COMBINED AXIAL LOAD - BENDING MOMENT BEHAVIOUR OF REINFORCED CONCRETE SHORT COLUMNS

by

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Submitted in partial fulfillment
of the requirements for the degree of
Master of Applied Science

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June, 1972

ABSTRACT

Extensive and sometimes comprehensive research on the ultimate strength of reinforced concrete columns has been carried out almost since the first use of this structural material. This thesis reviews briefly some of the more significant experimental and analytical work in this field.

An experimental investigation has been carried out on 6" by 6" square section concrete columns, with symmetrical reinforcement. Loads were applied at eccentricities e/t equal to 0.00±0.01; 0.10±0.01; 0.30±0.01; 0.50±0.01; 0.75±0.01 and ∞. A total of 100 column specimens were tested, divided into 4 groups, each having 25 specimens. The percentage of reinforcement in each group was: 1.24; 2.27; 5.14 and 7.14 respectively. The variations of concrete strength considered in the project were 4,000 and 6,000 psi.

The scope of this thesis was purposely divided into three groups depending on the predominant type of stress to which the columns were subjected, namely: (1) pure flexure; (2) axial compression; (3) combined axial compression and flexure.

From the comparison of the results, it was found that the experimentally determined mean strength of the column specimens was in good agreement with a mean value of 1.04 and a standard deviation of 0.09 in comparison with the loads calculated from the formulae in the American Concrete Institute Building Code Requirement ACI 318-63 and a mean value of 0.99 and a standard deviation of 0.02 for the strain gradient method.

A computer program using the strain gradient method is presented. The input requires only the geometry of the member, stress-strain characteristics of the concrete, the longitudinal reinforcement and the magnitude and location of the applied loading. Final results of the calculated maximum axial loads and maximum external moments are obtained and printed out.

This thesis analysed and discussed the test results, reviewed the ultimate strength theories, thus gave an overall picture of the developement and scope of the research on ultimate strength of reinforced concrete short columns.

ACKNOWLEDGEMENTS

The author wishes to express his gratitude to Dr. N. J. Gardner whose continued aid and guidance accounted for the success of this project.

The author is grateful to the Faculty Technical Staff who helped in the preparation and testing.

Thanks are due to the Structure Laboratory of the National Research Council and the Laboratory personnel.

Thanks to those friends in the Department, who gave help in the preparation, casting and testing work.

For the careful typing of this thesis, the author extends his sincere appreciation to his wife.

This investigation was carried out under the finanicial assistance of the National Research Council under Grant No. 5645.

TABLE OF CONTENTS

PAGE NO.

PAGE NO.

LIST OF TABLES

LIST OF FIGURES

NOTATION

A_s = area of tension steel, sq.in.

A_s' = area of compression steel, sq.in.

a = depth of equivalent rectangular stress block = $K_1 c$, in.

a_b = depth of equivalent rectangular stress block = $K_1 c_b$, in.

b = width of rectangular member, in.

c = distance from extreme compression fiber to neutral axis at ultimate strength, in.

C_b = distance from extreme compression fibre to neutral axis for balanced condition = $d(87{,}000)/(87{,}000+f_y)$

d = distance from extreme compression fiber to centroid of tension reinforcement

d' = distance from extreme compression fibre to centroid of compression reinforcement

d" = distance from plastic centroid to centroid of tension reinforcement

E_s = modulus of elasticity of reinforcement

E_c = modulus of elasticity of concrete

e' = eccentricity of load with respect to centroid of tension reinforcement

e = eccentricity of load with respect to plastic centroid of section

e_b = eccentricity of load P_b measured from plastic centroid of section

f_c = concrete strength

f_c' = compression strength of 6" by 12" concrete cylinders, psi.

f_c'' = flexural compressive strength of concrete member

f_y = yield strength of reinforcement, psi.

f_{s1} = compressive steel stress, psi.

f_{s2} = tensile steel stress, psi.

K_1 = the ratio of average stress to maximum stress

Kd = distance from extreme compression fibre to neutral axis at ultimate strength

L = span length, ft.

M_u' = ultimate moment, kip-in.

M_u^T = ultimate moment of column under pure bending, kip-in.

m = $\frac{f_y}{0.85f_c'}$

P_u' = ultimate load, kips.

P_u^T = ultimate lateral load of column under pure bending, kips.

p = $\frac{A_s}{bd}$

p' = $\frac{A_s'}{bd}$

P_b = axial load capacity at simultaneous crushing of concrete and yielding of tension steel, kips. (balanced conditions)

P_o = ultimate load of concentrically loaded column, kips.

q = $\frac{A_s}{bd} \cdot \frac{f_y}{f_c'}$

t	=	over-all depth of a rectangular section in the direction of bending
V	=	coefficient of variation
δ	=	deflection
ϵ	=	strain
$\epsilon_{s1}, \epsilon_{s2}$	=	steel strains
ϵ_c	=	compressive strain in concrete
ϵ_o	=	compressive strain in concrete corresponding to maximum stress (Fig. 4.1.a)
ϵ_s	=	steel strain
ϵ_x	=	strain in the X-direction
ϵ_u	=	useful limit of compressive strain in concrete (Fig. 4.1.a)
ρ	=	radius of curvature
ϕ	=	angle change per unit length, curvature
σ	=	standard deviation

CHAPTER I

INTRODUCTION

The basic function of columns in most normal frameworks is to carry the vertical load down to some lower level. In ordinary construction, there are many cases in which members are subjected to a combination of bending moments and axial loads. Generally, in reinforced concrete members, the direct axial load in such a combination is a compressive force. Lateral earth pressures which act upon foundation walls, columns connected to beams eccentrically, together with the effect of the rigidity of the connections, will cause bending moments to be applied to the columns. Wind loads acting upon a building also force the columns to bend sideways. In other cases, internal columns in the framework of a symmetrical structure, are subjected to out of balance bending moments, which also cause the column to bend. All of these, are ordinary causes of combined compressive force and flexure in the members that are affected by the above conditions.

Generally, problems involving compression and bending come into one of the two classes: the first includes those members under compression failure, with the crushing of concrete before yielding of the tension steel; the second covers those under tension failure, with the tension steel yield first.

In the past forty years, the strength of reinforced concrete columns, their behaviour under various conditions of loading, and proper procedures for their design have been of intense interest among Structural and Civil Engineers.

In 1638, Galilei's work regarding flexure of beams, was exclusively devoted to ultimate strength. Hooke's Law was formulated 40 years later, and Navier developed the fundamental theorems of the theory of elasticity in 1821. By 1930, the phenomena of failure and ultimate strength of columns in axial compression were fairly well covered, and theories for ultimate strength had also been developed. Tests and theories for eccentrically loaded columns were developed sometime later.

All present knowledge regarding ultimate strength of reinforced concrete columns is based on the results of tests. Consequently, ultimate strength methods give an indication of being both accurate and useful, while the straight-line method does not produce either consistent or satisfactory results.

The investigation described in this thesis was carried out to determine the effects of concrete quality, percentage and yield strength of reinforcement, and eccentricity on the actual failure load of the reinforced concrete tied columns and to the predicted ultimate strength, while the length and the nominal dimensions of the cross-section remained unchanged. The validity of the ACI Design Formulae and the strain gradient method were also checked.

The experimental results were analysed and compared with the theoretical results obtained from the equations of the American Concrete Institute Building Code, and the strain gradient method.

CHAPTER II

REVIEW OF LITERATURE

The subject of 'Mechanics of Materials' has been a science of empirical character from the earliest developments due to the nature of the problem involved. This is particularly the case with concrete and reinforced concrete. The first attempts to establish a mathematical design procedure for reinforced concrete members were characterised by loading to failure a number of specimens and trying to establish an equation which correlated with the test results. In current terminology, these early theories are referred to as inelastic or ultimate theories.

In 1638, the work done by Galilei, attempted to describe the behaviour of beams in flexure following ultimate load tests on masonry cantilever beams.

In 1821, Navier's theory of bending was developed, based on the earlier's work of Bernoulli and Hooke's Law.

In 1894, the Straight-Line theory, was presented by Coignet and tédesco (3)*, to explain the behavior of concrete beams and columns. The theory was mathematically simple and was accurate enough for design purpose at that time. This method assumed a linear distribution of strain and that the stresses of both the concrete and steel

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* Numerals in parentheses refer to corresponding items in the List of References.

were directly proportional to strain (the concrete was assumed to have no tension capability). No account was taken of the plastic behavior of the material nor was there any specific determination of the masimum load that could be carried.

From 1920 to 1930, another important development was taking place in the philosophy of column design. Before that time, bending stresses were generally neglected in the design of concrete building columns, and such stresses together with other effects, were assumed to be allowed for in an overall factor of safety. The Slope-Deflection and the Moment-Distribution Methods developed to analyse monolithic structures, enabled the bending moments as well as the axial loads on columns in frames to be determined easily. Consequently, current design codes required columns to be designed for both the axial load and bending moment obtained from a rational analysis.

Members subjected to axial compression have been represented in tests by axially-loaded columns. Although this type of load may never encountered in actual construction, it represents a limiting case for the columns under combined axial load and bending.

About 1900, Considére (4) proposed the following equation for the allowable load of concentrically loaded reinforced concrete columns, using appropriate safety factors to determine the allowable unit stresses for concrete and steel. Strain compatability was not considered.

$$P = A_c f_c + A_{st} f_s$$

where - f_c is allowable stress in extreme fiber of concrete, psi.

- f_s is allowable tensile stress of steel, psi.

- A_c and A_{st} are areas of concrete and steel respectively.

The Straight-Line Theory (Standard Theory) was established later, by using the transformed area formula, and the strain compatability was taken into account.

$$P = A_c f_c \left[1 + (n - 1)\, p_{st} \right]$$

where - n is the modular ratio (E_s/E_c)

- P_{st} is the ratio of total longitudinal reinforcement area to the gross area of concrete.

In 1921, McMillan (5) studied column test data, and found that building columns under load may develop steel stresses, due to plastic action, considerably higher than those predicted by the Straight-Line theory. This important study led to a number of ACI column research investigations, carried out by Slater & Lyse (66), Slater (7), and Richart (8), and others in the 1930's. From their work, rational equations for the ultimate load of concentrically loaded reinforced concrete columns were developed. The ultimate load for tied columns is simply the sum of the strengths of the longitudinal reinforcement and the concrete. The ultimate strength of tied columns were shown to be as follows:

$$P = 0.85A_c f_c' + A_{st} f_y$$

where - f_c' is the ultimate concrete cylinder strength

- f_y is the yield point stress of longitudinal reinforcement.

In 1938, Richart and Olsen (11), carried out an investigation of 82 eccentrically-loaded columns, and presented an expression relating the ultimate load on an eccentrically-loaded column, to that of an axially-loaded column. In which,

$$\frac{P_{ecc.}}{P_{axial}} = \frac{1}{1 + C\frac{ec}{k^2}}$$

where - e is the eccentricity of the load with respect to the centroid of the column cross-section,

- c is the distance from the centroid to the extreme fiber in compression,

- C is an empirical constant less than unity,

- k is the radius of gyration of the column section.

This empirical equation, based on experimental tests, was capable of predicting the ultimate strengths for failures in compression, but was not capable of predicting either the ultimate strengths for failures by yielding of the reinforcement nor if a member would fail in tension or by crushing of the concrete.

In 1937 and 1943, Whitney (12) (37) made an important contribution on eccentrically loaded columns. He presented ultimate theories for the compression and tension failures of members subjected to axial compression and bending. Whitney assumed that the non-linear concrete compressive stress distribution could be approximated to a rectangular stress block with a uniform stress intensity of $0.85f_c'$ over some depth 'a' such that equilibrium is satisfied. The following equation for compression failures in combined bending and axial load was developed.

$$P = \frac{2A_s'f_y}{\frac{2e}{d'} + 1} + \frac{btf_c'}{\frac{3te}{d^2} + 1.178}$$

where - A_s' is the area of compression reinforcement,

- d' is the distance from extreme compression fibre to centroid of compression reinforcement,
- d is the distance from centroid of tension reinforcement to compression face of member,

- b is the width of a rectangular member,

- t is the total depth of section.

For compression failures, this equation reduces to the ultimate load for a centrically-loaded column, with e = 0.

In 1944, Jensen (13) made an attempt to use the inelastic properties of the stress-strain curve for concrete. He assumed a trapezoidal stress distribution and derived the properties of this trapezoid as a function of cylinder strength by analysis of the observed ultimate strength of reinforced concrete beams.

Followed by a number of extensive studies of Jensen's theory, the ultimate strength theories for eccentrically loaded columns were developed.

Many additional tests have been reported between the years 1900 to 1950, from both European and North American investigators. Among these, are included those of Von Thullie (14), 1896; Bach and Graf (15), 1913; Baumann (16), 1934; Bittner (17), 1935; Brandzaeg (18), 1935; Thomas (19), 1938; The Russian Specifications (20), 1938; Anderson (21), 1941; and Richart (22), 1947.

In 1951, Hognestad (25) published a more refined ultimate strength theory, based on more realistic, and more complex assumptions regarding the stress-strain relation of concrete, derived empirically from his tests. His investigation not only predicted the types of failure and the ultimate strength of the columns, but also the behavior of the column throughout the entire loading range. In his paper, Hognestad also introduced the dimensionless form of interaction diagram; with the application of this diagram; with the application of this diagram, one can determine the behaviour in which the strength varied between the limits of axial compression to pure flexure of the column, simply and accurately.

Both the ACI (American Concrete Institute) Building Code (ACI - 318 - 71) (23) and the NBC (National Building Code of Canada, 1965) (24) permit the ultimate strength method to be used in the design of reinforced concrete columns, using a ϕ factor to reduce the predicted actual strength capacity of the column, to obtain the design ultimate strength. These ϕ factors were intended to take into account the importance of the member and the accuracy of the predicted equation and allow for any errors in construction techiques. The ACI code specifies $\phi = 0.7$ for tied columns and $\phi = 0.75$ for spirally hooped columns; while the NBC code specifies $\phi = 0.75$ for both.

CHAPTER III

EXPERIMENTAL INVESTIGATION

3.1 Object and Scope of Investigation

The object of this investigation was to experimentally study the basic behavior of reinforced concrete short columns subject to combined bending and axial load.

The major variables studied in this program were the percentage of reinforcement, steel yield stress, concrete strength and eccentricity of the applied loading, while the length and nominal dimensions of the columns remained unchanged.

Details of the nominal dimension of the reinforced concrete columns is given in Figure (3.2.a).

3.2 Material

3.2.1 Concrete Mixtures

The concrete used for all specimens was supplied by Dominion Building Materials Ltd. The concrete was transit mixed during the transportation from the batch plant to the laboratory of the University of Ottawa. Two design compression strengths, of 4000 psi. and 6000 psi. at 28 days, were used in the project as being typical of concrete strengths presently used in construction. The mix proportions of the concrete are shown in Table (3.2.1.a) and Table (3.2.1.b) below. The grading curves of coarse and fine aggregates are shown in Fig. (3.2.1).

TABLE (3.2.1.a) Mix Proportion by Weight with Compression Strength of 4000 psi. (Slump = 3")

MATERIAL	WEIGHT PER CUBIC YARD OF CONCRETE
Standard Portland Cement	500 lbs.
Sand	1,530 lbs.
3/4" Coarse Aggreate	1,910 lbs.
Water	294 lbs.

W/C = 294/500 = 0.59

TABLE (3.2.1.b) Mix Proportion by Weight with Compression Strength of 6000 psi. (Slump = 3")

MATERIAL	WEIGHT PER CUBIC YARD OF CONCRETE
Standard Portland Cement	700 lbs.
Sand	1,140 lbs.
3/4" Coarse Aggreate	1,960 lbs.
Water	286 lbs.

W/C = 286/700 = 0.41

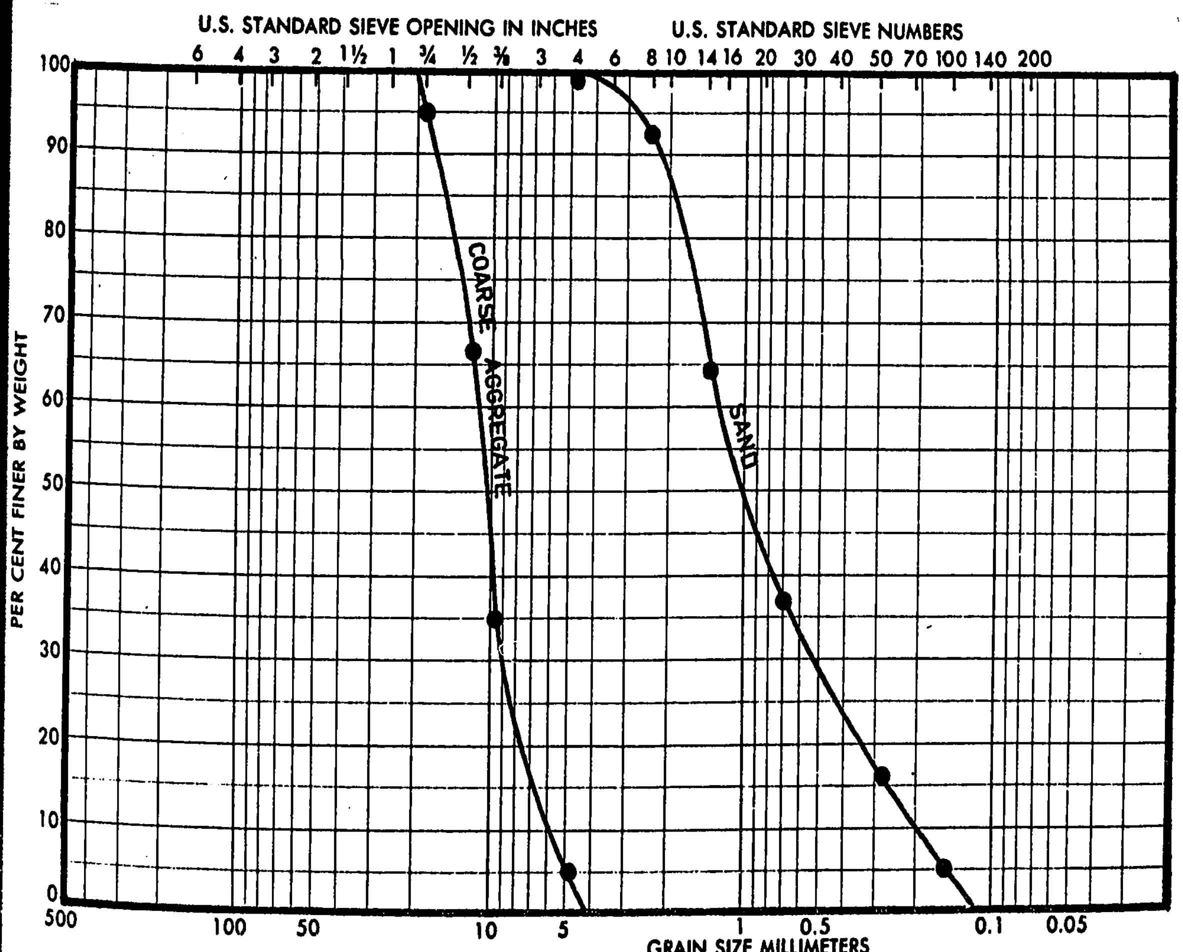


FIGURE (3.2.1) GRADING CURVES OF COARSE AGGREGATE & SAND

3.2.2 Reinforcing Steel

Three sizes of deformed bars of Nos. 7, 6, and 4, and one size of plain bar of No. 3, were used in the tests. The ties for all the specimens were manufactured by hand in the laboratory from #2, 1/4 in. plain bars. Average properties of the various lots of reinforcement as determined from tension tests in the laboratory are given in Table (3.2.2.a), and used for the calculations in both the ACI Design Formulae and the Strain Gradient Method.

All longitudinal reinforcement was specified to be Grade 60 steel.

TABLE (3.2.2.a)

AVERAGE TENSILE PROPERTIES OF REINFORCING STEEL

BAR SIZE (in.)	NO. OF TESTS	AVERAGE YIELD POINT (ksi.)		AVERAGE ULTIMATE STRENGTH (ksi.)	USED IN SERIES NO.
		MEAN	C.of V.		
3/8	25	54.40	1.6%	76.00	4
4/8	25	65.50	2.9%	92.80	3
6/8	25	52.60	1.4%	110.00	2
7/8	25	67.40	0.5%	112.10	1

* The yield point is defined by a residual strain of 0.2% instead of 0.5% of the total strain under load; the yielding stress was found to be very close in both cases.

Typical stress-strain curves for various size of reinforcement are shown in Appendix A, from Fig. (A.1) page 107 to Fig. (A.4) page 110.

3.3 Types of Specimens

The 100 test specimens was divided into four groups, all of them being tied columns. Since the investigation was confined to combined axial and bending problems, the specimens were kept fairly short, ($\frac{L}{r}$ =30) so that the results would not be affected by the occurrence of buckling failures.
An outline of the test program is given in Table (3.3.a), which indicates that the major variables were: size and yield stress of reinforcement, concrete strength and the eccentricity of applied loading.

Columns were completely identified by a numeral, a capital letter and followed by another numeral. The first numeral -- 1 through 4 -- indicates the size of the reinforcement; the capital letter -- A, B, C, D, DE or E -- indicates the eccentricity of load; the final numeral -- 1 to 5 -- indicates one of five companion specimens.

For example:-

1B - 1 is a column in group one, with 7/8 in. longitudinal reinforcement, loaded with a eccentricity of e/t = 0.1, and the first of this group of specimens.

The four groups of specimens with the different sizes of longitudinal reinforcement are shown in Fig. (3.2.a). Concrete cylinders, 6 by 12 in. were made as control specimens.

TABLE (3.3.a)

OUTLINE OF TESTS

GROUP NO.	ECCENTRICITY e/t	SIZE OF REINFORCEMENT (in.)	CONCRETE STRENGTH (psi.)	$P \frac{f_y}{0.85f_c'}$	TOTAL NO. OF COLUMNS	TESTED*
1	A (0.0)	7/8	4000	1.26	5	V
	B (0.1)				5	V
	C (0.3)				5	V
	D (0.5)				5	V
	E (∞)				5	H
2	B (0.1)	6/8	6000	0.404	5	V
	C (0.3)				5	V
	D (0.5)				5	V
	DE (0.75)				5	V
	E (∞)				5	H
3	A (0.0)	4/8	4000	0.479	5	V
	B (0.1)				5	V
	C (0.3)				5	H
	D (0.5)				5	H
	E (∞)				5	H
4	B (0.1)	3/8	6000	0.097	5	V
	C (0.3)				5	H
	D (0.5)				5	H
	DE (0.75)				5	V
	E (∞)				5	H

* H = Horizontally tested specimens
V = Vertically tested specimens

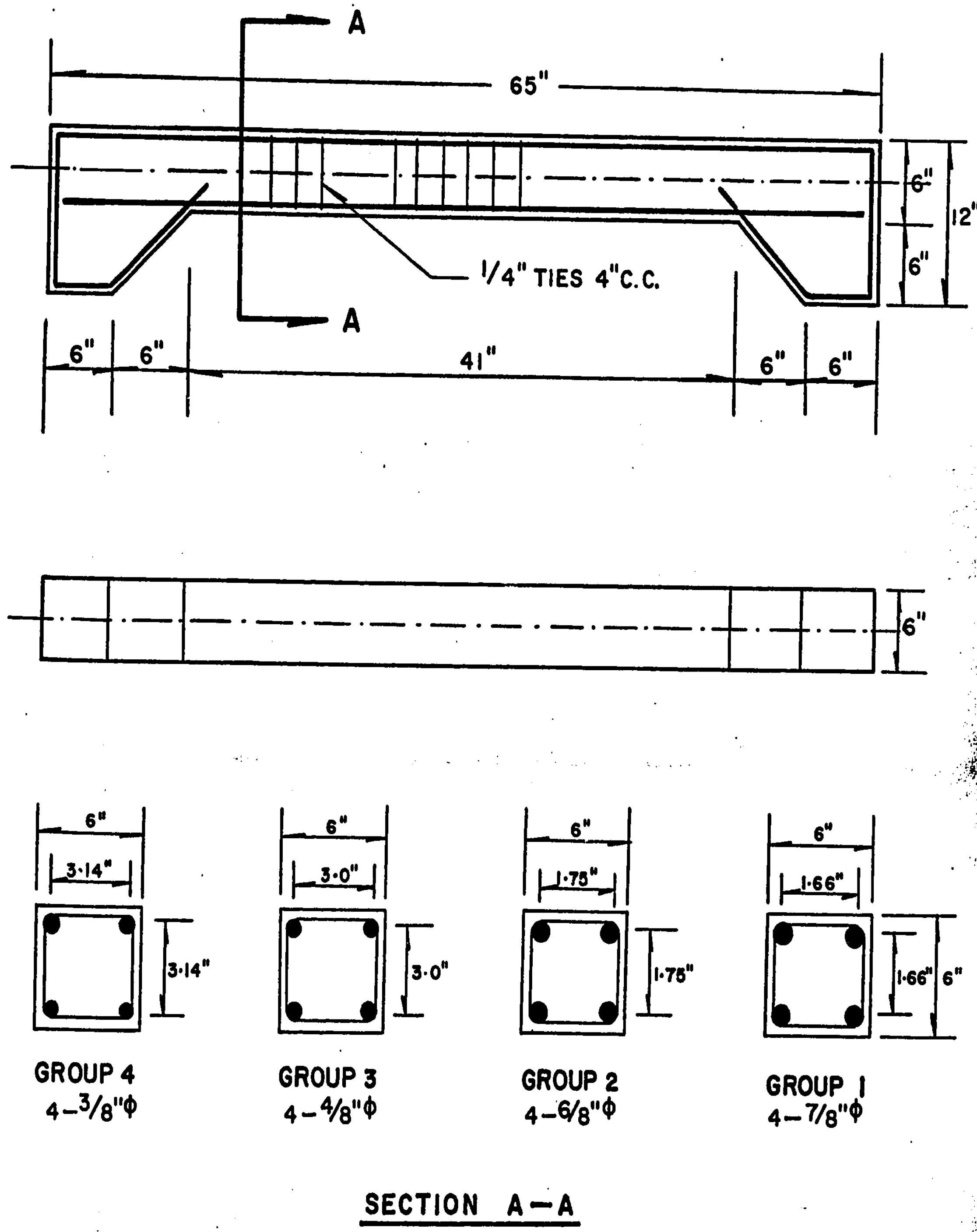


FIG. (3.2.a) DETAIL OF COLUMN SPECIMENS

3.4 Fabrication and Curing

The concrete was mixed and supplied by the Dominion Building Material Limited, and the specimens were cast in the structures laboratory of the University of Ottawa throughout the tests. The total time from mixing of concrete to the final placing and compacting operations, was kept within one hour and thirty minutes.

The reinforcement was assembled into a cage before it was placed in the forms for casting. The longitudinal bars were placed inside the ties and welded to them. Spacing blocks were used to provide accurate spacing and protecting coverings of the longitudinal reinforcement. Wooden blocks were inserted in the center portion of the steel forms before casting. Steel forms, as shown in Fig. (3.4.a), were used for each casting for the entire project.

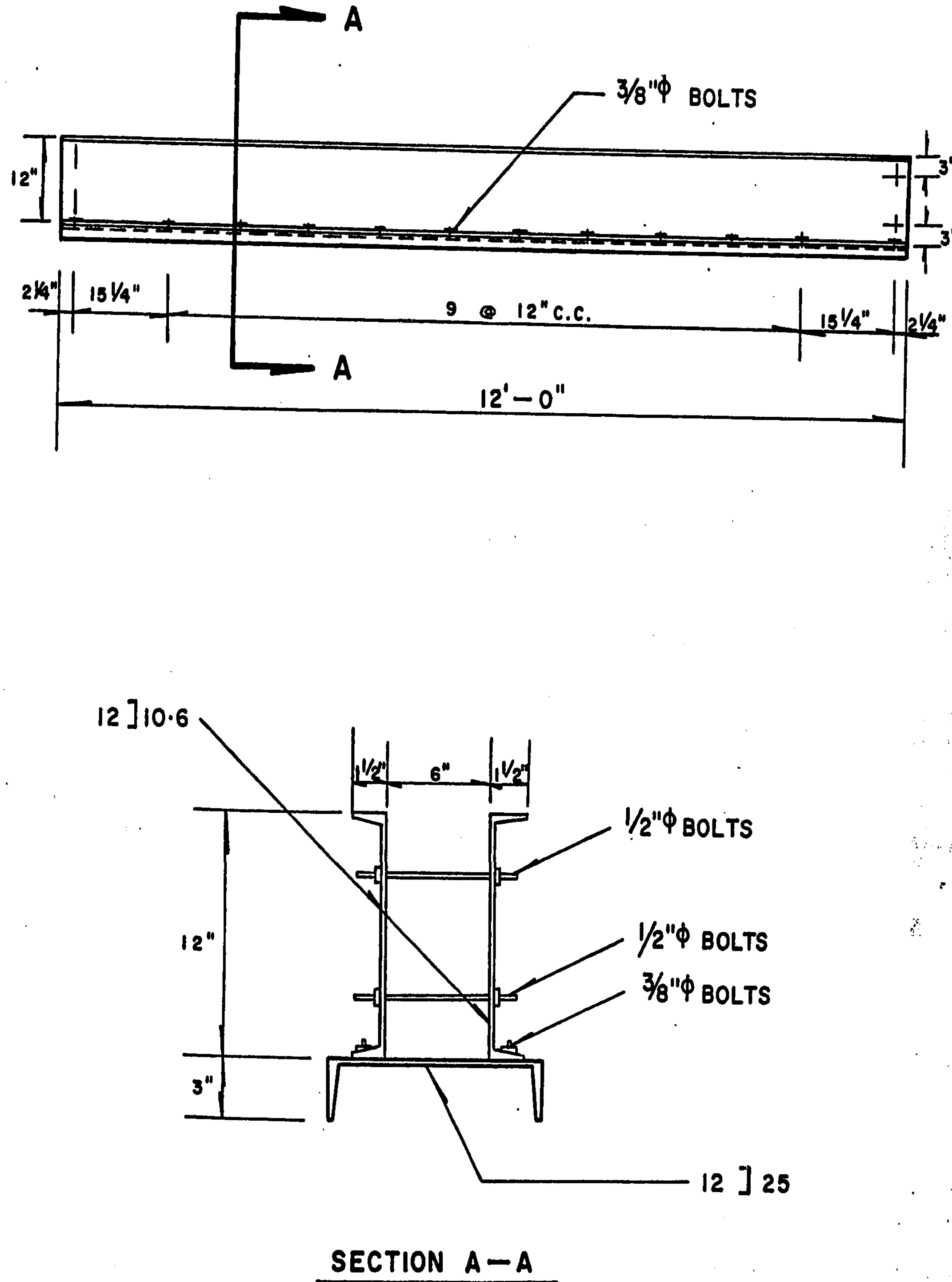


FIG. (3.4.a) TYPICAL DRAWING FOR STEEL FORM

3.5 Testing Methods and Procedure

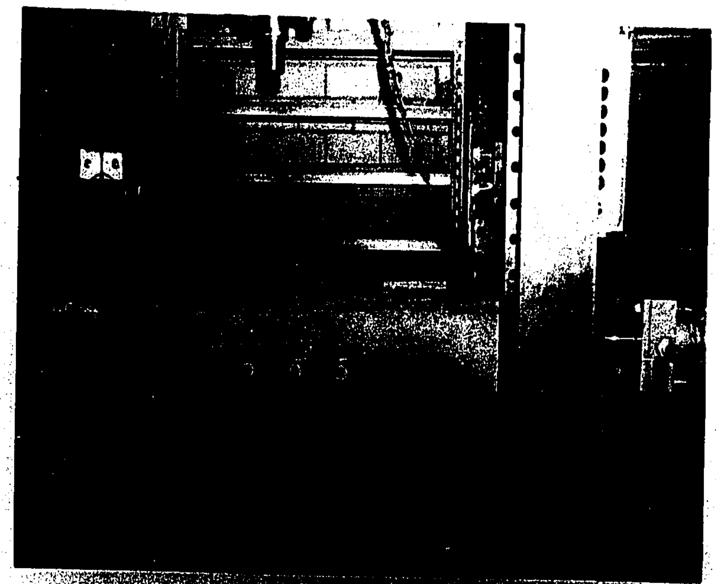
All five test specimens of a group of reinforced concrete columns and the twenty associated 6 by 12 in. concrete cylinders, were cast at the same time, and were tested within a 24 hour period, at the age of 28 days. The columns were subjected to short term loading, the time between the first and last increments of the applied load was approximately 30 minutes.

3.5.1 Reinforced Concrete Columns

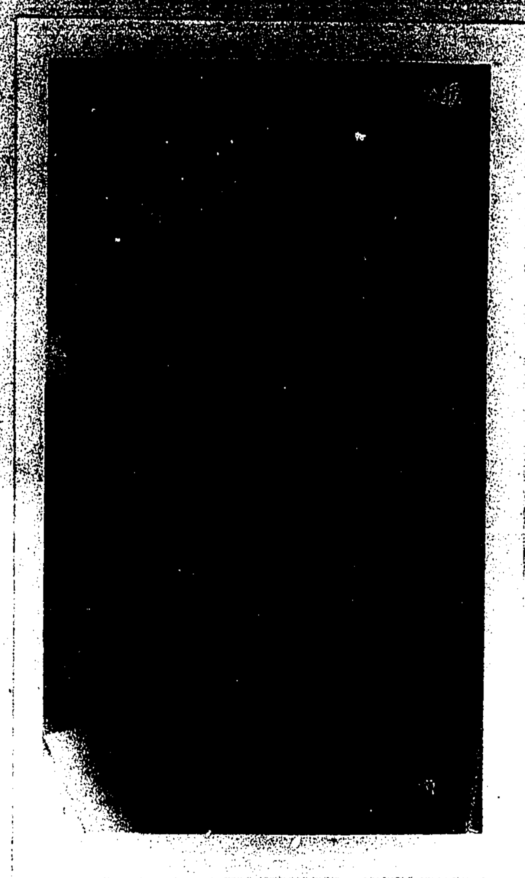
3.5.1.a Columns under Combined Axial and Eccentric Loadings

A total of 75 column specimens were assigned to be tested under axial and eccentric loadings, with eccentricities e/t equal to 0.00±0.01; 0.10±0.01; 0.30±0.01; 0.50±0.01 and 0.75±0.01 etc.; of which 25 were tested horizontally in a special testing frame, with a 100 kips capacity. The other 50 specimens were tested vertically in a 600,000-lb. capacity Baldwin Southwark Tate-Emery hydraulic testing machine. A view of both the test set ups during a column test are shown in Fig. (3.5.1.a) and (b), and a sketch of the testing equipment for each is given in Fig. (3.5.2.a) and (b), respectively. The test method is indicated by an horizontal and vertical tested specimen, respectively in Figs. (3.5.2.a) and (b).

The columns were cast in a horizontal position in order to avoid a differential in concrete quality along the column length, but, on the other hand, this would cause a strength differential through the cross-section of the column.

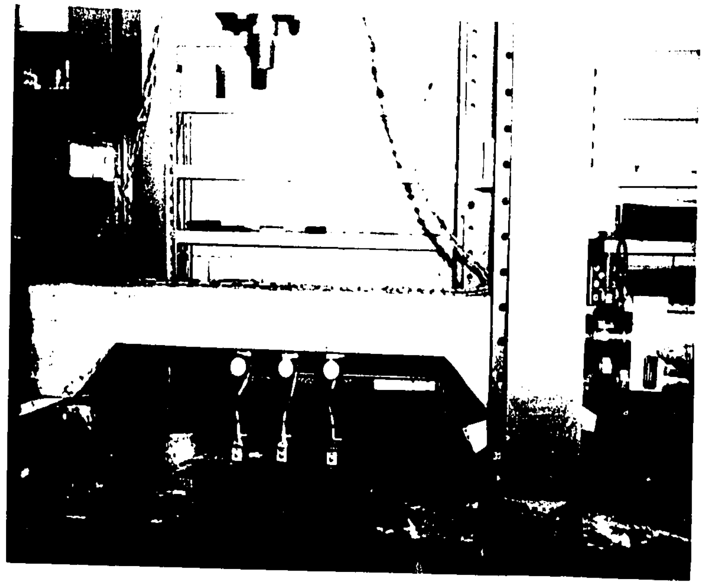


(a)

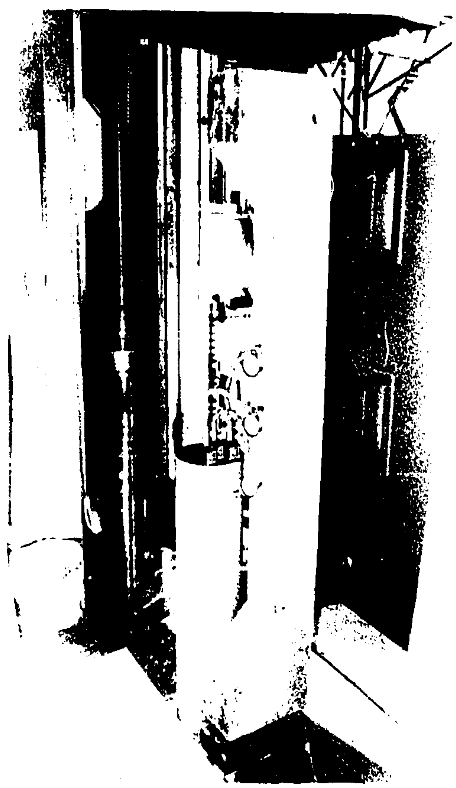


(b)

FIG.(3.5.1) GENERAL VIEW OF A TESTED SPECIMEN UNDER ECCENTRIC LOADING



(a)



(b)

FIG.(3.5.1) GENERAL VIEW OF A TESTED SPECIMEN UNDER ECCENTRIC LOADING

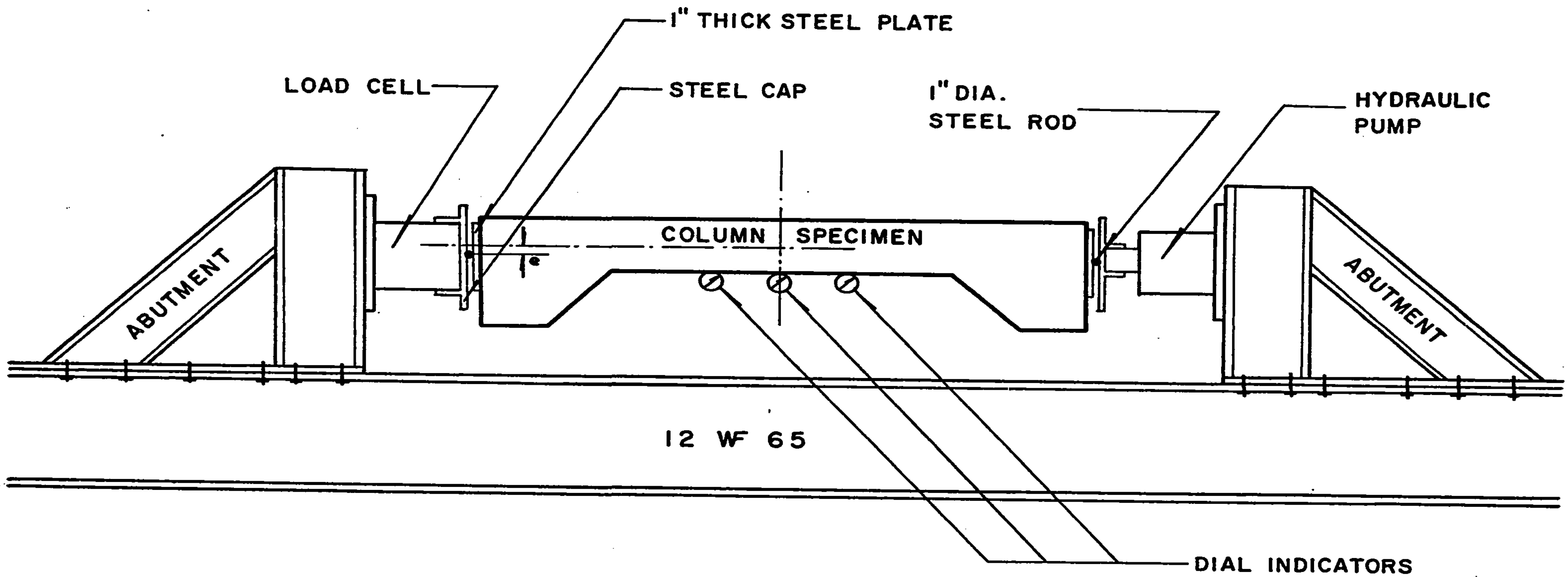


FIG.(3.5.2.a) ARRANGEMENT OF TESTING

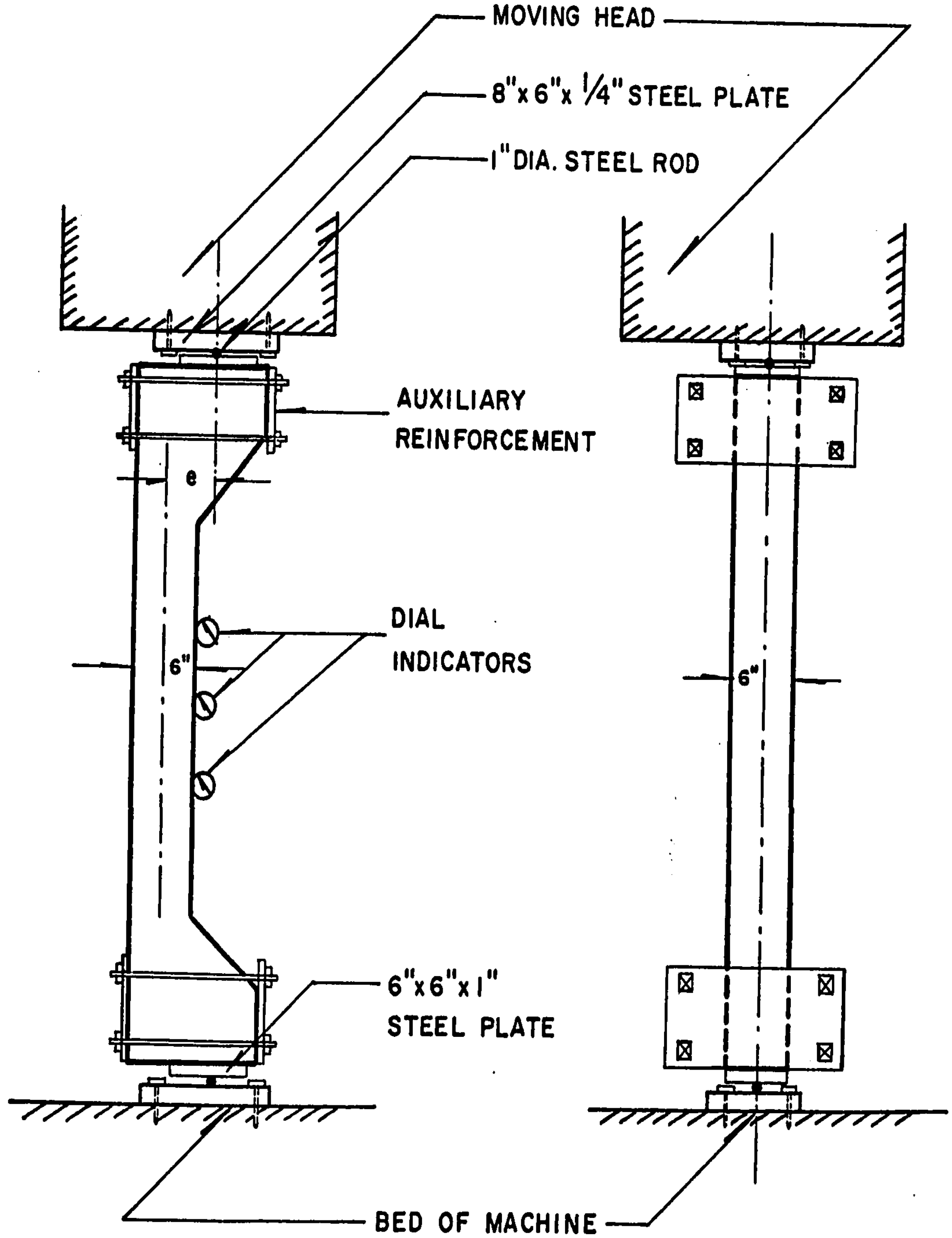


FIG. (3.5.2.b) ARRANGEMENT OF TESTING

Five or ten test specimens were cast at one casting, and the concrete was compacted in the forms by means of a small electrically driven pencil vibrator.

The day after the columns were cast, the forms were removed, and the columns and the control cylinders (20 for each casting), were cured under wet sacks in the laboratory for 28 days before testing.

The method and procedure used with both testing apparatuses were the same. In order to avoid the crushing of the concrete at both ends of the specimen, two 1" thick steel bearing plates were used, one at each end face of the specimen.

For the specimens tested horizontally in the testing frame, a steel cap was used for both the ram and the load cell. To obtain single axis hinged ends at both ends of the specimen, two 1" diameter steel rollers were put in between the end bearing plates and the steel caps, both of the which were provided with semi-circular grooves to keep the steel rollers in the desired positions.

For the specimens tested vertically in the 600,000-lb. Baldwin Southwark Tate-Emery hydraulic machine, two $1\frac{1}{4}$" steel bearing plates, screwed to the movable head and the bed of the machine respectively, were provided with semi-circular grooves, to keep the 1" roller in place. Two - 12" x 6" x 1/2" steel plates were clamped to both capitals of the columns by 4 - 1/2" thread steel bars (which can be seen in Fig. (3.5.2.b). These steel bars were placed on each capital of the column and prestressed before the test, by tightening the nuts, to reduce the possibility of diagonal tension failure in the capitals, because it was found impracticable to reinforce the capitals sufficiently with only embedded bar reinforcement.

The lateral deflection of the column was measured with three 0.001 in. dial indicators; one located at the midspan of the specimen, and the other two at 6" to the left and right of the midspan of the specimen respectively, in the plane of eccentricity.

All columns were carefully centered and levelled or plumbed in the testing machine. Initial readings of all gages were taken immediately before application of any loading. Loading proceeded to failure in 15 to 20 increments for every specimen.

3.5.1.b Columns under Pure Flexure

Twenty five specimens were tested under flexure, ie, with the eccentricity e/t equal to infinity. The method of testing was as for a simply-supported beam; loaded at two points located symmetrically about midspan. A view of the testing frame during the test of a specimen is shown in Fig. (3.5.3.a) and a sketch of the testing equipment is given in Fig. (3.5.3.b).

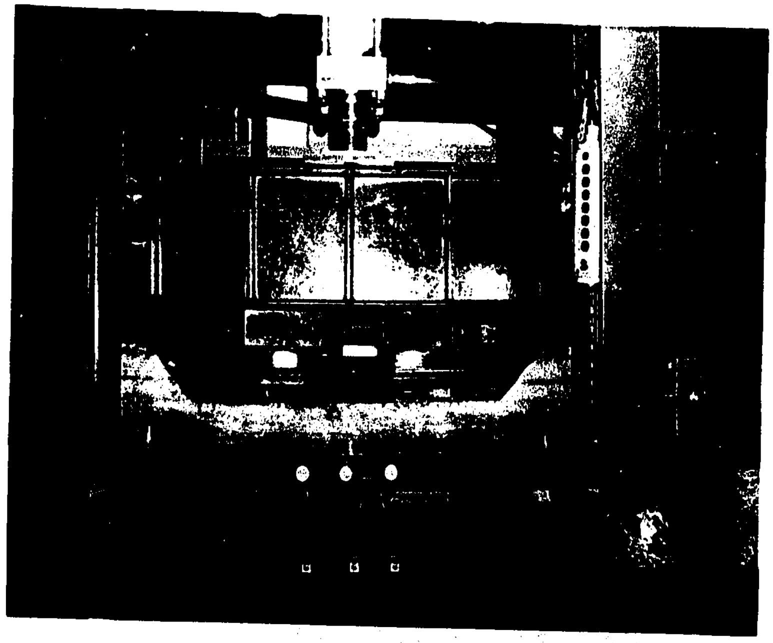


FIG.(3.5.3.a) GENERAL VIEW OF TESTED SPECIMEN UNDER PURE BENDING

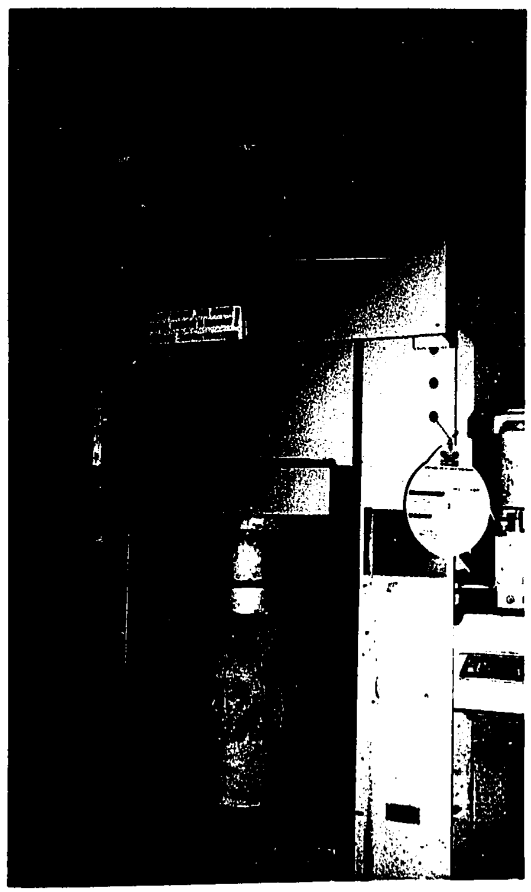


FIG.(3.5.4.a) 6" BY 12" CONCRETE CYLINDER UNDER TEST

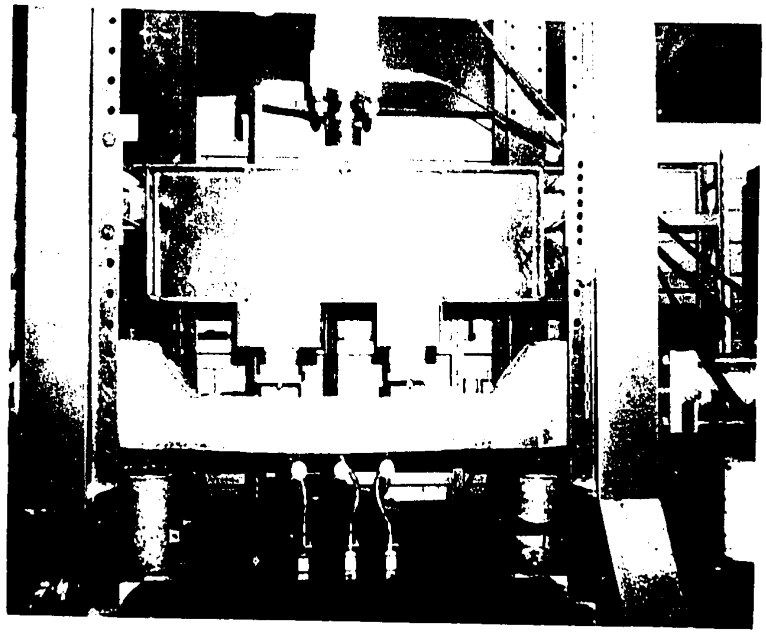


FIG.(3.5.3.a) GENERAL VIEW OF TESTED SPECIMEN UNDER PURE BENDING

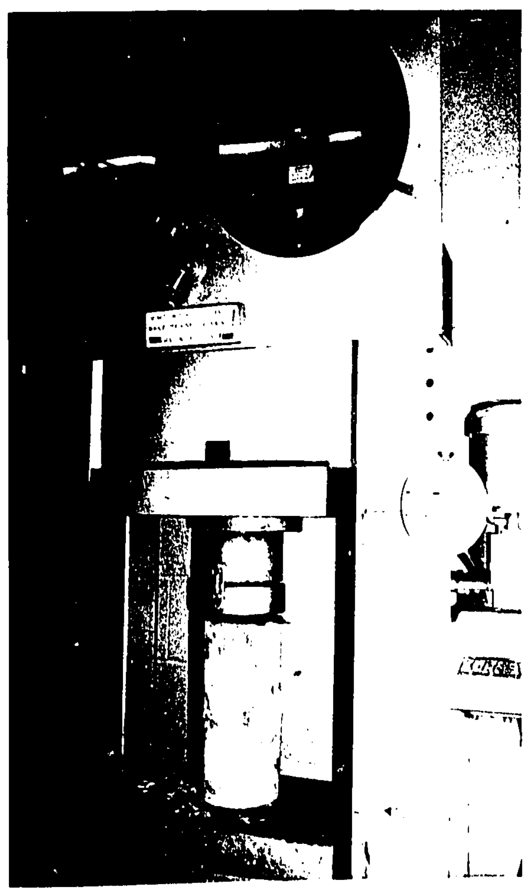


FIG.(3.5.4.a) 6" BY 12" CONCRETE CYLINDER UNDER TEST

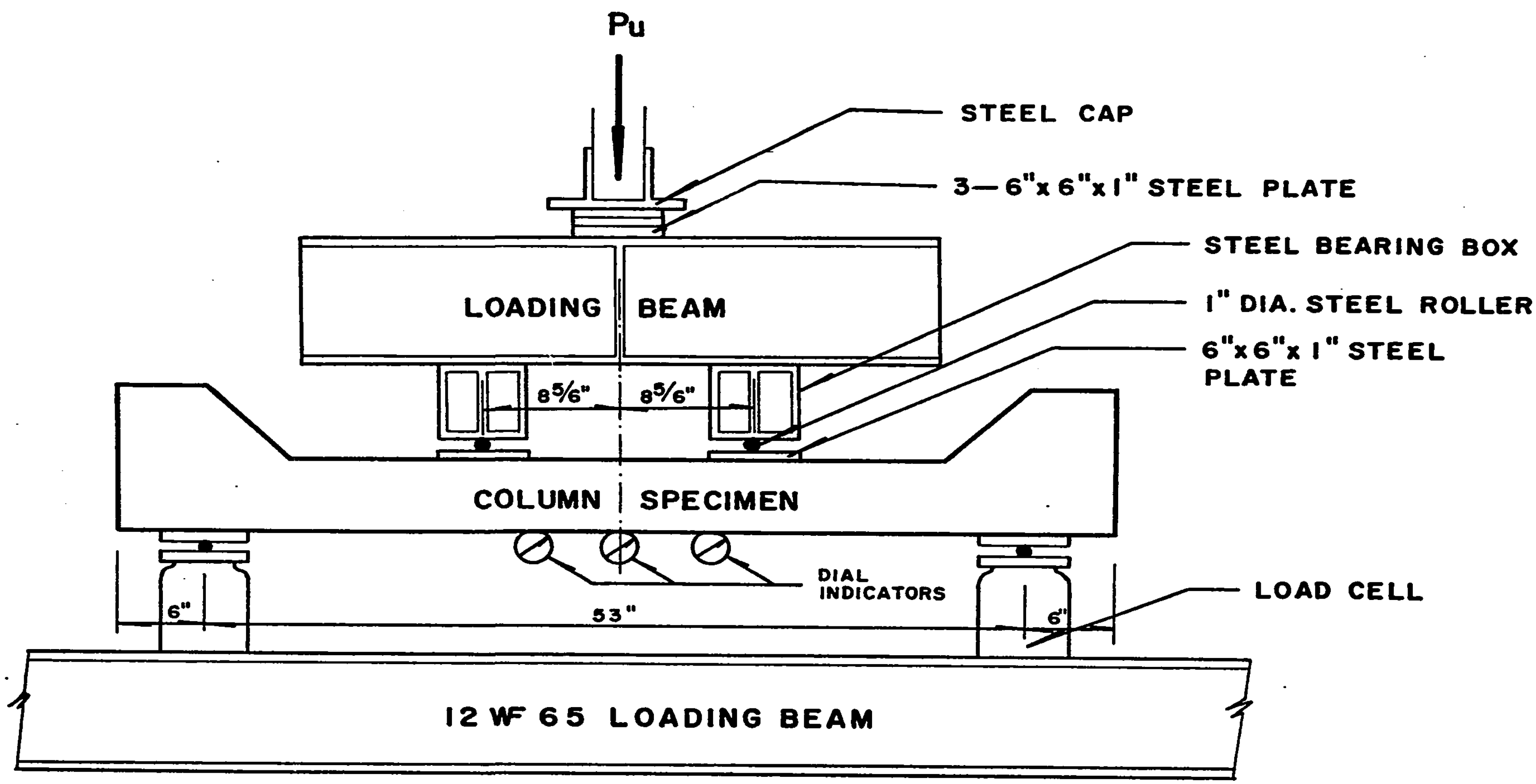


FIG. (3.5.3.a) SKETCH OF TEST SETUP FOR BEAM COLUMN

The steel loading beam, located under the ram, was supported symmetrically by two load transfer blocks, both with 1" thick steel bearing plates resting directly on the upper surface of the specimen. Two 1" diameter steel rollers were put in between the steel blocks and the bearing plates. The load was applied by the ram, and transfered to the test specimen through the steel beam. The concrete beam-column was supported by two load cells each with a capacity of 50 kips, 6" from each end of the column. The load was measured from the reactions of the load cells.

The lateral deflections of the specimen, and the increments of the applied loading, were measured and applied the same way as for a column tested under combined axial and bending.

3.5.2 Concrete Cylinders

The compression strength of the concrete cylinders were tested on the same day as the specimen and thus gave the actual concrete cylinder compression strength at that age. A total of 20 test cylinders were cast at the same time as the corresponding columns, hence no differential time effects would be considered between the testing specimens and the cylinders.

Both end faces of the concrete cylinder were capped with standard suphur capping compound and tested to failure in a 300,000-lb. capacity Forney hydraulic concrete testing machine. A view of a 6" by 12" concrete cylinder under test is shown in Fig. (3.5.4.a).

CHAPTER IV

THEORETICAL INVESTIGATION

4.1 Ultimate Strength Analysis of Reinforced Concrete Columns as for ACI 318 - 56 or NBC 1965 or CSA - A23.3 - 1970

The ultimate strength analysis of reinforced concrete member presented herein is based on the following assumptions:-

(1) A linear distribution of strain over the cross section is assumed at all load levels.

(2) The stress-strain relationship assumed for concrete in compression used in the strain-gradient method is that derived by Hognestad as shown in Fig. (4.1.a).

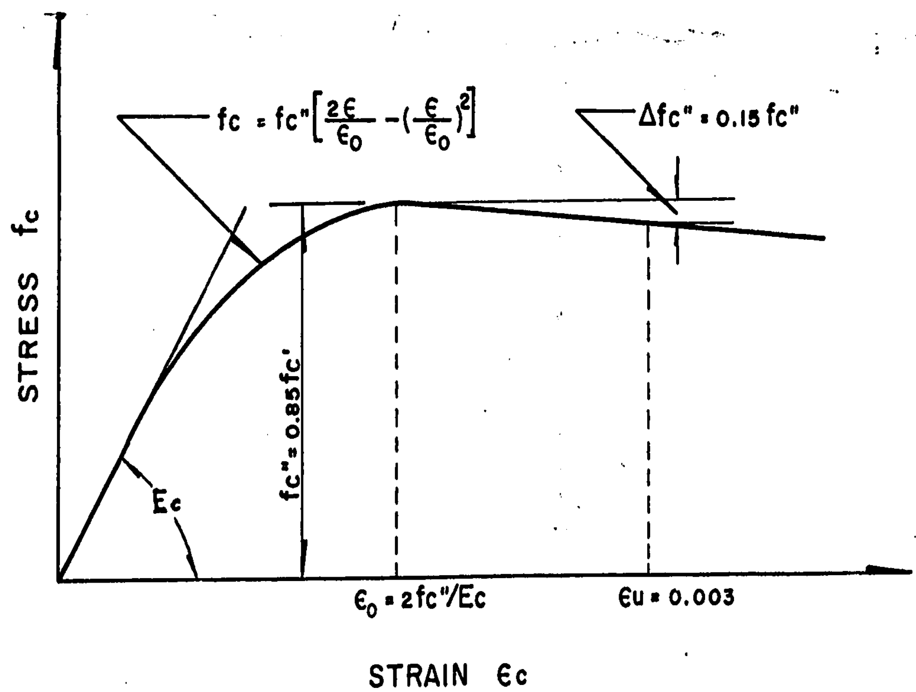


FIG. (4.1.a) ASSUMED STRESS—STRAIN RELATIONSHIPS FOR CONCRETE IN COMPRESSION

The initial modulus of elasticity, E_c is assumed in accord with Inge Lyse's Equation.

$$E_c = 1,800,000 + 460f_c' \quad \ldots\ldots\ldots\ldots\ldots(4.1.1)$$

(3) It is assumed in the present analysis that no tension stresses can be taken by the concrete.

(4) It is assumed that no general slip occurs between concrete and reinforcing steel, although local slip must be present at tension cracks in the concrete.

(5) In the ACI Design Formulae, the maximum strain at the extreme compression fiber at ultimate strength shall be assumed equal to 0.003.

(6) At ultimate strength the concrete stress is not proportional to strain. The variation of compressive concrete stress can be assumed to be uniform at an intensity of $0.85f_c'$ over an equivalent compression zone bounded by the edge of the cross section and a straight line located parallel to the neutral axis at a distance $a = K_1c$ from the fiber of maximum compressive strain. The distance "c" from the fiber of maximum strain to the neutral axis is measured in a direction perpendicular to that axis. The fraction "K_1" shall be taken as 0.85 for $f_c' \leq 4000$ psi., and decreased by 0.05 for every 1000 psi. above 4000.

In the above assumptions, Nos. 1, 3 and 4 are common for both the ACI and strain gradient method.

All the theoretical calculations presented use the final eccentricity of the columns taken from the experimental results.

4.1.1 Reinforced Concrete Columns under Balanced Failure

The balanced condition is the loading condition which produces at ultimate strength, simultaneously, a strain of 0.003 in the extreme fiber of concrete and yield strain in the tension steel, and assume that the compression steel reaches its yield point stress, from the strain relationship is shown in Fig. (4.1.1.a) :-

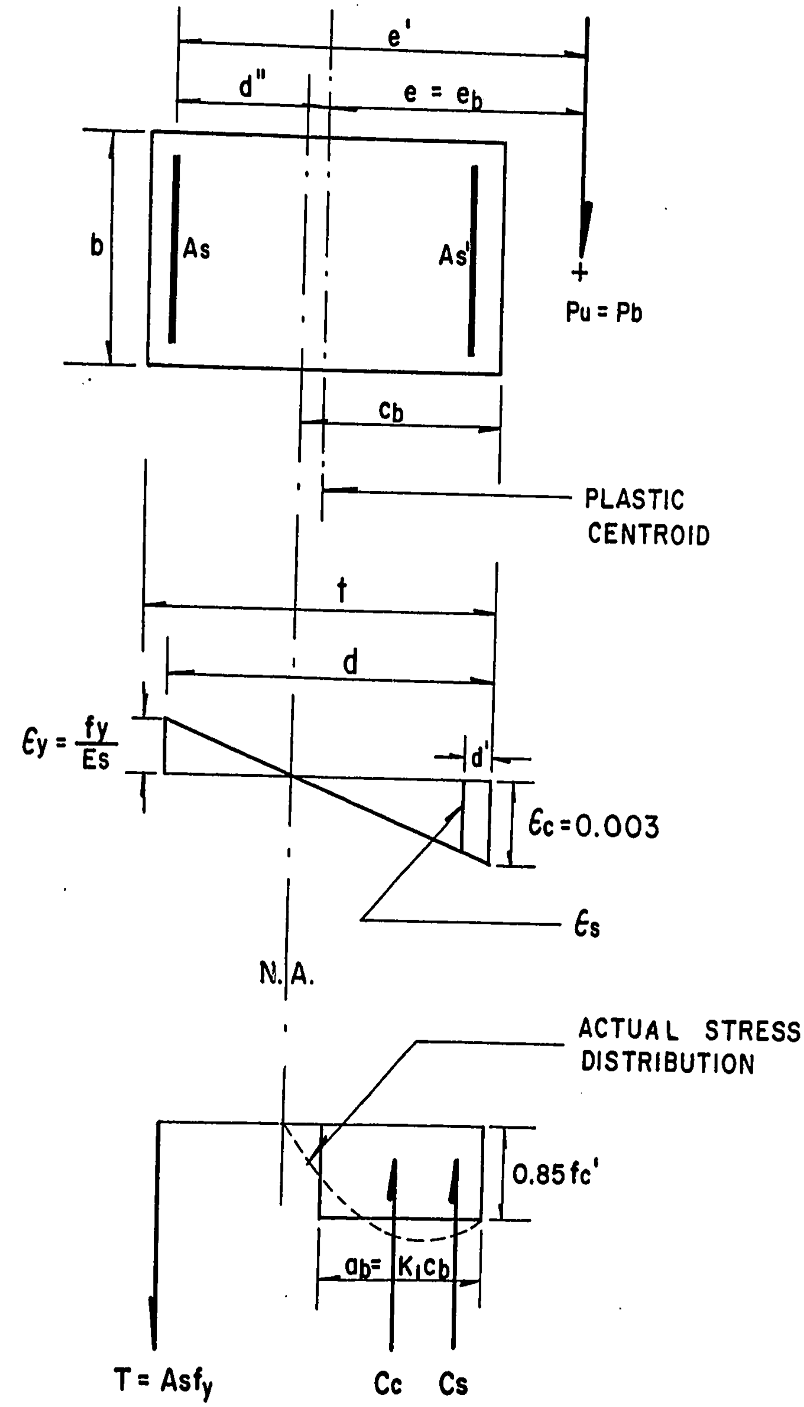


FIG. (4.1.1.a) BALANCED CONDITION— RECTANGULAR SECTION

$$\frac{c_b}{d} = \frac{\epsilon_u}{f_y / E_s + 0.003}$$

$$c_b = \frac{0.003}{f_y / (29 \times 10^6) + 0.003} \cdot d$$

$$= \frac{87{,}000d}{f_y + 87{,}000} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.1.1.a)$$

$$C_c = 0.85f_c'K_1c_bb$$

$$= 0.85f_c'a_bb \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.1.1.b)$$

$$C_s = A_s' (f_y - 0.85f_c') \quad \ldots\ldots\ldots\ldots\ldots (4.1.1.c)$$

$$T = A_sf_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.1.1.d)$$

Force equilibrium requires.

$$P_b = C_c + C_s - T$$

$$= 0.85f_c'K_1bc_b + A_s'(f_y - 0.85f_c') - A_sf_y \quad \ldots\ldots\ldots (4.1.1.e)$$

Taking moments about the plastic centroid,

$$P_ee_b = C_c\ (d - a_b/2 - d'') + C_s\ (d - d' - d'') + Td''$$

$$P_ee_b = M_b = 0.85f_c'K_1c_b \cdot b \cdot (d - 0.5K_1c_b - d'') + A_s'(f_y - 0.85f_c') \cdot$$

$$(d - d' - d'') + (A_sf_y)\ d'' \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.1.1.f)$$

4.1.2 Reinforced Concrete Columns under Compression Failure

When the ultimate eccentric load P_u' exceeds the balanced value P_b or when the eccentricity e is less than the balanced values e_b, the capacity of the section is controlled by compression failure of the concrete. In this case, the tensile force T will be based on a stress less than the yield strength and may actually be compressive.

Using the principles of statics, considering the actual strain variation as the unknown, the ultimate eccentric load can be found. However, two approximate procedures are provided in the ACI Code to simplify the calculations.

In one approach, P_u' is assumed to decrease linearly from P_o to P_b as the moment increases from zero to $M_b = P_b \cdot e_b$, as shown in an interaction diagram of Fig. (4.1.2.a).

$$\frac{P_o - P_u'}{P_o - P_b} = \frac{P_u' e}{P_b e_b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.1.2.a)$$

Solving Equation (4.1.2.a) for P_u' gives,

$$P_u' = \frac{P_o}{1 + \left[\left(P_o / P_b\right) - 1\right] e/e_b} \quad \ldots\ldots\ldots (4.1.2.b)$$

or

$$P_u' = P_o - (P_o - P_b)\frac{M_u'}{M_b} \quad \ldots\ldots\ldots\ldots\ldots (4.1.2.c)$$

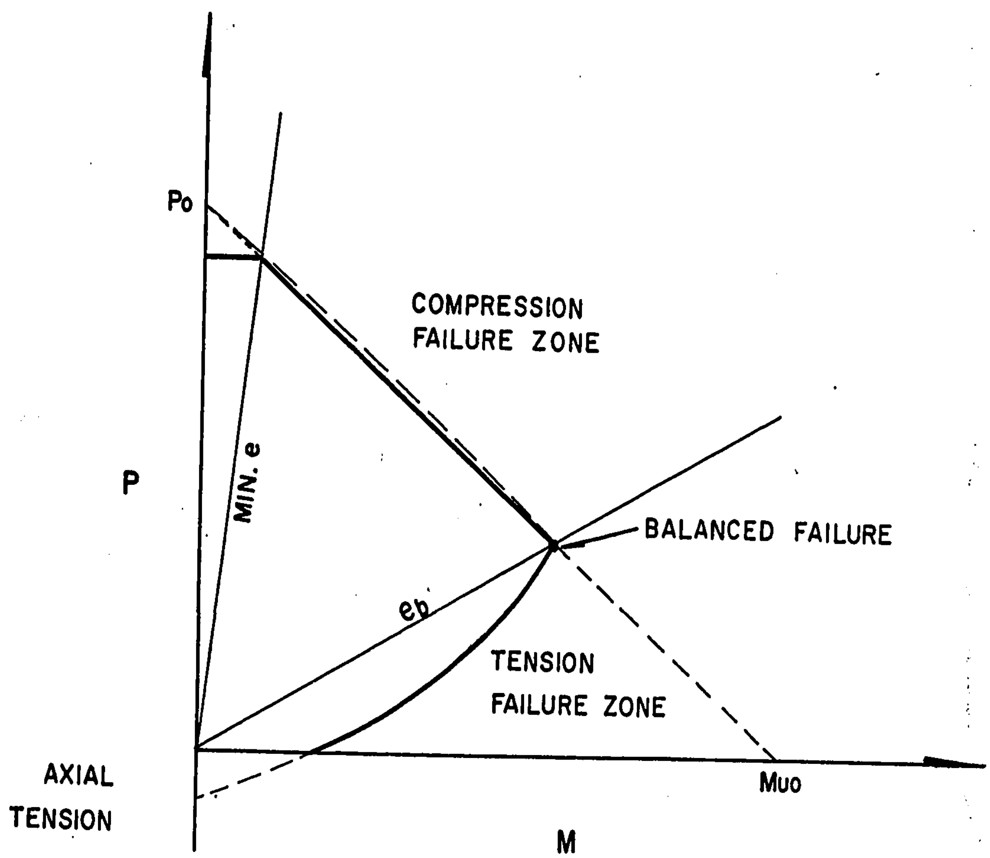


FIG. (4.1.2.a) INTERACTION DIAGRAM, COMPRESSION PLUS BENDING, ULTIMATE STREGTH Po AND Muo

A second approximate procedure may be applied for symmetrical reinforcement in terms of the column dimensions. Considering the moment equilibrium with respect to the tension steel, in Fig. (4.1.2.b) gives,

$$P_u' \left(e + \frac{d - d'}{2} \right) = C_c \left(d - \frac{a}{2} \right) + C_s \left(d - d' \right)$$

....................(4.1.2.a)

where

$$C_c = 0.85 \; f_c' \; b \; a \qquad \text{........(4.1.2.b)}$$

When compression controls, compression steel yields at ultimate strength, thus

$$C_s = A_s' \; f_y \qquad \text{........................(4.1.2.c)}$$

Substituting Equations (4.1.2.b) and (4.1.2.c) in Equation (4.1.2.a) gives,

$$P_u' = \frac{f_c' \; b \; t}{\dfrac{3te}{d^2} \quad \dfrac{3\,(d-d')\,t}{2d^2}} + \frac{A_s' \; f_y}{\left(\dfrac{e}{d-d'} + \dfrac{1}{2} \right)}$$

....................(4.1.2.d)

With the boundry condition of $P_u' = P_o$ and $e = 0$, then

$$P_o = 0.85 \; f_c' \; b \; t + 2 \; A_s' \; f_y \qquad \text{.........(4.1.2.e)}$$

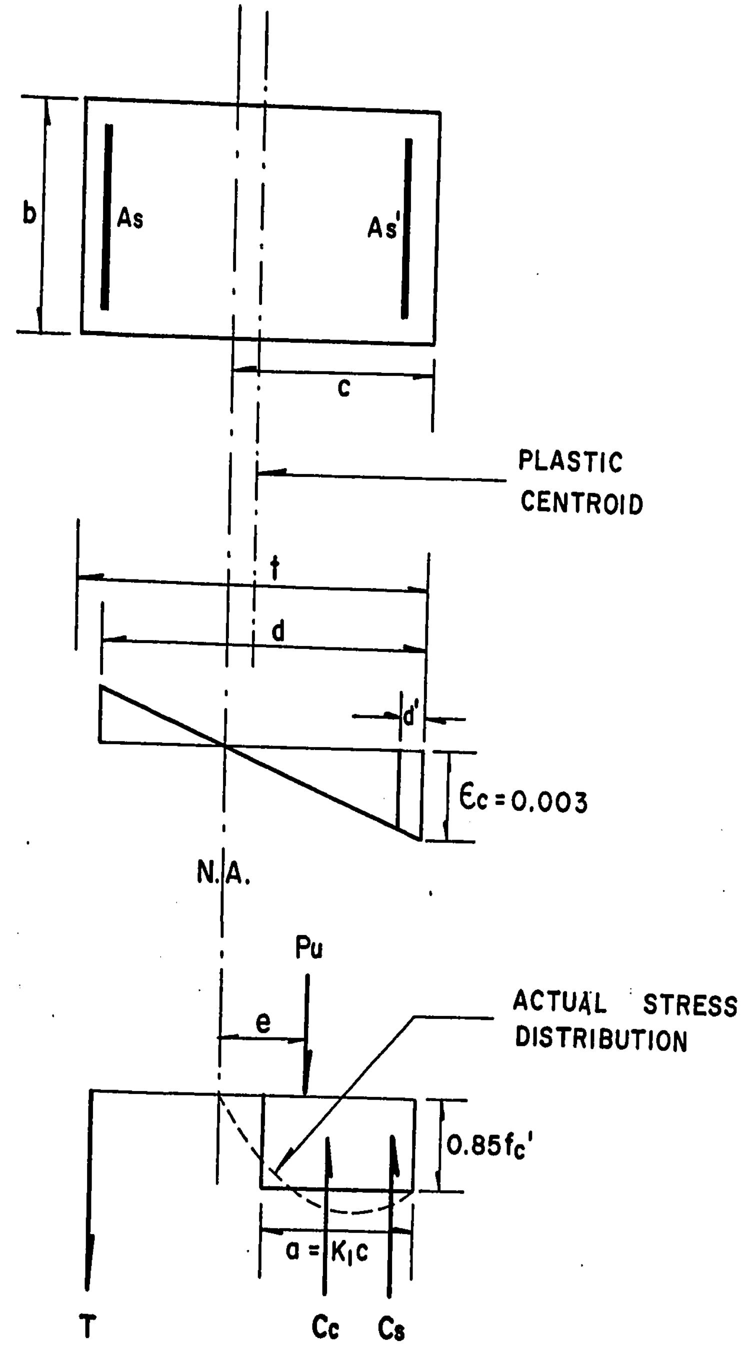


FIG.(4.1.2.b) ULTIMATE CAPACITY CONTROLLED BY COMPRESSION— RECTANGULAR SECTION

Substituting the above boundary condition in Equation (4.1.2.d), and neglecting the displacement of concrete by compression steel, it becomes,

$$P_u' = \frac{b\,t\,f_c'}{\left(\frac{3te}{d^2} + 1.18\right)} + \frac{A_s'\,f_y}{\left(\frac{e}{d - d'} + 0.5\right)}$$

....................(4.1.2.f)

It is to be observed that the direct use of statics gives probably the more accurate answer; and the result will be the same if the reduction for concrete displaced by compression steel is being applied. The first approach of straight line interaction gives the most conservative value.

