Search Space Analysis and Efficient Channel Assignment Solutions for Multi-interface Multi-channel Wireless Networks

by

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Abstract

This thesis is concerned with the channel assignment (CA) problem in multi-channel multi-interface wireless mesh networks (M^2WNs). First, for M^2WNs with general topologies, we rigorously demonstrate using the combinatorial principle of inclusion/exclusion that the CA solution space can be quantified, indicating that its cardinality is greatly influenced by the number of radio interfaces installed on each router. Based on this analysis, a novel scheme is developed to construct a new reduced search space, represented by a lattice structure, that is searched more efficiently for a CA solution. The elements in the reduced lattice-based space, labeled *Solution Structures* (SS), represent groupings of feasible CA solutions satisfying the radio constraints at each node. Two algorithms are presented for searching the lattice structure. The first is a greedy algorithm that finds a good SS in polynomial time, while the second provides a user-controlled depth-first search for the optimal SS. The obtained SS is used to construct an unconstrained weighted graph coloring problem which is then solved to satisfy the soft interference constraints.

For the special class of full M^2WNs (fM^2WNs), we show that an optimal CA solution can only be achieved with a certain number of channels; we denote this number as the characteristic channel number and derive upper and lower bounds for that number as a function of the number of radios per router. Furthermore, exact values for the required channels for minimum interference are obtained when certain relations between the number of routers and the radio interfaces in a given fM^2WN are satisfied. These bounds are then employed to develop closed-form expressions for the minimum channel interference that achieves the maximum throughput for uniform traffic on all communication links. Accordingly, a polynomial-time algorithm to find a near-optimal solution for the channel assignment problem in fM^2WN is developed.

Experimental results confirm the obtained theoretical results and demonstrate the performance of the proposed schemes.

Acknowledgements

Closely working with my supervisor, Dr. Nancy Samaan, has been both professionally rewarding and enjoyable. Her advise and support have been invaluable and are present through all the present work.

Essential support has been always found in my sister, without whom this research would not have even started, and my mother, with her love and light, that makes challenges look like games.

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Chapter 1

Introduction

1.1 An Overview

Multi-channel multi-interface wireless mesh networks (M^2WNs) represent a relatively new networking paradigm envisioned to provide cost effective high-speed broadband networking solutions. This can be attributed to their ease of deployment, low upfront costs, and low maintenance requirements.

Generally, M^2WNs are comprised of two different types of nodes: mesh routers and mesh clients. The former nodes are usually equipped with multiple radio interfaces, have no limitations regarding energy consumption, and are either nomadic or have fixed locations. These routers are connected together to form a high speed wireless backbone to which the mesh clients, mostly mobile, are connected. Moreover, some mesh routers can be additionally connected to an Internet gateway in order to provide the mesh clients with access to services located outside the M^2WN. The radio interfaces installed on each router allow it to simultaneously communicate with several neighboring nodes by tuning these interfaces to different channels. On the other hand, mesh clients represent users using their devices (e.g., laptops and PDAs) to communicate together or to the gateway via mesh routers or other mesh clients.

The performance of a M^2WN heavily depends on several key design factors: the adopted mesh topology, locations of the routers, the chosen network technology (e.g., IEEE802.11a), setting routers configuration parameters (emission power, antenna height, tilt, azimuth, etc.), assigning appropriate channels to communication links and developing appropriate link scheduling and routing schemes.

The work presented in this thesis is mainly concerned with the channel assignment

(CA) problem for M^2WNs which deals with the efficient assignment of frequency channels to each of the radio interfaces on each router and in turn to the network communication links in such way that the M^2WN's performance is optimized. As a CA scheme must use no more than the available network channels such that links adjacent to each router cannot be assigned more distinct channels than the available number of radios on that router, the main challenges facing any CA scheme are: the limited number of available radios on each router, the limited number of channels in the network and the resulting interference among links assigned the same channel, which in turn affects the overall network capacity.

The number of radio interfaces is mostly a design issue and is determined by the network designer (e.g., based on commercial device availability). On the other hand, the number of available channels is dependent on the network access technology (e.g., 3 channels for IEEE 802.11b/g and and 12 channels for IEEE 802.11a). Finally, interference is a byproduct of the above limitations, hence, a *good* CA scheme aims at minimizing such a phenomenon.

1.2 Motivation

The CA problem has been the focus of a large number of research efforts in the literature. From a network perspective, these schemes can be characterized along several axis; For example static CA schemes perform the CA task only when there are major changes to the network such as a node failure while dynamic schemes continuously adapt their assignments to achieve a better performance but at the expense of switching delays as the radio interface is switched from using one channel to the other. Similarly, topology preserving schemes assign a channel to every possible communication link in the network while non-topology preserving schemes selectively choose the communication links that must be assigned a network channel in order to reduce the experienced network interference. This comes at the expense of affecting the complexity of the employed routing protocols.

In spite of these different characteristics among existing CA schemes, they usually approach CA as an optimization problem solved using linear programs or through heuristic algorithms. Linear programming solutions define the problem in terms of an objective function in a system with restrictions. Adopted objective functions usually vary according to the parameters being optimized (e.g., interference vs. network traffic and channel utilization). A general limitation to these approaches is the need to relax some of the problem constraints due to the NP-complexity of the problem and the size of the solution space. While heuristics-based schemes have shown some promising results particularly for smaller networks, the performance of these approaches can not be guaranteed for general networks. This can be attributed to the lack of knowledge of the structure or the size of the CA solution space which highly affect the performance of these schemes. Another notable observation is the lack of theoretical bounds for interference values for optimal CA solutions. This fact limits the ability to evaluate the performance of the developed schemes to comparisons with solutions of relaxed integer linear problems or the use of simulation results that depend heavily on the adopted topologies.

This lack of knowledge of the structure of the solution space, the effect of radio constraints on the space, and the lack of theoretical results, are the main motivation of this work.

1.3 Contributions

To bridge the aforementioned gap, we provide a comprehensive analysis of the size of the solution space and the effects of the number of radio interfaces on the exponential reduction of this space. We show how this original search space can be represented by a much smaller solution space that can be searched in a more efficient manner. We then develop two novel schemes for searching the reduced space for an efficient CA solution.

In order to evaluate our solution for the general CA problem, we considered two scenarios: small and large networks while varying the number of radios per node. The presented work is the first to demonstrate that, for typical networks characterized with smaller number of radio interfaces the solution space can be dramatically reduced. Taking advantage of this feature, the proposed CA algorithms achieve a time complexity closer to linear. Optimum or close to optimum solutions are found when the problem is less constrained in terms of the number of radios. This is due to efficient structures that represent optimum CA solutions that share some common characteristics, and thus, are easier to find. In our developments, we compare the performance of the developed schemes with other existing approaches and show the achieved enhancements with respect to achieved total network interference, solution time and goodput.

Additionally, for full M^2WNs, (fM^2WN), where each node can communicate directly with all other nodes, we develop closed form lower bounds for the optimal interference values. We then introduce a polynomial time CA scheme based on this analysis. The scheme for full WMNs is compared to the lower bounds, and empirically verified through a comparison with results obtained from an exhaustive search procedure when possible.

More precisely, the main contributions of the presented work can be summarized in the following points:

1. For general topologies, we quantify the actual size of the CA solution space demonstrating the dependency of this space on the number of radio interfaces installed on the network routers using the combinatorial concept of inclusion and exclusion [9], [10] .

2. We develop two fast CA algorithms that reduce the solution space and then find an efficient CA by solving a much smaller Graph Coloring (GC) problem using a well known symmetry-avoiding GC algorithm [9], [10].

3. We provide closed form expressions for the optimal number of channels and a lower bound for network interference in full Wireless Mesh Networks [11].

4. Finally, we develop a polynomial time algorithm that finds a near-optimal solution for the CA problem for full Wireless Mesh Networks [11].

1.4 Organization of the Thesis

The remainder of the thesis is organized as follows.

- Chapter 2 provides an overview of the characteristics of wireless mesh networks, the channel assignment problem and the adopted network interference model. A literature review of relevant approaches and their characteristics is then provided.

- Chapter 3 introduces our main contribution for the analysis of the CA solution space for general wireless mesh networks, describes the new reduced search space and introduces two novel CA schemes for these networks.

- Chapter 4 develops various theoretical results for the CA problem in full wireless mesh networks and introduces a near-to-optimal polynomial algorithm for CA in these networks.

- Chapter 5 presents some conclusions and outlines some future research directions.

Chapter 2

Literature Review

In Section 2.1, we first explore the characteristics of multi-channel multi-interface wireless mesh networks (M^2WNs) and illustrate their common architectures and different applications. The main factors and challenges to be considered while designing protocols for M^2WNs are also discussed. Next, in Section 2.2, we describe the channel assignment problem for M^2WNs networks. The main performance measure for any CA scheme is channel interference, therefore, different approaches to model network interference are also described. Finally, a comprehensive analysis of existing CA algorithms is presented in Section 2.3.

2.1 M^2WNs: An Overview

As described before, M^2WNs represent a promising technology due to its ease of deployment, economy and the promise of higher-speeds and extendable coverage. M^2WNs differ significantly from both their wired and wireless predecessors; a M^2WN is comprised of mesh routers connected wirelessly to each other and, possibly, to a backbone infrastructure. Each mesh router is equipped with multiple radio interfaces and can use multiple frequency channels in order to provide connectivity to mesh clients. Mesh clients usually comprise end-users with their laptops, desktop computers, PDAs, phones, etc. Mesh routers are usually characterized with low mobility profiles and have no energy constraints, whereas mesh clients may have higher mobility, energy constraints, and may not be always available. Usually, some mesh routers will be connected to a backbone infrastructure in order to provide Internet access to the M^2WNs clients. These nodes are known as gateways. In the following sections, we review some of the characteristics of

M^2WNs.

2.1.1 M^2WNs Architectural Models

There are three main architectural models for M^2WNs [2]:

- The Backbone M^2WN model:

  In this model, mesh clients are connected to an infrastructure formed by the mesh routers. This infrastructure is itself a mesh of wireless links among the mesh routers. Some routers provide gateway functionality through a wired connection to the Internet. Figure 2.1 shows an example of such a model.

  Mesh clients can directly connect to mesh routers if they are using the same technology. Additionally, integration with other technologies is possible. In such cases, mesh clients must connect to appropriate base stations, which are, in turn, connected to the mesh routers using a common technology like Ethernet.

- The Client M^2WN Model:

  A M^2WN can also be formed by interconnected mesh clients. In this model, mesh clients also perform the role of mesh routers, including functions like routing as traffic can follow multiple hops (mesh clients) before reaching its destination. Therefore, end-user client devices may have additional requirements in this architecture. Figure 2.2 depicts an example of a Client M^2WN. Although this model is closer to the concept of ad hoc networks, the main difference is the ability to simultaneously communicate to more than one neighbor due to the multiple radios installed on each node.

- The Hybrid M^2WN Model:

  Combining the previous two architectures yields Hybrid M^2WNs. Mesh clients can connect to the network by connecting to mesh routers or to other mesh clients. While the infrastructure architecture provides interoperability with other technologies like Wi-Fi or WiMAX, the client architecture enhances the connectivity and coverage of the M^2WN. Figure 2.3 represents an example of a hybrid M^2WN architecture.

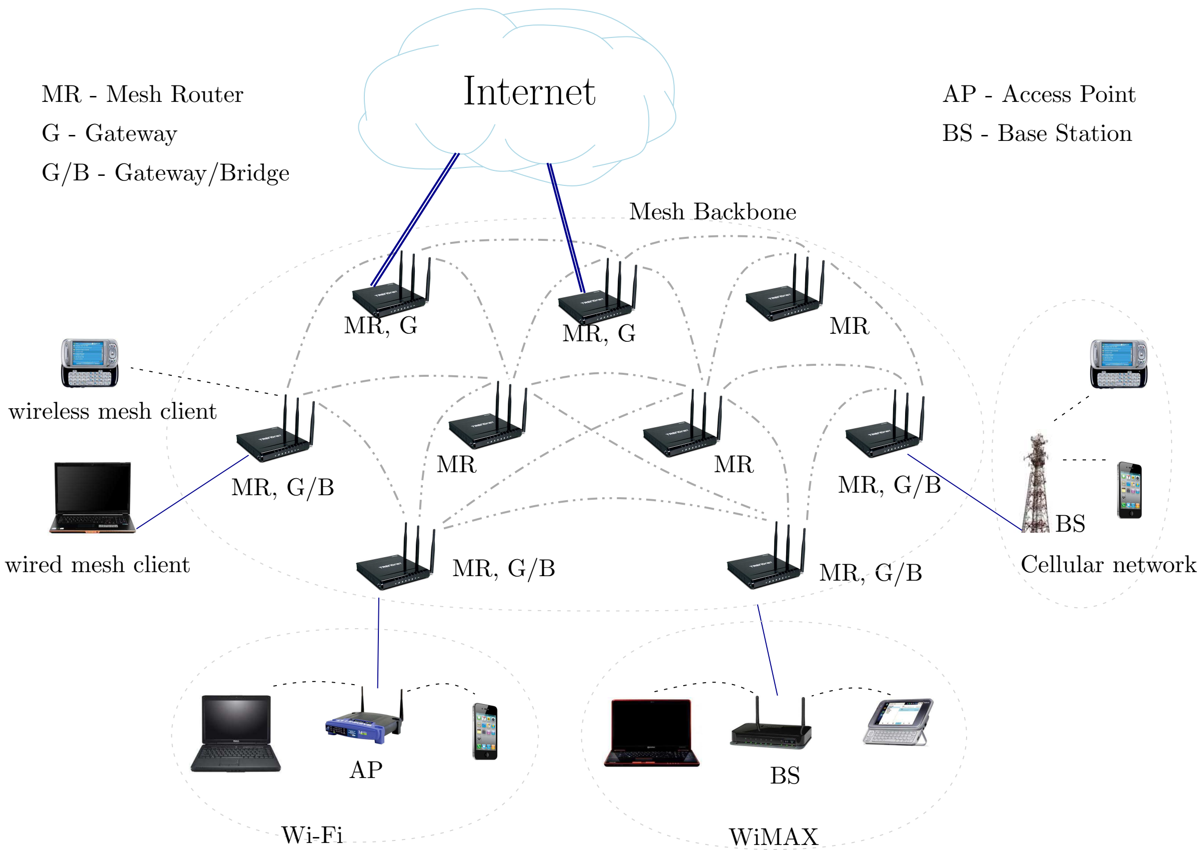


Figure 2.1: M^{2}WN Infrastructure/Backbone Architecture

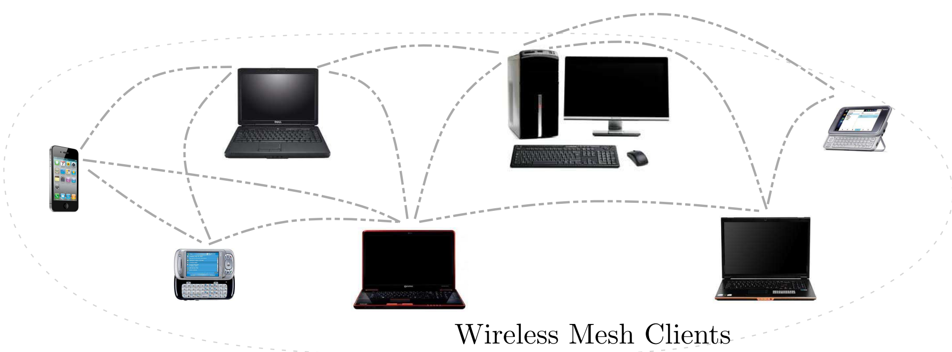


Figure 2.2: M^{2}WN Client Architecture

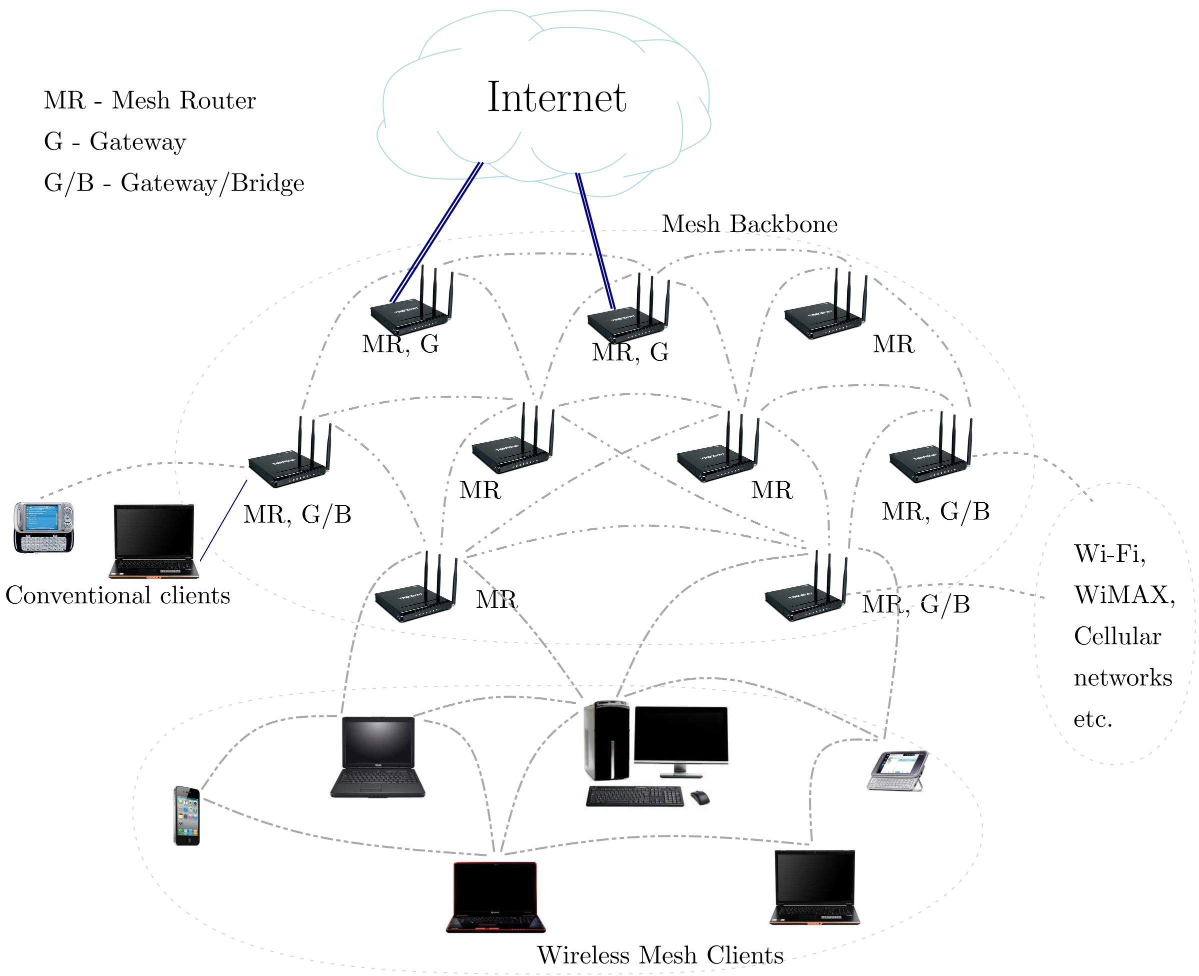


Figure 2.3: M^{2}WN Hybrid Architecture

2.1.2 Characteristics

Except for the case of full M^2WNs, traffic in M^2WNs with general topologies follow multi-hop routes, which means that packets will be forwarded by several nodes before reaching their final destination. Therefore, the current network coverage range is extended without sacrificing the channel capacity. Frequency channels are reused in different sections of the network, and users with non-line-of-sight (NLOS) connectivity or outside communication range of each other communicate effectively through the routers.

M^2WNs automatically respond to changes in the network such as node failures or new node additions. This implies that M^2WNs are *self-configuring*, *self-organizing* and *self-healing* networks [1]. These features provide M^2WNs with such flexibility that made them easy to deploy and configure, fault-tolerant, and able to grow if required, with clear economic advantages.

Mobility and *energy-constraints* are related to the type of mesh nodes. While mesh clients may be mobile and subject to energy constraints, mesh routers are usually stationary and have no restrictions in terms of power consumption.

In hybrid networks, clients in M^2WNs are provided with *multiple types of network access* or *traffic models*. For instance, a client can have both backhaul Internet communication as well as Peer-to-Peer communication (P2P) [29]. In the former, the client's traffic is directed outside the M^2WN through the mesh gateways. Whereas in P2P communication, multi-hop traffic occurs with clients at both ends. Additionally, services from other wireless networks are provided to clients thanks to the integration and inter-operability of M^2WNs with other types of networks.

These characteristics bring many benefits to M^2WNs. Low up-front costs come from the fact that M^2WNs are self-configuring and self-healing, thus, the network's coverage can be extended as required, with low configuration or maintenance. Ease of deployment and fault tolerance are other direct benefits.

Mesh connectivity, which means multipoint-to-multipoint communication, makes the architecture more robust and, by using appropriate protocols for tasks like routing, network's throughput, network reliability and quality of service (QoS) are highly improved compared to other network architectures.

Additionally, the use of *multiple radio interfaces* per node is a key factor for the efficient use of the wireless medium, which constitutes one of the main differences between M^2WNs and ad-hoc networks.

2.1.3 M^{2}WN Access Technologies

M^{2}WNs are envisioned to be *compatible and inter-operable* with different standards. For example, conventional Wi-Fi clients must be supported, together with mesh Wi-Fi clients, in M^{2}WNs that use different 802.11 standards. Other technologies, like WiMAX and Zig-Bee, also have the same compatibility requirements as Wi-Fi.

Several IEEE 802 task groups have been developing new standards aiming for mesh mode operation [56]. For example, 802.11s is dedicated to mesh Local Area Networks (LANs), while 802.15.5 is employed in mesh Personal Area Networks (PANs). Mesh mode networking has also been specified for the 802.16 standard [45], also known as WiMAX for Worldwide Inter-operability for Microwave Access. This standard provides broadband wireless communication in the frequency of 2.3 to 3.5 Ghz, and can come in fixed (802.16d) or mobile (802.11e) variations.

2.1.4 M^{2}WNs versus Ad-Hoc networks

Ad-hoc networks also provide a mesh connectivity, resembling a mesh architecture similar to client M^{2}WNs. However, several additional features uniquely distinguish M^{2}WNs over Ad-Hoc networks.

While in Ad-hoc networks clients can form a mesh network by connecting among them, these networks lack a true backbone infrastructure dedicated to perform heavy tasks like routing. These tasks are performed by client nodes, imposing additional resource consumption, like energy, memory and processing, in the clients, which could be low performance devices like Wi-Fi enabled mobile phones. On the other hand, their mobility and intermittent availability cause the network topology to vary over time, and create additional requirements on these protocols so they adapt to topology changes.

However, M^{2}WNs are mostly provided with a dedicated wireless backbone, where mesh nodes have low energy constraints and (mostly) static locations. This represents a stability feature that can be wisely used by various configuration protocols, like routing and channel assignment, and to give strength to other important features like QoS and security.

Integration with other technologies is possible in M^{2}WNs through bridge functionalities in mesh routers and the existence of multiple interfaces. Client nodes that operate in different technologies than these routers, can connect to a base station that is connected to one of these routers using such bridge functionality. For example, a client using its cellular phone connects to a cellular base station, which in turns is connected to a mesh router in the M^2WN probably by using an Ethernet connection. Such a client is then able to access services provided by the M^2WN, even when such services may be hosted by a network using a third technology standard. See Figure 2.3.

Additionally, mesh routers are generally equipped with several radio interfaces tuned to different radio frequencies. The collision domain of the wireless links is broken into several smaller collision domains, lowering the resulting interference and improving the physical medium spectrum use.

2.1.5 M^2WNs Applications

Using the wired Ethernet for *broadband home networking* has some obvious drawbacks, high cost for setting and configuring the infrastructure and user restricted mobility are two of them. On the other hand, the wireless alternative may suffer from blind spots, and having multiple access points are expensive due to they must be connected through a wired medium to the hub or router.

M^2WNs easily cope with these issues. Using mesh routers for the *broadband home networking* makes it easy to achieve the desired coverage. Additional mesh routers can be added, or the existing ones relocated or removed to modify such coverage, solving blind spot issues, etc. This impose little or no reconfiguration nor wired infrastructure setting.

M^2WNs can be successfully applied in *community and neighborhood networking*. Internet is usually provided through a DSL cable from the Internet Service Provider (ISP), which is connected to a wireless or Ethernet router at every home. This approach has several disadvantages.

Internet data accessed by different users must be downloaded every time from the ISP. For new users requiring the service from the ISP, a new DSL cable must be installed, with additional installation costs. Generally, such DSL cable represents the only path to the ISP. No path is created between clients in the same neighborhood, thus, if two users want to establish some communication, this must go through the ISP even when the users are geographically close. Also, once a client leaves his/her home, and even when remaining in the same neighborhood, it is unlikely he/she will still have access to the service.

