TURBULENT HEAT TRANSFER IN A CONCENTRIC ANNULUS: Under Simultaneous Development of Boundary Layers with Constant Wall Temperature.

By

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Submitted to the Faculty of Science and Engineering
of the University of Ottawa
in partial fulfillment of the requirements
for the degree of
MASTER OF APPLIED SCIENCE
in Mechanical Engineering
March, 1971.

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ACKNOWLEDGEMENT

The author is sincerely grateful to Dr. Y.Lee who initiated and provided valuable instructions regarding the present problem.

The author wishes to express his thanks to Dr. J.L.Howland, Mr. Bob Molloy and Mr. S.D.Park for their co-operation during the computation. He also extends his gratitude to the authority of the Computing Center, University of Ottawa, for their kind co-operation.

Sincere thanks are extended to his wife for her encouragement and co-operation. Lastly, the financial support of the Department of University Affairs, Ontario Government, and the Department of Mechanical Engineering, University of Ottawa, is gratefully acknowledged.

SUMMARY

An analytical solution has been obtained for the problem of heat transfer in a concentric annulus when both velocity and temperature are simultaneously developing. The solution is given both for the developing region and for the fully developed situation with inner wall heated and outer wall insulated and vice versa. The heat is being supplied to (or extracted from) the heated wall in such a way that temperature on the heated wall remains constant. The reason for choosing these boundary conditions is that with the solution for these two basic problems , one can obtain solution for any problem with mixed boundary conditions by superposition. For the present problem , fortunately , the method of superposition is applicable. The present work also includes an investigation of the effect of past history of the fluid flow on the entrance length. Necessary eigen-values have been estimated from the energy equation by the Runge-Kutta method. Eigen-constants are then evaluated from the Sturm-Liouville equation after applying the suitable boundary condition.

Results are presented to show the effect of Prandtl number , Reynolds number and radius ratio on the heat transfer in a concentric annulus. Calculations have been made for Prandtl numbers of 0.001 , 0.01 , 0.7 , 50 and

1000 with Reynolds numbers ranging from 10,000 to 1,000,000 and radius ratios of 1.01 , 2.31 and 5.625. It appears that no result is available to date which deals with the simultaneous development of turbulent velocity and temperature profiles in a concentric annulus with constant wall temperature boundary condition. Therefore , it has not been possible to provide a direct test of the present work at each stage. However , comparisons have been made whenever possible and the results found to be reasonably in agreement with those appearing in the existing literature.

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NOMENCLATURE

1. GENERAL

A	cross sectional area of the annulus
c	specific heat
c'	a constant defined by Equation 4.2
C	eigen-constant defined by Equation 3.20
D	equivalent diameter of the annulus
h	convective heat transfer coefficient
k	thermal conductivity
K	mixing length constant
P	fluid pressure in the flow direction
q	heat flux
r	radial distance from the center line of the annulus
r_m	radius of maximum velocity
R'	a radial distance parameter defined by Equations 3.2 & 3.3
t	temperature of fluid
u	velocity in the axial direction
u_τ	shear velocity
x	distance from entrance in the axial direction
y	radial distance from one of the annular walls

2. GREEK SYMBOLS

α	radius ratio (r_2/ r_1)
δ	hydrodynamic boundary layer thickness
ϵ	eddy diffusivity
r	thermal diffusivity ($k / \rho c_p$)
ρ	density
ν	kinematic viscosity (μ/ρ)
τ	time averaged shear stress
ϕ	a function of R given by Equation 3.18
ψ	a function of x given by Equation 3.17
λ^2	eigen-value
σ	eddy diffusivity ratio (ϵ_h/ ϵ_m)
μ	absolute viscosity
θ	time
η^*	$(\delta^+ - y^+)/ \delta^+$
η	$(\delta^+ - 26)/ \delta^+$

3. NONDIMENSIONAL PARAMETERS

Nu	Nusselt number (hD/k)
Pr	Prandtl number ($c_p\mu /k$)
Re	Reynolds number ($u_b D/\nu$)
R	$r/(r_2-r_1)$
D^+_j	Du_{τ_j}/ν
R^+_j	$R'_j\, u_{\tau_j}/\nu$

r_j^+	$r_j u_{\tau_j} / \nu$
T	$(t-t_w)/(t_e-t_w)$
u_j^+	u_j/u_{τ_j}
δ_j^+	$\delta_j u_{\tau_j} / \nu$
x^+	x/D
x^{++}	x/x_d
y_j^+	$y_j u_{\tau_j} / \nu$

4. SUBSCRIPTS

1	inner wall or inner region of the annulus
2	outer wall or outer region of the annulus
b	bulk value
d	fully developed value
e	entrance
h	thermal
j	refers to region 1 or 2
m	momentum, unless otherwise specified
max	maximum value
n	number
p	constant pressure
x	local value at an axial distance of x
w	wall

CHAPTER 1

INTRODUCTION

Heat transfer to fluids flowing through a duct has been the subject of many investigations because of its wide engineering applications, and the annulus is of considerable importance among duct geometries.

Starting from the energy equation, a theoretical analysis may be made for turbulent heat transfer in a concentric annulus. Such analyses have been presented for a concentric annulus, but most of these are confined to either hydrodynamically or both hydrodynamically and thermally fully developed flow. It has also been observed that, because of the variation of bulk temperature gradient ($\frac{\partial t_b}{\partial x}$) with the axial distance along the flow direction, analysis with constant wall temperature boundary condition is more difficult than that with constant heat flux boundary condition. In addition to this difficulty, when the flow is developing both hydrodynamically and thermally, it makes the problem extremely complicated. The classical method of Seban and Shimazaki (1)* based on the integral equation of momentum and energy is complicated enough even for a circular tube.

* Numbers in the parenthesis refer to the number in the reference.

A simultaneously developing flow means both the generalised velocity and the temperature profiles change shape with respect to the axial distance even when the flowing fluid is assumed to have constant properties. The shapes of the generalised velocity and temperature profiles are dictated mainly by the boundary conditions and past history of the flowing fluid. At the entrance , the fluid has to attain the free stream temperature within a very short height and therefore the temperature gradient in the fluid is infinite and correspondingly the local heat transfer coefficient is also infinite. As the fluid flows away from the entrance section , the heat transfer coefficient gradually decreases and ultimately converges to a constant fully developed value at an axial location far downstream from the entrance. It is however assumed that the fluid properties are constant with respect to time and geometric positions.

The term 'fully developed entrance length' has been defined in many ways by different authors. Most commonly , the thermal entrance length is defined as the distance required for the local Nusselt number to approach to within a few percent of the fully developed value when an 'exact method' is used. For an approximate solution , such as integral method , however , the thermal entrance length is the distance from the entrance where the local

Nusselt number is very near to the fully developed value with practical meaning. The reason is that in an integral method one can calculate the thermal boundary layer thickness at any cross section of the annulus.

The entrance length for simultaneously developing flow is not necessarily the same as the entrance length for purely thermally developing flow. This difference in entrance length is due to the effect of Prandtl number. When the turbulent Prandtl number as well as Prandtl number are unity and the boundary conditions are symmetric, both hydrodynamic and thermal boundary layers will have the same thickness. However, for an assymmetric boundary conditions the situation is not so simple.

The thermal entrance length in a simultaneously developing flow is defined in the present study in terms of the length of the duct needed from the entrance to a cross section perpendicular to the flow direction where both the generalised velocity and temperature profiles are invarient in the axial direction. Since the entrance length is solved by an 'exact method', it is not possible to obtain the entrance length based on $(Nu_x/Nu_d)\sim 1.00$. Therefore, in actual computation, a certain distance very far from the entrance is chosen and the Nusselt number corresponding to that particular section is increased by 5% to obtain the fully developed criterion of Nusselt number with practical meaning. The axial distance from the entrance section where the local Nusselt number is equal to or just below this value is accepted as the entrance length. It is apparent that the entrance length obtained from an 'exact solution' is largely dependent on the definition used. For the present work, however, the definition of thermal entrance length is based on (Nu_x/Nu_d) $\leq$ 1.05. The number 1.05 has been chosen to corelate the present results with those available in the existing literature.

CHAPTER 2

PREVIOUS WORK

Because of the wide engineering application , a considerable amount of work has been carried out on turbulent flow and heat transfer in a concentric annulus in recent years. Most of these are either fully developed or purely thermally developing cases. Very little work has been done on the simultaneously developing flow condition , although there is more likely to be a simultaneous development of both hydrodynamic and thermal boundary layers rather than thermal boundary layer alone.

The case of purely thermal entrance length problem in a concentric annulus has been studied analytically and/or experimentally by Reynolds et al(2) , Kays and Leung(3) , Quarmby(4) , Lee(5) , Quarmby and Anand(6) and Chen and Yu(7). Kays and Leung(3) utilized an empirical approach to obtain the temperature profiles for some specific values of radius ratio and Reynolds number. Quarmby and Anand (6) investigated the thermal entrance length problem with constant heat flux boundary conditions. They compared their results with the experimental results of reference 4. Lee(5) solved the problem where the heat flux on the annulus wall is constant . His results which are obtained by an approximate solution show that the thermal entrance length depends on radius ratio, Reynolds number and Prandtl number. The work by Chen and Yu(7) is mainly concerned with very low Prandtl number. They have studied the case of uniform heat flux and axially varying heat flux boundary conditions. The entrance region solution for purely thermally developing flow with constant wall temperature boundary condition can be found in reference(8) by Quarmby and Anand. They have used Deissler's velocity profile for the sublayer region and Von Karman's hypothesis for the core region velocity profile. The eddy diffusivity ratio was the same as that proposed by Jenkins. They have assumed that the mixing length constant is the same for both the inner and the outer wall regions. However, Barrow et al(9) have shown that the latter assumption is basically wrong because the annulus flow characteristics for the inner wall region are not the same as those in the pipe flow. Later on, it has been found by Lee and Park(10) that the mixing length constant for the inner wall region of an annulus is a function of Reynolds number, radius ratio and axial distance along the duct. The present author does not agree with Quarmby and Anand's(8) expression for the eigen-constant and the final form of the energy differential equation (equations and respectively) from which they have evaluated the eigen-values. The reason is that they have defined the nondimensional velocities u_1^+ and u_2^+ on the basis of the inner wall and the outer wall shear stresses respectively, whereas, it seems that neither in computing the eigen-constants nor in solving the energy differential equation they have considered the velocities for the inner wall and the outer wall regions separately.

