Commodity Prices and Currency Exchange Rates between North America and Greater China Region

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Content

1 Introduction

Exchange rate movements are important to individuals and firms' investment decisions, as a concise understanding of currency exchange rates may increase the value of currently held currency in the future. Due to the importance of potential benefits resulting from currency exchange rate movements, numerous economists specialize in exchange rate theory and policy and the forecasting of exchange rates.

At the end of their important investigation of the forecasting relationship between North America exchange rates and commodity prices, Chen, Rogoff and Rossi (2010) suggest that future economists should carry out a similar analysis for the Asia currencies. Until then, modeling of the relationship between commodity prices and exchange rates in Asia was a topic in which few people had shown interest. Therefore, one contribution of this paper is that it investigates this possible relationship in Asia

Padmanabhan (2008) discusses the roles played by mainland China and the United States (U.S.) in his review of global economy of the 21st century. He claims that mainland China has turned into one of the leading international manufacturing centers of the world, as it currently produces an extensive proportion of the worlds' manufacturing output. Later, Song, Marchant, Reed and Xu (2009) suggest that China is a significant player in global agricultural markets, as mainland China's advanced biotechnologies of soybeans have raised the role of agricultural products in the global economy more than ever. In addition, Agriculture and Agri-Food Canada (2011) claim that in 2010 Canada's exportation of agricultural products to China accounted for 7.41% of Canada's total exports, which makes mainland China is the third largest export market for Canada. Therefore, when we try to explore the possible relationship between commodity prices and currency exchange rates in Asia, we should definitely take mainland China into consideration .

We also choose Taiwan and Hong Kong because these two regions are part of the Economical Miracle which has been improving a great number of Asian economies over the past two decades. Hong Kong's position as a financial and commercial business hub in the world economy is well known, while Taiwan has demonstrated its economic ability as an export-driven economy worldwide. Both economies are tightly related to mainland China in several ways. One example is that quite a few firms whose head offices are in Taiwan or Hong Kong employ labour, machines and land to locate their manufacturing plants in mainland China. Since the 1990s, mainland China has shown its dominance in Hong Kong in a group of fields including regulation-making and government policies. Accordingly, it seems natural to include Taiwan and Hong Kong in a study of mainland China.

Since previous research has investigated the influence of commodity prices on exchange rates between the U.S. Dollar and the Canadian Dollar (Chen, Rogoff, and Rossi 2010; Ferraro, Rogoff, and Rossi 2011), this paper focuses on the exchange rates of the U.S. and Canada with the Greater China Region currencies. Normally the Greater China Region is considered to include four areas – mainland China, Hong Kong, Taiwan and Macao – but Macao is a smaller and less vibrant economy compared to the first three economies, so, I do not include Macao in this paper.

This paper is based on previous research in which commodity prices/indices are used to predict exchange rates. In particular, Ferraro, Rogoff, and Rossi (2011)'s very recent paper discovers that the oil price can predict the U.S./Canada exchange rate, and that this predictive ability also exists in the inverse direction. Mainland China is the biggest consumer of crude oil after the United States in the world oil market and its undeniable role in the consumption of oil suggests a potential interaction between the China Dollar and the crude oil price. This paper investigates whether oil prices contain supportive information for forecasting currency exchange rates for the chosen regions, thus real-time data will be employed in the empirical section.

Previous research has shown that the Markov Switching Model is one of the best models for predicting exchange rates (Engel 1994; Cheung and Erlandsson 2005), but it has some limitations. To obtain ever better forecasts of the exchange rates, one could consider an advanced Markov Switching Model, in particular, the Hidden Markov Model (HMM). Six specific Hidden Markov Models are estimated in this paper. Each one involves the real-time oil price and an exchange rate between a North American country and a Greater China Region country. The price of oil, based on daily, weekly, and monthly frequencies, will be used to examine its interrelation with the in-sample and out-of-sample exchange rates of Greater China Region and North America.

In this paper, the results suggest that despite clean data, there is little methodical relation between oil prices and daily currency exchange rates for Hong Kong and Taiwan, and the relation becomes even weaker when considering the rate between mainland China and North America countries. In all the models estimated, the short-term relationship between commodity prices and exchange rates is only robust when we consider the exchange rate between the U.S. Dollar (USD) and the Taiwan Dollar (TWD). Furthermore, when I attempt to estimate the HMM prediction model using weekly and monthly data, the HMM model cannot be estimated successfully due to the lack of some transition probabilities at the original states. This incapacity of HMM model also exists in the experiment of gold price's relationship with currency exchange rates.

The remainder of this paper is organized as follows. Section 2 describes research thus far on the relationship between commodity prices and exchange rates, while presenting a short organization structure of entire paper. Section 3 shortly describes the basic structure and characteristics of Markov Switching Model and Hidden Markov Model. Section 4 will introduce the data which are used in this paper, including the data source, number of observations and the sample period. Section 5 is the empirical analysis of the HMMs of the relationship between commodity prices and the exchange rates. Finally, section 6 presents a conclusion for this paper.

2 Literature Review

Research on the forecasting of currency exchange rates began as early as the 1970s. In the earliest research, economists tended to develop univariate forecasting models using currency exchange rate data only. Meese and Rogoff (1983) investigate the currency exchange rate forecasting models of the 1970s and compare structural models and time series models with respect to their out-of-sample predictor accuracy. The authors consider three specific structural models, the flexible-price monetary (Frenkel-Bilson) model, the sticky-price monetary (Dornbusch-Frankel) model, and the sticky-price asset

(Hopper-Morton) model. The three models can be interpreted within a general specification:

$$s = a_0 + a_1(m - \dot{m}) + a_2(y - \dot{y}) + a_3(r_s - \dot{r}_s) + a_4(\pi^e - \dot{\pi}^e) + a_5\overline{TB} + a_6\dot{\overline{TB}} + u\,,$$

where in the paper of Meese and Rogoff (1983), "s is the logarithm of [U.S.] dollar price of foreign currency, $m - \dot{m}$ is the logarithm of the ratio of the U.S. money supply to the foreign money supply, $y - \dot{y}$ is the logarithm of the ratio of U.S. to foreign real income, $r_s - \dot{r}_s$ is the short-term interest rate differential and $\pi^e - \dot{\pi}^e$ is the expected long-run inflation differential. $\overline{TB}$ and $\dot{\overline{TB}}$ represent the cumulated U.S. and foreign trade balances, and u is a disturbance term."

Meese and Rogoff choose an unconstrained vector autoregression (VAR) as the representative multivariate time series model, where the general form is:

$$s_t = a_{i1}s_{t-1} + a_{i2}s_{t-2} + \cdots + a_{in}s_{t-n} + a_{in}s_{t-n} + B'_{i1}X_{t-1} + B'_{i2}X_{t-2} + \cdots + B'_{in}X_{t-n} + u_{it}\,.$$

X_{t-j} is a vector of explanatory variables lagged j periods. After comparing the two kinds of models, Meese and Rogoff suggest that structural models fail to offer accurate predictions of out-of-sample data, while the time series and random walk models yield more accurate results.

Mark (1995) introduces dynamic forecasting equations as an improvement for forecasting currency exchange rates. The dynamic equations in his paper uses the general form:

$$\Delta_k e_{t+k} = \alpha_k + \beta_t (f_t - e_t) + v_{t+k}$$

where e_t is the exchange rate at time t and f_t is the exchange rate fundamental suggested by the monetary class of models. Mark finds that the out-of-sample forecasted results are better than those of driftless random walks at the longer horizons, while the random walk model is often used as the reference model for exchange rate prediction.

Engel (1994) confirms the random walk model's outperformance of most prediction models up to that point in time, as the random walk generates a lower mean squared error based on out-of-sample data. Later on, Taylor (1995) summarizes the exchange rate prediction models of the previous two decades and presents some of his results from tests of model efficiency. His conclusion that the application of time-series methods increases the econometric sophistication of tests of foreign exchange market efficiency definitely encourages the use of innovative methods in exchange rates forecasting models.

By this point of time, the research on exchange rate forecasting models was well developed, but another corresponding problem needed be taken into consideration, the determinants of forecasting models. Hopper (1997) presents a summary of the economic factors and fundamentals which could affect exchange rates. Among all the factors, the most impressive ones result from two categories, the macroeconomic fundamentals such as money supplies, output levels, price levels and the literature factors such as economic news and announcements, etc.

Many of the exchange rate forecasting models discussed above tend to produce a prediction with data for exchange rates only. However, after introducing a developed research on the determinants of exchange rates, it is natural to raise the question of whether there exists a better method of prediction of currency exchange rates with assistance from these determinants; moreover, an investigation of the effectiveness of certain types of models and the chosen determinants would be worthwhile.

Chen, Rogoff and Rossi (2010) explore the causal relationship between commodity price movements and exchange rate fluctuations in both directions. To verify the assumed relationship, the authors carry out dual-direction tests of Granger-causality and out-of-sample forecasting ability tests. Regression models with derivatives of commodity prices as the dependent variable and exchange rate movements as independent variables are undertaken to test the null hypothesis that no constant or exchange rate is needed in the model, while a corresponding regression with the roles of commodity prices and exchange rates reversed takes place later. In their empirical analysis, the economists study the USD/CAD exchange rate and the prices of metals, i.e. gold and copper, they surprisingly discover a satisfying forecasting capacity of exchange rates for commodity prices, yet they do not find an inverse relationship, suggesting that commodity prices could forecast exchange rates with robustness.

Further, Lizardo and Mollick (2010) claim that oil prices make a significant contribution to movements of the US dollars in long-run forecasting, and that the interaction moves in the opposite direction for the oil exporter countries like Canada, Mexico and Russia. This result comes from a composite model which consists of a three-stage analysis, where the first stage tests for existence of a stable long-run relationship among price level, demand for money and real income; the second stage tests for a linear relationship between the three factors; and the third stage compares the forecasting performance of different models.

In addition, in Chen, Rogoff and Rossi's (2010) consideration of a regression model approach for the exchange rates of foreign countries (Australia, Canada, New Zealand, South Africa, etc.), the authors also include foreign demand for money and purchasing power parity as explanatory variables. However, since the authors fail to provide numerical testing of the robustness of their model, it is hard to compare their result with previous ones, but it is definite that recently introduced explanatory variables play a positive role in the model.

Another recent study of specific currency exchange rates, in particular the USD/CAD rate, by Ferraro, Rogoff and Rossi (2011) re-estimates the relationship between oil price and exchange rates. This time, the authors investigate the forecasting ability of oil prices on USD/CAD exchange rates. Recall that in the paper published by Chen, Rogoff and Rossi (2010), it is clear that the USD/CAD exchange rate and certain commodity prices, such as metal and oil, share an interactive effect with each other. According to the 2011 paper, this relationship and effect could be truly strong to some extent. Actually, in contrast to the widespread use of oil and metal prices, Dong and Nam (2011) find that the prices of individual goods (i.e., seafood, juice, laundry, etc.) could serve as effective fundamentals for long-run predictability of exchange rate for the United States, Japan, and the United Kingdom.

As one can see from this short review of econometric models used for exchange rate prediction, the choice of models varies based on the country examined. Among all the widely used models, the random walk model and other time series models produce a more accurate prediction than structural models out-of-sample; time series models could effectively increase econometric sophistication in foreign exchange markets. The random walk model produces a general satisfying result, but it is not the optimal choice for some specific cases; for example, dynamic forecast equations outperform the random walk given a longer horizon. General regression models do not necessarily reveal a highly impressive result; nevertheless, when one chooses proper explanatory variables, the prediction results are robust.