M^2WNs come very handy in this scenario; users can benefit from multiple high speed broadband paths to the ISP, while having the added ability of sharing information between them. No additional hardware might be required for new subscribers that are under the coverage of the M^2WN. For those without this option, new mesh routers may be installed, which not only extends the coverage of the network, but could provide new alternative paths in the presence of failures. Users may enjoy greater mobility, where they could still connect to the M^2WN as long as they remain under its coverage, i.e., within the neighborhood.

The same principle applies in the case of *Metropolitan Area Networks* (MAN). Installation of wired MANs is expensive, and M^2WNs offer a good alternative for broadband networking. Deployment is cheaper as no wiring is required, and little or no manual configuration must be performed even in the presence of failures. M^2WNs can cover large areas, and this can be done gradually lowering initial investment; they also can make a better use of the wireless medium compared to other wireless technologies like cellular networks, achieving higher data rates [2]. Scalability is a main concern in these scenarios as the network may even cover entire cities.

M^2WNs are also applied in health and medical systems. Project Emergency Room Link-Tucson (ER Link-Tucson) "allows doctors to be virtually transported from a hospital emergency room into an ambulance to support assessment and assignment of emergency responders" [4]. Another example is in Wireless Internet Information System for Medical Response in Disasters (WIISARD) [13].

On the other hand, the project One Laptop Per Children [18] also uses wireless mesh networks, closely adhering to IEEE 802.11s WLAN mesh standard, to provide networking in a classroom environment [52].

M^2WNs are convenient for other scenarios like enterprise networking, transportation systems, security systems, building automation, etc. [6]. Common advantages are extended coverage, lower implementation costs, broadband networking, reliability and low configuration. However, in order to achieve these benefits, critical design factor must be considered.

2.1.6 Design Goals for M^2WNs

Several design goals are critical to the success of deployed M^2WNs.

- One example is the extend of the network coverage area which is highly affected by the topology of the deployed network.

- On the other hand, *compatibility and interoperability* are key goal when designing various protocols for M^2WNs. They should be compatible with conventional and mesh client nodes. This way, existing clients and networks may connect to newly implemented networks with obvious benefits.

- *Security* is a key goal for every day communication. As there may be no security-dedicated nodes, security in a mesh must be peer-to-peer. Issues like authentication and authorization to access the network must be solved at a peers level, securing links and ensuring the network to be resistant to active and passive attacks, etc.

- The ability of M^2WNs to Self-configure is driven by an *ease-of-use* goal. Protocols developed for M^2WNs must be autonomous, self-configurable and able to adapt to topology changes. Additionally, manual network configuration should still be possible, allowing monitoring and management of the M^2WNs.

- *Scalability* is also a concern for multi-hop communication, and thus in M^2WNs. As the network size increases, communication protocols and network performance decreases. Routing may fail finding reliable route paths, connection may be lost by transport protocols, and throughput can decrease for MAC protocols.

- Finally, *QoS* in M^2WN must be considered as many broadband applications will be using the network with different requirements regarding priority, throughput, delay, etc.

2.1.7 Challenges

With the above design goals in mind, there are several challenges that hurdles the achievement of such goals. At the Physical Layer, new algorithms other than OFDM or Ultra Wideband (UWB) are required to increase the transmission rate in large area networks [2]. On the other hand, issues related to the different types of antenna configuration must be addressed, for example, better directional antennas, practical and inexpensive antenna system and software, frequency agile radios, multi-channel antenna arrays, etc.

Data Link Layer protocols are highly dependent on this radio configurations, having to address single-radio single-channel, multi-channel single-radio, and multi-radio scenarios. In multi-channel or multi-radio M^2WNs, MAC and routing become closer and could be treated as an integral problem. Additionally, bridge capabilities must be added to the layer for the sake of inter-operability of multiple technologies within the M^2WN.

Data Link layer protocols must be designed carefully, and having cross-layer characteristics to further improve network throughput.

Routing is considered as a Network Layer problem despite its inter-layer dependency on other protocols in M^2WNs. Routing protocols for ad-hoc networks, like ad-hoc on demand distance vector (AODV) and dynamic source routing (DSR) are usually valid in M^2WNs. However, optimal routing for M^2WNs depends on M^2WNs specific characteristics, like the use of multiple channels, changing topology, low mobility and no energy constraints of mesh routers, traffic load variations, etc. Therefore, developing routing strategies for M^2WNs faces issues like dynamic and adaptive routing, trade-off between cross-layer design and single layer solutions, distributive algorithms, scalability, QoS, and mesh routers and clients support [2].

The focus of this thesis is the channel assignment (CA) problem which is presented in details in the next section. 2.2.

2.2 Channel Assignment (CA) problem

The channel assignment (CA) problem in M^2WNs is concerned with the efficient assignment of a frequency channel to each communication link in the network. The objective of channel assignment, is to increase network capacity by allowing concurrent transmission of links that would not be able to be active at the same time when using the same channel due to channel interference. On the other hand, a valid channel assignment must also satisfy several constraints related the the used channels. These constraints can be summarized as follows:

- The number of used frequency channels in the CA solution must not use more than the number of channels available for a given network technology. For example, for IEEE802.11a and b/g networks, their standards provide 12 and 3 orthogonal channels, respectively.
- Since radio interfaces are employed by the router to communicate to neighboring nodes and or to clients. The number of radio interfaces installed on each router limits the number of distinct channels that can be assigned to links adjacent to a given router.
- Finally, channel interference which results when two interfering links simultaneously use the same channel, must be minimized.

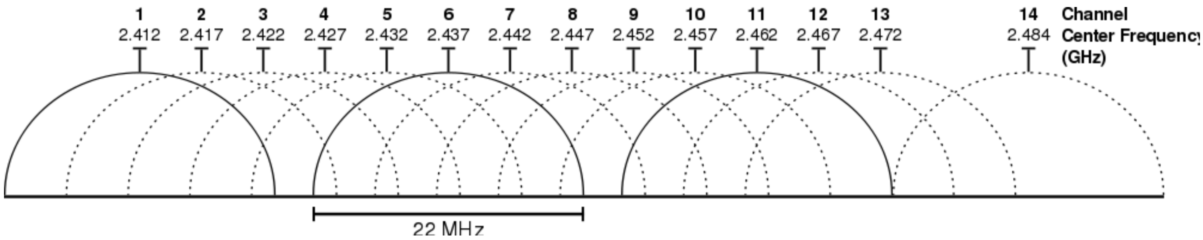


Figure 2.4: Graphical representation of Wi-Fi channels in 2.4 GHz band [62]

Since the CA problem is closely related to the phenomenon of interference, we will dedicate the next section to describe it.

2.2.1 Network Interference

Communication through a wireless medium is carried out by means of radio signals, where each signal is transmitted by a radio interface over a given frequency channel. A channel is defined by a Center Radio-Frequency and a sub-spectrum in the standard's spectrum. For example, Figure 2.4 shows the 802.11 channels on the 2.4 GHz spectrum. Even though the center frequency is different for each channel, their spectra can overlap as can be seen for consecutive channels.

Let us consider two signals emitted by two nodes v_1 and v_2 that use the same channel. Proper reception of each of the signals at a third node v_3 may be impossible if both signals are strong enough and received simultaneously on v_3 such that their signal-to-noise ratio (SNR) is too high. This phenomenon is known as *link Interference*. Interference not only happens for signals using the same channel, but it is also present for overlapping channels scenarios. This phenomenon makes it impossible for both communications to happen at the same time. Thus, only one node can communicate with v_3 at any given moment.

However, if node v_3 can receive signals on two different orthogonal channels, i.e., non-overlapping channels, both communications can happen simultaneously. Thus, the wireless spectrum is used more efficiently and the capacity of the network is increased when v_3 has multiple radios. This is the main goal when providing mesh routers with multiple radios or frequency agile radios. Channel assignment in mesh networks desires to improve network's capacity by a better use of the radio spectrum [1].

[1]Network capacity is an important topic in wireless communications. More information about network capacity can be found in Chapter 8 in [2], and other works ([29], [34]).

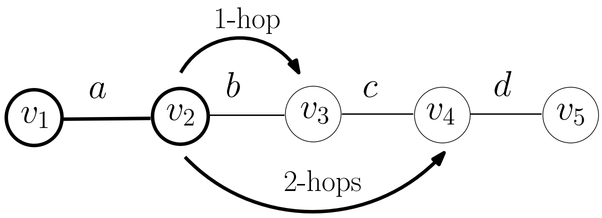


Figure 2.5: n-hop interference model (1-hop and 2-hop).

Interference models

Several models have been considered in the literature as approximation models to represent network interference. The most widely used ones are the n-hops model and the protocol model.

In the n-hops model, two links are said to interfere with each other if the vertices they are adjacent on are at an n-hops or less distance from each other. An example for the 1 and 2 -hops interference model is depicted in Figure 2.2.1. In a 1-hop model link a interferes with b, while in a 2-hop model it also interferes with c

One of the advantages of this model is its simplicity, even though it is not accurate for some instances. In [46], the authors deployed an actual indoor 802.11-based network testbed with 22 nodes and found that the 1-hop model is too optimistic as it fails to predict some of the actual interfering link pairs. While the 2-hop interference model showed better results, it was still considered too pessimistic as it predicted interference in some link pairs that did not show interfering in the testbed.

Figure 2.2.1 illustrates the concept behind the Protocol model. This model bases its prediction on the physical distance between the nodes, a transmission range and an interference range. Two nodes can be connected by a link if the receiver is located within the transmission range of the transmitter. Two links $e(v_1, v_2)$ and $e(v_4, v_5)$ interfere with each other if the receiver v_4 falls inside the interference range of transmitter v_2, where usually the interference range is twice of that of the transmission range.

Both models are simple and discrete, i.e., two links either totally interfere or not. While the n-hop model does not consider the physical distance between the nodes, the protocol model may not be suitable for indoor environments where walls and other obstacles affect signal propagation [46]. Additionally, interference for the specific case of Wireless Mesh Networks can be affected by *multi-way interference* as shown in [21]. More sophisticated models estimate signal degradation and consider Signal-to-Noise ratio (SNR) but their complexity represent new challenges.

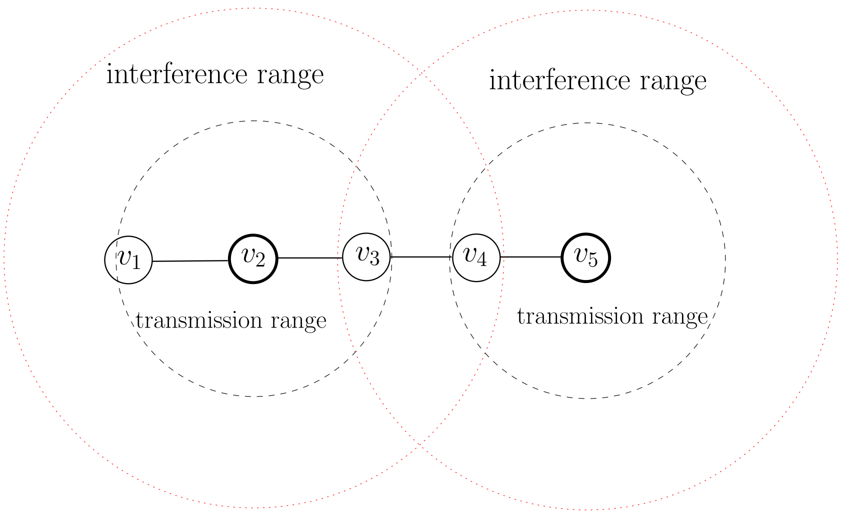


Figure 2.6: Protocol interference model.

2.2.2 Channel Assignment versus graph coloring

At a first glance, the CA problem looks similar to that of the **Graph Coloring** problem. We are trying to assign channels (colors) to links in such a way no two interfering links are assigned the same channel (color). However, in CA we have three constraints that are not present in Graph Coloring problem:

1. The number of channels on any given node cannot be higher that the number of radio interfaces of the node. Therefore, the number of available channels for the set of links incident on any given node has an upper bound.

2. The total number of channels is limited by the standard.

3. Channel pairs may expose different interference degrees due to spectrum overlapping. Thus, there can be a significant difference between two solutions that assign different channel pairs to two interference links as depicted in Figure 2.7.

Also, an interference-free channel assignment is unlikely to exist in practical scenarios. Therefore, we want to find a CA that minimizes the interference.

But other factors must be considered. For example, depending on the traffic scheme of the network, i.e., peer-to-peer or between client nodes and gateways, some links will carry more traffic than others. This even suggests the interdependency of the routing and channel assignment problems: the routing protocol additionally stresses link's load properties. Therefore, different link priorities would be desirable.

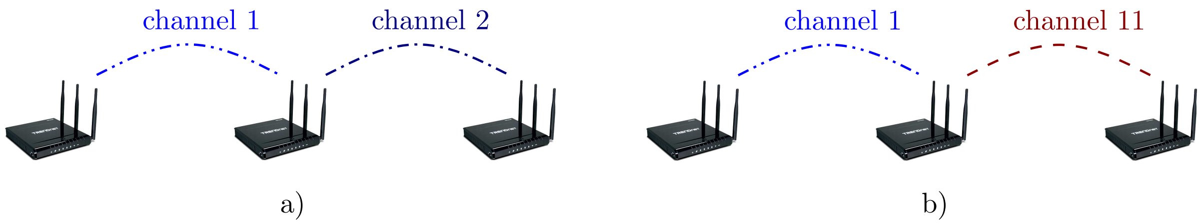


Figure 2.7: In a), two overlapping Wi-Fi channels produce cross-channel interference, whereas in b), two Wi-Fi orthogonal channels do not.

2.3 Existing CA approaches

2.3.1 CA Solution Characteristics

The Channel Assignment problem has been addressed extensively in literature (e.g., [20, 23, 48, 51]). Various CA algorithms are often categorized with respect to several characteristics:

- Static or Dynamic channel assignment.
- Topology preserving or non-preserving.
- Interference and traffic aware or not.
- Joint/Separate from Routing.

Static CA ([51], [34], [59], [48], [33], [58]) occurs when the network's CA does not change over time, unless there is a major topology change like a node failure or addition. In such cases, the CA algorithm must be rerun and a new CA is calculated for the network's new properties. These methods are usually appropriate for M^2WNs' backbone as mesh routers are mostly static and topology changes are not common. However, for M^2WNs where mobility is to be considered, these methods will have to be executed more often.

Dynamic CA ([50], [34], [42], [31]) algorithms allow CA to evolve over time, adapting to changes in network's topology or more subtle changes like link load or interference. This lowers CA's impact on other protocols like routing, and the M^2WN benefits from a better cross-layer integration and design. These methods could yield to a better network throughput in the long term for scenarios where traffic scheme varies frequently. However, frequently switching channels in a radio interface adds delays to the transmission, which might not deem them practical using current hardware [58]. These algorithms impose additional overhead in order to monitor such changes in link's load and interference. Finally, continuous execution of the algorithm will not yield new CA when such changes are not relevant, wasting mesh nodes' resources which may be scarce for mesh clients.

Topology-preserving CA ([51], [34], [20], [42], [48], [33]) algorithms assign channels to every link in the M^2WN. The possible higher interference in a topology preserving scheme solution is compensated by a higher network connectivity. This may result in more and shorter paths depending on the traffic scheme. Some CA schemes use a common control channel in order to maintain network's topology. However, the interference in this channel is higher, and mesh nodes must dedicate a radio interface only for this purpose, basically lowering the number of radios per node.

Non topology preserving CA ([50], [40], [59], [17]) schemes may return solutions where no channel has been assigned to some links. However, some connectivity is mandatory, for example to ensure every node can access directly or indirectly some gateway. A reduction in network's interference is achieved by lowering network's density. In these schemes, CA and routing are heavily dependent and must be solved jointly.

Traffic aware CA ([50], [34], [42] , [48], [58]) algorithms consider the link's traffic load. Links with higher traffic are usually assigned channels with lower interference than those assigned to links with less traffic. However, link load depends not only on the traffic scheme, but also in the routing protocol. Therefore, a proper interaction between a traffic aware CA and routing protocols is very important for network's capacity.

Non-traffic aware CA ([20], [40], [42], [48], [33], [58]) schemes do not consider network's traffic, they base their decision on other metrics like interference or number of hops to gateways instead. Therefore they work independently of routing algorithms and traffic scheme, and are more suitable for general cases. However, when link load differences are high, the M^2WNs may suffer from bottlenecks were some channels are heavily used while others might be inactive, yielding to sub-utilization of part of the frequency spectrum.

Joint CA and routing. ([50], [34], [47]) CA and routing problems can be tackled in a single algorithm, in which not only the channels are chosen for links, but also the routes for the network traffic. This approach offers higher flexibility regarding achieving high network performance: not only traffic information is available for channel assignment, but channel utilization is available for routing. However, this implies a much more complex problem as both problems must be solved at once, and metrics from both CA and Routing problems must be considered; their interdependency is even more evident.

CA in isolation. ([58], [40], [49], [33]) Addressing only the CA problem is easier than solving CA and Routing problems together. The CA scheme to be used can be decided at any point in time and several combinations of CA and Routing algorithms are possible, and hopefully we can find the more convenient to the actual scenario. However, both problems are still closely related, and a CA algorithm cannot ignore key assumptions or criteria used by the routing algorithm such as the required QoS. In this sense, an interesting research problem would be to compare different CA schemes in combination with different routing algorithms and traffic schemes.

2.3.2 A Comparison between CA approaches

In the following we describe and analyze some of the most prominent approaches for CA in the literature:

CA via Linear Programming

Several research efforts (e.g., [20, 23, 27, 48]) have approached channel assignment as an optimization problem. The objective function in these schemes varies depending on whether they consider topology control, interference, traffic scheme, number of hops to gateways, common radios, etc. This consideration also affects the constraints of the linear problem formulation. Commonly used constraints are:

- Every link must have a channel assigned (topology preserving).
- The number of channels in use by a node's links cannot be higher than the number of said node's radios (radio constraint).

But other constraints are also used. For example, [20] additionally considers "potentially interfering cliques of edges". However, solving most of these types of problems is still NP-hard and therefore, some of those constraints must be relaxed before a solution is found. These approaches suffer from the nearly endless variety of relaxations that one can consider.

Das et al. [20] formulate CA problem as a semi-definite programming problem (SDP) and solve it in non-polynomial time. Rad et al. [48] reduces the problem to an optimization problem using signal to interference and noise ratio. In [41] nodes are prioritized using an adaptive priority algorithm in order to reflect the lack of flexibility in nodes resulting from assigning channels to neighboring links. A comprehensive review of optimization based solutions is proposed in [3].

Heuristics-based CA

Heuristics based approaches have also shown promising results (e.g., [58], [8]). They have proven to be the most successful in fixed channel assignment, in particular for large networks. The performance of these algorithms is heavily influenced by the solution space structure and the fine tuning of their parameters. The most common approaches are Greedy, Tabu-based and Genetic Algorithm searches. Other heuristics have been also proposed, for instance transforming the problem into another thoroughly studied problem [9]. However, due to the uncountable number of issues that can be considered (topology control, traffic control, interference-aware, joint CA and routing, etc.), there is still no consensus on any prevailing CA scheme.

Centralized Tabu CA [58]

A centralized Tabu-based and a distributed greedy approaches are presented in [58].

The centralized algorithm is a topology preserving, static CA scheme that uses the Tabu search technique. In this scheme, the conflict graph is used and finding the CA solution is done by coloring the nodes of the conflict graph. They objective function is to minimize the interference, which is represented by the conflict graph edges between nodes with the same color (equivalent to interference links using the same channel). Therefore, the problem is translated in finding a solution for the Max K-cut problem plus the radio constraints, where the Max K-cut problem consists in partitioning a graph in K-sets such that the number of edges between the sets is maximized.

Therefore, the algorithm works in two steps: to solve the Max K-cut problem first (without including the radio constraint), and then modifying such a solution as such the radio constraints are met. The first step is done using the Tabu search technique. Links are assigned channels randomly, and next, links with higher interference are visited and a new channel is found such that it yields the lower conflicts in the network. This channel is chosen from the set of available channels minus a Tabu list of size m, which contains the last m channels already used by such link.

As a result of ignoring the radio constraints in the first step, the links of some nodes may be using more channels than the number of radios of said node. this nodes are said to violate the radio constraint. In the second step, these nodes are ordered in decreasing order of high such violation is. The nodes with higher difference are visited first, and iteratively two channels c_1 and c_2 are chosen and merged together, which reduces the number of channels being used by said node, until the number of channels is equal to the number of radios. Choosing c_1 and c_2 is done such that the overall interference is minimized. However, merging channels implies that links in c_1 channel will start using c_2 channel, which could break the constraint in the other endpoint of such link. Therefore, the links in said other endpoint using c_1 are changed as well to channel c_2, and so on. In other words, each time a new node in *fixed*, a ripple effect could be triggered. After visiting a violating node, such node is fixed and, because no new nodes violate the radio constraint, eventually the algorithm fixes all the nodes and the result is a valid channel assignment.

The Tabu algorithm is simple and intuitive and can yield results close to a lower bound as shown by authors in [58]. However, in addition to a possible ripple effect due to the last step, Tabu search may suffer from slow convergence when the number of local optima increases in the solution space, which is a typical case with a smaller number of available channels.

D-Hyacinth [50]

In [50], Raniwala et al. address the joint problem of CA and routing, and propose a load-aware distributed scheme with topology control, where routing trees rooted on the gateways are built. Every node is directly or indirectly connected to a gateway, and share the cost to reach the gateways by means of ADVERTISE messages. Based on this information, nodes may decide joining a different routing tree, which is accomplished by means of JOIN/ACCEPT/LEAVE messages. This tree structure establishes a parent-child relation between nodes, which is used when performing CA. In order to avoid ripple effect, a restriction is imposed to the nodes: NICs are marked as UP-NIC or DOWN-NIC for each node. UP-NICs are used to communicate with parent nodes, while DOWN-NICs are used by links with children nodes. Each node is responsible for assigning channels to their DOWN-NICs, where UP-NICs use the channel assigned to the corresponding DOWN-NIC in the parent. The decision is made based on the load information shared by means of messages.

This scheme addresses the joint problem of CA and routing, successfully avoiding the ripple effect. Additionally, it provides a solution for the self forming and self healing characteristics of M^2WNs; it describes how new nodes joints the network by means of hello packets and how the network recovers in the presence of a node failure. This protocol assumes that traffic goes from mesh routers to gateways only, and the creation of routing trees hinders traffic inside the network.

CLICA [40]

Connected Low Interference Channel Assignment (CLICA) is a greedy heuristic algorithm presented in [40]. This scheme performs topology control without requiring a dedicated radio for connectivity. It aims to find a low interference CA solutions visiting each node based on a priority function.

Nodes are visited in decreasing order of their priorities, which is initially calculated according to some criteria like distance to a wired gateway. Each time a node n_i is visited, channels are assigned to its links from a set of feasible channels in such way that the maximum conflict weight for each n_i adjacent link is minimized. Assigning channels to radios of n_i's neighbors reduces the options for coloring their links in future steps. To cope with this lack of flexibility, the priority function is continuously updated in every step of the algorithm in order to give higher priorities to nodes with less flexibility.

In this work, an ILP formulation of the problem is also presented and used to find a lower bound in order to evaluate the performance of the heuristic algorithm.

Among the advantages of this algorithm, we can mention it directly tackles the flexibility issues caused by the radio constraints, and it is a polynomial algorithm simple to understand and implement. As drawbacks, this algorithm may fail to fairness consideration as CA is performed considering only local information. Additionally, still has to deal with the undesirable ripple effect.

Minimum INterference Survivable Topology Control (INSTC) [59]

Tang et al. [59] present a topology control heuristic for the problem of Minimum INterference Survivable Topology Control. This problem's goal is to find a CA such that: 1. the resulting network topology is K-connected so as survivability is ensured, and (2) it has the minimum interference of all the K-connected topologies. The interference function used, Link Co-channel Interference (LCI), tries to model the co-channel interference by considering the spectral distance between the channels and the spatial distance between the nodes.

Similar to CLICA, all the links are visited sequentially, however, the order in which the links are visited is fixed, as opposed to CLICA's dynamic priority function. Links are visited in a non-increased order according to their respective Link Potential Interference (LPI) values, where LPI for each link is defined as the potential interference of such link with the links in its interference range. We note that LCI is not available at the beginning of the algorithm as no topology is defined yet, therefore the use of LPI to order the links instead of using LCI for this purpose. The channel for a given link is chosen in a similar way to CLICA, choosing from the locally best available channels.

The main advantage of INTSC algorithm is that it is a polynomial algorithm that ensures K-connectivity, being time-efficient while yielding robust topologies. Similar to CLICA, one of the main disadvantages is that fairness can be affected by the using only local information while choosing a channel for a node.

NSGA-II [17]

In [17], Non-dominated Sorting Genetic Algorithm II (NSGA-II), a topology control CA algorithm, is proposed after the CA problem is formulated as a Multi-objective Optimization Problem (MOP). Authors define two objective functions for a given channel assignment of the network: network connectivity and network interference. This functions are used to evaluate the fitness of any CA.