4.1.3 Reinforced Concrete Columns under Tension Failure

When the ultimate capacity P_u' is less than the balanced value P_b or the eccentricity e is greater than the balanced value e_b, the capacity of the section is controlled by tension in the steel. A tension failure is initiated by yielding of the tension steel, this produces a movement of the neutral axis towards the compression zone. In this situation the ultimate strain in the tension steel will be greater than the yield strain. Referring to Fig. (4.1.3.a) and assuming that the strain in the compression steel is greater than the yield strain, one can obtain,

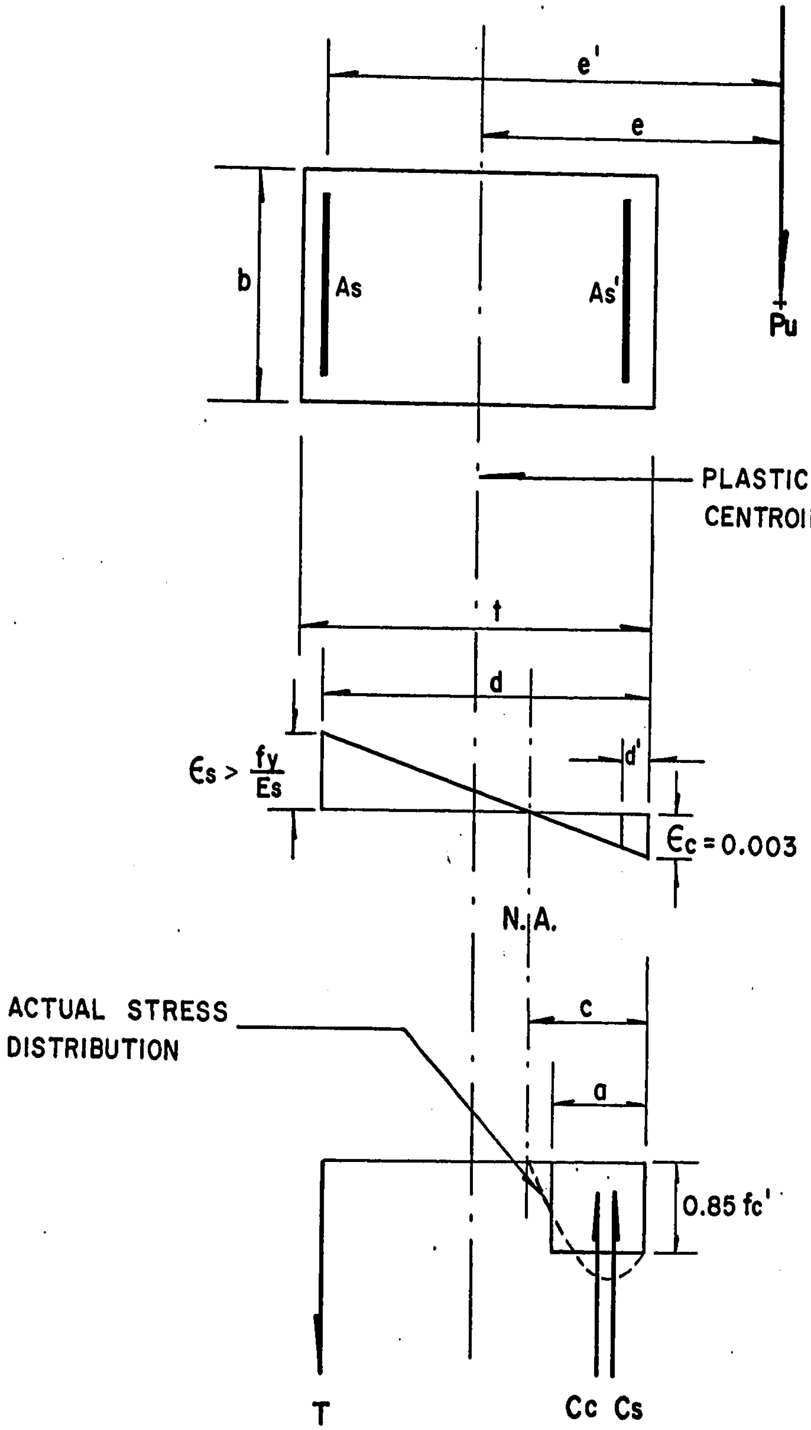


FIG. (4.1.3.a) ULTIMATE CAPACITY CONTROLLED BY TENSION — RECTANGULAR SECTION

$$T = A_s f_y$$

$$C_s = A_s' (f_y - 0.85f_c')$$

$$C_c = 0.85f_c'K_1 cb$$

From force equilibrium,

$$P_u' = 0.85f_c'K_1 cb + A_s'(f_y - 0.85f_c') - A_s f_y \quad \text{........(4.1.3.a)}$$

Let

$$\left.\begin{aligned} p &= \frac{A_s}{bd} \\ p' &= \frac{A_s'}{bd} \\ m &= \frac{f_y}{0.85f_c'} \end{aligned}\right\} \quad \text{..................(4.1.3.b)}$$

From moment equilibrium with respect to the tension steel,

$$P_u'e' = 0.85f_c'(K_1 c)\, b\, (d - \frac{K_1 c}{2}) + A_s'(f_y - 0.85f_c')\,(d - d')$$

$$\text{..................(4.1.3.c)}$$

Substituting Equations (4.1.3.a) and (4.1.3.b) in Equation (4.1.3.c), and rearrange,

$$P'_u = 0.85f_c'bd \left\{ p'(m - 1) - pm + (1 - e'/d) + \sqrt{(1 - e'/d)^2 + 2\left[(e'/d)(pm - p'm + p') + p'(m - 1)(1 - d'/d)\right]} \right\}$$

....................(4.1.3.d)

In our case, $A_s = A_s'$ and $p = p'$, Equation (4.1.3.d) reduced to

$$P'_u = 0.85f_c'bd\left\{ - p + 1 - e'/d + \sqrt{(1 - e'/d)^2 + 2p\left[(m - 1)(1 - d'/d) + e'/d\right]} \right\}$$

....................(4.1.3.e)

4.1.4 Reinforced Concrete Beams with Equal Tension and Compression Steel

When eccentricity 'e' becomes very large so that bending is predominant, the effect of the direct compression caused by P'_u is small. The member acts as a beam column and can be analyzed as a flexural member when the tensile steel limits the strength. When e becomes inifinite, it is approximately the case of pure moment resistance, with the moment capacity M_u^T in simple bending, and $P_u^T = 0$.

From force equilibrium as shown in Fig. (4.1.4.a) requires that

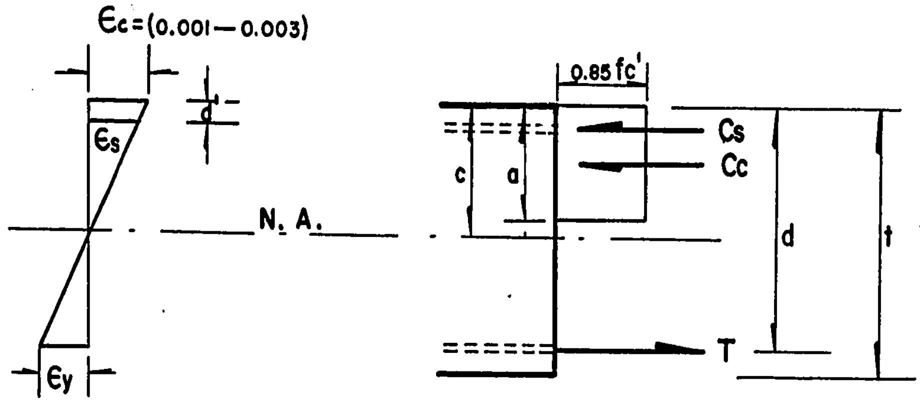


FIG. (4.1.4.a) ULTIMATE STRENGTH, FLEXURE —— RECTANGULAR SECTION

$$A_s f_y = A_s' f_s + 0.85 f_c' ba \quad \text{.................(4.1.4.a)}$$

where

$$a = K_1 \cdot c$$

From strain diagram

$$\epsilon_s = \frac{\epsilon_c \times (c - d')}{c} \quad \text{.................(4.1.4.b)}$$

where ϵ_c varied from 0.001 to 0.003

From stress-strain relationship of reinforcement, and a assigned value of ϵ_c (varied from 0.001 to 0.003), f_s can be determined.

Using Equations (4.1.4.a) and (b), and considering force equilibrium, c can be obtained by trial and error.

From moment equilibrium about the centroid of the tension steel, the moment is found to be :-

$$M_u^T = A_s' f_s (d - d') + 0.85 f_c' \times b \times K_1 c (d - K_1 c/2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.1.4.c)$$

The maximum load capacity of a structural member is attained when a mechanism forms. A two point loaded beam column collapes by forming a plastic hinge in the centre of the span, as shown in the Fig. (4.1.4.b).

Using either the equilibrium method or the mechanism method, the ultimate load P_u^T can be found,

$$P_u^T = \frac{6M_u^T}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.1.4.h)$$

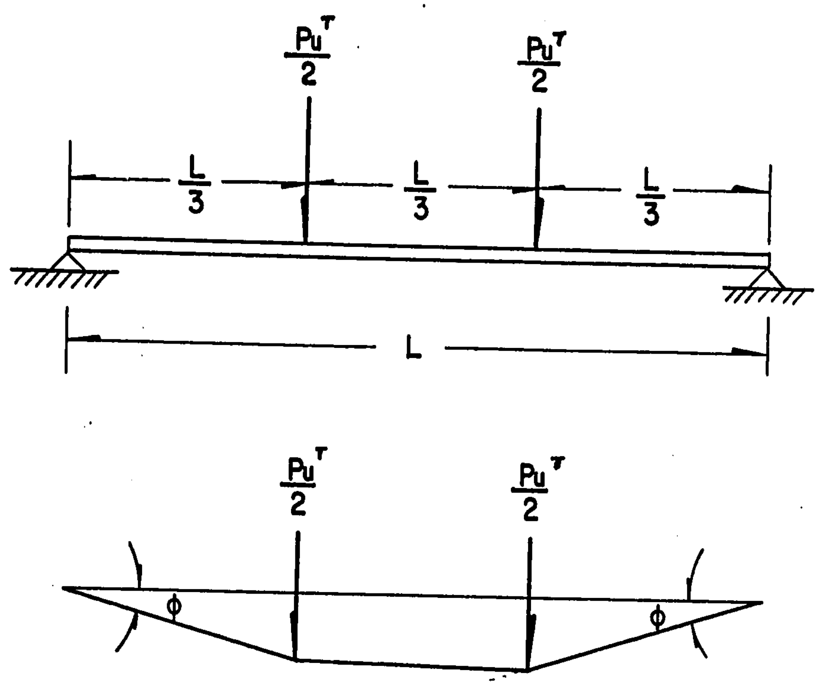


FIG. (4.1.4.b) COLLAPSE MECHANISM OF A BEAM COLUMN

4.2 Strain Gradient Method

For cross sections consisting of linear materials, it is generally possible, and usually most convenient to write a single continuous expression relating axial load, moment and curvature. Unfortunately, the response of reinforced concrete to external effects is very difficult to evaluate, due to the nonlinear behavior of the concrete. Instead, it is more convenient to define the relationship between P, M and ϕ by a number of discrete points—The strain-gradient method.

The determination of the relation between P, M and ϕ evolves directly from the application of ordinary principles of mechanics of materials. If a linear distribution of strain across the section is assumed and the stress-strain relationships known or assumed for both concrete and steel, the elemental force on each element of material can be determined. By algebraically summing these elemental forces, the external moment and force necessary are found.

The method assumes a linear strain distribution over the depth of the section is as shown in Fig. (4.2). A generalized stress-strain characteristic for concrete, was derived by Hognestad as given in Sec. 4.1. The characteristics for the steel were taken directly from the experimental results. From the stress-strain relationships for the concrete and the reinforcement, the distribution of stresses over the cross-section can be determined as shown in Fig. (4.2.c) and (d).

From these stresses, the external force and moment necessary to keep the member in equilibrium is found. For instance, the external moment is the algebraic sum of the external applied moment plus the axial load deformation moment. The moment curvature relationship will be obtained by varying the strain gradient and keeping the axial load equal to the desired value by moving the position of the neutral axis.

It is obvious that, the moment capacity which is available to carry external applied moment is equal to the member moment capacity less the moment capacity required by the axial load-lateral deformation moment.

Strain-Gradient Method :-

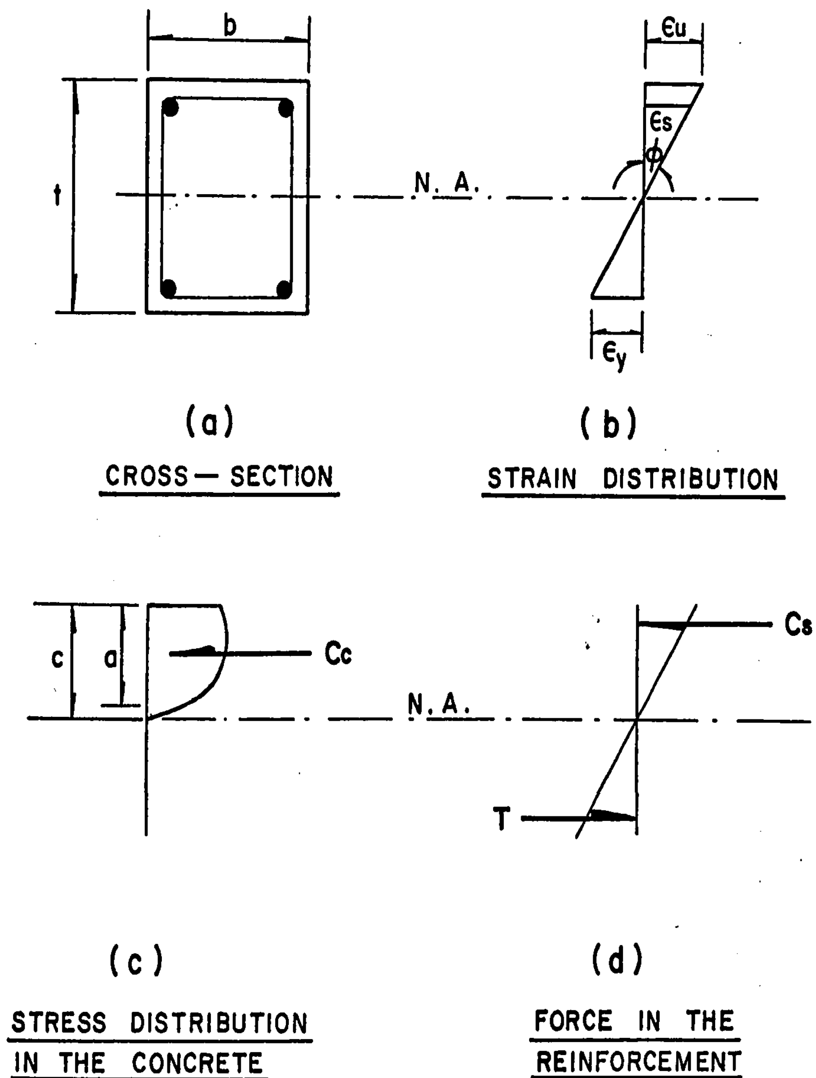


FIG. (4.2.) METHOD OF ANALYSIS

The method is iterative, it is convient to list the steps as follows:-

(1) Specify an axial load.

(2) Select a strain gradient.

(3) Select the position of the neutral axis.

(4) Determine the curvature by computing the angle between the line representing the strain distribution and the assumed position of the neutral axis.

(5) Sum algebraically the stress times the area over which they act to obtain the calculated axial load under equilibrium. If the calculated axial load and the chosen load agree with 1% — go to next step. If not, repeat step (3).

(6) Sum the bending moment caused by the normal stress to obtain the resulting bending moment.

(7) Increase strain gradient and obtain the maximum external moment until the maximum strain in the concrete reaches 0.003.

(8) Write out the calculated results of maximum axial load and maximum external moment for that curvature.

(9) Assign a new value of axial load until all experimental results have been considered.

If a sufficient number of discrete points relating P, M and ϕ are determined, the load-moment-curvature relationship for the section will be described.

4.2.1 Reinforced Concrete Columns under Balanced Failure

As shown in Fig. (4.2.1.1), the balance point 'b' is a common point to both the failure and yield interaction diagram, simultaneously, a strain of 0.003 in the extreme fibre of concrete and the yield strain in the tension steel. From the stress-strain diagram in Fig. (4.2.1.2.a) gives,

$$T = A_s f_{s1}$$

$$C_c = 0.85 f_c' ab$$

$$C_s = A_s f_{s2}$$

See notation

where

$$a = K_1 \cdot c$$

$$P_u' = C_c - T + C_s \quad \text{......................(4.2.1.1)}$$

$$M_u' = C_c \,(t/2 - a/2) + T\,(t/2 - d')$$

$$+ C_s\,(t/2 - d') \quad \text{..................(4.2.1.2)}$$

Solve Equations (4.2.1.1) and (4.2.1.2) by trial and error method.

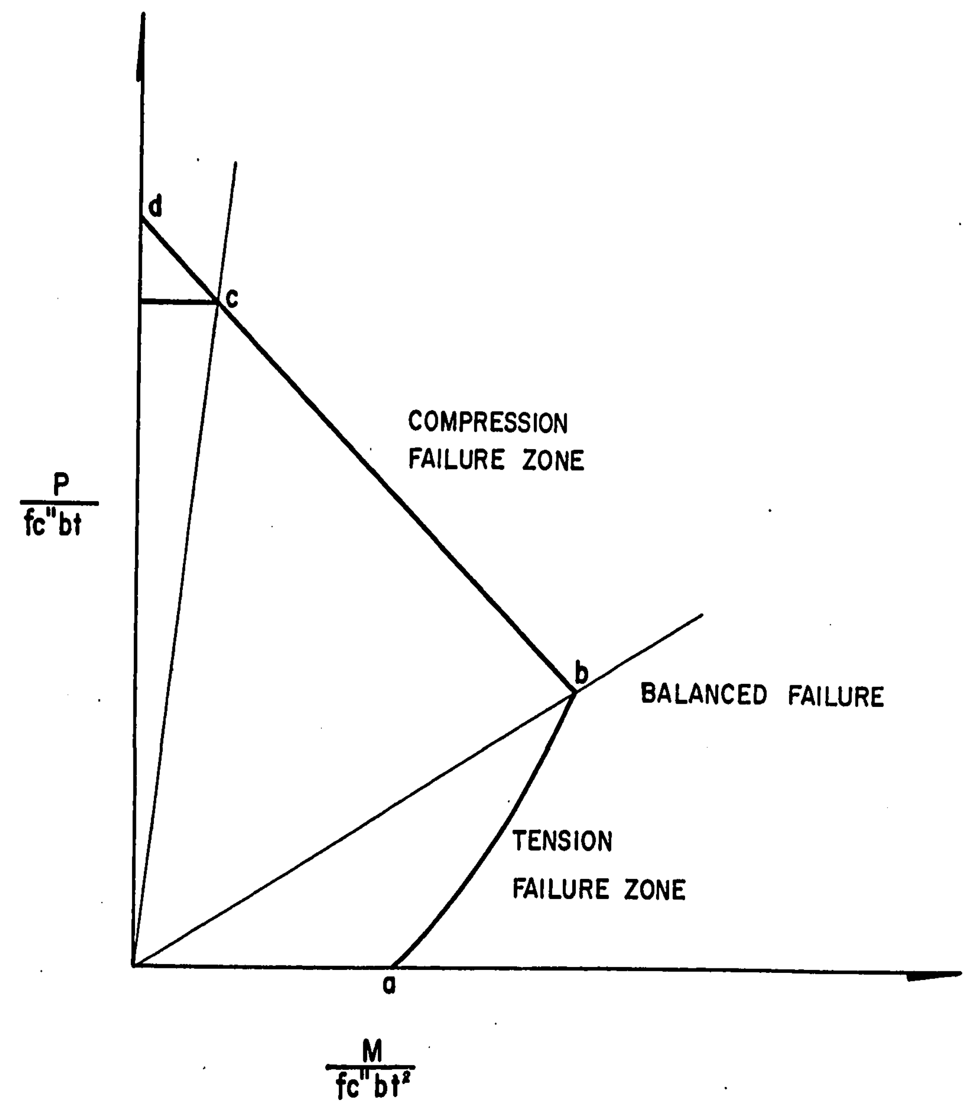


FIG.(4.2.1.1) INTERACTION DIAGRAM

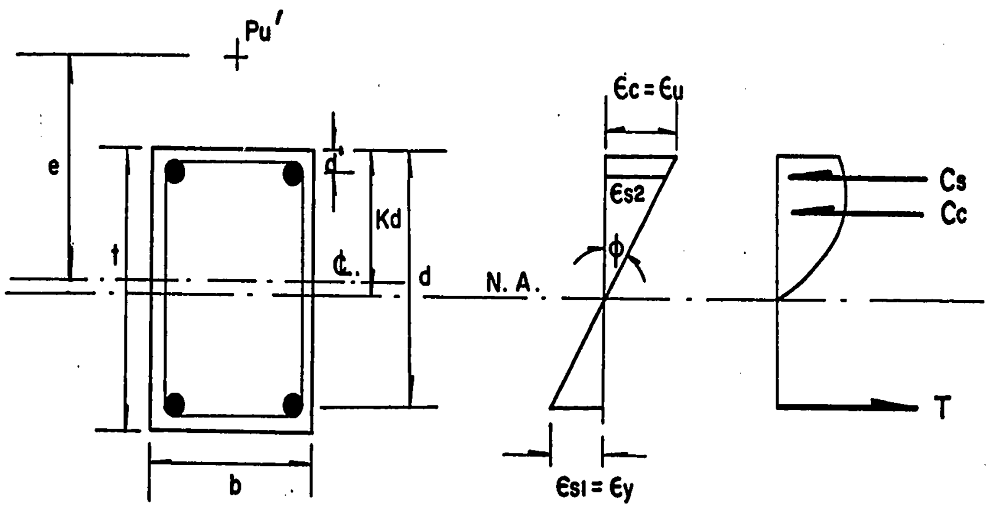


FIG. (4.2.1.2.a) POINT b (FIG. 4.2.1.1.)

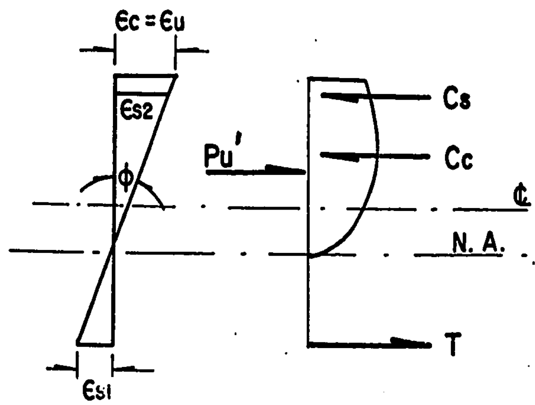


FIG. (4.2.1.2.b) LINE b—c

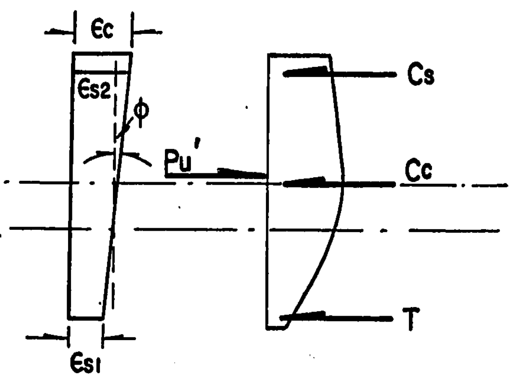


FIG.(4.2.1.2.c) LINE c—d

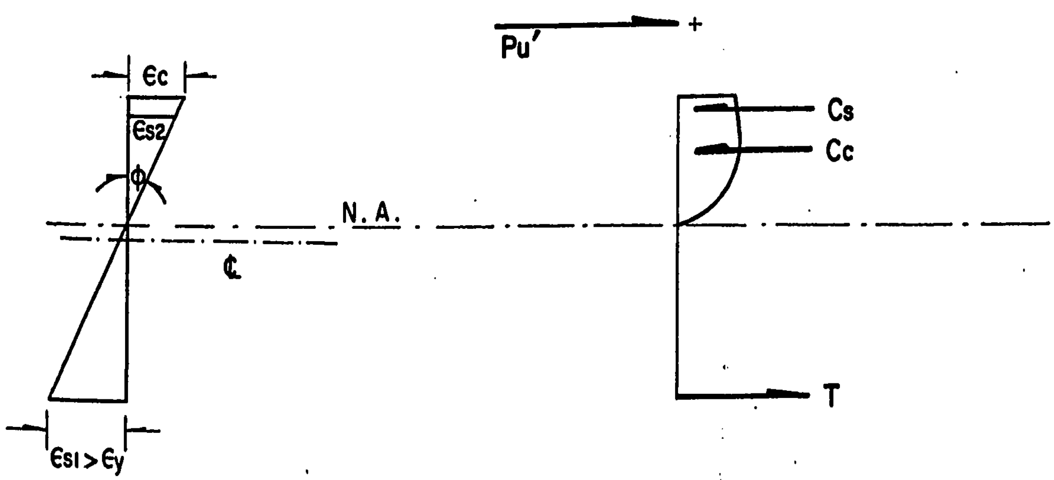


FIG. (4.2.1.2.d) LINE a—b

FIG.(4.2.1.2) STRESS — STRAIN RELATIONSHIP

4.2.2 Reinforced Concrete Columns under Compression Failure

For moderate eccentricities, when compression governs, the behavior of the column is represented by the line b-c in the interaction diagram in Fig. (4.2.1.1).

From the stress-strain diagram in Fig. (4.2.1.2.b) the equilibrium equation applies

$$P_u' = C_c - T + C_s \quad \text{.............................(4.2.1.3)}$$

$$M_u' = C_c\,(t/2 - a/2) + C_s\,(t/2 - d') + T\,(t/2 - d') \quad \text{....................(4.2.1.4)}$$

For zero or small eccentricities, as the interval line d-c, in the interaction diagram, and considering equilibrium as in Fig. (4.2.1.2.c), gives

$$P_u' = C_c + C_s + T \quad \text{.............................(4.2.1.5)}$$

$$M_u' = C_c\,(t/2 - c/2) - T\,(t/2 - d') + C_s\,(t/2 - d') \quad \text{....................(4.2.1.6)}$$

4.2.3 Reinforced Concrete Beam-columns under Tension Failure

For large eccentricities, when tension governs, as represented by the curve line a-b in the interaction diagram, and from the Fig. (4.2.1.2.d), we have,

$$P_u' = C_c + C_s - T \quad \text{.............................(4.2.1.7)}$$

$$M_u' = C_c\ (t/2 - a/2) + T\ (t/2 - d') + C_s\ (t/2 - d')$$

$$\text{.....................(4.2.1.8)}$$

When eccentricity become infinity, as the point a in the interaction diagram, we have,

$$T = C_c + C_s \quad \text{...............................(4.2.1.9)}$$

$$M_u^T = C_c\ (d - a/2) + C_s\ (d - d') \quad \text{................(4.2.1.1.0)}$$

and

$$P_u^T = \frac{6M_u^T}{L}$$

where L = span length (53")

CHAPTER V

DISCUSSION AND SUMMARY OF TEST RESULTS

5.1 General Behaviour and Modes of Failure of Specimens

All specimens with the eccentricity equal to 0.00±0.01; 0.10±0.01; 0.30±0.01; 0.50±0.01 and 0.75±0.01, failed in two modes: compression failure and tension failure. Only a few columns had the combination of the variables that the mode of failure was close enough to the balanced conditions.

All those specimens, which were designed as beam-columns, with the eccentricity equal to infinity failed by tension failure.

The observed modes of failure of the individual test columns are listed in Table (5.1), page 53, and the various typical failure phenomena are shown in Figs. (5.1.1) to (5.1.4), on page 56 to page 59. The load-deflection curves for each member are presented in Appendix B, from Fig. (B.1) page 112 to Fig. (B.85) page 196, the maximum load in each curve indicates the failure load for the member.

FIG. (5.1) TEST RESULT OF GROUP I, II, III, & IV

SPECIMEN NO.	f_c' (psi.)	REINFORCEMENT TYPE	REINFORCEMENT f_y(psi.)	e/t	$P_{exp.}$ (Kips)	MODE OF FAILURE*
1A-1 1A-2 1A-3 1A-4 1A-5	———	Deformed	———	0.0	———	———
1B-1 1B-2 1B-3 1B-4 1B-5	———	Deformed	———	0.1	———	———
1C-1 1C-2 1C-3 1C-4 1C-5	3,500 c.of v. (3.5%)	Deformed	67,400	0.3	112.00 110.00 110.50 112.12 115.00	C C C C C
1D-1 1D-2 1D-3 1D-4 1D-5	3,500 c.of v. (3.5%)	Deformed	67,400	0.5	74.50 72.50 68.00 69.00 70.00	C C C C C
1E-1 1E-2 1E-3 1E-4 1E-5	4,300 c.of v. (7.5%)	Deformed	67,400	∞	25.00 24.94 25.00 22.00 24.98	T T T T T
2B-1 2B-2 2B-3 2B-4 2B-5	———	———	———	0.1	———	———
2C-1 2C-2 2C-3 2C-4 2C-5	4,600 c.of v. (5.0%)	Deformed	52,600	0.3	100.00 104.00 112.50 105.50 106.50	C C C C C

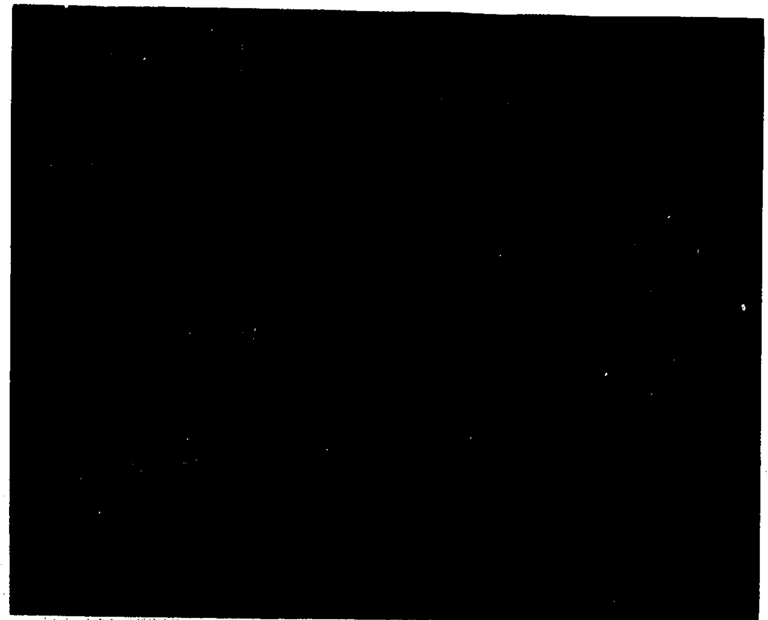
Continued

SPECIMEN NO.	f_c' (psi.)	REINFORCEMENT TYPE	f_y(psi.)	e/t	$P_{exp.}$ (Kips)	MODE OF FAILURE*
2D-1 2D-2 2D-3 2D-4 2D-5	4,000 c.of v. (5.1%)	Deformed	52,600	0.5	59.85 61.77 57.43 61.90 61.10	C C C C C
2DE-1 2DE-2 2DE-3 2DE-4 2DE-5	4,600 c.of v. (5.0%)	Deformed	52,600	0.75	45.10 47.70 45.45 45.50 43.50	T T T T T
2E-1 2E-2 2E-3 2E-4 2E-5	4,000 c.of v. (5.1%)	Deformed	52,600	∞	18.00 19.60 19.96 19.30 19.70	T T T T T
3A-1 3A-2 3A-3 3A-4 3A-5	4,000 c.of v. (4.6%)	Deformed	65,500	0.0	165.00 156.50 164.60 162.00 171.00	C C C C C
3B-1 3B-2 3B-3 3B-4 3B-5	3,700 c.of v. (5.3%)	Deformed	65,500	0.1	103.10 103.90 104.00 113.00 111.00	C C C C C
3C-1 3C-2 3C-3 3C-4 3C-5	3,700 c.of v. (5.3%)	Deformed	65,500	0.3	71.33 69.00 75.82 67.50 67.50	C C C C C
3D-1 3D-2 3D-3 3D-4 3D-5	3,500 c.of v. (4.3%)	Deformed	65,500	0.5	42.66 41.71 43.98 43.15 41.55	C & T C & T C & T C & T C & T
3E-1 3E-2 3E-3 3E-4 3E-5	3,500 c.of v. (4.3%)	Deformed	65,500	∞	11.50 11.74 12.39 11.92 11.37	T T T T T

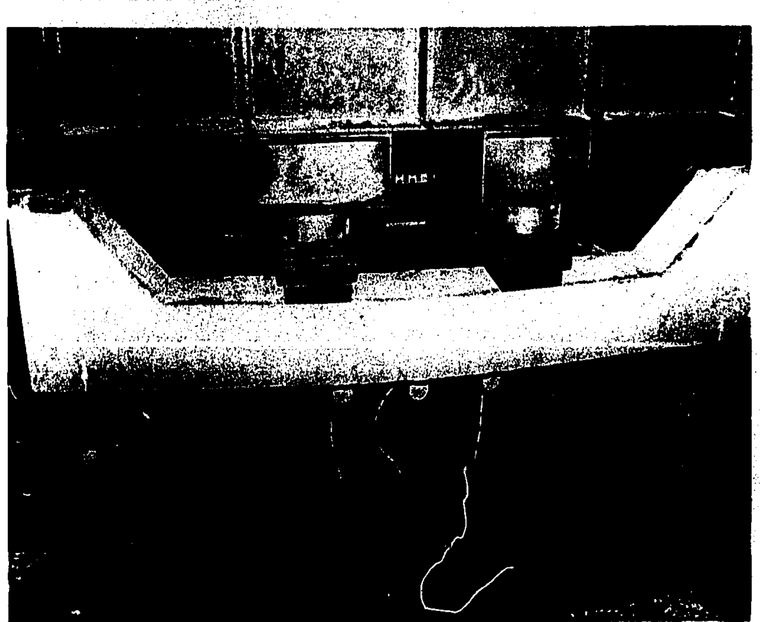
Continued

SPECIMEN NO.	f_c' (psi.)	REINFORCEMENT TYPE	REINFORCEMENT f_y(psi.)	e/t	$P_{exp.}$ (Kips)	MODE OF FAILURE*
4B-1 4B-2 4B-3 4B-4 4B-5	3,200 c.of v. (7.3%)	Plain	54,400	0.1	88.25 80.92 82.11 88.51 82.01	C C C C C
4C-1 4C-2 4C-3 4C-4 4C-5	4,600 c.of v. (7.9%)	Plain	54,400	0.3	61.00 65.30 55.00 60.00 67.00	C & T C & T C & T C & T C & T
4D-1 4D-2 4D-3 4D-4 4D-5	4,900 c.of v. (6.0%)	Plain	54,400	0.5	30.50 30.74 30.50 30.57 31.77	T T T T T
4DE-1 4DE-2 4DE-3 4DE-4 4DE-5	4,100 c.of v. (4.9%)	Plain	54,400	0.75	18.63 16.30 18.35 17.85 21.95	T T T T T
4E-1 4E-2 4E-3 4E-4 4E-5	4,900 c.of v. (6.0%)	Plain	74,500	∞	8.32 7.85 8.55 8.15 8.54	T T T T T

* C, T and C & T indicate compression, tension and near balance failure, respectively.

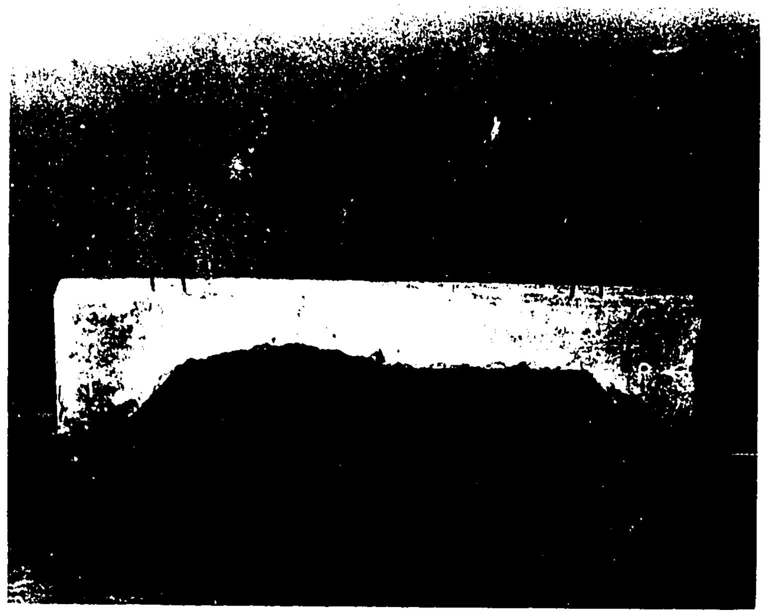


COMPRESSION FAILURE

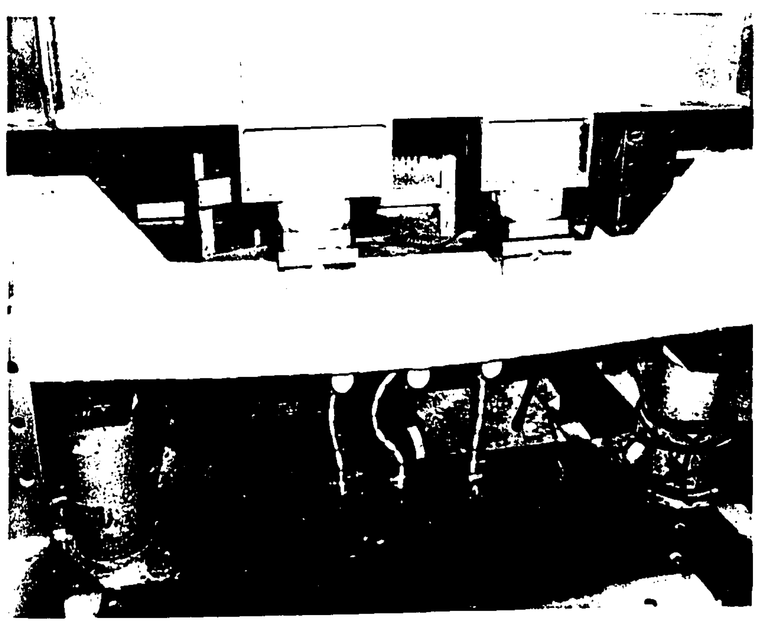


TENSION FAILURE

FIG.(5.1.1) TYPICAL FAILURE PHENOMENA OF GROUP I

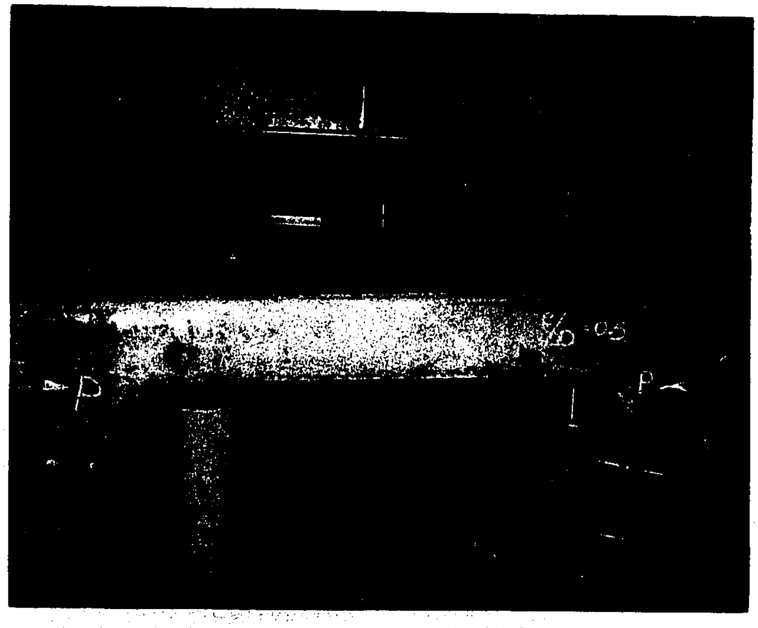


COMPRESSION FAILURE

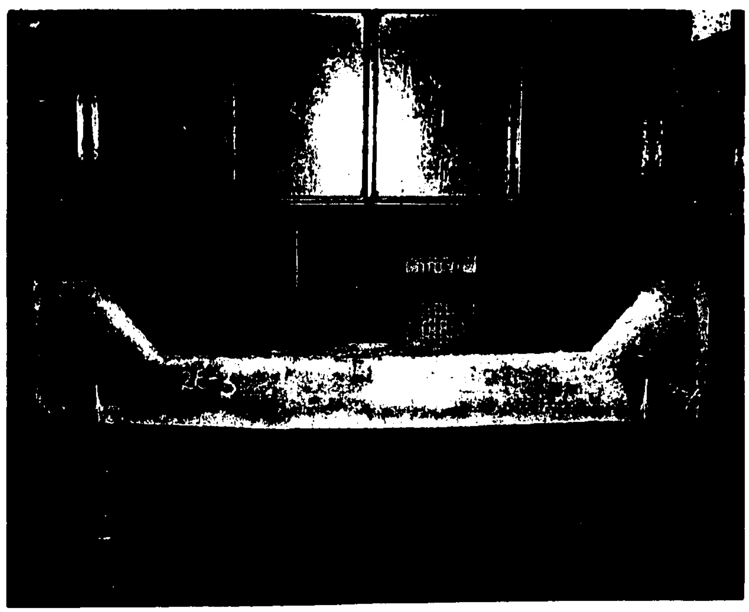


TENSION FAILURE

FIG. (5.1.1) TYPICAL FAILURE PHENOMENA OF GROUP I

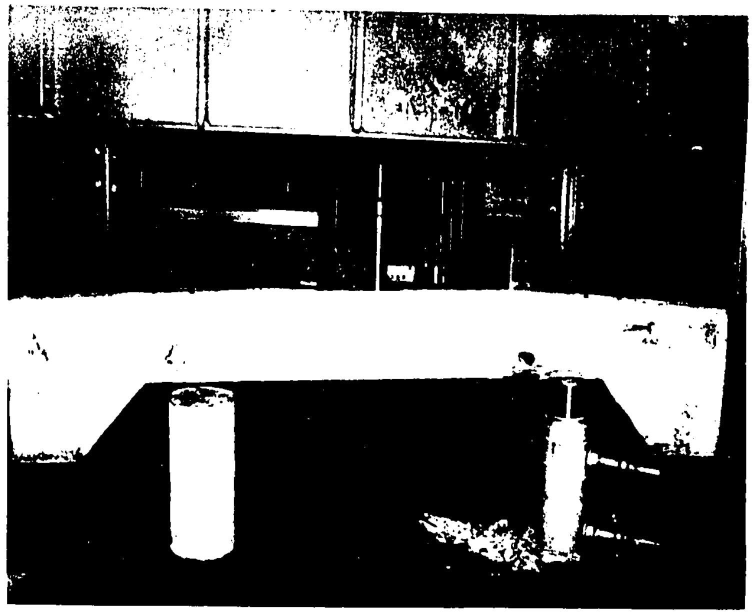


COMPRESSION FAILURE



TENSION FAILURE

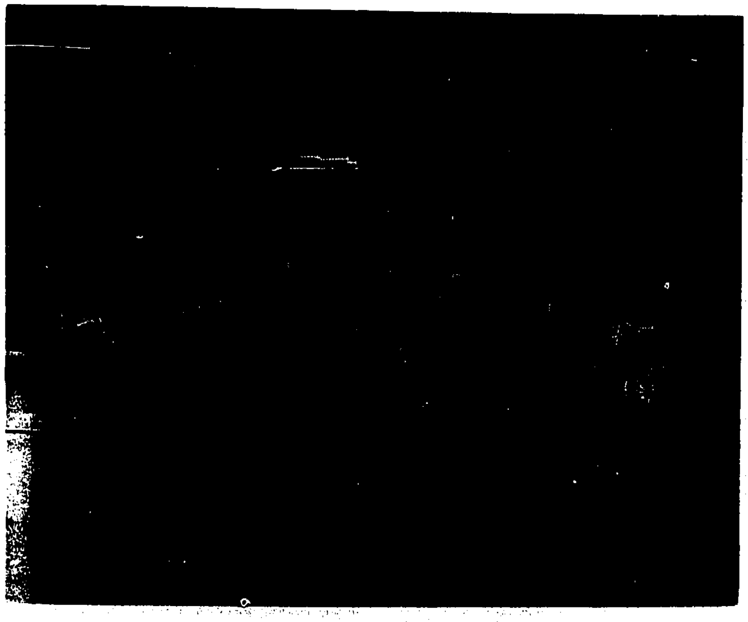
FIG. (5.1.2) TYPICAL FAILURE PHENOMENA OF GROUP 2



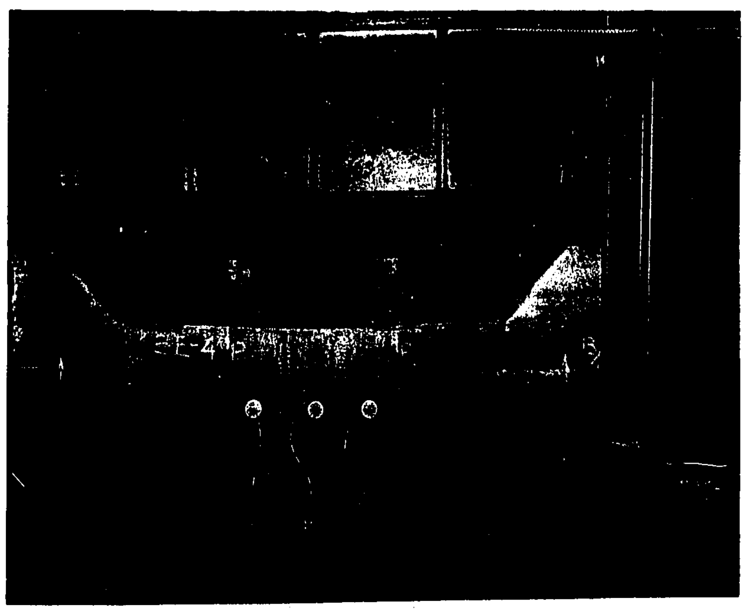
COMPRESSION FAILURE

TENSION FAILURE

FIG. (5.1.2) TYPICAL FAILURE PHENOMENA OF GROUP 2

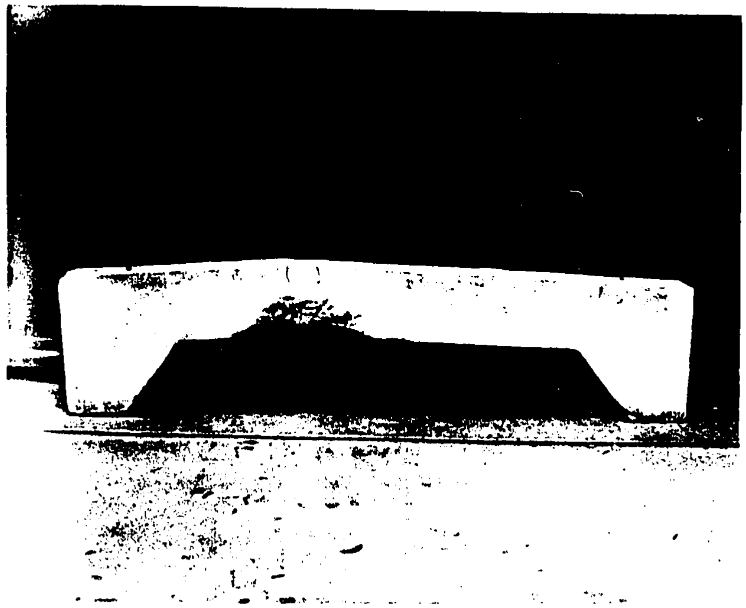


COMPRESSION FAILURE

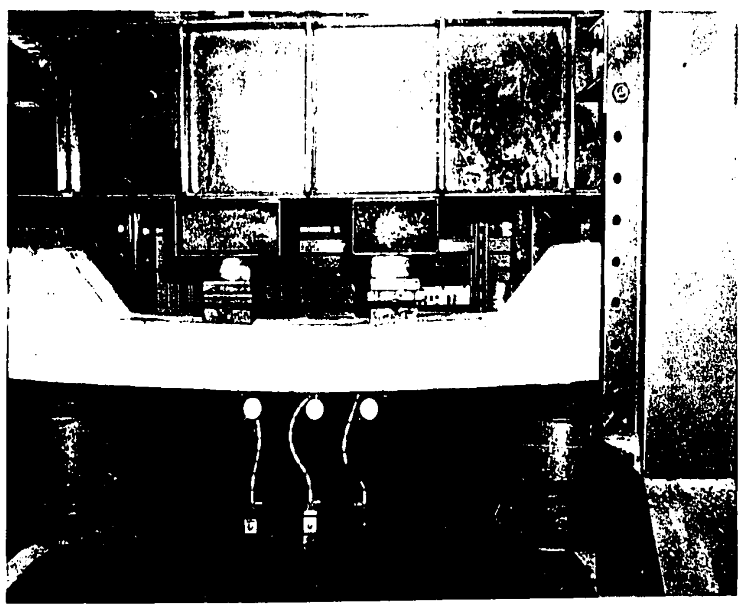


TENSION FAILURE

FIG.(5.1.3) TYPICAL FAILURE PHENOMENA OF GROUP 3



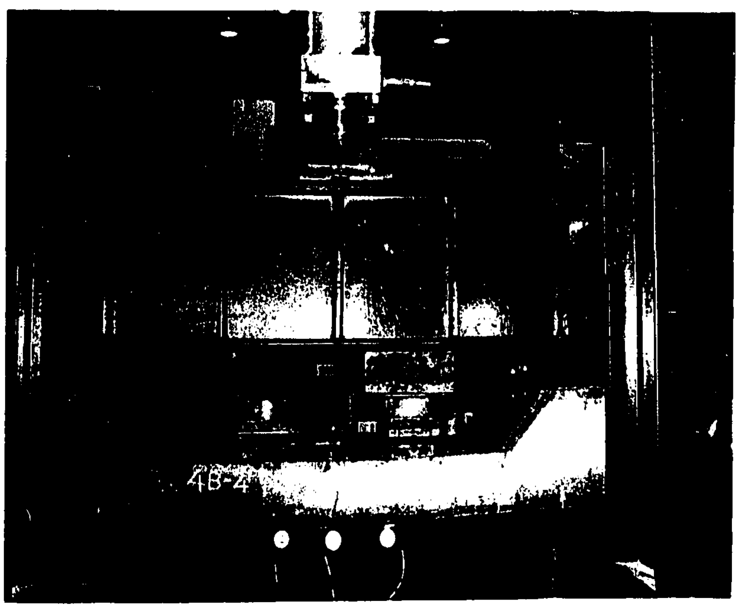
COMPRESSION FAILURE



TENSION FAILURE

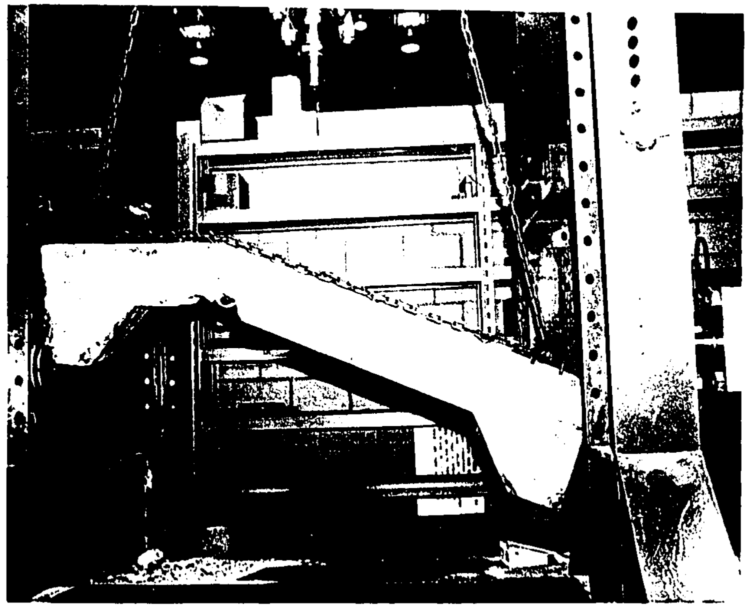
FIG.(5.1.3) TYPICAL FAILURE PHENOMENA OF GROUP 3

COMPRESSION FAILURE

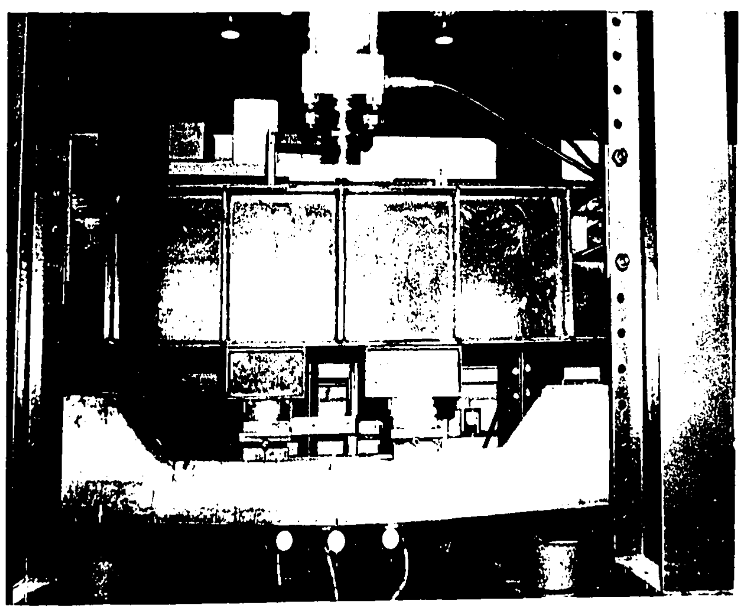


TENSION FAILURE

FIG.(5.1.4) TYPICAL FAILURE PHENOMENA OF GROUP 4



COMPRESSION FAILURE



TENSION FAILURE

FIG.(5.1.4) TYPICAL FAILURE PHENOMENA OF GROUP 4

5.2 Analysis of Experimental Results

Among the 100 specimens tested, 15 failed in shear through the capitals of the specimens. The reason was insufficient embedded bar reinforcement to reinforce the capitals against shear failure. To solve this problem, four auxiliary tension rods were used to clamp plates around each capital through the rest of the tests.

The results of the remaining 85 tests are believed to be significant, and yet are subjected to various errors as indicated below.

(1) Small initial eccentricities due to errors in centering and adjusting the specimens in the testing machines, and the nonhomogenous properties of concrete along the transverse direction.

(2) The position of the reinforcing bars in the cross-section of the member may vary ±1/8 in. from the nominal values. This error is not important for compression failure members, but for tension failure members, it changes the interal moment arm and thus affects the ultimate loads.

(3) The rate of applying load increments, and the total time that used for testing each column specimen can not be controlled to be identical.

Hence, it is possible but not too significant that the ultimate loads, may have been influenced by the uncontrolled time factors.

5.3 Analysis of Theoretical Results

Both the results of the American Concrete Institute design equations and the strain gradient method were obtained by computer programs. Table (5.3) on page 62 shows the results obtained from the ACI Design Formulae, and from the strain gradient method.

5.4 Comparision of Test Results with the ACI (American Concrete Institute) Design Formulae

The test results show general agreement with the design formulae for the full range of conditions from axially-loaded columns to beam-columns under pure bending. The actual strengths for all specimens under all cases are slightly larger than the calculated values as shown in Table (5.4) on page 64. The arithmetic mean of the ratios of measured to computed ultimate load is 1.04, the overall standard deviation is 0.09, and the overall coefficient of variation is 8.94 per cent.

Due to the known variations of concrete strength, steel yield stress and dimension, the measured results will not necessarily be equal to the theoretical results but still may be significant.

From the measured variabilities of the concrete cylinder stress (8%) and the steel yield stress (3%) and assuming a dimensional variability of 3% it can be shown that only deviations of measured to calculated loads greater than 6% are significant.

Hence within the accuracy of the experimental results the ACI design formulae accurately predicts the failure load of columns under combined axial load and moment.

TABLE (5.3) CALCULATED VALUES P_{ult} OF ACI DESIGNED FORMULARS AND STRAIN GRADIENT METHOD (KIPS)

SPECIMEN NO.	e/t	P_{exp}	P_{ACI}	P_{SGM}^{*}	SPECIMEN NO.	e/t	P_{exp}	P_{ACI}	P_{SGM}^{*}
4E-1		8.32	8.83	8.15	3D-1		42.66	44.11	4.164
4E-2		7.85	8.83	8.15	3D-2		41.71	38.79	41.64
4E-3	∞	8.55	8.83	8.92	3D-3	0.5	43.98	43.49	43.95
4E-4		8.15	8.83	8.15	3D-4		43.15	43.92	43.38
4E-5		8.54	8.83	8.92	3D-5		41.55	43.83	41.64
4D-1		30.50	28.09	30.39	3C-1		71.33	59.32	71.44
4D-2		30.75	28.18	30.83	3C-2		69.00	58.24	68.75
4D-3	0.5	30.50	25.90	30.39	3C-3	0.3	75.82	63.82	75.13
4D-4		30.57	28.73	30.83	3C-4		67.50	60.79	67.08
4D-5		31.77	26.95	31.94	3C-5		67.50	61.05	66.93
4C-1		61.00	55.68	61.37	3B-1		103.10	98.56	102.92
4C-2		65.38	55.68	64.73	3B-2		103.90	98.45	102.92
4C-3	0.3	55.00	54.47	54.76	3B-3	0.1	104.00	98.64	103.53
4C-4		60.00	54.11	59.74	3B-4		113.00	98.79	112.33
4C-5		67.00	58.47	67.03	3B-5		111.00	98.84	110.34
4DE-1		18.63	15.84	19.50	3A-1		165.00	159.60	164.52
4DE-2		16.30	15.90	16.40	3A-2		165.00	159.60	156.44
4DE-3	0.75	18.35	15.90	18.34	3A-3	0.0	164.60	159.40	163.29
4DE-4		17.85	16.00	17.74	3A-4		162.00	159.70	159.31
4DE-5		21.95	16.16	22.03	3A-5		171.00	159.50	169.84
4B-1		88.25	69.54	87.96	2E-1		18.00	19.47	17.11
4B-2		80.92	73.18	80.69	2E-2		19.60	19.59	19.61
4B-3	0.1	82.11	69.50	82.46	2E-3	∞	19.96	19.59	19.59
4B-4		88.51	68.99	88.04	2E-4		19.30	19.59	19.92
4B-5		82.01	71.49	82.07	2E-5		19.69	20.15	19.57
3E-1		11.50	11.77	12.01	2DE-1		45.10	46.23	45.21
3E-2		11.74	11.77	12.20	2DE-2		47.70	46.43	47.54
3E-3	∞	12.39	12.11	12.78	2DE-3	0.75	45.45	45.25	45.79
3E-4		11.92	12.11	12.51	2DE-4		45.30	43.38	45.67
3E-5		11.37	11.77	11.89	2DE-5		43.50	44.88	45.18

Continued

SPECIMEN NO.	e/t	P_{exp}	P_{ACI}	P_{SGM}*	SPECIMEN NO.	e/t	P_{exp}	P_{ACI}	P_{SGM}*
2D-1		59.85	59.13	60.10	1D-1		74.50	74.07	73.94
2D-2		61.77	58.85	62.36	1D-2		72.50	72.96	72.02
2D-3	0.5	57.43	55.85	57.83	1D-3	0.5	68.00	71.98	68.10
2D-4		61.90	59.22	62.36	1D-4		69.00	73.42	68.59
2D-5		61.10	60.59	60.90	1D-5		70.00	72.61	69.51
2C-1		100.00	93.51	99.81	1C-1		112.00	110.67	110.98
2C-2		104.00	94.51	102.99	1C-2		111.00	114.20	110.83
2C-3	0.3	112.50	95.62	112.82	1C-3	0.3	110.50	106.93	110.83
2C-4		105.50	97.20	105.58	1C-4		112.12	106.69	115.65
2C-5		106.50	96.36	106.73	1C-5		115.00	104.91	114.44
1E-1		25.00	27.62	25.63					
1E-2		24.94	27.62	25.45					
1E-3	∞	25.00	27.62	26.12					
1E-4		22.00	27.28	21.71					
1E-5		24.98	27.62	25.76					

NOTE: Results obtained from ACI (American Concrete Institute) Design Formalae.

* Results obtained from Strain Gradient Method.

TABLE (5.4) COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS BY ACI FORMULAE (KIPS)

SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$
4E-1		8.32	8.83	0.94	3D-1		42.66	44.11	0.97
4E-2		7.85	8.83	0.89	3D-2		41.71	38.79	1.04
4E-3	∞	8.55	8.83	0.97	3D-3	0.5	43.98	43.49	1.01
4E-4		8.15	8.83	0.92	3D-4		43.15	43.92	0.98
4E-5		8.54	8.83	0.97	3D-5		41.55	43.83	0.94
4DE-1		18.63	15.84	1.17	3C-1		71.33	59.32	1.20
4DE-2		16.30	15.90	1.02	3C-2		69.00	58.24	1.18
4DE-3	0.75	18.35	15.90	1.15	3C-3	0.3	75.82	63.82	1.18
4DE-4		17.85	16.00	1.11	3C-4		67.50	60.79	1.10
4DE-5		21.95	16.16	1.35	3C-5		67.50	61.05	1.10
4D-1		30.50	28.09	1.08	3B-1		103.10	98.56	1.04
4D-2		30.75	28.18	1.09	3B-2		103.90	98.45	1.05
4D-3	0.5	30.50	25.90	1.17	3B-3	0.1	104.00	98.64	1.05
4D-4		3057	28.73	1.06	3B-4		113.00	107.60	1.05
4D-5		31.77	26.95	1.17	3B-5		111.00	104.30	1.05
4C-1		61.00	55.68	1.09	3A-1		165.00	159.60	1.04
4C-2		65.30	55.68	1.17	3A-2		156.60	159.60	0.99
4C-3	0.3	55.00	54.47	1.01	3A-3	0.0	164.60	159.40	1.04
4C-4		60.00	54.11	1.09	3A-4		162.00	159.70	1.02
4C-5		67.00	58.47	1.14	3A-5		171.00	159.50	1.00
4B-1		88.25	69.54	1.26	2E-1		18.00	19.47	0.93
4B-2		80.92	73.18	1.10	2E-2		19.60	19.59	1.00
4B-3	0.1	82.11	69.50	1.18	2E-3	∞	19.96	19.59	1.02
4B-4		88.51	68.99	1.28	2E-4		19.30	19.59	0.99
4B-5		82.01	71.49	1.14	2E-5		19.69	20.15	0.97
3E-1		11.50	11.77	0.98	2DE-1		45.10	46.23	0.97
3E-2		11.74	11.77	1.00	2DE-2		47.70	46.43	1.02
3E-3	∞	12.39	12.11	1.02	2DE-3	0.75	45.45	45.25	1.02
3E-4		11.92	12.11	0.98	2DE-4		45.30	43.38	1.04
3E-5		11.37	11.77	0.97	2DE-5		43.50	44.88	0.96

Continued

SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$
2D-1		59.85	59.13	1.01	1D-1		74.50	74.07	1.00
2D-2		61.77	58.85	1.04	1D-2		72.50	72.96	0.99
2D-3	0.5	57.43	55.85	1.03	1D-3	0.5	68.00	71.98	0.95
2D-4		61.90	59.22	1.04	1D-4		69.00	73.42	0.93
2D-5		61.10	60.59	1.01	1D-5		70.00	72.61	0.96
2C-1		100.00	93.51	1.06	1C-1		112.00	110.67	1.01
2C-2		104.00	94.51	1.10	1C-2		111.00	114.20	0.97
2C-3	0.3	112.50	95.62	1.08	1C-3	0.3	110.50	106.93	1.03
2C-4		105.50	97.20	1.08	1C-4		112.12	106.69	1.05
2C-5		106.50	96.36	1.10	1C-5		115.00	104.91	1.09
1E-1		25.00	27.62	0.90					
1E-2		24.94	27.62	0.90					
1E-3	∞	25.00	27.62	0.90					
1E-4		22.00	27.28	0.81					
1E-5		24.98	27.62	0.90					

Mean value of P_{exp}/P_{cal} = 1.04

Standard deviation = 0.09

Coefficient of variation = 8.94%

5.4.1 Effects of Concrete Quality; Strength of Reinforcement and Eccentricity

Figs. (5.4.1) to (5.4.3) show the curves of ratio of measured* to computed ultimate load against concrete strength, reinforcement ratio and eccentricity ratio, respectively. It may be observed that the ratio of measured to computed ultimate load varied about ±10 percent from unity.

5.4.2 Interaction Diagram

Interaction diagrams are presented in Figs. (5.4.2.1) to (5.4.2.4), from page 68, to page 71; comparing the maximum measured load capacity of the specimens with the theoretical ultimate strengths. The material properties of the cross-section with a variation of concrete strength from 3400 to 4600 psi.; percentage of reinforcement varied from 1.27 (plain bar) to 7.14 percent; and steel yield strength from 43,000 to 67,400 psi.

For concentrically-loaded columns, small initial eccentricities due to errors in centering the specimens, as well as the inhomogeneous properties of the concrete along the lateral direction, will generally cause lateral deflections, and thus reduce the column strength. For eccentrically-loaded columns, the errors in entering may increase or reduce the column strength, depending on the direction of the error. The ultimate loads of concentrically-loaded columns are far more sensitive due to such centering errors, than those of eccentrically-loaded columns.

* each point represent the average ratio P_{exp}/P_{ACI} of each group.

