On the Association Between Modes of Mental Representation and Mathematics Experience in Pre-Service Education Students

by

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Abstract

The purpose of this study is to examine one aspect of the cognitive development of pre-service education students; i.e. the ability to utilize different modes of mental representation. This study attempts to provide a basis for understanding the relationship between the degree of experience in mathematics and the ability to utilize different modes of mental representation.

The selected instrumentation illustrates different aspects of mental representation. The "Modes of Thought Questionnaire" (MOTQ) of Aylwin (1985) is allied to thinking itself, the "Knowledge Accessing Modes Inventory" (KAMI) of Rancourt (1989) is allied to knowledge accessing, and the Diehl and England (1958) version of the "Griffitts Test of Mental Imaging" is allied to mental imaging.

In the MOTQ, associations were found between the level of experience in mathematics and both the ability to utilize each mode of mental representation and the overall use of the preferred modes. Specifically, a lack of experience in mathematics appears to be related to an inability to utilize correctly all three modes of mental representation when directed to do so. A lack of mathematics experience also appears to be related to a lower utilization of the verbal mode of mental representation, and a higher use of the visual mode, than expected.

In the KAMI test, likewise, an association was found between levels of experience in mathematics and the dominant mode of knowledge accessing. A significantly higher percentage of subjects with no experience in mathematics have the noetic mode as dominant mode, whereas a significantly higher percentage of subjects with a high level of experience in mathematics have the rational mode as dominant mode.

In the Griffitts test, however, no association was found between the level of experience in mathematics and the use of the three modes of mental imaging.

This study has implications both for future research and for education. First, it gives a clear indication that differences do exist between the mental representation modes preferred by individuals with no mathematics experience as compared to those who have even a small level of experience in mathematics. Second, it provides implications regarding teacher education in Ontario, which arise from these differences in the utilization of all of the modes of mental representation and knowledge acquisition.

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CHAPTER I : NATURE OF THE STUDY

Introduction

A closed cycle of issues relating to a lack of success in the teaching and learning of mathematics exists today in various schools in the provinces of Canada. At the core of the cycle lies the problem that a young student who has difficulty in learning certain topics in mathematics may avoid this subject in secondary and post-secondary education. As Usiskin (1987) noted with respect to geometry, for example, some school counsellors even attempt to dissuade weak students, and students not intending to study mathematics at the university level, from the study of mathematics courses deemed difficult in secondary school. Yet some of these individuals eventually enter teacher training (or pre-service education) institutions. Moreover, after graduation, they may then find themselves teaching mathematics, and will, perhaps, pass on their fears, dislikes, and inabilities to their students. Hence the closed cycle continues.

There is a window of opportunity in the academic year of pre-service teacher education, in which it may be possible to break the cycle. If this opportunity is to be seized, however, preliminary research must be carried out, by faculties of education, into the existing situation regarding mathematics and the pre-service education students. One such area of research which has not previously been addressed sufficiently lies in the cognitive domain.

This study addresses this area. More specifically, it attempts to compare the degree of mathematics experience of the pre-service education students to their utilization of different modes of mental representation. Modes of thinking, and ways of thinking, are alternative terminologies used in referring to the modes of mental representation with which cognitive theorists are concerned (Glucksberg, 1988). All of these terms refer to the different ways in which people actually represent what they think about, mentally. For example, an individual thinking about a tiger might, depending on the circumstances, have a mental visual picture of the animal, or imagine a hierarchical chart of the feline family with the tiger's location marked, or imagine being a tiger stalking something in the jungle. All of these are modes of mental representation.

The search of relevant literature presented in this study, therefore, covers not only research studies dealing with mathematics education, but also literature in the area of modes of mental representation, which lies in the cognitive domain. The discussion of this literature is found in the second chapter. The instrumentation used in this study consists of three individual measures of mental representation, which are discussed in the third chapter. The study closes with a presentation and discussion of the findings.

Nature of the Study

The learning of mathematics and the teaching of mathematics together arouse simultaneously more fear and interest in the pre-service education student population than does almost any other subject. The interest, on the one hand, is aroused by the fascinating and innovative techniques which now exist for teaching mathematics and which are readily available in today's classrooms. The fear, on the other hand, is derived from the anxiety felt by many of the student teachers, who have previously not been successful in the study of mathematics in secondary school and beyond, but will be required eventually to teach the subject none the less. Park (1990) stated, for example, that some pre-service education students may not have had any mathematics training since grade ten. In his words,

> They don't like math. They avoid it at all costs. Their training in math when they get here is practically nil. ... They teach mathematics and science because they are required to, not because they like it or because they have special training. (1990, p. D2)

Many pre-service teachers are, as a result, anxious about having to teach this subject, and, as Tobias (1978, p. 26) suggested, mathematics anxiety leads to poor performance and to mathematics avoidance.

Mathematics anxiety is not restricted, however, to pre-service education students. It is also common amongst both pre-university and university-level students. Even students who enjoy mathematics, occasionally find particular topics which they dislike and fear. Students who are considered to be mathematically anxious are known often to avoid courses in their academic studies which contain such topics (Robitaille, 1989; Senk, 1985). Kuendiger (1987) noted, however, that little is known about the mathematical learning histories of individual pre-service education students, nor of the impact of such history on the confidence of the students in teaching mathematics.

Many students have not been successful in one or more areas of mathematics. It is common knowledge, for example, that secondary school geometry is a major stumbling block for students. Senk (1985) noted in her discussion of an American national project in cognitive development and achievement in secondary school geometry, that fewer than 15% of high school graduates in the United States have mastered the writing of geometric proof. This finding might help one to understand why many students do not attempt geometry courses. Furthermore, some school guidance counsellors even attempt to dissuade weak students from the study of geometry, even though there is no evidence to show that a student who is unsuccessful in algebra will also be unable to succeed in geometry (Usiskin, 1987).

Geometry is not the only difficult area of mathematics for students. Algebra is also considered difficult. For example, in a study (Leitzel, 1989) of some 6,430 freshmen who entered Ohio State University, 1,543 had followed a three-year college mathematics preparation course, including two years in the study of algebra. Of this number, however, close to 400 could not use a variable under any circumstance, and almost half of the 1,543 students were placed in a remedial mathematics program because of their marks on a placement test. Leitzel suggests that large numbers of students have not mastered the material taught in their algebra courses in secondary school, despite receiving credit for two years of secondary algebra. The situation is similar for other areas of mathematics such as computational arithmetic and problem solving. One of the conclusions of the Second "National Assessment of Educational Progress" (NAEP) was that a large number of seventeen-year-olds demonstrated a lack of understanding of fractions, decimals, and percentage (Carpenter, Corbitt, Kepner, and Lindquist, 1982). Furthermore, both the report on the 1986 NAEP (Dossey, Mullis, Lindquist, and Chambers, 1988) and the Second International Mathematics Study (McKnight et al., 1987), funded by the International Association for the Evaluation of Educational Achievement (IEA), conclude that few senior secondary school mathematics students have achieved proficiency in multi-step problem solving.

Mathematics courses in North America have a reputation of being very difficult. As Ernest found in her investigation of teacher and student beliefs concerning mathematics, both boys and girls have considerable difficulty doing mathematics, and most of them do not like the subject very much (Ernest, 1977, p. 72). Parents and teachers are not necessarily surprised, then, when students fail to succeed in individual units. Yet many of these students who do not succeed in mathematics are very successful in other fields of study. Hence the reasons for lack of success in the study of mathematics do not appear to be clear-cut, and are most often not obvious to either student or teacher. There is, however, considerable anecdotal evidence from teachers in the field concerning possible reasons for lack of success in mathematics. Tobias, for example, stated that ambiguities in mathematical language play a role in lack of success in algebra (Tobias, 1978). Kieran was more specific, stating that one major ambiguity which confuses students of algebra is the multiplicity of uses of alphabetic characters in the subject, namely as variables, generalized numbers, specific unknowns, objects, and specific values (Kieran, 1989). This, according to Kieran (pp. 42-44), is derived from an inability to understand the underlying structure of algebraic expressions. Booth, on the other hand, attributes this confusion to lack of "cognitive readiness" (Booth, 1984).

The premise has also been brought forward that girls, but not boys, are allowed to fail mathematics (Tobias, 1978). In other words, both boys and girls think that mathematics is a subject which can be learned by boys, but is too difficult for girls (Cassie, 1980). As a result, if a girl has difficulty with mathematics, or fails a mathematics test, there is no stigma attached; the result is expected. Boys, on the other hand, are under pressure to do well in mathematics, and are encouraged to persevere. The perception of both boys and girls is that mathematics is a male domain (Boswell, cited in Fox, 1980). The result of this perception is that girls have been allowed, by themselves, by parents, and by school counsellors, to avoid mathematics study regardless of ability in the subject (Burton, 1979; Tobias, 1978). The attitudes of both students and their teachers towards the study of mathematics have also been brought forward as a possible reason for lack of success in mathematics (Reyes, 1980). Furthermore, many teachers of elementary school mathematics have not studied mathematics beyond the basic secondary school requirements, and some teachers of geometry have not studied this subject at all (Usiskin, 1987). This leads to what Usiskin refers to as the "performance dilemma" (p. 19), where lack of success in geometry, on the part of some, discourages other students from this study.

The performance dilemma also encourages school counsellors to direct students away from this study, thus influencing future teachers to stay away from the study of geometry and from the desire to teach topics in geometry. This leads to poor performance. In one study, Meserve and Meserve (1986) list the set of skills which teachers of mathematics consider to be basic to the study of geometry. They suggest that in order to develop the ability to solve problems in geometry, students of this subject require mastery in visual skills, verbal skills, drawing, logical analysis, and in the ability to apply the first four mentioned, and that a deficiency in any one of these skills may degrade progress in learning.

Usiskin's dilemma as a consequence is a circle of issues, which must be broken at some location on the circle, if success in geometry and other mathematical topics is to become more common. The pre-service education students form one location on the circle. If fear and anxiety on the part of some pre-service education students towards mathematics can be overcome during the training period of one academic year, then the current situation in the schools may improve.

Purpose of the Study

The main purpose of this study is to examine the pre-service education students, with a view towards breaking Usiskin's lack of success circle in mathematics. The window of opportunity of the one year of pre-service education in many institutions is the last chance for change, and it can and should be utilized. Before any changes are made to the pre-service year, however, a clear understanding must be developed both of the existing situation in the faculties of education, and of what has to be done. The initial stage in this process is the examination of the population of pre-service education students, in terms of the comparison of various cognitive factors of those students who have studied and succeeded in mathematics, with those of the students who have not studied mathematics.

Pre-service education students registered in the one-year program are aware that certification in Ontario, as well as elsewhere, presupposes competence in mathematics to the grade 10 level, and hence know that the teaching of mathematics will be an integral part of their teaching responsibilities. Even further, some of these students may be required to teach mathematics content which they themselves avoided in school, such as the solution of equations, geometry, or percentage. This presents a challenge both to the teacher training program in the university as well as to the pre-service teachers themselves. As Porter, Floden, Freeman, Schmidt, and Schwille stated:

> ...students with a teacher who fails to cover geometry or who gives little attention to estimation or measurement application are unlikely to have those omissions compensated for by other teachers in other grades. (1988, p.105)

Hence some students have gaps in mathematical knowledge which may lead to failure in later mathematics study, since much of mathematics depends on earlier mathematical knowledge.

A different problem is that of mathematics avoidance. Mathematics avoidance refers to the observed behaviour on the part of students, of selecting courses outside the mathematics domain whenever they are given the choice of selecting a mathematics or a non-mathematics course. For example, a study of women Ph.D. candidates in political science found that the one deciding factor in choice of graduate school was the requirement (or non-requirement) of a mathematics or statistics course (Merrill, B., 1976, cited in Tobias, 1978, p. 96). A review of the literature in the area of mathematics avoidance has not revealed any study which deals specifically with the situation faced by pre-service education students. Some authors of mathematics education studies, however, have suggested that there is a need to investigate the reasons why certain students specialize in mathematics, while others seek to avoid it. For example, Steen (1990) described the study of mathematics as a pipeline, where 50% of students are drained away from this discipline each year in the United States, from grade 9 through the Ph.D. level. He concludes that there is a strong need to examine the whole process, in order to develop a basic understanding of the system, and hence to bring about necessary changes.

Thomas (1991) noted the same fact concerning the dropout rate, and attributed it to a lack of interest on the part of the students. Robitaille and Garden (1989) and Winer and Bellando (1989), on the other hand, report on the second international study of mathematics, sponsored by the *International Association for the Evaluation of Educational Assessment* (IEA). They point in The IEA Study of Mathematics II to the dismal performance of students around the world on various topics in mathematics. Yet, as Good and Biddle (1988) and Hoyles (1987) state, few studies exist of individual student perceptions of what is deemed difficult, or not difficult in mathematics topics.

In a recent study of the relationship between mathematics anxiety and mathematics avoidance, a sample of 261 first year students in Georgia State University was examined. The majority in the sample (211) were in a remedial non-credit mathematics class which had to be passed before these students could enroll in regular studies, while the remainder were registered in a college algebra course for credit. The study noted that, overall, 64% of the entire sample felt that they suffered from mathematics anxiety, and that 65% reported having avoided mathematics courses in the past. Indeed, nearly half of the entire sample (48%) reported both mathematics anxiety and mathematics avoidance. The author concluded that there appeared to be a relationship between mathematics anxiety and mathematics avoidance (Cope, 1988). It must be realized, moreover, that very few of the freshmen who were not required to take the remedial course, were examined in this study. Little is known, therefore, about the history of mathematics anxiety and mathematics avoidance of those students who have generally succeeded in mathematics. Other studies have shown that mathematics anxiety "per se" has no measurable effect on performance when the anxious individual cannot avoid a mathematics course, and has to study the topics involved (Harriss-Dew, K. M., Galassi, J., and Galassi, M.D., 1984; Sime, W., Ansorge, C., Olson, J., Parker, C., and Lukin, M., 1987).

Furthermore, a review of the literature concerning mathematics anxiety and mathematics avoidance is inconclusive and sometimes contradictory in nature. Tobias, for example, brought forward the view of others that mathematics anxiety is not so much a "cause" of mathematics avoidance as an "effect" (Tobias, 1978, p. 96). She concluded, however, that although mathematics anxiety affected the scores achieved on tests in mathematics by both women and men, it was more disabling for women. At the other end of the spectrum, the Toronto Board of Education's report on mathematics avoidance and mathematics anxiety concluded that there is a straight line from bewilderment and confusion in the classroom to mathematics anxiety, and forward from that to mathematics avoidance (Wiggan et al., 1982). Borasi, however, stated that mathematics avoidance is a direct result, not of mathematics anxiety alone, but also of incorrect notions of the nature of the discipline (Borasi, 1990, p. 174).

The underlying causes of mathematics anxiety seem complex. Some researchers attribute blame for mathematics anxiety to the teachers of mathematics and the confusing methods they use (Gattuso, Lacasse, Lemire, and Van der Maren, 1989), while others state that parents and school counsellors are to blame (Lazarus, 1975; Casserly, 1978). Cultural beliefs about mathematics, such as the idea that there is only one right way to solve a problem, have also been identified as contributing factors (Kogelman & Warren, 1978). Various intervention techniques have been developed to attempt to relieve mathematics anxiety, and although little research into the effectiveness of such interventions has been carried out to date (Wiggan et al., 1980), it appears that the learning of mathematics is possible for even the most anxious (Tobias & Weisbrod, 1980).

One common note in the literature is the description of the confusion and frustration of some students in the mathematics classroom. As Gattuso et al. found, most students attributed this difficulty to something wrong in themselves - "les mathétiques, tu l'as ou tu ne l'as pas" (1989, p. 204). Tobias also stated that many students believe that even when they succeed in mathematics, they are frauds, and cheat their teachers (1978, p. 46). This feeling of inadequacy on the part of students should be investigated further. As Steen noted, "sound policy for mathematics education must look as much at the students being educated as at the mathematics being taught" (1990, p. 130). Thus closer looks at those individuals who succeed in mathematics, as well as those individuals who do not succeed in mathematics, appear to be warranted. These problems of mathematics anxiety, of mathematics avoidance, and of feelings of inadequacy which are prevalent in mathematics education, may be symptoms of a deeper problem in the individual. One possible area of research concerning the individual is the area of cognition. It has been suggested by some authors (Fuys, Geddes and Tischler, 1988; Krutetskii, 1976) that research is necessary in this area.

The main purpose of this study, therefore, is to examine one aspect of the cognitive development of pre-service education students, with a view towards breaking the lack of success circle in mathematics. Since there is, at present, little understanding of the cognitive strengths and weaknesses of these students with respect to mathematics, this study will examine one aspect of the current situation of these students - that is, their existing cognitive strengths and weaknesses, and their experience history in mathematics education.

In other words, each pre-service education student comes to teacher training with a history of mathematics experiences. A student may have had positive learning experiences or negative ones, and may have been successful or not successful in the mastery of various mathematical topics or concepts, as presented in academic courses. Each student has also developed an individual approach to learning, one aspect of which is the utilization of modes of mental representation. The cognitive development of an individual, including the use of modes of mental representation, may be related in part to the positive or negative nature of the experiences in mathematics which have occurred. It is this association, therefore, which the present research attempts to explore. In other words, this study intends to provide a basis for understanding the relationship between the degree of experience in mathematics and the ability to utilize different modes of mental representation.

CHAPTER II : REVIEW OF RELATED LITERATURE

The literature examined in this review can be divided into two main categories: the first falls under the category of the history of mathematics education in the Western world, and the second under the category of cognition. The area of cognition with which this study is concerned is that of mental representation, from the perspective of the school of constructivism.

The History of Mathematics Education in the Western World

The mathematics curricula used in schools today did not arise in a vacuum; there is a long history of curriculum development in mathematics. Today's subject matter is based on the subject matter taught in the past, although it reflects the pressures of the changing needs in society, and of the apparent inadequacies of past curricula. Hence it is necessary to study the history of mathematics teaching in the different components of the discipline, in order to understand the present situation. It is to be noted, however, that mathematical history herein described is examined from a Western perspective, and more particularly, from that which has led to the approaches used currently in North America.

The history of the teaching and learning of mathematics is examined in this chapter from the perspectives of basic computation, geometry, and algebra. Each of these areas has a specific history in education, and together, these three areas are associated with a wide range of problems which students encounter during the learning process. The search of the cognitive literature is limited to the relationship between mathematics and cognition, where the constructivist view of cognition is of particular importance. In this domain, the theories of Bruner and Aylwin are found to be the most complete in the area of cognitive development.

Computational Perspective

Basic computation in the nineteenth century was considered to be "arithmetic", not "mathematics" (Griffiths & Howson, 1974, p. 15). Computational methods were taught in elementary schools by means of considerable repetition, since arithmetic consisted of skills to be mastered. Johnson and Rising (1967), in their substantive review of the literature in the field of basic computation, found many studies that describe the ability of students to master these skills. One of these studies, by Schorling (1931, reported in Johnson and Rising, 1967) reported on the results obtained on a sample of 200,000 students in grades 5-12. In the study, Schorling remarked that only 2 in 10 grade 12 students could calculate a simple percent. Johnson and Rising also reported in their review that, a few years after Schorling's study, Taylor concluded from a study of 2000 freshmen in teachers' colleges, that fewer than half of them could divide 175 by 0.35. In 1942, Admiral Nimitz reported that 68% of the freshman at 27 universities in the U.S. could not pass an arithmetical-reasoning test; and in 1943, Brueckner (as reported by Johnson and Rising, 1967) found arithmetical competence throughout the United States as a whole to be even worse than this.

More recently, Hart (1981) and Lovell (1972) reported that children of various ages have considerable difficulty in understanding mathematical concepts, as well as in selecting the correct operations when attempting to solve problems. Furthermore, some recent reports confirm that children have low levels of mastery, not only in arithmetic calculations, but also in the concepts involved in such ideas as place value, in fractions, and in the relationships between operations (Assessment of Performance Unit: Mathematical Development: Primary and

Secondary, nos 1-3, 1980, 1981, 1982; Brown & Burton, 1978). The conclusion that can be reached from the above studies suggests that basic computation has for a long time remained beyond the reach of many students.

Geometric Perspective

Geometry has a somewhat different history from that of basic computation, in that the former was taught at the secondary level, mainly as a "mental discipline" (Brooks, 1883, p. 88, reprinted in Bidwell & Clasen, 1970). The documented history of instruction in geometry goes back more than a century, as does also the record of lack of success of many of the students who study the subject. In 1901, John Perry, in his report to the Royal Society concerning proposed objectives for the teaching of mathematics (reprinted in Bidwell & Clasen, 1970), complained that the students were not, in general, able to perform geometric proof, and that the teaching of mathematics was geared mostly to the passing of examinations, and not to the betterment of logical thought. Furthermore, Mercer (1912) stated in his course of study for mathematics, that "Many boys ... will never make much headway with Deductive Geometry" (reprinted in Griffiths & Howson, 1974, p. 170). More recently, in Ontario, geometry remains a part of the secondary school mathematics courses, but not all of the students are expected to master the content. In the Ontario Ministry of Education's Curriculum Guideline for Mathematics: Intermediate and Senior Divisions Part Three, for example, grade ten level teachers are reminded that:

> In the evaluation of a student's knowledge of proof during the Intermediate Division, it should be remembered that the ability to think hypothetically, although encouraged by experience, seems to be a product of maturation. (1985, p. 41)

That is, officials of the Ministry of Education acknowledge that many students will be unable to succeed in geometric proof because they are cognitively not ready for the concept. Students who have reached the requisite level of cognitive maturity, however, have only the potential to do well, and not the actual ability itself. The transformation of potential into true ability is not an automatic process, but depends rather on the interest and effort displayed by the student. The Ministry appears to recognize this reality when it states that "there may be few students, even in the Senior Division, who can be expected to demonstrate all these attributes [aspects of knowledge of proof] consistently" (Curriculum Guideline for Mathematics: Intermediate and Senior Divisions Part Three, 1985, p. 18).

Hence, although the study of geometry and geometric proof has been an important part of the school curriculum in the western world for many years, it is well known that the lack of success in learning and teaching geometry is not new at all. As Senk (1985, p. 448) pointed out, the writing of proofs has been seen to be an important objective of the geometry curriculum, while at the same time, it has been perceived as being one of the most difficult topics for students to learn. This duality of importance combined with difficulty has been accepted as true, but little appears to have been done over the years to change the curriculum. For example, the Euclidian method of proof (the method of reasoning deductively from the given so as to arrive at the solution) which was discussed in the report on mathematics presented to the "National Educational Association" of the United States (1893), is still used in modern textbooks of mathematics (Ebos, Tuck, Schofield, and Hamaguchi, 1987).

