

**Total Factor Productivity and Its Components:  
The Canadian Dairy Industry**

by

Som-at Wirattigowit

059290

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Supervisor: Dr. K. Day

ECO 7997

Ottawa, Ontario

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## CHAPTER 1 INTRODUCTION

### 1.1 Introduction

The Canadian dairy industry has been regulated since the mid 1960s. This regulation mainly comes in the form of a supply management system that restricts production through quotas given at the provincial and farm levels and imposes import controls (Canadian Dairy Commission, 1994: 6-7). Like many other regulated industries, the dairy industry is generally considered to be inefficient and uncompetitive. In particular, it lacks a comparative advantage in dairy production and its product prices are set above marginal costs. This in turn implies that the industry may enjoy its excess profits at the expense of consumers and taxpayers (Moschini, 1988: 187-188).

However, it is unlikely that it can continue to do so as the Canadian economy moves toward more dynamic and competitive economic conditions, in particular the form of trade liberalization that has been initiated by the General Agreement on Tariffs and Trade (GATT). For example, under GATT Canada has to reduce export subsidy dollars by 36 percent, and the volume of subsidized exports by 21 percent.<sup>1</sup> As well, it is required to provide other countries with moderate new access to its dairy market (Canadian Dairy Commission, 1994: 4). Evidently, firms in the Canadian dairy industry cannot forever continue to carry on their business in the same way as before. They have to be able to compete effectively with firms not only in Canada but also in the external market. Otherwise, many of them will be

forced out of the industry (Jeffrey, 1992: 653).

Apart from regulations, a number of other factors affect the economic performance of the dairy industry. Productivity advancement has been a major contributing factor to the dairy industry's growth. In general, this is because improving productivity can enhance firms' profit and strengthen the competitive position of the industry (Weersink, Turvey and Godah, 1990: 439-440). In particular, on the supply side, productivity advancement can be an effective measure against the pressures of inflation and resource shortages (Cowing and Stevenson, 1981: 4). On the demand side, it can increase production to meet expanding domestic and external markets.

## **1.2 Objectives of the Study**

The objectives of this study are to examine the total factor productivity growth (TFPG) of the Canadian dairy industry under regulation, and to analyse the behaviour of output residuals not accounted for by the input growth rate. This will help us better understand the contribution of productivity growth. More precisely, the study will focus on an analysis of total factor productivity growth and its components, namely, (i) scale economies, (ii) technological progress and (iii) the residual effect. The analysis will have implications for how to improve TFPG by managing one or more of its components. Such an improvement can benefit society as a whole by increasing the firm or industry's efficiency and permitting consumers

to buy dairy products at reasonable prices.

This framework of analysis applies relatively new ideas on TFPG and its components to the Canadian dairy industry. It utilizes time series data at the industry level in comparison to the recent studies of Moschini (1988) and Weersink, Turvey and Godah (1990), which used cross-section data at the provincial firm level. Moschini (1988) studied the cost structure of Ontario dairy farms in the years 1978 and 1983 with 612 observations. Some important features of his results are that the estimated cost function exhibited increasing long-run returns to scale for a wide range of output levels particularly for larger firms. Empirically, the prices of fluid and industrial milk were set above marginal costs. In the latter study, particular attention was paid to technical efficiency, using a non-parametric programming approach with 105 observations for 1987. Their analysis showed that the majority of Ontario dairy firms exhibit technical efficiency and increasing or constant returns to scale.

### **1.3 General Assumptions and Outline of the Study**

The model used to derive TFPG and its components is mainly drawn from Denny, Fuss and Waverman (1981: 179-218). The data used in calculating TFPG and its components have been obtained from the KLEMS database, Statistics Canada.<sup>2</sup> Except as otherwise noted, the basic assumptions used in this study are:

- (i) the inputs and outputs can be combined so as to aggregate them into a single

composite input and output for measuring productivity and (ii) this industry is isolated from the rest of the business sector of the Canadian economy such that its productivity is not influenced by the productivity gains of other industries. Alternatively, the model can be viewed as a tool to assess productivity in a static partial equilibrium (Statistics Canada, 1994: 123-124, 127).

The analysis of this study will proceed as follows. Section 2 presents the structure and performance of the Canadian dairy industry as well as its input-output utilization in the 1961-1990 period. Section 3 explains productivity concepts and measurement. The relevant concepts and models for calculating TFPG and decomposing it into its components are developed in section 4. Section 5 presents various aspects of the empirical results i.e., the appropriate specification of the model, its estimated coefficients, the elasticity of total cost with respect to output, estimates of technological progress, the estimation of TFPG and its decomposition. The final section offers some concluding remarks and reviews the limitations of this study.

## **CHAPTER 2 THE CANADIAN DAIRY INDUSTRY**

This section will briefly describe some important features of the Canadian dairy industry. Thereafter this study will utilize dairy industry data on gross output and capital, labour, energy, material and service inputs from the KLEMS database to examine some interesting features of this industry.

### **2.1 Structure and Performance**

The Canadian dairy industry is comprised of establishments that produce dairy products from mainly cow's milk. Generally, that milk can be classified into two principal types: fluid and industrial milk. Sixty percent of raw milk is utilized for industrial purposes and the remainder is sold as fluid milk. The first type is primarily used to produce pasteurized milk and creams. Other fluid-type products include fresh milk, chocolate-flavoured milk, buttermilk and fresh creams. The second type is called industrial milk because it is processed into value-added dairy products; for example, natural and processed cheese, creamery butter, condensed and evaporated milk, ice cream, yogurt, whole and skim milk powder, frozen desserts such as sherbet, and milk and yogurt-based fruit drinks (Food Product Branch, 1991:1).

In 1990, dairy processing was the second-largest component of the food processing sector in Canada after red meat processing. Dairy processing accounted

for approximately \$8.8 billion or about one-fifth of the total value of shipments of the food processing sector (Statistics Canada, 1994b: 4-5). The dairy industry used about 73 million hectolitres of milk with a total value of \$3.4 billion at the farm level. About \$8.6 billion or 98 percent of shipments of dairy products was sold in the domestic market. The remaining \$200 million or 2 percent was exported to external markets, mainly developing countries such as Algeria, Mexico, the Caribbean and some countries in Southeast Asia. However, some dairy products, particularly cheese, were imported. These imports accounted for about \$100 million (Food Product Branch, 1991:1 and Statistics Canada, 1994b: 4-5).

Dairy processing plants are located in every region of the country.

Establishments of fluid milk are situated in every province, in or near urban centers, whereas industrial establishments are located mainly in rural, milk-producing areas. In 1990, Statistics Canada (1994b: 4-5) found that there were 339 establishments engaged in the dairy industry. Those establishments employed 25,328 people, which did not include related jobs in other sectors such as transportation, packaging, marketing and advertising.

There are three organizational structures in the Canadian dairy processing industry. About 50 percent of the industry is made up of co-operatives, i.e., organizations with professional management teams appointed by owner-farm groups. Thirty-five percent of the industry is made up of public corporations and the remaining 15 percent is composed of small private firms (Food Product Branch, 1991: 2).

## **2.2 Features of Regulation in the Canadian Dairy Industry**

The Canadian dairy industry has been regulated since the proclamation of the Canadian Dairy Commission Act on October 31, 1966. Under the Act and the National Marketing Plan (NMP), the Canadian Dairy Commission (CDC) works closely with the provinces, its industry partners and related organizations in setting the many regulatory measures of the Canadian dairy industry. There are three important types of regulations: national milk supply management, import controls, and target prices.

### **2.2.1 National Milk Supply Management**

The objective of this measure is to provide a balance between the supply of industrial milk and the demand for dairy products. A national production target or Market Sharing Quota (MSQ) is set and adjusted to reflect changes in demand; then the MSQ is distributed to provinces according to the shares specified by the NMP. Each province allocates its share to producers according to its own policies.

Table 2-1 shows the MSQ entitlement at the provincial level. The MSQ declined approximately 4.0 percent from 1980 to 46.057 million hectolitres in 1990 and about 7.0 percent from 1990 to 42.905 million hectolitres in 1994, mainly due to decreases in demand for dairy products. However, the percentage market shares for milk quota for each province did not change much. Quebec received the largest share of the MSQ -- about 47-48 percent; second was Ontario at 31 percent; the West received 16-17 percent; and the remainder went to the

Atlantic provinces.

This measure directly imposes a certain level of national and provincial milk production. In order to ensure that the target price for milk is achieved, it is extremely important that the dairy firms and provinces stick to their given quotas. But as a result, milk producers are no longer competitive because the price level of milk is higher than its corresponding competitive price.

**Table 2-1 Market Sharing Quota (MSQ) Entitlement at the Provincial Level**

Province	1980		1990		1994	
	million hL	percent	million hL	percent	million hL	percent
Prince Edward Is.	0.908	1.9	0.865	1.9	0.860	2.0
Nova Scotia	0.580	1.2	0.606	1.3	0.564	1.3
New Brunswick	0.636	1.3	0.602	1.3	0.534	1.2
Quebec	22.996	48.0	21.858	47.4	20.424	47.6
Ontario	15.020	31.3	14.311	31.1	13.188	30.7
Manitoba	1.870	3.9	1.783	3.9	1.625	3.8
Saskatchewan	1.247	2.6	1.187	2.6	1.060	2.5
Alberta	3.218	6.7	3.062	6.6	2.782	6.5
British Columbia	1.485	3.1	1.783	3.9	1.886	4.4
Canada	47.960	100.0	46.057	100.0	42.905	100.0

Note: hL is hectolitres

Source: Canadian Dairy Commission. *Canadian Dairy Commission Annual Report*. Various Issues.

### **2.2.2 Import Controls on Dairy Products**

Canada has applied a number of measures to monitor and control imports of dairy products such as ice cream, yogurt, buttermilk powder, condensed milk and cheese in order to ensure the success of its national milk supply management system. The Canadian government has established import controls which are administered by Foreign Affairs and International Trade Canada under the provisions of the Export and Import Permits Act.

As a result, this measure protects similar dairy products produced domestically from competitive imports. In addition, such restrictions cause the related processing firms to be uncompetitive and leave consumers with no option but to buy the controlled dairy products at higher prices.

### **2.2.3 Target Price for Industrial Milk**

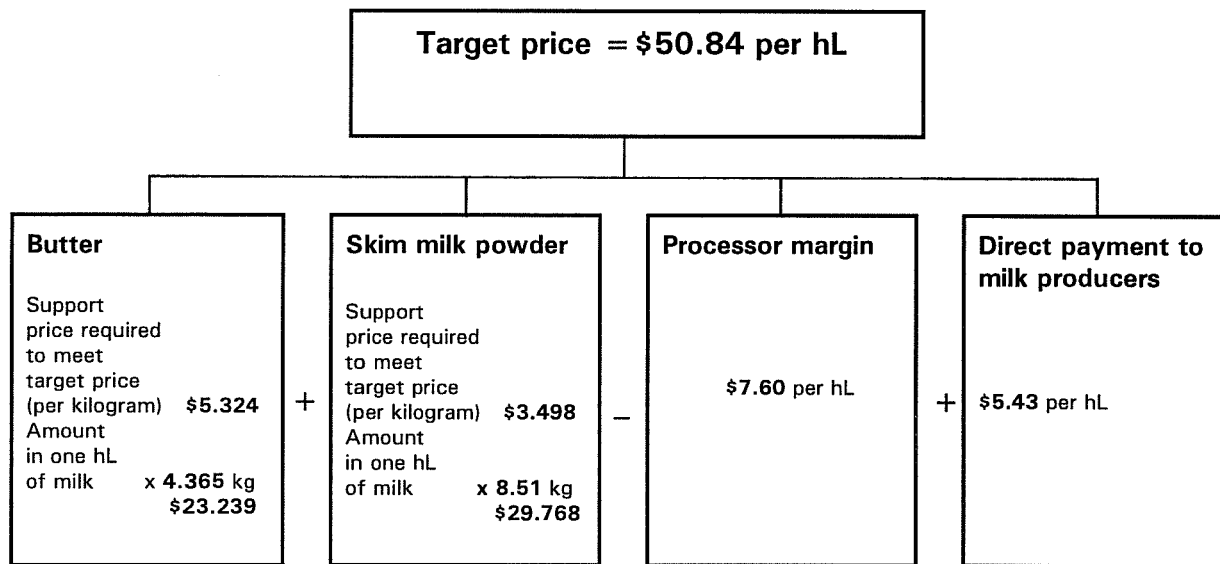
The target price is set at a level intended to provide efficient milk producers with a return which will cover their labour, investment and cash costs related to the production of industrial milk. The federal government has given the responsibility for setting dairy target and support prices to the CDC since 1990. This regulation is provided as a different but supporting measure for milk supply management measure so that dairy firms can ensure stable levels of dairy product prices covering the cost of production and an appropriate profit.