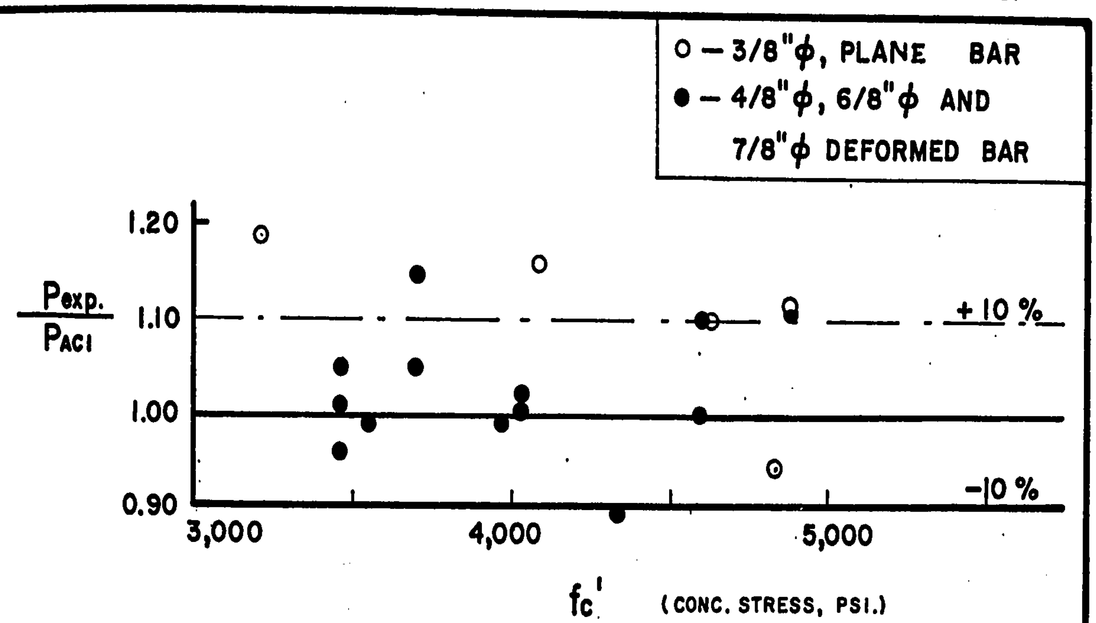


FIG.(5.4.1) Pexp./Pcal. VERSUS ULTIMATE CONCRETE STRESS

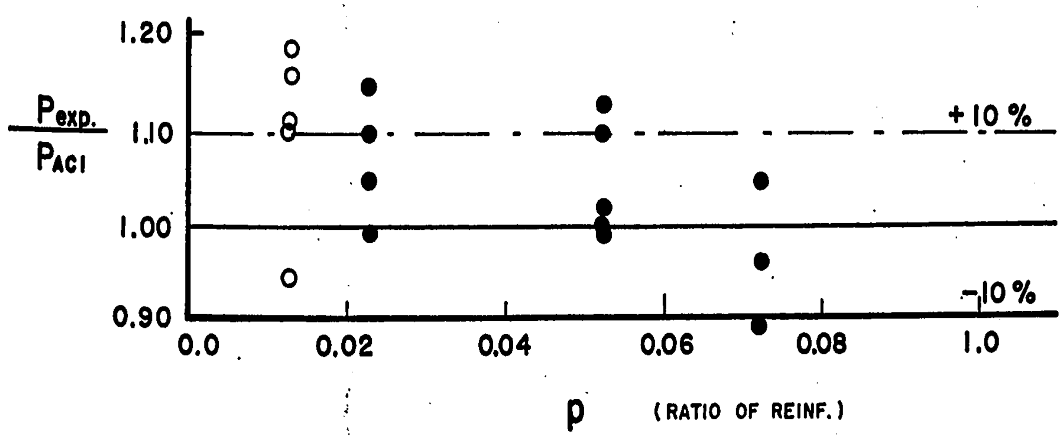


FIG.(5.4.2) Pexp./Pcal. VERSUS RATIO OF REINFORCEMENT

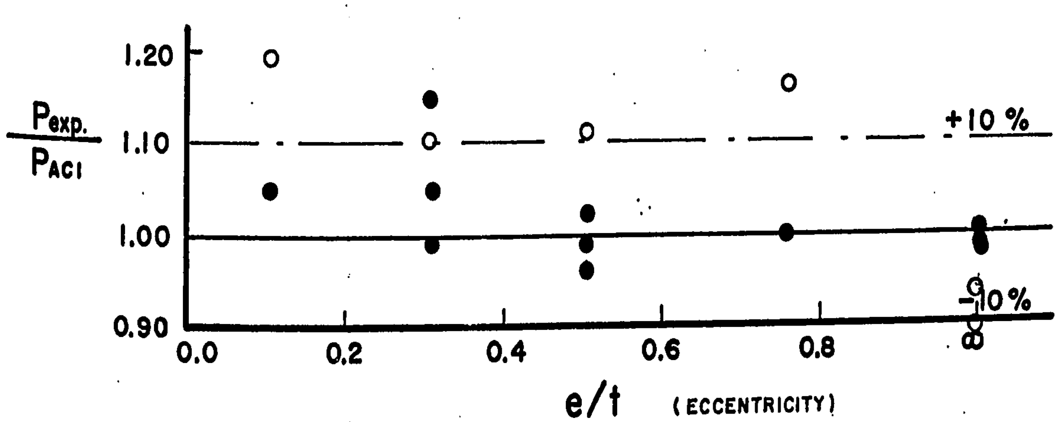


FIG.(5.4.3) Pexp./Pcal. VERSUS ECCENTRICITY RATIO OF LOAD (ACI DESIGN FORMULAE)

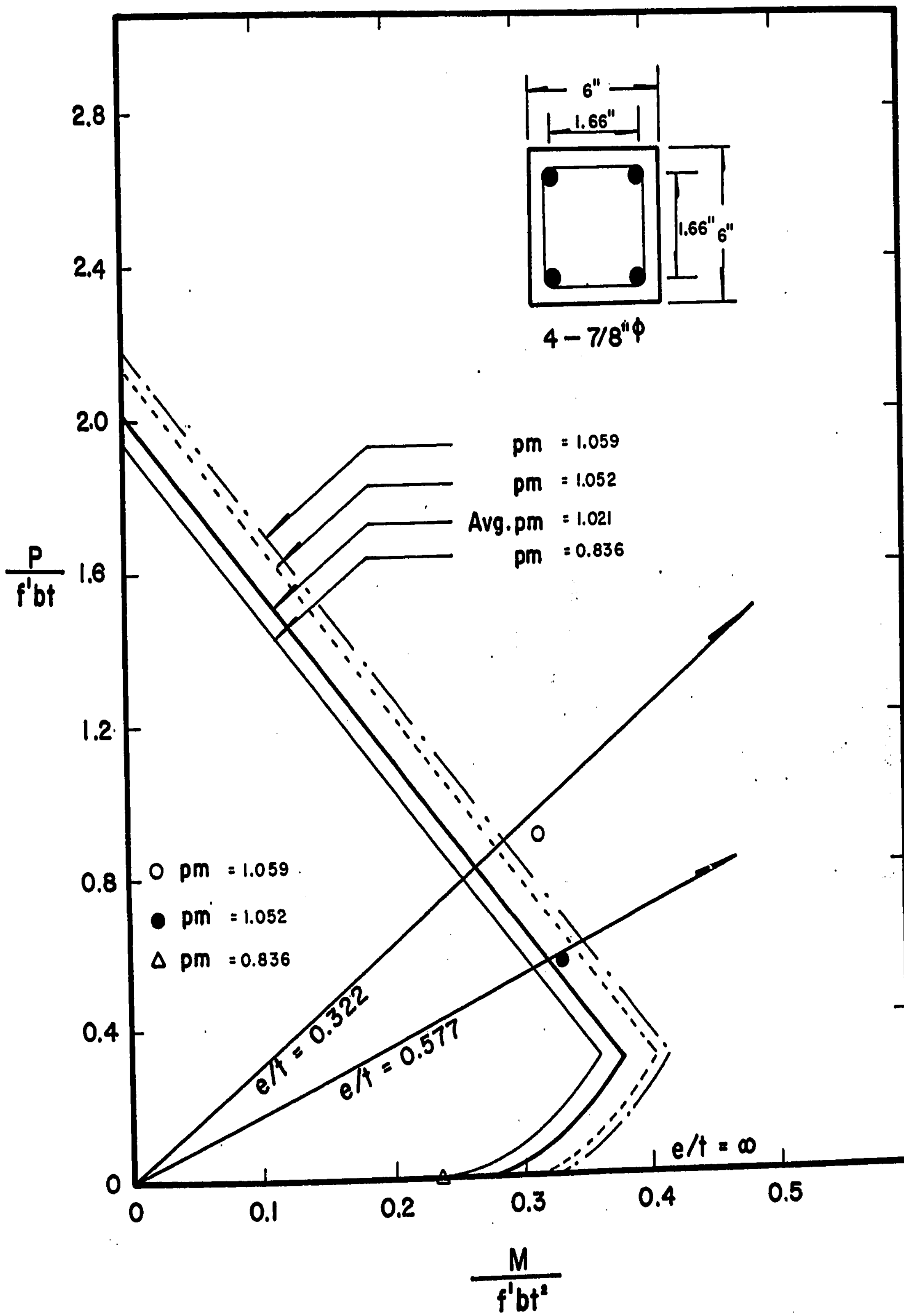


FIG. (5.4.2.1) INTERACTION DIAGRAM FOR GROUP I (ACI DESIGN FORMULAE)

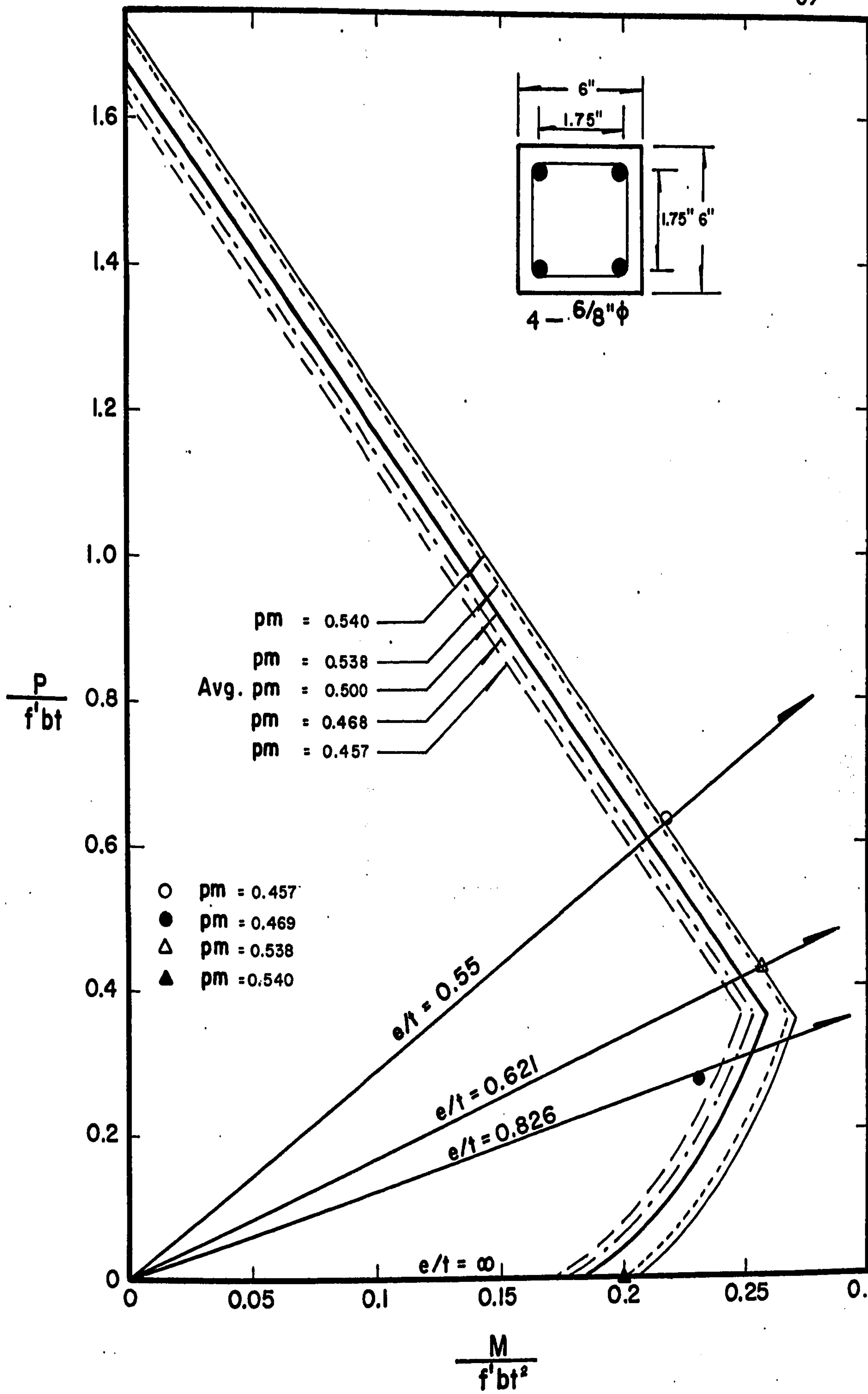


FIG.(5.4.2.2) INTERACTION DIAGRAM FOR GROUP 2 (ACI DESIGN FORMULAE)

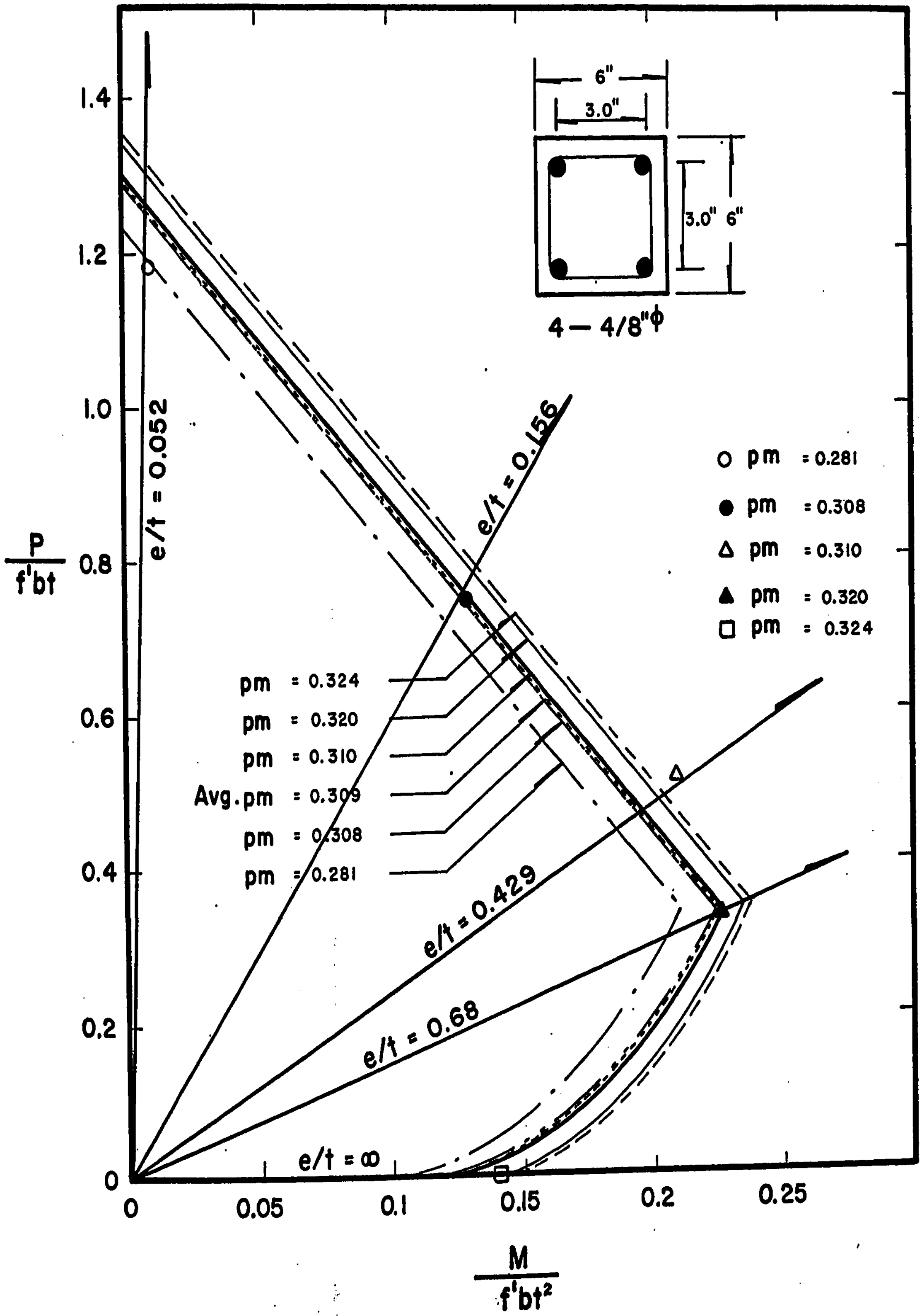


FIG. (5.4.2.3) INTERACTION DIAGRAM FOR GROUP 3

(ACI DESIGN FORMULAE)

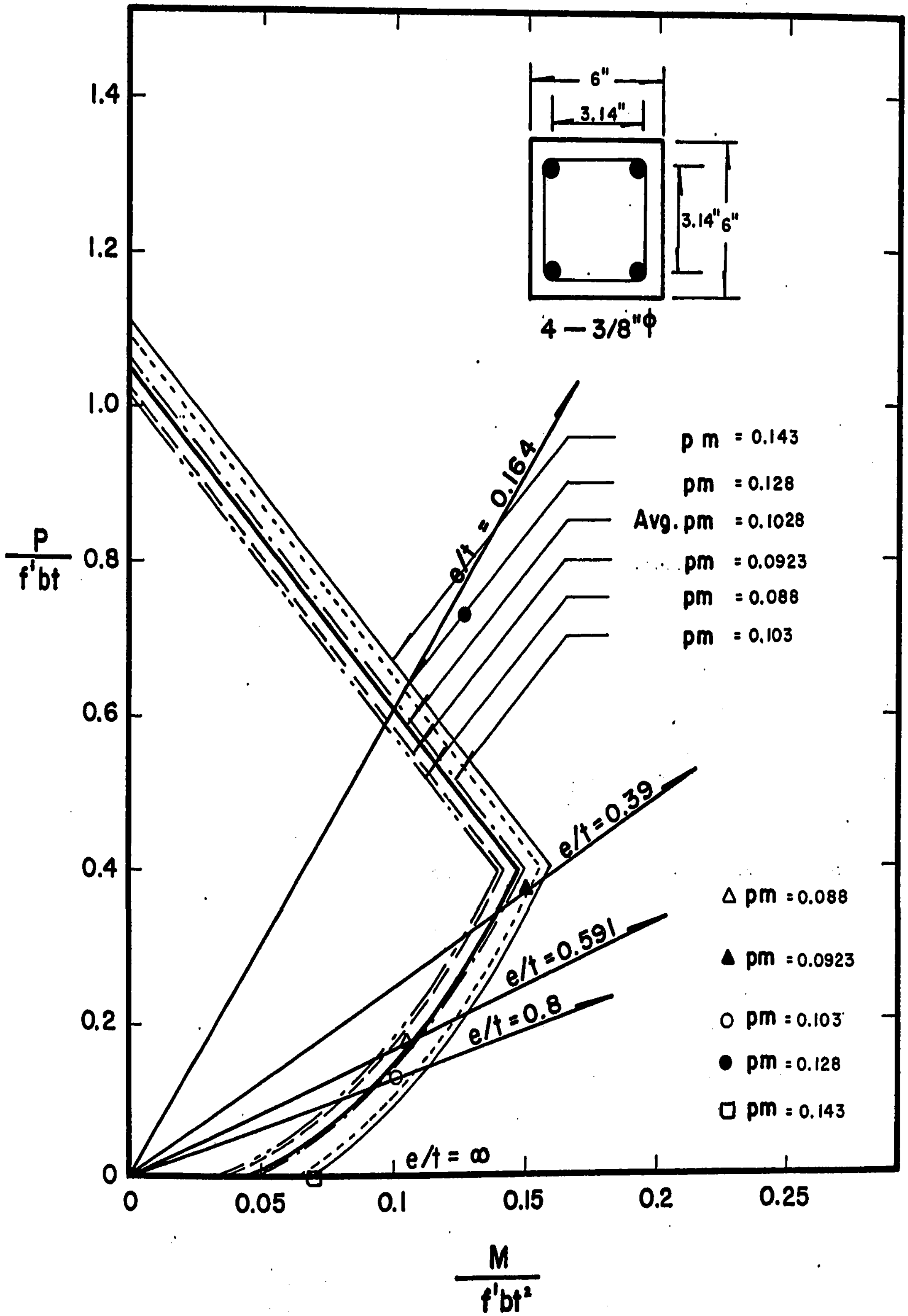


FIG.(5.4.2.4) INTERACTION DIAGRAM FOR GROUP 4

(ACI DESIGN FORMULAE)

From Figs. (5.4.2.1) to (5.4.2.4), it can be seen that the accuracy of the ACI Design formulae will generally be within ±10 percent.

5.4.3 Variation of $P_{exp.}/P_{cal.}$

A histogram and the fitted normal curve of the ratios of the experimental results to the theoretical results using the ACI formulae are shown in Fig. (5.4.3.1) of page The total population of 85 ratios was subjected to a statistical analysis.

The arithmetic mean of the total propulation is 1.04, the standard deviation of the ratios between the measured and calculated ultimate loads is 0.09 and the coefficient of variation is 8.94 percent. The probability of failure, calculated from the theoretical distribution of the ultimate load capacity, when using a ϕ factor of 0.70, under the ACI Building Code was found to be 0.03% and was 0.19% when using a ϕ factor of 0.75 under the NBC Building Code.

The test results appear to be distributed at random and no systemetic trends of variation were found. The column design formulae, using the ϕ reduction factor were so conservative that there is a very low probability of the maximum column capacity would be less than the ultimate design strength.

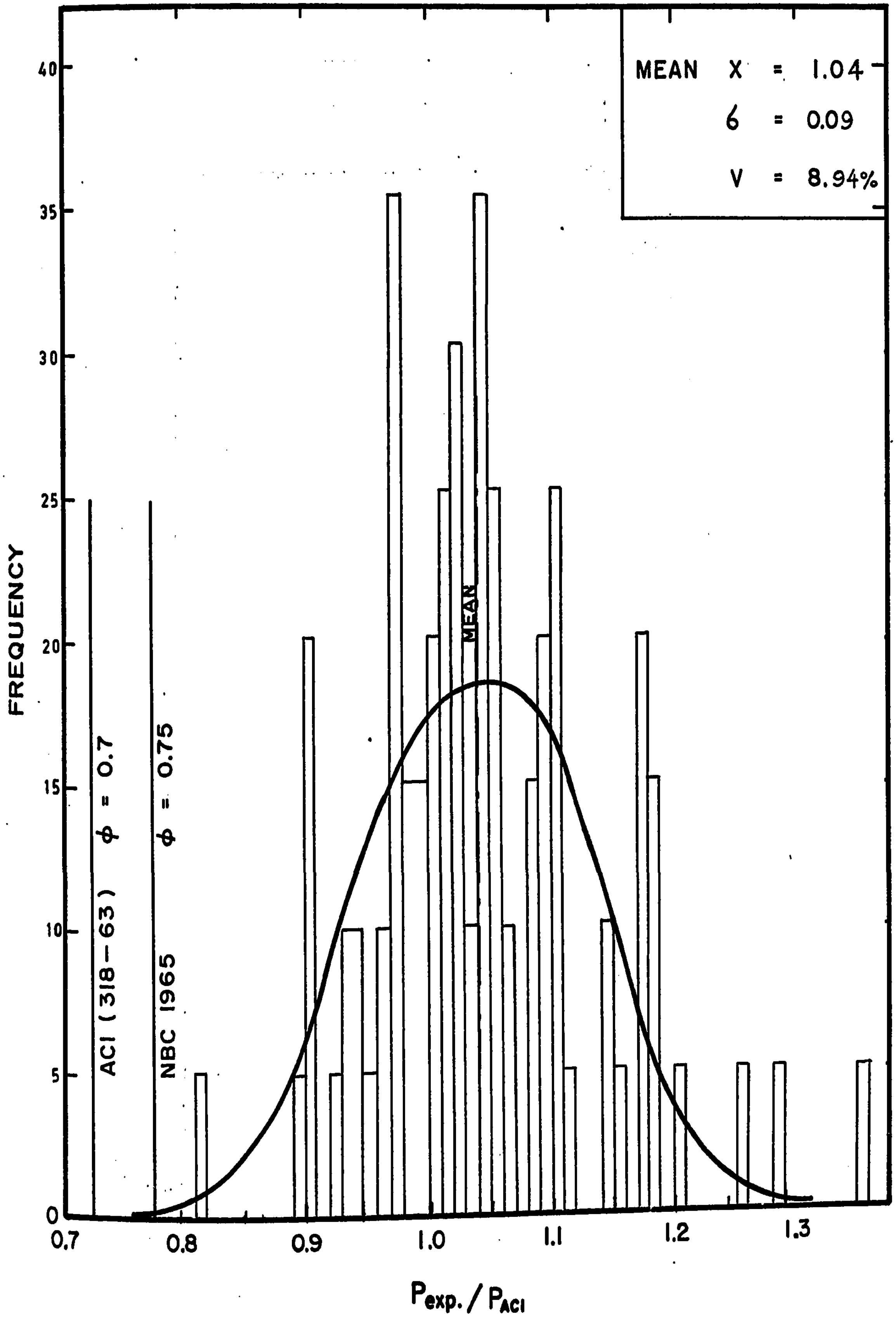


FIG. (5.4.3.1) HISTOGRAM AND FITTED NORMAL CURVE

5.5 Comparision of Test Results with the Strain Gradient Method

The test results showed close agreement with the results obtained from the Strain-Gradient Method for the full range of conditions from axially-loaded columns to beam columns under pure flexure. The actual strengths for all specimens under all cases are slightly less than the calculated values as shown in Table (5.5) on page 75. The arithmetic mean of the ratios of measured to computed ultimate load is 0.99, the overall standard deviation is 0.02, and the overall coefficient of variation is 1.94 percent.

TABLE (5.5) COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS BY STRAIN GRADIENT METHOD (KIPS)

SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$
4E-1		8.32	8.15	1.02	3D-1		42.66	41.64	1.02
4E-2		7.85	8.15	0.95	3D-2		41.71	41.64	1.00
4E-3	∞	8.55	8.92	0.96	3D-3	0.5	43.98	43.95	1.00
4E-4		8.15	8.15	1.00	3D-4		43.15	43.38	1.00
4E-5		8.54	8.92	0.96	3D-5		41.55	41.64	1.00
4DE-1		18.63	19.50	0.96	3C-1		71.33	71.44	1.00
4DE-2		16.30	16.40	0.99	3C-2		69.00	68.75	1.00
4DE-3	0.75	18.35	18.34	1.00	3C-3	0.3	75.82	75.13	1.01
4DE-4		17.85	17.74	1.01	3C-4		67.50	67.08	1.00
4DE-5		21.95	22.03	0.99	3C-5		67.50	66.93	1.01
4D-1		30.50	30.39	1.00	3B-1		103.37	102.92	1.00
4D-2		30.75	30.83	1.00	3B-2		105.56	102.92	1.03
4D-3	0.5	30.50	30.39	1.00	3B-3	0.1	106.53	103.53	1.03
4D-4		30.57	30.83	0.99	3B-4		117.08	112.33	1.04
4D-5		31.77	31.94	0.99	3B-5		106.53	110.34	0.97
4C-1		61.00	61.37	0.99	3A-1		165.00	164.52	1.00
4C-2		65.30	64.73	1.01	3A-2		156.00	156.44	1.00
4C-3	0.3	55.00	54.76	1.00	3A-3	0.0	166.00	163.29	1.02
4C-4		60.00	59.74	1.00	3A-4		162.00	159.31	1.02
4C-5		67.00	67.03	1.00	3A-5		171.00	169.84	1.00
4B-1		88.25	87.96	1.00	2E-1		18.00	17.11	1.05
4B-2		80.92	80.69	1.00	2E-2		19.60	19.61	1.00
4B-3	0.1	82.11	82.46	0.97	2E-3	∞	19.96	19.59	1.00
4B-4		88.51	88.04	1.00	2E-4		19.30	19.92	0.97
4B-5		82.01	82.07	1.00	2E-5		19.69	19.57	1.00
3E-1		11.50	12.01	0.96	2DE-1		45.10	45.21	1.00
3E-2		11.74	12.20	0.96	2DE-2		47.70	47.54	1.00
3E-3	∞	12.39	12.78	0.97	2DE-3	0.75	45.45	45.79	1.00
3E-4		11.92	12.51	0.95	2DE-4		45.30	45.67	0.99
3F-5		11.37	11.89	0.96	2DE-5		43.50	45.18	0.96

Continued

TABLE (5.5) COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS BY STRAIN GRADIENT METHOD (KIPS)

SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$
4E-1	∞	8.32	8.15	1.02	3D-1	0.5	42.66	41.64	1.02
4E-2		7.85	8.15	0.95	3D-2		41.71	41.64	1.00
4E-3		8.55	8.92	0.96	3D-3		43.98	43.95	1.00
4E-4		8.15	8.15	1.00	3D-4		43.15	43.38	1.00
4E-5		8.54	8.92	0.96	3D-5		41.55	41.64	1.00
4DE-1	0.75	18.63	19.50	0.96	3C-1	0.3	71.33	71.44	1.00
4DE-2		16.30	16.40	0.99	3C-2		69.00	68.75	1.00
4DE-3		18.35	18.34	1.00	3C-3		75.82	75.13	1.01
4DE-4		17.85	17.74	1.01	3C-4		67.50	67.08	1.00
4DE-5		21.95	22.03	0.99	3C-5		67.50	66.93	1.01
4D-1	0.5	30.50	30.39	1.00	3B-1	0.1	103.37	102.92	1.00
4D-2		30.75	30.83	1.00	3B-2		105.56	102.92	1.03
4D-3		30.50	30.39	1.00	3B-3		106.53	103.53	1.03
4D-4		30.57	30.83	0.99	3B-4		117.08	112.33	1.04
4D-5		31.77	31.94	0.99	3B-5		106.53	110.34	0.97
4C-1	0.3	61.00	61.37	0.99	3A-1	0.0	165.00	164.52	1.00
4C-2		65.30	64.73	1.01	3A-2		156.00	156.44	1.00
4C-3		55.00	54.76	1.00	3A-3		166.00	163.29	1.02
4C-4		60.00	59.74	1.00	3A-4		162.00	159.31	1.02
4C-5		67.00	67.03	1.00	3A-5		171.00	169.84	1.00
4B-1	0.1	88.25	87.96	1.00	2E-1	∞	18.00	17.11	1.05
4B-2		80.92	80.69	1.00	2E-2		19.60	19.61	1.00
4B-3		82.11	82.46	0.97	2E-3		19.96	19.59	1.00
4B-4		88.51	88.04	1.00	2E-4		19.30	19.92	0.97
4B-5		82.01	82.07	1.00	2E-5		19.69	19.57	1.00
3E-1	∞	11.50	12.01	0.96	2DE-1	0.75	45.10	45.21	1.00
3E-2		11.74	12.20	0.96	2DE-2		47.70	47.54	1.00
3E-3		12.39	12.78	0.97	2DE-3		45.45	45.79	1.00
3E-4		11.92	12.51	0.95	2DE-4		45.30	45.67	0.99
3E-5		11.37	11.89	0.96	2DE-5		43.50	45.18	0.96

Continued

SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P_{exp}	P_{cal}	$\frac{P_{exp}}{P_{cal}}$
2D-1	0.5	59.85	60.10	1.00	1D-1	0.5	74.50	73.94	1.01
2D-2		61.77	62.36	0.99	1D-2		72.50	72.02	1.01
2D-3		57.43	57.83	0.99	1D-3		68.00	68.10	1.00
2D-4		61.90	62.36	0.99	1D-4		69.00	68.59	1.00
2D-5		61.10	60.90	1.00	1D-5		70.00	69.51	1.00
2C-1	0.3	100.00	99.81	1.00	1C-1	0.3	112.00	110.98	1.01
2C-2		104.00	102.99	1.01	1C-2		111.00	110.83	1.00
2C-3		112.50	112.82	1.00	1C-3		110.50	110.83	1.00
2C-4		105.50	105.58	1.00	1C-4		112.12	115.65	0.98
2C-5		106.50	106.73	1.00	1C-5		115.00	114.44	1.00
1E-1	∞	25.00	25.63	0.97					
1E-2		24.94	25.45	0.98					
1E-3		25.00	26.12	0.96					
1E-4		22.00	21.71	1.01					
1E-5		24.98	25.76	0.97					

Mean value of P_{exp} / P_{cal} = 0.99

Standard deviation = 0.02

Coefficient of variation = 1.94%

5.5.1 Effects of Concrete Quality; Strength of Reinforcement and Eccentricity

Figs. (5.5.1) to (5.5.3) show the curves of ratio of measured* to computed ultimate load against concrete strength, reinforcement ratio and eccentricity ratio, respectively. It can be seen from these curves that the ratio of measured to computed ultimate load varied within the limit of ±3 percent from unity.

5.5.2 Interaction Diagram

From Figs. (5.5.2.1) to (5.5.2.4), a series of interaction curves, compare the maximum load capacity of the columns to the theoretical ultimate strengths. The material properties of the cross-section are: concrete strength from 3400 to 4600 psi.; percentage of reinforcement varied from 1.27 (plain bar) to 7.14 percent; and steel yield strength from 43,000 to 67,400 psi.

It can be seen from Figs. (5.5.2.1) to (5.5.2.4) that the experimental values lie well in between ±10 percent of the theoretical curves.

* each point represent the average ratio P_{exp}/P_{ACI} of each group.

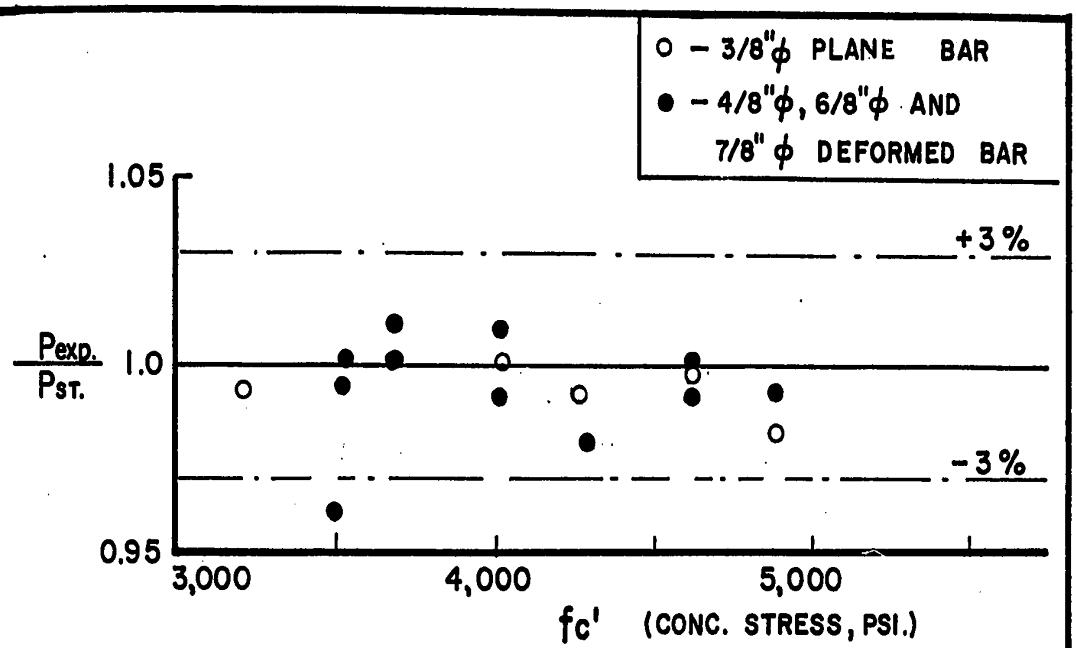


FIG.(5.5.1) $P_{exp.}/P_{cal.}$ VERSUS ULTIMATE CONCRETE STRESS

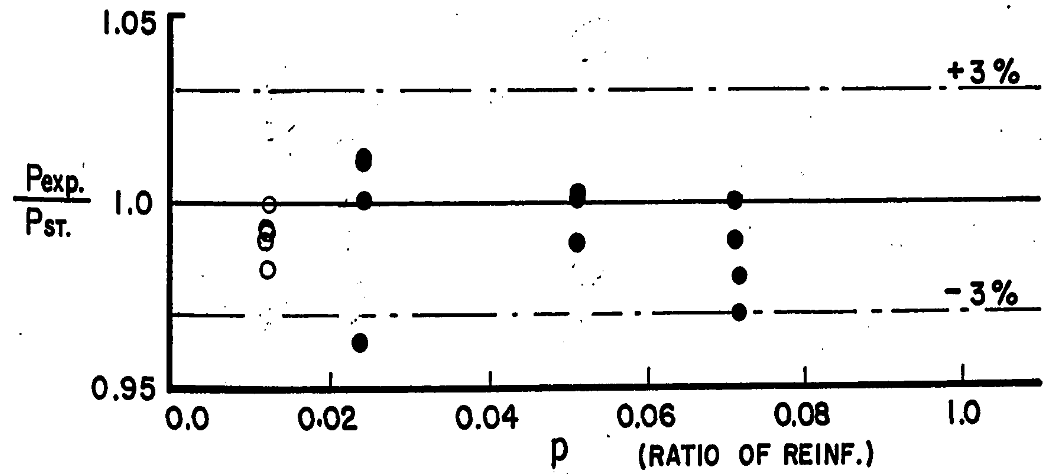


FIG.(5.5.2) $P_{exp.}/P_{cal.}$ VERSUS RATIO OF REINFORCEMENT

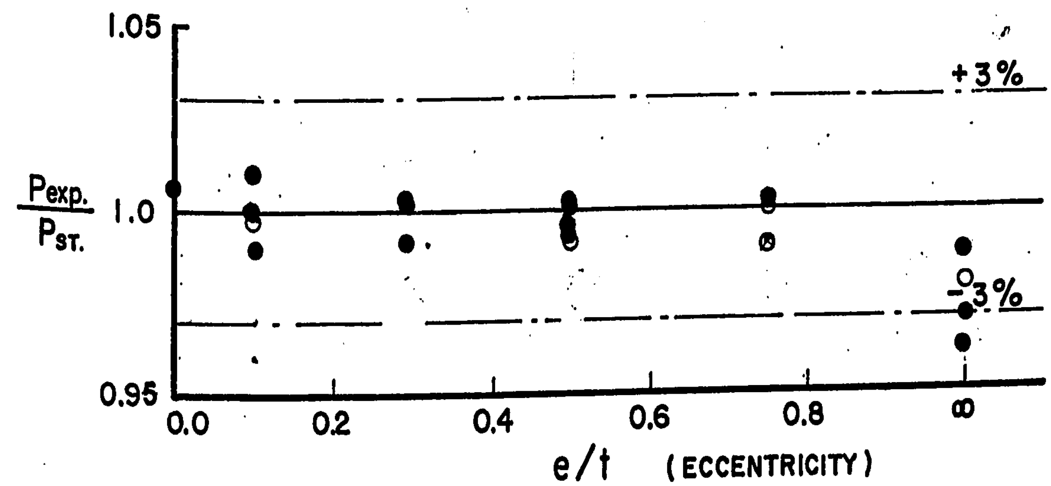


FIG.(5.5.3) $P_{exp.}/P_{cal.}$ VERSUS ECCENTRICITY RATIO OF LOAD (STRAIN GRADIENT METHOD)

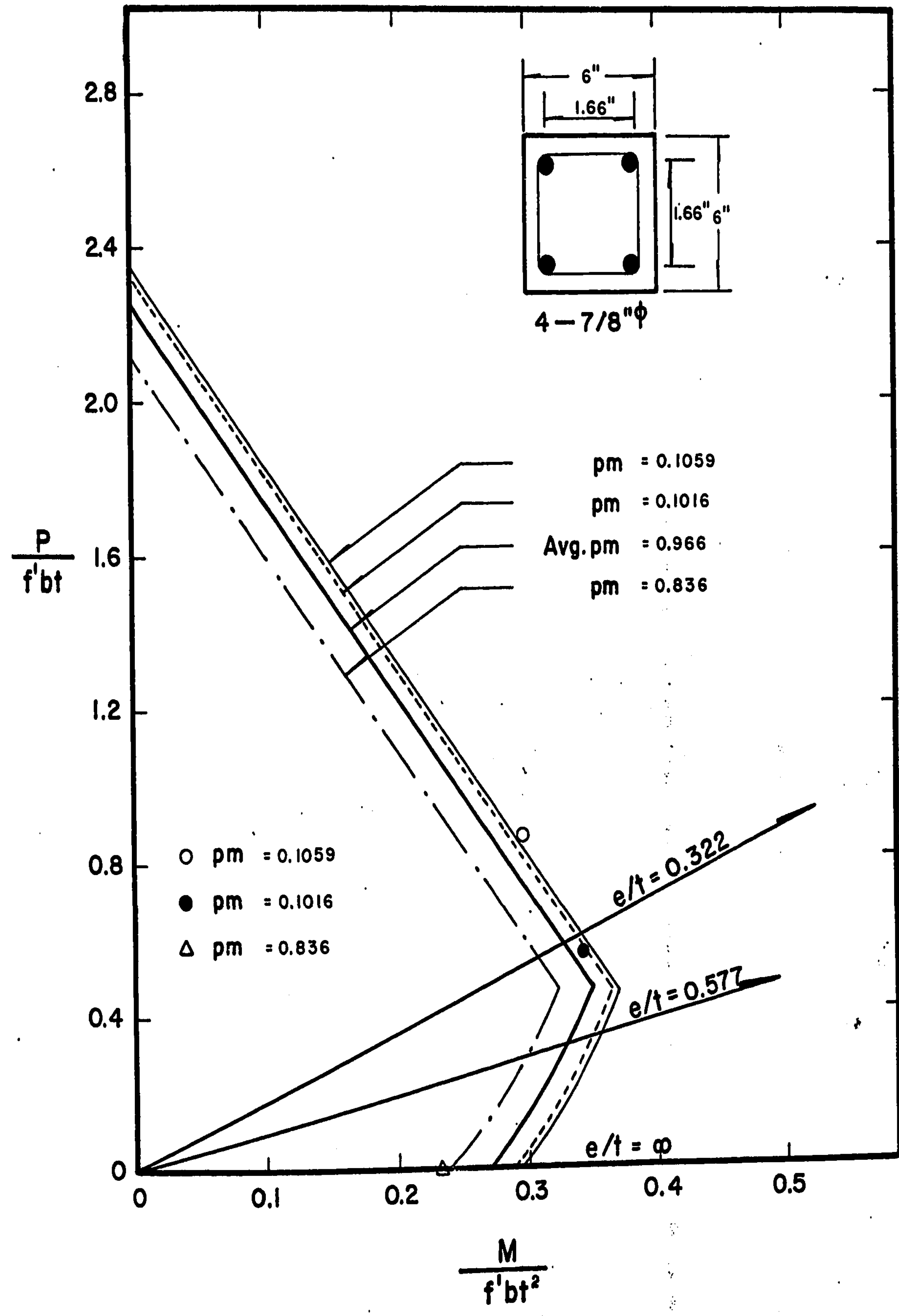


FIG. (5.5.2.1) INTERACTION DIAGRAM FOR GROUP I

(STRAIN GRADIENT METHOD)

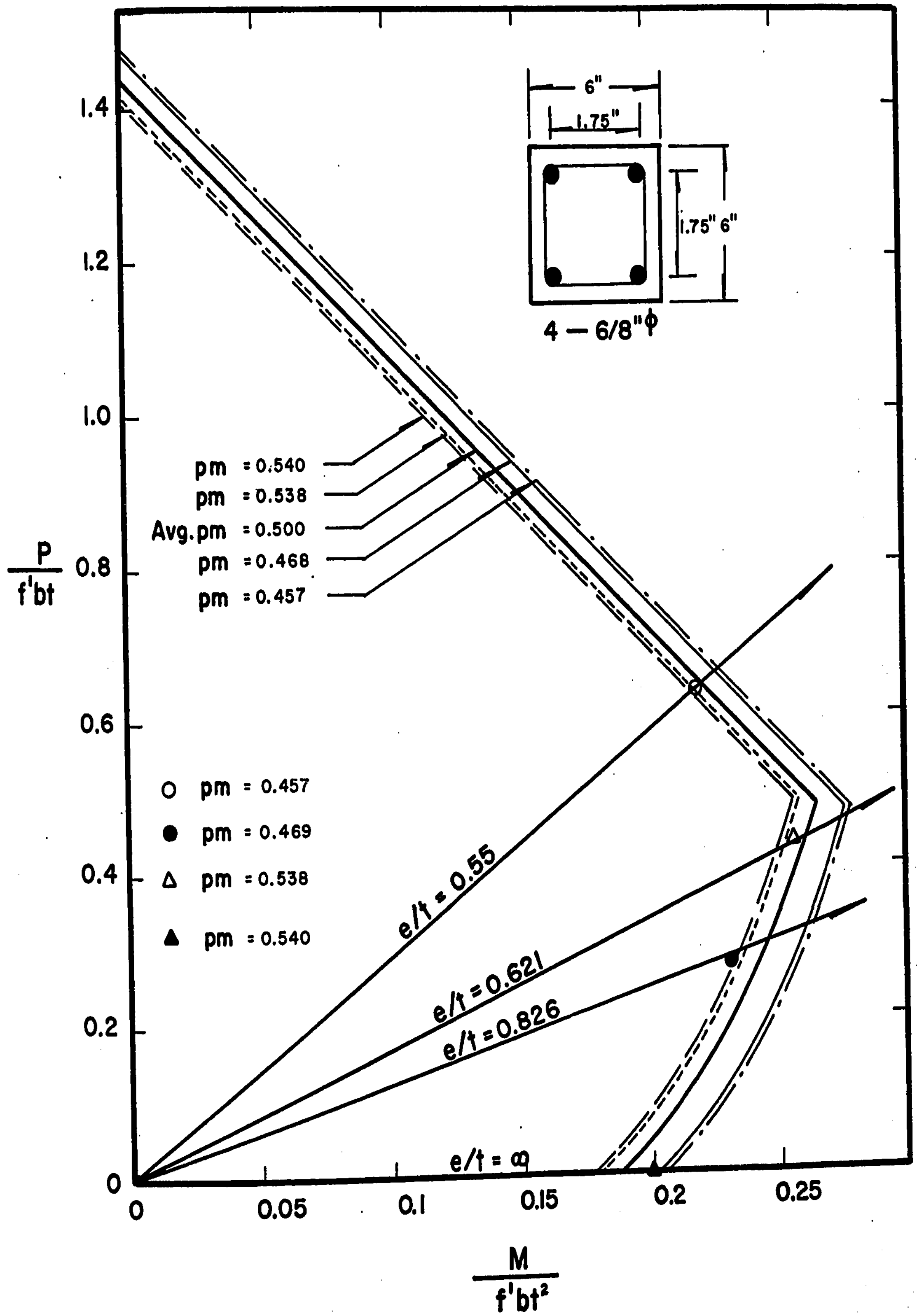


FIG. (5.5.2.2) INTERACTION DIAGRAM FOR GROUP 2
(STRAIN GRADIENT METHOD)

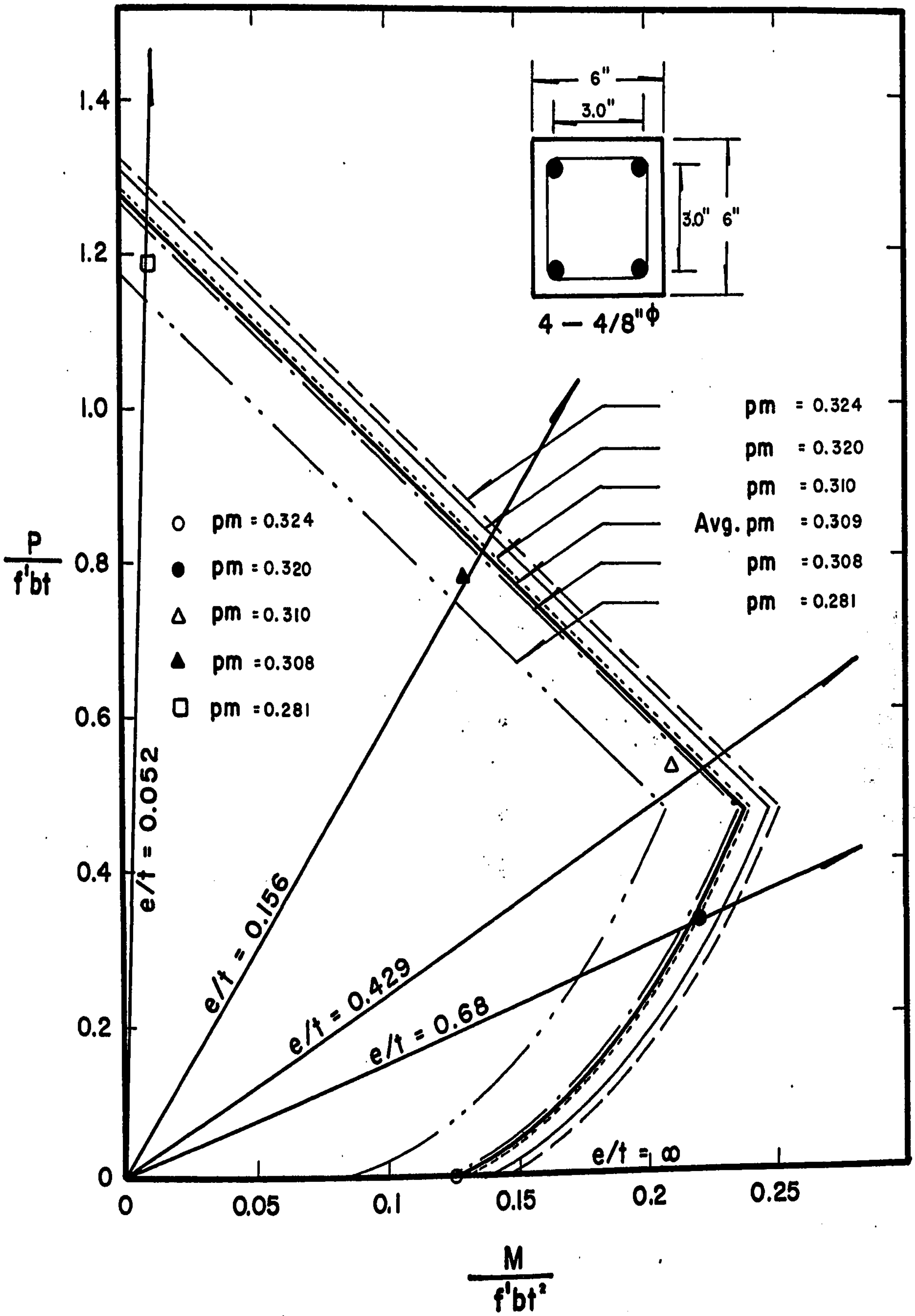


FIG. (5.5.2.3) INTERACTION DIAGRAM FOR GROUP 3

(STRAIN GRADIENT METHOD)

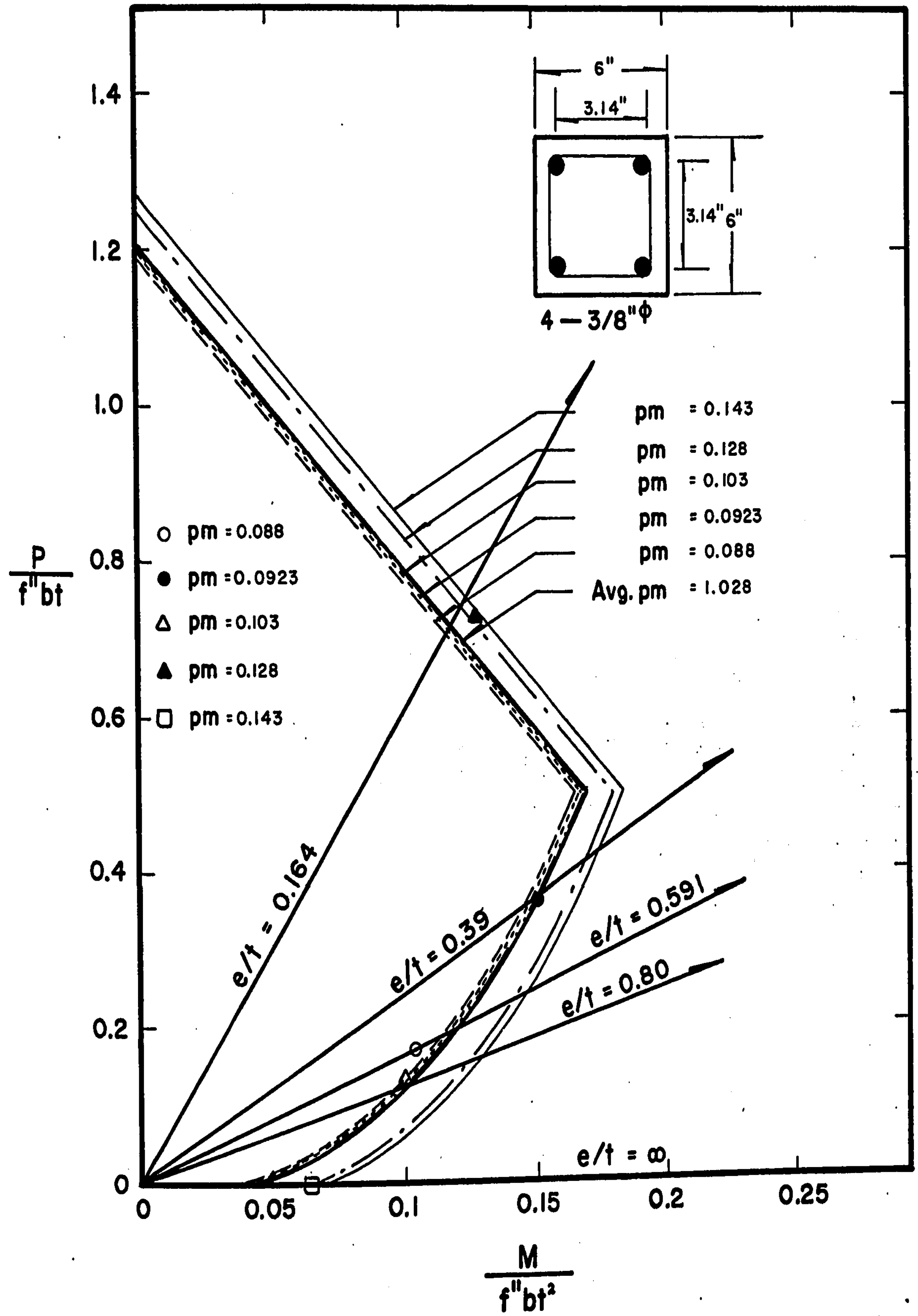


FIG. (5.5.2.4) INTERACTION DIAGRAM FOR GROUP 4

(STRAIN GRADIENT METHOD)

5.5.3 Variation of Pexp/Pcal

In Fig. (5.5.3.1) of page 84, shows the histogram and fitted normal curve of the ratios of the experimental results to the theoretical results given by the Strain Gradient Method. The total population of 85 ratios was studied and subjected to a statistical analysis.

The arithmetic mean of the total population is 0.99, the standard deviation of the ratios between the measured and calculated ultimate loads is 0.02, and the coefficient of variation is 1.94 percent.

Within the scope of the tests reported herin, the ultimate loads computed from the Strain Gradient Method slightly overestimate the column capacities, within one percent on the average.

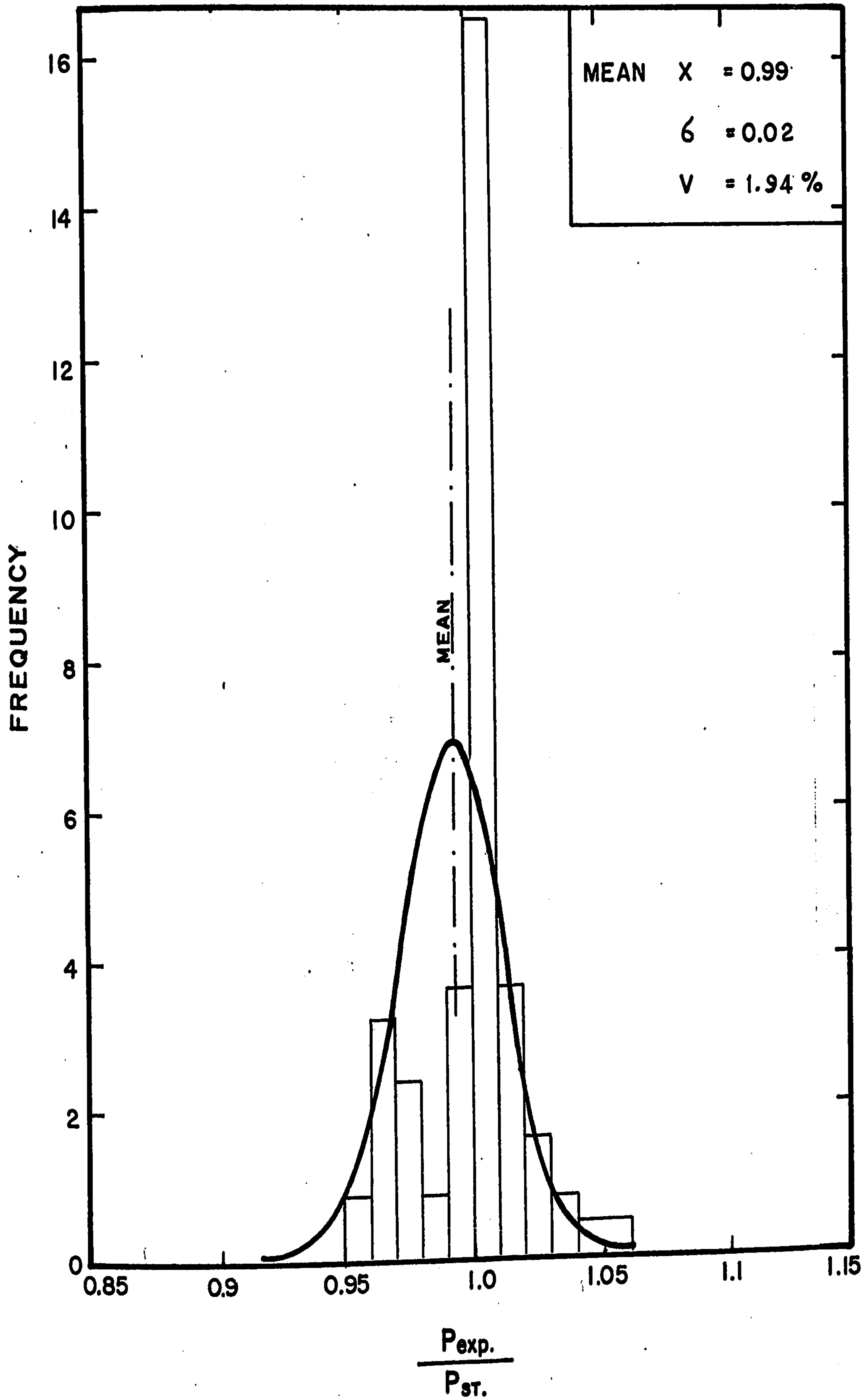


FIG.(5.5.3.1) HISTOGRAM AND FITTED NORMAL CURVE

5.6 Comparisions with Previous Investigations

The ratios of the computed values from the ACI Design Formulas and the test results are compared with the ratios presented by Chow (26), Hognestad (27), The ACI Column Investigation (27), and Bach and Graf (15).

In Table (5.4), on page 64, shows the ratios of the measured to the calculated results obtained in the project, with the mean value of 1.04 , the standard deviation of 0.09 , and the coefficient of variation of 8.94 percent.

In Table (5.6.2), on page 86, shows the ratios of the measured to the calculated results presented by Chow, with the mean value of 1.15, the standard deviation of 0.0931 and coefficient of variation of 8.3 percent.

Table (5.6.3) on page 87, shows the ratios of the measured to the calculated results presented by Hognestad, with a mean value of 0.97, standard deviation of 0.059, and coefficient of variation of 6.08 percent.

Table (5.6.4) on page 90, gives the ratios of the measured to the calculated results presented by The ACI Column Investigation, with a mean value of 1.00, standard deviation of 0.074, and the coefficient of variation of 7.40 percent.

Table (5.6.5), on page 91, gives the ratios of the measured to the calculated results by Bach and Graf, with a mean value of 1.035, standard deviation of 0.0703, and coefficient of variation of 6.79 percent.

Similiar results were found with the specimens reported by Hognestad, The ACI Column Investigation and Bach and Graf, and are in close agreement with results presented in Table (5.4).

The ratios of the computed values from the Strain Gradient Method and the test results are also compared with the ratios presented by Chow. The ratios of the measured to the calculated results presented by Chow, with the mean value of 0.97, the standard deviation of 0.076, and coefficient of variation of 7.9%, were compared to the results obtained in Table (5.5), with the mean value of 0.99, standard deviation of 0.02 and coefficient of variation of 1.94%.

Since the agreement between Chow's results and the ACI design formulae is much worse than that between Hognestad, The ACI Column Investigation, the Bach and Graf and the ACI design formulae, it is concluded that Chow's results must contain some experimental errors.

TABLE (5.6.2) COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS FOR UNDER ECCENTRICAL LOADS —— BY CHOW

SPECIMEN	f_c' (psi.)	f_y (ksi.)	% OF REINF.	P_{exp} (Kips)	P_{ult} (Kips)	$\frac{P_{exp}}{P_{cal}}$
B = 6 in. T = 6 in.	4174.50	49.899	3.44	30.00	26.30 (mean load)	1.14
	5482.00			37.80		1.45
	4379.55			34.50		1.31
	4411.50			33.00		1.25
	4164.70			31.00		1.17
	4574.50			29.40		1.11
	4123.10			32.20		1.22
	4414.10			31.20		1.18
	3596.90			30.27		1.15
	4866.85			33.00		1.25
	4313.00			33.00		1.25
	5249.25			29.18		1.11
	4548.55			29.00		1.10
	5492.85			33.16		1.26
	4249.40			30.00		1.14
	4063.00			28.00		1.06
	4116.90			29.20		1.11
	4630.70			30.00		1.14
	3479.20			28.30		1.07
	4405.25			29.30		1.11

Mean value of P_{exp} / P_{cal} = 1.15

Standard deviation = 0.0931

Coefficient of variation = 8.30%

TABLE (5.6.3) COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS UNDER ECCENTRICAL LOADS —— BY HOGNESTAD

SPECIMEN	f_c' (psi.)	f_y (ksi.)	% OF REINF.	P_{exp} (Kips)	P_{cal} (Kips)	$\frac{P_{exp}}{P_{cal}}$
	5280.00			388.00	452.00	0.86
	5660.00			441.00	481.00	0.92
	4250.00			343.00	372.00	0.92
	4070.00			352.00	356.00	0.99
B = 10 in.	2270.00			222.00	212.00	1.05
	2020.00	60.00	2.48	191.00	192.00	1.00
T = 10 in.	5280.00			239.00	240.00	1.00
	5830.00			253.00	260.00	0.97
	4250.00			213.00	206.00	1.03
	4070.00			190.00	197.00	0.96
	2270.00			118.50	116.00	1.02
	1970.00			100.00	103.00	0.97
	5660.00			133.50	154.00	0.87
	4830.00			140.00	158.00	0.89
	4630.00			125.90	134.30	0.94
	4290.00			116.00	129.40	0.90
	1880.00			60.50	66.00	0.92
	1690.00			64.00	52.00	1.03
	4810.00			84.50	85.70	0.99
	5600.00			81.00	92.10	0.88
	3800.00			80.00	77.70	1.03
	4290.00			81.00	81.10	1.00
	1690.00			50.50	48.00	1.05
	1730.00			52.00	49.00	1.06
	4810.00			48.20	45.60	1.06
	5600.00			42.80	46.90	0.91
	4290.00			46.10	44.50	1.04
	4590.00			45.50	45.00	1.01
	2310.00			39.00	37.70	1.04
	1770.00			32.80	34.00	0.96
	4080.00			456.00	437.00	1.04
	4040.00			420.00	436.00	0.96
	2020.00			225.00	268.00	0.84
	1520.00			202.00	222.00	0.91

Continued

TABLE (5.6.3) Continued

SPECIMEN	f_c' (psi.)	f_y (ksi.)	% OF REINF.	P_{exp} (Kips)	P_{cal} (Kips)	$\frac{P_{exp}}{P_{cal}}$
	5240.00			274.00	254.00	1.08
	5810.00			284.00	312.00	0.91
	4080.00			256.00	245.00	1.04
	4040.00			248.00	244.00	1.02
B = 10 in.	1970.00			141.00	147.00	0.96
	1520.00	60.00	2.48	126.80	135.00	0.94
T = 10 in.	5520.00			162.00	177.90	0.91
	5810.00			152.00	179.50	0.85
	4700.00			156.00	164.90	0.95
	4260.00			146.00	158.16	0.92
	1820.00			99.00	103.00	0.96
	1820.00			99.00	102.00	0.97
	5100.00			89.00	99.10	0.90
	5170.00			91.20	99.10	0.92
	4700.00			94.00	97.70	0.96
	4370.00			89.50	96.60	0.93
	1880.00			73.00	75.80	0.96
	1730.00			65.50	74.10	0.88
	5100.00			46.10	47.80	0.97
	5170.00			44.00	48.00	0.92
	4260.00			43.50	47.20	0.92
	4370.00			44.00	47.20	0.93
	2300.00			44.50	44.30	1.00
	1770.00			45.00	42.60	1.06
	3870.00			500.00	513.00	0.98
	4070.00			485.00	517.00	0.94
	2070.00			353.00	376.00	0.94
	4150.00			315.00	306.00	1.03
	5050.00			326.00	343.00	0.95
	4300.00			303.00	318.00	0.95
	4010.00			284.00	298.00	0.95
	2300.00			252.00	224.00	1.12

Continued

TABLE (5.6.3) Continued

SPECIMEN	f_c' (psi.)	f_y (ksi.)	% OF REINF.	P_{exp} (Kips)	P_{cal} (Kips)	$\frac{P_{exp}}{P_{cal}}$
	2200.00			230.00	218.00	1.05
	5350.00			220.00	227.00	0.97
	4850.00			210.00	216.00	0.97
	3580.00			180.00	188.00	0.96
B 10 in.	4290.00			206.00	206.00	1.00
	2300.00	60.00	2.48	151.00	153.00	0.99
T 10 in.	2070.00			137.00	148.00	0.93
	5350.00			142.00	159.00	0.89
	5100.00			153.00	155.50	0.98
	3580.00			138.80	141.00	0.98
	1950.00			115.50	108.00	1.07
	2070.00			104.00	111.00	0.94
	5100.00			88.00	82.80	1.06
	4850.00			79.00	83.00	0.95
	3800.00			74.00	80.30	0.92
	4630.00			84.00	82.00	1.03
	1950.00			72.50	74.00	0.97
	2070.00			74.50	73.50	1.01

Mean value of P_{exp} / P_{cal} = 0.97

Standard deviation = 0.059

Coefficient of variation = 6.08%

TABLE (5.6.4) COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS UNDER ECCENTRIC LOADS---by ACI COL. INVESTIGATION

SPECIMEN	f_c' (psi.)	f_y (psi.)	% OF REINF.	P_{exp} (Kips)	P_{cal} (Kips)	$\frac{P_{exp}}{P_{cal}}$
a	2860.00	50,000	4	219.00	231.00	0.95
b	3090.00	50,000	4	255.00	242.00	1.05
a	2650.00	50,000	4	253.00	222.00	1.14
b	2850.00	50,000	4	238.00	231.00	1.03
a	4700.00	44,700	1.5	225.00	246.00	0.92
b	4150.00	44,700	1.5	227.00	222.00	1.02
a	4670.00	50,000	4	285.00	310.00	0.92
b	4730.00	50,000	4	320.00	313.00	1.02
a	4225.00	50,000	4	293.00	291.00	1.01
b	4570.00	50,000	4	309.00	306.00	1.01
a	4215.00	42,200	6	317.00	315.00	1.01
b	4985.00	42,200	6	291.00	348.00	0.84
a	5870.00	50,000	4	353.00	363.00	0.97
b	6950.00	50,000	4	387.00	410.00	0.94
a	6245.00	50,000	4	410.00	379.00	1.08
b	6530.00	50,000	4	420.00	391.00	1.08

Mean value of P_{exp} / P_{cal} = 1.00

Standard deviation = 0.074

Coefficient of variation = 7.40%

TABLE (5.6.5) COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS UNDER ECCENTRIC LOADS ----- BY BACHAND GRAF

SPECIMEN	f_c' (psi.)	f_y (psi.)	% OF REINF.	P_{exp} (Kips)	P_{cal} (Kips)	$\frac{P_{exp}}{P_{cal}}$
B = 16 in. T = 16 in.	2830.00	53,700	1	735.00	729.00	1.01
	2830.00	53,700		447.00	423.00	1.05
	2830.00	53,700		273.00	257.00	1.06
	2830.00	53,700		153.40	148.00	1.03
	2830.00	53,700		71.30	68.50	1.04
	2830.00	53,700	1.9	892.00	835.00	1.07
	2830.00	53,700		496.00	486.00	1.02
	2830.00	53,700		347.00	343.00	1.01
	2830.00	53,700		231.50	219.00	1.05
	2830.00	53,700		118.00	116.40	1.01

Mean value of P_{exp} / P_{cal} = 1.035

Standard deviation = 0.0703

Coefficient of variation = 6.79%

5.7 Studies of Deflection

The deflections of all columns tested were observed by means of three dial indicators: at centre span and 6" to the left and right of the centre of the column along the longitudinal direction. These dial indicators were attached to the compression face of the columns as shown in Figs. (3.5.2.a) and (b). The load-deflection curves for all columns presented in Figs.(B.1) to (B.85), on page 112 to page 196.

The deflections at mid-height with respect to the hinged ends were used to calculate the actual eccentricities at failure. All the ACI Design Equations for ultimate loads of eccentrically-loaded members reported in this thesis were refered to the actual eccentricities at failure.

The deflections at mid-height of the columns were also considered theoretically. They can be obtained easily by introducing the simple assumption that the deflected shape was a cosine wave, as shown in Fig. (5.7.1).

$$Y = \delta \cos \frac{\pi x}{L} \quad ....(1)$$

$$\frac{d^2y}{dx^2} = -\delta \left(\frac{\pi}{L}\right)^2 \cos \frac{\pi x}{L}$$

but

$$\frac{d^2y}{dx^2} = \frac{-1}{\rho}$$

$$\therefore \frac{1}{\rho} = \delta \left(\frac{\pi}{L}\right)^2 \cos \frac{\pi x}{L} \quad ....(2)$$

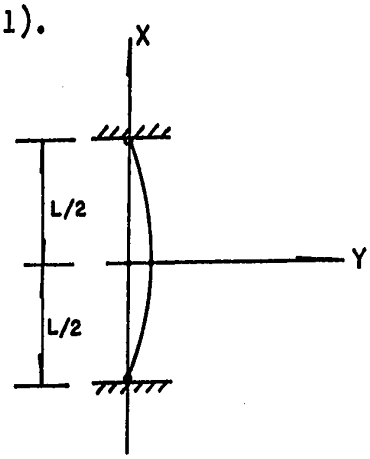


FIG. (5.7.1) DEFLECTED SHAPE OF A COLUMN

Apply boundary condition, when X = 0, Y = δ

Thus

$$\frac{1}{\rho} = \delta \left(\frac{\pi}{L} \right)^2 \quad ......(3)$$

From Fig. (5.7.2), consider column under deflection.

$$\frac{1}{\rho} = \frac{\epsilon x}{Y} = \frac{\epsilon_c}{Kd} = \phi \quad ......(4)$$

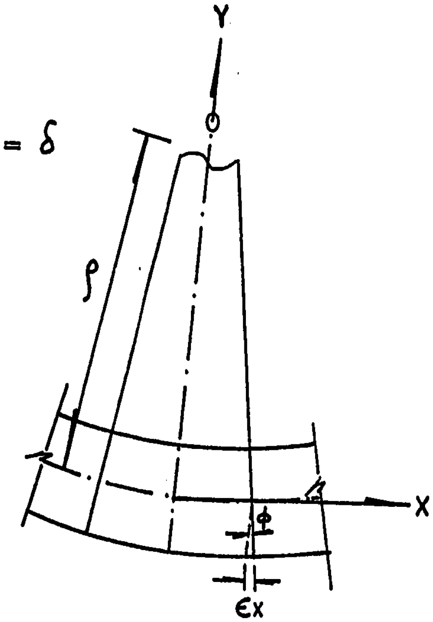


FIG.(5.7.2) COLUMN UNDER DEFLECTION

Substitute Eq. (4) in Eq. (3), we have,

$$\delta \left(\frac{\pi}{L} \right)^2 = \phi$$

$$\therefore \delta = \phi \cdot \left(\frac{\pi}{L} \right)^2 \quad ......(5)$$

In our case, as shown in Fig. (5.7.3) gives,

$$(\epsilon_c + \epsilon_{s1}) = d \cdot \phi$$

$$\therefore \phi = \frac{\epsilon_c + \epsilon_{s1}}{d} \quad ......(6)$$

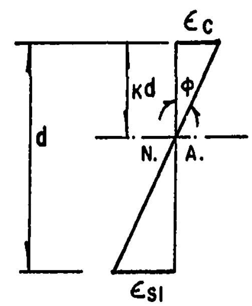


FIG.(5.7.3) STRAIN RELATIONSHIP

Substitute Eq. (6) in Eq. (5), we obtain,

$$\delta = \left(\frac{\pi}{L} \right)^2 \quad \left(\frac{\epsilon_c + \epsilon_{s1}}{d} \right)$$

$$= K \cdot (\epsilon_c + \epsilon_{s1})$$

where K = $\pi^2 / L^2 d$, and ϵ_c, and ϵ_{s1} can be obtained for each column, from the Strain Gradient Method.

The test results of the deflections at mid-height were compared with the theoretical results as shown in Table (5.7.1), and were found to be in a satisfactory agreement. The errors are probably due to the centering of the specimen; the beam-column effect under various eccentricities; the assumptions of the maximum concrete strain equal to 0.003 and the assumption that the deflected shape of the specimens is a cosine curve.

TABLE (5.7.1) COMPARISON BETWEEN OBSERVED AND CALCULATE CENTRE DEFLECTIONS

SPECIMEN *	INITIAL ECCENTRICITY e/t	DEFLECTION AT FAILURE		
		δ exp.		δ cal.
		MEAN	C.of V.	
4E (1 - 5)		0.481	11.1%	0.437
4DE (1 - 5)	0.75	0.261	7.7%	0.320
4D (1 - 5)	0.5	0.601	12.6%	0.642
4C (1 - 5)	0.3	0.537	9.6%	0.420
4B (1 - 5)	0.1	0.431	14.4%	0.448
2E (1 - 5)		0.363	16.5%	0.324
2DE (1 - 5)	0.75	0.463	12.9%	0.418
2D (1 - 5)	0.5	0.614	9.1%	0.550
2C (1 - 5)	0.3	0.380	5.5%	0.305
3E (1 - 5)		0.359	1.4%	0.384
3D (1 - 5)	0.5	1.050	6.3%	0.964
3C (1 - 5)	0.3	0.725	15.7%	0.734
3B (1 - 5)	0.1	0.413	2.0%	0.470
3A (1 - 5)	0.0	0.033	6.4%	0.034
1E (1 - 5)		0.460	4.8%	0.390
1D (1 - 5)	0.5	0.449	11.4%	0.415
1C (1 - 5)	0.3	0.312	10.3%	0.330

* Average Values of 5 Specimens in a Same Group.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

From this investigation, some main conclusions may be summarised as follows:-

1) The short column design formulae given in the ACI 318-56, NBC 1965 and CSA.A23.3-1970 Building Codes predicts the measured failure loads of columns within ±10%, provided that the actual eccentricity of the load at failure is used in the calculations. The average ratio of measured load / predicted load is 1.04 with a standard deviation of 0.09.

2) The ACI - CSA Design Formulae together with the recommended "ϕ" factor of 0.70 implies a probability of the ultimate section capacity of a laboratory specimen being less than the design ultimate load of 0.03%.

3) The probability of the actual load capacity being less than the design ultimate load is 0.19%, using a "ϕ" factor of 0.75 as recommended by the Canadian Standard (CSA.A23-1970), together with the ACI - NBC - CSA prediction formulae.

4) The strain gradient method using the measured steel stress-strain curve and Hognestad stress-strain curve for the concrete predicts the experimental values within ±3%. The average measured load / predicted load ratio is 0.99 with a standard deviation of 0.02.

5) Calculations of the lateral deflection from the strain gradient method assuming that the deflected shape of an eccentrically loaded long column deflects laterally in the form of a cosine agree reasonably well with the measured lateral deflections of the laboratorial specimens.

6) The accuracy of the results obtained from the ACI Design Formulae, increases as the percentage of the longitudinal reinforcement increases.

7) The ratio of the measured to computed ultimate load using the ACI Design Formulae, becomes close to unity as the ratio e/t approaching infinity.

8) The ratio of the measured to computed ultimate load using the strain gradient method, approaches close to unity as the ratio e/t increases.

9) The moment with respect to the balanced condition is increased with an increase of longitudinal reinforcing steel.

10) The ductility of the column section is decreased with the increase of the longitudinal reinforcing steel.

11) The experimental results using deformed bars rather than plain bars, give more consistant agreement with the predicted results from both the strain gradient method and ACI - NBC - CSA methods.

6.2 Recommendations

1) Longitudinal stirrups are recommended in the capitals to transmit the eccentric load to the prismatic section of the column.

2) Test the Strain Gradient Method experimentally, by installing strain gages in the specimen to check the correspondance of strain / strain gradient and load.

3) To develop a general computer program to represent the behavior of long column under uni-axial and bi-axial bending, by the Strain Gradient Method, for the analysis of the reinforced concrete columns.

