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Far-field Predictions from Near-field Measurements using the Three Dimensional Finite Element and Boundary Integral Methods

by

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A thesis
submitted to the School of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Master of Applied Science

Ottawa-Carleton Institute for Electrical Engineering

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Rédouane Laroussi, Ottawa, Canada, 1992



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Abstract

Characterizing antenna designs in their operational modes has been a classic problem for electromagnetic engineers for many years. The far-field pattern is of great importance in antenna characterization, but in many cases, it is impossible or at least impractical to measure directly.

This thesis presents a numerical technique whereby near-field measurements can be transformed into far-field solutions. The finite element method(FEM), and an integral formulation, solution to the scattering problem, are used separately to solve the open boundary wave propagation problem in three dimensions. The near-field of a known source is sampled over a mesh of points and used as data for both methods. The integral formulation yields the far-field solution for homogeneous problems without the need for space discretization. The finite element method uses tetrahedral elements for discretizing the domain of interest and second order interpolation functions. It is complemented by the application of an absorbing boundary condition(ABC) which allows truncation of the domain. Classical ABCs of the local type are applied to the three dimensional problem. A new non-local ABC is obtained from the integral formulation and is coupled to the finite element approach. Computed field strengths obtained with the different approaches are compared with the analytical results obtained from the excitation source's analytical expression. The agreement was very good with maximum error percentages of about 10% for the FEM method and 2.5% for the integral formulation.

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Chapter 1

Introduction

1.1 Motivation

The problem of characterizing antenna designs in their operational modes has been a classical problem in electromagnetics for quite some years. Measurement of the far-field pattern directly in the far-field is in many cases impractical, if not impossible to obtain. It may be that the far-field distance is too great and unachievable, or that the antenna cannot be moved about easily.

In the scope of controlling electromagnetic interference (EMI), a quick and efficient way of estimating radiated emissions of electrical equipment is needed. To comply with radiation regulations for electromagnetic interference and compatibility (EMI/EMC), signal levels at different distances and frequencies have to be specified for all electronic equipment.

Many algorithms have been developed to obtain far-field information from near-field measurements. These have been generally based on simplified schemes that take advantage of problem symmetries, or make assumptions that limit the range of applicability of the results. The amount of work done to model the phenomenon of electromagnetic(EM) wave propagation in 3-D is very limited and has been possible today only with the advent of powerful computers.

The motivation of the work that follows was to provide a way of transforming near-field data to far-field information by exact emulation of the wave propagation in space without introducing any simplifications. The FEM is not generally considered a suitable technique for antenna or radiation problems because they are unbounded problems; however, the FEM has many positive benefits over other techniques. The problem of using the finite element method on open problems and the errors due

to domain truncation present an added difficulty, the handling of which, still needs much improvement. What is known in the literature as an absorbing boundary condition(ABC) has been used quite extensively as a remedy to this problem. The ABC simulates the far-field response at the boundary of the truncated mesh. Applications of ABCs in 3-D are relatively new and publications in scientific journals presenting quantitative results to demonstrate the efficiency or inefficiency of the implementation of ABCs in numerical analysis are hard to find.

1.2 Contribution

The results presented in this thesis offers two approaches to tackle the problem of far-field predictions from near-field measurements using the finite element method and an integral equation solution to the scattering problem.

A meshing program was developed to discretize the 3-D domain of the problem solved. The meshing program MESHMAIN creates a mesh made up of tetrahedral elements. It allows finite elements to have different electrical characteristics (permittivity/conductivity). Boundaries of the domain can be identified as any one of three boundary types used in EM modeling: Dirichlet, Neumann or open boundary. The meshing program can be executed independently of the electromagnetic solver, and the mesh created can be modified without the need for re-executing the mesher with the use of other auxiliary programs.

The electromagnetic solver, the program MAINCOMPSG, solves the wave equation using the FEM based on a Galerkin formulation for the three Cartesian components of the magnetic field. For the open boundary sections, there is a choice of three ABCs to be applied: i) the Sommerfeld ABC, ii) the first order Bayliss-Turkel ABC, and iii) an integral type ABC derived from the boundary integral method (BIM) and adapted to the FE approach. For a homogeneous domain, an integral equation solution to the EM propagation (BIM) is utilized to obtain the field values anywhere in space. The program BEMRAD uses the mesh created by MESHMAIN to solve the wave equation with a boundary integral. BIM results are faster to obtain and the program itself requires much less memory than MAINCOMPSG for a similar configuration. Results are given for the two methods, BIM and FEM with different ABCs, and comparisons are made between the numerical approaches and analytical values.

Chapter 2

Literature Review

2.1 Introduction

Numerical methods have become important tools in solving practical electromagnetic problems. Problems that can be handled analytically have simple geometries or use assumptions that impose restrictions on generality and in most cases do not represent real or practical problems. The main issue to using this wide range of numerical tools is choosing a good numerical algorithm. The selection criteria will be those that deal with feasibility and flexibility considerations. An algorithm that is inflexible and solves only one problem configuration may be too difficult to modify for different configurations, and thus has low productivity. Another consideration is that every numerical algorithm must make use of a computer. Machine requirements in terms of memory and execution time weigh heavily when deciding which method to use. Reaching a decision which takes into account all aspects of the problem is an impossible task because there is never a unique solution and there will always be room for new arguments and improvements as long as scientific and technologic advancements continue.

2.2 3-D EM Propagation and FEM

Electromagnetic field propagation problems can be classified into two groups: i) bounded and ii) unbounded problems. These two situations have the same approach around the source region and inhomogeneities; however for the unbounded situation, an extra complication arises from the need to deal with an infinite space. The methods of dealing with this situation will be discussed in detail in the next section.

The finite element method despite its own shortcomings, remains the most appropriate method when the problem has discrete inhomogeneities and complex geometries. Integral equation methods are difficult to formulate for inhomogeneous problems, but are very easily adapted to solve unbounded problems. They can be used to solve large problems very quickly, but they lose their advantage over differential methods such as FEM when the inhomogeneities are not centralized. The Green functions involved become very complicated to find and integrate[1].

Several approaches have been developed specifically to solve 3-D EM problems with the finite element method. Many algorithms have been elaborated and tested on 2-D situations and parallels drawn to their use in 3-D. Webb[2] used a variational approach with FEM to solve a vector equation for either the magnetic field $\vec{H}$ or electric field $\vec{E}$, and derived the scattering parameters from the field values in waveguide structures. The results obtained were very good, but at the expense of sophisticated trial functions. Some of the problems solved were inherently 2-D problems. Webb did not mention any difficulties with spurious modes.

The occurrence of non-physical solutions (spurious modes) arises very frequently in the solution of 3-D EM problems[3, 4, 5]. Many approaches have been taken to eliminate this shortcoming of the FEM. In another paper[4], Webb promoted the use of a penalty factor associated with the divergence of the field quantity to be added to the wave equation in order to eliminate spurious modes based on the assumption that spurious modes are not divergence-free. Among the advantages mentioned for the penalty factor, was the preservation of the sparsity of the matrix system produced with FEM. The selection of the factor itself, however, was not systemized. In his paper, Webb implemented a test where the divergence is repeatedly checked until satisfactory levels of error were reached.

In [6], Konrad substituted the system of equations obtained from the divergence directly into the system of equations derived from the wave equation, and thus reduced its order. The final system is smaller, and quicker to solve, but the substitution itself is not straightforward[4]. In [3], Svedin enforced explicitly the boundary conditions between adjacent elements. The approach was tested on several situations including anisotropic media with magnetic and dielectric losses. The results obtained compare very well with other methods and did not present any spurious behaviour.

MacNeal[5] replaced the polynomial shape functions used in the classical FEM approach by modal basis functions. The modal functions were the eigenfunctions of

the associated eigenvalue problem. The field was rewritten as an expansion of modal functions and the modal amplitudes were solved for. The idea was that if the basis functions used were divergence free, the solution would be divergence free.

In [7], G.Mur was more concerned with the continuity of the tangential components and normal flux of the electromagnetic quantities than with spurious modes. The work relied on basis functions that provide linear interpolation on the whole finite element(FE) and not only in the interior, as was the case with Nedelec[8] in his work on continuity of the same quantities between finite elements. Mur reported excellent results for 3-D situations but did not mention any spurious mode problems.

The most complete work done on the subject of spurious modes has been conducted by Lynch and Paulsen[9, 10]. Lynch and Paulsen studied the dispersion relation for four versions of the Maxwell equation:

- The double curl equation: $\nabla \times \nabla \times \vec{E} - k^2 \vec{E} = 0$
- The Helmholtz equation: $\nabla^2 \vec{E} + k^2 \vec{E} = 0$
- The penalty equation: $\nabla \times \nabla \times \vec{E} - p \nabla(\nabla \cdot \vec{E}) - k^2 \vec{E} = 0$
- The primitive coupled Maxwell equations: $\nabla \times \vec{H} = -j\omega\epsilon\vec{E}$; $\nabla \times \vec{E} = j\omega\mu\vec{H}$

The findings were that the double curl, the penalty equation and the primitive coupled equations supported spurious modes. Some remedies are possible, but not always and not for all three approaches. The Helmholtz scheme supports physical modes that are divergence free. The enforcement of divergence free boundary conditions eliminates the potential for spurious modes.

This quick overview of what has been done with FE in 3-D illustrates some of the shortcomings that the finite element method has. There is a consensus, however, that it is one of the most promising methods in terms of versatility and generality for covering a wide range of problems. The ability of the finite element method to model any geometry or electrical property distribution, currently, gives it an edge over some of the other popular numerical techniques.

2.3 3-D Space Discretization

A finite element mesh lends itself very efficiently to modelling of all kinds of inhomogeneities in electrical characteristics and geometries. The cost is a huge amount

of mesh data that has to be stored and manipulated. The result is slower execution time and the need for large storage capabilities. This can be remedied by using a coarse mesh.

The finite element mesh, if any value is to be given to results, has to satisfy two requirements. The distance between two nodes of the finite element mesh has to be at most smaller than $\lambda/2$. λ is the wavelength of the frequency of investigation. Although the $\lambda/2$ limit stems from Nyquist's sampling theorem and theoretically should work; good results with reasonable accuracy require finer meshes in the order of $\lambda/5$ [11]. The second requirement is that the finite element mesh has to be fine enough so as to describe the smallest electrical or geometrical features in the structure. An adaptive algorithm which could create a very fine mesh around the detailed features and a coarse mesh in the other areas would optimize the number of finite elements needed for a problem.

In [12], J.D. Zhou proposed to use a function over the solution domain that returns a value proportional to the size of the finite element to be created. The same author in [13], proposed, after first creating a coarse mesh, to go through all the finite elements and subdivide them repeatedly until the mesh is fine enough. The disadvantage of such an approach is that the details of the structure are lost in the first triangularization given that the smaller finite elements are confined in the larger ones.

These schemes for mesh density are easily applied to 2-D as well as 3-D problems. The actual triangularization in 2-D and 3-D is not so straightforward. For the 2-D case, any improvised algorithm will work when enough overlapping tests are included. In [14] Nelson gives an algorithm that will triangularize any planar surface without the need to check for overlapping of finite elements during meshing. It is based on the selection of a third point to be associated with another two, such that the three points are the ones best spread on the circumference of a circle. This algorithm could not be easily extended to 3-D meshing.

Most commercially available packages that solve 3-D problems are still limited to structures with axial symmetries, or ones that use a uniform mesh. The approach of discretizing with symmetrical finite elements such as cubes which fit together nicely is easy, but does not provide much flexibility in following curved surfaces. The natural extension of the 2-D triangular finite element is the four faced tetrahedron, which allows an accurate modelling of the structure features and provides the ability to use

simplex coordinates(to be explained), thus simplifying the bases functions and the evaluation of volume and surface integrals[15].

2.4 Open Boundary Problem and the Absorbing Boundary Condition (ABC)

2.4.1 Introduction

The finite element method cannot be used to solve open boundary problems, unless a means of simulating the condition at infinity is achieved. The simple approach is to place the outer boundary very far away and mesh the interior. However, the demand on computer storage will be too great if a large domain is discretized, especially for a 3-D problem. With most problems, the area of interest is limited to a small region and solutions for the surrounding regions is not necessary. One approach has been to terminate the mesh before the fields go to zero and impose an absorbing boundary condition to represent the response of an infinite space. Another approach has been to combine the finite element method with an integral method to take care of the infinite space. This has resulted in methods called hybrid methods.

2.4.2 Local Type ABC

Local type absorbing boundary conditions are the ones that impose a value for the variation of the field quantity, in the direction normal to the outer boundary, evaluated from field values at the position of interest. These types are relatively simple to derive and implement, however, they have some limitations with regards to accuracy and general purpose usage.

Most of these ABCs are derived from the expression given by the expansion theorem for electromagnetic fields found in [16]. The expression is given here for convenience:

$$\vec{A}(r) = \frac{e^{jkr}}{r} \sum_{n=0}^{\infty} \frac{\vec{A}_n(\theta, \phi)}{r^n}, \quad (2.1)$$

where $\vec{A}$ is a vector radiating function, and the expression is valid in a region $r > \text{constant}$. Based on this equation, many authors have arrived at absorbing boundary conditions of varying accuracy and complexity using derivations and algebraic manipulations.

The most basic ABC is the one known as Sommerfeld's[17] given by:

$$\lim_{r \rightarrow \infty} \frac{\partial \vec{A}}{\partial r} = -jk\vec{A}, \quad (2.2)$$

where r is the direction of propagation.

This boundary condition is valid only when imposed very far from the sources, and hence defeats the purpose of early truncation of the domain to reduce the mesh size. J.J. Stoker[18] states that unique solutions of radiation problems cannot be determined by Sommerfeld type ABCs. The Sommerfeld condition is obtained by dropping all, except the first term of the derivative of Eq. (2.1) with respect to r . Bayliss and Turkel[19] derived a higher order ABC by using Eq. (2.1) and the wave equation. For one component u of $\vec{A}$, they obtained the expressions:

$$\frac{\partial u}{\partial r} = \alpha_2(r)u + \beta_2(r)\frac{\partial^2 u}{\partial \phi^2} \quad \text{in 2 dimensions,} \quad (2.3)$$

and

$$\frac{\partial u}{\partial r} = -jk[\alpha_3(r)u + \beta_3(r)Du] \quad \text{in 3 dimensions,} \quad (2.4)$$

where $\alpha_2, \alpha_3, \beta_2$ and β_3 are polynomial functions of r , and D is the operator defined as:

$$Df = \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial f}{\partial \theta}) + \frac{1}{\sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}. \quad (2.5)$$

There are three sources of error in expressions of the type of Eqs. (2.2), (2.3) and (2.4): i) the truncation of the series, ii) the direction of the normal to the outer boundary is not the direction of the coordinate r [20], and iii) the difference between r and $r - r_0$ is not negligible, r_0 being the position of the source. In the far-field, all sources behave as source points and furthermore, when the source is centered about the origin of the coordinate system, $r - r_0$ tends towards r . The closer the observation point is to the source, the more inaccurate this assumption becomes. In case (ii), many authors have used circular or spherical outer boundaries to be able to replace $\frac{\partial u}{\partial n}$ by $\frac{\partial u}{\partial r}$ where n is the variable in the direction of the normal, and r is the radial component. This assumption's validity depends on how close to a circular shape is the outer boundary.

Many attempts have been made to minimize the effects of these sources of error. In [21], Khebir, Kouki and Mittra derived an ABC from a modified version of Eq. (2.3). They used the progression $n = 0, kr/2, kr, \dots$ in Eq.(2.1), instead of $n = 1, 2, 3, \dots$, to obtain a higher order ABC equivalent to using more terms in the series. To simplify

the derivation, the artificial boundary was chosen such that the normal coincided with the spherical coordinate r . Some researchers have attempted to apply these differential ABCs to general boundaries without any clear advantages[22] over spherical boundaries.

An ABC that can be classified in either category, local or non-local, is the ABC developed by Ramahi[23]. Ramahi represented the field by a summation of Hankel functions in 2-D as:

$$u(r, \phi) = \sum a_n H_n^{(2)}(kr) e^{jn\phi}, \quad (2.6)$$

and a normal derivative of the field on the outer boundary written as a weighted combination of field values lying in the interior of the problem as:

$$\frac{\partial u}{\partial n} \Big|_{r_0} = c_0 u_0 + c_1 u_1 + c_2 u_2 + \dots \quad (2.7)$$

where: 0^* refers to the position where the derivative is to be evaluated,
and 0, 1, 2 refer to the positions of the nodes chosen for the evaluation.

Equation (2.7) was rewritten using (2.6) and enforced at as many harmonics as there were c coefficients in (2.7). This ABC could thus evaluate the normal variation with values of the field that are distributed all around the boundary and thus give a non-local estimate. The major drawback was the selection of the harmonics since there was no a priori knowledge of the dominant ones.

2.4.3 Non-Local Type ABC

The non-local ABC is based on Eq. (2.6). In [24], Keller and Givoli computed the coefficients a_n with a Fourier integral. Eq. (2.6) is then used to estimate the field everywhere outside an outer boundary of radius r on which the Fourier analysis was performed. A circular boundary was chosen to simplify the integration with respect to r . The derivative of the final expression was taken to obtain $\frac{\partial u}{\partial n}$. Keller and Givoli obtained some promising results in 2-D, however, a potential source of inaccuracies is that the field varies in space and the Fourier representation is based only on the surface variation of the field. Only surface field values are used to compute the Fourier coefficients and the variation of the field with respect to the normal is not considered.

2.4.4 Hybrid Methods

Hybrid methods with the FEM are often used to solve open boundary problems. Although, they cannot be called non-local ABC, they are closely related to them. The methods hybridized with FEM can be used to solve the homogeneous portion of the problem and at the same time provide an expression to terminate the FEM mesh. The finite element method has been coupled with several other numerical techniques: the multiple multipole method, the moment method and the boundary integral method. Each of these combinations, were an attempt to overcome the weakness of the FEM in tackling unbounded spaces.

Sroka, in [25], was the first to try to couple the generalized multiple multipole technique with the finite element method. The major findings were that the outer boundary did not need to be as far as when other techniques were used. The hybrid worked very effectively but more work needs to be done to improve the time efficiency of the algorithm.

The FE method has also been combined with the moment method. Xingchao[26] used the FEM to solve the interior region of a problem, the part with all the inhomogeneities, and the moment method for the exterior part. The two parts were coupled together using the equivalence principal and the continuity of the tangential components of $\vec{E}$ and $\vec{H}$.

Another hybrid method where extensive work has been done is the FEM/BEM method[1, 27], a combination of finite element and boundary element methods. B.H. McDonald in [1] used the FEM to solve the problem in the bounded region including all inhomogeneities and applied an integral constraint on the outer boundary derived from the application of Green's theorem on the boundary values. Paulsen[27] used a similar approach to solve an electromagnetic propagation problem in 3-D.

When all the inhomogeneities are enclosed in a surface on which the field values are known, the boundary integral methods can be used alone to solve the propagation problem. The formulation provided in [1, 10] can be taken up without the FEM part. The only problem is to find the appropriate Green's function. In [28], Poggio explains the derivation of the integral equation solution to the wave equation.

2.5 The Problem of Predicting Far-Field Patterns from Near-Field Measurements

Much research has been done in the attempt to solve the problem of determining far-field radiation levels and patterns from near-field information, for EMI/EMC compliance tests and antenna radiation pattern measurements. In both these cases, it is sometimes impossible or impractical to provide an open range, or obtain the far-field measurements directly in the far-field, either due to the electrical size of the apparatus or its physical size. One way to bypass this problem has been to collect near-field measurements and transform them to the far-field.

In [29], the field was expanded into a series expression and the coefficients are calculated via a Fourier analysis of the measured near-field data. The results obtained in 2-D situations compared successfully with measured values. In [30], Johnson summarized the three main techniques for obtaining far-field information. The techniques are: i) the transformation of near-field measurements, ii) making measurements on the test antenna irradiated by a plane wave, or iii) focusing the antenna and assuming that the near-field measurements are the same when the antenna is refocused to infinity. Johnson also gives guidelines to conduct measurements.

R.J. Luebbers [31] developed an algorithm to obtain far-field information from a transformation of near-field measurements using the FDTD method. T.K. Sarkar[32] used an algorithm where equivalent currents on the measurements surface are derived from the field values and plugged back into an integral expression to have the solution in the far-field. The disadvantage of this approach is that a matrix equation system has to be solved. In [33], Bucci elaborated on the sampling of the field. He proposed an optimal sampling algorithm where the effect of parameters was studied. Bucci looked at optimal sampling densities and optimal measurement surfaces in terms of shapes and positions in order to improve and simplify the near-field to far-field transformation. He used the Fourier analysis method to transform the measurements to the far-field.

Chapter 3

Mathematical Formulation of the Problem

3.1 The Model

The problem to be solved is of interest to the EMI/EMC community and has extensions into antenna testing applications. The solution is a transformation of field values measured in the vicinity of an electromagnetic source to corresponding far-field values.

In practical situations, the source would be an antenna under test or a piece of electrical equipment with parasitic radiations to be characterized. The apparatus is located above the ground and radiates into open space. This arrangement does not imply any a priori symmetries except that of the ground plane. The source of EMI is represented by an arbitrary distribution of point sources that provide a general and asymmetric field variation.

The magnetic field $\vec{H}$ in spherical coordinates created by a point source with an elementary current directed along the Cartesian $\hat{z}$ component is [33]:

$$\vec{H}(r, \theta, \phi) = \frac{I\Delta z}{4\pi} \left[\frac{j\beta}{r} + \frac{1}{r^2} \right] e^{-j\beta r} \sin \theta \hat{\phi}, \quad (3.1)$$

where: r, θ and ϕ are the coordinates of the observation point when the point source is located at the origin of the coordinate system, β is the phase velocity in free space, and $\Delta z \ll \lambda$ is the length of the filament of current amplitude I .

The contributions to the $\vec{H}$ field in Cartesian coordinates for current filaments directed in each of the three directions are given in Table 3.1 with A defined as

$$A = -\frac{I\Delta z e^{-j\beta r}}{4\pi r} \left[j\beta + \frac{1}{r} \right]. \quad (3.2)$$

	$\hat{x}$ directed current	$\hat{y}$ directed current	$\hat{z}$ directed current
H_x	0	$-A \cos \theta$	$A \sin \theta \sin \phi$
H_y	$A \cos \theta$	0	$-A \sin \theta \cos \phi$
H_z	$A \sin \theta \cos \phi$	$A \sin \theta \sin \phi$	0

Table 3.1: Contribution to the field H_i from the current filament I_i

The total contribution to the field $\vec{H}_x$ can be written as follows:

$$H_x(r, \theta, \phi) = \sum_{i=0}^{n_x} A(r, r_{oi}) 0 - \sum_{i=0}^{n_y} A(r, r_{oi}) \cos \theta' + \sum_{i=0}^{n_z} A(r, r_{oi}) \sin \theta' \sin \phi', \quad (3.3)$$

where n_x , n_y , n_z are the numbers of point sources directed respectively along $\hat{x}$, $\hat{y}$ and $\hat{z}$ axis. r_{oi} is the position of the i^{th} point source. θ' and ϕ' are functions of θ and θ_{oi} , and ϕ and ϕ_{oi} .

Similar, expressions for H_y and H_z can be written out. The point sources positioned in a cluster above ground are sketched in Fig.3.1. For clarity the sources are regrouped according to their orientation. The point source has been chosen because it has an analytical solution valid for both the near and far-field regions. It allows the construction of an EMI source of any desired degree of variation and the analytical solution can be used as a means for validation of the numerical algorithms. The total source field is obtained by superposition of the analytical solution of the individual point sources. Figure 3.2 identifies the different regions and limiting boundaries associated with Fig.3.1

The surface S in Fig.3.2 enclosing the solution domain V is made up of three segments: S_1 , S_2 , and S_3 .

$$S = S_1 \cup S_2 \cup S_3. \quad (3.4)$$

S_1 is the portion of S which encloses all sources in Fig.3.1 and is the surface on which measurements of the field values are made. In the model, these values are obtained by sampling Eq.(3.3) on S_1 for H_x , H_y and H_z . S_2 lies on the ground. In the model the ground is represented by an ideal ground plane and in the mathematical formulation to follow will be accounted for by using image theory. The last segment, S_3 , has no physical meaning but is necessary to limit the solution domain.

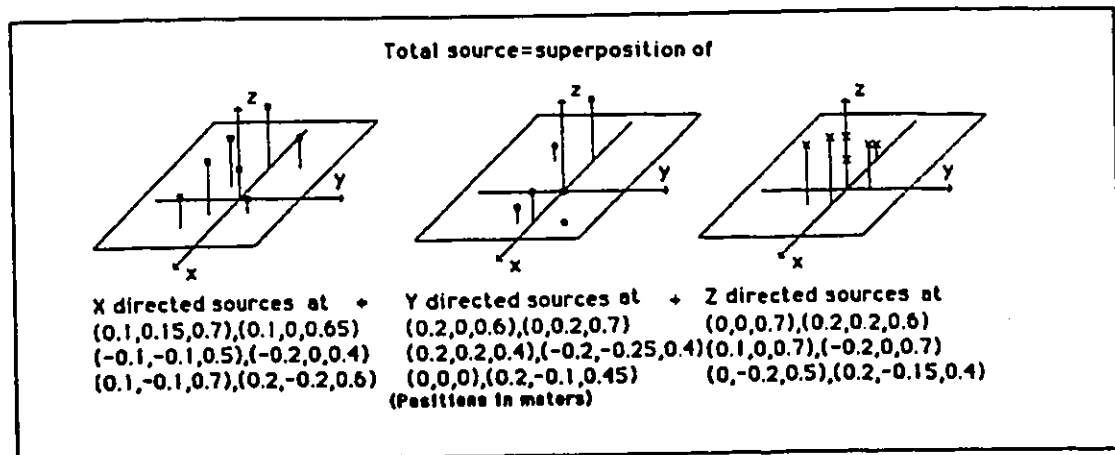


Figure 3.1: Source distribution above ground plane

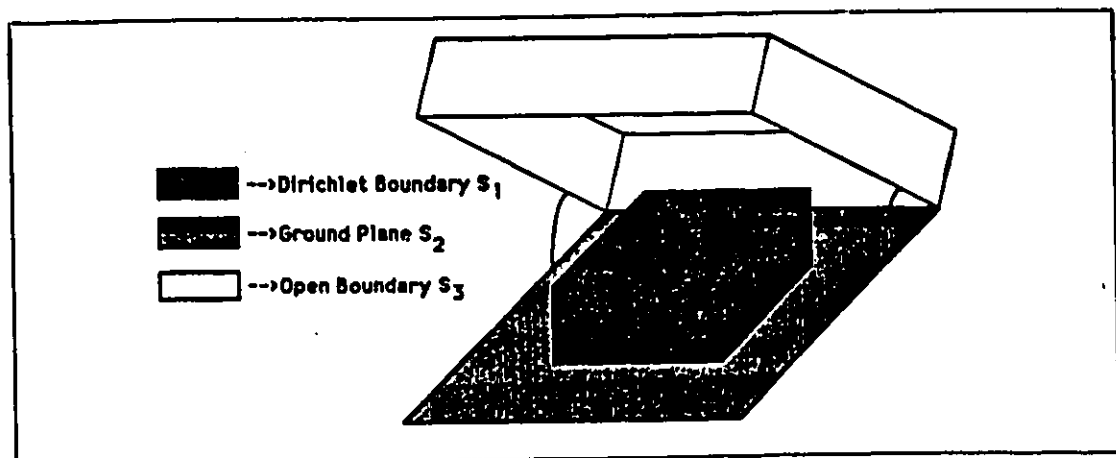


Figure 3.2: Domain and boundaries of the problem

3.2 FEM formulation

3.2.1 The Equations to Solve

In the volume V , the wave propagates in a homogeneous space. In the frequency domain, the Helmholtz wave equation describes the variation of the field components. For the magnetic field, the equation for the phasor $\underline{\vec{H}}$, is given by

$$\nabla^2 \underline{\vec{H}} + \omega^2 \mu_0 \epsilon_0 \underline{\vec{H}} = 0 \text{ in } V, \quad (3.5)$$

where ω is the phase velocity defined as $2\pi \times \text{frequency}$ and μ_0 and ϵ_0 are respectively, the magnetic permeability and dielectric constants of free space. Equation (3.5) is arrived at by assuming a time variation as $e^{j\omega t}$.

For the special case of Cartesian coordinates, the Laplacian of the magnetic field (or any vector field) can be written as:

$$\nabla^2 \underline{\vec{H}} = \hat{x} \nabla^2 H_x + \hat{y} \nabla^2 H_y + \hat{z} \nabla^2 H_z, \quad (3.6)$$

where the underlining bar representing phasors has been omitted to simplify notation. Since there are no inhomogeneities in V , Eq.(3.6) permits the uncoupling of the three Cartesian components of the magnetic field and permits the rewriting of Eq.(3.5) as three independent scalar equations:

$$\nabla^2 H_i + \omega^2 \mu_0 \epsilon_0 H_i = 0 \text{ in } V, \quad i = \hat{x}, \hat{y} \text{ or } \hat{z}. \quad (3.7)$$

This will allow solving problems three times larger than if Eq.(3.5) was used directly. However, it will limit the generality of the solution since the field components are decoupled and the domain must remain homogeneous. For more generality, one would have to replace the Laplacian by the double-curl,

$$\nabla^2 \underline{\vec{H}} = -\nabla \times \nabla \times \underline{\vec{H}}, \text{ simplified, here, because of the divergence free magnetic field.} \quad (3.8)$$

The second approach, however, is not spurious-mode free because of the cross-derivatives that arise in the double-curl[9].

A differential equation problem is not properly posed until all initial values are all specified and all boundary conditions are fixed. For the model in Fig.3.2, boundary conditions on S_1, S_2 and S_3 have to be specified to obtain the exact solution and unique solution of the problem at hand.

S_1 is the surface on which measurements will be made for the magnetic field components H_x , H_y and H_z . It will thus be a Dirichlet type boundary. S_3 is open with no constraints. To fully define the problem and obtain a unique solution a condition derived from an a priori knowledge of the solution and/or the governing equations of the problem must be applied at S_3 . This condition describes the variation of the field in the direction normal to the surface S_3 . Mathematically, it is $\frac{\partial H}{\partial n} = f$ where n is the normal to S_3 . This expression known as an absorbing boundary condition is discussed in the literature review and the forms implemented in this work are derived in Section 3.4, namely, the Sommerfeld ABC, the Bayliss-Turkel ABC and a integral type ABC. S_2 is on the ground plane; it is assumed to be a perfectly conducting surface. If the ground plane is in the $x-y$ plane, then

$$H_z = 0 \quad \text{since } H_z = \frac{j}{\omega\mu_0} \left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right], \quad (3.9)$$

and E_y, E_x , being the tangential components of the electric field on a metallic plate, are constant on the $x-y$ plane and identically zero.

From

$$\vec{E} = -\frac{j}{\omega\epsilon} \nabla \times \vec{H} = \left(-\frac{j}{\omega\epsilon}\right) \left[-\frac{\partial H_y}{\partial z} \hat{x} + \frac{\partial H_x}{\partial z} \hat{y} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{z} \right], \quad (3.10)$$

simplified using the fact that $H_z = 0$, it is deduced that

$$\frac{\partial H_y}{\partial z} = 0 \quad \text{and} \quad \frac{\partial H_x}{\partial z} = 0, \quad (3.11)$$

which for a ground in the $x-y$ plane becomes:

$$\frac{\partial H_y}{\partial n} = 0 \quad \text{and} \quad \frac{\partial H_x}{\partial n} = 0 \quad (3.12)$$

given that the normal $\hat{n} = \hat{z}$. A condition such as the one given in Eq.(3.12) is known as a homogeneous Neumann boundary condition. In table format, the summary of the boundary condition for the whole boundary S is shown in Table 3.2.1.

3.2.2 The Weighted Residual Method

Solving the problem using the Finite Element Method consists of the following: solving Eq.(3.7) on small patches, called finite elements(FE), which when combined form the domain V ; and, satisfying the boundary conditions given in Table 3.2.1. This scheme effectively breaks down the problem in a multitude of smaller problems to be

	S_1 Measuring surface	S_2 Ground plane	S_3 Open boundary
H_x	Dirichlet	$\frac{\partial H_x}{\partial n} = 0$	ABC
H_y	Dirichlet	$\frac{\partial H_y}{\partial n} = 0$	ABC
H_z	Dirichlet	$H_z = 0$	ABC

Table 3.2: Boundary conditions for the problem

solved simultaneously and simplifies the search for a solution by reducing the requirements to be met on the whole of V to requirements on the small FE patches. It is easier to find an expression, called interpolation or trial function, that fits the exact solution on a smaller domain, than on a larger one. The fitting process translates itself via an integral formulation into a set of linear algebraic equations for every patch. The variables solved for are parameters in the trial functions that make these functions a best fit. The method through which one can obtain the system of algebraic equations is one of two choices. The variational approach or the weighted residual approach. The variational approach is based on minimizing a functional; an integral expression derived from the governing equations of the problems and its boundary conditions. In some cases, this process turns out to be a minimization of the energy in the system in accordance with the state of minimum energy principle of physics. The disadvantage is that the functional has to be derived and varies from one problem to the other. The second approach is the weighted residual method. Here, the integral over the domain of the weighted error expression defined as the difference between the right hand side of the governing equation and the left hand side when the approximate solution is used, is forced to zero. This procedure usually will yield the same solution as the variational approach.

As it applies to the problem described in Section 3.1; if only one of the components is considered, say H_x , the problem is formulated as:

$$\nabla^2 H_x + k^2 H_x = 0 \text{ in } V, \quad (3.13)$$

$$H_x = g \text{ on } S_1,$$

$$\frac{\partial H_x}{\partial n} = 0 \text{ on } S_2,$$

$$\frac{\partial H_x}{\partial n} = ABC \text{ on } S_3.$$

(3.14)

The function g is the function describing the values of the field H_x on S_1 and $k^2 = \omega^2 \mu_0 \epsilon_0$.

Assume an approximate solution over a FE i as $\tilde{H}_{xi}$; the expression for the error E is then

$$\nabla^2 \tilde{H}_{xi} + k^2 \tilde{H}_{xi} = E \quad \text{in } V_i. \quad (3.15)$$

For a weighting function W , the weighted integral becomes

$$\int_{V_i} W (\nabla^2 \tilde{H}_{xi} + k^2 \tilde{H}_{xi}) dV_i = \int_{V_i} W E dV_i \quad (3.16)$$

where V_i is the domain of the i^{th} FE. When $\tilde{H}_{xi}$ tends towards the solution, E tends to zero.

Equation (3.16) is implemented for all the finite elements that form the domain of the solution. The patch work of $\tilde{H}_{xi}$ solutions will make up the global approximate problem solution $\tilde{H}_x$.

In the case of open problems, the domain V is infinite. To limit V to the region of interest and explicitly include the boundary condition defining the problem in Eq.(3.16), Green's first theorem is used. Green's first theorem will also reduce the requirements on the continuity of the derivatives of the patch solutions $\tilde{H}_{xi}$. As it is now, Eq.(3.16) contains a Laplacian term which involves second degree derivatives. To have well defined integrals of the Laplacian, $\tilde{H}_{xi}$ has to have a continuous first derivative; if the Laplacian is reduced to a gradient, $\tilde{H}_{xi}$ need only be continuous. Green's first theorem is as follows[35]:

$$\int_V (\nabla u \cdot \nabla v) dV = \oint_S u \frac{\partial v}{\partial n} dS - \int_V u \nabla^2 v dV \quad (3.17)$$

where: V is the domain,

S is the boundary enclosing V ,

u and v are two continuous functions with continuous second derivatives.

When Eq.(3.17) is substituted in Eq.(3.16) with u replaced by W and v by $\tilde{H}_{xi}$, the weighted residual integral becomes

$$\int_{V_i} (\nabla W_i \cdot \nabla \tilde{H}_{xi}) dV_i - k^2 \int_{V_i} W \tilde{H}_{xi} dV_i - \int_{S_i} W_i \frac{\partial \tilde{H}_{xi}}{\partial n} dS_i = 0 \quad (3.18)$$

where: V_i is the domain of a FE,

S_i is the boundary enclosing V_i ,

and E , the error, has been set to zero.

3.2.3 Tetrahedral Element and Interpolation Functions

The tetrahedral element in Fig.3.3 has been chosen to subdivide the domain V into finite subvolumes V_i . This choice, as will be seen in the following development, is the best one when it comes to representing irregular geometries. The mathematical properties the tetrahedral shape possesses, allow a high degree of automation in performing derivations and integrals analytically.

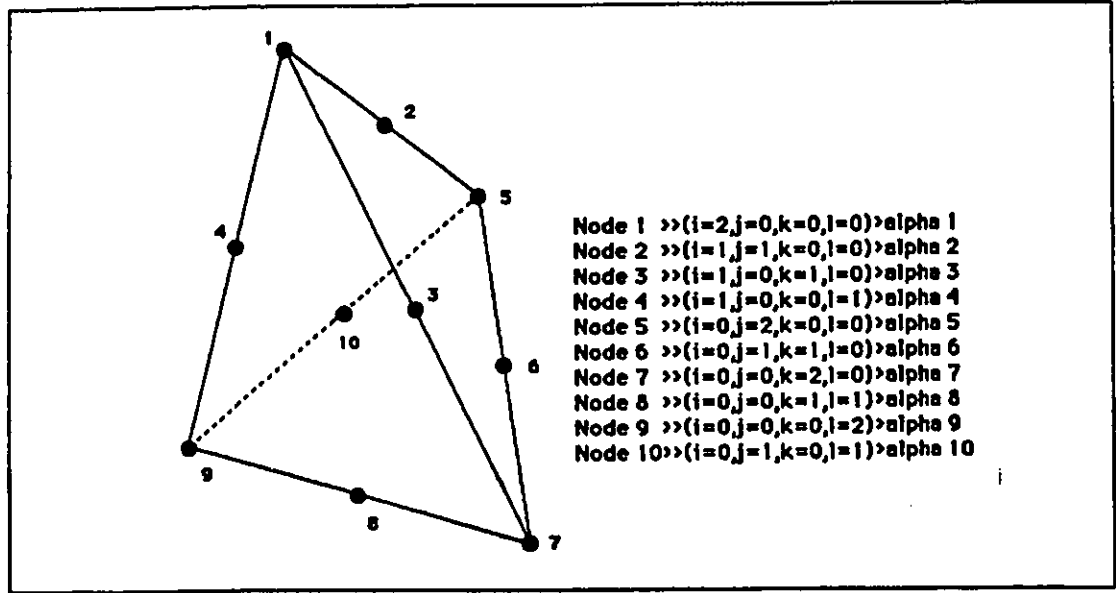


Figure 3.3: The tetrahedral element and its node indexing

The Simplex and Simplex Coordinates

A simplex S in N -dimensional space is the minimal non-trivial geometric figure in that space. In 2-D, it's a triangle defined by 2+1 vertices. In 3-D, it's a tetrahedron defined by 3+1 vertices.

When a point P is positioned inside an N space simplex, it divides it into $N + 1$ subsimplexes each of which has P as a vertex and the remaining N chosen from the $N + 1$ vertices of the original simplex. See Figure 3.4. The ratio of the size of the $N + 1$ subsimplexes to the size of the original simplex define $N + 1$ local coordinates called simplex coordinates. These coordinates define exactly the position of the point P in the simplex S . The size of a simplex in 1-D refers to its length, in 2-D to its area and in 3-D to its volume.

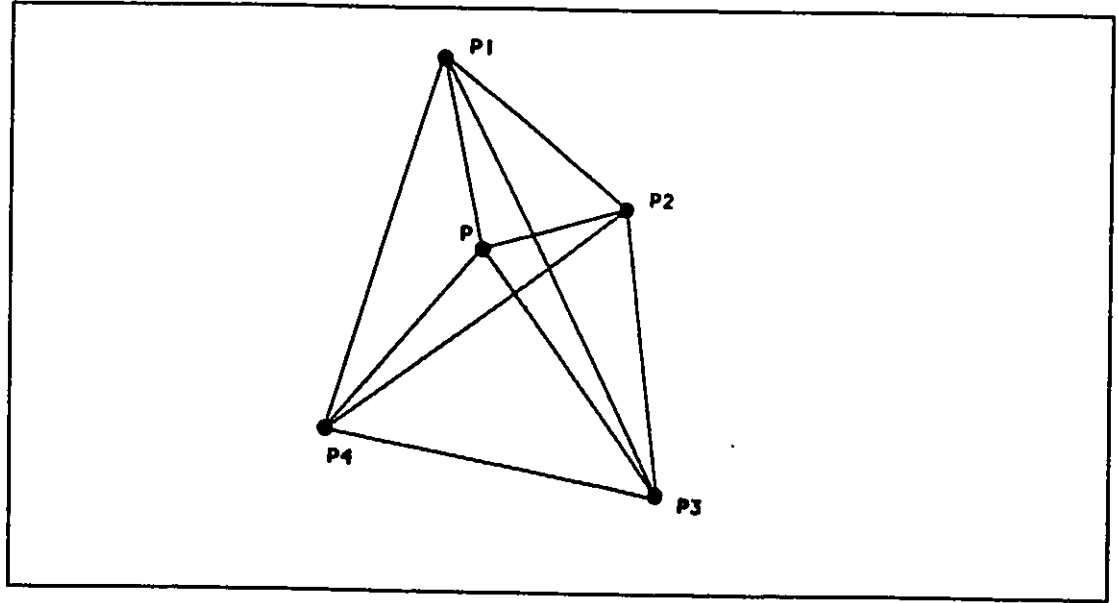


Figure 3.4: One simplex divided in four subsimplices

Let $\delta_1, \delta_2, \delta_3, \delta_4$ denote the four simplex coordinates and P_1, P_2, P_3, P_4 represent the four vertices of a simplex S . The coordinates of a point P inside S are given by:

$$\begin{aligned}\delta_1 &= \frac{V_1}{V} & \delta_2 &= \frac{V_2}{V} \\ \delta_3 &= \frac{V_3}{V} & \delta_4 &= \frac{V_4}{V}\end{aligned}\tag{3.19}$$

where: V is the volume of the simplex S

V_1 is the volume of the subsimplex having P, P_2, P_3, P_4 as vertices,

V_2 is the volume of the subsimplex having P, P_1, P_3, P_4 as vertices,

V_3 is the volume of the subsimplex having P, P_1, P_2, P_4 as vertices,

V_4 is the volume of the subsimplex having P, P_1, P_2, P_3 as vertices.

To note that when P coincides with one of the vertices P_1, P_2, P_3 or P_4 , say P_1 , then $\delta_1 = 1$. When P falls on a face defined by say P_1, P_2, P_3 then $\delta_4 = 0$. A point P on an edge travelling from one vertex P_1 to P_2 will see the simplex coordinate δ_1 vary from 0 to 1. When P is halfway between P_1 and P_2 , $\delta_1 = 1/2$.

The volume of the tetrahedron simplex defined by four vertices P_1, P_2, P_3, P_4 such that the coordinates of vertex P_i is

$$P_i \text{ at } (x = x_i, y = y_i, z = z_i)\tag{3.20}$$

is given by

$$V = \frac{1}{3!} \begin{vmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix}. \quad (3.21)$$

When the point P at (x, y, z) is inside S , we can define the coordinate simplexes $\delta_1, \delta_2, \delta_3, \delta_4$ as follows:

$$\begin{aligned} \delta_1 &= \frac{1}{3!V} \begin{vmatrix} 1 & x & y & z \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix} & \delta_2 &= \frac{1}{3!V} \begin{vmatrix} 1 & x & y & z \\ 1 & x_1 & y_1 & z_1 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix} \\ \delta_3 &= \frac{1}{3!V} \begin{vmatrix} 1 & x & y & z \\ 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix} & \delta_4 &= \frac{1}{3!V} \begin{vmatrix} 1 & x & y & z \\ 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \end{vmatrix} \end{aligned}$$

Using the cofactor expansion along the first row, the simplex coordinates can be rewritten as

$$\begin{aligned} \delta_1 &= A_1 + B_1x + C_1y + D_1z \\ \delta_2 &= A_2 + B_2x + C_2y + D_2z \\ \delta_3 &= A_3 + B_3x + C_3y + D_3z \\ \delta_4 &= A_4 + B_4x + C_4y + D_4z \end{aligned} \quad (3.22)$$

where: A_i is the cofactor of entry e_{11} in the definition of δ_i ,
 B_i that of entry e_{12} ,
 C_i that of entry e_{13} ,
 D_i that of entry e_{14} .

Equation (3.22) will define a point P as a function of local simplex coordinates $\delta_1, \delta_2, \delta_3, \delta_4$, themselves being functions of the global Cartesian coordinates.

Interpolation in a Tetrahedron

To be able to approximate the solution $\tilde{H}_{xi}$ inside a tetrahedral FE, one must use interpolation functions that are defined inside the FE. The variation they will provide should represent the variation of the real solution H_{xi} over the FE. The solution $\tilde{H}_{xi}$

will be locally defined by values of the field H_x at special positions in the FE called nodes.

The choice of the order of these interpolation functions depends on many factors. In 2-D, it has been found that it is better to use a coarse meshing, i.e. larger finite elements, and higher order interpolation functions than a fine mesh, and lower (first) order interpolation[14]. For a situation of similar node numbers, the former case will achieve a smoother fit due to the interpolation functions curvatures as opposed to a linear segment fitting. No similar result has been documented for 3-D problems. A complete second degree polynomial interpolation function set defined by 10 nodes values has been chosen for this work. The polynomials, called alpha functions and denoted as α can be determined via the following expression:

$$\begin{aligned} R_m(n_0, \delta) &= \frac{1}{m!} \prod_{k=0}^{m-1} (n_0 \delta - k) \quad m > 0 \\ \alpha_{ijkl} &= R_i(n_0, \delta_1) R_j(n_0, \delta_2) R_k(n_0, \delta_3) R_l(n_0, \delta_4) \\ R_0(n_0, \delta) &= 1 \end{aligned} \quad (3.23)$$

where: $i + j + k + l = n_0$, n_0 being the order of the polynomial.

The alpha functions are obtained by permutating the values 0,1 and 2 between i, j, k and l and keeping the sum $n_0 = 2$. The ten functions in sequence are:

$$\begin{aligned} \alpha_1 &= 2\alpha_1^2 - \alpha_1 & \alpha_6 &= 4\alpha_2\alpha_3 \\ \alpha_2 &= 4\alpha_1\alpha_2 & \alpha_7 &= \alpha_3^2 - \alpha_3 \\ \alpha_3 &= 4\alpha_1\alpha_3 & \alpha_8 &= 4\alpha_3\alpha_4 \\ \alpha_4 &= 4\alpha_1\alpha_4 & \alpha_9 &= 2\alpha_4^2 - \alpha_4 \\ \alpha_5 &= 2\alpha_2^2 - \alpha_2 & \alpha_{10} &= 4\alpha_2\alpha_4. \end{aligned} \quad (3.24)$$

These functions are associated with their respective node values of $\tilde{H}_{xi}$ as displayed on Fig.3.3. The node positions are such that the alpha function is identically one on its corresponding node, zero on all the other nodes and outside the FE, and different then zero everywhere else. The interpolation polynomials have one important advantage. They are expressed in terms of local coordinates and thus do not depend on global coordinate transforms. The work involved in evaluating Eq.(3.16) with these functions can easily be programmed because the integration variables $\delta_1, \delta_2, \delta_3, \delta_4$ always vary from zero to one.

The approximate solution $\tilde{H}_{xi}$ written with the functions given in Eq. (3.24) is:

$$\tilde{H}_{xi} = \sum_{i=0}^{10} \alpha_i(\delta_1, \delta_2, \delta_3, \delta_4) h_{xi} \quad (3.25)$$

where: h_{xi} is the value of H_x at node i .

The solution $\tilde{H}_{xi}$ will match h_{xi} exactly at the node positions and will be interpolated in between.

3.2.4 Single Element Contribution

The single element contribution is obtained from the evaluation of the integral in Eq.(3.16) for one FE and the use of the expanded expression of Eq. (3.25).

When W , the weighting function used in Eq.(3.16) is defined as

$$W = \frac{\partial \tilde{H}_{xi}}{\partial h_{xi}} = \alpha_i, \quad (3.26)$$

there will be ten possible weighting functions consisting of the ten interpolation alpha functions. This manner of defining the weighting functions is known as the Galerkin approach of the weighted residual method.

