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Identifying Permanent and Transitory Components in Latin-American Real Exchange Rates¹

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Abstract

The behaviour of the long-run real exchange rate for four Latin-American countries is investigated for the period 1957-2002. The long-run real exchange rate is derived from an unobserved component model which divides the real exchange rate into a permanent and a transitory component. The permanent component is modeled as a homoskedastic random walk while the transitory component is modeled as having variances which switch according to a Markov-Switching process, between low and high variance states. Following Engel and Kim (1999), we estimate this model using the Gibbs-sampling methodology with monthly data for Argentina, Brazil, Chile and Mexico. The analysis of results allows us to estimate different expected duration of the low and high-variance regimes; as well as the behavior of the permanent and transitory components and the detection of important events in the four countries related to the peaks of the high-variance state.

Keywords: Real Exchange Rates, Gibbs-Sampling, Permanent and Transitory Components, Low and High variance Regimes.

JEL Classification: C2, C5, F3.

Résumé

Ce travail analyse le comportement à long terme du taux de change de quatre pays d'Amérique latine pour la période de 1957 à 2002. Le taux de change à long terme est dérivé d'un modèle formé de deux composantes inobservées, soit une composante transitoire et une permanente. La composante permanente est modélisée en marche aléatoire avec homoscélasticité, alors que la composante transitoire est modélisée de façon à ce que la variance passe de faible à élevée selon le régime de Markov dans lequel elle se trouve. Suivant Engel et Kim (1999), nous estimons ce modèle utilisant la méthode d'échantillonnage à la Gibbs avec des données mensuelles pour l'Argentine, le Brésil, le Chili et le Mexique. L'analyse des résultats nous permet d'estimer différentes durées prévues pour les régimes de variance faible et élevée. Les résultats nous permettent aussi d'observer le comportement des composantes permanente et transitoire, ainsi que de détecter les événements importants (chocs) dans les quatre pays selon l'état maximal de variance élevée.

Mots-clés: Taux de change réel, échantillonnage à la Gibbs, composantes permanente et transitoire, régimes à variance faible et élevée.

Code JEL: C2, C5, F3.

1 Introduction

Nominal and real exchange rates always play a crucial role in any country, since their movements affect key economic variables. This role is particularly important for developing countries, given the external restriction they face, specially after the second half of 1970 with the breakdown of the Bretton Woods system. Unlike the industrialized countries, the Latin-American economies have experienced large devaluations and inflationary episodes. Not surprisingly, the evolution of the real exchange rate in these countries has showed periods of large appreciations, severe depreciations, and periods of stability.

In fact, exchange rates have been used extensively as an economic policy tool by many Latin-American countries. In some cases, exchange rates were kept fixed under inflationary episodes, while under restrictions of foreign capital, a policy of devaluation was often followed. Generally, an “intermediate” exchange rate policy is adopted given a context of abundant foreign capital and low inflation. Currently, the major Latin-American countries have a floating exchange rate as their exchange rate policy.

A vast amount of literature has been devoted to the analysis of the behavior of the real exchange rate, in particular to test the Purchasing Power Parity (hereafter PPP) theory. For this purpose, a variety of econometric methods have been used. The most common methods have been the application of unit-root tests, and cointegration analysis. The results from these methods have been so different³ that researchers have appropriately named it the “PPP puzzle” (Rogoff, 1996). However, there is little disagreement about the high persistence of the real exchange rate. It seems that “shocks” to the real exchange rates are of a long duration, causing its deviation -for long periods of time- from what is considered its PPP value.

In this paper, following Engel and Kim (1999), the real exchange rate is seen as consisting of two components, a non-stationary component and a stationary component and the model is estimated using the technique of Gibbs sampling⁴. We apply this approach to four real exchange rates of four Latin-American countries which are Argentina, Brazil, Chile, and Mexico. Monthly data is used and it spans the period from January 1957 to April 2002.

³ As surveyed in Froot and Rogoff (1995), while early research suggests the presence of a unit root in the real exchange rate, the recent evidence supports stationarity.

⁴ Gibbs-sampling, a Markov chain Monte Carlo method, is a technique for generating random variables from a distribution indirectly, without having to calculate the density. See the appendix for additional details.

The rest of this paper is organized as follows. Section 2 presents the characteristics of the model to be estimated. In section 3, we present the data and the estimation results. Conclusions are drawn in the section 4. An appendix includes some technical explanations about the Gibbs sampler.

2 The Model

The estimation of the real exchange rate is based on the unobserved component model used by Engel and Kim (1999), who applied it to the real exchange rate between US and United Kingdom. The real exchange rate is defined by

$$Q_t = E_t \left(\frac{P_t^*}{P_t} \right) \quad (1)$$

or in logarithm form, by

$$q_t = e_t + p_t^* - p_t, \quad (2)$$

where e_t is the logarithm of the nominal exchange rate (E_t), p_t^* is the logarithm of the foreign price level (P_t^*) and p_t represents the logarithm of the home country price level (P_t).

Let assume that the domestic and foreign prices are weighted averages of the prices of traded (T) and non-traded goods (N), that is

$$p_t = (1 - \pi)p_t^T + \pi p_t^N \quad (3)$$

$$p_t^* = (1 - \lambda)p_t^{T*} + \lambda p_t^{N*}, \quad (4)$$

which allows us to write the logarithm of the real exchange rate, q_t , as the sum of two components

$$q_t = y_t + x_t \quad (5)$$

where y_t and x_t are defined by

$$y_t = (p_t^{N*} - p_t^{T*}) - (p_t^N - p_t^T) \quad (6)$$

$$x_t = e_t + p_t^{T*} - p_t^T. \quad (7)$$

Thus, the real exchange rate, q_t , is assumed to consist of a permanent component, y_t , and a transitory component, x_t . Notice that x_t can be considered as the relative price of traded goods, x_t , which is likely to be a stationary random variable. In other terms, if all goods in the traded goods

price indices have the same weight at home and abroad, then changes in x_t occur only because of deviations from the law of one price (Engel, 1999). Even if the deviations from the law of one price are large and persistent, they are almost certainly stationary⁵.

