Seasonal Variation in a Prey-Predator-Predator Model

Noah Bolohan, under the supervision of Dr. Frithjof Lutscher

Faculty of Sciences, University of Ottawa

Motivation

The goal of this project is to study how climate change may affect oscillating populations.

Predator-prey systems often exhibit remarkably stable oscillations over long time spans. These species are also affected by environmental variables such as temperature, snow cover and others. One prediction of global change is that summers will be longer and winters will be shorter, and that show cover could decline. This project aims to find how global change could affect the regular oscillations of predator prey systems.

Tyson & Lutscher (2016) developed and analyzed a 2-species model that takes into account the snowshoe hare and the great horned owl in the Canadian Boreal forest. In this work, we extend the model to include the snowshoe hares most important predator, the Canadian lynx.

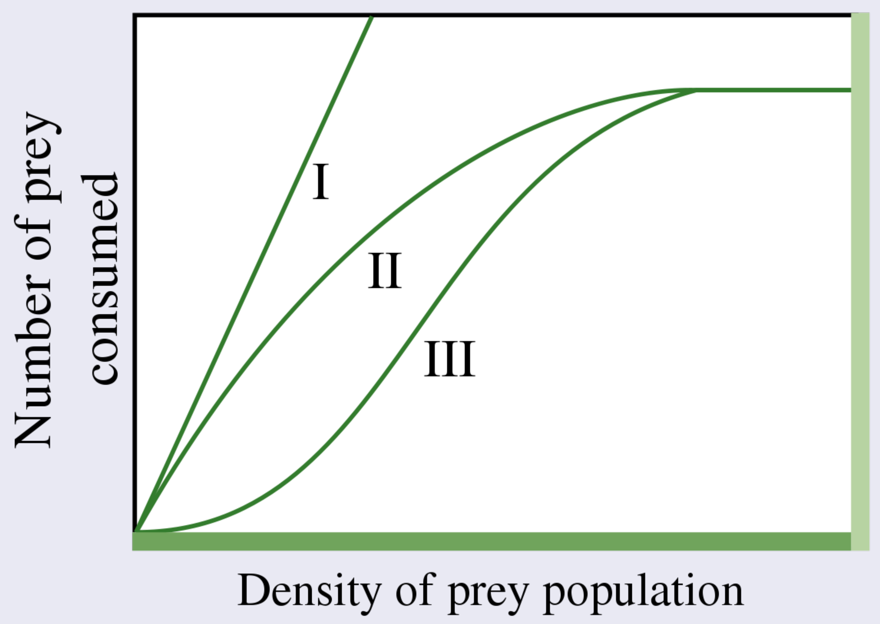

Figure 1: Functional responses (Wikipedia, Professor Moorcroft)

In ecology, the functional response of a predator describes the rate at which it consumes its prey. Figure 1 illustrates the consumption patterns of each functional response.

- Type I response indicates that consumption is a linear function of prey density, with no upper bound to consumption.
- Type II response indicates that the rate on consumption decelerates, as a result of the predators limited capacity to process food. These predators are classified as specialists.
- Type III response indicates high consumption when prey density is high but at low prey densities, the predator will search for other sources of food. These predators are classified as generalists.

The lynx is a specialist predator of the hare and exhibits a type II functional response throughout the entire year. On the other hand, the owl undergoes a switch in predation pattern, from a generalist in the summer to a specialist in the winter (Krebs, C. *et al*, 2001). This project analyzes the dynamics of coexistence of these 3 species as we vary the length of summer in a year.

Model

The seasonal model includes two separate sets of equations, one for summer and one for winter. We denote the hare density by x, the lynx density by y, and the owl density by z. The non-dimensionalized sets of summer and winter equations are

Summer

$$\frac{dx}{dt} = x(1-x) - \frac{xy}{b+x} - \frac{x^2 z}{B+x^2}$$
$$\frac{dy}{dt} = \frac{gxy}{b+x} - my$$
$$\frac{dz}{dt} = \frac{hx^2 z}{B+x^2} + \frac{sz}{A+z} - uz$$

Winter

$$\frac{dx}{dt} = -\frac{xy}{b+x} - \frac{cxz}{D+x}$$
$$\frac{dy}{dt} = \frac{gxy}{b+x} - my$$
$$\frac{dz}{dt} = \frac{fxz}{D+x} - uz$$

There are two main differences in the sets of equations:

- there is no hare growth in the winter.
- the functional response of the owl switches from type III to type II.

