Indirect Tax Reforms and Poverty in KwaZulu-Natal

By Yang Zhou

Student Number: 4850290

Major paper presented to the
Department of Economics of the University of Ottawa
in partial fulfillment of the requirements of the M.A. Degree

Supervisor: Professor Paul Makdissi

ECO 7997

Ottawa, Ontario

August, 2009

Indirect Tax Reforms and Poverty in KwaZulu-Natal

Abstract

This paper applies the methodology of the normalized consumption dominance curve derived by Makdissi and Wodon (2002) and improved by Duclos, Makdissi and Wodon (2008) to identify whether an indirect tax reform would reduce poverty and improve social welfare in the KwaZulu-Natal province of South Africa. The study utilizes the dataset from the KwaZulu-Natal Income Dynamics Study of 2004.

Keywords: Poverty, Social welfare, Indirect tax reform, Normalized consumption dominance curve

I. Introduction

Most governments in both developed and developing countries are always striving to find economic instruments or policies to reduce poverty and promote equality in a social community, while not decreasing government revenues. For example, they make certain essential goods and services exempt from taxes, and impose a high tariff on some imported goods that belong to industries under trade protection in the domestic market. In the last few decades, economists and researchers have proposed many measures to address poverty, social welfare and inequality such as the Lorenz Curves, the S-Gini indices and the Foster-Greer-Thorbecke (FGT) indices. Although these measures are very useful in evaluating the extent of poverty in a society, it is challenging for a nation to set a reasonable and precise poverty line in the process of computing the values of these indices. As a result, researchers have begun developing methods that do not require a poverty line to identify the poverty and the social welfare in a country or region.

Based on the previous results from Yizhaki and Thirsk (1990), Yizhaki and Slemrod (1991) first introduced the concept of concentration curves as a new method for identifying whether tax reforms obey Dalton's principles. This method negates the need for a specified poverty line. The methodology of concentration curves was extended by Mayshar and Yitzhaki (1996). However, considering that concentration curves only apply to Dalton-improving tax reforms, this approach limits the scope of the application in the second order of stochastic dominance. In an attempt to solve this

limitation, Makdissi and Wodon (2002) presented a new methodology of consumption dominance curves for any order of stochastic dominance.

In this paper, the methodology of the normalized consumption dominance curves derived by Makdissi and Wodon (2002) and extended by Duclos, Makdissi and Wodon (2008) is applied to investigate whether an indirect tax reform would reduce poverty and improve social welfare in the KwaZulu-Natal province of South Africa. Using the dataset from the KwaZulu-Natal Income Dynamic Study (2004), a comparison is made between the normalized consumption dominance curves of food expenditures with those of nonfood expenditures at the s^{th} order of stochastic dominance (where s=1, 2, 3, 4). The types of goods that should be taxed and those that should be subsidized in an indirect tax reform are also identified through discussing the normalized consumption dominance curves for pairs of commodities (grains, transport, healthcare and energy) at the first and second orders of stochastic dominance. In addition, an approach to the critical economic efficiency ratios and the critical poverty lines, as presented by Duclos, Makdissi and Wodon (2008), is applied to the data.

Section 2 presents a brief review of the literature on poverty and inequality in the recent decades, particularly those relevant to this study.

Section 3 describes the theoretical framework and methodology of the normalized

consumption dominance curves proposed by Makdissi and Wodon (2002) and developed by Duclos, Makdissi and Wodon (2008).

Section 4 applies the methodology to the dataset from the KwaZulu-Natal Income Dynamics Study of 2004. By comparing the normalized consumption dominance curves on food expenditures with those on nonfood expenditures at different order of stochastic dominance, it is found that subsidizing food is more beneficial to the poorest people. Further investigation using pairs of specified items reveals that a marginal increase in taxes on healthcare or energy expenditure, as opposed to grain or transport expenditure, is socially improving. The above analysis is also combined with a discussion on the corresponding critical economic efficiency ratios and critical poverty lines.

Section 5 concludes this study, and concisely outlines the potential impact of an indirect tax reform on poverty and social welfare in KwaZulu-Natal .

II. Background and Related literature

In early studies, authors only used Lorenz curves, and conventional summary statistics, such as variance, coefficient of variation and Gini coefficients, to measure inequality in income. Little attention was given to the theoretical foundation of the inequality measures. Atkinson (1970) pointed out two main problems when using conventional summary statistics. Firstly, he argued that "the use of these measures

often serves to obscure the fact that a complete ranking of distributions cannot be attained without a fully specifying the form of the social welfare function" (1970). Secondly, he demonstrated that "the properties of the social welfare functions implicit in these measures are unlikely to be acceptable in most cases" (1970). Therefore, he proposed a new approach which directly adopted the form of a social welfare function to measure inequality in terms of equally distributed equivalent income. Atkinson pioneered the modern literature on the measurements of inequality in income distributions.

With respect to poverty, little work had been done on constructing an index of poverty before 1976. Sen (1976) put forward a series of basic axioms related to poverty on the basis of the Monotonicity Axiom and Transfer Axiom. These are Axiom E (Relative Equity), Axiom R (Ordinal Rank Weights), Axiom M (Monotonic Welfare), respectively. He also proposed "a new ordinal approach to measure poverty by comparing the welfare, which avoided some shortcomings of the measures that were widely used at that time" (1976). These axioms made a major contribution to the definition of a general poverty index.

Subsequently, some economists and researchers gained awareness of the relationship between the subgroup poverty and total poverty in a population. Foster, Greer and Thorbecke (1984) presented a simple decomposable poverty measure that "(i) is additively decomposable with population-share weights, (ii) satisfies the basic

properties proposed by Sen, and (iii) is justified by a relative deprivation concept of poverty." The Foster-Greer-Thorbecke (FGT) poverty indices is written specifically as $\bar{P}(z;\alpha)=\int_0^q(\frac{g(p;z)}{z})^{\alpha}dp$, where the parameter α is non-negative, and represents the extent of aversion to poverty. When α=0, $\bar{P}(z;\alpha=0)$ is the simplest and most commonly used poverty index, which is called the poverty headcount ratio index. It implies the proportion of all the individuals in poverty. When $\alpha=1$, $\bar{P}(z;\alpha=1)$ represents the average poverty gap of the population. For $0<\alpha<1$, the FGT poverty indices show a positive relationship with inequality in the distribution of poverty gaps. On the contrary, for $\alpha>1$, these indices exhibit social approval for equality in the distribution of poverty gaps.

Referring to stochastic dominance, this concept has been widely used in the fields of economics and finance, and its definition has been continuously developed and improved in the following decades after 1960. Specifically, Quirk and Saposnik (1962), Fishburn (1964), Hardar and Russell (1969, 1971) gave a plausible interpretation on the first-degree stochastic dominance and the second-degree stochastic dominance. In a further study, Fishburn (1976) explored the generalized n_{th} order of stochastic dominance. According to Fishburn (1976), "stochastic dominance relations are first defined based on fractional integrals of probability distribution functions, and then convex cones of utility functions that are congruent with these relations are identified in the distributions of income. It is noted that this process can be reversed." He gave the initial version of the sufficient condition to improve social

welfare at higher order of stochastic dominance.

Subsequently, Atkinson (1998) pointed out that "most of the existing papers have been written either on poverty or inequality, but rarely on both topics simultaneously." Thus, it is hard to find the ethical links between the measurement of poverty, inequality and social welfare. Duclos and Makdissi (2004) defined the classes of additive poverty indices and the classes of social welfare indices, and attempted to find a relation between two distributions of income, F and G, in terms of poverty and social welfare at every order of stochastic dominance. They examined the difference in absolute poverty indices between the two distributions of income at any order of stochastic dominance, and proposed that "a necessary and sufficient condition for a poverty reduction, for any poverty line $z \in [0, z^+]$ and any absolute poverty index $P \in \Pi^s$, is $D_F^s(y) - D_G^s(y) \geq 0, \forall y \leq z^+$."

Considering the case of social improvement, Duclos and Makdissi (2004) assert that a movement from a distribution F to a distribution G does not decrease social welfare if the difference in the classes of utility functions between the distribution F and distribution G is non-negative. According to Duclos and Makdissi, "a sufficient condition for welfare improvement, for all $U \in \Omega^s$, is

$D_F^s(y) - D_G^s(y) \geq 0, \forall y \in [0, a]$ and if $s \geq 3$

$D_F^i(a) - D_G^i(a) \geq 0, \forall i \in \{2, 3, ..., s-1\}$" (2004).

When the s^{th}-order stochastic dominance curves for the two distributions of income, F

and G, satisfy the above two conditions, a movement from the distribution F to the distribution G is deemed to improve social welfare as well, not only reduce poverty. It should be noted that for s=1 and s=2, the dominance conditions of reducing poverty are identical to that of improving social welfare.[1]

On the other hand, through examining the differences in poverty classes or utility classes for the pairs of distribution functions in terms of the stochastic dominance at a given order, researchers and analysts come up with many ethical principles in the study of the transfers occurring in the distributions of income at different orders of dominance.