The lack of consensus among educators, as to how to change the course of instruction in geometry, has been suggested as a reason for this anomaly. As Allendoerfer stated, "there is not even agreement as to what the subject is about" (1969, p. 165). Even components of geometry courses upon which agreement has been achieved, such as "thinking skills" and "application of geometric facts", appear to have different meanings for different mathematics educators, depending on whether one is speaking philosophically or pragmatically. Hence it has seemed impossible until recently to come to a consensus on what kinds of changes are needed in the geometry courses. As a result, the general approach to geometry, in content and methodology, has remained virtually unchanged in North America since 1912. It must be pointed out, however, that some minor changes have been made. For example, the memorization of Euclid has been replaced by memorization of a smaller set of axioms, and the number of required proofs to be written has been decreased. That is, geometry teachers in fact teach the same content as did geometry teachers in the past, but cover fewer of the topics. The lack of agreement as to the desired nature of instruction in geometry may also be one of the reasons for the limited amount of research involved in the study of poor performance in geometry. It is noteworthy to emphasize, however, that although the study of geometry has changed a great deal in countries outside North America, the results obtained by students in those countries in various geometry tests remain unsatisfactory.

Algebraic Perspective

Algebra has been a required part of the curriculum of secondary schools for over 100 years. The syllabus has changed over the years from the original intellectual "mathematics discipline" approach of 1900 (Wagner, S. and Kieran, C. , 1989) to more integrated approaches, with each change in the syllabus making algebra more relevant and useful. Nevertheless, enrollments in algebra courses have declined sharply with the elimination of algebra from the generally-accepted list of compulsory courses.

Since the second world war, the approach to the teaching of algebra has changed to reflect the new and powerful ideas emerging in mathematics. The "New Math" movement, for example, attempted to introduce new ideas and new approaches into algebra instruction (Thorpe, 1989). Furthermore, research has begun to focus on some cognitive problems connected with the teaching and learning of mathematics in general, and of algebra in particular. Many of these studies incorporate the conceptual model of Piaget's genetic epistemology. In their studies, the researchers apply his concept of stages of intellectual development, and relate the various stages to the learning of algebraic concepts (Kieran, 1989). Yet, as noted by Herscovics (1989), the study of algebra still remains a major stumbling block in secondary school, in spite of the many changes which have been made in the approach to the subject. In this regard, recent researchers have examined specific topics, such as the use of variables in problem solving, and the solution of two equations in two unknowns, which are subsumed within the subject, in order to attempt to develop an understanding of the problems encountered by students (Chaiklin, 1989; Herscovics, 1989). Unfortunately, for each topic examined by these researchers, a significant proportion of students did not achieve mastery during classroom study and practice.

The concept "variable" is one such topic. To be specific, a variable may represent an unknown, or an argument, or again a parameter. The ambiguity inherent in this variety of meanings was found to constitute a problem for students by Thorndike in 1923, and this ambiguity has continued to perplex students, according to more recent researchers (Carpenter, Corbitt, Kepner, Lindquist, and Reys, 1981; Kuchemann, 1981). In particular, the use of the concept of variable as a specific unknown is confusing to the student.

The structure of an algebraic expression appears in itself to be a major stumbling block (Kieran, 1989).

Several studies, in fact, (Davis, Jockush, and McKnight, 1978; Resnick, L., 1987) have noted that some secondary level students have difficulty in understanding the structure of an equation such as "20x + 11 = 51". They also demonstrate difficulty in parsing. A study by Carry, Lewis and Bernard (1980) has indicated that the very same types of parsing errors are made also by college students studying algebra. One example of such a parsing error described in the study is the confused reduction of "20x-10" to the incorrect term "10x", or the incorrect movement of a term "across the equal sign", with or without additional errors in the use of the distributive property, in the false reduction of "4x + 2(x - 5) = 2" to the equation "4x + 2x = 2 - 5". The lack of mastery of the concept variable is illustrative of the lack of mastery of other algebraic concepts, not only in students at the secondary school level, but also in students of mathematics at the college level.

The Global Perspective

The "IEA Mathematics Study" (IMS) is the most recently published comprehensive individual study of various aspects of secondary school mathematics. The study examines the curricula, the instructional practices, and the student outcomes in mathematics, in twenty school systems throughout the world. The findings were published in 1989, in two texts, the first edited by Travers and Westbury, and the second, by Robitaille and Garden. The IMS study is a follow-up to a similar study done in 1964 by the same group, the "International Association for the Evaluation of Educational Achievement". A third study of this type was conducted in 1991; the preliminary findings have not yet been analyzed, but the results appear to be similar to those of the first two studies. In both studies, patterns of attitudes to mathematics and achievement are described, and characteristics found in teachers, schools, and systems of education around the world are recorded. An attempt was made to relate attitudes and achievement of students with the varying characteristics of different educational systems.

The IMS sample involved some 125,000 high-school level students, some 6,000 teachers, and some 4,000 principals and department heads of mathematics. The findings, in general, concluded that, in spite of the dramatic growth in the numbers of students completing secondary school in some areas of the world, there nevertheless has been a decline in the proportions of students studying mathematics at the pre-university level, in virtually every country's school system. In fact, the study shows that most educational systems, including those found in North America, deliberately provide alternative academic paths for those who wish to avoid courses in mathematics.

The test items used in comparisons among the results of different populations, are those which are common to the curricula of all participating groups. Overall performance in arithmetic is shown in this study to be relatively low, and is measured lower still, in all school systems, than the performance recorded in the 1964 study (Robitaille & Taylor). In the questions dealing with arithmetic, including whole numbers, fractions, ratio, and other items, the average student is correct 50 percent of the time, while the achievement scores (mean percent correct) for the school systems range from 60 down to 32 percent.

In algebra, the range of achievement for individual students is similar to that found in basic computation. The average student internationally is able to answer 43 percent of the questions correctly. The level of performance on the manipulation of simple equations, however, is so poor that the authors suggest that this material may be beyond the capability of most students

(Robitaille, 1989). The only promising note for algebra is that the level of achievement of 43 percent is higher than in the 1964 evaluation data (Robitaille & Taylor, 1989).

The performance in geometry was found, however, to be relatively unchanged from the study of 1964, despite changes in the geometry curriculum in the intervening years in some school systems. The study also demonstrates wide variations in approaches to geometry internationally, varying from relatively formal to highly intuitive, and from traditional Euclidian to transformational. There is, furthermore, a wide variation in the topics covered (Robitaille, 1989; Westbury & Wolfe, 1989). In geometry, no school system shows an achievement score in excess of 50% in any test item dealing with formal transformational geometry (Robitaille, 1989). In fact, the correct results for all students in these items range from 14 to 23 percent. The IEA studies of mathematics achievement have not only reinforced the reported findings of the unsatisfactory nature of achievement in mathematics as found over the last 80 years: they show that unsatisfactory achievement in mathematics is also pervasive, and note furthermore that the phenomenon is, regrettably, worldwide.

The results that are reported in the IEA study are poor, particularly when one considers that the percentage of students actually studying mathematics throughout their secondary school careers, ranges from 6 percent to 50 percent of the target population (Travers & Westbury, 1989). There is a further finding which arises from the study, however, in that student achievement is acknowledged to be poor in mathematics regardless of differing approaches, different teaching methods, and varying curricula among the twenty school systems in the eighteen countries involved in the IEA study.

The history of learning and teaching mathematics has not been entirely positive in nature. Basic computation, algebra, and geometry have all presented problems for students in the past, and continue to perplex students today. New approaches to algebra, new curricula in geometry, and new methods of teaching basic computation, have not improved the situation either in North America or, more generally, worldwide. The historical perspective, as discussed to this point, is important for it now appears clear that unsatisfactory performance in mathematics has not been ameliorated in the past by changing the teaching methods or approaches.

In summary, the brief examination of the history of mathematics education illustrates the long-term and continuing lack of success of students in mathematics. Despite the variety of different teaching methods used both in North America and in other areas of the world, this lack of success on the part of students persists in all of the branches of mathematics, from basic computation to algebra and geometry. It thus appears that in a search for reasons behind unsatisfactory performance, one might reasonably argue that personality variables might provide a new and meaningful avenue of research. And one dimension of personality that is drawing more attention from mathematics researchers is the area of cognition. Indeed, Hershkowitz, Bruckheimer, and Vinner (1987) and Herscovics (1989) suggest that the study of cognitive factors might well provide some important clues toward an understanding of student avoidance and unsatisfactory student performance in mathematics. The investigation of cognitive factors is the area of concern of this study.

Cognitive Factors in Mathematics

The approach taken in this literature review leads from basic definitions of cognition itself, through models of cognition, to the construction of personal models of reality. The concept of modes of mental representation in the construction process of personalized models of reality is clarified. The connection between cognition and mathematics is made through a study of epistemological obstacles and cognitive factors in the solution of mathematics problems.

Cognition, according to many authors (Bruner, 1966; Good & Brophy, 1990; Piaget, 1970), is defined to be the action or the faculty of knowing. Theorists in cognition are concerned with human learning, and the mental processes used by individuals in the learning process (Slavin, 1991, p. 99). Since these mental processes can only be inferred from actions of the individual, cognitive models are developed in an attempt to explain how we acquire, represent, process and transmit knowledge. Most models deal "de facto" with how the perceived outside world is mentally or symbolically represented. They examine how the continuous barrages of sensory stimuli that assault the individual are configured into understandable forms which become personal models of reality (Agnew & Brown, 1989). That is, the models suggest possible explanations as to how individuals construct a personal model of reality (Howson & Kahane, 1990).

Modes of mental representation are the means by which we transduce these barrages of stimuli presented to the senses into mental models of reality (Martin, 1976). Michael Polanyi further suggests that all knowledge is personal, resting on belief. That is: what we know is in some way shaped by our experience and attitudes; there is no objectivity. Personal knowledge, which he labels *tacit knowing*, is made up of the basic assumptions or beliefs with which we operate: it is *the knowledge of* a thing, rather than the thing itself (Polanyi, 1967). The structure of this knowledge is similar to that of a metaphor, in which analogies are made between reality and our personal model of it.

Some of the literature in cognition suggests that there may be cognitive or epistemological obstacles to the learning of new things. By obstacles, are meant those pieces of knowledge, or methods of acquiring pieces of knowledge, which have been useful in the past for solving simple problems, but which are counterproductive in face of more complex problems, and yet are difficult to overcome (Bachelard, 1983; Herscovics, 1989). An example of this type of obstacle is the notion that the result of division is a smaller value, an idea which no longer holds when division by decimals or fractions is considered.

As Herscovics suggests, an obstacle may be considered in the light of Piaget's concept of accommodation. The obstacle can be seen to force a modification in the existing cognitive structure and the knowledge base of the individual, and hence force an accommodation to be made in them. It may, therefore, be feasible to consider that cognitive obstacles to the learning of various topics in mathematics may exist. Similarly, there may be other cognitive factors at play, as well as obstacles, in the learning of complex tasks such as those found in mathematics. Such considerations are within the realm of this study.

Mathematical problems often have many different solutions, and each solution may perhaps call forth the utilization of a different set of cognitive factors on the part of the student. A teacher cannot infer from a correct answer what are the thought processes of a student that led to a given solution. The various mental processes which lead to a correct answer may be essentially different, and these differences can be very valuable in judging the abilities of the students (Krutetskii, 1976). This set of alternate pathways to a solution is of importance to the solution of problems in algebra, for example. The thought processes of the student who, when faced with the equation "$2x + 3 = 7$" knows that the first step in the solution is to "move the 3 to the other side and change the sign", are very different from the thought processes of the student who knows that "to subtract 3 from both sides will not change the equality of the relation". The former has memorized a rule; the latter has understood the fact behind the rule. Further instruction for each of these two students should be different, but will differ only if the teacher has recognized the thought processes involved.

There have been some attempts to isolate a general factor in mathematical ability. Early studies (Brown, 1910; Johnson & Rising, 1967; Oldman, 1938) concentrated on identifying that factor in arithmetic, algebra, and geometry. In general, however, it was found that there is no one specific factor uniting the three areas of mathematics, but that there are specific cognitive factors tied to each branch. As described by Krutetskii (1976), the number and type of factors recognized as making a difference in mathematics achievement varies from researcher to researcher, but include some or all of: general intelligence, numerical facility, spatial ability, verbal ability, and reasoning ability. In general, then, the study of the cognitive processes involved in the learning of mathematics has not progressed very far beyond the theoretical stage; all is possible, but little is known.

Cognitive Factors in Geometry

A comprehensive model of cognition, known as the "van Hiele Model", has been brought forward for geometry (Pegg, 1985). This model is an attempt to classify students according to their ability to master the basic concepts of geometry which are present in most secondary school geometry courses. According to the model, there are five levels of ability involved in developing a comprehension of concepts in geometry; these are provided in Table 1.

According to available studies published in English concerning these van Hiele levels (Driscoll, 1982; Crowley, 1987; Fuys, Geddes & Tischler, 1988; Thomas, 1991), a student who, at the beginning of a geometry course, functions at level 0 or 1, stands little chance of succeeding. These studies, as reported by Senk (1989), do not investigate the reasons behind failure to succeed. The placement of a student at a particular level accordingly gives only an indication of the chances of success, but does not provide any information of methods of improving the chance of success.

TABLE 1: Van Hiele Levels

Level	Characteristics of Student
0	identify, name, compare, and operate on geometric figures and shapes according to appearance only
1	analyze figures in terms of components and relationships among components, establish properties empirically
2.	formulate and use definitions, logically interrelate previously known properties using informal arguments
3	prove theorems deductively and establish interrelationships among networks of theorems
4.	establish theorems in different postulational systems, analyze and compare these systems

(Fuys, Geddes, and Tischler, 1988, Pages 5, 58-71)

The van Hiele model asserts that the sequential progress of the student from level to level depends on instruction, and not on maturity, as Piaget suggested. The suggestion is made, however,

> ... that certain methods of teaching do not permit the attainment of the higher levels, so that methods of thought used at these levels remain inaccessible to the student. (Van Hiele, 1959a, p. 246).

This statement implies that there is an internal development of potential for progression toward a more abstract, logical, rational mode of thought, but that proper instruction is required for mastery. Furthermore, it appears that the "higher levels" of skill in geometry require more specific modes of thought. This, however, is speculative at present, since no study has yet examined this hypothesis.

The van Hiele levels of ability in geometry provide some interesting clues for the investigation of possible factors behind success and failure in geometry. In particular, deductive thinking is said to begin at level two, an idea which suggests that there may be a cognitive factor behind some of the difficulties in mastering geometry, since deductive thinking is itself a well-known term to cognitive-style investigators. Fuys, Geddes &

Tischler (1988) suggest that certain other cognitive factors, such as inductive thinking, spatial awareness, categorization, and analytical thinking, may affect performance in geometry. They suggest further that research is needed into the relationships between individual methods of modeling reality, (or modes of mental representation), and success in geometry. Furthermore, teachers in the field suggest that the way in which a student thinks has a bearing on achievement in mathematics. That is to say, teachers should be able to identify the various modes of thinking of individual students, in order to direct their learning experiences most effectively. As Davis stated:

> "Doing mathematics" is a process of *thinking*. ... "Doing mathematics" means creating, in one's own mind, a mental representation of the problem, and a mental representation of some relevant knowledge that can be used in creating a solution. (1986, p. 274)

Different individuals use different types, or modes, of mental representation when they are representing problems such as those in mathematics. In geometry, the collection of mental representations which an individual has of a given concept is known as a "concept image"

(Hershkowitz, Bruckheimer, and Vinner, 1987). This notion also allows for partial concept images (which are incomplete), and incorrect concept images.

In the consideration of cognitive factors in geometry, it has become evident that performance in geometry may well be affected by specific cognitive factors. The van Hiele model provides some clues to the type of factors involved, but individual differences in internal mental representations of mathematical problems are thought by some to play a role.

Cognitive Factors in Algebra

In an attempt to develop algebra courses which have more appeal to students, some researchers are examining algebraic thinking (Wheeler, 1989). It is suggested, for example (Kaput 1989), that there is a distinct mental representation system for algebra, involving a mathematical system of language and symbols. Kaput suggests that this system must first be mastered and internalized by individuals, in order that both communication and thought in algebraic terms be possible. On the other hand, Kirshner (1989), reacting to Kaput, is not as optimistic, and points out that features of mental representation used by an individual do not necessarily correspond to the mathematical representations employed, and that Kaput does not show how to develop such representations.

These two studies illustrate the divergence of opinion concerning mental representation models in mathematics. Kintsch & Greeno (1985) suggest, furthermore, that the type of mental representation used by an individual will differ according to the type of problem being solved. That is, most simple algebraic problems require a representation of the solution method for correct solution, while word problems will require that the situation itself be represented mentally, before a method of solution can be selected. Whether one interprets the actual solution as a set of schema, as Chaiklin (1989) does, or as a series of production rules, as does Larkin (1989), mental representations of the problem states are assumed. For Chaiklin, it is the content of the representation that is vital, while for Larkin, the construction of the representation is more important. If the internal representation is incorrect, she points out, then algebraic rules will be applied to invalid items, and hence the solution will not be correct.

Constructivist View of Cognition

This section reviews the literature in the area of cognition that is firmly anchored to the individual, and to the acquisition process of those cognitive abilities. The concept of selective attention is explored, and the resulting creation of mental models by the individual brings the reader to the theory of constructivism. Important constuctivist theorists, and their ideas concerning individual differences in internal representations are introduced. This in turn leads to the concept of modes of mental representation, and the exploration of various models of mental representation presented by prominent theoreticians. Lastly, mathematics is brought into the picture, as skills required in mathematics are examined in terms of modes of mental representation.

It is necessary at this time to expand the search of the cognitive literature, in order to examine the contributing factors responsible for inability to master skills in geometry, algebra, and other areas of mathematics. The cognitive domain, however, is broad. It encompasses more aspects than can be adequately dealt with in this study. Accordingly, only a small subset of the cognitive field will be examined; namely, the constructivist view of cognition. This view ascribes more importance to the learner than to the content to be acquired. For example, Sternberg writes: "The fundamental unit of analysis in most cognitive theories is the information-processing component." (1988, p. 267). In attempting to understand human abilities from a cognitive point of view, Sternberg considers "verbal ability, quantitative ability, inductive and deductive reasoning abilities, and spatial ability" (1988, p.270) to be among those aspects of human intelligence which vary considerably amongst individuals. Each of these components has been the focus of attention of many researchers. However, these abilities can be subdivided still further.

The various cognitive components do not appear in the individual in a fully-developed form, but appear to be developmentally related. Individuals appear, furthermore, to acquire only the knowledge which they are ready to learn. Readiness to learn does not refer only to intellectual maturation, but also to selective attending (or selective encoding). Selective attending is the term given to the process by which we pay attention to only some of the available stimuli at any one time. The stimuli to which we do pay attention are those which "fit" our expectations at that moment. For example, if we are waiting for a bus, we will be looking for buses, and may not notice what other people, or cars, are doing. We can also be fooled into attending to the wrong things, as is attempted by the designers of many puzzles (Sternberg, 1989). Selective attending is considered to be essential for effective processing of information, since there is a limit to the number of stimuli which we can attend to at the same time. The effective limit of approximately seven items, has not been changed by researchers since Miller first investigated it in 1956. This means, as a result, that we do not experience all of the possible stimuli. Selective attending may also be a plausible explanation for the inability to see, for example, a pair of lost glasses sitting in full view on a table, since we do not expect to see them there and hence are not really attending to the contents of the table top when we look there.

Selective attention, and the creation of mental models of reality, form the basis of constructivist theories, in which:

> learners are seen as constructing meaning from input by processing it through existing cognitive structures and then retaining it in long-term memory where it remains open to further processing and possible reconstruction. (Good & Brophy, 1990, p. 209)

One important theorist in the development of constructivist theory was George Kelly. He suggested that individuals have mechanisms, or constructs, which automatically restrict our attention to stimuli (Kelly, 1955). These mechanisms operate within a restricted frame of reference, and are selected to be relevant to our expectations. Agnew and Brown (1989a) suggest that Kelly's constructivist theory is important to learning, and support the hypothesis that we construct our own reality. In their model, individuals use selective attending to shrink complex problems to manageable size, and then apply a limited set of rational and empirical tools to take care of any residual problems. In a further study, they suggest (1989b) that feedback from the results of deciding on and following a course of action provides the individual with a validation for the actions taken. For example, it appears that individuals approach truth through successive approximations.

The constructivist model may explain part of maturation, in that to state that one is intellectually ready to master a skill may simply mean that one is cognitively prepared to attend to the concepts required for the skill. This model may also be applied to describe individual differences in learning among students presented with the same stimuli. According to the model, each student may well attend to different sets of these stimuli. For example, a student of geometry may not appreciate the importance of a given diagram, and hence may not attend to this aspect of geometry, and thereby be unable, or find very difficult, the solution to problems which rely on the use of a diagram.

There is, furthermore, a basic underlying assumption to most cognitivist theories. Objects, symbols, and sensory inputs can be represented internally in more than one way, and these representations can be transformed one into another. In other words, the assumption is that stimuli are present in the environment for each of our senses, and that we obtain information from all of the senses as well as from reasoning. When we imagine a railway train, for example, we may imagine in turn the sight of the train, the sound of the whistle, the sensation of movement, and the emotion felt when a friend once left on a train to go to a new city. We may also think of a list of famous trains, the location of the train station, or the Toronto to Montreal train schedule. Each of these images is different, yet each can be evoked in some people by the expression "railway train".

The concept of different modes of mental representation, and of the capability of transformation from one mode to another, is considered important to the understanding of individual differences among students who confront new learning such as that found in mathematics (Fischler & Firschein, 1986; Horowitz, 1972; Larkin, 1989). For example, an algebraic solution might be thought of as a good example of a set of IF..THEN rules, so that, when involved in solving such a problem, an individual might reproduce one or more sets of such IF..THEN rules (Thompson, 1989). These condition-action rules, then, provide the framework for the mental model which has been developed for algebra (Holland, Holyoak, Nisbett, and Thagard, 1989, p. 15). The student may attempt to apply this model to other topics such as geometry, but there is no guarantee that a model which "works" for one type of problem will naturally work for another. This model in particular may not be directly applicable to geometric proof, since interpretation of geometric problems often requires more "visual" analysis than does the interpretation of algebraic problems.