So far the problem of purely thermal entrance length has been discussed. In designing actual heat transfer systems it is often necessary to know the heat transfer results for the entrance region where the flow is developing both hydrodynamically and thermally. However, when the length of the flow passage is very long, this may not make a significant contribution.

The simultaneously developing entrance length problem has been studied analytically by Kays(11), Sparrow(12), Heaton et al(13). Kays(11) obtain the solution for laminar flow in a circular tube. Sparrow(12) extended the available results by taking a rectangular duct. He considered the duct as two parallel planes with heat being transfered through each plane. Heaton et al(13) solved the problem of laminar heat transfer in circular tube and in a concentric annulus with constant heat flux on the boundary wall.

Very recently the simultaneously developing entrance length problem in a concentric annulus with turbulent flow has been studied by Roberts and Barrow(14), Wilson and Medwell(15) and Park and Lee(16). Roberts and Barrow(14) made a theoretical analysis for the case where the core wall of the annulus is heated and the other wall is adiabatic. They have compared their results with the experimental findings for air and with annuli having radius ratio of 2.1 and 4.0 . Wilson and Medwell(15) made a purely theoretical investigation for the uniform heat flux boundary condition where the core wall was heated. They found that the flow becomes hydrodynamically fully developed within 10 equivalent diameters of the annulus, while a further 30 equivalent diameters are required for the thermal boundary layer to reach the outer wall of the annulus. They considered a radius ratio range of 1.25 to 5.0 over a Reynolds number ranging from 10,000 to 300,000 for a Prandtl number of 0.7 . However, they made an assumption that the mixing length constants for both the walls are the same and equal to 0.4 . But this assumption is wrong according to the findings of Barrow et al(9). The most recent results which are available to the present author are due to Park and Lee(16). They have dealt with the case in which the annulus walls are at uniform heat flux condition.

CHAPTER 3

ANALYSIS

3.1 FLUID FLOW

The present work is a continuation of the research being conducted in the Mechanical Engineering Department at the University of Ottawa. The following details on the fluid flow are mainly due to investigation by Lee and Park(10).

3.1.1 EDDY DIFFUSIVITY VARIATION

The basic equation governing the momentum transfer within the turbulent boundary layer of a steady, incompressible fluid flow is given by Kays(17) as,

$$\frac{\tau}{\rho} = (\nu + \epsilon_m) \frac{\partial u}{\partial y} \tag{3.1}$$

On the basis of experimental observation, Reichardt proposed the following expression for the eddy diffusivity of momentum in the turbulent core region of a smooth pipe :

$$\frac{\epsilon_m}{\nu} = \frac{K R^+}{6} \left[1-(r/R')^2\right]\left[1+2(r/R')^2\right] \quad , K=0.4 \tag{3.2}$$

Equation (3.2) can be modified for regions remote from the walls as follows :

$$\left(\frac{\epsilon_m}{\nu}\right)_j = \frac{K_j}{6}\left|R_j^+ - r_j^+\right|\left[1 - \left(\frac{R_j^{'}-r}{R_j^{'}-r_j}\right)^2\right]\left[1 + 2\left(\frac{R_j^{'}-r}{R_j^{'}-r_j}\right)^2\right] \qquad (3.3)$$

where $R_j^{'} = r_1+\delta_1$ or $r_2-\delta_2$ and $r = r_1+y_1$ or r_2-y_2

For regions close to the walls, however, Deissler's expression for the eddy diffusivity is most commonly used. This is given by,

$$\left(\frac{\epsilon_m}{\nu}\right)_j = n^2 u_j^+ y_j^+ \left[1-\exp(-n^2 u_j^+ y_j^+)\right] \quad , \quad n^2=0.0154 \qquad (3.4)$$

The eddy diffusivity of heat is obtained by multiplying the eddy diffusivity of momentum with a factor σ, called the eddy diffusivity factor.

Now, in an annulus, the radius of maximum velocity is analogous to the center line of a circular pipe. Since this radius of maximum velocity is not exactly half way between the two boundary walls of the annulus, it is important to have an accurately determined eddy diffusivity throughout the entire section of the annulus. However, in the case of a pipe flow, because of symmetrical heating (or cooling) the effect of discontinuities in the eddy diffusivity variation has been found to have very little influence on the heat transfer(17).

Most of the previous investigators have assumed that the turbulent intensity at the radius of maximum velocity is zero. But recently it has been found to be a most unrealistic assumption. Equation (3.3), which is originally due to Reichardt, is, however, in good agreement with the experimental evidence.

So far the eddy diffusivity of momentum has been discussed. As mentioned earlier, the eddy diffusivity of heat is obtained by multiplying it by a factor. Direct measurement of the eddy diffusivity of heat and that of momentum shows that the former is somewhat greater than the latter for air. Jenkin's result, however, gives an opposite effect. Some investigators, such as Sleicher et al (18), prefer to use Jenkin's eddy diffusivity ratio multiplied by a constant factor.

In the present analysis the eddy diffusivity has been taken on the basis of some recent experimental observations(19). This ratio is given by,

$$\sigma = \frac{\epsilon_h}{\epsilon_m} = 0.968\,\alpha^{0.045}\,y_1^{+0.031} \qquad (3.5)$$

for $0 \leqslant y_1^+ < \delta_1^+$ and $\alpha \leqslant 30$

For the region above the hydrodynamic boundary layer, σ is taken as constant and it is given by,

$$\sigma = 0.968\,\alpha^{0.045}\,\delta_1^{+0.031} \qquad (3.6)$$

Therefore, the whole cross section of the annulus can be taken as a combination of the following:

1) Sublayer region: where Deissler's equation (3.4) together with Equation (3.5) for the inner wall region and Equation (3.6) for the outer wall region is to be used for heat transfer calculations.
2) Core region: where modified Reichardt's equation (3.3) together with Equation (3.5) or Equation (3.6) is used.
3) Potential flow region: where Equation (3.3) with $y_j = \delta_j$ is to be used in conjunction with Equation (3.6).

3.1.2 VELOCITY DISTRIBUTION

For the regions near the walls of the annulus, velocity is given by,

$$u_j^+ = \int_0^{y_j^+} \frac{dy_j^+}{1 + n^2 u_j^+ y_j^+ \left[1 - \exp(-n^2 u_j^+ y_j^+)\right]} \quad , \; n^2 = 0.0154 \quad (3.7)$$

Equation (3.7) which is for the sublayer region ($0 \leq y_j^+ \leq 26$) neglects the variation of shear stress.

For the regions remote from the walls, the following velocity profile is used

$$u_j^+ = -3\,\delta_j^+ \cdot \frac{1}{A_j} \cdot \ln\left|\frac{K_j\,\delta_j^+ - 4K_j\,\delta_j^+\,\eta_j^{*2} - A_j}{K_j\,\delta_j^+ - 4K_j\,\delta_j^+\,\eta_j^{*2} + A_j}\right| + c_j \qquad (3.8)$$

where $A_j = (9K_j^2\delta_j^{+2} + 48K_j\delta_j^+)^{\frac{1}{2}}$, and $\eta_j^* = (\delta_j^+ - y_j^+)/\delta_j^+$

The constant c_j in Equation (3.8) is obtained by forcing the velocity profile to go through a fixed point as mentioned in reference (10). The above equation for velocity distribution is based on the assumption of linear shear stress distribution within the hydrodynamic boundary layers.

Since the mechanism of flow outside the radius of maximum velocity is very similar to that occuring in pipe flow, a value of 0.4 for K_2 has been prescribed for evaluating the outer region velocity, while the value of K_1 has to be determined depending on radius ratio, Reynolds number and a particular cross section of the annulus. Now, in the potential flow region the eddy diffusivity is constant. Therefore equating the two eddy diffusivities at the edges of the hydrodynamic boundary layers,

$$\left(\frac{K_2}{K_1}\right)_x = \left(\frac{\delta_1}{\delta_2}\sqrt{\frac{\tau_{w_1}}{\tau_{w_2}}}\right)_x \qquad (3.9)$$

Again from a force balance,

$$\left(\frac{\tau_{w_1}}{\tau_{w_2}}\right)_x = \frac{r_2(2r_1\delta_1 + \delta_1^2)}{r_1(2r_2\delta_2 - \delta_2^2)} \tag{3.10}$$

Therefore,

$$K_1 = K_2 \cdot \frac{\delta_2}{\delta_1} \cdot \frac{1}{\sqrt{\alpha}} \cdot \sqrt{\frac{2r_2\delta_2 - \delta_2^2}{2r_1\delta_1 + \delta_1^2}} \tag{3.11}$$

3.2 HEAT TRANSFER

3.2.1 ENERGY EQUATION

With the usual assumptions* of constant fluid properties and negligible axial conduction , the general energy equation (ref.17) reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial t}{\partial r}\right) = \rho cu\frac{\partial t}{\partial x} \qquad (3.12)$$

On the basis of the model of the turbulent heat transfer as above , it can be assumed that the thermal conductivity in the above equation , which is a steady flow equation based on molecular conduction alone , is now a turbulent conductivity as defined by Kays(17). Although turbulent conductivity arises from velocity fluctuations and the flow in reality is not steady , all the effects of fluctuations are lumped together in this term and then the flow is treated as a steady flow.

