# A TEST OF PIAGET'S THEORY OF NUMBER EXTENDED TO DECIMAL NUMERALS

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# CURRICULUM STUDIORUM

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# INTRODUCTION

Jean Piaget, the Swiss child psychologist, has spent over fifty years formulating a theory of cognitive development, and demonstrating how this theory functions in different cognitive areas. His research into the child's concept of number is extensive. He has, however, limited this study to the investigation of the child's concept of smaller numbers, usually those less than ten. These are the number concepts that a child develops from his experiences with small sets of objects such as coins or counters.

Many researchers have used Piaget's work with number as a basis for further investigations of the child's smaller number concepts. However, little has been done to extend Piaget's theory to investigate other number concepts. Specifically, there are the concepts of larger numbers. These number concepts are developed differently than the concepts of smaller numbers. The child develops the concept of large numbers mainly through his experiences with numbers represented by decimal numeration.

A study of the child's concept of number represented by decimal numeration is important, as numerous researchers have noted the relationship between the child's understanding of decimal numeration and his ability to succeed at basic arithmetical skills. As well, they have discovered that children in the junior grades do not understand decimal numeration, and furthermore, their teachers do not know that this weakness exists.

Thus, there is a need for a review of the factors involved in the understanding of smaller numbers and their representation. These factors must then be re-examined as they relate to the acquisition of the concept of larger numbers represented by decimal numeration. Also required, is the formation of an evaluation instrument to assess what the child knows about these larger numbers. This paper is directed to these ends.

This report is divided into five chapters: Piaget's Theory of Cognitive Development, Related Research, the Experimental Design, the Presentation and Discussion of the Results, and the Summary and Conclusions.

In the first chapter, Piaget's main theory of cognitive development is outlined. In the second chapter, a discussion of the studies relevant to the formation of number concepts has been undertaken. A specific problem is identified in this chapter.

In the third chapter, the experimental design is outlined. The results of the experiment are presented and dicussed in Chapter Four.

The summary and conclusions to this study are presented in Chapter Five.

# CHAPTER I

## PIAGET'S THEORY OF COGNITIVE DEVELOPMENT

The outline of the theory in this chapter is more detailed than is usually the case in a paper of this type. There are three reasons for this.

First, previous Piagetian researchers have examined only particular aspects of Piaget's theory. Failing to see the relationships between the parts and the whole theory, they have, therefore, been limited in what they could contribute to enriching the theory. More important, some research based on aspects of the theory taken out of context had led to misinterpretations of the theory. Thus, this chapter provides a framework for relating previous research findings concerning number, to the main theory of cognitive development.

Secondly, the majority of the previous studies into the child's concept of number has been based on one book written by Piaget, and published in 1941. Piagetian theory has been refined in the years following this publication. The theory, as it stands today, is outlined in this chapter.

Finally, if any reasonable extension is to be made to the theory, the overall theory of cognitive development must be understood.

According to Piaget's theory of cognitive development, there are two different aspects of cognition developing simultaneously from birth onward.[1]

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1 Jean Piaget, "Development and Learning", Journal of Research in Science Teaching, Vol. 2, 1964, p. 186.

Each of these aspects, the figurative and the operative, are discussed separately in the first section of this paper.

However, in reality, these two aspects cannot be dissociated. As Sinclair states, these aspects, at all stages of development, are no more than "two sides of the same coin".[2] The simultaneous development of both aspects enables the child to advance to more complex stages of cognition. In the second section, the figurative and operative cognition characteristics, and the concepts of the child at different stages of development, are examined.

As the child develops, he makes transitions between the different stages. The processes involved in transitions between the stages are discussed in the third section of this chapter.

Much of Piaget's theory concerns aspects of the development of cognition which are common to all children. However, when considering any group of children of the same age, individual differences among children are evident. The issue of individual differences is discussed in the fourth section, "Décalages".

## 1. Figurative and Operative Aspects of Cognition

The figurative and operative aspects of Piaget's theory of cognition refer to two very different aspects of cognition. The distinction between figurative and operative aspects is difficult to

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2 Hermine Sinclair, "Piaget's Theory of Development: The Main Stages", in Myron F. Rosskopf, Leslie P. Steffe, Stanley Taback, (eds.), Piagetian Cognitive Development Research and Mathematical Education, Reston, Virginia, National Council of Teachers of Mathematics, 1971, p. 3.

explain. Explanation is further hampered by the fact that many terms used in reference to the two aspects have names similar to names used to identify terms in North American cognitive psychology. Although the names are similar, the terms that they represent are sometimes extremely different.

The purpose of this section is to clarify the distinction between the figurative and operative aspects of cognition. In addition, the terms relevant to each aspect are explained.

### i. The Figurative Aspect of Cognition

The figurative aspect of cognition refers to the knowledge derived from objects in their static states.[3] These static states may best be compared with the viewing of the successive individual frames of a moving picture film. The action component is lost as each frame presents only a different static configuration of objects.

The term "objects" refers to more things than the concrete materials Piaget uses in his different tasks. In its broader context, objects refers to such things as drawing, letters, numbers, actions and gestures. With reference to Mathematics, Piaget states:

> Just as arithmetical algebraic, or geometrical reasoning consists in combining objects (numbers, signs or figures) by means of the operations [...].[4]

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3 Jean Piaget, Genetic Epistemology, New York, Columbia University Press, 1970, p. 14.

4 --------, The Child's Conception of Number, London, Routledge and Kegan Paul Ltd., 1952, p. 180.

Henceforth, the term "object" is used in this broader context.

Objects in their static states are known through two functions, one called the perceptual function, the other the semiotic function. The perceptual function, or sensory perception, refers to the knowledge gained by using the basic senses of hearing, seeing, smelling, tasting and touching. Through the senses a child perceives that ice is cold, sugar is sweet, and the sunlight is bright.

Two classes of objects can be perceived. In one class, the objects represent only themselves. In the second class, the objects represent or stand for something else. For example, the sign "=" is perceptually only two small parallel lines. However, these same two little lines are also a signal indicating to the child that the objects on either side of the sign are equal. The second class of objects is the type used and understood through the semiotic function. This semiotic function is a process whereby the child uses objects to represent his thoughts to himself and other people.[5]

The semiotic function includes many processes of representation. Deferred imitation, symbolic play, drawing and internal representation are processes which a child may use to represent his thoughts.[6] With these processes, the objects used by the child always bear some resemblance to the things they are representing. These objects of representation are called symbols by Piaget.[7]

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5 J. Piaget, Genetic Epistemology, p. 45.

6 Ibid., p. 45.

7 J. Piaget and Barbel Inhelder, The Psychology of the Child, New York, Basic Books, 1969, p. 57.

Other semiotic processes involve the use of social languages such as English, Algebraic notation and the sign language of the deaf. The objects of a language (words, numerals, hand signs) bear no resemblance to the things they represent. These are arbitrary objects of representation imposed on the child by society. The term "sign" is used by Piaget for these arbitrary objects of representation.[8]

The semiotic processes: deferred imitation, symbolic play, drawing, internal representation and language, are explained briefly to illustrate how they are used to represent thoughts.

Deferred imitation refers to the child's ability to use his physical actions as a symbol to represent some specific incident he has seen or performed previously. Piaget often recounts the time his young daughter witnessed a temper tantrum of a friend, then several hours later, she imitated his behavior and then laughed.[9] This is deferred imitation, with her imitated tantrum being the physical action symbol, referring to the incident of her friend's tantrum.

In symbolic play a child may pretend to cry, or pretend to sleep using a book for a pillow. He is representing no particular incident. Instead, he is creating new situations by imitating previous actions. The symbols used are physical action imitations and articles, such as the book which serves as a pillow.

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8 Ibid., p. 57.

9 Ibid., p. 53.

Drawing differs from these two previous processes in the type of imitation used as a symbol. The drawing a child makes to represent his thoughts is a type of perceptual imitation.[10] As in deferred imitation, the child may be representing a specific incident. However, as in symbolic play, the child also draws new scenes for his own personal satisfaction.

Internal imitation to Piaget is also considered a kind of perceptual imitation which has mental images as symbols.[11] Unlike other semiotic processes, internal imitation is a personal type of representation. However, through other semiotic processes such as drawings, gestures and language, the child can reveal the nature of his mental images to some extent.

In the use of language, the child represents his thoughts using signs. As with the other four semiotic processes discussed, imitation is involved in language acquisition.[12] As stated previously, symbols have an imitative resemblance to the things that they represent, either obvious as in deferred imitation, or obscure as in drawings and internal imitation. Signs themselves bear no imitative resemblance to the things they signify. Therefore, the imitation with respect to language occurs when the child imitates the various signs of the particular language. For example, a young child can imitate people counting a set of pennies.

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10 Ibid., p. 63.

11 Ibid., p. 79.

12 Ibid., p. 55.

This is only an auditory imitation if the child is unaware that the counting has a meaning and a purpose. This imitation is to Piaget a prefiguration[13] of language. When the child wants to know how many pennies he has, and assigns a different numeral to each successive coin he sees, he is using counting in the true sense of language. Thus, for words to be true verbal language signs, the child must note what is also common in the various settings when he hears the words used.

The prefiguration discussed with respect to language is an occurrence which is also evident in the formation of other semiotic processes. For example, if Piaget's daughter had witnessed the tantrum and had represented the actions then, it would have been just a simple case of immediate imitation. It would have been a prefiguration of representation. Likewise, if she had feigned the tantrum only in the other child's presence, this still would be considered a prefiguration of deferred imitation. With true representation, the child can recall and represent the incident at his own free will, without promptings of any type.

The scribbles of a young child are prefigurations to representation through drawing. Once the child makes an effort to reproduce something, prefiguration ends and representation commences.

"Knowing" with reference to the figurative aspect of cognition is a superficial type of knowing. Through the perceptual function, or the semiotic function, a child collects, stores, and retrieves information.

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13 Ibid., p. 53.

The child does not have any logical understanding of information processed through the figurative processes of cognition alone. A child may be able to learn grammatical rules and express them if asked. The fact that he does not use the rules in his written work, indicates that he has only a figurative level knowledge of the rule. In effect, figurative knowing is a prefiguration to true knowledge. Piaget himself acknowledges being misled in his earlier works by this phenomenon.[14]

The figurative aspect of cognition, therefore, refers to the knowledge derived from objects in their static states. Objects, in the broad sense of the term, are known through perception and the various processes of the semiotic function. Imitation plays a major role in the semiotic function. Symbols bear an imitative resemblance to the things they signify, and the signs of language are learned in a context of imitation. There is, however, with the signs of language, an arbitrary meaning which must also be acquired. With the semiotic processes, there is prefiguration of representation which must be distinguished from true representation. Thus, through perception, the child knows the world directly. Through the symbols of deferred imitation, symbolic play, drawing, internal imitation, and the signs of social language, the child exhibits his present knowledge of things, and in this representation, he can learn more about those things. The knowledge derived, however, is superficial. It involves the collection, storage, and retrieval, but not the logical understanding, of information.

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14 Jean Piaget in Richard I. Evans, Jean Piaget, The Man and His Ideas, A Dialogue With Piaget, New York, E. P. Dutton and Co., Inc., 1973, p. 70.

### ii. The Operative Aspect of Cognition

According to Piaget, in physical reality there are states, and there are transformations from one state to another state.[15] It was noted previously that the figurative aspect of knowledge is concerned with the knowledge derived from states. The operative aspect of knowledge is concerned with the knowledge abstracted from the transformations between successive states. Before explaining the terminology relevant to the operative aspect more fully, it is important to clarify the distinction between the figurative and operative aspects of cognition.

In Piaget's famous task concerning the conservation of continuous quantities,[16] there were two tall, narrow cylinders, and a short, wide beaker. First, a child had to help equalize the amounts of water in the two cylinders. Then, as the child watched, the water from one cylinder was poured into the beaker. The child was questioned concerning the equality of the quantities of water in the cylinder and the beaker.

The children who performed this task observed two static states. In one, there were two cylinders with equal amounts of water, and an empty beaker. In the second, there was still one cylinder containing water, a beaker with water, and another cylinder which was empty. From the figurative aspect of knowing, a child could state some pertinent information with respect to the task. For instance, the columns of water in the cylinders were the same height. As well, the water in the

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15 J. Piaget, "Development and Learning", p. 186.

16 --------, The Child's Conception of Number, p. 4-24.

cylinder was taller and narrower than the water in the beaker. This is basically all that can be known by observing these two static states.

The children also saw the transformation. Using language or gesturing processes of the semiotic function, most children examined expressed quite clearly that they had witnessed the change. They could tell all about it, and used imitating gestures to indicate the pouring.

The unusual result was that not all children realized the equality of the quantities of water in the cylinder and in the beaker. Those who did not understand the implications of the transformation thought that there was more water in the cylinder if they looked at the heights of the water levels. When they looked at the widths of the containers, many then thought that there was more water in the beaker. They were totally unconcerned that they were giving inconsistent responses. As well, they thought the water level would be lower than before if the beaker were emptied back into the cylinder.

On the other hand, the children who had abstracted knowledge from the pouring transformation itself, realized there were equal quantities in the cylinder and the beaker. Also, they were aware that the difference in height of the columns of water in the cylinder and the beaker was compensated by the difference in width. Furthermore, they had no doubts that if the water in the beaker was returned to the cylinder, the water level would equal that in the other cylinder.

It is important to recall at this point that the only way for a child to represent what he has observed or what he thinks and understands, is to make use of the various processes of the semiotic function. He also represents his own thoughts to himself using the semiotic function. According to Piaget:

> Actions can be represented in a number of different ways, of which language is only one. Language is certainly not the exclusive means of representation. [...] In addition to language the semiotic function includes gestures, either idiosyncratic or, as in the case of the deaf and dumb language, systematized. It includes deferred imitation [...] drawing, painting, modelling. It includes mental imagery, which I have characterized elsewhere as internalized imitation. In all these cases there is a signifier which represents that which is signified, and all these ways are used by individual children in their passage from intelligence that is acted out to intelligence that is thought.[17]

However, a child is limited in what he can know through the perception of a stage. He is also limited in what he can know just by observing an action and representing it using the various processes of the semiotic function. Perception and the semiotic function are the main functions of the figurative aspect of knowing. Thus, there is a limit to what can be known through the figurative aspect of cognition alone.

Piaget states:

> Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy or image of it. To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed.[18]

Therefore, before the child can understand the equality of the quantities of water in the above problem, he must act. He must pour water back and fourth from one container to another. With figurative

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17 Jean Piaget, Genetic Epistemology, p. 45.

18 --------, "Development and Learning", p. 176.

knowing, the child can observe and report the events which he performs. Through processes of the operative aspect of cognition, the child acts on objects and abstracts knowledge from these actions. Through operative knowing, a child is aware of the type of transformations which he may perform in seeking solutions to questions. Once a solution is reached, the child uses the figurative aspect of cognition to report and store the knowledge abstracted from the actions.

Thus, knowledge involves both a figurative and an operative aspect of cognition. Without the operative aspect of cognition, the child knows without a true understanding. Without the figurative aspect of cognition, a child lacks information to transform for the abstraction of knowledge. Hence, both figurative and operative knowing are required in knowledge formation.

In the process of extending his knowledge, the child through the operative aspect of cognition, transforms existing information and abstracts new knowledge. There are several types of tranformations or actions from which knowledge may be abstracted. Piaget has given terms for each of the different actions. There are physical actions, generalized actions, internalized actions, and interiorized actions better known as operations. An explanation of each of these actions follows.

Physical actions refer to a child's actions on physical objects. For example, a child can have a set of coins stretched out in a line. One possible action he might perform is to push the coins into a cluster. This is a physical action on physical objects.

Physical actions are of two main types. Piaget calls them individual and co-ordinated actions.[19] Individual actions are single actions such as touching, pushing or grasping. Co-ordinated actions are actions which are composed of several other actions. For instance, to enjoy a mobile is a co-ordinated action. It is co-ordinated because the child must carry out several actions to enjoy the mobile. He must be able to see it, reach up to it, swing his arm, strike it, and laugh. These are a few of the actions that can be performed in the co-ordinated action of enjoying a mobile. In a sense, all actions are co-ordinated to some extent.

Another type of action is a generalized action. As stated above, different physical actions may be joined to form co-ordinated actions. Generalized actions are a particular type of co-ordinated action.[20] The co-ordination in this case is accomplished through noting the similarity in different actions. For example, a child may note that a toy just beyond reach can be pulled closer by means of a string which is attached to the toy, and is within reach. Also, a toy sitting on a blanket may be pulled into reach by pulling the blanket on which it is sitting. The generalized action from these two different actions is "pulling closer by means of an intermediary object".[21] This generalization

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19 Jean Piaget, Genetic Epistemology, p. 18.

20 Ibid., p. 18.

21 J. Piaget in Richard I. Evans, Jean Piaget, The Man and His Ideas, A Dialogue with Piaget, p. 18.

may then be applied to different situations where he wants other things beyond his reach. Thus, if he wants his bottle sitting on the table, on a tablecloth, he needs only to pull the cloth to make the bottle come within reach. Piaget also refers to such a generalized action as a scheme.[22]

Internalized actions are another type of action. In this case, the action is abstracted from the physical objects associated with it, and is taken within the child. The child represents some of his internalized actions by using the semiotic function. For instance, Piaget's daughter saw a cat meowing as it walked along the top of a fence. Later, the child was observed pushing a shell across the top of a box, and meowing.[23] This imitation indicated that she had separated the walking and the meowing from the cat, and the fence. She internalized this action, then later represented it to herself. Thus, internalized actions are derived from physical actions on physical objects, but retain some contextual similarities with the real event.[24]

Interiorized actions, or operations, are different from internalized actions. Interiorized[25] actions are abstracted from internalized actions and the contextual similarities associated with them. Piaget often recounts the actions of a mathematician friend.[26] As a child, his

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22 J. Piaget, Genetic Epistemology, p. 42.

23 J. Piaget and B. Inhelder, The Psychology of the Child, p. 54.

24 Hans Furth, Piaget and Knowledge, p. 263.

25 Ibid., p. 263.

26 J. Piaget, "Development and Learning", p. 179-180.

friend had some pebbles. He arranged them in a circle and counted them. He changed the circle to a pile, a line and so forth, and counted them after each change. An internalized action at this point might have been--changing the arrangement of the pebbles does not change the amount. This action is still closely linked with the actual events. By further abstraction, his friend concluded that summing is independent of the order of the addends. This is an interiorized action or operation. It is the bare action abstracted from the pebbles and the particular event.[27]

Internalized actions and operations both are actions which are within the child. The child reports these actions to others by using processes of the semiotic function. It is important to be able to distinguish between the two actions. What a child knows about an event helps to make this distinction. In the task concerning the pouring of water from the cylinder to the beaker, all children expressed clearly the change that they had observed. Many also used gestures to describe the pouring action. From this behavior, it was evident that the children had formed internalized actions from observing the event.

However, not all had formed operations. A child who has formed operations is aware of the action, and is also aware of the implications of the action. Therefore, only those children who realized that the pouring did not affect the quantities of water involved had formed operations. Those who did not understand the equality had only formed internalized actions. Thus, internalized actions can be distinguished

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27 J. Piaget, Ibid., p. 180.

from interiorized actions or operations by observing whether or not the child is aware of the implications of such actions.

Piaget is particularly interested in the role of logic with respect to the operative aspect of cognition. The co-ordinated physical actions of a child demonstrate a type of logical behavior which parallels true logical operations.[28] There are four basic ways a child can co-ordinate his physical actions. Actions can be co-ordinated through addition, sequencing, correspondence, and generalization. These various co-ordinations parallel logical addition, logical sequencing, logical correspondence and logical generalization respectively. Explanations of the type of co-ordinated actions and their logical parallels are required to clarify Piaget's view.

By an additive co-ordination of actions, several actions are performed at the same moment. For example, the child can be grasping his rattle, looking at it, and scratching it with the fingers of his second hand, all at one time. Thus, he has additively co-ordinated his actions to investigate his rattle. In this additive co-ordination of actions, the child exhibits aspects of a logic of class inclusion. That is, the grasping, looking and scratching schemes are included in the investigation scheme. Piaget studied logical class inclusion in his research on the child's concept of number. He had a set of wooden beads. More than half these beads were brown, and the rest were white.

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28 J. Piaget, Genetic Epistemology, p. 42, 43.

He asked the children which made the longer necklace-- the brown beads, or the wooden beads.[29] If a child had formed his operations, he knew that the class of brown beads and the class of white beads made up the class of wooden beads. As well, he realized the whole set was larger than either of the parts separately. Thus, he could answer correctly. Children without operations noted only that there were more brown beads than white beads, and never really understood that both subsets were wooden beads. They could not answer correctly.

The child may also co-ordinate actions by sequencing so that several actions follow each other to achieve some end. For instance, the child may see his mobile above him. He raises his hand, strikes at the mobile and laughs as it bobs around. Therefore, to enjoy his mobile he must carry out several actions in a particular sequence. To strike, and then reach is neither fun nor logical. This sequencing of actions parallels the logical sequencing involved in ordering a set of numerals from largest to smallest. Piaget studied the child's ability to order dolls and sticks from largest to smallest.[30] As with logical class inclusion, a child can only understand the ordering process after he has formed his operations.

Actions may also be co-ordinated through correspondences. A mother may clap her hands, and the child performs the corresponding action of clapping his own hands. Actions co-ordinated by correspondence have a logical parallel called logical correspondence. According to

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29 J. Piaget, The Child's Conception of Number, p. 168.

30 Ibid., p. 97.

Piaget, a child using logical correspondence can observe a line of counters, and can set out a second line of counters in a linear one-one correspondence with a first set. Without logical correspondence, the child is capable of making only a global correspondence.[31] In this instance, the distribution of the counters the child sets out is perceptually similar to the first set. However, the numerical quantities in each line may not correspond. See figure 1a and 1b to clarify the distinction between a global (perceptual) correspondence, and a linear one-one correspondence.

Finally, actions may be co-ordinated through generalization, that is, through noting the similarity in different actions. This was discussed earlier with respect to generalized actions or schemes. One such generalized action or scheme is the pulling closer of an object by means of an intermediary object. This scheme may be applied by the child to pull a toy closer by means of its pull string, or to pull a tablecloth to reach a spoon sitting on it. This co-ordination of actions parallels the logic involved in determining the elements in the intersection of sets, or subsets of a set in set theory mathematics.

Although Piaget has noted that the sensorimotor actions demonstrate a kind of logic, it is not until the child abstracts his operations that any true logic is evident. Thus, if a child can clap his hands in physical correspondence with his mother's clapping, it does not mean he

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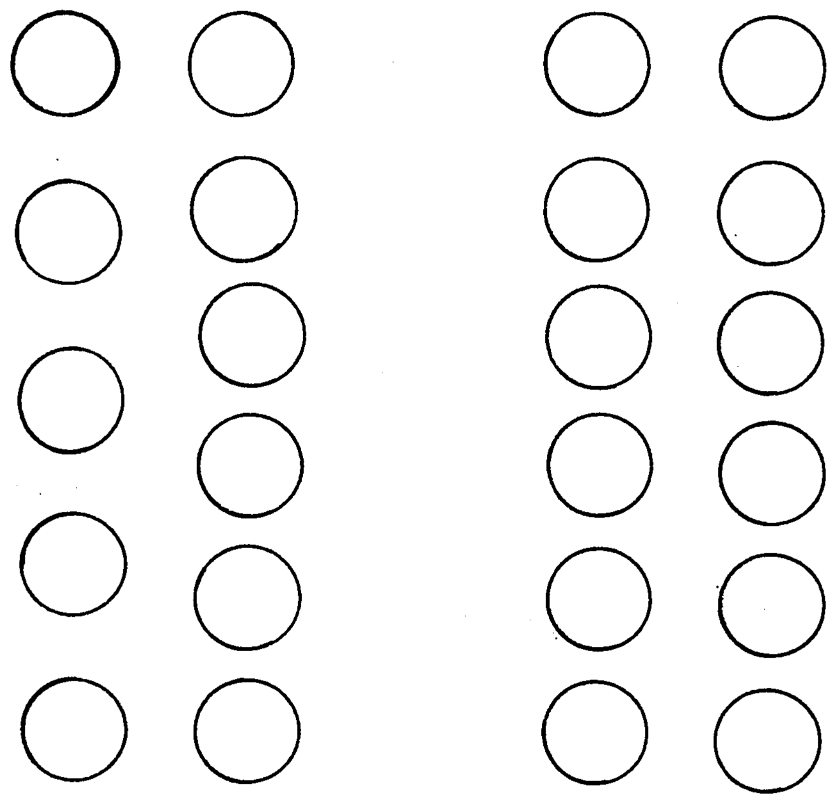
31 J. Piaget, The Child's Conception of Number, p. 41-94.



1a. A global (or perceptual) correspondence. The lines of counters look equally long, but the number in each differ.

1b. A linear 1-1 correspondence

Figure 1. Two Types of Numerical Correspondence

can make a numerical one-one correspondence. According to Piaget, this physical correspondence is just a beginning, or a basis on which higher forms of correspondence can develop.[32]

Piaget often describes the operations as logical operations. These logical operations are of two types: concrete operations and formal operations.

The concrete operations involve the basic logical operations of addition, sequencing, correspondence and generalization. They are labelled concrete operations because the child uses them to reason with respect to present events and objects, or with respect to events and objects which he has actually encountered. The child is limited in his ability to consider future events, or events not previously encountered. Furthermore, these concrete operations are used with respect to physical objects or to symbols representing physical objects.[33]

Piaget has used many physical objects such as counters and flowers in his study of the concrete operations. Thus, many researchers such as Beilin,[34] and Ginsberg and Opper[35] have misinterpreted the term "concrete operations". They believe that the concrete operations are

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32 J. Piaget in reply to H. Beilin re the "Developmental Stages and Developmental Processes", in Donald Ross Green, M. P. Ford, G. B. Flamer (eds.), Measurement and Piaget, New York, McGraw-Hill, 1971, p. 194.

33 J. Piaget, Genetic Epistemology, p. 14.

34 Harry Beilin, "Stimulus and Cognitive Transformation in Conservation", in David Elkind and John H. Flavell, (eds.), Studies in Cognitive Development: Essays in Honor of Jean Piaget, New York, Oxford University Press, 1969.

35 Herbert Ginsberg and Sylvia Opper, Piaget's Theory of Intellectual Development: An Introduction, New Jersey, Prentice-Hall, Inc., 1969, p. 151.

only used with physical objects. However, at one point, Piaget was discussing the addition of two numerals, e.g. 2+3=5. He acknowledged questions might be raised because they were numerical signs and not physical objects. Nevertheless, he clarified the point that the concrete operation of addition was as genuine in this situation as it was when two physical objects were added to other objects.[36]

Hermine Sinclair, one of Piaget's co-workers at the University of Geneva, has also tried to clarify this misunderstanding about the term "concrete operations". She says:

> However, this term does not mean that the child can think logically only if he can at the same time manipulate objects. Even less does it coincide with the (rather difficult to define) distinction between abstract and concrete. Concrete, in the Piagetian sense, means that the child can think in a logically coherent manner about objects that do exist and have real properties, and about actions that are possible; he can perform the mental operations involved both when asked purely verbal questions and when manipulating objects. The latter situation is far preferable to the former, mainly for reasons of clarity, but the actual presence of objects is no intrinsic condition.[37]

Thus, the concrete operations can be performed whether or not physical objects are present. However, as Sinclair states, using objects helps to clarify the problem involved.

Formal operations differ from concrete operations. They differ both in the types of logic and in the type of events which are involved

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36 Jean Piaget, "La langue et les opérations intellectuelles", in Problèmes de Psycholinguistiques, Neuchâtel Symposium de l'association de Psychologie scientifique de langue française, 1962, Paris, Presses Universitaires de France, 1963, Reading 5, p. 56.

37 Ibid., p. 5.

in the reasoning process. Whereas concrete operations exhibit only the simple logical operations such as logical addition, formal operations exhibit more complex logical operations. Combinatorial analysis[38] is one of the complex formal operations. This particular analysis is the type used to determine the number of ways that one can select two people from a group of four. If a child can perform simple logical operations, but not more complex logical operations, he is considered as having formed concrete operations only. If he can perform more complex logical operations, Piaget would consider him as having formed his formal operations. Thus, concrete and formal operations differ in the complexity of the logical operations involved in a reasoning process.

The types of events and objects involved in the reasoning of a child with concrete operations differ from those involved in the reasoning of a child with formal operations. In concrete operations, the child is restricted primarily to reasoning with respect to real events and objects which he has encountered. In formal operations, the child reasons with respect to all events or objects, either real or hypothetical. As well, the reasoning may also involve future events which have not been experienced. Hence, the objects and events involved in formal operations comprise a wider class than those in concrete operations.

Thus, the distinction between concrete and formal operations is easily made. One merely considers the complexity of the logic and the objects involved in the reasoning process.

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38 Jean Piaget, in Richard I. Evans, Jean Piaget, The Man and His Ideas, A Dialogue with Piaget, p. 27.

In this section, it was noted that with figurative cognition a child collects, stores and retrieves information. In contrast, with operative cognition the child acts on the information, transforms it, and abstracts new knowledge from the transformations. In the formation of new knowledge, the child requires both aspects. Through the figurative aspects the child collects information which must be transformed by the operative aspect. Only with both aspects combined can the child know with understanding.

There are different types of actions a child can perform to help in his construction of knowledge. First, there are physical actions, either individual or co-ordinated. With these, the child can construct generalized actions also known as schemes. Internalized actions and interiorized actions, or operations, are actions a child has taken within himself. The child uses various processes of the semiotic function to represent these actions.

Internalized actions are abstracted from physical actions on physical objects. Operations are, in turn, abstracted from co-ordinated internalized actions. When internalized actions are formed, the child expresses his awareness of the action. When operations are formed, the child expresses his awareness of the action and its implications.

Operative knowing exhibits a logical structure. Although co-ordinated physical actions demonstrate a type of logical behavior, it is not until operations are formed that true logical behavior appears. Operations are further classified by the type of logic they exhibit. Concrete operations are characterized by simple logical processes.

Furthermore, the term concrete operations refers to the child's ability to reason with respect to physical objects, or with respect to signs or symbols representing physical objects. In the latter case, the physical objects represented need not be present for the child to perform his concrete operations.

Formal operations are characterized by more complex logical processes. In addition, formal operations refer to the child's ability to reason with respect to a wide class of objects and events. Even hypothetical events may be involved in the reasoning characteristic of formal operations.

In conclusion, the distinction was made between figurative and operative aspects of cognition. Figurative aspects of cognition refer to the knowing involved in the observation, storage, and retrieval of information. Operative aspects refer to transformations of this information and abstractions of knowledge from these transformations. To extend his knowledge, the child uses both aspects of cognition.

Terms relevant to each aspect of cognition were explained as they are used in Piagetian theory. It was shown that some terms differ considerably from terms in North American psychology, even though the names for the terms were similar. The clarification of these terms is important for the understanding of further aspects of Piaget's theory of the cognitive development of the child.

## 2. Cognitive Development of the Child

Piaget postulates that cognitive development progresses through a series of stages. These stages are inclusive stages. That is, the lower stages are included as a basis for the higher level stages. When a child advances to a higher level stage, he does not abandon his previous forms of knowing. At the new stage, the child builds on the previous cognitive abilities and improves them. Thus, the scientist studying samples of moon rock for the first time, uses investigative processes similar to those of a young child investigating a new toy. The scientist, however, has refined his research skills to the extent that they bear little resemblance to the lower level investigating processes on which they were formed.

There are four main stages of cognitive development through which the child progresses. These stages are: the sensorimotor stage, the preoperational stage, the concrete operational stage, and the formal operational stage. The name for each stage reflects the type of operative cognition which is possible at the different levels. A discussion of each of the four stages follows.

In the discussion of the stages of development, a particular format is followed. First, the approximate ages at which a child may pass into a particular stage are indicated. Next, the types of figurative and operative cognition exhibited at each stage are explained. Finally, a description of the types of concepts and thoughts which are possible at the particular stage is examined.

At various points, examples are presented to illustrate different aspects of Piaget's stage theory. Often the same example has been examined at different stages, and with respect to both figurative and operative aspects of cognition. Thus, distinctions between the different stages and aspects of cognition can be made easily.

### i. The Sensorimotor Stage

The sensorimotor stage, the first of the Piagetian stages, lasts approximately until the child is one and a half years old.[39]

Of the two figurative processes, only perception is involved in the knowing at the sensorimotor stage. The child perceives his surroundings through the various senses of hearing, feeling, tasting, smelling and seeing. The child in this period has no semiotic processes functioning, although toward the end of this stage, some prefigurations to representation may be observed. Lacking the semiotic processes, a child during the sensorimotor stage lives only in the immediate present. He is unable to gain access to previous events by recalling them himself. On the other hand, he can recall events if there is some immediate perceptual cue. For example, a child may have a bouncing swing which he enjoys. If the swing is out of sight, the sensorimotor child cannot recall the fun that he had bouncing in it. However, if he sees his swing, he then can recall the fun he had previously. He may, perhaps, indicate his recognition of the swing by bouncing himself in his parent's arms. This

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39 Hermine Sinclair, Op. Cit., p. 2.

bouncing is only a prefiguration to representation, as the perceptual cue, the swing, was required to initiate the recall process.

During the sensorimotor stage of development, the child is only capable of performing physical actions on physical objects. At birth, the child can perform a few individual actions such as sucking, swallowing, grasping and crying. Very early, the child starts to co-ordinate the individual actions. At his first feeding, the child has to learn how to co-ordinate sucking and swallowing if he is to satisfy his hunger. Anyone who has fed a newborn is aware of the sputtering and confusion that exists until sucking and swallowing are co-ordinated in the proper sequence.

A child at the sensorimotor stage has no verbal concepts. There are two reasons for this. First, the sensorimotor child is just in the process of developing object permanence.[40] As well, he does not have language.

Near the end of the sensorimotor stage, the child develops knowledge of object permanence. That is, the child realizes that things exist even if he cannot see them. The child indicates his awareness of object permanence by hunting for a toy that has dropped out of sight, or has been hidden from him. It is only after the child is aware of object permanence that he can begin to know the objects and form concepts about them.[41]

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40 Jean Piaget, Psychology and Epistemology, New York, The Viking Press, Inc., 1971, p. 16-17.

41 Hermine Sinclair, Op. Cit., p. 3.

Language is also necessary to form verbal concepts. The use of words to name objects indicates that the child is able to recall past events at will.[42] Until the child can recall objects and events freely, he cannot compare and contrast them to form concepts. As well, language is useful in the formation of the concepts themselves. For example, a child learns about softness by feeling soft objects. He may have experienced the touch of a soft blanket, or the soft fur of a kitten. When he knows the term "soft", he can use it to describe what he feels when he touches other soft objects. Therefore, the word "soft" describes the feel of the new object, and links this experience with the previous experiences with soft objects. Thus, the child uses language to store and retrieve relevant information.[43] This ability to store and retrieve relevant information enables the child to form verbal concepts. Being unable to store and retrieve information, the child at the sensorimotor stage is unable to form verbal concepts.

Although the child is unable to form verbal concepts at this stage, he is capable of forming sensorimotor schemes. These schemes, or generalized actions, are in a sense sensorimotor concepts.[44] These schemes require neither the notion of object permanence nor language.

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42 Jean Piaget in Richard I. Evans, Jean Piaget, The Man and His Ideas, A Dialogue with Piaget, p. 24.

43 Bärbel Inhelder, Magali Bovet, Hermine Sinclair, C. D. Smock, "On Cognitive Development", in the American Psychologist, Vol. 21, No. 1, 1966, p. 163.

44 Jean Piaget in Richard I. Evans, Jean Piaget, The Man and His Ideas, p. 18.

To form sensorimotor schemes, the child has only to perceive relationships between his actions and the objects in his environment. As stated previously, the child becomes aware of the relationships "resting upon" and "moving a distant object from one place to another". In turn, the child can combine these relationships to construct the scheme of "pulling closer by means of an intermediary object".

Therefore, at the sensorimotor stage of cognitive development, the child knows the world through perception and through his physical actions. Lacking the semiotic function, the child can deal only with his immediate environment and the present time.[45] Lacking language and object permanence, the child cannot form verbal concepts. However, at this stage the child can form sensorimotor schemes which indicate that he can make effective generalizations.

### ii. The Preoperational Stage

The second stage, the preoperational stage, follows the sensorimotor stage and lasts until the child is approximately seven years of age.[46]

The child at the preoperational stage of development continues to know his world through perception as he did in the first stage. In addition, he also begins to develop some of the semiotic functions such

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45 Jean Piaget in Richard I. Evans, Ibid., p. 24.

46 Hermine Sinclair, "Piaget's Theory of Development: The Main Stages", p. 5.

as deferred imitation, symbolic play, drawing, internal imitation, and language. Having formed some of his semiotic processes, the preoperational child is no longer restricted to the immediate present. He can recall previous events, and, on the basis of previous experiences, he can make some limited predictions about the near future. If the child is accustomed to going for a walk after his father puts on a particular jacket, he anticipates the walk as soon as he sees his father going for his jacket. Thus, the preoperational child can extend his thoughts from the present, to the past, and to the future in a very limited manner.

During the period when the child is developing his semiotic processes, he makes use of imitation. In a sense, imitation is a type of correspondence.[47] For example, the mother may teach the child to clap with her as she says the action verse "Patta Cake". The child at first will imitate or make a physical correspondence with the mother's hand clapping. At another time, the child can use deferred imitations to let his mother know he wants to play the game. Thus, in the formation of the semiotic processes, the child can be observed going through a stage of imitation (or correspondence-making). The imitation may be less than accurate at first, but the child improves. Finally, the child can use what he has learned through imitation to represent thoughts to himself and others.

With the development of the semiotic function, the child begins to form schema for the representation of different objects. Whereas schemes are generalized actions, schema[48] are generalizations about representations.

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47 Jean Piaget, Genetic Epistemology, p. 43.

48 Hans Furth, Piaget and Knowledge, p. 264.

That is, the child co-ordinates different types of representation of a particular object. A child may have a schema for "boat". Included in this schema are mental images, and different words such as boat, ship, vessel. As well, a child has no difficulty in representing a ship using an old grocery box, or other physical objects. He also may use drawings to represent a boat. Thus, the child has many types of representation included in his schema for "boat".

While at the preoperational stage, the child attempts to develop a schema for the representation of number. However, at this stage, the child is only capable of imitating the representation of number. Piaget studied the development of the child's ability to represent number. In this study, he was interested in the child's ability to represent number, not by numerals, but by using counters. To do so, Piaget set out a line of counters and asked the child to make another line with the same number of counters.[49]

A preoperational child is not successful at this task. He imitates the model line by putting down a line of counters the same length as the model, but he may not have the same number of counters. To demonstrate that the child was imitating and not truly representing the same number shown by the model line, Piaget clustered the child's line. He then asked him if there were the same number of counters in his cluster as in the model line. The preoperational child typically thinks that there are then different numbers in the two sets. Thus, in representing number

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49 Jean Piaget, The Child's Conception of Number, p. 66.

using counters, the preoperational child is only at the level of imitating the model line. As stated previously, Piaget called this representation a global or perceptual correspondence.[50]

The preoperational child is influenced by perceptual factors. With respect to the representation of number, Piaget noted that some particular situations can provoke the child into making a numerical correspondence earlier than is possible using plain counters.[51] In one such task, Piaget used eggs and egg cups. These were considered provoking materials because only one egg can go in each cup. The task was similar to the counter task, except the child had to set out enough eggs to fill the cups.

In addition to using eggs and egg cups to provoke the child into making more accurate representation of number, Piaget noted another provoking situation. If the eggs were clustered near the cups where there were still perceptual connections, the child would consider the numbers of cups and eggs equal. However, the same cluster, just moved a little farther away from the cups, made the child think that there were different numbers of cups and eggs. Thus, perceptual factors can deter or provoke the child as he is developing his semiotic processes.[52]

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50 Ibid., p. 48.

51 Ibid., p. 42.

52 Alina Szeminska, "The evaluation of thought: Some applications of research findings to educational practice", in P. H. Mussen (ed.), European Research in Cognitive Development, Monographs of the Society for the Society for Research in Child Development, Vol. 30, No. 2, 1965, p. 48.

With respect to the operative aspect of knowing, the preoperational child is in the process of internalizing his physical actions to the level of mental thought. The child demonstrates that internalized actions have been formed by using processes of the semiotic function. This has been explained previously.

According to Piaget,[53] once the child internalizes his actions he has to co-ordinate them on the mental level, as he co-ordinated actions on the physical level. However, the preoperational child is at the stage where this co-ordination has not yet taken place. To illustrate this, Piaget used many tests. In the one with the identical cylinders with equal amounts of water, Piaget had the child empty one cylinder into a shorter, wider beaker. Then, he questioned the child on the equality of the quantities of water in the two containers. The preoperational child is aware of the pouring procedure. Also, he is aware that the water can be returned to the cylinder. If these two actions were internalized and co-ordinated, the child would realize that the quantities were equal. The child at the preoperational stage denies this equivalence, and furthermore, believes that the quantities will not even be equal if the water is returned to the cylinder. Thus, his internalized actions are not yet co-ordinated.

The preoperational child has difficulty co-ordinating his internalized actions. As well, he has difficulty co-ordinating ideas. He can consider different ideas successively, but he cannot consider two or

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53 Jean Piaget in Richard I. Evans, Op. Cit., p. 24.

more ideas simultaneously. For this reason, the child at the preoperational stage is limited in the information which he can abstract from his actions. By way of illustration, when the child was shown two lines of counters in one-one correspondence, he was asked if there were equal numbers of counters in each line. If he felt that they were not equal, he could make them equal. Thus, the child was aware that both lines had equal numbers of counters. When the one line was clustered, the preoperational child considered it to have different number of counters than the other line had. Piaget then would question the child, trying to have him realize that the two sets were equal in number. The preoperational child would make a series of contradictory statements. At one moment, the cluster seemed larger. Then the next moment the line seemed to have more counters. The preoperational child is unaware of his contradictory responses. He cannot co-ordinate them simultaneously to know that they are contradictory, so he is not bothered by them. Therefore, the child cannot learn from making these contradictory statements.

Also, the child at this stage cannot consider two qualities of an object simultaneously. In the same task, the preoperational child would talk about the cluster being large, or being short. The line would be long, or it would be thin. The child could not consider both the length and width simultaneously and say, for example, that the line was long and narrow.

Sinclair,[54] a linguist, studied the language of the preoperational child. She found that the child at this stage uses sentences known to linguists as scalars (e.g., that line is long; that cluster is short). Sinclair and others[55] attempted to teach preoperational children to use vector sentences (e.g., the cluster is wider and shorter than the line). Their efforts were not very successful. Preoperational children just cannot co-ordinate ideas simultaneously.

The preoperational child is capable of forming concepts. The concepts at this stage are in Piagetian terms, qualitative concepts as opposed to quantitative concepts.[56] For instance, a child knows what plasticene is. It is an oily clay-like substance which can be modelled into different shapes. These are some of the qualities a child can observe as he plays with the plasticene. On the other hand, if the preoperational child sees two round balls of plasticene the same size, he considers them to have the same amount of plasticene. When one ball is rolled into a cylinder, the child thinks there are different amounts of plasticene in each shape. If the cylinder is remade into a ball, the child considers the plasticene in each shape to be equal again. The preoperational child does not realize that the quantity of plasticene is not changed if its shape is changed. Thus, the child at the preoperational stage has a qualitative concept of plasticene but not a quantitative concept of it.

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54 Jean Piaget, Genetic Epistemology, p. 48.

55 Bärbel Inhelder et al., Op. Cit., p. 163.

56 Hermine Sinclair, Op. Cit., p. 3.

Unaware of the quantitative aspects of objects, the preoperational child cannot have a concept of number. He judges quantity only through perceptual qualities. If two sets of counters look equal, the child thinks the quantities are equal. If the two sets look unequal, they are, according to the preoperational child.

The preoperational child's concepts of familiar objects are very restrictive concepts. For example, Elkind[57] was interested in the child's concepts of "car". Therefore, he presented children with a car from which different parts could be removed. He told the child that parts would be removed from the car. The child was to tell him when the object was no longer a car. A child at the preoperational stage stopped him after very minor transformations, such as the removal of lights or doors. Once the car looked different from those with which he was familiar, the child thought it was not a car anymore. Thus, the preoperational child forms qualitative concepts which are rather restrictive. If there are too many perceptual differences, the preoperational child will form a new concept, rather than include the object in the existing concept.

There are major differences in the cognitive characteristics of sensorimotor and preoperational children. With reference to the figurative aspect of cognition, children at both stages rely primarily on perception to know the world. However, the preoperational child is also developing his semiotic functions. As a result he is no longer limited to knowing only the present. He has, at the preoperational stage, a limited access to prior and future knowing.

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57 David Elkind, "Conservation and Concept Formation", in David Elkind and John Flavell (eds.), <u>Studies in Cognitive Development: of Jean Piaget</u>, New York, Oxford University Press, 1969, p. 181.

In the development of the semiotic processes, the preoperational child imitates. The preoperational child imitates the actions and languages which he encounters throughout his day. With further reference to the semiotic processes, the preoperational child is also developing schema for the representation of objects. Since the child at this stage is highly dependent on perceptual information, representations of objects are often less precise than adults suspect. The child's representation schema for number, for example, was shown to be only a global or perceptual representation.

With respect to the operative aspect of cognition, the child is, with the use of the semiotic function, in the process of internalizing his physical actions. As a result, the preoperational child is raising his thought from the physical level, to the level of internal representation.

The preoperational child, however, has not co-ordinated his internalized actions. He is limited, therefore, in the knowledge which he can abstract from his internalized actions. Different ideas cannot be co-ordinated simultaneously; hence, the child is self-contradictory, and unaware that he is this way. Being unable to consider two variable quantities simultaneously, the preoperational child can only form qualitative concepts as opposed to quantitative concepts. Due to the child's use of imprecise representations, however, the qualitative concepts formed usually are very restrictive concepts.

iii. The Concrete Operational Stage

The concrete operational stage follows the preoperational stage and continues until the child is twelve to fourteen years of age.

Whereas the appearance of the semiotic processes indicates the commencement of the preoperational stage, the appearance of interiorized actions indicates that the child has advanced to the concrete operational stage. Interiorized actions have been described previously as concrete operations which have been abstracted from co-ordinated internalized actions.

With the formation of the concrete operations, there is a change in the figurative and operative aspects of cognition. The perceptual function and the semiotic function, comprising the figurative aspect, undergo considerable development. Prior to the concrete operational stage, the child knew the world through the perceptual function. The child completely accepted what he perceived as being true. In contrast, the child at the concrete operational stage realizes that perception can be misleading and he makes efforts to moderate his perception. When confronted with the correspondence task between the two sets of counters, he does not base his responses completely on his perception of the sets. When one set is clustered and appears larger, the concrete operational child discounts this illusion. He knows that if no counters are added or removed, the sets must be equal despite appearances. Thus, at the concrete operational stage the child continues to use perception to gather information. However, the child moderates perception with reasoning to avoid being misled by perceptual illusions.