On an annual basis, the estimation of the target price per hectolitre will be based on four factors: (i) a support price for butter, (ii) a support price for skim milk powder, (iii) a processor margin and (iv) a direct payment to milk producers.

Support prices for butter and skim milk powder, and a processor margin are set to cover production and marketing costs, and a moderate return on manufacturers' investment whereas a direct payment to milk producers is subsidized by the government based on deliveries of industrial milk and cream in response to domestic requirements. Table 2-2 shows that in 1993, the target price for industrial milk was \$50.84 per hectolitre with the shares of butter and skim milk powder support prices, the processor margin and the direct payment being 45.7, 58.5, - 14.9 and 10.7 percent respectively.

It should be noted that although regulation may help to deal with unstable markets, uncertain supplies and highly unstable revenues (product prices) for dairy firms (Canadian Dairy Commission, 1994: 6), it may also create a set of new problems. The regulatory system seems to allow the Canadian dairy industry to act like a cartel since it gives the CDC power to raise the target price for industrial milk and to control milk production (implying a restriction on entry of dairy firms) and dairy products imported. As a result, the target price for industrial milk and related dairy product prices are set above their corresponding average costs.<sup>3</sup> Noticeably, there are many criticisms of the regulations. Opponents have argued that such a regulation system has become very costly to consumers and that the efficiency losses are becoming unacceptably large (Blomqvist, Wonnacott and Wonnacott, 1982: 524-527 and 559-565).

**Table 2-2 Target Price Structure for Industrial Milk, August 1, 1993**



**Source:** Canadian Dairy Commission (1994: 12).

## 2.3 The Analysis of the Canadian Dairy Industry Using the KLEMS Database

### 2.3.1 The Growth Rates of Gross Output, Inputs and Their Prices

To characterize the growth of the dairy industry over the 1961-1990 period, the growth rates of gross output and all inputs have been calculated for three decades and the overall period and are shown in Table 2-3. The average growth rate of gross output over the period 1961-1990 was 1.26 percent, about one-third of the comparable average of output growth rates in all manufacturing industries (Johnson, 1994b: 34). The output growth rate was strongest at 2.07 percent a year for the 1960s. Thereafter, the growth rate tended to slow to 1.50 percent in

the 1970s and 0.30 percent in the 1980s.

In contrast to the decreasing growth rate of output quantity, the annual growth rate of output price increased between 1961 and 1990. In the 1960s, the period of mildest inflation (2.84 percent per year) exhibited the highest output growth rate. Whereas during the seventies and eighties, the inflation rates were 10.63 percent per year and 5.60 percent per year respectively while output growth rates decreased.

**Table 2-3 Average Annual Percentage Rates of Change for Input-Output Quantities and Implicit Prices**

Period	Gross Output		All Inputs	
	Quantities	Prices	Quantities	Prices
1961-1970	2.07	2.84	1.80	3.07
1971-1980	1.50	10.63	0.78	11.28
1981-1990	0.30	5.60	0.53	5.36
1961-1990	1.26	6.48	1.01	6.69

Turning to the growth rate of all inputs combined, their annual growth rate was 1.01 percent per year less than that of gross output over the 1961-1990 period as a whole. The annual growth rate of input quantity exhibited a pattern similar to that of gross output: it was highest in the 1960s, at 1.80 percent per year, lower in the 1970s at 0.78 percent per year and lowest in the 1980s at 0.53

percent per year. The annual rate of change in input prices also behaved in a similar manner to that of output prices. As the rate of change in input utilization tended to decline, the rate of change in input prices increased.

### 2.3.2 The Input Value Shares

Table 2-4 presents the average input value shares during the 1961-1990 period. Over the three decades, material inputs were the most dominant, accounting for approximately 66 percent of all costs of the dairy industry. Labour inputs were the next most important input at 12 percent. Capital and service inputs each accounted for about 10 percent of all costs. The input with the lowest cost share was energy at 1.3 percent. Notably, the value share of labour inputs tended to decline from approximately 15 percent in the 1960s to 12 percent in the 1980s, whereas the share of capital inputs increased from 8 percent to 12 percent during the same period. This implies that the dairy industry moved toward replacing labour inputs by increasing capital inputs.

**Table 2-4 Percentage Average Input Value Shares**

Period	Capital	Labour	Energy	Materials	Services
1961-1970	7.87	14.80	1.49	66.08	9.76
1971-1980	7.87	12.77	1.24	69.12	9.00
1981-1990	11.71	11.82	1.29	64.86	10.32
1961-1990	10.27	12.40	1.30	66.11	9.92

Table 2-3 indicates that the average growth rate of aggregate input prices over the period 1961-1990 was 6.69 percent a year. In Table 2-5, input prices are disaggregated. The average annual growth rates of the prices of capital and labour inputs were almost the same: 8.19 and 8.22 percent respectively. For all inputs except capital, the rate of growth of prices in the 1970s was between 8.39 and 13.92 percent.

**Table 2-5 Average Annual Percentage Rates of Change of Implicit Input Prices**

Year	Capital	Labour	Energy	Material	Service
1961-1970	0.13	6.55	0.93	2.96	2.78
1971-1980	11.49	11.67	13.92	11.72	8.39
1981-1990	12.13	6.26	6.26	4.38	4.95
1961-1990	8.19	8.22	7.24	6.47	5.47

**Table 2-6 Average Annual Percentage Rates of Growth of Input Quantities**

Period	Capital	Labour	Energy	Material	Service
1961-1970	6.88	-1.20	0.35	1.98	1.20
1971-1980	1.22	-2.18	-1.01	1.19	2.20
1981-1990	3.11	-0.12	-1.63	-0.26	3.57
1961-1990	3.63	-1.17	-0.80	0.93	2.61

These growth rates were likely influenced by the cost-push inflation due to the energy crises during that period. Yet, they led to substantially lower use of almost all inputs as shown in Table 2-6. Further, the inflation rates for inputs kept increasing in the 1980s, causing the growth of input quantities to decrease to 0.12, 0.26 and 1.63 percent a year for labour, material and energy inputs respectively. Capital and service inputs were exceptions with annual growth rates of 3.11 and 3.57 percent. This performance was possibly due to the replacement of labour, energy and materials by capital and service inputs.

## CHAPTER 3 PRODUCTIVITY: MEANING AND MEASUREMENT

This section will briefly describe the meaning of productivity and how to measure it.

### 3.1 The Meaning of Productivity

Productivity is generally defined as the relationship between the output produced and the input used to achieve that output. In a production process, a firm or an industry may use the services of multiple inputs to produce multiple outputs (Silver, 1984: 1). Mathematically, the relationship can be written as:

$$(3-1) \quad Y = f(X)$$

where  $Y = [Y_1 \dots Y_M]$  is the vector of output produced;  $X = [x_1 \dots x_N]$  is the vector of services provided by, for example, capital, labour, land, materials and energy. Equation (3-1) says that outputs are determined by multiple inputs. Such a relationship is popularly called a "production function".

By relating the outputs to any input factor or a subset of inputs utilized, one can obtain measures of *partial productivity*. For example, if  $x_1$  stands for capital in the production process,  $\frac{Y}{x_1}$  is the partial productivity of capital or capital productivity. In a similar manner, one can derive the partial productivities of labour,

materials, energy and other inputs.

Although the partial productivity may be appropriate for many analytical uses, it does not explain the contributions of all inputs used. Often it is difficult to distinguish the contribution of any one input to total output from the contributions of the other inputs used in the production process. This leads to the use of the concept of "*multifactor productivity*" or "*total factor productivity*" to measure the contributions of all productive inputs in a certain production process (Statistics Canada, 1994a: 119, 123).

Moreover, one may be interested in *total factor productivity growth* (TFPG), which refers to the residual growth in outputs not accounted for by the growth in factor inputs (Denny, Fuss and Waverman, 1981). Equivalently, it is the difference between the rate of growth of aggregate output and the rate of growth of aggregate input (Jorgenson and Griliches, 1967: 249). Some of this residual is attributable to the quality of factor inputs, the intensity and flexibility of resource use, capacity utilization, the quality of management, product mix, scale economies, market imperfections, the quality of the work environment, labour-management relations and so on (Rao and Lempriere, 1992: 4). This measure allows us to see how the rate of change in aggregate output responds to the rate of change in aggregate input. TFPG is positive if the rate of change in aggregate output grows faster than the rate of change in aggregate input and is negative otherwise. As well, it can be related to some particular functional form for the production and cost functions and decomposed into a number of components.<sup>4</sup>

## **3.2 The Measurement of Productivity<sup>5</sup>**

The measurement of productivity can sometimes become difficult, as different approaches often give different measurements. For the case of one homogeneous output and one homogeneous input, there are six different approaches to productivity measurement that have often been suggested in the literature. All of them can be shown to be equivalent. In this study, the single output and multiple input case is taken into consideration. Therefore, no detailed discussion of the simple case is given here.

There are different methodologies for measuring productivity in a single output framework and a brief description of them will be given here. Generally speaking, there are three main approaches. They are: (i) the production function approach, (ii) the cost function approach and (iii) the index number approach. The third approach contains at least six different alternatives and one of them is utilized in this study. No detailed discussion of the three will be given here, however, the relevant discussion will be provided later in related contexts.

### **3.2.1 The Production Function Approach**

The production function can be linked to productivity measurement in the sense that output responds to the advancement of production technologies. If a firm or an industry is able to produce more output per unit of input in the present than in the past, it is obviously experiencing a productivity improvement.

### **3.2.2 The Cost Function Approach**

This approach examines any change in output due to a change in the input price or total cost when a new technology is introduced. Productivity is said to have increased if the average cost of production is currently lower than it was previously. This implies that a higher level of output can be produced at the lower level of total cost. According to Diewert (1991: 20-21 and 1992: 172-177), the use of the cost function has a major advantage over the production function approach in that estimation of the unknown parameters that characterize technology is much more accurate when using the cost function technique. This is because the production function approach involves too many unknown parameters to estimate given the number of degrees of freedom available. The same is true for the cost function approach, but the problem can be resolved by imposing on the cost function the a priori applicable pattern between inputs and outputs to deal with such problem.

### **3.2.3 The Index Number Approach**

This approach provides several alternatives for productivity measurement. The conventional continuous Divisia index, a weighted sum of growth rates where the weights are the components' share in total value (Hulten, 1973: 1017) is well known in empirical studies. Its productivity measure is constructed by taking the ratio of the Divisia output quantity index and the Divisia input quantity index.<sup>6</sup> However, since the Divisia index is a continuous function, it must be empirically approximated using discrete data. There are four methods commonly used to

approximate the Divisia index, namely, the Laspeyres, the Paasche, the Tornqvist and the Fisher ideal indexes. For each index, the productivity measure can be defined as the ratio of an output quantity index to an input quantity index.

Defined mathematically,

$$(3-2) \quad TFP = \frac{Q_Y(p^0, p^1, Y^0, Y^1)}{Q_X(w^0, w^1, X^0, X^1)}$$

where TFP is total factor productivity;  $Q_Y$  is the output quantity index calculated from the prices  $(p^0, p^1)$  and quantities  $(Y^0, Y^1)$  of output in periods 0 and 1;  $Q_X$  is the input quantity index calculated from the prices  $(w^0, w^1)$  and quantities  $(X^0, X^1)$  of input.

The different indexes apply different weights in calculating the output and input quantity indexes in the two periods 0 and 1. Thus while the Laspeyres productivity index uses the period 0 output and input prices as weights, the Paasche index uses the period 1 prices as weights. The Fisher ideal productivity index is simply the geometric average of the Laspeyres and the Paasche productivity indexes, whereas in the Tornqvist productivity index the output and input quantity indexes are weighted by their average value share in the two succeeding periods.<sup>7</sup>

Now the question is which method should be used in estimating productivity? Diewert (1991 and 1992) suggests that from the theoretical point of view the Fisher productivity index is the best index number. However, since the

Tornqvist productivity index is not only commonly used in the literature but also corresponds exactly to the translog function (Diewert, 1993: 9-13), it will be used in this study.

## CHAPTER 4 METHODOLOGY

Apart from the difficulties in measuring productivity mentioned in section 3, the particular approach applied in this study mainly follows that of Denny, Fuss and Waverman (1981). The first step of this approach is to construct the Divisia productivity index. The index is then linked to the cost function under nonconstant returns to scale utilizing the information on the cost-output elasticity and technological progress. This allows us to decompose the index into three components, namely, scale economies, technological progress and the residual effect. The methodology will be explained below.