Pen (1971) combined the Pareto Principles with symmetry in the distribution of income, and came to a judgement as to the first-order welfare improving tax reform which is known as the Pen-improving tax reform. In addition, the symmetry indicates that interchanging any two individual's income does not have an impact on poverty and social welfare. According to Pen (1971), "ordering two distributions for the first order of dominance is equivalent to making living standards 'parade'." The first order principle not only states that poverty will weakly decrease, but also emphasizes that social welfare will slightly increase when raising an individual's income without any changes in the others' income (Pen, 1971).

[1] Duclo, Jean –Yves, Paul Makdissi (2004) 'Restricted and unrestricted dominance for welfare, inequality and poverty orderings.' *Journal of Public Economic Theory* 6, 145-164

With respect to the second-order judgments on poverty alleviation, David Donaldson and John A. Weymark (1986) first presented that "the strong downward transfer axiom strengthens this condition by further requiring that a transfer from a rich person to a poor person, which makes the latter rich, decreases the value of a poverty index." It forms the Pigou-Dalton principle. Considering the fact that the second-order index of poverty is concave, the Pigou-Dalton principle implies that a mean-preserving transfer of income from a higher income person to a lower income person leads to a poverty reduction. On the other hand, a transfer of income from the richer to the poorer can be deemed to have a positive impact on the social welfare because of the convex cone of the social welfare function. Additionally, Yizhaki and Slemrod (1991) suggested a new methodology to determine whether or not an indirect tax reform with only two commodities was a Dalton-improving tax reform without the need for a specific social welfare function. Mayshar and Yitzhaki (1995) developed this methodology to identify a Dalton-improving indirect tax reform with multiple commodities.

The third-order stochastic dominance of poverty indices states that poverty will be reduced when a beneficial Pigou-Dalton transfer at the bottom of the distribution is dominant over an adverse Pigou-Dalton transfer at the top of the distribution.[2] It implies a transfer-sensitivity to favourable composite transfers. According to Kolm

[2] Kolm, Serge-Christophe (1976) 'Unequal inequality:II.' *Journal of Economic Theory* 13, 82-111

(1976), "supposing that a Dalton transfer is more beneficial at lower income than at higher income among the individuals with a fixed income difference, the third-order indices of social welfare function are sensitive to favourable composite transfers. The social welfare will be markedly improved." Thus, the tax reforms adhering to the transfer-sensitivity principle are treated as the Kolm-improving tax reforms.

Turning to the generalized higher order transfer principle, Kolm (1976) had briefly noted this idea, but the concept was specifically defined by Fishburn and Willing (1984). They found that "the relative ethical weight assigned to the effect of income changes occurring at the bottom of the distribution increases, as the order of stochastic dominance increases"(1984). As an illustration, the fourth-order indices respond favourably to a combination that consists of two exactly opposite and symmetric composite transfers with the first one being favourable and occurring within a lower part of the distribution, and the second one being unfavourable and occurring within a higher part of the distribution.

Concerning the distributive effects of a tax reform, Yitzhaki and Slemrod (1991) and Mayshar and Yitzhaki (1995) developed a methodology for identifying which specific items are beneficial and which are harmful in an indirect tax reform, known as the concentration curve. As mentioned previously, this methodology does not require a specific social welfare function, which makes it applicable to a wider range of functions. According to Mayshar Yitzhaki, "supposing the government's revenue

neutrality, an indirect tax reform can be regarded as a welfare-improving tax reform when it obeys the Dalton-improving principle" (1995). However, there exists one limitation when using this methodology, that is, the concentration curves can only be employed to the second order of stochastic dominance.

Considering this limitation, Makdissi and Wodon (2002) presented a new graphical tool, the "Consumption Dominance Curve (CD-curve)," to determine the impact of an indirect tax reform on poverty for any order of restricted stochastic dominance, not only for the second-order dominance. They illustrated the use of consumption dominance curves for interprovincial public transport expenditure and medicine expenditure at the second and third order of stochastic dominance, using the dataset from Bolivian survey of 1999.

Duclos, Makdissi and Wodon (2008) extended the methodology of consumption dominance curves proposed by Makdissi and Wodon (2002) to the analysis of social improvement. In Duclos, Makdissi and Wodon's (2008) work, the concept of consumption dominance curves is clearly distinguished from that of normalized consumption dominance curves. Specially, "consumption dominance curves represent the ethically weighted cost of taxing commodities, while normalized consumption dominance curves show the ethically weighted cost of taxing certain commodity as a proportion of its average welfare cost. It can be derived from the consumption dominance curve for certain commodity normalized by the average consumption of

that commodity" (Duclos, Makdissi and Wodon, 2008). According to Duclos, Makdissi and Wodon (2008), the normalized consumption dominance curve can be applied to determine whether an indirect tax reform will reduce poverty and improve social welfare by observing the impact of price changes on commodities. Supposing the vector of pre-reform prices equals the vector of reference prices, Duclos, Makdissi and Wodon (2008) proposed that "a necessary and sufficient condition for a marginal tax reform, $dq_j = -\gamma(\frac{X_i}{X_j})dq_i > 0$, to be s^{th}-order poverty improving, that is, to reduce poverty weakly for all $P(z) \in \Pi^s(z)$, for all $z \in [0, z^+]$ and for a given $s \in \{1,2,3,...\}$, is that $CD_i^s - \gamma CD_j^s \geq 0, \forall y \in z^+$." They also presented that "a sufficient condition for a marginal tax reform, $dq_j = -\gamma(\frac{X_i}{X_j})dq_i > 0$, to be s^{th}-order welfare improving, that is, to increase social welfare weakly for all $W \in \Omega^s$ and for a given $s \in \{1,2,3,...\}$, is that $\overline{CD}_i^s - \gamma \overline{CD}_j^s \geq 0, \forall y \in [0, \infty]$"(2008).

Additionally, Duclos, Makdissi and Wodon (2008) introduced the concept of critical efficiency ratios and critical poverty lines, and defined "the critical efficiency ratio as $\delta^s(z) = \frac{\overline{\mathrm{CD}}_i^s(z)}{\overline{\mathrm{CD}}_j^s(z)}$, which represents the estimated distributive benefit of taxing good j instead of taxing good I". It should be noted that "the distributive benefit ratio tends to be infinite for $\overline{\mathrm{CD}}_j^s(z) = 0$, and thus the critical efficiency ratio $\delta^s(z)$ can written as γ^{++}"(Duclos, Makdissi and Wodon, 2008). When an estimated critical efficiency ratio is greater than the actual efficiency ratio, an indirect tax reform can be considered to be welfare-improving at a given order of stochastic dominance. In practice, policy makers often adopt this method to assess whether tax reforms will

make the poor better off and improve people's living standards. Moreover, Duclos, Makdissi and Wodon (2008) also suggested an alternative method to find out whether poverty is being improved. "When the estimated critical poverty lines are sufficiently high to encompass all plausible poverty line estimates, an indirect tax reform can be deemed to be poverty improving" (Duclos, Makdissi and Wodon, 2008). Policy markers adopt these estimated critical poverty lines to determine whether the poverty line they set is feasible or not. In the empirical part of their work, Duclos, Makdissi and Wodon (2008) applied the methodology of consumption dominance curves, critical poverty lines and critical economic efficiency ratios to analyze the impact of an indirect tax reform in terms of some categories and specific commodities, using the household-data from Mexico's 1996 National Income and Expenditure (ENGIH) Survey.

III. Methodology

Generally, a poverty index aggregates all the individuals' contribution to poverty. Mathematically, the additive poverty indices may be written as follows:

$$P(z) = \int_0^a p(y^E(q, y), z) dF(y)$$

Where F(.) is the cumulative distribution function of nominal incomes within the interval between 0 and a. z represents the poverty line, which is defined in real income space. y^E is the equivalent income, but y represents the nominal income. According to King (1983), "the equivalent income y^E is derived from the indirect utility function $v(q^R, y_E) = v(q, y)$." Thus, it can be expressed in as $y_E = y_E(q^R, q, y)$.

Here, q^R represents the vector of reference prices and q represents the vector of pre-reform prices. What's worth mentioning is that Besley and Kanbur (1988) assumed that p=q for the purpose of convenient proceeds, and applied the concept of equivalent income in their paper of "Food subsidies and poverty alleviations." Therefore, y_E can take the form of $y_E(q, y)$. Additionally, $p(y^E(q, y), z)$ is a function that measures the contribution to total poverty for an individual with an equivalent income y^E. There are two assumptions about the poverty measure in that: (1) it must be differentiable continuously in the range of 0 to a, mathematically expressed as: $(-1)^i p_1^i(y^E(q,y),z) \geq 0, \forall i=1,2,...,s$[3], (2) it is nonnegative for all income lower than the poverty line and equals zero for those incomes above the poverty line (Makdissi and Wodon, 2002).