The concrete operational child is also refining his use of the semiotic function. During the preoperational stage, the child had developed some facility with oral language. Until the concrete operational stage, however, the child's use of language is primarily to collect, store and retrieve perceived information. With the formation of interiorized actions, the child is able to transform information and abstract knowledge. This knowledge is also processed by the semiotic function. As a result, the language used by the concrete operational child expresses more understanding of the information being discussed than the language used by the preoperational child.

At the preoperational stage, the child commenced the formation of schema for the representation of different concepts. The schema developed, however, were most precise for the representation of physical objects which the child encountered in his environment. At the concrete operational stage, the child begins forming schema for the representation of more abstract concepts such as number. The concept of number is more abstract than the concept of physical objects such as "car". That is, a car exists as a physical object. Number does not exist as a physical object. It can, however, be represented by objects such as counters. Thus, a major achievement for the concrete operational child is in his ability to form precise schema for the representation of concepts for both physical and more abstract concepts.

The child's schema are more precise because the concrete operational child understands and can focus on important information. In representing number, the child realizes that the shape of the set of counters does not change the number of counters. Hence, to make a correspondence between two sets of counters, the concrete operational child disregards the shape of the sets involved.

With respect to the operative aspect of cognition, the concrete operational child is able to perform different actions in the abstraction of new knowledge. The child at this stage can co-ordinate his internalized actions. Furthermore, he is capable of co-ordinating at least two different ideas simultaneously. This enables the child to abstract further information from his actions. For example, a concrete operational child may, by error, make a contradictory statement with reference to the one-one correspondence task. Unlike the preoperational child, who cannot realize that he has made contradictory statements, the concrete operational child can. Furthermore, he has the ability to examine the problem logically and determine, on the basis of his actions, which statement is correct. He can also explain what led him to make the error. Thus, the child learns from his actions when he can co-ordinate ideas simultaneously.

The concrete operational child can also consider two variables of a set or object. With the cluster and line counters, the child considers both the length and the width of the sets simultaneously. This is evident in his speech patterns. Instead of using scalar sentences, the child at the concrete operational stage uses vectorial sentences[58] to express the qualities of the sets of counters. For example, the

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58 Jean Piaget, Genetic Epistemology, p. 48.

cluster is wider and shorter than the line of counters. Thus, once the child forms operations, his thinking becomes, in a sense, two-dimensional. This contrasts with the thought of a child at the preoperational stage. The preoperational child can only consider ideas successively. Therefore, his thought is, in comparison, one-dimensional.[59]

It was stated previously that the child abstracts operations from internalized actions which he has co-ordinated. In turn, the child co-ordinates his operations to form what Piaget calls a structure. At the level when the child has only formed concrete operations, this structure is described as a 'grouping' structure.[60] This 'grouping' structure refers to the fact that the child at this stage is concerned with different types of groups, and can work with them. For example, a child with the 'grouping' structure is capable of putting objects together into a group, of separating a group into subgroups, and of ordering elements of a group. To perform these actions, the child must co-ordinate his operations using logical generalization, addition, seriation and correspondence.

Operations are also co-ordinated with their reverse operation. This enables the child to think reverse thoughts. A distinction was made between internalized actions and operations. A similar distinction can be made between the child's awareness of reversible actions and his ability to think reverse thoughts. A child without operations is only

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59 Jean Piaget, The Child's Conception of Number, p. 10.

60 Hermine Sinclair, Op. Cit., p. 7.

aware of reverse actions. Therefore, he is aware that pouring water from a beaker to a cylinder is the reverse of pouring the water from the cylinder to the beaker. The preoperational child is unaware of the implications of the reverse actions. Thus, he does not realize that the quantity of water involved remains constant. On the other hand, an operational child is aware of the reverse actions and the implications arising from them.

When the child has co-ordinated his operations into a structure, there are three further changes in his thinking processes. First, the child can easily reverse his thoughts.[61] Thus, if a concrete operational child can think: $3 + 2 = \_\_$, he can also easily think: $5 = 3 + \_\_$. In the situation where the child made a one-one correspondence between two sets of counters, Piaget clustered one set. A child at the concrete operational stage often defended his view, by stating that it was possible to uncluster the counters to prove the equality still existed. Thus, by reversing his thoughts about the clustering, the child could solve the task.

This ability to reverse thoughts is important. If a child can reverse his thoughts with respect to something he knows, it is an indication that the child truly understands. In colloquial terms one often indicates the understanding of something by stating, "I know it backwards and forwards." or "I know it inside out." Thus, if the child can easily answer that $3 + 2 = \_\_$, but has some difficulty answering that $5 = 3 + \_\_$,

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61 Jean Piaget, "Development and Learning", p. 177.

there is an indication that he is not fully aware of the relationship between 3, 2 and 5. Therefore, the first important change in the child's thought as a result of forming the grouping structure, is in his ability to reverse his thoughts easily. Such flexibility in thinking indicates true understanding of the situation involved.

Another important change is in the child's ability to employ different operations to solve a problem. If the child wants to make a correspondence between two sets of counters, he may use several operations. For example, seriation, addition and even subtraction may be used to make the correspondence. The seriation used in making such a correspondence is not according to some characteristic, such as the size of the counters. The seriation involved is in the fact that the child must choose one counter to be first, another second, and so on, until the correspondence is made. Piaget refers to this as the vicariant[62] ordering of objects.

In another sense, the addition operation may be used. The child may put down three counters, and then note that four more are still required to make a correspondence with a second set of seven counters. As well, the subtraction operation may be used. The child may notice that there is a difference of four between the two sets of counters, therefore, four counters are required to complete the correspondence. Thus, with operations co-ordinated into a structure, the child is never restricted to using only one operation to solve a problem. There are always other operations which can be used to help solve the problem.[63]

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62 Jean Piaget, The Child's Conception of Number, p. 96.

63 --------, "Development and Learning", p. 177.

Finally, the child is able to generalize his operations.[64] If a child had realized from playing with a set of counters, that summing is independent of the order of the addends, he would be able to generalize that:

$$(2+1) = (1+2)$$
$$(A+B+C) = (C+A+B)$$
$$\text{John} + \text{Bill} = \text{Bill and John}$$

In the cylinder task, Piaget examined the ability of children to generalize. He did so by extending the task by pouring the water from the beaker into several containers, rather than just one.[65] Again, the children with operations could generalize about the continued equality of quantities of water. Thus, with the formation of operations into a structure, the child is able to generalize his operations to other situations requiring similar operations.

At the concrete operational stage, the child can form two types of concepts: qualitative and quantitative concepts.[66] Qualitative concepts at the preoperational stage differ from those at the concrete operational stage. From his experiment with the concept of "carness", Elkind[67] noted that the younger child's concept of "carness" is based on his experiences with cars. If he is familiar with seeing lights on all the cars he observes, he is inclined to think that a car must have lights.

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64 Jean Piaget, The Child's Conception of Number, p. 218.

65 Ibid., p. 4.

66 --------, Genetic Epistemology, p. 53.

67 David Elkind, Op. Cit., p. 171-189.

Likewise, the motor is hidden under a hood most of the time. Thus, a preoperational child, seldom seeing a motor, may be inclined to think that it is not necessary. In a sense, then, preoperational concepts are subjective concepts. The subject concludes, from his own experience with objects, what their concept involves.

At the concrete operational stage, the child begins to form qualitative concepts which are more objective. That is, the child becomes aware that objects have a particular identity that is not solely dependent upon his previous experiences with the objects.[68] In Elkind's experiment, the older child realized that a car is a motorized vehicle. Thus, even if the car looked unusual without doors, lights, or a windshield, he knew it was a car as long as it was a motorized vehicle.

In addition to qualitative concepts, the concrete operational child can also form quantitative concepts. Quantitative concepts are called logico-mathematical concepts.[69] Qualitative concepts of physical objects, such as a car, are different from logico-mathematical concepts. Qualitative concepts of physical objects are abstracted from the objects themselves, whereas logico-mathematical concepts are abstracted from actions on particular characteristics of an object. The concept of number is a logico-mathematical concept. Number is one of the characteristics of a set of counters. Number is not the counters themselves. Furthermore, number does not even exist as an object in the physical

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68 Hermine Sinclair, Op. Cit., p. 3.

69 J. Piaget, Genetic Epistemology, p. 16, 17.

world. Number can only be represented by different objects in the real world. To have the concept of number, the child must abstract it from the numerical characteristics of the objects representing number.

To form concepts, the child must be able to abstract the object's basic identity. Piaget considers this ability to be extremely important. He says,

> It is unnecessary to stress the importance in everyday life of the principle of identity; any attempt by thought to build up a system of notions requires a certain permanence in their definitions.[70]

In forming qualitative concepts, the child can abstract the object's basic identity by distinguishing between the permanent and impermanent qualities of the object.[71] Permanent qualities are the qualities an object must possess to be a particular object. Impermanent qualities are qualities which are not essential to the particular object. Thus, in Elkind's experiment, the older child could distinguish between the permanent and impermanent qualities of "carness".

A logico-mathematical concept also has a basic identity. This identity cannot be abstracted by distinguishing between permanent and impermanent qualities, because such a concept does not exist as a real object. Logico-mathematical concepts can only be represented. Particular characteristics of objects are used to represent the logico-mathematical concept. Thus, the concept's basic identity must be abstracted from these particular characteristics.

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70 Jean Piaget, The Child's Conception of Number, p. 3.

71 Hermine Sinclair, Op. Cit., p. 3.

However, the abstraction of a logico-mathematical concept requires some action to be performed on the object possessing the characteristic relating to the concept. The type of transformation must be one which leaves the characteristic invariant, or unchanged. Piaget examined the child's concept of number. He had the child make a one-one correspondence between two sets of counters. That is, he had the child use a set of counters to represent the same number represented by a model line of counters. Both sets had a numerical characteristic which was related to the concept of number. One set of counters was transformed into a cluster. The clustering transformation left the numerical characteristic invariant. Only one characteristic of both the cluster and the model line was invariant. That was the numerical characteristic. If the child were aware of this invariance, he was considered to have formed the concept of number.

Whenever an object, or a set of objects, can be transformed in such a way that a characteristic remains invariant, Piaget refers to this as conservation. He has defined conservation:

> We call "conservation" ... the invariance of a characteristic despite <u>transformations</u> of the object or a collection of objects possessing this characteristic.[72]

When a child can identify the invariant characteristic in a transformation, he is called a conserver. Piaget uses the term "conserver" primarily to refer to a child who has formed a logico-mathematical concept.

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72 Jean Piaget, "Quantification, Conservation, and Nativism", <u>Science</u>, Vol. 162, 1968, p. 978.

Elkind[73] suggests that qualitative concepts of objects can also be conserved by a child. Qualitative concepts are logical, but not mathematical. The child uses the logical identity and conservation principles to form such concepts. Qualitative concepts are not mathematical, as only qualitative, not quantitative, aspects are involved in their formation. Thus, to Elkind, a conserver is a child who has formed either a logico-mathematical concept, or a qualitative concept of an object.

The ability to conserve indicates more than just that the child has formed a concept. The ability to conserve indicates that the child has formed his operations and organized them into a logical thinking structure. In addition, the ability to conserve indicates that the child has also integrated the particular representational mode into his thinking structure.

With the organization of a logical thinking structure, the child is aware of logically related actions, and is able to draw logical conclusions. The operational child's thought is flexible enough that different ideas may be co-ordinated simultaneously so that any perceived contradictions may be resolved. Only when a child has formed and structured his operations, is he capable of conserving an invariant characteristic.[74] Thus, the ability to conserve indicates such a structure has been formed.

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73 David Elkind, Op. Cit., p. 171-189.

74 Jean Piaget, Genetic Epistemology, p. 22.

The ability to conserve also indicates that the child has integrated the particular representational mode into his thinking structure. In learning a representational mode, specifically a mode imposed on him by society, the child must first internalize it. Through use of the particular mode, the child refines it until it appears that he understands the mode. Using counters to represent number, the child passes through a global representational stage when he is content if his set looks similar to the model set in length and density. Later, however, he reaches a stage when he very easily makes a one-one correspondence between the sets. One may even be misled and conclude the child must understand number as represented with counters. By clustering one set, Piaget found a number of children who could make the one-one correspondence, but could not conserve. Hence, such children did not fully understand number represented with counters.

To advance past this stage, the child must integrate the particular representational mode into the logical thinking structure. The ability to conserve indicates such an integration has occurred.

As Sinclair stated, the figurative and operative aspects of cognition are no more than the two sides of the same coin. That is, to form a concept, both figurative and operative aspects are involved. Lacking either one, the child cannot form a concept. Thus, the ability to conserve indicates that the figurative and operative aspects of cognition have been integrated into the thinking structure resulting in the formation of a concept.

With the advancements in cognition made at the concrete operational stage, the child understands more of what he experiences than he did at the preoperational stage. This, in turn, enables the child to recall past events more clearly, and anticipate future events more accurately.

With the ability to consider events simultaneously, the child benefits greatly from recalling and anticipating events. In anticipating future events, the concrete operational child is, however, only able to predict concrete events. These are events which he may have experienced before, or which he knows are possible. He is still very limited in anticipating future hypothetical events which the formal operational child can consider.

In conclusion, the thought of the concrete operational child is very different from that of the preoperational child. Whereas the preoperational child has only internalized his actions, the concrete operational child has internalized and co-ordinated them. The concrete operational child, therefore, can perform more complex thinking. This is possible because of the changes in both figurative and operative aspects of cognition. Whereas, until this stage, the child relies on perceived information as completely true information, the concrete operational child realizes that he must modify perception at times. Thus, the concrete operational child avoids being misinformed by misleading perceptual cues.

With respect to the semiotic function, the child is refining his use of languages. His language expresses more logic. The concrete operational child develops schema for the representation of concepts of physical objects as well as more abstract objects such as number. In the representation of objects, the child is far more precise than he was at the preoperational level.

At the concrete operational stage, the child has the ability to perform two-dimensional thought. This is evident in his ability to consider two thoughts simultaneously. As a result, he is able to recognize and resolve contradictory information. As well, his thought is more flexible. This is evident in his ability to reverse his thoughts, to try different approaches in finding solutions to questions, and to generalize his understanding to new situations.

The concepts formed by the concrete operational child are also better than those of the preoperational child. First, the qualitative concepts of the concrete operational child are more objective than the very subjective concepts of the preoperational child. Secondly, the concrete operational child can form some of the logico-mathematical concepts such as the concept of number. It was impossible for the preoperational child to form any logico-mathematical concept.

The child indicates that he has formed logical mathematical concepts by being able to conserve. That is, he is able to identify an invariant characteristic despite changes in the object possessing this characteristic. Conservation of a concept indicates that the child has integrated both operations and the representational mode into the thinking structure. Thus, at the concrete operational stage, there are major changes in the child's thinking and knowing.

iv. The Formal Operational Stage

The child enters the formal operational stage between the ages of eleven and fourteen years.[75]

One observes further developmental changes in the child's figurative aspect of cognition. Through the perceptual function the child continues to know the world. However, information collected through this function is not accepted as being completely true information. At the concrete operational stage, the child had begun to screen out misleading perceptual cues. At the formal operational stage, the child is able to screen out stronger perceptual cues. In his drawings a child indicates how well he understands depth perception. The formal operational child draws objects in the foreground larger than similar objects in the background. Younger children are confused when similar-sized objects are drawn different sizes. As a result, younger children draw similar objects the same size despite their location. Thus, at the formal operational stage, the child regulates perception to avoid being misled by stronger perceptual illusions.

The formal operational child's use of language is also more sophisticated than that of the concrete operational child. Prior to the formal operational stage, the child was learning to use words correctly. By the end of the concrete operational stage, the child had a good literal understanding of many of the words in his vocabulary.

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75 Bärbel Inhelder and Jean Piaget, The Growth of Logical Thinking From Childhood to Adolescence, United States, Basic Books Inc., 1958, p. xxii.

At the formal operational stage, the child's understanding of words transcends their literal meanings. A formal operational child appreciates parables and allegories which have both a superficial literal story and an underlying message. The concrete operational child appreciates only the literal story. Thus, the formal operational child's use of language is becoming highly sophisticated.

At the concrete operational stage, the child was developing schema to represent more abstract concepts. This continues at the formal operational stage. In addition, the child at this stage is also able to use more abstract languages such as algebraic and chemical equations. With such languages, the child is reporting transformations and reactions that he cannot perceive. Thus, as the child develops, both the objects being represented and the languages used are progressively more abstract and removed from the perceptual field.

With respect to the operative aspect of development, the formal operational child is able to consider several variables in a task.[76] This is in contrast with the child at the concrete operational stage, who can consider only two variables in a task. The formal operational child, in a sense, has a multidimensional thinking structure.

In actual reasoning with the variables in a task, the formal operational child can reason with only two variables at any one time. The concrete operational child is also able to reason with two variables at one time. Children at both stages are, therefore, able to reason logically with two variables at one time.

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76 Ibid., p. 282.

The formal operational child differs from the concrete operational child in his ability to consider the different variables in a multifactor task. First, he is logically aware that more than two variables may be simultaneously involved in the task. The concrete operational child is not aware of this.[77] Secondly, the formal operational child understands that he must be systematic in considering the effects of each variable in the task. To do so, he begins reasoning with variables one and two, while holding variables three to n constant. Then he considers the next variable with the product of the first reasoning process. This successive reasoning is repeated systematically until all relevant variables are considered. Whereas the child at the concrete operational stage is logical when two variables must be co-ordinated, the formal operational child is both logical and systematic when more than two variables must be co-ordinated.[78]

Being both logical and systematic, the formal operational child is able to perform more complex logical operations. Combinatorial analysis is one such operation. To decide how many different ways two people can be selected from a group of four, the child must consider the different people successively. This requires both a logical and systematic thinking structure. The concrete operational child, although logical, is not sufficiently systematic in his thinking to perform such complex logical operations.

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77 Ibid., p. 347.

78 Ibid., p. 283.

Being able to consider several variables at a time, the formal operational child is able to think about his thinking.[79] At the concrete operational stage, the child only thinks about what he is doing. Thus, when classifying a set of objects, he is involved completely with the process and cannot think how he classifies. In contrast, the formal operational child can perform the operation, and at the same time, think about why he is classifying some objects into one set and others into another set. As a result of thinking about his thinking, the formal operational child can form rules to direct his thinking in the future.

The objects about which the formal operational child reasons are not restricted as they were at earlier stages. The concrete operational child could only reason with respect to objects he was experiencing or with respect to objects he had encountered previously. The formal operational child can reason about these objects and, as well, he can reason about hypothetical objects. In the following syllogism, one premise is invalid. However, if a child were asked to judge the conclusion from these premises, different responses would be made depending on the child's level of development.

If Toronto is a city, hens have four legs.
Toronto is a city.
Therefore, hens have four legs.

The concrete operational child would consider the conclusion false because hens only have two legs. In comparison, a formal operational child would judge that the conclusion is true, and following logically from the premise. He is not impeded from thinking by the

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79 Ibid., p. 340.

invalid premise as the concrete operational child is. Thus, the formal operational child reasons with reference to all types of objects either real or hypothetical.

The concepts formed by the formal operational child also differ from those formed at the lower levels. At lower stages, the child's concepts were based on qualitative aspects of different objects. Concepts formed at the formal operational stage, such as freedom and justice, can not be represented by the qualitative or quantitative characteristics of objects. These concepts are only represented by the quality of the behavior of people in particular situations. To identify what is similar to behaviors labelled as 'free' and as 'just' is a complex abstraction. Thus, such concepts are formed only after a child has reached the formal operational stage.

In conclusion, the formal operational child's thought differs from that of the concrete operational child. Whereas the difference between the preoperational and concrete operational child's thought was extreme, the change between the concrete and formal operational stages is less obvious. With respect to the figurative aspect of cognition, the formal operational child uses perception to know, while at the same time, he regulates perception to avoid being misled by perceptual illusions. The child, at this stage, is becoming more sophisticated in his use of verbal languages. He is also able to use more abstract languages to express thoughts concerning events which lack physical referents.

In the area of operative cognition, the formal operational child has formed a multidimensional thinking structure. The major difference between the formal operational child, and the concrete operational child who has a two-dimensional thinking structure, is in the formal child's ability to be systematic. Being both logical and systematic, the formal operational child is able to perform complex logical operations and to systematize the world about himself. He is, however, not limited to the physical world as the concrete operational child is. He systematizes everything either real or hypothetical.

In this section as a whole, Piaget's stages of cognitive development have been examined. It was found that the stages of development are inclusive. The lower stages are the basis required for the formation of the abilities at the higher stages of development. As the child progresses through the four stages, the sensorimotor, the preoperational, the concrete operational and the formal operational, the child's figurative and operative aspects of cognition become increasingly more complex, as are the concepts which he forms.

With reference to the figurative aspect of cognition, the sensorimotor child knows the world through perception. He is, however, unable to store and retrieve information except through a simple recall process which is possible when an event is associated with particular perceptual cues. At the preoperational stage, the child develops some of his semiotic functions. Then the child can begin the internalizing of information from the perceptual level to the level of thought. Therefore, the preoperational child can remember even if perceptual cues are not present. At the concrete operational stage, the child begins to realize perception is fallible and he must regulate perception to control simple perceptual illusions. The concrete operational child is more precise in his use of words and language. At this stage, the child is able to represent more abstract concepts, such as number, for the first time. At the formal operational stage, the child is able to regulate information from stronger perceptual illusions. The formal operational child's use and comprehension of verbal language is sophisticated. He can understand both literal and allegorical messages of a passage. The child at this stage is able to use abstract languages. In doing so, he releases his thought from specific events to reason about general events. Thus, as the child develops his figurative aspect of cognition, he is increasingly able to collect, store, and retrieve information at more abstract levels of thought.

As the child is developing figurative aspects of cognition, he is also developing his operative aspect of cognition. At the sensori-motor stage, the child develops an ability to co-ordinate actions at the physical level. Lacking any semiotic processes, the child is unable to consider his actions at the level of thought. At the preoperational stage, however, the child begins the internalizing of actions from the physical level, to the level of thought. Just as the child had to learn to co-ordinate physical actions, the child must learn to co-ordinate internalized actions. The preoperational child cannot co-ordinate internalized actions; hence, he can only consider thoughts one at a time. Consequently, he has only a one-dimensional thinking structure. At the concrete operational stage, the child succeeds at co-ordinating internalized actions and is able to form logical operations to abstract new knowledge. He is, nevertheless, only able to consider two thoughts simultaneously. Therefore, he has a two-dimensional thinking structure. At the formal operational stage, the child's thought is multidimensional. He is logical and systematic in the consideration of several variables in a task, and thus is able to abstract higher levels of knowledge. As with figurative development, the child's operative development becomes increasingly complex as he proceeds through the stages.

As a result of figurative and operative development, different concepts are formed at the different stages. At the sensorimotor stage, the concepts, referred to as schemes, are merely generalizations at the physical action level. At the preoperational stage, the child is able to form internalized qualitative concepts of objects. These concepts are very subjective, and in a sense, are pseudo-concepts. At the concrete operational stage, the child is able to form more objective qualitative concepts. Furthermore, the child is able to abstract logico-mathematical concepts. Finally, at the formal operational stage, the child can form highly abstract concepts. As well, he can abstract general relationships between concepts to develop rules to direct further thinking. Thus, as the child proceeds through the Piagetian stages, the concepts he forms are progressively more complex.

### 3. Transition Between Stages

The stages through which a child proceeds have been discussed. At some point, however, the child makes a transition from one stage to another. The processes involved in the transition between stages are explained. The transition processes are similar at each of the transition periods; hence, the explanation here is with particular reference to the transition between the preoperational and concrete operational stages.

The child has two processes functioning at all times. They are assimilation and accommodation. Assimilation[80] refers to the process of internalizing external data and relating it to the thinking structure. Accommodation[81] refers to the process of applying the thinking structure to external data. Together these processes account for any changes a child makes to his thinking structure. Therefore, they account for adaptation[82] of the thinking structure.

No adaptation of the thinking structure is made as long as the child perceives no difficulty in relating external data to his thinking structure, nor any problem in applying his structure to external events. When such a state of affairs exists, Piaget describes the child as being in a stable state of equilibrium.[83]

At the preoperational stage, the child has a one-dimensional thinking structure, and relies on perceptual information as being true. Thus, the child cannot consider two ideas at the same time, nor does he realize that he is self-contradictory. Furthermore, he has no difficulty concluding that if the set of counters looks larger after clustering, it must be more numerous. Such a preoperational child is in a stable state of equilibrium, and is making no adaptation to his thinking structure.

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80 Jean Piaget, The Child and Reality, New York, Grossman Publishers, 1973, p. 166.

81 Ibid., p. 166.

82 Ibid., p. 166.

83 Ibid., p. 167.

Eventually the child's state of equilibrium is disturbed. That is, the child becomes aware that he has some difficulty in relating external data to his thinking structure, and some problem in applying his structure to external events. This awareness may result from maturation, from social-educational transmission, or from knowledge derived from experience.[84] Whatever the reason for the disequilibrium, the child seeks to restore a balance. He does so by adapting his thinking structure until he resolves any discrepancies between assimilation and accommodation.

During the period when the child is adapting his thinking structure from the preoperational to the concrete operational level, he is in a brief stage of transition. Piaget calls this transition stage the semilogical stage.[85] While at this stage, the child shows evidence of having partially adapted his thinking structure. It is the semilogical child who easily makes a correct one-one correspondence between a set of counters, but denies the equivalence of the sets when one is clustered. The semilogical child is more logical than the preoperational child who can make only a global correspondence. On the other hand, the semilogical child is less logical than a concrete operational child who realizes the continuing equivalence of the sets, before and after the clustering. Thus, Piaget uses the term "semilogical" to describe children who have reached the end of the preoperational stage, and are about to enter the concrete operational stage.

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84 Jean Piaget, The Child and Reality, p. 26-28.

85 --------, Genetic Epistemology, p. 50, 51.

The processes of assimilation, accommodation and adaptation continue until the child eventually develops a thinking structure which reduces any discrepancies between assimilation and accommodation. When this occurs, the child is at the concrete operational stage and is in the next stable state of equilibrium.

With respect to the concepts he forms, the child proceeds through the same stages of development. In evaluating some of these concepts (e.g. number, area, density), Piaget constructed tasks to elicit the different behaviors of children at the preoperational, semilogical and concrete operational stages.

At the preoperational stage, the child is in a stable state of equilibrium. No matter what the question is, the child does not change his thinking. He consistently makes contradictory responses of which he is unaware. His thinking structure is stable.

When the child is at the concrete operational stage, no question will sway his responses from conservation. He understands the concept, and makes consistently correct responses. He, like the preoperational child, is in a stable state of equilibrium.

Semilogical children, however, respond to questions concerning the conservation to particular concepts in ways quite different from the ways preoperational or concrete operational children do. Usually, semilogical children make incorrect responses at the start of questioning. With further questioning, they become confused and vacillate between correct and incorrect responses. Occasionally, some children are able to adapt their thinking structure, and pass into the concrete operational stage. If this happens, the child can give correct responses to all further questions, or else he can justify his responses. Quite often, the child is unable to re-establish equilibrium but indicates that he knows something is amiss. On further questioning, he gives some correct and some incorrect responses.

Thus, the child's responses help the interviewer to classify the child at his proper development level. The preoperational child gives consistently contradictory responses. The transitional semilogical child gives vacillating responses. The concrete operational child gives consistently correct responses.

The processes of assimilation, accommodation and adaptation are ongoing at all times. The child is constantly having new experiences and developing new concepts. Although he may be at a concrete operational level with respect to one concept, he may be at the preoperational stage in the formation of another concept.[86] A child may conceivably never proceed beyond the preoperational level in the formation of some concepts. This is possible when the child has had no exposure to events relating to such concepts. Hence, to say a child is at a concrete operational stage, one must clarify the statement in reference to a specific concept.

Occasionally, one wishes to classify a child as being at a particular stage of development irrespective of the formation of any specific concept. This is required, perhaps, to indicate the readiness of a child to form particular concepts. To classify a child in this situation, it is best to refer to the type of operative thinking

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86 Jean Piaget in Richard I. Evans, Op. Cit., p. 27.

structure a child has formed. To identify a child who has formed a two-dimensional concrete operational thinking structure, a researcher would administer to the child a set of concrete operational conservation tasks. If the child can answer at least one of the tasks correctly, one can conclude that the child has a concrete operational thinking structure. Furthermore, if the operational structure is formed, one realizes that the child just requires exposure to experiences related to the concept.

Thus, one may consider the level of adaptation in thinking with respect to concepts or with respect to the thinking structure itself. One must be precise in reference to the particular adaptation with which he is concerned.

In conclusion, the child seeks to adapt his thinking structure whenever he encounters a discrepancy between assimilation and accommodation. That is, the child perceives some difficulty in internalizing external information and relating it to this thinking structure, or in applying his thinking structure to external events. In adaptation, the child attempts to restore the equilibrium between assimilation and accommodation.

In the specific transition between the preoperational and concrete operational stages, the child is semilogical in his behavior. He is beyond the preoperational stage, yet not to the concrete operational stage. Eventually the child completes his adaptation and enters the concrete operational stage. Adaptation between other stages is similar. In each case, the child enters a transition stage when he has partially adapted, but has not yet entered the next stage of development.

It was noted that the child's behavior at different stages accounts for Piaget's lack of concern about using standardized questions in his tasks. At the preoperational stage, no matter what question is used, the child is consistently self-contradictory. At the concrete operational stage, the child responds correctly to any question posed. The semilogical child is different. Like the preoperational child, he still makes some incorrect responses. However, unlike the preoperational child, he is aware that some of his responses are inappropriate. Thus, Piaget is concerned with the child's behavior rather than with specific responses to specific questions.

Assimilation, accommodation and adaptation are ongoing processes. One may wish to classify a child with respect to his formation of concepts or with reference to his operative thinking structure. One must be precise. In the former case, one identifies the level of development with reference to the particular concept. In the latter case, one must clarify that he is concerned with the adaptation of the operative thinking structure itself.

## 4. Décalages

Piaget's theory of cognitive development through a specific sequence of stages has been described. However, when observing a group of children the same age, one is confronted with a variety of individual differences. Piaget has accounted for some of the individual differences that exist in the child's cognitive development.

Piaget uses the term "décalage" to refer to differences which he observes in the cognitive development of children. The term "décalage" does not have a simple translation into North American psychological terminology. It has been variously described as a "time lag"[87] in learning, or as a "learning differential".[88] It refers to the variations in the rates at which children develop, or in the sequence of concepts which the children form.[89]

There are two types of décalages. They are: vertical and horizontal décalages. A vertical décalages refers to differences in the rates at which children develop cognitively. Such a décalage may be observed between children at the same age, or within a child at different ages. The differences are observed in abilities to perform different tasks of a particular cognitive function at different operative levels.

With reference to the correspondence function, there are physical correspondences a child begins making at the sensorimotor stage of development. There are global or perceptual correspondences that the child makes at the preoperational stage. At the concrete operational stage, the child can make a numerical one-one correspondence between two sets of counters. If a series of correspondence tasks to elicit such behaviors were presented to a group of five-year olds, the children would differ in the number of tasks they could perform. If a child could only make

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87 Jean Piaget in Donald R. Green, Op. Cit., p. 10, 11.

88 Kenneth Lovell, "Intellectual Growth and Understanding", in Arithmetic Teacher, Vol. 19, April 1972, p. 278.

89 Jean Piaget, The Child and Reality, p. 27.

a physical correspondence, he would be classified as being at the sensorimotor stage of development. If a child were successful at the first two tasks, he would be classified as being at the preoperational stage. Finally, if he were successful at all three levels, the child would be assessed as being at the concrete operational stage. Since these tasks are related to one cognitive function at different operative levels of development, children are expected to be restricted to the above response patterns. They would, however, not be restricted in the number of successes on these tasks.

Essentially, a vertical décalage is observed in the child's operative development. Because the sequence of development through different operative stages is fixed, the variations in such development are with respect to rate only. Some children develop through the stages rapidly. Others develop more slowly. Some may even remain at one stage longer than most, yet proceed through the next stage more rapidly than most children. Vertical décalages, therefore, are differences between children the same age, or within a child at different ages, which result through differences in a child's rate of development through the operative stages.

In contrast, horizontal décalages are differences observed in children in the concepts which they have formed, and in the order in which specific concepts are formed. Such a décalage may be observed when a group of children at the same operative stage of development is presented a set of tasks. To observe horizontal décalages, the tasks must involve similar operative behavior. Piaget constructed several concrete operational tasks to evaluate the child's formation of different concepts. Some of the concepts examined were the concepts of number, area, and volume. To form these concepts a child requires a two-dimensional concrete operational thinking structure.

If children with a concrete operational thinking structure were presented with a set of tasks involving the concepts of number, area, and volume, they may differ in the number of items that they can solve. They also may differ in the specific tasks that they have correct.

In effect, horizontal décalages are observed in the child's figurative development. Szeminska[90] has stated that horizontal décalages are observed when more effort is required to overcome content, perceptual or representational obstacles. The concepts of number, area and volume differ in content. Depending upon his previous experiences, a child will form one of these concepts earlier than the others. Another child with different experiences may form another of the concepts first.

Similarly, horizontal décalages are observed when a concept is represented with different representational modes. Representing a correspondence between sets of eggs and eggcups involves a different mode than representing a correspondence between sets of poker chips. Piaget has observed a décalage between these tasks. Usually the child solves the egg-eggcup correspondence earlier than he solves the one-one counter correspondence task.[91] Likewise, décalages may be observed when

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90 Alina Szeminska, "The Evolution of Thought: Some Applications of Research Findings to Educational Practice", in P. H. Mussen (ed.), European Research in Cognitive Development, Monographs of the Society for Research in Child Development, Vol. 30, No. 2, 1965, p. 4-8.

91 Jean Piaget, The Child's Concept of Number, p. 65.

children are presented with tasks differing in perceptual factors. Thus, horizontal décalages are differences between children, or within a child, at a particular stage of development which result when the child must overcome different content, representational and perceptual obstacles in the formation of concepts.

Piaget has acknowledged that horizontal décalages present some problems from the theoretical viewpoint.[92] He has not examined these décalages more closely because he believes that it is not possible to form a general theory about them. It is impossible because the causes of horizontal décalages are numerous. As well, they may be quite specific.[93] For example, the languages and objects that a child uses to represent his thoughts are imposed on him by his particular society. Thus, what gives rise to a horizontal décalage in one society, may not pose problems in another. Conceivably, therefore, a child from a primitive society which does not use eggs and eggcups, may find this task more difficult than a European or North American child does.

Price-Williams[94] encountered such problems while studying conservation of number among primitive children in Central Nigeria. He wondered what effect the child's perception of length had on his ability to remain a conserver. He tried Piaget's task concerning the length of necklaces made from equal sets of beads.[95] Problems arose with the beads.

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92 Jean Piaget in Donald Ross Green, Op. Cit., p. 10, 11.

93 Ibid., p. 11.

94 D. R. Price-Williams, "Conservation Among Primitive Children", in Acta Psychologica, Vol. 18, 1961, p. 297-305.

95 Jean Piaget, The Child's Conception of Number, p. 28.

The children were preoccupied with the beads and were not interested in the task. He finally had to resort to using lines of nuts, with which they were familiar, to answer his question. Despite the confusion which arose with the objects used to represent number, Price-Williams observed the existence of the Piagetian stages of operative cognitive development.[96]

Thus, individual differences in the cognitive development of children may be the result of vertical or horizontal dĕcalages.

Vertical dĕcalages are differences observed among children the same age, or within a child at different ages. These dĕcalages result through differences in a child's rate of development through the Piagetian stages of operative development.

Horizontal dĕcalages are differences among children or within a child at one specific stage of development. Horizontal dĕcalages occur when the child must overcome different content, representational, and perceptual obstacles in the formation of concepts.

Vertical dĕcalages occur in the area of operative development, thus, they are theoretically easy to explain. Horizontal dĕcalages occur in the area of figurative development. Any differences in this area may be quite specific to the particular society or environment in which the child lives. Therefore, theoretical explanations of horizontal dĕcalages tend to be more difficult to make than explanations of vertical dĕcalages.

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96 D. R. Price-Williams, Op. Cit., p. 302.

## 5. Conclusion

Piaget's theory of cognitive development is a broad theory explaining most aspects of cognitive development. It is, at times, a difficult theory to understand. Thus, it has been misinterpreted, and the risk of further misinterpretation exists. Much of this difficulty is in the use of Piagetian terms. Therefore, in the first section, some of the basic Piagetian terms were clarified.

In the first section, it was noted that cognition from birth onwards involves both a figurative and operative aspect. The relationship between these aspects is interdependent. Both are required in the processes of knowing and understanding any concept.

In the second section, the characteristic figurative and operative development at the four Piagetian stages were discussed. It was shown that the type of concepts a child forms is affected by his level of figurative and operative development. However, as the child develops, the figurative and operative aspects of cognition, and the concepts he forms, are increasingly more complex and abstract.

In the third section the processes involved in the transition between stages were discussed. It was noted that the child adapts his concepts just as he adapts his operative thinking structure. Thus, in the formation of specific concepts, the child proceeds through the stages of cognitive development. Furthermore, a child may not be at the same stage of development in the formation of all his concepts.

In the fourth section, individual differences in cognitive development were examined. Vertical décalages refer to observed differences in children's rate of operative development. Horizontal dēcalages refer to observed differences in children's figurative and conceptual development. Whereas with vertical dēcalages, the development sequence is fixed, with horizontal dēcalages, particular representational modes and specific concepts are learned as the child is exposed to them.

Wherever possible in the theoretical review, references were made to the child's formation of number concepts. However, the concept of number which Piaget examined was a very basic concept of number. Piaget has stated, he was only concerned with examining the genesis of the number concept. Essentially, Piaget examined the child's formation of small number concepts. The representation involved, usually, was with sets of physical objects. In the formation of the concept, the child had to make an operative correspondence between two sets of objects. As a result, the child was required to have a two-dimensional operational thinking structure to make the co-ordination. Piaget observed both vertical and horizontal dēcalages in the formation of this basic number concept. Children with preoperational and semilogical structures were not successful. Semilogical children understood more than preoperational children. A horizontal dēcalage was observed between tasks using sets of eggs and eggcups, and tasks using sets of counters.

The adult's concept of number is more complex and abstract than the basic concept Piaget examined. The child will also form this broader concept of number. The child will develop the understanding of both small and large numbers. He will expand his number representation schema to include various representational modes. As the representational modes become more complex, the correspondences a child makes between two modes will also be more complex.

In the formation of the broader concept of number, children will be observed to differ. There will be vertical dēcalages as children develop operative aspects at different stages. There will be horizontal dēcalages as children develop different figurative aspects.

The empirical review in the second chapter is directed to examining literature relevant to the child's development of aspects involved in the formation of the broader concept of number.

# CHAPTER II

## NUMBER CONSERVATION STUDIES

As the child develops, he continues to broaden and deepen his concept of number. No researcher has examined the child's formation of this broader and deeper number concept. Various researchers have examined the child's formation of specific aspects of number. From a synthesis of their findings, it is possible to understand more concerning the child's formation of the concept of number. This synthesis is undertaken in this chapter.

The chapter is divided into three sections. In the first section, there is an examination of the different representational modes which a child may learn to use to represent his thoughts about number.

In the second section, different correspondences between the various representations are identified and discussed in depth. The discussions are in depth because it is with respect to the specific correspondence that certain aspects of the formation of the number concept are relevant. For example, the representation of large numbers is not easy with sets of counters. Hence, the child's inclusion of large number concepts into his total number concept is not meaningfully discussed when one representation in a correspondence is with counters. Aspects such as this, are discussed with respect to the relevant correspondence.

In the third section, there is a summary of the chapter as a whole, a statement of the research problem and the basic research hypotheses to be examined.

## 1. The Number Representation Schema

An adult is often unaware of the variety of representation modes which he has accumulated into his number representation schema. The child, however, must learn these modes and organize them into his own number representation schema. Therefore, several researchers have examined the child's formation of number concepts using different representational modes.

In the numerous studies into the child's formation of number concepts, several for the representation of number were observed. These different modes may, however, be classified into five broad categories. These are: the material mode, the counting name mode, the pictorial mode, the numerical mode, and the verbal mode.

The "material" mode refers to the use of physical objects to represent number. Piaget has used this mode primarily. He has used eggs and eggcups, flowers and vases, dolls and hurdles, counters, coins, and candies.[1]

Dodwell[2] and Hood[3] carried out replication studies of Piaget's conservation tasks, hence they used materials identical to those which Piaget used. In other studies, different physical objects have been

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1 Jean Piaget, The Child's Conception of Number, London, Routledge and Kegan Paul Ltd., 1952.

2 P. C. Dodwell, "Children's Understanding of Number and Related Concepts", Canadian Journal of Psychology, Vol. 14, No. 3, 1960, p. 191-205.

3 Blair H. Hood, "An Experimental Study of Piaget's Theory of Development of Number in Children", British Journal of Psychology, Vol. 53, No. 3, 1962, p. 273-286.

used. Gruen[4] and Beilin[5] have used corks, while Lapointe and O'Donnell[6] used oval cough drops. Toys have been used in many studies. Zimiles[7] used toy trucks, and like Rothenberg and Courtney[8], also used toy blocks. Smithers, Smiley, and Rees[9] used marbles. Thus, there has been a variety of physical objects used in the material representation of number.

A second mode of number representation is a type referred to in this paper as the "counting name" mode. This is defined as the child's use of an oral counting name, or a standard numeral, to represent a

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4 G. E. Gruen, "Experiences Affecting the Development of Number Conservation in Children", Child Development, Vol. 36, No. 4, 1965, p. 963-979.

5 Harry Beilin, "Stimulus and Cognitive Transformation in Conservation", in David Elkind and John J. Flavell (eds.), Studies in Cognitive Development: Essays in Honor of Jean Piaget, New York, Oxford University Press, 1969, p. 317-339.

6 Karen Lapointe and James P. O'Donnell, "Number Conservation in Children Below Age Six-- Its Relationship to Age, Perceptual Dimensions, and Language Comprehension", Developmental Psychology, Vol. 10, No. 3, 1974, p. 422-428.

7 H. Zimiles, "The Development of Differentiation and Conservation of Number", Monograph Society for Research in Child Development, Vol. 31, No. 6, 1966, p. 1-41.

8 Barbara Rothenberg and Rosalea Courtney, "Conservation of Number in Very Young Children", Developmental Psychology, Vol. 1, No. 5, 1969, p. 493-502.

9 Suzanne Smithers, Sandra S. Smiley, and Rod Rees, "The Use of Perceptual Cues for Number Judgment by Young Children", Child Development, Vol. 45, No. 3, 1974, p. 693-699.

particular number. Several researchers (Piaget,[10] Dodwell,[11] Scandura and McGee,[12] Wallace,[13] and Wohlwill and Lowe[14]) have taken an interest in the child's use of counting and his ability to conserve. Gréco et al.[15] have examined the child's use of counting more specifically. They studied the child's use of particular counting names to represent specific numbers.

Wohlwill and Lowe,[16] and Siegel,[17] have studied the ability of the child to conserve using standard numerals. Specifically, Siegel examined the child's understanding of the first three real numbers (one, two, three), using the standard numerals: 1, 2, 3. On the other hand, Wohlwill and Lowe studied the child's use of the numerals: 6, 7, 8.

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10 Jean Piaget, The Child's Conception of Number, p. 61-64.

11 P. C. Dodwell, Op. Cit., p. 203.

12 J. M. Scandura and Robert McGee, "An Exploratory Investigation of Basic Mathematical Abilities in Kindergarten Children", Educational Studies in Mathematics, Dordrecht, Holland, D. Reidel Publishing Co., Vol. 4, No. 3, 1972, p. 331-345.

13 J. G. Wallace, "Some Issues Raised by a Non-verbal Test of Number Concepts", Educational Review, Vol. 18, No. 2, 1966, p. 122-135.

14 J. F. Wohlwill and R. C. Lowe, "Experimental Analysis of the Development of the Conservation of Number", Child Development, Vol. 33, No. 1, 1962, p. 153-167.

15 P. Gréco, J. B. Grize, S. Papert, and J. Piaget, "Problêmes de la construction du nombre", Etudes d'épistémologie génétique, Paris, Presses Universitaires, Vol. 11, 1960, p. 29-30, cited in John H. Flavell, The Developmental Psychology of Jean Piaget, New York, Van Nostrand, 1963, p. 360.

16 Wohlwill and Lowe, Op. Cit., p. 153-167.

17 Linda S. Siegel, "Heterogeneity and Spatial Factors as Determinants of Numeration Ability", Child Development, Vol. 45, No. 2, 1974, p. 532-534.

The child's understanding of larger numbers represented by standard numerals (e.g. 384) has been studied by Smith.[18] Smith's study differs from the previous studies in another way than just number size. The previous studies have been based on Piaget's theory of conservation of number. If the child conserved number, he was considered to have formed the concept of number. Smith, however, has made no reference to the child's ability to conserve as the criterion for the formation of larger number concepts. Furthermore, his study is essentially an empirical study without any theoretical basis.

A third broad category of number representation is the "pictorial" mode. This mode uses sets of pictured objects to represent number instead of sets of physical objects. The pictorial mode has been used frequently in conservation-based studies. Beard[19] and Dodwell[20] have used pictures of eggs and eggcups. Their use of pictures differs, however. Beard has two sets of cards. One set had pictures of eggcups, while the other set had pictures of eggs. Thus, a set of eggs (or eggcups) could be laid out in a line, or transformed into a cluster quite easily. On the other hand, Dodwell's pictures are static. That is, on one card there is a set of eggs and eggcups in one-one correspondence. Below this picture is

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18 G. W. Smith, Jr., A Study of Constant Errors in Subtraction and in the Application of Selected Principles of the Decimal Numeration System Made by Third and Fourth Grade Students, unpublished doctoral thesis presented to the Graduate Division of Wayne State University, Detroit, Michigan, Ann Arbor, University Microfilms, 1972, p. 1-172.

19 R. M. Beard, "The Order of Concept Development Studies in Two Fields", Educational Review, Vol. 10, No. 2, 1963, p. 105-117.

20 P. Dodwell, "Children's Understanding of Number Concepts: Characteristics of an Individual and of a Group Test", Canadian Journal of Psychology, Vol. 15, No. 1, 1961, p. 29-36.

another. In it, the cups are in the same position but the set of eggs is clustered. Thus, Dodwell indicates a transformation, but the child does not see it. Hence, his pictorial mode of representation is static as are the examples that follow.

