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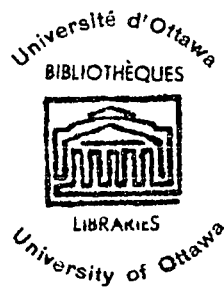
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STATISTICAL VARIATION OF REINFORCED
CONCRETE STRUCTURE STRENGTH

by

WAI - YUEN CHOW



A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Applied Science in the Faculty
of Graduate Studies of the University of
Ottawa, Ottawa, Canada.

December, 1969.

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ABSTRACT

This investigation involved the fabrication and testing to failure of a large number of nominally identical reinforced concrete members such as 20 simple beams, 20 two-span continuous beam and 20 columns. The simple beams were of rectangular section 5" wide by 8" deep with 2-#5 bars as tension reinforcement only. The simple beams were loaded to failure using equal loads at the third points. The two-span continuous beams, also of rectangular section 5" x 8" had both tension (2-#5 bars) and compression (2-#5 bars) reinforcement, equal loads were applied at the centre of each span. The column was rectangular in section (6" x 6") with symmetrical reinforcement of 4-#5 bars. The column was loaded to failure by an eccentric load.

The distribution of load carrying capacity of the members was determined by finding the joint probability of a given concrete strength and steel yield stress occurring and calculating the ultimate load using a prediction formulae.

From the comparison of the results, it was found that the experimentally determined mean strength of the beams and columns were in good agreement with the ultimate loads which were calculated from the prediction formulae of the National Building Code of Canada and the building code requirement of the American Concrete Institute.

From the predicted distribution of ultimate load capacity the probability of the ultimate load being less than the design load was determined. Using the ϕ factors recommended by the NBC, the probability of failure for the columns was less than 0.01% and 7% for the beams,

ACKNOWLEDGMENT

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NOTATION

- A_s = Area of tension steel, sq. inc.
 A_s' = Area of compression steel, sq. inc.
 a = Depth of equivalent rectangular stress block = $k_1 c$
 a_b = Depth of equivalent rectangular stress block = $k_1 c_b$
 b = Width of rectangular member
 c = Distance from extreme compression fibre to neutral axis at ultimate strength
 c_b = Distance from extreme compression fibre to neutral axis for balanced condition
 $\quad = d(87,000)/(87,000 - f_y)$
 d = Distance from extreme compression fibre to centroid of tension reinforcement
 d' = Distance from extreme compression fibre to centroid of compression reinforcement
 d'' = Distance from plastic centroid to centroid of tension reinforcement
 ϵ_u = Ultimate concrete strain in flexure
 ϵ_y = Strain in reinforcement
 E_s = Modulus of elasticity of reinforcement
 E_c = Modulus of elasticity of concrete
 e' = Eccentricity of load with respect to centroid of tension reinforcement
 e = Eccentricity of load with respect to plastic centroid of section

e_b = Eccentricity of load P_b measured from plastic centroid of section

f_c' = Compressive strength of 6" x 12" concrete cylinders, psi

f_y = Yield strength of reinforcement steel, psi

k_1 = The ratio of average stress to maximum stress

k_2 = The ratio of depth to resultant of compressive stress

k_3 = The ratio of maximum stress to cylinder strength

L = Span, ft.

M_{u1} = Moment at yielding of tension reinforcement at midspan, ft-kips

M_{u2} = Moment at yielding of tension reinforcement at midsupport, ft-kips

M_u = Ultimate moment, kip - in.

P_u = Ultimate load, Kips

$$p = \frac{A_s}{bd}$$

$$p' = \frac{A_s'}{bd}$$

P_b = Axial load capacity at simultaneous crushing of concrete and yielding of tension steel (balanced conditions)

p_b = Reinforcement ratio producing balanced conditions at ultimate strength

$$q = \frac{A_s f_y}{b d f_c'}$$

$\bar{x}$ = The average strength of all test specimen's strength

σ = Standard deviation

V = Coefficient of variation

N_1, N_2 = Number of samples

t = A constant depending upon the proportion of tests that may fall below f'_c and the number of samples

v = Degree of freedom

Chapter 1.

INTRODUCTION

The construction industry is one of the major industries in Canada, as well as its products having a major impact on the lives of both today's and tomorrow's citizens. A large portion of major construction uses reinforced concrete as its structural medium. Any improvement in the accuracy of the methods of analysis would result in greater economy, or more consistent factors of safety.

Fortunately in reinforced concrete construction, the ultimate load carrying capacity of any structural member is not in general, equal to the design value. This is due to inaccuracies in the prediction formulae, variations in quality of workmanship employed, and the fact that the strength of steel and concrete are statistical variables.

Naturally, as the properties of both steel and concrete are statistically variable, and the quality of workmanship is a random variable, then the load capacity of a reinforced concrete member is a statistically varying phenomena.

This project involved the fabrication and testing to failure of a number of nominally identical reinforced concrete members. From the measured experimentally determined failure loads, the mean value and standard deviation of the failure loads were determined. The reinforced concrete members chosen were a simple beam, a two-span continuous beam, and a column. Using the measured statistical variation of the concrete strength and the steel strength, the theoretical

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statistical properties were calculated. The experimental and theoretical characteristics were compared with each other and with the design value, using the method recommended by the National Building Code of Canada 1965.

As the load carrying capacity of any reinforced concrete member is a random variable, then there is a definite probability that the strength of any single member may fall below a specified value. The load imposed upon any structural member may also be a random variable, and a probability exists that the load may be greater than specified value. Naturally if the high load and the low strength occur together, then the member will fail. This is shown graphically in Figure (1-1).

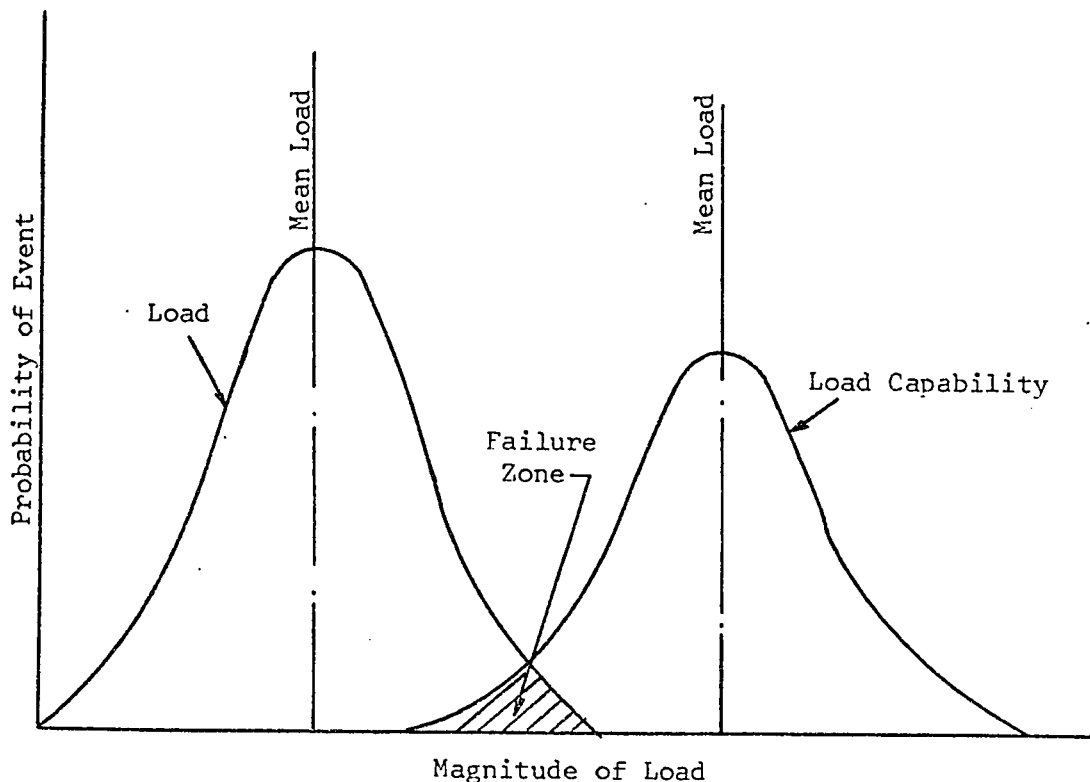


Figure (1-1). Probability Distribution of Load and Strength.

3.

The complete investigation to determine the probability of structural failure is extremely difficult, as obtaining data on the load spectrum for a given type of structural member is expensive and time consuming. This project only involved determining if the load capacity distribution could be found by calculation from a knowledge of the statistical properties of the concrete, steel, and the member geometry.

1-1 Object of Investigation

The object of this investigation was to determine experimentally the ultimate loads of a number of nominally identical simple beams, continuous beams and columns. The experimentally determined mean failure loads and the standard deviation were to be calculated. They were compared with the calculated mean and standard deviation, using the prediction formulae and the measured material properties.

From the comparison of the results, it would be possible to obtain the accuracy of the prediction formulae and the probability of a member failing at a load lower than the design load.

Chapter 2.

PREVIOUS RESEARCH

The analysis of reinforced concrete members has been attempted in two distinct ways: elastic analysis which assumes that the concrete is a linear elastic material and ultimate strength analysis.

The elastic analysis of reinforced concrete members assumes that the concrete is linear and elastic, and stress is proportional to strain. The analysis is carried out in the conventional manner for a reinforced concrete member.

The ultimate strength design method is fundamentally based on the geometry of the nonlinear concrete stress block in compression zone of reinforced concrete member. It is clear, however, that at or near the ultimate load stresses are no longer proportional to strains, thus, the distribution of concrete stress in the compression zone of reinforced concrete member subject to flexure is of fundamental importance in theories regarding the ultimate strength and behaviour of such structural members.

An nonlinear theory based upon a parabolic stress distribution was presented by Ritter^{(1)*} in 1899. Talbot's work^{(2)*} in 1904 also presented a flexural theory based upon a parabolic stress distribution for reinforced concrete beams. Suenson⁽³⁾ appears to have pioneered the use of a rectangular stress block for rectangular beams and slabs in 1912. Stussi⁽⁴⁾ 1932 suggested that the stress block could be characterised by two constants k_1 and k_2 (the ratio of average stress to maximum stress, and the ratio of depth to resultant of compressive stress

* Number in paranthesis refer to the list of references appended to this thesis.

and depth to neutral axis respectively).

Among all the theories relating stress distribution to ultimate strength that due to Whitney is the outstanding one. Whitney suggested that the stress distribution in a reinforced concrete beam at failure has the shape of the cylinder stress-strain curve (see Figure (2-1)).

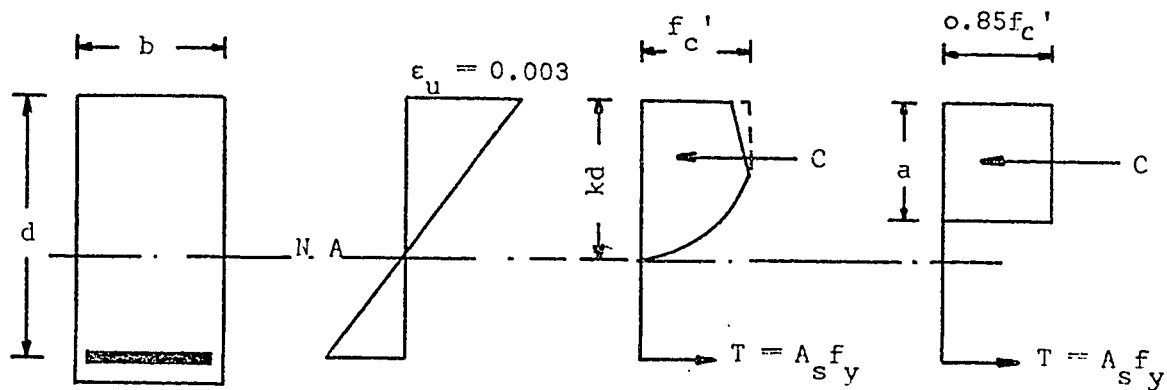


Figure (2-1). The Whitney Compressive Stress Block.

Whitney further suggested that the actual stress distribution could be replaced by an equivalent rectangular stress block with a stress level equal to $0.85 f'_c$ and a depth equal to "a". The location of the centroid of this rectangle corresponds closely to that of the actual area. The depth "a" is obtained by equilibrium considerations of the section and a knowledge of the material properties. The analysis of tests on 364 under-reinforced concrete beams showed good agreement with the results predicted using Whitney's assumptions.

A study by McMillan (5) in 1921 showed that elastic analysis

could not predict the stress in reinforced concrete columns, this led to a large research program on reinforced concrete columns carried out by Richart, Lyse, and Slater^{(6), (7), and (8)}.

In the Building Codes, both the ACI (American Concrete Institute Building Codes Requirements 318-63) and the NBC (National Building Code of Canada 1965) have tried to take account of the statistical variation in the mechanical properties of steel and concrete. The ACI 318-63 and NBC (1965) Codes use a ϕ factor to reduce the predicted actual load capacity of a reinforced concrete member, to obtain a safe consistent design capacity. These ϕ factors are given different values for different members depending upon the importance of the member. The ϕ factors specified are: $\phi = 0.90$ for flexure members, $\phi = 0.85$ for diagonal tension, bond and anchorage, $\phi = 0.75$ for spirally reinforced concrete column, and $\phi = 0.70$ for a tied column. A lower factor is used for columns because the consequences of a column failure in a multi-story building would be more catastrophic than of that of a single beam in a floor system of the same building.

According to the working stress design method. A maximum service load is specified as the design load. Elastic theory is used in the computation of internal stresses, which must be less than specified allowable values. In following such a procedure, a margin of safety is provided in the allowable stress. While no allowance is made directly for the possibility of abnormally heavy loads. In ultimate strength design, ultimate stresses are used instead of allowable stresses. To provide a margin of safety, especially to take care of extra heavy loads, it is necessary that the specified service loads be multiplied by load

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factors and the products be used in design. Both the ACI (1963) and the NBC (1965) Building Codes specify load factors to be used for each type of loading taken either singly or in combination with other loads. For example if only live load and dead load act upon a member the live load must be multiplied by 1.8 and the estimated dead by 1.5. Thus the ultimate design load is equal to:-

Ultimate design load = $1.5 \times \text{dead load} + 1.8 \times \text{live load}$.

The load factors for other combinations of load can be found by reference to the appropriate building code.

Chapter 3.

Theoretical Investigation

Two methods exist for designing reinforced concrete members: working stress design and ultimate strength design. The working stress design method assumes that the concrete component behaves in a linear elastic manner. This is obviously not correct and the method is gradually being discarded in favour of the ultimate strength design method. The ultimate strength method considers the equilibrium of the section at failure. Only ultimate strength methods of analysis are considered here.

3-1 Ultimate Load Capacity of Simple Beam

The maximum load capacity of a structural member is achieved when the member degenerates into a mechanism. A two point load simple beam collapses by forming a continuous hinge between the loads as shown in Figure (3-1).

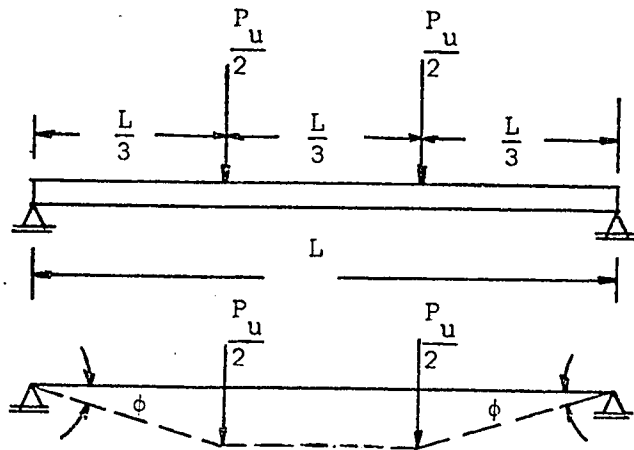


Figure (3-1). Simple Beam's Collapse Mechanism.

Using the equilibrium method the ultimate load can be found to be;

$$P_u = \frac{6M_u}{L}$$

It is generally assumed that the strains in a reinforced concrete member under flexure are linear over the depth of the section. A knowledge of the distribution of concrete stress, or the geometry of the stress block is fundamental to the theories which calculate the strength and behaviour of such members.

The properties of the inelastic stress distribution are defined by the coefficients, (see Figure (3-2)), Hognested⁽⁹⁾; k_1 , k_2 , k_3 - the ratio of average stress to maximum stress, the ratio of depth to resultant of compressive stress and the depth to neutral axis, and the ratio of maximum flexural stress to cylinder compressive strength (f'_c) respectively. These quantities are shown in Figure (3-2).

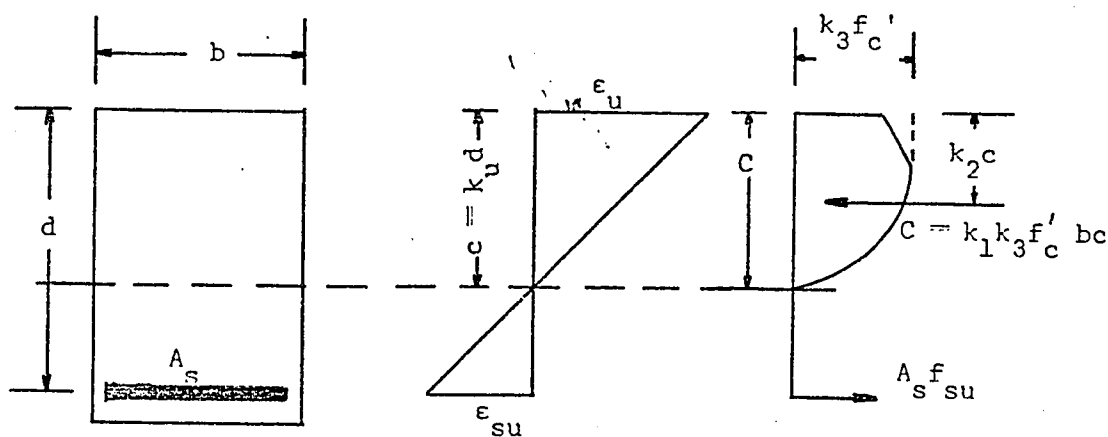


Figure (3-2). Flexural Analysis for a Rectangular Section of Beam at Ultimate Load Condition.

10.

For a rectangular section with tension reinforcement, subjected to pure flexure, equilibrium of moments and forces can be shown to be:

$$M_u = A_s f_{su} (d - k_2 c) \quad (3-1)$$

$$A_s f_{su} - k_1 k_3 f'_c b c = 0 \quad (3-2)$$

Failure may be governed by the steel yield first, or the concrete crushing first. All beams in practice are designed to yield in tension before crushing occurs, to give a ductile type of failure. If the failure is initiated by yielding of the tension reinforcement, the steel stress at the ultimate moment f_{su} is equal to the yield stress f_y . Thus the equations (3-1) and (3-2) can be rewritten as;

$$M_u = A_s f_y (d - k_2 c) \quad (3-1a)$$

$$A_s f_y - k_1 k_3 f'_c b c = 0 \quad (3-2a)$$

From equation (3-2a);

$$c = \frac{A_s f_y}{k_1 k_3 f'_c b} \quad (3-2b)$$

The ultimate moment can be obtained by solving the equation (3-1a) and (3-2b);

$$M_u = A_s f_y \left(d - \frac{k_2}{k_1 k_3} \frac{A_s f_y}{b f_c'} \right)$$

$$= A_s f_y d \left(1 - \frac{k_2}{k_1 k_3} q \right) \quad (3-3)$$

$$\text{or} \quad = b d^2 f_c' q \left(1 - \frac{k_2}{k_1 k_3} q \right) \quad (3-3a)$$

$$\text{where} \quad q = \frac{A_s f_y}{b d f_c'}$$

It may be seen from equation (3-3) that the effect of the assumed concrete stress distribution on ultimate strength controlled by tension, is represented by ratio $\frac{k_2}{k_1 k_3}$; for design purpose, this value is assumed to be independent of concrete strength. By studying the test results of large groups of beam test data $\frac{k_2}{k_1 k_3}$ can be found. For instance, Whitney⁽¹⁰⁾ found $\frac{k_2}{k_1 k_3} = 0.59$; on the basis of column tests, Hognestad⁽⁹⁾ found $\frac{k_2}{k_1 k_3} = 0.55$. The National Building Code of Canada⁽¹¹⁾ recommends the use of the more conservative value $\frac{k_2}{k_1 k_3} = 0.59$. For design purposes therefore the equation (3-3) can be rewritten as;

$$M_u = A_s f_y d (1 - 0.59 q) \quad (3-4)$$

$$\text{or} \quad M_u = b d^2 f_c' q (1 - 0.59 q) \quad (3-4a)$$

In design cases, Whitney⁽¹⁰⁾ suggested that it is convenient to replace the actual stress distribution by a fictitious equivalent rectangular stress distribution, as shown in Figure (3-3). Provided that this fictitious distribution results in the same total compressive force "C", applied at the same location as the actual stress distribution little error will result.

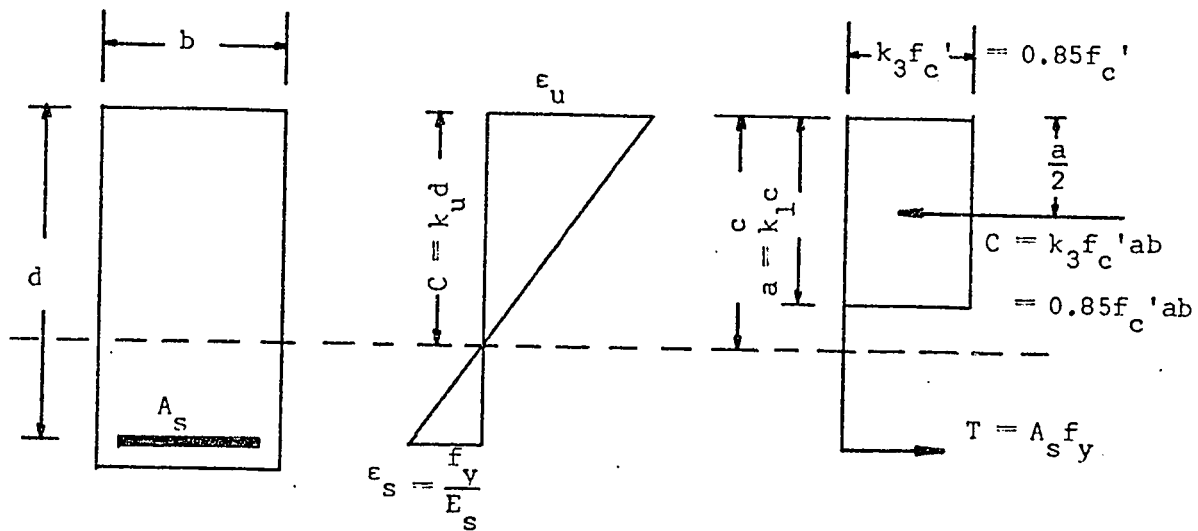


Figure (3-3). Whitney's Rectangular Stress Distribution Block.

Loads calculated by the rectangular stress block, or the more exact stress distribution are identical. The value of "a" in Figure (3-3) is expressed in terms of $k_1 c$, where $k_1 = 0.85$ for $f_c' \leq 4000$ psi. and decreases by 0.05 for every 1000 psi. above 4000 psi.. From the equilibrium of moments and forces, by letting $k_3 = 0.85$ and $k_1 = 2k_2^{(9), (10)}$, the ultimate moment capacity for a rectangular section with tension reinforcement, subjected to pure flexure is the same as equation (3-4) (3-4a).

3-2 Ultimate Load Capacity of Two-Span Continuous Beam

Concentrated Load on Each Span Figure (3-4) shows the two-span continuous beam which was investigated in this project.

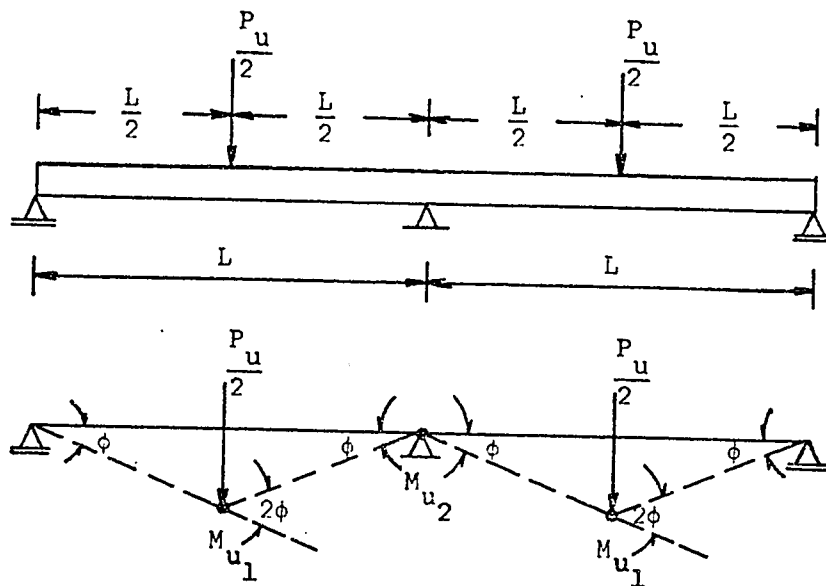


Figure (3-4). Collapse Mechanism of Two-Span Continuous Beam.

The beam forms a collapse mechanism when hinges form under each load point and about the centre support; provided the support hinge possesses enough ductility to allow the mid-span hinges to develop. The magnitude of P_u can be computed by application of the principle of virtual displacements

to the collapse mechanism of the beam. Referring to Figure (3-4).

$$\text{External work} = 2 \times \frac{P_u}{2} \times \phi \times \frac{L}{2} = \frac{P_u \phi L}{2}$$

$$\text{Internal work} = 4M_{u1}\phi + 2M_{u2}\phi$$

Equating the external work and internal work, the ultimate load is obtained :

$$P_u = \frac{4M_{u2}}{L} \left(\frac{2M_{u1}}{M_{u2}} + 1 \right) \quad (3-5)$$

in which M_{u1} and M_{u2} are the ultimate moment capacities of the beam in the mid-span section and at the intermediate support section respectively. All the test specimens of the two-span continuous beam were made with the #5 top and bottom bars. The bottom bars were extended throughout the full length of the beam, but the top bars were cut off in accordance with the minimum requirements of the ACI Code. Details of the reinforcing can be found in Figure (4-1). Therefore the M_{u1} can be computed from equation (3-4) or (3-4a), and the moment M_{u2} has to be established. To calculate the moment M_{u2} , it is necessary to determine whether the section is controlled by the concrete crushing or the tension steel yielding and if the compression steel yields. Referring to Figure (3-5), and making the following conventional assumptions;

- (1) The strains of the concrete on the extreme compressive face $\xi_c = \xi_u = 0.003$
- (2) The tension steel has yielded and hence the stresses in the compression steel can be expressed as;

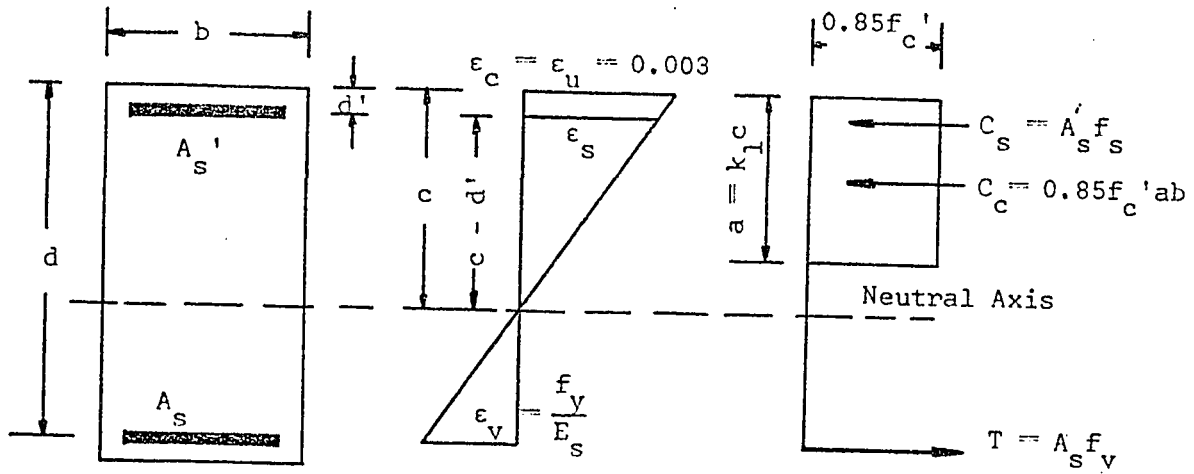


Figure (3-5). Analysis of Double-Reinforced Beam.

$$f'_s = 29,000,000 \times 0.003 \frac{(c - d')}{c} \leq f_y \quad (3-6)$$

where $E_s = 29,000,000$ psi

From the condition of equilibrium of forces,

$$A'_s f'_s + 0.85 f'_c a b = A_s f_y \quad (3-7)$$

Taking moments about the tension steel

$$M_u = A'_s f'_s (d - d') + 0.85 f'_c k_1 c b (d - \frac{k_1 c}{2}) \quad (3-8)$$

In equation (3-8), only "c" is the unknown value and can be obtained by trial and error from equation (3-7). Knowing the value of "c", the ultimate moment in the intermediate support section of the two-span continuous beam can be calculated from equation (3-8).

3-3 Ultimate Load Capacity of Column

A reinforced concrete short column can fail in two ways^{(12), (13)}:

- (i) An over-reinforced cross section fails by primary crushing of the concrete before the yield point stress is reached in the tensile reinforcement.
- (ii) An under-reinforced cross-section fails when the yield stress is reached in the tensile reinforcement.

In both these cases, the stress in the compressive reinforcement can either be at the yield stress in compression, or remain lower than this limit. In what follows, it is assumed that the stress in the compressive reinforcement reaches the elastic limit in compression before failure.

To establish the ultimate load capacity of the column section, subjected to an eccentric load, a knowledge of the balanced load condition on the cross section is necessary. The balanced condition is eccentric load which produces simultaneously, a strain of 0.003 in the extreme fiber of the concrete and the yield strain in the tension steel. The compressive stress diagram in a rectangular cross-section, subjected to an eccentric load, is assumed to be an equivalent rectangular stress distribution. The strain relationship is shown in Figure (3-6)⁽¹³⁾.

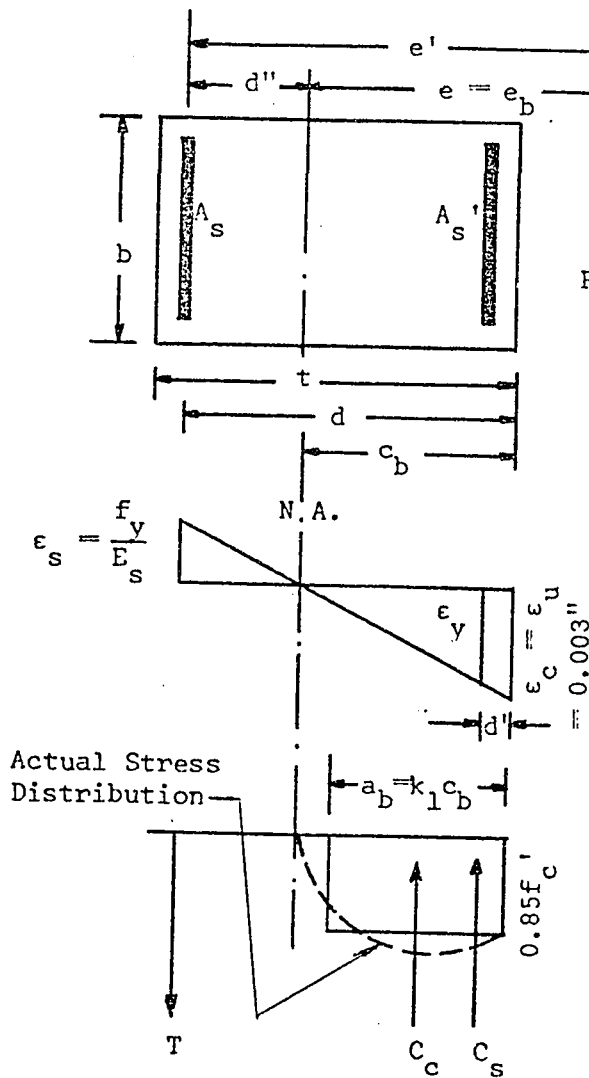


Figure (3-6). Balanced Condition for a Rectangular Section Subject to Combined Flexure and Axial Compression Load.

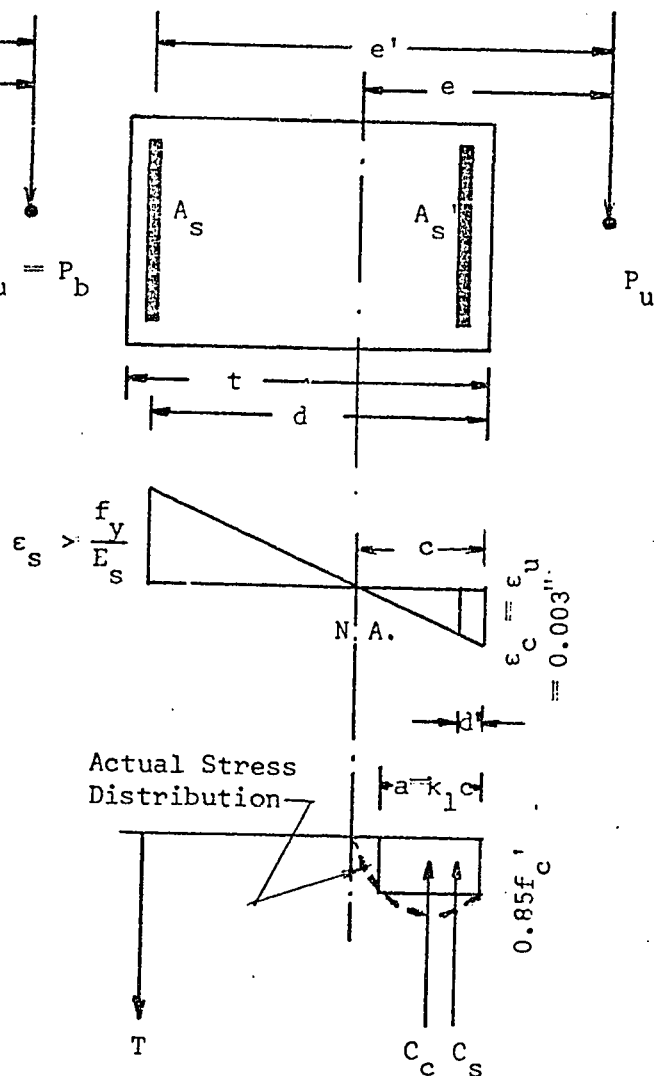


Figure (3-7). Ultimate Capacity Controlled by Tension for a Rectangular Section Subject to Combined Flexure and Axial Compression Load.

Consider the strain distribution;

$$c_b = \frac{\epsilon_u}{\epsilon_y + \epsilon_u} d$$

$$= \frac{0.003}{\left(\frac{f_y}{29 \times 10^6} \right) + 0.003} d$$

$$= \frac{87,000 d}{f_y + 87,000} \quad (\text{psi units})$$

Force equilibrium requires

$$P_b = C_c + C_s - T$$

$$= 0.85 f'_c k_1 c_b b + A_s' (f'_y - 0.85 f'_c) - A_s f_y \quad (3-9)$$

where:

$$k_1 = 0.85, \text{ If } f'_c \leq 4000 \text{ psi}$$

$$k_1 = 0.80, \text{ If } 4000 \leq f'_c \leq 5000 \text{ psi}$$

$$k_1 = 0.75, \text{ If } 5000 \leq f'_c \leq 6000 \text{ psi}$$

From the moment equilibrium with respect to the plastic centroid,

$$\begin{aligned}
P_b e_b &= C_c \left(d - \frac{k_1 c_b}{2} - d'' \right) + C_s (d - d' - d'') + T d'' \\
&= 0.85 f_c' k_1 c_b b \left(d - \frac{k_1 c_b}{2} - d'' \right) \\
&\quad + A_s' (f_y' - 0.85 f_c') (d - d' - d'') + A_s f_y d'' \quad (3-10)
\end{aligned}$$

The value of e_b can be obtained with equation (3-9) and (3-10) when the eccentricity " e " of an ultimate eccentric load P_u is less than the balanced value e_b , the capacity of the section is controlled by compression on the concrete, otherwise the capacity of the section is controlled by tension in the steel. This project, is only concerned with the tensile failure. Therefore, in what follows, the expression for the ultimate load of the eccentrically loaded columns, in the case where the section is controlled by tension will be derived using the assumptions given at the beginning of this paragraph, and referring to Figure (3-7).

From force equilibrium;

$$\begin{aligned}
P_u &= C_s + C_c - T \\
&= A_s' (f_y' - 0.85 f_c') + 0.85 f_c' (k_1 c) b - A_s f_y \\
&= 0.85 f_c' b d \left[\frac{k_1}{d} c + p' (m-1) - pm \right] \quad (3-11)
\end{aligned}$$

in this formulae where $m = \frac{f_y}{0.85 f_c'}$, $p = \frac{A_s}{bd}$, $p' = \frac{A_s'}{bd}$ are constants but the term $\frac{k_1}{d} c$ has to be determined. From moment equilibrium with respect to the tension steel,

$$P_u e' = C_s (d - d') + C_c (d - \frac{a}{2})$$

$$= A_s' (f_y - 0.85 f_c') (d - d') + 0.85 f_c' k_1 c b (d - \frac{a}{2})$$

$$= 0.85 f_c' b d \left[k_1 c - \frac{(k_1 c)^2}{2d} + p' (d - d') (m - 1) \right]$$

..... (3-12)

substituting equation (3-11) into equation (3-12) gives;

$$0.85 f_c' b d e' \left[\frac{k_1 c}{d} + p' (m - 1) - p m \right]$$

$$= 0.85 f_c' b d \left[k_1 c - \frac{(k_1 c)^2}{2d} + p' (d - d') (m - 1) \right]$$

Rearranging the above equation gives;

$$\frac{c}{d} = \frac{1 - \frac{e'}{d}}{k_1} + \sqrt{\left(\frac{1 - \frac{e'}{d}}{k_1} \right)^2 + \frac{2 \left[p' (m - 1) \left(1 - \frac{d'}{d} \right) + \frac{e'}{d} m (p - p') + p' \frac{e'}{d} \right]}{k_1^2}}$$

or

$$\frac{k_1 c}{d} = 1 - \frac{e'}{d} + \sqrt{\left(1 - \frac{e'}{d} \right)^2 + 2 \left[p' (m - 1) \left(1 - \frac{d'}{d} \right) + \frac{e'}{d} m (p - p') + p' \frac{e'}{d} \right]}$$

.... (3-13)

Substituting equation (3-15) into equation (3-13) to calculate the P_u ,

$$P_u = 0.85 f_c' b d \left[1 - \frac{e'}{d} + p'(m-1) - pm \right. \\ \left. + \sqrt{\left(1 - \frac{e'}{d}\right)^2 + 2 \left[p'(m-1) \left(1 - \frac{d'}{d}\right) + \frac{e'}{d} m (p - p') + p' \frac{e'}{d} \right]} \right] \dots (3-14)$$

In our column test specimen design, $A_s = A_s'$ i.e. $p' = p$ therefore the equation (6) can be simplified to;

$$P_u = 0.85 f_c' b d \left[1 - \frac{e'}{d} - p + \sqrt{\left(1 - \frac{e'}{d}\right)^2 + 2p \left[(m-1) \left(1 - \frac{d'}{d}\right) + \frac{e'}{d} \right]} \right] \dots (3-15)$$

All the theoretical calculations presented use the final eccentricity of the columns estimated from the experimental results.

3-4 Design Calculations

To allow for the variations of materials strength and workmanship found in practice the ACI - NBC Codes specify under-strength (ϕ) factors to reduce the calculated ultimated loads to design values. The ϕ factors recommended by the NBC are 0.90 for flexure, 0.85 for diagonal tension and 0.75 for columns. The ϕ factor recommended by the ACI are 0.90 for flexure, 0.85 for diagonal tension 0.75 for spiral columns and 0.70 for tied columns.

To allow for the effect of load capacity being reduced due to slenderness effects both the ACI-NBC Building Codes reduce the calculated load by a slenderness factor R.

For a pin-ended column bent in single curvature the following expressions are used.

$$R = 1.07 - 0.0075 \frac{h}{r} \leq 1.0 \quad \text{N.B.C.}$$

$$R = 1.07 - 0.008 \frac{h}{r} \leq 1.0 \quad \text{A.C.I.}$$

The predicted magnitudes of the failure loads of the specimens, using an assumed crushing strength $f'_c = 4000$ psi, steel yield stress $f_y = 50.000$ psi. are given below. The detailed calculations are given in the Appendix A. and the results summarised in Table (3-1).

TABLE (3-1). DESIGN LOADS OF TEST SPECIMENS

Member	Simple Beam	Two Span Continuous Beam	Columns	
			ACI	NBC
Predicted Load (kips)	14.92	32.94	29.9	29.9
ϕ Factor	0.90	0.90	0.70	0.75
Length Load Reduction Factor	Not Applicable	Not Applicable	0.73	0.75
Design Load (kips)	13.42	29.64	15.30	16.80

Chapter 4.

Experimental Determination of Failure Loads4-1 Experimental Investigation

The object of this investigation was to determine experimentally, the variation in the strength of three types of reinforced concrete members, namely: simple beams, two-span continuous beams and columns. Details of the nominal dimensions of the reinforced concrete members are shown in Figures (4-1), (4-2) and (4-3).

The simple beams were tested under two symmetrical point loads which were acting separately at one-third of span from both end supports.

The two-span continuous beams were tested under concentrated loads at the mid-point of each span, the loads being equal.

The columns were tested under eccentric loading, the eccentricity being 4.68" from the plastic centroid of the column section.

The specified strength and grades of concrete and reinforcing steel used in the preparation of these specimens were the same, but the delivery of concrete mix from Dominion Building Material Co., was made in twenty instalments, over a period of $1\frac{1}{2}$ months.

The columns as well as the beams were cast in the horizontal position. Thus the strength of the column will be uniform throughout the length and the widest crack will occur in the mid-point of the column length.

All the specimens were cured at room temperature (75°F) and stored under wet sacks to provide water for hydration.

Twenty-eight days after casting, the strength of beam specimens and the crushing strength of the 20 concrete 6" x 12" test cylinders, which had been cast at the same time as columns and beams, were obtained from tests.

4-2 Fabrication

The concrete was mixed and delivered by the transit-mix truck from Dominion Building Material Co., Ottawa and the test specimens were cast at the concrete laboratory of the University of Ottawa. The total time from batching to end of casting was one hour to one hour fifteen minutes.

The specimen forms were made of steel. This was advantageous because it was possible to use the same forms throughout the tests. These steel forms were available in the laboratory are shown in Figure (4-4). The two vertical sides of each form consisted of a steel channel section and was bolted solidly to the base. To prevent adhesion between the inside surface of the form and the specimen, the inside surfaces of the forms were coated with form oil.

A total of 60 test specimens were required. The specimens were to be as follows:

- (a) 20 identical simple beams
- (b) 20 identical two-span continuous beams
- (c) 20 identical columns

plus a total 400 of 6" x 12" cylinders. One simple beam, one two-span continuous beam, and one column plus twenty cylinders being cast from each batch of concrete supplied.

Twenty-four hours after the casting, the specimens were moved from the forms, and cured under wet sacks in the laboratory.

4-3 Materials

4.3.1. Concrete

Concrete with a specified design compressive strength of 4000psi at twenty-eight days (3" slump), was designed and supplied by the Dominion Building to the laboratory. Mixing was done while the truck was en route from the batch plant to the laboratory. The cement used in these projects was standard portland cement (type I). The sand had an average fineness modulus of 2.64, the coarse aggregate was well graded and had a maximum size of 3/4". Both of these aggregates passed the usual specification tests. The proportion per cubic yard by weight is listed as following.

