

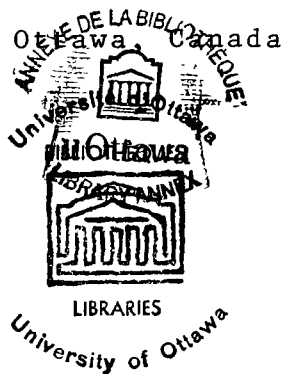
HYBRID COMPUTER SOLUTION OF THE SAINT VENANT EQUATIONS

by

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Submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

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PREFACE

A great variety of engineering problems are associated with unsteady nonuniform flow in rivers. A general approach to these problems involves the derivation of partial differential equations representing the process. The equations governing the process are the St. Venant equations. A continuous-space-discrete-time method of lines solution of the St. Venant equations is utilized in the study. The resulting ordinary differential equations are then solved on a hybrid computer.

Based on a comparison between mathematical model predictions and prototype observations it is concluded that the model is a good representation of the process. Its use for steady nonuniform flow is clearly established and the use for unsteady nonuniform flow is discussed.

The hybrid computer provides an ideal environment for interactive modelling and has speed of solution advantages over digital solutions. Such advantages can result in significant economic savings when a number of simulation experiments are carried out.

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The work of the Hydraulics Laboratory, National Research Council, in the supplying of data for the model application is gratefully acknowledged.

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NOTATIONS

a	2x2 matrix
A	cross section area
b	vector with components b_1 and b_2
C	Chezy coefficient
C_p	coefficient embodying effects of nonuniformity, unsteadiness and viscosity
C_t^*	turbulence correction
C_{wp}	coefficient relating wetted perimeter to water level
D_ℓ	losses due to lateral inflow and outflow
f	focal length of parabolic channel
$\overline{F}$	general continuous vector function of time and space
g	acceleration due to gravity
h	height above reference (channel bottom)
H	water level head
k	channel shape coefficient
m	maximum value of the variable subscripted
$\overline{n}$	unit vector directed normal to the surface S
N	total number of time steps
P	mean pressure
q	net lateral outflow per unit length
q_ℓ	lateral inflow
q_s	lateral outflow due to seepage and evaporation
Q	discharge rate
R	hydraulic radius

s	primary space dimension
S	surface bounding Ψ
S^{-1}	eigenvalue matrix
S_o	channel bottom slope
S_f	overall frictional losses
t	time
T	channel top width
u	velocity in x direction
$\bar{u}$	time mean velocity in the x direction
u'	turbulent fluctuations about mean velocity in the x direction
u_ℓ	longitudinal velocity of lateral inflow
U	vector of velocity and height
$\hat{u}$	vector of characteristic variables
v	velocity in the y direction
$\bar{v}$	time mean velocity in the y direction
v'	turbulent fluctuation about mean velocity in the y direction
V	magnitude of the velocity vector
Ψ	volume
$\bar{V}$	velocity vector
w	velocity in the z direction
$\bar{w}$	time mean velocity in the z direction
w'	turbulent fluctuation about the mean velocity in the z direction
W	dW/dx

W_1	characteristic forward variable
W_2	characteristic backward variable
X	component of exterior forces in x direction
Y	component of exterior forces in y direction
Z	component of exterior forces in z direction
α, α_1	velocity correction coefficients
γ	specific gravity of water
δ_s	incremental distance in the s direction
Δt	incremental step in time
ϵ	energy dissipation per unit area
η	general water level
θ	angle of inclination of the channel bottom
$\lambda_1 \lambda_2$	eigenvalues of the matrix a
ρ	density of water
σ_x^1	deviation from mean pressure in x direction
σ_y^1	deviation from mean pressure in y direction
σ_z^1	deviation from mean pressure in z direction
τ_{ij}	tangential stresses where i is the direction normal to the plane of action and j is the direction of the action
τ_o	average boundary shear stress
$\overline{\omega}$	velocity vector of the surface S

CHAPTER 1

INTRODUCTION

A number of empirically based techniques have been developed by hydrologists for the analysis of flood waves, (Dooge, 1972; Chow, 1959). Because of their empirical nature, problems arise in comparing the performance of alternate models for the same process (Kisiel, 1972). Therefore, there is a need for research effort in developing more advanced and general models (Dooge, 1972).

A general approach to the above problem involves the direct analysis of the partial differential equations governing the flow process (Henderson, 1966; Freeze, 1969). This approach will be pursued in this study since it has the potential of solving all unsteady, nonuniform flow problems in open channels rather than a small number of problems that fit a specialized, often idealized category.

The study of unsteady flow problems is usually undertaken by utilizing a mathematical model of the process and prototype (scaled replicas of the prototype are of use for special applications). The analyst utilizing a computer for the analysis must select the type of computer, that is digital, analog or hybrid, which is best suited to the given circumstances. In light of the modelling objectives, very careful considerations of the advantages and disadvantages of a particular computer must be made.

Analog computers utilize currents and voltages that vary continuously as the problem variables. A direct analogy of an ordinary differential equation can be constructed on the analog. The most important advantages of analog computation are the speed of the solution and the interactivity of the user. All components of a circuit act simultaneously and the solution appears as soon as the input is applied. The solution is available at many locations in the circuit allowing for the display or output of particular sub-circuits, and by analogy, the behaviour of prototype components. The value of analog computation in hydrology is recognized by its relatively widespread use (IASH Symposium, 1968; Riley et al, 1966).

Digital computers on the other hand have the advantages (Katell, 1969) of being able to store large volumes of data and retrieve these very quickly, and being able to compute algebraic expressions very precisely. The digital computer must, however, do all operations sequentially as opposed to the parallel operation of the analog.

Finally, the hybrid computer consists of both an analog and digital computer. The strength of the hybrid computer lies in solutions beyond the speed and economic scope of third generation digital machines, and in extending the accuracy of analog machines (Katell, 1969). Katell cites a particular application comparing an all digital with a hybrid simulation by the Central Electricity Generating

Board of Great Britain. As well as important fidelity advantages, a time saving of 200 to 1 was achieved on the hybrid computer as compared to the digital solution. The significance of time saving translated into economic terms could result in reducing the costs of computing time of a large simulation problem from, say 100 dollars per solution on an IBM 360/75 to less than 50 cents per solution on the hybrid. This could have an enormous impact on computer simulation studies in engineering.

In addition to the speed of solution and associated economic savings, the hybrid computer maintains the high degree of man-to-problem interfacing characteristic of the analog computer. The user of the hybrid is often physically surrounded by the computer and can investigate the response of his model by displaying the output of model components on scopes, plotters, or voltmeters. He can also take action to stop an erroneous solution from wasting computing time by observing intermediate output.

Thus the solution of complex problems on the hybrid computer, as opposed to a straight digital or straight analog, has proven tremendously rewarding in a number of applications in terms of time and economic savings. It is expected that similar advantages may result from the use of the hybrid for the solution of the unsteady flow equations (Vermuri, 1973; Chubb, 1970); hence it was selected as the most appropriate tool for this study.

The flow in natural rivers is a three-dimensional, time varying phenomena; however, the computer modelling of large scale, long duration events makes simplification of the process description mandatory. A one-dimensional approximation of the three-dimensional process is utilized. These one-dimensional equations are somewhat deficient in describing the results of irregularities in geometry observed in natural river channels and some smoothing of the observed geometry is required. As well the factors involved in the dissipation of energy and momentum are not precisely predictable from existing knowledge and some experimental adjustment of calibration is necessary to make the model predictions compare favourably with observations.

The one-dimensional partial differential equations must be changed to ordinary differential equations, which can then be solved on the analog computer. This requires a numerical approximation, which to date has not been documented in the literature. The continuous-space-discrete-time method of lines is thus developed for the St. Venant equations and the resulting ordinary differential equations form the basis for the hybrid computer solution.

The hybrid computer model is calibrated using observations from the St. Lawrence River hydraulic model located in the Hydraulics Laboratory, National Research Council. Once calibrated the computer model is then used to predict water levels under a variety of conditions which could arise in engineering problems.

1.1 Objectives

The objectives of this study are to:

1. construct a hybrid computer representation of the St. Venant equations,
2. experimentally determine the applicability of this one-dimensional simplification of a three-dimensional process,
3. provide guidelines for the use of these equations in engineering practice.

CHAPTER 2

THE SAINT VENANT EQUATIONS OF OPEN CHANNEL FLOW

The governing equations of open channel flow can be derived from the three basic principles of mass conservation, and momentum and energy flux and dissipation. The development simply involves the averaging of the point forms of these basic principles over the desired channel configuration and expressing the stress terms in a form amenable to evaluation.

2.1 The point forms of equations of continuity, energy and momentum

The continuity equation may be derived by considering the mass flux through a differential control volume. The resulting equations are derived (Whitaker, 1968) for a continuous incompressible conservative fluid to be

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad 2.1$$

where u , v and w represent velocities in the x , y and z directions respectively. The velocities are assumed averaged over a sufficiently large time as to allow passage of a statistically significant number of eddies past a point in space but not so large as to damp out gradual time variations of the flow (Strelkoff, 1969).

The point forms of the energy and momentum equations are derived by summing the forces acting on a fluid element in each of the component directions (Rouse, 1959), that is,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = X - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \left(\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \quad 2.2.a$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = Y - \frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{1}{\rho} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma'_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) \quad 2.2.b$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = Z - \frac{1}{\rho} \frac{\partial P}{\partial z} + \frac{1}{\rho} \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma'_z}{\partial z} \right) \quad 2.2.c$$

where X, Y, and Z are the components of exterior forces on the element

P and σ'_x , σ'_y and σ'_z are the mean pressure and deviations from this mean in the x, y and z directions, and τ_{ij} are the tangential stresses where i indicates the direction normal to the plane of action and j the direction of action.

If each of the terms in equation 2.2 are multiplied by the fluid density the equation takes on the form of momentum change per unit volume per unit time. If the only exterior terms are assumed to be those due to gravity and h is defined as the height above a horizontal reference then the momentum equations are

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} = - \frac{\partial (P + \rho gh)}{\partial x} + \left(\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \quad 2.3.a$$

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \rho w \frac{\partial v}{\partial z} = - \frac{\partial (P + \rho gh)}{\partial y} + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma'_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) \quad 2.3.b$$

$$\rho \frac{\partial w}{\partial t} + \rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} = - \frac{\partial (P + \rho gh)}{\partial z} + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma'_z}{\partial z} \right) \quad 2.3.c$$

The energy equation is derived by multiplying Eqn. 2.2 by the density and corresponding components of velocity. Adding all components of the equation and denoting $u^2 + v^2 + w^2$ by V^2 (magnitude of the velocity vector) the energy equation becomes (Rouse, 1959)

$$\begin{aligned} \frac{\rho}{2} \frac{\partial V^2}{\partial t} + \frac{\rho}{2} u \frac{\partial V^2}{\partial x} + \frac{\rho}{2} v \frac{\partial V^2}{\partial y} + \frac{\rho}{2} w \frac{\partial V^2}{\partial z} = \\ - u \frac{\partial (P + \rho gh)}{\partial x} - v \frac{\partial (P + \rho gh)}{\partial y} - w \frac{\partial (P + \rho gh)}{\partial z} + u \left(\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \\ + v \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma'_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) + w \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma'_z}{\partial z} \right) \end{aligned} \quad 2.4$$

2.2 The general macroscopic forms

Equations 2.1, 2.3 and 2.4 must now be averaged over some volume in space. Two general theorems are applied to convert the equations to a more tractable form. The first is the divergence theorem converting volume integrals to surface integrals. It may be stated as (Strelkoff, 1969)

$$\int_V \text{div } \bar{F} dV = \int_S \bar{F} \cdot \bar{n} dS \quad 2.5$$

where $\text{div } \bar{F}$ is the divergence of any continuous vector function of space and time, S is the surface bounding V , and $\bar{n}$ is the unit vector directed outward normal to S .

The second theorem involves a general equation for the time derivative of a volume integral and is called the general transport theorem. The transport of any continuous scalar function (such as density or volume) f may be written as (Whitaker, 1968)

$$\int_V \frac{\partial f}{\partial t} dV = \frac{d}{dt} \int_V f dV - \int_S f \bar{\omega} \cdot \bar{n} dS \quad 2.6$$

where $\bar{\omega}$ is the local velocity of the surface.

By making use of the divergence theorem and then the general transport theorem the continuity equation is converted to

$$\int_S (\bar{V} - \bar{\omega}) \cdot \bar{n} dS + \frac{dV}{dt} = 0 \quad 2.7$$

where $\bar{V}$ is the velocity vector, and $\bar{\omega}$ the vector of motion of the volume V .

Similarly the macroscopic form of the momentum equation is derived by integrating over a volume. The volume integrals are converted to surface integrals in the momentum flux terms and part of the stress term. The general transport theorem is used to rewrite the time derivatives yielding

$$\begin{aligned} \frac{d}{dt} \int_V \rho u dV + \int_S \rho u (\bar{V} - \bar{\omega}) \cdot \bar{n} dS = \\ - \int_V \frac{\partial}{\partial x} (P + \rho gh + \sigma') dV + \int_S (\tau_{yx} \frac{\partial y}{\partial n} + \tau_{zx} \frac{\partial x}{\partial n}) dS \end{aligned} \quad 2.8$$

for the x direction component. The y and z directional components are obtained by replacing u by v and w and the partial derivatives and stresses to the appropriate directions.

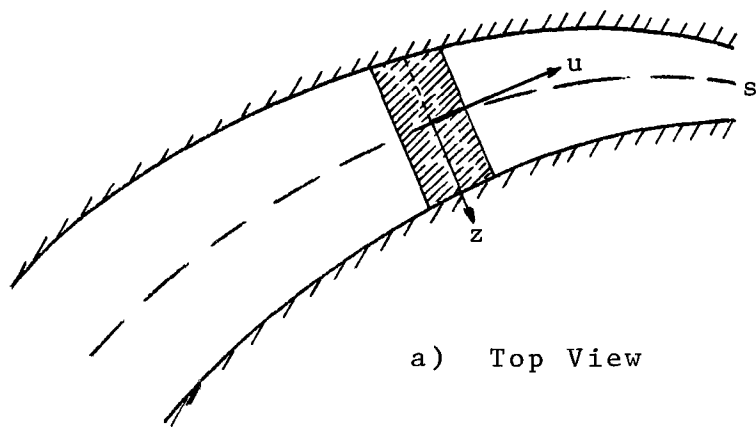
Similarly the macroscopic form of the energy equation is obtained by applying the general transport theorem to the first term and the divergence theorem to the energy flux term, the $(P + \rho gh)$ terms and part of the stress term to yield

$$\begin{aligned}
 & \frac{\rho}{2} \frac{d}{dt} \int_V v^2 dV + \frac{\rho}{2} \int_S v^2 (\bar{V} - \bar{\omega}) \cdot \bar{n} dS = \\
 & - \int_S (P + \rho gh) \bar{V} \cdot \bar{n} dS + \int_S [u(\tau_{yx} \frac{\partial y}{\partial n} + \tau_{zx} \frac{\partial z}{\partial n} - \sigma'_x \frac{\partial x}{\partial n} + v(\tau_{xy} \frac{\partial x}{\partial n} + \tau_{zy} \frac{\partial z}{\partial n} + \sigma'_y \frac{\partial y}{\partial n})) \\
 & + w(\tau_{xz} \frac{\partial x}{\partial n} + \tau_{yz} \frac{\partial y}{\partial n} + \sigma'_z \frac{\partial z}{\partial n})] dS - \int_V (\sigma'_x \frac{\partial u}{\partial x} + \tau_{yx} \frac{\partial u}{\partial y} + \tau_{zx} \frac{\partial u}{\partial z} + \sigma'_y \frac{\partial v}{\partial y} \\
 & + \tau_{zy} \frac{\partial v}{\partial z} + \tau_{xz} \frac{\partial w}{\partial x} + \tau_{yx} \frac{\partial w}{\partial y} + \sigma'_z \frac{\partial w}{\partial z}) dV
 \end{aligned} \tag{2.9}$$

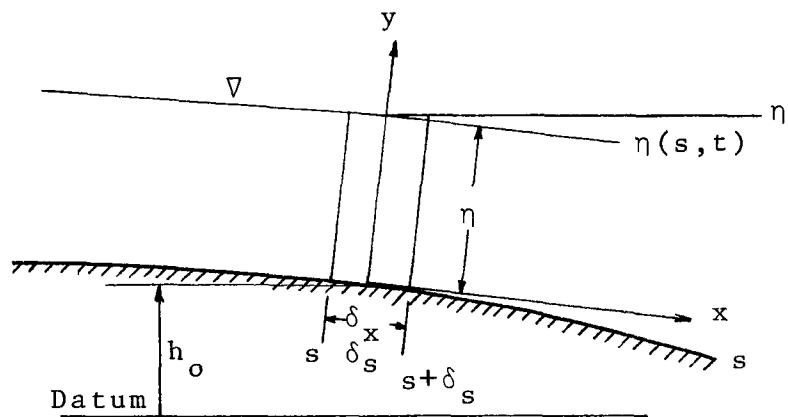
Equations 2.7, 2.8 and 2.9 apply to any moving deforming volume and hence most applications in hydraulics and hydrodynamics can be derived from these forms. The special application of interest here is the application of these equations to a natural river channel of general form and alignment. For most problems of practical interest in hydrology, one dimensional equations are used (Eagleson, 1970).

2.3 One dimensional flow equation

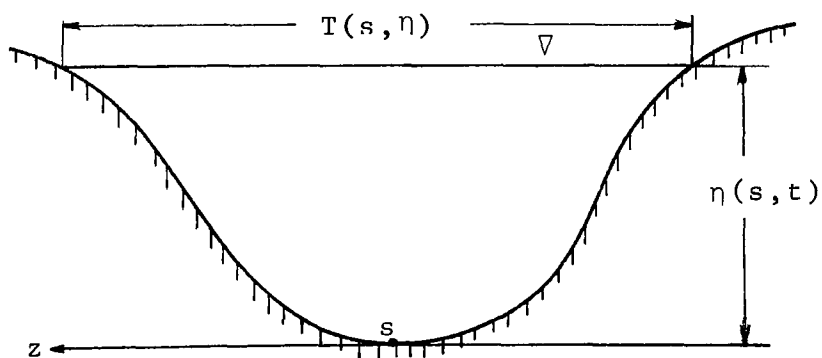
Firstly, one dimensionality is defined as conditions where the mean components of velocity are zero except in the primary space dimension S. Turbulent fluctuations are present in all coordinate directions. The channel of interest is sketched in Fig. 2.1, where V is chosen as a slice across the flow of incremental thickness δ_s and A is taken as the area at the midsection normal to the channel bottom and with its top always coincident with the fluctuating



a) Top View



b) Side Sectional View



c) Cross-sectional View

Fig. 2.1 One Dimensional Channel (Strelkoff, 1969)

water surface. The integration is carried out under conditions of lateral outflow at the rate of q per unit length of s .

The continuity equation is derived by breaking the total surface into three regions; areas of entrances and exits through which fluid may enter or leave the system, area of solid moving surfaces and, area of solid fixed surfaces. On the solid moving surfaces $\bar{V} - \bar{\omega} = 0$ and on the solid fixed surfaces $\bar{V} = \bar{\omega} = 0$. Denoting discharge by Q , then $(\bar{V} - \bar{\omega}) \cdot \bar{n} dS = dQ$. Furthermore, $\frac{dV}{dt}$ may be written as $\delta_s \frac{dA}{dt}$. Applying 2.7 with the above assumptions yields

$$\delta_s \frac{dA}{dt} + dQ + q \delta_s = 0 \quad 2.10$$

or letting $\delta_s \rightarrow 0$ results in

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial s} + q = 0 \quad 2.11$$

Application of the momentum and energy equations to the particular channel involves a closer analysis of the y and z portions of 2.8 and 2.9. Assuming all terms with velocity components in y and z can be neglected and that the atmospheric pressure is zero allows the writing of a pressure equation in the form

$$P(s, y, x, t) = \rho g(\eta(s, t) - y) \cos \theta(\delta) + C_t^*(s, y, z, t) + C_p(s, y, z, t) \quad 2.12$$

where θ is the angle of inclination of the channel bottom, C_t^* embodies all turbulence correction factors, and C_p embodies effects of nonuniformity, unsteadiness and viscosity.

At high Reynolds numbers (Streeter, 1966) usually encountered in open channel flow the turbulence effects are much larger than viscous effects. Furthermore, the expression for C_t^* can be simplified to (Strelkoff, 1969)

$$C_t^*(y) = - \overline{\rho v'v'}(y) \quad 2.13$$

where v' is the turbulent fluctuation about the mean velocity v , and $\overline{v'v'}$ is the average cross-product. Inherent in equation 2.13 are the assumptions that $\overline{\rho w'w'} = \overline{\rho v'v'}$, and that derivatives of $\overline{\rho v'w'}$ are negligible.

The most important contributions of C_p usually stem from streamline curvatures in the xy and yz planes.

Applying equations 2.8 and 2.9 to the one-dimensional channel in Fig. 2.1 yields (Strelkoff, 1969; Keulegan, 1943)

$$\begin{aligned} \frac{1}{g} \frac{\partial \bar{u}}{\partial t} + \frac{\partial}{\partial s} \left[\beta \frac{\bar{u}^2}{2g} + \eta \cos \theta + h_o \right] + \frac{\tau_o}{\gamma R} = - \frac{\bar{u}^2}{2g} \frac{\partial \beta}{\partial s} - \frac{1}{A \rho g} \\ \int_A \frac{\partial C_p}{\partial s} dA + \frac{1}{A \rho g} \int_A \frac{\partial C_T}{\partial s} dA - \frac{1}{Ag} \left\{ \beta \bar{u} \frac{\partial Q}{\partial s} + \bar{u} \frac{\partial A}{\partial t} + u_\ell q \right\} \end{aligned} \quad 2.14$$

and

$$\begin{aligned} \frac{\alpha_1}{g} \frac{\partial \bar{u}}{\partial t} + \frac{\partial}{\partial s} \left[\alpha \frac{\bar{u}^2}{2g} + \eta \cos \theta + h_o \right] + \frac{1}{Q\gamma} \int \epsilon dA = \\ - \frac{\bar{u}}{2g} \frac{\partial \alpha_1}{\partial t} - \frac{1}{\gamma} \frac{\partial C_p}{\partial s} + \frac{1}{\gamma} \frac{\partial C_t}{\partial s} - \frac{1}{Q} \left\{ \bar{H} \frac{\partial Q}{\partial s} + H_1 \frac{\partial A}{\partial t} + H_\ell q \right\} \end{aligned} \quad 2.15$$

where $C_t = \tau_{xx} - C_t^*$ is the total nondissipative contribution of fluid turbulence. Combining equation 2.13 with the assumption that viscous effects are small yields

$$C_t = - \overline{\rho u'u'} + \overline{\rho v'v'} \quad 2.16$$

As is evidenced in both equations 2.14 and 2.15 it is only the gradient of C_t which enters into the equations of motion rather than the magnitude itself.

The correction coefficients in equations 2.14 and 2.15 are evaluated from

$$\begin{aligned}\beta &= \frac{\int u^2 dA}{Q\bar{u}}, & \alpha_1 &= \frac{\int V^2 dA}{Q\bar{u}}, \\ \alpha &= \frac{\int V^2 u dA}{Q\bar{u}^2}, & \overline{C_p} &= \frac{\int C_p u dA}{Q}, \\ \overline{C_t} &= \frac{\int C_t u dA}{Q},\end{aligned}\tag{2.17}$$

where $\bar{u} = Q/A$ is the discharge mean velocity.

The terms involving head in the energy equation are defined as

$$\bar{H} = \eta \cos \theta + \xi_0 + \frac{\overline{C_p} - \overline{C_t}}{\gamma} + \frac{\alpha \bar{u}^2}{2g}\tag{2.18a}$$

$$H_1 = \eta \cos \theta + \xi_0 + \frac{\overline{C_p} - \overline{C_T}}{\gamma} + \frac{\alpha_1 \bar{u}^2}{2g}\tag{2.18b}$$

$$H_\ell = \eta \cos \theta + \xi_0 + \frac{C_{p\ell} + C_{T\ell}^* - \tau_{\ell\ell}}{\gamma} + \frac{V_\ell^2}{2g}\tag{2.18c}$$

where ℓ refers to conditions at the point of lateral inflow and $\tau_{\ell\ell}$ represents deformation stresses at the outflow in the direction V_ℓ and ξ_0 is the vertical reference elevation.

Equations 2.11, 2.14 and 2.15 are thus the general one-dimensional flow equations. They are both exact and complete and apply to laminar or turbulent flow and unsteady,

nonuniform, spatially varied flow surmounted by an atmosphere of negligible density in a fixed bed channel of arbitrary form and alignment.

For applications to open channels in natural settings several problems arise which usually lead to a further simplification of the equations before use. Firstly, it is very difficult and costly to measure the correction coefficients. Secondly, even when measurements are taken very often a degree of randomness remains in the data which is of little interest when studying a large scale process. Thus a number of physical assumptions will be made to yield simpler equations, representing unsteady nonuniform flow.

For the case of steady uniform flow all the characteristics of the cross-section are independent of s and a relationship for the dissipation terms from the momentum and energy equations becomes

$$-\gamma \frac{d\xi_o}{ds} = \frac{\overline{\tau_o}}{R} = \frac{1}{Q} \int_A \epsilon \, dA \quad 2.19$$

The assumption are made that θ is small and that $\cos \theta \approx 1$ and that the bulk of the streamlines are oriented in the s direction and uniformly distributed across a section so that departures from hydrostatic pressure are negligible. Thus all partials of the correction coefficients with respect to s will disappear.

Two cases of lateral discharge are of interest in hydrological problems, the case of seepage of fluid through

the channel wall and the case of inflow due to overbank flow or precipitation on the channel.

For the case of channel seepage it is assumed that u_ℓ , V_ℓ and $C_{p\ell}$ in equations 2.14 and 2.15 all vanish since the seepage occurs at essentially zero velocity in the longitudinal direction. With the above assumptions the equations of momentum and energy for the seepage case become

$$\frac{1}{g} \frac{\partial V}{\partial t} + \frac{\partial H}{\partial s} = - \frac{\bar{\tau}_o}{\gamma R} + \frac{Vq}{Ag} \quad 2.20$$

and
$$\frac{1}{g} \frac{\partial V}{\partial t} + \frac{\partial H}{\partial s} = - \frac{1}{Q\gamma} \int \epsilon \, dA + \frac{Vq}{2Ag} \quad 2.21$$

where
$$H = \xi_o + \frac{V^2}{2g} .$$

For equations 2.20 and 2.21 to remain compatible, $\bar{\tau}_o$ must decrease from what it is in equation 2.19. The energy equation is the convenient one to use in this case since it is assumed that the dissipation is the same as without lateral outflow.

In the case of lateral inflow the boundary shear τ_o increases only slightly due to increased turbulence of the flow as compared to uniform flow (Strelkoff, 1969). For this reason it is convenient to use the momentum equation for such cases in which $\bar{\tau}_o$ is independent of the process, that is

$$\frac{1}{g} \frac{\partial V}{\partial t} + \frac{\partial H}{\partial s} = - \frac{\bar{\tau}_o}{\gamma R} + \frac{Vq}{Ag} \left(1 - \frac{u_\ell}{V}\right) \quad 2.22$$

Again for the energy equation to remain compatible with equation 2.19, the dissipation must increase over that of

uniform flow. This agrees with the observed physical fact that cross flow generates increased dissipation of energy.

2.4 Momentum and energy losses

As is evident in equation 2.19 a simple relationship exists between the loss term in the energy and momentum equations for the special case of steady uniform flow. The mechanisms for the losses are quite different. In the case of the momentum equation the loss stems from the boundary stresses opposing the motion of fluid while in the case of the energy equation the loss measures the dissipation of energy that occurs throughout the fluid mass at a given section (Farell, 1966).

For this special case a simple relationship exists between head loss per unit length and boundary shear force per unit length. This relationship may be assumed to hold approximately for gradually varied unsteady flow problems (Farell, 1966; Strelkoff, 1969; Keulegan, 1943).

The relationship between wall shear and momentum dissipation is a function of relative roughness, roughness geometry and Reynolds number (Keulegan, 1943). Defining $S_f = \frac{\bar{\tau}_o}{\gamma R}$ leads to several empirical or semi-empirical relationships between S_f and flow parameters.

The frequently used Chezy equation (Daily and Harleman, 1966) evaluates S_f by

$$S_f = \frac{|\bar{u}| |\bar{u}|}{C^2 R} \quad 2.23$$

where C is the coefficient.

Another common form of the dissipation term is Manning's formulation (Schwab, 1966) which may be stated as

$$S_f = \frac{|\bar{u}|^2}{R^{4/3} n^2} \quad 2.24$$

Assuming that the dissipation terms as defined for steady uniform flow also hold for unsteady nonuniform flow leads then to one equation of motion. Changing the notation slightly at this stage in order to conform to that most frequently used yields

$$\frac{1}{g} \frac{\partial V}{\partial t} + \frac{\partial H}{\partial x} = - S_f + D_\ell \quad 2.25$$

where V is the discharge mean velocity, $H = \zeta_o + \frac{V^2}{2g}$, S_f is described by equations 2.23 or 2.24, $D_\ell = \frac{Vg}{2gA}$ for seepage outflow, $D_\ell = \frac{V - u_\ell}{gA} q$ for lateral inflow where u_ℓ is the inflow velocity in the x direction, and x is the primary space dimension (previously s). If the H term is taken relative to the channel bottom then $\frac{d\xi_o}{dx}$ is defined by $- S_o$.

Therefore, the assumptions leading to equations 2.25 and 2.11 have been established, and these equations are the basic mathematical expression of the physical flow process. These equations are usually called the Saint Venant equations after the French engineer who first utilized them (Stoker, 1957).

CHAPTER 3

THEORETICAL DEVELOPMENTS

The Saint Venant equations derived in the previous chapter cannot be solved explicitly except in some simplified cases. In this chapter the approximate method of lines will be applied to the Saint Venant equations and the close relationship between the mathematical developments and the physical flow process will be outlined. Some consideration of properly posing the boundary conditions from both physical and mathematical reasoning is also given.

3.1 One dimensional equations and boundary conditions

Equations 2.11 and 2.25 belong to a class of differential equations named hyperbolic partial differential equations. The general class of such equations can be put in the form (Ames, 1965; Mufti, 1973)

$$\frac{\partial U}{\partial t} + a \frac{\partial U}{\partial x} = b. \quad 3.1$$

where U is a 2-vector, a is 2x2 matrix, b is a 2-vector and a and b are in general functions of x , t and U . If the coefficient a is independent of U and if b depends linearly on U the equation is called linear; if a is independent of U and b depends on U , but not linearly, the equation is called semilinear; if a depends on U , as is the case in the one dimensional open channel flow equations, the equation is called quasi-linear.

A number of methods exist for the approximate solution of the hyperbolic equations (Strelkoff, 1970), including finite difference schemes, methods based on the theory of characteristics, and solution by the method of lines. The finite differencing schemes, either implicit or explicit, involve the discretization of both space and time. Usually little physical reasoning goes into such schemes although extensive considerations as to their stability and convergence are available (Isaacson, 1956).

The method of characteristics is based on the physics of flow by breaking down the partial differential equations into four ordinary ones that correspond to a backward and forward wave observed in flow problems (Amein, 1966). The method provides solutions along characteristics only, which is of some disadvantage when the solution is required at a particular location in space. However, a discussion of the method of characteristics can give considerable insight into the flow process and how boundary conditions are to be stated.

The method of lines consists of discretizing either time or space and solving continuously along the other dimension. As in the method of characteristics, the partial differential equations are changed to ordinary differential equations. The solution by the method of lines can yield the solution at specific space locations and at specific time without further interpolation. A close relationship

is maintained between the method of characteristics and the method of lines which will be further developed in this chapter.

3.1.1 Characteristics of hyperbolic equations

To introduce the idea of characteristics consider the case where U is a scalar. Assuming that U satisfies Equation 3.1, then since U is a function of x and t , its total differential is

$$dU = \frac{\partial U}{\partial t} dt + \frac{\partial U}{\partial x} dx \quad 3.2$$

or the total derivative of U with respect to t is

$$\frac{dU}{dt} = \frac{\partial U}{\partial t} + \frac{\partial U}{\partial x} \frac{dx}{dt} \quad 3.3$$

By comparing Equations 3.1 and 3.3 it becomes evident that U which satisfies Equation 3.1 also satisfies

$$\frac{dU}{dt} = b \quad 3.4$$

provided that

$$\frac{dx}{dt} = a. \quad 3.5$$

Conversely, if U satisfies Equations 3.4 and 3.5 it also satisfies Equation 3.1. To show this let U satisfy Equations 3.4 and 3.5. Then

$$b = \frac{dU}{dt} = \frac{\partial U}{\partial t} + \frac{\partial U}{\partial x} \frac{dx}{dt} = \frac{\partial U}{\partial t} + a \frac{\partial U}{\partial x} \quad 3.6$$

or U satisfies Equation 3.1.

Thus Equations 3.4 and 3.5 form an equivalent set of ordinary differential equations which have special physical

and geometric meaning (Mufti, 1973). Physically if an observer is moving with a velocity a , the time rate of change of U is equal to b . Geometrically, the equations state that on a trace of Equation 3.5 in the x, t plane the values of U are given by Equation 3.4 for points on the curve. The traces of Equation 3.5 in the x, t plane are called characteristics or characteristic curves and the direction is called the characteristic direction (Kells, 1965). The solution at any point, P , on the characteristic curve depends on the initial value at the point where the characteristic meets the initial curve (called the domain of dependence of P).

For the Saint Venant equations there are a pair of characteristics; a backward characteristic and a forward characteristic (see Fig. 3.1).

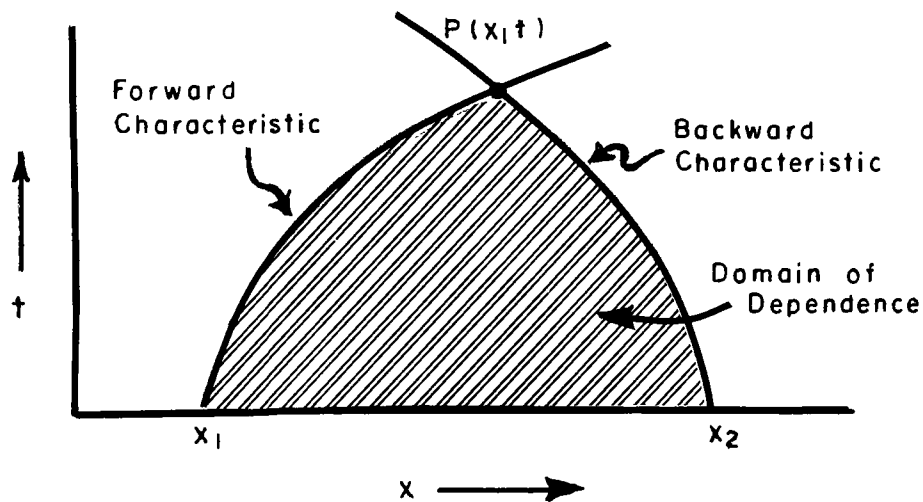


Fig. 3.1 Characteristic of the Saint Venant Equations

The theory of characteristics states that the solution at P is determined solely by the initial data on

the segment x_1-x_2 . This segment, or the region enclosed by the characteristics through P is called the domain of dependence of the point $P(x,t)$ (Henderson, 1966). This knowledge of characteristics will now be used as a guide in specifying initial and boundary conditions.

3.1.2 Initial and boundary conditions

The solution of the Saint Venant equations must start from known conditions and proceed from there, forward in time. Supplying conditions in velocity and water level for all of x at some initial time t_0 , is required information. If the initial conditions are supplied from data observations the necessary information can be obtained from continuous space observations or from combining discrete observations with an interpolation procedure.

In cases where observations are very scarce, the mathematical model may be used to assist in the interpolation procedure to obtain initial conditions (Prandle, 1972). A case where velocity and water level are known only at $x = 0$ and at $x = x_{\max}$ is cited. An initial coarse estimate of the continuous velocity and height distribution might be obtained from linear interpolation between known values. A better estimate of the initial conditions is then obtained by assuming that the prototype is at steady state and running the mathematical model until it reaches a state of equilibrium.