The majority of researchers using the pictorial mode of representation have used dots. (Beilin,[21] Brainerd,[22] Dodwell,[23] Lawson _et al._,[24] Stock _et al._,[25] Siegel,[26] Wallace[27] and Zimiles.[28]) The number of dots, and the color of dots used, have varied somewhat in the different conservation studies.

Besides using dots, Siegel[29] and Wohlwill[30] have used other shapes for variety. Siegel used different colored plane-shaped geometrical designs, while Wohlwill used bars, triangles and squares.

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21 Beilin, _Op. Cit._, p. 420-423.

22 Charles J. Brainerd, "The Origins of Number Concepts", _Scientific American_, Vol. 228, No. 3, 1973, p. 101-109.

23 Dodwell, _Op. Cit._, p. 33.

24 Glen Lawson, Jonathan Baron, Linda Siegel, "The Role of Number and Length Cues in Children's Qualitative Judgments", _Child Development_, Vol. 45, No. 3, 1974, p. 731-736.

25 William Stock and June Flora, "Cardination and Ordination Learning in Young Children", paper presented to the American Educational Research Association, 1975, 20 p.

26 Linda Siegel, _Op. Cit._, p. 532-533.

27 Wallace, _Op. Cit._, p. 125-126.

28 Zimiles, _Op. Cit._, p. 11.

29 Linda Siegel, _Op. Cit._, p. 532-533.

30 Joachim F. Wohlwill, "A Study of the Development of the Number Concept by Scalogram Analysis", _Journal of Genetic Psychology_, Vol. 97, 2nd Half, December 1960, p. 345-377.

In his study of the child's understanding of larger numbers, Smith[31] also used a pictorial mode. In some pictures, he also used dots. However, he could not just place his sets of dots in lines or clusters as the above researchers had. Representing larger numbers, he had to organize his dots so that the child could easily determine the represented number. The standard numerals belong to a decimal numeration system. Hence, Smith has organized the dots into groups of tens and ones. An example is shown in Figure 2.

Smith has used other pictures to represent larger numbers. He has used pictures of dimes and pennies, and bundles of sticks. To represent numbers in the hundreds, he has used bundles of sticks (ten in a bundle) which were, in turn, packed into boxes (ten bundles to a box). Drawing pictures of bundles of sticks, and boxes full of bundles, requires time, patience, and drawing skill. Larger numbers are more easily represented using drawings of place-value boxes (see Figure 3c) or drawings of an abacus (see Figure 3b). In both of these representations, Smith indicated the number of units (e.g. one, tens, hundreds etc.) by using particular markers. With the place-value boxes, sticks were drawn inside the boxes, while in the abacus pictures, markers were drawn on the place-value wires.

Smith[32] has used two other modes of representation of number. They are the "numerical" mode and the "verbal" mode. The following

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31 G. W. Smith, <u>Op. Cit.</u>, p. 157-158.

32 <u>Ibid.</u>, p. 159-166.

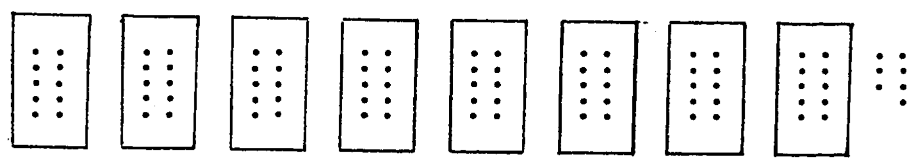

Figure 2.- Representation of a Larger Number Using Groups of Dots[33]

33 Ibid., p. 157.

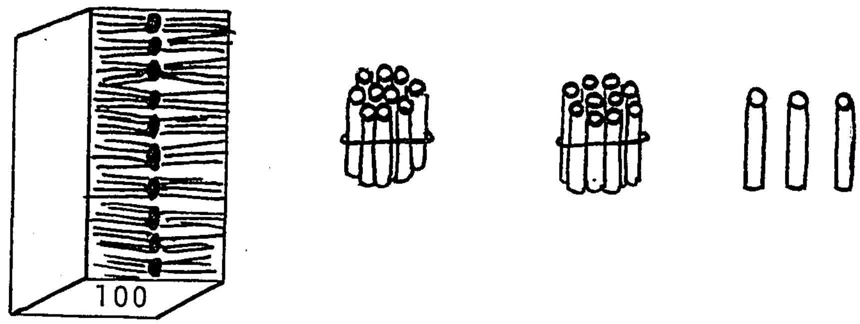


Figure 3a

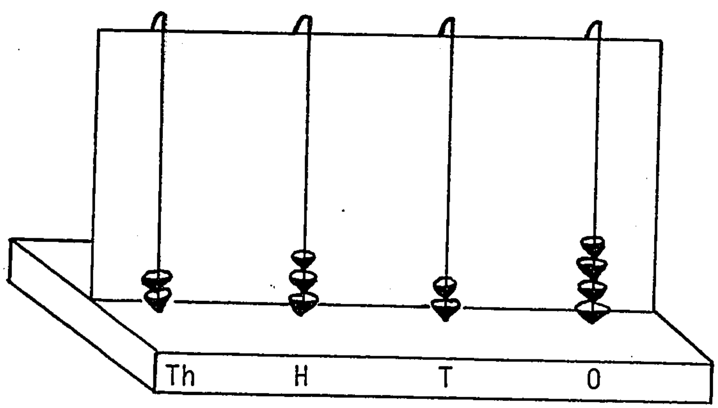


Figure 3b

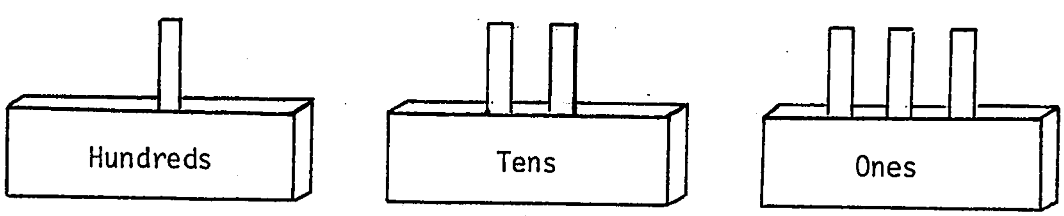


Figure 3c

Figure 3.- Pictorial Representations of Large Numbers[34]

34 Ibid., p. 157-158.

are examples of the numerical mode of representation:

(a) 7000 + 800 + 20 + 8

(b) 8000 + 30 + 4

The next two examples are represented by the verbal mode:

(c) 7 thousands, 8 hundreds, 2 tens, 8 ones

(d) 8 thousands, 0 hundreds, 3 tens, 4 ones

The distinction between these two modes is made by the type of elements used. In the numerical mode, only numerals and numerical signs (the "+" sign) are used. On the other hand, in the verbal mode, numerals and words are used. As well, the verbal mode tends to emphasize the place-value concept of a number, especially as in example (d) where there is a zero frequency of hundreds.

There is another mode of representation which has not been examined directly in previous research into the child's concept of number. This is the internal representation of number where a child uses mental images to represent number. It is a personal mode of number representation, hence, it is a difficult mode to examine. Furthermore, it can only be examined indirectly through another mode of representation of number.

There are, then, five different modes for the representation of number which have been used in previous research. They are: the material mode, the counting name mode, the pictorial mode, the numerical mode, and the verbal mode. The material mode is limited in its use. It is suitable for the representation of small numbers, whereas, larger numbers are more easily represented using the other four modes. The pictorial, numerical, and verbal modes may also be organized to stress the decimal numeration system.

One further mode, the internal representation mode, has not been examined directly in previous research. It is a personal mode which can only be examined in reference to other modes of number representation.

2. The Correspondence Function

Whenever a number is represented, it is put in correspondence with another representation of number. The child may think of the number "seven", and set down seven counters. Thus, in this case, there is a correspondence between an internal and a material representation of number.

In testing or learning situations, the type of correspondence described above is not used. In these situations, the experimenter must be able to ascertain whether the child can make a correct correspondence. This is impossible to determine whenever one representation is an internal representation.

Previous researchers have used a variety of different types of correspondences. The major types of correspondences used are: the material-material type, the counting name-pictorial type, the counting name-numerical type and the counting name-verbal type.

An explanation of each of these types of correspondence follows. In each explanation, there is a description of the type of correspondence tasks used, a discussion of the transformations possible with the correspondence, an examination of the basis on which the child conserves with a particular correspondence, and an examination of the décalages which have occurred with the correspondence type.

It must be noted here, that the majority of researchers who have studied the child's conception of number were concerned only with the operative aspect of cognition. Therefore, they used different modes of representation and different types of correspondences without analyzing the effects such changes would make. Hence, the data reported in this literature review concerning different correspondences had to be derived from data presented in studies designed to test other hypotheses.

i. The Material-Material Correspondence

A material-material correspondence is a correspondence which is made between two material representations of number. This is the type of correspondence Piaget has used primarily. Generally, in this type of correspondence, one set of objects is a model set. The child may be asked to set out a second set with as many objects as the first set has. Sometimes two sets are already set out in one-one correspondence, and the child can change either set if he feels that they are unequal. After establishing a prior equality, the child observes the transformation of one set, and is again asked about the equality of the sets.

The majority of researchers examining the child's conception of number have used the material-material type of correspondence. Hood[35] and Dodwell[36] replicated Piaget's studies as closely as possible.

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35 Hood, Op. Cit., p. 275, 276.

36 P. C. Dodwell, "Children's Understanding of Number and Related Concepts", p. 195.

Rothenberg,[37] Rothenberg and Courtney,[38] Scandura and McGee,[39] and Pufall et al.[40] have presented children with tasks similar to Piaget's task, although the objects used in making the material representations were different.

Zimiles[41] varied his tasks in another way. Instead of only using equal sets and transforming one, he used unequal sets and questioned the child about the equality of the sets before and after a transformation. Beilin[42] also had another variation. Instead of using only two lines, he had three lines of corks. The centre line was on a frame which enabled the row of corks to be stretched or contracted. As well, two rows were equal (one of which was the centre line), and one unequal. Thus, he was able to look at equal and unequal sets simultaneously.

Some researchers have presented tasks using a material-material correspondence without a transformation. That is, the child was shown two sets of counters and had to judge their equality. Usually, the sets were prearranged so that they were spatially different. As well, the

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37 Rothenberg, Op. Cit., p. 387.

38 Rothenberg and Courtney, Op. Cit., p. 494-495.

39 Scandura and McGee, Op. Cit., p. 333-334.

40 Peter B. Pufall, Robert E. Shaw, Ann Syrdal-Lasky, "Development of Number Conservation: An Examination of Some Predictions from Piaget's Stage Analysis and Equilibration Model", Child Development, Vol. 44, No. 1, 1973, p. 21-27.

41 Zimiles, Op. Cit., p. 11-14.

42 Beilin, Op. Cit., p. 421.

two sets were not always equal in number. Beard,[43] Beilin,[44] Lapointe and O'Donnell,[45] Pufall and Shaw,[46] and Smithers et al.[47] have presented tasks using such variations to the material-material type of correspondence.

When a material-material correspondence is used, there are three types of transformations which may be performed. There are: spatial transformations, subgrouping transformations, and numerical transformations.

The transformation Piaget has used most frequently may be called a "spatial"[48] transformation. That is, one set may be arranged spatially to form a cluster, a circle, a triangle etc. Peters and Rubin[49] performed three specific types of spatial transformations. They referred to them as linear, horizontal and vertical transformations.

In the linear transformation, the one set was transformed in ways that the resulting set was still a straight line parallel to the second set.

With horizontal transformations, one set was transformed so that the resulting set was in some respect, at right angles to the second set.

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43 Beard, Op. Cit., p. 107-109.

44 Beilin, Op. Cit., p. 421.

45 Lapointe and O'Donnell, Op. Cit., p. 423.

46 Pufall and Shaw, Op. Cit., p. 23.

47 Smithers et al., Op. Cit., p. 694.

48 Rothenberg, Op. Cit., p. 387-390.

49 D. L. Peters and Kenneth Rubin, "The Effects of Cued Materials and Transformation Variations in Conservation of Number Performance", Alberta Journal of Educational Research, Vol. 15, No. 1, March 1969, p. 47-56.

In vertical transformations, the elements of the transformed set were stacked in piles opposite the second set.

In all spatial transformations, the number of elements in the set is unchanged.

The second type of transformation which may be performed may be referred to as a "subgrouping" transformation. This is a type of transformation in which a set is divided into subsets. Piaget[50] used a subgrouping transformation. In Piaget's task the child was shown two sets of candies set in one-one correspondence. After the child had agreed that the sets were equal, one set was divided into two subsets. One subset had one candy, the other subset had seven. The second set was divided into subsets. However, these subsets both contained four sweets. The child was then to judge the equality of the sets. In subgrouping transformations, the total number of objects in the set is unchanged.

The third type of transformation, the numerical transformation, has been identified by Wallace.[51] To perform the transformation, the experimenter adds objects to the sets, after the child has previously judged the equality of the sets. There may be an addition of equal numbers of objects to both sets, as well as an addition of unequal numbers of objects to the sets. The child is then asked to judge the equality of the resulting sets. These transformations of the number of elements of a set are called "numerical" transformations.

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50 Piaget, The Child's Conception of Number, p. 185-202.

51 J. G. Wallace, Op. Cit., p. 125-126.

To conserve a material-material correspondence a child must act. There are different actions or operations the child may perform. One action is to make a one-one correspondence between two sets. When the sets are lined up opposite each other, this is easy. It is less easy when one or both the sets are clustered. If there is a tranformation of one set, and the child sees it, he can also conserve by reversing his thoughts about the seen tranformation. That is, he reasons that if the sets were equal before the transformation, they would be if the transformations were reversed. Hence, they are equal even if they do not appear to be.

Another procedure a child may use is to make compensations. He reasons that a change in the length of the sets is compensated by a change in the density. Hence, the sets are still equal.

Conservation judgments may be made on the basis of any of these three operations, whether or not the child has any facility with counting and the counting-names to describe a set. To use counting as a basis for conservation, the child must master the material-counting name correspondence.

Within the material-material type of correspondence, several décalages have been observed. A vertical décalage has been observed as a result of differences between children in the type of operative thinking structures they have formed. The other observed décalages were horizontal décalages. Such décalages occurred when different types of objects were used to represent number and when different sized sets of objects were used in the tasks. Other horizontal décalages resulted between tasks with, and without, the child observing a transformation, and between tasks in which different transformations were performed. Finally, a horizontal décalage was observed between tasks to examine the child's ability to conserve the equality, and the inequality, of sets of objects.

The vertical décalage arises due to differences in the child's operative thinking structure over time. Piaget states: "Psychologically to make a correspondence is merely to systematize judgments of resemblance and difference."[52] At the preoperational stage, a child has only a one-dimensional thinking structure. Hence, he can only consider thoughts successively. As a result, he cannot systematize judgments of resemblance and difference. He can, however, make a global correspondence. That is, the set which he makes resembles the model set in only one aspect. His line, for instance, is only like the model set with respect to length.

At the concrete operational stage, the child is capable of two-dimensional thought. Thus, he can consider simultaneously both the differences in the lengths and the densities of two materially represented sets. Therefore, he has an operative thinking structure which enables him to systematize judgments of resemblance and difference to make a true correspondence.

Since different thinking structures are involved in making global and true correspondences, there is a vertical décalage between the child's ability to perform the two tasks.

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52 Jean Piaget, The Child's Conception of Number, p. 88.

Toward the end of the preoperational stage, the child's global correspondences are replaced by more precise but not yet true correspondences. The child at this level makes a correspondence between two sets. However, the child thinks that both the number of elements, and their arrangement, is important. If the spatial configuration of one set is changed, the child considers that the correspondence is thus destroyed. In the presentation of his data, Hood[53] has taken this semilogical stage into consideration. Consequently, one is able to observe the vertical dēcalage between Stage I (the global correspondence stage) and Stage II (the semilogical stage), and between Stage II and Stage III (the concrete operational stage) across different age groups.

A horizontal dēcalage arises when different types of objects are used to represent number. Piaget[54] noted children could conserve a correspondence between a set of eggs and eggcups before they could conserve a correspondence between two sets of plain counters. Hood[54] verified Piaget's findings. Specifically, he noted 100% of children between the ages of 7.1 and 8 years could conserve with eggs and eggcups. However, 100% mastery of conservation with plain counters was not achieved until children were between 8.1-9 years of age.

Dodwell's[56] findings also confirm the existence of his horizontal dēcalage. He found that 60% of a group of children aged 5 years-10 months

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53 Hood, Op. Cit., p. 277.

54 Jean Piaget, The Child's Conception of Number, p. 41-64.

55 Hood, Op. Cit., p. 277.

56 Dodwell, "Children's Understanding of Number and Related Concepts", p. 201.

could conserve with an egg-eggcup correspondence. None of this same group could conserve with plain counters. Scandura[57] found that 81% of a group of kindergarten children could conserve with a correspondence between a set of dolls and a set of candies. With a correspondence between sets of plain counters, only 25% of the group conserved.

The existence of a horizontal dĕcalage resulting from the type of objects used to represent number, therefore, is confirmed in the literature. To describe the type of correspondence which gives rise to such a dĕcalage, a specific term has been used. Piaget[58] refers to such a correspondence as a "provoking" correspondence. That is, there exists some functional relationship between the objects which facilitates the making of the correspondence. Since the correspondence is facilitated, the term "provoking" is applied to such a correspondence.

Another horizontal dĕcalage results from differences in the number of elements in the sets. Zimiles[59] looked at conservation with correspondences between sets containing three and four elements, and correspondences between sets containing seven and nine elements. A group of Kindergarten and Grade One children were given the tasks. Of the group, 66.3% conserved on tasks with small sets, while 60.7% conserved on tasks with larger numbers in the sets.

Smithers, Smiley and Rees[60] studied children's ability to make correspondences between sets of 3-5 objects and sets of 7-9 objects. They

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57 Scandura and McGee, Op. Cit., p. 336-343.

58 Jean Piaget, The Child's Conception of Number, p. 42.

59 Zimiles, Op. Cit., p. 22.

60 Smithers, Smiley and Rees, Op. Cit., p. 698.

noted that younger children were only successful with correspondences between small sets of objects. Older children, however, were successful with both large and small sets.

Piaget has not examined this problem to any extent. He has, nevertheless, used sets having four or more objects primarily. Like Winer,[61] Piaget[62] states that the child can recognize the first few numbers through perception alone, without any understanding of the concept of number. Furthermore, a spatial transformation of very small sets (e.g. 2 or 3 elements) cannot really change the appearance of the set much. Hence, the child will still act as if he understands the concept of number. Winer[63] found that children conserved sets of 2-3 objects earlier than they conserved sets of 5-6 objects. Thus, a second horizontal décalage resulting from the number of elements in a set has been observed and substantiated.

The third horizontal décalage which has been noted in material-material correspondence results from differences in a child's ability to conserve on tasks with and without transformations.

Beilin[64] investigated the effects of seeing or not seeing a transformation on the child's ability to conserve both area and number. He expected that the child would find tasks with transformations more

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61 Gerald Winer, "Conservation of Different Quantities Among Preschool Children", Child Development, Vol. 45, No. 3, 1974, p. 841.

62 Piaget, The Child's Conception of Number, p. 67.

63 Winer, Op. Cit., p. 839-842.

64 Beilin, Op. Cit., p. 421-424.

difficult than tasks without transformations. He felt that asking the same question before and after the transformation would tend to confuse the child.[65] He found the reverse. Seeing the transformation, the child was able to conserve both area and number more easily. With respect to number specifically, Beilin[66] found that 32.5% of kindergarten children conserved on tasks with transformations. Only 8.8% of the children conserved on tasks without transformations.

Pufall _et al._[67] presented tasks, with and without showing transformations, to a group of children between the ages of 2 years 11 months and 5 years. They also observed a horizontal décalage between the two types of tasks. Of this group of children, 14.8% conserved consistently when they saw one set transformed. When the transformation was not seen, 9.3% of the group conserved on the task.

The existence of a horizontal décalage resulting from seeing or not seeing the transformation has been confirmed. Beilin has also tried to account for it. He suggested that observing a transformation, the child is better able to invoke strategies to overcome the perceived differences between the sets.[68] Not observing a transformation, the child has to decide upon a suitable strategy on his own. Hence, a décalage occurs.

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65 _Ibid._, p. 417.

66 _Ibid._, p. 422.

67 Pufall _et al._, _Op. Cit._, p. 24.

68 Beilin, _Op. Cit._, p. 422.

The types of transformations a child sees may affect his ability to conserve. In Hood's[69] presentation of the data from several tasks, it is possible to compare tasks with a spatial transformation (clustering) and tasks with a subgrouping transformation. From the age 5.1 years to 8.0 years, the task with a spatial transformation is somewhat easier than the task with a subgrouping transformation. However, both tasks were completely mastered by the children between 8.1-9.0 years of age.

There was, however, an unusual finding with reference to this horizontal dēcalage. One of the groups of children tested was between 4.1 and 5.0 years of age. Of this group, 24% were at Stage III-- the conservation stage with the subgrouping transformation task. On the other hand, only 4% of this group conserved on the task with the spatial transformation. Hood[70] also indicated the percentage of the group who were at Stage II-- the semilogical stage, on the tasks. When both stages are considered, there is no difference between the tasks for this age level. Thus, the unusually large percentage of conservers for the task with a subgrouping transformation may be the result of a problem in classifying children correctly, a problem in tabulating data, or just a result of chance. Considering the whole age range from 4.1-9.0 years of age, one cannot conclude if a dēcalage exists. Theoretically, a horizontal dēcalage is not expected. A subgrouping transformation is, in effect, just a particular type of spatial transformation.

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69 Hood, Op. Cit., p. 277.

70 Ibid., p. 277.

Peters et al.[71] have examined the child's ability to conserve with different types of spatial transformations. Specifically, they presented tasks with linear, horizontal and vertical transformations. They concluded that there was no difference in the child's ability to conserve on tasks with these different spatial transformations. In addition, they found that the test results for the different spatial transformations were found to be highly correlated. They interpreted this to indicate a stability in performance on tasks with different transformations.

Thus, on the basis of theory and the study by Peters et al., a horizontal décalage is not expected between tasks with spatial and sub-grouping transformations. However, considering Hood's results, one would require further research to resolve the issue.

Rothenberg[72] compared material-material correspondence tasks with spatial transformations and those with numerical transformations. Numerical transformations were transformations which resulted in a change in the number of elements in the material sets. In tasks with an "equal addition" numerical transformation, equal numbers of counters were added to each set. In tasks with "unequal addition" numerical transformations, unequal numbers of counters were added to each set.

Between tasks with spatial transformations and tasks with equal addition numerical transformations Rothenberg[73] found no décalage.

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71 Peters et al., Op. Cit., p. 55.

72 Rothenberg, Op. Cit., p. 383-404.

73 Ibid., p. 397.

This was confirmed in a second study by Rothenberg and Courtney.[74] A significant horizontal décalage was found, however, between tasks with a spatial transormation and tasks with an unequal addition transformation. Similarly, Rothenberg found a significant difference between tasks with the two different types of numerical transformations.[75] That is, tasks with equal addition transformations were more difficult than tasks with unequal addition transformations. In the study by Rothenberg and Courtney,[76] this décalage was again noted, but it was not found to be significantly different. They found 9.4% of the preschool group of children conserved on tasks with spatial transformations, 7.7% conserved on tasks with equal addition transformations and 17.1% conserved on tasks with unequal addition transformations.

When there are unequal numbers of counters added to two sets of counters, in effect, the child must judge the inequality of the sets. Beilin has referred to this type of task as a conservation of inequality task. Similarly, adding equal numbers of counters to both sets is, in fact, still a conservation task in the sense Piaget used. That is, the task still involves a conservation of the equality of sets. Thus, Rothenberg,[77] and Rothenberg and Courtney[78] have presented data which suggests that a horizontal décalage exists between conservation of equality and conservation of inequality tasks.

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74 Rothenberg and Courtney, Op. Cit., p. 495-497.
75 Rothenberg, Op. Cit., p. 397.
76 Rothenberg and Courtney, Op. Cit., p. 495-497.
77 Rothenberg, Op. Cit., p. 383-404.
78 Rothenberg and Courtney, Op. Cit., p. 493-502.

Logically, such a dĕcalage is not expected. If a child can judge the inequality of the number of elements in two sets, he must understand what the equality of the two sets means. If he does not understand equality of the number of sets, the child is at the preoperational stage of development, and therefore must be basing his judgment on the perceptual aspects of the two sets.

Some researchers have presented data which may answer questions concerning the existence of a horizontal dĕcalage between inequality and equality conservation tasks. Beilin[79] looked at two conservation of inequality tasks. One he called the classic inequality task. In this task there are two rows of counters. They differed in both the number of counters and in length. In a second task, a conservation of inequality task, the lengths of the two rows were equal, but there were different numbers in each set. Of the group of kindergarten children, 64% conserved on the classic inequality task, whereas only 39% conserved on the conservation of inequality tasks. Thus, on tasks where both the length and the number of elements in the set may be larger than those of a second set, the child is more successful. This is to be expected. For example, a longer, more numerous row looks larger perceptually, as well as being larger in fact. Hence, a child may be making perceptual judgments which just happen to coincide with the numerical judgment.

Pufall <u>et al.</u>[80] have examined this problem more closely. In two tasks, unequal numbers of elements were added to the sets. The result

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79 Beilin, <u>Op. Cit.</u>, p. 421-424.

80 Pufall <u>et al.</u>, <u>Op. Cit.</u>, p. 21-27.

was that the longer, more numerous line was opposite a shorter, less numerous line. In one of these tasks, 78% of the children were successful, while on the second task, 91% were successful.

In another task, there was an unequal addition to the sets, but the more numerous row was shorter than the less numerous row. In this situation, 32% of the group conserved. In comparison, Pufall et al.[81] found 15% conserved on the regular conservation of equality task.

From these results, it is difficult to interpret whether a horizontal décalage really does exist between conservation of equality and inequality tasks. However, if one is to study this problem, it is necessary to present tasks in which the perceptual judgment is not the same as the numerical judgment of the numerosity of the sets.

Thus, the type of correspondence used most frequently by researchers to examine the child's formation of number concepts was the material-material correspondence. In these tasks, sets of physical objects are used to represent a particular number.

In administering the task, the researcher presents the child with two sets of objects laid out in a one-one correspondence. The child is questioned concerning the equality of the sets. Then, after one set is transformed by a spatial, subgrouping or numerical transformation, the child is questioned concerning the equality of the sets. Variations of this basic material-material correspondence task have been discussed.

To solve these tasks, a child requires a two-dimensional concrete operational thinking structure to co-ordinate the similarities and differences between the sets.

81 Ibid., p. 24.

With reference to the material-material correspondence, there is evidence which supports the existence of a vertical décalage between tasks which require different operative thinking structures. Horizontal décalages were observed between tasks in which different objects were used to represent number, between tasks involving sets of different sizes, and between tasks in which transformations are seen as opposed to unseen.

Other horizontal décalages have been reported. There are décalages between tasks with different types of transformations, and décalages between conservation of inequality and equality tasks. The observations of these horizontal décalages have not been consistent and methodological problems have been noted. Hence, further studies are required to support the existence of these décalages.

### ii. The Material-Counting Name Correspondence

Piaget claimed that the ability to make a material-counting name type of correspondence followed the development of an ability to make a material-material correspondence.[82] Gréco _et al._,[83] and Piaget[84] have examined the child's ability to make the material-counting name type of correspondence. In doing so, their technique was to have the child count the number of elements in each of two sets. From this, the child was

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82 Jean Piaget, _The Child's Conception of Number_, p. 74.

83 Gréco _et al._, _Op. Cit._, p. 360.

84 Piaget, _Op. Cit._, p. 61-64.

asked to judge the equality of the sets. Then, one set was transformed. The child was allowed to recount the elements in the set and make a judgment on the equality of the sets.

Wohlwill and Lowe[85] used this type of correspondence with some variations. In their task, the child was shown a set of stars fastened on a frame which enabled the line to be expanded or contracted. In front of the child, there were three cards with a different numeral on each card (e.g. 6, 7, or 8). The child counted the stars and matched it with the appropriate card. The line of stars was lengthened or shortened, and again the child matched it with the appropriate card.

With this type of correspondence, transformations may only be performed on materially represented sets. That is, the standard numeral remains unchanged.

The transformations which may be performed on the material set are the same as those for material-material correspondences. That is, spatial transformations, subgrouping transformations, and numerical transformations may be performed. However, only spatial transformations have been discussed in the literature.

To conserve a material-counting name correspondence, the child performs actions. As with material-material correspondence, the child must make a one-one correspondence. With these tasks the correspondence is between objects and the counting names, whereas with the previous tasks, the correspondence was between the objects in the different sets.

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85 Wohlwill and Lowe, Op. Cit., p. 157-158.

After the transformation of the material set, the child can conserve the correspondence on the basis of reversibility and compensation, as is possible after transformations in tasks with material-material correspondences. The child, however, cannot reverse his thought or make compensations until he has formed a two-dimensional concrete operative thinking structure.

With respect to the material-counting name correspondence, one vertical décalage has been noted. Gréco et al[86] observed that a child can conserve the name of the numeral to describe the numerosity of a set before he can conserve the quantity which the term represents. Thus, a child may be able to count the number of elements in each of two equal sets, yet claim that a clustered set is larger than a line of counters.

Wohlwill and Lowe[87] were primarily concerned about training procedures. However, they acknowledged that there was a considerable décalage between the child's ability to enumerate collections by counting, and then, on this basis, to conserve the numerosity of the set after a transformation.

This vertical décalage is similar to the vertical décalage between the semilogical material-material correspondence and the true material-material correspondence. Both the semilogical material-material correspondence and the conservation of the name of the numeral which describes the numerosity of the set are prefigurations to the true correspondences.

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86 Gréco et al., Op. Cit., p. 360.

87 Wohlwill and Lowe, Op. Cit., p. 153-157.

These prefigurations indicate that the child has internalized these processes. However, he has not integrated them into his concrete operational thinking structure. Hence, he can neither represent number properly, nor can he logically judge the equality of the represented numbers. Integrating these representational processes into the thinking structure requires some time. Thus, there is a décalage which is observed between the semilogical material-material correspondence and the true material-material correspondence, as well as between rote and true counting as shown with these material-counting name correspondence tasks.

In conclusion, the material-counting name correspondence has not been used frequently in studies of the child's formation of number concepts. Furthermore, researchers who have used this correspondence have developed tasks which are quite differently administered. The tasks, however, are similar in that a child does observe a transformation of the material set.

As with the material-material correspondence, the child requires a two-dimensional concrete operational thinking structure to succeed.

One vertical décalage was observed between the child's ability to make a concrete operational correspondence. That is, children were observed to count accurately before they realized that counting was an enumeration process as such.

iii. The Material-Pictorial Correspondence

Wallace[88] is one of the few researchers to study the child's ability to make a material-pictorial correspondence. His task was as follows. He had a set of three choice cards having six, seven, or eight dots. He placed a set of counters in front of the child in an arrangement that was identical to one of the choice cards. The child was asked to select the matching choice card. Then, the set of counters was transformed by either a spatial or a numerical transformation. The child was then to select the choice card which showed the same number of dots. To make the task provoking, he used choice cards having pictures of eggcups, and the material set was a set of toy eggs.

In the material-pictorial correspondence, only the material representation can be transformed in front of the child. Of the three transformations discussed previously, Wallace used two-- the spatial transformation and the numerical. To perform the numerical transformation, he either added one object or removed (subtracted) one object from the material set.

The actions required of the child to make a material-pictorial correspondence are similar to the actions required in the previous types of correspondences. The initial one-one correspondence is facilitated by the experimenter arranging the material set in the same pattern as the pictorial set of objects. After the spatial transformation of the material set, the child may remake the one-one correspondence. However,

88 Wallace, Op. Cit., p. 124-126.

this correspondence is more difficult as the spatial arrangements of the sets differ. Similarly, with numerical transformations, the child makes correspondences between the pictorial and material sets until he finds matching sets. In both transformations, the child may use counting to compare sets.

Children who thoroughly understand the numbers involved in these tasks may bypass the correspondence and counting, to base their conservation responses on reversibility or compensation. As with the material-material and material-counting name correspondence tasks, these tasks discussed here also require a two-dimensional concrete operational thinking structure.

Only one horizontal dêcalage was noted with the material-pictorial correspondence. It was the same provoking type dêcalage observed with the material-material correspondence tasks. Of the children given the provoking and unprovoking tasks with a spatial transformation, 54% answered the provoking task correctly, while 47% answered the unprovoking task correctly.[89]

Wallace did not, however, observe the provoking dêcalage with the numerical transformation. This may be the result of methodological problems with this type of task itself. There were three choice cards from which the child had to select one to match the material set. With the numerical transformation of the material set (either adding or subtracting one object), the resultant set was different from the original

89 Ibid., p. 124-126.

set. Thus, to the child the set appeared different, and in fact, was different. Therefore, he only had two cards from which to select. That is, if the set equalled the set of seven dots before the transformation, then the answer must be either the six or eight card. Even if the child did not understand number, he had a 50% chance of being correct. Wallace reported that 57% of the children conserved on provoking items with a numerical transformation, and 55% conserved on the unprovoking tasks. Thus, there was only a slight difference between the tasks. Further research is required to determine if this décalage exists with numerical transformations.

Researchers have made little use of material-pictorial correspondence tasks. Essentially, the task is administered the same way a material-material task is administered. Similar transformations may also be used in the two types of correspondence.

As with the previous two correspondences, the child requires a two-dimensional concrete operational thinking structure to solve material-pictorial correspondence tasks.

One horizontal décalage was observed between tasks with provoking materials and tasks with neutral objects in the sets. This décalage was observed only when spatial transformations were performed on the material set. In tasks with numerical transformations, methodological problems were identified which may have obscured the existence of this horizontal décalage.

### iv. The Pictorial-Pictorial Correspondence

The second most frequently used type of correspondence, after the material-material correspondence, is the pictorial-pictorial correspondence.

In the majority of the pictorial-pictorial tasks, dots were used (Lawson et al.,[90] Wallace,[91] Dodwell,[92] Zimiles,[93] Stock et al.,[94] and Brainerd[95]). Beard and Dodwell also used pictures of eggs and eggcups.

In most tasks, the child was shown two sets of pictures and was questioned concerning the equality or inequality of the sets. Since the sets were pictured, most researchers could not transform the sets. Beard,[96] however, had his pictures of eggs and eggcups on individual cards. Thus, he could cluster one set. Dodwell[97] could not perform transformations on his sets. Therefore, he showed the child pictures of the sets before and after an implied transformation.

Usually the two sets of dots which the child had to consider were separated from each other. Stock et al.[98] used only one line of dots.

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90 Lawson et al., Op. Cit., p. 732.

91 Wallace, Op. Cit., p. 124-126.

92 Dodwell, "Children's Understanding of Number and Related Concepts", Characteristics of an Individual and Group Test, Op. Cit., p. 33-35.

93 Zimiles, Op. Cit., p. 11.

94 Stock et al., Op. Cit., p. 7-8.

95 Brainerd, Op. Cit., p. 104.

96 Beard, Op. Cit., p. 107-108.

97 Dodwell, Ibid., p. 33.

98 Stock et al., Op. Cit., p. 7-8.

To indicate different sets, different colored dots were used. The different colored dots were arranged so that the two colors alternated. Specifically, if one set of dots was red and the other blue, the line would be arranged with every other dot red, with blue dots in between.

To conserve on pictorial-pictorial correspondence tasks, different actions could be performed according to the particular task.

With Beard's task, the child had several alternative actions. He could make a one-one correspondence, or use counting. Seeing a transformation, he could also use reversibility and compensation just as a child who is presented with material-material correspondence tasks. With Dodwell's task, the child would have basically the same alternative actions which he could perform.

Without seeing a transformation, or an implied transformation, the child could only make a one-one correspondence, or use counting. The arrangement of the different colored sets of dots into one line may make it easier for the child to make a one-one correspondence between the sets.

Brainerd[99] disallowed counting in his tasks. Thus, the child could only conserve on the basis of one-one correspondence.

In these tasks, there are differences in two variables which the child must co-ordinate. In some tasks, there are differences in the lengths and in the densities of the elements in the two sets. In others, there are differences in the lengths and shapes of the sets. Therefore, a two-dimensional concrete operational thinking structure is also required for these pictorial-pictorial correspondence tasks.

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99 Brainerd, Op. Cit., p. 104.

Several possible décalages were noted in the literature review. These possible décalages were the result of differences in the types of correspondence tasks, differences between numerically large and small sets, differences in the arrangement of sets in one or two lines, and differences between the groups studied.

Wallace has presented information concerning possible horizontal décalages between two different types of correspondence tasks, the material-material and pictorial-pictorial correspondence tasks. From Table I, it is noted that the material-pictorial correspondence was further classified by the type of transformations performed and the type of objects used. The material-pictorial correspondence with a spatial transformation of unprovoking objects was only slightly easier than the pictorial-pictorial correspondence. The remaining three types of material-pictorial correspondences were easier to perform than the pictorial-pictorial correspondence. As stated previously, Wallace had possible methodological problems with his material-pictorial correspondence tasks, particularly the ones with numerical transformations. Thus, while it appears that there may be a horizontal décalage between material-pictorial correspondences, and pictorial-pictorial correspondences, further research is required to support the existence of such a décalage.

Logically, a horizontal décalage between these two types of correspondence is expected. The representational modes involved in the two correspondences are different. A materially represented set is in front of the child, and usually transformed while the child watches. With most pictorially represented sets such transformations are not

Table I.-

Percentage of Conservers on Wallace's[100] Tasks

| | |
|---|---|
| A. Pictorial-Pictorial Correspondences- | 46% |
| B. Material-Pictorial Correspondences | |
| -With Spatial Transformations | |
| of Provoking objects- | 54% |
| of Unprovoking objects- | 47% |
| -With Numerical Transformations | |
| of Provoking objects- | 57% |
| of Unprovoking objects- | 55% |

100 Wallace, Op. Cit., p. 127.

possible. When a material-pictorial correspondence task is presented, the child has four possible actions which he can perform. He can make a one-one correspondence, he can count the sets, and he can base his judgment on reversibility or compensation. When a pictorial-pictorial task is presented, the child can only make a one-one correspondence or count. Because there are these limitations to the pictorial-pictorial correspondence tasks, a décalage is expected between them and material-pictorial correspondence tasks.

A second horizontal décalage was noted in Zimiles' study. Zimiles had conservation tasks with large numbers of dots in the sets (7-9 dots), and tasks with smaller numbers of dots in the pictured sets (3-5 dots). For his group of Kindergarten and Grade One children, he found that 86.3% conserved on the small number tasks, while only 66.3% conserved on the large number tasks.[101]

A similar horizontal décalage was theoretically expected, and observed in the material-material correspondence tasks. Thus, it may be concluded that a horizontal décalage between tasks differing in the size of the sets involved is a phenomenon common to different correspondence tasks.

A third horizontal décalage appears to exist between conservation of equality and conservation of inequality tasks. Brainerd, Stock *et al.*, and Zimiles have all reported observing this décalage.

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101 Zimiles, *Op. Cit.*, p. 11.

Specifically, Brainerd[102] tested a group of children between five and seven years of age. He found that 40.6% conserved on the conservation of inequality tasks, while only 8.3% conserved on the equality tasks.

Stock _et al._,[103] using Brainerd's tests, found similar results with a group of Preschool, Kindergarten and Grade One children. They found that 30% conserved on the inequality tasks, while only 8% conserved on the equality tasks.

Zimiles[104] found that of his group of children, 80.7% conserved on the inequality tasks, while only 71.3% conserved on the equality tasks.

Thus, one might conclude from this evidence found on the pictorial-pictorial correspondence tasks, and the results with the material-material correspondence tasks, that a dēcalage exists between conservation of equality and inequality tasks. This evidence conflicts with theoretically expected results. That is, a child can only understand equality, if he knows inequality. Fortunately, Zimiles presented results from every pictorial-pictorial task in his study. A closer examination of these tasks follows.

In conservation of inequality tasks, three different arrangements of the elements are possible. The more numerous line can be the longer line, the more numerous line can be the shorter line, or the more numerous line can be equal in length to the less numerous line (see Figure 4). Perceptually, the first two arrangements would tend to

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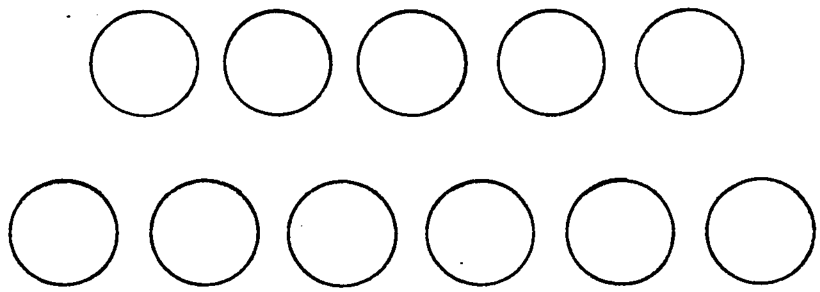
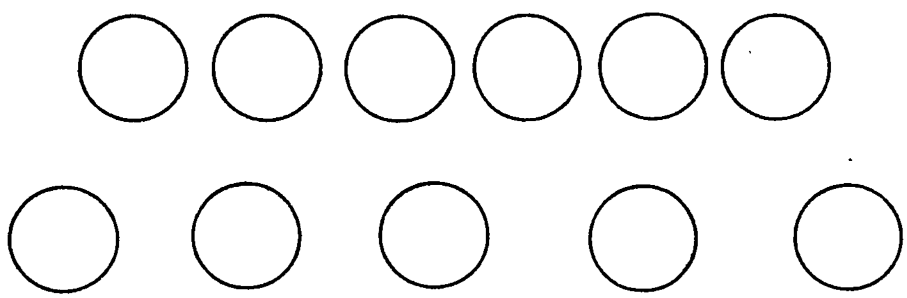
102 Brainerd, _Op. Cit._, p. 107-108.

103 Stock, _et al._, _Op. Cit._, p. 18.

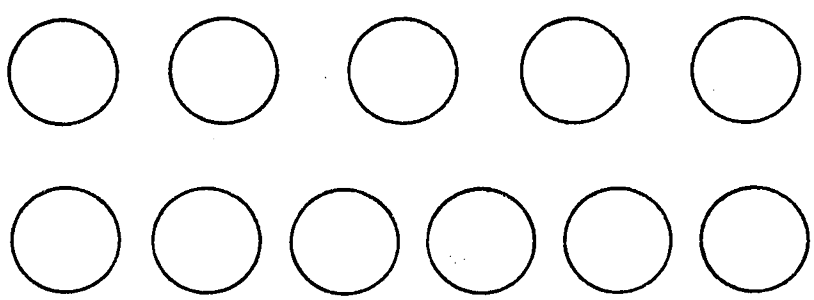
104 Zimiles, _Op. Cit._, p. 30.



(a) Longer, more numerous opposite shorter, less numerous set.



(b) Shorter, more numerous opposite longer, less numerous set.



(c) Equally long but numerically different sets.

Figure 4.- Set Arrangements for Conservation of Inequality

facilitate a correct response. In the first, the larger set looks larger because it is longer. In the second, the larger set looks larger because its elements are more crowded making it appear very dense. In the third arrangement, neither the length nor the density of the larger set is extremely obvious. Hence, of the three, it should be the most difficult arrangement.

From Zimiles' data, the following results were derived.[105] With the first arrangement, 94% of the children conserved. With the second arrangement, 80% conserved, while only 71.7% conserved with the third arrangement.

As expected, the first two arrangements did make the tasks easier. Furthermore, the tasks with the third arrangement were considerably more difficult. In fact, these tasks were as difficult as the conservation of equality tasks on which 71.3% conserved.

The results from the tasks with the third arrangement suggest the solution to the problem involved with inequality tasks. To test for conservation of inequality, one should make the sets look equal. This is done by having the sets equal in length. If this is not done, and the first two arrangements are used, a problem arises. Of the group answering these tasks correctly, not all are true conservers. There are some children at the semilogical stage who are partially making a perceptual judgment, but the perceptual judgment happens to coincide with the numerical judgment. Similarly, in direct one-one correspondence tasks,

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105 Ibid., p. 30.

both semilogical and concrete operational children make correct responses, but for different reasons. However, when the equal sets are made to look unequal by making one set different in length, only operative children conserve. Hence, to examine if a décalage does exist between conservation of equality and inequality, the arrangement of the sets is important. Perceptually, the sets should be arranged to look the reverse of what they are numerically. Sets for conservation of equality should look unequal. Sets for conservation of inequality should look equal.

Incidentally, the differences in the child's ability to succeed at the inequality tasks involving the three different arrangements is a vertical décalage.

Another horizontal décalage was observed between tasks in which the sets of dots were arranged in different ways. Stock et al.[106] had two groups of tasks involving conservation of equality and conservation of inequality. In the one group of tasks, the sets were arranged in two parallel lines. In the second group of tasks, the two sets of dots were arranged in a single line. As described previously, the two sets had different colored dots and the arranged line of dots alternated in color from one to the other. Unequal sets, therefore, could only have an extra element.

In presenting the tasks, the researcher asked the child to judge the equality of the sets. Furthermore, the child was requested not to

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106 Stock et al., Op. Cit., p. 7-8.

use counting in making his decision. In effect, the child could only succeed by making a visual one-one correspondence.

A considerable horizontal dĕcalage was observed between the group of tasks with the sets of dots in two lines, and the group of tasks with the dots in one line. Specifically, for the conservation of equality, 66% conserved on the tasks with the sets in one line, while only 8% conserved on the tasks with the sets in separate lines. For conservation of inequality, 67% conserved on the tasks with the sets in one line, while 30% conserved on tasks with the sets in separate lines.[107] Having both sets in one line facilitated the making of a one-one correspondence. In contrast, having the sets in separate lines perceptually deterred the child from making the correspondence. Thus, a dĕcalage exists between these two types of tasks.

The final dĕcalage for pictorial-pictorial tasks results when different groups of children are tested on the same set of tasks. In the study by Zimiles, there were three different groups.[108] There was one group composed of Kindergarten children between 5.3 and 6.3 years of age. One third of the group came from white middle class schools. The second third of the group were Negro children from slum neighborhoods, while the last third of the children came from lower-middle to middle class Jewish homes.

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107 Ibid., p. 18.

108 Zimiles, Op. Cit., p. 6.

A second group of children were Grade One students between 6.3 and 7.3 years of age. They came from lower-middle to middle class backgrounds. The third group was another group of Grade One students between 6.3 and 7.3 years of age. The socio-economic status of this group was similar to that of the Kindergarten group.