We adopt the approach of Engel and Kim (1999), who consider the shocks to the transitory component as coming primarily from a nominal demand sources, such as monetary shocks, see Mussa (1986).

On the other hand, permanent shocks to productivity could impart a non-stationary component to the relative price of non-traded to traded goods, thus y_t could have a unit root⁶. Hence, it is believed that the permanent component represents the shocks to tastes and technologies that cause permanent movements in relative price levels. See, for example, Stockman and Tesar (1995).

Given the above explanation, the permanent component, y_t , is modeled as a homoskedastic random walk, that is,

$$y_t = y_{t-1} + v_t \tag{8}$$

where $v_t \sim N(0, \sigma_v^2)$.⁷ The transitory component, x_t , is assumed to follow an $AR(2)$ process with a heteroskedastic error term, that is,

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \epsilon_t \tag{9}$$

where $\epsilon_t \sim N(0, \sigma_{\epsilon_t}^2)$. In particular, the error term ϵ_t depends on a discrete-valued first-order Markov switching variable, s_t , ($t = 1, 2, 3$) which evolves independently of v_t and ϵ_t , according to transition probabilities defined by $Pr[s_t = j | s_{t-1} = i] = p_{ij}$, for $i, j = 1, 2, 3$; and with conditions such as $\sum_{j=1}^3 p_{ij} = 1$; $\sigma_1^2 < \sigma_2^2 < \sigma_3^2$ and $\sigma_{\epsilon_t}^2 = \sigma_1^2 s_{1t} + \sigma_2^2 s_{2t} + \sigma_3^2 s_{3t}$, where $s_{kt} = 1$ if $s_t = k$ ($k = 1, 2, 3$), and $s_{kt} = 0$ otherwise. One last assumption in the model is that shocks v_t and ϵ_t are independent. This assumption is necessary in helping to distinguish the temporary from the permanent component. This means that the aggregate nominal shocks, which correspond to the transitory component, and the sectoral real shocks, which correspond to the permanent component, can be thought of as independent.⁸

⁵ Almost any theory of international price determination implies that deviations from the law of one price for traded goods are stationary.

⁶ See, for example, Balassa (1964) and Samuelson (1964).

⁷ We obtain no improvements when an hetetoskedastic random walk was used.

⁸ In words of Engel and Kim (1999), nominal shocks (affecting the transitory component) can be thought of as independent of the sectoral shocks that drive the permanent component.

3 Data and Estimation Results

Real exchange rates are calculated using (2) multiplied by 100. For our purpose, P_t^* represents the consumer (or producer) price index of United States; P_t is the consumer (or producer) price indices of Argentina, Brazil, Chile or Mexico and the logarithm of the nominal exchange rate is represented by e_t .⁹ Because the time span matters in this econometric estimation, a consumer or producer price index was chosen according to availability of information. Thus, for Argentina, Chile and Mexico, consumer price indices were used, and for Brazil, a producer price index was chosen¹⁰. Monthly data is used and it corresponds to the period from January 1957 until April 2002¹¹. Information on price indices and nominal exchange rates are taken from the “International Financial Statistics”, the CD-ROM of the International Monetary Fund (IMF).

In the estimation of this model, the Gibbs sampler is run for twelve thousand iterations, and we discard the first two thousand. The distributions of $\{X^T\}$, $\{Y^T\}$ and $\{S^T\}$ ¹² are based on the last ten thousand iterations. For the distribution of $\Theta = \{\phi_1, \phi_2, \sigma_v^2, \sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2, p_{11}, p_{22}\}$, it is taken every fifth observation from the final ten thousand observations (because of potential serial correlation across iterations). Observe that the parameters of the model are treated as random variables with prior distributions in the Bayesian setting. Unlike the model estimated by Engel and Kim (1999), we consider only two states for the variance of the transitory component. Therefore, it evolves according to a Markov switching process between a low and high variance states while the variance of the permanent component is kept homoskedastic.

In all tables of results, p_{11} is the probability of staying in the state 1 (low-variance state), p_{22} is the probability of staying in the state 2 (high-variance state), ϕ_1 is the coefficient associated to the first-order autoregressive term, ϕ_2 is the coefficient associated to the second-order autoregressive term; σ_v^2 is

⁹ A preliminary version of this paper included a historical background about the exchange rate policies in the four Latin-American countries. Most important references for the historical evolution are Diaz and Schamis (1999), Hamilton and Getulio (2002), Bonomo and Terra (1999), De Gregorio (1998), and Morandé and Matias (2002). Details can be requested from the authors.

¹⁰ It is important to point out that the behaviour of the real exchange rate under both indices is very similar; comparisons were made under equal periods.

¹¹ Information previous to 1957 is not available.