From a strictly mathematical point of view one boundary condition in each of velocity and water level should

be adequate knowledge for the unique solution of the partial differential equation. The theory of characteristics implies that if one of the variables is specified, say at $x = 0$, the other is implicitly known from the combination of the initial condition and the characteristic passing through the point on the boundary.

Thus if both boundary conditions are to be specified at the same space location, care must be exercised that the physical laws of flow are not violated by such specification.

3.2 Solution by the method of lines

As becomes evident from Fig. 3.1 an approximate solution based strictly on the theory of characteristics will result in the knowledge of the solution only along characteristic lines which are in general parallel to neither the x nor t axis. In most applications it is desired that the solution be known at specific points in space and time, thus a solution on characteristic lines must be combined with a two-dimensional interpolation scheme in order to obtain the desired information (Ames, 1965; Paul et al, 1970).

The interpolation can be avoided and much of the physical meaning associated with characteristics can be utilized by choosing lines along which the solution is obtained either parallel to the x or t axis. The continuous-space-discrete-time (C.S.D.T.) solution consists of lines

parallel to the x axis while the continuous time-discrete space solution consists of lines parallel to the time axis. For the present application the C.S.D.T. solution is preferred since the geometry can then be represented continuously in space.

In order to facilitate the utilization of the theory of characteristics, the Saint Venant equations are first written in the characteristic normal form (Ames, 1965). For the Saint Venant equation, Equation 3.1 is applicable where U is a 2-vector of velocity and height, a is a 2x2 matrix and b a 2-vector which depends on U , x and t .

Equation 3.1 is first premultiplied by S^{-1} to obtain

$$S^{-1} \frac{\partial U}{\partial t} + S^{-1} a \frac{\partial U}{\partial x} = S^{-1} b \quad 3.7$$

where the non-singular matrix S is such that

$$S^{-1} a S = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad 3.8$$

where λ_1 and λ_2 are the eigenvalues of the matrix a and they are assumed to be real and distinct.

Postmultiplying both sides of Equation 3.8 by S^{-1} on the right yields

$$S^{-1} a = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} S^{-1} \quad 3.9$$

which can be substituted into Equation 3.7 to give

$$s^{-1} \frac{\partial U}{\partial t} + \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} s^{-1} \frac{\partial U}{\partial x} = s^{-1} b \quad 3.10$$

Equation 3.10 is called the characteristic form of equation 3.1 where λ_1 and λ_2 are the characteristic directions. The equations of the characteristics are given by

$$\frac{dx}{dt} = \lambda_1 \quad 3.11a$$

and

$$\frac{dx}{dt} = \lambda_2 \quad 3.11b$$

For linear and semilinear problems the characteristic form can be simplified by introducing a new dependent vector $\hat{U}$ whose elements are called characteristic variables (Isaacson and Keller, 1966) and which is given by

$$\hat{U} = s^{-1} U \quad 3.12$$

Although Equation 3.12 is not applicable directly to the quasi-linear equation its equivalent can be obtained by making an assumption to be demonstrated later.

In the general form the Saint Venant equation becomes

$$\frac{\partial U}{\partial t} + a \frac{\partial U}{\partial x} = b \quad 3.13$$

where

$$U = \begin{pmatrix} h \\ v \end{pmatrix} \quad 3.13a$$

$$a = \begin{pmatrix} v & \frac{A}{T} \\ g & v \end{pmatrix} \quad 3.13b$$

$$b = \begin{pmatrix} -q - v \frac{dA}{dx} \\ \frac{T}{g(S_o - S_f + D_\ell)} \end{pmatrix} \quad 3.13c$$

The notation in Equation 3.13c may be shortened by writing the components of the vector b as b_1 and b_2 .

The eigenvalues of a are obtained from

$$\det(a - \lambda I) = 0 \quad 3.14$$

$$\text{or} \quad (v - \lambda)^2 - \left(\sqrt{\frac{A}{gT}}\right)^2 = 0 \quad 3.15$$

$$\text{from which} \quad \lambda_1 = v + \sqrt{\frac{A}{gT}} \quad 3.16a$$

$$\text{and} \quad \lambda_2 = v - \sqrt{\frac{A}{gT}} \quad 3.16b$$

The corresponding eigenvector matrix S^{-1} becomes (Mufti, 1973)

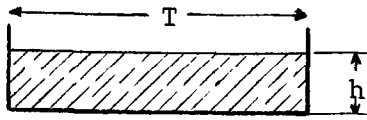
$$S^{-1} = \begin{pmatrix} \sqrt{\frac{T}{gA}} & 1 \\ -\sqrt{\frac{T}{gA}} & 1 \end{pmatrix} \quad 3.17$$

Now, utilizing Equations 3.16a, 3.16b and 3.17 yields the following characteristic normal form of the Saint Venant equations

$$\sqrt{\frac{T}{gA}} \frac{\partial h}{\partial t} + \frac{\partial v}{\partial t} + (v + \sqrt{\frac{A}{gT}}) \left(\sqrt{\frac{T}{gA}} \frac{\partial h}{\partial x} + \frac{\partial v}{\partial x} \right) = \sqrt{\frac{T}{gA}} b_1 + b_2 \quad 3.18a$$

$$-\sqrt{\frac{T}{gA}} \frac{\partial h}{\partial t} + \frac{\partial v}{\partial t} + (v - \sqrt{\frac{A}{gT}}) \left(-\sqrt{\frac{T}{gA}} \frac{\partial h}{\partial x} + \frac{\partial v}{\partial x} \right) = -\sqrt{\frac{T}{gA}} b_1 + b_2 \quad 3.18b$$

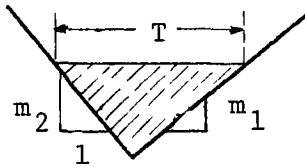
The equivalent of the characteristic variable will now be found by assuming that for the channels of interest a single relationship between area A and top width, T , can be found. Fig. 3.2 summarizes some frequently used channel shapes and their relationships for A and T .



Rectangular channel

$$A = Th \quad \text{or} \quad \frac{A}{T} = kh$$

where $k = 1.0$

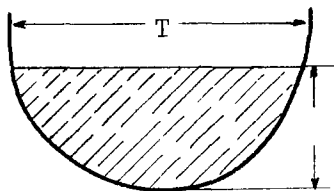


Triangular channel

$$A = \frac{h^2}{2m_1} + \frac{h^2}{2m_2} \quad T = \frac{h}{m_1} + \frac{h}{m_2}$$

$$\frac{A}{T} = \frac{\frac{h^2}{2} \left(\frac{1}{m_1} + \frac{1}{m_2} \right)}{h \left(\frac{1}{m_1} + \frac{1}{m_2} \right)} = kh$$

where $k = 1/2$



Parabolic channel

$$A = \frac{2}{3} Th \quad \text{or} \quad \frac{A}{T} = kh$$

where $k = 2/3$

Figure 3.2 Summary of Channel Shapes and A/T Ratios.

As becomes evident from Fig. 3.2 a variety of channel shapes have a simple relationship between A and T in the form

$$\frac{A}{T} = kh \quad 3.19$$

Substituting Equation 3.19 into Equation 3.18 yields

$$\frac{\partial}{\partial t}(v + 2\sqrt{\frac{g}{k}}h) + (v + \sqrt{gkh}) \frac{\partial}{\partial x}(v + 2\sqrt{\frac{g}{k}}h) = \hat{b}_1 \quad 3.20a$$

$$\frac{\partial}{\partial t}(v - 2\sqrt{\frac{g}{k}}h) + (v - \sqrt{gkh}) \frac{\partial}{\partial x}(v - 2\sqrt{\frac{g}{k}}h) = \hat{b}_2 \quad 3.20b$$

where $\hat{b}_1 = \sqrt{\frac{g}{kh}} b_1 + b_2$, and $\hat{b}_2 = -\sqrt{\frac{g}{kh}} b_1 + b_2$.

By inspecting Equations 3.20a and 3.20b and introducing new dependent variables of the form

$$W_1 = v + 2\sqrt{\frac{g}{k}}h \quad 3.21a$$

and

$$W_2 = v - 2\sqrt{\frac{g}{k}}h \quad 3.21b$$

results in simplifying Equation 3.20 to

$$\frac{\partial W_1}{\partial t} + \lambda_1 \frac{\partial W_1}{\partial x} = \hat{b}_1 \quad 3.22a$$

and

$$\frac{\partial W_2}{\partial t} + \lambda_2 \frac{\partial W_2}{\partial x} = \hat{b}_2 \quad 3.22b$$

W_1 and W_2 may be called characteristic variables as introduced in Equation 3.12. The physical

significance of W_1 and W_2 can be seen by inspecting the characteristic equivalent of Equation 3.22.

The characteristic equations of the Saint Venant equations are (Strelkoff 1970, Mufti 1973).

$$\frac{dv}{dt} + \sqrt{g \frac{T}{A}} \frac{dh}{dt} = \hat{b}_1 \quad \text{or} \quad \frac{dW_1}{dt} = \hat{b}_1 \quad 3.23a$$

along the characteristic $\frac{dx}{dt} = \lambda_1$, and

$$\frac{dv}{dt} - \sqrt{g \frac{T}{A}} \frac{dh}{dt} = \hat{b}_2 \quad \text{or} \quad \frac{dW_2}{dt} = \hat{b}_2 \quad 3.23b$$

along the characteristic $\frac{dx}{dt} = \lambda_2$.

From Equation 3.23a it can be seen that along the forward characteristic the rate of change of W_1 is $\hat{b}_1$. Thus the association between W_1 and λ_1 , the forward characteristic, becomes evident. Similarly W_2 is associated with λ_2 the backward characteristic.

The transformations from W_1 and W_2 back to v and h are

$$v = \frac{W_1 + W_2}{2} \quad 3.24a$$

$$h = k \frac{(W_1 - W_2)^2}{16g} \quad 3.24b$$

The next step in the method of lines is to discretize the time derivative in Equation 3.22. Using a backward difference approximation yields the following definition

$$\frac{\partial W_1}{\partial t} (x, j\Delta t) = \frac{W_1(x, j\Delta t) - W_1(x, (j-1)\Delta t)}{\Delta t} \quad 3.25a$$

$$\frac{\partial W_2}{\partial t} (x, j\Delta t) = \frac{W_2(x, j\Delta t) - W_2(x, (j-1)\Delta t)}{\Delta t} \quad 3.25b$$

where $j = 1, 2, \dots, N$.

Substituting Equation 3.25 into Equation 3.22 and shortening the notation yields

$$\frac{dW_1^j}{dx} = - \frac{W_1^j - W_1^{j-1}}{\lambda_1^j \Delta t} + \frac{\hat{b}_1^j}{\lambda_1^j} \quad 3.26a$$

$$\text{and} \quad \frac{dW_2^j}{dx} = - \frac{W_2^j - W_2^{j-1}}{\lambda_2^j \Delta t} + \frac{\hat{b}_2^j}{\lambda_2^j} \quad 3.26b$$

The expressions for λ_1^j , λ_2^j , $\hat{b}_1^j$ and $\hat{b}_2^j$ in terms of W_1^j and W_2^j are

$$\lambda_1^j = W_1^j \left(\frac{2+k}{4} \right) + W_2^j \left(\frac{2-k}{4} \right) \quad 3.27a$$

$$\lambda_2^j = W_1^j \left(\frac{2-k}{4} \right) + W_2^j \left(\frac{2+k}{4} \right) \quad 3.27b$$

$$\hat{b}_1^j = \frac{4g}{kT} \frac{(-q - \frac{W_1^j + W_2^j}{2} \frac{dA}{dx})}{W_1^j - W_2^j} + g(S_o - S_f + D_\ell) \quad 3.28a$$

$$\hat{b}_2^j = - \frac{4g}{kT} \left(\frac{-q - \frac{W_1^j + W_2^j}{2} \frac{dA}{dx}}{W_1^j - W_2^j} \right) + g(S_o - S_f + D_\ell) \quad 3.28b$$

Furthermore, the relationship for frictional losses and dynamic effects of lateral inflow in terms of W_1^j and W_2^j become

$$S_f = \frac{\frac{W_1^j + W_2^j}{2} \left| \frac{W_1^j + W_2^j}{2} \right|}{C^2 R} \quad 3.29$$

$$D_{\ell} = \frac{W_1^j + W_2^j}{4Ag} q \quad 3.30a$$

for seepage outflow

$$D_{\ell} = \left(\frac{\frac{W_1^j + W_2^j}{2} - u_{\ell}}{Ag} \right) q \quad 3.30b$$

for lateral inflow.

Since Equation 3.26a is associated with the forward characteristics, W_1^j must be given at the upstream location ($x = 0$). Similarly, W_2^j being associated with the backward characteristic, must be specified at the downstream location ($x = x_{\max}$). Hence, if Equations 3.26a and 3.26b respectively were independent of W_2^j and W_1^j , the natural way to solve these equations numerically for W_1^j and W_2^j would be to integrate Equation 3.26a in the increasing x direction, and Equation 3.26b in the decreasing x direction. Numerical stability considerations also demand that Equations 3.26a and 3.26b be integrated in the increasing and decreasing directions respectively (Vichnevetsky, 1969; Fromm, 1972).

Unfortunately, however, Equations 3.26a and 3.26b are coupled equations and with W_1^j specified at $x = 0$ and W_2^j at $x = x_{\max}$ form a two-point boundary value problem which if solved as a sequence of coupled initial value problems may lead to numerical instabilities (Ames, 1965). To avoid the numerical instability problem and make use of the preferred direction of integration the following algorithm is developed.

For convenience the forward equation may be written in the general form

$$\frac{dW_1^j}{dx} = f_1(W_1^j, W_2^j, x) \quad 3.31a$$

and the backward equation as

$$\frac{dW_2^j}{dx} = f_2(W_1^j, W_2^j, x) \quad 3.31b$$

The computation algorithm proceeds by first estimating W_2^{*j} (estimate of W_2^j) and integrating Equation 3.31a with this estimate in the forward direction, yielding values of W_1^{*j} . These values of W_1^{*j} are then used in the backward integration of Equation 3.31b to yield updated values of W_2^{*j} . The process is then repeated with these new values of W_2^{*j} until the differences between subsequent estimates is nonsignificant.

3.3 Boundary condition approximations

The relationship between velocity v , height h , and W_1 and W_2 as specified by Equations 3.21a and 3.21b requires both v and h at $x = 0$ and $x = x_{\max}$ to determine boundary conditions in W_1 and W_2 . This imposes some practical and economic restrictions on the use of the model; hence the following boundary condition approximations are possible to minimize these restrictions.

At $x = 0$ a boundary condition must be specified in W_1^j which may be obtained from both v and h by

Equation 3.21a or by combining v and W_2^{*j} in the form

$$W_1^{*j} = 2v^j - W_2^{*j} \quad 3.32$$

or by combining h and W_2^{*j} in the form

$$W_1^{*j} = W_2^{*j} + 4\sqrt{\frac{g}{k}h^j} \quad 3.33$$

Similarly at $x = x_{\max}$ the boundary condition in W_2^j may be specified using both v and h as in Equation 3.21b or by combining v and W_1^{*j} through the relationship

$$W_2^{*j} = 2v - W_1^{*j} \quad 3.34$$

or from h and W_1^{*j} by

$$W_2^{*j} = W_1^{*j} - 4\sqrt{\frac{g}{k}h^j} \quad 3.35$$

For practical problems, then, it is possible to combine a boundary condition estimation procedure with one measurement in either velocity or water level at both $x = 0$ and $x = x_{\max}$.

CHAPTER 4

MODEL DEVELOPMENT

The mathematical basis for the hybrid computer model is given by Equations 3.26 to 3.30. These equations are programmed on the hybrid computer. The mathematical model is applied to a particular prototype channel, hence a geometric description and the computer application of it are given. The mathematical model is then calibrated, verified, and employed in simulation experiments.

4.1 Hybrid computer solution

Equations 3.26 to 3.30 were solved on an EAI 690 Hybrid computer at the Analysis Laboratory, National Research Council, Ottawa. The system consists of a digital computer (EAI 640), an analog computer (EAI 680) and a digital-analog interface (Gagne, 1971). The approach of the solution is to design an analog circuit to represent the differential equations and let the digital computer perform the tasks of initializing the analog, storing and playing back current and past solutions, inputting the representation of the geometrical variables and outputting information in digital form wherever desired. During the initialization and input-output stages, the analog is in an initial condition (I.C.) or waiting mode. During the

integration the analog is continuously in an operate mode and the digital computer operates intermittently as signals from the analog dictate. Figs. 4.1 and 4.1a demonstrate the general program flow. The direction of the arrow across the interface indicates the origin of a command and hence the computer in control at that particular time.

The program for the digital computer was written in the standard scientific Fortran (FORTRAN IV, 1972) language. The interface operations are performed by library programs (Gagne, 1972). The analog program consists of patched components on a standard patchboard (680 Handbook, 1970). A special language (H.O.I.) is available (Gagne, 1972) to check the analog patchboard. This was very convenient during the analog program development stage as well as a fast tool for checking connections and components prior to each computing session.

4.1.1 Analog computer representation of ordinary differential equations

The basic important components available on the analog computer are (680 Handbook, 1970) summers, integrators, multipliers, potentiometers, function generators, comparators, switches and counters. These components can be arranged in a desired order by patching wires from the output of one component to the input of the next component. Most components have a dynamic range from -10 to +10 volts representing

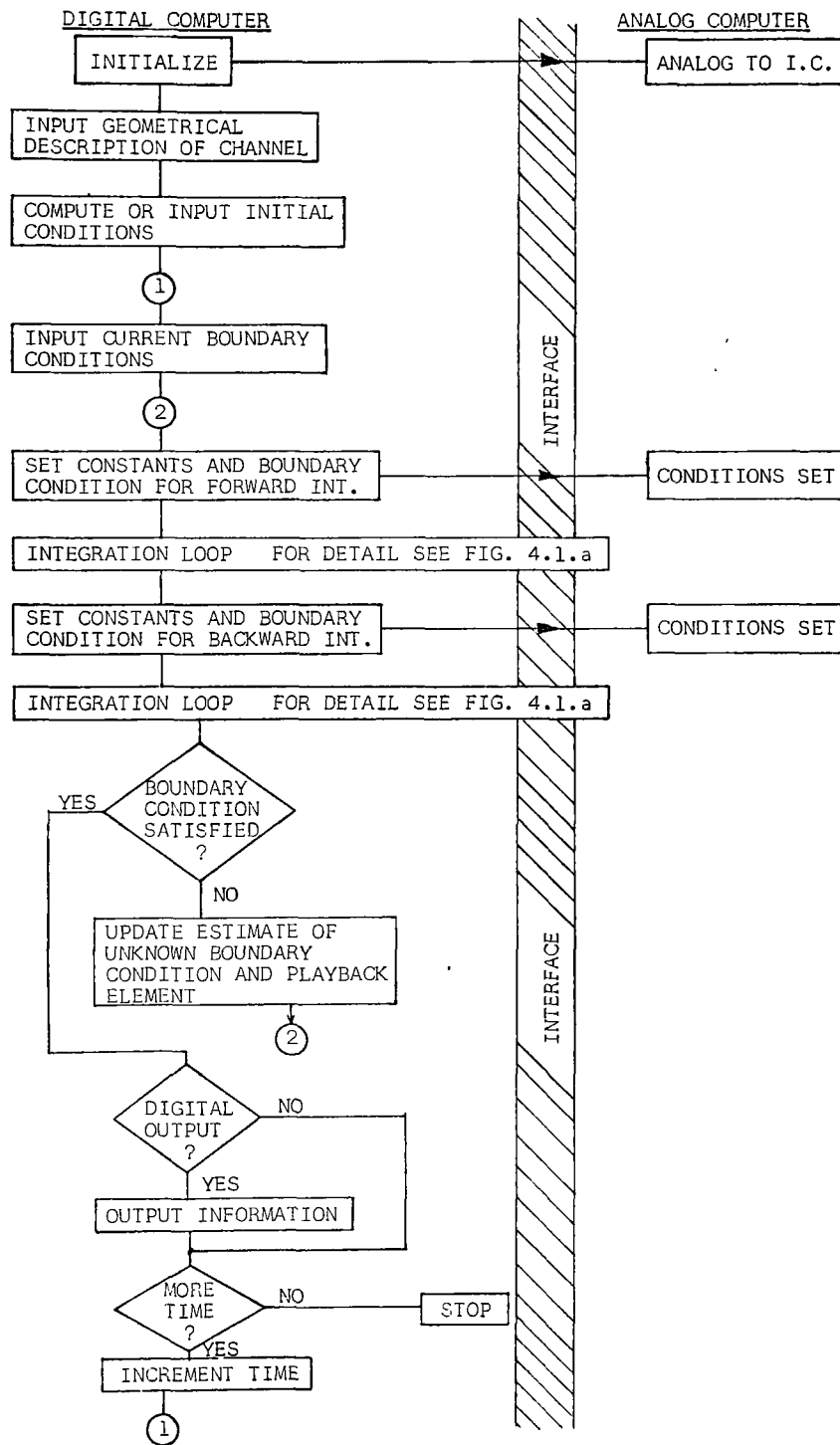


FIG. 4.1: GENERAL PROGRAM FLOW

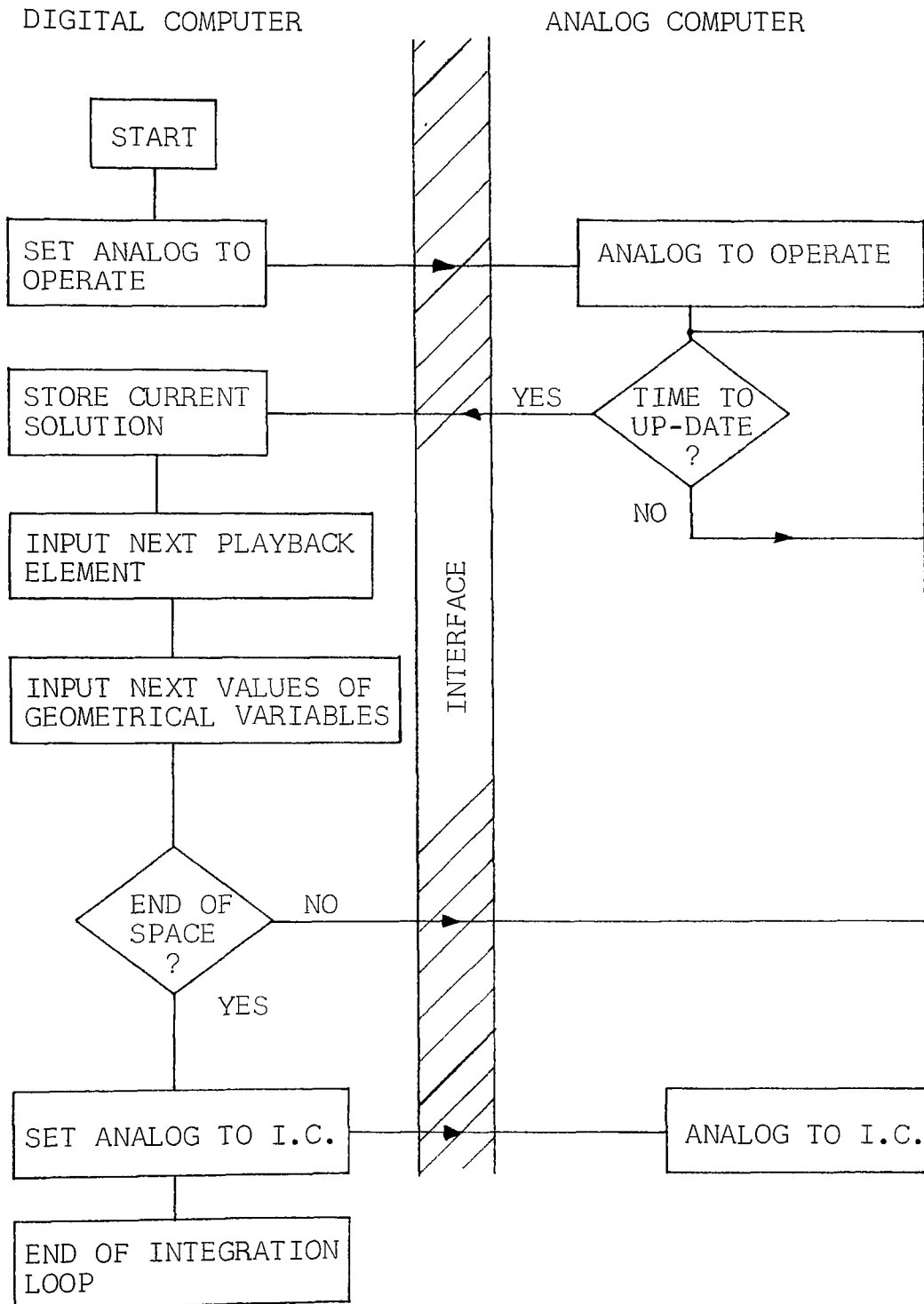


FIG. 4.1.a: INTEGRATION LOOP

numbers from -1.0 to +1.0. For this reason the analog representation of the differential equations must be scaled (Gagne, 1971) in such a way that none of the components have values beyond this range.

4.1.1.1 Magnitude scaling

The forward differential equation is (see Equation 3.26)

$$\frac{dW_1^j}{dx} = - \frac{W_1^j - W_1^{j-1}}{\lambda_1^j \Delta t} + \frac{\hat{b}_1^j}{\lambda_1^j} \quad 4.1$$

where $\lambda_1^j = + W_1^j \left(\frac{2+k}{4}\right) + W_2^j \left(\frac{2-k}{4}\right)$ 4.1a

$$\hat{b}_1^j = \frac{4g}{k} \frac{(-q - \left(\frac{W_1^j + W_2^j}{2}\right) \frac{dA}{dx})}{T(W_1 - W_2)} + g (S_o - S_f + D_\ell) \quad 4.1b$$

$$S_f = \frac{\frac{W_1^j + W_2^j}{2} \left| \frac{W_1^j + W_2^j}{2} \right|}{C^2 R} \quad 4.1c$$

$$D_\ell = \frac{(W_1^j + W_2^j)}{4gA} q_s \quad 4.1d$$

for seepage outflow

$$D_\ell = \frac{\frac{W_1^j + W_2^j}{2} - u_\ell}{Ag} q_i \quad 4.1e$$

for lateral inflow.

The variables in Equation 4.1 are W_1^j , W_1^{j-1} , W_2^j , q_s^j , q_ℓ^j , T^j , A^j , C^j , R^j , u_ℓ^j , S_o^j and $\dot{W}_1^j \left(\frac{dW_1^j}{dx}\right)$. Each of these

variables must be scaled to its maximum value. A subscript m will be used to denote these maxima, that is W_{1m} , W_{2m} , q_{sm} , etc. As well as the individual variables, whole terms must be scaled where there is a chance that these may exceed an absolute value of one.

Firstly, the left hand side of Equation 4.1 is scaled to $\dot{W}_m$ yielding

$$\left[\frac{\dot{W}}{\dot{W}_m} \right] = \frac{1}{\dot{W}_m} \left(\frac{W_1^j - W_1^{j-1}}{\lambda_1^j \Delta t} \right) - \frac{1}{\dot{W}_m} \left(\frac{b_1^j}{\lambda_1^j} \right) \quad 4.2$$

where the square brackets indicate that the contained quantity is scaled.

The right hand side of Equation 4.2 will be analyzed term by term. The top of the first term yields

$$- \left[\frac{W_1^j}{W_{1m}} \right] + \left[\frac{W_1^{j-1}}{W_{1m}} \right] \quad 4.3$$

Since both quantities are of the same sign their difference will never exceed one.

The bottom of the first term becomes

$$\left[\frac{\lambda_1^j \Delta t}{(\lambda \Delta t)_m} \right] = + \left(\frac{(2+k)}{4} \frac{\Delta t W_{1m}}{(\lambda \Delta t)_m} \right) \left[\frac{W_1^j}{W_{1m}} \right] + \left(\frac{(2-k)}{4} \frac{\Delta t W_{2m}}{(\lambda \Delta t)_m} \right) \left[\frac{W_2^j}{W_{2m}} \right] \quad 4.4$$

Thus the scaled equivalent of the first term becomes

$$\frac{\left(\frac{W_{1m}}{\dot{W}_m (\lambda \Delta t)_m} \right) \left(\left[\frac{W_1^j}{W_{1m}} \right] - \left[\frac{W_1^{j-1}}{W_{1m}} \right] \right)}{- \left(\frac{(2+k)}{4} \frac{\Delta t W_{1m}}{(\lambda \Delta t)_m} \right) \left[\frac{W_1^j}{W_{1m}} \right] - \left(\frac{(2-k)}{4} \frac{\Delta t W_{2m}}{(\lambda \Delta t)_m} \right) \left[\frac{W_2^j}{W_{2m}} \right]} = \frac{1}{\dot{W}_m} \left(\frac{W_1^j - W_1^{j-1}}{\lambda_1^j \Delta t} \right) \quad 4.5$$

Similarly the top and bottom of the second term can be analyzed. The bottom of the second term becomes

$$\frac{\lambda_1^j}{\lambda_m} = + \left(\frac{(2+k)}{4} \frac{W_{1m}}{\lambda_m} \right) \left[\frac{W_1^j}{W_{1m}} \right] + \left(\frac{(2-k)}{4} \frac{W_{2m}}{\lambda_m} \right) \left[\frac{W_2^j}{W_{2m}} \right] \quad 4.6$$

The top of the second term can again be analyzed in parts as

$$\frac{b_1^j}{\lambda_m \dot{W}_m} = \frac{4g}{k\lambda_m \dot{W}_m} \left(\frac{-q - \frac{W_1^j + W_2^j}{2} \frac{dA}{dx}}{T(W_1^j - W_2^j)} \right) + \frac{g S_o - g S_f + g D_\ell}{\lambda_m \dot{W}_m} \quad 4.7$$

The constituents of q will be broken down to outflow due to seepage and evaporation (q_s) and inflow due to precipitation on the channel and lateral inflow (q_ℓ). The first term in scaled form then becomes

$$\frac{4g}{k\dot{W}_m \lambda_m} \left(\frac{-q_{sm} \left[\frac{q_s}{q_{sm}} \right] - q_{\ell m} \left[\frac{q_\ell}{q_{\ell m}} \right] - A_m \left[\frac{A}{A_m} \right] \frac{W_{1m}}{2} \left[\frac{W_1^j}{W_{1m}} \right] + \frac{W_{2m}}{2} \left[\frac{W_2^j}{W_{2m}} \right]}{T_m \left[\frac{T}{T_m} \right] \left(W_{1m} \left[\frac{W_1^j}{W_{1m}} \right] - W_{2m} \left[\frac{W_2^j}{W_{2m}} \right] \right)} \right) \quad 4.8$$

Multiplying both top and bottom by $\frac{2W_{1m}}{2W_{1m}}$ and collecting the premultiplying factors yields

$$\begin{aligned} & \left(\frac{2g q_{sm}}{k\dot{W}_m \lambda_m W_{1m} T_m} \right) \left[\frac{q_s}{q_{sm}} \right] - \left(\frac{2g q_{\ell m}}{k\dot{W}_m \lambda_m W_{1m} T_m} \right) \left[\frac{q_\ell}{q_{\ell m}} \right] - \left(\frac{g \dot{A}_m}{k\dot{W}_m \lambda_m T_m} \right) \left[\frac{\dot{A}}{\dot{A}_m} \right] \left[\frac{W_1^j}{W_{1m}} \right] - \\ & \left[\frac{T}{T_m} \right] \left((0.5) \left[\frac{W_1^j}{W_{1m}} \right] - \left(\frac{W_{2m}}{2W_{1m}} \right) \left[\frac{W_2^j}{W_{2m}} \right] \right) \\ & \frac{\left(\frac{g \dot{A}_m W_{2m}}{k\dot{W}_m \lambda_m W_{1m} T_m} \right) \left[\frac{\dot{A}}{\dot{A}_m} \right] \left[\frac{W_2^j}{W_{2m}} \right]}{\left[\frac{T}{T_m} \right] \left((0.5) \left[\frac{W_1^j}{W_{1m}} \right] - \left(\frac{W_{2m}}{2W_{1m}} \right) \left[\frac{W_2^j}{W_{2m}} \right] \right)} \quad 4.9 \end{aligned}$$

The channel bottom slope term simply becomes

$$\left(\frac{g S_{om}}{\lambda_m \dot{W}_m} \right) \left[\frac{S_o}{S_{om}} \right] \quad 4.10$$

The frictional losses term becomes

$$\frac{g S_f}{\dot{W}_m \lambda_m} = \frac{\left(\frac{W_{1m}^2 g}{2 C_{m m}^2 R_{m m} \lambda_m} \right) \left[\frac{W_1^j}{W_{1m}} \right] + \left(\frac{W_{1m} W_{2m} g}{2 C_{m m}^2 R_{m m} \lambda_m} \right) \left[\frac{W_2^j}{W_{2m}} \right] \left| \left(\frac{1}{2} \right) \left[\frac{W_1^j}{W_{1m}} \right] + \left(\frac{W_{2m}}{2 W_{1m}} \right) \left[\frac{W_2^j}{W_{2m}} \right] \right|}{\left[\frac{R}{R_m} \right] \left[\frac{C}{C_m} \right]^2} \quad 4.11$$

The dynamic effects of lateral inflow and outflow term becomes

$$\begin{aligned} \frac{g D_\ell}{\dot{W}_m \lambda_m} = & \frac{\left(\frac{q_{sm} W_{1m}}{4 W_m \lambda_m A_m} \right) \left[\frac{W_1^j}{W_{1m}} \right] + \left(\frac{q_{sm} W_{2m}}{4 W_m \lambda_m A_m} \right) \left[\frac{W_2^j}{W_{2m}} \right] \left[\frac{q_s}{q_{sm}} \right]}{\left[\frac{A}{A_m} \right]} \\ & + \frac{\left(\frac{q_{\ell m} W_{1m}}{2 W_m \lambda_m A_m} \right) \left[\frac{W_1^j}{W_{1m}} \right] + \left(\frac{q_{\ell m} W_{2m}}{2 W_m \lambda_m A_m} \right) \left[\frac{W_2^j}{W_{2m}} \right]}{\left[\frac{A}{A_m} \right]} \left[\frac{q_\ell}{q_{\ell m}} \right] \end{aligned} \quad 4.12$$

Because of the limit to the number of variables that can be transferred quickly on digital to analog channels it was assumed in Equation 4.12 that the lateral inflow enters with zero longitudinal velocity, that is, U_ℓ in Equation 4.1e is zero.

Of course, the backward equations must be scaled in similar fashion, however, details of it will not be shown.

4.1.1.2 Analog wiring arrangement

Components on the analog computer are arranged to represent the scaled equations. Premultiplying scaling constants are set on potentiometers. Values of variables are transferred from the digital computer on digital to analog (D/A) conversion channels. Interpolation is used to reconstruct data to continuous form from the finite points in the digital by means of an analog reconstruction circuit (Bekey, 1968). A typical reconstruction circuit is shown in Fig. 4.2 and a detailed description of its operation is given in Appendix I as an example of analog computation.

The development of the analog circuitry follows the development of the scaled equations. Firstly, a circuit of the overall equation, Equation 4.2, is given in Fig. 4.3. The forcing term of Equation 4.2 is shown as a module in Fig. 4.3 with details given in Fig. 4.3a as they are represented by Equation 4.7. Again, further definition of the components of the forcing term are presented in Figs. 4.3a.1, 4.3a.2 and 4.3a.3 as they are given in Equations 4.12, 4.11 and 4.9, respectively.

