials and the recall of life experiences than by giving vocabulary to abstract ideas which she felt had happened in the fraction unit. From the choices she made I inferred that Yasmine saw her role as the person responsible for creating a setting that would most effectively support the students' learning using a variety of resources.

Yasmine's efficacy beliefs

Yasmine talked with conviction about her pedagogical choices in the classroom. She talked about reflecting and re-reflecting on her teaching practices. She showed strong efficacy beliefs when she acted autonomously in making pedagogical choices that would affect her students' learning. Yasmine gave a number of examples of this autonomy which emerged during our discussion:

> We had a text book for the school. That school is a private school. They did have a text book but I didn't use it. Like, I rarely use a text book. I come up with the stuff that goes with the class.

Yasmine's descriptions of how she taught suggested that she was confident and secure in her ability to decide what was best for the class based on her observations and interactions with the students. This strong sense of efficacy was seen again when Yasmine made a decision about changing the way she had been teaching. Her decision to change her lessons suggested that Yasmine did not feel bound by any one way of teaching and that she was comfortable and

prepared to make changes as she saw necessary:

> After that I decided we will do measurement with a totally different approach to see if that would work and I saw it working better.

By saying that she would "see" if a totally different approach would work Yasmine gave the impression that she was prepared to take a risk in teaching a lesson differently with the belief that it would be better for the students. It is likely that Yasmine did not believe there was one way to learn or one way to teach. As Yasmine talked it becomes obvious to me that she spent much time considering her pedagogical practices and what changes she could make in order to support the students with their learning.

While she has an overall strong sense of herself as a teacher, she viewed herself negatively for not being able to speak French to her French speaking students and this appeared to create a sense of inadequacy in her. She was confident enough to allow the students to converse and explain things to each other in French because her goal was the students' understanding but she was still critical of herself for not knowing French and she said "I wish I knew French… It was my deficiency". At the same time the issue was further complicated because English is not her first language, and as she said, "I find it difficult to explain and difficult to understand in a second language". Her anxiety about communication came across when she repeated part of her interaction with the multilingual students:

> How am I saying that they don't understand it? What is it that we are not in the same plane now? We don't have the same understanding of things. I am not feeling very comfortable.

From these examples I inferred that Yasmine's efficacy beliefs were not strong when it came to her feeling competent about communicating with the students. While she stressed the deep

importance of effective communication this was one area where she felt less effective. One the one hand she questioned the students as to why they had not understood what she has been teaching, and on the other hand she took responsibility for not being able to speak French to them.

At the beginning of the discussion Yasmine was somewhat hesitant so I suggested it might be easier for her if I asked her questions instead, and she agreed to this. At the end of the discussion, Yasmine said that she was trying to come up with something exciting for me. This comment made me think that she was concerned that what she had told me may not be valuable to my research. Yet as I reviewed the data later I sensed there was some evidence for strong efficacy beliefs. After consideration, perhaps the hesitancy I perceived may be understood as cultural differences between how the two of us would present ourselves to others when talking about our areas of expertise.

Summary of Yasmine's epistemological and efficacy beliefs

My discussion with Yasmine led me to consider that she believed learning took place when a common understanding was reached between the students or between the students and her as they participated in a shared activity. Yasmine viewed understanding as a process that required time. She said that it may be a slower process for the multilingual students when they are doing their mathematics because she experienced the same slower process herself when doing mathematics. That Yasmine believed, epistemologically, that understanding was shared, has implications for her pedagogy in the classroom and Yasmine's classroom practices, as she described them, appeared to reflect her epistemological beliefs. The mutual understanding that was the goal of Yasmine's practice meant she had to focus on effective communication between the students and herself in order for this understanding to occur. For Yasmine, however,

communication was not limited to words. She used other resources on hand to support the students' learning. These resources were the students' own language, their own history, and materials and artifacts in the classroom such as drawings, models, and the use of clay.

While Yasmine saw herself as a co-learner with the students she did not see herself as necessarily always being party to their learning. They could learn without her, since meaning was made by the students as they worked together coming to an understanding. The students were allowed a degree of autonomy and we see this when she allowed them to talk together using French, which she could not understand.

Yasmine was reflective and flexible in her teaching and she mentioned this often. She reflected on lessons she had given and experiences she has had and used these reflections to adjust her teaching to best support her multilingual students learning. For example, her shared experience of learning in a second language enabled her to reflect and thereafter adjust her teaching. Yasmine gave me the impression that she had thought a lot about how learning takes place and that she felt she had a good understanding of the process. Her confidence in her understanding of how students learn was reflected in her sense of efficacy in herself as a teacher in the classroom. She made the students' understanding the focus of her pedagogical practices and was comfortable making any changes she saw as necessary in order to promote the students' learning.

Geri and Melissa

Background

Geri and Melissa teach at the same alternative public elementary school. They both have a multi-age class of students who, in the regular elementary school system, would either be in a grade 1 or a grade 2 classroom. The alternative program differs from that of the regular elementary schools' program in its philosophy which calls for multi-age groupings of students

within each classroom. A multi-age classroom is a philosophical educational choice as opposed to a split grade classroom where for economic, logistical or other usually practical reasons, two grade levels are combined. A philosophical premise of the program is that collaborative work is an effective means of supporting students' learning. Importance is placed on students mentoring each other, as well as on students' interpersonal relationships and on ensuring opportunities for student leadership. The teacher fosters collaboration by encouraging discussion between the students and by giving students opportunities to express ideas and interests. The teacher evaluates the students through an on-going process of narrative reporting, observations, conferences with students with their work portfolios, assessment of students' projects. There are also opportunities for the students to use self-evaluation as a form of assessment for themselves. There is less emphasis on marking, grades, and traditional testing than in the regular schools. Assessment is anecdotal.

Within the public school board to which Geri and Melissa's school belongs, multilingual students are categorized according to stages of proficiency in English. There are 4 stages; beginner, high beginner, intermediate, and advanced. Following Cummins (2001) the public school board suggests that it takes students between one to two years to develop adequate everyday communication skills in English, and between five and eight years to develop the language and skills needed for success in school subjects.

Geri

Background

Geri is an experienced teacher with 12 years' teaching experience. She has twenty children in her classroom. Of those twenty six students are multilingual. The previous year Geri also had twenty students, and nine of them were multilingual. Geri refers to her multilingual students as ESL learners.

Epistemological beliefs

After our introductory chat Geri immediately started talking about her students and their mathematical abilities. It occurred to me that possibly Geri wanted to assure me that all her students were doing well in mathematics. I asked if we could backtrack in the conversation a little because I realized I did not have a clear idea about the makeup of students in the classroom. Geri described it to me:

> Yes, okay, well, I have a grade one two class and so they're from six to eight, some-, very few eights, but a few eights by the end of the year. Actually, more five to seven, I guess, five to seven.

Geri used the word "grade", and later on when she used the words "split grade", I began to think that Geri categorized the students as either in one grade or the other and that she used the idea of grades to make a distinction between the students when thinking about them. She also said that she taught both grades at the same time which strengthened the impression Geri gave of her thinking that she has two distinct student cohorts in her class – two cohorts that Geri saw herself as having to teach simultaneously with each presumably having its own requirements. Geri explained, for instance, that her expectations were different for each group of students that she had categorized:

> My grade twos will have to do that right from the start. My grade ones generally start, as this boy did, with pictures and then words and numbers will come.

