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Price Caps, Commitment and Innovation[*]

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Abstract
We analyze innovation incentives under price cap regulation by examining scenarios with endogenous price caps, both with and without regulatory commitment. In a setting without informational imperfections, our analysis reveals two principal conclusions. First, there is no trade-off between static and dynamic efficiency. Strengthening firm incentives by allowing it to charge higher prices, and thus realize greater profits, leads to less innovation because it reduces output. The optimal strategy to boost innovation and maximize welfare is to set a low price (and thus, a low profit) target, as innovation incentives are proportional to output. Second, the benefits of regulatory commitment for innovation and welfare are not unambiguously clear: commitment neither consistently outperforms nor underperforms non-commitment.

Under demand uncertainty, when the firm is risk-averse, the static-dynamic efficiency trade-off reappears, and the firm may prefer non-commitment due to risk-shielding. Under asymmetric information about firm demand type, the trade-off between static and dynamic efficiency becomes inherent (due to information rents and contract distortions), and commitment becomes unambiguously crucial for fostering innovation by preventing the "ratchet effect".
Key words: *Price cap regulation; Regulation; Innovation; R&D; Dynamic efficiency; Commitment.*
JEL Classification: D42, L12, L43, L51, O31, O38.

1 Introduction

Under price cap regulation (PCR), a regulator sets a cap on the price of a basket of a monopoly's goods and services. PCR is particularly popular in regulating the telecommunications industry and is widespread in both transition and advanced economies. One of its core advantages is that it simplifies the regulatory process by reducing regulatory administrative/transaction costs and micro-management. The literature generally concludes that this type of regulation has been more successful in fostering innovation and/or reducing prices compared with traditional forms of regulation, such as rate of return regulation (ROR)[1]. Another important benefit of PCR is its adaptability to a changing environment. As Sappington and Weisman (2012:20) point out: "The popularity of price cap regulation stems in part from its ability to adapt readily to changing industry conditions as competitive forces develop."

In this paper we revisit innovation incentives under PCR. We study how enhancing firm incentives, by allowing them to charge higher prices and earn more profit, affects innovation and welfare under different regulatory regimes. Additionally, we investigate how changes in exogenous parameters (demand, production costs, innovation costs, etc.) influence the regulatory contract, output, innovation, and other endogenous variables.

We model a game that involves a benevolent regulator and a multiproduct monopolist regulated by a price cap. In the first stage, the regulator sets up the regulatory contract to maximize welfare. In the second stage, the firm chooses cost-reducing innovation investments and prices to maximize profit while complying with the price cap constraint. We analyze two settings: one in which the regulator cannot commit to the regulatory contract, resulting in potential revisions upon environmental changes; and another where the regulator commits to the regulatory contract, ensuring the contract remains unchanged (until the subsequent review) regardless of changes in the operating environment. This initial analysis is conducted under the simplifying assumptions of symmetric information and certainty regarding all model parameters.

This paper makes two main findings. First, there is no trade-off between static and dynamic efficiency. That is, providing the firm with stronger incentives by allowing it to charge higher prices and make higher profits, with the hope that this will spur innovation and boost social welfare, actually leads to less innovation. This outcome hurts both static and dynamic efficiency. The best way to boost innovation is to choose a low price target (which is associated with a low profit target). This is because innovation incentives are proportional to output, and are thus negatively related to prices. This is sometimes referred to in the literature as the Arrow Effect (Arrow, 1962). High prices increase profit but reduce output, innovation, and welfare. The regulator need not choose between static and dynamic efficiency: she can maximize both with "strict" PCR, which means low prices and profit. This is true whether the regulator can commit to the regulatory contract or not.

The second main finding is that commitment is not always better (or worse) than non-commitment for innovation and welfare: which regime is superior depends on which parameter is changing, and in what direction. The presence of commitment alters many of the comparative statics of the model. In addition, without commitment the firm is mostly

[1] See, for example, Sappington and Weisman (2010).

shielded from changes in the environment; the effects, positive or negative, are mainly absorbed by consumers. Whereas with commitment, consumers are more shielded from the effect of exogenous changes (except for changes in demand parameters), and the firm must absorb most of the effects of exogenous changes.

Lastly, we consider comparative statics under non-commitment and commitment. Under non-commitment a change in any parameter moves prices and innovation in opposite direction because the value of innovation increases with output. Furthermore, we incorporate product differentiation, considering both substitution and complementarity between products. We show that the Arrow effect is more pronounced with complements than with substitutes. This is because the equilibrium output is higher under complements. While the symmetry assumption on demand simplifies the solution of the multi-product model to a single price for each good, the inclusion of multiple products remains crucial for analyzing the implications of demand interactions (substitutes and complements) and R&D spillovers across different product lines, which are pertinent features in many regulated industries.

While these findings provide fundamental insights under ideal informational conditions, regulatory environments are often characterized by informational imperfections. To address this, we extend our framework to explore how uncertainty and asymmetric information modify these conclusions. We first analyze scenarios where the firm is risk-averse and faces demand uncertainty. This extension reveals that the trade-off between static and dynamic efficiency re-emerges, as the regulator may need to allow higher prices to compensate a risk-averse firm for bearing uncertainty, which in turn reduces innovation. Furthermore, the firm might actually prefer a non-commitment regime if it offers greater risk-shielding. Second, we introduce asymmetric information regarding the firm's demand type. This more complex setting demonstrates that the static-dynamic efficiency trade-off becomes inherent, as the regulator must distort contracts (e.g., allow higher prices for a high demand firm) to induce truthful revelation and limit information rents. Crucially, under asymmetric information, commitment becomes unambiguously vital for fostering innovation, contrasting our initial result, because it prevents the opportunistic "ratchet effect" that would otherwise discourage long-term investments.

Literature review: Our paper is related to the literature modelling the interaction between price caps and innovation. Schmalensee (1989) compares PCR with cost-plus regulation; however his model incorporates only one good, and thus cannot address the effects of demand interactions or R&D spillovers; moreover, he does not consider the effect of commitment. Cabral and Riordan (1989) analyze the effect of price caps on innovation incentives. However, in their model the price cap is exogenous, and there is only one product; moreover, they do not consider the effect of commitment versus non-commitment by the regulator. Clemenz (1991) shows that even with an infinite horizon, PCR provides superior innovation incentives to ROR. Heyes and Liston-Heyes (1998) study PCR in the presence of an outside firm which invests in innovation, in addition to the innovation investment of the regulated firm. They find that a tighter price cap reduces innovation by the regulated firm, but has an ambiguous effect on innovation by the external developer; they also show that the optimal price cap may be higher or lower than in the absence of an external developer. Goel (2000) extends their framework by incorporating uncertainty and spillovers. He finds that a tighter price cap reduces innovation by both the regulated firm and the external engineering firm; however, he does not attempt to compute the welfare-maximizing price cap. Crew and Kleindorfer

(2001) develop a model of PCR, taking the issue of commitment into account. Their model comprises only one good, but allows for a rate hearing when profit lies outside a certain range.

Vives (2008) and López and Vives (2019) consider cost-reducing R&D investment in presence of competition. Vives (2008) in a setting without spillovers found that increasing the number of firms tends to decrease cost reduction expenditure per firm, whereas increasing the degree of product substitutability increases this expenditure.

In cases with informational imperfections, Earle et al. (2007) show that under demand uncertainty, the standard welfare-enhancing properties of price caps may reverse. Their results challenge the robustness of deterministic price-cap analysis, demonstrating that stochastic environments can yield non-monotonic responses in output and welfare. Sibley (1989) develops a mechanism that handles uncertainty and asymmetric information by inducing truthful revelation of demand through lagged profit-based instruments. His ISS-R mechanism closely parallels price-cap regulation and yields efficient outcomes even when the regulator lacks full information.

The current paper contributes to these literatures by analyzing the trade-off between static and dynamic efficiency with an endogenous price cap and endogenous innovation in the presence of product differentiation and R&D spillovers. And it examines the specific effects of changes in exogenous parameters on all equilibrium variables. Both questions are addressed with and without commitment by the regulator, and in different informational structures.

Structure of the paper: The paper is structured as follows. Section 2 presents the core model. The model is solved and equilibrium values of variables are derived in section 3. Section 4 introduces commitment and non-commitment to regulatory contracts. The effect of price and profit incentives on innovation (i.e., the trade-off between static and dynamic efficiency) is examined in section 5 (without commitment), along with comparative statics. The effect of commitment on this trade-off and on comparative statics is analyzed in section 6. Section 7 compares commitment versus non-commitment in the core model. Section 8 then extends the analysis to incorporate informational imperfections, specifically risk (uncertainty) and asymmetric information. Finally, Section 9 concludes.

2 The Model

There are $n \geq 2$ products, where the quantity of product i is denoted y_i. The vector of quantities is $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)$ and $\mathbf{p} = (p_1, \ldots, p_n)$ is the vector of prices. We assume that consumers have the same quasi-linear, twice-differentiable, strictly concave utility function $u(\boldsymbol{y}) + m$. From the utility maximization program we obtain the vector of demand functions $\mathbf{y}(\mathbf{p}) = (y_1(\mathbf{p}), y_2(\mathbf{p}), ..., y_n(\mathbf{p}))$ that satisfies the properties: $\frac{\partial y_i}{\partial p_i} < 0$, $\frac{\partial y_i}{\partial p_j} = \frac{\partial y_j}{\partial p_i}$ for all i, j.[2] The inverse demand functions satisfy the first-order conditions: $p_i(\mathbf{y}) = \frac{\partial u(\mathbf{y})}{\partial y_i}$ for all i. We assume that the own-price effect is greater than the cross-price effect: $\sum_{j=1}^{n} \frac{\partial y_j}{\partial p_i} < 0$.

[2]Since the utility function is quasi-linear, demand functions do not depend on income. Differentiability of demand functions follows from twice-differentiability of the utility function. Since the Hessian function is negative definite we have $\frac{\partial y_i}{\partial p_i} < 0$. Finally, the cross-symmetry of derivatives follows from Young's Theorem.

The surplus of the representative consumer (the indirect utility function) $CS(\boldsymbol{p}) = u(\boldsymbol{y}(\mathbf{p})) - \mathbf{p}\mathbf{y}(\mathbf{p})$ is a convex, decreasing function of prices.[3]

There are two players in the model: a regulator and a firm. The firm is a multi-product monopolist in all n markets. For each product i the firm can invest x_i to reduce marginal cost (cost-reducing R&D); the unit cost of i is $c_i(\mathbf{x})$, where, $\mathbf{x} = (x_1, x_2, ..., x_n) \in \mathbb{R}^n_+$, $\frac{\partial c_i(\mathbf{x})}{\partial x_i} < 0$.

