Scattering upon a Mixed Dielectric - Conductor Body A Time - Domain Approach

by

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ABSTRACT

A time-domain integral equation technique is presented for calculating the behaviour of a mixed dielectric - conductive body subjected to a transient plane wave. The incident pulse is a smooth Gaussian pulse, approximating a delta function. We develop an original formulation capable of calculating the electric and magnetic fields on or outside the surface of the scatterer from the moment the incident wave hits the target. The result of the calculations is an impulse response providing information, via the Fourier transform, on the distribution of fields over a broad range of frequencies.

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LIST OF SYMBOLS

E	electric field
H	magnetic field
J	electric current density
K	magnetic current density
$\mathbf{J}_s$	surface current density
ρ	electric charge
m	magnetic charge
ε	permittivity
μ	permeability
ω	angular frequency
c	speed of light
k	wave number
τ	time delay

1. INTRODUCTION

The description of the propagation of electromagnetic waves through matter may be attempted in a variety of ways. Spatially, a "local" description in terms of the well-known curl equations may be employed. It provides an approach sensitive to the properties of the media the waves go through. The solution associated with this approach leads to equations with operators having the same dimension as that of the geometry under consideration.

At the other end of the spectrum, Green's function "global" approach leads to integral equations. Such equations are more difficult to evaluate than the differential equations specific to the previous case. The computational effort however is compensated by a solution formalism having a smaller dimension than that of the space in which the propagation occurs. It is often the case that propagation of waves through a three-dimensional space is expressed in terms of surface integrals, thereby reducing the order of the solution from three to two. In addition, time-domain approaches do not involve matrix calculations, but simple additions, as described later in the body of this thesis under the name "marching-on-in-time" method.

The description of the propagation may be attempted either in the frequency- or in the time-domain. The classical Maxwell's equations, with a time-harmonic variation are generally suitable for narrow frequency band calculations. To estimate the response of a system under the influence of transient fields, the time-domain approach has to be considered. Such an approach constitutes the preferred basis for evaluating the behaviour of systems under a variety of electromagnetic stimuli. The technique can be applied to problems in electromagnetic interference, compatibility, antennas etc.

1.1 Rationale for Present Work

Practical reasons dictate the need to develop integral equations that describe the behaviour of hybrid bodies under the influence of electromagnetic waves. It is most common that electronic components, printed circuit boards and devices react to an external field as a combination of a dielectric, often lossy material and a conductive component with a more or less perfect conductivity. The need to address the issue of the response of such structures constitutes the first motivation for this work.

The second motivation is of a theoretical nature. It involves the attempt to expand the current level of knowledge beyond the treatment of homogeneous, isotropic and linear media, into media where those features have a "local" quality, but not a "global" one. Equivalently, the bodies we propose to study and develop descriptive equations for, are not homogeneous, isotropic and linear in their entirety, but only in parts. Scattering upon a body whose external surface consists of a dielectric part and a conductive one separated by a line boundary lying on that external surface, where the electrical and magnetic characteristics change abruptly, is a situation that has not been dealt with before in a time-domain formulation. This work attempts to fill that void.

1.2 State of the Art

The range and depth of frequency-domain approaches and techniques preclude our attempt to summarize them here. We will therefore restrict our attention to time-domain formulations.

Historically, the methods of solving electromagnetic problems in the time-domain started with analytical techniques in simpler geometries and configurations, of which examples are the works of Brundell (1960) [1] on transient wave propagating around a cylinder or of Francescheti and Papas (1974) [2] on pulsed antennas. For more complex structures, a numerical approach was necessary. Early works included magnetic integral equations for surface objects, Bennett and Weeks (1968) [3] and electrical field integral equations for wire objects, Sayre and Harrington (1968) [4].

Miller, Poggio and Burke (1973) [5] expanded the latter approach to a generalized wire treatment. An excellent early review of the field was Poggio and Miller's (1973) [6] chapter in Computer Techniques for Electromagnetics, later followed by E. K. Miller's (1987) [7] article. Experimentation in time-domain has also played a role in the development of the field as illustrated in Time-Domain Measurements in Electromagnetics edited by E.K. Miller (1986) [8].

These pioneers' works were expanded to account for the solution of specific geometries and media characteristics. Mieras and Bennett (1982) [9] starting from the simple Green's function for an infinite, homogeneous, isotropic, linear, non-dispersive medium, developed expressions for the magnetic and electric fields outside a surface S, at time t, as functions of the fields on the surface S at the retarded time t' = t - R/c, where R is the distance between the observation and source points and c is the speed of light in the medium outside the body.

By applying boundary conditions, the above workers developed a system of equations for dielectric, penetrable bodies. They wrote surface integrals for the electric and magnetic fields both inside and outside the body and then allowed the observation points to approach the surface. This method, suggested earlier by Poggio and Miller (1973) [6], will also be used in the present work.

Special attention was paid by workers in the time-domain field to perfectly-conductive bodies. For closed-surface conductors under the impact of an exterior field, Bennett and Weeks (1968) [3] developed the expression of the induced surface current density as a function of the incident magnetic field on cylindrical scatterers.

An electric field integral equation approach was proposed for plates or open structures by Bennett and Mieras (1981) [10] under the same conditions. Closed conducting surfaces were discussed by Bennett and DeLorenzo (1969) [11] and Bennett and Ross (1978) [12]. The latter also discussed thin wires, open thin surfaces, flat plates, bodies with wire antennas and struts and bodies with fins attached. Rao and Wilton (1991) [13] have used triangular patches to model both open- and closed-surfaced conductive objects, using electrical field integral equations.

Regarding the above workers' numerical approach, it consisted of dividing the surface into elements or patches for a spatial method of moments integration and a "marching-in-time" technique for the temporal integration. This latter technique takes into account the fact that the unknown fields could be expressed as functions of the same fields at some earlier moment in time, as noted above.

Presumably, the fields were already calculated at those earlier moments, during previous time steps. It is this special circumstance which the time-domain approaches use, that enables the performing of calculations without need for matrix inversion. The necessity for matrix inversion remains one of the characteristics of frequency-domain techniques.

1.3 Geometrical Considerations

The geometry to which the theory developed in this study applies consists of structures with surface discontinuities. A dielectric body partly covered by conductive areas of arbitrary thickness may be treated under the formulation developed here. If however the body consists of a number of dielectrics, the equations developed below will not be sufficient in describing the scattering of a plane wave incident upon it.

It could therefore be stated that the structures addressed here are those with surface, as opposed to volume discontinuities. By surface or volume discontinuities, we mean discontinuities of the electric and magnetic fields to changes in material characteristics. The only exception to the rule above is in the case of one or more conductor bodies adjoining one dielectric body. Such situations too could be analyzed with the formulation developed here, since volume discontinuities have no bearing upon the results, given the null values of the fields inside the conductor.

In this study we consider (A): a single dielectric with a ground plane, covered by a conductive strip and (B): a combination of one dielectric and one solid conductor.

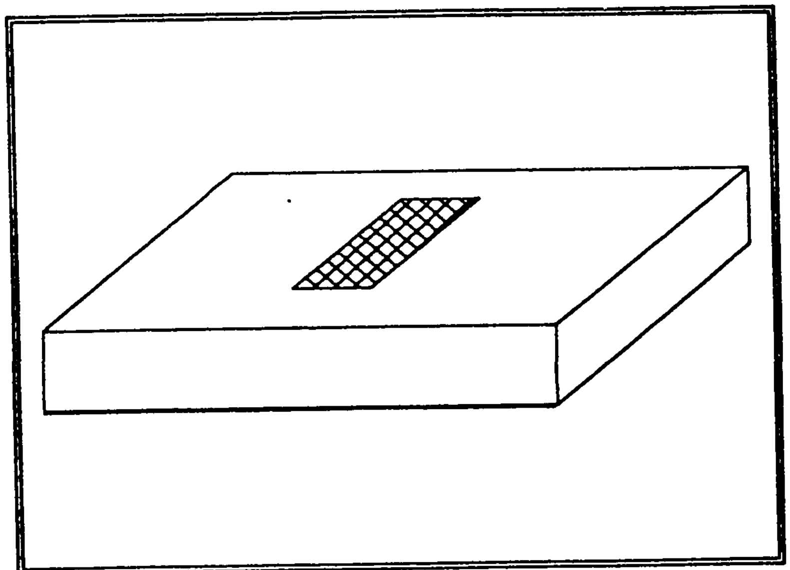

Figure 1 - Dielectric scatterer with ground plane and a face partially covered with conductive material

More precisely, we consider two scatterers: (A) a dielectric slab with a ground plane and a portion of one of its faces covered with a thin layer of conductive material of infinite conductivity (Fig.1).

(B) a metallic box in the shape of a cube with thick walls (Fig. 2). One of the walls has an rectangular opening. That opening is partly covered by a dielectric plug. The quantities we estimate are the magnetic and electric field at the center of the box. That location coincides with the centre of a rectangular system of coordinates. The fields at any location inside, outside or on the surface of the box may be calculated. The practical purpose of this endeavor is to estimate the electromagnetic interference fields to which an electronic component, device or instrument would be subjected inside such a testing box.

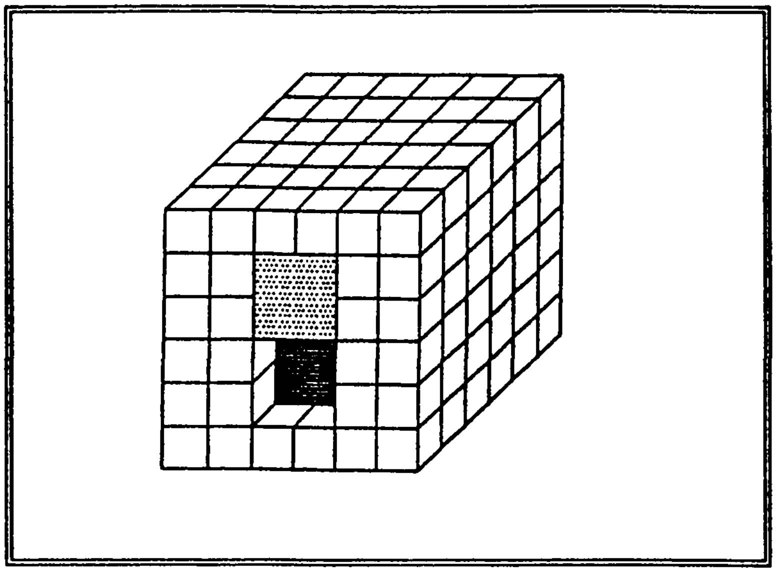

Figure 2 - Conductive box with opening partially covered by dielectric

In the time-domain literature, as discussed in the State of the Art chapter, both conductive and dielectric bodies have been considered separately as possible scatterers. However, a dielectric body partly covered with conductive material has not. Aside from the composition of the scatterer, the expansion of previous models to the current problem is not straightforward.

The derivation of the integral equations for the hybrid case is not a linear superposition of the formulations for the fully dielectric and the fully conductive bodies. When joined, these materials create an electrical and a magnetic discontinuity that cannot be addressed with the surface integral equations alone. Additional terms, allowing and accounting for the discontinuity, have to be included.

The derivation of these terms, their particular composition and form, within the framework of the time-domain formulation, constitutes the theoretical side of the original contribution of this work. Our configuration has important practical applications. The response of the structure to the incident impulse, as described above, provides some means to evaluate the behaviour of the scatterer in a transient field.

1.4 The Incident Field

Ideally, the transient field should be in the shape of a Dirac delta function. In that case, the response of the system will cover the infinite range of frequencies. In practice however, the numerical implementation of a delta function is not possible. A substitute for the incident pulse excitation must have a fast decay to zero with time. Such a substitute is the Gaussian pulse function. Replacing the delta function with the Gaussian pulse will limit the frequency range over which the calculations presented here are valid. An upper frequency bound will be determined later in the body of this work.

A Gaussian-pulse excitation with time dependence of the form:

$$f(t) = \frac{a}{\sqrt{\pi}} e^{-a^2 t^2} \qquad (1)$$

describes a very narrow pulse whose similarity to a delta function increases as parameter "a" tends towards infinity. This impulse rapidly decays to zero with time. Its Fourier transform is also a Gaussian pulse:

$$FT\left[\frac{a}{\sqrt{\pi}} e^{-a^2 t^2}\right] = e^{-\frac{\omega^2}{4a^2}} \qquad (2)$$

which exhibits a rapidly decreasing amplitude with increasing frequency. The larger the parameter "a", the better the impulse approximates the Dirac delta function and the larger is its high-frequency content.

In this work, an electromagnetic excitation in the form of a plane wave hits the target of an arbitrary shape. Both the electric and magnetic fields are represented by functions behaving in time like this:

$$f(t,z) \sim \exp\left[-a^2\left(t+\frac{z}{c}\right)^2\right] \qquad (3)$$

The exponential is modified by the appropriate scaling factor for the electric and magnetic fields. In addition, the non-flatness of the incident Gaussian spectrum has important consequences when calculating the fields in the frequency-domain, after appropriate Fourier transforms of the time-domain results. Specifically, the fields at a certain frequency f have to be adjusted for the decay with frequency of the initial Gaussian impulse.

1.5 Initial Parameters

We divided the surface of the scatterers into squares and considered the fields over each square roughly uniform. This assumption permitted us to evaluate the fields at the centre of each square only. The size of the side of the square constituted the space step ΔR. The time step Δt is restricted by the relation $c\ \Delta t < \Delta R$, where c is the speed of light in vacuum. The parameter "a" in the Gaussian-pulse expressions determines how narrow the initial temporal impulse is or, equivalently, how flat in the frequency domain. Assuming the arbitrary decay of say, -20 dB, between the peak and the signal value at f_{max}, as suggested by Miller (1987) [7], one arrives at the relation

$$a \approx \frac{\pi f_{max}}{\sqrt{\ln 10}} \tag{4}$$

A safe, better than the Nyquist sampling rate, assumption about the minimum discernable wavelength λ_{min} is

$$\lambda_{min} \sim 3\ \Delta R \tag{5}$$

and accordingly

$$f_{max} = c/\lambda_{min} = c/3\ \Delta R \tag{6}$$

We determine "a" for the configuration of interest.

2. THEORY

To solve the three-dimensional scattering of an electromagnetic impulse on a target, we attempt to reduce the problem in terms of unknown surface functions, a two-dimensional problem. The solution is thereby formulated over the surface of the scatterer. This formulation will lead to integral equations. The reduction in the number of independent variables from three to two is expected to compensate for the complicated aspect of the formulation.

The unknown functions need satisfy only the integral equations; the boundary conditions are already built in. The unknown field vector functions are functions of both space coordinates and time. The integral equations we develop contain not only the unknown functions, but also the time and space derivatives of the functions on the surface. From that point of view, the system of equations we develop is an integro-differential system. In this chapter the derivation of the frequency-domain integral equations will be demonstrated, followed by the corresponding time-domain solution.

2.1 Frequency-Domain Equations

The approach here is similar to that developed by Poggio and Miller (1973) [6], which in turn contained an original input from Stratton (1941) [14]. The technique allows integral equations to be written on surfaces whose tangents may not be differentiable functions of position at all points.

With suppressed time variation $e^{j\omega t}$, Maxwell's equations for a homogeneous, linear and isotropic space are written:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{K} \qquad \nabla \bullet \mathbf{E} = \rho/\varepsilon \tag{7}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \qquad \nabla \bullet \mathbf{H} = m/\mu \tag{8}$$

with the conservation of the electric and magnetic charges written as:

$$\nabla \bullet \mathbf{J} = -j\omega\rho \quad \text{and} \quad \nabla \bullet \mathbf{K} = -j\omega m \tag{9}$$

where **E** and **H** are the electric and magnetic field vectors, **J** and **K** the electric and magnetic current densities and ρ and m the electric and magnetic charge densities. For a linear, homogeneous, isotropic medium, both ε and μ are scalar quantities, so the vector wave equations for **E** and **H** can be written as:

$$\nabla \times \nabla \times \mathbf{E} - k^2\mathbf{E} = -j\omega\mu\mathbf{J} - \nabla \times \mathbf{K} \tag{10}$$

$$\nabla \times \nabla \times \mathbf{H} - k^2\mathbf{H} = -j\omega\varepsilon\mathbf{K} + \nabla \times \mathbf{J} \tag{11}$$

where $k = \omega/c$ and c is the velocity of light.

To solve these equations in a space containing sources, as well as in regions where the constitutive parameters differ from those of the medium in the surrounding space, we use the vector Green's theorem:

$$\int_V (\mathbf{Q}\cdot\nabla\times\nabla\times\mathbf{P} - \mathbf{P}\cdot\nabla\times\nabla\times\mathbf{Q})dv = \int_\Sigma (\mathbf{P}\times\nabla\times\mathbf{Q} - \mathbf{Q}\times\nabla\times\mathbf{P})\cdot\mathbf{n}da \tag{12}$$

where **P** and **Q** are vector functions of position with continuous first and second derivatives inside the domain V and on its boundary Σ. The element of area da has a normal directed outward from the volume V.

Q can be an arbitrary vector satisfying the above conditions. For convenience we choose

$$\mathbf{Q} = \hat{\mathbf{a}}\varphi = \hat{\mathbf{a}}\frac{e^{-jk|x-x'|}}{|x-x'|} \tag{13}$$

where $\hat{\mathbf{a}}$ is a unit vector in an arbitrary direction and $\mathbf{x}$ and $\mathbf{x'}$ are the observation and source position vectors, respectively. If the electric field $\mathbf{E}$ is substituted for $\mathbf{P}$ in (12), one obtains:

$$\int_V \left\{ j\omega\mu\mathbf{J}\varphi + \mathbf{K}\times\nabla\varphi - \left(\frac{\rho}{\varepsilon}\right)\nabla\varphi \right\} dv = \int_\Sigma \{ j\omega\mu(\mathbf{n}\times\mathbf{H})\varphi - (\mathbf{n}\times\mathbf{E})\times\nabla\varphi - (\mathbf{n}\cdot\mathbf{E})\nabla\varphi \} da \tag{14}$$

The vector operations above are performed in the source coordinates, while φ accounts for the relationship between observation and source coordinates. $\mathbf{Q}$ was chosen as a function directly related to the Green's function representing the magnetic vector potential in a homogeneous space due to a point current source given by $4\pi\hat{\mathbf{a}}\delta(\mathbf{x}-\mathbf{x'})$. While one could restrict the representation to a particular geometry, the general form of Green's function enables more flexibility.

The restriction that the observation and source points $\mathbf{x}$ and $\mathbf{x'}$ should not coincide, as it is apparent from the definition of $\mathbf{Q}$, is circumvented by surrounding the point **x** with a sphere and following the procedure of reducing, at the limit, the radius of the sphere to zero.

After extensive mathematical manipulation (Poggio and Miller 1973) [6], one arrives at the expression of the field at the observation point **x** as a function of the fields at the source **x'**:

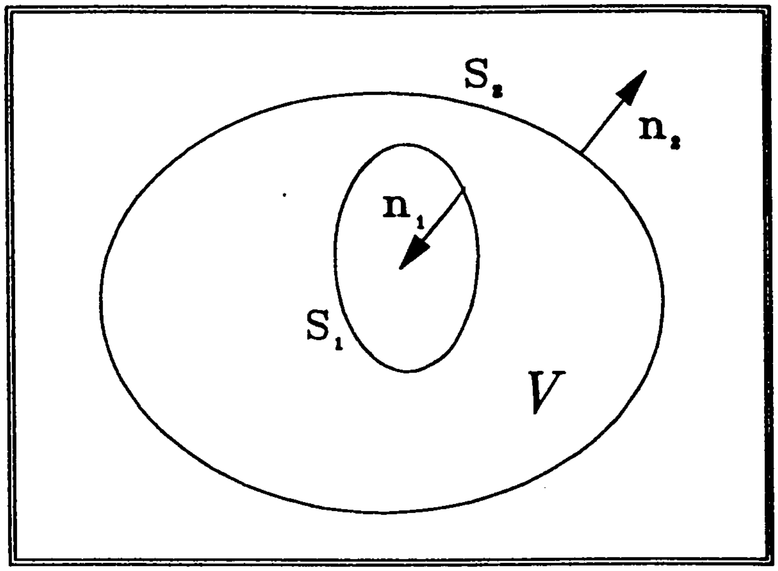

Figure 3 - Volume V bounded by surfaces S_1 and S_2.

$$\mathbf{E}(\mathbf{x}) = -\frac{T}{4\pi}\int_V \Big[j\omega\mu \mathbf{J}(\mathbf{x}')\varphi(\mathbf{x},\mathbf{x}') + \mathbf{K}(\mathbf{x}')\times\nabla\varphi(\mathbf{x},\mathbf{x}') - \left(\frac{\rho}{\varepsilon}\right)\nabla\varphi(\mathbf{x},\mathbf{x}')\Big] dv - \frac{T}{4\pi}\int_{S_1\cup S_2} \Big[j\omega\mu[\mathbf{n}\times\mathbf{H}(\mathbf{x}')]\varphi(\mathbf{x},\mathbf{x}') - [\mathbf{n}\times\mathbf{E}(\mathbf{x}')]\times\nabla\varphi(\mathbf{x},\mathbf{x}') - [\mathbf{n}\cdot\mathbf{E}(\mathbf{x}')]\nabla\varphi(\mathbf{x},\mathbf{x}')\Big] da \qquad (15)$$

where the normal **n** is oriented toward the domain V. The two surfaces S_1 and S_2, with normals $\mathbf{n}_1$ and $\mathbf{n}_2$ oriented opposite $\mathbf{n}$ in (15), are the inside and outside closed boundaries of the volume V (Fig. 3).

T is a quantity defined as $(1-\Omega/4\pi)^{-1}$, where Ω is the solid angle at the observation point. On a smooth surface, $\Omega = 2\pi$ and if $\mathbf{x}$ belongs to the volume V, obviously $\Omega = 4\pi$.