From this it did not appear to me that Geri viewed her classroom as one class, but rather as two separate classes in the same room.

As mentioned earlier at the beginning of my discussion with Geri, she was eager to address what she thought was my area of interest and also eager to assure me that her multilingual

students were doing well in mathematics:

> I mean, their first language isn't English but they are strong English speakers and to be honest, they have been really-, all of the ones I can think of, except for one little boy, were really, really high in their math skills.

Her saying this and repeating the word "really" made me think that Geri thought it important that I understood that all her students did well in mathematics, including the multilingual students. A little later when I asked Geri if she had any challenges teaching mathematics to multilingual students she again answered that her ESL (English as a second language) students were strong, and that she actually had no experience teaching multilingual students:

> I haven't really had the experience-, since I have been teaching in the-, at the grade 1, 2 level using this method with ESL learners. They have always been strong.

I gathered two things from this part of the discussion: First when Geri referred to a method she used as she taught the multilingual students mathematics I understood that she followed specific, clearly articulated steps for teaching mathematics. Second, when Geri said she had no experience teaching multilingual students I saw that she did not equate the expression I was using, *multilingual students* with the one she was using, *ESL learners.* We, therefore, talked again about what I meant by the term multilingual and I explained that for the purposes of my research it referred to any student whose first or home language was not English whether or not the student had been born in Canada. After this clarification of the term Geri recalled her first year of teaching when she had a student who did not speak English:

> No, but like I said, I have had-, I haven't had anybody who is-, you know my first year teaching I had a little boy who didn't speak a word of English and-, it was kindergarten-

> so I don't you know-. It's been so many years. I don't know what I did, but I remember, you know, that was really difficult. And at that time it was an issue for me.

Geri remembered the challenges she experienced teaching this young boy who spoke no English but that was the first and last time she believed she has had this teaching experience. When she said it was her first year of teaching I thought that Geri ascribed the difficulties she experienced to her being a new teacher but now that she was a more experienced teacher she felt competent supporting the ESL learners. As Geri went on to count the number of students in her class whose first language was other than English it became clear that she had taught multilingual students for at least two years and probably more:

> There are-, there are twenty children. There are probably six whose first language is not English, of those twenty this year, and last year, I had more. I had probably nine of the twenty.

These comments reinforced my interpretation that Geri did not equate multilingual students with ESL learners and in retrospect it would have been interesting to hear Geri's thoughts on what she thought was the difference between an ESL learner and a multilingual learner. Returning to the question of whether Geri thought she had any challenges teaching her multilingual students I commented that it sounded like it was just not an issue for her and she agreed. Geri did not believe she had taught multilingual students and she believed that her ESL students did well in mathematics. So I had the impression that the idea of multilingual students having difficulties in mathematics was not something Geri had considered. This is seen, I think, in the discourse below, where Geri said:

> I have three little girls in particular, that, that do struggle a little bit with language but they were all really, really successful with math and it is funny because I didn't think of

> it, but last week one of the mom's came to talk to me about having her daughter receive more ESL support, which isn't available obviously, and she was saying when she translates the math problems that I give her, she understands them a lot better.

Geri began this quote by stating that the three girls are "really, really successful in math" yet she continued speaking as if she has just come to an understanding about one of the girls that she had not had before; that a word problem would be easier for the student to understand in her first language. This idea about her student and mathematics seemed to evolve:

> I don't give her homework but the mom asks for the work that we have done in class to go over it with her daughter and she seems to-, she feels that she understands it a lot better in her first language and I can-, I can believe that.

Geri's tentative language might have signalled that this was a new idea for her.

Geri became animated when she talked about her mathematics curriculum which consisted of a program she called three part math. Geri told me that it was "a very high in problem solving approach". Geri explained how she initially presented the students with a problem and then showed them how to solve it. The students were then given another problem with enough difference from the first problem that "they have to do something". I understood this to mean that the variables in the mathematical question she showed the students would be changed. This new problem they would have to solve amongst themselves. While continuing to talk about her mathematics program, Geri explained to me how she started off the school year in September when she was getting to know her students:

> I am wandering around the room, and the first few we do with-, in a group so they are solving it together and I hear the dialogue between the children and I am walking around talking to them and things like that so that will help me to know.

Geri used the dialogue she heard between the students as they worked to solve mathematical problems as a way for her to know where the students were in their mathematical understanding. She did not test the students nor did she give them homework. For assessment she listened to the students:

> What I do is I walk around while they are doing it and I try to find examples of people thinking differently and so I never have more than two people present with the same idea.

When Geri talked about looking for students "thinking differently" I understood that she wanted the students to come up with different answers to a question and that these different answers were important to her as a sign that the students were learning. My inference that Geri placed value on what she called thinking differently was reinforced when she added that she restricted presentations by students to only those who had thought differently: if students independently came up with the same answer, only two could share their work with the rest of the class:

> I use different ways to show each other different ways of thinking and also, too, that gives them a little bit of experience talking about their math which I think is really important but it could be difficult for an ESL learner.

Geri emphasized thinking differently and said she herself gave the students examples of thinking differently. With her emphasis on this I realized that this was important to Geri, but I was unclear as to what she meant by it. Geri seemed tentative when she considered that communicating mathematically could be difficult for multilingual students and I got the impression again that Geri might have been considering this as a new thought. Geri continued telling me about her mathematics curriculum: The students had games involving mathematics once a week and direct lessons twice a week. The games were to reinforce memory skills. Geri gave me an example:

> So, here's one from October: *Help the Witches*. And I will read it out so that-, There are ten witches who need to ride on a broom stick. There are only two brooms. Show as many ways as you can to put the witches on the brooms. So you would need to have enough of-, enough language obviously to answer that and the children that I have do.

Geri believed her ESL students had enough vocabulary to make sense of this question.

Geri's efficacy beliefs

Geri gave me the impression of feeling comfortable and efficient teaching her students and running her classroom. I understood this by the way she excitedly discussed her mathematics curriculum and by how she taught the students mathematics:

> And this isn't my entire math program- but I do this once a week at least and then we also have math games once a week. So of those two- then the other three days will be lessons and follow up activities.

By having her mathematics schedule clearly planned for the week it suggested to me that Geri had a clear idea of what would be taught that week and she was prepared for it. At the beginning of the year Geri made plans for getting to know her students and she took time to listen to them to get a sense of what they may or may not know mathematically. I inferred that Geri's desire to have an overview of her students' mathematical knowledge indicated that she felt competent about her ability to understand the students and teach accordingly. Geri also talked enthusiastically about some of the mathematics lessons she had conducted with the students and from this I inferred that had she not felt effective as a teacher she would not have been able to relax enough to joke with them and tease them. She gave me an example of a fun filled lesson and this one was called the Chicken Jump Game which Geri said was hilarious.

In the Chicken Jump Game the students had to jump as far as they could as if they were

chickens. Then they had to measure the distance using bowling pins as the unit of measurement. The students were given large bowling pins with which to measure, while Geri measured her jump with tiny bowling pins. As a result her jump appeared much greater because she could add up the most bowling pins:

> Then I would say "Oh! I am the winner!" because I used the teeny ones and measured mine, and I had the most bowling pins. They were all screaming "That's not fair! That's not fair!" so then they had to come up with the rules that would make the game fair and figure out why.