A representation of the individuals (CA assignments) for the genetic algorithm is proposed. An individual is represented by $|V|$ blocks, each representing one node in the network. Then, each block is made of $|C|$ bits, where a bit i is 1 if channel C_i is in use by the corresponding node, or 0 otherwise. This results in a $C^{|V|}$ bits-length representation. The usual Mutation and Recombination operations are defined for these CA individuals. The recombination is performed by choosing two random crossover points, and then interchanging some blocks from each node to the other. Both operations ensure the resulting CA individual obey the radio constraints. The algorithm starts with an initial random population of size N. On each iteration, the Recombination operation duplicates the population, then a Replacement operation is performed selecting N individuals based on their fitness functions.

This is an genetic approach that succeeds in finding CA solutions with high connectivity and low interference. However, non-topology schemes should include a link priority function in order to reflect link importance in the network, based on expected traffic, proximity to a gateway, etc. The algorithms reports a low convergence rate as shown by the number of iterations reported in the experimental results. This is a main limitation of such algorithms as that they can be easily trapped in a local optima if the population is not diverse enough. But in the presence of topology changes, this scheme could benefit from previous executions by updating and reusing individuals from previous executions.

BFS-CA [49]

An interference-aware centralized static algorithm that uses a Breadth First Search is presented in [49]. A radio is used on a default channel in every router in order to maintain connectivity. In this scheme, interference from the own network and co-located networks is estimated at each router. This interference information is sent to a Channel Assignment Server (CAS) which runs the algorithm. The CAS chooses a channel for the default radios based on the interference information. Channels are ranked for each link based on the increasing number of interfering radios and the increasing channel utilization for on each of the link radios. This rank is used, along with other information like distance to a gateway and delay, to assign the channels to the links. A multi-radio conflict graph (MCO) is defined in the paper and used by the algorithm. MCO is based on the Conflict Graph (CG) [28] and its goal is to also include the radio constraint information in the CG. In MCO, a link is represented by several nodes (compared to only one in the original CO). These nodes are the possible configurations of such link over the radios of its adjacent nodes.

The algorithm assign channels to the links in a breath first order, starting from links adjacent to a gateway. Links at a same hop-distance from the gateway are ordered based on their delays so to give higher priority to better links. Then the algorithm assigns to the link the channel with the best rank for that link that does not conflicts with its neighbor links. If there is no non-conflicting channel, a random channel is chosen for the link (the default channel is never assigned). Then, nodes that represent such link's radios are removed from the MCO to reflect that links using those radios can be assigned.

In this scheme, channel assignment is readjusted periodically, being such period not too short to affect performance due to channel switching delays.

JOCAC [47]

In [47], Mohsenian et al. present the problem from a different perspective. They formulate the problem of Joint Optimal Channel Assignment and Congestion Control (JOCAC) where channel assignment is performed such as to minimize the congestion in the wireless links. They idea is that interference can potentially cause congestion in the wireless links, which affects the network throughput for data connections using the Transmission Control Protocol (TCP). The formulation of the problem takes into account the number of available channels, the number of radios in each router, the congestion on the links, transmission power, wireless path loss, and channel frequency response.

JOCAC extends the distributed unity maximization for congestion control, a general analytical model presented in [32]. In this model, each link has a capacity, and a there are S sources in the network with given transmission rates r_s and a utility function $U_s(r_s)$, $s \in [1, S]$. Also, each link l have a congestion price λ_l (e.g., link delay in TCP Vegas [39]). The goal of the congestion control is to adjust the transmission rates of the sources so the aggregated utility of the sources is maximized, i.e., $\sum_{s\in[1,S]} U_s(r_s)$. JOCAC extends this model and existing definitions of $U_s(r_s)$ and λ_l for TCP Vegas, by additionally considering the parameters of the CA problem mentioned above. In the distributed version of JOCAC, nodes share signal to noise ratio information (SINR) as well as λ_l with other nodes, and then calculate their channel assignment using JOCAC formulation with the information received from other nodes.

This approach as the advantage of considering both orthogonal and overlapping channels in order to make better use of the available spectrum. This is possible as the SINR is used, thus neighboring links can use non-orthogonal channels as long the the SINR remains under a given threshold and interference from neighbor transmissions is not significant. The main limitation of this distributed scheme is that nodes must share the SINR and their congestion price as nodes need this information in order to assign channels to its links.

Self-Stabilizing CA (SS-CA) [33]

Self-Stabilizing CA (SS-CA) [33] is a traffic-unaware, distributed channel assignment scheme oriented to large scale networks. It is inspired on authors' previous theoretical work on a distributed self-stabilizing protocol for replica placement presented in [30], whose main idea is to place replicas 'far' from each other. SS-CA consider wireless channels as replicas while adopting this replica placement behavior.

SS-CA considers the use of non-orthogonal channels where the interference between channels c_1 and c_2 is given by an interference cost function, $f(c_1, c_2) \in [0, 1] \in \Re$. This interference function depends on the spectral distance between the channels, and ranges from 0 for orthogonal channels up to 1 when both channels are the same. According to SS-CA, each node in the network greedily chooses a channel for its incident links based on the channels used by the links in its interference range, aiming to minimize the total interference cost in such range. Due to the delay in message exchange, the information on the link channels is not always up to date, therefore, a distributed stabilizing protocol is required by the algorithm. Authors hold on the work presented in [30] to design a distributed stabilizing protocol that runs the channel selection algorithm, exchanges channel information among nodes in the same interference range, and coordinates the channel changes using mutual-exclusion operations. Authors refer the readers to [30] for more details in the stabilization protocol.

This algorithms has the advantage of not imposing a high overhead, as each node selects channels based on a local information and an simple interference cost function is used as metric. Additionally, the algorithm employs a protocol for self-stabilizing and authors implement 14-nodes testbed to validate its suitability. However, the algorithm requires a radio on each node tunned to a default channel in order to guarantee connectivity. Also, the interference function is still naive as it does not consider the spatial distance among the nodes, and there is no traffic awareness, which can make the use of such function no always practical as a metric function.

Probabilistic Channel Usage based Channel Assignment (PCU-CA) [35]

PCU-CA[2], presented in [35] by Kyasanaur et al., is a distributed CA scheme that avoids the ripple effect by classifying the interfaces of each node in fixed and switchable. Nodes choose a channel for the fixed interfaces according to the channel usage of its two-hop neighbors. These interfaces' channel remains fixed for the duration of a period of time and are referred to as fixed channels. On the other hand, nodes change switchable interfaces' channel based on the traffic flow, choosing such among the fixed channels of the neighbor the transmission is intended to. The ripple effect is avoided as fixed interfaces are used for receiving traffic through a fixed channel. Then, when a node need to transmit some traffic to a neighbor node, it chooses a channel from the fixed channels of said neighbor. Whether a channel is to heavily used and what probability to use, are configurable criteria. The first one is intended to distribute the channel usage among all the channels, and the later aims to avoid channel oscillation, and thus convergence of the algorithm. The advantages of this schemes are (1) the network is stabilized as both the ripple effect and channel oscillation are addressed, and (2) the metric defined as channel usage is simple and easy to obtain. The limitations of the protocol is that it may not be optimal for the network performance due to the traffic information is not considered and the probabilistic model used.

Balancing Static Channel Assignment (BSCA), and Packing Dynamic Channel Assignment (PDCA) [34]

In [34], Kodialam et al. formulate the problem of joint routing, CA, and link scheduling in terms of the multi-commodity flow problem, a classic network flows problem. Traffic, sent from a set of source nodes to a set of destination nodes, is regarded as a commodity where its rate is defined in a rate vector. Authors propose a flexible LP model, that can include parameter information like different channel bandwidths, and

[2]Kyasanaur et al. do not provide the scheme with a name, therefore we use Probabilistic Channel Usage based Channel Assignment (PCU-CA) as suggested by Si et al. in [53]

also can specify different objective functions such as maximizing the network throughput or specifying fairness constraints. Also, necessary conditions to evaluate the feasibility of rate vectors are provided, using them to calculate upper bounds for the capacity of the network in terms of throughput.

First, the feasibility of the traffic scheme is determined and the routing is performed based on this traffic scheme. Next, the CA and link scheduling problems are solved. Authors propose one static and one dynamic CA schemes: BSCA and PDCA. BSCA is static, thus CA is calculated once and it does not change over time, while PDCA is a dynamic scheme where the CA can be calculated to react to traffic pattern changes. Both algorithms use a Greedy technique to deal with the NP complexity of the problem.

BSCA introduces the concept of 'constraint set', and aims to minimize the maximum traffic in any constraint set when assigning a channel to a link. Once the CA is performed, links within the same constraint set and using the same channel, will use different time slots for transmission to avoid interference.

In PDCA, time is divided in time slots, and CA is performed at the beginning of each time slot. This scheme follows a greedy approach where links are ordered in the descending order of their traffic loads, and channels are chosen so as to provide the highest bandwidth share. PDCA solves CA ad link scheduling problem simultaneously, whereas BSCA solves them separately.

The main advantage of this work is that the model is flexible enough so as to include other objective functions and constraints, for example, heterogeneous radios and directional antennas. However, due to the hardness of the problem, relaxations to the model need to be done before the problem is solved. The dynamic algorithm shows better performance compared to the static algorithm, yet, it suffers the delays of channel switching on the radios.

A summary of some of the approaches for CA problem is presented in Table 2.1.

Algorithm	**Static/ Dynamic**	**Centralized (c)/ Distributed (d)**	**Topology preserving**	**Traffic aware**	**Technique**	**Year**
C-Hyacinth [51]	Static	c	x	x	Greedy	2004
D-Hyacinth [50]	Dynamic	d		x	Greedy	2005
BSCA [34]	Static	c	x	x	Greedy	2005
PDCA [34]	Dynamic	c	x	x	Greedy	2005
Two ILP models [20]	Static	c	x		LP	2005
CLICA [40]	Static	c			Greedy	2005
INSTC [59]	Static	c			Greedy	2005
JOCAC [42]	Dynamic	both versions	x	x	LP	2006
MesTiC [54]	Static	c	x	x	Greedy	2007
TiMesh [48]	Static	c	x	x	LP	2007
GA [8]	Static	c	x	x	GA	2007
SS-CA [33]	Static	d	x		Greedy	2007
Tabu-based [58]	Static	c	x	x	Tabu	2008
Greedy [58]	Static	d	x	x	Greedy	2008
MCAR [7]	Static	c	x		Heuristic	2008
NSGA-II [17]	Static	c			GA	2009
CCAS [31]	Hybrid	c	x			2010
E^2CARA [5]	Static	c		x	Heuristic	2010

Table 2.1: Summary of channel assignment algorithms.

2.3.3 Channel Assignment Solution Evaluation

Evaluating existing and new CA schemes is mostly performed via simulations or prototyping [12] due to the lack of a theoretical analysis of the CA problem or the optimal interference values. Furthermore, links may have different traffic loads and network's capacity is even more difficult to model for wireless networks.

In an attempt to fill this gap, the authors in [24] considered interference in a single-channel full M^2WN with constraints on signal transmission power. They show that the number of nodes in the network is bounded in order to preserve the network topology and signal constraints.

As a result, the most common approaches when evaluating a channel assignment solution are:

1. Software simulations.

   Simulations allow to measure network's capacity, the actual goal. Takes additional time but provides more information on the network. It is possible to estimate the performance of the CA solution in the presence of different MAC protocols, routing algorithms, etc.

2. Testbeds/Prototypes.

   More accurate than simulations, but more expensive and with lower flexibility in topology configurations than software simulations.

3. Theoretical analysis.

   Although this approach is the most accurate, it is seldomly used due to the complexity of the CA problem. Nonetheless, there have been several attempts to model network capacity as there is usually, an inverse relation between network capacity and interference. However, this is not always as will be shown later on.

Chapter 3

Channel Assignment for M^2WNs

3.1 Introduction

In this chapter, we present novel fast and accurate algorithms for channel assignment (CA) in multi-interface multi-channel wireless networks (M^2WNs). The proposed work uses the fact that the number of radios per node is typically small and, as a consequence, reduces the size of the feasible CA solutions dramatically. This fact is first rigorously demonstrated using the combinatorial principle of inclusion/exclusion, where it is shown that the feasible solution space can be quantified, indicating that its cardinality is greatly influenced by the number of radios. Based on this observation, the proposed work develops a scheme to construct a reduced search space, represented by a lattice structure, that is searched more efficiently for a CA solution. The elements in the reduced lattice-based space, labeled *Solution Structures* (SS), represent groupings of feasible CA solutions satisfying the radio constraints at each node. Two algorithms are presented for searching the lattice structure. The first is a greedy algorithm that finds a good SS in polynomial time, while the second provides a user-controlled depth-first search for the optimal SS. The obtained SS is used to construct an unconstrained weighted graph coloring problem which is then solved to satisfy the soft interference constraints. By decoupling the hard constraints imposed by the radio interfaces from the soft constraints that minimize channel interference, the proposed algorithms succeed in reaching higher quality CA solution in a very short time. Empirical evaluation shows substantial speedup and accuracy of the proposed schemes.

This chapter is organized as follows. A mathematical formulation of the channel assignment problem and used notations are provided in Section 3.2. Section 3.3 provides an analysis of the solution space. Based on the obtained analysis, a new reduced search space is constructed in Section 3.4. Section 3.5 is dedicated to describing the proposed schemes while Section 3.6 discusses some extensions of our problem model and proposed techniques. Experimental results are presented in Section 3.7. Finally, Section 3.8 concludes the chapter.

3.2 Notation and Problem Formulation

An M^{2}WN can be modeled as an undirected communication graph $G = (V, E)$, where $V = \{v_1, \cdots, v_n\}$ is the set of n routers (nodes), $E = \{e_1, \cdots, e_m\}$ is the set of wireless communication links (edges), and n and m are known as the order and size of G, respectively. An edge $e \in E$ indicates that two nodes v_i and v_j can communicate using a common channel. Such an edge is typically denoted by $e(v_i, v_j)$. Also $E(v_i) \subseteq E$ is used to refer to the links incident on node $v_i \in V$.

In addition to G, an M^{2}WN defines the set $R = \{r_1, \cdots, r_n\}$ as the set of radio interfaces installed on each router, where $1 \leq r_i \leq d(v_i)$ is the number of radios installed on $v_i \in V$ and $d(v_i)$ is its degree, i.e., the number of edges incident on v_i. Finally, there is a set of k orthogonal frequency channels, $\mathcal{C} = \{c_1, \cdots, c_k\}$, available to all nodes in the network. Similar to other approaches, we assume that these channels are homogeneous in terms of the channel capacity [31]. Throughout this chapter, we will simply use $G(V, E, R, \mathcal{C})$ to refer to an M^{2}WN modeled by $G(V, E)$, R and $\mathcal{C}$.

A CA solution assigns a channel from $\mathcal{C}$ to each link in E while maintaining minimal interference. Hence, a CA solution S defines a mapping from E to $\mathcal{C}_s \subseteq \mathcal{C}$. We will use the notation $S(e_i)$ to identify the channel assigned to link $e_i \in E$. In a similar manner, $S(G)$ and $S(X)$ stand for the channels assigned to the sets E and $X \subseteq E$, respectively. Among all possible CA solutions, this chapter is concerned with the *topology-preserving* ones defined by:

Definition 1. *A topology preserving CA solution $S(G)$ for an M^{2}WN, $G(V, E, R, \mathcal{C})$, is a mapping $S(G) : E \rightarrow \mathcal{C}_s \subseteq \mathcal{C}$ such that*

$$\begin{aligned} \forall e \in E \quad \exists c_j \in \mathcal{C}_s \textit{ where } S(e) &= c_j \textit{ and} \\ |S(E(v_i))| = |\{c_j : S(e) = c_j, e \in E(v_i)\}| &\leq r_i, \\ \forall v_i \in V & \end{aligned} \tag{3.1}$$

We refer to a solution as topology preserving since each link in the network is assigned a channel, and hence, maintains its connectivity. The condition in (3.1) ensures that $S(G)$

satisfies the so called *radio constraint*, or the constraint that the number of different channels used by any node $v_i \in V$ is at most equal to the number of radio interfaces r_i installed on this node.

The interference between the communication channels arising from given CA solution $S(G)$ is usually defined by link interference model; to maintain lucidity in our presentation, we will consider a binary model defined by a function $I(e_i, e_j)$, which is equal to 1 if links e_i and e_j interfere when using the same channel but 0, otherwise. The generalization to other interference models is discussed in Section 3.6. The total interference arising from $S(G)$ is then given by a cost function defined as follows,

$$I_t(S(G)) = \frac{1}{2} \sum_{\substack{e_i, e_j \in_{i \neq j} E, \\ S(e_i) = S(e_j)}} I(e_i, e_j) \tag{3.2}$$

We refer to the notion that $S(G)$ needs to avoid assigning the same channel to two links marked as interfering ($I(e_i, e_j) = 1$) as the *soft constraint*. By contrast, the radio constraint in definition 1 is called the *hard constraint*. Clearly, a CA solution must minimize the number of unsatisfied soft constraints if it were to produce minimal interference. Nonetheless, it must satisfy all of the hard constraints if it were to be physically attainable. We also define, $I_t^f(S(G))$, the fractional interference for $S(G)$ [58], as the ratio between $I_t(S(G))$ and the interference resulting from using a single channel, which will be used as a normalized measure in our experimental evaluations. Finally, we define, $\Omega^R(G)$, the *CA solution space* for a M^{2}WN, $G(V, E, R, \mathcal{C})$, i.e., the set of all *topology preserving solutions* $S(G)$.

3.3 Quantifying the cardinality of the topology preserving CA solution space

To quantify the effects of the hard constraints on the CA problem, this section derives an estimate for the size of $\Omega^R(G)$. The presented derivations provide the central argument for the algorithms presented in the next section. For brevity, topology-preserving (non-preserving) solutions will be termed as valid (invalid) solutions.

3.3.1 CA as a Graph Coloring Problem

Graph-based approaches to the problem of CA typically poses the problem as that of coloring a communication graph $G(V, E)$ using no more than k colors with the objective of minimizing the number of node-adjacent edges (edges incident on a single node) having the same color. Naturally, a CA problem formulated in this manner, with possible k^m solutions, does not incorporate neither the hard nor *all* soft constraints of the CA problem, since not all the communication links marked as interfering are necessarily node-adjacent.

To embed all of the soft constraints in the problem of graph coloring, the problem is transformed into that of vertex coloring of the so-called Conflict Graph (CG) [38, 58], denoted by $G^c(V^c, E^c)$ and constructed as follows. For every edge $e_i \in E$ in $G(V, E)$, add a vertex v_{e_i} to V^c, and for any two edges $e_i, e_j \in E$ having $I(e_i, e_j) = 1$, add a link between v_{e_i} and $v_{e_j} \in V^c$. Since, $|V^c| = |E| = m$, the solution space Ω^{CG} of the CG coloring problem has a cardinality $\left|\Omega^{CG}\right| = k^m$. Hence, embedding the soft constraints in the solution space does not reduce its size.

3.3.2 Effect of Hard Constraints

This section shows that the exponential growth in the solution space (k^m) is dramatically curtailed when the hard constraints are taken into account. In fact, we use the notion of inclusion-exclusion [57] to derive an estimate for the number of topology-preserving CA solutions. To this end, we use d and r to denote the average number of radio interfaces and node degree, respectively, and assume that $r_i \simeq r$, and $d(v_i) \simeq d, \forall v_i \in V$. Clearly $m \simeq nd/2$. The following lemma describes the main results on the size of $\Omega^R(G)$.

Lemma 1. *Neglecting the effects of special subgraphs (e.g., triangles, squares, etc.) in G, the cardinality of the space, $\Omega^R(G)$, of all topology preserving CA solutions for an M² WN, $G(V, E, R, \mathcal{C})$, is given by*

$$\left|\Omega^R(G)\right| \simeq \frac{\eta^n}{k^m} \tag{3.3}$$

where

$$\eta = \sum_{i=1}^{r} \binom{k}{i} \left\{ \begin{matrix} d \\ i \end{matrix} \right\} i! \tag{3.4}$$

and $\left\{ \begin{smallmatrix} d \\ i \end{smallmatrix} \right\}$ is the Stirling number of the second kind [1].

[1] $\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$ enumerates partitions of an n set into k non-empty subsets [57]

Proof. The proof of the above lemma is carried out on several stages, where each stage examines the combined effect of a number of links and the radio constraints imposed by the vertices, on which they are incident, on the size of the solution space.

Solution Space Without Links constraints (i.e., constraints needed to preserve topology))

Here we assume that each node $v_j \in V$ is free to assign channels to its d links without any restrictions. This makes each node free to choose i channels $(i = 1, \cdots, r)$ out of the available k channels with $\binom{k}{i}$ possible choices, order those selections in $i!$ ways, and partition them in $\left\{ {d \atop i} \right\}$ ways along its d links. The number of possible ways of achieving that can be $\binom{k}{i}\left\{ {d \atop i} \right\} i!$, and for all possible values of i results in η choices given in (3.4). Consequently, the size of the solution space, without considering the constraints imposed by the communication links is η^n. We denote this space by Ω_0.

Effect of a single link

Assume that link $e \in E$ connects nodes v_i and v_j, and let node v_i be the one free to choose a CA for its links. Clearly, each choice for v_i on e eliminates $\bar{\eta}$ possible choices for v_j, where

$$\bar{\eta} = (k-1) \sum_{i=0}^{r-1} \binom{k-1}{i} \left\{ {d \atop i+1} \right\} i! \tag{3.5}$$

(3.5) can be simply explained by noting that all other $k-1$ channels become invalid choices for v_j to assign to e and, as a result, the choices made by v_j to its remaining links are also invalid. Those invalid choices range from $i = 0$, where node v_j uses the same channel used for e, and up to $r-1$ out of the remaining $k-1$ available channels to assign to its remaining $r-1$ radio interfaces. Hence, a single link in a network with n nodes entails $\eta^{n-1}\bar{\eta}$ invalid CA solutions. We denote the set of solutions eliminated by all m links as $\bar{\Omega}_1$. Thus $\left|\bar{\Omega}_1\right| = m\eta^{n-1}\bar{\eta}$.

Nevertheless, $\bar{\Omega}_1$ contains many repeated instances of the same invalid solutions. This is because the sets of CA solutions made invalid by individual links are not disjoint, and, therefore, grouping all of them in $\bar{\Omega}_1$ overestimates the actual number of invalid solutions. For example, in a network with 3 nodes and only 2 links, the sets of invalid solutions arising from each link share a common subset of size $\eta\bar{\eta}^2$. This is the set obtained by letting the node with two incident links be free to make η choices, while leaving the other two nodes with $\bar{\eta}$ invalid solutions each. Therefore, this set will appear twice in $\bar{\Omega}_1$ and must be subtracted once to reach a more accurate estimate.

To overcome this difficulty, we use the notion of inclusion exclusion [57] to progressively correct $\bar{\Omega}_1$ via studying joint sets of invalid solutions arising from more than one link.

Effect of two links

The example presented above can be used in a general setting, by counting the number of ways of choosing 2 out of m links, where a node on each link is allowed the free choice of η CA solutions leaving the other two nodes with $\bar{\eta}$ invalid solutions, and thereby rendering the whole CA configuration invalid. We call the arising subset $\bar{\Omega}_2$, and note that $\left|\bar{\Omega}_2\right| = \binom{m}{2}\eta^{n-2}\bar{\eta}^2$. Therefore, we need to exclude the duplicate instance of $\bar{\Omega}_2$ in $\bar{\Omega}_1$. Hence, a more realistic estimate for number of invalid solutions should be $m\eta^{n-1}\bar{\eta} - \binom{m}{2}\eta^{n-2}\bar{\eta}^2$.

Effect of 3 links

The act of excluding $\bar{\Omega}_2$ has not only eliminated the extra instance of $\bar{\Omega}_2$ in $\bar{\Omega}_1$, but it also removed all the subsets of invalid solutions at the intersection of 3 link-generated invalid solutions. This subset, denoted by $\bar{\Omega}_3$, appears 3 times in both of $\bar{\Omega}_1$ and $\bar{\Omega}_2$, and hence must be re-included in $\bar{\Omega}_1/\bar{\Omega}_2$.