The research above mainly focuses on the interaction between commodity prices and foreign currency exchange rates in North America, therefore, it is natural to ask whether this interaction may exist in other parts of the global market. Previous research proves the corresponding interaction is effective in developed countries such as United States, Canada and Germany; therefore, people may be curious whether this interaction still holds in developing countries. This paper discusses the commodity price-exchange rate interaction between the Greater China Region (including mainland China, Hong Kong and Taiwan) and North America Countries. China is one of the largest exporting areas in the world. Though economic research on the Greater China Region never ends, few have investigated the possible relationship between currency exchange rates and commodity prices in Greater China Region to date. This paper focuses on the possible relationship between commodity price and exchange rate between North America and Greater China Region.

3 The Markov Switching and Hidden Markov Models

The previous section describes the regular econometric models used in the prediction of exchange rates, but besides these, there is one particular type of model applied in this field, the Markov Switching Model. Engel (1994) estimates a two-state Markov switching model to forecast currency exchange rates, namely the dollar/mark, dollar/pound and dollar/french ratios. The methodology of this Markov switching model investigates currency exchange rates' moves based on quarterly and monthly data, the parameters of which are estimated by maximum likelihood methods. According to Engel, the assumption that currency exchange rates follow a general stochastic process is not supported due to a relatively poor prediction.

Another impressive contribution of Engel (1994) is that he confirms the random walk model's outperformance among most prediction models up to that point in time. The reason for this is that the random walk generates lower mean squared error based on out-of-sample forecasts. The author concludes that the failure of the Markov switching model is due to the limited number of states. Given more states - for instance, three states of currency exchange rates - it is probable that the Markov switching model would perform better with the same data.

Cheung and Erlandsson (2005) estimate Markov Switching Models for three dollar-based exchange rates (Deutsche mark, British pound, and French franc). Their comparisons of the Markov Switching and random walk models suggest that there is no strong evidence to reject the Markov Switching given existing usage of the random walk model. The prediction results of the two types of models vary with the choice of data frequency and the foreign country. Thus, the Markov Switching model is not worse than the random walk. As the previous literature has already shown that the random walk is a popular model for the prediction of exchange rates, consequently the Markov Switching is a relatively good method.

The Markov Switching model is one kind of univariate model, and its key characteristic is that the model only requires exchange rate data; it does not ask for other data to assist in prediction. As there are quite a number of determinants (e.g, the commodity prices) that could affect future moves of exchange rates, the current Markov Switching model seems too simple to predict exchange rates without the necessary consideration of determinants. Therefore, given a Markov Switching model with instrumental variables, the prediction result of exchange rates could be promising.

Wu, Ganapathiraju and Picone (1999), in their report on re-estimation methods for Hidden Markov Models (HMMs), propose a possible relationship between observation sequences and actual states, with estimated parameters of the Baum-Welch training module. [1] Hidden Markov Models are intended to describe the dependence between the independent variables, which are also known as unobserved actual states, and the observation sequences. In light of the widespread assumption that economic determinants contribute to the movements of currency exchange rates, the Hidden Markov Model

[1] The Baum-Welch training module will be discussed in later sections.

serves as a promising method for research on the relationship between the unobserved actual states and the observation sequences.

Hidden Markov Models have been applied in a great number of disciplines, such as bioinformatics (Eddy 1998), customer relationship dynamics (Netzer, Lattin and Srinivasan 2007), and quite a few have discovered a causal dependence among the chosen sequences. Considering the relatively unsatisfying prediction from regular Markov model, we may want to know whether an extended Markov model such as the Hidden Markov Model could produce a better result in explorations of currency exchange rates.

Since the Hidden Markov Models require two sequences, the actual states and observation sequences, a successful application requires a proper choice of actual states, given the currency exchange rates as predetermined observation sequences. The remainder of this section outlines in more details the regular Markov Switching model and the Hidden Markov Model, with the discussion of the latter in a discussion of how to identify the states.

3.1 The Regular Markov Switching Model

The Markov Model, also known as the Markov Chain, is a popular tool for forecasting time-series. One key assumption of the Markov Model is that the current state only depends on the previous state, independent of other earlier states. Previous research, however, suggests that the Markov Model does a relatively poor job of predicting currency exchange rates. One significant research paper on the Markov Model by Engel

(1994) only includes two states in the switching set, and this limited number definitely decreases predictive accuracy. In order to improve this limitation, this paper increases the elements within the set of states for both the state and observation sequences in the estimation of Hidden Markov Models.

The basic Markov Model requires three elements:

1. A discrete states set $S=(s_1,s_2,\ldots,s_N)$, where N is the number of states

2. A discrete time set $t=(1,2,\ldots,T)$, where T is the number of time periods

3. An assumption: The prediction of the next state and its associated characteristics depends only on the current state.

When the transition from one state to the next takes place within a Markov chain, a successful prediction requires the probabilities for each step and the initial states. This requirement then leads to the most noteworthy characteristics of the Markov Model: the transition probability matrix $A=[a_{ij}]$, where a_{ij} is the probability of moving from state i to state j, and the distribution of initial states $\pi=[\pi_i]$, which indicates the possibilities of the states observed in period 1. Here x_t denotes the movement at time t.

$$A=[a_{ij}], where\ a_{ij}=P(x_{t+1}=s_j \mid x_t=s_i)$$

$$\pi=[\pi_i],\ \pi_i=P(x_1=s_i)$$

The traditional Markov Model considers all states and movements to be visible. For example, suppose we have a five-state Markov Model as shown in Figure 1. In this figure, the numbered circles represent the possible states, while the arrow joining them represent all the possible transitions from one state to another.

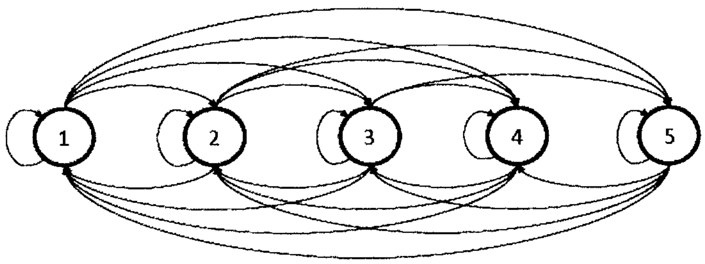

Figure 1 Five-state Markov Model

Now suppose the observed transition sequence is $Q=\{2,1,3,5,4\}$. Given the transition probability matrix $A=[a_{ij}]$ and the initial probability distribution $\pi=[\pi_i]$, then the probability of observing this sequence can be expressed as:

$$\begin{aligned} P(X \mid A,\pi) &= P(s_2)P(s_1 \mid s_2)P(s_3 \mid s_1)P(s_5 \mid s_3)P(s_4 \mid s_5) \\ &= \pi_2 a_{21} a_{13} a_{35} a_{54} \end{aligned}$$

More generally, for a regular Markov Model with state sequence $X=(x_1, x_2, \ldots, x_T)$, the sequence transition probability is:

$$\begin{aligned} P(X \mid A,\pi) &= P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_2)\cdots P(X_T \mid X_{T\text{-}1}) \\ &= \prod_{t=1}^{T-1} \pi X_t X_{t+1} \end{aligned}$$

For the traditional Markov Model with five states, we could observe all movements of this Markov chain, with $\sum \alpha_{ij} = 1$. However, in empirical research, sometimes the Markov Model is restricted due to the lack of observation of every movement. Mostly, one will see the observation sequence only, instead of the states from which the observations are generated. Therefore, although the Markov Model seems to be useful in evaluating and forecasting a sequence of time series data, however, in empirical research, one will often prefer the Hidden Markov Model when considering these unobserved characteristics. Obviously, an application of the regular Markov Model only needs a data on x_t, and it is clearly impossible in the model to investigate the possible relationship between sequences of values for x_t and hidden determinants of x_t. Considering the great number of factors which may affect currency exchange rates, the failure to introduce those factors may be the cause of the poor forecasting result in the earlier literature (Engel 1994, Cheung and Erlandsson 2005).

3.2 The Hidden Markov Model

Rabiner (1989) first introduced the Hidden Markov Model (HMM), in which a hidden stochastic process solves a hidden state Markov process based on its corresponding observation sequence. To illustrate the idea of the HMM, Rabiner (1989) suggests two popular examples, the Coin Toss Model and the *Urn and Ball Model.* [2] When we investigate the interaction between oil prices and currency exchange rates, the unobserved impact of oil prices on exchange rates suggests the application of the Hidden Markov Model. Section 4 will use the Hidden Markov Model to estimate the predictive capacity of oil prices on currency exchange rates.

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[2] The Urn and Ball Model was first introduced by J. Ferguson in the 1970s.

3.2.1 Characteristics of The Hidden Markov Model

First, we will introduce the basic structure and characteristics of the Hidden Markov Model. When we extend a traditional two-state Markov Model to a two-state Hidden Markov Model (Figure 2), the structure appears as in Figure 3. Figure 3 shows a two-state HMM with latent states s_1 and s_2 and their three identical possible observations v_i, $i = 1,2,3$.

In Figure 2, the move from state 1 at time t to the next state at time t+1 involves only two possibilities: one can stay in state 1 with probability a_{11} or move to state 2 with probability a_{12}. Compared with the regular Markov Model, in the HMM it is harder to derive the real state sequence.

For example, suppose one observes a sequence as $\{v_1, v_2\}$ in the traditional Markov Model. Clearly this observation sequence indicates a state sequence of $\{s_1, s_2\}$; however, when one sees this observation sequence based on the HMM, there are four possible latent state sequences: $\{s_1, s_1\}$, $\{s_1, s_2\}$, $\{s_2, s_1\}$, $\{s_2, s_2\}$.

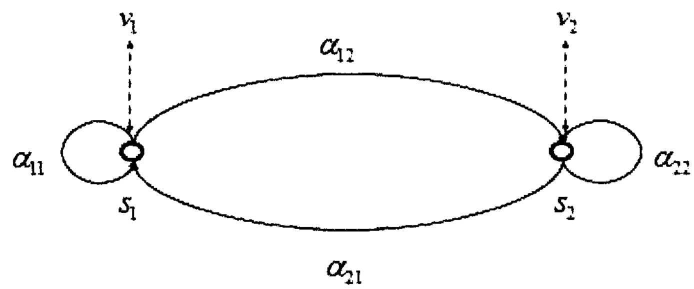

Figure 2 Two-state Markov Model

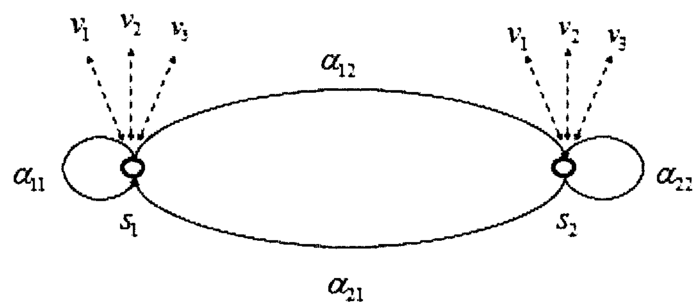

Figure 3 Two-state Hidden Markov Model

A HMM within time length T is defined by three parameters:

$$\lambda=(A,B,\pi)$$

Prior to defining of $A, B,$ and π, we introduce the definitions of the state set and state sequence. This HMM contains N latent states and M observations in the entire process. We define the latent state set as S and observation state set as V, where

$$S=\{s_1,s_2,\ldots,s_N\}$$

$$V=\{v_1,v_2,\ldots,v_M\}$$

Meanwhile, within time length T, the latent state sequence is defined as Q and a corresponding observation sequence is defined as O, where

$$Q = q_1, q_2, \dots, q_T$$

$$O = o_1, o_2, \dots, o_T$$

Q is related to S as the value of q_t must be chosen from the set S, and O is related to V as the value of o_t must be chosen from the set V.