By duality to (15), we have a similar relationship for the magnetic field vector $\mathbf{H}(\mathbf{x})$:

$$\mathbf{H}(\mathbf{x})=\frac{T}{4\pi}\int_V\left[-j\omega\varepsilon\mathbf{K}(\mathbf{x}')\varphi(\mathbf{x},\mathbf{x}')+\mathbf{J}(\mathbf{x}')\times\nabla\varphi(\mathbf{x},\mathbf{x}')-\left(\frac{m}{\mu}\right)\nabla\varphi(\mathbf{x},\mathbf{x}')\right]dv+\frac{T}{4\pi}\int_{S_1\cup S_2}\left[j\omega\varepsilon[\mathbf{n}\times\mathbf{E}(\mathbf{x}')]\varphi(\mathbf{x},\mathbf{x}')-[\mathbf{n}\times\mathbf{H}(\mathbf{x}')]\times\nabla\varphi(\mathbf{x},\mathbf{x}')-[\mathbf{n}\cdot\mathbf{H}(\mathbf{x}')]\nabla\varphi(\mathbf{x},\mathbf{x}')\right]da \quad (16)$$

where all functions of space coordinates are also functions of ω. A situation of interest is when one of the surfaces bordering the domain V, say S_2, recedes to infinity. It can be shown that for spatially-limited sources in V, the contribution of the integral over S_2, as it recedes towards infinity, is due to sources outside S_2. We shall refer to that contribution as an incident field and write the two equations above as:

$$\mathbf{E}(x,\omega)=\mathbf{E}^{inc}(x,\omega)-\frac{T}{4\pi}\int_V \left[j\omega\mu\mathbf{J}(x',\omega)\varphi(x,x',\omega)+\mathbf{K}(x',\omega)\times\nabla\varphi(x,x',\omega)-\left(\frac{\rho}{\varepsilon}\right)\nabla\varphi(x,x',\omega)\right]dv-\frac{T}{4\pi}\int_S \left[j\omega\mu[\mathbf{n}\times\mathbf{H}(x',\omega)]\varphi(x,x',\omega)-[\mathbf{n}\times\mathbf{E}(x',\omega)]\times\nabla\varphi(x,x',\omega)-[\mathbf{n}\cdot\mathbf{E}(x',\omega)]\nabla\varphi(x,x',\omega)\right]da \tag{17}$$

$$\mathbf{H}(x,\omega)=\mathbf{H}^{inc}(x,\omega)+\frac{T}{4\pi}\int_V \left[-j\omega\varepsilon\mathbf{K}(x',\omega)\varphi(x,x',\omega)+\mathbf{J}(x',\omega)\times\nabla\varphi(x,x',\omega)-\left(\frac{m}{\mu}\right)\nabla\varphi(x,x',\omega)\right]dv+\frac{T}{4\pi}\int_S \left[j\omega\varepsilon[\mathbf{n}\times\mathbf{E}(x',\omega)]\varphi(x,x',\omega)-[\mathbf{n}\times\mathbf{H}(x',\omega)]\times\nabla\varphi(x,x',\omega)-[\mathbf{n}\cdot\mathbf{H}(x',\omega)]\nabla\varphi(x,x',\omega)\right]da \tag{18}$$

where S is the remaining closed boundary surface, formerly S_1. In the above we assumed that all inhomogeneities in space are excluded from the volume V by the surface S. For a source-free region, the volume integrals above vanish and the surface integrals extend only over the boundary of the scatterer. The problem dealt with in this work is different from those considered in the past in that, among other things, the surface of the scatterer is not uniform in its electrical characteristics.

For a scatterer made up of a dielectric partially covered with conductive material of infinite conductivity, the above treatment is insufficient. The difficulty, as pointed out by Stratton (1941) [14] and Kottler (1923) [15], is that the field vectors **E** and **H** are not continuous and do not have continuous first derivatives over the entire surface. There is a discontinuous change in the tangential components of **E** and **H** in passing from the conductive to the dielectric regions on the surface S. The surface is divided into two separate areas A_1 (conductive) and A_2 (dielectric). The two areas A_1 and A_2 satisfy the following relationships:

$$A_1 \cup A_2 = S \tag{19}$$

$$A_1 \cap A_2 = C \tag{20}$$

where S is the total surface of the scatterer and C a contour line bordering area A_1, like in Figure 4.

An abrupt change in the surface current density can be accounted for by an accumulation of line charges on the contour. Let **ds** be an element of length along the contour in the positive direction as determined by the normal **n** to the surface. Also, $\mathbf{n}_1$ is the unit vector lying on the surface and normal to both **n** and **ds** and directed towards A_1.

The line densities of the electric and magnetic charges, designated by λ and λ^* are related to the electric and magnetic fields by the following relationships:

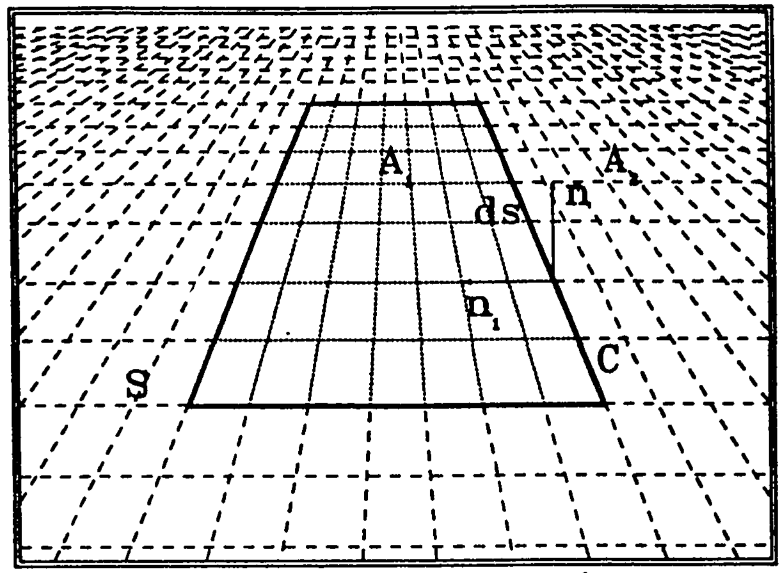

Figure 4 - Contour C separating conductive zone A_1 from dielectric A_2

$$-j\omega\lambda = \mathbf{n}_1 \bullet \left[(\mathbf{n} \times \mathbf{H}_2) - (\mathbf{n} \times \mathbf{H}_1) \right] = (\mathbf{H}_2 - \mathbf{H}_1) \bullet (\mathbf{n}_1 \times \mathbf{n}) = (\mathbf{H}_2 - \mathbf{H}_1) \bullet \mathbf{ds} \qquad (21)$$

$$j\omega\lambda^* = \mathbf{n}_1 \bullet \left[(\mathbf{n} \times \mathbf{E}_2) - (\mathbf{n} \times \mathbf{E}_1) \right] = (\mathbf{E}_2 - \mathbf{E}_1) \bullet (\mathbf{n}_1 \times \mathbf{n}) = (\mathbf{E}_2 - \mathbf{E}_1) \bullet \mathbf{ds} \qquad (22)$$

As described in Stratton (1941) [13], line integrals of the type shown below, along the contour C, are introduced. They assure that the fields along the two sides of the contour respect Maxwell's equations. Expressions (17) and (18) are therefore modified in the absence of sources thus :

$$\mathbf{E}(x,\omega)=\mathbf{E}^{inc}(x,\omega)+ \frac{T}{4\pi j\omega\varepsilon}\oint_C \nabla\varphi(x,x',\omega)[\mathbf{H}_1(x',\omega)-\mathbf{H}_2(x',\omega)]\cdot\mathbf{ds}- \frac{T}{4\pi}\int_{A_1\cup A_2}\Big[j\omega\mu[\mathbf{n}\times\mathbf{H}_i(x',\omega)]\varphi(x,x',\omega)- [\mathbf{n}\times\mathbf{E}_i(x',\omega)]\times\nabla\varphi(x,x',\omega)-[\mathbf{n}\cdot\mathbf{E}_i(x',\omega)]\nabla\varphi(x,x',\omega)\Big]da \tag{23}$$

and

$$\mathbf{H}(x,\omega)=\mathbf{H}^{inc}(x,\omega)- \frac{T}{4\pi j\omega\mu}\oint_C \nabla\varphi(x,x',\omega)[\mathbf{E}_1(x',\omega)-\mathbf{E}_2(x',\omega)]\cdot\mathbf{ds}+ \frac{T}{4\pi}\int_{A_1\cup A_2}\Big[j\omega\varepsilon[\mathbf{n}\times\mathbf{E}_i(x',\omega)]\varphi(x,x',\omega)- [\mathbf{n}\times\mathbf{H}_i(x',\omega)]\times\nabla\varphi(x,x',\omega)-[\mathbf{n}\cdot\mathbf{H}_i(x',\omega)]\nabla\varphi(x,x',\omega)\Big]da \tag{24}$$

where the subscripts 1 and 2 imply the values of the fields along the contour C, immediately inside the areas A_1 and A_2. Subscript "i" in the surface integrals will also be assigned values 1 or 2, depending on whether we perform the integrals over one or the other of the subdivisions A_1 and A_2.

Also, it should be noted that the left sides of equations (23) and (24) represent the fields calculated at the observation point **x**, where **x** could be either within the volume V or on the surface S. In the latter case however, the surface integrals are calculated in the principal value sense, i.e. eliminating, at the limit, the observation point. A rigorous definition of principal-value integrals can be found elsewhere. An excellent reference is Smirnov (1972) [16].

2.2 Time-Domain Integral Equations

Equations (23) and (24) will be used in the derivation of the time-domain integral equations with the aid of Fourier Transforms (FT). A short discussion of pertinent features of the Fourier transforms follows. If f(ω) is a frequency-dependent function, its Fourier transform is a function f(t) of time given by the relation:

$$f(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt \quad \text{or} \quad FT[f(\omega)]=\frac{1}{2\pi}\int_{-\infty}^{+\infty} f(\omega)e^{j\omega t}d\omega=f(t) \qquad (25)$$

A few special cases of the transform above are:

$$FT[e^{-j\omega\tau}]=\delta(t-\tau) \qquad (26)$$

$$FT[j\omega f(\omega)] = \frac{d}{dt} f(t) \tag{27}$$

also

$$FT\ [\varphi(x,x',\omega)] = FT\left[\frac{e^{-j\frac{\omega}{c}|x-x'|}}{|x-x'|}\right] = \frac{\delta[t-(|x-x'|/c)]}{|x-x'|} \tag{28}$$

$$FT\ [\nabla\varphi(x,x',\omega)] = \delta\left(t-\frac{|x-x'|}{c}\right)\frac{(x-x')}{|x-x'|^3} + \frac{1}{c}\delta\left(t-\frac{|x-x'|}{c}\right)\frac{(x-x')}{|x-x'|^2} \tag{29}$$

The mathematical representation of the convolution theorem for two functions is:

$$FT\ [f(\omega)\ g(\omega)] = f(t) \otimes g(t) = \int_{-\infty}^{+\infty} f(t_1) g(t-t_1) dt_1 \tag{30}$$

The equation (30) above applies also to the cross product of two functions:

$$FT\ [f(x,\omega) \times g(x,\omega)] = \int_{-\infty}^{+\infty} f(x,t_1) \times g(x,t-t_1) dt_1 \tag{31}$$

The convolution theorem for three functions is:

$$FT[a(\omega)\ b(\omega)\ c(\omega)] = a(t)\otimes b(t)\otimes c(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} a(t_1)b(t_2-t_1)c(t-t_2)dt_1dt_2 \tag{32}$$

The Fourier transforms of equations (23) and (24), after (26)-(32) have been taken into account, yield the electric and magnetic fields **E**(**x**,t) and **H**(**x**,t). In the above mathematical derivation, special consideration is given to the transformation of the contour integrals. The derivation of those terms, whose formulation constitutes one of the original theoretical contributions of this work, follows.

2.3 Hybrid Body Formulation

The treatment of a hybrid body involves the surface integrals specific to homogeneous bodies, with the added ingredient of line integrals, accounting for the transition between dielectric and conductive zones. This term opens the opportunity of solving scattering problems for bodies which are uniform in parts, as opposed to being so as a whole. This is in fact the situation most often found in day-to-day reality. This work addresses this concern, both from the theoretical and the practical points of view.

For $a(\omega) = 1/j\omega$; $b(\omega) = \mathbf{H}(\omega)$ and $c(\omega) = \nabla\varphi$, we obtain for their respective transforms:

$$FT\left[\frac{1}{j\omega}\right] = \begin{cases} 1 & \text{if } t>0 \\ 0 & \text{if } t<0 \end{cases} \tag{33}$$

$$FT\left[\mathbf{H}(\omega)\right] = \mathbf{H}(t) \tag{34}$$

By applying (29), (32), (33) and (34) we obtain:

$$FT\left[\frac{1}{j\omega}\mathbf{H}(\omega)\ \nabla\varphi(\omega)\right] = \int_0^{t-\frac{R}{c}}\left[\mathbf{H}(t-\frac{R}{c}-t_1)\frac{1}{R^2} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{H}(t-\frac{R}{c}-t_1)\frac{1}{R}\right]\hat{\mathbf{R}}dt_1 \tag{35}$$

where $R = |\mathbf{x}\text{-}\mathbf{x}'|$ and $\hat{\mathbf{R}}$ the unit vector. Equation (35) above was obtained under the assumption that the magnetic field is null before a predetermined time chosen as the origin, **H**(t) = 0 for t<0.

The same reasoning applies to the electric field.

$$FT\left[\frac{1}{j\omega}\,\mathbf{E}(\omega)\ \nabla\varphi(\omega)\right] = \int_0^{t-\frac{R}{c}}\left[\mathbf{E}(t-\frac{R}{c}-t_1)\frac{1}{R^2} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}(t-\frac{R}{c}-t_1)\frac{1}{R}\right]\hat{\mathbf{R}}dt_1 \tag{36}$$

By applying the Fourier transform to (23) and (24), one obtains:

$$\begin{aligned}\mathbf{E}(x,t)=&T\mathbf{E}^{inc}(x,t) + \frac{T}{4\pi\varepsilon}\\ &\int_0^{t-\frac{R}{c}} dt_1 \oint_C \hat{\mathbf{R}}\left(\frac{1}{R^2}+\frac{1}{cR}\frac{\partial}{\partial t}\right)\left[\mathbf{H}_1(x',t-\frac{R}{c}-t_1)-\mathbf{H}_2(x',t-\frac{R}{c}-t_1)\right]\mathbf{ds}-\\ &\frac{T}{4\pi}\int_{A_1\cup A_2}\left\{\mu\frac{1}{R}\frac{\partial}{\partial t}[\mathbf{n}'\times\mathbf{H}(x',\tau)]-\frac{1}{R^2}[\mathbf{n}'\times\mathbf{E}(x',\tau)]\times\hat{\mathbf{R}}-\right.\\ &\frac{1}{cR}\left[\mathbf{n}'\times\frac{\partial}{\partial t}\mathbf{E}(x',\tau)\right]\times\hat{\mathbf{R}}-\\ &\left.-\hat{\mathbf{R}}[\mathbf{n}'\cdot\mathbf{E}(x',\tau)]\frac{1}{R^2}-\hat{\mathbf{R}}\left[\mathbf{n}'\cdot\frac{\partial}{\partial t}\mathbf{E}(x',\tau)\right]\frac{1}{cR}\right\}da_{\ \tau=t-\frac{R}{c}}\end{aligned} \tag{37}$$

and

$$\mathbf{H}(x,t)=T\mathbf{H}^{inc}(x,t) - \frac{T}{4\pi\mu}$$

$$\int_0^{t-\frac{R}{c}} dt_1 \oint_C \hat{\mathbf{R}}\left(\frac{1}{R^2}+\frac{1}{cR}\frac{\partial}{\partial t}\right)\left[\mathbf{E}_1(x',t-\frac{R}{c}-t_1)-\mathbf{E}_2(x',t-\frac{R}{c}-t_1)\right]\cdot\mathbf{ds}+$$

$$\frac{T}{4\pi}\int_{A_1 \cup A_2}\left\{\varepsilon\frac{1}{R}\frac{\partial}{\partial t}[\mathbf{n}'\times\mathbf{E}(x',\tau)]+\frac{1}{R^2}[\mathbf{n}'\times\mathbf{H}(x',\tau)]\times\hat{\mathbf{R}}+\right. \tag{38}$$

$$\frac{1}{cR}\left[\mathbf{n}'\times\frac{\partial}{\partial t}\mathbf{H}(x',\tau)\right]\times\hat{\mathbf{R}}+$$

$$\left.\frac{1}{R^2}[\mathbf{n}'\cdot\mathbf{H}(x',\tau)]\hat{\mathbf{R}}+\frac{1}{cR}\left[\mathbf{n}'\cdot\frac{\partial}{\partial t}\mathbf{H}(x',\tau)\right]\hat{\mathbf{R}}\right\}da_{\,\tau=t-\frac{R}{c}}$$

where **n'** is the normal to the surface at the source point **x'** and $\hat{\mathbf{R}}$ the unit vector from **x'** to **x**.

Equations (37) and (38) are expressions of the total scattered fields **E(x**,t) and **H(x**,t) as functions of the incident fields $\mathbf{E}^{inc}(\mathbf{x},t)$ and $\mathbf{H}^{inc}(\mathbf{x},t)$ and a combination of fields and field derivatives integrated over the surface of the scatterer and on the line separating the dielectric and conductive regions. The observation point **x** may be located within the volume V or on the surface bordering same.

The expressions of the fields and of their derivatives on the surface and along the contour C are calculated at a time τ behind the current time by the amount it takes the electromagnetic wave at velocity c to cross the distance R between the observation and the source points. The light velocity is characteristic to the medium contained in the volume V and bordered by the surface S.

Following a technique proposed by Poggio and Miller (1973) [6] equations (37) and (38) are written for observation points both inside and outside the scatterer. Then, the two observation points are brought close together towards the surface, thereby obtaining expressions of the total fields at one point on the surface as functions of the fields on the rest of the surface.

In this work however, we address the difficulty that the surface of the scatterer is not homogeneous, i.e. it is partly dielectric and partly conductor. Therefore instead of writing only one pair of equations for each of the electric and magnetic fields, just inside and outside the body, we write two such pairs for each. One pair of observation points will be allowed to come together towards the conductive area A_1 and the other towards the dielectric area A_2.

Further, there are different material characteristics for the two dielectrics: ε_o, μ and c for the outside dielectric and ε_i, μ and c_i for the inside dielectric. In addition, each pair of equations for the fields has to satisfy appropriate boundary conditions at the dielectric/dielectric or dielectric/conductor interfaces. A further complication from the fact that the speed of light has different values inside and outside the body, is that the delays of the form

$$\tau = t - \frac{R}{c} \cdot \tag{39}$$

have different values inside and outside the body, according to c or c_i, respectively.

The boundary conditions are:

$$\mathbf{n} \times (\mathbf{E}_{do} - \mathbf{E}_{di}) = 0 \tag{40}$$

$$\mathbf{n} \times (\mathbf{H}_{do} - \mathbf{H}_{di}) = 0 \tag{41}$$

and

$$\mathbf{n} \bullet (\varepsilon_o \mathbf{E}_{do} - \varepsilon_i \mathbf{E}_{di}) = 0 \tag{42}$$

at the interface between the two dielectric materials: "do" (dielectric outside) and "di" (dielectric inside).

Because the electric and magnetic fields immediately inside the conductor are null, the boundary conditions at the conductor/dielectric interface are:

$$\mathbf{n} \times \mathbf{E}_{do} = 0 \tag{43}$$

$$\mathbf{n} \times \mathbf{H}_{do} = \mathbf{J}_s \text{ (surface current density)} \tag{44}$$

The integrals over the surface, after application of the appropriate boundary conditions will be split in integrals over the two subdivisions, A_1 (conductor of infinite conductivity) and A_2 (dielectric).

After extensive mathematical manipulation we obtain the expressions of the components of the electric and magnetic fields on the surface (see Appendix A). The fields expressions are obtained following the application of the boundary conditions for the normal and tangential components of the electric and magnetic fields at the dielectric - dielectric and dielectric - conductor interfaces. The tangential component of the electric field on the dielectric area A_2 of the surface S is:

$$\mathbf{n}\times\mathbf{E}(t) = \mathbf{n}\times\mathbf{E}^{inc}(t)+\frac{1}{4\pi}\mathbf{n}\times\Big\langle\frac{1}{\varepsilon_o}\int_0^{\tau_o}dt_1\oint_C\hat{\mathbf{R}}L_o[\mathbf{H}_1(\tau_o-t_1)-\mathbf{H}_2(\tau_o-t_1)]\cdot\mathbf{ds}'-$$

$$-\frac{1}{\varepsilon_i}\int_0^{\tau_i}dt_1\oint_C\hat{\mathbf{R}}L_i\mathbf{H}_2(\tau_i-t_1)\cdot\mathbf{ds}'-\iint_{A_1}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}\right)da'+ \tag{45}$$

$$+\iint_{A_2}\Big(\mathbf{n}'\{L_o\mathbf{E}(\tau_o)-L_i\mathbf{E}(\tau_i)\}\hat{\mathbf{R}}+[\mathbf{n}'\times[L_o\mathbf{E}(\tau_o)-L_i\mathbf{E}(\tau_i)]]\times\hat{\mathbf{R}}-$$

$$-\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial[\mathbf{H}(\tau_o)-\mathbf{H}(\tau_i)]}{\partial t}\Big)da'\Big\rangle$$

The equation corresponding to the tangential component of the magnetic field on the dielectric side is:

$$\mathbf{n}\times\mathbf{H}(t) = \mathbf{n}\times\mathbf{H}^{inc}(t)+\frac{1}{4\pi}\mathbf{n}\times\Big\langle\int_{A_1}[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)]\times\hat{\mathbf{R}}\,da' +$$

$$\int_{A_2}\left(\frac{1}{R}\mathbf{n}'\times\left[\varepsilon_o\frac{\partial\mathbf{E}(\tau_o)}{\partial t}-\varepsilon_i\frac{\partial\mathbf{E}(\tau_i)}{\partial t}\right]+\mathbf{n}'\cdot[L_o\mathbf{H}(\tau_o)-L_i\mathbf{H}(\tau_i)]\hat{\mathbf{R}}+\right. \tag{46}$$

$$\left.+[\mathbf{n}'\times[L_o\mathbf{H}(\tau_o)-L_i\mathbf{H}(\tau_i)]]\times\hat{\mathbf{R}}\right)da'\Big\rangle$$

On the conductive side, the tangential component is null and this fact would preclude expressing of the electric field at time "t" as a function of the fields at times prior to "t". Instead, we derive the equation for the normal component of the electric field.

$$\mathbf{n}\cdot\mathbf{E}(t) = 2\mathbf{n}\cdot\mathbf{E}^{inc}(t)+\frac{1}{2\pi}\mathbf{n}\cdot$$
$$\left\langle\frac{1}{\varepsilon_o}\int_0^{\tau_o}dt_1\oint_C\hat{\mathbf{R}}L_o[\mathbf{H}_1(\tau_o-t_1)-\mathbf{H}_2(\tau_o-t_1)]\cdot\mathbf{ds}'-\right.$$
$$-\int_{A_1}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}\right)da'- \tag{47}$$
$$\left.-\int_{A_2}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}-[\mathbf{n}'\times L_o\mathbf{E}(\tau_o)]\times\hat{\mathbf{R}}\right)da'\right\rangle$$

For the tangential component of the magnetic field on the conductive side we have:

$$\mathbf{n}\times\mathbf{H}(t) = 2\mathbf{n}\times\mathbf{H}^{inc}(t)+\frac{1}{2\pi}\mathbf{n}\times\left\langle\int_{A_1}[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)]\times\hat{\mathbf{R}}\,da'+\right. \tag{48}$$
$$\left.+\int_{A_2}\left(\varepsilon_o\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{E}(\tau_o)}{\partial t}+[\mathbf{n}'\cdot L_o\mathbf{H}(\tau_o)]\hat{\mathbf{R}}+[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)]\times\hat{\mathbf{R}}\right)da'\right\rangle$$

All the fields in equations (45) - (48) are outside fields and $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{H}_1$ and $\mathbf{H}_2$ are calculated on the surface of the scatterer, on the two sides of the contour C. We also use the notations:

$$L_o=\frac{1}{R^2}+\frac{1}{Rc}\frac{\partial}{\partial t} \quad ; \quad L_i=\frac{1}{R^2}+\frac{1}{Rc_i}\frac{\partial}{\partial t} \tag{49}$$

where $\hat{\mathbf{R}}$ is the unit vector aiming from the source $\mathbf{x}'$ to the observation point $\mathbf{x}$ and R is the distance between those two points. Expressions (45) - (48), together with similar expressions for the normal components of the electric and magnetic fields on the dielectric, constitute our final system of equations. For clarity, all six equations are listed in Appendix B. This system has to be solved numerically for a particular geometry of the scatterer and incident field.