Cost of investment is $\Gamma(\mathbf{x}) \in \mathbb{R}_+$. $c_i(\mathbf{x}), \Gamma(\mathbf{x})$ are differentiable, convex functions with positive second cross-derivates. Investment in good i has also positive spillover effects on other goods' costs: $\frac{\partial c_j(\mathbf{x})}{\partial x_i} \leq 0$. This assumption implies that R&D efforts for one product can beneficially reduce the marginal costs of other products within the multi-product firm, reflecting internal knowledge diffusion or shared technological advancements.[4]

The firm's profit is

$$\pi(\mathbf{p}, \mathbf{x}) = \sum_{i=1}^{n} (p_i - c_i(\mathbf{x}))\, y_i(\mathbf{p}) - \Gamma(\mathbf{x}) = (\mathbf{p} - \mathbf{c}(\mathbf{x}))\, \mathbf{y}(\mathbf{p}) - \Gamma(\mathbf{x}),$$

where $\mathbf{p}, \mathbf{x} \in \mathbb{R}^n_+$.

The regulator maximizes welfare, which is the sum of consumer surplus and profit:

$$W(\mathbf{p}, \mathbf{x}) = CS(\mathbf{p}) + \pi(\mathbf{p}, \mathbf{x}) = u(\mathbf{y}(\mathbf{p})) - \mathbf{c}(\mathbf{x})\mathbf{y}(\mathbf{p}) - \Gamma(\mathbf{x}).$$

The participation constraint of the firm is

$$\pi(\mathbf{p}, \mathbf{x}) \geq T. \tag{2.1}$$

The target $T \geq 0$ is exogenous to the regulator and to the model. In practice, incentive regulation often takes the form of a hybrid between pure PCR and ROR, which is consistent with the presence of the profit target T in (2.1). We assume that this target is not too large, i.e., $\max_{\mathbf{p},\mathbf{x}} \pi > T$. [5]

The price-cap constraint takes the form,

$$\mathbf{w}\mathbf{p} = \sum_{i=1}^{n} w_i p_i \leq \overline{p}, \tag{2.2}$$

where $\mathbf{w} = (w_1, w_2, ..., w_n)$ is a vector of weights, and $\overline{p} \in \mathbb{R}_{++}$ is the "average weighted price".[6]

Timing of the model. In the first stage the regulator chooses $(\mathbf{w}, \overline{p})$. In the second stage, after observing the regulator's choice, the firm chooses $(\mathbf{p}, \mathbf{x})$.

[3]From the Envelope theorem we have: $\frac{\partial CS}{\partial p_i} = -y_i(\mathbf{p})$.

[4]While negative spillovers are possible in other contexts (e.g., if R&D for one product diverts resources or creates complications for others), we focus on positive spillovers, which are commonly observed in integrated multi-product firms. Exploring the implications of negative spillovers would introduce additional complexities and is left for future research.

[5]Note that if $\max_{\mathbf{p},\mathbf{x}} \pi \leq T$, then the equilibrium will be the second-best and the price cap constraint is irrelevant.

[6]A higher value of $\overline{p}$ can be due to a higher initial price cap, a lower value of the X factor (in $RPI - X$, where RPI is the Retail Prices Index and X is the expected rate of cost-reducing technological progress), or both.

3 Firm Behavior and Determining the Optimal Price Cap

We are looking for a subgame perfect Nash equilibrium. In the second stage the firm chooses $\mathbf{p}, \mathbf{x}$ while taking $\mathbf{w}, \overline{p}$ as given. The firm maximizes its profit subject to the weighted price cap constraint:

$$\max_{\mathbf{p},\boldsymbol{x}\in\mathbb{R}^n_+} \pi\left(\mathbf{p},\mathbf{x}\right) \tag{3.1}$$

$$\text{s.t.} \sum_{i=1}^{n} w_i p_i \leq \overline{p}. \tag{3.2}$$

Consider the Lagrangian: $L = \pi\left(\mathbf{p},\mathbf{x}\right) + \lambda\left(\overline{p} - \sum_{i=1}^{n} w_i p_i\right)$, where $\lambda \geq 0$ is the Lagrange multiplier of the price-cap constraint. We assume that the profit function is strictly concave, ensuring that the second-order conditions for profit maximization are satisfied.[7]

The first-order conditions with respect to p_i and x_i are given by:

$$y_i + \sum_{j=1}^{n}(p_j - c_j)\frac{\partial y_j}{\partial p_i} - \lambda w_i = 0, \forall i \text{ and} \tag{3.3}$$

$$-\sum_{j=1}^{n} y_j \frac{\partial c_j}{\partial x_i} - \frac{\partial \Gamma}{\partial x_i} = 0, \forall i. \tag{3.4}$$

Conditions (3.2)–(3.4) determine the best response functions in stage 2, $\mathbf{p}\left(\mathbf{w},\overline{p}\right), \mathbf{x}\left(\mathbf{w},\overline{p}\right)$.[8]

The first-order conditions with respect to prices are the solutions of the multi-product monopoly pricing under the price cap constraint, where the innovation levels $\mathbf{x}$ play the role of parameters of this problem. We can re-write these equations as

$$\frac{p_i - c_i\left(\mathbf{x}\right)}{p_i} = \frac{1}{\varepsilon_{ii}} - \sum_{j\neq i}\frac{\left(p_j - c_j\left(\mathbf{x}\right)\right)\varepsilon_{ij} y_j\left(\mathbf{p}\right)}{p_i \varepsilon_{ii} y_i\left(\mathbf{p}\right)} - \frac{\lambda w_i}{\varepsilon_{ii} y_i\left(\mathbf{p}\right)}, \quad i = 1, 2, ..., n, \tag{3.5}$$

where $\varepsilon_{ji} = -\frac{\partial y_j(\mathbf{p})}{\partial p_i}\frac{p_i}{y_j(\mathbf{p})}$ is the cross-price elasticity of demand for good j with respect to price p_i and $\frac{p_i - c_i(\mathbf{x})}{p_i}$ is the Lerner index. The second term in (3.5) reflects the fact that the monopolist internalizes the effects that sales of goods have on the demand of other goods. The third term, $-\frac{\lambda w_i}{\varepsilon_{ii} y_i(\mathbf{p})}$, is a price-cap effect which is always negative.

Consider firm's incentives to innovate. We have

$$\frac{\partial \pi\left(\mathbf{p},\mathbf{x}\right)}{\partial x_i} = \underbrace{-y_i\frac{\partial c_i}{\partial x_i}}_{\text{size effect}} \underbrace{-\sum_{j\neq i} y_j \frac{\partial c_j}{\partial x_i}}_{\text{spillover effect}} \underbrace{-\frac{\partial \Gamma}{\partial x_i}}_{\text{cost effect}}, \quad i = 1, ..., n. \tag{3.6}$$

[7]Given the assumptions that: (1) demand functions are derived from a strictly concave quasi-linear utility function, (2) marginal cost functions $c_i(\mathbf{x})$ are convex, and (3) the R&D cost function $\Gamma(\mathbf{x})$ is convex, it is generally assumed in this class of models that the firm's profit function $\pi(\mathbf{p},\mathbf{x})$ is concave. For common functional forms (e.g., linear demand, quadratic costs), this concavity is readily verifiable.

[8]In vector form these conditions can be re-written as $\partial_p \pi = \lambda \mathbf{w}$ and $\partial_x \pi = \mathbf{0}$, where $\partial_p \pi = (\partial_{p_1}\pi, \cdots, \partial_{p_n}\pi)$ and $\partial_x \pi = (\partial_{x_1}\pi, \cdots, \partial_{x_n}\pi)$ are the gradients of $\pi(\mathbf{p},\mathbf{x})$.

Innovation x_i reduces costs by $-\frac{\partial \mathbf{c}(\mathbf{x})}{\partial x_i}$. This reduction applies to all produced quantities of good i - the size effect.[9] Because of spillovers, innovation x_i reduces costs for all goods $j \neq i$ and has a positive profit effect – the spillover effect. Finally, innovation is costly – the cost effect. The firm chooses innovations such that the marginal benefits of innovations are equal to the marginal costs.

The equilibrium profit in the subgame is $\pi(\overline{p}, \mathbf{w}) = \pi(\mathbf{p}(\mathbf{w}, \overline{p}), \mathbf{x}(\mathbf{w}, \overline{p}))$. Using the Envelope Theorem we obtain comparative statics for the profit with respect to the price-cap:

$$\frac{\partial \pi(\overline{p}, \mathbf{w})}{\partial \overline{p}} = \lambda \geq 0. \tag{3.7}$$

Therefore, profit is an increasing function of $\overline{p}$. Since the price cap $\overline{p}$ enters this program only in constraint (3.2), increasing $\overline{p}$ can only (weakly) increase profit.

We consider a symmetric set-up; the firm faces symmetric demands and costs and has identical levels of R&D, output, marginal cost, and interaction effects.[10] Therefore, in Stage 2 we have $p_i(\overline{p}) = p(\overline{p})$ and $x_i(\overline{p}) = x(\overline{p})$. Thus, in Stage 1, it is optimal to set equal weights: $w_i = \frac{1}{n}$. Consequently, welfare is a function of $\overline{p}$ only: $W(\overline{p}) = W(\mathbf{p}(\overline{p}), \mathbf{x}(\overline{p}))$, where $\mathbf{p}(\overline{p}) = (p(\overline{p}), \cdots, p(\overline{p}))$ and $\mathbf{x}(\overline{p}) = (x(\overline{p}), \cdots, x(\overline{p}))$.

Notice that constraint (3.2) is binding, implying $p_i(\overline{p}) = \overline{p}$ and $p_i'(\overline{p}) = 1$. A regulator's choice of the price cap $\overline{p}$ induces a subgame in which the firm optimally chooses $(\mathbf{p}(\overline{p}), \mathbf{x}(\overline{p}))$. Hence, we can define consumer surplus $CS(\overline{p})$, profit $\pi(\overline{p})$ and welfare $W(\overline{p})$ as functions of $\overline{p}$ only. Then we have

Lemma 1. *a) Consumer surplus and welfare are strictly decreasing in $\overline{p}$,*

b) Profit is increasing in $\overline{p}$,

c) $\mathbf{x}(\overline{p})$ is a decreasing function of $\overline{p}$.

Proof. See Appendix 10.1. □

In the first stage the regulator solves the following problem:

$$\max_{\overline{p}} W(\overline{p}) = CS(\mathbf{p}(\overline{p})) + \pi(\mathbf{x}(\overline{p})) \tag{3.8}$$

$$\text{s.t.} \quad \pi(\overline{p}) = (\mathbf{p}(\overline{p}) - \mathbf{c}(\mathbf{x})(\overline{p}))\, \mathbf{y}(\mathbf{p}(\overline{p})) - \Gamma(\mathbf{x}(\overline{p})) \geq T. \tag{3.9}$$

It follows from Lemma 1 that (3.9) is binding:

$$\pi(\overline{p}) = (\mathbf{p}(\overline{p}) - \mathbf{c}(\mathbf{x})(\overline{p}))\, \mathbf{y}(\mathbf{p}(\overline{p})) - \Gamma(\mathbf{x}(\overline{p})) = T. \tag{3.10}$$

This constraint determines the equilibrium price $\overline{p}$ in the first stage.