Substituting the ten W_i 's and the expression of Eq.(3.25) for $\tilde{H}_x$, one obtains ten integral equations of the form

$$\int_{V_i} [\nabla \alpha_i \cdot \nabla (\sum_{j=1}^{10} \alpha_j h_{xj}) - k^2 \alpha_i (\sum_{j=1}^{10} \alpha_j h_{xj})] dV_i - \int_{S_i} \alpha_i \frac{\partial H_x}{\partial n} dS_i = 0 \quad i = 1, 2, \dots \quad (3.27)$$

where V_i and S_i are ,respectively, the volume and surface of the i^{th} FE.

It is possible to take all constants and summation signs outside the integrals because of the linearity property:

$$\sum_{j=1}^{10} h_{xj} \int_{V_i} (\nabla \alpha_i \cdot \nabla \alpha_j - k^2 \alpha_i \alpha_j) dV_i - \int_{S_i} \alpha_i \frac{\partial H_x}{\partial n} dS_i = 0 \quad i = 1, 2, \dots \quad (3.28)$$

If the first integral is denoted by $E1_{ij}$, the second by $E2_{ij}$ and the last by $E3_{ij}$, the full system of linear algebraic equations can be written out in matrix form:

$$\begin{bmatrix} A_{1,1} & \dots & A_{1,10} \\ \vdots & & \\ A_{10,1} & & A_{10,10} \end{bmatrix} \begin{bmatrix} H_{x1} \\ H_{x2} \\ \vdots \\ H_{x10} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (3.29)$$

where $A_{ij} = E1_{ij} - E2_{ij} - E3_{ij}$.

The evaluation of the E terms can easily be systemized due to the type of interpolation functions used. For the first term $E1_{ij}$, the expression to be evaluated works

out to be:

$$\int_{V_i} (\nabla \alpha_i \cdot \nabla \alpha_j) dV_i = \int_{V_i} \left(\frac{\partial \alpha_i}{\partial x} \frac{\partial \alpha_j}{\partial x} + \frac{\partial \alpha_i}{\partial y} \frac{\partial \alpha_j}{\partial y} + \frac{\partial \alpha_i}{\partial z} \frac{\partial \alpha_j}{\partial z} \right) dV_i \quad (3.30)$$

With the use of the chain rule and Eq.(3.22), the derivatives with respect to the global Cartesian coordinates can be expressed in terms of the derivatives with respect to the local simplex coordinates as,

$$\begin{aligned} \frac{\partial \alpha_i}{\partial x} &= \frac{\partial \alpha_i}{\partial \delta_1} \frac{\partial \delta_1}{\partial x} + \frac{\partial \alpha_i}{\partial \delta_2} \frac{\partial \delta_2}{\partial x} + \frac{\partial \alpha_i}{\partial \delta_3} \frac{\partial \delta_3}{\partial x} + \frac{\partial \alpha_i}{\partial \delta_4} \frac{\partial \delta_4}{\partial x} \\ &= B_1 \frac{\partial \alpha_i}{\partial \delta_1} + B_2 \frac{\partial \alpha_i}{\partial \delta_2} + B_3 \frac{\partial \alpha_i}{\partial \delta_3} + B_4 \frac{\partial \alpha_i}{\partial \delta_4} = \sum_{k=1}^4 B_k \frac{\partial \alpha_i}{\partial \delta_k}. \end{aligned} \quad (3.31)$$

Similarly for the other terms in (Eq.3.30) so that E1 becomes:

$$\begin{aligned} \int_{V_i} (\nabla \alpha_i \cdot \nabla \alpha_j) dV_i &= \int_{V_i} \left(\sum_{k=1}^4 B_k \frac{\partial \alpha_i}{\partial \delta_k} \right) \left(\sum_{k=1}^4 B_k \frac{\partial \alpha_j}{\partial \delta_k} \right) + \\ &\quad \left(\sum_{k=1}^4 C_k \frac{\partial \alpha_i}{\partial \delta_k} \right) \left(\sum_{k=1}^4 C_k \frac{\partial \alpha_j}{\partial \delta_k} \right) + \left(\sum_{k=1}^4 D_k \frac{\partial \alpha_i}{\partial \delta_k} \right) \left(\sum_{k=1}^4 D_k \frac{\partial \alpha_j}{\partial \delta_k} \right) dV_i. \end{aligned} \quad (3.32)$$

The second term $E2_{ij}$ does not need to be manipulated and can be integrated directly.

The last term $E3$ is where the ABC is applied. The different ABC's in this work are derived in Section 3.4 of this chapter. The following section explains how the ABC's are incorporated in the FE method via the term $E3$.

The actual evaluation of the integrals in simplex coordinates is worked out in Appendix B for volume and surface integrals.

3.2.5 Implementation of ABC's

The first ABC to be implemented is the one known as the Sommerfeld's ABC. It approximates the variation of the field in the normal direction to the boundary as

$$\frac{\partial H_z}{\partial n} = -jkH_z. \quad (3.33)$$

When Eq.(3.33) is substituted for the term $\frac{\partial H_z}{\partial n}$ in $E3$, it becomes

$$\int_{S_i} \alpha_i \frac{\partial H_z}{\partial n} dS_i = -jk \sum_{j=1}^{10} h_{zi} \int_{S_i} \alpha_i \alpha_j dS_i \quad (3.34)$$

This term is now similar to $E2$ and its evaluation is straightforward. Eq.(3.34) differs from $E2$ only by a constant factor jk and by the fact that $E2$ involves a volume integral and Eq.(3.34) is a surface integral.

A higher order ABC similar to Sommerfeld's is the one derived by Bayliss and Turkel[18]. It defines the normal derivative of the field as

$$\frac{\partial H_x}{\partial n} = -jkH_x - \frac{H_x}{r}. \quad (3.35)$$

With the Bayliss-Turkel ABC, $E3$ becomes

$$\int_{S_i} \alpha_i \frac{\partial H_x}{\partial n} dS_i = - \sum_{j=1}^{10} h_{xj} \int_{S_i} (jk + \frac{1}{r}) \alpha_i \alpha_j dS_i \quad (3.36)$$

The term with r inside the integral can be taken out if the outer boundary of the problem on which S_i falls is chosen spherical. r , then, refers to the distance from the source to the boundary and can be considered constant.

A different ABC, more involved in computation, but more accurate theoretically, is an ABC that defines

$$\frac{\partial H_x(\vec{r})}{\partial n} = \frac{1}{4\pi} \oint_{S'} (H_x(\vec{r}') \frac{\partial^2 \phi(\vec{r}, \vec{r}')}{\partial n \partial n'} - \frac{\partial \phi(\vec{r}, \vec{r}')}{\partial n} \frac{\partial H_x(\vec{r}')}{\partial n}) dS'. \quad (3.37)$$

This ABC is derived in detail in the Section 3.3.2. The surface integral defining $E3$, once (3.37) is used, becomes

$$\int_{S_i} \alpha_i \frac{\partial H_x(\vec{r})}{\partial n} dS_i = \frac{1}{4\pi} \int_{S_i} \oint_{S'} \alpha_i [H_x(\vec{r}') \frac{\partial^2 \phi(\vec{r}, \vec{r}')}{\partial n \partial n'} - \frac{\partial \phi(\vec{r}, \vec{r}')}{\partial n} \frac{\partial H_x(\vec{r}')}{\partial n}] dS' dS_i. \quad (3.38)$$

The first integral is computed over the face S_i of the tetrahedral element using a Riemman sum with $n = 4$. The second integral is evaluated similarly; the kernel is described in more detail in Section 3.2.2.

The use of this last ABC will reduce the sparsity of the system S obtained with the finite element method. Unlike the two previous ABC's whose contributions are functions of the nodes on the finite element being handled, the new ABC has contributions in terms of all the nodes on a whole layer of finite elements enclosing the source.

3.2.6 Notes on The Global System Matrix S

After computing all the expressions for the single element contribution, the matrix system is incorporated into a larger linear system of equations S representing the whole domain of the problem. The rows in the smaller single element contribution system are added to rows in S corresponding to the same global node numbering.

Rows corresponding to Dirichlet nodes in the problem are all zeroed except for the diagonal entry which is set to 1. The entry on the right hand side is set to the value of the Dirichlet node. This scheme eliminates the need for row and column shifting and node number tracking when a row is deleted from the system S . In matrix form, the system is represented as

$$[S][H_x] = [B] \quad (3.39)$$

where: $[S]$ is the square system of linear algebraic equations describing the whole problem,
 $[H_x]$ is the column vector of nodal unknowns,
 $[B]$ is the right-hand side filled with the Dirichlet values.

The same procedures are repeated to obtain solutions for H_y and H_z .

3.3 Boundary Integral Formulation(BIM)

3.3.1 Introduction

The FEM, with today's computing capabilities, does not permit a solution for a very large domain to be computed. The limitation encountered when working with open problems may be unacceptable and other methods have to be considered. When the inhomogeneities are enclosed by a surface S' , integral methods become more suitable. They are, in these situations easier to implement, less demanding in computing requirements and provide a faster solution. One such method is the Boundary Integral Method or BIM.

The BIM will provide an alternative to the FEM and allow a comparison between two numerical methods. It will also be combined with the FEM to yield an alternative ABC. In the next section, the integrals to be used for BIM are derived and the adaptation of the method to the situation described in Section 3.2.1 is described.

3.3.2 Derivation of the Integral Expression

The expression needed is a solution to a boundary value problem in a three-dimensional domain. Starting with Green's first identity in vector form,

$$\int_V (\vec{Q} \cdot \nabla \times \nabla \times \vec{P} - \vec{P} \cdot \nabla \times \nabla \times \vec{Q}) dV = \int_{S'=S'_1 \cup S'_2} (\vec{P} \times \nabla \times \vec{Q} - \vec{Q} \times \nabla \times \vec{P}) d\vec{S}' \quad (3.40)$$

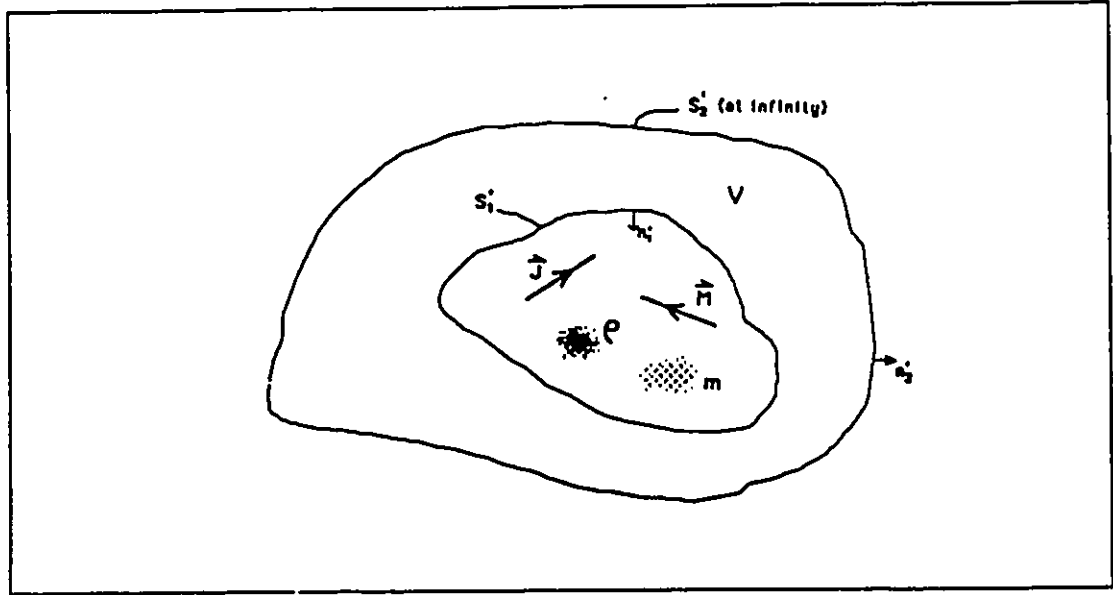


Figure 3.5: General volume enclosed by a surface

where: V is an arbitrary volume,
 S' is the surface enclosing V ,

$\vec{Q}$ and $\vec{P}$ are two vector functions with continuous 1st and 2nd derivatives.
 As it applies to our situation, V is homogeneous and source free. It is also infinite.
 Figure 3.5 illustrates the extent of V , S_1 and the terminology used in the equations to follow.

The choice of the vector functions $\vec{P}$ and $\vec{Q}$ is completely arbitrary. Since the problem is solved in a homogeneous open domain, one can replace $\vec{Q}$ by

$$\vec{Q} = \hat{a}\phi(\vec{r}, \vec{r}') = \hat{a} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \quad (3.41)$$

where: $\hat{a}$ is a unit vector of arbitrary orientation,

$\vec{r}, \vec{r}'$ are respectively the observation and source point position vectors.

$\vec{Q}$ defined as in Eq.(3.41) is recognized to be the Green's function representing the magnetic vector potential due to a point source in a homogeneous domain. Any other $\vec{Q}$ satisfying the particular boundary conditions of a problem can be used. $\vec{P}$ is chosen to be the magnetic field $\vec{H}$.

In the frequency domain the following equations will hold and will be useful for derivation:

$$\begin{aligned}\nabla \times \vec{E} &= -j\omega\mu\vec{H} - \vec{M}; & \nabla \cdot \vec{E} &= \frac{\rho}{\epsilon} \\ \nabla \times \vec{H} &= j\omega\epsilon\vec{E} + \vec{J}; & \nabla \cdot \vec{H} &= \frac{m}{\mu} \\ \nabla \cdot \vec{J} &= -j\omega\rho; & \nabla \cdot \vec{M} &= -j\omega m\end{aligned}$$

$$\nabla^2\phi - K^2\phi = 0 \quad (3.42)$$

$$\nabla \times \nabla \times \vec{E} - k^2\vec{E} = -j\omega\mu\vec{J} - \nabla \times \vec{M} \quad (3.43)$$

$$\nabla \times \nabla \times \vec{H} - k^2\vec{H} = -j\omega\epsilon\vec{M} + \nabla \times \vec{J} \quad (3.44)$$

where: ω is equal to 2π *frequency

$\vec{J}$ and $\vec{M}$ are the electric and fictitious magnetic current densities

ρ and m are the electric and fictitious magnetic current densities

ϵ and μ are the scalar permittivity and permeability for a homogeneous isotropic media.

If the magnetic field $\vec{H}$ is substituted for $\vec{P}$ and the Green's function $\phi\hat{a}$ for $\vec{Q}$, Eq.(3.40) becomes

$$\int_V (\phi\hat{a} \cdot \nabla \times \nabla \times \vec{H} - \vec{H} \cdot \nabla \times \nabla \times (\phi\hat{a})) dV = \int_{S'_1=S'_1} (\vec{H} \times \nabla \times \phi\hat{a} - \phi\hat{a} \times \nabla \times \vec{H}) d\vec{S}' \quad (3.45)$$

The surface integral over S'_2 has been reduced to zero since it extends to infinity and the H-field becomes null.

The first term in the integral can be, with the help of Eq.(3.44), written as

$$\int_V (\phi\hat{a} \cdot \nabla \times \nabla \times \vec{H}) dV = \int_V \hat{a} \cdot [k^2\vec{H}\phi - j\omega\epsilon\vec{M}\phi + (\nabla \times \vec{J})\phi] dV \quad (3.46)$$

By making use of the vector identities A1, A2, A3, A4 and A5 (in Appendix A), the divergence theorem, and the fact that the Green's function satisfies Helmholtz equation as in Eq.(3.42), the second term in Eq.(3.45) can be transformed into

$$\begin{aligned}\int_V (\vec{H} \cdot \nabla \times \nabla \times (\phi\hat{a})) dV &= \int_V [\nabla \cdot [(\hat{a} \cdot \nabla \phi)\vec{H}] - (\hat{a} \cdot \nabla \phi)(\nabla \cdot \vec{H}) + \vec{H} \cdot \hat{a}k^2\phi] dV \\ &= \int_V [\vec{H} \cdot \hat{a}k^2\phi - (\hat{a} \cdot \nabla \phi)\frac{m}{\mu}] dV + \\ &\quad \oint_{S'_1} (\hat{a} \cdot \nabla \phi)\vec{H} \cdot \hat{n} dS'.\end{aligned} \quad (3.47)$$

The right-hand side of Eq.(3.45), with the identity A6 becomes

$$\oint_{S'_1} (\vec{H} \times \nabla \phi\hat{a} - \phi\hat{a} \times \nabla \vec{H}) d\vec{S}' = \oint_{S'_1} \hat{a} \cdot [(\hat{n}'_1 \times \vec{H}) \times \nabla \phi + \phi\hat{a} \cdot (\hat{n}'_1 \times (\nabla \times \vec{H}))] dS'_1 \quad (3.48)$$

The last terms in Eqs.(3.46) and (3.48) can be rewritten as

$$\int_V (\nabla \times \vec{J})\phi \, dV = \int_V (\vec{J} \times \nabla\phi) \, dV + \oint_{S'_1} (\hat{n}'_1 \times \vec{J}\phi) \, dS'_1 \quad (3.49)$$

and

$$\oint_{S'_1} \phi \hat{a} \cdot (\hat{n}'_1 \times (\nabla \times \vec{H})) \, dS'_1 = \oint_{S'_1} \hat{a} \cdot [j\omega\epsilon(\hat{n}'_1 \times \vec{E})\phi + \hat{n}'_1 \times \vec{J}\phi] \, dS'_1 \quad (3.50)$$

with the the help of A6, A7 and Eq.(3.53).

By recombining Eqs.(3.46),(3.47),(3.48) and replacing terms defined by the identities (3.49) and (3.50), Eq.(3.45) becomes:

$$\int_V (-j\omega\epsilon\vec{M}\phi + (\vec{J} \times \nabla\phi) + \frac{m}{\mu} \nabla\phi) \, dV = \oint_{S'_1} [j\omega\epsilon(\hat{n}'_1 \times \vec{E})\phi + (\hat{n}'_1 \times \vec{H} \times \nabla\phi + (\hat{n}'_1 \cdot \vec{H})\nabla\phi)] \, dS'_1 \quad (3.51)$$

The volume integral will reduce to zero if S'_1 encloses all the sources and V becomes a homogeneous domain. Equation (3.51) then simplifies to

$$\oint_{S'_1} [j\omega\epsilon(\hat{n}'_1 \times \vec{E})\phi + (\hat{n}'_1 \times \vec{H} \times \nabla\phi + (\hat{n}'_1 \cdot \vec{H})\nabla\phi)] \, dS'_1 = 0 \quad (3.52)$$

On the imaginary surface S'_1 , there will be no physical current density $\vec{J}$ and thus,

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E}. \quad (3.53)$$

Equation (3.53) can be used to write EQ.(3.52) in terms of the magnetic field $\vec{H}$ only;

$$\oint_{S'_1} [\hat{n}'_1 \times (\nabla \times \vec{H})\phi + (\hat{n}'_1 \times \vec{H} \times \nabla\phi + (\hat{n}'_1 \cdot \vec{H})\nabla\phi)] \, dS'_1 = 0 \quad (3.54)$$

This last equation has a simplified form in the case of Cartesian coordinates and can be reduced to

$$\oint_{S'_1} [\vec{H} \frac{\partial\phi}{\partial n'_1} - \phi \frac{\partial\vec{H}}{\partial n'_1}] \, dS'_1 = 0 \quad (3.55)$$

Identities A2, A3, A8 and A9 are used for the transformation.

The definition of the Green's function as in Eq.(3.41) will have a singularity when r' , the integration variable, is equal to r . The singularity has to be eliminated since the H-field is differentiable and continuous everywhere and Eq.(3.55) is a finite integral. The discontinuity in ϕ will occur when the observation point r lies on the surface S'_1 . To circumvent this problem, the integral over S'_1 is partitioned into three integrals

$$\oint_{S'_1} [\vec{H} \frac{\partial\phi}{\partial n'_1} - \phi \frac{\partial\vec{H}}{\partial n'_1}] \, dS'_1 + \oint_{S_2} [\dots] \, dS_2 + \oint_{S_s} [\dots] \, dS_s = 0 \quad (3.56)$$

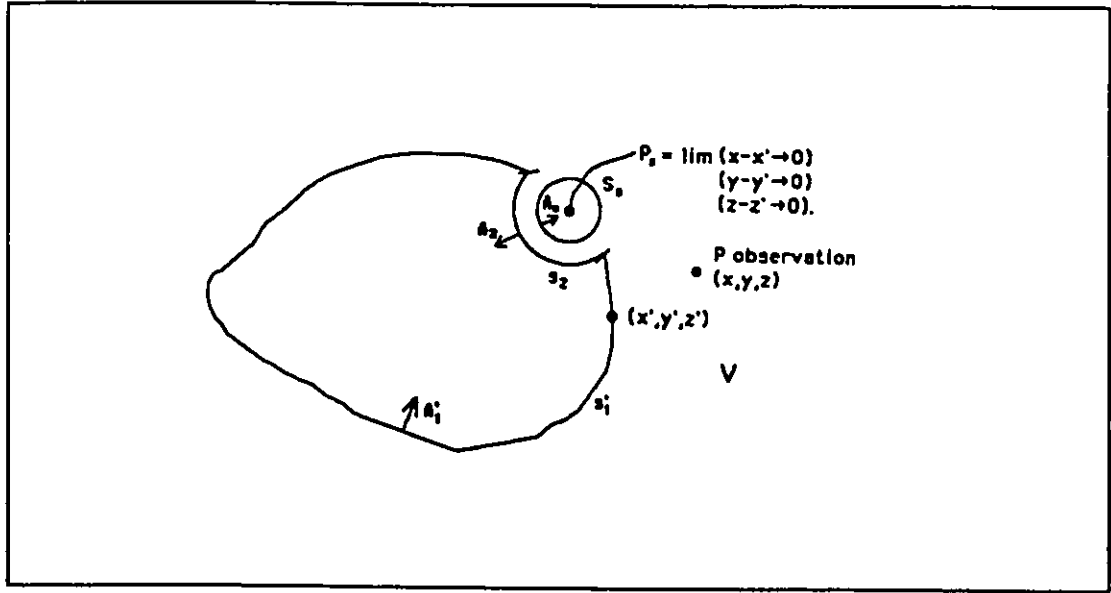


Figure 3.6: Surfaces of integration around the singularity

where S_1 , S_2 and S_s are depicted on Fig.3.6.

The center of S_s is at $P_s = |r - r'| = 0$, and because $\hat{n}_s$ is directed inward,

$$\frac{\partial \phi}{\partial n_s} \simeq -\frac{\partial \phi}{\partial r}. \quad (3.57)$$

Over S_s , chosen as a sphere, the variable of integration is constant and is denoted by r_s .

$$\frac{\partial \phi}{\partial n_s} = -\frac{\partial \phi}{\partial r}|_{r=r_s} = -\frac{e^{-jkr_s}}{r_s} \left[jk + \frac{1}{r_s} \right] \quad (3.58)$$

The integrals over S_s and S_2 , as r_s tends to 0, become two values to be added to the result of the integral over S_1^1 . If $\bar{H}_x$ and $\frac{\partial \bar{H}_x}{\partial n}$ are defined as the average values, respectively, of the x-component of $\bar{H}$ and $\frac{\partial \bar{H}}{\partial n}$ at r_s , Eq.(3.56) can be written as

$$\lim_{r_s \rightarrow 0} \left[-\bar{H}_x \left(\frac{jk}{r_s} + \frac{1}{r_s^2} \right) - \frac{1}{r_s} \frac{\partial \bar{H}}{\partial n} \right] 4\pi r_s^2 - \left[-\bar{H}_x \left(\frac{jk}{r_s} + \frac{1}{r_s^2} \right) - \frac{1}{r_s} \frac{\partial \bar{H}}{\partial n} \right] \Omega r_s^2 + \oint_{S_1^1} [\dots] dS_1^1 = 0 \quad (3.59)$$

with a negative sign before S_2 since $\hat{n}_2 = -\hat{n}_s$. Ω is defined as the solid angle subtended by S_2 . The sphere S_s subtends a solid angle of 4π steradians and the half sphere S_2 an angle of 2π steradians. If the limit is taken, Eq.(3.59) is simplified to

$$\bar{H}_x(r_s) = \frac{1}{(4\pi - \Omega)} \oint_{S_1^1} \left[H_x \frac{\partial \phi}{\partial n'_1} - \phi \frac{\partial H_x}{\partial n'_1} \right] dS_1^1. \quad (3.60)$$

If the observation point does not lie on S_1 then $\Omega = 0$. The boundary integral equation for all three Cartesian coordinates at an arbitrary observation point r is given by:

$$\vec{H}(\vec{r}) = \frac{1}{4\pi} \oint_{S'_1} [\vec{H}(\vec{r}') \frac{\partial \phi(\vec{r}, \vec{r}')}{\partial n'_1} - \phi(\vec{r}, \vec{r}') \frac{\partial \vec{H}(\vec{r}')}{\partial n'_1}] dS'_1 \quad (3.61)$$

where $r = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$. Equation (3.61) appears sometimes in the more popular BEM formulation, however, in this instance, the integral in Eq.(3.61) is simply used, because the kernel is available for computation, to obtain the magnetic field $\vec{H}$ anywhere outside the surface S' . No systems of equations are required to obtain the fields. This same equation is also used in the FEM to evaluate $\partial H / \partial n$ by taking its derivative (see Section 3.2.5).

3.3.3 Adaptation to the Model

Equation (3.61) shows that, if a field on a closed surface is known and the derivative in the direction of the normal to the enclosing surface is known, the field can be computed anywhere outside the surface.

The adaptation of the integral in Eq.(3.61) to the problem depicted in Fig.3.7 starts by defining the surface S' as

$$S' = S_1 \cup S'_1 \quad (3.62)$$

where S'_1 is the image of S_1 with respect to the ground plane. This trick has to be used since the domain, through out the derivation of Eq.(3.61), has been assumed homogeneous. Image theory takes care of the effect of the ground plane and provides the homogeneous domain necessary for Eq.(3.61) to hold. Another approach would have been to derive the Green's function ϕ for an infinite space divided by an infinite perfectly conducting plane and integrate (3.61) over S_1 only. This would have the same result; however, S_1 would have had to be closed by a surface S_g (see Fig.3.8). The surface S_g is just above the ground plane and would extend beneath any apparatus under test resulting in awkward situations for the purpose of measurements.

For the integral kernel in Eq.(3.61), the magnetic field $\vec{H}$ will be measured and is thus known on S_1 . To obtain an evaluation of the normal derivative $\frac{\partial H}{\partial n}$, an approximation is made via the first order forward-difference formula

$$\frac{dH}{dn} = \frac{H(x + \Delta n) - H(x)}{\Delta n} \quad (3.63)$$

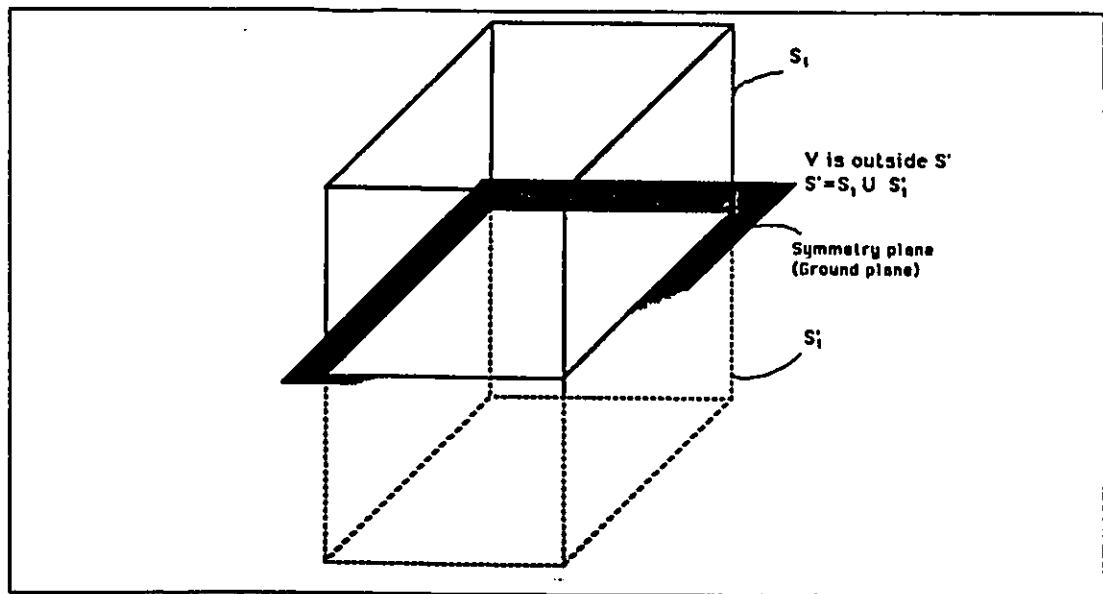


Figure 3.7: Domain and boundaries for BIM solution

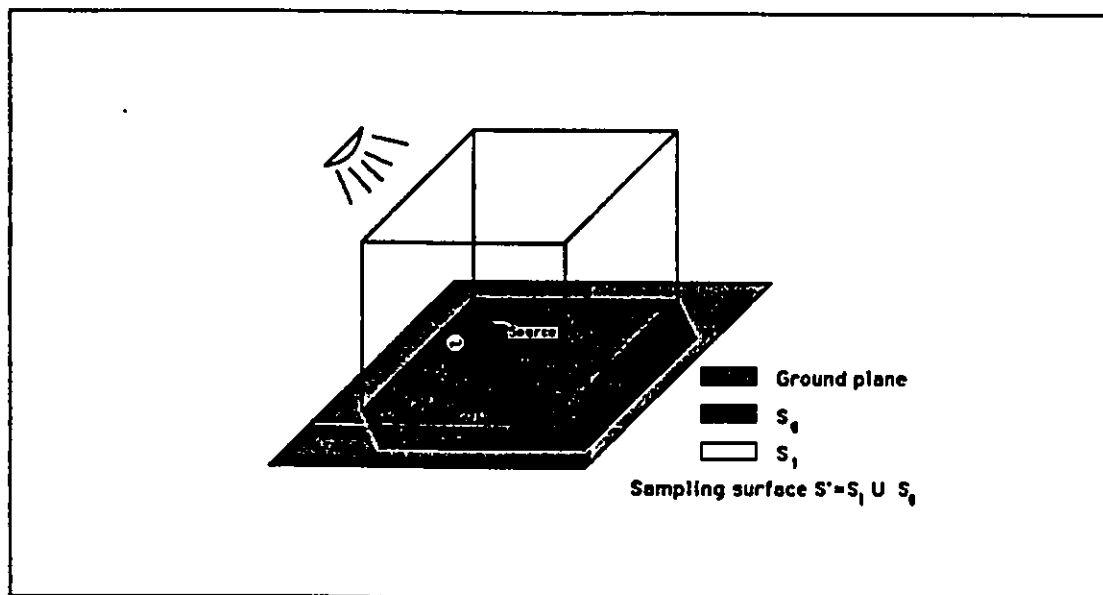


Figure 3.8: Domain and boundaries to be considered when image theory is not used

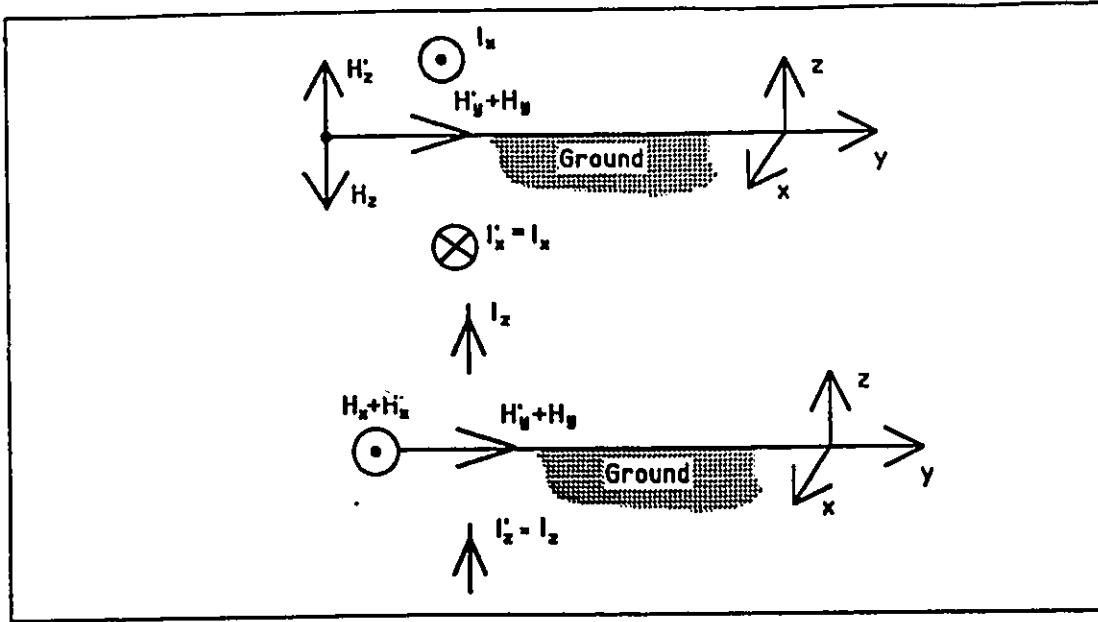


Figure 3.9: Fields generated by a current filament

The step size Δn is defined as a fraction of the wavelength the problem is being solved at. The normal on S_1 as depicted on Fig.3.7 depends on which face of the cubic cover is being considered.

By making use of image theory and analyzing the magnetic field generated by a short current filament above ground, as in Fig.3.9, the relations between $\vec{H}$ and $\vec{H}'$ (the image field), and $\frac{\partial \vec{H}}{\partial n}$ and $\frac{\partial \vec{H}'}{\partial n}$ can be derived from the requirement that the normal component of the magnetic field on a perfect conductor is zero. Table 3.3 summarizes these relations for all possible normal vectors $\hat{n}$ and field components.

To evaluate the integral over S_1 , a numerical representation of the surface is needed. This is already available from the mesh that was obtained in the FE method. However, only the first layer of finite elements is used. The mesh does not need to be extended far. The nodes generated by the one layer of finite elements define the positions where measurements are to be made (see Fig.3.10). The basis functions used in FEM are used here to obtain interpolation values for use in the integration. The values of $H(x + \Delta n)$ are obtained in this fashion.

The integral itself is evaluated as a summation of four points on the surface of the bases of every tetrahedral element lying on the Dirichlet boundary. Each triangular base is divided in four equal patches, as in Fig.3.11. It was observed that a finer subdivision of the triangles only slowed the execution without any appreciable gain

	$\hat{n} = \hat{x}$	$\hat{n} = -\hat{x}$	$\hat{n} = \hat{y}$	$\hat{n} = -\hat{y}$	$\hat{n} = \hat{z}$	$\hat{n} = -\hat{z}$
H_x	1	1	1	1	1	1
H_y	1	1	1	1	1	1
H_z	-1	-1	-1	-1	-1	-1
$\frac{\partial H_x}{\partial n}$	1	1	1	1	1	1
$\frac{\partial H_y}{\partial n}$	1	1	1	1	1	1
$\frac{\partial H_z}{\partial n}$	-1	-1	-1	-1	-1	-1

Table 3.3: Correction factor for obtaining the image quantity

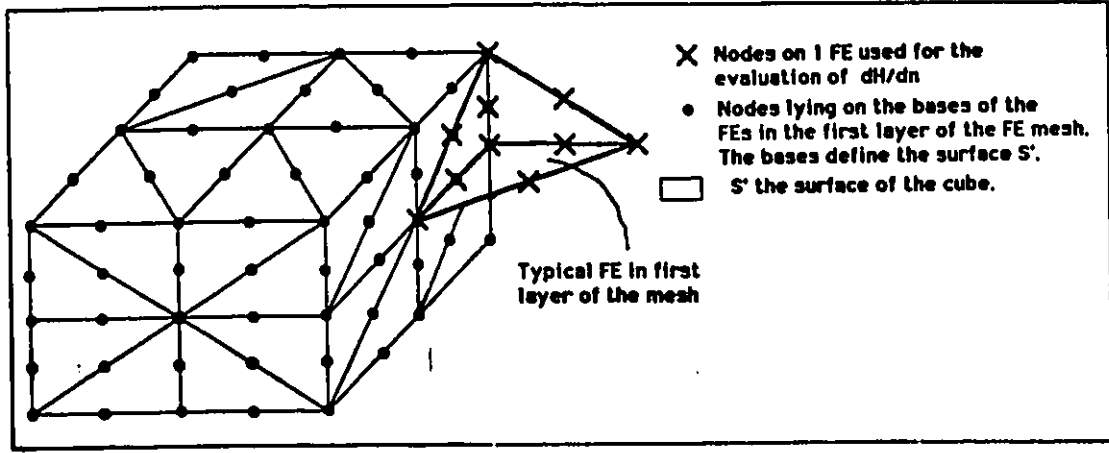


Figure 3.10: Definition of the integration surface with bases of FE's tetrahedrals

in accuracy.

The contribution of every integral over a given triangle is accounted for twice in order to integrate over the image surface as well. The values of the images of H_x and $\frac{\partial H_x}{\partial n}$ were obtained by implementing the sign corrections dictated by Table 3.3 without the need for the involved calculations rising from the interpolation of the field values between nodes.

A clarification about the necessity of using an appropriate evaluation expression for the derivative of the field is necessary. Unlike the finite element approach where derivatives were obtained directly by derivating the interpolation functions, the same cannot be done to obtain a good derivative of the field in the BIM situation. In the finite element method, derivatives at a vertex and in any direction add up to derivatives at the same vertex in parallel direction from other adjacent elements to give a valid final evaluation. This process occurs when single element contributions

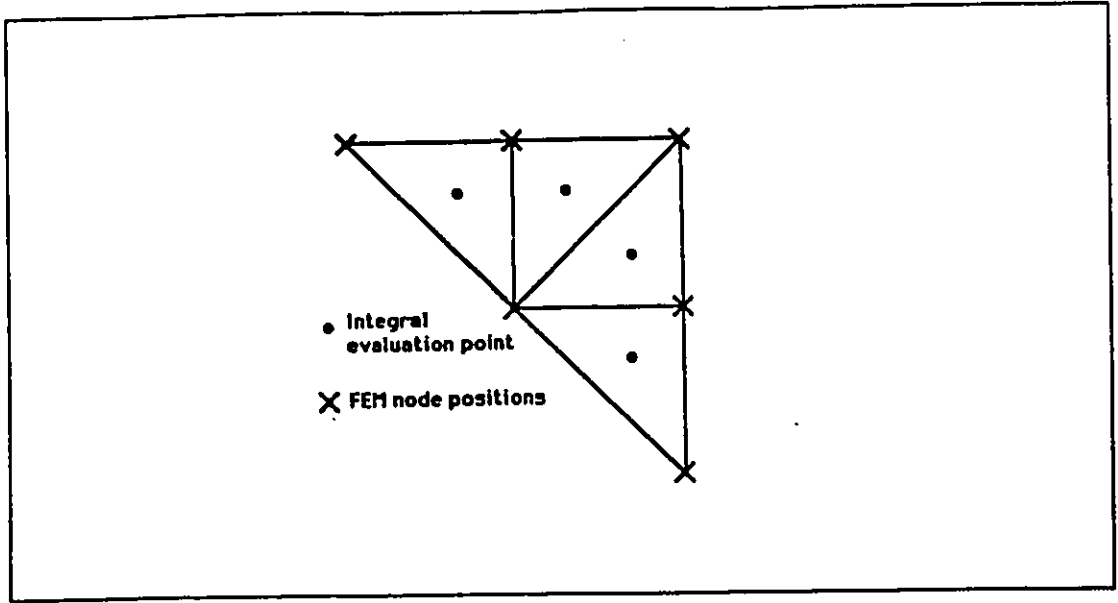


Figure 3.11: Triangle surface subdivision

are added to the global system of equations. Thus, an expression obtained from the derivation on a single element at a given vertex and in a direction parallel to the opposite edge direction will be incomplete because at that position the finite element has no depth. As the point is moved closer to the opposite edge, the derivative becomes more accurate. A diagram describes the situation in Fig.3.12. This means that in the BIM integral a complex algorithm would have to be devised to determine the location of a point in a finite element and catalog all the adjacent finite elements. The alternate route of forward difference formula was chosen to reduce execution time.

3.4 ABC's Derived

3.4.1 Derivation of the Local Type ABC

Two local ABC's are used in this work. They are derived from Wilcox Expansion theorem for electromagnetic fields[16]. The theorem states that:

Theorem 1 *If $\vec{A}(r)$ is a vector radiation function for a given $r > c$ where (r, θ, ϕ) are spherical coordinates, then $\vec{A}(r)$ has an expansion:*

$$\vec{A}(r) = \frac{e^{jkr}}{r} \sum_{n=0}^{\infty} \frac{\vec{A}_n(\theta, \phi)}{r^n} \quad (3.64)$$

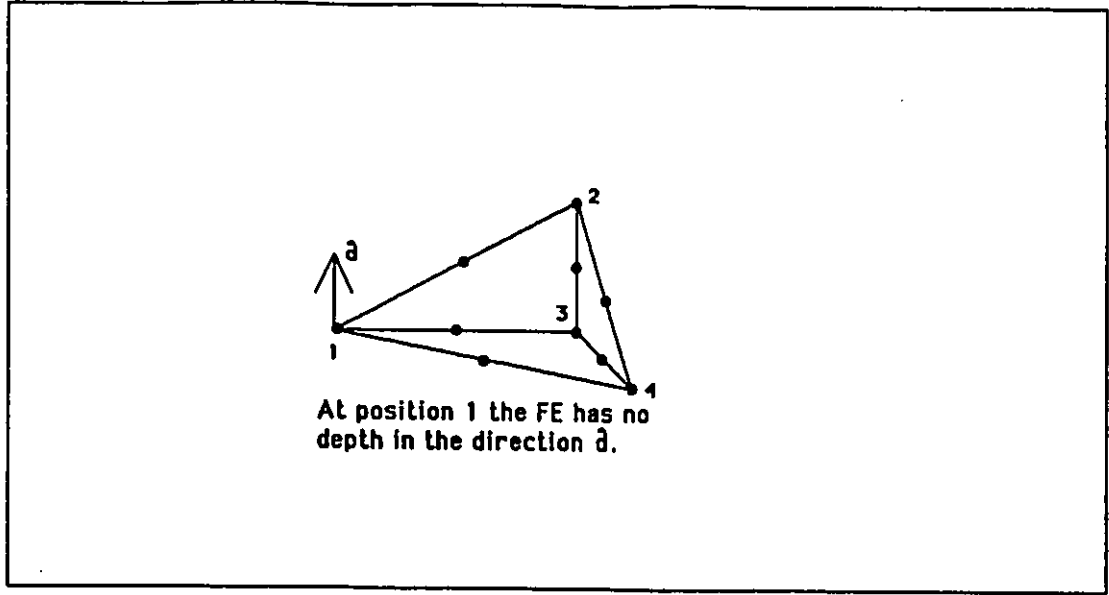


Figure 3.12: Position of points that provide innacurate derivation estimates

which is valid for $r > c$ and which converges absolutely and uniformly in the parameters r, θ and ϕ in any region $r \geq c + \epsilon > c$. The series can be differentiated term by term with respect to r, θ and ϕ any number of times and the resulting series all converge absolutely and uniformly.

A definition of a vector radiating function is given in theorem 2.

Theorem 2 *If $\vec{A}$ is a vector field defined in an exterior domain D . $\vec{A}$ is a vector radiation function for D if and only if the Cartesian components of $\vec{A}$ are scalar radiation function for D and*

$$\lim_{r \rightarrow \infty} e^{im\{kr\}} \hat{r} \cdot \vec{A}(r) = 0 \text{ or } \nabla \cdot \vec{A} = 0 \quad (3.65)$$

where $\hat{r} = r\hat{r}$.

From these two theorems, it can be assumed that the Cartesian components of $\vec{H}$ can be expressed as in Eq.(3.64) given that $\vec{H}$ is a vector radiation function satisfying the Helmholtz and radiation condition, and that $\nabla \cdot \vec{H} = 0$. For H_x the series is

$$H_x = \frac{e^{jkr}}{r} \sum_{n=0}^{\infty} \frac{H_{zn}(\theta, \phi)}{r^n}. \quad (3.66)$$

If the normal $\hat{n}$ is chosen in the direction of the spherical coordinate r , as is the case when the outer boundary of the FE mesh is spherical, and if the boundary is far

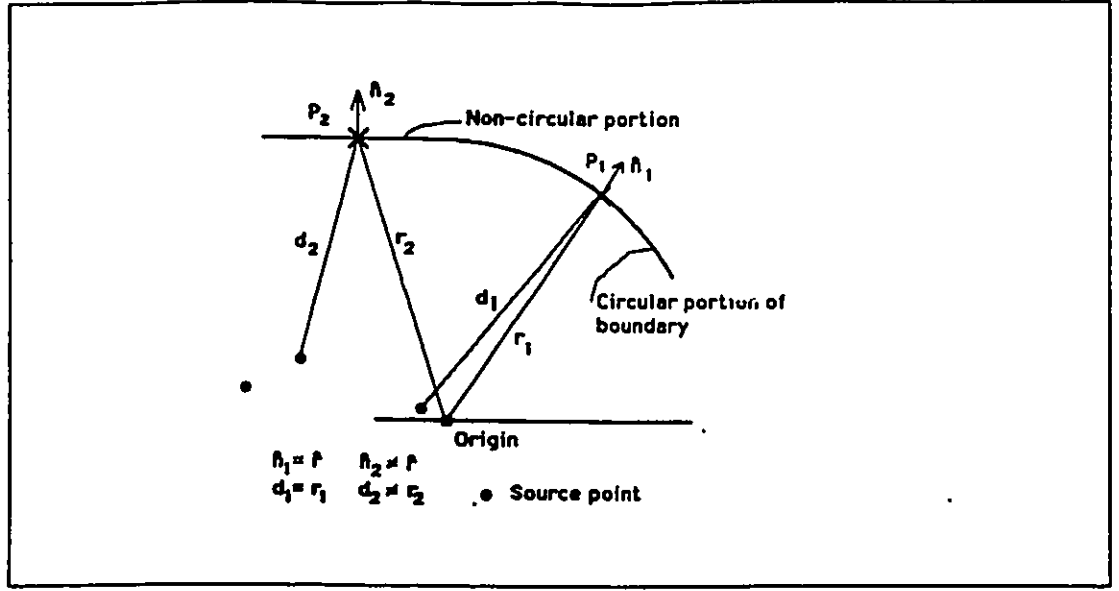


Figure 3.13: Sketch of spherical boundary and the relation between r , r_i and the source position

"enough" (see Fig.3.13 for details of such a configuration), the derivative with respect to n can be replaced by a derivative with respect to r .

$$\frac{\partial H_z}{\partial n} \simeq \frac{\partial H_z}{\partial r} = jkH_z - \frac{1}{r}H_z + \text{Error}(O^2). \quad (3.67)$$

The above equation is known as the 1st order Bayliss-Turkel ABC. If only the first term of Eq.(3.67) is retained, a simpler but less accurate expression is obtained.

$$\frac{\partial H_z}{\partial r} = jkH_z \quad (3.68)$$

is known as Sommerfeld's ABC.

3.4.2 Integral Type ABC

In Section 3.3.2, an integral solution to the wave equation was derived. The solution rewritten here for convenience is

$$\bar{H}(\vec{r}) = \frac{1}{4\pi} \oint_{S'} (\bar{H}(\vec{r}') \frac{\partial \phi(\vec{r}, \vec{r}')}{\partial n'} - \phi(\vec{r}, \vec{r}') \frac{\partial \bar{H}(\vec{r}')}{\partial n'}) dS' \quad (3.69)$$

The absorbing boundary condition must provide an evaluation of $\frac{\partial H_z}{\partial n}$. If the derivative is applied to Eq.(3.69), it becomes

$$\frac{\partial \bar{H}(\vec{r})}{\partial n} = \frac{1}{4\pi} \frac{\partial}{\partial n} \oint_{S'} (\bar{H}(\vec{r}') \frac{\partial \phi(\vec{r}, \vec{r}')}{\partial n'} - \phi(\vec{r}, \vec{r}') \frac{\partial \bar{H}(\vec{r}')}{\partial n'}) dS' \quad (3.70)$$

The normal n is defined by the outer boundary of the problem. n' is the normal with respect to the surface where H_x would be already known and can be any interior surface up to the Dirichlet boundary; however, it must still enclose all inhomogeneities. The two surfaces are distinct and are described by two separate coordinate systems; one for the observation point and the other for the Dirichlet surface. By Leibnitz' rule[37], the derivation can be moved inside the integral and performed before the integral.

$$\frac{\partial \vec{H}(\vec{r})}{\partial n} = \frac{1}{4\pi} \oint_{S'} (\vec{H}(\vec{r}') \frac{\partial^2 \phi(\vec{r}, \vec{r}')}{\partial n \partial n'} - \frac{\partial \phi(\vec{r}, \vec{r}')}{\partial n} \frac{\partial \vec{H}(\vec{r}')}{\partial n'}) dS' \quad (3.71)$$

As in the BIM integral of Eq.(3.71), the evaluation is done in the same fashion as in Eq.(3.51). The two equations are similar except for an extra derivative on the Green's function. An analytic expression is easily obtain for these derivatives once the normals are identified.