Consider the classes of aggregated poverty indices $P(z) \in \Pi^s(z)$ as follows:

$$\Pi^s(z) = \left\{ P(z) \middle| \begin{array}{ll} p(y^E, z) = 0 & if\ y^E > z, p(y^E, z) \in \hat{C}^s(z) \\ (-1)^i p^{(i)}(y^E, z) \geq 0 & for\ i = 0,1,...,s \\ p^{(t)}(z, z) = 0 & for\ t = 0,1,...,s-2\ when\ s \geq 2 \end{array} \right\}$$

Where $\hat{C}^s(z)$ is the set of functions that are s-time piecewise differentiable[4] over the range between 0 and z (Duclos and Makdissi, 2004). Specifically, for s=1, the first-order classes of poverty indices obey the Pareto principle that reduces the

[3] Paul Makdissi and Quentin Wodon (2002) 'Consumption dominance curves: testing for the impact of indirect tax reforms on poverty.' *Economics Letters* 75(2), 227-235

[4] When the (s-1)th derivative is a piecewise differentiable function, the function and its (s-2) first derivatives are differentiable everywhere. The $\hat{C}^s(z)$continuity assumption is made for analytical simplicity since it could be relaxed to include indices whose (s-1)th derivatives is discontinuous(and which are therefore not s-time piecewise differentiable).

poverty weakly due to $p^1(y^E, z) \leq 0$. For s=2, the second-order classes of poverty indices comply with the Pigou-Dalton principle that exhibits a greater ethical preference for the poorer than for the richer. To deduce, the s^{th}-order classes of poverty indices undoubtedly conform to the generalized higher-order transfer principles that have been mentioned earlier.

Considering the social welfare functions such that $U = \int_0^a u(y^E) dF(y^E)$. Duclos and Makdissi's (2004) defined the classes Ω^s of the social welfare indices as

$$\Omega^s = \left\{ U \mid u(y^E) \in \hat{C}^s \, and \, (-1)^i u^{(i)} \leq 0, \forall i = 1, 2, ..., s \right\}$$

In a marginal tax reform, taking the first-order partial differential in respect to commodity i and commodity j, the first order total differential equation of poverty indices for an individual with an income y is

$$dp(y^E(q, y), z) = p_i^1(y^E(q, y), z) \frac{\partial y^E(q, y)}{\partial t_i} + p_j^1(y^E(q, y), z) \frac{\partial y^E(q, y)}{\partial t_j} \qquad (1)$$

Next, supposing that the vector of prices before a tax reform is equivalent to the vector of reference prices, $q = q^R$, Besley and Kanbur (1988) adopt Roy's identity to demonstrate that the negative of the demand for commodity i is equivalent to the first order differential in equivalent income in respect to a marginal change in its own tax rate, $\frac{\partial y^E(y, \alpha, q, q^R)}{\partial t_i} \Big|_{q=q^R} = -x_i(y, \alpha, q^R)$. (2)

Concerning the government's revenue neutrality, we define total indirect tax revenue as $R=\sum_{K=1}^{K} t_K X_K(q)$, where X_K is the aggregate total consumption of the K[th] good, namely $X_K = \int_0^{\bar{I}} x_K(y,q)dF(y)$. Considering the government's need for a balanced budget, revenue neutrality is insured by $dR(q)=0$. Mathematically, it can be rewritten as follows;

$$dR(q)=[X_j(q)+\sum_{k=1}^{K} t_k \frac{\partial X_k(q)}{\partial q_j}]dq_j+[X_i(q)+\sum_{k=1}^{K} t_k \frac{\partial X_k(q)}{\partial q_i}]dq_i=0 \qquad (3)$$

Turning to the definition of economic efficiency cost γ. As assumed by Yitzhaki and Slemrod (1991), it states that "a marginal tax reform leads to no change on the government tax revenue." They also presented that holding producer prices constant, the change in the consumer price of commodity j equals the change in the tax rate of commodity j ($dq_j = dt_j$).

Where, $$\gamma = \frac{\frac{X_i+\sum_{K=1}^{K} t_K \frac{\partial X_K}{\partial q_i}}{X_i}}{\frac{X_j+\sum_{K=1}^{K} t_K \frac{\partial X_K}{\partial q_j}}{X_j}} \qquad (4)$$

According to Wildasin (1984), γ can be interpreted as "the differential efficiency cost of raising one dollar of public funds, by taxing the j[th] commodity and subsidizing the i[th] commodity." Furthermore, Yizhaki and Slemrod (1991) argued that the deadweight loss in the economy would be expanded after carrying out an indirect tax reform for $\gamma > 1$.

Substituting equation (4) into equation (3), we obtain the simplified expression about

revenue neutrality that $dq_j = -\gamma(\frac{X_i}{X_j})dq_i$. Furthermore, using equation (2) and (4) into equation (1), the first order normalized consumption dominance curves can be derived. Makdissi and Wodon (2002) find that "$\overline{CD_k^1}(z)$ indicates the ratio of consumption of good k for an individual with income y divided by the aggregate consumption of the good".

To extend the first-order consumption dominance curves to generalized consumption dominance curves for any order of stochastic dominance, consider the generalized stochastic dominance defined by Fishburn (1976) such that

$$D^s(z) = \frac{1}{(s-1)!}\int_0^z [z-y]^{(s-1)} dF(y)$$

According to Duclos, Makdissi and Wodon (2008), consumption dominance curves reflect the changes in stochastic dominance curves affected by changes in prices. Take the first-order derivative on $D^s(z)$ in respect to the tax rate on commodity k,

$$CD_k^s(z) = \frac{\partial D^s(z)}{\partial t_k}\bigg|_{q=q^R} = \begin{cases} x_k(z,q^R)f(z), \ if \ s=1 \\ \frac{1}{(s-2)!}\int_0^z x_k(y,q^R)(z-y)^{s-2}dF(y), \ if \ s=2,3,... \end{cases}$$

As mentioned by Duclos, Makdissi and Wodon (2008), $CD_k^s(z)$ measures the ethically weighted cost of taxing commodity k. But, in this paper, we adopt normalized consumption dominance curves, which are consumption dominance curves for commodity k normalized by the average consumption of that commodity. Mathematically, it can be written as $\overline{CD_k^s}(z) = \frac{CD_k^s(z)}{X_k(q)}$. $\overline{CD_k^s}(z)$ curves represent the ethically weighted cost of taxing commodity k as a proportion of the average welfare

cost[5]. It should be noted that normalized consumption dominance curves for second order of dominance indicates the share in total consumption of commodity k for those individuals holding income less than y. When s=3, 4,..., normalized consumption dominance curves for higher order of stochastic dominance represent that "greater weight is assigned to the shares of those individuals with higher poverty gaps"(Duclos, Makdissi and Wodon, 2008).

Applying "Theorem 1" and "Theorem 2" in Duclos, Makdissi and Wodon's work (2008), we can determine whether a marginal tax reform is s-order poverty improving, or s-order welfare improving.

IV. Empirical Analysis

This section will describe the dataset that is taken from the KwaZulu-Natal Income Dynamics Study of 2004 and present characteristics of the variables that will be used in the analysis. Subsequently, the normalized consumption dominance curves methodology will be applied to the household-level data on food, non-food, and certain specified goods and services. Recommendations will then be made on which commodity should be taxed and which should be subsidized, when a government has a goal of reducing poverty and improving social welfare with revenue neutrality.

[5] Duclos, J. –Y., P. Makdissi, and Q. Wodon (2008) 'Socially improving tax reforms.' *International Economic Review* 49, 1505-1537

1. KwaZulu-Natal Data Description

According to the United-Nation classification, South Africa is identified as a middle-income country with an abundant supply of resources, with well-developed financial, legal, communications, energy, and transport sectors. However, South Africa is still marked as a developing country even though it was ranked 25th in the world in terms of GDP in 2007.[6] There are large differences in income distribution across races, genders and locations. Except for the four economic centers of Cape Town, Port Elizabeth, Durban, and Pretoria/Johannesburg, the other areas are suffering from large income gaps, high unemployment rates, and severe income inequality. Consequently, the vast majority of South Africans are in poverty.

This paper will analyze poverty and social welfare using the methodology of sth order consumption dominance curves, and panel data from the KwaZulu-Natal province in South Africa. These data originate from a collaborative project between the International Food Policy Research Institute, the University of KwaZulu-Natal, the University of Wisconsin-Madison and the South African Labor Development Research Unit at Cape Town University. The 2004 survey only involved Africans and Indians in KwaZulu-Natal, since the population of other groups are negligible. From the third wave of the survey, there are 1,426 observations at the household level.