[9] Audretsch and Belitski (2020) show that spillovers have a strong effect on firm productivity in the U.K.
[10] See for example, Vives (2008), López and Vives (2019).

3.1 The Arrow Effect

We calculate the effect of a higher price on the marginal profitability of innovation. Differentiating (3.4) with respect to price p_i, we obtain[11]

$$\frac{\partial^2 \pi}{\partial x_i \partial p_i} = -\frac{\partial c_i(\mathbf{x})}{\partial x_i}\frac{\partial y_i(\mathbf{p})}{\partial p_i} - \frac{\partial c_j(\mathbf{x})}{\partial x_i}\frac{\partial y_j(\mathbf{p})}{\partial p_i} < 0. \tag{3.11}$$

Thus, the higher price reduces the marginal profitability of innovation to the firm, since it reduces output. Contrary to conventional wisdom, to provide more incentives for innovation actually calls for lower prices (and thus lower profit). In this setting, there is no trade-off between static and dynamic efficiency: higher static and dynamic efficiency are achieved by a lower profit target, and thus a lower price target.[12]

Remark 1. *Note that the Arrow effect follows directly for complements, since* $\frac{\partial y_j(\mathbf{p})}{\partial p_i} < 0$. *For substitutes, additional assumptions that direct effects dominate cross effects are required. As shown in Section 5, the Arrow effect is indeed more pronounced with complements than with substitutes, as equilibrium output is higher in the case of complements.*

The result that innovation incentives increase with output is well known in the literature (see for example Vives, 2008). Cabral and Riordan (1989) also find that a (marginal) reduction in the price cap increases innovation (although in their model the price cap is exogenous). This result goes back to Arrow (1962), who was the first to clarify that incentives for cost reduction are higher under perfect competition than under monopoly because of the lower monopoly output (and because of the replacement effect). Innovation incentives are determined at the margin, not by the total level of profit. Low profit may fail to attract firms to a market, but once a firm is in a market, it is volume, not the level of profit, which provides incentives for innovation (as long as the participation constraint is satisfied).

4 Commitment and no commitment to the regulatory contract

The operating environment of the firm can be described by the vector of exogenous parameters $\boldsymbol{\nu} = (T, \nu_d, \nu_c, \nu_\Gamma)$, where $\nu_i, i = d, c, \Gamma$ are parameters affecting demand, variable costs and fixed costs respectively. We denote by $RC(\boldsymbol{\nu})$ the regulatory contract: $RC(\boldsymbol{\nu}) = (w(\boldsymbol{\nu}), \overline{p}(\boldsymbol{\nu}))$ and by $d(RC(\boldsymbol{\nu}), \boldsymbol{\nu})$ the firm's decision vector: $d(RC(\boldsymbol{\nu}), \boldsymbol{\nu}) = (p(RC(\boldsymbol{\nu}), \boldsymbol{\nu}), x(RC(\boldsymbol{\nu}), \boldsymbol{\nu}))$. Depending on the regulatory environment, the regulatory contract may change after a change in the operating environment; the regulator reviews the conditions of the contract and sets a new contract. However, frequent changes in contractual relationship may not be possible in case of regulatory commitment. To capture both settings, with commitment and non-commitment, we consider the timing of the game as follow:

[11] Note that $\left|\frac{\partial c_i(\mathbf{x})}{\partial x_i}\right| > \left|\frac{\partial c_j(\mathbf{x})}{\partial x_i}\right|$ and $\left|\frac{\partial y_i(\mathbf{p})}{\partial p_i}\right| > \left|\frac{\partial y_j(\mathbf{p})}{\partial p_i}\right|$ is satisfied because $\frac{\partial y_j(\mathbf{p})}{\partial p_i} = \frac{\partial y_i(\mathbf{p})}{\partial p_j}$.

[12] This result is obtained in a context where the market structure is fixed: there is no entry or exit. It may not hold when the number of firms is allowed to vary.

1. In the first stage, the regulator chooses $RC(\boldsymbol{\nu})$ to maximize $W\left(d(RC(\boldsymbol{\nu}), \boldsymbol{\nu})\right)$ subject to (2.1).

   - Exogenous change occurs $\boldsymbol{\nu} \rightarrow \boldsymbol{\nu}'$,
   - *No commitment case:* the regulator reviews the regulatory contract and changes it to $RC(\boldsymbol{\nu}')$.
   - *Commitment case:* the initial contract $RC(\boldsymbol{\nu})$ is in place despite the changed environment.

2. In the second stage, the firm chooses $d(RC(\boldsymbol{\nu}'), \boldsymbol{\nu}')$ in case of no commitment and $d(RC(\boldsymbol{\nu}), \boldsymbol{\nu}')$ in case of commitment to maximize $\pi\left(\mathbf{p}, \mathbf{x}\right)$ subject to (3.2).

Figure 1 illustrates the commitment and no commitment cases.

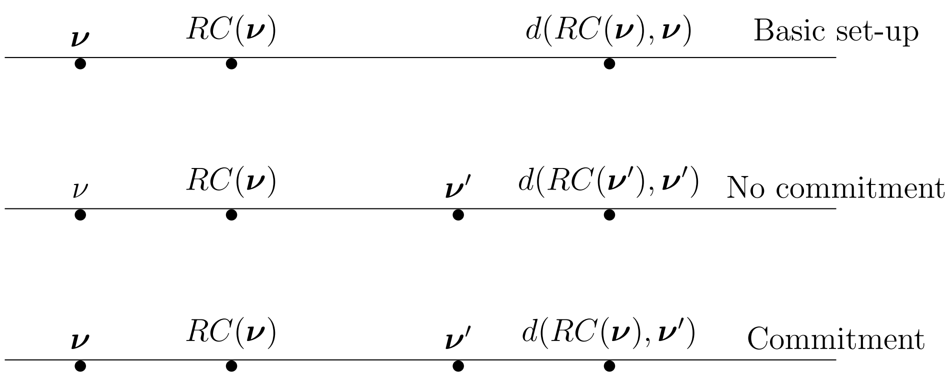

Figure 1: Commitment vs. no commitment

Because the regulator acts before the firm, she anticipates how the regulatory contract she designs in the first stage will affect firm's behavior at the second stage. The firm is the residual claimant to any private benefits from innovation, and is free to choose its innovation investments. This freedom is at the heart of PCR and of incentive regulation in general. In reality, the regulator does not guarantee that the firm will achieve the profit target T. Rather, the regulator designs the regulatory contract such that it is feasible for the firm to achieve this target. After that, it is up to the firm to make optimal decisions.

Commitment to the regulatory contract depends on the timing of regulatory hearings. In our symmetric information setting commitment to regulatory contract is sub-optimal given the negligible costs of such hearings. However, in practice the price-cap reviews may have regular dates; in the UK it is typically 4-5 years (see Guthrie, 2006). In developing countries exogenous shocks on demand or costs led to the early renegotiation of 38% regulatory price-cap contracts (Guasch et al., 2003). It is worth noting that we use a strong notion of commitment. Most price cap plans in practice have an exogenous factor (generally, a Z – factor) that allows for changes in the price cap for events outside of the firm's control. These may be tax changes, changes in the laws governing its operation, etc. Moreover, the regulatory contract may be complex. For example, the regulator may commit not to reduce the price cap, but makes no commitment regarding the level of wholesale prices. Hence, the regulator could circumvent the price cap constraint through artificially low wholesale (input) prices that render a price below the cap more profitable for the regulated firm.[13]

Our notion of commitment implies that the regulator sets a price cap $\overline{p}(\nu)$ based on the initial environment ν, and this cap remains fixed even if the environment changes to ν'. The firm then makes its innovation and pricing decisions given this pre-set (fixed) price cap. A critical aspect not fully explored in this deterministic setting is the firm's participation constraint under commitment if a severe adverse shock (ν') leads to potential losses or bankruptcy. For the scope of this paper, we assume that the range of exogenous parameter changes considered does not lead to a violation of the firm's participation constraint under commitment.

Discussion on Commitment and Asymmetric Information:

The analysis of commitment and non-commitment in this paper is conducted under the assumption of *symmetric information and certainty* regarding all model parameters. This implies that both the regulator and the firm perfectly observe and anticipate any exogenous changes in demand, costs, or the profit target. This framework allows us to isolate the effects of the regulator's commitment to the price cap itself on firm behavior and welfare, particularly concerning innovation incentives. Uncertainty and asymmetric information will be considered in Section 8.

5 Comparative statics under no commitment

We aim to clarify what values of parameters of the price cap mechanism should be used to maximize welfare while taking into account innovation incentives, how the optimal regulatory contract is affected by changes in the environment, and what are the consequences of those changes on other equilibrium variables. In this section, to fix ideas, we analyze how changes in a separate exogenous parameter affect the regulatory contract, innovation, and other endogenous variables.

1. the profit target (T);
2. parameters ν_d affecting demand functions $\mathbf{y}(\mathbf{p},\nu)$;
3. parameters ν_c affecting variable costs functions $\mathbf{c}(\mathbf{x},\nu)$;
4. parameters ν_Γ affecting fixed costs function $\Gamma(\mathbf{x},\nu)$.

In the case of any exogenous change of a parameter $\nu \in \{T,\nu_d,\nu_c,\nu_\Gamma\}$, the regulatory contract adjusts to take these changes into account. Differentiating $\pi\left(\mathbf{p},\mathbf{x};\nu\right) = T$ with respect to ν and using the first-order conditions (3.3) and (3.4) we have the following simple relation between changes in prices and the effect of the parameter.

$$\lambda\overline{p}'(\nu) + \partial_v\pi = \frac{dT}{dv}, \tag{5.1}$$

where $\frac{dT}{dv} = 1$ for $\nu = T$ and $\frac{dT}{dv} = 0$ otherwise.

[13] We thank Dennis Weisman for pointing this out.

5.1 The effect on price and innovation

Profit target (T)*:* Whereas under pure PCR profit should have no effect on the price cap, in practice many if not most plans involve adjustments to the cap if profit or the rate of return fall outside a predetermined range (sliding-scale regulation) (Braeutigam and Panzar, 1993). A change in T can be due to changes in the outside opportunities of the firm, and/or to lobbying by the firm. When such a change occurs the regulator adjusts the contract so that the price $\overline{p}$ is determined by the binding participation constraint (3.9) with a new profit target. Thus, we can determine the price cap $\overline{p}$ as a function of T and consider comparative statics with respect to T. Empirical evidence also supports the link between regulatory parameters and investment; for instance, Cambini and Rondi (2010) investigate the relationship between investment and regulatory regimes, finding that investment rates are negatively affected by the level of the X-factor in PCR. From (5.1) we obtain

$$\lambda \overline{p}'(T) = 1. \tag{5.2}$$

Denoting by $CS(T) = CS(\boldsymbol{p}(\overline{p}(T)))$, $W(T) = W(\boldsymbol{p}(\overline{p}(T)))$ and using Lemma 1 we have

Proposition 1. *Consider an exogenous increase in the profit target T. At the no-commitment equilibrium:*

(a) The optimal price cap $\overline{p}$ (the optimal innovation level x) increases (decreases);

(b) Consumer surplus and welfare decrease.

