ECONOMIC GROWTH AND INCOME DISTRIBUTION IN AN OVERLAPPING-GENERATION FRAMEWORK

By

Ekaterina Zviaguintseva

Department of Economics

University of Ottawa

Major Paper presented to the Department of Economics, University of Ottawa, in partial fulfillment of the requirements of M.A. Degree

April 2005

Student number: 3495925

Supervisor: Professor N. V. Quyen

1. INTRODUCTION

In the last few decades, the processes of economic growth in various countries have been intensively studied by many economists, who are fascinated by their growth experience. Of specific interests are the elements that drive the growth process in these countries and that can explain why some economies grow while others stagnate. Although several theories have been formulated over time to identify the sources of growth, none of these theories has provided a completely satisfactory answer to these questions. Despite the efforts by researchers in this area to determine the theoretical and empirical basis behind the observed worldwide output movement, a satisfactory explanation of the elements behind these growth processes remains a challenge.

Modern growth theories are rich with analyses of the many plausible determinants of the growth process. They have also renewed interest in the relationship between economic growth and the distribution of income. The relationship between income inequality and growth is profound, intriguing, and, at the same time, controversial. In trying to understand why two countries with similar characteristics, except for their income distribution, have evolved along different paths in their development, some researchers have tried to link economic growth to the distribution of income, like, for example, a comparative study of the economic growth between South Korea and the Philippines, carried out by Lucas (1993) and Benabou (1996). These two countries began economic development under similar initial conditions, but with different wealth distributions, and experienced phenomenally different growth rates. South Korea started with a more equal initial distribution of wealth than the Philippines and had noticeably higher growth.

The views of Lucas, op. cit., and Benabou, op. cit., about the links between income inequality and economic growth as well as the exact nature of these interactions are not without controversy. The views expressed by these two researchers have generated debates on the existence of the causality and the direction of the causality between income inequality and growth. Lack of a strong and unambiguous empirical support for any of the opposing views adds to the controversy.

In the past, it was presumed that a trade-off between income inequality and efficiency existed. Income and wealth inequality, it was argued, is more conducive to economic growth on the basis that productive activities are more efficient without resource redistribution. However, the recent literature points toward the possibility that historical income inequality may have negative long-run effects on a country's economic performance. A number of studies relating income inequality and growth suggest that a high level of inequality may hurt overall economic growth and that initial historical inequalities may persist and even increase during the course of development. See, for example, Galor and Zeira (1993), Benabou (1996), and Piketty (1997). Although these studies consider different channels of interactions through which growth could be affected by inequality, most of them, in one way or another, also consider the accumulation of physical and human capital and emphasize the interactions between these factors and income inequality. The fundamental basis of this strand of research in economic growth is the premise that in the presence of imperfections in the capital market, income inequality may adversely affect individual savings, investments, and occupational decisions. The end results are underinvestment in education, which will affect the quality of the work force available in the future, and the accumulation of physical capital. In addition, researchers are interested not only in the extent of income inequality, but also in the sources of these inequalities. If income inequality prevents individuals from realizing their potential, then inequality may have serious implications for a country's economic performance. Lack of social mobility perpetuates existing inequality and could be one of the main obstacles in realizing one's potential. While doubts regarding the negative effects of income and wealth inequality on growth still exist, there is agreement on the importance of inequality persistence across generations for the growth process.

At the present time, there is no consensus on the links between persistent income inequality across generations and economic growth. Some growth theories suggest that if heritability of abilities is high, then low social mobility is efficient and any intervention in choices made by individuals and market arrangements may compromise economic efficiency. Other growth theories contend that if abilities are only moderately heritable then substantial social mobility is highly desirable. Within the latter group of growth theories, some view the market system as being able to allocate efficiently individual talents. Others in this same group adopt the view that capital market imperfections might make inequality more persistent than it ought to be; that is, if credit markets are imperfect, then individuals with little initial wealth may face limited investment opportunities, so that initial wealth inequality is translated into persistent income inequality.

One common feature of the theoretical studies that investigate the relationships between inequality and growth is an unchanging economic environment setting. See, for example, Aghion and Bolton (1997), Galor and Zeira (1993), Benabou, op. cit., Banerjee and Newman (1993), and Aghion and Howitt (1998). Such a setting does not allow for agents' movement up or down the income distribution ladder and this is clearly at odds with reality. Further developments along this line of inquiry suggest that opportunities for mobility should be incorporated in models used to study relationships between growth and inequality. An interesting question to ask would be "how the income and wealth distribution together with growth rates would evolve over time in settings where individuals have opportunities for positive upward mobility."

In an attempt to find an answer the above question, the present paper addresses the evolution of growth and the wealth distribution in a life-cycle model with endogenous growth, which allows for intergenerational mobility and in which individuals face borrowing constraints. Human capital accumulation through education is the engine of growth. In the paper, individuals must self-finance their education. The recent literature on endogenous growth has pointed to the importance of human capital formation as a determinant of economic growth. See, for example, Romer (1986), Lucas (1988), Barro and Lee (1994). A number of studies, such as Benabou, op. cit., Glomm and Ravikumar (1992), De Gregorio (1996), de la Croix and Monfort (2000), argue persuasively that unequal income distribution, combined with borrowing constraints, may be viewed as obstacles to an efficient allocation of resources and can have a negative effect on growth through sub-optimal investment in education. In the present model, growth is driven by privately financed human capital accumulation and credit market imperfections may limit agents' ability to invest in education and, consequently, their ability to accumulate human capital.

The model of the present paper is formulated under the overlapping-generations framework. In the model, agents live for three periods and differ in their initial wealth and innate ability. Parental altruism is used to link individuals within a dynasty. All the generations in a dynasty are assumed to have the same preferences and in any period, a young individual is responsible for making her educational investment decisions, given her inheritance and her innate ability. To allow for positive economic mobility, I assume that in each period the innate abilities of the young individuals are randomly and independently distributed across dynasties. Furthermore, innate abilities, which are assumed to be realized at the beginning of the period and revealed to the young individuals who possess them, complement educational investment. Consequently, the same level of investment in education will yield different levels of human capital for agents with different innate abilities. It is through part of her inheritance that a young agent acquires the real resources to pay for the educational investment needed to accumulate human capital.

Due to the model's complexity, an analytical solution does not seem to be possible. Hence a simulation of the theoretical model has been carried out. In the simulated economy, the initial income inequality seems not to persist in the long run. The stochastic nature of abilities, by creating opportunity for mobility, allows individuals to move within the income distribution ladder. Inequality in initial asset holdings tends to decrease with time. The distribution of human capital along the economy's growth path appears to support the theory that a merit-based distribution is not necessarily an equal one. The results of the simulation indicate that the growth of the economy is determined largely by the availability of resources; how the available resources are distributed plays a lesser part in determining the growth of the economy. It also appears that in the long run the market economy is able to allocate resources efficiently.

The paper is organized as follows. In Section 2, a review of the recent developments in the subject is presented. The model of the paper is presented in Section 3. The model is simulated in Section 4. The results of the simulation are discussed in Section 5. Some concluding remarks are given in Section 6.

2. A REVIEW OF RECENT DEVELOPMENTS

Existing theories relating growth, income distribution, and wealth distribution can be classified into two groups. The first group consists of the classical theories relating inequality and growth. In the second group are found more recent theories, which suggest a negative correlation between inequality and growth.

The classical school of economics, which began with Adam Smith (1776), was the pioneer in analyzing the interaction between income distribution, wealth distribution, and the growth process. Classical economists saw growth as a saving-driven process, and considered income distribution especially relevant to capital accumulation. According to the classical theory, savings rise with wealth,[1] and wealth comes mainly from profits that classical economists attributed entirely to capital owners. Therefore, a higher concentration of income and wealth in the hands of individuals with a higher propensity to save – in other words a more unequal distribution of income – would increase capital accumulation and enhance growth. The classical economists also offered two additional arguments in favor of income inequality as the engine of growth. The first argument rests on the premise that in the presence of investment indivisibilities, greater income and

[1] Few studies attempted to verify the belief that wealthy agents have higher savings rates and that the saving rate is positively correlated to income level, i.e., the higher is the income the higher is the proportion of income saved. The argument in support of this belief is not so straightforward. Only the top 1% exhibited a higher propensity to save out of income. For more details, see Carroll (1998), Dynan, Skinner, and Zeldes (2004), Huggett and Ventura (2000).

wealth concentration makes it easier for these lumpy investments to be carried out and thus for the economy to grow. In the second argument, it is claimed that inequality creates an incentive to promote efficiency in the use of resources. In the first half of the twentieth century, Keynes (1936), and later Neo-Keynesians, such as Kaldor (1956), Kalecki (1971), and Pasinetti (1974), adhered to the same premise that unequal income and wealth distribution stimulates economic growth.

The second half of the twentieth century witnessed an explosive development of "neoclassical" growth theories. Growth and industrialization had become the focal point for economists, and distributional issues, for a while, were regarded as of less importance. Early neoclassical growth theories rely heavily on the assumptions that markets are perfectly competitive and complete; economic agents are identical; and there are decreasing returns to production factors. Such a specification makes income and wealth distribution neutral to macroeconomic outcomes: it eliminates distribution from the set of variables of interest and makes it impossible to investigate any relationship between income distribution and growth. On the other hand, the theories that considered the interaction between inequality and growth focused either on the reverse causality – growth is the source of income inequality – or emphasized the inevitable trade-off between efficiency and equity. See, for example, Lewis (1954), Kuznets (1955), and Kaldor (1956). Another strand of the theoretical literature in the post World War II period propagated the view that a well-functioning market system can promote efficiency and that there exists an unavoidable trade-off between growth and income inequality; that is more justice in distribution requires a sacrifice in output. See, for example Okun (1975). This position suggests that economic progress will eventually trickle down to the poor if wealthy individuals are unimpeded in their pursuit of profits.

2.1. *The Kuznets Curve*

In his seminal paper, Kuznets, op cit., proposed an inverted-U shaped function to describe the relationship between the level of per-capita income and the income distribution. This researcher suggested that income distribution evolves according to stages in the course of an economy's development. In the initial stage – when per-capita income is low and the economy is just starting to grow – the income distribution widens and inequality rises. With further growth of per-capita income and at some level of development, the economy reaches a point where inequality ceases to widen and starts to narrow, creating thus the inverse U-shaped relation between income distribution and growth.

There is fairly large body of empirical work – with different degrees of complexity and sophistication – that attempt either to derive or to test the "Kuznets hypothesis." See, for example, Campano and Salvatore (1988), Clarke (1995), Ram (1988, 1995), Fields and Jakubson (1994), and Jha (1996). In general, these studies use an aggregate specification in reduced form to link inequality to per capita income. To test the Kuznets hypothesis, these researchers used different cross-sectional data sets, different measures of inequality, and different econometric methodologies. Despite the enormous efforts spent on this endeavor, the "Kuznetsian" empirical literature failed to find a systematic relationship between growth and inequality.

One of the main shortcomings of these studies involves the compatibility and quality of the data used. Another shortcoming resides in the excessive sensitivity of the results to models' specifications. Many estimation results do not stand up to robustness tests. See, for example, Saith (1983), Deininger and Squire (1996a, 1996b, 1996c), Anand and Kanbur (1993a, 1993b). And last, but not least, is the criticism that the "Kuznetsian" empirical literature has a strong tendency to minimize the role of policy by treating the distribution-development relationship as a law; that is, policy makers need not worry about any systematic negative effects of growth on income distribution, since inequality will eventually fall.

Despite the inconclusive results of the empirical literature, several theoretical studies attempted to reproduce Kuznets' curve. In their models, Perotti (1993) and Aghion and Bolton (1997) showed that as the economy grows, the initial rise in income inequality will turn around later. Galor (2000) attempted to show that inequality may first rise in the course of development then start declining, as time goes on, when human capital accumulation becomes the main driving force behind growth. However, other studies claimed that in some cases inequality might limit an individual's opportunities to accumulate human and physical capital, and that initial inequality may persist and even worsen, leading the economy into a poverty trap.

2.2. *Endogenous Growth and Income Distribution*

Contrary to the claims made by theories asserting that there is a trade-off between income equality and growth, in the 1960s and 1970s one witnessed not just a worsening distribution of income but also a drastic rise in poverty, despite the rapid growth in some developing countries[2] and the "growth-equity" miracles of several East Asian countries. Following the debt crisis of the 1980s, the experience of several East Asian countries, and the increase in inequality in developed countries, the research in growth and income inequality – with the help of the endogenous growth theories – shifted its focus and now concentrates on searching for alternative interaction mechanisms between income distribution and growth, leading to the development of the recent theories relating economic growth and inequality. This strand of research theories challenges the classical views at both the theoretical and the empirical levels.[3]

First, some empirical studies disputed the notion that redistribution adversely effects growth. Easterly and Rebelo (1993) and Perotti (1996) examined the effect of fiscal policy variables on growth for a large cross-section of countries and found positive and significant effects of fiscal policies on economic growth. The experience of some East Asian countries also demonstrated that redistribution in the form of land and educational reforms has a positive impact on growth.[4]

[2] See Chenery et al. (1974) and Fishlow (1972).
[3] For an excellent overview of this subject, see Benabou (1996a), Solimano (1998), and Aghion, Caroli and Garcia-Penalosa (1999).

[4] See Chu (1995), You (1998), Bourguignon et al. (2001).

Second, despite the statement that inequality leads to greater savings, the experience of the East Asian countries demonstrated that even low-income individuals might have high saving rates. Some empirical findings even suggest that there might be no relationship between inequality and aggregate savings.[5]

Third, the remarkable growth with sustainable and even improving income and wealth distribution in the East Asian countries has put the trade-off between equity and growth into question. Early work in the 1990s suggested that there is little empirical evidence that initial inequality in income and wealth has a positive impact on growth. Some of these studies examined the impact of inequality upon economic growth by using cross-sectional data and different econometric techniques to estimate a reduced-form linear equation linking average GDP growth rate against a measure of inequality and found a negative correlation between these two variables.[6]

The findings of the 1990s have been challenged by the recent empirical literature on the relationship between inequality and growth, which suggests that the experiences of developing and developed countries may differ; that is, empirical studies using the data for poor countries support the negative correlation between inequality and growth, while those using the data for rich countries either find no such relation at all[7] or find a nonlinear relationship between inequality and growth.[8] Some authors, such as Forbes (2000) and Li and Zou (1998), even find a positive correlation between inequality and growth. Unfortunately, the estimations obtained from these cross-country studies have also been subjected to the same criticisms leveled against those trying to find empirical evidence for the "Kuznets hypothesis," due to their poor data quality, ad hoc specifications, and high sensitivity of the estimation results to model specifications. However, despite the diversity of results, these studies represent a challenge to the classical theories and suggest leads for further investigation of the relation between inequality and growth.