### 4.1 The Divisia Productivity Index and Its Approximation

A number of researchers have derived the Divisia productivity index.<sup>8</sup> A simple derivation of this index is presented by Diewert (1991: 42-46) as follows. Suppose that at any point in time the firm or industry uses  $N$  inputs to produce  $M$  outputs. The quantities and prices of inputs are  $x_i$  and  $w_i$  respectively, where  $i = 1, \dots, N$ . The quantities and prices of outputs are  $y_j$  and  $p_j$  respectively, where  $j = 1, \dots, M$ . It is assumed that the quantities and prices of inputs and outputs are continuous and differentiable.

Let  $TR$  be the firm or industry's total revenue and  $TC$  be its total cost. By definition,

$$(4-1) \quad TR = \sum_{j=1}^M p_j Y_j$$

$$\text{and } (4-2) \quad TC = \sum_{i=1}^N w_i X_i.$$

Now differentiate both sides of (4-1) with respect to time, and divide them by TR to obtain

$$(4-3) \quad \frac{dTR}{dt} \frac{1}{TR} = \sum_{j=1}^M p_j \frac{\partial Y_j}{\partial t} \frac{1}{TR} + \sum_{j=1}^M Y_j \frac{\partial p_j}{\partial t} \frac{1}{TR}$$

$$\text{or } (4-4) \quad \frac{\dot{TR}}{TR} = \sum_{j=1}^M p_j \dot{Y}_j \frac{1}{TR} \frac{Y_j}{Y_j} + \sum_{j=1}^M Y_j \dot{p}_j \frac{1}{TR} \frac{p_j}{p_j},$$

where a dot over a variable indicates a derivative with respect to time. After some simplification, it can be shown that

$$(4-5) \quad \frac{\dot{TR}}{TR} = \sum_{j=1}^M \frac{p_j Y_j}{TR} \frac{\dot{Y}_j}{Y_j} + \sum_{j=1}^M \frac{p_j Y_j}{TR} \frac{\dot{p}_j}{p_j}$$

$$\text{or } (4-6) \quad \frac{\dot{TR}}{TR} = \sum_{j=1}^M s_j^R \frac{\dot{Y}_j}{Y_j} + \sum_{j=1}^M s_j^R \frac{\dot{p}_j}{p_j},$$

where  $s_j^R = \frac{p_j Y_j}{TR}$  is the  $j$ th output's revenue share. Equation (4-6) says that the rate of change in total revenue is equal to a revenue share weighted sum of rates of change in output quantities plus a revenue share weighted sum of rates of change in output prices. If we define

$$(4-7) \quad \frac{\dot{Y}}{Y} = \sum_{j=1}^M s_j^R \frac{\dot{Y}_j}{Y_j},$$

$$(4-8) \quad \frac{\dot{P}}{P} = \sum_{j=1}^M s_j^R \frac{\dot{P}_j}{P_j},$$

then (4-6) can be rewritten as

$$(4-9) \quad \frac{\dot{TR}}{TR} = \frac{\dot{Y}}{Y} + \frac{\dot{P}}{P}.$$

Equation (4-9) says that the rate of change in total revenue is equal to the rate of change in aggregate output quantities plus the rate of change in aggregate output prices. Alternatively, the Divisia total revenue growth rate is equal to the Divisia aggregate output growth rate plus the Divisia aggregate output price growth rate.

In a similar fashion, the rate of growth in total cost can be derived by differentiating both sides of (4-2) with respect to time, dividing them by TC and rearranging to yield

$$(4-10) \quad \frac{\dot{TC}}{TC} = \sum_{i=1}^N \frac{w_i X_i}{TC} \frac{\dot{X}_i}{X_i} + \sum_{i=1}^N \frac{w_i X_i}{TC} \frac{\dot{w}_i}{w_i}$$

or (4-11) 
$$\frac{\dot{TC}}{TC} = \sum_{i=1}^N s_i^c \frac{\dot{X}_i}{X_i} + \sum_{i=1}^N s_i^c \frac{\dot{w}_i}{w_i},$$

where  $s_i^c = \frac{w_i X_i}{TC}$  is the  $i$ th input share in total cost. Equation (4-11) says that the rate of change in total cost is equal to a cost share weighted sum of rates of

change in input quantities plus a cost share weighted sum of rates of change in input prices. Now define

$$(4-12) \quad \frac{\dot{X}}{X} = \sum_{i=1}^N s_i^c \frac{\dot{X}_i}{X_i}$$

and (4-13) 
$$\frac{\dot{W}}{W} = \sum_{i=1}^N s_i^c \frac{\dot{W}_i}{W_i},$$

and substitute these definitions into (4-11) to obtain

$$(4-14) \quad \frac{\dot{TC}}{TC} = \frac{\dot{X}}{X} + \frac{\dot{W}}{W}.$$

Equation (4-14) says that the rate of change of total cost equals the rate of change in aggregate input quantities plus the rate of change in aggregate input prices.

Equivalently, the Divisia total cost growth rate is equal to the Divisia aggregate input growth rate plus the Divisia aggregate input price growth rate.

Recall the definition of total factor productivity growth in section 3.1: it is the difference between the rate of change in aggregate output quantities and the rate of change in aggregate input quantities. Equivalently, the Divisia productivity index (TFP) is equal to the Divisia aggregate output index,  $\left(\frac{\dot{Y}}{Y}\right)$  minus the Divisia aggregate input index,  $\left(\frac{\dot{X}}{X}\right)$ . The mathematical form of the definition can be written as <sup>9</sup>

$$(4-15) \quad \text{TFP} = \frac{\dot{Y}}{Y} - \frac{\dot{X}}{X}.$$

There are some problems in utilizing equation (4-15) to obtain the Divisia productivity index since its application requires continuous data. Since most economic data are discrete, this productivity measure has to be estimated in empirical studies using some approximation to (4-15). As do many other studies, this study uses the Tornqvist productivity index for a number of reasons: (i) it regularly adjusts its weights over time, thus being an appropriate discrete approximation to the Divisia index, which continually adjusts its weights over infinitesimal points of continuous time (Silver, 1984: 81); (ii) it conserves all the information contained in the weighted variables (Nadiri and Schankerman, 1981: 236); and (iii) it can be related to the production function (Diewert, 1991:33) and its dual, the cost function.<sup>10</sup> The index can be applied to measure total factor productivity growth as follows.

Define the rate of change of aggregate output in logarithms as <sup>11</sup>

$$(4-16) \quad \Delta \ln Y = \ln \left( \frac{Y_t}{Y_{t-1}} \right) = \frac{1}{2} \sum_{j=1}^M \left[ (s_{j,t}^R + s_{j,t-1}^R) \ln \left( \frac{Y_{j,t}}{Y_{j,t-1}} \right) \right],$$

where  $Y_{j,t}$  and  $Y_{j,t-1}$ ,  $p_{j,t}$  and  $p_{j,t-1}$ , and

$$s_{j,t}^R = \frac{p_{j,t} Y_{j,t}}{\sum_{j=1}^M p_{j,t} Y_{j,t}} \quad \text{and} \quad s_{j,t-1}^R = \frac{p_{j,t-1} Y_{j,t-1}}{\sum_{j=1}^M p_{j,t-1} Y_{j,t-1}}$$

are the quantities, prices and revenue shares of output  $j$  at time  $t$  and  $t-1$

respectively; and the rate of change of aggregate input is

$$(4-17) \quad \Delta \ln X = \ln \left( \frac{X_t}{X_{t-1}} \right) = \frac{1}{2} \sum_{i=1}^N \left[ (s_{i,t}^C + s_{i,t-1}^C) \ln \left( \frac{X_{i,t}}{X_{i,t-1}} \right) \right],$$

where  $x_{i,t}$  and  $x_{i,t-1}$ ,  $w_{i,t}$  and  $w_{i,t-1}$ , and

$$s_{i,t}^C = \frac{w_{i,t} X_{i,t}}{\sum_{i=1}^N w_{i,t} X_{i,t}} \quad \text{and} \quad s_{i,t-1}^C = \frac{w_{i,t-1} X_{i,t-1}}{\sum_{i=1}^N w_{i,t-1} X_{i,t-1}}$$

are the quantities, prices and cost shares of input  $x_i$  at time  $t$  and  $t-1$  respectively.

The corresponding rate of growth of total factor productivity (TFPG) can be derived as follows:

$$(4-18) \quad \text{TFPG} = \Delta \text{TFP} = \Delta \ln Y - \Delta \ln X.$$

Equation (4-18) is the Tornqvist productivity index that approximates the Divisia productivity index in equation (4-15). Equation (4-18) says that total factor productivity growth measured by the Tornqvist approach is equal to the difference between the aggregate output quantities weighted by the revenue shares of output in two succeeding periods and the aggregate input quantities weighted by the input cost shares in those two periods. For econometric purposes equation (4-18) can be denoted as

$$(4-19) \quad \text{TFPG} = \text{TFPG} + e$$

where  $TF\hat{P}G$  is the estimate of  $TFPG$  derived from the sum of the approximations from the cost function and  $e$  is a residual. Now we can use equation (4-19) to calculate total factor productivity growth using any time-series data.

## 4.2 The Nonhomothetic Box-Cox Cost Function (NBC)

The measure in (4-19) does not allow us to decompose the growth rate of the output residual into various contributing factors. This is because it simply defines those growth rates rather than relating them to any meaningful components. Also, such measurement requires econometric estimation of the production or cost functions. Therefore, it has to be related to a functional form applicable to those circumstances. This study focuses mainly on the cost function rather than the production function. This is because the cost function can be used to calculate the required information (the cost-output elasticities and technological progress) directly and simplifies analysis of the relationship between total factor productivity growth and its components. However, the application of a single output and multiple input framework does not permit one to quantify the effect of non-marginal cost pricing or the gap between marginal cost and output price (Denny, Fuss and Waverman, 1981: 192-196 and Kiss, 1983: 88-91).

The methodology will derive the elasticity of cost with respect to output and technological progress respectively and then relate them to the productivity decomposition later. Now the methodology can be described as follows. Suppose

that five inputs -- capital (K), labour (L), energy (E), materials (M), and services (S) -- are used to produce one homogeneous output: the dairy product (Y). The cost function of the dairy industry can be written in the general form

$$(4-20) \quad TC = g(w, Y, t)$$

where TC is the total cost of the industry;  $w = [w_K, w_L, w_E, w_M, w_S]$  is the vector of input prices of capital, labour, energy, materials and services respectively; Y is the quantity of dairy product; and t is a time trend -- a proxy for technological progress.

To estimate the parameters of the cost function, it is necessary to specify a functional form for equation (4-20). Further, it is desirable to obtain a form which is flexible and applicable to the cost structure of the Canadian dairy industry. ***Here, the nonhomothetic Box-Cox cost function (NBC) is chosen. It uses natural logarithms as the metric for total cost, input prices and output quantity and the Box-Cox metric for the time trend.***<sup>12</sup> Caves, Christensen and Tretheway (1979) indicate that it is as good as other flexible cost functions such as the Generalized Leontief and translog cost functions. The general conditions on TC are that it be nonnegative, real valued, non-decreasing and concave for w, and linearly homogeneous in w for Y (Fuss, 1987: 996 and Varian, 1992: 72). Thus, if the Box-Cox transformation of the time trend can be denoted as

$$(4-21) \quad T = \frac{t^{\lambda_t} - 1}{\lambda_t}$$

where  $\lambda_t$  is strictly positive, NBC can be written as

$$(4-22) \quad \ln TC = \alpha_o + \sum_{i=1}^5 \alpha_i \ln w_i + \alpha_Y \ln Y + \alpha_t T$$

$$+ \frac{1}{2} \left[ \sum_{i=1}^5 \sum_{k=1}^5 \gamma_{ik} \ln w_i \ln w_k + \gamma_{YY} (\ln Y)^2 + \gamma_{tt} T^2 \right]$$

$$+ \sum_{i=1}^5 \gamma_{iY} \ln w_i \ln Y + \sum_{i=1}^5 \gamma_{it} \ln w_i T + \gamma_{Yt} \ln Y T.$$

Using Shephard's lemma, one obtains

$$(4-23) \quad \frac{\partial \ln TC}{\partial \ln w_i} = \frac{\frac{\partial TC}{TC}}{\frac{\partial w_i}{w_i}} = \frac{\partial TC}{\partial w_i} \frac{w_i}{TC}$$

or (4-24) 
$$\frac{\partial \ln TC}{\partial \ln w_i} = \frac{w_i X_i}{TC} = M_i = \alpha_i + \sum_{k=1}^5 \gamma_{ik} \ln w_k + \gamma_{iY} \ln Y + \gamma_{it} T,$$

where  $M_i$  is the cost share of the  $i$ th input.<sup>13</sup>

Since  $TC = \sum_{i=1}^5 w_i X_i$ , the share equations (4-24) satisfy the adding-up condition:

$$(4-25) \quad \sum_{i=1}^5 M_i = 1,$$

implying that

$$(4-26) \quad \sum_{i=1}^5 \alpha_i = 1; \quad \sum_{k=1}^5 Y_{ik} = \sum_{i=1}^5 Y_{iY} = \sum_{i=1}^5 Y_{it} = 0.$$

Moreover, the linear homogeneity property of the cost function gives rise to the restriction:

$$(4-27) \quad \sum_{i=1}^5 Y_{ik} = 0.$$

Since NBC is a second-order approximation, it implies the additional constraints:

$$(4-28) \quad \frac{\partial^2 \ln TC}{\partial \ln w_i \partial \ln w_k} = Y_{ik} = Y_{ki}; \quad \forall i \neq j.$$

In addition, the cost function must be tested to ensure that monotonicity and concavity conditions are satisfied for all input price combinations. An increase in an input price must lead to an increase in total cost, implying non-negativity of input cost shares in (4-24) at each data point i.e., monotonicity; and concavity in input prices requires the Hessian matrix to be negative semidefinite which can be tested using the Allen partial elasticities (Fuss, 1987 and Berndt, 1991: 476).