As T increases, it is necessary to increase $\overline{p}$ to achieve a higher profit target. However, as T increases, x decreases. This is because the higher price associated with a higher T reduces output. This reduction in output reduces the value of cost reduction to the firm, reducing innovation. Thus, providing the firm with stronger incentives by allowing it to charge a higher price and make more profit actually *reduces* innovation.

It is straightforward to understand how the increase in T affects the other endogenous variables of the model. The increase in T, by increasing p, reduces output, consumer surplus, and welfare, all while increasing profit. Moreover, the decrease in x increases production costs. Hence, if the regulator could choose T, she should set it as small as possible, ideally $T = 0$ (considering the firm must at least break-even).

Demand parameters, $\boldsymbol{y}(\boldsymbol{p}, \nu)$*:* Demands for products may change gradually between reviews. Liston (1993) notes that the price cap and the X−factor must be adjusted not only for technological progress, but also for growth in demand. Factors affecting demand include a change in substitutability of goods, a general shift of demand, change in elasticity, etc.[14]

Consider a factor ν affecting the demand function. In this case, $\pi(\mathbf{p}, \mathbf{x}; \nu) = (\mathbf{p}(\nu) - c(\mathbf{x}(\nu)))\mathbf{y}(\mathbf{p}(\nu), \nu) - \Gamma(\mathbf{x}(\nu))$ and from (5.1) we obtain

$$\lambda \overline{p}'(\nu) + (\mathbf{p}(\nu) - c(\mathbf{x}(\nu)))\partial_v \mathbf{y}(\mathbf{p}(\nu), \nu) = 0,$$

where $\partial_v \mathbf{y}(\mathbf{p}(\nu), \nu) = (\partial_\nu y_1, \cdots, \partial_\nu y_n)$. Thus, we have:

[14] Brennan and Crew (2016) study PCR under declining demand, focusing on the postal sector.

Proposition 2. *Consider an exogenous change in a demand parameter ν. At the no-commitment equilibrium an increase in demand (e.g., an upward shift of the demand curve) leads to a lower optimal price cap ($\overline{p}$) and more innovation x.*

Suppose the increase of parameter ν increases the demand for all prices. As demand increases, the regulator can afford to reduce prices while meeting the profit target. Output increases because of the higher demand and lower prices, increasing the value of cost reduction, leading the firm to increase x.

Example 1. *In the case of two products consider the linear inverse demand functions*

$$p_i\left(y_i, y_j\right) = A - y_i - by_j, i, j = 1, 2. \tag{5.3}$$

We invert the system in (5.3) to obtain the demand functions:

$$y_i\left(p_i, p_j\right) = \frac{A(1-b) - p_i + bp_j}{1-b^2}.$$

A change in parameter ν can represent the shift of the intercept A or changes in the degree of product substitutability b. An increase in the intercept A implies higher demand for all prices. An increase in the degree of product substitutability b (for substitutes, i.e., $b > 0$) implies a decrease in demand for a given price. Thus, an increase in A leads to lower prices and more innovation, whereas an increase in b leads to higher prices and less innovation.

Variable costs parameters, $\mathbf{c}(\mathbf{x}, \nu)$: Let a factor ν affect the cost function. In this case, $\pi(\mathbf{p}, \mathbf{x}; \nu) = (\mathbf{p}(\nu) - c(\mathbf{x}(\nu)), \nu)\mathbf{y}(\mathbf{p}(\nu)) - C(\mathbf{x}(\nu))$ and from (5.1) we obtain

$$\lambda\overline{p}'(\nu) - \partial_\nu c(\mathbf{x}(\nu), \nu)\mathbf{y}(\mathbf{p}(\nu)) = 0,$$

where $\partial_\nu \mathbf{c}(\mathbf{p}(\nu), \nu) = (\partial_\nu c_1, \cdots, \partial_\nu c_n)$. Thus, an increase in variable costs (e.g., an upward shift of the cost function) leads to a higher optimal price cap ($\overline{p}$). An increase in variable costs leads to less innovation (x).

As production costs increase, the regulator is forced to raise the price cap to achieve the profit target. This increase in prices reduces output, inducing the firm to reduce innovation. The opposite is true in case of spillovers. As spillovers increase, the equilibrium price decreases. This is because the higher spillovers reduce production costs, making it possible to meet the profit target at a lower price. This lower price increases output, inducing an increase in innovation. Thus, as spillovers increase, price decreases and innovation increases, while c decreases.[15] Spillovers have a beneficial effect in this model because there is no competition: the firm is a monopolist and all spillovers are intra-firm. As Linhart and Radner (1992:22) note: "the incentive to innovate might actually be greater in a pure monopoly with price-cap regulation."If spillovers were accruing to a competitor, they might reduce innovation and call for higher prices and lower output.[16] The positive nature of spillovers, as assumed, is critical for these beneficial effects.

[15] We will show that CS and W increase.

[16] This basic idea traces its origins back to Schumpeter.

Example 2. *Suppose marginal cost functions have the form*

$$c_i(x_i, x_j) = \alpha - x_i - \beta x_j,$$

where $\beta \in [0,1]$ is a spillover rate and α is initial marginal cost. Changes in parameters α and β represent exogenous changes in marginal costs having the opposite effects on prices and innovations.

Fixed costs parameters, $\Gamma(\boldsymbol{x}, \nu)$: A fixed cost parameter can represent, among other things, fixed production costs, or the cost of servicing the debt and equity capital. As Joskow (2006) notes, capital costs play an important role in determining the initial level of the price cap and the X factor.

Let ν be a fixed cost parameter. In this case, $\pi(\mathbf{p}, \mathbf{x}; \nu) = (\mathbf{p}(\nu) - c(\mathbf{x}(\nu)))\mathbf{y}(\mathbf{p}(\nu)) - \Gamma(\mathbf{x}(\nu), \nu)$ and from (5.1) we obtain

$$\lambda \overline{p}'(\nu) - \partial_\nu \Gamma(\mathbf{x}(\nu), \nu) = 0.$$

Thus, an increase in fixed costs leads to a higher optimal price cap ($\overline{p}$) and less innovation (x).

The higher fixed cost forces the regulator to raise the price cap to allow the firm to achieve the profit target. The higher prices reduce output and the value of cost reduction. This induces the firm to reduce its $R\&D$ spending, increasing production costs and also calling for higher prices. The higher prices and higher costs translate into lower output, consumer surplus and welfare.

Example 3. $\Gamma(x; \gamma, F) = \gamma \sum_{i=1}^{2} x_i^2 + F$, *where the parameter $F > 0$ is a fixed production cost and the parameter $\gamma > 0$ is the research cost.*

A higher F forces the regulator to increase the price cap to achieve the profit target. It is interesting that an exogenous increase in F increases equilibrium variable costs, even with constant returns to scale.

5.2 The effect on welfare, consumer surplus and profit

Let a parameter $\nu \in \{T, \nu_d, \nu_c, \nu_\Gamma\}$. For consumer surplus $CS(\nu) = CS(\mathbf{p}(\overline{p}(\nu)))$ we have

$$CS'(\nu) = CS'(\mathbf{p}(\overline{p}(\nu)))\overline{p}'(\nu).$$

By Lemma 1 we have $CS'(\overline{p}(\nu)) < 0$. Thus, the marginal change of the parameter affects the consumer surplus opposite to its effect on price.

To consider the effect of changes of the parameter $\nu = \nu_d$ on consumer surplus we use the Marshallian representation of the consumer surplus that directly incorporates the effect of a parameter ν on the demand function:

$$CS(\mathbf{p}, \nu) = \sum_{i=1}^{n} \int_{p_i}^{\hat{p}} y_i(t, \mathbf{p}_{-i}, \nu) dt, \tag{5.4}$$

where $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$ and $y_i(\hat{p}, \mathbf{p}_{-i}, \nu) = 0$.[17] Then we have for equilibrium consumer surplus

$$CS(\nu) = CS(\mathbf{p}(\overline{p}(\nu)), \nu) = \sum_{i=1}^{n} \int_{p_i(\overline{p}(\nu))}^{\hat{p}} y_i(t, \mathbf{p}_{-i}(\overline{p}(\nu)), \nu) dt. \tag{5.5}$$

Thus, the marginal change of the parameter affects the consumer surplus opposite to its effect on price.

For profit we assume that the change in parameter is $\nu' = \nu + \Delta\nu$. We denote by $\pi_{NC}(\nu, \nu + \Delta\nu)$ the optimal profit under non-commitment when the parameter ν changes to $\nu + \Delta\nu$. We also denote for future references by $\mathbf{x}^*_{NC}(\nu, \nu + \Delta\nu)$ the optimal level of innovations under non-commitment. We have from (3.10)

$$\frac{\partial \pi}{\partial T} = 1 \text{ and } \frac{\partial \pi}{\partial \nu} = 0,$$

for $\nu \in \{\nu_d, \nu_c, \nu_\Gamma\}$. Under non-commitment, the regulator instantaneously adjusts profit so that the constraint (3.10) is binding.

For the welfare function $W(\nu) = W(\overline{p}(\nu), \nu)$, using the Envelope Theorem, we obtain:

(a)
$$\frac{dW(T)}{dT} = -\mu;$$ [18]

(b) If $\nu = \nu_d$ then
$$\frac{dW(\nu)}{d\nu} = \frac{dCS(\overline{p}(\nu), \nu)}{d\nu};$$

(c) If $\nu \in \{\nu_c, \nu_\Gamma\}$, then
$$\frac{dW(\nu)}{d\nu} = (1 + \lambda) \frac{d\pi(\overline{p}(\nu), \nu)}{d\nu}.$$

Summarizing, we have

Proposition 3. *Suppose ν is a parameter affecting the operating environment (profit target or demand)*[19]*. Then, in the no-commitment case:*

(a) Consumer surplus and welfare change in the opposite direction to the optimal price cap;

(b) Welfare decreases or remains unchanged with an increase in the profit target T;

(c) Welfare increases as demand increases.

Consumer surplus and welfare increase from the lower prices and costs and higher output (apart from the direct effect on CS and W, for given $\mathbf{p}$ and $\mathbf{y}$). Except for T, profit is not affected by changes in any other parameter, since it is determined by the constraint $\pi = T$. In terms of welfare, the gains and losses from any parameter change are all absorbed by consumers.

[17] Under the assumption of quasi-linearity the Marshallian consumer surplus coincides with the equivalent and compensated variations, and, therefore, it is a proper measure of consumer surplus. Particularly, the integral in (5.4) is path-independent (see for example Vives, 2001).