The special checkout language for the analog (H.O.I.) simply checks in a systematic fashion the value after each component by comparing the value read on the analog to that computed from the equation the circuit represents. If differences between computed values and observed values exceed a specified tolerance (usually 6 parts

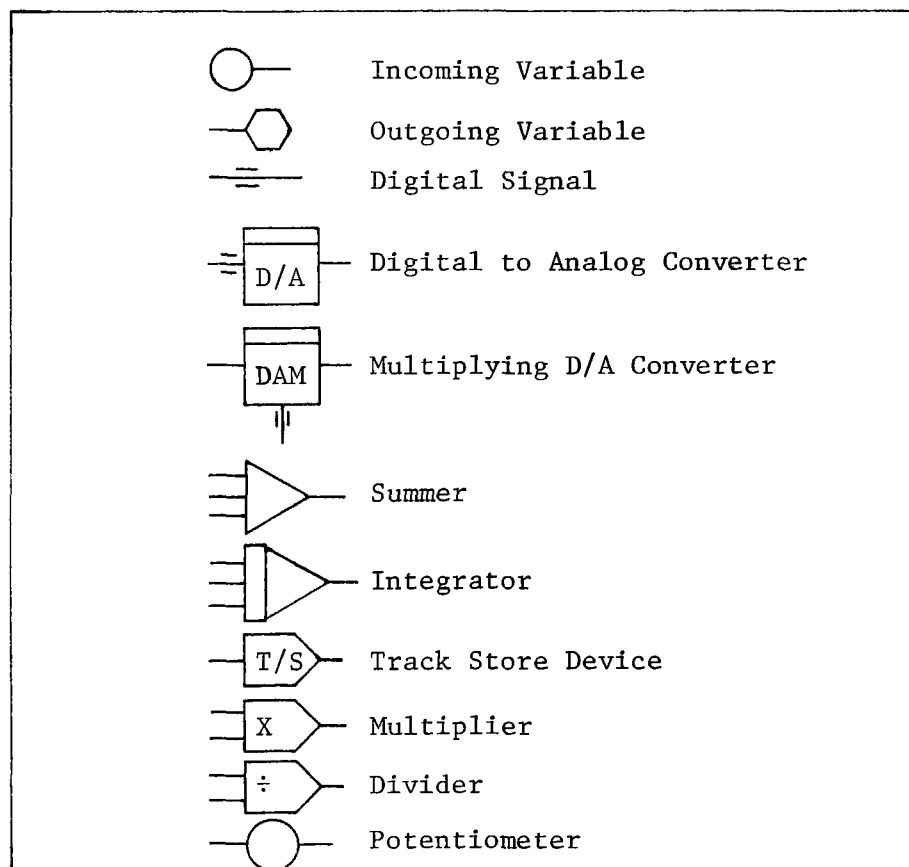
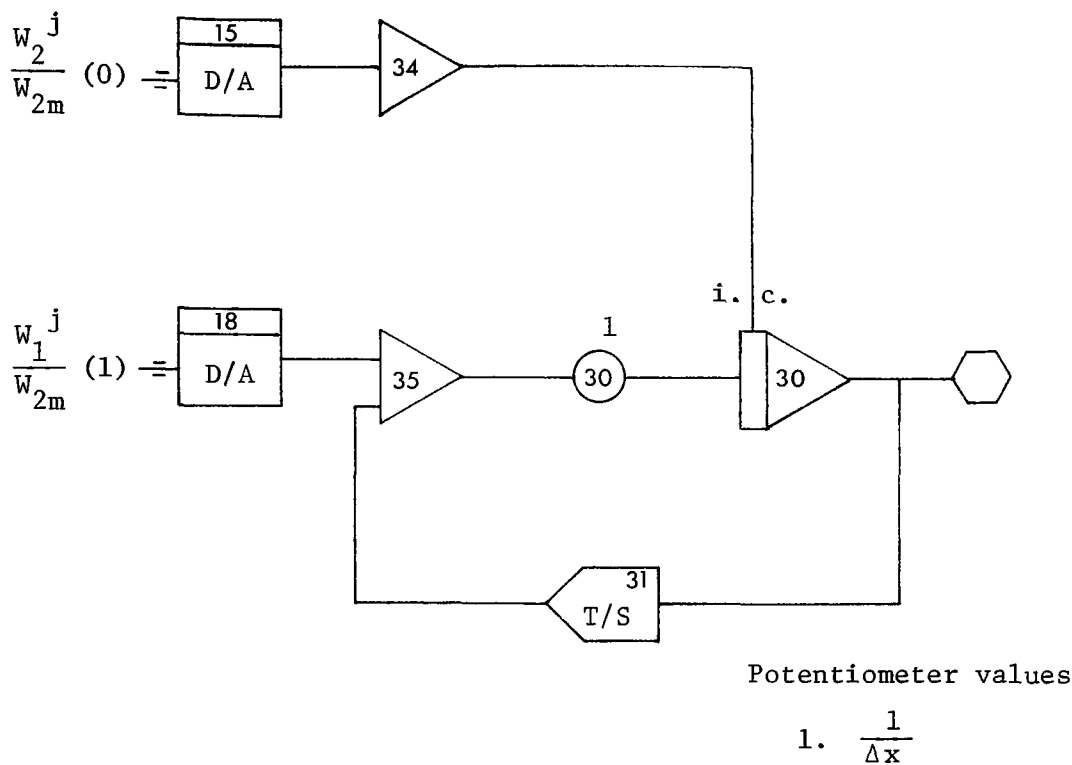


Figure 4.2 Data Reconstruction Circuit

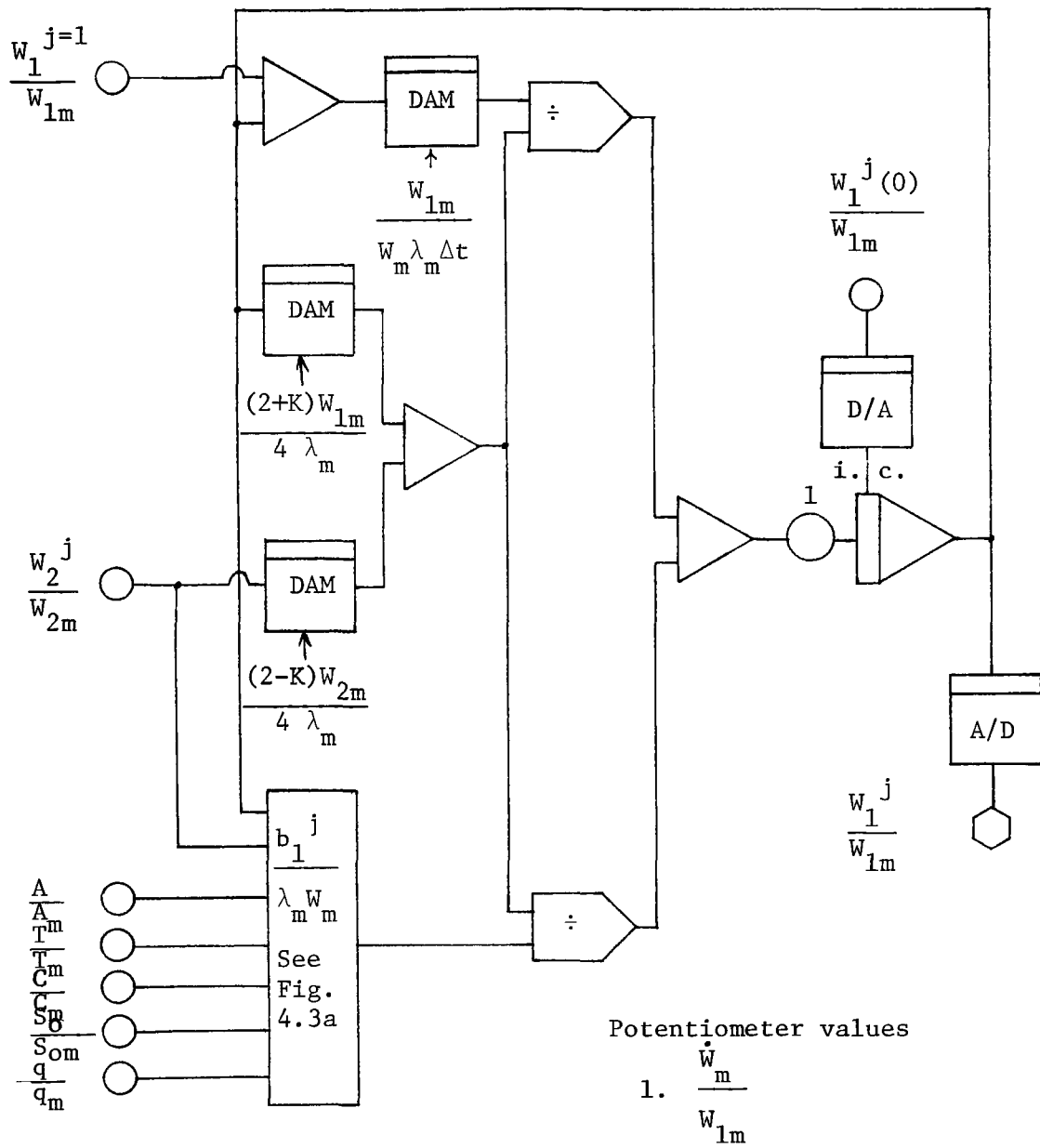
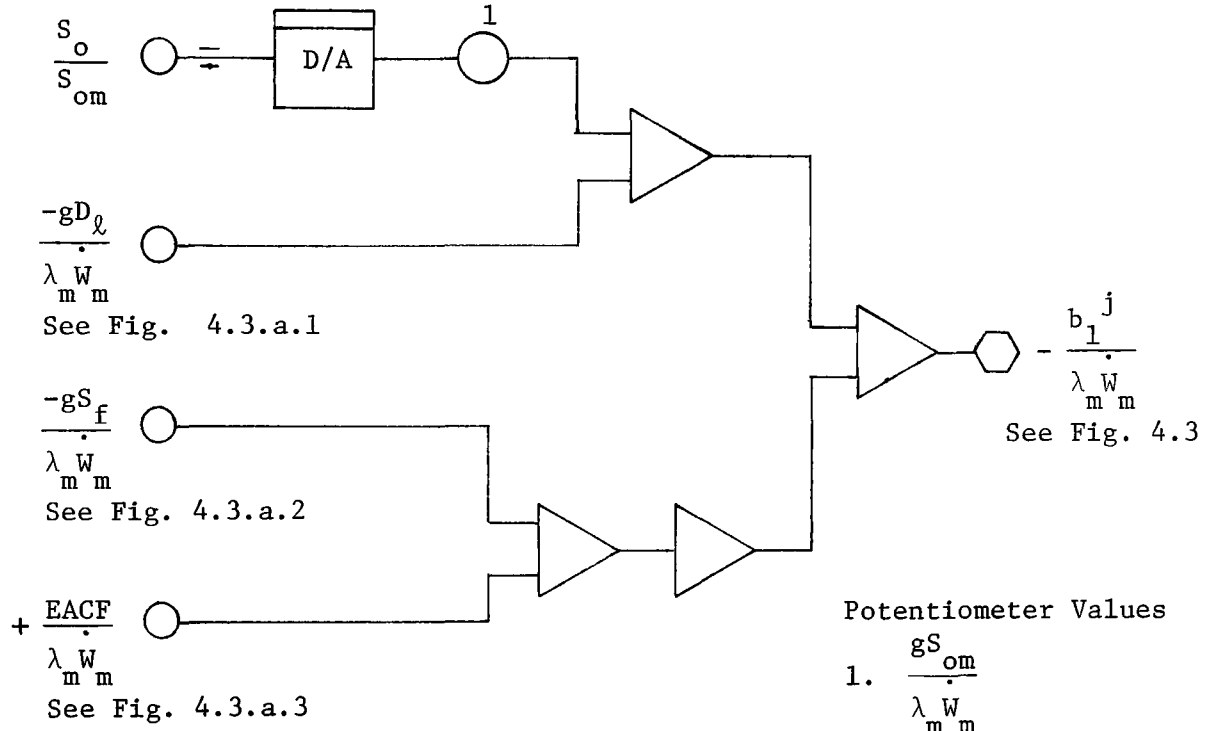
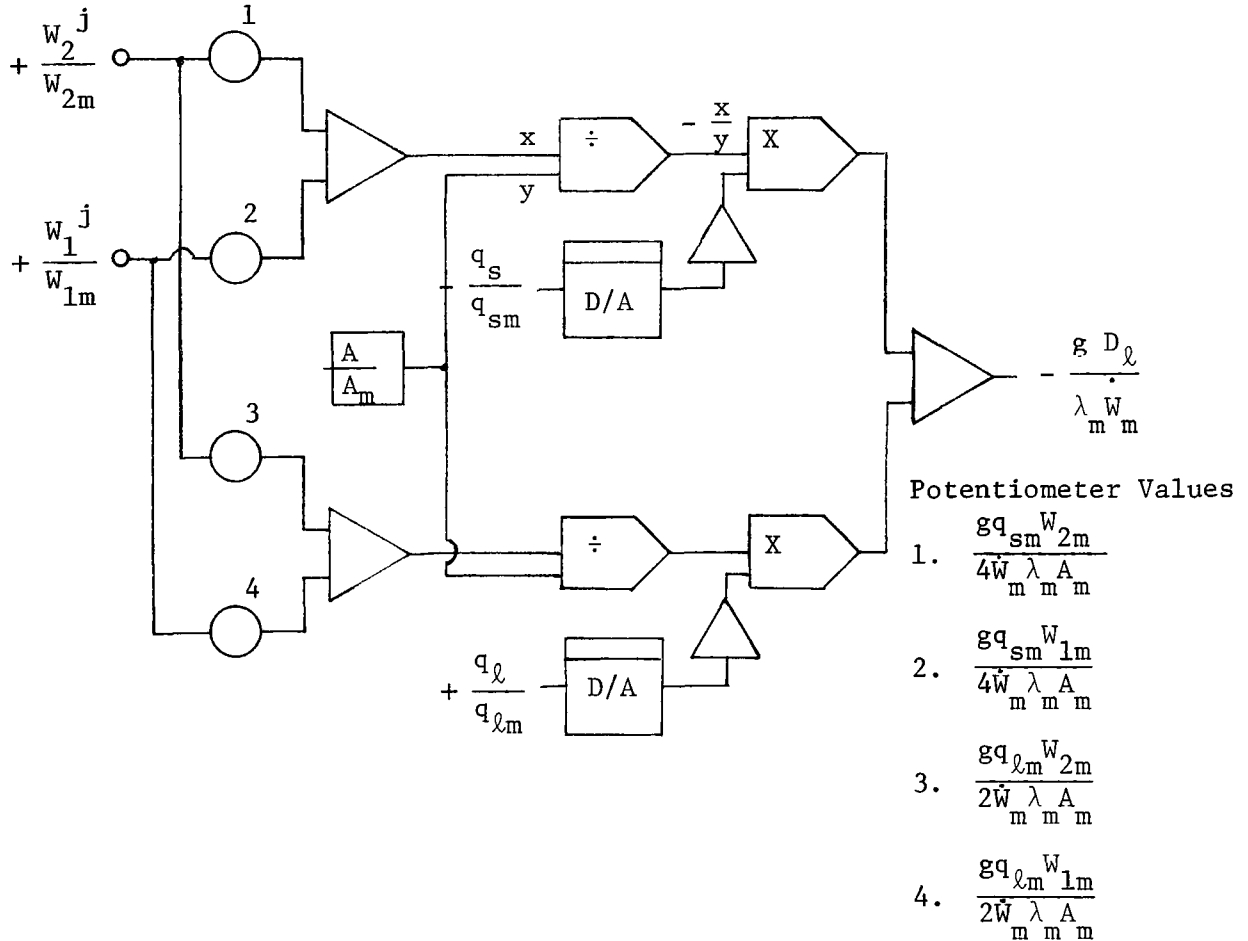


Figure 4.3 General Arrangement of Forward Integration Circuit
(Representation of Eq. 4.2)

Figure 4.3.a Summary of Forcing Terms (b_1)
(Representation of Equation 4.7)



Detailed Circuit of the D_ℓ Fig. 4.3.a.1 (represents Eq. 4.12)



Detailed Circuit of S_f Fig. 4.3.a.2 (represents Eq. 4.11)

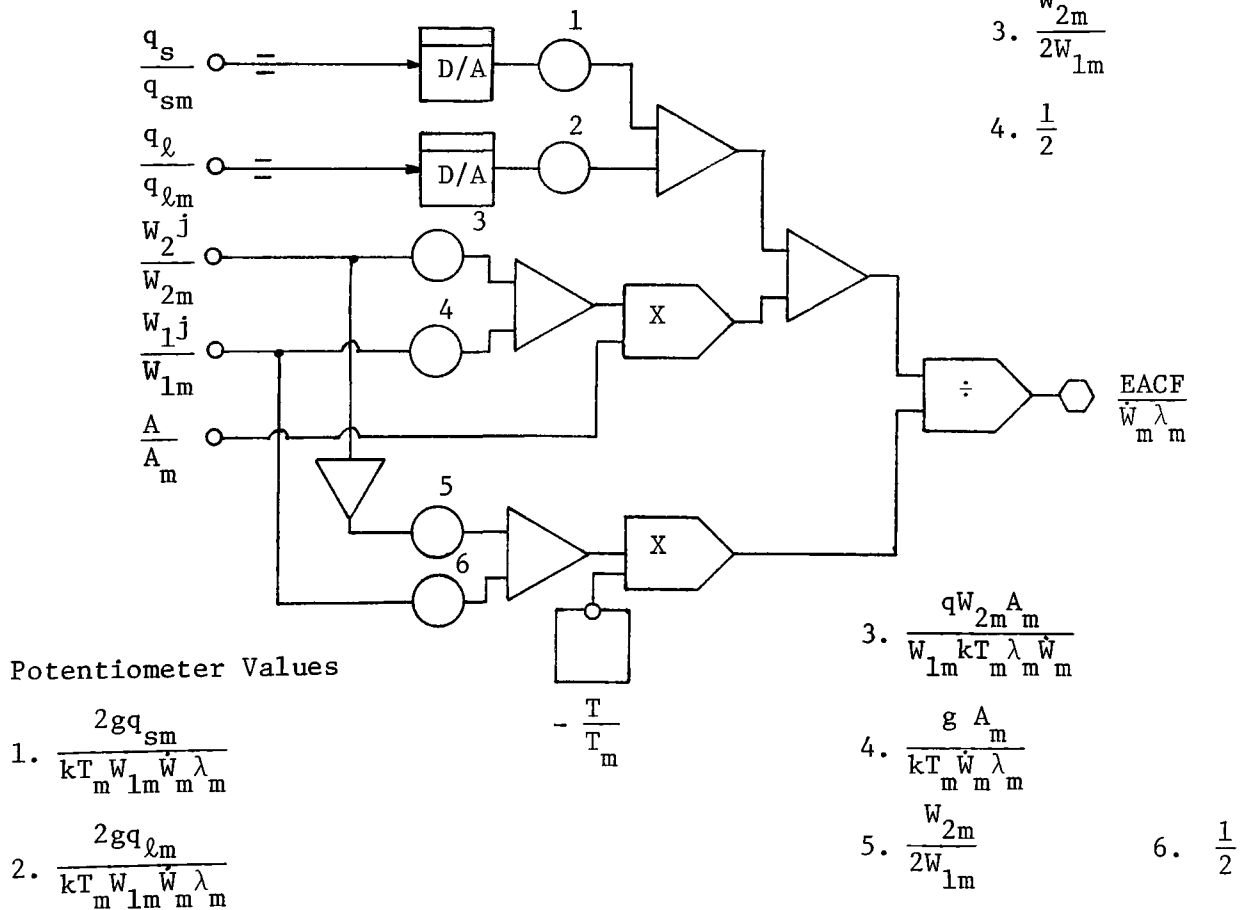
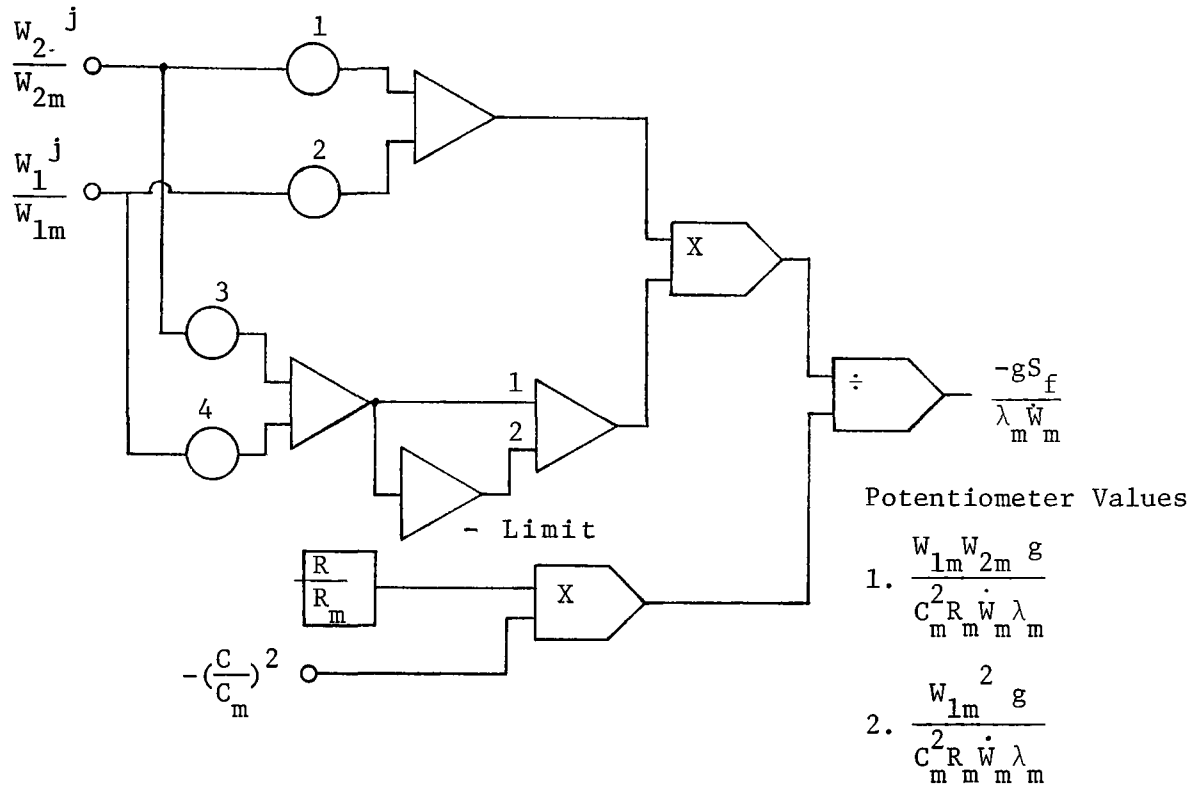


Figure 4.3.a.3 Detailed Circuit of Forcing Due to Expanding or Contracting Channel. (represents Eq. 4.9)

in 10^4) an error is printed out on the typewriter. Errors due to incorrect wiring or faulty components can be localized very quickly by such a program. Before each computing session this checkout program is run to ensure that all components used are still operating correctly and that no wires have been disconnected since the last session.

Only the solution to the forward questions has been described here. The backward equation is very similar with the exception of several signs and constants. The new values of the constants associated with the eigenvalues are input via D/A multipliers so that the digital computer simply inputs the values associated with the backward integration. The sign changes are obtained by switching in additional inverters for the backward integration.

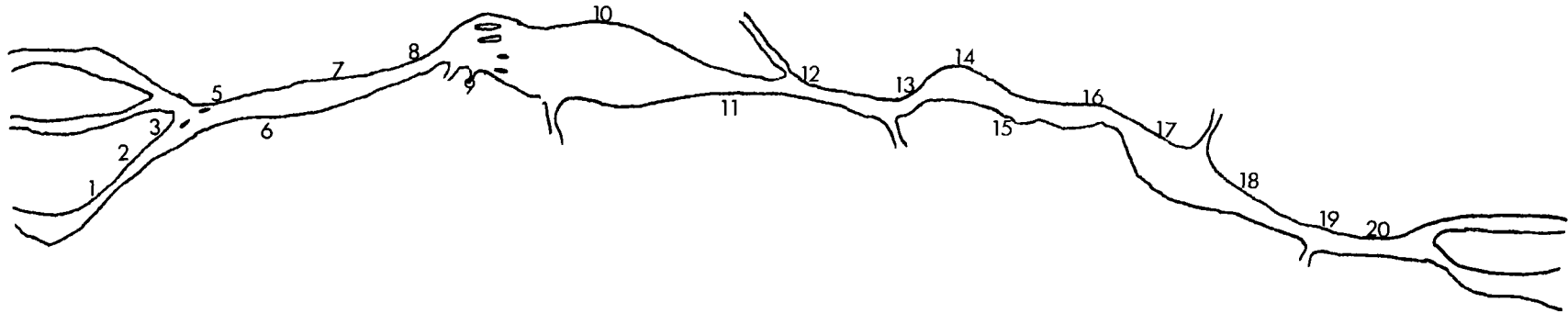
A variety of accessory equipment exists for the output of results (Gagne, 1971). On the analog side, results can be displayed on a screen or plotted on X-Y plotters or strip chart recorders. The response of any part of the circuit can be studied by plotting its output in a desired fashion. On the digital side output is available on a teletype typewriter, a line printer, a Calcomp plotter or punched paper tape.

CHAPTER 5

APPLICATION OF THE HYBRID COMPUTER MODEL

Several factors governed the selection of the particular prototype to which the model was applied. The most important was the ability to control the prototype such that tests could be specified without waiting for their occurrence in nature. Furthermore, it is much more convenient to gather extensive amounts of data in an environment where delicate instruments and extensive recording equipment are available. For these reasons the artificial channel of the St. Lawrence River Tidal Model at the Hydraulics Laboratory, National Research Council was chosen for the application of the mathematical model. This channel will henceforth be called the prototype, and is shown in Fig. 5.1.

The total prototype channel length is 750 feet of which a 413-foot section was chosen which best fits the assumptions of one dimensionality. The flow at the upstream end is controlled by a weir and head-tank and can be controlled through a range of 0 to 0.5 c.f.s. The downstream level can be controlled by a variable pitch turbine pump which can be operated to simulate tidal conditions at Father Point. Lateral inflow through tributaries is possible, however, this capability was not utilized for the purpose of the present tests.



Station No.	Name	Location (Ft. from Frontenac)	Station No.	Name	Location
1	Frontenac St.	0.0	11	Port-St. Francois	193.78
2	Longue Pte.	11.62	12	Trois Rivieres	209.62
3	Pte.-aux-Trembles	21.30	13	Champlain	241.30
4	Varennnes	30.63	14	Batiscan	261.36
5	Repentigny	39.87	15	Cap-à-la Roche	278.52
6	Verchères	53.07	16	Grondines	290.40
7	Lavaltrie	74.98	17	Portneuf	319.71
8	Lanoraie	91.08	18	Neuvville	358.78
9	Sorel	114.84	19	Quebec Bridge	395.21
10	Louiseville	154.18	20	Quebec City	412.37

Figure 5.1 Plan view of Laboratory Channel of St. Lawrence River

The control system for the prototype is centered on an E.A.I. 640 similar to the digital computer used in the mathematical model. Most operations can be controlled from the teletype input of the computer. Data gathering from the channel can be either preprogrammed or manually operated.

Float type water level gauges are located along the channel at various locations. In the selected reach there are 20 water level gauges. Fig. 5.1 illustrates the location of the gauges along with the midstream distances downstream from a gauge called Frontenac Street at what is Montreal in the natural St. Lawrence.

5.1 Geometry of the prototype

Data on the cross-sections were obtained by measuring depth below a known reference at distances across the stream. Readings were taken at a minimum of 30 points in the cross-section but at minimum spacings of 0.1 feet. Assuming the channel bottom to vary linearly between points of measurement allowed the computation of various channel parameters from the basic data by a simple computer program. Appendix 2 contains sketches of the observed cross-section shapes along with the required variable values as a function of water level in the channel in tabular form. Values of depth relative to a standard reference (I.G.L.D.), depth relative to the channel bottom and the corresponding values of cross-sectional area, wetted perimeter and top width all measured in inches and decimals are recorded in the tables in Appendix 2.

5.1.1 Hybrid computer representation of geometry

As discussed in the next chapter, it was found that the shape of the channel cross-section could best be represented by a parabola and the channel bottom slope by a Fourier series. The computing of the parabola focal length f and $\frac{df}{dx}$ and the channel bottom slope S_0 took place on the digital computer from the algebraic expressions derived in the next chapter. The digital input the variable values at frequent locations along x and a continuous representation was obtained on the analog by a linear reconstruction circuit as previously demonstrated. It may have been possible to construct a complete analog representation of the geometry, however, insufficient hardware was available for this purpose.

The geometric variables used in the differential equations are S_0 , A , T , and $\frac{dA}{dx}$. Fig. 5.2 illustrates the relationship between area A , top width T , and height of water above the channel bottom h , for the parabolic channel. The analog circuit deriving the geometric variables from S_0 , f , and $\frac{df}{ds}$ are shown in Fig. 5.3

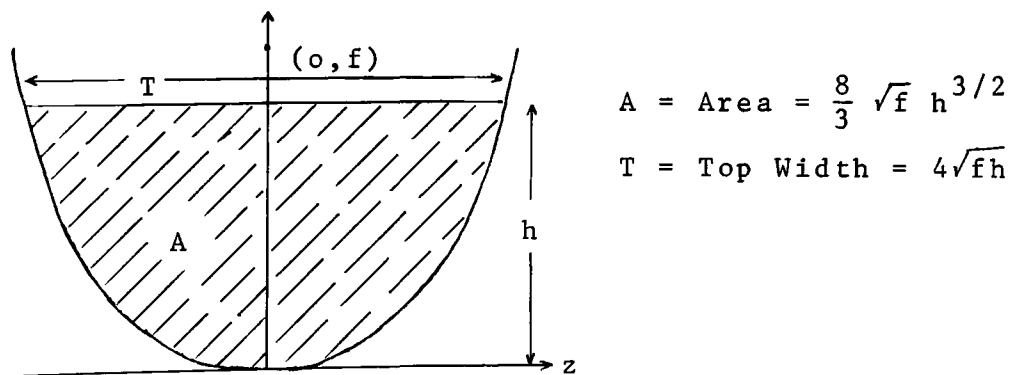
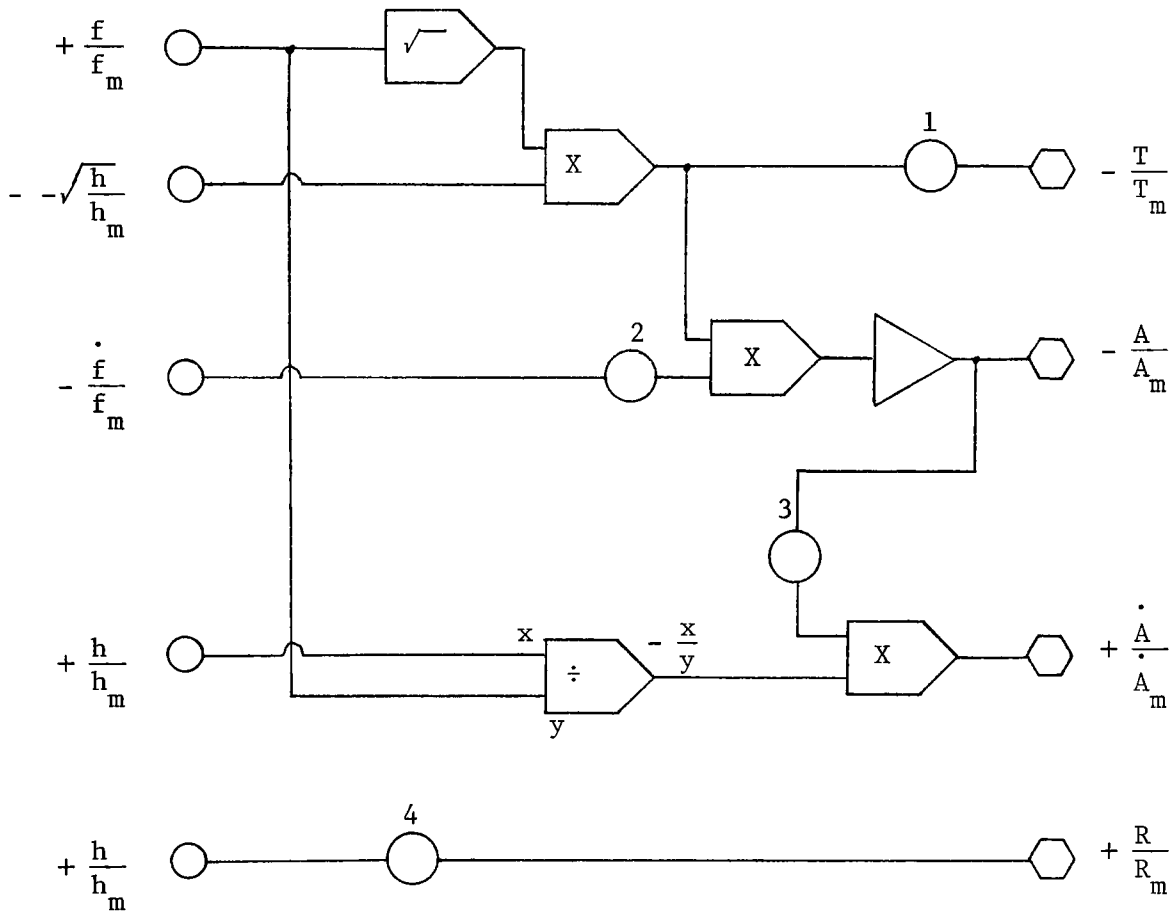


Figure 5.2 The Parabolic Channel



Potentiometer values

$$1. \quad 4 f_m^{1/2} h_m^{1/2}$$

$$3. \quad A_m \dot{f}_m$$

$$2. \quad \frac{8}{3} \frac{h_m^{3/2} f_m^{1/2}}{A_m}$$

$$4. \quad \frac{h_m}{C_{wp} R_m}$$

Figure 5.3 Analog Representation of Geometry

5.2 Prototype experiments

The prototype was run under steady state conditions with the aim of calibrating and verifying the mathematical model under conditions of steady nonuniform flow. Five tests were run and observations on water level and discharge were recorded after steady conditions had been reached. (Several hours elapsed between the time the controls were set for a given test until a steady state was reached.) The results of these tests are summarized in Table 5.4

5.3 Calibration of the mathematical model

The process of calibration of a mathematical model involves the adjustment of model parameters or coefficients in such a way as to minimize the differences between observation and model computations (Bekey, 1970). For the mathematical model developed in this study the Chezy coefficient, C , is used for calibration. The calibration procedure involves the selection of Chezy's C such that the observed values of water level correspond closely to the model output under invariant boundary conditions.

Keulegan (1943) has shown that C is a function of height, h , and distance, x . The problem is thus one of determining C in two dimensions.

For steady state conditions the governing equation of motion (see Equation 2.25) reduces to

$$\frac{V}{g} \frac{dV}{dx} + \frac{dh}{dx} = S_o - \frac{V|V|}{C^2 R}. \quad 5.1$$

Table 5.4 Steady State Tests on Prototype

Station	Location	Depth (feet)					
		Bottom IGLD	Test #1	Test #2	Test #3	Test #4	Test #5
1	0.0	-0.048	0.236	0.236	0.235	0.235	0.235
2	11.62	-0.104	0.286	0.285	0.284	0.289	0.284
3	21.38	-0.179	0.355	0.355	0.353	0.353	0.353
4	30.63	-0.247	0.416	0.415	0.414	0.414	0.414
5	39.87	-0.312	0.474	0.472	0.471	0.470	0.470
6	53.07	-0.371	0.529	0.527	0.525	0.524	0.524
7	74.98	-0.349	0.499	0.498	0.495	0.494	0.494
8	91.08	-0.323	0.469	0.467	0.464	0.463	0.462
9	114.84	-0.123	0.267	0.263	0.261	0.259	0.258
10	154.18	-0.233	0.362	0.360	0.355	0.354	0.352
11	193.78	-0.468	0.586	0.581	0.575	0.572	0.570
12	209.62	-0.443	0.558	0.554	0.547	0.543	0.542
13	241.30	-0.260	0.360	0.353	0.342	0.337	0.334
14	261.36	-0.200	0.292	0.283	0.269	0.262	0.257
15	278.52	-0.243	0.323	0.308	0.290	0.274	0.264
16	290.40	-0.321	0.396	0.382	0.354	0.334	0.318
17	319.71	-0.526	0.591	0.573	0.539	0.513	0.488
18	358.78	-0.472	0.536	0.517	0.481	0.453	0.425
19	395.21	-0.288	0.351	0.331	0.294	0.265	0.237
20	412.37	-0.313	0.375	0.356	0.318	0.289	0.260

Discharge at $x = 0$ 0.1521 c.f.s.

Solving for C gives

$$C = \left(\frac{V|V|}{(S_o - \frac{V}{g} \frac{dV}{dx} - \frac{dh}{dx})R} \right)^{1/2} \quad 5.2$$

where all terms are defined previously. Values of C can be computed from the steady state observations (Fig. 5.4) by assuming that $\frac{dh}{dx}$ can be approximated by a finite difference form. These values of C were then used as an initial guide in the two dimensional optimization problems.