Geri told me that she got a lot of her ideas for her mathematics curriculum from a workshop she had attended during the summer. She commented that she had not followed through in making use of all the ideas she had taken from the workshop and from this comment I thought that Geri found her work demanding but manageable:

> All of the ones I can think of except for one little boy were really, really high in their math skills.

Geri did not see her ESL students as having any difficulties when they were doing mathematics. She noticed that some struggled with language a little but did not see this as affecting their mathematical comprehension. Her frequent use of the qualifying word *really* led me to think that she either was confident about the students' progress in mathematics or that she wished to assure me that there were no problems with the ESL students' mathematics. I was not sure which.

Summary of Geri's epistemological and efficacy beliefs

Geri appeared to have a clear understanding of what her students should be capable of and this clarity, I thought, was a result of Geri's following the definitions of what a grade 1 and a grade 2 student should know, and what a bilingual student at certain level of proficiency should

be capable of. Despite the alternative program fostering collaborative learning in a multi-age classroom I thought Geri did not see her students as one learning community. I understood that Geri saw her students in two categories that were differentiated by age, with the result that she made an age distinction between the students when teaching and learning. Geri accepted and used the stages of bilingualism because possibly this fitted in with how she saw knowledge and learning – as compartmentalized according to ability and age. I understood Geri to think that learning was a result of the students repeating concepts that have been taught. Geri did not appear to hold beliefs that were consistent with the philosophy of her school.

From her obvious joy in teaching and her excitement in her mathematics program I understood that Geri had a strong sense of efficacy in herself as a teacher. She did not show any signs of uncertainty in the choices she made but rather was proud to show me examples of her ideas, and the students' work. She invited me to come back any time I pleased to observe in her classroom. This showed me that she was satisfied and confident with herself as a teacher and that she did not mind answering any questions I might have.

Melissa

Background

This was Melissa's first year with a class of her own to teach, however the position was a temporary one. Melissa was certified as a teacher in 2008 and had worked in supply teaching since then. She was not sure where she would be teaching next but hoped to find a permanent position at the school where she is now. Melissa's mixed age classroom has 18 students who are either in grade 1 or grade 2. Melissa has three students whom she refers to as ESL learners**.**

Epistemological beliefs

Melissa started off our discussion by describing her students to me:

> I teach a grade one two classroom and I have-, I started the year with 20 students and halfway through I went down to 18. I have 2-, 3 students that are ESL learners. One is in stage one, one is in stage two, and the other is stage four.

Melissa's reference to her classroom as being a grade one two classroom, which she later also referred to as a split class, led me to understand that she saw the class as comprising two distinct groups rather than an integrated multi-age classroom. That she viewed the class as divided into two separate entities was clear when she said that "I did find that challenging - having a one two split". After speaking about the grade levels Melissa immediately went on to distinguish the multilingual students by categories of proficiency in English. There are 4 levels of proficiency in English with her school board. Her students were in stages 1, 2, and 4 of bilingualism. That she used these categories of grade and bilingualism made me think that compartmentalizing students according to age and language proficiency was important to her and to her understanding of how learning occurred and this was especially so since one of the goals of the alternative program was to not use grade level categorizations. This clear demarcation of levels by Melissa was seen again when she described her multilingual students by saying "I have another student from Israel and she is at stage 2, so she is beginning to use-." When she discussed this student from Israel Melissa fitted the student to the category by saying that because the student was at stage 2 a certain proficiency could be expected. This created an impression for me that the particular circumstances of this student did not come into play much in Melissa's considerations of the student. This impression of the student fitting the category and not vice versa was clearer when Melissa said:

> Her writing is coming along well so she is progressing well, and stage four is-, basically if they speak English pretty well they're ready pretty much to be not an ESL learner anymore.

To me, Melissa appeared to mean that this student was ready to move from one category to the next because the young student spoke English fairly well. By saying this Melissa showed that using the stages of bilingualism were critical to her thinking about and understanding of her students. That these distinctions were valuable for Melissa was also apparent when she reconsidered what she said earlier about having three ESL students and corrected it to saying she had two ESL students. This was because as Melissa said "One student doesn't really count." The student whom Melissa discounts as no longer being an ESL learner had been categorized as a stage 4 bilingual student which for Melissa seemed to mean that the designation was not appropriate anymore. This suggested also that there was a clear distinction between the levels of proficiency that Melissa relied on and used in her pedagogical choices. This also suggested to me that how Melissa referred to her multilingual students was a reflection of her epistemological beliefs.

When Melissa described her mathematics curriculum to me I was able to think about how she approached her teaching of mathematics to her students:

> So what it is-, usually I start math lessons on the carpet and we'll-, we'll do an activity or demonstration together, and then we take what we have learned from there and bring it to a table and seat work generally applies to work sheets.

What I thought from the above was that when teaching mathematics Melissa liked to get together with the students on the carpet and present a concept through an activity or a *demonstration*, as she called it. I gathered this would be considered the students' group work. After the presentation

Melissa required the students to work on worksheets doing operations that reinforced what she had just shown in the group presentation:

> We do a lot of math centers so it is a lot of hands on activities using manipulatives. We have real clocks, real coins, things like that which is excellent because then the kids see each other working with them and build on each other.

Melissa seemed to be thinking that it was a good thing when the students observed other students using the artifacts because then they "build on each other". I understood this to mean that Melissa believed the students would learn from each other when they watched each other using the manipulatives, but their learning was individual. Melissa generally taught by grade level but occasionally would put some of the stronger grade 1 students in with the grade 2 students for a lesson:

> But not usually vice versa because I find the grade 2 feel-, they can sense the divide and that makes them feel bad.

When Melissa talked about the "divide" I imagined she was referring to the distinction she made in her class between the grade 1 students and the grade 2 students and that she judged that the students would perceive it better to be in grade 2 than in grade 1. That was why she would not put a grade 2 student in a grade 1 lesson because presumably the student would have felt as if they were being put back or demoted.

In the alternative program where Geri and Melissa taught, text books were not used for fulfilling the Ontario mathematics curriculum so Melissa told me how she created her own lessons drawing on books such as *Hands on Math* and *Jump Math*, and by getting ideas from the Internet and speaking with her colleagues:

> How I always start planning my lessons is I went to the curriculum, took out which expectation I wanted to teach that time, then thought, "How can I make this interesting for the children and something that is going to help them learn it easily and in a fun way."

Melissa used the curriculum to guide her pedagogy and what she thought the students should know. The expectation was what she anticipated the students would learn from the lesson. Melissa voiced her thinking that she wanted the lesson to be interesting, fun, and easy to learn and by using the word "easy" Melissa seemed to imply that there are different types of learning – easy, and hard.

Melissa discussed teaching mathematics to her multilingual students with me. The two students she categorized as ESL learners received support in the morning when they learned new vocabulary from an assistant who gave them words relevant to that day's mathematics lesson. Melissa said that this helped a little because the words would be familiar to the students when they came to the lesson. Melissa spoke a few times about one of her students, a young boy from Russia:

> I would have sight words of the mathematical terms we were going to use, so if we were talking about-, well, if we were talking about *probability*, talking about *unlikely* and *likely* those would be words he would find difficult, so we tried our best to label the words and explain to him what they mean.