Firstly, "household size" is a crucial variable in the analysis of normalized

[6] https://www.cia.gov/library/publications/the-world-factbook/rankorder/2001rank.html#

consumption dominance curves. According to Lanjouw and Ravallion (1994), "there is a negative relationship between household size and consumption (or income) per capita in developing countries. In other words, larger families tend to be poorer, especially in developing countries." This variable is created by means of excluding records that show those who are "dead" or "not applicable" to living status, and then collapsing the remaining unedited raw data by household. In this paper, the variable "household size" (hhsize) is composed of the aggregated number of residents and non-residents in one household, who lived in KwaZulu-Natal in 2004.

Secondly, the two variables "aggregated household monthly expenditure on food" (foodexp) and "aggregated household monthly expenditure on non-food" (nfoodexp) will be used to determine the effect of a marginal tax reform on these two categories. The variable "foodexp" can be directly retrieved from the dataset. The "nfoodexp" variable is derived from adding regular non-food expenditure and occasional non-food expenditure together.

Lastly, there will be a discussion of normalized consumption dominance curves for any order of dominance on four kinds of specified commodities, which are grains, transport, healthcare and energy. "Grain" is an essential and necessary good for impoverished individuals in a developing country like South Africa. "Transport" and "energy" are two main components of non-food expenditures. Additionally, expenditure on "health care" is also considered, as it is another indispensible expense

in people's daily lives, due to the alarming prevalence of HIV/AIDS in South Africa.[7] Using these four variables, an investigation can be conducted to determine which specified items should be subsidized and which should be taxed in an indirect tax reform with multiple commodities.

South Africa follows a value-added tax (VAT) regime. A VAT is an indirect tax based on the consumption of goods and services. South Africa first introduced the VAT in 1991 to replace the General Sales Tax. It is presently levied on a very broad base at the standard rate of 14 percent, or zero-rate for low-income households.[8] The government raises revenue by requiring traders or vendors to charge VAT on some taxable supplies of goods and services. It also conducts many tax initiatives to reduce poverty and improve social welfare in South Africa. For example, certain food and non-food goods and services are exempt from VAT.

2. *S^{th}-order Consumption Dominance Curves*

Using the dataset obtained from the KwaZulu-Natal Income Dynamics Study of 2004, the methodology of normalized consumption dominance curves will be applied. The focus will be on an indirect tax reform that increases the marginal tax rate on non-food expenditures and decreases the marginal tax rate on food expenditures. To

[7] The Centre for Acturial Research, The Burden of Disease Research Unit and The Actuarial Society of South Africa (2006, November) 'The demographic impact of HIV/AIDS in South Africa'.

[8] Annual Conference of The International Bar Association (2002, October 24) Address by the Honourable Trevor A Manuel, MP, Minister of Finance.

simplify the analysis, it is assumed that the economic efficiency cost γ=1. This implies that there is no relative economic efficiency advantage in the process of taxing non-food compared to food. Table 1 exhibits the maximum value of a poverty line which can be attained at the first crossing of $\overline{\mathrm{CD}}^{1}_{food}(z)$ and $\overline{\mathrm{CD}}^{1}_{nonfood}(z)$. It also represents the first-order critical poverty line $z_1^+(\gamma)$ at the bottom panel of Table 1 for γ=1, $z_1^+(1)$=R5588.896 per capita per month. The standard error of the sampling distribution of $z_1^+(\gamma)$ is estimated to be 762.09. It can be interpreted that the true value of $z_1(1)$ lies in the interval of poverty lines between R4,826.806 and R6,350.986 at a 5% critical level. In other words, an indirect tax reform decreases poverty weakly for any poverty lines below R4,826.806 per capita per month at a 95% confidence level when marginally increasing the tax rate on non-food expenditures, decreasing the tax rate on food expenditures (or providing a subsidy for food commodities) and holding the government's revenue balanced at the same time.

Figure 1 shows that the normalized consumption dominance curve $\overline{\mathrm{CD}}^{1}_{food}(z)$ always lies above the normalized consumption dominance curve $\overline{\mathrm{CD}}^{1}_{nonfood}(z)$ for any poverty line under a maximum poverty line. Table 1 says that for γ=1 and for the first order of dominance, the maximum poverty line $z_1^+(\gamma)$ equals R5,588.896 per capita per month. It can be concluded that increasing taxes on non-food expenditures and decreasing taxes on food expenditures is first-order poverty improving in KwaZulu-Natal. However, it should be noted that these two $\overline{\mathrm{CD}}^{1}(z)$ curves do intersect at a critical poverty line. That implies that there exists a maximum poverty

line for the first order of dominance. Consequently, the poverty line does not extend infinitely. According to the theory of Pen-improving tax reform[9], this indirect tax reform is deemed not Pen-improving when the efficiency cost $\gamma=1$. In other words, taxing on non-food items and subsidizing food items does not improve social welfare at first-order of ethical dominance with $\gamma=1$. The outcome obtained above is consistent with the outcomes that Duclos, Makdissi and Wodon achieved in their paper "Socially improving tax reforms" (2008). It says that a Pen-improving tax reform is not possible when the efficiency cost $\gamma=1$.

Figure 2 presents the normalized CD curves of order two on food commodities and non-food commodities with the assumption of $\gamma=1$. As shown in figure 2, $\overline{CD}^2_{food}(z)$ always lies above $\overline{CD}^2_{nonfood}(z)$ within the range of income per capita per month between R0 and R15,000. Thus, we can conclude that an indirect tax reform obeys the Pigou-Dalton principle and can be deemed as a Dalton-improving tax reform. Specifically, any transfer of income from a richer person to a poorer person will reduce the poverty weakly. Meanwhile, social welfare will be improved at the second order of dominance in an indirect tax reform where there is a marginal increase in the tax on non-food expenditures and a marginal decrease in the tax on food expenditures. This is because of the absence of the intersection between $\overline{CD}^2_{food}(z)$ and $\overline{CD}^2_{nonfood}(z)$ within the whole range of poverty lines between R0 and

[9] Pen (1971) stated that a first-order tax reform is welfare improving when an individual's income increases with no other's change under the condition that the maximum poverty line extends to infinity.

R15,000 per capita per month.

The focus will now turn to critical efficiency ratios at different order of stochastic dominance. It can be called the distributive benefit ratio as well since it represents the estimated distributive benefit of taxing food items instead of taxing non-food items. The theory of critical efficiency ratios says that if the economic efficiency cost γ for the s^{th} order of dominance is lower than the estimated distributive benefit ratio $\delta^s(z)$ up to a certain point of the maximizing poverty line, an indirect tax reform of marginally increasing the taxes on non-food expenditures and decreasing the taxes on food expenditures is beneficial. Table 1 presents the critical poverty lines $z_s(\gamma)$ for different ratios of economic efficiency costs γ at different orders of dominance. The indirect tax reform is s^{th}-order poverty improving for any poverty lines under the critical value $z_s(\gamma)$. To be specific, referring to the first-order stochastic dominance, there is no critical poverty line only occurring at γ=0.25. Figure 5 further demonstrates this result based on the fact that the first-order distributive benefit ratio is always above the economic efficiency cost $\gamma = 0.25$ within the interval of income between R0 and R15,000 per capita, per month. However, in the other three cases of γ=0.5, γ=1 and γ=1.5, the critical poverty lines do exist. Those figures (1,6,7,8) of the first-order normalized CD curves of food and non-food expenditures distinctly indicate that the larger the economic efficiency cost γ is assumed at the first order, the smaller the corresponding critical poverty line that is estimated.

Referring to the critical efficiency ratio of order two $\delta^2(z)$, Table 1 shows that an indirect tax reform, which complies with the Pigou-Dalton principle, is beneficial for any poverty lines when γ=0.25, γ=0.5 and γ=1 due to the absence of a maximizing poverty line z. However, when $\gamma = 1.5$, Table 1 shows that there is a critical value $z_2(\gamma)$=R5,084.274 per capita per month with a large standard error of 125038.604 at a 95% confidence level. Whereas, for any order greater than two, there is no critical poverty line whenever γ=0.25, γ=0.5, γ=1 or $\gamma = 1.5$. Figure 2, 3 and 4 clearly demonstrate that no intersection exists between the two sth-order normalized CD curves of food and non-food expenditures (where $\overline{CD}^s_{food}(z)$ and $\overline{CD}^s_{nonfood}(z)$, $s \in \{2,3,4\}$) when $\gamma = 1$ in the range of income between R0 and R15,000 per capita, per month. This implies that the distributive benefit ratio $\delta^s(z)$ is always above the economic efficiency cost γ at the sth-order of dominance, where $s > 2$ and in the whole range. We can conclude that an indirect tax reform will narrow the poverty gap and expands social welfare when taxing non-food items and subsidizing food items for any values of poverty lines at the order of stochastic dominance greater than two.