¹² Where X^T refers to the vector of T values of the transitory component, whereas Y^T refers to the vector of T values of the permanent component. S^T is the vector of T values of the state S_t .

the error term variance of the permanent component, $\sigma_{\epsilon_1}^2$ is the error term variance of the transitory component in state 1, and $\sigma_{\epsilon_2}^2$ is the error term variance of the transitory component in state 2. Finally, the row denoted by *eigen* refers to the largest eigenvalue of the transitory component. Using the estimates of p_{11} and p_{22} , we can calculate the expected duration of each regimen. In the case of the regimen of low variance, the expected duration is $1/(1 - p_{11})$. In a similar way, the expected duration for the regimen of high variance is calculated as $1/(1 - p_{22})$.

The results for Argentina are presented in Table 1. The expected duration of the low-variance state is of 71.4 months, approximately 5.9 years. In comparison, the expected duration of the high-variance state is 0.5 years. Both autoregressive coefficients seem to be significant. Table 2 presents the results for Brazil. In this case, the expected duration of both regimes is shorter than for the case of Argentina. In fact, the expected duration of the low-variance and high-variance state are 37.03 and 2.07 months, respectively -it is equivalent to approximately 3.1 and 0.17 years. In this case, the second autoregressive coefficient is not significant, the same happens with the error term variance of the transitory component in the state 2.

Table 3 shows the results for Chile. In this case, the expected duration of the low-variance state is 83.3 months, equivalent to 6.9 years, the highest among all the countries. The expected duration of the high-variance state is however, approximately, 0.32 years a value between the case of Argentina and Brazil. Moreover, the second autoregressive coefficient and the error term variance of the transitory component in the state 2 seem to be non-statistically significant. The results for Mexico are shown in Table 4. According to the mean, the expected duration of the low-variance and high-variance state is 0.99 and 0.25 years respectively, but according to the median values of p_{11} and p_{22} , the expected duration of the low-variance and high-variance state is of 5.95 and 0.28 years.

The time-series behavior of the different components of the real exchange rate is shown in Figures 1-4, where each figure represents each country; they contain four panels. In the first panel, we present the evolution of the real exchange rates together with the evolution of the permanent and the transitory components. The second panel shows the evolution of the permanent component with its upper and lower band. Panel 3 presents similar information as in the second panel but for the transitory component. Finally, the fourth panel shows the probability of the high-variance state (state 2). After observing the graphs, we can say that for Argentina, all components seems to be correlated with the original real exchange time series, although it is easy to observe that most of time of the complete

period, the transitory component seems to drive the evolution of the real exchange rate. The permanent component is closely related to the real exchange rate until 1975 and in the period 1990-1991.

In the case of Brazil, the situation is more complicated although, overall, both components seem to drive the real exchange rate. However, for the period 1995-1999, it seems to be the permanent component the more important to explain the evolution of the real exchange rate.

For Chile, it is clear the importance of the transitory component to explain the behavior of the real exchange rate. We observe clearly that the transitory component is larger than the permanent component for the period 1960-1981. An opposite situation is observed for the subsequent period. We also observe that the permanent component shows a strong persistent behavior with an upward trend in all the period. In particular, it is more notorious after the period 1974-1975. The permanent component resembles the evolution of the real exchange rate only until 1971. As in the case of Chile, the evolution of the real exchange of Mexico seems to be driven for the transitory component. However, the permanent component also follows closely the behavior of the real exchange rate, in particular since 1976.

Some additional comparisons between countries are possible from the results of Tables 1-4. The error term variance of the permanent component is higher for Argentina. It is followed by Brazil, Chile and Mexico, which present the lower variance. Same result is observed for the error term variance of the transitory component in the state 1 (low-variance regime). The error term variance of the transitory component in the high-variance state (state 2) is also higher for Argentina, the coefficient of this element is statistically different from zero. Then, overall, the real exchange rate for Argentina seems to have higher variability in the permanent and transitory components (in both states).

Higher probabilities for the low-variance state are observed for Chile. It is followed by Argentina, Brazil and Mexico. It is equivalent to say that Chile and Argentina present the highest expected durations of the low-variance regimes. For p_{22} the higher value is observed for Argentina, followed by Chile, Mexico and Brazil. It is equivalent to say that Argentina and Chile show the highest expected duration of the high-variance regime. Then, both last results, seem to indicate that Argentina and Chile present higher expected duration of both states of the real exchange rates. Mexico presents the shortest expected duration for a low-variance regime and Brazil presents the shortest expected duration for a high-variance regime.

Regarding the value of the autoregressive coefficients, the eigenvalue is higher for Argentina. It is the only case where the second autoregressive

coefficient seems to be significant. In the rest of the cases, it seems that only the first autoregressive coefficient is important; the most persistent case is Brazil followed by Chile, Mexico and Argentina.

The evolution of the probabilities of the high-variance state (fourth panel in Figures 1-4) allows us to point out some important facts. Overall, peaks observed in these Figures are related to important events that happened in the four countries. In the case of Argentina, the period 1975:04-1975:07 corresponds to an episode of strong devaluations. The second period (1982:06-1982:09) corresponds to a floating exchange rate adjusted passively to inflation. The third period (1989:04-1990:07) is related to the collapse of the *Primavera Plan*. Notice that the full hyperinflation took place in 1990 implying a quarter inflation of 334% and a devaluation of 267%. Finally, the last period of high-variance (2002:02-2002:04) is related to the abandon of the currency board system and the adoption of a flexible exchange rate system.

In the case of Brazil, we find two peaks for the high-variance probabilities with three short periods (duration of only one month). The period 1964:03-1964:05 is related to the intent of unification of the exchange rate with the characteristic of infrequent but large devaluations. The period 1999:02-1990:04 corresponds to the adoption of a floating exchange rate policy. The period 1983:03 corresponds to a devaluation of 30% with a subsequent return of the maxi-devaluations system. The peak corresponding to April, 1990 is related to another important devaluation and the subsequent intent of adopting a fix exchange rate. Finally, the peak related to August 1994 correspond to the *Real Plan* with an intent to maintain a fix exchange rate.