Melissa discussed here how she attempted to help the Russian student understand the meaning of the mathematical concept being taught by first giving him vocabulary, but she acknowledged when she said that "we tried our best", that it was probable the student did not understand what she was trying to convey. For Melissa, I assumed, this was the best way to support her student in his mathematical thinking. She observed to me that when the Russian student worked with his

classmates he worked really well:

> He would be able to see what they are doing and then feel confident to try it himself. So he worked really well in small groups. It gave him confidence to try that on his own.

I gathered that Melissa felt it was an issue of confidence that was making the work more challenging for the student, and that the confidence he gained working in a group allowed him to progress. I also inferred that Melissa believed learning was an individual process, and cooperative work helped the individual learning. Melissa had said the student from Russia was good at mathematics and I asked her what made her think so:

> Well, any time we were doing lessons and-, or went to do seat work, he is whipping through his math getting everything done and correct. So where I thought-, and higher than a grade-, he is a grade 1 student, he could do grade 2, no problem, in math. And I am not sure, but I think in Russia that they are a little bit ahead in their math curriculum, so that could be why.

I believed the above quotation supported some earlier inferences I had made about Melissa's epistemological belief that learning was an individual process and hence the students did seat work, and secondly that speed and accuracy in completing a task were indicators of learning taking place**.** Melissa felt that the Russian student could do the next level mathematics of the curriculum and she surmised that the Russian student has been exposed to a curriculum that is "a little bit ahead". On another matter, Melissa talked about how the Russian student was good at reading and writing:

> I know that my little boy, his printing is very good. He can phonetically sound out much anything. His spelling is very good. He can read-, he can read anything, but he has zero comprehension about what he is reading. So that is very interesting.

Melissa sounded surprised that the student could read and write yet have no comprehension about what he was doing. When she said that it was very interesting that this was the case, I understood her to mean that she had not come across such a situation before and was unsure how to make sense of it. Melissa talked about the multilingual student from Russia at length so I remarked that she did not speak about her other multilingual student very much. Melissa answered that she felt she had more challenges with the Russian student because he hardly spoke English but went on to speak about the other multilingual student:

> My girl from Israel – she is not strong in math but she tries very hard and if she doesn't understand something she is very good about asking for clarification. She did benefit from having things-, the words written out for her on a card, so that she could go and look back. Lots of visuals were good especially when we were doing shapes and money.

Melissa appeared to have made an assessment that the student from Israel was not good at mathematics and that Melissa accepted that as a fact. This could mean possibly that for Melissa, students' mathematical understanding depends on innate ability. I further got the impression that Melissa did not think there was much she could do about the matter and left it up to the student to decide whether she needed help or not.

At the close of our discussion I asked Melissa about her training as a teacher, specifically with regards to the teaching of mathematics. She recalled that she had two courses in her training which related to mathematics:

> One of them was awesome and I think it was because basically the professor was very enthusiastic and he taught in a way where we did hands on things, and did it from the perspective of the students.

Melissa implied that the work was enjoyable because they used manipulatives and because she

gained some insights into students' thinking. She also talked about her practicum which she had thoroughly enjoyed:

> Phenomenal! …Some kids just have different ways of seeing things. They still did to need to learn that way but it came easier, I found, after they could see it a different way.

Saying that some students may see things differently suggested that for Melissa there were students who would benefit from being shown a different ways to come to an understanding of what was being taught. When she said that the students still did need to learn "that way" I understood her to mean that she believed there was a correct way to know something but that approaching the problem from a variety of ways helped some students. From Melissa's reflections on her training as a teacher, I inferred that Melissa had experienced more exciting and rewarding ways to teach and learn mathematics but this was not how she taught. In her discussions with me it was clear that Melissa preferred using procedures and algorithms to teach mathematics to the students.

Melissa's efficacy beliefs

Melissa enjoyed her first year of teaching but found the work challenging. To me she came across as a new teacher finding her way. As Melissa described her experiences I began to think that she was unsure of her efficacy as a teacher particularly when she said that "For me it's been a really huge learning experience and I am very grateful to have colleagues to turn to." Melissa explained how she relied on her colleagues for advice when she was unsure about how to make an assessment of one of her multilingual students:

> And assessing was a little bit tricky because I wasn't sure how to measure his progress. Do I measure it based on everybody else's expectations or do I have different ones? So then I went to my colleagues and they helped me with that.

It appeared as if Melissa was unsure of her own abilities and she definitely valued the support of her colleagues' opinions. Also if one of Melissa's epistemological beliefs was that knowledge is discrete and measurable, she realized the challenge of measuring her multilingual students' progress because it would not make sense to use the usual methods of measurement due to the student's challenges of communicating in English.

One of Melissa's biggest concerns was managing the class. She had two support teachers who worked with her in the classroom. They came more frequently at the beginning of the year to help the two multilingual students build up their vocabulary. That Melissa was not confident about managing the class was suggested when she described her mathematics schedule:

> Math is always at the same time every day. Unfortunately, with my schedule this year we don't-, I don't necessarily get to choose it, so mine is at the end of the day. I would prefer to have it in the morning in the middle block because at the end of the day they are tired. They come in from recess and it is harder to learn.

I was unclear what Melissa meant about having to schedule mathematics lessons for the end of the day because I understood that she had the students all day and could conceivably teach them mathematics when it suited her. I was further confused by her teaching mathematics at the end of the school day when she said on a number of occasions that the students were tired then.
When I asked Melissa why she taught mathematics then she replied that the support teachers were only available at certain times of the day and that was usually in the morning. I understood, then, that for Melissa, it was more important to have the support teachers in the classroom where they would help manage students' behaviour than teach mathematics at a time she would prefer and at a time she felt would be better for the students' learning because she was worried about managing the class on her own:

> Their behaviour has been wonderful, so in that respect-, always, the only issues sometimes would be excitement, and you know, moving around on the carpet and things like that, but you know, very respectful children.

She mentioned a few times that the students were tired. She found "the afternoon, just too long, so I had to break that up" in order to give the students a break. She referred to the need for constant awareness and for keeping an eye on the students. I inferred that Melissa's lack of confidence in herself as a teacher came from her concerns about her ability to manage the class.

Summary of Melissa' epistemological and efficacy beliefs

After our discussion together I inferred that Melissa viewed learning taking place whereby knowledge was accumulated through a process of repeated exercises taught by the teacher. Melissa, I believe, saw knowledge as a body of information that could be measured by a variety of categories, such as grade, or language proficiency. I understood that she saw her role as the person responsible for managing that knowledge both for how it was delivered and how it was assessed.

Melissa did not have a strong sense of efficacy in herself as a teacher, and I believed this was for two reasons. First because she viewed herself as a new and inexperienced teacher even though she had worked as a supply teacher for over 5 years and second, I believed Melissa was unsure of her ability to manage the classroom on her own. Both Melissa's efficacy beliefs and her epistemological beliefs appear to have been influenced by the epistemological beliefs of her peers at the school and this is seen when relies heavily on her peers for advice and direction.

Linda

Background

Linda had just completed a year of teaching in France when I met her last summer. In our

discussion she recalled her experiences teaching mathematics with multilingual students in Ontario over the course of 12 years. She is certified as a Montessori teacher. Linda remembered her classrooms as always having a number of multilingual students in them. She remembered having students with Greek, Korean, and Hindi as first languages.