This model is difficult to analyze due to a discontinuous switch of seasonal equations. For this reason, we use a technique called *averaging* to study the dynamic behavior. If we let Ts denote the length of summer in a year, we can compute the weighted average of the two previous sets of equations to create our final model.

$$\frac{dx}{dt} = Ts\left(x(1-x) - \frac{xy}{b+x} - \frac{x^2 z}{B+x^2}\right) + (1-Ts)\left(-\frac{xy}{b+x} - \frac{cxz}{D+x}\right)$$
$$\frac{dy}{dt} = \frac{gxy}{b+x} - my$$
$$\frac{dz}{dt} = Ts\left(\frac{hx^2 z}{B+x^2} + \frac{sz}{A+z} - uz\right) + (1-Ts)\left(\frac{fxz}{D+x} - uz\right)$$

Results

We simulated the averaged model using Matlab, and observed that solutions could oscillate or approach steady states. We found parameter values from the literature and adjusted them so that we would get the coexistence of all 3 species for intermediate values of Ts. We then fixed this set of parameters and varied the value of Ts. We present the results for four different values of Ts below.

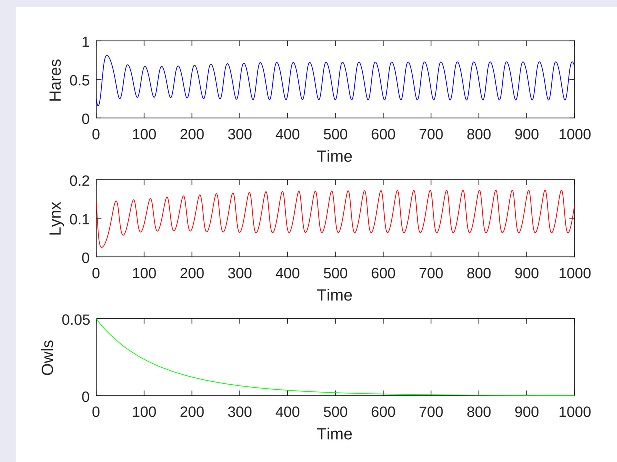

Figure 2: Ts=0.45

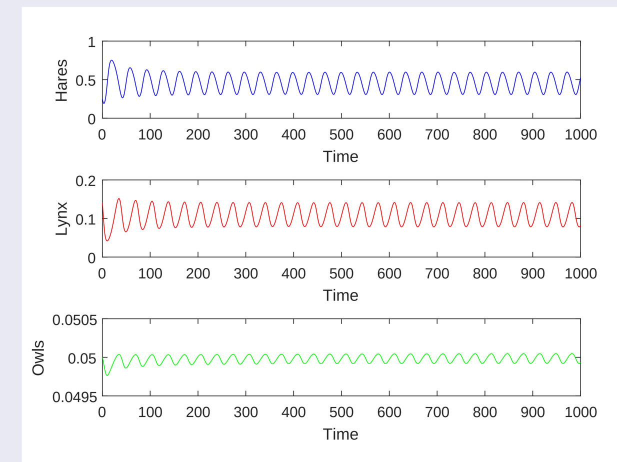

Figure 4: Ts=0.51122

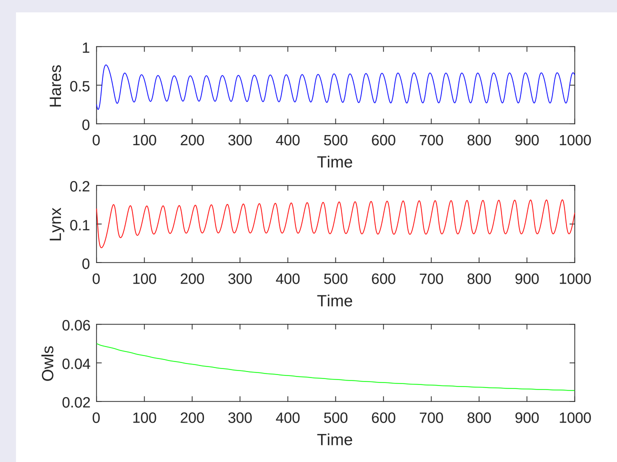

Figure 3: Ts=0.5

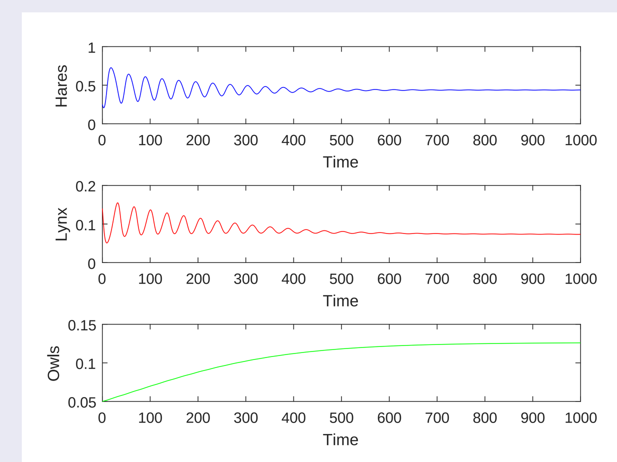

Figure 5: Ts=0.54

When summer is short, the owls die out and the hares oscillate with the lynx. As the length of summer increases, owls are more likely to persist and when there are enough owls, the hare and lynx cycles become steady population states.

Parameters

The previous results use the following parameters:

$b = 0.0625$
$B = 0.078$
$g = 1.52$
$m = 1.33$
$h = 0.003$
$s = 0.18$

$A = 1.3$
$u = 0.07$
$D = 0.0625$
$c = 0.4$
$f = 0.00175$

Moreover, we use the initial conditions $(x_0, y_0, z_0) = (0.24, 0.14, 0.05)$. These values are taken from various sources in the literature, namely Tyson & Lutscher (2016) as well as Vitense *et al* (2016).

Figure 6: Snowshoe Hare (Wikipedia, D. Robertson)

Figure 7: Great Horned Owl (Wikipedia, brendan.lally)

Figure 8: Canadian Lynx (Wikipedia, Keith Williams)

Discussion

Tyson & Lutscher (2016) showed how changes in season length affect the dynamics of one prey (hares) and one predator (owls) whose predation behaviour switched between seasons. They found a number of scenarios and abrupt transitions between them that demonstrate that climate change can stabilize oscillating populations or destabilize coexisting species, or lead to species extinction.

In this work, we extended their model and included the Canadian lynx as a third species. The lynx is the most important predator of the hare and it behaves as a specialist year round. As a first step, we considered the scenario where hares and lynx coexist via a stable limit cycle for intermediate summer length, and we asked how owls may enter the system, depending on summer season length.