As discussed earlier, in the Geometrical Considerations subchapter, bodies that have volume discontinuities are not covered by the approach presented in this thesis. For those scatterers, the fields' discontinuities are accounted for by introducing a distribution of surface densities of charge and currents. Their presence has to be addressed in a manner similar to the line integrals for line densities above. Accordingly, one must introduce surface integrals of the surface charge and current densities over the contact areas between homogeneous parts of the body.

3. NUMERICAL APPROACH

Once the integral equations are obtained, the numerical solution involves two phases: a) spatial and b) temporal calculations. The surface of the scatterer is divided into square patches and the surface integrals are replaced by sums of integrals over patches. In turn, the integrals over patches are approximated by the product between the area of the patch and the average of the integrand. That average value is assigned to the center of the patch. Time is similarly divided into intervals Δt, starting with $t_0 = 0$, so that $t_1 = t_0 + \Delta t$; $t_2 = t_0 + 2\Delta t$ and so on.

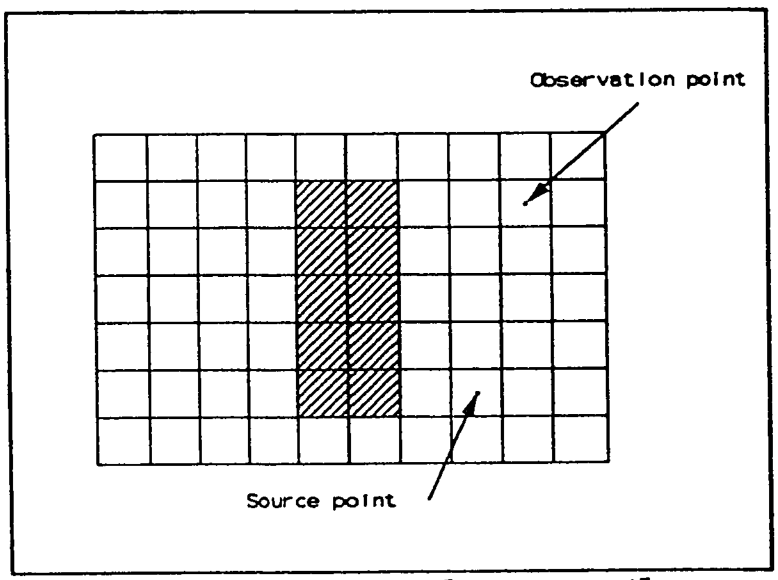

Figure 5 - Square patches over the scatterer for numerical calculations

The values of the integrals are, as noted above, principal value integrals. They may contain singular integrands and for purposes of numerical implementation, the patch at which the observation point is located, is eliminated from calculation (Poggio and Miller 1973) [6], (Fig. 5).

The above approach appears to be quite straightforward. However, considering our integral equations, their solutions are affected by a number of factors.

i) To take full advantage of the marching-in-time approach, it is necessary to properly separate the fields at one sample point from the adjacent point in space. To ensure that condition, the following relationship between the minimum space step ΔR and the time step Δt must be satisfied:

$$c\,\Delta t \leq \Delta R \tag{50}$$

A stricter requirement is observed in this work.

$$c\,\Delta t \leq \Delta z \tag{51}$$

where Δz is the distance between patch centres measured along the z axix. Similarly to the spatial patches, we assume that the values of the fields and of their derivatives do not vary over a time interval, but change discretely at the end of such a time interval. As noted before, the values of the fields at time t depend upon the values of the fields at some time in the past, called retardation time. In our formulation, there are however, two such retardation times:

$$\tau_o = t - R/c \tag{52}$$

and

$$\tau_i = t - R/c_i \tag{53}$$

where t is the current time, R is the distance between observation and source points and c and c_i are light velocities outside (in the air) and inside the dielectric, respectively. Because the fields depend upon values in the past, any calculated values should be kept in the computer memory for future use.

ii) In contrast to the situations described in the State of the Art chapter, the scattering body in the current problem is not uniform, neither totally dielectric, nor totally conductive, but a combination thereof. To the integrals accounting for the dielectric and the conductive parts, a circular line integral together with a temporal integral are added, as seen in (37) and (38).

The implication of the time integrals is that for the points in space implicated in the circular integral, i.e. points along the bordering line between the dielectric and the conductive parts of the surface, the retardation in time is not limited to calculating the fields at times expressed in (37) and (38) above.

For those points, the fields at time t depend on the fields at **all times** prior to that time t. To numerically solve our system of equations, we wrote a custom FORTRAN program for each configuration.

3.1 Implementation

(i) Regarding the spatial coordinates, all domains A of surface integrals of the form:

$$\int_A L\ \mathbf{F}(x,x')\ da \tag{54}$$

contained in our system of equations, where L has been defined before in (49), **F** is the tangential or normal component of either the electric or the magnetic field or a combination thereof. A is the surface of either the conductive or the dielectric zone and da the element of surface. All zones are divided into square patches of side ΔR.

Using collocation, the integral above is replaced by the sum:

$$\sum_{i=1}^{N} L\ \mathbf{F}_{avg}(x,x') \ (\Delta R)^2 \qquad (55)$$

where N is the total number of patches over the area A above, $\mathbf{F}_{avg}$ is the average value of one of the six field components we calculate, $\mathbf{E}_t$, $\mathbf{E}_n$, $\mathbf{H}_t$ or $\mathbf{H}_n$ on the dielectric or $\mathbf{E}_n$ or $\mathbf{H}_t$ on the conductor.

Solving the system of six equations and six unknowns, presented in Appendix B, does not however require the inversion of a matrix. The reason is that the left-hand side of the equations and the sums on the right-hand side are not calculated at the same point in time. The unknown field components on the left, at time t, are expressed in terms of the known quantities on the right, already calculated at earlier times.

(ii) The line integrals present in some of the equations are of the form:

$$\oint_C L\ \mathbf{H} \cdot d\mathbf{s} \qquad (56)$$

where L has been already defined in (49), **H** is the tangential component of the magnetic field on the dielectric or conductive sides of the contour C separating same and **ds** is the oriented element of length along the contour. The contour C has an orientation and is divided into vectors of length ΔR, also oriented.

Such integrals are replaced by sums of the form:

$$\sum_{k=1}^{N_s} L\ \mathbf{H}_{avg} \cdot \mathbf{s}\ \Delta R \tag{57}$$

where N_s is the total number of segments along the contour, $\mathbf{H}_{avg}$ is the average of the tangential component of the magnetic field in the patch bordering that particular element of length, **s** is a unit vector oriented along the contour C and ΔR is the space step.

(iii) Time is involved in the solving of the system of equations in two ways:

(a) In the line integrals, time appears under the integral sign. A time integral with an upper limit of say t - R/c is replaced by a sum over all time steps preceding that limit. In other words, for a specific time t and two points (observation and source) separated by a distance R, the delay R/c and subsequently $t_1 = t - R/c$ are calculated first. Because time varies discretely in steps Δt, the value t_1 is assigned to the closest multiple of that time step. The integral is replaced by a summation from time t = 1 to that calculated time t_1.

(b) All six equations express one of the six unknowns at time t as a function of a combination of those same unknowns at an earlier time, say t - R/c. Those values of the fields have already been calculated at previous time steps, so that at each instant only the appropriate sums have to be performed. This is in fact the essence of the "marching-on-in-time" method. One notices that there is no need here for any matrix operation.

The only difficulty arises at the initial time step $t = \Delta t$, where no previous field values are known. There, the fields are expressed with the aid of the Gaussian function:

$$f(n,m) \sim \frac{d}{\sqrt{\pi}} \exp\left[-d^2 \left(n\Delta t - \frac{m\Delta R}{c}\right)^2\right] \tag{58}$$

where n and m are integers. Because time derivatives of the fields are also needed, the time derivative of the above expression is used. Following the procedure described above, the values of the field components and of their derivatives at the initial time are known. At subsequent time steps, the time derivative of the fields is replaced by the difference between the values at two consecutive times divided by the value of the time step Δt.

3.2 Fast Fourier Transform (FFT)

The results calculated by the program designed to model equations in Appendix B are the fields **E**(x,t) and **H**(x,t) at all points on the surface of the scatterer or at any location around it, at a number of time steps. The values obtained constitute the Fourier transform of the fields **E**(x,f) and **H**(x,f), where f is the frequency of the signal within the range of frequencies over which that transform is valid.

To obtain the frequency-domain values of the fields, one has to Fourier transform the time-domain results at the location of interest. The frequency results provide information about the behaviour of the fields over the entire range of frequencies.

The text of Brigham [17] has constituted an excellent source of inspiration on the topic. We used the FFT subroutine program contained in the book above to convert the data from the time-domain into the frequency domain. For completeness, we list the program in Appendix C. A particularly fast transform takes place where the number of time steps is a power of 2. We used sixteen (N=16) such time steps throughout the programs associated with this work.

3.2 Program Description

The program consists of a number of individual subroutines emulating the various functions that appear in the six equations for the tangential and normal components of the electric and magnetic fields on the surface of the dielectric and the normal component of the electric field and tangential component of the magnetic field on the conductive side of the body.

Such basic functions are the dot and cross products, operators L_0 and L_i, as shown in Appendix A, the shape functions of the Gaussian impulses of the electric and magnetic fields, the derivatives of these fields at the initial moment, the distance between the observation and the source points, the vector pointing from the source to the observation point and so on.

In addition to the above basic functions, four more subroutines implement the actual six equations, which calculate the fields on the surface of the body. For clarity, these six equations are shown in Appendix B.

For the box configuration, two additional subroutines calculate the electric and magnetic fields at a location outside the surface of the body. The main program calls for the fields in the center of the box, but the fields may be calculated at any location.

An important component of the main program is the numerical description of the scattering body. Quite simply, the coordinates of all the patch centres and their associated normal vectors are defined. Also, the curve(s) lying between the dielectric and the conductive regions of the body are defined by means of vectors.

Finally, the program calculates the sum of the normal components of the magnetic vectors over the entire surface of the body for checking puposes and stores all relevant values in arrays carried in common by the subroutines. A listing of the program appears in Appendix D.

3.3 Accuracy checks

The most useful accuracy check would be, of course, a direct comparison of the computed results to those obtained at a time-domain measuring facility. There we would have direct access to measured scattered and radiated pulses. In the absence of access to such a facility, a direct comparison with frequency-domain calculations, after appropriate Fourier transform, would provide useful information. The paper by Bernardi and Cicchetti (1990) [18] dealt with the response of a planar microstrip line excited by an external field. Our results confirm that paper's findings.

To verify the accuracy of the results, we attempted to check at every time step whether Maxwell's law stating that, for μ constant, the flux of the magnetic field through a closed surface is zero.

Accordingly, an imaginary surface was constructed around

$$\int \mathbf{H} \cdot \mathbf{n} \, da = 0 \tag{59}$$

the body of interest, be it the dielectric slab with conductive microstrip or the metallic box with dielectric plug. The surface completely surrounds the structure in such a way that its inside surface coincides with the the surface of the body and its outside surface follows the inside surface in a parallel manner. The lateral walls of the surface are perpendicular to the outside and inside surfaces.

One could visualize this surface, say in the case of the slab, as six layers of air of a certain, arbitrary thickness, laid on the six sides of our slab, toward the outside. The magnetic flux through the surface of the group of six layers should, according to Maxwell, be zero.

It could be easily seen that, due to symmetry, the flux through the outside surface of this blanket of air and that through the lateral walls, is always zero. There, only the incident, exciting field has any value. The only difficulty rests with calculating the flux through the inner surface of the blanket, which coincides with the outer surface of our structure. The integral in (59) is replaced by a sum over all the patches of the body. We calculate:

$$\sum_{\text{all body}} \mathbf{H} \cdot \mathbf{n} \, (\Delta R)^2 \tag{60}$$

at every time step. Closeness to zero would indicate the goodness of our numerical results. The findings are presented in the Results chapter.

4. RESULTS

The results for the two configurations, i) dielectric slab with ground plane and a perfectly-conductive microstrip on the opposite face and ii) metallic box with thick walls are discussed below, together with their geometries and electrical characteristics.

4.1 Dielectric Slab with Ground Plane and Microstrip

We calculated the current induced in the metallic microstrip for the relative permittivity of the dielectric substrate $\varepsilon_r = 10.0$. The length of the microstrip is 4 cm and its width 4 mm. The incident field is normal to the body, with the electric field along the microstrip and the magnetic field perpendicular to it.

The overall dimensions of the body in mm are 160 X 20 X 8. Its surface was divided into 580 squares of side length 4 mm. Two hundred squares constitute the ground plate on the bottom of the slab. An additional 10 squares on top simulate the microstrip. Of the 580 total number of squares, 210 are on the conductive side and 370 on the dielectric.

The program calculates at each time step all the field components $\mathbf{E}_t$, $\mathbf{E}_n$, $\mathbf{H}_t$ and $\mathbf{H}_n$ on the dielectric and $\mathbf{E}_n$ and $\mathbf{H}_t$ on the conductor. The current intensity along the microstrip (in the y direction) is calculated with the formula:

$$I(y) = \Delta R\ \bar{H}_y \tag{61}$$

where I(y) is the current at location y (center of one of the ten squares), ΔR is the width of the microstrip and H_y is the amplitude of the y-component of the (**n** X **H**) product or surface current density after FFT at frequency f. Bernardi and Cicchetti (1990) [18] studied a similar case of a microstrip line and a ground plane, on a dielectric substrate. The line was terminated into its characteristic impedance at both ends (ours is open-ended). The frequency of calculation was 3 GHz at the same relative permittivity of the substrate.

The above authors calculated, using a transmission line model and assuming a quasi-TEM dominant mode, that a current in the range of a few microamperes was induced in the microstrip. We extrapolated their results, by taking the limit of those end impedances to infinity, and compared them to those obtained with our program after the appropriate FFT.

More specifically, the above authors developed a formula expressing the induced current along the microstrip as function of geometry and the electrical characteristics of both the incident field and those of the scatterer (slab with ground plane and microstrip). Of interest to us is the dependence of that induced current upon the value of the loads at the end of the microstrip.

The induced current is estimated by the following formula:

$$I(y) = F\left[1 - \frac{\cos(\beta y)}{\cos\left(\beta \frac{l}{2}\right)}\right] \tag{62}$$

where $\beta = \omega/c\sqrt{\varepsilon_{eff}}$, l is the length of the microstrip, y is a position along the microstrip and ε_{eff} is the effective relative permittivity of the quasi-TEM mode in the microstrip structure.

The factor F is given by the following expression:

$$F = -\frac{2E_0 c}{\omega Z_0} \frac{\sin\left(\frac{\omega}{c}\sqrt{\varepsilon_r}\, d\right)}{j\sin\left(\frac{\omega}{c}\sqrt{\varepsilon_r}\, d\right) + \sqrt{\varepsilon_r}\cos\left(\frac{\omega}{c}\sqrt{\varepsilon_r}\, d\right)} \tag{63}$$

where E_0 is the amplitude of the incident electric pulse, Z_0 is the characteristic impedance of the microstrip with ground plane with the dielectric substrate removed, d is the distance between microstrip and ground plane and ε_r is the relative permittivity of the substrate.

The complex factor F does not depend on the position y along microstrip. It may by put under the form $F = Ae^{i\Psi}$. In our work we estimated the values of the induced current. We compare separately a) the behavior of the current along the microstrip, b) the magnitude A of the factor F for five frequencies and c) the phase ϑ of the factor F for our geometry and compared them with our results for the same. The characteristic impedance for our microstrip was evaluated with the formula [19]:

$$Z_0 \sim \frac{\eta}{4} \frac{K(k)}{K(\sqrt{1-k^2})} \tag{64}$$

where $\eta = \sqrt{\mu/\varepsilon}$, K is the complete elliptical integral of the first degree, and k its argument. k is given by:

$$k = \left[\cosh\left(\frac{\pi w}{4d}\right)\right]^{-1} \tag{65}$$

where w is the width of the microstrip and d the distance between it and the ground plane. Numerically, $Z_0 = 139.83\ \Omega$.

Figure 6 displays an example of the induced magnetic field time-domain transform at all sixteen time steps. A good time response may be noticed. The wave passes over our body and the induced field reacts to it. No instabilities are noticed.

The graphs in Figures 7-11 present the relative behaviour of the current across the microstrip at five frequencies. FFT was performed on the magnetic field results along the microstrip at all 10 locations. The frequencies are multiples of the fundamental value of 4.6875 GHz. The values of the currents obtained in this work and in that of Bernardi and Cicchetti are normalized to a common maximum value. Figures 12 and 13 compare the amplitude and phase of the factor F as obtained by Bernardi and

Cicchetti and by us. We notice the good agreement.

Figure 14 depicts the behaviour of the integral present in Maxwell's second law over a surface comprising the body and a parallel surface just outside it. The closeness to a null value reflects the fact that the calculated fields respect that law.

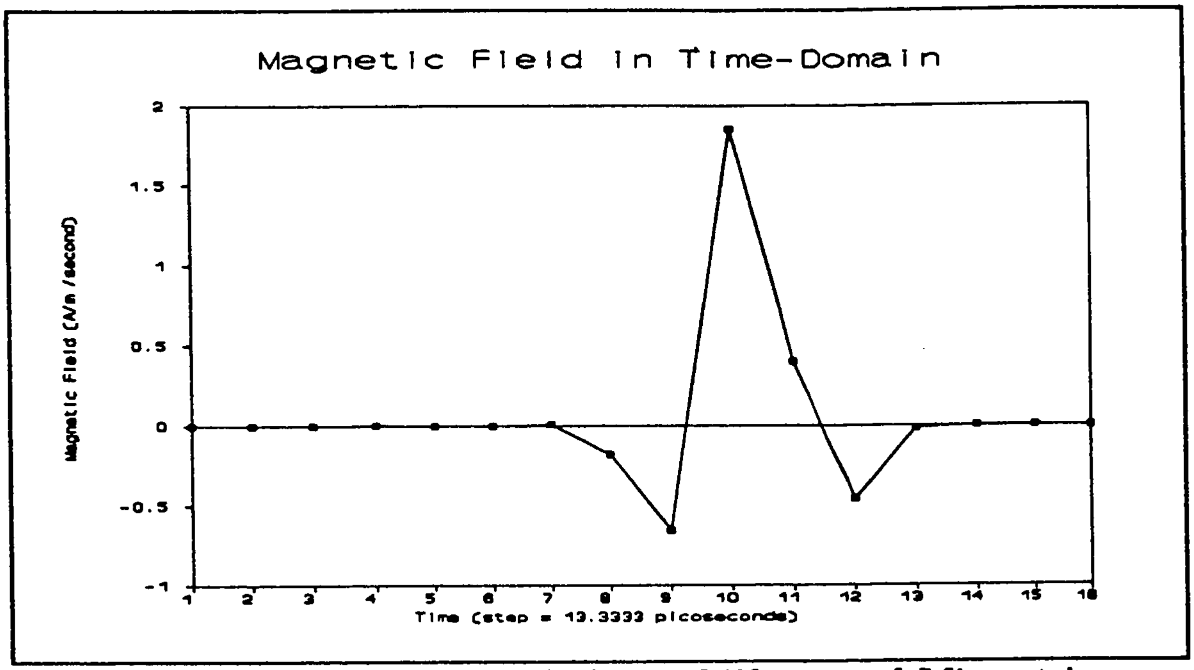

Figure 6 - Dielectric Slab with Ground Plane and Microstrip: Time-Domain Magnetic Field versus Time

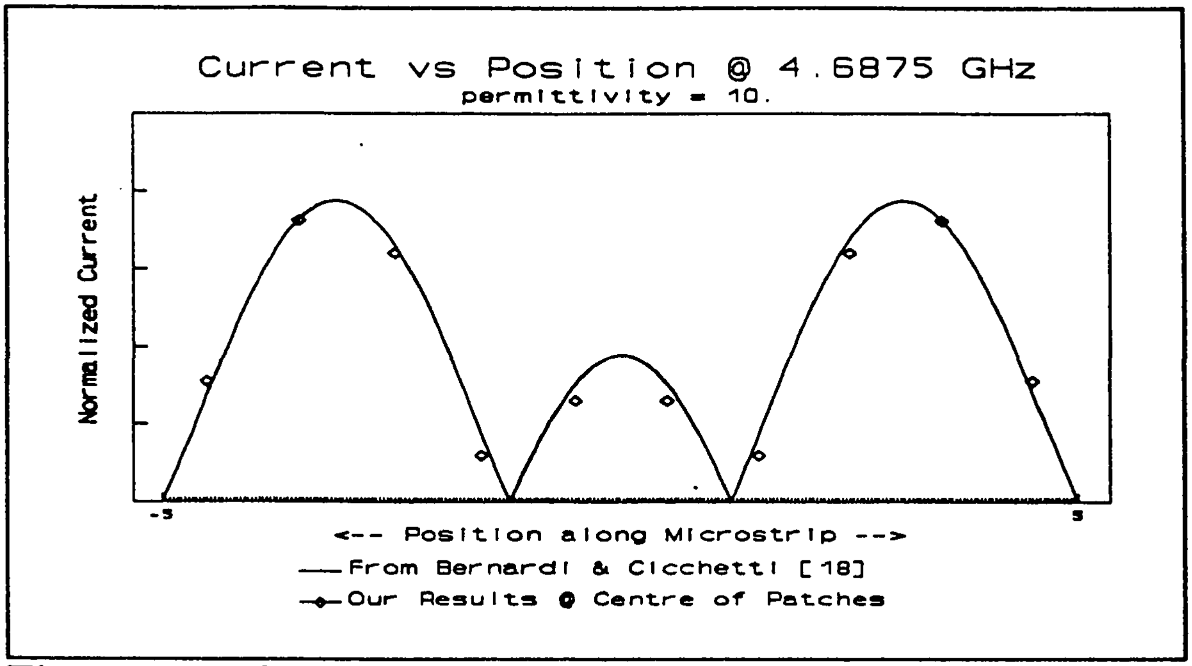

Figure 7 - Relative Current (µA) along the microstrip vs. position: a comparison of this work's results with Bernardi & Cicchetti's [18]