5.4 Simulation experiments, steady flow

Once the mathematical model has been calibrated it was then tested against observations not used in the calibration. It was concluded that the model was a reasonable representation of the actual flow process and it was then used to simulate what might happen under conditions for which it was not calibrated. A number of simulation tests are formulated as they might arise in engineering problems.

5.4.1 Height boundary conditions

Under natural conditions it is difficult to determine discharge mean velocity while it is relatively easy to obtain water levels; hence it would be desirable to have a model which utilizes water levels only, in the specification of the boundary conditions. Thus the model was run for test #1 to determine if a specification of

water level at $x = 0$ and $x = x_{\max}$ was sufficient for good predictions.

For the forward integration the coupled variable W_2^j was obtained by linear extrapolation from the previous two time values. This estimate of W_2^j (W_2^*) was then combined with water level h to yield the boundary condition for W_1 in the form

$$W_1^{*j} = \sqrt{\frac{16 g h^j}{k}} + W_2^{*j} \quad 5.3$$

For the backward integration the current solution of W_1^j was combined with h^j to yield the boundary condition on W_2^j at $x = x_{\max}$. No further approximation is required at this point since the current value of $W_1^j(x)$ is played back, and $W_1^j(x_{\max})$ combined with $h^j(x_{\max})$ uniquely determines all requirements for the backward integration. The predicted water level was recorded at 3 locations in the channel and compared to those obtained from using both velocity and water level in the boundary conditions.

5.4.2 Sensitivity of the model to the Chezy coefficient

It was observed in the Hydraulic Laboratory Model (Ploeg, 1973) that the laboratory channel was very sensitive to changes in roughness at certain locations while in other areas a change in roughness did not affect the flow regime. It was decided to test if this was also the case in the mathematical model.

For this test, it would appear that if there are areas where the calibration is particularly sensitive, they would likely correspond to locations where the depth is at a minimum. These conditions occur at $x = 127$ feet, 264 feet and 401 feet. The Chezy coefficient was increased significantly (by 20%) over the calibrated value for a length of 14 feet surrounding these points, and the model was run for steady state test #1.

Similarly, the deepest sections were tested by increasing the Chezy coefficient by 20% at $x = 58$, 195 and 332 feet. The results of both of the tests are compared with those of the original simulation.

5.5 Simulation experiments, unsteady flow

5.5.1 Effect of changing downstream control

The model was permitted to attain the steady state condition of test #5, and then a control was introduced to raise the water level at the downstream end to that corresponding to test #1. The water level was actually changed in 5 discrete evenly spaced increments from 0.260 to 0.375 feet. The mathematical model was then run continuously to see if it could simulate the unsteady flow and eventually settle to the water levels observed in test #1. From a computational point of view it is desirable to take large time steps, providing accuracy is not sacrificed. Therefore, the same simulation was repeated 3 times with $\Delta t = 5$,

10 and 20 seconds to study the effect of Δt on the observed solution.

5.5.2 Flood routing

In order to investigate the usefulness of the mathematical model in flood routing, flood conditions were simulated using water level boundary conditions (see section 5.4.1 and 6.5.2). The mathematical model was started from a water level corresponding to that of steady state test #5, and then subjected to unsteady flow at $x = 0$ according to a triangular flood peak.

The boundary condition at $x = 0$ was supplied by Equation 5.3, using the triangular flood wave at $x = 0$. The boundary condition at $x = x_{\max}$ was supplied from the steady state (test #5) values, which is only valid until the flood wave reaches $x_{\max}$. Water levels at $x = 20, 40$ and 60 feet were recorded and plotted in order to study the flood propagation. The solution was obtained for a period of 70 seconds with $\Delta t = 1$ second and 60 playback locations.

5.6 Time scaling

In terms of computing time, the present model utilized a time scaling of one computer second representing 10 feet of space. This the time to integrate 413 feet was 41.3 computer seconds or approximately 83 seconds for each time step. Several possibilities exist for the speeding up the solution such as selecting a different time scale

(Wright, 1971), increasing Δt , and utilizing more analog capability. In general the limit to the solution speed was governed by the rate at which the digital could perform its share of the calculations and perform the data transfer functions. The speed of solution is, of course, important in economic terms and it can be critical in special problems where model predictions are used to control a physical system.

CHAPTER 6

RESULTS AND DISCUSSION

6.1 Representation of the geometry

An early version of the mathematical model utilized the geometry directly by a table look-up scheme to get values of the geometric variables. Assuming that the one-dimensional equations accurately described the process over all of the channel cross-section, the model was then run with what appeared to be reasonable first estimates of the Chezy coefficients. The results of this study were grossly at variance with the flow as it appeared in the prototype. Two particular sections were especially troublesome; the very wide section at Louiseville and the very deep section at the Quebec bridge.

At the very deep section the velocity would increase very rapidly with the steep slope leading to the deep section because of the size of the gravity acceleration force. Past the deep section the bottom slope had a large negative value and a rapid deceleration of the velocity would take place. In fact it was difficult to maintain positive values of velocity in the section from 395 to 413 feet.

Similarly the velocities and water levels in the very wide channel section could not be accurately modelled. In order to maintain water levels of the model output as they were observed in the prototype very large values of the Chezy friction coefficient had to be used. The very large

frictional coefficients and geometric discontinuities would then result in the model predicting the propagation of a wave in the downstream direction during the forward integration and in the upstream direction for the backward integration.

It was concluded at that time that the one-dimensional equations could not adequately describe the total flow process in the channel. However, the model may be adequate for the description of the flow in a portion of the channel cross-section. It was observed in the prototype that in very wide sections a portion of the channel cross-section has negligible velocity and does not contribute to the conveyance of water. Furthermore a close inspection of the water levels observed in the steady state tests (Table 5.4, and Fig. 6.1) demonstrated that the water levels are governed mainly by shallow portions of the channel at distances from Montreal of about 120, 270 and 440 feet. Small slope changes in the deeper channel sections appeared to have no influence on the flow whatsoever suggesting that the deepest portion of the channel acted simply as dead water storage in these regions.

If the conveyance portion of the channel could be separated from the non-conveyance portion, the one-dimensional model may be a reasonable representation of the process in that portion of the channel. Furthermore the waves observed to propagate in the model output near channel discontinuities

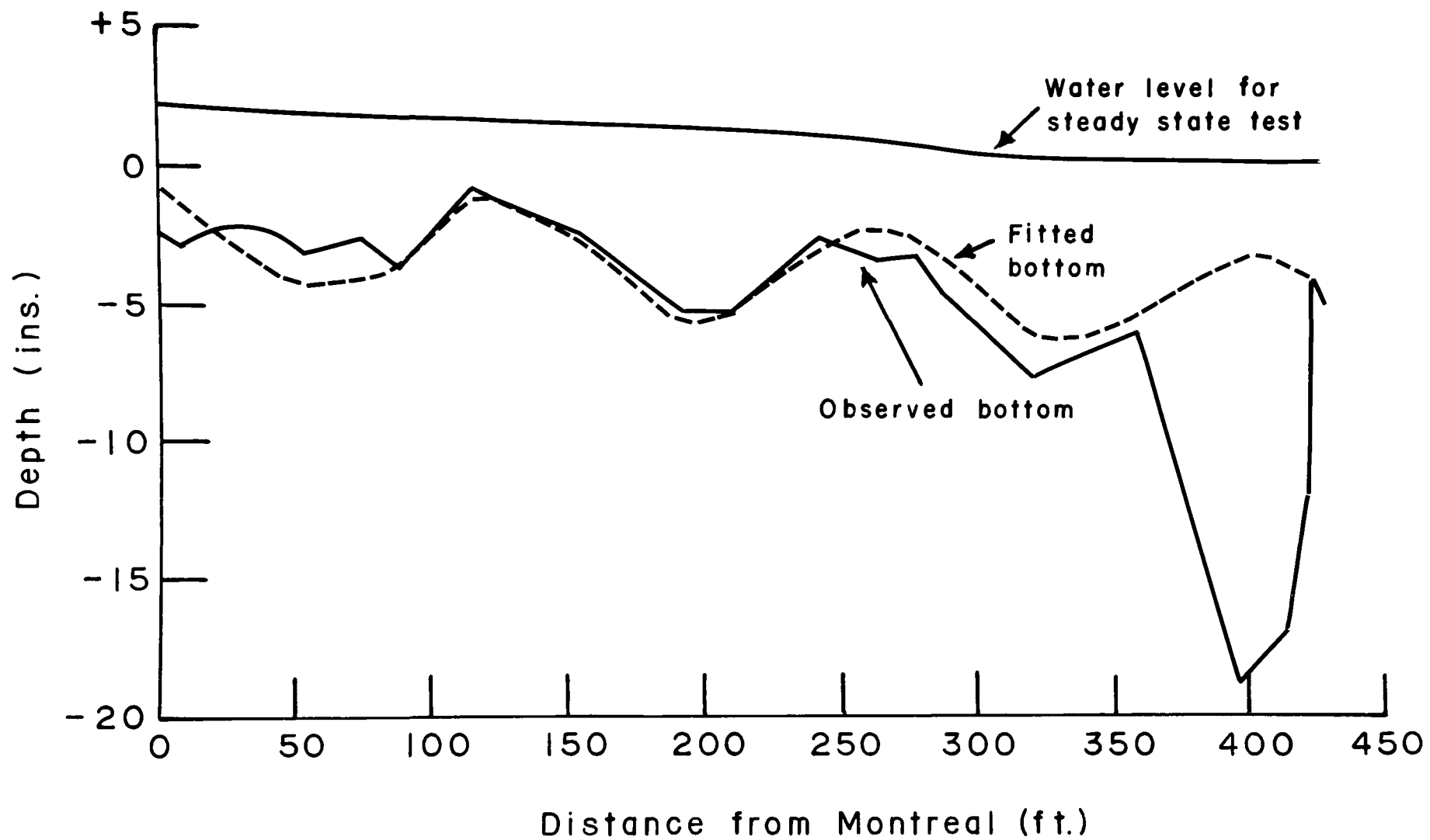


Fig. 6.1 Sectional side view of channel

suggested that sudden changes in geometry should be smoothed since these waves are not observed in the prototype flow. Therefore, it was decided to represent the conveyance portion of the channel by an algebraic model such that local irregularities in the channel bottom slope could be smoothed out, nonconveying portions of cross-sections eliminated and transitions made gradual and smooth.

A linear trend was fitted to the channel bottom and then removed from the data by subtracting. The remainder was subjected to a Fourier analysis (Jenkins, 1969) with the resulting spectrum shown in Fig. 6.2. Padding the data by adding zeros (Henry, 1971) improved the resolution of the computed spectrum significantly.

The spectrum (Fig. 6.2) demonstrates the existence of a major cycle with a period of 137 feet. (There is likely a good geomorphological explanation for the existence of such waves on the bottom of the St. Lawrence (Lane, 1955)). The channel bottom was thus represented by the following equation

$$z = -2.186 - 0.00785x + 1.913 \cos(0.046x + 0.458) \quad 6.1$$

where z is the bottom elevation in inches from (below) reference (IGLD), x is the distance in feet downstream of Montreal and the angle is measured in radians. Fig. 6.1 illustrates the actual channel bottom and that represented by Equation 6.1.

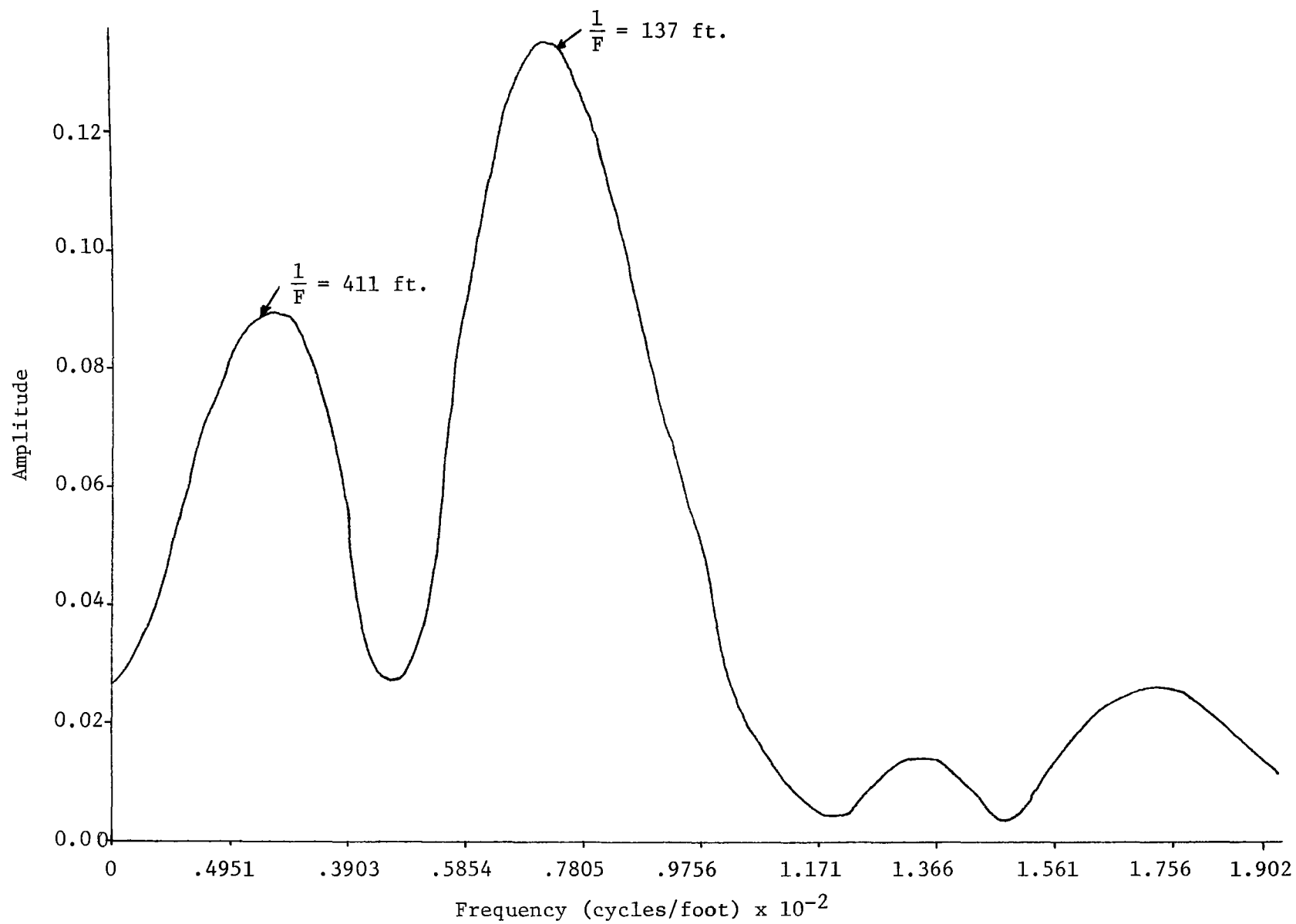


Figure 6. 2 Spectrum of Channel Bottom Space Series

The channel bottom slope S_o is defined as $-dz/dx$ and thus becomes

$$S_o = 0.00785 + 0.088 \sin(0.046x + 0.458) \quad 6.2$$

where S_o is the bottom slope in inches per foot.

Equations 6.1 and 6.2 are the mathematical representations of the channel bottom and are called the "effective" reference. When this effective bottom is above the observed bottom it is assumed that the area below this effective bottom does not contribute to the flow but acts simply as dead storage.

Further, an assumption was made in the theory leading to Equation 3.26, that the cross-sectional ratio A/T could be described by the relationship

$$\frac{A}{T} = kh \quad 6.3$$

The A/T ratios were computed for the observed cross-sections. It was found that in the range of water levels of interest, the average value of k for all sections was equal to 0.65. The theoretical value of k for the parabolic shape is $2/3$ or 0.66. Therefore, the parabola was chosen to be the shape which best preserved the A/T ratio at the surface water level. Thus the approach was to choose a parabola focal length f , which best fitted the observed cross-section data; with particular emphasis on area and top width and their ratio at the mean water surface elevation. In order to obtain a continuous representation of the channel geometry

along x it becomes necessary to interpolate between sections at which the focal length was estimated. To eliminate portions of the cross-section not contributing to water conveyance the parabola focal length values were smoothed along x. The approach to fitting was to first remove the trend in focal length with a straight line and then subject the deviations to a Fourier analysis. The resulting expression for f as a function of x is

$$f = 22.07 + 0.02324x + 5.044 \cos(0.0139x + 2.274) \\ + 1.706 \cos(0.0337x - 2.232) \quad 6.4$$

where f is the parabola focal length in inches.

Although the wetted perimeter can be derived from the focal length the expression for it becomes very complex and somewhat difficult to program. It was thus decided that it would be more appropriate to independently establish a relationship between height h and hydraulic radius R from the observed data in the form

$$R = \frac{A}{P} = \frac{h}{C_{wp}} \quad 6.5$$

where A is the cross-sectional area, P the wetted perimeter, and C_{wp} is the coefficient relating h to R. The average value of C_{wp} at the mean water level for all sections was 2.2.

Thus Equations 6.2, 6.4 and 6.5 represent a continuous space model of the conveyance section of the channel and it was then implemented along with the one-dimensional equations.

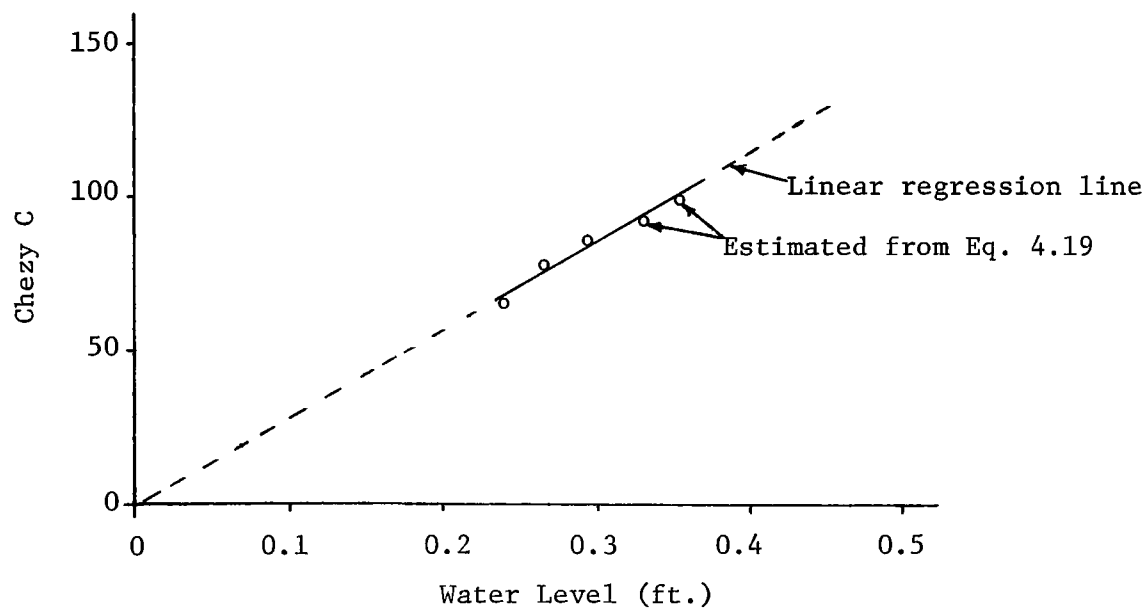


Figure 6.3 Chezy Coefficient-Depth Relationship at Station 19

6.2 Calibration of the mathematical model

As indicated in the previous chapter, the model calibration involves a two-dimensional selection of Chezy's C . The problem was simplified to a one-dimensional selection by establishing a relationship between C and height from the steady state observations.

The values of C computed from Equation 5.2 were plotted as a function of h at each of the observation stations (see Fig. 6.3 for an example). For the particular range of water levels tested, a correlation established that C may be treated as a linear function of h . The correlation coefficients ranged from a low of 0.94 to 1.00 for cases where the range of h was small. It was also found that the intercept passed through the origin, thus the calibration simply became one of determining the slope of the C versus h relationship for all of x . This was done for one of the steady state experiments (test #5).

The starting values of slope C were obtained from Equation 5.2 and then the mathematical model was run and a comparison made between prototype observations and model output. The differences between these two were minimized by a trial and error process; and the final values of slope C versus x are shown in Fig. 6.4. The agreement between observed and computed values of water level are shown in Fig. 6.5.

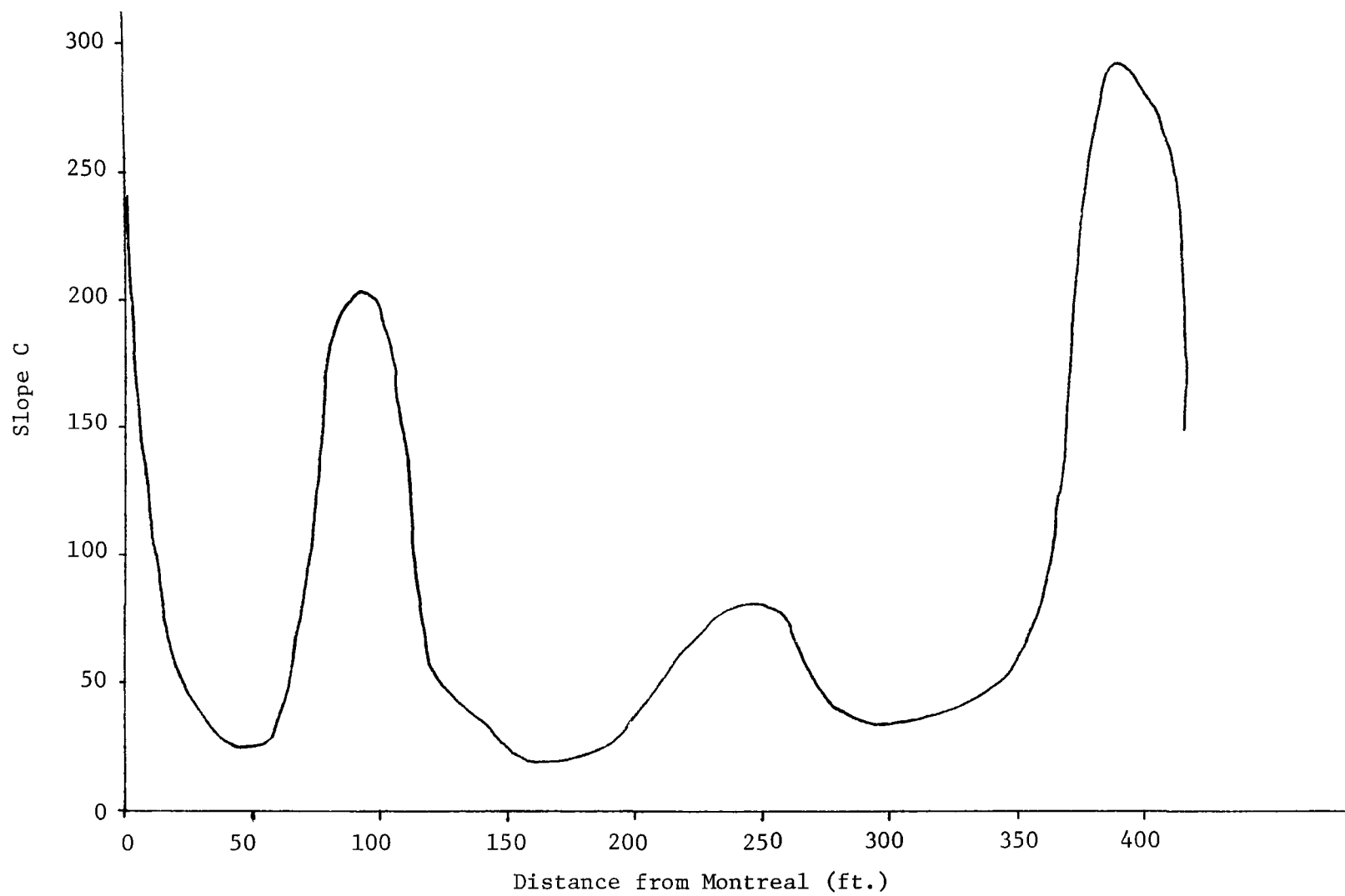


Figure 6.4 Slope C as a Function of Distance

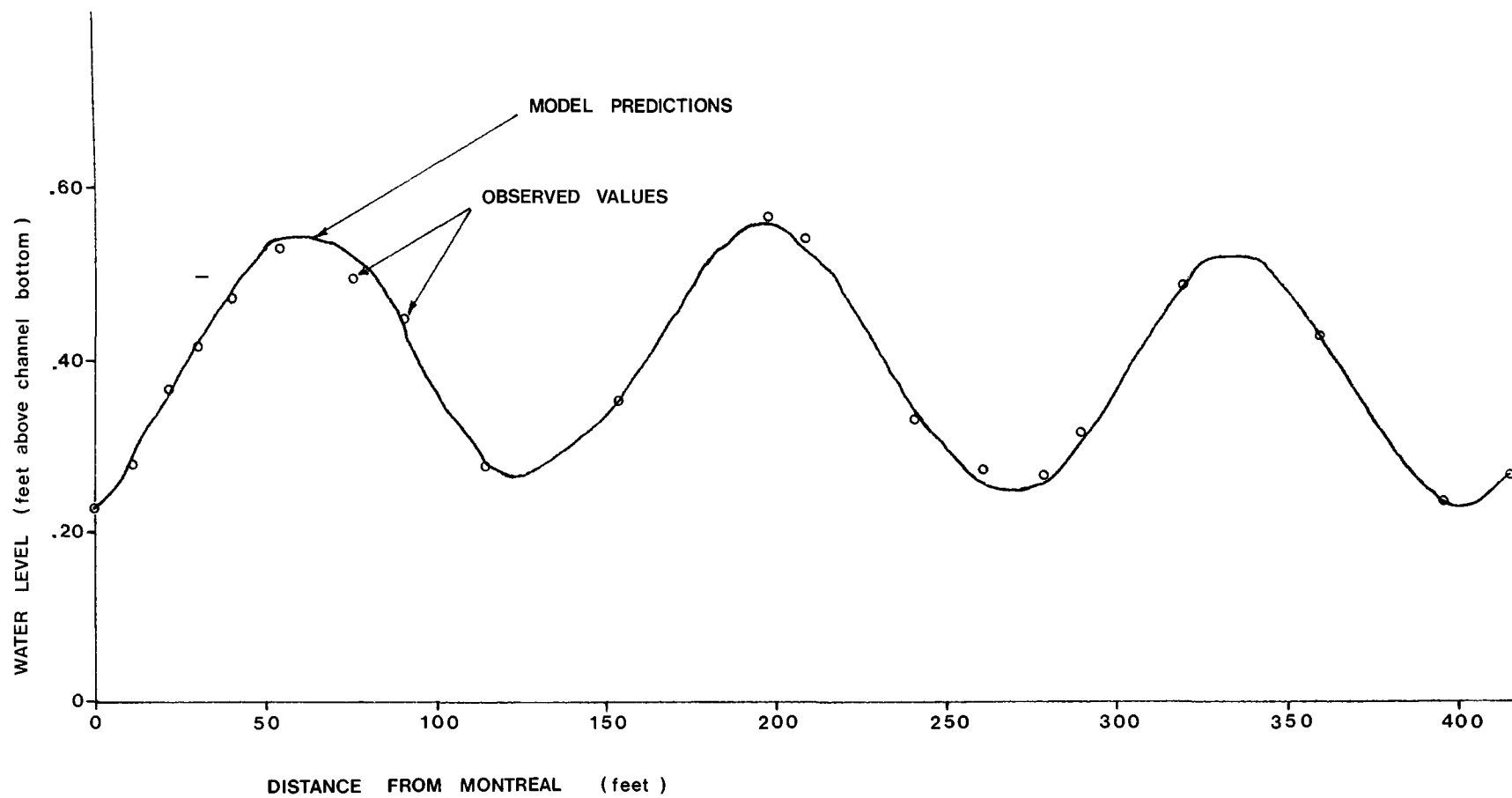


Figure 6.5 Calibration (Test #5)

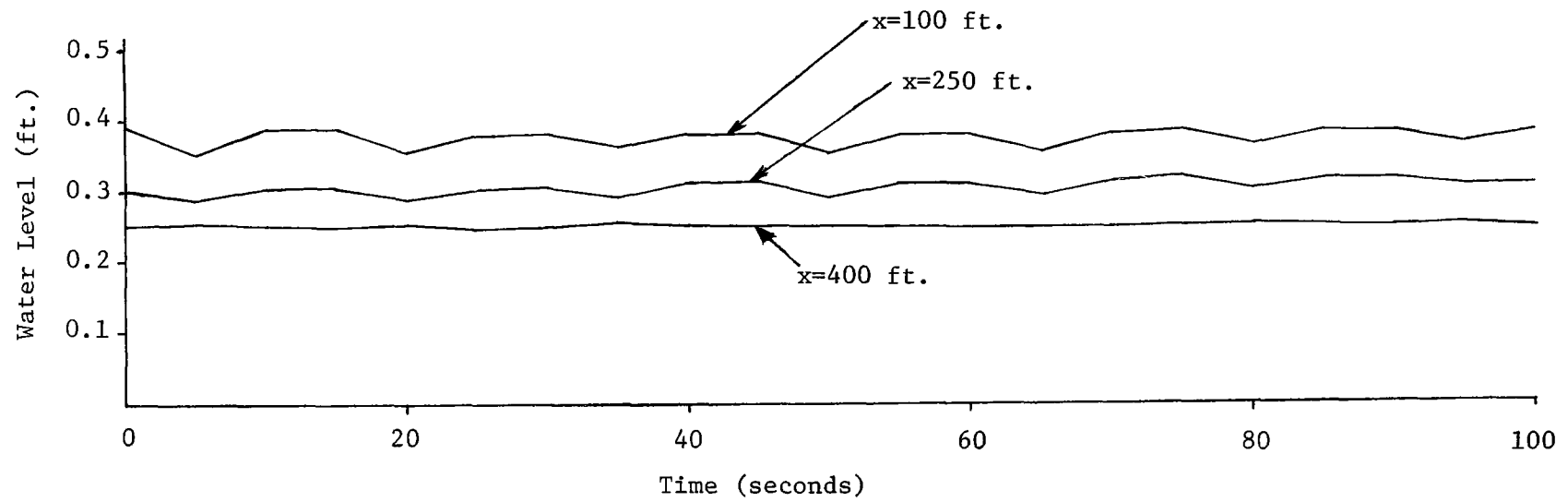


Figure 6.6 Mathematical Model Output for Steady State Conditions

6.2.1 Test of the stability of the solution

The stability of the solution was tested by integrating for some time with the boundary conditions in test #5. The solution was accepted and time incremented with each completed forward and backward integration. The results are displayed in Fig. 6.6.

Some cycling is observed in the solution. Because the cycles were similar to those observed when a large imbalance of forces existed due to poor calibration, it was decided that the cause of these cycles was some residual errors in the calibration. Since it was practically impossible to eliminate the cycles completely and since they do not appear to magnify with increasing time, a filtering process is suggested where the cycles in the solution confuse a desired result.

6.3 Verifying the model for steady state conditions

The remaining steady state runs (tests #1-#4) were used to verify the calibration. These simulations were run with $\Delta t = 5$ seconds and sixty playback locations. For each test the model was run a significant length of time (at least 10 time steps) to see if the water levels would change from their initial values. The results are illustrated in Fig. 6.7 and 6.8 which show the continuous space model predictions and discrete observations. An inspection of the results establish that the model predicts accurately

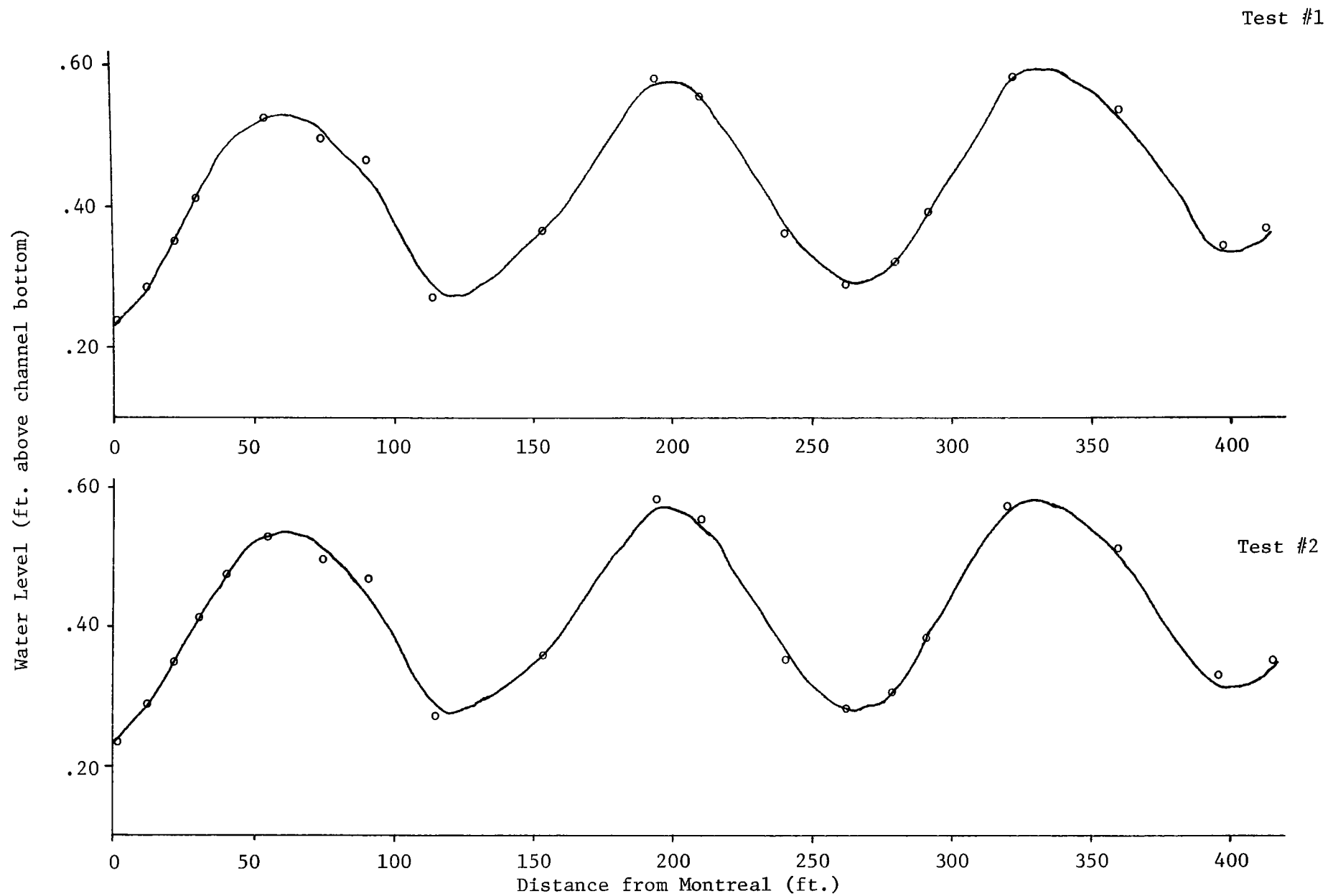


Figure 6.7 Model Test Runs 1 and 2

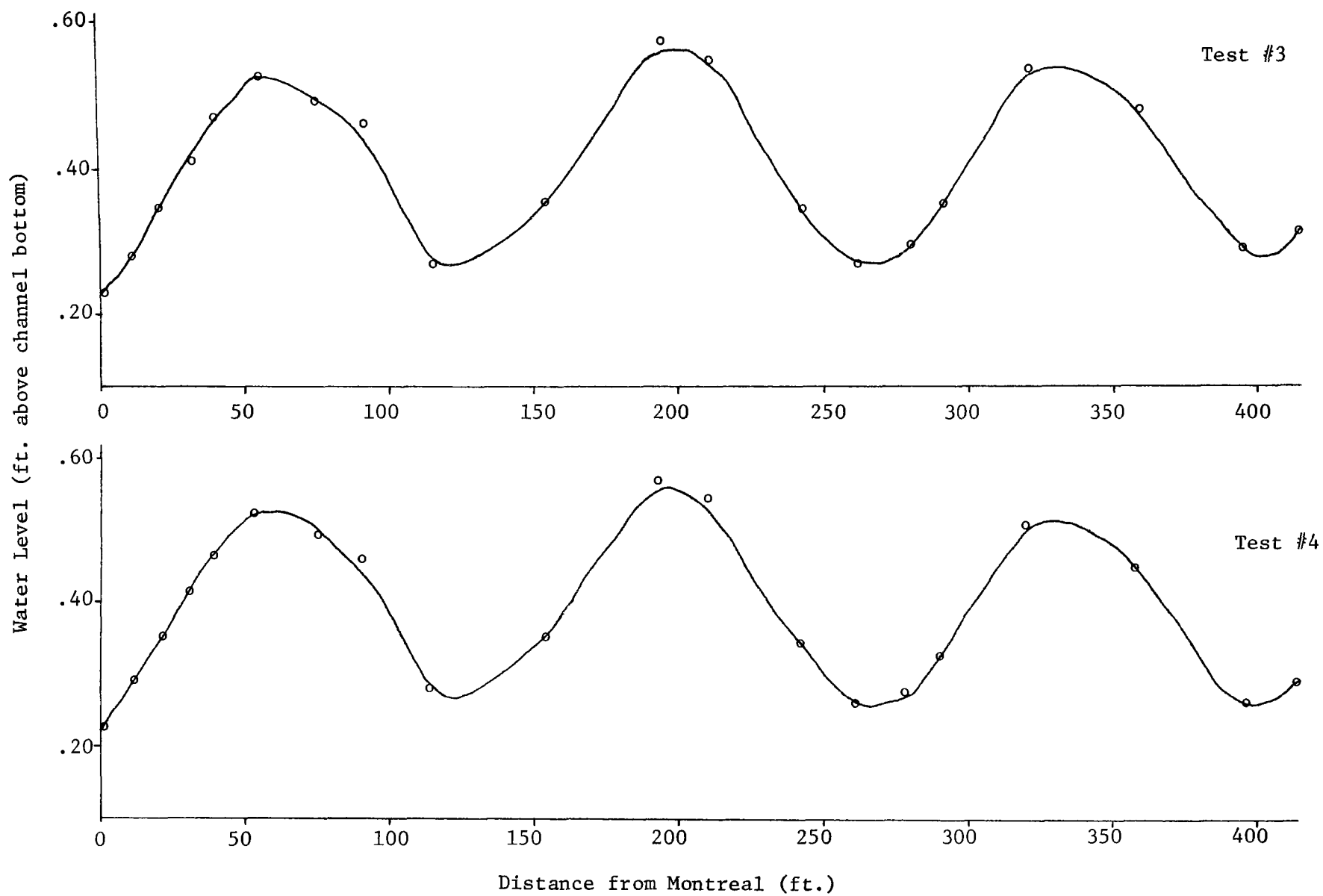


Figure 6.8 Model Test Runs 3 and 4

the water levels observed in the prototype. Of note is the excellent agreement obtained from 250 feet to 400 feet where the water levels change significantly from one test to the next. The good predictions verify the calibration procedure and establish the utility of the model in problems associated with steady nonuniform flow.