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APPENDIX A

STEEL PROPERTIES

FIGURE (A.1) to FIGURE (A.4)

STRESS-STRAIN CURVES OF

LONGITUDINAL REINFORCEMENTS

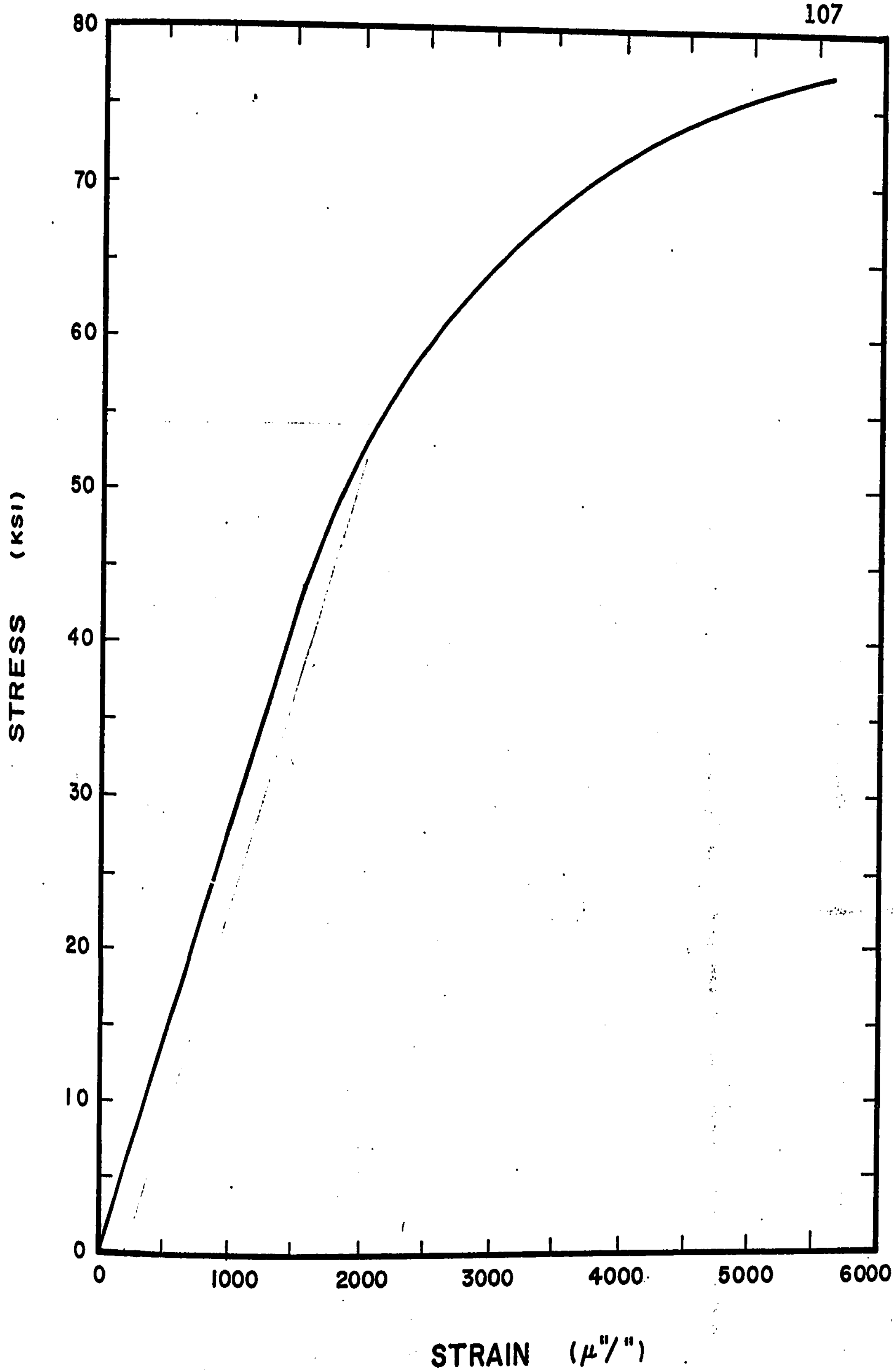


FIG.(A.1) STRESS — STRAIN CURVE OF #3 LONGITUDINAL REINFORCEMENT (PLAIN BAR)

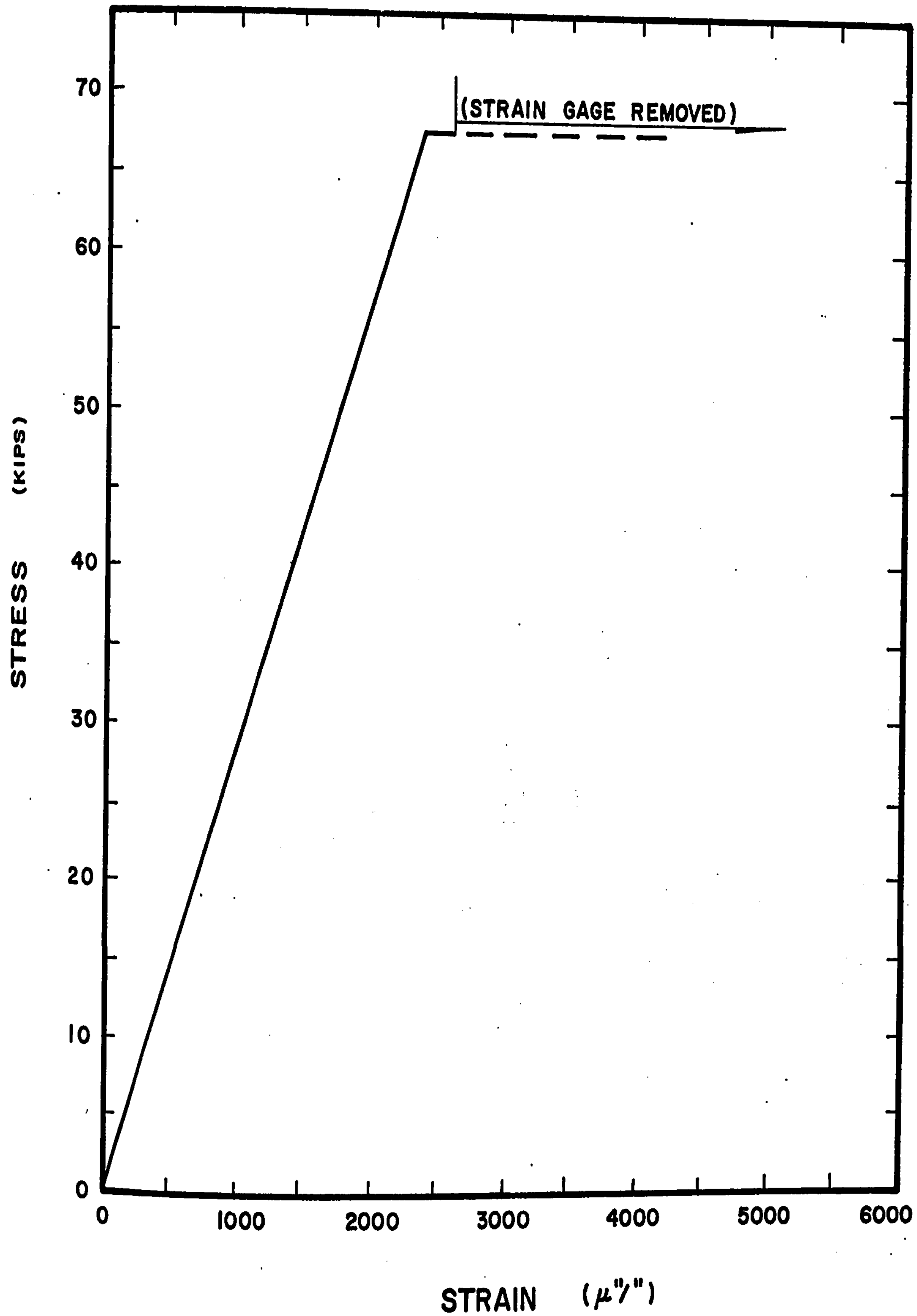


FIG. (A.2) STRESS — STRAIN CURVE OF #4 LONGITUDINAL REINFORCEMENT (DEFORM BAR)

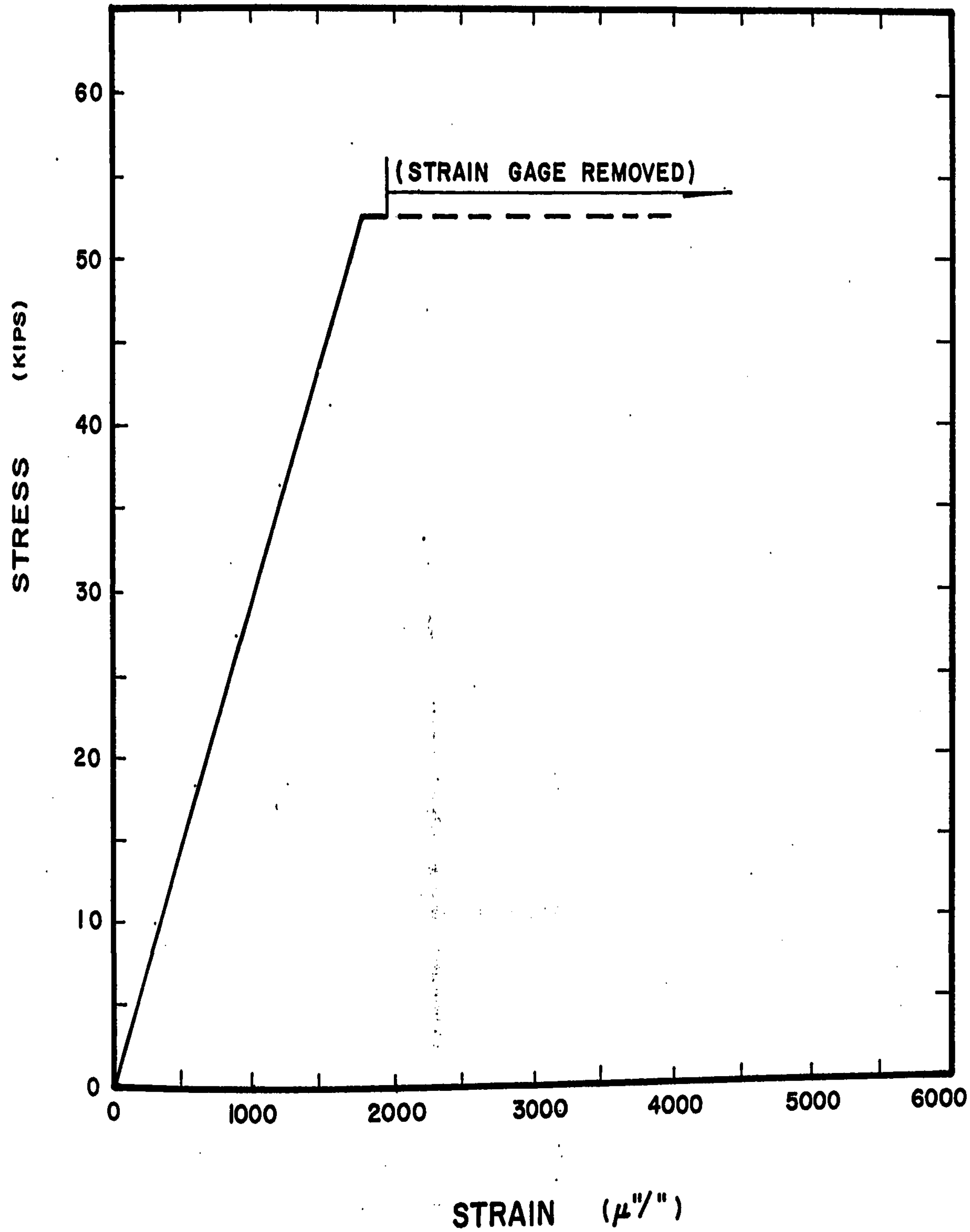


FIG.(A.3) STRESS—STRAIN CURVE OF #6 LONGITUDINAL REINFORCEMENT (DEFORM BAR)

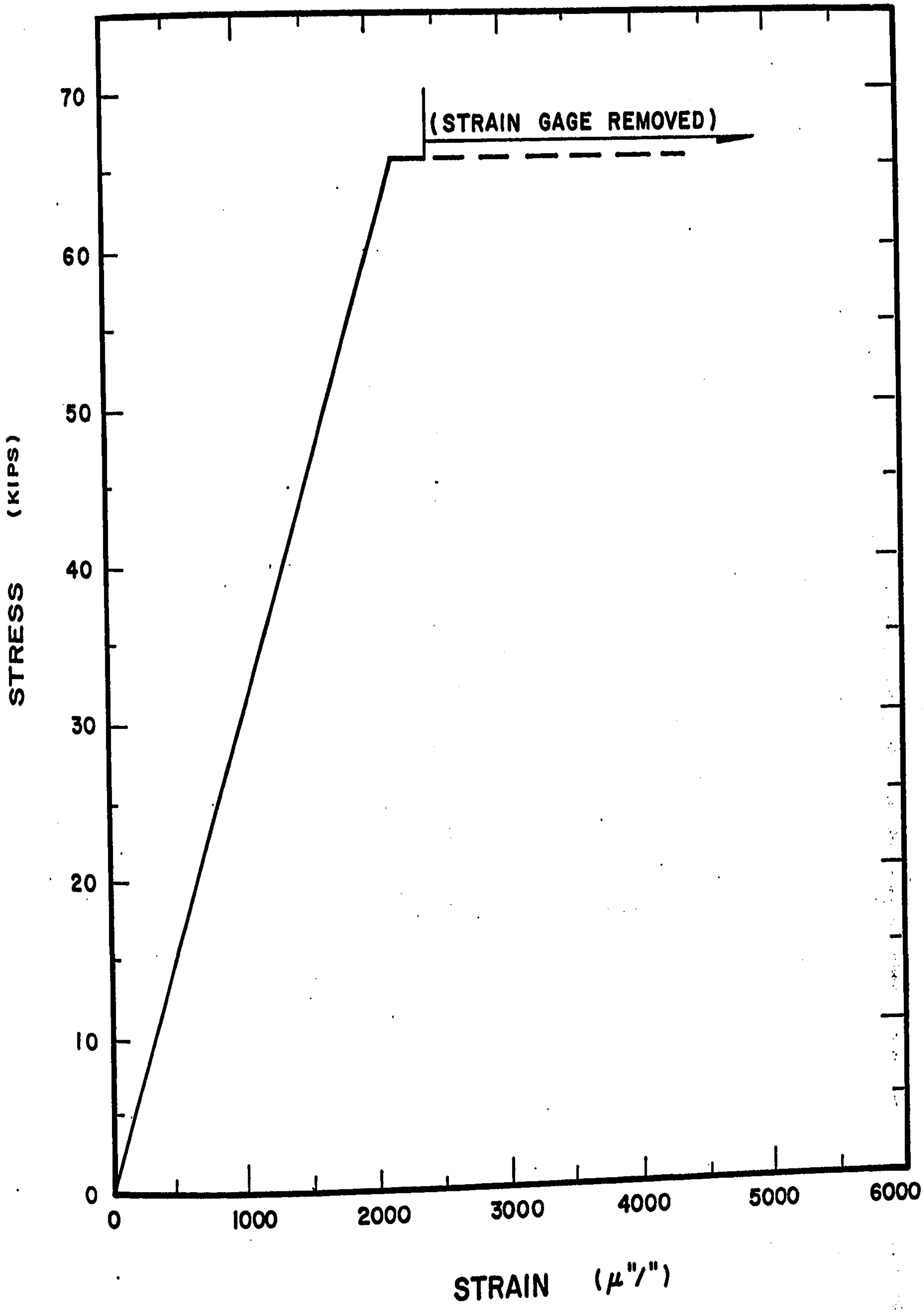


FIG. (A.4) STRESS — STRAIN CURVE OF #7 LONGITUDINAL REINFORCEMENT (DEFORM BAR)

APPENDIX B

INDIVIDUAL GRAPHS OF SPECIMENS

FIGURE (B.1) to FIGURE (B.85)