[18] Where μ is the Lagrange multiplier of the profit constraint.

[19] Similar conclusions are valid for changes in variable or fixed costs

5.3 Discussion

From Section 5.1 we obtain the following:

Corollary 1. *A change in any parameter $\nu \in \{T, \nu_d, \nu_c, \nu_\Gamma\}$ moves prices and innovation in opposite directions.*

This is because a change in price moves output in the opposite direction, and the value of innovation increases with output.

Note that equilibrium prices in Example 1 are always higher when the goods are substitutes than when they are complements. This is because with complements, lower prices are needed to achieve the profit target (given the higher output). This follows directly from Proposition 2, where p increases with the parametrized level of substitutability b. This is the opposite of what an unregulated firm would do, since in that case p would decrease with b.

To compare the equilibrium R&D for complements and substitutes consider again the linear demands from Example 1. Notice that from Proposition 2 it follows that equilibrium R&D is higher under complements than under substitutes. This is because equilibrium output is higher under complements. It follows that the marginal profitability of R&D decreases as b increases. To see this, note that

$$\frac{\partial^2 \pi}{\partial x_i \partial b} = -\partial c\left(\mathbf{x}\right).\partial_b \mathbf{y} < 0.$$

As goods become closer substitutes, price is higher (because of PCR) and output is lower; this reduces the marginal gain from innovation, leading to lower R&D as b increases. An unregulated firm would also reduce R&D as b increases (as long as it is a monopolist on both products), but it would reduce prices and outputs in the process.

Changes in a firm's cost structure, encompassing both variable and fixed costs, significantly impact optimal price caps and innovation incentives under no-commitment regulation. An increase in variable costs (such as a higher base marginal cost) or fixed costs (like higher fixed production costs, or increased research costs) compels the regulator to raise the price cap to ensure the firm meets its profit target. This higher price reduces output, which, due to the Arrow effect, diminishes the firm's incentive for cost-reducing innovation. Conversely, beneficial changes such as increased R&D spillovers reduce variable costs, allowing the regulator to set a lower price cap. This lower price, in turn, boosts output and stimulates innovation. These dynamics highlight how regulatory adjustments in response to cost changes are driven by maintaining the firm's financial viability while simultaneously influencing its investment behavior.

6 Comparative statics under commitment

In this section, we assume that the regulator can fully commit to the contract. When an exogenous parameter ν changes, the firm will have to adjust to those changes until the next review, but cannot change prices. A higher T is a safeguard against unforeseen circumstances which may unduly reduce profit. For unchanged parameters, the level of innovation, and all other endogenous variables (including p), are identical under commitment and non-commitment.

We assume such change $\nu' = \nu + \Delta\nu$ occurs after the regulator sets $\overline{p}$, but before the firm chooses innovation. In this setting, $\overline{p} = \overline{p}(\nu)$ is determined by (3.9) based on the "old" parameter value ν, while innovation levels are solutions of the following program,

$$\pi_C(\nu, \nu + \Delta\nu) = \max_{\mathbf{x}} \pi(\mathbf{p}(\overline{p}(\nu)), \mathbf{x}; \nu + \Delta\nu),$$

where we denote by $\pi_C(\nu, \nu + \Delta\nu)$ the optimal profit under commitment when the parameter ν changes. We also denote by $\mathbf{x}^*_C(\nu, \nu + \Delta\nu)$ the optimal level of innovations under commitment. Output is based on the $\overline{p}$ chosen by the regulator, and the firm has to serve all demand at the ongoing price; the firm is not allowed to increase prices after the regulator sets $\overline{p}$ (and given the symmetry of the model and the assumption that the profit target T is not high enough, we always have $p_i = \overline{p}$). Note that the profit target T cannot be changed under commitment. All other variables are measured at the "new" parameter values. The following Proposition summarizes the effects of marginal changes in parameters on innovations:

Proposition 4. *Under commitment, if the regulatory contract remains unchanged an increase in demand leads to more innovation and an increase in consumer surplus, profit, and welfare.*

Proof. See Appendix.

In the linear demand example, when goods become closer substitutes, quantity decreases (even with p unchanged). The firm responds by decreasing innovation, which increases costs.

Conversely, for changes in other parameters like production costs or fixed costs, the dynamics under commitment are distinct. If variable production costs decrease (e.g., due to lower raw material prices), the firm's profitability improves. With the price cap fixed, the firm has an incentive to invest more in innovation to further reduce costs and boost its profits, leading to increased welfare. Similarly, if research costs decrease, this directly reduces the cost of innovation, leading the firm to undertake more R&D, which again translates into higher profits and welfare, without affecting consumer surplus or output directly (as prices are held constant). In essence, under commitment, the firm largely absorbs the effects of cost-related exogenous changes, adjusting its innovation efforts and profit levels, while consumers remain shielded by the fixed price.

7 Comparison between commitment and non-commitment

Comparing Propositions 1 – 4, we see that the effects of changes in parameters are quite different between the commitment and non-commitment cases. Changes in parameters under non-commitment always affect price and consumer surplus, and generally have no effect on profit (except for T). Such changes are mostly absorbed by the consumer, the firm is shielded from them. Under commitment, changes have much less effects on the consumer or on output (except for parameters of demand), but always affect profit and welfare. Such changes are mostly absorbed by the firm; the consumer is more shielded from them.

It is useful to compare welfare variations following changes in parameters with and without commitment. Comparing Propositions 3 and 4 shows that the direction of change in welfare following a change in parameters is the same with and without commitment.

Consider a parameter that affects profit under *commitment* in a positive way. More precisely, let us define:

Definition 1. *Fix $\nu \in \{v_d, v_c, v_\Gamma\}$. A change of parameter $\nu' = \nu + \Delta\nu$ is called π_C-positive if*

$$\pi_C(\nu, \nu + \Delta\nu) \geq \pi_{NC}(\nu, \nu) = T.$$

If the change of a parameter is π_C-positive, then the commitment profit is greater than the initial profit.

Example 4.

- *An upward shift of the demand curve $\nu_d + \Delta\nu$ is π_C-positive if $\frac{\partial y(\mathbf{p},\nu)}{\partial \nu_d} \geq 0$ for all $\nu \in [\nu_d, \nu_d + \Delta\nu]$. For example an increase in A for the linear demand is π_C-positive,*
- *A decrease in research costs, such as decrease in α or increase in the spillover rate β, is π_C-positive,*
- *A decrease in fixed costs, such as decrease in constant fixed cost F or decrease in research cost γ, is π_C-positive.*

Under *non-commitment*, following a π_C-positive change in a parameter, the regulator can set the initial price. In that case, everything will be the same for the firm as under commitment. It will choose optimally the innovation levels and the profit will be larger than T. That is, the constraint (2.1) is satisfied. Therefore, the welfare level under *non-commitment* will be the same as under *commitment*. However, the regulator is not constrained to choose a specific price in the case of non-commitment. Therefore, optimal welfare under non-commitment will be at least as good as under commitment.[20]

Proposition 5. *Consider a parameter $\nu \in \{\nu_d, \nu_c, \nu_\Gamma\}$ and a π_C-positive change of this parameter, denoted as $\nu' = \nu + \Delta\nu$. In this scenario, we observe the following:*

- *Welfare under non-commitment is greater than or equal to welfare under commitment, $W_{NC}(\nu + \Delta\nu) \geq W_C(\nu, \nu + \Delta\nu)$;*
- *The optimal level of innovation under non-commitment is greater than or equal to the optimal level of innovation under commitment, $\mathbf{x}^*_{NC}(\nu, \nu + \Delta\nu) \geq \mathbf{x}^*_C(\nu, \nu + \Delta\nu)$.*

There is a general view that commitment is good, and lack of commitment is bad, under PCR. For example, Sappington (2002:244) writes: "the potential gains from regulatory policies like price cap regulation may be minimal in settings where regulators cannot credibly promise to abide by the terms of the announced policy". The discussion in the "commitment" literature focuses on settings with asymmetric information. In such settings, a regulator's expected payoff is always (weakly) higher when she has unlimited commitment powers than

[20] Note that this argument is very general and does not depend on the assumptions of a specific model.

when she has no commitment powers.[21] The idea that lack of commitment is detrimental to innovation incentives under PCR is widespread, and probably holds in many circumstances. However, the model shows that in some cases lack of commitment may yield higher innovation incentives following a change in (some) parameters. Consider for example the effect of an increase in spillover rate β. In both cases innovation increases following an increase in spillovers. For the linear example the increase in innovation following the increase in β is greater in the absence of commitment. This is because absent commitment, prices are pushed downward by the regulator, which increases output. However, under commitment, prices and hence outputs do not change. The increase in output under non-commitment gives an additional incentive for the firm to increase innovation.

Consider a change in a parameter ν such that the commitment profit is smaller than the initial profit. Then, in the case of non-commitment the regulator cannot set the old price because the profit would be lower than T. It is always optimal, in the case of non-commitment, to set the profit equal to T. This can be achieved by increasing the price cap which will lead by Lemma 1 to lower consumer surplus. In the case of commitment, following the increase of the parameter, consumer surplus remains constant, but profit decreases. In both cases welfare decreases. Which welfare decreases more may depend on the parameters of the model.[22]

8 Informational Imperfections

8.1 Incorporating Risk (Uncertainty)

Our current model assumes that all exogenous parameters are known with certainty. In reality, future demand, production costs, or R&D effectiveness can be uncertain. Introducing uncertainty necessitates a shift from a deterministic to a stochastic framework, where agents maximize expected values. This section explores how the model's dynamics change when the firm faces demand uncertainty and is risk-averse, while the regulator remains risk-neutral.

8.1.1 Model Setup

For analytical clarity, we simplify our framework to focus on a single representative product. The market demand for this product is characterized by a linear function, $y(p, \tilde{\theta}) = A - p + \tilde{\theta}$, where $A > 0$ denotes a baseline demand level. A critical feature of this setting is the presence of demand uncertainty, introduced through $\tilde{\theta}$, a random variable representing an exogenous demand shock. We assume $\tilde{\theta}$ follows a normal distribution with an expected value of zero ($E[\tilde{\theta}] = 0$) and a variance of $\sigma^2 > 0$, capturing the inherent volatility in market conditions.

The firm's production involves a unit cost, $c(x) = c_0 - x$, which can be reduced through cost-reducing innovation investments, x. The cost associated with these innovation efforts is modeled as $\Gamma(x) = \frac{1}{2}\gamma x^2$, with c_0 and γ being positive constants. Consequently, the firm's total profit, $\pi(p, x, \tilde{\theta})$, is given by the expression: $(p - c_0 + x)(A - p + \tilde{\theta}) - \frac{1}{2}\gamma x^2$.

[21] We thank David Sappington for pointing this out.

[22] For concrete example in the linear demand case see Appendix 10.3.