The advantages of a continuous space solution become particularly evident for problems in which the spatial mean velocity is of interest, that is, problems such as determining the travel time of a decaying pollutant. For such problems the analog computer can perform an integration to yield the time to travel through a section.

6.4 Results of simulation experiments - steady flow

6.4.1 Height boundary conditions

The predicted water level based on boundary conditions in height only was recorded at 3 locations along the channel and compared with those obtained using both velocity and water levels as boundary conditions (Fig. 6.9). With $\Delta t = 5$ seconds the solution after 40 time steps was not significantly different from that obtained by using both velocity and water level for boundary conditions. Thus water level predictions are reasonably well determined using height boundary conditions.

However, it was observed that the velocity predictions under height boundary conditions were less than

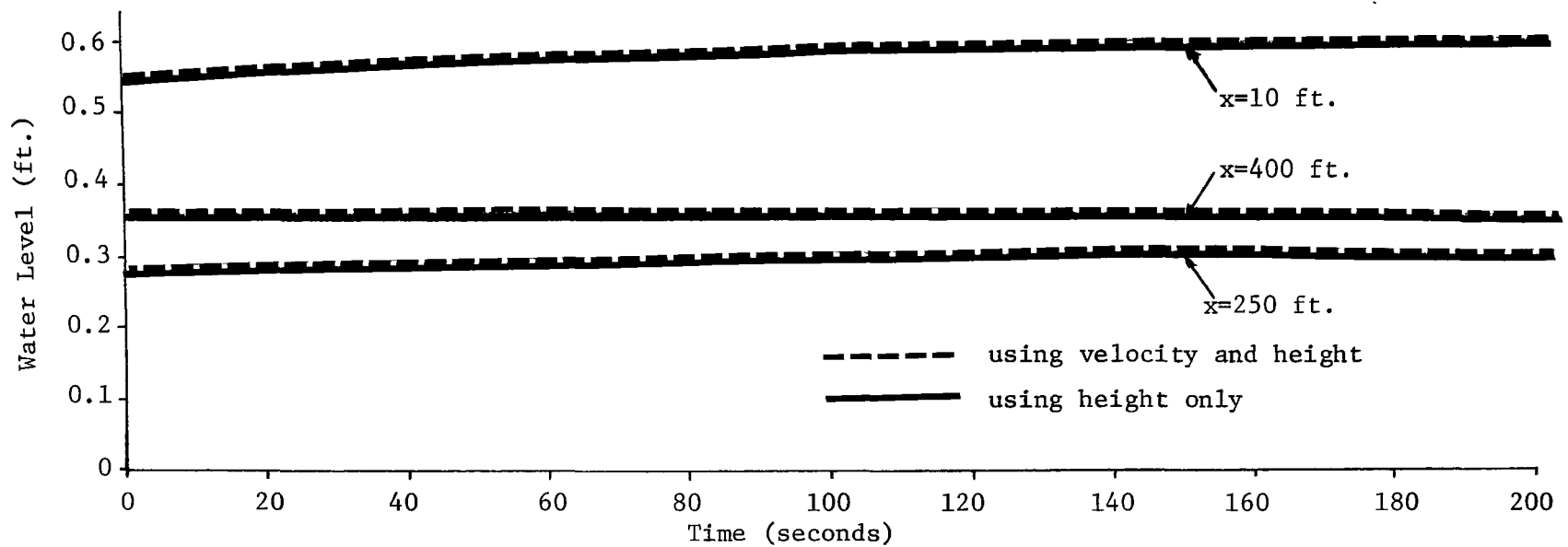


Figure 6.9 Water Level Predictions Based on Boundary Conditions in Height Only

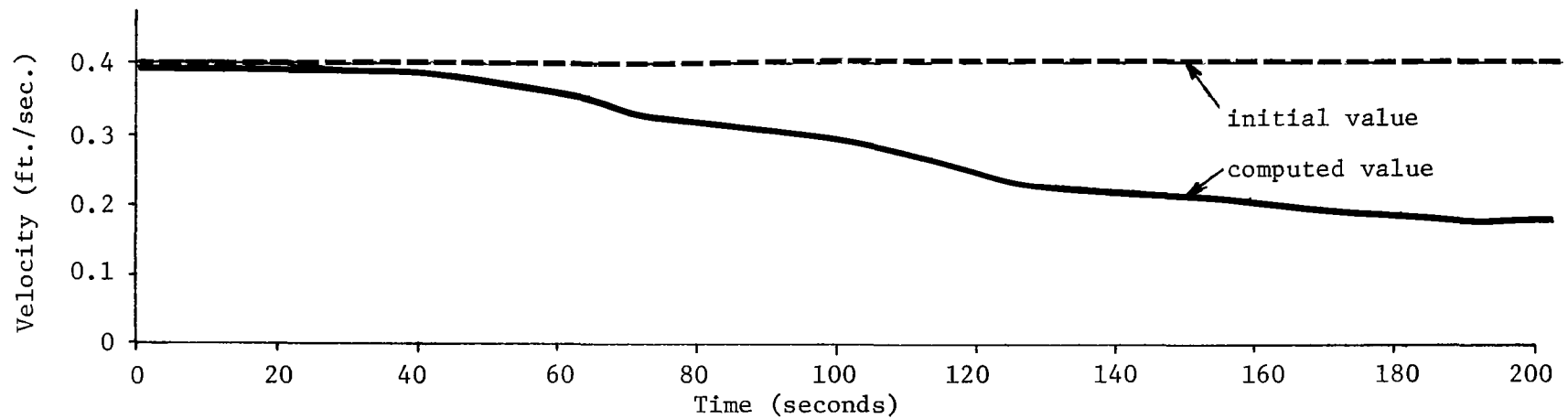


Figure 6.10 Velocity Predictions at $x = 0$ Based on Boundary Conditions in Height Only

satisfactory, especially near $x = 0$ (Fig. 6.10). This can be explained by going through an estimation cycle. The values of v , h , W_1 and W_2 at $x = 0$ for steady state test #1 are $v = 0.409$ ft./sec., $h = 0.236$ ft., $W_1 = 7.1446$ and $W_2 = -6.326$. If the estimate of W_2 is accurate, then combining h and W_2^* in Equation 5.3 yields the correct value of W_1 . However, if a 5% deviation exists in W_2^* from the correct value, that is $W_2^* = -6.642$, then the boundary condition in W_1^* becomes 6.829. Combining W_1^* and W_2^* to yield velocity results in $v^* = \frac{W_1^* + W_2^*}{2} = 0.093$, which is a gross deviation from the correct value of 0.409. If the errors are defined as $\epsilon_1 = W_1 - W_1^*$ and $\epsilon_2 = W_2 - W_2^*$, then a positive ϵ_2 will always result in a positive ϵ_1 by Equation 5.3. Since h is derived from the difference of W_1 and W_2 (see Equation 3.24) and v from the sum, the errors tend to cancel in the computation of h , while they accumulate in computations of v . Compounding the problem is the fact that the relative error in v is $\left| \frac{W_2}{v} \right|$ times the relative error in W_2 , hence the errors are magnified.

It may be concluded from this simulation study that using boundary conditions in water level can yield reasonable predictions of water level. However, if velocity is a required model output, at least one of the boundary conditions must be specified in velocity.

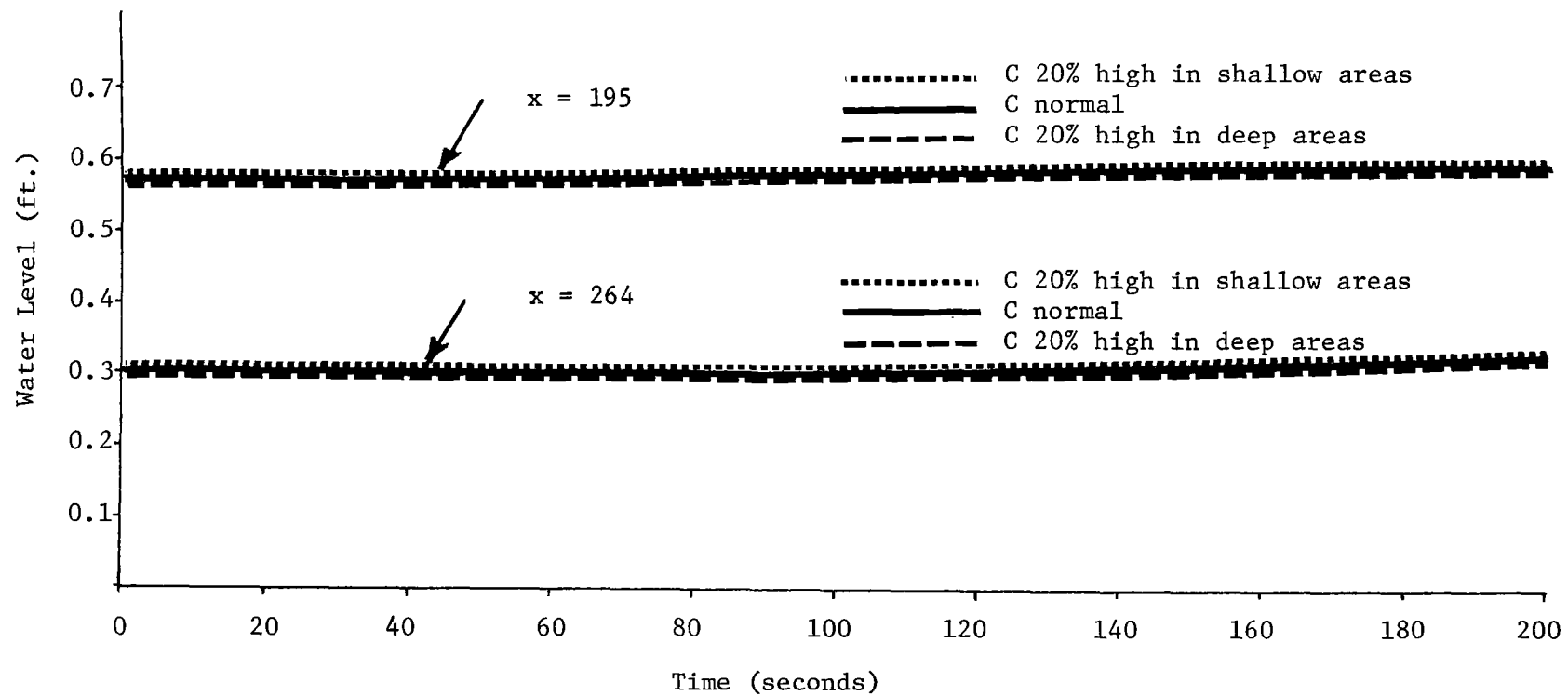


Figure 6.11 Sensitivity of Water Level to Changes in Chezy C

6.4.2 Sensitivity of the model to the Chezy coefficient

A typical result from the simulation to determine the sensitivity of the solution to changes in Chezy's C is illustrated in Fig. 6.11. After 200 seconds ($40 \times \Delta t$) no significant difference existed in predicted water levels. It was thus obvious that a local variation of 20% in the Chezy coefficient has no significant effect on model behaviour under steady state conditions. It also becomes evident that adding artificial roughness (i.e. a brick or a metal strip) to a section in the hydraulic model affects the Chezy coefficient by much more than 20%. Unfortunately it was not possible to change the Chezy coefficient beyond this range without substantial rescaling of the circuit.

6.5 Unsteady flow simulation

6.5.1 Changing downstream control

The simulated water level at $x = 400$ ft. is plotted for Δt values of 5, 10 and 20 seconds in Fig. 6.12. The results indicate that the model does predict the correct final steady state value (test #1). The water level predictions during the period of unsteady flow are dependent upon Δt , hence the accuracy of the simulation is a function of Δt . The results of plotting the unsteady flow period, using the beginning of water level changes at $x = 400$ as a starting time, is shown in Fig. 6.13 to reduce the effect of differing total times of change of the boundary conditions. Here again the

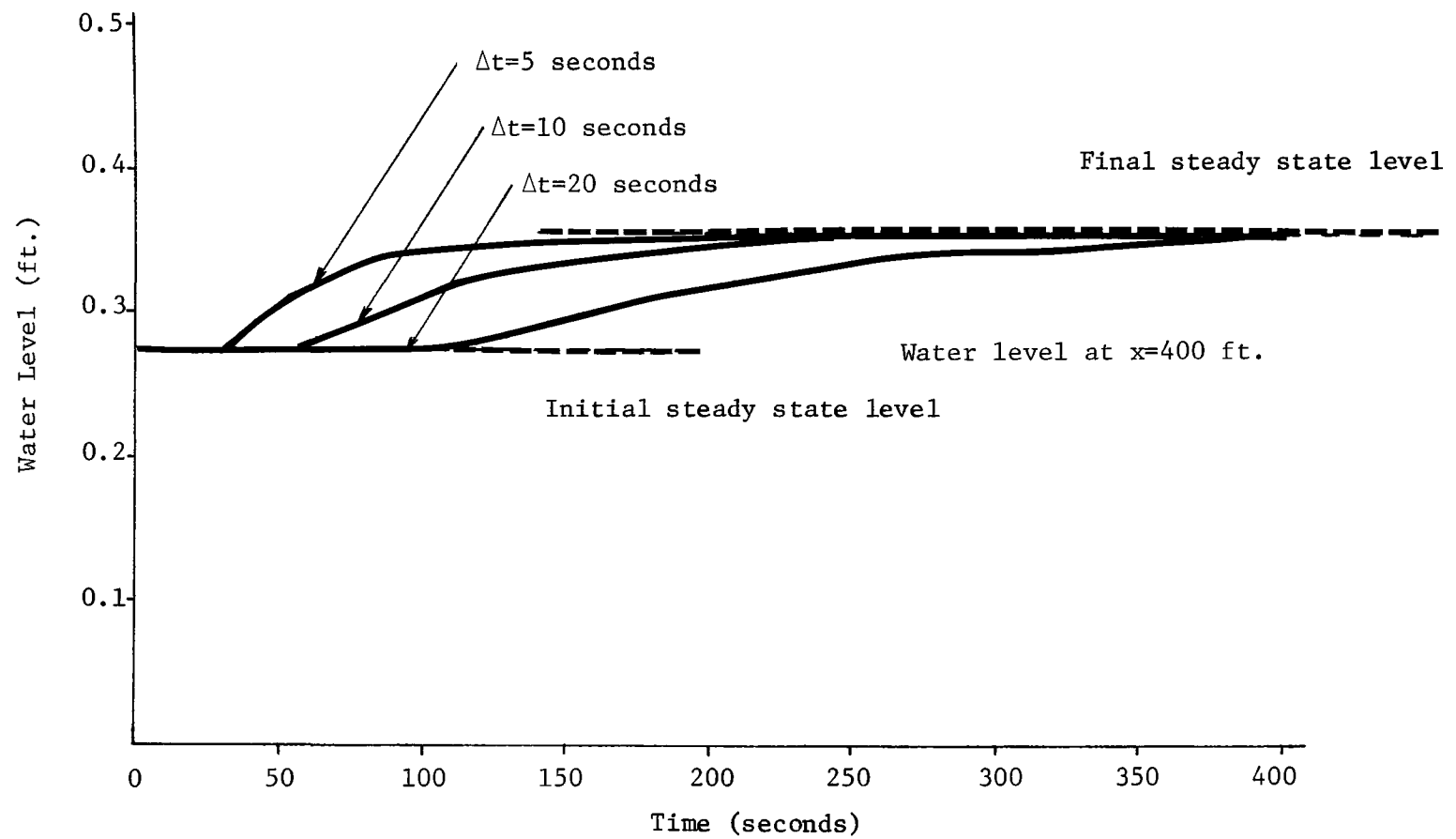


Figure 6.12 Effect of Changing Downstream Control

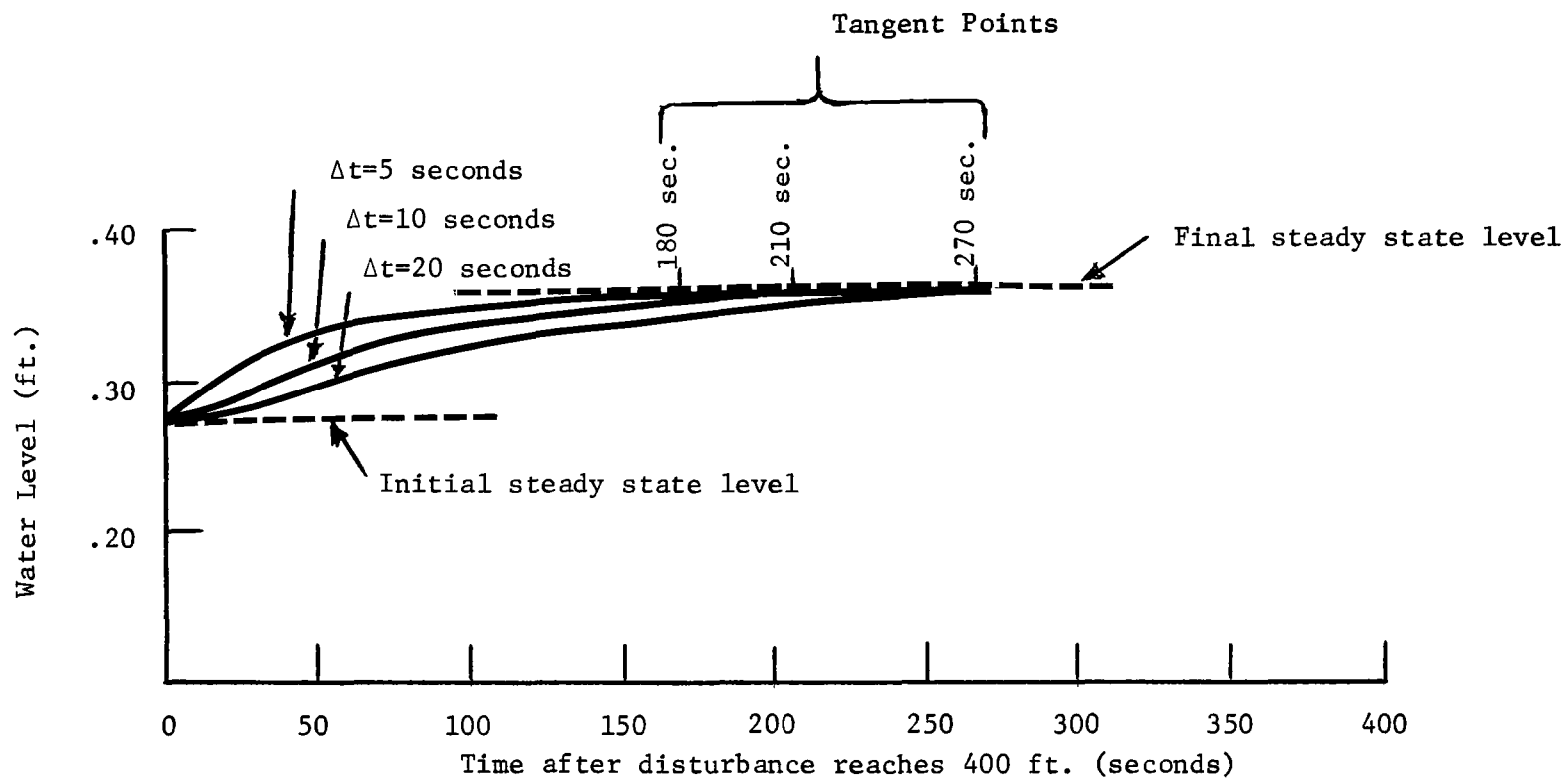


Figure 6.13 Time Profile of Water Level with a Change in Downstream Control

differences between various Δt values are evident. The time required to settle to a new steady state value is distinctly a function of Δt .

In general the shorter time steps result in solutions closer to the actual than longer time steps. It was observed that Δt values of 20 seconds resulted in very small values of $\frac{w_1^j - w_1^{j-1}}{\lambda_1 \Delta t}$, in fact only slightly larger than the accuracy of the analog equipment (about 1 part in 10^3). The appropriate value of Δt appears to be a function of the physical process which in this case is probably in the order of 5 seconds.

6.5.2 Flood routing

The results of the flood routing simulation are presented in Fig. 6.14. The figure illustrates that the front of the disturbance has not yet reached $x = 410$ ft. at 70 seconds, so the downstream boundary condition is still valid. The water level at all locations settles down to the steady state value after the disturbance has passed; and the sharpness of the peak is reduced as the wave is transmitted down the channel. As may be expected (Stoker, 1957) there is some indication that an instability is forming at the front of the wave. This is a hydrodynamic instability formed when the peak of the flood wave travels more rapidly than the beginning of the wave.

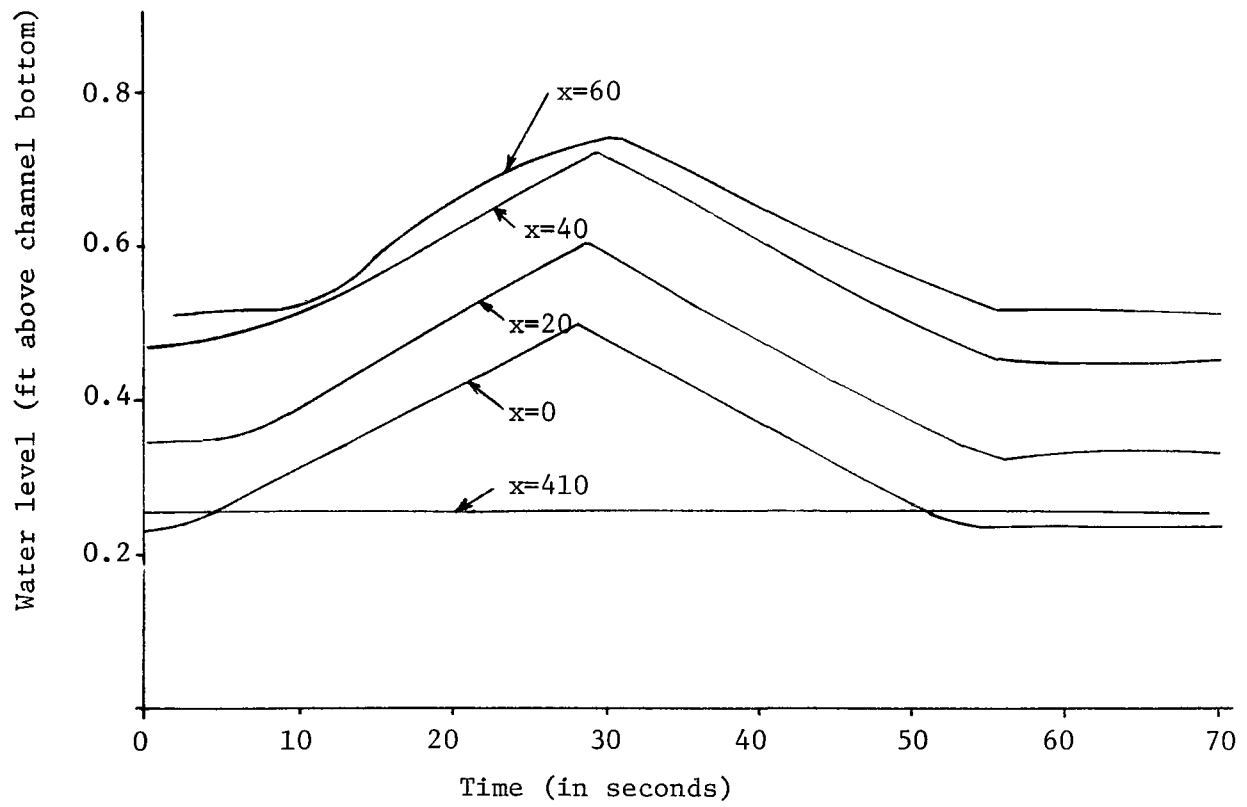


Figure 6.14 Flood Routing

It would appear that the flood wave is propagated more rapidly than what is expected for the prototype suggesting that some further adjustment of the calibration or the geometry may be necessary for successful modelling of flood flows.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

1) It has been demonstrated for the first time that the St. Venant equations can be solved on a hybrid computer.

2) Most of the advantages which are generally accredited to the hybrid computer were verified; namely:

a) continuous prediction of the output variables is provided.

b) interactive modelling which provides insight into the basic physical processes,

c) rapid solutions resulting in economic advantages for the user.

3) The numerical method used in the solution, the method of lines, offers advantages over other methods of solution because a knowledge of the flow process is utilized in the derivation of the equations, the specification of boundary conditions, and the direction of integration.

4) A technique has been presented for calibrating the channel geometry which is physically realistic in that it detects storage areas which do not contribute to flow.

5) Simulations utilizing boundary conditions which are specified in terms of water level and velocity can provide satisfactory water level and velocity predictions. However, if the boundary conditions are specified only in

terms of water levels then, while the water level predictions will be satisfactory, the velocities could be in error.

6) Under steady state conditions local variations in Chezy's C of up to 20% will have little or no influence on water level predictions. However, the upper bound on the % variation of C, which has no significant influence on water level predictions, has not been determined.

7) For non-steady state conditions the model is more difficult to calibrate, and it appears to be more sensitive to Chezy's C. A recommendation has been made for further research.

7.2 Recommendations for further research

During this study several factors became evident which may provide topics for further research.

1) The time steps used in the simulation of unsteady flow ranged from 5 to 20 seconds. Such time steps are very small for studying large scale problems requiring long computer runs. This problem may be overcome by writing the equations in dimensionless form before discretization. The integration can then take place in dimensionless time and space. Time steps can be made appropriately small by proper selection of the time scale.

2) For unsteady flow the channel calibration procedure needs to be reexamined to more adequately account for changes in the channel geometry as a function of water level. For example, it may be desirable to more accurately describe the width of the channel in order to provide adequate storage volumes. The sensitivity of the predictions to variations in Chezy's C, and the variation of Chezy's C as a function of water level in unsteady flow also requires further investigation.

REFERENCES

- Amein, M., "Streamflow Routing by Characteristics", Water Resources Research, Vol. 2, No. 1, First Quarter, 1966.
- Ames, W.F., "Nonlinear Partial Differential Equations in Engineering," Academic Press Inc., New York, 1965.
- Bekey, G.A. and Karplus, W.J., "Hybrid Computation," John Wiley and Sons Inc., 1968.
- Bekey, G.A., "System Identification - An Introduction and a Survey," Simulation, October, 1970.
- Baltzer, R.A. and Rai, C., "Computer Simulation of Unsteady Flows in Waterways," Journal of the Hydraulics Divisions, A.S.C.E., Vol. 94, No. HY4, July, 1968.
- Bruce, J.P. and Clarke, R.H., "Introduction to Hydrometeorology," Pergamon Press Ltd., London, 1966, p.197-202.
- Chow, Ven Te, "Flood Routing" in "Open Channel Hydraulics," McGraw-Hill Book Company, Inc., 1959, Chapter 20.
- Chubb, B.A., "Economic Evaluation of the C.S.M.P. Digital Simulation Language," Simulation, Vol. 14, No. 3, March, 1970.
- Daily, J.W. and Harleman, D.R.F., "Fluid Dynamics," Addison-Wesley Publishing Co. Inc., Reading, Mass., 1966, Chapter 13.
- Dooge, J.C.I., "Mathematical Models of Hydrologic Systems," International Symposium on Modelling Techniques in Water Resources Systems, Ottawa, Published by Environment Canada, May 1972.
- Eagleson, P.S., "Dynamic Hydrology," McGraw-Hill Book Co., 1969.
- "EAI 640 Fortran IV Language Reference Manual," Analysis Section, Division of Mechanical Engineering, National Research Council, Ottawa, Report No. AC-100(R), 2nd printing, April 21, 1972.
- Farell, C., "Discussion of Critical Analysis of Open Channel Resistance," by Hunter Rouse, Journal of the Hydraulics Division, A.S.C.E., Vol. 92, No. HY2, Page 200, Paper 4387, March 1966.
- Freeze, R.A. and Harlan, R.L., "Blueprint for a Physically Based, Digitally Simulated Hydrologic Response Model," Journal of Hydrology, 9, pp.237-258, 1969.

Fromm, N.C., "Hybrid Simulation of River Hydraulics Equations," Scientific Computation Department, Electronics Associates Inc., West Long Branch, Report 72-2, February 1972.

Gagne, R.E., "General Facilities in the Analysis Laboratory," Division of Mechanical Engineering News Letter, National Research Council, Computers, Vol. 1, No. 1, January 1971.

Gagne, R.E., "User Response to HOI," Analysis Section, Division of Mechanical Engineering, National Research Council, Report AC-117, May 27, 1971.

Gagne, R.E., "HOI Reference Handbook," Analysis Section, Division of Mechanical Engineering, National Research Council, Ottawa, Report AC-108, 2nd Printing, April 1972.

Gagne, R.E., "EAI 690 Fortran Linkage Routines," Analysis Section, Division of Mechanical Engineering, National Research Council, Ottawa, Report AC-104R, Revised, April 1972.

Henderson, F.M., "Open Channel Flow," McMillan Book Company, New York, 1966.

Henry, R.F. and Graefe, P.W.U., "Zero Padding as a Means of Improving Definition of Computed Spectra," Manuscript Report Series #20, Marine Sciences Branch, Environment Canada, 1971.

Hinze, J.O., "Turbulence, an Introduction to its Mechanisms and Theory," McGraw-Hill Book Company, 1959.

Isaacson, E., Stoker, J.J., and Froesh, A., "Numerical Solution of Flood Prediction and River Regulation Problems," Report III, Courant Institute of Mathematical Sciences, No. 1MM-NYU 235, October, 1956.

Isaacson, E. and Keller, B.H., "Analysis of Numerical Methods," John Wiley and Sons Inc., 1966.

Jenkins, G.M. and Watts, D.G., "Spectral Analysis and its Applications," Holden-Day, San Francisco, 1969.

Katell, E., "Hybrid Computation Comes of Age," EAI Applications Reference Library, General Section 1-4.15 H, January 1969.

Kells, L.M., "Elementary Differential Equations," McGraw-Hill Book Company, Sixth Edition, 1965.

Keulegan, G.H. and Patterson, G.W., "Effect of Turbulence and Channel Slope on Translation of Waves," U.S. Dept. of Commerce, National Bureau of Standards, Research Paper R.P. 1544, Vol. 30, June 1943.

Kisiel, C.C. and Duckstein, L., "Economics of Hydrologic Modelling," International Symposium on Modelling Techniques in Water Resources Systems, Ottawa, Published by Environment Canada, May 1972.

Landauer, P.J., "Spectrum of Applications for the Modern Hybrid Computer," Applications Reference Library, Electronic Associates, West Long Branch, New Jersey, General Section 1.1.1H, July 1966.

Lane, E.W., "The Importance of Fluvial Morphology in Hydraulic Engineering," A.S.C.E. Proc., Vol. 81, No. 746, 1955.

Lighthill, M.J. and Whitham, G.B., "On Kinematic Waves: I. Flood Movement in Long Rivers," Proc. Roy. Soc., London, (A) Vol. 229, No. 1178, p.281, May 1955.

MacLean, A.A., "Agriculture and Environmental Quality - Saint John River Valley," Proceedings of the Work Planning Meeting on Agriculture and Environmental Quality, Research Branch, Canada Agriculture, January 1973.

Morley, H.V., "U.N. Environmental Conference Recommendations Relating to Agriculture," Proceedings of the Work Planning Meeting on Agriculture and Environmental Quality, Research Branch, Canada Agriculture, January 1973.

Mufti, I.H., "Characteristics and the Numerical Solution of Hyperbolic Equations," Division of Mechanical Engineering, National Research Council, Ottawa, Report AC141, Sept., 1973.

Paul, R.J.A. and Ahmad, A.I.S., "Numerical Solutions of Second Order Hyperbolic Partial Differential Equations by a Method of Continuous Characteristics," Proc. I.E.E.E., Vol. 117, No. 6, June 1970.

Ploeg, J., discussed in personal conversation, August 1973.

Prandle, D. and Crookshank, N., "Numerical Model Studies of the St. Lawrence River," Division of Mechanical Engineering, National Research Council, Canada, Report MH-109, July 1972.

Riley, J.P., Chadwick, D.G. and Bagley, J.M., "Application of an Electric Analog Computer to Solution of Hydrologic and River-Basin Planning Problems," Utah Simulation Model II, Utah Water Research Laboratory, Utah State University, Logan, Utah, PR WG 32-1, 1966.

Rouse, H., "Advanced Mechanics of Fluids," Wiley Book Company, New York, 1959.

Schwab, G.O., Frevert, R.K., Edminster, T.W. and Barnes, K.K., Soil and Water Conservation Engineering," John Wiley and Sons Inc., Second Edition, 1966.

, "680 Scientific Computing System Reference Handbook," Electronics Associates Inc., Publ. No. 00800-2048-3, March 1970.

Stoker, J.J., "Water Waves," Interscience Publishers, New York, 1957.

, "Stream Gauging Procedure," U.S. Geological Survey, Paper 888, 1960.

Streeter, V.L., "Fluid Mechanics," McGraw-Hill Book Company, Fourth Edition, 1966.

Strelkoff, T., "One Dimensional Equations of Open Channel Flow," Journal of Hydraulics Division, Proc. A.S.C.E., Vol. 95, No. HY3, Proc. Paper 6557, May 1969.

Strelkoff, T., "Numerical Solution of Saint Venant Equations," Journal of Hydraulics Division, Proc. A.S.C.E., Vol. 96, No. HY1, January 1970.

, "Symposium on 'The Use of Analog and Digital Computers in Hydrology," Tucson Symposium, IASH-UNESCO Publication #80, December 1968 (14 papers on analog applications).

Vermuri, K., "Hybrid Computation in Hydrosystems," Advances in Hydrosience No. 6, Academic Press Inc., New York, 1973.