From Zimiles'[109] presentation of his data, it was difficult to see any particular pattern which would support the assumption of a dĕcalage resulting from the group differences in Grade and socio-economic background. Specifically, Zimiles gave the number of conservers and non-conservers on each task for each of the three groups. To make the data more meaningful, the number of conservers was converted to a percentage for each task. Then, the percentages for similar tasks were combined and averaged to yield an overall percentage for a particular type of task. That is, the percentages for all the conservation of equality tasks were combined and averaged to yield an overall percentage of conservers on conservation of equality tasks.

One must be cautioned that the resulting percentages are only very rough estimates. Zimiles' study was not designed for this type of analysis, thus, group sizes are unequal, as are the numbers of similar tasks. The results of this analysis, limited as they are, are shown in Table II.

For every task in Table II the results are similar. The lowest percentage of conservers is in the Kindergarten group. The Grade One

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109 <u>Ibid.</u>, p. 30.

Table II.-

Group Percentages of Conservers on Different Tasks
--A Reorganization of Zimiles' Data

| Type of Tasks | Kindergarten | Grade One (Lower-middle to middle-class) | Grade One |
|---|---|---|---|
| Small Sets | .79 | .85 | .95 |
| Large Sets | .55 | .68 | .76 |
| Equal Sets | .63 | .71 | .80 |
| Unequal Sets | .71 | .81 | .91 |
| Larger Set in the Larger Line | .86 | .97 | .99 |
| Larger Set in the Shorter Line | .72 | .78 | .90 |
| Larger Set equal in length to Smaller | .53 | .78 | .84 |
| Small Unequal Sets | .86 | .89 | .98 |
| Small Equal Sets | .66 | .78 | .90 |
| Large Unequal Sets | .51 | .70 | .74 |
| Large Equal Sets | .60 | .65 | .71 |

group, with a similar socio-economic status among the children as for the Kindergarten group, consistently had the highest percentage of conservers. The Grade One group composed of children from lower-middle to middle-class backgrounds consistently had percentages between the other two groups.

The explanation for such a décalage is complicated. The décalage between the Kindergarten and Grade One groups of children with similar socio-economic backgrounds appears to be the result of maturation and school experience.

Insufficient description of the three groups was provided. Hence, it is impossible to explain why the Grade One group from lower-middle to middle class backgrounds consistently yielded percentages between the other two groups.

Theoretically, the ability to conserve depends upon maturation and experience. Maturation would occur in every socio-economic area. The experiences a child has would, however, differ from area to area. For instance, lower middle to middle class children may spend many inactive hours watching television. Slum area children may not have a television, or if they do, it may not be functioning because their parents cannot afford the repair costs. Hence, they may engage in more active and varied pastimes. Furthermore, Jewish children attending Jewish schools, and middle class children, may have home environments that encourage a variety of activities. According to Piaget, a child understands through his actions. Through perception a child knows without true understanding. Hence, children with more active and varied lifestyles may be expected to be more successful on logico-mathematical tasks than children spending many inactive hours watching a television. Thus, décalages between groups from different environments are expected.

In conclusion, the second most frequently used correspondence is the pictorial-pictorial correspondence. In most of these correspondence tasks, there are pictures of two sets of objects (usually dots). The child is questioned concerning the equality of the two sets. It is impossible to transform static pictures of sets, hence, researchers using this mode present the child with several of these tasks, with the sets in various tasks arranged differently.

To conserve, the child has to make a visual one-one correspondence between the sets, or use counting correctly. The child requires a two-dimensional concrete operational thinking structure to succeed at these tasks.

Several horizontal décalages were observed. First, there was some evidence to support the existence of a horizontal décalage between tasks with material-pictorial and pictorial-pictorial correspondences. Another was observed between tasks with numerically large and numerically small sets of objects. A third horizontal décalage was observed between tasks with two separate sets of objects, and tasks with two sets of objects combined in one line.

A décalage between conservation of equality and inequality tasks was also examined. There are some questions about this décalage. Theoretically, it is not expected, and some methodological problems were identified. A solution to the problem has been suggested. Further research incorporating this recommendation is required to settle questions concerning this décalage.

Evidence was also presented which suggested that dĕcalages may exist between groups from different socio-economic backgrounds. However, further research is required to support the existence of these dĕcalages, and to account for them.

v. The Counting Name-Pictorial Correspondence

Siegel[110] and Smith[111] are the primary researchers to use the counting name-pictorial correspondence. Siegel studied the child's understanding of the first three numerals (1, 2, 3). Smith, on the other hand, was interested in the child's understanding of larger numerals.

Siegel's tasks were as follows. The child was presented with a numeral (1, 2 or 3). The child was then shown four pictures. Some pictures consisted of dots in either a linearly or randomly arranged pattern. Other pictures consisted of heterogeneous sets of geometric forms varying in both shape and color. The heterogeneous sets were arranged only in lines. The number of objects in the pictures ranged between one and nine. The child's task was to find the picture which corresponded with the numeral.

In this type of correspondence, it is not possible to perform transformations. The numeral is always written the same way, and the pictures, once constructed, are static.

To make this type of counting name-pictorial correspondence, the child must be able to count. He must count the number of dots in each

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110 Linda Siegel, Op. Cit., p. 533-534.

111 Smith, Op. Cit., p. 172.

of the four sets, and select the set which matches the numeral. Since there is no transformation for the child to see, it is impossible for the child to conserve on the basis of reversibility and compensation as he could on correspondence tasks involving transformations.

Siegel[112] observed a horizontal décalage in the counting name-pictorial correspondence. Specifically, she found that the correspondence tasks between the counting name numerals and the heterogeneous pictorial sets were significantly more difficult than the other two tasks. Thus, there is a horizontal décalage between tasks involving different modes of representation. That is, representing a number with a set of heterogeneous pictures is a more difficult task for the child, than representing number with a set of homogeneous pictures. This is similar to the horizontal décalage found between correspondences with homogeneous, neutral objects (poker chips), and correspondences with homogeneous, provoking objects (eggs and eggcups).

One must be cautioned against thinking that it is only figurative aspects of representation giving rise to this horizontal décalage between homogeneous and heterogeneous sets. More important is a subtle conceptual difference between the tasks. That is, it is easy to apply a number to describe a homogeneous set. Thus, four counters plus two more counters make six counters.

However, if the elements are heterogeneous, an obstacle is created. The child has to determine a common bond to link them into one set. If

112 Linda Siegel, Op. Cit., p. 534.

there are, for example, four apples and two oranges, the common bond to unite the objects is the fact that all are fruit. Once the child establishes a bond, he can easily apply the number six to describe the set.

Thus, the difference in difficulty of the tasks may be due to figurative aspects alone, conceptual aspects alone, or to a combination of both.

One might expect to find a horizontal dêcalage between the linearly and randomly arranged sets. That is, it is perceptually easier to count objects in a line, than scattered about. Siegel's data indicated that making correspondences with linearly arranged sets was easier than making correspondences with randomly arranged sets.[113] The difference in difficulty, however, was not significant. This finding may be the result of the use of small numerals, 1, 2, 3. As Piaget has stated, the small numbers may be identified through perception primarily. That is, the child can easily recognize sets of one, two, or three dots. As well, the first three numerals are familiar to young children. The child, therefore, may be successful on these tasks and not really understand number and numeration. Thus, one would be adivsed to use larger sets (e.g. 5-10 dots) to examine the child's understanding of the counting name-pictorial correspondence with linearly and randomly arranged sets.

In these tasks devised by Siegel, one may question the level of operational thought being examined. That is, if the child can identify the correct set by counting, does that indicate he is at the concrete

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113 Ibid., p. 533.

operational stage of conservation of number? As Gréco _et al._,[114] Piaget,[115] and Wohlwill and Lowe[116] have observed, the child at the end of the preoperational stage (the semilogical stage) is able to count the elements of a set correctly. However, if the set is transformed, the semilogical child only conserves the name of the numeral to describe the numerosity of the set. He does not conserve the quantity represented by the numeral. In contrast, the concrete operational child can count and can conserve the quantity represented by the numeral. Siegel only examines the child's ability to count and identify a set with the number of elements represented by the numeral. She does not present tasks to determine if the child can conserve the quantity represented by the numeral. Consequently, it is concluded that her tasks may be solved by children at the semilogical stage as well as by children at higher levels of development. Siegel's task would have to be redesigned to differentiate between children at semilogical and concrete operational levels of development.

Smith has also used a counting name-pictorial correspondence in his tasks. He was, however, concerned with the child's understanding of larger numbers. Specifically, Smith constructed tasks with numbers ranging between 34 and 657.[117] The task in Figure 5 illustrates the type of correspondence used by Smith.

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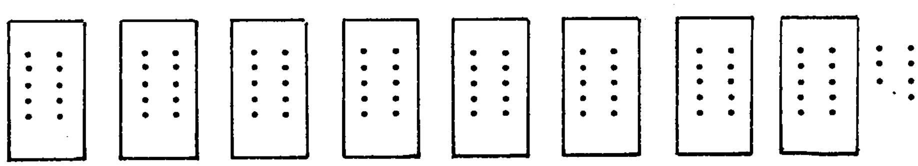
114 Gréco et al., _Op. Cit._, p. 360.

115 Piaget, _The Child's Conception of Number_, p. 74.

116 Wohlwill and Lowe, _Op. Cit._, p. 153-167.

117 Smith, _Op. Cit._, p. 157-158.

How many dots?



a. 7

b. 16

c. 8107

d. 87

Figure 5.- Smith's[118] Counting Name-Pictorial Correspondence

118 Ibid., p. 157.

In these tasks, the child was required to select the best response from a set of alternatives. In this example, the pictorial representation is in the same order as the numeral. That is, the groups of ten are to the left of the set of ones, just as the tens' column of the numeral is to the left of the ones' column. Smith referred to such items as "in order" items. In some items the pictorial representation of the hundreds', tens', and ones' groups was "out of order". That is, the sets of ones may have been placed in the tens' position. Smith referred to these items as "out of order" items. In another set of tasks, regrouping was required. See Figure 6.

Regrouping is defined herein, as the exchange of ten units of one denomination (e.g. tens) to one unit of the next higher denomination (ie. hundreds). Correspondingly, regrouping is also defined as the exchange of one unit of one denomination (e.g. hundreds) to ten units of the next lower denomination (ie. tens). The first type of regrouping is commonly referred to as "carrying" as is done when adding columns of numbers. The second type of regrouping is often described as "borrowing". Thus, regrouping involves either a carrying or a borrowing process.

In constructing his tasks, Smith was attempting to emphasize different principles of the decimal numeration systems. The following is a list of some of the principles of decimal numeration compiled by Flournoy _et al._,[119] and used by Smith in the construction of his tasks.

1. There are ten basic numerals, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, in the decimal system.

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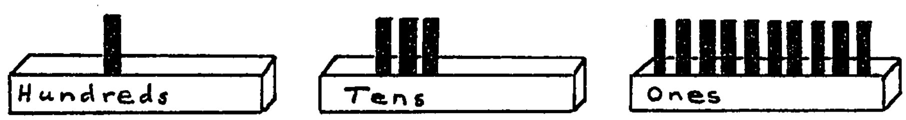
119 Frances Flournoy, Dorothy Brandt and Johnnie McGregor, "Pupil Understanding of the Numeration System", _Arithmetic Teacher_, Vol. 10, February 1963, p. 88-91.

What number does the place-value chart show?




a. 140

b. 1310

c. 130

d. 1003010

Figure 6.- A Counting Name-Pictorial Correspondence With Regrouping[120]

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120 Smith, Op. Cit., p. 158.

2. The symbol, 10, means one group of the base or one group of ten in the decimal system.

3. The Hindu-Arabic system of numeration which we use is a place value system as well as a decimal system.

4. In the numeration system zero indicates no frequency or no count in a place.

5. The value of a digit depends on its position in the numeral.

6. An additive principle and the place value principle are applied in determining the value of a number which a numeral represents, as 333 means 300 + 30 + 3.

7. Either an "absolute" or a "relative" grouping may be used in interpretation. For example, 2346 may be interpreted as 2 thousands, 3 hundreds, 4 tens, 6 ones. This is an "absolute" interpretation by individual digits and places. A "relative" interpretation, as 23 hundreds, 46 ones or 234 tens, 6 ones, makes use of varied ways of grouping rather than by the individual places.

8. Beginning with the ones position and going to the left in a numeral, the value of each place is 10 times greater than the value of the previous place. For example, the value of a 4 in the thousands position is 10 times the value of a 4 in the hundreds position; this is a 10 to 1 relationship.

9. Beginning at the left and going to the right in a numeral, the value of each place is 1/10 as great as the value of the previous place. For example, the value of a 4 in the hundreds place is 1/10 the value of a 4 in the thousands place; this is a 1 to 10 relationship.

10. One hundred should be understood as 10x10, one thousand as 10x10x10, ten thousand as 10x10x10x10, hundred thousand as 10x10x10x10x10, and so on for other powers of ten.

11. Exponential notation should be understood as a short way to represent each power of ten for each place in the numeration system.

Thus, in using "in order" and "out of order" tasks, Smith attempted to differentiate between children who could or could not apply principles one, three, five and six.

In solving tasks with and without regrouping, each of principles one to seven was involved.

As with Siegel's tasks, it is not possible to perform any transformations with this type of correspondence. That is, both the pictorial representation and the counting name representation are static.

The actions a child must perform to conserve with this type of correspondence are more complex than actions required in Siegel's task. As with Siegel's task, the child must count. He must count the number of ones in the ones' space, the number of tens in the tens' space, and the number of hundreds in the hundreds' space. Then the child may make a correspondence between the number of hundreds, tens, and ones in the pictures, and in the columns of the standard numeral.

If there is no regrouping involved, the task is easy. The number of hundreds, tens and ones in both the picture and the numeral are identical, hence, the correspondence is easily made.

In tasks involving regrouping, the child is forced to exhibit more understanding of the decimal numeration system to make the correct correspondence. After determining the number of hundreds, tens and ones in the picture, the child notes that there appear to be discrepancies between the pictured set and the numeral.

To make the correct correspondence, the child must adopt a particular sequence. That is, he must first make a correspondence between the number of ones in the picture and the numeral. Then, he must proceed to the tens' and the hundreds' columns.

In addition, the child must understand the relationship between the columns. Specifically, he must understand that ten units in one column (e.g. tens column) equals one unit of the next higher unit (e.g. hundreds column).

Finally, when regrouping is required, the child must consider the two involved columns simultaneously as opposed to independently. That is, the child must consider how the actual regrouping will affect both columns.

Thus, as the correspondence tasks are extended to larger numbers, the child is required to perform more complex actions to conserve.

From Smith's analysis of the data, it is possible to derive information about two possible vertical décalages. One vertical décalage was observed between the different grades. The second vertical décalage was between correspondence tasks with and without regrouping. Table III shows the proportion of students making constant errors for the different grades and tasks.

A constant error was defined by Smith as the occurrence of three or more errors in a particular type of item.[121] From Table III therefore, one observes a difference between Grade 3 and Grade 4 students on counting name-pictorial correspondence tasks without regrouping. This difference of 0.0735 was significant at the 0.01 level of significance.

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121 Ibid., p. 9.

Table III.-

Proportions of Third and Fourth Grade Students Making Constant Errors on Smith's[122] Place Value Test

| Pictorial-Counting Name Correspondences | Third Grade | Fourth Grade |
|---|---|---|
| Without Regrouping | 0.1241 | 0.0506 |
| With Regrouping | 0.4624 | 0.3813 |

122 *Ibid.*, p. 96.

Smith used a simple t-test to determine this significance. As well, he used the same test to determine the significance between grades on nine other contrasts.[123] Thus, there is a high probability that some decisions of significance may be in error. Sufficient data do not exist to perform a correct analysis. Hence, one may conclude only that a trend exists which cannot be significantly supported. Thus, there appear to be differences in the proportion of Grade Three and Grade Four students making constant errors on counting name-pictorial correspondence tasks without regrouping.

A vertical dēcalage between students in Grade Three and Grade Four in their ability to perform counting name-pictorial correspondence tasks with regrouping was also observed. Although there was a difference of .08 between the proportions of students making constant errors, it was not significant.[124]

Theoretically, a dēcalage is expected between children in different grades. The longer a child attends school, the more experience he will have with number and its representation. Hence, Grade Four children are expected to make fewer errors than Grade Three children on the two sets of tasks, with and without regrouping.

The second dēcalage appears to exist between correspondence tasks with regrouping and those without regrouping. For the Third Grade students, there is a difference of 0.3383 between the proportions of students

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123 Ibid., p. 96.

124 Ibid., p. 96.

making constant errors on the two types of tasks. For the Fourth Grade students, the difference is almost as large.

This décalage between tasks with regrouping and tasks without regrouping is expected from a theoretical point of view. The tasks with regrouping require more understanding of number and its representations. Also important, the child must be able to consider two columns simultaneously to co-ordinate the changes which need to be made to complete the regrouping. Only when the child is at the operative stage of development, is he capable of comparing two thoughts simultaneously. Thus, a two-dimensional concrete operational thinking structure is required to solve these tasks.

In contrast, tasks without regrouping can be performed by a child with relatively little understanding of decimal numeration. The child just has to consider the number of elements for a particular column in a picture with the corresponding column in the numeral. He need not even be systematic. He may choose to look at the ones' columns first, then the hundreds', and then the tens' columns. Therefore, these tasks can be solved by children who are at, or past the semilogical stage. That is, these tasks require at most a one-dimensional preoperational thinking structure.

Since different levels of operative thought are required for tasks with and without regrouping, a vertical décalage is expected. That is, the child will succeed on tasks without regrouping before he will succeed on tasks with regrouping.

To conclude, few researchers have used the counting name-pictorial correspondence to examine the child's formation of number concepts. Researchers using this correspondence developed tasks to examine either very small, or large numbers.

To examine the small number concepts, the researcher showed the child a numeral. Then the child selected a pictured set, from a group of four, to match the numeral. The pictured elements and their arrangements varied. To succeed on these tasks, the child had to be able to count.

One horizontal décalage was observed. Tasks involving pictured sets of heterogeneous objects were more difficult than tasks with pictured sets of homogeneous objects. A second horizontal décalage was expected, but not observed. The use of very small numerals precluded the observation of any décalage between linearly and randomly arranged sets.

These tasks appear to require at most a semilogical thinking structure. The tasks require some revision if the purpose is to differentiate between preoperational and semilogical children, and between semilogical and concrete operational children.

The large number tasks were different. There was only one pictorial representation per task. The principles of decimal numeration were used in the pictorial representation. The child had to select the standard numeral from a set of four to match the pictured number. Some tasks involved regrouping between place-value columns. To identify the correct numeral, the child had to make a correspondence between the various place-value columns in the two representations.

Two vertical décalage were observed. The first vertical décalage was between the different grades. The second vertical décalage was observed between tasks without regrouping and tasks with regrouping. A child requires a one-dimensional thinking structure for tasks without regrouping. For tasks with regrouping, the child requires a two-dimensional thinking structure. Hence, a vertical décalage was observed.

vi. The Counting Name-Numerical Correspondence

Smith has used tasks with counting name-numerical correspondence to examine the child's understanding of numbers in the hundreds. Since Smith's tasks were pencil and paper tasks, he was concerned only with a written counting name representation. Specifically, he used standard numerals for the counting name representation. The following is an example[125] of Smith's tasks:

523 means. . . .
a. 5 + 2 + 3
b. 300 + 20 + 5
c. 200 + 30 + 5
d. 500 + 20 + 3

In this task, the child was required to select the best response from the set of alternatives to correspond with the standard numeral.

In the above task, the numerical representations on the right are "in order". Smith also employed "out of order" counting name-numerical correspondence tasks.

In addition, Smith also had counting name-numerical correspondence tasks involving a "single" regrouping. In these tasks, the regrouping

125 Ibid., p. 161.

was usually from the hundreds' to the tens' column. Furthermore, in some tasks there was a "double" regrouping. That is, there was one regrouping from the ones' to the tens' column, and another regrouping from the tens' to the hundreds' column. The following example[126] is a counting name-numerical correspondence task with a single regrouping:

| | | |
|---|---|---|
| 637 means. . . . | a. | 500 + 130 + 7 |
| | b. | 600 + 30 + 17 |
| | c. | 600 + 130 + 7 |
| | d. | 500 + 130 + 17 |

With the counting name-numerical correspondence tasks, the child does not see a transformation. Both representations involved are static. There is, however, a transformation suggested to the child to make the correspondence. The addition signs "+" suggest to the child that he can add to make the correspondence.

To solve these tasks, the child may perform two actions. One action is to make a correspondence between the elements of the numerical representation and the columns of the standard numeral. However, this becomes more complex when tasks are out of order or involve regrouping. Another method is to total the elements for each of the four numerically represented alternatives. Thus, if the elements of the (a) alternative above are totalled the sum is: 637. The child, then, must compare this sum with the standard numeral on the left to see if it corresponds.

Although Smith had a variety of tasks in his study, from the data presented, only one décalage was observed. This was a horizontal décalage

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126 _Ibid._, p. 161.

between counting name-numerical correspondence tasks, and counting name-verbal tasks. This décalage is discussed after the counting name-verbal correspondence tasks are explained.

In conclusion, only one researcher has used the counting name-numerical correspondence. He studied large number concepts. He included non-regrouping, single regrouping, and double regrouping tasks. In addition, some were "in order" and others "out of order". No transformations were possible, but the "+" signs in the numerical representation suggested a mental operation.

To succeed on these tasks, the child had to make a correspondence between the numerical representation and one of four numerals. He could make a correspondence between columns, or convert the numerical representation to numerals and add. Both require a thorough understanding of decimal numeration principles.

One horizontal décalage was observed between different correspondence tasks.

vii. The Counting Name-Verbal Correspondence

Smith also used counting name-verbal correspondence tasks to examine the child's understanding of numbers in the hundreds. The following is an example[127] of such tasks.

368 means. . . .
a. 9 hundreds 6 tens 3 ones
b. 300 hundreds 60 tens 8 ones
c. 36 hundreds and 8 ones
d. 3 hundreds 6 tens 8 ones

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127 Ibid., p. 160.

Tasks such as this are similar to the counting name-numerical correspondence tasks. Again the child was required to select the best response from the set of alternatives to correspond with the standard numeral.

The above task was an "in order" correspondence task. There were two other variations in the counting name-verbal correspondence tasks. Some were "out of order" tasks while others were "regrouping" tasks. These two variations are similar to those for the counting name-numerical correspondence tasks and pictorial-counting name tasks.

Both representations involved in this task are static. Hence, the child does not see any transformations. Unlike the counting name-numerical correspondence tasks, there are no addition signs in the verbal representation. Thus, there is no indication in the verbal representation to suggest to the child that he can add to make the correspondence.

With counting name-verbal correspondence tasks, the child has two actions he can perform. One is to make a correspondence by comparing each element in the verbal representation with the numerals in each column of the standard numeral. "Regrouping" tasks, and "out of order" tasks make this approach more difficult as the child has to co-ordinate changes in two columns simultaneously. Specifically, if there is a regrouping from the hundreds' to the tens' column, the child has to consider the changes in both simultaneously to see if they balance.

Furthermore, the child can resort to adding the elements in the verbal representation. This is more complicated for the verbal representation than for the numerical representation because the child must remember such relationships as: 13 tens is the same as 130.

A horizontal décalage was observed between counting name-numerical and counting name-verbal correspondence tasks. For Grade Four students in Smith's study the proportion of students making constant errors on counting name-numerical correspondence tasks was 0.6654. For counting name-verbal correspondence tasks the proportion was 0.7899.[128]

For the Grade Three group the proportions of students making constant errors were 0.8158 and 0.8722[129] for the respective tasks.

This horizontal décalage is expected for two reasons. In the numerical representation of number, there are addition signs used. This would suggest to a child that he should add the elements in this representation to find the answer. In the verbal representation, there are no addition signs. This difference in the two representations would affect the child's ability to make the correspondence involving these representations.

Secondly, with the verbal representation, the child has to understand the words involved. Furthermore, to add the elements in the verbal representation, the child must change the words to numeral values (e.g. 13 tens = 130). These difficulties do not arise with numerical representations.

Therefore, a horizontal décalage is expected between these two correspondence type tasks because there are two different representations being used which emphasize different principles of decimal numeration.

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128 Ibid., p. 96.

129 Ibid.

It was impossible to compare the counting name-pictorial tasks and the counting name-verbal tasks. The set of counting name-pictorial correspondence tasks was composed of ten items, five of which were regrouping tasks. In comparison, the set of counting name-verbal tasks was composed of seven items of which only two involved regrouping. Therefore, although results are presented for both types of correspondences, the comparison is not justified.

Attempts were made to examine the existence of other possible dẽcalages. There was a vertical dẽcalage noted between non-regrouping and single regrouping tasks involving counting name-pictorial correspondences. The data presented by Smith for the counting name-numerical, and counting name-verbal correspondence tasks did not reveal if this dẽcalage also existed with these correspondences.

Smith also used double regrouping tasks. The relative difficulty of these tasks to non-regrouping and single regrouping tasks was of particular concern. The non-regrouping tasks seem to require at most a one-dimensional semilogical thinking structure. Thus, these tasks can be solved by children at the end of the preoperational stage.

In single regrouping tasks, discrepancies in two columns must be co-ordinated. This requires a two-dimensional thinking structure. In double regrouping tasks, discrepancies in three columns must be co-ordinated. Hence, double regrouping tasks require a three-dimensional thinking structure. This three-dimensional structure is formed when the child enters the formal operational stage between twelve to fourteen years of age. Children in Smith's study were between seven and ten years of age.

Theoretically, double regrouping tasks would have been difficult for these children. The relative difficulty of these tasks could, however, not be ascertained in Smith's study. Research in this area is required to settle the issue.

The difficulty with regrouping processes which Smith observed is not an isolated observation, The child's difficulty with regrouping concepts has been observed by several researchers. Flournoy, Brandt and McGregor[130] developed a set of twenty-five items to assess the child's understanding of sixteen principles of decimal numeration. The most difficult item involved a sequence of two regroupings. Of a group of one hundred six Grade Seven students with an average IQ of 107, 83.02% answered incorrectly.

Pace[131] developed a test to evaluate the child's basic understanding of arithmetic. The test was administered to a group of British children and to groups of Grade Five and Six children in New York. The two most difficult items in the section on operations with whole numbers involved decimal numeration concepts. One of these tasks involved a double regrouping. Only 25% of the English children were successful. For the Grade Five New York children, 13% were successful while for the Grade Six group, 22% succeeded.

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130 Frances Flournoy, Dorothy Brandt and Johnnie McGregor, Op. Cit., p. 88-91.

131 Angela Pace, "Understanding of Basic Concepts of Arithmetic: A Comparative Study", Journal of Educational Research, Vol. 60, November 1966, p. 107-120.

Other researchers, Dutton,[132] Logan[133] and Pincus et al.[134] have also observed that regrouping in arithmetic tasks poses problems for children. Pincus et al. suggested recommendations for the remediation of such problems. Specifically, they stated:

> Provide practice in decomposing numbers in a variety of ways. (463 is the same as 4 hundreds 6 tens 3 ones, or 46 tens 3 ones, or 4 hundreds 63 ones, or 4 hundreds 5 tens 13 ones.) Use many numbers with zeros.[135]

The suggestion that numbers with zero be used is noteworthy. Zero is a difficult number for people in general to understand and use correctly. The major problem with zero is that it cannot be treated materially as other numbers are. It is easy to show sets of one, or six, but not easy to show a set of zero. Hence, when zero appears in different algorisms, problems arise.

Osburn[136] stated that in his study of arithmetic errors made by children in Grades Three to Eight, the most frequent errors occurred in items involving regrouping and the use of zero.

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132 Wilbur H. Dutton, "New Mathematics for Ethiopian Elementary Schools", Arithmetic Teacher, Vol. 15, February 1968, p. 115-123.

133 Bayne Logan, On the Learning of Mathematics - A Cross-Sectional Study of the Relative Effects of Maturation and Instructional Procedures in the Learning of Mathematics at the Junior Grade Level, University of Ottawa, Ottawa, Canada, 1973, p. 48.

134 Morris Pincus, Margaret Coonan, Harold Glasser, Lillian Levy, Frances Morgenstern, Herbert Shapiro, "If You Don't Know How Children Think, How can You Help Them?", Arithmetic Teacher, Vol. 22, No. 7, November 1975, p. 580-585.

135 Ibid., p. 581.

136 W. J. Osburn, "Errors in Fundamentals of Arithmetic", Journal of Educational Research, Vol. 5, April 1922, p. 348-349.

Dutton[137] found that 95% of Ethiopian students, and almost as many of their teachers, did not understand the use of zero, and place value concepts in arithmetic algorisms.

Reys[138] asked a group of twelve teachers what was the answer to 6 ÷ 0. Eleven of the twelve teachers gave inappropriate responses. Pincus _et al._[139] identified the use of zero as a source of errors in the four basic operations.

Considering the statements made in the literature, one may conclude that place value concepts involving regrouping are difficult concepts for all children. Furthermore, tasks involving numerals with zero are observed to complicate place-value tasks even more than tasks involving numerals composed of the other nine digits. From Osburn's 1922 article, it may also be concluded that children, then and now, are not unlike in finding these tasks very difficult.

Therefore, counting name-verbal tasks were similar to the counting name-numerical and counting name-pictorial tasks. Although there was a variety of tasks, only one décalage could be observed in the data presented. This was a horizontal décalage between counting name-verbal and counting name-numerical correspondences.

A vertical décalage was expected between non-regrouping, single regrouping and double regrouping tasks. From isolated correspondence

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137 Dutton, _Op. Cit._, p. 115-123.

138 Robert Reys, "Division and Zero, An Area of Needed Research", _Arithmetic Teacher_, February 1974, p. 153-156.

139 Pincus _et al._, _Op. Cit._, p. 581-585.

tasks in other studies, double regrouping tasks were found to be very difficult. There were no other tasks to make relative comparisons. The general conclusion was that regrouping tasks are difficult, particularly when a place holder zero is involved. Further research is required in this area.

### 3. Summary and Basic Hypotheses

Number is known and understood through the different modes of representation. Five different modes of representation have been identified in the literature. They are: the material mode, the counting name mode, the pictorial mode, the numerical mode, and the verbal mode.

Confusion over the term "concrete" has led to the use in research of the material and pictorial modes primarily. The use of concrete objects, or pictures representing concrete objects is not a requirement for the child to perform concrete operational thought. Therefore, other modes of representation should be utilized in further number studies.

Different correspondences can be made between the various modes of representation. Such correspondences are shown in Figure 7. Most research has involved material-material, and pictorial-pictorial correspondence tasks. Very little is known about the child's use of the other correspondences. Further research should be directed to the investigation of the other correspondences.

In some correspondences, physical transformations are possible, while in others, the representations are static. Tasks with static modes were found to be more complex, as any actions on these modes have to be on the level of operational thought alone.

| Representation | Material | Counting Name | Pictorial | Numerical | Verbal |
|---|---|---|---|---|---|
| Material | * | 0 | 0 | | |
| Counting Names (oral counting and standard numerals) | | | 0<br>S | S | S |
| Pictorial | | | * | | |
| Numerical (more than just standard numerals) | | | | | |
| Verbal (written words) | | | | | |

* Areas of extensive Piagetian-based research
0 Areas of limited Piagetian-based research
S Areas studied by Smith, a Non-Piagetian researcher

Figure 7.- Representational Modes and Possible Correspondences For Use in Conservation of Number Studies

Although some correspondences were more complex than others, the logical co-ordinations required to solve most tasks could be performed by a child with a two-dimensional concrete operational thinking structure. Some large number tasks involving double regrouping seemed to require a three-dimensional formal operational structure. Further investigation of such tasks is indicated.

Within each type of correspondence, dêcalages were observed. However, very little is known about the child's formation of the complex number concept. In the studies examined, most research has been focused on the child's understanding of small whole numbers less than ten. Only one study has examined larger whole numbers. From the illustration of the numbers involved in the complex number concept (Figure 8), it is seen that research has been concentrated on one small aspect of the complex number concept. Hence, research should be directed to the examination of the formation of other number concepts.

A number of problems could be examined. One particular problem requiring immediate investigation is described below.

Children have been observed to experience difficulty in performing their basic arithmetical operations. Researchers have suggested that this seems to be the result of the child's failure to understand decimal numeration concepts. According to Piagetian theory, the representation is part of the concept itself. Thus, if the child does not understand the decimal numeration representation of a number, he does not have the full concept of the number. Hence, it may be concluded that the child has difficulty with arithmetical operations on large numbers because he does not understand the numbers involved.

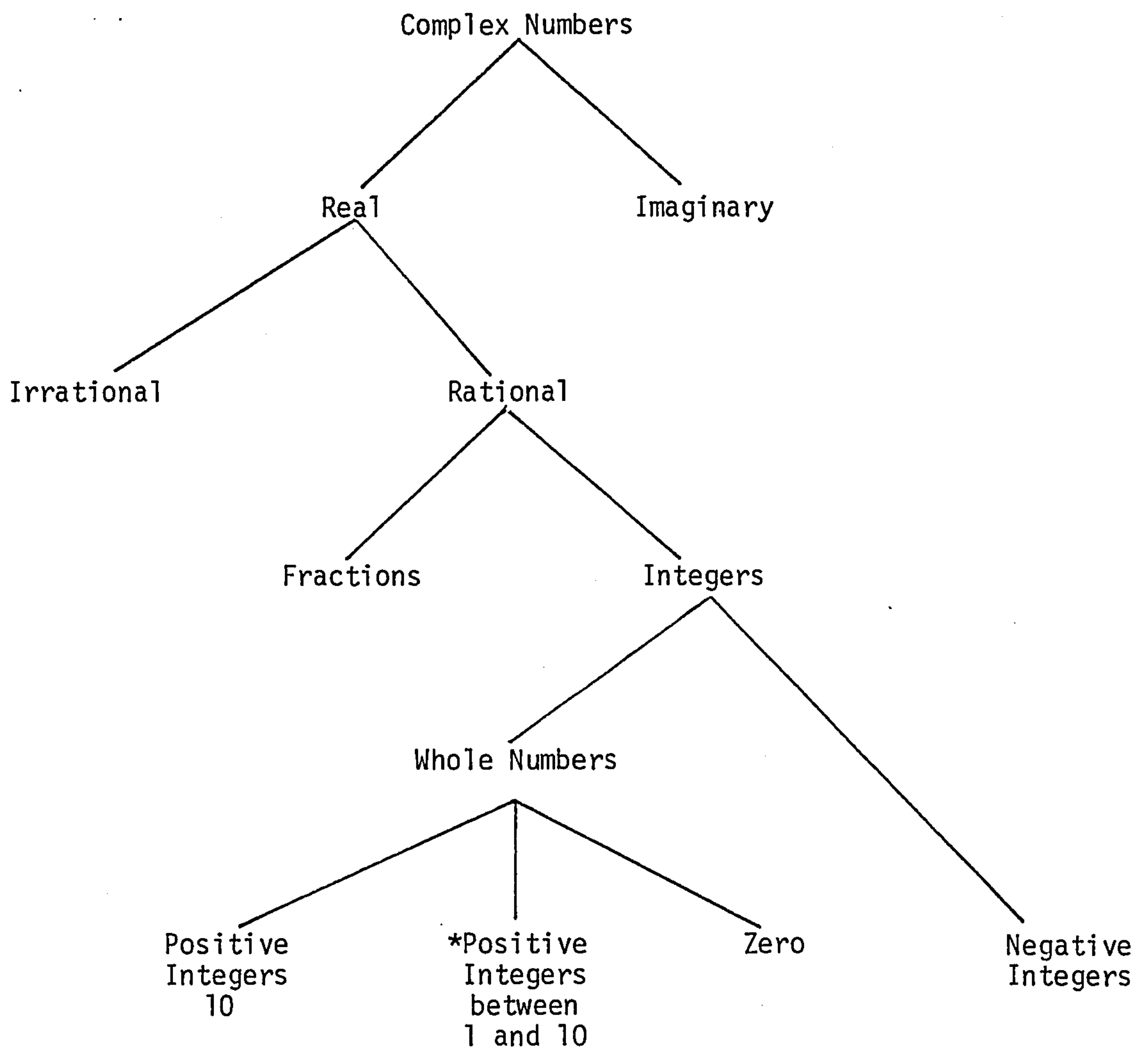


Figure 8.- The Classification of Number

* Area of conceptual development studied by Piaget, and Piagetian-based researchers

If a child is to be assisted in mastering large numbers, it is important to understand how children develop these concepts. In this paper, therefore, the child's formation of large number concepts when numbers are represented with a decimal numeration system is examined. With reference to the formation of large number concepts two vertical, and two horizontal dēcalages are expected.

If the review of the literature particular areas of difficulty with decimal numeration concepts were identified. Specifically, children in different countries, and at different times have been observed to experience difficulty when they have to perform a regrouping process in arithmetical tasks, or in tasks concerning the decimal numeration representation of number.

The results of Smith's study support the expectation that non-regrouping correspondence tasks requiring a one-dimensional preoperational thinking structure are easier than single regrouping tasks requiring a two-dimensional thinking structure. Double regrouping tasks seem to require a three-dimensional formal operational thinking structure. Consequently, they are expected to be the most difficult of these three types of tasks. The expectation of a vertical dēcalage between these tasks may be expressed in the following hypothesis:

> Children find non-regrouping preoperational tasks less difficult than single regrouping concrete operational tasks, which they find less difficult than double regrouping formal operational tasks.

A second vertical dēcalage is expected. According to Piaget, reversibility of thought is an important operative characteristic. Different versions of a task can be constructed to evaluate if the child has developed reversibility of thought. A non-regrouping verbal-counting name correspondence task may be presented in the following two ways:

A. 9 hundreds, 6 tens, 5 ones = _____ (Forward)

B. 3 hundreds, _ tens, 4 ones = 364 (Reverse)

Two versions of a single regrouping task with the verbal-counting name correspondence are:

C. 2 hundreds, 16 tens, 4 ones = _____ (Forward

D. 3 hundreds, __ tens, 8 ones = 478 (Reverse)

Tasks such as (A) and (C) are defined herein as forward tasks. That is, the child has to provide the numeral to complete the correspondence.

Tasks such as (B) and (D) are reverse tasks. In these, the child has to complete one of the several elements of the verbal, pictorial or numerical representations.

In a sense, reverse and forward tasks are opposite versions. Success on both versions of a specific task would indicate that the child had mastered the task. Success on only one version would indicate partial mastery of the task.

According to Piaget, children develop the operative aspects of cognition in a particular sequence. Thus, children should master forward and reverse versions in a specific order. To determine which version is more likely to be learned first, the versions of a task must be compared.

For the non-regrouping tasks, it appears that the forward version, (A), would be more difficult than the reverse version, (B). The child is given no suggestion in the forward task, as to how to solve it. In contrast, with the reverse version, the child will observe the three and the four in both the verbal and the counting name representation. Even if he does not understand decimal numeration concepts, the child will probably conclude that a six is required. Hence, reverse versions of non-regrouping tasks are expected to be mastered before forward versions are.

With single regrouping tasks, however, the opposite is more likely. Reverse versions appear more difficult because there are apparent discrepancies that the child must resolve. That is, in the verbal representation there are three hundreds, while in the numeral there are four hundreds. There are no such discrepancies in the forward tasks. Thus, for single regrouping, and for double regrouping tasks as well, the forward versions are expected to be mastered before the reverse versions.

The expected vertical décalage between versions of the non-regrouping, single regrouping and double regrouping tasks may be expressed as follows which implies statistical interaction.

> Children find forward versions of non-regrouping tasks more difficult than reverse versions.
>
> Children find reverse versions of single regrouping and double regrouping tasks more difficult than forward versions.

The first horizontal décalage is expected between tasks involving different content. The digits, one to nine, differ from the digit, zero, with respect to content. The digits one to nine represent varying amounts of something and can also be represented materially. The digit, zero, represents an absence of something and cannot be represented materially.

Numerals are composed of digits. Numerals composed of non-zero and zero digits usually result when there is 'no count' in a particular column. In the task below, the child would have to supply a zero for the tens' column of the numeral to indicate that there are no tens in the verbal representation.

3 hundreds, 7 ones = 307

Because zero cannot be represented materially, it is expected that the child develops the concept of zero after he develops the concept of the other nine digits. Hence, tasks involving numerals composed of non-zero digits are expected to be easier than tasks involving numerals composed of non-zero and zero digits. This horizontal décalage is examined with reference to non-regrouping tasks. The hypothesis is:

> Children find non-regrouping tasks involving numerals with non-zero and zero digits, more difficult than non-regrouping tasks involving numerals with non-zero digits.

The second horizontal décalage is expected when perceptual factors exist which facilitate a correct solution to a task type earlier than expected. Consider the following double regrouping tasks:

A. 3 thousands, 17 hundreds, 15 tens, 6 ones = 4856

B. 3 thousands, 9 hundreds, 15 tens, 6 ones = 4056

Task (A) is a double regrouping task which differs perceptually from Task (B). On first inspection, Task (B) appears to be a single regrouping task. Actually it is a double regrouping task because the group of ten tens is regrouped into one hundred, and this is combined with the existing nine hundreds to make two hundreds. Thus a second regrouping is required.

Incidentally, it must be noted that in Task (B) there are zero and non-zero digits in the numeral. In the previously discussed tasks, zero had to be supplied to indicate "no count" in a column. In Task (B), this is not so. Although there is a zero in the hundreds' column of the numeral, there are nine hundreds in the verbal representation. When a child regroups, he usually leaves the right-most digit in the column of interest, and carries the left-most digit to the next column. The child would then have ten hundreds. Repeating the process, he would leave the right-most digit, the zero, in the hundreds' column and carry the left-most digit to the thousands' column. Conceivably, the child may generate a zero this way and not even be aware of it. Thus, in these tasks the child is at all times working with "some count" in the hundreds' column of the verbal representation.

Task (A) is expected to be the more difficult task. The seventeen hundreds and fifteen tens side by side, tend to confuse the child about the proper regrouping procedures in such a situation.

In contrast, tasks such as Task (B) do not tend to disorient the child to the same extent. Seeing only one regrouping, the child will attempt the task and continue regrouping automatically. Tasks such as Task (B) are referred to as provoking double regrouping tasks because they facilitate correct solutions.

The expection of a décalage between provoking double and double regrouping tasks is expressed as follows:

> Children find double regrouping tasks more difficult than provoking double regrouping tasks.

In the décalages discussed, five sets of tasks (including forward and reverse versions) were considered. They were non-regrouping tasks involving numerals with non-zero digits, non-regrouping tasks involving numerals with zero and non-zero digits, single regrouping tasks, provoking double regrouping tasks, and double regrouping tasks. The expected dēcalage may be consolidated and expressed into two major hypotheses:

> There will be differences among the difficulty levels of the five task types.

> There will be an interaction between task direction and task type with difficulty level as the dependent variable.

As children develop and proceed through school, they master more aspects of the complex number concept. Therefore, children in higher grades are expected to be more successful on the regrouping sets of tasks than children in lower grades. The third major hypothesis follows:

> Children in higher grades have greater success on the regrouping sets of tasks than children in lower grades.

In the next chapter, the research methods used for testing these hypotheses in the null form will be presented.

# CHAPTER III

## THE EXPERIMENTAL DESIGN

The experimental aspect of this study was directed towards the evaluation of the research hypotheses previously outlined. Appropriate tasks were developed. Research subjects were selected and administered the set of tasks. The children's responses on the tasks were then analyzed. In this chapter, the above topics are discussed.

### 1. Task Construction

Five sets of tasks (including forward and reverse versions) were required to test the research hypotheses. These tasks were: non-regrouping tasks with numerals composed of non-zero digits, single regrouping tasks, double regrouping tasks, non-regrouping tasks with numerals involving zero and non-zero digits, and provoking double regrouping tasks. The single and double regrouping tasks involve only numerals with non-zero digits. Hence, the first three sets of tasks involve similar content, and differ with respect to the level of operative thought required.

Four-digits numerals are used in all tasks. This was felt justified since, in the texts recommended for use in Ontario, children are expected to exhibit mastery of such numerals in addition and subtraction from Grade Four onwards.

Thirty-two tasks were constructed. The composition of the collection of tasks is shown in Table IV. Smith found non-regrouping tasks very easy for Grade Three and Four children, while regrouping tasks were very difficult.[1] Hence, fewer items were allocated for the non-regrouping tasks than for regrouping tasks.

Two correspondences, pictorial-counting and verbal-counting, were used to provide opportunities for the child to generalize his thoughts. Tasks were randomly arranged within each correspondence type to enable the child to demonstrate flexibility in his thought with respect to the task concepts.

A completion format is used for all tasks except the reverse pictorial-counting name correspondences. In these tasks, the child has to circle, from a set of pictured monetary bills, the number required to complete a correspondence. In effect, these are multiple choice items.

The tasks were combined into one instrument entitled "Number and Numeration". The tasks are found in Appendix 1.

The specifications for the pictorial-counting name and verbal-counting name tasks are found in Appendices 2 and 3 respectively. In these specifications, the type of regrouping, the direction, the operational level and the analysis label are indicated for each task.

For practical reasons, a group administration of the tasks was selected.

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1 G. W. Smith, Jr., A Study of Constant Errors in Subtraction and in the Application of Selected Principles of the Decimal Numeration System Made by Third and Fourth Grade Students, unpublished doctoral thesis presented to the Graduate Division of Wayne State University, Detroit, Michigan, Ann Arbor University Microfilms, 1972, p. 96.

Table IV.-

The Number of Items, for Each Correspondence, Direction, Task Type included in "Number and Numeration", a Test for the Conservation of Large Decimal Numerals

| Task Type | Correspondence: Pictorial-Counting Name — Direction: Forward | Pictorial-Counting Name — Direction: Reverse | Verbal-Counting Name — Direction: Forward | Verbal-Counting Name — Direction: Reverse |
|---|---|---|---|---|
| 1. Non-regrouping without-zero | 1 | 1 | 1 | 1 |
| 2. Non-regrouping with zero | 1 | 1 | 1 | 1 |
| 3. Single regrouping | 2 | 2 | 2 | 2 |
| 4. Provoking double regrouping | 2 | 2 | 2 | 2 |
| 5. Double regrouping | 2 | 2 | 2 | 2 |

Information regarding the reliability and the validity of "Number and Numeration" was required. Hence, the set of tasks was administered by a classroom teacher to a group of twenty-five Grade Four students.

The difficulty index for each item for this group is reported in Appendices 2 and 3. The Kuder-Richardson Formula 20 reliability estimate for the set of tasks was 0.915.

The content validity of the tasks was examined by consulting different texts and tests. In textbooks permitted in Ontario schools, all contained verbal correspondence tasks in the lessons concerning decimal numeration place value concepts. In addition, pictures of money, (coins) were used in the Copp Clark text, "Mathematics Book Four".[2]

Verbal-counting name correspondence tasks were also identified on three standardized achievement tests. These tests were:

1) The Metropolitan Achievement Tests, Form G, items 14 and 27.
2) The Metropolitan Achievement Tests, Form F, item 5.
3) The Canadian Test of Basic Skills, Form 1, item 16.