Epistemological beliefs

Linda told me what she thought was the biggest challenge in the classroom for English language learners and that was being able to interact socially with the other students in the class:

> Socially it is hard for them to communicate. But being around children they have a great desire to interact and as long as they are around other students that are understanding and that, they will be included, and they start playing together and working together.

I thought that Linda was saying that the multilingual students would like very much to interact with the others students but that relying solely on language made it difficult for them to communicate and socialize. When Linda used the phrase "as long as they are around other students" who were learning, I surmised that she was confident that learning would take place and that learning did take place in the social interactions of the students as they worked and played together. Linda explained that her classroom was designed in such a way as to create opportunities for the students to work together in teams:

> The other children are a big support. I depend on the other students in the class. They are the teachers in the classroom.

When Linda referred to the other students as teachers it suggested that for Linda there were other ways for the students to learn in addition to being taught by her. I inferred that, for Linda, teaching and learning was the coming together in a mutual understanding with the students on whatever was being taught at the time. While Linda says it was harder for the multilingual

students to communicate when they were still busy building up their proficiency in English, she showed a lot of confidence in the value of the manipulatives the students used in their mathematics:

> They can still do the work on their own without having to say the language. So when they are ready they will-, and feel confident enough- they will start using vocabulary. So they are not hindered by the vocabulary to progress in the mathematics area.

The above example seemed to intimate that Linda believed there are ways of communicating and understanding that did not use spoken or written language and Linda gave an example of this when the students used the manipulatives. Through watching them work, Linda said she was able to know whether they understood what they were doing. I gathered that Linda believed language was not always necessary to the students' communication and to the students' understanding. Linda acknowledged that the manipulatives allowed the students to work on their own which gave the students a degree of autonomy and decentralized control because Linda did not always need to be present in order for the students to learn. Thus there was a shared control of learning.

As Linda reflected about her teaching mathematics to multilingual learners she pondered about what this experience could be like for her students, and explained that "I have never really thought until now, how different it could possibly be in their own language". Linda clearly said that she has not thought about what it would be like for a multilingual student to be learning mathematics in a second or third language. At this point she was thinking specifically about her Korean students and how different their language was from English. The implication of this, I imagined for Linda, was that for the Korean multilingual students there was an additional challenge when they are doing their work in English:

> It is only recently that I have noticed that in different countries they may write a problem slightly differently but it doesn't really matter. They are still getting the process of-, if you are multiplying they are still understanding that units times units will give you units. And they still understand distributive division.

Linda, through her discussions with me, was considering that there may be differences learning mathematics in a language other than one's primary language. Continuing along this line of thinking Linda considered the manipulatives and explained that she believed they transcended language, so that they can be understood in the same way by all the students including the multilingual students:

> You know, it is almost as if it is a neutral language – teaching with the material. I might be speaking English to them, they might not understand what I am saying, but they see the material in front of them and they still can count. Counting is universal.

Linda referred to teaching with the manipulatives as similar to teaching with a neutral language and from this I thought that Linda saw the manipulatives as being free of cultural meaning. Linda also suggested that mathematics was free of context as well when she says that counting was universal:

> Math and geometry – they are so manipulative and it doesn't always require having the vocabulary in English. The beauty of the material is that everything is there.

When Linda said "the beauty of the material is that everything is there" I understood her to mean that for her the manipulatives contained all that was necessary for the students to learn and that the manipulatives would be understood the same way by the students. Linda went on to discuss with me how she tried to understand what her multilingual students had already learned in mathematics. She acknowledged that the lack of a common language was a barrier for both her

understanding and the students' understanding:

> I didn't know they knew as much about math to start with as they did know because of the language barrier. So sometimes, if they don't speak a lot of English you are focusing on the-, you think they don't know certain things because they are not communicating with you. But as long as you are being observant-, you watch them work with the material you start realizing, "Oh! They know how to do that!"

While Linda said earlier that the students were able to learn mathematics without proficiency in English by using the manipulatives she found that she sometimes underestimated what they knew because they were not able to communicate in English. When I asked Linda about what she thought of the students communicating mathematically she outlined her expectations:

> For them to explain to you why it is-, you know showing them how a format-, writing it down to justify it-, is probably the first way of doing-, they are speaking-, it is going to take a lot longer. You know, I think two years is a good bench mark-, a sort of goal-, to think that in two years they might be able to do a little explaining.

Linda had an expectation that the students would communicate mathematically. She acknowledged that the mathematics vocabulary would take a long time for the students to learn. That she had an expectation for the students to communicate their understanding suggested that Linda saw mathematics learning as a process of making meaning between students as they engaged in mathematical activities.

Linda told me about a fractions lesson she was giving to the multilingual students. They were having trouble understanding what was being taught and Linda was having trouble trying to understand why the students were not understanding:

> I still don't know where the language breakdown was there. The person who is doing it is fine now because they were working with some other students and it sort of has come together.

Linda clearly could not identify the reasons why the students had not understood what was being discussed. Linda thought it could be a breakdown in communication. Linda turned to the other students to help the multilingual students which they did. Linda said that the students who were struggling were now fine and that "it sort of has come together" which suggested that the frustrated students have gained their understanding through working together with another student.

Linda's efficacy beliefs

Linda came across as a thoughtful and calm teacher who respects herself and her teaching. She recounted her class sizes over the years:

> Anywhere from one year I had 12 students, another year 20 students. Another one 25, to 28 students. So, a wide range-, and have been doing that now for about 12 years.

When Linda talked about the different numbers of students she has had in her classroom, the different ages that she has taught, the years she has taught and the different locations she has taught, it gave me the impression that she valued all her experiences and possibly saw herself as flexible as a teacher. She carefully considered her experience with multilingual students, and although "multilingual" was not a term she used at all she did not appear confused or uncomfortable with my use of it. This gave me a sense that she was relaxed and open to new ideas. Linda spoke positively about her experience with multilingual students:

> I have always had success with them...I have always experienced them developing well on their own.

Linda spoke about always being successful with the students and finding that they progressed well. Linda identified strongly with the Montessori environment and Montessori pedagogy which seemed to give her confidence when she made her pedagogical choices and I think this led to a strong sense of efficacy. Linda illustrated in the way she talked that she was a reflective teacher and she showed a willingness to entertain new ideas. She told me she had reread her evaluations of her multilingual students in preparation for our meeting and I took this as a desire on her part to be prepared and confident during our discussion. She mentioned that "I was reading over my past evaluations of them and at the end of the year they were pretty integrated" and I believed that here Linda was reaffirming her original assessments that she made of the multilingual students. I took it as a strong sense of efficacy that Linda could speak about the challenges posed by the multilingual students, yet do this in a confident and serene manner. She said that it took her more time working with these students because she had to "figure out...where they are at" but she did not appear to begrudge this time.

Linda did have times of feeling at a loss when teaching her multilingual students:

> In personal frustration, you when I am not getting a point across, like that division question, going "Oh! Somebody help me!" and making sure you go back because you know that language development and vocabulary is very important so you have to be more diligent in helping those.

From this quote I got a sense that Linda suffered frustration when she found it a challenge to communicate with her multilingual students as she would see a lack of communication as a barrier to her students' learning. Her words about making sure that she did her work diligently suggested Linda had a strong commitment to her work with a sense of obligation to her students. Despite any challenges, Linda had an open generous manner towards her multilingual students.