Therefore , for the turbulent flow , the above differential equation can be modified to

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\rho c_p r(\kappa+\epsilon_h)\frac{\partial t}{\partial r}\right] = \rho u c_p\frac{\partial t}{\partial x} \qquad (3.13)$$

* A detailed discussion of the assumptions is given in App.I

Under the assumption of constant properties, ρc_p cancel out from both sides and the Equation (3.13) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r(\kappa+\epsilon_h)\frac{\partial t}{\partial r}\right] = u\frac{\partial t}{\partial x} \qquad (3.13a)$$

To express this equation in the nondimensional form, the following parameters are defined :

$$R = r/(r_2-r_1) \; ; \qquad x^+ = x/D$$

$$u_j^+ = u_j/u_{\tau_j} \; ; \qquad T = (t-t_w)/(t_e-t_w)$$

Substituting these in the Equation (3.13a),

$$\frac{1}{R(r_2-r_1)^2}\cdot\frac{\partial}{\partial R}\left[R(\kappa+\epsilon_h)\frac{\partial T}{\partial R}\right] = u_{\tau_j} u_j^+ \frac{\partial T}{\partial x^+}\cdot\frac{1}{2(r_2-r_1)}$$

$$\text{or,} \quad \frac{1}{\nu R}\frac{\partial}{\partial R}\left[R(\kappa+\epsilon_h)\frac{\partial T}{\partial R}\right] = \frac{r_2-r_1}{2}\cdot\frac{u_{\tau_j}}{\nu}\cdot u_j^+\frac{\partial T}{\partial x^+} \qquad (3.14)$$

Equation (3.14) can be split in two parts as follows :

inner wall region: $\quad r_1 \leq r \leq r_m$

$$\frac{1}{R}\frac{\partial}{\partial R}\left[R\left(\frac{\epsilon_h}{\nu}+\frac{1}{Pr}\right)\frac{\partial T}{\partial R}\right] = \frac{\alpha-1}{2}\, r_1^+\, u_1^+\frac{\partial T}{\partial x^+} \qquad (3.14a)$$

outer wall region: $\quad r_m < r \leq r_2$

$$\frac{1}{R}\frac{\partial}{\partial R}\left[R\left(\frac{\epsilon_h}{\nu}+\frac{1}{Pr}\right)\frac{\partial T}{\partial R}\right] = \frac{\alpha-1}{2\alpha}\, r_2^+\, u_2^+\frac{\partial T}{\partial x^+} \qquad (3.14b)$$

3.2.2 TEMPERATURE PROFILE

From the energy equation, it is seen that when Prandtl number, Reynolds number and radius ratio are fixed temperature is a function of x^+ and R. Because of the homogeneous character of the linear differential energy equation and possibility of forcing the product solution to satisfy initial and boundary conditions it may be assumed that

$$T = \phi(R).\psi(x^+) \qquad (3.15)$$

Substituting this expression for T in Equation(3.14a),

$$\frac{1}{R}\cdot\frac{d}{dR}\left[R\left(\frac{\epsilon_h}{\nu}+\frac{1}{Pr}\right)\psi\frac{d\phi}{dR}\right] = \frac{\alpha-1}{2}\, r_1^+ u_1^+ \phi\,\frac{d\psi}{dx^+}$$

or,

$$\frac{1}{R}\cdot\frac{d}{dR}\left[R\left(\frac{\epsilon_h}{\nu}+\frac{1}{Pr}\right)\frac{d\phi}{dR}\right]\cdot\frac{2}{(\alpha-1)r_1^+u_1^+\phi} = \frac{1}{\psi}\frac{d\psi}{dx^+} \qquad (3.16)$$

Now, since it is known that the convective heat transfer depends on Reynolds number of flow, both sides of Equation (3.16) are multiplied by (Re/8) and then equated to the separation constant $-\lambda^2$. Thus,

$$\frac{1}{\psi}\frac{d\psi}{dx^+}(Re/8) = -\lambda^2 \qquad \text{or,} \qquad \frac{d\psi}{dx^+} = -\frac{8}{Re}\lambda^2\psi$$

The solution of the above equation is

$$\psi(x^+) = C'\exp\left(-\frac{8\lambda^2}{Re}x^+\right) \qquad (3.17)$$

where C' is an integration constant.

Since λ can have an infinite number of values viz. λ_1, λ_2, λ_3 etc. Equation (3.17) is written as,

$$\psi_n(x^+) = C_n' \exp(-8\lambda_n^2 x^+/Re) \qquad (3.17a)$$

and the left side of Equation (3.16) gives

$$\frac{1}{R}\frac{d}{dR}\left[R\left(\frac{\epsilon_h}{\nu}+\frac{1}{Pr}\right)\frac{d\phi_n}{dR}\right]+\left[4\ (\alpha-1)\frac{r_1^+}{Re}\right]u_1^+\lambda_n^2\phi_n = 0 \qquad (3.18a)$$

for the inner wall region, and similarly for the outer wall region

$$\frac{1}{R}\frac{d}{dR}\left[R\left(\frac{\epsilon_h}{\nu}+\frac{1}{Pr}\right)\frac{d\phi_n}{dR}\right]+\left[4\ \frac{(\alpha-1)}{\alpha}\frac{r_2^+}{Re}\right]u_2^+\lambda_n^2\phi_n = 0 \qquad (3.18b)$$

Once $\phi_n(R)$ and $\psi_n(x^+)$ are established, temperature is determined from

$$T = \sum_{n=1}^{\infty} C_n\ \phi_n(R)\ \psi_n(x^+) \qquad (3.19)$$

where the eigen-constants (C_n) are evaluated from

$$C_n = \frac{\int_{R_1}^{R_2} r_j^+ u_j^+ R\,\phi_n(R)\ dR}{\int_{R_1}^{R_2} r_j^+ u_j^+ R\,\phi_n(R)^2 dR} \qquad (3.20)$$

3.2.3 NUSSELT NUMBER

Once the temperature profile is established, the calculation of Nusselt number is relatively simple. By definition,

$$Nu = \frac{h\,D}{k} = -\left.\frac{\partial t}{\partial r}\right|_w \cdot \frac{D}{t_w - t_b} \qquad (3.21)$$

where the bulk temperature (t_b) is defined as,

$$t_b = \frac{\int_A \rho u t\, dA}{\int_A \rho u\, dA} = \frac{\int_{R_1}^{R_2} u_j^+ R t\, dR}{\int_{R_1}^{R_2} u_j^+ R\, dR} \qquad (3.21a)$$

and $\left.\frac{\partial t}{\partial r}\right|_w$ is derived from Equation (3.19) and is given by

$$\left.\frac{\partial t}{\partial r}\right|_w = \frac{t_e - t_w}{r_2 - r_1} \sum_{n=1}^{\infty} C_n \exp(-8\lambda_n^2 x^+/Re) \qquad (3.21b)$$

From Equations (3.21), (3.21a) and (3.21b)

$$Nu = \frac{2\sum_1^{\infty} C_n \exp(-8\lambda_n^2\, x^+/Re)}{\int_{R_1}^{R_2} u_j^+ RT\, dR \Big/ \int_{R_1}^{R_2} u_j^+ R\, dR} \qquad (3.22)$$

3.2.4 BOUNDARY CONDITIONS OF THE PROBLEM

Before proceeding any further it is necessary to specify the boundary conditions of the problem. The solution of Equations (3.18a) and (3.18b) requires two boundary conditions and the Sturm-Liouville system needs one more boundary condition. These are given under two separate headings as follows :

3.2.4.a INNER WALL IS AT CONSTANT TEMPERATURE AND OUTER WALL IS INSULATED

In this case , since heat is being supplied to (or extracted from) the inner wall of the annulus , the definition of the nondimensional temperature will be

$$T = (t-t_1)/(t_e-t_1)$$

The boundary values are as follows :

$$\text{i)} \quad \phi_n(R_1) = 0 \; ; \quad \text{ii)} \quad \phi_n'(R_2) = 0$$

3.2.4.b OUTER WALL IS AT CONSTANT TEMPERATURE AND INNER WALL IS INSULATED

Since the outer wall is at a constant temperature , T is given by

$$T = (t-t_2)/(t_e-t_2)$$

The boundary conditions are given by,

$$\text{i)}\ \phi_n'(R_1) = 0\ ;\quad \text{ii)}\ \phi_n(R_2) = 0$$

Apart from the boundary conditions mentioned above, the third boundary condition which is valid for both the cases is

$$T(0,R) = 1$$

The solution of Equations (3.18a) and (3.18b) involves a trial and error procedure because the value of λ is unknown. Therefore, it is necessary to derive one more condition on either wall of the annulus. This can be developed on the wall which is at a constant temperature and is given by

$$\phi_n'(R_1) = 1 \text{ ,for heating on the inner wall}$$

and

$$\phi_n'(R_2) = 1 \text{ ,for heating on the outer wall.}$$

CHAPTER 4

METHOD OF COMPUTATION

4.1 FLUID FLOW (Details taken from ref.(31))

4.1.1 COMPUTATION OF REYNOLDS NUMBER

Before starting any numerical work on the heat transfer part of the problem, it is necessary to obtain a corelation between r_2^+ (or r_1^+) and Reynolds number for some specific values of radius ratio considered in this analysis. In doing so, r_2^+ has been taken as an independant parameter and Reynolds number is then evaluated for that particular r_2^+. The Reynolds number is defined as,

$$Re = \frac{u_b D}{\nu} = \frac{D}{\nu}\frac{\int_A u \, dA}{\int_A dA} = \frac{4\int_{r_1}^{r_2} ur \, dr}{\nu(r_2+r_1)}$$