In the Canadian Test of Basic Skills, Item 16 is classified as testing place value, and zero as a place holder skills.

Several-non-standardized tests were also found to use verbal-counting name correspondences. In a set of Topic Tests developed by Morrison,[3] five verbal-counting name tasks were noted. The table of specifications described these tasks as examining the child's ability to regroup.

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2 W. W. Bates, Florence Roliff, W. B. MacLean, Mathematics Book Four, Toronto, Copp Clark Publishing Co., 1966, p. 4 and 41.

3 F. E. Morrison, Development and Use of Mathematics Topic Tests, Elementary Series, Research Report 74-04, June 1974, p. 29.

Survey tests developed by Smith,[4] Flournoy et al.,[5] and Pace[6] also included verbal-counting name tasks. Smith used pictorial-counting tasks as well.

Pincus et al.[7] in advising procedures to overcome basic skill problems in arithmetic, suggested that teachers give the children tasks to strengthen decimal numeration understanding. The tasks suggested were verbal-counting name correspondence tasks involving regrouping and zero concepts.

Hence, there is considerable support for the content validity of "Number and Numeration".

The construct validity of "Number and Numeration" was examined. Tasks requiring more complex thinking were expected to be more difficult than tasks requiring less complex thinking. In Table V, the difficulty levels for the directional versions of the Task Types for the pre-study group are presented. The differences among these difficulty indices are in the directions expected. The fact that the forward preoperational task with zero was quite difficult may be due to the fact that in the verbal-counting name task, the subject must understand the concept of

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4 G. W. Smith, Op. Cit., p. 157.

5 Frances Flournoy, Dorothy Brandt and Johnny M. McGregor, "Pupil Understanding of the Numeration System", Arithmetic Teacher, Vol. 10, February 1963, p. 88-89.

6 Angela Pace, "Understanding of Basic Concepts of Arithmetic: A Comparative Study", Journal of Educational Research, Vol. 60, November 1966, p. 107-120.

7 Morris Pincus, Margaret Coonan, Harold Glasser, Lillian Levy, Frances Morgenstern, Herbert Shapiro, "If you Don't Know How Children Think, How can You Help Them?", Arithmetic Teacher, Vol. 22, No. 7, November 1975. p. 580-585.

Table V.-

Difficulty Levels for the Directional Versions of the Task Types for the Pre-study Group

| | Task Type* | | | | |
|---|---|---|---|---|---|
| | Task 1 | Task 2 | Task 3 | Task 4 | Task 5 |
| Forward | 0.80 | 0.36 | 0.70 | 0.45 | 0.42 |
| Reverse | 0.86 | 0.82 | 0.34 | 0.26 | 0.16 |

* Task 1 - Preoperational Tasks Without Zero

Task 2 - Preoperational Tasks With Zero

Task 3 - Concrete Operational Tasks

Task 4 - Provoking Formal Operational Tasks

Task 5 - Formal Operational Tasks

zero as a place holder. In addition this result may be unique to the specific group. The more complex tasks were more difficult than less complex tasks.

Tasks involving different levels of operative development were expected to be mastered in a specific sequence. A scalogram analysis[8] of eight verbal-counting name tasks requiring different levels of operative thought was performed. Information concerning the tasks selected, the task difficulty indices and the task type is reported in Appendix 4. The scoring patterns for the children in the pre-study group are shown in Appendix 5. Non-regrouping tasks involving zero were not included since they are more appropriate to a sequence of figurative development rather than operative development.

The resultant sequence of task development from the scalogram analysis is shown in Figure 9. In this sequence, preoperational tasks, concrete operational tasks, provoking formal operational tasks and formal operational tasks are mastered in succession. The coefficient of reproducibility was 0.945. The critical coefficient is 0.90. Hence, these results are interpreted as supporting Piaget's theory of concept development, as well as, supporting the construct validity of the tasks in "Number and Numeration".

On the basis of the above, it was concluded that the validity of "Number and Numeration" was adequate for research purposes.

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8 Louis Guttman, "The Basis for Scalogram Analysis", in S. A. Stouffer et al., Measurement and Prediction, Princeton, Princeton University Press, 1950, p. 60-90.

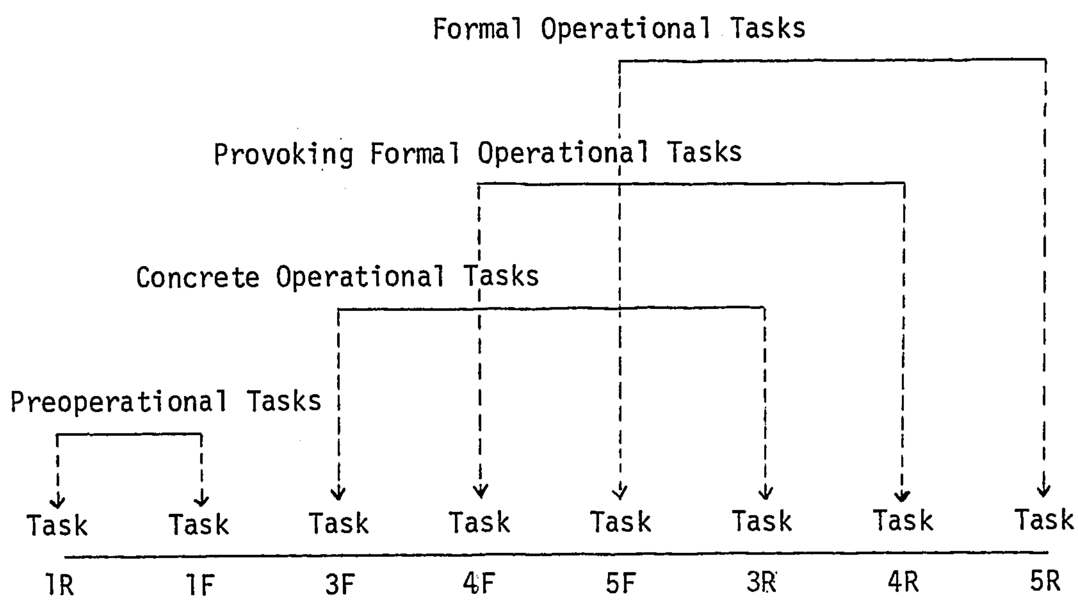


Figure 9.- The Resultant Scalogram Task Sequence in the Development of the Operative Aspects of Decimal Numeration Concepts* for 25 Grade Four Students

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* "R" indicates a reverse direction, while "F" indicates a forward direction.

Other questions relating to the construct validity of "Number and Numeration" could not be answered with the limited number of subjects in the pre-study group. Specifically, it is necessary to examine if the tasks on "Number and Numeration" involve figurative and operative aspects in concept formation. As well, it is important to examine if the different number concepts tested in "Number and Numeration" are related to an overall number concept. Finally, it is relevant to ascertain if the sequence in the development of the operative aspects of decimal numeration is invariant for all groups as Piaget claims. These questions are examined in this study.

## 2. The Research Subjects

Although concept development is best observed in a longitudinal study, a cross-sectional study of children at five successive grade levels was deemed more appropriate because of time limitations.

The target population included children in Grades Four through Eight. All would have encountered four-digit numerals. Furthermore, all children were expected to be at, or beyond, the concrete operational stage of development, though not necessarily at these levels with respect to decimal numeral concepts.

At least one class from each grade level was tested in the selected schools. This was done to minimize differences in experiences which might obscure the observation of concept development if different grades in different schools were examined.

Schools with Grades Four through Eight were required. In the Ottawa Board of Education, there were sixteen Kindergarten to Grade Eight schools. Eight schools were involved in intensive longitudinal research studies. From the remaining eight schools, three were selected to represent as varied a population as possible.

School A was located in a middle to high socio-economic area. At the Grade Seven level, children from middle socio-economic areas fed into the school, and remained until they completed Grade Eight. This school was the largest with at least 1½ classes at each grade level.

School B was situated in a low to middle socio-economic area. At the Grade Seven level, children from middle socio-economic areas fed into the school. This school had very small classes at the Grades Four through Six levels.

School C was located in a low socio-economic area in a small town on the fringe of the city. It was the smallest of the three schools, although the classes at the Grade Four through Six levels were larger than the same classes in School B.

The three principals agreed to provide at least one teacher-volunteered classroom at each grade level. In two schools, tests for more than one classroom per grade were requested. Tests were provided and the results included in the analyses.

In Table VI, the age range, the average age, and the distribution of subjects by age for each grade level are presented. The age range for each grade spans between 3 to 4 years. There is, on the average, one year difference between the ages at successive grade levels.

Table VI.-

The Age Range, the Average Age, and the Distribution of Subjects by Age for Each Grade Level

| Grade Level | Age Range | Average Age | Numbers of Subjects by Age | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Grade 4 | $8^9$ - $12^0$ | $9^{11}$ | 4 | 26 | 19 | - | 1 | - | - | - |
| Grade 5 | $9^3$ - $12^{11}$ | $11^0$ | - | 3 | 39 | 31 | 9 | - | - | - |
| Grade 6 | $10^{11}$ - $14^4$ | $11^{10}$ | - | - | 1 | 43 | 17 | 4 | 1 | - |
| Grade 7 | $11^3$ - $14^{11}$ | $12^{11}$ | - | - | - | 2 | 44 | 30 | 4 | - |
| Grade 8 | $11^{11}$ - $15^{11}$ | $13^{10}$ | - | - | - | 1 | 4 | 68 | 37 | 6 |

* Ages as of February 11, 1976

In Appendix 6, the same age data are presented for each classroom group in the three schools. The age ranges for the classes in School C are generally larger than those in the other two schools, however, in all schools, the average ages are remarkably similar at the different grade levels.

In all, 404 subjects were tested. In Table VII there is a record of the number of subjects by grade and sex who completed or failed to complete all tasks. Ten subjects failed to complete all tasks. They were well distributed among the grades and between the sexes. They were eliminated from all further analyses. They are not included in the age analyses above.

### 3. Administration Procedures

The classroom teacher administered the set of tasks in one of their regular mathematics periods during the week of February 11, 1976. Teachers were instructed to allow the children sufficient time to complete all tasks, and to check papers to rule out omissions by oversight. After recording the data, marked tests were returned to the teachers by the researchers.

### 4. The Analyses of the Data

The analysis of the data involved two major stages, a descriptive, and an inferential. In the descriptive analysis, further reliability and validity analyses were undertaken.

Table VII.-

Numbers of Subjects by Grade and Sex, Completing or Failing to Complete All Tasks in "Number and Numeration"

| Grade Level | Completed All Tasks | | Failed to Complete Tasks | |
|---|---|---|---|---|
| | Males | Females | Males | Females |
| Grade 4 | 28 | 22 | 1 | - |
| Grade 5 | 45 | 37 | 2 | 2 |
| Grade 6 | 32 | 34 | 1 | 1 |
| Grade 7 | 41 | 39 | 1 | 1 |
| Grade 8 | 65 | 51 | - | 1 |
| Totals | 211 | 183 | 5 | 5 |

The internal consistency of the tasks to measure the child's ability to work with decimal numeration concepts at one point in time was the primary reliability concern. Hence, the Kuder-Richardson Formula 20 reliability estimate was considered the most appropriate estimate.

A factor analysis of the results on the individual tasks in "Number and Numeration" was performed to examine if different figurative and operative aspects are involved in concept formation. With tasks varying in correspondence type, and operative levels, several factors were expected.

Although the tasks vary in correspondence type and operative levels, they all involve aspects of the complex number concept. Thus, a second-order factor analysis of the results on the individual tasks was performed to assess if the factors at the first level are related to one overall factor.

A factor analysis of the results on the five task types was also performed. Each task type involved a different decimal numeration concept. These concepts were expected to be related to an overall large number concept. Therefore, one major factor was expected.

In these factor analyses, a principal factoring analysis with iteration was performed. The diagonal of the correlation matrix was replaced by communality estimates. The initial estimate was the squared multiple correlation between a given variable and the rest of the variables in the matrix. Since factors were considered to be related, an oblique rotation was performed.

The minimum eigenvalue for factors to be extracted was 1.0. The maximum number of iterations permitted was 25. The degree of obliqueness of the rotation was set at Delta equal to 0. Data were analyzed by computer utilizing the SPSS[9] programme.

Scalogram analysis was performed to examine if there was a specific sequence in the development of the operative understanding of large decimal numerals. According to Wohlwill,[10] this is the appropriate analysis for the verification of developmental sequences.

Scalogram analysis is not without methodological problems. Thus, special considerations were necessary, and these are discussed briefly.

Tasks included in the analysis were not selected arbitrarily so as to inflate the coefficient of reproducibility, the scalogram statistic. Only tasks relevant to the operative development of decimal numeration concepts were included. The non-regrouping tasks involving zero digits were not included. They tested the child's ability to represent zero, and thus, were more appropriate to a sequence of figurative development.

A decision was made to perform scalogram analysis with verbal-counting name correspondence tasks. For those not possessing the skill, the multiple choice response in the reverse pictorial-counting name tasks would introduce more random error than the reverse verbal-counting name

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9 Norman H. Nie, C. Hadlai Hull, Jean G. Jenkins, Karin Steinbrenner, Dale H. Bent, Statistical Package for the Social Sciences, Second Edition, New York, McGraw-Hill, 1975, p. 499.

10 Joachim Wohlwill, The Study of Behavioral Development, Academic Press, New York, 1973, p. 116.

tasks. Scalogram analysis is based on a deterministic model with no allowance made for random error.

Eight tasks were included. They were the forward and reverse versions of non-regrouping tasks (without zero), single regrouping tasks, provoking double and double regrouping tasks. When more than one example of these tasks existed, one task was selected as follows. The average difficulty for the set of examples (including both the pictorial and verbal tasks) was determined. The verbal-counting name task with a difficulty index closest to the average difficulty was selected for the analysis.

Subjects with zero or perfect scores also inflate the coefficient of reproducibility. To reduce this problem, the responses of children in Grades Four to Six were analyzed. The majority of these children were expected to have a partial, but not a full mastery of decimal numeration concepts.

In the scalogram analysis the specific sequence of tasks was based on the theoretical expectations of the developmental sequence. In certain instances this sequence was slightly different than a sequence which is based on task difficulty.

Two inferential analyses were performed. In the first inferential analysis, the following null hypotheses were tested:

Null Hypothesis I

> There are no differences among the difficulty levels of five task types.

Null Hypothesis II

> There is no interaction between task direction and task type with difficulty as the dependent variable.

A two-way univariate design with measures repeated on both variables is the most appropriate design for the analysis of these hypotheses.[11] The design is illustrated in Figure 10.

The unit of analysis was each child's proportion of successes on forward and reverse versions of each of the five task types. Thus, for each child, there was a set of ten repeated measurements.

For all the repeated measurements, the number of items was limited to two or four. According to Hsu and Feldt,[12] this limitation need be of little concern, especially in samples involving fifty or more subjects. With 394 subjects in this analysis, the scale limitation was considered of no further concern.

From the pre-study group data, it was concluded that the assumption of homogeneous variances and covariances would not be tenable. Hence, the decision was made to test the hypotheses with the Greenhouse-Geisser conservative F statistic.[13]

The assumption of compound symmetry of the pooled variance-covariance matrix is not required in the Greenhouse-Geisser conservative test.[14] Thus, this assumption was not tested.

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11 Geoffrey Keppel, Design and Analysis: A Researcher's Handbook, Englewood Cliffs, New Jersey, Prentice-Hall Inc., 1973, p. 423-433.

12 T. C. Hsu, and L. S. Feldt, "The Effect of Limitations on the Number of Criterion Score Values on the Significance Level of F-test", American Educational Research Journal, 1969, Vol. 6, p. 515-527.

13 Roger E. Kirk, Experimental Design: Procedures for the Behavioral Sciences, Belmont California, Brooks-Cole Publishing Company, 1968, p. 142-143.

14 Ibid.

| Task Type | Task 1 | | Task 2 | | Task 3 | | Task 4 | | Task 5 | |
|---|---|---|---|---|---|---|---|---|---|---|
| Direction | Forward | Reverse | Forward | Reverse | Forward | Reverse | Forward | Reverse | Forward | Reverse |
| Subject 1 | | | | | | | | | | |
| Subject 2 | | | | | | | | | | |
| Subject 3 | | | | | | | | | | |
| Subject 394 | | | | | | | | | | |

Figure 10.- The Two-way Repeated Measures Design to Test Null Hypotheses I and II

Null Hypothesis III

> There are no differences among grades in the difficulty levels of single regrouping, provoking double regrouping and double regrouping tasks.

A two-way design with measures repeated on one variable was required to test Hypothesis III. The design is illustrated in Figure 11. The major interest in this test was the differences between grades on each of the tasks.

The unit of analysis was each child's proportion of successes on each of the tasks.

The major assumption in this design is the assumption of the equality of the variance-covariance matrices among grades. This was tested with the Box Test as outlined by Kirk.[15]

According to Noe,[16] this assumption is frequently violated in developmental studies, particularly if unequal groups are used. A decision was made to equalize groups by randomization if the assumption of equal variance-covariance matrices among grades was not met with all subjects.

Symmetry of the pooled variance-covariance matrix is not an assumption in the multivariate design.[17]

In this chapter procedures for the evaluation of the research hypotheses were discussed. Task development, the research subjects,

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15 Ibid., p. 285-260.

16 Michael J. Noe, A Monte Carlo Study of Several Test Procedures in the Repeated Measures Design, Paper presented to the 1976 Annual Meeting of the American Educational Association, San Francisco, California, April 19-23, 1976 p. 56.

17 Ibid., p. 8.

| | | Task Type | | |
|---|---|---|---|---|
| Grade | Subjects | Single Regrouping | Provoking Double Regrouping | Double Regrouping |
| Grade 4 | $S_1$<br>$S_2$<br>⋮ | | | |
| Grade 5 | $S_{51}$<br>⋮ | | | |
| Grade 6 | S<br>⋮ | | | |
| Grade 7 | S<br>⋮ | | | |
| Grade 8 | S<br>⋮ | | | |

Figure 11.- The Two-way Design with Measures Repeated on One Variable to Test Null Hypothesis III

administration procedures, and the analysis of the data were outlined. The results of the experiment are reported and discussed in the following chapter.

# CHAPTER IV

## PRESENTATION AND DISCUSSION OF THE RESULTS

This chapter is composed of six sections. In the first section, the reliability estimate of Number and Numeration is presented. In the second section, the results of the factor analysis of the children's responses on the individual tasks are presented and discussed. The results of the second-order factor analysis, and the factor analysis of the children's scores on the sets of tasks types are discussed in the third section. In the following section, the results of the scalogram analyses are examined. In the fifth section, the results of the testing of Null Hypotheses I and II are discussed. The results of the tests of Null Hypothesis III are discussed in the sixth section.

To facilitate the presentation of the results, analysis labels for the tasks have been used. The analysis labels for the pictorial-counting name tasks are presented in Appendix 2, whereas those for the verbal counting name tasks are presented in Appendix 3.

### 1. Reliability Results.

The Kuder-Richardson Formula 20 reliability estimates for Number and Numeration at Grades Four to Eight respectively were: 0.932, 0.942, 0.914, 0.917, 0.888. The fact that the Grade Eight group achieved a 75% or more level of mastery on nineteen tasks no doubt contributed to the lower reliability for this group.

The difficulty index for each task in Number and Numeration is reported for the total group of subjects in Table VIII. The total test variance for this group was 62.708. The resultant Kuder-Richardson Formula 20 reliability estimate for the total group on the full set of tasks was 0.933.

2. Concepts - Their Components.

The correlation coefficients between the tasks are reported in Appendix 7. It is noted that there is a positive correlation between all tasks.

In Table IX, the resultant factor pattern in the individual task factor analysis is presented. Six related factors were extracted according to the criteria stated in the previous chapter. These factors accounted for 58.8% of the variance. The largest loading for each task on the six factors is indicated with an asterisk.

In Table X, the matrix of correlations between the six resultant factors is presented. It is noted that some of the correlations between the factors are negative. Since all the correlations between the individual tasks were observed to be positive in Appendix 7, it is concluded that it is the position of the factor axes which is causing these negative correlations.

There is in the results of this factor analysis support for Piaget's contention that both figurative and operative aspects are involved in concept formation. Two different figurative aspects exist in the set of tasks. Specifically, pictorial-counting name, and

Table VIII.-

The Difficulty Indices for the Pictorial-Counting Name and Verbal-Counting Name Correspondence Tasks in Number and Numeration

| Correspondence Mode | | | | | |
|---|---|---|---|---|---|
| Pictorial-Counting Name | | | Verbal-Counting Name | | |
| Task | Type | Difficulty | Task | Type | Difficulty |
| A | 1(R) | 0.926 | A | 3(R) | 0.391 |
| B | 1(F) | 0.876 | B | 3(F) | 0.698 |
| C | 5(R) | 0.399 | C | 3(F) | 0.731 |
| D | 5(F) | 0.614 | D | 4(R) | 0.406 |
| E | 3(R) | 0.635 | E | 5(F) | 0.640 |
| F | 4(R) | 0.531 | F | 3(R) | 0.614 |
| G | 3(F) | 0.731 | G | 4(R) | 0.465 |
| H | 2(F) | 0.739 | H | 1(F) | 0.957 |
| I | 3(R) | 0.642 | I | 1(R) | 0.957 |
| J | 4(F) | 0.635 | J | 4(F) | 0.703 |
| K | 4(R) | 0.558 | K | 5(F) | 0.657 |
| L | 4(F) | 0.624 | L | 2(F) | 0.612 |
| M | 5(F) | 0.642 | M | 4(F) | 0.673 |
| N | 3(F) | 0.739 | N | 5(R) | 0.307 |
| O | 5(R) | *** | O | 2(R) | 0.896 |
| P | 2(R) | 0.871 | P | 5(R) | 0.294 |

*** Reproduction problems resulted in the loss of this task.

Table IX.-

Factor Pattern of the Tasks in Number and Numeration

| Pictorial | Type | Factor 1 | Factor 2 | Factor 3 | Factor 4 | Factor 5 | Factor 6 |
|---|---|---|---|---|---|---|---|
| A | 1(R) | ¯0.04190 | 0.07690 | ¯0.06353 | ¯0.00892 | ¯0.00269 | 0.61995* |
| B | 1(F) | 0.00387 | 0.04497 | 0.07183 | 0.15109 | ¯0.15904 | 0.29561* |
| C | 5(R) | ¯0.08135 | ¯0.01049 | ¯0.05361 | 0.60119* | 0.03309 | 0.02648 |
| D | 5(F) | 0.28427* | ¯0.03036 | ¯0.06790 | 0.21893 | ¯0.07456 | 0.11322 |
| E | 3(R) | 0.01132 | 0.01104 | 0.04716 | 0.73766* | ¯0.09660 | 0.01365 |
| F | 4(R) | 0.05123 | 0.00291 | ¯0.08366 | 0.66655* | 0.02178 | 0.03068 |
| G | 3(F) | 0.48948* | 0.06260 | 0.03776 | 0.13206 | ¯0.14636 | ¯0.00650 |
| H | 2(F) | 0.22672 | ¯0.05099 | ¯0.07318 | 0.07817 | ¯0.05170 | 0.27718* |
| I | 3(R) | 0.22681 | ¯0.07952 | ¯0.16511 | 0.28074* | 0.09122 | 0.16164 |
| J | 4(F) | 0.48846* | ¯0.00893 | ¯0.02266 | 0.06108 | ¯0.18771 | 0.07138 |
| K | 4(R) | 0.15561 | 0.06601 | ¯0.04472 | 0.57659* | 0.01572 | ¯0.06088 |
| L | 4(F) | 0.52439* | ¯0.08409 | ¯0.14466 | ¯0.02017 | ¯0.10405 | 0.14636 |
| M | 5(F) | 0.56879* | ¯0.00910 | 0.02313 | 0.15646 | ¯0.08087 | 0.03502 |
| N | 3(F) | 0.77668* | 0.06716 | ¯0.03219 | ¯0.02777 | ¯0.03663 | ¯0.03008 |
| P | 2(R) | 0.08265 | 0.12092 | 0.05232 | 0.02509 | 0.00628 | 0.43419* |

Variance accounted for: 58.8%

* Largest loading for each item

Table IX.- (Continued)

Factor Pattern of the Tasks in Number and Numeration

| Verbal | Type | Factor 1 | Factor 2 | Factor 3 | Factor 4 | Factor 5 | Factor 6 |
|---|---|---|---|---|---|---|---|
| A | 3(R) | 0.12299 | ¯0.02621 | ¯0.36117* | 0.00154 | ¯0.05238 | 0.04484 |
| B | 3(F) | 0.14465 | 0.06923 | ¯0.02031 | 0.02468 | ¯0.72144* | ¯0.07043 |
| C | 3(F) | 0.15559 | 0.12066 | ¯0.00969 | 0.04139 | ¯0.70562* | ¯0.08552 |
| D | 4(R) | ¯0.00562 | 0.03565 | ¯0.67720* | 0.11748 | ¯0.07580 | ¯0.07486 |
| E | 5(F) | 0.04557 | ¯0.01542 | ¯0.11572 | 0.02133 | ¯0.67901* | 0.04119 |
| F | 3(R) | 0.01094 | 0.10326 | ¯0.39540* | 0.14124 | ¯0.33838 | ¯0.06962 |
| G | 4(R) | ¯0.00168 | 0.05992 | ¯0.71992* | 0.09611 | ¯0.11185 | ¯0.10905 |
| H | 1(F) | ¯0.11594 | 0.52024* | 0.03707 | 0.00995 | ¯0.18725 | 0.11679 |
| I | 1(R) | ¯0.03442 | 0.93089* | 0.01318 | 0.04620 | ¯0.03369 | ¯0.03195 |
| J | 4(F) | 0.14827 | 0.06025 | ¯0.07652 | ¯0.02706 | ¯0.68833* | 0.02418 |
| K | 5(F) | ¯0.08710 | ¯0.03023 | ¯0.13562 | 0.01493 | ¯0.76672* | 0.08001 |
| L | 2(F) | 0.14160 | 0.03001 | 0.00547 | 0.22304 | ¯0.25940* | 0.13655 |
| M | 4(F) | 0.05421 | ¯0.06931 | ¯0.09225 | 0.00132 | ¯0.70575* | 0.14376 |
| N | 5(R) | ¯0.01042 | 0.00265 | ¯0.66821* | ¯0.04135 | 0.00410 | 0.10623 |
| O | 2(R) | 0.13051 | 0.53343* | ¯0.12317 | ¯0.02343 | 0.13589 | 0.09500 |
| P | 5(R) | ¯0.03954 | 0.02167 | ¯0.61955* | 0.03254 | ¯0.02275 | ¯0.02380 |

Variance accounted for: 58.8%

* Largest loading for each item

Table X.-

Correlations Between the Factors Resultant from the Analysis of Responses to the Tasks in Number and Numeration

| | Factor 1 | Factor 2 | Factor 3 | Factor 4 | Factor 5 | Factor 6 |
|---|---|---|---|---|---|---|
| Factor 1 | 1.00000 | 0.10288 | -0.39252 | 0.54217 | -0.51948 | 0.40250 |
| Factor 2 | | 1.00000 | -0.17244 | 0.16154 | -0.26719 | 0.36810 |
| Factor 3 | | | 1.00000 | -0.51684 | 0.51674 | -0.21988 |
| Factor 4 | | | | 1.00000 | -0.45176 | 0.41093 |
| Factor 5 | | | | | 1.00000 | -0.31368 |
| Factor 6 | | | | | | 1.00000 |

verbal-counting name correspondences are used to present the number concepts. Factors one, four and six account for the pictorial-counting name correspondence tasks. Factors two, three and five account for all the verbal-counting name tasks. Hence, there is a distinction made in this analysis between number concepts presented with different correspondence tasks.

It was noted that the majority of the children performed rough calculations in the test booklet. In the rough work for the pictorial-counting name tasks, children did what is normal to do with money. They put down the amounts in numerals, then added (or subtracted if necessary) to arrive at their response. In contrast, there were relatively few calculations suggesting that addition processes were being used with verbal-counting name tasks. For these tasks, the children seemed to be using a correspondence approach. If different approaches were used, this would explain why different sets of factors account for the pictorial-counting name and verbal-counting name tasks.

The tasks in Number and Numeration require different levels of operative thought. Of the three pictorial-counting name factors (see Table IX), factor six best explains all the preoperational tasks, factor one explains the forward operational tasks, and factor four explains the reverse operational tasks. Results for the verbal-counting name factors are similar. All the preoperational tasks except the verbal-counting task, Task 2(F), are best explained by factor two. Factor five best explains all the forward operational tasks, while factor three explains all the reverse operational tasks. Thus, within each correspondence there is a preoperational factor, a forward operational factor, and a reverse operational factor.

The distinction between preoperational and operational tasks may be explained by the fact that different processes are involved in these tasks. In preoperational tasks, the child has to make a comparison between the corresponding columns of the two representations involved in the task. In the operational tasks the child must make a correspondence after making some transformation in the presented data. Hence, the operational correspondence is not direct as the preoperational correspondence is, and a factorial distinction between these tasks results.

The distinction between forward and reverse operational tasks can also be explained. To solve forward operational pictorial-counting name tasks, the child has to total the amounts presented. In solving reverse operational pictorial-counting name tasks, the child has to total the amounts presented, then determine the amount of money still required to complete the correspondence. Usually the required amount was determined by subtraction. Thus in reverse operational pictorial-counting name tasks there is an extra step required and a factorial distinction results between forward and reverse operational pictorial-counting name tasks.

To solve forward operational verbal-counting name tasks, the child has to make a correspondence by starting with a comparison of the ones' columns. Then he must proceed to the thousands' columns and complete any regroupings necessary in the process. In contrast, to solve the reverse operational versions, it is preferable to commence these tasks with a comparison of the thousands' columns and proceed towards the ones' columns. In this approach, the borrowing transformation is easily identified. In order to adopt the correct procedures to solve these forward and reverse operational tasks, the child must make a distinction between them. This distinction is also evident in the results of the factor analysis, with forward tasks loading on one factor and reverse tasks loading on another.

There was no distinction observed between concrete and formal operational tasks. According to Piaget, the contrast between the preoperational and the concrete operational child is more distinct than that between the concrete and formal operational child. At the preoperational stage, the child is illogical in considering two variables in a problem. In contrast, the concrete operational child is logical and systematical in considering two variables. The formal operational child, in turn, is logical and systematical in managing several variables in a task. In these preoperational, concrete, and formal operational tasks, the content is the same. That is, the content involves four-digit decimal numerals. The figurative aspects also are the same. The operative aspects involve regrouping. In the preoperational tasks, there is no regrouping. In the concrete tasks, there is a single regrouping, and in the formal tasks there is a sequence of two regroupings. Thus, between concrete and formal operational tasks the only difference is that the child must be logical enough to realize there could be more regroupings, and systematical enough to carry out regroupings in succession. Hence, in this factor analysis, a distinction between preoperational and concrete operational tasks is expected, whereas one between concrete and formal operational tasks is not.

One verbal-counting name task, the forward non-regrouping task involving zero and non-zero digits does not have its largest loading with the other preoperational verbal-counting name tasks. In all, there are four non-regrouping tasks constructed to examine the child's ability to use zero as a digit to represent "no count" in a particular column. Of these tasks, the forward verbal-counting name task is by for the most difficult. Comparisons between this and the other three tasks were made. In solving this task, the child is forced to confront the concept of zero as a place holder indicating "no count". The fact that the only task to truly confront the child with the concept of zero as a place holder does not load on the same factor as the other non-regrouping tasks is an indication that there may be more than a subtle conceptual difference between such tasks.

In this study, it was considered that there is only a subtle content difference between the concept of "non-zero" and the concept of "zero" digits. That is, non-zero digits indicate an amount of something and can be represented materially, whereas zero indicates an absence of something and cannot be represented materially. Preoperational pictorial-counting name tasks involving both zero and non-zero digits, do load on the same factor as the preoperational pictorial-counting name tasks with non-zero digits. This can be explained from the child's rough calculations. The children appeared to solve the forward pictorial task "H" by adding \$8000 + \$900 + \$6 to get \$8906. For the reverse pictorial task "P", children appeared to add the shown bills: \$6000 + \$4 to get \$6004. Then they subtracted this from the total, \$6104, to conclude that one \$100 bill was still required. Using this approach a child may easily be unaware of the meaning, and the necessity of zero in these tasks. Similar procedures were used to solve the non-regrouping tasks with non-zero digits. Thus, with all the pictorial-counting name non-regrouping tasks being solved the same way it follows that they are explained by the same factor.

The reverse verbal-counting name non-regrouping task involving zero and non-zero digits has its largest loading on the same factor as the non-regrouping versions with non-zero digits. In contrast with the pictorial-counting name tasks, there were few calculations for these non-regrouping tasks. It is assumed, therefore, that the children used a correspondence procedure for these tasks. Using a correspondence procedure, a child may easily arrive at a solution to this reverse non-regrouping task without noting the zero. Since similar procedures are used with this reverse task, and with both versions of the non-regrouping tasks with non-zero digits, these tasks tend to be explained by the same factor.

Thus, three of the non-regrouping tasks with zero and non-zero digits have their highest loadings on the non-regrouping factors. One task, the verbal-counting name, forward non-regrouping task with zero and non-zero digits does not load on the preoperational verbal-counting name factor. Further research is required to clarify this finding. Specifically, it is important to ascertain if tasks involving a supplied zero to indicate "no count" are described by the same, or different factors than tasks involving only non-zero digits. Since this forward verbal-counting name task has its highest loading on the forward operational verbal-counting name factor, it is also necessary to examine this result more carefully.

In the individual factor analysis, therefore, there is support for the theoretical view concerning the components of a concept. That is, a concept involves both figurative and operative aspects. The content of the concepts included in this analysis involves only digits interpreted as indicating "some count" in each place column. The content does not seem to include the digit "zero" which the child has to perceive, and has to supply to indicate "no count". The existence of different conceptual content, however, requires further substantiation.

### 3. Concepts - Their Relationships.

The factor correlation matrix in Table X was factor analyzed to obtain the second-order factor pattern of the individual tasks. The resultant factor pattern is reported in Table XI. Two factors were extracted.

In Table XII, the correlations between the five task types are presented. In the factor analysis of the children's responses to these sets of items, one factor was extracted previously. The factor pattern of the task types is presented in Table XIII.

According to Piaget, a child begins the formation of concepts at the preoperational stage. These concepts are limited and in a sense, are psuedo or pre-concepts. They are, however, the basis upon which

Table XI.-

The Second-Order Factor Pattern of the Tasks in Number and Numeration

| Variable** | Factor 1 | Factor 2 |
|---|---|---|
| Forward Operational Pictorial-Counting Name Factor | 0.61439* | 0.11594 |
| Preoperational Verbal-Counting Name Factor | 0.04343 | 0.37400* |
| Reverse Operational Verbal-Counting Name Factor | 0.75583* | ¯0.12354 |
| Reverse Operational Pictorial-Counting Name Factor | 0.66527* | 0.10565 |
| Forward Operational Verbal-Counting Name Factor | 0.69868* | 0.03049 |
| Preoperational Pictorial-Counting Name Factor | ¯0.03462 | 0.90157* |

Variance accounted for: 64.9%

* Largest loading for each variable

** Variables were the factors derived from the first-order factoring

Table XII.-

Correlations Between the Five Task Types
in Number and Numeration

| | Task 1 | Task 2 | Task 3 | Task 4 | Task 5 |
|---|---|---|---|---|---|
| Task 1 | 1.00000 | 0.49626 | 0.41978 | 0.39719 | 0.37150 |
| Task 2 | | 1.00000 | 0.56217 | 0.55416 | 0.55822 |
| Task 3 | | | 1.00000 | 0.83474 | 0.79793 |
| Task 4 | | | | 1.00000 | 0.81182 |
| Task 5 | | | | | 1.00000 |

Table XIII.-

The Factor Pattern of the Five Task Types in Number and Numeration

| Task Type | Factor 1 |
|---|---|
| Task 1 | 0.49293 |
| Task 2 | 0.66267 |
| Task 3 | 0.90068 |
| Task 4 | 0.89803 |
| Task 5 | 0.86993 |

Variance accounted for: 67.4%

higher level concepts are formed. Hence, the preoperational and the operational concepts examined in Number and Numeration are expected to be related to one overall number concept. In both the second-order factor analysis, and the task type factor analysis, there is support for this expectation.

In the second-order factor analysis two factors were extracted with a correlation of 0.538 between them. From the original factor analysis the preoperational verbal-counting name factor and the preoperational pictorial-counting name factor have major loadings on one of the second-order factors. Each of the remaining four factors from the first-order analysis has its largest loading on the second second-order factor. Thus, in this second-order factor analysis, a distinction is made between preoperational and operational level concepts. There is, however, a sizable correlation between these factors which is interpreted as indicating that these factors have aspects of the overall number concept in common.

In the task type factor analysis, all the task types loaded on one factor. The loadings for the preoperational tasks, (Task 1 and Task 2) are lower than those for the operational tasks. This results because preoperational concepts are related to the overall number concept, but not to the same extent as the operational concepts. It is noteworthy that the preoperational tasks with zero and non-zero digits (Task 2), have a higher loading than the preoperational tasks with only non-zero digits. That is, the Task 2 concepts are broader number concepts than Task 1 concepts, therefore a larger loading is understandable.

In the factor analyses of task types no distinction is made between concrete and formal operational concepts. This is expected. If a child cannot solve concrete tasks, he will not be able to solve the formal tasks. If he can solve formal tasks, he will also be able to solve the concrete tasks. Hence, in these two situations, the results on these two sets of tasks will be highly correlated.

If one were interested in attempting to observe a distinction between concrete and formal operations tasks in a factor analysis, a special population would be required. That is, children who had only mastered the concrete tasks would be required. Only in such a situation would the children's results on the concrete and formal tasks be less correlated.

To conclude, in the second-order factor analysis and in the task type factor analysis, there is evidence to suggest that the different number concepts evaluated in Number and Numeration are related to an overall number concept.

4. Concepts - Their Sequence of Development.

In Table XIV, the selected scalogram tasks and their difficulty indices for different groupings of Grade Four to Six subjects are presented. The order of the selected tasks in this presentation, is the task sequence assessed by the scalogram analysis. The difficulty indices decrease from task to task with one exception for each group. For the Grade Four group, the forward provoking double regrouping task "M", was more difficult than the forward double regrouping task "E". This was

Table XIV.-

The Difficulty Indices of the Selected Verbal-Counting Name Tasks for Different Groupings of the Grade Four to Six Subjects

| Verbal-Counting Name Task | I | H | C | M | E | F | G | N |
|---|---|---|---|---|---|---|---|---|
| Task Type | 1R | 1F | 3F | 4F | 5F | 3R | 4R | 5R |
| Difficulty Indices for All* Subjects: | | | | | | | | |
| In Grade Four | 0.940 | 0.920 | 0.500 | 0.280 | 0.340 | 0.280 | 0.160 | 0.080 |
| In Grade Five | 0.939 | 0.963 | 0.671 | 0.622 | 0.585 | 0.561 | 0.378 | 0.232 |
| In Grade Six | 0.924 | 0.970 | 0.788 | 0.697 | 0.667 | 0.651 | 0.500 | 0.273 |
| In Grades Four to Six | 0.934 | 0.955 | 0.667 | 0.561 | 0.551 | 0.520 | 0.364 | 0.207 |
| Difficulty Indices for Informative** Subjects: | | | | | | | | |
| In Grade Four | 0.978 | 0.957 | 0.500 | 0.261 | 0.326 | 0.261 | 0.130 | 0.043 |
| In Grade Five | 0.969 | 1.000 | 0.631 | 0.569 | 0.523 | 0.492 | 0.262 | 0.077 |
| In Grade Six | 0.941 | 1.000 | 0.765 | 0.647 | 0.608 | 0.588 | 0.392 | 0.098 |
| In Grades Four to Six | 0.963 | 0.988 | 0.636 | 0.506 | 0.494 | 0.457 | 0.265 | 0.074 |

* Subjects with scores of 0 to 8 on the set of scalogram tasks.

** Subjects with scores of 1 to 7 on the set of scalogram tasks.

contrary to theoretical expectations. In the remaining groups, task "M" was easier than task "E", hence, these tasks were placed according to theoretical expectations and not according to their difficulty index.

Beyond the Grade Four levels, children found the forward non-regrouping task "H" easier than the reverse non-regrouping task "I". Since this was contrary to theoretical expectations, task "I" was placed before task "H".

The results of the scalogram analyses are presented for each group in Table XV. According to Piaget, the sequence of concept development with respect to vertical décalages is fixed. The child masters preoperational concepts first, then concrete, and finally, formal operational concepts. In the scalogram analyses, there is support that a specific sequence of development is followed in the child's formation of decimal numeration concepts.

In each analysis, the coefficient of reproducibility is beyond the criterion level of 0.90. Furthermore, the coefficients of scalability for each analysis are beyond the critical level of 0.60. The analyses with the informative subjects only, indicate the lowest reproducibility and scalability for a particular group. The analyses with all subjects (both informative and non-informative) indicate the maximum possible coefficients for a specific group. These are the maximum coefficients assuming that the non-informative subjects actually developed their concepts as the informative subjects did. Thus, as Piaget states, there is a specific sequence in the development of the operative aspects of concepts.

Table XV.-

Scalogram Analyses of Selected Verbal-Counting Name Tasks for Grade Four to Six Subjects

| | Grade | | | |
|---|---|---|---|---|
| Analysis with all Subjects* | Grade 4 | Grade 5 | Grade 6 | Grade 4-6 |
| Coefficient of Reproducibility | 0.940 | 0.936 | 0.932 | 0.936 |
| Minimum Marginal Reproducibility | 0.778 | 0.717 | 0.741 | 0.702 |
| % Improvement | 16.3 | 22.0 | 19.1 | 23.4 |
| Coefficient of Scalability | 0.730 | 0.774 | 0.737 | 0.784 |
| Number of Subjects | 50 | 82 | 66 | 198 |
| Analysis with Informative Subjects** | | | | |
| Coefficient of Reproducibility | 0.935 | 0.919 | 0.912 | 0.921 |
| Minimum Marginal Reproducibility | 0.798 | 0.733 | 0.758 | 0.725 |
| % Improvement | 13.7 | 18.7 | 15.4 | 19.6 |
| Coefficient of Scalability | 0.679 | 0.698 | 0.636 | 0.714 |
| Number of Subjects | 46 | 65 | 51 | 162 |

* Subjects with scores of 0-8 on the set of scalogram tasks.

** Subjects with scores of 1-7 on the set of scalogram tasks.

The resultant sequence of task development is illustrated in Figure 12. This is the same sequence observed for the pre-study group. Development proceeds from a preoperational to an operational stage. The operational stage, however, is composed of both forward and reverse versions of the operational tasks.

Within both the forward operational and the reverse operational sets of tasks, operational concepts are formed in a specific sequence. Tasks 3F and 3R comprise the concrete operational concept of single regrouping. Tasks 4F and 4R comprise the provoking formal operational concept of double regrouping. Tasks 5F and 5R comprise the formal operational concept of double regrouping. From Figure 12, it is seen that the preoperational, concrete operational, provoking formal operational and formal operational concepts are mastered in succession. This is the sequence of development outlined by Piaget.

Although no distinction was made between concrete and formal operational tasks in the factor analyses, a distinction is made between them in the scalogram analyses. Specifically, in the scalogram analyses, concrete versions were found to be mastered before the formal operational versions. Therefore, the distinction made is in their sequence of development.

One may question the logic in including preoperational tasks in a scale of operative development. That is, the behavior of the illogical preoperational child is not compatible with the behavior of the logical operational child. To advance to the concrete operational stage, the child must exchange his preoperational behavior for operational behavior.

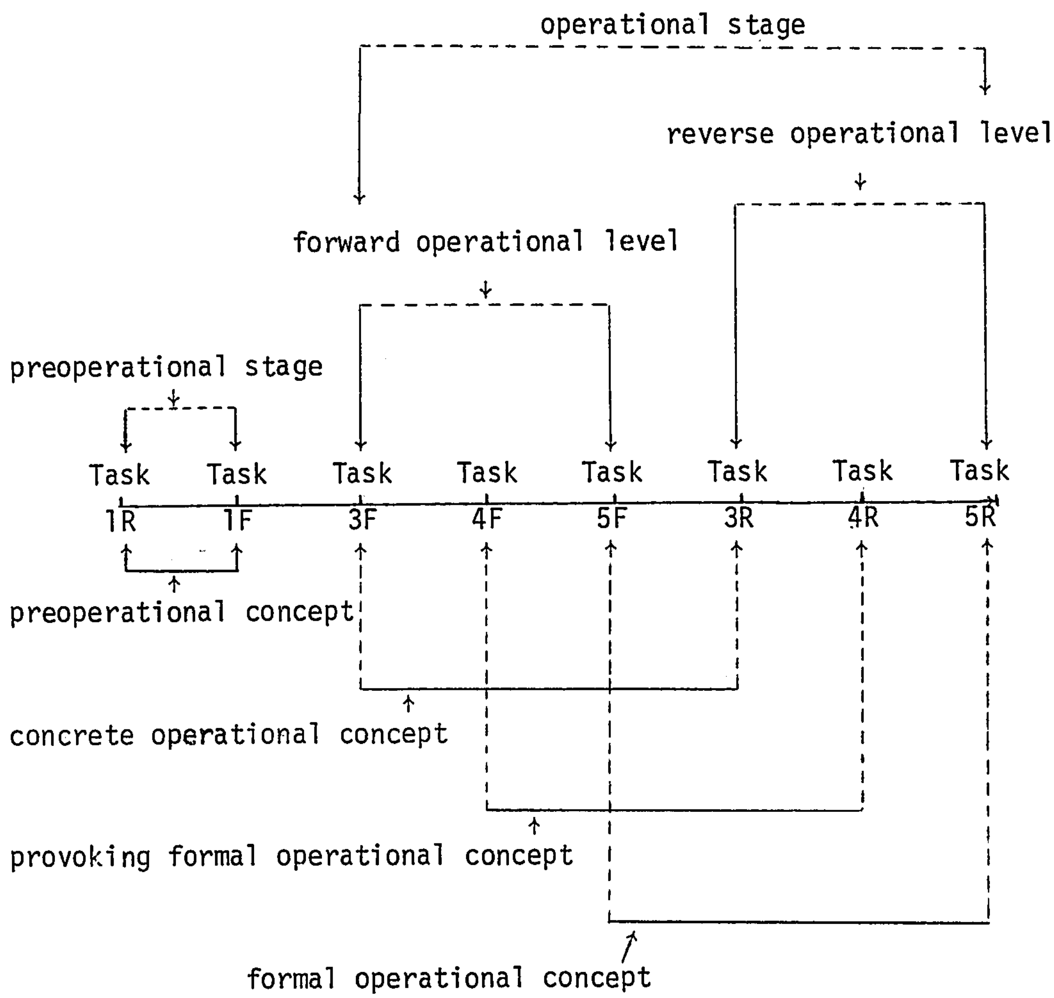


Figure 12.- The Task Sequence in the Development of the Operative Aspects of Decimal Numeration Concepts

Success on the preoperational tasks in Number and Numeration indicates that the child is at the semilogical level of the preoperative stage, and is about to enter the concrete operational stage. Failure on these tasks indicates that the child is somewhere other than at the end of the preoperational stage with respect to the decimal numeration concepts of large numerals. At the semilogical level the child can succeed in making simple correspondences but still has no logical understanding of a concept. This is similar to Piaget's observation that children proceed from making global correspondences to making one-one correspondences without realizing that the sets remain equal in number if one set of counters is clustered. Thus, whereas preoperational behavior is incompatible with operative behavior, the semilogical level of preoperative behavior is compatible and is a necessary basis for further development in the understanding of a concept. Hence, success on the preoperational tasks is relevant in the scale of operative development of decimal numeration concepts.