Once the system of equations consisting of the cost function (4-22) and four of the five cost-share equations in (4-24) are estimated simultaneously, then we can estimate of the cost-output elasticity and technological progress. Define the elasticity of cost with respect to output as <sup>14</sup>

$$(4-29) \quad e_{cY} = \frac{\partial \ln TC}{\partial \ln Y} = \frac{\frac{\partial TC}{TC}}{\frac{\partial Y}{Y}}$$

$$\text{or } (4-30) \quad e_{cY} = \alpha_Y + \gamma_{YY} \ln Y + \sum_{i=1}^5 \gamma_{iY} \ln w_i + \gamma_{Yt} T.$$

It should be noted that  $e_{cY}$  is the percentage change in cost due to a one percent increase in the scale of output. It can be related to returns to scale (RTS) (Diewert, 1991: 49)<sup>15</sup> as follows:

$$(4-31) \quad \text{RTS} = \frac{1}{e_{cY}}$$

where RTS measures the degree of returns to scale -- the effect of a one percent increase in total cost on the percentage increase in outputs. If  $\text{RTS} = 1$ , the production function exhibits constant returns to scale and it shows increasing (decreasing) returns to scale when  $\text{RTS} > 1$  ( $< 1$ ) (Moschini, 1988: 201).

Now define technical progress (TP) at time  $t$  by (Diewert, 1991: 47-48):

$$(4-32) \quad \text{TP} = - \left[ \frac{\partial \ln TC}{\partial t} \right] = - \left[ \frac{\partial \ln TC}{\partial T} \frac{\partial T}{\partial t} \right]$$

$$\text{or } (4-33) \quad \text{TP} = - \left[ \alpha_t + \gamma_{tt} T + \sum_{i=1}^5 \gamma_{it} \ln w_i + \gamma_{Yt} \ln Y \right] (t^{\lambda_t - 1}),$$

where  $\frac{\partial \ln TC}{\partial t}$  is the rate of change of the cost function with respect to time. TP has a negative sign since technological improvement should reduce costs, ceteris paribus the rate of technological progress varies over time and responds to variations in the time trend, factor prices and output levels through the coefficients  $\alpha_t$ ,  $\gamma_{it}$  and  $\gamma_{yt}$  respectively. The coefficient  $\alpha_t$  can be interpreted as the average rate of change in factor utilization. Technological progress is said to be all factor-saving (using) if  $\alpha_t$  is less (greater) than one (Norsworthy and Jang, 1992: 48-49). Factor prices influence technical progress through  $\gamma_{it}$  which is called biased (non-neutral) technological progress if  $\gamma_{it} \neq 0$ . Technological progress is input-using (saving) if  $\gamma_{it}$  is greater (less) than zero; and is said to be neutral if  $\gamma_{it}$  is equal to zero (McKillop and Glass, 1991: 194 and Norsworthy and Jang, 1992:48-49). Finally, the parameter  $\gamma_{yt}$  measures the relation between output and technological progress. For example, if  $\gamma_{yt}$  is negative, it implies a larger scale of production with cost reduction resulted from technical progress.

Equations (4-30) and (4-33) can be used to decompose TFPG into different effects. This decomposition is presented in the next section.

### 4.3 Total Factor Productivity and Cost Theory

Now the results in section 4.2 must be related to TFPG. Suppose that the production function for a firm or an industry is homogeneous and twice differentiable such that the single output,  $Y$ , is related to the services of multiple

inputs,  $X$ , at each point in time. Then the single output transformation function is represented by

$$(4-34) \quad F(Y, X, t) = 0,$$

where  $Y$  is an output quantity;  $x = [x_1, \dots, x_N]$  is the vector of input quantities; and  $t$  is a time variable that represents technical progress. Let us also assume that factor prices,  $w$ , and output,  $Y$ , are exogenous and that the firms are cost-minimizers. Then duality theory associates with the production function in (4-34) a cost function of the form

$$(4-35) \quad TC = g(w, Y, t),$$

where  $w = [w_1, \dots, w_N]$  is the vector of input prices; and  $Y$  and  $t$  are as defined above.

Now totally differentiate both sides of (4-35) with respect to time, divide them by  $TC$  and rearrange to obtain

$$(4-36) \quad \frac{dTC}{dt} \frac{1}{TC} = \sum_{i=1}^N \frac{\partial g}{\partial w_i} \frac{\partial w_i}{\partial t} \frac{1}{TC} + \frac{\partial g}{\partial Y} \frac{\partial Y}{\partial t} \frac{1}{TC} - \frac{\partial g}{\partial t} \frac{1}{TC}$$

or

$$(4-37) \quad \frac{\dot{TC}}{TC} = \sum_{i=1}^N \frac{\partial g}{\partial w_i} \frac{1}{TC} \dot{w}_i \frac{w_i}{w_i} + \frac{\partial g}{\partial Y_j} \frac{1}{TC} \dot{Y} \frac{Y}{Y} - \dot{g} \frac{1}{TC}.$$

It should be noted that  $TP = -\frac{\partial g}{\partial t} \frac{1}{TC}$  has a negative sign as mentioned in section 4.2.

According to Shephard's lemma, the partial derivative of the cost function with respect to the  $i$ th input price is equal to the  $i$ th input demand function,  $x_i$ :

$$(4-38) \quad \frac{\partial g}{\partial w_i}(w, Y, t) = X_i(w, Y, t); \quad i = 1 \dots N.$$

Substituting (4-38) into (4-37) and rearranging yields

$$(4-39) \quad \frac{\dot{TC}}{TC} = \sum_{i=1}^N \frac{w_i X_i}{TC} \frac{\dot{w}_i}{w_i} + e_{cy} \frac{\dot{Y}}{Y} - TP.$$

To relate TP to the rate of change in total factor productivity, we first substitute (4-10) into (4-39) to obtain

$$(4-40) \quad TP = -\sum_{i=1}^N \frac{w_i X_i}{TC} \frac{\dot{X}_i}{X_i} + e_{cy} \frac{\dot{Y}}{Y}$$

or (4-41) 
$$TP = -\frac{\dot{X}}{X} + e_{cy} \frac{\dot{Y}}{Y}.$$

To relate  $T\dot{F}P$  to TP, one can substitute (4-15) into (4-41) to yield

$$(4-42) \quad TP = T\dot{F}P - \frac{\dot{Y}}{Y} + e_{cy} \frac{\dot{Y}}{Y}$$

or (4-43) 
$$TFP = (1 - \epsilon_{CY}) \frac{\dot{Y}}{Y} + TP.$$

Equation (4-43) says that the Divisia total factor productivity index,  $TFP$ , is equal to one minus the cost-output elasticity times the output growth index,  $\frac{\dot{Y}}{Y}$ , plus technological progress,  $TP$ . Denny, Fuss and Waverman (1981: 197) interpret this to mean that the rate of growth in total factor productivity is composed of: (i) a movement along the cost function or scale economies,  $(1 - \epsilon_{CY}) \frac{\dot{Y}}{Y}$ ; and (ii) a shift in the cost function or technological progress,  $TP$ .

However, since  $TFP$  in (4-43) has to be approximated in a discrete form ( $TFPG = TFPG + e$ ), the variables on the right hand side of (4-43) should be consistently estimated in the same manner as  $TFP$ . According to Kiss (1983: 85-113), those variables can be approximated by using equation (4-30) and the output growth rate as follows. The approximation of scale economies,  $SE$ , is

$$(4-44) \quad SE = \left[ 1 - \frac{1}{2} (\epsilon_{CY, t} + \epsilon_{CY, t-1}) \right] \ln \left( \frac{Y_t}{Y_{t-1}} \right)$$

where  $\epsilon_{CY, t}$  and  $\epsilon_{CY, t-1}$  are elasticities of cost with respect to output at time  $t$  and  $t-1$  respectively.

To proxy for technological progress in equation (4-32), the Box-Cox transformation of technology,  $T$ , shifts the cost function by  $\phi = \frac{\partial \ln TC}{\partial T}$ . Since it is

associated with the cost function and a time trend, equation (4-32) can be approximated by

$$(4-45) \quad TP = - \left[ \frac{1}{2} (\phi_t + \phi_{t-1}) \frac{\Delta T}{\Delta t} \right]$$

or

$$(4-46) \quad TP = - \left[ \frac{1}{2} (\phi_t + \phi_{t-1}) \left( \frac{t^{\lambda_t} - t^{\lambda_{t-1}}}{\lambda_t} \right) \right]$$

where  $\frac{\partial \ln TC}{\partial T} = \alpha_t + \gamma_{tt} T + \sum_{i=1}^5 \gamma_{it} \ln w_i + \gamma_{yt} \ln Y$ ;  $\Delta T = \frac{t^{\lambda_t} - t^{\lambda_{t-1}}}{\lambda_t}$  and  $\Delta t = 1$ .

Eventually, using equations (4-19), (4-44) and (4-46), the approximations of TFPG and its components can be rewritten as

$$(4-47) \quad TFPG = SE + TP + e.$$

Equation (4-47) says that the growth rate of total factor productivity is equal to the sum of the growth rates of scale economies (SE), technological progress (TP) and a residual effect ( $e$ ). In the next section, the relevant models from this section will be used to estimate the desired variables.

## CHAPTER 5 EMPIRICAL RESULTS

This section will present the estimates of TFPG and its components. The application is to the Canadian dairy industry for the 1961-1990 period. The choice of functional form for the cost function is explained first. Thereafter, this study will present estimates of the nonhomothetic Box-Cox cost function (NBC), the elasticity of cost with respect to output, technological progress, TFPG and its decomposition.

### 5.1 The Reasons for Choosing NBC

The choice of the NBC model was made on empirical grounds using maximum likelihood estimation and likelihood ratio tests (LR) <sup>16</sup> for single output models of the Canadian dairy industry. At first, allowing for nonhomotheticity, the NBC models were examined by applying identical Box-Cox metric values ( $\lambda_Y = \lambda_t$ ) to both output  $\left( \frac{Y^{\lambda_Y} - 1}{\lambda_Y} \right)$  and the time trend  $\left( \frac{t^{\lambda_t} - 1}{\lambda_t} \right)$ . The resulting functional form for the cost function was

$$\begin{aligned}
(4-48) \quad \ln TC = & \alpha_o + \sum_{i=1}^5 \alpha_i \ln w_i + \alpha_Y \left( \frac{Y^{\lambda_Y - 1}}{\lambda_Y} \right) + \alpha_t \left( \frac{t^{\lambda_t - 1}}{\lambda_t} \right) \\
& + \frac{1}{2} \left[ \sum_{i=1}^5 \sum_{k=1}^5 \gamma_{ik} \ln w_i \ln w_k + \gamma_{YY} \left( \frac{Y^{\lambda_Y - 1}}{\lambda_Y} \right)^2 + \gamma_{tt} \left( \frac{t^{\lambda_t - 1}}{\lambda_t} \right)^2 \right] \\
& + \sum_{i=1}^5 \gamma_{iY} \ln w_i \left( \frac{Y^{\lambda_Y - 1}}{\lambda_Y} \right) + \sum_{i=1}^5 \gamma_{it} \ln w_i \left( \frac{t^{\lambda_t - 1}}{\lambda_t} \right) \\
& + \gamma_{Yt} \left( \frac{Y^{\lambda_Y - 1}}{\lambda_Y} \right) \left( \frac{t^{\lambda_t - 1}}{\lambda_t} \right),
\end{aligned}$$

while the corresponding share equations were

$$(4.49) \quad \frac{\partial \ln TC}{\partial \ln w_i} = \frac{w_i X_i}{TC} = M_i = \alpha_i + \sum_{k=1}^5 \gamma_{ik} \ln w_k + \gamma_{iY} \left( \frac{Y^{\lambda_Y - 1}}{\lambda_Y} \right) + \gamma_{it} \left( \frac{t^{\lambda_t - 1}}{\lambda_t} \right).$$

The  $\lambda_Y$  and  $\lambda_t$  values can be estimated by doing a grid search for the system of equations consisting of the total cost equation (4-48) and four of the five share equations (4-49).<sup>17</sup> Here,  $\lambda_Y$  and  $\lambda_t$  values were started from 0 with an increment of 1 and narrowed down until the log likelihood function (LF) was maximized. The iterative Zellner estimation procedure was applied to the stochastic version of the system of equations using SHAZAM version 7.0. This procedure gives estimates which are asymptotically equivalent to maximum likelihood estimates.<sup>18</sup>

Table 5-1 presents some values of the log likelihood function under

alternative identical values of  $\lambda$  for output and the time trend, estimated from the system of equations in (4-48) and (4-49). At  $\lambda_y = \lambda_t = 3$ , the LF was maximized at 626.297. On the basis of that LF, the NBC function in (4-48) with the Box-Cox metric  $\lambda_y = \lambda_t = 3$  was initially accepted as the appropriate form for the cost function.