LOAD-DEFLECTIONS CURVES

OF SPECIMENS

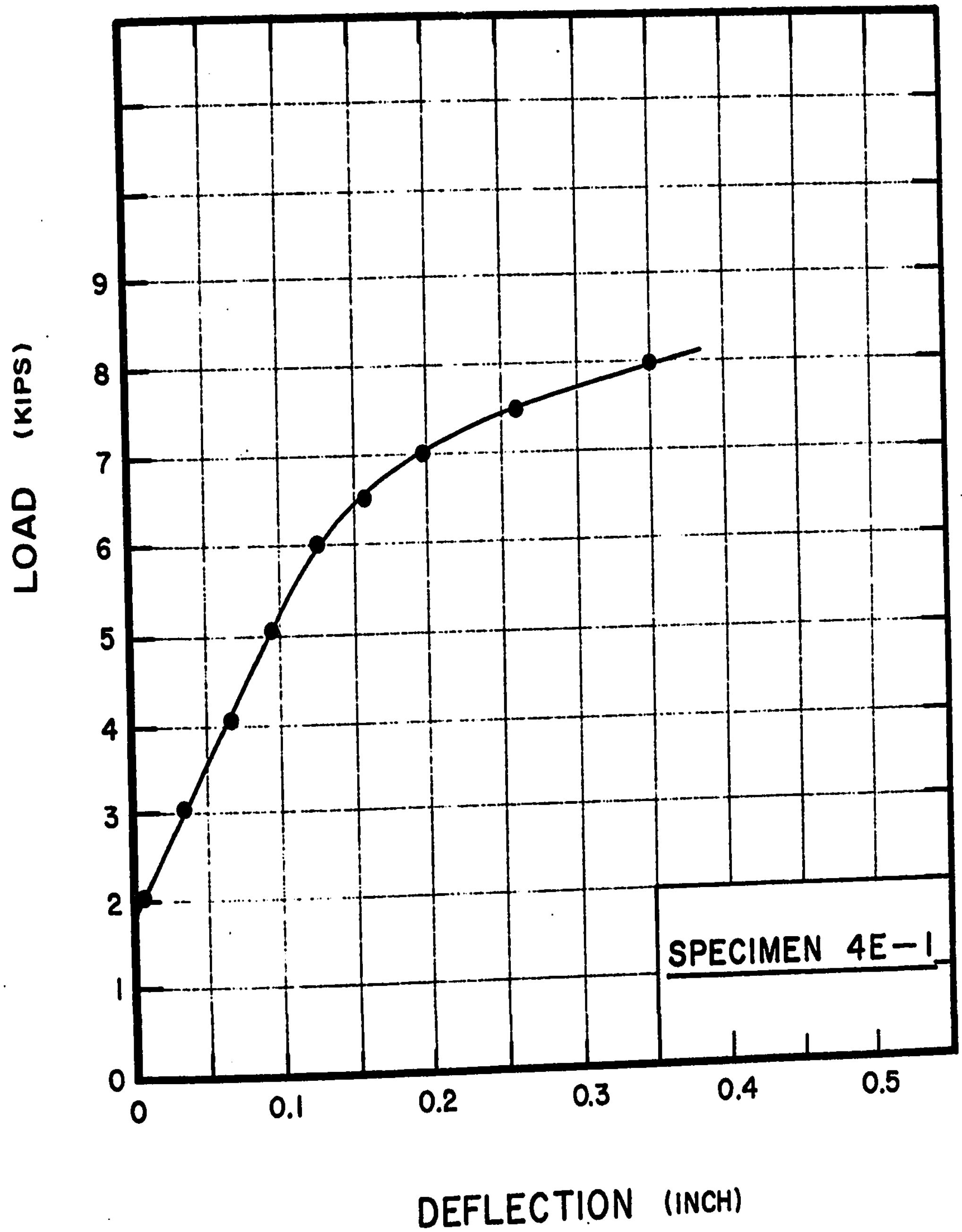


FIG. (B. 1) LOAD—DEFLECTION CURVE OF SPECIMEN 4E-1

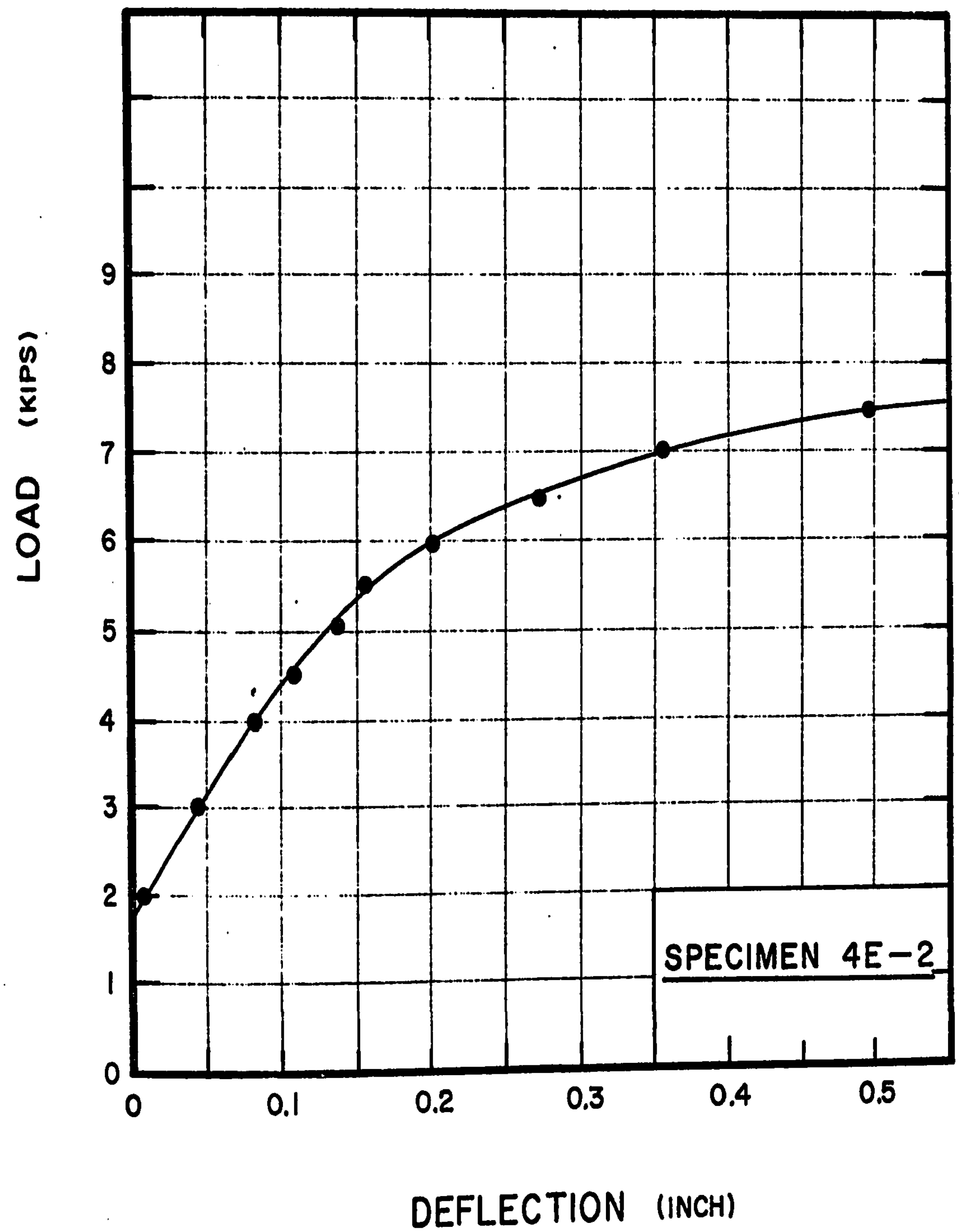


FIG. (B. 2) LOAD—DEFLECTION CURVE OF SPECIMEN 4E-2

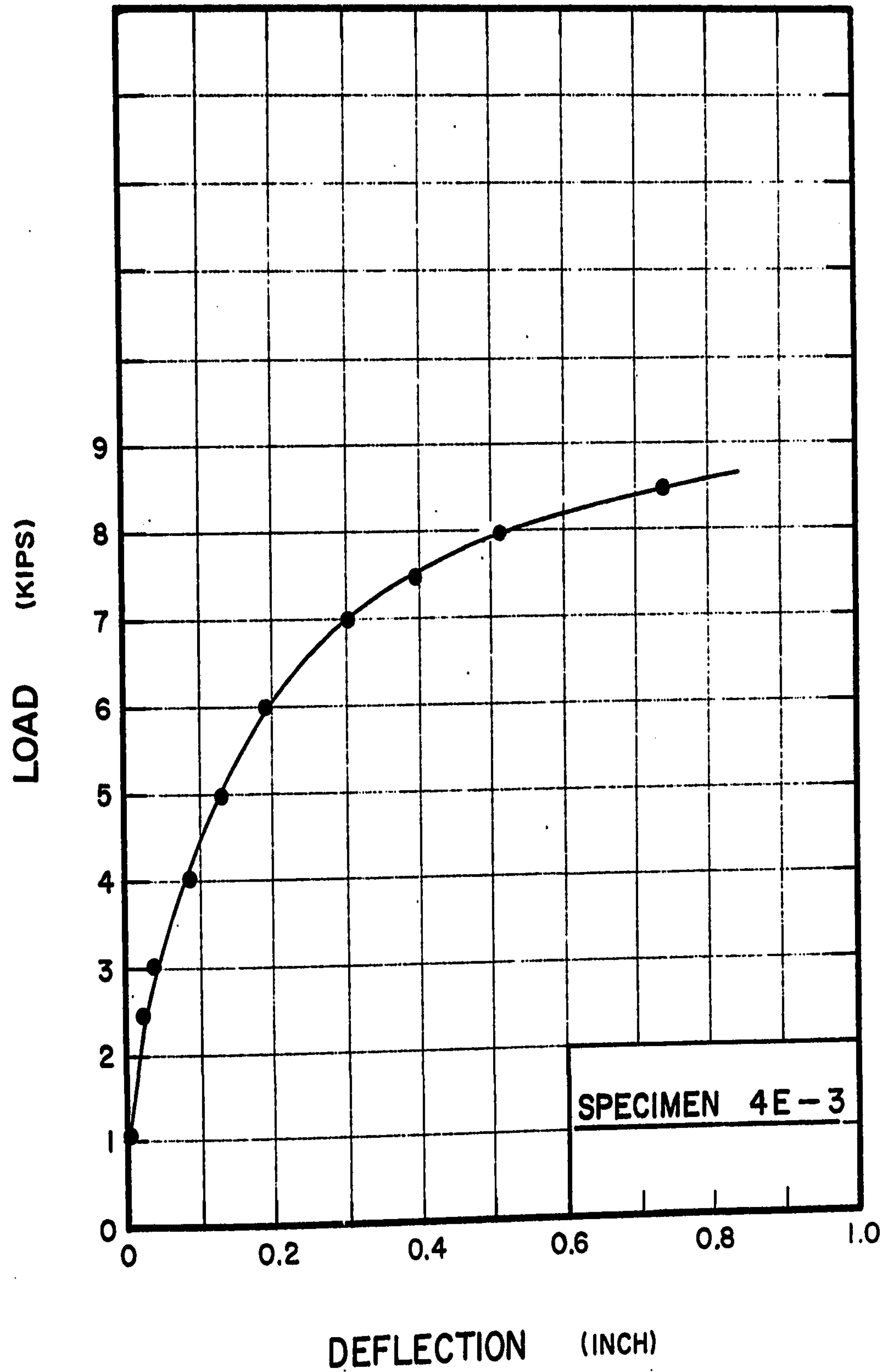


FIG.(B. 3) LOAD—DEFLECTION CURVE OF SPECIMEN 4E-3

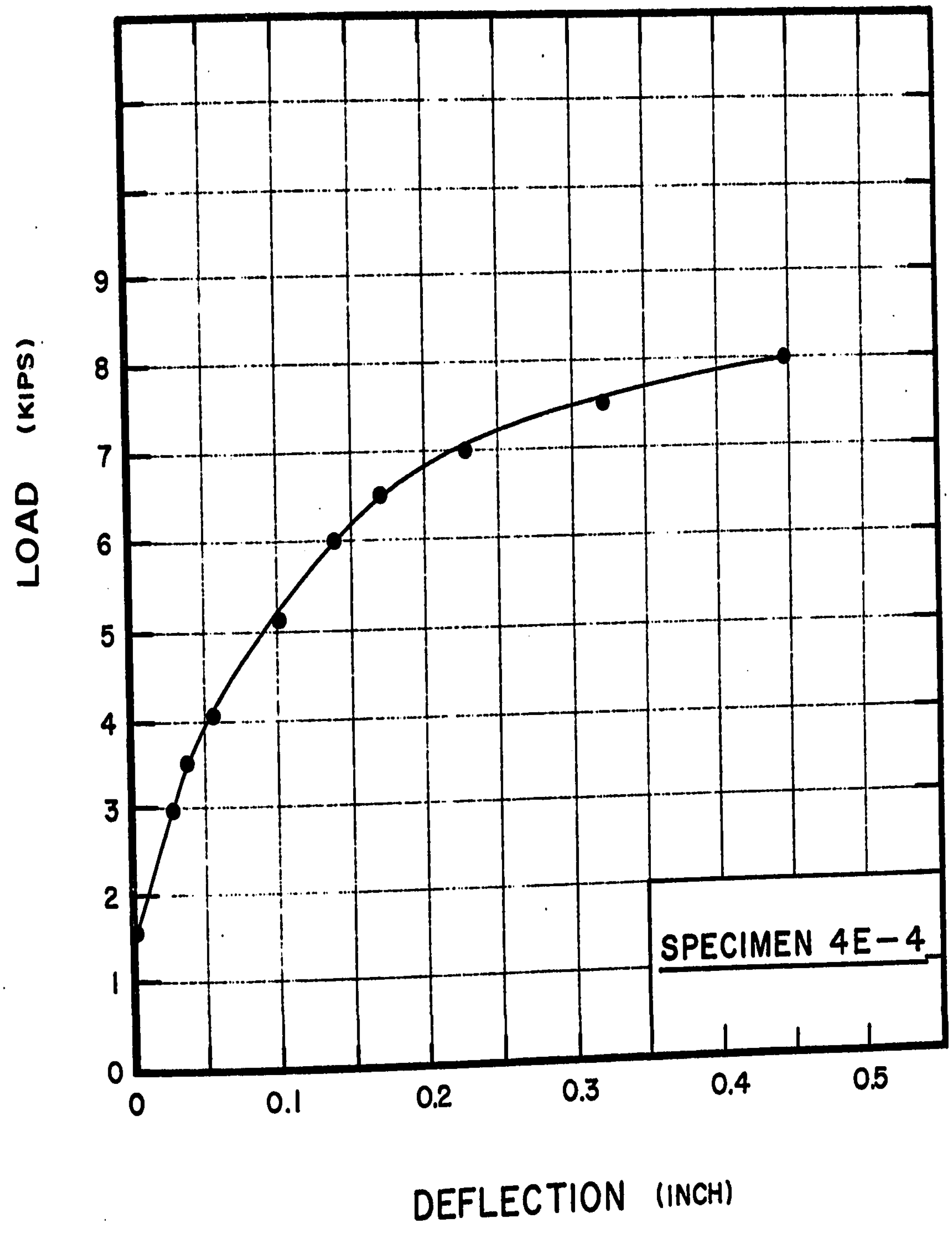


FIG. (B. 4) LOAD—DEFLECTION CURVE OF SPECIMEN 4E-4

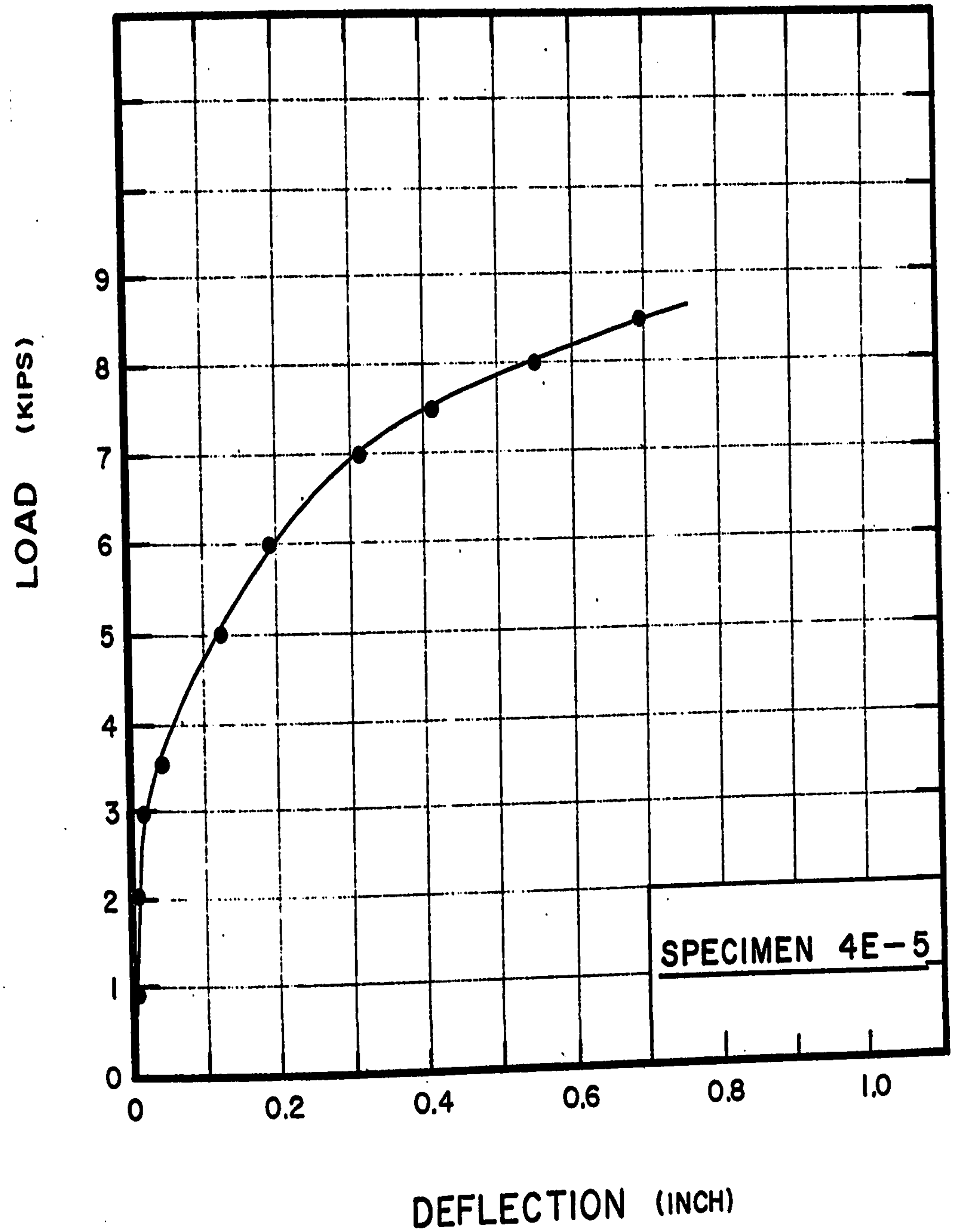


FIG. (B. 5) LOAD—DEFLECTION CURVE OF SPECIMEN 4E-5

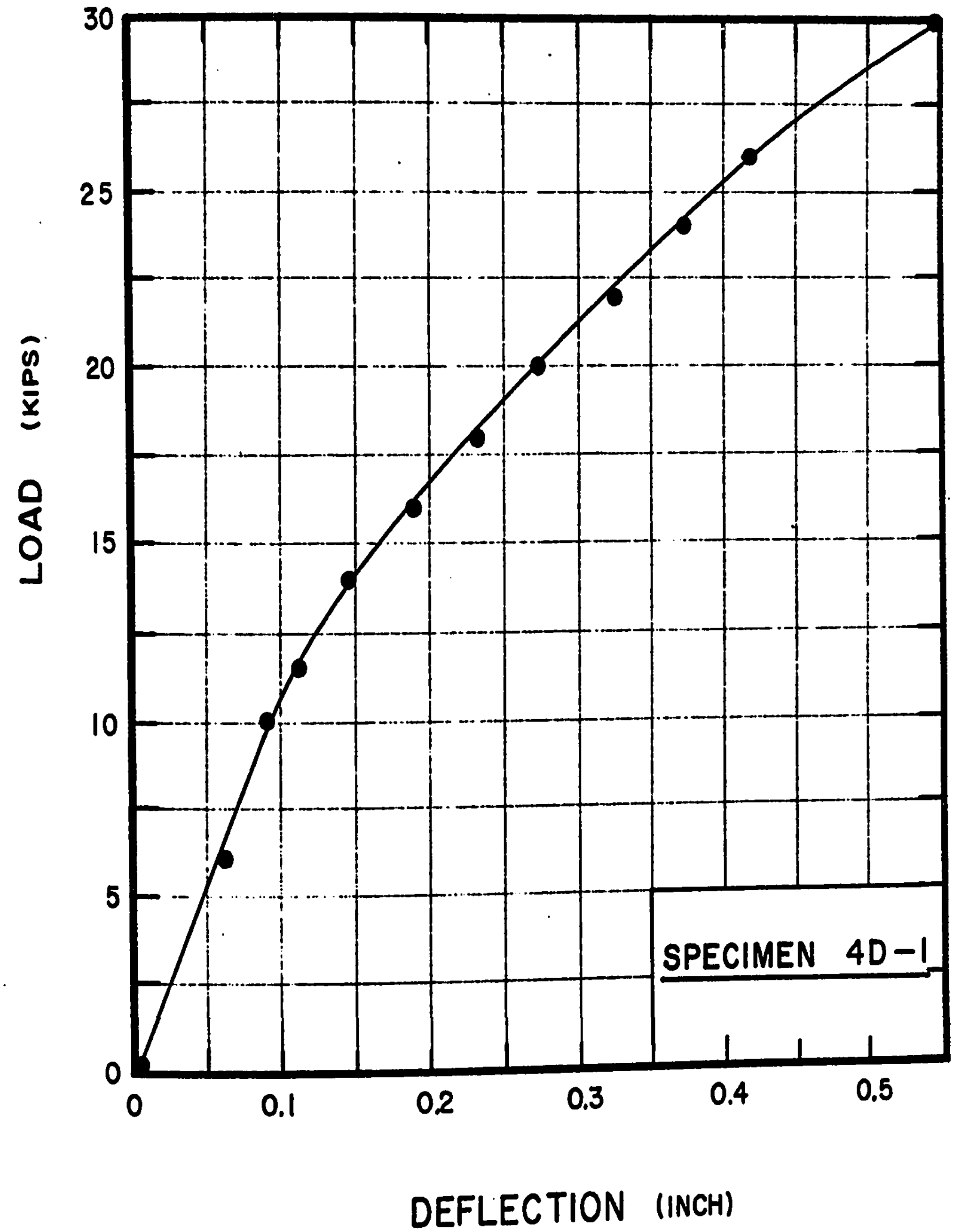


FIG. (B. 6) LOAD—DEFLECTION CURVE OF SPECIMEN 4D-1

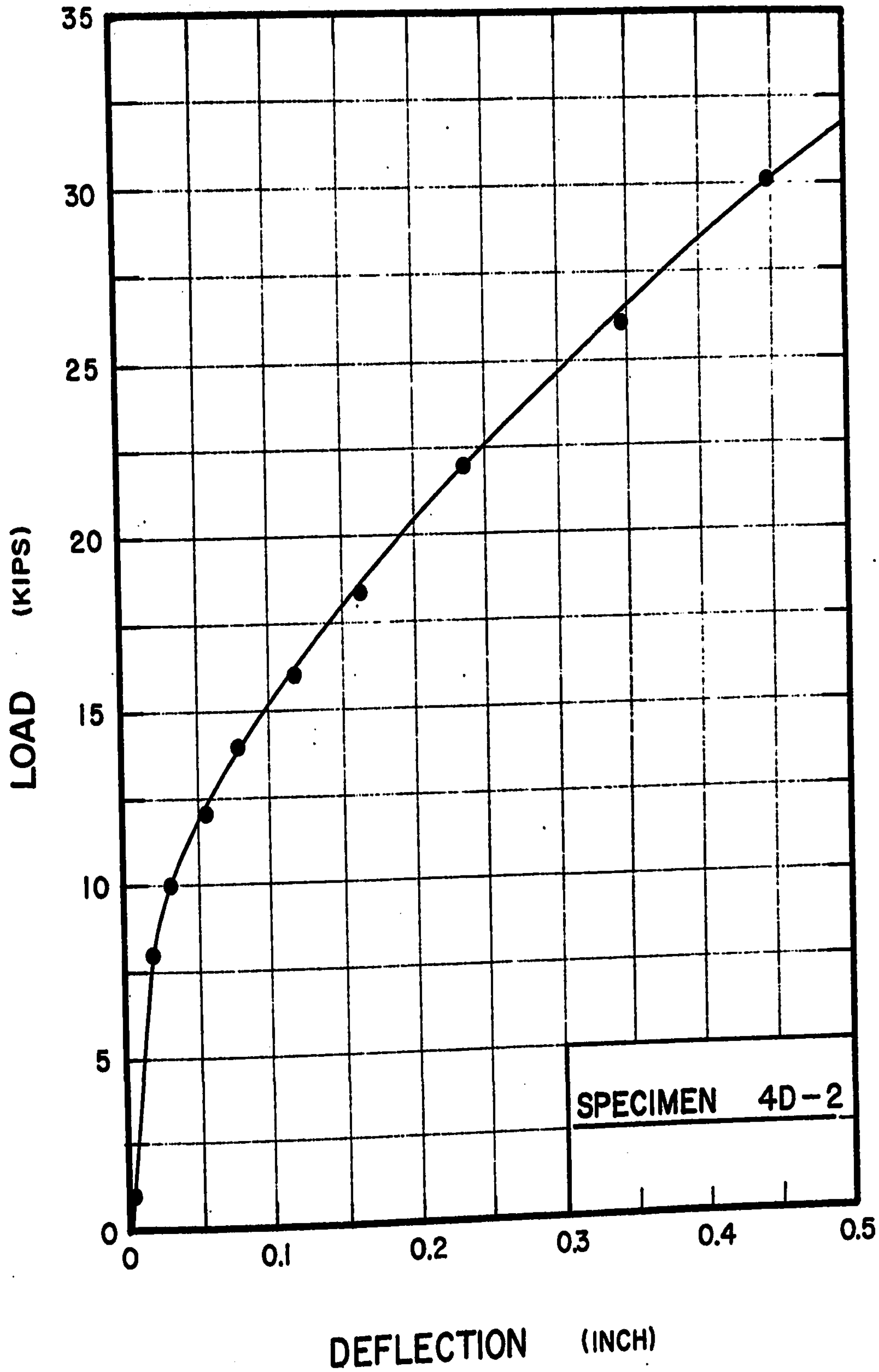


FIG.(B. 7) LOAD—DEFLECTION CURVE OF SPECIMEN 4D—2

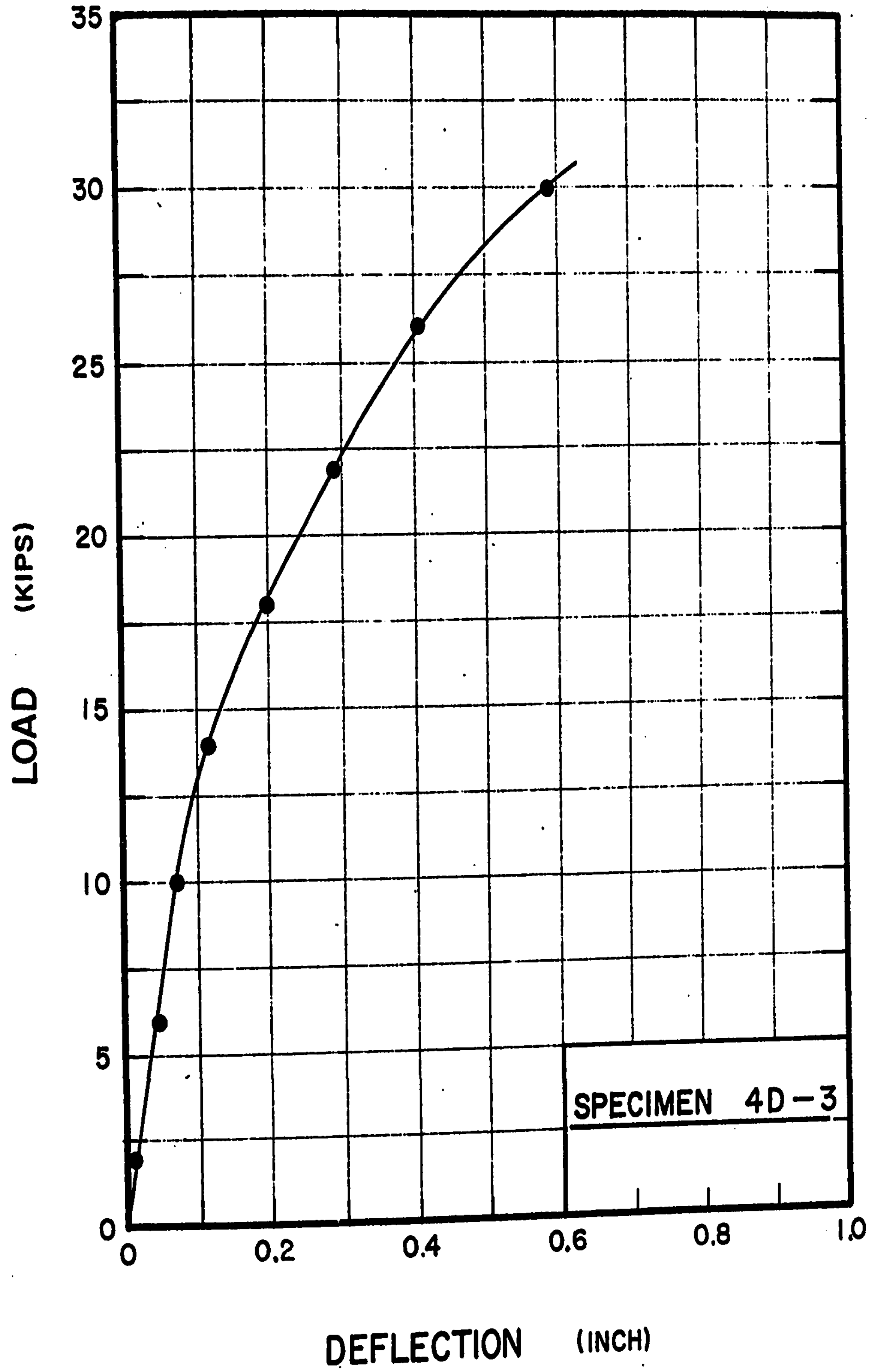


FIG.(B. 8) LOAD—DEFLECTION CURVE OF SPECIMEN 4D-3

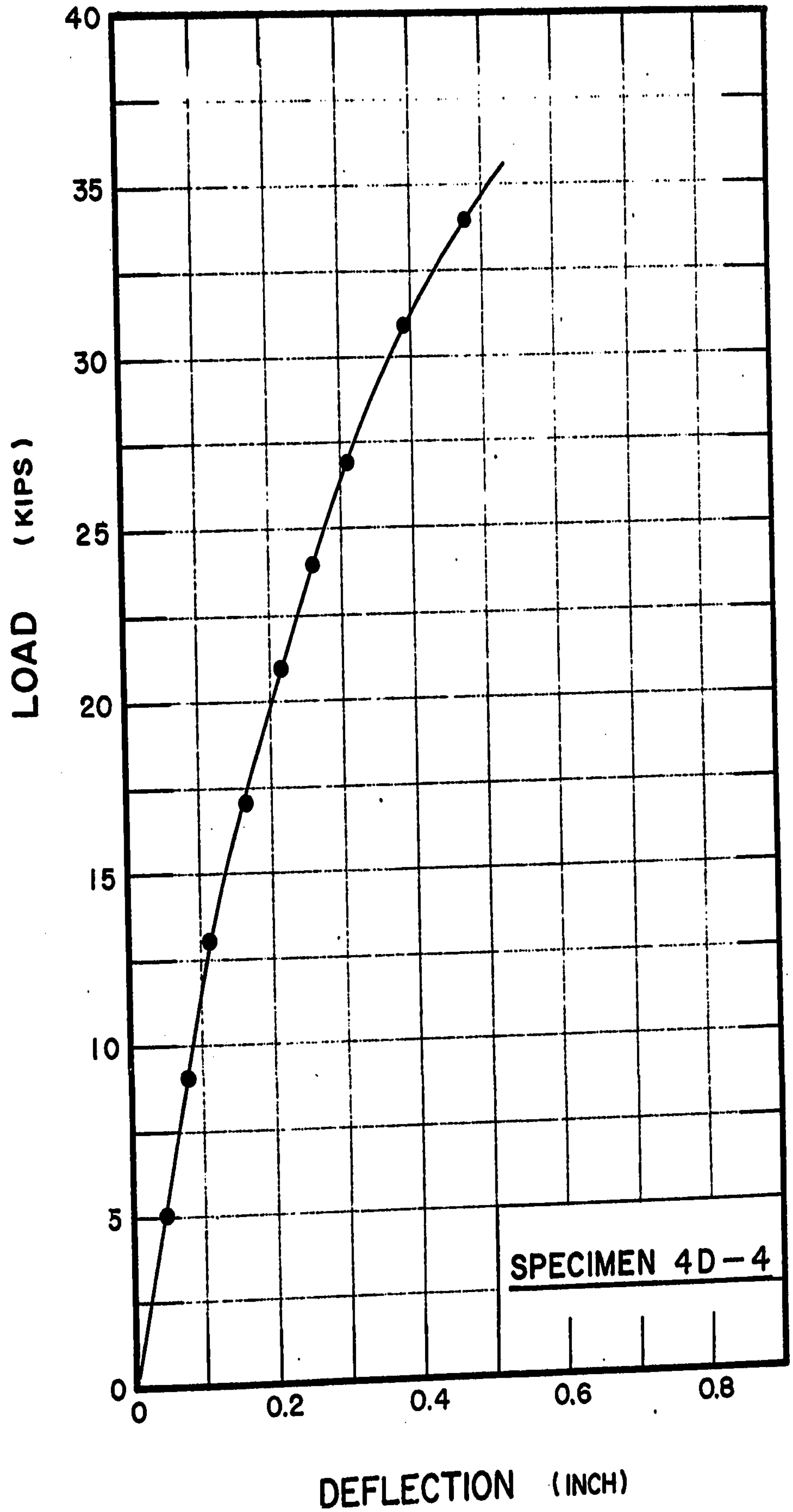


FIG.(B. 9) LOAD-DEFLECTION CURVE OF SPECIMEN 4D-4

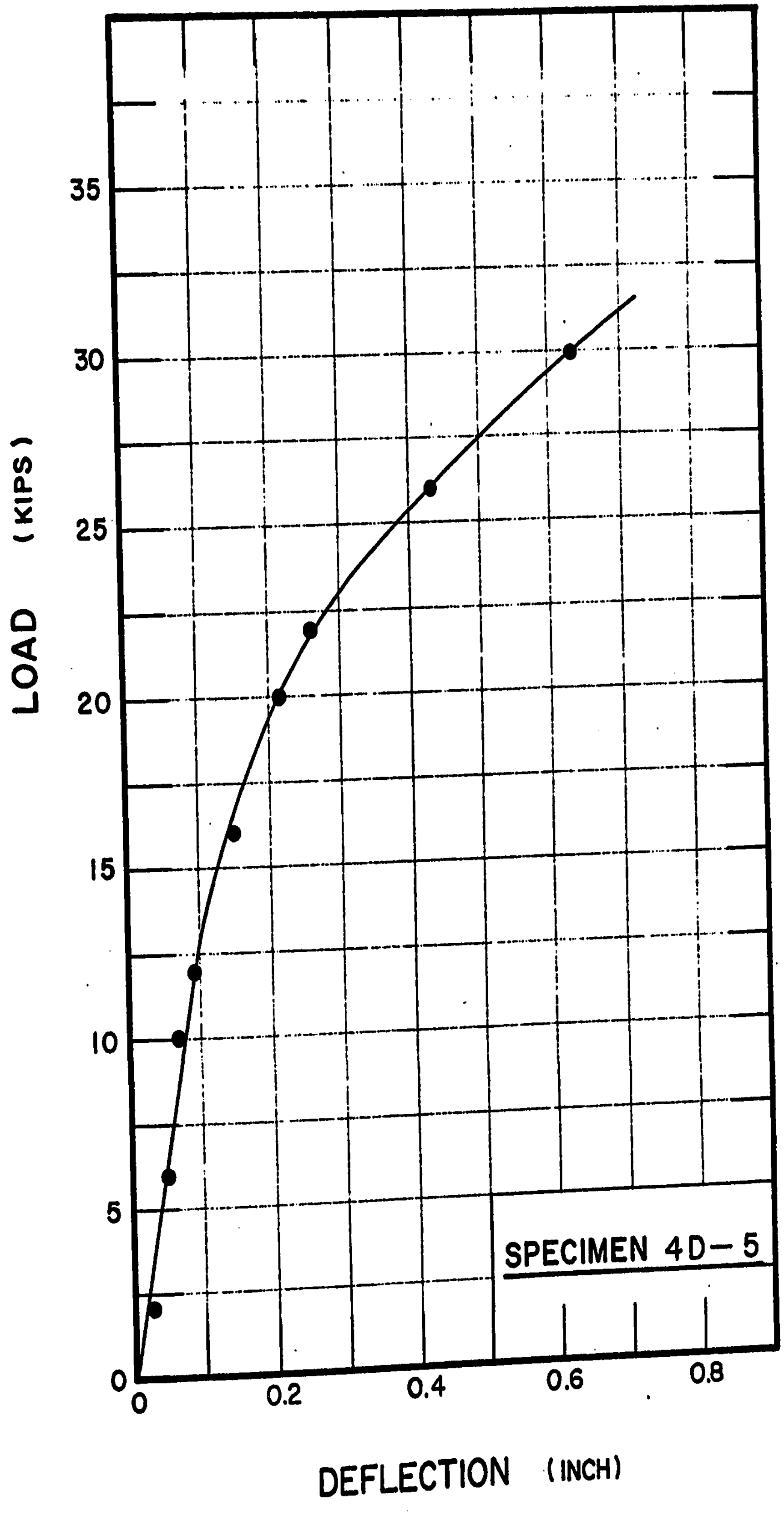


FIG.(B.10) LOAD-DEFLECTION CURVE OF SPECIMEN 4D-5

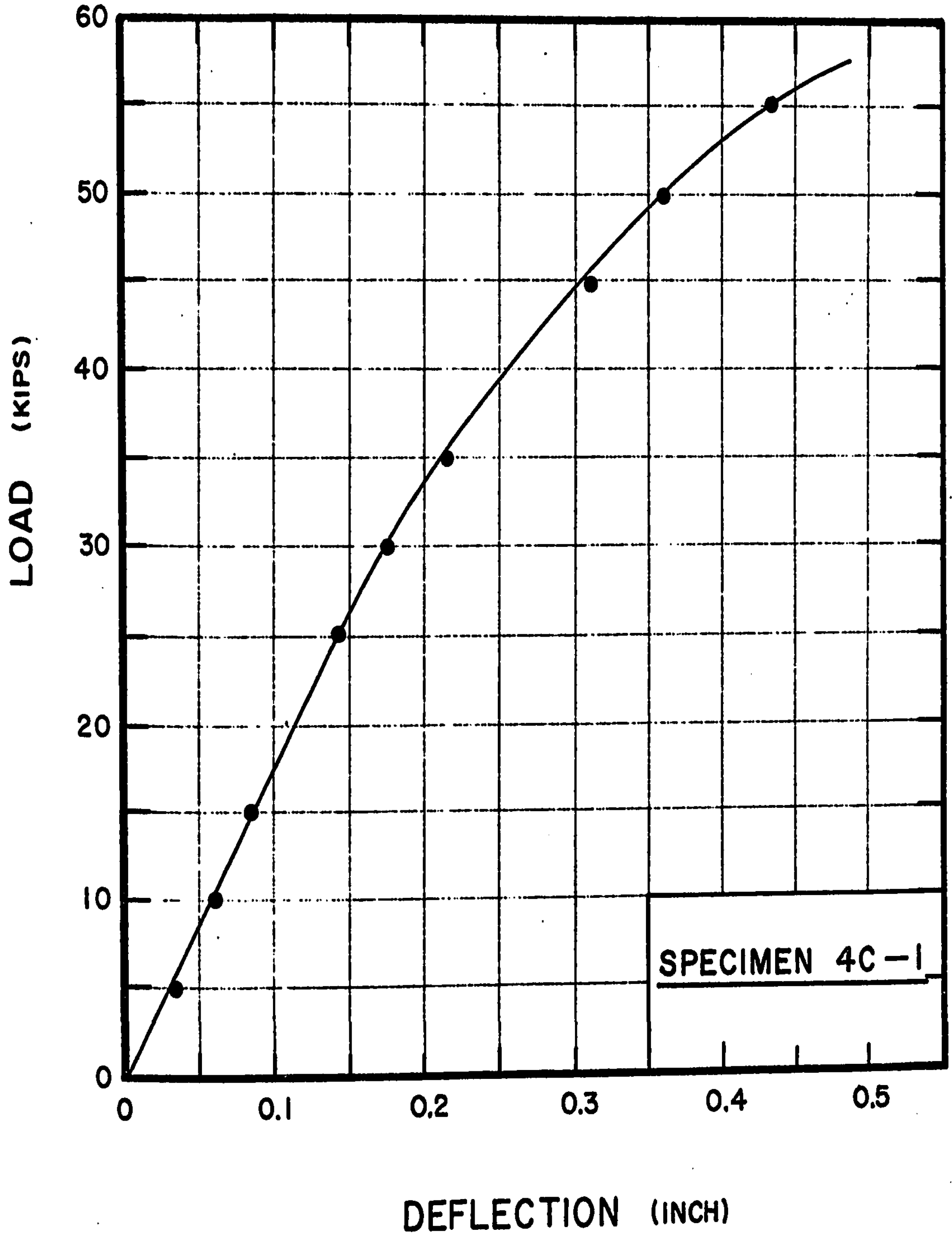


FIG. (B. 11) LOAD—DEFLECTION CURVE OF SPECIMEN 4C-1

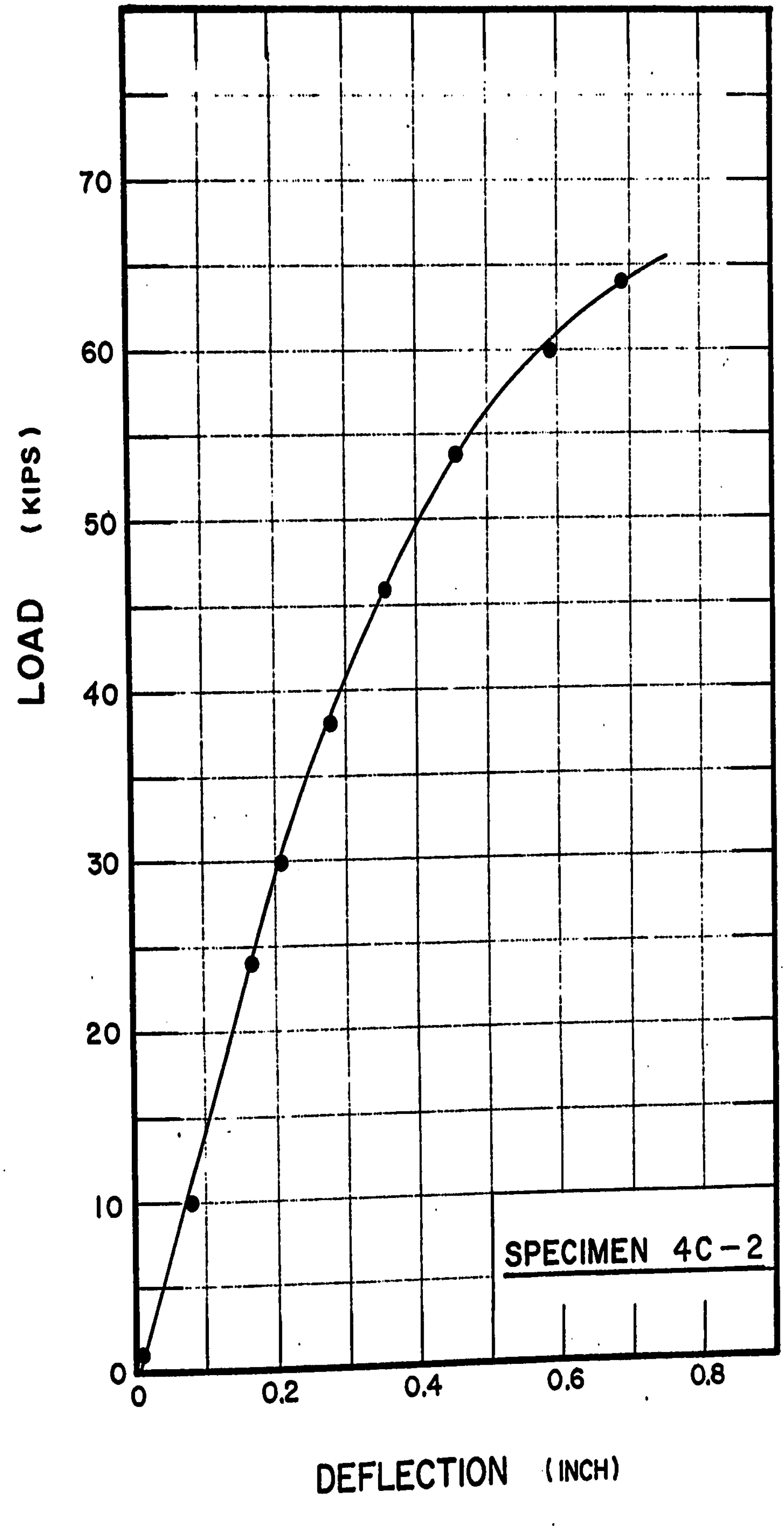


FIG.(B.12) LOAD-DEFLECTION CURVE OF SPECIMEN 4C-2

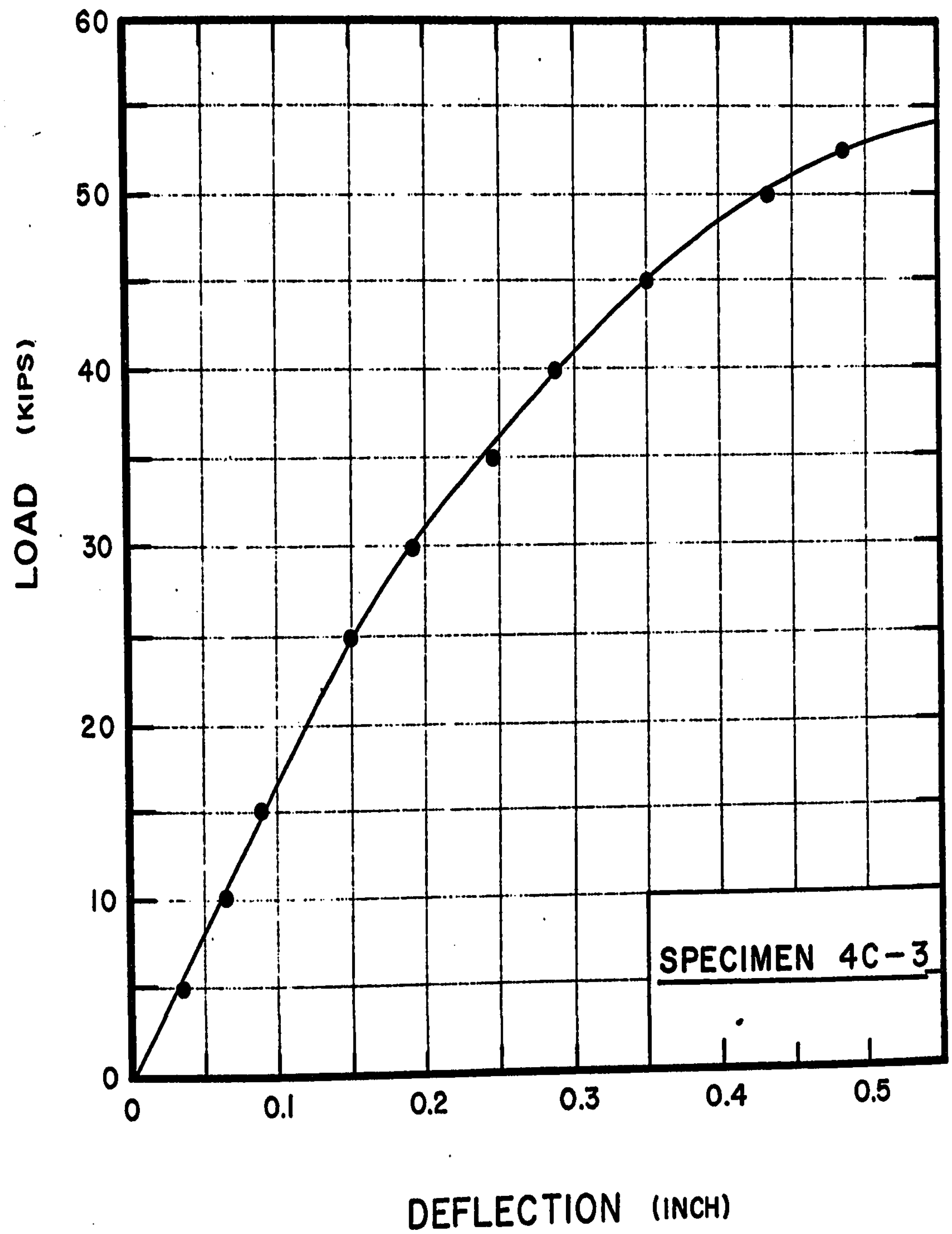


FIG. (B. 13) LOAD—DEFLECTION CURVE OF SPECIMEN 4C-3

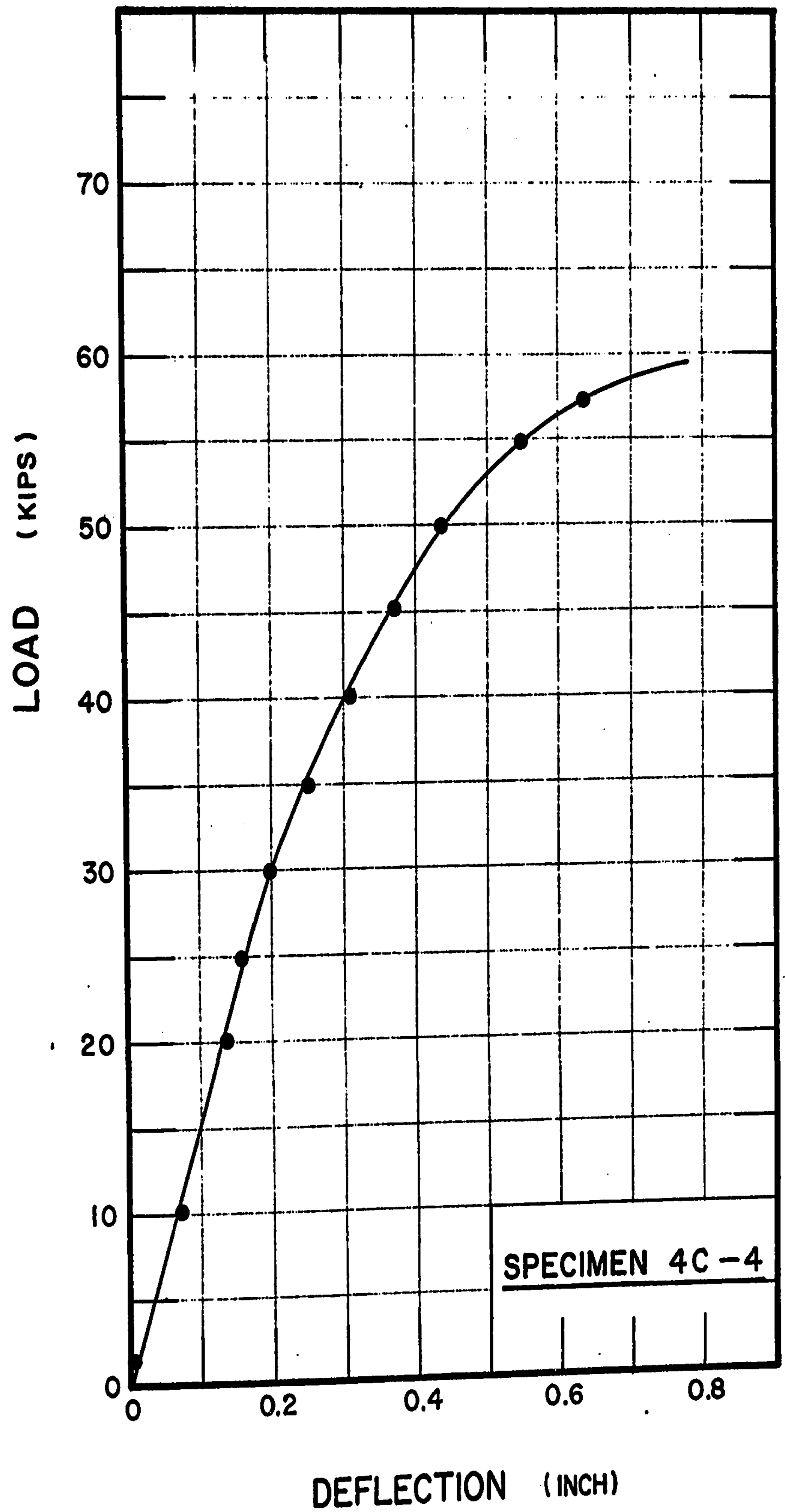


FIG.(B. 14) LOAD-DEFLECTION CURVE OF SPECIMEN 4C-4

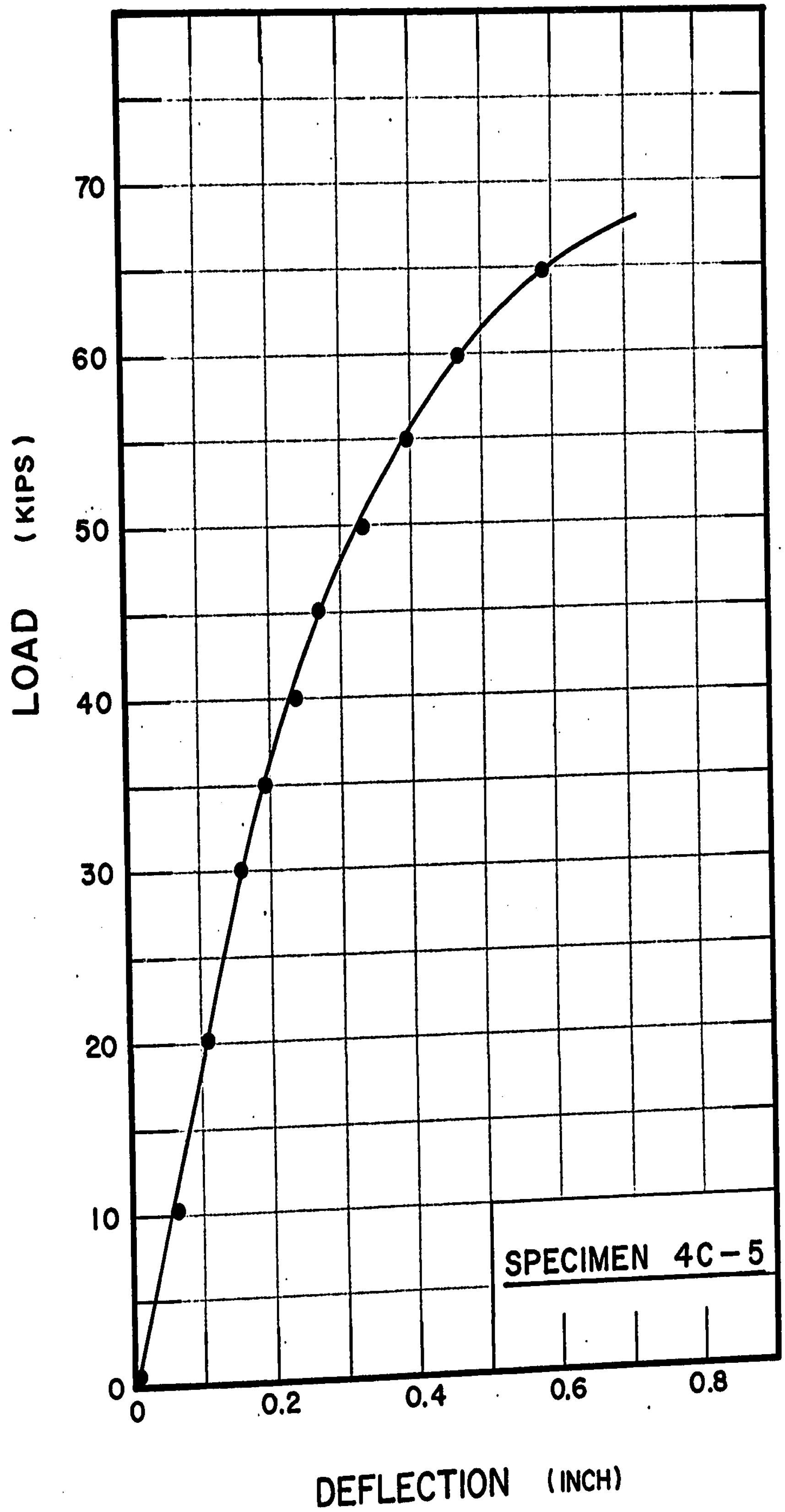


FIG.(B.15) LOAD-DEFLECTION CURVE OF SPECIMEN 4C-5

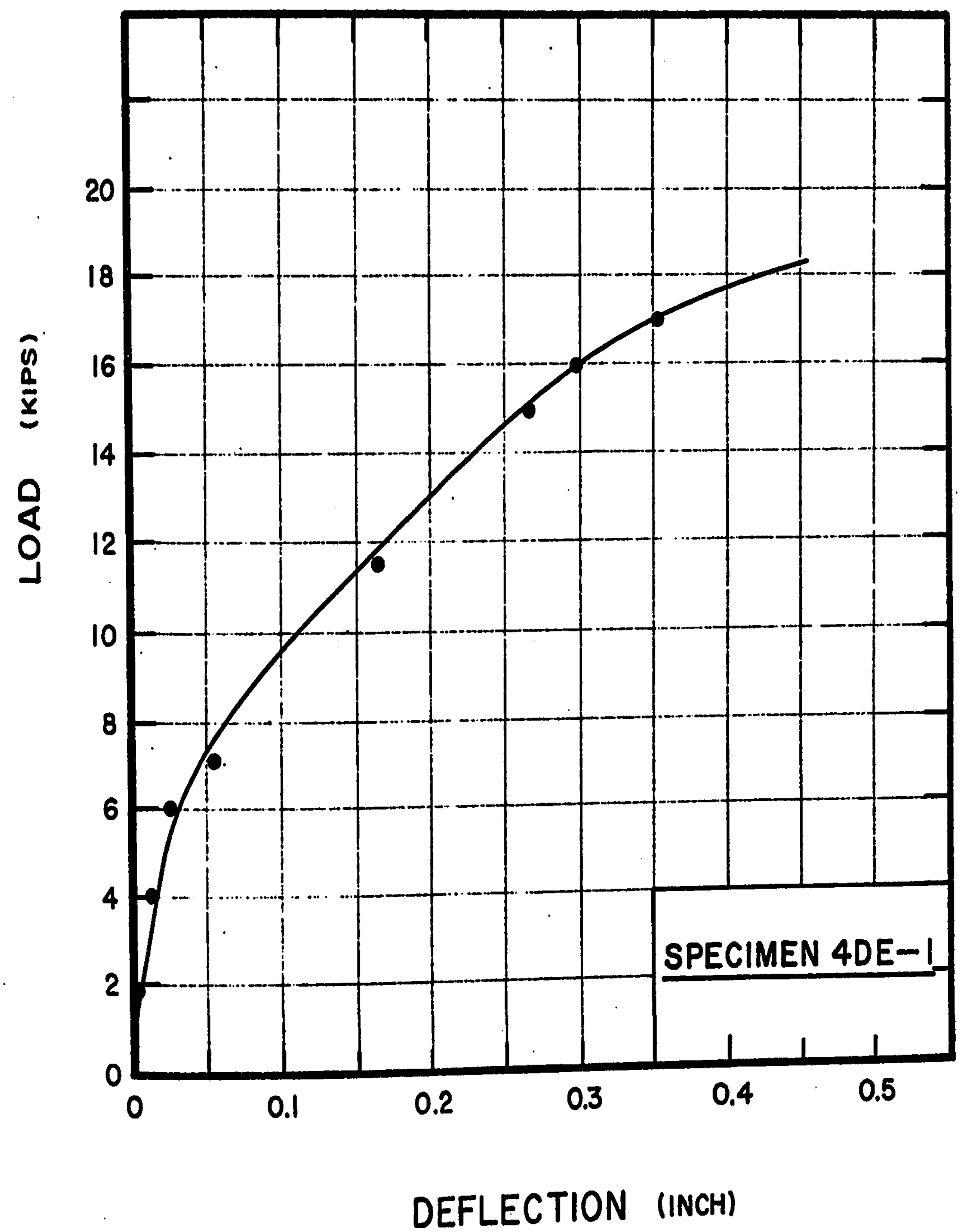


FIG. (B. 16) LOAD—DEFLECTION CURVE OF SPECIMEN 4DE-1

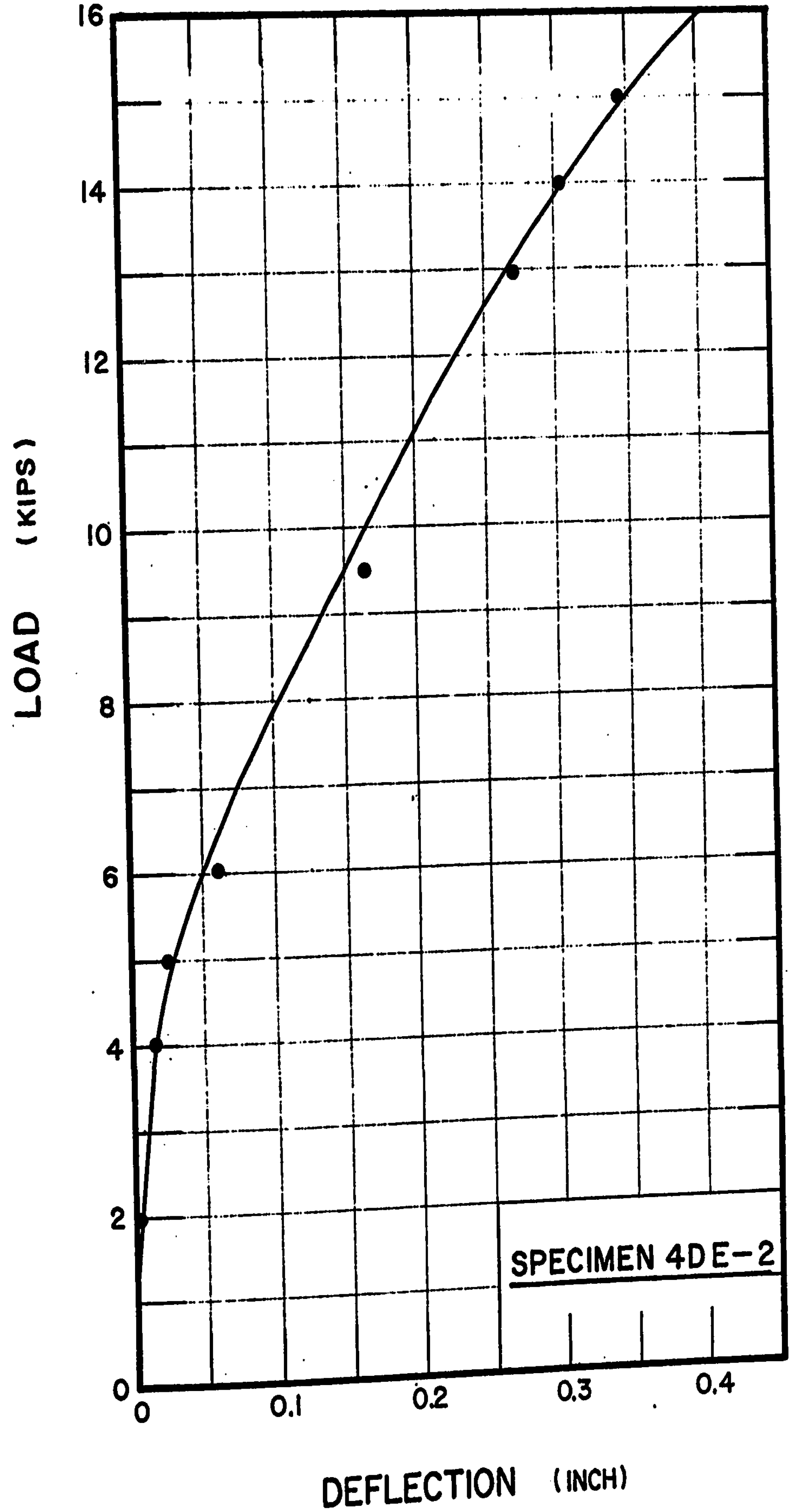


FIG.(B.17) LOAD-DEFLECTION CURVE OF SPECIMEN 4DE-2

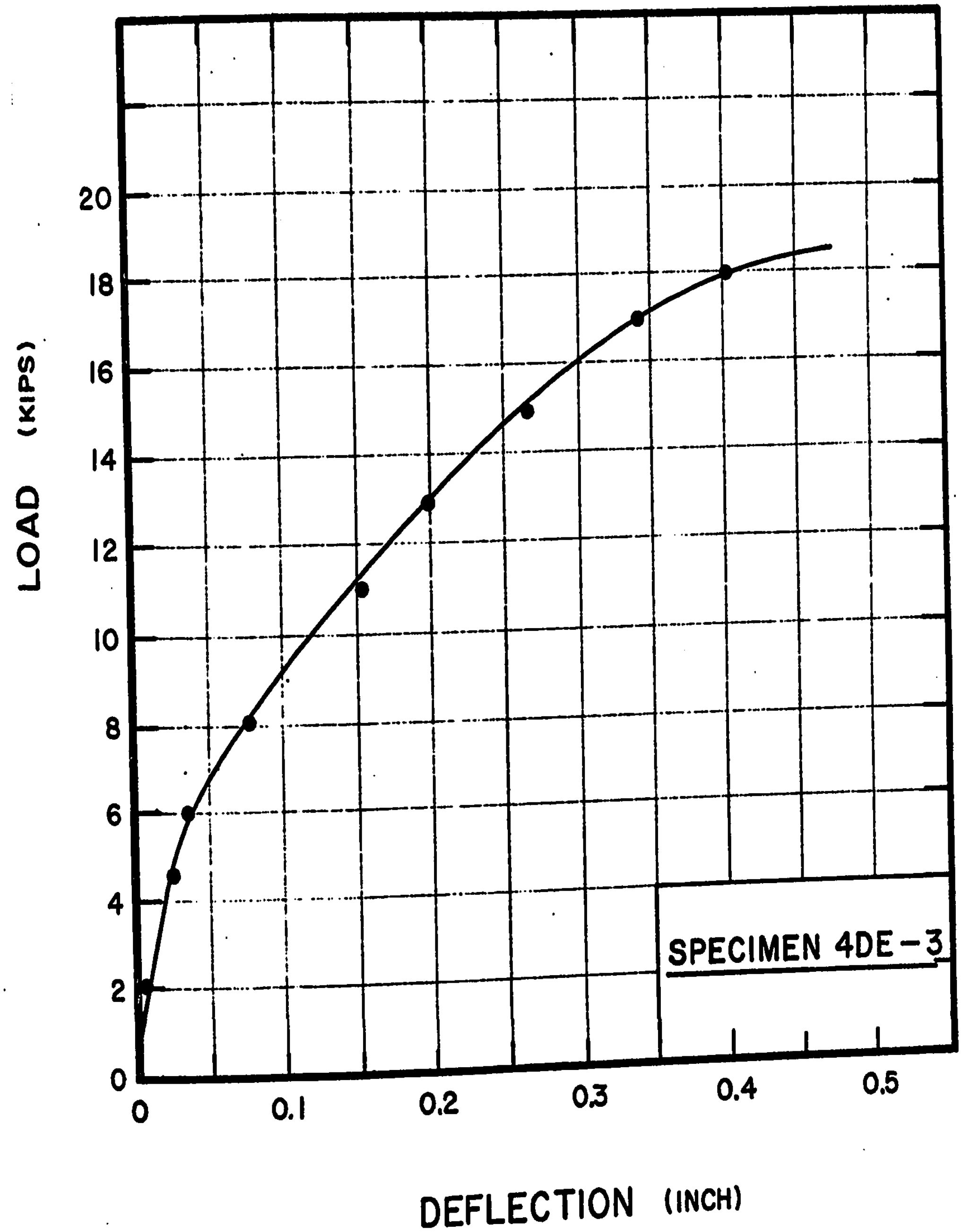


FIG. (B. 18) LOAD—DEFLECTION CURVE OF SPECIMEN 4DE-3

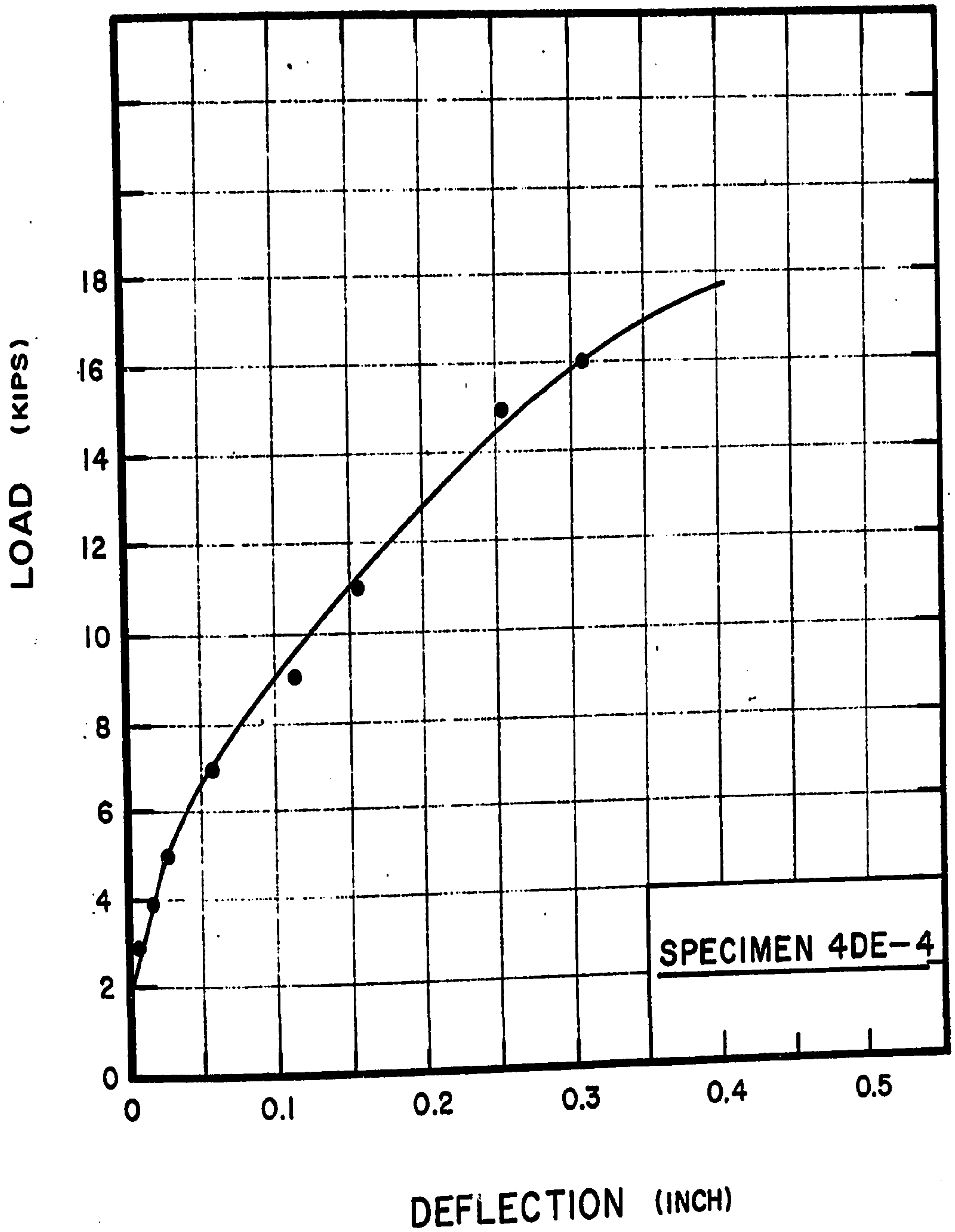


FIG. (B. 19) LOAD—DEFLECTION CURVE OF SPECIMEN 4DE-4

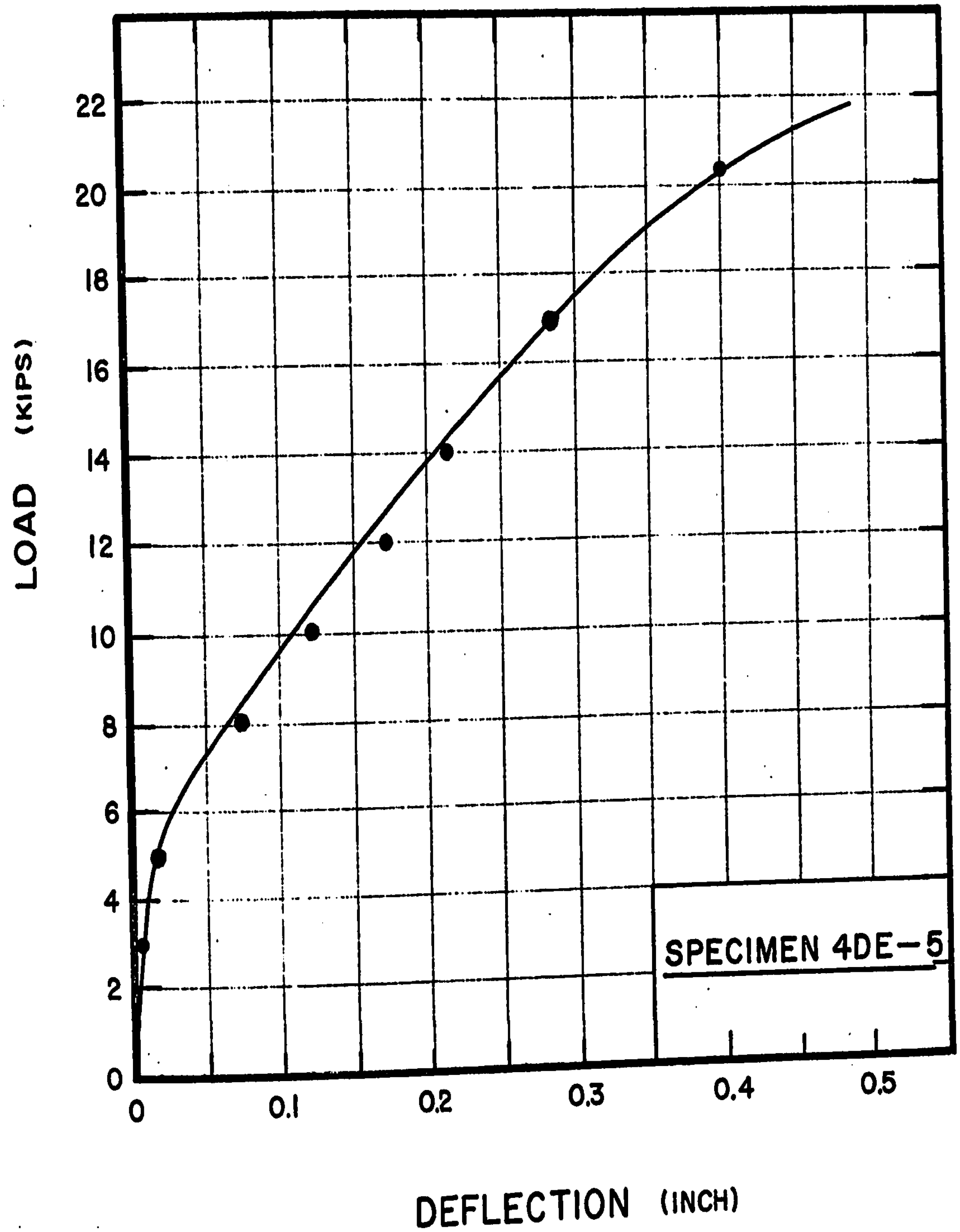


FIG. (B. 20) LOAD—DEFLECTION CURVE OF SPECIMEN 4DE-5

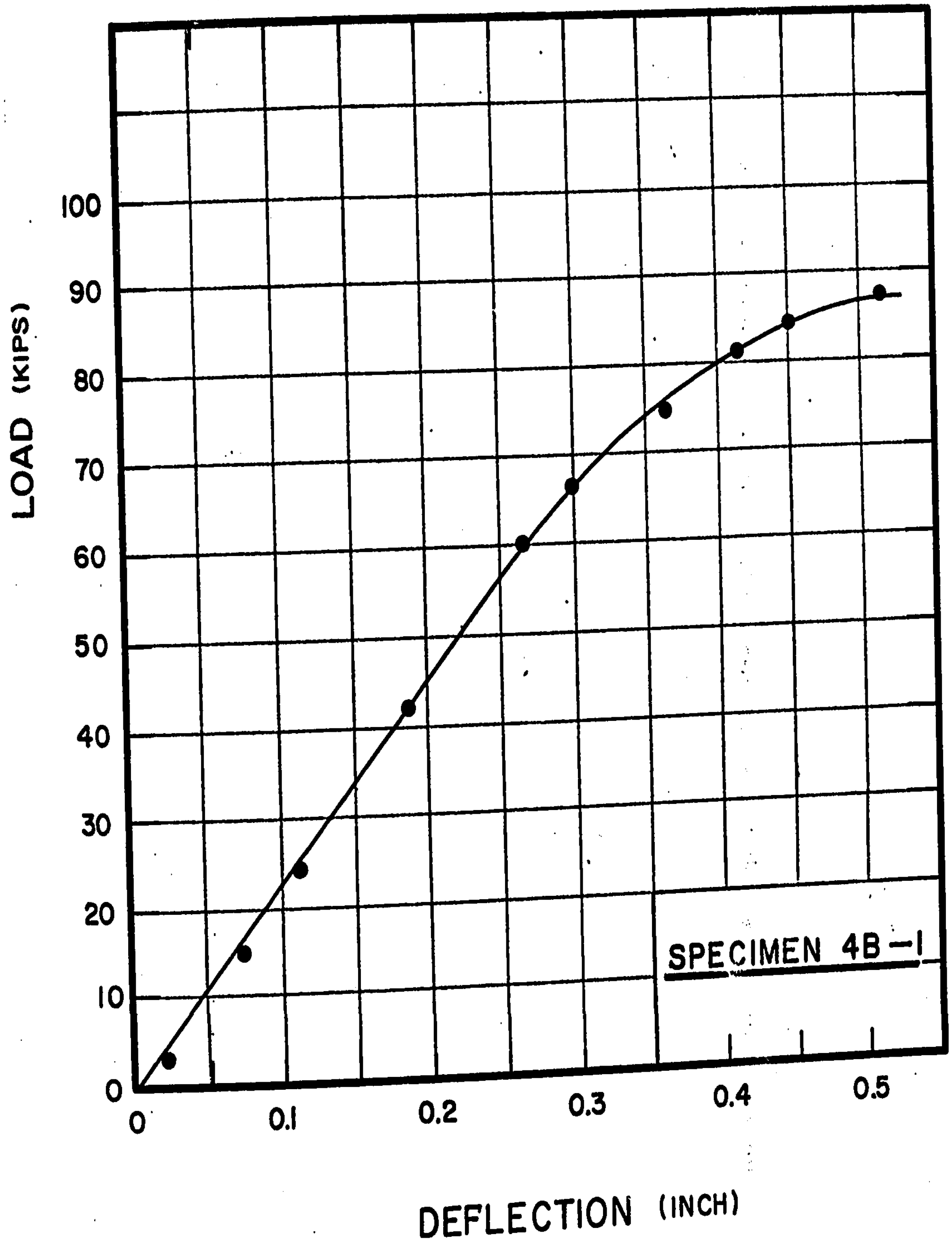


FIG.(B. 21) LOAD—DEFLECTION CURVE OF SPECIMEN 4B—1

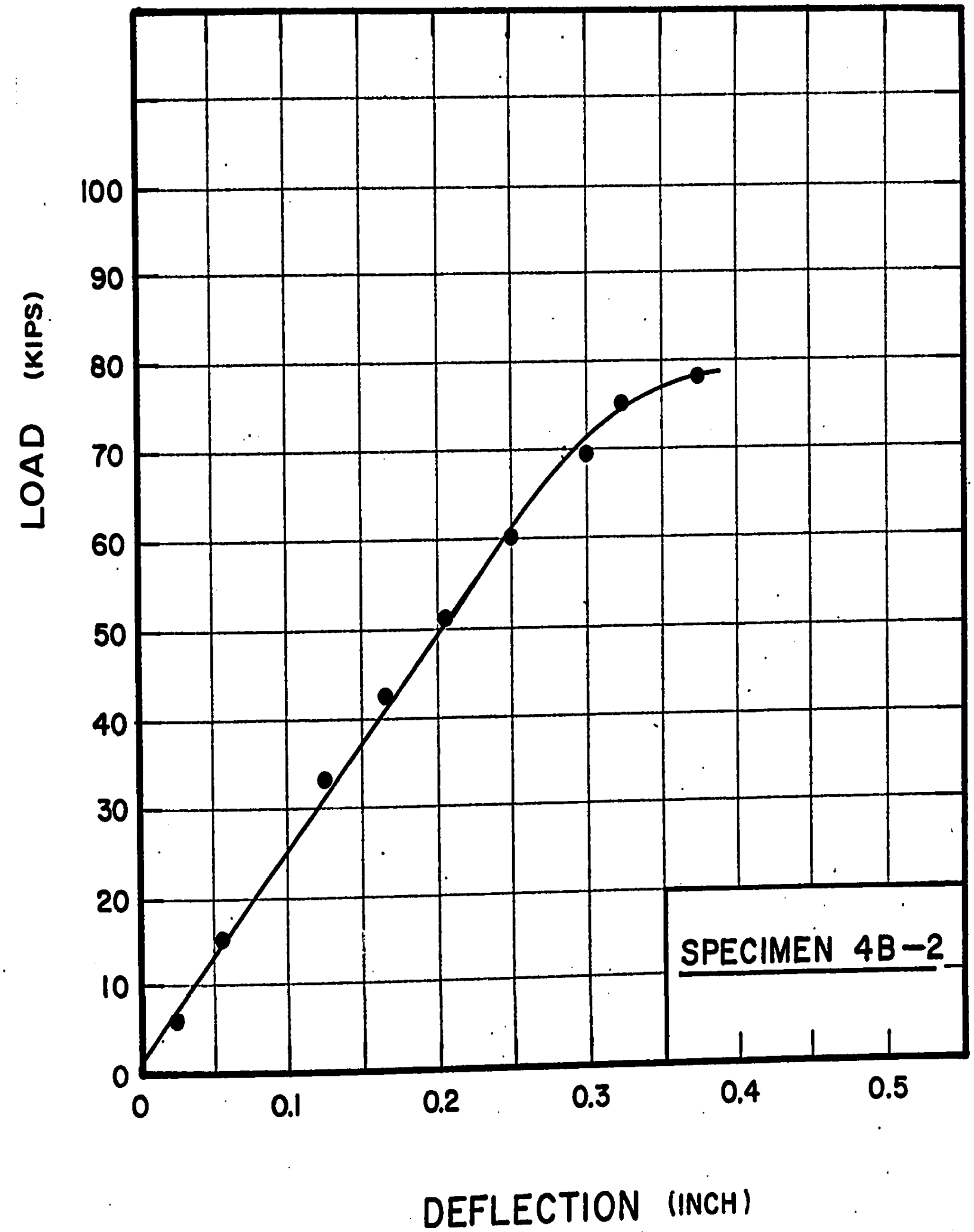


FIG. (B. 22) LOAD — DEFLECTION CURVE OF SPECIMEN 4B-2

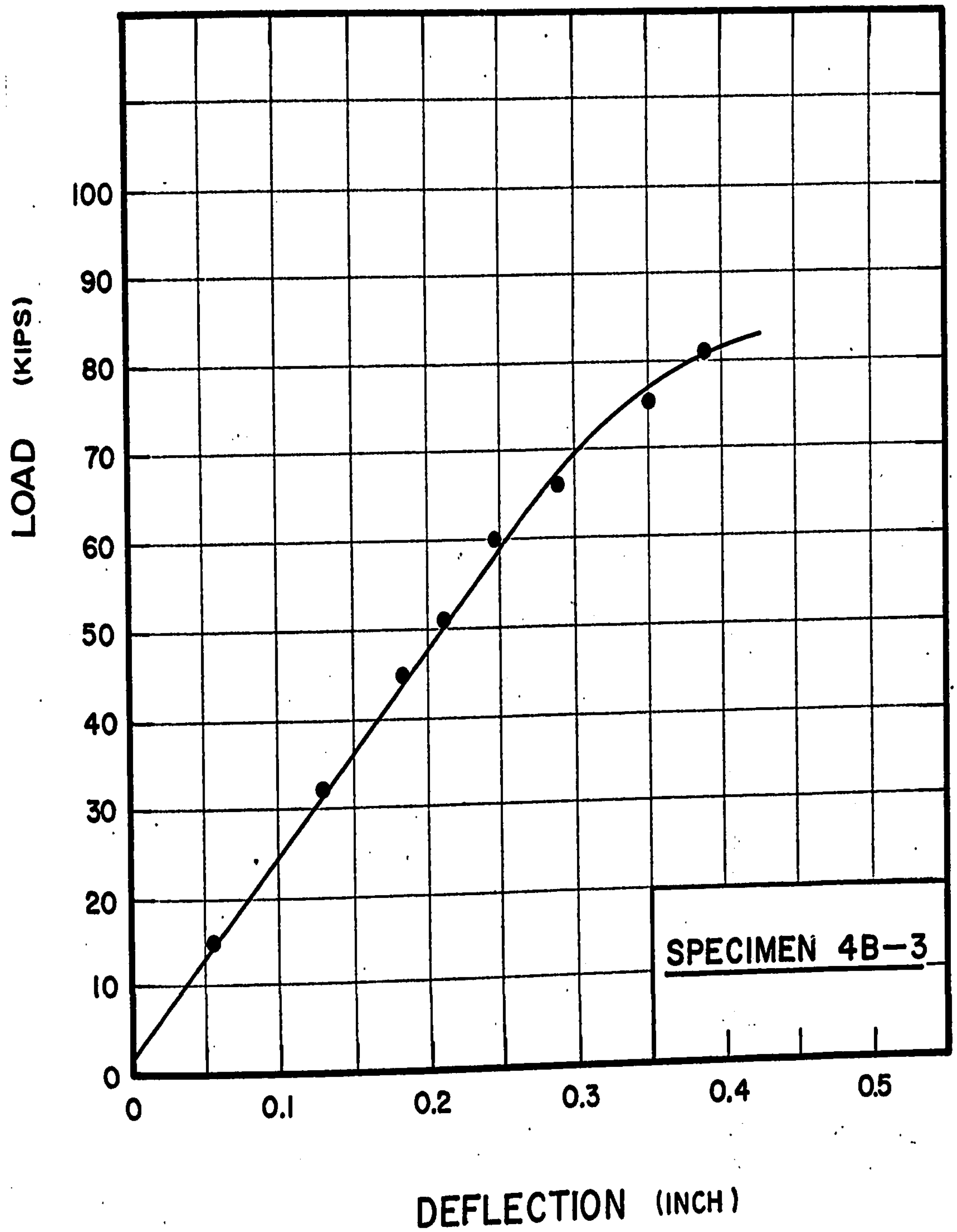


FIG. (B. 23) LOAD — DEFLECTION CURVE OF SPECIMEN 4B-3

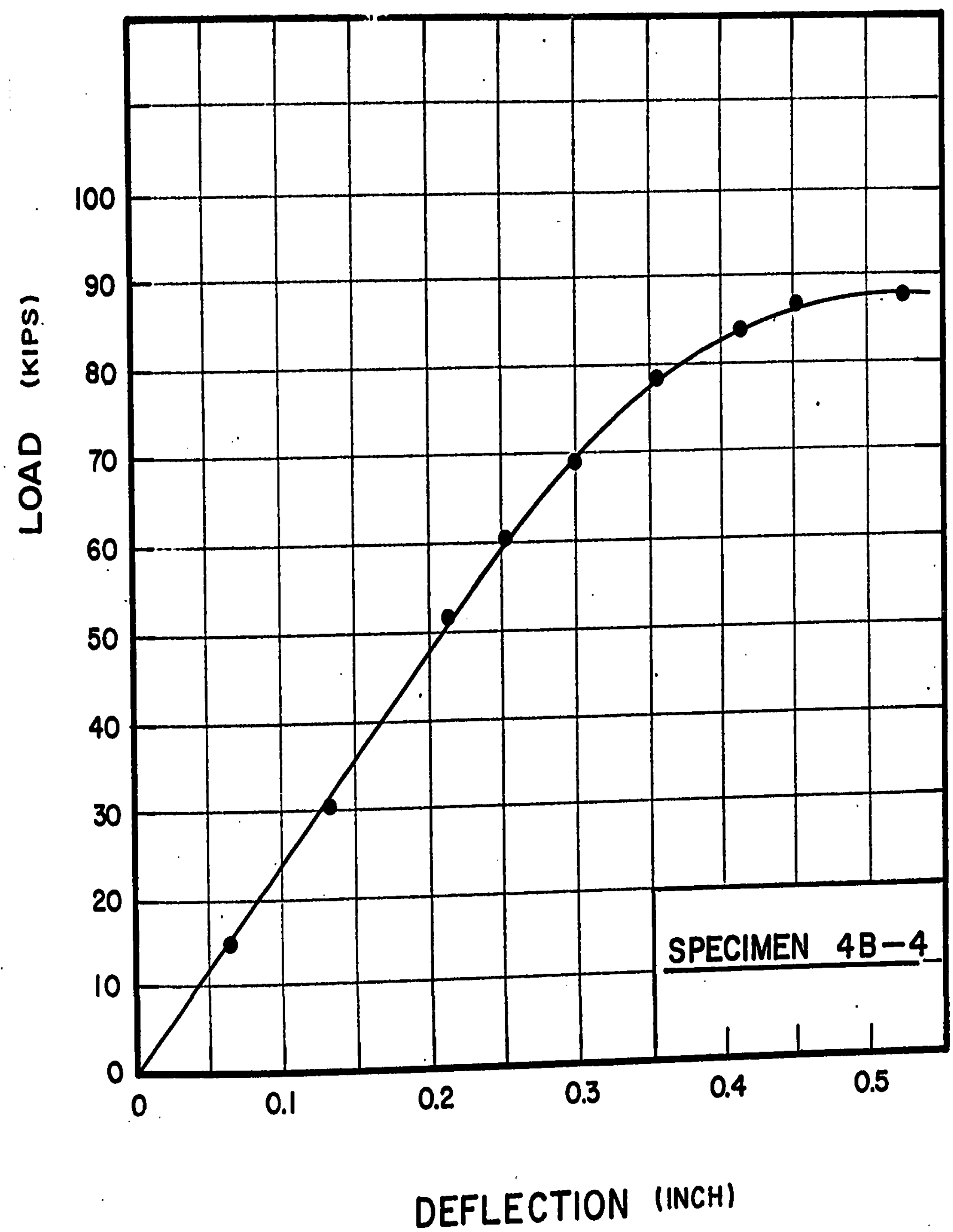


FIG. (B. 24) LOAD — DEFLECTION CURVE OF SPECIMEN 4B — 4

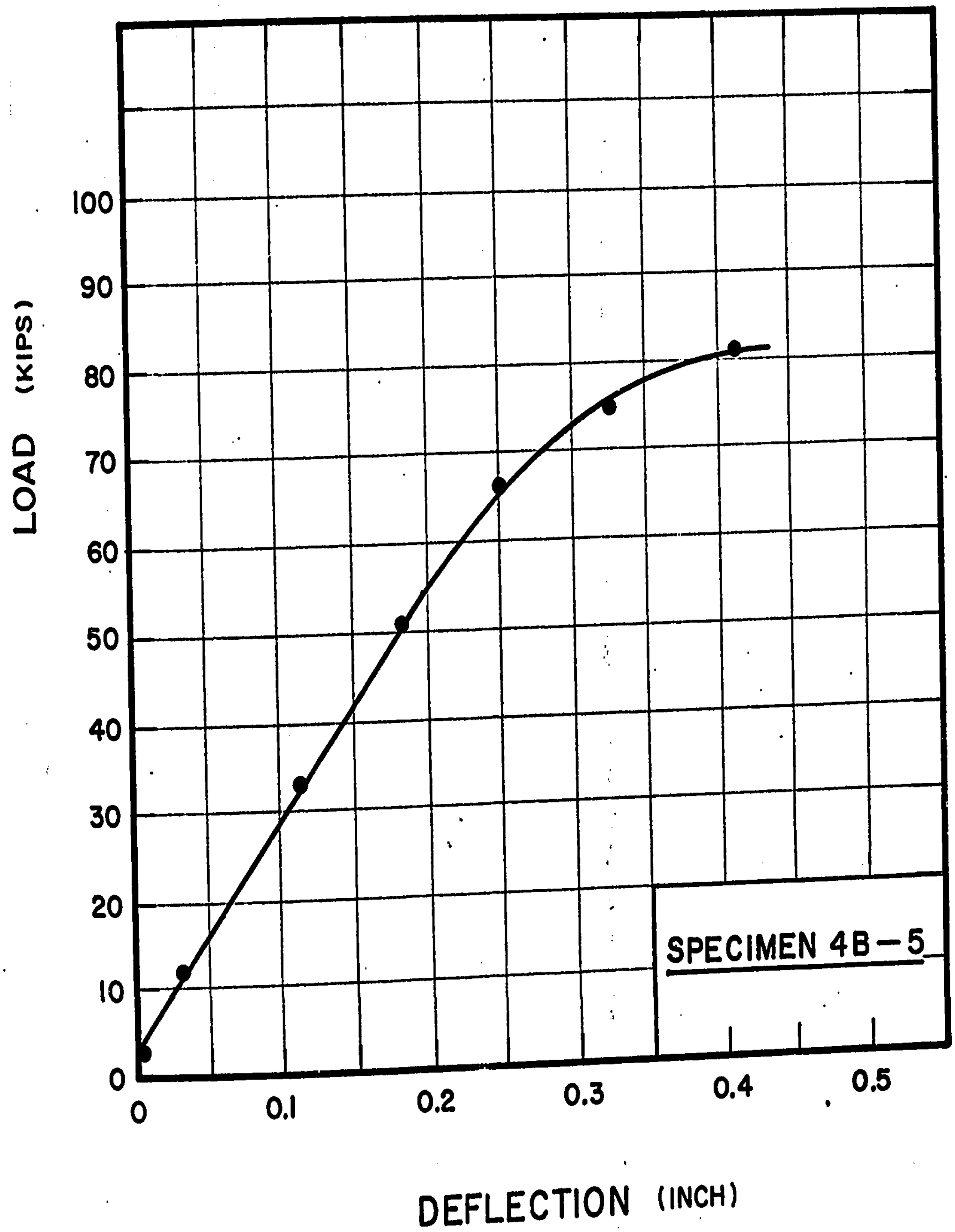


FIG.(B. 25) LOAD—DEFLECTION CURVE OF SPECIMEN 4B-5

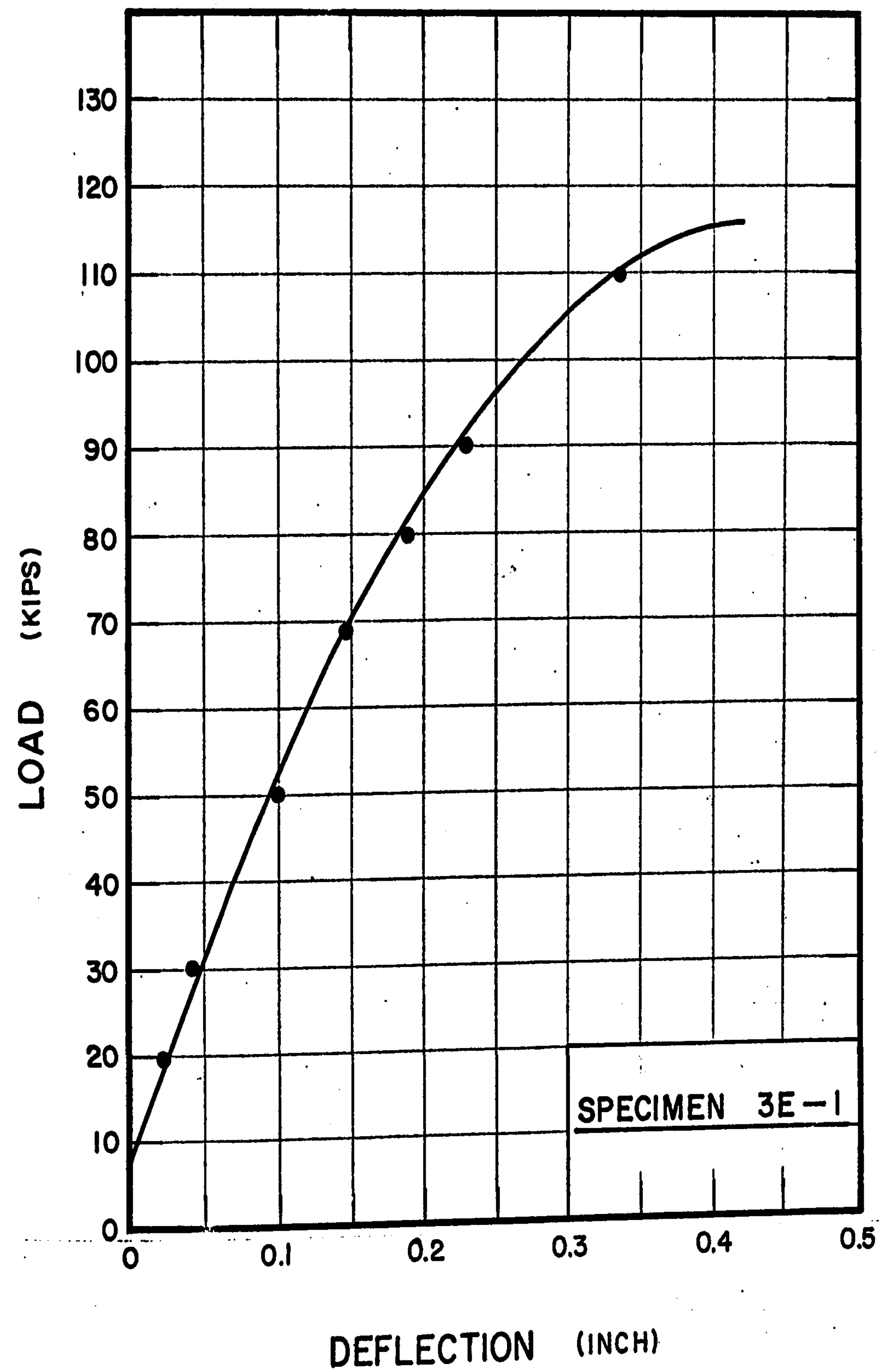


FIG.(B.26) LOAD-DEFLECTION CURVE OF SPECIMEN 3E-1

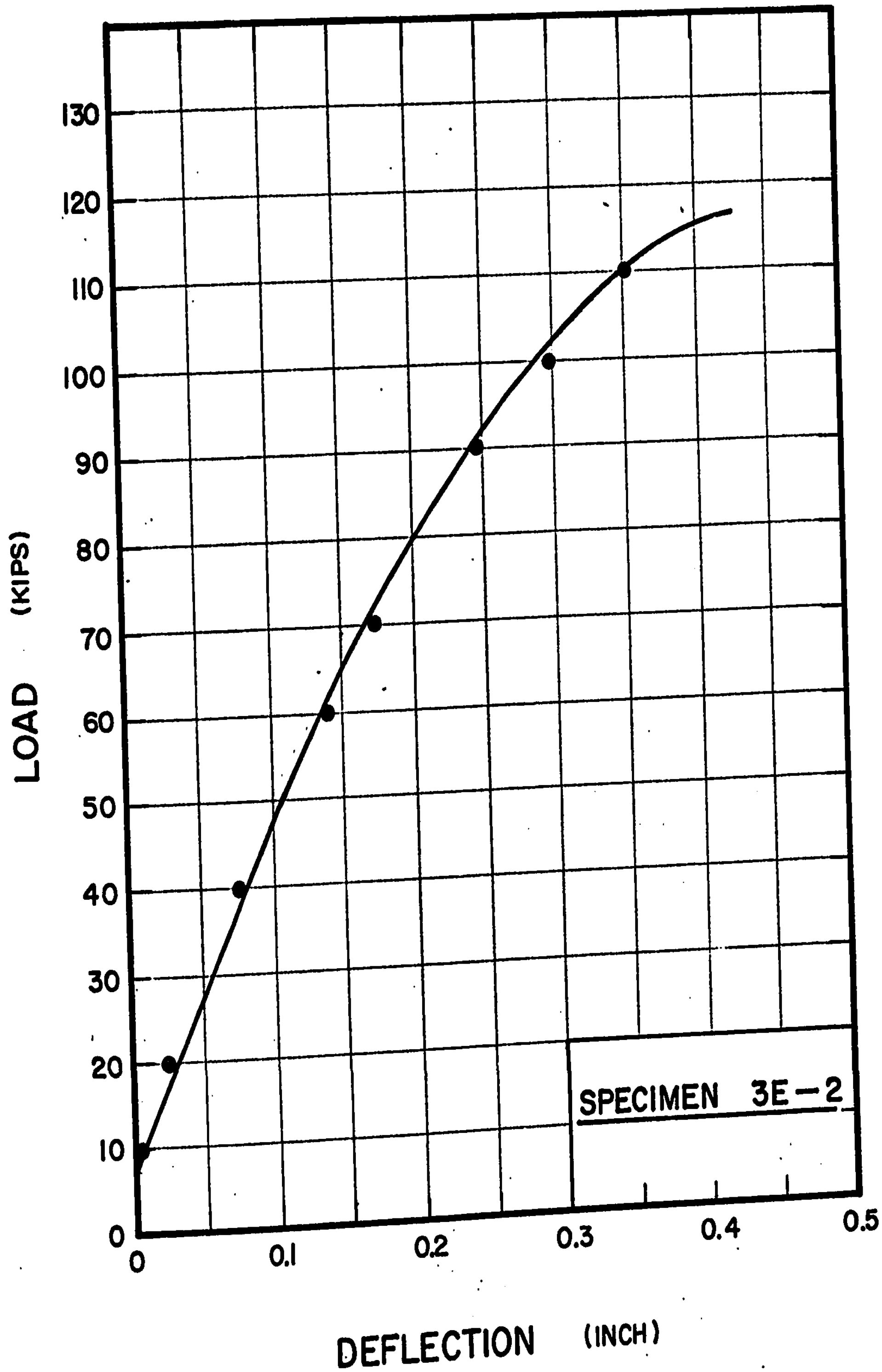


FIG.(B.27) LOAD-DEFLECTION CURVE OF SPECIMEN 3E-2

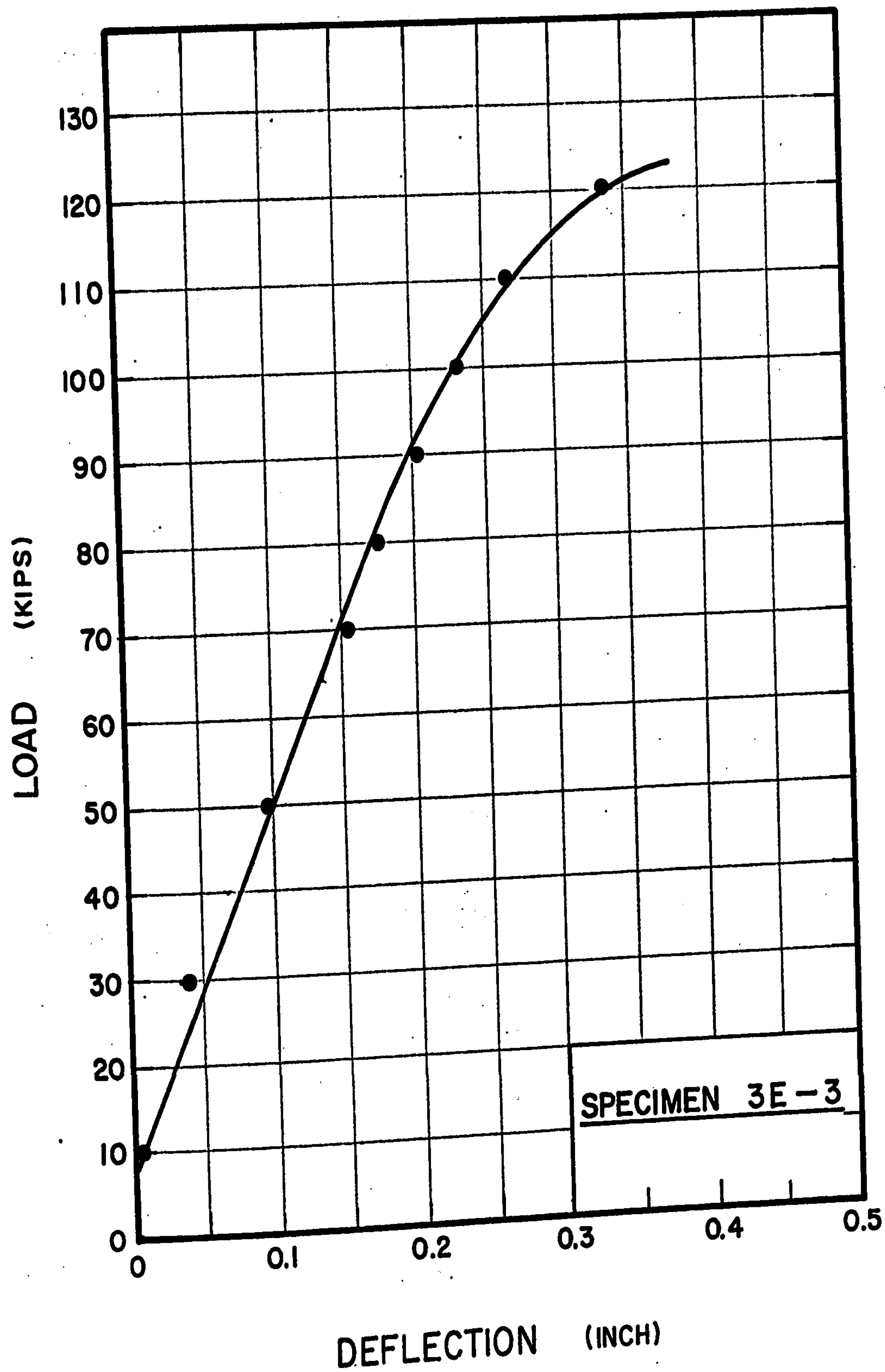


FIG.(B.28) LOAD-DEFLECTION CURVE OF SPECIMEN 3E-3

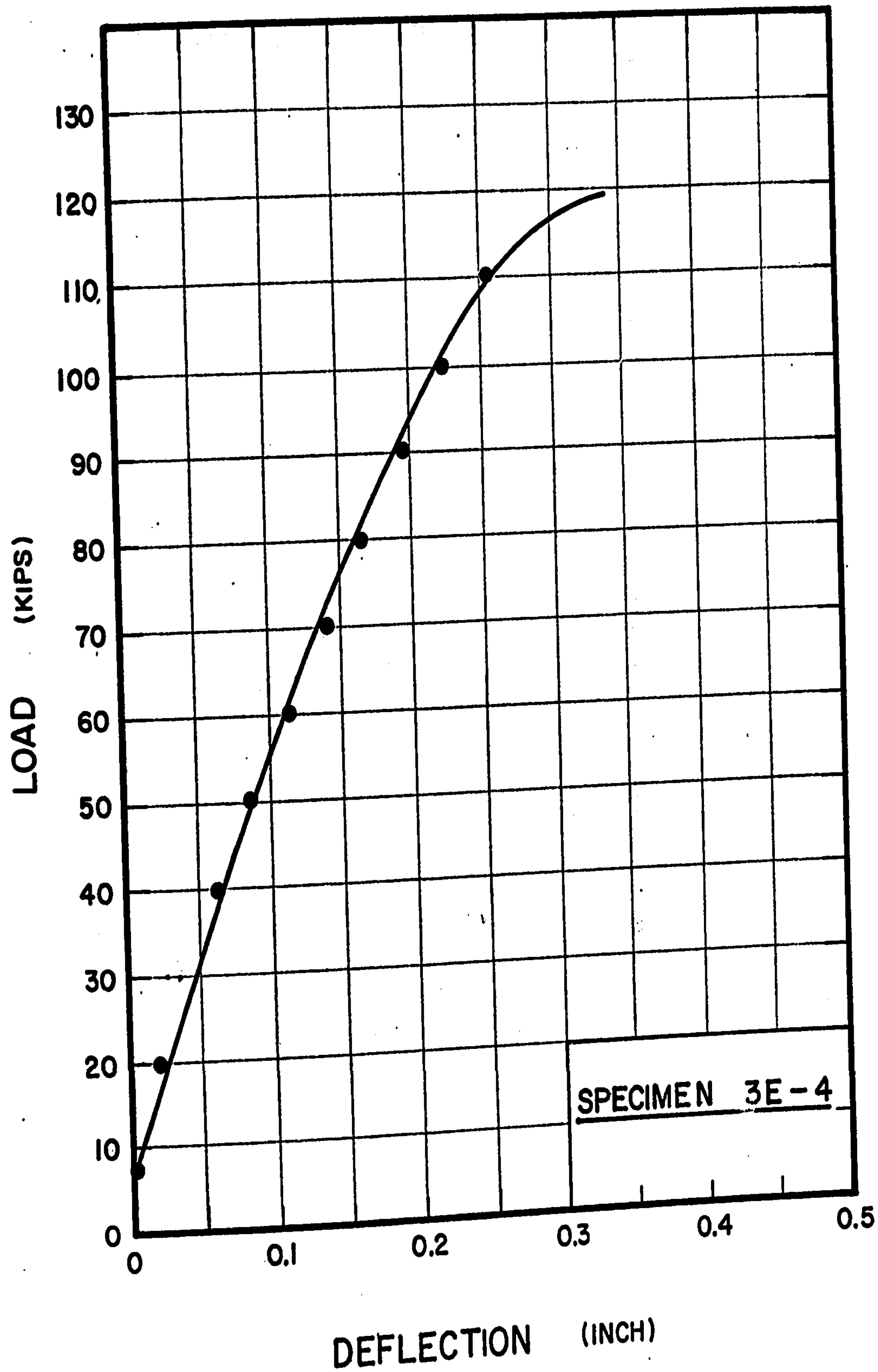


FIG.(B.29) LOAD-DEFLECTION CURVE OF SPECIMEN 3E-4

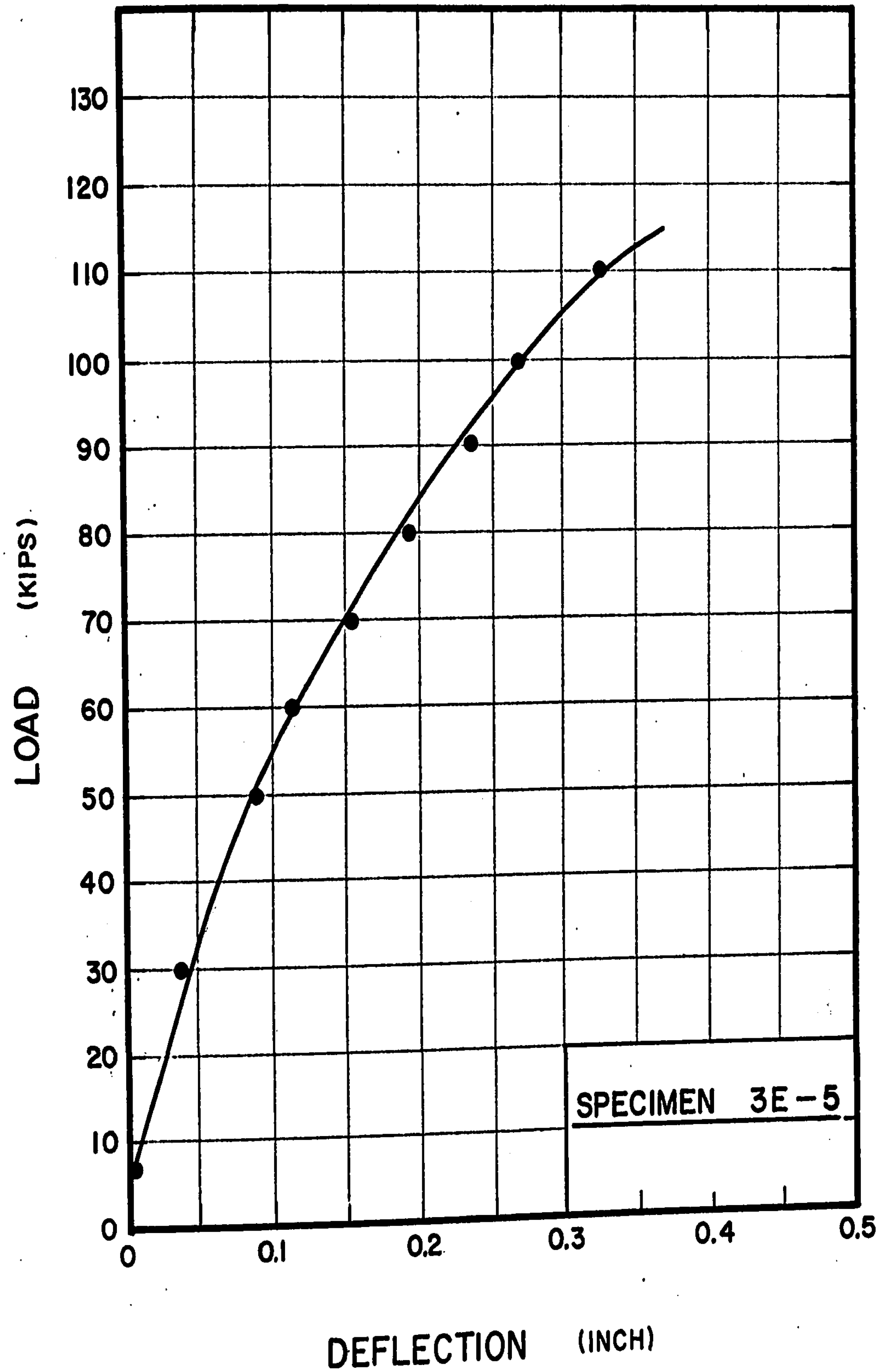


FIG.(B.30) LOAD—DEFLECTION CURVE OF SPECIMEN 3E-5

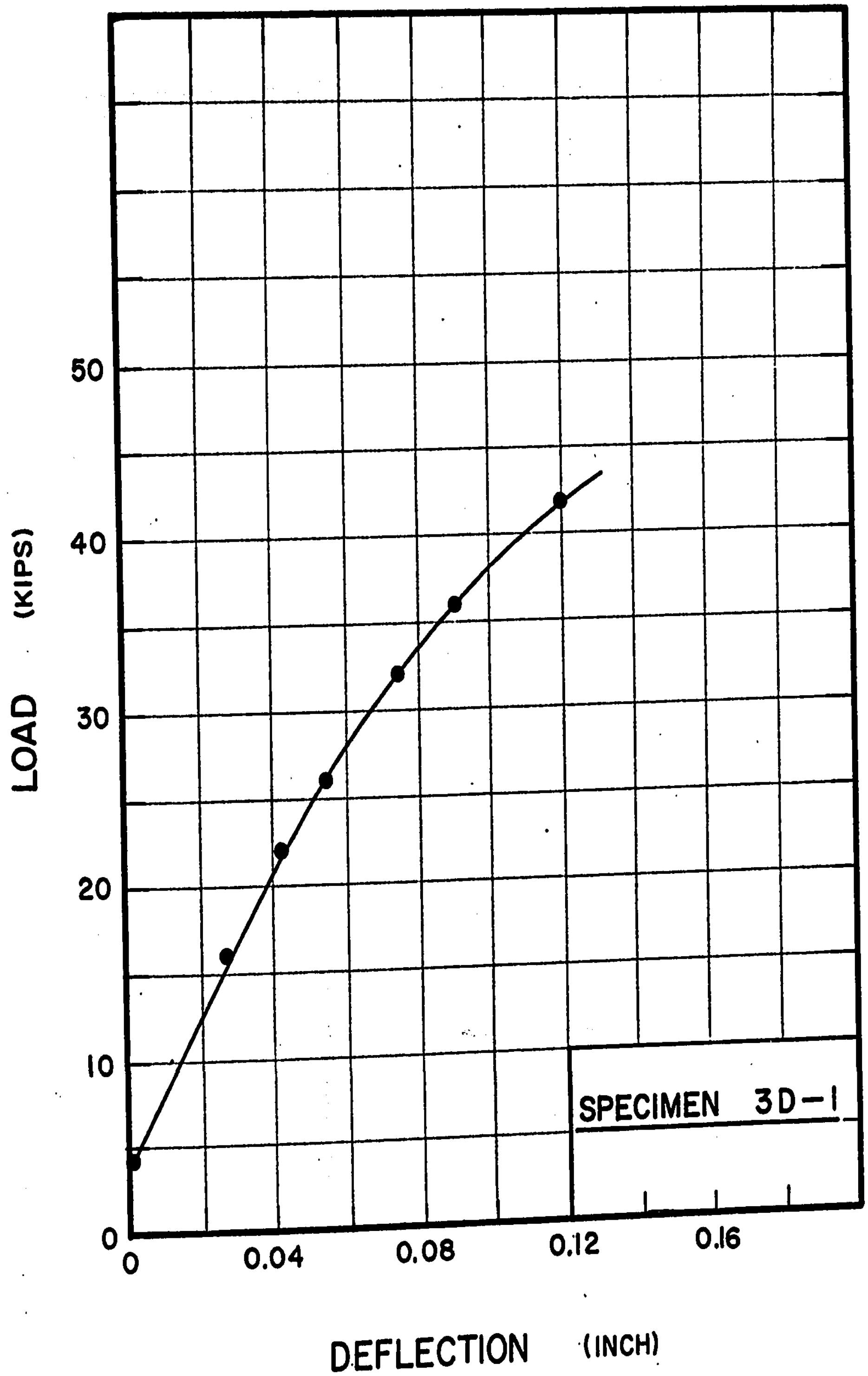


FIG.(B. 31) LOAD—DEFLECTION CURVE OF SPECIMEN 3D-1

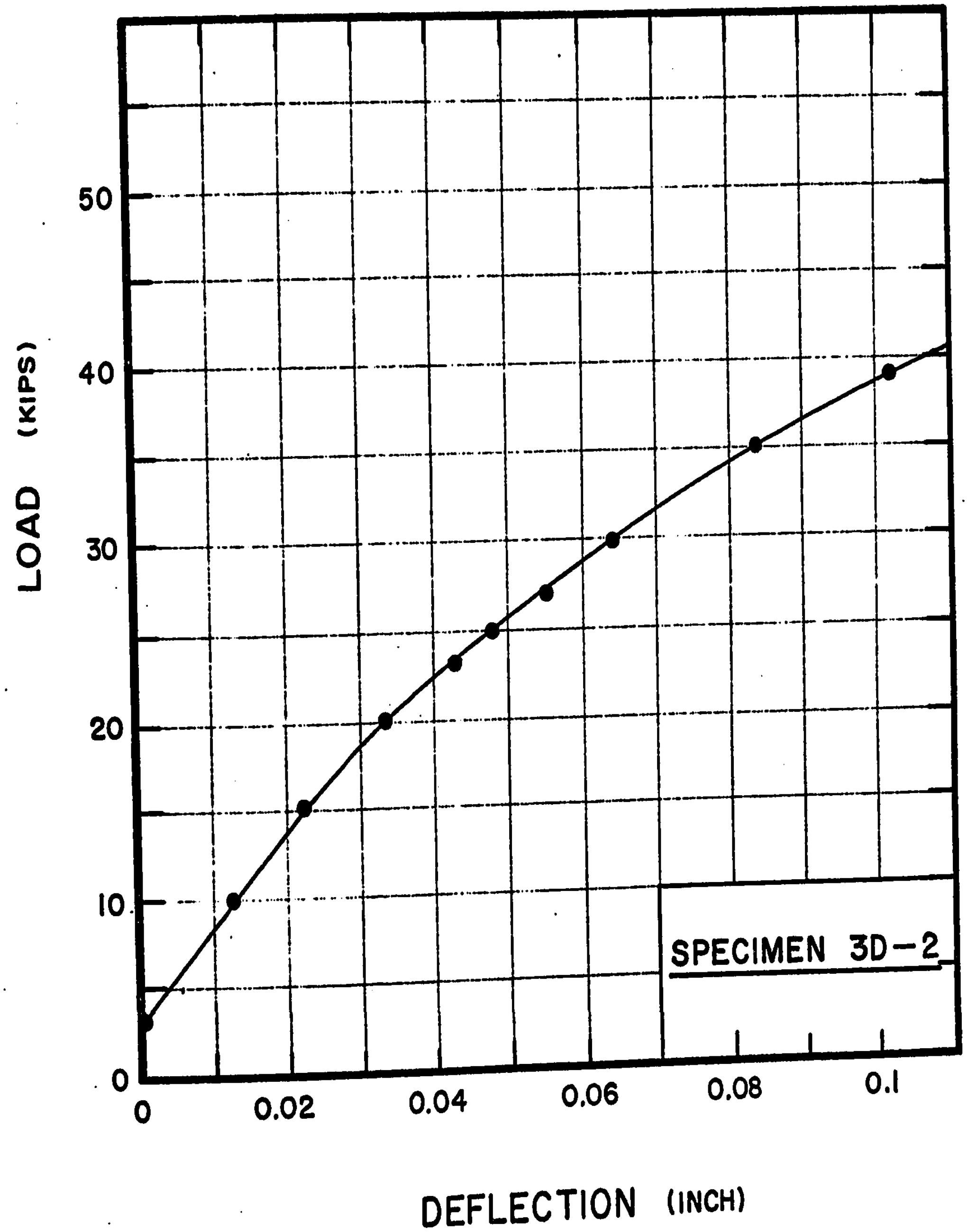


FIG. (B. 32) LOAD—DEFLECTION CURVE OF SPECIMEN 3D-2

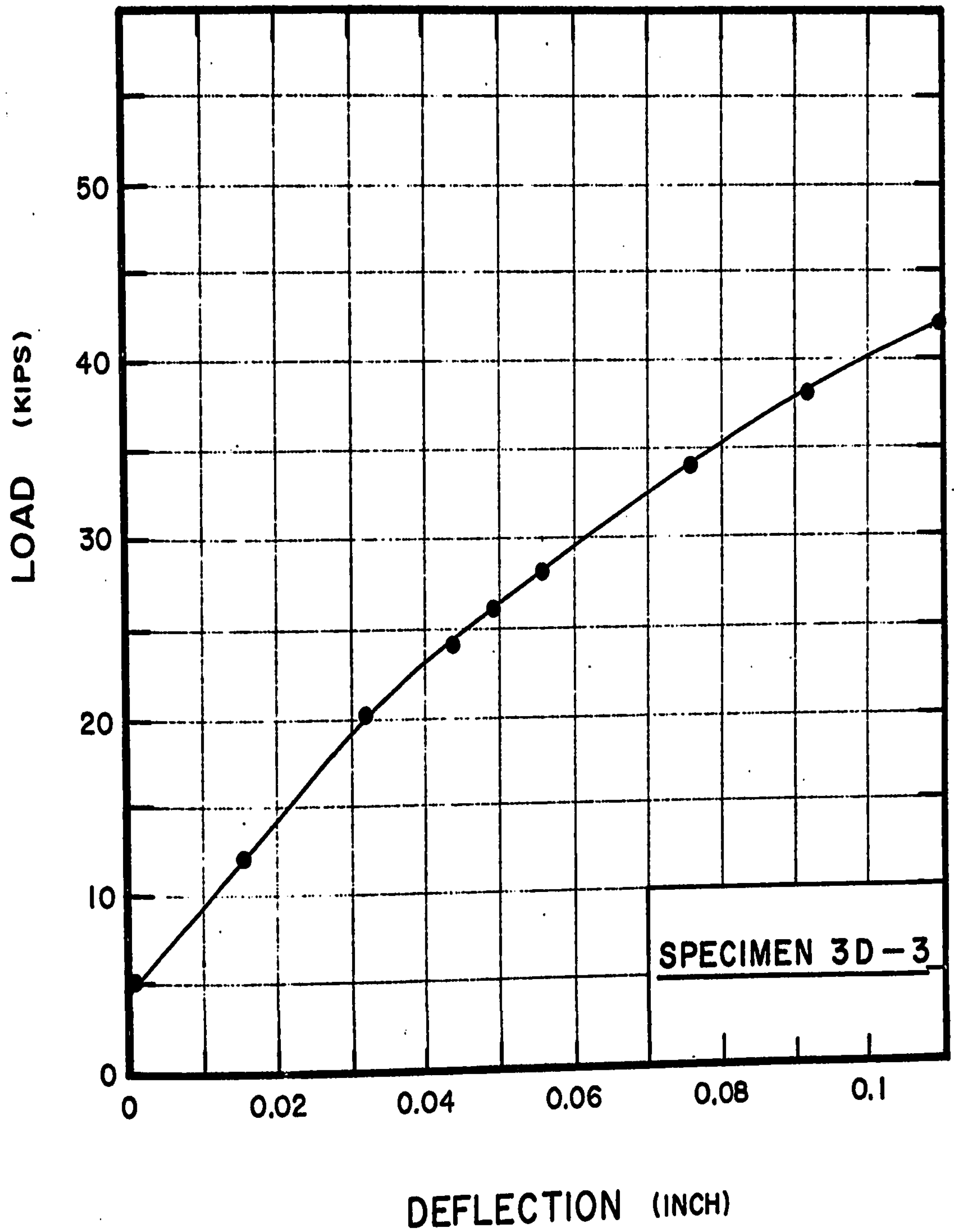


FIG. (B. 33) LOAD—DEFLECTION CURVE OF SPECIMEN 3D-3

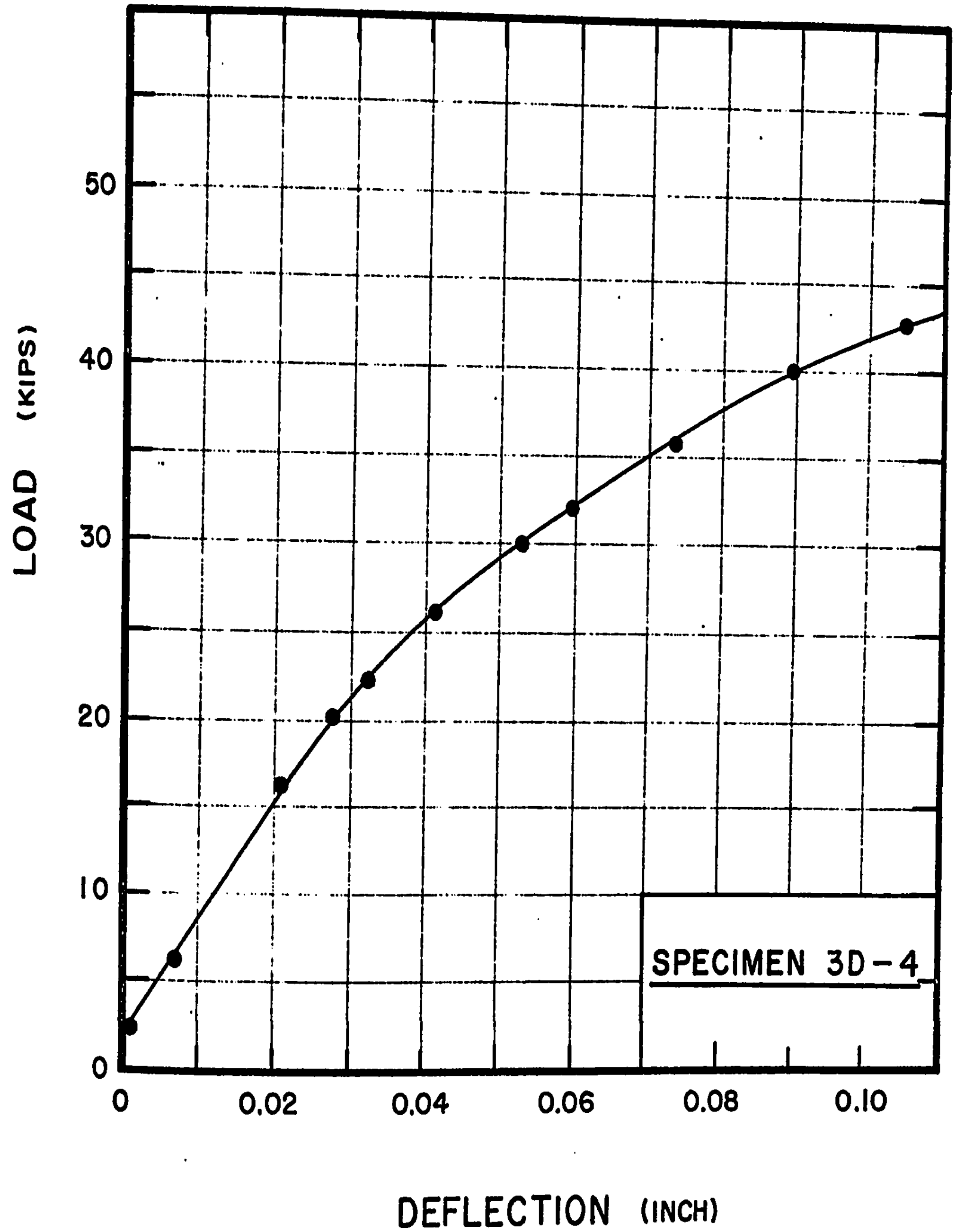


FIG. (B. 34) LOAD—DEFLECTION CURVE OF SPECIMEN 3D-4

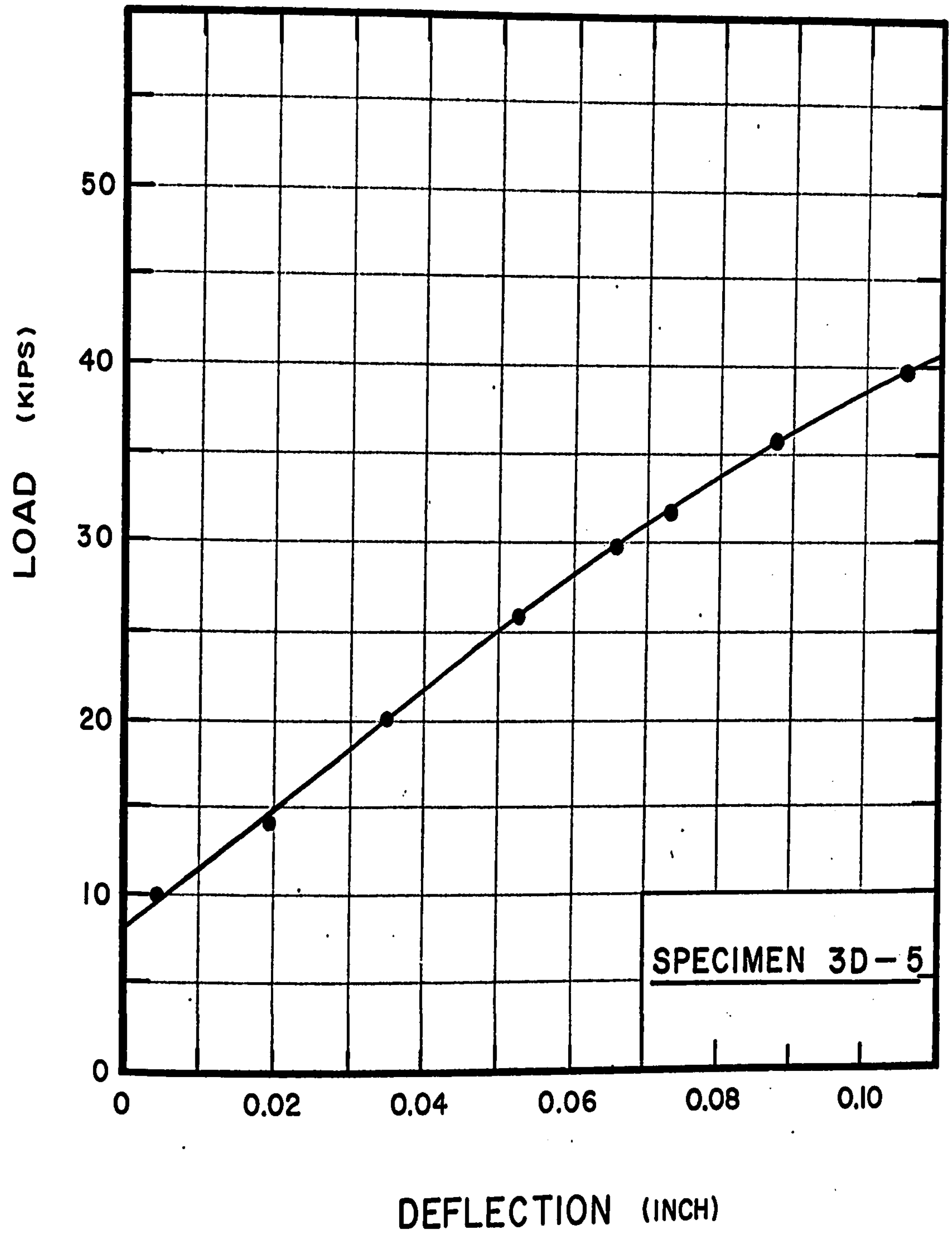


FIG. (B. 35) LOAD—DEFLECTION CURVE OF SPECIMEN 3D-5

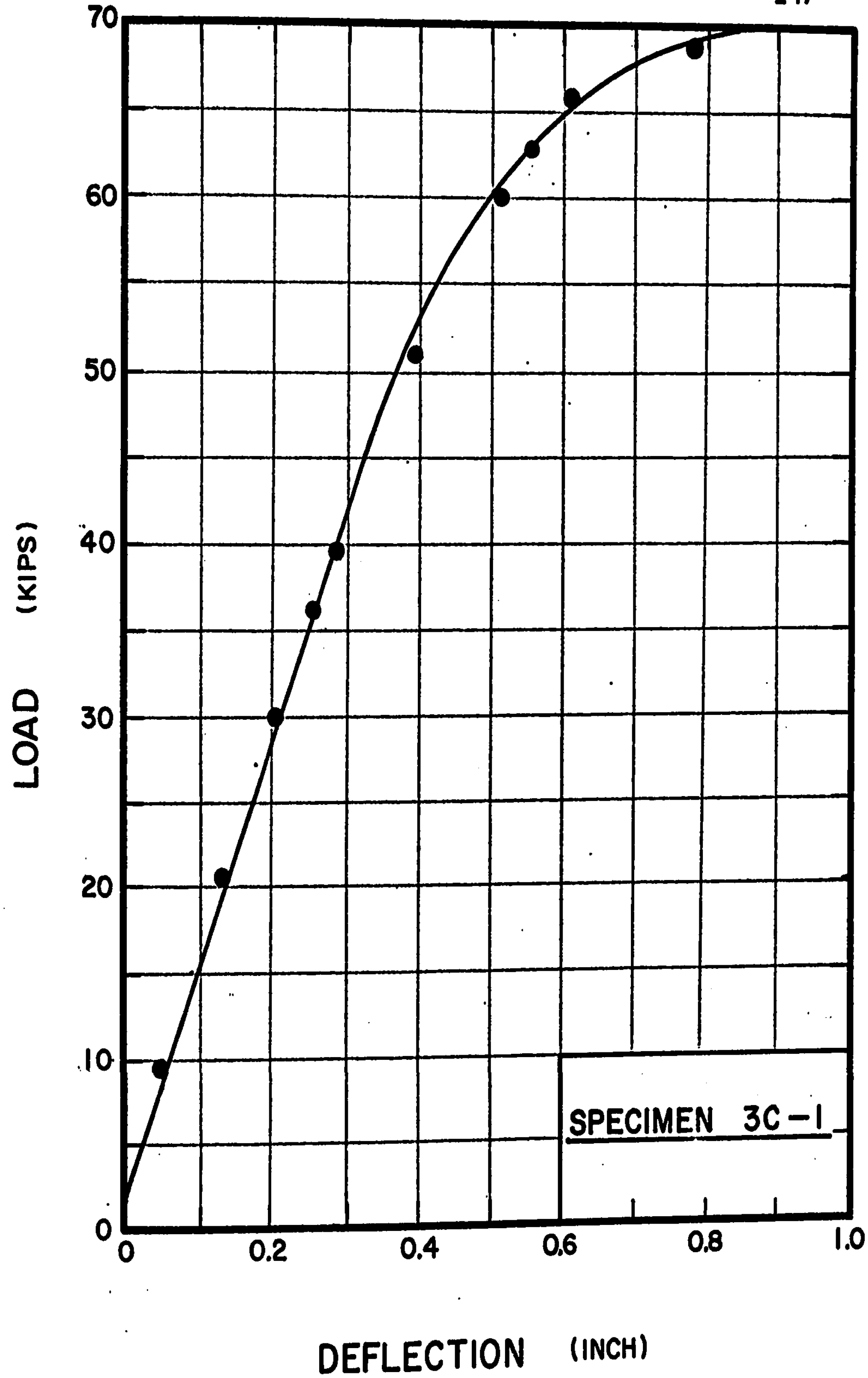


FIG. (B.36) LOAD-DEFLECTION CURVE OF SPECIMEN 3C-1

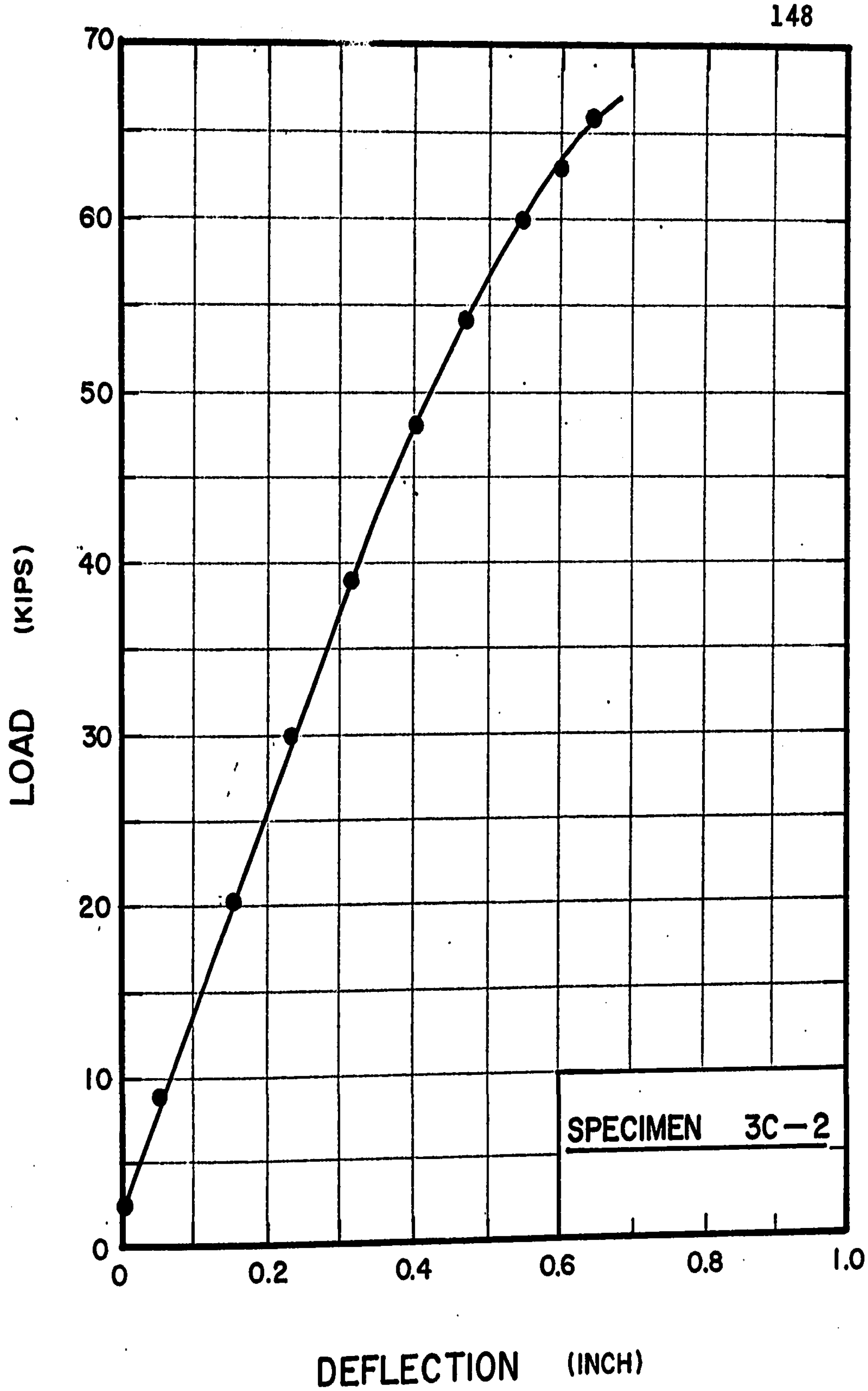


FIG. (B.37) LOAD—DEFLECTION CURVE OF SPECIMEN 3C-2

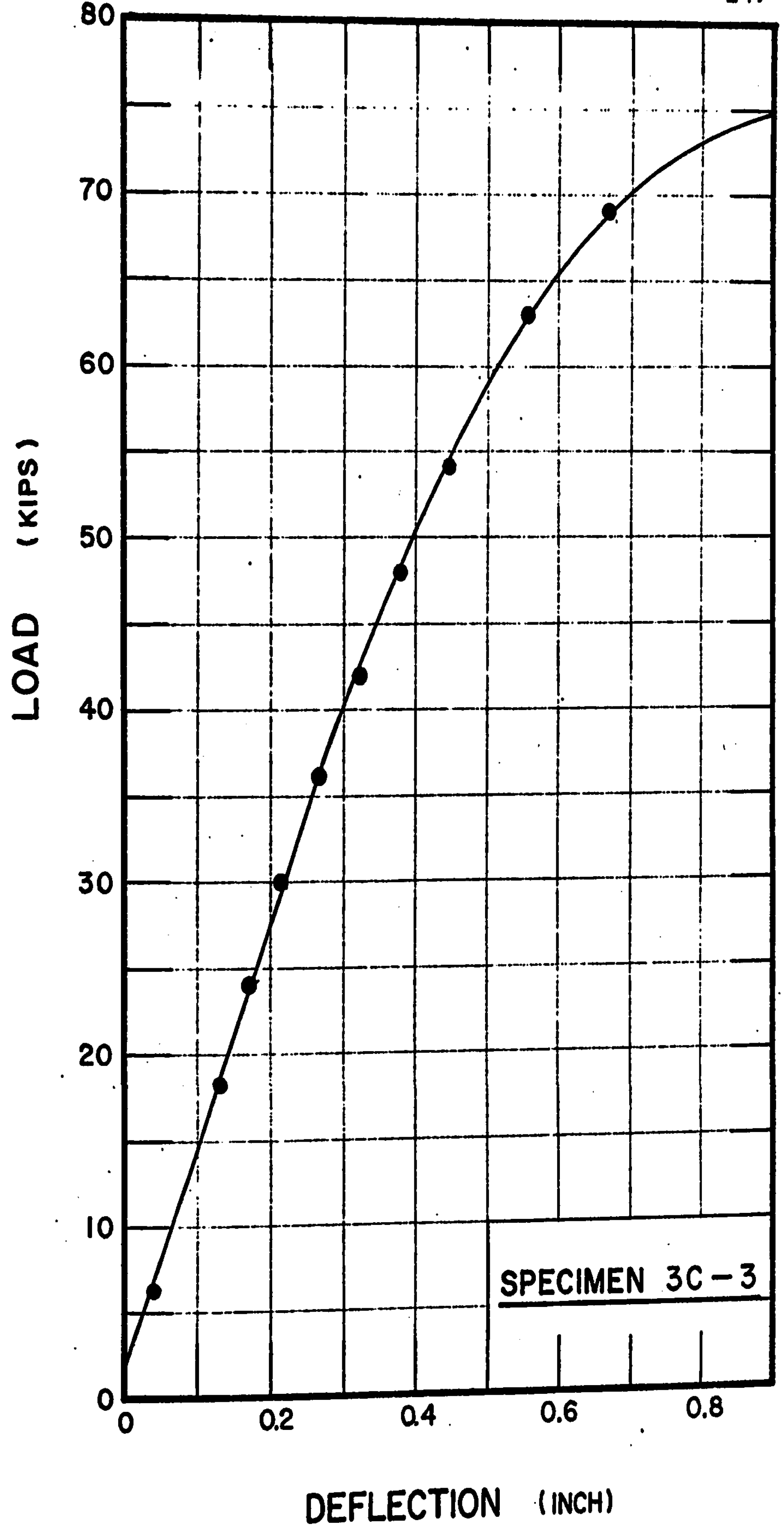


FIG.(B.38) LOAD-DEFLECTION CURVE OF SPECIMEN 3C-3

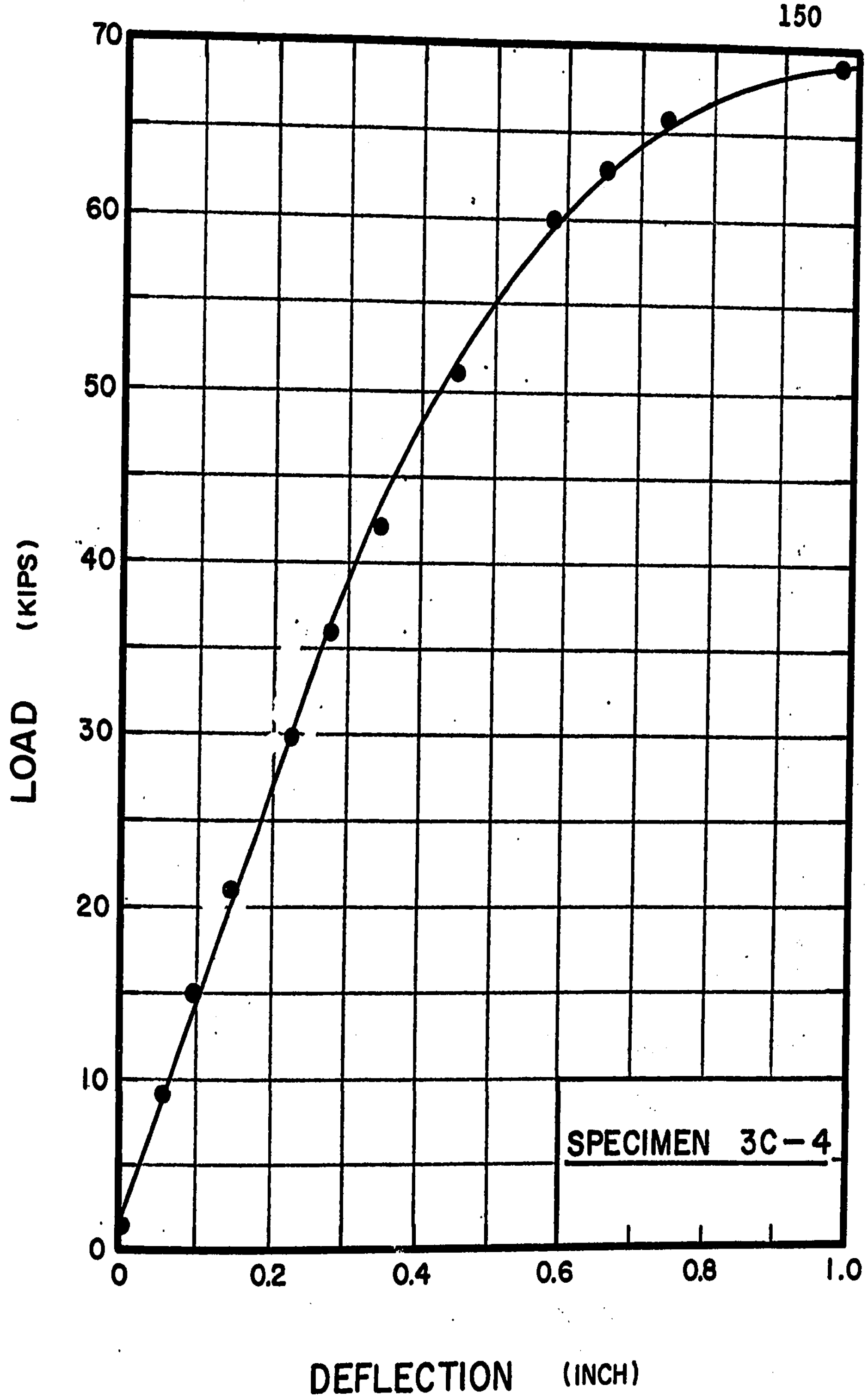


FIG. (B.39) LOAD-DEFLECTION CURVE OF SPECIMEN 3C-4

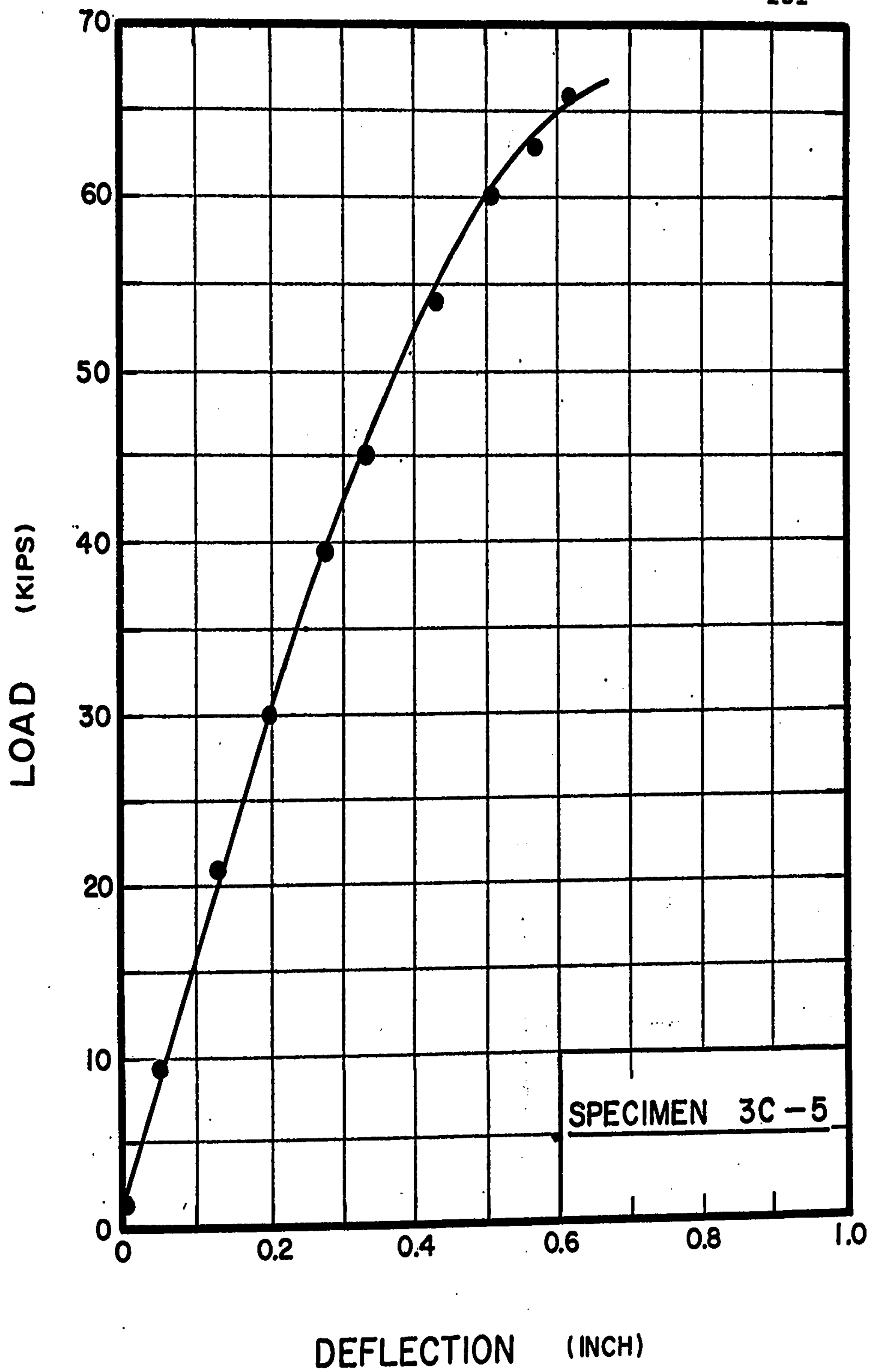


FIG. (B.40) LOAD-DEFLECTION CURVE OF SPECIMEN 3C-5

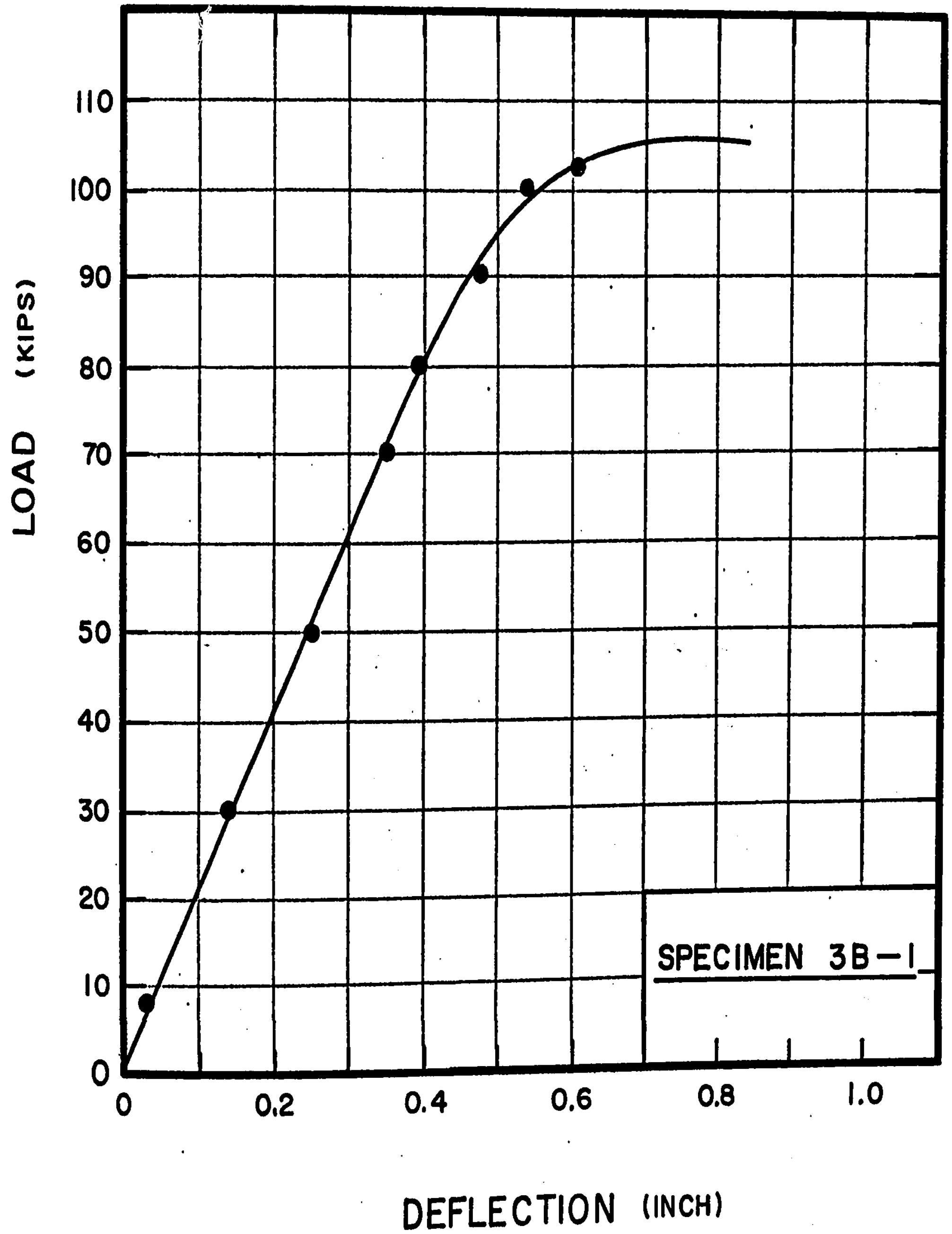


FIG.(B. 41) LOAD—DEFLECTION CURVE OF SPECIMEN 3B-I

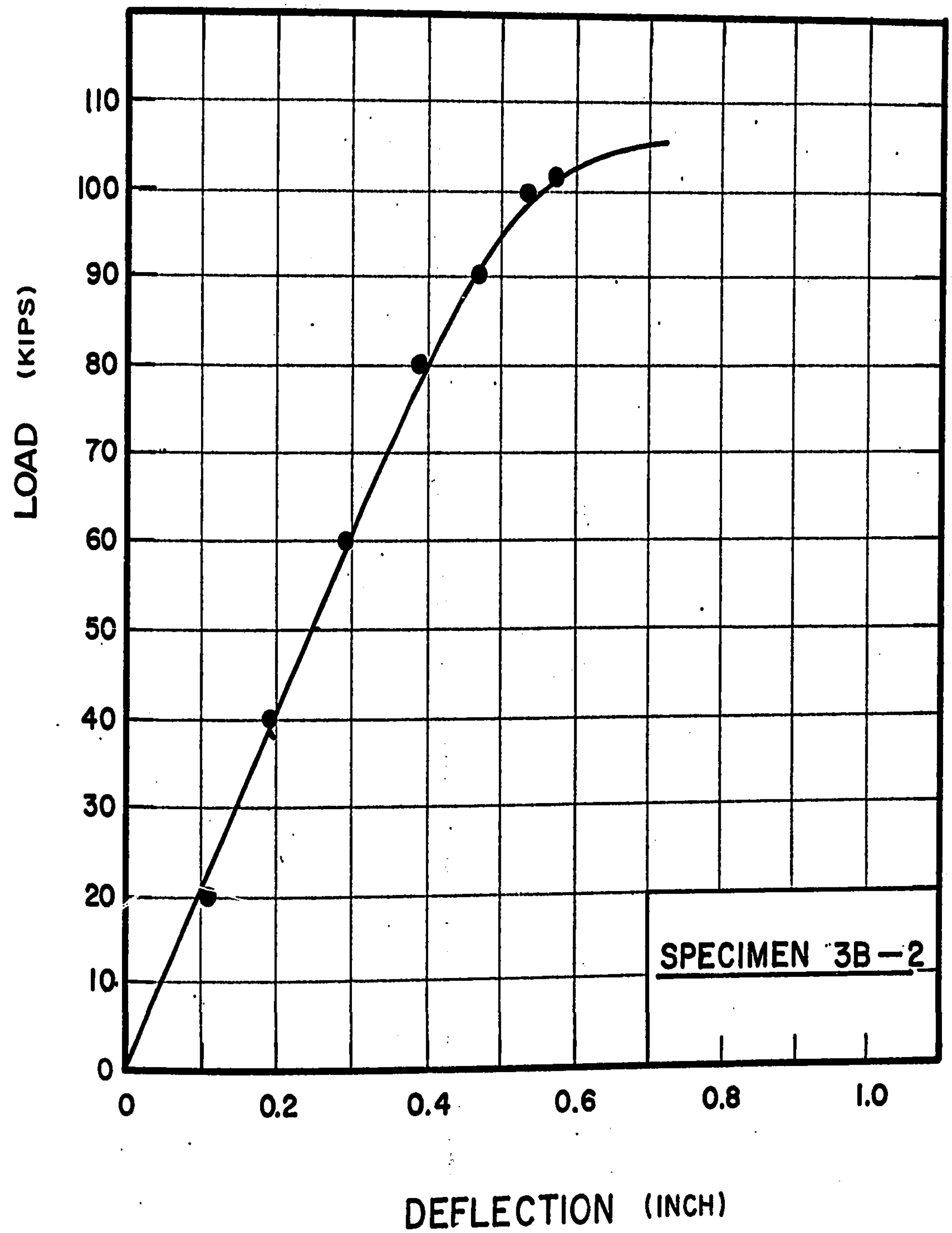


FIG. (B. 42) LOAD — DEFLECTION CURVE OF SPECIMEN 3B—2

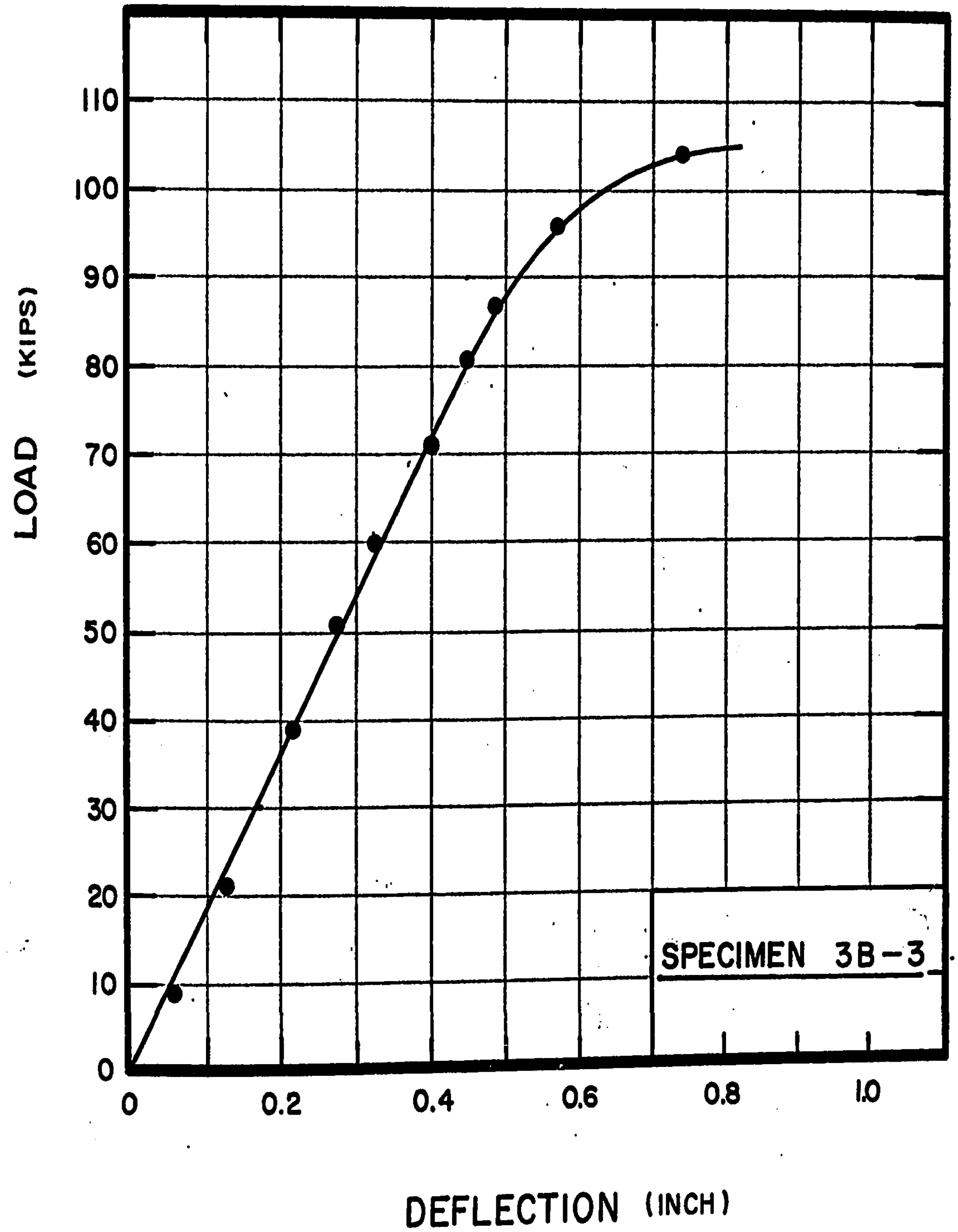


FIG. (B. 43) LOAD — DEFLECTION CURVE OF SPECIMEN 3B-3

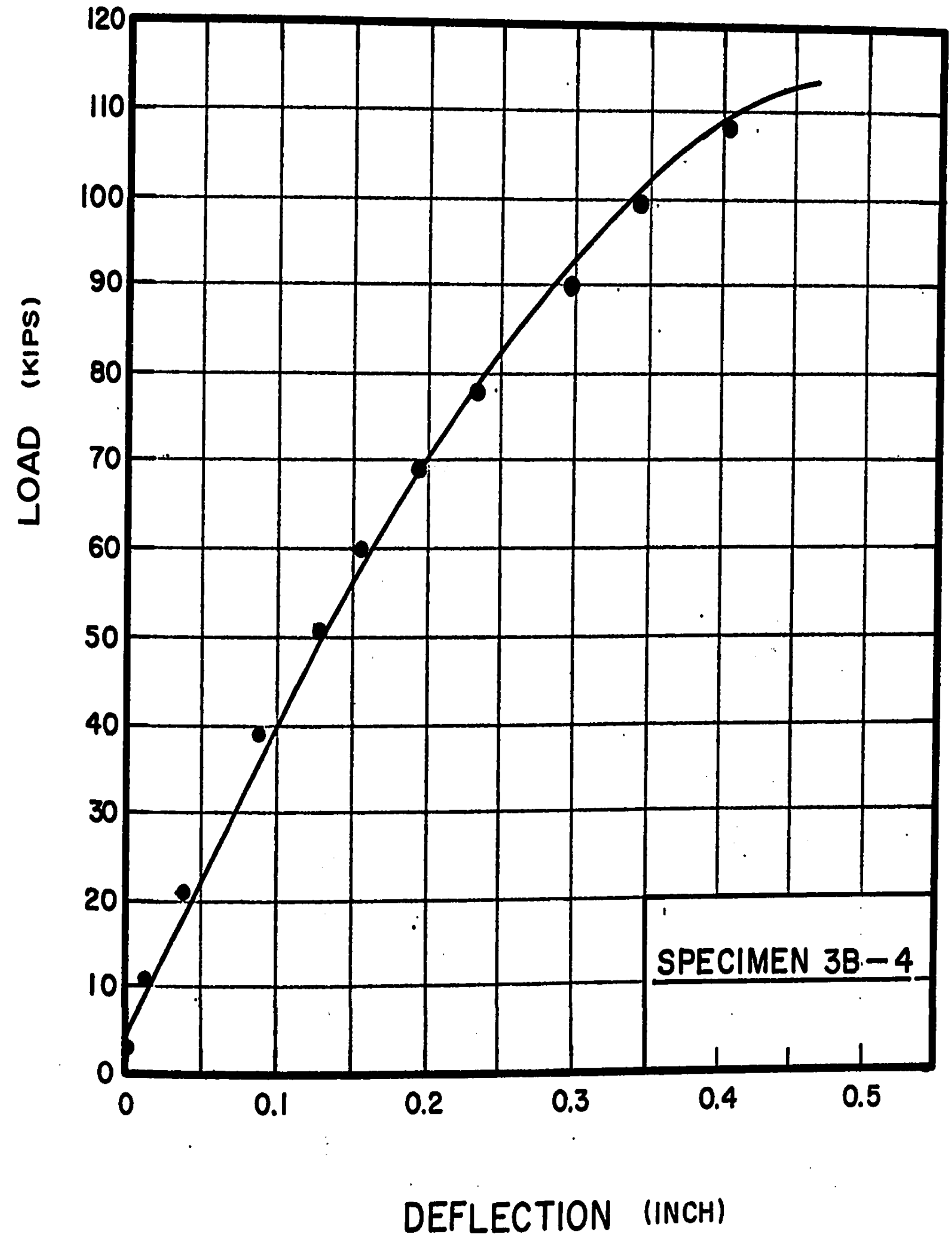


FIG. (B.44) LOAD — DEFLECTION CURVE OF SPECIMEN 3B-4

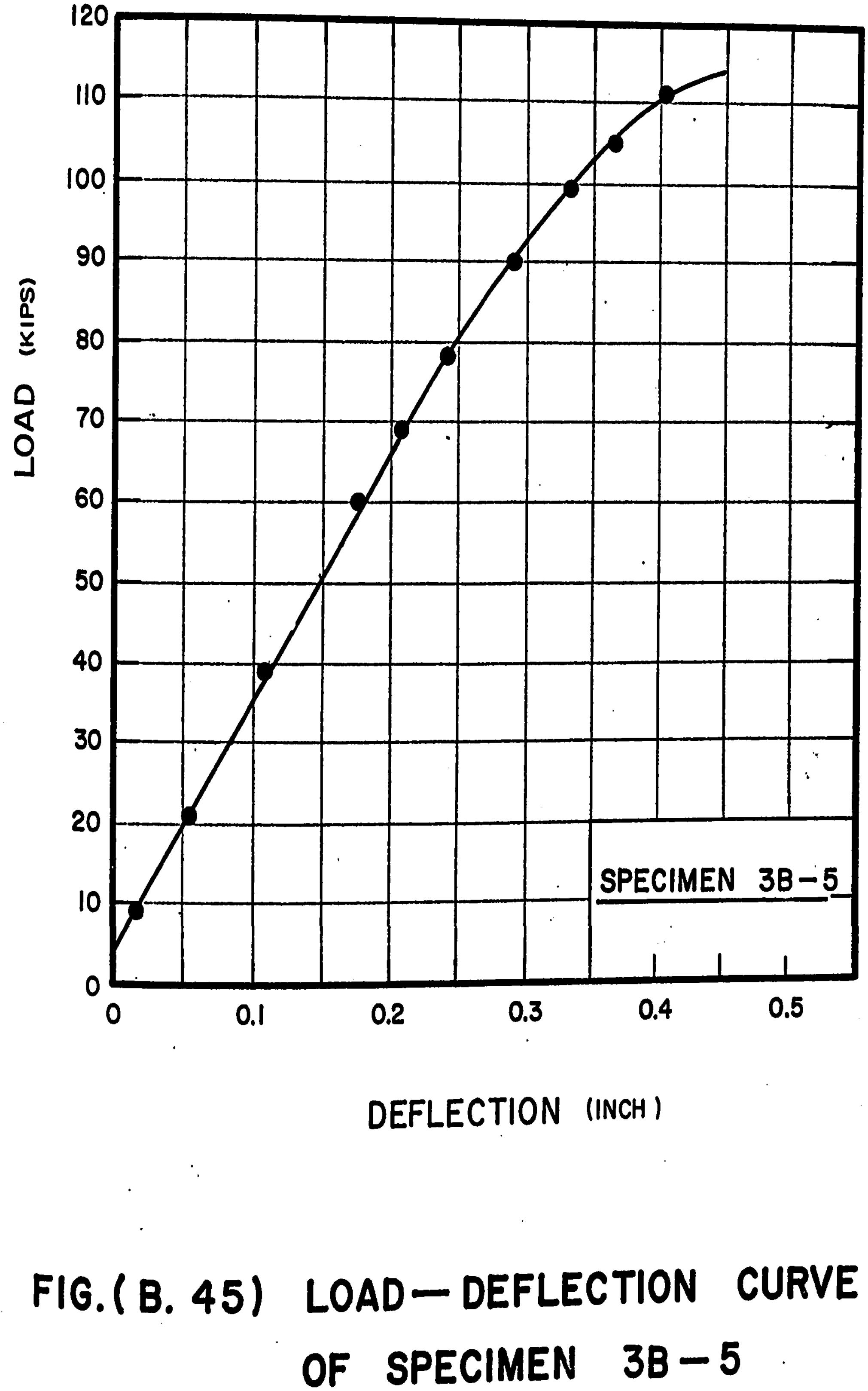


FIG. (B. 45) LOAD—DEFLECTION CURVE OF SPECIMEN 3B-5

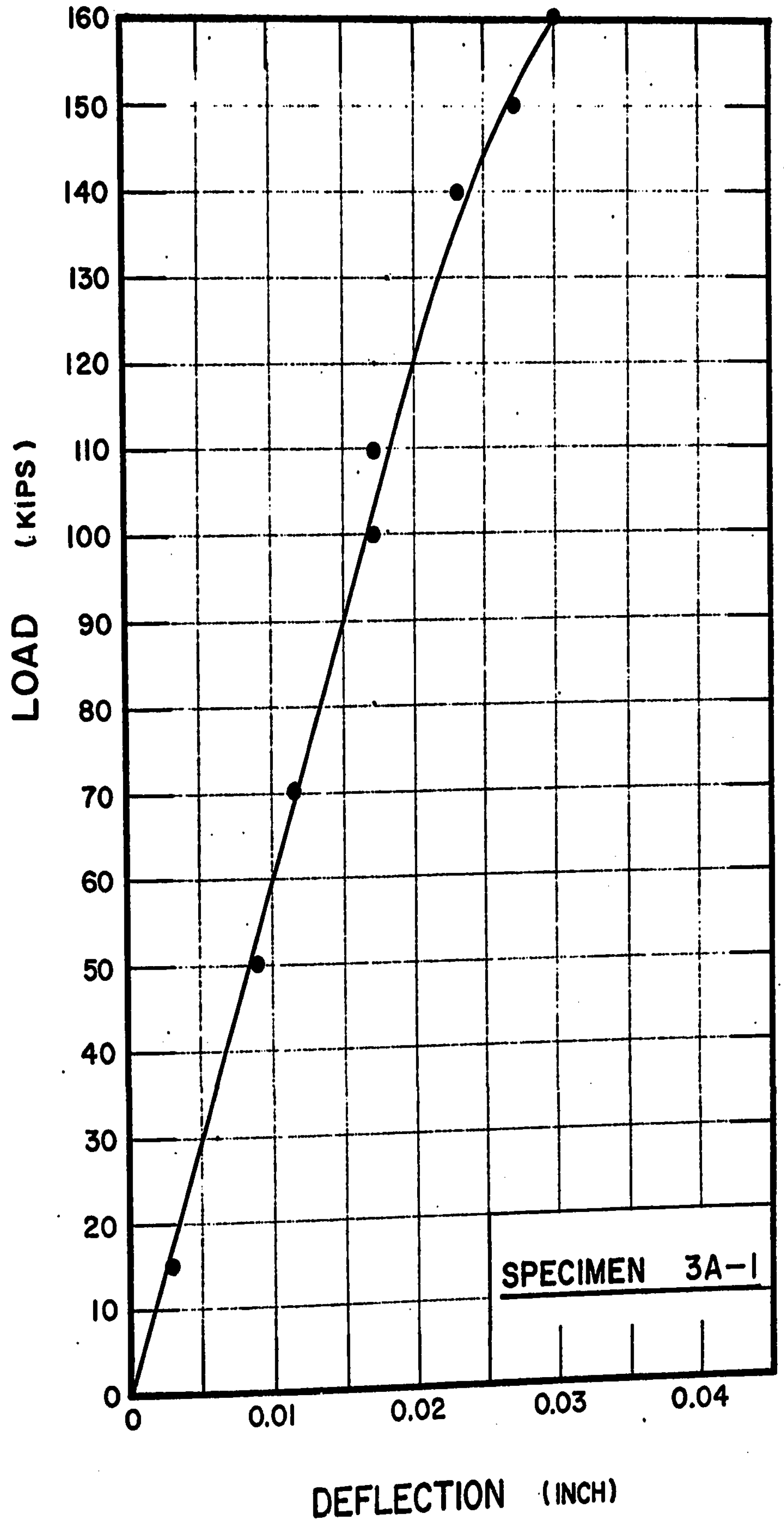


FIG.(B.46) LOAD-DEFLECTION CURVE OF SPECIMEN 3A-1

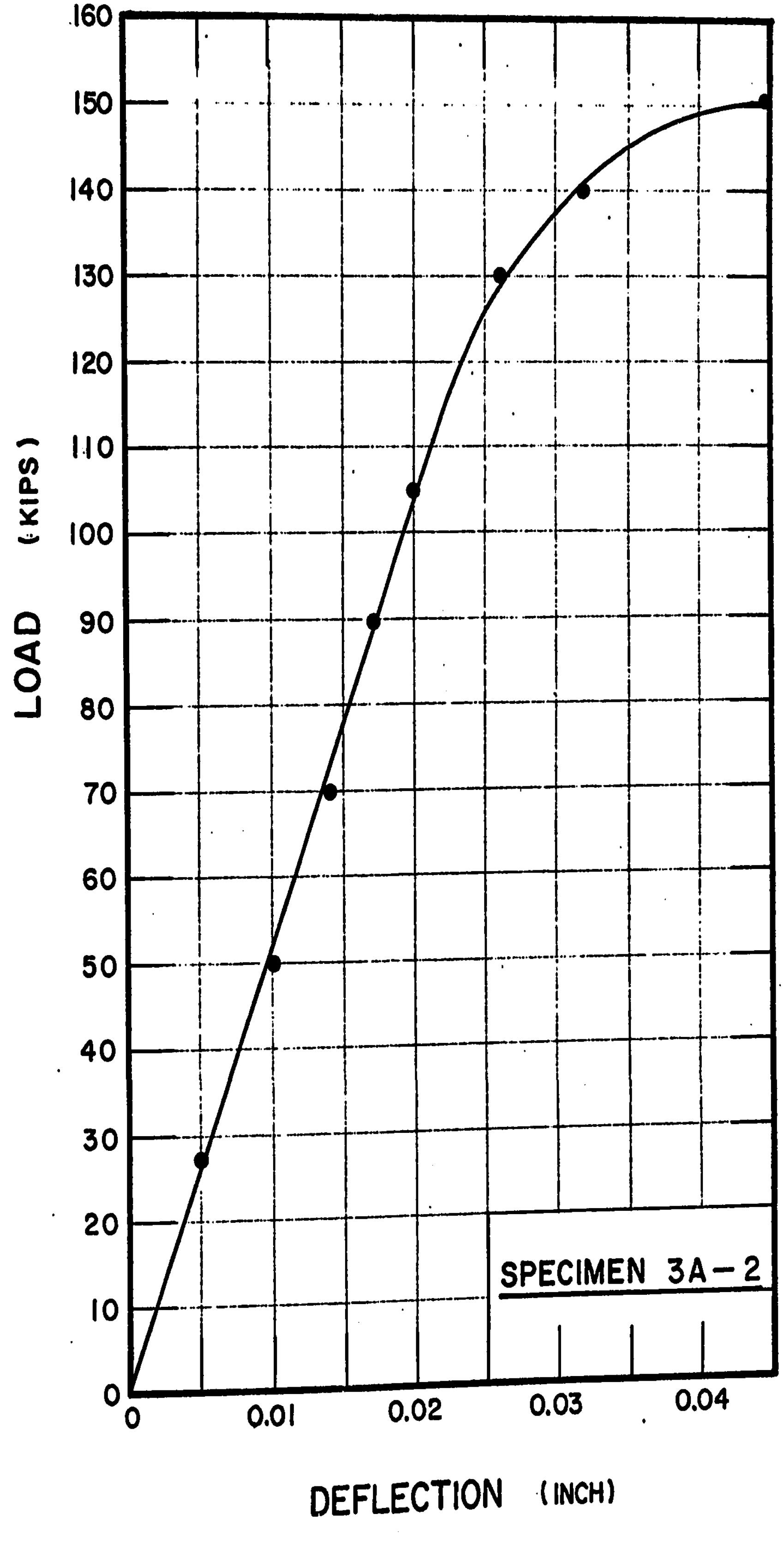


FIG.(B.47) LOAD-DEFLECTION CURVE OF SPECIMEN 3A-2

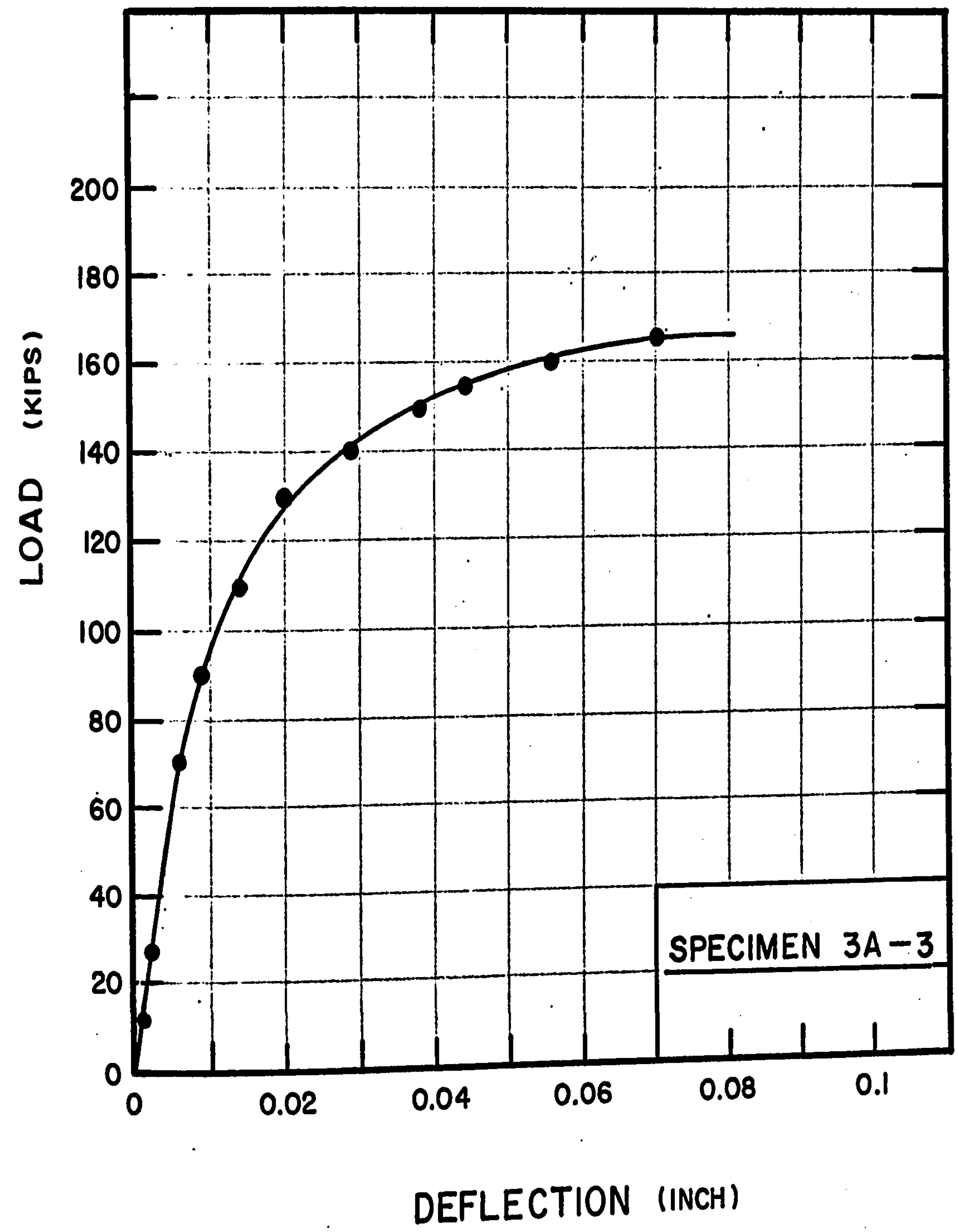


FIG. (B.48) LOAD—DEFLECTION CURVE OF SPECIMEN 3A—3

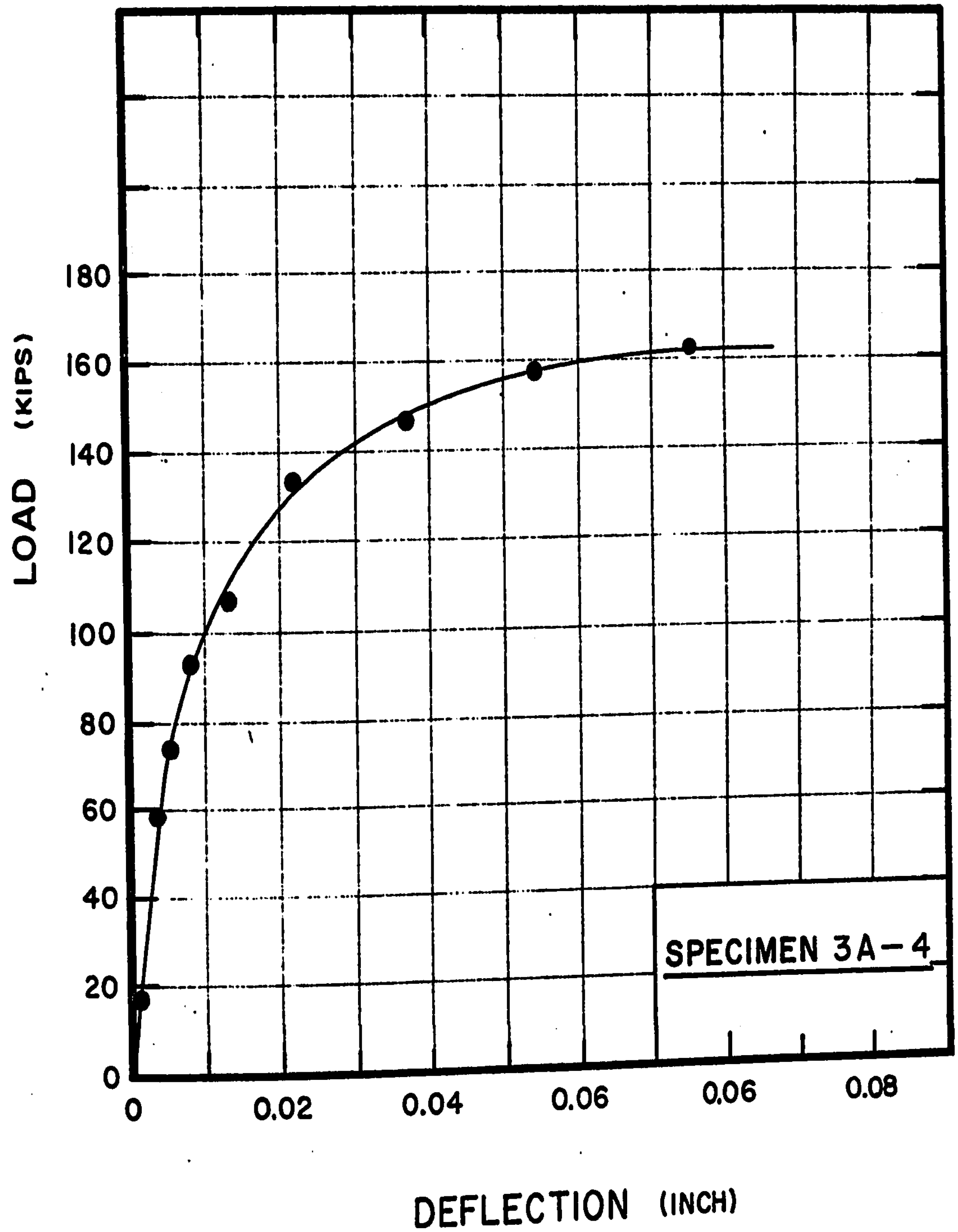


FIG. (B. 49) LOAD—DEFLECTION CURVE OF SPECIMEN 3A-4

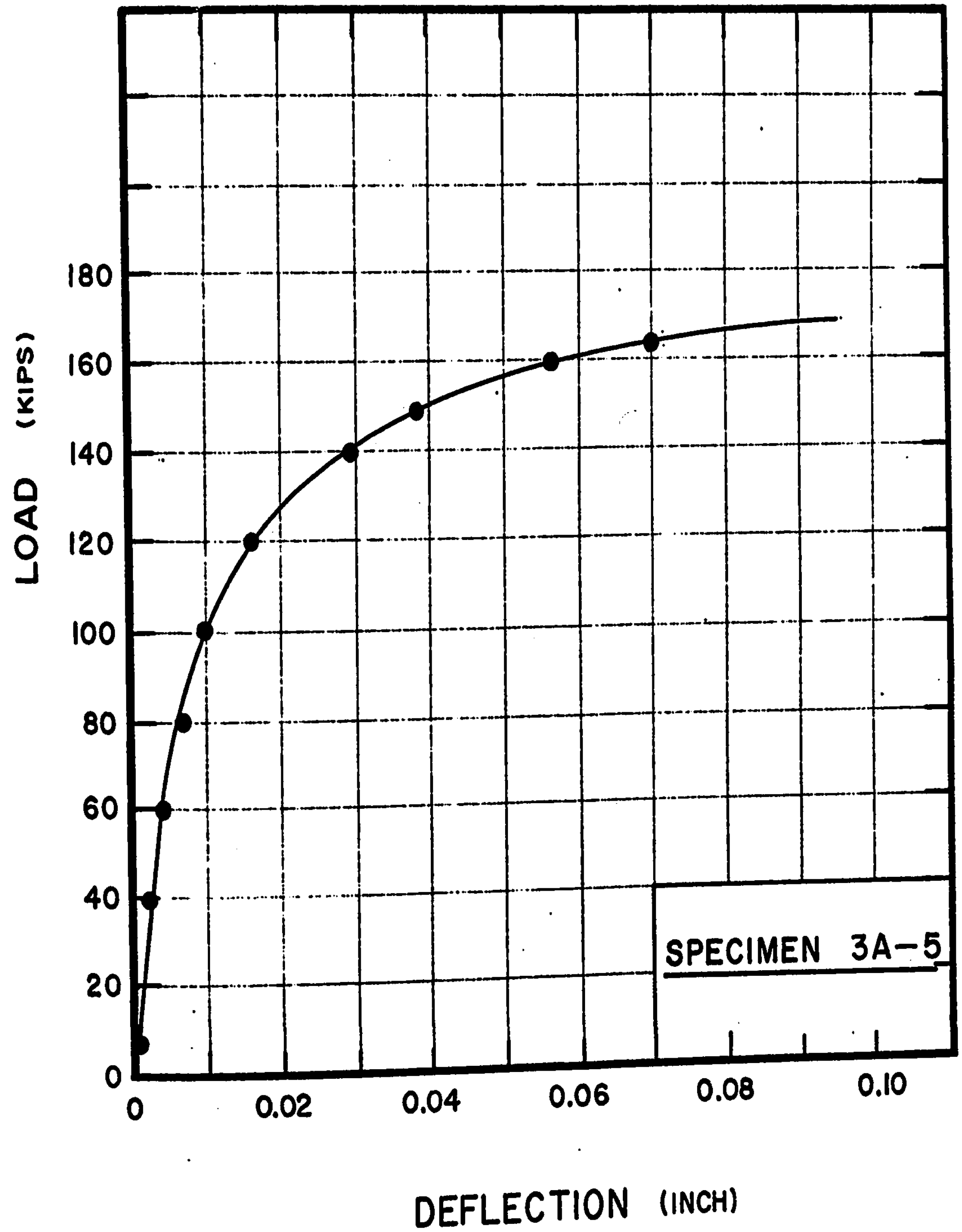


FIG. (B. 50) LOAD—DEFLECTION CURVE OF SPECIMEN 3A-5

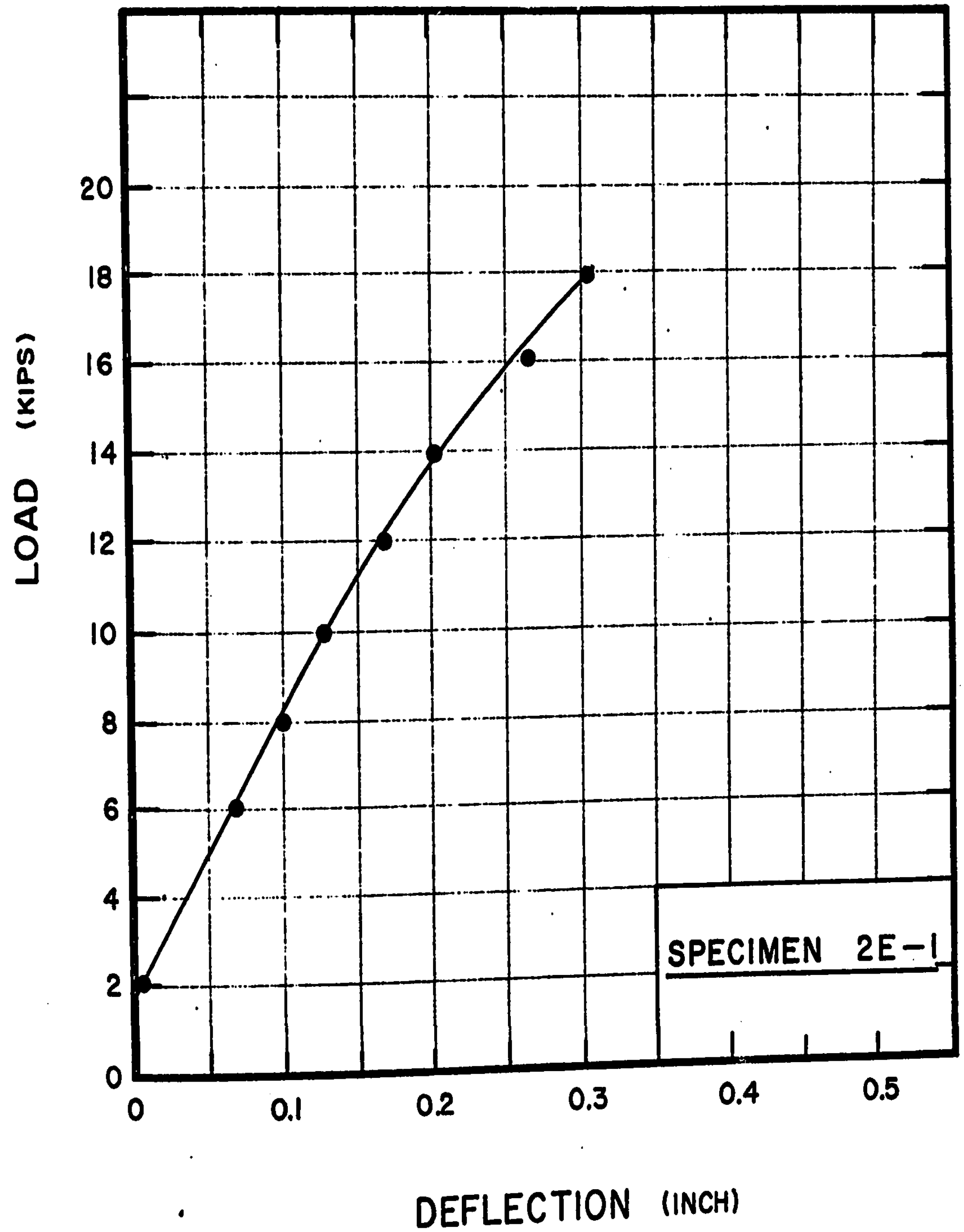


FIG. (B. 51) LOAD—DEFLECTION CURVE OF SPECIMEN 2E-1

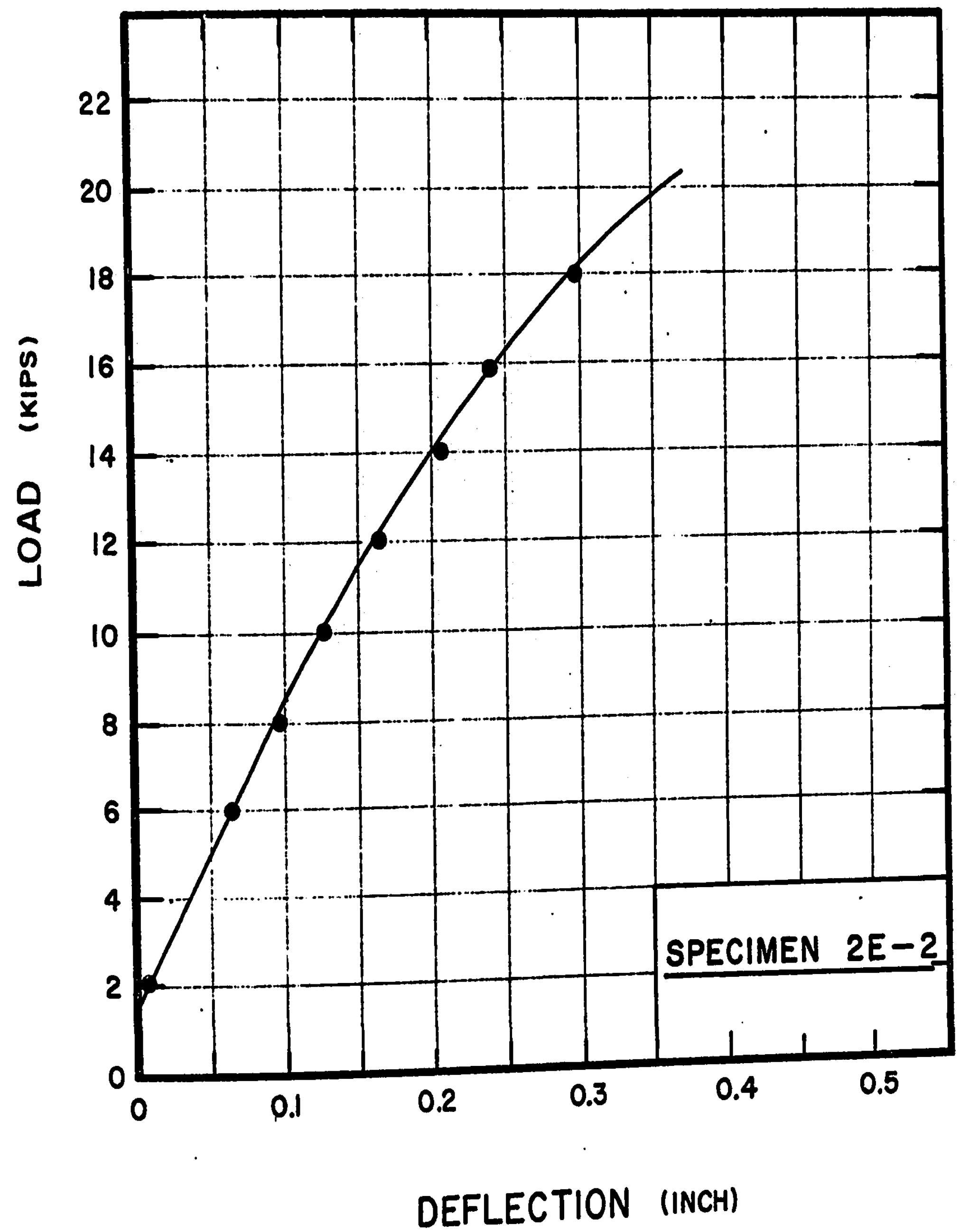


FIG. (B. 52) LOAD—DEFLECTION CURVE OF SPECIMEN 2E-2

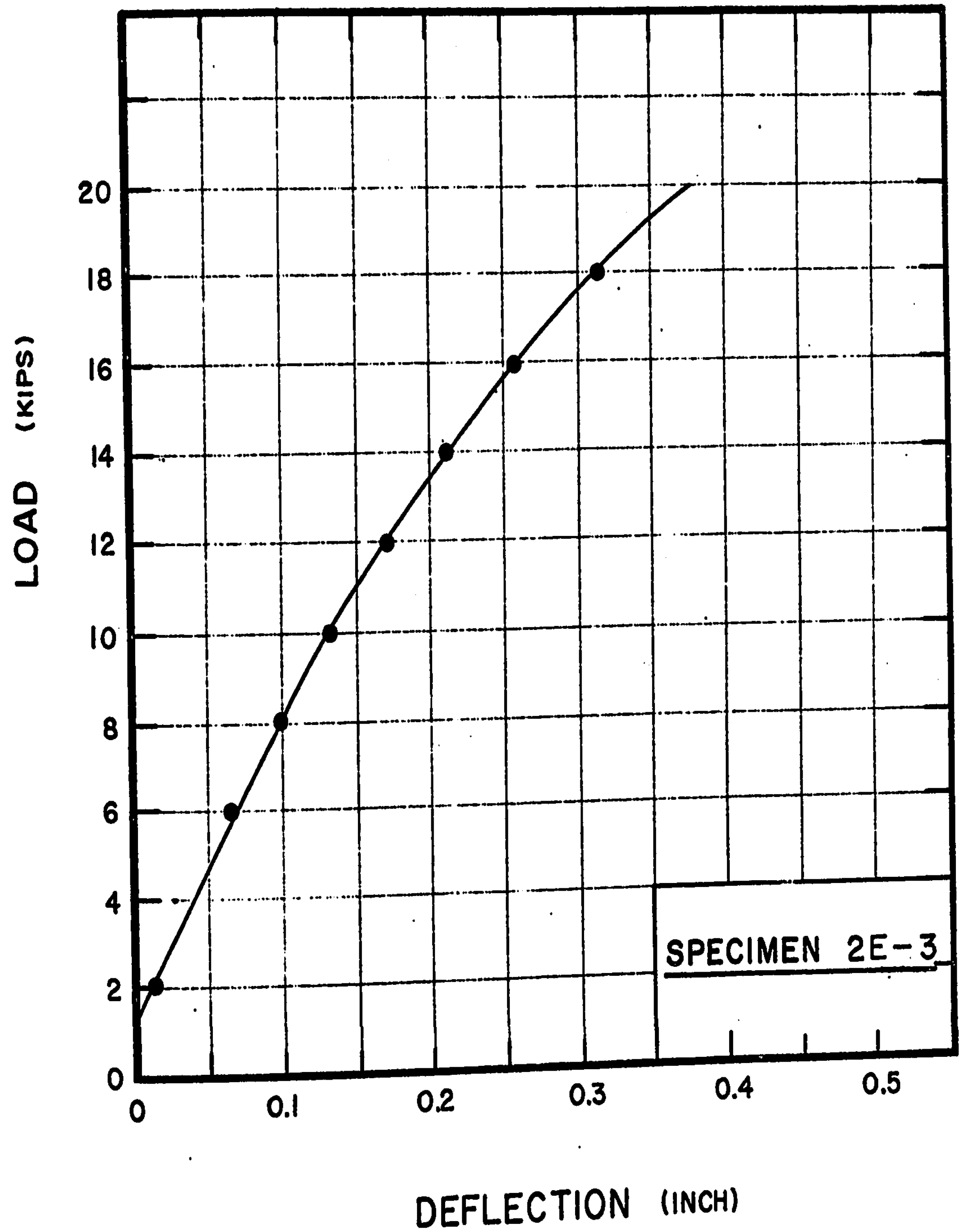


FIG. (B. 53) LOAD—DEFLECTION CURVE OF SPECIMEN 2E-3

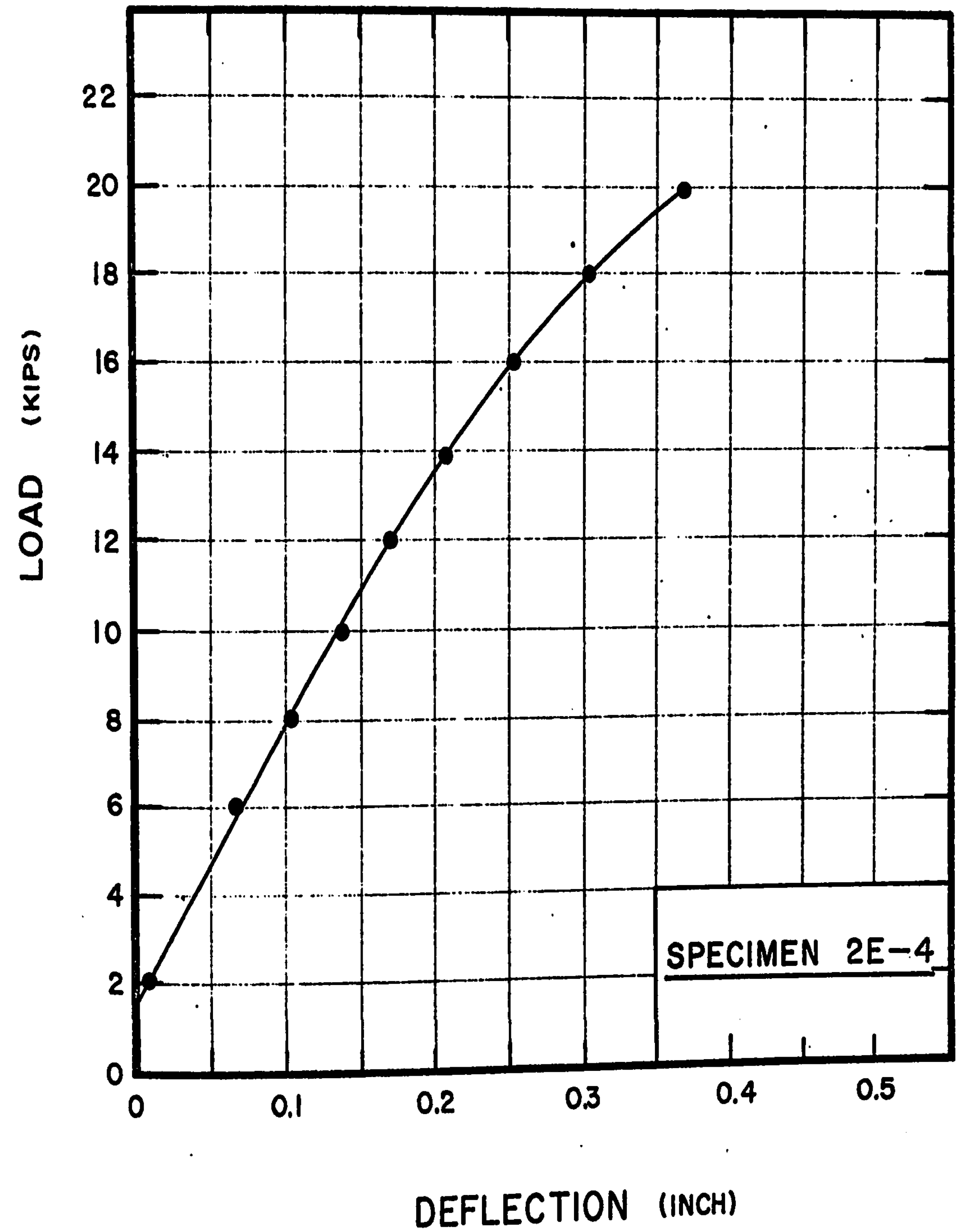


FIG. (B. 5 4) LOAD—DEFLECTION CURVE OF SPECIMEN 2E-4

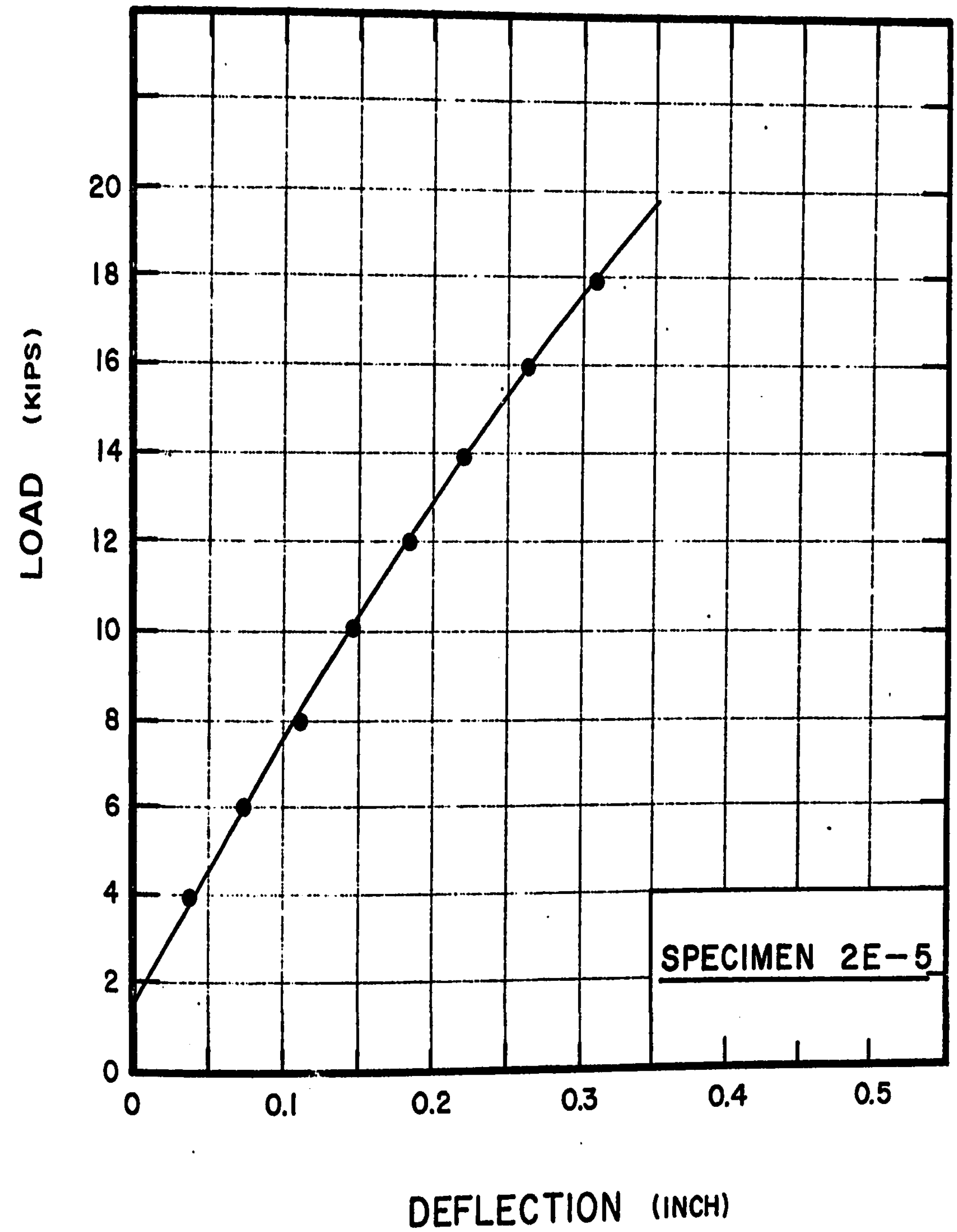


FIG. (B. 55) LOAD—DEFLECTION CURVE OF SPECIMEN 2E-5

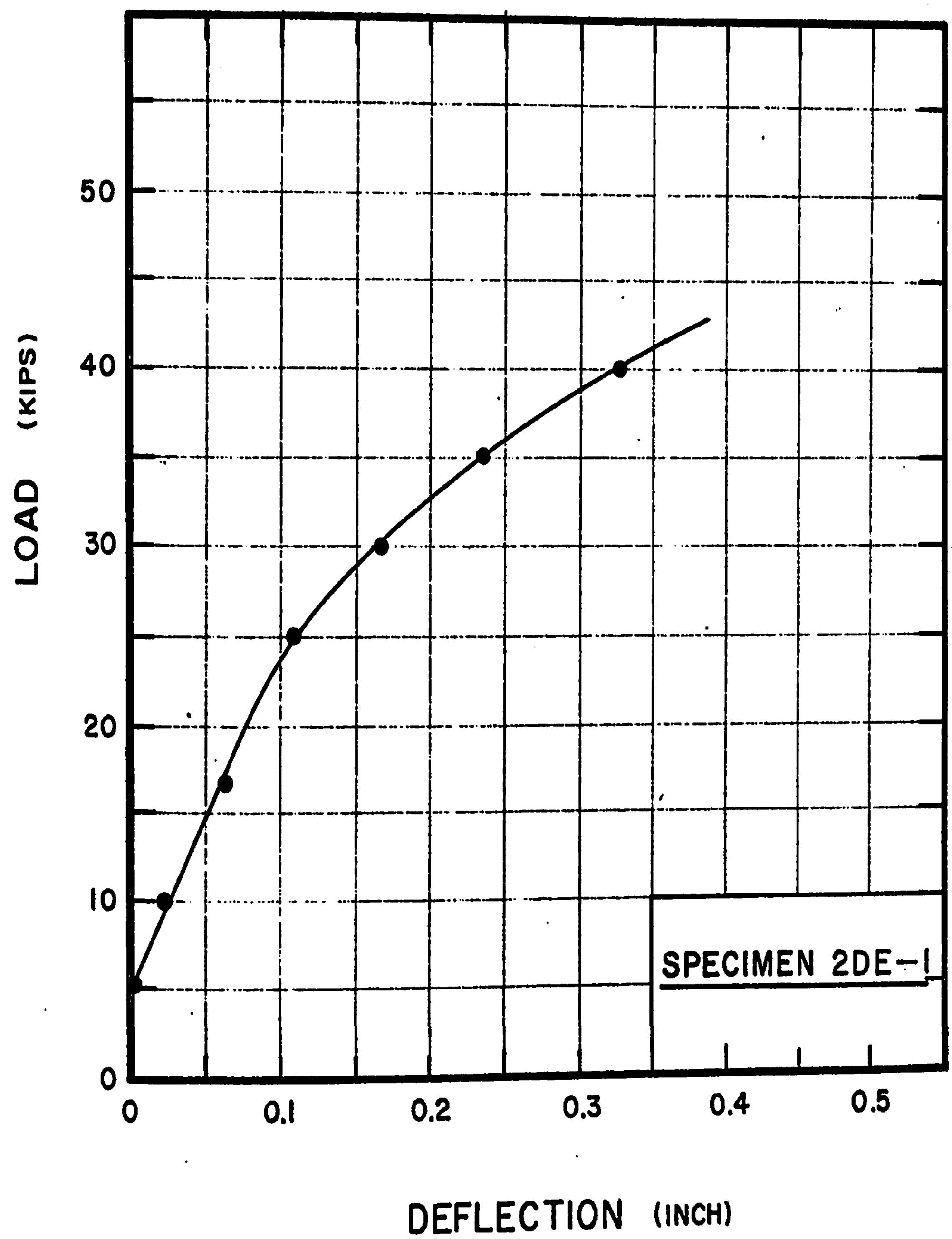


FIG. (B. 56) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE-1

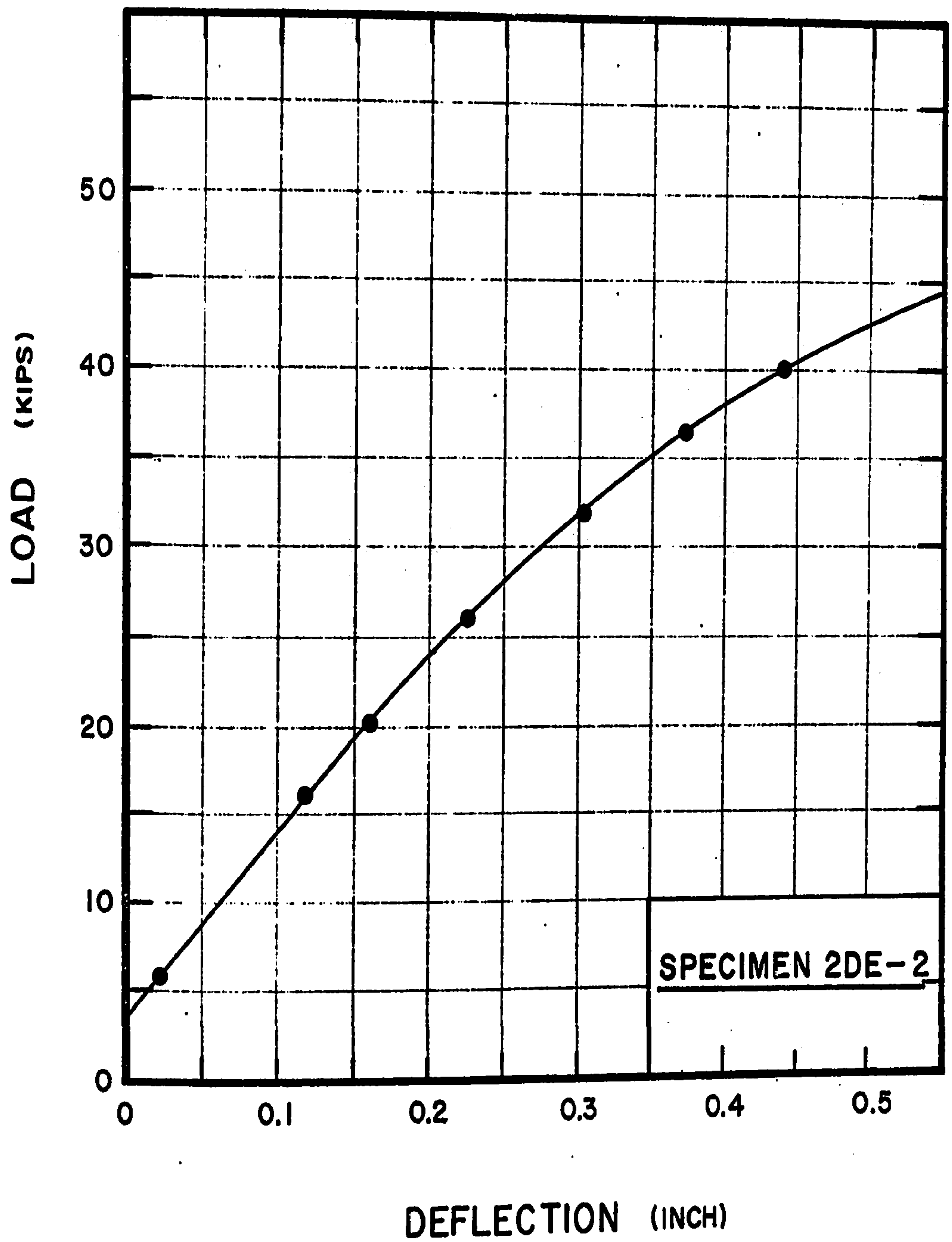


FIG. (B. 57) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE-2

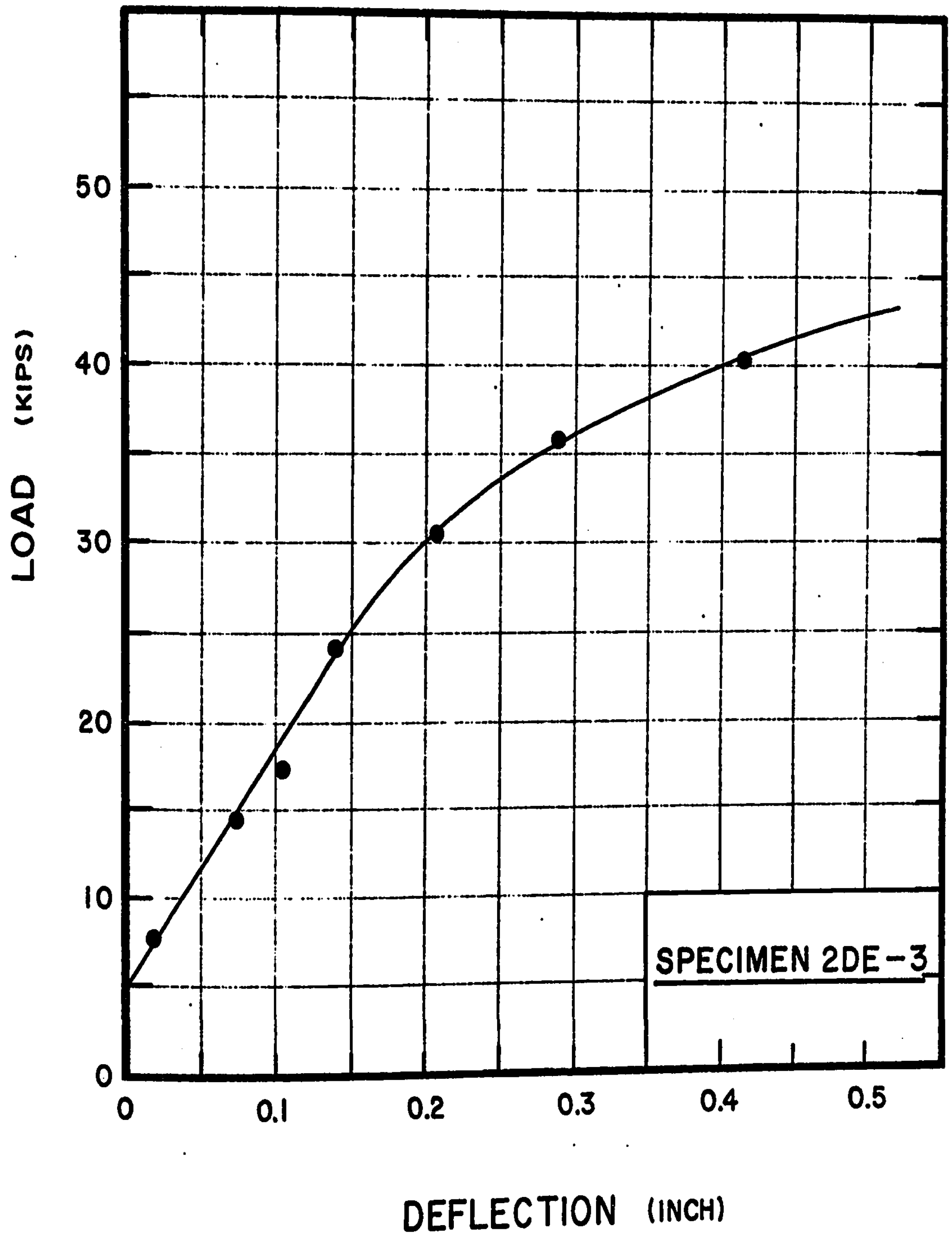


FIG. (B. 58) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE-3

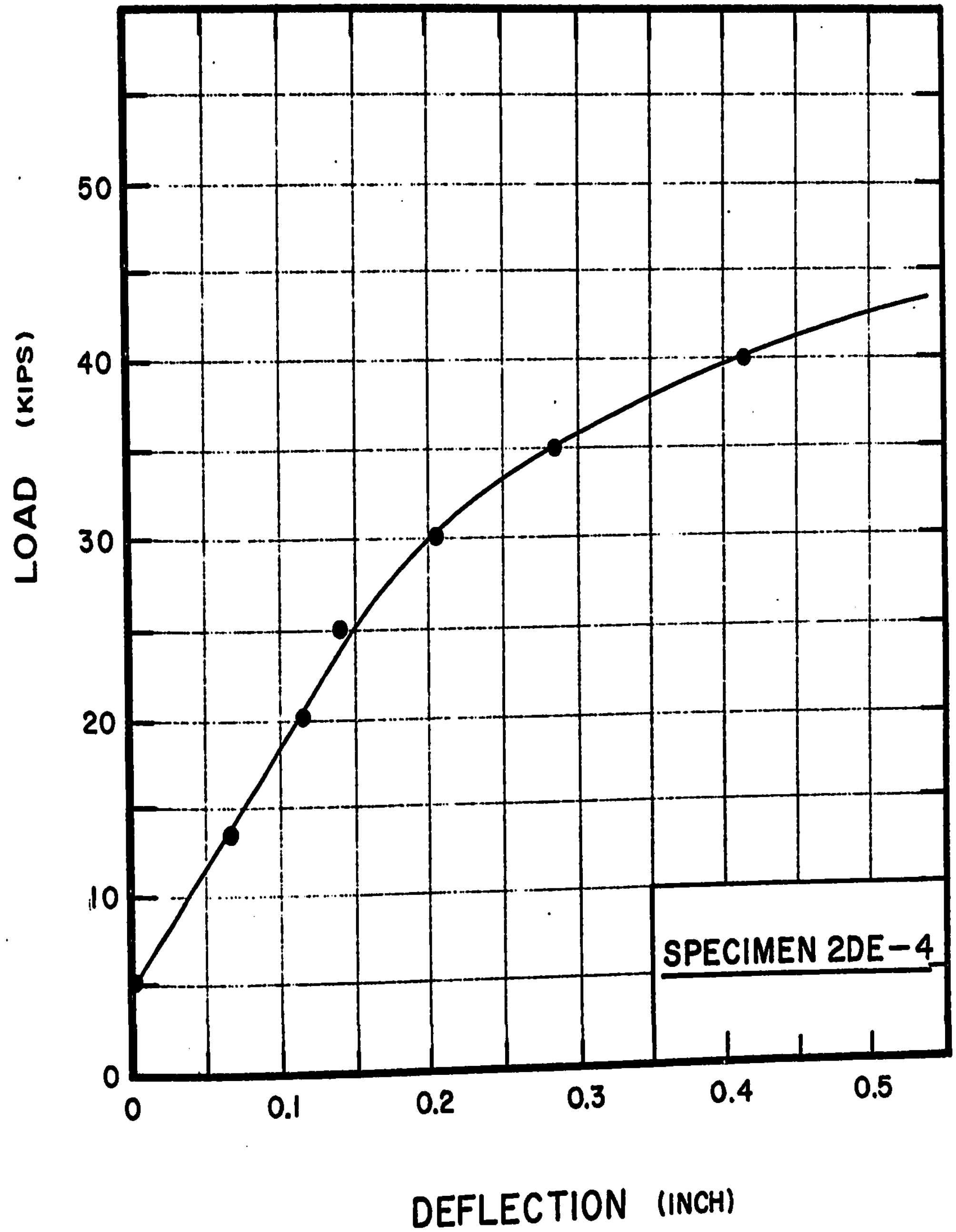


FIG. (B. 59) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE-4

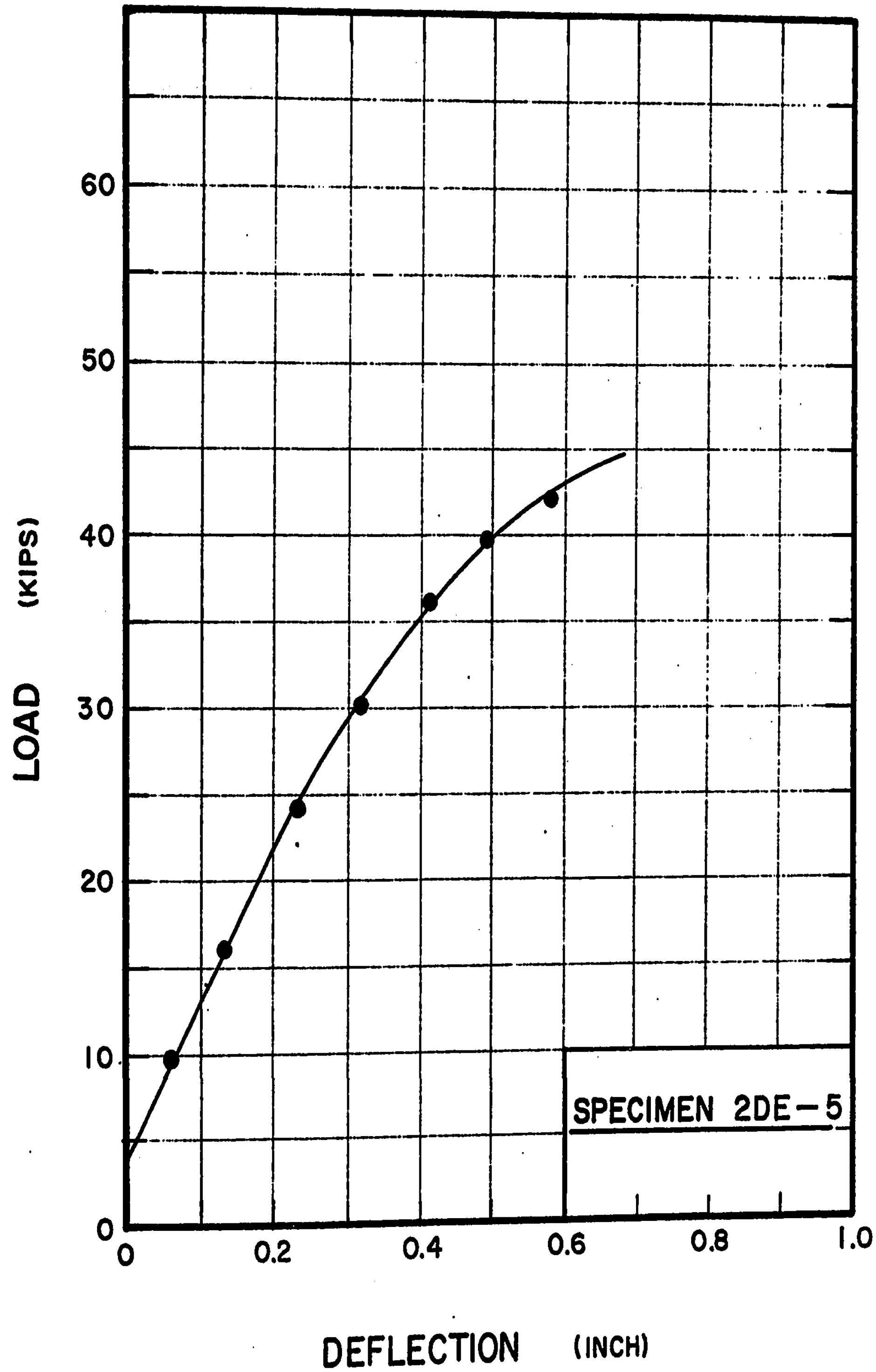


FIG.(B.60) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE-5

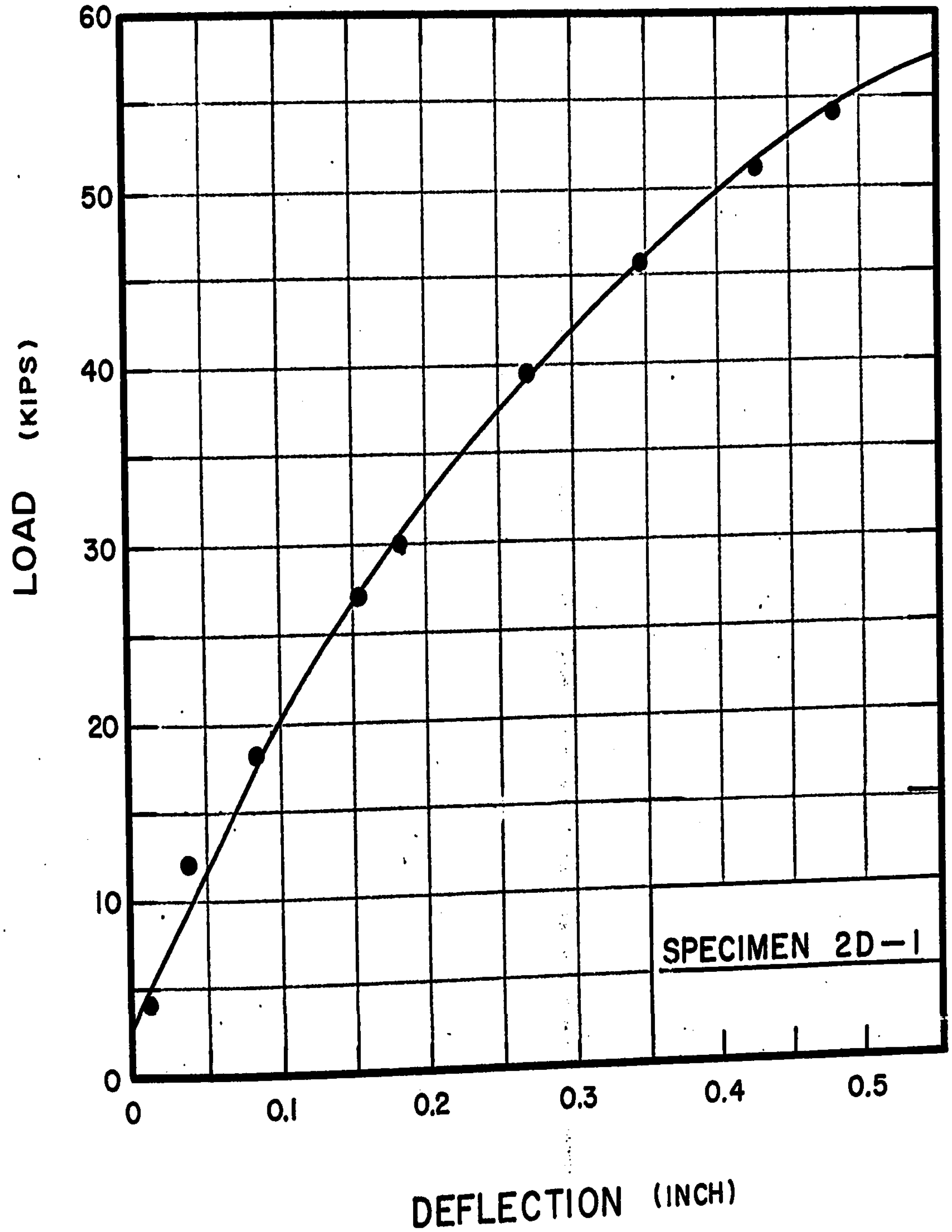


FIG. (B. 61) LOAD—DEFLECTION CURVE OF SPECIMEN 2D—1

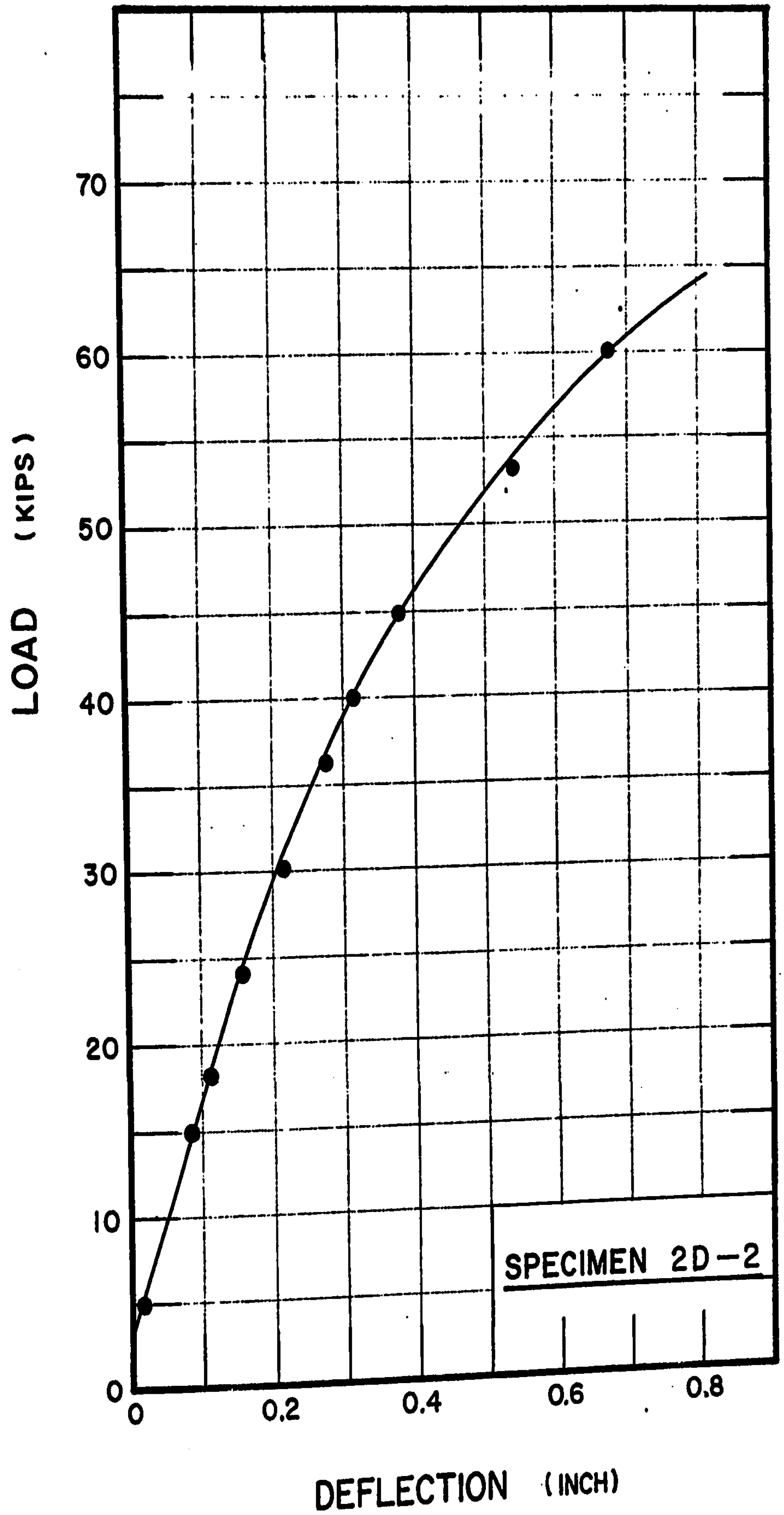


FIG.(B. 62) LOAD-DEFLECTION CURVE OF SPECIMEN 2D-2

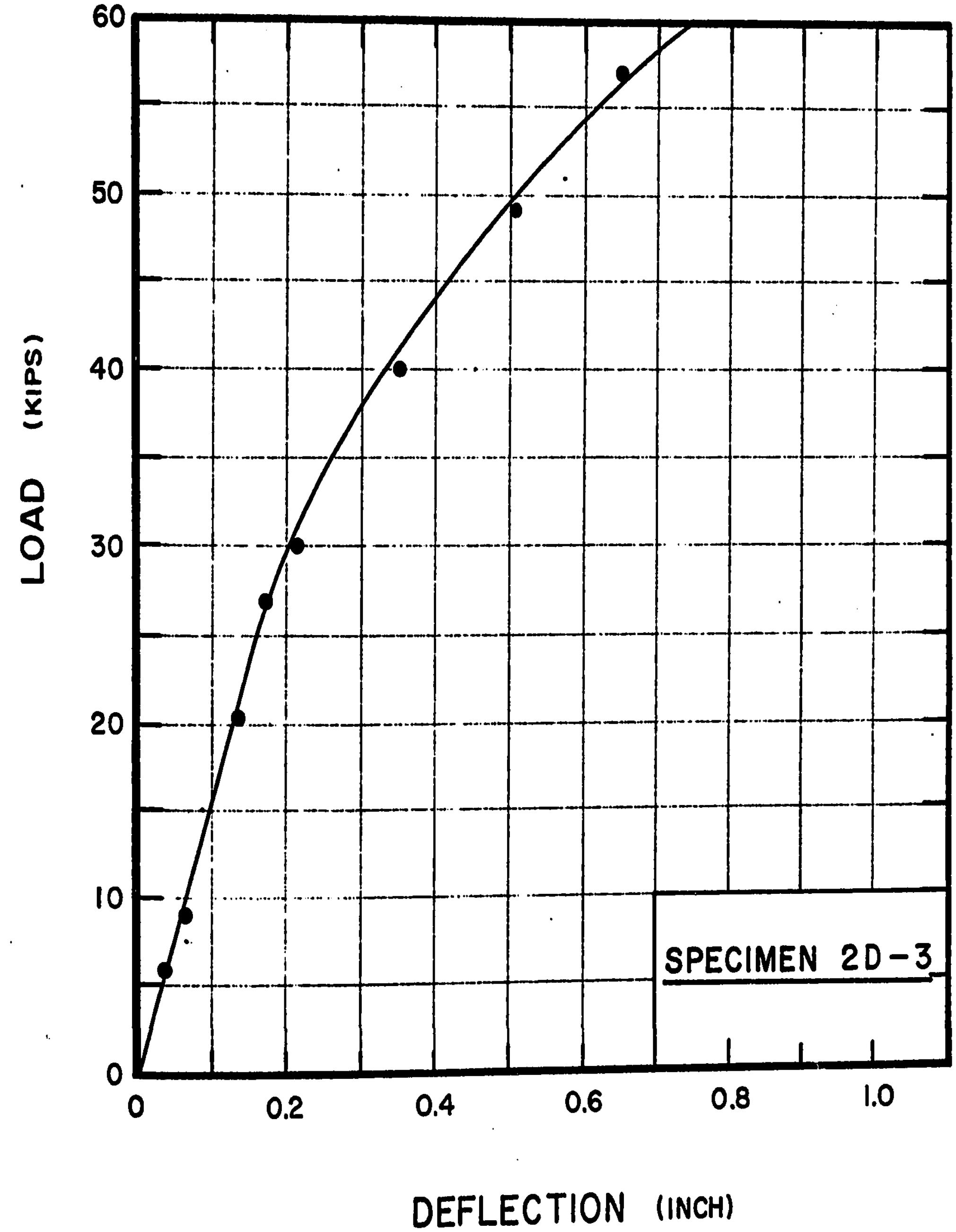


FIG. (B. 63) LOAD—DEFLECTION CURVE OF SPECIMEN 2D-3

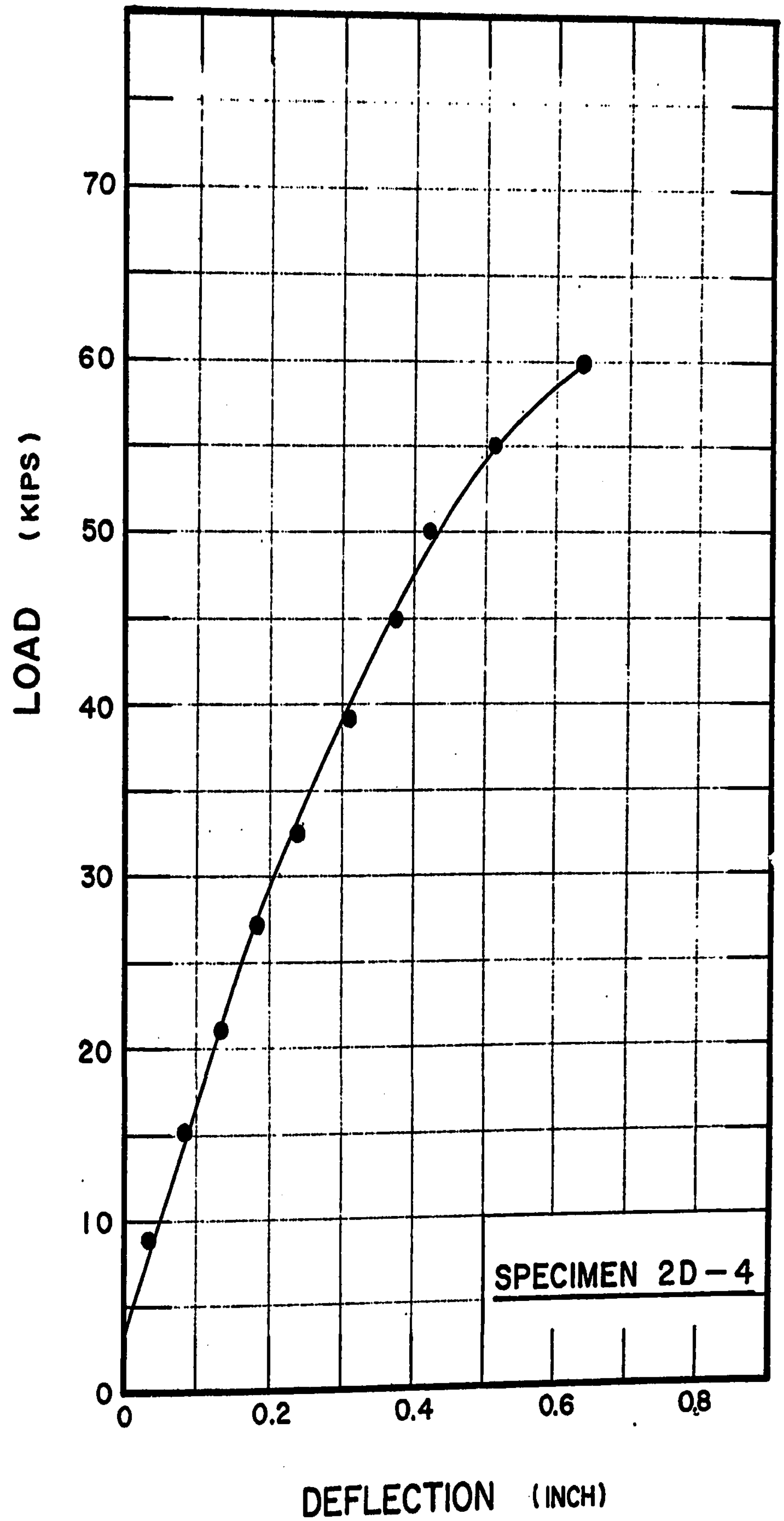


FIG.(B.64) LOAD-DEFLECTION CURVE OF SPECIMEN 2D-4

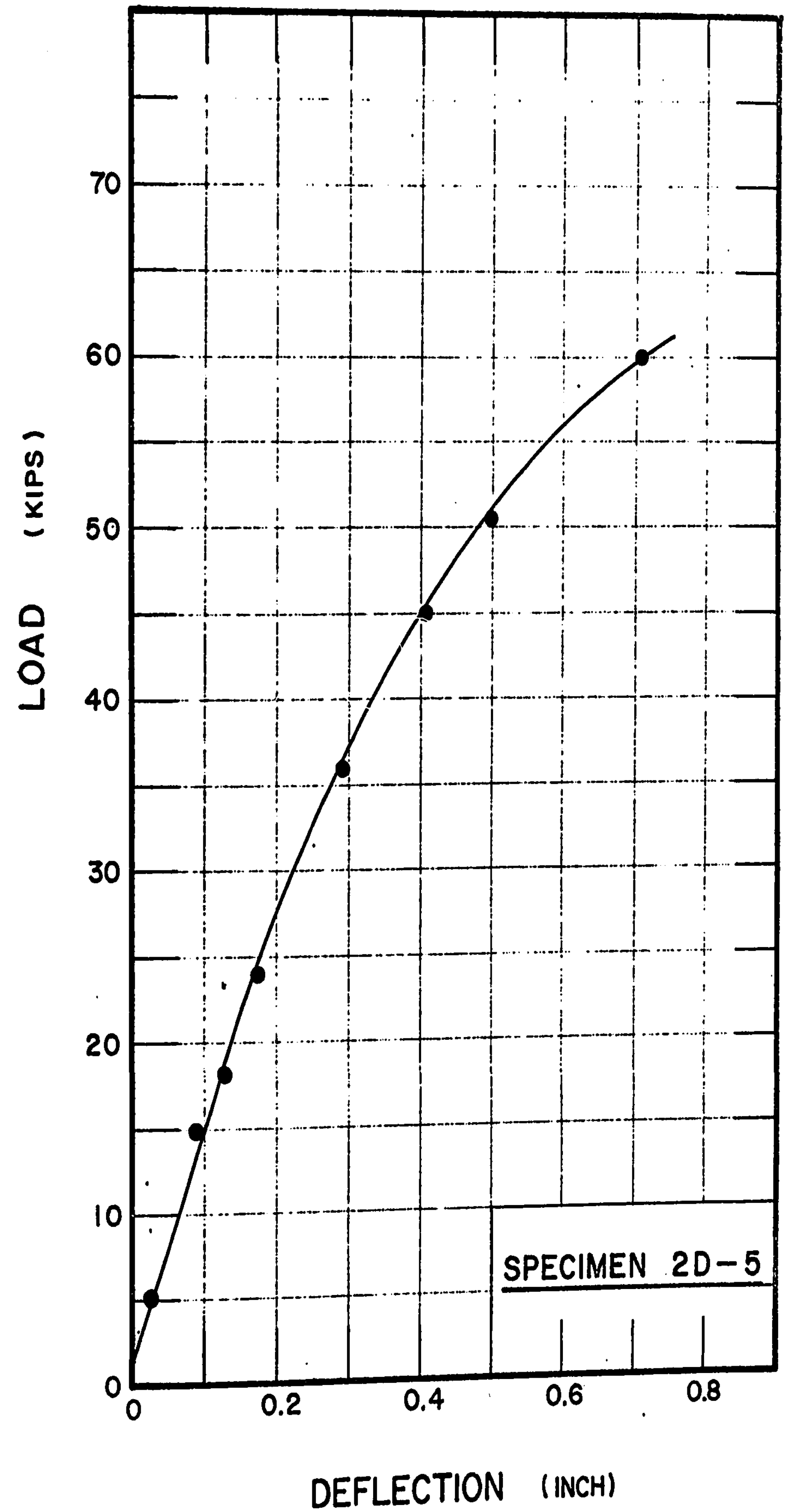


FIG.(B.65) LOAD-DEFLECTION CURVE OF SPECIMEN 2D-5

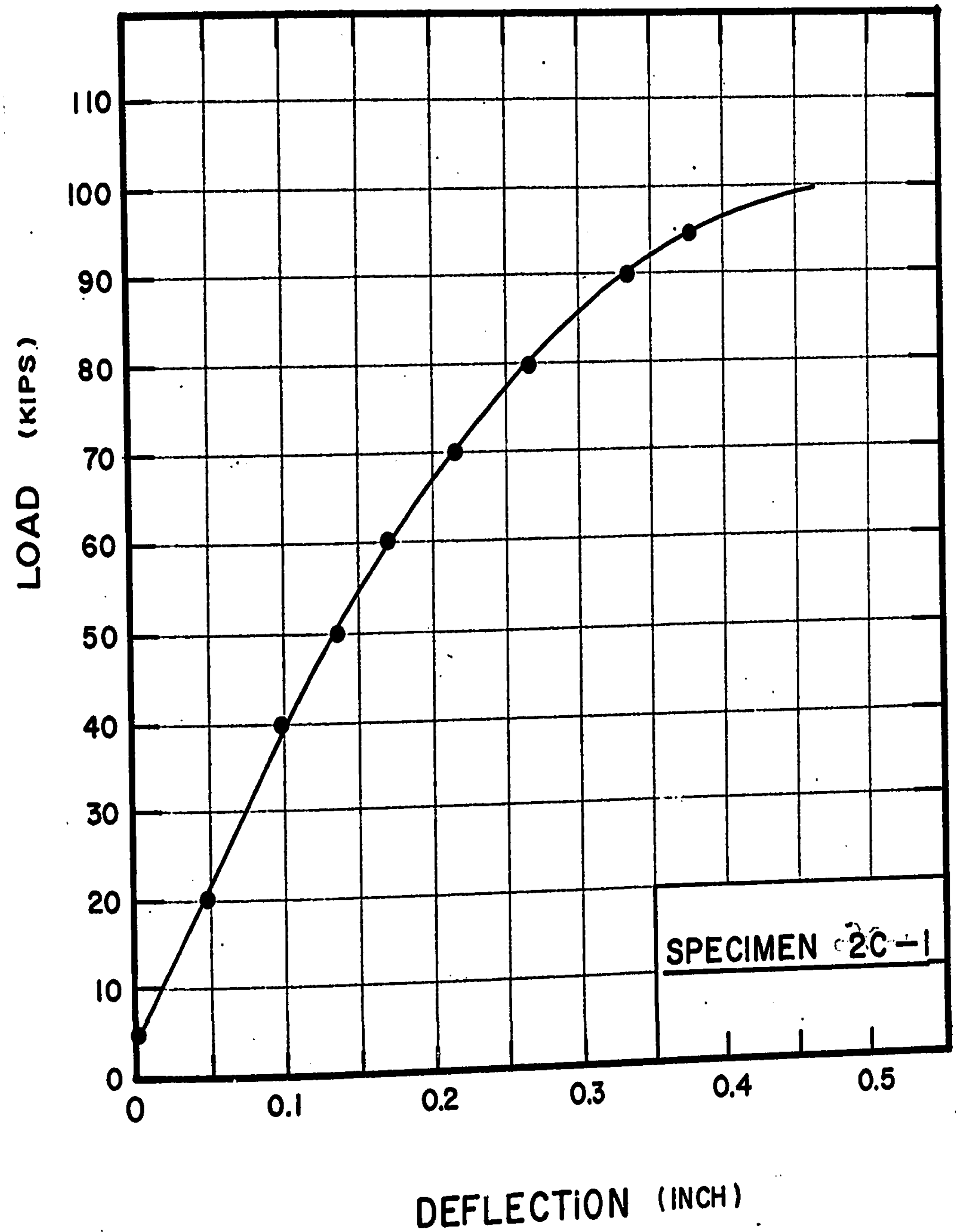


FIG. (B. 66) LOAD—DEFLECTION CURVE OF SPECIMEN 2C-1

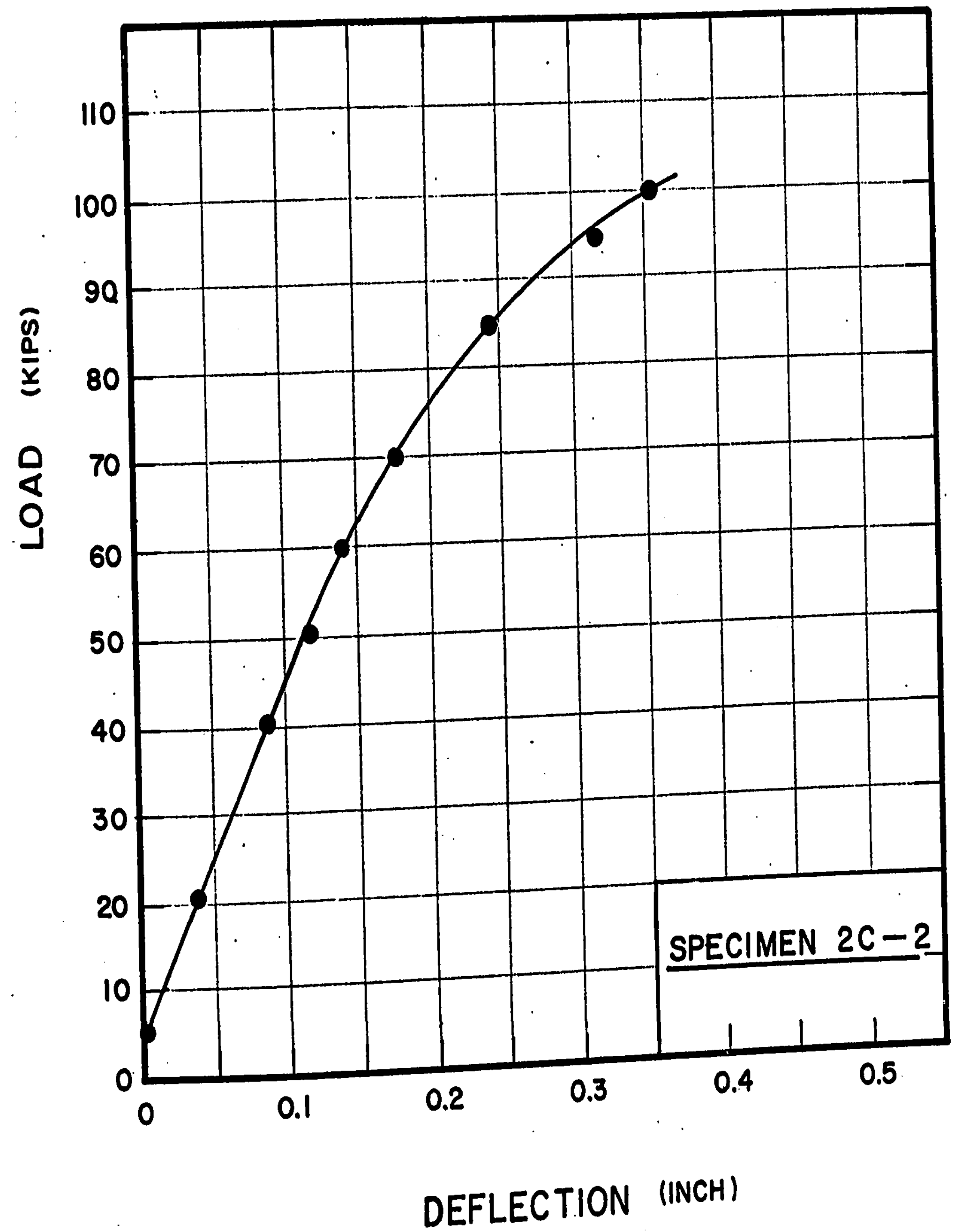


FIG. (B. 67) LOAD — DEFLECTION CURVE OF SPECIMEN 2C-2

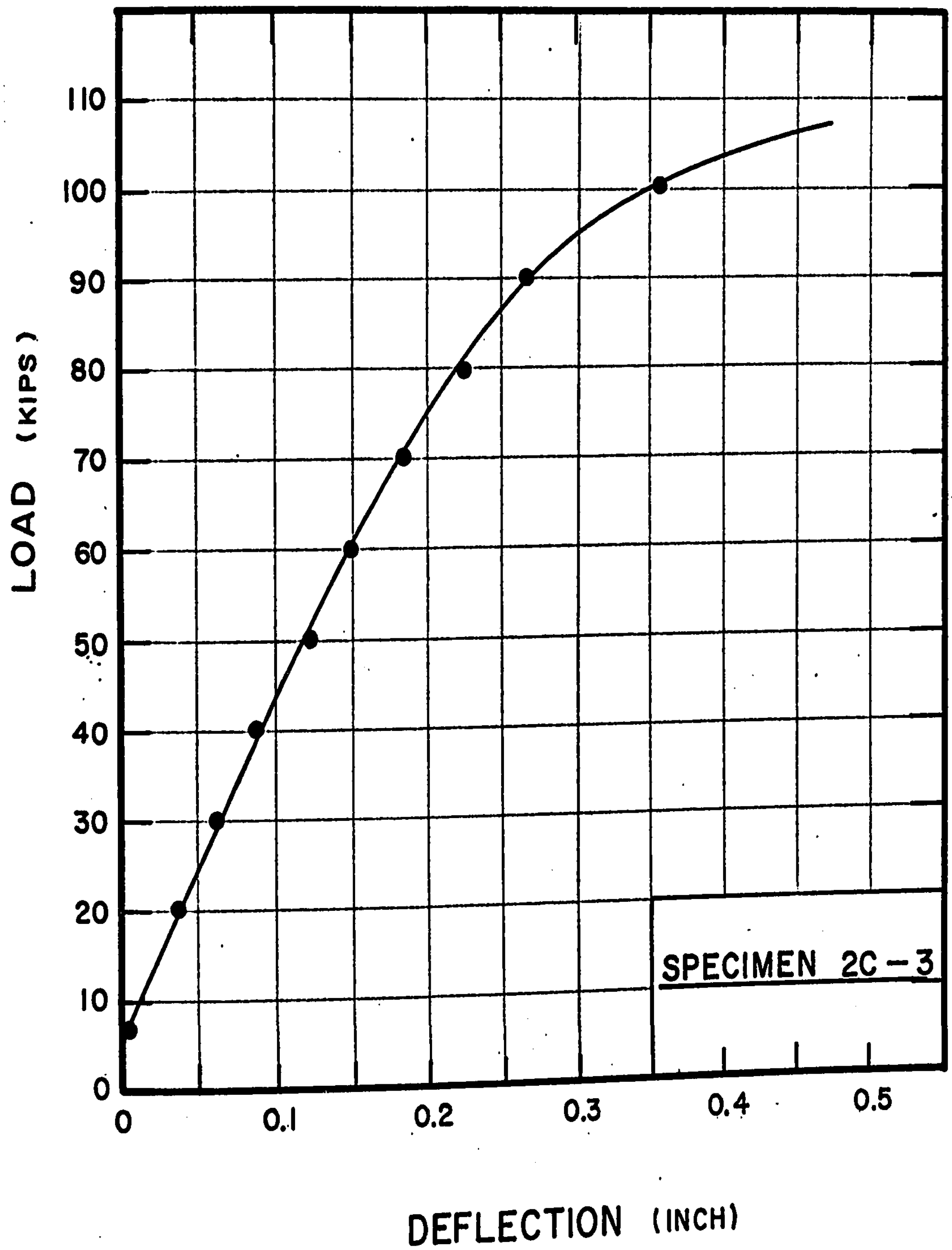


FIG.(B.68) LOAD-DEFLECTION CURVE OF SPECIMEN 2C-3

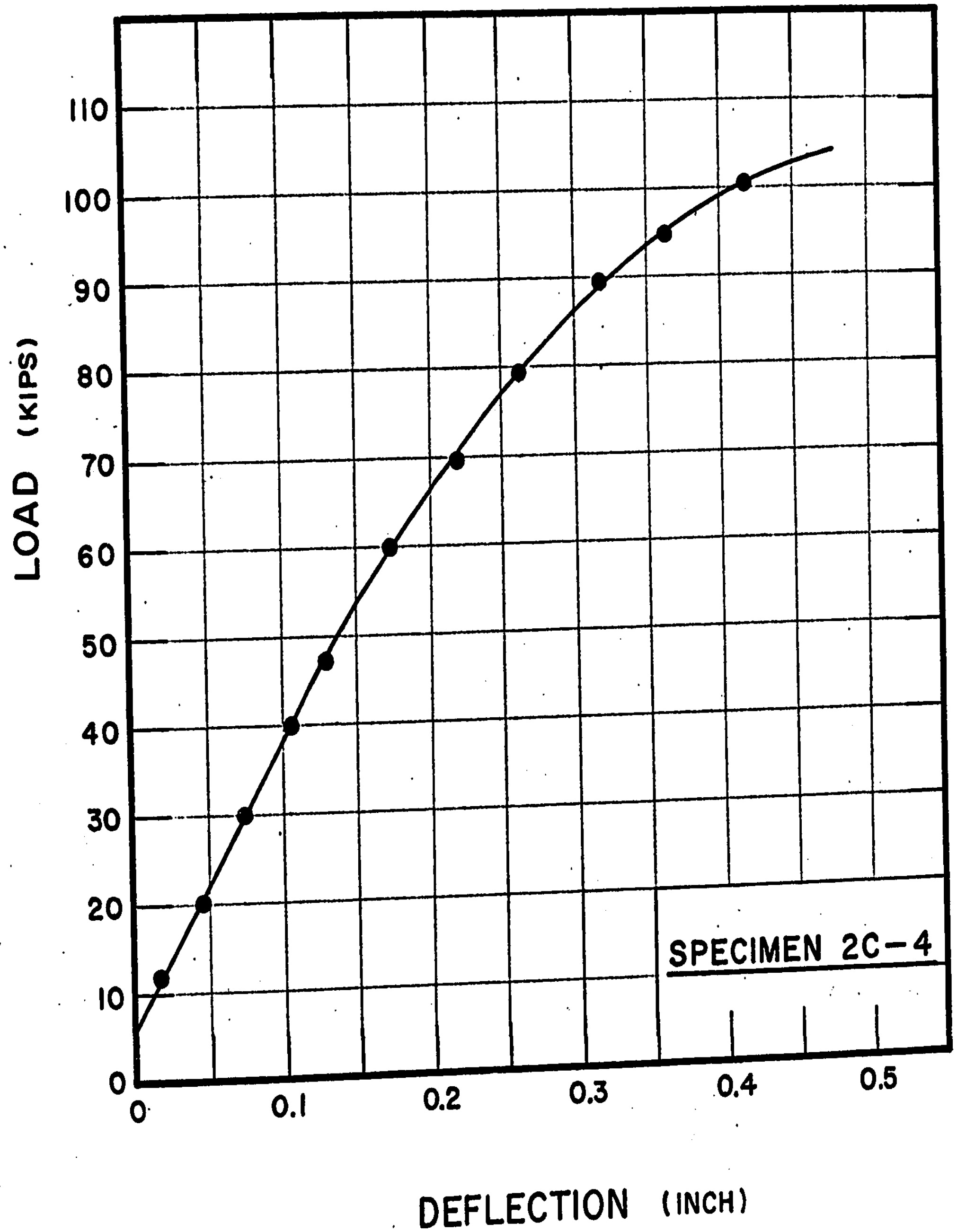


FIG. (B.69) LOAD—DEFLECTION CURVE OF SPECIMEN 2C-4

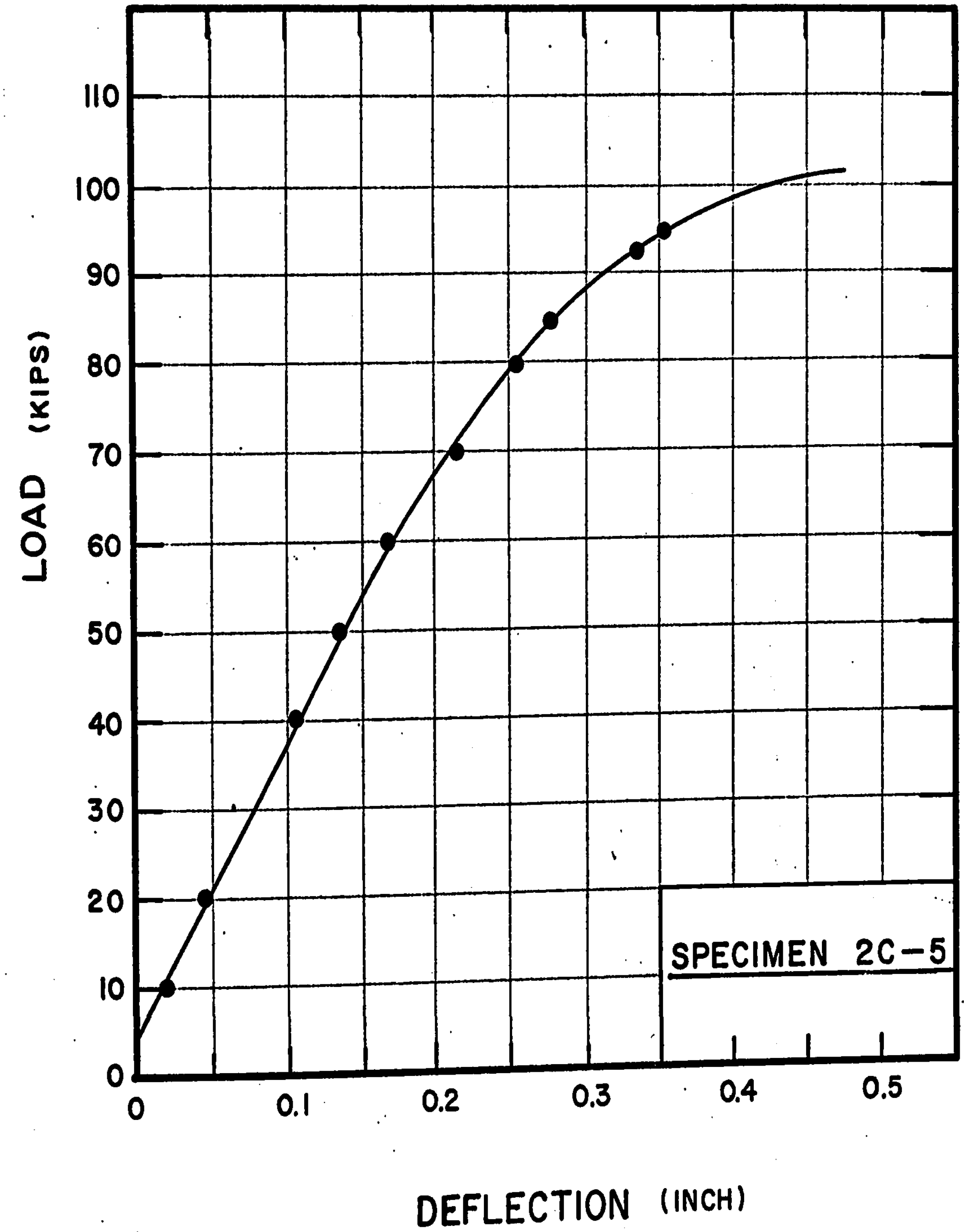


FIG.(B.70) LOAD—DEFLECTION CURVE OF SPECIMEN 2C-5

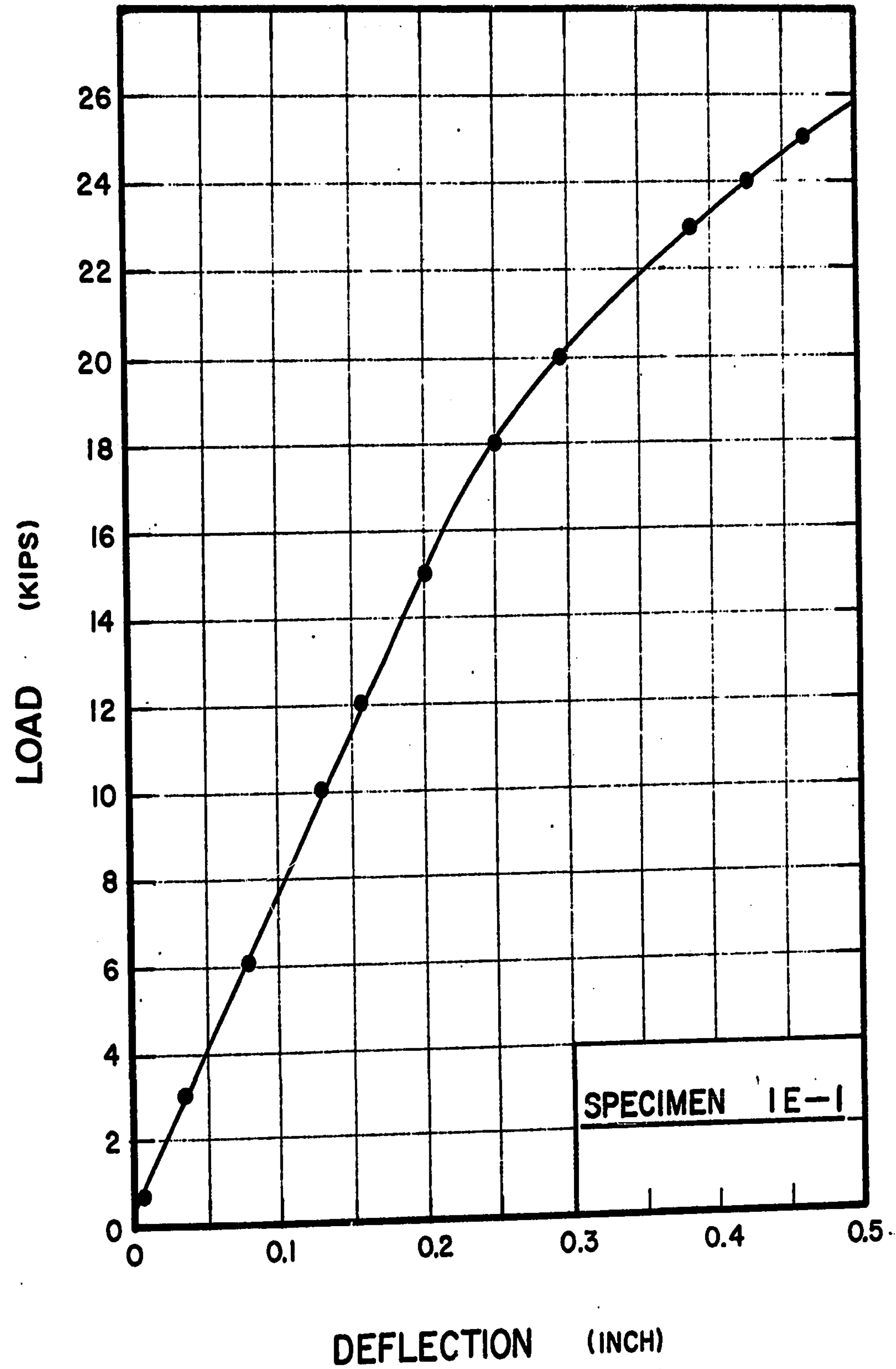


FIG.(B. 71) LOAD—DEFLECTION CURVE OF SPECIMEN 1E-1

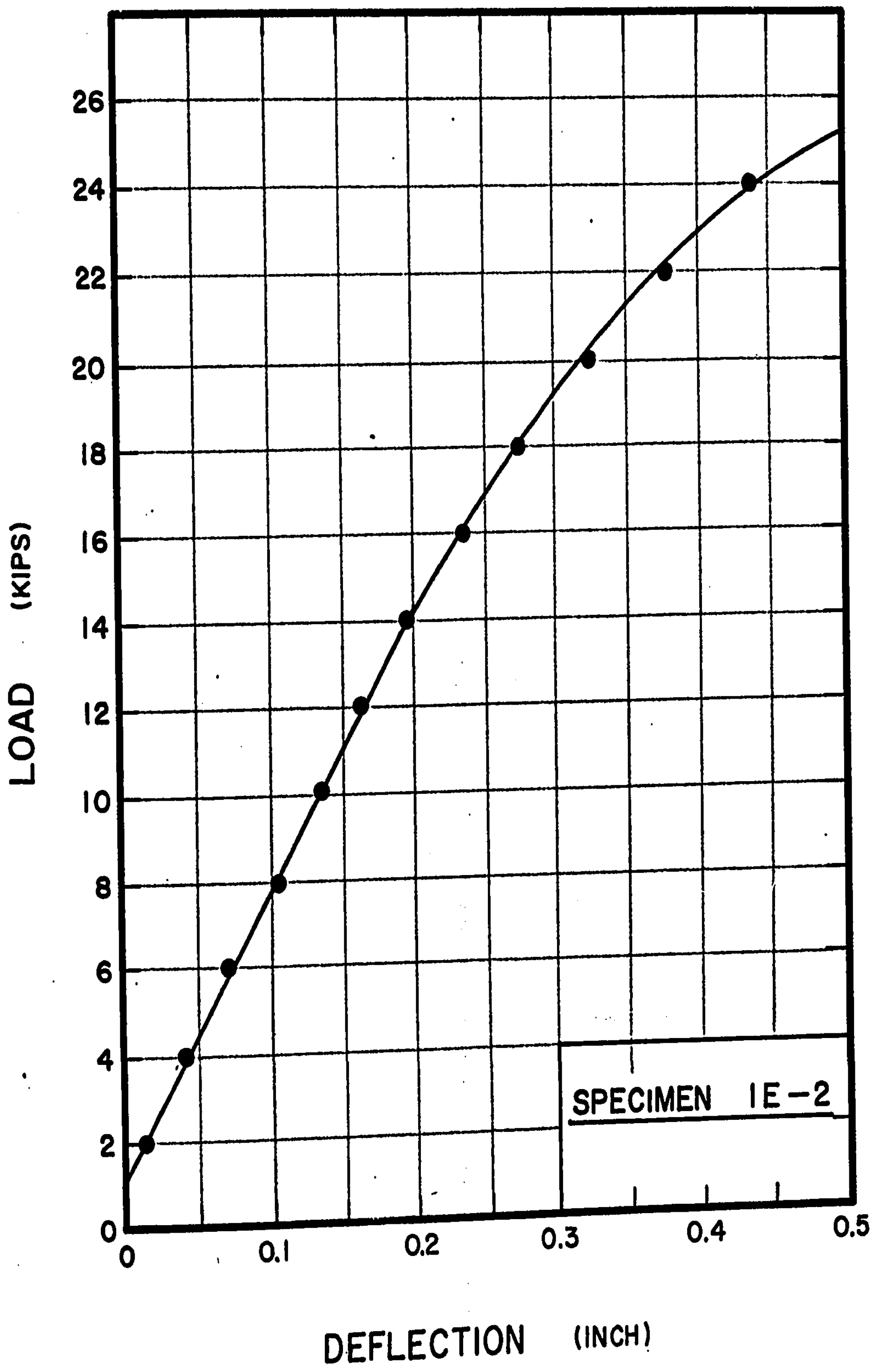


FIG.(B.72) LOAD-DEFLECTION CURVE OF SPECIMEN IE-2

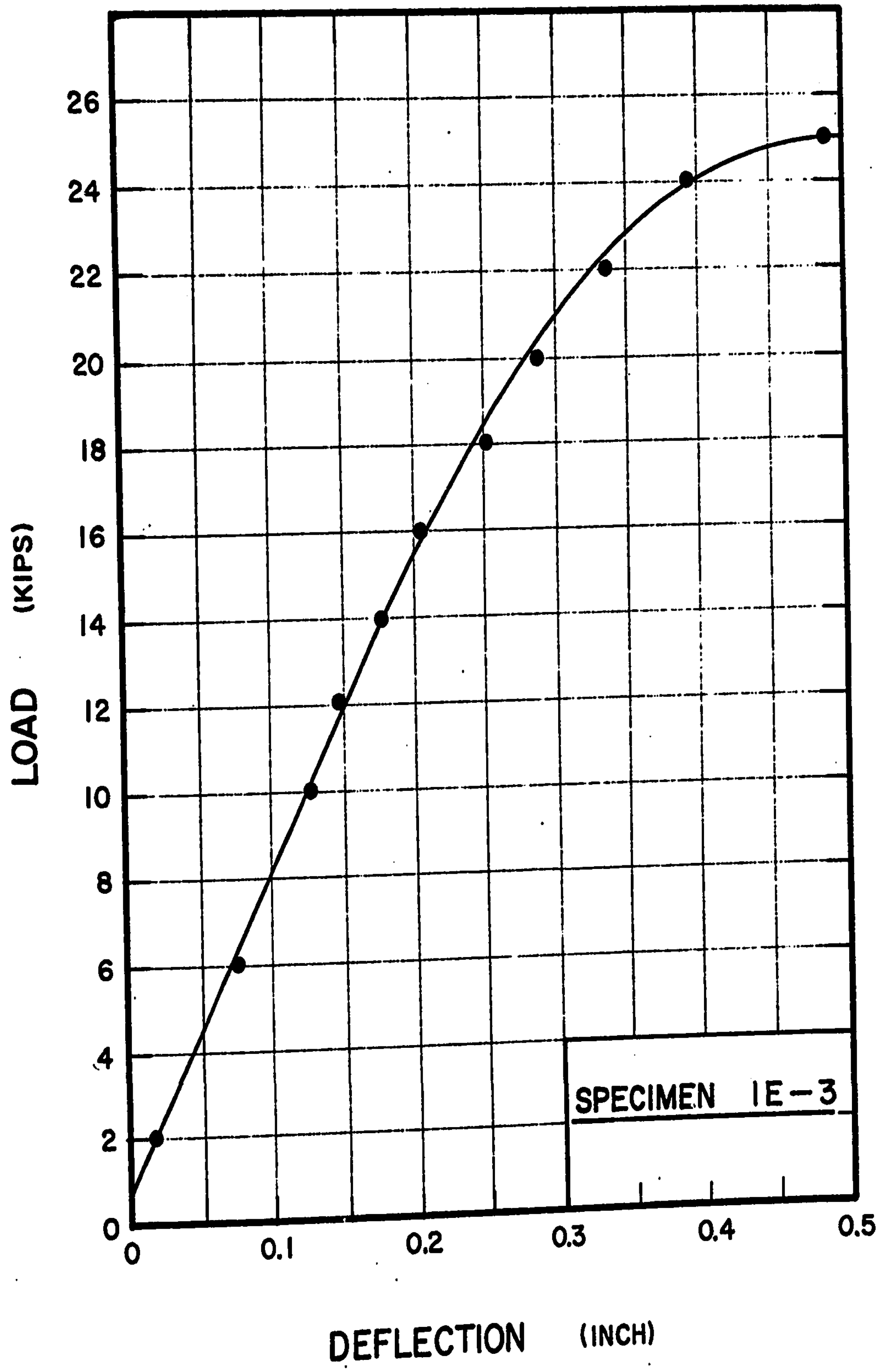


FIG.(B.73) LOAD—DEFLECTION CURVE OF SPECIMEN IE-3

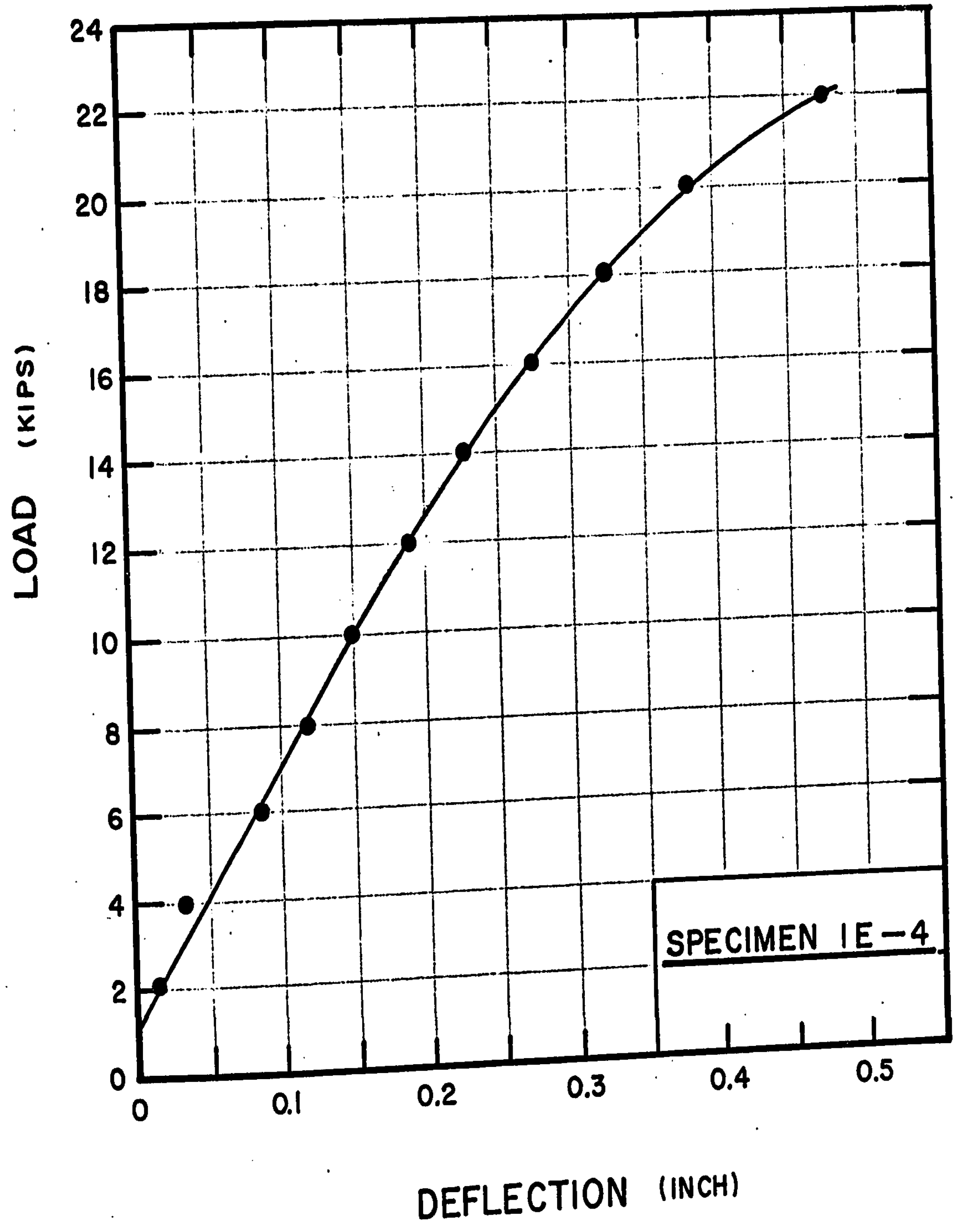


FIG. (B. 74) LOAD — DEFLECTION CURVE OF SPECIMEN IE — 4

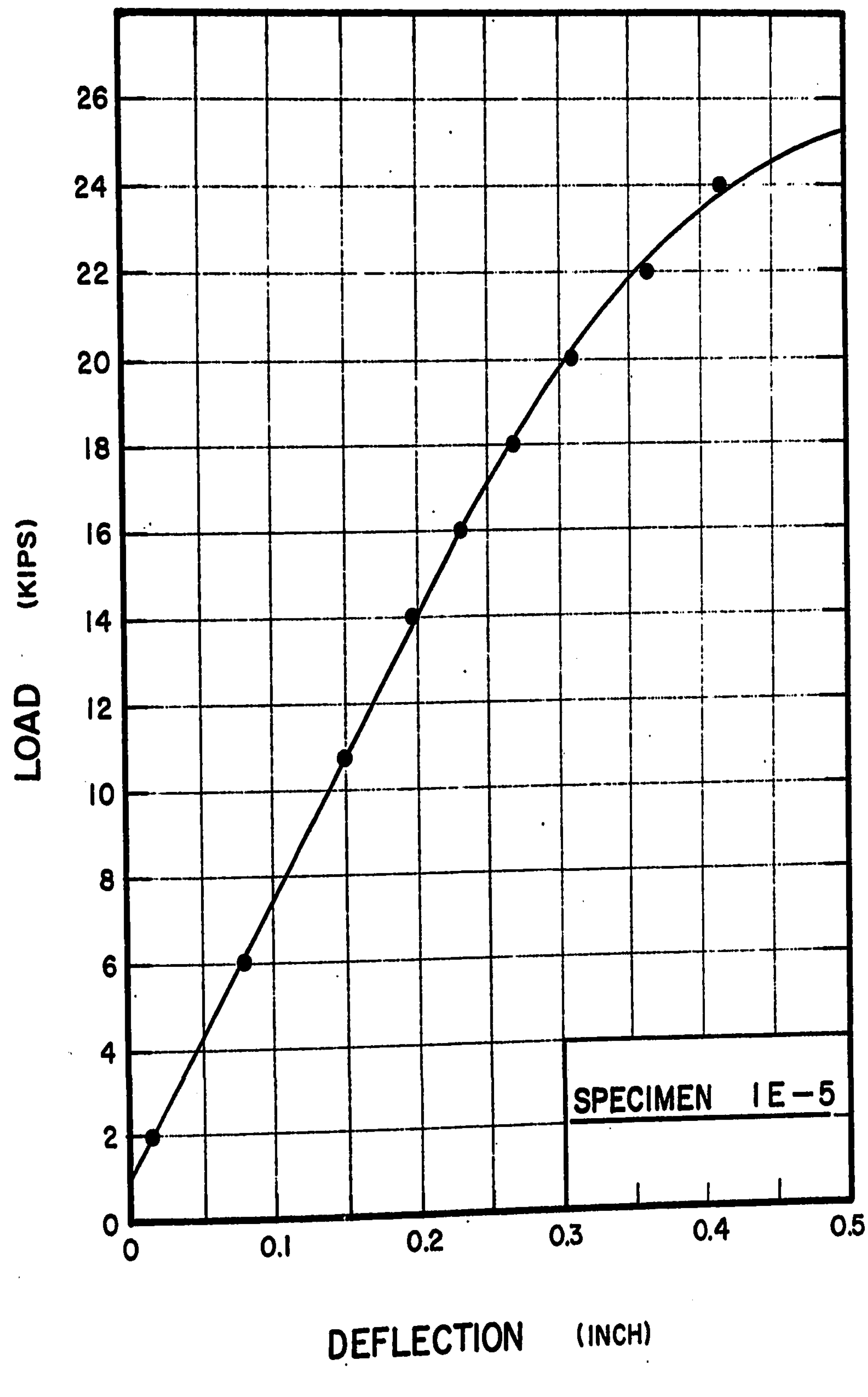


FIG.(B.75) LOAD—DEFLECTION CURVE OF SPECIMEN 1E-5

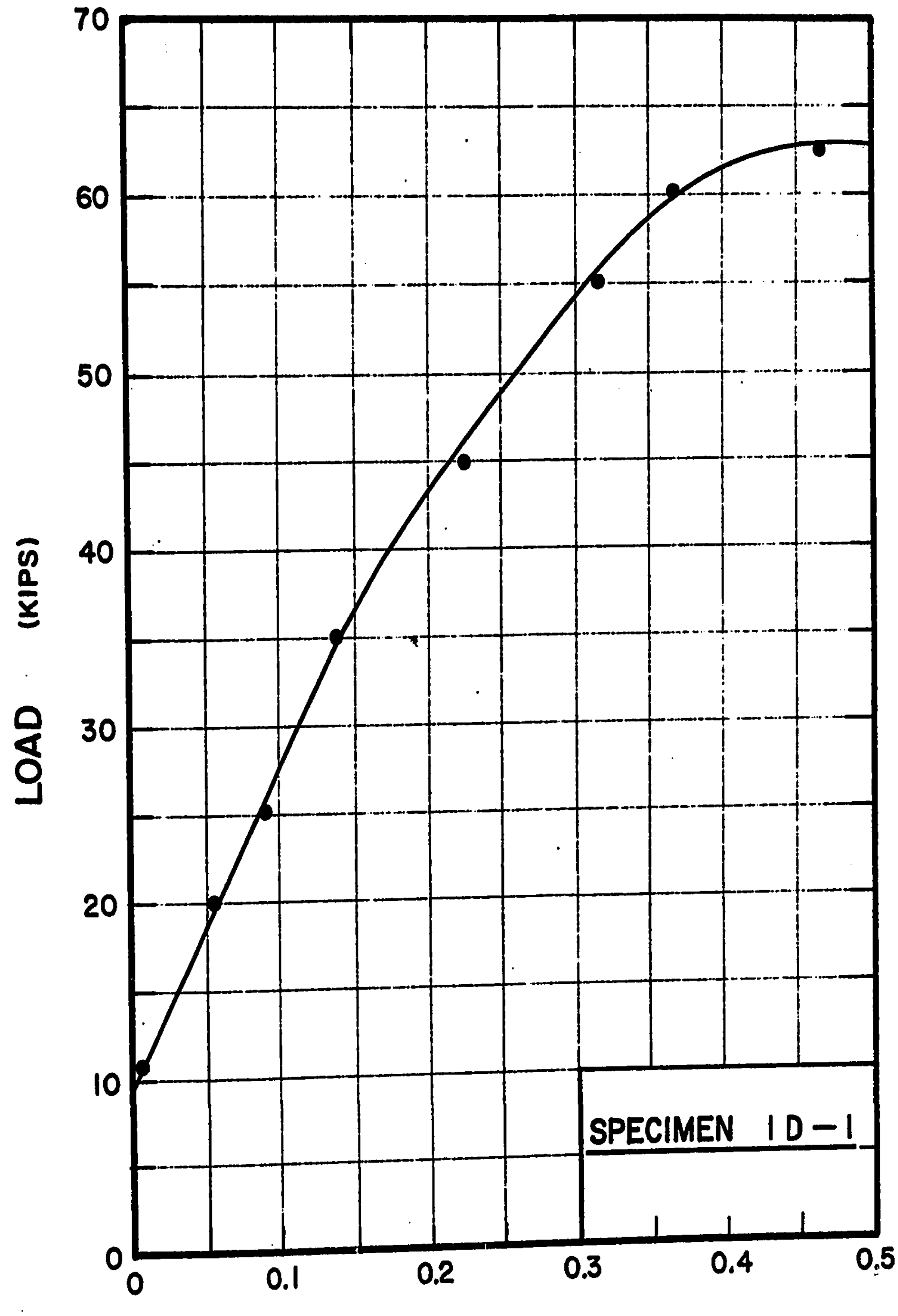


FIG.(B.76) LOAD—DEFLECTION CURVE OF SPECIMEN ID-I

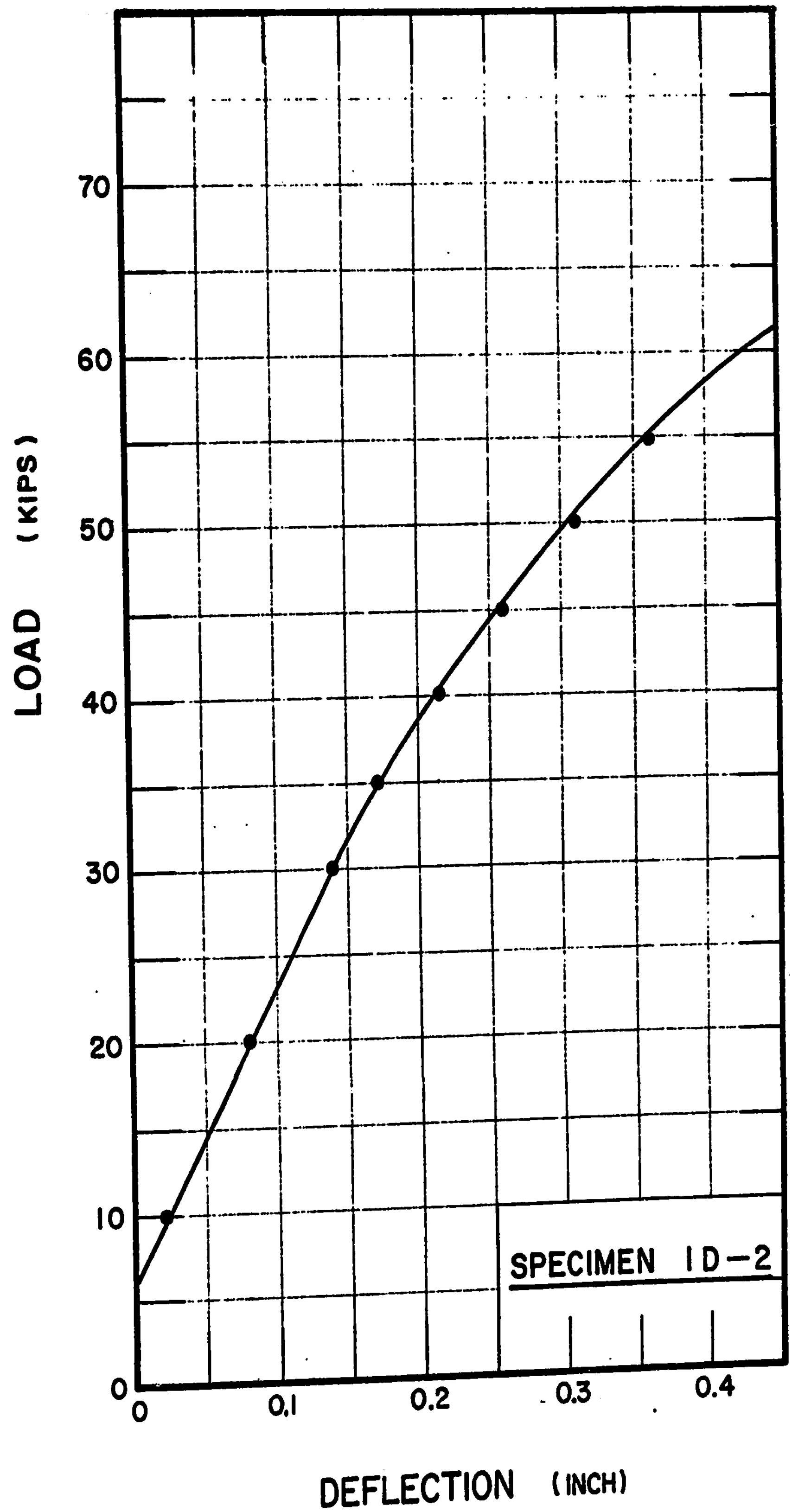


FIG.(B.77) LOAD-DEFLECTION CURVE OF SPECIMEN ID-2

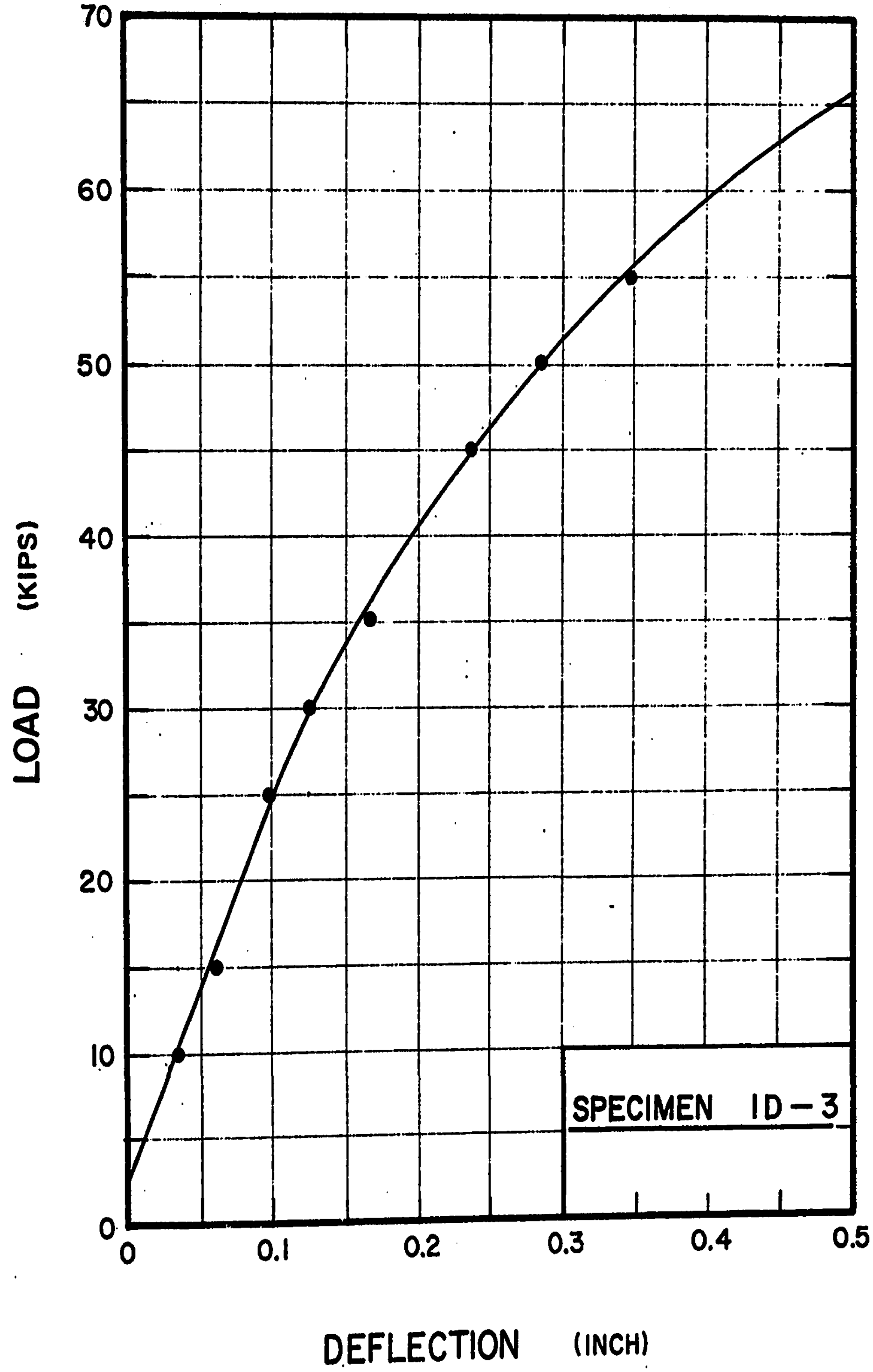


FIG.(B.78) LOAD—DEFLECTION CURVE OF SPECIMEN ID-3

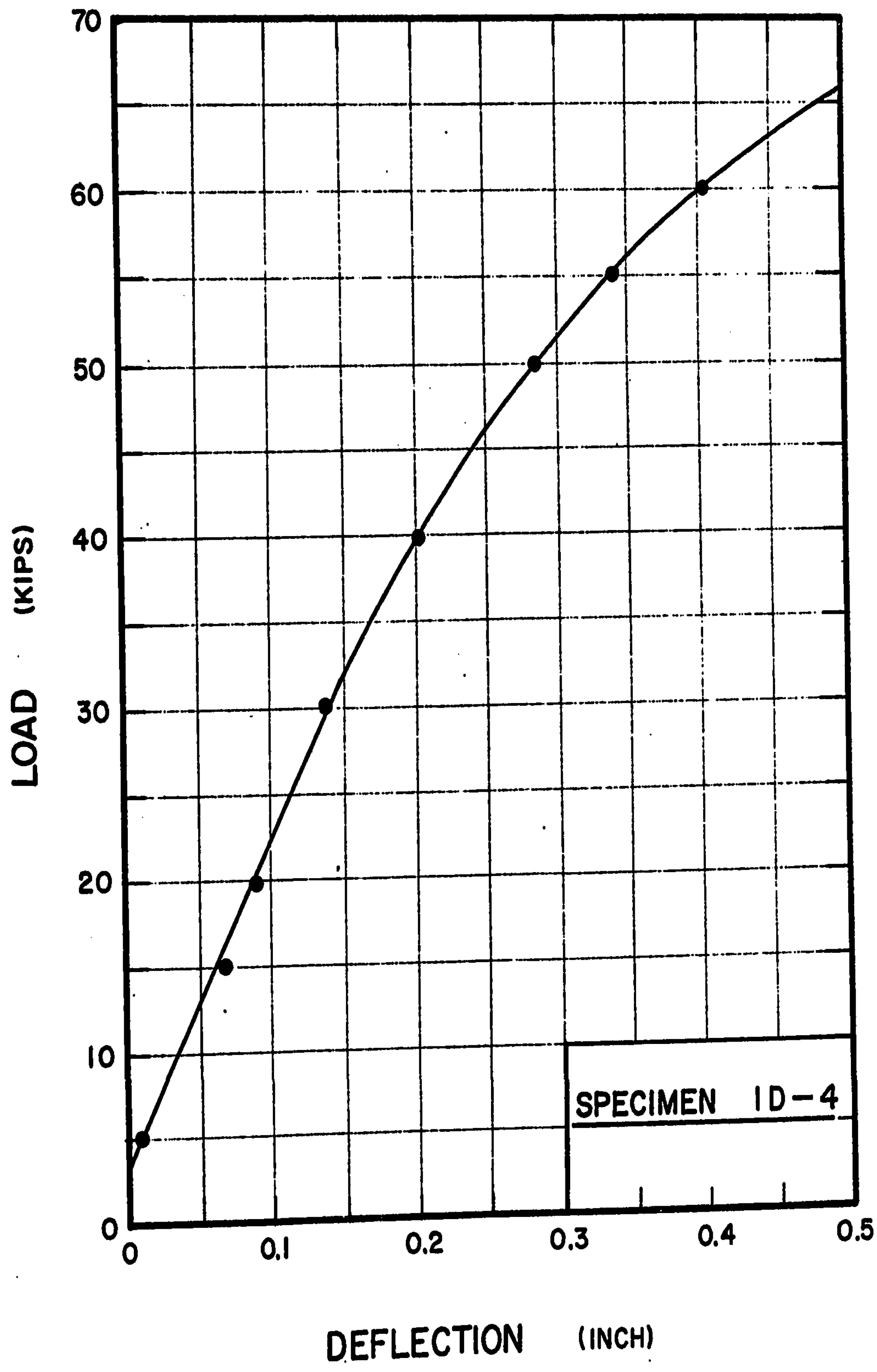


FIG.(B.79) LOAD—DEFLECTION CURVE OF SPECIMEN ID-4

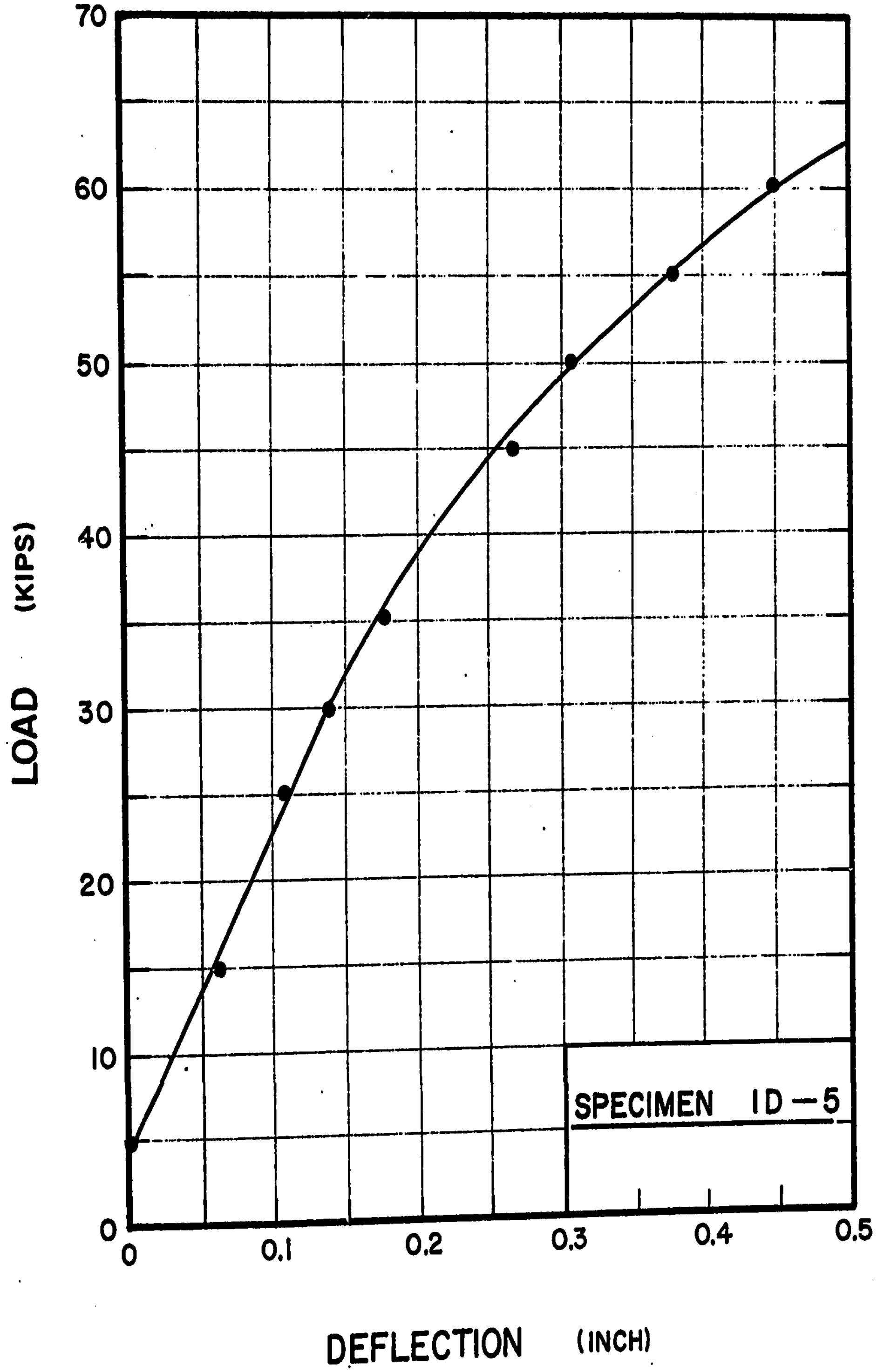


FIG.(B.80) LOAD—DEFLECTION CURVE OF SPECIMEN ID—I

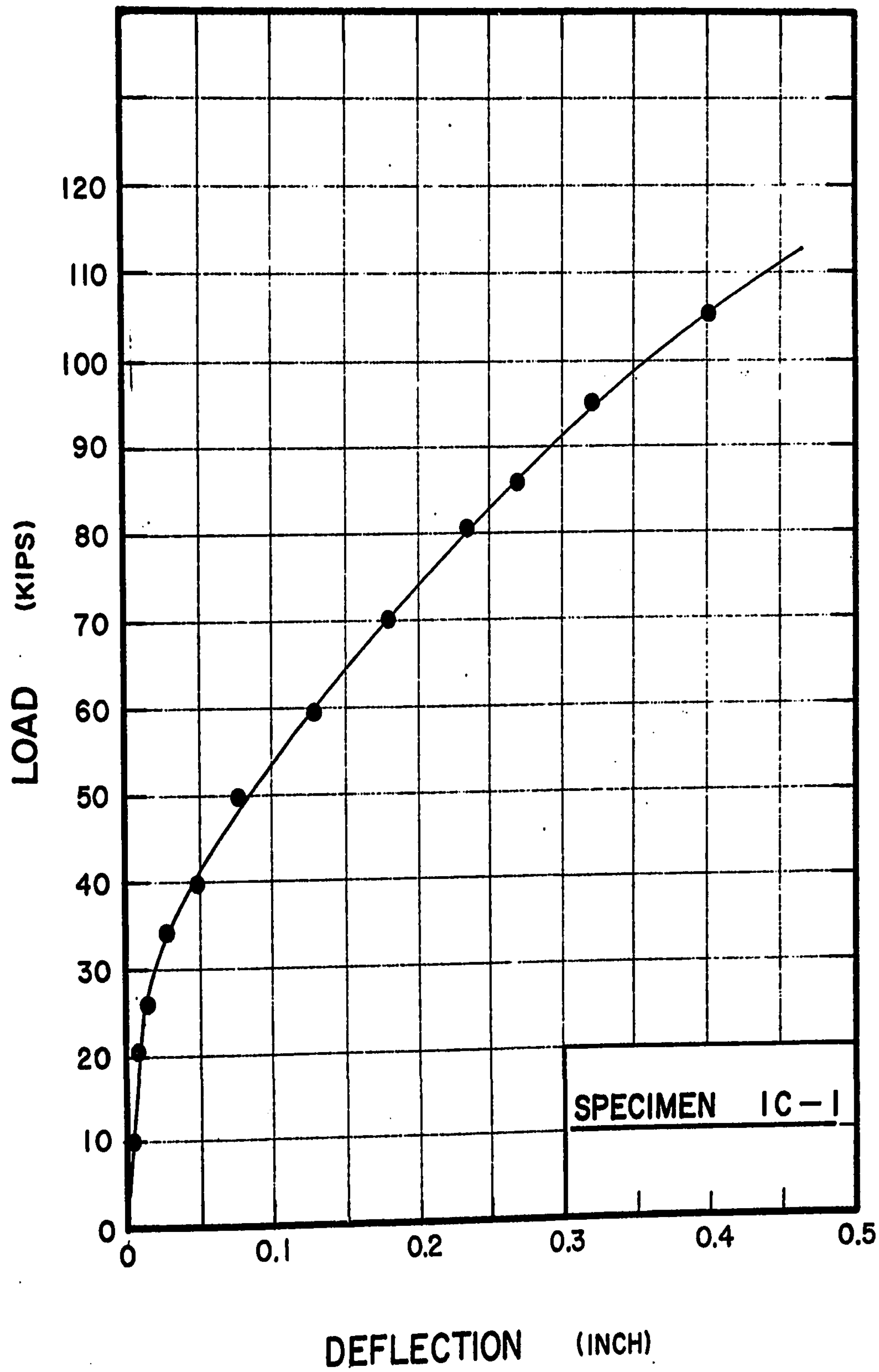


FIG.(B. 81) LOAD—DEFLECTION CURVE OF SPECIMEN IC-I

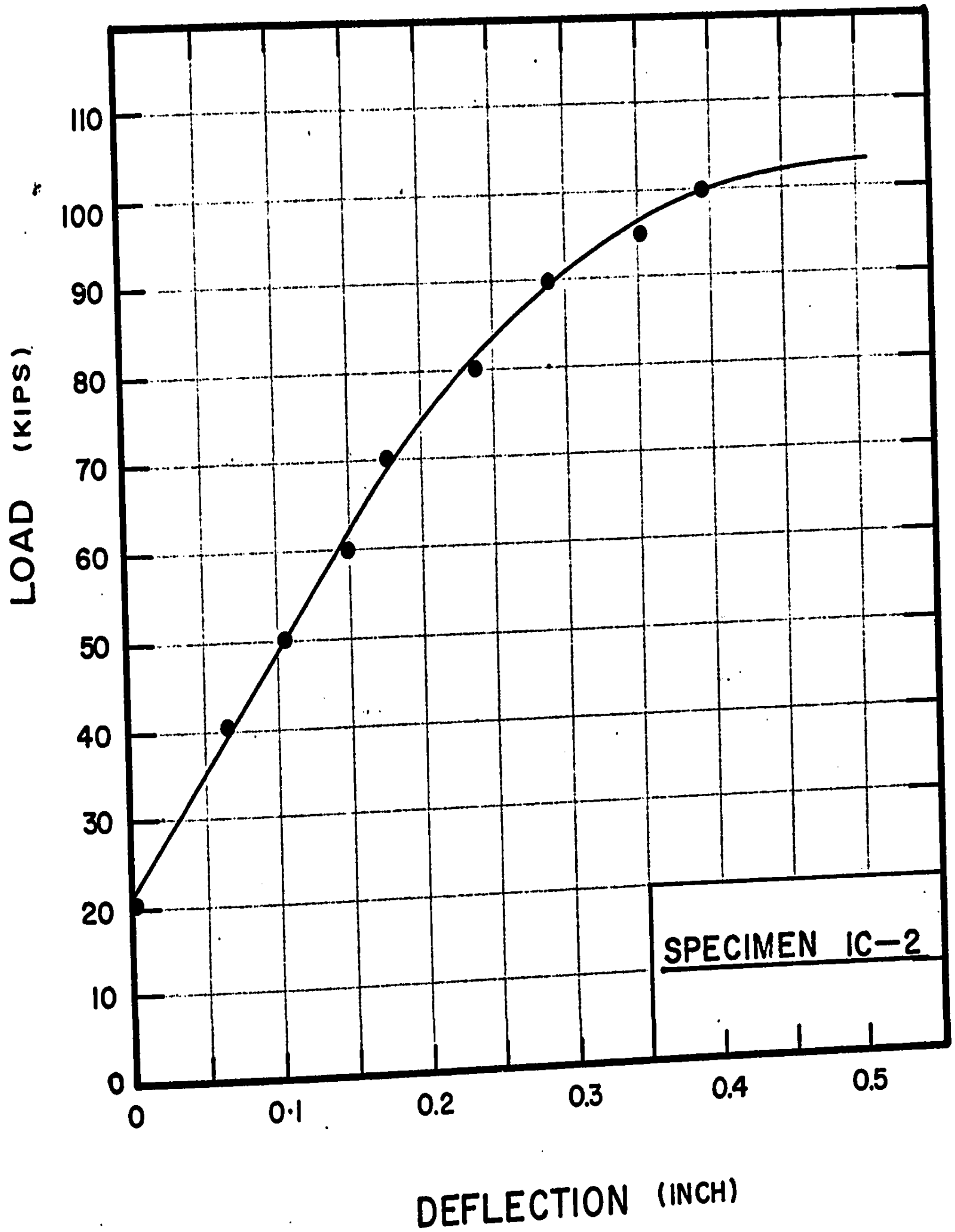


FIG.(B.82) LOAD—DEFLECTION CURVE OF SPECIMEN IC—2

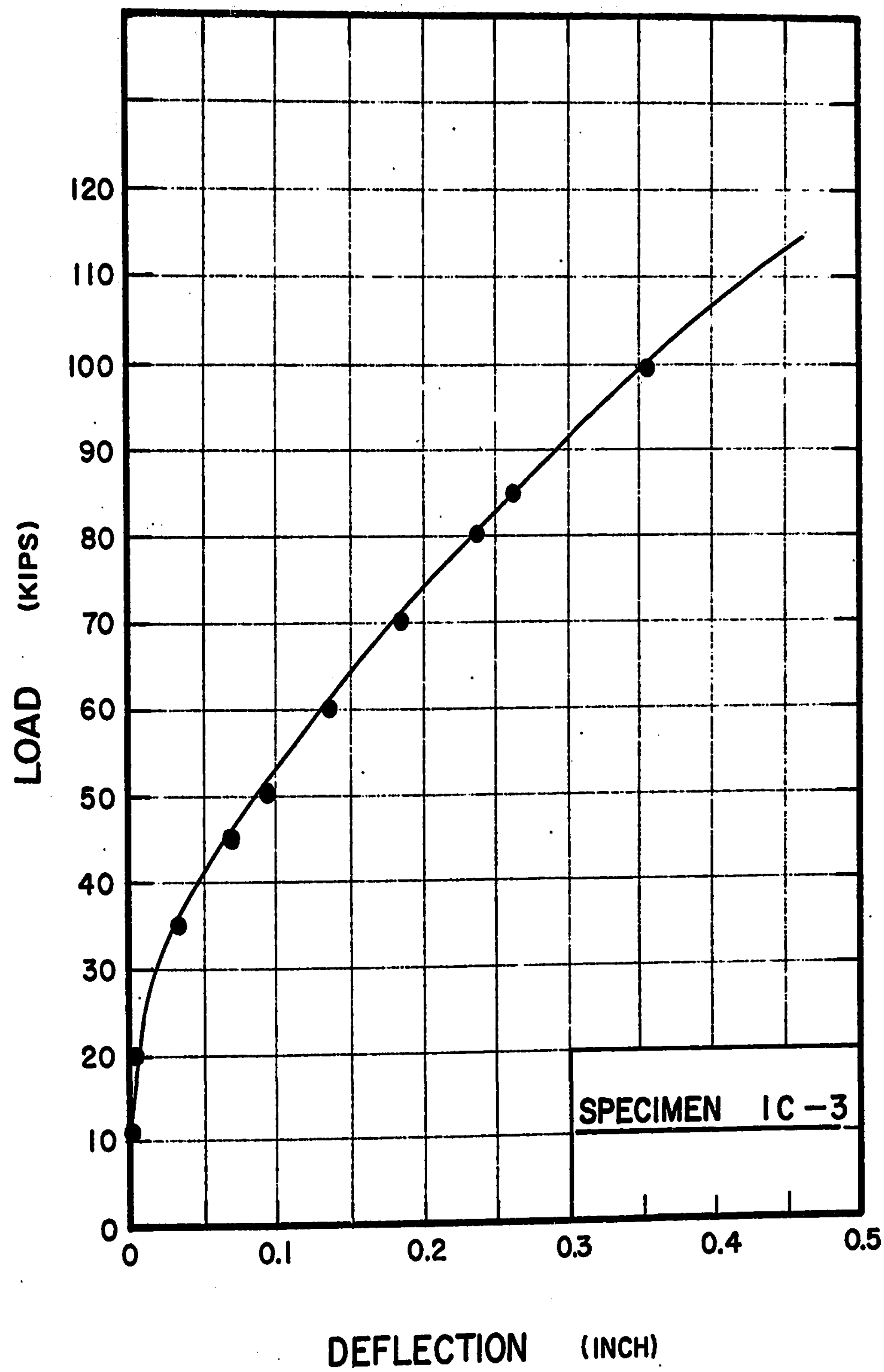


FIG.(B.83) LOAD—DEFLECTION CURVE OF SPECIMEN IC-3

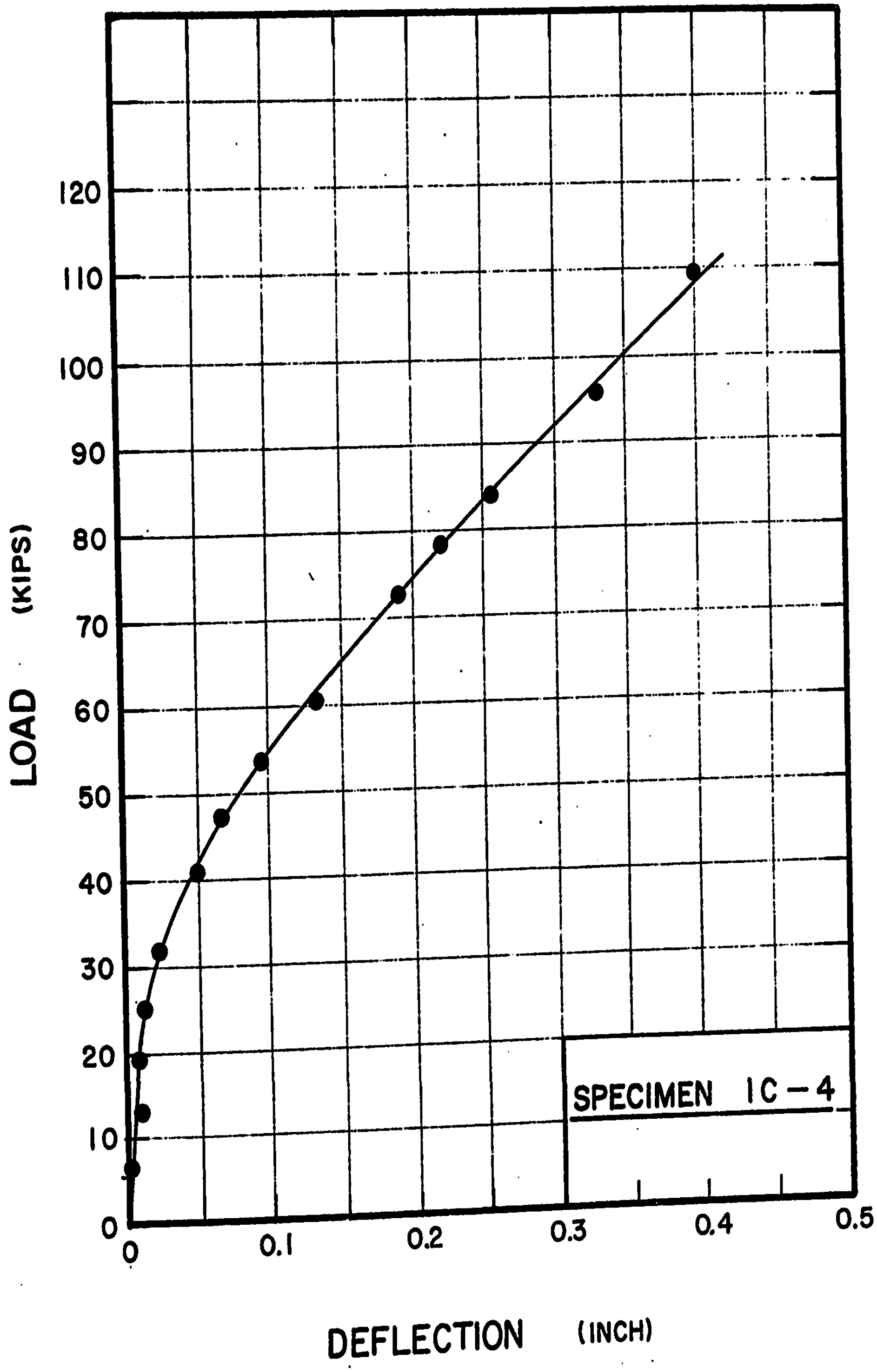


FIG.(B.84) LOAD—DEFLECTION CURVE OF SPECIMEN IC-4

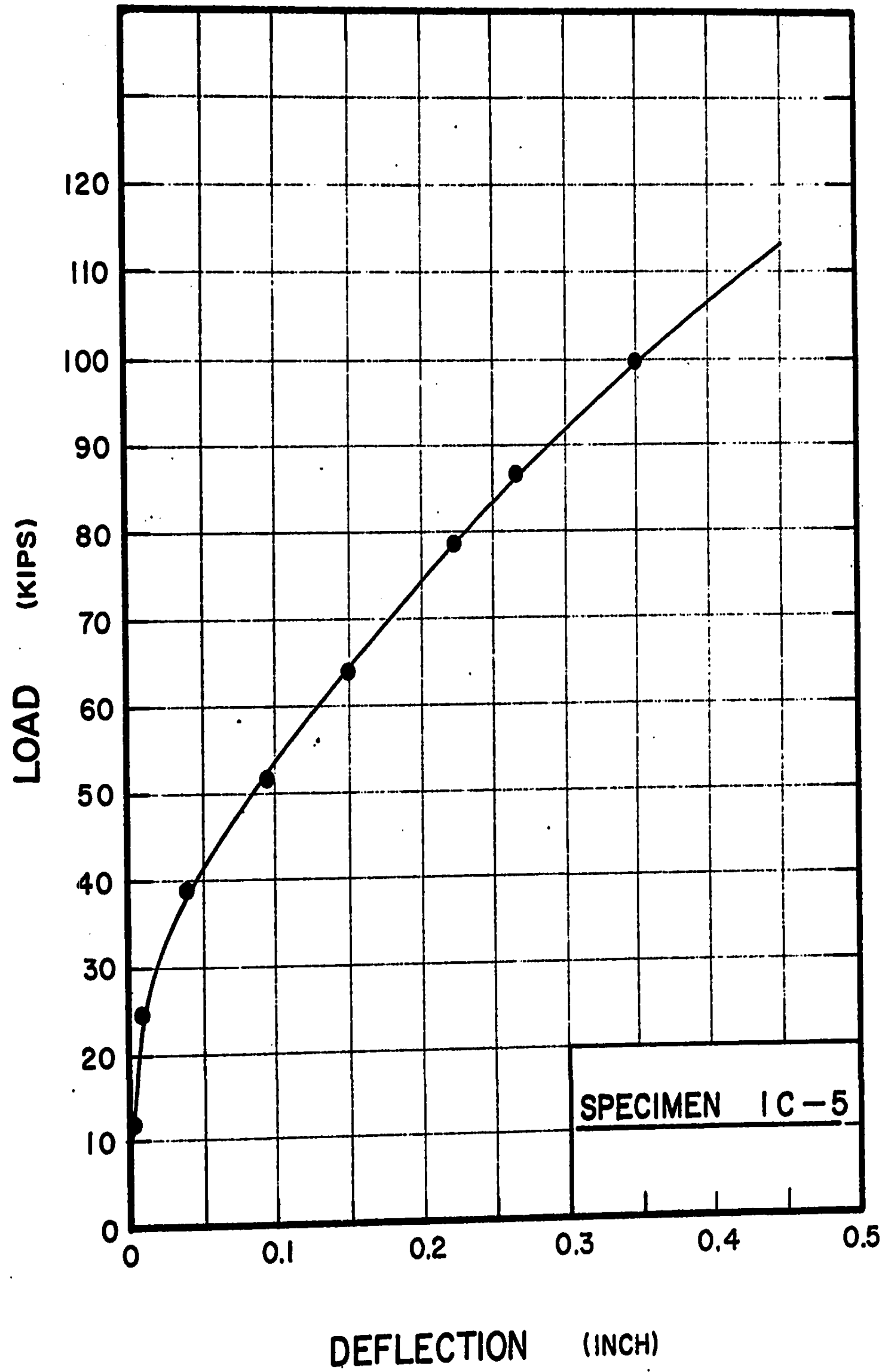


FIG.(B.85) LOAD—DEFLECTION CURVE OF SPECIMEN IC-5

APPENDIX C

E X A M P L E S

EXAMPLE A

COLUMN DUE TO ECCENTRIC LOADING
BY
ACI DESIGN FORMULAE

EXAMPLE B

COLUMN DUE TO PURE BENDING
BY
STRAIN-GRADIENT METHOD

(A) Reinforced Concrete Columns due to Eccentric Loading

Example: Column 4D-4

Given: $e/t = 0.3$

$f_c' = 48,708$ psi.

$f_y = 45,386$ psi.

$d = 4.57"$

$d' = 1.43"$

$d" = 1.57"$

$A_s = A_s' = 0.11$ sq.in.

$e = 3.38"$

Refer to Figs. (4.1.1.a) and (4.1.3.a)

$$C_b = \frac{87d}{87 + f_y} = \frac{87 \times 4.57}{87 + 45.386}$$

$$= \frac{87 \times 4.57}{132.386} = 3.0"$$

$$a = K_1 C_b = 0.85 \times 3 = 2.55"$$

$$C_c = 0.85 f_c' ab$$

$$= 0.85 \times 4.871 \times 2.55 \times 6$$

$$= 63.4 \text{ kips.}$$

$$C_s = A_s (f_y - 0.85f_c')$$
$$= 0.22 (45.39 - 0.85 \times 4.871)$$
$$= 9.08 \text{ kips.}$$

$$T = A_s f_y = 0.22 \times 45.386 = 9.99 \text{ kips.}$$

$$P_b = C_s + C_c - T = 9.08 + 63.4 - 9.99$$
$$= 62.49 \text{ kips.}$$

Substitute in Eq. (4.1.3.c)

$$P_b e_b = C_c (d - \frac{a}{2}) + C_s (d - d')$$
$$= 63.4 (f.57 - \frac{2.55}{2}) + 9.08 (4.57 - 1.43)$$
$$= 173.5 \text{ k-in.}$$

$$e_b = \frac{173.5}{62.49} = 2.78'' < e = 3.38''$$

Therefore tension controls.

$$p = \frac{AS}{bd} = \frac{0.22}{6 \times 4.95} = 0.0074$$

$$m = \frac{f_y}{0.85f_c'} = \frac{45.386}{0.85 \times 4.871} = 1.097$$

Substitute into Eq. (4.1.3.e)

$$P_u = 0.85 f_c' bd \left\{ \left(-p + 1 - e'/d \right) + \sqrt{\left(1 - \frac{e'}{d}\right)^2 + 2P\left[(m-1)\left(1 - \frac{d'}{d}\right) + \frac{e'}{d} \right]} \right\}$$

$$= 0.85 \times 4.87 \times 6 \times 4.57$$

$$\left\{ -0.0074 + 1 - 4.95/4.57 + \sqrt{(1 - 4.95/4.57)^2 + 2 \times 0.0074 \left[(9.97)\left(1 - \frac{1.43}{4.57}\right) + \left(\frac{4.95}{4.57}\right) \right]} \right\}$$