In the case of Chile, we have to note that in April 1965 a crawling peg system was established with two devaluations per month. Therefore, the first peak observed in the high-variance regime for Chile is related to important deluations. The period 1971:06-1971:08 is also related to a devaluation with the intent (after that) to keep a fix exchange rate. The peak of 1973:04-1974:04 corresponds to a strong devaluation to restore competitiveness. The government established a crawling peg with four devaluations per month with the objective of arriving to an unification of exchange rates.

In the case of Mexico, there are three peaks in the probabilities of the high-variance state. The first period (1982:02-1982:05) is related to a strong devaluation to correct the current account deficit. The following peak (1982:11-1983:02) is also related to a devaluation. Notice that in this time Mexico suffered the so called “debt crisis” because of that Mexico was outside of the international financial system. The last peak is observed at 1994:12-1995:04. Note that Mexico had adopted a band system. In April

1994 the dollar was often at the ceiling of the band (3.4 pesos per dollar). Then, in December 20, 1994, the government adjusted the ceiling of the band in 15%.

Tables 1-4 also included mean and medians for the different coefficients estimated. In making the comparison between both values, we note that the case of Mexico shows clear differences between the mean values respect to the median values. It is a possible indicator of a skewed distribution for this country.

4 Conclusions

Following Engel and Kim (1999), we use Gibbs-Sampling approach to estimate a model for the long-run real exchange rate of four Latin-American countries (Argentina, Brazil, Chile and Mexico) covering the period 1957-2002 using monthly data. A permanent and a transitory components are estimated. Using these estimations, we calculated smoothed probabilities of both states of the variance in the transitory component. In general, the results show a significant importance of the transitory component to explain the behavior of the real exchange rates. Brazil presented the most persistent case.

5 Appendix

Some details about the Gibbs-sampling methodology are shown here. For further details, see Engel and Kim (1999).

5.1 Gibbs-Sampling Methodology

Gibbs sampling is a Markov chain Monte Carlo (MCMC) simulation method for approximating joint and marginal distributions by sampling from conditional distributions. It was introduced to the statistic literature by Geman and Geman(1984). In order to gain a better understanding of this algorithm we begin with a brief review of the theory of Markov Chains, the random processes that form the mathematical backbone of MCMC method.

5.1.1 Markov Chains

A Markov Chain is a random process $(X_n)_{0 \leq n \leq N}$, (i.e. a sequence of random variables¹³ X indexed by n) satisfying the Markov property:

$$\begin{aligned} \mathbf{P}(X_m = i_m, \dots, X_{m+n} = i_{m+n} | X_0 = i_0, \dots, X_m = i) \\ = \mathbf{P}(X_m = i_m, \dots, X_{m+n} = i_{m+n} | X_m = i) \end{aligned} \quad (10)$$

In other words, the distinguishing feature of a Markov Chain is that the evolution of the process after period m depends solely on the state of the process at period m and is independent of the states prior to period m . The index is normally associated with time, and Markov Chains are often understood in this context. Markov Chains can be described in terms of a transition matrix P and an initial distribution λ . A transition matrix has elements p_{ij} , the probability that the process is in state j given that it was in state i in the previous period, which can be represented by

$$p_{ij} = \mathbf{P}(X_{n+1} = j | X_n = i) \quad (11)$$

In another hand, the initial distribution λ is a vector describing the distribution for X_0 :

$$\lambda_i = \mathbf{P}(X_0 = i) \quad (12)$$

From this initial distribution and the transition matrix, we can determine the distribution of X_n for $n = 1, 2, 3$ recursively as follows:

$$\begin{aligned} \mathbf{P}(X_1 = i_1) &= \sum_{i_0} \mathbf{P}(x_0 = i_0) p_{i_0 i_1} = (\lambda P)_i \\ \mathbf{P}(X_2 = i_2) &= \sum_{i_0} \mathbf{P}(x_1 = i_1) p_{i_1 i_2} = (\lambda P^2)_i \\ &\vdots \\ \mathbf{P}(X_n = i_n) &= \sum_{i_{n-1}} \mathbf{P}(x_{n-1} = i_{n-1}) p_{i_{n-1} i_n} = (\lambda P^n)_i \end{aligned} \quad (13)$$

then, Markov Chain Monte Carlo methods rely on the limiting distribution of a Markov Chain as $n \rightarrow \infty$.

¹³In the case of a Markov chain, the random variable is assumed to take integer values and frequently this is limited to finite subsets of the integers.

5.1.2 The Gibbs Sampler in Bayesian Inference

Suppose that we have a random sample $X = (X_1, \dots, X_T)$ from a distribution $f(X_i|\theta)$ where the parameter $\theta = (\theta_1, \dots, \theta_k)$ is unknown but prior beliefs about θ follow a distribution $\pi(\theta)$. Then the posterior distribution is represented by

$$\pi(\theta | X = x) \propto \pi(\theta) f(X | \theta) \quad (14)$$

where

$$f(X | \theta) = \prod_i f(X_i | \theta) \quad (15)$$

which can be estimated by sampling from θ , given X . In fact, the Gibbs sampler creates a sample as follows:

1. Take an initial vector $\theta^{(0)} = (\theta_1^{(0)}, \dots, \theta_k^{(0)})$.
2. Generate a sample $\theta_1 = \theta_1^{(1)}$ from the conditional distribution $\pi(\theta_1^{(1)} | \theta_2^{(0)}, \dots, \theta_k^{(0)}, X)$. This can be done by generating a random number, p , uniformly between 0 and 1 and then taking the value t that satisfies

$$\mathbf{P}(\pi(\theta_1^{(1)} | \theta_2^{(0)}, \dots, \theta_k^{(0)}, X) \leq t) = p \quad (16)$$

3. Generate samples for $\theta_2^{(1)}, \dots, \theta_k^{(1)}$ using the conditional distribution:

$$\pi(\theta_i^{(1)} | \theta_1^{(1)}, \dots, \theta_{i-1}^{(1)}, \theta_{i+1}^{(0)}, \dots, \theta_k^{(0)}, X)$$

4. Repeat steps 2 and 3 for $\theta^{(n)} = (\theta_1^{(n)}, \dots, \theta_k^{(n)})$ using $\theta^{(n-1)}$
5. For large s and t , record the n vectors $\theta^{(s)}, \theta^{(s+t)}, \dots, \theta^{(s+(n-1)t)}$.

These n vectors should be independent random samples from the distribution $\pi(\theta|X)$.

We now have a set of n vectors¹⁴, $\theta^{[1]}, \dots, \theta^{[n]}$ assumed to be samples from $\pi(\theta|X)$. We can now use this sample to approximate the mean of θ given the data X

$$\theta = n^{-1} \sum_i \theta^{(i)}$$

¹⁴ Here we define $\theta^{[i]} \equiv \theta^{(s+it)}$.

Alternatively, we can form confidence intervals for the true value of the vector θ by assuming that for large n , the $\theta^{[i]}$ are approximately normally distributed with mean θ and sample variances matrix

$$\hat{\sigma}^2 = (n - k)^{-1} \sum_i (\theta^{(i)} - \hat{\theta})(\theta^{(i)} - \hat{\theta})' \quad (17)$$

then the confidence intervals are given by

$$\mathbf{P}[(\theta - \hat{\theta})' \hat{\sigma}^2 (\theta - \hat{\theta}) \leq F_{n,m}^\alpha] = 1 - \alpha \quad (18)$$

Calculation of $\pi(\theta|X)$ is a specific application of a more general calculation of $f(\theta|\phi)$ where the θ are unknown parameters and the ϕ are parameters specified in full. Notice that the Gibbs sampler, when used in Bayesian inference, requires an explicit calculation of the conditional distribution of θ_i given the other elements of θ and X . Often, the prior distribution $\pi(\theta)$ is chosen such that the conditional posterior distribution takes on a standard form like a normal or beta distribution.

5.1.3 A simple example of Gibbs-Sampling in Econometrics

This example is adapted from Kim and Nelson (1999). Consider an $AR(2)$ model for an economic time series:

$$\begin{aligned} y_t &= \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t \\ \epsilon_t &\sim i.i.d. \quad N(0, \sigma^2) \end{aligned} \quad (19)$$

where the roots of $(1 - \phi_1 L - \phi_2 L^2) = 0$ lie outside the complex unit circle. In matrix notation we have

$$\begin{aligned} Y &= X\beta + \epsilon \\ \epsilon &\sim N(0, \sigma^2 I_T) \end{aligned} \quad (20)$$

To apply the Gibbs sampling algorithm, we need conditional posterior distributions of β and σ^2 (the unknown “parameters”), given appropriate conditional prior distributions. We need the following two conditional distributions for Gibbs-sampling which are the conditional distribution of β given σ^2 and the conditional distribution of σ^2 given β .

5.1.4 Conditional Distributions of β , given σ^2

The prior distribution can be represented by

$$\beta|\sigma^2 \sim N(\beta_0, \Sigma_0)_{1[s(\phi)]}, \quad (21)$$

where β_0 and Σ_0 are known and $1[s(\phi)]$ is an indicator function to indicate that the roots of $\phi(L) = 0$, are outside the unit circle. In another hand, the posterior distribution will be

$$\beta|\sigma^2, Y \sim N(\beta_1, \Sigma_1)_{1[s(\phi)]}, \quad (22)$$

where

$$\beta_1 = (\Sigma_0^{-1} + \sigma^{-2}X'X)^{-1}(\Sigma_0^{-1}\beta_0 + \sigma^{-2}X'Y) \quad (23)$$

$$\Sigma_1 = (\Sigma_0^{-1} + \sigma^{-2}X'X)^{-1} \quad (24)$$

5.1.5 Conditional Distribution of σ^2 , Given β

In this case, the prior distribution can be represented by

$$\sigma^2|\beta \sim IG(v_0/2, \delta_0/2), \quad (25)$$

where IG is the inverted gamma distribution and v_0 and δ_0 are known. The posterior distribution will be

$$\sigma^2|\beta, Y \sim IG(v_1/2, \delta_1/2), \quad (26)$$

$$v_1 = v_0 + T,$$

$$\delta_1 = \delta_0 + (Y - X\beta)'(Y - X\beta)$$

Given the conditional posterior distributions in (22) and (26). we can proceed to implement the Gibbs-sampling algorithm. It is possible to start the iteration of Gibbs sampling with an arbitrary starting value for $\sigma^2 = \{\sigma^2\}^0$. Then the following is iterated for $j = 1, 2, \dots, L + M$:

1. Conditional on $\sigma^2 = \{\sigma^2\}^{j-1}$, a generated value of σ^2 at the previous iteration generate β^j from the conditional posterior distribution in [25],
2. Conditional on $\beta = \beta^j$, a generated value of β from step 1, generate $\{\sigma^2\}^{j-1}$ from the conditional posterior distribution in (26).