The size of $\bar{\Omega}_3$ depends on whether the 3 links form a triangle or not, where three links can be formed with 3-6 nodes. With three nodes and three links we have a triangle subgraph in which one node will have η choices, and all solutions where the other two has $\bar{\eta}$ choices become invalid, hence, the third link becomes redundant and it does not eliminate any solutions. To account for this special case, we can calculate the expected number of triangles in a graph, ω, as the product of the number of ways of selecting three nodes and the probability p of having three links connecting them. More precisely, we have $\omega \simeq \begin{pmatrix} n \\ 3 \end{pmatrix} p^3$ with $p = \frac{m}{\frac{n(n-1)}{2}} = \frac{d}{n-1}$. now, we can accurately calculate the cardinality of $\bar{\Omega}_3$

$$\begin{aligned} |\bar{\Omega}_3| &= \begin{pmatrix} m \\ 3 \end{pmatrix} \eta^{n-3}\overline{\eta}^3 + \omega(\eta^{n-2}\overline{\eta}^2 - \eta^{n-3}\overline{\eta}^3) \\ &= \begin{pmatrix} m \\ 3 \end{pmatrix} \eta^{n-3}\overline{\eta}^3 + \omega\eta^{n-2}\overline{\eta}^2(1 - \frac{\overline{\eta}}{\eta}) \end{aligned} \tag{3.6}$$

Substituting in (3.6), we have

$$|\bar{\Omega}_3| = \begin{pmatrix} m \\ 3 \end{pmatrix} \eta^{n-3}\overline{\eta}^3 + \omega\eta^{n-2}\overline{\eta}^2(1 - \frac{\overline{\eta}}{\eta}) \tag{3.7}$$

In a similar manner we can include the effects of other special subgraphs such as squares. Noting that these structures only affect the solution space when the graph is very dense, i.e., when d, and in turn, p are very large.

As the lemma statement neglects the effects of subgraphs such as triangles, squares, etc, this proof will proceed without considering their effects. Thus, similar to the above case, we will have $\left|\bar{\Omega}_3\right| \simeq \binom{m}{3} \eta^{n-3}\overline{\eta}^3$.

Effect of 4 or more links

The powerful idea of inclusion-exclusion used often in combinatorics can be utilized here to get closer to the actual number of possible invalid solutions arising from the communication topology by continuing to add (include) and subtract (exclude) subsets of solutions eliminated by larger number of links sequentially. We first state this theory:

Theorem 1 (Inclusion-exclusion Principle [57])**.** *Let $A_1, A_2, \cdots, A_n$ be finite sets,then*

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{\substack{J\subseteq[n] \\ J\neq\pi}} (-1)^{|J|-1} \left|\bigcap_{i\in J} A_i\right| \tag{3.8}$$

In our case, for the prove of lemma 1, we can set $A_1{=}\bar{\Omega}_1$, which reflects the set of invalid solutions due to one link. Similarly, $A_2{=}\bar{\Omega}_2$ for the set of invalid solutions due to two links. For the case of three links, the cardinality of $A_3{=}\bar{\Omega}_3$ depends on the number of nodes connected by these three links which can range from 3 up to 6 nodes.

Thus, the set of valid CA solutions, $\Omega^R(G)$, will have a size that is asymptotically given by

$$\left|\Omega^R\right| = \Omega_0 - \sum_{j=1}^{m}(-1)^{j-1}\bar{\Omega}_j \tag{3.9}$$

where $|\bar{\Omega}_j| \simeq \binom{m}{j} \eta^{n-j}\overline{\eta}^j$.

Hence,

$$\begin{aligned}\left|\Omega^R\right| &\simeq \sum_{j=0}^{m}(-1)^{j-1}\binom{m}{j}\eta^{n-j}\overline{\eta}^j \\ &\simeq \eta^n\sum_{j=0}^{m}\binom{m}{j}\left(\frac{-\overline{\eta}}{\eta}\right)^j \\ &\simeq \eta^n\left(1-\frac{\overline{\eta}}{\eta}\right)^m \end{aligned} \quad (3.10)$$

Noting that $\left(\begin{array}{c}k-1\\ i-1\end{array}\right)=\frac{i}{k}\left\{\begin{array}{c}k\\ i\end{array}\right\}$ and $(i-1)!=\frac{i!}{i}$, $\overline{\eta}$ can be rewritten as

$$\begin{aligned}\bar{\eta} &= (k-1)\sum_{i=0}^{r-1}\left(\begin{array}{c}k-1\\ i\end{array}\right)\left\{\begin{array}{c}d\\ i+1\end{array}\right\}i! \\ &= (k-1)\sum_{i=1}^{r}\left(\begin{array}{c}k-1\\ i-1\end{array}\right)\left\{\begin{array}{c}d\\ i\end{array}\right\}(i-1)! \\ &= \frac{k-1}{k}\eta \end{aligned} \quad (3.11)$$

Substituting in (3.10),we have

$$\begin{aligned}\left|\Omega^R\right| &\simeq \eta^n\left(1-\frac{k-1}{k}\right)^m \\ &\simeq \frac{\eta^n}{k^m}\end{aligned} \quad (3.12)$$

□

It should be noted that the special case of having $r = d$ corresponds to having no hard radio constraints. In this case, we have $\eta = k^d$ (see [26]), thereby making $|\Omega^R(G)| = k^m$ which is the size of the original graph coloring problem indicated earlier. More generally, it can be shown that for any r, we have $|\Omega^R| = \mathcal{O}(k^{rn-m})$.

Figure 3.1 shows the size of valid solutions, $\left|\Omega^R\right|$, versus the degree d of a network with $n = 30$ nodes and assuming $k = 3, 12$ and $r = 2, 3$. These values represent a typical configurations for IEEE802.11a and b/g networks, where their standards provide 12 and 3 orthogonal channels, respectively. This figure also compares this size versus the size of the solution space without taking the hard constraints into account, $\left|\Omega^{\mathrm{CG}}\right|$. The drop in the number of solutions below 1 for large values of d is explained by the fact that at higher values of d the network becomes dense as every node becomes connected to more other nodes. As a result, the communication graphs become dominated by special subgraphs, e.g., triangles, squares, and cubes, which have been neglected in our derivations but can be incorporated through a more complex formulation.

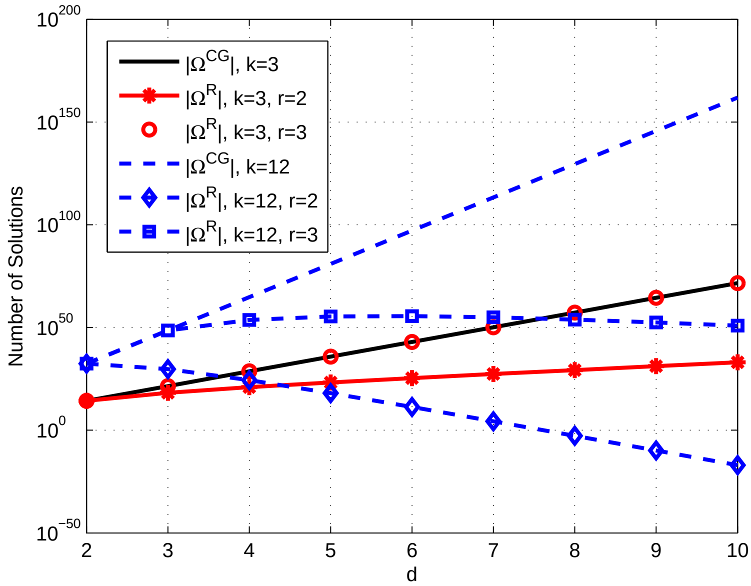


Figure 3.1: Effect of the hard radio constraints on the size of the solution space, for a network with $n = 30$ nodes.

3.4 A Reduced Solution Space for CA

The previous section demonstrated the central role played by the number of radio interfaces, r, and the hard constraints that it imposes, in reducing the size of the feasible topology-preserving CA solutions for a communication graph $G(V, E)$ for the typical networks with small values of r (typically, 2-4 radios). Motivated by this observation, we construct a new, much smaller search space, which we denote by $\Pi_c(G)$, guided by the hard radio constraints. This reduced space can be efficiently searched for elements that represent CA solutions satisfying the radio constraints. Once a candidate element in $\Pi_c(G)$ is identified, a more focused search is initiated to find an optimal CA solution within the space corresponding to this element. The key idea is, therefore, based on decoupling the original space into subsets with much smaller size that satisfy the hard radio constraints and then searching them for an optimal CA solution.

We show that $\Pi_c(G)$ is naturally endowed with a lattice structure and can be searched efficiently for small n, or using a greedy algorithm for large n, for a CA solution that satisfies these constraints. That solution is used to construct a *weighted* conflict graph which is colored to obtain the CA solution. This section is dedicated to the construction and characterization of the reduced space $\Pi_c(G)$.

3.4.1 Elements and Structure of the Reduced Space, $\Pi_c(G)$

Here we show that the CA solution space can be modeled as a subset of a partition-lattice. Each of the nodes (vertices) of the partition lattice represents groups of CA solutions that are obtained by partitioning $G(V, E)$ into edge disjoint link partitions that satisfy one or more hard radio constraints. We call those nodes *Solution Structures* and define them formally as follows.

Definition 2. *A partition $\pi = \{p_1 \dots p_\ell\}$, $p_i \subseteq E$, $i = 1, \cdots, \ell$, is a solution structure (SS) for a M²WN, $G(V, E, R, \mathcal{C})$, such that:*

1. $\ell \leq |E|$.
2. $\bigcup_i p_i = E$ *and* $p_i \cap p_j = \phi$, $\forall i, j = 1 \cdots \ell$, $i \neq j$. p_i *is referred to as a block of* π.
3. *all* p_i *are connected, i.e., there is a path between any two edges* e_1, $e_2 \in p_i$.

We denote the subsets $p_i \in \pi$ as the blocks of π and the number of these blocks by $|\pi|$. The set of all solution structures of G is denoted by $\Pi_c(G)$. $\Pi_c(G)$ can be endowed with a lattice structure [60] by defining a refinement relation between the nodes π_i as follows.

Definition 3. *An SS π_i is a refinement of another π_j, written as $\pi_i \preceq \pi_j$, if and only if they are not equal and every block in π_i is contained in a block in π_j. This relation can be written as:*

$$\pi_i \preceq \pi_j \Longleftrightarrow \pi_i \neq \pi_j \wedge \forall p_h \in \pi_i \ \exists p_k \in \pi_j \ s.t.\ p_h \subseteq p_k.$$

According to the above definition, $\preceq$ induces a partial order on $\Pi_c(G)$; there is a minimal partition with respect to this relation, $\underline{\pi} = \{p_i : p_i = \{e\} \quad \forall e \in E\}$, which is the bottom root for $\Pi_c(G)$. Similarly, there is a maximal partition $\overline{\pi} = \{E\}$, which is the top root for $\Pi_c(G)$. The *rank* of a partition $\pi \in \Pi_c(G)$ is defined as $m - |\pi|$. Thus, $\text{rank}(\underline{\pi}) = 0$ and $\text{rank}(\overline{\pi}) = m - 1$.

Finally, we note an *upward* path as a path from a more to a least refined elements in $\Pi_c(G)$. For example, the path from π_i to π_ℓ is an upward path whenever $\pi_i \preceq \pi_j \preceq \pi_\ell$.

An illustrative example

Figure 3.2 shows a simple network with 4 nodes and its partition Lattice. Let π_1=$\{\{l_1, l_2, l_3, l_4\}\}$, π_2=$\{\{l_1, l_2, l_3\}, \{l_4\}\}$ and π_3= $\{\{l_1, l_2\}, \{l_3\}, \{l_4\}\}$, then we have $\pi_3 \preceq \pi_2 \preceq \pi_1$. Also, note that from the figure that the most refined solution structure is $\{\{l_1\}, \{l_2\}, \{l_3\}, \{l_4\}\}$.

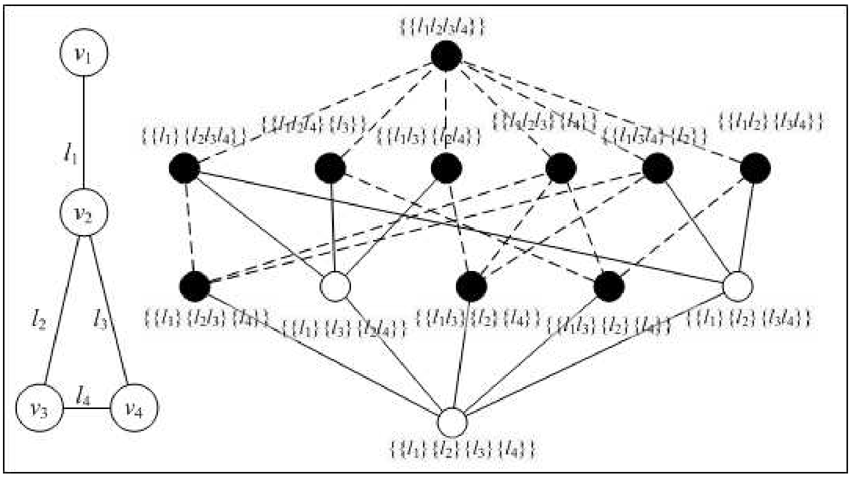


Figure 3.2: A simple network (left) and its lattice partition (right).

3.4.2 Relation between $\Pi_c(G)$ and $\Omega^R(G)$

As mentioned earlier, $\Pi_c(G)$ is searched for an SS that is used in a second level search to find an optimal CA solution in $\Omega^R(G)$. Therefore, we need to establish a relation between the CA solutions and interference levels in $\Omega^R(G)$ and $\Pi_c(G)$. The following two subsections are dedicated for this purpose.

CA solutions in $\Pi_c(G)$

The relation between an SS $\pi \in \Pi_c(G)$ and a CA solution in $\Omega^R(G)$ is expressed by a function $\Theta(\pi)$ which maps π into a number of solutions in $\Omega^R(G)$ through assigning a common channel to links in each block, or more formally,

$$\Theta(\pi) := \{S(G) : S(e_i) = S(e_j)\ \forall e_i, e_j \in p, p \in \pi\} \tag{3.13}$$

We refer to the set of CA solutions represented by $\Theta(\pi)$ as the *solution space* of π.

It should be noted at this point that not all $\pi \in \Pi_c(G)$ will have solution spaces, $\Theta(\pi)$, that are feasible in the sense that they satisfy the hard radio constraints. The following definition highlights the basic characteristics of those π that correspond to feasible solution spaces, where all the hard radio constraints are satisfied.

Definition 4. *An SS π is a feasible SS (FSS) if*

$$|\{p_j : E(v_i) \cap p_j \neq \phi, \forall p_j \in \pi\}| \leq r_i, \qquad \forall v_i \in V \tag{3.14}$$

Furthermore, the set of all FSS in $G(V, E)$ is denoted by $\Pi_c^{\mathrm{FSS}}(G)$

For example, in the lattice partition of Figure 3.2, and assuming that $r_1 = 1$ and $r_2 = r_3 = r_4 = 2$, $\pi = \{\{l_1\}, \{l_2\}, \{l_3\}, \{l_4\}\}$ does not represent an FSS since it violates the radio constraints of v_2. Similarly, all the SSs represented by the white circles in the figure are not FSS ones while those represented with the black circles are FSS and correspond to collections of feasible CA solutions.

An important property of the lattice structure that will be useful in searching $\Pi_c^{\mathrm{FSS}}(G)$ is described by the following lemma.

Lemma 2. *For any two FSSs $\pi_i, \pi_j \in \Pi_c^{\mathrm{FSS}}(G)$, we have*

$$\pi_i \preceq \pi_j \Rightarrow \Theta(\pi_j) \subseteq \Theta(\pi_i) \tag{3.15}$$

Proof. From definition 3, for each $p_i \in \pi_i$, there exists a block $p_j \in \pi_j$, such that $p_i \subseteq p_j$. Since every solution $S_j(G) \in \Theta(\pi_j)$ assigns the same channel to all edges in p_j, the refinement relation implies that the same channel is assigned to all edges in p_i and hence $S_j(G) \in \Theta(\pi_i)$. □

The above lemma indicates that once an FSS is identified, one does not need to examine the remaining FSSs on all upward paths from that solution, since its solution space dominates that of these structures. The dashed lines in Figure 3.2 provide examples of these upward paths.

It should be noted that the total number of different CA solutions represented by any SS π is $|\Theta(\pi)| = k^{|\pi|}$, as each block $p_i \in \pi$ can be assigned any of the k channels where

$|\pi|$ is dependent on the rank of π in the lattice. For example, for $\underline{\pi}$ we have $|\Theta(\underline{\pi})| = k^m$, which represents the solution space of the original conflict graph coloring problem when $r = d$. On the other extreme, $|\Theta(\overline{\pi})| = k$, which is the size of the solution space when $r = 1$ for all nodes (assuming that the communication graph is connected).

Interference Levels of FSS

Next, we quantify the level of interference arising from a single FSS π. Since links inside a block in $p_i \in \pi$ use the same channel in any $S(G) \in \Theta(\pi)$, pairs of edges within the same block will interfere if they do in the conflict graph. We refer to this source of interference as *intra-interference* of π, $I_A(\pi)$, as it results from links within the same block. Thus,

$$I_A(\pi) = \frac{1}{2} \sum I(e_i, e_j), \forall e_i, e_j \in p_\ell, \forall p_\ell \in \pi \tag{3.16}$$

In addition, we also have the interference arising between block pairs, or *inter-interference* $I_I(p_i, p_j)$, given by

$$I_I(p_i, p_j) = \frac{1}{2} \sum I(e_i, e_j), \forall e_i \in p_i, e_j \in p_j, \tag{3.17}$$

These interference values can be used to calculate the total interference corresponding to a CA solution $S(G) \in \Theta(\pi)$,

$$I_t(S(G)) = I_A(\pi) + \frac{1}{2} \sum_{\substack{p_i, p_j \in \pi, i \neq j \\ S(p_i) = S(p_j)}} I_I(p_i, p_j) \tag{3.18}$$

3.4.3 Upper bound on the number of FSS

Finally, in order to provide some insights on the size of the reduced search space and its impact on the efficiency of the search, this section derives an upper bound for the number of FSS in $\Pi_c(G)$, or $\left|\Pi_c^{\text{FSS}}(G)\right|$. If we assume that each node decides independently on partitioning its links, and again ignoring the effects of special subgraphs (e.g., triangles), then it has $\sum_{i=1}^{r} \left\{ {d \atop i} \right\}$ choices. Hence, the cardinality of the solution space for the FSSs becomes,

$$|\Pi_c^{\text{FSS}}(G)| \leq \left(\sum_{i=1}^{r} \left\{ {d \atop i} \right\} \right)^n \tag{3.19}$$

Using the reduction formula of partial sums of stirling numbers obtained from [43], we have,

$$|\Pi_c^{\mathrm{FSS}}(G)| \leq \left(\frac{r^d}{r!} + \sum_{\ell=1}^{r-2} \frac{\ell^d}{\ell!} \left(\sum_{j=2}^{r-\ell} \frac{(-1)^j}{j!} \right) \right)^n \tag{3.20}$$

Table 3.1 shows values of the obtained upper bound for $|\Pi_c^{\mathrm{FSS}}(G)|$ as r varies from 1 to 4.

Table 3.1: Upper bound on the size of $\Pi_c^{\mathrm{FSS}}(G)$ for various values of r

r	$\lvert\Pi_c^{\mathrm{FSS}}(G)\rvert$
1	1
2	$2^{(d-1)n}$
3	$\frac{1}{2^n}\left(3^{d-1}+1\right)^n$
4	$\left(\frac{4^d}{24}+\frac{2^d}{4}+\frac{1}{3}\right)^n$

Table 3.1 indicates that, for example, when $r = 2$ there are at most $2^{(d-1)n}$ FSS to examine for the optimal CA which is much smaller than the typical CA solution space of k^m with $k = 12$. Moreover, lemma 2, indicates that as these structures are related through the refinement relationship, a search scheme does not need to exhaustively visit all of them, which further reduces the search space. The reduced size of this space allows for developing efficient search techniques for an FSS as will be shown in the next section.

3.5 Using $\Pi_c^{\mathrm{FSS}}(G)$ to find optimal CA solutions

This section presents the algorithmic flow of the proposed approaches using a pseudo-code description. The algorithms proceed in two mains stages; in the first stage, the lattice partition space $\Pi_c(G)$ is searched for an FSS, π. Then in the second stage, the solution space $\Theta(\pi)$ is explored for an optimal CA solution through converting π into a weighted conflict graph and running a weighted coloring scheme. The first stage is implemented using two different algorithms. The first algorithm uses a greedy search method while the second algorithm performs an more exhaustive search on $\Pi_c^{\mathrm{FSS}}(G)$ to find an optimal π, subject to a user-specified precision control parameter.

3.5.1 A Greedy Search for a FSS

The algorithm starts with the bottom root SS in the lattice, $\underline{\pi}$, as its initial solution, π, and traverses the lattice greedily in an upward path until it finds an FSS. To achieve that, each node v is visited once in a breadth-first order starting with the node with the highest degree, i.e., the highest number of incident links $E(v)$. Blocks of π containing $E(v)$ are gradually merged so that the radio constraint (described by Equation (3.14)) of that node is satisfied while minimizing the intra-interference for the resulting blocks. Each time two blocks are merged, the algorithm proceeds one step higher along an upward path in the partition lattice (Algorithm 2 line: 5). Once the radio constraint is satisfied for the current node, all its neighbors $N(v)$ are also visited, beginning with nodes of the highest degree. This ordering in traversing the lattice allows nodes to use as many of their radios as possible, distributing their links more efficiently among these radios. As each node is visited, an additional constraint is satisfied, and eventually all the constraints will hold, thereby constructing a new FSS, whose solution space $\Theta(\pi)$ has only feasible CA solutions. It should be noted that, according to lemma 2, once an FSS has been constructed, the search does not need to explore the upward paths of that FSS.

Algorithm 1 describes a pseudo-code used to find an FSS π. It calls Algorithm 2 (see Algorithm 1: line 10) in order to ensure that the radio constraint holds for each vertex of the communication graph. This algorithm considers all the blocks of the node's links. If there are enough radios, no operation is required. However, if the number of blocks exceeds the number of radios, it will join pairs of them until there are r blocks and, thus, the hard constraint of r radios for that node holds. Each pair is selected aiming to construct r blocks with minimum intra-interference.

An illustrative example

Figure 3.3 illustrates the various stages of building a solution structure for a sample communication network, where each node is equipped with $r = 2$ radio interfaces. Initially, we have $\pi = \{\{l_1\}, \cdots, \{l_6\}\}$. Starting with a node with the highest degree, in this case v_2 (alternatively, v_3 or v_5) the algorithms merges two of its blocks to satisfy its radio constraints. In this case $\{l_1\}$ and $\{l_2\}$ were joined resulting in $\pi = \{\{l_1, l_2\}, \{l_3\}, \cdots, \{l_6\}\}$ as shown in Figure 3.3 (b). Next, all v_2's neighbors must be visited, performing the same action as for v_2. Hence, v_3 is visited, and again, to satisfy v_3's constraint, $\{l_3\}$ and $\{l_6\}$ are merged resulting in π=$\{\{l_1, l_2\}, \{l_3, l_6\}, \{l_4\}, \{l_5\}\}$ as shown in Figure 3.3 (c). Similarly, v_5 is visited and $\{l_4\}$ and $\{l_5\}$ are joined (Figure 3.3 (d)). Finally, v_1 and v_4

Algorithm 1 Greedy Search for a FSS π

Input: A connected communication graph $G = (V, E, R)$

Output: A FSS π satisfying the hard radio constraints R

```
initialize π = π̲
initialize an empty queue Q_toVisit = null
initialize an empty set V_visited = φ
ENQUEUE(Q_toVisit, v), where d(v) ≥ d(v_i), ∀v_i ∈ V, v ∈ V
while Q_toVisit is not empty do
    v ← DEQUEUE(Q_toVisit)
    for all v_i ∈ N(v) − V_visited visiting v_i with the highest degree first do
        ENQUEUE(Q_toVisit, v_i)
    end for
    π ← Satisfy Constraint (G, π, v)
    V_visited ← V_visited ∪ {v}
end while
return π
```

Algorithm 2 Satisfy Constraint (for a vertex $v \in V$)

Input: Communication graph $G = (V, E, R)$, an SS π, possibly infeasible thus far, vertex $v \in V$

Output: a partial SS π satisfying r_i.

```
π^v ← {p_i : p_i ∩ E(v) ≠ φ, p_i ∈ π}
//repeat merging blocks until the radio constraint r is satisfied for v
while |π^v| > r do
    p_i, p_j ← min_{p_i,p_j ∈ π^v} I_A(π − {p_i} − {p_j} ∪ (p_i ∪ p_j))
    π ← π − {p_i} − {p_j} ∪ {p_i ∪ p_j}
    π^v ← {p_i : p_i ∩ E(v) ≠ φ, p_i ∈ π}
end while
return π
```

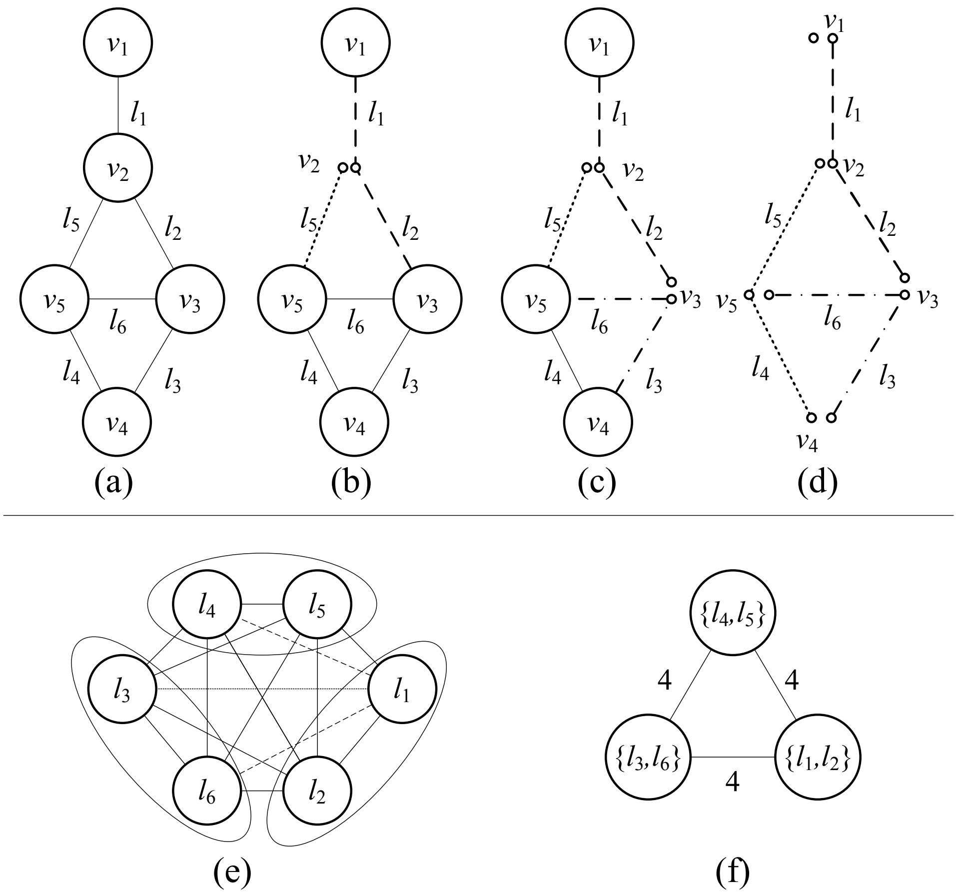


Figure 3.3: An example of the process of building a feasible solution structure (FSS) (a)-(d) and the conflict graph (e) and (f).

are visited respectively, but in both cases the number of corresponding blocks is smaller than or equal to 2, the number of radios. So no merge is required. The final solution structure becomes $\pi = \{\{l_1, l_2\}, \{l_3, l_6\}, \{l_4, l_5\}\}$. This set represents a solution structure that, for any channel assignment of its blocks, the radio constraints for each node will be satisfied.