Task Committee on Development of Research Projects in Surface Water Hydrology "Needed Research Projects in Surface Water Hydrology," Journal of the Hydraulics Division, A.S.C.E., Vol. 92, No. HY6, November 1966.

Vichnevetsky, R., "Hybrid Computer Integration of Hyperbolic Partial Differential Equations by a Method of Lines," Proceedings of Fourth Australian Computer Conference, Adelaide, South Australia, 1969.

Vichnevetsky, R., "Incremental Hybrid Methods for First Order Partial Differential Equations," TR 70-26, Electronics Associates Inc., West Long Branch, New Jersey, October 1970.

Wigham, J.M., "Sediment Transportations - Handbook on the Principles of Hydrology," D.M. Gray, Editor-in-Chief, Canadian National Committee for the I.H.D., National Research Council, Canada, 1970.

Whitaker, S., "Introduction to Fluid Mechanics," Prentice Hall, 1968.

Wright, E.J., "Generalized Algorithm for the Simulation of Unsteady Fluid Flow in Mechanical Systems," Simulation Council Inc., April 1971.

APPENDIX I

ANALOG DATA RECONSTRUCTION

APPENDIX 1

ANALOG DATA RECONSTRUCTION

The operation of the analog data reconstruction circuit of Fig. 4.2 is described as an example of analog computation.

The digital computer supplies values of $\frac{W_2^j}{W_{2m}}$ at $x = 0$ ($\frac{W_2^j(0)}{W_{2m}}$) and at $x = 1$. Δx ($\frac{W_2^j(1)}{W_{2m}}$) to D/A channels 15 and 18 respectively. On the analog computer side of the conversion channel appears an inverted signal. The signal on D/A channel 15 is passed through an inverter to the initial condition for integrator 30. The resulting signal on the output of integrator is $-\frac{W_2^j(0)}{W_{2m}}$. The output of the integrator is patched to track-store device No. 31. This track store (T/S) device will transmit the signal from the integrator in the track mode and hold a constant value in the store mode. The logic signal L_1 is set up such that the T/S device is in-store for all of the time corresponding to Δx except for a small period δ at the beginning of the time period when it is in track. The output of the T/S device is added to the value on D/A channel 18 by means of adder 35. The output of adder 35 will have the difference between subsequent values in space of W_2^j . Multiplying this by $\frac{1}{\Delta x}$ yields the slope between subsequent values. The output of potentiometer 30 is connected to the derivative terminal of integrator 30.

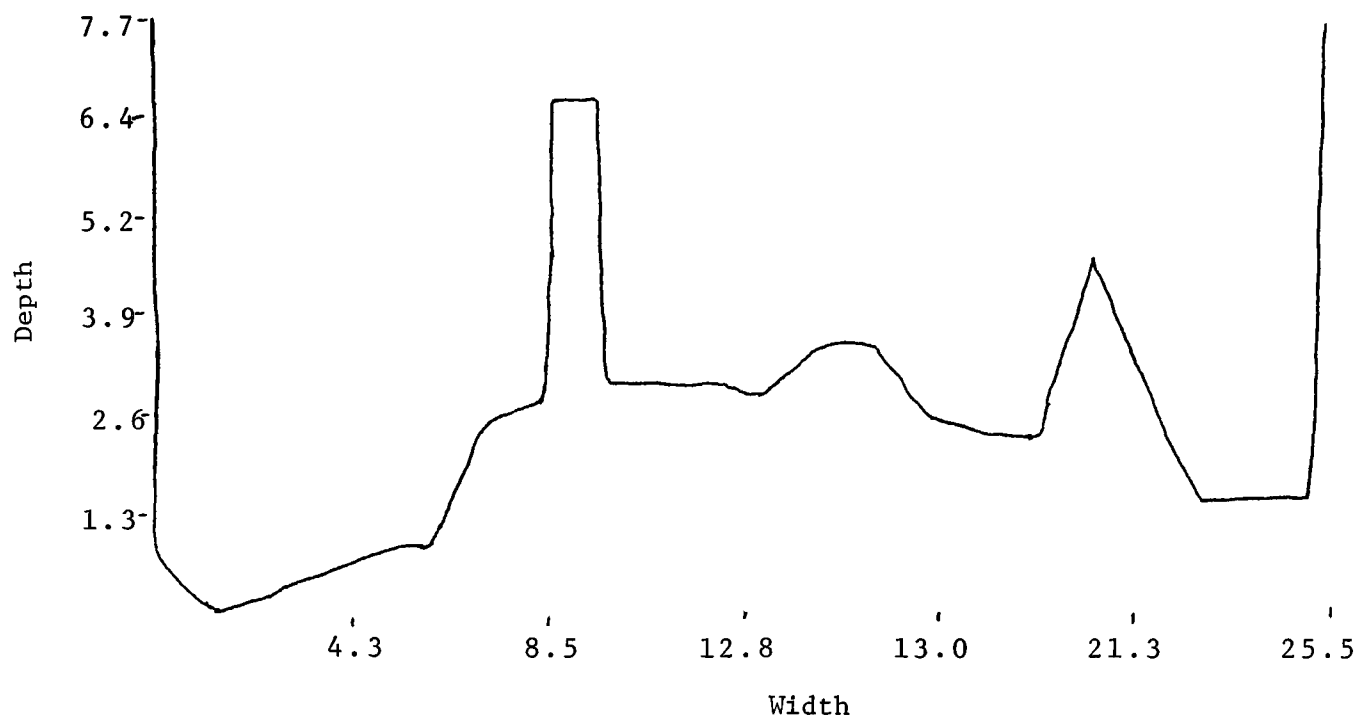
When the computer is in the operate mode the value

of the derivate will be integrated and added to the existing output of integrator 30. When Δx has elapsed the output on integrator 30 will be $\frac{W_2^J(1)}{W_{2m}}$. A logic signal L_1 then sets T/S device 31 to track such that it will now transmit $\frac{W_2^J(1)}{W_{2m}}$ and at the same time send a signal to the digital to input $\frac{W_2^J(2)}{W_{2m}}$ on D/A channel 18. Upon completion of these tasks T/S device 31 is again set to store and the derivative being integrated is that corresponding to the second Δx . Thus the output of integrator 30 will have a continuous value of $-\frac{W_2^J}{W_{2m}}$ obtained by linear interpolation between subsequent points. This continuous representation is then used in subsequent computations.

This circuit has very stable error propagation properties since the difference integrated will always be formed by subtracting the current solution from the desired solution at the end of that Δx . Thus despite errors in the current solution the solution after Δx will always tend to the correct value.

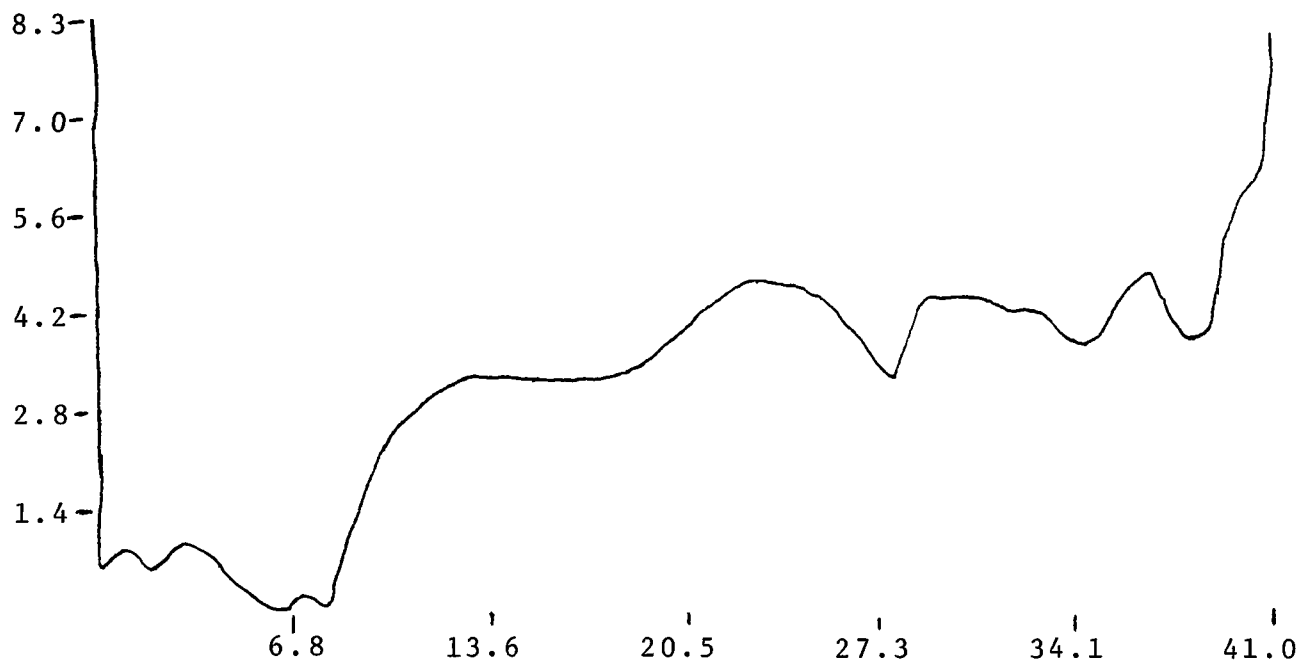
APPENDIX II

CHANNEL CROSS SECTIONS



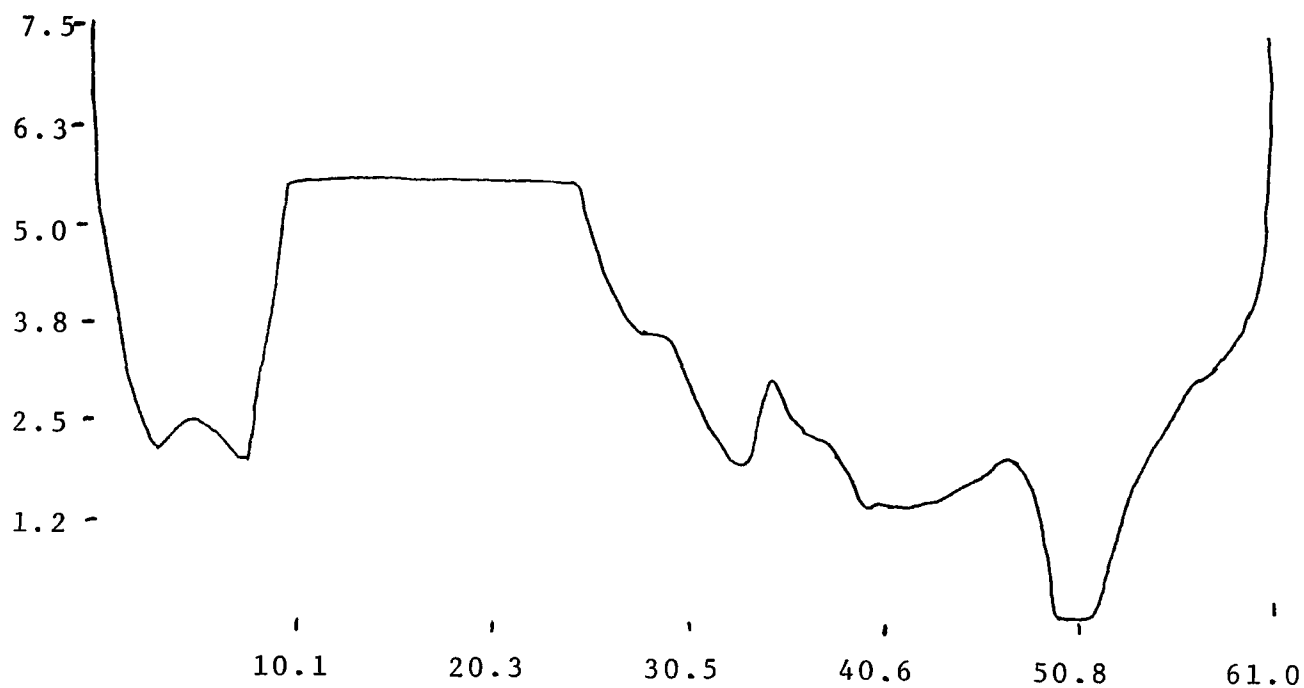
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-1.9	.3	.5	2.6	2.7
-1.5	.7	2.0	4.7	5.0
-1.1	1.1	4.2	6.1	7.0
-.7	1.5	7.1	8.9	10.6
-.3	1.9	10.7	9.5	12.4
.0	2.3	14.6	11.7	15.6
.4	2.7	19.8	14.5	19.8
.8	3.1	26.3	19.8	27.0
1.2	3.5	34.4	22.6	32.0
1.6	3.9	43.3	23.1	34.5
1.9	4.2	52.5	23.6	36.9
2.3	4.6	61.8	24.1	39.4
2.7	5.0	71.2	24.2	41.0
3.1	5.4	80.7	24.2	42.6
3.5	5.8	90.1	24.3	44.1
3.9	6.2	99.6	24.3	45.7
4.3	6.6	109.2	24.4	47.3
4.7	7.0	118.9	25.4	49.4
5.1	7.4	128.8	25.4	50.2
5.5	7.8	138.8	25.5	50.9

Fig. II.1 Cross Section at Station 1, Frontenac St.



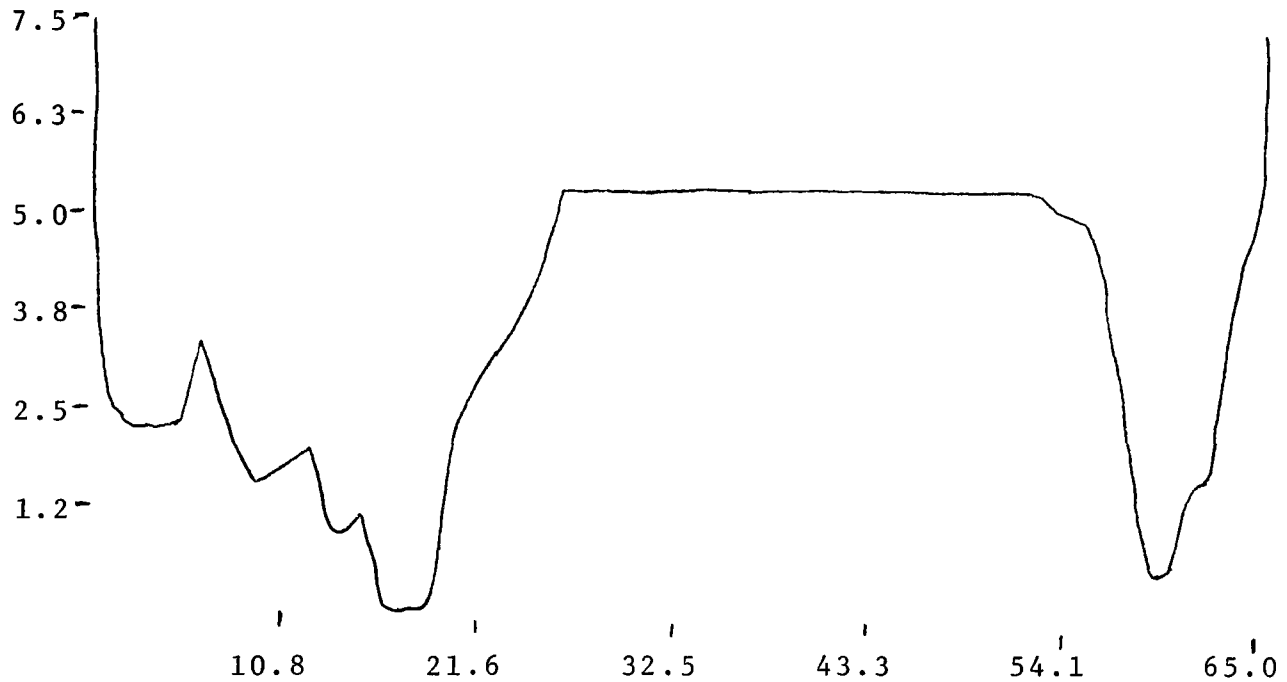
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-2.4	.4	1.0	3.4	3.6
-2.0	.8	3.1	7.4	8.4
-1.6	1.2	6.7	8.8	10.6
-1.2	1.6	10.6	9.2	11.6
-.8	2.1	14.5	9.6	12.6
-.3	2.5	18.7	10.2	13.7
.0	2.9	23.2	11.1	15.2
.4	3.3	28.5	17.7	22.2
.8	3.7	37.1	21.7	27.1
1.3	4.2	47.5	27.1	33.8
1.7	4.6	60.6	36.6	44.7
2.1	5.0	76.9	39.5	48.5
2.5	5.4	93.6	39.8	49.4
2.9	5.8	110.4	40.1	50.3
3.4	6.3	127.4	40.8	51.6
3.8	6.7	144.6	40.9	52.5
4.2	7.1	161.8	41.0	53.4
4.6	7.5	179.0	41.0	54.2
5.0	7.9	196.2	41.0	55.0
5.5	8.4	213.5	41.0	55.9

Fig. II.2 Cross Section at Station 2, Longue Pt.



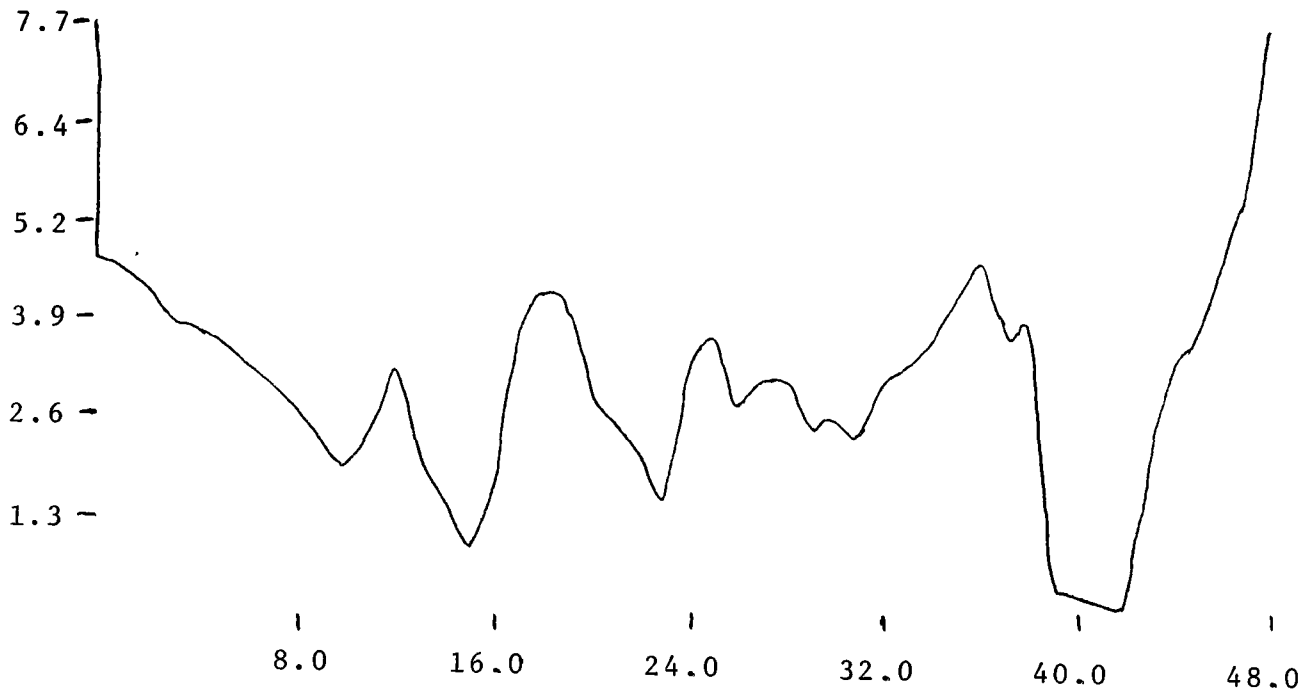
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-1.7	.3	.9	2.7	3.1
-1.3	.7	2.1	3.5	4.2
-.9	1.1	3.5	4.1	5.1
-.5	1.5	5.5	9.1	10.6
-.2	1.9	9.7	13.2	14.9
.1	2.2	16.5	22.5	25.0
.5	2.6	27.0	30.2	33.4
.9	3.0	39.0	33.0	37.1
1.3	3.4	52.3	36.5	41.2
1.7	3.8	67.0	40.4	45.7
2.0	4.1	82.7	42.1	48.1
2.4	4.5	99.1	43.7	50.4
2.8	4.9	115.9	44.6	52.2
3.2	5.3	133.0	45.5	54.0
3.6	5.7	152.0	60.9	70.4
3.9	6.0	175.2	60.9	71.2
4.3	6.4	198.3	61.0	71.9
4.7	6.8	221.5	61.0	72.7
5.1	7.2	244.7	61.0	73.4
5.5	7.6	267.9	61.0	74.2

Fig. II.3 Cross Section at Station 3, Pte-aux-Trembles



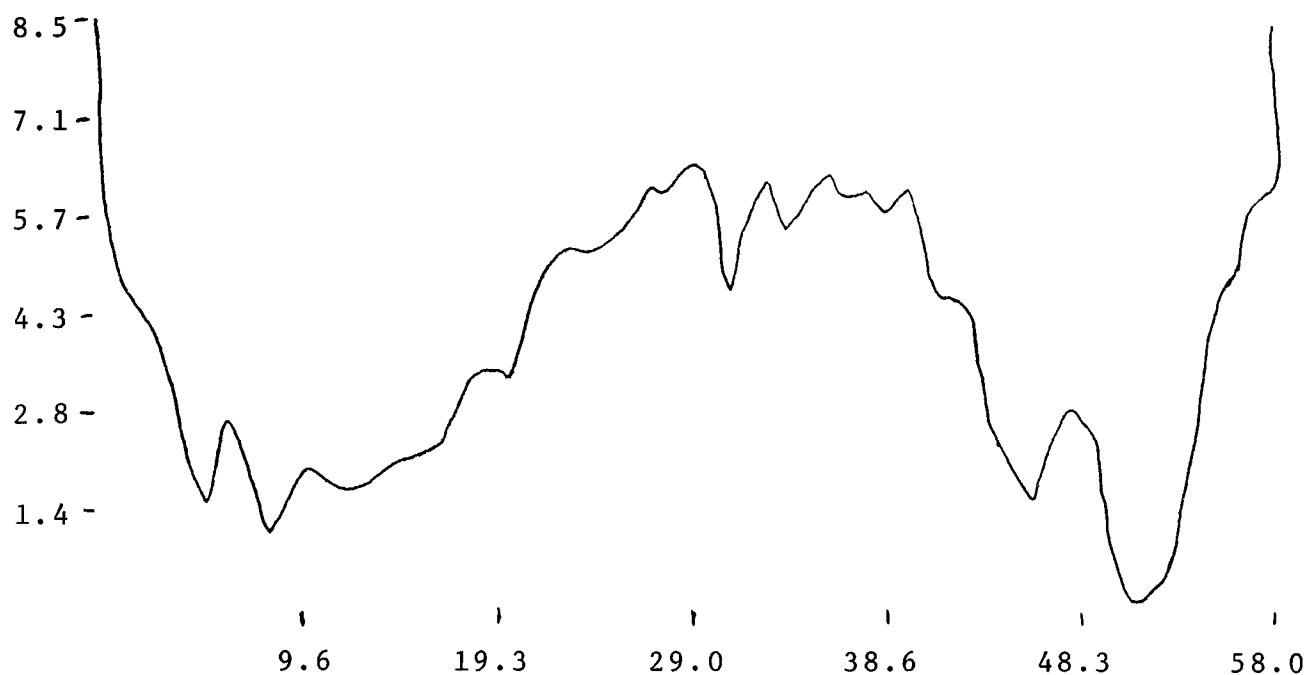
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-1.7	.3	1.1	3.3	3.6
-1.3	.7	2.7	5.2	6.2
-.9	1.1	5.1	8.1	9.8
-.5	1.5	8.6	9.9	12.5
-.2	1.9	13.2	14.6	18.1
.1	2.2	19.3	17.1	21.5
.5	2.6	27.2	23.1	28.4
.9	3.0	36.4	25.4	31.7
1.3	3.4	46.5	28.0	35.3
1.7	3.8	57.6	29.8	38.0
2.0	4.1	69.2	31.1	40.1
2.4	4.5	81.2	32.3	42.0
2.8	4.9	93.8	34.1	44.5
3.2	5.3	107.5	37.8	48.8
3.6	5.7	127.4	64.9	76.8
3.9	6.0	152.1	64.9	77.5
4.3	6.4	176.8	65.0	78.3
4.7	6.8	201.5	65.0	79.1
5.1	7.2	226.2	65.0	79.8
5.5	7.6	250.9	65.0	80.6

Fig. II.4 Cross Section at Station 4, Varennes



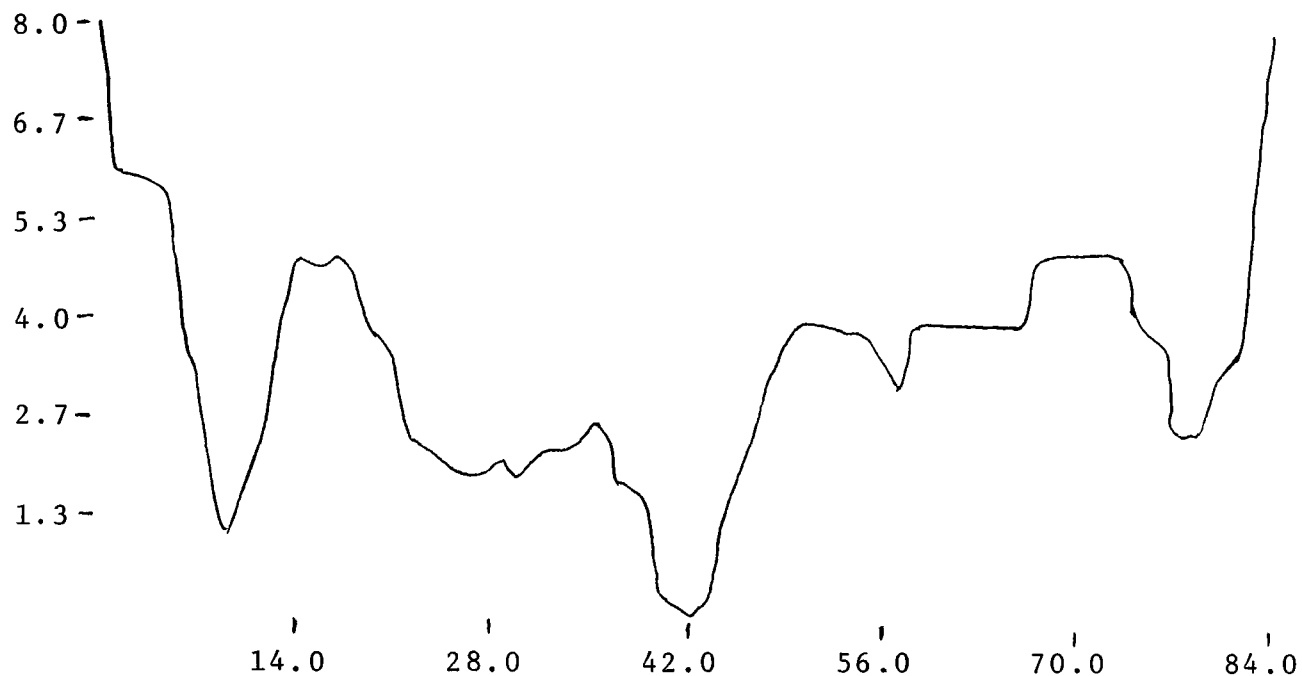
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-1.9	.3	.7	3.2	3.5
-1.5	.7	2.0	3.5	4.3
-1.1	1.1	3.7	4.9	6.5
-.7	1.5	5.9	6.7	9.3
-.3	1.9	8.9	8.7	12.5
.0	2.3	13.2	13.5	18.8
.4	2.7	19.7	19.5	26.6
.8	3.1	28.4	26.1	35.3
1.2	3.5	39.8	32.1	42.8
1.6	3.9	53.6	38.3	50.1
1.9	4.2	69.1	42.2	54.8
2.3	4.6	86.2	45.7	58.7
2.7	5.0	104.3	46.4	60.0
3.1	5.4	122.5	46.9	61.0
3.5	5.8	140.8	47.1	61.8
3.9	6.2	159.2	47.3	62.6
4.3	6.6	177.7	47.4	63.4
4.7	7.0	196.3	47.6	64.3
5.1	7.4	214.9	47.8	65.1
5.5	7.8	233.6	48.0	65.9

Fig. II.5 Cross Section at Station 5, Repetigny



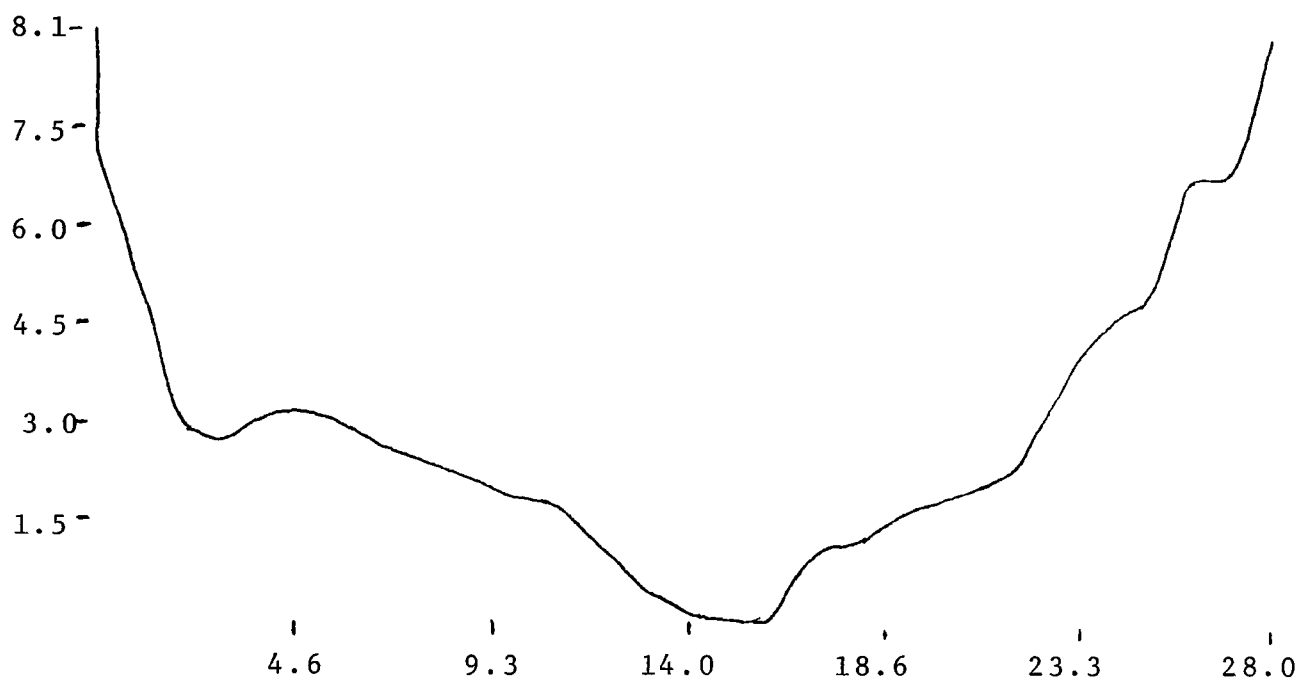
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-2.6	.4	.5	2.2	2.4
-2.2	.8	1.8	3.2	3.8
-1.8	1.2	3.4	4.5	5.8
-1.3	1.7	5.8	7.5	9.9
-.9	2.1	10.8	15.2	19.3
-.5	2.5	18.8	21.1	26.5
-.1	3.0	28.8	24.7	31.3
.3	3.4	39.8	27.4	34.9
.7	3.8	52.2	29.4	37.8
1.2	4.3	65.2	31.2	40.4
1.6	4.7	79.7	35.7	45.7
2.0	5.1	95.5	38.0	49.3
2.4	5.5	113.3	43.5	56.0
2.9	6.0	133.3	50.4	64.6
3.3	6.4	156.8	57.3	72.9
3.7	6.8	181.7	57.9	74.4
4.2	7.3	206.7	57.9	75.2
4.6	7.7	231.6	58.0	76.1
5.0	8.1	256.5	58.0	76.9
5.5	8.6	281.5	58.0	77.8

Fig. II.6 Cross Section at Station 6, Verchères



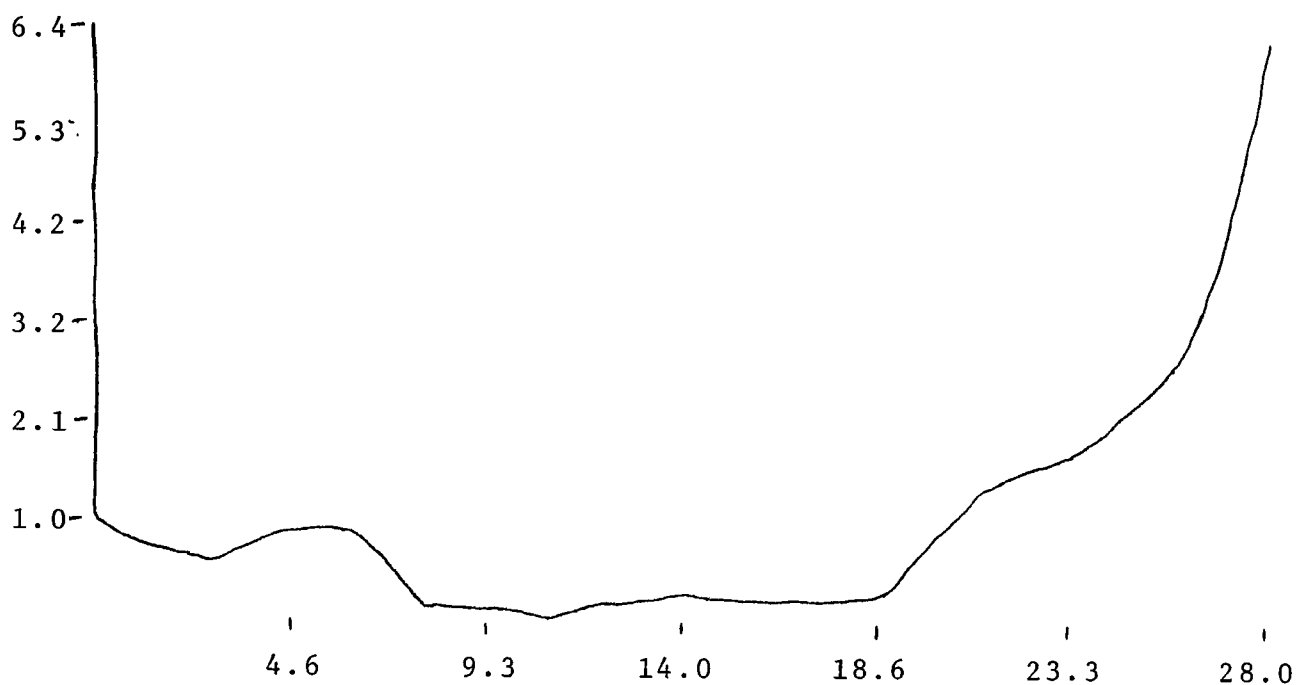
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-2.1	.4	.9	4.0	4.2
-1.7	.8	2.7	4.7	5.2
-1.3	1.2	4.8	5.9	7.0
-.9	1.6	7.6	7.9	9.6
-.5	2.0	12.0	15.1	17.3
-.1	2.4	20.4	26.3	29.3
.2	2.8	32.4	32.7	36.6
.6	3.2	46.3	36.0	41.1
1.0	3.6	61.9	41.3	47.4
1.4	4.0	80.3	50.9	58.1
1.8	4.4	105.0	63.5	71.8
2.2	4.8	131.3	67.1	76.5
2.6	5.2	160.9	77.5	87.5
3.0	5.6	192.4	78.1	88.6
3.4	6.0	224.6	82.1	92.9
3.8	6.4	257.9	82.4	93.8
4.2	6.8	291.4	82.8	94.7
4.6	7.2	325.0	83.2	95.6
5.0	7.6	358.8	83.6	96.5
5.5	8.1	392.7	84.0	97.4