A mental model is constructed from the stimuli impinging upon the individual's cognitive structure (Agnew & Brown (1989a). One important set of environmental stimuli to be examined is that of sensory input, that is, the stimuli resulting from sight, touch, hearing, smell, and taste. Sensory input can arise both directly, from the senses, and indirectly, from associations, as in "smelling" smoke when one hears a fire engine's wail. These different kinds of sensory input lead to differences in the activation of modes of mental representation described for human cognitive systems, since these systems are related to the senses. Of all the modes of mental representation described in the literature, the visual, verbal (or auditory), and kinaesthetic (or enactive) are most often described. The visual mode arises from the sense of sight, the auditory from hearing, and the kinaesthetic from the other senses.

These modes of mental representations, and the resulting thinking processes used, may be of importance in the study of mathematics learning. The implication suggests that there are different ways of thinking (Agnew & Brown, 1989b). Concomitantly, it should be possible to outline a model of learning which accommodates these individual ways of thinking. For example, an individual can imagine being in a footrace, and winning the race. The image of upraised arms breaking the tape might be conjured up, or the time taken to run the race might be remembered. These two, the image and timing fact, would represent different ways of thinking.

Paivio (1971, 1986) and Anderson (1980) suggest that the main forms of mental representation include only visual and verbal representations, and that, although other modes do exist, they are of minor importance. Verbal representation, for Paivio (1986, p. 53), involves abstract and logical thought, whereas visual representation involves concrete, representational thought. Anderson (1980, p. 122) goes further in his suggestion that these two representations are translated into an abstract representational form for long-term memory. Both Paivio and Anderson consider visual and verbal representations, however, to be internalized products of experience.

On the other hand, Horowitz (1972, p. 796) suggests that there are three main modes of representation: enactive, image, and lexical. She suggests further that the image mode of representation has a separate organizing system of the image for each sense, including vision, touch, and so on. An image, to Horowitz, is a thought representation of a sensory experience, and she considers vision to be the most important of the image representations. Visual imagery refers to visual mental images, or mental pictures, which can be very vivid in colour and detail; while by enactive imagery is meant the idea of imagining motion in a mental image which one is visualizing. By "lexical", Horowitz means that specific form of representation which is actionless and imageless: that is, a lexical representation is any representation which is not an example of an enactive or an image mode.

When one examines more closely the necessary skills for different areas of mathematics, there appears to be a relationship between some of these skills and what is known as "spatial intelligence". This form of intelligence, for Michael, Guilford, Fruchter, and Zimmerman (1957), is made up of visual imagery, kinaesthetic imagery, and spatial relations or orientation. That is, in terms of mental representations, the addition of the spatial aspect to the visual and kinaesthetic, produces the notion of spatial intelligence. This involves a visual mental image which is not static, but which is mentally moved around or otherwise manipulated. Hence two forms of mental representation may be involved - the enactive and the image; and the ability to perform the task as a whole may depend on the ability to utilize both modes of representation, possibly simultaneously.

That there are wide variances in spatial ability in the population is well known. In describing an experiment in which subjects were asked to rotate objects mentally, to see what the final appearance would be without actually physically moving the objects, Shepard and Cooper state:

> As we have come to expect for tasks involving spatial abilities, there were wide differences among the performances of individual subjects. Of the ten subjects, three...had error rates...that were so high... that it seemed best to exclude their data in computing the mean reaction time. (1982, p. 194)

Whether or not an individual has each of the distinct modes of representation, and has, furthermore, developed each mode to a high level, can only be ascertained by testing. It is reasonable, however, to suggest that a person who has not developed the ability to create vivid visual mental images, and then to manipulate them mentally, and to make deductions based on them, might not be able to handle such tasks as drawing a diagram to represent a problem as competently as those who can do so. Holland et al. (1989) also point out that, in order to be able to begin to solve any kind of problem, an individual must be able to attend selectively to the proper stimuli. That is, the actual question being asked, or the issue of concern, has to be identified and understood, or, like the hungry frog whose brain cannot recognize a motionless fly, the individual will not be able to see how to solve it.

In other words, the lexical or verbal representation of the problem is important, in that the individual has to be able to read the question and understand what is asked. In the process of understanding the question, the person must be able, in algebra, geometry, or in problem-solving in general, to visualize an image of the situation. If the problem cannot be understood, then the situation can resist encoding, and a mental model which would permit a solution to be attempted, might not be built. The individual is then defeated even before beginning the task.

Gallagher (1987) found a high level of correlation between the mathematics scores on the Scholastic Achievement Test for Mathematics (SAT-M), and high visual-spatial ability. Although her analysis is interesting in its conclusion, some caution must be exercised. Three points can be made. First, the sample used, unfortunately, consisted only of students from a school for the gifted. Second, no other modes of representation were tested, hence the fact that the high achievers were gifted spatially may or may not have much significance, since the students could have had high abilities in other modalities also. Third, the degree of dominance of the visual-spatial modes of representation was not measured; only the ability to use this form of reasoning was examined. Thus generalizations of results are seriously compromised. Moreover, different areas of mathematics were not tested separately but lumped together as a general score, so the specific relevance to the current study is difficult to ascertain. A relationship, however, between high visual-spatial ability and success in mathematics does raise the possibility that there is a relationship between the ability to utilize visual-spatial modes of representation, and success in individual topics of mathematics. The necessity for further research in this area is indicated.

In a related study, Wheatley, Grayson, and Talsma (1982) studied learning in geometry by pre-service teachers. They found that student-teachers who functioned in mathematics at the formal operations level were better achievers in a geometry unit than those who did not. The researchers compared the achievements of those at different levels of cognitive development, according to Piaget's model, with those of differing abilities in spatial-visualization, and found that the cognitive development level was more important. This finding appears to contradict Gallagher's research, and illustrates the need for further study. The contradiction is, of course, perhaps more apparent than real, since the cognitive development levels of the students in Gallagher's research were not investigated.

The constructivist theorists presented to this point, although informative and persuasive, are found lacking in one aspect or other of their argumentation, research, or instrumentation. For example, although the model presented by Horowitz (1972) is elegant and thorough, there is no instrumentation available with which to examine it. On the other hand, those such as Shepard and Cooper (1982) who are investigating spatial intelligence, have considerable instrumentation and research findings to report, but do not have complete theories concerning all modes of mental representation. Hence it is necessary to focus on those models which have considerable research support, and for which instrumentation has been developed. One of these models of cognitive development is that of Susan Aylwin, who has used the theories of Bruner as a basis from which to build her own model of utilization of modes of mental representation.

Bruner and Aylwin

There is no consensus as to the number of modes of mental representation, or as to their nature, or as to how and when the ability to utilize them arises. Many theorists, however, appear to present views which are consonant with Jerome Bruner's theory of cognitive development. One of these theorists, Susan Aylwin, has written a clear, concise description of modes of representation and how they develop. The work of Bruner and Aylwin is important for two main reasons: first, both share a concept of mental representation which may have a bearing on the differing abilities of students to master the concepts required in the study of mathematics; and second, their models complement each other in their approaches. Bruner's views are well-known amongst theorists in the cognitive field. Aylwin, however, is not as well known in North America. Her major work has emerged from data gathered in Australia and the British Isles, and her journal articles and book were mostly published abroad. She has developed, along with a cognitive model, the empirical instruments required to evaluate the different forms of mental representation used by individuals.

Bruner (1964) argues that children begin their cognitive development with movement and touch, and develop a combined way of thinking and its related form of mental representation which he terms "enactive". Thinking enactively refers to anticipatory tensing of muscle groups, and to the mimicking responses to another individual's motor activity (Bruner, p. 16). This process is also known as action thinking. Bruner suggests that children represent objects first in terms of the sensations which the objects themselves arouse, that is to say that the child learns by physically manipulating objects in the environment, and thinks concretely. As an example of this, the child who is attempting to add numbers might imagine herself moving objects and counting them.

The child progresses, early in development, from this enactive mode to what Bruner terms the "iconic" mode. The iconic mode is a form of mental representation in which mental images from all of the senses - sight, taste, touch, smell, and taste, are used to store information. It differs from the enactive mode in that there is no muscle tension involved, but rather a set of static mental images which come from the senses. That is, the process of thinking is one step removed from the enactive, in that only the image exists; the child is no longer there manipulating it.

The third form available to the child, and the most advanced, according to Bruner, is the "symbolic". This mode consists of symbols, such as numbers, words, or pictographs, which are representations even further removed from the original sensory input than are the other two types. Using this mode of representation, one learns to classify objects, and form hierarchies of categories. One also learns to reason without constant reference to the concrete: that is, to master "the self-conscious use of logic and reasoning" (Bruner, 1971, p. 20).

Aylwin uses slightly different terms to describe much the same ways of thinking, or modes of mental representation, as Bruner. She considers, however, the three forms of representation posited by her, to be mental images (1985). She calls the first form of representation "enactive imagery", and states that this form is kinaesthetic in nature, involving muscular tension in the same way as Bruner's enactive form of mental representation. It involves, according to Aylwin, imagined action and role playing. She considers it to be the foundation of all subsequent intelligence.

Aylwin's second form of representation to emerge in children is hypothesized to be "visual imagery". She uses the "pictures in the mind's eye" metaphor to describe this repres ntation, and suggests that there are parallels between the mental processing of a stimulus being viewed by the eye, and of mental images, either of things seen previously and remembered, or of completely imaginary things. These parallels arise from the idea that they use the same processing system, that an individual can sometimes mistake one for the other, and that they may interfere one with the other. The experiments of Shepard and Cooper (1982) give credence to this view, since they found that a mental rotation of an object through 180 degrees, for example, took longer in timed tests than a mental rotation of 45 degrees. Aylwin argues, further, that there are some things which cannot be done with images, such as picturing a "running yellow", thus illustrating a belief in some structure for visual imagery.

The last emerging form of mental representation is, for Aylwin, "verbal imagery". This is a form of inner speech, with the same properties as outer speech. It is used to code information, and to handle conceptual and abstract thought. She states Vygotsky's view, that there are structural differences between inner and outer speech, in that inner speech is highly contracted, and contains mostly predicates without subjects. Verbal imagery, by this definition, consists of words and sound, and is structured categorically and hierarchically.

Aylwin makes the conjecture that the three modes of mental representation may use different cognitive structures, and hence may have different kinds of affects and feelings associated with them. For example, visual imagery may use sets of mental pictures, while verbal imagery may use a hierarchical structure of categories. In terms described by Rumelhart and Norman (1985), the verbal structure may be propositional in nature, while visual imagery may be analogical. Whatever form the structures may take, Aylwin suggests that there are systematic structural relationships and interconnections among the three types of representation.

There are both similarities and differences between the views of Bruner and Aylwin. Bruner and Aylwin share the same view of the enactive form of representation, but differ in their interpretations of the image form. Aylwin includes only visual images in the image mode, whereas Bruner maintains that there is an image for each of the five senses: smell, taste, touch, hearing, and sight, although he does concede that vision is the most important of the image modes of representation. The third form of representation for both theorists is the most abstract form. Aylwin assumes that there is an auditory aspect to the verbal form, and that the individual in inner speech hears words being spoken internally. Bruner, on the other hand, does not make this assumption, and allows symbols other than words to be used. For both, inner speech is used for abstract thinking.

It is interesting to note that in Bruner's case, the development of these forms of mental representation is considered to be "in very considerable measure dependent upon growth from the outside in" (1971, p. 21). In other words, Bruner suggests that mental growth does not occur automatically, but is a result of experiences: it occurs when the individual is forced to accommodate these experiences with earlier modes of thought. He also describes a process of "logical necessity", in which the individual develops the skill of logical reasoning only when it becomes necessary to do so. That is, although the potential may exist in the individual, it is not realized unless there is some motivation for this to occur. The mature individual has the potential to develop three systems of thinking skills: "for the hand, for the distance receptors, and for the process of reflection" (1971, p. 28). These three systems of thinking skills appear very similar to the three modes of mental representation. The enactive mode of mental representation appears related to the "thinking for the hand" system, the visual mode appears to coincide with the "thinking for distance receptors" system (or that of the eyes), and the verbal mode of mental representation appears to be similar to the "process of reflection" system.

Therefore Bruner and Aylwin both distinguish carefully between three modes of mental representation, and suggest that these are sequentially acquired in the developmental process of individuals. Both suggest that there may be different cognitive structures tied to each of the modes of representation, and that they all arise separately. Bruner suggests that earlier theories of learning arise from at least two of these three forms of representation of thought. He suggests (1971, p. 18) that the stimulus-response theory may be a reasonable description of learning from the standpoint of the enactive form of thought, and that Gestalt theory describes very well the iconic mode. Aylwin does not make this comparison to earlier learning theories in her work.

The application of their models of mental representation to novel situations is the main point at which Bruner and Aylwin differ. Bruner's focus is mainly on the developmental process in children, and has little to do with mature individuals. The closest Bruner comes to the adult situation occurs in his concept that in each new learning situation, the individual progresses through the three modes of mental representation: from enactive to iconic to symbolic. All beginners in a learning situation, for example, would use the enactive mode (1964), and would progress to the other modes when motivated to do so.

Aylwin agrees with Bruner's model as it relates to children, but has developed different ideas concerning adults. In her model, she has operationalized Bruner's three categories, and she uses them in a different context for mature individuals than they were used for children; they are no longer developmental in nature. Aylwin states that an individual presented with a new situation would tend to use whichever mode of mental representation seems to fit best. Thus two different individuals might use different modes, given the same new stimulus, even though both of them were able to use all three modes. This implies that there are individual differences in utilization of the modes of mental representation.

The actual proficiency level demonstrated in the use of each representation differs amongst individuals. Some use more enactive levels of thought, some use more iconic, and some use more symbolic. The majority of people do use all three forms in different circumstances, but to differing degrees. Aylwin (1985) suggests, moreover, that the differing degrees of use of the three modes of mental representation give rise to three personality styles, namely verbalizers, visualizers, and enactive imagers. This marked preference for one of the modes of mental representation, however, may indicate that the individual who makes predominant use of one mode of representation, may not have developed one or both of the other two modes of representation sufficiently to be able to attend to the stimuli demanding them. As a result, the individual may not be able to demonstrate mode-flexing abilities. The statement is made by Krutetskii that:

> Under identical conditions for the perception of mathematical material, pupils with different mathematical abilities obtain (or more precisely, actively procure) different information. (1976, p. 233)

This approach to mental representations is fundamental to the present research.

The individual differences Aylwin proposes for use of different modes of mental representation may not be restricted to adults, but may occur in younger children. Krutetskii, for example, found that some young children (aged 8 to 10 years old), consistently reasoned analytically when solving problems, while others used only visual-pictorial representations to solve problems (1976, pp. 320-321). He suggested that the mathematical development of these students was one-sided, since these students were in many cases using methods which required considerably more work, in order to utilize the representation they preferred. This finding did not appear to be a matter of developmental delay, since some of those children who consistently reasoned analytically, could have solved certain problems much more easily using visual methods. The preferred use of a particular mode of mental representation thus appears to be common to both children and adults.

Aylwin's model of modes of mental representation points to individual differences in mode utilization, or to individual preferences in such utilization, as a possible reason for the finding above. There may also be an inability to mode-flex, or transfer a problem which was represented in one mode, to another for solution.

Krutetskii's finding, together with the other evidence presented to this point, suggests that a better understanding of some of the differing degrees of success of students in mathematics might result from examining differing abilities in the utilization of the three modes of mental representation. More precisely, the preference of some students for the use of visual representations in solving problems, for example, might be related to a lower ability to reason verbally, or to a difficulty in the transferal of a visual representation to a verbal representation. The preference of other students to limit themselves to verbal reasoning may be related to a lower ability in visualization, or in mode transfer.

The Modes of Thought Questionnaire

Aylwin (1985) states that adults, in general, have personal preferences in the utilization of modes of representation to solve specific tasks. She has developed an instrument to assist in her research into mental representation, called the "MOTQ" or "Modes of Thought Questionnaire". The purpose of the instrument is to evaluate individual preferences in modes of representation, and hence to assess representational biases through their "characteristic cognitive structures" (p. 66). In other words, Aylwin measures, with the MOTQ, the ability of an individual to comply with directions to utilize particular modes of mental representation, and observes the actual modes used in the process.

Aylwin uses the MOTQ to examine the bias evinced towards the modes of representation by students preparing for different careers. A strong bias for verbalization is found in commerce students; visualizing is found to be associated with arts and social science students, particularly women; and engineering students are found to have a bias for enactive imagery. She supports these findings with references to research by Witkin, Ashton, Richardson, and others. Witkin and Goodenough, for example, argued (1981, pp. 44-46), that field-dependent people tend to select vocations such as nursing and clinical psychology, while field-independent people tend to select the medical sciences, engineering, and mathematics. The nursing field has changed in recent years in that the prerequisites for entry into the field are now similar to those for science or engineering, so Aylwin's comparison would no longer be expected to hold, at least in North America. In a recent study, for example, Rancourt and Noble (1991), found nurses to have characteristics which are generally associated with field-independency, a fact which is at odds with Witkin's earlier finding.

The relationship between preferred modes of representation, and careers, is interesting in light of the current question, concerning the possibility that the relative ability to utilize the three modes of mental representation has a bearing on the ability of a student to master mathematics in general, and specific topics within the discipline, such as geometric proof, or the solution of three equations in three unknowns. To examine this relationship in greater depth, it is necessary to examine the three modes of mental representation more closely.

Aylwin considers each mode of representation to be a way of thinking, in that a particular representation of reality directs one's thinking into a path related to the representation; the reality cannot exist to the thinker without the representation. Aylwin compares and contrasts the three modes of thought, with reference to cognitive structures, general features, value or gain, and social aspects. Each mode of thought can be understood as being a way of making sense of the world. Paying attention to the same object, but using different modes of representation, may result in different outcomes.

Verbal representation, for Aylwin, provides the basis for a traditional orientation in which people worry about status and conventional values, and see relationships with others in terms of subordinate, superior, and opposition. Visual representation leads to a social affiliation, in which people worry about what others think about them. In enactive imagery, action and motive underlie a sense of self-efficacy, and the self is seen in terms of agent and experiencer. These three modes do not act in isolation, and most people can move from one to another to some degree. Each mode of representation, however, is connected to its own version of reality, with its own facts and values. Thus Aylwin suggests that each mode of thought involves a particular way of attending, which in turn affords a general social orientation.

Riesman, in his book The Lonely Crowd: A Study of the Changing American Character (1950), suggests that there are three social personality types, namely tradition-directed, inner-directed, and outer-directed. Aylwin (1985) suggests that there may be a connection between the personalities of those with specific dominant modes of mental representation, and Riesman's three social personality types. A connection is suggested between the verbalizer and Riesman's tradition-directed personality type, the enactive imager is thought to connect to Riesman's inner-directed type, and the visualizer is thought to connect to Riesman's outer-directed personality type.

The connection is also suggested by findings from a series of psychometric studies carried out by Aylwin, Hammond, and Dunne (reported in Aylwin, 1985), in which Aylwin examined connections between her personality types, and those of other researchers. The studies used the MOTQ in conjunction with many well-known instruments, including the Wilson-Patterson Attitude Inventory (WPAI), Rotter's Internal-External Locus of Control Scale (IE), and others. It was found that verbalizers tended to illustrate traditional values, in which, for example, women stay at home while men go out and are assertive. Male visualizers tended to be introverted, while the correlation between social desirability and visual scores was very high for both men and women - in other words, visualizers were attempting to be liked by others. Male enactive imagers tended to lack spontaneity, and have a low self-acceptance and self regard. Female enactive imagers tended to be self-sufficient, reserved, and shy.

These findings support the personality traits suggested by the MOTQ itself - that a particular dominance in use of the modes of mental representation may colour the ways in which one deals with the world. That is, a particular way of thinking may cause one to interact with the world in a particular way, and furthermore, a person with a particular personality type may be influenced to think in a certain way. This may be one source of the gender differences found in personality types and in modes of representation. Men, for example, are encouraged to stand on their own, to lead, to be aggressive; women are encouraged to be socially oriented, to nurture, to be dependent. It is more socially acceptable, according to Aylwin (1985), for a woman to be a visualizer, and for a man to be an enactive imager or a verbalizer.

Aylwin (1985) examines the three modes of mental representation, in the light of how an individual, with one of the three modes dominant, makes sense of the world and the place for the self in it. An individual who is dominant in the verbal mode fits events, objects, and people into hierarchies, in the same way that a large corporation places its employees into positions within a bureaucratic hierarchy. Such an individual tends to be competitive, and feel superior. The verbalizer is placed in a social hierarchy and a work hierarchy, and is concerned with obtaining higher positions in both of them.

The visualizer, however, does not look at the societal structure in this way, but rather is more self-interested, in that an individual of this dominance has a need for affiliation, and tends to focus more strongly on personal interactions with others. In doing so, the visualizer may go along with others, in order to be liked. The enactive individual, in comparison with the other two, however, is not concerned with anyone else, but seems to be driven to do whatever has to be done without wanting or asking for assistance. Such individuals are more interested in things rather than people.

These different ways of making sense of the world by individuals who use different modes of representation have an important implication for the present research. Namely, when different individuals are biased in how they think of themselves in general, a bias which may be due to their particular dominant modes of representation, then they may well approach other facets of life differently. In school-based learning in particular, it may be that the mode of representation used by the student influences the understanding of and response to different kinds of lessons presented by teachers, as well as the attention paid to different types of subject material. Moreover, the quality of learning effectiveness taking place in classrooms may be influenced differentially by the modes of mental representation used by the students and by the teacher.

Aylwin argues (1985, pp. 172-174) that it is possible to distinguish evidence of three epistemological strata within scientific activity, each stratum being identified with one of the three modes of thought. She finds that verbal thought provides the logical foundation in which conceptual organization takes place, postulating that the process of defining basic entities, classifying them into a hierarchical system, and generating a nomenclature, is characteristic of verbal thought. Visual imagery, on the other hand, contributes the use of metaphors, and hence the use of models in analysis of phenomena. Enactive imagery, in comparison, states Aylwin, provides the process of scientific method, and the causal agents for transformations which occur. It is the flexible interactions among the three which provide the successive re-articulation and refinement of scientific theories.

Others in the field of cognition believe that an individual uses his or her cognitive system to construct a mental model of a problem space which is then mentally manipulated in various ways in order to solve the problem (Holland et al., 1989; Johnson-Laird, 1983). One such mental model is that of a script, or schema (Rumelhart, 1980; Schank & Abelson, 1977). In a script, one imagines a series of actions which must be followed under certain situations, in much the same fashion that a movie script dictates the actions which a character must take on the stage. This model appears to be related to the verbal, kinaesthetic and visual modes of mental representation, in that the verbal mode controls the action of the kinaesthetic mode, and one mentally uses the visual mode to view the action taking place.