When the process moves from latent state s_i at time $t-1$ to latent state s_j at time t, we define the transition probability to be a_{ij}, which leads to a corresponding transition probability matrix A:

$$A = [a_{ij}], where\ a_{ij} = P(q_t = s_j \mid q_{t-1} = s_i)$$

But, instead of witnessing the real state transition process, we can only see an observation transition process with probability matrix B:

$$B = [b_i(k)],\ b_i(k) = P(o_t = v_k \mid q_t = s_i) \text{ where } 1 \le i \le N, 1 \le k \le M$$

Finally, the initial state probability distribution is π:

$$\pi = [\pi_i], \pi_i = P(q_t = s_i)$$

Blunsom (2004) suggests three pre-assumptions for the application of HMM.

1. Observation Probability $b_i(k)$ indicates the probability that latent state q_t determines observation o_t at time t. However, this process is independent of time t.

2. Current state q_t depends only on the previous state q_{t-1}.

$$P(q_t \mid q_1^{t-1}) = P(q_t \mid q_{t-1})$$

3. Observation o_t at time t is dependent only on the current state q_t.

$$P(o_t \mid o_1^{t-1}, q_1^t) = P(o_t \mid q_t)$$

Figure 4 describes a more general HMM process. The HMM in Figure 4 contains more states and observations than that in Figure 3. In Figure 4, circles represent observations, squares represent real states and lines represent possible transitions. The HMM will choose the next state based on its observation probability. The main characteristic of the HMM is that even though an observer could not tell the true state sequence behind the observations, it is still possible, as will be discussed below, to estimate the model given the observations. This characteristic effectively reduces the requirements to apply the HMM in empirical research.

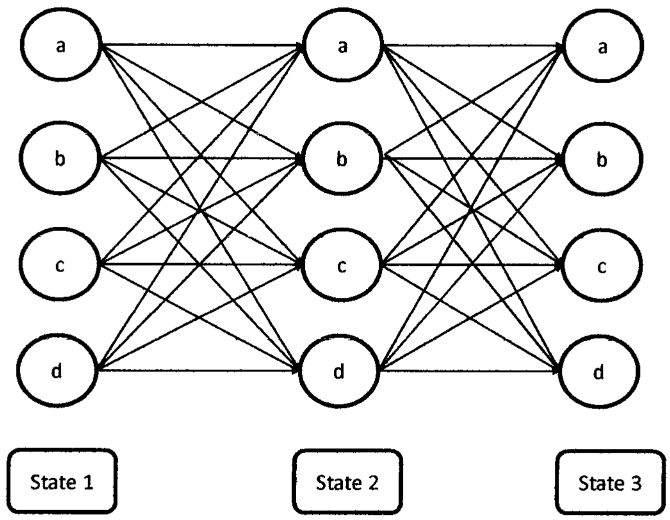

Figure 4 Regular Hidden Markov Model Process with States and Observations

HMMs are mainly used to solve three basic problems.

1. Given the model λ=(A, B, π) and the observation sequence $O = o_1, o_2, \ldots, o_T$, the problem is to calculate the observation sequence probability $P(O \mid \lambda)$.

2. Given the model λ=(A, B, π) and observation sequence $O = o_1, o_2, \ldots, o_T$, the problem is to choose the most plausible state sequence which produces the observation sequence $O = o_1, o_2, \ldots, o_T$.

3. Given the observation sequence $O = o_1, o_2, \ldots, o_T$ and state sets $S = \{s_1, s_2, \ldots, s_N\}$, the problem is to find the model which could maximize $P(O \mid \lambda)$.

Problem 3, which is also known as the "Learning Task" (Blunsom 2004), aims to find the optimal choice of parameters λ=(A, B, π) in order to explain the relationship between a latent state sequence $Q = q_1, q_2, \ldots, q_T$ and an observation sequence $O = o_1, o_2, \ldots, o_T$. Here we only focus on Problem 3 due to the requirements of our problem.

Recall that our objective is to examine the relationship between oil prices and currency exchange rates. We are looking for a reasonable explanatory impact from oil prices to currency exchange rates. Therefore, in our analysis, we treat the exchange rate sequence as an observation sequence $O = o_1, o_2, \ldots, o_T$, while the oil price sequence appears as the latent state sequence $Q = q_1, q_2, \ldots, q_T$.

Problem 3 could be interpreted as maximizing the joint probability of an observation sequence, thus:

$$\max_{\lambda} P(O \mid \lambda) = \sum_{Q} P(O, Q \mid \lambda)$$

There are a few methods to estimate the parameters in this maximization problem, but most economists tend to use one of two algorithms, the Forward Algorithm and the Back-Forward Algorithm (Baum-Welch Algorithm, Blunsom 2004). The Baum-Welch algorithm, which is also a Maximum Likelihood Estimator, effectively improves the process of solving the maximization problem since it only depends on the observation sequence. In the following paragraphs, we introduce the basic steps of the Baum-Welch algorithm and then present the corresponding HMM for our topic.

The characteristics of the Baum-Welch algorithm include:

1. Forward Probability

$$a_t(i) = P(s_t = i, o_1 \dots o_t \mid \lambda)$$

$a_t(i)$ is the probability being in state s_i at time t, based on an observation sequence $O = o_1, o_2, \dots, o_t$ so far. We define the method of calculation $a_t(i)$ as:

$$a_1(i) = \pi_i b_i(o_1)$$

$$a_{t+1}(j) = b_j(o_{t+1}) \sum_{i=1}^{N} a_t(i) a_{ij}$$

2. Backward Probability

$$\beta_t(i) = P(o_{t+1} \dots o_T \mid s_t = i, \lambda)$$

$\beta_t(i)$ is the probability of observing a future observation sequence $O = o_{t+1}, \dots, o_T$, based on the fact that one is in state s_i at time t. Correspondingly, the calculation methods are:

$$\beta_T(i) = 1$$

$$\beta_j(t) = \sum_{i=1}^{N} \beta_j(t+1) a_{ij} b_j(o_{t+1})$$

3. Transition Probabilities

The probability of being in state s_i at time t, and then moving to s_j at time t+1 is

$$\xi_t(i,j) = P(q_t = i, q_{t+1} = j \mid O, \lambda) = \frac{a_i(t) a_{ij} \beta_j(t+1) b_j(o_{t+1})}{\sum_{i=1}^{N} \sum_{j=1}^{N} a_i(t) a_{ij} \beta_j(t+1) b_j(o_{t+1})}$$

The probability of staying in state s_i during the entire period of time is

$$\gamma_i(t) = P(q_t = i \mid O, \lambda) = \frac{a_i(t) \beta_i(t)}{\sum_{j=1}^{N} a_j(t) \beta_j(t)}$$

Blunsom (2004) shows that maximizing the original Maximum Likelihood Function is equivalent to maximizing the function:

$$Q(\lambda, \bar{\lambda}) = \sum_i P(S = s_i \mid O = o_i, \lambda) \ln P(S = s_i, O = o_i \mid \bar{\lambda})$$

Note that the statement $P(O \mid \bar{\lambda}) \geq P(O \mid \lambda)$ could be interpreted as $Q(\lambda, \bar{\lambda}) \geq Q(\lambda, \lambda)$ (Blunsom, 2004). Therefore, the MLE problem we are working with is:

$$\begin{aligned}
\max_{\bar{\lambda}} Q(\lambda, \bar{\lambda}) &= \sum_i P(S = s_i \mid O = o_i, \lambda) \ln P(S = s_i, O = o_i \mid \bar{\lambda}) \\
s.t. \quad & \sum_{i=1}^{N} \bar{\pi}_i = 1 \\
& \sum_{j=1}^{N} \bar{a}_{ij} = 1 \qquad where\ 1 \leq i \leq N \\
& \sum_{k=1}^{M} \bar{b}_j(k) = 1 \quad where\ 1 \leq j \leq N
\end{aligned}$$

By solving this maximization problem, we will obtain estimators for our model, $\bar{\lambda} = (\bar{A}, \bar{B}, \bar{\pi})$. The estimates are:

1. The expected frequency of state s_i at the initial time, period 1:

$$\bar{\pi}_i = \gamma_i(1)$$

2. The ratio of expected transitions from state s_i to state s_j over expected transitions from state s_i:

$$\bar{a}_{ij} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \gamma_t(i)}$$

3. The ratio of expected transitions in state s_i considering observations k over expected transitions from state s_i:

$$\bar{b}_i(k) = \frac{\sum_{t,o_t=k} \gamma_t(i)}{\sum_{t=1}^{T} \gamma_t(i)}$$

So far, we have provided a short introduction to HMM. With proper information on the observation sequence only, we will discover a reasonable HMM estimating current data. By re-defining the observation sequence and the state sequence, we will have estimators as the solution to Problem 3 in our model.

3.2.2 Clustering of States in The Hidden Markov Model

States are the key to a successful application of the HMM. In our model, both commodity prices and currency exchange rates are continuous time-series data, and dividing them into discrete states is the first step of HMM process. As the assumption in this paper is that commodity prices determine currency exchange rates, we treat commodity prices as an unobserved state sequence and currency exchange rates as the observation sequence.

There are a great number of ways to justify the number of states we should use in our model, among which the most popular ones are K-means (Ding and He 2004), hierarchical clustering (Defays 1977), the AIC/BIC criteria method (Jones 2011), etc. Nevertheless, all these methods can only assist in determining the number of states, instead of offering one global preferable state set directly. Actually, there is still no clear method offering this global solution.

Compared with the other two popular algorithms, Hierarchical Clustering reallocates objects within a given number of groups (Sibson 1973). The hierarchical clustering algorithm creates clusters by moving from the leaves to the root of an entire dendrogram (a tree diagram).

With detailed steps, Hierarchical clustering starts with every single element of the entire set, and then merges all the elements to form clusters of a higher level in to a hierarchical structure. This process will be repeated until all the elements are gathered into one single cluster, i.e. the highest level cluster.

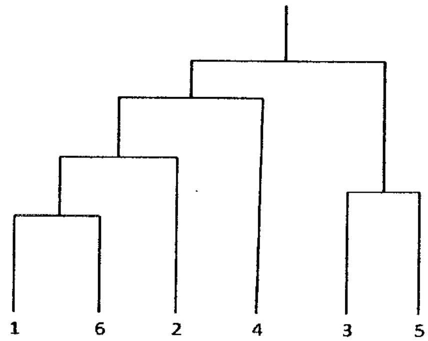

Figure 5 Example of a Dendrogram of Hierarchical Clustering

Figure 5 describes a dendrogram of Hierarchical Clustering. Therefore, it is easy to conclude that the dendrogram itself describes the clustering results. However, the clustering results do not necessarily follow the time sequence of the observations. In other words, given a sequence $S = \{s_1, s_2, s_3, s_4 \ where \ s_1 < s_2 < s_3 < s_4\}$, we will not necessarily obtain a group with $G = \{g_1 = s_1, g_2 = s_2, g_3 = s_3, g_4 = s_4\}$. Even if the clustering sequence does not correspond to the observation sequence, it will not affect the clustering analysis result. In fact, we focus only on methods for clustering.

In our model, we use commodity prices and currency exchange rates as the state sequence and observation sequence respectively for the HMM, so that two clustered sequence sets are required for HMM analysis. Note that the HMM application requires discrete sequences for both observations and states.