Dry Weights

Cement	500 lbs	
Sand	1410 lbs	
Stone (3/4")	1880 lbs	$\frac{w}{c} = \frac{290}{500} = 0.58$
Water	290 lbs	

4.3.2. Steel Reinforcement

Hard grade bars (minimum specified yield stress = 50,000 psi) were used throughout. The reinforcement cages were fabricated by a commercial supplier G. & H. Co. Ltd., Ottawa.

A typical stress-strain curve is shown in Figure (4-5) and the measured yield strength are listed in Table (5-3). The strains were measured using a NRC designed mechanical strain gauge and the loads were applied by a 120,000 lbs. Olsen Hydraulic testing machine.

4-4 Test Procedure

The tests were carried out at the concrete laboratory of

the University of Ottawa. Test specimens for each of the group included one simple beam, one two-span continuous beam, one column and twenty 6" x 12" cylinders. All of these were cast at the same time, and were tested as closely as practicable at the same time. All of these specimens were subjected to fast loading to failure. The time for the column, simple beam and two-span continuous beam test, from the beginning of the testing to the failure of the specimens, was approximately 30 minutes, 30 minutes and one hour respectively. The cylinders were tested within 24 hours after completion of the member tests.

4.4.1. Column

All columns were tested in a special testing frame. A view of this special testing frame, during a test, is shown in Figure (4-6). A sketch of the test set up is given in Figure (4-7).

To avoid crushing of the concrete at the ends of the specimen, 1" thick bearing steel plates with a semi-circular groove were used. The ram and the load cell had a steel cap which also had semi-circular grooves. Between the semi-circular groove of the cap and the steel plate, there was a 1" diameter steel roller, the roller being at the required eccentricity of the column.

The load was applied in increments of 2000 lbs up to 20,000 lbs, and thereafter in increments of 1000 lbs to failure. The magnitude of the applied load could be measured from a digital indicator which was connected directly to a 100 kips capacity load cell.

Besides the ultimate load, the deflection at the centre of the column was also measured with 0.001 in. dial gauge. In addition to these measurements, the location and extent of cracks were recorded immediately after the application of each increment of load.

Twenty reinforced concrete columns have been tested. Typical failure mode of the columns are shown in Figure (4-8).

4.4.2. Simply Supported Beams

The specimens were supported over a span of 6'0" @ c/c. A view of the testing frame during the test of simply supported beam is shown in Figure (4-9). A sketch of the test set-up is given in Figure (4-10). The steel loading beam was supported symmetrically by two steel plate boxes, and two 1" thick steel bearing plates (with a 1" diameter steel roller in between the steel plate box and bearing plate) which directly rested on the test beam. The test beam was supported by two 50 kips capacity load cells. The load was applied by the ram through the steel loading beam. Then, it was transferred to the concrete beam and the actual applied load was measured as the reaction by the load cells.

The load was applied in increments of 1000 lbs up to 8000 lbs, and thereafter in increments of 500 lbs to failure. The required measurements of this test were applied loads and the corresponding deflections at the centre of span, ultimate load of this specimen and the location and extend of cracks.

Twenty of these identical simply supported beams were tested. Typical failure mode of the simply supported beams after test, are shown in Figure (4-11).

4.4.3. Two-span Continuous Beam

The test beam arrangement and the testing equipment were all the same as the arrangement and equipments for simply supported beams. A view of a test is shown in Figure (4-12). A sketch of the test set-up is given in Figure (4-13).

Twenty identical reinforced concrete two-span beams were tested. A typical failure mode of this test is shown in Figure (4-14).

4.4.4. Cylinders

Twenty test cylinders, cast at the same time as the corresponding beam and column, were tested to determine the compressive strength. Total casting time was less than one hour so no differential time effects would have been present between the members and the cylinders.

The compressive strength of the test cylinders was determined in a hydraulic concrete testing machine having a capacity of 300 kips. The test specimens were placed in the testing machine and tested to failure. The end faces of the test cylinders were capped with standard sulphur compound.

Chapter 5.

Discussion of Results

Twenty sets of nominally identical specimens were tested to failure. Each set consisted of a simple beam, a two-span continuous beam, a eccentrically loaded column and twenty concrete control cylinders. Load-deflection curves were taken for every reinforced concrete member and are presented in Appendix B.

5-1 Statistical Relations

The discussion involves comparing the experimental and the theoretical magnitudes of the mean and the standard deviation of the member strengths. The statistical quantities used in the discussion are described below:

- (i) Mean ($\bar{X}$), the average strength of all test specimen's strength

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{N} = \frac{\sum_{i=1}^N X_i}{N} \quad (5-1)$$

where $X_1, X_2, \dots, X_n$ are the results of individual strength tests, and N is the number of tested specimens.

- (ii) Standard deviation σ , or the root mean square deviation, is the

$$\sigma = \left[\frac{\sum_{i=1}^N (X_i - \bar{X})^2}{N - 1} \right]^{\frac{1}{2}} \quad (5-2)$$

- (iii) Coefficient of variation (V) - the standard deviation expressed as a percentage of the average strength, is called the coefficient of variation.

$$V = \frac{\sigma}{\bar{X}} \times 100\% \quad (5-3)$$

5-2 Test Results of Cylinders

Table (5-1) contains results of the tests on the 28 day cylinders, which show the compressive crushing strength in psi of each of the 400 cylinders tested.

A student t_0 test was used to compare the between batch variation of concrete with the variation within the batches and showed that the different batches are statistically independent. However for the purpose of engineering design the concrete must be considered to belong to the same population. For the purpose of analysis, the mean result of the cylinders strength for each batch was considered as one test. The mean value $\bar{X}$, standard deviation and coefficient of variation of the concrete means were determined using equations (5-1), (5-2) and (5-3).

Mean value	= 4440 psi
Standard deviation	= 530 psi
Coefficient of variation	= 12%

Figure (5-1) shows the histogram of the 28 day cylinder strengths; the test results are grouped in interval of 300 psi.

The distribution of values falls in same pattern similar to a normal frequency distribution curve. The exact theoretical frequency distribution, the student 't' distribution with 19 degree's of freedom, was fitted to the experimental result and is shown in Figure (5-1).

The Chi-Square test was used to check the 'goodness of fit' of the student 't' distribution to the experimental results. The detailed calculation is shown in Appendix C. and the Chi-Square test showed a 99% level of significance between the fitted curve and the measured results.

TABLE (5-1). 28 DAY CYLINDER STRENGTHS

	Compressive Strengths (psi)		Compressive Strength (psi)		Compressive Strength (psi)		Compressive Strength (psi)	
	Series 1.		Series 2.		Series 3.		Series 4.	
	4014	5528	4386	4244	4386	4492	4244	4280
	4103	4103	4492	4368	4492	4368	4368	4368
	3979	5394	4227	3714	4227	3714	4227	4227
	3937	5695	4421	4227	4421	4227	4227	4227
	4350	5606	4297	4563	4297	4563	4563	4563
	4174	5642	4297	4333	4297	4333	4333	4333
	4598	5801	4527	4669	4527	4669	4669	4669
	4227	5305	4545	4386	4545	4386	4386	4386
	4085	5659	4368	4439	4368	4439	4439	4439
	4191	5871	4262	4474	4262	4474	4474	4474
	4333	5624	4457	4368	4457	4368	4368	4368
	4280	5482	4474	4315	4474	4315	4315	4315
	3837	5341	4350	4527	4350	4527	4527	4527
	3837	5818	4510	4262	4510	4262	4262	4262
	4297	5783	4244	4457	4244	4457	4457	4457
	4386	5341	4439	4633	4439	4633	4633	4633
	4209	5801	4439	4863	4439	4863	4863	4863
	4209	4492	4333	4368	4333	4368	4368	4368
	4280	5695	4120	4740	4120	4740	4740	4740
	4103	5659	4403	4280	4403	4280	4280	4280
Mean Value	4174.5	5482.0	4379.55	4411.50	4379.55	4411.50	4411.50	4411.50
Standard Deviation	187.01	443.41	113.20	238.44	113.20	238.44	238.44	238.44
Coefficient of Variation	4.48%	8.09%	2.58%	5.40%	2.58%	5.40%	5.40%	5.40%

TABLE (5-1). 28 DAY CYLINDER STRENGTHS (CONTINUOUS)

	Compressive Strength (psi)		Compressive Strength (psi)		Compressive Strength (psi)		Compressive Strength (psi)	
	Series 5.	Series 6.	Series 6.	Series 7.	Series 7.	Series 8.	Series 8.	
Mean Value	4138 4174 4386 4014 4510 4244 4315 4368 3979 4050 4280 4403 4633 3944 4032 3926 3855 4174 4191 3678	4510 4598 4580 4881 4952 4297 4174 4989 4333 3979 4421 4740 4863 4563 4474 4828 4368 4669 4616 4457	4510 4598 4580 4881 4952 4297 4174 4989 4333 3979 4421 4740 4863 4563 4474 4828 4368 4669 4616 4457	4156 4067 3873 4280 4209 4067 4244 4297 4067 4333 3891 4191 4050 4209 4067 4085 4032 4227 4014 4103	4156 4067 3873 4280 4209 4067 4244 4297 4067 4333 3891 4191 4050 4209 4067 4085 4032 4227 4014 4103	4616 4616 4120 4474 4616 4775 4333 4886 4439 4191 4616 4156 4510 4350 4350 4457 3837 4262 4386 4492	4616 4616 4120 4474 4616 4775 4333 4886 4439 4191 4616 4156 4510 4350 4350 4457 3837 4262 4386 4492	
Standard Deviation	235.77	265.12	265.12	126.03	126.03	225.20	225.20	
Coefficient of Variation	5.66%	5.81%	5.81%	3.06%	3.06%	5.10%	5.10%	

TABLE (5-1). 28 DAY CYLINDER STRENGTHS (CONTINUOUS)

	Compressive Strength (psi)		Compressive Strength (psi)		Compressive Strength (psi)	
	Series 9.	Series 10.	Series 11.	Series 12.	Series 12.	Series 12.
	3448 3325 3855 3625 3731 3731 3307 3484 3625 3802 3360 3501 3325 3749 3784 3643 3873 3519 3555 3696	5111 5182 4934 5040 5022 4952 4916 4421 4863 4846 4863 4510 4987 4934 4828 4863 5040 4386 4846 4793	4439 4315 4368 4174 4085 4050 4080 4563 4510 4563 4527 4492 3837 4474 4191 4280 4403 4156 4350 4403	4952 5376 5182 4952 5040 4916 5005 5022 5465 4616 5235 5093 5553 5412. 5748 5748 5500 5305 5376 5129		
Mean Value	3596.90	4866.85	4313.00	5249.25		
Standard Deviation	182.46	210.51	200.78	313.66		
Coefficient of Variation	5.07%	4.33%	4.66%	5.98%		

TABLE (5-1). 28 DAY CYLINDER STRENGTHS (CONTINUOUS)

	Compressive Strength (psi)		Compressive Strength (psi)		Compressive Strength (psi)	
	Series 13.	Series 14.	Series 14.	Series 15.	Series 16.	
	4457 4421 4793 4280 4439 5005 4686 4262 4403 4333 4722 4333 4987 4757 4775 4032 4863 4209 4669 4545	5376 5659 5305 5376 5765 5571 5111 5677 5642 5730 5447 5588 5818 5058 5217 5394 5535 5447 5588 5553	4315 4333 4191 4138 4209 3979 4315 4350 4032 4616 4067 4421 4032 4333 4315 4171 4085 4297 4580 4209	4244 4156 4386 4227 4067 3555 3926 4174 4085 4014 4191 4156 4244 4191 3961 3855 3961 4174 3979 3714		
Mean Value	4548.55	5492.85	4249.40	4063.00		
Standard Deviation	271.96	209.98	172.60	197.87		
Coefficient of Variation	5.98%	3.82%	4.06%	4.87%		

TABLE (5-1). 28 DAY CYLINDER STRENGTHS (CONTINUOUS)

	Compressive Strength (psi)		Compressive Strength (psi)		Compressive Strength (psi)	
	Series 17.	Series 18.	Series 19.	Series 20.		
	4315 4014 3891 3784 4156 4333 3997 4156 4244 3873 4280 4085 4403 4545 4120 3767 4350 4244 3820 3961	4474 4527 4421 4686 4262 4474 4439 4863 5005 4580 5005 4686 4563 4227 4952 4633 4598 5359 4527 4333	3501 3731 3413 3519 3696 3448 3519 3501 3378 3466 3395 3466 3325 3466 3501 3342 3625 3802 3236 3254	4439 4474 4474 4474 4386 4297 4174 4403 4510 4474 4262 4315 4457 4669 4457 4421 4262 4474 4103 4580		
Mean Value	4116.90	4630.70	3479.20	4405.25		
Standard Deviation	223.59	282.97	147.99	135.25		
Coefficient of Variation	5.43%	6.11%	4.25%	3.07%		

For total cylinders: $\bar{X} = 4435.3867$ kips, $\sigma = 559.8936$, $V = 12.62\%$

Figure (5-2) shows a plot of the fraction cumulative frequency against the class mean concrete cylinder strength. The fraction cumulative frequency is the percentage of test values that fall below a particular strength value. According to the provisions of section 504C-Strength Tests of Concrete of ACI 318-63⁽¹⁵⁾.

- (1) For structures designed in accordance with part IV-A (working design) of this code. The average of any five consecutive strength tests of the laboratory-cured specimens representing each class of concrete shall be equal to or greater than the specified strength f'_c , and not more than 20 percent of the strength tests shall have values less than the specified strength.
- (2) For structures designed in accordance with part IV-B (ultimate strength design) of this code, and for prestressed structures the average of any three consecutive strength tests of the laboratory-cured specimens representing each class of concrete shall be equal to or greater than the specified strength, f'_c , and not more than 10 percent of the strength tests shall have values less than the specified strength.

From Figure (5-2), it can be seen that the 10% level of the fraction cumulative frequency curve is equivalent to a cylinder strength of 3620 psi which is lower than the specified strength. The 20% level is 3975 psi, therefore the strength of these concretes which have been used in this experiment is not up to the requirement of ACI Code. The required strength in this case will be equal to:

$$f_{cr} = \frac{f'_c}{(1 - tV)}$$

$$= \frac{4000}{(1 - 1.325 \times 0.12)} = 4740 \text{ psi}$$

where f_{cr} = Required average strength

f'_c = Design strength specified

t = A constant depending upon the proportion of tests that may fall below f'_c and the number of samples used to establish "V" see Table (5-2) (used $N = 20$)

V = Forecasted value of the coefficient of variation

This required strength f_{cr} is higher than the average strength supplied, but the measured coefficient of variation (12%) shows that concrete manufacturing to be well controlled. (See Table (5-2a))⁽¹⁴⁾.

TABLE (5-2)⁽¹⁴⁾. Values of t.

No. of samples minus 1*	Percentage of tests falling within the limits $\bar{X} \pm t \sigma$							
	50	60	70	80	90	95	98	99
	Chances of falling below lower limit							
	1 in 4	1 in 5	1 in 6.7	1 in 10	1 in 20	1 in 40	1 in 100	1 in 200
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576

*Degrees of freedom.

Because the specified mean strength was not satisfactory, to allow the ultimate strength design with $f'_c = 4000$ psi, the ultimate load calculations for the members were recalculated using $f'_c = 3620$ psi (See Appendix A.). The concrete would then meet the requirement that not more than 10% of the strength tests be less than the specified strength.

TABLE (5-2a). RELATION OF VARIATION IN STRENGTH TO QUALITY OF WORKMANSHIP

Coefficient of Variation	Quality of Workmanship
5	Attainable only in well controlled laboratory work
10	Excellent - approaches laboratory precision
12	Excellent
15	Good
18	Fair
20	Fair - Minus
25	Bad

5-3 Reinforcement Bars Strength

The stress-strain curve of all the reinforcing bars demonstrated a long yield plateau giving a well defined yield stress. Table (5-3) gives the yield stress of twenty pieces of #5 bars which were used throughout in this project. Figure (5-3) shows the frequency distribution for the steel bars tensile test. The stepped diagram representing the actual test data is based on class intervals of 3 ksi.

Again a student 't' distribution with 19 degree's of freedom was fitted to the experimental results. The Chi-Square test was

used and showed that the use of the student 't' distribution was justified with a 99% level of significance. The detailed results are given in Appendix C.

TABLE (5-3). PROPERTIES OF #5 REINFORCING STEEL

Test No	Yield Point Stress * (ksi)	Test No	Yield Point Stress (ksi)	Test No	Yield Point Stress (ksi)
1	46.65	8	51.50	15	48.50
2	49.60	9	50.60	16	48.80
3	51.25	10	54.50	17	49.40
4	41.90	11	53.00	18	53.20
5	43.60	12	51.00	19	52.60
6	41.90	13	51.90	20	51.60
7	53.20	14	53.20		

* Steel showed a distinct yield stress.

5-4 Test Results of Simple Beams, Two-Span Continuous Beams and Columns

Table (5-4) represents the results of simple beams, continuous beams and columns.

As discussed Chapter III, the ultimate strength of beam is controlled by yielding of the tension reinforcement, the ratio $\frac{k_2}{k_1 k_3}$ in equation (3-3) or (3-3a) for design purposes is equal to 0.59. At present, it may be determined by statistical methods from test results of reinforced concrete beams. For this purpose equation (3-3a) was expressed by R.C. Elstner⁽⁹⁾ as;

$$S = \Sigma \left[\frac{M_u}{bd^2 f'_c} - q \left(1 - \frac{k_2}{k_1 k_3} q \right) \right]^2 \quad (5-5)$$

TABLE (5-4). MAXIMUM APPLIED LOADS ON SIMPLE BEAM,
CONTINUOUS BEAM AND COLUMN.

Specimen No	Simple Beam (kips) P_u	Continuous Beam (kips) P_u	Column (kips) P_u
1	15.30	32.61	30.00
2	15.40	34.48	37.80
3	15.00	32.60	34.50
4	15.00	33.00	33.00
5	15.80	32.40	31.00
6	14.93	33.50	29.40
7	15.00	32.41	32.20
8	15.125	34.10	31.20
9	14.25	29.31	30.27
10	14.50	30.48	33.00
11	15.35	32.50	33.00
12	15.00	34.00	29.18
13	15.35	33.70	29.00
14	14.25	32.77	33.16
15	15.60	34.30	30.00
16	15.85	32.60	28.00
17	15.965	32.50	29.20
18	15.70	34.07	30.00
19	14.90	30.00	28.30
20	15.10	32.00	29.30

Mean value = 15.16
Standard variation = 0.48

Mean value = 32.66
Standard deviation = 1.40

Mean value = 30.92
Standard variation = 2.63

let $\frac{dS}{k} = 0$ A least squares solution gives :-
 $d\left(\frac{k_2}{k_1 k_3}\right)$

$$\frac{k_2}{k_1 k_3} = \frac{-\Sigma \frac{M_u}{bd^2 f_c'} q^2 - \Sigma q^3}{\Sigma q^4} \quad (5-6)$$

Table (5-5) contains the procedure for the calculation of $\frac{k_2}{k_1 k_3}$ with the test results of simple beams. The calculations give the ratio of $\frac{k_2}{k_1 k_3} = 0.56$ which is quite near to the value (0.59), suggested by the National Building Code of Canada.

Figures (5-4), (5-5) and (5-6) summarize graphically the results of the tests carried out on the test specimens of simple beam, two-span continuous beam and column.

Figures (B-1) to (B-10), (B-11) to (B-20) and (B-21) to (B-40) represent graphically the result of the load against the deflection for columns, simple beams and continuous beams respectively. Theoretically, the curves of each group should coincide, but as can be seen from these curves, they do not. This is due to the following facts:

- (1) The concrete is not a homogeneous material. As is known the deflection of reinforced concrete structure members is a function of E_c ($E_c = 33 w^{1.5} \sqrt{f_c'}$), weaker strength concrete has a small E_c value, and consequently greater deflections result.
- (2) The yield point stress of reinforcement steel is not constant. The cross sections which contain the lower yield point stress steel will deform more than a similar section with higher yield point stress steel.

No	P(kips)	f'_c (psi)	$bd^2 f'_c$	M_u	q	q^2	q^3	q^4	$\frac{M_u}{bd^2 f'_c} \cdot q^2$
1	15.300	4174.4	934.14	183.60	0.2215408	0.0490803	0.0108733	0.0024089	0.0096464
2	15.400	5482.0	1226.3	184.80	0.1686975	0.0284589	0.0048009	0.0008099	0.0042871
3	15.000	4379.5	980.4	180.00	0.2111657	0.0445909	0.0094161	0.0019884	0.0081898
4	15.000	4411.5	987.20	180.00	0.2096339	0.0439464	0.0092126	0.0019313	0.0080129
5	15.800	4164.7	931.97	189.60	0.2220568	0.0493092	0.0109494	0.0024314	0.0100314
6	14.930	4564.5	1021.44	179.16	0.2026070	0.0410496	0.0083169	0.0016851	0.0072001
7	15.000	4123.1	922.66	180.00	0.2242972	0.0503092	0.0112842	0.0025310	0.0098147
8	15.125	4414.1	987.78	181.50	0.2095104	0.0438946	0.0091964	0.0019267	0.0080654
9	14.250	3596.9	804.91	171.00	0.2571102	0.0661057	0.0169964	0.0043700	0.0140438
10	14.500	4866.8	1089.09	174.00	0.1900222	0.0361084	0.0068614	0.0013038	0.0057689
11	15.350	4313.0	965.16	184.00	0.2144215	0.0459766	0.0098584	0.0021068	0.0087688
12	15.000	5429.0	1174.62	180.00	0.1761859	0.0310415	0.0054691	0.0009636	0.0047568
13	15.350	4548.5	1017.86	184.00	0.2033198	0.0413389	0.0084050	0.0017089	0.0074810
14	14.250	5492.8	1229.17	171.00	0.1683658	0.0283471	0.0047727	0.0008036	0.0039436
15	15.600	4249.4	950.93	187.20	0.2176307	0.0473631	0.0103077	0.0022433	0.0093239
16	15.850	4063.0	909.21	190.20	0.2276151	0.0518086	0.0117924	0.0026841	0.0108379
17	15.965	4116.9	921.27	191.58	0.2246350	0.0504609	0.0113353	0.0025463	0.0104933
18	15.700	4630.7	1036.25	188.40	0.1997106	0.0398843	0.0079653	0.0015908	0.0072513
19	14.900	3479.2	778.57	178.80	0.2858082	0.0706540	0.0187804	0.0049920	0.0162257
20	15.100	4405.2	985.79	181.20	0.2099337	0.0440722	0.0092522	0.0019424	0.0081009

TABLE (5-5). $b = 5"$ $d = 6.69"$ f_y (Average) = 49.899 ksi.

The Chi-Square test was used to check that the observed ultimate loads could be represented by student 't' distribution. The calculations are given in Appendix C.

Simple beam	99% Confidence
Two-span continuous beam	99% Confidence
Column	99% Confidence

5-5 Theoretical Results

The calculated strength of a structure depends upon what design theory is used, the variation of material properties and the quality of the the workmanship. For these tests, the design of the simple beam, two-span continuous beam and column test specimens were based on the same specified strength of concrete and reinforcements. Therefore, theoretically, the ultimate load carrying capacity should be the same for each specimen in the same group. But in reinforced concrete construction, no matter how well a job is controlled, there will be some variation in the strength. In addition to the variations which exist in the strengths of reinforcement steel and concrete itself, variations may be introduced in fabrication, testing and the curing of the test specimens. Thus, the observed mean of each group may be lower or higher than the calculated mean. The term calculated mean is used in this connection to designate the mean ultimate load carrying capacity calculated using the mean measured strengths of the steel and concrete. The difference between these two means may or may not have significance. Thus, it is necessary to know the calculated mean and also to know the relation between the calculated mean and that of the observed mean by means of statistical tests.

The distribution of load carrying capacity of the reinforced concrete members was predicted by finding the joint probability of a given

concrete strength and steel yield strength occurring and calculating the ultimate load capacity of the member using the given steel and concrete strength. Thus the member capacity and its probability of occurrence was determined. By considering all the joint probabilities of various concrete and steel strengths occurring, the distribution of member ultimate load capacity was determined. If there is no significant difference in the observed mean and calculated mean, it can be concluded that the experiment agrees with the theory.

It must be noted that the theoretical results only consider the steel yield stress and concrete cylinder strengths as random variables. No allowance has been made in the theoretical calculations of variations in dimensions of the member or positioning of the reinforcement. Thus the calculated distribution of results will be optimistic (less spread) compared to a calculated result considering variations in dimensions and position of reinforcement.

The calculations were performed by dividing the distributions of both f'_c and f_y into a number of portions as shown in Figure (5-7). The probability of a cylinder strength in a specified band is the area of the band. The value of the cylinder strength for any given band was taken as the average of the bands upper and lower limits.

Thus the probability of a cylinder strength being 4506 ± 66 psi is 10% (shown in Figure (5-7 a)).

Similarly the probability of a steel yield stress being $49,000 \pm 460$ psi is 10%. Hence the probability of a steel yield stress of $49,440 \pm 460$ psi and a concrete cylinder strength of 4506 ± 66 psi occurring together is the product of the individual probabilities.

46.

$$\text{Joint probability of } \begin{cases} f_y = 49,440 \pm 460 \text{ psi} \\ f'_c = 4506 \pm 66 \text{ psi} \end{cases} = 0.010$$

Considering the simple beam described in Figure (4-2);

$$M_u = A_s f_y (d - 0.5a)$$

$$M_u = 180,480 \text{ lbs. ins.}$$

$$\therefore P_u = 15,040 \text{ lbs.}$$

Thus there is a probability of 0.01 that $f'_c = 4506 \pm 66$ psi and $f_y = 49,440 \pm 460$ psi occur simultaneously which would give an ultimate load of 15,040 lbs. By consideration of all the combinations of f'_c and f_y occurring together the distribution of P_u (simple beam) can be found.

Table (5-7) shows the combinations of f'_c , f_y used and their corresponding joint probability. The calculated values of the ultimate loads for columns, continuous beams and simple beams are also shown.

The theoretical distribution of ultimate loads for the column was calculated using an eccentricity of 5.25" in equation (3-15). This value was taken as being the initial eccentricity of the load plus the lateral deformation of the column (estimated at 0.57" from the measured deformations).

For convenience normal distributions have been fitted to results given in Table (5-7) and are shown as dotted curves on Figures (5-8), (5-9) and (5-10). For comparison between the measured and calculated results, a student 't' distribution curve has been used to replace the experimental result and is also plotted in Figures (5-8), (5-9) and (5-10), (shown by the solid line).

For convenience the mean predicted loads and the mean measured loads are given in Table (5-7).

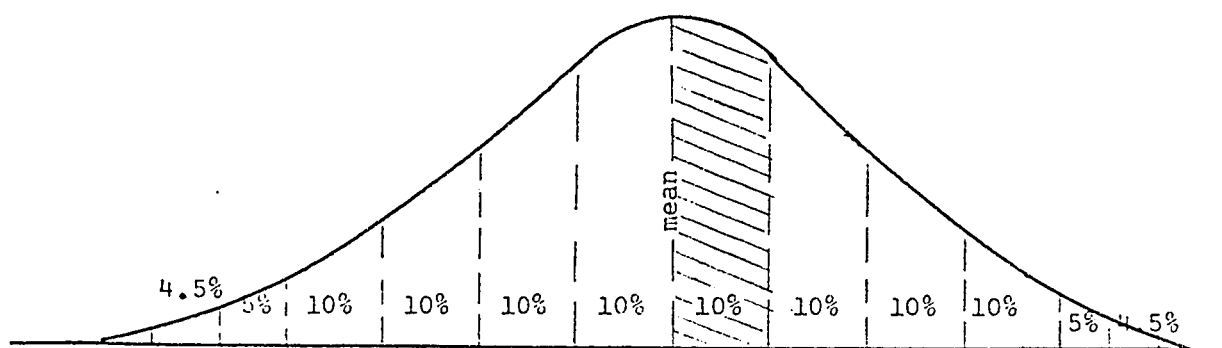
TABLE (5-7).

	Measured Mean Load (kips)	Predicted Mean Load (kips)
Simple	15.2	15.06
Two-Span Continuous Beam	32.7	33.25
Column	31.9	26.30

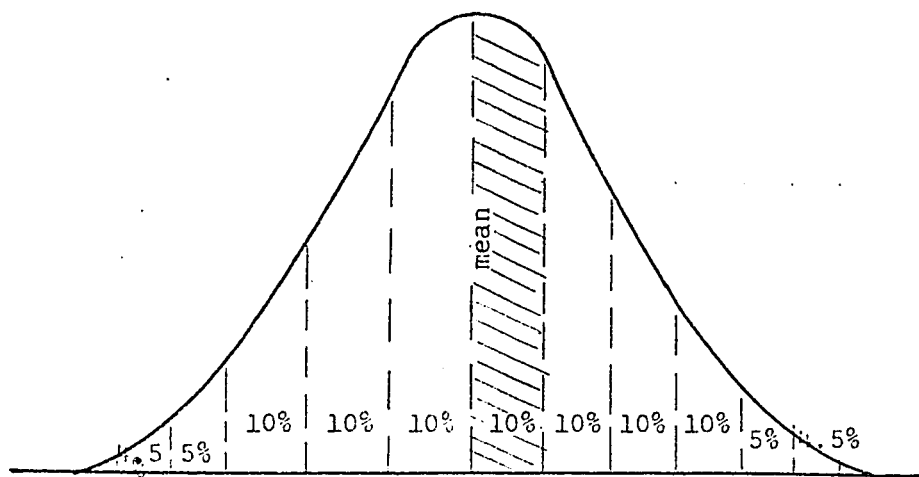
To check the theoretical results, a test of significance was used to prove that the experimental results came from the same population as the theoretical results. The test of significance applied to this case is the student 't' test. The test procedure is to calculate;

f'_c (psi)	f_y (ksi)	probability of f'_c	probability of f_y
3,220	41.30	0.045	0.045
3,630	44.19	0.050	0.050
3,860	45.80	0.100	0.100
4,070	47.28	0.100	0.100
4,230	48.42	0.100	0.100
4,374	49.44	0.100	0.100
4,506	50.36	0.100	0.100
4,650	51.38	0.100	0.100
4,810	52.52	0.100	0.100
5,020	54.00	0.100	0.100
5,250	55.61	0.050	0.050
5,660	58.50	0.045	0.045

TABLE(5-6). Values of f'_c , f_y and the corresponding probability extracted from figures (5-1) and (5-3).



(a). f'_c (see figure (5-1)).



(b). f_y (see figure (5-3)).

Figure (5-7). Values of f'_c and f_y extracted from figures (5-1) and (5-3).

TABLE (5-8). THE COMBINATION f'_c , f_y AND THEIR CORRESPONDING
PROBABILITY ULTIMATE LOADS FOR COLUMNS, CONTINUOUS BEAMS AND SIMPLE BEAMS.

(CONTINUOUS)

Combination No	f'_c (ksi)	f_y (ksi)	Probability of the Combination of f'_c and f_y	Corresponding Ultimate Load Column (kips)	Corresponding Ultimate Load of Continuous Beam (kips)	Corresponding Ultimate Load of Simple Beam (kips)
1.	3.220	41.30	0.0020	21.25	27.32	12.27
2.	3.220	44.19	0.0022	21.80	28.84	12.98
3.	3.220	45.80	0.0045	22.08	29.66	13.37
4.	3.220	47.28	0.0045	22.32	30.41	13.72
5.	3.220	48.42	0.0045	22.50	30.98	13.99
6.	3.220	49.44	0.0045	22.66	31.49	14.22
7.	3.220	50.36	0.0045	22.79	31.94	14.43
8.	3.220	51.38	0.0045	22.94	32.43	14.66
9.	3.220	52.52	0.0045	23.10	32.98	14.92
10.	3.220	54.00	0.0045	23.30	33.68	15.25
11.	3.220	55.61	0.0022	23.51	34.43	15.60
12.	3.220	58.50	0.0020	23.86	35.75	16.21
13.	3.630	41.30	0.0022	22.40	27.79	12.50
14.	3.630	44.19	0.0025	22.99	29.37	13.24
15.	3.630	45.80	0.0050	23.29	30.23	13.65
16.	3.630	47.28	0.0050	23.55	31.02	14.02
17.	3.630	48.42	0.0050	23.74	31.61	14.30
18.	3.630	49.44	0.0050	23.91	32.14	14.55

TABLE (5-8). THE COMBINATION f'_c , f_y AND THEIR CORRESPONDING
PROBABILITY ULTIMATE LOADS FOR COLUMNS, CONTINUOUS BEAMS AND SIMPLE BEAMS

(CONTINUOUS)

Combination No	f'_c (ksi)	f_y (ksi)	Probability of the Combination of f'_c and f_y	Corresponding Ultimate Load Column (kips)	Corresponding Ultimate Load of Continuous Beam (kips)	Corresponding Ultimate Load of Simple Beam (kips)
19.	3.630	50.36	0.0050	24.05	32.61	14.77
20.	3.630	51.38	0.0050	24.21	33.13	15.01
21.	3.630	52.52	0.0050	24.38	33.70	15.28
22.	3.630	54.00	0.0050	24.59	34.44	15.63
23.	3.630	55.61	0.0025	24.81	35.23	16.01
24.	3.630	58.50	0.0022	25.19	36.62	16.66
25.	3.860	41.30	0.0045	23.04	28.03	12.61
26.	3.860	44.19	0.0050	23.65	29.64	13.36
27.	3.860	45.80	0.0100	23.96	30.51	13.78
28.	3.860	47.28	0.0100	24.23	31.31	14.15
29.	3.860	48.42	0.0100	24.43	31.92	14.44
30.	3.860	49.44	0.0100	24.60	32.46	14.70
31.	3.860	50.36	0.0100	24.75	32.94	14.93
32.	3.860	51.38	0.0100	24.91	33.47	15.18
33.	3.860	52.52	0.0100	25.08	34.06	15.46
34.	3.860	54.00	0.0100	25.30	34.81	15.81
35.	3.860	55.61	0.0050	25.53	35.62	16.20
36.	3.860	58.50	0.0045	25.92	37.05	16.87

TABLE (5-8). THE COMBINATION f'_c , f_y AND THEIR CORRESPONDING
PROBABILITY ULTIMATE LOADS FOR COLUMNS, CONTINUOUS BEAMS AND SIMPLE BEAMS.

(CONTINUOUS)

Combination No	f'_c (ksi)	f_y (ksi)	Probability of the Combination of f'_c and f_y	Corresponding Ultimate Load Column (kips)	Corresponding Ultimate Load of Continuous Beam (kips)	Corresponding Ultimate Load of Simple Beam (kips)
37.	4.070	41.30	0.0045	23.62	28.22	12.69
38.	4.070	44.19	0.0050	24.25	29.85	13.46
39.	4.070	45.80	0.0100	24.57	30.74	13.88
40.	4.070	47.28	0.0100	24.85	31.55	14.27
41.	4.070	48.42	0.0100	25.05	32.17	14.56
42.	4.070	49.44	0.0100	25.22	32.72	14.82
43.	4.070	50.36	0.0100	25.38	33.21	15.05
44.	4.070	51.38	0.0100	25.55	33.75	15.31
45.	4.070	52.52	0.0100	25.72	34.35	15.59
46.	4.070	54.00	0.0100	25.95	35.12	15.96
47.	4.070	55.61	0.0050	26.19	35.94	16.35
48.	4.070	58.50	0.0045	26.59	37.40	17.05
49.	4.230	41.30	0.0045	24.06	28.36	12.75
50.	4.230	44.19	0.0050	24.70	30.00	13.53
51.	4.230	45.80	0.0100	25.03	30.90	13.96
52.	4.230	47.28	0.0100	25.32	31.72	14.35
53.	4.230	48.42	0.0100	25.52	32.35	14.64
54.	4.230	49.44	0.0100	25.70	32.90	14.91

TABLE (5-8). THE COMBINATION f'_c , f_y AND THEIR CORRESPONDING
PROBABILITY ULTIMATE LOADS FOR COLUMNS, CONTINUOUS BEAMS AND SIMPLE BEAMS.

(CONTINUOUS)

Combination No	f'_c (ksi)	f_y (ksi)	Probability of the Combination of f'_c and f_y	Corresponding Ultimate Load Column (kips)	Corresponding Ultimate Load of Continuous Beam (kips)	Corresponding Ultimate Load of Simple Beam (kips)
55.	4.230	50.36	0.0100	25.86	33.40	15.14
56.	4.230	51.38	0.0100	26.03	33.94	15.40
57.	4.230	52.52	0.0100	26.22	34.55	15.69
58.	4.230	54.00	0.0100	26.45	35.33	16.06
59.	4.230	55.61	0.0050	26.69	36.17	16.46
60.	4.230	58.50	0.0045	27.10	37.64	17.17
61.	4.374	41.30	0.0045	24.45	28.48	12.80
62.	4.374	44.19	0.0050	25.11	30.14	13.59
63.	4.374	45.80	0.0100	25.45	31.05	14.02
64.	4.374	47.28	0.0100	25.74	31.87	14.41
65.	4.374	48.42	0.0100	25.95	32.50	14.71
66.	4.374	49.44	0.0100	26.13	33.06	14.98
67.	4.374	50.36	0.0100	26.30	33.57	19.22
68.	4.374	51.38	0.0100	26.47	34.12	15.48
69.	4.374	52.52	0.0100	26.65	34.73	15.77
70.	4.374	54.00	0.0100	26.89	35.52	16.15
71.	4.374	55.61	0.0050	27.14	36.36	16.55
72.	4.374	58.50	0.0045	27.55	37.86	17.27

TABLE (5-8). THE COMBINATION f'_c , f_y AND THEIR CORRESPONDING
PROBABILITY ULTIMATE LOADS FOR COLUMNS, CONTINUOUS BEAMS AND SIMPLE BEAMS.

(CONTINUOUS)

Combination No	f'_c (ksi)	f_y (ksi)	Probability of the Combination of f'_c and f_y	Corresponding Ultimate Load Column (kips)	Corresponding Ultimate Load of Continuous Beam (kips)	Corresponding Ultimate Load of Simple Beam (kips)
73.	4.506	41.30	0.0045	24.80	28.58	12.89
74.	4.506	44.19	0.0050	25.47	30.25	13.64
75.	4.506	45.80	0.0100	25.82	31.16	14.07
76.	4.506	47.28	0.0100	26.11	32.00	14.47
77.	4.506	48.42	0.0100	26.33	32.63	14.77
78.	4.506	49.44	0.0100	26.52	33.20	15.04
79.	4.506	50.36	0.0100	26.68	33.70	15.28
80.	4.506	51.38	0.0100	26.86	34.26	15.55
81.	4.506	52.52	0.0100	27.05	43.88	15.84
82.	4.506	54.00	0.0100	27.29	35.67	16.22
83.	4.506	55.61	0.0050	27.54	36.53	16.63
84.	4.650	58.50	0.0045	27.96	38.04	17.35
85.	4.650	41.30	0.0045	25.19	28.68	12.89
86.	4.650	44.19	0.0050	25.87	30.36	13.69
87.	4.650	45.80	0.0100	26.22	31.29	14.13
88.	4.650	47.28	0.0100	26.53	32.13	14.53
89.	4.650	48.42	0.0100	26.75	32.77	14.83
90.	4.650	49.44	0.0100	26.94	33.34	15.10

TABLE (5-8). THE COMBINATION f'_c , f_y AND THEIR CORRESPONDING
PROBABILITY ULTIMATE LOADS FOR COLUMNS, CONTINUOUS BEAMS AND SIMPLE BEAMS.

(CONTINUOUS)

Combination No	f'_c (ksi)	f_y (ksi)	Probability of the Combination of f'_c and f_y	Corresponding Ultimate Load Column (kips)	Corresponding Ultimate Load of Continuous Beam (kips)	Corresponding Ultimate Load of Simple Beam (kips)
91.	4.650	50.36	0.0100	27.11	33.85	15.35
92.	4.650	51.38	0.0100	27.29	34.41	15.62
93.	4.650	52.52	0.0100	27.48	35.03	15.91
94.	4.650	54.00	0.0100	27.72	35.84	16.30
95.	4.650	55.61	0.0050	27.98	36.70	16.71
96.	4.650	58.50	0.0045	28.41	38.22	17.44
97.	4.810	41.30	0.0045	25.62	28.79	12.93
98.	4.810	44.19	0.0050	26.32	30.49	13.74
99.	4.810	45.80	0.0100	26.68	31.42	14.18
100.	4.810	47.28	0.0100	26.99	32.27	14.59
101.	4.810	48.42	0.0100	27.21	32.91	14.89
102.	4.810	49.44	0.0100	27.41	33.49	15.17
103.	4.810	50.36	0.0100	27.58	34.00	15.41
104.	4.810	51.38	0.0100	27.76	34.57	15.69
105.	4.810	52.52	0.0100	27.96	35.20	15.99
106.	4.810	54.00	0.0100	28.21	36.01	16.38
107.	4.810	55.61	0.0045	28.47	36.88	16.79
108.	4.810	58.50	0.0050	28.91	38.42	17.53

TABLE (5-8). THE COMBINATION f'_c , f_y AND THEIR CORRESPONDING
PROBABILITY ULTIMATE LOADS FOR COLUMNS, CONTINUOUS BEAMS AND SIMPLE BEAMS.

(CONTINUOUS)

Combination No	f'_c (ksi)	f_y (ksi)	Probability of the Combination of f'_c and f_y	Corresponding Ultimate Load Column (kips)	Corresponding Ultimate Load of Continuous Beam (kips)	Corresponding Ultimate Load of Simple Beam (kips)
109.	5.020	41.30	0.0045	26.17	28.93	12.99
110.	5.020	44.19	0.0050	26.89	30.64	13.80
111.	5.020	45.80	0.0100	27.26	31.58	14.25
112.	5.020	47.28	0.0100	27.58	32.44	14.66
113.	5.020	48.42	0.0100	27.81	33.09	14.97
114.	5.020	49.44	0.0100	28.01	33.67	15.25
115.	5.020	50.36	0.0100	28.19	34.19	15.50
116.	5.020	51.38	0.0100	28.38	34.77	15.77
117.	5.020	52.52	0.0100	28.58	35.40	16.08
118.	5.020	54.00	0.0100	28.84	36.22	16.47
119.	5.020	55.61	0.0050	29.11	37.11	16.90
120.	5.020	58.50	0.0045	29.56	38.67	17.65
121.	5.250	41.30	0.0022	26.77	29.07	13.05
122.	5.250	44.19	0.0025	27.51	30.80	13.87
123.	5.250	45.80	0.0050	27.89	31.75	14.32
124.	5.250	47.28	0.005	28.22	32.62	14.73
125.	5.250	48.42	0.0050	28.46	33.28	15.05
126.	5.250	49.44	0.0050	28.67	33.86	15.33

TABLE (5-8). THE COMBINATION f'_c , f_y AND THEIR CORRESPONDING
PROBABILITY ULTIMATE LOADS FOR COLUMNS, CONTINUOUS BEAMS AND SIMPLE BEAMS.