3.5.2 Best First local Search (BFL) for an FSS

Performance evaluation results in Section 3.7 show that an FSS constructed by the above algorithm typically leads to CA solutions with acceptably good interference values. However, in order to allow a user-enabled search in $\Pi_c(G)$ for more FSSs with a further improved CA solutions, this section presents an algorithm controlled by a user-specified parameter β that determines the number of *best* upward paths, at each node along the path, that need to be locally searched and explored before the best FSS found thus far is returned. In the extreme case, a large β results in an almost exhaustive search for an FSS. As will be demonstrated through the developed experiments, this is feasible for a small number of radio interfaces and a small average node degree.

The scheme, maintains the current best FSS found so far, denoted π_{best}, which is initialized to $\overline{\pi}$, the partition with one block containing all the edges in the network resembling a CA with a single channel(Algorithm: 3, line: 2). The scheme terminates when all possible FSS in β paths for each node are examined and the FSS with the minimal intra-interference is returned. As shown in Algorithm 4, a recursive branch-and-bound depth first search (BDFS) is employed to achieve this goal. The partition lattice $\Pi_c(G)$ is traversed starting from the bottom root ($\pi = \underline{\pi}$), each node is visited once where β possible partitioning choices for its incident edges on the r radios are selected and stored in the set Σ_v. Consequently, another node is visited and the process is repeated in a depth-first sequence until one FSS is reached. A backtracking is then employed to examine the remaining $\beta - 1$ paths for each of the traversed nodes. To speed-up the search process, we rely on two main procedures; first, at each node v, the *best* β partitioning choices are heuristically generated such that the resulting partitions are both balanced with respect to their sizes and achieve minimum interference (Algorithm 4: line 6). Secondly, an efficient branch-and-bound technique is employed to prune the branches of $\Pi_c(G)$ as follows. The best FSS π_{best} found thus far with the minimum interference value, is maintained and applied to eliminate all the search branches for which the expected FSS will not achieve a better value (algorithm 4 line: 8). Since, at each step only one radio constraint is satisfied, the current SS are obtained with the hard constraints satisfied only for the already visited nodes, a lower bound on the interference is used for the remaining nodes to calculate the potential interference for the expected FSS. This value is obtained, by assuming that each radio on a node v_i is assigned a different channel and that all the links incident on v_i are evenly distributed on these radios, and interfere only with its other incident links, hence obtaining $\underline{I}_{v_i}$. Finally, two critical decisions that can affect the performance of the search are the selected current node and the order of partitions of its adjacent edges. Experimentally, we have found that selecting a node with the highest degree first and then repeating the same process for in a breadth-first order speeds up the search process by pruning more search paths. This can be attributed to the ability to detect conflicts among adjacent nodes sooner. On the other hand, selecting balanced blocks with almost equal number of edges for each node first speeds up the process of reaching a better FSS and hence prunes more branches in the search process.

Algorithm 3 Best first local search (BFL) for a FSS π_{best}

Input: a connected communication graph $G(V, E, R)$, the control parameter β
Output: A FSS π_{best}.

1: initialize Q_{toVisit} with all $v \in V$ in a breadth first order
2: $\pi_{\text{best}} \leftarrow \overline{\pi}$
3: $\pi_{\text{best}} \leftarrow$ BDFS($\underline{\pi}$,π_{best},Q_{toVisit}, β)
4: **return** π_{best}

3.5.3 Solving the reduced problem

Once an FSS π is constructed, it is employed to build a new weighted conflict graph $\tilde{G}_c$=($\tilde{V}_c, \tilde{E}_c, \tilde{W}^e_c$) as follows. A vertex $\tilde{v}_i \in \tilde{V}_c$ corresponding to each block $p_i \in \pi$ is created. Similarly, an edge $\tilde{e}(\tilde{v}_i, \tilde{v}_j) \in \tilde{E}_c$ with weight $\tilde{w}^e_c = I_I(p_i, p_j)$ is added to represent the interference between every $\tilde{v}_i, \tilde{v}_j \in \tilde{V}_c$. Figures 3.3(e) and 3.3(f) show the original conflict graph G_c of the previous example using 1-hop interference model and marked with the constructed blocks, and the new resulting weighted graph $\tilde{G}_c$, respectively.

Once $\tilde{G}_c$ is built, the original CA problem is transformed into an unconstrained weighted graph coloring problem with a much smaller solution space $k^{|\pi|}$. In our example, the size of the original and the reduced solution spaces, when using 12 channels are 12^6 and 12^3, respectively. The new weighted graph coloring problem can be solved using any efficient coloring scheme. In this work we employ the algorithm developed in [25], which is described here for completeness. The scheme uses a genetic algorithm to find good solutions and then employs a local tabu search to enhance the obtained solutions. As described in Algorithm 5, initially, several solutions for coloring $\tilde{G}_c$ are generated and used as an initial population. A new solution is then generated through a crossover step between pairs of solutions and then enhanced by searching neighboring solutions with less interference. The algorithm terminates after it successfully finds a conflict free vertex coloring solution or after a predetermined timer expires. For details pertaining to this process, the reader is referred to [25]. Finally, the solution of the original CA problem is obtained by mapping the links within each block p_i to the channel (color) assigned to the corresponding vertex $\tilde{v}_i$ in the graph coloring solution of $\tilde{G}_c$.

Algorithm 4 Branch-and-bound depth first Search (BDFS) for π_{best}

Input: Current SS π, not necessarily an FSS, the best solution thus far, π_{best}, Q_{toVisit} queue of the nodes remaining to visit, the control parameter β

Output: a FSS π_{best}.

if Q_{toVisit}=null **then**
    return π_{best}
else
    $v \leftarrow$ DEQUEUE(Q_{toVisit})
    $\Sigma_v \leftarrow$ generate good β feasible blocks of edges incident on v, $E(v)$, π_i^v=$\{p_{i1}, p_{i2}, \cdots\}$, $i = 1, \cdots, \beta$, s.t. $|\pi_i^v| \leq r_i \wedge \cup_j p_{ij} = E(v)$
    for all $\pi_i^v \in \Sigma_v$ **do**
        /**proceed with the merge into the current SS if the expected interference is better than the current, using* $\underline{I}_{v_i}$=$r_i \binom{d(v_i)/r_i}{2}$ *as an estimated interference for nodes* $v_i \in V_{\text{toVisit}}$ *otherwise, prune the branch**/
        if $I_A(\text{merge}(\pi, \pi_i^v))+ \sum_{v_j \in Q_{\text{toVisit}}} \underline{I}_{v_j} < I_A(\pi_{\text{best}})$ **then**
            $\pi \leftarrow$ BDFS(merge(π, π_i^v), π_{best},Q_{toVisit},β)
            if $I_A(\pi) < I_A(\pi_{\text{best}})$ **then**
                $\pi_{\text{best}} \leftarrow \pi$
            end if
        end if
    end for
end if

Algorithm 5 Channel Assignment for $\tilde{G}_c$

Input: Weighted conflict graph $\tilde{G}_c = (\tilde{V}_c, \tilde{E}_c, \tilde{W}_c)$ of $G(V, E, R, \mathcal{C})$, number of channels k

output: CA solution $S(G)$

```
𝒮 ←Initialize a population of solutions
s.t. each S_i ∈ 𝒮 is a solution for G̃
while not stop condition() do
    (S_i, S_j) ← ChooseParents(𝒮)
    //create a new solution S as a combination of two older solutions
    S ← create a new solution using a crossover operation on S_i,S_j
    // perform a local search trying to enhance the solution S by changing single vertex
    assignments
    S ← LocalSearch(S)
    // add S to the solution population
    𝒮 ← UpdatePopulation(𝒮, S)
end while
return  S ∈ 𝒮 s.t. I_t(S(G)) < I_t(S_i(G))∀S_i ∈ 𝒮
```

When channels are symmetric, the solution space of the coloring problem of the new graph $\tilde{G}_c$ can be further reduced by posing it as a partitioning problem of subgraphs with minimal edges in each subgraph rather than distinguishing the colors, and hence eliminating the permutation symmetry [25].

3.6 Generalization of the proposed scheme

Thus far, we have focused on developing fast CA techniques through the reduction of the search space size. To maintain lucidity in our illustrations, we have adopted several assumptions with regard to the uniform traffic for all links and non-overlapping channels. In this section, we sketch some possible extensions to the proposed schemes to relax these assumptions.

To accommodate different traffic loads on various links, the developed schemes can be extended by adding weights to the communication links similar to the approach employed in [61] as follows. If we assume that we have an a-prior knowledge about the set of flows in the network, then the weight of each communication link e, $\omega(e)$, can be set to $\frac{\sum_{f_i:e\in path(f_i)} f_i}{c(e)}$, where $c(e)$ is the capacity of link e and $path(f_i)$ is the path of flow f_i determined by the routing algorithm in the network. These weights are used to calculate a weighted interference value for each pair of links as $I^{\omega}(e_i,e_j)= I(e_i,e_j)\omega(e_i)\omega(e_j)$ to replace the binary interference value $I(e_i,e_j)$ in our calculations.

The case for overlapping channels affects the last step of assigning channels to the blocks of the FSS π. Let $0 \leq \lambda(c_i,c_j) \leq 1$ denote the level of interference between two channels c_i and c_j. Then, we can substitute the link weight used in the weighted conflict graph in the final step, $I_I(p_1,p_2)$, i.e., the inter-interference between two partitions p_1, p_2 for a FSS π, $p_1,p_2 \in \pi$, with a more accurate value $I_I(p_1,p_2)\lambda(c_i,c_j)$ when c_i and c_j are the colors assigned for p_1 and p_2.

3.7 Performance Evaluation

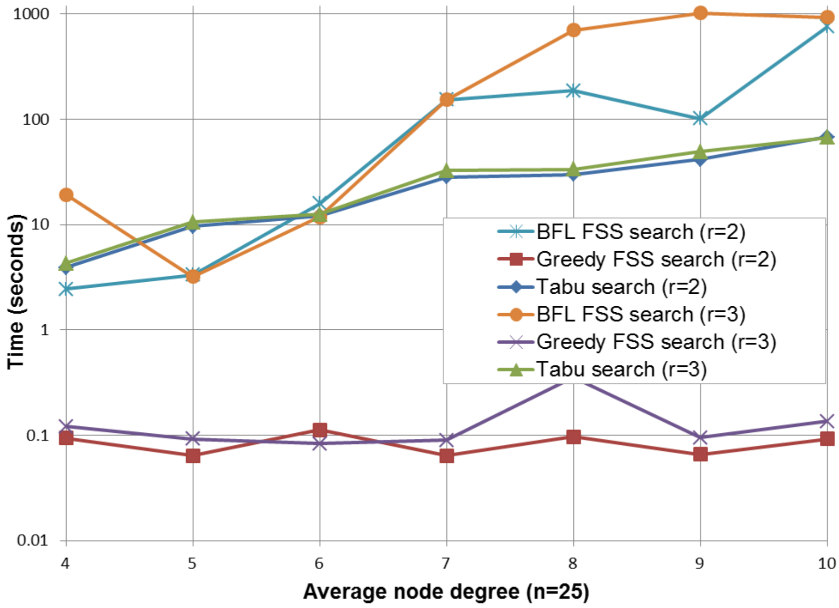
The performance of the two proposed schemes described by the algorithms in Section 3.5 is evaluated and compared against the tabu search presented in [58]. The performance comparison between the three algorithms first shows the time taken by each one (Section 3.7.1) and then shows the levels of fractional interference attained by the CA solutions reached, by each algorithm, at the end of the search time (Section 3.7.2). Subsequently, Section 3.7.3 provides a comparison for the level of throughput obtained from the CA solutions in each algorithm.

Following [58], the experimental setup for the three schemes was simulated using a small and a large mesh networks with $n = 25$ and $n = 50$ nodes, respectively, randomly placed in an initial area of $500 \times 500\text{m}^2$, with a transmission range initially set to 150m yielding an average node degree, d, of 5 and 10, respectively. The user-specified control parameter, β, was set to 6 and 2 for the small and the large network, where, it was experimentally observed that these values achieve a balance between the search speed and the obtained interference values.

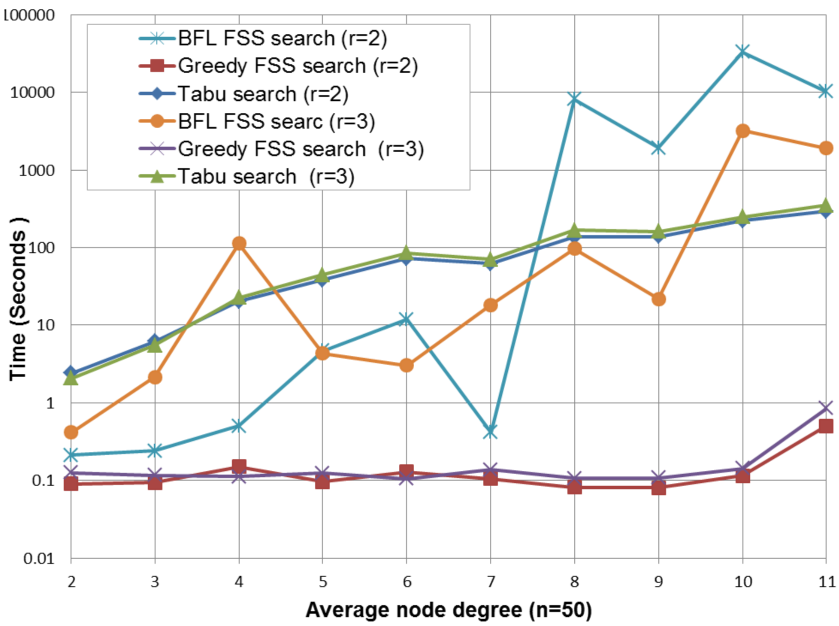
3.7.1 Search Time and Reduction of Solution Space

Figure 3.4 shows the search time taken by the proposed schemes and compares that against the time taken by the tabu search. This comparison was carried out versus an average node degree, d, ranging from 4 to 10, for the small ($n = 25$) and large ($n = 50$) networks, with the results shown in Figures 3.4(a), and 3.4(b), respectively.

The first observation arising from Figure 3.4 is that the greedy FSS search is several



(a) Small network $n = 25$.



(b) Dense network $n = 50$.

Figure 3.4: Algorithms search time versus average node degree.

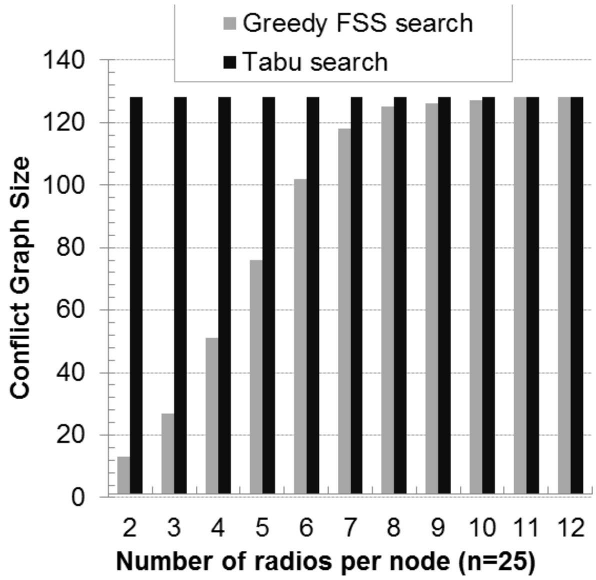
orders-of-magnitude faster than the tabu search. This fact can be easily attributed to the reduction in the search space explored by the proposed scheme and can be demonstrated by comparing the size of the conflict graph obtained by the former and used by the latter as shown in Figure 3.5. As this figure illustrates, for the practical number of radios available in M^{2}WN, the size of the conflict graph in the greedy FSS is much smaller than that of the tabu search. In fact, it was observed that for 2 to 4 radios, the resulting reduced graph was almost always colorable with less than 12 colors, concluding that a good CA solution does not necessary need to use all the available channels.

Another important observation arising from Figure 3.4 is related to the search time taken by the BFL FSS which appears to be comparable to the time taken by the tabu search. However, this observation must be considered against the fact that the BFL FSS visits a much larger number of candidate CA solutions than the tabu search. This fact evidently impacts the quality of the CA solution obtained by the BFL FSS and substantially lowers the level of interference in the final CA solution as will be shown in the next section.

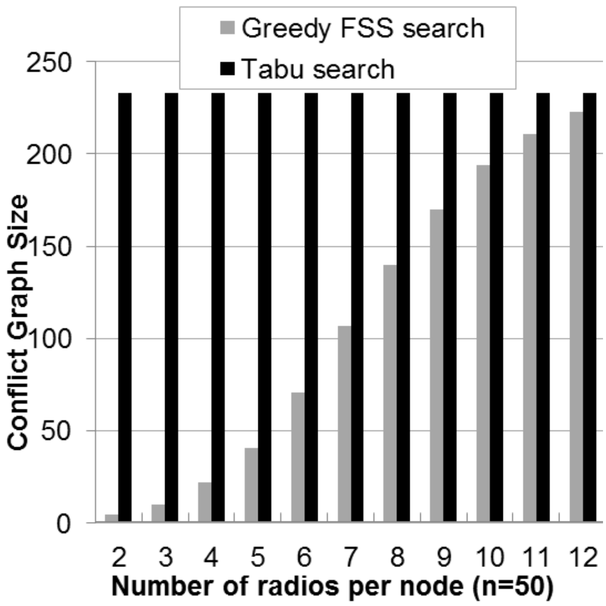
3.7.2 Achieved Fractional Interference

This section reports the levels of interference achieved by the CA solutions found in the search times reported in the previous section. As illustrated by Figures 3.6(a) and 3.6(b), the proposed schemes achieve smaller interference values compared to the tabu search scheme, where the BFL FSS search scheme achieves a remarkably lower values for the case of the small network. In fact, it is worth noting here that, for the small network with $n = 25$, we did not achieve any significantly better results when increasing β up to 20.

Figures 3.7(a), and 3.7(b), gives more insights by showing the ratio between the intra- and inter-interference values for the obtained FSSs for small and large networks, respectively, against the number of radios per node. For small values of r (1-4), the intra-interference is the dominant source of interference. This observation clearly demonstrates that the inter-interference effects can be ignored in networks with $r \leq 4$,which is the practical number of radio interfaces available in M^{2}WN. Hence, using the reduced space Π_c^{FSS} as the search space, where the focus is on the intra-interference, has proved to be the main key in reaching a CA solution much faster. Finally, Figures 3.8(a) and 3.8(b) demonstrate the obtained interference values versus the number of radios as it increases from 2 to 5 for the small and large networks with a fixed node degrees at 5 and 10,

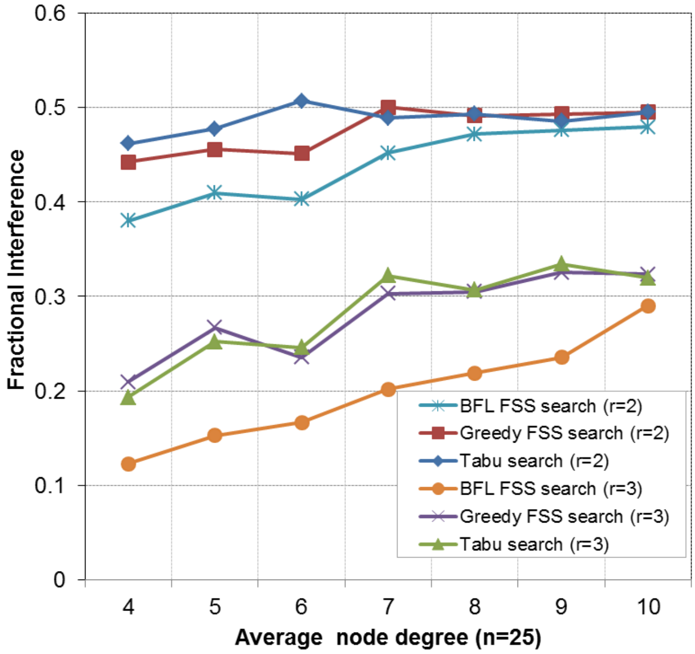


(a)

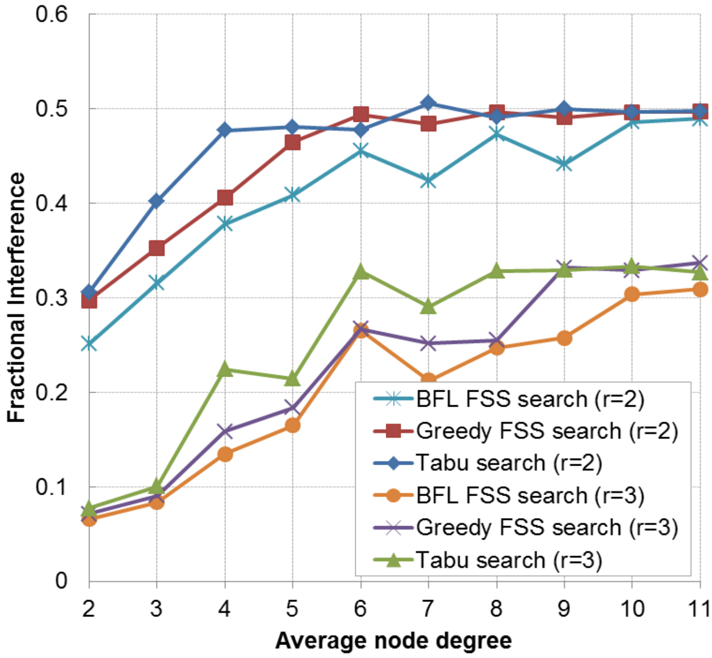


(b)