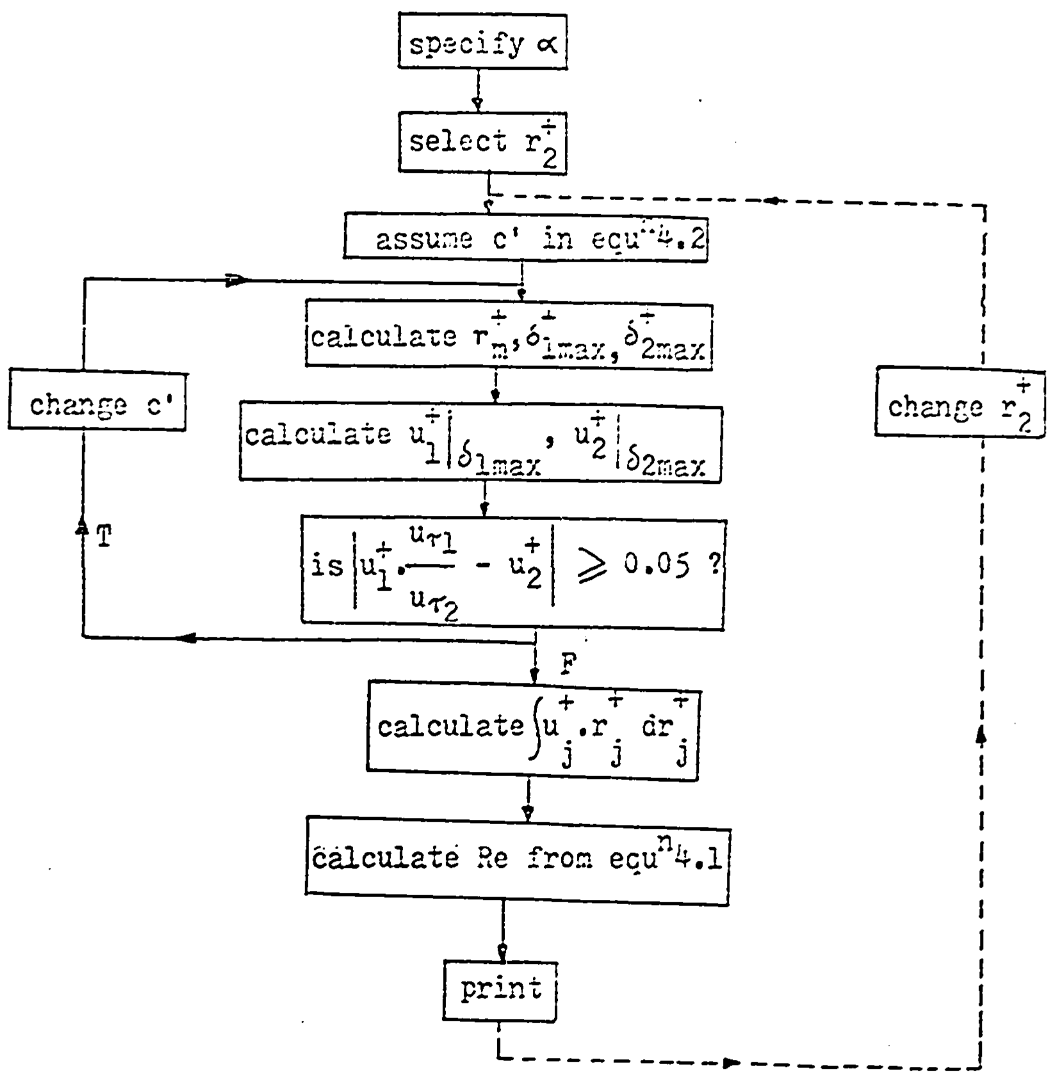
$$\text{or} \qquad Re = \frac{4}{r_2^+(1+1/\alpha)}\left[\left\{\int_0^{26} u_1^+(r_1^+ + y_1^+)\,dy_1^+ + \int_{26}^{\delta_{1max}^+} u_1^+(r_1^+ + y_1^+)\,dy_1^+\right\}\frac{u_{\tau_1}}{u_{\tau_2}} + \left\{\int_0^{26} u_2^+(r_2^+ - y_2^+)\,dy_2^+ + \int_{26}^{\delta_{2max}^+} u_2^+(r_2^+ - y_2^+)\,dy_2^+\right\}\right] \tag{4.1}$$

In calculating δ_{1max} and δ_{2max} it is necessary to obtain the radius of maximum velocity (r_m) and this is given by

$$r_m^+ = r_1^+\left[\frac{1+\alpha^{1-c'}}{1+\alpha^{-c'}}\right] \tag{4.2}$$

where c' is a constant and to be determined by matching the two velocities u_1 and u_2 at the edges of the hydrodynamic boundary layers for a particular cross section along the annulus.

A simplified flow diagram for the calculation of Reynolds number is given below



4.1.2 COMPUTATION OF MIXING LENGTH CONSTANT(K_1) AND BOUNDARY LAYER THICKNESSES AS A FUNCTION OF x^+

The calculation of the mixing length constant at the inner wall region requires the estimation of the hydrodynamic boundary layer thickness for the particular cross section under consideration. Assuming that the ratio of the hydrodynamic boundary layer thicknesses in the developing region is the same as that in the hydrodynamically developed region and utilising Equation (3.11), this constant (K_1) at a particular cross section can be easily evaluated.

Starting from the momentum equation, Lee and Park(10) have developed an expression corelating the hydrodynamic boundary layer thicknesses and x^+. This is given by,

$$x_j^+ = \int_{\frac{Re}{2}}^{E^+} \left[\frac{\delta_j^+ (R_j^+ + r_j^+)}{r_j^+ (D_j^+)^2} \; u_{\delta_j}^+ - \frac{2}{r_j^+ (D_j^+)^2} \int_0^{\delta_j^+} u_j^+ r^+ \, dy_j^+ \right] dE^+$$

$$+ \int_0^{D^{++}} \frac{dD^{++}}{r_j^+ (D_j^+)^2} . \qquad (4.3)$$

where $2E^+ = u_{\delta_j}^+ D_j^+$ and $D^{++} = \int_0^{\delta_j^+} (u_{\delta_j}^+ - u_j^+) u_j^+ r \, dy_j^+$

($u_{\delta_j}^+$ means nondimensional velocity at the edge of the hydrodynamic boundary layer)

Equation (4.3) has been derived on the basis of the following assumptions :

i) the flow outside the hydrodynamic boundary layers is potential flow.

ii) the boundary layer thicknesses are zero at $x^+=0$.

iii) the flow within the hydrodynamic boundary layers is entirely turbulent.

From Equation (4.3) it is apparent that the computation of the boundary layer thicknesses for a certain cross section of the annulus is a lengthy operation. Instead, one of the boundary layer thicknesses is assumed and the other one is then evaluated from the known boundary layer thickness ratio which is given below

$$\left.\frac{\delta_1}{\delta_2}\right|_x = \frac{\delta_{1max}}{\delta_{2max}} = \frac{r_m - r_1}{r_2 - r_m} \qquad (4.4)$$

Once the hydrodynamic boundary layer thicknesses are determined, the corresponding distance from the entrance section (x^+) is evaluated from Equation(4.3). Before starting any numerical work, it is also necessary to obtain an expression for the ratio of wall shear stresses. From a force balance on a certain elemental cross section at a distance of x (wherethe hydrodynamic boundary layer thicknesses are δ_1 and δ_2, axial pressure is P and wall shear stresses are τ_1 and τ_2), one can obtain the following :

$$\tau_{w_1}\Big|_x = \frac{dP}{dx} \cdot \frac{(r_1 + \delta_1)^2 - r_1^2}{2r_1} \tag{4.5a}$$

for the inner wall region, and for the outer wall region this is given by

$$\tau_{w_2}\Big|_x = \frac{dP}{dx} \cdot \frac{r_2^2 - (r_2 - \delta_2)^2}{2r_2} \tag{4.5b}$$