The resultant sequence of development through the series of forward and reverse operational tasks is noteworthy. To solve the forward operational tasks, the child must have some operative understanding of the processes involved in performing a regrouping. Once he has mastered the forward concrete operational task, the child has the logical understanding to solve the forward provoking formal and formal operational tasks. Hence, development from the forward preoperational task, to the forward concrete operational task indicates the development of some logical flexibility in the child's thinking. Development from the forward concrete operational task to the forward provoking formal and formal tasks indicates progress in the child's ability to systematically apply this logical flexibility as often as necessary to reach a solution.

Development from the forward formal operational task to the reverse concrete operational task indicates a further degree of logical flexibility in the child's thought. That is, the child discovers that solutions to the reverse tasks are reached easily by commencing with a comparison of the thousands' columns instead of the ones' columns. Finally, progress through the reverse provoking formal and formal operational tasks marks the development in the child's ability to apply this new logical flexibility systematically, and as often as necessary.

Thus, in the scalogram analyses, there is evidence to support the hypothesis that a specific sequence of decimal numeration concept formation occurs. Furthermore, this sequence is in keeping with Piaget's view that preoperative, concrete operative and formal operative concepts are developed in succession. The child learns first to represent the concept, then to logically manipulate it. Also, there is evidence to support the view that, within the operational stage of development, the child is becoming more systematical and logically more complex in his thinking.

5. Décalages in Concept Formation.

The mean difficulty indices and the variances for the forward and reverse versions of each task type are presented in Table XVI. As expected, the variances for the directional versions of the task types were heterogeneous. In Table XII the correlations between the task types were observed to differ considerably. Hence, it was concluded that the symmetry of the variance-covariance was not tenable. Thus, Null Hypotheses I and II were assessed with the Greenhouse-Geisser Conservative F-test.

The summary of the analysis of variance of the task type and directional variables is presented in Table XVII. There is a significant difference ($p < 0.05$) between the mean difficulty indices of the forward and reverse versions of the task types. A significant difference is also observed among the mean difficulty indices of the five task types. The task direction by task type interaction is significant.

On the basis of this analysis, Null Hypotheses I and II are rejected in favour of the first two major research hypotheses.

Further analyses of the significant differences among the difficulties of the task types were made. Symmetry of the variance-covariance matrices is not a requirement in the Greenhouse-Geisser Conservative F-test of the overall null hypotheses, hence, Kirk[1] and Keppel[2] state that specific comparisons among the repeated measures must be made with an F-ratio

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1 Roger E. Kirk, Experimental Design: Procedures for the Behavioral Sciences, Belmont California, Brooks-Cole Publishing Company, 1968, p. 263.

2 Geoffrey Keppel, Design and Analysis: A Researcher's Handbook, Englewood Cliffs, New Jersey, Prentice-Hall Inc., 1973, p. 430.

Table XVI.-

The Mean Difficulty and the Variance for the Directional Versions of the Task Types in Number and Numeration

| Direction | | Task Type<br>Task 1 | Task 2 | Task 3 | Task 4 | Task 5 |
|---|---|---|---|---|---|---|
| Forward | Mean | 0.916 | 0.675 | 0.725 | 0.659 | 0.638 |
| | Variance | 0.0438 | 0.1397 | 0.1253 | 0.1440 | 0.1319 |
| Reverse | Mean | 0.942 | 0.883 | 0.570 | 0.490 | 0.333 |
| | Variance | 0.0347 | 0.0640 | 0.1154 | 0.1488 | 0.1227 |

Task 1 Types are non-regrouping preoperational tasks with non-zero digits.

Task 2 Types are non regrouping preoperational tasks with zero and non-zero digits.

Task 3 Types are concrete operational tasks.

Task 4 Types are provoking double regrouping provoking formal operational tasks.

Task 5 Types are double regrouping formal operational tasks.

Table XVII.-

The Summary of the Analysis of Variance of the Task Type and Directional Variables

| Source | Sums of Squares | df | Mean Square | F |
|---|---|---|---|---|
| D | 6.1474 | 1 | 6.1474 | 71.054* |
| S | 212.1470 | 393 | 0.0025 | |
| D × S | 34.0012 | 393 | 0.0865 | |
| T | 95.9551 | 4 | 23.9888 | 356.124* |
| T × S | 105.8911 | 1572 | 0.0674 | |
| D × T | 31.1939 | 4 | 7.7985 | 178.895* |
| D × T × S | 68.5275 | 1572 | 0.0436 | |
| Total | 553.8632 | 3939 | | |

D = Directional variable

T = Task Type variable

S = Subjects

* Significant at the Greenhouse-Geisser Conservative Level of Significance: $_{.95}F_{(1,393)} = 3.87$

involving only those data relevant to the comparison. Thus, there is a different error term for each specific comparison.

To examine possible décalages in the child's development of large number concepts when represented with a decimal numeration system, four specific contrasts among the five task type difficulties were performed. Two contrasts were made to assess the existence of a vertical décalage in the child's development of an operative understanding of large decimal numerals. The first contrast involved the testing, in the null form, of the significance of the difference in difficulty between the pre-operational non-regrouping tasks with non-zero digits (Type 1), and the concrete operational regrouping tasks (Type 3). In the second contrast, the significance of the difference in difficulty between the concrete operational single regrouping tasks (Type 3), and the formal operational double regrouping tasks (Type 5) was tested in the null form.

Questions might arise concerning the omission of the preoperational non-regrouping tasks with zero and non-zero digits (Type 2), and the provoking formal operational double regrouping tasks (Type 4) in the assessment of contrasts relevant to a vertical décalage in the development of the operative understanding of large decimal numerals. Task types 1, 3 and 5 have similar content. These tasks test the child's understanding of four-digit numerals composed of non-zero digits. The representations of the numerals in these tasks are similar. These sets of tasks differ only with respect to the level of operative thought required to solve them. Hence, if differences are observed among these tasks, they may be attributed to the differences in the operative levels in the tasks.

To compare task types 2 and 3 to assess a vertical décalage between preoperational and concrete operational tasks, one would find an additional variable confounding the results. Task types 2 and 3 differ in the operative level required to solve them. In addition, the contents of these sets of tasks differ. The preoperational tasks (Type 2) involve numerals composed of zero and non-zero digits, whereas in the concrete operational tasks (Type 3), the numerals consist of non-zero digits. In the pictorial, and the verbal representation of the type 2 tasks, there is no count in one column, therefore, only three of the four columns are represented. In the concrete operational tasks, there is some count in each column, and all four columns are represented in the pictorial and verbal representations. Thus, if differences are observed between task types 2 and 3, it is impossible to determine if it is the operative differences, the content differences or both contributing to the results.

Similarly a comparison of the difficulty of concrete operational single regrouping tasks (Type 3) and provoking formal operational double regrouping tasks (Type 4) is not appropriate to assess the existence of a vertical décalage between concrete and formal operational tasks. That is, there are other differences in these tasks than just operational differences. Hence, it would be impossible to determine which variable is affecting the observed results. For a vertical décalage between tasks, it must be the operational differences contributing to the results.

When tasks vary in content, perceptual or representational aspects, horizontal décalages may occur. Two contrasts relevant to horizontal décalages were made. In the comparison of the difficulties of the preoperational non-regrouping tasks with non-zero digits (Type 1) and the preoperational non-regrouping tasks with zero and non-zero digits (Type 2), the operational levels of the tasks are the same, as are the correspondences used to present the tasks. These tasks differ only with respect to the digit content in the represented numbers in the tasks. In the type 1 tasks, there is some count in each column, therefore, all four columns are represented in the pictorial and verbal representations. In the type 2 tasks, there is no count, or a zero, in one column with the result that only three of the four columns are shown in these representations. Since there are content differences in these tasks, any observed dĕcalage would be horizontal.

In the fourth comparison, the operational levels of the provoking formal operational double regrouping tasks (Type 4) and the formal operational double regrouping tasks (Type 5), are the same, as are the correspondences. There are zeros in the counting name representations of the type 4 tasks; however, there is in both the type 4 and 5 tasks, some count in each column of the pictorial and verbal representations. Thus, the content of these tasks is similar. The difference between these tasks is perceptual, thus, a horizontal dĕcalage is expected.

The results for the testing of these comparisons in the null form are reported in Table XVIII. Each contrast is significant ($p < 0.01$). There is, therefore, support for Piaget's theory of concept formation. One vertical and two horizontal dĕcalages are identified in the child's formation of the complex concept of large numbers involving decimal numeration.

Table XVIII.-

Summary Analysis of the Planned Contrasts of the Task Type Differences in Difficulty

| Source | Sums of Squares | Degrees of Freedom | Mean Square | F |
|---|---|---|---|---|
| $T_1T_3$ Contrast | 31.221 | 1 | 31.221 | 379.75* |
| $T_1T_3$ Error | 32.311 | 393 | 0.082 | |
| $T_3T_5$ Contrast | 10.340 | 1 | 10.340 | 244.08* |
| $T_3T_5$ Error | 16.649 | 393 | 0.042 | |
| $T_1T_2$ Contrast | 8.865 | 1 | 8.865 | 171.30* |
| $T_1T_2$ Error | 20.389 | 393 | 0.052 | |
| $T_4T_5$ Contrast | 3.121 | 1 | 3.121 | 71.43* |
| $T_4T_5$ Error | 17.170 | 393 | 0.044 | |

The $T_1T_3$ Contrast is between the preoperational tasks with non-zero digits and the concrete operational tasks.

The $T_3T_5$ Contrast is between the concrete operational and the formal operational tasks.

The $T_1T_2$ Contrast is between the preoperational tasks with non-zero digits and the preoperational tasks with zero and non-zero digits.

The $T_4T_5$ Contrast is between the provoking formal and the formal operational tasks.

* Significant at the Greenhouse-Geisser Conservative Level of Significance: $_{.99}F_{(1,393)} = 6.71$

The vertical décalage exists among tasks requiring different levels of operative thinking. The non-regrouping preoperational tasks with non-zero digits, the single regrouping concrete operational tasks, and the double regrouping formal operational tasks, differ in the level of operative thinking required. To solve non-regrouping tasks the child has only to compare corresponding columns in the two representations. Since the child does not have to make a transformation of the digits in the columns of the representations, the child requires at most a semilogical preoperational level thinking structure to succeed at the non-regrouping tasks.

In solving single regrouping tasks, the child must perform a carrying regrouping in forward versions, or a borrowing regrouping in the reverse versions before completing the correspondence. In both versions, the regrouping involves changes in two columns which must be co-ordinated, thus, a two-dimensional concrete operational structure is required to solve single regrouping tasks.

The difference in difficulty between the non-regrouping and single regrouping tasks is significant, and, from the scalogram analysis these tasks were observed to be mastered successively. These observations considered together are support for Piaget's contention that the child must make major changes in his thinking structure in the advancement from the semilogical preoperational to the concrete operational level of understanding concepts.

In the double regrouping tasks, there are two successive re-groupings affecting three columns. The child must learn to repeat the regrouping process systematically to be successful. In the reverse double regrouping tasks this is particularly complex. In these tasks, the blank to be completed is at the second of the three columns affected by the double regrouping. Whereas in the reverse single regrouping tasks, the value in the blank is affected by the column to the one side, in these double regrouping tasks the value in the blank is affected by the columns on either side. Therefore, when one regrouping is completed, the child must be aware that the third column may also affect his response, and he must continue examining all columns. This is the characteristic behavior of the formal operational child.

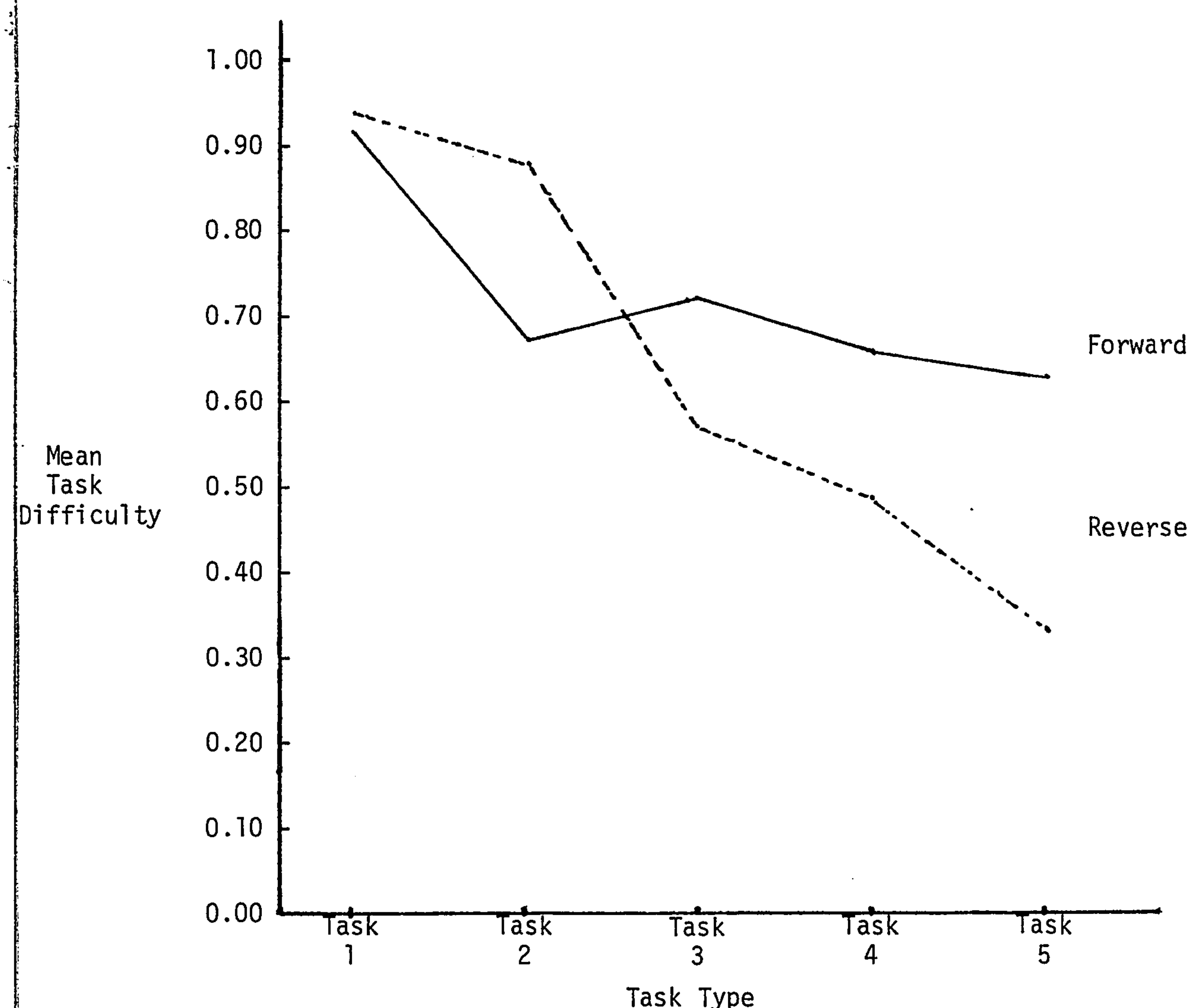
In the scalogram analysis, single regrouping concrete operational and double regrouping formal operational tasks were mastered in succession. The modification of the thinking structure required in the child's advancement from the concrete to the formal level of understanding does not appear to be very great. However, the significant difference in difficulty between the tasks is an indication that for the child in the process of development, it is not easy to anticipate that other columns may affect his response.

It must be noted that the décalage between the non-regrouping tasks may have been affected by the instructions. That is, with non-regrouping examples the child may not have expected regrouping tasks even though he was warned that the actual tasks were more difficult.

Thus, preoperational non-regrouping tasks are easier than concrete operational single regrouping tasks, and in turn, the concrete operational tasks are easier than the formal operational double regrouping tasks. In Appendix 8, the difficulty indices for these tasks are reported for each class grouping. In every instance the preoperational non-regrouping tasks are the easiest tasks, while the formal operational double regrouping tasks are the most difficult. This consistency of observed differences, and the observed sequence in the mastery of preoperational, concrete operational and formal operational tasks are support for the conclusion that a vertical décalage exists in the child's ability to succeed at these tasks. That is, the child must modify his operative thinking structure to solve these tasks.

The first horizontal décalage examined was between tasks differing in content. Preoperational tasks with numerals composed of zero and non-zero digits are significantly more difficult than preoperational tasks with numerals containing only non-zero digits. From the graph in Figure 13, the major contribution to this significant difference between the tasks appears to be from the forward versions of the preoperational non-regrouping tasks with zero and non-zero digits. With only three of the four columns being represented in these tasks, the child must supply a zero in the correct column of the counting name numeral to indicate that there is no count in one column of the pictorial or verbal representations. The reverse versions of the preoperational tasks with zero and non-zero digits also complicate the non-regrouping tasks. Hence, children do find that zero in a numeral does tend to make mathematical operations more complex.

This finding stresses the importance of using tasks involving numerals with zero. The number concept includes all ten digits. Therefore, to form this complex concept, the child requires experiences with numerals composed of all ten digits.




Forward refers to the forward directional task versions.
Reverse refers to the reverse directional task versions.

Figure 13.- The Mean Task Difficulty for the Directional Versions of the Five Task Types in "Number and Numeration"

Thus, a horizontal dēcalage exists between tasks with numerals that do or do not include the digit zero. This is a horizontal dēcalage arising from content differences.

It may be noted that with horizontal dēcalages, Piaget is not interested in the sequence of task mastery and in the consistency of the observed results. He considers some horizontal dēcalages such as those arising from content differences to be affected by societal influences. Thus, influences such as classroom instruction may affect the sequence and the consistency of certain horizontal dēcalages from group to group.

In this particular horizontal dēcalage, it would seem that there may be a specific sequence in task mastery, and consistency in the observation of this dēcalage. That is, to learn zero to indicate an absence of something, the child should be aware of other numerals indicating an amount of something. For all class groupings, the non-regrouping tasks with zero and non-zero digits were more difficult than those with non-zero digits. Although a specific sequence seems reasonable, and there is consistency from group to group, this dēcalage is horizontal. It arises from content differences as opposed to operative differences.

In the second horizontal dēcalage, different perceptual factors in two sets of formal operational tasks were expected to affect the child's results. The provoking formal operational tasks were significantly easier than the formal operational tasks.

The explanation of the perceptual factors appears more complex than expected. The forward provoking formal tasks with a double regrouping seem similar to the forward concrete tasks with a single regrouping. They were expected to be solved similarly. That is, the child would carry the one to be regrouped, add it to the nine in the adjacent column, then repeat the process automatically.

In contrast, to solve the forward double regrouping formal tasks, the child was expected to be confused by the observations of the two required regroupings. Familiar with single regrouping tasks (and four-digit numerals), the child would be expected to add the quantities (16+14) to be regrouped as in the task that follows:

3 thousands 16 hundreds 14 tens 5 ones = 3305

This was the most frequently used inappropriate approach observed in all grades. Thus, the child appears to be affected by the perceptual factors in the forward versions as expected.

In the reverse tasks, the child was expected to find the provoking tasks easier to check, hence, he would make fewer errors. This expectation was held because a situation similar to that in the forward tasks would exist in the checking process. The difference in difficulty between the reverse tasks was not expected to be greater than that between the forward versions. From the graph in Figure 13, the results were contrary to expectations. The difference in difficulty between forward versions of the formal tasks was less than that between the reverse versions. It is concluded that some other perceptual factor is involved in the reverse versions.

In an examination of the mathematical exercises of several texts, numerals such as "4000" were found to be used in numerous subtraction exercises. This was especially true in money problems when a child had to tell how much change a person would receive in a particular transaction. Such numerals were used much less frequently in other operations.

From classroom observation, the child very soon develops techniques to solve such problems. His work usually is as follows:

$$\begin{array}{r} 2\ 9\ 9\ \ \ \\ \not{3}\ \not{0}\ \not{0}\ {}^{1}0 \\ -1\ 8\ 6\ 7 \\ \hline 1\ 1\ 3\ 3 \end{array}$$

It would appear that the child understands what he is doing. However, the child frequently cannot explain what he is doing. Furthermore, if presented a task such as "a", the child often responds as in "b".

$$\text{(a)}\quad \begin{array}{r} 3\ 0\ 0\ 9 \\ -1\ 8\ 7\ 6 \\ \hline \end{array} \qquad\qquad \text{(b)}\quad \begin{array}{r} 2\ 9\ 9\ \ \ \\ \not{3}\ \not{0}\ \not{0}\ 9 \\ -1\ 8\ 7\ 6 \\ \hline 1\ 1\ 2\ 3 \end{array}$$

In this situation, one is aware that the child is using a technique to solve the tasks with a minimum of understanding involved. That is, the child knows he has to regroup, but not how to regroup when there are zeros.

Thus, it is concluded that the décalage between these reverse formal tasks is the result of the child adopting a response technique which just happens to be correct in the provoking tasks. The child using the correct response technique would have both provoking formal and formal tasks correct. This is still a horizontal décalage, as the perception of the multiple of ten results in the child selecting a specific technique.

The two horizontal décalages examined stress the elusiveness and the complexity in the study of the concept of zero. In the previous horizontal décalage, zero with no count in one column of the verbal and pictorial representations complicated the operation. In this décalage, the zero with some count in all columns of the verbal and pictorial representations made the tasks easier. Hence, the child's response to zero is dependent upon the specific situations in which it occurs.

In teaching the use of zero, these décalages are an indication that when zero is used in numerals, there should be variety in both the number and the location of the zeros. Then the child's generalization will be inclusive for all tasks.

The explanation of this second horizontal décalage is in part based on the retrospection of classroom observations and therefore, further investigation is required.

An examination of the significant task direction by task type interaction was made using the graph in Figure 13.

Reversibility is to Piaget an important operative characteristic of the child's thought. If the child can reverse and return to previous thoughts, his thinking is flexible. The more flexible a child is in reversing his thinking, the more advanced he is in the level of operative development.

For all task types, there are forward and reverse versions. If a child is successful on both versions, he is flexible in considering the concept. If the child can only solve one version of a task type, he is considered partially flexible in understanding the concept. Finally, non-success on each version is an indication that the child has no mastery of the concept.

There are three sets of operative tasks in Number and Numeration. In the forward versions of the concrete operational single regrouping tasks, the child must add the quantities represented to determine the amount represented. In the reverse versions of these tasks, the total amount is presented. As well, the amount for three of the four columns is shown. To determine the amount which should be in the blank column, the child requires some reversibility in his thought. He has to determine the difference between the amount presented and the total amount. The child may add the presented amounts and subtract it from the total to find the solution. Thus, between the forward and reverse versions of the concrete operational tasks there is a difference in the level of flexible thought required of the child. In the reverse versions, more reversible flexibility is required. Similarly, with the reverse versions of the provoking formal operational double regrouping, and the formal operational double regrouping tasks, more reversible flexibility is required of the child, than in the forward versions of these tasks.

It was noted that there are observed discrepancies between the two representational modes in the reverse operational tasks which may confuse the child. These discrepancies do not occur in the forward operational tasks as there is only one representation shown. Unlike the perceptual factors in the horizontal décalage discussed above, these forward and reverse tasks differ in ways other than the perceptual differences. The reverse versions require a more complex level of operative thought than the forward versions. Furthermore, these discrepancies are inherent in the reverse tasks. Thus, any differences between the two versions of the operative tasks is the result of operative rather than perceptual differences. As expected, the child finds the reverse operational tasks considerably more difficult than the forward operational versions. It is easy for the child to total the amounts shown in one representational mode. To total the amounts in one, and determine the amount required to complete a correspondence is difficult, particularly when the child must be able to account for the discrepancies between the representations. The more flexible a child is in his thinking, the better he is in overcoming any perceived confusion. Many of the younger children resorted to changing the digits in the numeral so that there was no discrepancy. As well, they indicated to their teacher that there was a mistake in the question which necessitated their alteration. Obviously they did not have reversible flexibility, and they had to adopt an inappropriate technique.

The forward and reverse versions of the operational tasks were found to differ in difficulty. Furthermore, in the scalogram analysis, forward versions of these tasks were found to be mastered before the reverse versions. The conclusion is that there is a vertical décalage between the directional versions of the operational regrouping tasks.

For the non-regrouping preoperational tasks, the reverse versions are easier than the forward versions. In the reverse versions of these tasks, the correspondence for three of the four columns is made and the child has only to complete the correspondence for the fourth column. In the forward versions, the child has to make the correspondence in all four columns. Hence, the difference in difficulty is as expected.

If a vertical dĕcalage exists between the directional versions of these non-regrouping preoperational tasks, there must be a specific sequence in their mastery. For the non-regrouping tasks with non-zero digits, it is difficult to determine if there is any specific sequence. These tasks were very easy. A younger age group of children is required if one wishes to assess this sequence of task mastery.

To determine if the reverse versions of the non-regrouping tasks with zero and non-zero digits are mastered before forward versions, the proportion of students for all possible scoring patterns is shown in Table XIX. From this grid, 55.3% of the children found reverse and forward tasks equally difficult. Another 38.4% found reverse tasks easier than forward tasks. Only 6.4% found forward tasks easier. This analysis is not ideal, however, some data exist to support the contention that reverse versions are mastered before forward versions.

Although, there is some limited support for a specific sequence in the mastery of the directional versions of some preoperational tasks, a vertical dĕcalage cannot be inferred to exist between them. The interaction of the task direction and task type is an indication that the child may not be responding operatively to these tasks. Otherwise,

Table XIX.-

Proportions of Students for Each Scoring Pattern on Forward and Reverse Non-regrouping Tasks with Zero and Non-zero Digits

| Number of Correct Forward Task Responses | Number of Correct Reverse Task Responses | | |
|---|---|---|---|
| | 0 | 1 | 2 |
| 0 | 0.020 | 0.033 | 0.107 |
| 1 | 0.018 | 0.069 | 0.244* |
| 2 | 0 | 0.046 | 0.464 |

* The proportion of students with two reverse and one forward response correct is 0.244.

the child should find the reverse versions more difficult than the forward versions, as he does for the concrete operational, provoking formal and formal operational tasks. In the preoperational tasks, it is possible for the child to respond either perceptually or operatively. This is similar to the younger child who can make a one-one correspondence between a set of parallel counters, yet deny the equivalence of the sets after a clustering of one set. Such a child is making a perceptual correspondence, just as an older child may do in solving the preoperational tasks above.

It could not be ascertained in this study, if all the children are reacting perceptually to the preoperational non-regrouping tasks, or only the children with a very limited understanding of decimal numeration concepts. It would be informative to examine if the more operatively advanced child changes his approach to the preoperational tasks, or whether he persists in solving these tasks at a perceptual level. Some mathematical textbooks have exercises which consist primarily of preoperational non-regrouping tasks. If a teacher knows the child will continue to solve these tasks at a perceptual level, he can be more selective in choosing exercises which require more advanced-operative behavior.

The dĕcalage between the directional versions of the preoperational non-regrouping tasks cannot be classified as vertical or horizontal. In fact, it may differ for different groups of children. The younger children may only respond perceptually and for them, the dĕcalage would be horizontal. For older children responding operatively, it may be a vertical dĕcalage.

Thus, in the results of the testing of Null Hypotheses I and II, there is support for the existence of both vertical and horizontal décalages in concept formation. Vertical décalages occur between tasks requiring different levels of operative thinking, whereas horizontal décalages were observed between tasks involving different content and perceptual aspects. Flexible reversibility of thought appeared to differ for preoperational and operational tasks. For the preoperational tasks, the required flexibility seemed to be at a perceptual level.

6. Concept Development by Grade.

The variance-covariance matrices among the three regrouping task types for all subjects in each of the five grades, and the pooled variance-covariance matrix are presented in Appendix 9. The summary data for the test of the assumption of the equality of the variance-covariance matrices are reported in Appendix 10. The result of this test is significant.

This significant result is interpretable. Both the grade and the regrouping task variables are in a sense graduated. Subjects in higher grades are expected to be more highly developed than subjects in lower grades. In addition, the three tasks differ in difficulty. Hence, it follows that there may not be equality of the variance-covariance matrices at different grade levels. That is, the variance of the formal operational double regrouping tasks for the Grade Four subjects may be small. These children are just being introduced to large numerals, and will find these double regrouping tasks very difficult. Similarly, the variance for the concrete operational single regrouping tasks at the

Grade Eight level may be small. In this situation, the children will have a good mastery of these easiest regrouping tasks. In other situations such as at the Grade Six level, variances for the three tasks may be large. At this level, some will have full mastery, and others will have partial mastery. With a wider range in the children's mastery of the tasks, a larger variance is expected.

The correlations among tasks will also vary at the different grade levels. Correlations will be smaller when tasks have smaller variances, and greater when tasks have larger variances. As a result, the covariances among tasks will differ at various grade levels.

Although, this heterogeneity of the variance-covariance matrices is meaningful in itself, it makes the interpretation of the significance of Null Hypothesis III very difficult. According to Noe,[3] it is impossible to determine the effect of heterogeneity on the level of significance when there are unequal numbers of subjects at each level. This renders any interpretation of significant results impossible. He has shown, however, that when there are equal numbers of subjects at each level, the nominal level of significance tended to be overestimated slightly when the assumption of the equality of the variance-covariance matrices was violated. For the interaction,[4] and the non-repeated main effect tests[5] the largest overestimation for a nominal level of significance of 0.01 was 0.020. For a nominal level of significance of 0.05, the largest

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3 Michael J. Noe, <u>A Monte Carlo Study of Several Test Procedures in the Repeated Measures Design</u>, a Paper presented at the 1976 Annual Meeting of the American Educational Research Association, San Francisco, California, April 19-23, 1976, p. 52.

4 <u>Ibid.</u>, p. 39.

5 <u>Ibid.</u>, p. 29.

overestimation for the multivariate interaction test was 0.074. For the grade test, the largest overestimate was 0.076. Therefore, to make a meaningful interpretation of the testing of Null Hypothesis III, it was decided to equalize the number of subjects at each level by a random procedure. The nominal level of significance may be overestimated, but with equal numbers of subjects, there is some limit to the amount of overestimation expected. A decision was also made to test Null Hypothesis III at the nominal 0.01 level of significance to limit the expected overestimate to 0.02 approximately.

The difficulty indices for the three regrouping tasks at each grade level are presented graphically in Figure 14, and in tabular format in Table XX. It is noted in Figure 14 that the change in difficulty across the grades is similar for the three regrouping tasks. If the changes for each task are parallel, the multivariate test of Null Hypothesis III essentially reduces to a univariate analysis of the differences among grades in the average regrouping task difficulty.[6] Thus, the average regrouping task difficulty at each grade is of interest, and these indices are reported in Figure 14 and Table XX.

The criteria for the test of the parallelism of the changes in the difficulties of the regrouping tasks across the grades are reported in Table XXI. The hypothesis of parallelism is retained. Consequently, the test of Null Hypothesis III is reduced to the univariate analysis of grade differences in the average regrouping task difficulty.

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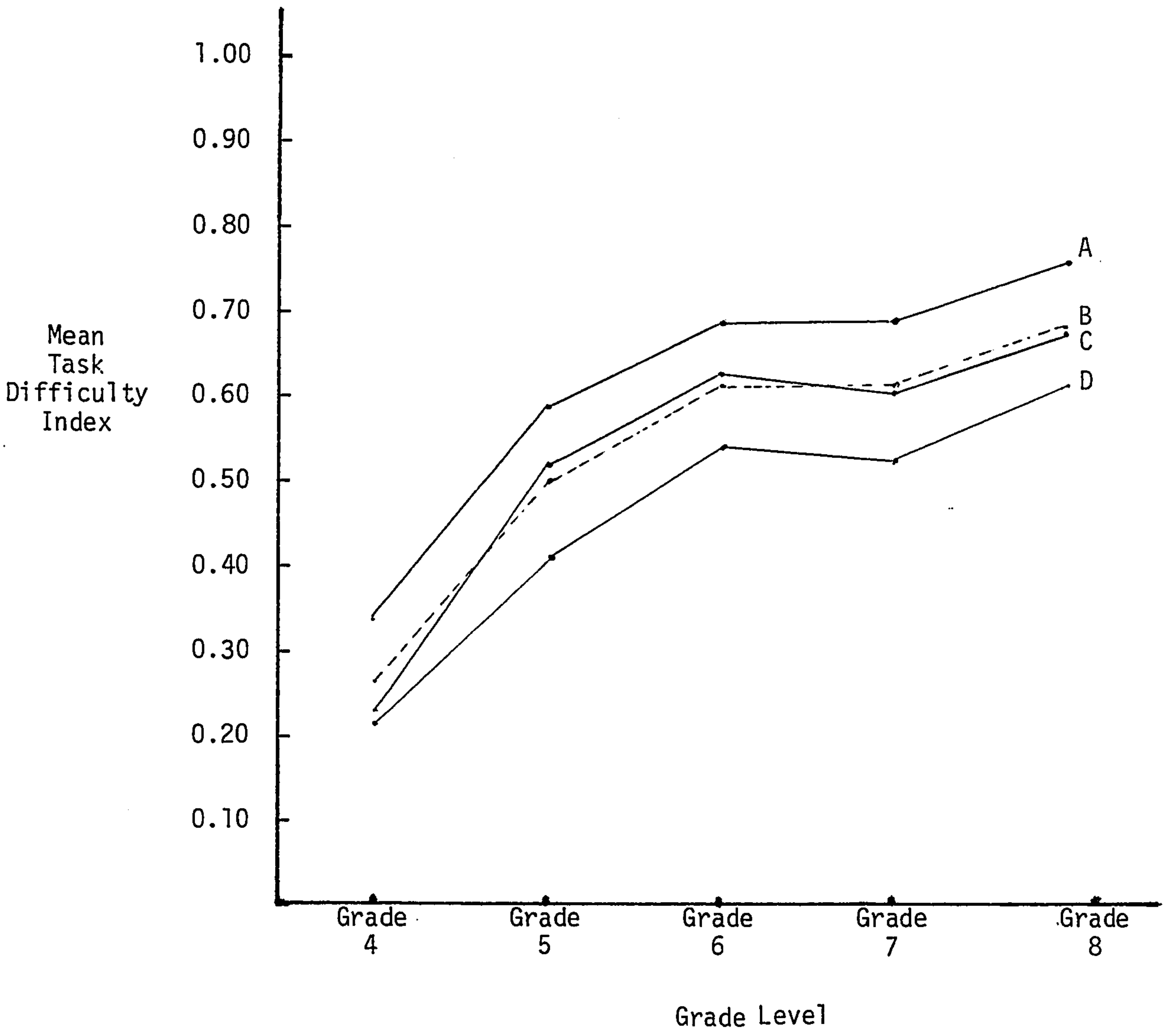
6 Ibid., p. 10.




A indicates Task 3.
B indicates the Regrouping Task Average.
C indicates Task 4.
D indicates Task 5.

Figure 14.- The Change in the Level of the Difficulty of the Regrouping Tasks and the Regrouping Task Average from Grade Four to Eight

Table XX.-

The Regrouping Task Difficulties, and the Average Regrouping Task* Difficulty for Subjects in Grades Four to Eight

| Grade | Task 3 | Task 4 | Task 5 | Regrouping Task Average* |
|---|---|---|---|---|
| Four | 0.338 | 0.228 | 0.214 | 0.260 |
| Five | 0.588 | 0.520 | 0.409 | 0.505 |
| Six | 0.690 | 0.625 | 0.540 | 0.618 |
| Seven | 0.690 | 0.603 | 0.531 | 0.608 |
| Eight | 0.755 | 0.675 | 0.614 | 0.681 |

* Average Regrouping Task difficulty is the average difficulty of the three regrouping tasks at each grade level.

Table XXI.-

Criteria for the Testing of the Parallelism of the Regrouping Task Types and the Grade Variables

| Criterion | Observed | | Critical $\alpha = 0.01$ | Significant |
|---|---|---|---|---|
| Roy's Largest Root | 0.0253 | < | 0.0453 | no |
| Hotelling's Trace Criterion | 0.0273 | < | 0.0841 | no |
| Wilk's Lambda | 0.9727 | > | 0.9197 | no |
| Pillai's Trace Criterion | 0.0266 | < | 0.0801 | no |

The summary of the univariate analysis of variance is presented in Table XXII. Null Hypothesis III is rejected in favour of the third major research hypothesis. A post hoc analysis of the differences among grades in the average regrouping task difficulty was made using Scheffé's comparison procedure as outlined by Kirk.[7] The results of this analysis are presented in Table XXIII.

An assumption made in this study was that all the children in these Grade Four to Eight classes had at least a concrete operational thinking structure. That is, it was assumed that all the children were capable of reasoning concerning changes in two related variables in some concept area. This is an easy assumption to make as any child without this structure would easily be recognized as progressing very slowly. Furthermore, such a child would not likely be in a regular classroom.

Although all the children are assumed to have formed a concrete operational thinking structure, they may not be at the operational stage in the understanding of large number concepts involving numerals from a decimal numeration system. If a child has no experience with the representation of these concepts, he cannot form an operational concept of large numbers.

To develop the concept, the child must experience it through its representation. This experience enables the child to internalize the basic decimal numeration representation of number. The child's ability

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7 Roger E. Kirk, <u>Op. Cit.</u>, p. 90, 91.

Table XXII.-

Summary of the Analysis of Variance of the Differences Among Grades in the Average Regrouping Task Difficulty

| Source | Sums of Squares | Degrees of Freedom | Mean Square | F |
|---|---|---|---|---|
| Grades | 5.5175 | 4 | 1.3794 | 18.42* |
| Error | 18.3450 | 245 | 0.0749 | |

* Significant at: $_{0.01}F_{(4,245)} = 3.39$

Table XXIII.-

Summary of the Post Hoc Analysis of the Differences in the Average Regrouping Task Difficulty Between Successive Grades using Scheffé's Procedure

| Contrast between: | Observed Scheffé "F" | Confidence Interval |
|---|---|---|
| Grades Four and Five | 20.035* | 0.245 ± 0.202* |
| Grades Five and Six | 4.262 | 0.113 ± 0.202 |
| Grades Six and Seven | 0.033 | ⁻0.010 ± 0.202 |
| Grades Seven and Eight | 1.779 | 0.073 ± 0.202 |

* Significant at: $(k-1)_{0.01}F_{(4,245)} = 13.56$

where $k = 5$, the number of Grade Levels to solve preoperational tasks is an indication that this internalization of the basic representation has occurred.

To understand more about the representation, and hence, more about the number concept, the child must integrate the representational schema into his concrete operational thinking structure. The child's ability to perform both forward and reverse versions of the concrete operational tasks is an indication that this integration has occurred.

Between Grades Four and Five there is a significant increase in the average regrouping task difficulty level. That is, between Grades Four and Five, the child is involved primarily with experiencing the decimal numeration representation of large number concepts, internalizing the representation and integrating it into the concrete operational thinking structure. Hence, between these grades, the child makes great advances in his ability to solve the three regrouping tasks.

To advance past the concrete operational level of development of the decimal numeration concepts, the child must change his thinking structure. He must develop a formal operative thinking structure. This development, unlike the development which occurs between Grades Four and Five, cannot be accelerated by presenting the child with learning experiences. Instead, the child must adapt his thinking so that he can be logical and systematical in considering several variables in a task.

Children at the Grade Six level are about eleven to twelve years of age. At this grade, a few children may begin to develop the formal operational thinking structure. The majority, however, still have a concrete operational structure. Not until about fourteen or fifteen years of age will children, on the whole, have formed the formal operational structure.

The results of the testing of Null Hypothesis III support this development. In Figure 14, it is noted that between Grades Four to Six the development of decimal numeration concepts is steadily increasing. The change between Grades Five and Six, however, is not significant. The children at this point are still integrating the decimal numeration representation into their concrete operational structure, thus they can manipulate the large number concepts more easily. Some children, though, will be reaching a plateau where further improvement in the tasks cannot be achieved until they develop a formal operational thinking structure. Consequently, with some children still in the process of integrating, and with some reaching a plateau, there is an increase in the average regrouping task level of difficulty, but not a significant one.

Between Grades Six and Seven, most of the children have integrated as much from their experiences of the concept as they can with a concrete operational thinking structure. From Figure 14, the plateau between Grades Six and Seven is observable.

By Grade Eight, mid-term, when many of the students would be into the formal operational stage, there is a general increase in the understanding of the regrouping tasks. Some pupils, no doubt, will still be at the plateau awaiting the advancement into the formal operational stage. Hence, the increase in the level of difficulty of the regrouping task average is not significant.

The general increase in the three regrouping tasks is meaningful. The child with a formal operational structure is able to develop procedures to solve the formal operational tasks. These procedures were discussed, and were shown to differ somewhat from procedures which the child develops to solve concrete and preoperational tasks. Whereas preoperational and concrete operational procedures are inappropriate for solving formal operational tasks, formal procedures may be used to solve all the tasks. That is, formal operational procedures are generalizable. When used, these procedures help the child to reduce errors. As a result, there is at the Grade Eight level an overall improvement in the three regrouping tasks.

The fact that there was not a significant difference between the average regrouping task difficulties at Grade Seven and Eight, is an indication that perhaps it may have been informative to have extended this study to the Grade Nine level. This is reinforced by the observed average regrouping task difficulty. Specifically, there is only about a 70% mastery of these regrouping tasks at the Grade Eight level. For the formal operational double regrouping tasks, the level of mastery is about 60%. This is not particularly high, considering these tasks assess the child's understanding of four-digit numerals. By Grade 9 more students will have developed a formal operational structure and will have integrated these concepts into it.

Nevertheless, from the analysis, evidence is observed to support Piaget's theory of concept development. Children in Grades Four to Eight have at least a concrete operational thinking structure. They are ready, then, to benefit from experience in the decimal numeration representation of number. These experiences are internalized and integrated into their concrete operational structure. A steady increase is evident during the Grade Four to Six level. Experience cannot generate further development. The children must, at this point, await the development of the formal operational thinking structure. By mid-term Grade Eight, an increase in the difficulty of the regrouping tasks indicates that many children are developing this advanced and generalizable thinking structure.

In this chapter, the presentation and the discussion of the results were undertaken. In these six sections, insights were furnished into particular aspects involved in the child's formation of large number concepts when numbers are represented with a decimal numeration system. Furthermore, these aspects were found to support Piaget's generalized theory of concept development.

## SUMMARY AND CONCLUSIONS

In this study, the concept of number was found to be complex. To understand the number concept, the child must master many specific concepts. Two of the many specific concepts comprising the complex number concept involve the child's understanding of small and large numbers. These specific concepts differ. Small number concepts are derived primarily through experiences with sets of physical objects. Large number concepts, in contrast, are encountered through experiences with decimal numerals. Hence, these specific concepts comprise two of the numerous different aspects of the overall number concept.

According to Piaget, in the formation of the small number concepts, figurative and operative processes of knowing are involved. In fact, these processes are involved in the formation of any concept, including the concept of large numbers. These processes themselves are developing. Thus, the concept that a child forms at any particular stage is dependent upon his level of figurative and operative development.

The figurative aspect of knowing refers to the child's ability to collect, store and retrieve information. Specifically, it refers to the child's ability to represent his concepts. In the formation of the number concept, the child may use several modes of representation. As the child develops, he is capable of using more complex modes of representation. Whereas physical objects may be used early to represent number, the child also learns to use other representations such as decimal numeration.

The operative aspect of knowing refers to the child's ability to logically understand the concept. As the child develops, he is increasingly more flexible in understanding transformations of variables concerning the concept. At the semilogical level of the preoperational stage, the child understands transformations involving single variables. At the concrete operational stage, the child understands transformations involving two variables, while at the formal operational stage transformations involving several variables are understood.

In the development of the figurative and operative aspects of a concept, décalages may be observed. Décalages in the figurative process are called horizontal décalages. These décalages result when different content, representational or perceptual obstacles must be mastered. Décalages in the operative process are vertical décalages. These décalages result when different levels of operative understanding are required to solve tasks.

Piaget and other researchers have examined horizontal and vertical décalages in the child's formation of small number concepts. Little research has been performed to examine the child's formation of large number concepts. Thus, tasks were constructed to elicit the existence of horizontal and vertical décalages in the formation of large number concepts involving decimal numerals.

Significant décalages were observed. In vertical décalages, tasks requiring different levels of operative thinking were significantly different in difficulty. Furthermore, tasks requiring different operative levels of thinking were mastered in a specific sequence. Basically, preoperational tasks were mastered first, then concrete and formal operational tasks. As well, forward versions of the operational level tasks were mastered before the reverse versions. Hence, Piaget's postulated sequence of development was observed.

In horizontal décalages, tasks requiring different figurative aspects differed significantly in difficulty. One horizontal décalage was observed between tasks differing in the digit content of the numerals. If the digit zero had to be used to indicate no count in a column, a task was more difficult than a similar task with numerals composed of non-zero digits. A second horizontal décalage occurred between tasks differing in perceptual aspects.

The development of large number concepts across grades indicates strong support for Piaget's theory of cognitive development. That is, the child progresses through specific stages in which he develops increasingly more complex and generalizable thinking structure. A child, however, is not able to use a specific structure in all situations. That is, he requires experiences with the different content, representational, and perceptual aspects. When exposed to these aspects, development is rapid, and limited only by the child's level of operative thinking. Further development is possible when the thinking structure is adapted. Since this adaptation is towards a more generalizable structure, the child may be observed to improve in all areas in which he has experience.

Further research is indicated. In the discussion of the results, the use of the digit zero as a placeholder was found to create difficulties for the child. Further examination of this elusive concept is required.

It may be preferable to use a clinical approach in the early investigation stages.