Combination No	f'_c (ksi)	f_y (ksi)	Probability of the Combination of f'_c and f_y	Corresponding Ultimate Load Column (kips)	Corresponding Ultimate Load of Continuous Beam (kips)	Corresponding Ultimate Load of Simple Beam (kips)
127.	5.250	50.36	0.0050	28.85	34.39	15.58
128.	5.250	51.38	0.0050	29.05	34.97	15.86
129.	5.250	52.52	0.0050	29.25	35.62	16.17
130.	5.250	54.00	0.0050	29.52	36.45	16.57
131.	5.250	55.61	0.0025	29.80	37.34	17.00
132.	5.250	58.50	0.0022	30.26	38.92	17.76
133.	5.660	41.30	0.0020	27.83	29.30	13.13
134.	5.660	44.19	0.0022	28.61	31.06	13.97
135.	5.660	45.80	0.0045	29.01	32.02	14.43
136.	5.660	47.28	0.0045	29.36	32.90	14.85
137.	5.660	48.42	0.0045	29.61	33.57	15.17
138.	5.660	49.44	0.0045	29.83	34.17	15.46
139.	5.660	50.36	0.0045	30.02	34.71	15.71
140.	5.660	51.38	0.0045	30.23	35.30	16.00
141.	5.660	52.52	0.0045	30.45	35.96	16.31
142.	5.660	54.00	0.0045	30.72	36.80	16.72
143.	5.660	55.61	0.0022	31.01	37.71	17.16
144.	5.660	58.50	0.0020	31.50	39.33	17.94
Mean ($\bar{X}$)						
				26.32	33.25	15.06
Standard Deviation (σ) (kips)				2.15	2.68	1.27
Coefficient of Variation (V) %				8.17	8.06	8.45
Student's t_o				8.66	0.97	0.35

$$t_o = \frac{\bar{X}_1 - \bar{X}_2}{S \left[\frac{1}{N_1} + \frac{1}{N_2} \right]^{\frac{1}{2}}} \quad \text{in which } S = \left[\frac{N_1 \sigma_1^2 + N_2 \sigma_2^2}{N_1 + N_2 - 2} \right]^{\frac{1}{2}}$$

where the $\bar{X}_1$, $\bar{X}_2$, σ_1 , σ_2 , N_1 and N_2 in this case are the mean, the standard deviation and the number of samples for observation and calculation respectively; t is student's distribution with degree of freedom $v = N_1 + N_2 - 2$.

Before using the significance test, a hypothesis for this test is necessary. Let μ_1 , μ_2 denote the population mean and the observed mean respectively. There are two possible hypotheses;

$H_o: \mu_1 = \mu_2$, and there is essentially no difference between the means.

$H_i: \mu_1 \neq \mu_2$ and there is a significant difference between the means.

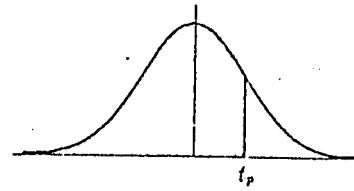
Now according to this hypothesis let us formulate the following rule of decision:

- (a) Reject the hypothesis at 0.02 level of significance, if the distribution t_o is computed to be greater than the tabulated values (see Table (5-8)) for the 0.02 level of significance selected.
- (b) Accept the hypothesis, otherwise.

The t_o values have been computed and given at the bottom of Table (5-7). The t_o calculation for the simple beam is given in the following as a typical calculation of t_o .

TABLE (5-9).

PERCENTILE VALUES (t_p)
for
STUDENT'S t DISTRIBUTION
with ν degrees of freedom
(shaded area = p)



ν	$t_{.995}$	$t_{.99}$	$t_{.975}$	$t_{.95}$	$t_{.90}$	$t_{.80}$	$t_{.75}$	$t_{.70}$	$t_{.60}$	$t_{.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	.727	.325	.158
2	9.92	6.96	4.30	2.92	1.89	1.061	.816	.617	.289	.142
3	5.84	4.54	3.18	2.35	1.64	.978	.765	.584	.277	.137
4	4.60	3.75	2.78	2.13	1.53	.941	.741	.569	.271	.134
5	4.03	3.36	2.57	2.02	1.48	.920	.727	.559	.267	.132
6	3.71	3.14	2.45	1.94	1.44	.906	.718	.553	.265	.131
7	3.50	3.00	2.36	1.90	1.42	.896	.711	.549	.263	.130
8	3.36	2.90	2.31	1.86	1.40	.889	.706	.546	.262	.130
9	3.25	2.82	2.26	1.83	1.38	.883	.703	.543	.261	.129
10	3.17	2.76	2.23	1.81	1.37	.879	.700	.542	.260	.129
11	3.11	2.72	2.20	1.80	1.36	.876	.697	.540	.260	.129
12	3.06	2.68	2.18	1.78	1.36	.873	.695	.539	.259	.128
13	3.01	2.65	2.16	1.77	1.35	.870	.694	.538	.259	.128
14	2.98	2.62	2.14	1.76	1.34	.868	.692	.537	.258	.128
15	2.95	2.60	2.13	1.75	1.34	.866	.691	.536	.258	.128
16	2.92	2.58	2.12	1.75	1.34	.865	.690	.535	.258	.128
17	2.90	2.57	2.11	1.74	1.33	.863	.689	.534	.257	.128
18	2.88	2.55	2.10	1.73	1.33	.862	.688	.534	.257	.127
19	2.86	2.54	2.09	1.73	1.33	.861	.688	.533	.257	.127
20	2.84	2.53	2.09	1.72	1.32	.860	.687	.533	.257	.127
21	2.83	2.52	2.08	1.72	1.32	.859	.686	.532	.257	.127
22	2.82	2.51	2.07	1.72	1.32	.858	.686	.532	.256	.127
23	2.81	2.50	2.07	1.71	1.32	.858	.685	.532	.256	.127
24	2.80	2.49	2.06	1.71	1.32	.857	.685	.531	.256	.127
25	2.79	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
26	2.78	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
27	2.77	2.47	2.05	1.70	1.31	.855	.684	.531	.256	.127
28	2.76	2.47	2.05	1.70	1.31	.855	.683	.530	.256	.127
29	2.76	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
30	2.75	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
40	2.70	2.42	2.02	1.68	1.30	.851	.681	.529	.255	.126
60	2.66	2.39	2.00	1.67	1.30	.848	.679	.527	.254	.126
120	2.62	2.36	1.98	1.66	1.29	.845	.677	.526	.254	.126
∞	2.58	2.33	1.96	1.645	1.28	.842	.674	.524	.253	.126

Source: R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research* (5th edition), Table III, Oliver and Boyd Ltd., Edinburgh, by permission of the authors and publishers.

For given:

$$\bar{X}_1 = 15.16 \text{ kips}$$

$$\bar{X}_2 = 15.06 \text{ kips}$$

$$\sigma_1 = 0.48 \text{ kips}$$

$$\sigma_2 = 1.27 \text{ kips}$$

$$N_1 = 20$$

$$N_2 = 144$$

$$S = \left[\frac{N_1 \sigma_1^2 + N_2 \sigma_2^2}{N_1 + N_2 - 2} \right]^{\frac{1}{2}}$$

$$= \left[\frac{20(0.48)^2 + 144(1.27)^2}{20 + 144 - 2} \right]^{\frac{1}{2}}$$

$$= 1.21$$

$$(t_o) = \frac{\bar{X}_1 - \bar{X}_2}{S \left[\frac{1}{N_1} + \frac{1}{N_2} \right]^{\frac{1}{2}}} = \frac{15.16 - 15.06}{1.21 \left[\frac{1}{20} + \frac{1}{144} \right]^{\frac{1}{2}}}$$

$$= 0.35 < t_{.98, 162}$$

Comparing the computed t_o and tabulated t_p values, it can be seen that t_o for both the simple beam and the continuous beam is smaller than the t_p ($= 2.10$) at 0.02 level of significance. Therefore it is concluded that there is no significant difference between the observed mean and the theoretically calculated mean for simple beam and continuous beam. In other words, the experimental results are from the same population as the theoretical results. However the student 't' showed a significant difference between means of the measured and theoretical distributions

for the columns. Thus the experimental results and theoretical results cannot be considered as coming from the same population. Hence it must be concluded that the column prediction formulae is overly conservative.

In summary the beam prediction formulae given by the ACI-NBC accurately predict the measured failure loads. However the column prediction formulae is conservative.

5-6 Comparison of Results with Design Formulae

The calculated design failure loads for the beams, using a cylinder strength of 3620 psi. (which meets the specifications), are shown in Figures (5-9) and (5-10) using both $\phi = 1$ and $\phi = 0.90$.

Similarly the column design failure loads using a cylinder strength of 3620 psi. and the appropriate ϕ and slenderness strength reduction factors is shown in Figure (5-8).

It can be seen that for both simple and continuous beams a possibility of a member strength less than the design is possible. Thus it must be assumed that, as not many beams fail in practice, the design value of the load must be unduly conservative.

The column design formulae, using both the ϕ factor and the length reduction factor, is so conservative that there is a very low probability of a column capacity less than the design ultimate load. The eccentric load column prediction formulae needs to be modified to predict the actual column behaviour of short columns. Further the length strength reduction factors need to be improved to represent more accurately long column behaviour.

The probabilities of failure, calculated from the theoretical distribution of ultimate load capacity, are summarised in Table (5.10).

TABLE (5-10). PROBABILITY OF FAILURE

Member	Design Method	Probability of Failure
Tied Column	NBC $\phi = 0.75, R = 0.75$	<0.01%
	ACI $\phi = 0.70, R = 0.73$	<0.01%
Simple Beam	NBC & ACI $\phi = 0.90$	6.3%
Two-Span Continuous Beam	NBC & ACI $\phi = 0.90$	7.35%

Chapter 6.

Conclusions

The main conclusion which can be drawn from this investigation may be summarised as follows:

1. The ACI-NBC prediction formulae ($\phi = 1$) accurately predict the actual failure loads of both simple and continuous beams providing the measured mean values of steel yield stress and cylinder strength are used in the calculations.
2. The probability of the reinforced concrete beams failing at a load less than the design ultimate load, using a ϕ factor of 0.90, is 6.3% and 7.35% for the simple beam and continuous beam respectively.
3. The ACI-NBC column prediction formulae, allowing for the effect of increased lateral deformation with length, is some 15% conservative compared with the experimental results.
4. The probability of a column failure at a load less than the design ultimate load is less than 0.01% using the NBC ϕ factor of 0.75 and a length reduction factor of 0.75. The probability of failure using the ACI ϕ factor of 0.70 and length reduction factor of 0.73 is less than 0.01%.
5. Using ϕ factors of 0.9 for the beam and 0.7 for the column gives a lower probability of failure (ie. greater margin of safety) for the columns. This was the intention of the Building Code Committees as column failures are more catastrophic than beam failures.

6. It is recommended that a more complete experimental investigation be carried out to improve the accuracy of the column prediction formulae and to derive realistic length-strength reduction factors to allow for slenderness effects.

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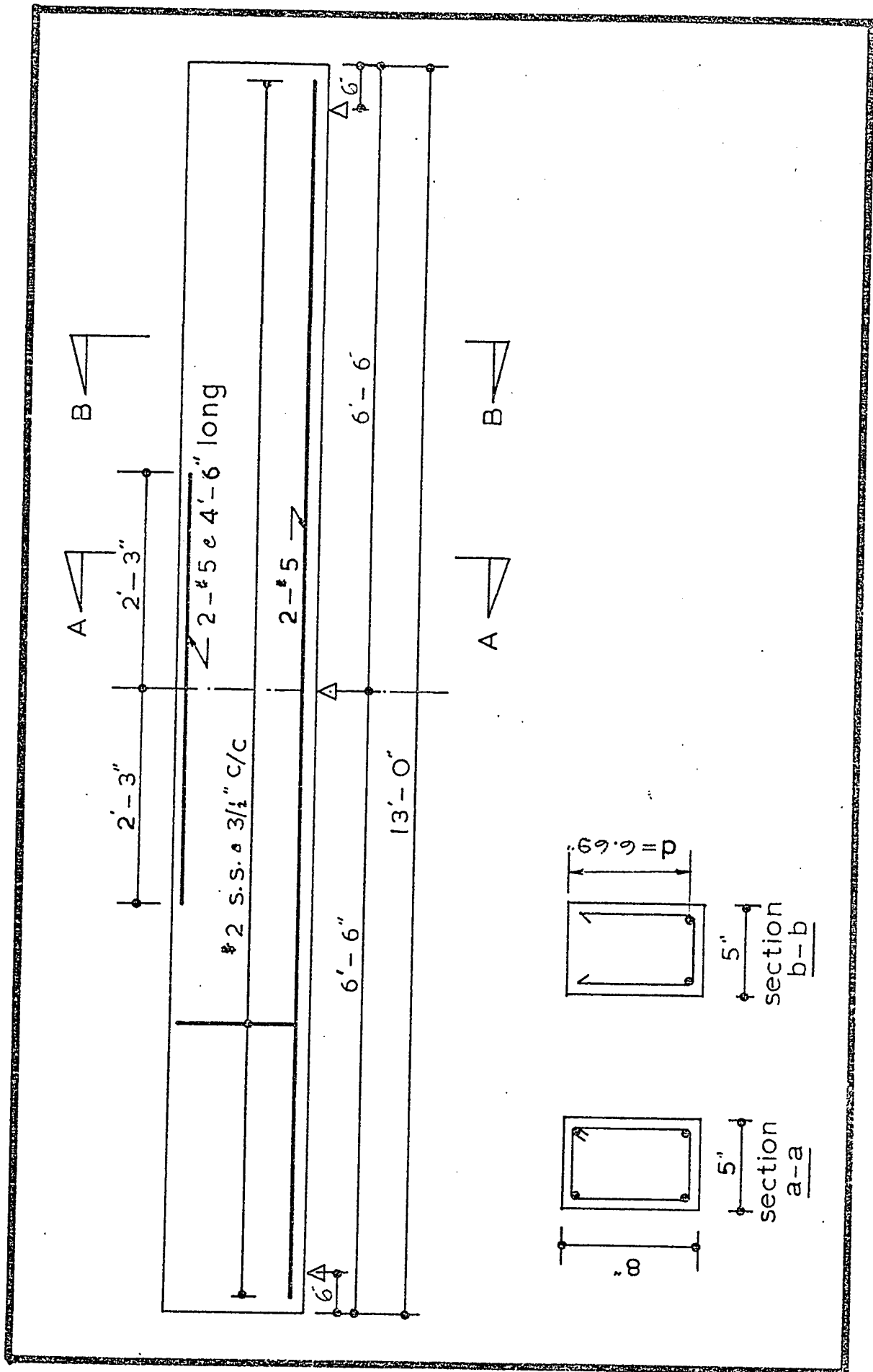


FIG.(4-1). DETAIL OF TWO-SPAN CONTINUOUS BEAM

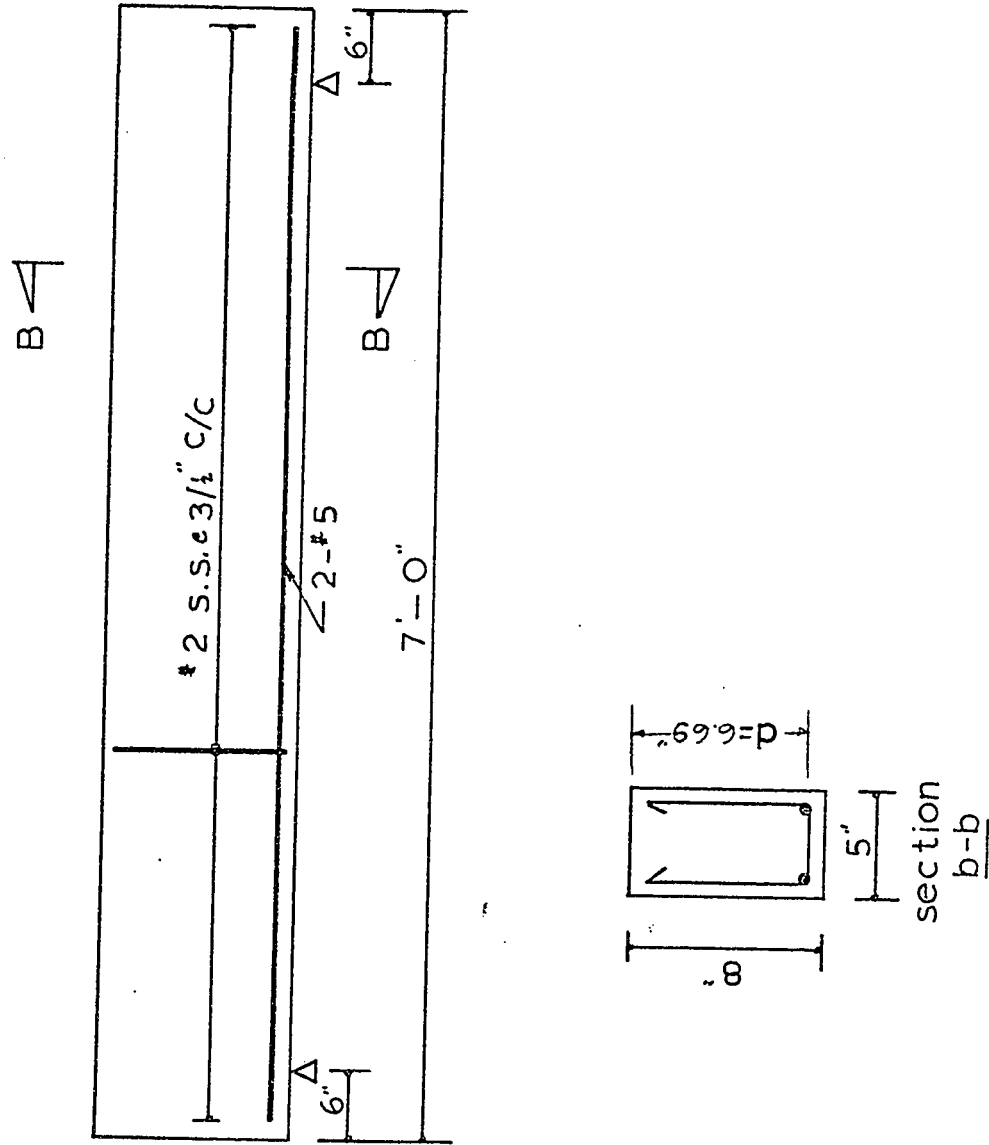


FIG.(4-2). DETAIL OF
SIMPLY SUPPORTED BEAM

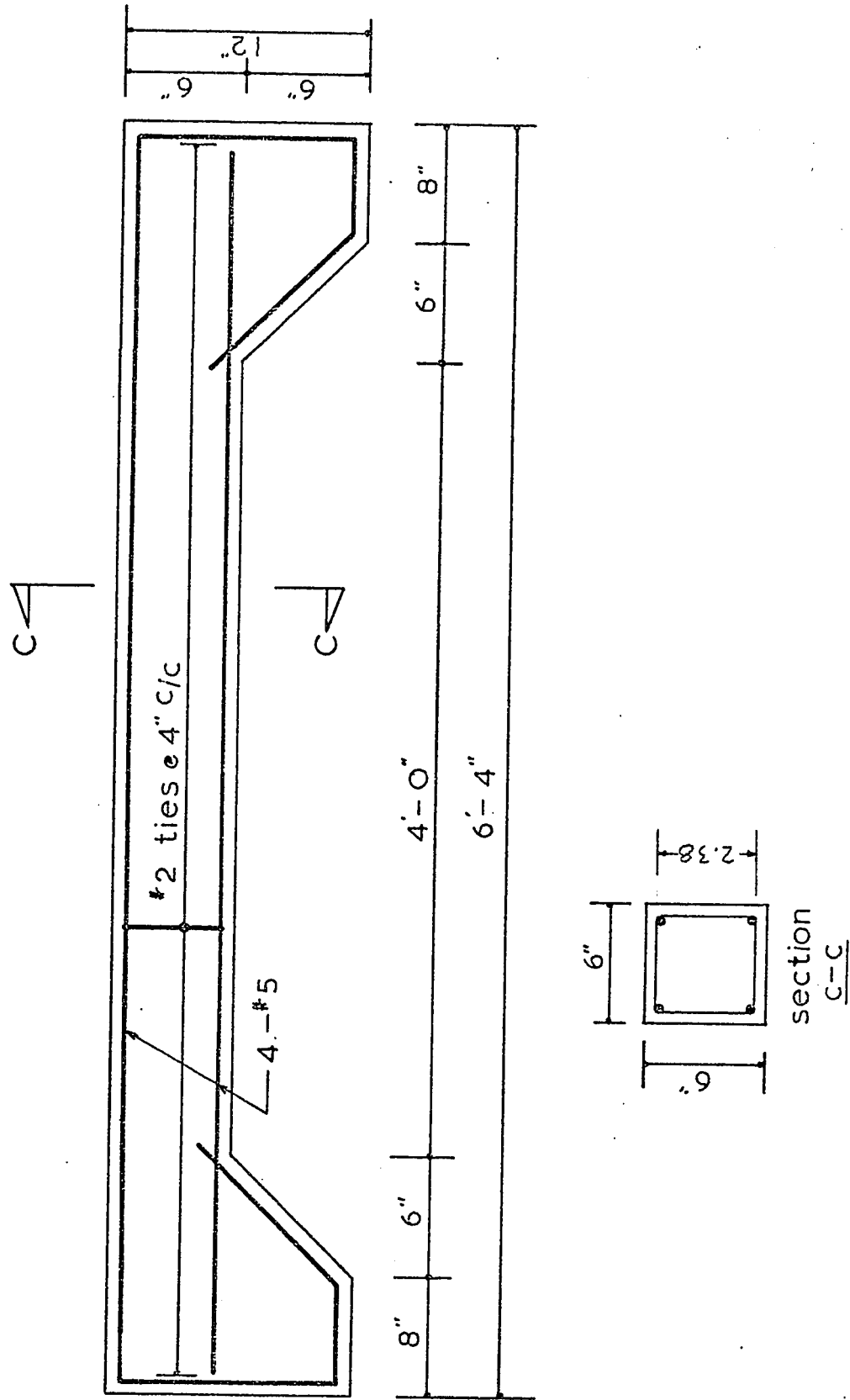


FIG. (4-3). DETAIL OF
COLUMN

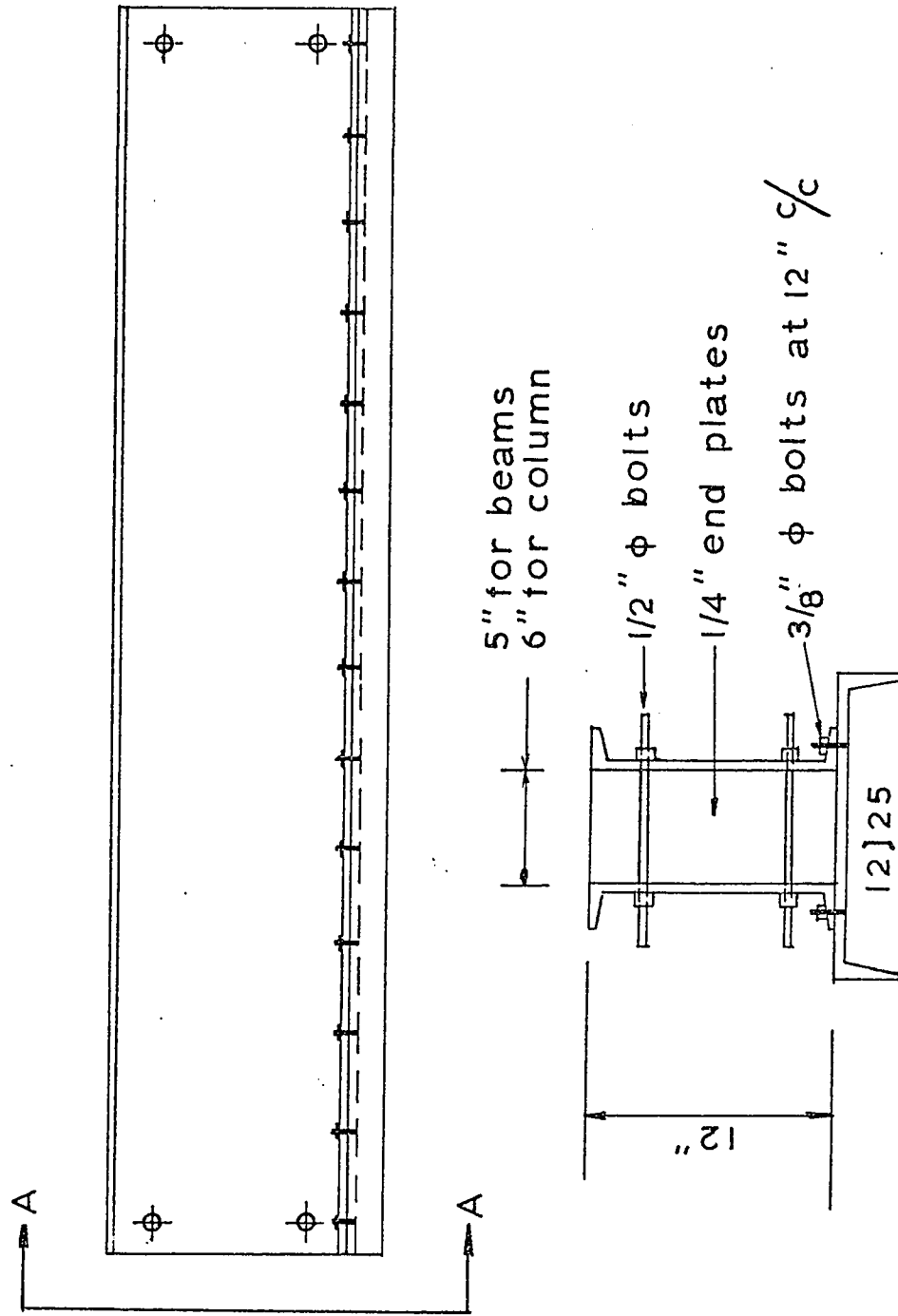


FIG.(4-4) TYPICAL STEEL FORMS

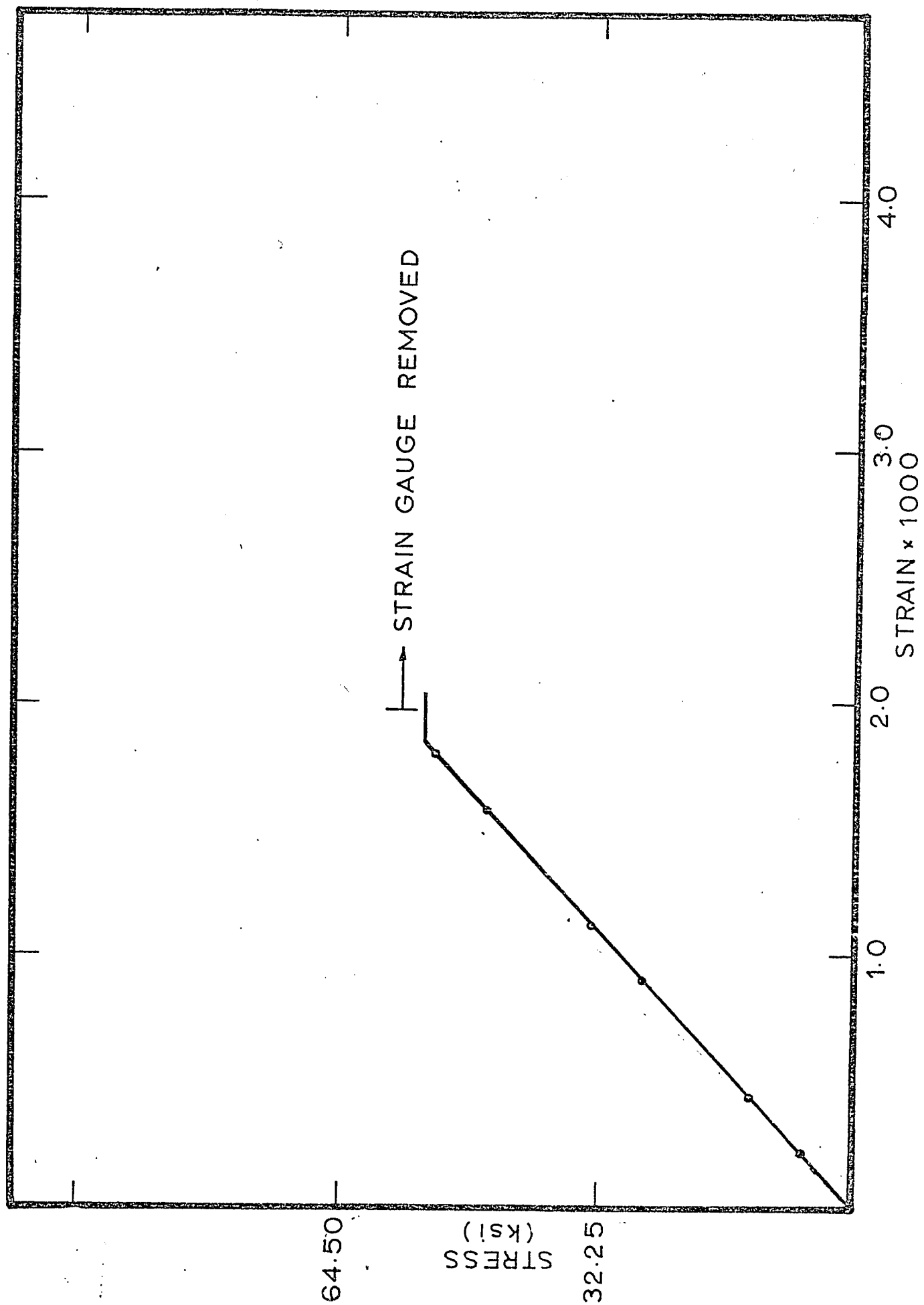


FIG.(4-5) — STRESS-STRAIN CURVE FOR REINFORCING BARS (#5)

71.

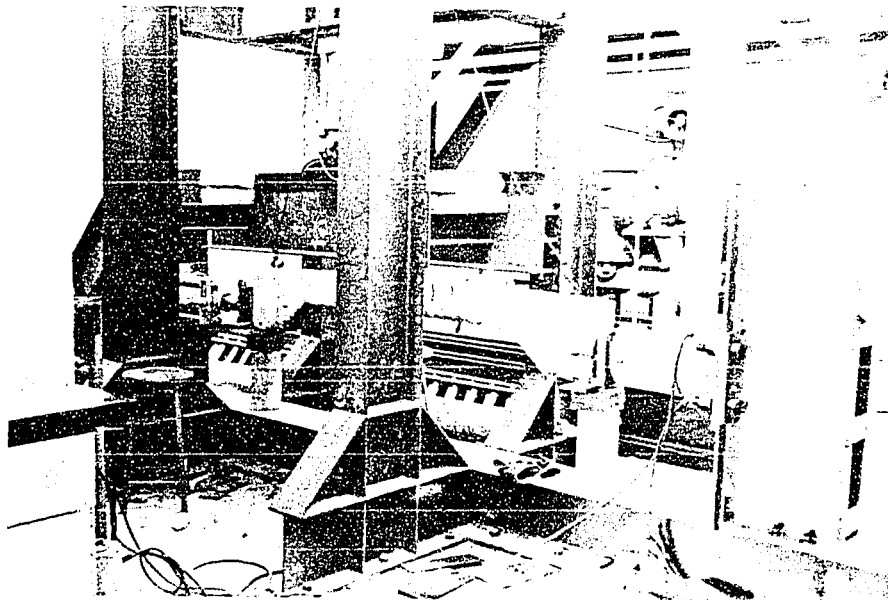
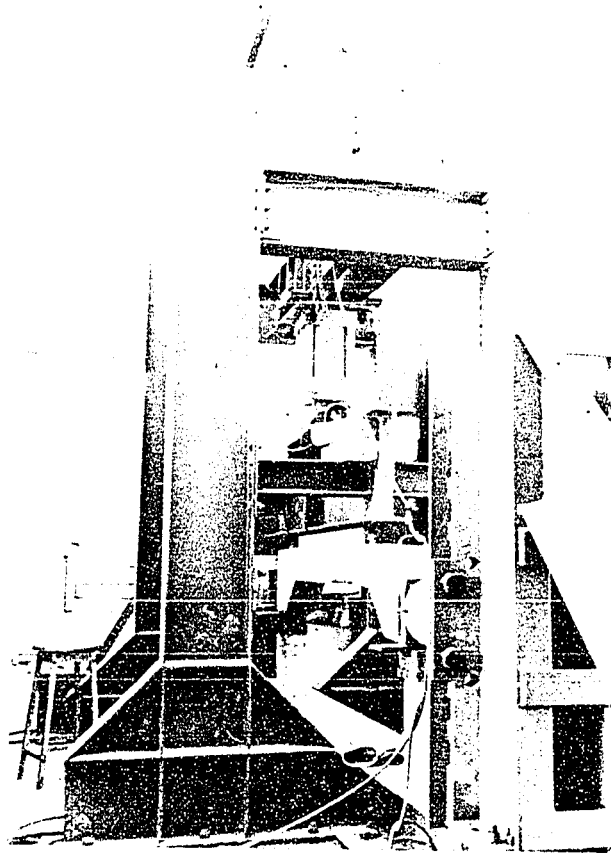