Figure 3 exhibits a Kolm-improving tax reform at the third order of dominance when a government increases the tax on non-food expenditures and provides a subsidy for food expenditures. As shown by the normalized CD curves at the third order for these two goods, $\overline{CD}^3_{food}(z)$ lies above $\overline{CD}^3_{nonfood}(z)$ throughout the whole interval of income R0-R15,000 per capita, per month. In this case, a tax reform is more sensitive to favourable composite transfers occurring at the bottom of income distribution than

unfavourable composite transfers occurring at the top of income distribution. Due to the fact that the critical poverty line does not exist in the x-axis of poverty lines, it can be deduced that the indirect tax reform has the impact of improving social welfare and reducing the poverty gaps at third order, when taxing non-food commodities and subsidizing food commodities.

Figure 4 exhibits the two normalized consumption dominance curves of food and non-food items at the fourth order of dominance for γ=1. They are steeper than those at the third order of dominance for γ=1. This implies that, as the order of dominance increases, more weight is assigned to the effect of income changes occurring at the bottom of the income distribution. Specifically, at the fourth-order of dominance, two exactly opposite and symmetric composite transfers jointly operate when increasing the taxes on the food category and reducing the taxes on the non-food category. The composite transfers occurring within a lower part of income distribution commonly affect more favourably than those composite transfers occurring within a higher part of income distribution. Thus, the combination of the two composite transfers tends to the poverty reduction and the improvement of social welfare. As shown in Figure 4, $\overline{CD}^{4}_{food}(z)$ is located on top of $\overline{CD}^{4}_{nonfood}(z)$ within the whole interval of poverty lines between R0 and R15,000 in the x-axis, and therefore no critical poverty line appears at the fourth order of dominances.

The methodology of consumption dominance curves is equally applicable to certain

specified items. For this approach, the focus will be on "aggregated monthly household expenditure on grain (grains)", "aggregated monthly household expenditure on transport (transport)", "aggregated monthly household expenditure on health care (heacare)" and "aggregated monthly household expenditure on energies (energy)". It is noted that the variable "energy" here is a mixed bundle of essential energies for a household to use (including a household's water and electricity payment).

According to Figure 9, the normalized CD curves of grains, transport, health care and energy expenditures, $\overline{\mathrm{CD}}^{1}_{grain}(z)$ and $\overline{\mathrm{CD}}^{1}_{transpo}(z)$ both lie above $\overline{\mathrm{CD}}^{1}_{health}(z)$ and $\overline{\mathrm{CD}}^{1}_{energy}(z)$ before the crossing of any two CD curves. This illustrates that the tax reform has the effect of reducing poverty at the first order of dominance while having a neutral impact on government revenue, when marginally increasing the subsidies on grain or transport expenditures and financing this through a marginal increase in taxes on health care or energy (electricity and water).

For simplicity, I compute the critical poverty line for any pair of the four items, assuming that the economic efficiency cost γ=1. Table 2 shows that there is at least one critical poverty line with a standard error at a 95% confidence level between any pair of the goods and services at the first order of dominance. To illustrate, focusing on the normalized CD curves of grain and transport expenditures, $\overline{CD}^{1}_{grain}(z)$ and $\overline{CD}^{1}_{tranpo}(z)$ intersects at the first critical poverty line $z^{+}(\gamma)$=R2,828.219 per capita

per month with a standard error of 486.721 for γ=1, when marginally increasing the tax on grain expenditures and marginally decreasing the subsidy on transport expenditures, As shown in Figure 9, $\overline{CD}^{1}_{grain}(z)$ is above $\overline{CD}^{1}_{tranpo}(z)$ up to the poverty line $z^{+}(\gamma=1)$=R2,828.219 per capita per month, it is reasonable to conclude that an indirect tax reform where there is a marginal increase in the tax on transport expenditures and a marginal decrease in the tax on grain expenditures is first-order poverty reducing for any poverty lines below $z^{+}_{grain-transpo}(\gamma=1)$ -1.96*486.721=R1,874.256 per capita per month at a 95% confidence level. However, it is not first-order welfare improving due to the presence of a critical poverty line. An indirect tax reform is not Pen-improving tax reform when the first-order normalized CD curves cross at a certain critical poverty line for $\gamma=1$. This conclusion applies equally in all cases of VAT reforms for any pair of the four items.

Additionally, it should be noted that the critical poverty line of $\overline{CD}^{1}_{grain}(z)$ and $\overline{CD}^{1}_{tranpo}(z)$ is equal to R2,828.219 per capita per month, which is smaller than the critical poverty line $z^{+}(\gamma=1)$=R4,064.869 per capita per month of $\overline{CD}^{1}_{grain}(z)$ and $\overline{CD}^{1}_{healthcare}(z)$, and the critical poverty line of $z^{+}(\gamma=1)$=R4,173.144 per capita per month for $\overline{CD}^{1}_{grain}(z)$ and $\overline{CD}^{1}_{energy}(z)$. As the critical poverty line increases, more benefits is assigned to reduce the poverty gap in a marginal tax reform.

As shown is Figure 9, the intersection of $\overline{CD}^{1}_{transpo}(z)$ and $\overline{CD}^{1}_{healthcare}(z)$ in an

indirect tax reform of subsidizing transport and taxing on healthcare places on the right hand side of the intersection of $\overline{CD}^{1}_{transpo}(z)$ and $\overline{CD}^{1}_{energy}(z)$ in a tax reform that marginally increases the taxes on energy and decreases the taxes on transport. Table 2 specified the results that $z^{+}_{transpo-health}(\gamma)$=R8,965.130 per capita per month (with a standard error of 347.704) is much higher than $z^{+}_{transpo-energy}(\gamma)$=R5,271.667 per capita per month (with a standard error of 1,447.217) for γ=1 at the first order of dominance. It implies that a tax reform is first-order poverty reducing for any poverty lines below $z^{+}_{transpo-health}(\gamma)$ -1.96*347.704=R8,283.630 per capita per month at a 95% confidence level, when increasing a tax on health care and reducing a tax on transport. This conclusion is roughly similar as the above where an indirect tax reform of a marginal increase in energy expenditures and a marginal subsidy for transport expenditures is first-order poverty improving for any poverty lines up to the critical level $z^{+}_{transpo-energy}(\gamma)$ -1.96*1447.217=R2,435.122 per capita per month at a 5% critical level.

With respect to the normalized CD curves for the four items at the second order of dominance, Figure 10 shows that $\overline{CD}^{2}_{grain}(z)$ and $\overline{CD}^{2}_{transpo}(z)$ always lies above $\overline{CD}^{2}_{healthcare}(z)$ and $\overline{CD}^{2}_{energy}(z)$ for all poverty lines within the range of income between R0 and R15,000 per capita, per month on the x-axis. This implies that the indirect tax reform of increasing marginal subsidies on grains or transport and financing this by a marginal increase in the tax on health care or energy satisfies the Pigou-Dalton principle for all poverty lines. Thus, any transfers from the richer to the

poorer reduce poverty within the interval of all poverty lines.

Figure 10 demonstrates that there are no crossings between any two of the four $\overline{CD}^2(z)$ curves. Specifically, $\overline{CD}^2_{grain}(z)$ lies above $\overline{CD}^2_{transpo}(z)$, $\overline{CD}^2_{transpo}(z)$ above $\overline{CD}^2_{healthcare}(z)$ and $\overline{CD}^2_{healthcare}(z)$ above $\overline{CD}^2_{energy}(z)$ for all poverty lines in the range from R0 to R15,000 per capita per month on the x-axis. In other words, it clearly shows that there's no critical poverty line existing within the interval of poverty lines on the x-axis when applying normalized CD curves. The upper $\overline{CD}^2(z)$ dominates the lower $\overline{CD}^2(z)$ for all poverty lines. When marginally subsidizing the upper $\overline{CD}^2(z)$ and taxing on the lower $\overline{CD}^2(z)$, the VAT ref orm is deemed to improve social welfare as well, not only to reduce poverty at the second-order dominance. This conclusion is in line with the theory of a Dalton-improving tax reform.

V. Conclusion

This study shows how the s^{th}-order normalized consumption dominance curves derived by Makdissi and Wodon (2008) can be used to assess the impact of an indirect tax reform on poverty and social welfare. The findings conclude that a marginal tax reform where food is taxed and non-food is subsidized will reduce poverty and improve social welfare in KwaZulu-Natal. Furthermore, examining the normalized consumption dominance curves for multiple commodities in an indirect tax reform shows that subsidizing grains or transport is more beneficial in comparison with

healthcare or energy. Additionally, the critical poverty lines and the critical economic efficiency ratios can also be used to determine whether or not KwaZulu-Natal policies would be socially improving.