She spoke positively of them and recognized the attributes they brought to the classroom:

> I find the ones that have English as a second language or third language, overall they're more confident, easier to socialize with other people. Just-, they have a lot more gifts. They may be more culturally aware.

She acknowledged that having multilingual students in the classroom had benefitted her and that they had made her more opened minded in her thinking. Linda appeared to act as an advocate for the multilingual students which I did not think she would do if she had low efficacy beliefs in herself. She explained the importance of being close to the multilingual students' families and said "You have to really be on side" with them. She explained the importance of parent education in helping parents understand the challenges their children experienced learning in a second or third language:

> So it is a lot of parent education as well in that respect. Just asking them to have faith that it is going- that they're going to be there when they are ready is not enough.

Summary of Linda's epistemological and efficacy beliefs

I inferred that Linda believed much mathematical learning could be accomplished in the classroom when she taught and the students worked together, talking, socializing, and using the manipulatives. Linda's focus on the social aspect of teaching and learning aligns her beliefs with the philosophy of the Montessori approach where there is a strong focus on the importance of understanding the classroom as a community, especially at the elementary level. Linda also tried to understand the background and history of her multilingual students. She acknowledged that students were taught differently in other countries and classrooms and that this difference might have impacted their learning in her classroom. On the other hand she did not consider the manipulatives as expressions of a cultural background. With regards to mathematics Linda saw

learning as a social cultural activity and yet she viewed numbers as universal and abstract.

Linda came across as having a strong sense of her efficacy as a teacher possibly because of her clear epistemological beliefs. She was confident in her pedagogical choices and in her role, which I understood her to see as observing her students and setting up the environment that would best facilitate their learning. Where ideas were new for Linda, she did not respond with insecurity or anxiety, but rather saw that there was now something different to consider which she could possibly integrate into her teaching. I inferred this ability to entertain new ideas came from a strong sense of confidence in herself.

Discussion

What emerged from my findings was the power of the teachers' established beliefs when considering mathematics and multilingual students. I discuss this below looking at different areas where these established beliefs appear to conflict with other sets of beliefs or philosophies. I found, in addition, that teachers appeared to hold beliefs inconsistently within their own belief system. The findings further revealed that teachers do experience real challenges as they try to support their multilingual students in their learning of mathematics and that these challenges may cause additional stresses for them as they manage competing demands on their time and expertise. Finally, I discuss how the teachers view their multilingual learners and consider what this could mean in terms of their pedagogical practices.

Teachers' established beliefs and their own training

The contrast between teachers' established beliefs and the philosophical approach of their own teacher training is visible when we compare Lindy's and Linda's teaching. Lindy and Linda were trained as Montessorians according to the Association Montessori Internationale (AMI) requirements. Both teachers consider themselves Montessori teachers although at present Lindy

is not teaching in a Montessori school. Each of them referred to the Montessori philosophy when talking about their current teaching practices and they clearly saw it as a methodology they were practicing in their classrooms. Montessori theory is usually considered a constructivist theory similar to Piaget's (Chattin-McNichols, 1992, p. 3). It also views learning as developmental with the student having certain key times, or "sensitive periods" as providing optimal opportunities for learning. The classroom environment is a very important aspect of the Montessori philosophy insofar as it offers the means through which students can construct their learning with freely chosen activities. These activities are motivated by their interests and by lessons they have received from the teacher.

Despite their Montessori training in common Lindy and Linda had very different epistemological beliefs about learning and these were revealed through their descriptions of their teaching practices. I found that Lindy's teaching practices, as discussed with me during our meeting, differed from her training and this led me to consider her a traditional teacher who viewed learning as an accumulation of knowledge over a period of time. I understood Lindy to see knowledge as facts. It was important to Lindy that the students worked quickly and efficiently and from this I inferred that Lindy thought that the quicker the students could do their work the more they would learn. Hence Lindy focused on giving the multilingual students as much English vocabulary as they could manage in order to increase their understanding of what is being asked and therefore to further their learning. Lindy assessed the success of the students' learning by how quickly they learned, as mentioned, and also by their answers. Her goal was for the students to manage to get the correct answers to their work and she attributed the multilingual students' initial slow progress in mathematics to their not knowing their number facts. Lindy's premise was that if the students knew their number facts it would be easier for them to get the

correct answers. Montessori theory sees learning differently: Error, according to Montessori, is not only unavoidable, it is the source of the student's learning (Chattin-McNichols, 1992, p. 94). We see here the stability of Lindy's epistemological beliefs about learning which differ from the Montessori philosophy as discussed by Chattin-McNichols.

Linda, the other Montessori trained teacher, focused on the sociocultural setting of the classroom believing it an important contributor to the students' learning. This is in keeping with Montessori philosophy. Linda pointed out that in the Montessori environment students were encouraged to work together and she saw part of her role as the teacher was to,

> Try to selectively… put that student with somebody who speaks English and they will work together on it and so they are hearing the language more.

But, she noted, that without a common language it was harder for the multilingual students to take part in activities and discussions with the other students.

Whereas Lindy believed the multilingual students were being held back by their lack of vocabulary in English, Linda believed the Montessori manipulatives "override the language barrier" because the multilingual students were able to make use of the manipulatives without using language. Linda acknowledged that vocabulary was important and that she was always giving the students vocabulary but she believed the biggest challenge for the multilingual students was developing bonds with the other students in the classroom. Lindy, as we saw, believed the students needed the vocabulary in order to learn faster and more efficiently, whereas Linda believed the multilingual students needed vocabulary in order to socialize with their classmates and to learn with them. How Lindy and Linda viewed the role of vocabulary in supporting the multilingual students learning reflected their epistemological beliefs.

Teachers' established beliefs and their institution's philosophy

Geri and Melissa embraced their alternative school's philosophy and both spoke proudly of how things were done and were not done at their school, yet their teaching practices reflected a philosophy different from that of the school. The key factor of the alternative school was the multi-age classrooms which allowed for,

> Interaction among children of different ages and at different stages of development [which] provides an environment where students learn from each other. (retrieved from http://www.ocdsb.ca/programs/ele/alt/Pages/default.aspx, para 5)

The multi-age classroom groupings were considered a vehicle for implementing the school's philosophy with regards to learning and a place where students can develop their interests in a non- competitive, more family-like setting free of the pressure of standardized tests and the more traditional forms of assessment. The alternative school's philosophy specifically stated that the primary focus of assessment was the personal growth of the student. The goal of the multi-age classroom, then, would appear to be a deliberate breaking down of the concept of grade levels to allow for a mix of ages and learning. In the findings, Geri and Melissa seemed to share similar beliefs about knowledge and learning. When Geri and Melissa used terms such as "split grades", and a "grade one two class" it was clear that they did not see their classrooms as multi-age. Melissa pointed out that she did not mix the grades when she taught as she believed the older students would feel uncomfortable if they shared a lesson with the younger students, presumably because they would not feel very smart. Geri said that she taught both grades at the same time illustrating that she made a distinction by grade level. Geri had different expectations of the students depending on their grade level, as did Melissa. Geri and Melissa saw their classrooms as divided into two groups of students – the grade 1 students and the grade 2 students. By making

this distinction according to ages they revealed their epistemological beliefs that there was learning that was age appropriate. On another matter, part of their school's philosophy was to allow the students' interests to direct their work and to inform part of the teachers' lesson planning, however both Melissa and Geri followed the curriculum to create lessons and did not appear to allow the students much autonomy in deciding what they would like to do.