FIG.(4-6). GENERAL VIEW OF COLUMN TEST

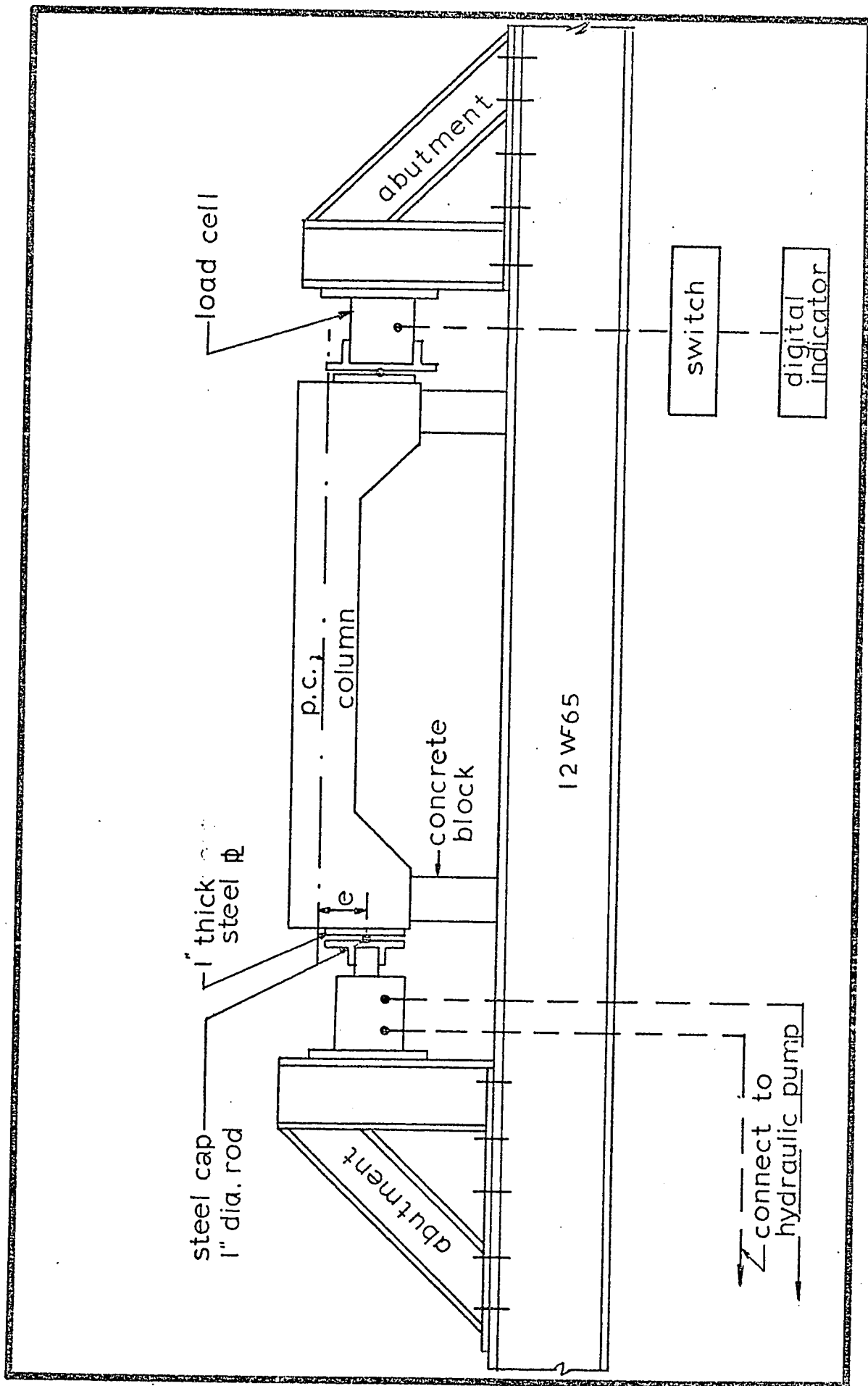


FIG.(4-7). SKETCH OF TEST SET UP FOR COLUMN

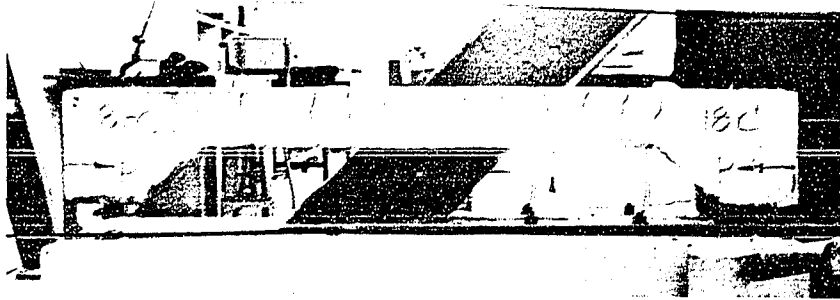


FIG.(4-8). TYPICAL FAILURE MODE OF COLUMNS

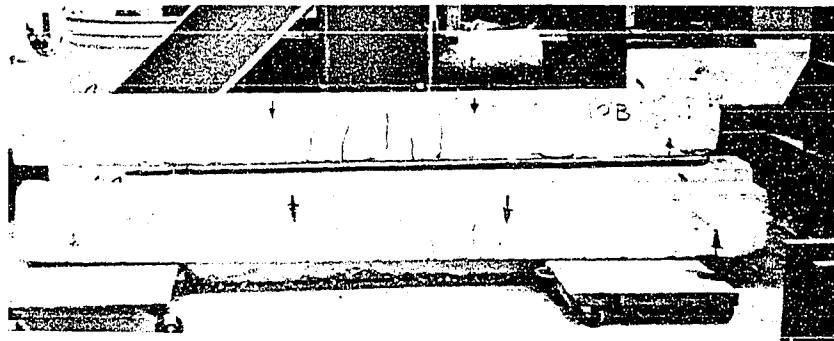


FIG.(4-11). TYPICAL FAILURE MODE OF SIMPLY SUPPORTED BEAMS

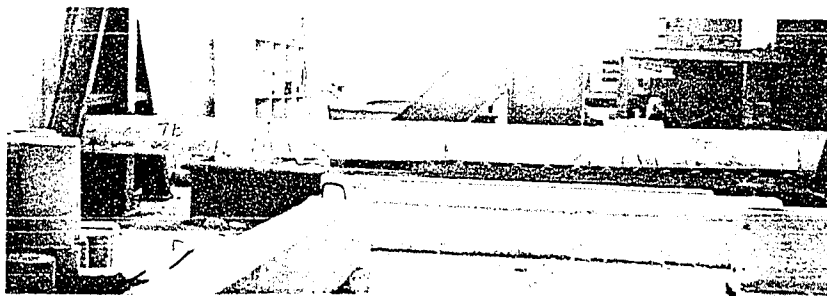


FIG.(4-14). TYPICAL FAILURE MODE OF TWO-SPAN CONTINUOUS BEAMS

FIG.(4-9). GENERAL VIEW OF SIMPLY SUPPORTED BEAM TEST

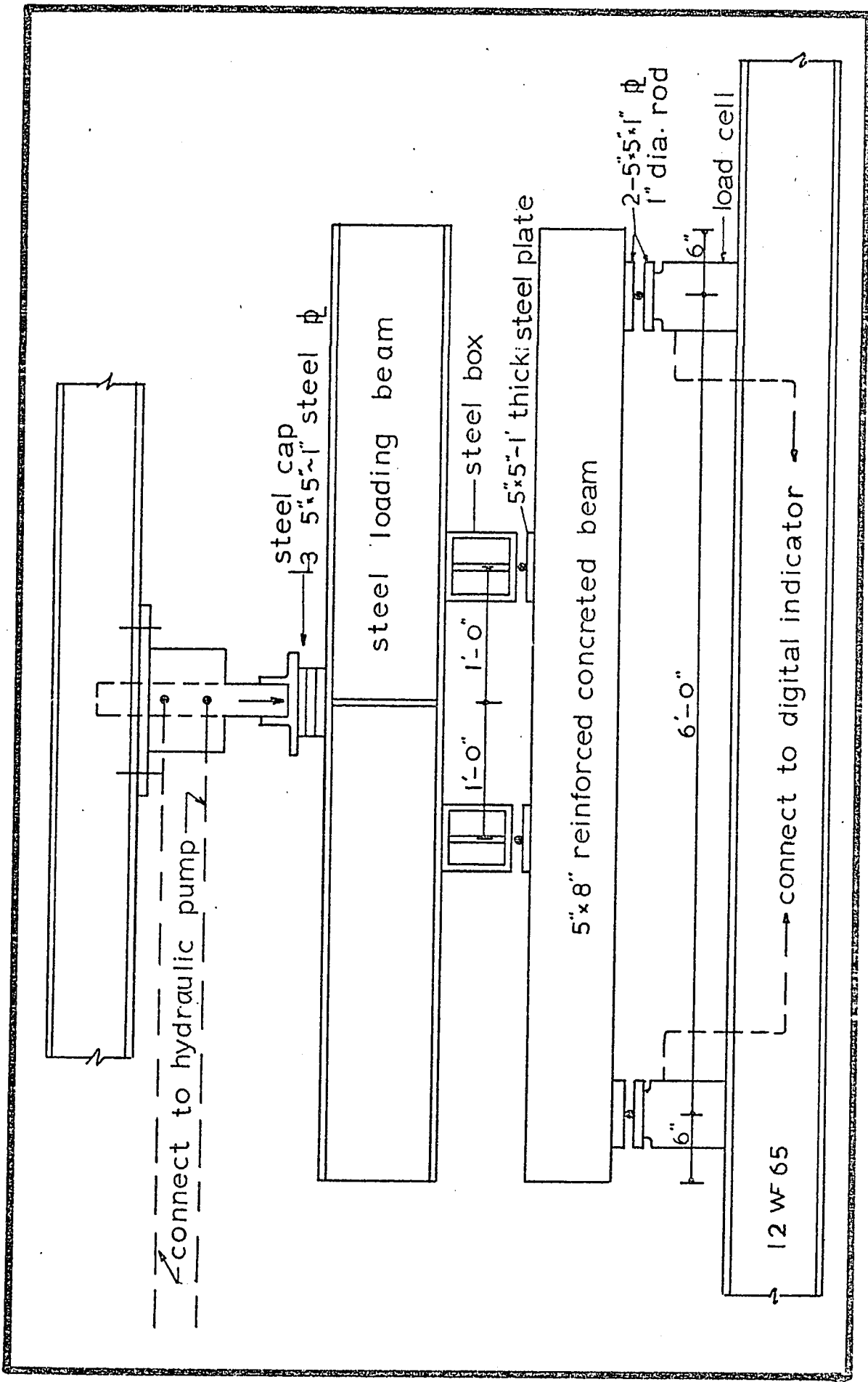


FIG. (4-10). SKETCH OF TEST SET UP
FOR SIMPLY SUPPORTED BEAM

FIG.(4-12). GENERAL VIEW OF TWO-SPAN CONTINUOUS BEAM TEST

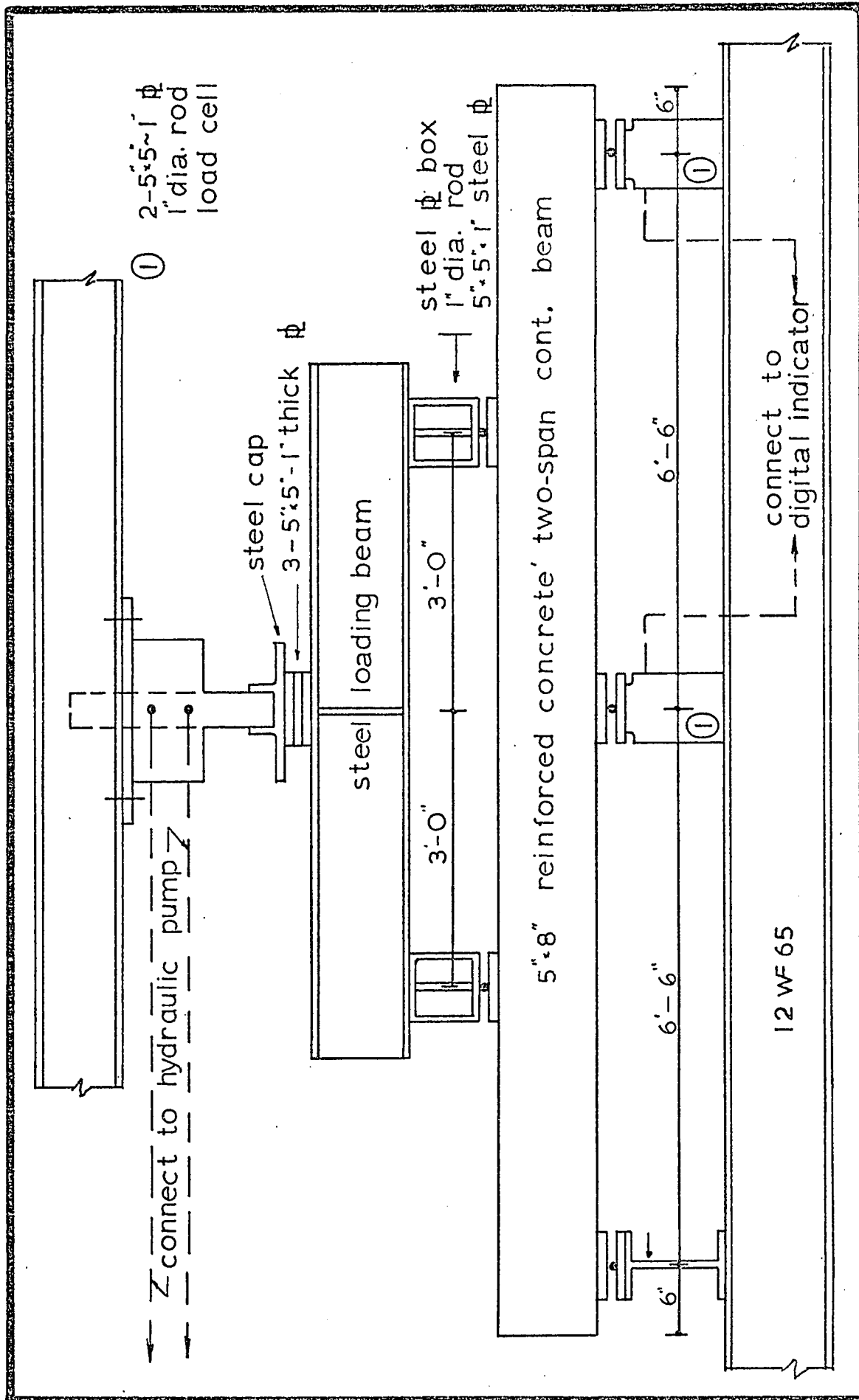


FIG.(4-13). SKETCH OF TEST SET UP
FOR TWO-SPAN CONTINUOUS BEAM

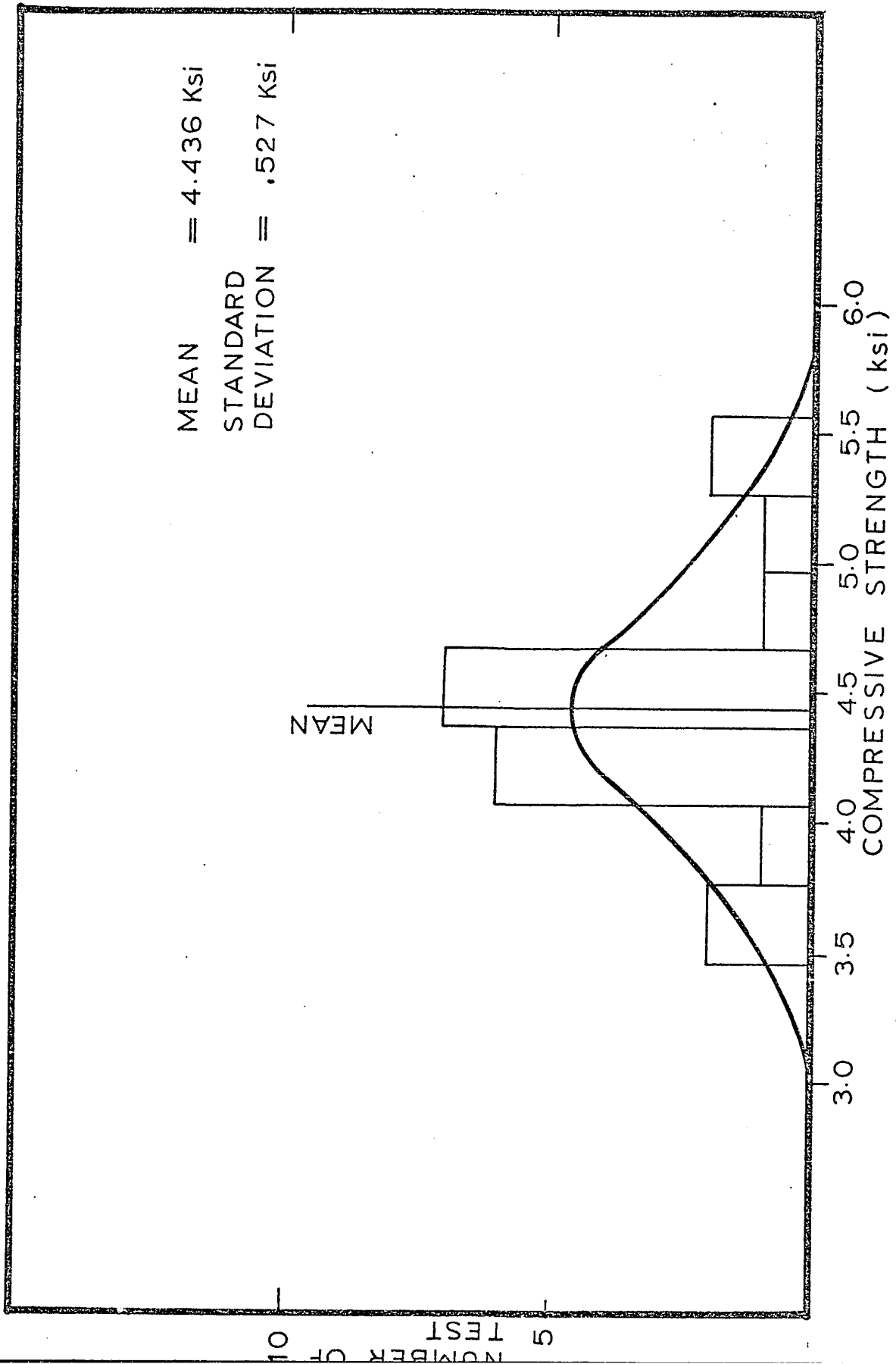


FIG.(5-1) HISTOGRAM & FITTED CURVE FOR 28 DAY CYLINDERS COMPRESSIVE STRENGTH

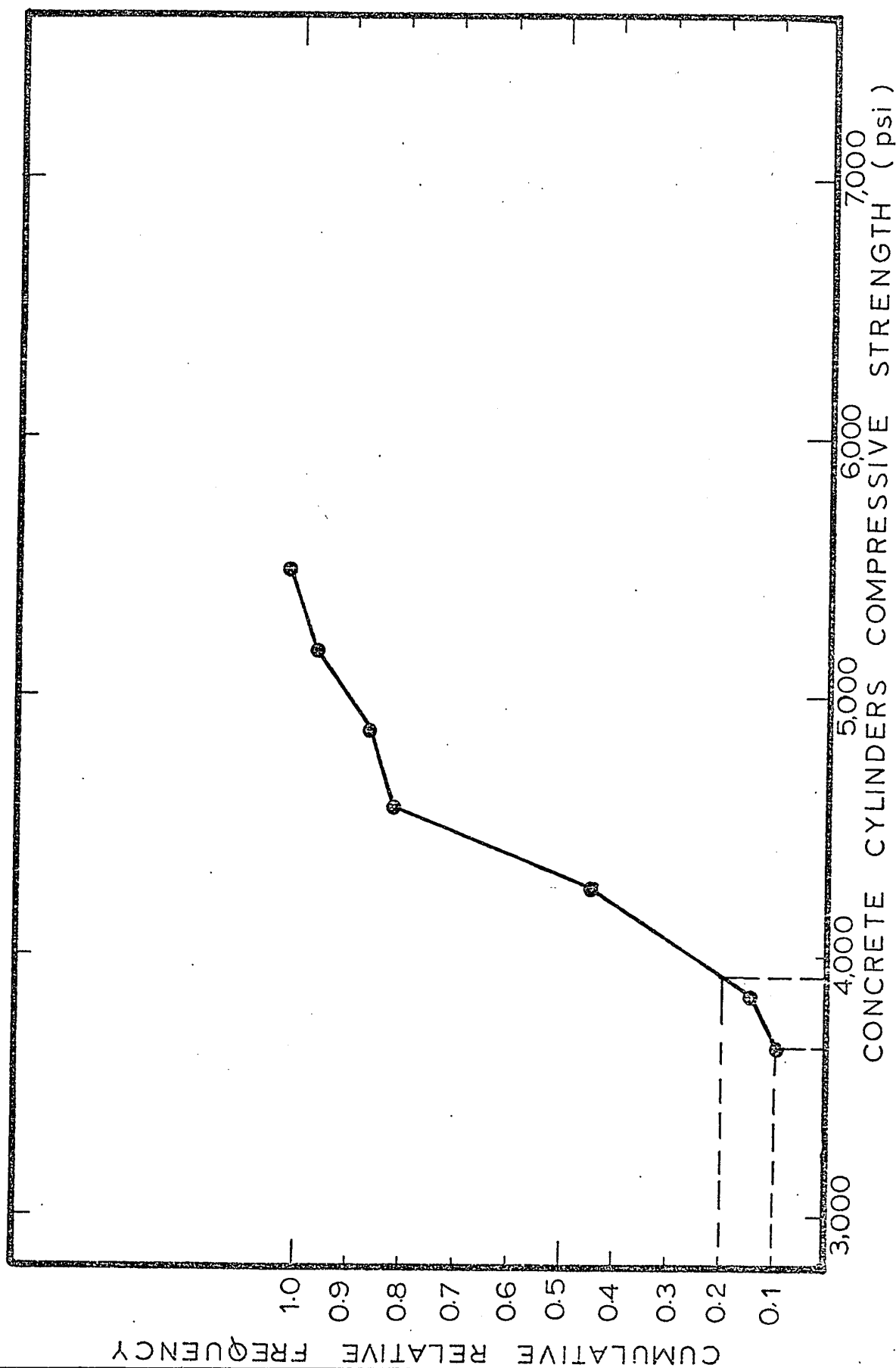


FIG.(5-2) CUMULATIVE RELATIVE FREQUENCY POLYGON FOR CONCRETE CYLINDERS COMPRESSIVE STRENGTH

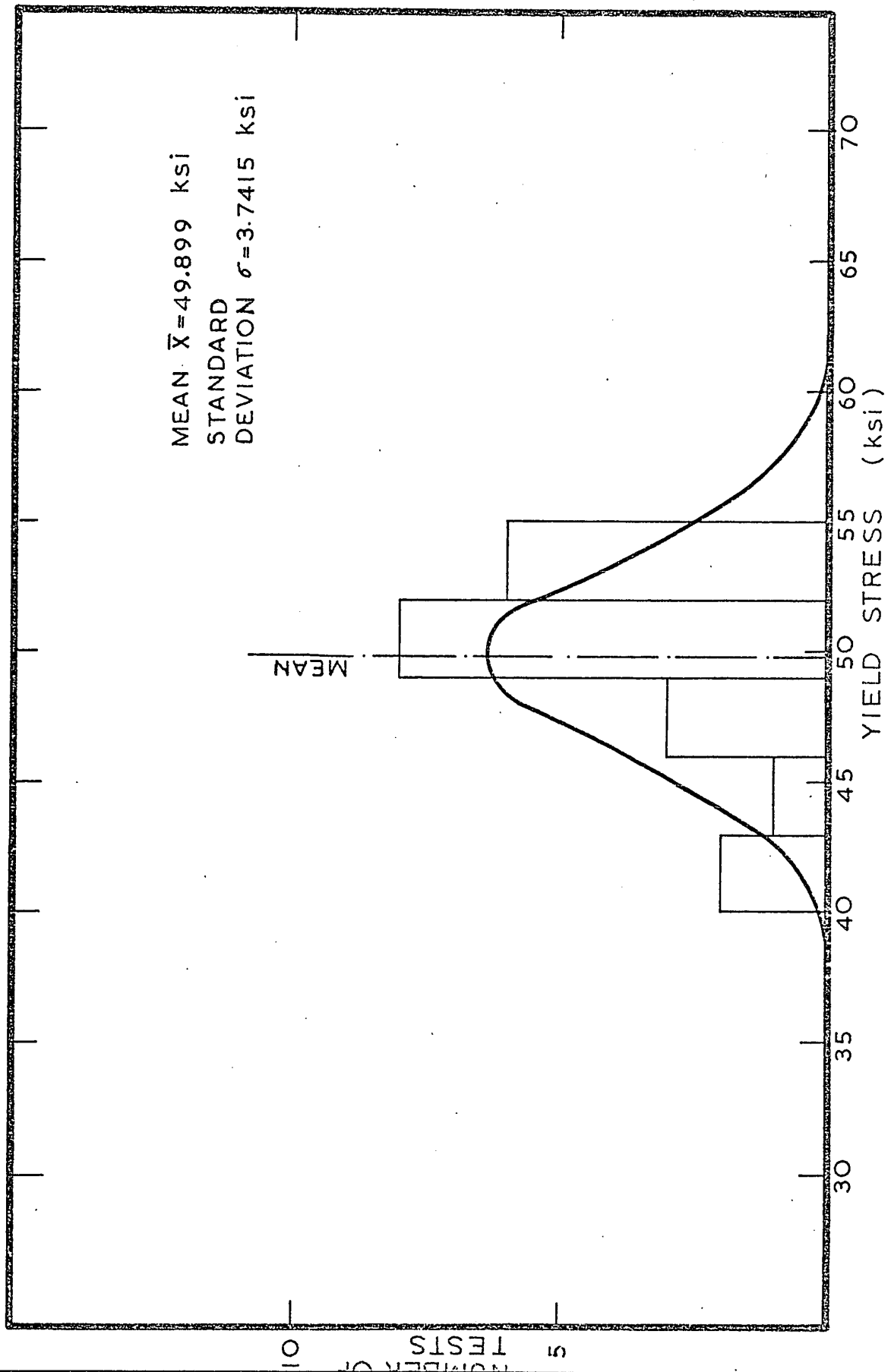


FIG.(5-3) HISTOGRAM & FITTED CURVE OF REINFORCING STEEL YIELD STRESS

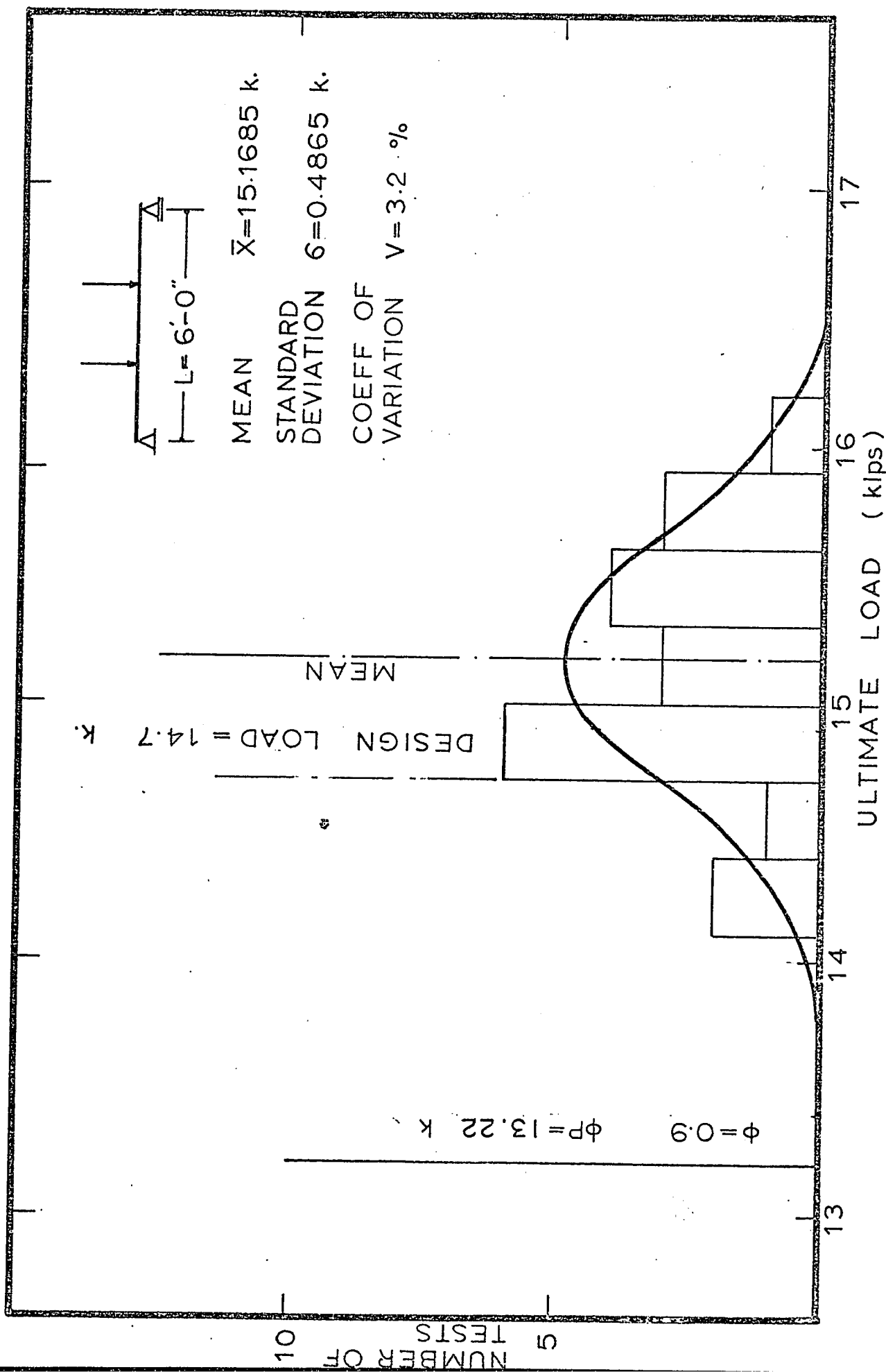


FIG. (5-4). HISTOGRAM & FITTED CURVE OF EXPT. RESULTS FOR 20 SIMPLY SUPPORTED BEAMS

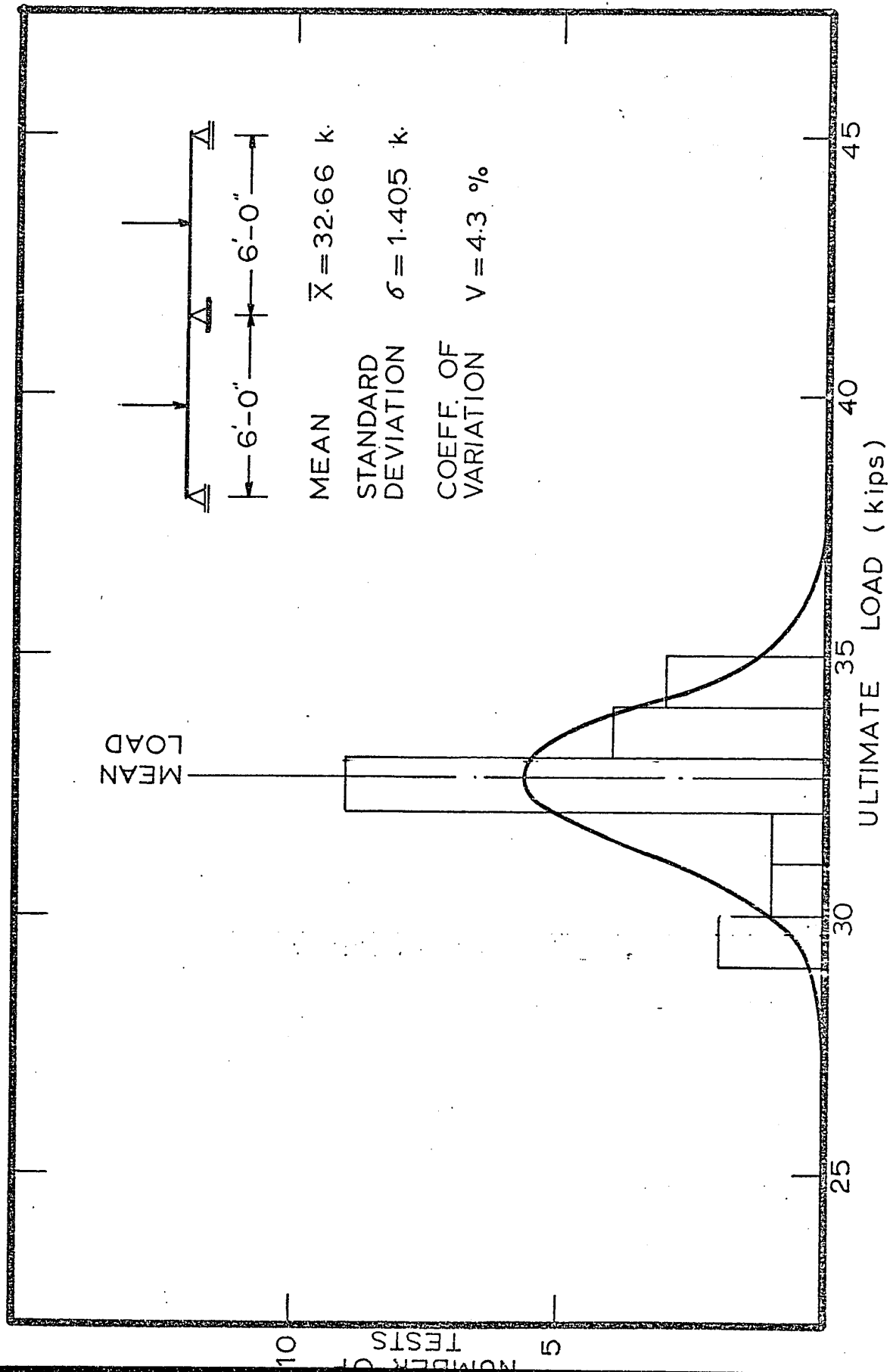


FIG.(5-5). HISTOGRAM & FITTED CURVE OF EXPT. RESULTS FOR 20 CONTINUOUS BEAMS

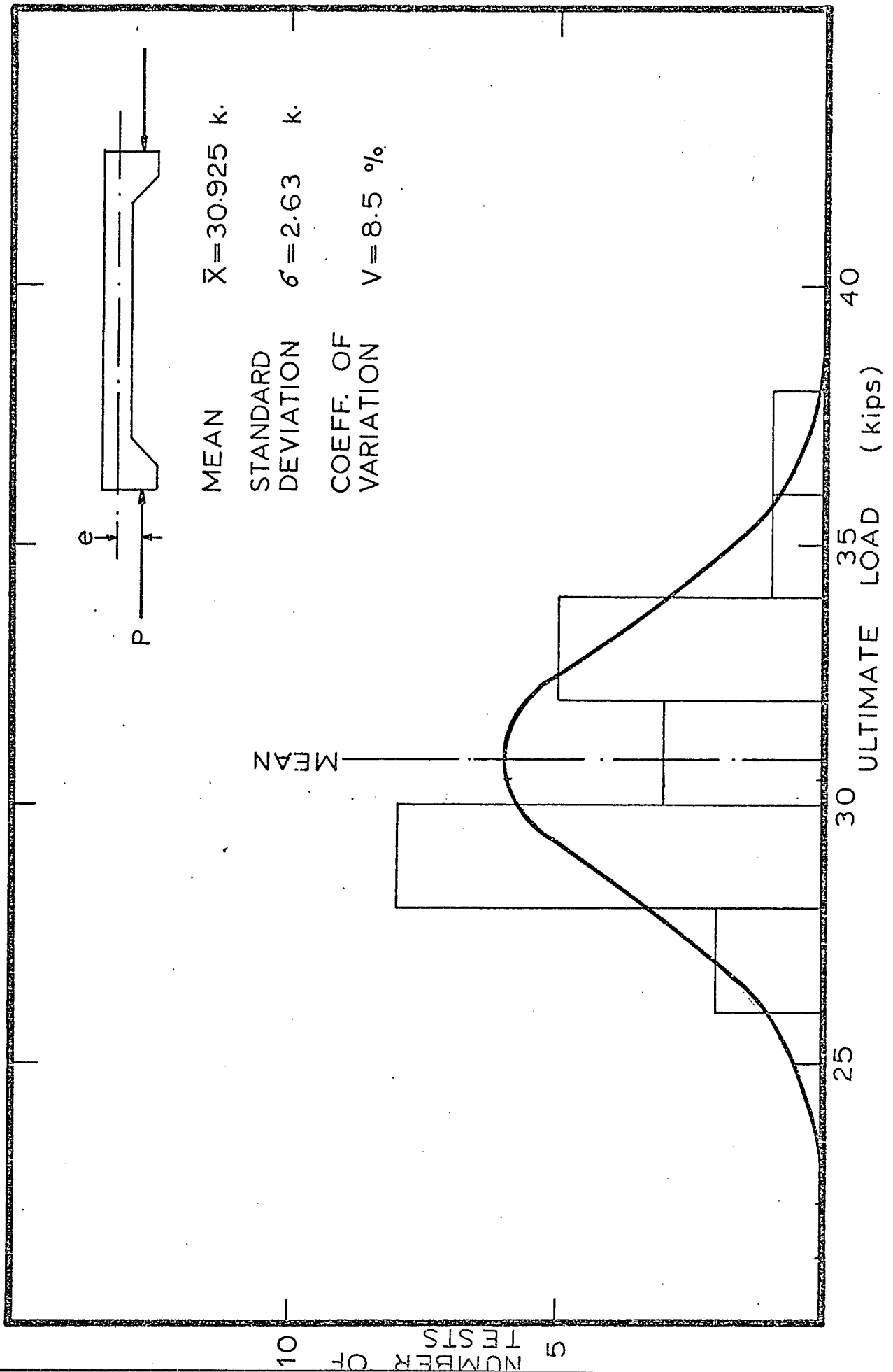


FIG. (5-6). HISTOGRAM & FITTED CURVE OF EXPT. RESULTS FOR 20 COLUMNS

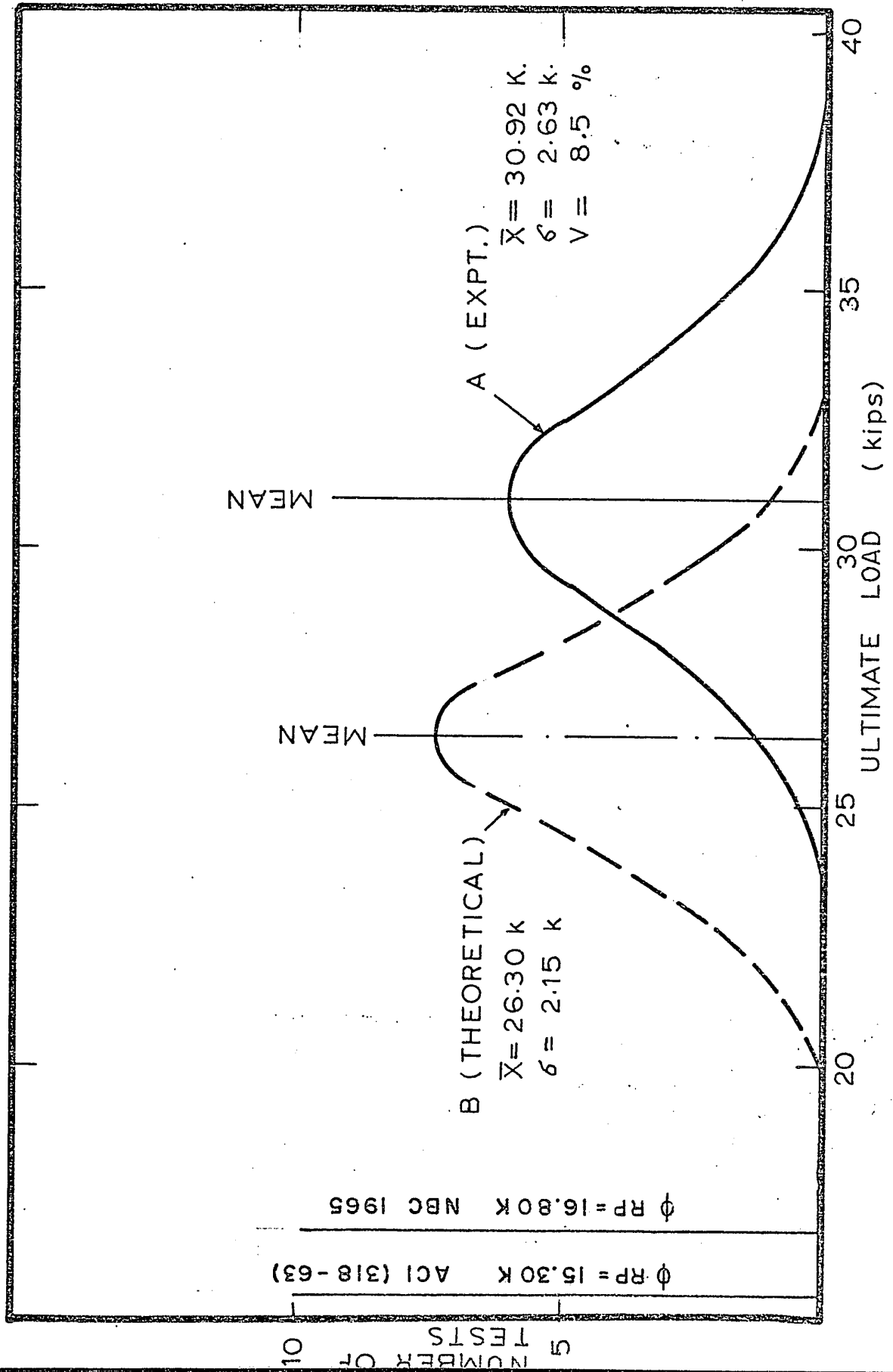


FIG. (5-8). COMPARISON OF EXPT. & THEORETICAL DISTRIBUTION FOR COLUMN

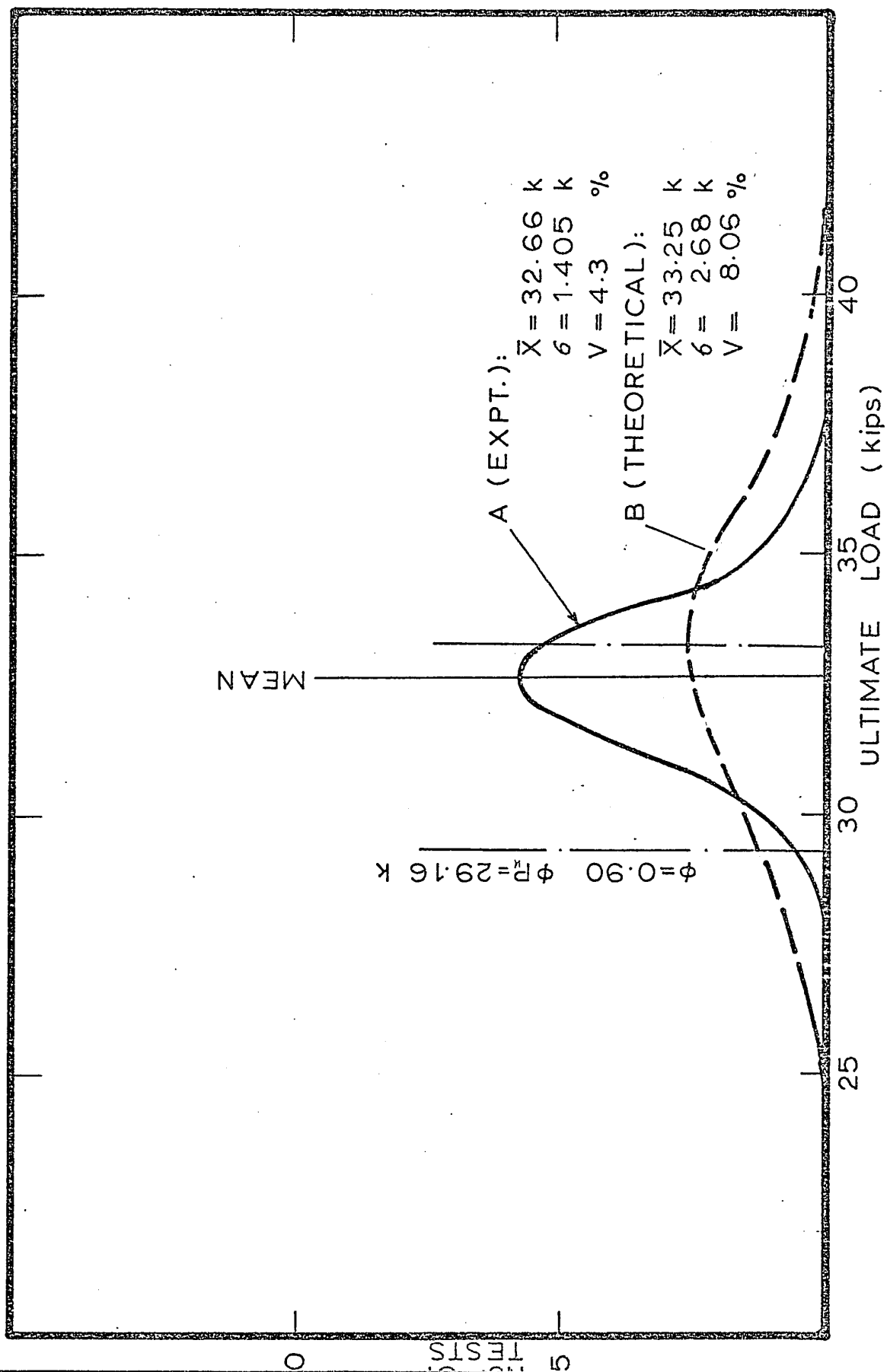


FIG. (5-9) COMPARISON OF EXPT. & THEORETICAL DISTRIBUTIONS FOR CONTINUOUS BEAM

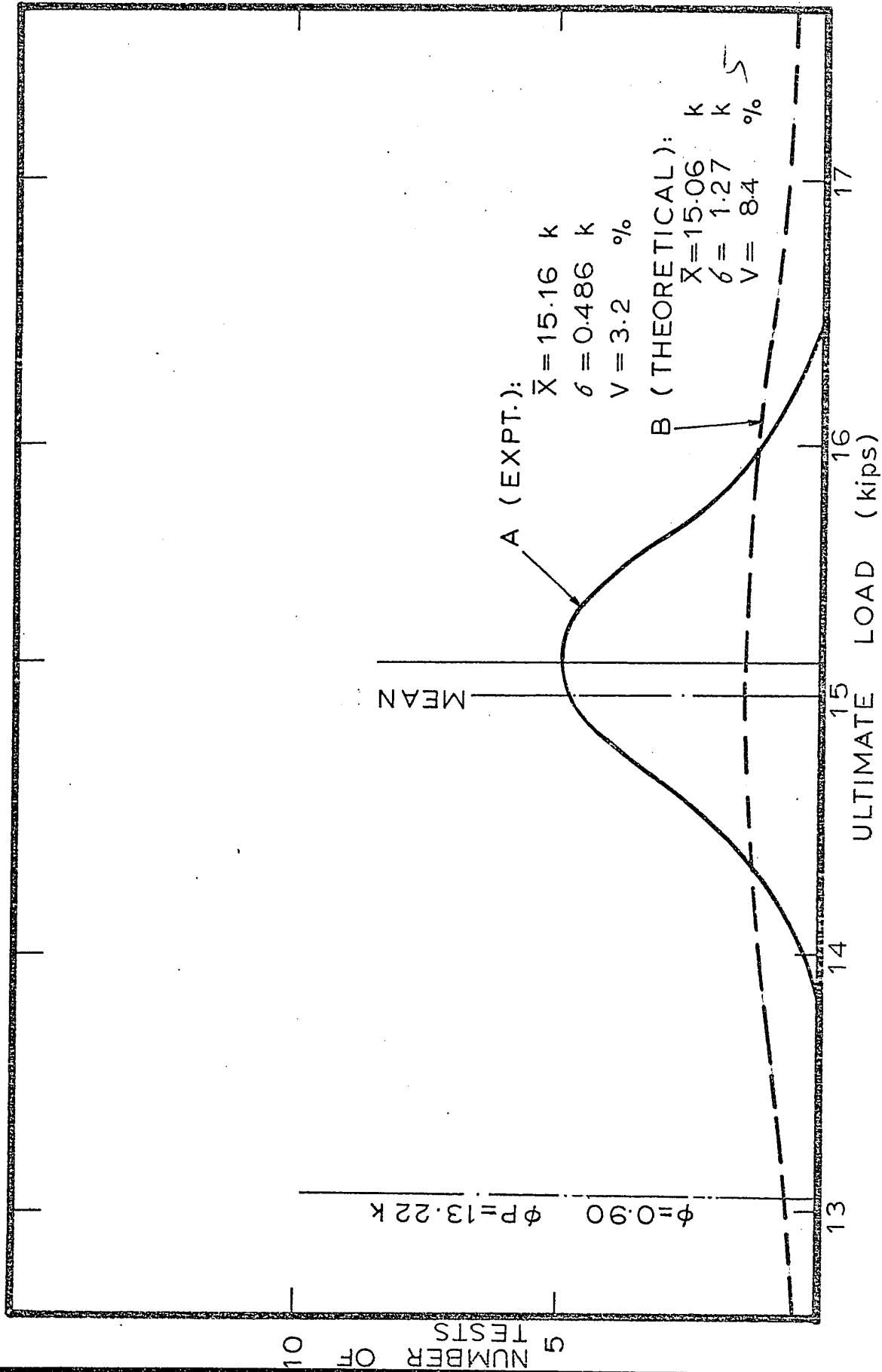


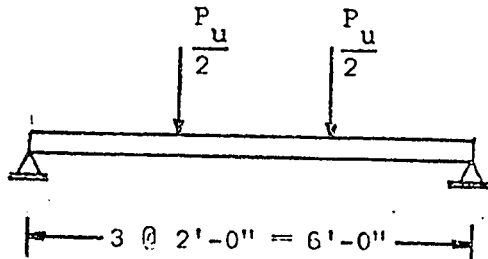
FIG. (5-10). COMPARISON OF EXPT. & THEORETICAL DISTRIBUTIONS FOR SIMPLY SUPPORTED BEAM

APPENDIX A.
(CALCULATIONS)

PRELIMINARY CALCULATIONS

Ultimate load capacity calculations of test specimens

(1) Simply supported beam



For more details see Figure (4-2).

For given: $f'_c = 4,000$ psi

$f_y = 50,000$ psi

$A_s = 0.62$ in²

$b = 5$ in

$d = 6.69$ in

Solution:

$$p_{\max} = 0.75 \times 0.85 k_1 \frac{f'_c}{f_y} \frac{87}{87 - f_y}$$

$$= 0.75 \times 0.85 \times 0.85 \frac{4}{50} \times \frac{87}{137}$$

$$= 0.0274$$

$$p = \frac{A_s}{bd} = \frac{0.62}{5 \times 6.69}$$

$$= 0.0186 < 0.0274$$

Therefore tension controls

$$q = \frac{A_s}{p_f c} = 0.0186 \times \frac{50}{4}$$

$$= 0.2316$$

Substitute into equation (3-4a) with all given values;

$$M_u = b d^2 f_c' a (1 - 0.59q)$$

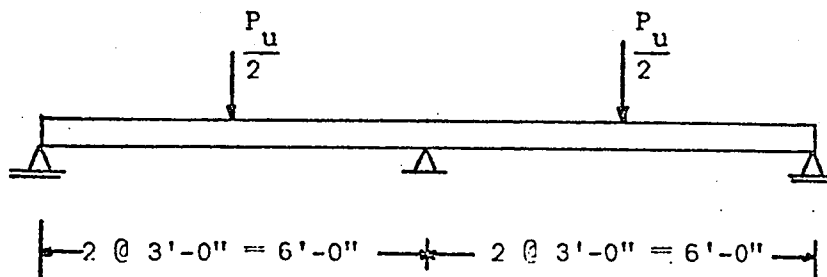
$$= 5 \times 6.69^2 \times 4 \times 0.2316 (1 - 0.59 \times 0.2316)$$

$$= 178.99 \text{ in-kips}$$

$$P_u = \frac{M_u}{12} = 14.92 \text{ kips}$$

$$\phi P_u = 0.90 \times 14.92 = 13.42 \text{ kips}$$

(2) Two-span continuous beam



For more details see Figure (4-1).

For given; $f_c' = 4,000$ psi

$f_y = 50,000$ psi

$A_s = A_s' = 0.62 \text{ in}^2$

$b = 5 \text{ in}$

$d = 6.69 \text{ in}$

$d' = 1.31 \text{ in}$

Solution: Referring to Figure (3-5), let us try

$c = 2.9048 \text{ in}$, from equation (3-6)

$$f_s = 29,000 \times 0.003 \frac{2.9048 - 1.31}{2.9048}$$

$$= 47.763 \text{ ksi}$$

$$M_u' = A_s f_s (c - d') + 0.85 f_c' k c b (c - \frac{k c}{2})$$

$$= 0.62 \times 47.763 (2.9048 - 1.31)$$

$$+ 0.85 \times 4 \times 0.85 \times 2.9048 \times 5$$

$$\times (2.9048 - \frac{2.9048 \times 0.85}{2})$$

$$= 117.336 \text{ in-kips}$$

$$M_u'' = A_s f_y (d - c) = 117.33 \text{ kips}$$

$$M_u' = M_u'' \quad \text{O.K.}$$

$$\text{Therefore } M_2 = M_u' + M_u'' = 234.677 \text{ in-kips}$$

91.

$M_{u1} = 178.991$ in-kips (see the calculations of simple beam)

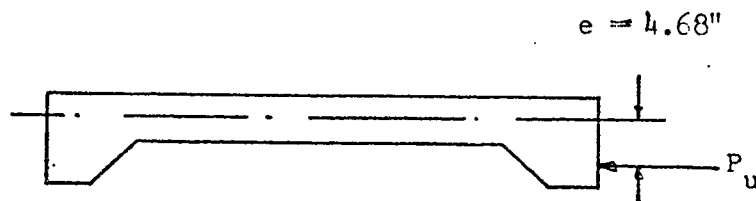
From equation (2-5) obtained

$$P_u = \frac{4M_{u2}}{L} \left(2 \frac{M_{u1}}{M_{u2}} + 1 \right)$$

$$= 4 \times \frac{234.677}{6 \times 12} \left(2 \times \frac{178.911}{234.677} + 1 \right)$$

$$= 32.94 \text{ kips} \quad \phi P_u = 0.90 \times 32.94 = 29.64 \text{ kips}$$

(3) Column



For more details see Figure (4-3).