Figure 3.5: The size of the conflict graph

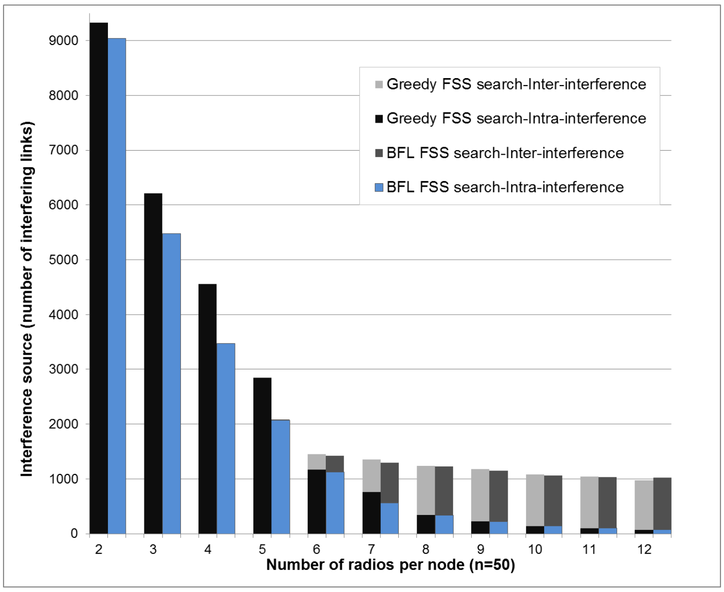


(a) $n = 25$

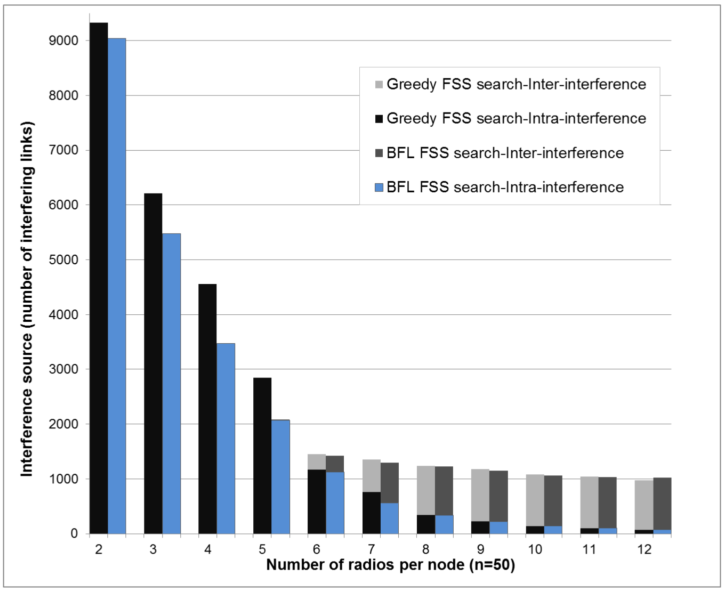


(b) $n = 50$

Figure 3.6: The fractional interference versus the average node degree



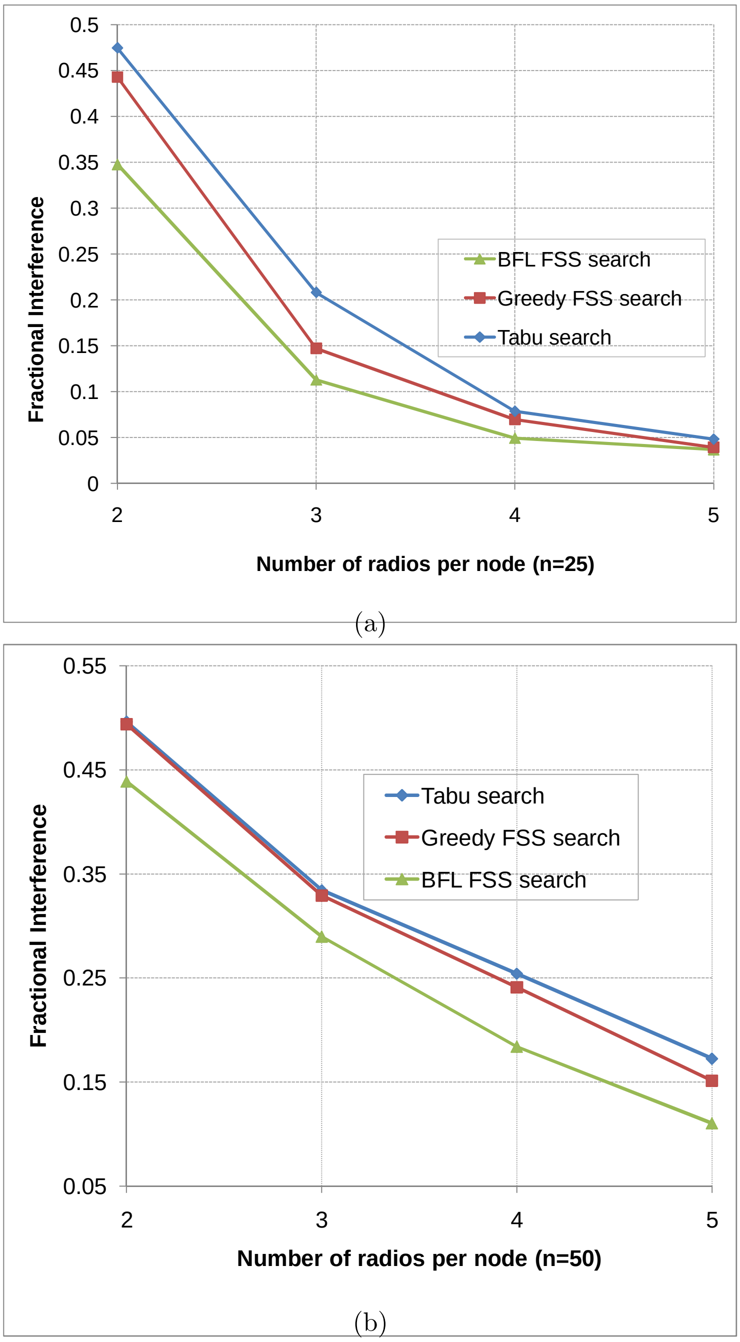
(a)



(b)

Figure 3.7: Comparison between the Intra- and inter- interference values

respectively. In general, both our algorithms perform extremely well compared to the Tabu-based scheme. The BFL algorithm always performs better that than the Greedy and Tabu scheme.



(a)

(b)

Figure 3.8: The fractional interference against the number of radios per node

3.7.3 Achieved Throughput

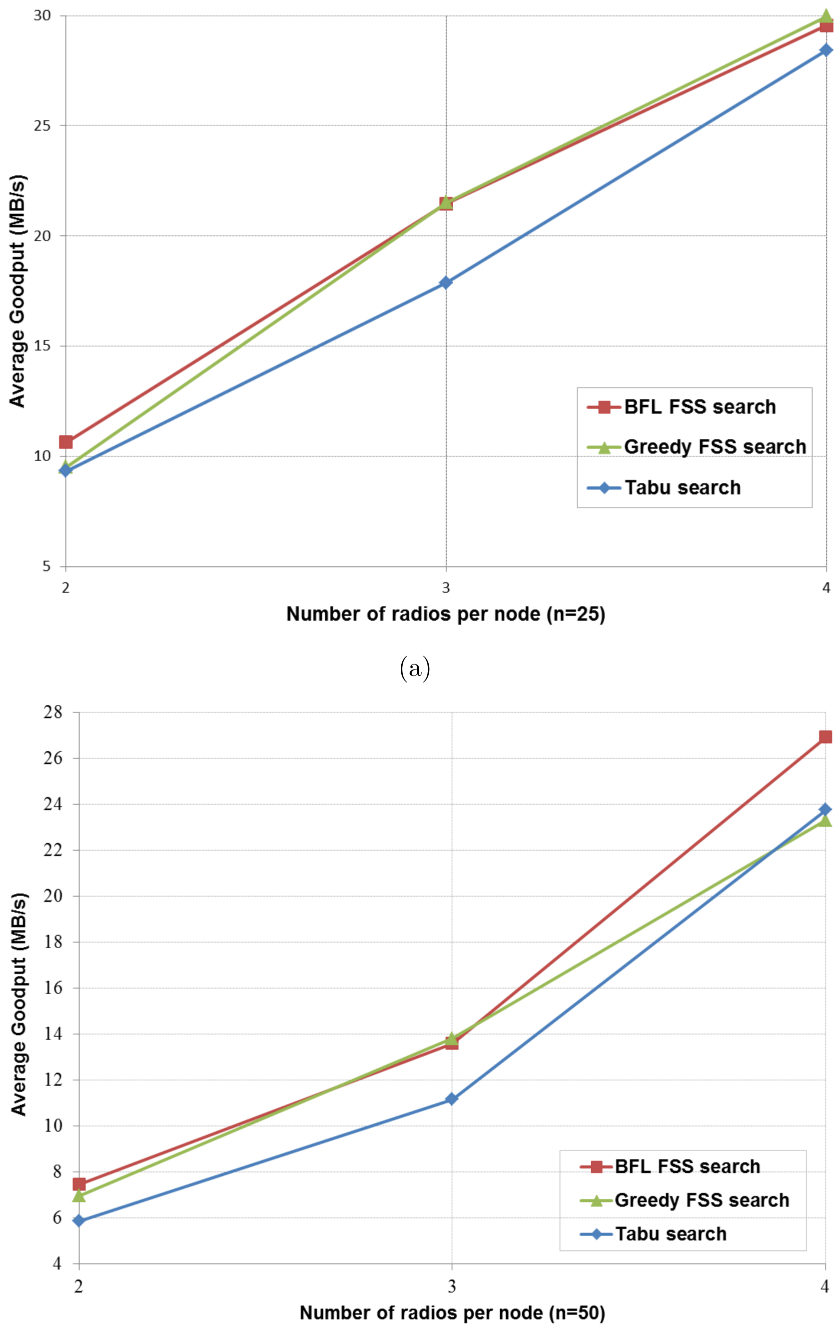
Figures 3.9 compares the achieved Goodput of the proposed Greedy FSS scheme and Tabu based search [58], performed using Network Simulator 3 (ns3) [44]. The adopted traffic scheme is a constant data rate of 8 MB/s for every link in the network. The propagation model used by ns3 is the logarithmic propagation model. The network standard is 802.11a, and all the wireless channels are orthogonal, i.e., there is no cross-channel interference. We measure the network Goodput, i.e., the traffic successfully delivered, varying the number of radios per node from 2 to 4. Both schemes make a better use of the wireless medium the more radios per node are available. However, the proposed schemes achieve noticeably higher values for the case of the large network. This can be attributed to the large size of the Tabu search space which makes it harder to find an optimal solution. On the other hand, the proposed scheme takes advantage of the reduced space where the effects of the hard constraints dramatically reduces the space of feasible solutions.

3.7.4 Complexity analysis

In this section, we sketch the time complexity of the proposed schemes. For the Greedy FSS algorithm, as each link in the communication graph is visited twice, when each of its vertices are visited, the algorithm requires $\mathcal{O}(|E|)$ time steps. For the BFL scheme, the worst case solution requires $\mathcal{O}(\beta^n)$ time steps. In the second step, solving the graph coloring problem heuristically is $\mathcal{O}(|\pi|^2)$. In our experiments, in a typical large network (up to 100 nodes) using a small number (2-4) of radios, the order of the algorithm becomes closer to $\mathcal{O}(|E|)$.

3.8 Summary

In this chapter, we have presented a characterization of the solution space of the channel assignment (CA) problem for wireless networks with multiple interfaces and multiple channels. We have provided a closed form expression for the number of feasible channel assignment solutions for a given graph and showed that the size of the search space for the CA is reduced substantially when the number of radio interfaces installed on each router is small. We have demonstrated that it is possible to significantly reduce the solution space before solving the problem.



(a)

(b)

Figure 3.9: Goodput comparison for small and large networks.

Based on our analysis, we have developed two novel schemes that transform the CA problem into a much smaller unconstrained weighted graph coloring problem. Performance evaluations illustrated the efficiency and the speed up achieved by the proposed schemes in the particular case of smaller number of radio interfaces and for larger networks.

Chapter 4

Channel Assignment for Full Wireless Mesh Networks

4.1 Introduction

The knowledge of theoretical bounds for the optimal interference values is considered an indispensable asset in measuring the success of a CA algorithm or to estimate the quality of a given CA solution. Unfortunately, and to the best of our knowledge, the literature lacks such a theoretical analysis.

This chapter addresses the aforementioned gap by formulating the CA problem for fM^{2}WNs as a complete graph decomposition (CGD). In general, the CGD problem [15, 63] addresses the existence, or lack thereof, of edge-disjoint partitions of a complete graph K_n that are isomorphic[1] to some predefined graph(s) or a complete graph K_p, $p < n$. Nonetheless, the CA for fM^{2}WN imposes two additional constraints: the radio-interface constraint which implies that each router[2] with r radio interfaces, uses no more than r channels and, hence, can participate in no more than r subgraphs. The second constraint is that the subgraphs are degree-constrained in order to obtain an optimal CA with minimum interference and fairness in channel usage among all routers. Another difficulty over the classical CGD is that the resulting subgraphs are not necessarily isomorphic to a predefined graph. These differences make adapting results reported in this area to handle the CA problem far from being straightforward. The main goal of this chapter

[1] Two graphs which contain the same number of graph vertices connected in the same way are said to be isomorphic.

[2] the words router and node are used interchangeably

is to address this difficulty. More precisely, the following are the contributions presented in this chapter:

1. We derive a new formulation for the CA problem for $f\text{M}^2$WNs as a degree-constrained CGD and identify the necessary conditions for such a decomposition to reflect optimal and fair CA solutions,

2. We show that the minimum network interference, and hence, maximum throughput for uniform traffic on communication links, can only be achieved with a specific number of channels, denoted as the *characteristic channel number for n routers with r radios in a $f\text{M}^2$ WN*. We then derive upper and lower bounds for that number for any $f\text{M}^2$WN and exact values for a class of $f\text{M}^2$WN with special relationships between n and r.

3. We develop closed-form expressions for the minimum channel interference in $f\text{M}^2$WNs.

4. We present a polynomial-time algorithm for a near-optimal CA solution for $f\text{M}^2$WNs.

The remainder of this chapter is organized as follows. The channel assignment problem formulation and adopted notations are provided in Section 4.2. Sections 4.3 and 4.4 provide an analysis for the optimal CA solution and derive values for the optimal interference, respectively. Section 4.5 is dedicated to describing the proposed CA scheme. Experimental results are presented in Section 4.6. Finally, Section 4.7 concludes the chapter.

4.2 Problem formulation and definitions

We consider a $f\text{M}^2$WN comprised of n routers such that each router is equipped with r radio interfaces. Typically, such routers have a longer transmission range than traditional mesh nodes [37] where each pair of routers can communicate directly when using a common channel. Moreover, these routers are either nomadic or have a low mobility degree [36,37]. Similar to other works [58], we assume uniform traffic for all communication links where the amount of traffic is roughly the same between all pairs of routers. This assumption is also motivated by the aforementioned applications of $f\text{M}^2$WNs. Figure 4.1 demonstrate an example of a military $f\text{M}^2$WN. A $f\text{M}^2$WN can be modeled by an undirected complete graph $K_n(V, E, r, k)$, where $V = \{v_1, \cdots, v_n\}$ is the set of n routers

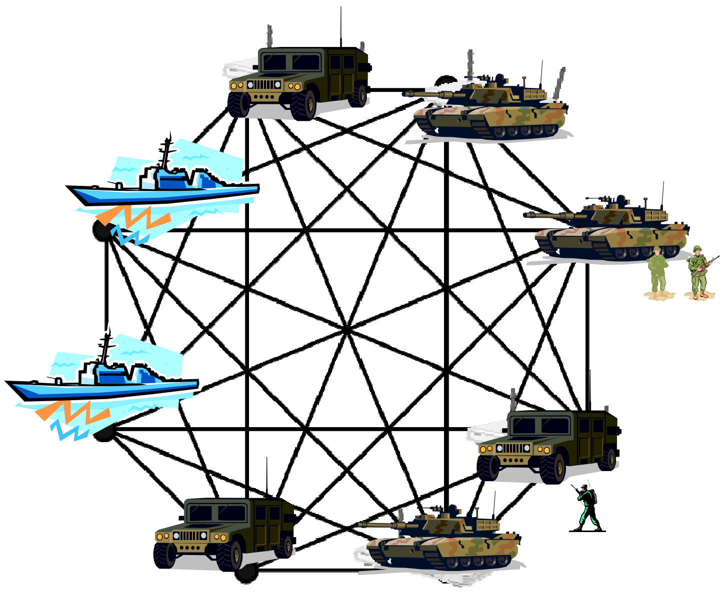


Figure 4.1: An example of a full military M^2WN with 8 routers

and E is the set of links, such that $m = |E| = n(n-1)/2$. Similar to the previous chapter, we assume that each node has $r \leq n-1$ radio interfaces and there are k orthogonal channels, represented by $\mathcal{C} = \{c_1, \cdots, c_k\}$ and are available to all nodes. Two communication links are said to interfere if they cannot successfully transmit data simultaneously using the same channel. Link interference is usually modeled through an interference model; we consider a discrete symmetric interference model $I(e_i, e_j)$, where $I(e_i, e_j) = 1$ if $e_i, e_j \in E$ are interfering and $I(e_i, e_j) = 0$ otherwise. Since in K_n any two links are at one-hop distance, any two links interfere, i.e., $I(e_i, e_j) = 1$, if they use a common channel. Throughout this chapter, we are only concerned with *topology preserving CA solutions*, defined as follows:

Definition 5. *A topology preserving CA solution $S(K_n)$ for a fM^2WN $K_n(E, V, r, k)$ is a mapping $\mathcal{S}(K_n) : E \rightarrow \mathcal{C}_s$ such that:*

1. $\mathcal{C}_s = \{c_1, \cdots, c_{\sigma_s}\}$, $\mathcal{C}_s \subseteq \mathcal{C}$,
2. *$S(K_n)$ uses $\sigma_s \leq k$ channels.*
3. *$|S(v)| \leq r$, where $S(v)$ is the set of all channels used by $v \in V$ in $S(K_n)$.*

The last condition ensures that $S(K_n)$ satisfies the so called *radio constraint*; the constraint that the number of different channels used by any node is at most equal to the number of radio interfaces r. We refer to such a solution as topology preserving since each link in the network is assigned a channel, and hence, maintains its connectivity. $S(K_n)$ partitions the links of K_n, into the sets $E_1, \cdots, E_{\sigma_s}$,where E_i is the set of links using channel c_i in K_n. We associate with every solution $S(K_n)$ a total cost $I_t(S(K_n))$ that measures the total network interference. More precisely,

$$I_t(S(K_n)) = \frac{1}{2} \sum_{e_i, e_j \in_{i \neq j} E} I(e_i, e_j) \tag{4.1}$$

We also define, $I_t^f(S(K_n))$, the fractional interference for $S(K_n)$ [58], as the ratio between $I_t(S(K_n))$ and the interference resulting from using a single channel and we use $I_t^f(S(K_n))$ to define an optimal CA solution as follows:

Definition 6. *An optimal CA solution for a fM*^{2}*WN* $K_n(E, V, r, k)$*,* $S^*(K_n)$*, is one that satisfies that for any other solution* $S(K_n)$*, we have:* $I_t^f(S^*(K_n)) \leq I_t^f(S(K_n))$*.*

Since we are considering symmetric interference and traffic models, we use the local or intra- interference, $I_i(S(v))$ as a measure of fairness and calculate it as follows [55]:

Definition 7. *A fair CA solution for a fM*^{2}*WN,* $K_n(E, V, r, k)$*,* $S(K_n)$*, is one that satisfies the condition that for any other solution* $\acute{S}(K_n)$*, we have:* $I_i(S(v)) \leq I_i(\acute{S}(v))$*,* $\forall v \in V$ *such that :*

$$I_i(S(v)) = \frac{1}{2} \sum_{v,u,\acute{u}_{u \neq \acute{u}} \in V} I_i(e(v, u), e(v, \acute{u})), v \in V \tag{4.2}$$

Typically, different fairness models are adopted in literature (e.g., max-min flow fairness in [64]). In this work, since delay is the critical metric to our target applications, we focus on fairness with respect to the experienced link delays introduced by contention among adjacent links due to sharing a common channel which results from minimizing the node's intra-interference [55].

Finally, $K_n(V, E, r, .)$ to the CA problem with no restriction on the number of available channels, whereas, $K_n(V, E, r, k)$ represents the case where the number of available channels is k.

4.3 Theoretical Results

In this section, we are mainly concerned with answering the following question: What is the number of channels that if used can achieve an optimal CA for a fM^{2}WN,

$K_n(V, E, r, .)$? To facilitate following the developments in this section, the following list briefly summarizes the main stages used in our derivations:

1. We commence with establishing the relation between the CA problem in $f\mathrm{M}^2\mathrm{WN}$ and the CGD problem in Section 4.3.1.
2. Next, we identify the additional conditions that must be imposed on the latter such that its solution (graph decomposition) would represent an *optimal* and *fair* solutions to the former.
3. Section 4.3.4 builds on the existing graph theory through presenting new propositions, and proofs, to find the maximum number of *balanced* and *regular* CGD, where balanced and regular decompositions will be defined precisely in Section 4.3.1. The results obtained therein are used to demonstrate that the number of channels needed to find the optimal CA (or equivalently, the number of subgraphs in K_n) can be known exactly when certain relations between n and r are satisfied.
4. The case for general n and r is then addressed in Section 4.3.5, where lower and upper bounds for that number of channels that leads to optimal CA are presented. The obtained results provide interesting insights showing that the number of radios imposes a bound on the number of channels in an optimal CA.
5. Finally, Section 4.3.6 uses the aforementioned results to address CA when the number of available channels is given a-priori.

4.3.1 CA as a graph decomposition problem

In order to clearly identify the similarities and differences between the classical CGD problem and that of the optimal CA, we state the former problem precisely through the following definition:

Definition 8. *[19] The H-decomposition problem of K_n is stated as follows: Can an input graph K_n be represented as an edge disjoint union of subgraphs, $G_1, \cdots, G_\ell$, such that:*

1. $\bigcup_{i=1}^{\ell} G_i = K_n$,
2. *G_i is isomorphic to H, $\forall i = 1, \cdots, \ell$.*

It should be emphasized here that the number of subgraphs ℓ is not pre-specified or restricted by the decomposition problem, but is rather determined upon reaching the decomposition. It is also worth noting here that the H-decomposition problem for K_n is NP-complete whenever H is connected and has at least 3 edges [19]. Next, we establish the mapping between the two problems.

Definition 9. *A CA for solution $S(K_n)$ for a fM^{2}WN, $K_n(V, E, r, k)$, is equivalent to a decomposition of K_n into subgraphs $G_1, \cdots, G_{\sigma_s}$, where links assigned to a common channel c_i, $i = 1, \cdots, \sigma_s$, are represented by a subgraph G_i such that:*

1. $\bigcup_{i=1}^{\sigma_s} G_i = K_n$, $\sigma_s \leq k$,
2. $|\{G_i : v \in V(G_i),\, i = 1 \cdots, \sigma_s\}| \leq r \quad \forall v \in V$.
3. *Furthermore, an optimal CA $S^*(K_n)$ is a decomposition of K_n that, in addition to satisfying the above conditions, must also satisfy the minimality condition of $I_t^f(S^*(K_n))$ specified in definition (6).*

The second condition in definition (9) guarantees the satisfaction of the radio constraint on each router. Definitions (8) and (9) show that the CA problem departs from classical CGD in several aspects. First, rather than starting with an input graph H, the CA problem seeks to decompose K_n into unknown subgraphs, one per channel, with the additional objective of achieving optimal interference. In addition, the number of subgraphs (channels) is restricted to be less than k. Furthermore, the decomposition must satisfy the radio constraints. Finally, the restriction that all the subgraphs must be isomorphic is eliminated. Next, we provide two definitions that will be useful in our derivations.

Definition 10. *A regular (almost-regular) σ-decomposition of K_n is a decomposition of K_n into σ subgraphs such that for subgraph $G_i(V_i, E_i)$ we have $d_i(w) = d_i(v)$ ($|d_i(w) - d_i(v)| \leq 1$, respectively) $\forall v, w \in V_i$, where $d_i(v)$ is the degree of v in G_i and $\bigcup_i G_i = K_n$, $i = 1, \cdots, \sigma$.*

Definition 11. *A balanced (nearly-balanced) σ-decomposition of K_n is a decomposition of K_n into σ subgraphs such that for each subgraph $G_i(V_i, E_i)$ we have $|m_i - m_j| \leq 1$ ($|m_i - m_j| \leq 2$, respectively) $\forall i, j = 1, \cdots, \sigma$, where $m_i = |E_i|$ and $\bigcup_i G_i = K_n$.*

In the following, the next two propositions demonstrate the equivalence between optimal CA and balanced decompositions, on one side, and fair CA and almost regular-decompositions, on the other.

4.3.2 Properties of optimal CA solutions

The following proposition identifies the additional condition that needs to be imposed on the original CGD problem to cast it as an optimal CA solution.