$$= 28.8 \text{ kips.}$$

For NBC design load, $\phi = 0.75$

P design = 0.75 x 28.8
= 21.6 kips.

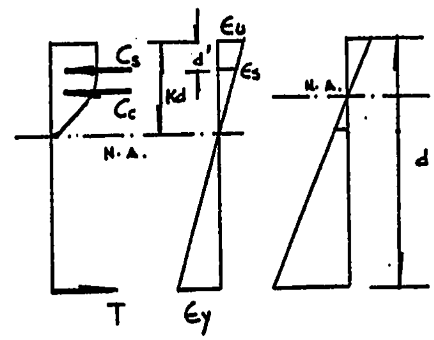
For ACI design load, $\phi = 0.7$

P design = 0.7 x 28.8
= 20.2 kips.

(B) Reinforced Concrete Columns due to Pure Bending

Example: Column 3E-5

From Strain Diagram as Shown below.



$$\epsilon_y = \epsilon_u \left(\frac{1}{K} - 1\right)$$

$$\epsilon_s = \left(\frac{Kd - d'}{Kd}\right) \epsilon_u$$

if compression steel below N.A. then

$$\epsilon_s = \left(\frac{d' - Kd}{Kd}\right) \epsilon_u$$

Given: f_c' = 3.461 ksi.

f_y = 65.525 ksi.

d = 4.69"

d' = 1.31"

$A_s = A_s'$ = 0.2 sq.in.

$e/t = \infty$

1st TRIAL :

Assume: $\begin{cases} \epsilon_u = 0.003 \\ K = 0.4 \end{cases}$

$$\therefore Kd = 0.4\ (4.69) = 1.876''$$

$$\epsilon_y = \left(\frac{4.69}{1.876} - 1\right) 0.003$$

$$= 0.003\ (1.5)$$

$$= 0.0045$$

From Stress-Strain diagram of steel

$$f_y = 65,525 \text{ psi.}$$

$$T = 2 \times 0.2 \times 65.525$$

$$= 26.20 \text{ kips.}$$

$$\epsilon_s = 0.003\ \left(\frac{1.876 - 1.31}{1.876}\right)$$

$$= 0.003\ (0.218)$$

$$= 0.000646$$

$$C_s = 20 \times 0.2 \times 2 = 8.0 \text{ k}$$

From Stress-Strain diagram of concrete

$f_c' = 2.492$ ksi.

$C_c = 0.85 f_c' a \times B$

$= 0.85 \times 0.85 \times 2.492 \times 1.876 \times 6$

$= 28.05$ k

$C_c + C_s = 28.05 + 8.0 = 34.05 \text{ k} > T = 26.20$

(NO GOOD)

2nd TRIAL :

Assume: $\begin{cases} \epsilon_u = 0.0026 \\ K = 0.34 \end{cases}$

$\therefore Kd = 1.60$

From Stress-Strain diagram of concrete, obtains,

$f_c' = 2.720$ psi.

$\epsilon_y = 0.0026 \left(\frac{1}{0.34} - 1\right) = 0.0026 \times 1.94$

$= 0.00505$

$T = 2 \times 0.2 \times 65.525 = 26.20$ k.

$$C_c = 0.72 \times 2.72 \times 1.60 \times 6 = 18.80 \text{ k.}$$

$$\epsilon_s = 0.0026 \left(\frac{1.60 - 1.31}{1.31}\right) = 0.0026\ (0.221)$$

$$= 0.000575$$

From Stress-Strain diagram of steel, gives,

$$f_s = 17{,}000 \text{ psi.}$$

$$C_s = 3.4 \times 2 = 6.8 \text{ k.}$$

$$\therefore C_c + C_s = 18.80 + 6.8 = 25.60 \doteq T = 26.20 \text{ k.}$$

For

$$P_u = \frac{6M_u}{L}$$

Take moment about the centroid of Rectangular concrete block,

$$M_u = M_1 \div M_2$$

$$M_1 = 26.20\ (d - a/2) = 26.20\ (4.69 - \frac{1.36}{2}) = 105 \text{ k}'$$

$$M_2 = 6.8\ (1.31 - 0.68) = 4.28 \text{ k}'$$

$$\therefore M_u = -4.28 + 105.0 = 100.72 \text{k}'$$

Therefore,

$$P_u = 6M/L = \frac{6 \times 100.72}{53} = 11.41 \text{ k}' \doteq 11.37 \text{ k}'$$

(O.K.)

APPENDIX D

COMPUTER PROGRAMS

PROGRAM A

PROGRAM FOR ACI DESIGN FORMULAE

PROGRAM B

PROGRAM FOR STRAIN-GRADIENT METHOD

A). PROGRAM FOR ACI DESIGN FORMULAE

The arguments used in the program are defined as follows:-

TM = m

PUA = Ultimate load of column under concentric loading

PUB = Ultimate load of column under pure bending

PUBB = Ultimate load of column under balanced condition

PUC = Ultimate load of column under eccentric loading (compression failure)

PUT = Ultimate load of column under eccentric loading (tension failure)

D = d

DP = d'

E = Initial eccentricity

EI = δ (deflection)

FY = f_y

FC = f_c'

AS = A_s

EC = e = E + δ

EB = e_b

AB = a_b

EP = e'

MUB = Balanced moment, kip-in.

$$p = p = \frac{A_s}{bd}$$

```
      C
      C
      C
      C     ******************************************************************
      C     GENERAL SOLUTIONS FOR SHORT REINFORCED CONCRETE COLUMNS
      C     SUBJECTED TO AXIAL AND BENDING---BY ACI DESIGN FORMULAE
      C     ******************************************************************
      C
      C
      C     ******************************************************************
      C     COMMON AND DIMENSION STATEMENT
      C     ******************************************************************
      C
0001        DIMENSION TM(100),PUA(100),          PUC(100),PUBB(100),PUT(100),
           &     D(100),DP(100),E(100),EI(100),FY(20),FC(20),AS(20),
           &EC(100),EB(100),AB(100),EP(100),CK(10),   P(100),PP(20),MUB(1000),
           &TT(100),CC(100),CS(100),EBB(100),ABB(100),AK(100),AKM(100),
           &AP(100),AM(100),BP(100),          CP(100),CM(100),DDP(100),BM(1000),
           &DDM(100),EEP(100),EM(100), C(1000),DK(1000),PUB(1000),
           &FORCE(100),AXP(100),AXM(100),BXP(100),BXM(100),CXP(100),CXM(100),
           &A0(20),A1(20),A2(20),A3(20),A4(20),STC(5),STA(1000),FD(1000),
           &DXP(100),DXM(100),EXP(100),EXM(100),XBM(100),FS(1000)
      C
      C     ******************************************************************
      C     INPUT DATA
      C     ******************************************************************
      C
0002        M=100
0003        N=20
0004        READ (1,100) (E(I),I=1,M)
0005        READ (1,100) (D(I),I=1,M)
0006        READ (1,100) (DP(I),I=1,M)
0007        READ (1,100) (EI(I),I=1,M)
0008        READ (1,100) (FC(J),J=1,N)
0009        READ (1,100) (FY(J),J=1,N)
0010        READ (1,100) (AS(J),J=1,N)
0011        READ (1,100) (FORCE(I),I=1,M)
0012        READ (1,209) (STC(K),K=1,5)
0013        READ (1,209) (CK(KK),KK=1,10)
0014        READ (1,200) (A0(J),J=1,N)
0015        READ (1,200) (A1(J),J=1,N)
0016        READ (1,200) (A2(J),J=1,N)
0017        READ (1,200) (A3(J),J=1,N)
```

```
  100 FORMAT (8F10.5)
  209 FORMAT (5F10.5)
  200 FORMAT (5E13.6)
      WRITE (3,909) (E(I),I=1,M)
  909 FORMAT (///,5X,10F12.5//)
      WRITE (3,908) (D(I),I=1,M)
  908 FORMAT (//,5X,10F12.5//)
      WRITE (3,908) (DP(I),I=1,M)
      WRITE (3,908) (EI(I),I=1,M)
      WRITE (3,908) (FC(J),J=1,N)
      WRITE (3,908) (FY(J),J=1,N)
      WRITE (3,908) (AS(J),J=1,N)
      WRITE (3,908) (FORCE(I),I=1,M)
      B=6.0
      T=6.0
      I=1
      DO 110 J=1,N
      DO 900 II=1,5
      WRITE (3,11) I
   11 FORMAT (I3)
      R=E(I)/T
      WRITE (3,505)
  505 FORMAT (1H1,15X,'E/D'//)
      PP(J)=0.
      PP(J)=AS(J)/(B*T)
      WRITE (3,506) R
  506 FORMAT (12X,F6.2)
      P(I)=0.
      P(I)=AS(J)/(E*D(I))
      WRITE (3,710)
  710 FORMAT (//,5X,'THE VALUE OF P'//)
      WRITE (3,720) P(I)
  720 FORMAT (12X,F17.6)
      AB(I)=0.
      AB(I)=(87.0/(87.0+FY(J)))*0.85*D(I)
      EC(I)=0.
      EC(I)=E(I)+EI(I)
      WRITE (3,410)
  410 FORMAT (//,5X,'THE VALUES OF EC')
      WRITE (3,500) EC(I)
      TM(J)=0.
      TM(J)=FY(J)/(0.85*FC(J))
```

```
      AK(I)=0.
      AK(I)=(P(I)*FY(J))/FC(J)
      AKM(I)=0.
      AKM(I)=P(I)*TM(J)
      WRITE (3,810)
  810 FORMAT (///,5X,'THE VALUES FO P*FY/FC')
      WRITE (3,500) AK(I)
      WRITE (3,820)
  820 FORMAT (///,5X,'THE VALUE OF P*M')
      WRITE (3,500) AKM(I)
      IF (E(I).LE.0.) GO TO 120
      IF (E(I).GE.  1000.) GO TO 220
      EB(I)=0.
      TT(J)=0.
      CC(I)=0.
      CS(J)=0.
      PUBB(I)=0.
      TT(J)=AS(J)*FY(J)
      CS(J)=AS(J)*(FY(J)-0.85*FC(J))
      CC(I)=0.85*FC(J)*AB(I)*B
      PUBB(I)=CC(I)+CS(J)-TT(J)
      EB(I)=
     &   (TT(J)*(T/2-DP(I))
     &+CC(I)*(T/2-AB(I)/2)
     &+CS(J)*(T/2-DP(I)))/PUBB(I)
      WRITE (3,430)
  430 FORMAT (//,5X,'THE VALVES OF EB')
      WRITE (3,500) EB(I)
C
C     ****************************************************************
C     DEFINE FAILURE CONDION
C     ****************************************************************
C
      IF (EC(I)-EB(I)) 301,302,303
  120 PUA(J)=0.
      AP(J)=0.
      AM(J)=0.
C
C     ****************************************************************
C     COMPUTE ULTIMATE LOAD DUE TO CONCENTRIC LOADING
C     ****************************************************************
C
      PUA(J)=           0.85*FC(J)*(B*T-2*AS(J))+2*AS(J)*FY(J)
```

```
      AF(J)=PUA(J)/(FC(J)*B*T)
      WRITE (3,510)
  510 FORMAT (//,5X,'THE VALUES OF PUA')
      WRITE (3,500 ) PUA(J)
  500 FORMAT (///,5X,F17.6)
      WRITE (3,560)
  560 FORMAT (///,3X,'THE VALUEOF P/PU',2X,'THE VALVE FO M/MU')
      WRITE (3,601) AP(J),AM(J)
  601 FORMAT (///,10X,2F10.2)
       AXP(I)=0.
      AXM(I)=0.
      AXP(I)=FORCE(I)/(FC(J)*B*T)
      WRITE (3,599)
  599 FORMAT (//,3X,'THE VALUE OF PEXP./PU',2X,'THE VALVE OF MEXP/PU')
      WRITE (3,601) AXP(I),AXM(I)
      GO TO 400
  220 BF(I)=0.
C
C     ******************************************************************
C     COMPUTE ULTIMATE MOMENT & LOAD DUE TO PURE BENDING
C     ******************************************************************
C
      DC 221 K=1,5
      KK=1
  281 C(KK)=CK(KK)*D(I)
      CK(KK)=0.85*C(KK)
      STA(KK)=     ABS(STC(K)*(C(KK)-DP(I))/C(KK))
      FS(KK)=        A0(J)+A1(J)*STA(KK)+A2(J)*STA(KK)**2
     &+A3(J)*STA(KK)**3+A4(J)*STA(KK)**4
      FC(KK)=        AS(J)*FS(KK)+0.85*FC(J)*B*DK(KK)-AS(J)*FY(J)
      IF (ABS(FO(KK)).GT.(0.01*AS(J)*FY(J))) GO TO 227
      MUB(KK)=AS(J)*FS(KK)*(D(I)-DP(I))
     &        +C.85*FC(J)*B*DK(KK)*(D(I)-DK(KK)/2.)
      PUB(KK)=            6.*MUB(KK)/53.
      BM(KK)=MUB(KK)/(FC(J)*B*T*T)
      WRITE (3,520)
  520 FORMAT (//,5X,'THE VALUES OF PUB')
      WRITE (3,500) PUB(KK)
      WRITE (3,560)
      WRITE (3,601) BP(I),BM(KK)
  227 KK=KK+1
      IF (KK.LE.10) GO TO 281
  221 CONTINUE
```

```
      BXP(I)=0.
      BXM(I)=0.
      XBM(I)=0.
      XBM(I)=FORCE(I)*53./6.
      BXM(I)=XBM(I)/(FC(J)*B*T*T)
      WRITE (3,599)
      WRITE (3,601) BXP(I),BXM(I)
      GO TO 400
  301 PUC(I)=0.
      CP(I)=0.
      CM(I)=C.
C
C     ****************************************************************
C     COMPUTE THE ULTIMATE LOAD DUE TO ECCENTRIC LOADING(COMP.FAILURE)
C     ****************************************************************
C
      PUC(I)=AS(J)*FY(J)/((EC(I)/(D(I)-DP(I)))+0.5)
     &+B*T*FC(J)/((3*B*EC(I)/(D(I)**2))+1.18)
      CP(I)=
     &         PUC(I)/(FC(J)*B*T)
      CM(I)=CP(I)*EC(I)/T
      WRITE (3,530)
  530 FORMAT (//,5X,'THE VALUES OF PUC')
      WRITE (3,500 )   PUC(I)
      WRITE (3,560)
      WRITE (3,601) CP(I),CM(I)
      CXP(I)=0.
      CXM(I)=0.
      CXP(I)=FORCE(I)/(FC(J)*B*T)
      CXM(I)=CXP(I)*EC(I)/T