For given: Cross section 6" x 6"

$$f'_c = 4,000 \text{ psi}$$

$$f_y = 50,000 \text{ psi}$$

$$d = 4.19"$$

$$d' = 1.81 \quad d'' = 1.19"$$

$$A_s = A'_s = 0.62 \text{ in}^2$$

$$e = 4 \frac{11}{16} = 4.688"$$

Referring to Figures (3-6) and (3-7)

$$c_b = \frac{87d}{87 + f_y} = \frac{87 \times 4.9}{50 + 87}$$

$$= 2.66''$$

$$a = k_1 \cdot c_b = 0.85 \times 2.66 = 2.26''$$

$$C_c = 0.85 f_c' ab$$

$$= 0.85 \times 4 \times 2.26 \times 5 = 38.42 \text{ kips}$$

$$C_s = A_s (f_y - 0.85 f_c')$$

$$= 0.62(50 - 0.85 \times 4) = 35.09 \text{ kips}$$

$$T = A_s f_y = 0.62 \times 50 = 31 \text{ kips}$$

$$P_b = C_s + C_c - T = 42.51 \text{ kips}$$

From equation (3-12)

$$P_b e_b = C_c (d - \frac{a}{2} - d'') + C_s (d - d' - d'') + T d''$$

$$= 38.42 \times 1.87 + 35.09 \times 1.19 + 31 \times 1.19$$

$$= 150.492 \text{ in-kips}$$

$$\therefore e_b = \frac{150.492}{42.51} = 3.54'' < e = 4.688''$$

Therefore tension controls.

$$m = \frac{f_y}{0.85 f_c'} = \frac{50}{0.85 \times 40} = 14.7$$

$$e' = \frac{t}{2} - d' + e = (3 - 1.81 + 4.688) = 5.878''$$

$$p = \frac{0.62}{6 \times 4.19} = 0.0246$$

Substitute into equation (3-17) with all given values,

$$p_u = 0.85f_c'bd \left\{ \left(-p + 1 - \frac{e'}{d} \right) + \sqrt{\left(1 - \frac{e'}{d} \right)^2 + 2p \left[(m - 1) \left(1 - \frac{d'}{d} \right) + \frac{e'}{d} \right]} \right\}$$

$$= 0.85 \times 4 \times 6 \times 4.19 \left\{ \left(-0.0246 + 1 - \frac{5.878}{4.19} \right) + \sqrt{\left(1 - \frac{5.878}{4.19} \right)^2 + 2 \times 0.0246 \left[(14.7 - 1) \left(1 - \frac{1.81}{4.19} \right) + \frac{5.878}{4.19} \right]} \right\}$$

$$= 30.42 \text{ kips}$$

The NBC design load is the predicted load multiplied by the ϕ factor and length reduction factor.

$$R = 1.07 - 0.0075 \frac{kh}{r} = 0.75$$

$$\phi = 0.75$$

$$P_{\text{design}} = 17.1 \text{ kips}$$

Similarly the ACI design load is the predicted load reduced by a ϕ factor and a length reduction factor

$$R = 1.07 - 0.008 \frac{kh}{r} = 0.73$$

$$\phi = 0.70$$

$$P_{\text{design}} = 15.5 \text{ kips}$$

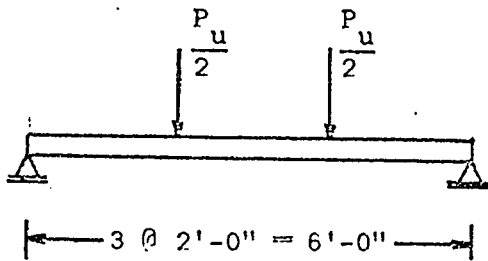
94.

REVISED CALCULATION

REVISED CALCULATIONS

Ultimate load capacity calculations of test specimens

(1) Simply supported beam



For more details see Figure (4-2).

For given: $f'_c = 3,620$ psi $f_y = 50,000$ psi $A_s = 0.62$ in² $b = 5$ in $d = 6.69$ in

Solution:

$$p_{\max} = 0.75 \times 0.85 k_1 \frac{f'_c}{f_y} \frac{87}{87 - f_y}$$

$$= 0.75 \times 0.85 \times 0.85 \frac{3.62}{50} \times \frac{87}{137}$$

$$= 0.0248$$

$$p = \frac{A_s}{bd} = \frac{0.62}{5 \times 6.69}$$

$$= 0.0186 < 0.0248$$

Therefore tension controls

$$q = \frac{A_s}{p_{f_c}} = 0.0186 \times \frac{50}{3.62}$$

$$= 0.257$$

Substitute into equation (3-4a) with all given values;

$$M_u = bd^2 f_c' \phi (1 - 0.59q)$$

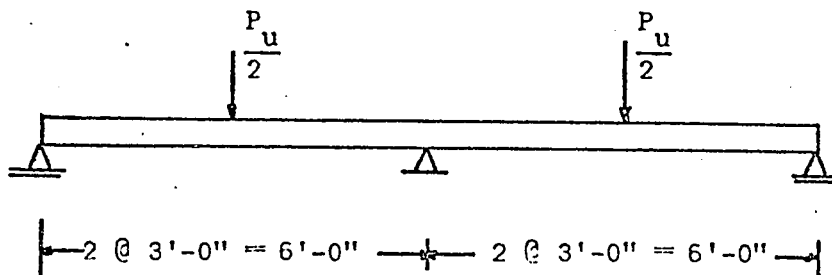
$$= 5 \times 6.69^2 \times 3.62 \times 0.257 (1 - 0.59 \times 0.257)$$

$$= 1716.5 \text{ in-kips}$$

$$P_u = \frac{M_u}{12} = 14.70 \text{ kips}$$

$$\phi P_u = 0.90 \times 14.70 = 13.22 \text{ kips}$$

(2) Two-span continuous beam



For more details see Figure (4-1).

For given; $f_c' = 3,620$ psi

$f_y = 50,000$ psi

$A_s = A_s' = 0.62$ in²

$b = 5$ in

$d = 6.69$ in

$d' = 1.31$ in

Solution: Referring to Figure (3-5), let us try

$c = 2.96$, from equation (3-6)

$$f_s = 29,000 \times 0.003 \frac{2.96 - 1.31}{2.96}$$

$$= 48.5 \text{ ksi}$$

$$M_u' = A_s f_s (c - d') + 0.85 f_c' k c b (c - \frac{k c}{2})$$

$$= 0.62 \times 48.5 (2.96 - 1.31)$$

$$+ 0.85 \times 3.62 \times 0.85 \times 2.96 \times 5$$

$$\times (2.96 - \frac{2.96 \times 0.85}{2})$$

$$= 115.55 \text{ in-kips}$$

$$M_u' = A_s f_y (d - c) = 0.62 \times 50 \times 3.73 = 115.55 \text{ in-kips}$$

$$M_u' = M_u''$$

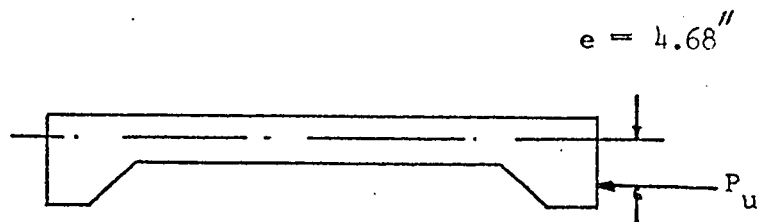
$$\text{Therefore } M_2 = M_u' + M_u'' = 231.10 \text{ in-kips}$$

$$M_{u1} = 176.5 \text{ in-kips (see the calculations of simple beam)}$$

From equation (2-5) obtained

$$\begin{aligned} P_u &= \frac{4M_{u2}}{L} \left(2 \frac{M_{u1}}{M_{u2}} + 1 \right) \\ &= 4 \times \frac{231.1}{6 \times 12} \left(2 \times \frac{176.5}{231.1} + 1 \right) \\ &= 32.4 \text{ kips} \quad \phi P_u = 0.90 \times 32.94 = 29.16 \text{ kips} \end{aligned}$$

(3) Column



For more details see Figure (4-3).

For given: Cross section 6" x 6"

$$f'_c = 3,620 \text{ psi}$$

$$f_y = 50,000 \text{ psi}$$

$$d = 4.19"$$

$$d' = 1.81 \quad d'' = 1.19"$$

$$A_s = A'_s = 0.62 \text{ in}^2$$

$$e = 4 \frac{11}{16} = 4.688"$$

Referring to Figures (3-6) and (3-7)

$$c_b = \frac{87d}{87 + f_y} = \frac{87 \times 4.9}{50 + 87}$$

$$= 2.66''$$

$$a = k_1 \cdot c_b = 0.85 \times 2.66 = 2.26''$$

$$C_c = 0.85 f_c' ab$$

$$= 0.85 \times 3.62 \times 2.26 \times 5 = 34.8 \text{ kips}$$

$$C_s = A_s (f_y - 0.85 f_c')$$

$$= 0.62 (50 - 0.85 \times 3.62) = 29.10 \text{ kips}$$

$$T = A_s f_y = 0.62 \times 50 = 31 \text{ kips}$$

$$P_b = C_s + C_c - T = 32.90 \text{ kips}$$

From equation (3-12)

$$P_b e_b = C_c (d - \frac{a}{2} - d'') + C_s (d - d' - d'') + T d''$$

$$= 34.8 \times 1.87 + 29.10 \times 1.19 + 31 \times 1.19$$

$$= 136.5 \text{ in-kips}$$

$$\therefore e_b = \frac{136.5}{32.9} = 4.15'' < e = 4.688''$$

Therefore tension controls.

$$m = \frac{f_y}{0.85 f_c'} = \frac{50}{0.85 \times 3.62} = 16.2$$

100.

$$e' = \frac{t}{2} - d' + e = (3 - 1.81 + 4.688) = 5.878''$$

$$p = \frac{0.62}{6 \times 4.19} = 0.0246$$

Substitute into equation (3-17) with all given values,

$$P_u = 0.85f_c'bd \left\{ \left(-p + 1 - \frac{e'}{d} \right) + \sqrt{\left(1 - \frac{e'}{d} \right)^2 + 2p \left[(m - 1) \left(1 - \frac{d'}{d} \right) + \frac{e'}{d} \right]} \right\}$$

$$= 0.85 \times 3.62 \times 6 \times 4.19 \left\{ \left(-0.0246 + 1 - \frac{5.878}{4.19} \right) + \sqrt{\left(1 - \frac{5.878}{4.19} \right)^2 + 2 \times 0.0246 \left[(16.2 - 1) \left(1 - \frac{1.81}{4.19} \right) + \frac{5.878}{4.19} \right]} \right\}$$

$$= 29.9 \text{ kips}$$

NBC Design Load

$$R = 0.75$$

$$\phi = 0.75$$

$$P_{\text{design}} = 16.80 \text{ kips.}$$

ACI Design Load

$$R = 0.73$$

$$\phi = 0.70$$

$$P_{\text{design}} = 15.30 \text{ kips.}$$

101.

APPENDIX B.