Fig. II.7 Cross Section at Station 7, Lavaltrie



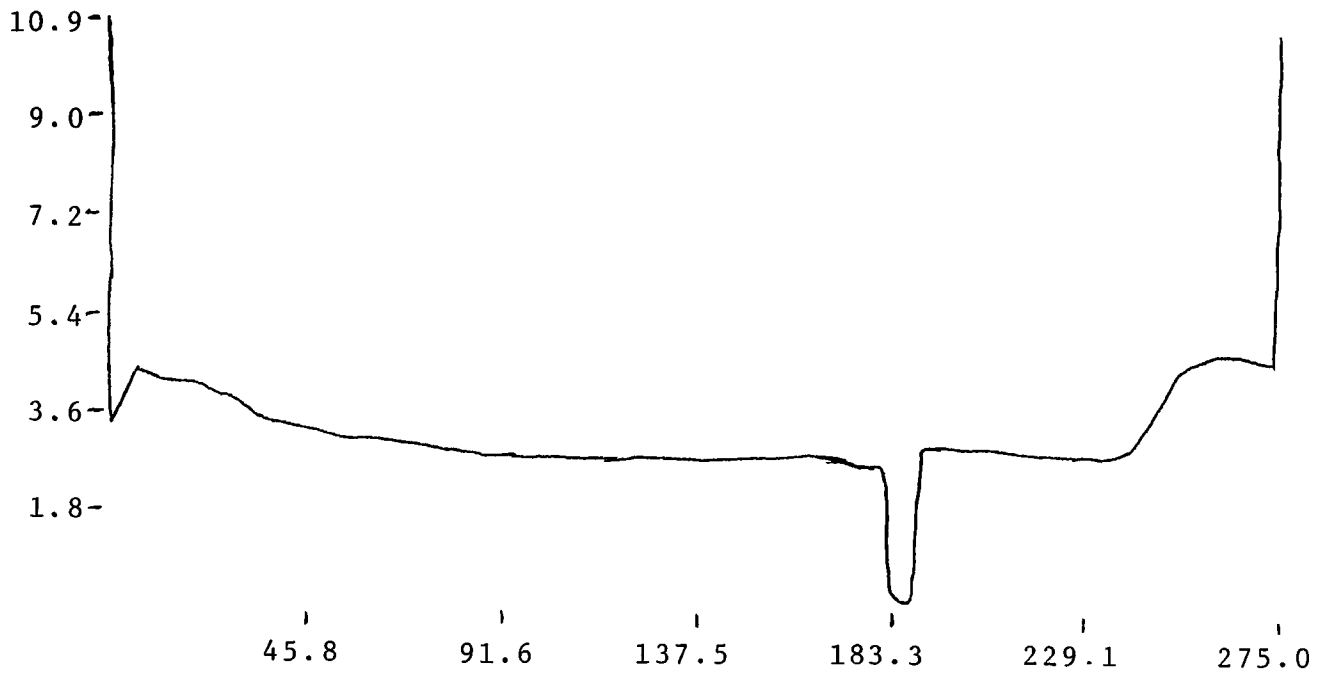
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-3.1	.4	1.1	3.5	3.7
-2.6	.9	3.0	4.6	5.2
-2.2	1.3	5.6	6.8	7.7
-1.7	1.8	9.1	9.2	10.2
-1.3	2.2	14.2	13.0	14.2
-.8	2.7	20.8	15.8	17.2
-.4	3.1	29.0	19.5	21.5
.0	3.6	38.5	21.1	23.7
.4	4.0	48.3	21.8	24.8
.9	4.5	58.4	22.5	26.0
1.4	5.0	68.9	23.9	27.8
1.8	5.4	79.9	24.3	28.8
2.3	5.9	91.1	24.8	29.8
2.7	6.3	102.5	25.3	30.8
3.2	6.8	114.1	25.8	31.9
3.6	7.2	126.3	27.1	33.9
4.1	7.7	138.7	27.3	34.8
4.5	8.1	151.2	27.5	35.8
5.0	8.6	163.8	27.7	36.7
5.5	9.1	176.5	28.0	37.7

Fig. II.8 Cross Section at Station 8, Landraie



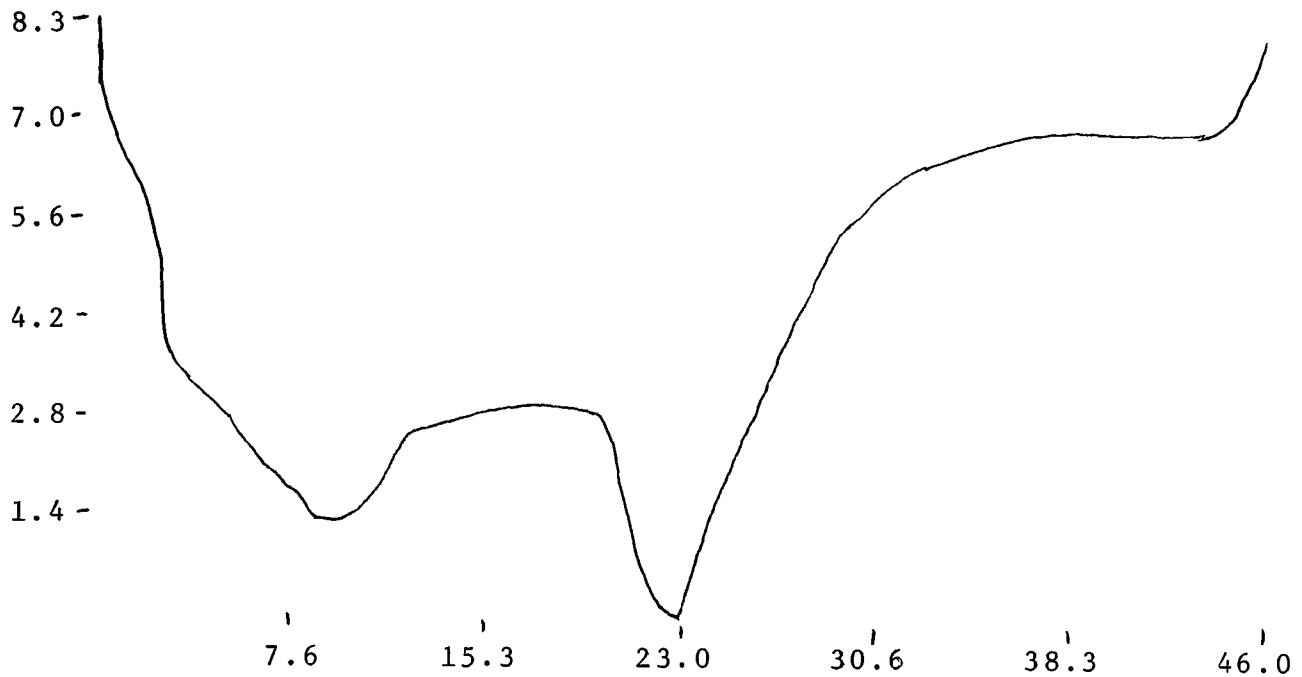
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
- .5	.3	1.5	11.4	11.5
- .2	.6	5.4	13.2	13.4
.1	.9	10.4	18.1	18.6
.3	1.2	16.9	20.7	21.5
.7	1.6	23.7	21.9	23.1
1.0	1.9	31.1	23.7	25.2
1.3	2.2	38.8	24.5	26.4
1.6	2.5	46.8	25.3	27.5
1.9	2.8	55.0	25.9	28.6
2.3	3.2	63.4	26.2	29.3
2.6	3.5	71.8	26.5	30.1
2.9	3.8	80.3	26.7	30.8
3.2	4.1	88.9	27.0	31.5
3.5	4.4	97.6	27.1	32.2
3.9	4.8	106.3	27.3	32.9
4.2	5.1	115.1	27.4	33.5
4.5	5.4	123.9	27.5	34.2
4.8	5.7	132.7	27.7	34.9
5.1	6.0	141.6	27.8	35.5
5.5	6.4	150.6	28.0	36.2

Fig. II.9 Cross Section at Station 9, Sorel



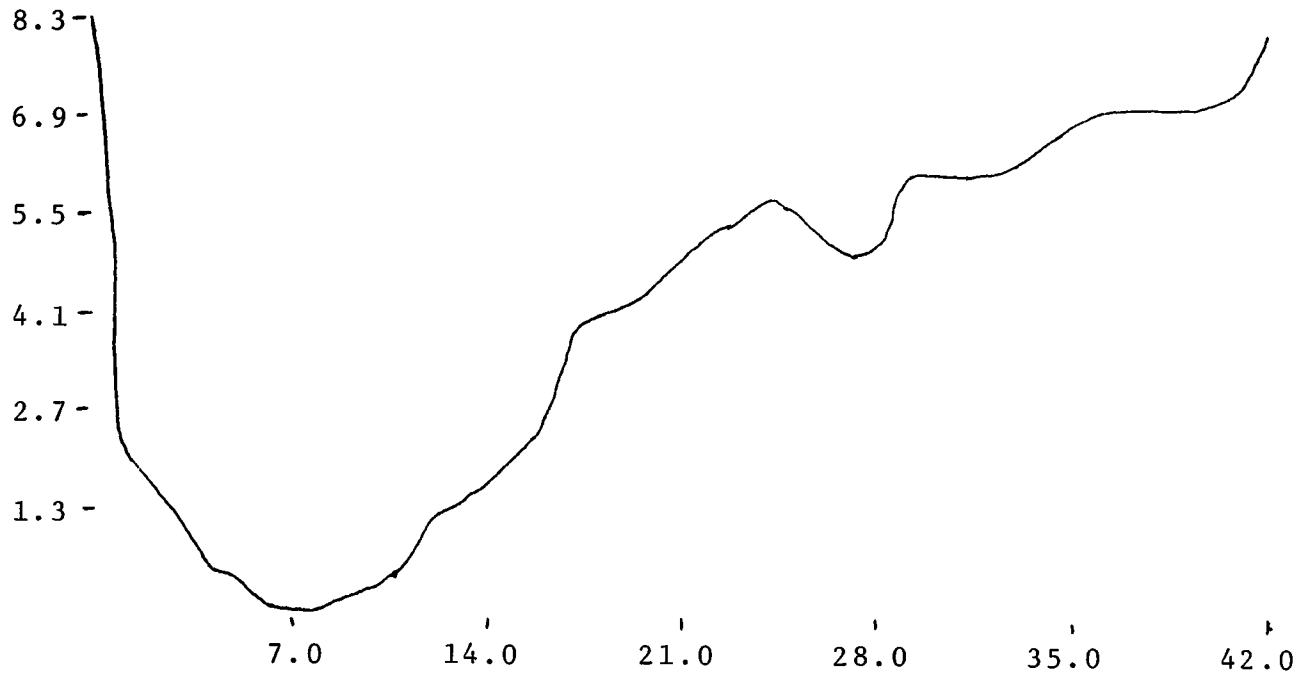
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-1.7	.5	2.1	4.8	5.1
-1.2	1.0	5.0	5.6	6.5
-.6	1.6	8.2	6.1	7.7
-.1	2.1	11.7	6.8	9.0
.4	2.7	16.6	15.6	18.4
.9	3.2	78.4	190.4	193.3
1.5	3.8	191.9	219.2	222.7
2.0	4.3	318.5	248.7	252.9
2.6	4.9	459.0	274.0	279.0
3.1	5.4	608.4	274.1	280.1
3.6	5.9	757.8	274.2	281.2
4.2	6.5	907.3	274.3	282.3
4.7	7.0	1056.8	274.3	283.4
5.3	7.6	1206.4	274.4	284.5
5.8	8.1	1356.0	274.5	285.6
6.4	8.7	1505.6	274.6	286.7
6.9	9.2	1655.3	274.7	287.8
7.5	9.8	1805.1	274.8	288.9
8.0	10.3	1954.9	274.9	290.0
8.6	10.9	2104.8	274.9	291.1

Fig. II.10 Cross Section at Station 10, Louiseville



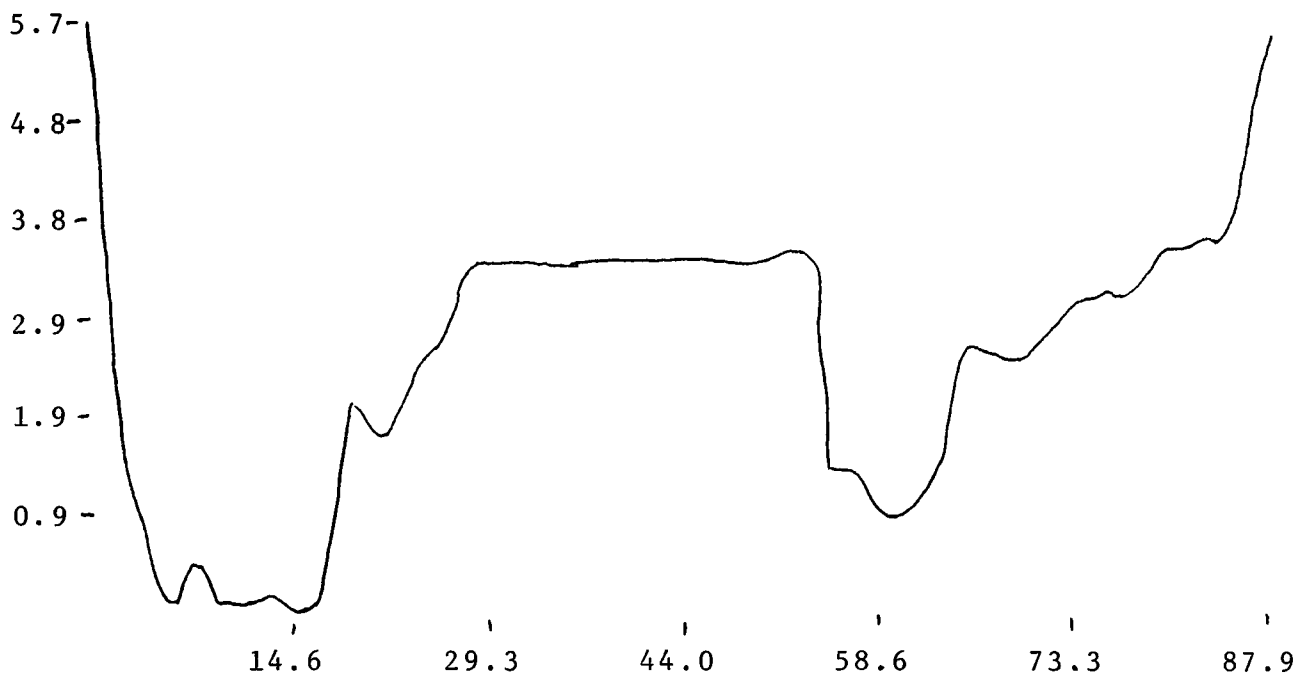
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-4.9	.4	.4	1.4	1.8
-4.5	.8	1.1	2.0	2.8
-4.1	1.2	2.1	2.6	3.8
-3.7	1.6	4.2	6.7	8.5
-3.3	2.1	7.5	9.0	11.3
-2.8	2.5	11.8	11.5	14.4
-2.4	2.9	17.7	17.8	21.1
-2.0	3.3	26.8	22.9	26.5
-1.6	3.7	36.7	23.9	27.8
-1.2	4.2	46.9	24.7	29.0
- .7	4.6	57.5	25.4	30.2
- .3	5.0	68.3	26.1	31.3
.1	5.4	79.4	26.7	32.4
.4	5.8	91.0	28.3	34.3
.9	6.3	103.3	30.1	36.3
1.3	6.7	116.5	33.3	39.7
1.7	7.1	132.3	44.1	50.6
2.1	7.5	151.0	44.9	52.0
2.5	7.9	170.0	45.4	53.0
3.0	8.4	189.3	46.0	54.1

Fig. II.11 Cross Section at Station 11, Port-St-François



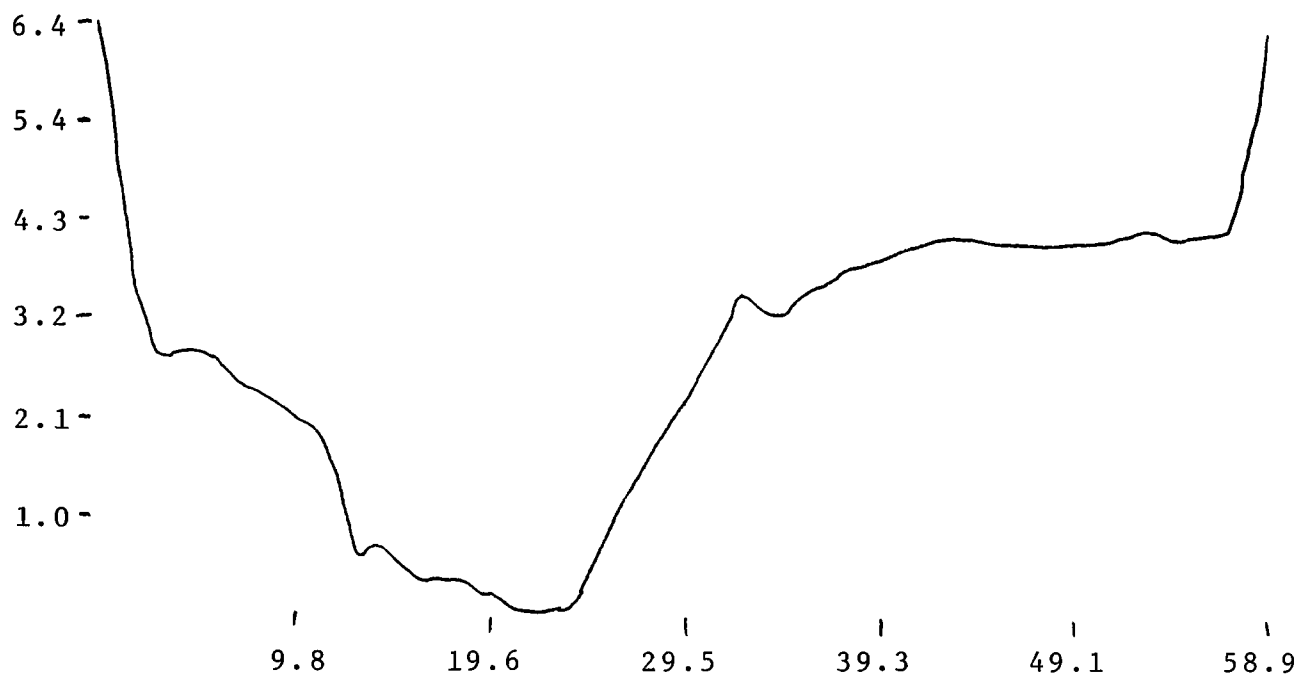
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-4.8	.4	1.3	4.8	4.9
-4.4	.8	4.0	7.7	7.9
-4.0	1.2	7.5	9.0	9.5
-3.6	1.6	11.8	11.4	12.1
-3.2	2.0	17.0	13.6	14.5
-2.8	2.4	22.9	14.6	15.9
-2.3	2.9	29.1	15.2	17.0
-1.9	3.3	35.6	15.6	17.9
-1.5	3.7	42.1	16.0	18.9
-1.1	4.1	48.9	17.0	20.4
- .7	4.5	56.5	19.2	23.0
- .3	4.9	64.8	20.4	24.5
.1	5.3	74.2	24.1	28.9
.5	5.8	85.0	27.5	33.0
.9	6.2	96.9	31.7	37.8
1.3	6.6	110.4	33.3	39.8
1.7	7.0	124.6	35.0	41.9
2.1	7.4	140.5	40.9	48.2
2.5	7.8	157.6	41.4	49.2
3.0	8.3	174.9	42.0	50.3

Fig. II.12 Cross Section at Station 12, Trois Rivières



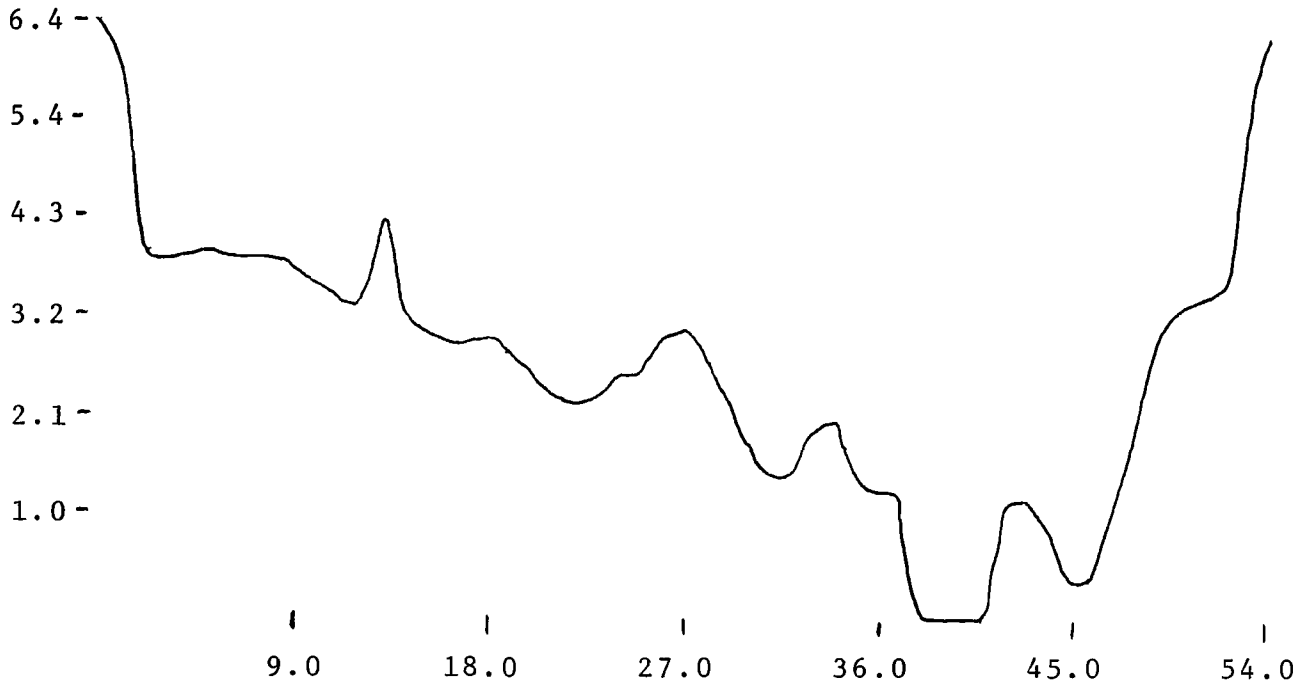
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-2.5	.2	1.9	10.6	10.8
-2.2	.5	5.4	13.4	13.9
-1.9	.8	9.4	14.2	14.8
-1.6	1.1	14.4	18.9	19.8
-1.3	1.4	20.4	23.8	25.0
-1.0	1.7	27.6	25.9	27.5
-.7	2.0	35.7	29.4	31.7
-.4	2.3	44.6	31.8	34.6
-.1	2.6	54.4	40.3	43.5
.1	2.9	66.6	42.9	46.5
.3	3.1	79.7	49.5	53.6
.6	3.4	94.7	53.5	58.0
.9	3.7	117.5	84.5	89.2
1.2	4.0	142.1	85.5	90.4
1.5	4.3	167.0	85.8	91.1
1.8	4.6	192.0	86.2	91.8
2.1	4.9	217.0	86.6	92.5
2.4	5.2	242.2	86.8	93.2
2.7	5.5	267.4	87.3	94.0
3.0	5.8	292.9	88.0	95.0

Fig. II.13 Cross Section at Station 13, Champlain



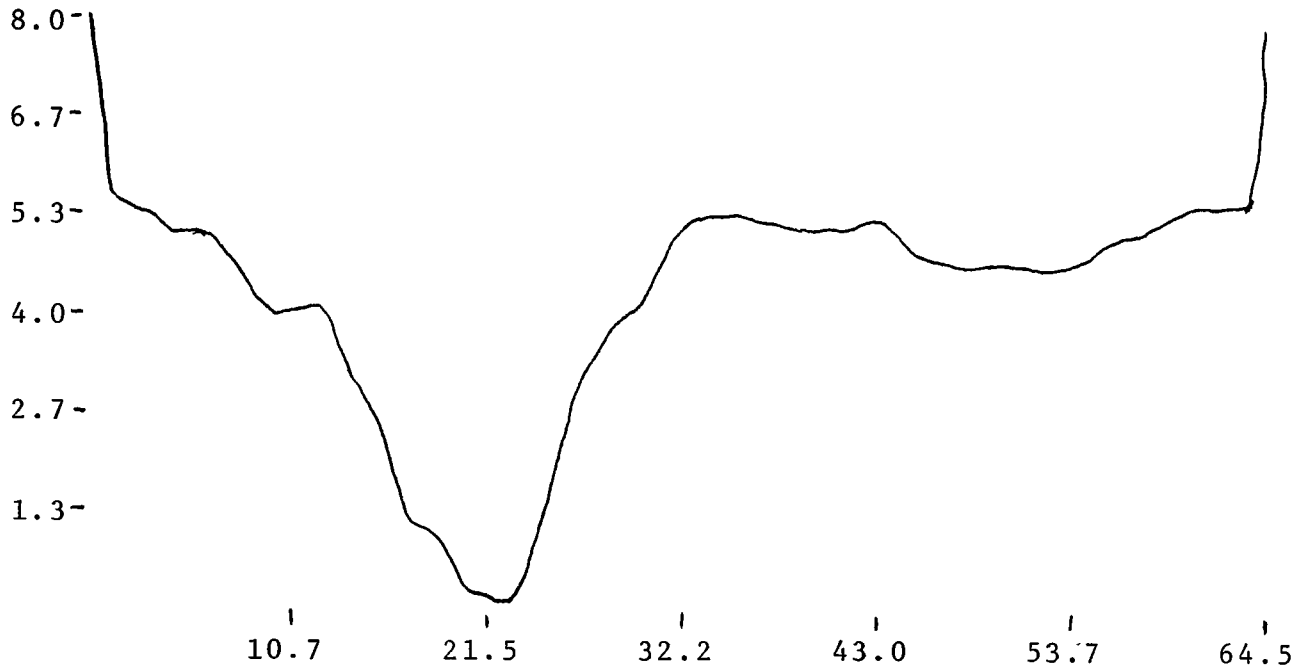
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-3.1	.3	1.3	6.0	6.1
-2.8	.6	4.1	10.4	10.7
-2.5	.9	8.1	12.9	13.4
-2.2	1.3	12.4	14.1	14.7
-1.8	1.6	17.3	15.5	16.3
-1.5	1.9	22.6	17.0	17.9
-1.2	2.2	28.6	19.8	20.8
- .9	2.6	35.6	23.6	24.8
- .5	2.9	43.7	27.8	29.2
- .2	3.2	53.0	28.9	30.4
.1	3.5	63.1	33.4	35.2
.4	3.9	74.4	36.2	38.2
.7	4.2	87.2	50.2	52.4
1.0	4.5	105.0	56.0	58.4
1.3	4.8	123.3	56.6	59.3
1.7	5.2	141.8	57.1	60.2
2.0	5.5	160.5	57.6	61.0
2.3	5.8	179.3	58.0	61.7
2.6	6.1	198.2	58.5	62.5
3.0	6.5	217.3	59.0	63.3

Fig. II.14 Cross Section at Station 14, Batiscan



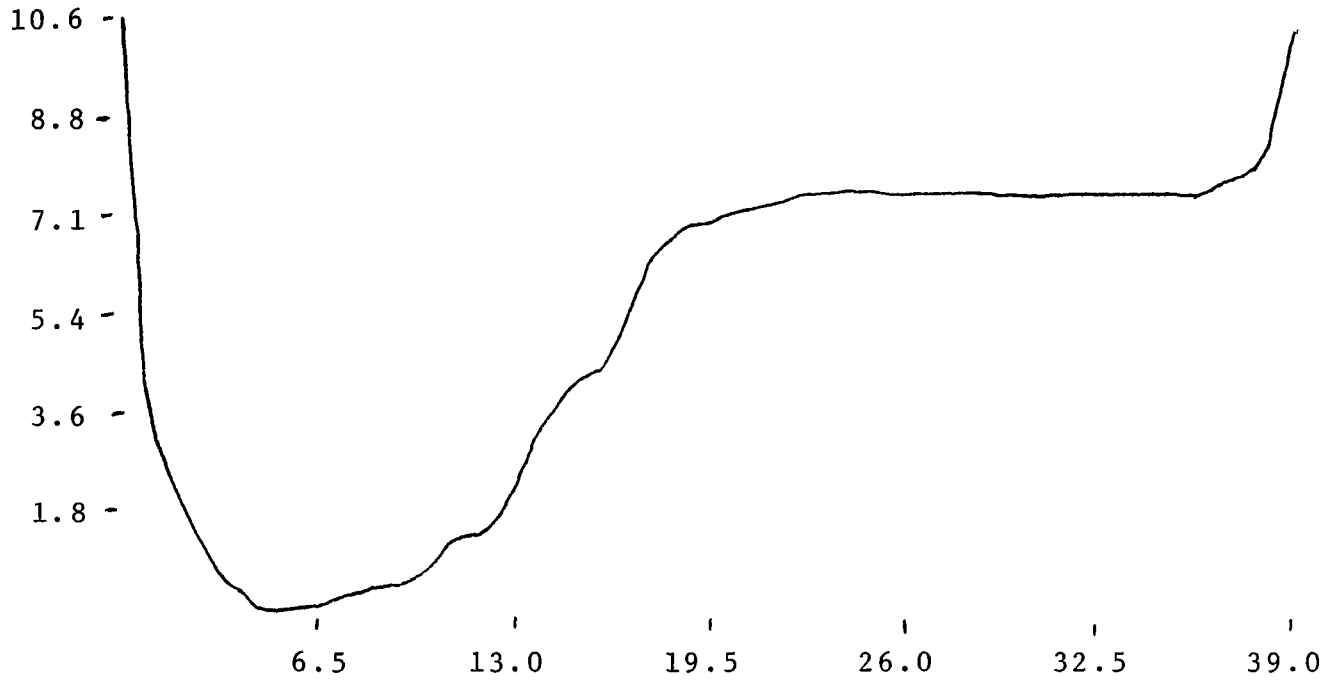
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-3.1	.3	1.0	3.4	3.8
-2.8	.6	2.5	5.5	6.3
-2.5	.9	4.5	6.8	8.2
-2.2	1.3	7.0	8.7	10.5
-1.8	1.6	10.8	13.9	16.1
-1.5	1.9	15.8	16.3	18.8
-1.2	2.2	21.5	19.0	21.8
-.9	2.6	28.5	23.6	26.7
-.5	2.9	36.9	27.7	31.1
-.2	3.2	47.3	34.9	38.5
.1	3.5	59.1	38.9	42.8
.4	3.9	72.5	43.0	47.3
.7	4.2	88.5	50.0	55.0
1.0	4.5	104.9	50.7	56.2
1.3	4.8	121.5	51.0	56.9
1.7	5.2	138.1	51.3	57.6
2.0	5.5	154.8	51.6	58.3
2.3	5.8	171.7	52.0	59.1
2.6	6.1	188.7	52.7	60.1
3.0	6.5	206.0	54.0	61.5

Fig. II.15 Cross Section at Station 15, Cap-à-la Roche



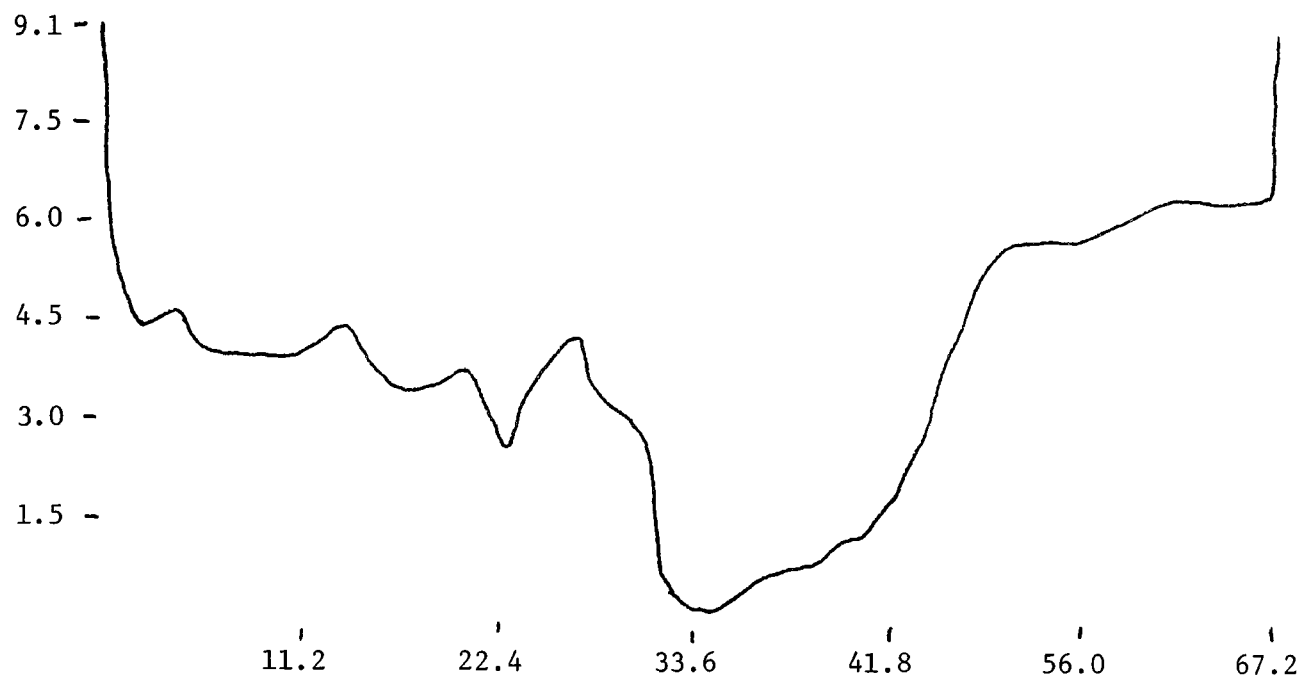
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-4.6	.4	1.0	3.6	3.8
-4.2	.8	2.7	4.9	5.3
-3.8	1.2	5.2	7.5	8.1
-3.4	1.6	8.5	8.3	9.3
-3.0	2.0	12.0	9.1	10.4
-2.6	2.4	15.9	10.0	11.6
-2.2	2.8	20.2	11.0	12.9
-1.8	3.2	25.0	12.7	14.8
-1.4	3.6	30.5	14.3	16.6
-1.0	4.0	36.7	17.1	19.6
-.6	4.4	44.9	21.8	24.5
-.2	4.8	55.5	32.9	35.8
.1	5.2	70.6	46.1	49.2
.5	5.6	93.3	63.0	66.3
.9	6.0	118.8	63.2	67.2
1.3	6.4	144.5	63.5	68.0
1.7	6.8	170.3	63.7	68.9
2.1	7.2	196.2	64.0	69.7
2.5	7.6	222.2	64.2	70.6
3.0	8.1	248.2	64.5	71.4