References:

Atkinson, Anthony B. (1970) 'On the measurement of inequality.' *Journal of Economic Theory* 2, 244-263

Atkinson, Anthony B. (1987) 'On the measurement of poverty.' *Econometrica* 55, 759-764

Atkinson, Anthony B. (1998) 'Poverty in Europe' (*Oxford: Blackwell Publishers*)

Besley, Timothy, and Ravi Kanbur (1988) 'Food subsidies and poverty alleviation.' *Economic Journal* 98, 701-719

Dalton, Hugh (1920) "The measurement of the inequality of income." *Economic Journal* 30, 348-461

Donaldson, David, and John A. Weymark (1986) 'Properties of fixed-population poverty indices.' *International Economic Review* 27, 667-688

Duclos, Jean-Yves, and Paul Makdissi (2004) 'Restricted and unrestricted dominance for welfare, inequality and poverty orderings.' *Journal of Public Economic Theory* 6, 145-164

Duclos, Jean-Yves, Paul Makdissi, and Quentin Wodon (2008) 'Socially improving tax reforms.' *International Economic Review* 49, 1505-1537

Fishburn, Peter C. (1976) 'Continua of stochastic dominance relations for bounded probability distributions.' *Journal of Mathematical Economics* 3, 295-311

Fishburn, Peter C. (1980) 'Continua of stochastic dominance relations for unbounded probability distributions.' *Journal of Mathematical Economics* 7, 271-285

Fishburn, Peter C., and Robert D. Willing (1984) 'Transfer principles in income

redistribution.' *Journal of Public Economics* 25, 323-328

Foster, James, Joel Greer, and Erik Thorbecke. (1984) 'A class of decomposable poverty measures.' *Econometrica* 52, 761-776

King, Mervyn A. (1983) 'Welfare analysis of tax reforms using household data.' *Journal of Public Economics* 21, 183-214

Kolm, Serge-Christophe (1976) 'Unequal inequalities: I.' *Journal of Economic Theory* 12, 416-442

Kolm, Serge-Christophe (1976) 'Unequal inequalities: II.' *Journal of Economic Theory* 13, 82-111

Makdissi, Paul, and Quentin Wodon (2002) 'Consumption dominance curves: Testing for the impact of indirect tax reforms on poverty.' *Economics Letters* 75, 227-235

Makdissi, Paul, and Quentin Wodon (2007) 'Poverty-reducing and welfare-improving marginal public price and price cap reforms.' *Journal of Public Economic Theory* 9, 683-698

Mayshar, Joram, and Shlomo Yitzhaki (1995) 'Dalton-improving tax reform.' *American Economic Review* 85, 793-807

Pen, Jan (1971) 'Income distribution: Facts, Theories, Policies' (*New York: Preaeger Publishers*)

Pigou, Arthur Cecil (1912) 'Wealth and Welfare' (*London: Macmillan Publishers*)

Sen, Amartya (1976) 'Poverty: An ordinal approach to measurement.' *Econometrica* 44, 219-231

Shorrocks, Anthony F., and James E. Foster (1987) 'Transfer sensitive inequality

measures.' *Review of Economic Studies* 54, 485-497

Wildasin, David E. (1984) 'On public good provision with distortionary taxation.' *Economic Inquiry* 22, 227-243

Yitzhaki, Shlomo, and Joel Slemrod (1991) 'Welfare dominance: An application to commodity taxation.' *American Economic Review* 81, 480-496

Appendix:

Figure 1: Normalized Consumption Dominance Curves, 1st order with γ=1

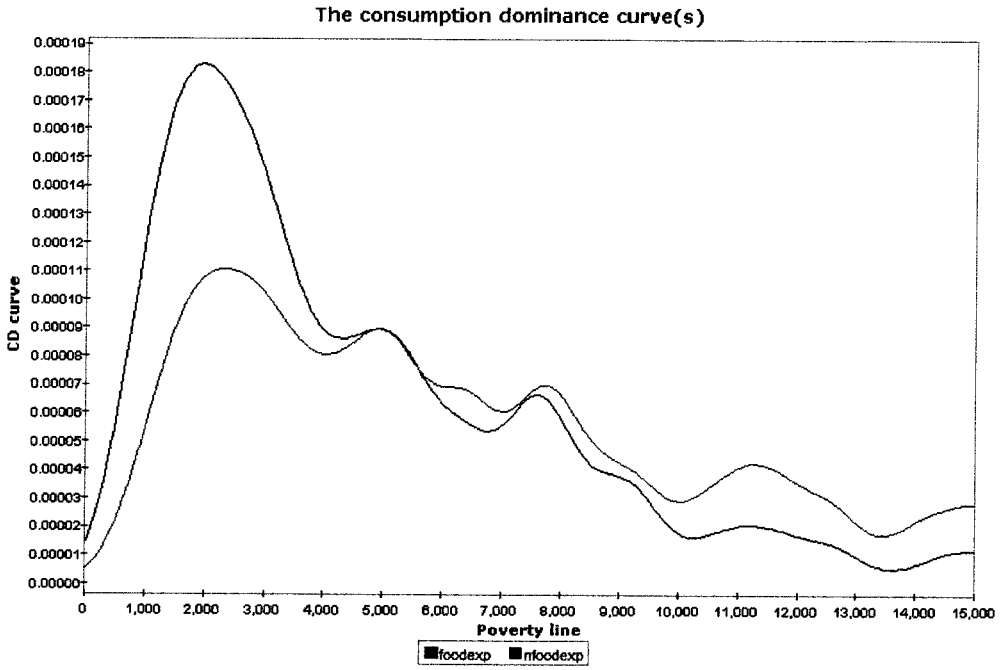

Figure 2: Normalized Consumption Dominance Curves, 2nd order with γ=1.

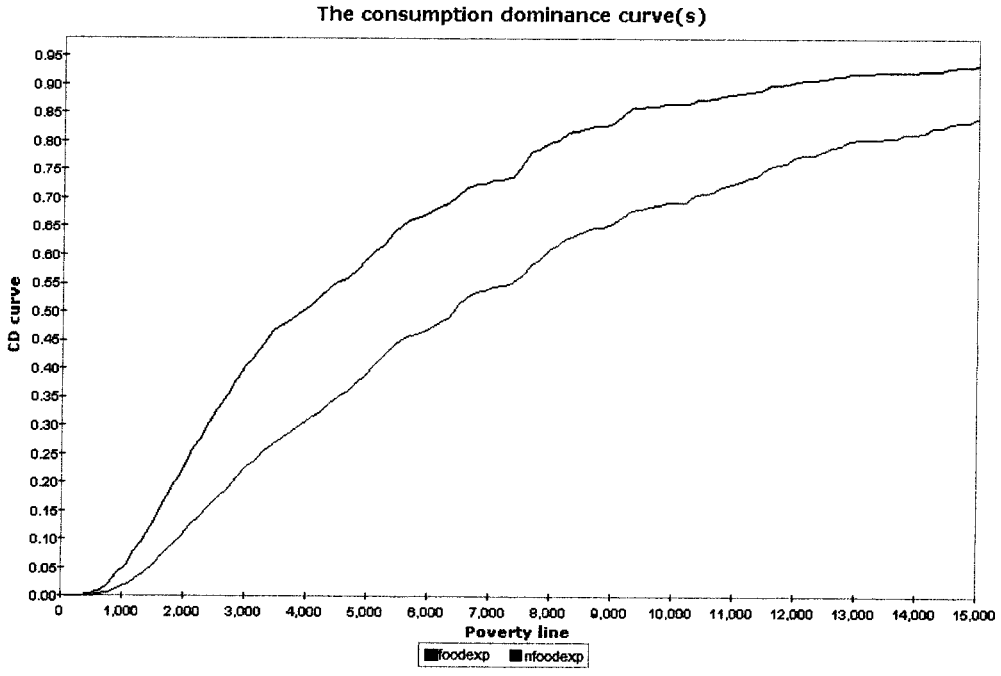

Figure 3: Normalized Consumption Dominance Curves, 3rd order with γ=1.

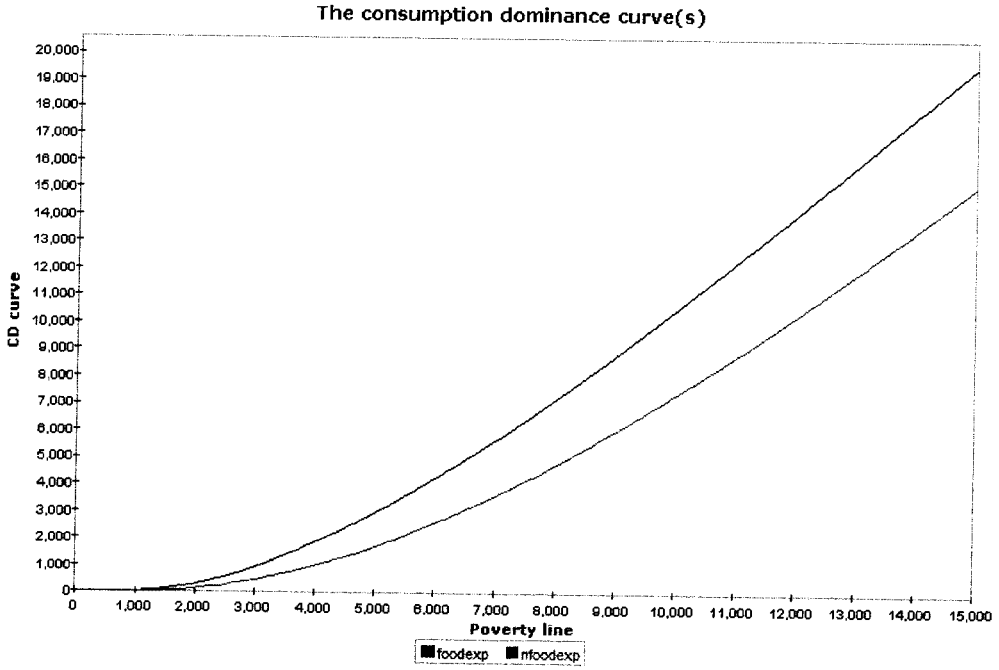

Figure 4: Normalized Consumption Dominance Curves, 4th order with γ=1.