3. Set $j = j - 1$, and go to step (1).

In generating $\beta' = (\mu, \phi_1, \phi_2)'$ from (22), it is employed rejection sampling in order to ensure that roots of $(1 - \phi_1 L - \phi_2 L^2) = 0$ lie outside the complex unit circle. As a result, we have the following sets of generated values for β and σ^2 :

$$\beta^1, \beta^2, \beta^3, \dots, \beta^{L+M} \quad (27)$$

$$\{\sigma^2\}^1, \{\sigma^2\}^2, \{\sigma^2\}^3, \dots, \{\sigma^2\}^{L+M} \quad (28)$$

where the first L generated values are discarded to ensure the convergence of the Gibbs sampler, then the inferences on β and σ^2 (mean, median, standard deviation, posterior probability bands) are made based on the remaining M generated values. These remaining values of β and σ^2 provide us with the marginal and joint distributions.

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Table 1. Results for Argentina

Parameter	Mean	SD	Median	90% Posterior Band	95% Posterior Band	97.5% Posterior Band
p_{11}	0.986	0.006	0.987	[0.978,0.993]	[0.975,0.994]	[0.971,0.995]
p_{22}	0.830	0.082	0.843	[0.716,0.923]	[0.679,0.940]	[0.6450,0.951]
ϕ_1	0.786	0.055	0.786	[0.714,0.855]	[0.695,0.878]	[0.677,0.897]
ϕ_2	0.192	0.054	0.192	[0.122,0.261]	[0.103,0.279]	[0.083,0.293]
$\phi_1 + \phi_2$	0.978	0.011	0.981	[0.964,0.987]	[0.956,0.987]	[0.948,0.988]
σ_v^2	11.544	5.983	11.149	[4.296,19.422]	[2.896,22.302]	[1.809,25.309]
$\sigma_{\epsilon_1}^2$	36.733	7.243	36.170	[28.081,46.452]	[26.072,49.434]	[24.362,52.233]
$\sigma_{\epsilon_2}^2$	1497.734	601.110	1370.739	[912.842,2213.878]	[816.954,2548.949]	[733.369,2922.721]
<i>eigen</i>	0.982	0.009	0.985	[0.969,0.989]	[0.964,0.989]	[0.957,0.989]

Table 2. Results for Brazil

Parameter	Mean	SD	Median	90% Posterior Band	95% Posterior Band	97.5% Posterior Band
p_{11}	0.973	0.025	0.978	[0.955,0.991]	[0.945,0.993]	[0.921,0.995]
p_{22}	0.519	0.176	0.532	[0.276,0.738]	[0.206,0.795]	[0.137,0.828]
ϕ_1	1.008	0.063	1.008	[0.928,1.088]	[0.904,1.114]	[0.886,1.132]
ϕ_2	-0.032	0.061	-0.033	[-0.109,0.043]	[-0.133,0.067]	[-0.156,0.086]
$\phi_1 + \phi_2$	0.975	0.018	0.980	[0.955,0.989]	[0.945,0.989]	[0.931,0.990]
σ_v^2	18.058	3.651	17.857	[13.514,22.996]	[12.199,24.212]	[11.127,25.41]
$\sigma_{\epsilon_1}^2$	8.808	1.624	8.676	[6.876,10.881]	[6.346,11.648]	[5.955,12.363]
$\sigma_{\epsilon_2}^2$	167.354	145.087	129.140	[44.697,323.648]	[29.569,424.265]	[22.398,562.210]
<i>eigen</i>	0.974	0.019	0.979	[0.953,0.988]	[0.942,0.989]	[0.927,0.989]

Table 3. Results for Chile

Parameter	Mean	SD	Median	90% Posterior Band	95% Posterior Band	97.5% Posterior Band
p_{11}	0.988	0.005	0.989	[0.981,0.995]	[0.978,0.996]	[0.975,0.997]
p_{22}	0.740	0.124	0.760	[0.573,0.884]	[0.508,0.906]	[0.429,0.926]
ϕ_1	0.969	0.073	0.971	[0.875,1.065]	[0.846,1.091]	[0.822,1.114]
ϕ_2	0.009	0.072	0.008	[-0.083,0.101]	[-0.109,0.129]	[-0.130,0.152]
$\phi_1 + \phi_2$	0.979	0.009	0.982	[0.964,0.989]	[0.959,0.989]	[0.954,0.990]
σ_v^2	8.802	3.091	8.308	[5.193,13.111]	[4.573,14.472]	[4.051,15.706]
$\sigma_{\epsilon_1}^2$	20.017	4.229	19.648	[14.867,25.539]	[13.736,27.522]	[12.753,29.345]
$\sigma_{\epsilon_2}^2$	823.427	517.239	708.517	[349.983,1422.069]	[285.286,1757.157]	[242.226,2186.345]
<i>eigen</i>	0.979	0.009	0.982	[0.965,0.989]	[0.959,0.989]	[0.954,0.989]

Table 4. Results for Mexico

Parameter	Mean	SD	Median	90% Posterior Band	95% Posterior Band	97.5% Posterior Band
p_{11}	0.916	0.178	0.986	[0.680,0.993]	[0.495,0.995]	[0.311,0.996]
p_{22}	0.664	0.199	0.705	[0.383,0.876]	[0.221,0.910]	[0.140,0.937]
ϕ_1	0.934	0.086	0.929	[0.827,1.049]	[0.797,1.081]	[0.774,1.113]
ϕ_2	0.043	0.085	0.047	[-0.070,0.149]	[-0.103,0.177]	[-0.130,0.198]
$\phi_1 + \phi_2$	0.978	0.008	0.979	[0.966,0.987]	[0.962,0.988]	[0.958,0.989]
σ_v^2	0.308	0.680	0.023	[0.001,1.052]	[0.000,1.420]	[0.000,2.048]
$\sigma_{\epsilon_1}^2$	10.432	2.266	10.252	[7.746,13.433]	[7.059,14.357]	[6.453,15.426]
$\sigma_{\epsilon_2}^2$	213.504	215.305	179.474	[9.924,424.331]	[8.711,545.790]	[7.754,711.482]
<i>eigen</i>	0.979	0.008	0.981	[0.967,0.988]	[0.963,0.989]	[0.958,0.989]