Proposition 1. *An optimal CA solution $S^*(K_n)$ for a fM²WN, $K_n(V, E, r, .)$, is equivalent to a balanced decomposition of K_n.*

Proof. Let σ_s be the number of channels in $S^*(K_n)$, then, since any two links in the same channel interfere, the total interference is given by:

$$I_t(S^*(K_n)) = \sum_{i=1}^{\sigma_s} \binom{m_i}{2} = \frac{1}{2}\sum_{i=1}^{\sigma_s}(m_i^2 - m_i) \tag{4.3}$$

where $m_i = |E_i|$. Let $\hat{m}$ be the smallest number of $m_1, m_2, \cdots, m_{\sigma_s}$, where $m_1+m_2+\cdots+m_{\sigma_s}= m$. Then

$$I_t(S^*(K_n)) = \frac{1}{2}\sum_{i=1}^{\sigma_s} m_i^2 - \frac{m}{2} \tag{4.4}$$

Since

$$\begin{aligned}\sum_{i=1}^{\sigma_s} m_i^2 &= \sum_{i=1}^{\sigma_s}(\hat{m} + (m_i - \hat{m}))^2 \\ &= \sum_{i=1}^{\sigma_s}(\hat{m}^2 + 2(m_i - \hat{m})\hat{m} + (m_i - \hat{m})^2)\end{aligned} \tag{4.5}$$

we get,

$$\begin{aligned}I_t(S^*(K_n)) &= \frac{1}{2}\sum_{i=1}^{\sigma_s}\hat{m}^2 - \frac{m}{2} + \frac{1}{2}\sum_{i=1}^{\sigma_s}(2(m_i - \hat{m})\hat{m} + (m_i - \hat{m})^2) \\ &= \frac{\sigma_s}{2}\hat{m}^2 - \frac{m}{2} + \frac{1}{2}\sum_{i=1}^{\sigma_s}({x_i}^2 - 2x_i\hat{m})\end{aligned} \tag{4.6}$$

where $x_i = (\hat{m} - m_i)$. Obviously, $I_t(S^*(K_n))$ is minimized when $x_i = 0$, i.e., when $|\hat{m} - m_i| = 0$. Hence, the optimal CA is achieved when all the channels have the same number of links. Since, the number of used channels σ_s may not always divide m, the optimal configuration would have p subgraphs with $\lceil \frac{m}{\sigma_s} \rceil$ links and $\sigma_s - p$ subgraphs with $\lfloor \frac{m}{\sigma_s} \rfloor$ links, where p is the remainder of m/σ_s resulting in a balanced decomposition of K_n. □

The above proposition states that the optimal interference can be obtained by simply dividing the links equally among the used channels. For example, for $K_5(\{1,2,3,4,5\}, E,2,2)$, an optimal solution can be achieved with the decomposition: $E_1 = \{(1,2), (1,3), (1,4), (1,5), (2,3)\}$ and $E_2 = \{(2,4), (2,5), (3,4), (3,5), (4,5)\}$, each representing links allocated to a common channel where nodes are numbered from 1 to 5 and (v,u) represents a link between v and u. It is clear that while this solution provides optimal interference value, it is not a fair solution, since node 1 is using one channel. Next we will formally define a mapping of fair CA solutions.

4.3.3 Properties of fair CA solutions

We can follow the same steps to identify the additional condition that must be imposed on the original CGD problem to cast it as a fair CA.

Proposition 2. *A fair CA solution for a fM^{2}WN, $K_n(V,E,r,.)$, is equivalent to an almost-regular decomposition of K_n.*

Proof. Denote by c_1, c_2, $\cdots$, c_r, the channels utilized by the r radios in $v \in V$ within a fair CA solution $S(K_n)$. Let $c_i(v)$ denote the set of links incident on v using channel c_i, then,

$$I_i(v) = \sum_{i=1}^{r} \binom{|c_i(v)|}{2} = \frac{1}{2}\sum_{i=1}^{r} |c_i(v)|^2 - \frac{1}{2}\sum_{i=1}^{r}(|c_i(v)|$$

Defining $\tilde{c} = \min(|c_1(v)|,|c_2(v)|,\cdots,|c_r(v)|)$ and $x_i = \tilde{c} - |c_i(v)|$, and noting that $\sum_{i=1}^{r} |c_i(v)| = n-1$, the following manipulation:

$$\sum_{i=1}^{r} |c_i(v)|^2 = \sum_{i=1}^{r}(\tilde{c} + (|c_i(v)| - \tilde{c}))^2 \quad (4.7)$$

$$= \sum_{i=1}^{r}(\tilde{c}^2 + 2(|c_i(v)| - \tilde{c})\tilde{c} + (|c_i(v)| - \tilde{c})^2)$$

$$= \sum_{i=1}^{r} \tilde{c}^2 + \sum_{i=1}^{r}((\tilde{c} - |c_i(v)|)^2 - 2(\tilde{c} - |c_i(v)|)\tilde{c})$$

$$= \sum_{i=1}^{r} \tilde{c}^2 + \sum_{i=1}^{r}(x_i^2 - 2x_i\tilde{c})$$

$$= \sum_{i=1}^{r} \tilde{c}^2 + \sum_{i=1}^{r} x_i(x_i - 2\tilde{c}) \quad (4.8)$$

demonstrates that $I_i(v)$ is minimized when $x_i = 0$, i.e., when $\tilde{c} = |c_i(v)|$ for all c_i. Since r may not always divide $n-1$, $I_i(v)$ is minimized when $(|c_i(v)| - \tilde{c}) \in \{0, 1\}$. In other words, $I_i(v)$ is minimized when the subgraphs, each corresponding to a channel, decompose K_n into almost-regular subgraphs with degrees $\lceil \frac{n-1}{r} \rceil$ and $\lfloor \frac{n-1}{r} \rfloor$ such that each node v has a degree $\lceil \frac{n-1}{r} \rceil$ in l subgraphs and a degree $\lfloor \frac{n-1}{r} \rfloor$ for $(r-l)$ subgraphs, where l is the remainder of $(n-1)/r$. $\square$

Revisiting the CA problem for $K_5(\{1,2,3,4,5\},\ E, 2, 2)$, the optimal solution $E_1 = \{(1,2), (1,3), (2,4),\ (3,5), (4,5)\}$, $E_2 = \{(1,4), (1,5), (2,3), (2,5), (3,4)\}$ is both optimal and fair.Figure 4.2 provides some examples of optimal and fair CA solutions.

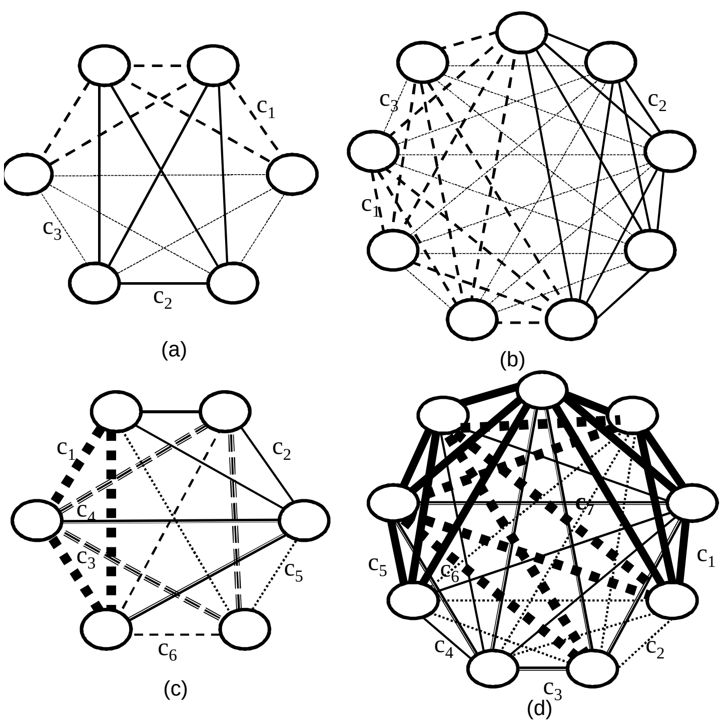


Figure 4.2: An example of almost-regular and balanced optimal CA for $n = 6, 9$ with $r = 2$(a),(b) and $r = 3$ (c),(d)

In the next section, we use the above derived properties to find the number of channels needed to achieve an optimal CA for $K_n(V, E, r, .)$ and show that this CA is fair.

4.3.4 Exact values for the number of channels for optimal CA

In this section, we address the problem of finding an optimal CA for $K_n(V, E, r, .)$. We will show that when certain relations between n and r exist, one can calculate a closed-form expression for the used number of channels in this solution according to following proposition:

Proposition 3. *If there exists a K_{q+1}-decomposition of K_n with $q = \frac{n-1}{r}$, then this decomposition is equivalent to the minimum interference CA for a fM*^{2}*WN, $K_n(V, E, r, .)$. Furthermore, this solution uses exactly $\frac{nr^2}{n+r-1}$ channels.*

Proof. Let σ_s be the number of channels used in the optimal solution $S^*(K_n)$. Proposition 1 indicates that the resulting subgraphs in a minimum interference CA are either of size $\lceil \frac{m}{\sigma_s} \rceil$ or $\lfloor \frac{m}{\sigma_s} \rfloor$. Assume that σ_s divides m, then the size of all subgraphs is $m_i = \frac{m}{\sigma_s} = \frac{n(n-1)}{2\sigma_s}$. Hence, substituting for $\hat{m} = m_i = \frac{m}{\sigma_s}$, $i = 1, \cdots, \sigma_s$ and $x_i = 0$ in (4.6), then $I_t(S^*(K_n))$ can be written as:

$$I_t(S^*(K_n)) = \frac{\sigma_s m_i^2}{2} - \frac{m}{2} = \frac{m^2}{2\sigma_s} - \frac{m}{2} \tag{4.9}$$

indicating that $I_t(S^*(K_n))$ is minimized when the number of subgraphs (channels), σ_s, is maximized. Maximizing the number of subgraphs entails minimizing the number of nodes, n_i, participating in each subgraph. Given that a graph with m_i links and n_i nodes must satisfy the inequality $m_i \leq \frac{n_i(n_i-1)}{2}$, it follows that (through solving the quadratic for n_i) the minimum number of nodes $n_i = \frac{1+\sqrt{1+8m_i}}{2}$.

In the absence of the radio constraints, this problem is trivial, and solved by setting $m_i = 1$, $n_i = 2$ and $\sigma_s = \frac{n(n-1)}{2}$, which is the maximum possible value, where each node will use $n-1$ channels (i.e., participates in $n-1$ subgraphs). However, the radio constraints necessitate that each node can use at most $r \leq n-1$ channels. Therefore, the total number of nodes in all the subgraphs cannot exceed r occurrences of each of the n nodes, or more precisely, $n_i \sigma_s \leq nr$. Substituting for n_i in the previous inequality with $n_i = \frac{1+\sqrt{1+8m_i}}{2} = \frac{1+\sqrt{1+\frac{4n(n-1)}{\sigma_s}}}{2}$ and proceeding as shown below

$$\frac{1+\sqrt{1+\frac{4n(n-1)}{\sigma_s}}}{2} . \sigma_s \leq nr$$

$$1 + 4\frac{n(n-1)}{\sigma_s} \leq \left(\frac{2nr}{\sigma_s} - 1\right)^2 \tag{4.10}$$

yields

$$\sigma_s \leq \frac{nr^2}{n+r-1} \tag{4.11}$$

Hence, the maximum possible number of subgraphs that satisfy the radio constraints is $\sigma_s = \frac{nr^2}{n+r-1}$. The size and order of each subgraph are:

$$m_i = \frac{1}{2}\frac{n-1}{r}\left(\frac{n-1}{r} + 1\right) = \frac{q(q+1)}{2}, \quad n_i = \frac{n-1}{r} + 1 = q + 1, \quad q = \frac{n-1}{r} \tag{4.12}$$

Given m_i and n_i in (4.12), the optimal CA decomposes K_n into σ_s complete graphs K_{q+1}. If such a decomposition exists, we can construct the optimal solution. $\square$

In the remainder of this chapter, we denote the number of channels that leads to optimal interference in $K_n(V, E, r, .)$ by $\bar{\sigma}(n, r)$ and call it the characteristic number of channels for a given n and r.

Proposition 3, thus, shows that the existence of a K_{q+1}-decomposition of K_n, where $q = \frac{n-1}{r}$ is a sufficient condition for the knowledge of $\bar{\sigma}(n, r)$. The next step is to identify the sufficient conditions for the existence of such decomposition. To this end, we rely on the following theorem from [63].

Theorem 2. *[63] Let H be a simple graph with k vertices and m edges with a degree sequence $d_1, d_2, \cdots, d_k$. Then for sufficiently large n satisfying the following two conditions:*

1. $(n-1) \equiv 0 \mod D$, *where D is the greatest common divisor for $d_1, d_2, \cdots, d_k$,*
2. $n(n-1) \equiv 0 \mod 2m$, *m is the size of H, there exists an H-decomposition of K_n.*

The above two necessary conditions are also sufficient provided that $n > n_0$, where n_0 is a sufficiently large constant.

Applying the result of theorem 2 to proposition 3, we have, by analogy, $D = q = \frac{n-1}{r}$ as all subgraphs are complete, and $m = \frac{(q+1)q}{2} = \frac{(n-1)(n+r-1)}{2r^2}$. Hence, the two conditions in theorem 2 become:

(c1) r divides $n-1$, and

(c2) $n+r-1$ divides nr^2.

As the statement of theorem 2 indicates, the above two necessary conditions (c1) and (c2) become sufficient conditions for large values of n to have a K_{q+1} decomposition and, in turn, to use proposition 3 to find $\bar{\sigma}(n, r)$ as well as the optimal CA.

It should be noted, however, that in $f\text{M}^2$WNs, n is, typically, not large. Thus, it may seem that this fact represents a hurdle against using the results of theorem 2. Fortunately, it has been proved, through various individual efforts [15], that, the two conditions of theorem 2 (and equivalently, (c1) and (c2)) are sufficient for almost all values of n and r of typical $f\text{M}^2$WNs. The following lemma uses these results to identify $\bar{\sigma}(n, r)$ for several values of n and r of interest.

Lemma 3. *For a $f\text{M}^2$WN, $K_n(V, E, r, .)$, where n and r satisfy the relations denoted in Table 4.1, the optimal CA uses exactly the corresponding $\bar{\sigma}(n, r)$ values.*

Table 4.1: Exact values for $\bar{\sigma}(n, r)$

n	r	$\bar{\sigma}(n, r)$
all n	$n - 1$	$\frac{n(n-1)}{2}$
$n \equiv 1$ or $3 \pmod 6$	$\frac{n-1}{2}$	$\frac{n(n-1)}{6}$
$n \equiv 1$ or $4 \pmod{12}$	$\frac{n-1}{3}$	$\frac{n(n-1)}{12}$
$n \equiv 1$ or $5 \pmod{20}$	$\frac{n-1}{4}$	$\frac{n(n-1)}{20}$
$n \equiv 1$ or $6 \pmod{20}$	$\frac{n-1}{5}$	$\frac{n(n-1)}{30}$
$n \equiv h^2 + h + 1$, $h = 1, 2, \cdots$	$\frac{n-1}{h}$	$r^2 - r + 1 = n$

Proof. We can provide a sketch of the proof for lemma 3 as follows. The results of the above lemma are obtained by mapping equivalent results in graph decomposition. We provide the following examples: when $r = n - 1$, where clearly there are enough radio interfaces to assign a channel to each link and hence the number of channels is equal to $\frac{n(n-1)}{2}$. To prove the relation in the second row of Table 3, let $n \equiv 1$ or $3 \pmod 6$ it has been shown that K_3 decomposes K_n [15]. This is equivalent to finding a CA for K_n with subgraphs, representing different channels, each with degree $q = 2$, i.e., for every $\frac{n-1}{r} = 2$, or equivalently, $r = \frac{n-1}{2}$. We can then calculate $\bar{\sigma}(n, r)$ using proposition 3. To prove the last relation in Table 3, we use the theorem of De Bruijn and Erdos [14] which states that one can decompose K_n into subgraphs that are all complete and of the same degree, if and only if K_n is partitioned into exactly n complete subgraphs each with some degree h. They showed that such a decomposition can be attained if and only if n can take the form $n = h^2 + h + 1$. Their theorem accords with proposition 3 by setting $h = (n - 1)/r$ and obtaining $\bar{\sigma}(n, r) = n = r^2 - r + 1$ (as for the case of K_7 in Figure 4.3). The other relations in Table 3 can be proved in a similar manner. □

Finally, we use the following lemma to show that the optimal solutions obtained in proposition 3 and lemma 3 are always fair solutions.

Lemma 4. *The optimal solutions satisfying proposition 3 or lemma 3 are always fair.*

Proof. It is straightforward to show that since all the resulting subgraphs in the decomposition of proposition 3 are regular decompositions with degree $\frac{n-1}{r}$ then the resulting solution provides the minimum value for Equation (4.7). In other words, there is no other solution that can further minimize the local interference for any node $v \in V$ and hence this solution achieves fairness. □

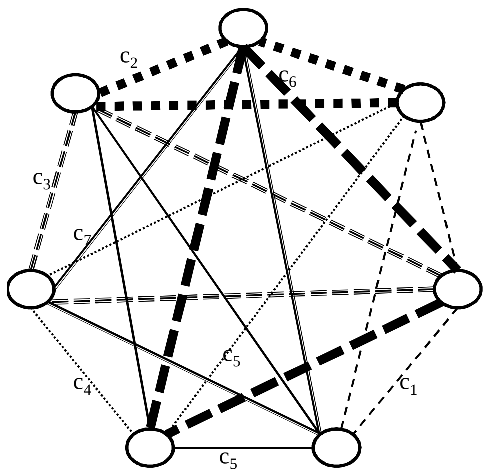


Figure 4.3: Optimal CA for K_7, $r = 3$ with $\bar{\sigma}(n, r) = r^2 - r + 1$

4.3.5 Bounds for $\bar{\sigma}(n, r)$ for general n and r

This section addresses the general case where n and r do not satisfy the conditions (c1) and (c2). This case has been characterized in graph theory by Werra et al. [22] (Theorem 7.1), where it was shown that K-edge partitioning of a complete graph (or equivalently regular decomposition) is NP-complete when the number of subgraphs $k \geq 3$. This means that the general CA problem for a fM^{2}WN is NP-complete if the number channels exceeds three. Nonetheless, the results obtained in the previous section can be used to obtain upper and lower bounds on the characteristic channel number as shown in the next proposition.

Proposition 4. *The optimal CA for a fM^{2}WN, $K_n(V, E, r, .)$, uses $\bar{\sigma}(n, r)$ channels, such that*

$$\frac{r^2 + r}{2} \leq \bar{\sigma}(n, r) < r^2 \tag{4.13}$$

Proof. Consider a fM^{2}WN, $K_{n=r+1}(V, E, r, .)$, with $r + 1$ nodes. From lemma 3, $\bar{\sigma}(r + 1, r) = \frac{n(n-1)}{2} = \frac{(r+1)r}{2} = \frac{r^2+r}{2}$. In other words, one channel is dedicated to each link. For example, in K_2, each node has one radio and can use one channel; similarly, in K_3 there exists three nodes with $r = 2$ radios, and each node can use two channels, etc. Since $r \leq n - 1$, $K_{n=r+1}$ is the smallest fM^{2}WN to use r radios and for any $K_{\acute{n} \geq n}(V, E, r, .)$, we have $\bar{\sigma}(\acute{n}, r) \geq \bar{\sigma}(n, r)$.

Now, to obtain an upper bound, assume that theoretically we can have a fM^{2}WN with $n > n_0$, n_0 is exponentially large and n satisfy the necessary conditions in Theorem 2 for a given r. We can guarantee the existence of K_{q+1} decomposition of K_n and we obtain $\bar{\sigma}(n,r) = \frac{nr^2}{n+r-1}$ by proposition 3. Taking the limit of $\bar{\sigma}(n,r)$ as $n \longrightarrow \infty$, and since, $\bar{\sigma}(n,r)$ is nondecreasing in n, we obtain $\bar{\sigma}(n,r) < r^2$. □

The above results indicate that $\bar{\sigma}(n,r)$ is bounded as a function of r. This lemma represents some interesting results for typical IEEE802.11a and b/g, where their standards provide 12 and 3 orthogonal channels, respectively. Since most commercial routers are usually equipped with $2-3$ radios, the above lemma indicates that not all the available channels should be used. For example, when $r = 2$, we use 3 channels for any $n \geq 3$ for optimal interference. Similarly, for $r = 3$, we achieve the minimum interference by using at most 8 channels. On the other hand, using the results of proposition 3, we know that for $r = 4$, 12 channels are not sufficient to guarantee minimum interference for all $f\mathrm{M}^2$WNs with any n.

We can still improve the above bounds using the following proposition:

Proposition 5. *The optimal CA for fM²WN $K_n(V,E,r,.)$ uses $\bar{\sigma}(n,r)$ such that*

$$\bar{\sigma}(n_1,r) \leq \bar{\sigma}(n,r) \leq \bar{\sigma}(n_1,r) + \frac{r\ell(r-1)}{(q+1)(q+\ell+1)} \tag{4.14}$$

where n_1, q and ℓ satisfy the following two conditions:

1. *n_1 is the largest number $n_1 < n$ such that $n_1 - 1 = rq$ and $n_1 + r - 1$ divides $n_1 r^2$.*
2. *n_2 is the least number $n_2 \geq n$ such that $n_2 - 1 = r(q+\ell)$ and $n_2 + r - 1$ divides $n_2 r^2$.*

Proof. An optimal CA for $K_{n_1}(V,E,r,.)$ must satisfy proposition 3. Hence, the size of any subgraph in such a solution must be $m_{i1} = \binom{q(q+1)}{2}$, the smallest graph with degree q. For $n > n_1$, the resulting subgraphs for each channel must be of size $m_i \geq m_{i1}$, as n increases from $n_1 + 1$ to n_2, the complete subgraphs become regular ones with the same degree and we have $\bar{\sigma}(n_1,r) \leq \bar{\sigma}(n,r)$.

Similarly, for $K_{n_2}(V,E,r,.)$, m_{i2}=$\binom{(q+\ell)(q+\ell+1)}{2}$ is the number of links of the smallest subgraphs with degree $(q+\ell)$. Removing one node from K_{n_2}, reduces r complete subgraphs $K_{q+\ell+1}$ into $K_{q+\ell}$ while leaving $\bar{\sigma}(n_2,r) - r$ complete subgraphs $K_{q+\ell+1}$ and the links must be rearranged to obtain balanced decompositions resulting in $\bar{\sigma}(n_2-1,r)$= $\bar{\sigma}(n_2,r)$ or $\bar{\sigma}(n_2,r) - 1$. Hence, $\bar{\sigma}(n,r) \leq \bar{\sigma}(n_2,r)$. We can recursively remove one node at a time from the resulting graph up to n_1 nodes, such that the resulting subgraphs

graphs have degree $q \leq n_i - 1 \leq (q+\ell)$, with $\bar{\sigma}(n-i,r) \leq \bar{\sigma}(n_2,r)$, $i = 1\cdots, n_2 - n_1$. Let $\bar{\sigma}(n_2,r) = \bar{\sigma}(n_1,r) + \delta$. Proceeding with the manipulation: $\delta = \frac{n_2 r^2}{n_2+r-2} - \frac{n_1 r^2}{n_1+r-2}$, substituting for $n_1 = rq+1$ and $n_2 = r(q+l)+1$ and simplifying, we obtain

$$\delta = \frac{r\ell(r-1)}{(q+1)(q+\ell+1)} \tag{4.15}$$

and finally, substituting for δ in $\sigma(n,r) \leq \sigma(n_1,r)+\delta$, we prove the proposition. □

Table 4.2 demonstrates upper and lower bounds for $\bar{\sigma}(n,r)$ for various values of n, r.

Table 4.2: Exact values and upper and lower bounds for $\bar{\sigma}(n,r)$ various values of n, r

n	3	4	5	6	7	8	9	10	11	12	13	14
$r=2$	3	3	3	3	3	3	3	3	3	3	3	3
$r=3$	-	6	6	6	7	7,8	7,8	7,8	7,8	7,8	7,8	7,8
$r=4$	-	-	10	10,12	10,12	10,12	12	12,13	12,13	12,13	13	13,14

4.3.6 Optimal CA for a given number of channels k

Thus far, we were mainly interested in identifying the minimum interference in the problem of $K_n(V,E,r,.)$ when the number of channels is not given or restricted. Now, we consider the case where the number of available channels is k. This case could be addresses using the next corollary.

Corollary 1. *The optimal interference for a fM^{2}WN, $K_n(V,E,r,k)$, uses* $\min(k,\bar{\sigma}(n,r))$ *channels.*

Proof. When $\bar{\sigma}(n,r) < k$, proposition 3 states that the optimal interference uses $\bar{\sigma}(n,r)$ channels and no increase in the number of channels is possible due to the radio constraints. On the other hand, when $\bar{\sigma}(n,r) > k$ we use k channels and proposition 1 indicates that the minimum interference is obtained by simply equally dividing all the links on the channels in a balanced decomposition. □

4.4 Bounds on fM^{2}WN minimum interference

In the previous section, we identified the number of channels $\bar{\sigma}(n,r)$ necessary to achieve the optimal interference. In this section, we use $\bar{\sigma}(n,r)$ to derive the optimal values for channel interference.