A second model is one in which there is a set of rules for each situation - that is, "if 'A' then 'B'" (Anderson, 1983). This model appears to be allied to the verbal form of mental representation, being more abstract and logical than the script form. The relationship between the representation and the cognition models appears to be that the former is the manner in which thoughts are stored and accessed, while the latter is the means of processing the stored information.

Aylwin considers each mode of thought to be a way of making sense of the world, which means that, if different individuals pay attention to the same object, but use different forms of representation, their perceptions may result in different outcomes. This difference in use of the three modes of thought may be related to the encoding concept, in which we attend only to those aspects of an object to which we are ready to attend, but ignore those which as yet have no meaning for us.

One can consider, for example, the mathematical problem of finding the distance across a lake, without employing direct measurement. The visualizer might focus on the lake itself, and look in depth at a mental image of the lake in question, to find clues to the problem solution. The verbalizer, on the other hand, might imagine a hierarchy of problem-solving techniques, in which "lake, distance across" would be a category, and various methods of solution would be written in a table. The enactive imager might imagine the physical process of taking measurements, walking, and so on, to determine a method of solution. Hence a different preference in mode of mental representation between individuals may reflect a different approach to problems, in addition to a different set of interests, and thus potentially different subject matter specialties. These different preferences in modes of mental representation may therefore lead to different preferences in courses selected at university, and hence eventually to different careers.

Aylwin's MOTQ instrument, then, attempts to measure the preferred modes of mental representation of the individual, with a view towards the development of an understanding of individual differences. For internal representations may well bias an individual towards or away from a particular approach to problems, both in the academic sense and in the social sense of day-to-day living. This internal bias may be one of the factors which affect mathematics achievement.

Mathematics and Models of Mental Representation

In Aylwin's model of mental representation, it is assumed that different careers make different types of cognitive demands, and hence attract individuals with different cognitive biases (1985, p. 68). In her analysis of university students in different faculties, Aylwin found differences in utilization of the three modes of mental representation. Students in commerce, for example, were found to have a greater utilization of the verbal mode of mental representation than did other students. Such bias would, in her view, be of benefit in this career, where organization and management are of great importance. The arts and social sciences (mainly social work) students, on the other hand, were found to have a bias towards visualization. One faculty in which students used the enactive mode of mental representation more than did other students was that of engineering, where spatial flexibility is thought to be important. Thus for Aylwin, as for Phenix (1964), the career choice, and the university training preparatory to the career, have different epistemological structures. In her words:

> Each mode of thought is related to specific cognitive abilities which may interact positively with career choice. (Aylwin, 1985, p. 71)

For Aylwin, however, it is not clear whether the difference in utilization of the modes of mental representation comes first, and particular biases of the individual lead to a particular career choice, or whether training for a particular career will promote the use of one mode of mental representation over another, and lead to the bias.

According to Phenix (1964), different subject matters have their own epistemological structures. He notes that the realm of mathematics is different in structure from that of the arts or of the sciences. He suggests that the methods of teaching and learning must be likewise different, and that students will have greater ability in one area such as mathematics, or the arts, or the sciences, than in another. Scheffler (1967) further states that in mathematics and in science:

> ... observation is not at all a bare apprehension of pure sense content, but rather an active process in which we anticipate, interpret and structure in advance what is to be seen. (p. 13)

More recently, Fischler and Firschein (1986), in a discussion of the central role of representation to intelligence, both for humans and computers, point out that the frames of reference of an observer will colour what is understood to occur:

> an observed phenomenon is interpreted in accordance with a stored framework (model, metaphor, representation) that is used by a person to deal with the outside world. Different areas of human intellectual and emotional activities access different representations of the world with different attributes - they construct different realities. (p. 44)

In other words, according to Scheffler and to Fischler and Firschein, two individuals may be talking, and hearing each other speak, yet have no meaningful communication, since each interprets the other's words in terms of personal understanding. In agreement with Bruner (Bruner, Goodenow, and Austin, 1962), Scheffler states that the perceiver of an event will tend to fill in any missing attributes, will change colours if it is necessary or relevant, and will rectify attributes which differ from expectations, all to conform to an internal representation of what should be present. Fischler and Firschein agree with this view, adding that people cannot deal with anything, even on a superficial basis, without reference to the appropriate mental construct.

There is a second model of mental representation, which uses terminology similar to that of Aylwin, but the two models are different in nature. This second model is that of Griffitts, in which many individuals show a decided preference for the auditory, the visual, or the kinaesthetic form of mental imagery. Griffitts examines a different aspect of imagery from Aylwin, however. In the form of the Griffitts test used by Diehl and England (1958) and by Richardson (1969), an attempt is made to find out which kinds of sensory input are attended to by an individual, when a stimulus is presented. This is done by means of a word-association test, in which a subject is presented with a stimulus such as "fire engine" or "bicycle", which could be thought of in terms of any of the senses, and is required to state which of the senses was used in the formation of a mental image of the item. Hence this test is complementary to that of Aylwin; she is interested in the modes of mental representation used by individuals, while Griffitts examines the input of sensory data to the individual.

Diehl and England (1958) used the Griffitts test in research with both young adults and children. With young adults in university, they investigated the sensory modes of mental imagery used by individuals training for different occupations; with children, they attempted to find a relationship between the modes of mental imagery and the process of learning to read. Some of the findings suggest that the imagery type of young children may be a factor in learning how to read, in that some children learn best through phonics (auditory method), whereas others learn more easily by tracing the outlines of words with their fingers (kinaesthetic method). In this model, imagery type is understood by Diehl and England to be an

> ...attitude of mind, through which one most easily acquires new experiences and knowledge and most readily recalls memories of facts or words. (1958, p. 268)

The Griffitts test was originally designed for disabled individuals. It was an attempt aimed at the improvement of effective teaching methods for them. However, its use has been enlarged to include those who want to understand individual differences in the general population. The model underlying the Griffitts test has been used, as reported by Richardson (1969), to investigate the mental imagery used by individuals in different populations. Diehl and England (1958), for example, utilized the Griffitts test to examine the dominant mode of mental imagery of students majoring in art, music, and physical education. The Griffitts test is essentially a word list, of concrete rather than abstract words such as *fire*, *radio*, and *car*, which can be thought of as having stimulus value in all of the major imagery categories.

In the study carried out by Diehl and England, the art majors were found to have a higher mean dominance score for visual imagery, while the kinaesthetic mean score was higher for the physical education majors. The scores of the music students, however, were not significantly different from those of the control group on any of the three modes. Richardson (1969) reported on a similar study conducted by Roe, where the sample consisted of scientists. Roe's study found significantly more verbalizers among psychologists and anthropologists, and significantly more visualizers among biologists and physicists.

This makes the test appropriate for this study. It seems probable that the sensory input to which one pays attention, might be related to the mode of mental representation which one prefers to use. Since it has been found that both Aylwin's MOTQ and Griffitts' Test of Mental Imagery distinguish between populations having different areas of study, or different occupations, then any possible relationship between them should be explored. Hence the Griffitts test is used in this research to allow the examination of not only the mode of mental representation utilized by the individual, but also of the preferred sensory input.

It must be noted that there is a potential problem with the Griffitts test, as with other instruments of this type. Although the psychometric properties of these tests are reported to be satisfactory, the sample sizes used in studies relying on these instruments appear small. The researcher should not, therefore, rely solely on them.

They do, however, provide information of interest to researchers, and therefore the Griffitts test, and variations on it, are still commonly utilized today (Boutcher & Rotella, 1987; Kohl, Roenker, and Turner, 1985).

A third model of mental representation focuses not on the modes of mental representation themselves, but on modes of knowledge accessing. Royce and Powell (1983) are concerned with the different ways of knowing that an individual uses to gain knowledge, and they associate cognitive styles, and affective styles, with epistemological styles. Their model presents three modes of knowledge accessing, each of which is committed to a different way of processing information. Empirical knowing is related to the selection of perceiving modes of processing, rational knowing is related to the conceptual modes of processing, and metaphoric knowing is related to symbolizing as a mode of processing. Each way of knowing is thus ultimately tied to a different view of reality.

In a similar fashion to that of the modes of mental representation and of mental imagery, the modes of knowledge accessing provide a different frame of reference for each individual in dealing with the environment. Similarly, Phenix (1964) suggests that different subject matters have their own epistemological structures. Each individual can deal with subject matter from an individualized frame of reference, which may or may not match the structure of the subject matter. And furthermore, whether one examines the subject matter, or the student in interaction with the subject, some students will have greater ability and success in such interactions than will others (Rancourt, 1978).

Rancourt and Dionne (1982), using a model of knowledge accessing based in part on Royce's writing, found that there are differences in modes of knowledge accessing among teachers of different subject areas in the schools of Ontario. In the study, 485 teachers who had identified a preferred teaching subject, were examined as to their preferred modes of knowledge accessing.

Rancourt and Dionne posit three such modes: the Empirical, the Rational, and the Noetic. The Empirical mode is defined by a commitment to sensory perceptions as the primary data source, and is related to empirical or inductive reasoning; the Rational mode is defined by a commitment to knowledge of rules, principles, and concepts as a primary data source, and is related to deductive reasoning; and the Noetic mode is defined by a commitment to subjective experience as a primary data source, and is related to metaphorical or intuitive reasoning. The epistemic orientation profile that was found for teachers of mathematics has Rational as the major mode, Empirical as the secondary mode, and Noetic as the minor mode (REN). This differed markedly from the profile of language teachers (NRE), for example.

More recently, in a study of 308 adolescents from Ontario, Quebec, Tunisia, and the Ivory Coast, Rancourt and Deschênes (1990) found that students whose favourite subject of study was mathematics have a profile similar to that of teachers of mathematics, namely REN. The study involved francophone secondary school students between the ages of 13 and 16, with sample sizes ranging from 60 for Tunisia to 98 for the Ivory Coast. The students involved, completed "l'Inventaire des modalités d'accès à la connaissance" (Rancourt, 1988), an inventory of knowledge accessing modes. The percentages of subjects with mathematics as the preferred subject, and with the REN profile, range from 52% for those from Tunisia, to 77% for those from the Ivory Coast.

Thus, both students who prefer mathematics, and teachers of mathematics, appear to rely more on the Rational mode of knowledge accessing than on other modes.

Royce called this reliance on one mode, a form of mental encapsulation (Royce, 1964). In teachers, the dominant mode of teachers who are not teachers of mathematics, is not the Rational mode (Rancourt and Dionne, 1982). There is, accordingly, some probability that the dominant modes of mental representation used by mathematics students may well differ from the dominant modes of mental representation of non-mathematics students.

Although the potential exists in each individual to use effectively all three modes of mental representation, the evidence presented thus far suggests that the actual use of each of the three modes differs amongst individuals. It should be possible, therefore, to determine preferences towards particular modes of mental representation in different populations.

More specifically, the review of the literature to this point suggests that it is plausible to suggest that achievement in school subjects preferred by students may depend upon their ability to represent the content in the appropriate mental modality. Overuse of a preferred mode of mental representation, according to Aylwin (1985), Rancourt (1978), and Royce (1964), may bring a student to handle best only one of the possible three modes. Hence students who do not succeed in mathematics may show a particular hierarchy of use of the three modes of mental representation.

In a pilot study conducted by the author, a small sub-sample of twenty pre-service education students with different backgrounds in mathematics was examined as to preferred modes of mental representation and profiles of knowledge accessing. In the sub-sample, students with considerable experience in mathematics were found to have the knowledge accessing profile of ERN. That is, Empirical was the dominant mode, Rational was the secondary mode, and Noetic was the minor mode. The Rational and the Empirical scores, however, were very close, suggesting that the ranking might change with a larger sample. The students who had no mathematics experience beyond secondary school were found to have a knowledge accessing profile of NER.

Furthermore, the students with considerable experience in mathematics were found to have a greater use of the verbal mode of mental representation than those students with little or no mathematics experience. Similarly the students with considerable experience in mathematics were found to have a greater use of the auditory mode of mental imaging than those students with little experience. Thus the pilot study provided some further evidence that there may be a relationship between mathematics experience and the degree of utilization of particular modes of mental imaging, knowledge acquisition, and mental representation. The number of subjects in this sample was small, however, so that no generalizations can be drawn from these data.

Statement of Problem

The review of the literature to this point suggests that the question of learning mathematics should be examined from the perspective of the process of acquiring knowledge. Suggestions of a possible specific underlying cognitive profile of those who specialize in mathematics ade by Herscovics (1989) and Hershkowitz et al. (1987), are indicative of the direction which this research will follow. Furthermore, anecdotal evidence from teachers in the field (Kieran, 1989; Meserve & Meserve, 1986; Usiskin, 1987) suggests that specific cognitive factors, such as visual recognition of properties, verbal descriptions of properties, and hierarchical classification, may be involved in the study of mathematics.

These types of anecdotal evidence exist in the form of either speculations on the part of teachers and students, or of case studies, or again of criticisms of current and past curricula or teaching methods. Some of these case studies tend to point out individual differences among students, and speculate on possible reasons for these differences (Krutetskii, 1976). Others are presented in an attempt to advance an argument. The idea that children construct their own mathematics (Yackel, Cobb, Wood, Wheatley, and Merkel, 1990), or that cooperative learning leads to better cognitive rehearsal of an individual's viewpoint, and hence to better mastery of topics in mathematics (Lees, 1991), are examples of the latter.

Since cognitive factors are involved in all forms of knowledge acquisition, including mathematics, the literature review to this point suggests that it is timely to introduce cognitive factors into the research framework of mathematics education. The ability to utilize different modes of mental representation, modes of knowledge acquisition, and modes of mental imaging, may well be related to the effective acquisition of knowledge in mathematics, and such ability might well be measured by means of tests. Hence this study can be seen as a first step in the process of understanding the relationship between modes of mental representation and mathematics education.

Statement of Hypotheses

The hypotheses that arise from the theoretical framework and the review of the research literature on the construction of mental representation can be stated as follows:

Major Hypothesis

1. There is an association between levels of experience in mathematics and the degree of effective utilization for each of the three modes of mental representation, as measured by Aylwin's (1985) Modes of Thought Questionnaire.

Minor Hypotheses

2. There is an association between levels of experience in mathematics and the degree of preference for each of the three modes of knowledge accessing, as measured by Rancourt's (1988) Knowledge Accessing Modes Inventory.

3. There is an association between levels of experience in mathematics and the degree of preference for each of the three modes of mental imaging, as measured by the revised Griffitts Test of Mental Imaging (Richardson, 1969).

CHAPTER III : METHODOLOGY

This chapter presents information concerning the methods by which the hypotheses were tested. In order to accomplish this testing, a representative sample from a population of subjects and valid instrumentation are required. Both are presented in this chapter. The data accumulated during the research, and the methodology utilized in their analysis also appear here.

Instrumentation

This study examines the mental representation of pre-service education students in Ontario. The instrumentation was selected to illustrate different aspects of mental representation: namely the mental representation allied to thinking itself, to the process of knowledge accessing, and to mental imaging. It may be argued that these aspects may not be actually different; that the same entity is simply being examined from different conceptual frames of reference. However, the body of literature on modes of mental representation does not, at present, show how these different aspects of representation might be related. Consequently the instrumentation is assumed to be representative of the different research avenues being explored at the present time.

Three instruments are used in this study. These are Aylwin's (1985) Modes of Thought Questionnaire, Rancourt's (1988) Knowledge Accessing Modes Inventory and the updated version of the Griffitts Test of Mental Imaging (Richardson, 1969).

Modes of Thought Questionnaire

Aylwin's Modes Of Thought Questionnaire (MOTQ) is designed to determine individual preferences in the utilization of the three modes of mental representation. The questionnaire is concerned with how a subject responds to the associations between ideas. It examines the underlying assumption that thinking in different modes results in different associations.

A subject is directed to think in a particular way for words that provide a specific stimulus. For example, to encourage an individual to use the visual form of mental representation, instructions are given to the subject to see a picture in the mind's eye after being presented with a stimulus word, and then to write down the first association to the image which comes to mind. The subject is asked to use the specified mode of representation, even if it seems unnatural or tiring, and to give the association as soon as it is possible after the specified form of representation is made.

Each response is then coded as to mode of mental representation utilized. For example, given the stimulus word "walrus", an indication of the enactive mode would be a phrase such as "eating fish", which indicates action, or "exhausted", which implies affect. An indication of the visual mode would be a phrase such as "grey with whiskers", indicating an attribute of "walrus", or "on the beach", indicating environment. The verbal mode would be indicated by a phrase such as "walrus and the carpenter", a phrase completion which is a line from a popular song, or "a mammal related to seals", indicating the position of "walrus" in a category. Some associations, however, particularly synonyms, cannot be coded, since it is not possible to distinguish the mode of mental representation employed in making the association.

The data resulting from the MOTQ consist of numerical scores of the utilization of each of the three modes of mental representation, for each section of the questionnaire. There are data concerning the utilization of a mode when told to do so, and also the utilization of one mode when requested to use a different mode. There are 24 stimulus words in the MOTQ for each of the three modes of mental representation. A subject who is very adept at all three ways of mental representation can theoretically achieve a score of 24 on each of the three modes. In practice, this does not occur. Rather, one mode of mental representation tends to dominate to some extent, and scores on any one mode may vary, theoretically, from 0 to 72. The dominant mode of thought is identified by use of the questionnaire, as are also the degrees of use of the two minor modes of thought.

The MOTQ has been used previously to determine the preferred hierarchies of modes of mental representation of Australian university students in the fields of engineering, commerce, and the arts. In all, more than 2,000 students participated in studies using the MOTQ. The results have been found to be reliable. The findings correlate well with research from Witkin, Ashton, J. Richardson, and others (Aylwin, 1985; Witkin, Cox, Goodenough, and Moore, 1977). The test-retest reliability ranges from .69 to .84 on the three aspects of the instrument, for a seven week period, while the Cronbach alpha measures range from .83 to .89 on the three aspects of the instrument. Construct validity was tested using principal axis factor analysis on the subscales. The highest loading subscales on each factor are, indeed, those appropriate to the relevant mode of representation.

There are actually two different versions of MOTQ questionnaires used in this study: the MOTQ-1 and the MOTQ-2. A small pilot was undertaken by the author in which both the MOTQ-1 and the MOTQ-2 were examined over a six-month period. Both tests provide a measure of the utilization of the three modes of mental representation. They differ in that the MOTQ-1, the original instrument, requires approximately one hour to complete, and consists of 180 value judgements to be made by the subjects, while the MOTQ-2 requires less than 20 minutes for the subjects to write down 72 word associations. In the MOTQ-1, the subjects are asked to judge the probability level which they would give to particular word associations. For example, they are asked how probable would be the association of "stumbling" with "embarrassment", or "apparatus" with "scientist".

Many of the phrases used in the MOTQ-1, however, are not judged to be suitable for North American audiences. For example, terms such as badgers, hedgehogs, green grocers, and traffic wardens are not common words in the vocabulary of North Americans. The MOTQ-2, on the other hand, allows a subject to make any desired association, and hence is more acceptable. Since the psychometric data concerning the MOTQ instrument were evaluated using the MOTQ-1(Aylwin, 1985), a small sub-sample of approximately 35 subjects completed this questionnaire in addition to the MOTQ-2. A delay of six months was allowed between the completion of the two instruments, since it was felt that there would be a risk of remembered responses.

The results provided by the two MOTQ questionnaires were compared. Pearson product-moment correlation coefficients were calculated between the scores on the three modes of mental representation of the two instruments.

The Pearson product-moment correlations between the two MOTQ questionnaires were .60 for the Verbal mode, .62 for the Visual mode, and .53 for the Enactive mode. The correlations between the two MOTQ questionnaires appear not to be as high as one might expect for two tests which purport to measure the same modes of mental representation. The range of correlations found, however, must be considered in the light of the following two factors. First, the vocabulary of the MOTQ-1 was often confusing to the subjects, since many terms were judged to be more familiar in Britain. Second, the mode of delivery is different, in that the MOTQ-1 provides pairs of associations, and requires that a subject estimate the relative likelihood of such associations coming to mind. The MOTQ-2, on the other hand, simply requires one to provide the association to the stimulus word, with this association later being evaluated as to type by the examiner.

The combination of these two factors meant that the majority of subjects, who were not of British backgrounds, found that some of the associative pairs in the MOTQ-1 were very unlikely to occur, although such associations might well occur to a subject from Britain. For some subjects, educated in countries in the Middle East or Eastern Europe with both different cultures and different languages, many of the MOTQ-1 associative pairs were meaningless. The MOTQ-2 was considerably more free from such cultural bias, since any association made by the subjects could be coded regardless of language used, providing that a possible coding existed. Hence the MOTQ-2 was selected for the purposes of this study.

Knowledge Accessing Modes Inventory

The Knowledge Accessing Modes Inventory (KAMI), developed by Rancourt (1988), is used to identify the ways of knowing of an individual. These ways of knowing are called knowledge accessing modes. They provide the set of internal rules which are used in the process of selectively attending to stimuli. KAMI identifies three ways of knowing by means of an inventory whereby a subject is presented with a set of twenty sentence beginnings, each with three possible endings. The task for the subject is to rank the three endings from most applicable to least applicable, for each of the sentences. Since each ending corresponds to a different knowledge accessing mode, the order of ranking will give an indication of the knowledge-accessing mode used by the subject for each sentence. Since every statement in every group has a rank, no ambiguities can arise in the coding of the responses on this instrument.

The data resulting from the KAMI test consist of a score on each of the three modes of knowledge accessing. The highest value is that of the dominant mode, providing that the score achieved in this mode is at least 4 greater than the scores achieved in each of the other two modes.

If the scores are closer than that, there is said to be joint dominance in the modes concerned. The second highest score is referred to as the secondary mode, and the lowest score is labeled the minor mode.

From an epistemological perspective, knowledge can arise from the senses, from reasoning, or from intuition (Rancourt, 1988). An empirical knowledge accessing mode is allied to both sensory, and hands-on stimuli, and hence is posited to be possibly related to Aylwin's enactive mode of mental representation. A rational knowledge accessing mode is allied to deductive reasoning, and, in turn, may be related to Aylwin's verbal mode of mental representation. The third knowledge accessing mode, known as noetic, is allied to intuition and metaphoric thinking, and it may be related to Aylwin's visual mode of mental representation.