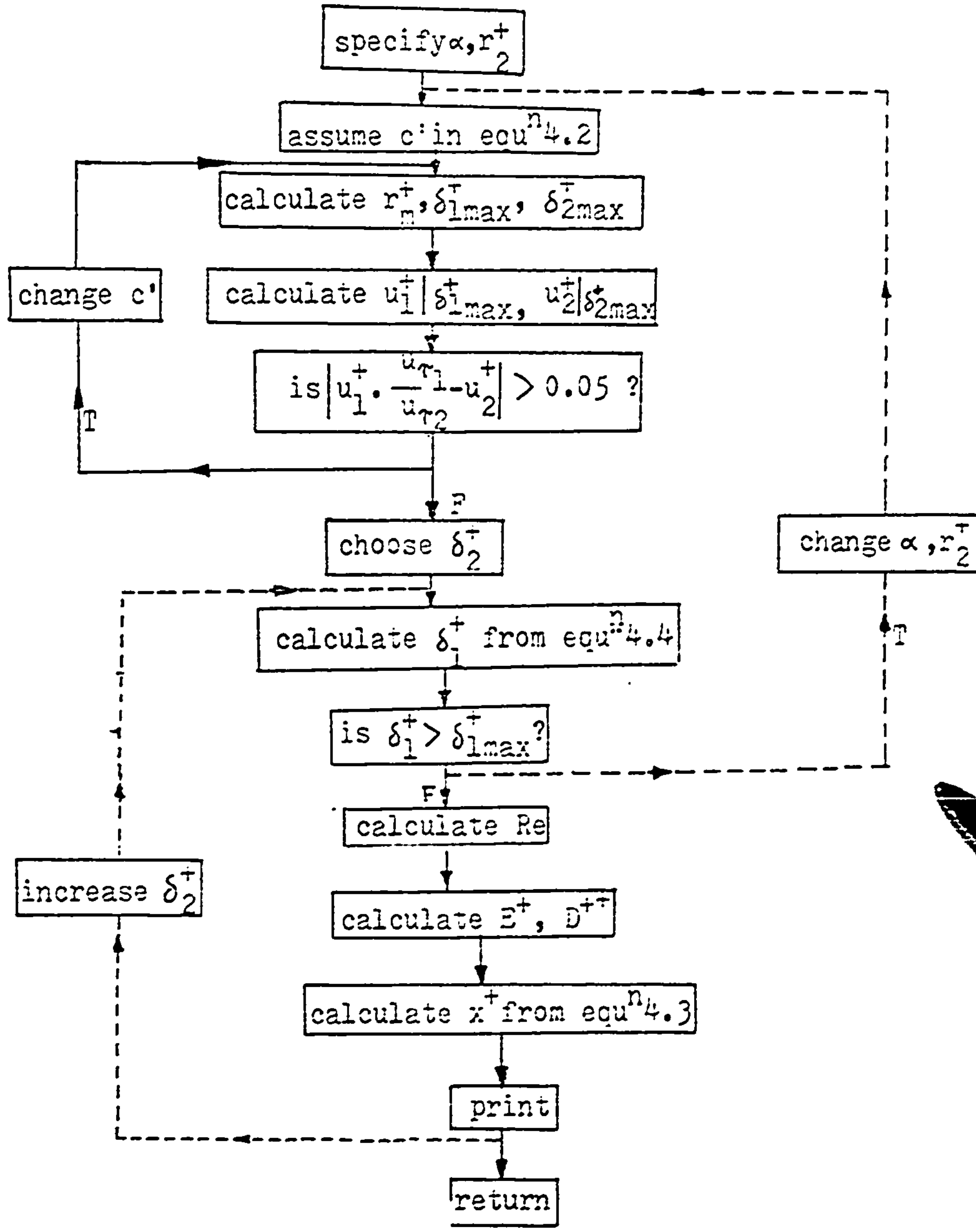
In writing Equations (4.5a) and (4.5b), it has been assumed that there is no variation of pressure in the radial direction. From Equations (4.5a) and (4.5b) it follows that

$$\frac{\tau_{w_1}}{\tau_{w_2}}\Bigg|_x = \frac{r_2}{r_1} \cdot \frac{2r_1 \delta_1 + \delta_1^2}{2r_2 \delta_2 - \delta_2^2} \tag{4.6}$$

which is the same as Equation (3.10) mentioned in Chapter 3. For the hydrodynamically developed region, Equation becomes

$$\frac{\tau_{w_1}}{\tau_{w_2}} = \frac{r_2}{r_1} \cdot \frac{r_m^2 - r_1^2}{r_2^2 - r_m^2} \tag{4.7}$$

A simplified flow diagram for the calculation of the mixing length constant (K_1) and the hydrodynamic boundary layer thicknesses corresponding to a particular cross section of the annulus is given on the next page.


specify α, r₂⁺
assume c' in equⁿ 4.2
calculate r_m⁺, δ₁max⁺, δ₂max⁺
calculate u₁⁺|δ₁max⁺, u₂⁺|δ₂max⁺
is |u₁⁺ · u_τ1/u_τ2 − u₂⁺| > 0.05 ?
T
change c'
F
choose δ₂⁺
calculate δ₁⁺ from equⁿ 4.4
is δ₁⁺ > δ₁max⁺?
T
change α, r₂⁺
F
calculate Re
calculate E⁺, D⁺⁺
calculate x⁺ from equⁿ 4.3
print
return
increase δ₂⁺

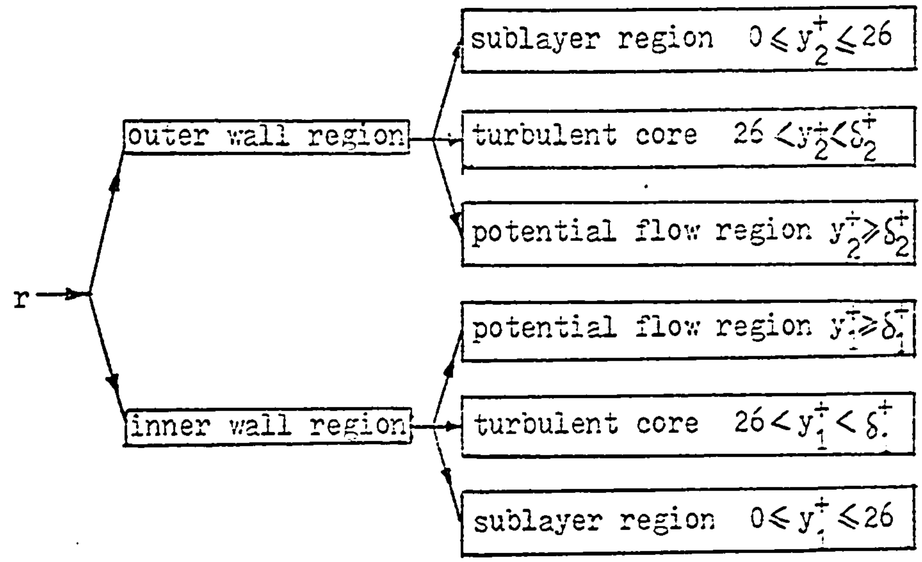
4.2 HEAT TRANSFER

4.2.1 COMPUTATION OF EIGEN-VALUES

The computation for the temperature profile and hence Nusselt number requires a fair estimation of the eigen-values. It has been observed during computation work that one has to evaluate about five eigen-values from the begining to obtain a reasonably good heat transfer result. However , to be on the conservative side , the first seven eigen-values have been estimated. It is also obvious from Equation (3.17a) that when λ is very large , even for high Reynolds number and small x^+ , ψ becomes very small thereby making its contribution to the final temperature profile practically negligible. Thus , any eigen-value which is more than 150 (this number has been chosen after a careful study of the effect of magnitude of the higher order eigen-values on the Nusselt number) is considered as insignificant so far as the heat transfer result is concerned. The above two limitations are important from the economic point of view of the computer time. It is also necessary because the calculation of eigen-values involves a lengthy trial and error procedure. However , exception has been made in certain cases where the spread of the eigen-values is large.

The actual method involves integrating of Equations (3.18a) and (3.18b). First the eigen-value is assumed and the two differential equations are integrated. The integration is conducted using the classical method of Runge-Kutta and the step size is varied until the result is convergent. Since both the eigen-function and its slope are known on the wall which is at a constant temperature, all integrations are started from the heated wall. Then the slope of the eigen-function on the insulated wall is calculated. If it does not satisfy the required boundary condition on the insulated wall the magnitude of the assumed eigen-value is modified until it fits the specific boundary condition. This procedure is repeated until the desired accuracy of the eigen-value is obtained. The original method by which the eigen-values are traced, was developed by Larson (23) to find the 'zero of a function'.

It is important to mention at this stage that before starting the numerical work, it is necessary to simplify Equations (3.18a) and (3.18b). It is also important to evaluate the local velocity and the eddy diffusivity of heat properly depending on the radial location. The whole radial cross section has been divided in some regions and subregions as given on the next page.



Depending on the boundary conditions two cases may arise and these are presented below.

4.2.1.a INNER WALL IS AT CONSTANT TEMPERATURE AND OUTER WALL IS INSULATED

Under this boundary condition both the eigen-function and its slope (these two are the starting solutions for the Runge-Kutta integration) are known on the inner wall of the annulus. Hence the integration is started from the inner wall. Therefore , with an assumed eigen-value , when the slope of the eigen-function on the outer wall will be equal to or very near to zero , that eigen-value will be the one corresponding to which the eigen-function satisfies all the boundary conditions and the applicable energy equation. The classical Runge-Kutta method has been used for this forward integration and the boundary conditions on the inner wall are taken as the starting solution of the problem.

In general, it has been observed that the effect of step size of integration is more pronounced for high Prandtl number cases. With increase of step size the error becomes larger whereas the truncation error becomes smaller. Therefore, in all the cases the step size has been varied in a wide range and the most reliable result (which is less fluctuating with respect to step size of integration) is accepted for further computational work. Special care has also been taken to refine the first two eigen-values since it has been found that these have a large effect on the final heat transfer result.

4.2.1.b OUTER WALL IS AT CONSTANT TEMPERATURE AND INNER WALL IS INSULATED

In this case the numerical method is basically the same as the previous case except that one has to apply the backward integration method. The reason is quite obvious from the nature of the boundary conditions. The integration is started from the outer wall and as a result the Runge-Kutta formulations have to be modified for the backward integration. It has been seen, in general, that in this case the eigen-values are higher than those in the former case.