Teachers' own belief systems – inconsistent beliefs

Cross (2009) points out that beliefs can be held inconsistently by teachers and I found this to be the case as well. The Montessori teacher, Linda, in her teaching focused on creating opportunities for the students to work together in a social setting as she believed this was when learning took place, or at the least, that a social setting supported learning. Her energies were directed towards trying to understand what her Korean students knew mathematically since she wished to make her teaching relevant to their current mathematical understanding. On the other hand she held a belief that mathematics was a body of knowledge free of cultural influence. She said that:

> When they first came in…just trying to see what they know. I gave a lot of different varieties of stuff, like math on paper, numbers- what I thought would be universal.

By this statement Linda seemed to imply that the sociocultural aspect of the students' mathematical past did not have a bearing on their current learning because numbers were universal.

In Geri's classroom when the students were working on their mathematics problems she walked around looking for examples of students "thinking differently" and then allowed some students to present their work to the class. She pointed out, however, that she allowed no more than two students with the same idea to present their work. Geri spoke about the value of

allowing the students to see alternate ways to solve problems, but by not allowing more than two students who have come up with the same way of solving the problem to present their work, it implied to me that Geri saw the presentations by the students as more of a check list of requirements for the alternative program than the outcome of a shared learning activity. Geri did not allow for the discussion that could arise if all the students, working in their separate groups, or independently, had all come up with the same way of solving the problem.

Yasmine placed a lot of importance on the students' understanding. She spent time reflecting on lessons she had given, how the students had responded to them and what she could have improved on for the next lesson. This approach to teaching suggested, as mentioned in the findings, that Yasmine believed learning occurred when the students worked together discussing their thoughts and ideas. Yasmine however, directed the students quite often even when the students could conceivably have made decisions for themselves:

> We decided to make a house and then each team get a room. Like one to make a kitchen, one to make a bathroom, or bedrooms.

Although Yasmine talked about "we" meaning the students and her, she appeared to make most of the decisions. Yasmine called the groups "teams" which suggested an edge of competition. Later, when she was having difficulties getting the multilingual students to understand the concept of fractions she repeated to me what she had said to the students:

> If I tell you, "Find a common denominator", can you? … What is it that you can't? I just told you how.

When she said she told them how to find the common denominator and now is confused by their lack of comprehension, it sounded like Yasmine believed the students should be able to understand this concept because she had explained it to them and that should have been enough

for them. This is in contrast to her other expression of learning as making use of other resources available, “not just through words”.

Drawing distinctions – What’s in a name?

ESL students or multilingual students

My findings indicate that how the teachers referred to their multilingual students reflected their epistemological beliefs. I saw this with my use of the term *multilingual* when referring to students whose first language was other than English. Despite explaining my use of this term in my research, it did not resonate with some of the teachers as a term they would use when talking about their students. So much so that Geri said that she had not had any experience teaching multilingual students. Lindy said the same thing. The teachers whom I would view as having a more traditional approach to teaching, that is, Lindy, Geri, and Melissa referred to their multilingual students as ESL learners. For Geri and Melissa this was in keeping with their school board’s identifying and naming learners according to stages of bilingualism.

When multilingual students are considered ESL learners there is a different emphasis than if they are viewed as multilingual learners. An ESL designation was considered a temporary one and this was seen when Melissa pointed out when talking about some of her multilingual students that “They’re ready pretty much not to be an ESL learner anymore”. The designation of multilingual learner is a more permanent one which does not disappear as the student learns English.

The above teachers appeared to see their multilingual students’ lack of proficiency in English as a problem to be solved. This was seen in a few examples. Lindy in her pedagogical practice does not use her own proficiency in French to communicate with her French speaking students. She explicitly told me that she did not allow the students to speak to her in French. This

is in stark contrast with Yasmine who allowed her French speaking students to speak French together even though she did not speak the language and did not, therefore, understand what they were saying. For the French speaking students to use French, then, is a logical choice on Yasmine's part given her epistemological beliefs that learning is a social activity where students use the resources available, which include their first language. For Lindy with the more traditional view of learning, if the students, or she, used French to communicate it would be at the expense of the students' acquisition of English, and Lindy's goal was for the students' to learn English and to do their mathematics using English. Therefore her choice of not letting the students use French, and not speaking to them in French, was a logical one.

Yasmine and Linda whose epistemological beliefs I understand as more in keeping with a sociocultural theory of learning did not use either the term multilingual or ESL. Yasmine referred to "two French students" and the rest of the time in her discussion with me referred to "them". Linda, in her discussion with me, said:

> I have had a number of students that have had English as their second language, their first language being either Korean, or Indian.

Linda referred to her students by their first language, such as "the Greek girl" or "the Koreans". Yasmine's and Linda's naming of their multilingual students was a little more nuanced and contextual than that of the other teachers which may be a result of their epistemological beliefs.

That the teachers viewed their multilingual students as ESL learners matters because it argues that in order to support their students in mathematics the teachers' focus need only be vocabulary. As we have learned from the review on the literature, especially the work done by Barwell (2009) and Moschkovich (2002), giving vocabulary is not considered enough to support the students learning of mathematics.

Cultural diversity or multilingual students

Reeves (2006) found in her study that teachers believed it was a good idea and positive thing to have English language students in the classroom but when it came to having them in their own classrooms, the teachers were not as positive. In keeping with Reeves (2006) my findings showed that most of the teachers felt that the multilingual students brought something extra to the atmosphere of the classroom. Geri told me how it was enriching for all the students to have students from diverse backgrounds in the classroom:

> I have seen a lot of pride at this age in having their own country and their own traditions and because part of our curriculum for social studies is traditions and celebrations. That's been really nice.

Linda, reflecting on her Korean students, said:

> I think it is definitely an advantage to a class to have people that have different languages. I think you want to be global. You want to be thoughtful of other people, instead of closed minded.

Melissa also talked about the advantages of having a culturally diverse classroom. Referring to her Israeli student she said:

> My little girl from Israel-, she would often share if there were special celebrations or things like that. Her parents would come in and share those with the class and she enjoyed talking about that. It gave her a chance to talk about something she knew about and there would different special treats that are from their country or things like that. So the kids enjoyed that part.

When considering what the teachers thought about the positive aspects of having multilingual students in the classroom it appeared that they valued the obvious, more visible

cultural expressions that the students and families brought, such as the various celebrations and accompanying treats, rather than considering the language of the multilingual students as a resource for those students, and as contributing to the richness of the classroom experience for all the students.

Reeves (2006) also found that the teachers, while liking the idea in general about having multilingual students in their class, believed that the students should be fairly proficient in English first before they were admitted to the classroom. They thought it would take two years to attain proficiency (p. 136). It was not as clear from my findings that the teachers believed the students should be proficient in English before coming to the classroom, although Linda also believed proficiency would take two years. It may be the case that when viewing multilingual students in a positive light, the teachers are considering their cultural diversity and not their proficiency in their first language.

Teaching multilingual students - A mixed blessing

Work and efficacy

Despite the teachers' stated advantages of having multilingual students in the classroom their presence also created more work for them. Yasmine rewrote all her mathematics lesson for her multilingual students and expressed discomfort and frustration when her students did not understand what she was trying to teach in the fractions lesson. Linda also talked about the increased amount of work the multilingual students generated:

> Because I have to try and figure out what-, figure out more easily where they are at. It takes more time. Yes. It takes a lot more time to find out where they are than with an English speaker.