One great benefit of Hierarchical clustering is that it offers choices for pairwise distance and linkage distance. Pairwise distance is used to compute distance between pairs of objects in a matrix. Linkage distance is used to compute the distance between different clusters. Sometimes it is hard to determine the best method for clustering data before any empirical experiment, and thus it is best to compare the results of different methods before making a choice. In the next section, the paper introduces five Pairwise distance options (Chebychev, Euclidean, Mahalanobis, Minkowski, Seuclidean[3]) for measuring distance between elements in a data set, and seven linkage distance options (Unweighted Average Distance (UPGMA), Centroid Distance (UPGMC), Furthest Distance, Weighted Center of Mass Distance (WPGMC), Shortest Distance, Inner Squared Distance, Weighted Average Distance (WPGMA)[4]) for measuring distance between clusters.[5]

4 Data Description

This paper chooses three Greater China Region currencies - the Chinese Yuan (CNY, also known as the Renminbi), the Hong Kong Dollar (HKD) and the Taiwan Dollar (TWD) - and investigates their exchange rates with two North America currencies, the United States Dollar (USD) and the Canadian Dollar (CAD). For commodity prices, oil prices and gold prices are used in this paper. Both the currency exchange rate sequences and commodity price sequences are available in daily, weekly, and monthly frequencies, which provide sufficient information for time-series models.

The currency exchange rates between the Greater China Region and North America were observed for the time-period January 1, 1996 to December 31, 2010. The data for

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[3] These are popular distance options in empirical research.

[4] These are popular distance options in empirical research. The paper uses short names for the seven distance options: single, complete, average, weighted, centroid, median and ward. For future details on each method, see the empirical analysis part.

[5] The definitions of distances could be found at http://www.mathworks.com/help/toolbox/stats/linkage.html.

commodity prices cover the same period of time. The paper's in-sample experimental data ranges from September 26, 1998 to December 31, 2010, while the out-of-sample experimental data are for the period January 1, 2011 to December 31, 2011.

For the nominal exchange rates, the paper chooses the mid-point exchange rate from OANDA's Historical Exchange Rate for the entire empirical analysis. [6] The choice of mid-point results from a consideration of the need to smooth unnecessary fluctuations. Exchange rates may vary greatly within a short period of time like one week or even one day; therefore, if we choose the upper or lower bound of the exchange rate, the exchange rate sequences would be too extreme to be modeled.

The oil price data are for West Texas Intermediate (WTI), which is regarded as a benchmark for crude oil prices, of the U.S. Energy Information Administration. [7] For other commodity prices, the paper uses the London Afternoon (PM) Gold Price Fix from the USAGOLD database as the price of gold.[8] Gold has long been regarded as the oldest and most respected currency worldwide, especially during wars or upheavals. Banks still consider gold to be the first option for investment, and its role is still significant in the economy.

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[6] OANDA is a private firm which provides an financial internet-based trading and currency information service and filtered currency databases. The data were downloadd from *http://www.oanda.com/currency/historical-rates/* on January 20, 2012.

[7] U.S. Energy Information Administration. The data were downloaded from *http://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=pet&s=rwtc&f=d* on January 20, 2012.

[8] USAGOLD is one of the top gold firms in United States, providing data and information for gold-based research and analysis. The data were downloaded from *http://www.usagold.com/reference/prices/history.html* on January 20, 2012.

5 Empirical Analysis of Commodity Prices and Currency Exchange Rates

This section introduces specific clustering methods for each currency exchange rate. In total there are seven observations: the Oil Price, the CAD/CNY, the CAD/HKD, the CAD/TWD, the USD/CNY, the USD/HKD, and the USD/TWD. Then a further analysis of prediction results will be included.

5.1 Clustering of Oil Price and Currency Exchange Rates

The matrix in Table 1 presents the correlation indexes between the Oil Price Daily series and its clustering methods. Elements in matrix are correlation coefficients for hierarchical cluster tree of Oil Price Daily, and it is often regarded as a method to evaluate the fitness of clustering method. We regard the best combination of clustering distance and linkage distance to be the one with the greatest correlation coefficient. The bigger the coefficient is, the better its method fits real data sequence.

Here, the greatest element in the correlation Table is 0.897275393, which implies that we should choose [Centroid, Euclidean] as the method for Oil Price Daily clustering for the HMM analysis. Next, we consider effective clustering for currency exchange rates using same approach for CAD/CNY daily exchange rate, obtaining the correlations in Table 2.

Table 1 Oil Price Daily Clustering Correlations[a]

Distance between Clusters	Distance Between Elements				
	Chebychev	Euclidean	Mahalanobis	Minkowski	Seuclidean
Single	0.845262935	0.845262935	0.845262935	0.845262935	0.845262935
Complete	0.894419129	0.894419129	0.894419129	0.894419129	0.894419129
Average	0.897274015	0.897274015	0.897274015	0.897274015	0.897274087
Weighted	0.890133961	0.890133961	0.890121715	0.890133961	0.896279889
Centroid		**0.897275393**			
Median		0.89013197			
Ward		0.760630478			

[a] Centroid, Median and Ward only work for the distance option Euclidean. The correlation in bold is the largest value for this sequence. The settings above apply to all correlation tables in this paper

Table 2 CAD/CNY Clustering Correlations[a]

Distance between Clusters	Distance Between Elements				
	Chebychev	Euclidean	Mahalanobis	Minkowski	Seuclidean
Single	0.411045083	0.411045083	0.411045083	0.411045083	0.411045083
Complete	0.725702243	0.725702243	0.725702243	0.725702243	0.725702243
Average	0.725577803	0.725577803	0.725577803	0.725577803	0.725577803
Weighted	0.707434221	0.707434221	**0.73318028**	0.707434221	0.707434221
Centroid		0.725577804			
Median		0.728448936			
Ward		0.711669299			

[a] See note to Table 1

Here, the greatest correlation index is 0.73318028 for [Weighted, Mahalanobis]. Therefore, we will choose this set as our setting for CAD/CNY Daily clustering.

Similarly, as Table 3 indicates we should choose [Weighted, Seuclidean] for the CAD/HKD daily exchange rate.

Table 3 CAD/HKD Daily Clustering Correlations[a]

Distance between Clusters	**Distance Between Elements**				
	Chebychev	Euclidean	Mahalanobis	Minkowski	Seuclidean
Single	0.626244779	0.626244779	0.626244779	0.626244779	0.626244779
Complete	0.815066405	0.815066405	0.815066405	0.815066405	0.815066405
Average	0.814186638	0.814186638	0.814186638	0.814186638	0.814186638
Weighted	0.815382891	0.815382891	0.815382891	0.815382891	**0.815384973**
Centroid		0.814186638			
Median		0.815382828			
Ward		0.811135924			

[a] See note to Table 1

In contrast to the previous cases, Table 4 shows that there is more than one pair of distance measures with the same greatest value for the correlation coefficient. This means that there is no way to choose better options, so we choose setting of CAD/TWD Daily to be [Complete, Euclidean].

Table 4 CAD/TWD Daily Clustering Correlations[a]

Distance between Clusters	Distance Between Elements				
	Chebychev	Euclidean	Mahalanobis	Minkowski	Seuclidean
Single	0.645319499	0.645319499	0.645319499	0.645319499	0.645319499
Complete	**0.815927762**	**0.815927762**	**0.815927762**	**0.815927762**	**0.815927762**
Average	0.810407107	0.810407107	0.810407107	0.810407107	0.810407107
Weighted	0.767238057	0.767238057	0.800758219	0.767238057	0.767266056
Centroid		0.810407107			
Median		0.767266075			
Ward		0.79941726			

[a] See note to Table 1

Table 5 USD/CNY Daily Clustering Correlations[a]

Distance between Clusters	Distance Between Elements				
	Chebychev	Euclidean	Mahalanobis	Minkowski	Seuclidean
Single	0.756752435	0.756752435	0.756752435	0.756752435	0.756752435
Complete	0.964625749	0.964625749	0.964625749	0.964625749	0.964625749
Average	0.969411236	0.969411236	**0.969411392**	0.969411236	0.969411392
Weighted	0.952378435	0.952378435	0.952378435	0.952378435	0.952378435
Centroid		0.969411389			
Median		0.952378435			
Ward		0.963642834			

[a] See note to Table 1

A similar problem arises in Table 5 and we choose the setting for USD/CNY Daily to be [Average, Mahalanobis] because of the greatest value 0.969411392.

Table 6 USD/HKD Daily Clustering Correlations[a]

Distance between Clusters	**Distance Between Elements**				
	Chebychev	Euclidean	Mahalanobis	Minkowski	Seuclidean
Single	0.198291509	0.198291509	0.198291509	0.198291509	0.198291509
Complete	0.640868757	0.640868757	0.640868757	0.640868757	0.640868757
Average	0.829652207	0.829652207	**0.829807981**	0.829652207	0.829652424
Weighted	0.735998272	0.735998272	0.735998272	0.735998272	0.735998272
Centroid		0.829652207			
Median		0.735998298			
Ward		0.76060776			

[a] See note to Table 1

In USD/HKD Daily Correlation table, we have 0.82980798 as the greatest value and the setting of USD/HKD Daily is [Average, Mahalanobis].

When we consider clustering of USD/TWD Daily Correlation table, Table 7 shows that there is more than one pair of distance measures with the same greatest value 0.887782512. This means that there is no way to choose better options, so we choose setting of USD/TWD Daily to be [Average, Euclidean].

Table 7 USD/TWD Daily Correlations[a]

Distance between Clusters	Distance Between Elements				
	Chebychev	Euclidean	Mahalanobis	Minkowski	Seuclidean
Single	0.813340182	0.813340182	0.813340182	0.813340182	0.813340182
Complete	0.709103873	0.709103873	0.709103873	0.709103873	0.709103873
Average	**0.887782513**	**0.887782513**	**0.887782513**	**0.887782513**	**0.887782513**
Weighted	0.676341941	0.676341941	0.594223182	0.676341941	0.676341941
Centroid		0.887782513			
Median		0.676342918			
Ward		0.851089715			

[a] See note to Table 1

So far, we have discussed the clustering settings for all currency exchange rates and Oil Price Daily. It is interesting to discover that exchange rates between the USD and all the Greater China Region currencies share [Average] as the same option. However, the settings for exchange rates between the CAD and the Greater China Region currencies do not exhibit the same consistency as those with the USD. The CAD settings vary with the clustering distance options of Euclidean, Mahalanobis and Seuclidean, and linkage distance options of Weighted and Centroid. This variation merely implies that CAD exchange rates are more likely to be affected by the choice of distance option. To sum up, different preferable choices of distance and linkage distance options often lead to further consideration in mathematics, but in economic research the distance options only serve as methods for measurement, so we will use the distance and linkage options as descriptive methods rather than discuss their mathematical definitions.

We also apply the same clustering method to Real Gold Price to define the state sequence over the same time period. However, we find the Baum-Welch Algorithm cannot estimate a transmission matrix using Gold Price Daily. The simulation process stops when some zero transition probabilities appear for the originating states.

Recall that our HMMs for currency exchange rates and commodities price are based on the assumption that there exists a relationship between commodity prices and currency exchange rates. Since we now have the clustering outcome that define the states observed, we can move on to the estimation of the HMM. We use the Hidden Markov Model toolbox from Matlab as the programming method. We choose 30 as the number of clusters (i.e., states) for the Oil Price and exchange rates. Matlab toolbox uses the number 30 as the default maximum number of clusters in its programming, so we choose this number. Thus we have 30 states for every variable in the HMMs, including commodity prices and exchange rates.