---

[5] See Schmidt-Hebbel and Serven (1997).
[6] See Perotti (1996), Alesina and Rodrick (1994), Persson and Tabellini (1994), and Clarke (1995).
[7] See Barro (1999, 2000) and Quah (2001).
[8] See Banerjee and Duflo (2003).

2.3. *Income Inequality, Economic Growth, and Political Economy*

One strand of the recent literature on economic growth and income inequality looks at the problem from the combined perspective of endogenous growth and political economy. Political economy models on inequality and growth are usually based on two premises. The first premise asserts that a highly unequal income and wealth distribution would inflame social conflict, endanger property rights, depress investment, and, as a consequence, reduce growth.[9] The second premise asserts that pronounced inequality, by disturbing social and political stability, would result in higher macroeconomic volatility. See, for example, Alesina and Perotti (1996). Researchers such as Haussmann and Gavin (1996) and Breen and Garcia-Penalosa (1998) found a positive correlation between income inequality and the growth rate's volatility. In their cross-country regressions, Ramey and Ramey (1995) and Haussmann and Gavin (1996) find that greater volatility of growth rates negatively affects the average growth rate. Therefore, unequal income distribution negatively affects growth through higher macroeconomic volatility.

Another strand of the literature employs the median voter theorem and starts with the premise that an unequal income distribution would lead to redistribution, which in turn, would hamper economic growth. That is, in democratic societies where government policies are determined through majority voting, the majority of the population – in the presence of a highly unequal income distribution – would choose to carry out an active program of redistribution. A more active redistribution of income would discourage investment by depressing its returns and, consequently, would lower growth. See, for example, the works of Perotti (1992), Betrola (1993), Alesina and Rodrik (1994). Unfortunately, this strand of literature is not supported by the data, as it has already been mentioned that empirical research finds support for the premise that income redistribution has a positive effect on growth.

[9] See Bellettini (1996), Benhabib and Rustichini (1996), and Alesina et al. (1996).

2.4. *Innovation Goods and Imperfect Capital Markets*

Some studies suggest that inequality might affect growth by altering the demand for various goods, especially the demand for innovation goods.[10] If demand for goods is hierarchic and growth is driven by innovations, an unequal income distribution could lead to smaller markets for innovation, which in turn means a weak demand for innovations. An economy that exhibits these characteristics will produce fewer innovations and have slower growth.[11] Inequality also may affect demand for manufacturing goods, causing slower productivity progress.[12]

Some researchers have advanced the idea of capital market imperfections to explain the link between inequality and growth. In economies where credit markets are less than perfect due to information or enforcement problems, agents could be limited in their borrowing capacity. Unequal income or wealth distribution coupled with borrowing constraints could limit agents' investment opportunities and lead to underinvestment in human or physical capital, thus negatively affecting growth.

Perotti (1992) finds that greater credit availability positively affects growth. Researchers such as Aghion and Bolton (1992, 1997), Galor and Zeira (1993), Benabou (1996), and Piketty (1997) propound the view that redistribution could help agents to overcome markets imperfections and undertake the correct kind and the right amount of investments necessary for economic growth. According to Aghion, Banerjee, and Piketty (1999), inequality combined with capital market imperfections may generate macroeconomic fluctuations, which impede growth. Suppose that inequality takes the form of unequal access to investment opportunities; that is, only a fraction of economic agents have access to high-profit investments. Consequently, there will be a separation of savers and investors in such an economy. Inequality of this type combined with a high degree of capital market imperfections can generate persistent investment volatility, which would

[10] See Zweimuller (2000).
[11] See Murphy, Shleifer and Vishny (1989).
[12] See Eswaran and Kotwal (1993).

result in lower production and hence lower economic growth. Haussmann and Gavin (1996) and Breen and Garcia-Penalosa (1998) also found that income inequality is positively correlated with growth the rate's volatility. And the empirical work of Ramey and Ramey (1995) and Haussmann and Gavin (1996) using cross-country regressions, in their turn, confirmed the proposition that greater volatility of growth rates negatively effects the average growth rate.

2.5. *Social Status, Fertility, and Income Inequality*

Some researchers have gone beyond economics and looked at the *pursuit of social status* as a factor that might link income inequality and economic growth. Their findings indicate that the effects on growth of the desire for status are ambiguous. The ambiguity stems from the different measures of status. Fershtman et al. (1996) showed that if status, measured as level of human capital or occupation, is a normal good, and demand for status is increasing with wealth, then in the presence of credit market imperfections higher inequality may result in an inefficient resource allocation and consequently in slower growth. Their model predicts that in their pursuit of status wealthy, but low-ability, agents over-invest in education while poor, but higher-ability, agents under-invest in education, and this results in a misplacement of talents and, consequently, in a lower average level of human capital for the economy. Corneo and Jeanne (2001) also showed that if status arises from wealth, more inequality leads to slower growth. On other hand, Frank (1999) and Hopkins and Kornienko (2004) demonstrated that if status arises from "*conspicuous consumption*," then there is a possibility that greater equality may result in slower growth through higher present consumption and lower savings.

According to Knell (1999), social comparison is also a possible channel through which inequality may effect economic growth. This researcher suggested that individuals, especially in developed countries, might be tempted to "keep up with the Jones" by adopting a level of consumption that is higher then they can afford by lowering savings and investment in human capital. Consequently in economies where social comparisons are particularly pronounced, higher inequality results in lower physical and human capital accumulation and, consequently, in slower growth.

Some researchers consider endogenous fertility as a possible channel for mediating the relationship between inequality and growth. Galor and Zang (1997), Kremer and Chen (2000), and later de la Croix and Doepke (2003) demonstrated that higher fertility rates negatively affect economic growth and that economies with a more unequal distribution of income tend to have significantly higher fertility rates. Theories relating endogenous fertility, inequality, and growth generally argue that parents face a trade-off between the quality and the quantity of their children and that inequality, combined with credit market imperfections, distorts parents' decision regarding number of offspring they want to raise and the investment in education for these children. In their models, Dahan and Tsiddon (1998), Kremer and Chen (2000), and de la Croix and Doepke (2003) assumed that children are normal goods and that parents obtain utility from the number and the quality – with quality measured by the education level obtained – of the children they raise. Raising a child is costly in terms of time; educating a child is costly in terms of real resources. High-income parents thus face a higher opportunity cost of raising a child, and this induces them to substitute quality for quantity. Less wealthy parents, on the contrary, face a lower opportunity cost and choose to have more children, but spend less on their education. As a result, everything else equal, higher inequality results in higher fertility differentials, which, in turn, results in less human capital accumulation and, consequently, in slower growth. In models with endogenous fertility and credit market imperfections, economic growth would thus be determined by the initial income distribution.[13]

2.6. *Income Inequality, Economic Growth, and Intergenerational Mobility*

The latest research on income inequality and economic growth takes the channels through which inequality could affect growth, and vice versa, focuses attention on the changes in inequality over time, especially whether initial inequalities dissipate with time or

[13] See Dahan and Tsiddon (1998), Galor and Weil (2000).

continue to persist across generations. This strand of research has intergenerational mobility as its object of study and is composed of several competing theories, which have been motivated by conflicting empirical evidence on the extent of inequality persistence.

Early empirical studies on intergenerational mobility estimated the intergenerational income correlation in the United States and several western European countries as very low – in the range of 0.18-0.25 – suggesting that there is very little persistence in economic status in free market economies. See, for example, Behrman and Taubman (1995), Becker and Tomes (1986). However, the problem with these estimates is that they used non-representative samples with poorly measured data. More recent studies, such as Solon (1992) and Zimmerman (1992), demonstrate that these estimates are far too low and produce significantly higher estimates for different measures of status,[14] demonstrating that social mobility in developed countries is not as high as it was thought before.

Transmission of wealth through inheritance is often referred to as one of the most obvious channels through which inequality may persist over time. Unfortunately, the data on inheritance is scarce, and only a small number of empirical studies have attempted to estimate the correlation between intergenerational mobility and income. See, for example, Menchik (1979) and Tomes (1981). The recent work of Mulligan (1997) finds estimates of total income intergenerational correlation in the range of 0.7-0.8. This suggests that family transmission of wealth could make inequality very persistent across generations. Most of the theoretical works on the long-run dynamics of wealth inequality support this proposition. Only under the assumptions of a concave inheritance function and equality of labor earnings could initial wealth inequality disappear with time.[15] However, if any of these assumptions were relaxed the result would not hold. For instance, if (i) the saving function is convex, (ii) economic agents differ in their fertility

[14] See N. Stokey (1998) for a great review of recent empirical work on intergenerational mobility.

[15] See Stiglitz (1969).

behavior, (iii) credit markets are imperfect, and (iv) there is earnings inequality, then wealth inequality will persist in the long run.[16]

Although wealth transfer is a very powerful transmission mechanism, empirical studies show that intergenerational income is in large part due to the persistent inequality of labor earnings.[17] Piketty (2000) suggests several possible sources of the high persistency of earnings inequality:

(i) within family transmission of abilities, ambitions, and tastes;

(ii) imperfect capital markets;

(iii) local segregation into unequal communities;

and

(iv) self-fulfilling beliefs.

The theories that attribute the persistence of earnings inequality to family transmission of abilities, ambitions, tastes, and efficient human capital investment suggest that an agent's future earnings capacity is in large part determined by family background either through transmission of genetic abilities or through transmission of cultural characteristics. The implication of this point of view is that inequality is the inevitable result of efficient market choices by economic agents and that low economic status mobility is justified. There is not much empirical evidence regarding genetic abilities transmission,[18] but there is an impressive number of studies that find significant positive effect of family background (characteristics) on individuals' earning capacity.[19]

The persistence of earnings inequality has also been attributed by some theories to credit market imperfections. The basic idea of these theories is that credit market imperfections limit the investment opportunities of economic agents. The families that are constrained by their low initial wealth are not able to find enough funds to realize their optimal

[16] See, for example, Bourguignon (1981), Dahan and Tsiddon (1998), and Piketty (1996, 1997).

[17] See Mulligan (1997).

[18] See Herrnstein and Murray (1994).

[19] See Becker and Tomes (1986) and Mulligan (1997).

investment and thus will remain poor. That is especially relevant to efficient human capital investment. With credit market imperfections, initial wealth inequality could be translated into subsequent earnings inequality, making inequality across generations more persistent than it ought to be.[20]

Despite the existence of extensive support for credit constraints at the micro level,[21] the empirical evidence regarding credit market imperfections relevant to aggregate intergenerational mobility is unclear. Mulligan (1997), Cameron and Heckman (1998), and Carneiro and Heckman (2002) argue that only small portion of agents faces short-term borrowing constraints. However, it is possible that the importance of credit constraints for social mobility may vary over time and across countries.

Segregation – a sociological factor – has also been advanced to explain inequality persistence. The quality of the community in which the agent chooses to live often affects her income. Benabou (1993, 1996b) and Durlauf (1996) demonstrated that local segregation could reinforce income inequality and lead to lower income mobility. Borjas (1992, 1995), Mulligan (1993) and some other researchers provide evidence on the impact of local environment on intergenerational mobility. However, the inefficiency of local segregation depends on the channels through which intergenerational mobility is affected. If social mobility is affected through peer effects, then integrated neighborhoods could improve mobility prospects for poor individuals. However, if the quality of local education is responsible for low mobility, then the negative effects of local segregation could be overcome by sufficient redistribution of resources.

Some theories attribute persistence of earnings inequality to self-fulfilling beliefs. If an agent views upward mobility as a result of luck and not as a result of one's abilities or hard work, this may reduce her incentives to move up. If individuals in the present society believe that their economic success is primarily determined by social origins, then poor agents might be skeptical regarding their chances for success and exert less effort

[20] See, for example, Loury (1981), Banerjee and Newman (1993), and Piketty (1997).

[21] See, for example, Evans and Jovanovic (1989) and Blanchflower and Oswald (1998),

than is optimal. Such self-fulfilling beliefs could perpetuate the initial inequality. Persistence of income inequality generated by self-fulfilling beliefs could be closely tied with persistence in economic status and family transmission of ambition and tastes. Economic agents, by comparing their social achievements to the ones of their parents or some "reference group", could choose to act according to society's expectations. Consequently, inequality would persist simply because a gents expect this inequality to persist.[22]

3. THE MODEL

The foundation of the present model is built upon two distinguished strands of literature. The first strand of literature examines the relationship between inequality and economic growth, and the second strand of literature focuses on intergenerational mobility. The model most closely resembles the works of Loury (1981) and Owen and Weil (1998). Loury, op cit., developed a model with parental altruism and where the productivity of an individual depends on her educational investment and innate abilities, with innate abilities assumed to be independently and identically distributed across young individuals. The main differences between my model and the former model reside in the fact that Loury, op cit., studied a stagnant economy, while my model incorporates endogenous growth. Another difference is that this author assumed that an individual does not know her endowments at the time of educational investment. More specifically, the model of this researcher features uncertainty at the individual level. Furthermore, in his model, parents transfer assets through time by means of investments in their children's education, not through bequests; that is, in his model, no storage technology is available to young individuals, which eliminates the possibility of borrowing and limits their investment options. Also, all decisions in the model are made by mature individual; that is, only parents may decide on consumption and educational investment. The model of Owen and Weil (1998), which was inspired by the works of Becker and Tomes (1986), Galor and Zeira (1993), and Galor and Tsiddon (1997), also featured random abilities that

[22] See Piketty (1998, 2000).

are identically and independently distributed across young individuals, as well as asset transfers from parents to offspring. Their model also incorporates borrowing constraints and allows young individuals to make decisions on investment in education. However, in their model abilities, which are measured in efficiency units, are constant over time and just reallocated among individuals; that is, their model shares a common feature with that of Loury's model in the sense that uncertainty at the individual level is generated by different combinations of the same abilities. Our model, on the other hand, features ability endowments that change over time, which in turn creates uncertainty at the aggregate level. Another feature that differentiates our model from that of Owen and Weil (1998) is that in their model the cost of education is fixed, and although young individuals differ in their abilities, and, consequently, may receive different labor incomes, based on initial asset endowments, they have only two options: either to invest a fixed amount, become educated, and work as a skilled worker or not to invest at all, remain uneducated, and work as an unskilled worker. The present model has a less restrictive structure of human capital accumulation;[23] that is, every young agent has an opportunity to acquire education, although investment in education may differ across agents because of their difference in innate ability.