**Table 5-1 Sample Log-Likelihoods under Alternatives Values of  $\lambda_y$  and  $\lambda_t$  for Nonhomothetic Box-Cox Cost Function**

$\lambda_y$	0	1	2	3	4	5
$\lambda_t$	0	1	2	3	4	5
<b>lnL</b>	582.517	581.325	603.794	626.297	604.479	591.001

As a next step, the accepted model was verified in the following two cases: the restriction  $\lambda_y = 0$  was imposed but  $\lambda_t$  was allowed to vary between 1.0 and 5.0, and the restriction  $\lambda_t = 0$  was imposed but  $\lambda_y$  was allowed to vary between 1.0 and 6.0. The variations of Box-Cox transformation variables  $\lambda_t$  and  $\lambda_y$  were narrowed down until the LF in the corresponding cases was maximized. The results for these two cases are shown in Tables 5-2 and 5-3. In the first case, the LF was maximized at  $\lambda_y = 0$  and  $\lambda_t = 3$  whereas in the second case, its LF = 598.880 at  $\lambda_y = 4.9$  and  $\lambda_t = 0$  was the maximum value.

**Table 5-2 Sample Log-Likelihoods under  $\lambda_y = 0$  and Alternative  $\lambda_t$  Values**

$\lambda_y$	0	0	0	0	0	0	0	0
$\lambda_t$	1.0	2.0	2.5	2.9	3.0	3.1	4.0	5.0
lnL	578.620	604.727	623.184	629.268	629.541	629.457	618.806	608.314

**Table 5-3 Sample Log-Likelihoods under  $\lambda_t = 0$  and Alternative  $\lambda_y$  Values**

$\lambda_y$	1.0	2.0	3.0	4.0	4.8	4.9	5.0	6.0
$\lambda_t$	0	0	0	0	0	0	0	0
lnL	598.497	584.118	597.083	598.526	598.878	598.880	598.875	598.435

To ensure that there were no other models with a greater LF value than that of the specification with  $\lambda_y = 0$  and  $\lambda_t = 3$ , other two verifications were done: the restriction  $\lambda_t = 3$  was imposed but  $\lambda_y$  was allowed to vary over the range 1 and 6, and the restriction of  $\lambda_y = 4.9$  was imposed but  $\lambda_t$  was allowed to vary over the same range until the corresponding LFs were maximized. The related results are shown in Tables 5-4 and 5-5. Those results show that there is no other LF value greater than that of the specification with  $\lambda_y = 0$  and  $\lambda_t = 3$ .

**Table 5-4 Sample Log-Likelihoods under  $\lambda_t = 3$  and Alternative  $\lambda_y$  Values**

$\lambda_y$	1.0	2.0	3.0	4.0	4.9	6.0
$\lambda_t$	3.0	3.0	3.0	3.0	3.0	3.0
lnL	629.126	628.111	626.297	623.494	620.045	614.740

**Table 5-5 Sample Log-Likelihoods under  $\lambda_y = 4.9$  and Alternative  $\lambda_t$  Values**

$\lambda_y$	4.9	4.9	4.9	4.9	4.9	4.9
$\lambda_t$	1.0	2.0	3.0	4.0	5.0	6.0
lnL	584.391	600.419	620.045	601.611	591.226	587.299

As a final step, to verify which model should be selected for further analysis, several alternative specifications of equations (4-48) and (4-49) were selected for testing using likelihood ratio tests: (i) a model with the Box-Cox transformation applied to the time trend but not output ( $\lambda_y = 0, \lambda_t = 3$ ); (ii) a model with the Box-Cox transformation applied to the output but not the time trend ( $\lambda_y = 4.9, \lambda_t = 0$ ); and (iii) a translog cost specification ( $\lambda_y = \lambda_t = 0$ ).<sup>19</sup> These model specifications were compared to the unconstrained model with  $\lambda_y \neq 0$  and  $\lambda_t \neq 0$  (Incidentally, the values of  $\lambda_y$  and  $\lambda_t$  which maximize the log likelihood function in this case were  $\lambda_y = 0$  and  $\lambda_t = 3$ , which is also the solution for model (i).). The resulting test statistics are shown in Table 5-6.

**Table 5-6 Likelihood Ratio Test Statistics for Various Cost Functional Forms**

$H_1$	NBC
$H_0$	$\lambda_Y \neq 0$ and $\lambda_t \neq 0$
$\lambda_Y = 0$	0(1)
$\lambda_t = 0$	61.322(1)
$\lambda_Y = \lambda_t = 0$	94.048(2)

**Note:**  $H_0$  = Null hypothesis.

$H_1$  = Alternative hypothesis.

Numbers in parentheses indicate the number of restrictions imposed.

**Table 5-7 Critical Values of the Chi-Squared Distribution at Levels 0.01 and 0.05**

Number of restrictions	$\chi^2_{0.05}$	$\chi^2_{0.01}$
1	3.84	6.63
2	5.99	9.21

In case (i), the null hypothesis of  $\lambda_Y = 0$  could not be rejected at significance level 0.05 with 1 constraint since the absolute value of the computed LR (0) is less than the corresponding critical value in Table 5-7. This result implies that one must be indifferent between the two models. In cases (ii) and (iii), the two null hypotheses prove to be significantly rejected at both the 0.01 and 0.05 significance levels. Statistically, this implies that the time trend variable does not enter the model in a

logarithmic form. As a result, the nonhomothetic Box-Cox cost function used here involves the logarithms of total cost, output quantity and input prices and the Box-Cox metric of a time trend variable as shown in equation (4-22).

## 5.2 Estimation of the Cost Model

The empirical results are based on time series data for the Canadian dairy industry during the 1961-1990 period. The data in Appendix Tables A 1-1, A 1-2 and A 1-3 were used to construct the total cost, TC; the prices of inputs,  $w_K$ ,  $w_L$ ,  $w_E$ ,  $w_M$  and  $w_S$  (the prices of capital, labour, energy, materials and services respectively); and output quantity,  $Y$ .<sup>20</sup> As indicated in the previous section, the iterative Zellner estimation procedure is applied to a system of five equations -- the total cost equation (4-22) and the four of five cost share equations in (4-24) so as to produce estimates which are asymptotically equivalent to maximum likelihood estimates and invariant to the cost share equation dropped.<sup>21</sup>

The NBC model fits the data quite well as indicated by the values of  $R^2$  and the log of likelihood function presented in Table 5-8.<sup>22</sup> Overall the explanatory power of NBC is quite high at 0.9981. The R-squareds for the individual equations are 0.9636, 0.9839, 0.4508 and 0.9794 for the cost shares of capital ( $M_K$ ), labour ( $M_L$ ), energy ( $M_E$ ) and materials ( $M_M$ ) respectively. The low  $R^2$  for the energy share equation reflects low explanatory power of the independent variables relative to  $M_E$  in that equation.

All the estimated coefficients of the NBC model as shown in Table 5-9 meet the required conditions (4-25) - (4-28). The estimated model performs moderately well since 25 out of the 35 coefficients (not including the intercept term) are statistically significant.<sup>23</sup> Also, 15 out of the 17 of coefficients  $\gamma_{ik}$  are statistically significant confirming the influence of logarithmic input prices on the logarithm of total cost. However, it should be noted that there is evidence of positive autocorrelation in the system of equations as shown in Table 5-8 and consequently, the reported standard errors in Table 5-9 may be underestimated.<sup>24</sup>

To be consistent with cost theory, the NBC must be well-behaved for all input price combinations. Consistently, the fitted values of the cost share equations are positive at sample points which satisfies monotonicity.<sup>25</sup>

**Table 5-8 Some Statistics of System of Equations**

System of Equations	TC	$M_K$	$M_L$	$M_E$	$M_M$
Durbin-Watson	1.2560	1.2418	0.9738	1.3088	1.6372
R-Squared	0.9981	0.9636	0.9839	0.4508	0.9794
Log of Likelihood Function = 629.541					

Table 5-9 The Estimation of Nonhomothetic Box-Cox Cost Function (NBC)

Parameters	Estimated Coefficients	Standard Errors
$\alpha_0$	-25.8540	88.1900
$\alpha_K$	-0.1536	0.1637
$\alpha_L$	0.4472	0.1396
$\alpha_E$	0.0919	0.1058
$\alpha_M$	-0.1391	0.1679
$\alpha_S$	0.7535	0.2179
$\alpha_Y$	6.4116	20.5600
$\alpha_t$	0.0063	0.0008
$\gamma_{KK}$	0.0399	0.0045
$\gamma_{\kappa}$	0.0200	0.0108
$\gamma_{EE}$	0.0072	0.0035
$\gamma_{MM}$	0.2277	0.0145
$\gamma_{SS}$	0.0409	0.0127
$\gamma_{YY}$	-0.5938	2.3960
$\gamma_{tt}$	-0.00000005	0.00000004
$\gamma_{KL}$	-0.0058	0.0023
$\gamma_{KE}$	-0.0084	0.0022
$\gamma_{KM}$	-0.0367	0.0037
$\gamma_{KS}$	0.0110	0.0048
$\gamma_{\leq}$	-0.0235	0.0032
$\gamma_{LM}$	-0.0758	0.0097
$\gamma_{LS}$	0.0850	0.0081
$\gamma_{EM}$	0.0231	0.0050
$\gamma_{ES}$	0.0015	0.0054
$\gamma_{MS}$	-0.1384	0.0097
$\gamma_{KY}$	0.0266	0.0190
$\gamma_{LY}$	-0.0377	0.0158
$\gamma_{EY}$	-0.0097	0.0122
$\gamma_{MY}$	0.0956	0.0192
$\gamma_{SY}$	-0.0748	0.0250
$\gamma_{Kt}$	0.0000066	0.0000008
$\gamma_{Lt}$	0.0000005	0.0000004
$\gamma_{Et}$	0.0000009	0.0000004
$\gamma_{Mt}$	-0.0000096	0.0000007
$\gamma_{St}$	0.0000016	0.0000009
$\gamma_{Yt}$	-0.000674	0.000098

### 5.3 Elasticity of Cost with Respect to Output

Estimates of the cost-output elasticity ( $\epsilon_{CY}$ ) for the whole period and various sub-periods are shown in Table 5-10. These estimates were constructed using equation (4-30). As can be seen, the average elasticity of cost with respect to output over the 1961-1990 period was -0.3834 per year. This negative elasticity indicates that a one percent increase (decrease) in the output of all dairy products resulted in a 0.38 percent decrease (increase) in total cost per year. This in turn implies that the Canadian dairy industry exhibits increasing returns to scale since a one percent decrease in total cost resulted in a 2.61 percent increase in all dairy products per year.