5.2 Hidden Markov Models of Currency Exchange Rates

The HMMs for the Greater China Region exchange rates are estimated using the Hidden Markov Model toolbox in Matlab version 2007. Given the latent state variables and the observation sequence, the HMM toolbox imputes the corresponding parameters of the HMM. Later, with the estimated parameters, we will examine the effectiveness of the HMM using in-sample and out-of-sample forecasts. The method of evaluation is to calculate the matching percentages. Recall the HMM's assumption that the commodity price determines exchange rates. After we obtain estimates of the parameters of the HMMs using the Baum-Welch Algorithm, we will take an in-sample or out-of-sample series of exchange rates as a given observation sequence to predict the state sequence of commodity prices with the estimated parameters. If the HMMs describe the correlation between commodity prices and exchange rates correctly, we should observe a high percentage of matches between the predicted state sequence and the real state sequence. The test we use involves predicting the oil price sequence given the observation sequence of exchange rates, but predictions could be done in the inverse direction as well. It is not necessary to do predictions in both directions to evaluate the models.

5.2.1 Oil Price and Currency Exchange Rates

These HMMs produce predicted exchange rates based on a given state series. Given both a state series and an observation series, we can obtain the parameters of the HMMs. The test which is used to examine the accuracy of the HMMs is to compare the predicted state series with real state series based on the observation series and estimated parameters.

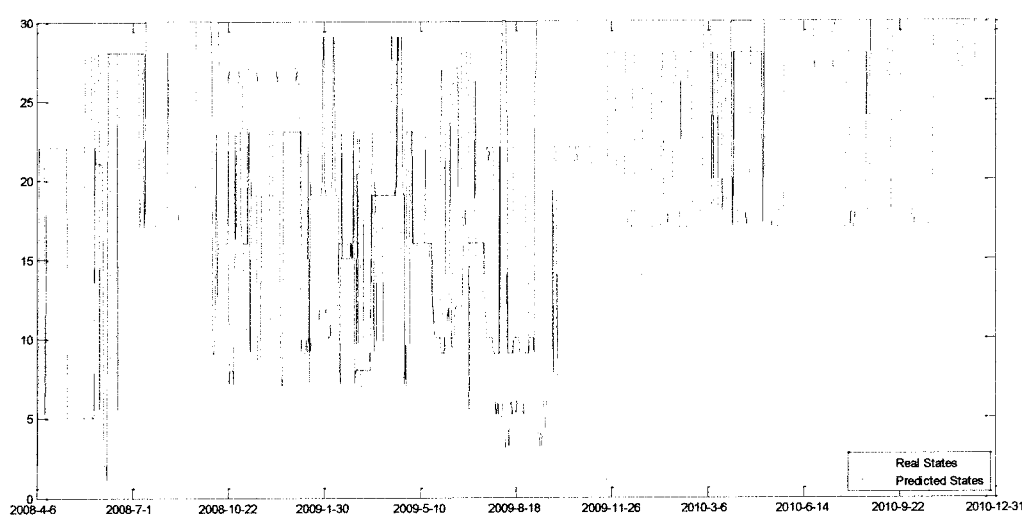

Figure 6 Actual and Predicted States of Oil Price Daily Given CAD/CNY

In Figure 6 above, the blue line shows the forecasted state movements of Oil Price Daily given the observation sequence of the CAD/CNY Daily exchange rate between 2008 and 2010, and the red line describes Oil Price Daily's real state movements during the same period of time. From April 2008 to Oct 2008, the forecasted states do not appear to fit the real states of Oil Price Daily very well; major differences in both the magnitudes of fluctuations and the direction are observed.

One representative example is the difference around July 1st, 2008, where the error is as large as 28 units. We can conclude that in the first six months of the sample, the HMM exhibits little capacity to forecast the states of Oil Price Daily. Later on, from late October 2008 to Jan 2009, the predictability increases slightly for future state movements. To be more specific, the HMM can forecast trends in next-step movements of states, but, it fails to predict the exact locations where the states will be later. It is noteworthy that around June 2009, the predictability of the HMM is so precise that it predicts exactly the same movements as the real state sequence; while around August 2009, the HMM moves simultaneously with the real state movements, but with a smaller magnitude.

However, during the rest of the period from Jan 2009 to August 2009, the HMM actually predicts movements almost totally opposite to the real state movements. After that, the HMM's predictions do not make sense for the rest of the observations. Hence, in the first sixteen months of the sample period, we can observe a reasonable correlation with the real state sequence: the same tendency but not the same magnitude, an opposite trend with the same magnitude, or the same tendency with same magnitude. But the rest of the forecasted states do not fit the real data. We will discuss possible reasons for this chaotic behaviour later.

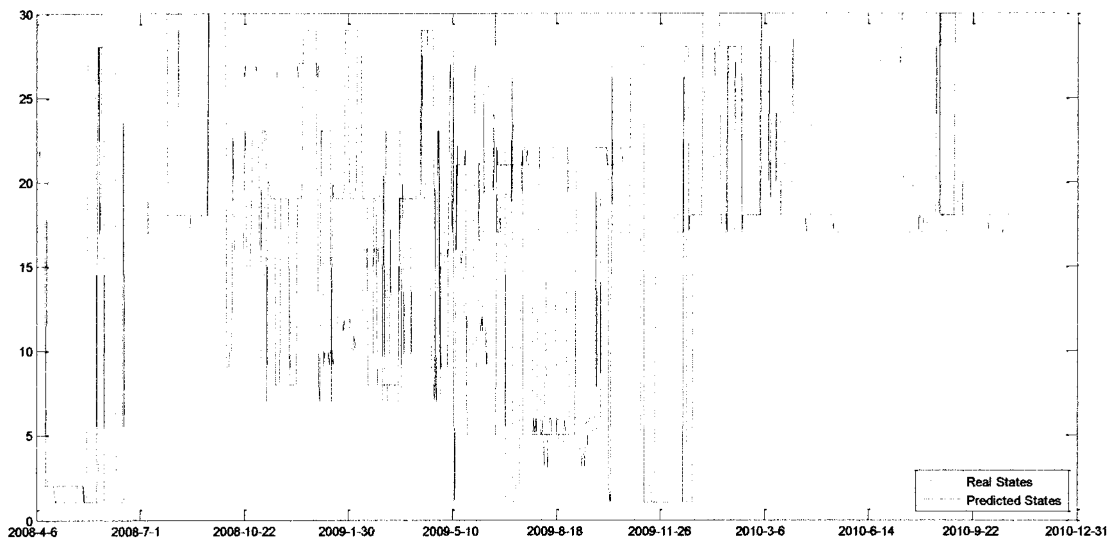

Figure 7 Actual and Predicted States of Oil Price Daily Given CAD/HKD

Figure 7 shows that compared with the previous results for the CAD/CNY Daily, the HMM of the forecasted CAD/HKD exchange rate does a better job of fitting the data. From April 2008 to Oct 2008, the HMM successfully forecasts the exact trend of Oil Price Daily real states, though it fails to describe some detailed fluctuations. Then from October 2008 to March 2010, the forecasted state sequence no longer seems to correspond to the actual sequence. Around January 2009, May 2009, November 2009 and March 2010, the HMM presents precise prediction results as the distances from real states are quite small. Meanwhile, the predictions around October 2008, February 2009 and June 2009 describe meaningful results in the opposite direction but almost same the magnitude as real state movements. During the rest of the time period, the HMM seems to forecast the correct trend of real state movements, but fails to accurately predict the level of the oil prices. Overall, the HMM of the CAD/HKD Daily outperforms that of CAD/CNY Daily in accuracy, but it still includes some confusing segments with incorrect predictions of real state movements. This raises the question what drives the opposite predictions in the HMM.

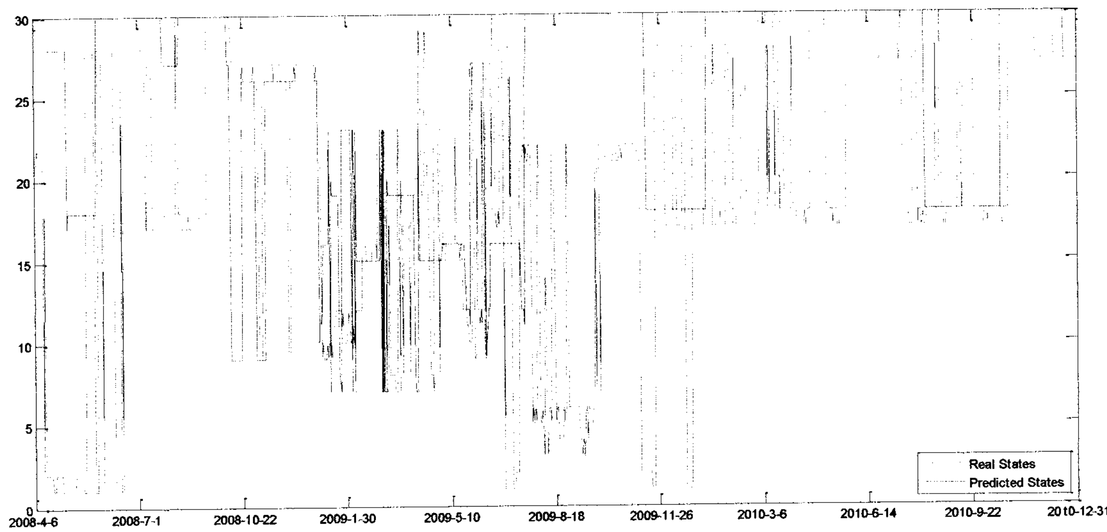

Figure 8 Actual and Predicted States of Oil Price Daily Given CAD/TWD

The HMM of the Canada/Taiwan Daily (CAD/TWD Daily) exchange rate has a relatively short effective forecasting length, as its predictive effectiveness declines after August 2009. In the CAD/TWD Daily's effective forecasting time period, the predicted state sequence still lacks precision. One can observe that during the first six months, the HMM predicts the approximate trend of the real state sequence; however, Figure 8 shows that the detailed fluctuations and magnitudes of the two lines do not match. Later on, from January 2009 to August 2009, the HMM prediction reflects several sharp fluctuations in the real state movements, but after that the HMM prediction does not make much sense in forecasting. Therefore, it is easy to conclude that the HMM's predictability by the CAD/TWD Daily is lower than that of the two HMMs discussed above.

Thus far, we have reviewed the predictions of HMMs of exchange rates between the Canadian dollar and Greater China Region currencies. The model of the CAD/HKD Daily undoubtedly outperforms those of the other two currency exchange rates in terms of its ability to predict trends and exact movements within the sample. But all three HMMs share the same sharply decreasing predictability in the last several months of our sample, which raises the question of why does the HMM's predicitability declines within later years of the data set. Also, predictions around October 2008 and May 2009 suggest a high forecasting capacity of all three models; undoubtedly the HMMs are most effective during this seven-month period. In addition, the forecasts that move in the wrong direction do not occur randomly, as they appear during the same time period, October 2008 and May 2009, for all three currencies. Therefore, it would be desirable to investigate the reason for this uniformity of errors

Before attempting to answer these questions, however, we will examine HMMs of the U.S. dolar exchange rates of the three Greater China Region currencies. As discussed above, one interesting feature of the clustering of the USD/Greater China Region exchange rates is that the exchange rates all have the same clustering setting, [Average]. Consequently, we may consider the possibility that states which are generated by exactly the same clustering method may share some consistency in HMM prediction results.