In the model, time is discrete and denoted by $t, t = 0,1,...$ In the economy I consider, there is a single good which can either be consumed or used as an investment good to augment the stock of physical capital. The good is produced from labor and capital according to a standard neoclassical production function by competitive firms. There are m dynasties in the model, and in each period a dynasty is represented by three individuals: a young individual, a mature individual, and an old individual. A young individual does not work; the only activity, besides consumption, that she engages in is an education program. An old individual is retired and does not work, either. The mature generation in any period thus constitutes the labor force in that period. Furthermore, I shall not model the leisure and labor supply choice of a mature individual by assuming that each mature individual

[23] The modified human capital production function used in the present work was inspired by the general function used by Glomm and Ravikumar (1992), De Gregorio (1996) and Hendricks (1999).

has one unit of time that she in-elastically offers for sale on the labor market at the prevailing wage rate.

To concentrate on the dynamics of growth and income distribution, we shall abstract from fertility decisions and population growth. Thus we assume that at the end of each period the old individual of a dynasty dies, the mature individual becomes old, and the young individual becomes mature, and a new young individual appears in the following period as the youngest member of the dynasty of that period. Furthermore, there is a transfer of wealth from the old individual to the new young individual in the form of a bequest, and it is with this inheritance that the new young individual begins her economic life.

3.1. *Profit Maximization*

Let r_t and ω_t denote, respectively, the rental rate of capital and the wage rate of *effective labor* that prevail in period t. The consumption good is taken to be the numeraire in each period. In period t, the representative firm produces the consumption good using effective labor L_t and physical capital K_t according to the following Cobb-Douglas production function:

(1) $$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},$$

where $A_t > 0$ is the technological level of the firm in period t and $\alpha, 0 < \alpha < 1$, is a parameter. In what follows, I assume that capital depreciates completely at the end of each period. Expression (1) thus represents output net of depreciation in each period.

Facing the rental rate of capital r_t and the wage rate paid to effective labor ω_t, the representative firm solves the following profit maximization problem:

(2) $$\max_{(K_t, L_t)} \left[A_t K_t^{\alpha} L_t^{1-\alpha} - r_t K_t - \omega_t L_t \right]$$

The following first-order conditions characterize the solution of (2):

(3) $$\alpha A_t \left[\frac{K_t}{L_t}\right]^{-(1-\alpha)} = r_t,$$

and

(4) $$(1-\alpha)A_t \left[\frac{K_t}{L_t}\right]^{\alpha} = \omega_t.$$

3.2. *The Life Cycle of a Young Individual*

Consider a young individual of dynasty j who begins her economic life in period t with a_{jt}^0 as her asset holdings. These asset holdings – which are her inheritance passed on by the old individual of the dynasty in the preceding period – constitute her initial wealth. Out of her initial wealth, such a young individual has to decide how much to consume in the current period, how much to invest in education to accumulate human capital, and how much to save. To fix ideas, I shall assume that production and education activities are carried out during the period and that the remuneration to the two factors of production are made at the end of the period, at which time consumption and saving decisions are made.

Let e_{jt}^0 be the amount of real resources devoted to education activities by a young individual of dynasty j in period t. The part of the inheritance that remains is then equal to $a_{jt}^0 - e_{jt}^0$, which is loaned on the capital market at the gross rate of return r_t. At the end of period t, the wealth of this young individual is $r_t(a_{jt}^0 - e_{jt}^0)$, part of which is used for current consumption, c_{jt}^0, and the remaining part, namely $s_{jt}^0 = r_t(a_{jt}^0 - e_{jt}^0) - c_{jt}^0$, is saved. The saving constitutes her assets in the following period, when she becomes a mature individual, i.e.,

(5) $$a_{j,t+1}^1 = r_t(a_{jt}^0 - e_{jt}^0) - c_{jt}^0, \qquad (j = 1,\dots,m, t = 0,1,\dots).$$

Obviously, we must have $c_{jt}^{0} \geq 0, e_{jt}^{0} \geq 0$. As for $a_{j,t+1}^{1}$, it can be negative if the individual is allowed to borrow. However, following the strand of literature on income inequality and growth that insists on imperfect capital market as a driving force behind persistent income inequality, I shall constrain $a_{j,t+1}^{1}$ to be non-negative.

The human capital that such a young individual accumulates obviously depends on her investment in education, namely e_{jt}^{0}, and her innate ability, say θ_{jt}^{0}. I shall assume that a young individual knows her innate ability. However, before a young individual appears on the scene, her innate ability is not known and is assumed to be uniformly distributed on the interval $[\theta_{\min}, \theta_{\max}]$, where $\theta_{\min}$ and $\theta_{\max}$ are two parameters satisfying $0 \leq \theta_{\min} < \theta_{\max}$. The value θ_{jt}^{0} thus represents the realized innate ability of the young individual of dynasty j in period t. I shall assume that the random variables θ_{jt}^{0}'s are independent through time and across dynasties. Furthermore, to allow for the externalities of the economy's stock of knowledge on individual human capital accumulation, I shall also assume that the human capital accumulated by a young individual of period t depends on the average human capital of the mature generation, i.e., of the mature individuals of the m dynasties, in that period. More specifically, the human capital accumulated by a young individual in period t and is at her disposal in the following period is

(6) $$h_{j,t+1}^{1} = \eta + \chi\left[\theta_{jt}^{0} e_{jt}^{0}\right]^{\gamma}\left[\varepsilon \bar{h}_{t}^{1}\right]^{1-\gamma},$$

where $\gamma, 0 < \gamma < 1$, is a parameter representing the elasticity of the level of human capital attained with respect to educational investment; $\varepsilon > 0$ is a parameter representing the weight accorded to the existing stock of human capital in the human capital of an individual in the economy; $\eta > 0$ is a parameter representing the base level of human capital of any individual when she does not make any investment in education; and $\chi > 0$ is the parameter representing the technological level of the technology of individual human capital accumulation. Also, in (6) I have let

(7) $$\bar{h}_t^1 = \frac{1}{m}\sum_{j=1}^{M} h_{jt}^1$$

denote the average human capital of all the mature individuals who exist in period t. The expression $\theta_{jt}^0 e_{jt}^0$, which is the product of innate abilities and the amount of real resources devoted to educational activities can be interpreted as the *effective educational investment.* As specified by (6), investment in education raises human capital above the base level η by an amount that is a linear homogenous function of individual effective educational investment and the *effective average human capital level of the current working generation,* namely $\varepsilon\bar{h}_t$. The specification of a base level of human capital prevents the economy from degenerating. The small weight accorded to the average human capital level of the current working generation ensures that the endogenous growth driven by the externalities generated by the economy's accumulation of knowledge is not too explosive. In particular, note that because innate ability is an input in the production of human capital, the same level of educational investment will result in different levels of human capital, depending on the innate ability of the young individual. To translate human capital into *effective labor*, I shall assume that an individual with the level of human capital h who works one hour will supply h *hours of effective labor.*

At the end of period $t+1$, the young individual of dynasty j, who is now mature, receives a labor income of $\omega_{t+1}h_{j,t+1}^1$ and a capital income of $r_{t+1}a_{j,t+1}^1$. If $c_{j,t+1}^1$ is her consumption at the end of this period, then her assets at the beginning of the following period – when she is old – is given by

(8) $$a_{j,t+2}^2 = r_{t+1}a_{j,t+1}^1 + \omega_{t+1}h_{j,t+1}^1 - c_{j,t+1}^1.$$

Let $c_{j,t+2}^2$ be the consumption in the last period of the life cycle of the young individual of dynasty j in period t. Then the bequest she leaves behind at the end of period $t+2$, which constitutes the inheritance of the young individual of her dynasty in period $t+3$, is

(9) $$a_{j,t+3}^0 = r_{t+2}a_{j,t+2}^2 - c_{j,t+2}^2.$$

The list $\left(c_{jt}^{0}, c_{j,t+1}^{1}, c_{j,t+2}^{2}, a_{j,t+3}^{0}\right)$ thus represents a realization of the life cycle of the young individual of dynasty j in period t. The lifetime utility associated with such a realized life cycle is assumed to be given by

(10) $u\left(c_{jt}^{0}, c_{j,t+1}^{1}, c_{j,t+2}^{2}, a_{j,t+3}^{0}\right) = u\left(c_{jt}^{0}\right) + \delta u\left(c_{j,t+1}^{1}\right) + \delta^{2} u\left(c_{j,t+2}^{2}\right) + b\left(a_{j,t+3}^{0}\right)$

In (10), $\delta, 0 < \delta < 1$, is the discount factor; u is the single-period utility function; and b is the sub-utility function of bequest. The sub-utility function b is intended to capture "*the joy of giving*" that parents enjoy by transferring resources to their children. For analytical tractability, I shall assume that both u and b are logarithmic and take on the following forms $u(c) = Log[c]$ and $b(a) = \beta Log[a]$, respectively, where $\beta > 0$ is a parameter representing the *degree of altruism*.

3.3. *The Price System and Decision Rules*

I shall now attempt to define a competitive equilibrium for the economy in question. Because of the stochastic nature of the innate abilities of the young individuals as time progresses, the relevant equilibrium concept is that of rational expectations equilibrium. To find a rational expectations equilibrium, we thus have to find the prices – the rental rate of capital and the wage rate – and the decisions made by each agent in each period in terms of the state of the system at the beginning of that period. Furthermore, because each young individual of a dynasty lives three periods, we also need to specify her expectations concerning future prices such that the choices she makes in each phase of her life cycle are optimal, given these expectations. Finally, the choices made by all the economic agents in each period must be mutually coherent in the sense that markets clear in each period.

In what follows, $x_t = \left(\left(a_{jt}^{0}, \theta_{jt}^{0}\right), \left(a_{jt}^{1}, h_{jt}^{1}\right), a_{jt}^{2}\right)_{j=1}^{m}$ denotes the state of the economy at the beginning of period $t, t = 0,1,\ldots$ The initial state of the system, namely $x_0 = \left(\left(a_{j0}^{0}, \theta_{j0}^{0}\right), \left(a_{j0}^{1}, h_{j0}^{1}\right), a_{j0}^{2}\right)_{j=1}^{m}$, is assumed to be given.

By *a stationary price system*, I mean a function $\hat{r}: x_t \to \hat{r}(x_t)$ that specifies the gross rental rate of capital and a function $\hat{\omega}: x_t \to \hat{\omega}(x_t)$ that specifies the wage rate paid to effective labor – both in period t – given that x_t is the state of the economy at the beginning of that period.

Next, let $(\hat{K},\hat{L}): x_t \to (\hat{K}(x_t),\hat{L}(x_t))$ be the *stationary decision rule* used by the representative firm in choosing its capital and effective labor inputs in each period, as functions of the state of the economy at the beginning of that period.

To continue, let

$$\left(\left(\hat{e}_j^0,\hat{c}_j^0\right),\hat{c}_j^1,\hat{c}_j^2\right)_{j=1}^m : x_t \to \left(\left(\hat{e}_j^0(x_t),\hat{c}_j^0(x_t)\right),\hat{c}_j^1(x_t),\hat{c}_j^2(x_t)\right)_{j=1}^m$$

be *a collection of stationary decision rules* for all the individuals who exist in any period. Here $\hat{e}_j^0(x_t)$ and $\hat{c}_j^0(x_t)$ represents, respectively, the educational investment and the consumption of the young individual of dynasty j in that period, while $\hat{c}_j^1(x_t)$ and $\hat{c}_j^2(x_t)$ represent, respectively, the consumption of the mature and old individuals of dynasty j – also in that period.