Chapter 4

Results and Discussions

4.1 The Source Field

The source used to generate the excitation fields in all the approaches has been described in Sec 3.1. Figures 4.1-4.6 illustrate the complexity of the source field achieved with 16 point sources. The real and imaginary part of the Cartesian components of the magnetic fields are given in the near-field at a distance of 1.5m. The signal is a monochromatic signal at 40 MHz with a wavelength of 7.5m. Because the numerical algorithms solve for the real and imaginary parts of the Cartesian components of the magnetic field, the results are plotted in this format to avoid any attenuation of improvements due to the transformation to spherical or magnitude and phase formats.

The expression for the source field has been sampled at positions specified by the mesh generation. In the case of the FEM solution, the positions coincide mainly with nodes at the extremities of, and half-way on all edges of faces of tetrahedrons lying on S_1 , the Dirichlet surface. The values of the three Cartesian components of the magnetic field are required. The boundary integral method requires field values for all nodes in the first layer of finite elements adjacent to S_1 . Usually, that is the only layer present, however, if the mesh has been generated for use with the FEM, then that may not be the case anymore.

4.2 The Mesher Results

The solution domain described in Fig.3.2 has been discretized using a tetrahedral mesh of finite elements. The discretization required 680 tetrahedrons. The domain mesh was created from 243 primary node points and 1106 secondary nodes defined

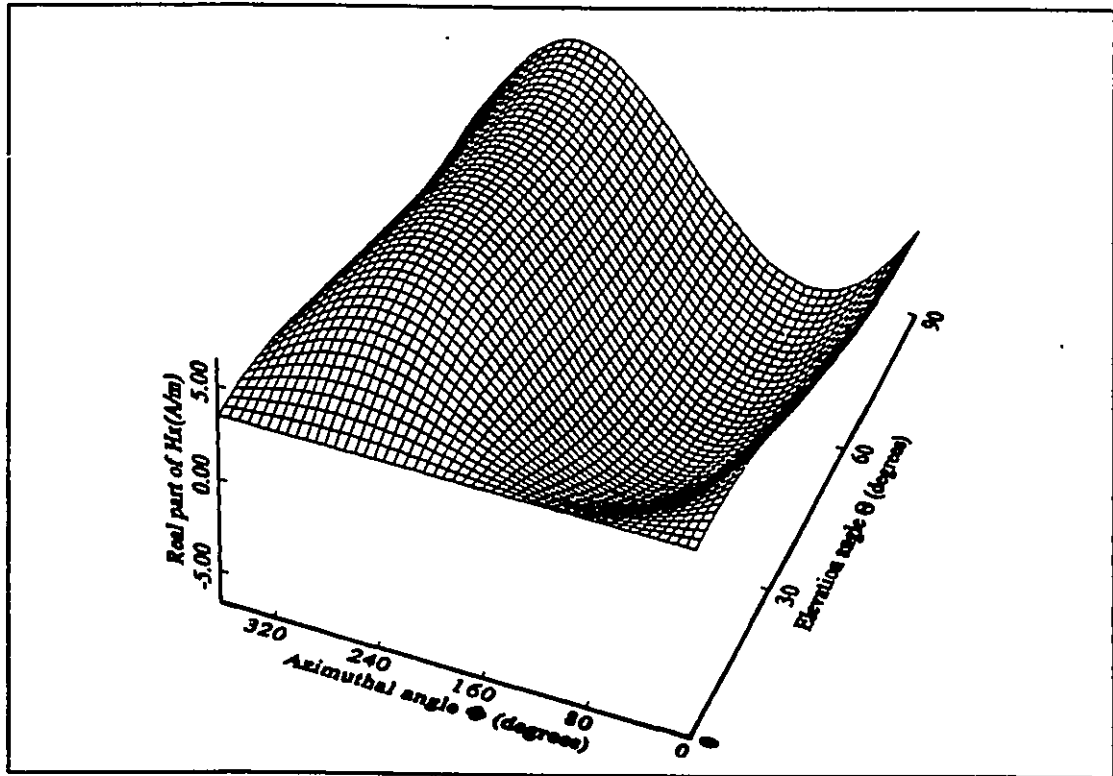


Figure 4.1: Real part of the x-component of the magnetic field created by the validation source at a distance $r=1.5\text{m}$ from the origin

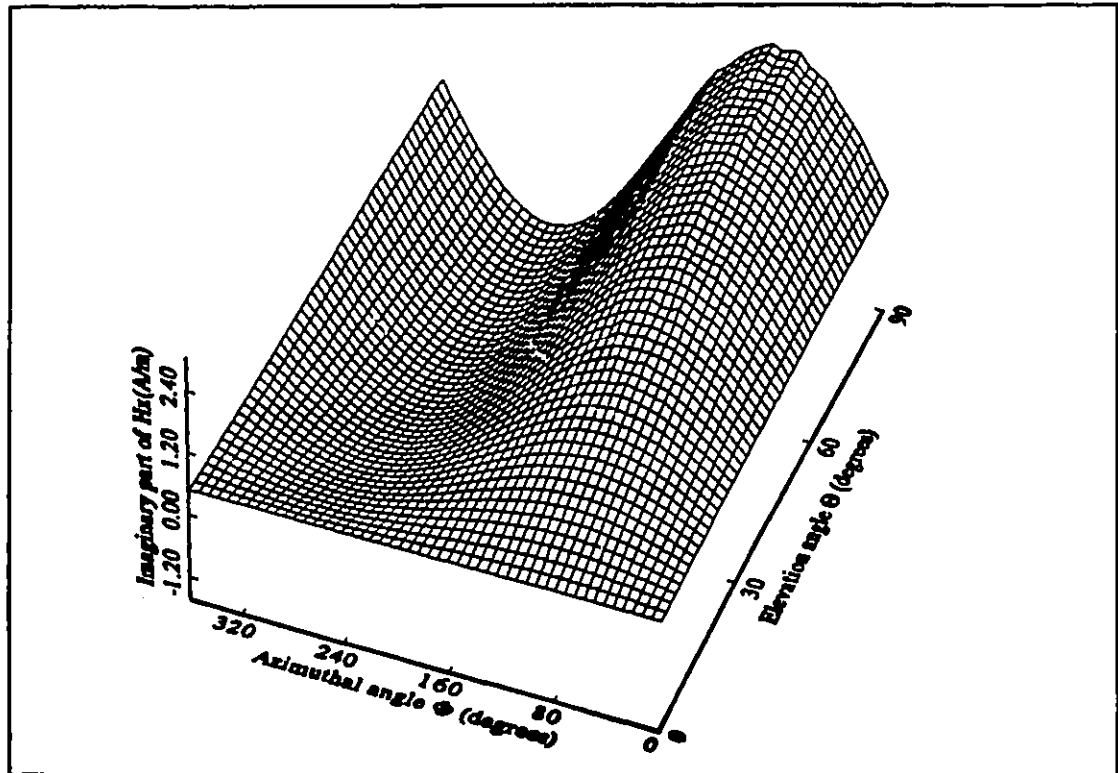


Figure 4.2: Imaginary part of the x-component of the magnetic field created by the validation source at a distance $r=1.5\text{m}$ from the origin

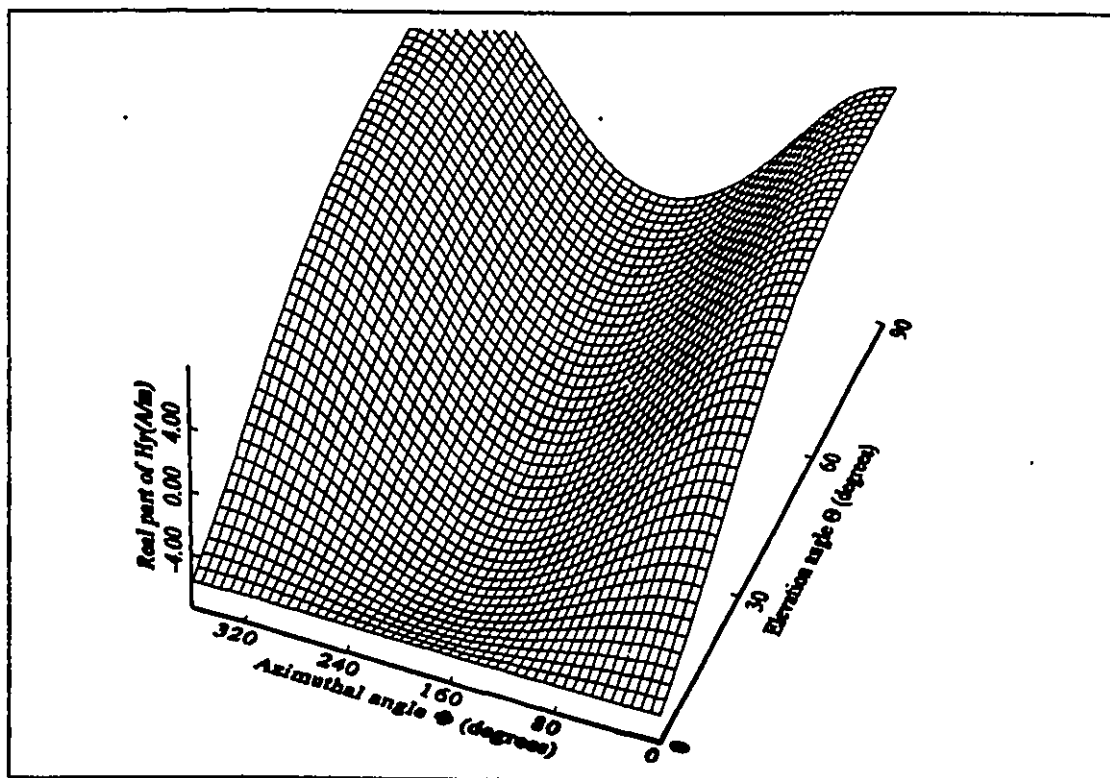


Figure 4.3: Real part of the y-component of the magnetic field created by the validation source at a distance $r=1.5\text{m}$ from the origin

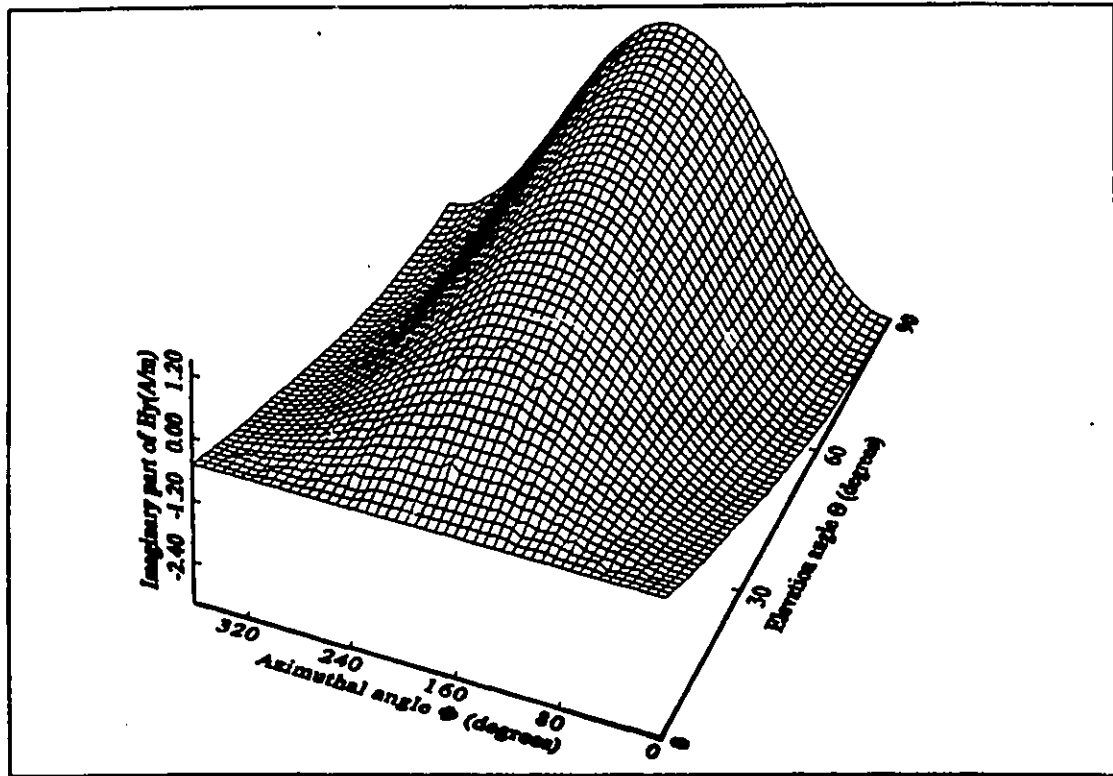


Figure 4.4: Imaginary part of the y-component of the magnetic field created by the validation source at a distance $r=1.5\text{m}$ from the origin

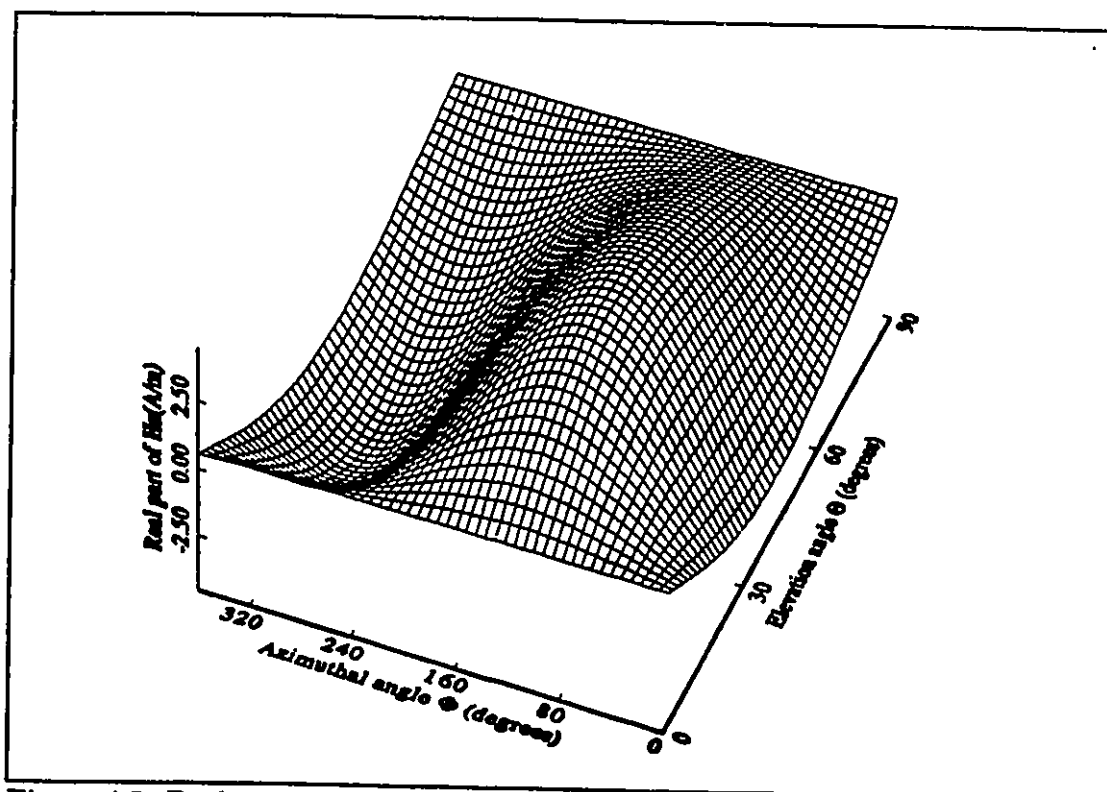


Figure 4.5: Real part of the z-component of the magnetic field created by the validation source at a distance $r=1.5\text{m}$ from the origin

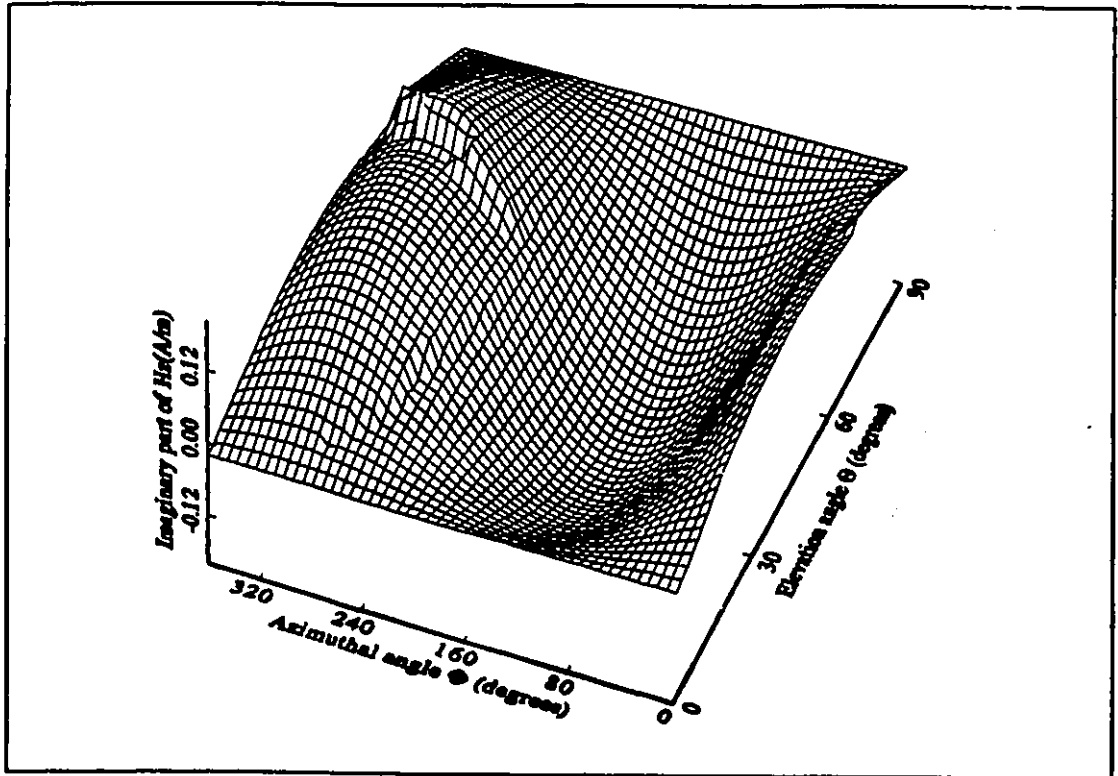


Figure 4.6: Imaginary part of the z-component of the magnetic field created by the validation source at a distance $r=1.5\text{m}$ from the origin

halfway on the 1106 edges needed to form the tetrahedrons. These numbers are the ones that set the size of the final system of equations $[S]$ defined in Eq.(3.39). The mesh defines 243+1106 complex unknowns. Figure 4.9 shows the outer tetrahedron faces delimiting the solution domain. It represents S_3 , the outer boundary. On Fig.4.8, one can see S_1 and S_2 as defined in Fig.3.2. To be able to apply the first Bayliss-Turkel ABC (BT1), which is more efficient on spherical boundaries, the outer surface S_3 has been mapped onto a spherical dome as in Fig.4.11. The corresponding surfaces S_1 and S_2 are given in Fig.4.10.

The boxes in Figs.4.8 and 4.10 representing S_1 have dimensions $1.5\text{m} \times 1.5\text{m} \times 1.5\text{m}$ and the center of its base is located at the origin of the coordinate system. The plate surrounding the box in Fig.4.8 represents S_2 and has dimensions $3.0\text{m} \times 3.0\text{m}$. Figure 4.9 shows the cover, S_3 , that bounds the solution domain. Its dimensions are $3.0\text{m} \times 3.0\text{m}$ and a height of 2.0m . In Figures 4.10 and 4.11, the spherical dome(S_3) and its base(S_2) have a radius $r = \sqrt{0.5^2 + 0.5^2 + 2^2} = 2.91\text{m}$. All the faces of the tetrahedral elements making up the surfaces S_1 , S_2 and S_3 are identified with respect to which type of boundary they represent for proper computation of the system $[S]$. The tetrahedral shapes are the ones obtained if a cube of $.5\text{m} \times .5\text{m} \times .5\text{m}$ is divided into five tetrahedrons as in Fig.4.7. The distance between nodes, because the interpolation is a second order interpolation, is 0.25m on two thirds of the edges and $0.25\sqrt{2}\text{m}$ for the rest or, 21 nodes/wavelength in the worst case. The second layer of FE suffers a deterioration of the number of nodes per wavelength because it has been stretched to map it onto a spherical dome. In some rare case, the ratio can decrease to 15 nodes/wavelength.

Another mesh has been generated for the purpose of executing the boundary integral algorithm. It has the same shape as the mesh in Figs.4.8 and 4.9 but is limited to only one layer of FE with a ratio of nodes per wavelength of 42. This second mesh has 580 tetrahedrons connecting 258 points via 1093 edges.

4.3 ABC Results

The application of the FEM method for an open problem has necessitated the use of absorbing boundary conditions. The choice has been restricted to one local ABC, the first order Bayliss-Turkel(BT1) ABC, and an integral ABC. The accuracy of each of the absorbing boundary conditions has been evaluated directly before being

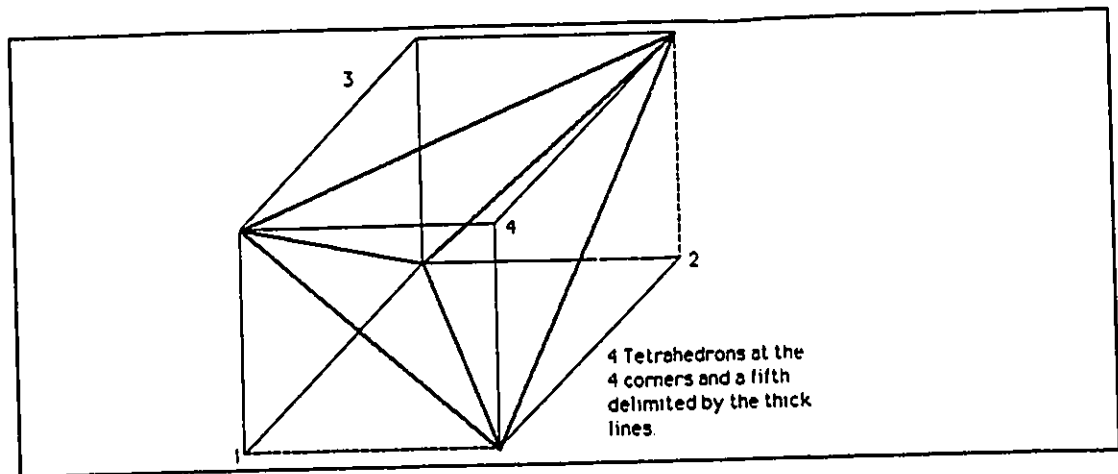


Figure 4.7: Actual shapes of tetrahedrons used in the FEM mesh

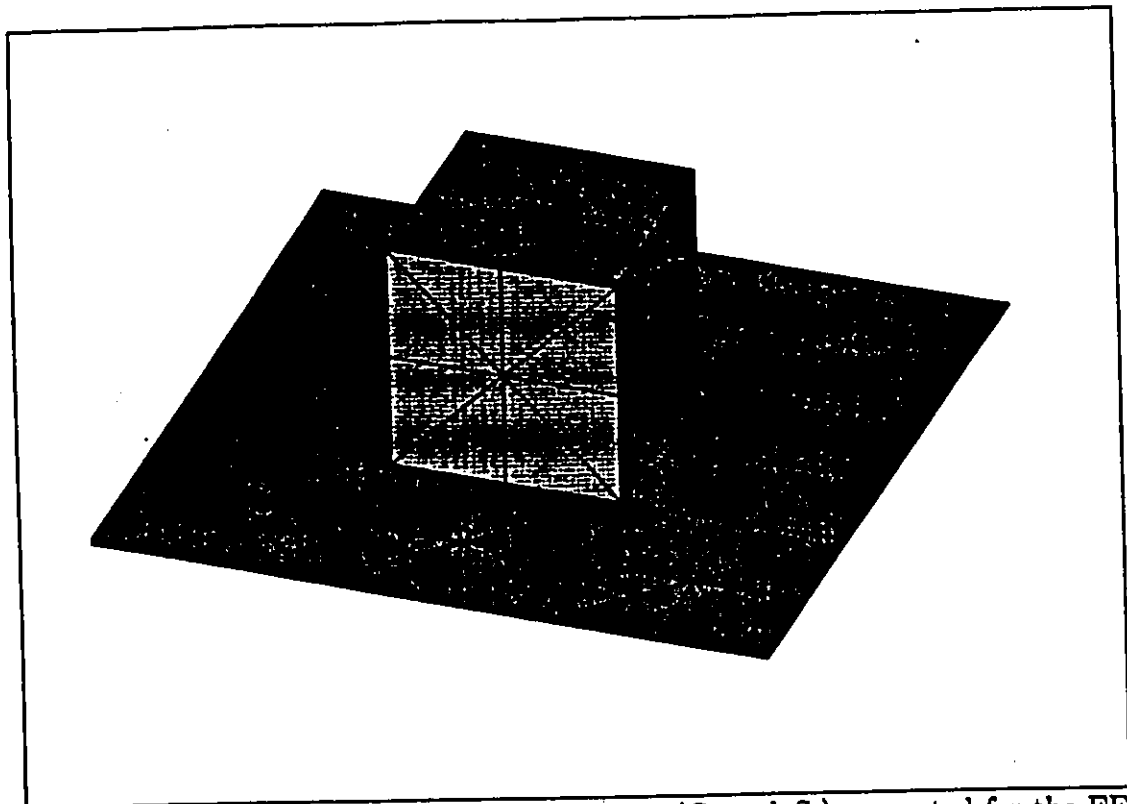


Figure 4.8: Dirichlet and ground plane surfaces (S_1 and S_2) generated for the FEM mesh and integral ABC

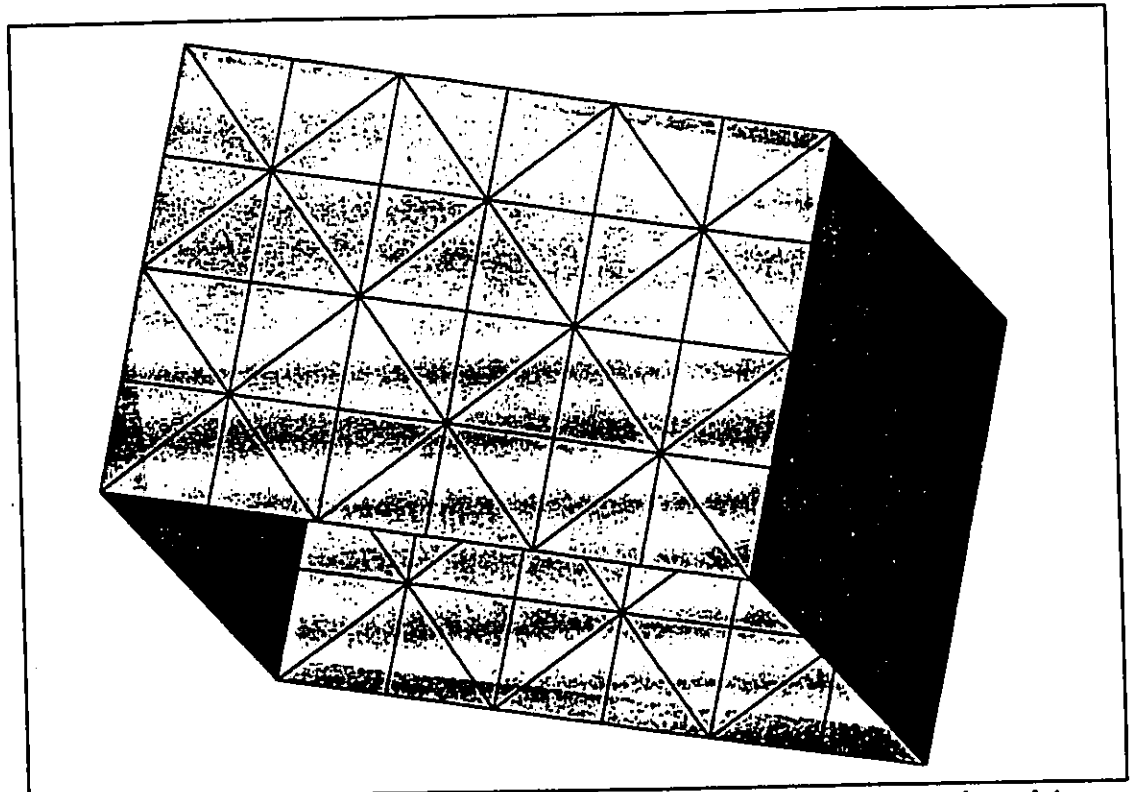


Figure 4.9: Open boundary surface (S_3 generated for the FEM mesh and integral ABC

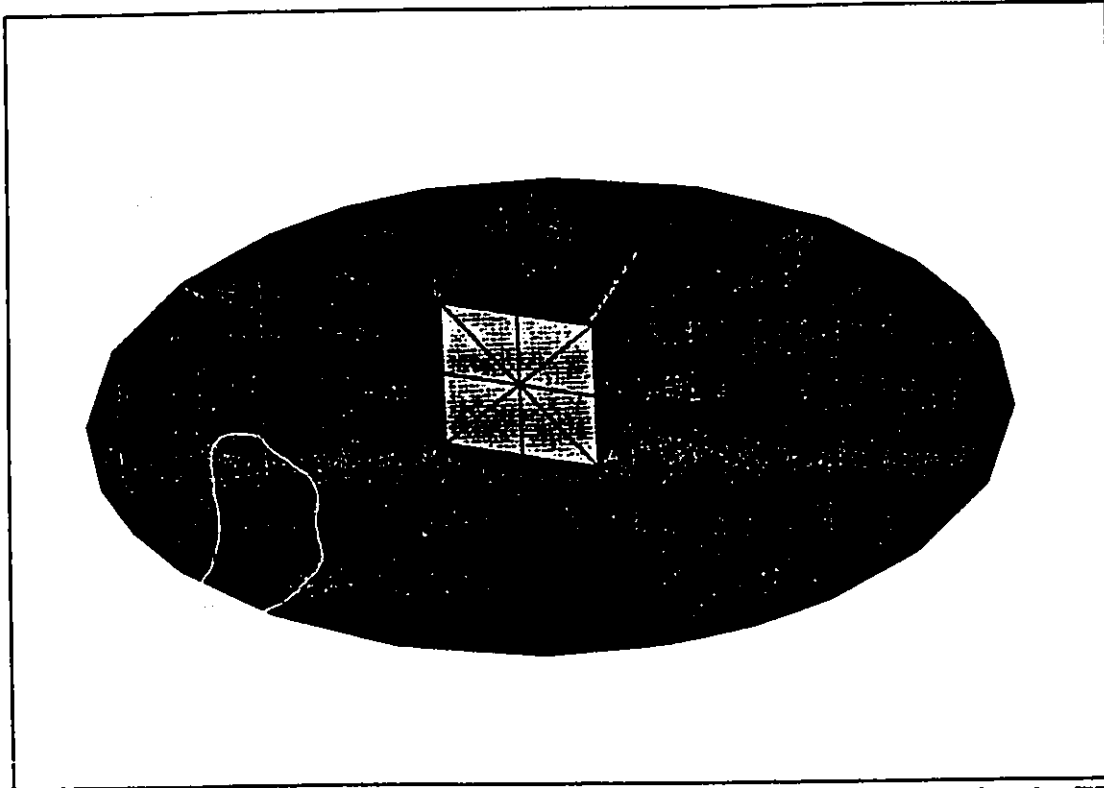


Figure 4.10: Dirichlet and ground plane surfaces (S_1 and S_2) generated for the FEM mesh and BT1 ABC

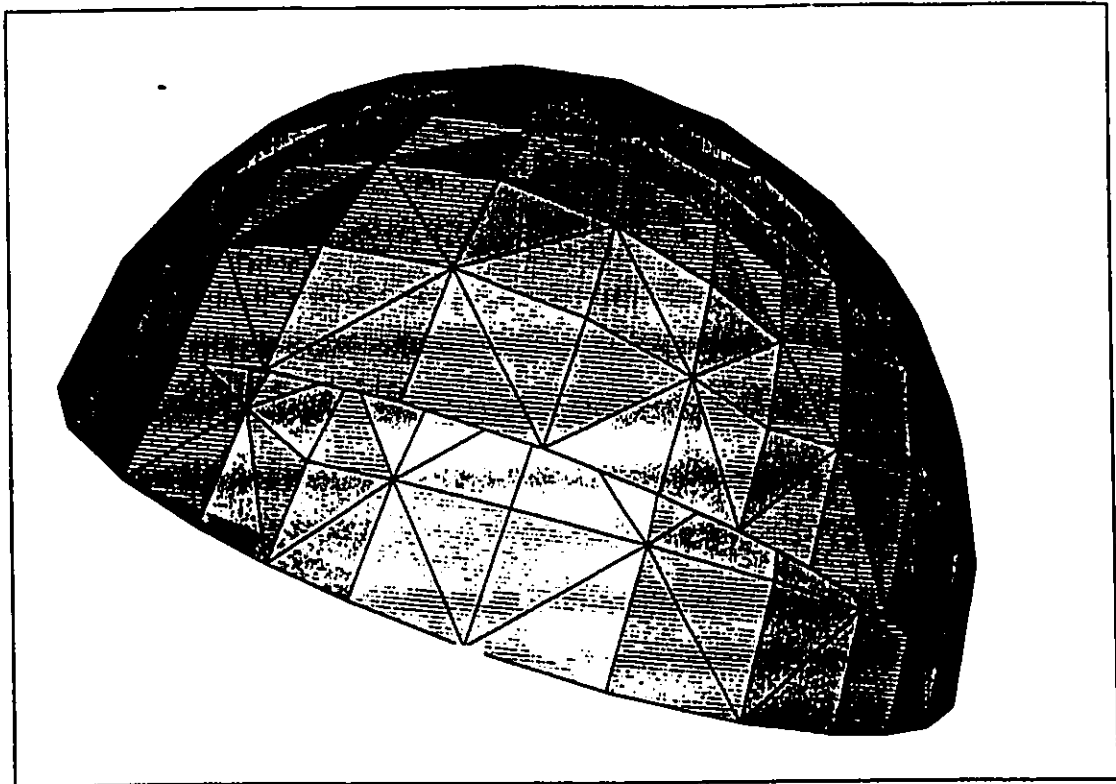


Figure 4.11: Open boundary surface (S_3 generated for the FEM mesh and BT1 ABC

incorporated in the FEM method.

The analytical values of the derivative of the source field in the radial direction were computed using the two point forward formula. The values obtained have been compared with the BT1 and integral ABCs' values. The evaluation was done at a distance of $r=2.91\text{m}$ which is the extent of the FEM mesh. A distance of 21m was also used to show the improvements of the local type ABC as one moves away from the source. The elevation angle θ was in both case equal to 37° .

Figures 4.12 to 4.21 show the real and imaginary part of the derivative with respect to the radial variable for the three Cartesian components of the magnetic field. The improvement is most noticeable in the imaginary part of the ABC. As the observation point is removed further away. The two ABCs converge toward the analytical values. For the 21m distance only the imaginary parts are given since that is where the change occurs.

The improvement with the integral ABC compared to the BT1 ABC can be even more drastic if a more complex source is chosen. In this study, the BT1 has been advantaged by the fact that the derivative with respect to r of the point source analytical field expression is very well approximated by the BT1. One has only to compare the Bayliss-Turkel ABC,

$$\frac{\partial H_z}{\partial r} = -jkH_z - \frac{H_z}{r} = \frac{I\Delta z}{4\pi} e^{-jkr} \left[\frac{k^2}{r} - \frac{2jk}{r^2} - \frac{1}{r^3} \right] \quad (4.1)$$

and the derivative with respect to r of H_z .

$$\frac{\partial H_z}{\partial r} = \frac{I\Delta z}{4\pi} e^{-jkr} \left[\frac{k^2}{r} - \frac{2jk}{r^2} - \frac{2}{r^3} \right] \quad (4.2)$$

In fact the error of approximation decays as O^3 . Furthermore, there is no series truncation error in this instance. The derivation with respect to r itself does not introduce much error given that the sources are relatively close to the origin. As the observation point is removed, the convergence of the two ABCs to the analytical values becomes perfect.

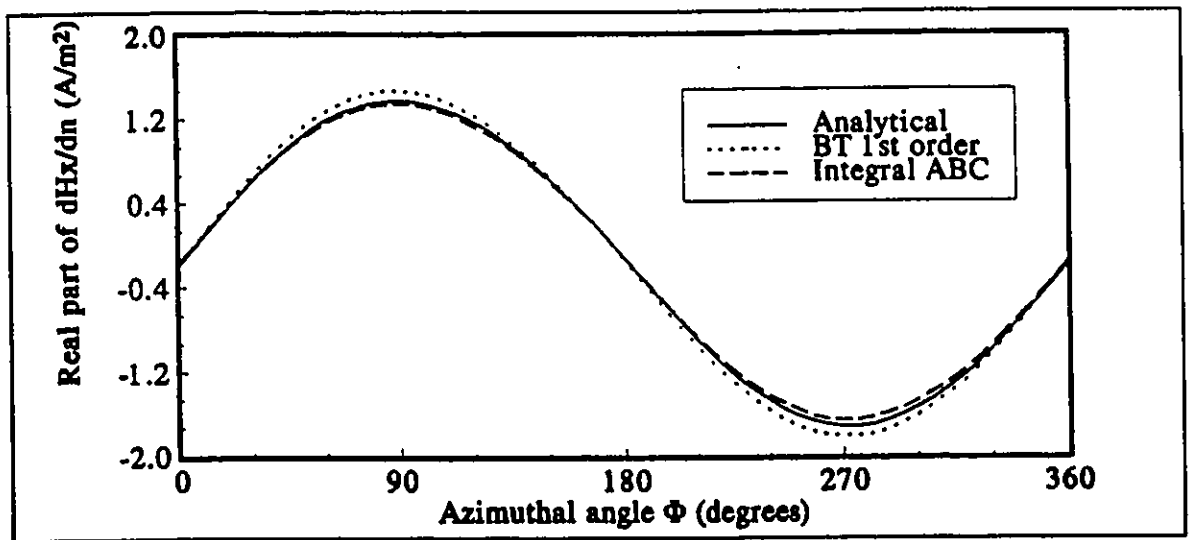


Figure 4.12: Real part of $\partial H_x / \partial r$ at $r=2.91\text{m}$ and $\theta = 37^\circ$

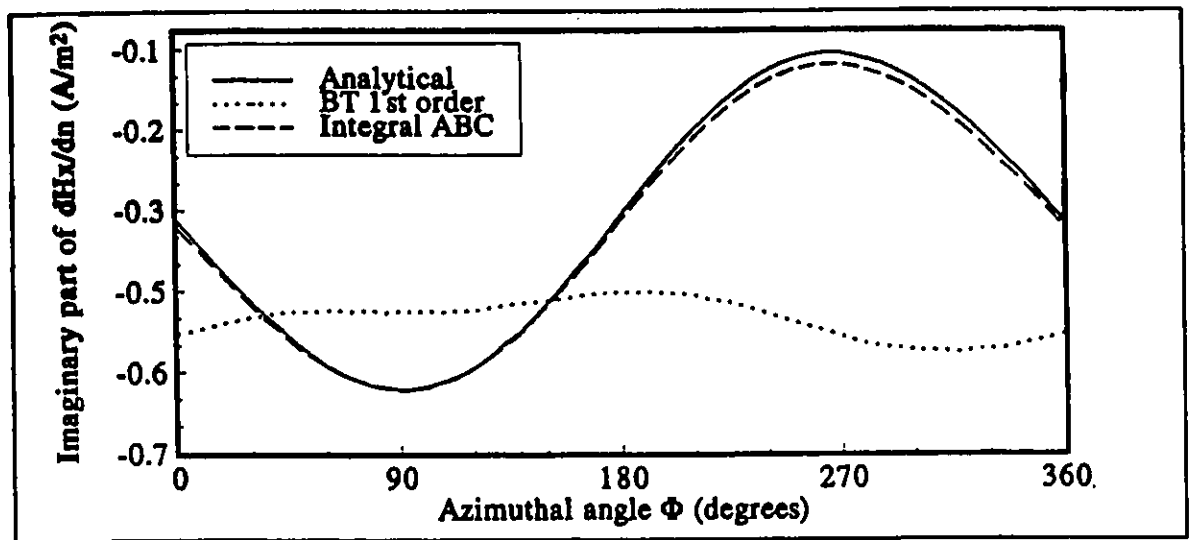


Figure 4.13: Imaginary part of $\partial H_x / \partial r$ at $r=2.91\text{m}$ and $\theta = 37^\circ$

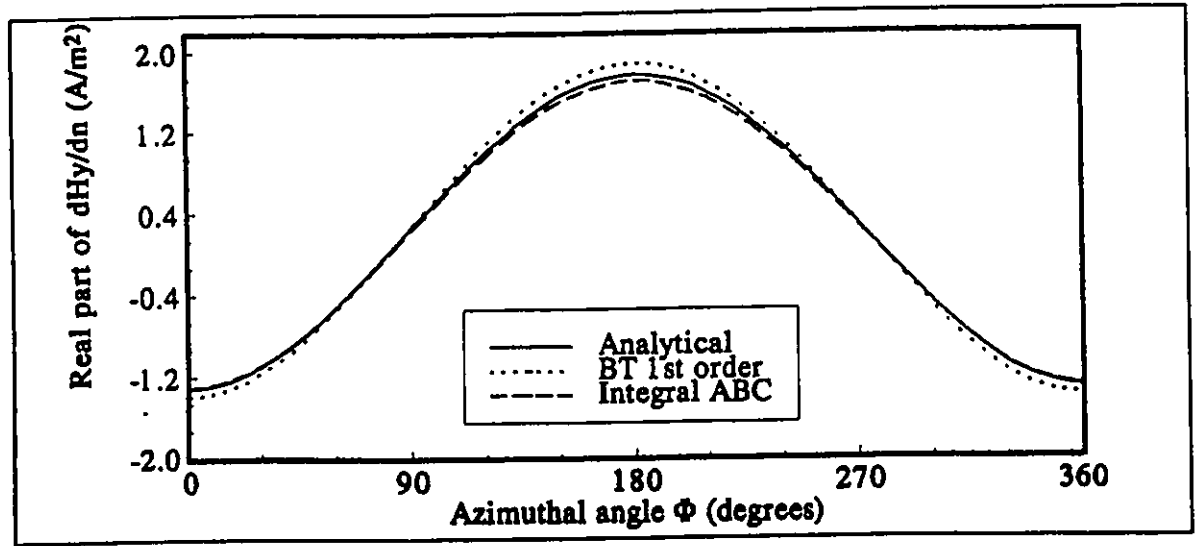


Figure 4.14: Real part of $\partial H_y / \partial r$ at $r=2.91\text{m}$ and $\theta = 37^\circ$

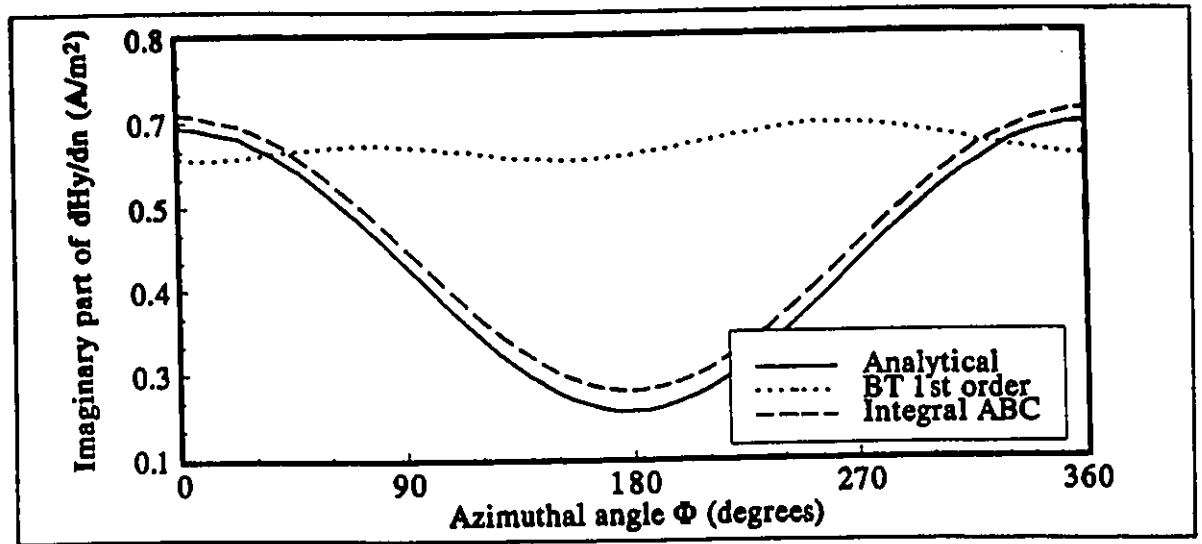


Figure 4.15: Imaginary part of $\partial H_y / \partial r$ at $r=2.91\text{m}$ and $\theta = 37^\circ$

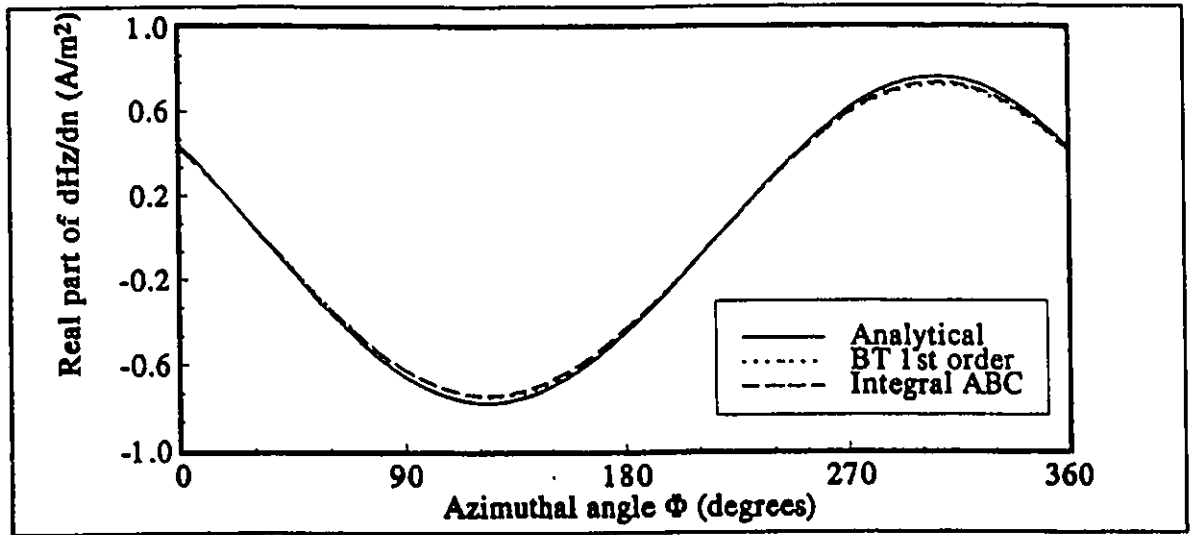


Figure 4.16: Real part of $\partial H_z / \partial r$ at $r=2.91\text{m}$ and $\theta = 37^\circ$