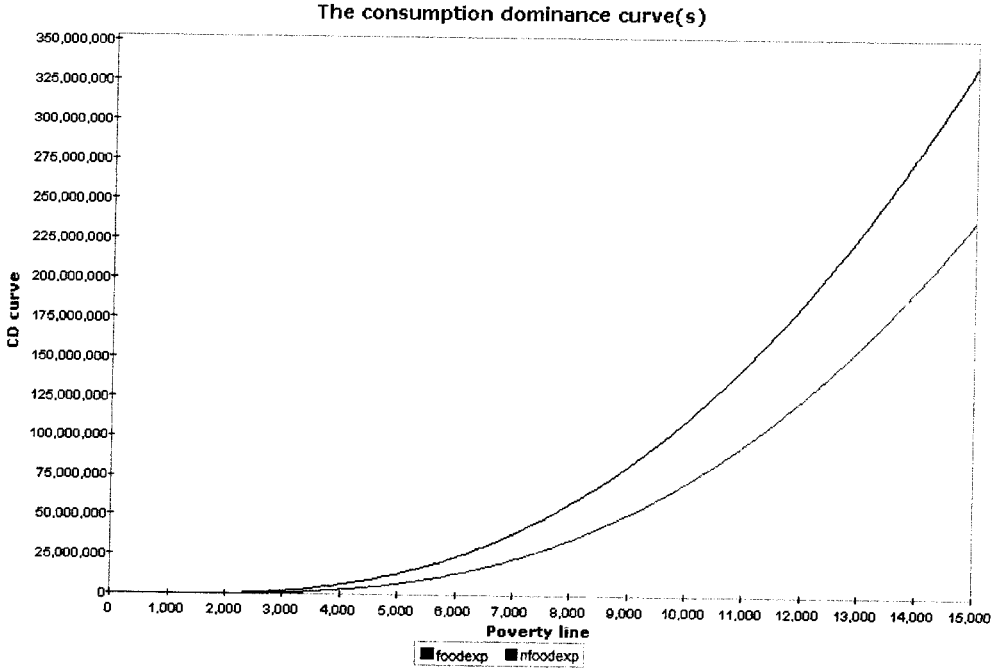

Figure5: Distributive Benefit Curve $\delta^1(food\ \exp, nfood\ \exp)$ for 1st order Consumption Dominance