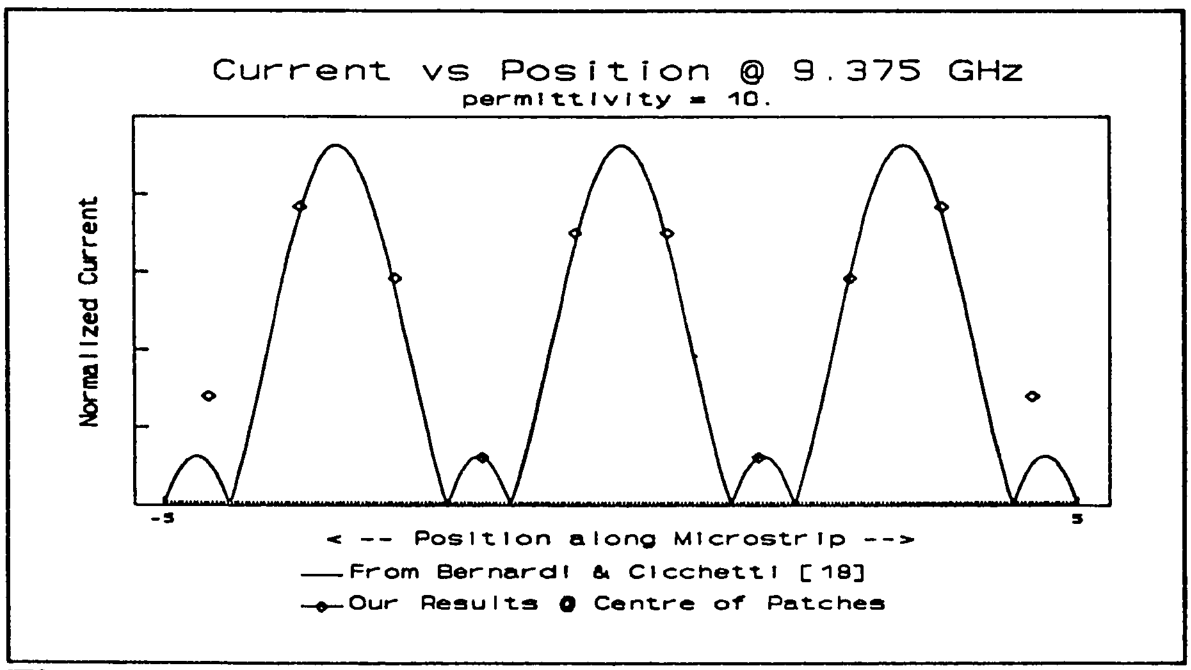

Figure 8 - Relative Current (µA) along microstrip vs. position: a comparison of this work's results with Bernardi & Cicchetti's [18]

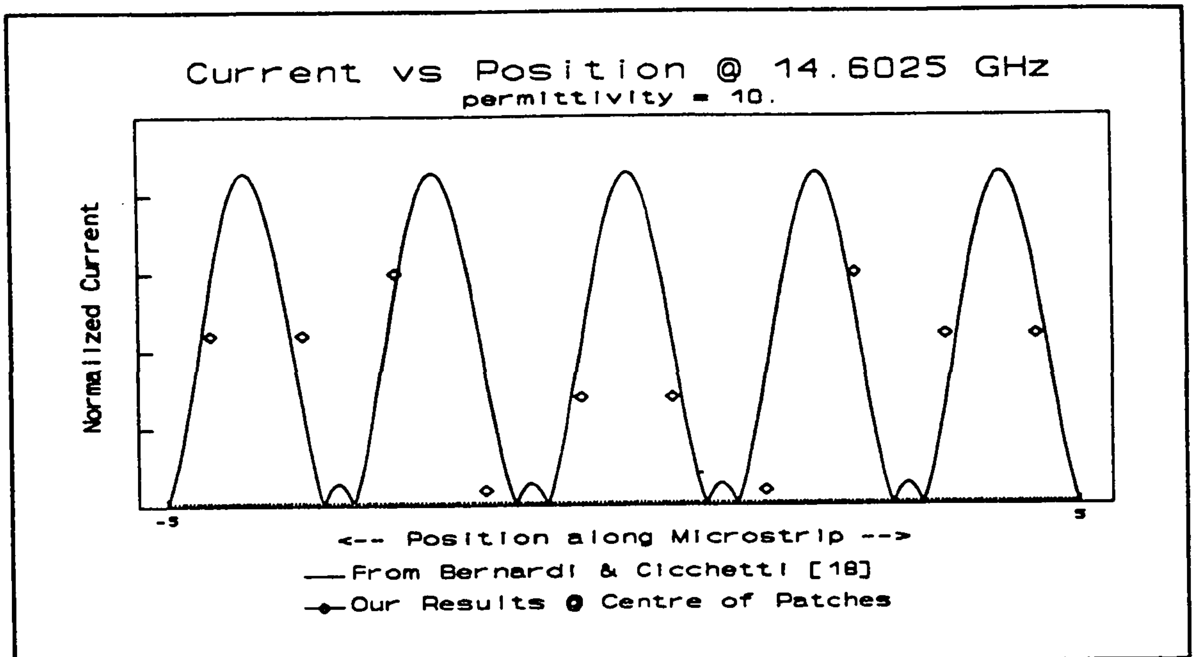

Figure 9 - Current (µA) along microstrip vs. position: a comparison of our results with Bernardi and Cicchetti's [18]

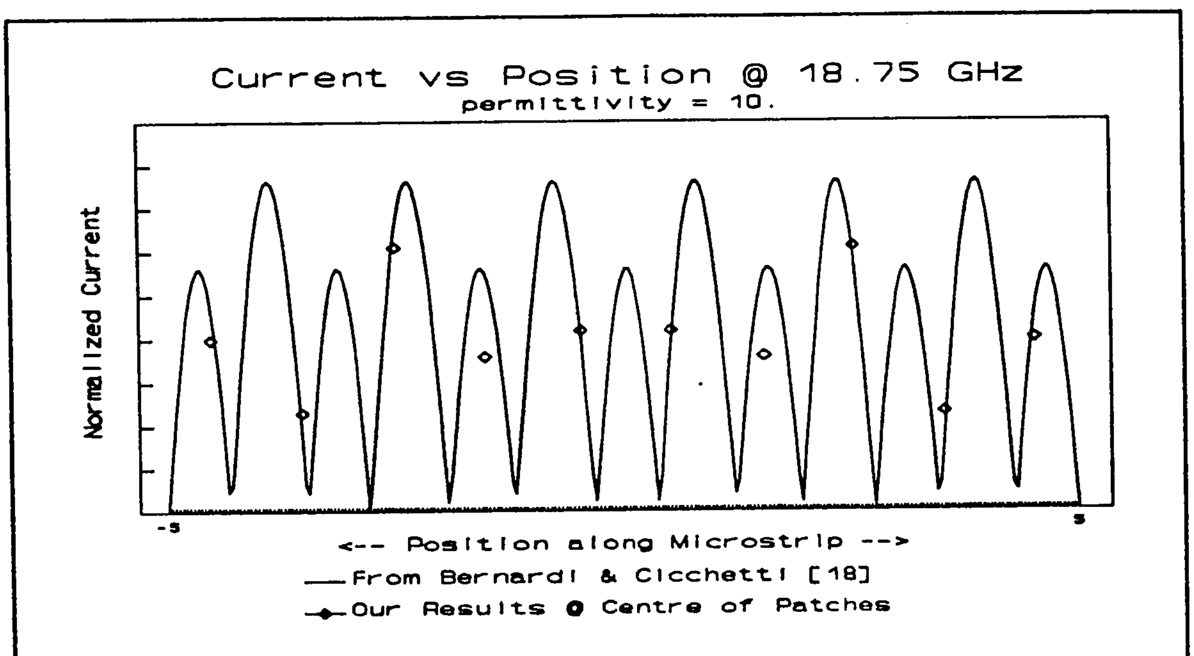

Figure 10 - Current (µA) along microstrip vs. position: a comparison of our results with Bernardi & Cicchetti's [18]

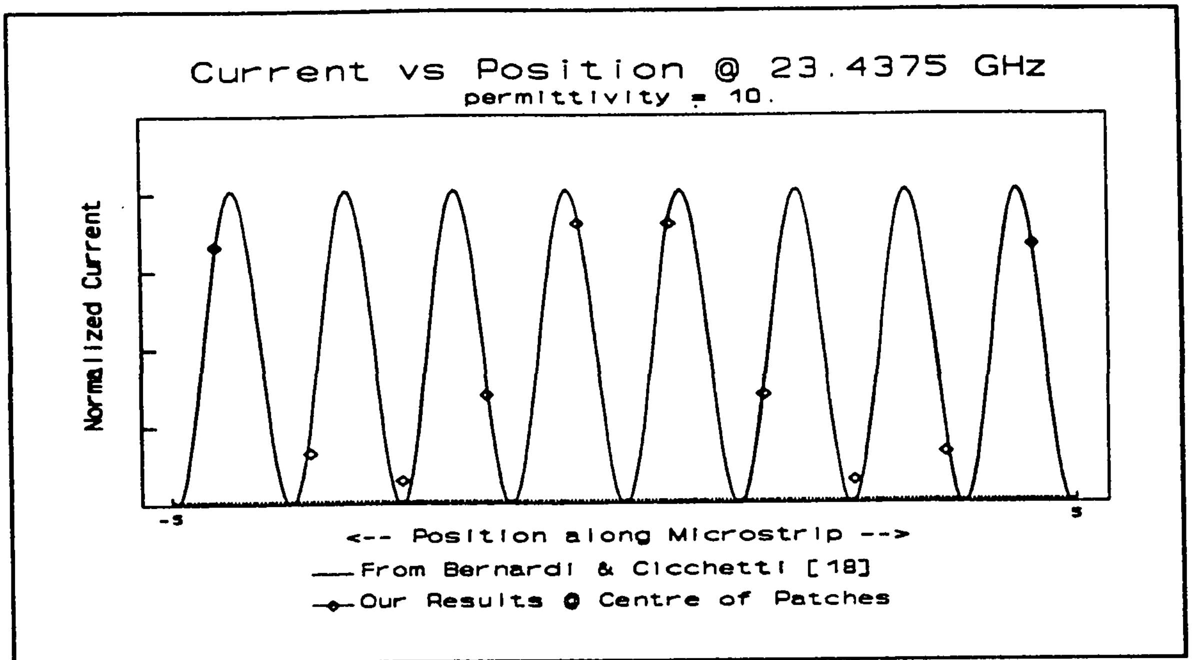

Figure 11 - Current (µA) along microstrip vs. position: a comparison of our results with Bernardi & Cicchetti's [18]

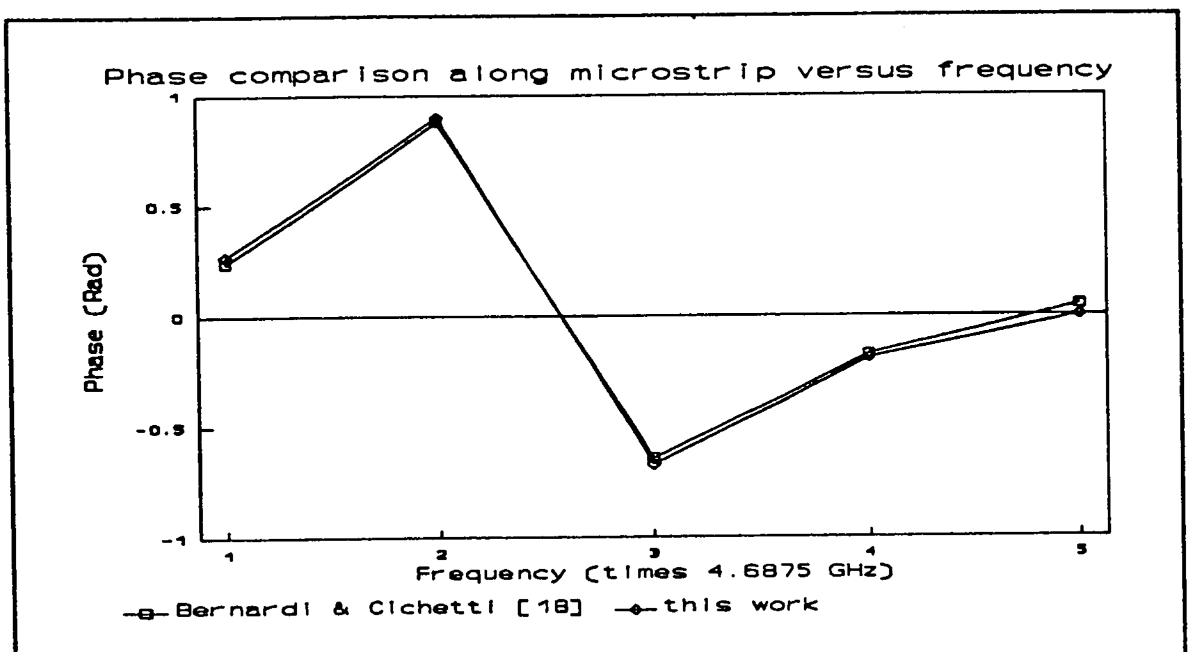

Figure 12 - Dielectric Slab with Ground Plane and Microstrip: Phase of Factor F versus Frequency

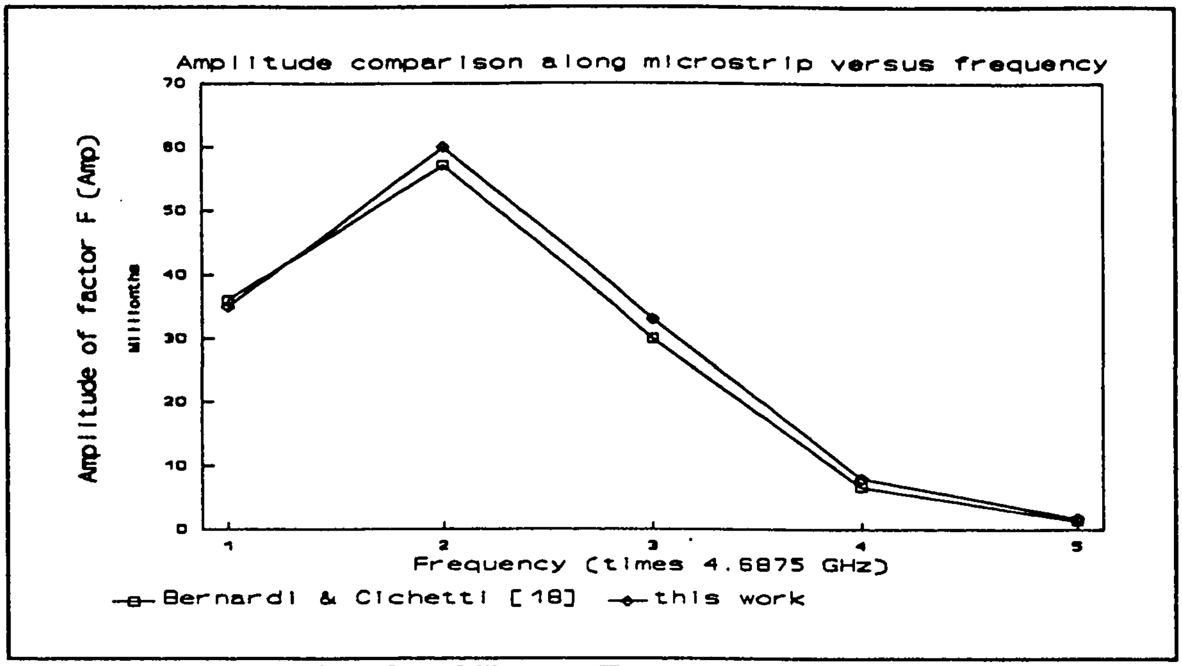

Figure 13 - Amplitude of Factor F versus Frequency for the microstrip

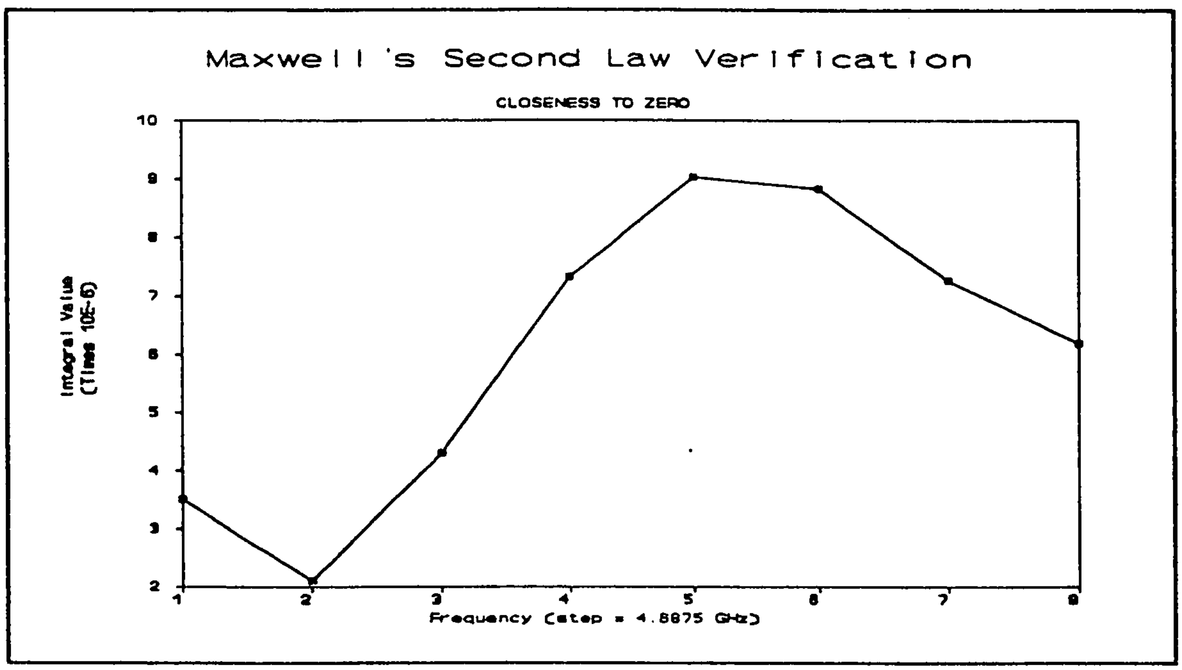

Figure 14 - Maxwell's Second Law Closeness to Zero

The patterns observed in comparing our results to Bernardi and Cicchetti's extrapolated data confirm the establishment of a stationary wave along the microstrip. The integral appearing in Maxwell's Second Law, calculated at eight frequency steps, departs from zero by a maximum of 9 10^{-6} A·m.

4.2 Results of the Conductive Box with Dielectric Plug

The box has a side of 30 cm and a thickness of the walls of 5 cm. The opening in the front wall is 10 cm wide and 20 cm long. The dielectric plug has permittivity $\varepsilon_r = 10.0$. The length of the opening is varied between 5 and 15 cm and the effect of that variation studied. The incident wave was normal to the opening of the box.

The graphs in Figures 15 and 16 show the behaviour of the electric and magnetic fields at the centre of the box in the time-domain. Figure 17 depicts the behaviour of the integral present in Maxwell's Second Law. The closeness to a null value reflects the fact that the calculated fields respect that law. This constitutes an accuracy check of our calculations.