In what follows, let $\mathcal{P} = (\hat{r},\hat{\omega})$ denote a price system and $\mathcal{D} = \left((\hat{K},\hat{L}),\left(\left(\hat{e}_j^0,\hat{c}_j^0\right),\hat{c}_j^1,\hat{c}_j^2\right)_{j=1}^m\right)$ a collection of decision rules. *The collection of decision rules* $\mathcal{D}$ *is said to be consistent with the price system* $\mathcal{P}$ if the following market-clearing conditions hold for capital and labor:

$$(11) \quad \hat{K}(x_t) = \sum_{j=1}^{m}\sum_{i=0}^{2} a_{jt}^i - \sum_{j=1}^{m} \hat{e}_j^0(x_t),$$

$$(12) \quad \hat{L}(x_t) = \sum_{j=1}^{m} h_{jt}^1.$$

3.4. *Transition Probabilities*

Consider a price system $\mathcal{P}$ and an array of decision rules $\mathcal{D}$ that is consistent with $\mathcal{P}$. Under the pair $(\mathcal{P},\mathcal{D})$, the education investment made by the young individual of dynasty j in period t is $\hat{e}_j(x_t)$, and her human capital – when she becomes mature – is given by

$$(13) \quad \hat{h}^1_{j,t+1} = \eta + \chi\left[\theta^0_{jt}\hat{e}^0_j(x_t)\right]^{\gamma}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}.$$

Furthermore, in period t the rental rate of capital and the wage rate paid to effective labor are given, respectively, by

$$(14) \quad \hat{r}(x_t) = \alpha A_t\left[\sum_{j=1}^{m}\sum_{i=0}^{2} a^i_{jt} - \sum_{j=1}^{m}\hat{e}^0_j(x_t)\right]^{\alpha-1}\left[\sum_{j=1}^{m} h^1_{jt}\right]^{1-\alpha},$$

and

$$(15) \quad \hat{\omega}(x_t) = (1-\alpha)A_t\left[\sum_{j=1}^{m}\sum_{i=0}^{2} a^i_{jt} - \sum_{j=1}^{m}\hat{e}^0_j(x_t)\right]^{\alpha}\left[\sum_{j=1}^{m} h^1_{jt}\right]^{-\alpha}.$$

For the young individual of dynasty j in period t, the asset she owns at the beginning of the following period, when she is mature, is given by

$$(16) \quad \hat{a}^1_{j,t+1} = r_t\left(a^0_{jt} - \hat{e}^0_j(x_t)\right) - \hat{c}^0_j(x_t).$$

As for the mature individual of dynasty j in period t, her assets at the beginning of period $t+1$, when she is old, are given by

$$(17) \quad \hat{a}^2_{j,t+1} = r_t a^1_{jt} + \omega_t h^1_{jt} - \hat{c}^1_j(x_t).$$

The wealth that is left as a bequest by the old individual of dynasty j in period t is

$$(18) \quad \hat{a}^0_{j,t+1} = r_t a^2_{jt} - \hat{c}^2_j(x_t).$$

If $\theta^0_{j,t+1}$ is the realized innate ability of the young individual of dynasty j who appears on the scene in period $t+1$, then the state of the economy at the beginning of that period is $\hat{x}_{t+1} = \left(\left(\hat{a}^0_{j,t+1},\theta^0_{j,t+1}\right),\left(\hat{a}^1_{j,t+1},\hat{h}^1_{j,t+1}\right),\hat{a}^2_{j,t+1}\right)^m_{j=1}$. Note that given x_t and the pair $(\mathcal{P},\mathcal{D})$, all the elements of x_{t+1}, except for $\theta^0_{j,t+1}, j=1,\ldots,m,$ are completely determined. Theof randomness in x_{t+1} comes from $\theta^0_{j,t+1}, j=1,\ldots,m,$ the randomness of the innate abilities of the young individuals in period $t+1$. The density function that describes the transition from x_t to x_{t+1} is given by

(19)

$$f(x_{t+1}|x_t,(\mathcal{P},\mathcal{D})) = f\left(\left(\left(a^0_{j,t+1},\theta^0_{j,t+1}\right),\left(a^1_{j,t+1},h^1_{j,t+1}\right),a^2_{j\,t+1}\right)^m_{j=1}\middle|x_t,(\mathcal{P},\mathcal{D})\right)$$

$$=\begin{cases}\dfrac{1}{[\theta_{\max}-\theta_{\min}]^m} & if\ \left(a^0_{j,t+1},\left(a^1_{j,t+1},h^1_{j,t+1}\right),a^2_{j\,t+1}\right)^m_{j=1}=\left(\hat{a}^0_{j\,t+1},\left(\hat{a}^1_{j,t+1},\hat{h}^1_{j,t+1}\right),\hat{a}^2_{j,t+1}\right)^m_{j=1},\\ 0, & \text{otherwise.}\end{cases}$$

The transition density $f(x_{t+1}|x_t,(\mathcal{P},\mathcal{D}))$ describes completely the evolution of the economy from any given initial condition under the assumption that $\mathcal{P}$ is the prevailing price system and $\mathcal{D}$ is the collection of decision rules used by all the agents in each period. Furthermore, if x_t is the state of the economy at the beginning of period t, then the prevailing price system in that period is $(r_t,\omega_t)=(\hat{r}(x_t),\hat{\omega}(x_t)), t=0,1,\ldots,$ and the probability distribution, say $G((r_{t+1},\omega_{t+1})|(r_t,\omega_t))$, that describes the transition from (r_t,ω_t) to (r_{t+1},ω_{t+1}) can be computed from $f(x_{t+1}|x_t,(\mathcal{P},\mathcal{D}))$ and $\mathcal{P}$.

3.5. *Expected Lifetime Utility Maximization*

Suppose that $x_t=\left(\left(a^0_{jt},\theta^0_{jt}\right),\left(a^1_{jt},h^1_{jt}\right),a^2_{jt}\right)^m_{j=1}$ is the state of the economy at the beginning of period t and that the price system prevailing in this period is (r_t,ω_t). Furthermore, the price system in the next period, given (r_t,ω_t), is described by the transition distribution function $G((r_{t+1},\omega_{t+1})|(r_t,\omega_t)), t=0,1,\ldots,$ where the initial price system is given by $(r_0,\omega_0)=(\hat{r}(x_0),\hat{\omega}(x_0))$. In this subsection, I solve the expected utility maximization problems of the young, mature, and old individuals of each dynasty in period t, under the assumption that the expectations of these economic agents concerning future prices are described by the transition distribution function $G((r_{t+1},\omega_{t+1})|(r_t,\omega_t)), t = 0,1,...$

3.5.1. *Utility Maximization by the Old Individual*

At the beginning of period t, the asset of the old individual of dynasty j is a_{jt}^2. She does not work, and her wealth at the end of that period is $r_t a_{jt}^2$, which must be allocated between current consumption and a bequest. In her old age, this individual solves the following utility maximization problem:

(20) $\max_{(c_{jt}^2, a_{j,t+1}^0)} Log[c_{jt}^2] + \beta Log[a_{j,t+1}^0]$

subject to

(21) $c_{jt}^2 + a_{j,t+1}^0 = r_t a_{jt}^2$.

The solution of the utility maximization problem constituted by (20) and (21) is given by

(22) $c_{jt}^2 = \dfrac{r_t a_{jt}^2}{\beta + 1}$

and

(23) $a_{j,t+1}^0 = \dfrac{\beta r_t a_{jt}^2}{\beta + 1}$.

Observe that the consumption and bequest of the old individual are constant fractions of her wealth, and are independent of her beliefs about future prices. Substituting (22) and (23) into the objective function (20), we obtain the following expression for the utility of the old individual of dynasty j in period t:

$$\begin{aligned} v_{jt}^2(a_{jt}^2, r_t) &= Log\left[\frac{r_t a_{jt}^2}{\beta+1}\right] + \beta Log\left[\beta\frac{r_t a_{jt}^2}{\beta+1}\right] \\ &= (1+\beta)(Log[a_{jt}^2] + Log[r_t]) \\ &\quad + \beta Log[\beta] - (1+\beta)Log[1+\beta] \\ &= (1+\beta)(Log[a_{jt}^2] + Log[r_t]) \\ &\quad + m_2, \end{aligned} \tag{24}$$

where

$$m_2 = \beta Log[\beta] - (1+\beta)Log[1+\beta].$$

3.5.2. *Expected Utility Maximization by the Mature Individual*

The mature individual of dynasty j in period t solves the following expected utility maximization problem:

(25) $$\max_{\left(c^1_{jt}, a^2_{j,t+1}\right)|c^1_{jt}} Log[c^1_{jt}] + \delta \int v^2_{j,t+1}\left(a^2_{j,t+1}, r_{t+1}\right) dG\left[\left(r_{t+1}, \omega_{t+1}\right)|\left(r_t, \omega_t\right)\right].$$

subject to

(26) $$c^1_{jt} + a^2_{j,t+1} = r_t a^1_{jt} + \omega_t h^1_{jt}.$$

Using (24), we can rewrite the objective function (25) as

(27) $$\begin{aligned} & Log[c^1_{jt}] \\ & \quad + \delta(1+\beta) \int Log\left(a^2_{j,t+1}\right) dG\left[\left(r_{t+1}, \omega_{t+1}\right)|\left(r_t, \omega_t\right)\right] \\ & \quad\quad + \delta(1+\beta) \int Log\left(r_{t+1}\right) dG\left[\left(r_{t+1}, \omega_{t+1}\right)|\left(r_t, \omega_t\right)\right] \\ & \quad\quad\quad + \delta m_2 \\ & = Log[c^1_{jt}] + \delta(1+\beta) Log\left(a^2_{j,t+1}\right) \\ & \quad\quad + \delta(1+\beta) \int Log\left(r_{t+1}\right) dG\left[\left(r_{t+1}, \omega_{t+1}\right)|\left(r_t, \omega_t\right)\right] \\ & \quad\quad\quad + \delta m_2. \end{aligned}$$

Maximizing (27) under the constraint (26) yields

(28) $$c^1_{jt} = \frac{r_t a^1_{jt} + \omega_t h^1_{jt}}{1+\delta(1+\beta)},$$

(29) $$a^2_{j,t+1} = \frac{\delta(1+\beta)}{1+\delta(1+\beta)}\left(r_t a^1_{jt} + \omega_t h^1_{jt}\right)$$

According to (28) and (29), the current consumption and the saving of a mature individual are constant fractions of her wealth at the end of the period.

Substituting (28) and (29) into the objective function (27), we obtain the following expression for the expected discounted utility – for the remaining two periods of her life cycle – of the young individual of dynasty j in period t:

$$v_{jt}^1\left(\left(a_{jt}^1, h_{jt}^1\right),\left(r_t, \omega_t\right)\right)$$

(30)
$$= Log\left[\frac{r_t a_{jt}^1 + \omega_t h_{jt}^1}{1+\delta(1+\beta)}\right] + \delta(1+\beta) Log\left[\frac{\delta(1+\beta)}{1+\delta(1+\beta)}\left(r_t a_{jt}^1 + \omega_t h_{jt}^1\right)\right]$$
$$+\delta(1+\beta)\int Log[r_{t+1}]dG\left(\left(r_{t+1}, \omega_{t+1}\right)\middle|\left(r_t, \omega_t\right)\right)$$
$$+\delta m_2$$

$$= [1+\delta(1+\beta)] Log\left[r_t a_{jt}^1 + \omega_t h_{jt}^1\right]$$
$$+\delta(1+\beta)\int Log[r_{t+1}]dG\left(\left(r_{t+1}, \omega_{t+1}\right)\middle|\left(r_t, \omega_t\right)\right)$$
$$+\delta(1+\beta) Log[\delta(1+\beta)] - [1+\delta(1+\beta)] Log[1+\delta(1+\beta)]$$
$$+\delta m_2$$

$$= [1+\delta(1+\beta)] Log\left[r_t a_{jt}^1 + \omega_t h_{jt}^1\right]$$
$$+\delta(1+\beta)\int Log[r_{t+1}]dG\left(\left(r_{t+1}, \omega_{t+1}\right)\middle|\left(r_t, \omega_t\right)\right)$$
$$+m_1,$$

where $m_1 = \delta(1+\beta) Log[\delta(1+\beta)] - [1+\delta(1+\beta)] Log[1+\delta(1+\beta)] + \delta m_2$.

3.5.3. *Expected Lifetime Utility Maximization by the Young Individual*

The young individual of dynasty j in period t solves the following expected lifetime utility maximization problem:

(31)
$$\max_{\left(e_{jt}^0, a_{j,t+1}^1\right)}\left(\begin{array}{l} Log[r_t(a_{jt}^0 - e_{jt}^0) - a_{j,t+1}^1] \\ +\delta\int v_{j,t+1}^1\left[\left(a_{j,t+1}^1, h_{j,t+1}^1\right),\left(r_{t+1}, \omega_{t+1}\right)\right]dG\left(\left(r_{t+1}, \omega_{t+1}\right)\middle|\left(r_t, \omega_t\right)\right)\end{array}\right)$$

subject to

(32) $e^0_{jt} \geq 0,,$

(33) $a^1_{j,t+1} \geq 0,$

(34) $r_t(a^0_{jt} - e^0_{jt}) - a^1_{j,t+1} \geq 0,$

and

(35) $h^1_{j,t+1} = \eta + \chi\left[\theta^0_{jt} e^0_{jt}\right]^{\gamma}\left[\varepsilon \bar{h}^1_t\right]^{1-\gamma}.$

Observe that (32) requires education investment to be nonnegative. Inequality (33) reflects capital market imperfections, which preclude her from borrowing. Inequality (34) constrains current consumption to be non-negative. Finally, (35) represents her human capital in the next period, when she is mature.

Using (30), we can rewrite the objective function in (31) as

$$Log[r_t(a^0_{jt} - e^0_{jt}) - a^1_{j,t+1}] + \delta \int \begin{pmatrix} [1+\delta(1+\beta)]Log(r_{t+1}a^1_{j,t+1} + \omega_{t+1}h^1_{j,t+1}) \\ + \delta(1+\beta)\int Log[r_{t+2}]dG((r_{t+2},\omega_{t+2})|(r_{t+1},\omega_{t+1})) \\ + m_1 \end{pmatrix} dG((r_{t+1},\omega_{t+1})|(r_t,\omega_t))$$

(36)
$$\begin{aligned} &= Log[r_t(a^0_{jt} - e^0_{jt}) - a^1_{j,t+1}] \\ &\quad + \delta[1+\delta(1+\beta)]\int Log(r_{t+1}a^1_{j,t+1} + \omega_{t+1}h^1_{j,t+1})dG((r_{t+1},\omega_{t+1})|(r_t,\omega_t)) \\ &\quad + \delta^2(1+\beta)\int\left[\int Log[r_{t+2}]dG((r_{t+2},\omega_{t+2})|(r_{t+1},\omega_{t+1}))\right]dG((r_{t+1},\omega_{t+1})|(r_t,\omega_t)) \\ &\quad + \delta m_1. \end{aligned}$$

Because the human capital production function and the single-period utility function both satisfy the Inada condition, the inequalities (32) and (34) will both be strict at the optimum. However, inequality (33), which embodies the assumption of capital market imperfections, can hold with equality at the optimum. The following first-order conditions characterize the solution of this maximization problem:

$$(37)\quad -\frac{r_t}{r_t(a^0_{jt}-e^0_{jt})-a^1_{j,t+1}} + \delta[1+\delta(1+\beta)]\int\frac{\omega_{t+1}\gamma\chi\theta^0_{jt}\left[\theta^0_{jt}e^0_{jt}\right]^{\gamma-1}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}}{r_{t+1}a^1_{j,t+1}+\omega_{t+1}\left(\eta+\chi\left[\theta^0_{jt}e^0_{jt}\right]^{\gamma}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}\right)}dG\left((r_{t+1},\omega_{t+1})\middle|(r_t,\omega_t)\right)=0,$$

and

$$(38)\quad -\frac{1}{r_t(a^0_{jt}-e^0_{jt})-a^1_{j,t+1}} + \delta[1+\delta(1+\beta)]\int\frac{r_{t+1}}{r_{t+1}a^1_{j,t+1}+\omega_{t+1}\left(\eta+\chi\left[\theta^0_{jt}e^0_{jt}\right]^{\gamma}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}\right)}dG\left((r_{t+1},\omega_{t+1})\middle|(r_t,\omega_t)\right)\le 0,$$

with equality holding in (38) if $a^1_{j,t+1}>0$.