The KAMI has been used extensively in Africa, Finland, Holland, and Canada. Considerable evidence has been gathered which provides a high degree of congruence between particular styles of knowing and various teaching specializations, as well as learning preferences. More than 17,000 subjects have participated in the validation of the KAMI. The validity of the instrument has been examined in several studies (Niday, 1987; Rancourt, 1986). In the Niday study, for example, a high correlation was found between the KOLB Learning Style Indicator (1976) converger dimension and the KAMI rational scale, and between the assimilator dimension and the KAMI empirical scale. Further, correlations between Royce's Psych-Epistemological Profile and the KAMI have been found to be significant at the 0.001 level (Rancourt, 1986). Reliability was measured in two ways by Noble and Rancourt (1987). The test-retest reliability was found to vary between .71 and .87 for a three-month period, while the split-half reliability coefficients varied from .78 to .82 for the three scales.

Griffitts Test of Imagery Dominance

The Griffitts Test of Imagery Dominance, developed by Griffitts in 1927, and modified by Diehl and England in 1958, is used to determine which of the senses, (visual, auditory, or kinaesthetic), dominates in mental images. The test consists of a set of 20 stimulus words, each of which could be experienced in different concrete ways. For example, given the stimulus word "bicycle", the individual might see the image of a bicycle, might hear the sound of the wheels, might feel the wind on the face, might imagine riding the bicycle, or experience a combination of images. The subject is directed to indicate which sense (or senses) is (are) used when the stimulus word is presented. The data resulting from this test consist of a set of three scores, one for each of the three modes of mental imaging. It is similar to the KAMI test, in that there are a dominant mode and two minor modes.

The Griffitts test is one of a set of tests which purports to examine the mental imagery of subjects, all of which go back to a common base, that of Galton (1883), but each of which has a different focus. Betts (see Richardson, 1969), for example, was interested in vividness of images in seven different modalities, and developed a test known as the *Questionnaire upon Mental Imagery*. The Sheehan version of this test (1967), also known as the Betts' QMI, is reprinted in Richardson's 1969 book. Gordon (1949), on the other hand, was interested in the ability of individuals to control their imagery, and developed an instrument known as the *Test of Visual Imagery*.

Several other researchers have examined the dominant modes of mental imaging, using subsets of Griffitts' large wordlist, and subsets of the seven sensory modalities (Richardson, 1977; Schmeidler, 1965; White, Sheehan, and Ashton, 1977). Others used very similar wordlists (Brower, 1947; Liebovitz, London, Cocper, and Tart, 1972; Lindauer, 1969). Even more researchers are currently using similar tests, in this era of renewed interest in mental imagery. For example, sports psychologists examine the relationship between mental imagery ability and performance (Boutcher & Rotella, 1987; Gal-Or, Tenenbaum, and Shimrony, 1986; Kohl, Roenker, and Turner, 1985). Psychologists and teachers of normal as well as learning-disabled children, are also looking for connections between imaging and ability to learn (Cotrell & Weaver, 1985; Langham-Johnson, 1984).

When White, Sheehan and Ashton (1977) surveyed major self-report measures of different aspects of mental imagery, they found that there were very few tests which could really be said to differ. These tests could be broken down into three main categories; that of imagery type, that of vividness, and that of control. The conclusions made in the survey were that the evidence concerning these tests demonstrates reliability and predictive efficiency for a variety of self-report tests of individual differences in imagery.

There are few modern instruments which examine mental imagery, and those which do exist, are not suitable for use in this study. For example, the tests used for young children are not applicable to the adult population. The sports studies are likewise not applicable, since the research in this area is very narrowly focused on athletic performance. Hence the search for an instrument on the modes of mental imaging was directed to the derivatives of Griffitts. Of these, the Diehl and England version (1958) used exactly three modalities, namely visual, auditory, and kinaesthetic, and a manageable list of twenty words. Furthermore, the population tested by Diehl and England consisted of university students with different subject majors.

The White, Sheehan, and Ashton (1977) survey of instruments testing mental imagery, found "a surprising degree of reliability for self-report tests of imagery both in terms of the internal consistency of the scales and their test-retest stability" (p. 162). The Diehl and England version of the Griffitts test is one of these. The test-retest stability was found to vary over the three modes from .73 to .89 (Diehl and England, 1958), while the reliability of the instrument was approved by Richardson (1969) as well.

Certain researchers (Richardson, 1969; Roe, 1951) suggest that the hierarchy of the sense modalities that is used in mental images is related to dominant modes of mental representation and also to academic specialization in university. Some teachers in the field also believe that students can be visual, auditory, or kinaesthetic learners in the sense that Griffitts understood, and that achievement in mathematics might be related to such types. Hence this inventory has been selected as a complement to the description of mental representation. There are, of course, problems with this type of instrument, in that the validity of such an instrument cannot be verified. That is, there is really no way actually to know whether imagery itself is being measured, or whether some similar function is being examined.

Limitations of Instrumentation

All of these instruments are suspect to some degree, since there is no way to accurately discern what they actually measure. An attempt is being made to look at thought through a filter; what is reported may not be exactly what is occurring. The KAMI and the MOTQ are known to be reliable, in that re-tests provide the same results (Aylwin, 1985; Rancourt, 1986), but what those results actually are, cannot be evaluated. In their description of tests of mental imagery, furthermore, White, Sheehan and Ashton (1977) conclude that the tests of mental imaging are also reliable, but they also express some concern over exactly what the tests measure. As they state, one problem is "the absence of any guarantee that the stimulus items on the scales mean the same thing to every subject" (p. 163). This is also a problem with the MOTQ, in that some subject responses have to be discarded due to ambiguity; it is not always possible to tell which mode of mental representation is being used to form an association. Further, all of the instruments are questionnaires, and this format may hinder the use of modes of mental representation or of mental imaging which are not readily accessible through this medium.

In the MOTQ, for example, the subject is asked to write down an association which is supposed to occur visually, verbally, or enactively. It is possible that the requirement to write the association down may change the nature of the association made; there is no way to know whether this occurs, by the very nature of the instrument. Despite these limitations, however, the instrumentation selected for this study appears best to address the association under investigation: namely, the roles of mental representation, knowledge accessing, mental imaging, and the degree of experience in mathematics.

Sample Description

The subjects who participated in this study were taken from the population of pre-service education students registered in various 1990-1991 Bachelor of Education programs in the province of Ontario. Permission to conduct the study was received from the various universities concerned, in the Eastern and Mid-North regions of Ontario. Participation in the study was on a volunteer basis, and was invited according to the general guidelines set by a university ethics committee.

A total of 223 subjects took part in the study. The responses of eight subjects were discarded because they either did not understand the instructions, did not complete all of the four questionnaires, or did not identify themselves on all of the questionnaires. The final sample consisted of 215 subjects. Of that total, eighty-six were planning to teach younger students in grade six or lower, while 129 were planning to teach older students in grade seven or higher. The gender breakdown of these sub-samples is provided in Table 2, as is the breakdown by mathematics experience, age, subject major at university, and experience in mathematics. The numbers in this table reflect the fact that four of the 215 subjects did not indicate gender during the testing session.

Table 2: Description of Subjects n=215

Breakdown by Gender and Program of Study

Program of study	Male N	Male %	Female N	Female %	Total N	Total %
Elementary (grades 1 to 6)	15	7.1	70	33.2	85	40.3
Secondary (grades 7 up)	56	26.5	70	33.2	126	59.7
Total [a]	71	33.6	140	66.4	211	100.0

Breakdown by Mathematics Experience and Age

Mathematics Experience	N	%	Age Range	Average Age	Median Age
None	60	27.9	22 - 54	29.2	27
Low	51	23.7	22 - 44	28.8	26
Moderate	53	24.7	21 - 50	27.2	25
High	51	23.7	20 - 54	27.8	24
Total	215	100.0			

Breakdown by University Major and Mathematics Experience

University Subject Major	N	%	Average Number of Courses taken Containing Mathematics: Math Only	Math+Related
Languages	26	12.5	0.10	0.9
Humanities	50	24.0	0.72	6.1
Physical Education	23	11.1	2.80	13.9
Life Science	39	18.7	4.17	18.4
Science	28	13.5	4.74	20.8
Mathematics	42	20.2	9.40	26.7
Total [b]	208		Average 3.80	14.8

[a] - 4 subjects did not indicate gender
[b] - 7 subjects did not have any of these majors of study

The pre-service education students completed four questionnaires in approximately one hour. Demographic evidence was obtained regarding gender, age, number of mathematics and other courses studied at a post-secondary level, as well as program of study in teacher education. Personal opinions concerning courses in different subject areas, studied at the post-secondary level were also requested on this questionnaire. The numbers of courses studied at university were adjusted in the coding process, to refer uniformly to semester courses rather than the original mix of full-year and semester courses. The other three questionnaires were the MOTQ, the KAMI, and the Griffitts, designed to identify the various modes of mental representation in the sample. A description of the MOTQ-2 (that used in this study) appears in Appendix B-1, followed by a description of the MOTQ-1 in Appendix B-2. A description of the KAMI appears in Appendix C, and the complete Griffitts test appears in Appendix D.

The ages of subjects ranged from 20 to 54, with a mean age of 28.3 years and a median age of 26. Approximately one third of the subjects came directly into pre-service education studies after graduation from university training (37.1% were under 25 years of age), while approximately one in every six subjects (16.7%) were more than 34 years of age, and hence had a considerable gap between university training and entrance into the faculty of education.

Various samples of subjects were formed from the data. The main set of sub-samples was formed on the basis of different levels of experience in mathematics. This set was used to test the hypotheses. Other sub-samples were also formed which were based on gender, age, and programs of study, both those followed during post-secondary education of subjects, and those followed during the year of teacher education. The sub-samples formed according to programs of study during post-secondary education, were used in a more specific analysis than that possible with the main sample. This analysis provides additional support for the findings from the main sample with respect to the hypotheses. The other sub-samples were not used in testing hypotheses, but rather were used to highlight the individual differences present among the sub-samples.

The criteria for the placement of subjects in the sample of "degree of mathematics experience" were as follows. The first sub-sample, the "no experience level in mathematics" sub-sample, consists of subjects who have no mathematics experience of any type at the post-secondary level. They have not studied mathematics, nor have they studied any related disciplines such as science, nursing, engineering, or kinanthropology which use mathematics in their program of study. The fourth sub-sample, the "high mathematics experience" sub-sample, consists of subjects who have taken at least 6 courses in mathematics at the post-secondary level. These two sub-samples comprise approximately half of the total. The remaining subjects in the sample are differentiated into two sub-samples by means of their mathematics experience, both in the study of mathematics, and in the study of courses which contain considerable mathematics respectively. Such courses are taken into consideration in the sub-sample composition, so as to provide a means by which the two middle sub-samples, the "low experience level" and "moderate experience level" mathematics experience sub-samples, could be determined. The university courses which are considered to involve mathematics include science and kinanthropology, among others. The breakdown of the sample with regard to mathematics experience is shown in Table 2. In this table, the four sub-samples are seen to be approximately equal in size, with percentages of subjects ranging from 23.7% to 27.9%.

The sub-samples formed with respect to mathematics experience are also examined with respect to age groupings. As can be seen in Table 2, each of the four sub-samples have a similar wide range of ages of subjects, with similar average ages and similar median ages. All sub-samples contain subjects who have proceeded to the program directly from post-secondary education. Furthermore, all sub-samples also contain subjects who have gained considerable experience in the workplace before entering the education program. The four sub-samples are thus shown to be similar in this respect.

The second method of grouping by mathematics experience involves the sub-samples formed according to university majors. These provide a different way of examining the effect of mathematics experience, since different university majors involve different numbers of courses in mathematics and mathematics-related courses, as illustrated in Table 2. A total of one subject in ten of the group who has studied languages, for example, has studied one course in the mathematics department, leading to an average of 0.1 course for the group. Nine out of ten of these individuals, then, have taken no courses at the post-secondary level which contain any mathematics at all, including science, nursing, and kinanthropology.

Whether one lists only the actual mathematics courses taken, or whether one includes the courses such as physics or geological science which involve the use of some mathematics, the programs of study sort into the same order, with the language majors having the least experience in mathematics, and the mathematics majors, the most experience in mathematics. In the third portion of Table 2, the total number of subjects is 208, since the remaining seven subjects, mainly music majors, are too few in number to form a viable group.

Statistical Methods Used in Data Analysis

Various statistical techniques are utilized in the analysis of data in this study. In the examination of categorized data such as that derived from frequencies of subjects with particular dominant modes of mental representation, of mental imaging, and of knowledge accessing, the nonparametric procedure of Chi-Square is used. Contingency tables are formed for the sub-samples formed by "levels of experience" in mathematics, by gender, and by program of study, for the dominant modes of mental representation, as measured by the MOTQ, for the dominant modes of knowledge accessing, as measured by the KAMI test, and for the dominant modes of mental imaging, as measured by the Griffitts test.

In a further analysis of individual scores achieved in the subscales of the different instruments, the scores are treated as continuous data, using the parametric ANOVA procedure. The ANOVA procedure is used primarily to test for significant differences in utilization of the three modes of mental representation as measured by the MOTQ, in the gender groups, the groupings by university major, and the mathematics experience groups. Wherever the null hypothesis is rejected, post hoc analysis is carried out by means of Scheffé's procedure, a robust method of analysis for unequal sample sizes such as those which are present here.

This chapter has discussed the sampling procedures, the instrumentation, and the statistical tools used in this study. In particular, the sample population of pre-service education students, and the instrumentation of the MOTQ, the KAMI, and the Griffitts test were described. The data collection and data analysis procedures used to test the three hypotheses that are the focus of this research, were also described in this chapter.

CHAPTER IV : RESULTS AND DISCUSSION

In this chapter, the results of the examination of data arising from the study are presented. The findings are organized around the three hypotheses, which in turn are related to the three instruments. The first hypothesis involves the Modes of Thought Questionnaire. In the analysis of data from this questionnaire, the overall use of the three modes of mental representation is examined, but the degree of compliance to the instructions to utilize a particular mode in certain portions of the test, is also investigated. The level of experience in mathematics is examined in terms of mathematics as a subject, and also in terms of mathematics studied in the content of other subjects, in all three hypotheses. All three hypotheses are tested using the null hypothesis form, and are accepted or rejected accordingly.

An exploratory analysis of the data collected in the study is completed and discussed in this chapter, providing additional support for the major hypothesis. Further analysis, not related to the hypotheses, but of interest to researchers in this field, is carried out. The relationships among the three instruments are also explored in this chapter.

Hypothesis I

The first hypothesis is

> "there is an association between levels of experience in mathematics and the degree of effective utilization for each of the three modes of mental representation, as measured by Aylwin's (1985) Modes of Thought Questionnaire".

With regards to the Modes of Thought Questionnaire (MOTQ), there are two aspects to consider in the examination of the "degree of effective utilization" of the modes of mental representation. In the MOTQ, the subjects are directed to utilize the visual mode for one-third of the test, the verbal mode for one-third of the test, and the enactive mode for the remaining third. If a subject is able to utilize a particular mode when directed to do so, then the subject should utilize the visual mode on the visual portion of the test, and so on. For this reason, the first aspect of effective utilization is the degree of compliance to the direction to use a particular mode, or the degree of individual control over modes of mental representation. The second aspect of effective utilization to consider is the personal preference of use of each mode of mental representation by subjects, regardless of direction.

The actual ability on the part of a subject to utilize a particular mode of mental representation when directed to do so, is not consistently demonstrated in the data from the Modes of Thought Questionnaire (MOTQ). In Table 3, the ability of the subjects to do this is recorded. If the subject uses each of the three modes as directed, then the correct dominance is said to occur; otherwise, the subject is said to have incorrect dominance. The threshold of correct dominance for each mode is set at 80% use of the directed mode. This was done because the percentage of subjects using the appropriate mode remains approximately the same (between 80% and 95%), while few subjects have greater accuracy than that.

Table 3: Utilization of Modes of Mental Representation

	Mathematics Experience Groups							
Degree of Compliance to Direction to Use Each Mode	1 NONE		2 LOW		3 MODERATE		4 HIGH	
	N	%	N	%	N	%	N	%
Uses directed mode	23	38.3	27	53.0	33	62.2	40	78.4
Does not use directed mode	37	61.7	24	47.0	20	37.8	11	21.6
Totals	60	100.0	51	100.0	53	100.0	51	100.0

In Table 3, the degree of direction compliance to utilize a particular mode of mental representation is presented in relation to the "levels of experience" in mathematics of the subjects. The results of testing for appropriate use of the three modes of mental representation are shown to be mixed, in that many subjects do not use the particular mode that they are directed to use. In this table, one can note that the percentage of subjects utilizing all three modes, as directed to do so, increases as the amount of experience in mathematics increases. The percentages of subjects utilizing these modes as directed, range in a steady progression from 38.3% for the sub-sample with no mathematics experience, to 78.4% for the sub-sample with considerable mathematics experience.

The findings displayed in Table 3 are examined by means of a four by two contingency table, comparing mathematics experience to correct and incorrect mode utilization. At the 0.01 level of significance, an association is found to exist between levels of mathematics experience and the appropriate utilization of the modes of mental representation (Chi-square = 15.73, df = 3). The difference is found in the two extreme sub-samples, the "no experience level" of mathematics, and the "high experience level" of mathematics. These sub-samples are different with respect to the appropriate utilization of the three modes of mental representation. In other words, fewer subjects in the sub-sample with no experience in mathematics (38.3%) use all three modes as directed, while significantly more subjects (78.4%) in the sub-sample with a high level of experience in mathematics are able to utilize the modes as directed.

The utilization of each of the three modes of mental representation is examined more closely, to see which specific modes are used by the subjects in the four mathematics sub-samples, when a particular mode is requested. The results of direction to use each separate mode of mental representation are provided in more detail in Table 4. In this table, as in Table 3, the threshold is set at the 80% / 20% level for the responses of each subject. That is, a subject is said to use the verbal mode (as directed) when such use by the individual exceeds 80% of responses, whereas a subject who uses an incorrect mode 20% or more of the time, is listed under this incorrect mode. Each subject is listed in only one category.

As noted in Table 4, there appears to be considerable confusion with the visual mode of mental representation. Specifically, many subjects use the visual mode when directed to use the verbal mode, and the

Table 4: Degree of Compliance to Direction to use Modes of Mental Representation

Modes Used When Directed to Use **VERBAL** Mode	Mathematics Experience: NONE N	%	LOW N	%	MODERATE N	%	HIGH N	%
Verbal	23	38.3	31	56.8	36	67.9	34	66.7
Visual	33	55.0	20	39.2	15	28.3	12	23.5
Enactive	0	0.0	0	0.0	0	0.0	0	0.0
Mix of Vis/Enac	4	6.7	0	0.0	2	3.8	5	9.8
Totals	60	100.0	51	100.0	53	100.0	51	100.0

Modes Used When Directed to Use **VISUAL** Mode	Mathematics Experience: NONE N	%	LOW N	%	MODERATE N	%	HIGH N	%
Visual	23	38.3	13	24.1	11	19.7	16	32.1
Enactive	22	36.7	20	40.7	20	35.7	22	41.5
Verbal	5	8.3	7	13.0	6	10.7	4	7.5
Mix of Enac/Verb	10	16.7	11	22.2	16	33.9	9	18.9
Totals	60	100.0	51	100.0	53	100.0	51	100.0

Modes Used When Directed to Use **ENACTIVE** Mode	Mathematics Experience: NONE N	%	LOW N	%	MODERATE N	%	HIGH N	%
Enactive	14	23.3	12	23.5	12	22.6	13	25.5
Visual	45	75.0	31	60.8	33	62.3	34	66.7
Verbal	0	0.0	0	0.0	0	0.0	0	0.0
Mix of Vis/Verb	1	1.7	8	15.7	8	15.1	4	7.8
Totals	60	100.0	51	100.0	53	100.0	51	100.0

majority of subjects use this mode when directed to use the enactive mode of mental representation. When directed, however, to use the visual mode itself, the majority of subjects use a different mode, usually the enactive. In many of these cases of inappropriate mode use, the subject follows direction for the first few items, but then switches to a different mode.

The correct utilization of the verbal mode ranges within the four mathematics experience sub-samples, from 38.3% for the sub-sample with no mathematics experience, to 67.9% of responses for the sub-sample with moderate experience in mathematics. The incorrect use of the visual mode in place of the verbal mode ranges from 23.5% for the "high experience level" mathematics sub-sample, to 55% for the sub-sample with no mathematics experience. None of the subjects in any of the sub-samples uses the enactive mode of mental representation in place of the verbal.

The visual-verbal situation is examined by means of a four by two contingency table containing the numbers in each of the four sub-samples using the verbal and visual modes of mental representation. At the 0.01 level of significance, an association is found to exist between "level of mathematics experience" and the appropriate use of the verbal mode of mental representation (Chi-square = 14.51, df=3). The sub-sample of subjects with "no mathematics experience" is found to differ from the other three subsamples, in that the subjects in this sub-sample seem less able to utilize the verbal mode on direction, than the subjects in the sub-samples with at least some experience in mathematics. These latter subjects, furthermore, use the visual mode more often in error, than do subjects with a higher level of experience in mathematics.

When the ability of the subjects to utilize the visual mode of mental representation is examined, it is found that the use of the visual mode on direction ranges widely in the four sub-samples formed with respect to experience in mathematics. The data reveal that the visual mode is used less often, despite directions to do so, than is the verbal mode, in three of the four sub-samples. The majority of subjects who do not consistently use the visual mode, most often use the enactive mode. The percentages of subjects who use the visual mode correctly range among the sub-samples from 19.7% to 38.3%, while the percentages of subjects who incorrectly use the enactive mode at least 20% of the time, range from 35.7% to 41.5%.

There are some subjects who do confuse the visual mode and the verbal mode. The percentages for those subjects who use the verbal mode in error range from 7.5% to 13% among the four sub-samples formed with respect to mathematics experience. The numbers involved, however, are judged to be small. Furthermore, from 16.7% to 33.9% of the subjects in each of the four "levels of mathematics experience" sub-samples, use a combination of the verbal mode and enactive mode, when directed to utilize the visual mode of mental representation.

When the visual-enactive situation is examined by means of contingency tables, no significant differences are found among the different sub-samples formed with respect to "levels of experience" in mathematics. The ability to discriminate between the visual and the enactive modes appears to be difficult for many subjects regardless of their experience in mathematics; the correct (visual) mode is used less than 40% of the time by any sub-samples, despite direction to do so.

The visual and enactive mode confusion is pointed out even more clearly when the subjects are directed to use the enactive mode. The majority of subjects in each "level of mathematics experience" sub-sample, from 60.8%

to 75.0%, use the visual mode 20% or more of the time when directed to use the enactive mode, while only between 22.6% and 25.5% of each sub-samples can correctly use the enactive mode with consistency. Not one subject, however, uses the verbal mode as much as 20% of the time in this situation, thus showing that one does not consistently confuse the verbal and enactive modes of mental representation, no matter which of the two is being requested. When the enactive mode situation is examined by means of contingency tables, no significant differences are found.