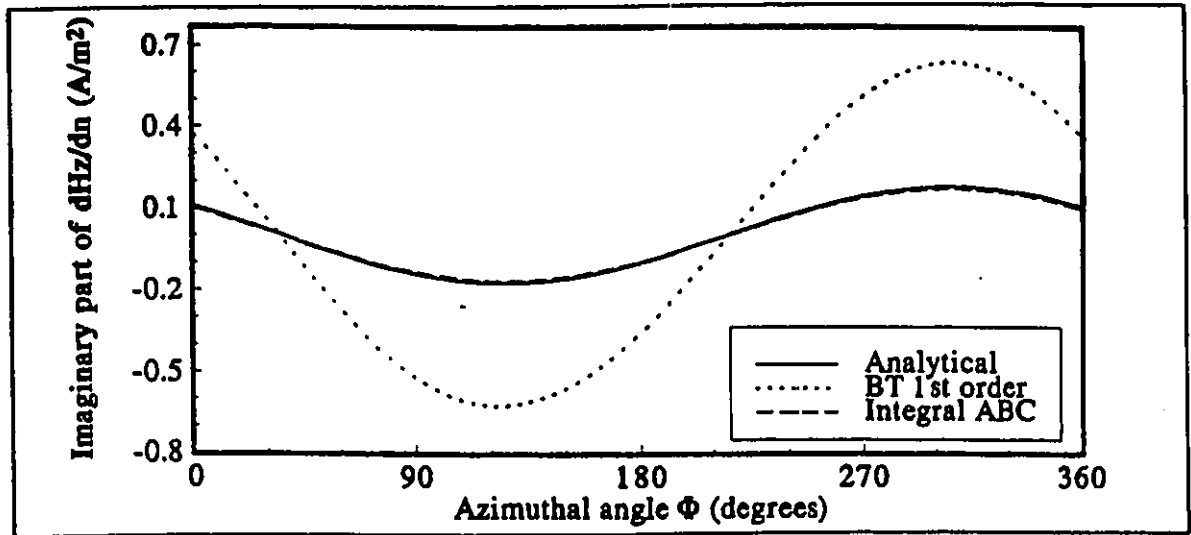


Figure 4.17: Imaginary part of $\partial H_z / \partial r$ at $r=2.91\text{m}$ and $\theta = 37^\circ$

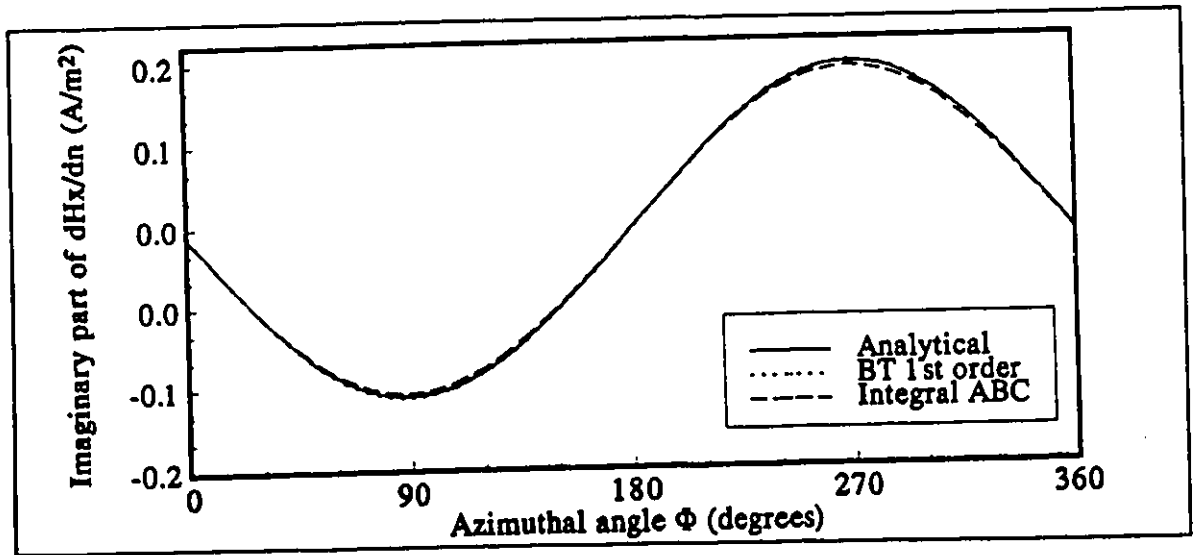


Figure 4.18: Imaginary part of $\partial H_x / \partial r$ at $r=21\text{m}$ and $\theta = 37^\circ$

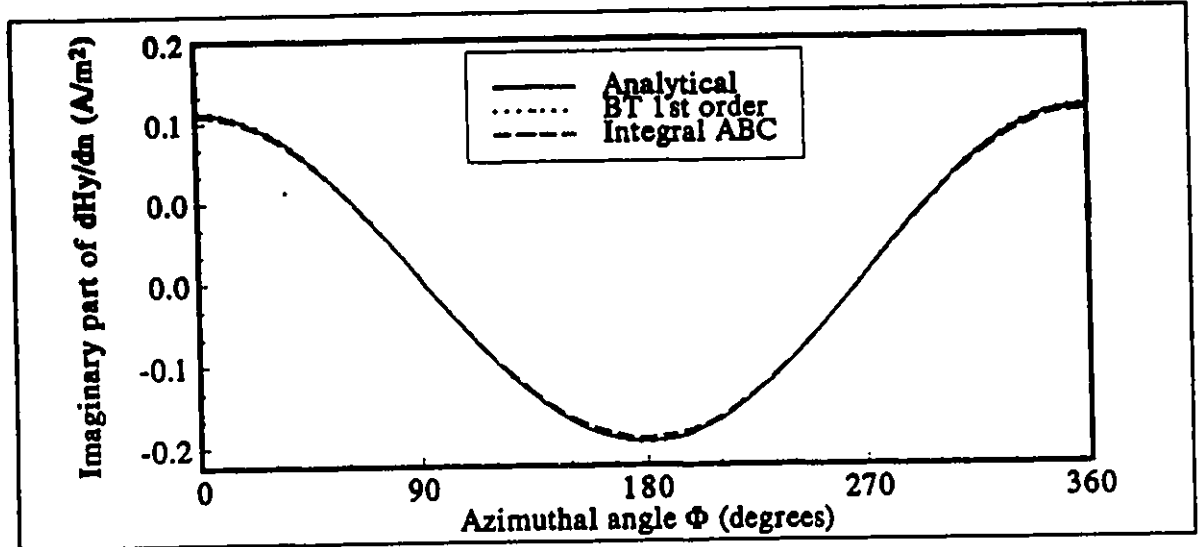


Figure 4.19: Imaginary part of $\partial H_y / \partial r$ at $r=21\text{m}$ and $\theta = 37^\circ$

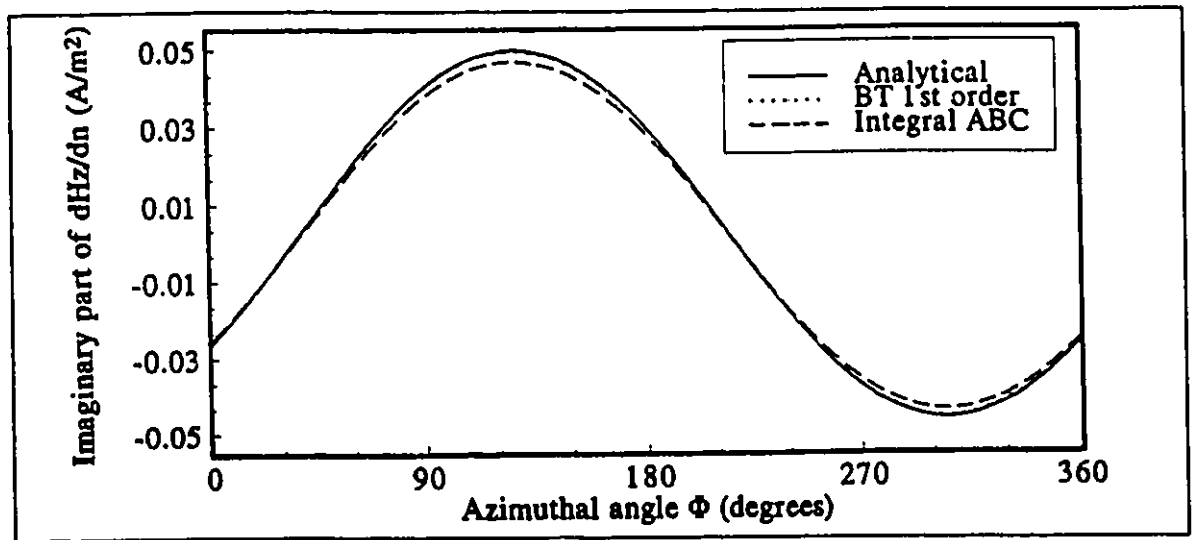


Figure 4.20: Imaginary part of $\partial H_z / \partial r$ at $r=21m$ and $\theta = 37^\circ$

4.4 FEM Results

The finite element method results have been obtained for the mesh depicted in Figs.4.8 and 4.9. The absorbing boundary conditions have been implemented in the solution to be able to use a truncated mesh. The outer boundary has been placed at one seventh of a wavelength from the Dirichlet boundary. The figures 4.21 to 4.26 display the results obtained with the best combination of mesh shape and ABC. The best results have been obtained with the BT1 local ABC when the outer boundary of the mesh is spherical. The reasons have been covered in Sec.3.3. The integral ABC gave better results when the mesh had a cubic outer boundary. The underlying reason is a simple question of resolution. Because the number of collocation points for the evaluation of the integral in Eq.(3.61) on every facet has been maintained constant and because the facets are stretched to form the spherical boundary, the resolution decreases resulting in better results with the cubic mesh.

Figures 4.21 to 4.26 show the real and imaginary part of the three Cartesian components of the magnetic field at an elevation angle of $\theta = 37^\circ$ and a distance $r = 2.91m$ (the furthest possible within the cubic mesh). The system of equations obtained with both numerical approaches had 1398 complex unknowns partitioned into 2698 real unknowns (real & imaginary). This was necessary because the package[37] to solve the system was available only for real systems. The system was solved using a di-

agonally scaled biconjugate gradient iterative method. The filling percentage was 0.911%, 0.911% and 0.83% for the three x, y and z components respectively with the local type ABC and 17.17% for the three components when the integral ABC was used. The execution time was of 72 seconds in the first case and 525 seconds in the second case on a Digital 3100 RISC station.

4.5 Discussion of FEM and ABC Results

Although the far-field to near-field transformation has not been achieved with the FEM -only a distance of $\lambda/7$ was possible-, the approach has allowed the study of the local ABC and the derivation of an integral ABC for early mesh truncation. The integral ABC has been shown to work extremely well for a very close outer boundary with the advantage of not being dependent on the source complexity or position. For even higher accuracy, a finer mesh would do the same for both types of ABCs, however a more accurate integral evaluation for the integral ABC can provide better results for the same mesh.

The degree of improvement shown from the integral ABC to the local ABC as shown in Figs.4.12 to 4.20 has been attenuated when the expressions are incorporated to the FEM system. One hypothesis, that can be advanced as an explanation for this is that an averaging of the error or improvement is occurring over all the nodal unknowns.

The mesh size has been limited by the computer memory. The use of the integral ABC puts an extra strain on memory requirements but provides very good results with more flexibility as to the choice of outer boundary shapes and proximity to the source. With the local ABC, the mesh can extend a little further, however the accuracy of the results would still depend on the complexity of the source and the degree of variation of the field.

With the advent of ever larger and faster computers most memory and execution time considerations will become obsolete. But a global ABC such as the integral ABC or any ABC that would provide accurate results for any source with the added choice of being able to truncate the solution domain to the area of interest would always keep the edge.

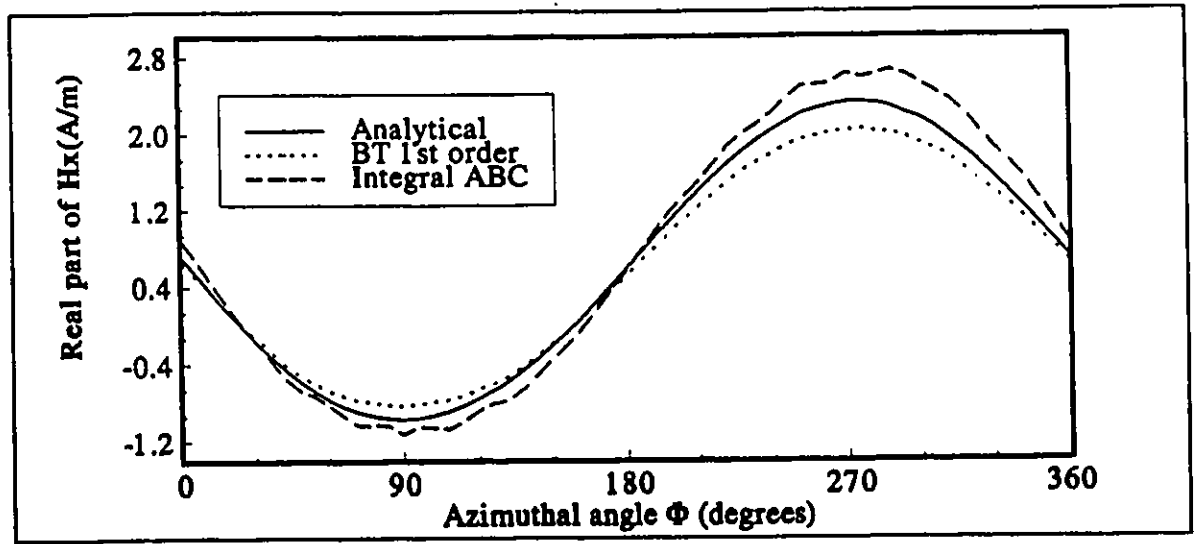


Figure 4.21: Real part of the x-component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

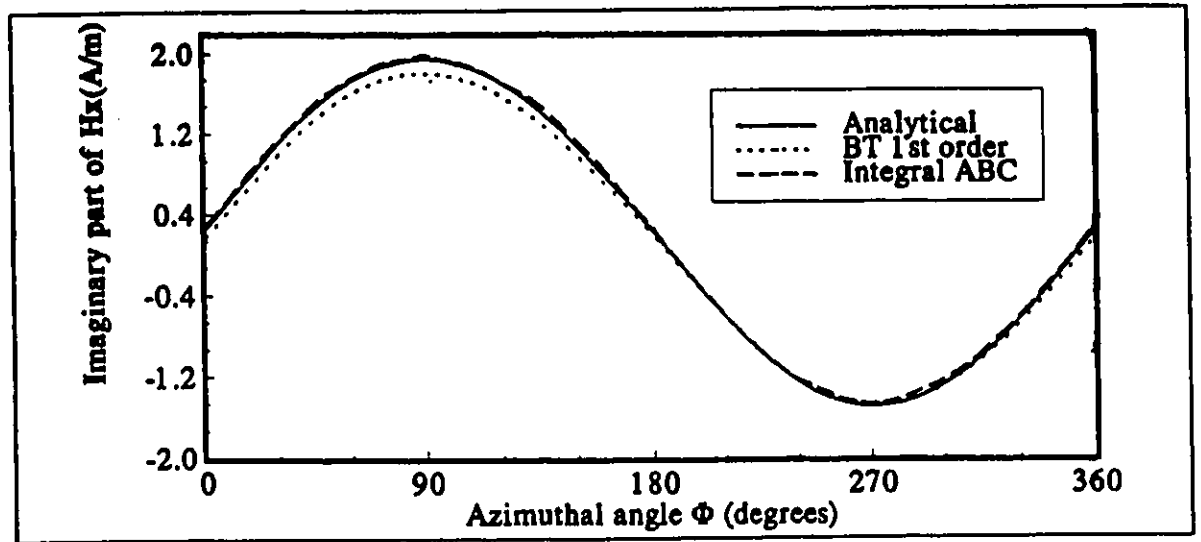


Figure 4.22: Imaginary part of the x-component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

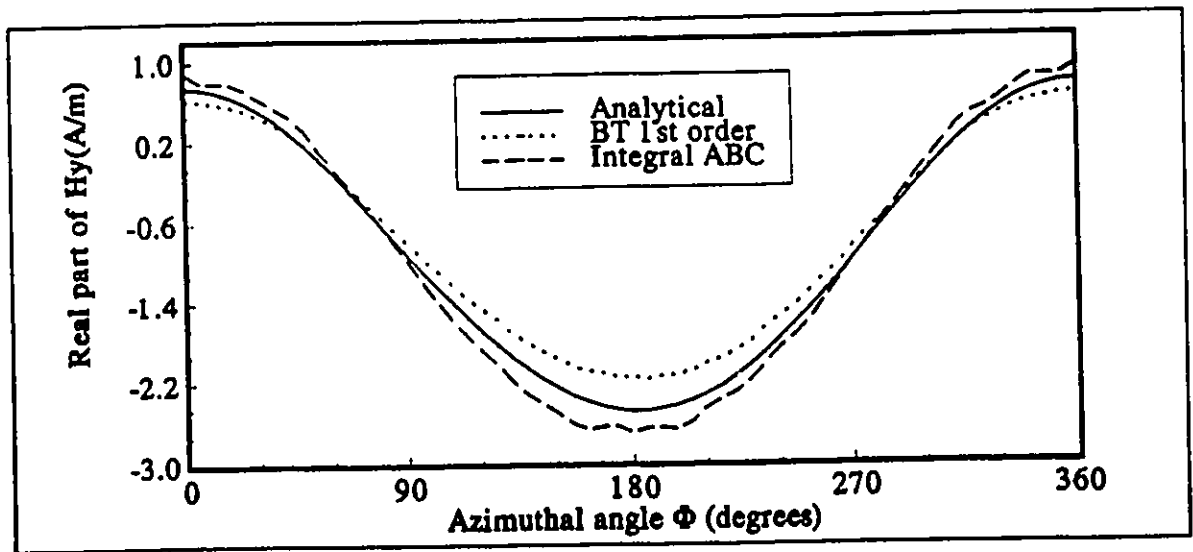


Figure 4.23: Real part of the y-component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

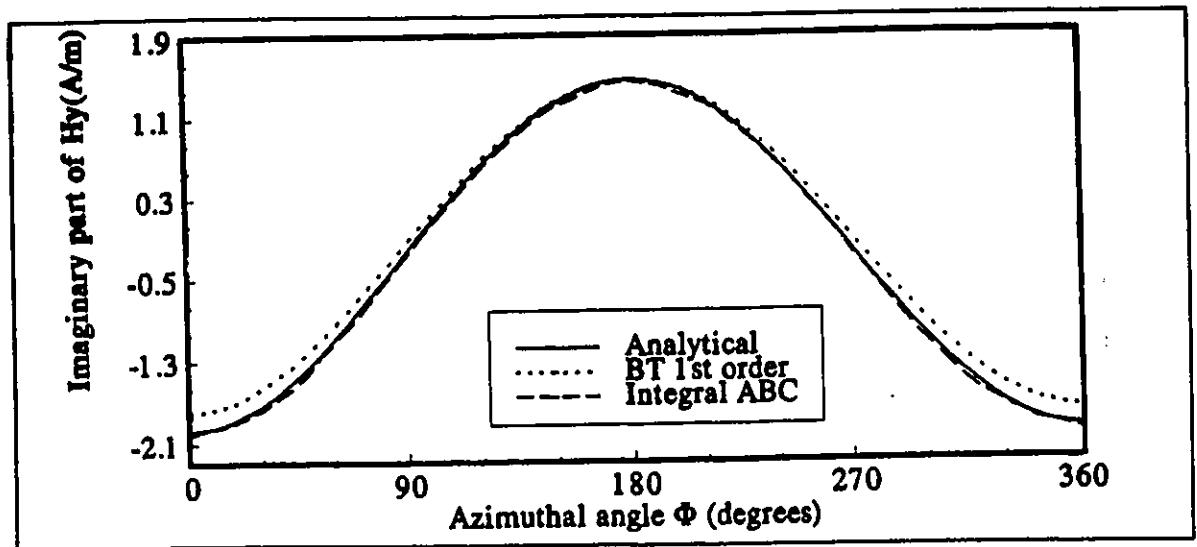


Figure 4.24: Imaginary part of the y-component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

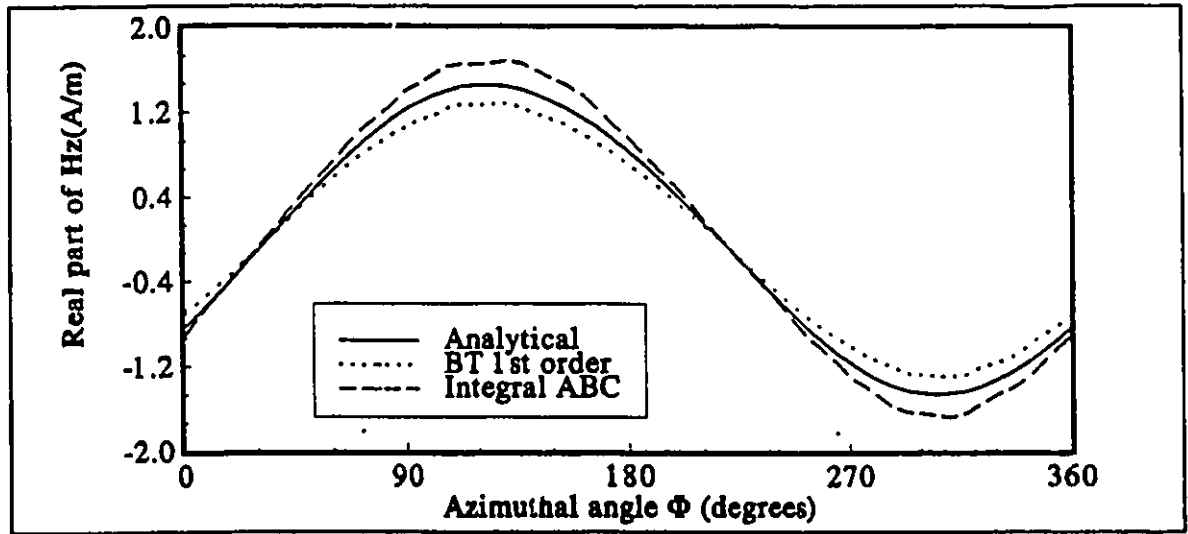


Figure 4.25: Real part of the z-component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

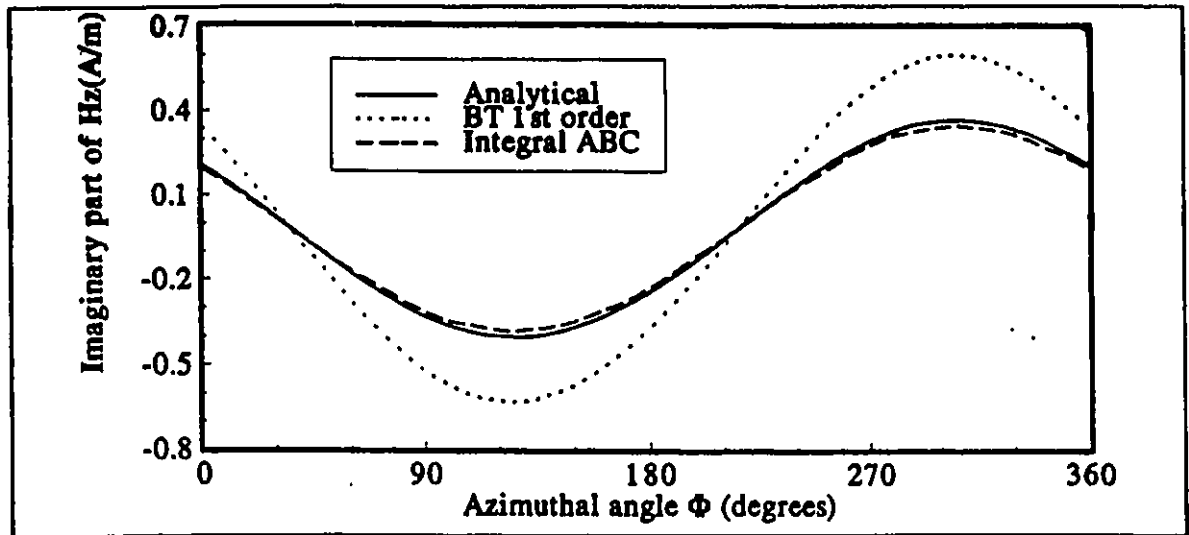


Figure 4.26: Imaginary part of the z-component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

4.6 BIM Results

As an alternative to the FEM use for the near-field to far-field transformation, the application of which faces technical limits, the integral boundary solution has been used to perform the transformation. The method provides field values at any desired range with a very short execution time. Figures 4.27 to 4.38 show the magnitude and phase of the spherical components of the magnetic field at a distance $r = 2.91m$ (same as FEM figures) and 30m (far-field). The elevation angle θ was 37° . The agreement between the analytical results and the boundary integral's is very good with maximum error in the order of 2.5%. A discrepancy in the radial component at $r = 30m$ can be noticed however. It can be explained by the fact that the magnitudes involved are very small and that numerical errors due to a low resolution in the integration method over the Dirichlet boundary starts to take over. A remedy would be to increase the number of collocation points in the integral as well as the use of a finer mesh. The mesh used, as mentioned before, is similar in shape to the one depicted in Figures 4.8 and 4.9. It has however a ratio of nodes/wavelength of 42; i.e. twice that used for the FEM. Unlike the FEM, the use of the mesh does not put any restrictions on the size of the problem or the range to which far-field transformation is needed because only a thin volume surrounding the Dirichlet boundary is discretized.

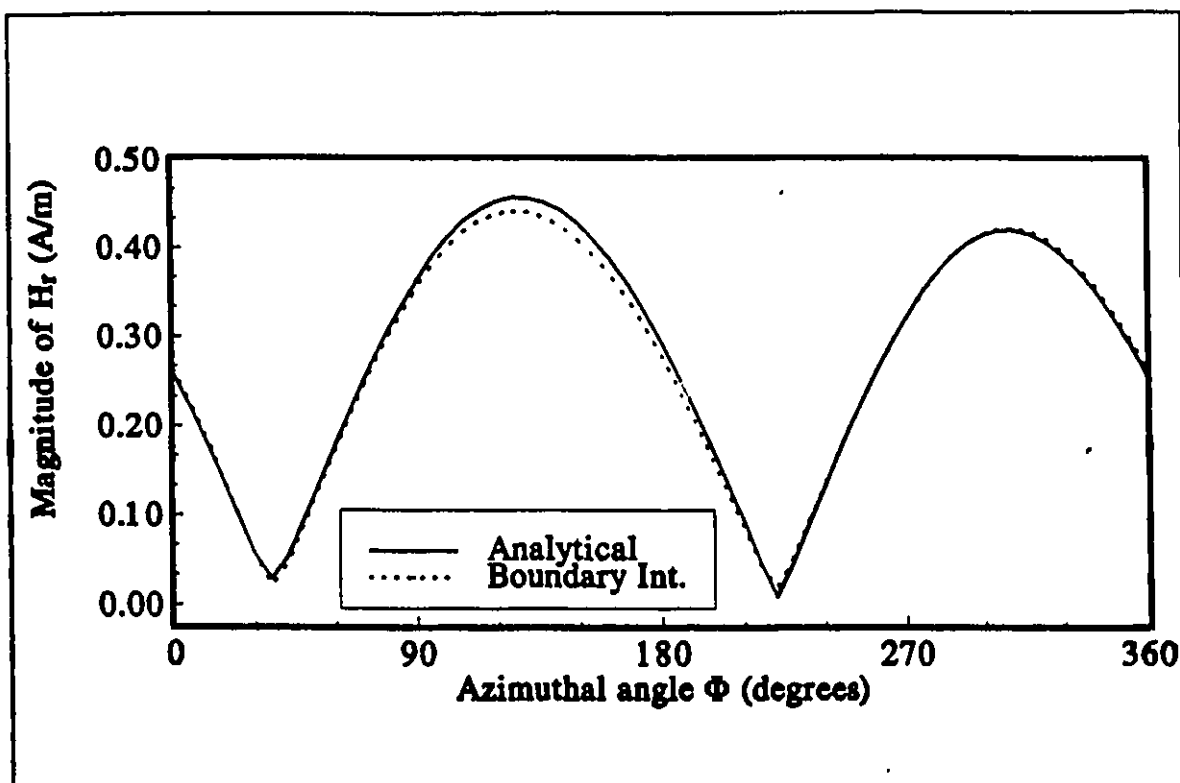


Figure 4.27: Magnitude of the r-component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

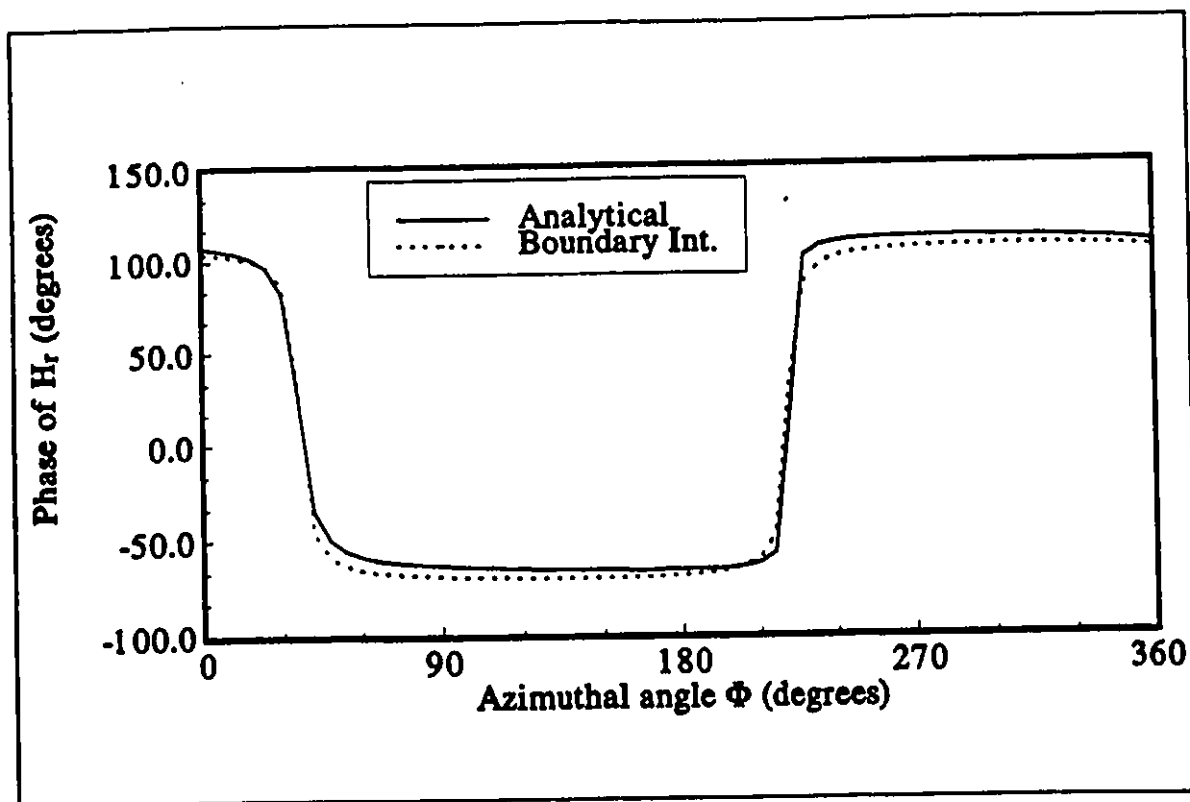


Figure 4.28: Phase of the r-component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

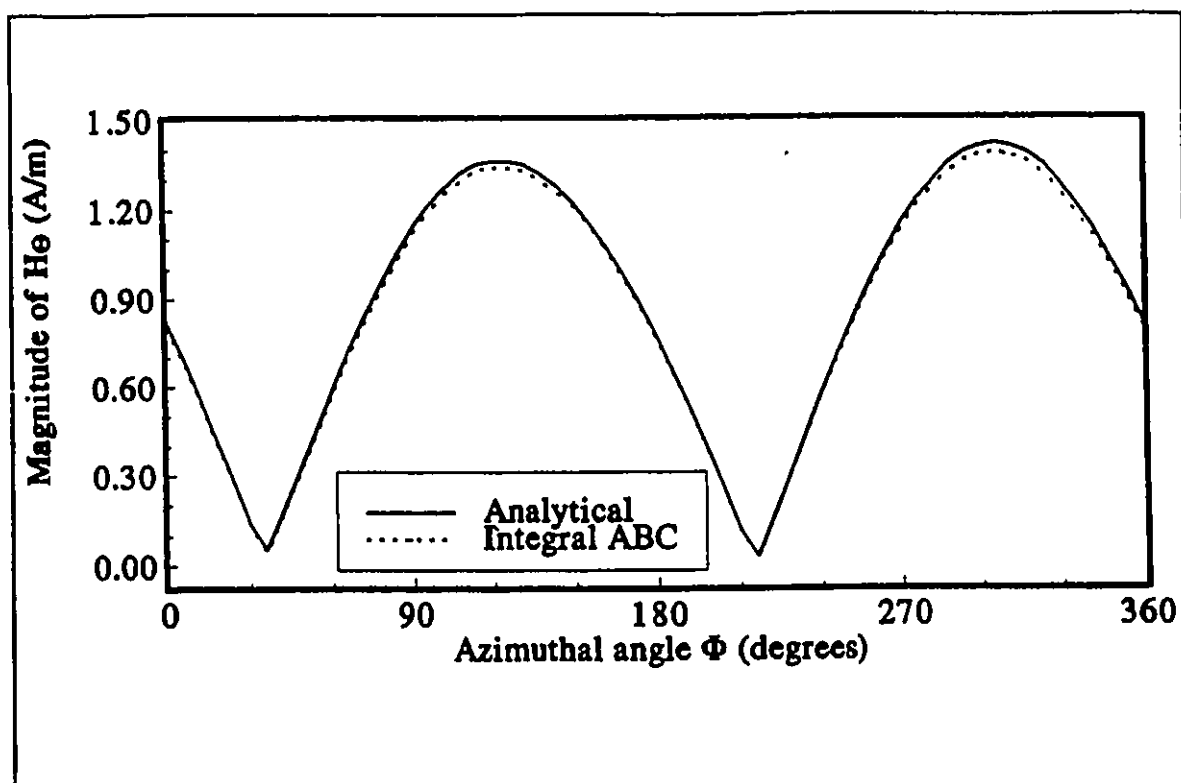


Figure 4.29: Magnitude of the θ -component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

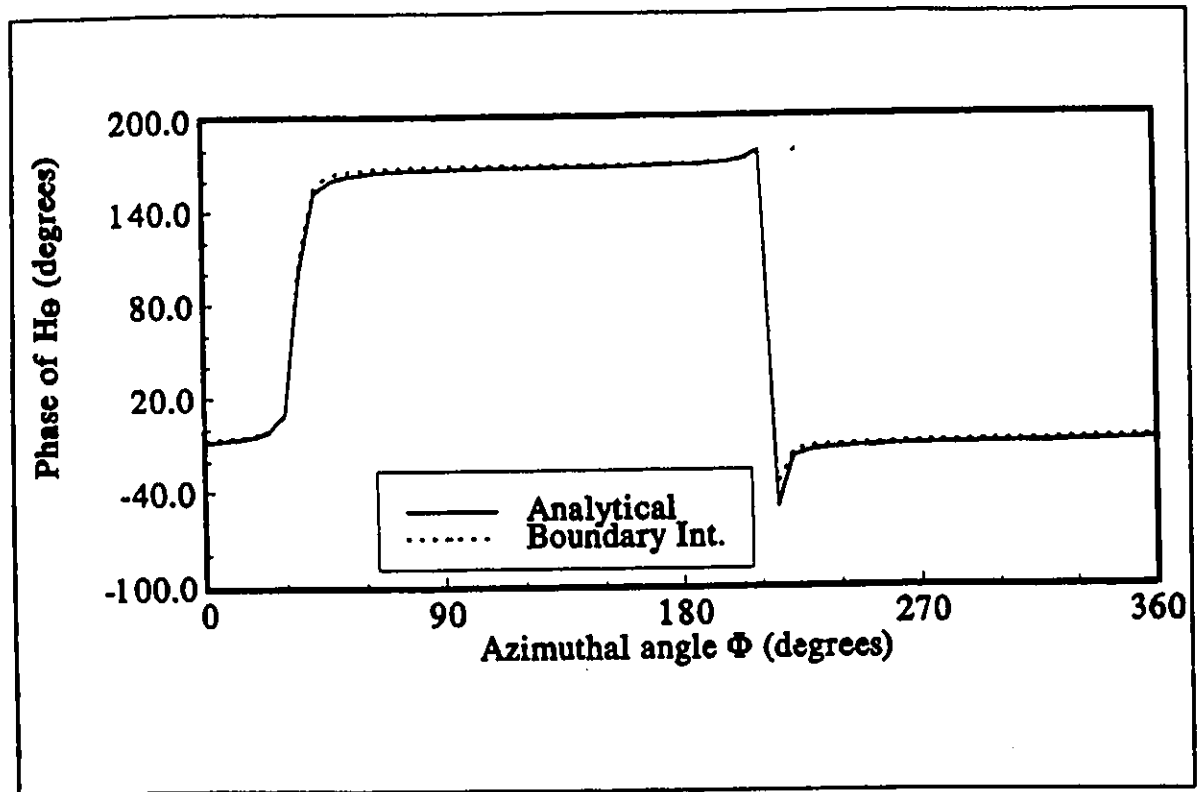


Figure 4.30: Phase of the θ -component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

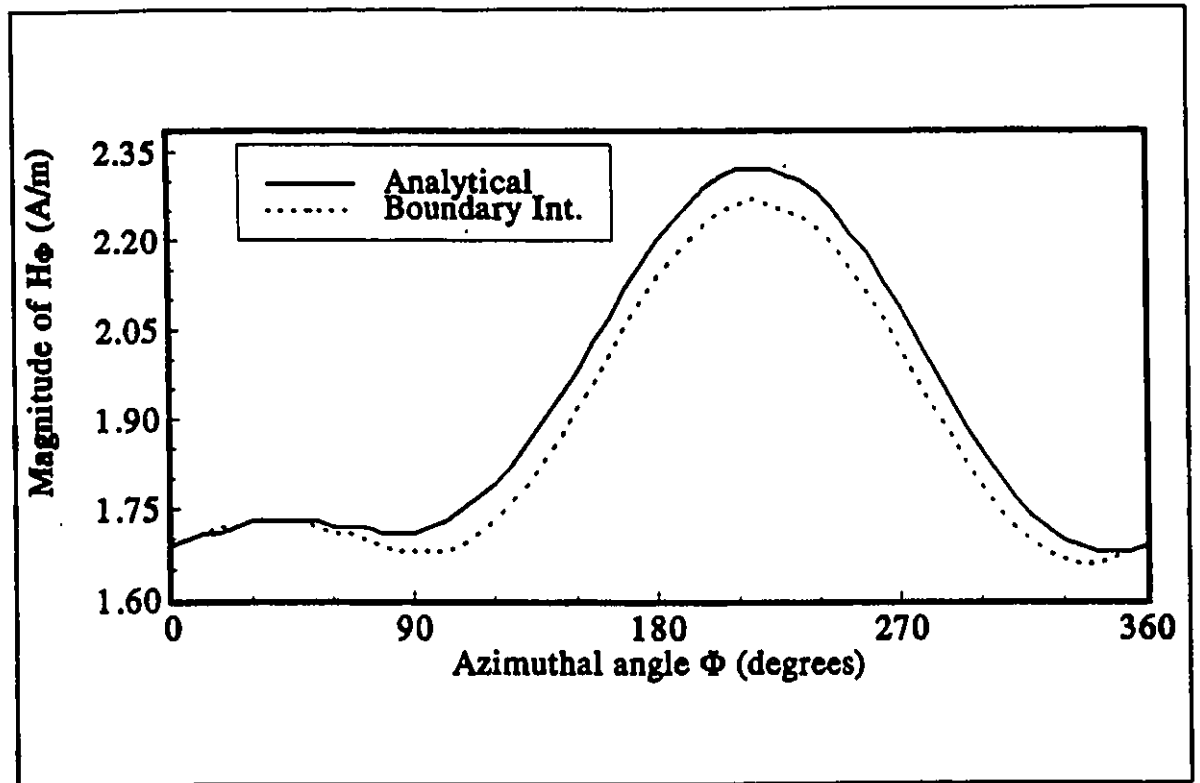


Figure 4.31: Magnitude of the ϕ -component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

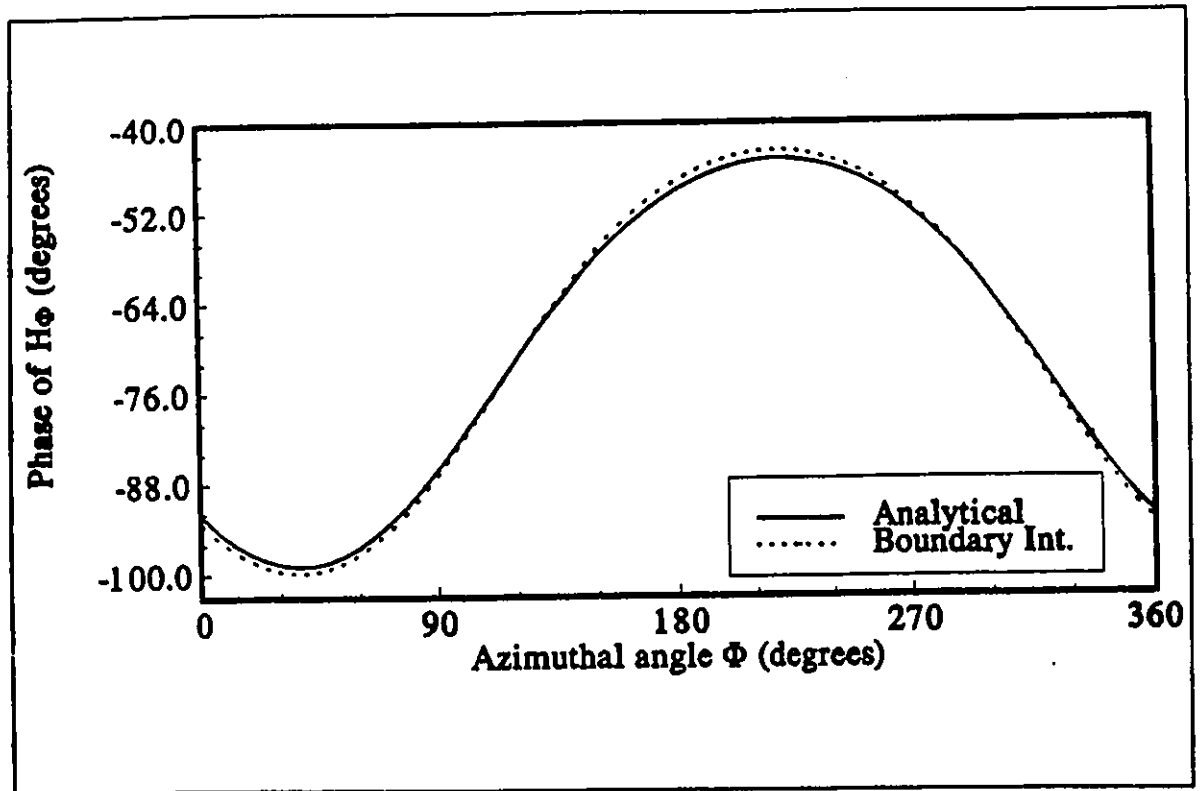


Figure 4.32: Phase of the ϕ -component of the magnetic field at $r=2.91\text{m}$ and $\theta = 37^\circ$

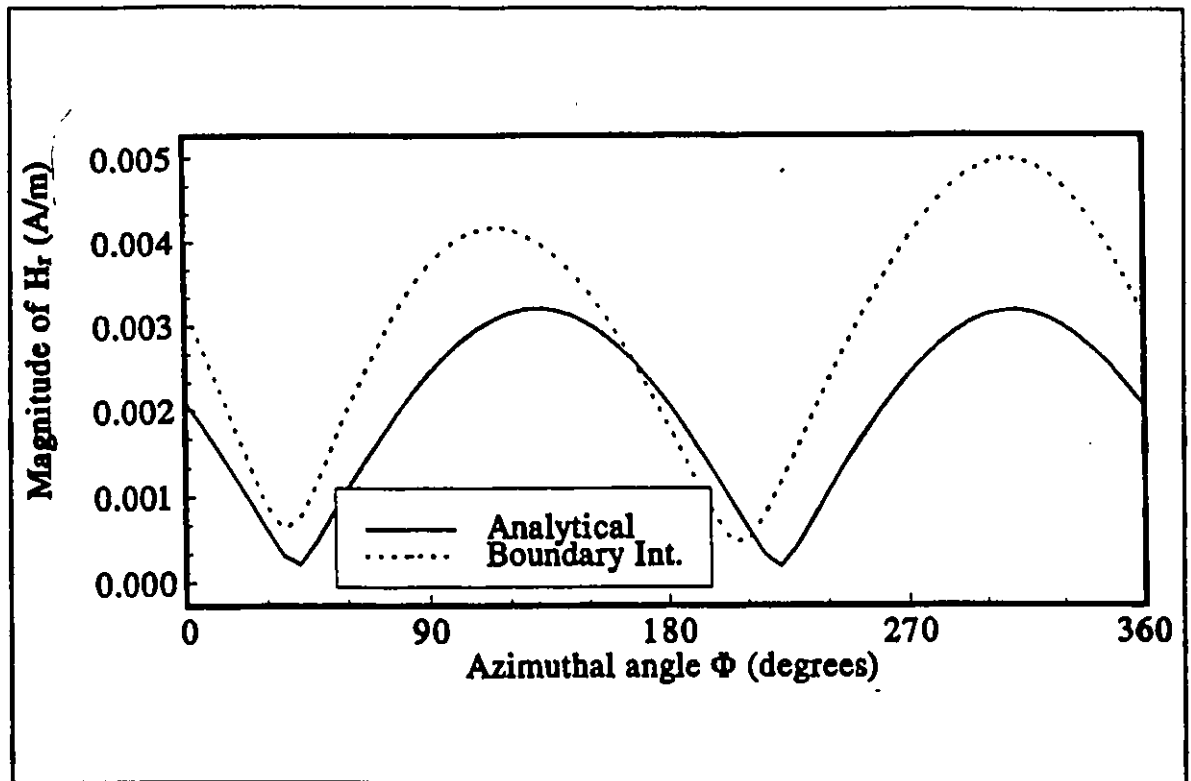


Figure 4.33: Magnitude of the r-component of the magnetic field at $r=30\text{m}$ and $\theta = 37^\circ$

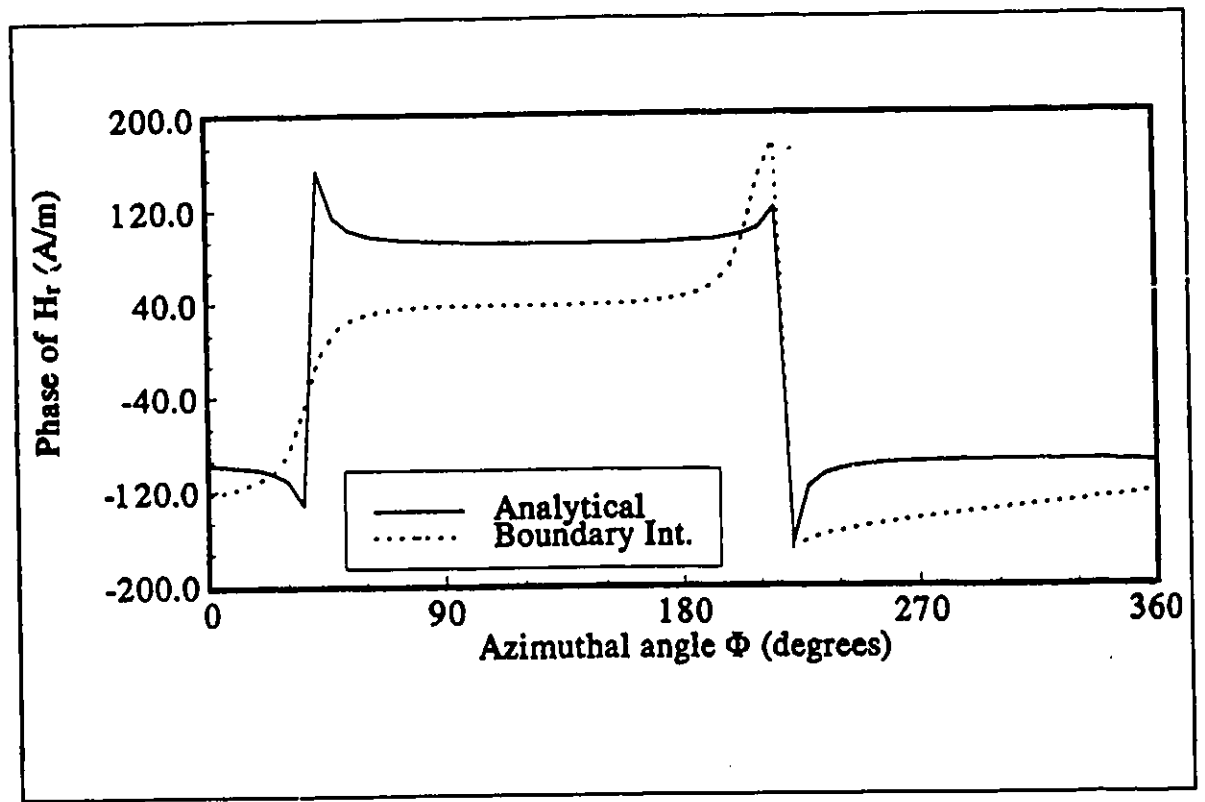


Figure 4.34: Phase of the r-component of the magnetic field at $r=30\text{m}$ and $\theta = 37^\circ$