A crucial departure from our core model is the firm's risk-averse nature. Its preferences are captured by a Constant Absolute Risk Aversion (CARA) utility function, $U(\pi) = -e^{-\rho\pi}$, where $\rho > 0$ signifies the coefficient of absolute risk aversion. For such a firm, the objective is to maximize the certainty equivalent (CE) of its profit, defined as $CE = E[\pi] - \frac{1}{2}\rho Var(\pi)$, which effectively penalizes profit volatility. In contrast, the regulator is assumed to be risk-neutral, aiming to maximize expected social welfare, $E[W]$, which is the sum of expected consumer surplus and expected firm profit ($E[W] = E[CS] + E[\pi]$). Consumer surplus for this linear demand is $CS(p, \tilde{\theta}) = \frac{1}{2}(A - p + \tilde{\theta})^2$. Finally, the firm's participation is contingent on achieving a minimum profit target, $T \geq 0$.

Timing:

1. Regulator sets the price cap $\overline{p}$ (under commitment) or a rule for adjustment (under non-commitment);

2. The demand shock $\tilde{\theta}$ is realized and observed by both;

3. The firm chooses its innovation level x. The price is fixed at $p = \overline{p}$.

Proposition 6. *With uncertainty we have:*

(a) ***Arrow Effect Persistence:*** *The Arrow effect, where a higher price cap reduces innovation incentives, still holds. However, the magnitude of this effect is modified by the firm's risk aversion and the level of uncertainty;*

(b) ***Re-emergence of Static-Dynamic Efficiency Trade-off:*** *A trade-off emerges between static efficiency (low prices) and dynamic efficiency (innovation). To ensure the risk-averse firm's participation and compensate it for bearing risk, the regulator may need to allow a higher price cap, which consequently reduces innovation;*

(c) ***Firm's Preference for Non-Commitment:*** *If the regulator operates under non-commitment and can perfectly adapt the price cap to absorb shocks and guarantee the firm's target profit, the risk-averse firm strongly prefers this regime as it eliminates profit volatility. This reinforces the notion that commitment is not universally superior, as certainty of profit under an adaptive regulator can be highly valuable to a risk-averse firm.*

Proof. See Appendix 10.4.

The core implications arise from how this structure affects the participation constraint and risk allocation:

- **Trade-off Between Static and Dynamic Efficiency Re-emerges:** The regulator must ensure the firm's participation ($CE \geq T$). For a risk-averse firm, compensation for bearing risk (which increases with output, as output amplifies the demand shock) might necessitate a higher price cap $\overline{p}$. This higher $\overline{p}$ would, due to the Arrow effect, lead to reduced innovation. Thus, a trade-off emerges between ensuring firm viability/risk compensation and achieving low prices (static efficiency) and high innovation (dynamic efficiency);

- **Firm's Preference for Non-Commitment (Risk Shielding):** Under a non-commitment regime, if the regulator can observe the shock $\tilde{\theta}$ and adjust $\overline{p}$ accordingly to guarantee the firm's profit exactly at T, the firm's profit becomes deterministic ($\pi = T$). In this scenario, the risk-averse firm bears no risk, and its risk aversion does not directly influence its innovation decision. A risk-averse firm would strongly prefer such a non-commitment regime because it completely shields it from profit volatility, even if it leads to different (potentially lower expected) outcomes for consumers who absorb the fluctuations through price adjustments.

8.2 Incorporating Asymmetric Information

Our current model assumes that all parameters are perfectly observable by both the firm and the regulator. However, in reality, firms often possess private information about their cost structure, R&D capabilities, and demand conditions. This section explores how introducing such asymmetric information, specifically about demand conditions, would alter the model and its implications. For analytical clarity in this discussion, we adopt a simplified setting with a single product and a risk-neutral firm, as combining asymmetric information with risk aversion simultaneously introduces excessive complexity beyond the scope of a conceptual discussion.

The challenge for the regulator is to design a regulatory contract that induces the firm to reveal its private information truthfully and to make socially desirable decisions, despite this informational asymmetry. This contrasts with our previous model which assumed symmetric information, simplifying the regulator's problem of anticipating firm behavior.

8.2.1 Model Setup with Asymmetric Information

We adopt a simplified single-product setting for analytical clarity. The demand function is linear: $y(p, \tilde{\theta}) = A - p + \tilde{\theta}$. The demand shock $\tilde{\theta}$ is now private information of the firm. It can take one of two values:

- $\theta_H = +\delta$ (high demand type), occurring with probability q.
- $\theta_L = -\delta$ (low demand type), occurring with probability $1 - q$.

We assume $A - \overline{p} > \delta > 0$ to ensure positive quantities for both types. The firm's unit cost is $c(x) = c_0 - x$ and the cost of innovation is $\Gamma(x) = \frac{1}{2}\gamma x^2$. The firm's profit for type θ_k is:

$$\pi(p, x, \theta_k) = (p - c_0 + x)(A - p + \theta_k) - \frac{1}{2}\gamma x^2.$$

The regulator is risk-neutral and maximizes expected social welfare, $E[W]$, where $W = CS + \pi$. Consumer surplus for linear demand is $CS(p, \theta_k) = \frac{1}{2}(A - p + \theta_k)^2$.

Timing of the Game with Asymmetric Information:

1. Nature draws the firm's demand type $\theta_k \in \{\theta_L, \theta_H\}$. The firm observes θ_k privately;
2. The regulator, knowing the possible types and their probabilities, designs a menu of price-cap contracts $\{(\overline{p}_L, T_L), (\overline{p}_H, T_H)\}$, where $\overline{p}_k$ is the price cap and T_k is the minimum profit level for type k. This menu is offered to the firm;

3. The firm, knowing its type, chooses its preferred contract from the menu;

4. The firm, now operating under its chosen price cap $\overline{p}_k$, optimally chooses its innovation level x_k^*. The price chosen by the firm will be equal to the price cap, $p_k = \overline{p}_k$.

8.2.2 Implications for the Paper's Main Lessons under Asymmetric Information

The introduction of asymmetric information fundamentally changes the economic mechanisms and has profound implications for the paper's main lessons.

Proposition 7. *In the second-best equilibrium, to induce truthful revelation and maximize expected welfare subject to the firm's participation:*

(a) ***Price Distortion:*** *The price cap for the high-demand firm ($\overline{p}_H^{SB}$) is set higher than its first-best (full information) optimal level ($\overline{p}_H^{FB}$). Conversely, the price cap for the low-demand firm ($\overline{p}_L^{SB}$) is set at its first-best optimal level ($\overline{p}_L^{FB}$);*

(b) ***Innovation Distortion:*** *The high-demand firm undertakes less innovation ($x_H^{SB} < x_H^{FB}$) than it would under full information, due to its higher price cap. The low-demand firm's innovation (x_L^{SB}) is equal to its first-best level (x_L^{FB});*

(c) ***Information Rents:*** *The high-demand firm earns a strictly positive "information rent" (profit above its participation constraint T), while the low-demand firm earns exactly its minimum required profit (T);*

(d) ***Welfare Loss and Trade-off Re-emergence:*** *Expected social welfare under asymmetric information is strictly lower than under full information. This outcome highlights an inherent trade-off between inducing truthful information revelation (which requires distortions and information rents) and achieving full static and dynamic efficiency.*

Proof. See Appendix 10.5.

These formal results lead to the following crucial implications:

- **Arrow Effect Still Holds (within contract choices):** The fundamental relationship that a higher price cap (which reduces output) discourages innovation remains valid. Given a specific regulatory contract, the firm's response to price incentives for innovation will still follow the Arrow effect. However, the choice of the price cap itself will now be influenced by asymmetric information;

- **Re-emergence of the Static-Dynamic Efficiency Trade-off:** This is a crucial divergence from our finding of no inherent trade-off under symmetric information. With asymmetric information, the regulator cannot directly observe the firm's true demand conditions. To induce truthful revelation and prevent the high-demand type from mimicking the low-demand contract, the regulator typically has to distort the contract for the high-demand type. This distortion often involves setting a higher price cap (and thus lower output) for the high-demand type than would be socially optimal under full information. While this limits information rents, it simultaneously reduces innovation by the high-demand firm. Therefore, the regulator faces a direct trade-off: pursuing greater static efficiency (lower prices) may come at the cost of higher information rents, while attempting to reduce information rents through contract distortion (e.g., higher prices) sacrifices static efficiency and, consequently, dynamic efficiency (innovation). This means that a trade-off between static and dynamic efficiency becomes inherent in the presence of private information.

- **Commitment Becomes Crucial and Unambiguously Superior:** In an environment with asymmetric information, the regulator's ability to commit to the announced menu of contracts is paramount:
  - **Under Commitment with Asymmetric Information:** The regulator announces the menu of contracts ex-ante and adheres to it. This commitment ensures that the firm, by revealing its type (implicitly through contract choice), can trust that the terms will not be retrospectively altered. This certainty is essential for the incentive compatibility constraints to hold and for firms to undertake long-term innovation investments;
  - **Under Non-Commitment with Asymmetric Information (Ratchet Effect):** If the regulator cannot commit, they would be tempted to revise the contract ex-post once the firm's type is revealed (e.g., by its contract choice or observed performance), potentially tightening the price cap or reducing the allowed profit for a high-demand firm. This opportunistic behavior, known as the "ratchet effect," (see for example Laffont and Tirole, 1993) would lead firms to strategically under-innovate or conceal their efficiency to avoid future disadvantageous revisions. Anticipating this, firms would have a strong disincentive to innovate. Thus, commitment becomes unambiguously superior for fostering dynamic efficiency (innovation) in the presence of asymmetric information, contrasting our initial finding in a symmetric information context that commitment is not always better.

Remark 2. *Proposition 7 reveals an interesting departure from the standard "no distortion at the top" principle in many adverse selection models. Typically, the efficient type (e.g., low-cost firm, or high-productivity agent) is offered a contract that induces its first-best quantity or effort level, while the inefficient type's quantity is distorted downwards to reduce the information rent of the efficient type (see for example Laffont and Tirole, 1993 and Bolton and Dewatripont, 2004). However, in this specific model, the distortion pattern is indeed reversed for quantities/innovation.*

This pattern is a valid outcome in certain mechanism design problems, and it arises due to the specific structure of this model:

1. *Nature of Private Information: The private information is about demand type, which directly affects the firm's revenue and output, rather than just its cost structure. A high-demand type is inherently more profitable for any given price;*

2. *Regulatory Instrument: The regulator controls the price cap, not directly the quantity or innovation. The firm then chooses quantity and innovation given the price cap;*

3. *Profit-Price Relationship: The firm's profit is an increasing function of the price cap;*
4. *Information Rent: The high-demand firm (efficient type) earns a positive information rent. To provide this rent, the regulator must offer a contract that yields a higher profit for the high-demand type than its reservation profit T. Since profit is increasing in the price cap, the regulator must set a higher price cap for the high-demand type to deliver this required profit (the information rent). This higher price cap, in turn, reduces output and innovation for the efficient firm.*

This model illustrates that the "no distortion at the top" principle for quantities is not universal and depends on the specific nature of asymmetric information and the regulatory instruments available. Here, the low-demand type receives its first-best contract, while the high-demand type's contract is distorted to manage its information rent.