The second aspect of "effective utilization" of the modes of mental representation from hypothesis I, namely that of overall utilizations of each mode of mental representation, is examined for the four sub-samples of subjects formed with respect to "levels of experience" in mathematics. In other words, the number of times that the verbal mode is actually used is recorded without regard to the requested mode. The average raw score of the majority of subjects is highest in the visual mode, as expected from the history of the MOTQ, since in Aylwin's (1985) work, students in the social sciences, and women in general, have been found more likely to have visual dominance. In this study, the sample contains many subjects who fall into these two categories.

The degree of utilization of the three modes of mental representation for each subject is therefore calculated by comparison with the average score on each particular scale, in order to accentuate the differences among the sub-samples of subjects. The dominant mode, then, is defined as the mode in which the score of the subject is farthest away from the mean score: in other words, it is the mode used most frequently in comparison to the mode used by the average subject. It should be noted, however, that this method of calculation for the dominant mode provides the same statistical results as does the use of raw scores, but the information is presented in a more accessible form.

The numbers and frequencies of subjects in each of the sub-samples formed with respect to levels of mathematics experience, who have each of the dominant modes of mental representation, are presented in Table 5. In this table, verbal dominance is observed to range from 11.1% in the "no experience level" sub-sample, to 44.4% in the "moderate experience level" sub-sample. Visual dominance ranges from 15% in the "low experience level" sub-sample to 62.2% in the "no experience level" sub-sample. Finally, enactive dominance ranges from 26.7% in the "no experience level" sub-sample, to 44.4% in the sub-sample with the "high experience level" in mathematics. The sub-sample with the "no experience level" in mathematics appears to stand apart from all other sub-samples. The difference does not appear to be related to the level of mathematics experience, or one would expect a steady progression in scores. This does not occur.

Table 5: <u>Mental Representation and Mathematics Experience</u>

Mathematics Experience Level	Dominant Mode of Mental Representation: VERBAL N	%	VISUAL N	%	ENACTIVE N	%	TOTAL N	%
None	5	11.1	28	62.2	12	26.7	45	100
Low	17	42.5	6	15.0	17	42.5	40	100
Moderate	20	44.4	7	15.6	18	40.0	45	100
High	12	33.3	8	22.2	16	44.4	36	100
Total	54	32.5	49	29.5	63	38.0	166	100

The results from Table 5 are examined by means of a four by three contingency table. At the 0.01 level of significance, an association is found to exist between "level of experience" in mathematics and dominant mode of mental representation (Chi-square = 34.5, df = 6). More specifically, the sub-sample of subjects with the "no experience level" in mathematics is found to differ from the other three sub-samples in terms of use of the verbal and visual modes of mental representation. No significant difference is found among the sub-samples regarding the enactive mode of mental representation.

Discussion of Findings for Hypothesis I

The first research hypothesis, that there is an association between levels of experience in mathematics and the utilization of different modes of mental representation, is confirmed. Specifically, a lack of experience in mathematics appears to be related to an inability to utilize correctly all three modes of mental representation when directed to do so (Table 3). Furthermore, a high level of experience in mathematics is related to a relatively high degree of ability to utilize all three modes of mental representation when directed to do so. The sub-sample of subjects with no mathematics experience is found, moreover, to have a lower utilization of the verbal mode of mental representation, and a higher use of the visual mode, than do the other sub-samples. In summary, then, the first research hypothesis is confirmed. There is a difference in degree of effective utilization of at least the verbal mode of the three modes of mental representation, as measured by the MOTQ, which is attributable to experience in mathematics.

Although the sub-sample of "no mathematics experience level" has significantly different scores from the subjects in all other sub-samples, the three "mathematics experience level" sub-samples which have some mathematics experience, whether at a low, moderate, or high level, do not have significantly different scores in any of the three modes of mental representation. Therefore any possible relationship between different degrees of experience in mathematics and the ability to use a particular mode of mental representation is not pointed out.

Even the subjects in the sub-sample with low experience in mathematics, however, have up to 15 post-secondary courses in subject majors which require some expertise in mathematics, such as physics, computer science, and kinanthropology. The subjects in the sub-sample of "moderate experience level" in mathematics have, moreover, taken from 16 to 34 courses in subjects which require a considerable degree of mathematics experience. The subjects in the sub-sample of "high experience level" in mathematics have accumulated a minimum of six mathematics courses, and have experience also in areas of study allied to mathematics.

It is therefore possible that the mathematics studied in a mathematics course, and mathematics studied within a physics, or other non-mathematics course, both provide similar practice in the uses of the different modes of mental representation. For example, the understanding required to calculate the trajectory of a javelin in a kinanthropology course may not differ from the understanding required to calculate velocity of a projectile in a calculus course. Only the frame of reference is changed; the problem is really the same.

The MOTQ verbal score does not appear to differentiate between mathematics expertise derived from courses in the actual discipline, and that derived from other courses. It is arguable, then, that mathematics experience as defined as being obtained solely in mathematics courses does not differ from mathematics experience in related courses. This was not examined by Aylwin, who was concerned only with comparisons among broad concentrations of study, such as arts, commerce, social science and engineering.

The other Aylwin measures, namely the visual and empirical, have a similar result, in that the sub-sample with no experience in mathematics stands out as being different from the other three sub-samples, but that these other three sub-samples are not differentiated by the measures. There may indeed be a relationship between "levels of experience" in mathematics and ability to utilize different modes of mental representation, but this relationship does not appear to be progressive in nature, since the sub-sample of "no experience level" in mathematics is different from the other three sub-samples, but no apparent differences can be noted among the other sub-samples formed with respect to mathematics experience. This relationship is examined in greater depth in the "exploratory analysis" section of this chapter.

Hypothesis II

The second research hypothesis is

> there is an association between levels of experience in mathematics and the degree of preference of each of the three modes of knowledge acquisition, as measured by the Knowledge Acquisition Modes Inventory of Rancourt (1988).

In order to examine this hypothesis, a sub-sample was formed of those subjects who have a dominant knowledge accessing mode. Those subjects with similar scores on different modes are said to be able to flex, that is, they appear able to comply to the mode called for. No one mode is dominant over the other. Table 6 includes only those subjects having a clear dominance in one of the three knowledge accessing modes. The criterion used in this study to determine the threshold for dominance is defined to be a score difference of at least six between the highest score and the second highest score of mode utilization. The criterion of six was selected because percentages with each dominant mode did not change for differences greater than six.

Table 6 considers the four sub-samples that reflect levels of experience in mathematics, and provides the numbers and percentages of subjects in each sub-sample who have as dominant mode, each one, respectively, of the particular modes of knowledge acquisition. The percentages of subjects who demonstrate a dominant noetic mode show a steady progression from 36.8% of the sub-samples with a moderate or high level of mathematics experience to 66.6% of the sub-sample with no experience in mathematics. Those who show a dominant empirical mode represent only 16.7% of the sub-sample with no mathematics experience, as compared to 34% or more in all of the other sub-samples. The percentages of those who show a dominant rational mode are generally low; but rational dominance is less in the sub-samples with no experience and low experience in mathematics, than in either the "high level of experience" or the "moderate level of experience" in mathematics sub-samples. The percentages of subjects in these four sub-samples who have a dominant rational mode vary from 10.3% to 29.0%.

Table 6 : Knowledge Accessing Modes and Mathematics Experience

Mathematics Experience	Noetic N	Noetic %dom	Empirical N	Empirical %dom	Rational N	Rational %dom	Total N	Total %
	Dominant Knowledge Accessing Mode							
None	28	66.6	7	16.7	7	16.7	42	26.8
Low	21	53.8	14	35.9	4	10.3	39	24.8
Moderate	14	36.8	16	42.1	8	21.1	38	24.2
High	14	36.8	13	34.2	11	29.0	38	24.2
Total	77		50		30		157	100.0

The data presented in Table 6 are examined by means of a three by four contingency table. At the 0.05 level of significance, an association is found to exist between "level of experience" in mathematics and the frequency of the dominance of the three knowledge accessing modes (Chi-square = 13.44, df = 6). The sub-sample with no experience in mathematics is found to differ from the other three sub-samples in both noetic dominance and empirical dominance, having more noetic and fewer empirical dominant subjects than do the other sub-samples.

A further difference exists in the sub-sample with a high level of experience in mathematics, in that this sub-sample has proportionally more rational dominant subjects than do the other three sub-samples.

Discussion of Findings For Hypothesis II

The second research hypothesis, suggesting an association between levels of experience in mathematics and the utilization of different knowledge accessing modes, is confirmed. Two thirds of the subjects with no mathematics experience have the noetic mode as dominant mode, a percentage which is significantly higher (at the 0.05 level) than that of any of the other three mathematics experience sub-samples. The second significant finding is in the frequency of rational mode dominance among the subjects with high levels of experience in mathematics. In this sub-sample, 29% of the individuals have rational mode dominance. This percentage is not as high as the 56% rational dominance found by Rancourt and Deschênes (1990) for French-speaking students in Ontario whose favourite school subject was mathematics, or the 40.7% rational dominance found by Rancourt (1991) for teachers of high school mathematics.

This finding is unexpected; it was assumed that the "high experience level" mathematics sub-sample would be similar in composition to the sample of mathematics teachers studied by Rancourt (1991), since the pre-service education students who plan to teach mathematics form a part of this sub-sample. These students, however, do not form the entire sub-sample, so it is possible that the presence of elementary school teaching candidates, and of future teachers who do not plan to teach mathematics, has contributed to the difference in the result. This possibility is supported by an examination of the date concerning the individual subjects in this sub-sample.

Nine subjects, for example, were found in this "high experience level" mathematics sub-sample who plan to teach elementary school; none of them has rational dominance. Hence some of the difference between the numbers in the two sub-samples with rational dominance may result from a basic difference in sub-sample makeup. There is no way to determine, at present, whether all of the difference in preferred knowledge accessing mode between the pre-service education students, and the practising teachers, occurs because of a basic difference in the sub-samples, or whether such differences may result from changes over time caused by the teaching experience itself. It might well be that neither of the two samples, that of the current study and that of the practising teachers, is representative of mathematics specialists.

It is also possible that the students in a pre-service education program are, indeed, different in profile from both teachers of secondary school mathematics and students who have mathematics as their favourite subject. This difference might well be the result of either self-selection or of the process of candidate selection for the teacher education program. One requirement for entrance to teacher education today is that the candidate have considerable experience with children. Only a small proportion of those who apply to the teacher-education program today are accepted today, while in the past, it was considered normal for that all applicants to the program were accepted into it. It is possible, therefore, that the students in the teacher education program today are a rather select subset of those students who have completed a university mathematics program. This should be examined further, to answer the question of whether the new mathematics teachers entering the field today are representative of the population of mathematicians. If they are different, the further question arises of whether this difference is a positive or negative factor in the teaching of mathematics.

Hypothesis III

The third research hypothesis is:

> there is an association between levels of experience in mathematics and the degree of preference for each of the three modes of mental imaging, as measured by the revised Griffitts Test of Mental Imaging (Richardson, 1969).

In order to examine this hypothesis, a sub-sample was formed of those subjects who have a preferred mode of mental imaging. Similar to the knowledge accessing inventory, a difference of six points between the highest score and the second highest score on the Griffitts test of Imagery Dominance is defined to be the threshold for dominance.

Table 7 outlines the composition of the four sub-samples with respect to experience in mathematics, and provides the frequencies and percentages of subjects in each sub-sample with each of the modes of mental imaging dominant. From an examination of the table, the results in general appear consistent from sub-sample to sub-sample. The only apparent difference is to be found in the kinaesthetic mode of mental imaging.

Table 7: Modes of Mental Imaging and Mathematics Experience

Mathematics Experience	Auditory N	Auditory %dom	Visual N	Visual %dom	Kinaesthetic N	Kinaesthetic %dom	Total N	Total %
None	18	32.7	25	45.5	12	21.8	55	28.2
Low	16	37.2	22	51.2	5	11.6	43	22.0
Moderate	18	31.6	27	51.9	7	13.5	52	26.7
High	16	35.6	23	51.1	6	13.3	45	23.1
Total	68		97		30		195	100.0

Dominant Mode of Mental Imaging (column heading spanning Auditory, Visual, Kinaesthetic)

The findings displayed in the table are analyzed by means of contingency tables, comparing the sub-sample with no mathematics experience to the other sub-samples. No significant differences are found.

Discussion of Findings Concerning Hypothesis III

The third research hypothesis, that there is an association between levels of experience in mathematics and the utilization of different modes of mental imaging, is not confirmed. The wide ranges of scores found in each mode of mental imaging, for each sub-sample formed with respect to experience in mathematics, together with the similar frequency of subjects in each sub-sample with each of the dominant modes of mental imaging, leads to the conclusion that, whatever the form of mental imaging, the level of experience in mathematics is not a factor.

Exploratory Analysis

The results described above, concerning the first hypothesis and the MOTQ, are those resulting directly from the research. There are, however, other findings of interest to this hypothesis, which come to light from the data generated from the MOTQ during the study. The MOTQ is not designed solely to identify the dominant mode of an individual. Rather, as Aylwin states, it "assumes that each person uses all the forms of representation, with the balance varying between individuals" (1985, p. 67). Hence, in the examination of the utilization of the modes of mental representation, the scores achieved on each of the subscales of the MOTQ were examined, rather than just the dominances. This examination may be considered to be exploratory, but the overall utilization of all of the modes of mental representation has been shown by Aylwin (1985) to be of relevance in highlighting differences in populations of university students in different faculties.

From an examination of scores achieved on the subscales of the MOTQ, this study reveals data which appear to illuminate major differences between subjects who have prior experience in mathematics, and those who do not. Table 8 provides the mean scores and their standard deviations, on each of the three modes, for each of the mathematics experience sub-samples. The average scores of the sub-sample with no experience in mathematics are lower on the verbal and enactive scales, and higher on the visual scale, than are the scores of the other three sub-samples. Table 8 indicates that there does not appear to be a progression in scores as experience in mathematics increases; the scores achieved on the three scales by the three sub-samples with at least some mathematics experience, are very similar.

Table 8: Means and Standard Deviations of the Mathematics Sub-samples on the MOTQ

		Mode of Mental Representation					
Mathematics		Verbal		Visual		Enactive	
Experience	N	Mean	SD	Mean	SD	Mean	SD
None	60	17.87	5.45	22.67	4.30	18.07	6.68
Low	51	21.30	5.05	18.71	3.30	20.41	7.28
Moderate	53	21.28	5.73	19.00	4.15	20.61	5.17
High	51	20.07	4.22	19.30	3.32	21.24	4.80

The three modes shown in Table 8 are examined separately for the four sub-samples of subjects organized with respect to mathematics experience. It appears that the sub-sample of subjects who have not studied any mathematics, either in mathematics courses, or indirectly, as part of other courses, portray a different profile than do the subjects in the sub-samples "low experience level", "moderate experience level", and "high experience level" in mathematics. The scores achieved in the "none level of experience" mathematics sub-sample appear lower on the verbal mode and enactive mode, and higher on the visual mode, than do the scores of the other three sub-samples.

The scores achieved on each of the three subscales, for each of the four mathematics experience sub-samples, are analyzed using a series of one-way ANOVAs, shown in Table 9. The one-way ANOVA of the verbal mode scores demonstrates that the means achieved by subjects among these four groups, formed with respect to levels of mathematics experience, are distinct at the 0.05 level of significance, and thus that the sub-samples are not all taken from the same population.

Table 9: ANOVAs of Scores on the MOTQ by Mathematics Sub-samples

ANOVA of MOTQ Verbal Scores

SOURCE	DF	SS	MS	F	P
Math Group	3	414.8	138.3	4.88	0.003
ERROR	211	5973.4	28.3		
TOTAL	214	6388.2			

```
Mathematics      95% Confidence Intervals for Group Means
Experience       --------+---------+---------+---------+-
   None           (-----*------)
   Low                             (------*-------)
   Moderate                       (------*------)
   High                      (------*------)
                 --------+---------+---------+---------+-
                        18.0      20.0      22.0
```

ANOVA of MOTQ Visual Scores

SOURCE	DF	SS	MS	F	P
Math Group	3	592.2	197.4	12.78	0.000
ERROR	211	3258.2	15.4		
TOTAL	214	3850.4			

```
Mathematics      95% Confidence Intervals for Group Means
Experience       --------+---------+---------+---------+---
   None                                      (----*----)
   Low                 (-----*----)
   Moderate              (----*----)
   High                   (-----*----)
                 --------+---------+---------+---------+---
                        18.0      20.0      22.0      24.0
```

ANOVA of MOTQ Enactive Scores

SOURCE	DF	SS	MS	F	P
Math Group	3	319.1	106.4	2.71	0.046
ERROR	211	8281.9	39.3		
TOTAL	214	8601.0			

```
Mathematics      95% Confidence Intervals for Group Means
Experience       --------+---------+---------+---------+---
   None          (-------*-------)
   Low                      (--------*--------)
   Moderate                   (--------*-------)
   High                         (--------*-------)
                 --------+---------+---------+---------+---
                        18.0      20.0      22.0      24.0
```

Modes of Mental Representation vs Mathematics Experience

Post hoc analysis, using Scheffé's procedure, reveals that the sub-sample with no mathematics experience has significantly lower scores on the verbal mode of mental representation than do the sub-samples with low or moderate levels of experience in mathematics. The difference between this sub-sample of subjects with no mathematics experience and the sub-sample with a high level of mathematics experience, however, is not statistically significant. This is shown in the set of confidence intervals in Table 9, which provides a visual illustration of the differences among the sub-samples. It can be seen that the two middle sub-samples, that with a low level of experience and that with a moderate level of experience in mathematics, are very similar, and both are distinct from the "no level of experience" sub-sample.

This is interesting in that most of the mathematics experience obtained by subjects in these two sub-samples has occurred outside of the mathematics department. That is, the mathematics studied by these subjects has been to a large extent, applied mathematics, such as that in kinanthropology, science, or similar areas. The finding illustrated in Table 9 supports the suggestion made in the discussion of the first hypothesis, namely that the mathematics learned in mathematics courses studied in a mathematics faculty does not appear to differ from the mathematics learned in a course in a different department or faculty, in relation to the dominant mode of mental representation.

The visual mode scores are examined in the same way as the verbal mode scores. The mean of the sub-sample with no experience in mathematics is statistically different from the means attained by each of the other sub-samples. The average scores of the subjects in this sub-sample are higher than those of the other sub-samples. These data are illustrated in the confidence intervals of the sub-sample means, found in Table 9.

When the enactive mode scores are examined, however, it is found that the mean score of the sub-sample of subjects with no mathematics experience is lower than the scores obtained by the individuals in the other three sub-samples, but that these differences are not found to be significant statistically. The confidence intervals for the mean scores on this mode are provided in Table 9. As shown in Table 8, the standard deviations for these means are quite large in comparison with the standard deviations of the means for the other modes. For example, the largest standard deviation for the visual mode is 4.30, while the largest standard deviation for the enactive mode is 7.28. Hence the confidence intervals are wider, and the steady progression in means, as illustrated in Table 9, is not significant statistically.

Mathematics Experience Sub-samples

The sub-sample of subjects with no experience in mathematics has mean scores in the visual mode and verbal mode which are significantly different from the mean scores achieved by the other three mathematics experience sub-samples. These findings support the position that mathematics in itself does not provide the only answer for differing uses of the modes of mental representation. Specifically, the mean achieved on the verbal mode of mental representation by the sub-sample with the highest level of mathematics experience is not the highest mean. The means of the two sub-samples with the low experience and the moderate experience levels are slightly higher, as shown in Table 8 and Table 9.

Furthermore, the means achieved on the visual mode of mental representation by the "low level of experience" and "moderate level of experience" mathematics sub-samples, are lower than the mean achieved by the high experience sub-sample. There are two possible explanations for this: first, that perhaps mathematics is not the only factor playing a role in the use of the three modes of mental representation; and second, that the composition of the sub-samples is perhaps too heavily weighted in favour of mathematics taught in mathematics courses, rather than in applied mathematics. Furthermore, these sub-samples are not adequately differentiated. Hence these data were examined from other points of reference.

Sub-Samples of Post-Secondary Areas of Concentration

The four sub-samples formed with respect to mathematics experience are, in one respect, not as satisfactory as one would expect, since the separation among the sub-samples is not clear-cut. A re-examination of subjects as to academic profile was undertaken. The analysis of the program of study of each subject at the post-secondary level was done. The results indicate that the academic concentrations clearly differentiate the mathematics experience of the subjects, and hence provide a different focus for an examination of the relationship between mathematics experience and of utilization of the three modes of mental representation.

Six sub-samples of subjects were formed according to the academic areas of concentration, or majors, at university. These areas of concentration are humanities, languages, mathematics, physical education, science, and life science. The subjects with other majors, such as music, are too few in number and hence are not included in this portion of the study. The results of the re-examination are provided in Table 10, wherein the results for the sub-samples are listed in increasing order of average number of courses in mathematics studied at the post-secondary level. The numbers of such courses range from an average of 0.10 of a mathematics course for the sub-sample of subjects with language majors, to an average of 9.40 courses for the sub-sample of mathematics majors.

The confidence intervals of Table 10 illustrate a steady progression of means for most areas of concentration. The progression for the verbal mode means is an increasing progression, while the progression for the visual means is a decreasing one. Post-hoc analysis on these findings, using Scheffé's procedure at a confidence level of 0.05, show that a significant difference between the scores of the language sub-sample and of the science sub-sample on both the verbal and the visual measures, and between the scores of the humanities sub-sample and the science sub-sample on the visual mode.