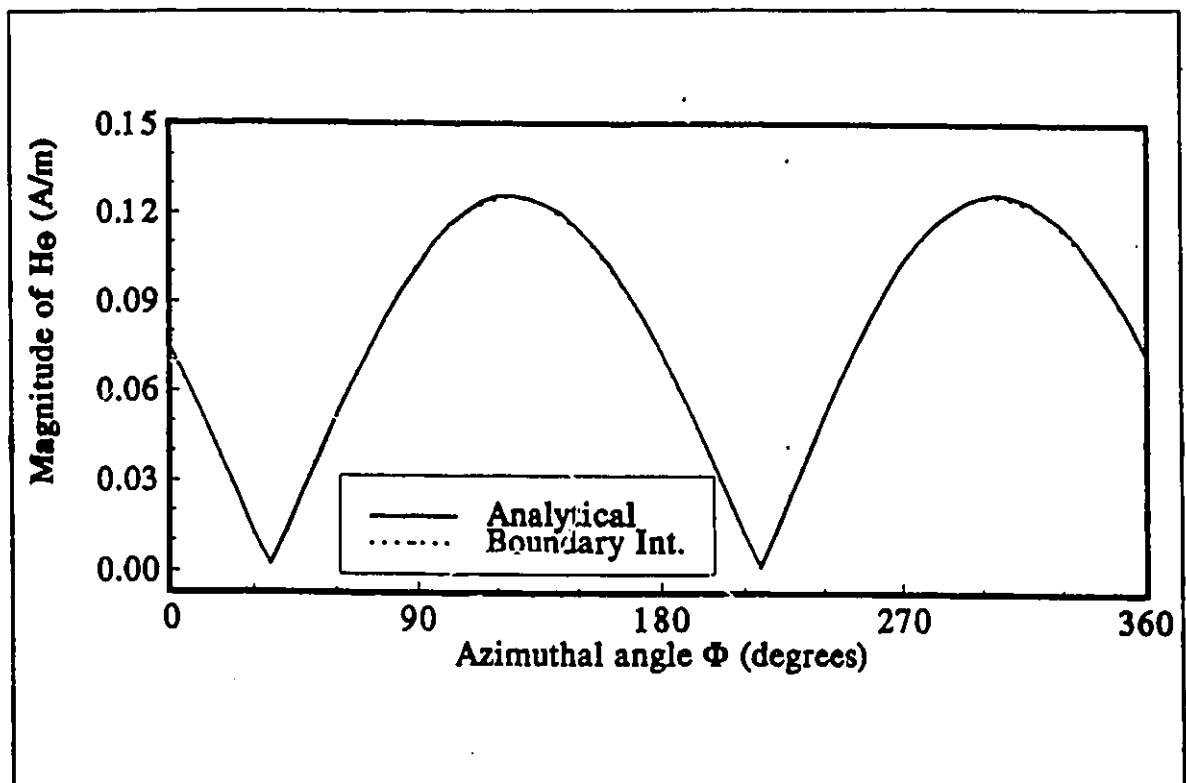


Figure 4.35: Magnitude of the θ -component of the magnetic field at $r=30\text{m}$ and $\theta = 37^\circ$

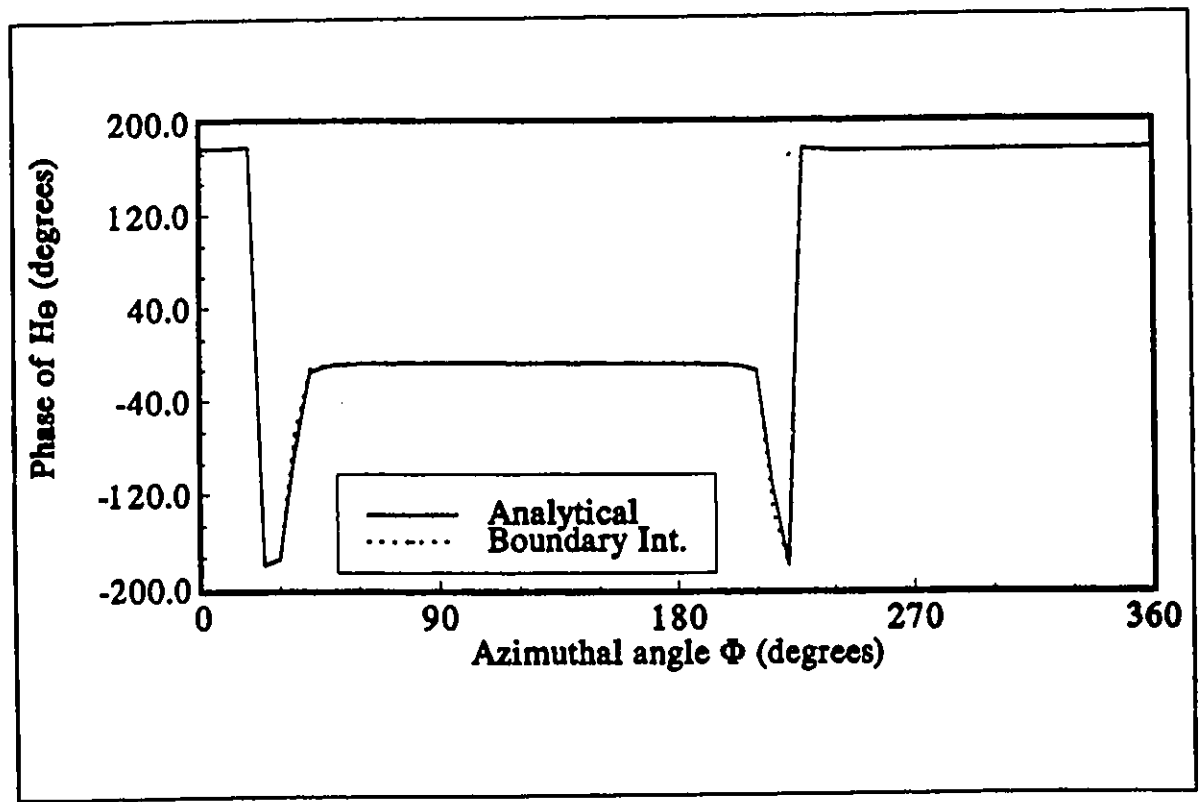


Figure 4.36: Phase of the θ -component of the magnetic field at $r=30\text{m}$ and $\theta = 37^\circ$

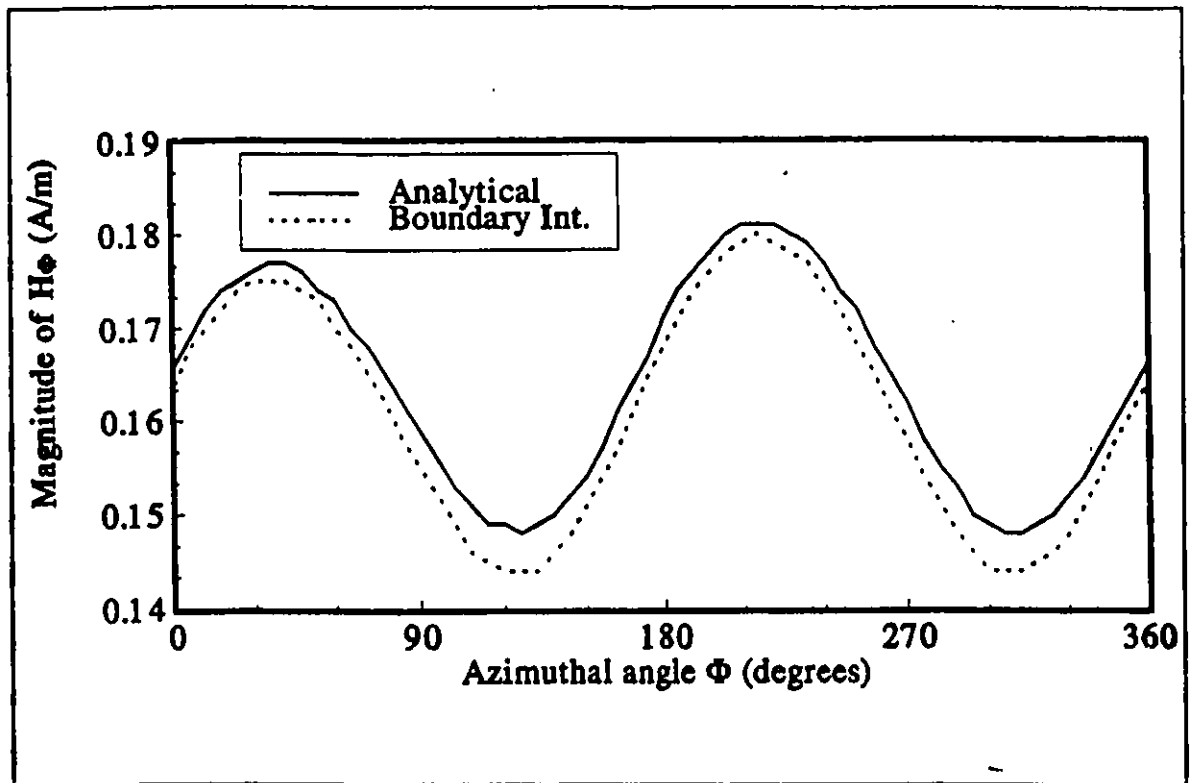


Figure 4.37: Magnitude of the ϕ -component of the magnetic field at $r=30\text{m}$ and $\theta = 37^\circ$

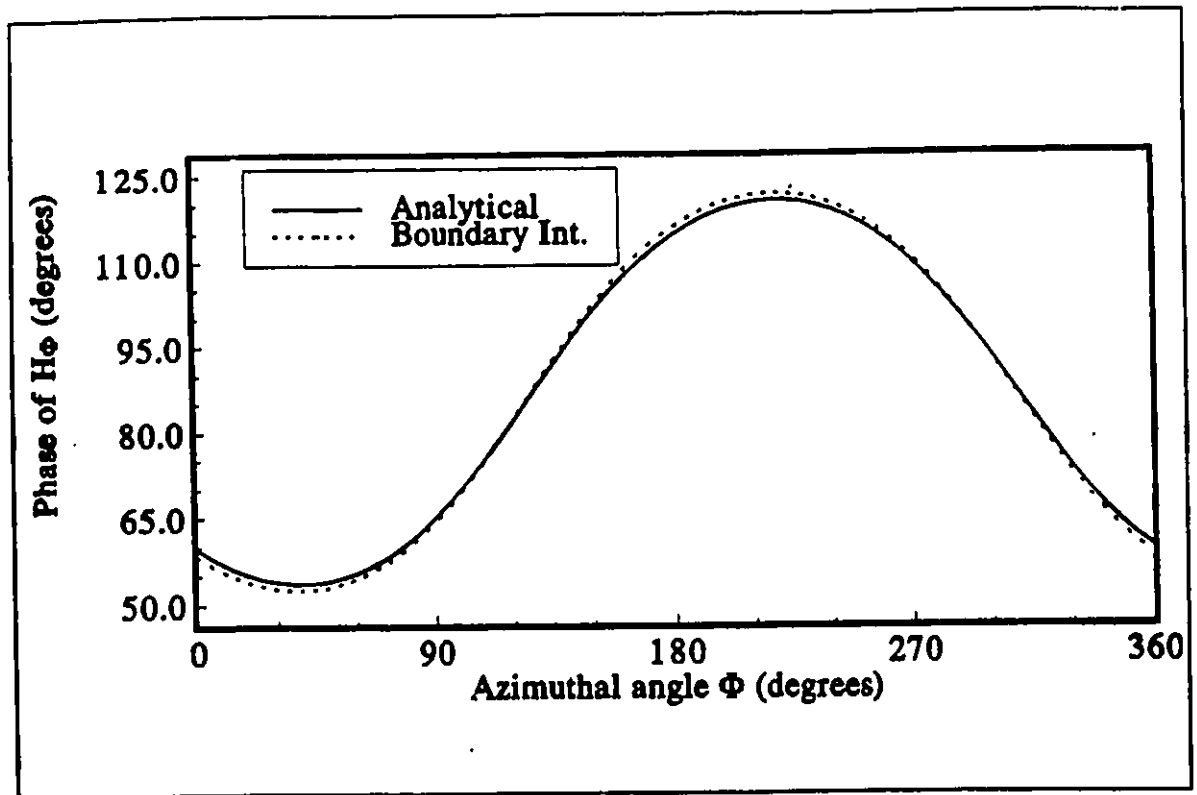


Figure 4.38: Phase of the ϕ -component of the magnetic field at $r=30\text{m}$ and $\theta = 37^\circ$

Chapter 5

Conclusions

The intent of this thesis has been to derive a method to transform near-field measurements to far-field values. The problem is a three dimensional wave propagation problem. The finite element method was used first. It required the generation of a finite element mesh to discretize the 3-D solution domain and the application of absorbing boundary conditions to truncate the domain. The approach was implemented via a series of FORTRAN programs (Appendix C).

The far-field distances were not achievable with the FEM because of technical limitations. Nevertheless, it was proven that the FEM method, in spurious mode free formulation, can be used to solve 3-D wave propagation problems and eventually, hardware allowing, be used for near-field-far-field transformations with the advantage of being able to add inhomogeneities to the solution domain.

To achieve far-field distances, a second method based on a boundary integral equation solution to the scattering problem has been used for near-field to far-field transformation. This method revealed itself to be more appropriate for the problem and provided quick and accurate results. One of its drawbacks was that it required a homogeneous domain. The same boundary integral solution was then modified to derive a global ABC for the FEM solution that proved to be better than existing ABCs.

The 3-D automatic mesh generator, one of the major problems with 3-D FEM, was written to incorporate as much versatility as possible and is in fact able to discretize a wide range of shapes and forms of solution domains. It is also able to introduce inhomogeneous media in the domain. To take advantage of this and solve the wave propagation in the inhomogeneous domain, the FEM formulation in Cartesian coordinates which uncouples the components would have to be modified.

Appendix A

Vector and Integral Identities

$$\nabla \times \nabla \vec{G} = \nabla(\nabla \cdot \vec{G}) - \nabla^2 \vec{G} \quad (\text{A1})$$

$$\nabla \cdot (f \vec{G}) = \vec{G} \cdot \nabla f + f(\nabla \cdot \vec{G}) \quad (\text{A2})$$

$$\nabla \times (f \vec{G}) = \nabla f \times \vec{G} + f(\nabla \times \vec{G}) \quad (\text{A3})$$

$$\nabla \times (\vec{F} \times \vec{G}) = \vec{F}(\nabla \cdot \vec{G}) - \vec{G}(\nabla \cdot \vec{F}) + (\vec{G} \cdot \nabla) \vec{F} - (\vec{F} \cdot \nabla) \vec{G} \quad (\text{A4})$$

$$\nabla(\vec{F} \cdot \vec{G}) = (\vec{F} \cdot \nabla) \vec{G} + (\vec{G} \cdot \nabla) \vec{F} + \vec{F} \times (\nabla \times \vec{G}) + \vec{G} \times (\nabla \times \vec{F}) \quad (\text{A5})$$

$$\vec{F} \cdot (\vec{G} \times \vec{H}) = \vec{G} \cdot (\vec{H} \times \vec{F}) = \vec{H} \cdot (\vec{F} \times \vec{G}) \quad (\text{A6})$$

$$\int_V (\nabla \times \vec{G}) dV = \int_S (\hat{n} \times \vec{G}) dS \quad (\text{A7})$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C} \quad (\text{A8})$$

$$\int_S [(\hat{n} \cdot \nabla)(\phi \vec{H}) + \hat{n} \times \nabla(\phi \vec{H}) - \hat{n} \nabla \cdot (\phi \vec{H})] dS = 0 \quad (\text{A9})$$

Appendix B

Analytical Integration in Simplex Coordinates

For the calculation of the contribution to the general system of equations from a single element, surface and volume integrals have to be performed over the FE in question. With the simplex coordinates, the integrals can be done analytical without introducing any numerical errors.

The 3-D domain of all the finite elements is defined by a variation of the simplex variables $\delta_1, \delta_2, \delta_3$ and δ_4 from zero to one. The integrals arising from the evaluation of the FEM system have all one of the following two forms:

$$\int_{V_e} \delta_1^i \delta_2^j \delta_3^k \delta_4^l dV_e \quad (.1)$$

or

$$\int_{S_e} \delta_1^i \delta_2^j \delta_3^k \delta_4^l dS_e \quad (.2)$$

where V_e is the volume of the e^{th} finite element and S_e is the surface of one of the four faces defining the tetrahedral FE.

Because the simplex coordinates satisfy the identity

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 = 1, \quad (.3)$$

only three variables are needed to describe a volume, and only two for a surface, one remaining constant.

For V_e ,

$$\delta_4 = 1 - \delta_1 + \delta_2 + \delta_3, \quad (.4)$$

and for one face of V_e (i.e. S_e), one of the simplexes will be the constant zero.

$$\delta_3 = 1 - \delta_1 + \delta_2, \quad \delta_4 = 0. \quad (.5)$$

The integrals are integrals of polynomials which have a closed form analytical expression and hence for a volume integral

$$\int_{V_e} \delta_1^i \delta_2^j \delta_3^k \delta_4^l dV_e = 3!V_e \left(\frac{i!j!k!l!3!}{(i+j+k+l+3)!} \right), \quad (.6)$$

and for a surface integral

$$\int_{V_e} \delta_1^i \delta_2^j \delta_3^k dS_e = 2!S_e \left(\frac{i!j!k!2!}{(i+j+k+2)!} \right). \quad (.7)$$

where the Jacobian has been used to change variables in the integral;

$$dV_e = 3!V_e d\delta_1 d\delta_2 d\delta_3 \quad (.8)$$

and

$$dS_e = 2!S_e d\delta_1 d\delta_2 \quad (.9)$$

The volume V_e and the surface S_e is readily calculated from the coordinates of the tetrahedron vertices. In the surface integrals the simplex variable which is constant is always dropped before evaluation.

Appendix C

The Computer Programs

All the code with the exception of the iterative solving routines has been written by the author in DEC FORTRAN77 for RISC Version 1.0 and executed on the Digital 3100 RISC DECstations(16Meg RAM, MIPS2000 CPU chip).

The first program POINTSFECREAT.F generates the list of node indexes, their coordinates and their boundary IDs to be used in the meshing program. If a mesh already exists, POINTSFECREAT can be used to generate the same list of points as was used for the mesh but modify the boundary IDs. This permits the user to have similar meshed structures but with different boundary conditions. The new boundary conditions are implemented in the mesh by running PROPERTIES.F. The mesh itself is generated by running MESHMAIN.F. Before a FEM problem is solved, all the Dirichlet and Neumann boundary values have to be defined according to the data in the FE mesh. This is done by running the program PREPROS.F. The system is solved then by running MAINCOMPSG.F which will ask all kind of questions about the type of ABC to be used, the maximum number of iterations for the iterative method, the maximum error tolerance and so on. The non-zero entries in the system solved is stored in one large column vector. If the number of non-zero entries in each individual column in the system is known it will speed up the building up of the system. For the local ABC, this number is around 70. If it is not known, it is good to overshoot if the computer memory allows it, otherwise it's a good idea to store this filling pattern for speedier subsequent runs. The program RESPROS.F will process the results and provide field values at the desired coordinates.

The program BEMRAD.F is based on the boundary integral method. It uses a smaller mesh and provides field results at any range. The mesh can be the same as the one used for FEM. There are beam versions of PROPERTIES.F, PREPROS.F and POINTSFECREAT.F that have to be rerun on the FEM mesh. They are not listed since they are very similar, with minor variations in the boundary conditions handling.

ANALYTICRAD.F and DERANALRAD.F are the two programs that give the analytical results for comparison. ANALYTICRAD.F gives the values of the field at any desired position and DERANALRAD.F gives the derivative of the field in the radial direction. The programs listed after DERANALRAD.F are listings of subroutines called by the main programs mentioned above. The code has been written with as many subroutines as possible with descriptive subroutine names. Further documentation is included in the code listings

POINTSFECEAT.F

```
C PROGRAM THAT GENERATES THE POINTS TO BE USED BY THE MESH GENERATOR
C AND THE VARIOUS MESH MODIFIERS
C THE ID CODE OF EVERY NODE IS DEFINED AS FOLLOWS
C CODED BY POSITION NXASCD
CX=1 OUTER BOUNDARY
CX=0 INNER BOUNDARY (NO SPECIAL PROPERTY)
CA OR B OR C=1 DIRICHLET
CA OR B OR C=2 NEUMANN
CA OR B OR C=3 UNKNOWN
CA OR B OR C=4 ON THE BOUNDARY OF A DIRICHLET AND A NEUMANN
CA FOR X COMPONENT
CB FOR Y COMPONENT
CC FOR Z COMPONENT
CD DIGIT CODE FOR THE MEDIUM USUALLY ONE FOR ALL HOMOGENEOUS
CN=5 NODE LYING ON A SURFACES NEEDED FOR THE INTEGRAL ABC
CNPS IS THE NUMBER OF NODES ON THE DIRICHLET SURFACE
CNPV IS THE TOTAL NUMBER OF NODES
C
C IMPLICIT NONE
REAL P(41000,3)
INTEGER NPS,NPV,PID(41000),NP,I
C
NP=0
CALL SURFACEPOINTGEN(NP,P,PID,1,1,3,0.5,0.5,0.5)
NPS=NP
CALL EXTRAPPOINTS(NP,P,PID,1,1,3,3,4,0.5,0.5,0.5)
NPV=NP
CALL WRITER(NPS,NPV,P,PID)
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
SUBROUTINE THAT GENERATES THE POINTS OFF THE DIRICHLET SURFACE.
C DEPENDS ON THE DIFFERENT IF STATEMENTS THE LAST OR ALL LAYERS ARE
CMAPPED ON A SPHERICAL DOME
SUBROUTINE EXTRAPPOINTS(NP,P,PID,LX1,LY1,LZ1,LX,LY,LZ,DX,DY,DZ)
INTEGER NP,PID(41000),LX,LY,LZ,LX1,LY1,LZ1
REAL P(41000,3),DX,DY,DZ
C
```

```

IMPLICIT NONE
REAL X,Y,Z,R,SX,SY,SZ
INTEGER I,J,K,RX,RY,RZ
LOGICAL LAST1,LAST,RLAST,LAY1
DO 10 I=1,LX,LX
DO 20 J=1,LY,LY
DO 30 K=0,LZ
RX=ABS(I)
RY=ABS(J)
RZ=ABS(K)
IF ((RX.GE.RY).AND.(RX.GE.RZ)) THEN
RY=RX
RZ=RX+1
ELSE IF ((RY.GE.RX).AND.(RY.GE.RZ)) THEN
RX=RY
RZ=RY+1
ELSE IF ((RZ.GE.RY).AND.(RZ.GE.RX)) THEN
RX=RZ-1
RY=RZ-1
END IF
R=SQRT((RX**2)+(RY**2)+(RZ**2))
IF ((ABS(I).LE.LX1).AND.(ABS(J).LE.LY1).AND.(ABS(K).LE.LZ1)) GOTO 30
RLAST=(ABS(J).EQ.LY).OR.(ABS(I).EQ.LX).OR.(K.EQ.LZ)
LAST=(ABS(J).GT.LY1).OR.(ABS(I).GT.LX1).OR.(K.GT.LZ1)
* last like this will convert all layers into down
LAY1=(ABS(J).GE.(LY-1)).OR.(ABS(I).GE.(LX-1)).OR.(K.GE.(LZ-1))
LAST1=(LAY1.AND.(NOT.(RLAST)))
NP=NP+1
X=I*DX
Y=J*DY
Z=DZ*K
P(NP,1)=X
P(NP,2)=Y
P(NP,3)=Z
PID(NP)=1
* IF (LAST) THEN
IF (RLAST) THEN
CALL CONVARTOSFE(X,Y,Z,SX,SY,SZ,R)
P(NP,1)=SX
P(NP,2)=SY
P(NP,3)=SZ
IF (RLAST) PID(NP)=10001
* IF (RLAST) PID(NP)=11111
END IF
IF (K.EQ.0) PID(NP)=12211
* IF ((K.EQ.0).AND.RLAST) PID(NP)=14411
IF (LAST1) PID(NP)=PID(NP)+800000
30 CONTINUE
20 CONTINUE
10 CONTINUE
5 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CONVARTOSFE(CX,CY,CZ,SX,SY,SZ,R)
REAL CX,CY,CZ,SX,SY,SZ,R
C
IMPLICIT NONE
REAL TET,FI,R1
C
R1=SQRT(CX**2+CY**2+CZ**2)
TET=ACOS(CZ/R1)
IF (R1.EQ.0) TET=0.0
FI=ATAN(CY/CX)
IF (CX.EQ.0) FI=0.0
IF ((CX.EQ.0).AND.(CY.GT.0)) FI=1.570796327
IF ((CX.EQ.0).AND.(CY.LT.0)) FI=-1.570796327
IF ((CX.GT.0).AND.(CY.EQ.0)) FI=0.0
IF ((CX.LT.0).AND.(CY.EQ.0)) FI=3.1415926
IF ((CX.LT.0).AND.(CY.LT.0)) FI=FI+3.1415926
IF ((CX.LT.0).AND.(CY.GT.0)) FI=FI+3.1415926
SX=R*SIN(TET)*COS(FI)
SY=R*SIN(TET)*SIN(FI)
SZ=R*COS(TET)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE THAT STORES THE POINTS GENERATED. IF THE POINTS ARE INTENDED FOR
CA NEW MESH unpoint.dat HAS TO BE COPIED INTO unmesh.dat
SUBROUTINE WRITER(NPS,NPV,P,PID)
INTEGER NPS,NPV,PID(41000)
REAL P(41000,3)
C
IMPLICIT NONE
INTEGER REK,OFFREC,I,SCP,SCF,SCE
C
OPEN(UNIT=2,FILE='point.dat',STATUS='UNKNOWN',ACCESS='SEQUENTIAL',
* FORM='FORMATTED')
OPEN(UNIT=3,FILE='unpoint.dat',STATUS='UNKNOWN',ACCESS='DIRECT',
* FORM='UNFORMATTED',RECL=11,RECORDTYPE='FIXED')
C
SCP=0
SCF=0
SCE=0
WRITE(2,FMT=100) NPS, THE NUMBER OF POINTS ON SURFACES'
WRITE(2,FMT=100) NPV, THE NUMBER OF ALL POINTS'
WRITE(2,FMT=100) SCP, THE NUMBER OF PYRAMIDS'
WRITE(2,FMT=100) SCF, THE NUMBER OF FACES'
WRITE(2,FMT=100) SCE, THE NUMBER OF EDGES'
WRITE(2,FMT=100) '---,POINTS ON SURFACE---'
WRITE(3,REC=1) NPS
WRITE(3,REC=2) NPV
WRITE(3,REC=3) SCP
WRITE(3,REC=4) SCF
WRITE(3,REC=5) SCE
* S=1000
OFFREC=5
DO 10 I=1,NPS
REK=OFFREC+I
WRITE(2,FMT=110) I,P(I,1),P(I,2),P(I,3),PID(I)
10 WRITE(3,REC=REK) P(I,1),P(I,2),P(I,3),PID(I)
OFFREC=REK

```

```

WRITE(2,FMT=120) '---INSIDE SURFACE---'
OFFREC=OFFREC-NPS
DO 20 I=NPS+1,NPV
  REK=OFFREC+I
  WRITE(2,FMT=110) I,P(1,1),P(1,2),P(1,3),PID(I)
20 WRITE(2,REC=REK) P(1,1),P(1,2),P(1,3),PID(I)
C
C
100 FORMAT(18,(A))
110 FORMAT(X,18,3X,F9.5,3X,F9.5,3X,F9.5,3X,17)
120 FORMAT((A))
130 FORMAT(X,F9.5,3X,F9.5,3X,F9.5,3X,18,2X,18,2X,18)
1000 CLOSE(2)
CLOSE(3)
END
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
SUBROUTINE THAT GENERATES THE POINTS ON THE DIRICHLET SURFACE
SUBROUTINE SURFACEPOINTGEN(NP,P,PID,LX,LY,LZ,DX,DY,DZ)
INTEGER NP,PID(41000),LX,LY,LZ
REAL P(41000,3),DX,DY,DZ
C
C ON THE CUBE
IMPLICIT NONE
INTEGER I,J
DO 10 I=LX,LX
  DO 20 J=LY,LY
    NP=NP+1
    P(NP,1)=I*DX
    P(NP,2)=J*DY
    P(NP,3)=LZ*DZ
20  PID(NP)=11111
10  CONTINUE
C
DO 30 I=LY,LY
  DO 40 J=0,(LZ-1)
    NP=NP+1
    P(NP,1)=LX*DX
    P(NP,2)=I*DY
    P(NP,3)=J*DZ
    PID(NP)=11111
    IF (J.EQ.0) PID(NP)=14411
    NP=NP+1
    P(NP,1)=LX*DX
    P(NP,2)=I*DY
    P(NP,3)=J*DZ
    PID(NP)=11111
    IF (J.EQ.0) PID(NP)=14411
40  CONTINUE
30  CONTINUE
C
DO 50 I=(LX-1),(LX-1)
  DO 60 J=0,(LZ-1)
    NP=NP+1
    P(NP,1)=I*DX
    P(NP,2)=LY*DY
    P(NP,3)=J*DZ
    PID(NP)=11111
    IF (J.EQ.0) PID(NP)=14411
    NP=NP+1
    P(NP,1)=I*DX
    P(NP,2)=LY*DY
    P(NP,3)=J*DZ
    PID(NP)=11111
    IF (J.EQ.0) PID(NP)=14411
60  CONTINUE
50  CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

```

MESHMAIN.F

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C PROGRAM CONNECTS POINTS IN A MESH CREATE TETRAHEDRAL FINITE C
C ELEMENTS C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C ANY TIME YOU CHANGE THE GEOMETRY OF THE PROBLEM YOU HAVE TO
C CHANGE THE PARAMTERS IN CENTervalIDS AND CENTervalIDV OR THEIR
C CALLS
C
IMPLICIT NONE
INTEGER FACE(85000,4),EDGE(80000,3),PYR(40000,5),NP,SCF,SCE,SCP
INTEGER EXTRAPYR(40000,6),PID(30000),NPS,NPV,EPV
REAL P(30000,3)
CHARACTER*24 fdate
EXTERNAL fdate
C
C
SCF=0
SCE=0
SCP=0
NP=0
WRITE(*,*) fdate()
CALL POINTREADER(NPS,NPV,PID,P)
WRITE(*,*) 'I JUST READ THE POINTS'
CALL FACECREATIONSURFACE(NP,NPS,SCF,SCE,P,PID,EDGE,FACE)
WRITE(*,*) 'I JUST CREATED THE SURFACES'
NPV=NPS+NPV
CALL PYRAMIDGENERATOR(NP,NPV,SCF,SCE,SCP,P,PID,EDGE,FACE,PYR)
C
WRITE(*,*) 'I JUST CREATED PYRAMIDS'
NP=NPV
CALL SCNDORDER(SCE,SCP,EDGE,FACE,PYR,EXTRAPYR)
WRITE(*,*) 'I JUST CREATED THE 6 EXTRA POINTS'
CALL MESHWRITER(NP,NPS,SCF,SCP,SCE,P,PID,EDGE,FACE,PYR,EXTRAPYR)
WRITE(*,*) 'I M DONE.'
WRITE(*,*) fdate()
C
END

```

PROPERTIES.F

```

C PROGRAM THAT MODIFIES THE PROPERTIES OF FACES IN AN ALREADY
C MESHED VOLUME. CREATES NEW POINTS ON ALL EDGES AND STORES THE
C ONE ON BOUNDARY CONDITIONS IN dirichletx.dat
C PROGRAM WORKS SUCCESSFULLY ONLY IF THE NUMBER OF NEW POINTS IS
C THE SAME AS THE NUMBER OF OLD POINTS AND THE COORDINATE TRANSFORMATION
C DOES NOT CREATE OVERLAPPING FINITE ELEMENTS
C
C IMPLICIT NONE
C
INTEGER NP,SCP,SCF,SCE,PID(30000),EDGE(80000,3),FACE(88000,4),NPS
INTEGER PYR(40000,6),EXTRAPYR(40000,6),FIL,NPS1,NP1,I,PIDN(21000,4)
INTEGER EDGEID,FACID,PYRID,IND(4),OCCUR,PYRCOM,PIDD(21000,4),NN,ND
INTEGER I1,I2,I3,I4
REAL P(30000,3),DUM1(3),DUM2(3),DUM(3)
C
OPEN(UNIT=4,FILE='dirichletx.dat',STATUS='UNKNOWN',
+ ACCESS='SEQUENTIAL',FORM='FORMATTED')
CALL GETBASINFO(NP,NP,SCP,SCF,SCF,FIL)
CALL MESHREADER(NP,NPS,SCP,SCF,SCE,P,PID,EDGE,FACE,PYR,EXTRAPYR,FIL)
CALL NEWPOINTREADER(NPS1,NP1,PID,P)
IF ((NPS.NE.NPS1).OR.(NP.NE.NP1)) THEN
  WRITE(*,*) 'YOU RE ABOUT TO SCREWUP THE OLD MESH !'
  WRITE(*,*) 'I M GONNA SAVE THE DAY!'
  GOTO 100
END IF
DO 20 I=1,SCP
  CALL BASE3NODE(EDGE,FACE,I,IND(1),IND(2),IND(3))
  CALL OCCURENCEP(I,SCP,OCCUR,PYR,PYRCOM)
  FACE(I,4)=FACID(PID(IND(1)),PID(IND(2)),PID(IND(3)))
20 CONTINUE
DO 10 I=1,SCE
  IND(1)=EDGE(I,1)
  IND(2)=EDGE(I,2)
  EDGE(I,3)=EDGEID(PID(IND(1)),PID(IND(2)))
  NP1=NP1+1
  CALL ONEDIM(P(IND(1),1),P(IND(1),2),P(IND(1),3),DUM1)
  CALL ONEDIM(P(IND(2),1),P(IND(2),2),P(IND(2),3),DUM2)
  CALL MIDPOI(DUM1,DUM2,DUM)
  P(NP1,1)=DUM(1)
  P(NP1,2)=DUM(2)
  P(NP1,3)=DUM(3)
  PID(NP1)=EDGE(I,3)
10 CONTINUE
CALL GETPIDW(PID,PIDD,PIDN,NP1,NN,ND)
WRITE(*,*) ND
DO 50 I=1,ND
  I1=PIDD(I,1)
  I2=PIDD(I,2)
  I3=PIDD(I,3)
  I4=PIDD(I,4)
50 WRITE(*,130) P(I1,1),P(I1,2),P(I1,3),I2,I3,I4
WRITE(*,*) NN
DO 40 I=1,NN
  I1=PIDN(I,1)
  I2=PIDN(I,2)
  I3=PIDN(I,3)
  I4=PIDN(I,4)
40 WRITE(*,130) P(I1,1),P(I1,2),P(I1,3),I2,I3,I4
C
DO 30 I=1,SCP
  CALL PYRANODE(EDGE,FACE,PYR,I,IND)
30 PYR(I,5)=PYRID(PID(IND(1)),PID(IND(2)),PID(IND(3)),PID(IND(4)))
CALL MESHWRITWO(NP,NPS,SCP,SCF,SCE,P,PID,EDGE,FACE,PYR,EXTRAPYR)
100 CONTINUE
130 FORMAT(X,F9.5,3X,F9.5,3X,F9.5,3X,I5,2X,I5,2X,I5)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C SUBROUTINE THAT READS THE SIZE OF THE ALREADY EXISTING MESH
SUBROUTINE GETBASINFO(NP,NP,SCP,SCF,SCF,FIL)
INTEGER NP,SCP,SCF,SCF,FIL,NPS
C
IMPLICIT NONE
OPEN(UNIT=2,FILE='unmesh.dat',STATUS='UNKNOWN',ACCESS='DIRECT',
+ FORM='UNFORMATTED',RECL=11,RECORDTYPE='FIXED')
READ(2,REC=1) NPS
READ(2,REC=2) NP
READ(2,REC=3) SCP
READ(2,REC=4) SCF
READ(2,REC=5) SCE
FIL=2
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C SUBROUTINE THAT READS THE MESH POINTS' NEW COORDINATE AND PROPERTY IDS
SUBROUTINE NEWPOINTREADER(NPS,NP,PID,P)
INTEGER NPS,NP,PID(30000)
REAL P(30000,3)
C
IMPLICIT NONE
INTEGER REK,OFFREC,I,SCP,SCF,SCE
C
OPEN(UNIT=3,FILE='unpoint.dat',STATUS='UNKNOWN',ACCESS='DIRECT',
+ FORM='UNFORMATTED',RECL=11,RECORDTYPE='FIXED')
C
READ(3,REC=1) NPS
READ(3,REC=2) NP
READ(3,REC=3) SCP
READ(3,REC=4) SCF
READ(3,REC=5) SCE
OFFREC=5
DO 10 I=1,NPS
  REK=OFFREC+I
10 READ(3,REC=REK) P(I,1),P(I,2),P(I,3),PID(I)
OFFREC=REK
OFFREC=OFFREC-NPS
DO 20 I=NP+1,NP
  REK=OFFREC+I
20 READ(3,REC=REK) P(I,1),P(I,2),P(I,3),PID(I)
C

```



```

END IF
WRITE(*,*) 'OK, HERE WE GO!'
WRITE(*,*)
DO I=1,3
  CALL BUILD5(A,AI,AJ,RHS,N,NZ,SIZ,NZERO,FIL3,I,DIR,NOI,
    * DIRN,NOIN,MDIR,
    * B5PYR,SIMP,NOR,NODES,MAXFAC,CO,ALF,H2OS,SS,SOB,NP,EPS,BET,
    * NPYR,PYRIND,NPS,NPOS,H,DH,HNODE,ABC,SPC,UPD)
  WRITE(*,*) 'I VE JUST BUILT 5 AND SIZE IS',SIZ
  PER=((NZERO*1.0)/SIZ**2)*100.0
  WRITE(*,*) 'PERCENT OCCUPANCY:',PER
  IF (GUESS.EQ.1) THEN
    CALL VFILLGUESS( SIZ, SOL, N, I)
  ELSE
    CALL VFILLDIR( SIZ, SOL, RHS, N)
  END IF
  WRITE(*,*) 'SOLVING FOR THE ',I,'th COMPONENT.'
  CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
  C CALL SOLVER(SIZ, RHS, SOL, NZERO, AI, AJ, A,
  C *TOL, ITMAX, ITER, ERR, IERR,IWORK,N,NZ)
  ISYM=0
  ITOL=1
  IUNIT=0
  *FOR MONITORING CONVERGENCE
  IUNIT=6
  LENW=N*9
  LENIW=20
  CALL DSDCGS(SIZ, RHS, SOL, NZERO, AI, AJ, A, ISYM,
    *ITOL, TOL, ITMAX, ITER, ERR, IERR, IUNIT,RWORK,LENW,IWORK,LENIW)
  CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
  REPEAT(I)=ITER
  WRITE(*,*) 'I VE JUST SOLVED 5 FOR I EQUAL',I
  WRITE(*,*) 'IT TOOK',ITER,'ITERATIONS!'
  WRITE(*,*) 'THE ERROR RESIDUE IS',ERR
  WRITE(*,*) 'STATUS IS',IERR
  WRITE(*,*) 'IT USED',IWORK(10),' STORAGE LOCATIONS FOR REAL.'
  WRITE(*,*) 'IT USED',IWORK(9),' STORAGE LOCATIONS FOR INTEGER.'
  CALL SOLWRITER(SOL,N,SIZ,I)
  WRITE(*,*) 'DONE WITH ONE COMPONENT'
  I CONTINUE
  WRITE(*,*) 'I M DONE.'
  CLOSE(FIL3)
  DO I=1,3
    I WRITE(*,*) 'COMPONENT ',I,'TOOK ',REPEAT(I),' ITERATIONS.'
    WRITE(*,*) (data())
    ELAPSE=ETIME(TIMEARRAY)
    WRITE(*,*) ELAPSE
  END
  C
  CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
  C
  C SUBROUTINE SOLVER(SIZ,RHS,SOL,NZERO,AI,AJ,A,TOL,ITMAX,ITER,ERR,IERR
  C * IWORK,N,NZ)
  C INTEGER SIZ,AI(NZ),AJ(N),NZERO,NZ
  C INTEGER IWORK(20),ITER,ITMAX,IERR,N
  C REAL*8 A(NZ),RHS(N),SOL(N),TOL,ERR
  CC
  C IMPLICIT NONE
  C INTEGER ITOL
  C INTEGER IUNIT,ISYM,LENW,LENIW
  C REAL*8 RWORK(600000)
  CC
  C ISYM=0
  C ITOL=1
  C IUNIT=6
  C *FOR MONITORING CONVERGENCE
  C IUNIT=0
  C LENW=N*10
  C LENIW=20
  C CALL DSDCGN(SIZ, RHS, SOL, NZERO, AI, AJ, A, ISYM,
  C *ITOL, TOL, ITMAX, ITER, ERR, IERR, IUNIT,RWORK,LENW,IWORK,LENIW)
  C END
  C
  CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

```

RESPROS.F

```

C PROGRAM THAT PROCESSES THE RESULTS IN THE FILE unresolution.dat (IT ASKS FOR
C THE COORDINATES WHERE THE FILE IS DESIRED
C
C
C IMPLICIT NONE
REAL AP(4),BP(4),CP(4),DP(4),POINT(1,3),COORD(4,3),I,J,Z,Y1,Z1
REAL SIMP(1,4),A(4),B(4),C(4),D(4),BET,IX,FX,IX,IV,IZ,FY,FZ,SY,SZ
COMPLEX EPSILON(10),EX,EY,EZ,EX0(10),EY0(10),EZ0(10),COMPUTE
INTEGER PYR,K,L,INDP(6),INDEX(4),NODE(10),NP,SCP,REK
INTEGER ALF(10,3,3),FIL,FIL3,SCF,SCF,EDGE(4,3),POINTS(4,3,2),MEDP
INTEGER MEDP(4),MEDE(4,3),SCREEN,VAR,SC
LOGICAL VAL(4),ABOVE
CHARACTER*2 AA
CHARACTER*20 FILENAME,FNAME,FNAME1
COMPLEX HR,HTET,HFI
C
C
TYPE *. 'ENTER DATA FILENAME?'
ACCEPT 130,FILENAME
TYPE *. 'ENTER MESH FILENAME?'
ACCEPT 130,FNAME
TYPE *. 'WAIT A SEC!'
OPEN(UNIT=4,FILE=FILENAME,STATUS='UNKNOWN',ACCESS='DIRECT'
  * ,FORM='UNFORMATTED',RECL=2,RECORDTYPE='FIXED')
CALL FANCTIONS(ALF,EPSILON,BET)
CALL DEFINEPYR(FIL,SCP,NP,FIL3,SCF,SCF,FNAME)
I TYPE *. 'ENTER OUTPUT VARIABLE(1 FOR SCREEN/2 FOR FILES) '
ACCEPT *,SCREEN
IF (SCREEN.EQ.3) THEN
  TYPE *. 'ENTER OUTPUT FILENAME?'
  ACCEPT 130,FNAME1
  OPEN(UNIT=20,FILE=FNAME1,STATUS='UNKNOWN',ACCESS=
    * 'SEQUENTIAL',FORM='FORMATTED')
  TYPE *. 'ENTER COORDINATE THAT VARIES(R/1,THETA/2,PHI/3) '
  ACCEPT *,VAR

```



```

REAL ABCD(2000,4,4),ABCDI(2000,4,4),NOR(2000,3),NVAL
REAL ABCD1(4,4),ABCDI1(4,4),NOR1(3),H2(30,4,10)
REAL BET,X,Y,Z,IX,IY,PY,SY,IZ,PZ,SZ,S(30,4,4),LW1,LH1,LW2,LH2
REAL A(2000,4),B(2000,4),C(2000,4),D(2000,4),Y1,Z1,Y2,Z2,X1
COMPLEX EPS(10),VAL(3),HF(3),ALLDIR(5000,3),DIR(10,3)
COMPLEX H(2000,30,3),DH(2000,30,3),HI(2000,30,3),DHI(2000,30,3)
COMPLEX HR,HTET,HFI
CHARACTER*2 AA
CHARACTER*20 FNAME
C
IDCOD=811111 ID CODE BY WHICH PYRLAYBAS RECOGNIZES THE PYRAMIDS
NEEDED FOR INTEGRATION
NPUS=4 NUMBER OF COLLOCATION POINT FOR THE RIEMMAN
ISUM(INTEGRAL)
SIZE=2000 (MAXIMUM NUMBER OF PYRAMIDS IN THE LAYER OF PE
NPI=30 (MAXIMUM NUMBER OF NPUS
MAXN=5000 (MAXIMUM NUMBER OF NODES IN THE FIRST LAYER OF PE
SCREEN=1 (PARAMETER FOR SCREEN/FILE OUTPUT
NVAL=0 (NUMBER OF POINTS COMPUTED IN THE FAR-FIELD
PASS=0 (PARAMETER TO BYPASS ROUTINES NECESSARY FOR FIRST RUN
ONLY
C THE NEXT 4 VALUES HAVE TO BE CHANGED IF THE MESH SIZE IS CHANGED
LW1=0.5 (WIDTH/LENGTH OF THE FIRST BOX1 ON WICH MEASUREMENTS
IARE MADE
LH1=1.0 (HEIGHT OF BOX1
LW2=0.75 (WIDTH/LENGTH OF THE SECOND BOX2 CONCENTRIC TO BOX1
DEFINING THE VOLUME TO BE DISCRITIZED BY PE
LH2=1.25 (HEIGHT OF BOX2
CALL PYRLAYBAS(LW1,LH1,LW2,LH2,NPYR,PYRIND,NODES,SIZE,CI,
* NOR,SIMP,BSPYR,IDCOD)
WRITE(*,*) '# OF INNER FACES=',CI
WRITE(*,*) '# OF BOUNDARY PYRAMIDS=',NPYR
CALL FUNCTIONS(ALF,EPS,BET)
CALL READDIRS(ALLDIR,ALLDIRN,MAXN,MAXDIR)
CALL GETSMPVALNP(S,NPI,NPUS)
CALL GETH2(ALF,H2,S,NPI,NPUS)
8 TYPE *, 'ENTER OUTPUT VARIABLE(1 FOR SCREEN/2 FOR FILES) '
ACCEPT *,SCREEN
IF (SCREEN.EQ.2) THEN
TYPE *, 'ENTER OUTPUT FILENAME?'
ACCEPT 140,FNAME
OPEN(UNIT=20,FILE=FNAME,STATUS='UNKNOWN',ACCESS=
* 'SEQUENTIAL',FORM='FORMATTED')
TYPE *, 'ENTER COORDINATE THAT VARIES(X/1,Y/2,Z/3) '
ACCEPT *,VAR
END IF
TYPE *, 'ENTER RADIUS '
ACCEPT *,IX
TYPE *, 'ENTER INITIAL THETA, FINAL THETA AND THE STEP '
ACCEPT *,IY,FY,SY
TYPE *, 'ENTER INITIAL PHI, FINAL PHI AND THE STEP '
ACCEPT *,IZ,PZ,SZ
TYPE *, 'ENTER 99 FOR SPHERICAL/ANYTHING ELSE FOR CARTESIAN'
ACCEPT *,SC
DO 10 X1=1,IX,IX
DO 20 Y1=1,IY,FY,SY
DO 30 Z1=1,IZ,PZ,SZ
HF(1)=CMPLX(0.0,0.0)
HF(2)=CMPLX(0.0,0.0)
HF(3)=CMPLX(0.0,0.0)
NVAL=NVAL+1.0
Y1=Y2*3.14159/180.0
Z1=Z2*3.14159/180.0
X=X1*SIN(Y1)*COS(Z1)
Y=X1*SIN(Y1)*SIN(Z1)
Z=X1*COS(Y1)
DO 40 I=1,CI
CALL SELECTDATA(BSPYR(I),I,SIMP,NOR,NODES,SIZE,NOD1,SMP1,
* ABCD1,ABCDI1,NOR1)
IF (PASS.EQ.0) THEN
CALL GETDIR10(DIR,NOD1,PYRIND(BSPYR(I)))
CALL INTEGRAL(X,Y,Z,BET,VAL,SMP1,ABCD1,ABCDI1,NOR1,
* ALF,H2,DIR,S,NPI,NPYR,PYRIND,NODES,SIZE,NPUS,BSPYR(I),
* I,H,DH,HI,DHI)
ELSE
CALL INTEGRAL1(X,Y,Z,BET,VAL,SMP1,ABCD1,ABCDI1,NOR1,
* S,NPI,SIZE,NPUS,I,H,DH,HI,DHI)
END IF
HF(1)=HF(1)+VAL(1)
HF(2)=HF(2)+VAL(2)
40 HF(3)=HF(3)+VAL(3)
PASS=1
IF (SC.EQ.99) THEN
HR=HF(1)*SIN(Y1)*COS(Z1)+HF(2)*SIN(Y1)*SIN(Z1)+HF(3)*COS(Y1)
HTET=HF(1)*COS(Y1)*COS(Z1)+HF(2)*COS(Y1)*SIN(Z1)-HF(3)*SIN(Y1)
HFI=HF(1)*SIN(Z1)+HF(2)*COS(Z1)+HF(3)*0.0
ELSE
HR=HF(1)
HTET=HF(2)
HFI=HF(3)
END IF
CALL CONVTOMAGFAZ(HR)
CALL CONVTOMAGFAZ(HTET)
CALL CONVTOMAGFAZ(HFI)
IF (SCREEN.EQ.3) THEN
IF (VAR.EQ.1) WRITE(20,110) X1,HR,HTET,HFI
IF (VAR.EQ.2) WRITE(20,110) Y2,HR,HTET,HFI
IF (VAR.EQ.3) WRITE(20,110) Z3,HR,HTET,HFI
END IF
IF (SCREEN.EQ.1) WRITE(*,100) X,Y,Z,HR,HTET,HFI
50 CONTINUE
60 CONTINUE
10 CONTINUE
IF (SCREEN.EQ.2) CLOSE(20)
TYPE *, 'OTHER VALUES (y/n) '
ACCEPT 130,AA
IF (AA.EQ.'y') GOTO 8
100 FORMAT('AT:(',F9.5,',',F9.5,',',F9.5,')',Hr=(',E12.5,
* ',F10.5,')',Htet=(',E12.5,',F10.5,')',Hphi=(',E12.5,',
* F10.5,')')
110 FORMAT(1X,F7.3,2X,E10.3,2X,F7.2,2X,E10.3,2X,F7.2,2X,E10.3,2X,F7.2)
120 FORMAT(A2)