9 Conclusions

This paper has explored the relationship between Price Cap Regulation (PCR), regulatory commitment, and innovation incentives. Our findings challenge two conventional wisdoms in regulatory economics, particularly when considering real-world complexities like informational imperfections.

Initially, our core model, assuming perfect information, suggested there's no inherent trade-off between static and dynamic efficiency. Counter-intuitively, we found that allowing firms to charge higher prices and earn greater profits actually reduces innovation. This is because higher prices lead to lower output, diminishing the firm's incentive to invest in cost-reducing R&D – a phenomenon consistent with the Arrow effect. We concluded that maximizing overall welfare and innovation is best achieved by targeting lower prices and, consequently, lower profits for the regulated firm.

However, recognizing that regulatory environments are rarely perfect, we extended our analysis to incorporate crucial informational imperfections. Under demand uncertainty where the firm is risk-averse, the familiar static-dynamic efficiency trade-off reappears. To ensure a firm's financial viability and compensate it for bearing risk, the regulator may need to permit higher prices, which, in turn, dampens innovation. Furthermore, in scenarios of asymmetric information about firm types, the trade-off becomes inherent. To encourage truthful information revelation, regulators often must distort contracts, leading to higher prices and reduced innovation for more efficient firms.

The role of regulatory commitment also presents a nuanced picture. While our symmetric information model showed its benefits to be ambiguous – sometimes better, sometimes not – commitment becomes unambiguously crucial for fostering innovation under asymmetric information. Without it, firms anticipate opportunistic regulatory adjustments (the "ratchet effect"), leading them to under-innovate to avoid future penalties. Conversely, a risk-averse firm might actually prefer a non-commitment regime if it offers greater risk-shielding from unpredictable demand shocks.

In essence, our work demonstrates that the optimal design of PCR, and the effectiveness of commitment, critically depends on the underlying informational environment. Policymakers must carefully consider the specific context of uncertainty and private information when designing regulatory frameworks to truly foster innovation and maximize social welfare.

10 Proofs

10.1 Proof of Lemma 1

a) Follows from (3.7).

b) Because $\frac{\partial CS}{\partial p_i} < 0$, we have $CS'(\overline{p}) = \partial_p CS(\mathbf{p}(\overline{p}))\mathbf{p}' = \sum_{i=1}^{n} \frac{\partial CS}{\partial p_i} < 0$, where $\mathbf{p}'= (1, 1, ..., 1)$. Differentiating $W(\overline{p}) = W(\mathbf{p}(\overline{p}), \mathbf{x}(\overline{p}))$ we obtain

$$W'(\overline{p}) = (\partial_p W)\mathbf{p}'(\overline{p}) + (\partial_x W)\mathbf{x}(\overline{p}) = \sum_{i=1}^{n} \frac{\partial W}{\partial p_i},$$

where $\partial_x W = \partial_x(CS + \pi) = \partial_x \pi = 0$ by the first-order condition.

We have:

$$\frac{\partial W}{\partial p_i} = \sum_{j=1}^{n}(p_j - c_j)\frac{\partial y_j}{\partial p_i} = (\overline{p} - c)\sum_{j=1}^{n}\frac{\partial y_j}{\partial p_i} < 0.$$

c) Differentiating (3.4) for $i = 1$ with respect to $\overline{p}$ and using $x_j(\overline{p}) = x(\overline{p})$, we obtain:

$$x'(\overline{p})\left(\sum_{j=1}^{n}\frac{\partial^2\Gamma}{\partial x_1 \partial x_j} + \sum_{k=1}^{n} y_k \sum_{j=1}^{n}\frac{\partial^2 c_k}{\partial x_1 \partial x_j}\right) = -\sum_{j=1}^{n}\frac{\partial c_j}{\partial x_1}\left(\sum_{k=1}^{n}\frac{\partial y_j}{\partial p_k}\right).$$

Since $\sum_{k=1}^{n} \frac{\partial y_j}{\partial p_k} < 0$, $\frac{\partial c_j}{\partial x_1} < 0$, and the second cross derivatives of cost functions are positive, we have $x'(\overline{p}) < 0$.

10.2 Proof of Proposition 4

For consumer surplus under commitment we have

$$CS(\nu) = CS(\mathbf{p}(\overline{p}), \nu) = \sum_{i=1}^{n}\int_{p_i(\overline{p})}^{\hat{p}} y_i(t, \mathbf{p}_{-i}(\overline{p}), \nu)dt. \tag{10.1}$$

Thus, the sign of $\frac{dCS(\nu)}{d\nu}$ depends on the direction of $\partial_\nu \mathbf{y}(\mathbf{p}, \nu)$.

For profit we have: $\pi(\mathbf{p}, \mathbf{x}; \nu) = (\mathbf{p} - \mathbf{c}(\mathbf{x}))\mathbf{y}(\mathbf{p}, \nu) - \Gamma(\mathbf{x})$ and by the Envelope theorem we have $\pi'(\nu) = (\mathbf{p} - \mathbf{c}(\mathbf{x}))\partial_\nu \mathbf{y}(\mathbf{p}, \nu)$.

10.3 Comparison between commitment and non-commitment in linear demand case

Consider settings from Examples 2 and 3. Table 1 quantifies welfare changes for a 10% increase in each parameter, with and without commitment, with both substitutes and complements.[23] The table shows that for all parameters, welfare changes in relative terms (and also in absolute terms, although this is not shown in the table) are *always larger under non-commitment than under commitment.*

Table 1: Relative welfare changes under commitment and non-commitment

	Substitutes (%)		**Complements** (%)	
	Non-commitment	Commitment	Non-commitment	Commitment
β	0.042	0.039	0.28	0.26
b	-0.87	-0.86	6.39	6.35
A	11.61	11.55	23.32	23.24
α	-5.03	-4.86	-10.07	-10
γ	-0.06	-0.05	-0.37	-0.35
F	-0.34	-0.31	-0.21	-0.20

The largest changes are induced by changes in the demand intercept and marginal cost, while the smallest changes are induced by spillovers and research costs.[24]

Comparing substitutes and complements, welfare changes are almost always larger under complements than under substitutes; except for the effect of F, where the opposite holds. This result holds both under non-commitment and under commitment. This is because with complements, the change in the cost or demand attributes of one good amplify the effect of the change in the cost or demand attributes of the other good. Whereas with substitutes, these changes tend to partially cancel each other. For F the result is different, because it is a purely fixed cost (although it does increase prices under non-commitment), and welfare is higher under complements, thus the relative change in welfare following an increase in F is larger under substitutes (this is confirmed by the fact that the change in absolute terms in welfare following an increase in F is larger under substitutes than under complements with non-commitment, and is exactly the same (equal to ΔF) under commitment).

From Table 1, it can be seen that the relative increase in welfare following an increase in β is higher under non-commitment than under commitment, under both substitutes and complements. A similar effect can be seen for the increase in A. A decrease in b or in γ (both increase innovation) would induce larger innovation and welfare increases under

[23]The base configuration is: $\beta = 0.5; b = 0.5$(with substitutes); $b = -0.5$(with complements); $A = 1000; \alpha = 500; \gamma = 60; F = 20,000$. Each parameter is increased by 10% in absolute value from the base configuration. For b, the effect is negative under substitutes because in that case b is increased to 0.55, while the effect is positive under complements because in that case b is "increased" to -0.55. Income is held fixed at 500,000.

[24]This observation may depend on the specific numbers chosen for some parameters; given the 10% increase in all parameters, a higher initial value for a parameter will result in a more significant change in welfare in percentage terms.

non-commitment than under commitment. There are some cases where the ratchet effect, surprisingly, boosts innovation.

To clarify this point further, Table 2 illustrates changes in innovation x following increases and decreases (10%) in parameters under commitment and non-commitment.[25]

Table 2: Comparison of innovation changes under commitment and non-commitment

	Non-commitment (%)	**Commitment (%)**
$\beta = 0.55$	**3.428**	3.333
$\beta = 0.45$	-3.419	**-3.333**
$b = 0.55$	-3.502	**-3.226**
$b = 0.45$	**3.746**	3.448
$A = 1100$	**22.595**	21.083
$A = 900$	-23.415	**-21.084**
$\alpha = 550$	-11.558	**0**
$\alpha = 450$	**11.364**	0
$\gamma = 66$	-9.203	**-9.091**
$\gamma = 54$	**11.279**	11.111
$F = 22000$	-0.730	**0**
$F = 18000$	**0.719**	0

In each row, the number in bold represents the choice (between commitment and non-commitment) which yields a larger increase in innovation or a smaller decrease in innovation, when an exogenous parameter changes. Consider for example the effect of a change in β. The increase in β induces a larger increase in innovation under non-commitment than under commitment, because under non-commitment prices are forced down, outputs increase, which increases the value of cost reduction and increases innovation "more". But when β decreases, by the same token, innovation decreases more under non-commitment than under commitment, because prices are allowed to increase, which reduces outputs, and thus reduces innovation incentives[26] "more" than under commitment. From the point of view of maximizing innovation incentives, non-commitment is better for increases in spillovers, while commitment is better for decreases in spillovers. Thus it cannot be said that commitment is universally better in fostering innovation. Even for a parameter like A, which changes output, the change in output is more important under non-commitment (because the price declines), inducing a greater increase in innovation under non-commitment than under commitment when A increases, but a smaller decrease in innovation under commitment than under non-commitment when A decreases.

The effects of the other parameters can be understood in the same manner. In table 2, innovation does not respond to changes in α or F under commitment because in that case, when those two parameters change, the marginal gain from innovation is not affected. Reading tables 1 and 2 jointly confirms that when innovation changes more, welfare changes more (in percentage terms). In table 1 only parameter increases are presented, while in table

[25]We use the same base configuration as in Table 1.

[26]Remember that absent parameter changes, all endogenous variables take exactly the same values with and without commitment.

2 both increases and decreases in parameters are presented to make the results more explicit. Thus, commitment is neither always better nor worse, for innovation, profit, consumers surplus, or welfare. The results in table 2 are presented only for substitutes, but qualitatively similar results hold for complements.

Many parameters are equally likely to increase or decrease between reviews (unlike demand, which is more likely to increase over time, at least for non-mature products and services), and it cannot be said that commitment is universally better than non-commitment. In many cases, the firm itself may prefer non-commitment to commitment.

Even if one builds a model where the level of innovation is positively related to prices, it will remain the case that when a change in a parameter induces a greater increase in innovation (say) under commitment (versus non-commitment), then the opposite change in the same parameter will induce a greater increase in innovation under non-commitment (versus commitment). Hence, it will still be true that commitment is not universally better than non-commitment.