      WRITE (3,599)
      WRITE (3,601) CXP(I),CXM(I)
      GO TO 400
  302 DCP(I)=C.
      DCM(I)=0.
C
C     ****************************************************************
C     COMPUTE THE ULTIMATE LOAD DUE TO BALANCED CONDITION
C     ****************************************************************
C
      DDP(I)=
     &         PUBB(I)/(FC(J)*B*T)
      DCM(I)=DDP(I)*EC(I)/T
```

```
  540 FORMAT (//,5X,'THE VALVES OF PUBB')
      WRITE (3,500 )  PUBB(I)
      WRITE (3,560)
      WRITE (3,601) DDP(I),DDM(I)
      DXP(I)=0.
      DXM(I)=0.
      DXP(I)=FORCE(I)/(FC(J)*B*T)
       DXM(I)=DXP(I)*EC(I)/T
      WRITE (3,599)
       WRITE (3,601) DXP(I),DXM(I)
      GC TO 400
  303 PUT(I)=C.
      EP(I)=0.
      EEP(I)=0.
      EM(I)=C.
      EP(I)=EC(I)+3.-DP(I)
C
C     *****************************************************************
C     COMPUTE THE ULTIMATE LOAD DUE TO ECCENTRIC LOADING
C                 (TENSION FAILURE)
C     *****************************************************************
C
       PUT(I)=C.85*FC(J)*B*D(I)*(-P(I)+1.0-EP(I)/D(I)
      &+SQRT((1.0-EP(I)/D(I))**2+
      &2.0*P(I)*((TM(J)-1.0)*(1.0-DP(I)/D(I))+EP(I)/D(I))))
       EEP(I)=PUT(I)/(FC(J)*B*T)
       EM(I)=EEP(I)*EC(I)/T
       WRITE (3,550)
  550 FCRMAT (//,5X,'THE VALVES OF PUT')
       WRITE (3,500 ) PUT(I)
       WRITE (3,560)
       WRITE (3,601) EEP(I),EM(I)
       EXP(I)=0.
       EXM(I)=C.
       EXP(I)=FORCE(I)/(FC(J)*B*T)
       EXM(I)=EXP(I)*EC(I)/T
       WRITE (3,599)
       WRITE (3,601) EXP(I),EXM(I)
       GC TO 400
  400 I=I+1
  900 CONTINUE
  110 CONTINUE
```

```
C192      STOP
C193      END
```

B). PROGRAM FOR STRAIN-GRADIENT METHOD

The arguments used in the program are defined as follows:-

STC = Strain of Concrete

AK = Factor

AS = Area of tension steel, sq.in.

EC = Eccentricity of load with respect to plastic centroid of section

DP = Distance from extreme compression fibre to centroid of tension reinforcement

D = Distance from extreme compression fibre to centroid of tension reinforcement

DK = Distance from extreme compression fibre to neutral axis at ultimate strength

STA = Strain in tension reinforcement

STB = Strain inccompression reinforcement

FY = Yield strength of tension reinforcement, psi.

FC = Compressive strength of concrete cylinders, psi.

FS = Tensile steel stress, psi.

AB = Depth of equivalent rectangular stress block = $K_1 c$

A0 — A4		
B1 — B4		Coefficients of equation
C1 — C4		

BM0 = Internal resisting moment, kip-in.

BM1 = Internal resisting moment in terms of the strength of steel, kip-in.

BM2 = Internal resisting moment in terms of the strength of compression steel, kip-in.

PU = P_u, Ultimate load, kips.

```
              C      CALCUTIONS OF ULTIMATE AXIAL LOAD NAD ULTIMATE MONENT
              C      FROM THE STRAIN GRACIENT METHOD  FOR REINFORCING CONCRETE
              C            COLUMNS SUBJESTED TO AXIAL AND BENDING
              C
              C
0001                 DIMENSION FORCE(50),STC(6),AK(45),AS(50),EC(50),DP(50),
                   & D(50),DK(3000),STA(3000),STB(3000),FCAL(3000),
                   & FCA(3000),FSB(3000),FSC(3000),MCAL(3000),
                   & MMAX(50),PEPS(50),MEPS(50),            FC1(3000),FC2(3000),
                   & FY(3000),FC(3000),FS(3000),AB(3000),FC3(3000),FC4(3000),
                   & A0(50),A1(50),A2(50),FC5(3000),FC6(3000),A3(50),FC10(3000),
                   & C0(50),C1(50),C2(50),C3(50),R(50),FC7(3000),FC8(3000),FC9(3000)
                   &,A4(50),        C4(50),F0(300),BM0(300),BM1(300),BM2(300),PJ(300)
0002                 REAL MCAL,MMAX,MEPS
              C
              C
0003                 READ(1,100) (FORCE(I),I=1,35)
0004             100 FORMAT(8F10.5)
0005                 READ(1,100) (D(I),I=1,35)
0006                 READ (1,100) (EC(I),I=1,35)
0007                 READ (1,100) (AS(I),I=1,35)
0008                 READ (1,100) (DP(I),I=1,35)
0009                 READ (1,100) (AK(J),J=1,42)
0010                 READ (1,100) (STC(K),K=1,6)
0011                 READ (1,100) (R(I),I=1,35)
0012                 READ (1,299) (A0(I),I=1,35)
0013             299 FORMAT (5E13.6)
0014                 READ (1,299) (A1(I),I=1,35)
0015                 READ (1,299) (A2(I),I=1,35)
0016                 READ (1,299) (A3(I),I=1,35)
0017                 READ (1,299) (A4(I),I=1,35)
0018                 READ (1,200) (C0(I),I=1,35)
0019                 READ (1,200) (C1(I),I=1,35)
0020                 READ (1,200) (C2(I),I=1,35)
0021                 READ (1,200) (C3(I),I=1,35)
0022                 READ (1,200) (C4(I),I=1,35)
0023             200 FORMAT (6E13.6)
0024                 WRITE (3,208)
0025             208 FORMAT(///,20X,'FORCE')
0026                 WRITE (3,202) (FORCE(I),I=1,35)
0027             202 FORMAT (///,2X,8F16.6)
0028                 WRITE(3,201)
0029             201 FORMAT (///,2X,'THEVALVES OF D(I)'/)
```

```
      WRITE (3,202) (D(I),I=1,35)
      WRITE(3,204)
  204 FORMAT (///,2X,'THE VALVES OF EC')
      WRITE (3,202) (EC(I),I=1,35)
      WRITE (3,205)
  205 FORMAT (///,20X,'AS')
      WRITE (3,202) (AS(I),I=1,35)
      WRITE(3,601)
  601 FORMAT (///,20X,'DP')
      WRITE (3,202) (DP(I),I=1,35)
      WRITE (3,206)
  206 FORMAT (///,20X,'AK')
      WRITE (3,202) (AK(I),I=1,42)
      WRITE (3,207)
  207 FORMAT (///,20X,'STC.')
      WRITE (3,202) (STC(K),K=1,6)
      WRITE (3,302)
  302 FORMAT (///,20X,'A0')
      WRITE(3,509) (A0(I),I=1,35)
      WRITE(3,303)
  303 FORMAT (///,'A1')
      WRITE(3,509) (A1(I),I=1,35)
      WRITE(3,304)
  304 FORMAT (///,'A2')
      WRITE(3,509) (A2(I),I=1,35)
      WRITE(3,305)
  305 FORMAT (///,'A3')
      WRITE(3,509) (A3(I),I=1,35)
      WRITE(3,306)
  306 FORMAT (///,'A4')
      WRITE(3,509) (A4(I),I=1,35)
      WRITE(3,406)
  406 FORMAT (///,'C0')
      WRITE(3,509) (C0(I),I=1,35)
      WRITE(3,407)
  407 FORMAT (///,'C1')
      WRITE(3,509) (C1(I),I=1,35)
      WRITE(3,408)
  408 FORMAT (///,'C2')
      WRITE(3,509) (C2(I),I=1,35)
      WRITE(3,409)
  409 FORMAT (///,'C3')
      WRITE(3,509) (C3(I),I=1,35)
```

```
      WRITE(3,501)
  501 FORMAT (///,'C4')
      WRITE(3,509) (C4(I),I=1,35)
  509 FORMAT (///,2X,8E13.6/)
C
C
      B=6.
      T=6.
C
C
      DO 1000 I=26,35
      PEPS(I)=0.
      PEPS(I)=PEPS(I)   +0.01*FORCE(I)
      MMAX(I)=0.
      MMAX(I)=MMAX(I)+FORCE(I)*EC(I)
      MEPS(I)=0.
      MEPS(I)=MEPS(I)   +0.01*MMAX(I)
C
C
      WRITE (3,19)
   19 FORMAT (///,5X,'E/D')
      WRITE (3,29) R(I)
   29 FORMAT (//,F8.4)
      DO 999 K=1,6
      J=1
   61 IF (STC(K).LT.0.0001) GO TO 999
      IF (STC(K).GT.0.003) GO TO 999
   26  IF (J.GT.42) GO TO 999
      DK(J)=AK(J)*D(I)
      STA(J)=0.
      STB(J)=0.
      FCA(J)=0.
      FSB(J)=0.
      FSC(J)=0.
      FCAL(J)=0.
      MCAL(J)=0.
      FO(J)=0.
      BMO(J)=0.
      BM1(J)=0.
      BM2(J)=0.
      PU(J)=0.
      AB(J)=0.
      FY(J)=0.
```

```
      FS(J)=0.
      FC(J)=0.
      AB(J)=0.85*DK(J)
      IF (DK(J)-D(I)) 50,150,250
C
   50 IF (DK(J)-DP(I)) 10,20,30
   10 STA(J)=STA(J)+STC(K)*(1./AK(J)-1.)
      STB(J)=STB(J)+STC(K)*(DP(I)-DK(J))/DK(J)
      IF (STA(J).GT.0.1  .OR.  STB(J).GT.0.1) GO TO 101
C
      FY(J)=FY(J)+A0(I)+A1(I)*STA(J)+A2(I)*STA(J)**2
     &+A3(I)*STA(J)**3+A4(I)*STA(J)**4
      FS(J)=FS(J)+A0(I)+A1(I)*STB(J)+A2(I)*STB(J)**2
     &+A3(I)*STB(J)**3+A4(I)*STB(J)**4
      FC1(J)= C0(I)+C1(I)*19/20*STC(K)+C2(I)*(19/20*STC(K))**2
     &+C3(I)*(19/20*STC(K))**3+C4(I)*(19/20*STC(K))**4
      FC2(J)= C0(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2
     &+C3(I)*(17/20*STC(K))**3+C4(I)*(17/20*STC(K))**4
      FC3(J)= C0(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2
     &+C3(I)*(15/20*STC(K))**3+C4(I)*(15/20*STC(K))**4
      FC4(J)= C0(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2
     &+C3(I)*(13/20*STC(K))**3+C4(I)*(13/20*STC(K))**4
      FC5(J)= C0(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2
     &+C3(I)*(11/20*STC(K))**3+C4(I)*(11/20*STC(K))**4
      FC6(J)= C0(I)+C1(I)* 9/20*STC(K)+C2(I)*( 9/20*STC(K))**2
     &+C3(I)*( 9/20*STC(K))**3+C4(I)*( 9/20*STC(K))**4
      FC7(J)= C0(I)+C1(I)* 7/20*STC(K)+C2(I)*( 7/20*STC(K))**2
     &+C3(I)*( 7/20*STC(K))**3+C4(I)*( 7/20*STC(K))**4
      FC8(J)= C0(I)+C1(I)* 5/20*STC(K)+C2(I)*( 5/20*STC(K))**2
     &+C3(I)*( 5/20*STC(K))**3+C4(I)*( 5/20*STC(K))**4
      FC9(J)= C0(I)+C1(I)* 3/20*STC(K)+C2(I)*( 3/20*STC(K))**2
     &+C3(I)*( 3/20*STC(K))**3+C4(I)*( 3/20*STC(K))**4
      FC10(J)=C0(I)+C1(I)* 1/20*STC(K)+C2(I)*( 1/20*STC(K))**2
     &+C3(I)*( 1/20*STC(K))**3+C4(I)*( 1/20*STC(K))**4
      FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)
     &+FC8(J)+FC9(J)+FC10(J))
C
      FCA(J)=FC(J)*B*DK(J)
      FSB(J)=FSB(J)+AS(I)*FY(J)
      FSC(J)=FSC(J)+AS(I)*FS(J)
      WRITE (3,7)
    7 FORMAT (//,10X,'FSB',10X,'FSC',10X,'FCA')
       WRITE (3,9) FSB(J),FSC(J),FCA(J)
```

```
    5 FORMAT (     3F17.6)
      IF (EC(I).LT.1000.) GO TO 310
      FO(J)=FO(J)+FSC(J)+FSB(J)-FCA(J)
      IF (ABS(FO(J)).LE.(  0.01*FCA(J))) GO TO 110
      J=J+1
        GO TO 26
C
  110 BM1(J)=BM1(J)+FSB(J)*(D(I)-AB(J)/2)
      BM2(J)=BM2(J)+FSC(J)*(DP(I)-AB(J)/2)
       BMO(J)=BMO(J)+BM1(J)+BM2(J)
      PU(J)=PU(J)+6.*BMO(J)/53.
      IF (ABS(PU(J)-FORCE(I)).LE.PEPS(I)) GO TO 210
      J=J+1
      GO TO 26
C
  210 WRITE (3,39)
   39 FORMAT (///,2X,'I',5X,'J',5X,'K',
     &5X,'FORCE',5X,'CAL.PU',5X, 'PURE BENDING MU')
      WRITE (3,500) I,J,K,FORCE(I),PU(J),BMO(J)
      GO TO 999
  310 FCAL(J)=FCAL(J)+FCA(J)-FSC(J)-FSB(J)
      MCAL(J)=MCAL(J)+FSB(J)*(T/2-DP(I))
     &-FSC(J)*(T/2-DP(I))
     &+FCA(J)*(T/2-AB(J)/2)
       GO TO 300
C
C
   20 STA(J)=STA(J)+STC(K)*(1./AK(J)-1.)
       STB(J)=STB(J)
      IF (STA(J).GT.0.1  .OR.  STB(J).GT.0.1) GO TO 101
C
       FY(J)=FY(J)+AO(I)+A1(I)*STA(J)+A2(I)*STA(J)**2
     &+A3(I)*STA(J)**3+A4(I)*STA(J)**4
       FS(J)=FS(J)+AO(I)+A1(I)*STB(J)+A2(I)*STB(J)**2