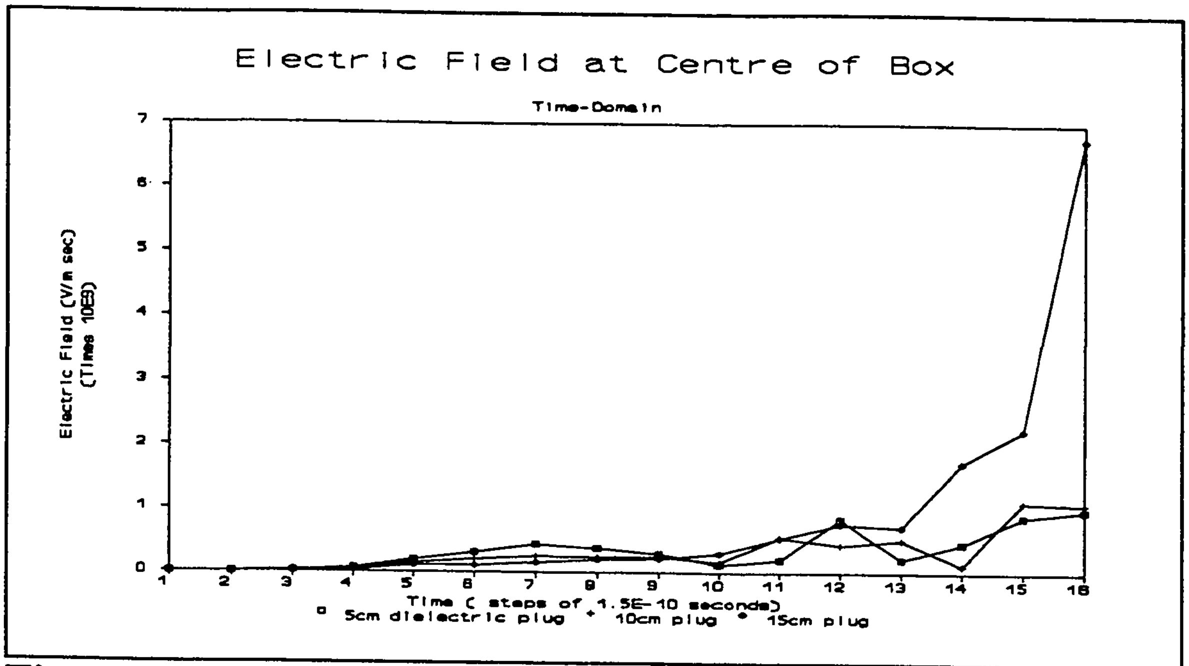

Figure 15 - Electric field in the Time-Domain at Centre of Box

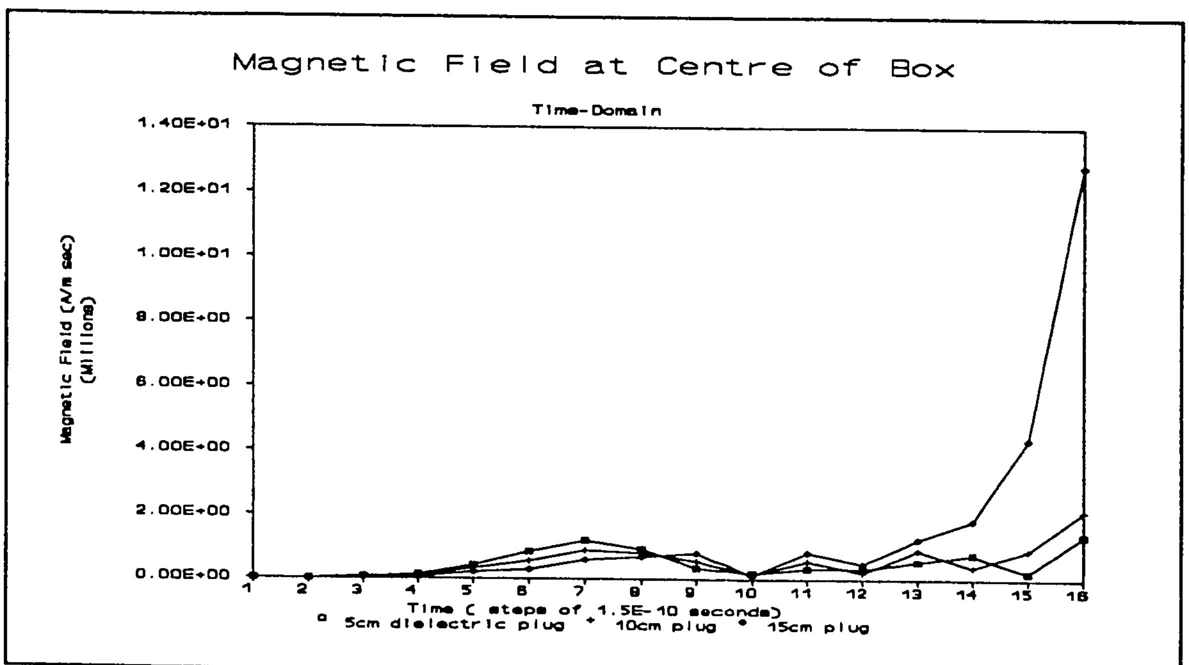

Figure 16 - Magnetic Field in the Time-Domain at Centre of Box

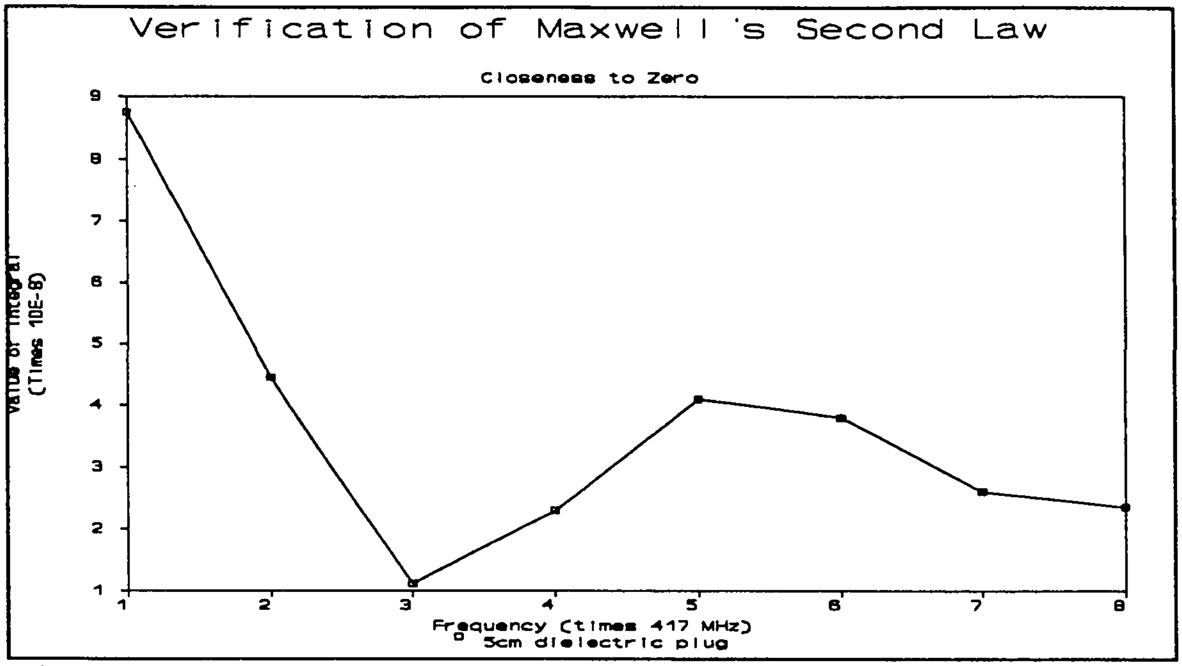

Figure 17 - Maxwell's Second Law Closeness to Zero

A study of the time behaviour of the electric and magnetic fields for several openings of the box, indicate that for the 5 or 10 centimetre plugs the results do not differ much. When the plug is 15 centimetres long however, the values of the fields increase significantly.

A possible explanation would be that some type of constructive interference effect occurs in that situation corresponding to a minimum opening size. Also, the possibility of numerical instability, should not be discounted. To estimate whether this unstable behavior was perpetrated past the cut-off threshold of 16 time steps, a 32 step run was attempted for this configuration. The aspect of the time dependence results of the fields do not change past the 16 step mark. A longer run, where the number of time steps would be in the hundreds might prove beneficial to determining whether we deal with an instability or not. Future work may undertake to perform such a long run. A powerful, more rapid computer is necessary.

The integral present in Maxwell's Second Law, like in the microstrip case, continues to remain close to zero, with a maximum departure from a null value of about $9\ 10^{-8}$ A·m.

5. CONCLUSIONS AND FUTURE WORK

The choice of the time-domain method used in this work requires some additional consideration. It has to be mentioned again that the frequency-domain approach is more suitable when the purpose of the calculation is to determine the response of the system to a few or a narrow range of frequencies. The broad frequency range however, is the main and most serious attraction of time-domain formulations. The closer the incident pulse shape is to a delta function, the closer is the response of the system to its "signature" over the largest frequency range.

The methods described herein and the results obtained are expected to provide a tangible means to model the behaviour of a mixed dielectric - conductor scatterer subjected to an incident plane wave. In the case of the microstrip, the current induced can be calculated and the expected interference effects estimated. In the case of the metallic box with thick walls, the fields at any location inside it could be estimated.

Such a box was originally conceived to serve as a testing environment for electronic equipment. The penetration of the outside field into the box is a measure of the expected interference upon the device or component undergoing testing. The equations developed so far are new and are expected to provide a solid basis for performing numerical time-domain predictions on even more complicated structures.

Future work may include the development of more general formulations that would describe the scattering of waves upon multi-dielectric bodies. Such integro-differential equations would take into account volume discontinuities, by introducing surface charges at the interface between the dielectrics. Such a development would greatly expand the number of structures which may be analyzed as possible scatterers.

APPENDIX A

Equations (37) and (38) are being multiplied by $\mathbf{n}^o$ and $\mathbf{n}^i$, the unit vectors normal on the outside and on the inside to the surface at the observation point $\mathbf{x}$. Like the source point normals $\mathbf{n}'^o$ and $\mathbf{n}'^i$, they are equal and of opposite sign:

$$\mathbf{n} = \mathbf{n}^o = -\,\mathbf{n}^i \tag{A.1}$$

$$\mathbf{n}' = \mathbf{n}'^o = -\,\mathbf{n}'^i \tag{A.2}$$

With the notations:

$$L_o = \frac{1}{R^2} + \frac{1}{Rc}\frac{\partial}{\partial t} \quad ; \quad L_i = \frac{1}{R^2} + \frac{1}{Rc_i}\frac{\partial}{\partial t} \tag{A.3}$$

where $\hat{\mathbf{R}}$ is the unit vector aiming from the source $\mathbf{x}'$ to the observation point $\mathbf{x}$, we obtain:

$$\mathbf{n}\times\mathbf{E}^o(x,t) = T\mathbf{n}\times\mathbf{E}^{inc}(x,t) - \frac{T}{4\pi}\mathbf{n}\times\{OEA_1 + OEA_2\} + \frac{T}{4\pi\varepsilon^o}\mathbf{n}\times\int_0^{\tau_o} dt_1 \oint_C \hat{\mathbf{R}} L_o[\mathbf{H}_1(x',\tau_o - t_1) - \mathbf{H}_2(x',\tau_o - t_1)]\cdot\mathbf{ds} \tag{A.4}$$

$$-\mathbf{n}\times\mathbf{E}^{i}(x,t)=-\frac{T}{4\pi}\mathbf{n}\times\{IEA_1+IEA_2\}+ \qquad \text{(A.5)}$$

$$\frac{T}{4\pi\varepsilon^{i}}\mathbf{n}\times\int_{0}^{\tau_i}dt_1\oint_{C}\hat{\mathbf{R}}L_i[\mathbf{H}_2(x',\tau_o-t_1)\cdot\mathbf{ds}]$$

where:

$$OEA_1=\int_{A_1}\left\{\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial}{\partial t}\mathbf{H}_1^{o}(x',\tau_o)-L_o[\mathbf{n}'\cdot\mathbf{E}_1^{o}(x',\tau_o)]\hat{\mathbf{R}}da\right\} \qquad \text{(A.6)}$$

$$OEA_2=\int_{A_2}\left\{\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial}{\partial t}\mathbf{H}_2^{o}(x',\tau_o)-L_o\left[\mathbf{n}'\times\mathbf{E}_2^{o}(x',\tau_o)\right]\times\hat{\mathbf{R}}-L_o\left[\mathbf{n}'\cdot\mathbf{E}_2^{o}(x',\tau_o)\right]\hat{\mathbf{R}}\right\}da \qquad \text{(A.7)}$$

$$IEA_2=\int_{A_2}\left\{\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial}{\partial t}\mathbf{H}_2^{i}(x',\tau_i)-L_i\left[\mathbf{n}'\times\mathbf{E}_2^{i}(x',\tau_i)\right]\times\hat{\mathbf{R}}-L_i\left[\mathbf{n}'\cdot\mathbf{E}_2^{i}(x',\tau_i)\right]\hat{\mathbf{R}}\right\}da \qquad \text{(A.8)}$$

$IEA_1 = 0$ (both **E** and **H** are null inside the perfect conductor) (A.9)

For the tangential components of the magnetic field we have ($\mathbf{E}_1\cdot ds = 0$):

$$\mathbf{n}\times\mathbf{H}^{o}(x,t)=T\mathbf{n}\times\mathbf{H}^{inc}(x,t)+\frac{T}{4\pi}\mathbf{n}\times\{OHA_1+OHA_2\} \tag{A.10}$$

$$-\mathbf{n}\times\mathbf{H}^{i}(x,t)=\frac{T}{4\pi}\mathbf{n}\times\{IHA_1+IHA_2\} \tag{A.11}$$

where:

$$OHA_1=\int_{A_1}\left\{L_o\left[\mathbf{n}'\times\mathbf{H}_1^{o}(x',\tau_o)\right]\times\hat{\mathbf{R}}\right\}da \tag{A.12}$$

$$OHA_2=\int_{A_2}\left\{\varepsilon_o\frac{1}{R}\mathbf{n}'\times\frac{\partial}{\partial t}\mathbf{E}_2^{o}(x',\tau_o)+L_o\left[\mathbf{n}'\times\mathbf{H}_2^{o}(x',\tau_o)\right]\times\hat{\mathbf{R}}+L_o\left[\mathbf{n}'\cdot\mathbf{H}_2^{o}(x',\tau_o)\right]\hat{\mathbf{R}}\right\}da \tag{A.13}$$

$$IHA_2=\int_{A_2}\left\{\varepsilon_i\frac{1}{R}\mathbf{n}'\times\frac{\partial}{\partial t}\mathbf{E}_2^{i}(x',\tau_i)+L_i\left[\mathbf{n}'\times\mathbf{H}_2^{i}(x',\tau_i)\right]\times\hat{\mathbf{R}}+L_i\left[\mathbf{n}'\cdot\mathbf{H}_2^{i}(x',\tau_i)\right]\hat{\mathbf{R}}\right\}da \tag{A.14}$$

IHA_1 = 0 (both **E** and **H** are null inside the perfect conductor) (A.15)

Equations (A.4) and (A.5) for the tangential components of the electric field on one hand and (A.10) and (A.11) for the magnetic field on the other hand are values of the respective fields just outside and inside of the scatterer's surface.

If the observation point is on the dielectric, we obtain by subtracting (A.5) from (A.4) and for T=2:

$$\mathbf{n}\times\mathbf{E}(t) = \mathbf{n}\times\mathbf{E}^{inc}(t)+\frac{1}{4\pi}\mathbf{n}\times\Big\langle\frac{1}{\varepsilon_o}\int_0^{\tau_o}dt_1\oint_C\hat{\mathbf{R}}L_o[\mathbf{H}_1(\tau_o-t_1)-\mathbf{H}_2(\tau_o-t_1)]\cdot\mathbf{ds}'-$$
$$-\frac{1}{\varepsilon_i}\int_0^{\tau_i}dt_1\oint_C\hat{\mathbf{R}}L_i\mathbf{H}_2(\tau_i-t_1)\cdot\mathbf{ds}'-\int_{A_1}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}\right)da'+ \qquad (A.16)$$
$$+\int_{A_2}\Big(\mathbf{n}'\{L_o\mathbf{E}(\tau_o)-L_i\mathbf{E}(\tau_i)\}\hat{\mathbf{R}}+\big[\mathbf{n}'\times[L_o\mathbf{E}(\tau_o)-L_i\mathbf{E}(\tau_i)]\big]\times\hat{\mathbf{R}}-$$
$$-\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial[\mathbf{H}(\tau_o)-\mathbf{H}(\tau_i)]}{\partial t}\Big)da'\Big\rangle$$

Similarly, by subtracting (A.11) from (A.10), we obtain:

$$\mathbf{n}\times\mathbf{H}(t) = \mathbf{n}\times\mathbf{H}^{inc}(t)+\frac{1}{4\pi}\mathbf{n}\times\Big\langle\int_{A_1}\big[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)\big]\times\hat{\mathbf{R}}\,da' +$$
$$\int_{A_2}\left(\frac{1}{R}\mathbf{n}'\times\left[\varepsilon_o\frac{\partial\mathbf{E}(\tau_o)}{\partial t}-\varepsilon_i\frac{\partial\mathbf{E}(\tau_i)}{\partial t}\right]+\mathbf{n}'\cdot[L_o\mathbf{H}(\tau_o)-L_i\mathbf{H}(\tau_i)]\hat{\mathbf{R}}+\right. \qquad (A.17)$$
$$\left.+\big[\mathbf{n}'\times[L_o\mathbf{H}(\tau_o)-L_i\mathbf{H}(\tau_i)]\big]\times\hat{\mathbf{R}}\right)da'\Big\rangle$$

We notice from (A.16) and (A.17) above that the electric and magnetic fields respectively are expressed at time t as functions of the fields over the surface of the scatterer at times prior to time t.

If the observation point is on the conductor, the derivation is different from the one described above for the dielectric side of the scatterer. More specifically, the tangential component of the electric field is zero and we cannot express the field at time t as a function of all the other fields at times prior to that. The left hand side of equation (A.16) is null on the conductor. It is therefore necessary to use the normal component of the field expressed by equation (37) and to follow a similar procedure to the one described.

$$\begin{aligned}\mathbf{n}\cdot\mathbf{E}(t) = {} & 2\mathbf{n}\cdot\mathbf{E}^{inc}(t)+\frac{1}{2\pi}\mathbf{n}\cdot \\ & \left\langle\frac{1}{\varepsilon_o}\int_0^{\tau_o}dt_1\oint_C\hat{\mathbf{R}}L_o[\mathbf{H}_1(\tau_o-t_1)-\mathbf{H}_2(\tau_o-t_1)]\cdot\mathbf{ds}'-\right. \\ & -\int_{A_1}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}\right)da'- \\ & \left.-\int_{A_2}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}-[\mathbf{n}'\times L_o\mathbf{E}(\tau_o)]\times\hat{\mathbf{R}}\right)da'\right\rangle\end{aligned} \tag{A.18}$$

For the magnetic field on the conductive portion of the scatterer, we obtain from (A.10) and because the left side of (A.11) is zero:

$$\mathbf{n}\times\mathbf{H}(t) = 2\mathbf{n}\times\mathbf{H}^{inc}(t)+\frac{1}{2\pi}\mathbf{n}\times\left\langle\int_{A_1}\left[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)\right]\times\hat{\mathbf{R}}\,da'+\right.$$
$$\left.+\int_{A_2}\left(\varepsilon_o\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{E}(\tau_o)}{\partial t}+\left[\mathbf{n}'\cdot L_o\mathbf{H}(\tau_o)\right]\hat{\mathbf{R}}+\left[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)\right]\times\hat{\mathbf{R}}\right)da'\right\rangle \tag{A.19}$$

APPENDIX B

Tangential component of the electrical field on the dielectric

$$\mathbf{n}\times\mathbf{E}(t) = \mathbf{n}\times\mathbf{E}^{inc}(t)+\frac{1}{4\pi}\mathbf{n}\times\Big\langle\frac{1}{\varepsilon_o}\int_0^{\tau_o}dt_1\oint_C\hat{\mathbf{R}}L_o[\mathbf{H}_1(\tau_o-t_1)-\mathbf{H}_2(\tau_o-t_1)]\cdot\mathbf{ds}'-$$
$$-\frac{1}{\varepsilon_i}\int_0^{\tau_i}dt_1\oint_C\hat{\mathbf{R}}L_i\mathbf{H}_2(\tau_i-t_1)\cdot\mathbf{ds}'-\iint_{A_1}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}\right)da'+ \tag{B.1}$$
$$+\iint_{A_2}\Big(\mathbf{n}'\{L_o\mathbf{E}(\tau_o)-L_i\mathbf{E}(\tau_i)\}\hat{\mathbf{R}}+[\mathbf{n}'\times[L_o\mathbf{E}(\tau_o)-L_i\mathbf{E}(\tau_i)]]\times\hat{\mathbf{R}}-$$
$$-\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial[\mathbf{H}(\tau_o)-\mathbf{H}(\tau_i)]}{\partial t}\Big)da'\Big\rangle$$

Normal component of the electrical field on the dielectric

$$\mathbf{n}\cdot\mathbf{E}(t) = \mathbf{n}\cdot\mathbf{E}^{inc}(t)+\frac{1}{4\pi}\mathbf{n}\times\Big\langle\frac{1}{\varepsilon_o}\Big\{\int_0^{\tau_o}dt_1\oint_C\hat{\mathbf{R}}L_o[\mathbf{H}_1(\tau_o-t_1)-\mathbf{H}_2(\tau_o-t_1)]\cdot\mathbf{ds}'-$$
$$-\int_0^{\tau_i}dt_1\oint_C\hat{\mathbf{R}}L_i\mathbf{H}_2(\tau_i-t_1)\cdot\mathbf{ds}'\Big\}-\iint_{A_1}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}\right)da'+ \tag{B.2}$$
$$+\iint_{A_2}\Big(\mathbf{n}'\left[L_o\mathbf{E}(\tau_o)-\frac{\varepsilon_i}{\varepsilon_o}L_i\mathbf{E}(\tau_i)\right]\hat{\mathbf{R}}+\left[\mathbf{n}'\times\left[L_o\mathbf{E}(\tau_o)-\frac{\varepsilon_i}{\varepsilon_o}L_i\mathbf{E}(\tau_i)\right]\right]\times\hat{\mathbf{R}}-$$
$$-\mu\frac{1}{R}\mathbf{n}'\times\left[\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-\frac{\varepsilon_i}{\varepsilon_o}\frac{\partial\mathbf{H}(\tau_i)}{\partial t}\right]\Big)da'\Big\rangle$$

Tangential component of the magnetic field on the dielectric

$$\mathbf{n}\times\mathbf{H}(t) = \mathbf{n}\times\mathbf{H}^{inc}(t)+\frac{1}{4\pi}\mathbf{n}\times\left\langle \int_{A_1}\left[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)\right]\times\hat{\mathbf{R}}\,da' + \right.$$
$$\int_{A_1}\left(\frac{1}{R}\mathbf{n}'\times\left[\varepsilon_o\frac{\partial\mathbf{E}(\tau_o)}{\partial t}-\varepsilon_i\frac{\partial\mathbf{E}(\tau_i)}{\partial t}\right]+\mathbf{n}'\cdot\left[L_o\mathbf{H}(\tau_o)-L_i\mathbf{H}(\tau_i)\right]\hat{\mathbf{R}}+ \right. \tag{B.4}$$
$$\left.\left.+\left[\mathbf{n}'\times\left[L_o\mathbf{H}(\tau_o)-L_i\mathbf{H}(\tau_i)\right]\right]\times\hat{\mathbf{R}}\right)da'\right\rangle$$

Normal component of the magnetic field on the dielectric

$$\mathbf{n}\cdot\mathbf{H}(t) = \mathbf{n}\cdot\mathbf{H}^{inc}(t)+\frac{1}{4\pi}\mathbf{n}\cdot\left\langle \int_{A_1}\left[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)\right]\times\hat{\mathbf{R}}\,da' + \right.$$
$$\int_{A_1}\left(\frac{1}{R}\mathbf{n}'\times\left[\varepsilon_o\frac{\partial\mathbf{E}(\tau_o)}{\partial t}-\varepsilon_i\frac{\partial\mathbf{E}(\tau_i)}{\partial t}\right]+\mathbf{n}'\cdot\left[L_o\mathbf{H}(\tau_o)-L_i\mathbf{H}(\tau_i)\right]\hat{\mathbf{R}}+ \right. \tag{B.4}$$
$$\left.\left.+\left[\mathbf{n}'\times\left[L_o\mathbf{H}(\tau_o)-L_i\mathbf{H}(\tau_i)\right]\right]\times\hat{\mathbf{R}}\right)da'\right\rangle$$