10.4 Proof of Proposition 6

Firm's Problem and Optimal Innovation: In the second stage, given $\overline{p}$ and observed $\tilde{\theta}$, the firm chooses x to maximize its certainty equivalent (CE). The profit function (with $p = \overline{p}$) is:

$$\pi(\overline{p}, x, \tilde{\theta}) = (\overline{p} - c_0 + x)(A - \overline{p} + \tilde{\theta}) - \frac{1}{2}\gamma x^2.$$

The expected profit is:

$$E[\pi] = (\overline{p} - c_0 + x)(A - \overline{p}) - \frac{1}{2}\gamma x^2.$$

The variance of profit is:

$$Var(\pi) = Var((\overline{p} - c_0 + x)\tilde{\theta}) = (\overline{p} - c_0 + x)^2 Var(\tilde{\theta}) = (\overline{p} - c_0 + x)^2 \sigma^2.$$

The firm maximizes:

$$CE = (\overline{p} - c_0 + x)(A - \overline{p}) - \frac{1}{2}\gamma x^2 - \frac{1}{2}\rho(\overline{p} - c_0 + x)^2 \sigma^2.$$

The first-order condition with respect to x is:

$$\frac{\partial CE}{\partial x} = (A - \overline{p}) - \gamma x - \rho(\overline{p} - c_0 + x)\sigma^2 = 0.$$

Rearranging terms to solve for x, the optimal innovation level is:

$$x^*(\overline{p}) = \frac{A - \overline{p} - \rho\sigma^2(\overline{p} - c_0)}{\gamma + \rho\sigma^2}.$$

The Arrow Effect under Risk Aversion: To evaluate the Arrow effect, we examine how innovation changes with the price cap $\overline{p}$:

$$\frac{\partial x^*}{\partial \overline{p}} = \frac{-1 - \rho\sigma^2}{\gamma + \rho\sigma^2}.$$

Since $\rho \geq 0$, $\sigma^2 \geq 0$, and $\gamma > 0$, we have $\frac{\partial x^*}{\partial \overline{p}} \leq 0$. This confirms that the Arrow effect, stating a higher price cap leads to less innovation, still holds.

Comparing the magnitude to the risk-neutral case ($\rho = 0$), where $\frac{\partial x^*}{\partial \overline{p}} = -1/\gamma$:

Let $K = \rho\sigma^2$. The magnitude of the change in innovation is $\left|\frac{-(1+K)}{\gamma+K}\right|$. The difference in magnitude compared to the risk-neutral case is $\left|\frac{1+K}{\gamma+K}\right| - \left|\frac{1}{\gamma}\right| = \frac{\gamma(1+K)-(\gamma+K)}{\gamma(\gamma+K)} = \frac{K(\gamma-1)}{\gamma(\gamma+K)}$.

- If $\gamma > 1$: The magnitude of the negative effect is larger with risk aversion. This implies risk aversion makes innovation more sensitive to price changes.
- If $\gamma < 1$: The magnitude of the negative effect is smaller with risk aversion.
- If $\gamma = 1$: The magnitude is the same.

Regulator's Problem: The risk-neutral regulator maximizes expected social welfare $E[W] = E[CS] + E[\pi]$. Expected consumer surplus is:

$$E[CS] = E\left[\frac{1}{2}(A - \overline{p} + \tilde{\theta})^2\right] = \frac{1}{2}((A - \overline{p})^2 + \sigma^2).$$

The regulator chooses $\overline{p}$ to maximize $E[W]$ subject to the firm's participation constraint, which now involves the certainty equivalent: $CE \geq T$.

10.5 Proof of Proposition 7

Firm's Optimal Innovation (Second Stage): Given a price cap $\overline{p}$ and demand type θ_k, the firm chooses its innovation level x to maximize profit:

$$\max_x \left[(\overline{p} - c_0 + x)(A - \overline{p} + \theta_k) - \frac{1}{2}\gamma x^2\right].$$

The first-order condition with respect to x is:

$$(A - \overline{p} + \theta_k) - \gamma x = 0.$$

Solving for x, the optimal innovation level for type θ_k at price $\overline{p}$ is:

$$x^*(\overline{p}, \theta_k) = \frac{A - \overline{p} + \theta_k}{\gamma}.$$

Substituting $x^*(\overline{p}, \theta_k)$ back into the profit function, we obtain the maximal profit for type θ_k at price $\overline{p}$:

$$\pi^*(\overline{p}, \theta_k) = (\overline{p} - c_0 + x^*(\overline{p}, \theta_k))(A - \overline{p} + \theta_k) - \frac{1}{2}\gamma(x^*(\overline{p}, \theta_k))^2.$$

Let $y_k(\overline{p}) = A - \overline{p} + \theta_k$, the quantity demanded at price $\overline{p}$ for type k. Substituting $x^*(\overline{p}, \theta_k) = y_k(\overline{p})/\gamma$ into the profit function:

$$\pi^*(\overline{p}, \theta_k) = (\overline{p} - c_0)y_k(\overline{p}) + \frac{1}{2\gamma}y_k(\overline{p})^2.$$

This expression $\pi^*(\overline{p}, \theta_k)$ is increasing in θ_k for a given $\overline{p}$. The derivative with respect to θ_k is $\frac{\partial \pi^*}{\partial \theta_k} = (\overline{p} - c_0)\frac{\partial y_k}{\partial \theta_k} + y_k \frac{\partial \overline{p}}{\partial \theta_k} + \frac{1}{\gamma} y_k \frac{\partial y_k}{\partial \theta_k} = \left(\overline{p} - c_0 + \frac{y_k}{\gamma}\right)\frac{\partial y_k}{\partial \theta_k} = (\overline{p} - c_0 + x_k^*) > 0$, since $\overline{p} - c_0 + x_k^*$ is the per-unit operating profit. This confirms the high-demand type is indeed more efficient at generating profit for any given price.

Second-Best Contract: The regulator chooses a menu of price caps $(\overline{p}_L, \overline{p}_H)$ to maximize expected welfare $E[W] = qW_H(\overline{p}_H) + (1-q)W_L(\overline{p}_L)$, subject to Individual Rationality (IR) and Incentive Compatibility (IC) constraints.

Constraints:

1. **IR-L**: $\pi_L^*(\overline{p}_L) \geq T$;
2. **IR-H**: $\pi_H^*(\overline{p}_H) \geq T$;
3. **IC-H**: $\pi_H^*(\overline{p}_H) \geq \pi^*(\overline{p}_L, \theta_H)$;
4. **IC-L**: $\pi_L^*(\overline{p}_L) \geq \pi^*(\overline{p}_H, \theta_L)$.

The binding constraints in the second-best optimal solution are:

1. **IR-L**: $\pi_L^*(\overline{p}_L^{SB}) = T$;
2. **IC-H**: $\pi_H^*(\overline{p}_H^{SB}) = \pi^*(\overline{p}_L^{SB}, \theta_H)$.

The other constraints (IR-H and IC-L) will be slack. The IR-H constraint is slack because the IC-H constraint implies the high-demand type earns more than T. The IC-L constraint is slack because the low-demand type would earn less by mimicking the high-demand type's contract, which is designed for higher output and lower profit margin to deter the high-demand type.

Second-Best Price Caps and Distortions:

1. **Optimal $\overline{p}_L^{SB}$:** Since IR-L binds, $\pi_L^*(\overline{p}_L^{SB}) = T$. This means the low-demand firm receives exactly its reservation profit. By setting the price such that the firm just breaks even, the regulator is achieving the first-best welfare outcome for the low type. Thus, $\overline{p}_L^{SB} = \overline{p}_L^{FB}$. There is no distortion for the low-demand type.

2. **Optimal $\overline{p}_H^{SB}$:** The key distortion arises from the binding IC-H constraint:

$$\pi_H^*(\overline{p}_H^{SB}) = \pi^*(\overline{p}_L^{SB}, \theta_H).$$

Let's expand $\pi^*(\overline{p}_L^{SB}, \theta_H)$, which is the profit of the high-demand type if it were forced to take the low-demand type's price cap $\overline{p}_L^{SB}$ (and optimize its x accordingly, i.e., $x^*(\overline{p}_L^{SB}, \theta_H)$). We know $\pi^*(\overline{p}, \theta_k) = (\overline{p} - c_0)y_k(\overline{p}) + \frac{1}{2\gamma} y_k(\overline{p})^2$. The profit for the high-demand type mimicking the low-demand type's contract is:

$$\pi^*(\overline{p}_L^{SB}, \theta_H) = (\overline{p}_L^{SB} - c_0)(A - \overline{p}_L^{SB} + \theta_H) + \frac{1}{2\gamma}(A - \overline{p}_L^{SB} + \theta_H)^2.$$

Since $\theta_H > \theta_L$, it can be shown that $\pi^*(\overline{p}_L^{SB}, \theta_H) > \pi^*(\overline{p}_L^{SB}, \theta_L) = T$. This difference, $\pi^*(\overline{p}_L^{SB}, \theta_H) - T$, represents the information rent that the high-demand type would earn if it chose the low-demand type's contract. To satisfy IC-H, the high-demand type must earn at least this profit by choosing its own contract. Therefore, the high-demand type's actual profit in the second-best solution is:

$$\pi_H^*(\overline{p}_H^{SB}) = T + \left[\pi^*(\overline{p}_L^{SB}, \theta_H) - \pi^*(\overline{p}_L^{SB}, \theta_L)\right].$$

The term in the square brackets is the information rent the high-demand type extracts due to asymmetric information.

The regulator chooses $\overline{p}_H^{SB}$ to fulfill this profit target. Critically, to reduce this information rent (and thus increase overall welfare), the regulator will distort $\overline{p}_H^{SB}$ upwards relative to its first-best counterpart, $\overline{p}_H^{FB}$. Setting $\overline{p}_H^{SB}$ higher makes the contract less attractive to the low-demand type, relaxing the IC-H constraint indirectly. This distortion implies:

$$\overline{p}_H^{SB} > \overline{p}_H^{FB}.$$

This higher price cap for the high-demand type directly leads to lower output ($y_H^{SB} < y_H^{FB}$) and, consequently, lower innovation ($x_H^{SB} < x_H^{FB}$) compared to the first-best.

We summarize: The solution under asymmetric information yields a separating equilibrium where:

- **Low-Demand Type (θ_L):** Receives a contract ($\overline{p}_L^{SB}, x_L^{SB}$) such that $\overline{p}_L^{SB} = \overline{p}_L^{FB}$. Its contract is efficient (undistorted), and it earns zero information rent ($\pi_L^{SB} = T$).

- **High-Demand Type (θ_H):** Receives a contract ($\overline{p}_H^{SB}, x_H^{SB}$) such that $\overline{p}_H^{SB} > \overline{p}_H^{FB}$. This contract is distorted upwards (higher price, lower output, reduced innovation) compared to its first-best counterpart. This type earns a positive information rent ($\pi_H^{SB} > T$).

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