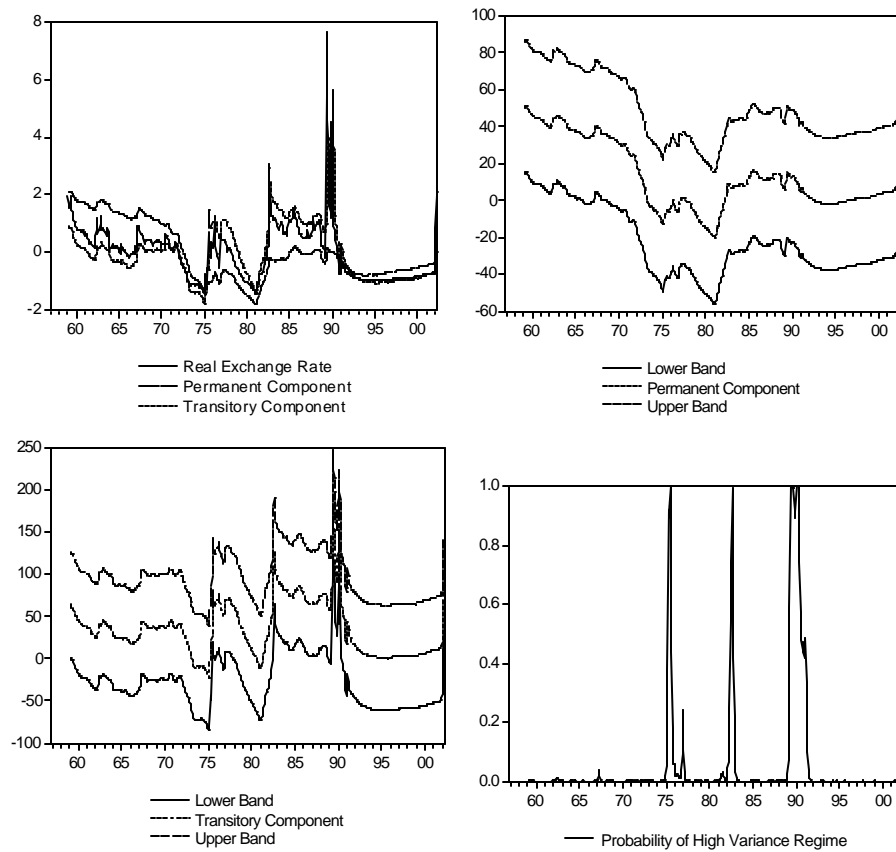


Figure 1. Argentina: Real Exchange Rate, Permanent Component, Transitory Component and Probability of High Variance Regime

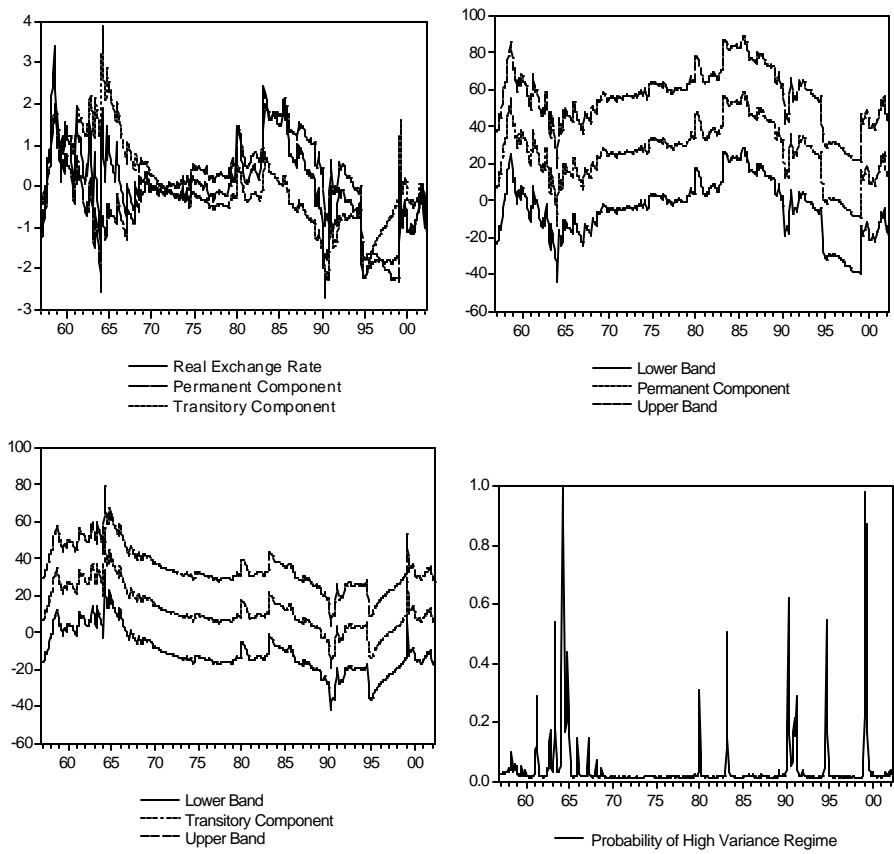


Figure 2. Brazil: Real Exchange Rate, Permanent Component, Transitory Component and Probability of High Variance Regime