When $a^1_{j,t+1}=0,$ the first-order condition (37) becomes

$$(39)\quad -\frac{1}{(a^0_{jt}-e^0_{jt})}+\delta[1+\delta(1+\beta)]\frac{\gamma\chi\theta^0_{jt}\left[\theta^0_{jt}e^0_{jt}\right]^{\gamma-1}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}}{\eta+\chi\left[\theta^0_{jt}e^0_{jt}\right]^{\gamma}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}}=0.$$

Letting

$$(40)\quad \kappa^0_{jt}=\frac{e^0_{jt}}{a^0_{jt}}$$

denote the fraction of her inheritance that the young individual of dynasty j in period t devote to education investment, we can rewrite (39) as follows

$$(41)\quad \frac{1}{a^0_{jt}(1-\kappa^0_{jt})}=\delta[1+\delta(1+\beta)]\frac{\gamma\chi\theta^0_{jt}\left[\kappa^0_{jt}\theta^0_{jt}a^0_{jt}\right]^{\gamma-1}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}}{\eta+\chi\left[\kappa^0_{jt}\theta^0_{jt}a^0_{jt}\right]^{\gamma}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}}.$$

Equations (42) – (46) represent successively simplified versions of (41).

$$(42)\quad \frac{\left(1-\kappa^0_{jt}\right)\left[\kappa^0_{jt}\right]^{\gamma-1}\left[\theta^0_{jt}a^0_{jt}\right]^{\gamma}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}}{\eta+\chi\left[\kappa^0_{jt}\right]^{\gamma}\left[\theta^0_{jt}a^0_{jt}\right]^{\gamma}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}}=\frac{1}{\delta[1+\delta(1+\beta)]\gamma\chi},$$

$$(43)\quad \frac{\left(1-\kappa^0_{jt}\right)\left[\kappa^0_{jt}\right]^{\gamma-1}}{\frac{\eta}{\left[\theta^0_{jt}a^0_{jt}\right]^{\gamma}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}}+\chi\left[\kappa^0_{jt}\right]^{\gamma}}=\frac{1}{\delta[1+\delta(1+\beta)]\gamma\chi},$$

$$(44)\quad \left(1-\kappa^0_{jt}\right)\left[\kappa^0_{jt}\right]^{\gamma-1}=\frac{1}{\delta[1+\delta(1+\beta)]\gamma\chi}\left(\frac{\eta}{\left[\theta^0_{jt}a^0_{jt}\right]^{\gamma}\left[\varepsilon\bar{h}^1_t\right]^{1-\gamma}}+\chi\left[\kappa^0_{jt}\right]^{\gamma}\right),$$

(45) $$\left[\kappa_{jt}^{0}\right]^{\gamma-1}=\frac{1}{\delta[1+\delta(1+\beta)]\gamma\chi}\left(\frac{\eta}{\left[\theta_{jt}^{0}a_{jt}^{0}\right]^{\gamma}\left[\varepsilon\bar{h}_{t}^{1}\right]^{1-\gamma}}+\chi\left[\kappa_{jt}^{0}\right]^{\gamma}\right)+\left[\kappa_{jt}^{0}\right]^{\gamma},$$

(46) $$\left(\frac{1}{\delta[1+\delta(1+\beta)]\gamma\chi}\right)\left(\frac{\eta}{\left[\theta_{jt}^{0}a_{jt}^{0}\right]^{\gamma}\left[\varepsilon\bar{h}_{t}^{1}\right]^{1-\gamma}}\right)\left[\kappa_{jt}^{0}\right]^{1-\gamma}+\left(1+\frac{\chi}{\delta[1+\delta(1+\beta)]\gamma\chi}\right)\kappa_{jt}^{0}=1.$$

Observe that the left side of (46) – when considered as a function of κ_{jt}^{0} – is strictly increasing. This function is takes on the value 0 when $\kappa_{jt}^{0}=0$ and a value greater than 1 when $\kappa_{jt}^{0}=1$. Hence there is a unique value of $\kappa_{jt}^{0}, 0<\kappa_{jt}^{0}<1$, that satisfies (46). Such a value represents the educational investment – as a proportion of inheritance – made by the young individual of dynasty j in period t. It is clear that κ_{jt}^{0} rises with innate abilities, inheritance and the stock of human capital of the working generation. In the simulation carried out in Section 5, equation (46) will be solved numerically.

As for the first-order condition (38), when $a_{j,t+1}^{1}=0$, it becomes

(47) $$-\frac{1}{r_t(a_{jt}^{0}-e_{jt}^{0})}+\frac{\delta[1+\delta(1+\beta)]}{\eta+\chi\left[\theta_{jt}^{0}e_{jt}^{0}\right]^{\gamma}\left[\varepsilon\bar{h}_{t}^{1}\right]^{1-\gamma}}\int\frac{r_{t+1}}{\omega_{t+1}}dG((r_{t+1},\omega_{t+1})|(r_t,\omega_t))\leq 0,$$

which, with the help of (40) and (41), can be reduced to

(48) $$\frac{\delta[1+\delta(1+\beta)]}{\eta+\chi\left[\theta_{jt}^{0}e_{jt}^{0}\right]^{\gamma}\left[\varepsilon\bar{h}_{t}^{1}\right]^{1-\gamma}}\int\frac{r_{t+1}}{\omega_{t+1}}dG((r_{t+1},\omega_{t+1})|(r_t,\omega_t))\leq\frac{1}{r_t(a_{jt}^{0}-e_{jt}^{0})}$$

When the young individual of dynasty j in period t spends all of her inheritance on education and current consumption, it is necessary that inequality (48) holds. Because the objective function is strictly concave, this inequality is also sufficient for her saving, namely $a_{j,t+1}^{1}$, to be equal to 0. Observe that (48) will hold if her inheritance is small or if her innate ability is high. It is intuitive that a high innate ability will induce her to favour human capital over physical capital accumulation. A small inheritance means less resources are available for current consumption as well as for the accumulation of capital, physical or human. Furthermore, a low level of investment favours human capital accumulation over physical capital accumulation because of the high marginal productivity of the former.

3.6. *Rational Expectations Equilibrium*

Let $\mathcal{P} = (\hat{r}, \hat{\omega})$ be a price system and $\mathcal{D} = \left((\hat{K}, \hat{L}), \left((\hat{e}_j^0, \hat{c}_j^0), \hat{c}_j^1, \hat{c}_j^2\right)_{j=1}^{m}\right)$ be a collection of decision rules that is consistent with $\mathcal{P}$. Furthermore, let (r_t, ω_t) be the price system that prevails in period t and suppose that all the agents – through time and across generations – believe that the evolution of the stochastic process $(r_t, \omega_t), t = 0,1,\ldots,$ is governed by the transition distribution function $G((r_{t+1}, \omega_{t+1})|(r_t, \omega_t)), t = 0,1,\ldots,$ that is defined in Subsection 3.4. The pair $(\mathcal{P}, \mathcal{D})$ is said to constitute *a rational expectations equilibrium* if, given these expectations, the choices made by each economic agent in each period are her optimal choices. More explicitly, the pair $(\mathcal{P}, \mathcal{D})$ constitutes *a rational expectations equilibrium* if the following three conditions are satisfied.

First, for each $j = 1,\ldots,m,$ $\hat{c}_j^2(x_t)$ is the current consumption chosen by the old individual of dynasty j in period t; that is,

$$(49) \qquad \hat{c}_j^2(x_t) = \frac{r_t a_{jt}^2}{\beta + 1},$$

where the right side of (49) is given by (22), which is the consumption level that solves the utility maximization problem constituted by (20) and (21).

Second, for each $j = 1,\ldots,m,$ $\hat{c}_j^1(x_t)$ is the current consumption chosen by the mature individual of dynasty j in period t; that is,

$$(50) \qquad \hat{c}_j^1(x_t) = \frac{r_t a_{jt}^1 + \omega_t h_{jt}^1}{1 + \delta(1 + \beta)},$$

where the right side of (50) is given by (28), which is the consumption level that solves the expected utility maximization problem constituted by (25) and (26).

Third, for each $j=1,\ldots,m$, $\hat{e}_j^0(x_t)$ is the educational investment and $\hat{c}_j^0(x_t)$ is the current consumption chosen by the young individual of dynasty j who in period t. More precisely, with $a_{j,t+1}^1 = r_t[a_{jt}^0 - \hat{e}_j^0(x_t)] - \hat{c}_j^0(x_t)$, the following versions of the first-order conditions (37) and (38) must hold:

(51)
$$\delta[1+\delta(1+\beta)]\int \frac{\omega_{t+1}\theta_{jt}^0\gamma\left[\theta_{jt}^0\hat{e}_j^0(x_t)\right]^{\gamma-1}\left[\varepsilon\bar{h}_{jt}^1\right]^{1-\gamma}}{r_{t+1}a_{j,t+1}^1+\omega_{t+1}\left(\eta+\chi\left[\theta_{jt}^0\hat{e}_j^0(x_t)\right]^{\gamma}\left[\varepsilon\bar{h}_{jt}^1\right]^{1-\gamma}\right)}dG\left((r_{t+1},\omega_{t+1})\middle|(r_t,\omega_t)\right) = \frac{r_t}{r_t[a_{jt}^0-\hat{e}_j^0(x_t)]-a_{j,t+1}^1},$$

and

(52)
$$\delta[1+\delta(1+\beta)]\int \frac{r_{t+1}}{r_{t+1}a_{j,t+1}^1+\omega_{t+1}\left(\eta+\chi\left[\theta_{jt}^0\hat{e}_j^0(x_t)\right]^{\gamma}\left[\varepsilon\bar{h}_{jt}^1\right]^{1-\gamma}\right)}dG\left((r_{t+1},\omega_{t+1})\middle|(r_t,\omega_t)\right) \le \frac{1}{r_t[a_{jt}^0-\hat{e}_j^0(x_t)]-a_{j,t+1}^1},$$

with equality holding in (52) if $a_{j,t+1}^1 > 0$.

4. THE DATA

To assess quantitatively economic growth and the evolution of income distribution, I simulate the theoretical model presented in Section 3. In order to perform this task, the initial state of the system and the values of the parameters of the model are required.

The parameters that require specification are (i) the production function parameter, α; (ii) the parameters of human capital accumulation function as elasticity of human capital level attained with respect to educational investment, γ, weight accorded to the existing stock of human capital, ε, and parameter representing technological level of the human capital accumulation function, χ; (iii) the discount factor, δ; and (iv) the degree of altruism, β. The production function parameter, α, is usually interpreted as the capital income share. I set α equal to 0.4. The value of the annual discount factor is conventionally set to be 0.98. In the set up of the model each period is interpreted as approximately 20 years, thus the resulting one period discount factor is $\delta = 0.98^{20} = 0.67$. The parameter γ in the human capital production function is adopted from Heckman (1976) and Haley (1976). In both studies, the elasticity of the human capital output with respect to the education investment is estimated to be around 0.55, with a confidence interval ranging approximately up to 0.7. Due to my specification of the theoretical model, $\gamma = 0.7$ appears to be an appropriate value. The values chosen for $\eta, \chi,$ and ε are somewhat arbitrary, but their values are chosen to make the results of the simulation more reasonable. The base level of human capital is taken to be equal to 10% of the human capital of the mature individual of dynasty 1 in period 0. Also, χ and ε are set to 1 and 0.5 respectively. The suggested specification of the human capital accumulation function and the chosen values of the parameters allow the model to escape from a potential poverty trap. In order to avoid an explosion of human capital accumulation and, consequently an explosive endogenous growth, β is conveniently set equal to 0.65. The rate of human capital depreciation is assumed to be zero, and physical capital is assumed to depreciate completely in every period. The values of the parameters of the model are summarized in Table I.

TABLE I

PARAMETER VALUES

Parameter	Values
Capital income share, α	0.4
Elasticity of human capital production, γ	0.7
Weight of the existing stock of human capital, ε	0.5
Technological level of human capital accumulation, χ	1.0
Rate of time preference, δ	0.67
Degree of altruism, β	0.65

The initial state of the system, $x_0 = \left(\left(a_{j0}^0, \theta_{j0}^0\right), \left(a_{j0}^1, h_{j0}^1\right), a_{j0}^2\right)_{j=1}^m$, requires specification of the initial asset holdings of the three generations of each dynasty and the human capital of mature individuals. I use the data provided by the Survey of Financial Security of 1999 (SFS 1999) on total asset holdings with pension plans valued on terminal basis to approximate asset holdings of young, mature, and old individuals.[24] SFS 1999 uses a sample that represents all families and individuals in Canada with several exceptions. Based on the assumption that each period in the model represents approximately 20 years, asset holdings of young, mature, and old individuals are expressed, respectively, as average total assets of Canadians in the three age groups (i) from 15 to 34, (ii) from 35 to 54, and (iii) 54 and above. To model intra-generational differences in asset holdings, I use average total assets held by each quintile, thus setting the number of dynasties equal to five. Table II shows average total assets and shares of total assets held by each quintile and age group.

To construct the human capital of mature individuals, I use the measure of the stock of human capital based on the view that the value of the human capital of an individual can be approximated by the present value of her future income stream.[25] The estimation has been carried out according to the following steps:

First, I formed five educational attainment groups based on the highest level of education. These groups are: (i) unqualified individuals; (ii) high school graduates; (iii) individuals with and without a non-university postsecondary certificate and individuals with or without a university certificate below the Bachelor's degree; (iv) Bachelor's degree holders; and (v) individuals with degrees or certificates above the Bachelor's degree.