Normal component of the electrical field on the conductor

$$\begin{aligned}\mathbf{n}\cdot\mathbf{E}(t) &= 2\mathbf{n}\cdot\mathbf{E}^{inc}(t)+\frac{1}{2\pi}\mathbf{n}\cdot \\ &\left\langle \frac{1}{\varepsilon_o}\int_0^{\tau_o}dt_1\oint_C \hat{\mathbf{R}}L_o[\mathbf{H}_1(\tau_o-t_1)-\mathbf{H}_2(\tau_o-t_1)]\cdot d\mathbf{s}'- \right. \\ &-\int_{A_1}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}\right)da'- \\ &\left. -\int_{A_2}\left(\mu\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{H}(\tau_o)}{\partial t}-[\mathbf{n}'\cdot L_o\mathbf{E}(\tau_o)]\hat{\mathbf{R}}-[\mathbf{n}'\times L_o\mathbf{E}(\tau_o)]\times\hat{\mathbf{R}}\right)da'\right\rangle\end{aligned} \tag{B.5}$$

Tangential component of the magnetic field on the conductor

$$\begin{aligned}\mathbf{n}\times\mathbf{H}(t) &= 2\mathbf{n}\times\mathbf{H}^{inc}(t)+\frac{1}{2\pi}\mathbf{n}\times\left\langle\int_{A_1}[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)]\times\hat{\mathbf{R}}\,da'+\right. \\ &\left.+\int_{A_2}\left(\varepsilon_o\frac{1}{R}\mathbf{n}'\times\frac{\partial\mathbf{E}(\tau_o)}{\partial t}+[\mathbf{n}'\cdot L_o\mathbf{H}(\tau_o)]\hat{\mathbf{R}}+[\mathbf{n}'\times L_o\mathbf{H}(\tau_o)]\times\hat{\mathbf{R}}\right)da'\right\rangle\end{aligned} \tag{B.6}$$

APPENDIX C

```
C                    FFT Subroutine from Brigham
C
      subroutine fft( xreal, ximag, n, nu)
      dimension xreal(n), ximag(n)
      PI = 3.141592653589
      n2 = n/2
      nu1 = nu-1
      k=0
      do 100 l=1,nu
102   do 101 i=1,n2
      p=ibitr(k/2**nu1,nu)
      arg=2*PI*p/float(n)
      c=cos(arg)
      s=sin(arg)
      k1=k+1
      k1n2 =  k1 + n2
      treal = xreal(k1n2)*c + ximag(k1n2)*s
      timag = ximag(k1n2)*c - xreal(k1n2)*s
      xreal(k1n2) = xreal(k1) - treal
      ximag(k1n2) = ximag(k1) - timag
      xreal(k1) = xreal(k1) + treal
```

```
      ximag(k1) = ximag(k1) + timag
101   k = k +1
      k = k + n2
      if(k.lt.n) go to 102
      k=0
      nu1=nu1-1
100   n2=n2/2
      do 103 k=1,n
      i=ibitr(k-1,nu)+1
      if(i.le.k) go to 103
      treal= xreal(k)
      timag=ximag(k)
      xreal(k)=xreal(i)
      ximag(k)=ximag(i)
      xreal(i)=treal
      ximag(i)=timag
103   continue
      return
      end
C
C
      function ibitr(j,nu)
```

```
      j1 = j
      ibitr = 0
      do 200 i = 1,nu
      j2 = j1/2
      ibitr = ibitr*2 + (j1 - 2*j2)
200   j1 = j2
      return
      end
```