4.2.2 COMPUTATION OF EIGEN-CONSTANT

Once the eigen-values are accurately determined, calculation of the corresponding eigen-constants is simple. For a particular eigen-value, eigen-functions are generated at some discrete points by a subprogram designed according to the Runge-Kutta formulations and finally the eigen-constant is calculated from Equation (3.20). The integration is performed using Simpson's rule and the accuracy is ensured by gradually reducing step size. It may be important to mention that Equation (3.20) has been derived after applying the suitable boundary condition. A detailed discussion on the Sturm-Liouville system appears in App.III. As in the case of computation of the eigen-values, the effect of step size has been given due consideration in computing the eigen-constants.

4.2.3 COMPUTATION OF TEMPERATURE DISTRIBUTION AND NUSSELT NUMBER

In previous sections the computational methods for the calculation of eigen-values and eigen-constants have been outlined. Once the eigen-values and the corresponding eigen-constants are determined, the calculation of temperature is relatively straight forward. For a certain cross section of the annulus, the function Ψ

is constant. So depending on the radial distance, temperature can be obtained from Equation (3.19). These temperatures corresponding to different radial distances are stored in computer. Next, the bulk temperature can be obtained from Equation (3.21a). Knowing the bulk temperature Nusselt number is calculated by solving Equation(3.22).

The whole procedure for evaluating Nusselt number as a function of the distance from the entrance section can be summarised as follows :

1) A relationship is obtained between hydrodynamic boundary layer thicknesses, mixing length constant (K_1) and distance along the annulus from the Equations (4.4), (3.11) and (4.3) respectively.
2) A rough estimation of the eigen-values is made by solving Equations (3.18a) and (3.18b).
3) The eigen-values obtained above are refined to the desired accuracy.
4) Eigen-constants are calculated from Equation(3.20).
5) Eigen-values and eigen-constants, obtained above are used to find the temperature distribution for that particular cross section.
6) Bulk temperature is evaluated corresponding to

the temperature distribution in step (5) by solving Equation (3.21a).

7) Nusselt number for the particular cross section is calculated from Equation (3.21).

8) Until the flow is hydrodynamically developed, another cross section is chosen and steps (2) to (7) are repeated.

9) Once the flow is hydrodynamically developed, x^+ is increased by a suitable amount. Temperature distribution, bulk temperature and Nusselt number are evaluated corresponding to the particular cross section.

10) x^+ is increased and step (9) is repeated until the thermally fully developed situation is attained.

CHAPTER 5

RESULTS AND DISCUSSION

The results of the present analysis are presented in both tabular and graphical forms whenever it seems pertinent. The whole set of results can be divided into two groups; the first group includes the results obtained from the hydrodynamic part of the problem and the second group of results consists of those computed from the energy equation.

5.1 FLUID FLOW

A similar technique to that of reference (10) has been used to obtain the results in this section. Reynolds numbers are calculated from Equation (4.1) corresponding to different values of radius ratio. The results appear in Table 1 and a plot of these is given in Fig.2 .

Once a relationship between Reynolds number and r_2^+ is established for a particular radius ratio , the hydrodynamic boundary layer thickness at the outer wall region is assumed. Then the hydrodynamic boundary layer thickness at the inner wall region is obtained from Equation (4.4). Next , the mixing length constant (K_1) and the distance along the annulus are evaluated from Equations (3.11) and (4.3) respectively. Six sections have been taken in the hydrodynamically developing region and the results are given in Table 2 listing all the results obtained from the hydrodynamic part of the problem for three different radius ratios and five different Reynolds number.

The hydrodynamic entrance length is defined in terms of the length of the duct needed for the local velocity profile to approach 100% of its limiting condition. According to the subdivisions made for the whole radial cross section in Chapter 4, it follows that with this definition there will be no potential flow region when the flow is hydrodynamically fully developed. For the purpose of experimental work some authors, however, prefer to use local pressure gradient or local shear stress as the criterion for determining the hydrodynamic entrance length. A detailed discussion on the validity of the hydrodynamic relationships, used in this present analysis, has been given in reference (10) and (19).

From Table 2 it is apparent that the mixing length constant (K_1) varies with radius ratio, Reynolds number and the axial distance along the annulus. It is also seen that when radius ratio is small K_1 tends to a value which is close to 0.4. Again, for a particular Reynolds number and axial location, K_1 increases with increase of radius ratio. This is probably due to the effect of radial convexity of the boundary wall of the annulus. It is observed that as the flow approaches its fully developed characteristics, K_1 tends to a constant value depending on radius ratio and Reynolds number. It has been observed by Lee and Park (10) that the fully developed values of K_1 obtained from their analysis agrees well with those obtained by Levy.

5.2 HEAT TRANSFER

A complete list of eigen-values and corresponding eigen-constants for different combinations of Prandtl number, Reynolds number and radius ratio is given in Tables 3 and 4. It is observed that at very low Prandtl number (e.g. 0.001) the eigen-values are widely dispersed. The effect of step size on the typical eigen-constants is shown in Fig.3. It is seen that when step size is large or in other words, number of steps is small (<500) the values of C_n are fluctuating. The reason is that with large step size the accuracy of calculation is not high. On the other hand, with very small step size the computation time becomes longer and also it is likely that the truncation error is large.

The variation of fully developed temperature profile with Prandtl number and Reynolds number is shown in Fig. 4. Dimensionless temperature profile has been plotted against a normalised wall distance parameter. It is seen that the effect of Prandtl number is very significant in predicting the shape of the temperature profile.

The entrance region Nusselt numbers for different values of Prandtl number , Reynolds number and radius ratio appear in Tables 5 and 6. Plots of these results are presented in Figs. 5 , 6 and 7. In all the cases it is seen that very near the entrance , Nusselt number has a very high value and it gradually decreases with increase of the distance along the annulus. This is quite evident as near the entrance the fluid has to attain the free stream temperature within a very short height from the heated wall of the annulus which means that the temperature gradient will be very steep.

The effect of Prandtl number on the entrance region heat transfer is shown in Figs. 5a and 5b. It is seen that Prandtl number is very significant in predicting the heat transfer rate. This seems to be logical because of the prounced effect of Prandtl number on the temperature distribution observed. Comparison with the nearest existing results has been made for Pr = 0.7 and 1000 and the results of the present analysis are found to be in reasonable agreement.

Figs. 6a and 6b show the effect of Reynolds number on the heat transfer rate for a radius ratio of 5.625 and Prandtl number of 0.7. As it is expected , it is seen that with increase of Reynolds number the heat transfer rate increases. It is apparent that with increase of flow velocity , the convective heat transfer coefficient will increase resulting in a larger heat transfer rate.

The effect of radius ratio on the heat transfer rate is shown in Figs.7a and 7b. It may be concluded that radius ratio does not have a very significant effect on the heat transfer rate. Although , it increases slightly with increase of radius ratio.

In all the Figs. (5 , 6 and 7) there is at least one common characteristics; for a particular choice of Prandtl number , Reynolds number and radius ratio the Nusselt number at a particular cross section along the annulus is greater for heating on the inner wall than that for heating on the outer wall. The reason is that the mixing length constant for the outer wall region is always 0.4 whereas that for the inner wall region is always greater than 0.4. Thus for heating on the outer wall the mixing length constant in the proximity of the heated wall is always 0.4, whereas, for heating on the inner wall the mixing length constant in the proximity of the heated wall is always greater than 0.4. This causes greater thermal diffusion on the inner wall region and hence greater heat transfer rate.

The fully developed Nusselt number as a function of Reynolds number is shown in Figs. 8a and 8b. The same trend has been observed as found by previous investigators (8).

The effect of Reynolds number on the entrance length for a Prandtl number of 0.7 and a radius ratio of 5.625 is shown in Fig.9 . Also shown is the effect of past history of the flowing fluid on the entrance length. It appears that the entrance length obtained by an 'exact method' is rather arbitrary. The reason is that it depends largely on the definition of fully developed flow. Nevertheless, a graph has been drawn to show the variation of entrance length with Reynolds number of flow. It is observed that for high Reynolds number the fluid will take a somewhat greater length than for low Reynolds number to attain fully developed characteristics. Although the present entrance length solution is based on $Nu_x/Nu_d=1.05$ it is still in reasonable agreement with the results of Lee(5) , Wilson and Medwell(15) and Park(19). Another very interesting conclusion can be drawn from Fig.9 . The effect of past history of the flowing fluid is very important in predicting the entrance length. For a Prandtl number of 0.7 and a radius ratio of 5.625 , it is seen from Fig.9 that the entrance length for a simultaneously developing flow is greater than that for a purely thermally developing flow by about ten equivalent diameters. Again , a change of 2% in the definition of fully developed flow may cause a change in the entrance length to even ten equivalent diameters.

CHAPTER 6

CONCLUSION

The following conclusions can be drawn from results of this analytical investigation :

1) The effect of increasing Prandtl number of fluid is to increase the heat transfer rate.

2) For a particular combination of Prandtl number and radius ratio , heat transfer rate increases with increase of Reynolds number of flow.

3) The effect of radius ratio is less important than that of Prandtl number or Reynolds number on the heat transfer rate , although it increases with increase of radius ratio.