[24] To avoid the poverty trap, only positive asset holdings were considered.
[25] This procedure was first developed by Jorgenson and Fraumeni (1989).

TABLE II

AVERAGE TOTAL ASSET HOLDINGS BY QUINTILE AND AGE GROUPS

Quintiles	Age groups		
	15 – 34	35 – 54	55 and above
1st quintile:			
Average total assets, (1999 $)	1896	14932	22265
Share, %	0.3	0.9	1.0
2nd quintile:			
Average total assets, (1999 $)	11708	102872	131344
Share, %	1.8	6.1	5.8
3rd quintile:			
Average total assets, (1999 $)	44560	207858	284295
Share, %	6.9	12.0	12.5
4th quintile:			
Average total assets, (1999 $)	134719	356087	512789
Share, %	21.0	21.0	22.5
5th quintile:			
Average total assets, (1999 $)	451120	1004369	1337394
Share, %	70.0	60.0	58.2

Source: Survey of Financial Security of 1999, Canada.

Second, I used the data from Survey of Labour and Income Dynamics (SLID) for 1999 on wages and salaries[26] of 35, 40, 45, 50, and 54 year-old persons to approximate labor income of mature individuals of the present model and to model their time-path of labor income stream, such that after a five-year interval, the income of a 35 years old unqualified individual can be approximated by that of a 40 year-old unqualified individual.

[26] Only positive wages and salaries were considered. All earnings were expressed in current 1999 dollars.

Third, I constructed estimates of the discounted values of future income streams for all the five educational attainment groups by using median labor income for each age and educational attainment group.[27]

Table III summarizes the stocks of human capital for the mature individuals of each of the five dynasties.

TABLE III
ESTIMATES OF MATURE INDIVIDUALS' HUMAN CAPITAL STOCK

Educational attainment groups	Stock of human capital (1999 $)
Unqualified individuals	100137
High school graduates	139951
Individuals with and without a non-university postsecondary certificate and individuals with or without a university certificate below the Bachelor's degree	148505
Bachelor's degree holders	216800
Individuals with a degree or certificate above the Bachelor's degree.	246931

5. SIMULATION

Given the numerous state variables, 25 in total, it does not seem possible to solve the model analytically. Hence a simulation of the theoretical model has been carried out.[28] A numerical solution of the model involves approximating all the decision rules – which are functions of the state variables. Again, the large number of state variables strains the

[27] I assumed a constant discount rate and used the 1999 average nominal interest rate on five year guaranteed investment certificates adjusted for core inflation, reported by Bruce *et al.* (2001), as the discount rate.

[28] The software package *Mathematica* was used to perform numerical simulation.

memory capacity of my personal computer, even when I tried only for a crude linear approximation of the decision rule involving education investment. Therefore, I have made the extra assumption that young individuals, besides not being able to borrow, are not allowed to lend, either. This assumption means that $a^1_{j,t+1} = 0$ for each $j = 1,...,m$ and for each $t = 0,...$. The extra assumption allows me to use equation (46) to compute the education investment – as a share of inheritance – of a young individual from her innate abilities and her inheritance, as well as the average stock of human capital of the current working generation. Consequently, mature individuals would have no assets except for their stock of human capital. The initial state of the economy, with assets of mature individuals being suppressed, is used to carry out the experiment.

The experiment consists of simulating the evolution of the state of the economic system through time. It allows a numerical assessment of the evolution of the income distribution and economic growth. In order to perform the simulation exercise, a calibration of the technological level of the representative firm, A , for the initial period was performed. In the calibration, the initial yearly gross interest rate is assumed to be $r = 1.10$. Furthermore, the assets of old individuals are assumed to be the assets at the end of the period. Thus the assets of old individuals discounted to the beginning of the period and the part of the assets of the young generation – after the education investments have been accounted for – constitute the stock of physical capital at the beginning of the given period. In the simulation, the innate abilities of young individuals are assumed to be uniformly distributed on the unit interval [0, 1].

6. DISCUSSION OF THE RESULTS

The economy has been simulated for 200 periods. This exercise has provided numerical series for asset holdings, stocks of human and physical capital, gross output, interest rate and wage rate.

To visualize the evolution of the income distribution, I used the results of the simulation to calculate the Gini coefficients[29] for asset holdings of young individuals and for mature individuals' human capital stock.[30] Figure 1 depicts the evolution of young individuals' asset holdings' Gini coefficient through time. It shows that although initially inequality of young individuals' asset holdings was high – the Gini coefficient of the first period is 0.63 – the simulated economy on average is characterized by low level of inequality, with an average Gini coefficient of 0.29. Figure 1 also reveals that with time the inequality slightly decreases. On the other hand, Figure 2 indicates that initially the distribution of the stock of human capital is more egalitarian, but becomes more unequal with time – although the economy maintains a conventionally low average Gini coefficient of 0.29.

According to the results of the simulation, for a time horizon that is long enough, income and wealth inequality appear not to persist. Figure 3 depicts the motion of the 1st dynasty young individual's asset holdings within the distribution ladder. Initially positioned at the very bottom of the distribution, she has experienced both positive upward mobility and negative downward shifts.

The evolution of the aggregate stock of human capital is depicted in the Figure 4. The evolutions of each dynasty's stock of human capital are illustrated in Figures 5, 6, 7, 8, 9 and 10. Since the investment in education of each dynasty is dictated by ability endowment and asset holdings, dynasty 1 initially could be constrained by low asset holdings. As shown in Figure 5, this dynasty accumulated little human capital in earlier periods. As time progressed, the dynasty was able accumulate more resources and base its education investment according to its ability endowment, which in following periods has resulted in much higher levels of stock of human capital. Figures 6, 7, and 8 shows that dynasties 2,

[29] Gini coefficient lies within [0, 1] interval, where zero Gini coefficient corresponds to perfect equality.
[30] Gini coefficients were calculated for every tenth period.

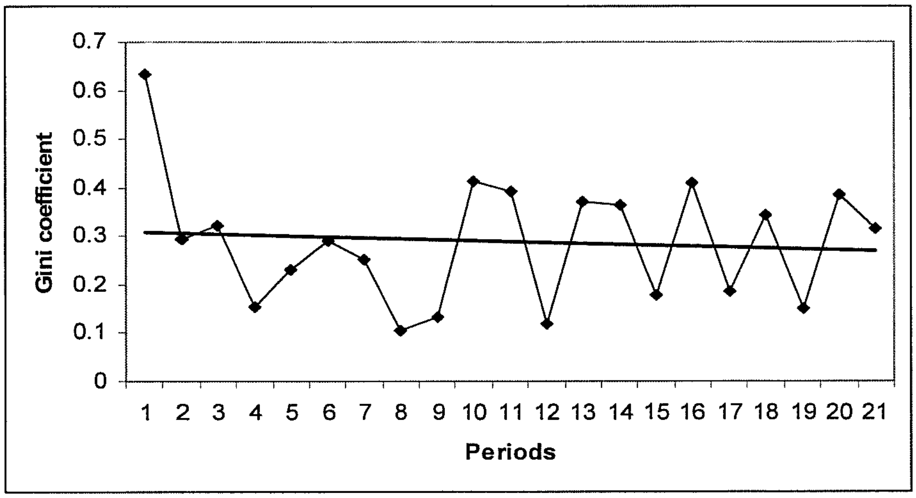

FIGURE 1.- The evolution of the Gini coefficient of young individuals' assets holdings

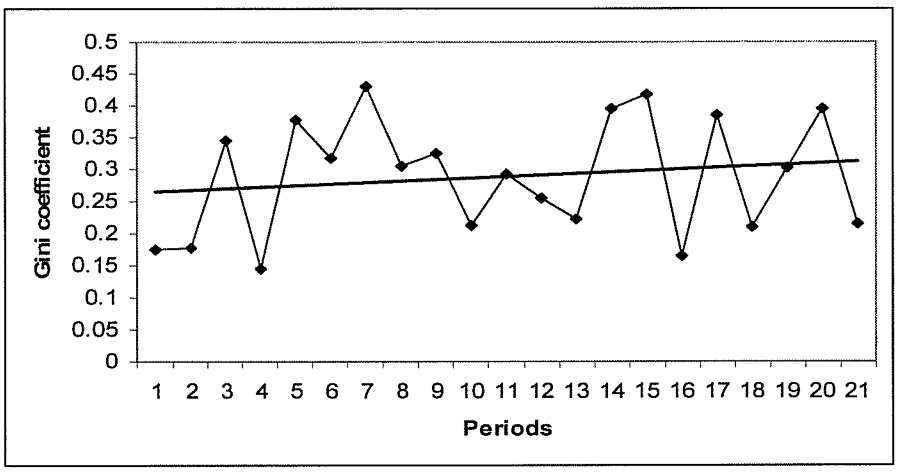

FIGURE 2.- The evolution of the Gini coefficient of mature individuals' stock of human capital

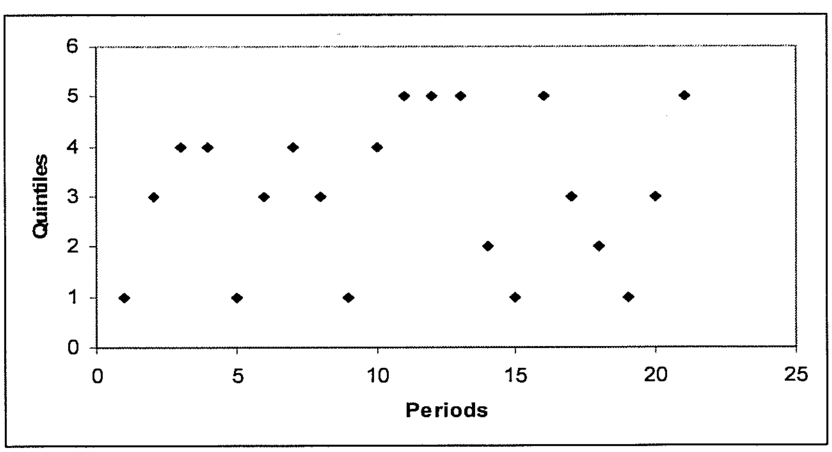

FIGURE 3.- Motion within the distribution ladder of the young individual of dynasty 1