The extra work was a challenge for Melissa as was the vigilance she believed necessary to keep

on top of the multilingual students' progress:

> You do have to prepare more. When you have ESL children you have specific things that you need to have ready for them to be able to work with the other children, so that adds a little bit more to your work load, and, you are always kind of keeping an eye on them a little bit. You are always aware of what they're doing to make sure they are not wandering and doing other things.

As mentioned above, Yasmine experienced frustration when attempting to teach her multilingual students the concept of a common denominator and it is possible that Yasmine's frustration and her inability to speak French created feelings of inefficiency for her as she believed it was her responsibility to be able to communicate effectively with her students. Some of the other teachers felt frustrated with their own efforts at communicating with and understanding their multilingual students. Linda talked about how it was a real challenge having multilingual students in the classroom because of difficulties with communication and commented that "You don't want to have too many people in the classroom that don't understand you." She expressed frustration when she was teaching her multilingual students fractions. She said, "Oh! Somebody help me!" Melissa had the same experience of frustration and suffered considerable anxiety about the progress of her multilingual students. Talking about a Russian student she has Melissa said:

> He spoke absolutely nothing and it was-, it was very hard because I wasn't sure if he understood anything I was saying.

She went on to say that:

> It's been a really huge learning experience and I am very grateful to have colleagues to turn to. I spent a lot of time working closely with them.

Melissa explained that assessment was difficult for her:

> And assessing was a little bit tricky because I wasn't sure how to measure his progress. Do I measure it based on everybody else's expectations or do I have different ones? So then I went to my colleagues and they helped me with that.

When reviewing the teachers' comments about the reality of having multilingual students in their classrooms, it would seem probable that their sense of efficacy as teachers could be affected negatively.

An unexpressed tension

While talking with the teachers and while reviewing my findings it became apparent to me that there was a tension in what they were saying to me about teaching mathematics to their multilingual students. Lindy, Linda, and Geri had each expressed to me that they had no problems teaching their ESL students mathematics. Geri said that her three ESL students "Were all really, really successful with math". Linda said when referring to her students who had English as their second language that "I have always had success with them" and finally, Lindy said "It was a good year". In the findings, however, it is revealed that the teachers do have challenges teaching multilingual students. The teachers' saying and believing that there were no challenges with their teaching of mathematics to multilingual students created a tension that perhaps went unrecognized in the classroom and in the teaching community.

Conclusion and possible contributions of this study

The power of established beliefs is widely reported in the literature (Brownlee, et al. 2002; Nespor, 1987; Pajares, 1992; Schommer, 1990) and is definitely clear in my findings. The fact that the power of the teachers' established beliefs can absolutely override both their own training

as teachers and their institutions' philosophies suggests that when educators or administrators consider bringing in any changes such as implementing new programs of professional development for teachers close attention needs to be paid to the role of those established beliefs. We should not simply try to bring forward initiatives while assuming the program would be readily embraced by all. If we do not address beliefs and help bring them to the consciousness of teachers, we do not have a way of building a conversation between one way of thinking and another way of thinking. The teachers' beliefs will very probably remain intact and any initiatives will not work.

This research shows the importance of terminology: The words the teachers used to name students reflected their beliefs. We see teachers using the term ESL learner and referring to grade levels and teaching accordingly. Using terms such as ESL learners creates a very narrow lens through which to view multilingual students and endorses the view that a lack of English is something that needs to be corrected. In addition, when viewing students through the lens of cultural diversity the focus is also limited and appears to be on what multilingual students bring to everyone else in the classroom, rather than what they bring as a resource to their own learning as well. We need to focus on what are we calling the communities we deal with. We do not wish to imply a deficit model which the term ESL does, while the term multilingual, which is more commonly found in research contexts, recognizes the richness of diversity.

In spite of their acknowledgement that the children bring something extra to the classroom the teachers' repeated concerns about some of the challenges they encounter really highlight areas of difficulties for them. It is a challenge for teachers to teach multilingual students because they appear unsure or unaware of how best to support them in some instances, and in other instances feel that vocabulary is enough support. It does take more time and effort to teach

multilingual students and teachers have to manage this additional work. These challenges, which may impact negatively on teachers' sense of efficacy, suggest the need for ongoing and extensive professional development. There is almost no research on teachers' beliefs and mathematics with multilingual students. My research contributes to the small body of literature to create further discussion on this relevant topic.

Implications for further research

As I understand beliefs as having been formed through the lived experiences of the teachers' lives, an investigation into the history of each teacher's formation experiences which have lead to their current beliefs would make sense in any future research. I expect this history would offer opportunities for me as a researcher to gain a picture of the specific incidents in a teacher's life that have helped contribute to present beliefs. I would anticipate collecting this history through a series of meetings with each teacher. This master's study has also made clear the necessity of making observations of the teachers' practices in their classrooms. While the interviews conducted in this research have given me some material to work with in understanding teachers' experiences, I do not think the material is adequate for the depth of understanding I would hope to gain through observing and collecting data on teachers practicing in their classrooms. Classrooms observations would allow me to witness teachers enacting their beliefs and would also allow me to investigate the coherence of those beliefs in action with the beliefs I would have inferred during the interviews. Furthermore, class observations in conjunction with interviews would allow me to enhance the credibility of my research through the triangulation of data sources (Krefting, 1991). Finally, as well as the interviews and classroom observations, I would conduct a focus group meeting with the research participants. I anticipate that teachers may well be interested in taking part in a focus group discussion with all

the research participants who have in common the experience of teaching mathematics to multilingual students.

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Appendix A: Recruitment text

My name is Alison Goss and I am a Master's of Arts student at the Faculty of Education at the University of Ottawa. My supervisor is Dr. Barbara Graves. I am conducting a study to explore how teachers view the teaching and learning of mathematics with multilingual students.

Our elementary school classrooms in Ontario are becoming increasingly multicultural with children coming from language groups other than the language of instruction. I am interested in meeting with elementary school teachers who teach mathematics in classrooms with children from these linguistically diverse backgrounds in order to discuss their views on teaching mathematics in this context.

I am looking for teachers for an in depth one on one interview about their experiences. The time allowed for the interview is 45 minutes to an hour. If you are an elementary teacher who teaches mathematics with multilingual children in your classroom and are interested in participating in an interview about your experiences please contact me.

Ms. Alison Goss
Master's of Arts student
Faculty of Education
University of Ottawa

Appendix B: Interview Guide

The initial in depth question to the teachers will be:

What are your experiences with teaching and learning of mathematics to multilingual students?

I am anticipating that the teachers will have a great deal to say in answer to this question when asked with genuine interest about their practice I anticipate that this question will generate a rich description of the teachers' pedagogical practices in the classroom, the teachers' interactions with the students, the teachers' observations of the multilingual students, their recognition of their learning and their difficulties, thoughts about solutions, strategies, steps taken, and perceived outcomes.

Topics I am hoping will be touched upon, and that have come out of the literature review are:

The nature of mathematical knowledge
The nature of language
The nature of learning and teaching
Teachers' understanding of their own teaching
Teachers' understanding of multilingual students