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```

140 FORMAT(A20)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CSUBROUTINE THAT COMPUTE THE INTEGRAL JUST LIKE THE SUBROUTINE 'INTEGRAL'
HOWEVER ONLY THE GREEN'S FUNCTIONS ARE RECOMPUTED. AFTER THE FIRST PASS
CTHE H,DH/DN AND THEIR IMAGES ARE STORED FROM THE SUBROUTINE 'INTEGRAL'
CTHIS SPEEDS UP THING TREMENDOUSLY
SUBROUTINE INTEGRAL1(X,Y,Z,K,VAL,SIMP,ABCD1,ABCD11,NOR,5,
* NP,SIZE,NPUS,BS,H,DH,HI,DHI)
COMPLEX VAL(3),H(SIZE,NP,3),DH(SIZE,NP,3),DHI(SIZE,NP,3)
COMPLEX HI(SIZE,NP,3)
INTEGER SIMP,NP,SIZE
INTEGER NPUS,BS
REAL ABCD1(4,4),ABCD11(4,4)
REAL NOR(3),S(NP,4,4),X,Y,Z,K
C
IMPLICIT NONE
COMPLEX GG(30,3),G(30),GGI(30,3),GI(30)
COMPLEX GN(30),GNI(30),DUM
REAL P(30,3),PI(30,3),NORI(3)
INTEGER AX,I
C
NORI(1)=NOR(1)
NORI(2)=NOR(2)
NORI(3)=-NOR(3)
CALL GET3POLOS(ABCD11,S,P,PI,SIMP,NP,NPUS)
CALL GETGRADG(PI,GGI,K,X,Y,Z,NP,NPUS)
CALL GETG(PI,GI,K,X,Y,Z,NP,NPUS)
CALL GETGRADG(P,GG,K,X,Y,Z,NP,NPUS)
CALL GETG(P,G,K,X,Y,Z,NP,NPUS)
CALL DOTGN(GG,NOR,GN,NP,NPUS)
CALL DOTGN(GGI,NORI,GNI,NP,NPUS)
DO 10 AX=1,3
DUM=CMPLX(0.0,0.0)
DO 20 I=1,NPUS
20 DUM=DUM+(H(BS,I,AX)*GN(I)-G(I)*DH(BS,I,AX))
DO 30 I=1,NPUS
30 DUM=DUM+(HI(BS,I,AX)*GNI(I)-GI(I)*DHI(BS,I,AX))
* minus sign because Nor was directed inward instead of outward
VAL(AX)=-(DUM/12.56637)*0.0078125
10 CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CSUBROUTINE THAT DOES THE INTEGRAL OVER A SURFACE
SUBROUTINE INTEGRAL(X,Y,Z,K,VAL,SIMP,ABCD1,ABCD11,NOR,ALP,HS,DIR,S,
* NP,NPYR,PYRIND,NODES,SIZE,NPUS,PYR,IPS,H,DH,HI,DHI)
COMPLEX VAL(3),DIR(10,3),H(SIZE,NP,3),DH(SIZE,NP,3),DHI(SIZE,NP,3)
COMPLEX HI(SIZE,NP,3)
INTEGER SIMP,NP,NPYR,PYRIND(SIZE),SIZE,ALP(10,2,8),NODES(SIZE,10)
INTEGER NPUS,PYR,BS
REAL ABCD1(4,4),ABCD11(4,4),HS(NP,4,10)
REAL NOR(3),S(NP,4,4),X,Y,Z,K
C
IMPLICIT NONE
COMPLEX GG(30,3),G(30),GGI(30,3),GI(30),GFN(30,3),GFNI(30,3)
COMPLX GN(30),GNI(30),DUM,P(30,3),PI(30,3)
REAL P(30,3),PI(30,3),NORI(3)
INTEGER AX,I
C
NORI(1)=NOR(1)
NORI(2)=NOR(2)
NORI(3)=-NOR(3)
CALL GET3POLOS(ABCD11,S,P,PI,SIMP,NP,NPUS)
CALL GETGRADG(PI,GGI,K,X,Y,Z,NP,NPUS)
CALL GETG(PI,GI,K,X,Y,Z,NP,NPUS)
CALL GETGRADG(P,GG,K,X,Y,Z,NP,NPUS)
CALL GETG(P,G,K,X,Y,Z,NP,NPUS)
CALL DOTGN(GG,NOR,GN,NP,NPUS)
CALL DOTGN(GGI,NORI,GNI,NP,NPUS)
CALL GETGRADH(P,NP,NPUS,GFN,NOR,NPYR,PYRIND,NODES,SIZE,ALP,
* PYR,DIR,ABCD1)
CALL GETGRADHI(GFN,GFNI,NOR,NORI,NP,NPUS)
CALL GETH(HS,DIR,SIMP,P,NP,NPUS)
CALL GETHI(P,PI,NP,NPUS)
DO 10 AX=1,3
DUM=CMPLX(0.0,0.0)
DO 20 I=1,NPUS
H(BS,I,AX)=P(I,AX)
DH(BS,I,AX)=GFN(I,AX)
20 DUM=DUM+(P(I,AX)*GN(I)-G(I)*GFN(I,AX))
DO 30 I=1,NPUS
HI(BS,I,AX)=PI(I,AX)
DHI(BS,I,AX)=GFNI(I,AX)
30 DUM=DUM+(PI(I,AX)*GNI(I)-GI(I)*GFNI(I,AX))
* minus sign because Nor was directed inward instead of outward
C0.0078125 DEPENDS ON THE SIZE OF THE FACES AND HAS TO BE CHANGED EVERY TIME
CTHE MESH SIZE IS MODIFIED(GIMME A BREAK)
VAL(AX)=-(DUM/12.56637)*0.0078125
10 CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CSUBROUTINE THAT RETURNS THE IMAGE OF DH/DN ABOUT THE XY PLANE FOR ALL 3
CCARTESIANCOMPONENTS.
SUBROUTINE GETGRADHI(F,PI,NOR,NORI,N,NS)
COMPLEX F(N,3),FI(N,3)
INTEGER AX,N,NS
REAL NOR(3),NORI(3)
C
IMPLICIT NONE
INTEGER I,DIR
C
DIR=NOR(1)*NORI(1)+NOR(2)*NORI(2)+NOR(3)*NORI(3)
DO 10 AX=1,3
DO 5 I=1,NS
FI(I,AX)=F(I,AX)
IF (AX.EQ.3) FI(I,AX)=-F(I,AX)
5 CONTINUE

```

[illegible]


```

COMPLEX EX,EY,EZ,DIRICHLETVALX,DIRICHLETVALY,DIRICHLETVALZ
COMPLEX HR,HTET,HFI
INTEGER SCREEN,VAR,SC
CHARACTER*2 AA
CHARACTER*20 PNAME
C
C
OPEN(UNIT=3,FILE='functions.dat',STATUS='UNKNOWN',ACCESS=
  * 'SEQUENTIAL',FORM='FORMATTED')
READ(3,*) FREQ
CLOSE(3)
5 TYPE *, 'ENTER OUTPUT VARIABLE(1 FOR SCREEN/2 FOR FILES) '
ACCEPT *, SCREEN
IF (SCREEN.EQ.2) THEN
  TYPE *, 'ENTER OUTPUT FILENAME?'
  ACCEPT 140, PNAME
  OPEN(UNIT=20,FILE=PNAME,STATUS='UNKNOWN',ACCESS=
    * 'SEQUENTIAL',FORM='FORMATTED')
  TYPE *, 'ENTER COORDINATE THAT VARIES(X/1,Y/2,Z/3,TWO/4) '
  ACCEPT *, VAR
END IF
8 TYPE *, 'ENTER RADIUS '
ACCEPT *, JX,FX, SX
TYPE *, 'ENTER INITIAL THETA, FINAL THETA AND THE STEP '
ACCEPT *, IY,FY, SY
TYPE *, 'ENTER INITIAL PHI, FINAL PHI AND THE STEP '
ACCEPT *, IZ,FZ, SZ
TYPE *, 'ENTER 99 FOR SPHERICAL/ANYTHING ELSE FOR CARTESIAN'
ACCEPT *, SC
DO 10 I=IX,FX,SX
  DO 20 J=IY,FY,SY
    DO 25 K=IZ,FZ,SZ
      Y1=J*3.14159/180.0
      Z1=K*3.14159/180.0
      X=I*SIN(Y1)*COS(Z1)
      Y=I*SIN(Y1)*SIN(Z1)
      Z=I*COS(Y1)
      NVAL=NVAL+1.0
      EX=DIRICHLETVALX(X,Y,Z,FREQ)
      EY=DIRICHLETVALY(X,Y,Z,FREQ)
      EZ=DIRICHLETVALZ(X,Y,Z,FREQ)
      IF (SC.EQ.99) THEN
        HR=EX*SIN(Y1)*COS(Z1)+EY*SIN(Y1)*SIN(Z1)+EZ*COS(Y1)
        HTET=EX*COS(Y1)*COS(Z1)+EY*COS(Y1)*SIN(Z1)-EZ*SIN(Y1)
        HFI=-EX*SIN(Z1)+EY*COS(Z1)+EZ*0.0
      ELSE
        HR=EX
        HTET=EY
        HFI=EZ
      END IF
      CALL CONVTOMAGFAZ(HR)
      CALL CONVTOMAGFAZ(HTET)
      CALL CONVTOMAGFAZ(HFI)
      IF (SCREEN.EQ.2) THEN
        IF (VAR.EQ.1) WRITE(20,110) I,HR,HTET,HFI
        IF (VAR.EQ.2) WRITE(20,110) J,HR,HTET,HFI
        IF (VAR.EQ.3) WRITE(20,110) K,HR,HTET,HFI
        IF (VAR.EQ.4) WRITE(20,130) J,K,HR,HTET,HFI
      END IF
      IF (SCREEN.EQ.1) WRITE(*,100) X,Y,Z,HR,HTET,HFI
    25 CONTINUE
  20 CONTINUE
10 CONTINUE
IF (SCREEN.EQ.2) CLOSE(20)
TYPE *, 'OTHER VALUES (y/n) '
ACCEPT 120, AA
IF (AA.EQ.'y') GOTO 5
100 FORMAT('AT:(',F8.5,',',F8.5,',',F8.5,')',H=(',E12.5,
  * ',F10.5,')',Htet=(',E12.5,')',Hphi=(',E12.5,')',
  * 'F10.5,')')
110 FORMAT(1X,F7.3,2X,E10.3,2X,F7.3,2X,E10.3,2X,F7.3,2X,E10.3,2X,F7.2,2X,E10.3,2X,F7.2)
130 FORMAT(1X,F7.3,2X,F7.3,2X,F7.3,2X,E10.3,2X,F7.3,2X,E10.3,2X,
  * 'F7.2,2X,E10.3,2X,F7.2)
120 FORMAT(A2)
140 FORMAT(A20)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

```

DERANALRAD.F

```

C PROGRAM THAT PRINTS ANALYTICAL DERIVATIVE OF FIELD IN THE RADIAL
C DIRECTION GIVEN FOR THE SOURCE DEFINED IN sourcecube.f.
C
C IMPLICIT NONE
C
COMPLEX H(1,3)
REAL IX,FX,SX,IY,FY,SY,IZ,FZ,SZ,P(1,3),NOR(3),PI,FREQ
REAL Y1,Z1,BET,I,J,K
CHARACTER*2 AA
INTEGER SCREEN,VAR
CHARACTER*20 PNAME
C
C
OPEN(UNIT=3,FILE='functions.dat',STATUS='UNKNOWN',ACCESS=
  * 'SEQUENTIAL',FORM='FORMATTED')
READ(3,*) FREQ
CLOSE(3)
PI=ACOS(-1.0)
BET= 2.0*PI*FREQ*SQRT(6.65E-12*1.25664E-6)
8 TYPE *, 'ENTER OUTPUT VARIABLE(1 FOR SCREEN/2 FOR FILES) '
ACCEPT *, SCREEN
IF (SCREEN.EQ.2) THEN
  TYPE *, 'ENTER OUTPUT FILENAME?'
  ACCEPT 140, PNAME
  OPEN(UNIT=20,FILE=PNAME,STATUS='UNKNOWN',ACCESS=
    * 'SEQUENTIAL',FORM='FORMATTED')
  TYPE *, 'ENTER COORDINATE THAT VARIES(R/1,THETA/2,PHI/3) '
  ACCEPT *, VAR
END IF
TYPE *, 'ENTER RADIUS '

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ACCEPT *IX,PX,EX
TYPE *, 'ENTER INITIAL THETA, FINAL THETA AND THE STEP '
ACCEPT *JY,PY,SY
TYPE *, 'ENTER INITIAL PHI, FINAL PHI AND THE STEP '
ACCEPT *IZ,PZ,SZ
TYPE *, 'DERIVATIVE IN THE R DIRECTION'
DO 10 I=IX,PX,EX
DO 20 J=JY,PY,SY
DO 30 K=IZ,PZ,SZ
Y1=J*3.14159/180.0
Z1=K*3.14159/180.0
P(1,1)=I*SIN(Y1)*COS(Z1)
P(1,2)=I*SIN(Y1)*SIN(Z1)
P(1,3)=I*COS(Y1)
NOR(1)=P(1,1)
NOR(2)=P(1,2)
NOR(3)=P(1,3)
CALL GETREALDHDN1(P,H,NOR,1,1,BET)
CALL CONVTOMAGFAZ(H(1,1))
CALL CONVTOMAGFAZ(H(1,2))
CALL CONVTOMAGFAZ(H(1,3))
IF (SCREEN.EQ.2) THEN
IF (VAR.EQ.1) WRITE(20,110) I,H(1,1),H(1,2),H(1,3)
IF (VAR.EQ.2) WRITE(20,110) J,H(1,1),H(1,2),H(1,3)
IF (VAR.EQ.3) WRITE(20,110) K,H(1,1),H(1,2),H(1,3)
END IF
IF (SCREEN.EQ.1) WRITE(*,100)
* P(1,1),P(1,2),P(1,3),H(1,1),H(1,2),H(1,3)
30 CONTINUE
20 CONTINUE
10 CONTINUE
IF (SCREEN.EQ.2) CLOSE(20)
TYPE *, 'OTHER VALUES (y/n)?'
ACCEPT 120,AA
IF (AA.EQ.'y') GOTO 5
100 FORMAT('AT:',F8.5,',',F8.5,',',F8.5,',',H(1,1),H(1,2),
* ',F10.5,',H(1,1),H(1,2),',F10.5,',H(1,1),H(1,2),',
* 'F10.5,',')
110 FORMAT(1X,F7.3,2X,E10.3,2X,F7.2,2X,E10.3,2X,F7.2,2X,E10.3,2X,F7.2)
120 FORMAT(A2)
140 FORMAT(A20)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

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DFDNCOMP.F

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C THIS LIST OF SUBROUTINES ARE USED BY MAINCOMP FOR THE IMPLEMENTATION
C OF THE ABC BASED ON BEM. MOST OF THE ROUTINES ARE SIMILAR TO THE ONE
C IN BEMRAD BUT NOT QUITE THE SAME.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE GETFULLVALONS(BSPYR,SIMP,NOR,NODES,SIZE,CO,ALP,H2,S,NP,
* NPYR,PYRIND,NPS,H,DH,HNODE)
INTEGER BSPYR(SIZE),SIMP(SIZE),SIZE,NODES(SIZE,10),CO,ALP(10,5,2)
INTEGER NP,NPYR,PYRIND(SIZE),NPS,HNODE(SIZE,10)
REAL NOR(SIZE,3),H2(NP,4,10),S(NP,4,4)
REAL H(SIZE,NP,10),DH(SIZE,NP,10)
C
IMPLICIT NONE
INTEGER NOD1(10),SMP1,I
REAL ABCD1(4,4),ABCD11(4,4),NOR1(3)
C
DO 40 I=1,CO
CALL SELECTDATA(BSPYR(I),I,SIMP,NOR,NODES,SIZE,NOD1,SMP1,
* ABCD1,ABCD11,NOR1)
CALL GETHDDHIDHI(SMP1,ABCD1,ABCD11,NOR1,
* ALP,H2,S,NOD1,NP,NPYR,PYRIND,NODES,SIZE,NPS,
* BSPYR(I),I,H,DH,HNODE)
40 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE GETHDDHIDHI(SIMP,ABCD1,ABCD11,NOR,ALP,H2,S,NODE,
* NP,NPYR,PYRIND,NODES,SIZE,NPUS,PYR,BS,H,DH,HNODE)
REAL H(SIZE,NP,10),DH(SIZE,NP,10)
INTEGER SIMP,NP,NPYR,PYRIND(SIZE),SIZE,ALP(10,2,5),NODES(SIZE,10)
INTEGER NPUS,PYR,BS,NODE(10),HNODE(SIZE,10)
REAL ABCD1(4,4),ABCD11(4,4),H2(NP,4,10)
REAL NOR(3),S(NP,4,4)
C
IMPLICIT NONE
REAL F(16,10),GFN(16,10)
REAL P(16,3)
INTEGER ND,I
C
CALL GET3PGLOSSING(ABCD11,S,P,SIMP,NP,NPUS)
CALL GETGRADH(P,NP,NPUS,GFN,NOR,NPYR,PYRIND,NODES,SIZE,ALP,
* PYR,ABCD1)
CALL GETH(HS,SIMP,P,NP,NPUS)
DO 10 ND=1,10
HNODE(BS,ND)=NODE(ND)
DO 20 I=1,NPUS
H(BS,I,ND)=(F(I,ND)/12.56637)*0.03125
20 DH(BS,I,ND)=(GFN(I,ND)/12.56637)*0.03125
10 CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE INTEGRAL1(POS,NPOS,K,VAL,VALNOD,NVAL,MXVND,SIMP,ELAREA,
* ABCD1,ABCD11,NOR,H2OS,S,NP,MXFAC,NPUS,NVEC,BS,H,DH,HNODE,
* AX,CRF,IN,SIMPLEX)
REAL H(MXFAC,NP,10),DH(MXFAC,NP,10)
INTEGER SIMP,NP,VALNOD(MXVND),MXVND,NVAL,NPOS,IN(5),SIMPLEX
INTEGER NPUS,BS,HNODE(MXFAC,10),AX,CRF(MXVND),MXFAC,IND
REAL ABCD1(4,4),ABCD11(4,4),NVEC(3),ELAREA
COMPLEX VAL(5,MXVND)

```



```

IMPLICIT NONE
INTEGER SP,P1,P2,P3,P4,ND
REAL VAL1,VAL
C
DO 10 ND=1,10
  VAL2(ND)=0.0
  VAL=0.0
  DO 20 SP=1,2
    P1=ALF(ND,SP,1)
    P2=ALF(ND,SP,2)
    P3=ALF(ND,SP,3)
    P4=ALF(ND,SP,4)
    VAL1=( (S(1)**P1)*(S(2)**P2)*(S(3)**P3)*(S(4)**P4) ) *ALF(ND,SP,5)
  20 VAL=VAL+VAL1
  VAL2(ND)=VAL
  IF (OP.EQ.0) VAL3(ND)=-3.0*VAL2(ND)
  IF (OP.EQ.1) VAL3(ND)=4.0*VAL2(ND)
  IF (OP.EQ.2) VAL3(ND)=-VAL2(ND)
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE GETGRADGN(P,GG,K,X,Y,Z,N,NS)
REAL X,Y,Z,K,P(N,3)
COMPLEX GG(N,3)
INTEGER N,NS
C
IMPLICIT NONE
C
REAL R
INTEGER I
COMPLEX FI,EX,FIP
C
DO 10 I=1,NS
  R=SQRT( (X-P(I,1))**2+(Y-P(I,2))**2+(Z-P(I,3))**2 )
  EX=CMPLX(0.0,-1.0)*K*R
  FI=CEXP(EX)/R
  FIP=-FI/(R**2)+CMPLX(0.0,1.0)*K*FI/R
  GG(I,1)=(X-P(I,1))*FIP
  GG(I,2)=(Y-P(I,2))*FIP
10 GG(I,3)=(Z-P(I,3))*FIP
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE GETGRADGNP(P,GG,K,X,Y,Z,NOR,N,NS)
REAL X,Y,Z,K,P(N,3),NOR(3)
COMPLEX GG(N,3)
INTEGER N,NS
C
IMPLICIT NONE
C
REAL R
INTEGER I
COMPLEX FI,EX,FIA,FIB,A,B,IMG,XX,YY,ZZ,XY,XZ,YZ
C
IMG=CMPLX(0.0,1.0)
DO 10 I=1,NS
  R=SQRT( (X-P(I,1))**2+(Y-P(I,2))**2+(Z-P(I,3))**2 )
  EX=IMG*K*R
  FI=CEXP(EX)/R
  A=IMG*K/R+1.0/R**2
  B=(K**2-3.0*IMG*K/R-3.0/R**2)/(R**2)
  FIA=FI*A
  FIB=FI*B
  XX=FIA+(X-P(I,1))**2*FIB
  YY=FIA+(Y-P(I,2))**2*FIB
  ZZ=FIA+(Z-P(I,3))**2*FIB
  XY=(X-P(I,1))*(Y-P(I,2))*FIB
  XZ=(X-P(I,1))*(Z-P(I,3))*FIB
  YZ=(Y-P(I,2))*(Z-P(I,3))*FIB
  GG(I,1)=NOR(1)*XX+NOR(2)*XY+NOR(3)*XZ
  GG(I,2)=NOR(1)*XY+NOR(2)*YY+NOR(3)*YZ
10 GG(I,3)=NOR(1)*XZ+NOR(2)*YZ+NOR(3)*ZZ
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE GETH(H,SP,P,N,NS)
REAL H(N,4,10),P(N,10)
INTEGER SP,N,NS
C
IMPLICIT NONE
C
INTEGER P,ND
REAL VAL(10)
C
DO 10 P=1,NS
  DO 20 ND=1,10
    F(P,ND)=H(P,SP,ND)
  20 CONTINUE
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

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DFDNSUBS.F

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C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE IDENTIFY(MEDP,IDCOD,INNER,BON,J)
INTEGER MEDP(4),IDCOD,J
LOGICAL INNER,BON
C
IMPLICIT NONE
BON=.FALSE.
IF (IDCOD.EQ.800000) THEN
  BON=(INNER.AND.(J.EQ.1)).OR.(MEDP(J).GE.IDCOD)
ELSE IF (IDCOD.EQ.11111) THEN
  BON=(INNER.AND.(J.EQ.1)).OR.(MEDP(J).EQ.IDCOD)

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      IF(A(N,N).EQ.0.)A(N,N)=TINY
      RETURN
    END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CSUBROUTINE THAT COMPUTES A PSEUDO CENTER OF A TRIANGULAR SURFACE DEFINED BY
CDUM1,DUM2,DUM3.
C
SUBROUTINE MIDCENTER(COR,DUM,DUM4)
REAL COR(4,3),DUM(3),DUM4(4)
C
IMPLICIT NONE
INTEGER I
REAL DUM1(3),DUM2(3),DUM3(3),MID(3)
C
DO 10 I=1,3
  DUM1(I)=COR(1,I)
  DUM2(I)=COR(2,I)
  DUM4(I)=COR(4,I)
10  DUM3(I)=COR(3,I)
  CALL MIDPOI(DUM1,DUM2,MID)
  CALL MIDPOI(MID,DUM3,DUM)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
LOGICAL FUNCTION INNERBOUND(DM,D4,V,LXY,LZ)
REAL DM(3),V(3),LXY,LZ,D4(3)
C
IMPLICIT NONE
REAL DLXY,DLZ,MD(3)
C
DLXY=LXY-0.001
DLZ=LZ-0.001
CALL MIDPOI(DM,D4,MD)
INNERBOUND=.FALSE.
IF ((ABS(V(1)).NE.1).XOR.(ABS(V(2)).NE.1).XOR.(ABS(V(3)).NE.1))
  * GOTO 10
IF ( ((ABS(DM(1)).LT.DLXY).AND.(ABS(DM(2)).LT.DLXY)).AND.
      (DM(3).LT.DLZ) ) GOTO 10
IF ( (ABS(DM(1)).GT.DLXY).OR.(ABS(DM(2)).GT.DLXY) ) GOTO 10
IF ( V(3).LT.0.0 ) GOTO 10
INNERBOUND=.TRUE.
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CSUBROUTINE THAT COMPUTES THE WEIGHT DUE TO THE ALPHA FUNCTIONS(INTERPOLATION)
CAT ALL THE COLLOCATION POINT, FOR THE FOUR POSSIBLE FE ORIENTATION AT ALL THE C10 NODES.
C
SUBROUTINE GETH2(ALF,H2,S,N,NS)
INTEGER ALF(10,2,8),N,NS
REAL S(N,4,4),H2(N,4,10)
C
IMPLICIT NONE
INTEGER P,SP,ROW,P1,P2,SPN,P3,P4
REAL VAL,VAL1
C
DO 10 P=1,NS
  DO 30 ROW=1,10
    DO 35 SPN=1,4
      VAL1=0.0
      DO 30 SP=1,2
        P1=ALF(ROW,SP,1)
        P2=ALF(ROW,SP,2)
        P3=ALF(ROW,SP,3)
        P4=ALF(ROW,SP,4)
        VAL=(S(P,1,SPN)**P1)*(S(P,2,SPN)**P2)*(S(P,3,SPN)**P3)
          * (S(P,4,SPN)**P4)
      VAL=VAL+ALF(ROW,SP,8)
30  VAL1=VAL1+VAL
      H2(P,SPN,ROW)=VAL1
35  CONTINUE
30  CONTINUE
10  CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CSUBROUTINE THAT COMPUTES THE SIMPLEX COORDINATES OF THE COLLOCATION POINTS
CFOR ALL FOUR POSSIBLE FACES.(VALID ONLY FOR NS=4 OR NS=1)
C5 IS THE MATRIX OF THE COORDINATES
CN IS MAXIMUM COLLOCATION POINT NUMBER
CNS IS NUMBER OF COLLOCATION POINTS
C
SUBROUTINE GETSMPVALNP(S,N,NS)
REAL S(N,4,4)
INTEGER N,NS
C
IMPLICIT NONE
INTEGER SPN,SP,P
REAL S1(100,4),TEMP
C
DO 30 P=1,NS
  S1(P,4)=0.0
30 CONTINUE
IF (NS.EQ.1) THEN
  S1(1,1)=1.0/3.0 !
  S1(1,2)=1.0/3.0 !3 LINES FOR NS=1
  S1(1,3)=1.0/3.0 !
ELSE IF (NS.EQ.4) THEN
  S1(1,1)=1.0/8.0 !
  S1(1,2)=1.0/4.0 !
  S1(1,3)=8.0/8.0 !
  S1(2,1)=1.0/8.0 !
  S1(2,2)=1.0/2.0 !12 LINES FOR NS=4
  S1(2,3)=3.0/8.0 !
  S1(3,1)=3.0/8.0 !
  S1(3,2)=1.0/2.0 !
  S1(3,3)=1.0/8.0 !
  S1(4,1)=8.0/8.0 !
  S1(4,2)=1.0/4.0 !
  S1(4,3)=1.0/8.0 !
ELSE

```



```

REAL DUMPI(3),DUMP2(3),DUMP3(3),MIDI(3),MID2(3)
C
CALL ONEDIM(COR(1,1),COR(1,2),COR(1,3),DUMPI)
CALL ONEDIM(COR(2,1),COR(2,2),COR(2,3),DUMP2)
CALL ONEDIM(COR(3,1),COR(3,2),COR(3,3),DUMP3)
CALL MIDPOI(DUMPI,DUMP2,MIDI)
CALL MIDPOI(MIDI,DUMP3,MID2)
CALL CONV CARTOSFE(MID2(1),MID2(2),MID2(3),R,FI,TET)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
SUBROUTINE CONV CARTOSFE(CX,CY,CZ,R,FI,TET)
REAL CX,CY,CZ,R,FI,TET
C
IMPLICIT NONE
C
R=SQRT(CX**2+CY**2+CZ**2)
TET=ACOS(CZ/R)
IF (R.EQ.0) TET=0.0
FI=ATAN(CY/CX)
IF (CX.EQ.0) FI=0.0
IF ((CX.EQ.0).AND.(CY.GT.0)) FI=1.570796327
IF ((CX.EQ.0).AND.(CY.LT.0)) FI=-1.570796327
IF ((CX.GT.0).AND.(CY.EQ.0)) FI=0.0
IF ((CX.LT.0).AND.(CY.EQ.0)) FI=-3.1415926
IF ((CX.LT.0).AND.(CY.LT.0)) FI=FI+3.1415926
IF ((CX.LT.0).AND.(CY.GT.0)) FI=FI+3.1415926
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
SUBROUTINE GETCONTALFNORM(BCONTS,NORM,NSID,AREA,SIMPLEX,IN10)
COMPLEX BCONTS(10),NSID(8)
REAL NORM(10,10,1,4),AREA
C REAL NORM(10,4,1,1),AREA
INTEGER SIMPLEX,IN10(8)
C
IMPLICIT NONE
INTEGER K,J,L,K1
DO 10 J=1,8
K=IN10(J)
DO 20 L=1,8
K1=IN10(L)
C IF (NSID(J).NE.999.0)
C BCONTS(K)=BCONTS(K)-NORM(K1,SIMPLEX,1,1)*AREA*NSID(L)
C BCONTS(L)=BCONTS(L)-NORM(L,SIMPLEX,1,1)*AREA*NSID(J)
BCONTS(K)=BCONTS(K)+NORM(K,K1,1,SIMPLEX)*AREA*NSID(L)
20 CONTINUE
10 CONTINUE
END
CC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CC
SUBROUTINE GETCONFIELDNORM(CONTS,FIELDN,AREA,B,C,D,IN,NV)
INTEGER IN(8)
COMPLEX CONTS(10,10)
REAL FIELDN(10,10,1,4),AREA,NV(3),FACX,FACY,FACZ,B(4),C(4),D(4)
C
IMPLICIT NONE
INTEGER J,K,L
DO 10 J=1,10
DO 20 K=1,10
FACX=0.0
FACY=0.0
FACZ=0.0
DO 30 L=1,4
FACX=FACX+FIELDN(J,K,1,L)*B(L)
FACY=FACY+FIELDN(J,K,1,L)*C(L)
30 FACZ=FACZ+FIELDN(J,K,1,L)*D(L)
20 CONTS(J,K)=CONTS(J,K)+(FACX*NV(1)+FACY*NV(2)+FACZ*NV(3))*AREA
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE GETDIR(DIR,DIRN,ALLDIR,ALLDIRN,N,ND,NODE)
INTEGER ALLDIR(N),NODE(10),DIRN(10),ND,N
COMPLEX DIR(10),ALLDIR(N)
C
IMPLICIT NONE
INTEGER I,J
C
J=1
DO WHILE (J.LE.END)
DO 10 I=1,10
IF (ALLDIRN(I).EQ.NODE(I)) THEN
IF (REAL(ALLDIR(I)).NE.999.0) THEN
DIR(I)=ALLDIR(I)
DIRN(I)=1
END IF
END IF
10 CONTINUE
J=J+1
END DO
20 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE READIR(ALLDIR,ALLDIRN,N,MAXDIR,FIL2,AX)
INTEGER FIL2,ALLDIRN(N),MAXDIR,N,AX
COMPLEX ALLDIR(N)
C
IMPLICIT NONE
INTEGER J,IND
REAL X1X,X1Y,Y1X,Y1Y,Z1X,Z1Y
C
REWIND(FIL2)
READ(FIL2,*) MAXDIR
DO 10 J=1,MAXDIR
READ(FIL2,*) IND,X1X,X1Y,Y1X,Y1Y,Z1X,Z1Y
IF (.AX.EQ.1.) THEN

```



```

IF (WHERE.EQ.0) WHERE=COLSTART+DEL
IF (AI(WHERE).NE.-999) CALL
* SHIFTOANDINSERT(A,AI,AJ,N,NZ,NZERO,WHERE,SCE,NP)
AI(WHERE)=INDX
END IF
IF (ABS(VALSG(J,K)-999.999).GT.2.0) THEN
A(WHERE)=A(WHERE)+VALSG(J,K)
ELSE
A(WHERE)=1.0
END IF
END IF
50 CONTINUE
40 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE ADSCONTSTOS(RHS,BSG,NODESG,DIRNSG,SCONTS,DIRSG,SIZE,A,
* AI,AJ,N,NZ,NZERO,SCE,NP,TOTNONZERO)
INTEGER NODESG(20),DIRNSG(20),N,AI(NZ),AJ(N),SIZE,NZERO,SCE,NP
INTEGER TOTNONZERO,NZ
REAL SCONTS(20,20),BSG(20),DIRSG(20)
REAL*8 RHS(N),A(NZ)
C
IMPLICIT NONE
INTEGER I,INDX,INDY,J,K,WHERE,NEW,COLSTART,DEL
C
DO 20 J=1,20
RHS(NODESG(J))=RHS(NODESG(J))+BSG(J)
IF (DIRNSG(J).NE.0) THEN
DO 30 K=1,20
30 SCONTS(J,K)=0.0
SCONTS(J,J)=999.999
RHS(NODESG(J))=DIRSG(DIRNSG(J))
END IF
IF (NODESG(J).GT.SIZE) SIZE=NODESG(J)
20 CONTINUE
DO 40 K=1,20
INDY=NODESG(K)
COLSTART=AJ(INDY)
DEL=AJ(INDY+1)-COLSTART-1
DO 50 J=1,20
IF (SCONTS(J,K).NE.0) THEN
INDX=NODESG(J)
WHERE=0
NEW=1
DO 60 I=COLSTART,(COLSTART+DEL)
IF (AI(I).EQ.INDX) THEN
WHERE=I
NEW=0
END IF
IF (NEW.EQ.0) GOTO 50
60 CONTINUE
50 IF (WHERE.EQ.0) TOTNONZERO=TOTNONZERO+1
IF ((WHERE.NE.0).AND.(A(WHERE).EQ.0.0))
TOTNONZERO=TOTNONZERO+1
IF (TOTNONZERO.GE.NZ) THEN
WRITE(*,*)
* NEXT PASS THE TOTAL SIZE ALLOCATED FOR A(NONZERO) WILL EXCEED
WRITE(*,*) 'IF I AM NOT STOPPED I M GONNA CRASH'
WRITE(*,*) 'TOTNONZERO=',TOTNONZERO
END IF
IF (NEW.EQ.1) THEN
DO 70 I=(COLSTART+DEL),COLSTART,-1
70 IF (AI(I).EQ.-999) WHERE=I
IF (WHERE.EQ.0) WHERE=COLSTART+DEL
IF (AI(WHERE).NE.-999) CALL
* SHIFTOANDINSERT(A,AI,AJ,N,NZ,NZERO,WHERE,SCE,NP)
AI(WHERE)=INDX
END IF
IF (ABS(SCONTS(J,K)-999.999).GT.2.0) THEN
A(WHERE)=A(WHERE)+SCONTS(J,K)
ELSE
A(WHERE)=1.0
END IF
END IF
50 CONTINUE
40 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE SHIFTOANDINSERT(A,AI,AJ,N,NZ,NZERO,WHERE,SCE,NP)
INTEGER AI(NZ),AJ(N),N,NZERO,WHERE,SCE,NP,NZ
REAL*8 A(NZ)
C
IMPLICIT NONE
C
INTEGER I,TOT
C
TOT=(SCE+NP)*2
IF ((NZ-NZERO).LE.1) WRITE(*,*) 'A VECTOR IS FULL, I M GONNA CRASH'
DO 10 I=NZERO,WHERE,-1
A(I+1)=A(I)
10 AI(I+1)=AI(I)
DO 20 I=1,TOT
20 IF (AJ(I).GT.WHERE) AJ(I)=AJ(I)+1
NZERO=NZERO+1
WHERE=WHERE+1
A(WHERE)=0.0D0
AJ(TOT+1)=NZERO+1
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CREATVALNOD(VALNOD,NVAL,BS,HNODE,CRF,MXVND,MXPAC,NODSG,
* NP,SCE)
INTEGER MXVND,CRF(MXVND),VALNOD(MXVND),NVAL,BS,HNODE(MXPAC,10),MXPAC
INTEGER NODSG(MXVND),NP,SCE
C
IMPLICIT NONE
INTEGER I,IND,INDC,TOT,OFFSET,J
C

```

```

IF (BS..Q.1) THEN
  DO 20 I=1,MXVND
  20 CRF(I)=0
  NVAL=0
END IF
DO 10 I=1,10
  IND=HNODE(BS,I)
  IF (CRF(IND).EQ.0) NVAL=NVAL+1
  IF (NVAL.GE.MXVND) WRITE(*,*) 'MAXIMUM NUMBER OF NODES IN GLOBAL
    * BOUNDARY (MXVND) IS EXCEED. NVAL=',NVAL
  IF (CRF(IND).EQ.0) VALNOD(NVAL)=IND
  IF (CRF(IND).EQ.0) CRF(IND)=NVAL
10 CONTINUE
INDC=0
TOT=NP+SCE
DO 30 I=1,2
  OFFSET=TOT*(I-1)
  DO 40 J=1,NVAL
    IND=J+INDC
    NODSG(IND)=VALNOD(J)+OFFSET
40 CONTINUE
INDC=IND
30 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CONVVALVLSQ(VAL,VALSQ,NODSG,MXVND,NVAL,NP,SCE,AX,
  * B6ND,B12ND)
COMPLEX VAL(6,MXVND)
REAL VALSQ(12,MXVND)
INTEGER MXVND,NODSG(MXVND)
INTEGER NP,SCE,AX,NVAL,B6ND(6),B12ND(12)
C
IMPLICIT NONE
INTEGER I,J,IND,INDC,TOT,OFFSET
INTEGER INDCX,INDCY,INDX,INDY,INDXI,INDYI
REAL VALR,VALI
C
INDC=0
TOT=NP+SCE
DO 10 I=1,2
  OFFSET=TOT*(I-1)
  DO 20 J=1,NVAL
    IND=J+INDC
    IF (J.LE.6) B12ND(J+6)=B6ND(J)+OFFSET
    IF (J.LE.6) B12ND(J)=B6ND(J)
20 CONTINUE
INDC=IND
10 CONTINUE
C
INDCX=0
INDCY=0
DO 40 I=1,6
  INDX=I+INDCX
  INDXI=INDX+6
  DO 50 J=1,NVAL
    INDY=J+INDCY
    INDYI=INDY+NVAL
    VALR=REAL(VAL(I,J))
    VALI=AIMAG(VAL(I,J))
    VALSQ(INDX,INDY)=VALR
    VALSQ(INDXI,INDYI)=VALR
    VALSQ(INDX,INDYI)=-VALI
    VALSQ(INDXI,INDY)=VALI
50 CONTINUE
40 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CONVCONTSBONTS(CONTS,BONTS,SCONTS,BSQ,NODE,NODESQ,DIRN,
  * DIRNSQ,DIR,DIRSQ,NP,SCE,AX)
COMPLEX CONTS(10,10),BONTS(10),DIR(10)
REAL SCONTS(20,20),BSQ(20),DIRSQ(20)
INTEGER NODE(10),NODESQ(20),NP,SCE,DIRN(10),DIRNSQ(20),AX
C
IMPLICIT NONE
INTEGER I,J,IND,INDC,TOT,VAL,OFFSET
INTEGER INDCX,INDCY,INDX,INDY,INDXI,INDYI
REAL VALR,VALI
C
INDC=0
TOT=NP+SCE
DO 10 I=1,2
  OFFSET=TOT*(I-1)
  DO 20 J=1,10
    IND=J+INDC
    VAL=NODE(J)+OFFSET
    IF (MOD(I,2).EQ.0) DIRSQ(IND)=AIMAG(DIR(J))
    IF (MOD(I,2).NE.0) DIRSQ(IND)=REAL(DIR(J))
    IF (DIRN(J).NE.0) DIRNSQ(IND)=DIRN(J)+INDC
    NODESQ(IND)=VAL
20 CONTINUE
INDC=IND
10 CONTINUE
C
INDCX=0
INDCY=0
DO 40 I=1,10
  INDX=I+INDCX
  INDXI=INDX+10
  DO 50 J=1,10
    INDY=J+INDCY
    INDYI=INDY+10
    VALR=REAL(CONTS(I,J))
    VALI=AIMAG(CONTS(I,J))
    SCONTS(INDX,INDY)=VALR
    SCONTS(INDXI,INDYI)=VALR
    SCONTS(INDX,INDYI)=-VALI
    SCONTS(INDXI,INDY)=VALI
50 CONTINUE
40 CONTINUE
BSQ(INDX)=REAL(SCONTS(I))

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      BSG(INDXI)=AIMAG(SCONTS(I))
40  CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE INITIALIZE(RHS,N1,N2)
COMPLEX RHS(N1,N2)
INTEGER N1,N2
C
IMPLICIT NONE
COMPLEX NIX
INTEGER I,J
C
NIX=CMPLX(0.0,0.0)
DO 10 I=1,N1
DO 20 J=1,N2
20  RHS(I,J)=NIX
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE INITIALIZE4(RHS,N1,N2)
REAL RHS(N1,N2)
INTEGER N1,N2
C
IMPLICIT NONE
INTEGER I,J
C
DO 10 I=1,N1
DO 20 J=1,N2
20  RHS(I,J)=0.0
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE INITIALIZE4D(RHS,N1,N2)
REAL RHS(N1,N2)
INTEGER N1,N2
C
IMPLICIT NONE
INTEGER I,J
C
DO 10 I=1,N1
DO 20 J=1,N2
20  RHS(I,J)=0.0
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE READNOI(ALLNOI,ALLNOIN,MAXN,MAXNOI,FIL2,AX)
INTEGER ALLNOI(MAXN),MAXN,MAXNOI,FIL2,AX
COMPLEX ALLNOI(MAXN)
C
IMPLICIT NONE
INTEGER ND,J,IND
REAL X1,X2,Y1,Y2,Z1,Z2
COMPLEX X(3)
C
REWIND(FIL2)
READ(FIL2,*) ND
READ(FIL2,100)
READ(FIL2,*) MAXNOI
DO 10 J=1,MAXNOI
READ(FIL2,*) IND,X1,X2,Y1,Y2,Z1,Z2
X(1)=CMPLX(X1,X2)
X(2)=CMPLX(Y1,Y2)
X(3)=CMPLX(Z1,Z2)
ALLNOI(J)=X(AX)
ALLNOIN(J)=IND
10 CONTINUE
100 FORMAT(<ND-1>(/))
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE GETNSID(NSID,IN,ALLNOI,ALLNOIN,N,MAXNOI)
INTEGER IN(6),ALLNOIN(N),N,MAXNOI
COMPLEX NSID(6),ALLNOI(N)
C
IMPLICIT NONE
INTEGER I,J
COMPLEX VAL
C
DO 5 I=1,6
5  NSID(I)=CMPLX(999.0,0.0)
I=1
VAL=CMPLX(999.0,0.0)
DO WHILE (I.LE.MAXNOI)
DO 10 J=1,6
IF (IN(J).EQ.ALLNOIN(I)) NSID(J)=ALLNOI(I)
10 CONTINUE
I=I+1
END DO
DO 30 I=1,6
IF (NSID(I).EQ.CMPLX(999.0,0.0)) THEN
WRITE(*,*) 'NODE ',IN(I), ' IS SUPPOSED TO BE A NEUMANN VALUE.'
WRITE(*,*) 'COULD NOT FIND IT! CHECK IT OUT, FOR ME LIFE GOES ON.'
END IF
30 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE SOLWRITER(BSIDE,N,SIZE,AX)
REAL*8 BSIDE(N)
INTEGER N,SIZE,AX
C
IMPLICIT NONE

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COMPLEX VAL
INTEGER I,BLK,REK,OFFREK
C
OPEN (UNIT=2,FILE='unresolution.dat',STATUS='UNKNOWN',ACCESS='DIRECT'
* ,FORM='UNFORMATTED',RECL=2,RECORDTYPE='FIXED')
C
BLK=SIZE/2
IF (AX.EQ.1) THEN
DO 10 I=1,BLK
VAL=DCMPLX(BSIDE(I),BSIDE(I+BLK))
WRITE(2,REC=1) VAL
10 CONTINUE
ELSE IF (AX.EQ.2) THEN
OFFREK=BLK
DO 20 I=1,BLK
REK=OFFREK+I
VAL=DCMPLX(BSIDE(I),BSIDE(I+BLK))
WRITE(2,REC=REK) VAL
20 CONTINUE
ELSE IF (AX.EQ.3) THEN
OFFREK=BLK*2
DO 30 I=1,BLK
REK=OFFREK+I
VAL=DCMPLX(BSIDE(I),BSIDE(I+BLK))
WRITE(2,REC=REK) VAL
30 CONTINUE
END IF
CLOSE(2)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE VFILLGUESS(SIZ,SOL,N,AX)
REAL*8 SOL(N)
INTEGER SIZ,N,AX
C
IMPLICIT NONE
INTEGER I,BLK,II,OFFREK,REK
COMPLEX VAL
C
OPEN(UNIT=4,FILE='unresolution.dat',STATUS='UNKNOWN',
* ACCESS='DIRECT',FORM='UNFORMATTED',RECL=2,RECORDTYPE='FIXED')
BLK=SIZE/2
IF (AX.EQ.1) OFFREK=0
IF (AX.EQ.2) OFFREK=BLK
IF (AX.EQ.3) OFFREK=SIZE
DO 10 I=1,BLK
REK=OFFREK+I
READ(4,REC=REK) VAL
II=BLK+I
SOL(I)=REAL(VAL)
SOL(II)=AIMAG(VAL)
10 CONTINUE
CLOSE(4)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE VFILLDIR(SIZ,SOL,RHS,N)
REAL*8 SOL(N),RHS(N)
INTEGER SIZ,N
C
IMPLICIT NONE
INTEGER I
C
DO 10 I=1,SIZ
SOL(I)=RHS(I)
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE REORDERNODES(NODES,BSIDE,SIZ,N)
COMPLEX BSIDE(N)
INTEGER NODES(N),N,SIZ
C
IMPLICIT NONE
INTEGER I,TEMP,ROW,IR
COMPLEX DUM
C
DO 15 ROW=1,SIZ
DO 20 I=1,SIZ
IF (NODES(I).EQ.ROW) IR=I
20 DUM=BSIDE(ROW)
BSIDE(ROW)=BSIDE(IR)
BSIDE(IR)=DUM
TEMP=NODES(ROW)
NODES(ROW)=NODES(IR)
NODES(IR)=TEMP
15 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