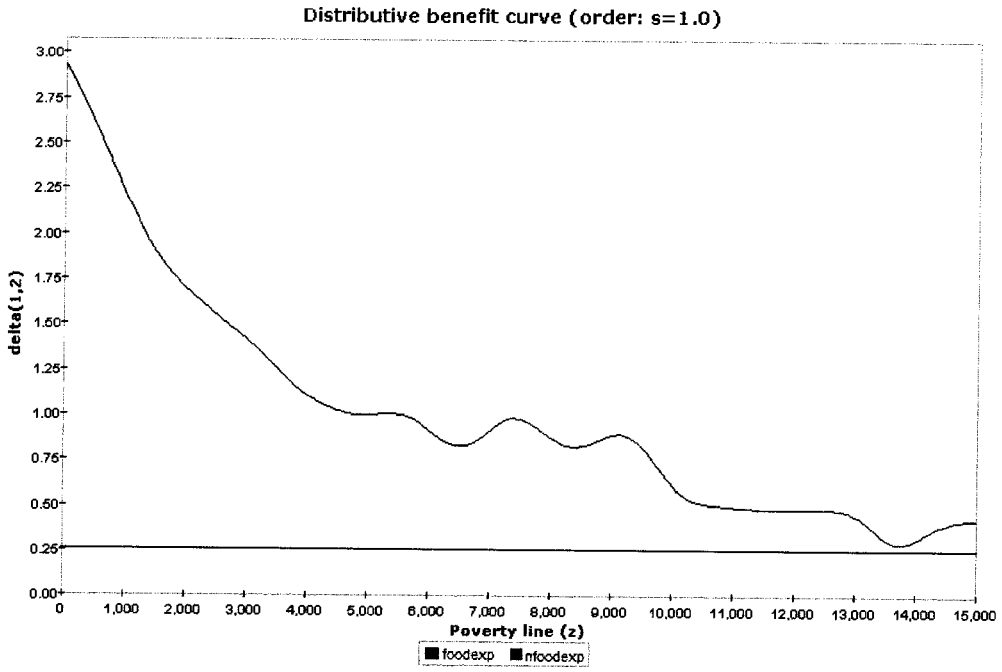

Figure6: Normalized Consumption Dominance Curves, 1st order with γ=1.5

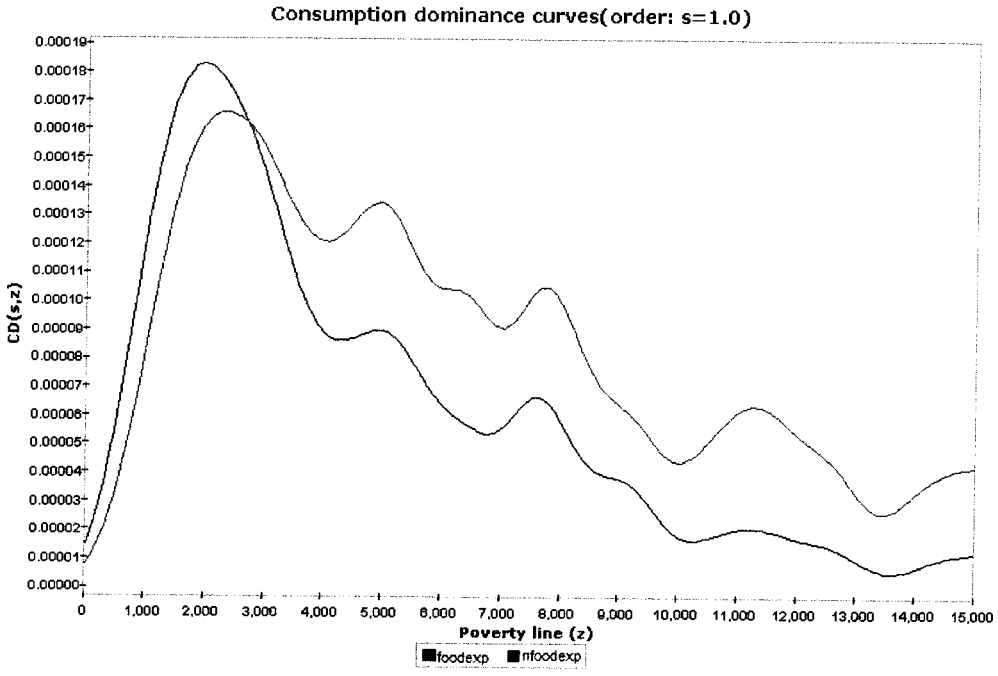

Figure 7: Normalized Consumption Dominance Curves, 1st order with γ=0.5

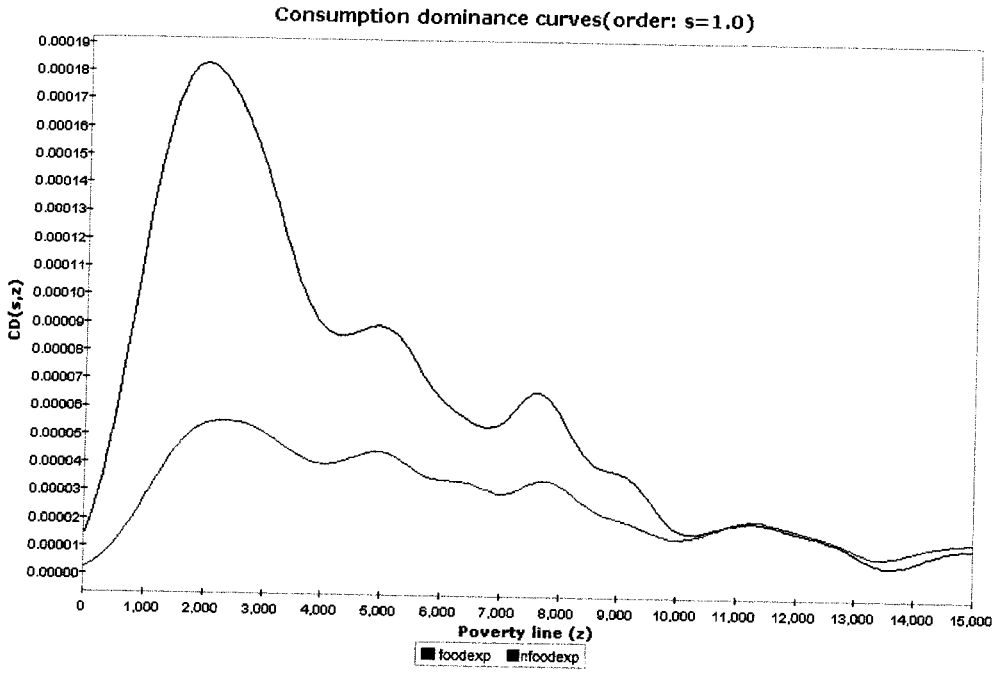

Figure 8: Normalized Consumption Dominance Curves, 1st order with γ=0.25

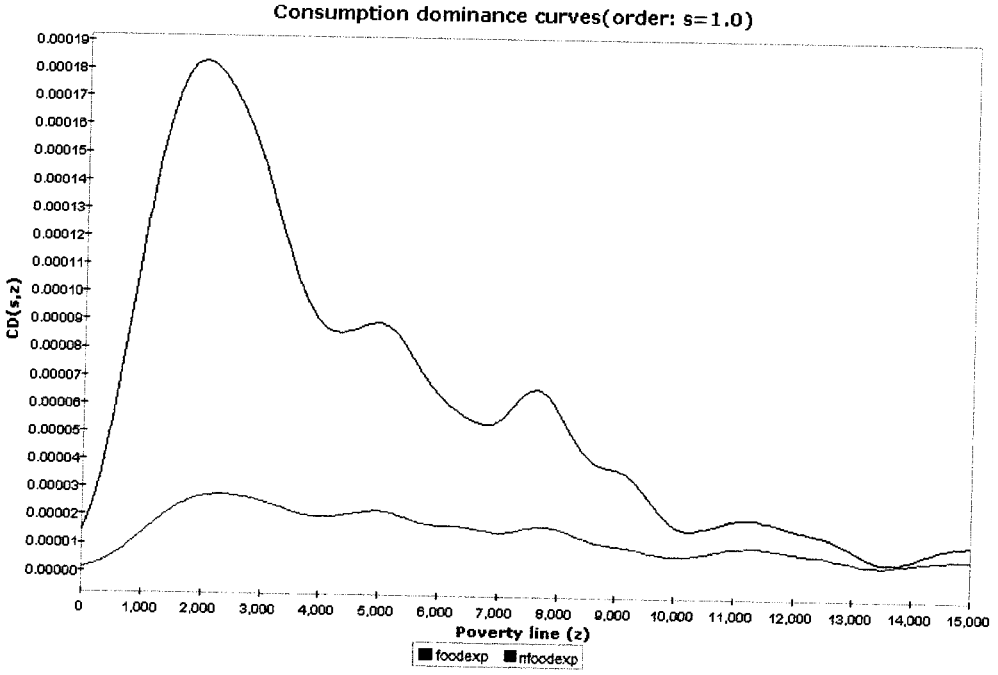

Figure9: Normalized Consumption Dominance Curves, 1st order with γ=1

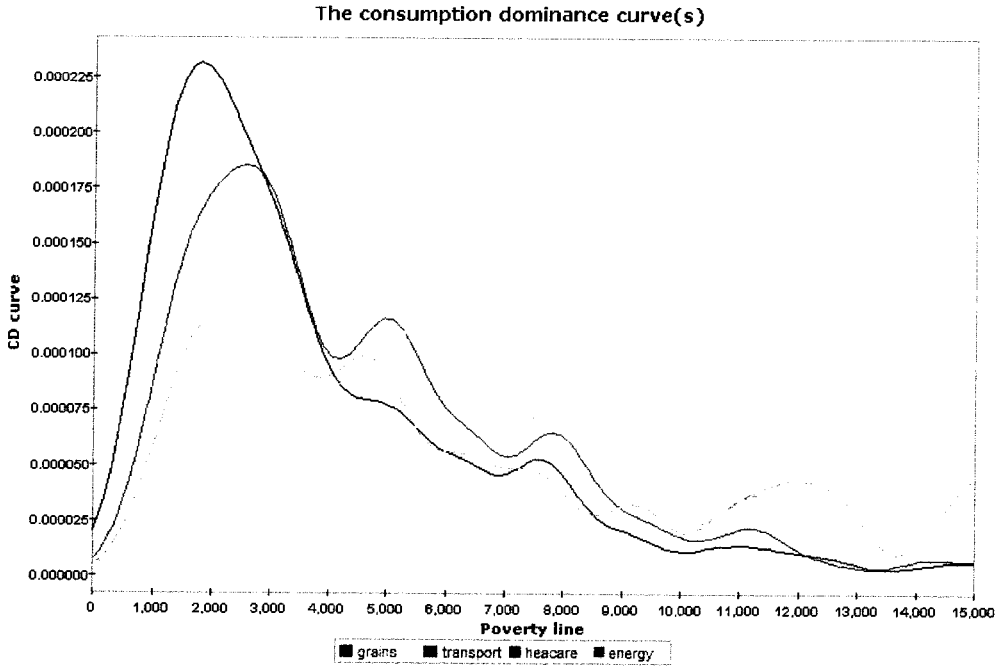

Figure 10: Normalized Consumption Dominance Curves, 2nd order with γ=1

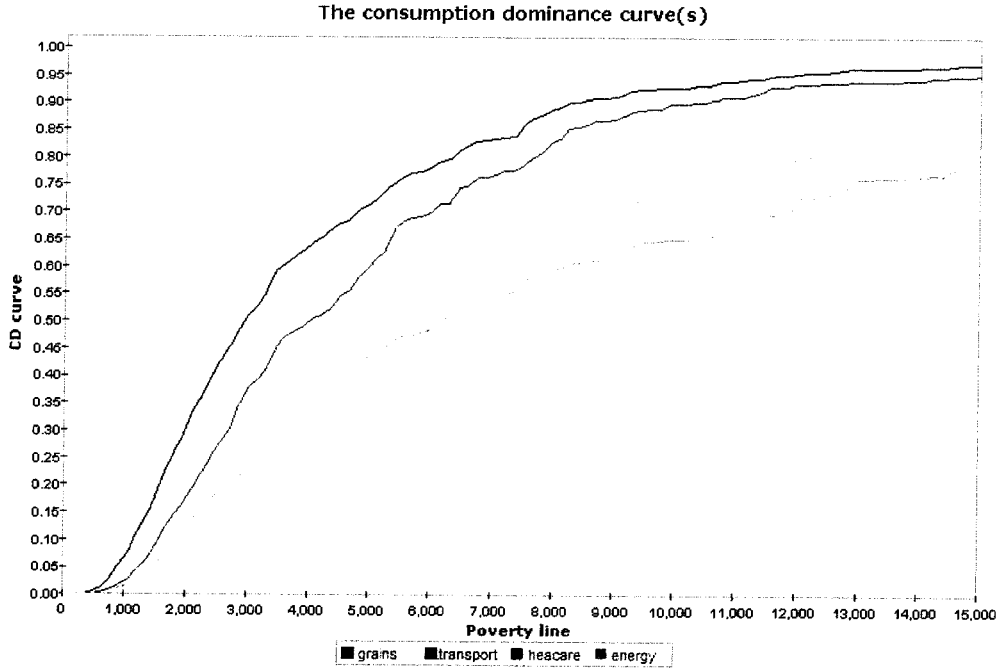

Table 1: Critical poverty lines $z_s(\gamma)$ for different ratios of economic efficiency costs γ, and for different orders of dominance s, (foodexp vs. nfoodexp).

	γ=0.25	γ=0.5	γ=1	γ=1.5
$z_1(\gamma)$	--	10836.036 (1268.159)	5588.896 (768.090)	2680.581 (287.331)
$z_2(\gamma)$	--	--	--	5084.274 (125038.604)
$z_3(\gamma)$	--	--	--	--
$z_4(\gamma)$	--	--	--	--

Note: Dataset is 1426 households from KIDS 2004.

Table 2: The critical poverty lines for any pairs of $\overline{CD}^{1}(z)$ curves of grains, transport, healthcare and energy

Tax items / Subsidize items	Grains	Transport	Health care	Energy(water&electricity)
Grains		2828.219 (486.721)	4064.86 (275.516)	4173.144 (143.425)
Transport			8965.130 (347.704)	5271.667 (1447.217)
Health care				4559.640 (643.355)
Energy(water&electricity)				

Note: The upper number means the first intersection of two $\overline{CD}^{1}(z)$ curves. The lower number means the standard error for the critical poverty line of these two $\overline{CD}^{1}(z)$ curves.