When summer is short (Ts <0.45), the owls will eventually die out whereas hares and lynx coexist in cycles (Fig. 4). Owl feeding rate in the winter is small and gain from alternate prey in the summer is insufficient since the summer is short.

When the summer season is long enough, the owls can persist in the system at low density while hares and lynx continue to exhibit stable oscillations. In the long run, the owl density appears constant, but exhibits very small amplitude variation (Fig 5).

As summer season length increases, the oscillations become clearly visible in all three species. The average owl density increases whereas the amplitude of the cycles of hares and lynx decreases (Fig 6).

Eventually, the cycles disappear and the three species coexist at a stable equilibrium (Fig 7). At this point, the owls gain substantial amounts of food from their alternate prey in the summer and they incur substantial predation on the hares in the winter. The low hare density then only allows a low lynx density to persist.

When summers get even longer (Ts >0.65), the owl population reaches such high levels on the alternate prey that they drive the hare population to extinction through predation in the winter. As a consequence, the specialist predator also dies out (plot not shown).

Our work shows that the effects of climate change may substantially alter species dynamics and community composition. In particular, switching of predation behaviour may lead to unexpected results where an important prey species can die out and take with it a specialist predator. But even less drastic effects, such as the loss of population oscillations, could have serious ecological consequences since oscillations are typically associated with higher species richness than equilibrium dynamics.

In the future, we aim to study more scenarios that could arise, based on the results by Tyson and Lutscher (2016), namely bistability and global bifurcations. Secondly, we also aim to compare simulations of the seasonal model with those of the seasonally averaged model.

References

1. Tyson, R. and Lutscher, F. 2016. Seasonally Varying Predation Behavior and Climate Shift Are Predicted to Affect Predator-Prey Cycles. The American Naturalist 5(188).

2. Vitense, K. *et al.* 2016. Theoretical impact of habitat loss and generalist predation on predator-prey cycles. Ecological Modelling 327:85-94.

3. Krebs, C. *et al.* Ecosystem Dynamics of the Boreal Forest: The Kluane Project, Volume 1. Oxford University Press, 2001.

Acknowledgements

I would like to thank the UROP and the University of Ottawa for the opportunity to undertake this project. I would also like to thank Dr. Frithjof Lutscher for his continuous support and guidance.

E-mail: nbolo094@uottawa.ca

uOttawa