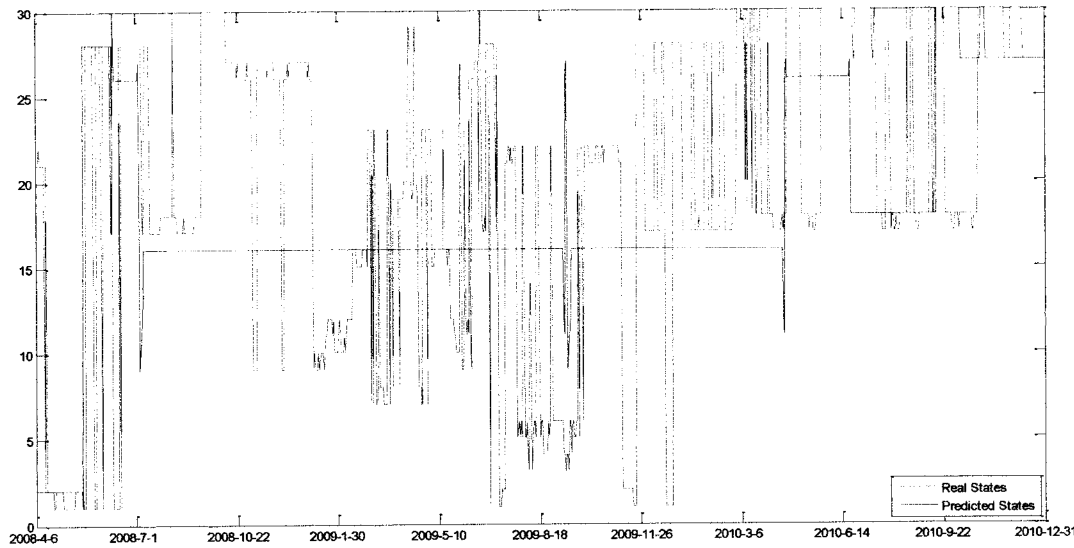

Figure 9 Actual and Predicted States of Oil Price Daily Given USD/CNY

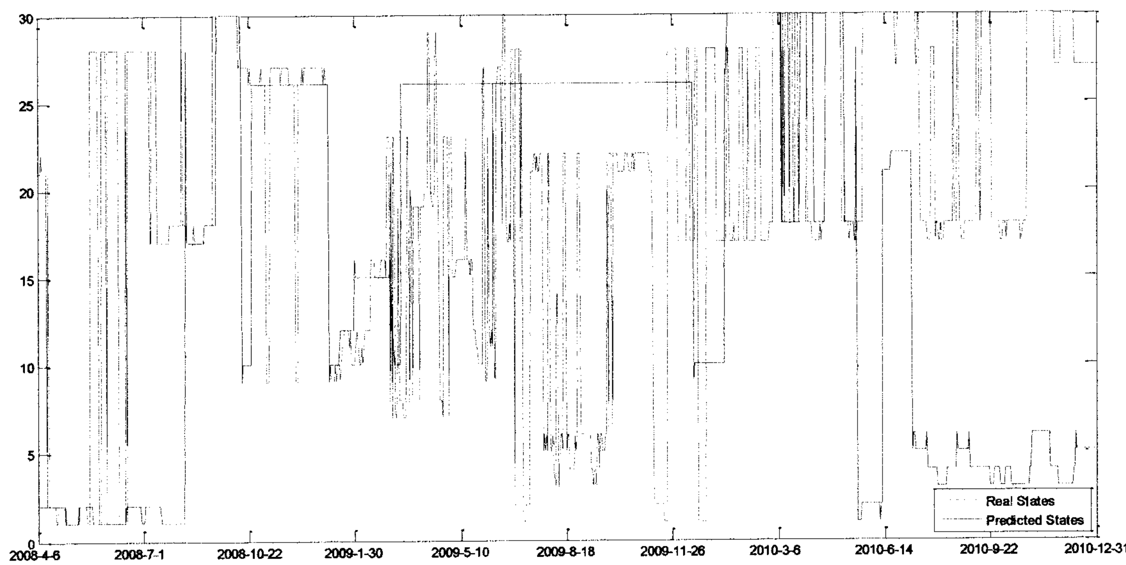

Figure 10 Actual and Predicted States of Oil Price Daily Given USD/HKD

Figure 9 and Figure 10 compare the actual and forecasted states of the USD/CNY and the USD/HKD exchange rates. Obviously, the two HMMs' predictive power is quite low during the sample period and neither trends nor fluctuations are reflected by the two HMMs. However, the model of the thrid regional currency, the USD/TWD does a better job predicting the state sequence. Though the period of greatest accuracy is still short (April 2008 to May 2009), the forecasted results are satisfying to some extent. From Figure 11, we can see that the HMM of USD/TWD performs best during the period between January 2009 and May 2009, while during the first nine months of this period the forecasted results are quite vague, with a lack of detailed fluctuations. This performance of the forecast of the USD/TWD exchange rate is similar to that of the CAD exchange rates discussed earlier. Thus even though the USD exchange rates display high consistency in clustering methods, this did not improve the HMMs forecasting capacity. Hence clustering consistency does not guarantee HMM accuracy.

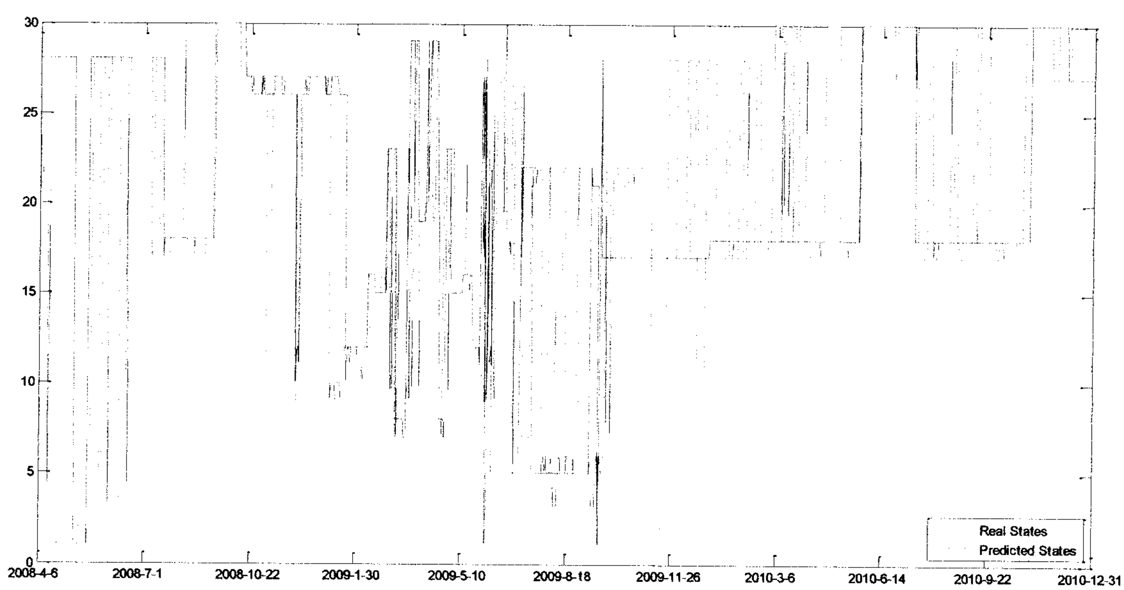

Figure 11 Actual and Predicted States of Oil Price Daily Given USD/TWD

Overall, we do not observe great success in predicting actual states of Oil Price by HMMs and the reasons for this failure will be discussed in secion 5.3. When we compare the predictiability ability of USD and CAD, the CAD exchange rates outperform the USD exchange rates in forecasting states, given the same sequences of currency exchange rates. Actually, despite the USD/TWD's forecasting results, the other two forecasted USD sequences do not contain too much information for our estimation of HMM. The USD/TWD forecasting results are somehow meaningful in explaining the relation between commodity prices and exchange rates, but it is still not as effective as an HMM for CAD exchange rates. The reported matching percentage of the six HMMs to be discussed in section 5.3 also confirm our results.

5.2.2 Other Commodities and Different Frequencies of Exchange Rates

We also apply the same HMM method using Real Gold Price to define the state sequence over the same time period. After applying the Hierarchical Clustering method to Gold Price Daily, we find that [Average, Euclidean] provides the best fit. Surprisingly, we find that Baum-Welch Algorithm cannot estimate a transmission matrix using Gold Price Daily. The stimulation process stops when some zero transition probabilities appear for the originating states.

In addition, when we apply previously estimated parameters to predict Oil Price states from the out-of-sample exchange rates, we find that the zero transition probabilities also take place when originating states. Unlike the in-sample data of exchange rates, the out-of-sample exchange rates have a much shorter period of time, which only contains 365 observations. This lack of data may be the reason for the failure of the HMM.

This paper estimates HMMs in the inverse direction; in other words, we tried to construct a HMM with commodity prices as the observation sequence and an exchange rate as the hidden state sequence. However, the inverse HMMs do not make sense in explaining the interaction between exchange rates and commodity prices, because the corresponding matching percentages are quite low.

When we use currency exchange rates at weekly and monthly frequencies, we cannot estimate an HMM process either. However, unlike the zero transition probability problem, this time the HMM fails because of a lack of sufficient data. The unsuccessful modeling of the Gold Price and exchange rates of other frequencies may be due to the use of an inappropriate method of clustering. Though we chose Hierarchical Clustering as our method, when it comes to the number of states, we chose 30, which is the value for all the HMM estimates in Matlab. It is probable that this value is not appropriate for them.

5.3 Discussion

This paper uses a totally original method to investigate the relationship between commodity prices and currency exchange rates. As a result, there are few research studies based on the same method to which we can compare our results, as the tests for effectiveness and methods are completely different.

But one can compare the experimental results with previous research in other ways. Table 8 below presents the matching percentage between actual and in-sample forecasts of state sequences for HMMs. A higher value represents a better matching case for the HMM. We can see that the USD/TWD HMM has the highest value, implying that it fits the real states best.

Table 8 Matching Percentage for HMMs

	OD-CAD/CNY	OD-CAD/HKD	OD-CAD/TWD	OD-USD/CNY	OD-USD/HKD	OD-USD/TWD
Matching Percentage	0.227	0.266	0.271	0.142	0.183	0.446

Engel (1994) uses the count of correct forecasts (out of sample) of the direction of change to compare the prediction results of the Random Walk, Forward Rate and Markov switching models with in sample. His comparison includes more than 10 currency exchange rates from 1973;3 to 1986:1, with out of sample forecasts generated for the period 1986:2 to 1991:1. In order to compare the results from the two papers, one can use the percentage of correctly forecasted directions or movements to evaluate his models. His results are shown in Table 9 below.

In Engel's research paper, the average percentage of correctly forecasted directions of change is 49.7%, while the minimum percentage is 15.7% and maximum percentage is 70.6%. This paper chooses six of Engel's samples, namely USFR, USIT, USUK, USWG, JASW and JAUK, to compare with the HMM prediction results. We choose the six samples because of their different capacities to predicate the correct direction in Engel's models. USWG and JAUK display a high level of prediction, while USIT and JASW displaying a low level of prediction USFR and USUK exhibit a medium level of prediction. We can see that even results for the same data set vary with the forecasting model. Therefore, when comparing Engel's results to those in Table 8, it is easy to find that our most accurate prediction (USD/TWD) reaches the average value of prediction of Engel's model, while the rest of the matching values of the HMMs are much lower than these of Engel's research.