Little is known about the child's formation of other concepts. There are, for example, other mathematical concepts, or language concepts which may be examined using Piaget's theory as a basis.

Many researchers have stressed the existence of a relationship between the child's understanding of decimal numerals and his success at the four mathematical operations. Experimental support is limited. This relationship should be examined further.

Finally, numerous measurement problems arise in the evaluation of Piagetian-based research. For example, to examine vertical décalages one is interested to know if the difference between tasks is significant. As well, one wants to know if a specific sequence is followed in the mastery of the tasks. No adequate test exists to examine both questions simultaneously, and the second question is difficult to answer with existing tests. A solution to measurement problems such as this, would be an invaluable contribution to the study of the child's formation of concepts.

## ANNOTED BIBLIOGRAPHY

Elkind, David, "Conservation and Concept Formation", in David Elkind and John H. Flavell (eds.), Studies in Cognitive Development: Essays in Honor of Jean Piaget, New York, Oxford University Press, 1969, p. 171-190.

Elkind compares the intensive and extensive aspects of a concept. Then he discusses the evaluation of the intensive aspect, the aspect with which Piaget is concerned. This article clarifies Piaget's purpose in selecting specific tasks in his evaluation of concept formation.

Flavell, J. H., The Developmental Psychology of Jean Piaget, New York, Van Nostrand Co., Inc., 1963, xvi-472 p.

Flavell explains Piagetian theory to the North American researcher. Included is an excellent bibliography of Piagetian publications to about 1960.

Furth, Hans G., Piaget and Knowledge, Englewood Cliffs, N. J., Prentice-Hall Inc., 1969, 232 p.

This is a psychological discussion of Piaget's theory. Of particular merit, is his glossary of Piagetian terms.

Jean Piaget, The Child's Conception of Number, London, Routledge and Kegan Paul Ltd., 1952, vii-243 p.

Piaget presents the results and discussion of his examination of the child's formation of the early number concept. This has been the basis for numerous studies concerning the child's understanding of number.

--------, "Development and Learning", Journal of Research in Science Teaching, Vol. 2, 1964, p. 176-186.

Piaget discusses the core of his theory of cognitive development, with a greater emphasis on the operative aspect. This was a speech, hence, Piaget is more explicit than he is in his volumous texts.

--------, Genetic Epistemology, New York, Columbia University Press, 1970, 1-78 p.

This book is concerned with the general Piagetian theory of knowledge. There is a more balanced discussion of the figurative and operative aspects in knowing. This is an excellent but complex work.

--------, "Quantification, Conservation and Nativism", Science, Vol. 162, November 1968, p. 976-979.

This is Piaget's response to researchers who had misinterpreted his theory, particularly concerning conservation. To clarify conservation, Piaget gives a precise definition.

Jean Piaget and Bärbel Inhelder, The Psychology of the Child, New York, Basic Books, 1969, xiv-173 p.

This is an insightful discussion of the child's use of the semiotic processes to represent his thoughts.

Sinclair, Hermine, "Piaget's Theory of Development: The Main Stages", in Myron F. Rosskopf, Leslie P. Steffe, Stanley Taback (eds.), Piagetian Cognitive Development, Research and Mathematical Education, Reston, Virginia, National Council of Teachers of Mathematics, 1971, p. 1-10.

Sinclair clarifies the meaning of the term "concrete operations". There has been more confusion over this than any other Piagetian term.

Smith, C. W. Jr., A Study of Constant Errors in Subtraction and in the Application of Selected Principles of the Decimal Numeration System Made by Third and Fourth Grade Students, unpublished doctoral thesis presented to the Graduate Division of Wayne State University, Detroit, Michigan, Ann Arbor, University Microfilms, 1972, 1-172 p.

This is the only study found which examined the child's understanding of the non-regrouping and regrouping principles of decimal numeration. The study is empirically, rather than theoretically based.

Wohlwill, Joachim F., "Piaget's System as a Source of Empirical Research", Merrill-Palmer Quarterly, Vol. 4, 1963, p. 253-262.

Wohlwill stresses the developmental aspect of Piaget's theory, and the general lack of research into this aspect.

## APPENDIX 1

## NUMBER AND NUMERATION

NUMBER and NUMERATION

| | |
|---|---|
| NAME: ______________________ | AGE: _____ |
| BIRTHDAY: ______________________ | GRADE: _____ |

This is a test to see how well you understand numbers. You may write in this booklet.

These are examples for you to try.

---

A. John set out some play money bills:

[$100] [$10] [$10] [$1] [$1]
[$1]

How much money did John set out? $ ________

---

B. Jill needs some $1 bills to finish showing $148.

[$100] [$10] [$10]
[$10] [$10]

Circle the number of $1 bills Jill needs.

[$1] [$1] [$1] [$1] [$1] [$1] [$1] [$1]
[$1] [$1] [$1] [$1] [$1] [$1] [$1] [$1] None

---

<u>Fill in the blanks</u>

C. 2 hundreds, 3 tens, 4 ones = ___

D. 1 hundred, 6 tens, __ ones = 162

---

The questions in this booklet are more difficult than these examples, because you already understand these small numbers. Try to figure out each question the best way you can. Do not spend too much time on any one question. If you do not understand a question, leave it until you finish the other questions.

Some children are playing with toy money.

They have some questions for you to answer:

---

A. Mary needs some $10 bills to finish showing $3641.

| | | | | |
|---|---|---|---|---|
| $1000 | $1000 | $100 | $100 | $1 |
| | $1000 | $100 | $100 | |
| | | $100 | $100 | |

Circle the number of $10 bills Mary needs.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | |
| $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | None |

---

B. John set out these bills:

| | | | | | | |
|---|---|---|---|---|---|---|
| $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| | | $100 | $10 | $10 | $1 | $1 |
| | | | | | $1 | $1 |
| | | | | | | $1 |

How much money did John set out? $ ________

---

C. Alice needs some $10 bills to finish showing $3644.

| | | | | | |
|---|---|---|---|---|---|
| $1000 | $1000 | $100 | $100 | $1 | $1 |
| | $1000 | $100 | $100 | $1 | $1 |
| | | | $100 | $1 | $1 |
| | | | | $1 | $1 |
| | | | | $1 | $1 |
| | | | | $1 | $1 |
| | | | | $1 | $1 |

Circle the number of $10 Alice needs.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | |
| $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | None |

---

D. Sally set out these bills:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| | $1000 | | | $10 | $10 | $1 | $1 |
| | | | | $10 | $10 | $1 | $1 |
| | | | | $10 | $10 | $1 | $1 |
| | | | | $10 | $10 | $1 | $1 |
| | | | | $10 | | $1 | $1 |

How much money did Sally set out? $ ________

---

E. Tina needs some $100 bills to finish showing $3756.

| | | | | | |
|---|---|---|---|---|---|
| $1000 | $1000 | $10 | $10 | $1 | $1 |
| | $1000 | $10 | $10 | $1 | $1 |
| | | $10 | $10 | $1 | $1 |
| | | $10 | $10 | | |
| | | $10 | $10 | | |
| | | $10 | $10 | | |
| | | $10 | $10 | | |
| | | $10 | | | |

Circle the number of $100 bills Tina needs.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $100 | $100 | $100 | $100 | $100 | $100 | $100 | $100 | |
| $100 | $100 | $100 | $100 | $100 | $100 | $100 | $100 | None |

---

F. Helen needs some $100 bills to finish showing $5038.

| | | | | | |
|---|---|---|---|---|---|
| $1000 | $1000 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | $10 | $10 | $1 | $1 |
| | | $10 | $10 | $1 | $1 |
| | | $10 | $10 | $1 | $1 |
| | | $10 | $10 | | |
| | | $10 | $10 | | |
| | | $10 | | | |

Circle the number of $100 bills Helen needs.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $100 | $100 | $100 | $100 | $100 | $100 | $100 | $100 | |
| $100 | $100 | $100 | $100 | $100 | $100 | $100 | $100 | None |

---

G. Sue set out these bills:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | $10 | $10 | | |
| $1000 | $1000 | $100 | $100 | $10 | $10 | | |
| | | $100 | $100 | $10 | $10 | | |
| | | $100 | $100 | | | | |
| | | $100 | $100 | | | | |
| | | $100 | $100 | | | | |

How much money did Sue set out? $ ________

---

H. Ann set out these bills:

| | | | | | |
|---|---|---|---|---|---|
| $1000 | $1000 | $100 | $100 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | | |
| | | | $100 | | |

How much money did Ann set out? $ ________

---

I. Mark needs some $100 bills to finish showing $7682.

| | | | | | |
|---|---|---|---|---|---|
| $1000 | $1000 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | $10 | $10 | | |
| $1000 | $1000 | $10 | $10 | | |
| | | $10 | $10 | | |

Circle the number of $100 bills Mark needs.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $100 | $100 | $100 | $100 | $100 | $100 | $100 | $100 | |
| $100 | $100 | $100 | $100 | $100 | $100 | $100 | $100 | None |

J. Kevin set out these bills:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| | | $100 | $100 | $10 | $10 | | |
| | | | $100 | $10 | $10 | | |
| | | | | $10 | $10 | | |
| | | | | $10 | $10 | | |

How much money did Kevin set out? $ ________

K. Karen needs some $10 bills to finish showing $9205.

| | | | | |
|---|---|---|---|---|
| $1000 | $1000 | $100 | $1 | $1 |
| $1000 | $1000 | | $1 | $1 |
| $1000 | $1000 | | $1 | $1 |
| $1000 | $1000 | | $1 | $1 |
| | $1000 | | $1 | $1 |
| | | | $1 | $1 |
| | | | $1 | $1 |
| | | | $1 | |

Circle the number of $10 bills Karen needs.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | |
| $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | None |

L. Fred set out these bills.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| | | $100 | $100 | $10 | $10 | $1 | $1 |
| | | | $100 | $10 | $10 | $1 | |
| | | | | $10 | $10 | | |

How much money did Fred set out? $ ________

---

M. Tim set out these bills:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| | | $100 | $100 | $10 | $10 | $1 | $1 |
| | | | $100 | $10 | $10 | $1 | $1 |
| | | | | $10 | $10 | $1 | $1 |
| | | | | $10 | $10 | $1 | $1 |
| | | | | $10 | $10 | | |
| | | | | $10 | | | |

How much money did Tim set out? $ ________

---

N. Sam set out these bills:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | $100 | $100 | $10 | $10 | $1 | $1 |
| $1000 | $1000 | | $100 | $10 | $10 | $1 | $1 |
| | | | | $10 | $10 | $1 | |
| | | | | $10 | $10 | | |
| | | | | $10 | $10 | | |
| | | | | $10 | $10 | | |

How much money did Sam set out? $ ________

---

O. David needs some $10 bills to finish showing $7243.

| | | | | |
|---|---|---|---|---|
| $1000 | $1000 | $100 | $1 | $1 |
| $1000 | $1000 | | $1 | $1 |
| $1000 | $1000 | | $1 | $1 |
| | $1000 | | $1 | $1 |
| | | | $1 | $1 |
| | | | $1 | $1 |
| | | | $1 | |

Circle the number of $10 bills David needs.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | |
| $10 | $10 | $10 | $10 | $10 | $10 | $10 | $10 | None |

---

P. Frank needs some $100 bills to finish showing $6104.

| | | | | |
|---|---|---|---|---|
| $1000 | $1000 | | $1 | $1 |
| $1000 | $1000 | | $1 | $1 |
| $1000 | $1000 | | | |

Circle the number of $100 bills Frank needs.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $100 | $100 | $100 | $100 | $100 | $100 | $100 | $100 | |
| $100 | $100 | $100 | $100 | $100 | $100 | $100 | $100 | None |

---

TURN OVER PLEASE ......

# APPENDIX 1

Fill in the blanks

A. 8 thousands, 6 hundreds, ___ tens, 4 ones = 8754

B. 5 thousands, 7 hundreds, 18 tens, 6 ones = _____

C. 1 thousand, 15 hundreds, 9 tens, 1 one = _____

D. 6 thousands, ___ hundreds, 16 tens, 4 ones = 7064

E. 1 thousand, 6 hundreds, 17 tens, 15 ones = _____

F. 3 thousands, ___ hundreds, 14 tens, 5 ones = 3845

G. 5 thousands, ___ hundreds, 18 tens, 3 ones = 6083

H. 6 thousands, 2 hundreds, 3 tens, 9 ones = _____

I. 3 thousands, 6 hundreds, ___ tens, 2 ones = 3692

J. 8 thousands, 9 hundreds, 13 tens, 6 ones = _____

K. 4 thousands, 12 hundreds, 19 tens, 5 ones = _____

L. 4 thousands, 5 tens, 7 ones = _____

M. 2 thousands, 5 hundreds, 9 tens, 16 ones = _____

N. 9 thousands, 3 hundreds, ___ tens, 12 ones = 9462

O. 3 thousands, ___ hundreds, 8 ones = 3408

P. 3 thousands, ___ hundreds, 17 tens, 6 ones = 4776

## APPENDIX 2

SPECIFICATIONS OF THE REGROUPING TYPE, DIRECTION, OPERATIONAL LEVEL, ANALYSIS LABEL AND DIFFICULTY FOR THE PICTORIAL-COUNTING NAME TASKS IN "NUMBER AND NUMERATION"

Specifications of the Regrouping Type, Direction, Operational Level, Analysis Label and Difficulty for the Pictorial-Counting Name Tasks in "Number and Numeration"

| Task | Type of Regrouping | Direction | Operational Level | Analysis Label | Difficulty Index* |
|---|---|---|---|---|---|
| A | Non-(without zero) | Reverse | Pre-(without zero) | Task 1 (R) | 0.76 |
| B | Non-(without zero) | Forward | Pre-(without zero) | Task 1 (F) | 0.72 |
| C | Double | Reverse | Formal | Task 5 (R) | 0.16 |
| D | Double | Forward | Formal | Task 5 (F) | 0.28 |
| E | Single | Reverse | Concrete | Task 3 (R) | 0.28 |
| F | Provoking Double | Reverse | Provoking Formal | Task 4 (R) | 0.24 |
| G | Single | Forward | Concrete | Task 3 (F) | 0.64 |
| H | Non-(with zero) | Forward | Pre-(with zero) | Task 2 (F) | 0.44 |
| I | Single | Reverse | Concrete | Task 3 (R) | 0.44 |
| J | Provoking Double | Forward | Provoking Formal | Task 4 (F) | 0.36 |
| K | Provoking Double | Reverse | Provoking Formal | Task 4 (R) | 0.32 |
| L | Provoking Double | Forward | Provoking Formal | Task 4 (F) | 0.32 |
| M | Double | Forward | Formal | Task 5 (F) | 0.40 |
| N | Single | Forward | Concrete | Task 3 (F) | 0.64 |
| O | Double | Reverse | Formal | Task 5 (R) | 0.12 |
| P | Non-(with zero) | Reverse | Pre-(whith zero) | Task 2 (R) | 0.80 |

* Index for the pre-study group (25 Grade Four students)

# APPENDIX 3

## SPECIFICATIONS OF THE REGROUPING TYPE, DIRECTION, OPERATIONAL LEVEL, ANALYSIS LABEL AND DIFFICULTY FOR THE VERBAL-COUNTING NAME TASKS IN "NUMBER AND NUMERATION"

# APPENDIX 3

Specifications of the Regrouping Type, Direction, Operational Level, Analysis Label and Difficulty for the Verbal-Counting Name Tasks in "Number and Numeration"

| Task | Type of Regrouping | Direction | Operational Level | Analysis Label | Difficulty Index* |
|---|---|---|---|---|---|
| A | Single | Reverse | Concrete | Task 3 (R) | 0.24 |
| B | Single | Forward | Concrete | Task 3 (F) | 0.72 |
| C | Single | Forward | Concrete | Task 3 (F) | 0.80 |
| D | Provoking Double | Reverse | Provoking Formal | Task 4 (R) | 0.24 |
| E | Double | Forward | Formal | Task 5 (F) | 0.52 |
| F | Single | Reverse | Concrete | Task 3 (R) | 0.40 |
| G | Provoking Double | Reverse | Provoking Formal | Task 4 (R) | 0.24 |
| H | Non-(without zero) | Forward | Pre-(without zero) | Task 1 (F) | 0.88 |
| I | Non-(without zero) | Reverse | Pre-(without zero) | Task 1 (R) | 0.96 |
| J | Provoking Double | Forward | Provoking Formal | Task 4 (F) | 0.64 |
| K | Double | Forward | Formal | Task 5 (F) | 0.48 |
| L | Non-(with zero) | Forward | Pre-(with zero) | Task 2 (F) | 0.28 |
| M | Provoking Double | Forward | Provoking Formal | Task 4 (F) | 0.48 |
| N | Double | Reverse | Formal | Task 5 (R) | 0.08 |
| O | Non-(with zero) | Reverse | Pre-(with zero) | Task 2 (R) | 0.84 |
| P | Double | Reverse | Formal | Task 5 (R) | 0.20 |

* Index for the pre-study group (25 Grade Four students)

APPENDIX 4

THE TASK TYPE AND DIFFICULTY INDEX FOR THE
VERBAL-COUNTING NAME TASKS SELECTED IN THE SCALOGRAM
ANALYSIS OF THE PRE-STUDY GROUP DATA

# APPENDIX 4

The Task Type and Difficulty Index for the Verbal-Counting Name Tasks Selected in the Scalogram Analysis of the Pre-study Group Data

| Verbal-Counting Name Task | I | H | B | J | E | F | D | P |
|---|---|---|---|---|---|---|---|---|
| Task Type | 1R | 1F | 3F | 4F | 5F | 3R | 4R | 5R |
| Difficulty Index | 0.96 | 0.88 | 0.72 | 0.64 | 0.52 | 0.40 | 0.24 | 0.20 |

# APPENDIX 5

## THE PRE-STUDY GROUP RESPONSE PATTERNS FOR THE TASKS SELECTED IN THE SCALOGRAM ANALYSIS

APPENDIX 5

The Pre-Study Group Response Patterns for
the Tasks Selected in the Scalogram Analysis

| Subject | I | H | B | J | E | F | D | P |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | X |
| 2 | | | | | | | | X |
| 3 | | | | | | | | X |
| 4 | | | | | | | X= | |
| 5 | | | X= | | | | | |
| 6 | | | | | | | X | X |
| 7 | | | | | | | X | X |
| 8 | | | | | | X | X= | |
| 9 | | | | | | X= | | X |
| 10 | | | | | X= | | X= | |
| 11 | | | | | | X | X | X |
| 12 | | | | | | X | X | X |
| 13 | | | | | | X | X | X |
| 14 | | | | | X= | | X | X |
| 15 | | | | | X= | | X | X |
| 16 | | | | | X | X | X | X |
| 17 | | | | X= | | X | X | X |
| 18 | | | X | X | X | X= | | |
| 19 | | | | X | X | X | X | X |
| 20 | | | | X | X | X | X | X |
| 21 | | | X | X | X | X | X | X |
| 22 | | | X | X | X | X | X | X |
| 23 | | X | X | X | X | X | X | X |
| 24 | | X | X | X | X | X | X | X |
| 25 | X | X | X | X | X | X | X | X |
| Task | I | H | B | J | E | F | D | P |

[ ] indicates a correct response to a task

[X] indicates an incorrect response to a task

= indicates an error in the response pattern

# APPENDIX 6

THE AGE RANGE, THE AVERAGE AGE, AND THE DISTRIBUTION OF SUBJECTS BY AGE FOR EACH CLASSROOM GROUP IN SCHOOL A, SCHOOL B AND SCHOOL C

# APPENDIX 6

The Age Range, the Average Age, and the Distribution of Subjects by Age for Each Classroom Group*

School A

| Classroom Group | Age Range | Average Age | Numbers of Subjects by Age | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Grade 4 | $9^{0}$ - $10^{11}$ | $10^{0}$ | - | 13 | 7 | - | - | - | - | - |
| Grade 5 | $9^{11}$ - $12^{10}$ | $11^{0}$ | - | 1 | 14 | 8 | 4 | - | - | - |
| Combined Grade 5/6: 5 | $10^{3}$ - $12^{1}$ | $11^{0}$ | - | - | 6 | 6 | 1 | - | - | - |
| Combined Grade 5/6: 6 | $10^{11}$ - $12^{4}$ | $11^{6}$ | - | - | 1 | 9 | 1 | - | - | - |
| Grade 6 | $11^{3}$ - $14^{4}$ | $11^{11}$ | - | - | - | 14 | 6 | 1 | 1 | - |
| Grade 7 | $12^{1}$ - $14^{1}$ | $12^{11}$ | - | - | - | - | 18 | 12 | 1 | - |
| Grade 8A | $12^{1}$ - $15^{0}$ | $13^{8}$ | - | - | - | - | 1 | 17 | 5 | 1 |
| Grade 8B | $12^{7}$ - $14^{9}$ | $13^{9}$ | - | - | - | - | 1 | 18 | 9 | - |
| Grade 8C | $13^{2}$ - $15^{1}$ | $14^{0}$ | - | - | - | - | - | 10 | 11 | 1 |

* Ages as of February 11, 1976

APPENDIX 6

The Age Range, the Average Age, and the Distribution of Subjects by Age for Each Classroom Group*

School B

| Classroom Group | Age Range | Average Age | Numbers of Subjects by Age | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Grade 4 | $8^{11}$ - $10^{11}$ | $9^{8}$ | 1 | 2 | 1 | - | - | - | - | - |
| Grade 5 | $10^{0}$ - $12^{2}$ | $11^{1}$ | - | - | 4 | 8 | 2 | - | - | - |
| Grade 6 | $11^{5}$ - $13^{11}$ | $12^{4}$ | - | - | - | 4 | 3 | 2 | - | - |
| Grade 7 | $12^{1}$ - $14^{11}$ | $12^{11}$ | - | - | - | - | 13 | 7 | 2 | - |
| Grade 8 | $12^{8}$ - $15^{0}$ | $13^{10}$ | - | - | - | - | 2 | 10 | 9 | 1 |

* Ages as of February 11, 1976

The Age Range, the Average Age, and the Distribution of Subjects by Age for Each Classroom Group*

School C

| Classroom Group | | Age Range | Average Age | Numbers of Subjects by Age | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Grade 4 | | $8^{9}$ - $12^{0}$ | $9^{11}$ | 1 | 8 | 4 | - | 1 | - | - | - |
| Combined Grade 4/5 | 4 | $8^{10}$ - $10^{10}$ | $9^{10}$ | 2 | 3 | 7 | - | - | - | - | - |
| | 5 | $9^{3}$ - $12^{11}$ | $11^{3}$ | - | 1 | 5 | 7 | 2 | - | - | - |
| Combined Grade 5/6 | 5 | $9^{9}$ - $11^{6}$ | $10^{7}$ | - | 1 | 10 | 2 | - | - | - | - |
| | 6 | $11^{4}$ - $13^{6}$ | $11^{11}$ | - | - | - | 3 | - | 1 | - | - |
| Grade 6 | | $11^{2}$ - $12^{11}$ | $11^{9}$ | - | - | - | 13 | 7 | - | - | - |
| Grade 7 | | $11^{3}$ - $14^{6}$ | $12^{11}$ | - | - | - | 2 | 13 | 11 | 1 | - |
| Grade 8 | | $11^{11}$ - $15^{11}$ | $13^{11}$ | - | - | - | 1 | - | 13 | 3 | 3 |

* Ages as of February 11, 1976

# APPENDIX 7

## CORRELATION COEFFICIENTS BETWEEN THE TASKS ON "NUMBER AND NUMERATION"

# APPENDIX 7

## Correlation Coefficients Between the Tasks on "Number and Numeration"

| Tasks | A* | B* | C* | D* | E* |
|---|---|---|---|---|---|
| A* | 1.00000 | 0.27668 | 0.15001 | 0.21589 | 0.23011 |
| B* | 0.27668 | 1.00000 | 0.14964 | 0.22272 | 0.33685 |
| C* | 0.15001 | 0.14964 | 1.00000 | 0.29360 | 0.44547 |
| D* | 0.21589 | 0.22272 | 0.29360 | 1.00000 | 0.37296 |
| E* | 0.23011 | 0.33685 | 0.44547 | 0.37296 | 1.00000 |
| F* | 0.20222 | 0.23105 | 0.40219 | 0.37223 | 0.57434 |
| G* | 0.13585 | 0.25698 | 0.25997 | 0.38925 | 0.34774 |
| H* | 0.27471 | 0.17837 | 0.22467 | 0.28791 | 0.28013 |
| I* | 0.25592 | 0.11976 | 0.28318 | 0.34375 | 0.38994 |
| J* | 0.23011 | 0.24103 | 0.25170 | 0.47041 | 0.36522 |
| K* | 0.17993 | 0.20693 | 0.40020 | 0.30316 | 0.50315 |
| L* | 0.26305 | 0.21589 | 0.19241 | 0.32195 | 0.39079 |
| M* | 0.21537 | 0.23207 | 0.20748 | 0.41988 | 0.45591 |
| N* | 0.18623 | 0.23088 | 0.21287 | 0.37098 | 0.35210 |
| P* | 0.32563 | 0.29000 | 0.11308 | 0.14484 | 0.16266 |
| A | 0.12619 | 0.17579 | 0.14486 | 0.27153 | 0.24070 |
| B | 0.19563 | 0.23788 | 0.18538 | 0.29629 | 0.37316 |
| C | 0.22352 | 0.23964 | 0.21321 | 0.34222 | 0.37151 |
| D | 0.17370 | 0.12370 | 0.24536 | 0.26251 | 0.34854 |
| E | 0.19330 | 0.21370 | 0.22226 | 0.37160 | 0.39630 |
| F | 0.21589 | 0.23852 | 0.28295 | 0.34662 | 0.41627 |
| G | 0.14557 | 0.16592 | 0.27108 | 0.31969 | 0.38977 |
| H | 0.27500 | 0.18495 | 0.09629 | 0.08832 | 0.15010 |
| I | 0.27500 | 0.18495 | 0.09629 | 0.11398 | 0.17604 |
| J | 0.22099 | 0.29372 | 0.21128 | 0.35219 | 0.40647 |
| K | 0.18562 | 0.27891 | 0.22714 | 0.33968 | 0.35159 |
| L | 0.25405 | 0.20471 | 0.28685 | 0.37417 | 0.36856 |
| M | 0.25901 | 0.26153 | 0.25855 | 0.39148 | 0.38021 |
| N | 0.16659 | 0.11752 | 0.25606 | 0.24505 | 0.26534 |
| O | 0.22227 | 0.12345 | 0.09062 | 0.15680 | 0.12109 |
| P | 0.07545 | 0.12532 | 0.23632 | 0.20307 | 0.24741 |

* Indicates Pictorial-Counting Name Tasks

APPENDIX 7

Correlation Coefficients Between the Tasks
on "Number and Numeration"

| Tasks | F* | G* | H* | I* | J* |
|---|---|---|---|---|---|
| A* | 0.20222 | 0.13585 | 0.27471 | 0.25592 | 0.23011 |
| B* | 0.23105 | 0.25698 | 0.17837 | 0.11976 | 0.24103 |
| C* | 0.40219 | 0.25997 | 0.22467 | 0.28318 | 0.25170 |
| D* | 0.37223 | 0.38925 | 0.28791 | 0.34375 | 0.47041 |
| E* | 0.57434 | 0.34774 | 0.28013 | 0.38994 | 0.36522 |
| F* | 1.00000 | 0.34666 | 0.28514 | 0.42217 | 0.35257 |
| G* | 0.34666 | 1.00000 | 0.36847 | 0.28733 | 0.47847 |
| H* | 0.28514 | 0.36847 | 1.00000 | 0.26677 | 0.28013 |
| I* | 0.42217 | 0.28733 | 0.26677 | 1.00000 | 0.31298 |
| J* | 0.35257 | 0.47847 | 0.28013 | 0.31298 | 1.00000 |
| K* | 0.52537 | 0.35946 | 0.28513 | 0.33832 | 0.36517 |
| L* | 0.36237 | 0.43943 | 0.39729 | 0.37210 | 0.51050 |
| M* | 0.39035 | 0.44254 | 0.31496 | 0.39254 | 0.42293 |
| N* | 0.34301 | 0.48570 | 0.31645 | 0.32701 | 0.53202 |
| P* | 0.24320 | 0.20936 | 0.21797 | 0.16953 | 0.19406 |
| A | 0.26379 | 0.26311 | 0.16879 | 0.26163 | 0.25150 |
| B | 0.35582 | 0.43610 | 0.28797 | 0.29306 | 0.45351 |
| C | 0.35813 | 0.44503 | 0.26427 | 0.26345 | 0.43093 |
| D | 0.40518 | 0.24527 | 0.24496 | 0.33700 | 0.29488 |
| E | 0.35299 | 0.37904 | 0.29930 | 0.27771 | 0.42923 |
| F | 0.37223 | 0.36574 | 0.26418 | 0.24586 | 0.38379 |
| G | 0.38673 | 0.28957 | 0.24136 | 0.29183 | 0.30523 |
| H | 0.10056 | 0.15285 | 0.10108 | 0.04993 | 0.12416 |
| I | 0.15062 | 0.15285 | 0.10108 | 0.04993 | 0.12416 |
| J | 0.34574 | 0.39486 | 0.28334 | 0.29119 | 0.47568 |
| K | 0.30659 | 0.36999 | 0.31288 | 0.25311 | 0.40712 |
| L | 0.36689 | 0.33865 | 0.29632 | 0.30685 | 0.35774 |
| M | 0.37312 | 0.40607 | 0.32343 | 0.28020 | 0.49252 |
| N | 0.29562 | 0.21779 | 0.20826 | 0.24450 | 0.24249 |
| O | 0.14571 | 0.14939 | 0.15667 | 0.14440 | 0.15561 |
| P | 0.29532 | 0.19097 | 0.13085 | 0.21505 | 0.23585 |

* Indicates Pictorial-Counting Name Tasks

# APPENDIX 7

## Correlation Coefficients Between the Tasks on "Number and Numeration"

| Tasks | K* | L* | M* | N* | P* |
|---|---|---|---|---|---|
| A* | 0.17993 | 0.26305 | 0.21537 | 0.18623 | 0.32563 |
| B* | 0.20693 | 0.21589 | 0.23207 | 0.23088 | 0.29000 |
| C* | ~~0.40020~~ | ~~0.19241~~ | ~~0.20748~~ | ~~0.21287~~ | ~~0.11308~~ |
| D* | 0.30316 | 0.32195 | 0.41988 | 0.37098 | 0.14484 |
| E* | 0.50315 | 0.39079 | 0.45591 | 0.35210 | 0.16266 |
| F* | ~~0.52537~~ | ~~0.36237~~ | ~~0.39035~~ | ~~0.34301~~ | ~~0.24320~~ |
| G* | 0.35946 | 0.43943 | 0.44254 | 0.48570 | 0.20936 |
| H* | 0.28513 | 0.39729 | 0.31496 | 0.31645 | 0.21797 |
| I* | ~~0.33832~~ | ~~0.37210~~ | ~~0.39254~~ | ~~0.32701~~ | ~~0.16953~~ |
| J* | 0.36517 | 0.51050 | 0.42293 | 0.53202 | 0.19406 |
| K* | 1.00000 | 0.37614 | 0.37030 | 0.37818 | 0.18997 |
| L* | ~~0.37614~~ | ~~1.00000~~ | ~~0.48143~~ | ~~0.54041~~ | ~~0.15367~~ |
| M* | 0.37030 | 0.48143 | 1.00000 | 0.59210 | 0.21685 |
| N* | 0.37818 | 0.54041 | 0.59210 | 1.00000 | 0.25238 |
| P* | ~~0.18997~~ | ~~0.15367~~ | ~~0.21685~~ | ~~0.25238~~ | ~~1.00000~~ |
| A | 0.18865 | 0.21318 | 0.25078 | 0.25166 | 0.10744 |
| B | 0.35006 | 0.42576 | 0.43144 | 0.41378 | 0.17450 |
| C | ~~0.37098~~ | ~~0.40398~~ | ~~0.44254~~ | ~~0.45965~~ | ~~0.19231~~ |
| D | 0.39194 | 0.35325 | 0.29388 | 0.30377 | 0.07252 |
| E | 0.31180 | 0.44383 | 0.39901 | 0.39554 | 0.16723 |
| F | ~~0.35566~~ | ~~0.28965~~ | ~~0.35462~~ | ~~0.37098~~ | ~~0.11377~~ |
| G | 0.35681 | 0.36507 | 0.30245 | 0.33401 | 0.08623 |
| H | 0.11300 | 0.09322 | 0.07599 | 0.10108 | 0.21581 |
| I | ~~0.13616~~ | ~~0.06743~~ | ~~0.10205~~ | ~~0.12951~~ | ~~0.21581~~ |
| J | 0.31690 | 0.49379 | 0.41865 | 0.48561 | 0.21272 |
| K | 0.32718 | 0.37865 | 0.35351 | 0.33723 | 0.16768 |
| L | ~~0.30865~~ | ~~0.33903~~ | ~~0.41549~~ | ~~0.36743~~ | ~~0.23573~~ |
| M | 0.32708 | 0.47513 | 0.41559 | 0.43421 | 0.19821 |
| N | 0.24860 | 0.30052 | 0.23302 | 0.22078 | 0.12559 |
| O | ~~0.18234~~ | ~~0.14759~~ | ~~0.14440~~ | ~~0.15667~~ | ~~0.24002~~ |
| P | 0.24927 | 0.25957 | 0.18020 | 0.20689 | 0.08320 |

* Indicates Pictorial-Counting Name Tasks

APPENDIX 7

Correlation Coefficients Between the Tasks
on "Number and Numeration"

| Tasks | A | B | C | D | E |
|---|---|---|---|---|---|
| A* | 0.12619 | 0.19563 | 0.22352 | 0.17370 | 0.19330 |
| B* | 0.17579 | 0.23788 | 0.23964 | 0.12370 | 0.21370 |
| C* | ~~0.14486~~ | ~~0.18538~~ | ~~0.21321~~ | ~~0.24536~~ | ~~0.22226~~ |
| D* | 0.27153 | 0.29629 | 0.34222 | 0.26251 | 0.37160 |
| E* | 0.24070 | 0.37316 | 0.37151 | 0.34854 | 0.39630 |
| F* | ~~0.26379~~ | ~~0.35582~~ | ~~0.35813~~ | ~~0.40518~~ | ~~0.35299~~ |
| G* | 0.26311 | 0.43610 | 0.44503 | 0.24527 | 0.37904 |
| H* | 0.16879 | 0.28797 | 0.26427 | 0.24496 | 0.29930 |
| I* | ~~0.26163~~ | ~~0.29306~~ | ~~0.26345~~ | ~~0.33700~~ | ~~0.27771~~ |
| J* | 0.25150 | 0.45351 | 0.43093 | 0.29488 | 0.42923 |
| K* | 0.18865 | 0.35006 | 0.37098 | 0.39194 | 0.31180 |
| L* | ~~0.21318~~ | ~~0.42576~~ | ~~0.40398~~ | ~~0.35325~~ | ~~0.44383~~ |
| M* | 0.25078 | 0.43144 | 0.44254 | 0.29388 | 0.39901 |
| N* | 0.25166 | 0.41378 | 0.45965 | 0.30377 | 0.39554 |
| P* | ~~0.10744~~ | ~~0.17450~~ | ~~0.19231~~ | ~~0.07252~~ | ~~0.16723~~ |
| A | 1.00000 | 0.32302 | 0.29830 | 0.31205 | 0.22212 |
| B | 0.32302 | 1.00000 | 0.77267 | 0.42013 | 0.64605 |
| C | ~~0.29830~~ | ~~0.77267~~ | ~~1.00000~~ | ~~0.38512~~ | ~~0.65322~~ |
| D | 0.31205 | 0.42013 | 0.38512 | 1.00000 | 0.42696 |
| E | 0.22212 | 0.64605 | 0.65322 | 0.42696 | 1.00000 |
| F | ~~0.37839~~ | ~~0.53477~~ | ~~0.57737~~ | ~~0.52794~~ | ~~0.46934~~ |
| G | 0.32824 | 0.42420 | 0.41581 | 0.75320 | 0.45529 |
| H | 0.06770 | 0.24120 | 0.26552 | 0.15016 | 0.23085 |
| I | ~~0.09330~~ | ~~0.24120~~ | ~~0.26552~~ | ~~0.15016~~ | ~~0.20483~~ |
| J | 0.31569 | 0.67342 | 0.68296 | 0.43561 | 0.63439 |
| K | 0.27144 | 0.61996 | 0.63530 | 0.45543 | 0.68331 |
| L | ~~0.12596~~ | ~~0.41728~~ | ~~0.42085~~ | ~~0.30892~~ | ~~0.41063~~ |
| M | 0.28178 | 0.64834 | 0.61341 | 0.42275 | 0.67036 |
| N | 0.36879 | 0.31811 | 0.32945 | 0.43540 | 0.38515 |
| O | ~~0.05154~~ | ~~0.17410~~ | ~~0.20562~~ | ~~0.16333~~ | ~~0.17700~~ |
| P | 0.28143 | 0.27939 | 0.31655 | 0.42968 | 0.34572 |

* Indicates Pictorial-Counting Name Tasks

Correlation Coefficients Between the Tasks
on "Number and Numeration"

| Tasks | F | G | H | I | J |
|---|---|---|---|---|---|
| A* | 0.21589 | 0.14557 | 0.27500 | 0.27500 | 0.22099 |
| B* | ~~0.23852~~ | ~~0.16592~~ | ~~0.18495~~ | ~~0.18495~~ | ~~0.29372~~ |
| C* | 0.28295 | 0.27108 | 0.09629 | 0.09629 | 0.21128 |
| D* | 0.34662 | 0.31989 | 0.08832 | 0.11398 | 0.35219 |
| E* | ~~0.41627~~ | ~~0.38977~~ | ~~0.15010~~ | ~~0.17604~~ | ~~0.40647~~ |
| F* | 0.37223 | 0.38673 | 0.10056 | 0.15062 | 0.34574 |
| G* | 0.36574 | 0.28957 | 0.15285 | 0.15285 | 0.39486 |
| H* | ~~0.26418~~ | ~~0.24136~~ | ~~0.10108~~ | ~~0.10108~~ | ~~0.28334~~ |
| I* | 0.24586 | 0.29183 | 0.04993 | 0.04993 | 0.29119 |
| J* | 0.38379 | 0.30523 | 0.12416 | 0.12416 | 0.47568 |
| K* | 0.35566 | 0.35681 | 0.11300 | 0.13816 | 0.31690 |
| L* | ~~0.28965~~ | ~~0.36507~~ | ~~0.09322~~ | ~~0.06743~~ | ~~0.49379~~ |
| M* | 0.35462 | 0.30245 | 0.07599 | 0.10205 | 0.41865 |
| N* | 0.37098 | 0.33401 | 0.10108 | 0.12951 | 0.48561 |
| P* | ~~0.11377~~ | ~~0.08623~~ | ~~0.21561~~ | ~~0.21561~~ | ~~0.21272~~ |
| A | 0.37839 | 0.32824 | 0.06770 | 0.09330 | 0.31569 |
| B | 0.53477 | 0.42420 | 0.24120 | 0.24120 | 0.67342 |
| C | ~~0.57737~~ | ~~0.41581~~ | ~~0.26552~~ | ~~0.26552~~ | ~~0.60296~~ |
| D | 0.52794 | 0.75320 | 0.15016 | 0.15016 | 0.43561 |
| E | 0.46934 | 0.45529 | 0.23085 | 0.20483 | 0.63439 |
| F | ~~1.00000~~ | ~~0.61262~~ | ~~0.21662~~ | ~~0.24228~~ | ~~0.54618~~ |
| G | 0.61262 | 1.00000 | 0.17271 | 0.17271 | 0.49388 |
| H | 0.21662 | 0.17271 | 1.00000 | 0.56967 | 0.27207 |
| I | ~~0.24228~~ | ~~0.17271~~ | ~~0.56967~~ | ~~1.00000~~ | ~~0.27207~~ |
| J | 0.54618 | 0.49388 | 0.27207 | 0.27207 | 1.00000 |
| K | 0.52645 | 0.47935 | 0.21517 | 0.24149 | 0.67781 |
| L | ~~0.40626~~ | ~~0.31392~~ | ~~0.13836~~ | ~~0.18962~~ | ~~0.41676~~ |
| M | 0.51370 | 0.46541 | 0.22450 | 0.22450 | 0.69475 |
| N | 0.42590 | 0.46113 | 0.08722 | 0.14137 | 0.30022 |
| O | ~~0.17388~~ | ~~0.16739~~ | ~~0.29582~~ | ~~0.54128~~ | ~~0.17874~~ |
| P | 0.40898 | 0.48148 | 0.08236 | 0.13717 | 0.29794 |

* Indicates Pictorial-Counting Name Tasks

# APPENDIX 7

## Correlation Coefficients Between the Tasks on "Number and Numeration"

| Tasks | K | L | M | N | O | P |
|---|---|---|---|---|---|---|
| A* | 0.18562 | 0.25405 | 0.25901 | 0.16659 | 0.22227 | 0.07545 |
| B* | ~~0.27891~~ | ~~0.20471~~ | ~~0.26153~~ | ~~0.11752~~ | ~~0.12345~~ | ~~0.12532~~ |
| C* | 0.22718 | 0.28685 | 0.25855 | 0.25606 | 0.09062 | 0.23632 |
| D* | 0.33968 | 0.37417 | 0.39148 | 0.24505 | 0.15680 | 0.20307 |
| E* | ~~0.35159~~ | ~~0.36856~~ | ~~0.38021~~ | ~~0.26534~~ | ~~0.12109~~ | ~~0.24741~~ |
| F* | 0.30659 | 0.36689 | 0.37312 | 0.29562 | 0.14571 | 0.29532 |
| G* | 0.36999 | 0.33865 | 0.40607 | 0.21779 | 0.14939 | 0.19097 |
| H* | ~~0.31288~~ | ~~0.29632~~ | ~~0.32343~~ | ~~0.20826~~ | ~~0.15667~~ | ~~0.13085~~ |
| I* | 0.25311 | 0.30685 | 0.28020 | 0.24450 | 0.14440 | 0.21505 |
| J* | 0.40712 | 0.35774 | 0.49252 | 0.24249 | 0.15561 | 0.23585 |
| K* | 0.32718 | 0.30865 | 0.32708 | 0.24860 | 0.18234 | 0.24927 |
| L* | ~~0.37865~~ | ~~0.33903~~ | ~~0.47513~~ | ~~0.30052~~ | ~~0.14759~~ | 0.25957 |
| M* | 0.35351 | 0.41549 | 0.41559 | 0.23302 | 0.14440 | 0.18020 |
| N* | 0.33723 | 0.36743 | 0.43421 | 0.22078 | 0.15667 | 0.20689 |
| P* | ~~0.16768~~ | ~~0.23573~~ | ~~0.19821~~ | ~~0.12559~~ | ~~0.24002~~ | 0.08320 |
| A | 0.27144 | 0.12596 | 0.28178 | 0.36879 | 0.05154 | 0.28143 |
| B | 0.61996 | 0.41728 | 0.64834 | 0.31811 | 0.17410 | 0.27939 |
| C | ~~0.63530~~ | ~~0.42085~~ | ~~0.61341~~ | ~~0.32945~~ | ~~0.20562~~ | 0.31655 |
| D | 0.45543 | 0.30892 | 0.42275 | 0.43540 | 0.16333 | 0.42968 |
| E | 0.68331 | 0.41063 | 0.67036 | 0.38515 | 0.17700 | 0.34572 |
| F | ~~0.52645~~ | ~~0.40626~~ | ~~0.51370~~ | ~~0.42590~~ | ~~0.17388~~ | ~~0.40898~~ |
| G | 0.47935 | 0.31392 | 0.46541 | 0.46113 | 0.16739 | 0.48148 |
| H | 0.21517 | 0.13836 | 0.22450 | 0.08722 | 0.29582 | 0.08236 |
| I | ~~0.24149~~ | ~~0.18962~~ | ~~0.22450~~ | ~~0.14137~~ | ~~0.54128~~ | ~~0.13717~~ |
| J | 0.67781 | 0.41676 | 0.69475 | 0.30022 | 0.17874 | 0.29794 |
| K | 1.00000 | 0.43427 | 0.70428 | 0.35312 | 0.15679 | 0.33729 |
| L | ~~0.43427~~ | ~~1.00000~~ | ~~0.44286~~ | ~~0.20306~~ | ~~0.20601~~ | ~~0.22904~~ |
| M | 0.70428 | 0.44286 | 1.00000 | 0.35897 | 0.11649 | 0.30829 |
| N | 0.35312 | 0.20306 | 0.35897 | 1.00000 | 0.20887 | 0.52362 |
| O | ~~0.15679~~ | ~~0.20601~~ | ~~0.11649~~ | ~~0.20887~~ | ~~1.00000~~ | ~~0.12896~~ |
| P | 0.33729 | 0.22904 | 0.30829 | 0.52362 | 0.12896 | 1.00000 |

* Indicates Pictorial-Counting Name Tasks

## APPENDIX 8

THE DIFFICULTY INDICES OF THE PREOPERATIONAL, CONCRETE OPERATIONAL AND FORMAL OPERATIONAL TASKS FOR EACH SCHOOL AND GRADE GROUPING

APPENDIX 8

The Difficulty Indices of the Preoperational, Concrete Operational and Formal Operational Tasks for Each School and Grade Grouping

| Task Type: Regrouping Type: Operative Level: | | | Type 1 Non- Preoperative | Type 3 Single Concrete | Type 5 Double Formal |
|---|---|---|---|---|---|
| School | Grade | N | | | |
| A | 4 | 20 | 0.81 | 0.55 | 0.35 |
| B | 4 | 4 | 0.88 | 0.38 | 0.25 |
| C | 4 | 12 | 0.83 | 0.18 | 0.06 |
| C | 4 | 14 | 0.79 | 0.15 | 0.14 |
| A | 5 | 27 | 0.95 | 0.63 | 0.51 |
| A | 5 | 13 | 1.00 | 0.85 | 0.68 |
| B | 5 | 14 | 0.80 | 0.56 | 0.45 |
| C | 5 | 13 | 0.92 | 0.48 | 0.30 |
| C | 5 | 15 | 0.83 | 0.38 | 0.21 |
| A | 6 | 22 | 0.99 | 0.80 | 0.66 |
| A | 6 | 11 | 0.96 | 0.81 | 0.48 |
| B | 6 | 9 | 0.86 | 0.68 | 0.54 |
| C | 6 | 20 | 0.91 | 0.54 | 0.46 |
| C | 6 | 4 | 0.94 | 0.34 | 0.14 |
| A | 7 | 31 | 0.94 | 0.82 | 0.65 |
| B | 7 | 22 | 0.99 | 0.71 | 0.64 |
| C | 7 | 27 | 0.91 | 0.56 | 0.35 |
| A | 8 | 24 | 0.98 | 0.86 | 0.73 |
| A | 8 | 28 | 0.96 | 0.72 | 0.60 |
| A | 8 | 22 | 0.98 | 0.75 | 0.62 |
| B | 8 | 22 | 0.99 | 0.81 | 0.68 |
| C | 8 | 20 | 0.98 | 0.70 | 0.58 |

# APPENDIX 9

## THE VARIANCE-COVARIANCE MATRICES AND THEIR DETERMINANTS FOR ALL SUBJECTS AT EACH OF THE GRADE FOUR TO EIGHT LEVELS AND FOR THE POOLED MATRIX

# APPENDIX 9

The Variance-Covariance Matrices and Their Determinants for All Subjects at Each of the Grade Four to Eight Levels and for the Pooled Matrix

| Grade | | Task 3 | Task 4 | Task 5 |
|---|---|---|---|---|
| Four | Task 3 | 0.102 | 0.082 | 0.054 |
| | Task 4 | | 0.094 | 0.067 |
| | Task 5 | | | 0.073 |
| | | $\|D\| = 7.04 \times 10^{-5}$ | | |
| Five | Task 3 | 0.115 | 0.103 | 0.092 |
| | Task 4 | | 0.130 | 0.096 |
| | Task 5 | | | 0.104 |
| | | $\|D\| = 1.11 \times 10^{-4}$ | | |
| Six | Task 3 | 0.087 | 0.074 | 0.063 |
| | Task 4 | | 0.094 | 0.069 |
| | Task 5 | | | 0.082 |
| | | $\|D\| = 7.75 \times 10^{-6}$ | | |
| Seven | Task 3 | 0.073 | 0.067 | 0.051 |
| | Task 4 | | 0.104 | 0.064 |
| | Task 5 | | | 0.075 |
| | | $\|D\| = 1.01 \times 10^{-4}$ | | |
| Eight | Task 3 | 0.050 | 0.042 | 0.044 |
| | Task 4 | | 0.066 | 0.047 |
| | Task 5 | | | 0.064 |
| | | $\|D\| = 3.39 \times 10^{-5}$ | | |
| Pooled | Task 3 | 0.085 | 0.074 | 0.061 |
| | Task 4 | | 0.098 | 0.068 |
| | Task 5 | | | 0.080 |
| | | $\|D\| = 8.46 \times 10^{-5}$ | | |

# APPENDIX 10

## SUMMARY DATA FOR THE TEST FOR THE EQUALITY OF THE VARIANCE-COVARIANCE MATRICES FOR THE REGROUPING TASKS AT EACH OF THE GRADE FOUR TO EIGHT LEVELS

# APPENDIX 10

Summary Data for the Test for the Equality of the Variance-Covariance Matrices for the Regrouping Tasks at Each of the Grade Four to Eight Levels

Analysis with All Subjects at Each Grade Level

$E = 0.0181$

$M = 83.8484$

$\chi^2 = 82.329$ *

$df = 24$

* Significant at: $_{0.05}\chi^2_{24} = 36.415$

APPENDIX 11

THE SCHOOL, GRADE, SEX, AGE AND SCORING PATTERN FOR EACH CHILD COMPLETING "NUMBER AND NUMERATION"

# APPENDIX 11

In presenting the scoring patterns of each child, a negative procedure was used. That is, errors were indicated with an "o". Blanks indicate successes.