APPENDIX D

```
C                       Program for Box
C
C
C       Declarations
C
      INTEGER*4 NTNT, MOMO
      DIMENSION X(308,3,2), AN(308,3,2)
      DIMENSION H1T(308,3,16), H2T(308,3,16), H2N(308,16)
      DIMENSION E2T(308,3,16), E1N(308,16), E2N(308,16)

      DIMENSION S1(18,3)
      DIMENSION ESUM(16), HSUM(16)
       dimension efinal(3,16), hfinal(3,16)
C
      COMMON/COORD/X
      COMMON/NORM/AN
      COMMON/CURVES/S1
       COMMON/TFIELD/E2T, H2T, H1T
      COMMON/NFIELD/E1N, E2N, H2N
COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      COMMON/CHECKS/ESUM, HSUM
      common/final/efinal, hfinal
C

         CONSTANTS

      DELTAR = 5.E-2
      DELTAT = 1.5E-10
      EPSILO = 8.854E-12
      EPSILI = EPSILO * 10.0
      CO = 3.E8
      CI = CO * SQRT ( EPSILO/EPSILI)
      AMU = 4. * 3.1415E-7
      D = 3.E9
C
C
         DESCRIPTION OF THE SCATTERERS

         X'S ARE CENTRES OF PATCHES
         AN'S ARE NORMALS
```

```
      do 101 i = 1, 6
      do 101 k = 1, 24
      l = 6 * (k-1) + i
            x(l,1,1)      = float(i) - 3.5
            an(l,1,1)     = 0.0
      if    (k.le.6)            then
            x(l,2,1)      = float(k) - 3.5
            x(l,3,1)      = 3.0
            an(l,2,1)     = 0.0
            an(l,3,1)     = 1.0
      elseif      (k.ge.7.and.k.le.12)    then
            x(l,2,1)      = 3.0
            x(l,3,1)      = 9.5 - float(k)
            an(l,2,1)     = 1.0
            an(l,3,1)     = 0.0
      elseif      (k.ge.13.and.k.le.18)   then
            x(l,2,1)      = 15.5 - float(k)
            x(l,3,1)      = -3.0
            an(l,2,1)     = 0.0
            an(l,3,1)     = -1.0
      else
            x(l,2,1)      = -3.0
            x(l,3,1)      = float(k) - 21.5
            an(l,2,1)     = -1.0
            an(l,3,1)     = 0.0
      endif
101   continue
C
      do 102 i = 1,6
      do 102 k = 1,6
      l = 144 + 6*(k-1) + i
C
            x(l,1,1)      = -3.0
            x(l,2,1)      = float(k) - 3.5
            x(l,3,1)      = float(i) - 3.5
C
            an(l,1,1)     = -1.0
            an(l,2,1)     = 0.0
            an(l,3,1)     = 0.0
C
102   continue
C
      do 103 i = 1, 6
```

```
        do 103 k = 1, 2
        l1 = 180 + 6*(k-1) + i
        l2 = 196 + 6*(k-1) + i
                x(l1,1,1)    = 3.0
                x(l2,1,1)    = 3.0

                x(l1,2,1)    = float(k) - 3.5
                x(l2,2,1)    = float(k) + 0.5
                x(l1,3,1)    = float(i) - 3.5
                x(l2,3,1)    = float(i) - 3.5
C
                an(l1,1,1)  = 1.0
                an(l2,1,1)  = 1.0
                an(l1,2,1)  = 0.0
                an(l2,2,1)  = 0.0
                an(l1,3,1)  = 0.0
                an(l2,3,1)  = 0.0
C
103     continue
C
        do 104 i = 193, 194
        l = i + 2
                x(i,1,1) = 3.0
                x(l,1,1) = 3.0
                x(i,2,1) = -0.5
                x(l,2,1) = 0.5
        if(i.eq.193) then
                x(i,3,1) = -2.5
                x(l,3,1) = -2.5
        else
                x(i,3,1) = 2.5
                x(l,3,1) = 2.5
        endif
C
                an(i,1,1) = 1.0
                an(l,1,1) = 1.0
                an(i,2,1) = 0.0
                an(l,2,1) = 0.0
                an(i,3,1) = 0.0
                an(l,3,1) = 0.0
C
104     continue
C
```

```
C
      do 105 i = 1, 4
      do 105 k = 1, 16
      l = 208 + 4 * (k-1) + i
C
            x(l,1,1)     = float(i) - 2.5
            an(l,1,1)    = 0.0
      if    (k.le.4)           then
            x(l,2,1)     = float(k) - 2.5
            x(l,3,1)     = 2.0
            an(l,2,1)    = 0.0
            an(l,3,1)    = -1.0
      elseif      (k.ge.5.and.k.le.8)      then
            x(l,2,1)     = 2.0
            x(l,3,1)     = 6.5 - float(k)
            an(l,2,1)    = -1.0
            an(l,3,1)    = 0.0
      elseif      (k.ge.9.and.k.le.12)     then
            x(l,2,1)     = 10.5 - float(k)
            x(l,3,1)     = -2.0
            an(l,2,1)    = 0.0
            an(l,3,1)    = 1.0
      else
            x(l,2,1)     = -2.0
            x(l,3,1)     = float(k) - 14.5
            an(l,2,1)    = 1.0
            an(l,3,1)    = 0.0
      endif
105   continue
C
      do 106 i = 1, 4
      do 106 k = 1, 4
      l = 272 + 4 *(k-1) + i
C
            x(l,1,1)     = -2.0
            x(l,2,1)     = float(k) - 2.5
            x(l,3,1)     = float(i) - 2.5
C
            an(l,1,1)    = 1.0
            an(l,2,1)    = 0.0
            an(l,3,1)    = 0.0
C
106   continue
```

```
C
      do 107 i = 1, 4
      l1 = 288 + i
      l2 = l1 + 4
         x(l1,1,1) = 2.0
         x(l2,1,1) = 2.0
         x(l1,2,1) = -1.5
         x(l2,2,1) = 1.5
         x(l1,3,1) = float(i) - 2.5
         x(l2,3,1) = float(i) - 2.5
C
         an(l1,1,1) = -1.0
         an(l1,2,1) = 0.0
         an(l1,3,1) = 0.0
C
         an(l2,1,1) = -1.0
         an(l2,2,1) = 0.0
         an(l2,3,1) = 0.0

 107  continue
C
      do 108 i = 1, 7, 2
      l = 298 + i
      k = l  + 1
         x(l,1,1) = 2.5
         x(k,1,1) = 2.5
         x(l,2,1) = -1.0
         x(k,2,1) =  1.0
         x(l,3,1) = (float(i)-4.)/2.
         x(k,3,1) = (float(i)-4.)/2.
C
         an(l,1,1) = 0.0
         an(k,1,1) = 0.0
         an(l,2,1) = 1.0
         an(k,2,1) = -1.0
         an(l,3,1) = 0.0
         an(k,3,1) = 0.0
C
 108  continue
C
      do 109 i = 1, 2
      l = 296 + i
      k = l + 10
```

```
C
           x(l,1,1) = 2.5
           x(k,1,1) = 2.5
           x(l,2,1) = float(i) - 1.5
           x(k,2,1) = float(i) - 1.5
           x(l,3,1) = -2.0
           x(k,3,1) =  2.0
C
           an(l,1,1) = 0.0
           an(k,1,1) = 0.0
           an(l,2,1) = 0.0
           an(k,2,1) = 0.0
           an(l,3,1) =  1.0
           an(k,3,1) = -1.0
C
109   continue
C
C
      nah = 3
C
      do 110 i = 1, 2
      do 110 k = 1, nah
      l1 = 4*(k-1) + i
      l2 = l1 + 2
      l3 = 12 + i
C
           x(l1,1,2) = 3.0
           x(l2,1,2) = 2.0
           x(l1,2,2) = float(i) - 1.5
           x(l2,2,2) = float(i) - 1.5
           x(l1,3,2) = 2.5 - float(k)
           x(l2,3,2) = 2.5 - float(k)

C
           an(l1,1,2) = 1.0
           an(l2,1,2) = -1.0
           an(l1,2,2) = 0.0
           an(l2,2,2) = 0.0
           an(l1,3,2) = 0.0
           an(l2,3,2) = 0.0
C
           x(l3,1,2) = 2.5
           x(l3,2,2) = float(i) - 1.5
```

```
            x(l3,3,2) = 2.0 - float(k)
C
            an(l3,1,2) = 0.0
            an(l3,2,2) = 0.0
            an(l3,3,2) = -1.0

110   continue

            VECTORS S ON CURVE C BETWEEN DIELECTRIC
                AND CONDUCTOR

C
      do 112 i = 1, 2
      l = i + 2
            s1(i,1) = 0.0
            s1(l,1) = 0.0
            s1(i,2) = 1.0
            s1(l,2) = -1.0
            s1(i,3) = 0.0
            s1(l,3) = 0.0

      do 112 k = 1, nah
      l1 = 4*k + i
      l2 = l1 + 2
            s1(l1,2) = 0.0
            s1(l2,2) = 0.0
            s1(l1,1) = 0.0
            s1(l2,1) = 0.0
      if(i.eq.1) then
            s1(l1,3) = 1.0
            s1(l2,3) = -1.0
      else

            s1(l1,3) = -1.0
            s1(l2,3) = 1.0
      endif
112   continue
C
      do 113 i = 17, 18
            s1(i,2) = 0.0
            s1(i,3) = 0.0
            s1(i,1) = -1.0
```

```
      if(i.eq.18) s1(i,1) = 1.0
113   continue
C
      do 114 l3 = 15, 300
             x(l3,1,2) = 0.0
             x(l3,2,2) = 0.0
             x(l3,3,2) = 0.0

C
             an(l3,1,2) = 0.0
             an(l3,2,2) = 0.0
             an(l3,3,2) = 0.0
114   continue
C
      do 115 l = 1 ,7, 2
      l1 = 300 + l
      l2 = l1 + 1
      x(l1,1,2) = 2.
      x(l2,1,2) = 2.
      x(l1,2,2) = -0.5
      x(l2,2,2) =  0.5
      x(l1,3,2) = (float(l) + 1.)/2. - 2.5
      x(l2,3,2) = x(l1,3,2)
      an(l1,1,2) = -1.
      an(l2,1,2) = -1.
      an(l1,2,2) = 0.0
      an(l2,2,2) = 0.0
      an(l1,3,2) = 0.0
      an(l2,3,2) = 0.0
115   continue

C

C
C           MAIN PROGRAM

C
      DO 4444 NTNT = 1, 16
C

      HSUM(NTNT) = 0.0
      WRITE (*,*) ' D TIME =', NTNT
      DO 1111 MOMO = 1, 14
```

```
C
      CALL PART1        (MOMO, NTNT)
      CALL PART2        (MOMO, NTNT)
C
            MAXWELL'S SECOND LAW

      HSUM(NTNT) = HSUM(NTNT) + H2N(MOMO,NTNT) *
DELTAR ** 2

C
1111   CONTINUE
C
      WRITE (*,*) ' C TIME =', NTNT
      DO 2222 MOMO = 1, 300
C
      CALL PART3        (MOMO, NTNT)
      CALL PART4        (MOMO, NTNT)
C

C
2222   CONTINUE
C
      OPEN(UNIT = 10, FILE = 'ESUM',ACCESS= 'DIRECT',
RECL = 20
    1        ,STATUS = 'UNKNOWN', FORM= 'FORMATTED')
      WRITE(UNIT = 10, FMT=300, REC=NTNT) ESUM(NTNT)
      CLOSE(UNIT = 10, STATUS = 'KEEP')
C
      OPEN(UNIT = 10, FILE = 'HSUM',ACCESS= 'DIRECT',
RECL = 20
    1        ,STATUS = 'UNKNOWN', FORM= 'FORMATTED')
      WRITE(UNIT = 10, FMT=300, REC=NTNT) HSUM(NTNT)
      CLOSE(UNIT = 10, STATUS = 'KEEP')


            CALCULATE FIELDS AT CENTRE OF BOX

C
      CALL EFIN(300,NTNT)
      CALL HFIN(300,NTNT)
C
      OPEN(UNIT = 10, FILE = 'ELECTRIC.FIN',ACCESS=
'DIRECT',
```

```
    1 RECL = 60,STATUS = 'UNKNOWN', FORM=
'FORMATTED')
      WRITE(UNIT = 10, FMT=500, REC=NTNT)
(EFINAL(J,NTNT),J=1,3)
      CLOSE(UNIT = 10, STATUS = 'KEEP')
C
      OPEN(UNIT = 10, FILE = 'MAGNETIC.FIN',ACCESS=
'DIRECT',
    1 RECL = 60,STATUS = 'UNKNOWN', FORM=
'FORMATTED')
      WRITE(UNIT = 10, FMT=500, REC=NTNT)
(HFINAL(J,NTNT),J=1,3)
      CLOSE(UNIT = 10, STATUS = 'KEEP')

C
4444   CONTINUE
C
300    FORMAT(1X,   E10.4)
500   FORMAT(1X, 3(E10.4,5X))
C
      STOP
      END
C
C
C
      SUBROUTINE EINCID (M, N, l, B)

             GAUSSIAN PULSE FOR ELECTRIC FIELD

      INTEGER*4 M, N, l
      REAL*4 B
      DIMENSION B(3), X(308,3,2)

 COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      COMMON/COORD/X
      B(1) = 0.0
      B(2) = 0.0
      B(3) = EXP(-D**2* (DELTAT*FLOAT(N-8) + DELTAR *
x(M,1,l)/CO)
     1          ** 2) * D/SQRT(3.1415)
      RETURN
      END
C
```

```
      SUBROUTINE EderIN (M, l, B)

            DERIVATIVE OF GAUSSIAN PULSE AT T = 0

      INTEGER*4 M, l
      REAL*4 B
      DIMENSION B(3), X(308,3,2)

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      COMMON/COORD/X
      call eincid(m,1,l,b)
      B(3) = -2.*d**2*b(3)* (-7.*DELTAT + DELTAR *
x(M,1,L)/CO)
      RETURN
      END

      SUBROUTINE LOEV (l, k, C, FIN, FOUT, MO, MS, N)

            CALCULATE L FOR ELECTRIC FIELD

      INTEGER*4 MO, MS, N, l, k
      REAL*4 FIN, FOUT, CON1, CON2, DIST
      DIMENSION FIN(308,3,21), FOUT(3), temp(3), X(308,3,2)

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      COMMON/COORD/X
      DIST = DISTAN(l, k, MO, MS)
      IF (N.LT.1) THEN
            DO 55 J = 1, 3
55           FOUT(J) = 0.0
            RETURN
C
      ELSEIF (N.EQ.1) THEN
            CALL EderIN (MS, k, FOUT)
            call cross (k, fout, temp, ms)
            CON1 = 1./(DIST**2*DELTAR**2)
            CON2 = 1./(C*DELTAR*DIST)
            DO 66 J = 1, 3
66    FOUT(J) = Fin(MS,J,N) * CON1 + CON2 * temp(j)
      RETURN
      ELSE
            CALL DERIVE (FIN,MS,N,FOUT,l)
            CON1 = 1./(DIST**2*DELTAR**2)
```

```
          CON2 = 1./(C*DELTAR*DIST)
          DO 2 I = 1,3
2      FOUT(I) = CON1 * FIN(MS,I,N) + CON2 * FOUT(I)
      ENDIF
      RETURN
      END
C
C
      SUBROUTINE LOES (l, k, C, FIN, FOUT, MO, MS, N)

           CALCULATE L APPLIED TO A SCALAR

      INTEGER*4 MO, MS, N, l, k
      REAL*4 FIN, FOUT, CON1, CON2, DIST
      DIMENSION FIN(308,21), temp(3), X(308,3,2)

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      COMMON/COORD/X
      DIST = DISTAN(l, k, MO, MS)
      IF (N.LT.1) THEN
           FOUT = 0.0
           RETURN
C
      ELSEIF (N.EQ.1) THEN
           CALL EderIN (MS, k, TEMP)
           CON1 = 1./(DIST**2*DELTAR**2)
           CON2 = 1./(C*DELTAR*DIST)
           FOUT = Fin(MS,N) * CON1 + CON2 *
DOTN(K,TEMP,MS)
           RETURN
      ELSE
           CON1 = 1./(DIST**2*DELTAR**2)
           CON2 = 1./(C*DELTAR*DIST)
           FOUT = CON1 * FIN(MS,N) +
     1          CON2 * (FIN(MS,N)-FIN(MS,N-1))/DELTAT
      ENDIF
      RETURN
      END
C
      SUBROUTINE DERIVE (A,MS,N,DERIV,l)

           CALCULATE DERIVATIVE OF ELECTRIC FIELD
```

```
      INTEGER*4 MS, N, l
      REAL*4 A, DERIV
      DIMENSION A(308,3,21), DERIV(3), temp(3), X(308,3,2)

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      COMMON/COORD/X
C
      IF    (N.LT.1) THEN
            DO 44 I = 1,3
            DERIV(I) = 0.0
44           CONTINUE
            RETURN
C
      ELSEIF    (N.EQ.1) THEN
            call ederin(ms, l, deriv)
            call cross (l, deriv, temp, ms)
            do 55 i = 1, 3
55          deriv(i) = temp(i)
       RETURN
C
      ELSE
            DO 45 I = 1,3
            DERIV(I) = (A(MS,I,N)-A(MS,I,N-1))/DELTAT
45           CONTINUE
      ENDIF
C
      RETURN
      END

C
      SUBROUTINE HINCID (M, N, l, B)

            GAUSSIAN PULSE FOR MAGNETIC FIELD

      INTEGER*4 M, N, l
      REAL*4 B
      DIMENSION B(3), X(308,3,2)

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      COMMON/COORD/X
      B(1) = 0.0
      B(3) = 0.0
      B(2) = EXP(-D**2* (DELTAT*FLOAT(N-8) + DELTAR *
```

```
x(M,1,l)/CO)
    1       **2) * SQRT(EPSILO/AMU) * D/SQRT(3.1415)
        RETURN
        END
C
        SUBROUTINE HderIN (M, l, B)

                DERIVATIVE OF GAUSSIAN PULSE FOR
                MAGNETIC FIELD AT T = 0


        INTEGER*4 M, l
        REAL*4 B
        DIMENSION B(3), X(308,3,2)

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
        COMMON/COORD/X
        call hincid(m, 1, l, b)
        B(2) = -2.*d**2*b(2) * (-7.*DELTAT + DELTAR *
x(M,1,l)/CO)
        RETURN
        END
C
        SUBROUTINE LOHV (l, k, C, FIN, FOUT, MO, MS, N)

                CALCULATE L APPLIED TO MAGNETIC FIELD

        INTEGER*4 MO, MS, N, l, k
        REAL*4 FIN, FOUT, CON1, CON2, DIST
        DIMENSION FIN(308,3,21), FOUT(3), X(308,3,2), TEMP(3)

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
        COMMON/COORD/X
        DIST = DISTAN(l, k, MO, MS)
        IF (N.LT.1) THEN
                DO 55 J = 1, 3
55                FOUT(J) = 0.0
                RETURN
C
        ELSEIF (N.EQ.1) THEN
        CALL HderIN (MS, k, FOUT)
        call cross (k, fout, temp, ms)
        CON1 = 1./(DIST**2*DELTAR**2)
```

```
      CON2 = 1./(C*DELTAR*DIST)
      DO 66 J = 1, 3
66     FOUT(J) = Fin(ms,J,1) * CON1 + CON2 * temp(j)

      RETURN
      ELSE
            CALL DERIVH (FIN,MS,N,FOUT,l)



            CON1 = 1./(DIST**2*DELTAR**2)
            CON2 = 1./(C*DELTAR*DIST)
            DO 2 I = 1,3
2     FOUT(I) = CON1 * FIN(MS,I,N) + CON2 * FOUT(I)

      ENDIF
      RETURN
      END
C

      SUBROUTINE LOHS (l, k, C, FIN, FOUT, MO, MS, N)

            CALCULATE L APPLIED TO A SCALAR
            CONTAINING MAGNETIC FIELD

      INTEGER*4 MO, MS, N, l, k
      REAL*4 FIN, FOUT, CON1, CON2, DIST
      DIMENSION FIN(308,21), temp(3), X(308,3,2)

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      COMMON/COORD/X
      DIST = DISTAN(l, k, MO, MS)
      IF (N.LT.1) THEN
            FOUT = 0.0
            RETURN
C
      ELSEIF (N.EQ.1) THEN
            CALL HderIN (MS, k, TEMP)
            CON1 = 1./(DIST**2*DELTAR**2)
            CON2 = 1./(C*DELTAR*DIST)
            FOUT = Fin(MS,N) * CON1 + CON2 *
DOTN(K,TEMP,MS)
            RETURN
```

```
      ELSE
            CON1 = 1./(DIST**2*DELTAR**2)
            CON2 = 1./(C*DELTAR*DIST)
            FOUT = CON1 * FIN(MS,N) +
     1      CON2 * (FIN(MS,N)-FIN(MS,N-1))/DELTAT
      ENDIF
      RETURN
      END
C

      SUBROUTINE DERIVH (A,MS,N,DERIV,l)

            DERIVATIVE OF MAGNETIC FIELD

             INTEGER*4 MS, N, l
      REAL*4 a, DERIV
      DIMENSION A(308,3,21), DERIV(3), temp(3), X(308,3,2)

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      COMMON/COORD/X
C
      IF    (N.LT.1) THEN
             DO 44 I = 1,3
             DERIV(I) = 0.0
44            CONTINUE
             RETURN
C
      ELSEIF    (N.EQ.1) THEN
            call hderin(ms, l, deriv)
            call cross(l, deriv, temp, ms)
            do 55 i = 1, 3
55          deriv(i) = temp(i)
            RETURN
C
      ELSE
             DO 45 I = 1,3
             DERIV(I) = (A(MS,I,N)-A(MS,I,N-1))/DELTAT
45            CONTINUE
      ENDIF
C
      RETURN
      END
```

```
C
      SUBROUTINE CROSS (l, B, C, MS)

           CROSS PRODUCT FOR NORMALS TO PATCHES

      INTEGER*4 MS, l
      REAL*4 B, C
      DIMENSION B(3), C(3), AN(308,3,2)
      COMMON/NORM/AN
      DO 1 I = 1,3
      J = I+1
      K = I+2
      IF(J.EQ.4) J=1
      IF(K.EQ.4) K=1
      IF(K.EQ.5) K=2
1      C(I) = an(MS,J,l)*B(K) - An(MS,K,l)*B(J)
      RETURN
      END
C
      SUBROUTINE CROSSA (A,B,C)

           GENERAL CROSS PRODUCT

      REAL*4 A, B, C
      DIMENSION A(3), B(3), C(3)
      DO 1 I = 1,3
      J = I+1
      K = I+2
      IF(J.EQ.4) J=1
      IF(K.EQ.4) K=1
      IF(K.EQ.5) K=2
1      C(I) = A(J)*B(K) - A(K)*B(J)
      RETURN
      END

C
C
      FUNCTION DOTN (l, B, MS)

           DOT PRODUCT FOR NORMALS TO PATCHES

      INTEGER*4 MS, l
```

```
      REAL*4 B, DOTN
      DIMENSION B(3), AN(308,3,2)
      COMMON/NORM/AN
      DOTN = 0.0
      DO 1 I=1,3
1      DOTN = DOTN + An(MS,I,l) * B(I)
      RETURN
      END

C
      FUNCTION DOTs (B, MS)

           DOT PRODUCT INVOLVING CURVES

      INTEGER*4 MS
      REAL*4 B, DOTS
      DIMENSION B(3), S1(18,3)
      COMMON/CURVES/s1
      DOT = 0.0
      DO 1 I=1,3
1      DOTS = DOTS + s1(MS,I) * B(I)
      RETURN
      END

      FUNCTION DISTAN (l, k, MO, MS)

           CALCULATE DISTANCE BETWEEN TWO CENTRES

      INTEGER*4 MO, MS, l, k
      REAL*4 DISTAN
      DIMENSION X(308,3,2)
      COMMON/COORD/X
      DISTAN = 0.0
      DO 10 I = 1,3
10     DISTAN =
DISTAN+(x(MO,I,l)-x(MS,I,k))*(x(MO,I,l)-x(MS,I,k))
      DISTAN = SQRT(DISTAN)
      RETURN
      END
C
```

```
      FUNCTION OFFSET(R,C)

            CALCULATE TIME DELAY

      REAL*4 R, C
      INTEGER*4 OFFSET

 COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
      OFFSET = IFIX(R*DELTAR/(C*DELTAT))
      RETURN
      END
C

      SUBROUTINE RHAT (l, k, MO, MS, RH)

            CALCULATE UNIT VECTOR R HAT

      INTEGER*4 MO, MS, l, k
      REAL*4 RH, DIST
      DIMENSION RH(3),X(308,3,2)
      COMMON/COORD/X
      DIST = DISTAN (l, k, MO, MS)
      DO 1 I = 1,3
1     RH(I) = (x(MO,I,l)-x(MS,I,k))/DIST
      RETURN
      END
C
C
C
C
      SUBROUTINE PART1(MO, NT)
C

                   CALCULATE ELECTRICAL FIELDS ON
                   DIELECTRIC

      INTEGER*4 OFFSET, MO, NT
      DIMENSION X(308,3,2), AN(308,3,2)
      DIMENSION H1T(308,3,16), H2T(308,3,16), H2N(308,16)
      DIMENSION E2T(308,3,16), E1N(308,16), E2N(308,16)

      DIMENSION EINC(3), S1(18,3)
      DIMENSION RH1(3), H1NEW(3), H2NEW(3), HM6(3)
```

```
      DIMENSION H1LO(3), H2LO(3), H2LOA(3)
      DIMENSION H2NEWA(3), DH1(3), HM5(3)
      DIMENSION E2TO(3), E2TI(3)
      DIMENSION ERHAT(3), DH2O(3)
      DIMENSION DH2I(3), E2LO4(3), E2LO3(3), EM4(3)
      DIMENSION HM2(3), EM2(3)
      DIMENSION EM3(3), TEMP1(3), TEMP2(3)
C
      COMMON/COORD/X
      COMMON/NORM/AN
      COMMON/CURVES/S1
      COMMON/TFIELD/E2T, H2T, H1T
      COMMON/NFIELD/E1N, E2N, H2N

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
C

C
C
C
C       EQUATION 1
C
C       MODULE 6
      DO 3301 J = 1,3
      H1NEW(J) = 0.0
      H2NEW(J) = 0.0
      H2NEWA(J) = 0.0
      HM2(J)=0.0
      HM5(J)=0.0
      HM6(J)=0.0
      EM2(J)=0.0
      EM3(J)=0.0
3301    EM4(J)=0.0
C
C
      IF(NT.EQ.1) GOTO 3434
      DO 1 MS1 = 189, 300

IF(MS1.NE.191.AND.MS1.NE.194.AND.MS1.NE.196.AND.MS1.N
E.201
    1 .AND.MS1.NE.299.AND.MS1.NE.300.AND.
    1
MS1.NE.190.AND.MS1.NE.200.AND.MS1.NE.291.AND.MS1.NE.2
```

```
95.AND.
    1
MS1.NE.189.AND.MS1.NE.199.AND.MS1.NE.290.AND.MS1.NE.2
94.AND.
    1
MS1.NE.292.AND.MS1.NE.216.AND.MS1.NE.220.AND.MS1.NE.2
96)
    1 GOTO 1
      DIST = DISTAN (2, 1, MO, MS1)
      ITANO = NT - OFFSET(DIST,CO)
      CALL RHAT (2,1,MO,MS1,RH1)
      DO 144 IT = 0, ITANO
      L = ITANO - IT
      CALL LOHV (2,1,CO,H1T,H1LO,MO,MS1,L)
      call cross(1, h1lo, temp1, ms1)
      IF (MS1.EQ.189) MSU = 13
      IF (MS1.EQ.190) MSU =  9
      IF (MS1.EQ.191) MSU =  5
      IF (MS1.EQ.194) MSU =  1
      IF (MS1.EQ.196) MSU =  2
      IF (MS1.EQ.199) MSU = 14
      IF (MS1.EQ.200) MSU = 10
      IF (MS1.EQ.201) MSU =  6
      IF (MS1.EQ.216) MSU =  3
      IF (MS1.EQ.220) MSU =  4
      IF (MS1.EQ.290) MSU = 15
      IF (MS1.EQ.291) MSU = 11
      IF (MS1.EQ.292) MSU =  7
      IF (MS1.EQ.294) MSU = 16
      IF (MS1.EQ.295) MSU = 12
      IF (MS1.EQ.296) MSU =  8
      IF (MS1.EQ.303) MSU = 18
      IF (MS1.EQ.304) MSU = 17
      DO 143 I = 1,3
143     H1NEW(I) = H1NEW(I) - RH1(I) * DOTS(temp1,MSU) *
DELTAR
144     CONTINUE
1     CONTINUE
C

      DO 2 MS1 = 1, 18
      IF (MS1.EQ. 1) MSU =  1
      IF (MS1.EQ. 2) MSU =  2
```

```
      IF (MS1.EQ. 3) MSU =  3
      IF (MS1.EQ. 4) MSU =  4
      IF (MS1.EQ. 5) MSU =  1
      IF (MS1.EQ. 6) MSU =  2
      IF (MS1.EQ. 7) MSU =  3
      IF (MS1.EQ. 8) MSU =  4
      IF (MS1.EQ. 9) MSU =  5
      IF (MS1.EQ.10) MSU =  6
      IF (MS1.EQ.11) MSU =  7
      IF (MS1.EQ.12) MSU =  8