Table 9 Count of Correct Forecast of Direction of Change[a]

Currency	M odel	Number of Forecast Periods		
		1	2	4
USFR	Random Walk	0.4	0.263158	0.352941
	Forwarded Rate	0.3	0.315789	0.352941
	Markov Model	0.65	0.526316	0.588235
USIT	Random Walk	0.35	0.263158	0.352941
	Forwarded Rate	0.35	0.263158	0.352941
	Markov Model	0.35	0.263158	0.352941
USJA	Random Walk	0.5	0.526316	0.588235
	Forwarded Rate	0.5	0.578947	0.588235
	Markov Model	0.45	0.473684	0.588235
USSW	Random Walk	0.6	0.684211	0.647059
	Forwarded Rate	0.5	0.578947	0.529412
	Markov Model	0.65	0.684211	0.647059
USUK	Random Walk	0.35	0.368421	0.352941
	Forwarded Rate	0.35	0.368421	0.352941
	Markov Model	0.55	0.368421	0.529412
USWG	Random Walk	0.55	0.684211	0.705882
	Forwarded Rate	0.6	0.631579	0.647059
	Markov Model	0.6	0.631579	0.529412
JACA	Random Walk	0.5	0.473684	0.411765
	Forwarded Rate	0.5	0.473684	0.411765
	Markov Model	0.55	0.473684	0.411765
JAFR	Random Walk	0.65	0.631579	0.529412
	Forwarded Rate	0.65	0.631579	0.470588
	Markov Model	0.65	0.578947	0.470588
JAIT	Random Walk	0.65	0.631579	0.470588
	Forwarded Rate	0.65	0.631579	0.470588
	Markov Model	0.65	0.631579	0.470588
JASW	Random Walk	0.4	0.473684	0.529412
	Forwarded Rate	0.25	0.157895	0.235294
	Markov Model	0.6	0.473684	0.470588
JAUK	Random Walk	0.55	0.578947	0.647059
	Forwarded Rate	0.55	0.578947	0.647059
	Markov Model	0.55	0.473684	0.647059
JAWG	Random Walk	0.65	0.631579	0.411765
	Forwarded Rate	0.45	0.368421	0.294118
	Markov Model	0.55	0.578947	0.352941
UKCA	Random Walk	0.45	0.421053	0.470588
	Forwarded Rate	0.4	0.421053	0.294118
	Markov Model	0.55	0.578947	0.647059

[a] The table is based on Table 6 of Engel (1994). The abbreviations in Table 9 stand for different countries: US (United States), FR (France), CA (Canada), IT (Italy), WG (German), UK (United Kingdom), and SW (Swiss).

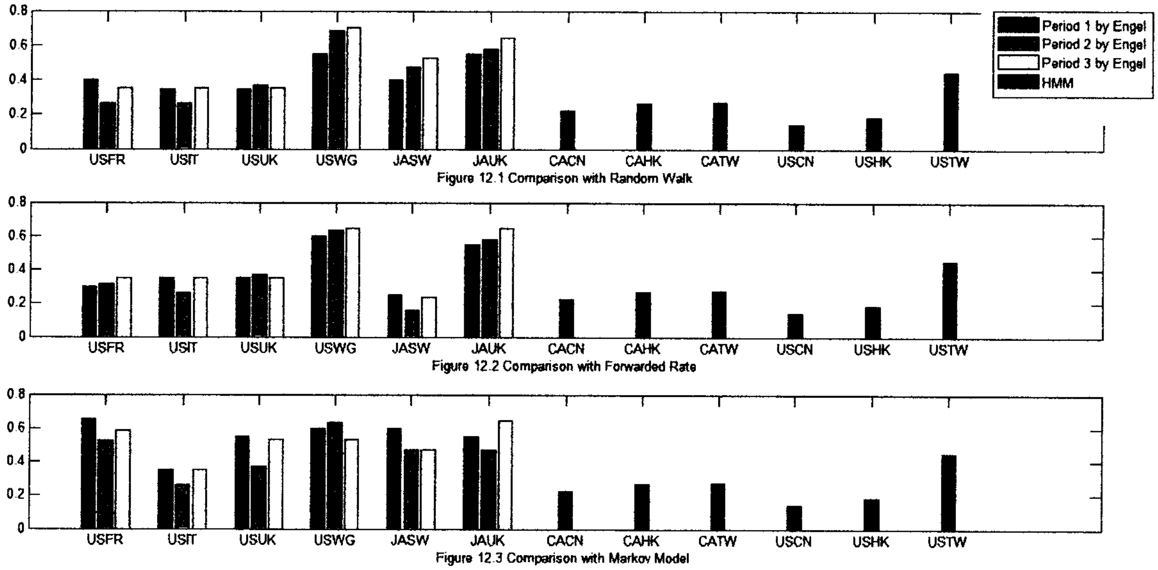

Figure 12 Comparison of HMM prediction results and Engel's results[9]

Other economists (Cheung and Erlandsson 2005, Lizardo and Mollick 2010) suggest using Root Mean Squared Error of the forecast as a measure of the model's accuracy. When consider the HMMs, to calculate the Root Mean Squared Error of a HMM, one must convert the discrete states into true oil prices. Further, the values should be normalized because the previous discrete states represent the ranges of real oil price values, so there will be error when converting states into real values.

[9] The first six columns represent prediction percentages from Engel (1994), where the red columns stand for correct-count percentages from first data set, the orange columns stand for correct-count percentages from second data set; the yellow columns stand for the correct-count percentages from the third data set. At the right hand side, the six purple columns stand for the matching percentages from HMMs.

Table 10 Root Mean Squared Errors of In-Sample Prediction results of HMMs

	OD-CAD/CNY	OD-CAD/HKD	OD-CAD/TWD	OD-USD/CNY	OD-USD/HKD	OD-USD/TWD
Matching Percentage	0.227	0.266	0.271	0.142	0.183	0.446
MSE[a]	0.453433	0.449426	0.888353	1.114092	1.06622	0.384564
RMSE[b]	0.673374	0.670392	0.942525	1.055506	1.032579	0.620132

[a] Mean Squared Error

[b] Root Mean Squared Error

Table 10 shows the RMSE and MSE values of the HMMs discussed in section 5.2. Both measures are computed with in-sample data. However, the HMMs' RMSE values are much larger than those of Lizardo and Mollick (2010), which are reproduced in Table 11. Lizardo and Mollick estimate models of exchange rates between the USD and other currencies. This comparison suggests that the HMM is not a better method for prediction, at least it is no better than the time series or regular Markov Switching model. However, since Lizardo and Mollick use data from Europe, while this paper uses data from east Asia, the unsatisfying prediction may be the result of different policies in the different areas.

Table 11 In-sample RMSE of Lizardo and Mollick's models

	Canada	Denmark	Euro	Japan	Mexico	Russia	Sweden	U.K.
Basic Model	0.048	0.046	0.067	0.102	0.097	0.024	0.102	0.057
Composite Model	0.042	0.043	0.063	0.093	0.078	0.023	0.089	0.053

In our experiments involving Oil Price Daily and the Greater China Region currency exchange rates, we find that the most unsatisfying result is the HMM for USD/CNY Daily. The percentage of matched states is as low as 14.2%, which can be observed directly in Table 8. In other words, the predicted states do not match the real states either in the trend or in their magnitudes. Even for the two periods of October 2008 and May 2009 in which most of the HMMs predict quite well, the HMM for the USD/CNY Daily still fails to provide a reasonable result. Therefore, due to the particular failure of USD/CNY Daily HMM, we will discuss determinants which could largely affect the U.S. and mainland China economies and the exchange rate.

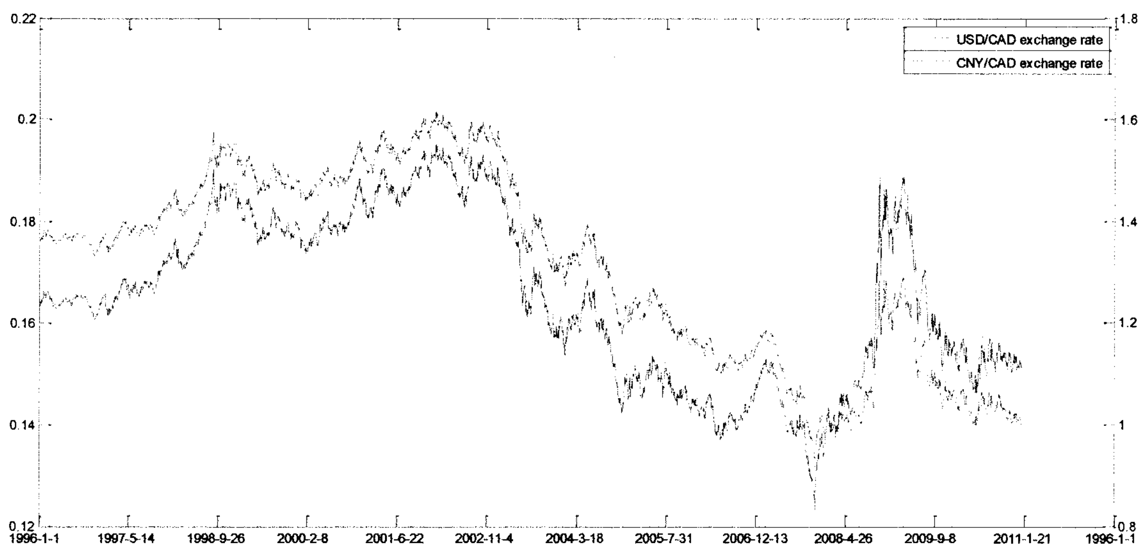

Figure 13 CNY/CAD and USD/CAD exchange rates[10]

Figure 13 portrays the CNY/CAD and USD/CAD exchange rates during the time period from 1996 to 2010. The figure shows that before 2008 their tendencies are similar to a great extent. However, after 2008, the CNY/CAD exchange rate displays much greater magnitude in its fluctuations. Meanwhile, we chose data for the period from 2008 to 2010 as our in-sample experimental data. This comparison suggests that the fluctuation

[10] The blue line describes the CNY/CAD's tendency and its relative scale in on the left side; while the green line describes the USD/CAD, of which the scale is on the right side.

magnitudes result from a change in the behaviour of the CNY stemming from the changes in CNY exchange rate policy announced by the People's Bank of China, and that this change may be responsible for the different prediction results for HMMs based on the CNY's exchange rates with the USD and CAD separately.

The two economies, the U.S. and mainland China, are tightly tied together since mainland China has loaned the U.S. a large amount and appears to be the largest foreign holder of U.S. Treasury debt. Meanwhile, U.S. consumer goods, including clothes and electronic goods, are produced in Chinese factories, which in turn has driven Chinese economic growth over the past few decades. It is easy to conclude that there is a close economic relationship between the two large countries.

When we consider the benchmark in our paper, the oil price, we find that both the U.S. and mainland China display a robust dependence on the supply of oil. Since 2009, the two major Chinese state oil companies, China National Petroleum Corporation (CNPC) and China Petrochemical Corporation (Sinopec) have been pursuing oil supply contracts from other foreign countries such as Canada, Russia, Iran, etc. Hayward (2009) predicts that by 2025 mainland China's annual oil imports will reach 53.6% of U.S. oil imports, with a constant annual increase in growth year-by year. So it is unlikely that the failure of in-sample prediction can be attributed to the choice of oil price as the commodity most closely linked to the exchange rate. However, two decades ago, the mainland China economy was closed off from the world economy, while Hong Kong and Taiwan and North America were participants in a fluctuating currency market. Not surprisingly, we do not expect to see a strong interaction between the CNY and world commodity prices during that period of time.

Excluding the benchmark choice, we will consider the possible problems of our exchange rate data sample. Unlike most exchange rates' policy, the mainland China government has maintained controls over its exchange rates for years. According to the latest announcement by The People's Bank of China (PBOC), the country's central bank, it is said that the mainland China government will continue the current monetary policies but with small detailed changes (Yang 2011). Thus, as long as the international portfolio is still under the control of the government, we cannot anticipate an unfixed exchange rate between the CNY and foreign currencies (Mckinoon 2000).