It is of interest that  $\epsilon_{CY}$  declined over the sample period from 1.3317 in the 1961-1965 period to -3.8043 in the 1986-1990 period.  $\epsilon_{CY}$  has been less than one (0.3028) since the 1971-1980 period, and substantially decreased to -2.6930 in the 1981-1990 period. This in turn reflects that returns to scale kept increasing in the 1970s and 1980s; i.e., from the averages of 3.3025 to -0.3713 per year in those two decades. In other words, the more dairy products the Canadian dairy industry produced, the lower its total cost of production.<sup>26</sup>

**Table 5-10 Average Annual Elasticity of Cost with Respect to Output ( $\epsilon_{CY}$ )**

Period	$\epsilon_{CY}$
1961-1965	1.3317
1966-1970	1.1486
1971-1975	0.7278
1976-1980	-0.1223
1981-1985	-0.5817
1986-1990	-3.8043
1961-1970	1.2402
1971-1980	0.3028
1981-1990	-2.6930
1961-1990	-0.3834

One possible explanation for the increase in the degree of returns to scale is that firms in the dairy industry substituted labour and materials for capital and service inputs since the average input value shares of capital and services increased but those of labour and materials declined.<sup>27</sup> Another reason may be that the small dairy firms increasingly exited the industry while their large-size counterparts which favour capital-using techniques were entering.<sup>28</sup> It should be noted that the estimates of increasing returns to scale were consistent with those earlier studies

using provincial firm-level data done by Moschini (1988) and Weersink, Turvey and Godah (1990) for the years 1978, 1983 and 1987.<sup>29</sup>

#### 5.4 Technological Progress

As mentioned earlier, the functional form used in this study, NBC, involves non-neutral technological progress (TP) if  $\gamma_{it} \neq 0$ . The estimates in Table 5-9 indicate that the average rate of technological progress for the Canadian dairy industry is all input using ( $\alpha_t = 0.0063$ ). The coefficient  $\alpha_t$  implies that a one percent increase in technological progress resulted in a 0.0063 percent increase in total cost each year.

The estimated coefficients  $\gamma_{it}$  are significant at the 0.05 level in capital, energy and material share equations. This means that at constant input prices, these input shares would have changed as a result of technological progress, which implies non-neutral technological change over the 1962-1990 period. The coefficients for the time trend in the capital and energy share equations are 0.0000066 and 0.0000009, therefore indicating capital and energy-using technological progress for those inputs respectively. On the other hand, the negative coefficient of the time trend in the material share equation (-0.0000096) implies material-saving technological progress.

Table 5-11 presents technological progress calculated from equation (4-46). The overall growth rate of TP was negative at 5.26 percent per year over the 1962-1990 period. This negative growth rate indicates an increase in total cost due

to regressive technological progress over that period for the Canadian dairy industry. The negative TP declined from an average rate of -0.53 percent per year in the 1962-1965 period to an average rate of -9.84 percent per year during 1976-1980 but thereafter the growth rates of technological change tended to increase to the average rate of -1.24 percent per year in the 1985-1990 period.

**Table 5-11 Average Annual Rates of Technological Progress (TP)**

<b>Period</b>	<b>TP</b>
1962-1965	-0.0053
1966-1970	-0.0282
1971-1975	-0.0653
1976-1980	-0.0984
1981-1985	-0.0967
1986-1990	-0.0124
1962-1970	-0.0181
1971-1980	-0.0818
1981-1990	-0.0546
1962-1990	-0.0526

The results in Table 5-11 might in turn reflect the fact that the Canadian dairy industry was protected from the consequences of technological improvement by

other factors such as price supports and supply management schemes. However, to investigate this further, one would have to develop a theory relating technological progress to market imperfections, something which is beyond the scope of this study.

## 5.5 Total Factor Productivity and Its Components

Given the cost-output elasticity and technological progress, TFPG and its contributing factors can be estimated using equation (4-47). The results are shown in Table 5-12 and Table 5-13. The average annual TFPG for the whole period, 1962-1990, was 0.23 percent which is about one-third of the comparable productivity, 0.68 percent, for all Canadian manufacturing industries (Johnson, 1994b: 38).

Overall the results indicate that TFPG was attributable to scale economies (SE), technological progress (TP) and a residual effect ( $\epsilon$  or RE) at the average rates of 0.23, -5.26 and 5.26 percent per year in the 1962-1990 period respectively. RE was the most positive substantial part of TFPG; SE was the second whereas TP was negative. The RE consists of many unexplained factors such as errors in optimization, measurement errors and market imperfections. The scale effect has been positive reflecting economies of scale in the 1970s at an average rate of 0.91 percent per year, which declined to 0.31 percent per year in the 1980s. The figures for economies of scale in those decades are consistent with increasing returns to

scale due to large dairy firms existing in those two decades as analysed in section 5.3, whereas the negative technological progress implies regressive improvement.

**Table 5-12 Average Annual Total Factor Productivity Growth and Its Components**

<b>Period</b>	<b>TFPG</b>	<b>Scale Effect</b>	<b>Technological Effect</b>	<b>Residual Effect</b>
1962-1965	0.0071	-0.0124	-0.0053	0.0248
1966-1970	-0.0014	-0.0012	-0.0282	0.0280
1971-1975	0.0068	-0.0021	-0.0653	0.0742
1976-1980	0.0068	0.0203	-0.0984	0.0849
1981-1985	0.0021	0.0201	-0.0967	0.0787
1986-1990	-0.0067	-0.0140	-0.0124	0.0197
1962-1970	0.0024	-0.0062	-0.0181	0.0266
1971-1980	0.0068	0.0091	-0.0818	0.0796
1981-1990	-0.0023	0.0031	-0.0546	0.0492
1962-1990	0.0023	0.0023	-0.0526	0.0526

Table 5-13 Annual Total Factor Productivity Growth and Its Components

Year	TFP		Scale Effect	Technology Effect	Residual Effect
	Index (1986=100)	TFPG			
1961	91.5357				
1962	92.8951	0.0147	-0.0175	-0.0008	0.0329
1963	94.6129	0.0183	-0.0156	-0.0036	0.0375
1964	93.1627	-0.0155	-0.0092	-0.0066	0.0004
1965	94.1686	0.0107	-0.0073	-0.0105	0.0286
1966	94.7147	0.0058	-0.0062	-0.0151	0.0271
1967	92.7687	-0.0208	0.0005	-0.0206	-0.0007
1968	91.4622	-0.0142	0.0022	-0.0275	0.0112
1969	90.9016	-0.0062	-0.0013	-0.0352	0.0303
1970	93.5064	0.0283	-0.0013	-0.0427	0.0722
1971	97.1734	0.0385	0.0006	-0.0494	0.0873
1972	97.6744	0.0052	0.0007	-0.0564	0.0609
1973	99.1288	0.0148	0.0007	-0.0650	0.0791
1974	102.4391	0.0329	0.0074	-0.0726	0.0981
1975	96.7395	-0.0573	-0.0199	-0.0832	0.0458
1976	99.5146	0.0283	0.0130	-0.0945	0.0197
1977	101.3015	0.0178	0.0276	-0.0987	0.0889
1978	101.7583	0.0045	0.0327	-0.1000	0.0719
1979	103.3078	0.0151	0.0409	-0.0989	0.0731
1980	100.1055	-0.0315	-0.0127	-0.0997	0.0810
1981	100.1814	0.0008	0.0038	-0.1027	0.0997
1982	98.7451	-0.0144	-0.0222	-0.1048	0.1125
1983	97.1823	-0.0160	-0.0017	-0.1054	0.0912
1984	99.0245	0.0188	0.1207	-0.0937	-0.0082
1985	101.1404	0.0211	0	-0.0772	0.0983
1986	100.0000	-0.0113	0.0443	-0.0618	0.0062
1987	100.8466	0.0084	0.0357	-0.0393	0.0120
1988	99.4471	-0.0140	0.0459	-0.0115	-0.0484
1989	98.7683	-0.0069	-0.1413	0.0131	0.1214
1990	97.8198	-0.0097	-0.0546	-0.0376	0.0074

It is notable that the negative rates of change of TP seem to vary conversely with the rates of the RE for the overall period and each sub-period. This in turn suggests that the regressive technological progress was offset by the residual effect. As mentioned earlier, the market imperfection component (a part of the residual effect) cannot be analysed since the single output framework does not permit one to quantify the gap between marginal cost and output price. In the sense of market imperfections, price supports which come in the form of a target price for industrial milk are likely to be an important element of the residual effect since they both consistently tended to increase over the 1962-1990 period.<sup>30</sup> A challenging question for further study is to examine the composition of the unexplained effect and how it affects TFPG.

## CHAPTER 6 CONCLUSIONS

In this study, total factor productivity growth (TFPG) and its components were calculated by applying an approximation to the Divisia index, the Tornqvist index, associated with the nonhomothetic Box-Cox cost function. The results indicate that the Canadian dairy industry has been characterized by increasing returns to scale together with on all average input-using techniques over the 1962-1990 period.

Decreasing returns to scale arose in the 1960s at average rate of 0.8063 per year. Thereafter, the 1970s and 1980s show increasing returns to scale at averages of 3.3025 and -0.3713 per year respectively. These figures are quite consistent with earlier studies of Moschini (1988) and Weersink, Turvey and Godah (1990). This finding is likely attributable to the fact that the number of dairy farms and plants is falling while the remaining and incoming firms were larger in size, positively affecting scale economies in the later periods.

During 1962-1990, technological progress averaged -5.26 percent per year indicating regressive technological change. It declined substantially from average rates of -1.81 to -8.18 percent per year between the 1960s and 1970s respectively, but slowed regressively at an average rate of -5.46 percent per year in the 1980s possibly reflecting a little technological improvement in that period.

Subsequently, the approximations of TFPG and its components were

estimated. During 1962-1990, the average annual TFPG was 0.23 percent which was about one-third of the comparable TFPG of all Canadian manufacturing industries. TFPG grew at average rates of 0.24, 0.68 and -0.23 percent per year in the 1960s, 1970s and 1980s respectively. The dominant positive contributions to those growth rates over the whole period were the residual and scale effects at average rates of 5.26 and 0.23 percent per year respectively whereas the technological effect was regressive at an average rate of -5.26 percent per year. The residual effect cannot be clearly explained; however the strong price supports and other market imperfections are likely to be the components of those unexplained effects. It is recommended that future studies explore this possibility.

Admittedly, this study is not complete and suggests some other important extensions related to model specification; for example, the concavity and various elasticities of the cost function were not examined. As mentioned in section 5, the technology variable significantly affects model specification. To obtain a meaningful measure of technological change is a difficult task.

This study simply applied a time trend in the form of a Box-Cox transformation,  $T = \frac{t^{\lambda_t} - 1}{\lambda_t}$  as a proxy for technology. But this variable and its related coefficients do not exactly explain technological changes since its related coefficients ( $\alpha_t$  and  $\gamma_{it}$ ) simply capture trends not explained by the other variables (Norsworthy and Jang, 1992: 48-49). If this is so, the positive and large residual effect may capture technological progress rather than market imperfections; i.e., the technological effect might be a part of the residual effect too whereas market

imperfections might conversely be a part of the technological effect in the model because of, for example, increases in price supports overtime. However, such problems can be remedied by using technology indicators incorporated into the model such as technical change-inducing innovative activity,<sup>31</sup> cumulative research and development expenditure and cumulative output for a learning curve effect (Norsworthy and Jang, 1992: 59-82 and Fuss, 1983: 21-22). As a result, the interpretations in this study associated with the time variable might not be appropriate.

In addition, the input-output data used in the analysis are rather aggregated and to some extent may bias the productivity measure.<sup>32</sup> The data were obtained from the KLEMS database which was generated initially using Statistics Canada Input-Output Tables. Undoubtedly, the level of reliability and validity of these data may be a source of estimation errors. Furthermore, all the output and input prices were calculated implicitly from the volume indices and the current price values, which from a theoretical point of view, do not reflect the values of those output and input correctly. As a result, the interpretation of the estimates obtained using such data may be problematic.

Lastly, as discussed in section 2.2, the Canadian dairy industry under the regulatory system is likely to act like a cartel since the Canadian Dairy Commission has the power to raise industrial milk prices and control milk production. Under such circumstances, dairy firms can enjoy the excess profits since the prices of dairy products are set above their average costs. If this is so, dairy firms may have

no incentive to be cost minimizers. As a result, the application of cost function to dairy industry may not be appropriate.

## Notes

1. The Canadian government signed this agreement in December 1993. It came into effect on January 1, 1995.

2. See the description of this data set in Appendix 1.

3. For example, Jeffrey (1992) used a farm management approach to analyse technical and economic efficiencies for milk production applied to representative Canadian dairy farms in comparison to their American counterparts at the region level in the 1989-1990 period. His results show that total costs per litre and physical efficiency of milk production in the Canadian provinces were higher and lower than those of their American counterparts respectively. The following Table shows that in term of efficiency, all American dairy farms in all the states examined have greater milk production per cow than British Columbia, which has the greatest milk production per cow (7164 litres) among the Canadian dairy farms. In term of total costs, all American dairy farms have lower costs per cow than Alberta, which has the lowest costs per litre (\$0.374) among Canadian farms.