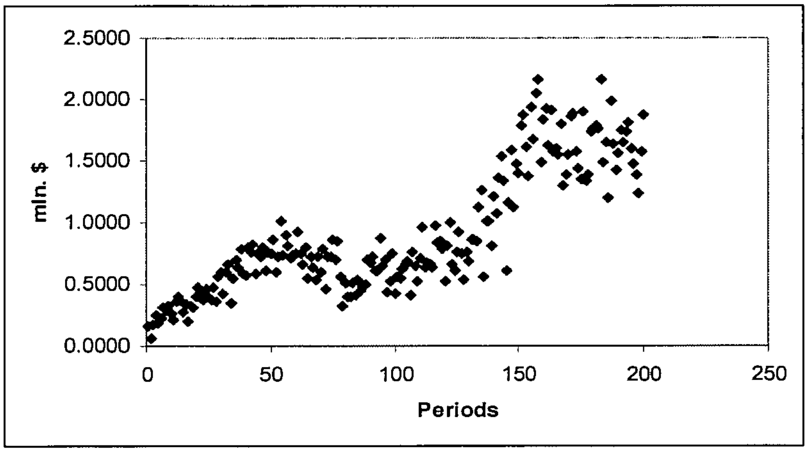

FIGURE 4.- The evolution of the aggregate stock of human capital

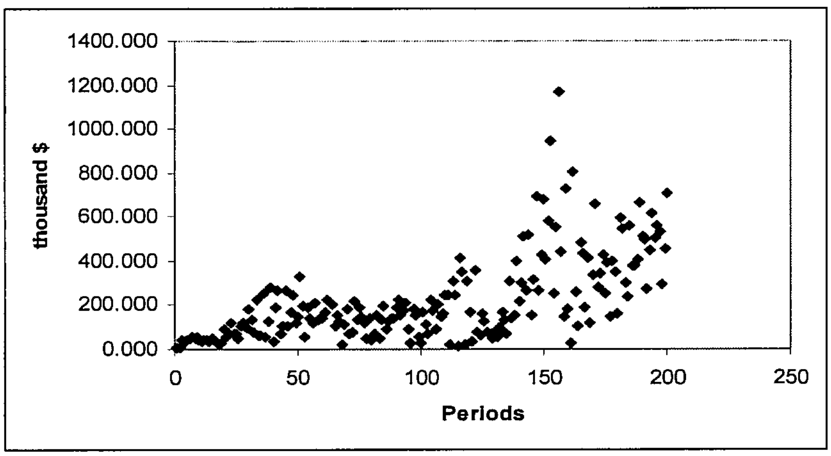

FIGURE 5.- The evolution of dynasty 1's human capital stock

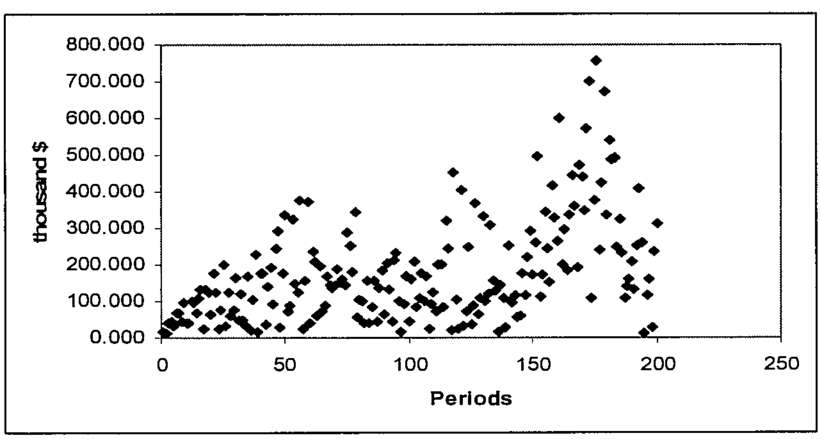

FIGURE 6.- The evolution of dynasty 2's human capital stock

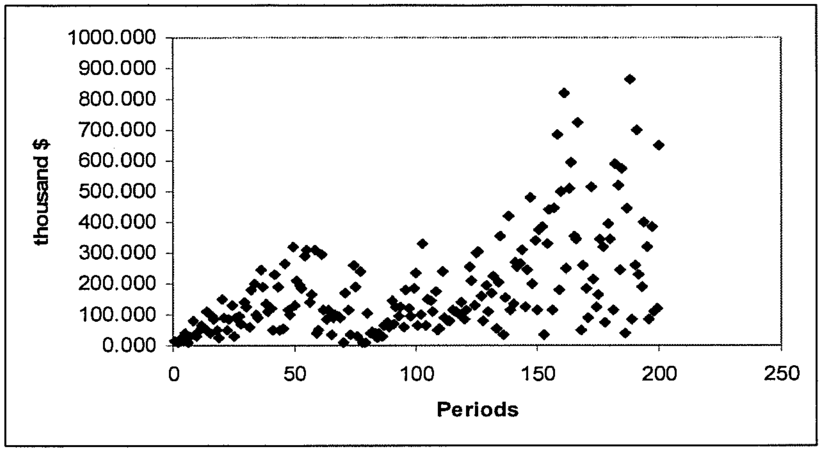

FIGURE 7.- The evolution of dynasty 3's human capital stock

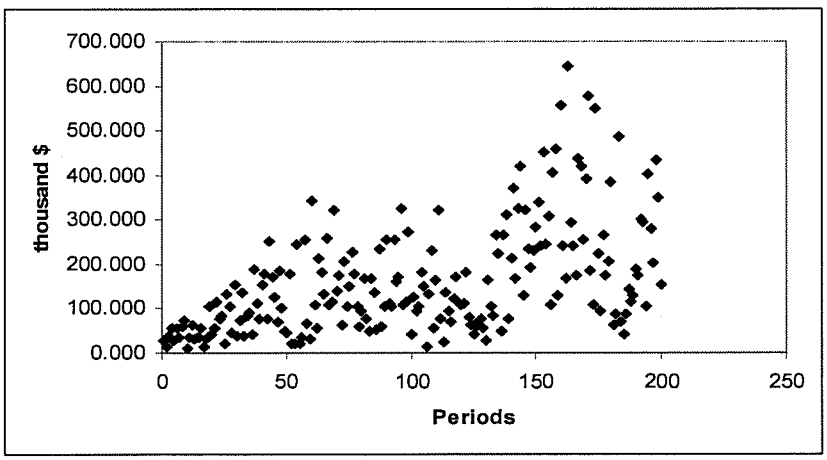

FIGURE 8.- The evolution of dynasty 4's human capital stock

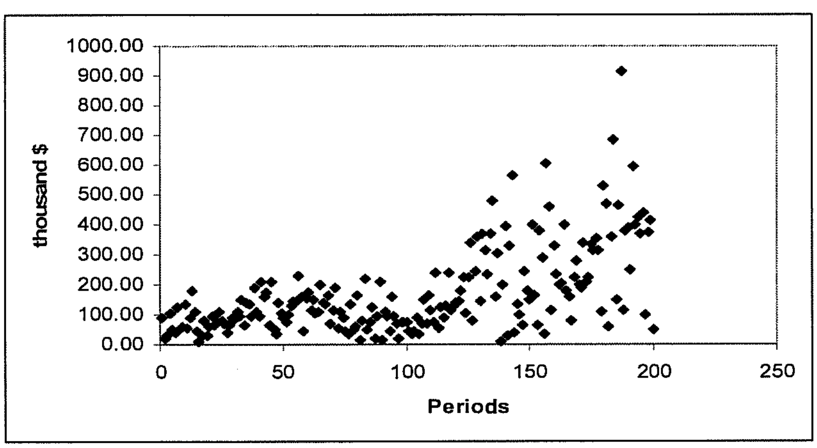

FIGURE 9.- The evolution of dynasty 5's human capital stock

3, and 4 were able to accumulate high levels of human capital in earlier periods. Since these dynasties were probably less constrained by asset holdings, resulted stocks of human capital could be attributed more to the path of abilities' endowments. Higher stocks of human capital were accumulated in periods when one's ability endowment was high. Dynasty 5's evolution of the stock of human capital is of particular interest. Initially this dynasty was the richest in terms of asset holdings and easily could afford sufficient educational investments, but as shown in Figure 9, the accumulated stock of human capital for the first 100 periods had surprisingly a flat profile, which could be explained by poor ability endowments. As was mentioned before, in our simulated economy, income inequality does not seem to persist and the figures depicting the dynasties' evolution of the stocks of human capital indicate that at some time or another all dynasties were able to climb to the top of the distributional ladder.

The evolutions of the stock of physical capital and GDP are depicted in Figures 10 and 11. Close examination of Figures 4, 10, and 11 reveals that the aggregate stocks of human and physical capital, as well as gross output, mimicked each other's evolution path. Such close correlation may indicate that in the simulated economy the evolution of gross output in the long run is mostly determined by the magnitudes of economy-wide resources and less by the distribution of those resources among the various economic agents, which in turn could indicate that the market economy is able to allocate resources efficiently in the long run. In our simulated economy – with the asset holdings of mature individuals being suppressed – the economy's stock of physical capital in each period consists of part of the asset holdings of young individuals – after educational investments have been accounted for – and the asset holdings of old individuals, which result from these individuals' utility maximization in the preceding two periods. In periods characterized either by uncommonly low ability endowments or mismatch of opportunities, i.e., individuals with higher ability were constrained by their insufficient asset holdings, lower levels of aggregate human capital stock and physical capital stock were accumulated, and, consequently, as shown in Figure 11, lower levels of growth in output were realized. Though on average the economy grew at 2.02% per period, growth rates varied significantly across periods. In the first 60 periods, the economy on average grew at a rate of 3.28% per period. In the following 25 periods, the economy experienced a fall in output, and the average growth rate was 0.48% per period. After these depressed periods, the simulated economy picked up and again grew at rates higher than average.

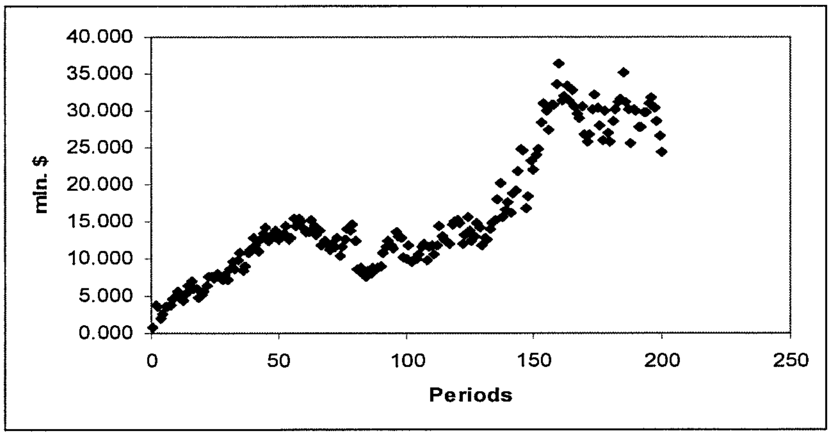

FIGURE 10.- The evolution of the stock of physical capital

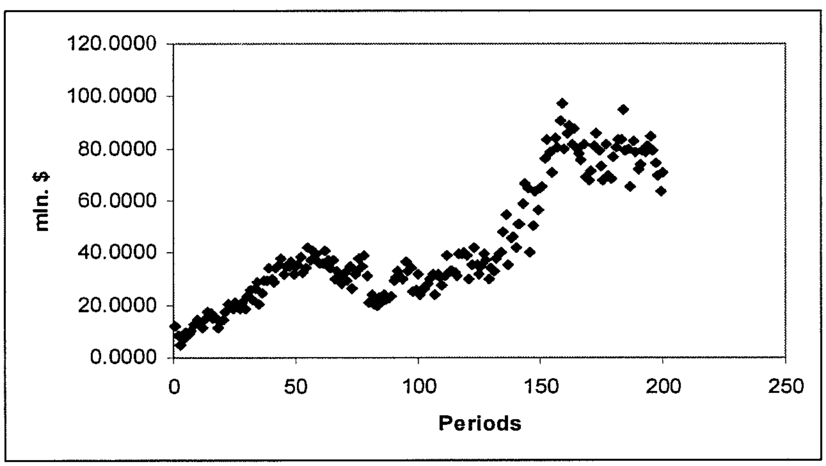

FIGURE 11.- The evolution of GDP

Although GDP might fluctuate in the short term, a rising trend can be observed in Figure 11. The rising tend in GDP can be attributed to the rising trends in both the stock of human and physical capital, as shown in Figures 4 and 10, respectively.

7. CONCLUDING REMARKS

The present paper has attempted to analyze the evolution of income distribution and economic growth in a model where initial asset distribution is unequal; credit markets are imperfect; and human capital accumulation is affected by individuals' random abilities. The conventional theory predicts that an initial unequal income distribution together with credit market imperfections and inheritability of assets could lower educational investment of agents with little initial assets, inducing the initial inequality to persist. However, as shown by the simulation of the present model, if the time horizon is long enough, randomness of abilities creates opportunities for human capital accumulation, even for individuals with low initial wealth. Consequently, mobility within the income distribution ladder becomes feasible. Furthermore, the results of the simulation exercise support the idea that merit-based distribution of human capital could be characterized by some degree of inequality.

More thorough numerical analysis of the present model could be performed if computing equipment with greater capacity were used. This would allow for a numerical simulation of the more complex analytical framework described in Section 3 of the present paper.

Abstracting from computational challenges, there are several directions in which the present analysis could be extended. A constructive extension of the present work would be to perform a sensitivity analyses. Since numerical evaluation of the analytical model is used in present paper, investigation of sensitivity of the findings to the changes in the parameter values is useful for the verification of the results. One possibility would be to introduce government into the analysis. Government intervention – in the form of taxes, public education, and education subsidies – could play important role in the real world. Therefore, a more detailed modeling of the economy could improve the analysis. Another possibility would be to extrapolate on the details of credit market imperfections and include endogenous determination of credit rationing by considering family and ability effects. Many economists suggest that in a market economy more able individuals in most cases could find sufficient education funding, irrespective of their economic status. Last but not least, a possible extension would be to include a more elaborate functional form for human capital accumulation by considering a combination of formal education, on job training, and experience gains.

REFERENCES

AGHION, P., A. V. BANERJEE, AND T. PIKETTY (1999): "Dualism and Macroeconomic Volatility," *Quarterly Journal of Economics*, 114 (4), 1359-1397.

AGHION, P. AND P. BOLTON (1997): "A Trickle-Down Theory of Growth and Development," *Review of Economic Studies*, 64 (2), 151-172.

AGHION, P., E. CAROLI, AND C. GARCIA-PENALOSA (1999): "Inequality and Economic Growth: The Perspective of the New Growth Theories," *Journal of Economic Literature*, 37 (4), 1615-1660.

AGHION, P. AND P. HOWITT (1998): *Endogenous Growth Theory*, Cambridge: MIT Press.

ALESINA, A. et al. (1996): "Political Instability and Economic Growth," *Journal of Economic Growth*, 1 (2), 189-211.

ALESINA, A. AND R. PEROTTI (1996): "Income Distribution, Political Instability, and Investment," *European Economic Review*, 40 (6), 1203-1228.

ALESINA, A. AND D. RODRIK (1994): "Distributive Politics and Economic Growth," *Quarterly Journal of Economics*, 109 (2), 465-490.

ANAND, S. AND R. KANBUR (1993a): "The Kuznets Process and the Inequality-Development Relationship," *Journal of Development Economics*, 40 (1), 42-50.

__________ (1993b): "Inequality and Development: A Critiques," *Journal of Development Economics*, 41 (1), 19-43.

BANERJEE, A.V. AND E. DUFLO (2003): "Inequality and Growth: What Can the Data Say?" *Journal of Economic Growth*, 8 (3), 267-299.

BANERJEE, A.V. AND NEWMAN (1993): "Occupational Choice and the Process of Development," *Journal of Political Economy*, 101 (2), 274-299.

BARRO, R.J., (1999): "Inequality, Growth, and Investment," NBER Working Paper No. 7038.

__________ (2000): "Inequality and Growth in a Panel of Countries," *Journal of Economic Growth*, 5 (1), 5-32.

BARRO, R. AND J.W. LEE (1994): "Sources of Economic Growth," *Carnegie-*

Rochester Conference Series on Public Policy, 40, 1-46.

BECKER, G. AND N. TOMES (1986): "Human Capital and the Rise and Fall of Families," *Journal of Labor Economics*, 4 (3), S1-S39.

BEHRMAN, J.R. AND P.TAUBMAN (1985): "Intergenerational Earnings Mobility in The United States: Some Estimates and Test of Becker's Interegenerational Endowments Model," *Review of Economics and Statistics*, 67 (1), 144-151.

BELLETTINI, G. (1996): "Redistribution and Growth in Two-Party System," *Economic Notes*, 25 (2), 207-224.

BENABOU, R. (1993): "Workings of a City: Location, Education, Production," *Quarterly Journal of Economics*, 108 (3), 619-653.

__________ (1996a): "Inequality and Growth," *NBER Macroeconomics Annual*, 11, 11-74.

__________ (1996b): "Heterogeneity, Stratification and Growth: Macroeconomic Implications of Community Structure and School Finance," *American Economic Review*, 86 (3), 584-609.

BENHABIB, J. AND A. RUSTICHINI (1996): "Social Conflict and Growth," *Journal of Economic Growth*, 1 (3), 125-142.

BERTOLA, G. (1993): "Factor Shares and Savings in Endogenous Growth," *American Economic Review*, 83 (5), 1184-1198.

BLANCHFLOWER, D. G. AND A. J. OSWALD (1998): "What Makes an Entrepreneur?" *Journal of Labor Economics*, 16 (1), 26-60.