Fig. II.16 Cross Section at Station 16, Grondines



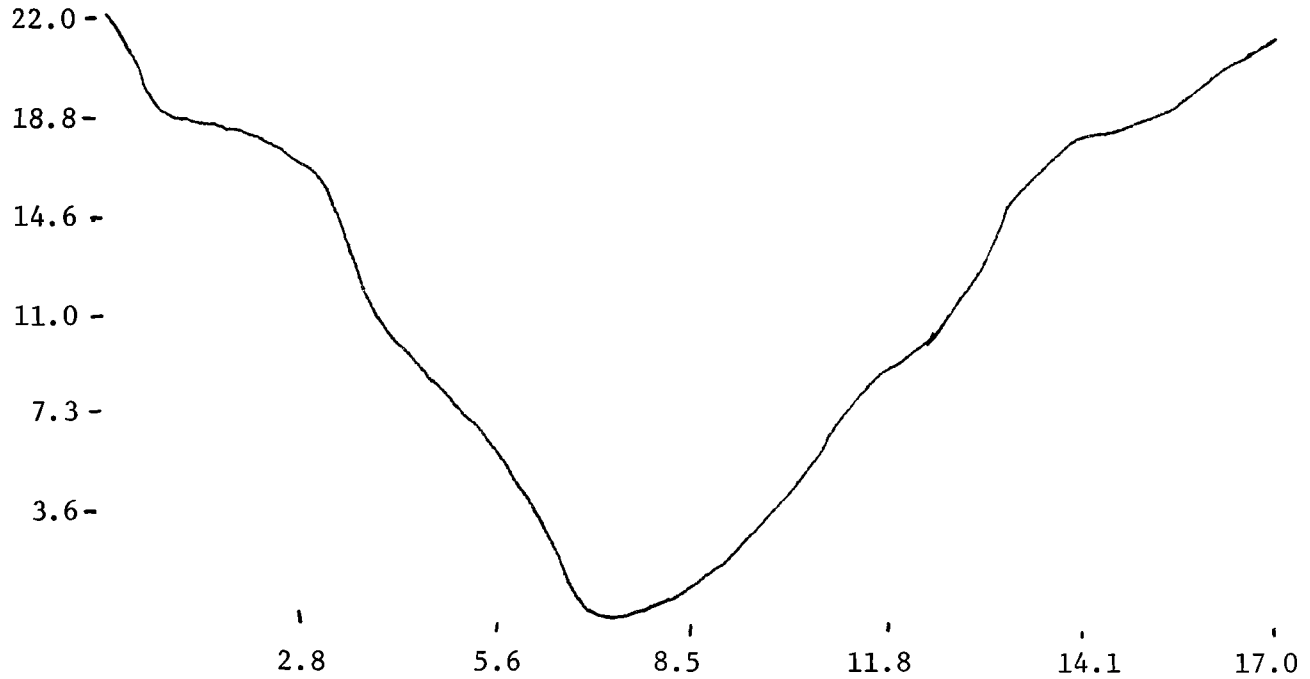
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-7.2	.5	1.7	5.8	6.0
-6.7	1.0	5.6	7.8	8.3
-6.1	1.6	10.3	9.9	10.9
-5.6	2.1	16.0	11.0	12.4
-5.1	2.7	22.2	11.9	13.8
-4.5	3.2	28.9	12.7	15.2
-4.0	3.7	36.0	13.5	16.6
-3.4	4.3	43.5	14.4	18.1
-2.9	4.8	51.6	15.3	19.7
-2.4	5.4	60.0	15.8	20.9
-1.8	5.9	68.7	16.2	22.1
-1.3	6.4	77.7	16.9	23.5
-.7	7.0	87.0	17.8	25.1
-.2	7.5	97.4	21.1	28.9
.3	8.1	113.5	36.6	44.9
.8	8.6	133.8	37.8	46.8
1.3	9.1	154.3	38.0	47.9
1.9	9.7	174.9	38.3	49.0
2.4	10.2	195.7	38.6	50.2
3.0	10.8	216.7	39.0	51.3

Fig. II.17 Cross Section at Station 17, Portneuf



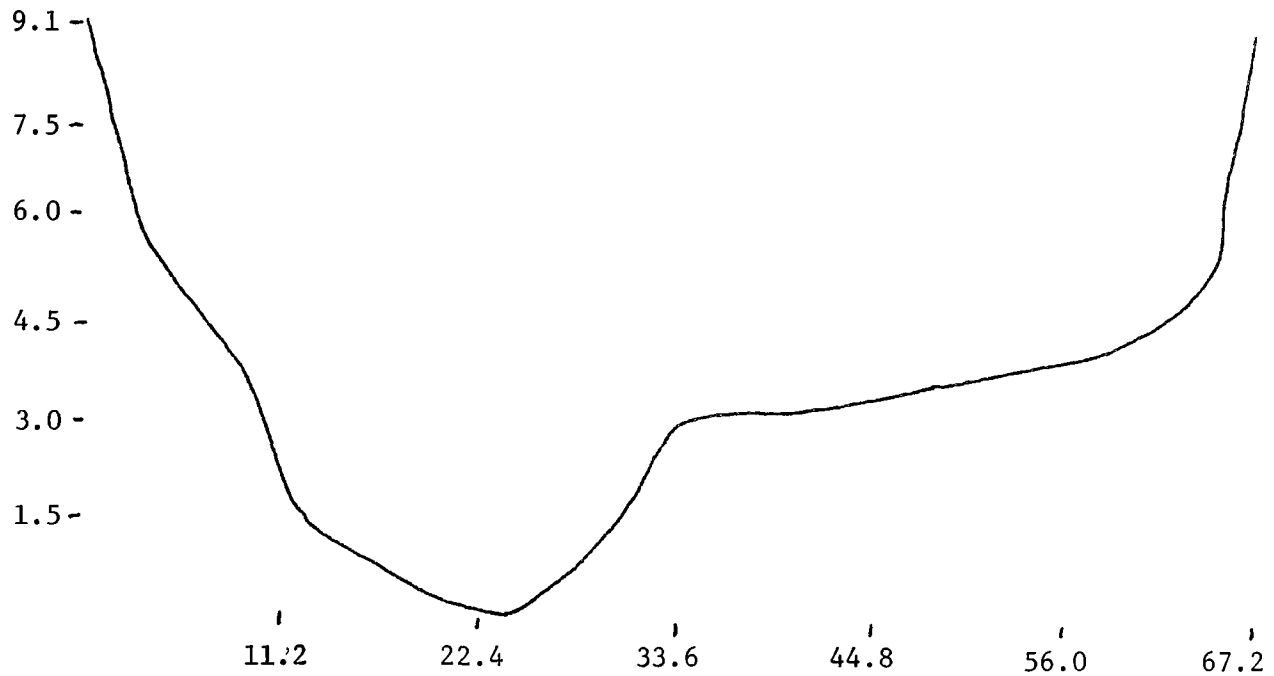
Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-5.6	.4	1.1	4.9	5.0
-5.1	.9	4.2	9.4	9.8
-4.7	1.3	9.1	12.2	13.0
-4.2	1.8	15.0	13.8	14.9
-3.8	2.2	21.5	14.7	16.3
-3.3	2.7	28.6	16.7	18.9
-2.9	3.1	36.9	19.8	22.5
-2.4	3.6	47.2	26.8	30.1
-2.0	4.0	61.3	37.8	41.7
-1.5	4.5	80.1	46.3	50.7
-1.0	5.0	102.0	48.9	53.7
-.6	5.4	124.5	49.9	55.1
-.1	5.9	148.1	56.0	61.5
.2	6.3	174.5	59.9	65.8
.7	6.8	204.0	66.8	73.3
1.1	7.2	234.5	67.0	74.3
1.6	7.7	265.0	67.1	75.2
2.0	8.1	295.5	67.1	76.1
2.5	8.6	326.1	67.2	77.0
3.0	9.1	356.7	67.2	77.9

Fig. II.18 Cross Section at Station 18, Neuville



Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-17.9	1.1	1.2	1.6	3.0
-16.8	2.2	3.4	2.4	5.4
-15.7	3.3	6.4	3.0	7.7
-14.6	4.4	10.1	3.6	10.0
-13.5	5.5	14.5	4.2	12.3
-12.4	6.6	19.5	4.9	14.6
-11.3	7.7	25.4	5.6	16.9
-10.2	8.8	31.9	6.3	19.2
- 9.1	9.9	39.5	7.4	21.6
- 8.0	11.0	48.2	8.1	24.0
- 6.9	12.1	57.4	8.5	26.2
- 5.8	13.2	67.1	8.9	28.4
- 4.7	14.3	77.2	9.3	30.7
- 3.6	15.4	87.8	9.7	32.9
- 2.5	16.5	98.8	10.3	35.2
- 1.4	17.6	110.9	11.5	37.7
- .3	18.7	124.9	14.1	41.6
.8	19.8	140.9	14.9	43.9
1.9	20.9	157.8	15.7	46.3
3.0	22.0	175.8	17.0	48.9

Fig. II.19 Cross Section at Station 19, Quebec Bridge



Depth Rel. to I.G.L.D. (Inches)	Depth Rel. to Chan. Bot. (Inches)	Area (Square Inches)	Top Width (Inches)	Wetted Perimeter (Inches)
-16.1	1.0	1.1	2.3	3.2
-15.1	2.0	4.2	3.7	5.7
-14.1	3.0	8.9	5.5	8.4
-13.1	4.0	15.0	6.3	10.6
-12.1	5.0	21.7	6.8	12.7
-11.1	6.0	28.9	7.3	14.8
-10.1	7.0	36.5	8.3	17.4
- 9.1	8.0	47.6	12.5	22.6
- 8.1	9.0	61.6	15.9	26.9
- 7.1	10.1	79.0	18.0	30.0
- 6.0	11.1	97.6	18.9	32.2
- 5.0	12.1	117.2	19.7	34.4
- 4.0	13.1	137.4	20.1	36.5
- 3.0	14.1	157.9	20.4	38.5
- 2.0	15.1	178.6	20.6	40.6
- 1.0	16.1	199.7	20.9	42.6
- .0	17.1	221.0	21.2	44.7
.9	18.1	242.5	21.4	46.7
1.9	19.1	264.4	21.7	48.7
3.0	20.2	286.4	22.0	50.8

Fig. II.20 Cross Section at Station 20, Quebec City

APPENDIX III

LISTING OF THE FORTRAN PROGRAM

```

COMMON/GDAT/XLOC(100), A(100), H(100), KMAX
COMMON/GENRL/VO(2), HO(2), SLOPE(150), HP(20), N4, XPLOC(20)
COMMON/PLBACK/W1(3, 150), W2(3, 150), C(150), QS(150), QL(150),
1  W1M, W2M, XOUT(18)
COMMON/SCALE/FM, FDM, SOM, CM, DELX, SHPF, QSM, QLM, JAY, JM1, JM2, G
SCALED FRACTION W1, W2, XOUT
LOGICAL LG, LOGIC, PLTFLG, SENSW, LSS, N5, LST, REFLAG
LOGICAL ENDFLG
INTEGER UPDAT

```

```

C
C MAIN PROGRAM FOR THE HYBRID COMPUTER
C DEVELOPED BY MARTIN OOSTERVELD, 1973

```

```

C *****

```

```

C INITIALIZE THE ANALOG

```

```

C CALL EX7MT
C CALL QSHYIN(IERR, 680)
C CALL QSSSECN(IERR)
C PLTFLG = .FALSE.
C N5 = .FALSE.

```

```

C
C CONSTANTS OF THE PROBLEM

```

```

C SS= SPACE SCALE -SS FT. = 1 SEC.
C SS = 10.
C G = 32.2
C NFREQ=NO. OF FEET BETWEEN PLAYBACK LOCATIONS
C NFREQ=7
C IDEC=1
C ICNT=1

```

```

C
C IITT=NUMBER OF ITERATIONS PER SOLUTION
C IITT=1

```

```

C *****

```

```

C DATA INPUT SECTION

```

```

C NO. OF TIME STEPS, NO. OF DATA INPUT LOCATIONS, DELTA T
C READ (6, 150) JMAX, KMAX, DELT

```

```

C SCALING INFORMATION
C READ(6, 675) SHPF, CM, FM, FDM, SOM, QSM, QLM, VMAX, HMAX

```

```

C STATION LOCATIONS
C READ (6, 111) (XLOC(K), K=1, KMAX)

```

```

C INITIAL WATER LEVELS
C READ(6, 90) (H(K), K=1, KMAX)

```

```

C COMPUTE SCALING CONSTANTS

```

```

C FAC = SQRT(G/SHPF)
C TEMP = 2. *SQRT(HMAX)*FAC
C W1M = TEMP + VMAX
C W2M=W1M

```

```

      WDM=2.0
      XLMM = VMAX + SQRT(G*SHPF*HMAX)
      XLMDT = XLMM*DELT
C
C *****
C
C      INITIALIZE ROTATING STORAGE COUNTERS
C
      902 JM2=1
          JM1=2
          JAY=3
C
      CALL GEOME
      EX = NFREQ
      XSPACE = 0.1*EX*SS
      DELX=XSPACE
C
C      FORM INITIAL VALUES OF W1 AND W2
C
      CALL CREAT(KKMAX)
      TYPE 820, KKMAX
C
C      INPUT CHEZY FRICTION COEFFICIENT INFORMATION
C
      READ(6,98)(SLOPE(K),K=1, KKMAX)
      JJ = 0
C
C *****
C
C      INPUT LATERAL INFLOW DATA
C
      DO 202 K=1, KLM
          KLM = KMAX-1
          QS(K) = 0.
          QL(K)=0.
      202 CONTINUE
C
C *****
      398 CONTINUE
C
C      FORM INITIAL ESTIMATE OF W2
      DO 901 K=1, KKMAX
          DUM1=W2(JM1, K)
          DUM2=W2(JM2, K)
          ANS=DUM1
      901 W2(JAY, K)=ANS
          TYPE 362, JAY, JM1, JM2
C
C *****
C
C      COMPUTE CURRENT VALUES OF CHEZY COEFFICIENT
      DO 116 K=1, KKMAX
          DUM1=W1(JM1, K)
          DUM2=W2(JM1, K)
          H(K)=((W1M*DUM1-W2M*DUM2)**2.)*SHPF/(16.*G)
      116 C(K)=SLOPE(K)*H(K)
          IF(SENSE(4)) WRITE(6,97)(K, H(K), C(K), K=1, KKMAX)
C
C *****
      389 CONTINUE

```

```

C
C INPUT BOUNDARY CONDITIONS
C
C BOUNDARY CONDITIONS FOR FLOOD ROUTING
C
C BOUNDARY CONDITIONS
C AT X=0
C IF(JJ.EQ.0) H0(1)=0.235
C IF(JJ.GT.0.AND.JJ.LE.25) H0(1)=H0(1)+0.0106
C IF(JJ.GT.25.AND.JJ.LE.50) H0(1)=H0(1)-0.0106
C IF(JJ.GT.50)H0(1)=0.235
C AT X=XMAX
C H0(2)=0.260
C
C *****
C
397 CONTINUE
C
C ESTIMATE W1(JAY,1)
C DUM1=W2(JAY,1)
C DUM2=SQRT(16.*G*H0(1)/SHPF)
C DUM3=(DUM2+DUM1*W2M)/W1M
C W1(JAY,1)=DUM3
C
C *****
C
C INITIALIZING FOR FORWARD INTEGRATION
C
C CALL QWJDAS(W1(JAY,1),09,IERR)
C DQS=QS(1)/QSM
C DQL=QL(1)/QLM
C CALL QWJDAR(DQS,17,IERR)
C CALL QWJDAR(DQL,18,IERR)
C DAM0 = (2.+SHPF)*W1M/(40.*XLMM)
C DAM1 = (2.-SHPF)*W2M/(4.*XLMM)
C DAM2 = W1M/(WDM*XLMDT)
C CALL QWJDAR (DAM0,00,IERR)
C CALL QWJDAR (DAM1,01,IERR)
C CALL QWJDAR (DAM2,02,IERR)
C
C PUT INITIAL VALUES INTO RECONSTRUCTION CIRCUITS
C K=1
C
C CALL GEOVAL(K)
C CALL QWJDAS(W1(JM1,1),04,IERR)
C CALL QWJDAS(W2(JAY,1),05,IERR)
C
C IND=0 FOR FORWARD INTEGRATE
C
C INPUT VALUES FOR NEXT SPACE ELIMENT
C
C IND=0
C K=2
C CALL PROUT(K,IND)
C CALL QSTDA
C CALL QWCLL(2,.FALSE.,IERR)
C
C *****
C
C DATA CHANGE AND INPUT SECTION

```

```

C      INPUT PLOTTING LOCATIONS FOR CALCOMP PLOTTER
C
C      11 IF (SENSW(1)) GO TO 50
C
C      CHANGE NO OF PLAYBACK LOCATIONS
C
C      51 IF (SENSW(2)) GO TO 52
C
C      *****
C
C      53 CALL QSIC(IERR)
C      CALL QSDLY(1000)
C
C      SENSE LINE 3 IS CONNECTED TO A PUSHBUTTON TO START INTEGRATION
C
C      CALL QRSLL (3,LSS,IERR)
C      CALL QRSLL (3,LSS,IERR)
C      IF (.NOT. LSS) GO TO 11
C
C      IF (.NOT. SENSW(5)) GO TO 12
C      IF (.NOT. N5) GO TO 13
C      CALL SBYSET
C
C      *****
C
C      INITIALIZE THE CALCOMP PLOTTER
C
C      CALL PLTINT (KKMAX,N4,XPLOC)
C
C      FIND CURRENT PLOTTING HEIGHTS
C
C      CALL HTPLT
C      N5 = FALSE.
C      PLTFLG = .TRUE.
C      13 TIME = JJ
C      XP=TIME*DELT
C      TYPE 86, (HP(JKN),JKN=1,N4)
C
C      PLOT POINTS
C
C      CALL STRPLT (N4,0.,XP,HP)
C      GO TO 14
C      12 IF (PLTFLG) CALL STNDEY
C      14 CONTINUE
C
C      *****
C
C      INITIALIZE CONTROL LINES AND SENSE LINES
C      CALL QWCLL(1, TRUE., IERR)
C      CALL QRSLL(2,LG,IERR)
C      CALL QRSLL(0,LOGIC,IERR)
C
C      START ON A CLOCK PULSE
C
C      CALL QRSLL(1,LST,IERR)
C      15 CALL QRSLL(1,LST,IERR)
C      IF (.NOT. LST) GO TO 15
C
C      *****

```

```
C
C
C      START OPERATION ON FORWARD INTEGRATE
C
C      CALL QSOP(IERR)
399 KK=K+1
C
C      INPUT NEXT SPACE DATA INFORMATION
C
C      CALL PROUT(KK, IND)
C
C      TIME TO UPDATE
C
141 CALL QRSLL(2, LG, IERR)
    IF(LG) GO TO 501
131 CALL QRSLL(2, LG, IERR)
    IF(.NOT. LG) GO TO 131
C
C      HAVE DIGITAL READ CURRENT VALUE OF W1
C
C      CALL QRBADS(W1(JAY, K), 1, 1, IERR)
C
C      TRANSFER VALUES FOR NEXT ELIMENT
C
C      CALL QSTDA
C
C      SET T/S DEVICES TO STORE
C
C      CALL QWCLL(1, TRUE., IERR)
C      K=K+1
C
C      HAS SPACE ELAPSED
C
C      IF(K-KKMAX)399, 141, 130
C
C      SET ANALOG TO I. C.
C
130 CALL QSIC(IERR)
C
C      *****
C
C      IF(KK. NE. KKMAX) GO TO 502
C
C      OUTPUT W1
C
C      IF (SENSW(6))  WRITE(6, 700) (W1(JAY, K), K=1, KKMAX)
C
C      WANT TO CHANGE CALIBRATION
C
C      IF(SENSW(3)) GO TO 507
C      TYPE 405, W1(JAY, KKMAX)
C
C      *****
C
C      BACKWARD INTEGRATION LOOP
C
C      INITIALIZATION FOR BACKWARD INTEGRATE
C
C      DUMVAR=SQRT(16. *G*H0(2)/SHFF)
C      DUM=W1(JAY, KKMAX)
```

```

      W2(JAY, KKMAX) = (W1M*DUM-DUMVAR)/W2M
137 TEMP = 4. *XLMM
      DAMO = (2. + SHPF)*W2M/(TEMP*10. )
      DAM1 = (2. - SHPF)*W1M/TEMP
      DAM2 = W2M/(WDM*XLMDT)
      CALL QWCLL(2, . TRUE. , IERR)
      CALL QWJDAR(DAMO, 00, IERR)
      CALL QWJDAR(DAM1, 01, IERR)
      CALL QWJDAR(DAM2, 02, IERR)

C
C      IND=1 FOR BACKWARD INTEGRATION
C
      IND=1
      K=KKMAX
      KK=K-1
      CALL GEOVAL(KK)
      CALL QWJDAS(W2(JAY, KKMAX), 09, IERR)
      CALL PROUT(KK, IND)
      CALL QSTDA
      CALL QWJDAS(W2(JM1, KKMAX), 04, IERR)
      CALL QWJDAS(W1(JAY, KKMAX), 05, IERR)

C
C
      CALL QWCLL(1, . TRUE. , IERR)
      CALL QWCLL(2, . TRUE. , IERR)
      CALL QSDLY(1000)
      K=KKMAX-1

C
C      WAIT FOR A CLOCK-PULSE
C
      CALL QRSLL(0, LOGIC, IERR)
      CALL QRSLL(2, LG, IERR)
      CALL QRSLL(1, LST, IERR)
341 CALL QRSLL(1, LST, IERR)
      IF(. NOT. LST) GO TO 341

C
C      *****
C
C      OPERATE ON BACKWARD LOOP
C
      CALL QSOP(IERR)

C
342 KK=K-1
      CALL PROUT(KK, IND)
434 CALL QRSLL(2, LG, IERR)
      IF(LG) GO TO 503
326 CALL QRSLL(2, LG, IERR)
      IF(. NOT. LG) GO TO 326
      CALL QRBADS(W2(JAY, K), 1, 1, IERR)
      CALL QSTDA
      CALL QWCLL(1, . TRUE. , IERR)
      K=K-1
      IF(K-1) 329, 434, 342
329 CALL QSIC(IERR)

C
C      *****
C
C      CORRECT NUMBER OF SEGMENTS
      IF(KK. NE. 1) GO TO 504

```

```
      IF(SENSW(6)) WRITE(6,700) (W2(JAY,K),K=1, KKMAX)
C
450 CONTINUE
C
C *****
C
C INCREMENT TIME
C
      IF(IDEQ.EQ.1) GO TO 400
      IF(IDEQ.EQ.0) GO TO 402
400 IDEQ=0
      GO TO 401
402 CONTINUE
      ICNT=ICNT+1
      IF(ICNT.GT.IITT) GO TO 404
401 CONTINUE
      IF(IDEQ-1)397,375,380
C
C *****
C
404 CONTINUE
      ICNT=1
      IDEQ=1
      GO TO 401
C
C
C *****
C INCREMENT TIME AND PLOT CURRENT VALUES
C
375 JM2=JM2+1
      JM1=JM1+1
      JAY=JAY+1
      CALL HTPLT
      IF(JM2.EQ.4) JM2=1
      IF(JM1.EQ.4) JM1=1
      IF(JAY.EQ.4) JAY=1
      JJ = JJ + 1
      IF (JJ-JMAX) 398,398,500
C
C *****
C
380 CONTINUE
      INPUT NEW VALUE INTO W2(JAY,1)
      TYPE 381
381 FORMAT(32H INPUT NEW ESTIMATE OF W2(JAY,1))
      ACCEPT 382,W2(JAY,1)
      GO TO 397
C
C *****
C
C TIMING ERROR PRINT OUT SECTION
C
501 TYPE 510
      GO TO 555
503 TYPE 511
      GO TO 555
502 TYPE 512, KK
      GO TO 389
504 TYPE 514, KK
      GO TO 389
```

```

505 CALL QSH(IERR)
    TYPE 515, KK
    GO TO 555
506 CALL QSH(IERR)
    TYPE 516, KK
555 CALL QSIC(IERR)
    PAUSE
    GO TO 398
510 FORMAT (29H SAMPLING RATE TOO HIGH (501)/)
511 FORMAT (29H SAMPLING RATE TOO HIGH (503)/)
512 FORMAT (25H AT THE END OF COUNTUP K=, I4/)
514 FORMAT (27H AT THE END OF COUNTDOWN K=, I3/)
515 FORMAT(35H UP COUNTER OVER MAX. VALUE, VALUE = , I4)
516 FORMAT(29H DOWN COUNTER TOO LOW , VALUE=, I3)
C *****
  50 TYPE 600
    TYPE 603
C
C   INPUT THE PLOT LOCATIONS
C
    ACCEPT 5, N4
    DO 604 I=1, N4
      TYPE 605, I
604 ACCEPT 606, XPLOC(I)
      NS = .TRUE.
      GO TO 51
C
C *****
C
600 FORMAT (40H RESET SENSW(A), INPUT N4, XPLOC(J), J=1, N4/)
603 FORMAT (24H HOW MANY PLOT LOCATIONS/)
605 FORMAT (15H INPUT LOCATION, I3/)
606 FORMAT (F8.2)
602 FORMAT (27H RESET SENSW(B), INPUT NFREQ)
  6 FORMAT (I10)
C
C *****
C
C   HOW MANY PLAYBACK LOCATIONS
C
  52 TYPE 602
    ACCEPT 6, NFREQ
    VAL = 1. /FLOAT(NFREQ)
    CALL QWPR(4HP000, VAL, IERR)
    CALL QWPR(4HP030, VAL, IERR)
    GO TO 53
507 ACCEPT 518, K, C(K)
C
C *****
C
  97 FORMAT(I3, 3X, F5.3, 3X, F6.0)
518 FORMAT(I3, F12.3)
    GO TO 398
500 STOP
  95 FORMAT (8X, F10.7)
  98 FORMAT(8F10.0)
  99 FORMAT (16X, 4F16.5)
111 FORMAT (5X, F10.6)
120 FORMAT (45H INPUT BOUNDARY VALUES OF VELOCITY AND HEIGHT)
150 FORMAT (I5, I5, F12.2)

```

```
362 FORMAT(I2)
651 FORMAT (11H VALUE READ, I10, 10H SET VALUE, I10)
675 FORMAT(10F8. 5)
688 FORMAT (2HK=, I5, 5HKMAX=, I5)
690 FORMAT (10H TIME LEFT, I10)
804 FORMAT (19H D/A CHANNEL VALUES)
810 FORMAT(5(S10, 2X)/)
820 FORMAT (25H NO. OF PLAYBACK ELEMENTS, I5)
661 FORMAT (17H PLAYBACK ELEMENT, I4/)
700 FORMAT ((10(S6, 2X)))
    5 FORMAT(I3)
382 FORMAT(S6)
405 FORMAT(S5)
    86 FORMAT(5F10. 4)
    90 FORMAT(5X, F10. 6)
    END
```

```
SUBROUTINE CREAT(KKMAX)
COMMON/GDAT/XLOC(100), A(100), H(100), KMAX
COMMON/SCALE/FM, FDM, SOM, CM, DELX, SHPF, QSM, QLM, JAY, JM1, JM2, G
COMMON/PLBACK/W1(3, 150), W2(3, 150), C(150), QS(150), QL(150),
1  W1M, W2M, XOUT(18)
SCALED FRACTION W1, W2, XOUT

C
C  THIS SUBROUTINE CREATES INITIAL VALUES OF W1 AND W2 FROM DATA ON
C  WATER LEVELS AND DISCHARGE
C

XSPACE=DELX
FAC=SQRT(G/SHPF)
Q=0. 1521
I = 1
K = 1
20 IP1 = I+1
TEMP = XLOC(IP1) - XLOC(I)
SLVEL = (Q/A(IP1) - Q/A(I))/TEMP
SLHGT = (H(IP1) - H(I))/TEMP
XXX = K-1
DIST = XXX*XSPACE - XLOC(I)
VEL = Q/A(I) + DIST*SLVEL
HEIGHT = H(I) + DIST*SLHGT
TEMP = 2. *SQRT(HEIGHT)*FAC
W1(JM1, K) = (VEL+TEMP)/W1M
W2(JM1, K)=(VEL-TEMP)/W2M
W2(JM2, K)=W2(JM1, K)
W1(JM2, K)=W1(JM1, K)
W1(JAY, K)=W1(JM1, K)
W2(JAY, K)=W2(JM1, K)
X = K
K = K+1
IF (X*XSPACE .GT. XLOC(IP1)) I=IP1
IF (I-KMAX) 20, 21, 21
21 W1(JM1, K)=(Q/A(I)+2. *SQRT(H(I))*FAC)/W1M
W2(JM1, K)=(Q/A(I)-2. *SQRT(H(I))*FAC)/W2M
W2(JM2, K)=W2(JM1, K)
KKMAX=K-1
RETURN
END
```

```
      SUBROUTINE PROUT(K,IND)
C
C      THIS SUBROUTINE INPUTS VALUES OF GEOMETRIC VALUES
C
      COMMON/SCALE/FM,FDM,SOM,CM,DELX,SHFF,QSM,QLM,JAY,JM1,JM2,G
      COMMON/PLBACK/W1(3,150),W2(3,150),C(150),QS(150),QL(150),
1      W1M,W2M,XOUT(18)
      SCALED FRACTION W1,W2,XOUT
      XK=K-1
      X=XK*DELX
      THETA1=0.01379*X+2.274
      THETA2=0.03372*X-2.232
      FOUR1=5.044*COS(THETA1)
      FOUR2=1.706*COS(THETA2)
      XOUT(13)=(22.07+0.02324*X+FOUR1+FOUR2)/(FM*12.)
      FOUR1=0.0696*SIN(THETA1)
      FOUR2=0.0575*SIN(THETA2)
      XOUT(14)=(0.02324-FOUR1-FOUR2)/(FDM*12.)
      THETA1=0.046*X+0.458
      XOUT(15)=(0.00785+0.088*SIN(THETA1))/(SOM*12.)
      XOUT(16)=(C(K))**2./(CM**2.)
      XOUT(11)=W1(JM1,K)
      IF(IND.EQ.1)XOUT(11)=W2(JM1,K)
      XOUT(12)=W2(JAY,K)
      IF(IND.EQ.1) XOUT(12)=W1(JAY,K)
      CALL QWBDA(XOUT(11),11,6,IERR)
      IF(IERR.EQ.2) GO TO 5
      RETURN
5 CALL QSH(IERR)
      TYPE 6,(XOUT(JK),JK=11,16)
6 FORMAT(F10.6)
      RETURN
      END
```

```

SUBROUTINE GEOVAL(I)
COMMON/SCALE/FM,FDM,SOM,CM,DELX,SHPF,OSM,OLM,JAY,JM1,JM2,G
COMMON/PLBACK/W1(3,150),W2(3,150),C(150),OS(150),OL(150),
1  W1M,W2M,XOUT(18)
SCALED FRACTION W1,W2,XOUT

C
C THIS SUBROUTINE DETERMINES VALUES OF F, FDOT, AND SO GIVEN X
C
C
X1=XI-1
X=XI*DELX
THETA1=0.01379*X+2.274
THETA2=0.03372*X-2.232
FOUR1=5.044*COS(THETA1)
FOUR2=1.706*COS(THETA2)
F=(22.07+0.02324*X+FOUR1+FOUR2)/(FM*12.)
FOUR1=0.0696*SIN(THETA1)
FOUR2=0.0575*SIN(THETA2)
FDOT=(0.02324-FOUR1-FOUR2)/(FDM*12.)
THETA1=0.046*X+0.458
SO=(0.00785+0.088*SIN(THETA1))/(SOM*12.)
C2=C(I)**2./CM**2
CALL OWJDAR(F,06,IERR)
CALL OWJDAR(FDOT,07,IERR)
CALL OWJDAR(SO,08,IERR)
CALL OWJDAR(C2,10,IERR)
1 FORMAT(F10.3)
RETURN
END

```

```
      SUBROUTINE GEOME
C
C      THIS SUBROUTINE COMPUTES VALUES OF AREA FROM DEPTH
C
      COMMON/GDAT/XLOC(100),A(100),H(100),KMAX
      DO 20 K=1,KMAX
      THETA1=0.01379*XLOC(K)+2.274
      THETA2=0.03372*XLOC(K)-2.232
      FOUR1=5.044*COS(THETA1)
      FOUR2=1.706*COS(THETA2)
      F=(22.07+0.02324*XLOC(K)+FOUR1+FOUR2)/12.
      A(K)=8./3.*SQRT(F)*H(K)**1.5
20  CONTINUE
      RETURN
      END
```

```
SUBROUTINE STRPLT(N,XINIT,X,Y)
C
C  NOTE'*****
C  THE PLOTTER MUST BE INITIALIZED, ANY AXES DRAWN, AND PLOTTING
C  SCALES ESTABLISHED BEFORE STRPLT IS EXECUTED
C  *****
C  SUBROUTINE FOR STRIP-PLOTTING Y(I) VS X
C  FOR I=1,2,...,N   WHERE N.LE. 10
C  PLOTTER INITIALIZES WHEN X.LE.XINIT
C  DIMENSION Y(10),PREVY(10)
C  CHECK FOR INITIALIZING
C  IF(X.LE.XINIT)GO TO 7
C  DO THE PLOTTING
C  DO 6 I=1,N
C  CALL PLOT(XPREV,PREVY(I))
C  CALL PENDN
C  CALL PLOT(X,Y(I))
C  CALL PENUP
6  CONTINUE
C  UPDATE PEN POSITIONS
7  XPREV=X
C  DO 8 I=1,N
C  PREVY(I)=Y(I)
8  CONTINUE
C  RETURN
C  END
```

```

SUBROUTINE PLTINT(JMAX,N4,XPLOC)
DIMENSION XPLOC(10)
*****
C
C SUBROUTINE WHICH SETS UP FOR PLOTTING HEIGHT VS TIME
C AT UP TO 10 SPACE LOCATIONS,
C DRAWS AND LABELS THE X-Y AXIS
C
C *****
C
DATA XSC,YSC/20.,0.25/,XREF,YREF/1.0,1.0/,XVAL,YVAL/0.,0./,XP,YF
120.,8./,XN,YN/0.0,0.0/
C
C CONVERT INPUTS FROM INTEGER TO REAL
C
FMAX=FLOAT(JMAX)+0.1
C SET UP PLOTTER MODE ARRAY, POSITION PLOTTER PAPER
C AND DRAW X-Y AXIS
C
CALL EJECT(20,0,0.0)
CALL PLTSET(XSC,YSC,XREF,YREF,XVAL,YVAL,XP,YF,XN,YN)
CALL STNDBY
CALL QSIZE(2)
CALL LINECD(0)
CALL INPLOT(0.0,YF)
CALL PENDN
CALL INPLOT(0.0,0.0)
CALL INPLOT(10.0,0.0)
CALL PENUP
C
C LABEL GRID Y-AXIS
C
CALL CHSIZE(0.125,90.0)
CALL INPLOT(-0.75,1.95)
CALL PLTEXT(18HWATER LEVEL (FEET))
CALL CHSIZE(0.125,0.0)
Y=0.8
YV=0.2
DO 2 I=1,10
CALL INPLOT(0.0,Y)
CALL YMARK
CALL INPLOT(-0.45,Y)
CALL PLTVAL(YV,1)
Y=Y+0.8
2 YV=YV+0.2
C
C LABEL GRID X-AXIS
C
X=1.0
XV=40.
DO 6 J=1,5
DO 4 I=1,2
CALL INPLOT(X,0.0)
CALL XMARK
4 X=X+1.0
Z=X-1.1
CALL INPLOT(Z,-0.25)
CALL PLTVAL(XV,-1)

```

```
6 XV=XV+40.  
  CALL INPLOT(3.9,-0.6)  
  CALL PLTEXT(17HTIME (IN SECONDS))  
C  
C  PRINT LEGEND  
  YST=5.75  
  XST=6.15  
  XS1=7.25  
  XS2=7.15  
  CALL INPLOT(XST,YST)  
  CALL PLTEXT(12HELEMENTS # )  
  CALL INPLOT(XS1,YST)  
  CALL PLTVAL(FMAX,-1)  
  DO 8 I=1,N4  
  YST=YST-0.3  
  MZ=2*I  
  CALL INPLOT(XST,YST)  
  CALL SYMBOL (MZ)  
  CALL PLTEXT(9H - LOC )  
  CALL INPLOT(XS2,YST)  
  CALL PLTVAL(XPLOC(I),-1)  
8 CONTINUE  
C  
C  INITIALIZE THE CURVE PLOT  
C  
  CALL STNDBY  
  RETURN  
  END
```

```
      SUBROUTINE HTPLT
      COMMON/GENRL/VO(2),HO(2),SLOPE(150),HP(20),N4,XPLOC(20)
      COMMON/SCALE/FM,FDM,SOM,CM,DELX,SHPF,OSM,OLM,JAY,JM1,JM2,G
      COMMON/PLBACK/W1(3,150),W2(3,150),C(150),OS(150),OL(150),
1      W1M,W2M,XOUT(18)
      SCALED FRACTION W1,W2,XOUT

C
C      THIS SUBROUTINE CALCULATES WATER LEVELS AT PLOT LOCATIONS
C      FROM CURRENT VALUES OF W1 AND W2
C
      XSPACE=DELX
      DO 40 J=1,N4
      I = 0
21  I = I+1
      X = I
      IF (XPLOC(J) - X*XSPACE) 20,20,21
20  XINC=XPLOC(J)-(X-1)*XSPACE
      IP1=I+1
      SD1=W1(JAY,IP1)-W1(JAY,I)
      SLW1=SD1/XSPACE
      SD2=W2(JAY,IP1)-W2(JAY,I)
      SLW2=SD2/XSPACE
      DUM1=W1(JAY,I)
      DUM2=W2(JAY,I)
      W2P=DUM2+XINC*SLW2
      W1P=DUM1+XINC*SLW1
      TEMP = W1P*W1M - W2P*W2M
      HP(J) = TEMP*TEMP*SHPF/(16 *G)
40  CONTINUE
      RETURN
      END
```