### Milk Production and Total Cost of Representative Canadian and American Dairy Farms

Province or state	Milk production (litres per cow)	Total cost (dollars per litre)
<b>Canada</b>		
Quebec	6070	0.422
Ontario	6027	0.453
Manitoba	5984	0.407
Saskatchewan	6252	0.486
Alberta	6945	0.374
British Columbia	7164	0.476
<b>U.S.A.</b>		
New York	7599	0.371
Minesota	7313	0.318
Washington	8626	0.351
California	8254	0.293

Source: Jeffrey (1992: 656).

4. See sections 4 and 5 for details on the derivation of TFPG, its components and applications.

5. See the derivation of productivity measurement using a Divisia index in section 4.

6. See details in Diewert (1991: 1-71 and 1992: 163-198).

7. More details on this index are provided in section 4.1.

8. For example Richter (1966: 739-755); Jorgenson and Griliches (1967: 249-283); Diewert (1980: 443-446 and 1991: 42-46).

9. Diewert (1991:46) further demonstrates that if the model satisfies constant returns to scale, then:

$$(1) \quad TR = TC,$$

which implies that

$$(2) \quad \frac{\dot{TR}}{TR} = \frac{\dot{TC}}{TC}.$$

Substituting (4.9) and (4.13) into (2) and rearranging, one obtains:

$$(3) \quad \frac{\dot{Y}}{Y} - \frac{\dot{X}}{X} = \frac{\dot{w}}{w} - \frac{\dot{p}}{p}.$$

Equation (3) says that the Divisia total factor productivity index is equal to the difference between the rate of change in the Divisia aggregate output index and the rate of change in the Divisia aggregate input index. But it also equals the difference between the rate of change in aggregate input prices and the rate of change in aggregate output prices.

10. Denny, Fuss and Waverman (1981: 192-197) relate this index to the cost function.

11. In the single output framework, this rate of change is simply defined as

$$\Delta \ln Y = \ln \left( \frac{Y_t}{Y_{t-1}} \right) = \ln \left( \frac{Y_{j,t}}{Y_{j,t-1}} \right)$$

since  $j = M$ , the  $j$ th output revenue share is equal to one.

12. A nonhomothetic cost function has the property that the input demands (the share equations) depend on the level of output. By contrast, the input demands associated with a homothetic cost function are independent of the level of output (Berndt, 1991: 469). The Box-Cox metric of a variable  $X$  can be defined as:

$$X^{(\lambda)} = \frac{X^\lambda - 1}{\lambda}.$$

In this study  $\lambda$  is strictly positive. In fact, the choice of  $\lambda_c = 3.0$  was made on empirical grounds by maximizing the log-likelihood function.

13. Econometrically, a factor share equation includes a disturbance term since an industry can make random errors in choosing its cost-minimizing inputs. Thus the cost share can be defined as

$$M_i = \alpha_i + \sum_{k=1}^5 \gamma_{ik} \ln w_k + \gamma_{iy} \ln Y + \gamma_{it} + e_i$$

where  $e$  is an error term and  $\sum_{i=1}^5 e_i = 0$ .

14. Further, one can derive different economic variables using NBC, for example elasticities of substitution and own and cross price elasticities. See more details in for example Berndt and Khaled (1979: 1220-1245) and Caves, Christensen and Tretheway (1979: 477-481).

15. Blomqvist, Wonnacott and Wonnacott (1983: 466-468) indicate that economically, economies of scale exist when an  $x$  percent increase in all inputs causes an increase of more than  $x$  percent in output. Although economies of scale are not formally defined in terms of a falling average cost, the two are closely related. In general, if the prices of inputs remain constant, economies of scale will result in a falling average cost. However, if the prices of inputs change, this simple relationship no longer holds.

16. The LR test statistics is computed as

$$LR = -2(\ln L_c - L_u)$$

where  $\ln L_c$  and  $\ln L_u$  denote the values of the sample log likelihood functions for the constrained and unconstrained models respectively (Berndt, 1991: 446-447).

17. Note that one of the share equations must be dropped because otherwise the system of equations would have a singular covariance matrix.

18. To ensure that the iterative Zellner estimation process would converge to the maximum likelihood estimates, the covariance matrix was computed using  $N$  as a divisor rather than  $N-1$ , where  $N$  is the number of observations.

19. When  $\lambda_Y$  and  $\lambda_t$  is close to zero,  $\frac{Y^{\lambda_Y} - 1}{\lambda_Y} = \ln Y$  and  $\frac{t^{\lambda_t} - 1}{\lambda_t} = \ln t$ . See Chiang (1984: 429-430) for more details..

20. The variables needed for estimation were constructed to fit the model as follows:

- (i) total cost in current dollars,  $TC$ , is equal to the sum of the current price values of capital, labour, energy, material and service inputs as shown in Table A 1-1;
- (ii) output quantity,  $Y$ , is equal to gross output at constant prices as shown in Table A 1-2; and
- (iii) input prices for capital ( $w_K$ ), labour ( $w_L$ ), energy ( $w_E$ ), materials ( $w_M$ ) and services ( $w_S$ ), are implicit input prices as shown in Table A 1-3.

21. For more details, see Berndt (1991: 469-476). In this study, the service share equation was dropped.

22. Here, the R-squareds are the R-squareds between observed and predicted values.

23. The critical t-value for 24 degrees of freedom is 2.064 at the 2.5 percent level of significance. Although the small sample distribution of the t ratios is unknown, the critical value for the t distribution was used because it is more conservative than the critical value for the normal distribution.

24. The bounds of Durbin-Watson  $d$  statistic with 6 explanatory variables (excluding the constant term) and 30 observations at the 0.05 level of significance are  $d_L = 1.071$  and  $d_U = 1.833$ . Gujarati (1988: 378-379) points out that if the D-W statistic values are less than the upper limit  $d_U$ , the model specifications exhibit positive autocorrelation. McKillop and Glass (1991: 195-196) cite from Berndt and Savin (1975) that if there is significant autocorrelation, parameter estimates will not be invariant to the equation dropped. As done in their study, the equation with the poorest D-W statistic -- i.e., the service share equation ( $M_S$ ) -- was dropped since it is difficult to deal with the autocorrelation problem using Zellner estimation for a system of equations. However, it is suggested that one should apply maximum likelihood estimation in this circumstance. See Berndt and Savin (1975) for more details.

25. For monotonicity, see the results in Appendix 2. The concavity of NBC was not tested.

26. It is reasonable to interpret the negative sign of the elasticity of cost with respect to output as measuring the percentage decrease in total cost as a result of a one percent increase in the output of all dairy products since the average output of all dairy products increased annually. To be consistent with that interpretation, returns to scale should be interpreted as the percentage increase of dairy products resulting from a one percent decrease in total cost.

27. See Table 2-4.

28. The number of dairy farms declined steadily from 153,000 in 1968-1969 to 36,500 in 1988-1989 (about 76 percent), whereas the number of dairy plants also fell by about 70 percent from 1308 plants in 1966 to 393 plants in 1986. The quantity of milk produced has remained the same; however, the volume of milk processed per plant increased by at least 200 percent (Canadian Dairy Commission, 1989: 5,7).

29. See Table A 4-1 for the corresponding years indicating increasing returns to scale.

30. When regressing the residual effect (RE) on the target price (P) using the corresponding data in Table 5-13 and Table A 5-1 for the 1962-1990 period, I obtained the following equation:

$$(1) \quad \begin{array}{l} \hat{RE} = 4.3595 + 0.0421 P, \\ SD \quad (1.628) \quad (0.0550), \text{ the R-squared} = 0.0220 \end{array}$$

where  $\hat{RE}$  is the estimate of RE; P is the target price; and SD is standard error. The coefficient of P = 0.0421 is not statistically significant at the 0.05 level with 27 degrees of freedom since the t - statistic = 0.7654 is less than the critical t value = 2.052. The above coefficient indicates that a one percent increase in the target price will cause the residual effect to increase in the same direction by 4.21 percent. This in turn implies that the target price was likely to be an important part of the residual effect. Further, regressing technological progress (TP) on the target price using data from the same Tables yields the equation below:

$$(2) \quad \begin{array}{l} \hat{TP} = -0.02734 - 0.001124 P, \\ SD \quad (0.01303) \quad (0.0004404) , \text{ the R-squared} = 0.2004 \end{array}$$

where  $\hat{TP}$  is the estimate of TP; and the other variables are defined as above. The coefficient of P (-0.001124) is statistically significant at the 0.05 level with 27 degrees of freedom since the t - statistics of that coefficient (-2.55222) is greater than the critical t - value (-2.052). That coefficient indicates that a one percent increase in the target price will regress technological improvement by 0.11 percent.

This in turn implies that market imperfections (in the form of the target price) contributed to the lack of technological progress.

31. For example, in their analysis of the measurement of total factor productivity in Canadian telecommunications, Denny, Fuss and Waverman (1981) used the following variables as innovation indicators: the percentage of telephones with access to direct distance dialling and the percentage of telephones connected to central offices with modern switching facilities .

32. Statistics Canada (1994a: 124-125) points out that the more disaggregated a set of input and output commodities are, the better the quality of productivity measurement will be. However, due to statistical limitations, the available data set consists only of those commodities presented.

33. Statistically, the value of the labour input of self-employed persons is estimated as an imputed value. The imputation is based on the assumption that the value of an hour worked by a self-employed person is the same as the value of an hour worked by an average paid worker in the same industry. However, an adjustment is made for some particular occupations such as doctors, lawyers and engineers.

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## Appendix 1 Description and Sources of Data

The data on output and inputs used in estimation have been mainly obtained from the KLEMS database provided by the Input-Output Division, Statistics Canada. Most of the components of that database are generated from another data source, the productivity database which is derived from the current and constant price of Input-Output Tables except capital and labour inputs which are obtained from other sources. Various dairy products are combined into one series of three-digit 104 SIC (Standard Industry Classification) and different input factors are grouped into five broad categories, i.e., capital (K), labour (L), energy (E), material (M) and service (S) (Johnson, 1994a: 19-32). The data set is comprised of a 30-year time series for the period 1961-1990.

On the output side, the gross output is empirically the basis for output measurement at the industry level (Norsworthy and Jang, 1992: 15). The composite output commodities in current and 1986 constant prices are shown in Tables A 1-1 and A 1-2 respectively. The constant price values can be used as estimates of quantities (Johnson, 1994a: 21). From these values, the implicit prices of output and inputs can be calculated as follows:

On the input side, all five categories of inputs in current and 1986 constant prices can be narrowly classified into two types, namely, primary and intermediate inputs. The intermediate inputs are disaggregated into three classifications of inputs: energy, materials and services. In general, they are considered to be those inputs which are produced and consumed during a period (usually a year) by the industry (Statistics Canada, 1994a: 124-125). The energy commodities are fuel and electricity used by the establishments in that industry for energy purposes only and the services correspond to actions performed by producers (Johnson, 1994a: 21), while materials are the commodities used in manufacturing operations (Statistics Canada, 1994b: 409).

Primary inputs include labour and capital inputs which are supplied from other sectors of the economy, such as the household sector. For the labour input, the value of labour services includes all payments in cash or in kind by domestic dairy producers to persons at work (or working hours) as remuneration for work, including wages, salaries and supplementary labour income of paid workers, plus an imputed labour income for the self employed.<sup>33</sup> Subsequently, the prices of labour inputs can be derived as the ratio between the labour compensation values and the working hours (Statistics Canada, 1994a: 121). The current values of capital inputs are assumed to be the income generated from those capital services, which is the residual income after paying for all other input costs while the constant price values are derived from data on the capital stock owned by the industry (Johnson, 1994a: 21). Computationally, the price of capital is the ratio between the current price

value (the cost) of capital and the stock of capital of the previous year in constant prices (Statistics Canada, 1994a: 138).