Table 10: MOTQ Subscale Scores VS Post-Secondary Area of Concentration

ANOVA of VERBAL Scores by Area of Concentration

SOURCE	DF	SS	MS	F	P
sub-sample	5	408.6	81.7	2.81	0.018
ERROR	205	5964.4	29.1		
TOTAL	210	6373.0			

```
  Area of             VERBAL       95% Confidence Intervals for Means
Concentration  N   Mean  Sdev   -+---------+---------+---------+-------
  Languages    25 16.94 5.42    (--------*-------)
  Humanities   46 18.99 5.40               (-----*-----)
  Phys.Ed.     25 20.09 6.42                 (-------*--------)
  Life Sci     43 20.76 5.47                      (-----*------)
  Science      28 21.88 6.47                         (-------*-------)
  Maths        43 20.83 3.53                      (-----*------)
                                -+---------+---------+---------+-------
                                15.0      17.5      20.0      22.5
```

ANOVA of VISUAL Scores by Area of Concentration

SOURCE	DF	SS	MS	F	P
sub-sample	5	377.4	75.5	4.53	0.001
ERROR	205	3418.7	16.7		
TOTAL	210	3796.1			

```
  Area of             VISUAL       95% Confidence Intervals for Means
Concentration  N   Mean  Sdev   -------+---------+---------+---------+--
  Languages    25 22.41 4.17                       (------*-----)
  Humanities   46 21.84 4.58                       (---*----)
  Phys.Ed.     25 20.30 4.30               (-----*-----)
  Life Sci     43 18.97 3.84           (----*----)
  Science      28 17.57 3.50     (------*------)
  Maths        43 19.33 3.43             (---*----)
                                -------+---------+---------+---------+--
                                     17.5      20.0      22.5      25.0
```

The subjects who studied languages at university are found to differ from those who studied science, on both the Aylwin verbal and visual measures. On the verbal measure, the mean of the language sub-sample is 16.94, while that of the science sub-sample is 21.88. On the visual measure, the mean of the language sub-sample is 22.41, while that of the science sub-sample is 17.57. The subjects who studied humanities differ from the science majors on the visual mode, but not on the verbal.

The enactive mode mean scores are also examined with respect to these sub-samples of subjects. There is neither a progression according to mathematics experience, nor any significant differences among the sub-samples formed with respect to post-secondary programs of study with respect to the enactive mode scores.

The relationship between experience in mathematics and utilization of modes of mental representation is more evident in Table 10 and the areas of concentration than it was evident when membership in one of the four sub-samples formed with respect to levels of experience in mathematics was used. Table 10 indicates that as mathematics experience increases, so too does the use of the verbal mode; it also suggests that as mathematics experience increases, the use of the visual mode decreases. The progression in each case, however, is not completely smooth, in that the sub-sample of science majors is out of step. These subjects, despite having studied slightly fewer mathematics courses than those in the sub-sample of mathematics majors, have higher verbal scores and have lower visual scores than do the mathematics majors.

The main difference between those science majors and mathematics majors suggests that science majors include mathematics and science experience, while mathematics majors do not, in general, have much science experience. Hence, although mathematics experience appears to be related to the ability to utilize different forms of mental representation, it is possible that such experience is not the only factor involved. That is, the application of mathematics to other subjects may also be important in the consideration of the ability to use the verbal mode of mental representation.

The study of science and the study of mathematics can be considered to be an intersection of two sets of topics. That is, science courses and mathematics courses sometimes contain identical content. For example, the study of vector forces is the same in both algebra and physics courses, and modern set theory is now an integral part of modern chemistry as well as of algebra. Each of the two disciplines, however, contains some distinct topics unique to the discipline. The fact that the subjects in the sub-sample of science majors were better able to utilize the verbal mode, and less able to utilize the visual mode of mental representation, than were the subjects in the sub-sample of mathematics majors, suggests that these discipline-specific topics are of importance.

The science majors may have had some experiences which enhanced their verbal ability. Such experiences might include the application of mathematics to scientific problems, the analysis of common phenomena to discover the scientific and mathematical laws governing them, and the categorization and hierarchy necessary for scientific understanding. Further research should be undertaken to find out more about these factors involved in science but not in mathematics. Research should also be undertaken to examine specific experiences in the study of different branches of mathematics, whether pure mathematics, applied mathematics, statistics and so on, as well as in the application of mathematics to other areas of study.

When the academic concentrations undertaken at post-secondary institutions by the subjects of this research are considered, other factors can be observed. The sub-sample of subjects with no experience in mathematics comprises individuals studying languages and humanities. The language major students, in particular, have the lowest mean scores on the verbal mode of mental representation of any sub-samples, and have the highest mean scores on the visual mode, of any sub-samples. The majority of subjects in this sub-sample have not studied any mathematics content in any post-secondary course; they have, furthermore, studied no science. Hence the relationship between mathematics and uses of the modes of mental representation perhaps might more accurately be examined as the relationships among mathematics, science, and the modes of mental representation.

Previous research findings for comparisons between areas of concentration or majors of study and the use of different modes of mental imaging for arts and physical education students are not borne out by this study (Richardson, 1969). The visual scores of arts students, and the kinaesthetic scores of the physical education students, are not significantly different from the average scores of the subject pool, whereas significant differences were found in the earlier study. It is possible, however, that the differences between the two studies lie in the populations themselves. The earlier study (Richardson, 1969) examined university students who were registered in particular programs of interest, whereas the current study contains the sub-set of those students who have decided to become teachers. This sub-sample may well be different from the general population, since those who enter teaching as a profession are precisely those people who wish to interact with children. At most faculties of education, one criterion for entrance into the program is experience with children. In Aylwin's studies (1985), a dominance in the visual mode of mental representation is associated with a caring, humanistic personality. Hence the high rating on the visual mode of these subjects may reflect the chosen profession generally, although this has never previously been reported for this population.

Findings Concerning Gender

During the data collection phase of this research, some information was gathered which does not directly relate to the hypotheses, but does appear to clarify some of the individual differences inherent in the sample. These data refer to the personal characteristics of subjects, such as gender, age, and program of study in teacher education. It has been suggested, for example, that success in mathematics might be gender related

(Tobias, 1978). Hence gender as a variable was examined.

The investigation of the gender variable, with specific reference to mathematics within the programs of study in teacher education, is addressed by examining male and female subjects registered in the elementary and secondary teaching options. Contingency tables are set up to examine the issue. In Table 11 below, the frequencies of male and female subjects in each mathematics experience sub-sample are presented. More than three quarters of the "no experience level" in mathematics and the "low experience level" in mathematics sub-samples are female, while the other two sub-samples have a more even number of male and female subjects.

Table 11: Frequency by Gender and Mathematics Experience

Mathematics Experience	Males N	Males %	Females N	Females %	Total N	Total %
None	13	22.03	46	77.97	59	100.0
Low	9	18.37	40	81.63	49	100.0
Moderate	27	52.94	24	47.06	51	100.0
High	21	41.18	30	58.82	51	100.0
Total	70	33.33	140	66.67	210	100.0

An association between gender and mathematics experience is found in the data in Table 10 at the 0.01

level of significance (Chi-square = 18.56, df=3). In particular, the smaller number of male subjects found in the sub-samples with no mathematics experience and low mathematics experience is found to be significant, as is the higher number of males in the moderate mathematics experience sub-samples.

When this issue is examined in relation to the three instruments, no significant differences are found between male and female subjects, in regards to Aylwin's MOTQ and Griffitts' test of mental imaging. There are, furthermore, no significant differences found between sub-sample scores of those in the elementary and secondary programs of study. Utilization of the three modes of mental representation, and of the three modes of mental imaging, does not therefore appear to be related either to gender or to program of study in pre-service education.

In the KAMI test, however, gender differences are found in both the noetic and the rational knowledge accessing modes. Table 12 provides the frequencies and percentages of subjects with each particular dominant mode, by gender and by program of study followed during the pre-service education program. The percentage of subjects with an empirical dominance is very different between men and women. For more than 40% of male subjects, this is the highest score achieved, while for female subjects, many more have a noetic dominance.

Table 12: <u>KAMI Dominant Modes VS Gender & Programs of Study</u>

	Dominant Mode							
Gender	Noetic		Empirical		Rational		Total	
	N	%	N	%	N	%	N	%
Male	15	28.85	24	46.15	13	25.00	52	100
Female	58	57.43	26	25.74	17	16.83	101	100
Total	73	47.71	50	32.68	30	19.61	153	100

Program	Noetic		Empirical		Rational		Total	
	N	%	N	%	N	%	N	%
Elementary	33	54.10	16	26.23	12	19.67	61	100
Secondary	43	44.79	35	36.46	18	18.75	96	100
Total	76	48.41	51	32.48	30	19.11	157	100

A gender difference can be seen in Table 12, in respect to the dominant mode of knowledge accessing. More than half of the women in both elementary and secondary programs are found to have noetic dominance, while fewer than thirty percent of male subjects have that dominance. On the other hand, the majority of male subjects in both elementary and secondary programs have an empirical dominance, while fewer than thirty percent of female subjects are dominant in that mode.

The elementary and secondary programs of study, however, show similar dominance profiles in knowledge accessing. The dominance profile in women is similar in the elementary and secondary programs of study. The dominance profile in men is likewise similar in the elementary and secondary programs of study. The two profiles, for men and women, however, are different.

Contingency tables are used to examine both the gender data and the program of study data illustrated in Table 12. An association is found to exist between gender and dominant knowledge accessing mode, at the 0.01 level of significance (Chi-square = 11.42, df = 2). No significant gender differences are found as regards to the rational mode of knowledge acquisition.

__Discussion of Findings Concerning Gender Differences__

Gender differences are found in the data obtained from the KAMI test but not in the data resulting from the Aylwin or Griffitts instruments. It is possible that an explanation for this lies in the fact that in the KAMI instrument, individuals rank statements as to their own perceived sense of the relative accuracy of each. This activity may well be influenced by preconceived notions of the relative importance of each of the statements, that is, by personal perceptions. It should also be noted that no statistically significant gender differences were found related to the area of concentration of the subjects, though the numbers of males in some areas of concentration were too small to allow inferences to be made.

The Aylwin instrument, on the other hand, attempts to measure how well an individual is able to use different modes of mental representation when requested to do so. The Aylwin results seem to imply that both males and females can use the different modes of mental representation equally well, but the KAMI results seem to imply that men and women perceive themselves to access knowledge in different ways. This suggestion is also supported by research by Gilligan (1982) and by Belenky, Clinchy, Goldberger, and Tarule (1986), in which women are found to work from a different frame of reference from that of men, in accessing knowledge.

It is also possible that cultural and social expectations lead to the measured differences between men and women on the KAMI instrument, since subjects are certainly aware of societal pressures. For example, a male student faced with completing the sentence "I understand things because I use:", might not rank "my intuition" very highly regardless of the truth of such as statement, since some men feel that logical thought is superior - that intuition is not as good as deductive reasoning, for men (Ferguson, 1980, p. 106). In this study, 7.4% of men ranked this completion in the highest position, whereas 36.8% of women did so. Of the 15 men who had noetic dominance, 3 were in the arts, while 5 were in mathematics and none in either science.

It is unclear from this fact whether men do indeed use intuition less, or whether they will not admit to using it. An analysis of this type of possible hidden bias could well form the basis of another study. Such a study should involve an analysis of the actual techniques used by individuals in problem-solving, using video and audio taping, to avoid the possibility of bias in self-reporting. The main result from such a study would be to either confirm or refute the societal perception that men use intuition less than women do. This study would be important for educators, in that the use of intuition in problem-solving might indeed be important for learning, but many texts discuss only non-intuitive techniques, thus discouraging students from utilizing other methods. With results from such a study in hand, teachers would at least know whether to "go by the book", or to encourage the use of intuitive techniques by students in the classroom.

Joint Dominances Among Instruments

The relationships among the different modes of mental representation, of knowledge accessing, and of mental imaging, can also be examined by means of the pairs of dominances, or joint dominances, among the different measures. In the comparison of the dominant mode of mental representation and the dominant mode of knowledge accessing, as illustrated in Table 13, two thirds of the MOTQ-visual subjects are also KAMI-noetic, a finding which is expected. Fewer than 10% of subjects, however, have a joint dominance of MOTQ-verbal/KAMI-rational, a result which is lower than expected. Almost half of the MOTQ-verbal subjects are KAMI-empirical, while less than one third of the MOTQ-enactive have the KAMI-empirical dominance which was expected to predominate.

The joint dominances of the MOTQ and KAMI shown in Table 13 are examined by means of a three by three contingency table. Although it appears that the high number of noetic/visual subjects is out of the ordinary, the analysis does not support this. At the 0.05 confidence level, no significant associations among the MOTQ and the KAMI modes are revealed; the critical Chi-square value is 9.48, whereas the Chi-square value found for this table is 9.14.

Table 13: Joint Dominances on MOTQ / KAMI / GRIFFITTS Tests

	Dominant Modes	KAMI Noetic N	%	Empirical N	%	Rational N	%	Total N	%
MOTQ	Verbal	17	39.53	21	48.84	5	11.63	43	100
	Visual	21	67.74	8	25.81	2	6.45	31	100
	Enactive	20	34.48	18	31.03	10	17.24	48	100
	Total	58	43.94	47	36.61	17	20.45	122	100

	Dominant Modes	MOTQ Verbal N	%	Visual N	%	Enactive N	%	Total N	%
Griffitts	Auditory	17	28.8	20	33.9	22	37.3	59	100
	Visual	29	41.4	20	28.6	21	30.0	70	100
	Kinaesthetic	4	22.2	6	33.3	8	44.5	18	100
	Total	50	34.0	46	31.3	51	34.7	147	100

	Dominant Modes	KAMI Noetic N	%	Empirical N	%	Rational N	%	Total N	%
Griffitts	Auditory	23	51.1	15	33.3	7	15.6	45	100
	Visual	23	39.0	18	30.5	18	30.5	59	100
	Kinaesthetic	11	64.7	5	29.4	1	5.9	17	100
	Total	57	47.1	38	31.4	26	21.5	121	100

In the comparison of the dominant mode of mental representation and the dominant mode of mental imaging, as illustrated in Table 13, the Griffitts-visual/MOTQ-verbal pair is most common; 19.7% of subjects have this dominance, while only 11.6% have the expected Griffitts-auditory/MOTQ-verbal pair, and only 13.6% have the visual/visual dominance pair.

The joint dominances of the MOTQ and the Griffitts tests of Table 13 were examined by means of a three by three contingency table. At the 0.05 confidence level, no significant associations were found among the MOTQ and Griffitts dominant modes (Chi-square = 3.70).

In the comparison of the dominant mode of mental imaging and the dominant mode of knowledge accessing, as illustrated in Table 13, the Griffitts-visual/KAMI-noetic and the Griffitts-auditory/KAMI-noetic pairs are most common; 19% of subjects have each of these dominance pairs, while only 4% of subjects had the joint dominance of Griffitts-kinaesthetic/KAMI-empirical.

The expected frequency of rational-kinaesthetic joint dominance is too small to allow the use of a contingency table to examine the associations among the different joint dominance pairs. The frequency of 18 subjects (14.9%) with rational-visual joint dominance, however, does appear to be at odds with the other frequencies, of 7 and 1 respectively (or 5.8% and 0.8%), in Table 13.

Discussion of Findings Concerning Joint Dominances

It appears reasonable to suggest, from the review of the literature, that the preferred mode of knowledge accessing should be matched to the preferred mode of mental representation, since both are concepts tied to the learning process. However, the suggestion is not borne out by the data. The frequencies of subjects with joint dominant modes of the MOTQ and KAMI, for example, illustrate primarily that there is no relationship between the two instruments. That is, none of the modes of knowledge accessing appear to have any relationship to any of the modes of mental representation. The rational mode of knowledge acquisition, for example, is related to deductive reasoning (Rancourt & Dionne, 1982), while the verbal mode of Aylwin is related to an analytical, structured approach to living and learning (Aylwin, 1985). The terms 'deductive reasoning' and 'analytical, structured approach', are often tied in mathematics and science, yet there are only five subjects (4% of the subsample) with this joint dominance.

A closer examination of knowledge accessing and of mental representation, however, reveals a basic difference in their natures which may partially explain the finding. Knowledge accessing modes describe the methods of reasoning preferred by an individual, while mental representation describes a way of conceptualization of an object or action. These are different aspects of thinking, in that the former is a purposeful process, while the latter is more a storage medium. Furthermore, the instrument for knowledge accessing is based on an individual's report of his or her own understanding of his personal preferences in thought, while mental representation is based on an analysis of the actual modes utilized. It is possible that some individuals might report preferences which they think are socially acceptable, rather than their own views.

Furthermore, no relationship appears to exist between the Griffitts auditory, visual, and kinaesthetic modes of mental imaging, and the MOTQ verbal, visual, and enactive modes of mental imagery. Despite similar terminology of names and concepts underlying the modes, the instruments do not appear to measure the same things. There are, for example, 20 subjects (13.6% of the subsample) with MOTQ-visual and Griffitts-visual joint dominance, the smallest number of any joint dominance pair including either visual mode.

The differences found between the MOTQ and the Griffitts test must now be analyzed. If the basic premise underlying the modes of mental imaging is examined more closely, one finds that when the senses of an individual are presented with a stimulus, one sense or another will attend to the stimulus. The Griffitts test provides a stimulus word such as "kettle" to a subject, and the subject is asked to decide on the sense which applies. In this case, the subject might see the kettle, hear the kettle's whistle, feel the heat of the kettle, smell the steam, or experience some combination of these. Each of the stimulus words can be imaged using any of the modes of mental imaging. No matter which sense or senses the individual uses, however, the reporter of the event is only examining the sense, not looking at the thinking which comes afterwards. The MOTQ examines this second stage, that of the mental representation of the thought or idea. As Kosslyn stated,

> You cannot know if the representation that produces the experience actually takes part in thinking; images might have no more to do with actual thinking than the heat given off by a light bulb has to do with reading. (1983, p. 32)

It appears, from the data presented in this study, that the modes of mental representation used in thinking, and the modes of knowledge accessing, are not dependent upon the type of sensory input. Hence the lack of association among the three instruments is not really surprising. However, the whole question of the process by which learning occurs, from the first impact on the senses, through to the construction of knowledge, should be the focus of research.

Furthermore, future studies should also examine the relationship between specific profiles of mental representation, and effective learning and teaching. One aspect of learning and teaching which can be measured quantitatively is student mastery of a topic, as measured by testing. Therefore a possible means of studying such relationships would be to have a completely new topic taught to different sub-samples of students, by teachers with different profiles of mental representation.

Grade 11 or grade 12 students in a large secondary school, for example, could be part of the study, and placed at random into sub-samples. In such a school there may well be 300 students of grade 11 mathematics, 200 students of grade 12 mathematics, and 6 to 12 teachers of this subject. It should be possible to locate teachers who have different profiles of mental representation. Pre-test and post-test results for the subjects could then be compared to the personal profiles of both the subjects and the teachers, to examine the relationships in question. Of particular interest would be the results of the teacher-student pairs with matching profiles of mental representation, compared to those of other teacher-student profile pairs.

IMPLICATIONS FOR THE FUTURE

This study has clear implications both for future research and for education. First, it gives a straightforward indication that differences do exist between the mental representation modes preferred by students enrolled in teacher education who have no mathematics experience, as compared to those who have even a small amount of experience in mathematics. Second, it implies that future research should concentrate on the differences in teaching which may arise from these differences in the utilization of all of the modes of mental representation and knowledge acquisition.

Implications of Lack of Mathematics Experience

That is, since we now know that student teachers with no mathematics experience differ from the rest of the student population both in their use of the modes of mental representation and in their preferred knowledge accessing modes, the next question to research is that of the effect, if any, of this difference on the required teaching of mathematics which these individuals must conduct in the performance of their duties. Further studies are necessary to determine whether such teaching is effective, or whether more attention has to be paid to the training of prospective teachers who possess no experience in mathematics. There is, further, a question as to whether individuals who wish to enter the profession should be encouraged to have a broader university education than is currently the norm, including at least a few courses which contain mathematics in one form or another.

Mental Representation Over Time

A further important question is that of individual changes in the preferred modes of mental representation and knowledge accessing. Do the preferred modes of knowledge accessing and the dominant mode of mental representation remain fixed entities throughout the teaching career of the individual, or do changes occur over time? In an unpublished study (Gurney, 1991), the author has examined the dominant modes of mental representation and the preferred modes of knowledge accessing in a set of 36 senior science consultants in the province of Ontario. The profile displayed by this sub-sample did not match that of the science pre-service education students. In particular, the consultants had significantly higher scores on the Aylwin visual measure (in which the score was higher than the mean visual score of any of the pre-service mathematics experience sub-samples), and considerably fewer consultants had a rational dominant mode of knowledge accessing.

Two possible reasons for this measured difference are: first, that it may be possible to change the profile of mental representation and knowledge accessing of an individual throughout his or her career; and second, that it is possible that consultants are specifically those individuals who are interested in other people, and hence have self-selected for this career path. If the first possibility is true, that change can occur during an individual's adult working career, then it may be possible to enhance the ability of teachers in the field to utilize all three modes of mental representation. Since it is important to know which of these two scenarios is more likely, further research in this area is recommended.

Implications of Individual Differences

One implication of the differences found in mental representations is that these future teachers may tend to teach using methodologies which are honed from their preferred modes of mental representations. Students who have different profiles of mental representation, however, may not be on the same "wavelength" as their teachers, and may, therefore, face problems in knowledge accessing. Specifically, they have to develop an understanding of the unexpected (for them) methods of presentation of new material as well as of the subject matter itself. This double learning situation may cause the student to be labeled, or to label himself or herself, as not being very clever in the learning of that subject.

Furthermore, the teacher who does not demonstrate effective use of the verbal mode of mental representation may not present learning experiences in the classroom which would encourage the growth in use of this mode by the students. Alternatively, a teacher who is highly verbal in preference might not provide the concrete and visual material which many students prefer. One direct advantage of this study, then, is that student teachers can be made aware of the potential profile mismatches between teacher and students, and learn to recognize and accommodate them. It may, further, be possible to introduce into the curriculum of both pre-service and in-service courses, exercises which encourage the utilization of all the modes of mental representation, and hence which might assist teachers in developing their own abilities to use any mode in mental representation, if these can, indeed, be learned.

Other Avenues of Research

An important aspect of these research findings is the set of questions arising from them, which should determine future avenues of study. In addition to those mentioned above, there are further directions of research that are of interest. In particular, the relationship between different concentrations of study and the use of the modes of mental representation should be examined more closely, since experience in mathematics did not appear to be the only factor in individual profiles of mental representation. It would be useful, further, to compare and contrast the profiles of the pre-service education students with those of students with similar experience, but who have not chosen to enter this profession. This would enable researchers to determine whether many potential teachers who self-select for this career have a particular profile of modes of mental representation and knowledge accessing.

This avenue of research leads into the general question of what might be the preferred modes of mental representation of teachers, in both elementary and secondary school. The fact that the majority of the pre-service education students prefer to use the visual mode of mental representation may be related to the preferred modes of mental representation of some of their teachers. The preferred modes of mental representation of teachers in the field are not known. It appears reasonable that the preferred mode of mental representation of a teacher may be related to the subject matter being taught and to the methods of teaching used in the classroom, but there are no data concerning this.