      IF (MS1.EQ.13) MSU =  9
      IF (MS1.EQ.14) MSU = 10
      IF (MS1.EQ.15) MSU = 11
      IF (MS1.EQ.16) MSU = 12
      IF (MS1.EQ.17) MSU = 14
      IF (MS1.EQ.18) MSU = 13
C
C
      IF(MO.EQ.MSU) GO TO 2
      DIST = DISTAN (2, 2, MO, MSU)
      ITANO = NT - OFFSET(DIST,CO)
      ITANI = NT - OFFSET(DIST,CI)
      CALL RHAT(2, 2, MO, MSU, RH1)
      DO 12 ITO = 0, ITANO
      DO 12 ITI = 0, ITANI
      L1 = ITANO - ITO
      L2 = ITANI - ITI
      CALL LOHV (2,2,CO,H2T,H2LO,MO,MSU,L1)
      call cross(2, h2lo,temp1, ms1)
      CALL LOHV (2,2,CI,H2T,H2LOA,MO,MSU,L2)
      call cross(2, h2loa, temp2, ms1)
      DO 22 I = 1,3
      H2NEW(I) = H2NEW(I) - RH1(I) * DOTS (temp1, MS1) *
DELTAR
22    H2NEWA(I) = H2NEWA(I) - RH1(I) * DOTS (temp2, MS1)
* DELTAR
12    CONTINUE
2     CONTINUE
      DO 3 I=1,3
      hm2(i) = 1./epsilo * (h1new(i)-h2new(i)+h2newa(i)) * deltat
3     HM6(I) = (1./EPSILO * (H1NEW(i)-H2NEW(i)) + 1./EPSILI
*
   1        H2NEWA(i)) * deltat
```

```
C
C
C
C      MODULE 5
C
       DO 51 MS1 = 1, 300
       DIST = DISTAN(2,1,MO,MS1)
       ITANO = NT - OFFSET(DIST,CO)
       if(itano.le.0) goto 51
       CALL DERIVH (H1T,MS1,ITANO,DH1,1)
       CALL LOES (2,1,CO,E1N,E1LO,MO,MS1,ITANO)
       CALL RHAT (2,1,MO,MS1,RH1)
       DO 521 I = 1,3
521    HM5(i) = HM5(i) + (AMU/(DIST*DELTAR) * DH1(i) -
     1        E1LO * RH1 (i)) * DELTAR ** 2
51     continue
C
C      MODULES 4, 3, 2
C
       DO 20 MS1 = 1, 14
       IF(MO.EQ.MS1) GOTO 20
       DIST = DISTAN(2,2,MO,MS1)
       ITANO = NT - OFFSET(DIST,CO)
       IF(ITANO.LE.0) GO TO 2555
       CALL LOES (2,2,CO,E2N,E2LO,MO,MS1,ITANO)
       CALL DERIVH (H2T,MS1,ITANO,DH2O,2)
       CALL LOEV (2,2,CO,E2T,E2TO,MO,MS1,ITANO)
2555   ITANI = NT - OFFSET(DIST,CI)
       IF(ITANI.LE.0) GOTO 20
       CALL RHAT (2,2,MO,MS1,ERHAT)
       CALL LOES (2,2,CI,E2N,E2LI,MO,MS1,ITANI)
       CALL LOEV (2,2,CI,E2T,E2TI,MO,MS1,ITANI)
       DO 2422 I = 1,3
       E2LO4(I) = E2TO(I) - EPSILI/EPSILO* E2TI(I)
2422   E2LO3(I) = E2TO(I) - E2TI(I)
       DO 2421 J = 1,3
2421   DH2I(J) =
AMU*DH2O(J)/(DIST*DELTAR)*(1.-EPSILI/EPSILO)
       CALL CROSSA (E2LO3,ERHAT,TEMP1)
       CALL CROSSA (E2LO4,ERHAT,TEMP2)
       DO 2520 I = 1,3
       EM3(I) = DELTAR ** 2 * ( DH2I(I) - ERHAT(I) *
(E2LO-E2LI)
```

```
    1       - TEMP2(I)) + EM3(I)
      EM2(I) = DELTAR ** 2 * ( ERHAT(I) *
(E2LO-EPSILO/EPSILI*E2LI)
    1       + TEMP1(I)) + EM2(I)
2520 CONTINUE
20      CONTINUE
C
C       MODULE 1
C
        DO 11 I =1,3
        EM3(I) = HM2(I) - HM5(I) - EM3(I)
11      EM2(I) = HM6(I) - HM5(I) + EM2(I)
C
C
        CALL CROSS (2, EM2, TEMP1, MO)
3434    CALL EINCID (MO,NT,2,EINC)
        CALL CROSS (2, EINC, TEMP2, MO)
        DO 3312 I =1,3
        EINC(I) = EINC(I) + EM3(I)/(4.*3.1415)
3312 E2T(MO,I,NT) = TEMP2(I) + TEMP1(I)/(4.*3.1415)
        E2N(MO,NT) = DOTN(2,EINC,MO)

        DO 987 J = 1, 3
C
        IF(J.EQ.2.OR.(J.EQ.3.AND.(MO.GE.1.AND.MO.LE.4)))
    1       E2T(MO,J,NT) = 0.0
C
987     CONTINUE
C
C
        RETURN
        END
C
        SUBROUTINE PART2(MO, NT)

                CALCULATE MAGNETIC FIELDS ON DIELECTRIC


        INTEGER*4 OFFSET
        DIMENSION X(308,3,2), AN(308,3,2)
        DIMENSION H1T(308,3,16), H2T(308,3,16), H2N(308,16)
        DIMENSION E2T(308,3,16), E1N(308,16), E2N(308,16)
```

```
      DIMENSION HINC(3), S1(18,3)
      DIMENSION RH1(3), H1NEW(3), H2NEW(3), HM6(3)
      DIMENSION H2NEWA(3), HM5(3)
      DIMENSION H2TO(3), H2TI(3)
      DIMENSION ERHAT(3)
      DIMENSION EM4(3)
      DIMENSION HM2(3), EM2(3), DE2O(3)
      DIMENSION EM3(3)
      DIMENSION H2LO3(3)
C
      COMMON/COORD/X
      COMMON/NORM/AN
      COMMON/CURVES/S1
      COMMON/TFIELD/E2T, H2T, H1T
      COMMON/NFIELD/E1N, E2N, H2N

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
C

C
C      EQUATION 3
      DO 3301 J = 1,3
      H1NEW(J) = 0.0
      H2NEW(J) = 0.0
      H2NEWA(J) = 0.0
      HM2(J)=0.0
      HM5(J)=0.0
      HM6(J)=0.0
      EM2(J)=0.0
      EM3(J)=0.0
3301    EM4(J)=0.0
C
C      MODULE 5
C
      IF(NT.EQ.1) GOTO 3434
      DO 351 MS1 = 1, 300
      DIST = DISTAN(2,1,MO,MS1)
      ITANO = NT - OFFSET(DIST,CO)
      IF(ITANO.LE.0) GOTO 351
      CALL LOHV (2,1,CO,H1T,H1NEW,MO,MS1,ITANO)
      CALL RHAT (2,1,MO,MS1,RH1)
      CALL CROSSA (H1NEW,RH1,HM2)
```

```
      DO 3851 I = 1,3
3851    HM5(I) = HM5(I) + HM2(I) * DELTAR ** 2
351   CONTINUE
C
C       MODULES 4, 3, 2
C
      DO 320 MS1 = 1, 14
      IF(MO.EQ.MS1) GOTO 320
      DIST = DISTAN(2,2,MO,MS1)
      ITANO = NT - OFFSET(DIST,CO)
      IF(ITANO.LE.0) GOTO 3520
      CALL LOHS (2,2,CO,H2N,H2LO,MO,MS1,ITANO)
      CALL DERIVE (E2T,MS1,ITANO,DE2O,2)
      CALL LOHV (2,2,CO,H2T,H2TO,MO,MS1,ITANO)
3520  ITANI = NT - OFFSET(DIST,CI)
      IF(ITANI.LE.0) GOTO 320
      CALL LOHS (2,2,CI,H2N,H2LI,MO,MS1,ITANI)
      CALL RHAT (2,2,MO,MS1,ERHAT)
      CALL LOHV (2,2,CI,H2T,H2TI,MO,MS1,ITANI)
      DO 322 I = 1,3
      DE2O(I) = (EPSILO - EPSILI)*DE2O(I)/(DIST*DELTAR)
      H2LO4 = H2LO -  H2LI
      H2LO3(I) = H2TO(I) - H2TI(I)
322     EM4(I) = ERHAT(I) * H2LO4
      CALL CROSSA (H2LO3,ERHAT,EM3)
      DO 1320 I = 1,3
1320    EM2(I) = EM2(I) + DELTAR**2. * ( DE2O(I) +
     1       EM3(I) + EM4(I))
320     CONTINUE
C
C       MODULE 1
C
      DO 311 I = 1, 3
311     EM4(I) = EM2(I) + HM5(I)
      CALL CROSS (2, EM4, EM3, MO)
3434    CALL HINCID (MO,NT,2,HINC)
      CALL CROSS (2, HINC, EM2, MO)
      DO 312 I =1,3
      HM6(I) = EM4(I)/(4.*3.1415) + HINC(I)
312     H2T(MO,I,NT) = EM2(I) + EM3(I)/(4.*3.1415)
      H2N(MO,NT) = DOTN (2,HM6,MO)
C
C
```

```
      RETURN
      END
C
C
      SUBROUTINE PART3(MO, NT)

            CALCULATE ELECTRIC FIELD ON CONDUCTOR


      INTEGER*4 OFFSET
      DIMENSION X(308,3,2), AN(308,3,2)
      DIMENSION H1T(308,3,16), H2T(308,3,16), H2N(308,16)
      DIMENSION E2T(308,3,16), E1N(308,16), E2N(308,16)

      DIMENSION EINC(3), S1(18,3)
      DIMENSION RH1(3), H1NEW(3), H2NEW(3), HM6(3)
      DIMENSION H1LO(3), H2LO(3)
      DIMENSION H2NEWA(3), DH1(3), HM5(3)
      DIMENSION E2TO(3)
      DIMENSION ERHAT(3), DH2O(3)
      DIMENSION EM4(3)
      DIMENSION HM2(3), EM2(3)
      DIMENSION EM3(3), temp1(3), temp2(3)
C
      COMMON/COORD/X
      COMMON/NORM/AN
      COMMON/CURVES/S1
      COMMON/TFIELD/E2T, H2T, H1T
      COMMON/NFIELD/E1N, E2N, H2N

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
C

C
C       EQUATION 5
C
C       MODULE 6
       DO 3301 J = 1,3
       H1NEW(J) = 0.0
       H2NEW(J) = 0.0
       H2NEWA(J) = 0.0
       HM2(J)=0.0
       HM5(J)=0.0
```

```
      HM6(J)=0.0
      EM2(J)=0.0
      EM3(J)=0.0
3301   EM4(J)=0.0
C
      IF(NT.EQ.1) GOTO 3434
C
      DO 501 MS1 = 189, 300
      IF(MO.EQ.MS1) GOTO 501

IF(MS1.NE.191.AND.MS1.NE.194.AND.MS1.NE.196.AND.MS1.N
E.201
    1 .AND.MS1.NE.299.AND.MS1.NE.300.AND.
    1
MS1.NE.190.AND.MS1.NE.200.AND.MS1.NE.291.AND.MS1.NE.2
95.AND.
    1
MS1.NE.189.AND.MS1.NE.199.AND.MS1.NE.290.AND.MS1.NE.2
94.AND.
    1
MS1.NE.292.AND.MS1.NE.216.AND.MS1.NE.220.AND.MS1.NE.2
96)
    1 GOTO 501


      DIST = DISTAN (1,1,MO, MS1)
      ITANO = NT - OFFSET(DIST,CO)
      CALL RHAT (1,1,MO,MS1,RH1)
      DO 11501 IT = 0, ITANO
      L = ITANO - IT
      CALL LOHV (1,1,CO,H1T,H1LO,MO,MS1,L)
      CALL CROSS(1, H1LO, TEMP1, MS1)
      IF (MS1.EQ.189) MSU = 13
      IF (MS1.EQ.190) MSU =  9
      IF (MS1.EQ.191) MSU =  5
      IF (MS1.EQ.194) MSU =  1
      IF (MS1.EQ.196) MSU =  2
      IF (MS1.EQ.199) MSU = 14
      IF (MS1.EQ.200) MSU = 10
      IF (MS1.EQ.201) MSU =  6
      IF (MS1.EQ.216) MSU =  3
      IF (MS1.EQ.220) MSU =  4
```

```
      IF (MS1.EQ.290) MSU = 15
      IF (MS1.EQ.291) MSU = 11
      IF (MS1.EQ.292) MSU =  7
      IF (MS1.EQ.294) MSU = 16
      IF (MS1.EQ.295) MSU = 12
      IF (MS1.EQ.296) MSU =  8
      IF (MS1.EQ.303) MSU = 18
      IF (MS1.EQ.304) MSU = 17
      DO 12501 I = 1,3
12501  H1NEW(I) = H1NEW(I) - RH1(I) * DOTS(TEMP1,MSU) *
DELTAR
11501  CONTINUE
501    CONTINUE
C
      DO 502 MS1 = 1, 18
      IF (MS1.EQ. 1) MSU =  1
      IF (MS1.EQ. 2) MSU =  2
      IF (MS1.EQ. 3) MSU =  3
      IF (MS1.EQ. 4) MSU =  4
      IF (MS1.EQ. 5) MSU =  1
      IF (MS1.EQ. 6) MSU =  2
      IF (MS1.EQ. 7) MSU =  3
      IF (MS1.EQ. 8) MSU =  4
      IF (MS1.EQ. 9) MSU =  5
      IF (MS1.EQ.10) MSU =  6
      IF (MS1.EQ.11) MSU =  7
      IF (MS1.EQ.12) MSU =  8
      IF (MS1.EQ.13) MSU =  9
      IF (MS1.EQ.14) MSU = 10
      IF (MS1.EQ.15) MSU = 11
      IF (MS1.EQ.16) MSU = 12
      IF (MS1.EQ.17) MSU = 14
      IF (MS1.EQ.18) MSU = 13
C
      DIST = DISTAN (1,2,MO,MSU)
      ITANO = NT - OFFSET(DIST,CO)
      CALL RHAT(1,2,MO,MSU,RH1)
      DO 11502 ITO = 0, ITANO
      L1 = ITANO - ITO
      CALL LOHV (1,2,CO,H2T,H2LO,MO,MSU,L1)
      CALL CROSS(2,H2LO,TEMP2,MS1)
      DO 12502 I = 1, 3
12502  H2NEW(I) = H2NEW(I) - RH1(I) * DOTS (TEMP2,MS1) *
```

```
DELTAR
11502  CONTINUE
502    CONTINUE
       DO 503 I=1,3
503    HM6(I) = 1./EPSILO * (H1NEW(I)-H2NEW(I)) * deltat
C
C
C
C      MODULE 5
C
       DO 551 MS1 = 1, 300
       IF(MO.EQ.MS1) GOTO 551
       DIST = DISTAN(1,1,MO,MS1)
       ITANO = NT - OFFSET(DIST,CO)
       IF(ITANO.LE.0) GOTO 551
       CALL DERIVH (H1T,MS1,ITANO,DH1,1)
       CALL LOES (1,1,CO,E1N,E1LO,MO,MS1,ITANO)
       CALL RHAT (1,1,MO,MS1,RH1)
       DO 5511 I = 1,3
5511   HM5(I) = HM5(I) + (AMU/(DIST*DELTAR)*DH1(I) -
    1     E1LO * RH1 (I)) * DELTAR ** 2
551    CONTINUE
C
C      MODULES 4, 3, 2
C
       DO 520 MS1 = 1, 14
       DIST = DISTAN(1,2,MO,MS1)
       ITANO = NT - OFFSET(DIST,CO)
       IF(ITANO.LE.0) GOTO 520
       CALL LOES (1,2,CO,E2N,E2LO,MO,MS1,ITANO)
       CALL RHAT (1,2,MO,MS1,ERHAT)
       CALL DERIVH (H2T,MS1,ITANO,DH2O,2)
       CALL LOEV (1,2,CO,E2T,E2TO,MO,MS1,ITANO)
       DO 522 I = 1,3
       HM2(I) = AMU*DH2O(I)/(DIST*DELTAR)
522    EM4(I) = ERHAT(I) * E2LO
       CALL CROSSA (E2TO,ERHAT,EM3)
       DO 5820 I = 1,3
5820 EM2(I) = EM2(I) + DELTAR**2 * (HM2(I) - EM3(I) -
EM4(I))
520  CONTINUE
C
C      MODULE 1
```

```
C
      DO 511 I =1,3
511     EM4(I) = -1./(2.*3.14) * (EM2(I) + HM5(I) - HM6(I))
3434    CALL EINCID (MO,NT,1,EINC)
      DO 9511 I=1,3
9511 EM4(I) = 2.*EINC(I) + EM4(I)
      E1N(MO,NT) = DOTN (1,EM4,MO)
C
C
C
      RETURN
      END
C
C
      SUBROUTINE PART4(MO, NT)

             CALCULATE MAGNETIC FIELD ON CONDUCTOR


      INTEGER*4 OFFSET
      DIMENSION X(308,3,2), AN(308,3,2)
      DIMENSION H1T(308,3,16), H2T(308,3,16), H2N(308,16)
      DIMENSION E2T(308,3,16), E1N(308,16), E2N(308,16)

      DIMENSION HINC(3), S1(18,3)
      DIMENSION RH1(3), H1NEW(3), H2NEW(3), HM6(3)
      DIMENSION H2NEWA(3), HM5(3)
      DIMENSION H2TO(3)
      DIMENSION ERHAT(3)
      DIMENSION EM4(3)
      DIMENSION HM2(3), EM2(3), DE2O(3)
      DIMENSION EM3(3)
C
      COMMON/COORD/X
      COMMON/NORM/AN
      COMMON/CURVES/S1
      COMMON/TFIELD/E2T, H2T, H1T
      COMMON/NFIELD/E1N, E2N, H2N

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
C

      DO 3301 J = 1,3
```

```
      H1NEW(J) = 0.0
      H2NEW(J) = 0.0
      H2NEWA(J) = 0.0
      HM2(J)=0.0
      HM5(J)=0.0
      HM6(J)=0.0
      EM2(J)=0.0
      EM3(J)=0.0
3301    EM4(J)=0.0
C
C
C
C      EQUATION 6
C
C
C      MODULE 5
C
      IF(NT.EQ.1) GOTO 3435
      DO 651 MS1 = 1, 300
      IF(MO.EQ.MS1) GOTO 651
      DIST = DISTAN(1,1,MO,MS1)
      ITANO = NT - OFFSET(DIST,CO)
      IF(ITANO.LE.0) GOTO 651
      CALL LOHV (1,1,CO,H1T,H1NEW,MO,MS1,ITANO)
      CALL RHAT (1,1,MO,MS1,RH1)
      CALL CROSSA (H1NEW,RH1,HM2)
      DO 6511 I = 1,3
6511    HM5(I) = HM5(I) + HM2(I) * DELTAR ** 2
651     CONTINUE
C
C      MODULES 4, 3, 2
C
      DO 620 MS1 = 1, 14
      DIST = DISTAN (1,2,MO,MS1)
      ITANO = NT - OFFSET(DIST,CO)
      IF(ITANO.LE.0) GOTO 620
      CALL LOHS (1,2,CO,H2N,H2LO,MO,MS1,ITANO)
      CALL RHAT (1,2,MO,MS1,ERHAT)
      CALL DERIVE (E2T,MS1,ITANO,DE2O,2)
      CALL LOHV (1,2,CO,H2T,H2TO,MO,MS1,ITANO)
      DO 622 I = 1,3
      HM2(I) = EPSILO*DE2O(I)/(DIST*DELTAR)
622     EM4(I) = ERHAT(I) * H2LO
```

```
      CALL CROSSA (H2TO,ERHAT,EM3)
      DO 6920 I = 1,3
6920    EM2(I) = EM2(I) + DELTAR**2 * (HM2(I) + EM3(I) +
EM4(I))
620   CONTINUE
C
C      MODULE 1
C
      DO 611 I =1,3
611     EM4(I) = EM2(I) + HM5(I)
      CALL CROSS (1, EM4, EM3, MO)
3435    CALL HINCID (MO,NT,1,HINC)
      CALL CROSS (1, HINC, EM2, MO)
      DO 612 I =1,3
612     H1T(MO,I,NT) = 2.* EM2(I) + 1./(2.*3.14)*EM3(I)
C
C
C
      RETURN
      END


C     THE FINAL SUBROUTINES

      SUBROUTINE EFIN(MO, NT)

            CALCULATE ELECTRIC FIELD INSIDE BOX (OFF
            WALLS
C
      INTEGER*4 OFFSET, MO, NT
      DIMENSION X(308,3,2), AN(308,3,2)
      DIMENSION H1T(308,3,16), H2T(308,3,16), H2N(308,16)
      DIMENSION E2T(308,3,16), E1N(308,16), E2N(308,16)

      DIMENSION EINC(3), S1(18,3), HINC(3)
      DIMENSION RH1(3), H1NEW(3), H2NEW(3), HM6(3)
      DIMENSION H1LO(3), H2LO(3)
      DIMENSION H2NEWA(3), DH1(3), HM5(3)
      DIMENSION E2TO(3), E2TI(3), E2LI(3)
      DIMENSION ERHAT(3), DH2O(3)
      DIMENSION DH2I(3), EM4(3)
      DIMENSION HM2(3), EM2(3)
      dimension efinal(3,16), hfinal(3,16)
```

```
      DIMENSION EM3(3), TEMP1(3), TEMP2(3)
C
      COMMON/COORD/X
      COMMON/NORM/AN
      COMMON/CURVES/S1
      COMMON/TFIELD/E2T, H2T, H1T
      COMMON/NFIELD/E1N, E2N, H2N

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
C
      common/final/efinal, hfinal
C
C
C
C      EQUATION 1
C
C      MODULE 6
       DO 3301 J = 1,3
       H1NEW(J) = 0.0
       H2NEW(J) = 0.0
       H2NEWA(J) = 0.0
       HM2(J)=0.0
       HM5(J)=0.0
       HM6(J)=0.0
       EM2(J)=0.0
       EM3(J)=0.0
3301   EM4(J)=0.0
C
C
       IF(NT.EQ.1) GOTO 3434
       DO 1 MS1 = 216, 296

IF(MS1.NE.216.AND.MS1.NE.220.AND.MS1.NE.292.AND.MS1.N
E.296
    1
.AND.MS1.NE.291.AND.MS1.NE.295.AND.MS1.NE.290.AND.MS1
.NE.294)
    1       GOTO 1
       DIST = DISTAN (2, 1, MO, MS1)
       ITANO = NT - OFFSET(DIST,CO)
       CALL RHAT (2,1,MO,MS1,RH1)
       DO 144 IT = 0, ITANO
       L = ITANO - IT
```

```
      CALL LOHV (2,1,CO,H1T,H1LO,MO,MS1,L)
      call cross(1, h1lo, temp1, ms1)
      IF (MS1.EQ.189) MSU = 13
      IF (MS1.EQ.190) MSU =  9
      IF (MS1.EQ.191) MSU =  5
      IF (MS1.EQ.194) MSU =  1
      IF (MS1.EQ.196) MSU =  2
      IF (MS1.EQ.199) MSU = 14
      IF (MS1.EQ.200) MSU = 10
      IF (MS1.EQ.201) MSU =  6
      IF (MS1.EQ.216) MSU =  3
      IF (MS1.EQ.220) MSU =  4
      IF (MS1.EQ.290) MSU = 15
      IF (MS1.EQ.291) MSU = 11
      IF (MS1.EQ.292) MSU =  7
      IF (MS1.EQ.294) MSU = 16
      IF (MS1.EQ.295) MSU = 12
      IF (MS1.EQ.296) MSU =  8
      IF (MS1.EQ.303) MSU = 18
      IF (MS1.EQ.304) MSU = 17
      DO 143 I = 1,3
143     H1NEW(I) = H1NEW(I) - RH1(I) * DOTS(temp1,MSU) *
DELTAR
144     CONTINUE
1     CONTINUE
C

      DO 2 MS1 = 1, 18
      if
(ms1.NE.3.AND.MS1.NE.4.AND.MS1.NE.7.AND.MS1.NE.8
    1
.AND.MS1.NE.11.AND.MS1.NE.12.AND.MS1.NE.15.AND.MS1.N
E.16)
    1  goto 2
       IF (MS1.EQ. 1) MSU =  1
       IF (MS1.EQ. 2) MSU =  2
       IF (MS1.EQ. 3) MSU =  3
       IF (MS1.EQ. 4) MSU =  4
       IF (MS1.EQ. 5) MSU =  1
       IF (MS1.EQ. 6) MSU =  2
       IF (MS1.EQ. 7) MSU =  3
       IF (MS1.EQ. 8) MSU =  4
       IF (MS1.EQ. 9) MSU =  5
```

```
      IF (MS1.EQ.10) MSU =  6
      IF (MS1.EQ.11) MSU =  7
      IF (MS1.EQ.12) MSU =  8
      IF (MS1.EQ.13) MSU =  9
      IF (MS1.EQ.14) MSU = 10
      IF (MS1.EQ.15) MSU = 11
      IF (MS1.EQ.16) MSU = 12
      IF (MS1.EQ.17) MSU = 14
      IF (MS1.EQ.18) MSU = 13
C
C
      IF(MO.EQ.MSU) GO TO 2
      DIST = DISTAN (2, 2, MO, MSU)
      ITANO = NT - OFFSET(DIST,CO)
      CALL RHAT(2, 2, MO, MSU, RH1)
      DO 12 ITO = 0, ITANO
      L1 = ITANO - ITO
      CALL LOHV (2,2,CO,H2T,H2LO,MO,MSU,L1)
      call cross(2, h2lo,temp1, ms1)
      DO 22 I = 1,3
22     H2NEW(I) = H2NEW(I) - RH1(I) * DOTS (temp1, MS1) *
DELTAR
12     CONTINUE
2     CONTINUE
      DO 3 I=1,3
3     HM6(I) = 1./EPSILO * (H1NEW(i)-H2NEW(i)) * deltat
C
C
C
C      MODULE 5
C
      DO 51 MS1 = 209, 296
      DIST = DISTAN(2,1,MO,MS1)
      ITANO = NT - OFFSET(DIST,CO)
      if(itano.le.0) goto 51
      CALL DERIVH (H1T,MS1,ITANO,DH1,1)
      CALL LOES (2,1,CO,E1N,E1LO,MO,MS1,ITANO)
      CALL RHAT (2,1,MO,MS1,RH1)
      DO 521 I = 1,3
521   HM5(i) = HM5(i) + (AMU/(DIST*DELTAR) * DH1(i) -
    1       E1LO * RH1 (i)) * DELTAR ** 2
51    continue
C
```

```
C      MODULES 4, 3, 2
C
       DO 20 MS1 = 3, 12
       IF(MS1.EQ.5.OR.MS1.EQ.6.OR.MS1.EQ.9.OR.MS1.EQ.10)
GOTO 20
       IF(MO.EQ.MS1) GOTO 20
       DIST = DISTAN(2,2,MO,MS1)
       ITANO = NT - OFFSET(DIST,CO)
       IF(ITANO.LE.0) GO TO 20
       CALL LOES (2,2,CO,E2N,E2LO,MO,MS1,ITANO)
       CALL DERIVH (H2T,MS1,ITANO,DH2O,2)
       CALL LOEV (2,2,CO,E2T,E2TO,MO,MS1,ITANO)
       CALL RHAT (2,2,MO,MS1,ERHAT)
       DO 2421 J = 1,3
2421 DH2I(J) = AMU*DH2O(J)/(DIST*DELTAR)
       CALL CROSSA (E2TO,ERHAT,TEMP2)
       DO 2520 I = 1,3
       EM3(I) = DELTAR ** 2 * (DH2I(I) - ERHAT(I) * E2LO -
TEMP2(I))
    1          + EM3(I)
2520 CONTINUE
20     CONTINUE
C
       do 5566 i = 1, 3
       temp1(i) = 0.
5566 temp2(i) = 0.
C      MODULE OF THE AIR WALL
C
3434 DO 45  MS1 = 301, 302
       DIST = DISTAN(2,2,MO, MS1)
       ITANO = NT - OFFSET(DIST, CO)
       CALL EINCID(MS1, ITANO, 2, EINC)
       CALL HINCID(MS1, ITANO, 2, HINC)
       CALL EINCID(MS1, ITANO-1, 2, TEMP1)
       CALL HINCID(MS1, ITANO-1, 2, TEMP2)
4445 CALL RHAT( 2, 2, MO, MS1, ERHAT)
       DO 451 J = 1, 3
       DH2I(J) =
AMU/(DIST*DELTAR)*(HINC(J)-TEMP2(J))/DELTAT
       E2TO(J) = EINC(J)/(DIST**2*DELTAR**2) +
(EINC(J)-TEMP1(J))/
    1          (CO*DIST*DELTAR*DELTAT)
451  CONTINUE
```

```
      CALL CROSS (2, E2TO, TEMP1, MS1)
      CALL CROSSA (TEMP1, ERHAT, E2LI)
      CALL CROSS (2, DH2I, DH2O, MS1)
      DO 452 J = 1, 3
      E2TI(J) = ERHAT(J) * DOTN (2, E2TO, MS1)
      EM2(J) = EM2(J) + (DH2O(J)-E2TI(J)-E2LI(J))*DELTAR**2
452   CONTINUE
45    CONTINUE
C
C      MODULE 1
C
      DO 11 I =1,3
11     EFINAL(I,NT) =
(HM6(I)+HM5(I)+EM2(I)+EM3(I))/(4.*3.1415)
C
C
C
      RETURN
      END
C
C
      SUBROUTINE HFIN(MO, NT)

              CALCULATE MAGNETIC FIELD INSIDE BOX
              (OFF WALLS)


      INTEGER*4 OFFSET
      DIMENSION X(308,3,2), AN(308,3,2)
      DIMENSION H1T(308,3,16), H2T(308,3,16), H2N(308,16)
      DIMENSION E2T(308,3,16), E1N(308,16), E2N(308,16)

      DIMENSION HINC(3), S1(18,3), EINC(3)
      DIMENSION RH1(3), H1NEW(3), HM6(3)
      DIMENSION HM5(3), TEMP1(3), TEMP2(3)
      DIMENSION H2TO(3), E2TO(3)
      DIMENSION ERHAT(3), E2TI(3)
      dimension efinal(3,16), hfinal(3,16)
      DIMENSION EM4(3), DH2O(3), E2LI(3)
      DIMENSION HM2(3), EM2(3), DE2O(3), DH2I(3)
      DIMENSION EM3(3)
C
      COMMON/COORD/X
```

```
      COMMON/NORM/AN
      COMMON/CURVES/S1
      COMMON/TFIELD/E2T, H2T, H1T
      COMMON/NFIELD/E1N, E2N, H2N
      common/final/efinal, hfinal

COMMON/CST/DELTAR,DELTAT,CO,CI,EPSILO,EPSILI,AMU,D
C

C
C      EQUATION 3
      DO 3301 J = 1,3
      H1NEW(J) = 0.0
      HM2(J)=0.0
      HM5(J)=0.0
      HM6(J)=0.0
      EM2(J)=0.0
      EM3(J)=0.0
3301    EM4(J)=0.0
C
C      MODULE 5
C
      IF(NT.EQ.1) GOTO 3434
      DO 351 MS1 = 209, 296
      DIST = DISTAN(2,1,MO,MS1)
      ITANO = NT - OFFSET(DIST,CO)
        IF(ITANO.LE.0) GOTO 351
      CALL LOHV (2,1,CO,H1T,H1NEW,MO,MS1,ITANO)
      CALL RHAT (2,1,MO,MS1,RH1)
      CALL CROSSA (H1NEW,RH1,HM2)
      DO 3851 I = 1,3
3851    HM5(I) = HM5(I) + HM2(I) * DELTAR ** 2
351     CONTINUE
C
C      MODULES 4, 3, 2
C
      DO 320 MS1 = 3, 12
      IF(MS1.EQ.5.OR.MS1.EQ.6.OR.MS1.EQ.9.OR.MS1.EQ.10)
      GOTO 320
      DIST = DISTAN(2,2,MO,MS1)
      ITANO = NT - OFFSET(DIST,CO)
        IF(ITANO.LE.0) GOTO 320
      CALL LOHS (2,2,CO,H2N,H2LO,MO,MS1,ITANO)
```

```
      CALL DERIVE (E2T,MS1,ITANO,DE2O,2)
      CALL LOHV (2,2,CO,H2T,H2TO,MO,MS1,ITANO)
      CALL RHAT (2,2,MO,MS1,ERHAT)
      DO 322 I = 1,3
      DE2O(I) = EPSILO * DE2O(I)/(DIST*DELTAR)
322     EM4(I) = ERHAT(I) * H2LO
      CALL CROSSA (H2TO,ERHAT,EM3)
      DO 1320 I = 1,3
1320    EM2(I) = EM2(I) + DELTAR**2 * (DE2O(I) + EM3(I) +
EM4(I))
320     CONTINUE
C
C       MODULE 1
C
      do 566 i = 1, 3
      temp1(i) = 0.
      em4(i) = 0.
566   temp2(i) = 0.
C
C     MODULE OF THE AIR WALL
C
3434 DO 45  MS1 = 301, 302
      DIST = DISTAN(2,2,MO, MS1)
      ITANO = NT - OFFSET(DIST, CO)
      CALL EINCID (MS1, ITANO, 2, EINC)
      CALL HINCID (MS1, ITANO, 2, HINC)
      CALL EINCID (MS1, ITANO-1, 2, TEMP1)
      CALL HINCID (MS1, ITANO-1, 2, TEMP2)
      CALL RHAT ( 2, 2, MO, MS1, ERHAT)
      DO 451 J = 1, 3
      DH2I(J) =
EPSILO/(DIST*DELTAR)*(EINC(J)-TEMP1(J))/DELTAT
      E2TO(J) = HINC(J)/(DIST**2*DELTAR**2) +
(HINC(J)-TEMP2(J))/
    1       (CO*DIST*DELTAR*DELTAT)
451   CONTINUE
      CALL CROSS (2, E2TO, TEMP1, MS1)
      CALL CROSSA (TEMP1, ERHAT, E2LI)
      CALL CROSS (2, DH2I, DH2O, MS1)
      DO 452 J = 1, 3
      E2TI(J) = ERHAT(J) * DOTN (2, E2TO, MS1)
      EM4(J) = EM4(J) +
(DH2O(J)+E2TI(J)+E2LI(J))*DELTAR**2
```

```
452  CONTINUE
45   CONTINUE
C
C     MODULE 1
C
      DO 11 I =1,3
11     HFINAL(I,NT) = -(HM5(I)+EM2(I)+EM4(I))/(4.*3.1415)
C
C
C
      RETURN
      END
C
C
```

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