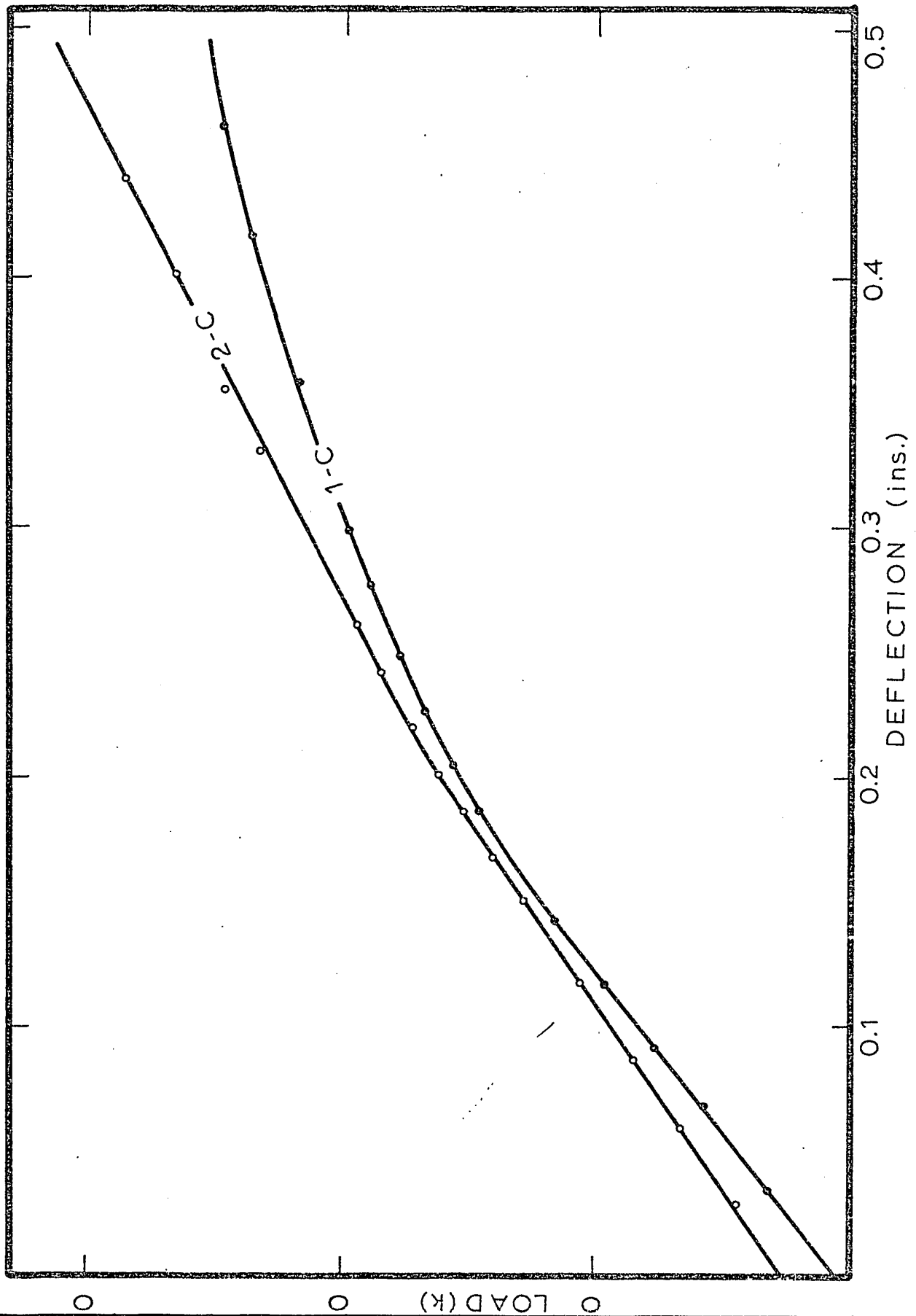


FIG.(B-1) LOAD-DEFLECTION CURVE FOR COLUMN-1 & 2

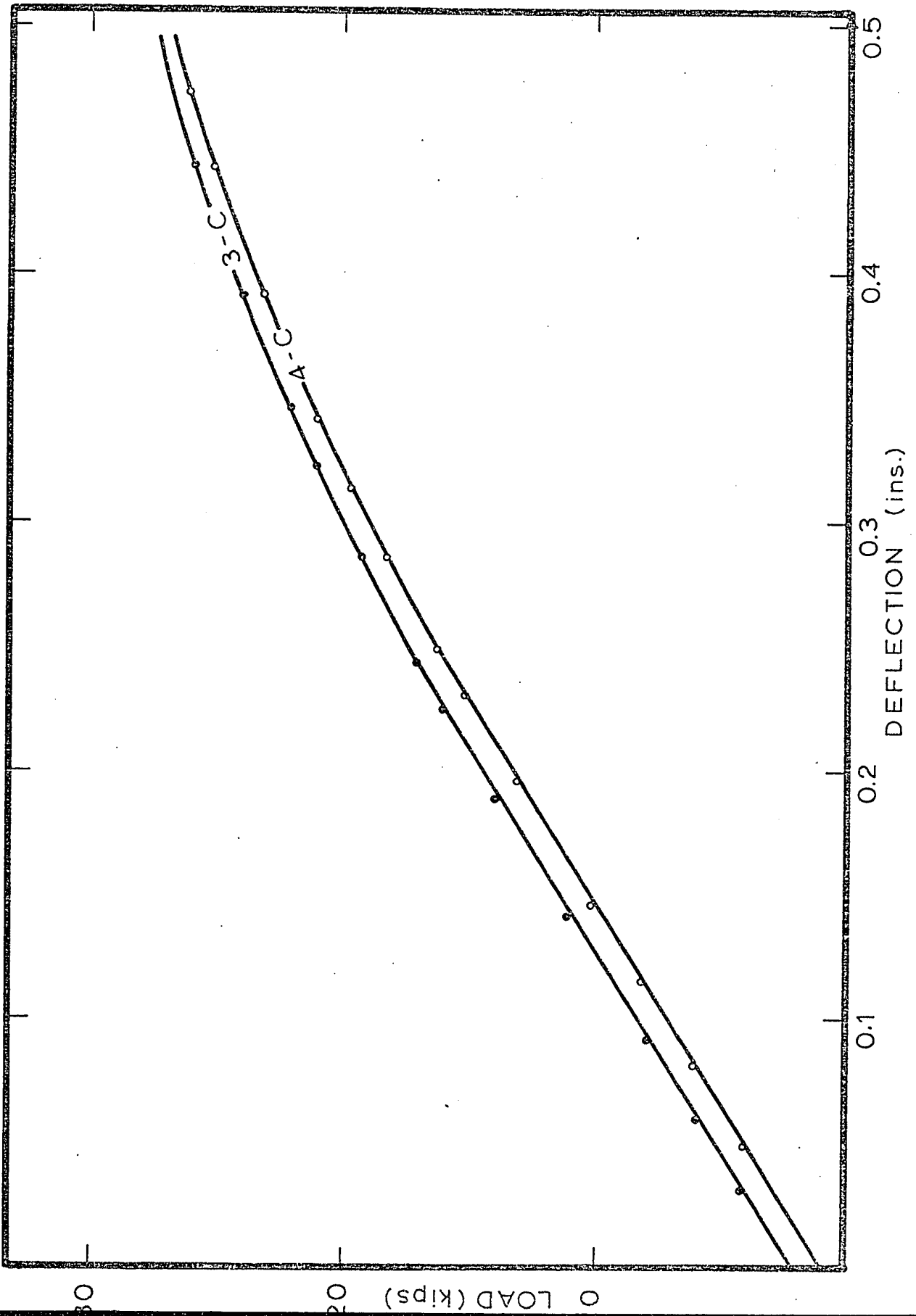


FIG.(B-2) LOAD-DEFLECTION CURVE FOR COLUMN 3 & 4

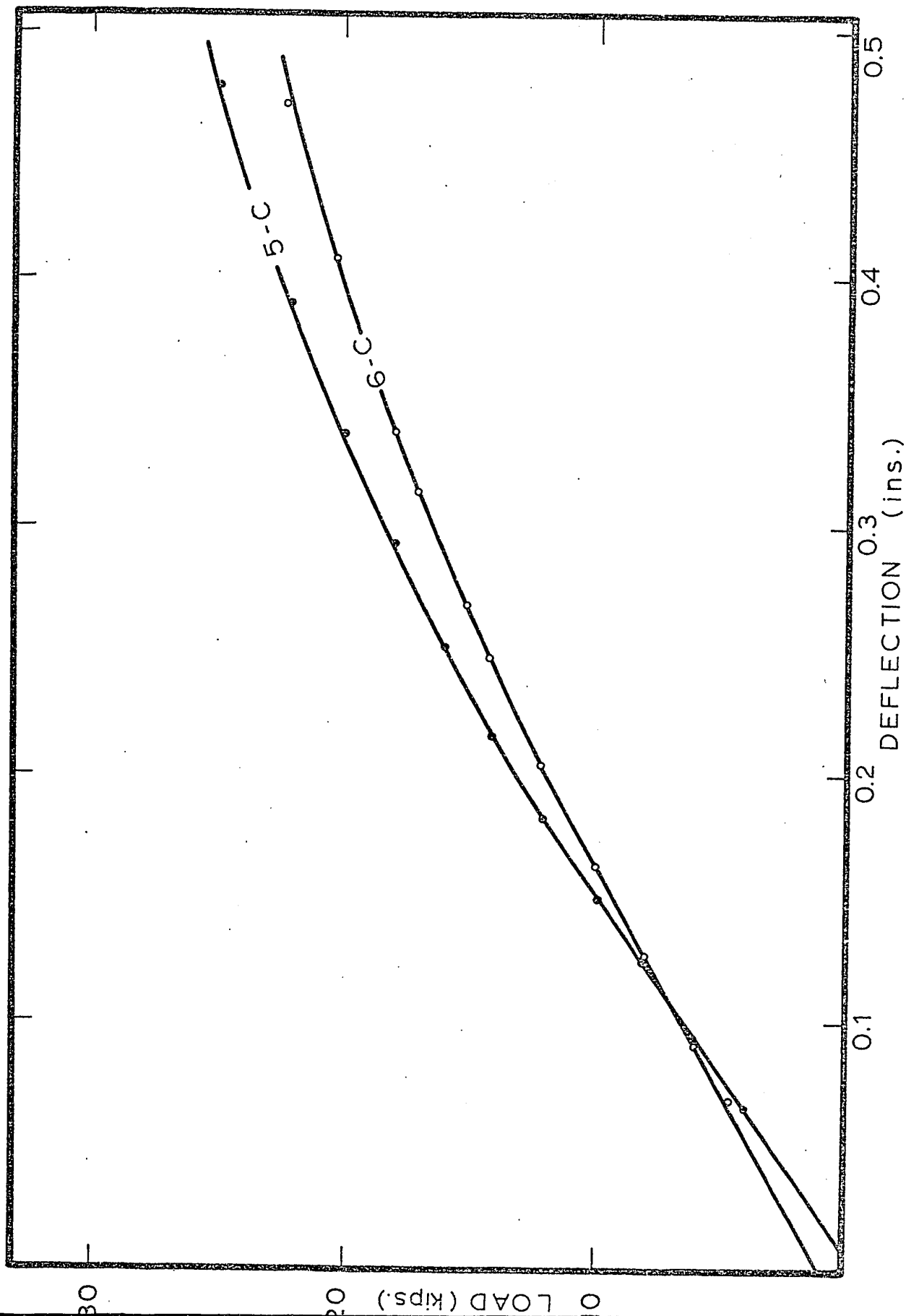


FIG. (B-3) LOAD-DEFLECTION CURVE FOR COLUMNS 5 & 6

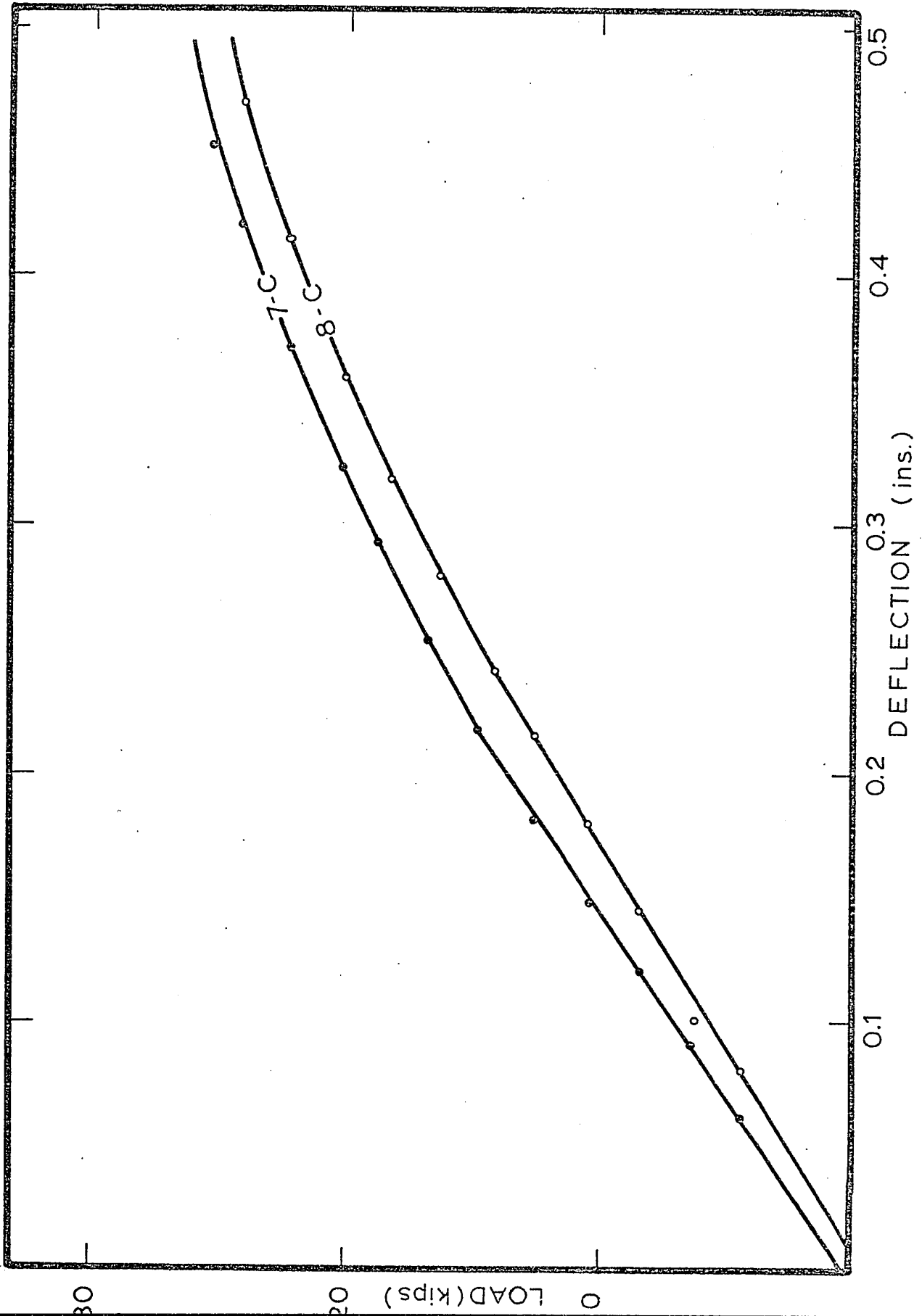


FIG.(B-4) LOAD-DEFLECTION CURVE FOR COLUMNS 7 & 8

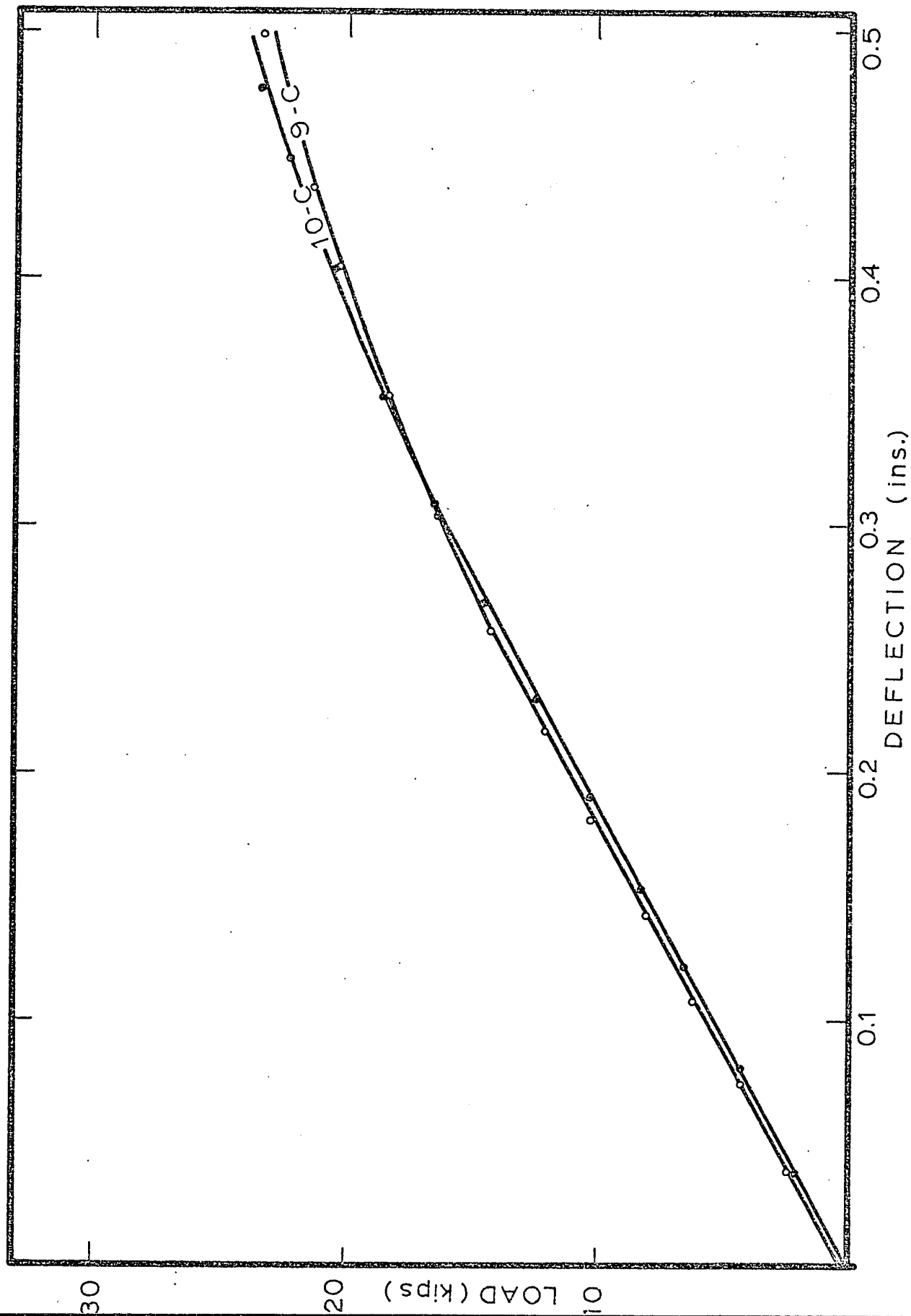


FIG.(B-5) LOAD DEFLECTION CURVE FOR COLUMNS 9&10

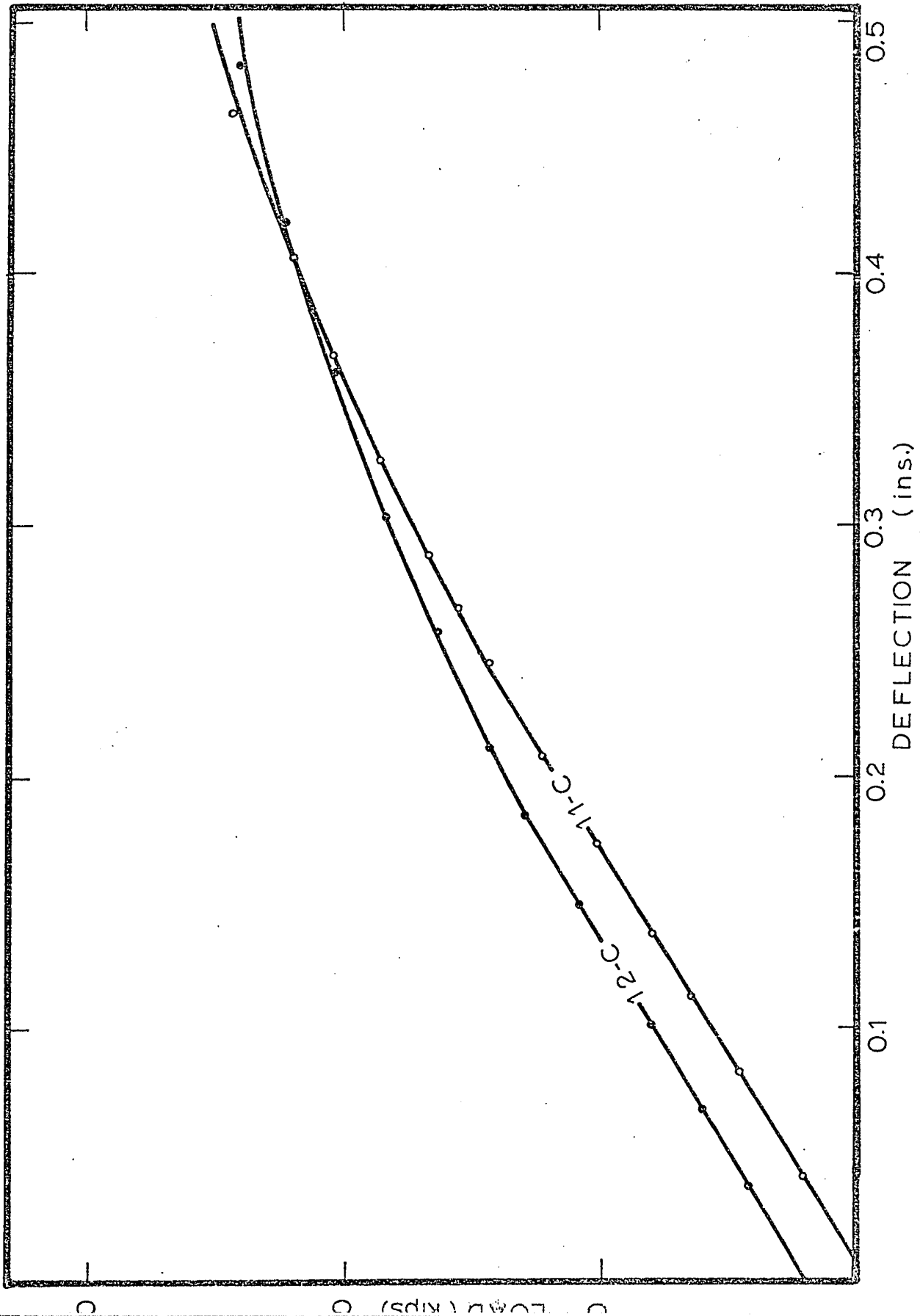


FIG.(B-6) LOAD-DEFLECTION CURVE FOR COLUMNS 11&12

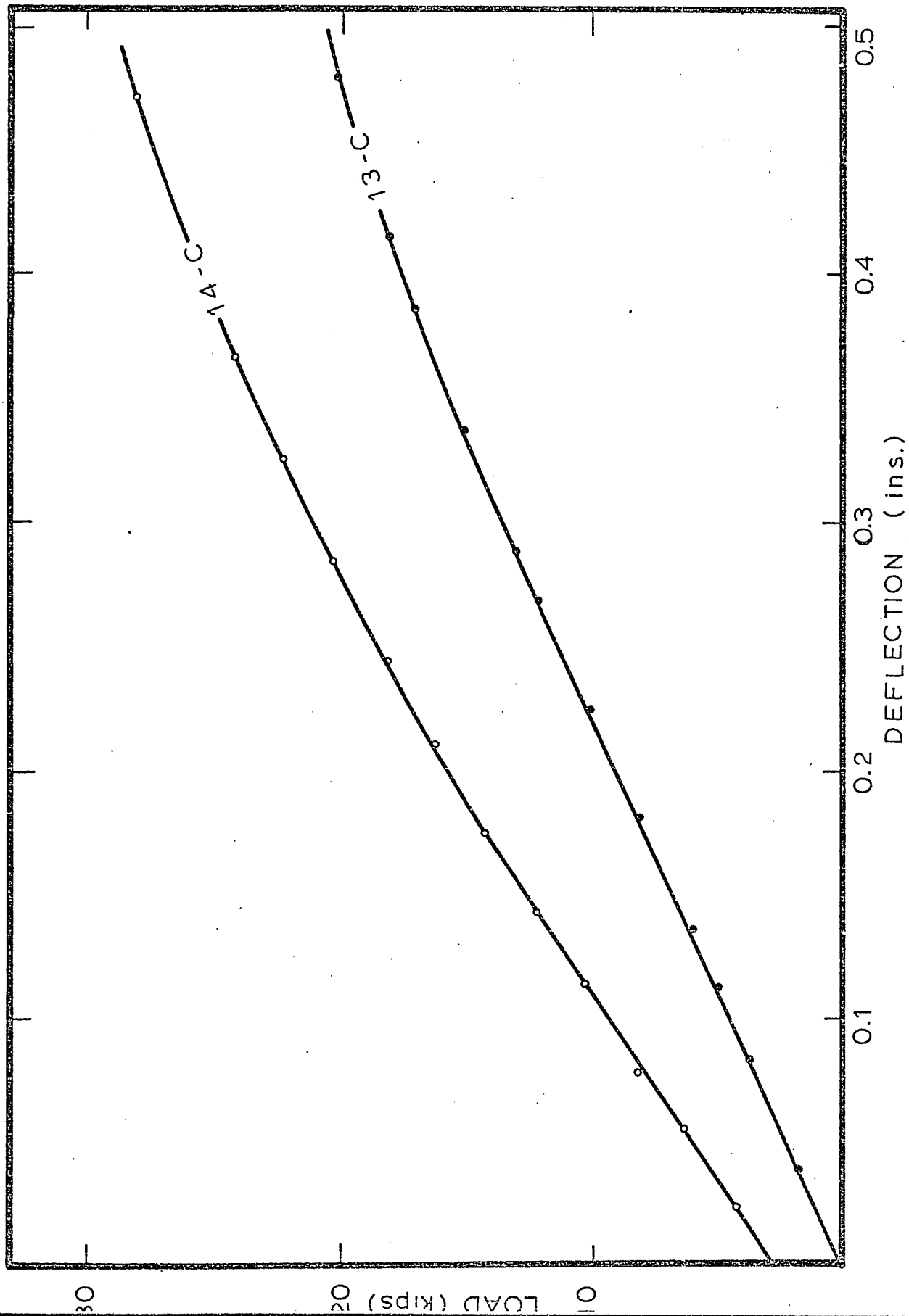


FIG.(B-7) LOAD DEFLECTION CURVE FOR COLUMNS 13& 14

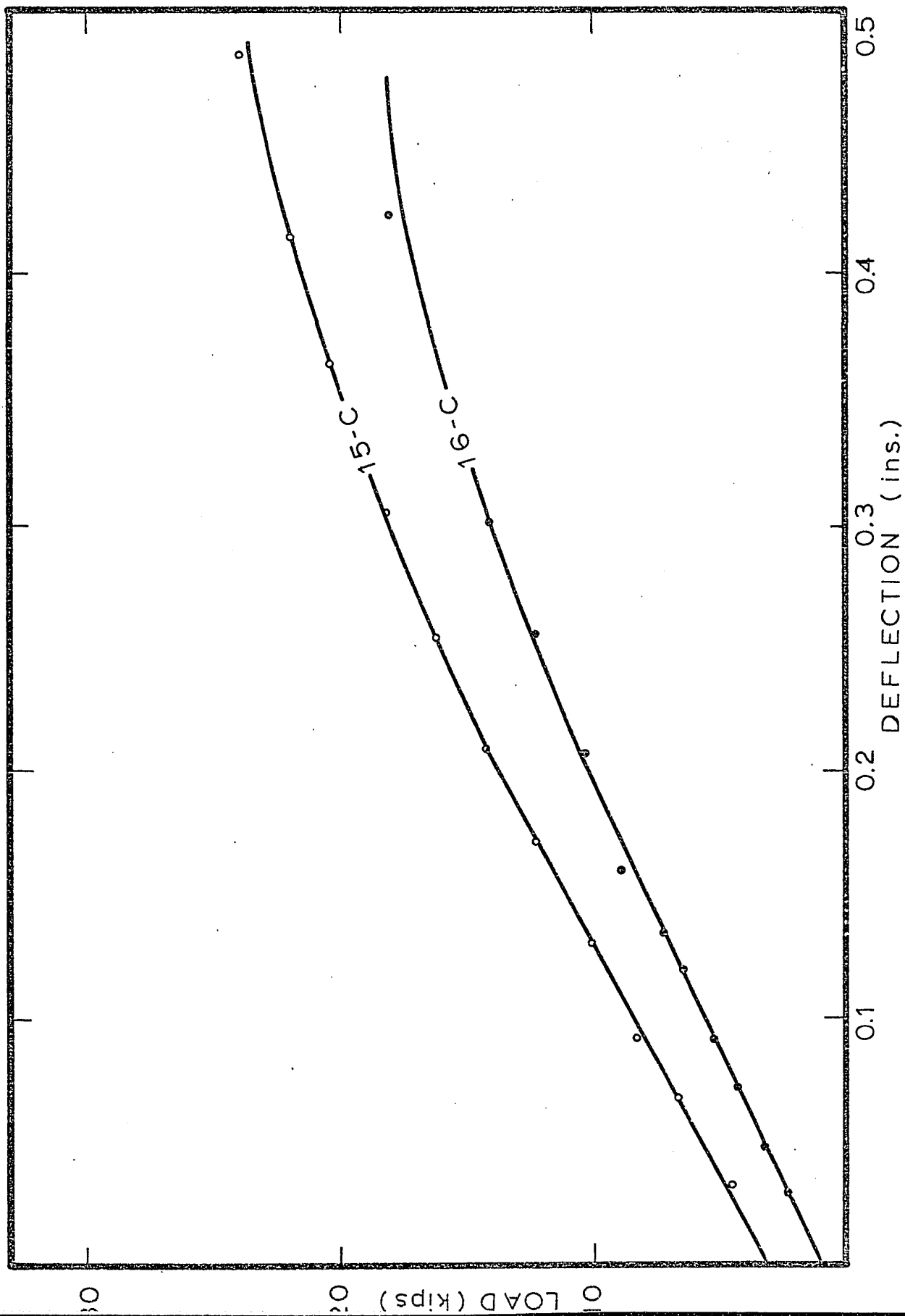


FIG.(B-8) LOAD-DEFLECTION CURVE FOR COLUMNS 15&16

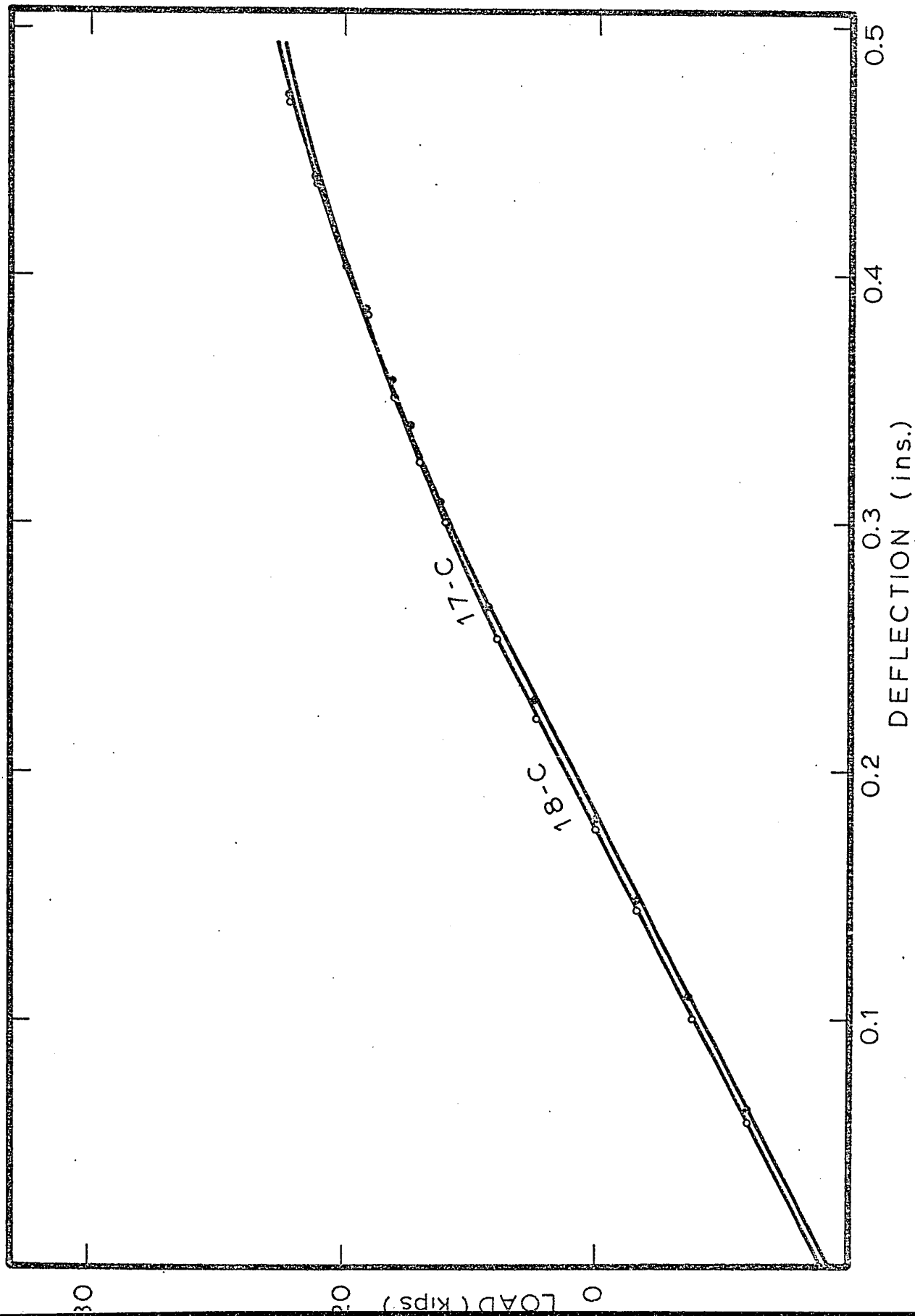


FIG.(B-9) LOAD-DEFLECTION CURVE FOR COLUMNS 17 & 18

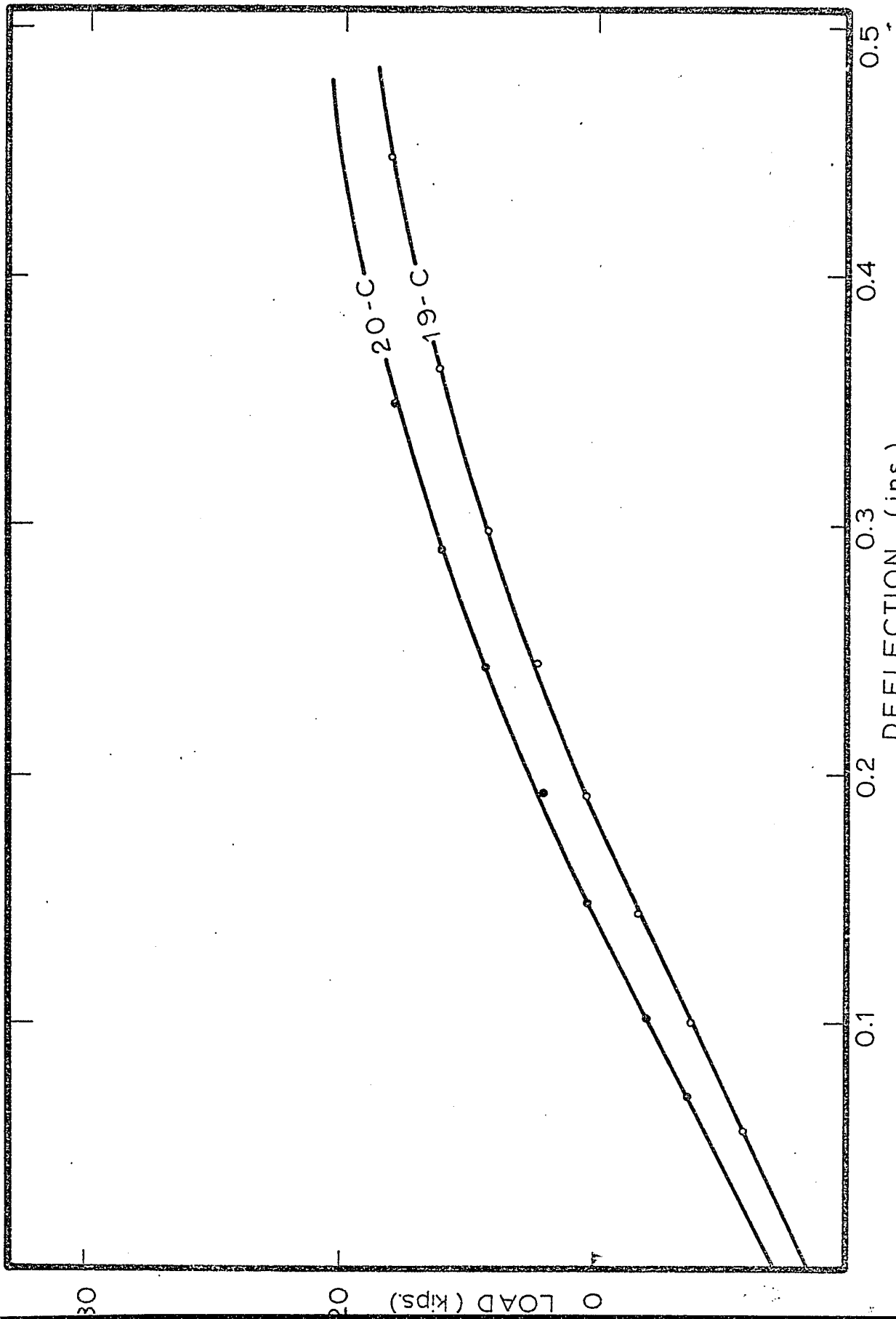


FIG.(B-10) LOAD DEFLECTION CURVE FOR COLUMNS 19 & 20

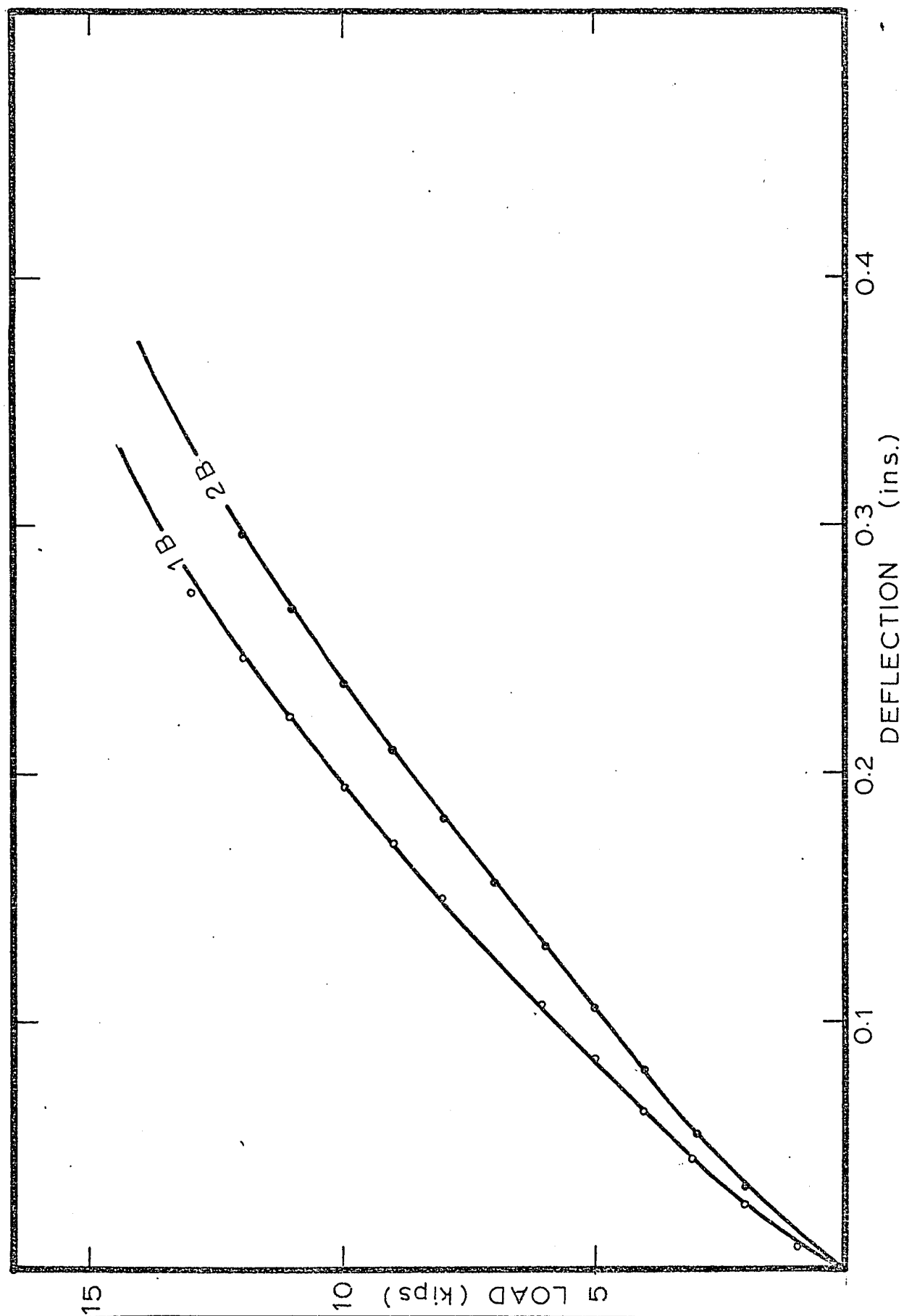


FIG.(B-11) LOAD-DEFLECTION CURVE FOR SIMPLE BEAMS 1 & 2

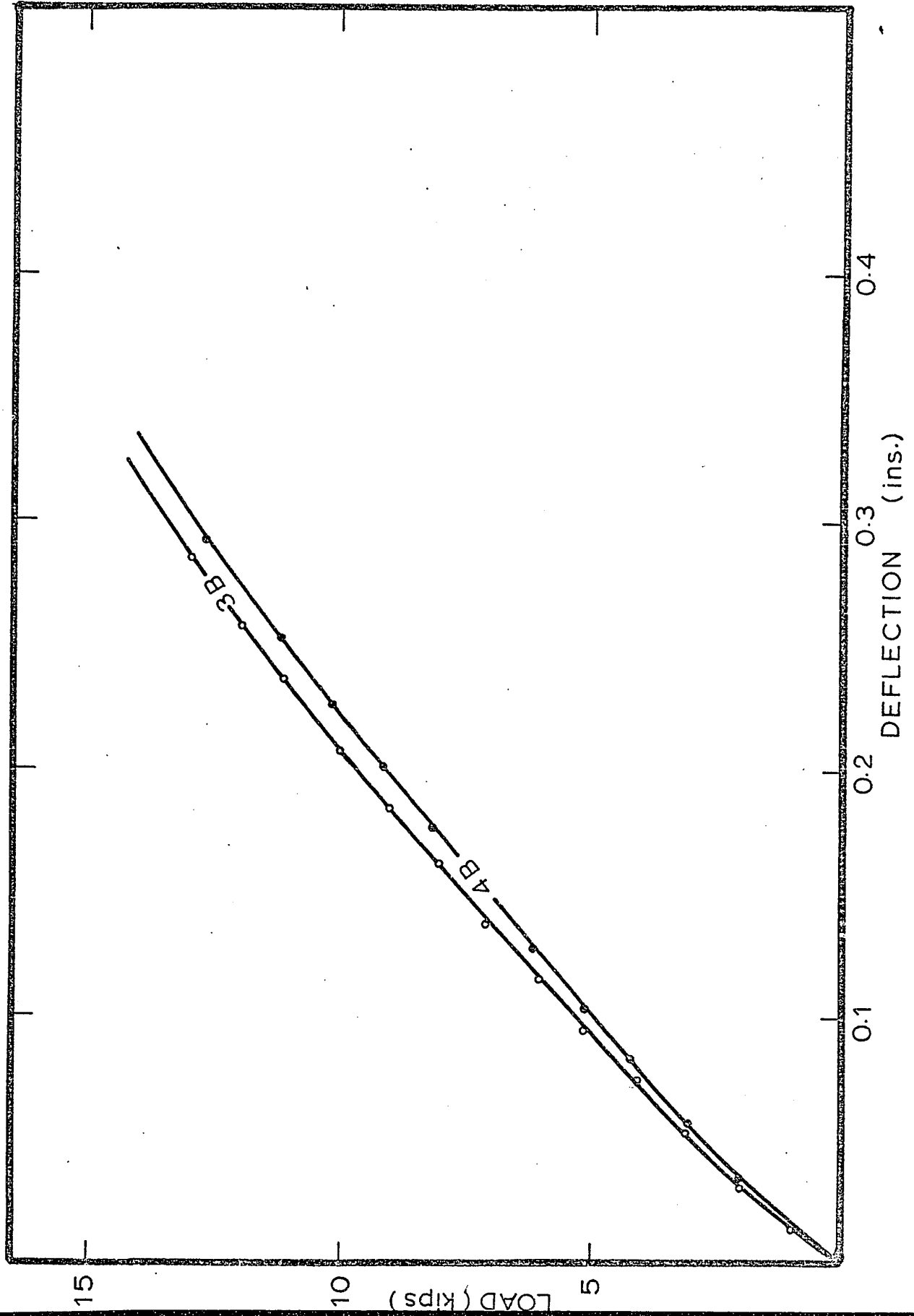


FIG.(B-12) LOAD-DEFLECTION CURVE FOR SIMPLE BEAMS 3 & 4

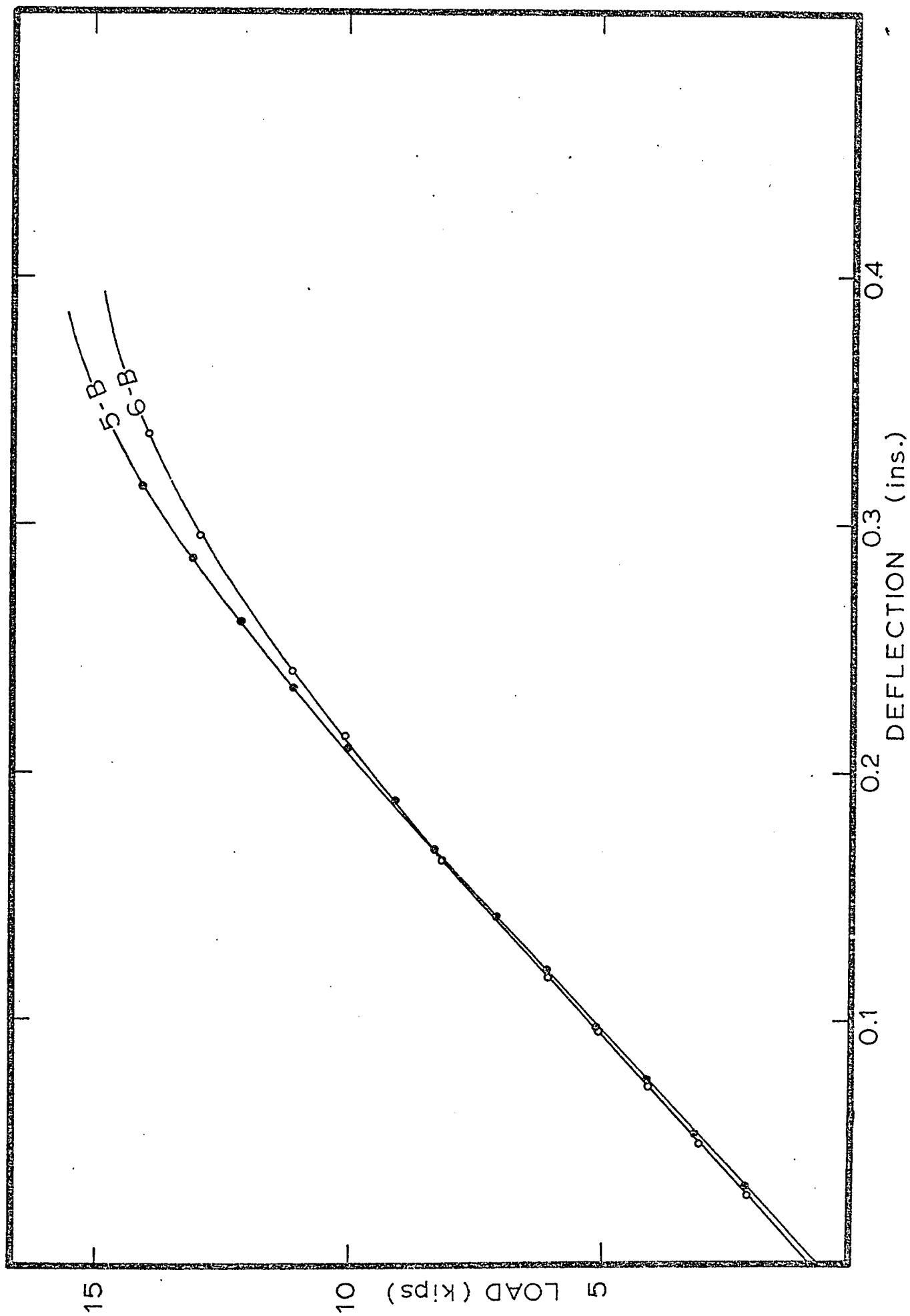


FIG.(B-13) LOAD-DEFLECTION CURVE FOR SIMPLE BEAMS 5 & 6

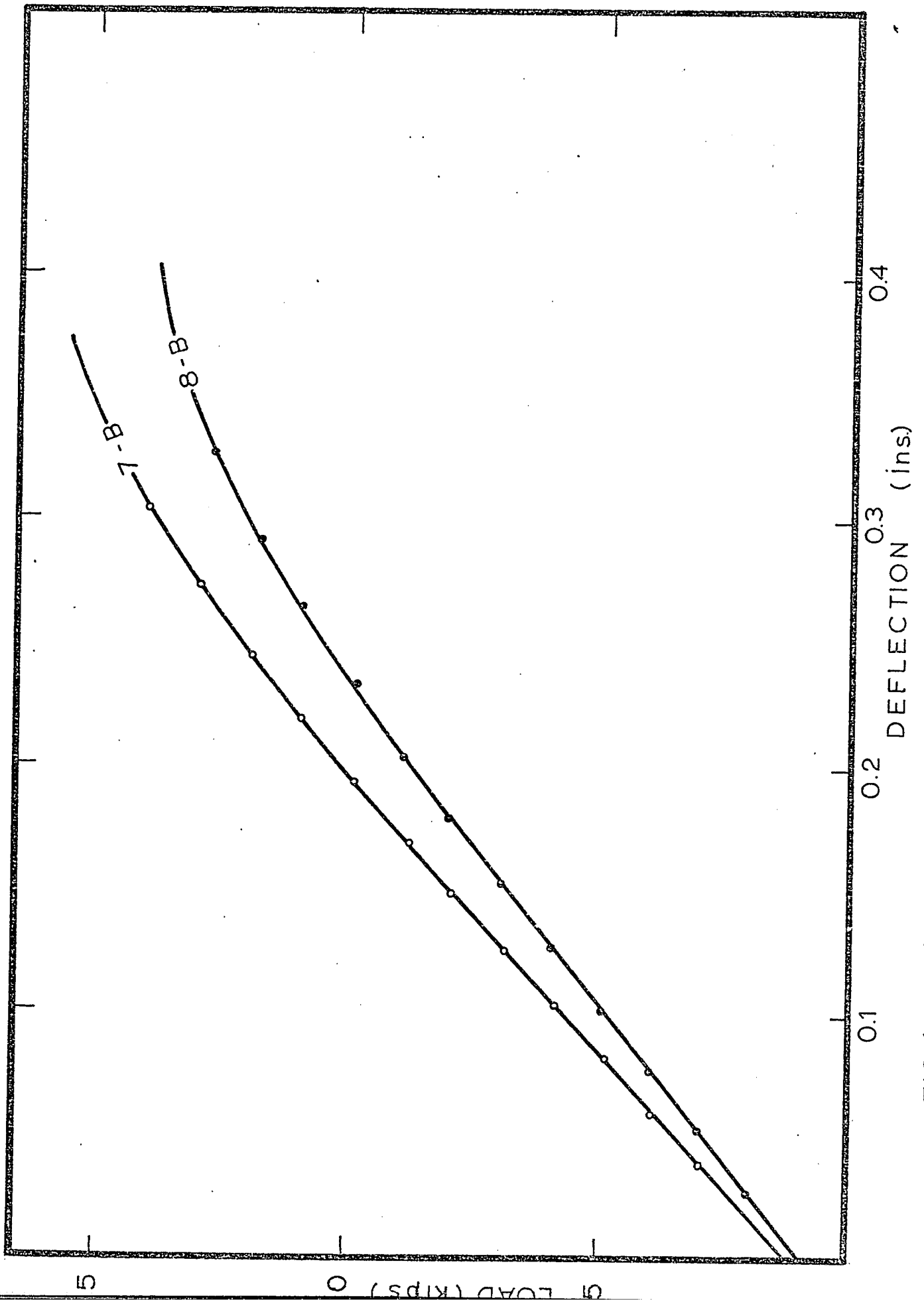


FIG.(B-14) LOAD DEFLECTION CURVE FOR SIMPLE BEAMS 7 & 8

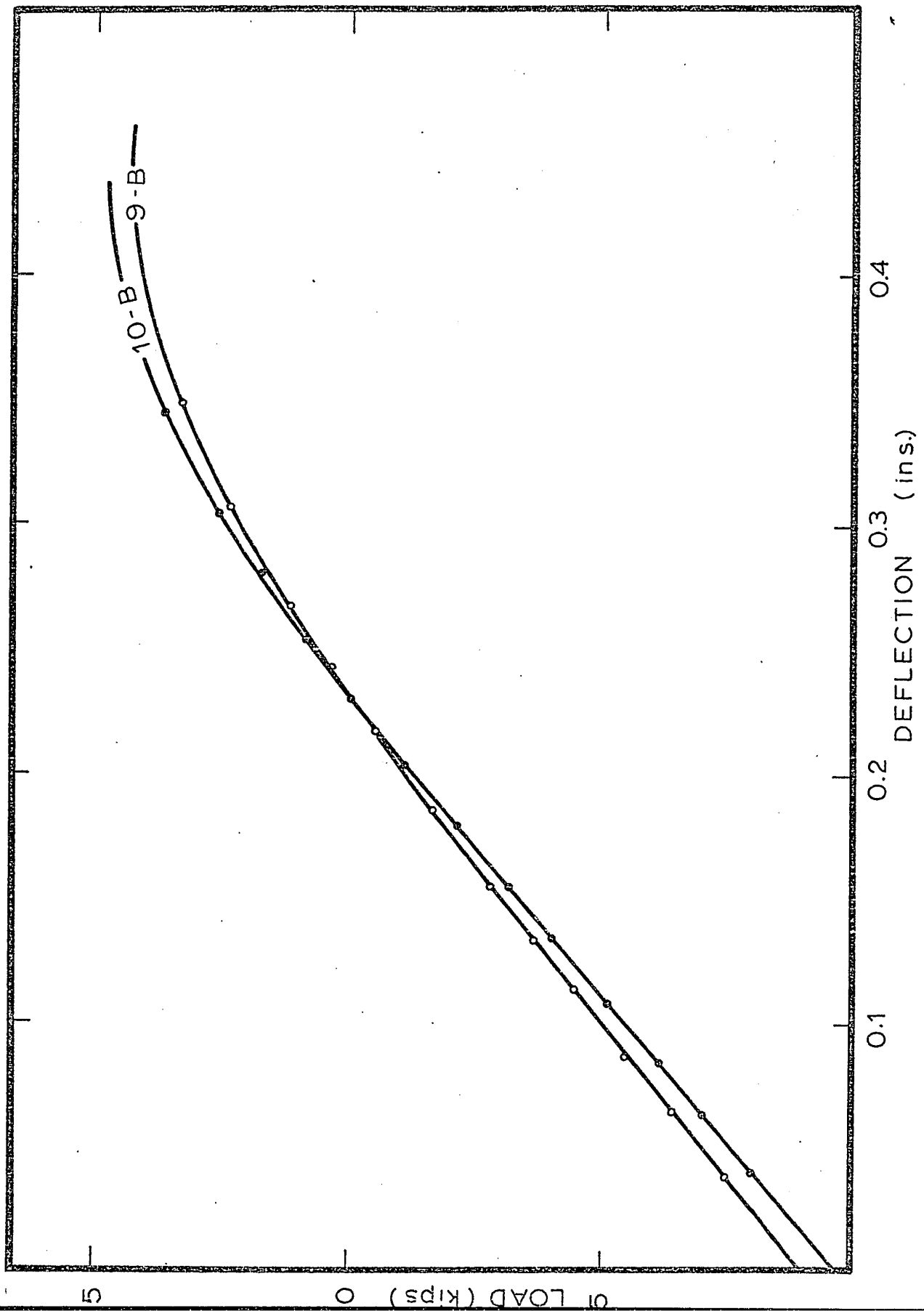


FIG.(B-15) LOAD-DEFLECTION CURVE FOR SIMPLE BEAMS 9&10

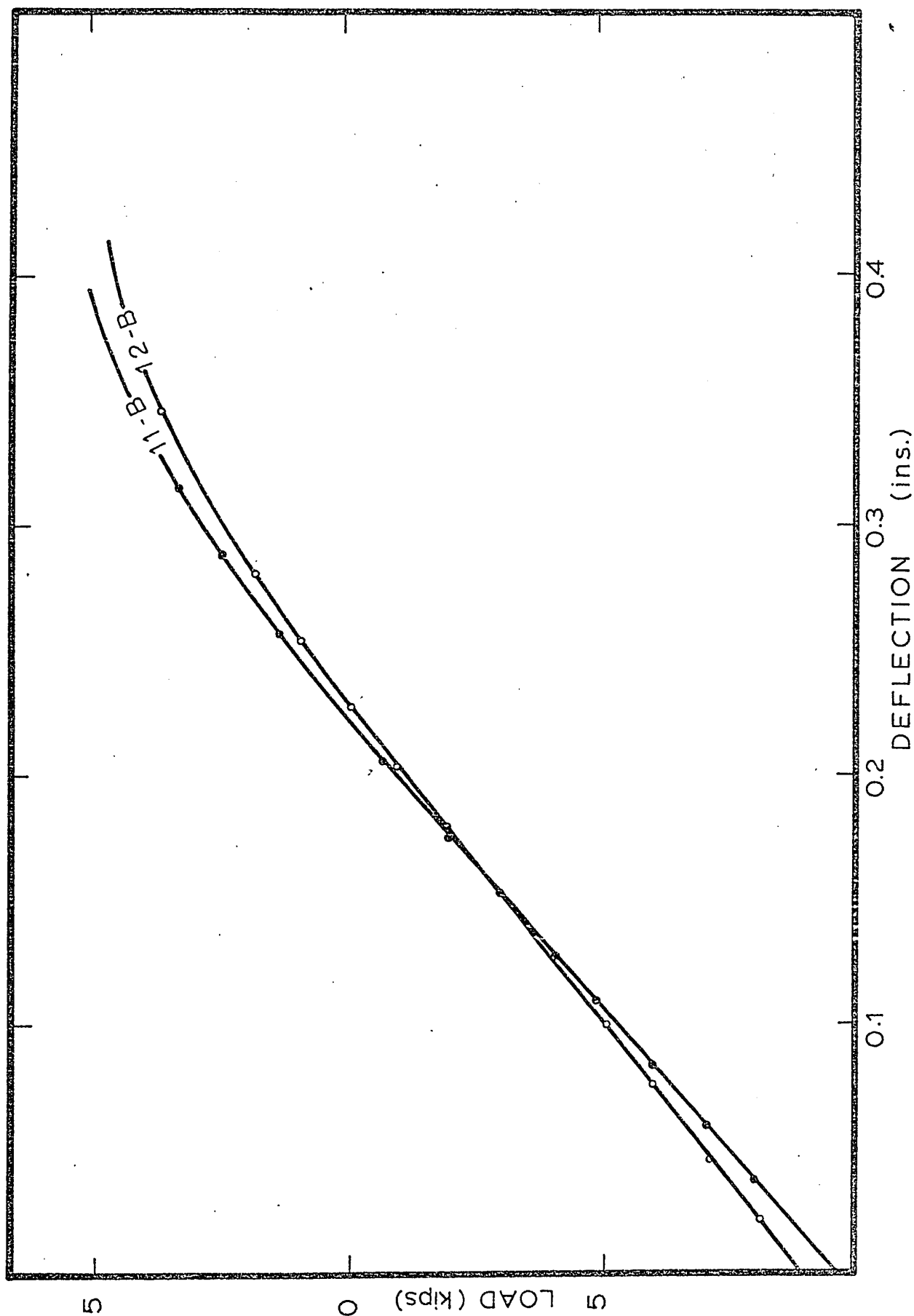


FIG.(B-16) LOAD-DEFLECTION CURVE FOR SIMPLE BEAMS 11&12

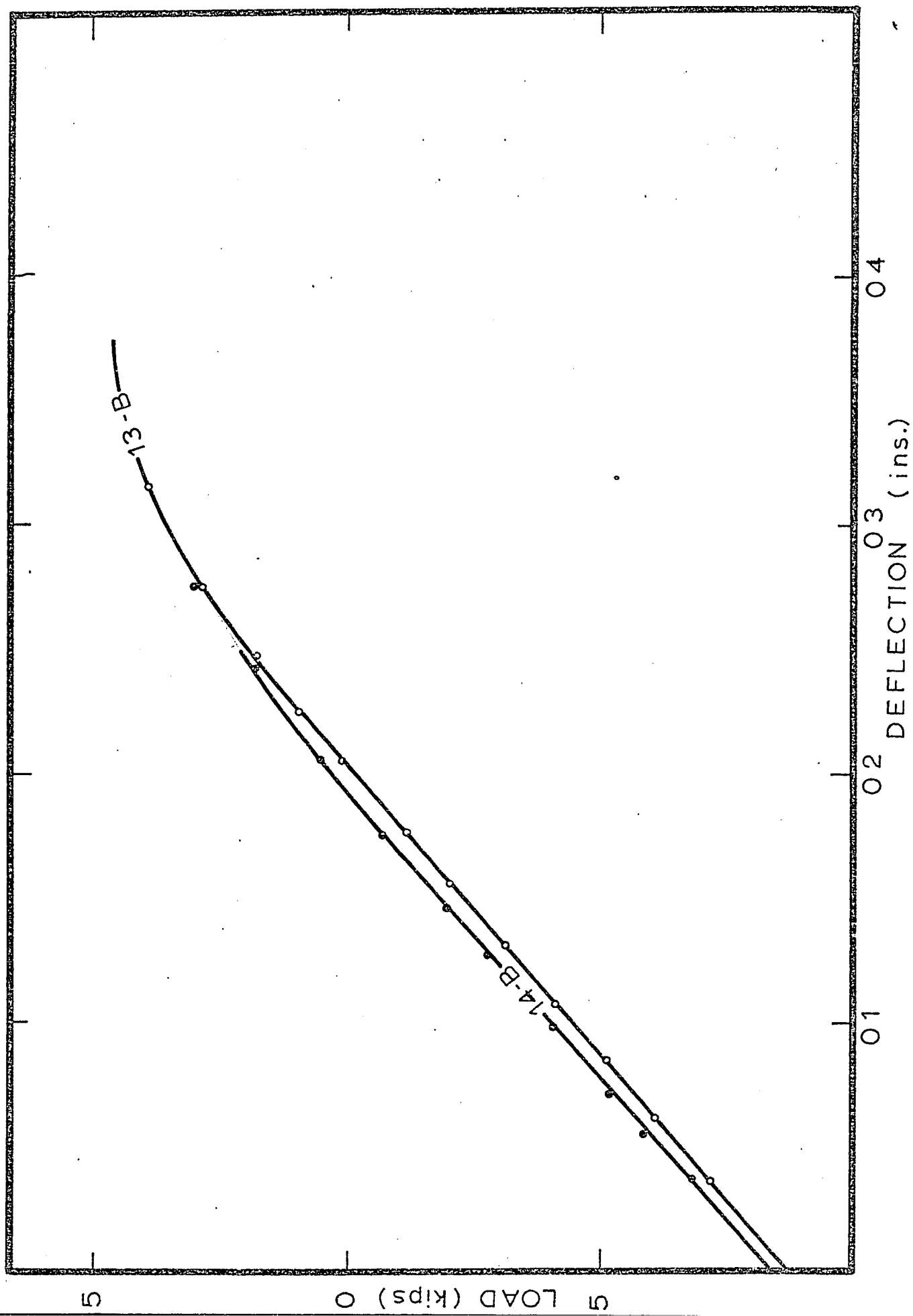


FIG.(B-17) LOAD DEFLECTION CURVE FOR SIMPLE BEAMS 13&14

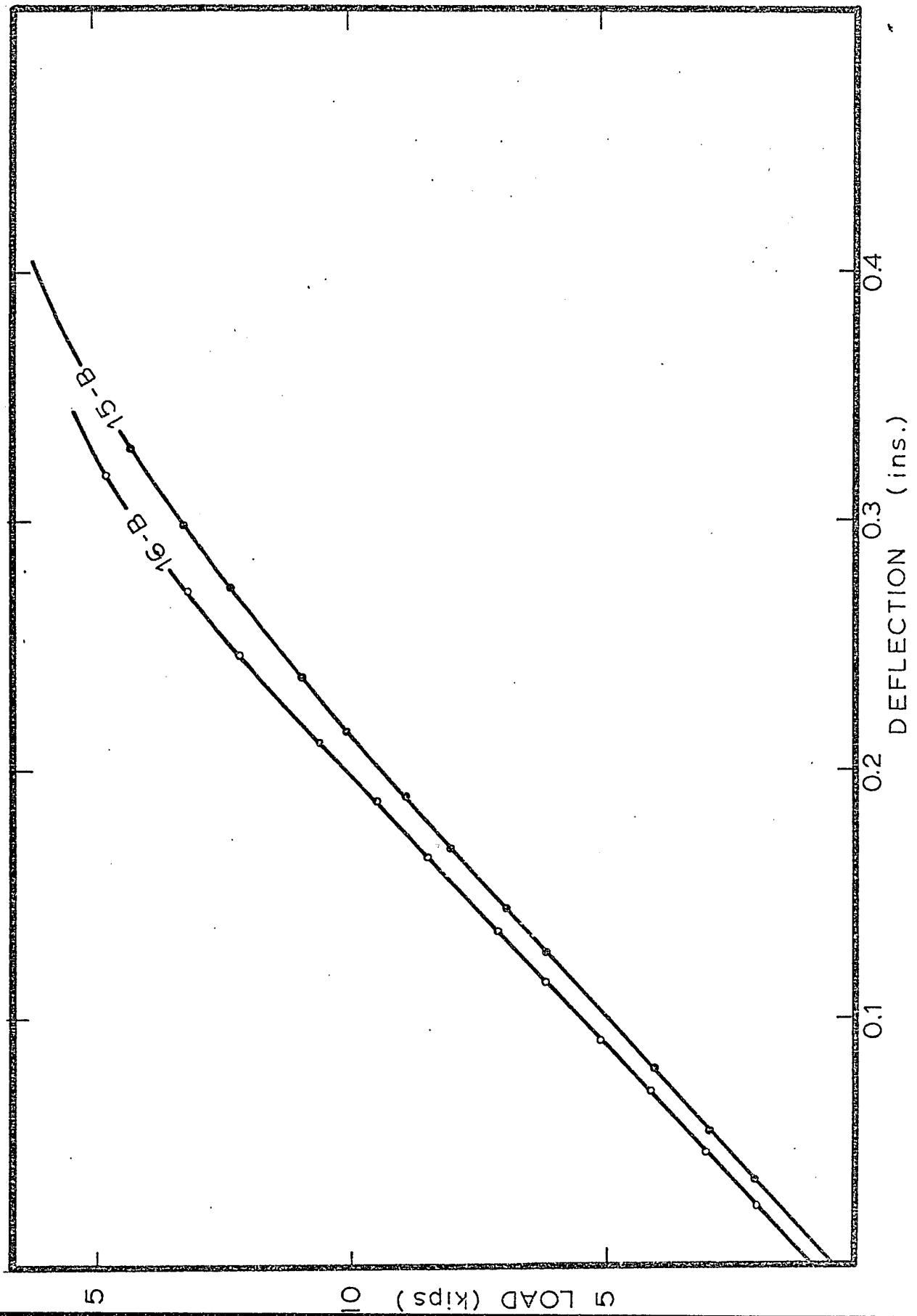


FIG.(B-18) LOAD-DEFLECTION CURVE FOR SIMPLE BEAMS 15 & 16

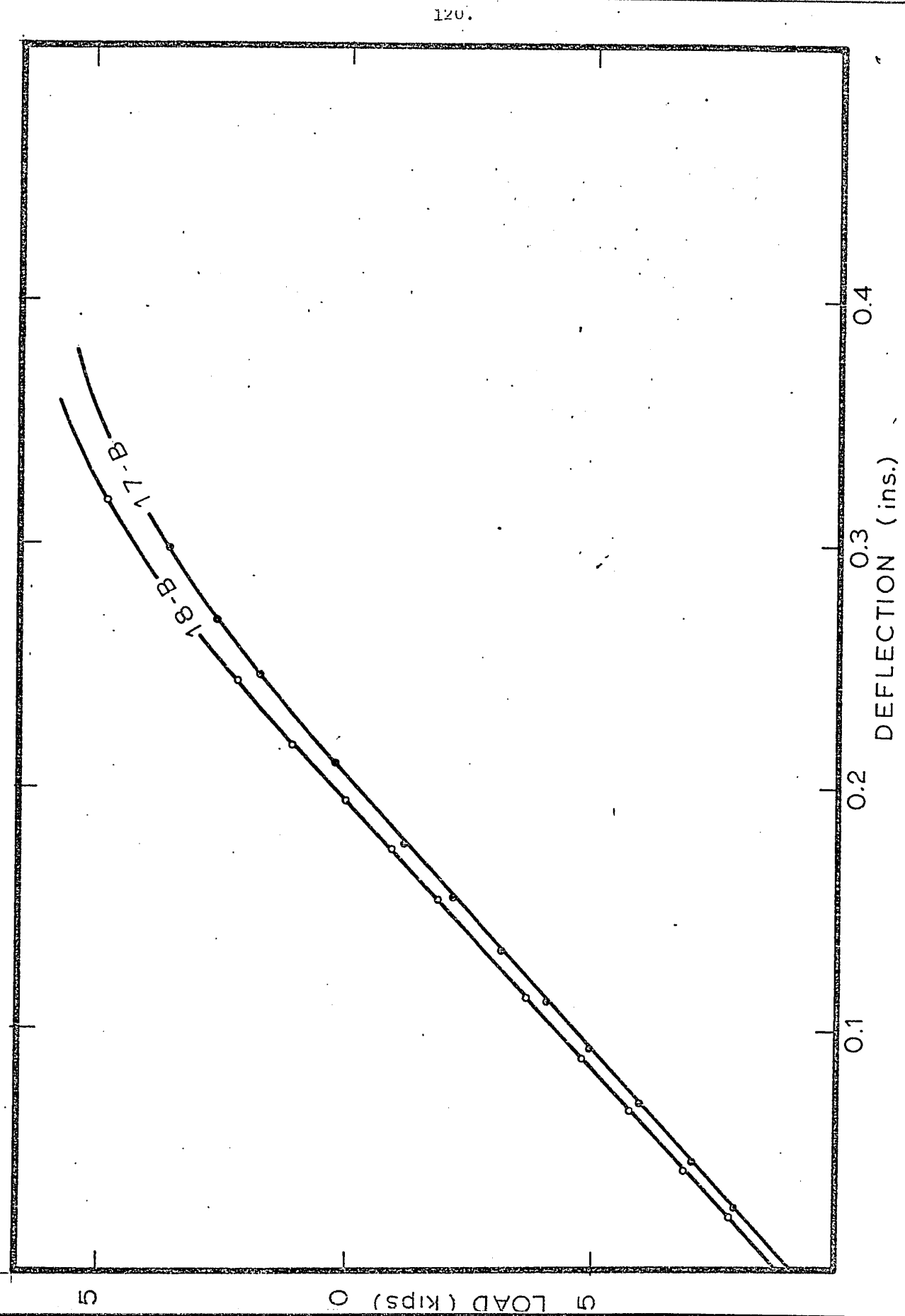


FIG.(B-19) LOAD-DEFLECTION CURVE FOR SIMPLE BEAMS 18 & 19

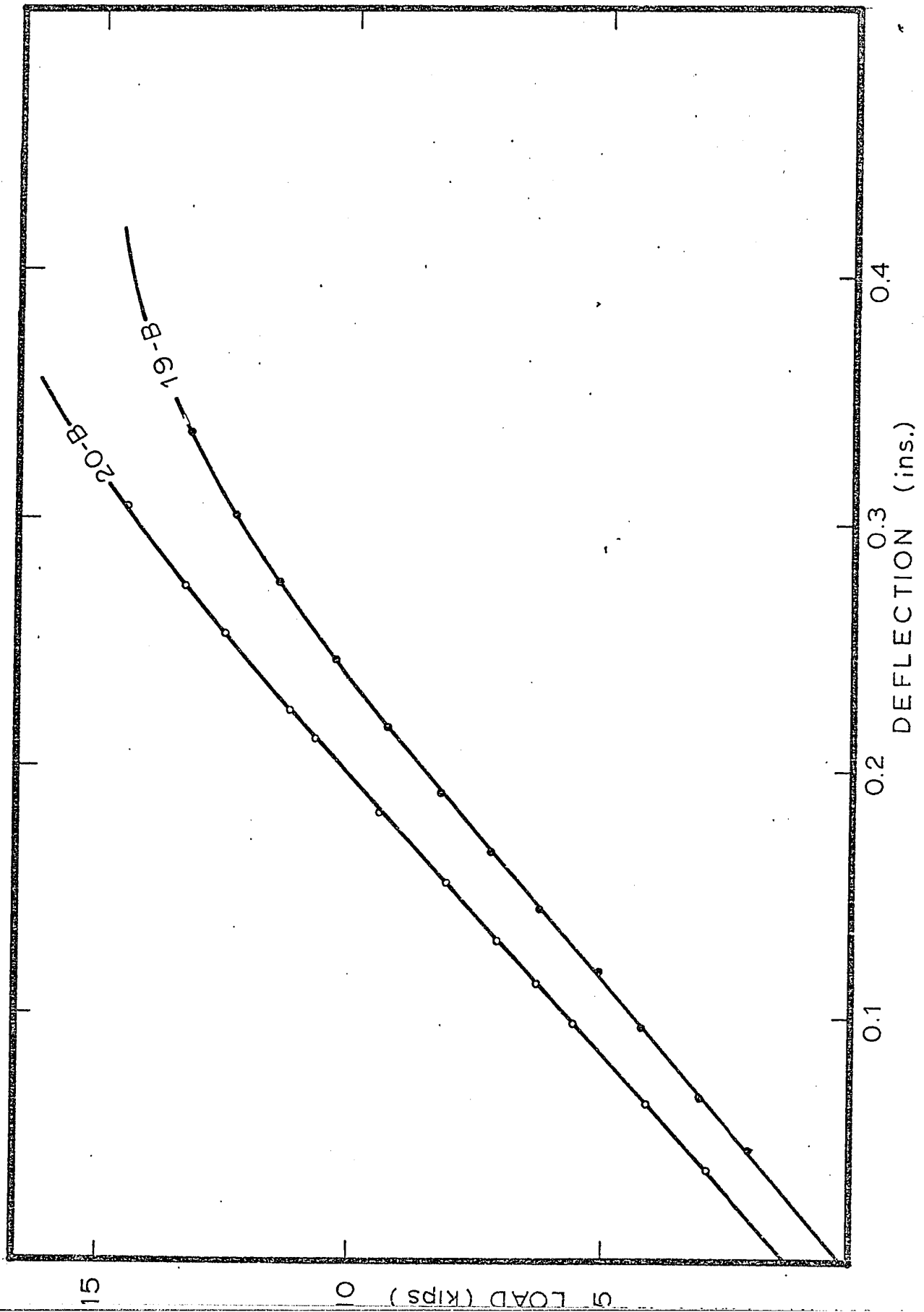


FIG.(B-20) LOAD-DEFLECTION CURVE FOR SIMPLE BEAMS 19

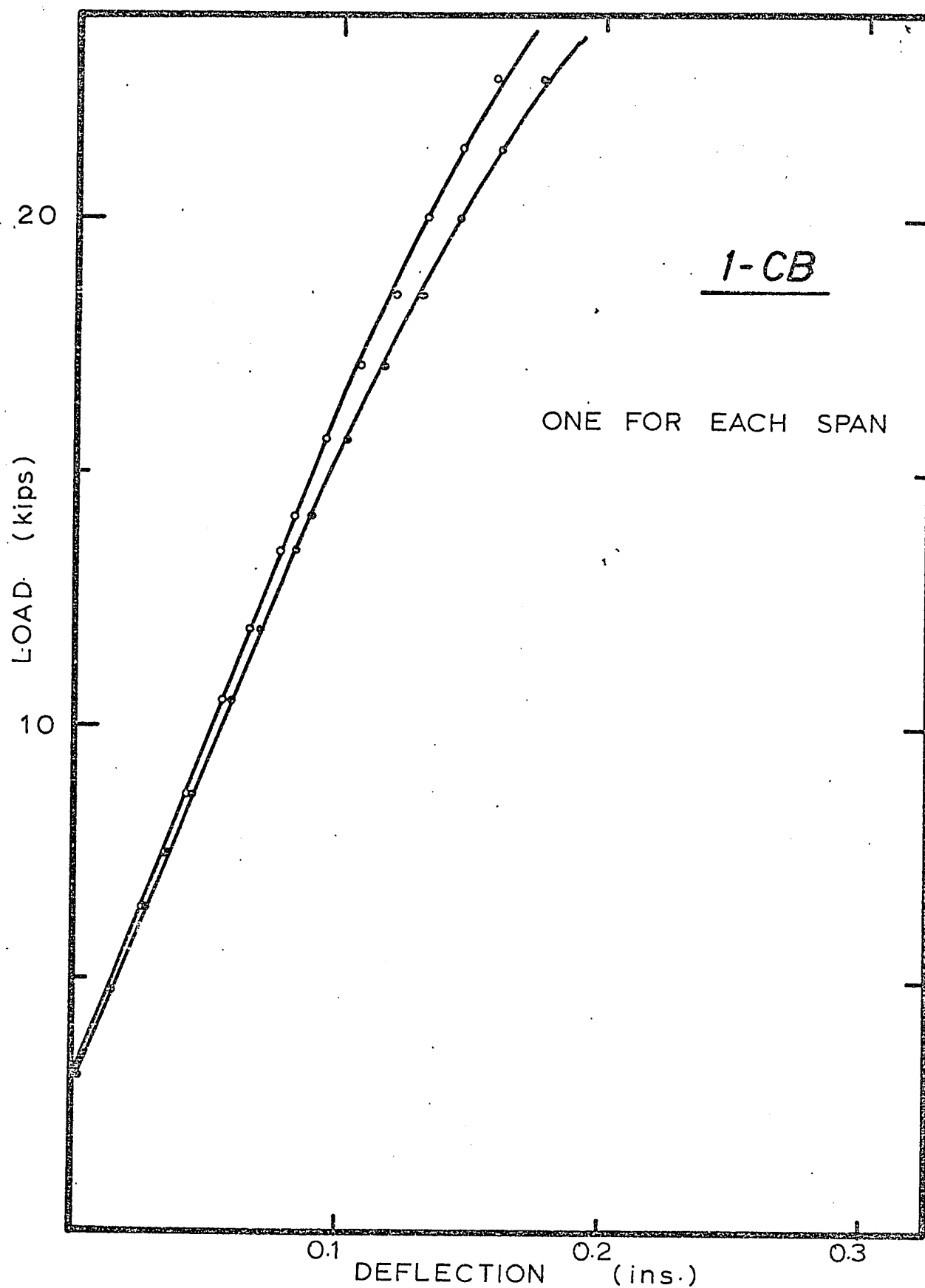


FIG.(B-21) LOAD-DEFLECTION CURVE

FOR TWO SPAN CONT. BEAM

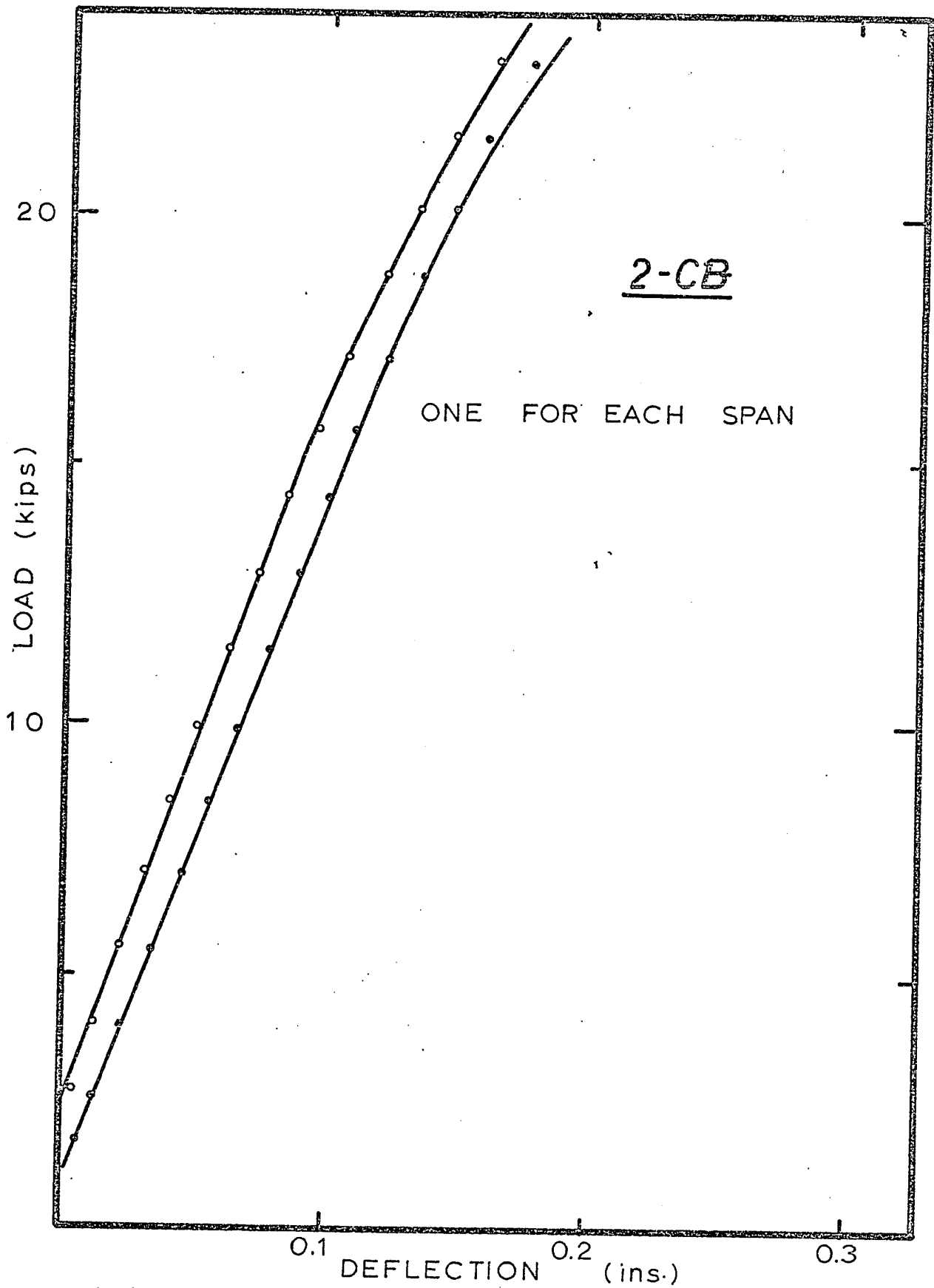


FIG.(B-22) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 2

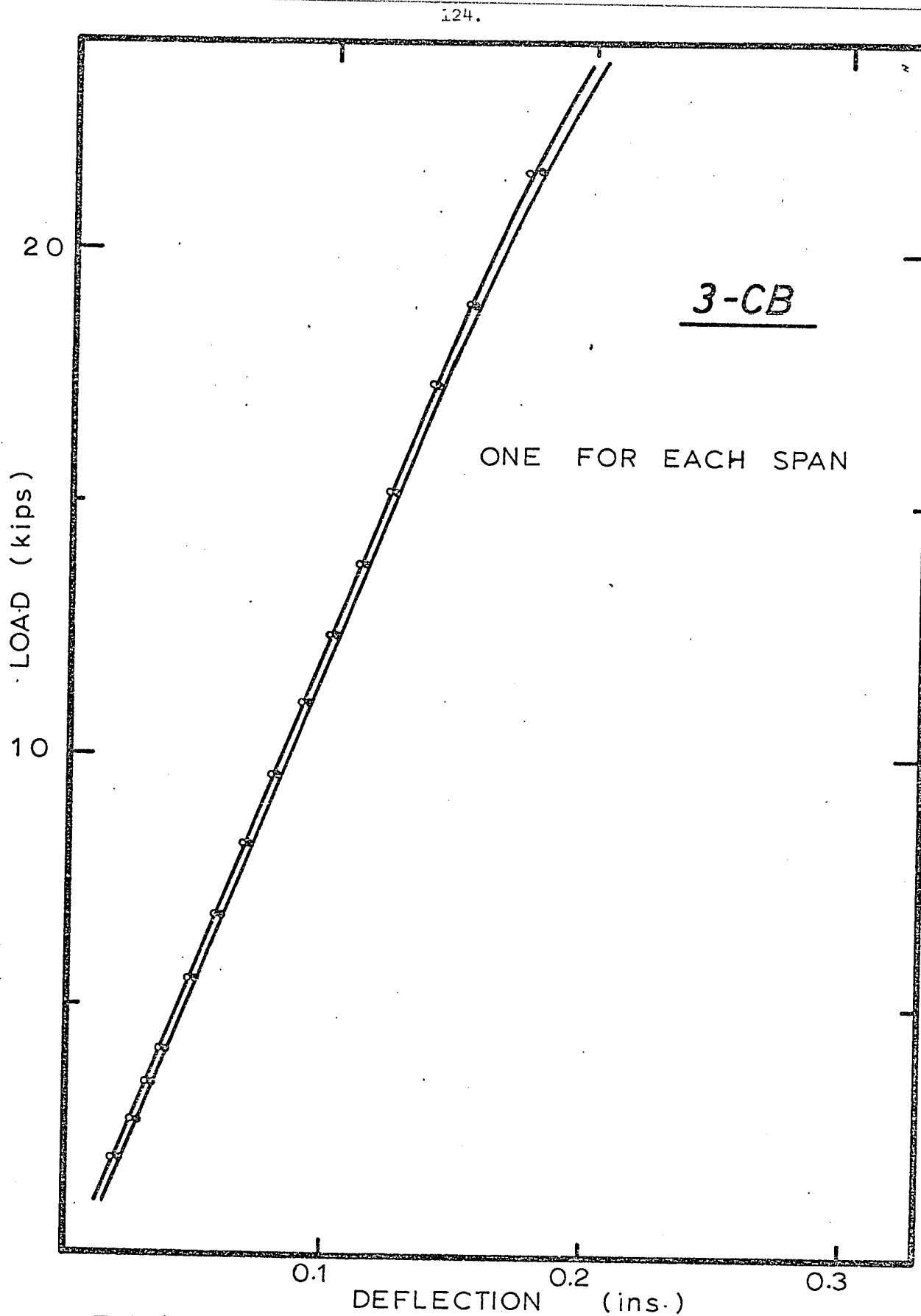


FIG.(B-23) LOAD-DEFLECTION CURVE FOR
TWO-SPAN CONTINUOUS BEAM 3

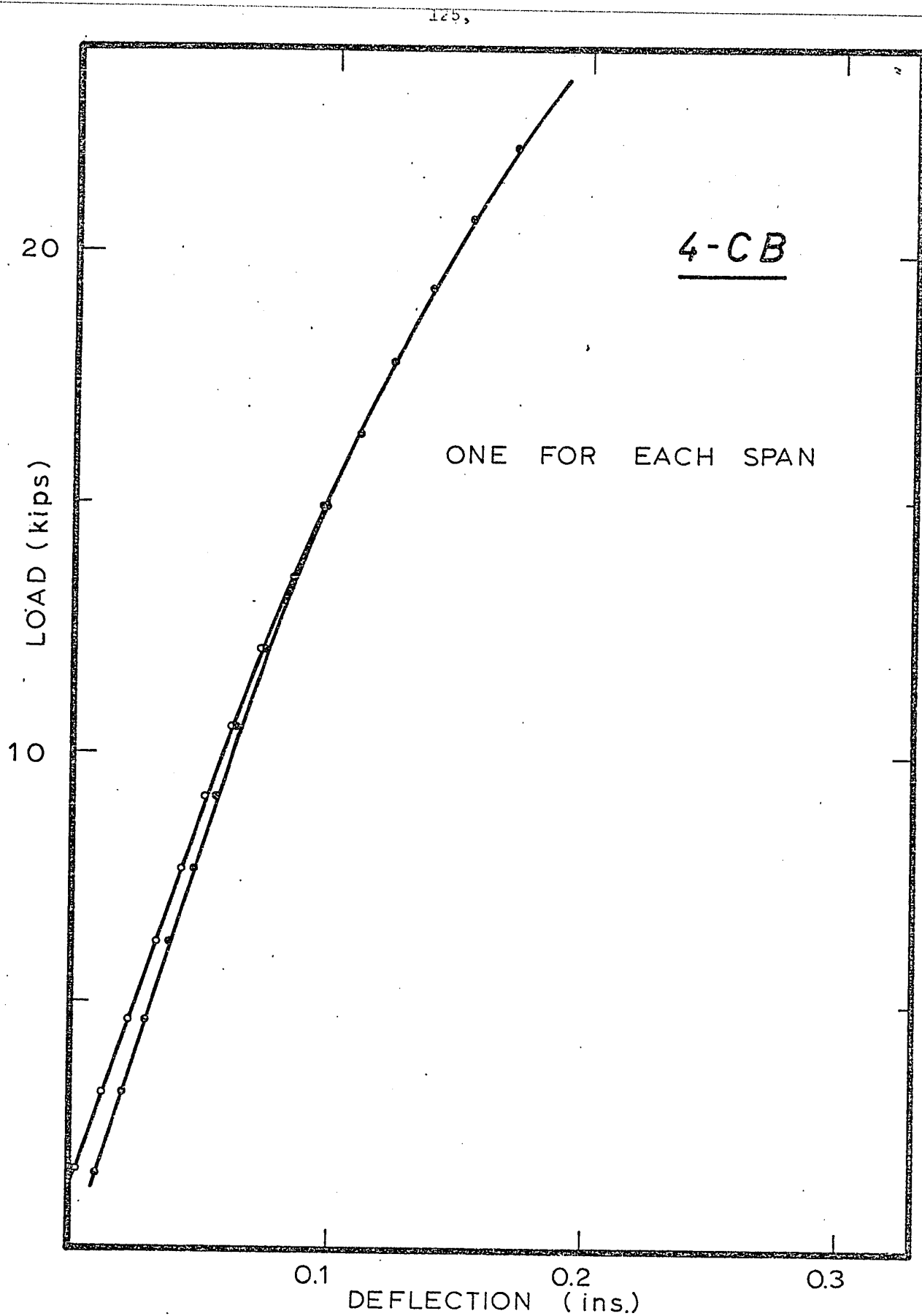


FIG.(B-24) LOAD-DEFLECTION CURVE FOR TWO SPAN CONTINUOUS BEAM 4

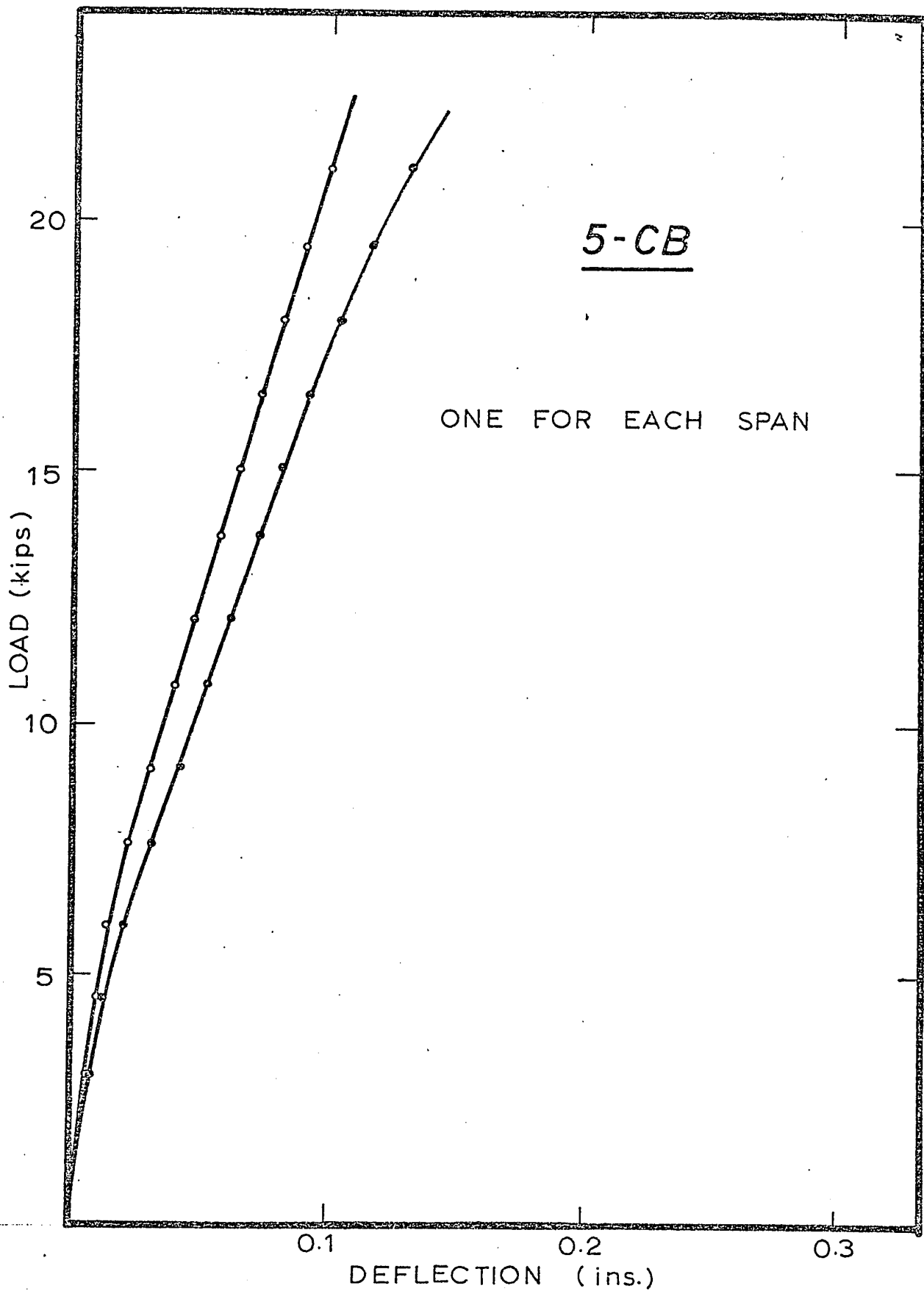


FIG.(B-25) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 5

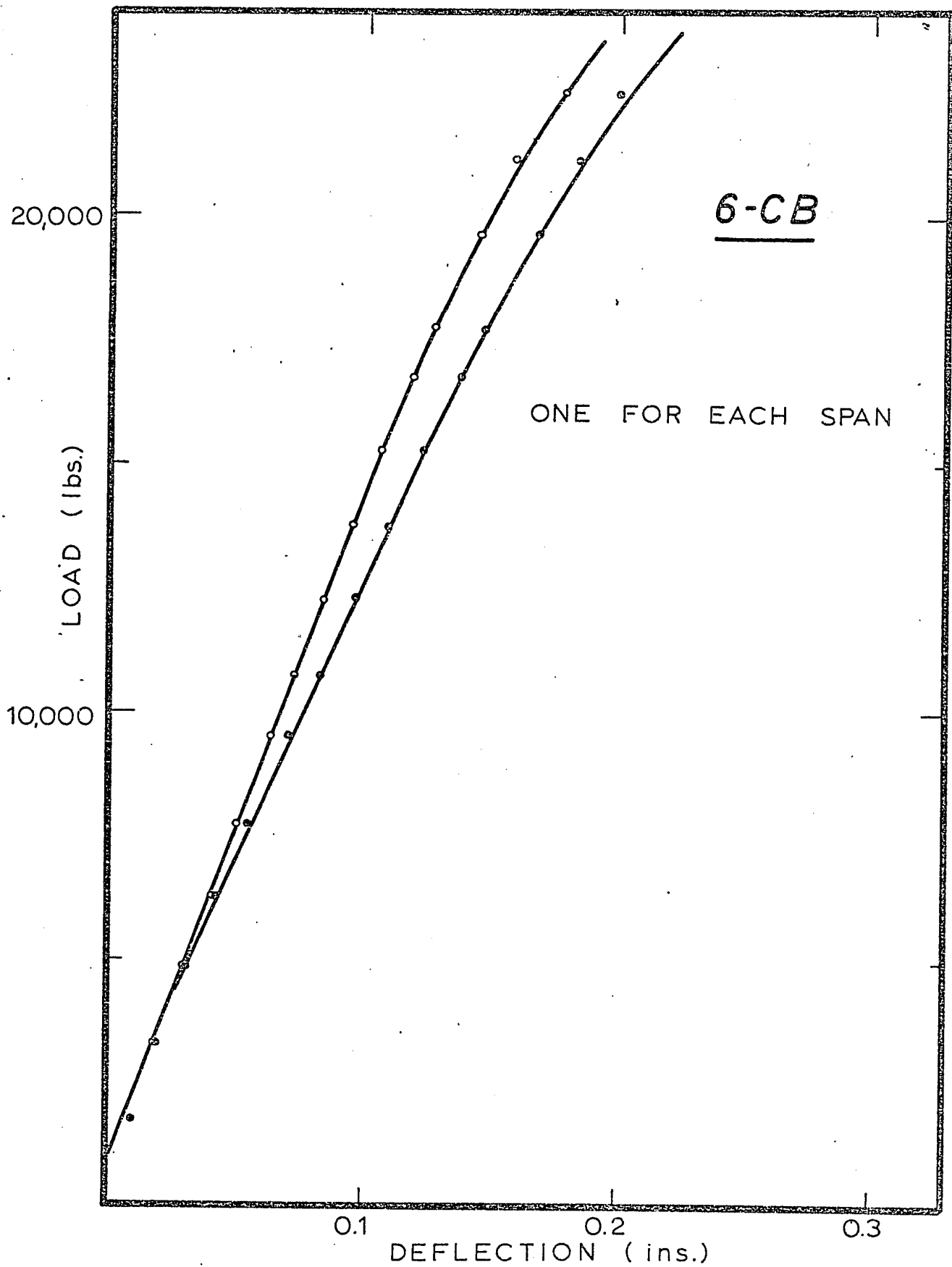


FIG.(B-26) LOAD-DEFLECTION CURVE FOR

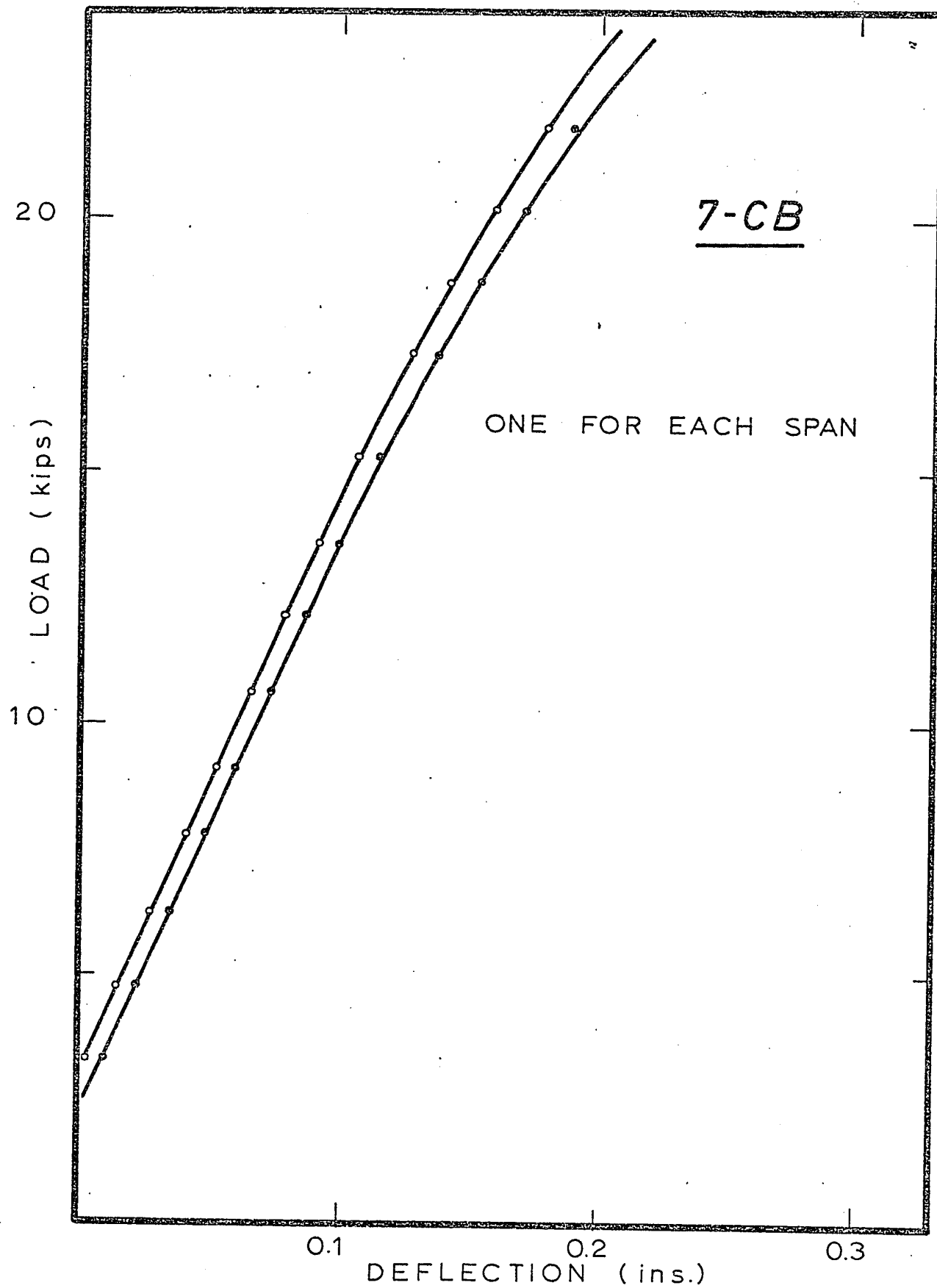


FIG.(B-27) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 7

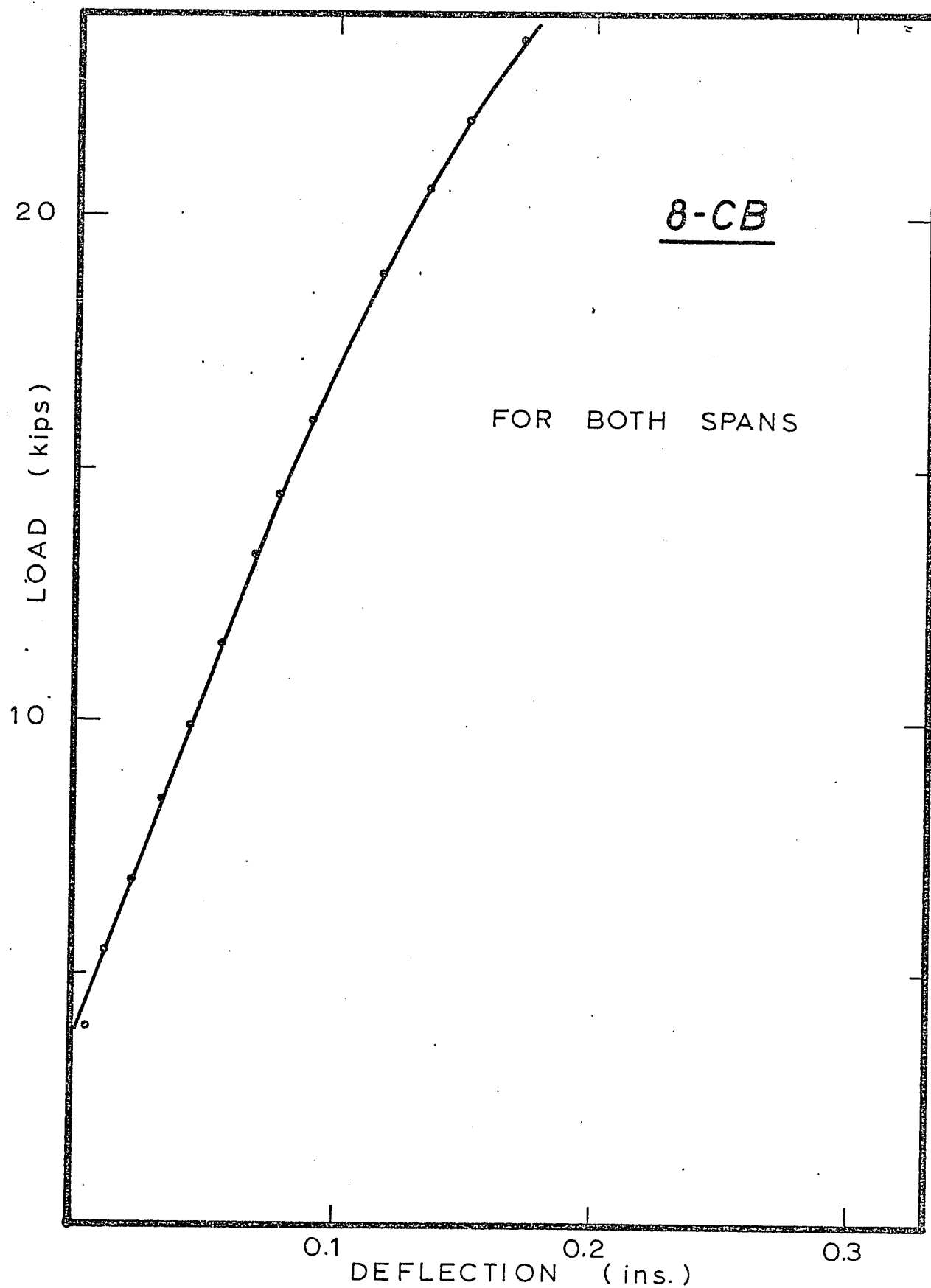


FIG.(B-28) LOAD-DEFLECTION CURVE FOR
TWO-SPAN CONTINUOUS BEAM 8

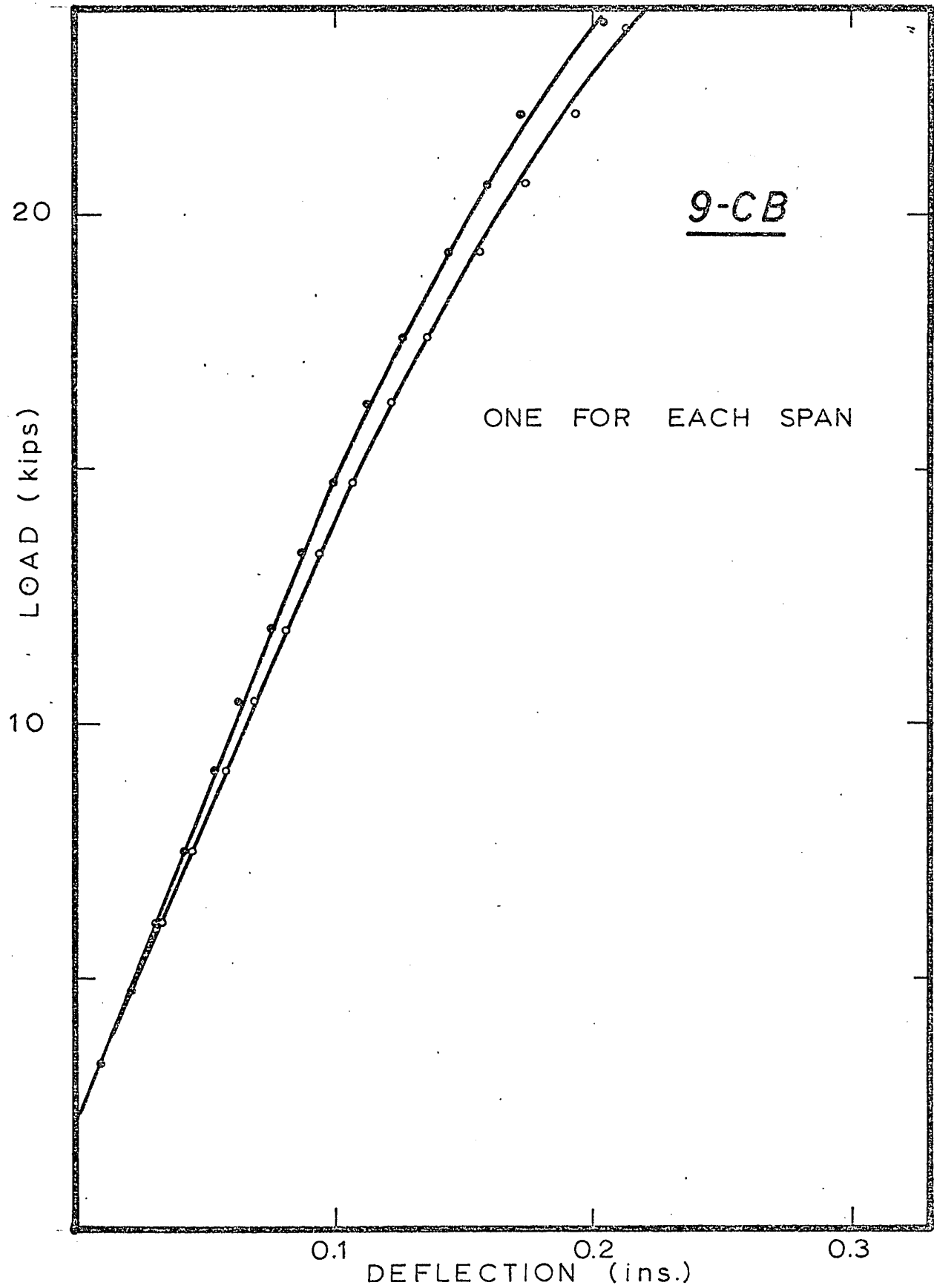


FIG.(B-29) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 9

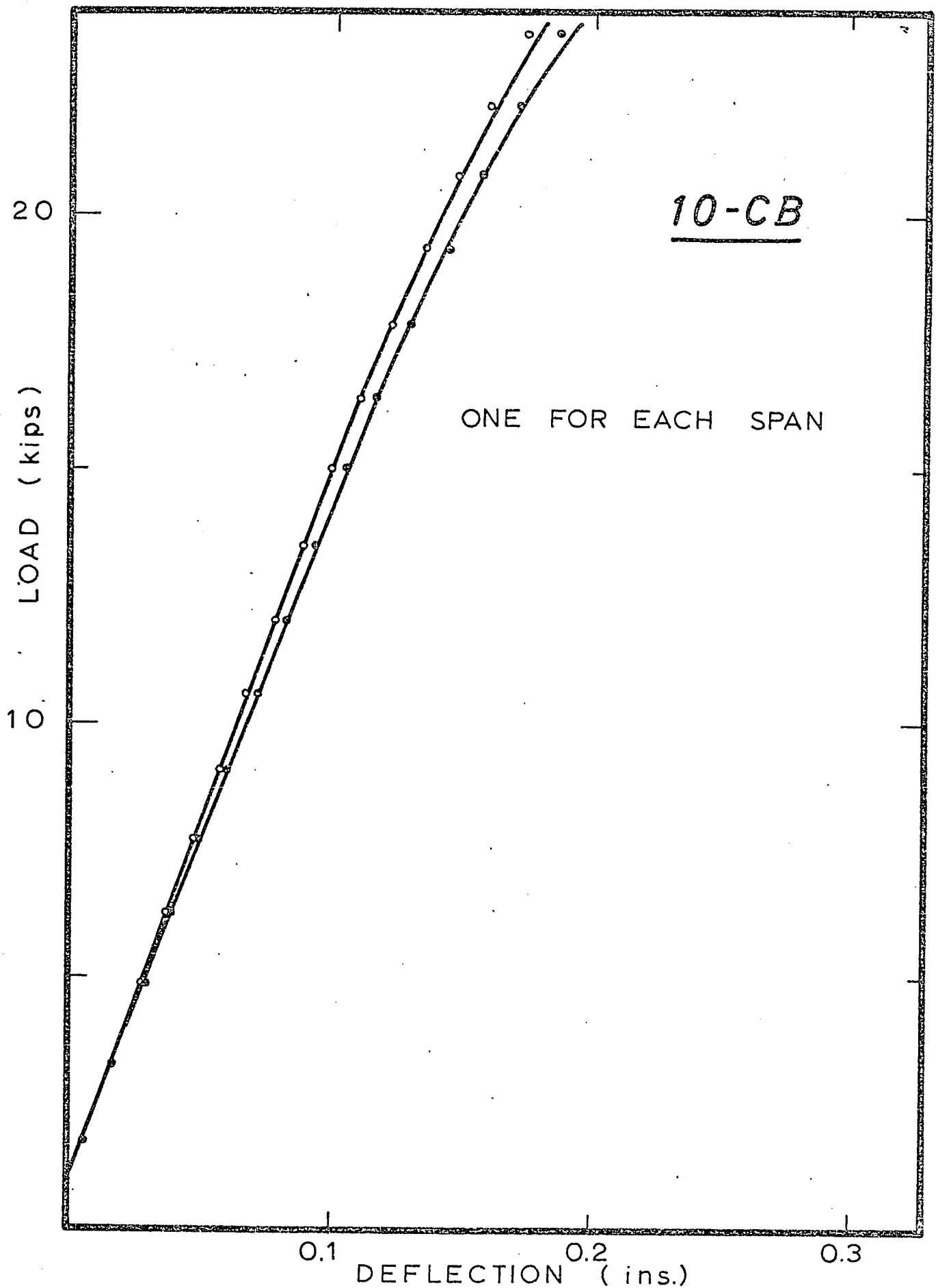


FIG.(B-30) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 10

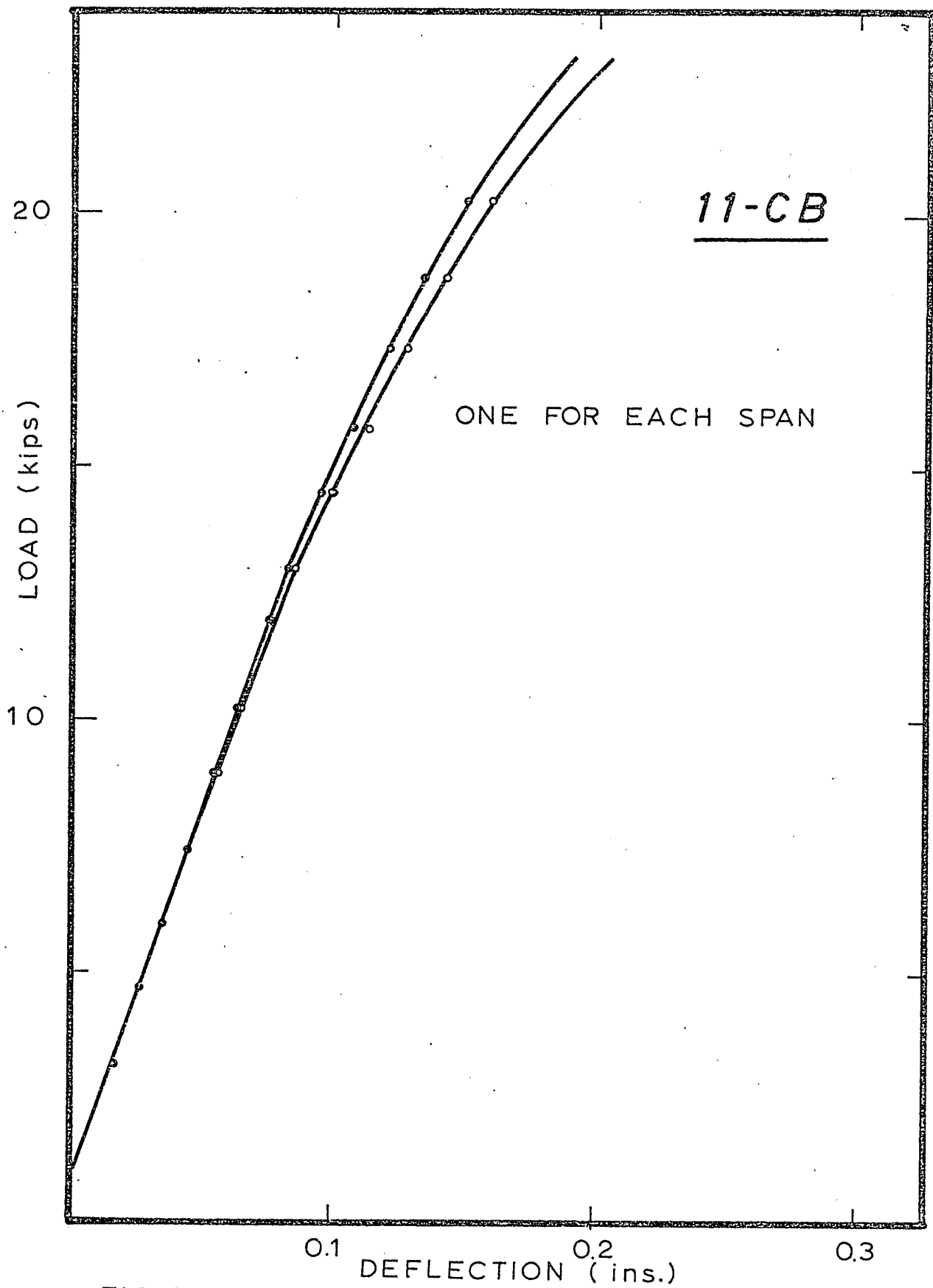


FIG.(B-31) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM

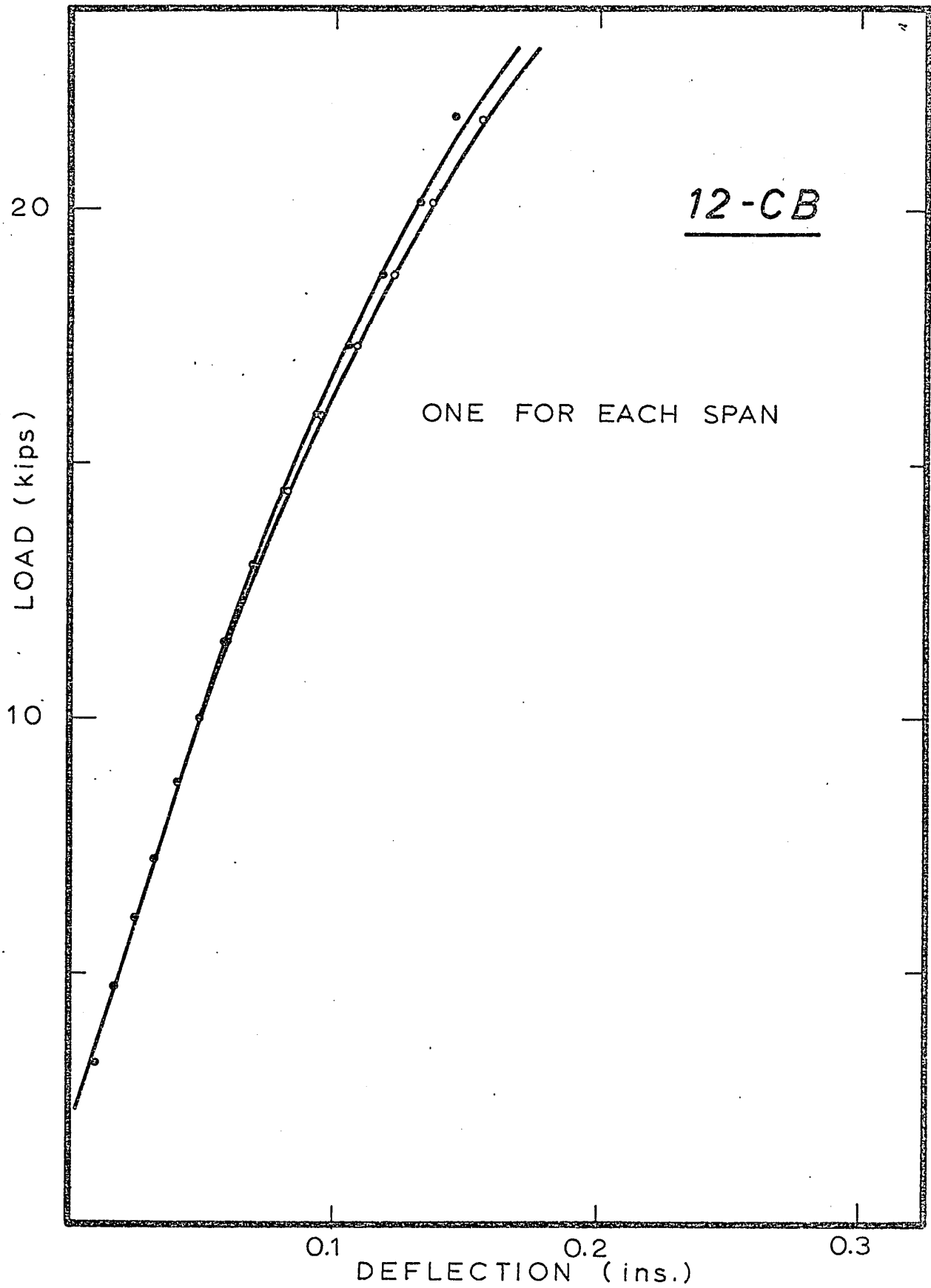


FIG.(B-32) LOAD-DEFLECTION CURVE FOR
TWO-SPAN CONTINUOUS BEAM 12

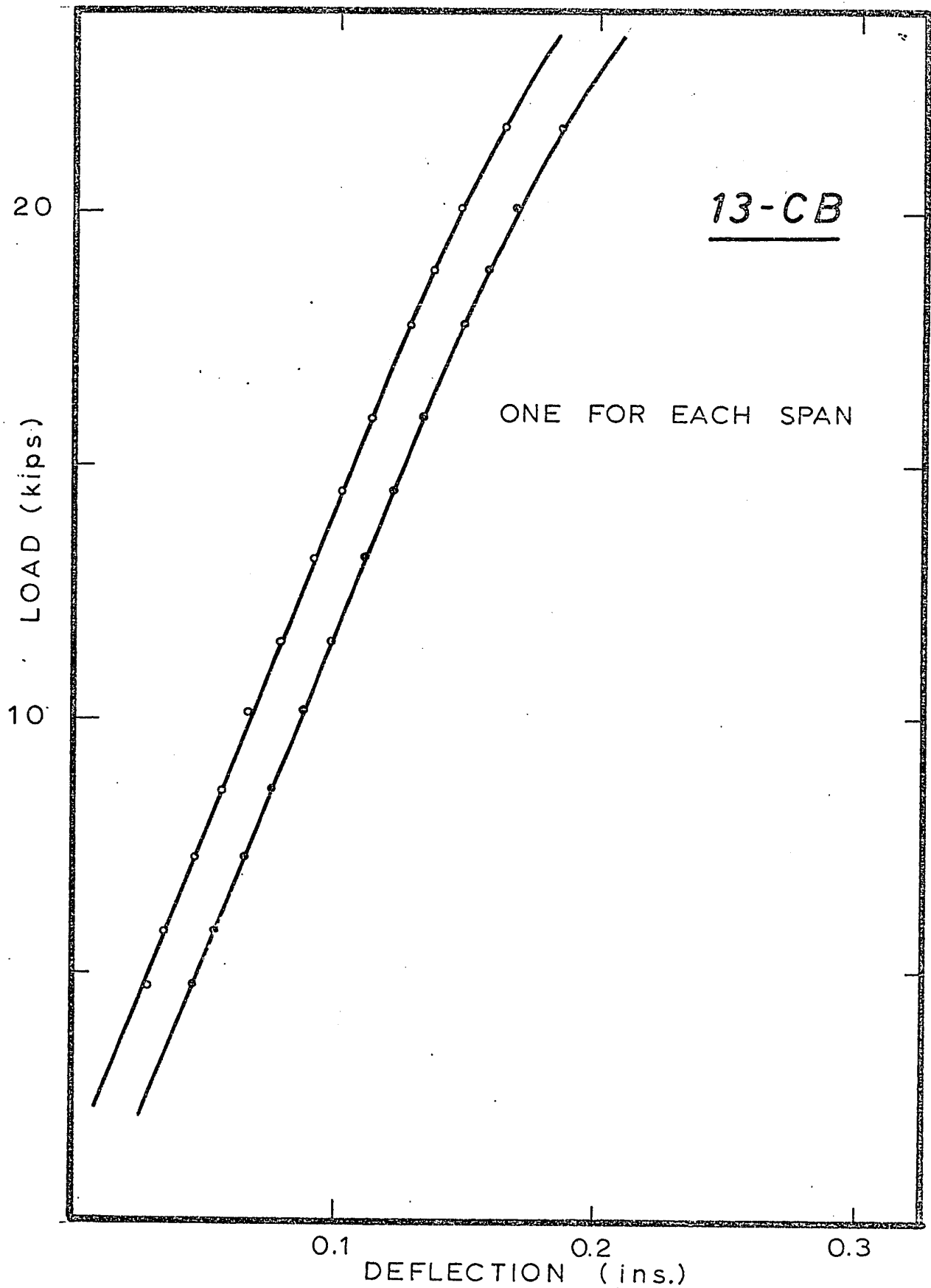


FIG.(B-33) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 13

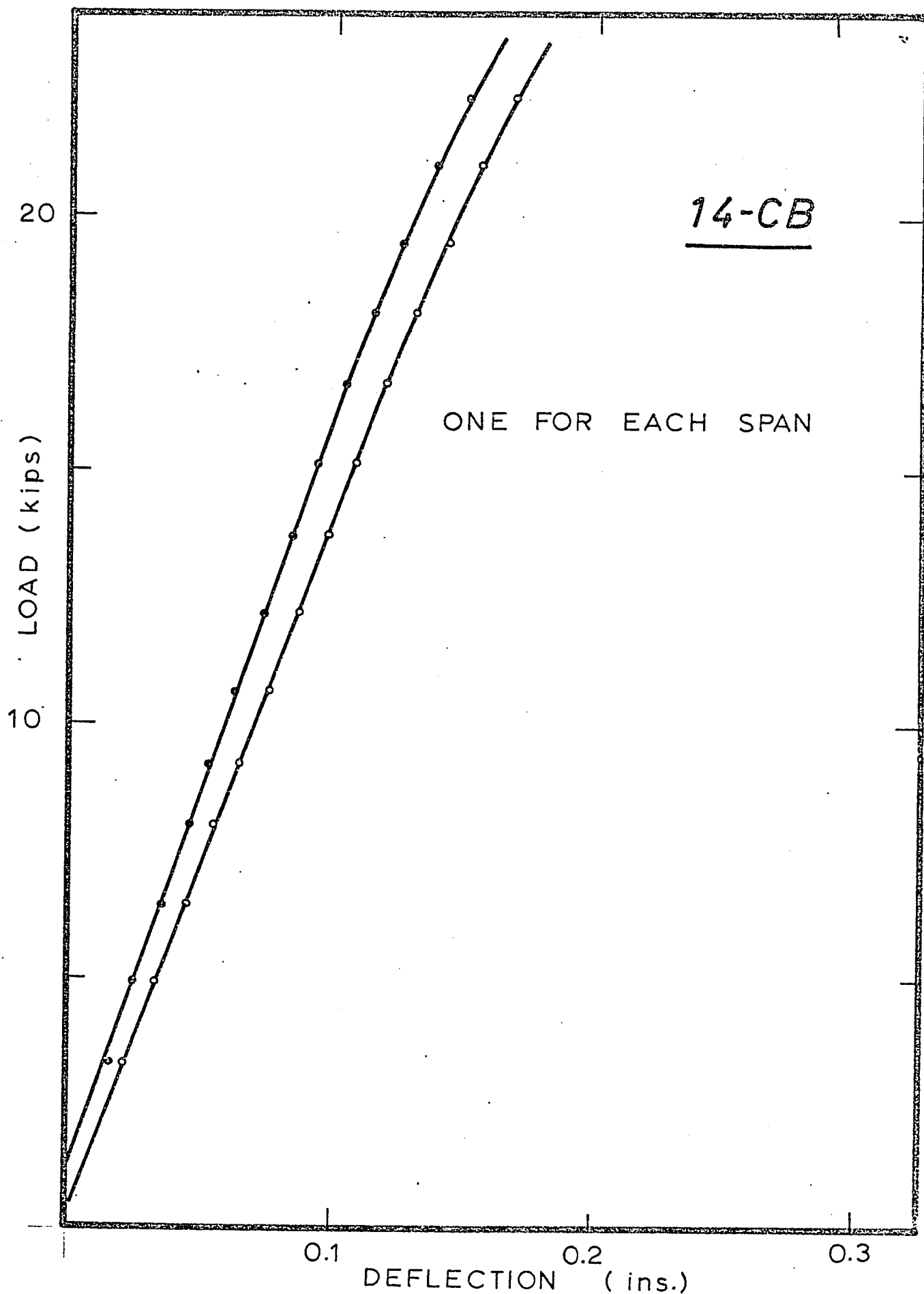


FIG.(B-34) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 14

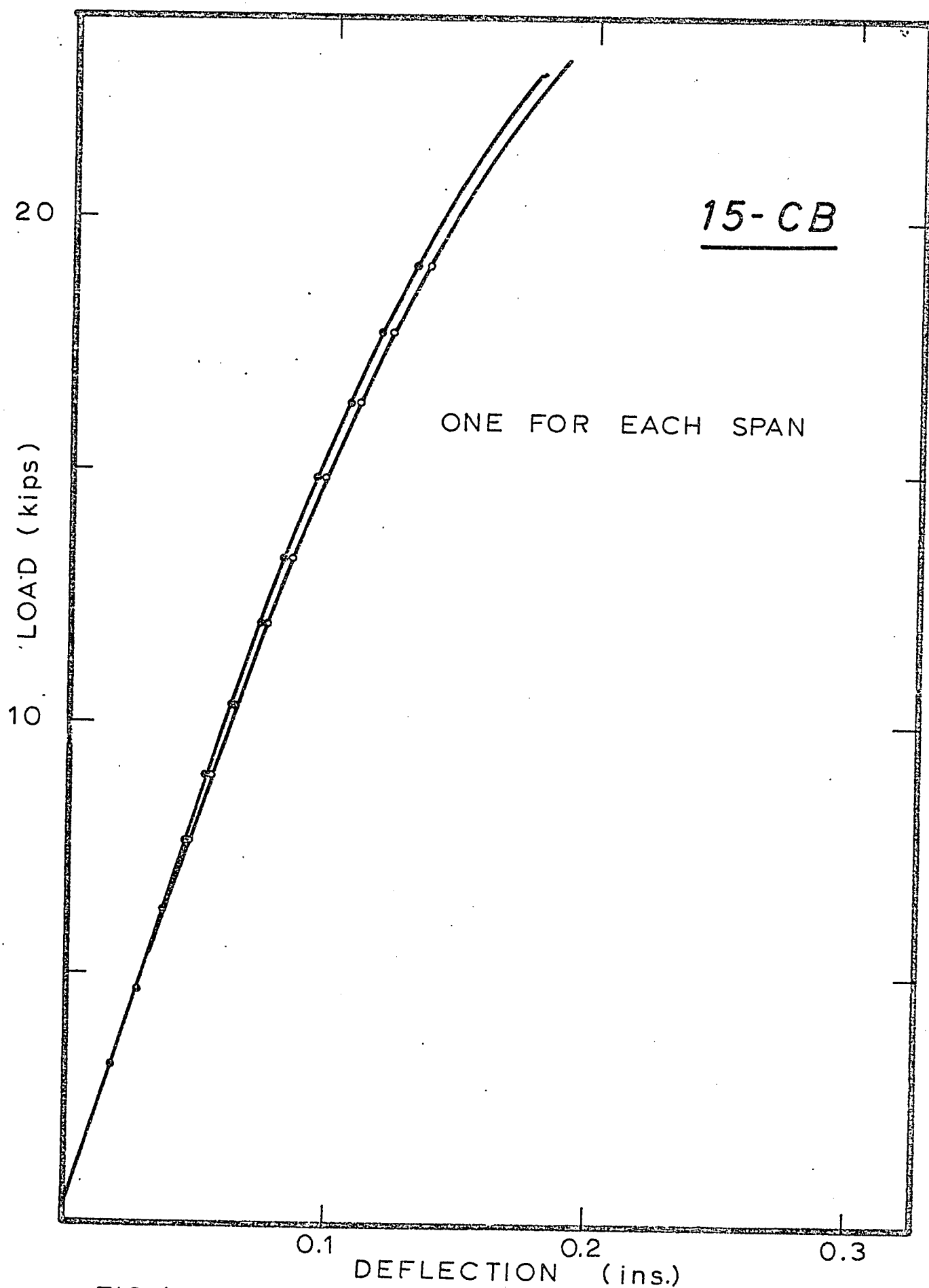


FIG.(B-35) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 15

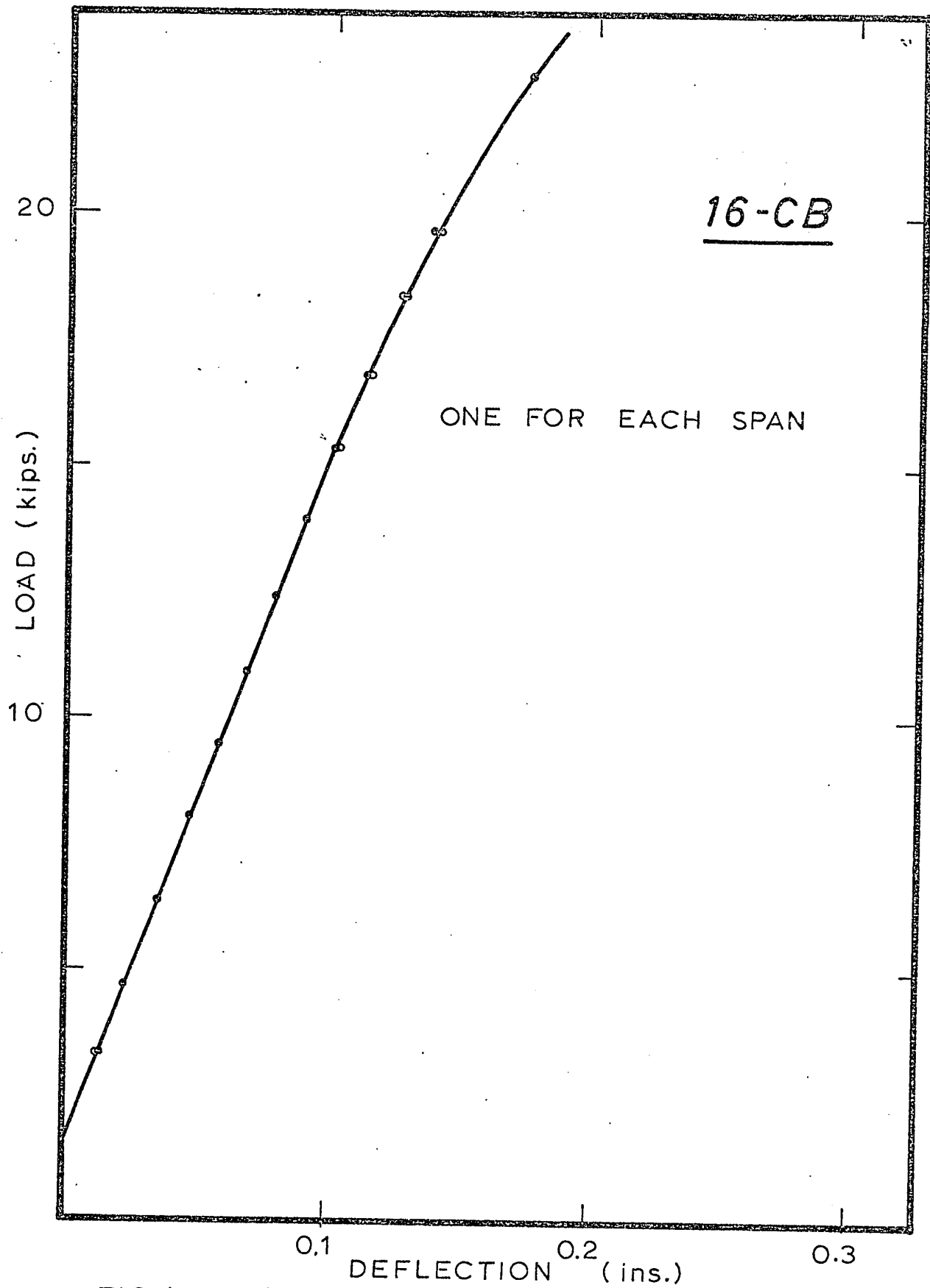


FIG.(B-36) LOAD-DEFLECTION CURVE FOR
TWO-SPAN CONTINUOUS BEAM 16

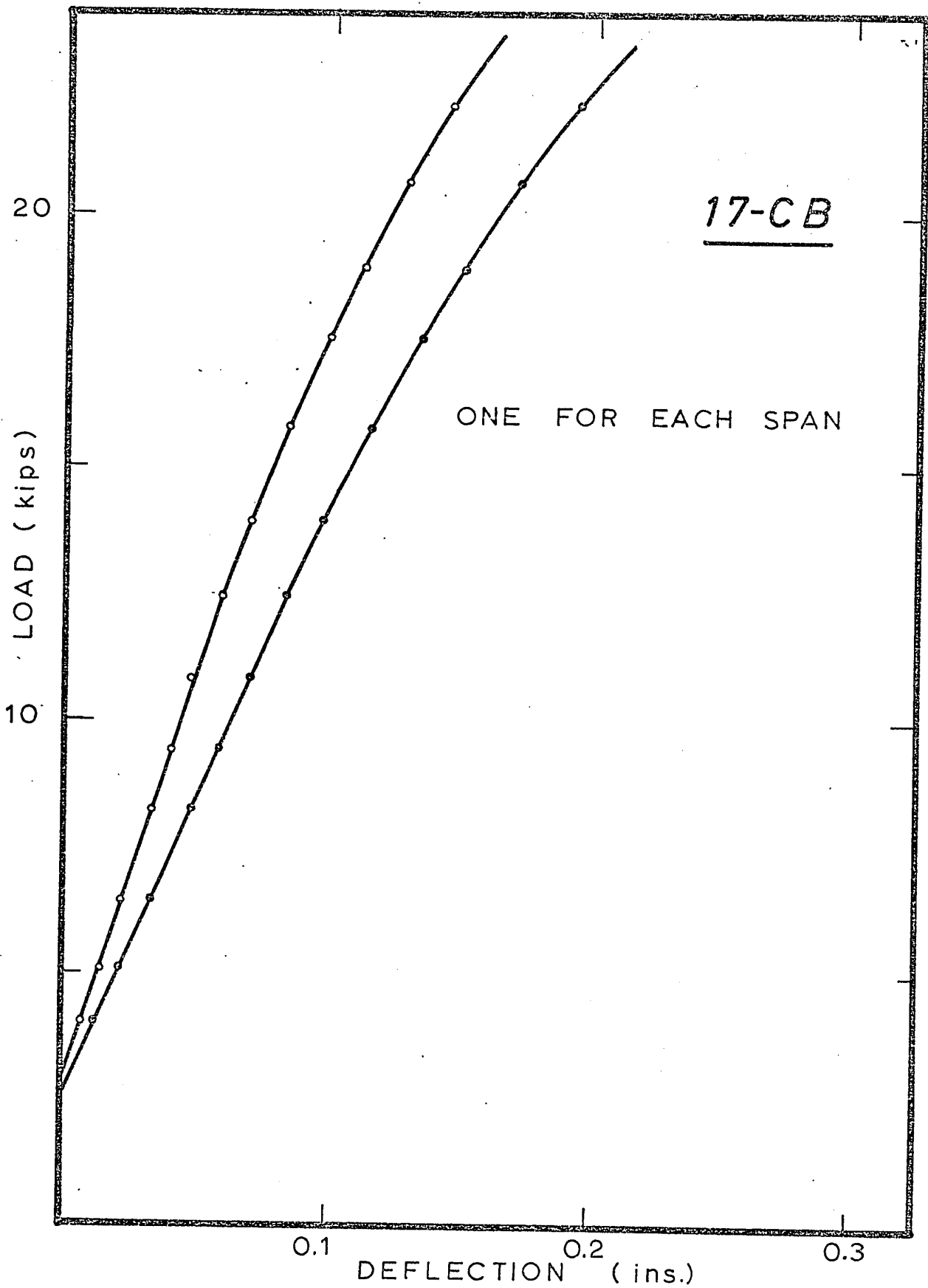


FIG.(B-37) LOAD-DEFLECTION CURVE FOR
TWO-SPAN CONTINUOUS BEAM 17

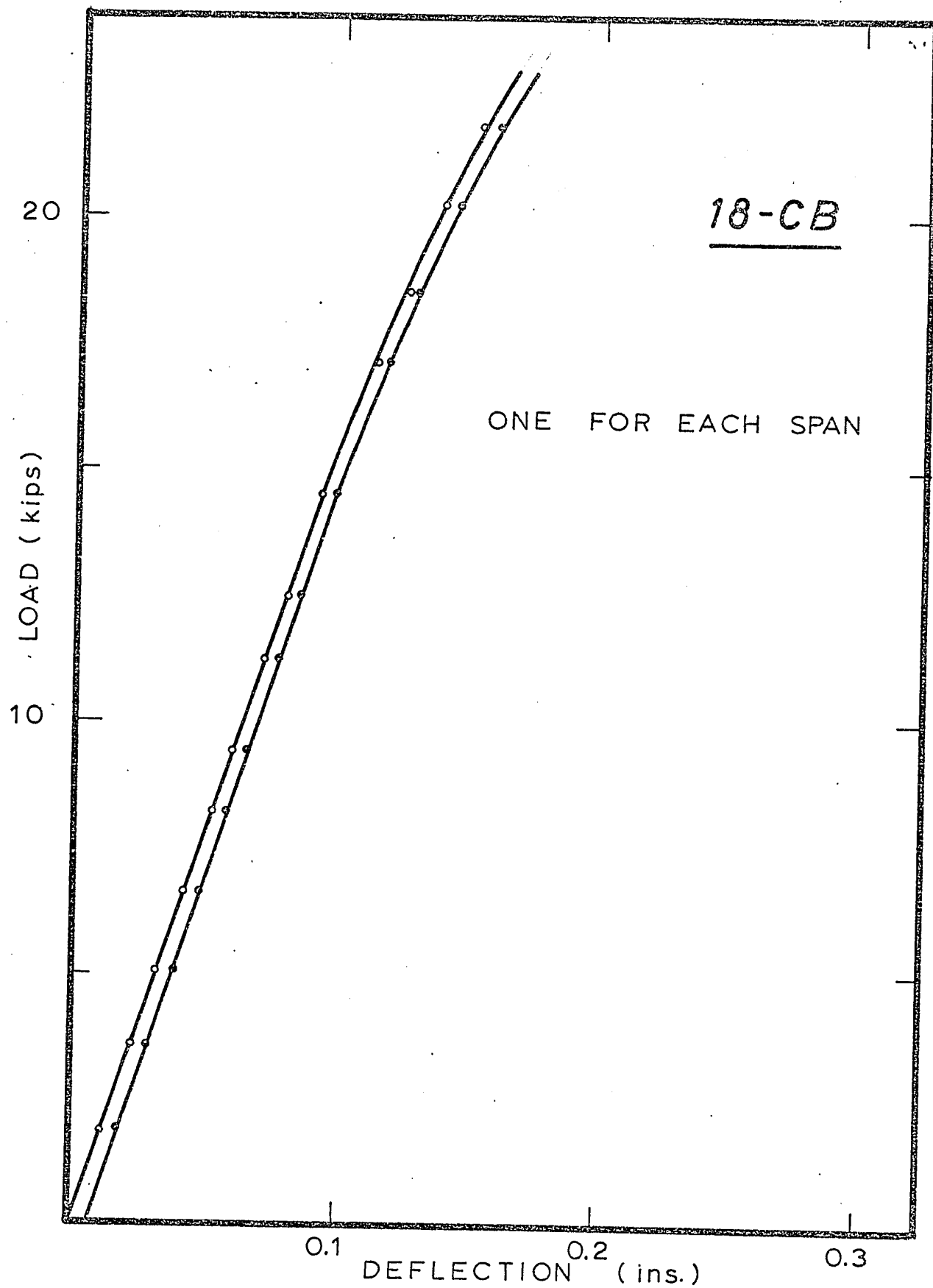


FIG.(B-38) LOAD-DEFLECTION CURVE FOR
TWO-SPAN CONTINUOUS BEAM 18