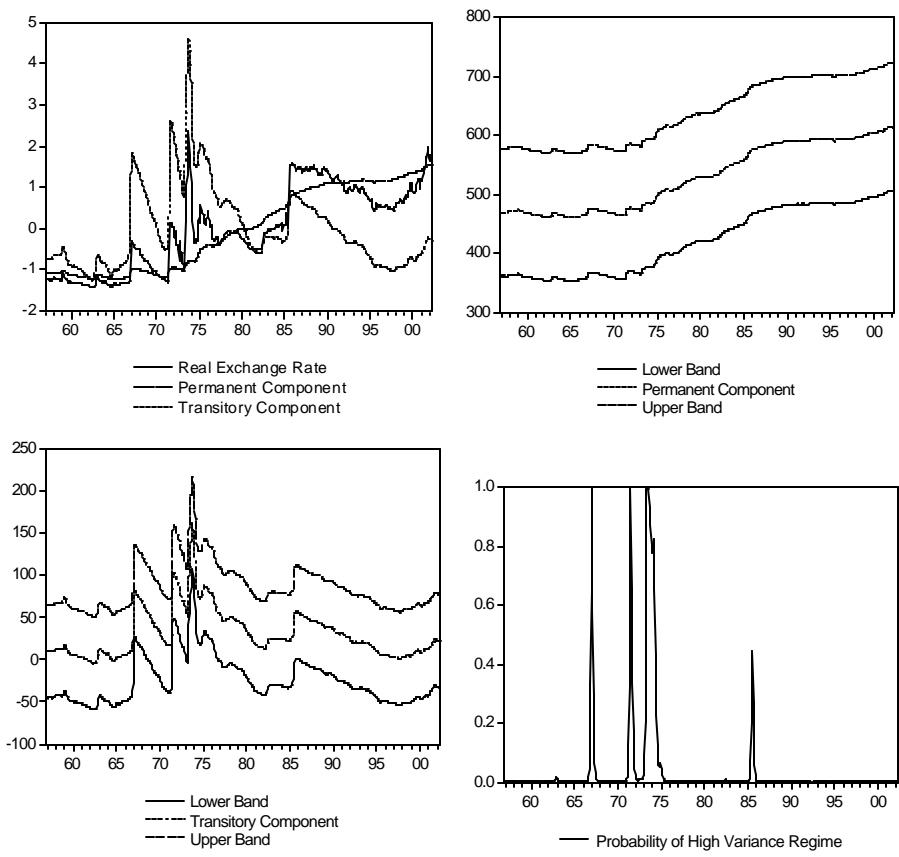


Figure 3. Chile: Real Exchange Rate, Permanent Component, Transitory Component and Probability of High Variance Regime

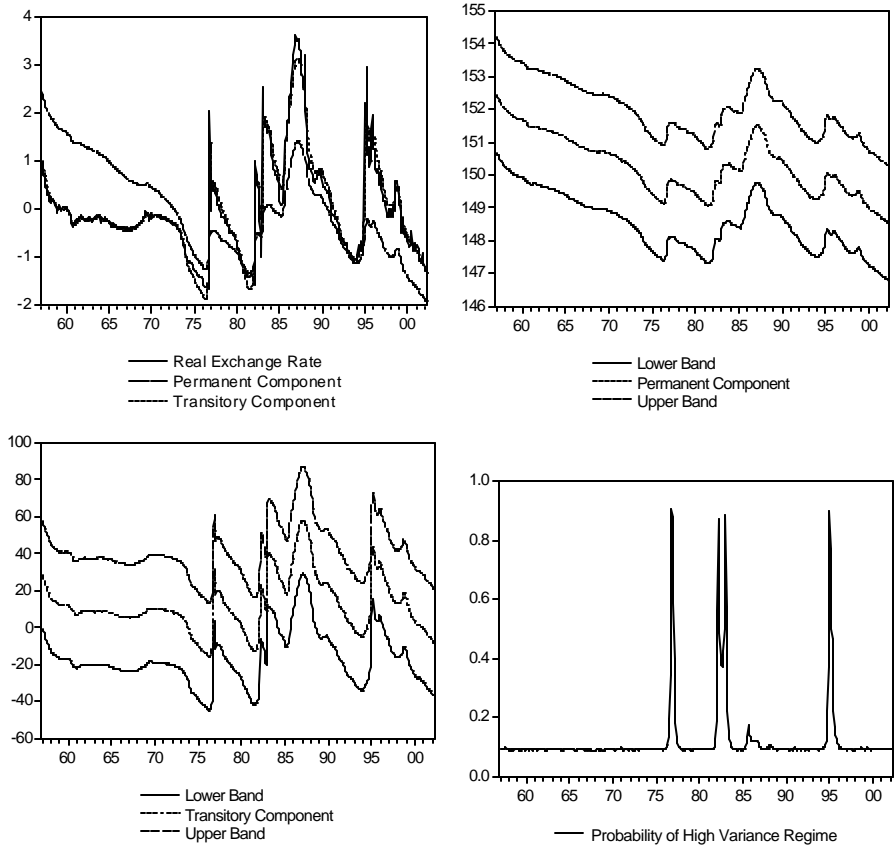


Figure 4. Meico: Real Exchange Rate, Permanent Component, Transitory Component and Probability of High Variance Regime