When the exchange rates are controlled by the government manually, it is possible that fluctuations in exchange rates do not reflect the potential impacts from other determinants. The training process of a HMM largely depends on the assumed relationship between the state sequence and the observation sequence. When an unanticipated factor restricts the freedom of exchange rates, that assumed relationship would be easily rejected since now the HMM contains three factor sequences; the state sequence, the observation sequence, and the unanticipated factor sequence. Actually, if the unanticipated factor appears to be a constant, a random distribution instead of a sequence, the HMM would not produce any reasonable prediction results, as the model structure is violated.

In 2008, the CNY was found to exhibit a general tendency to appreciate against the USD, the CAD and other major currencies. The move of the CNY was generated by an anti-inflation policy which was published by the People's Bank of China (PBOC), claiming that the government considered the accelerating inflation to be the greatest problem at that time. In July of 2008, the USD/CNY dropped to its lowest level (6.8456) since 2005.

The impact of the PBOC's fixed monetary policy on the exchange rates policy seems to be a possible explanation for the failure of the USD/CNY HMM.

Comparing the other, more effective HMMs, those for CAD/CNY, CAD/TWD, CAD/HKD and USD/TWD all have some particular time periods within which the HMM prediction results are reasonable and satisfying. The CAD/CNY and CAD/HKD HMMs suggest that the autumn of 2008 (October 2008 to January 2009) and May 2009 are time periods during which real state sequences can be effectively predicted. The CAD/TWD HMM's most effective time periods are the autumn of 2008 (October 2008 to January 2009) and March 2009. As for the USD/TWD, the effective time periods are a little different: the November 2008 and the spring of 2009 (February 2009 to May 2009).

If we look at the most effective time periods of exchange rates between the CAD and Greater China Region currencies, it is clear that the effective periods are almost the same for all the currencies. But we may need to take the relationship between mainland China and the other two regions into consideration. The economies of Hong Kong and Taiwan rely on their trading, financial, commercial and shipping business interactions with overseas countries and mainland China. Since the 1990s, the two smaller regions' economic interactions with mainland China have become stronger and more sophisticated, essentially through enhanced commercial association of private sector economies and a considerable housing sector. Consequently, when the currency exchange rate of mainland China fluctuates, a corresponding impact necessarily takes place in the currencies of the other two economies.

Considering the best HMM, which is USD/TWD, we find the matching percentage is as high as 44.6%. This relatively high percentage suggests that though we consider the interaction between mainland China and Taiwan to be robust, the two regions have greatly different economic impacts in the world economy.

Mainland China has held the world's largest stock of foreign exchange reserves for many years-$1.4 trillion in 2007-leading to the fact that mainland China and its related region Taiwan have emerged as a major power in the world economy (Dobson and Mason 2009). Since the 1990s, Taiwan has been reshaped into a significant financial and commercial service hub for mainland China, but before that the Taiwan economy was separated from that of mainland China for many decades, so the interactions between Taiwan and other foreign economies have a longer history compared to those with mainland China.

When considering the U.S.' relationship with Taiwan, one may find that Taiwan's political liberalization has obtained U.S. support till now. In addition to commodities, the U.S. is also a major supplier of arms to Taiwan, of which five programs are worth $6.4 billion in total (Kan and Morrison 2011). This robust relationship between Taiwan and the U.S. seems to suggest that a stronger international relationship, beyond mere economic interrelation, could improve the performance of an HMM.

6 Conclusion

This paper presents an original method, the HMM, to examine the correlations between commodity prices and exchange rates. We use the Oil Price and Gold Price as commodity prices, which are observed as observation sequences in the HMMs, and exchange rates between North America and the Greater China Region.

We use the matching percentage and the root mean squared error to compare the general model accuracy with that of previous research. From Figure 12, we can see that the most accurate prediction reaches the average value of other researchers' prediction results of the random walk and Markov Switching models. Since the performance of different currencies varies, it is natural to raise the conclusion that at least the HMM is not worse than the random walk and the Markov Switching models.

When we consider the root mean squared error to be the measure of performance, we find that the RMSE of the HMM is much larger than the RMSE of time series and the Markov Switching models. Consequently, we must conclude that the accuracy of the HMM ranks among other widely used models as follows: the time series model is the best, and then the Markov Switching or HMM or random walk. The latter three models do not have a certain ranking since their accuracies may vary when we choose different currencies. In this paper, we find that Europe currencies (Lizardo and Molllick, 2010) generate a better prediction result than that east Asia currencies. The weak in-sample efficiency indicates that the choices of geographic region plays a role in the models' estimates and predictions.

According to the tests above, the exchange rates between the CNY and North America currencies do not have a strong correlation with the Oil Price due to the low matching percentage and a corresponding high RMSE. Further, we find that the CAD/CNY model predicts better than the USD/CNY model. When investigating the economic characteristics during the in-sample period (2008 to 2009) of the model, we find that The

People's Bank of China published an announcement that the mainland China government would continue fixed monetary policies on China's currency. This control on exchange rates may be the reason for the poor predictive ability of exchange rates between the CNY and North America currencies, as the HMMs cannot produce any reasonable prediction results with controlled values. Consequently, since mainland China does affect Hong Kong, the prediction results for the Hong Kong and North America currencies are not very impressive either; however, the HMMs of the HKD are still better than those of the CNY. We attribute the better performance to Hong Kong's longer economic cooperation with North America.

The best HMM is generated from the Oil Price and the USD/TWD exchange rate. This high matching percentage suggests that the relationship between Oil Price and the USD/TWD exchange rate is quite robust. The U.S. has long been a political and commercial supporter of Taiwan, as the U.S. and Taiwan are enrolled in five programs worth billions of dollars. This robust relationship between Taiwan and the U.S. seems to suggest that a stronger international relationship, beyond mere economic interrelation, could improve the performance of an HMM.

In addition, the most effective forecast periods of HMMs for the same currency are almost the same. To be more specific, the most effective time periods of the CAD and all three Greater China Region currencies are autumn of 2008 and May 2009, while the effective time periods of the USD and Greater China Region currencies are November 2008 and the spring of 2009.

Overall, we find that HMM predictive ability relies on geographic regions, period of time, and most important one, the relationship between two regions. When there exists a closer relationship between two regions, the exchange rate between their currencies will produce a better HMM based on the Oil Price. Nevertheless, when we carry out HMMs based on the Gold Price of exchange rates with different frequencies, the estimating process cannot move on because of a lack of enough information, which is represented as zero transition probabilities at originating states. Therefore, we may conclude that the HMM requires data a high frequency. The state clustering of currencies and exchange rates may be another reason for the unsuccessful application of the HMM. But the clustering problem is hard to solve because there still does not exist a good way to choose the proper number of clustering.

7 References

Agriculture and Agri-Food Canada, "Agri-Food Comsumer Profile China", May 2011.

Blunsom, P., "Hidden Markov Models", 2004. http://digital.cs.usu.edu/~cyan/CS7960/hmm-tutorial.pdf

Chen, Y.C., K. Rogoff and B. Rossi, "Can Exchange Rates Forecast Commodity Prices?", *The Quarterly Journal of Economics*, 125(2010): 1145-1194.

Cheung, Y. W. and U. G. Erlandsson, "Exchange Rates and Markov Switching Dynamics", *Journal of Business & Economic Statistics*, 23(2005): 314-320.

Cheung, Y.W., M. D. Chinn and A. G. Pascual, "Empirical Exchange Rate Models of the Nineties: Are any fit to survive?", *Journal of International Money and Finance*, 24(2005): 1150-1175.

Defays, D., "An efficient Algorithm for a complete link method", The Computer Journal, 20(1977):364-366.

Ding, C. and X.He, "K-means Clustering via Principal Component Analysis", *Proceedings of The Twenty-First International Conference on Machine Learning*, 2004.

Dobson, W. and P. R. Masson, "Will The Renminbi Become A World Currency?", *China Economic Review*, 20(2009): 124-135.

Dong, W. and D. Nam, "Exchange Rates and Individual Good's Price Misalignment: Some Preliminary Evidence of Long-Horizon Predictability", Discussion Paper, Bank of Canada, (8)2011.

Eddy, S.R., "Profile Hidden Markov Models", *Bioinformatics Review*, 14(1998): 755-763.

Engel, C. and J. D. Hmilton, "Long Swings in the Dollar: Are They in the Data and Do Markets Know It?", *American Economic Review*, 80(1990): 689-713

Engel, C., "Can the Markov Switching Model Forecast Exchange Rates?", *Journal of International Economics*, 36(1994): 151-165.

Ferraro, D., K. Rogoff and B. Rossi, "Can Oil Prices Forecast Exchange Rates?", Working Papers, 11-34, 2011.

Hayward, D.L.O., "China's Oil Supply Dependence", *Journal of Energy Security*, 2009. http://www.ensec.org/index.php?option=com_content&view=article&id=197:chinas-oil-supply-dependence&catid=96:content&Itemid=345

Hopper, G.P., "What Determines the Exchange Rate: Economic Factors or Market Sentiment?", *Business Review*, September(1997): 17-29.

Jones, R.H., "Bayesian Information Criterion for Longitudinal and Clustered Data", *Statistics in Medicine*, 25(2011):3050-3056.

Kan, S.A. and W. M. Morrison, "U.S.-Taiwan Relationship: Overview of Policy Issues", Congressional Research Service, August 2011.

Lizardo, R.A. and A. V. Mollick, "Oil Price Fluctuations and U.S. Dollar Exchange Rates", *Energy Economics*, 32(2010): 399-408

Lum, T. and D. K. Nanto, "China's Trade with the United States and the World", CRS Report for Congress, Order Code RL31403.

Mark, N.C., "Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability", *American Economic Review*, 85(1995): 201-18.

Mckinnon, R., "China's Financial Policies Upon Accession to the WTO", *Overseas Young Chinese Forum*, Vol 2, 2000.
http://www.oycf.org/Perspectives2/7_083100/china_rm.htm

Meese, R.A. and K. Rogoff, "Empirical Exchange Rate Models of The Seventies", *Journal of International Economics*, 14(1983): 3-24.

Netzer, O., J. Lattin and V. S. Srinivasan, "A Hidden Markov Model of Customer Relationship Dynamics", Stanford GSB Research Paper No.1904, June 2007.

Padmanabhan, P., "The Role Of China And The United States In The Global Economy Of The 21st Century", *International Business & Economics Research Journal*, 7(2008): 33-38

Rabiner, L. R., "A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition", *Proceedings of the IEEE*, 2(1989): 257-286.

Sibson, R., "SLINK: An Optimally Efficient Algorithm For The Single-Link Cluster Method", *The Computer Journal*, 16(1973):30-34.

Song, B., M. A. Marchant, M. R. Reed and S. Xu, "Competitive Analysis and Market Power of China's Soybean Import Market", *International Food and Agribusiness Management Review*, 12(2009): 21-42.

Statistics Canada, "Canadian Economic Observer", Catalogue no. 11-010-XIB, June 2004.

Taylor, M. P., "The Economics of Exchange Rates", *Journal of Economic Literature*, XXXIII(1995): 13-47.

Wang, H.H and B. Ke, "Efficiency Tests of Agricultural Commodity Futures Markets in China", *Australian Journal of Agricultural and Resource Economics*, 49(2005).

Wu, Y., A. Ganapathiraju and J. Picone, "Report for Baum-Welch Re-estimation of Hidden Markov Model", *Institute for Signaland Information Processing*, 1999.

Yang, F., "China to Continue Prudent Monetary Policy Inflation Control", *People's Daily on Line*, November 17, 2011.