It is, furthermore, feasible to suppose that some students are dissuaded from the teaching profession, or indeed from continuation of particular programs of study, by teachers who do not encourage use of modes of mental representation which are preferred by those students. This question cannot be answered without further research into the preferred modes of mental representation of, and preferred teaching methods used by practising teachers. Such research should, if possible, be longitudinal in nature, testing teachers at various stages in their careers. This would enable the researchers to find out, not only the preferred modes of mental representation and teaching methods found in different school disciplines, but also whether or not there are changes in preferred modes of mental representation over time.

Implications for Curriculum

Another implication of the findings of this study which should be looked at more closely relates to the curricula of post-secondary humanities programs. The finding that the ability to use the verbal modes of mental representation was greater even in the subjects who had only a minimum degree of experience in mathematics, suggests that such ability might be enhanced by the addition to the required curricula of one or two courses which contain mathematics. This would be comparable to many university programs in the sciences which have a required arts course, designed to encourage writing skills. The additional courses need not, as it appears from the data, be courses taught in the mathematics department, but should have mathematics content. By this means, those individuals who intend to enter the teaching profession would have a greater opportunity to develop the three modes of mental representation before arriving at the faculties of education. It is not known, of course, whether a greater ability to utilize the verbal mode of mental representation would lead to better teaching of mathematics or of any other course. Research into this question is necessary before any changes to programs can be envisioned.

This finding may also have relevance to the curricula of the faculties of education. If the ability to use all three modes of mental representation is desirable for future teachers, then either entire courses or sections of courses may have to be modified in some way so as to provide experiences which will encourage the development of such abilities in the three modes. Whether it is even possible to increase the ability of an individual to use different modes of mental representation, or whether, at some point, the ability becomes fixed at a particular level, is unknown, and should be examined. If development is indeed possible, then the means by which such change can occur are also not entirely obvious. Further research will be necessary, in order to answer both parts of this question of ability to use all three modes of mental representation.

This study has provided a starting point for an area of investigation on different rates of success in mathematics. It is clear from the data that students who possess no experience in mathematics do not have the same profile of use of the modes of mental representation and of knowledge accessing as do those who have had experience. As described previously, however, the relationship between experience in mathematics and the ability to use the modes of mental representation is not yet understood. This problem warrants further study. For example, research should be carried out in the secondary school, with students who are studying in each area of mathematics, namely algebra, geometry, arithmetic, and functions. The ability of the students to utilize the different modes of mental representation should be measured before and after the courses are studied, and relationships between success in the area of mathematics, and mode utilization, should be examined. A study of this nature should also be carried out in a faculty of education, examining readiness to teach mathematics in an elementary classroom. That is, ability and experience in mathematics should be examined, together with the ability to utilize the modes of mental representation, to see whether there is any relationship between them.

It is hoped that this study will contribute to the development of curriculum by mathematics educators in faculties of education. When the members of one sub-sample of subjects in this study were requested (in the data collection phase) to list the words and phrases which, in their minds, are associated with mathematics, the sample as a whole was highly negative about the subject, although they were not negative about other subjects which were listed. Even students planning to teach mathematics did not express positive comments concerning their chosen fields. The bachelor of education program provides an excellent opportunity for the faculty to attempt to change the attitudes of these future teachers concerning mathematics, since these future teachers want to teach all subjects with equal success. If the negative attitudes are not changed, they may significantly affect the approaches to mathematics that these new teachers apply when they are in control of the curriculum in their own classrooms(Cobb, Yackel and Wood, 1989, Sowder, 1989). This might occur even inadvertently, since a subject which is not liked by a teacher in the elementary panel, may not be given adequate attention in the school day.

Results from this study may furthermore contribute to the on-going process of renewal in the field of mathematics education. Concomitantly, the findings may also have implications concerning the mathematics curriculum at both the elementary and the secondary levels. Many of the students at the pre-service level in the faculties of education dislike mathematics as a subject. This dislike was developed in the elementary and secondary schools, a fact which provides further support to the current drive for changes in the curriculum. The direction of change, however, should take into account two main points: first, the methods of teaching should provide greater enjoyment for the students (Good & Brophy, 1990). Secondly, the study suggests that the learning experiences of these students should encourage in them the utilizations of different forms of mental representation. If the teacher addresses all three modes of mental representation in teaching, then there is a better chance, not only that the student will be able to learn the topic more easily because of a match in learning and teaching mode (Garling & Frank, 1986), but also that the student will begin to appreciate the other two modes.

Both factors can be provided for at the same time, by offering more opportunities for hands-on manipulation, for co-operative learning, for computer assisted instruction, and for concrete applications in mathematics courses, rather than the blackboard to pencil methods currently in use. For example, the teaching of vectors might be better handled outside the school in the playing field, using aspects of football, soccer, and other games, to enable students to acquire a deeper understanding of forces, directions, and interactions of two or more forces. The ideas inherent in similar triangles might be developed by students in small groups, through building models and drawing pictures. Algebraic problems might also be more easily understood by means of small group discussions, together with models and diagrams. These methods generate more interest among students than does the traditional textbook approach, and also appear to require different kinds of thinking.

Contributions of this Study

This study has contributed to the advancement of knowledge in the cognitive domain. The association between the modes of mental representation and experience in mathematics has been demonstrated, as is the association between the modes of knowledge accessing and experience in mathematics. A connection has been made between lack of experience in mathematics and the ability to utilize particular modes of mental representation on direction.

This study has provided some information concerning the underlying models on which the instruments are based. Many subjects in the study, when directed to utilize a particular mode of mental representation, would, indeed, begin using the directed mode, but after perhaps 8 to 10 trials, would abruptly switch to a different mode for the remaining trials. This suggests that a fatigue factor is at play in the exercise, and that the preferred mode of mental representation will tend to be used much of the time, whether or not this use is most appropriate for the situation. This finding provides additional support for Aylwin's model of mental representation, contradicting the idea that individuals who do not utilize a particular mode of mental representation, have not developed the ability to utilize the mode. It appears that the ability exists, but that the individual finds it more difficult to use a non-dominant mode.

The findings in this study do not entirely fit expectations for two of the instruments. On the KAMI test, the frequency of rational mode dominance of subjects with high levels of experience in mathematics is not as high as expected. Furthermore, the Griffitts test did not differentiate among the sub-samples of subjects, although such differentiation was expected to occur. There appear to be two possible reasons for such divergent results. First, there may be difficulties with the underlying models. Second, the sample may not be truly representative of the population at large.

There may be difficulties with the underlying models, but the probability of this being true seems small, since there is a large body of research supporting the models. It is more likely that the sample used in this study differs from the general population. The pre-service education students are a rather select group, in that they have self-selected for a career in which they will have constant contact with many children. A requirement for entrance to the program is experience with children, so these subjects have demonstrated an understanding of the responsibilities involved in teaching children. Those mathematicians, scientists, and others who enter teaching as a profession, form a subset of the population.

In summary, this study has provided a valuable starting point for research and development in the modes of mental representation and of knowledge accessing in mathematics education. In pointing out the nature of individual differences in the utilization of modes of mental representation and of knowledge acquisition, the study has provided a focus for teachers and researchers, both in the on-going attempt to understand the learning process, and in the provision of a better learning environment for students in the school system.

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APPENDIX A PERSONAL PROFILE (condensed)

NAME or CODE______________________ **AGE**_______ **GENDER**_________

For the following **grade 13** (or last year of high school) courses, circle those which you studied (as far as you remember).

algebra functions geometry calculus trigonometry computer-science
art drama music English literature technology economics
French German Latin other-language geography history phys-ed
politics botany chemistry physics zoology other-science

===

For each of the following **university** or **post-secondary** areas of study, indicate the numbers (as best you can) of courses that you took in each of the disciplines. Describe all courses in terms of **half course** equivalences (i.e. a full course "equals" two half courses).

NAME	**NUMBER of COURSES TAKEN**													
mathematics	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
computer science	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
engineering tech	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
chemistry	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
physics	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
life sciences	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
geological sciences	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
psychology	0	1	2	3	4	5	6	7	8	9	10	11	12	>12

similarly for other subject areas in languages, history, ...

visual arts	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
dramatic arts	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
music	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
other humanities	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
phys-ed (kinanthro.)	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
economics / business	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
other____________	0	1	2	3	4	5	6	7	8	9	10	11	12	>12
other____________	0	1	2	3	4	5	6	7	8	9	10	11	12	>12

Teaching Subject(s) at Faculty of Education______________________

PART B Complete this where it is possible to do so.

1. Do you enjoy mathematics ? Yes No Sometimes

   Have you ever avoided studying a mathematics course? Yes No

   What words come to mind when you think of mathematics ?________

2. Do you enjoy science ? Yes No Sometimes

Have you ever avoided studying a science course? Yes No

What words come to mind when you think of science ?________

----

3. Do you enjoy the arts ? Yes No Sometimes

Have you ever avoided studying an arts course? Yes No

What words come to mind when you think of the arts ?________

----

4. Do you enjoy literature ? Yes No Sometimes

Have you ever avoided studying a literature course? Yes No

What words come to mind when you think of literature ?________

----

5. Do you enjoy foreign languages ? Yes No Sometimes

Have you ever avoided studying a language course? Yes No

What words come to mind when you think of languages?________

----

6. Do you enjoy kinanthropology or phys-ed ? Yes No Sometimes

Have you ever avoided studying a kin or phys-ed course? Yes No

What words come to mind when you think of kin or phys-ed?________

----

7. List the subjects you find most enjoyable:____________________

----

8. List the subjects you find least enjoyable:____________________

SUMMARY OF COMMENTS PROVIDED IN PART B

There are 44 subjects in the sub-sample which completed the PART B portion of the questionnaire above. This sub-sample includes members of each of the sub-samples formed with respect to experience in mathematics, and hence forms a representative sub-sample of the whole. Mathematics is an area of study unlike all others, for these subjects. This area was named as one of the "least enjoyable" areas of study by 41% of all subjects in this sub-sample, including six out of the ten in the "high mathematics experience" sub-sample; the next least popular course is physics, with 25%. The results for mathematics and for three other areas of study are presented in Table 14; The 'arts' area of study is not presented in the table, since only one subject in the entire sub-sample did not enjoy this area of study.

The number of subjects in the sub-sample who enjoy mathematics is lower than the number who enjoy the other areas of study. Only 36.4% of the subjects enjoy mathematics, while 52.3% enjoy science, and 68.2% enjoy literature or physical education. The most worrying item in Table 14 is that only three of the ten subjects who have a high level of experience in mathematics actually enjoy the subject, and seven out of the ten have avoided studying mathematics courses in the past. Furthermore, four out of these ten subjects provided only unfavourable comments concerning mathematics.

TABLE 14: Attitudes Towards Areas of Study, by Mathematics Experience

Mathematics Experience Sub-sample	N	MATH like	MATH avoid	LITERATURE like	LITERATURE avoid	PHYS. ED. like	PHYS. ED. avoid	SCIENCE like	SCIENCE avoid
1	12	3	7	12	1	7	2	3	9
2	14	6	7	8	5	12	7	8	7
3	8	4	4	3	4	4	1	5	3
4	10	3	7	7	3	7	3	7	7

Comments Concerning Courses

Mathematics Experience Sub-sample	MATH good	MATH bad	MATH mix	LITERATURE good	LITERATURE bad	LITERATURE mix	PHYS. ED. good	PHYS. ED. bad	PHYS. ED. mix	SCIENCE good	SCIENCE bad	SCIENCE mix
1	0	8	0	7	0	0	7	0	1	5	2	0
2	2	4	0	4	1	0	5	0	2	5	1	0
3	0	2	2	4	1	0	3	1	0	3	1	0
4	0	4	1	5	0	1	5	0	0	2	1	1

The number of subjects who have avoided taking courses in all areas of study is high, ranging from 29.5% for literature or physical education, to 56.8% for mathematics courses and 59.1% for science courses. These numbers are misleading, since, for example, six subjects who stated that they had never avoided taking a mathematics course, have not studied any mathematics courses at all at the post-secondary level, and four of these list mathematics on their lists of most enjoyable courses. No conclusions can be drawn from these data, since the reasons for avoiding courses are not provided.

The words which come to mind for subjects on presentation of the mathematics area of study, can be arranged into three main types. The first consists of favourable comments, such as "fun", "challenging", and "I like it". The second consists of unfavourable comments, such as "frustration", "confusing", and "meaningless". The third set of words consists of lists of concepts to be mastered in such courses, or titles of such courses, such as "addition, subtraction, ...", or "algebra, calculus, geometry,...". In mathematics, there are far fewer favourable, and far more unfavourable, comments than in any other type. This is true even among those in the "high experience" mathematics sub-sample.

APPENDIX B-1

This appendix describes the second modes of thought questionnaire. The entire questionnaire is not provided, but the introduction is given, together with a sampling of the different types of associations requested. The coding of the MOTQ-2 is also described, to enable this process to be better understood.

The Modes of Thought Questionnaire (MOTQ-2)

This questionnaire is concerned with associations between ideas, and in particular looks at whether thinking about ideas in different ways will lead to different associations. It is divided into three sections with each section involving the use of a different way of thinking. The three ways of thinking are:

Inner Speech: here you are asked to say each word to yourself under your breath, and then to write down the first word or phrase that comes to mind in association with the word.

Visual Imagery: here you are asked to see a picture in your mind's eye of what each given word refers to, and then to write down your first association to that image.

Enactive Imagery: here you are asked to imagine being or doing what each word refers to, and then to write down your first association to that felt image.

Try to use each way of thinking as instructed. Write down the first thing that comes to mind after you have represented each word as suggested. For your associations a word or a short phrase is all that is needed.

SECTION 1: Inner Speech. For the following words, **say each word to yourself under your breath** before thinking of an association.

uncle	walrus
April	up

.
.

SECTION 2: Visual Imagery. For the following words use visual imagery: **get a picture in your mind's eye** of what each word refers to before thinking of an association.

monkey	cottonwool
nurse	tree

.
.

SECTION 3: Enactive Imagery. For the following words use enactive imagery: imagine **being or doing** what each word refers to before thinking of an association.

lion	athlete
stumbling	jackal

.
.

Coding the MOTQ-2

For purposes of coding, disregard the fact that the questionnaire is divided into three sections.

verbal score is the sum of the attributes falling into the following categories:

Superordinate (e.g. April--spring, walrus--animal)
Phrase Completion (e.g. aluminum--window, leap--frog)
Rhyme (e.g. baboon--moon)
Opposite (e.g. red--blue(or green), long--short)

visual score is the sum of:

Environment (e.g. octopus--sea, kettle--stove)
Attribute (e.g. skyscraper--tall, butterfly--colour)
Part (e.g. face--nose, octopus--legs)

enactive score is the sum of:

Act (e.g. ox--ploughing, athlete--running)
Affective (e.g. fox--cunning, tasting--hungry; crying & smiling when not coded as an act)
Consequence (e.g. slamming--noise, slicing--cut; also including consequences which are acts.)
Affective Consequence (e.g. climbing--exhaustion, stumbling--ouch; including consequences which are acts.)

NOTE

A response that is actually a synonym cannot be coded, no matter what part of speech it may be. Some other associations will also be uncodable.

A person's dominant representational 'type' is the mode with the highest score, but the three scores can be examined separately as scores on the three modes.

Notes on scoring particular words:

stimulus - **alligator**
response - swamp is coded as environment (visual)
- teeth is coded as a part (visual)
- bite is coded as an act (enactive).

stimulus - **armchair**
response - comfort is coded as an attribute (visual)
- fireside, room are coded as environment (vis)
- relax is coded as affective (enactive)
- fire, dad cannot be coded.

APPENDIX B-2

The first modes of thought questionnaire is illustrated below. The entire questionnaire is not provided, but a sampling of the substance can be derived here. The coding information for the responses of the user are provided.

The Modes of Thought Questionnaire (MOTQ-1)

When we try to hold an idea in our minds we usually find that it naturally leads to some other associated idea. This questionnaire is on such associations between ideas, and in particular looks at whether thinking about the same idea in different ways will lead to different associations.

The experiment consists of pairs of items; for each pair you are asked to think about the first item in one of three ways - either (i) to **see** a picture in your mind's eye; (ii) to **say** the word or words to yourself under your breath; or (iii) to **be** the thing - to imagine that you are who or whatever the idea expresses. You are then asked to assess the likelihood of the second idea coming along sooner or later in association to the first. Here are three practice pairs, with explanation of what to do:

(A) **see** TRAIN ----- station ... ( )

Get a picture in your mind's eye of a **train**, allow ideas to come to mind in association with it, and then assess the likelihood of **station** being among them. The rating scale to use is as follows:

(1) no likelihood - wouldn't have thought of the second idea at all if it hadn't been suggested.

(2) unlikely - but it would perhaps come along in the end.

(3) eventually - not immediate but it would come along after some thought.

(4) likely - it would be in the first few ideas.

(5) certain - it would be the first idea that would come to mind.

Enter a number from 1 to 5 in the brackets following **station** to indicate your choice. These ratings are purely personal - there are no 'right' or 'wrong' answers, so just trust your intuition.

(B) **be** SCIENTIST ----- apparatus ... ( )

Imagine that you are a scientist, allow ideas to come to mind in association with this imagined role play; then rate **apparatus** for its likelihood of being among them.

(C) **say** TABLE ----- chair ... ( )

Say the word **table** to yourself under your breath, allow ideas to come to mind in association with it, and then rate **chair** for its likelihood of being among them.

Since the experiment is on different ways of thinking, it is important that you do your best to follow instructions about saying, seeing and being, even if you sometimes find it difficult. You may find that it helps to close your eyes to think about each pair of ideas.

Here are three more practice pairs:

(D)	**be**	CLOWN	-----	laughter	( )
(E)	**say**	LUGGAGE	-----	baggage	( )
(F)	**see**	BUS	-----	bus stop	( )

You should now be ready to go on to the main questionnaire.

PLEASE TRY NOT TO MISS ANY PAIRS OF WORDS

Many thanks for your help.

The SCALE:

no	unlikely	eventually	likely	certain
(1)	(2)	(3)	(4)	(5)

For the first group **say** the first item of each pair

1)	**say**	SCARLET	-----	fever ...	( )
2)	**say**	IN	-----	out ...	( )
3)	**say**	FLOPPY	-----	rag doll ...	( )

.
.
.

147)	**see**	BUSINESS	-----	office ...	( )
148)	**see**	HEDGEHOG	-----	snout ...	( )

.
.
.

154)	**be**	SWIMMING	-----	shivering ...	( )
155)	**be**	TEACHER	-----	chastises pupil ...	( )
156)	**be**	FORCED MATCH	-----	exhausted ...	( )
157)	**be**	SEAGULL	-----	could divebomb people...	( )

.
.
.

Scoring the MOTQ

The following is an extract from the scoring manual for the MOTQ-1 questionnaire, which is representative of the completeness of the manual.

It is scored as follows: there are 13 structural subscales, where the items are preceded by a representational instruction appropriate to the mode.

Verbal Subscales

$$\text{Verbal Score} = \left(\frac{\text{opposite}}{18} + \frac{\text{phrase}}{13} + \frac{\text{rhyme}}{6} \right) \times 10$$

Phrase Completion (thirteen items) Q numbers 1, 5, 7, 40, 52, 73, 77, 108, 111, 121, 164, 168

Rhyme (six items) 54, 75.105, 112, 122, 128

Opposite (eighteen items) 2, 4, 6, 8, 34, 36, 38, 50, 53, 56, 76, 80, 107, 109, 124, 126, 165, 169

Superordinate (eleven items) 33, 37, 49, 55, 74, 79, 106, 127, 163, 166, 170

Visual Subscales

- .
- .

Enactive Subscales

- .
- .

Cross-Modal Scale
This uses representational instructions crossed with cognitive structures. (twenty-four items) 14, 28, 68, 83; 24, 92, 137, 159; 3, 35, 51, 123; 42, 59, 94, 114; 39, 78, 110, 167; 97, 103, 135, 146.

buffer items 19, 102, 151

APPENDIX C

The Knowledge Accessing Modes Inventory

When presented with the KAMI instrument, the subject is directed to consider each numbered item separately. Each of the twenty items consists of the beginning of a sentence, followed by three potential completions for the sentence. The subject is to rank the three completions, from 1, for most likely to be true, to 3, for least likely to be true for the subject. Each of the three potential completions is associated with one of the three modes of knowledge accessing, so that the overall profile of use of the three modes of knowledge accessing can be determined from the responses to the instrument.

Permission to include the Inventory has been received from the author. Hence a portion of it is presented in this appendix, to enable the reader to understand the nature of the instrument.

KNOWLEDGE ACCESSING MODES INVENTORY

Response Sheet

1. I enjoy reading about subjects that are:
   ___ a) technical
   ___ b) artistic
   ___ c) theoretical

2. I prefer to be with people who are:
   ___ a) spontaneous
   ___ b) trustworthy
   ___ c) realistic

3. A good teacher helps me to:
   ___ a) improve my thinking skills
   ___ b) become more self-confident
   ___ c) learn useful and practical skills

4. I accept advice when it appeals to my:
   ___ a) logic
   ___ b) feelings
   ___ c) common sense

5. When playing a game or sport, I:
   ___ a) do what feels right
   ___ b) focus on strategy
   ___ c) demonstrate flexibility

6. In a dangerous situation, I:
   ___ a) imagine an escape
   ___ b) observe the situation
   ___ c) react on impulse

7. A job should offer the occasion for:
   ___ a) personal growth
   ___ b) an intellectual challenge
   ___ c) varied experimentation

8. I would feel hurt if someone accused me of:
   ___ a) being biased in my observation
   ___ b) reacting before thinking
   ___ c) not having any principles

APPENDIX D

The Griffitts' Test of Imagery Dominance
(visual, auditory, kinaesthetic)

Test Words (20 in all)

telephone	bicycle	piano	fire	electric fan
hammer	tennis	clock	whistle	scissors
music	steam	typewriter	car	fire engine
ball	bell	gun	radio	lawn mower

Instructions (to be spoken aloud by the tester)

When I say the word, tell me what comes into your mind. If you see the object, hear a sound, think of a muscular activity on your part, or a taste, a smell, a sensation of heat or cold, answer accordingly.

If a word has elicited more than one response, then out of a total of 7 points, how many points would be given to each?

Scoring: Each word has a possible 7 points, which will be assigned (in whole or in part) to Visual, Auditory, or Kinaesthetic. Add up all of the points for each type (A, V, K), and adjust the final scores.

$\frac{A}{140}$ $\frac{V}{140}$ $\frac{K}{140}$ ---> % scores