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MESHER3DSUB.F

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SUBROUTINE MESHWRITER(NP,NPS,SCP,SCF,SCE,P,PID,EDGE,FACE,PYR,EXTRAPYR)
INTEGER NP,SCP,SCF,SCE,PID(30000),EDGE(40000,3),FACE(86000,4),NPS
INTEGER PYR(40000,5),EXTRAPYR(40000,6)
REAL P(30000,3)
C
IMPLICIT NONE
INTEGER OFFREK,REK,I,IN1,IN2,IN3
OPEN(UNIT=1,FILE='edge.dat',STATUS='UNKNOWN')
OPEN(UNIT=2,FILE='mesh.dat',STATUS='UNKNOWN',ACCESS='SEQUENTIAL',
* FORM='FORMATTED')
OPEN(UNIT=3,FILE='unmesh.dat',STATUS='UNKNOWN',ACCESS='DIRECT',
* FORM='UNFORMATTED',RECL=11,RECORDTYPE='FIXED')
WRITE(2,FMT=100) NPS, ' THE NUMBER OF POINTS ON THE SURFACE'

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ELSE IF ((D1(I).EQ.3).OR.(D2(I).EQ.2).OR.(D3(I).EQ.2)) THEN
  IF (D1(I).EQ.4) D1(I)=2
  IF (D2(I).EQ.4) D2(I)=2
  IF (D3(I).EQ.4) D3(I)=2
END IF
IF ( (D1(I).EQ.D3(I)).AND.(D2(I).EQ.D3(I)) )
  *
  FACD=FACD+D1(I)*(10**J)
40 CONTINUE
IF ((D1(6).EQ.6).AND.(D2(6).EQ.5).AND.(D3(6).EQ.5)) FACD=FACD+500000
FACID=FACD
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL FUNCTION DISTP2PLAN(DCEN,DAP,DBP,DCP,DDP)
REAL DCEN(3),DAP,DBP,DCP,DDP
C
REAL DEN,NUM
DEN=SQRT( DAP**2+DBP**2+DCP**2)
NUM=DCEN(1)*DAP+DCEN(2)*DBP+DCEN(3)*DCP+DDP
DISTP2PLAN=ABS(NUM/DEN)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE PLANEPA3(DUM1,DUM2,DUM3,DAP,DBP,DCP,DDP)
REAL DUM1(3),DUM2(3),DUM3(3),DAP,DBP,DCP,DDP
C
IMPLICIT NONE
REAL DM1(3),DM2(3),DM3(3),MAT(3,3)
INTEGER I
C
DO 10 I=1,3
  DM1(I)=DUM1(I)
  DM2(I)=DUM2(I)
  DM3(I)=DUM3(I)
10 CALL MAKEMAT3(DM1,DM2,DM3,MAT)
CALL DETERMINANT(MAT,3,3,DDP)
DDP=-DDP
DM1(3)=1.0
DM2(3)=1.0
DM3(3)=1.0
CALL MAKEMAT3(DM1,DM2,DM3,MAT)
CALL DETERMINANT(MAT,3,3,DCP)
DM1(2)=DUM1(3)
DM2(2)=DUM2(3)
DM3(2)=DUM3(3)
CALL MAKEMAT3(DM1,DM2,DM3,MAT)
CALL DETERMINANT(MAT,3,3,DBP)
DBP=-DBP
DM1(1)=DUM1(3)
DM2(1)=DUM2(3)
DM3(1)=DUM3(3)
CALL MAKEMAT3(DM1,DM2,DM3,MAT)
CALL DETERMINANT(MAT,3,3,DAP)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CENTERSPHERE(DUM1,DUM2,DUM3,DUM4,CENT)
REAL DUM1(3),DUM2(3),DUM3(3),DUM4(3),CENT(3)
C
IMPLICIT NONE
REAL DM1(4),DM2(4),DM3(4),DM4(4),MAT(4,4)
REAL DAS,DOS,DES,DFS
INTEGER I
LOGICAL COPL,COPLANAR
C
COPL=COPLANAR(DUM1,DUM2,DUM3,DUM4)
IF (.NOT.(COPL)) THEN
  DO 10 I=1,3
    DM1(I)=DUM1(I)
    DM2(I)=DUM2(I)
    DM3(I)=DUM3(I)
  10 DM4(I)=DUM4(I)
  DM1(4)=1.0
  DM2(4)=1.0
  DM3(4)=1.0
  DM4(4)=1.0
  CALL MAKEMAT4(DM1,DM2,DM3,DM4,MAT)
  CALL DETERMINANT(MAT,4,4,DAS)
  DM1(1)=DUM1(1)**2+DUM1(2)**2+DUM1(3)**2
  DM2(1)=DUM2(1)**2+DUM2(2)**2+DUM2(3)**2
  DM3(1)=DUM3(1)**2+DUM3(2)**2+DUM3(3)**2
  DM4(1)=DUM4(1)**2+DUM4(2)**2+DUM4(3)**2
  CALL MAKEMAT4(DM1,DM2,DM3,DM4,MAT)
  CALL DETERMINANT(MAT,4,4,DOS)
  DOS=-DOS
  DM1(2)=DUM1(1)
  DM2(2)=DUM2(1)
  DM3(2)=DUM3(1)
  DM4(2)=DUM4(1)
  CALL MAKEMAT4(DM1,DM2,DM3,DM4,MAT)
  CALL DETERMINANT(MAT,4,4,DES)
  DM1(3)=DUM1(2)
  DM2(3)=DUM2(2)
  DM3(3)=DUM3(2)
  DM4(3)=DUM4(2)
  CALL MAKEMAT4(DM1,DM2,DM3,DM4,MAT)
  CALL DETERMINANT(MAT,4,4,DFS)
  DFS=-DFS
  CENT(1)=-DOS/DAS/2
  CENT(2)=-DES/DAS/2
  CENT(3)=-DFS/DAS/2
ELSE
  CENT(1)=9999.9
  CENT(2)=9999.9
  CENT(3)=9999.9
END IF
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

```

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C
SUBROUTINE PYRAMIDGENERATOR(NP,NPV,SCF,SCE,SCF,P,PID,EDGE,FACE,PYR)
INTEGER NPV,SCF,SCE,SCF,FACE(40000,4),EDGE(40000,3),PYR(40000,8)
INTEGER PID(30000),NP
REAL P(30000,3)
C
IMPLICIT NONE
REAL DUMP1(3),DUMP2(3),DUMP3(3),DUMP4(3),VOLUME,VOL,RAT
REAL AP,BP,CP,DP,DIST,MID1(3),COORD(4,3),COSINUS
REAL MINI,RATREF,MINREF1
REAL DIR1(3),DIR2(3),SURFACE,AREA,NEWP(3)
INTEGER VICIN(80),NV,NO,SCOLD,SCPOL,SCPOL,K,MAX
INTEGER I,J,SC,LSTAK(8000),SCE1,SCE2,SCE3,SCF4,P1,OLDSCEP
INTEGER IN1,IN2,IN3,OCCUR,SCF1,SCF2,SCF3,IND,I,PYRCON,NODE(4)
INTEGER NPYR,VICPYR(400),NEW,SNCFAS(40000),SNCFASC,ORDPOINT(30000,30)
INTEGER VICEDG(1000),VICFAC(2000),NEDG,NFAC,VICIN1(80),NV1
LOGICAL CANTUSE,PYRLAP,PYRAMOVERLAP,NOGD2
LOGICAL DONTUSE,NOGD1,NOTVALID,CENTERVALIDV,NOTVALID1,CENTERVALIDV1
CHARACTER*24 PDATE
C
C
NO=0
LSTAK(1)=1
SC=1
* CALL FILLSTAK(LSTAK,SC,SCF)
SNCFASC=0
VOL=0.0
SCE=0
SCF=0
MAX=30
CALL ORDER(ORDPOINT,NP,NPV,P,MAX)
WRITE(*,*) 'I JUST FINISHED ORDER'
WRITE(*,*) PDATE()
120 DO WHILE (SC.GT.0)
  SCF1=LSTAK(1)
  CALL UPDATESKAK(SC,LSTAK)
  DONTUSE=.FALSE.
100 IF (.NOT.DONTUSE) THEN
  CALL BASENODE(EDGE,FACE,SCF1,IN1,IN2,IN3)
  CALL ONEDIM(P(IN1,1),P(IN1,2),P(IN1,3),DUMP1)
  CALL ONEDIM(P(IN2,1),P(IN2,2),P(IN2,3),DUMP2)
  CALL ONEDIM(P(IN3,1),P(IN3,2),P(IN3,3),DUMP3)
  CALL PLANEPA3(DUMP1,DUMP2,DUMP3,AP,BP,CP,DP)
  CALL GETCORNER(DUMP1,DUMP2,DUMP3,IN1,IN2,IN3,IND,MID1)
  CALL VICINITY(VICIN,NV1,IND,MAX,ORDPOINT)
  NV=30
  IF (MAX.LT.NV) NV=MAX
  * this is the distance between 2 points
15  RATREF=0.25
  MINREF1=10000.0
  IND=0
  CALL GETVICPYR(VICPYR,NPYR,EDGE,FACE,PYR,SCF,SCF,SCE,SCF1,
  * NFAC,VICFAC,VICIN,NV1,P)
  CALL FACEVICINITY(NPYR,VICPYR,PYR,NFAC,VICFAC)
  CALL EDGEVICINITY(NFAC,VICFAC,FACE,NEDG,VICEDG)
  CALL OCCURENCEPVIC(SCF1,NPYR,OCCUR,PYR,PYRCON,VICPYR)
  DO 20 I=1,NV
  I=VICIN(I)
  IF ((IN1.EQ.I).OR.(IN2.EQ.I).OR.(IN3.EQ.I)) GOTO 20
  CALL ONEDIM(P(I,1),P(I,2),P(I,3),DUMP4)
  * the last 3 parameters change with size of problem
  NOTVALID=CENTERVALIDV(EDGE,FACE,SCF1,DUMP4,P,0.5,0.5,1.0)
  IF (NOTVALID) GOTO 20
  PYRLAP=PYRAMOVERLAP(EDGE,FACE,PYR,AP,BP,CP,DP,SCF1,PYRCON,I,P)
  IF (PYRLAP) GOTO 20
  CALL INSIDEPYR(IN1,IN2,IN3,I,DUMP1,DUMP2,DUMP3,DUMP4,
  * P,NV1,VICIN1,NO)
  IF (NO.NE.0) GOTO 20
  CALL OVERLAPWITHFACE(FACE,EDGE,VICEDG,VICFAC,NEDG,NFAC,SCF1,
  * I,P,NOGD1,NOGD2,NV,VICIN)
  IF (NOGD1) GOTO 20
  IF (NOGD2) GOTO 20
  CALL GETNORMALTOFACE(DUMP1,DUMP2,DUMP3,DUMP4,DIR1)
  DIR1(1)=-DIR1(1)
  DIR1(2)=-DIR1(2)
  DIR1(3)=-DIR1(3)
  CALL DIRECTION(DUMP4,MID1,DIR2)
  RAT=COSINUS(DIR1,DIR2(1),DIR2(2),DIR2(3))
  MINI=DIST(DUMP4,MID1)
  IF ((RAT+0.000001).LT.RATREF) GOTO 20
  IF ((RAT.LT.RATREF).AND.(MINI.GE.MINREF1)) GOTO 20
  *
  IF (MINI.GT.(1.5*MINREF1)) GOTO 20
  CALL NOOVERLAP
  * (EDGE,FACE,SCF1,DUMP4,P,PYR,SCF,CANTUSE,VICPYR,NPYR)
  IF (CANTUSE) GOTO 20
  IND=1
  RATREF=RAT
  MINREF1=MINI
20  CONTINUE
SCOLD=SC
IF (IND.NE.0) THEN
  CALL EXISTENCEEDGE(IN1,IN2,IND,NEDG,SCE1,SCE2,SCE3,VICEDG,EDGE)
  CALL CHECKEDGE(SCE1,SCE,SC,IN1,IN2,PID,EDGE,0,LSTAK,NEDG,VICEDG)
  CALL CHECKEDGE(SCE2,SCE,SC,IN2,IND,PID,EDGE,0,LSTAK,NEDG,VICEDG)
  CALL CHECKEDGE(SCE3,SCE,SC,IN1,IND,PID,EDGE,0,LSTAK,NEDG,VICEDG)
  CALL EXISTENCEFACE(SCE1,SCE2,SCE3,NFAC,SCPOL,VICFAC,FACE)
  CALL CHECKFACE(SCE1,SCE2,SCE3,IN1,IN2,IND,SCF,SCPOL,FACE,PID,
  * 1,LSTAK,SC,NFAC,VICFAC)
SCF1=SCPOL
  CALL EXISTENCEEDGE(IN1,IN3,IND,NEDG,SCE1,SCE2,SCE3,VICEDG,EDGE)
  CALL CHECKEDGE(SCE1,SCE,SC,IN1,IN3,PID,EDGE,0,LSTAK,NEDG,VICEDG)
  CALL CHECKEDGE(SCE2,SCE,SC,IN3,IND,PID,EDGE,0,LSTAK,NEDG,VICEDG)
  CALL CHECKEDGE(SCE3,SCE,SC,IN1,IND,PID,EDGE,0,LSTAK,NEDG,VICEDG)
  CALL EXISTENCEFACE(SCE1,SCE2,SCE3,NFAC,SCPOL,VICFAC,FACE)
  CALL CHECKFACE(SCE1,SCE2,SCE3,IN1,IND,IN3,SCF,SCPOL,FACE,PID,
  * 1,LSTAK,SC,NFAC,VICFAC)
SCF2=SCPOL
  CALL EXISTENCEEDGE(IN2,IN3,IND,NEDG,SCE1,SCE2,SCE3,VICEDG,EDGE)
  CALL CHECKEDGE(SCE1,SCE,SC,IN2,IN3,PID,EDGE,0,LSTAK,NEDG,VICEDG)
  CALL CHECKEDGE(SCE2,SCE,SC,IN3,IND,PID,EDGE,0,LSTAK,NEDG,VICEDG)
  CALL CHECKEDGE(SCE3,SCE,SC,IN2,IND,PID,EDGE,0,LSTAK,NEDG,VICEDG)
  CALL EXISTENCEFACE(SCE1,SCE2,SCE3,NFAC,SCPOL,VICFAC,FACE)
  CALL CHECKFACE(SCE1,SCE2,SCE3,IND,IN2,IN3,SCF,SCPOL,FACE,PID,

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      1,LSTAK,SC,NFAC,VICFAC)
SCF3=SCPOL
CALL EXISTENCEEDGE(IN1,IN2,IN3,NEDG,SC1,SC2,SC3,VICEDG,EDGE)
CALL CHECKEDGE(SC1,SC2,SC3,IN1,IN2,PID,EDGE,0,LSTAK,NEDG,VICEDG)
CALL CHECKEDGE(SC2,SC3,SC1,IN2,IN3,PID,EDGE,0,LSTAK,NEDG,VICEDG)
CALL CHECKEDGE(SC3,SC1,SC2,IN1,IN3,PID,EDGE,0,LSTAK,NEDG,VICEDG)
CALL EXISTENCEFACE(SC1,SC2,SC3,NFAC,SCPOL,VICFAC,FACE)
CALL CHECKFACE(SC1,SC2,SC3,IN1,IN2,IN3,SCF,SCPOL,FACE,PID,
      1,LSTAK,SC,NFAC,VICFAC)
SCF4=SCPOL
CALL EXISTENCEPYR(SCF1,SCF2,SCF3,SCF4,PYR,SCPOL,NPYR,VICPYR)
IF (SCPOL.EQ.0) THEN
  CALL CHECKPYR(SCF1,SCF2,SCF3,SCF4,IN1,IN2,IN3,IND,
      SCF,SCPOL,PID,PYR)
  call pyr4node(edge,face,pyr,scp,node)
  DO 51 K=1,4
    COOR(K,1)=P(NODE(K),1)
    COOR(K,2)=P(NODE(K),2)
51    COOR(K,3)=P(NODE(K),3)
    VOL=VOL+VOLUME(COOR)
    WRITE(*,1110) scp,node(1),node(2),node(3),node(4),volume(coor),vol
    IF (SCOLD.NE.SC) SC=SC-1
    ELSE
      DONTUSE=.TRUE.
      GOTO 100
  END IF
  ELSE
    DONTUSE=.TRUE.
    GOTO 100
  END IF
  ELSE
110 SC=SC-1
    NO=0
  END IF
  END DO
  C
  AREA=0.0
  F1=0
  DO 40 I=1,SCP
    CALL OCCURENCEP(I,SCP,OCCUR,PYR,PYRCOM)
    CALL BASE3NODE(EDGE,FACE,I,IN1,IN2,IN3)
    write(*,*) I,IN1,IN2,IN3,OCCUR
    IF (OCCUR.EQ.1) THEN
      CALL ONEDIM(P(IN1,1),P(IN1,2),P(IN1,3),DUMP1)
      CALL ONEDIM(P(IN2,1),P(IN2,2),P(IN2,3),DUMP2)
      CALL ONEDIM(P(IN3,1),P(IN3,2),P(IN3,3),DUMP3)
      AREA=AREA+SURFACE(DUMP1,DUMP2,DUMP3)
      F1=F1+1
    END IF
    IF (OCCUR.GE.2) CALL FACEIN(FACE(I,4))
40 CONTINUE
  VOL=0.0
  DO 1000 I=1,SCP
    CALL PYR4NODE(EDGE,FACE,PYR,I,NODE)
    DO 50 K=1,4
      COOR(K,1)=P(NODE(K),1)
      COOR(K,2)=P(NODE(K),2)
50    COOR(K,3)=P(NODE(K),3)
      VOL=VOL+VOLUME(COOR)
    WRITE(*,1110) I,node(1),node(2),node(3),node(4),volume(coor),vol
1000 CONTINUE
    WRITE(*,*) 'AREA IS',AREA,'FOR ',F1,' FACES'
    WRITE(*,*) 'VOLUME IS ',VOL
1100 FORMAT(810,3x,f12.7)
1110 FORMAT(810,3x,f12.7,f12.7)
  END
  C
  CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
  C
  SUBROUTINE GETCORNER(DUMP1,DUMP2,DUMP3,IN1,IN2,IN3,IND,MID1)
  REAL DUMP1(3),DUMP2(3),DUMP3(3),MID1(3)
  INTEGER IN1,IN2,IN3,IND
  C
  IMPLICIT NONE
  REAL DIR1(3),DIR2(3),ANG(3),COSINUS
  C
  CALL DIRECTION(DUMP2,DUMP1,DIR1)
  CALL DIRECTION(DUMP3,DUMP1,DIR2)
  ANG(1)=COSINUS(DIR1,DIR2(1),DIR2(2),DIR2(3))
  CALL DIRECTION(DUMP1,DUMP2,DIR1)
  CALL DIRECTION(DUMP3,DUMP2,DIR2)
  ANG(2)=COSINUS(DIR1,DIR2(1),DIR2(2),DIR2(3))
  CALL DIRECTION(DUMP2,DUMP3,DIR1)
  CALL DIRECTION(DUMP1,DUMP3,DIR2)
  ANG(3)=COSINUS(DIR1,DIR2(1),DIR2(2),DIR2(3))
  IF ((ANG(1).LE.ANG(2)).AND.(ANG(1).LE.ANG(3))) IND=IN1
  IF ((ANG(2).LE.ANG(1)).AND.(ANG(2).LE.ANG(3))) IND=IN2
  IF ((ANG(3).LE.ANG(1)).AND.(ANG(3).LE.ANG(2))) IND=IN3
  IF (IND.EQ.IN1) CALL MIDPOI(DUMP2,DUMP3,MID1)
  IF (IND.EQ.IN2) CALL MIDPOI(DUMP1,DUMP3,MID1)
  IF (IND.EQ.IN3) CALL MIDPOI(DUMP2,DUMP1,MID1)
  IF (IND.EQ.IN1) CALL MIDPOI(MID1,DUMP1,MID1)
  IF (IND.EQ.IN2) CALL MIDPOI(MID1,DUMP2,MID1)
  IF (IND.EQ.IN3) CALL MIDPOI(MID1,DUMP3,MID1)
  END
  C
  CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
  C
  SUBROUTINE GETNORMALTOFACE(DUMP1,DUMP2,DUMP3,DUMP4,ORT)
  REAL DUMP1(3),DUMP2(3),DUMP3(3),DUMP4(3),ORT(3)
  C
  IMPLICIT NONE
  REAL MID(3),MID1(3),DIR(3),FAC,MAQ,COSINUS,VAL
  C
  FAC=1.0
  CALL DIRECTION(DUMP2,DUMP1,MID)
  CALL DIRECTION(DUMP3,DUMP1,MID1)
  CALL DIRECTION(DUMP4,DUMP1,DIR)
  CALL NORMALVEC(MID,ORT,MID1(1),MID1(2),MID1(3))
  VAL=COSINUS(ORT,DIR(1),DIR(2),DIR(3))
  IF (VAL.GT.0.0) FAC=-1.0
  MAQ=SQRT(ORT(1)**2+ORT(2)**2+ORT(3)**2)
  ORT(1)=ORT(1)*FAC/MAQ

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AF(NF)=1
END IF
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE EDGEINCOMMONFROMPOINTS(FACE,EDGE,NEDG,NFAC,VICEDG,VICFAC,
*IND,SCF1,EDJ,NE)
INTEGER FACE(88000,4),EDGE(80000,3),IND,SCF1,NE,EDJ(1000)
INTEGER NEDG,NFAC,VICEDG(1000),VICFAC(2000)
C
IMPLICIT NONE
INTEGER E1,E2,E3,P1,P2,I,J,K,NF1,AF1(300),NEOLD,L,E
C
E1=FACE(SCF1,1)
E2=FACE(SCF1,2)
E3=FACE(SCF1,3)
NE=0
DO 10 I=1,NEDG
P1=EDGE(VICEDG(I),1)
P2=EDGE(VICEDG(I),2)
IF ((P1.EQ.IND).OR.(P2.EQ.IND)) THEN
IF ((I.NE.E1).AND.(I.NE.E2).AND.(I.NE.E3)) THEN
NE=NE+1
EDJ(NE)=VICEDG(I)
END IF
END IF
10 CONTINUE
C
NEOLD=NE
DO 20 L=1,NEOLD
CALL EDGEINCOMMON(VICFAC,NFAC,NF1,AF1,EDJ(L),FACE)
C
DO 50 I=1,NF1
DO 60 J=1,3
E=FACE(AF1(I),J)
IF ((I.EQ.E1).OR.(I.EQ.E2).OR.(I.EQ.E3)) GOTO 60
DO 70 K=1,NE
IF (E.EQ.EDJ(K)) GOTO 60
70 CONTINUE
NE=NE+1
EDJ(NE)=E
60 CONTINUE
80 CONTINUE
20 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE EDGEINCOMMONFROMPOINTSPECH(FACE,EDGE,SCB,SCF,SCF1,EDJ,NE)
INTEGER FACE(88000,4),EDGE(80000,3),SCF1,NE,EDJ(1000),SCB,SCF
C
IMPLICIT NONE
INTEGER P1,P2,I,K,NF1,AF1(300),NEOLD,L,E,EDJ1(1000),NE1
INTEGER E1,E2,E3
LOGICAL COTE1,COTE2,COTE3
C
NE1=0
NE=0
CALL BASE3NODE(EDGE,FACE,SCF1,E1,E2,E3)
DO 10 I=1,SCB
P1=EDGE(I,1)
P2=EDGE(I,2)
COTE1=(P1.EQ.P1).OR.(E1.EQ.P2)
COTE2=(E2.EQ.P1).OR.(E2.EQ.P2)
COTE3=(E3.EQ.P1).OR.(E3.EQ.P2)
IF (COTE1.OR.COTE2.OR.COTE3) THEN
DO 40 L=1,NE1
40 IF (EDJ1(L).EQ.I) GOTO 10
NE1=NE1+1
EDJ1(NE1)=I
END IF
10 CONTINUE
C
NEOLD=NE1
DO 40 L=1,NEOLD
CALL EDGEINCOMMON2D(FACE,SCF,NF1,AF1,EDJ1(L))
DO 60 I=1,NF1
E=AF1(I)
DO 70 K=1,NE
IF (E.EQ.EDJ(K)) GOTO 60
70 CONTINUE
NE=NE+1
EDJ(NE)=E
60 CONTINUE
80 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
LOGICAL FUNCTION SAMESIDE(EDGE,FACE,AP,BP,CP,DP,SCF1,IND1,IND2,P)
INTEGER EDGE(80000,3),FACE(88000,4),IND1,SCF1,IND2
REAL P(30000,3),AP,BP,CP,DP
C
IMPLICIT NONE
REAL VAL1,VAL3
LOGICAL OVER,UNDER
C
SAMESIDE=.FALSE.
VAL1=AP*P(IND1,1)+BP*P(IND1,2)+CP*P(IND1,3)+DP
VAL3=AP*P(IND2,1)+BP*P(IND2,2)+CP*P(IND2,3)+DP
OVER=(VAL1.GT.0).AND.(VAL3.GT.0)
UNDER=(VAL1.LT.0).AND.(VAL3.LT.0)
IF (OVER.OR.UNDER) SAMESIDE=.TRUE.
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL FUNCTION SURFACE(DUM1,DUM2,DUM3)
REAL DUM1(3),DUM2(3),DUM3(3)

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[illegible]

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NO=0
CALL ELIMINATORS(VICIN,NV,CENT1,DUMP1,P,NO,VIC1OL)
IF (NO.NE.0) THEN
CALL SPECIALORDER(CENT1,IND,IN1,IN2,NO,VIC1OL,P)
IN1=VIC1OL(1)
DO 30 I=2,(NO-1)
IN2=VIC1OL(I)
IN3=VIC1OL(I+1)
CALL EXISTENCEEDGE2D(IN1,IN2,IN3,SCE,SCE1,SCE2,SCE3,EDGE)
CALL EXISTENCEFACE2D(SCE1,SCE2,SCE3,SCF,SCPOL,FACE)
CALL CHECKEDGE2D(SCE1,SCE,SC,IN1,IN2,PID,EDGE,1,LSTAK)
CALL CHECKEDGE2D(SCE2,SCE,SC,IN2,IN3,PID,EDGE,1,LSTAK)
CALL CHECKEDGE2D(SCE3,SCE,SC,IN1,IN3,PID,EDGE,1,LSTAK)
CALL CHECKFACE2D(SCE1,SCE2,SCE3,IN1,IN2,IN3,SCF,SCPOL,FACE,PID,
0,LSTAK,SC)
30 CONTINUE
ELSE
SCE1=SCE1I
SCE2=SCE2I
SCE3=SCE3I
CALL CHECKEDGE2D(SCE1,SCE,SC,IN1,IN2,PID,EDGE,1,LSTAK)
CALL CHECKEDGE2D(SCE2,SCE,SC,IN2,IND,PID,EDGE,1,LSTAK)
CALL CHECKEDGE2D(SCE3,SCE,SC,IND,IN1,PID,EDGE,1,LSTAK)
CALL CHECKFACE2D(SCE1,SCE2,SCE3,IN1,IN2,IND,SCF,SCPOL,FACE,PID,
0,LSTAK,SC)
END IF
SC=SC-1
ELSE
NO=0
SC=SC-1
END IF
IF (sefold.ne.scf) then
call base3node(edge,face,scf,in1,in2,in3)
write(*,*) scf,in1,in2,in3
sefold=scf
end if
ELSE
SC=SC-1
END IF
END DO
DO 1000 I=1,SCF
call base1node(edge,face,I,in1,in2,in3)
1000 WRITE(*,1100) I,in1,in2,in3
1100 FORMAT(7I8)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE VICINITY2D(VICIN,NV,POI,NP,NPD,P,NUMB)
INTEGER VICIN(50),NV,NP,NPD,NUMB
REAL POI(3),P(30000,3)
C
IMPLICIT NONE
REAL DUM(3),DIST,LON(30000),FAR
INTEGER I,J,STAR,PIN,IND,K,VOISIN(30000)
C
NV=0
STAR=NP+1
PIN=NPD+NP
CALL ONEDIM(P(STAR,1),P(STAR,2),P(STAR,3),DUM)
LON(1)=DIST(DUM,POI)
VOISIN(1)=STAR
DO 10 I=1,(PIN-STAR)
IND=STAR+I
CALL ONEDIM(P(IND,1),P(IND,2),P(IND,3),DUM)
FAR=DIST(DUM,POI)
IF (FAR.GT.CRIT) GOTO 10
IF (FAR.LT.LON(I)) THEN
DO 20 J=1,I-1
VOISIN(J+1)=VOISIN(J)
20 LON(J+1)=LON(J)
LON(I)=FAR
VOISIN(I)=IND
ELSE
DO 30 J=1,I
IF (FAR.LT.LON(J)) THEN
DO 40 K=1,J-1
VOISIN(K+1)=VOISIN(K)
40 LON(K+1)=LON(K)
LON(J)=FAR
VOISIN(J)=IND
GOTO 10
END IF
30 CONTINUE
LON(I+1)=FAR
VOISIN(I+1)=IND
END IF
10 CONTINUE
NV=NUMB
DO 50 I=1,NV
50 VICIN(I)=VOISIN(I)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
INTEGER FUNCTION EDGEID(P1,P2)
INTEGER P1,P2
C
IMPLICIT NONE
INTEGER D1(5),D2(5),DP1,DP2,VAL,I,J,EDG
DP1=P1
DP2=P2
IF (DP1.GE.500000) DP1=DP1-500000
IF (DP2.GE.500000) DP2=DP2-500000
DO 10 I=1,5
VAL=INT(DP1/10)*10
D1(I)=DP1-VAL
10 DP1=VAL/10
DO 20 I=1,5
VAL=INT(DP2/10)*10
D2(I)=DP2-VAL
20 DP2=VAL/10

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ORDPOINT(L,K)=IND
IF (K.EQ.MAX) GOTO 1
60 CONTINUE
END DO
1 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE SCNDORDER(SCE,SCP,EDGE,FACE,PYR,EXTRAPYR)
INTEGER SCE,FACE(66000,4),SCP,PYR(40000,6),EXTRAPYR(40000,6)
INTEGER EDGE(60000,3)
C
IMPLICIT NONE
INTEGER SIDE(6),I,K,L,J,THAT,IND(6,2),TEMP,X
LOGICAL THERE
C
DO 20 I=1,SCP
DO 30 J=1,6
30 SIDE(J)=0
SIDE(1)=FACE(PYR(I,1),1)
SIDE(2)=FACE(PYR(I,1),2)
SIDE(3)=FACE(PYR(I,1),3)
K=3
DO 40 L=2,3
DO 50 J=1,3
THAT=FACE(PYR(I,L),J)
IF (THERE(SIDE,THAT,K,6)) THEN
K=K+1
SIDE(K)=THAT
END IF
60 CONTINUE
40 CONTINUE
DO 70 J=1,6
IND(J,1)=EDGE(SIDE(J),1)
70 IND(J,2)=EDGE(SIDE(J),2)
DO 80 J=2,6
IF ((IND(1,1).NE.IND(J,1)).AND.(IND(1,1).NE.IND(J,2))) THEN
DO 90 L=J+1,6
IF ((IND(L,1).EQ.IND(1,1)).OR.(IND(L,2).EQ.IND(1,1))) THEN
TEMP=SIDE(J)
SIDE(J)=SIDE(L)
SIDE(L)=TEMP
DO 95 K=1,6
IND(K,1)=EDGE(SIDE(K),1)
95 IND(K,2)=EDGE(SIDE(K),2)
GOTO 60
END IF
90 CONTINUE
95 END IF
80 CONTINUE
IF (IND(2,1).NE.IND(1,1)) THEN
TEMP=IND(2,2)
IND(2,2)=IND(2,1)
IND(2,1)=TEMP
END IF
IF (IND(3,1).NE.IND(1,1)) THEN
TEMP=IND(3,2)
IND(3,2)=IND(3,1)
IND(3,1)=TEMP
END IF
X=1
DO 115 L=4,6
DO 100 J=L,6
IF (L.EQ.6) X=3
IF ((IND(X,2).NE.IND(J,1)).AND.(IND(X,2).NE.IND(J,2))) THEN
IF ((IND(2,2).NE.IND(J,1)).AND.(IND(2,2).NE.IND(J,2))) THEN
TEMP=SIDE(J)
SIDE(J)=SIDE(X)
SIDE(X)=TEMP
DO 120 K=1,6
IND(K,1)=EDGE(SIDE(K),1)
120 IND(K,2)=EDGE(SIDE(K),2)
GOTO 115
END IF
END IF
100 CONTINUE
115 CONTINUE
C
DO 60 K=1,6
60 EXTRAPYR(I,K)=SIDE(K)
20 CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
LOGICAL FUNCTION THERE(SIDE,THAT,K,N)
INTEGER SIDE(N),THAT,K,N
C
IMPLICIT NONE
INTEGER I
THERE=.TRUE.
DO 10 I=1,K
10 IF (SIDE(I).EQ.THAT) THERE=.FALSE.
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL FUNCTION DISTLINPOI(CENT,DUM1,DX,DY,DZ)
REAL CENT(3),DX,DY,DZ,DUM1(3)
C
IMPLICIT NONE
REAL E1,E2,E3,E
C
E=DX**2+DY**2+DZ**2
E1=(CENT(1)-DUM1(1))*DY-(CENT(2)-DUM1(2))*DX
E2=(CENT(2)-DUM1(2))*DZ-(CENT(3)-DUM1(3))*DY
E3=(CENT(3)-DUM1(3))*DX-(CENT(1)-DUM1(1))*DZ
DISTLINPOI=SQRT((E1**2+E2**2+E3**2)/E)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

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C
SUBROUTINE ELIMINATORS(VICIN,NV,CENT,DUMP1,P,NO,VIC1OL)
INTEGER VIC1OL(20),VICIN(50),NV,NO
REAL CENT(3),DUMP1(3),P(30000,3)
C
IMPLICIT NONE
INTEGER I
REAL R,RD,DIST,DUM(3)
C
NO=0
R=DIST(DUMP1,CENT)
DO 10 I=1,NV
CALL ONEDIM(P(VICIN(I),1),P(VICIN(I),2),P(VICIN(I),3),DUM)
RD=DIST(DUM,CENT)
IF (ABS(RD-R).LT.0.0001) THEN
NO=NO+1
VIC1OL(NO)=VICIN(I)
END IF
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C THIS SUBROUTINE DEPENDS ON THE PROBLEM
LOGICAL FUNCTION CENTERVALIDS(CENT)
REAL CENT(3)
C
IMPLICIT NONE
REAL RLX,RLY,RLZ
INTEGER DUM(3),LX,LY,LZ
LOGICAL GX,GY,GZ,PX,PY,PZ,PZP,GZP,BZ,BZP,BAS
C
RLX=0.5
RLY=0.5
RLZ=1.0
LX=INT(RLX*10000)
LY=INT(RLY*10000)
LZ=INT(RLZ*10000)
DUM(1)=ABS(INT(CENT(1)*10000))
DUM(2)=ABS(INT(CENT(2)*10000))
DUM(3)=ABS(INT(CENT(3)*10000))
CENTERVALIDS=.TRUE.
GX=DUM(1).GT.LX
GY=DUM(2).GT.LY
GZ=DUM(3).GT.LZ
PX=DUM(1).LT.LX
PY=DUM(2).LT.LY
PZ=DUM(3).LT.0
PZP=DUM(3).GT.0
* PZP=.TRUE. SWITCH 2 LINES FOR EMI OR ANTE PROB
* PZ=.TRUE. WITH previous 2 lines
* BAS=((INT(CENT(3)*10000)).EQ.(-LZ))
GZP=DUM(3).LT.LZ
BZP=PZP.AND.GZP
BZ=PZ.OR.GZ
IF ((PX).AND.(PY).AND.(BZP)) CENTERVALIDS=.FALSE.
IF ((GX).AND.(GY).AND.(BZ)) CENTERVALIDS=.FALSE.
* IF ((PX).AND.(PY).AND.(BAS)) CENTERVALIDS=.FALSE.
* IF ((GX).AND.(GY).AND.(BAS)) CENTERVALIDS=.FALSE.
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C THIS SUBROUTINE DEPENDS ON THE PROBLEM
LOGICAL FUNCTION CENTERVALIDV(EDGE,FACE,SCF1,DUMP,P,RLX,RLY,RLZ)
REAL P(30000,3),RLX,RLY,RLZ,DUMP(3)
INTEGER EDGE(50000,3),FACE(50000,4),SCF1
C
IMPLICIT NONE
REAL DUM1(3),DUM2(3),DUMP1(3),DUMP2(3),DUMP3(3)
REAL CENT(3),DIR1(3),RLZ1
INTEGER IN1,IN2,IN3
LOGICAL LX1,LX2,LY1,LY2,LZ1,LZ2,L1,L2
C
RLZ1=-RLZ
CALL BASE3NODE(EDGE,FACE,SCF1,IN1,IN2,IN3)
CALL ONEDIM(P(IN1,1),P(IN1,2),P(IN1,3),DUMP1)
CALL ONEDIM(P(IN2,1),P(IN2,2),P(IN2,3),DUMP2)
CALL ONEDIM(P(IN3,1),P(IN3,2),P(IN3,3),DUMP3)
CALL MIDPOI(DUMP1,DUMP2,CENT)
CALL MIDPOI(CENT,DUMP3,CENT)
CALL DIRECTION(CENT,DUMP,DIR1)
DUM1(1)=ABS(DUMP(1)+0.002*DIR1(1))
DUM1(2)=ABS(DUMP(2)+0.002*DIR1(2))
DUM1(3)=DUMP(3)+0.002*DIR1(3)
DUM2(1)=ABS(DUMP(1)+0.998*DIR1(1))
DUM2(2)=ABS(DUMP(2)+0.998*DIR1(2))
DUM2(3)=DUMP(3)+0.998*DIR1(3)
CENTERVALIDV=.FALSE.
LX1=(DUM1(1).LE.RLX)
LX2=(DUM2(1).LE.RLX)
LY1=(DUM1(2).LE.RLY)
LY2=(DUM2(2).LE.RLY)
* LZ1=(DUM1(3).LE.RLZ).AND.(DUM1(3).GT.0.0) SWITCH THIS 3 LINE WITH
* LZ2=(DUM2(3).LE.RLZ).AND.(DUM2(3).GT.0.0) NEXT 3 FOR EMI OR ANT PROB
LZ1=(DUM1(3).LE.RLZ).AND.(DUM1(3).GE.RLZ1)
LZ2=(DUM2(3).LE.RLZ).AND.(DUM2(3).GE.RLZ1)
L1=LX1.AND.LY1.AND.LZ1
L2=LX2.AND.LY2.AND.LZ2
IF (L1.OR.L2) CENTERVALIDV=.TRUE.
END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
LOGICAL FUNCTION CENTERVALIDVI(EDGE,FACE,SCF1,DUMP,P,RLX,RLY,RLZ)
REAL P(30000,3),RLX,RLY,RLZ,DUMP(3)
INTEGER EDGE(50000,3),FACE(50000,4),SCF1
C
IMPLICIT NONE
REAL DUM1(3),DUM2(3),DUMP1(3),DUMP2(3),DUMP3(3)
REAL CENT(3),DIR1(3)

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INTEGER IN1,IN2,IN3
LOGICAL LX1,LX2,LY1,LY2,LZ1,LZ2,L1,L2
C
CALL BASE3NODE(EDGE,FACE,SCF1,IN1,IN2,IN3)
CALL ONEDIM(P(IN1,1),P(IN1,2),P(IN1,3),DUMP1)
CALL ONEDIM(P(IN2,1),P(IN2,2),P(IN2,3),DUMP2)
CALL ONEDIM(P(IN3,1),P(IN3,2),P(IN3,3),DUMP3)
CALL MIDPOI(DUMP1,DUMP2,CENT)
CALL MIDPOI(CENT,DUMP3,CENT)
CALL DIRECTION(CENT,DUMP,DIR1)
DUM1(1)=ABS(DUMP(1)+0.002*DIR1(1))
DUM1(2)=ABS(DUMP(2)+0.002*DIR1(2))
DUM1(3)=DUMP(3)+0.002*DIR1(3)
DUM2(1)=ABS(DUMP(1)+0.998*DIR1(1))
DUM2(2)=ABS(DUMP(2)+0.998*DIR1(2))
DUM2(3)=DUMP(3)+0.998*DIR1(3)
CENTERVALIDV1=.FALSE.
LX1=(DUM1(1).GT.RLX)
LX2=(DUM2(1).GT.RLX)
LY1=(DUM1(2).GT.RLY)
LY2=(DUM2(2).GT.RLY)
LZ1=(DUM1(3).LT.RLZ)
LZ2=(DUM2(3).LT.RLZ)
L1=(LX1.OR.LY1).AND.LZ1
L2=(LX2.OR.LY2).AND.LZ2
IF ( L1.OR.L2 ) CENTERVALIDV1=.TRUE.
END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE DIRECTION(DUM1,DUM2,DIRE)
REAL DUM1(3),DUM2(3),DIRE(3)
C
c points from dum2 to dum1
IMPLICIT NONE
INTEGER I
DO 10 I=1,3
10 DIRE(I)=DUM1(I)-DUM2(I)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE NORMALVEC(VEC,NVEC,AD,BD,CD)
REAL VEC(3),NVEC(3),AD,BD,CD
C
IMPLICIT NONE
C
NVEC(1)=VEC(2)*CD-VEC(3)*BD
NVEC(2)=VEC(3)*AD-VEC(1)*CD
NVEC(3)=VEC(1)*BD-AD*VEC(2)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CENTERSPACE(DUM1,DUM2,DUM3,DCEN,CHECK)
REAL DUM1(3),DUM2(3),DUM3(3),DCEN(3)
INTEGER CHECK
C
IMPLICIT NONE
REAL MID1(3),MID2(3),DIR1(3),DIR2(3),NDIR1(3)
REAL NDIR2(3),AP,BP,CP,DP,T,MID12(3),MID22(3)
LOGICAL COLINEAR,COLI
C
COLI=.FALSE.
IF (CHECK.EQ.1) COLI=COLINEAR(DUM1,DUM2,DUM3)
IF (.NOT.(COLI)) THEN
CALL PLANEPA3(DUM1,DUM2,DUM3,AP,BP,CP,DP)
CALL MIDPOI(DUM1,DUM2,MID1)
CALL MIDPOI(DUM1,DUM3,MID2)
CALL DIRECTION(DUM1,DUM2,DIR1)
CALL DIRECTION(DUM1,DUM3,DIR2)
CALL NORMALVEC(DIR1,NDIR1,AP,BP,CP)
CALL NORMALVEC(DIR2,NDIR2,AP,BP,CP)
MID12(1)=MID1(1)+NDIR1(1)
MID12(2)=MID1(2)+NDIR1(2)
MID12(3)=MID1(3)+NDIR1(3)
MID22(1)=MID2(1)+NDIR2(1)
MID22(2)=MID2(2)+NDIR2(2)
MID22(3)=MID2(3)+NDIR2(3)
CALL DIRECTION(MID1,MID12,DIR1)
CALL DIRECTION(MID2,MID22,DIR2)
CALL PARAMETERINT(MID1,MID2,DIR1,DIR2,T)
DCEN(1)=MID2(1)+T*DIR2(1)
DCEN(2)=MID2(2)+T*DIR2(2)
DCEN(3)=MID2(3)+T*DIR2(3)
ELSE
DCEN(1)=9999.9
DCEN(2)=9999.9
DCEN(3)=9999.9
END IF
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE PARAMETERINT(MID1,MID2,DIR1,DIR2,T)
REAL MID1(3),MID2(3),DIR1(3),DIR2(3),T
C
IMPLICIT NONE
REAL C1,C2,C3,A1,B1,A2,B2,A3,B3,D1,D2,D3
C1=MID2(1)-MID1(1)
C2=MID2(2)-MID1(2)
C3=MID2(3)-MID1(3)
A1=DIR1(1)
B1=DIR2(1)
A2=DIR1(2)
B2=DIR2(2)
A3=DIR1(3)
B3=DIR2(3)
D1=B1*A2-B2*A1
D2=B1*A3-B3*A1
D3=B2*A3-B3*A2
IF (D1.NE.0.0) THEN
T=(A2*C1-A1*C2)/D1
ELSE IF (D2.NE.0.0) THEN

```



```

OCCUR=OCCUR+1
PYRCOM=K
END IF
* IF (PYRCOM.NR.0) GOTO 30
10 CONTINUE
20 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CREATEFACE(SCE1,SCE2,SCE3,SCF,FACE,IN1,IN2,IN3,PID)
INTEGER SCE1,SCE2,SCE3,SCF,FACE(80000,4),PID(30000),IN1,IN2,IN3
IMPLICIT NONE
C
INTEGER FACEID
FACE(SCF,1)=SCE1
FACE(SCF,2)=SCE2
FACE(SCF,3)=SCE3
FACE(SCF,4)=FACEID(PID(IN1),PID(IN2),PID(IN3))
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CREATEEDGE(SCE,IND1,IND2,PID,EDGE)
INTEGER SCE,IND1,IND2,PID(30000),EDGE(80000,3)
C
IMPLICIT NONE
INTEGER EDGEID
EDGE(SCE,1)=IND1
EDGE(SCE,2)=IND2
EDGE(SCE,3)=EDGEID(PID(IND1),PID(IND2))
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CHECKEDGE(DSCE,SCE,SC,IN1,IN2,PID,EDGE,N,LSTAK,NEDG,VICEDG)
INTEGER DSCE,SCE,SC,IN1,IN2,PID(30000),EDGE(80000,3),N
INTEGER LSTAK(8000),NEDG,VICEDG(1000)
IMPLICIT NONE
C
IF (DSCE.EQ.0) THEN
  SCE=SCE+1
  DSCE=SCE
  NEDG=NEDG+1
  VICEDG(NEDG)=SCE
  CALL CREATEEDGE(DSCE,IN1,IN2,PID,EDGE)
  IF (N.EQ.1) THEN
    LSTAK(SC)=SCE
    SC=SC+1
  END IF
END IF
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CHECKEDGE2D(DSCE,SCE,SC,IN1,IN2,PID,EDGE,N,LSTAK)
INTEGER DSCE,SCE,SC,IN1,IN2,PID(30000),EDGE(80000,3),N
INTEGER LSTAK(8000)
IMPLICIT NONE
C
IF (DSCE.EQ.0) THEN
  SCE=SCE+1
  DSCE=SCE
  CALL CREATEEDGE(DSCE,IN1,IN2,PID,EDGE)
  IF (N.EQ.1) THEN
    LSTAK(SC)=SCE
    SC=SC+1
  END IF
END IF
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL FUNCTION ANGLE(VEC1,VEC2,VEC)
REAL VEC1(3),VEC2(3),VEC(3)
C
IMPLICIT NONE
INTEGER I
REAL SUM1,SUM2,MAG1,MAG2,COSINE2
REAL MAG,SUM,COSINE3,ANG
C
SUM1=0
MAG1=0
MAG2=0
SUM2=0
SUM=0
MAG=0
DO 10 I=1,3
  SUM1=SUM1+VEC1(I)*VEC1(I)
  SUM2=SUM2+VEC2(I)*VEC2(I)
  SUM=SUM+VEC(I)*VEC(I)
  MAG1=VEC1(I)**2+MAG1
  MAG2=VEC2(I)**2+MAG2
10 MAG=SUM/SQRT(MAG1)/SQRT(MAG2)
COSINE2=SUM2/SQRT(MAG1)/SQRT(MAG2)
COSINE3=SUM/SQRT(MAG1)/SQRT(MAG2)
ANG=ATAN2(COSINE3,COSINE2)*180.0/3.141592654
ANG=INT(ANG*100)/100
DO WHILE (ANG.LT.0.0)
  ANG=ANG+360.0
END DO
ANGLE=ANG
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE SPECIALORDER(CENT,IND,IN1,IN2,NO,VICIOR,P)
INTEGER IND,IN1,IN2,NO,VICIOR(20)
REAL P(30000,3),CENT(3)
C
IMPLICIT NONE
REAL DDIR(3),TET(20),DUM(3),DIR1(3),DIR(20,3),ANGLE