     &+A3(I)*STB(J)**3+A4(I)*STB(J)**4
       FC1(J)= CO(I)+C1(I)*19/20*STC(K)+C2(I)*(19/20*STC(K))**2
     &+C3(I)*(19/20*STC(K))**3+C4(I)*(19/20*STC(K))**4
       FC2(J)= CO(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2
     &+C3(I)*(17/20*STC(K))**3+C4(I)*(17/20*STC(K))**4
       FC3(J)= CO(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2
     &+C3(I)*(15/20*STC(K))**3+C4(I)*(15/20*STC(K))**4
       FC4(J)= CO(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2
     &+C3(I)*(13/20*STC(K))**3+C4(I)*(13/20*STC(K))**4
```

```
0166      FC5(J)= CO(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2
         &+C3(I)*(11/20*STC(K))**3+C4(I)*(11/20*STC(K))**4
0167      FC6(J)= CO(I)+C1(I)* 9/20*STC(K)+C2(I)*( 9/20*STC(K))**2
         &+C3(I)*( 9/20*STC(K))**3+C4(I)*( 9/20*STC(K))**4
0168      FC7(J)= CO(I)+C1(I)* 7/20*STC(K)+C2(I)*( 7/20*STC(K))**2
         &+C3(I)*( 7/20*STC(K))**3+C4(I)*( 7/20*STC(K))**4
0169      FC8(J)= CO(I)+C1(I)* 5/20*STC(K)+C2(I)*( 5/20*STC(K))**2
         &+C3(I)*( 5/20*STC(K))**3+C4(I)*( 5/20* STC(K))**4
0170      FC9(J)= CO(I)+C1(I)* 3/20*STC(K)+C2(I)*( 3/20*STC(K))**2
         &+C3(I)*( 3/20*STC(K))**3+C4(I)*( 3/20*STC(K))**4
0171      FC10(J)=CO(I)+C1(I)* 1/20*STC(K)+C2(I)*( 1/20*STC(K))**2
         &+C3(I)*( 1/20*STC(K))**3+C4(I)*( 1/20*STC(K))**4
0172      FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)
         &+FC8(J)+FC9(J)+FC10(J))
     C
0173      FCA(J)=FC(J)*B*DK(J)
0174      FSB(J)=FSB(J)+AS(I)*FY(J)
0175      FSC(J)=0.
0176      IF  (EC(I).LT.1000.) GO TO 330
0177      FO(J)=FO(J)+FSB(J)-FCA(J)+FSC(J)
0178      IF  (ABS(FO(J)).LE.(  0.01*FCA(J))) GO TO 130
0179      J=J+1
0180      GO TO 26
     C
     C
0181  130  BM1(J)=BM1(J)+FSB(J)*(D(I)-AB(J)/2)
0182      BM2(J)=BM2(J)+FSC(J)*(DP(I)-AB(J)/2)
0183      BMO(J)=BMO(J)+BM1(J)+BM2(J)
0184      PU(J)=PU(J)+6.*BMO(J)/53.
0185      IF  (ABS(PU(J)-FORCE(I)).LE.PEPS(I) ) GO TO 210
0186       J=J+1
0187      GO TO 26
     C
     C
0188  330  FCAL(J)=FCAL(J)+FCA(J)-FSC(J)-FSB(J)
0189       MCAL(J)=MCAL(J)+FSB(J)*(T/2-DP(I))
         &-FSC(J)*(T/2-DP(I))
         &+FCA(J)*(T/2-AB(J)/2)
0190        GO TO 300
     C
     C
0191   30  STA(J)=STA(J)+STC(K)*D(I)/DK(J)-STC(K)
0192       STB(J)=STB(J)+((DK(J)-DP(I))*STC(K))/DK(J)
```

```
0193          IF (STA(J).GT.0.1  .OR.  STB(J).GT.0.1) GO TO 101
      C
      C
0194          FY(J)=FY(J)+AO(I)+A1(I)*STA(J)+A2(I)*STA(J)**2
             &+A3(I)*STA(J)**3+A4(I)*STA(J)**4
0195          FS(J)=FS(J)+AO(I)+A1(I)*STB(J)+A2(I)*STB(J)**2
             &+A3(I)*STB(J)**3+A4(I)*STB(J)**4
0196          FC1(J)= CO(I)+C1(I)*19/20*STC(K)+C2(I)*(19/20*STC(K))**2
             &+C3(I)*(19/20*STC(K))**3+C4(I)*(19/20*STC(K))**4
0197          FC2(J)= CO(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2
             &+C3(I)*(17/20*STC(K))**3+C4(I)*(17/20*STC(K))**4
0198          FC3(J)= CO(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2
             &+C3(I)*(15/20*STC(K))**3+C4(I)*(15/20*STC(K))**4
0199          FC4(J)= CO(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2
             &+C3(I)*(13/20*STC(K))**3+C4(I)*(13/20*STC(K))**4
0200          FC5(J)= CO(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2
             &+C3(I)*(11/20*STC(K))**3+C4(I)*(11/20*STC(K))**4
0201          FC6(J)= CO(I)+C1(I)* 9/20*STC(K)+C2(I)*( 9/20*STC(K))**2
             &+C3(I)*( 9/20*STC(K))**3+C4(I)*( 9/20*STC(K))**4
0202          FC7(J)= CO(I)+C1(I)* 7/20*STC(K)+C2(I)*( 7/20*STC(K))**2
             &+C3(I)*( 7/20*STC(K))**3+C4(I)*( 7/20*STC(K))**4
0203          FC8(J)= CO(I)+C1(I)* 5/20*STC(K)+C2(I)*( 5/20*STC(K))**2
             &+C3(I)*( 5/20*STC(K))**3+C4(I)*( 5/20*STC(K))**4
0204          FC9(J)= CO(I)+C1(I)* 3/20*STC(K)+C2(I)*( 3/20*STC(K))**2
             &+C3(I)*( 3/20*STC(K))**3+C4(I)*( 3/20*STC(K))**4
0205          FC10(J)=CO(I)+C1(I)* 1/20*STC(K)+C2(I)*( 1/20*STC(K))**2
             &+C3(I)*( 1/20*STC(K))**3+C4(I)*( 1/20*STC(K))**4
0206          FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)
             &+FC8(J)+FC9(J)+FC10(J))
      C
0207          FCA(J)=FC(J)*B*DK(J)
0208          FSB(J)=FSB(J)+FY(J)*AS(I)
0209          FSC(J)=FSC(J)+FS(J)*AS(I)
      C
0210           IF (EC(I).LT.1000.) GO TO 630
0211          FO(J)=FO(J)+FSB(J)-FCA(J)-FSC(J)
0212          IF (ABS(FO(J)).LE.(  0.01*FSB(J))) GO TO 430
0213           J=J+1
0214          GO TO 26
      C
0215      430 BM1(J)=BM1(J)+(FSB(J)-FSC(J))*(D(I)-AB(I)/2)
0216          BM2(J)=BM2(J)+FSC(J)*(D(I)-DP(I))
0217           BMO(J)=BMO(J)+BM1(J)+BM2(J)
```

```
      PU(J)=PU(J)+6.*BMO(J)/53.
      IF (ABS(PU(J)-FORCE(I)).LE.PEPS(I)) GO TO 210
      J=J+1
      GO TO 26
C
  630 FCAL(J)=FCAL(J)+FCA(J)-FSB(J)+FSC(J)
      MCAL(J)=MCAL(J)+FCA(J)*(T/2-AB(J)/2)
     & +FSC(J)*(T/2-DP(I))
     &+FSB(J)*(T/2-DP(I))
      GO TO 300
C
C
  150 STA(J)=STA(J)
      STB(J)=STB(J)+(STC(K)/DK(J))*(DK(J)-DP(I))
      IF (STA(J).GT.0.1  .OR.  STB(J).GT.0.1) GO TO 101
C
C
      FY(J)=FY(J)+A0(I)+A1(I)*STA(J)+A2(I)*STA(J)**2
     &+A3(I)*STA(J)**3+A4(I)*STA(J)**4
      FS(J)=FS(J)+A0(I)+A1(I)*STB(J)+A2(I)*STB(J)**2
     &+A3(I)*STB(J)**3+A4(I)*STB(J)**4
      FC1(J)= C0(I)+C1(I)*19/20*STC(K)+C2(I)*(19/20*STC(K))**2
     &+C3(I)*(19/20*STC(K))**3+C4(I)*(19/20*STC(K))**4
      FC2(J)= C0(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2
     &+C3(I)*(17/20*STC(K))**3+C4(I)*(17/20*STC(K))**4
      FC3(J)= C0(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2
     &+C3(I)*(15/20*STC(K))**3+C4(I)*(15/20*STC(K))**4
      FC4(J)= C0(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2
     &+C3(I)*(13/20*STC(K))**3+C4(I)*(13/20*STC(K))**4
      FC5(J)= C0(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2
     &+C3(I)*(11/20*STC(K))**3+C4(I)*(11/20*STC(K))**4
      FC6(J)= C0(I)+C1(I)* 9/20*STC(K)+C2(I)*( 9/20*STC(K))**2
     &+C3(I)*( 9/20*STC(K))**3+C4(I)*( 9/20*STC(K))**4
      FC7(J)= C0(I)+C1(I)* 7/20*STC(K)+C2(I)*( 7/20*STC(K))**2
     &+C3(I)*( 7/20*STC(K))**3+C4(I)*( 7/20*STC(K))**4
      FC8(J)= C0(I)+C1(I)* 5/20*STC(K)+C2(I)*( 5/20*STC(K))**2
     &+C3(I)*( 5/20*STC(K))**3+C4(I)*( 5/20*STC(K))**4
      FC9(J)= C0(I)+C1(I)* 3/20*STC(K)+C2(I)*( 3/20*STC(K))**2
     &+C3(I)*( 3/20*STC(K))**3+C4(I)*( 3/20*STC(K))**4
      FC10(J)=C0(I)+C1(I)* 1/20*STC(K)+C2(I)*( 1/20*STC(K))**2
     &+C3(I)*( 1/20*STC(K))**3+C4(I)*( 1/20*STC(K))**4
      FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)
     &+FC8(J)+FC9(J)+FC10(J))
```

```
      C
0241        FSB(J)=FSB(J)
0242        FSC(J)=FSC(J)+FS(J)*AS(I)
0243        FCA(J)=FC(J)*B*DK(J)
      C
0244        FCAL(J)=FCAL(J)+FCA(J)+FSC(J)
0245        MCAL(J)=MCAL(J)+FCA(J)*(T/2-AB(J)/2)
           &+FSC(J)*(T/2-DP(I))
0246        GO TO 300
      C
      C
0247    250 STA(J)=STA(J)+(STC(K)/DK(J))*(DK(J)-D(I))
0248        STB(J)=STB(J)+(STC(K)/DK(J))*(DK(J)-DP(I))
0249        IF (STA(J).GT.0.1  .CR.  STB(J).GT.0.1) GO TO 101
      C
      C
0250        FY(J)=FY(J)+A0(I)+A1(I)*STA(J)+A2(I)*STA(J)**2
           &+A3(I)*STA(J)**3+A4(I)*STA(J)**4
0251        FS(J)=FS(J)+A0(I)+A1(I)*STB(J)+A2(I)*STB(J)**2
           &+A3(I)*STB(J)**3+A4(I)*STB(J)**4
0252        FC1(J)= C0(I)+C1(I)*19/20*STC(K)+C2(I)*(19/20*STC(K))**2
           &+C3(I)*(19/20*STC(K))**3+C4(I)*(19/20*STC(K))**4
0253        FC2(J)= C0(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2
           &+C3(I)*(17/20*STC(K))**3+C4(I)*(17/20*STC(K))**4
0254        FC3(J)= C0(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2
           &+C3(I)*(15/20*STC(K))**3+C4(I)*(15/20*STC(K))**4
0255        FC4(J)= C0(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2
           &+C3(I)*(13/20*STC(K))**3+C4(I)*(13/20*STC(K))**4
0256        FC5(J)= C0(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2
           &+C3(I)*(11/20*STC(K))**3+C4(I)*(11/20*STC(K))**4
0257        FC6(J)= C0(I)+C1(I)* 9/20*STC(K)+C2(I)*( 9/20*STC(K))**2
           &+C3(I)*( 9/20*STC(K))**3+C4(I)*( 9/20*STC(K))**4
0258        FC7(J)= C0(I)+C1(I)* 7/20*STC(K)+C2(I)*( 7/20*STC(K))**2
           &+C3(I)*( 7/20*STC(K))**3+C4(I)*( 7/20*STC(K))**4
0259        FC8(J)= C0(I)+C1(I)* 5/20*STC(K)+C2(I)*( 5/20*STC(K))**2
           &+C3(I)*( 5/20*STC(K))**3+C4(I)*( 5/20*STC(K))**4
0260        FC9(J)= C0(I)+C1(I)* 3/20*STC(K)+C2(I)*( 3/20*STC(K))**2
           &+C3(I)*( 3/20*STC(K))**3+C4(I)*( 3/20*STC(K))**4
0261        FC10(J)=C0(I)+C1(I)* 1/20*STC(K)+C2(I)*( 1/20*STC(K))**2
           &+C3(I)*( 1/20*STC(K))**3+C4(I)*( 1/20*STC(K))**4
0262        FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)
           &+FC8(J)+FC9(J)+FC1C(J))
      C
```

```
0263          IF (T.LE.AB(J)) AB(J)=T
0264          FSB(J)=FSB(J)+FY(J)*AS(I)
0265          FSC(J)=FSC(J)+FS(J)*AS(I)
0266          FCA(J)=FC(J)*B*DK(J)
        C
0267          FCAL(J)=FCAL(J)+FCA(J)+FSC(J)+FSB(J)
0268          MCAL(J)=MCAL(J)+FCA(J)*(T/2-AB(J)/2)
             &-FSB(J)*(T/2-DP(I))
             &+FSC(J)*(T/2-DP(I))
0269          GO TO 300
        C
        C
0270      300 IF (ABS(FCAL(J)-FORCE(I)).LE.PEPS(I)) GO TO 60
        C
0271      1C1 J=J+1
0272          GO TO 26
        C
        C
0273      60  WRITE (3,59)
0274      59  FORMAT(///,2X,'I',5X,'J',5X,'K',5X,'FORCE',
             &5X,'CAL.PU',5X,'CAL.MU')
0275          WRITE (3,500) I,J,K,FORCE(I),FCAL(J),MCAL(J)
0276      500 FORMAT (///,2X,3(I3,5X),3(F17.6)/)
        C
0277          WRITE (3,69)
0278      69  FORMAT (///,2X,'I',5X,'J',5X,'K',5X,'CAL.MU',5X,'EXTERNAL MU',
             &2X,'STC')
0279          WRITE (3,500) I,J,K,MCAL(J),MMAX(I),STC(K)
0280          WRITE (3,79)
0281      79  FORMAT (///,2X,'DK',5X,'D',5X,'STA',5X,
             &'STB',5X,'FCA',5X,'FSB',5X,'FSC')
0282          WRITE (3,600) DK(J),D(I),STA(J),STB(J),FCA(J),
             &FSB(J),FSC(J)
0283      600 FORMAT (///,5X,7F17.6)
        C
0284      999 CONTINUE
0285     1000 CONTINUE
C286          RETURN
0287          END
```