4) The entrance length depends on Reynolds number of flow , Prandtl number of fluid, radius ratio , past history of the flowing fluid and the definition of fully developed flow. The results show that the entrance length increases with increase of Reynolds number and decrease of Prandtl number and radius ratio. This is in agreement with Lee(5). He concluded that Reynolds number effect is relatively minor for Prandtl number greater than unity.

5) The analysis shows that for a Prandtl number

of 0.7 and a radius ratio of 5.625 , the entrance length for simultaneous growth of boundary layers is greater than that for purely thermally developing flow by about ten equivalent diameters.

6) Depending on the definition of entrance length used in a particular solution it varies.

7) Past history of the flowing fluid is very important in predicting the entrance length.

APPENDIX I

ON THE ASSUMPTIONS MADE IN DERIVING THE ENERGY EQUATION

Equation (3.9) in Chapter 3 has been derived on the basis of the following assumptions

1) Steady flow- Velocity, temperature and different fluid properties are independent of time.
2) No internal heat generation- There is no generation of heat inside the flow field due to chemical reactions or any other reason. By this assumption there will be no 'source function' in the energy equation.
3) No work is done by the external field.
4) Dufour effect on the heat transfer is negligible- This means that the effect of mass concentration gradient is neglected as it is assumed to be much smaller compared to temperature gradient.
5) Flow is axisymmetric- Velocity and temperature of the fluid particles at the same radius are independent of the angular orientation about the center line of the annulus.
6) Dissipation function is negligible- This function consists of the viscous part of the normal stresses and shear stresses. Dissipation

function depends on velocity, velocity gradient and Prandtl number. Viscous energy dissipation may be important for a high Prandtl number fluid (e.g. oil), even for rather moderate velocity and velocity gradient. For gases, on the other hand, where Prandtl number is near unity, velocity must approach the speed of sound before this term is significant (17).

7) Axial conduction is negligible compared to radial conduction- The condition under which this assumption is valid depends on the magnitude of Peclet number; large values of this number make this assumption a logical one. As a general rule, according to the findings of Singh(20), axial conduction can be neglected for Peclet number greater than approximately 100.

8) Boundary layer is thin relative to the dimensions of the annulus- A fundamental assumption of the boundary layer approximation is that the fluid immediately adjacent to the body surface is at rest relative to the body. This seems to be valid except for very low pressure gases. Now, if the boundary layer thickness is very small relative to the dimensions of the annulus velocity in the axial direction is much greater than that in

the radial direction and the pressure gradient term is negligible. It is also assumed that both hydrodynamic and thermal boundary layers start at the entrance section of the annulus.

9) Constant properties- The property values of the fluid and the material of the annulus are taken as constants.

10) The flow is entirely turbulent- In actual flow situations, perhaps, the flow is laminar before being fully turbulent.

APPENDIX II

FURTHER SIMPLIFICATION OF THE ENERGY EQUATION

Equations (3.18a) and (3.18b) in Chapter 3 have to be expressed in suitable forms before starting any numerical work. These two equations can be represented by one equation as follows:

$$\frac{1}{R}\frac{d}{dR}\left[R\left(\frac{\epsilon_h}{\nu}+\frac{1}{Pr}\right)\frac{d\phi_n}{dR}\right]+4\frac{\alpha-1}{S'}\cdot\frac{1}{Re}\cdot r_j^+u_j^+\lambda_n^2\phi_n=0$$

$$S'=\begin{cases}1\text{, for the inner region}\\ \alpha\text{, for the outer region}\end{cases}$$

or,

$$\frac{1}{R}\cdot\frac{d}{dR}\left[p(R)\frac{d\phi_n}{dR}\right]+q(R)\lambda_n^2\phi_n=0 \tag{II.1}$$

where $p(R)=\left(\frac{\epsilon_h}{\nu}+\frac{1}{Pr}\right)R$ and $q(R)=4\cdot\frac{\alpha-1}{S'}\cdot\frac{1}{Re}\cdot r_j^+u_j^+$

or,

$$p(R)\frac{d^2\phi_n}{dR^2}+\frac{dp(R)}{dR}\,\frac{d\phi_n}{dR}+q(R)R\lambda_n^2\phi_n=0$$

$$\frac{d^2\phi_n}{dR^2}+\frac{\frac{dp(R)}{dR}}{p(R)}\cdot\frac{d\phi_n}{dR}+\frac{q(R)}{p(R)}R\lambda_n^2\phi_n=0$$

or,

$$\boxed{\frac{d^2\phi_n}{dR}=-Z(R)\frac{d\phi_n}{dR}-S(R)\lambda_n^2\phi_n} \tag{II.2}$$

Functions Z and S in Equation (II.2) are given by

$$Z(R) = \frac{\frac{d}{dR}\left[\left(\epsilon_h/\nu + 1/Pr \right)R\right]}{\left(\epsilon_h/\nu + 1/Pr \right)R}$$

and

$$S(R) = 4(\alpha - 1)\frac{r_1^+}{Re}\;\frac{u_1^+}{\left(\epsilon_h/\nu + 1/Pr \right)}$$

for the inner wall region and for the outer wall region function S is given by

$$S(R) = 4 \cdot \frac{(\alpha - 1)}{\alpha} \cdot \frac{r_2^+}{Re} \cdot \frac{u_2^+}{\left(\epsilon_h/\nu + 1/Pr \right)}$$

Further details on functions Z and S appear in Appendix V.

APPENDIX III

ON THE EVALUATION OF EIGEN-CONSTANT

The derivation of the expression for the eigen-constant (sometimes called series-constant) requires the application of the orthogonality of the eigen-functions. A detailed description of the Sturm-Liouville system can be found in reference(20). The norm of the eigen-function is given by

$$\left\| \phi_n \right\| = \sqrt{\int_{R_1}^{R_2} \left[4 \frac{\alpha - 1}{S'} \cdot \frac{r_i^+ \, u_i^+}{Re} \cdot R \right] \phi_n^2 \, dR} \qquad \text{(III.1)}$$

where , $S' = \begin{cases} 1 & \text{for the inner region} \\ \alpha & \text{for the outer region} \end{cases}$

Now , if ϕ_1 , ϕ_2 etc. are the eigen-functions which are orthogonal on the interval $R_1 \leq R \leq R_2$ with respect to the weight function (the term within the bracket in Equation (III.1)) and if a given function f(R) can be represented by a series , $f(R) = \sum_{n=1}^{\infty} C_n \phi_n(R)$ then the eigen-constants are given by

$$C_n = \frac{1}{\left\| \phi_n \right\|^2} \int_{R_1}^{R_2} \left(4 \frac{\alpha - 1}{S'} \cdot \frac{r_i^+ \, u_i^+}{Re} R \right) f(R) \, \phi_n \, dR$$

or,

$$C_n = \frac{\int_{R_1}^{R_2} r_j^+ \; u_j^+ \; Rf(R)\,\phi_n \; dR}{\int_{R_1}^{R_2} r_j^+ \; u_j^+ \; R\,\phi_n^2 \; dR} \tag{III.2}$$

The function f(R) is obtained by applying the boundary condition $T(0,R) = 1$. From Equation (3.19) in Chapter 3 and this boundary condition it follows that $f(R) = 1$. Therefore,

$$C_n = \frac{\int_{R_1}^{R_2} r_j^+ \; u_j^+ \; R\,\phi_n \; dR}{\int_{R_1}^{R_2} r_j^+ \; u_j^+ \; R\,\phi_n^2 \; dR} \tag{III.3}$$

or,

$$C_n = \frac{\int_{R_1}^{R_m} r_1^+ \; u_1^+ \; R\,\phi_n \; dR \; + \; \int_{R_m}^{R_2} r_2^+ \; u_2^+ \; R\,\phi_n \; dR}{\int_{R_1}^{R_m} r_1^+ \; u_1^+ \; R\,\phi_n^2 \; dR \; + \; \int_{R_m}^{R_2} r_2^+ \; u_2^+ \; R\,\phi_n^2 \; dR} \tag{III.4}$$

where $R_m = r_m/(r_2 - r_1)$.

Once the eigen-constants are estimated, a suitable step size is chosen and the computation is continued to find the temperature distribution.

APPENDIX IV

BOUNDARY CONDITIONS OF THE PROBLEM

A. INNER WALL IS AT CONSTANT TEMPERATURE

In this case the core wall of the annulus is maintained at a constant temperature, different from the entrance fluid temperature, and the outer wall is insulated. The reason why the inner wall temperature should be different from the entrance fluid temperature is that the denominator in the definition of T becomes zero. Since heat is being supplied to (or extracted from) the inner wall, T is defined as $T = (t-t_1)/(t_e-t_1)$. The boundary conditions will be

$$t = t_1 \quad \text{on the inner wall}$$

and

$$\frac{\partial t}{\partial r} = 0 \quad \text{on the outer wall}$$

when expressed in nondimensional form, thes are as follows

$$T = 0 \text{ at } R=R_1 \quad \text{i.e.} \quad T(x^+,R_1)=0$$

and

$$\frac{\partial T}{\partial R} = 0 \text{ at } R=R_2 \quad \text{i.e.} \quad \left.\frac{\partial T}{\partial R}\right|_{R=R_2} = 0$$

These can be modified further and written as,

$$\phi_n = 0 \text{ at } R=R_1 \quad \text{i.e.} \quad \phi_n(R_1) = 0$$

and

$$\phi_n' = 0 \text{ at } R=R_2 \quad \text{i.e.} \quad \phi_n'(R_2) = 0$$

The solution of Equations (3.18a) and (3.18b) requires three known conditions - two for the second order differential equations and one for the unknown eigen-value. To be more specific, to estimate the eigen-value from the second order differential equations, one has to know the following :

1) the magnitude of the eigen-function on the wall which is at a constant temperature;

2) the magnitude of the eigen-function or its derivative on the insulated wall.