Lemma 5. *The minimum interference for a fM^{2}WN, $K_n(E,V,r,.)$, is*

$$I_t(S^*(K_n)) \geq \frac{m(m-\bar{\sigma}(n,r)) + p(\bar{\sigma}(n,r)-p)}{2\bar{\sigma}(n,r)} \tag{4.16}$$

where p is the remainder of $m/\bar{\sigma}(n,r)$.

Proof. Proposition 1 shows that the optimal CA results in subgraphs with sizes $m_i = \left\lceil \frac{m}{\bar{\sigma}(n,r)} \right\rceil$ or $\left\lfloor \frac{m}{\bar{\sigma}(n,r)} \right\rfloor$. Substituting for m_i in (4.3), we have:

$$I(S^*(K_n)) = \frac{1}{2}(p\left\lceil \frac{m}{\bar{\sigma}(n,r)} \right\rceil^2 + (\bar{\sigma}(n,r)-p)\left\lfloor \frac{m}{\bar{\sigma}(n,r)} \right\rfloor^2 - m) \tag{4.17}$$

Substituting $(\frac{m}{\bar{\sigma}(n,r)} + \frac{(\bar{\sigma}(n,r)-p)}{\bar{\sigma}(n,r)})$ for $\left\lceil \frac{m}{\bar{\sigma}(n,r)} \right\rceil$ and $(\frac{m}{\bar{\sigma}(n,r)} - \frac{p}{\bar{\sigma}(n,r)})$ for $\left\lfloor \frac{m}{\bar{\sigma}(n,r)} \right\rfloor$, and simplifying $I(S^*(K_n))$ produces the results. □

We can also obtain a closed form for the value of the optimal fractional interference as follows.

Corollary 2. *The optimal fractional interference for fM^{2}WN $K_n(V,E,r,.)$ is always less than $\frac{1}{\bar{\sigma}(n,r)}$.*

Proof. Let $n > n_o$ such that proposition 3 is satisfied, then we have:

$$\begin{aligned} I_t^f(S^*(K_n)) &= I_t(S^*(K_n))/\binom{m}{2} \\ &= \frac{(m-p)(m-\bar{\sigma}(n,r)+p)}{\bar{\sigma}(n,r)m(m-1)} \end{aligned} \tag{4.18}$$

Taking the limit of $I_t^f(S^*(K_n))$ as $m \to \infty$ and since it is non-decreasing in m, then $I_t^f(S^*(K_n)) < \frac{1}{\bar{\sigma}(n,r)}$. □

4.4.1 Bounds on fM^{2}WN minimum intra-interference

In this section, we also derive bounds for the total minimum intra-interference which we will use as a measure of fairness for the developed CA scheme in Section 4.5. This interference value is the direct result of the radio constraints. A closed form for the minimum interference can be derived as follows.

Lemma 6. *The minimum intra-interference in a CA for fM^{2}WN $K_n(V, E, r, .)$ is*

$$I_i(S^*(K_n)) = \frac{n(n-l-1)(n+l-r-1)}{2r} \tag{4.19}$$

Proof. Using proposition 2, we have $I_i(v) = l \begin{pmatrix} \lceil \frac{n-1}{r} \rceil \\ 2 \end{pmatrix} + (r-l) \begin{pmatrix} \lfloor \frac{n-1}{r} \rfloor \\ 2 \end{pmatrix}$. Substituting $(\frac{n-1}{r} + \frac{n-l}{r})$ for $\lceil \frac{n-1}{r} \rceil$ and $(\frac{n-1}{r} - \frac{\ell}{r})$ for $\lfloor \frac{n-1}{r} \rfloor$ and let $I_i(S^*(K_n)) = nI_i(v)$, we get the results. □

4.5 Proposed Channel assignment scheme

In this section, we develop an algorithm that achieves near-optimal values for the total network interference. As indicated in the introduction, most applications of fM^{2}WNs impose stringent delay requirements and may experience node failures. Hence, an appropriate scheme must take these two requirements into consideration. Constructing an optimal CA is not a trivial task, even when knowing $\bar{\sigma}(n, r)$. In fact, the authors in [16] have shown that it is not always possible to construct an almost regular decomposition for K_n and stated some necessary, but not sufficient, conditions for its existence. As a result, there is a tradeoff between the CA algorithm time complexity and optimality. In this section, we develop a polynomial time CA algorithm that requires a linear number of control messages. The algorithm assumes the existence of a centralized node that is expected to be more secure and less nomadic than other nodes, we refer to this node as the master node. Otherwise, this node can be selected dynamically as follows: when a node detects changes (e.g., new or failed neighbors or changes in neighbors channel set), the node broadcasts a control message indicating that it is performing the CA algorithm again and claims the role of the master node. The scheme uses $\sigma_s = \frac{r^2+r}{2}$ channels (for example, when $r = 2$, the scheme uses $\sigma_s = 3$ channels, similarly, for $r = 3$ we have $\sigma_s = 6$ channels). The rationale behind this is to speedup the CA process by making the number of used channels independent of the number of currently active nodes, and hence, omit the phase of calculating the optimal number of channels $\bar{\sigma}(n, r)$. As will be shown through the performance evaluation results, the resulting deviation from the optimal interference is not significant for $r = 2$ for any n and for $r = 3$ with small values for n.

The CA scheme works as follows, the master node broadcasts a message indicating that a new CA is being performed and probes existing nodes to acknowledge this message. This message exchange can either use a default channel or can be broadcast on all available channels similar to the approach presented in [50]. When the master node receives an acknowledgement from all its neighbors, it uses Algorithm 6, shown below, to compute the appropriate CA. Finally, the node broadcasts a message with the new CA. The following section describes the CA algorithm in more details.

4.5.1 Proposed Channel Assignment algorithm

Algorithm 6 constructs a CA solution by partitioning K_n into $\frac{r^2+r}{2}$ subgraphs. The algorithm starts at the master node v and constructs r complete graphs adjacent to v. Of these complete graphs, l graphs, where $l = n - 1 \mod r$, use $\lceil \frac{n-1}{r} \rceil$ links adjacent to v as well as all the links between all the vertices connected to v using these links. Similarly, each of the remaining $r - l$ complete graphs uses $\lfloor \frac{n-1}{r} \rfloor$ links adjacent to v and all the links connecting the vertices on their other end. Each of the constructed complete graphs is assigned a new channel. Hence, the intra-interference for node v is minimized by equally distributing its links over r channels.

Once these complete subgraphs are constructed, the remaining unassigned links will have endpoints belonging to different complete subgraphs. These links are assigned new channels by constructing almost-regular subgraphs that span the nodes of a pair of the previously constructed complete graphs. In other words, to construct a new subgraph (channel), the vertices of two of the already constructed complete graphs, excluding v, are selected as the vertices for this subgraph. Then, the links that span these two complete graphs, excluding those adjacent to v, are selected as the links for the new subgraph.

4.5.2 Proposed scheme Complexity

To compute the time complexity of Algorithm 6, we note that the algorithm assigns a channel for each edge sequentially resulting in $\mathcal{O}(|E|)$ complexity (quadratic in n). On the other hand, the message exchange between the master node and other nodes is linear

Algorithm 6 Almost regular Channel Assignment scheme for $K_n(V, E)$

let $v \in V$ be the master node
// *Assign the first* r *channels*
Partition $V - \{v\}$ into disjoint sets $V_1, V_2, \cdots, V_r$ such that $||V_i| - |V_j|| \leq 1 \; \forall \; i, j \in \{1, \cdots, r\}$
Construct complete subgraphs $K^{(1)}_{q+1}(V_1 \cup \{v\}, E_1)$, $K^{(2)}_{q+1}(V_2 \cup \{v\}, E_2)$, $\cdots$, $K^{(r)}_{q+1}(V_r \cup \{v\}, E_r)$ such that $q \in \{\lfloor \frac{n-1}{r} \rfloor, \lceil \frac{n-1}{r} \rceil\}$
for each i assign a new channel to E_i, $i = 1, \cdots, r$
// *Assign the remaining* $\binom{r}{2}$ *channels*
for $i = 1 \leftarrow r$ **do**
  for $j = i + 1 \leftarrow r$ **do**
    Construct a graph $G_{ij}(V_{ij}, E_{ij})$ s.t. $V_{ij} = V_i \cup V_j$ and $E_{ij} = \{e(v_x, v_y) | v_x \in V_i \wedge v_y \in V_j\}$
    Assign a new channel to E_{ij}
  end for
end for

$\mathcal{O}(n)$.

4.5.3 Relation to optimal CA

The solution $S^A(K_n))$, constructed by Algorithm 6, decomposes K_n into r complete subgraphs and $\frac{r(r-1)}{2}$ regular subgraphs. These $\frac{r^2+r}{2}$ subgraphs, each representing a channel, are either complete or almost-regular ones with degrees $\lceil \frac{n-1}{r} \rceil$ and $\lfloor \frac{n-1}{r} \rfloor$. Hence, the algorithm is fair according to proposition 2. The following lemma establishes a relationship between the interference resulting from the CA using Algorithm 6 and the optimal one.

Lemma 7. *Algorithm 6 has an approximation factor* $\alpha(n, r) = \frac{I_t(S^A(K_n))}{I_t(S^*(K_n))} < \frac{2r-1}{r}$ *of the optimal CA.*

Proof. The solution $S^A(K_n))$, constructed by Algorithm 6, has r complete graphs K_{q+1}, $q = \frac{n-1}{r}$ and $\frac{r(r-1)}{2}$ subgraphs with degree q and order $2q$. hence, the resulting total interference is given by:

$$I_t(S^A(K_n)) = r \begin{pmatrix} \frac{q(q+1)}{2} \\ 2 \end{pmatrix} + \frac{r(r-1)}{2} \begin{pmatrix} q^2 \\ 2 \end{pmatrix} \tag{4.20}$$

Using lemma 5, the inverse of optimal interference is $\frac{1}{I_t(S^*(K_n))} \leq \frac{2\bar{\sigma}(n,r)}{m(m-\bar{\sigma}(n,r))+p(\bar{\sigma}(n,r)-p)}$. Substituting for $I_t(S^A(K_n))$ and $I_t(S^*(K_n))$ in $\alpha(n,r)$, we obtain

$$\alpha(n,r) = \frac{I_t(S^A(K_n))}{I_t(S^*(K_n))} \leq \frac{2\bar{\sigma}(n,r)}{m(m-\bar{\sigma}(n,r))+p(\bar{\sigma}(n,r)-p)} \cdot \frac{qr(q-1)(2qr-q+2)}{8} \quad (4.21)$$

and taking the limit of $\alpha(n,r)$ as $n \to \infty$, we get $\alpha(n,r) < \frac{2r-1}{r}$. □

Hence, the resulting interference by the Algorithm 6 is at most $2 - \frac{1}{r}$ of the optimal interference. Note, that this is the worst case performance of the proposed algorithm, particularly, when $\bar{\sigma}(n,r)$ approaches r^2. As will be shown in our evaluation, $\alpha(n,r)$ is typically much smaller.

4.6 Empirical Evaluation

In this section, we compare the derived theoretical bounds against the results obtained from an exhaustive search procedure for the optimal CA. We also analyze the performance of the developed scheme.

4.6.1 Verifying derived bounds on $I_t^f(S^*(K_n))$

Due to the size of the solution space, an exhaustive search could not be performed beyond $n = 10$; Figures 4.4 and 4.5 plot the obtained optimal interference using an exhaustive search procedure against the theoretically derived values for the case of two and three radios, respectively. The theoretical values are obtained from (4.19) and (4.18), for the intra- and fractional interference, respectively, with $\bar{\sigma}(n,r)$ obtained from the corresponding lower bounds in Table 4.2.

Two remarks can be drawn from the figures; the first is that, the optimal interference obtained through the exhaustive search does not always coincide with the theoretical lower bounds (e.g., for $n = 4$ with two radios and $n = 5$ with three radios), this is attributed to the nonexistence of an almost regular and balanced decomposition of the complete graph in these cases. The second remark is that the intra-interference values obtained through both the theoretical and exhaustive search are identical.

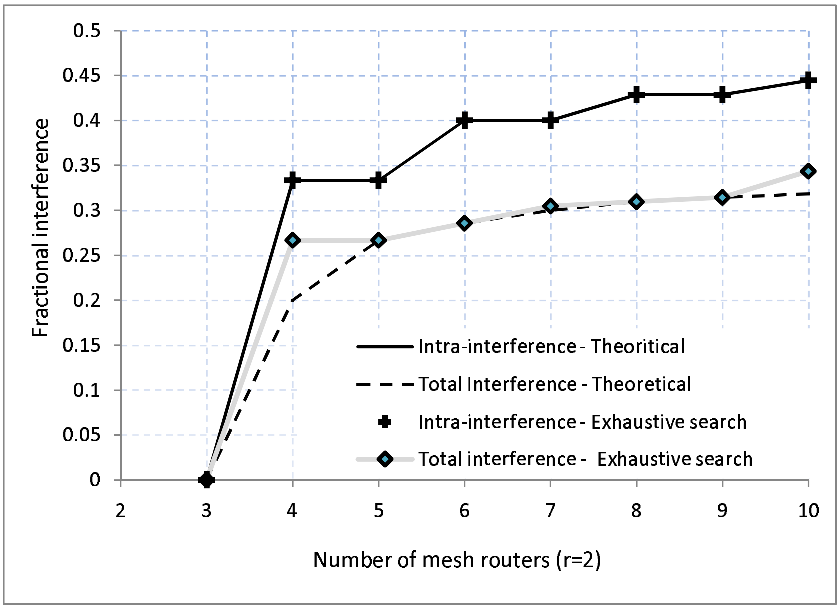


Figure 4.4: Exhaustive search results vs. obtained ($\bar{\sigma}(n,r) = 3$) theoretical bounds for two radios.

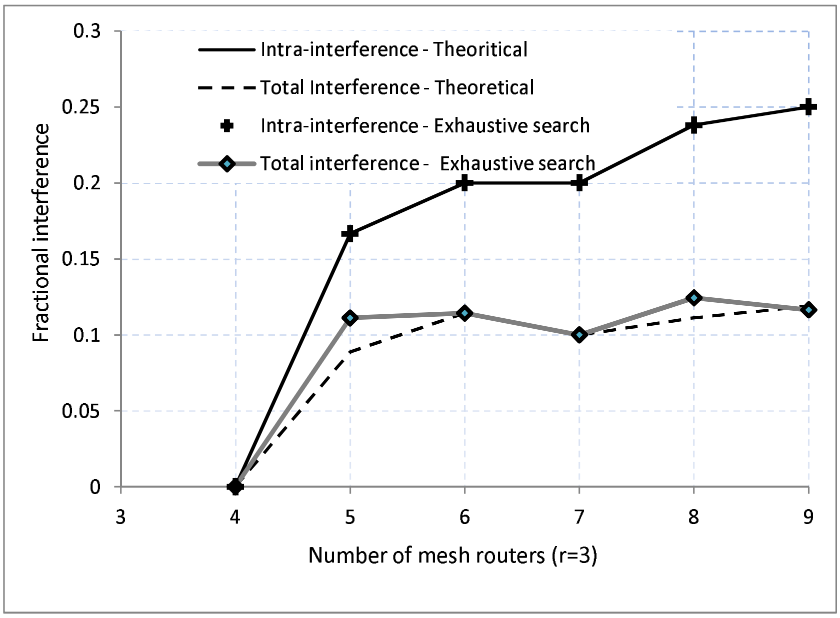


Figure 4.5: Exhaustive search results vs. obtained theoretical bounds when $\bar{\sigma}(n,r) = 6$ and $\bar{\sigma}(n,r) = 8$ for three radios.

4.6.2 Theoretical bounds on the fractional interference

Figures 4.6 and 4.7 provide the theoretically derived fractional interference for $r = 2, 3$, respectively. To confirm our derived bounds in corollary 2, we use proposition 4, stating that $\frac{1}{\bar{\sigma}(n,r)} \leq \frac{2}{r^2+r}$ to obtain $I_t^f(S^*(K_n)) \leq \frac{2}{r^2+r}$. As shown in the figures, the fractional interference approaches $\frac{2}{r^2+r} \simeq 0.333$, 0.1667 for $r = 2$, 3, respectively.

4.6.3 Performance of the proposed CA scheme

Figures 4.6 and 4.7 also compare the fractional interference resulting from the proposed CA scheme for 2 and 3 radios, respectively, against the theoretical bounds, while increasing n up to 100 nodes. As shown, the developed scheme achieves the optimal values for the intra-interference and near-optimal total interference. Figure 4.8 plots the approximation factor $\alpha(n, r)$ of Algorithm 6 for the theoretical interference values. As shown in the figure, when $r = 2$, the algorithm has an approximation $\alpha(n, 2) < 1.15$ of the theoretical interference value, since the number of channels used by the algorithm is the same as the characteristic channel number. For $r = 3$, the resulting interference values deviate from the theoretical ones as n increases. This is attributed to the increasing gap between the number of channels used by the algorithm (6 channels) and the characteristic channel number $\bar{\sigma}(n, r)$ (sample values are shown in Table 4.2). The increase in the interference occurs at each transitional increase for $\bar{\sigma}(n, 3)$, particularly at $n = 7$ where $\bar{\sigma}(7, 3) = 7$ and at $n = 16$ where $\bar{\sigma}(16, 3) = 8$. As shown in the figure, $\alpha(n, 3)$ is always less than 1.5.

4.7 Summary

In this chapter, we studied the channel assignment (CA) problem for full multi-interface multi-channel wireless networks. We have shown that the maximum number of channels that can be used in such a network is dependent on the number of radio interfaces for an optimal channel assignment. Lower and upper bounds for the optimal number of channels as well as for the minimum interference values were derived. Empirical evaluation results verified the theoretically obtained bounds. Finally, we have developed a polynomial-time CA scheme that achieves near-optimal interference values.

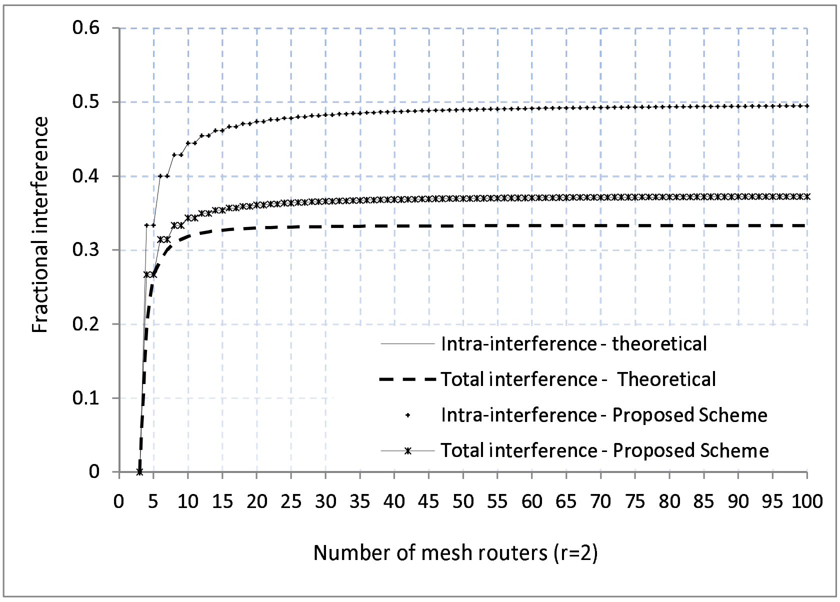


Figure 4.6: Fractional Interference obtained using Algorithm 6 plotted against the theoretical bounds for two radios.

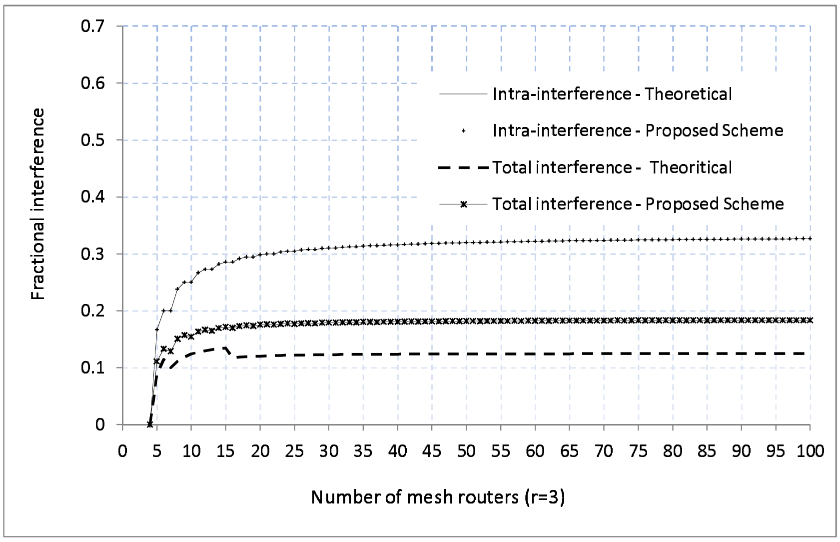


Figure 4.7: Fractional Interference obtained using Algorithm 6 plotted against the theoretical bounds for three radios.

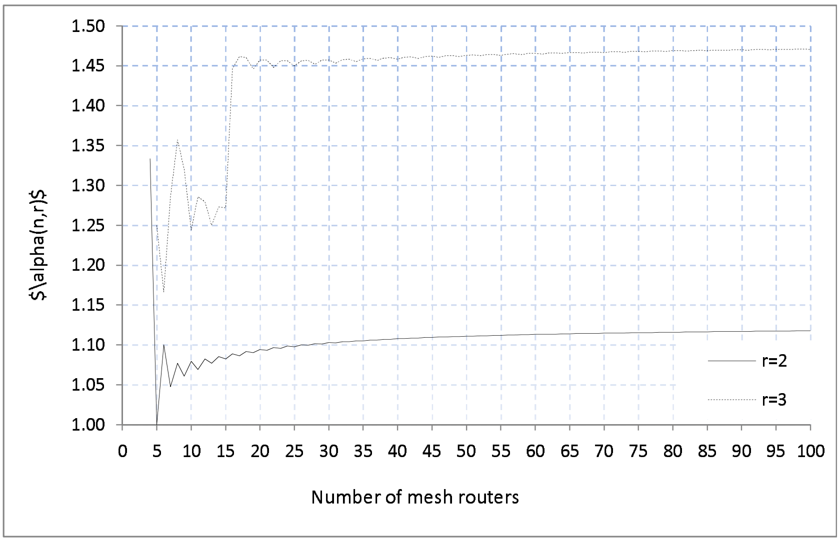


Figure 4.8: The approximation factor of Algorithm 6.

Chapter 5

Conclusions and Future Work

In this thesis, we have addressed the problem of channel assignment (CA) in multi-channel multi-interface wireless mesh networks (M^2WNs). We first considered the problem for general topologies, and later we addressed the special case of fully connected networks.

For general topologies, we have analyzed the characteristics of the solution space and the effects of the constraints inherent to the problem, i.e., the radio constraint and number of channels constraint, on the solution space. We have shown that the search space contains unfeasible solutions and a high degree of redundancy, and that it is therefore possible to significantly reduce said solution space before solving the problem. Based on our analysis, we have developed a novel scheme that transforms, in a first step, the CA problem into a much smaller unconstrained weighted graph coloring problem, successfully avoiding all the unfeasible solutions. Then, in a second step, this reduced problem is solved using a graph coloring algorithm. The achieved reduction of the search space was illustrated for some random topologies, where it is shown that this reduction is more evident the higher the constrained the graph is, i.e., lower number of radio interfaces and denser networks scenarios. Other performance evaluations illustrated the efficiency of the proposed scheme, including estimation of the interference reduction and network throughput, performing simulations using Network Simulator 3 [44].

However, some improvements must be addressed in future work:

- A good solution structure must be found in the first step of the algorithm, ideally a solution structure that includes optimal solutions, a task that is more difficult for such highly constrained graphs. We have developed two approaches and compared their results, but this area is still subject to further investigation.

- Another interesting problem is to develop an algorithm that can update an existing solution structure. This will allow to have a CA scheme that can respond to topology changes still using previous CA results.

- Another area that must be investigated is including non-orthogonal channels, therefore avoiding simplifications to the interference model.

- The presented solution considers traffic scheme, but it is a static CA scheme, i.e., does not react to traffic changes. Therefore, an interesting future work is to consider such changes.

An analysis of the problem for the special case of full networks was also driven. We showed that the maximum number of channels in an optimal CA depends on the number of radios. We developed lower and upper bounds for the optimal number of channels and interference values. Additionally, a polynomial algorithm that finds near-optimal solutions was developed. These theoretical results were verified with empirical evaluations by comparing to exhaustive search when possible.

Future research work involves:

- Developing a lower bound for the optimally reduced solution spaces.

- Investigating the effects of varying several constraints, such as the use of overlapping channels on the solution space, and non-uniform link traffic.

- Topologies that are almost fully connected networks can be also solved by this approach. An interesting research work is to find in which cases this scheme is still appropriate and when it is not, establishing a boundary between the two schemes presented in this work.

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