```

```

REAL AP,SP,CP,DP,DIR2(3),NDIR1(3),TEMP1
INTEGER I,J,I1,TEMP
C
DO 10 I=1,NO
  I1=VICIOL(I)
  CALL ONEDIM(P(I1,1),P(I1,2),P(I1,3),DUM)
  CALL DIRECTION(DUM,CENT,DDIR)
  DIR(I,1)=DDIR(1)
  DIR(I,2)=DDIR(2)
  DIR(I,3)=DDIR(3)
10 CONTINUE
CALL ONEDIM(DIR(1,1),DIR(1,2),DIR(1,3),DIR1)
CALL ONEDIM(DIR(2,1),DIR(2,2),DIR(2,3),DIR2)
CALL PLANEPAR3(DIR2,DDIR,DIR1,AP,SP,CP,DP)
CALL NORMALVEC(DIR1,NDIR1,AP,SP,CP)
DO 20 I=1,NO
  CALL ONEDIM(DIR(I,1),DIR(I,2),DIR(I,3),DUM)
  20 TET(I)=ANGLE(DIR1,DUM,NDIR1)
C
DO 30 J=1,(NO-1)
  DO 40 I=(J+1),NO
    IF (TET(I).LT.TET(J)) THEN
      TEMP1=TET(J)
      TET(J)=TET(I)
      TET(I)=TEMP1
      VICIOL(J)=VICIOL(I)
      VICIOL(I)=TEMP
    END IF
  40 CONTINUE
30 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE TRANSFER(VEC1,VEC2)
REAL VEC1(3),VEC2(3)
C
IMPLICIT NONE
INTEGER I
DO 10 I=1,3
  10 VEC2(I)=VEC1(I)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
LOGICAL FUNCTION COLINEAR(DUM1,DUM2,DUM3)
REAL DUM1(3),DUM2(3),DUM3(3)
C
IMPLICIT NONE
REAL NUM1,NUM2,NUM3,DEN1,DEN2,DEN3,RAT1,RAT2,RAT3,X(3)
INTEGER I
C
DO 10 I=1,3
  X(I)=0
  IF ((DUM1(I).EQ.DUM2(I)).AND.(DUM1(I).EQ.DUM3(I))) X(I)=1
10 CONTINUE
COLINEAR=.FALSE.
NUM1=DUM2(1)-DUM1(1)
NUM2=DUM2(2)-DUM1(2)
NUM3=DUM2(3)-DUM1(3)
DEN1=DUM3(1)-DUM1(1)
DEN2=DUM3(2)-DUM1(2)
DEN3=DUM3(3)-DUM1(3)
RAT1=NUM1/DEN1
RAT2=NUM2/DEN2
RAT3=NUM3/DEN3
IF ((X(1)+X(2)+X(3)).GT.1) THEN
  COLINEAR=.TRUE.
ELSE IF (X(1).EQ.1) THEN
  IF (ABS(RAT3-RAT2).LT.0.0001) COLINEAR=.TRUE.
ELSE IF (X(2).EQ.1) THEN
  IF (ABS(RAT1-RAT3).LT.0.0001) COLINEAR=.TRUE.
ELSE IF (X(3).EQ.1) THEN
  IF (ABS(RAT2-RAT1).LT.0.0001) COLINEAR=.TRUE.
ELSE IF ((ABS(RAT1-RAT2).LT.0.0001)
      .AND.(ABS(RAT1-RAT3).LT.0.0001)) THEN
  COLINEAR=.TRUE.
END IF
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE BASE3NODE(EDGE,FACE,SCF1,IN1,IN2,IN3)
INTEGER EDGE(50000,3),FACE(50000,4),SCF1,IN1,IN2,IN3
C
IMPLICIT NONE
INTEGER TRY,I,J
LOGICAL FOUND,NOUV
C
IN1=0
IN2=0
IN3=0
I=1
J=1
FOUND=((IN1.NE.0).AND.(IN2.NE.0).AND.(IN3.NE.0))
IN1=EDGE(FACE(SCF1,I),J)
DO WHILE (.NOT.(FOUND))
  IF (J.EQ.3) THEN
    J=0
    I=I+1
  END IF
  J=J+1
  TRY=EDGE(FACE(SCF1,I),J)
  NOUV=((TRY.NE.IN1).AND.(TRY.NE.IN2).AND.(TRY.NE.IN3))
  IF (NOUV) THEN
    IF (IN2.EQ.0) THEN
      IN2=TRY
    ELSE
      IN3=TRY
    END IF
  END IF

```



```

C
SUBROUTINE CHECKFACE(SCE1,SCE2,SCE3,IN1,IN2,IN3,SCF,SCPOL,
*      FACE,PID,N,LSTAK,SC,NFAC,VICFAC)
INTEGER SCE1,SCE2,SCE3,SCF,SCPOL,FACE(85000,4),PID(30000),N
INTEGER LSTAK(5000),SC,IN1,IN2,IN3,NFAC,VICFAC(3000)
C
IMPLICIT NONE
INTEGER I,J
IF (SCPOL.EQ.0) THEN
  SCF=SCF+1
  SCPOL=SCF
  NFAC=NFAC+1
  VICFAC(NFAC)=SCF
  CALL CREATEFACE(SCE1,SCE2,SCE3,SCF,FACE,IN1,IN2,IN3,PID)
  IF (N.EQ.1) THEN
    LSTAK(SC)=SCF
    SC=SC+1
  END IF
ELSE
  DO 10 I=1,SC
    IF (LSTAK(I).EQ.SCPOL) THEN
      DO 20 J=1,SC-1
        LSTAK(J)=LSTAK(J+1)
      SC=SC-1
    END IF
  10 CONTINUE
END IF
30 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE CHECKFACE2D(SCE1,SCE2,SCE3,IN1,IN2,IN3,SCF,SCPOL,
*      FACE,PID,N,LSTAK,SC)
INTEGER SCE1,SCE2,SCE3,SCF,SCPOL,FACE(85000,4),PID(30000),N
INTEGER LSTAK(5000),SC,IN1,IN2,IN3
C
IMPLICIT NONE
IF (SCPOL.EQ.0) THEN
  SCF=SCF+1
  SCPOL=SCF
  CALL CREATEFACE(SCE1,SCE2,SCE3,SCF,FACE,IN1,IN2,IN3,PID)
  IF (N.EQ.1) THEN
    LSTAK(SC)=SCF
    SC=SC+1
  END IF
END IF
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE MODIFYPOINTSUR(NPS,PID)
INTEGER NPS,PID(30000)
C
IMPLICIT NONE
INTEGER I
DO 10 I=1,NPS
  PID(I)=PID(I)-INT(PID(I)/10)*10
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE NOOVERLAP(EDGE,FACE,SCF1,DUMP,P,PYR,SCP,CANTUSE,
*      VICPYR,NPYR)
INTEGER EDGE(50000,3),FACE(85000,4),SCF1,PYR(40000,5),SCP
INTEGER VICPYR(400),NPYR
REAL P(30000,3),DUMP(3)
LOGICAL CANTUSE
C
IMPLICIT NONE
INTEGER I,K,IN1,IN2,IN3
INTEGER NODE(4),L
REAL DUMP1(3),DUMP2(3),DUMP3(3),DUMP4(3)
REAL AAP(4),BP(4),CP(4),DP(4),POINT(44,3)
REAL DIR1(3),CENT(3),POIN(1,3)
LOGICAL VAL(4),ABOVE
C
CANTUSE=.FALSE.
CALL BASE3NODE(EDGE,FACE,SCF1,IN1,IN2,IN3)
CALL ONEDIM(P(IN1,1),P(IN1,2),P(IN1,3),DUMP1)
CALL ONEDIM(P(IN2,1),P(IN2,2),P(IN2,3),DUMP2)
CALL ONEDIM(P(IN3,1),P(IN3,2),P(IN3,3),DUMP3)
CALL MIDPOI(DUMP1,DUMP3,CENT)
CALL MIDPOI(CENT,DUMP2,CENT)
CALL DIRECTION(CENT,DUMP,DIR1)
DO 50 I=0,10
  K=I+1
  POINT(K,1)=DUMP(1)+(0.002+I*0.0996)*DIR1(1)
  POINT(K,2)=DUMP(2)+(0.002+I*0.0996)*DIR1(2)
  50 POINT(K,3)=DUMP(3)+(0.002+I*0.0996)*DIR1(3)
  CALL MIDPOI(DUMP1,DUMP,CENT)
  CALL MIDPOI(CENT,DUMP3,CENT)
  CALL DIRECTION(CENT,DUMP3,DIR1)
DO 51 I=0,10
  K=I+11
  POINT(K,1)=DUMP3(1)+(0.002+I*0.0996)*DIR1(1)
  POINT(K,2)=DUMP3(2)+(0.002+I*0.0996)*DIR1(2)
  51 POINT(K,3)=DUMP3(3)+(0.002+I*0.0996)*DIR1(3)
  CALL MIDPOI(DUMP1,DUMP,CENT)
  CALL MIDPOI(CENT,DUMP3,CENT)
  CALL DIRECTION(CENT,DUMP2,DIR1)
DO 52 I=0,10
  K=I+21
  POINT(K,1)=DUMP2(1)+(0.002+I*0.0996)*DIR1(1)
  POINT(K,2)=DUMP2(2)+(0.002+I*0.0996)*DIR1(2)
  52 POINT(K,3)=DUMP2(3)+(0.002+I*0.0996)*DIR1(3)
  CALL MIDPOI(DUMP3,DUMP,CENT)
  CALL MIDPOI(CENT,DUMP3,CENT)
  CALL DIRECTION(CENT,DUMP1,DIR1)
DO 53 I=0,10
  K=I+31
  POINT(K,1)=DUMP1(1)+(0.002+I*0.0996)*DIR1(1)

```



```

POINT(K,2)=DUMP1(2)+(0.002+1*0.0996)*DIR1(2)
53 POINT(K,3)=DUMP1(3)+(0.002+1*0.0996)*DIR1(3)
DO 45 K=1,NPYR
CALL PYRANODE(EDGE,FACE,PYR,VICPYR(K),NODE)
CALL ONEDIM(P(NODE(1),1),P(NODE(1),2),P(NODE(1),3),DUMP1)
CALL ONEDIM(P(NODE(2),1),P(NODE(2),2),P(NODE(2),3),DUMP2)
CALL ONEDIM(P(NODE(3),1),P(NODE(3),2),P(NODE(3),3),DUMP3)
CALL ONEDIM(P(NODE(4),1),P(NODE(4),2),P(NODE(4),3),DUMP4)
CALL PARPLANES(DUMP1,DUMP2,DUMP3,DUMP4,AAP,BP,CP,DP)
DO 10 I=1,44
POIN(1,1)=POINT(1,1)
POIN(1,2)=POINT(1,2)
POIN(1,3)=POINT(1,3)
DO 60 L=1,4
60 VAL(L)=ABOVE(POIN,AAP(L),BP(L),CP(L),DP(L))
IF (VAL(1).AND.VAL(2).AND.VAL(3).AND.VAL(4)) CANTUSE=.TRUE.
IF (CANTUSE) GOTO 70
10 CONTINUE
45 CONTINUE
70 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
LOGICAL FUNCTION FACEOVERLAP(DUMP1,DUMP2,DUMP3,DUMP4)
REAL DUMP1(3),DUMP2(3),DUMP3(3),DUMP4(3)
C
IMPLICIT NONE
REAL VAL1,VAL2,MAT(3,3),DUM1(3),DUM2(3),DUM3(3),DUM4(3)
INTEGER I
LOGICAL OVER,UNDER
C
FACEOVERLAP=.FALSE.
DO 10 I=1,3
DUM1(I)=DUMP1(I)+0.0086474
DUM2(I)=DUMP2(I)+0.0086474
DUM3(I)=DUMP3(I)+0.0086474
10 DUM4(I)=DUMP4(I)+0.0086474
CALL MAKEMAT3(DUM1,DUM2,DUM3,MAT)
CALL DETERMINANT(MAT,3,3,VAL1)
CALL MAKEMAT3(DUM1,DUM2,DUM4,MAT)
CALL DETERMINANT(MAT,3,3,VAL2)
c i commented this and it worked!
4 IF (VAL2.GT.0) FACEOVERLAP=.TRUE.
OVER=(VAL1.GT.0).AND.(VAL2.GE.0)
UNDER=(VAL1.LT.0).AND.(VAL2.LE.0)
IF (OVER.OR.UNDER) FACEOVERLAP=.TRUE.
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE EDGEINCOMMON(VICFAC,NFAC,NF,AF,E,FACE)
INTEGER VICFAC(2000),NFAC,NF,AF(300),E,FACE(88000,4)
C
IMPLICIT NONE
INTEGER I,E1,E2,E3
LOGICAL SAME
NF=0
C DO 10 I=1,SCF
DO 10 I=1,NFAC
E1=FACE(VICFAC(I),1)
E2=FACE(VICFAC(I),2)
E3=FACE(VICFAC(I),3)
SAME=(E.EQ.E1).OR.(E.EQ.E2).OR.(E.EQ.E3)
IF ( SAME) THEN
NF=NF+1
AF(NF)=VICFAC(I)
END IF
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE EDGEINCOMMON2D(FACE,SCF,NF,AF,E)
INTEGER FACE(88000,4),SCF,NF,AF(300),E
C
IMPLICIT NONE
INTEGER I,E1,E2,E3
LOGICAL SAME
NF=0
DO 10 I=1,SCF
E1=FACE(I,1)
E2=FACE(I,2)
E3=FACE(I,3)
SAME=(E.EQ.E1).OR.(E.EQ.E2).OR.(E.EQ.E3)
IF ( SAME) THEN
NF=NF+1
AF(NF)=I
END IF
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
LOGICAL FUNCTION EQUALCENTER(CENT,CENT1)
REAL CENT(3),CENT1(3)
C
IMPLICIT NONE
REAL VAL(3)
INTEGER I
EQUALCENTER=.FALSE.
DO 10 I=1,3
10 VAL(I)=ABS(CENT(I)-CENT1(I))
IF ((VAL(1).LT.0.001).AND.(VAL(3).LT.0.001))
EQUALCENTER=.TRUE.
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE GETDUMPS(P,IN1,IN2,IN3,DUMP1,DUMP2,DUMP3)
INTEGER IN1,IN2,IN3
REAL P(30000,3),DUMP1(3),DUMP2(3),DUMP3(3)
C

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```

IMPLICIT NONE
CALL ONEDIM(P(IN1,1),P(IN1,2),P(IN1,3),DUMP1)
CALL ONEDIM(P(IN2,1),P(IN2,2),P(IN2,3),DUMP2)
CALL ONEDIM(P(IN3,1),P(IN3,2),P(IN3,3),DUMP3)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
LOGICAL FUNCTION EQUALFACE(F,FACE,EDGE,IN11,IN22,IN33)
INTEGER F,FACE(80000,4),EDGE(80000,3),IN11,IN22,IN33
C
IMPLICIT NONE
LOGICAL VAL1,VAL2,VAL3
INTEGER IN1,IN2,IN3
EQUALFACE=.FALSE.
CALL BASE3NODE(EDGE,FACE,F,IN1,IN2,IN3)
VAL1=(IN11.EQ.IN1).OR.(IN11.EQ.IN2).OR.(IN11.EQ.IN3)
VAL2=(IN22.EQ.IN1).OR.(IN22.EQ.IN2).OR.(IN22.EQ.IN3)
VAL3=(IN33.EQ.IN1).OR.(IN33.EQ.IN2).OR.(IN33.EQ.IN3)
IF ((VAL1).AND.(VAL2).AND.(VAL3)) EQUALFACE=.TRUE.
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL FUNCTION VOLUME(COOR)
REAL COOR(4,3)
C
IMPLICIT NONE
REAL COOR1(4),COOR2(4),COOR3(4),COOR4(4),VOL,MAT4(4,4)
C
CALL ONEDIM4(COOR(1,1),COOR(1,2),COOR(1,3),1.0,COOR1)
CALL ONEDIM4(COOR(2,1),COOR(2,2),COOR(2,3),1.0,COOR2)
CALL ONEDIM4(COOR(3,1),COOR(3,2),COOR(3,3),1.0,COOR3)
CALL ONEDIM4(COOR(4,1),COOR(4,2),COOR(4,3),1.0,COOR4)
CALL MAKEMAT4(COOR1,COOR2,COOR3,COOR4,MAT4)
CALL DETERMINANT(MAT4,4,4,VOL)
VOLUME=ABS(VOL)/6
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE DUMP4PROJ(DUM,AP,BP,CP,DP,DUMP6)
REAL DUM(3),AP,BP,CP,DP,DUMP6(3)
C
IMPLICIT NONE
REAL DEN,NUM,T
C
DEN=AP**2+BP**2+CP**2
NUM=DUM(1)*AP+DUM(2)*BP+DUM(3)*CP+DP
T=-NUM/DEN
DUMP6(1)=DUM(1)+T*AP
DUMP6(2)=DUM(2)+T*BP
DUMP6(3)=DUM(3)+T*CP
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
LOGICAL FUNCTION PYRAMOVERLAP(EDGE,FACE,PYR,AP,BP,CP,DP,SCF1,
PYRCOM,IND1,P)
INTEGER EDGE(80000,3),FACE(80000,4),IND1,SCF1
INTEGER PYRCOM,PYR(40000,5)
REAL P(30000,3),AP,BP,CP,DP
C
IMPLICIT NONE
INTEGER NODE(4),I,IN1,IN2,IN3,IND3
REAL VAL1,VAL2
LOGICAL OVER,UNDER
C
PYRAMOVERLAP=.FALSE.
IF (PYRCOM.EQ.0) GOTO 20
CALL BASE3NODE(EDGE,FACE,SCF1,IN1,IN2,IN3)
CALL PYR4NODE(EDGE,FACE,PYR,PYRCOM,NODE)
DO 10 I=1,4
10 IF ((NODE(I).NE.IN1).AND.(NODE(I).NE.IN2).AND.(NODE(I).NE.IN3))
IND3=NODE(I)
VAL1=AP*P(IND1,1)+BP*P(IND1,2)+CP*P(IND1,3)+DP
VAL2=AP*P(IND3,1)+BP*P(IND3,2)+CP*P(IND3,3)+DP
OVER=(VAL1.GE.0).AND.(VAL2.GT.0)
UNDER=(VAL1.LE.0).AND.(VAL2.LT.0)
IF (OVER.OR.UNDER) PYRAMOVERLAP=.TRUE.
20 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE PYR4NODE(EDGE,FACE,PYR,PYRCOM,NODE)
INTEGER EDGE(80000,3),FACE(80000,4),PYRCOM,NODE(4),PYR(40000,5)
C
IMPLICIT NONE
INTEGER F1,F2,IN1,IN2,IN3,IN4,IN5,IN6,IN7
F1=PYR(PYRCOM,1)
F2=PYR(PYRCOM,2)
CALL BASE3NODE(EDGE,FACE,F1,IN1,IN2,IN3)
CALL BASE3NODE(EDGE,FACE,F2,IN4,IN5,IN6)
IF ((IN4.NE.IN1).AND.(IN4.NE.IN2).AND.(IN4.NE.IN3)) IN7=IN4
IF ((IN5.NE.IN1).AND.(IN5.NE.IN2).AND.(IN5.NE.IN3)) IN7=IN5
IF ((IN6.NE.IN1).AND.(IN6.NE.IN2).AND.(IN6.NE.IN3)) IN7=IN6
NODE(1)=IN1
NODE(2)=IN2
NODE(3)=IN3
NODE(4)=IN7
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE FACEINCOMMON(PYR,SCP,FAS,SCF1,NP,AP)
INTEGER PYR(40000,5),SCP,FAS,NP,AP(18),SCF1
IMPLICIT NONE
INTEGER I,F1,F2,F3,F4
NP=0
C
DO 10 I=1,SCP
F1=PYR(I,1)

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F2=PYR(I,2)
F3=PYR(I,3)
F4=PYR(I,4)
IF ((FAS.EQ.F1).OR.(FAS.EQ.F2).OR.(FAS.EQ.F3).OR.(FAS.EQ.F4)) THEN
IF (FAS.NE.SCF1) THEN
NP=NP+1
AP(NP)=I
END IF
END IF
10 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE MESHREADER(NP,NPS,SCP,SCF,SCE,P,PID,EDGE,FACE,PYR,
*
EXTRAPYR,FIL)
INTEGER NP,SCP,SCF,SCE,PID(30000),EDGE(60000,3),FACE(66000,4),NPS
INTEGER PYR(40000,6),EXTRAPYR(40000,6),FIL
REAL P(30000,3)
C
IMPLICIT NONE
INTEGER OFFREC,REK,I
READ(FIL,REC=1) NPS
READ(FIL,REC=2) NP
READ(FIL,REC=3) SCP
READ(FIL,REC=4) SCF
READ(FIL,REC=5) SCE
OFFREC=5
DO 60 I=1,NPS
REK=OFFREC+I
60 READ(FIL,REC=REK) P(I,1),P(I,2),P(I,3),PID(I)
OFFREC=REK
DO 70 I=NPS+1,NP
REK=OFFREC+I-NPS
70 READ(FIL,REC=REK) P(I,1),P(I,2),P(I,3),PID(I)
OFFREC=REK
DO 20 I=1,SCR
REK=OFFREC+I
20 READ(FIL,REC=REK) EDGE(I,1),EDGE(I,2),EDGE(I,3)
OFFREC=REK
DO 30 I=1,SCF
REK=OFFREC+I
30 READ(FIL,REC=REK) FACE(I,1),FACE(I,2),FACE(I,3),FACE(I,4)
OFFREC=REK
DO 40 I=1,SCP
REK=OFFREC+I
40 READ(FIL,REC=REK) PYR(I,1),PYR(I,2),PYR(I,3),PYR(I,4),
*
PYR(I,5),EXTRAPYR(I,1),EXTRAPYR(I,2),EXTRAPYR(I,3),
*
EXTRAPYR(I,4),EXTRAPYR(I,5),EXTRAPYR(I,6)
C
CLOSE(FIL)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

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MESHCOMPSUPSG.F

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SUBROUTINE OPERATORGRADGRAD(GRADGRAD,ALF)
REAL GRADGRAD(10,10,4,4)
INTEGER ALF(10,2,5)
C
IMPLICIT NONE
INTEGER ROW,COL,ALFR(2,5),ALFC(2,5),ALPCD(2,5),SIMP1,SIMP2
INTEGER I,J,SIMP,DALFR(4,2,5),DALFC(4,2,5),ALFRD(2,5),ALFAD(4,5)
REAL INTEGRATIONVOL
C
DO 10 ROW=1,10
DO 20 I=1,2
DO 30 J=1,5
30 ALFR(I,J)=ALF(ROW,I,J)
20 CONTINUE
DO 60 SIMP=1,4
CALL DERIVATE(ALFR,ALFRD,SIMP,I)
DO 70 I=1,2
DO 80 J=1,5
80 DALFR(SIMP,I,J)=ALFRD(I,J)
70 CONTINUE
60 CONTINUE
DO 90 COL=1,10
DO 100 I=1,2
DO 110 J=1,5
110 ALFC(I,J)=ALF(COL,I,J)
100 CONTINUE
DO 120 SIMP=1,4
CALL DERIVATE(ALFC,ALPCD,SIMP,I)
DO 130 I=1,2
DO 140 J=1,5
140 DALFC(SIMP,I,J)=ALPCD(I,J)
130 CONTINUE
120 CONTINUE
DO 150 SIMP1=1,4
DO 160 I=1,2
DO 170 J=1,5
170 ALFRD(I,J)=DALFR(SIMP1,I,J)
160 CONTINUE
DO 180 SIMP2=1,4
DO 190 I=1,2
DO 200 J=1,5
200 ALPCD(I,J)=DALFC(SIMP2,I,J)
190 CONTINUE
CALL MULTIPLYALFA(ALFRD,ALPCD,ALFAD)
GRADGRAD(ROW,COL,SIMP1,SIMP2)=INTEGRATIONVOL(ALFAD)
180 CONTINUE
160 CONTINUE
90 CONTINUE
10 CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

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```

      DO 50 I=1,2
      DO 50 J=1,5
60      ALFC(I,J)=ALF(COL,I,J)
60      CONTINUE
      DO 70 SIMPX=1,4
      CALL DERIVATE(ALFC,ALPCD,SIMPX,1)
      CALL MULTIPLYALFA(ALFR,ALPCD,ALFAD)
      FIELDN(ROW,COL,1,SIMPX)=INTEGRATIONSURF(ALFAD,SIMPX)
70      CONTINUE
40      CONTINUE
10      CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE OPERATORALFNORMoriginal(NORM,ALF)
REAL NORM(10,4,1,1)
INTEGER ALF(10,2,5)
C
IMPLICIT NONE
INTEGER ROW,I,J,ALPC(2,5),ALFR(2,5),ALFAD(4,5)
REAL INTEGRATIONSURF
C
DO 10 I=1,2
DO 20 J=1,5
20      ALPC(I,J)=0
10      CONTINUE
ALPC(1,5)=1
DO 30 ROW=1,10
DO 40 I=1,2
DO 50 J=1,5
50      ALFR(I,J)=ALF(ROW,I,J)
40      CONTINUE
CALL MULTIPLYALFA(ALFR,ALPC,ALFAD)
DO 60 I=1,4
60      NORM(ROW,I,1,1)=INTEGRATIONSURF(ALFAD,I)
30      CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE OPERATORALFNORM(NORM,ALF)
REAL NORM(10,10,1,4)
INTEGER ALF(10,2,5)
C
IMPLICIT NONE
INTEGER ROW,I,J,ALPC(2,5),ALFR(2,5),ALFAD(4,5),COL
REAL INTEGRATIONSURF
C
DO 5 ROW=1,10
DO 10 I=1,2
DO 20 J=1,5
20      ALPC(I,J)=ALF(ROW,I,J)
10      CONTINUE
DO 30 COL=1,10
DO 40 I=1,2
DO 50 J=1,5
50      ALFR(I,J)=ALF(COL,I,J)
40      CONTINUE
CALL MULTIPLYALFA(ALFR,ALPC,ALFAD)
DO 60 I=1,4
60      NORM(ROW,COL,1,1)=INTEGRATIONSURF(ALFAD,I)
30      CONTINUE
5      CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL FUNCTION INTEGRATIONVOL(ALFAD)
INTEGER ALFAD(4,5)
C
IMPLICIT NONE
REAL SUM,INTEGRATEVOL
INTEGER I
SUM=0.0
DO 10 I=1,4
SUM=SUM+INTEGRATEVOL(ALFAD(I,1),ALFAD(I,2),
      ALFAD(I,3),ALFAD(I,4))*ALFAD(I,5)
10      CONTINUE
INTEGRATIONVOL=SUM
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL FUNCTION INTEGRATEVOL(A1,A2,A3,A4)
INTEGER A1,A2,A3,A4
C
IMPLICIT NONE
INTEGER SUM,FAC
SUM=A1++AA4+3
INTEGRATEVOL=5.0*FAC(A1)*FAC(A2)*FAC(A3)*FAC(A4)/FAC(SUM)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
INTEGER FUNCTION FAC(VAL)
INTEGER VAL
C
IMPLICIT NONE
INTEGER SUM,I
SUM=1
DO 10 I=VAL,1,-1
10      SUM=SUM*I
FAC=SUM
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE COEFFICIENT(DUM1,DUM2,DUM3,AD,BD,CD,DD,DEN)
REAL DUM1(3),DUM2(3),DUM3(3),AD,BD,CD,DD,DEN
C

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IMPLICIT NONE
REAL MAT3(3,3),DM1(3),DM2(3),DM3(3)
INTEGER I
C
DO 10 I=1,3
  DM1(I)=DUM1(I)
  DM2(I)=DUM2(I)
  DM3(I)=DUM3(I)
CALL MAKEMAT3(DM1,DM2,DM3,MAT3)
CALL DETERMINANT(MAT3,3,3,AD)
AD=AD/DEN
DM1(1)=1.0
DM2(1)=1.0
DM3(1)=1.0
CALL MAKEMAT3(DM1,DM2,DM3,MAT3)
CALL DETERMINANT(MAT3,3,3,BD)
BD=BD/DEN
DM1(2)=DUM1(1)
DM2(2)=DUM2(1)
DM3(2)=DUM3(1)
CALL MAKEMAT3(DM1,DM2,DM3,MAT3)
CALL DETERMINANT(MAT3,3,3,CD)
CD=CD/DEN
DM1(3)=DUM1(2)
DM2(3)=DUM2(2)
DM3(3)=DUM3(2)
CALL MAKEMAT3(DM1,DM2,DM3,MAT3)
CALL DETERMINANT(MAT3,3,3,DD)
DD=DD/DEN
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL FUNCTION INTEGRATESURF(ALPAD,SIMP)
INTEGER ALPAD(4,5),SIMP
C
IMPLICIT NONE
REAL SUM,INTEGRATESURF
INTEGER I,ALFAS(4,4)
SUM=0.0
CALL ALFSURF(ALPAD,4,ALFAS,SIMP)
DO 10 I=1,4
  SUM=SUM+INTEGRATESURF( ALFAS(I,1),ALFAS(I,2),ALFAS(I,3) )*ALFAS(I,4)
INTEGRATESURF=SUM
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL FUNCTION INTEGRATESURF(A1,A2,A3)
INTEGER A1,A2,A3
C
IMPLICIT NONE
INTEGER SUM,FAC
SUM=A1+A2+A3+3
INTEGRATESURF=2.0*FAC(A1)*FAC(A2)*FAC(A3)/FAC(SUM)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE ALFSURF(ALPAD,N,ALFAS,SIMP)
INTEGER ALPAD(N,5),SIMP,ALFAS(4,4),N
C
IMPLICIT NONE
INTEGER I,J
DO 10 I=1,N
  ALFAS(I,4)=ALPAD(I,5)
  10 IF ((ALPAD(I,SIMP).NE.0).OR.(ALPAD(I,5).EQ.0)) ALFAS(I,4)=0
DO 20 I=1,N
  DO 30 J=1,N-1
    IF (J.LT.SIMP) THEN
      ALFAS(I,J)=ALPAD(I,J)
    ELSE
      ALFAS(I,J)=ALPAD(I,J+1)
    END IF
  30 CONTINUE
  20 CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE TRANSPOSE(MAT1,MAT2,L,S)
REAL MAT1(L,L,1,S),MAT2(L,L,1,S)
INTEGER L,S
C
IMPLICIT NONE
INTEGER I,J,K
DO 10 I=1,L
  DO 20 J=1,L
    DO 30 K=1,S
      30 MAT2(J,I,1,K)=MAT1(I,J,1,K)
  20 CONTINUE
  10 CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE OPERATORLAPLACIAN(LAPLACE4,ALF)
REAL LAPLACE4(10,10,1,4)
INTEGER ALF(10,2,5)
C
IMPLICIT NONE
INTEGER ROW,COL,SIMP,ALFR(2,5),ALFC(2,5),ALFCD(2,5)
INTEGER ALPAD(4,5),I,J
REAL INTEGRATIONVOL
C
DO 10 ROW=1,10
  DO 20 I=1,2
    DO 30 J=1,5
      30 ALFR(I,J)=ALF(ROW,I,J)
  20 CONTINUE
  10 CONTINUE

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DO 40 COL=1,10
DO 50 I=1,3
DO 60 J=1,8
50 ALFC(I,J)=ALF(COL,I,J)
50 CONTINUE
DO 70 SIMP=1,4
CALL DERIVATE(ALFC,ALFCD,SIMP,2)
CALL MULTIPLYALFA(ALFR,ALFCD,ALFAD)
LAPLACE(RCOW,COL,I,SIMP)=INTEGRATIONVOL(ALFAD)
70 CONTINUE
40 CONTINUE
10 CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE BTLAPLACTAN(TANLAPLAC,ALF)
REAL TANLAPLAC(10,10,4,4)
INTEGER ALF(10,3,8)
C
IMPLICIT NONE
INTEGER ROW,COL,SIMP,ALFR(3,8),ALFC(3,8),ALFCD(3,8)
INTEGER ALFAD(4,8),I,J,SIMPX
REAL INTEGRATIONSURF
C
DO 10 ROW=1,10
DO 20 I=1,3
DO 30 J=1,8
30 ALFR(I,J)=ALF(ROW,I,J)
20 CONTINUE
DO 40 COL=1,10
DO 50 I=1,3
DO 60 J=1,8
60 ALFC(I,J)=ALF(COL,I,J)
50 CONTINUE
DO 70 SIMP=1,4
CALL DERIVATE(ALFC,ALFCD,SIMP,2)
CALL MULTIPLYALFA(ALFR,ALFCD,ALFAD)
DO 80 SIMPX=1,4
80 TANLAPLAC(ROW,COL,SIMP,SIMPX)=INTEGRATIONSURF(ALFAD,SIMPX)
70 CONTINUE
40 CONTINUE
10 CONTINUE
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE ALLCOEFFICIENT(COOR,A,B,C,D)
REAL COOR(4,3),A(4),B(4),C(4),D(4)
C
IMPLICIT NONE
REAL DUM1(3),DUM2(3),DUM3(3),MAT4(4,4),AA,BB,CC,DD,DEN
REAL DUM14(4),DUM24(4),DUM34(4),DUM44(4)
C
CALL ONEDIM4(1.0,COOR(1,1),COOR(1,2),COOR(1,3),DUM14)
CALL ONEDIM4(1.0,COOR(2,1),COOR(2,2),COOR(2,3),DUM24)
CALL ONEDIM4(1.0,COOR(3,1),COOR(3,2),COOR(3,3),DUM34)
CALL ONEDIM4(1.0,COOR(4,1),COOR(4,2),COOR(4,3),DUM44)
CALL MAKEMAT4(DUM14,DUM24,DUM34,DUM44,MAT4)
CALL DETERMINANT(MAT4,4,4,DEN)
CALL ONEDIM(COOR(2,1),COOR(2,2),COOR(2,3),DUM1)
CALL ONEDIM(COOR(3,1),COOR(3,2),COOR(3,3),DUM2)
CALL ONEDIM(COOR(4,1),COOR(4,2),COOR(4,3),DUM3)
CALL COEFFICIENT(DUM1,DUM2,DUM3,AA,BB,CC,DD,DEN)
A(1)=AA
B(1)=BB
C(1)=CC
D(1)=DD
CALL ONEDIM(COOR(1,1),COOR(1,2),COOR(1,3),DUM1)
CALL COEFFICIENT(DUM1,DUM2,DUM3,AA,BB,CC,DD,DEN)
A(2)=AA
B(2)=BB
C(2)=CC
D(2)=DD
CALL ONEDIM(COOR(2,1),COOR(2,2),COOR(2,3),DUM2)
CALL COEFFICIENT(DUM1,DUM2,DUM3,AA,BB,CC,DD,DEN)
A(3)=AA
B(3)=BB
C(3)=CC
D(3)=DD
CALL ONEDIM(COOR(3,1),COOR(3,2),COOR(3,3),DUM3)
CALL COEFFICIENT(DUM1,DUM2,DUM3,AA,BB,CC,DD,DEN)
A(4)=AA
B(4)=BB
C(4)=CC
D(4)=DD
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
INTEGER FUNCTION NORMALSIMPTOSURFACE(ND1,ND2,ND3,VERT)
INTEGER ND1,ND2,ND3,VERT(4)
C
IMPLICIT NONE
INTEGER I,V(4)
NORMALSIMPTOSURFACE=0
DO 10 I=1,4
V(I)=0
10 IF ((VERT(I).EQ.ND1).OR.(VERT(I).EQ.ND2).OR.(VERT(I).EQ.ND3)) V(I)=1
DO 20 I=1,4
20 IF (V(I).EQ.0) NORMALSIMPTOSURFACE=1
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE FANCTIONS(ALF,EPSILON,BET)
INTEGER ALF(10,3,8)
COMPLEX EPSILON(10)

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REAL BET
C
IMPLICIT NONE
INTEGER I,N,IND
REAL DX,DY,FREQ,MU,PI,EP
C
OPEN(UNIT=3,FILE='functions.dat',STATUS='UNKNOWN',ACCESS=
  * 'SEQUENTIAL',FORM='FORMATTED')
PI=ACOS(-1.0)
EP=8.88E-12
READ(3,100)
DO 10 I=1,10
  READ(3,*) ALF(1,1,1),ALF(1,1,2),ALF(1,1,3),ALF(1,1,4),ALF(1,1,5)
  READ(3,*) ALF(1,2,1),ALF(1,2,2),ALF(1,2,3),ALF(1,2,4),ALF(1,2,5)
10 CONTINUE
READ(3,*)
READ(3,*) FREQ
READ(3,*) MU
READ(3,*)
READ(3,*) N
BET=2*PI*FREQ*SQRT(MU*EP)
DO 20 I=1,N
  READ(3,*) DX,DY,IND
  EPSILON(IND)=CMPLX(DX,DY)*BET**2
20 CONTINUE
CLOSE(3)
100 FORMAT(8(/))
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE READBASINFO(NP,SCE,SCF,SCP,FIL)
INTEGER NP,SCE,SCF,SCP,FIL
C
IMPLICIT NONE
OPEN(UNIT=2,FILE='unmesh.dat',STATUS='UNKNOWN',ACCESS='DIRECT',
  * FORM='UNFORMATTED',RECL=11,RECORDTYPE='FIXED')
READ(2,REC=2) NP
READ(2,REC=3) SCP
READ(2,REC=4) SCF
READ(2,REC=5) SCE
FIL=2
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE READINFO(I,NP,SCE,SCF,EDGE,POINT,MEDP,
  MEDF,MEDE,INDP,COORD,INDEX,FIL)
REAL COORD(4,3)
INTEGER I,NP,SCE,SCF,EDGE(4,3),POINT(4,3,2),MEDP,MEDF(4)
INTEGER MEDE(4,3),INDP(6),INDEX(4),FIL
C
IMPLICIT NONE
INTEGER OFFSET,IND,J,K,FACE(4),IND1,IND2,ND
C
IND=0
OFFSET=NP+SCE+SCF+1+5
READ(FIL,REC=OFFSET) FACE(1),FACE(2),FACE(3),FACE(4),MEDP,INDP(1),
  * INDP(2),INDP(3),INDP(4),INDP(5),INDP(6)
DO 10 J=1,4
  OFFSET=NP+SCE+FACE(J)+5
  READ(FIL,REC=OFFSET) EDGE(J,1),EDGE(J,2),EDGE(J,3),MEDF(J)
DO 20 K=1,3
  OFFSET=NP+EDGE(J,K)+5
  READ(FIL,REC=OFFSET) POINT(J,K,1),POINT(J,K,2),MEDE(J,K)
20 CONTINUE
10 CONTINUE
ND=0
DO 40 J=1,4
  OFFSET=NP+5+INDP(J)
  READ(FIL,REC=OFFSET) IND1,IND2
DO 50 K=1,ND
  IF (INDEX(K).EQ.IND1) GOTO 55
  ND=ND+1
  INDEX(ND)=IND1
55 DO 60 K=1,ND
  IF (INDEX(K).EQ.IND2) GOTO 40
  ND=ND+1
  INDEX(ND)=IND2
IF (ND.EQ.4) GOTO 45
40 CONTINUE
45 DO 70 J=1,4
DO 80 K=1,3
  IND1=POINT(J,K,1)
  IND2=POINT(J,K,2)
  IF ( ((INDEX(1).EQ.IND1).AND.(INDEX(2).EQ.IND2)).OR.
    * ((INDEX(1).EQ.IND2).AND.(INDEX(2).EQ.IND1)) ) INDP(1)=EDGE(J,K)
  IF ( ((INDEX(1).EQ.IND1).AND.(INDEX(3).EQ.IND2)).OR.
    * ((INDEX(1).EQ.IND2).AND.(INDEX(3).EQ.IND1)) ) INDP(2)=EDGE(J,K)
  IF ( ((INDEX(1).EQ.IND1).AND.(INDEX(4).EQ.IND2)).OR.
    * ((INDEX(1).EQ.IND2).AND.(INDEX(4).EQ.IND1)) ) INDP(3)=EDGE(J,K)
  IF ( ((INDEX(2).EQ.IND1).AND.(INDEX(3).EQ.IND2)).OR.
    * ((INDEX(2).EQ.IND2).AND.(INDEX(3).EQ.IND1)) ) INDP(4)=EDGE(J,K)
  IF ( ((INDEX(3).EQ.IND1).AND.(INDEX(4).EQ.IND2)).OR.
    * ((INDEX(3).EQ.IND2).AND.(INDEX(4).EQ.IND1)) ) INDP(5)=EDGE(J,K)
  IF ( ((INDEX(4).EQ.IND1).AND.(INDEX(2).EQ.IND2)).OR.
    * ((INDEX(4).EQ.IND2).AND.(INDEX(2).EQ.IND1)) ) INDP(6)=EDGE(J,K)
80 CONTINUE
70 CONTINUE
DO 30 J=1,4
  OFFSET=INDEX(J)+1
30 READ(FIL,REC=OFFSET) COORD(J,1),COORD(J,2),COORD(J,3)
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
SUBROUTINE OPERATORS(ALF,COMPUTE)
INTEGER ALF(10,2,5),COMPUTE
C
IMPLICIT NONE
REAL LAPLACIAN(10,10,1,4),TRANS LAPLACIAN(10,10,1,4),FIELDN(10,10,1,4)
REAL ALFF(10,10,1,1),GRADGRAD(10,10,4,4),NORM(10,10,1,4)
C REAL ALFF(10,10,1,1),GRADGRAD(10,10,4,4),NORM(10,4,1,1)

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REAL TRANSFIELDN(10,10,1,4),TANLAPLAC(10,10,4,4),SURFALFF(10,10,1,4)
COMMON/OPERATEUR/ LAPLACIAN,TRANSLAPLACIAN,GRADGRAD,ALFF
      ,NORM,FIELDN,TRANSFIELDN,TANLAPLAC,SURFALFF
C
OPEN(UNIT=4,FILE='operateur.dat',STATUS='UNKNOWN',ACCESS=
      'SEQUENTIAL',FORM='FORMATTED')
IF (COMPUTE.EQ.1) THEN
CALL OPERATORLAPLACIAN(LAPLACIAN,ALFF)
CALL WRITER(LAPLACIAN,10,10,1,4,'LAPLACIAN ',4)
CALL TRANSPOSE(LAPLACIAN,TRANSLAPLACIAN,10,4)
CALL WRITER(TRANSLAPLACIAN,10,10,1,4,'TRANSLAPLN',4)
CALL OPERATORGRADGRAD(GRADGRAD,ALFF)
CALL WRITER(GRADGRAD,10,10,4,4,'GRADIENT ',4)
CALL ALFFIELD(ALFF,ALFF)
CALL WRITER(ALFF,10,10,1,1,'ALFFFIELD ',4)
CALL OPERATORALFNORM(NORM,ALFF)
CALL WRITER(NORM,10,10,1,4,'ALFFconst',4)
C CALL WRITER(NORM,10,4,1,1,'ALFFconst',4)
CALL OPERATORFIELDN(FIELDN,ALFF)
CALL WRITER(FIELDN,10,10,1,4,'ALFFdFI/dm',4)
CALL TRANSPOSE(FIELDN,TRANSFIELDN,10,4)
CALL WRITER(TRANSFIELDN,10,10,1,4,'FidALFF/dm',4)
CALL STLAPLACTAN(TANLAPLAC,ALFF)
CALL WRITER(TANLAPLAC,10,10,4,4,'TANLAPLAC',4)
CALL ALFFPOVSURF(SURFALFF,ALFF)
CALL WRITER(SURFALFF,10,10,1,4,'ALFFPOVSURF',4)
ELSE
CALL READER(LAPLACIAN,10,10,1,4,4)
CALL READER(TRANSLAPLACIAN,10,10,1,4,4)
CALL READER(GRADGRAD,10,10,4,4,4)
CALL READER(ALFF,10,10,1,1,4)
CALL READER(NORM,10,10,1,4,4)
CALL READER(FIELDN,10,10,1,4,4)
END IF
CLOSE(4)
C
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

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SOURCESUBS.F

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LIST OF SUBROUTINES USED COMMONLY BY SEVERAL PROGRAMS
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
SUBROUTINE THAT CALCULATES THE 3 CARTESIAN COMPONENTS OF THE MAGNETIC FIELD
SUBROUTINE GETREALH(P,F,NP,NS,K)
REAL K,P(NP,3)
INTEGER NP,NS
COMPLEX F(NP,3)
C
IMPLICIT NONE
INTEGER I,AX
COMPLEX DIRICHLETVALX,DIRICHLETVALY,DIRICHLETVALZ
REAL FREQ,PI
C
PI=ACOS(-1.0)
FREQ=K/(2.0*PI*SQRT(8.85E-12*1.35664E-6))
DO 5 AX=1,3
DO 10 I=1,NJ
IF (AX.EQ.1) F(I,AX)=DIRICHLETVALX(P(I,1),P(I,2),P(I,3),FREQ)
IF (AX.EQ.2) F(I,AX)=DIRICHLETVALY(P(I,1),P(I,2),P(I,3),FREQ)
IF (AX.EQ.3) F(I,AX)=DIRICHLETVALZ(P(I,1),P(I,2),P(I,3),FREQ)
10 CONTINUE
5 CONTINUE
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
SUBROUTINE THAT CALCULATES THE X-COMPONENT OF THE MAGNETIC FIELD
COMPLEX FUNCTION DIRICHLETVALX(X,Y,Z,FREQ)
REAL X,Y,Z,FREQ
C
IMPLICIT NONE
REAL MEASUREERROR,PHASE,ANG,MAG,FAC
COMPLEX FILCONT,VAL
VAL=FILCONT(X,Y,Z,0.0,0.0,0.7,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.0,0.0,-0.7,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.2,0.2,0.8,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.2,0.2,-0.8,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.1,0.0,0.7,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.1,0.0,-0.7,1,FREQ,1,3)
VAL=VAL+FILCONT(X,Y,Z,-0.2,0.0,0.7,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,-0.2,0.0,-0.7,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.0,-0.2,0.8,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.0,-0.2,-0.8,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.2,-0.18,0.4,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.2,-0.18,-0.4,1,FREQ,1,3)
VAL=VAL+FILCONT(X,Y,Z,0.2,0.0,0.6,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.2,0.0,-0.6,1,FREQ,-1,3)+
      * FILCONT(X,Y,Z,0.0,0.2,0.7,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.0,0.2,-0.7,1,FREQ,-1,3)+
      * FILCONT(X,Y,Z,0.2,0.2,0.4,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.2,0.2,-0.4,1,FREQ,-1,3)
VAL=VAL+FILCONT(X,Y,Z,-0.2,-0.28,0.4,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,-0.2,-0.28,-0.4,1,FREQ,-1,3)+
      * FILCONT(X,Y,Z,0.0,0.0,0.0,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.0,0.0,-0.0,1,FREQ,-1,3)+
      * FILCONT(X,Y,Z,0.2,-0.1,0.48,1,FREQ,1,3)+
      * FILCONT(X,Y,Z,0.2,-0.1,-0.48,1,FREQ,-1,3)
VAL=VAL+FILCONT(X,Y,Z,0.1,0.18,0.7,1,FREQ,1,1)+
      * FILCONT(X,Y,Z,0.1,0.18,-0.7,1,FREQ,-1,1)+
      * FILCONT(X,Y,Z,0.1,0.0,0.88,1,FREQ,1,1)+
      * FILCONT(X,Y,Z,0.1,0.0,-0.88,1,FREQ,-1,1)+
      * FILCONT(X,Y,Z,-0.1,-0.1,0.8,1,FREQ,1,1)+
      * FILCONT(X,Y,Z,-0.1,-0.1,-0.8,1,FREQ,-1,1)
VAL=VAL+FILCONT(X,Y,Z,-0.3,0.0,0.4,1,FREQ,1,1)+
      * FILCONT(X,Y,Z,-0.3,0.0,-0.4,1,FREQ,-1,1)+
      * FILCONT(X,Y,Z,0.1,-0.1,0.7,1,FREQ,1,1)+
      * FILCONT(X,Y,Z,0.1,-0.1,-0.7,1,FREQ,-1,1)+
      * FILCONT(X,Y,Z,0.2,-0.2,0.8,1,FREQ,1,1)+
      * FILCONT(X,Y,Z,0.2,-0.2,-0.8,1,FREQ,-1,1)

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REAL X,Y,Z,FREQ
INTEGER FLG
C
IMPLICIT NONE
COMPLEX DIRICHLETVALZ
IF (FLG.EQ.999) THEN
  DIRICHVALZPL=CMPLX(999.0,0.0)
ELSE
  DIRICHVALZPL=DIRICHLETVALZ(X,Y,Z,FREQ)
END IF
END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

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