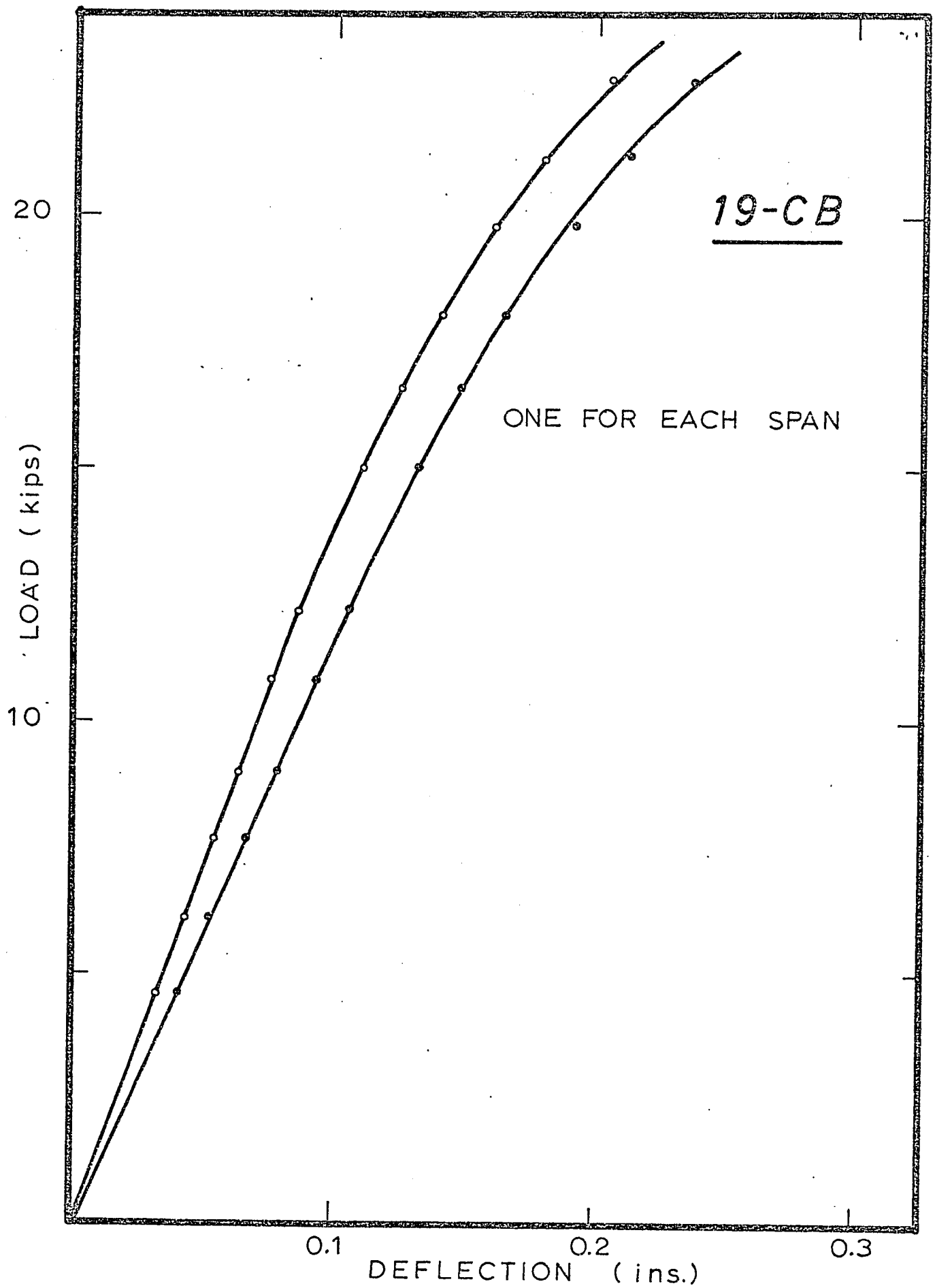


FIG.(B-39) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 19

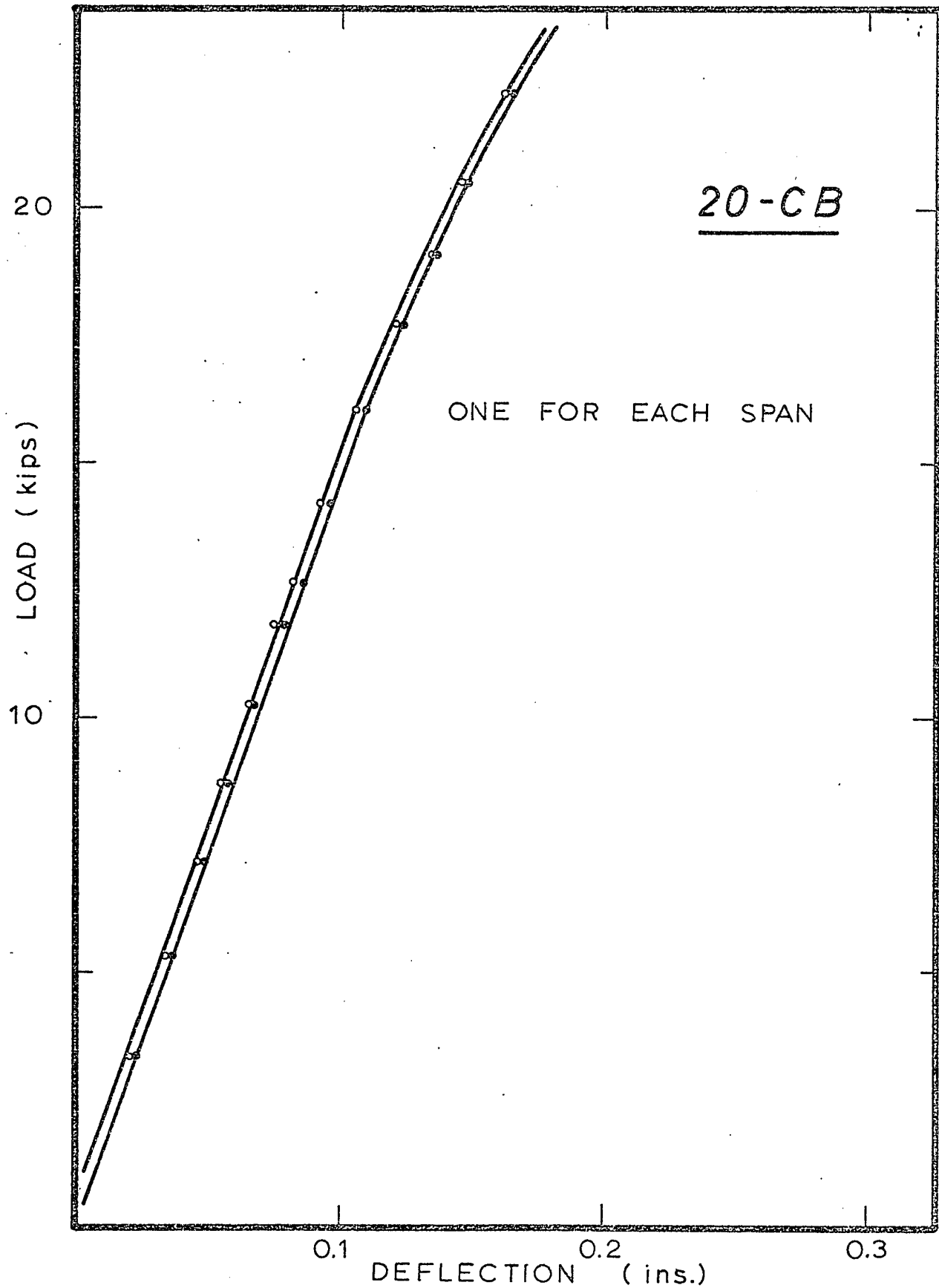


FIG.(B-40) LOAD-DEFLECTION CURVE FOR TWO-SPAN CONTINUOUS BEAM 20

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APPENDIX C.

(Chi Square Test for t-distribution)

TEST OF FIT OF STUDENT 't' DISTRIBUTION TO EXPERIMENTAL RESULTS USING CHI-SQUARED TEST

CONCRETE

 $\bar{X} = 4440$ PSI, $\sigma = 530$ PSI.

Compressive Crushing Strength (psi)	Class Boundary (psi)	$t = \left(\frac{X - \bar{X}}{\sigma}\right)$	Area for Each Class	Expected Frequency	Observed Frequency
3471-3770	3470.5	-1.83	0.068	1.36	2
3771-4070	3770.5	-1.26	0.135	2.70	1
4071-4370	4070.5	-0.70	0.198	3.96	6
4371-4670	4370.5	-0.13	0.227	4.54	7
4671-4970	4670.5	+0.43	0.170	3.40	1
4971-5270	4970.5	+1.00	0.09	1.80	1
5271-5570	5270.5	+1.57	0.045	0.90	2
	5570.5	+2.14			

$$\chi^2 = \frac{(2-1.36)^2}{1.36} + \frac{(1-2.7)^2}{2.7} + \frac{(6-3.96)^2}{3.96} + \frac{(7-4.54)^2}{4.54} + \frac{(1-3.4)^2}{3.4} + \frac{(1-1.8)^2}{1.8} + \frac{(2-0.9)^2}{0.9}$$

$$= 7.16$$

$$\chi^2_{.99,4} = 13.3 > 7.16$$

Thus we can conclude that the fit of the data is very good.

TEST OF FIT OF STUDENT 't' DISTRIBUTION TO EXPERIMENTAL RESULTS USING CHI-SQUARED TEST

REINFORCEMENT

$\bar{X} = 49.90$ KSI, $\sigma = 3.74$ KSI.

Yielding Strength (ksi)	Class Boundary (ksi)	$t = \frac{X - \bar{X}}{\sigma}$	Area for Each Class	Expected Frequency	Observed Frequency
40.001-43.000	39.995	-2.65	0.032	0.64	2
43.001-46.000	42.995	-1.84	0.111	2.22	1
46.001-49.000	45.995	-1.04	0.252	5.04	3
49.001-52.000	48.995	-0.24	0.252	6.30	8
52.001-55.000	51.995	+0.56	0.315	3.80	6
	54.995	+1.36	0.190		

$$\chi^2 = \frac{(2-0.64)^2}{0.64} + \frac{(1-2.22)^2}{2.22} + \frac{(3-5.04)^2}{5.04} + \frac{(8-6.30)^2}{6.30} + \frac{(6-3.8)^2}{3.8}$$

$$= 6.13$$

$$\chi^2_{.99,2} = 9.21 > 6.13$$

Thus we can conclude that the fit of the data is very good.

TEST OF FIT OF STUDENT 't' DISTRIBUTION TO EXPERIMENTAL RESULTS USING CHI-SQUARED TEST
COLUMN ($\bar{X} = 30.92$ KIPS, $\sigma = 2.63$ KIPS).

Ultimate Load (kips)	Class Boundary (kips)	$t = \frac{X - \bar{X}}{\sigma}$	Area for Each Class	Expected Frequency	Observed Frequency
26-27.9	25.95	-1.89	0.10	2.0	2
28-29.9	27.95	-1.13	0.22	4.4	8
30-31.9	29.95	-0.37	0.30	6.0	3
32-33.9	31.95	+0.39	0.21	4.2	5
34-35.9	33.95	+1.15	0.10	2.0	1
36-37.9	35.95	+1.91	0.03	0.6	1
	37.95	+2.67			

$$\chi^2 = 0 + \frac{(8-4.4)^2}{4.4} + \frac{(3-6)^2}{4.4} + \frac{(5-4.2)^2}{4.2} + \frac{(1-2)^2}{2} + \frac{(1-0.6)^2}{0.6}$$

$$= 5.38$$

$$\chi^2_{.99,3} = 11.3 > 5.38$$

Thus we can conclude that the fit of the data is very good.

TEST OF FIT OF STUDENT 't' DISTRIBUTION TO EXPERIMENTAL RESULTS USING CHI-SQUARED TEST
SIMPLE BEAM ($\bar{X} = 15.16$ KIPS, $\sigma = 0.48$ KIPS).

Ultimate Load (kips)	Class Boundary (kips)	$t = \frac{X - \bar{X}}{\sigma}$	Area for Each Class	Expected Frequency	Observed Frequency
14.101-14.400	14.095	-2.21	0.05	1.0	2
14.401-14.700	14.395	-1.59	0.10	2.0	1
14.701-15.000	14.695	-0.98	0.19	3.8	6
15.001-15.300	14.995	-0.36	0.25	5.0	3
15.301-15.600	15.295	+0.26	0.20	4.0	4
15.601-15.900	15.595	+0.88	0.12	2.4	3
15.901-16.200	15.895	+1.50	0.05	1.0	1
	16.195	+2.12			

$$\chi^2 = \frac{(2-1)^2}{1} + \frac{(1-2)^2}{2} + \frac{(6-3.8)^2}{3.8} + \frac{(3-5)^2}{5} + 0 + \frac{(3-2.4)^2}{2.4} + 0$$

$$= 3.72$$

$$\chi^2_{.99,4} = 13.3 > 3.72$$

Thus we can conclude that the fit of the data is very good.

TEST OF FIT OF STUDENT 't' DISTRIBUTION TO EXPERIMENTAL RESULTS USING CHI-SQUARED TEST
CONTINUOUS BEAM ($\bar{X} = 32.66$ KIPS $\sigma = 1.40$ KIPS).

Ultimate Load (kips)	Class Boundary (kips)	$t = \frac{X - \bar{X}}{\sigma}$	Area for Each Class	Expected Frequency	Observed Frequency
29.001-30.000	28.995	-2.610	0.03	0.6	2
30.001-31.000	29.995	-1.900	0.09	1.8	1
31.001-32.000	30.995	-1.190	0.19	3.8	1