BORJAS, G.L. (1992): "Ethnic Capital and Intergenerational Mobility," *Quarterly Journal of Economics*, 107 (1), 123-150.

__________ (1995): "Ethnicity, Neighbourhoods and Human Capital Expenditures," *American Economic Review*, 85 (3), 365-390.

BOURGUIGNON, F. (1981): "Pareto-Superiority of Unegalitarian Equilibria in Stiglitz' Model of Wealth Distribution with Convex Saving Function," *Econometrica*, 49 (6), 1469-1475.

BOURGUIGNON, F., M. FOURNIER, AND M. GUARGAND (2001): "Fast Development with a Stable Income Distribution: Taiwan, 1979 – 94," *Review of income and wealth*, 47(2), 139 – 163.

BREEN, R. AND C. GARCIA-PENALOSA (1998): "Income Inequality and Macroeconomic Volatility: An Empirical Investigation", Mimeo, Nuffield College, Oxford.

BRUCE C. et al. (2001): "Selecting the Discount Rate – An Update," *The Expert Witness Newsletter*, 6 (4), http://www.economica.ca/ew53p1.htm.

CAMERON, S.V. AND J.J. HECKMAN (1998): "Life Cycle Schooling and Dynamic Selection Bias: Models and Evidence for Five Cohorts," NBER Working Papers No. 6385, National Bureau of Economic Research.

CAMPANO, F. AND D. SALVATORE (1988): "Economic Development, Income Inequality and Kuznets' U-shaped Hypothesis," *Journal of Policy Modeling*, 10 (2), 265-280.

CARNIERO, P. AND J.J. HECKMAN (2002): "The Evidence on Credit Constraints in Post-Secondary Schooling," Manuscript, University of Chicago.

CARROL, C. (1998): "Why do the rich save so much?" In *Does Atlas Shrug? Economic Consequences of Taxing the Rich*, J. SLEMROD, eds., Cambridge: Cambridge University Press.

CHENERY, H. et al. (1974): *Redistribution with Growth*, Oxford: Oxford University Press.

CHU, Y.-P. (1995): "Taiwan's Inequality in the Postwar Era," Working Paper No. 96 -1, San Yat Sen Institute, Taiwan.

CLARKE, G. R. G. (1995): "More Evidence on Income Distribution and Growth," *Journal of Development Economics*, 47 (2), 403-427.

CORNEO, G. AND O. JEANNE (2001): "Status, the Distribution of Wealth, and Growth," *Scandinavian Journal of Economics*, 103 (2), 283-293.

DAHAN, M. AND D. TSIDDON (1998): "Demographic Transition, Income Distribution, and Economic Growth," *Journal of Economic Growth*, 3 (1), 29-52.

DE GREGORIO, J. (1996): "Borrowing Constraints, Human Capital Accumulation and Growth," *Journal of Monetary Economics*, 37 (1), 49-71.

DEININGER, K. AND S. SQUIRE (1996a): "A New Data Set for Measuring Income Inequality," *World Bank Economic Review*, 10 (3), 565-591.

__________ (1996b): "Does Inequality matter? Re-examining the

Links between Growth and Inequality," Mimeo, World Bank, Washington D.C.

__________ (1996c): "New Ways of Looking at Old Issues: Inequality and Growth," Mimeo, World Bank, Washington D.C.

DE LA CROIX, D. AND M. DOEPKE (2001): "Inequality and Growth: Why Differential Fertility Matters," Mimeo, UCLA.

DE LA CROIX, D. AND P. MONFORT (2000): "Education Funding and Regional Convergence," *Journal of Population Economics*, 13 (3), 403-424.

DURLAUF, S. (1996): "A Theory of Persistent Income Inequality," *Journal of Economic Growth*, 1 (2), 75-93.

DYNAN, K.E., J SKINNER AND S.P. ZELDES (2004): "Do the Rich Save More?" *Journal of Political Economy,* 112(2), 397-444.

EASTERLY, W. AND S. REBELO (1993): "Fiscal Policy and Economic Growth: An Empirical Investigation," *Journal of Monetary Economics*, 32 (3), 417-458.

ESWARAN, M. AND A.Y. KOTWAL (1993): "A Theory of Real Wage Growth in LDC's," *Journal of Development Economics*, 42 (2), 243-269.

EVANS, D.S. AND B. JOVANOVIC (1989): "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints," *Journal of Political Economy*, 97 (4), 808-827.

FERSHTMAN, C., K.M. MURPHY, AND Y. WEISS (1996): "Social status, Education, and Growth," *Journal of Political Economy*, 104 (1), 108-132.

FIELDS, G. AND G. JAKUBSON (1994): "New Evidence on the Kuznets curve," Mimeo, Cornell University, Ithaca.

FISHLOW, A. (1972): "Brazilian Size Distribution of Income," *American Economic Review*, 62 (1), 391-402.

FORBES, K. (2000): "A Reassessment of the Relationship between Inequality and Growth," *American Economic Review,* 90 (4), 869-887.

FRANK, R.H. (1999): *Luxury Fever: Why Money Fails to Satisfy in an Era of Excess*, New York: Free Press.

GALOR, O. (2000): "Income Distribution and Process of Development," *European Economic Review*, 44 (4), 706-712.

GALOR, O AND D. TSIDDON (1997): "The Distribution of Human Capital and Economic Growth," *Journal of Economic Growth*, 2 (1), 96-124.

GALOR, O. AND D.N. WEIL (2000): "Population, Technology and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond," *American Economic Review*, 9 (4), 806-828.

GALOR, O. AND H. ZANG (1997): "Fertility, Income Distribution, and Economic Growth: Theory and Cross-Country Evidence," *Japan and the World Economy*, 9 (2), 197-229.

GALOR, O. AND J. ZEIRA (1993): "Income Distribution and Macroeconomics," *Review of Economic Studies*, 60 (1), 35-52.

GLOMM, G. AND B. RAVIKUMAR (1992): "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy*, 100 (4), 818-834.

HALEY, W.J. (1976): "Estimation of the Earnings Profile from Optimal Human Capital Accumulation," *Econometrica*, 44 (6), 1223-1238.

HALL, J. AND C.I. JONES (1998): "Why Do Some Countries Produce So much More Output per Worker than Others?," *Quarterly Journal of Economics*, 114 (1), 83-116.

HAUSSMANN, R. AND M. GAVIN (1996): "Securing Stability and Growth in a Shock-Prone Region: The Policy Challenges for Latin America" in *Securing Stability and Growth in Latin America*, R. HAUSSMANN AND H. REISEN, eds., Paris: OECD, 23-64.

HECKMAN, J.J. (1976): "A Life-Cycle Model of Earnings, Learning and Consumption," *Journal of Political Economy*, 84 (4), S11-S44.

HENDRICKS L. (1999): "Taxation and Long-Run Growth," *Journal of Monetary Economics*, 43 (2), 411-434.

HERRNSTEIN, R. AND C. MURRAY (1994): *The Bell Curve: Intelligence and Class Structure in American Life*, New York: The Free Press.

HOPKINS, E. AND T. KORNIENKO (2004): "Status, Inequality and Growth," Working Paper, http://homepages.ed.ac.uk/ehk/growth.pdf.

HUGGETT, M. AND G. VENTURA (2000): "Understanding Why High Income Households Save More than Low Income Households," *Journal of Monetary Economics,* 45 (2), 361-397.

JHA, S. (1996): "The Kuznets Curve: A Reassessment," *World Development*, 24 (4), 773-780.

JORGENSON, D.W. AND B.M. FRAUMENI (1989): "The Accumulation of Human and Nonhuman Capital, 1948 – 1984," in *The Measurement of Saving, Investment, and Wealth*, R.E. LIPSEY AND H.S. TICE, eds., Chicago: University of Chicago Press.

KALDOR, N. (1956): "Alternative Theories of Distribution," *Review of Economic Studies*, 23 (2), 83-100.

KALECKI, M. (1971): *Selected Essays on the Dynamics of Capitalist Economy, 1933 – 1970*, Cambridge: Cambridge University Press.

KEYNES, J.M. (1936): *The General Theory of Employment, Interest and Money*, London: Macmillan.

KNELL, M. (1999): "Social Comparisons, Inequality and Growth," *Journal of Institutional and Theoretical Growth*, 155 (4), 664-695.

KREMER, M. AND D. CHEN (2000): "Income Distribution Dynamics with Endogenous Fertility," NBER Working Paper No. 7530, National Bureau of Economic Research.

KRUEGER, D. AND F. KUBLER (2004): "Computing Equilibrium in OLG Models with Stochastic Production," *Journal of Economic Dynamics and Control*, 28, 1411-1436.

KUZNETS, S. (1955): "Economic Growth and Income Inequality," *The American Economic Review*, 45 (1), 1-28.

LAROCHE, M. AND M. MERETTE (2000): "Measuring Human Capital in Canada," Department of Finance Working Paper No 2000-05.

LEWIS, W.A. (1954) "Economic Development with Unlimited Supplies of Labor," *Manchester School of Economics and Social Studies*, 22, 139-181.

LI, H. AND H. ZOU (1998): "Income Inequality is not Harmful for Growth: Theory and Evidence," *Review of Development Economics* 2(3), 318-334.

LOURY, G. (1981): "Intergenerational Transfers and the Distribution of Earnings," *Econometrica*, 49 (4), 843-867.

LUCAS, R.E. (1988): "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22 (1), 3 – 42.

__________ (1993): "Making a Miracle," *Econometrica*, 61, 251-272.

MENCHIK, P. (1979): "Intergenerational Transmission of Inequality: An Empirical Study of Wealth Mobility," *Economica*, 46 (184), 349-362.

MULLIGAN, C. (1993): "On Intergenerational Altruism, Fertility and the Persistence of Economic Status," PhD Dissertation, Department of Economics, University of Chicago.

MULLIGAN, C. (1997): *Parental Priorities and Economic Inequality*, Chicago: University of Chicago Press.

MURPHY, K.M., A. SHLEIFER, AND R.W. VISHNY (1989): "Income Distribution, Market Size and Industrialization," *Quarterly Journal of Economics*, 104 (4), 869-895.

OKUN, A.M. (1975): *Equality and Efficiency: The Big Tradeoff*, Washington: The Brookings Institution.

OWEN, A.L. AND D.W. WEIL (1998): "Intergenerational Earnings Mobility, Inequality and Growth," *Journal of Monetary Economics*, 41 (1), 71-104.

PASINETTI, L.L. (1962): "Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth," *Review of Economic Studies*, 29 (4), 267-79.

PEROTTI, R. (1992) "Fiscal Policy, Income Distribution, and Growth," Columbia University Working Paper No. 636.

PEROTTI, R. (1993): "Political Equilibrium, Income Distribution and Growth," *Review of Economic Studies*, 60 (4), 755-776.

PEROTTI, R. (1996): "Growth, Income Distribution, and Democracy: What the Data Say," *Journal of Economic Growth*, 1 (2), 149-187.

PIKETTY, T. (1995): "Social Mobility and Redistributive Politics," *Quarterly Journal of Economics*, 110 (3), 551-584.

PIKETTY, T. (1997): "The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing," *Review of Economic Studies*, 64 (2), 173-189.

PIKETTY, T. (1998): "Self-Fulfilling Beliefs about Social Status," *Journal of Public Economics*, 70 (1), 115-132.

PIKETTY, T. (2000): "Theories of Persistent Inequality and Intergenerational Mobility," In *Handbook of Income Distribution*, Vol. 1, A.B. ATKINSON AND F. BOURGUIGNON, eds., Elsevier: Amsterdam.

QUAH, D. (2001): "Some Simple Arithmetic on How Income Inequality and Economic Growth Matter," Mimeo, London School of Economics.

RAMEY, G. AND V.A. RAMEY (1995): "Cross-country Evidence on the Link Between Volatility and Growth," *American Economic Review*, 85 (5), 1138-1151.

RAM, R. (1988): "Economic Development and Income Inequality: Further Evidence on the U-curve Hypothesis," *World Development*, 16 (11), 1371-1376.

__________ (1995): "Economic Development and Inequality: An Overlooked Regression Constraint," *Economic Development and Cultural Change*, 43 (2), 425-434.

ROMER, P.M. (1986): "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94 (5), 1002-1037.

SAITH, A. (1983): "Development and Distribution: A Critique of the Cross-Country U Hypothesis," *Journal of Development Economics*, 13 (3), 367-382.

SCHMIDT-HEBBEL, K. AND L. SERVEN (1997): "Saving Across the World: Puzzles and Policies," World Bank Discussion Paper No. 354, World Bank.

SMITH, A. (1776): *An Inquiry into the Nature and Causes of the Wealth of Nations*, New York: Random House, 1937.

SOLIMANO, A. (1998): *Roadmaps to Prosperity. Essays on Growth and Development*, Chicago: University of Michigan Press.

SOLON, G. (1992): "Intergenerational Income Mobility in The United States," *American Economic Review*, 82 (3), 393-408.

STIGLITZ, J.E. (1969): "The Distribution of Income and Wealth among Individuals," *Econometrica*, 37 (3), 382-397.

STOKEY, N., (1998): "Shirtsleeves to Shirtsleeves: the Economics of Social Mobility," in *Frontiers of Research in Economic Theory: The Nancy L. Schwartz Memorial Lectures 1983-1997*, ed. By D. JAKOBS et al., Cambridge, Cambridge University Press.

TAMURA, R. (1991): "Income Convergence in Endogenous Growth Model," *Journal of Political Economy*, 99 (3), 522-40.

TOMES, N. (1981): "The Family, Inheritance, and Interegenerational Transmission of Inequality," *Journal of Political Economy*, 89 (5), 928-958.

YOU, J.-I. (1998): "Income Distribution and Growth in East Asia," *Journal of Development Studies*, 34 (6), 37-65.

ZIMMERMAN, D.J. (1992): "Regression toward Mediocrity in Economic Stature," *American Economic Review*, 82 (3), 409-429.

ZWEIMULLER, J. (2000): "Schumpeterian Entrepreneurs Meet Engel's Law: The

Impact of Inequality on Innovation-Driven Growth," *Journal of Economic Growth,* 5 (2), 185-206.