**Table A 1-1 Current Price Values of Output and Inputs**

<b>Year</b>	<b>Gross Output</b> ( \$ million)	<b>Capital</b> ( \$ million)	<b>Labour</b> ( \$ million)	<b>Energy</b> ( \$ million)	<b>Materials</b> ( \$ million)	<b>Services</b> ( \$ million)
1961	919.358	62.503	137.971	17.824	606.583	919.358
1962	980.731	84.658	142.887	14.818	642.975	95.393
1963	1049.115	107.190	148.960	15.218	677.078	100.669
1964	1103.700	99.368	156.325	16.312	727.464	104.231
1965	1141.775	95.519	165.091	17.017	755.486	108.662
1966	1204.379	94.343	179.582	22.094	793.670	114.690
1967	1278.050	87.871	191.036	17.985	857.050	124.108
1968	1324.726	98.402	198.834	18.074	879.971	129.445
1969	1399.539	101.092	211.334	18.647	931.364	137.102
1970	1417.996	98.761	218.351	18.197	938.344	144.343
1971	1537.167	126.959	229.215	18.744	1012.162	150.087
1972	1638.313	123.147	242.653	19.040	1091.752	161.721
1973	1780.169	130.362	253.565	20.369	1203.913	171.960
1974	2176.473	144.984	285.031	27.146	1518.364	200.948
1975	2714.039	178.070	342.200	31.603	1932.500	229.666
1976	2893.265	229.515	363.966	34.885	2015.244	249.655
1977	3202.541	278.893	397.020	40.606	2210.114	275.908
1978	3474.051	315.357	428.923	44.186	2376.082	309.503
1979	3910.569	358.670	472.115	48.935	2683.714	347.135
1980	4393.709	297.002	523.859	58.527	3115.669	398.652
1981	4998.806	384.358	590.014	71.786	3512.088	440.560
1982	6639.043	804.397	761.623	93.158	4318.872	660.993
1983	6846.264	792.623	823.796	89.953	4445.586	694.306
1984	6358.416	601.344	713.806	92.406	4324.506	626.354
1985	6639.416	601.344	713.806	92.406	4323.506	626.354
1986	6846.264	792.623	823.796	89.953	4445.586	694.306
1987	7109.762	947.518	844.156	81.564	4507.524	729.000
1988	7464.032	975.7130	891.132	84.206	4701.176	811.805
1989	7562.193	1072.735	918.808	78.790	4630.603	861.257
1990	7750.532	1192.198	944.371	84.315	4619.193	910.455

**Table A 1-2 Constant Price Values (1986 base) of Output and Inputs**

Year	Gross Output (\$ millions)	All Inputs (\$ millions)	Capital (\$ millions)	Labour (\$ millions)	Energy (\$ millions)	Materials (\$ millions)	Services (\$ millions)
1961	4702.104	5135.982	353.692	1126.944	111.604	3240.223	380.025
1962	4927.805	5305.469	345.557	1124.690	91.627	3437.877	375.085
1963	5153.506	5444.141	348.386	1115.674	91.404	3557.765	393.326
1964	5303.974	5690.668	403.562	1128.071	99.216	3697.094	402.827
1965	5440.335	5777.980	443.883	1128.071	103.346	3723.016	407.767
1966	5576.696	5885.835	470.410	1113.421	136.492	3768.379	423.348
1967	5562.589	5993.691	529.123	1115.674	109.484	3826.703	435.129
1968	5496.760	6009.099	591.372	1093.136	104.350	3816.983	433.989
1969	5548.483	6101.547	610.826	1048.058	110.265	3907.709	448.050
1970	5647.227	6039.915	635.584	1009.742	105.131	3859.106	453.370
1971	5891.737	6060.459	648.317	973.680	103.680	3907.709	452.610
1972	5929.354	6065.595	640.889	939.871	96.203	3946.592	460.591
1973	5948.162	5998.827	641.243	898.174	98.100	3910.949	464.391
1974	6084.523	5937.195	649.024	876.762	97.096	3855.865	475.032
1975	5816.503	6014.235	645.134	898.174	97.542	3901.229	481.492
1976	5948.162	5978.283	650.793	842.954	89.730	3923.910	472.371
1977	6164.459	6086.139	647.256	832.812	90.623	4021.117	487.952
1978	6371.351	6260.762	647.256	847.462	89.507	4150.726	514.934
1979	6587.648	6378.890	651.854	837.319	91.404	4238.212	541.916
1980	6531.223	6522.697	714.457	806.892	94.306	4338.659	562.057
1981	6545.329	6532.969	769.279	799.003	88.949	4332.178	556.357
1982	6474.798	6558.649	796.160	786.607	87.163	4325.698	582.199
1983	6470.096	6656.233	774.585	602.384	85.935	4377.541	628.182
1984	6761.626	6830.856	735.325	804.638	84.261	4539.552	663.524
1985	6761.626	6687.049	757.608	795.622	80.578	4380.782	671.885
1986	6846.264	6846.264	792.623	823.796	89.953	4445.586	694.306
1987	6907.391	6851.400	802.173	812.527	79.016	4445.586	709.507
1988	6977.923	7015.751	849.567	826.050	83.480	4487.709	765.371
1989	6785.137	6866.808	879.985	813.654	76.895	4309.497	778.672
1990	6719.307	6866.808	962.749	795.622	78.346	4215.530	795.393

Table A 1-3 Implicit Prices of Output and Inputs

Year	Gross Output	All Inputs	Capital	Labour	Energy	Materials	Services
1961	0.19552	0.17900	0.17672	0.12243	0.15971	0.18720	0.24861
1962	0.19902	0.18485	0.24499	0.12705	0.16172	0.18703	0.25432
1963	0.20357	0.19271	0.30768	0.13352	0.16649	0.19031	0.25594
1964	0.20809	0.19394	0.24623	0.13858	0.16441	0.19677	0.25875
1965	0.20987	0.19761	0.21519	0.14635	0.16466	0.20292	0.26648
1966	0.21596	0.20462	0.20055	0.16129	0.16187	0.21061	0.27091
1967	0.22976	0.21323	0.16607	0.17123	0.16427	0.22397	0.28522
1968	0.24100	0.22045	0.16640	0.18189	0.17321	0.23054	0.29827
1970	0.25110	0.23477	0.15539	0.21624	0.17309	0.24315	0.31838
1971	0.26090	0.25364	0.19583	0.23541	0.18079	0.25902	0.33160
1972	0.27631	0.27010	0.19215	0.25818	0.19792	0.27663	0.35112
1973	0.29928	0.29675	0.20330	0.28231	0.20763	0.30783	0.37029
1974	0.35771	0.36658	0.22339	0.32509	0.27958	0.39378	0.42302
1975	0.46661	0.45127	0.27602	0.38100	0.32400	0.49536	0.47699
1976	0.48641	0.48396	0.35267	0.43177	0.38878	0.51358	0.52851
1977	0.51952	0.52620	0.43089	0.47672	0.44808	0.54963	0.56544
1978	0.54526	0.55489	0.48722	0.50613	0.49366	0.57245	0.60105
1979	0.59362	0.61305	0.55023	0.56384	0.53537	0.63322	0.64057
1980	0.67272	0.67360	0.41570	0.64923	0.62061	0.71812	0.70927
1981	0.76372	0.76517	0.49963	0.73844	0.80705	0.81070	0.79187
1982	0.84784	0.83700	0.54639	0.81259	0.97559	0.88330	0.87508
1983	0.89584	0.87078	0.67520	0.84435	1.07339	0.89826	0.90936
1984	0.94037	0.93084	0.81779	0.88711	1.09667	0.95263	0.94398
1985	0.98187	0.99282	1.06176	0.95727	1.15612	0.98587	0.98379
1986	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1987	1.02930	1.03771	1.18119	1.03893	1.03225	1.01393	1.02747
1988	1.06966	1.06390	1.14848	1.07879	1.00870	1.04757	1.06067
1989	1.11452	1.10127	1.21904	1.12924	1.02464	1.07451	1.10606
1990	1.15347	1.12870	1.23833	1.18696	1.07619	1.09576	1.14466

## Appendix 2 Rates of Change of Aggregate Output and Aggregate Input

**Table A 2-1 Rates of Change of Aggregate Output,  $\ln(Y_t/Y_{t-1})$   
and Aggregate Input,  $\ln(X_t/X_{t-1})$**

Year	$\ln(Y_t/Y_{t-1})$	$\ln(X_t/X_{t-1})$
1962	0.0469	0.0321
1963	0.0448	0.0265
1964	0.0288	0.0442
1965	0.0254	0.0146
1966	0.0248	0.0190
1967	-0.0025	0.0182
1968	-0.0119	0.0023
1969	0.0094	0.0155
1970	0.0176	-0.0106
1971	0.0424	0.0039
1972	0.0064	0.0012
1973	0.0032	-0.0116
1974	0.0227	-0.0102
1975	-0.0450	0.0122
1976	0.0223	-0.0059
1977	0.0357	0.0179
1978	0.0330	0.0285
1979	0.0334	0.0183
1980	-0.0086	0.0229
1981	0.0022	0.0014
1982	-0.0108	0.0036
1983	-0.0007	0.0152
1984	0.0441	0.0253
1985	0	-0.0211
1986	0.0124	0.0238
1987	0.0089	0.0005
1988	0.0102	0.0241
1989	-0.0280	-0.0212
1990	-0.0097	-0.0001

### Appendix 3 Fitted Values of Cost Share Equations

Table A 3-1 Fitted Values of Cost Share Equations

Year	Capital share	Labour share	Energy share	Material share	Service share
1961	0.0556	0.1482	0.0196	0.6792	0.0974
1962	0.0889	0.1474	0.0166	0.6440	0.1032
1963	0.1085	0.1443	0.0142	0.6268	0.1062
1964	0.0933	0.1436	0.0152	0.6452	0.1026
1965	0.0842	0.1446	0.0157	0.6534	0.1020
1966	0.0786	0.1452	0.0140	0.6579	0.1042
1967	0.0664	0.1463	0.0151	0.6694	0.1028
1968	0.0719	0.1483	0.0154	0.6560	0.1083
1969	0.0727	0.1504	0.0129	0.6507	0.1133
1970	0.0680	0.1532	0.1326	0.6491	0.1164
1971	0.0780	0.1500	0.1063	0.6453	0.1161
1972	0.0743	0.1497	0.0114	0.6463	0.1183
1973	0.0741	0.1457	0.0099	0.6547	0.1155
1974	0.0614	0.1323	0.0129	0.6897	0.1036
1975	0.0719	0.1237	0.0101	0.6937	0.1005
1976	0.0864	0.1272	0.0113	0.6635	0.1116
1977	0.0929	0.1248	0.0116	0.6568	0.1139
1978	0.0963	0.1246	0.0124	0.6519	0.1148
1979	0.0951	0.1209	0.0112	0.6605	0.1123
1980	0.0679	0.1212	0.0149	0.6843	0.1117
1981	0.0738	0.1172	0.0176	0.6758	0.1156
1982	0.0780	0.1169	0.0201	0.6649	0.1201
1983	0.0953	0.1167	0.0201	0.6420	0.1259
1984	0.1008	0.1134	0.0180	0.6462	0.1217
1985	0.1206	0.1134	0.0155	0.6222	0.1283
1986	0.1167	0.1174	0.0117	0.6272	0.1270
1987	0.1293	0.1177	0.0105	0.6120	0.1305
1988	0.1258	0.1188	0.0096	0.6167	0.1291
1989	0.1360	0.1214	0.0087	0.5995	0.1345
1990	0.1394	0.1231	0.0090	0.5892	0.1393

**Appendix 4 Annual Elasticity of Cost with Respect to Output ( $\epsilon_{CY}$ )****Table A 4-1 Annual Elasticity of Cost with Respect to Output ( $\epsilon_{CY}$ )**

Year	$\epsilon_{CY}$
1961	1.3852
1962	1.3612
1963	1.3354
1964	1.3052
1965	1.2715
1966	1.2333
1967	1.2008
1968	1.1666
1969	1.1097
1970	1.0328
1971	0.9387
1972	0.8429
1973	0.7395
1974	0.6110
1975	0.5071
1976	0.3275
1977	0.1243
1978	-0.1025
1979	-0.3499
1980	-0.6110
1981	-0.8948
1982	-1.2025
1983	-1.5415
1984	-1.9343
1985	-2.3353
1986	-2.7829
1987	-3.2598
1988	-3.7770
1989	-4.3091
1990	-4.8928

## Appendix 5 Annual Target Price for Industrial Milk

Table A 5-1 Annual Target Price for Industrial Milk

Year	Target price
1962	5.87
1963	6.41
1964	7.08
1965	7.88
1966	9.18
1967	10.64
1968	10.86
1969	10.86
1970	10.86
1971	11.88
1972	12.90
1973	14.86
1974	19.29
1975	25.50
1976	25.97
1977	26.90
1978	28.17
1979	31.01
1980	34.61
1981	38.06
1982	41.02
1983	42.80
1984	44.65
1985	45.68
1986	46.48
1987	N/A
1988	47.06
1989	47.45
1990	48.69

**Note:** N/A means the data not available and for statistic estimation purpose, the sample mean of target price between 1986 and 1988 was replaced for the missig value in 1987.

**Source:** Canadian Dairy Commission. *Canadian Dairy Commission Annual Report*. Various Issues.