32.001-33.000	31.995	-0.476	0.28	5.6	9
33.001-34.000	32.995	+0.232	0.23	4.6	4
34.001-35.000	33.995	+0.950	0.12	2.4	3
	34.995	+1.670			

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$$\chi^2 = \frac{(2-0.6)^2}{0.6} + \frac{(1-1.8)^2}{1.8} + \frac{(1-3.8)^2}{3.8} + \frac{(9-5.6)^2}{5.6} + \frac{(4-4.6)^2}{4.6} + \frac{(3-2.4)^2}{2.4}$$

$$= 8.68$$

$$\chi^2_{.99,3} = 11.3 > 8.68$$

Thus we can conclude that the fit of the data is very good.

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APPENDIX D.

(programs)

149.

Column

0001		DIMENSION TM(40,40), PU(40,40), FY(40), FC(40)
0002		READ (1,90) M,N
0003		READ (1,91)(FY(I),I=1,N)
0004		READ (1,92)(FC(J),J=1,M)
0005	91	FORMAT (8F10.4)
0006	90	FORMAT (2I5)
0007	92	FORMAT (8F10.4)
0008		DO 10 I=1,N
0009		TM (I,J)=0.
0010		DO 10 J=1,M
0011		TM(I,J)=FY(I)/(0.85*FC(J))
0012		WRITE (3,911) TM(I,J)
0013	10	CONTINUE
0014	911	FORMAT (//,5X, 8(F12.6, 2X))
0015		P=0.0246
0016		E=6.44
0017		D=4.19
0018		DP=1.81
0019		B=6.0
0020		DO 20 I=1,N
0021		PU(I,J)=0.
0022		DO 20 J=1,M
0023		PU(I,J)=0.85*FC(J)*B*D*(-P+1.0-E/D+SQRT(((1.0-E/D)**2 1+2.0*P*((TM(I,J)-1.0)*(1.0-DP/D)+E/D)))
0024		WRITE (3,911)PU(I,J)
0025	20	CONTINUE
0026		STOP
0027		END

Where;

$$P = p = \frac{A_s}{b d}$$

$$E = e'$$

$$D = d'$$

$$TM = m$$

$$PU = P_u = \text{Ultimate Load}$$

$$f'_c = FC$$

$$FY = f_y$$

(for more details see figures (3-6) and (3-7))

Two-Span Continuous Beam.

```

0001      DIMENSION FC(15),FY(15)
0002      N=12
0003      M=12
0004      U=0.003
0005      V=1.31
0006      C=6.69
0007      READ (1,2) (FY(J),J=1,M)
0008      READ (1,2) (FC(I),I=1,N)
0009      2  FORMAT (8F10.4)
0010      DO 10 I=1,N
0011      DO 10 J=1,M
0012      T=0.62*FY(J)
0013      X=2.
0014      5  F=29000. * 0.003 *(X-V)/X
0015      CS=0.62*F
0016      IF (FC(I).LE.4.0)AA=0.85
0017      IF (FC(I).GT.4.0.AND.FC(I).LE.4.2) AA=0.84
0018      IF (FC(I).GT.4.2.AND.FC(I).LE.4.4) AA=0.83
0019      IF (FC(I).GT.4.4.AND.FC(I).LE.4.6) AA=0.82
0020      IF (FC(I).GT.4.6.AND.FC(I).LE.4.8)AA=0.81
0021      IF (FC(I).GT.4.8.AND.FC(I).LE.5.0) AA=0.80
0022      IF (FC(I).GT.5.0.AND.FC(I).LE.5.2) AA=0.79
0023      IF (FC(I).GT.5.2.AND.FC(I).LE.5.4) AA=0.78
0024      IF (FC(I).GT.5.4.AND.FC(I).LE.5.6) AA=0.77
0025      IF (FC(I).GT.5.6.AND.FC(I).LE.5.8) AA=0.76
0026      CC=0.85*FC(I)*AA*5.0*X
0027      T=0.62*FY(J)
0028      FX=0.85*FC(I)*AA*5.0*(X-0.5*AA*X)*X
0029      1+0.62*F*(X-V)-0.62*FY(J)*(C-X)
0030      F1X=0.85*FC(I)*AA*5.0*(1.0-0.5*AA)*2*X
0031      1+0.62*87.*(1.0-V**2/(X**2))+0.62*FY(J)
0032      XX=X-FX/F1X
0033      IF (ABS(XX-X)-0.001)4,4,5
0034      5  X=XX
0035      GO TO 6
0036      4  X=XX
0037      WRITE (3,7)FC(I),FY(J),AA,X
0038      7  FORMAT (/1X,4F20.4)
0039      AM1=0.62*FY(J)*(C-0.62*FY(J)*0.5/(0.85*FC(I)*5.))
0040      AM2=0.62*FY(J)*(C-X)*2.0
0041      PU=AM2*(2.*AM1/AM2+1.)/18.
0042      WRITE (3,12) AM1, AM2, PU
0043      12  FORMAT (/1X,3F20.4)

```

FORTRAN IV G LEVEL 18

MAIN

DATE = 70049

```

0042      10  CONTINUE
0043      RETURN
0044      END

```

Simply supported Beam

```

0001      DIMENSION FC(40), FY(40), TMU(40,40)
0002      READ (1,661) N,M
0003      READ(1,671)((FC(I),I=1,N)
0004      READ(1,671)((FY(I),I=1,M)
0005      DO 10 I=1,M
0006      DO 10 J=1,N
0007      A=0.62/4.25*FY(I)/FC(J)
0008      10  TMU(I,J)=(0.62*FY(I)*(6.69-0.5*A))/12.
0009      DO 20 I=1,M
0010      20  WRITE (3,961) (TMU(I,J),J=1,N)
0011      661  FORMAT (2I5)
0012      671  FORMAT (8F10.5)
0013      961  FORMAT (/10X,3F12.4)
0014      STOP
0015      END

```

Where;

$$FC = f'_c \quad FY = f_y$$

A = a , Depth of concrete stress block

TMU = P_u , Ultimate load

Notations for two-span continuous beam program;

$$FC = f'_c \quad FY = f_y$$

U = ϵ_u Ultimate strain of concrete

V = Concrete cover

D = d , Effective depth

$$AM1 = M_{u1}$$

$$AM2 = M_{u2}$$

PU = P_u , Ultimate load