Pictorial-counting name item "O" was lost due to reproduction difficulties.

| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| A | 4 | M | $9^{8}$ | | | | | | | | | o | | | | | | | | o | | | o | | | | | | | o | | | | | |
| A | 4 | M | $9^{8}$ | | o | o | o | | | | | | o | | | | | | | | | | | | | | | | | | | | o | | o |
| A | 4 | M | $10^{10}$ | | | o | | | | | o | | | | | o | | | o | | | | | | | | | | | | o | | o | | o |
| A | 4 | M | $9^{4}$ | | | | | | | o | | o | o | o | o | | | | | o | o | | | | | | | | | | | | | | |
| A | 4 | F | $9^{10}$ | | | o | | | | | | | | | | | | | | o | | | o | | o | o | | | | | o | | o | | o |
| A | 4 | M | $9^{6}$ | | | | | | o | | o | | | o | | | | | | o | | | o | | o | o | | | | | | | o | | |
| A | 4 | M | $9^{11}$ | | | o | o | | | | | | | | | | | | | | | | o | o | | o | | | | o | o | o | o | | o |
| A | 4 | M | $10^{11}$ | | | o | | o | o | | o | | | o | | | | | | | | | o | | | o | | | | o | | | o | | o |
| A | 4 | F | $10^{2}$ | | | o | o | o | o | | | | | | o | o | | | | | | | | | o | o | | | | | | | o | | o |
| A | 4 | F | $9^{4}$ | o | | o | o | o | o | | o | o | o | o | o | | | | | o | | | | | | | | | | | o | | o | o | o |
| A | 4 | F | $10^{6}$ | | o | o | | o | o | | o | o | o | o | o | | o | | | o | | | o | | o | o | | | | | o | | o | | o |
| A | 4 | F | $9^{9}$ | | | o | o | o | o | o | | o | o | o | o | o | o | | | o | | | o | | | o | | | | | o | | o | o | o |
| A | 4 | F | $9^{10}$ | | | o | o | o | o | | o | o | | o | o | o | | | | o | | | o | o | | o | | | | o | o | o | o | | o |
| A | 4 | F | $9^{10}$ | | o | o | o | o | o | | | o | | o | o | o | | | | o | | | o | o | o | o | | | o | o | o | o | o | | o |
| A | 4 | F | $10^{6}$ | | o | o | o | o | o | | | | o | o | | o | o | | o | o | | | o | | o | o | | | o | o | o | o | o | | o |
| A | 4 | F | $9^{0}$ | | | o | o | o | o | o | | | o | o | o | o | o | | | o | o | o | | o | o | | | | o | o | o | o | o | | |
| A | 4 | F | $9^{7}$ | o | | o | o | o | o | o | o | o | o | o | o | o | o | | | o | | | o | o | o | o | | | o | | | o | o | | o |
| A | 4 | F | $10^{8}$ | o | o | o | o | o | o | o | o | o | o | o | o | o | o | | o | o | | | o | | o | o | | | | | o | | o | o | o |
| A | 4 | M | $9^{10}$ | o | o | o | o | o | o | o | o | o | o | o | o | o | o | | o | o | o | o | o | o | o | o | o | | o | o | o | o | o | | o |
| A | 4 | M | $10^{6}$ | o | o | o | o | o | o | o | o | o | o | o | o | o | o | | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| B | 4 | M | $9^{7}$ | | o | o | | o | | o | | | | o | | o | o | | | o | | | o | | o | o | | | | | | | | | o |
| B | 4 | F | $8^{11}$ | | | o | o | o | o | o | o | o | o | o | o | o | o | | | | | | o | o | | o | | | o | | | o | o | | o |
| B | 4 | M | $9^{3}$ | | | o | o | o | o | o | o | o | o | o | o | | | | | o | o | o | o | o | | o | | | o | o | o | o | o | | o |
| B | 4 | F | $10^{11}$ | | o | o | o | o | o | o | o | o | o | o | o | o | o | | o | o | | | | | | o | | | o | o | o | o | o | | o |

| School | Grade | Sex | Age | Pictorial-Counting Name A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Verbal-Counting Name A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | 4 | M | $8^{11}$ | 0 | | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | | 0 | 0 | | | | | | | 0 | 0 | | 0 | | | | | 0 | 0 | 0 | | |
| C | 4 | M | $10^{10}$ | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | | | 0 | | | 0 | 0 | 0 | 0 | | | | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | F | $10^{8}$ | | | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | | 0 | 0 | | | | 0 | 0 | | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | F | $10^{4}$ | | | 0 | 0 | 0 | 0 | | 0 | | 0 | 0 | 0 | 0 | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $10^{1}$ | | | | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $10^{1}$ | | | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $8^{10}$ | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | F | $9^{6}$ | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $10^{2}$ | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 4 | M | $9^{8}$ | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $10^{1}$ | | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 |
| C | 4 | F | $9^{2}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $9^{5}$ | | | | | | | | 0 | | | | | | | | | 0 | 0 | | | | | | | | | | | 0 | 0 | | 0 |
| C | 4 | M | $8^{9}$ | | | 0 | 0 | | | | | | 0 | 0 | | | | | | 0 | 0 | | | | 0 | | | | | | 0 | | | | |
| C | 4 | M | $9^{11}$ | | 0 | | | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | F | $10^{1}$ | | | 0 | | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $9^{8}$ | 0 | | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | F | $10^{3}$ | | | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $9^{8}$ | 0 | | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $12^{0}$ | 0 | | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $10^{3}$ | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | F | $9^{10}$ | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $9^{8}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | M | $9^{9}$ | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | F | $10^{1}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 0 | 0 | 0 | | 0 |
| C | 4 | F | $9^{9}$ | 0 | | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| School | Grade | Sex | Age | Pictorial-Counting Name A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Verbal-Counting Name A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 5 | M | $12^{0}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 5 | M | $10^{10}$ | | | | | | | | | | | | | | | | o | | | | | | | | | | | | | | | | |
| A | 5 | M | $10^{10}$ | | | | | | | | | | | | | | o | | | | | | | | | | | | | | | | | | |
| A | 5 | F | $11^{0}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | o | | | | |
| A | 5 | F | $10^{3}$ | | | | | | | | | | | | | | | | | o | | | | | | | | | | | o | | o | | |
| A | 5 | M | $10^{9}$ | | | | o | | | | | | | o | | o | | | | o | | | | | | | | | | | | | o | | o |
| A | 5 | M | $12^{2}$ | | | | | | | | | | | | | | | | | o | | o | o | | | | | | | o | | | o | | |
| A | 5 | M | $10^{9}$ | | | o | | | o | | | | | o | | | | | | | | | o | | | o | | | | | | | | | o |
| A | 5 | M | $9^{11}$ | | | o | | | o | | | | | o | | | | | | o | | | | | | o | | | | | o | | | | o |
| A | 5 | M | $10^{10}$ | | o | | | o | | | | | | | | o | o | | o | | | | | | | | | | | | o | | | | o |
| A | 5 | F | $10^{2}$ | | | o | | | | | | | | | o | | | | | o | | | | | o | o | | | | | | | o | | o |
| A | 5 | F | $10^{9}$ | | | | | | | | | o | | | | o | | | | o | | | o | | | o | | | | | o | | o | | o |
| A | 5 | F | $10^{7}$ | | | o | | | o | | | | | o | | | | | | | | | o | | | o | | | | o | | | o | | o |
| A | 5 | F | $10^{3}$ | | | | o | | o | o | | | o | | o | o | o | | | | | | | | | | | | | | o | o | | | |
| A | 5 | F | $11^{0}$ | | | o | o | | o | | | | | o | | | | | o | o | | | | | | | | | | | o | o | o | | o |
| A | 5 | M | $11^{3}$ | o | o | o | o | o | o | | | o | | | o | o | | | | o | | | o | | | | | | | | | | o | | |
| A | 5 | M | $11^{3}$ | | | o | o | o | o | | | o | o | | o | | o | | | o | | | o | o | | | | | | | | | o | | o |
| A | 5 | M | $11^{5}$ | | | o | o | o | o | | | o | o | o | | | | | | o | | | o | | o | o | | | | | | | o | | o |
| A | 5 | F | $11^{0}$ | | | | o | | | o | | | o | o | o | o | o | | o | | | o | | o | | | | | | | o | | o | o | o |
| A | 5 | F | $10^{9}$ | | | o | o | o | o | | | o | o | o | | o | | | | o | | | o | | o | o | | | | | | | o | | o |
| A | 5 | F | $10^{10}$ | | | | | | o | o | o | | | o | o | o | o | | | o | o | o | o | o | | o | | | | | | | o | | o |
| A | 5 | F | $11^{4}$ | | | o | | | o | o | o | o | o | o | o | o | o | | o | o | | | o | | | o | | | | | o | | o | o | o |
| A | 5 | F | $10^{7}$ | | | o | o | o | o | o | o | o | o | o | o | o | o | | | o | | | o | | o | o | | | | | o | | o | | o |
| A | 5 | F | $12^{10}$ | | | o | o | | o | o | o | o | o | | o | o | o | | o | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| A | 5 | M | $11^{5}$ | | o | o | | o | o | o | | o | o | o | o | o | o | | o | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| A | 5 | F | $10^{6}$ | | | o | o | o | o | o | | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| A | 5 | M | $12^{8}$ | | o | o | o | o | o | o | o | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |

| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| A | 5 | M | 11 6 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 5 | F | 10 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | o | | |
| A | 5 | M | 11 5 | | | | | | | | | | | | | | | | | o | | | | | | | | | | | | | o | | o |
| A | 5 | M | 10 3 | | | | | | | | o | | | | | | | | | | | | | | | | | | | | o | | | | o |
| A | 5 | F | 10 5 | | | | o | | | | | | | | | | | | | o | | | o | | | | | | | | | | | | |
| A | 5 | M | 12 1 | | | o | | | | o | | | | | | | | | | o | | | | | | | | | | | | | | | |
| A | 5 | F | 11 1 | | | o | | | | | | | | | | | | | | o | | | | | | | | | | o | | | o | | o |
| A | 5 | F | 10 9 | | | o | o | | | o | o | | | o | | | | | | o | | | | | | | | | | | | | | | |
| A | 5 | F | 11 4 | | | o | | | | | | | | | | | | | | | | o | | o | o | | | | | o | o | o | o | | o |
| A | 5 | F | 11 1 | | | o | | | | o | | | | | | | | | | | | | o | | o | o | | | | | o | | o | | o |
| A | 5 | M | 10 9 | | | | | | o | | | | | | o | | o | | | | | | o | o | | o | | | o | | | | | | o |
| A | 5 | M | 11 6 | | | o | | | | | | | | | | | | | | o | | | o | | o | o | | | | | o | | o | | o |
| A | 5 | M | 10 11 | | | o | o | | | o | | o | o | o | | | | | o | | | | | o | | | | | | | | | o | | |
| B | 5 | M | 11 3 | | | o | | | | | | | | | | | | | | o | | | | | | | | | | | | | o | | o |
| B | 5 | F | 10 9 | | | o | | | | | o | | | | | | | | | | | | | | | | | | | | o | o | o | | |
| B | 5 | M | 10 0 | | | | | | | | o | | o | | o | | | | | o | | | o | | | o | | | o | | | o | | | |
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| B | 5 | F | 10 2 | | o | o | o | o | | | | o | o | | | | | | | | | | | | | o | | | | | | | o | | o |
| B | 5 | M | 12 2 | | | o | o | | | | | o | | | | | | | | o | | | o | | | o | | | | | | | o | o | o |
| B | 5 | M | 11 1 | | | | o | | | o | o | | o | o | o | o | o | | | o | | | o | | | | | | | o | o | | | | |
| B | 5 | M | 11 9 | o | | o | o | | o | | o | | | | o | | | | | | | | o | | o | o | | | | | o | o | o | | o |
| B | 5 | F | 12 0 | | | o | | o | o | o | | o | | o | | | | | | o | | | o | | o | o | | | | | o | | o | | o |
| B | 5 | F | 11 0 | | | | | | o | o | o | o | | o | o | o | | | o | o | | | o | | | o | | | | | | | o | o | o |
| B | 5 | F | 11 0 | | o | o | o | o | o | | o | o | | | | | | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| B | 5 | M | 11 2 | o | o | o | o | o | o | | o | | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| B | 5 | F | 10 5 | | o | o | o | o | o | o | | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| B | 5 | F | 11 1 | o | o | o | o | o | o | o | o | o | o | o | o | o | o | | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |

| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
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| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| C | 5 | F | 10 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | o |
| C | 5 | M | 10 5 | | | | | | | | | | | o | o | | | | | | | | | | | | | | | | | | o | | o |
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| C | 5 | M | 10 6 | | | o | | | | | o | | | o | | | | | | | | | o | o | | o | | | | o | | | o | | o |
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| C | 5 | M | 10 5 | | | o | o | o | o | | | o | | o | | o | | | | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| C | 5 | M | 11 4 | | | o | o | o | o | o | | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 5 | M | 10 1 | | | o | o | o | o | o | | o | o | o | o | o | o | | o | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 5 | M | 10 10 | | o | o | o | o | o | o | | | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 5 | M | 10 2 | | | o | o | o | o | o | o | o | o | o | o | o | o | | o | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
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| C | 5 | M | 12 11 | | | o | | | o | | | | | | | | o | | o | o | | | o | o | | o | | | o | | | | o | | o |
| C | 5 | F | 10 0 | | | | o | | | o | | o | | | | o | | | | o | o | | o | o | | o | | | | o | | o | o | | o |
| C | 5 | M | 11 11 | | | o | | o | o | | | | o | o | | o | | | | o | | | o | o | o | o | | | | | | o | o | | o |
| C | 5 | F | 10 3 | | | o | o | | | o | | | o | | o | o | o | | | o | o | o | o | o | o | o | | | o | o | | o | o | | o |
| C | 5 | F | 10 1 | | | o | | o | | | | | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | | o | o | | o |
| C | 5 | F | 11 4 | | | o | o | o | o | | o | | o | o | o | o | o | | | o | | | o | o | o | o | | | o | | o | o | o | | o |
| C | 5 | M | 12 8 | | | o | o | o | o | | | | o | o | o | o | | | | | o | o | o | o | o | o | | o | o | o | o | o | o | o | o |
| C | 5 | M | 9 3 | | | o | | o | o | o | o | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 5 | F | 11 7 | | | o | o | o | o | o | o | o | o | o | o | o | o | | o | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 5 | M | 11 2 | | o | o | o | o | o | o | | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 5 | M | 11 9 | o | o | o | o | o | o | o | o | | o | o | o | o | | | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| C | 5 | F | 11 5 | o | o | o | o | o | o | o | o | o | o | o | o | o | o | | o | o | o | o | o | o | o | o | | o | o | o | o | o | o | o | o |

| School | Grade | Sex | Age | Pictorial-Counting Name A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Verbal-Counting Name A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
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| A | 6 | M | 12 2 | | | | | | | | | | | | o | | | | | | | | | | | | | | | | | o | | | |
| A | 6 | F | 11 3 | | | | | | | | | | | | | | | | | | | | o | | | | | | | | | | | | o |
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| A | 6 | F | 11 7 | | | | | | | | | | | | | | | | | | | | | o | | | | | | | | | o | o | o |
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| A | 6 | F | 11 4 | | | o | | | | | | | o | o | o | | o | | | | | | | | | | | | | | | | | | |
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| A | 6 | F | 12 7 | | | o | o | o | o | | | o | o | o | o | | | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| A | 6 | M | 11 4 | | | o | o | o | o | o | | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| A | 6 | M | 11 6 | | | o | | | | | | o | | | | o | | | | o | | | | | | | | | | | | | o | | |
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| A | 6 | M | 11 1 | | | o | o | | o | | | o | | | | | | | | o | | | | | | | | | | | | | o | | o |
| A | 6 | M | 12 4 | | | | | | o | | | | o | | | | | | | o | | | o | | o | | | | | | | | o | | o |
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| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
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| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| A | 6 | M | $10^{11}$ | | | o | | | | | | | | | | | | | | o | | | o | | o | o | | | | | | | o | | o |
| A | 6 | F | $11^{5}$ | o | | | | | | | | | | | o | | | | | o | | | o | | o | o | | | | | | | o | | o |
| A | 6 | F | $11^{10}$ | | | o | | o | o | | | | | o | | | | | | | | | o | | o | o | | | | | o | | o | o | o |
| A | 6 | M | $11^{9}$ | | o | o | o | | o | | | | | o | | o | | | o | | | | o | | | | | | | | o | | o | o | |
| A | 6 | F | $11^{3}$ | | | o | o | o | o | | | | o | | | o | | | | o | | | o | o | | o | | | | o | o | | | | o |
| A | 6 | F | $11^{6}$ | | | o | o | | o | o | o | | o | o | o | o | o | | | | | | | | | | | | | | | | o | o | o |
| B | 6 | M | $11^{5}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | o | | o |
| B | 6 | F | $12^{0}$ | | | | | | | | | | | | | | o | | | o | | | | | o | | | | | | | | | | o |
| B | 6 | M | $11^{8}$ | | o | | | o | | | | | | | | o | | | | o | | | | | | | | | o | | | | | | |
| B | 6 | M | $12^{10}$ | | | o | | | | | | | | | | | o | | | o | | | | | | | | | | | | o | o | | o |
| B | 6 | M | $11^{7}$ | o | | o | | | | | o | o | | | | | | | | | | | o | | | | | | | | o | | o | | o |
| B | 6 | F | $11^{9}$ | | | | | | | | o | | | | o | | | | | o | | | o | o | | o | | | | o | o | | o | | |
| B | 6 | F | $12^{4}$ | | | o | | o | o | o | | | o | o | | | | | | | | | o | | o | o | | o | | o | o | | o | o | o |
| B | 6 | F | $13^{5}$ | | | o | o | o | o | | | o | | o | | | | | o | o | | | o | o | o | o | | | o | o | | o | o | | o |
| B | 6 | F | $13^{11}$ | | o | o | | o | o | o | | | | o | o | o | o | | | o | o | o | o | o | o | o | | o | o | o | o | o | o | o | o |
| C | 6 | F | $12^{5}$ | | | | o | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| C | 6 | M | $11^{6}$ | | | | o | | | | | | | o | | | | | | | | | | | | | | | | | | | | | |
| C | 6 | F | $11^{7}$ | | | o | | | o | | | | | | | | | | | | | | o | | | o | | | | | o | | | | |
| C | 6 | F | $11^{11}$ | | | o | | o | | o | | | o | | o | | | | | | | | | | | | | | | | o | | | | |
| C | 6 | M | $11^{2}$ | | | | | o | | | | | | | | | | | | o | | o | o | | | o | | | | | | | o | | |
| C | 6 | M | $12^{1}$ | | | | | | | | | o | | | | | | | | o | | | o | | o | o | | | | | | | o | | o |
| C | 6 | M | $11^{3}$ | | | o | o | o | o | | | | | o | o | | | | | | | | | | | | | | | | | | o | | o |
| C | 6 | F | $11^{4}$ | | | o | | | | | | | | | | | | | | | | | o | o | | o | | | | o | o | o | o | | o |
| C | 6 | M | $11^{3}$ | | | o | o | | | o | | | o | o | | | o | | | o | | | o | | | o | | | | | | | o | | |
| C | 6 | F | $11^{5}$ | | | o | | | | | o | o | | | | | | | | o | o | o | o | o | o | o | | | | o | | o | o | | o |
| C | 6 | F | $11^{7}$ | | | o | o | o | o | o | o | o | | o | o | o | | | | o | o | | o | | | | | | | | o | | | | |
| C | 6 | F | $12^{4}$ | | | | | | | | | | | o | | | | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |

| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
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| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
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| C | 6 | M | 12 0 | | o | | | | o | | | | | o | | | | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
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| C | 6 | M | 12 2 | | | o | | | o | o | | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | o | o |
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| A | 7 | M | 12 9 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 7 | M | 14 9 | | | | | | | | | | | | | | | | | | | | | | | | | | | | o | | | | |
| A | 7 | M | 13 6 | | | | | | | | | | | | | o | | | | | | | | | | | | | | | | | | | |
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| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
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| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| A | 7 | F | 12 8 | | | | | | | | | | | o | | | | | | | | | o | | o | | | | | | | | | | |
| A | 7 | M | 12 10 | | | o | | | o | o | | | | | | | | | | | | | | o | | | | | | | | o | o | | |
| A | 7 | F | 12 6 | | | | | | | | o | o | | | | o | | | | | | | | o | | | | | | o | o | | | | |
| A | 7 | F | 12 3 | | | | o | | | | | | o | | | | | | | | | | | o | | | | | | o | o | | o | | o |
| A | 7 | F | 12 0 | | | | | | o | | | o | | | | | | | | | | | o | | o | o | | | | | | | o | | o |
| A | 7 | F | 12 5 | | | | | | o | | | | | o | | | | | | o | | | o | | o | o | | | | | | | o | | o |
| A | 7 | F | 12 6 | | | o | | | | | | | | o | | | | | | | | | o | o | o | o | | | | o | | | o | | o |
| A | 7 | M | 13 0 | | | o | | | | | | | o | | o | | | | | | | | o | o | | | o | | o | | | | o | | o |
| A | 7 | M | 13 0 | | | o | | o | o | o | o | | | o | o | | | | | | | | o | | | | | | | | | | | | o |
| A | 7 | M | 12 8 | | | o | | | | | | o | | | o | | | | | | | | o | | | o | | | o | o | | o | o | | o |
| A | 7 | F | 13 0 | | | o | | o | o | | | o | | o | | o | o | | o | | | | o | | | | | | | | o | | o | | |
| A | 7 | M | 13 5 | | | o | | o | | | | | | o | | o | | | | o | o | o | o | | o | o | | | o | | | o | o | | o |
| A | 7 | F | 13 6 | | | o | o | o | o | | | o | | o | | | | | | | o | o | o | o | | o | | | | | | | o | | o |
| A | 7 | F | 13 5 | | | | | | | | | o | | | o | | o | | | | o | o | o | o | | o | | | o | o | o | o | | | o |
| A | 7 | M | 12 3 | | | o | | o | o | o | | | o | o | | | o | | | o | o | o | o | o | o | o | o | o | o | o | | o | o | | o |
| A | 7 | M | 12 7 | o | o | o | o | o | o | | o | o | o | o | o | | | | o | o | o | o | o | o | o | o | o | | o | o | o | o | o | | o |
| B | 7 | M | 13 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| B | 7 | M | 13 2 | | | | | | | | | | | | | | | | | o | | | | | | | | | | | | | | | |
| B | 7 | M | 12 3 | | | | | | | | | | | | | | | | | o | | | | | | | | | | | | | o | | |
| B | 7 | M | 13 5 | | | | | | | o | | | | | | | | | | o | | o | | | | | | | | | | | | | o |
| B | 7 | M | 12 9 | | | o | | | | | | | o | | | | | | | o | o | | | | | | | | | | | | o | o | |
| B | 7 | F | 12 1 | | | | | | o | | | o | | | | | | | | | | | o | | | o | | | | | | | o | | o |
| B | 7 | M | 12 7 | | | | | | | | o | | o | o | | | | | | o | | | | | | | | | | | | | o | | o |
| B | 7 | F | 12 2 | | | | | | | o | | | o | | | | | | | o | o | o | | | | | | | o | | | | | | |
| B | 7 | F | 13 4 | | | | o | | | | | | | | | o | | | | o | | o | | | | | | | | | | | o | | o |
| B | 7 | F | 13 6 | | | | | | | | | | | o | o | | | | | | o | | o | | | o | | | | | | | o | | o |
| B | 7 | M | 12 4 | | | o | | o | o | | | o | | o | | | | | | | | | | | | | | | | | o | | | | o |

| School | Grade | Sex | Age | Pictorial-Counting Name A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
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| B | 7 | M | 14 8 | | | | | | o | o | o | o | | o | o | | | | o |
| B | 7 | F | 12 11 | | | o | | o | o | | | o | | o | | | | | |
| B | 7 | M | 13 1 | | | | | | | | | | o | | | | | | |
| B | 7 | F | 12 9 | | | | | o | | | | | | | | o | | | |
| B | 7 | F | 13 2 | | | | | | o | | | | | | | | | | |
| B | 7 | F | 12 8 | | | o | | o | o | | | o | | o | o | | | | |
| B | 7 | F | 12 5 | | | o | | o | o | | | | | o | | o | | | |
| B | 7 | M | 14 11 | | | o | | | o | | o | | o | | | | | | o |
| B | 7 | F | 12 8 | | o | | | | | o | o | | | | | | | | |
| B | 7 | F | 12 7 | | | | o | | o | | | o | o | o | o | o | o | | |
| B | 7 | F | 12 7 | | | o | | | | | | o | o | | o | o | o | | |
| C | 7 | M | 13 4 | | | | | | | | | o | | | | | | | |
| C | 7 | M | 12 5 | | | | | | | | | | | | | o | o | | |
| C | 7 | M | 11 3 | | | o | | | | | | | | | | | | | |
| C | 7 | M | 12 11 | | | o | o | | | | | | | | | | | | o |
| C | 7 | F | 12 9 | | | | | | | o | | | | | | | | | |
| C | 7 | M | 13 5 | | o | | | | o | | | | | | o | | | | |
| C | 7 | F | 12 10 | | | | | | | | | | | | o | | | | |
| C | 7 | F | 12 10 | | | o | | | | | | | | o | | | | | |
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| C | 7 | M | 13 5 | | | o | o | | | | | | | | | | | | |
| C | 7 | M | 12 7 | | o | o | | | | o | | | o | | o | | | | |
| C | 7 | F | 14 6 | | | o | | | o | | o | | | o | | | | | |
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| C | 7 | F | 13 1 | | | o | o | | o | | | | o | o | | | | | |
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| School | Grade | Sex | Age | Verbal-Counting Name A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
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| B | 7 | F | 12 8 | o | | | o | | | o | | | | | | | o | | o |
| B | 7 | F | 12 5 | o | o | o | o | | o | o | | | | | | | o | | o |
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| B | 7 | F | 12 7 | o | o | o | o | o | | | | | | | | | o | o | o |
| B | 7 | F | 12 7 | o | o | o | o | o | o | o | | | o | o | | o | o | | o |
| C | 7 | M | 13 4 | | | | | | | | | | | | | | o | | o |
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| C | 7 | M | 11 3 | o | | | | | | | | | | o | | | o | | o |
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| C | 7 | M | 13 5 | o | | | o | | | o | | | | | | | o | | o |
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| C | 7 | F | 12 7 | | o | | o | o | | o | | | o | o | | o | | | o |
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| C | 7 | F | 13 1 | o | | o | o | | o | o | | | | o | | o | | | o |
| C | 7 | F | 12 5 | o | | | o | | | o | | | | | | o | | | o |

| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
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| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| C | 7 | F | $12^{11}$ | | | | o | | o | | | o | o | | o | o | | | | | o | o | o | o | o | o | | | o | o | o | | o | | o |
| C | 7 | F | $12^{4}$ | | | o | | | | | o | | | | | | | | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| C | 7 | F | $13^{5}$ | | | | o | | o | o | o | | o | | o | o | o | | | | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 7 | F | $13^{5}$ | | o | o | | o | o | | | | | o | o | | | | o | | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 7 | F | $12^{1}$ | | | | o | | | | o | | o | | o | | o | | o | o | o | o | o | o | o | o | | | o | o | o | o | o | o | o |
| C | 7 | M | $13^{2}$ | | | | o | | o | o | | | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 7 | M | $13^{8}$ | | | o | o | o | o | | | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 7 | F | $12^{2}$ | | | o | o | | o | o | o | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 7 | F | $12^{5}$ | | o | o | o | o | o | o | o | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | | o | o | | o |
| C | 7 | M | $13^{8}$ | | o | o | o | o | o | o | o | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| C | 7 | M | $13^{4}$ | o | | o | | o | o | | | o | o | o | o | o | o | | o | | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| A | 8 | M | $14^{7}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | F | $13^{6}$ | | | | | | | | | o | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | M | $15^{0}$ | | | | | | | | | | | | | o | | | | o | | | | | | | | | | | | | | | |
| A | 8 | F | $13^{6}$ | | | o | o | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | M | $13^{4}$ | | | o | | | | | | | | | | | | | | | | | | | | | | | | | o | | | | o |
| A | 8 | M | $13^{5}$ | | o | o | | o | o | | | | | | | | | | | | | | | | | | | | | | | o | | | |
| A | 8 | F | $14^{0}$ | | | o | | | o | | | o | | | | | | | | | | | o | | | | | | | | | | | | o |
| A | 8 | M | $13^{4}$ | | | o | | o | | | | | | | | | | | | | o | | o | | | | | | | | | | o | | |
| A | 8 | F | $13^{9}$ | | | | | | | | | | | | | | | | | o | | | o | | o | o | | | | | | | o | | o |
| A | 8 | F | $12^{1}$ | | | | | | o | | | | o | | | | | | o | o | | | o | | | | | | | | | | o | | |
| A | 8 | M | $13^{2}$ | | | | | | | | | | | | | | | | | o | | | o | | o | o | | | | | | | o | | o |
| A | 8 | M | $13^{3}$ | | o | o | o | | | | | | o | o | | | | | | | | | | | | | | | | | | | o | | o |
| A | 8 | M | $14^{6}$ | | | o | | | | | o | | | | | | | | | | | | o | | o | o | | | o | | | | o | | o |
| A | 8 | F | $13^{5}$ | | | o | | | | | | | o | o | o | | | | | | | o | o | o | | | | | | | | | | | o |
| A | 8 | F | $13^{8}$ | | | | o | o | o | | o | o | | | | o | | | o | o | | | | | | | | | | | | | | | |
| A | 8 | M | $13^{5}$ | | | o | | | o | | | o | | o | | | | | | o | | | o | | | | | | | | | | o | | o |

# APPENDIX 11



| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| A | 8 | F | 13 8 | | o | o | o | o | o | | | | | o | | | | | | o | | | | o | | | | | | o | | | | | |
| A | 8 | M | 13 6 | | | | o | | | | | o | o | | | o | | | | o | | | o | | o | o | | | | | o | | o | | o |
| A | 8 | M | 14 8 | | | o | | o | o | | o | | | o | | | | | | o | o | | o | | o | o | | | | | o | | | | o |
| A | 8 | M | 13 3 | | | o | o | o | o | | | | o | o | | | | | | o | | | o | | o | o | | | o | | | | o | | o |
| A | 8 | F | 13 3 | | | o | o | | o | | | o | | o | | o | | | | o | | | o | | o | o | | | | | o | | o | | o |
| A | 8 | F | 13 2 | | | o | | o | o | o | | o | | o | | o | | | | o | o | o | o | o | o | o | | | | | | | o | | o |
| A | 8 | F | 13 11 | | | o | o | | o | o | o | | o | o | o | o | o | | | | o | o | o | o | o | o | | | o | o | | o | o | | o |
| A | 8 | M | 14 6 | | | o | | o | o | o | | o | | o | o | o | o | | | o | o | o | o | o | o | o | o | | o | o | o | o | o | | o |
| A | 8 | M | 14 9 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | F | 12 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | F | 14 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | F | 13 6 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | M | 14 0 | | | | | | | | | | | | | o | | | | | | | | | | | | | | | | | | | |
| A | 8 | M | 13 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | o | | o |
| A | 8 | M | 13 10 | | | | o | | | | o | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | M | 13 4 | | | | | | | | | | | | | | | | | | | | o | | | o | o | | | | | | | | |
| A | 8 | M | 14 1 | | | o | | | | | | o | | | | | | | | | | | | | | | | | | | | | | | o |
| A | 8 | M | 13 10 | | | | | | | | | o | | | | | | | | | | | o | | | | | | | | o | | | | o |
| A | 8 | M | 13 4 | | | o | | | | | | | | | | | | | | | | | | | o | o | o | | | | | | | | |
| A | 8 | M | 14 3 | | | | | | | | | o | | | o | | | | | o | | | | | | | | | | | | | | | o |
| A | 8 | F | 13 8 | | | | | | | | o | | | o | o | o | | | | | | | | | | | | | | | | | | | o |
| A | 8 | M | 14 5 | | | o | | | o | | | o | | | | | | | o | o | | | | | | | | | | | | | | | |
| A | 8 | F | 13 5 | | | o | o | | o | | | | o | | | | | | | o | | | | | | | | | | | | | | | |
| A | 8 | F | 13 10 | | | | | | | | o | | | | | | | | | o | | | o | | o | | | | | | | | o | | |
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| A | 8 | M | 13 10 | | | o | o | | | o | | | | | | | | | | | | | o | | o | o | | | | | | | o | | o |
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| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
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| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| A | 8 | M | $13^{10}$ | | | o | | | | | | | | | | | | | | | | | o | | | | | | | | | | | | |
| A | 8 | F | $14^{2}$ | | | | o | o | | | | | | | | | | | | o | | | | | | o | | | | | | | o | | o |
| A | 8 | M | $13^{4}$ | | | o | | o | | | | | | | | | | | | | | o | | o | | | | | | | | | o | | o |
| A | 8 | M | $13^{9}$ | | | | | | | | | | o | | o | | | | | o | o | | o | | | o | | | o | | | | | | |
| A | 8 | F | $13^{7}$ | | | o | o | | | | | | | | | o | | | | o | | | o | | | | | | | | o | | | | o |
| A | 8 | F | $13^{4}$ | | | | | o | o | | | | | | | | | | | o | | | o | o | o | o | | | | | | | o | | o |
| A | 8 | M | $13^{4}$ | | | | | | | | | | | o | o | | | | | o | | | o | | | o | | | o | o | | o | o | | o |
| A | 8 | F | $13^{10}$ | | | o | | o | o | o | o | | | o | | o | | | | | | | o | | o | o | | | | o | o | | o | | o |
| A | 8 | F | $14^{2}$ | | | o | o | | | | | | o | | | | | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| A | 8 | F | $14^{1}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | F | $13^{8}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | M | $13^{9}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | M | $13^{11}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | 8 | M | $14^{11}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | o |
| A | 8 | F | $14^{8}$ | | | o | | | | | | | | | | | | | | | | | | | | | | | o | | | | | | |
| A | 8 | F | $13^{2}$ | | | | | | | | | | | | | o | | | | o | | | | | | | | | o | o | | | | | o |
| A | 8 | M | $14^{6}$ | | | o | | | | | | | | o | o | | | | | | | | o | | | o | | | | | | | | | |
| A | 8 | M | $13^{9}$ | | | | | | | | | | | | o | o | o | | | o | | | | | | | | | | | | | o | | |
| A | 8 | M | $14^{3}$ | | | o | | | | | | | | | | | | | | o | | | o | | | o | | | o | o | | | | | |
| A | 8 | M | $13^{4}$ | | | o | | | | | o | | | | | o | o | | | o | | | | | | | | | | | | | o | | |
| A | 8 | F | $14^{1}$ | | | o | o | | | | | o | | o | | | | | | | | | o | | | o | | | | | o | | | | o |
| A | 8 | F | $13^{11}$ | | | o | | | o | | o | o | | o | | | | | | o | | | | | | | | | | | | | o | | o |
| A | 8 | M | $14^{0}$ | | | | | | | | | | | o | | | | | | o | o | | o | o | | | | | o | o | | o | o | | o |
| A | 8 | F | $14^{0}$ | | | o | o | | o | | | o | | o | | | | | | o | | | o | | | o | | | | | | | o | | o |
| A | 8 | M | $14^{4}$ | | | | | o | | | | o | | | o | | | | | o | | | o | | | o | | | o | o | | | o | | o |
| A | 8 | M | $14^{0}$ | o | | | o | | | | | o | o | o | | | | | | o | | | o | | | | | | | o | | o | o | | o |
| A | 8 | M | $13^{6}$ | o | | o | | o | o | | | o | | o | | o | | | o | o | | | | | o | | | | | | | | o | | |

# APPENDIX 11



| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| A | 8 | M | $13^{11}$ | | | o | o | | | | o | o | o | | o | | o | | | o | | | o | | o | o | | | | | | | o | | o |
| A | 8 | F | $13^{6}$ | | | | | | o | | | o | o | | o | o | o | | | o | o | | o | | | o | | | | | | o | o | | o |
| A | 8 | F | $14^{1}$ | | | o | o | o | o | o | o | o | o | o | o | o | | | | | o | o | o | o | o | o | | | o | o | o | o | o | o | o |
| A | 8 | F | $15^{1}$ | | | o | o | o | o | o | o | o | o | o | o | o | o | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |
| B | 8 | F | $12^{9}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | o | | |
| B | 8 | M | $14^{3}$ | | | | | | | | | | | | | | | | | | | | | | | | | | o | | | | | | |
| B | 8 | M | $13^{4}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | o | | | | |
| B | 8 | M | $13^{5}$ | | | | | | | | | | | | o | | | | | | | | | | | o | | | | | | | o | | |
| B | 8 | M | $12^{8}$ | | | o | | | | | | o | | | | | | | | o | | | | | | | | | | | | | | | |
| B | 8 | M | $13^{4}$ | | | | | | | | | | o | | | | o | | | | | | | | | | | | | | o | | | | |
| B | 8 | M | $13^{7}$ | | | | | | o | | | | o | | | | | | | | | | | | | | | | | | | | | | o |
| B | 8 | F | $14^{7}$ | | | o | | | o | | | | | | | | | | | | | | | | | | | | | | o | | | | o |
| B | 8 | M | $13^{5}$ | | | o | | o | o | | | | | | o | | | | | | | | | | | | | | | | | | | | o |
| B | 8 | F | $15^{0}$ | | | o | | | | | | | | | o | | | | | o | | | | | | | | | | | | | o | | o |
| B | 8 | M | $14^{0}$ | | | | | | | o | | | o | | | | | | o | | | | | | | o | | | | | | | o | | o |
| B | 8 | F | $14^{5}$ | | | o | | | | | | o | | o | | | | | | | | | o | | | o | | | | | | | | | o |
| B | 8 | M | $13^{6}$ | | | | o | | o | | | | | | | | | | | o | | o | | | o | | | | | | | | o | | o |
| B | 8 | F | $13^{8}$ | | | o | | | o | | | | | o | | o | | | | | | o | o | | | | | | | | o | | | | o |
| B | 8 | F | $14^{1}$ | | | | | | o | | | | | o | | | o | | o | | | | o | | | o | | | | | | | o | | o |
| B | 8 | M | $13^{9}$ | | | o | | o | o | | | | | o | | | | | | o | o | | | | | | | | | | | | o | | o |
| B | 8 | M | $13^{9}$ | | | | | | | | | | | o | | | | | | | | | o | | o | o | | | | o | | o | o | o | o |
| B | 8 | M | $14^{7}$ | | | o | | o | | | o | | | | o | o | o | | | | | | o | | | | | | | | | | o | | o |
| B | 8 | F | $13^{2}$ | | | o | | o | o | | | o | | o | o | | | | | o | | | | o | | | | | | | | | o | | o |
| B | 8 | F | $14^{2}$ | | | o | o | o | o | o | | o | o | o | | o | | | | | o | | | o | | | | | | | o | o | | | |
| B | 8 | M | $14^{5}$ | | | | o | | o | | | o | | | | | o | | | o | | | o | | o | o | | o | o | | o | | o | o | o |
| B | 8 | F | $14^{3}$ | | | | o | o | o | | | | | | | | | | | o | o | o | o | o | o | o | | | o | o | o | o | o | | o |

APPENDIX 11



| School | Grade | Sex | Age | Pictorial-Counting Name | | | | | | | | | | | | | | | | Verbal-Counting Name | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| C | 8 | M | $13^{8}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| C | 8 | M | $13^{7}$ | | | | | | | | | | | | | | | | | o | | | | | | | | | | | | | | | |
| C | 8 | F | $13^{9}$ | | | o | | | | | | | | | | | | | | | | | | | | | | | | | | | o | | |
| C | 8 | M | $13^{11}$ | | | | o | | | | | | | | | o | | | | | | | | | | | | | | | | | | | |
| C | 8 | M | $13^{11}$ | | | o | | | | | | | | o | | | | | | | | | o | | | | | | | | | | | | |
| C | 8 | M | $14^{5}$ | | | o | | | | | | | | | | | | | | o | | | | | | | | | | | | | o | | o |
| C | 8 | M | $13^{11}$ | | o | | | | | | | | | | | | | | o | o | o | | | | | | | | | | | | | | |
| C | 8 | M | $14^{4}$ | | | o | | | | | | | o | | | | | | o | o | | | | | | | | | | | o | | | | o |
| C | 8 | F | $13^{3}$ | | | | | | | | | o | | | o | | | | | o | | | o | | | | | | | | | | o | o | o |
| C | 8 | M | $14^{6}$ | | | o | | | | | | o | o | | | | | | | o | | | | | | | | | | | o | | o | | o |
| C | 8 | F | $11^{11}$ | | | o | | o | o | | | o | | o | | | | | | o | | | o | | | | | | | | | | o | | |
| C | 8 | F | $13^{2}$ | | | o | | o | | | | o | o | o | o | o | o | | | | | | o | | | o | | | | | | | o | | o |
| C | 8 | M | $15^{2}$ | | | o | o | o | o | | | | | o | | | o | | | o | | | o | | o | o | | | | | o | | | | o |
| C | 8 | F | $13^{11}$ | | | o | o | | | o | | | | | | | | | | o | o | | o | o | o | o | | | | o | | o | o | | o |
| C | 8 | M | $15^{11}$ | | | | | | | | | o | o | | o | | | | | | o | | o | o | o | | | | o | o | o | o | o | | o |
| C | 8 | M | $13^{2}$ | | | o | o | o | | | | o | | | | | | | | o | | o | | o | o | o | | | o | o | | | o | o | o |
| C | 8 | M | $13^{10}$ | | | | o | o | o | | | o | | o | o | o | o | | | o | | | o | o | | o | | | o | | o | | o | | o |
| C | 8 | F | $13^{7}$ | | | o | | | o | | | | | | | | | | | o | o | o | o | | o | o | | | o | o | | o | o | | o |
| C | 8 | F | $13^{1}$ | | | o | | o | o | | | | o | o | | | | | | o | o | o | o | o | o | o | | | o | o | | | o | | o |
| C | 8 | F | $15^{1}$ | o | | | | | o | | | o | | o | o | o | | | o | o | o | o | o | o | | o | | | o | o | o | o | o | | o |

APPENDIX 12

ABSTRACT OF

A Test of Piaget's Theory of Number Extended to Decimal Numerals

APPENDIX 12

ABSTRACT OF

A Test of Piaget's Theory of Number Extended to Decimal Numerals

Considerable Piagetian-based research has been performed to examine the child's formation of small number concepts. There has been little research into the child's formation of large number concepts involving numbers represented by decimal numerals. The child's formation of large number concepts has been examined in this paper.

In the first section, concepts formation was found to involve operative and figurative aspects. In the development of these aspects, vertical and horizontal dêcalages may be observed.

Studies relevant to the formation of number concepts were examined. Several vertical and horizontal dêcalages in the formation of small number concepts were noted. Possible vertical and horizontal dêcalages in the formation of large number concepts were identified.

Tasks were devised to elicit the existence of these dêcalages, and were presented to 396 students in Grades Four through Eight.

Significant vertical dêcalages were observed between tasks requiring different levels of operative thought. These dêcalages were consistent with Piaget's hypothesized sequence of cognitive development.

Significant horizontal dêcalages were also observed between tasks involving different figurative aspects.

Considerable development of large number concepts involving regrouping occurs between Grades Four to Six. However, full development

cannot be expected until the child reaches the formal operative stage. This stage is reached by many sometime between Grade Seven and Eight. These results are also in accord with Piaget's general theory of cognitive development.