Now, on the heated wall temperature is maintained constant and the heat transfer rate is also constant at a particular distance along the annulus. The reason is like this : under steady state condition, at a particular x^+, the fluid film adjacent to the heated wall is at a constant temperature and therefore, $\left.\frac{dT}{dR}\right|_{R=R_1}$=constant. This means that on the heated wall ϕ_n' = constant. This constant will be different for different axial locations along the annulus. But it may be taken as unity and the multiplying factor, which is different for different cross sections along the annulus, is absorbed in the eigen-constants for that particular cross section.

Therefore, the boundary conditions can be written as,

$\phi_n = 0$, on the inner wall i.e. $\phi_n(R_1)=0$

and $\phi_n' = 0$, on the outer wall i.e. $\phi_n'(R_2)=0$.

And the third relationship which has been derived for carrying out the numerical work conveniently is

$\phi_n' = 1$, on the inner wall i.e. $\phi_n'(R_1)=1$.

B. OUTER WALL IS AT CONSTANT TEMPERATURE

Proceeding in the similar way as before, the known boundary values for this case can be written as,

$$\phi_n(R_2) = 0$$

$$\phi_n'(R_1) = 0 \quad ,$$

and the third relationship which is needed for the numerical integration is,

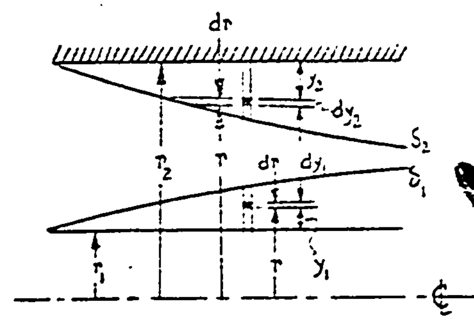
$$\phi_n'(R_2) = 1.$$

APPENDIX V

DETAILS OF GENERATION OF THE FUNCTIONS S AND Z

Before going into the details of the functions S and Z, it is necessary to establish a relationship between y_j^+ and R. This is done as follows.

INNER WALL REGION



From the diagram,

$$r = r_1 + y_1$$

$$\text{or, } R(r_2-r_1) = r_1+y_1$$

$$\text{or, } y_1 = R(r_2-r_1) - r_1 = Rr_1(\alpha -1) - r_1$$

$$\text{or, } y_1^+ = r_1^+\left[(\alpha -1)R - 1\right] \text{ and } \frac{dy_1^+}{dR} = r_1^+(\alpha -1) \qquad (V.1)$$

OUTER WALL REGION

In this case,

$$r = r_2 - y_2$$

$$\text{or, } R(r_2-r_1) = r_2 - y_2$$

$$\text{or, } y_2 = r_2 - R(r_2-r_1) = r_2 - r_2R(1 - 1/\alpha)$$

$$\text{or, } y_2^+ = r_2^+\left[1 - R(1-1/\alpha)\right] \text{ and } \frac{dy_2^+}{dR} = -r_2^+(1-1/\alpha) \qquad (V.2)$$

A. FUNCTION S

The function S as appeared in Appendix II is given by,

$$S = \frac{4(\alpha - 1)\, r_j^+}{Re\, S'} \cdot \frac{u_j^+}{(\epsilon_h/\nu + 1/Pr)} \qquad (V.3)$$

INNER WALL REGION

1. SUBLAYER REGION $0 \leq y_1^+ \leq 26$

In this region velocity and eddy diffusivity relationships are given by

$$u_1^+ = \int_0^{y_1^+} \frac{dy_1^+}{1 + n^2 u_1^+ y_1^+ \left[1 - \exp(-n^2 u_1^+ y_1^+)\right]} \qquad (V.4a)$$

$$\left(\frac{\epsilon_m}{\nu}\right)_1 = n^2\, u_1^+\, y_1^+ \left[1 - \exp(-n^2\, u_1^+\, y_1^+)\right] \qquad (V.4b)$$

$$\sigma = 0.968\, \alpha^{0.045}\, y_1^{+\;0.031} \qquad (V.4c)$$

And finally S is evaluated from,

$$S = \frac{4(\alpha - 1)\, r_1^+}{Re} \cdot \frac{u_1^+}{(\epsilon_h/\nu + 1/Pr)} \qquad (V.5)$$

2. TURBULENT CORE REGION $26 < y_1^+ < \delta_1^+$

In the turbulent core region velocity and eddy diffusivity are given by,

$$u_1^+ = -\frac{3\,\delta_1^+}{A_1}\cdot \ln\left[\frac{K_1\,\delta_1^+ - 4K_1\,\delta_1^+\,\eta_1^{*2} - A_1}{K_1\,\delta_1^+ - 4K_1\,\delta_1^+\,\eta_1^{*2} + A_1}\right] + 12.8$$

$$+\frac{3\,\delta_1^+}{A_1}\cdot \ln\left[\frac{K_1\,\delta_1^+ - 4K_1\,\delta_1^+\,\eta_1^{2} - A_1}{K_1\,\delta_1^+ - 4K_1\,\delta_1^+\,\eta_1^{2} + A_1}\right] \qquad (V.6a)$$

where, $A_1 = (\,9K_1^2\,\delta_1^{+2} + 48K_1\,\delta_1^+\,)^{\frac{1}{2}}$.

$$\left(\frac{\epsilon_h}{\nu}\right)_1 = \frac{0.968\;K_1\,\alpha^{0.045}\;\delta_1^+}{6}\,y_1^{+0.031}\left(1+\eta_1^{*2} - 2\,\eta_1^{*4}\right) \qquad (V.6b)$$

where η_1^* and η_1 in Equations (V.6a) and (V.6b) are defined as

$$\eta_1^* = (\,\delta_1^+ - y_1^+)/\,\delta_1^+ \quad \text{and} \quad \eta_1 = (\,\delta_1^+ - 26)/\,\delta_1^+$$

Once velocity and eddy diffusivity are evaluated S is calculated from Equation (V.5). It has been found that it is convenient to calculate these two values by separate calling programs and then substitute these in the expression for S.

3. POTENTIAL FLOW REGION $\delta_1^+ \leq y_1^+ \leq (r_m^+ - r_1^+)$

For a particular combination of radius ratio, Reynolds number and Prandtl number, the function S has a constant value in the potential flow region as far as a certain cross section is concerned. This is given by,

$$S = \frac{4(\alpha - 1)r_1^+}{Re} \cdot \frac{1}{(0.968\, K_1\, \alpha^{0.045}\, \delta_1^{+1.031})/6 + 1/Pr} \cdot$$

$$\left[12.8 + \frac{3\,\delta_1^+}{A_1} \left\{ \ln\left(\frac{K_1\,\delta_1^+ - 4K_1\,\delta_1^+\,\eta_1^2 - A_1}{K_1\,\delta_1^+ - 4K_1\,\delta_1^+\,\eta_1^2 + A_1} \right) - \ln\left(\frac{K_1\,\delta_1^+ - A_1}{K_1\,\delta_1^+ + A_1} \right) \right\} \right] \qquad (V.7)$$

OUTER WALL REGION

Depending on the radial distance of a particular point, an expression for S can be established in the same way as in the case of inner wall region except with the folling changes:

i) function S for the outer wall region is defined as

$$S = \frac{4(\alpha - 1)r_2^+}{Re} \cdot \frac{u_2^+}{(\epsilon_h/\nu + 1/Pr)}$$

ii) eddy diffusivity ratio for the outer wall region is constant throughout the radial cross section and is given by,

$$\sigma = 0.968\,\alpha^{0.045}\,\delta_1^{+0.031}$$

iii) subscript 1 in all the expressions for velocity and eddy diffusivity of momentum is to be replaced by 2 for the outer wall region.

B. FUNCTION Z

The function Z has been defined in Appendix II as follows:

$$Z = \frac{\frac{d}{dR}\left[R(\epsilon_h/\upsilon + 1/Pr)\right]}{R(\epsilon_h/\upsilon + 1/Pr)}$$

or,

$$Z = \frac{1}{R} + \frac{\frac{d}{dR}\left[(\epsilon_h/\upsilon + 1/Pr)\right]}{(\epsilon_h/\upsilon + 1/Pr)}$$

or,

$$\boxed{Z = \frac{1}{R} + \frac{\frac{d}{dy_j^+}(\epsilon_h/\upsilon)}{(\epsilon_h/\upsilon + 1/Pr)}\,\frac{dy_j^+}{dR}} \qquad \text{(V.8)}$$

INNER WALL REGION

1. SUBLAYER REGION $0 \leq y_1^+ \leq 26$

In this case velocity and eddy diffusivity relationships are the same as those given by Equations (V.4a), (V.4b) and (V.4c). In actual numerical work, velocity in the sublayer region has been obtained by a separate subprogram which solves a trancedental equation. The accuracy in the velocity obtained by the trial and error method is specified as 0.01% . For a certain y_1^+, once velocity and eddy diffusivity are determined, these values are substituted in the Equation (V.8) to obtain the value of Z.

2. TURBULENT CORE REGION $26 < y_1^+ < \delta_1^+$

In this region eddy diffusivity relationship is given by,

$$\left(\frac{\epsilon_m}{\nu}\right)_1 = \frac{K_1 \delta_1^+}{6} (1 - \eta_1^{*2}) (1 + 2 \eta_1^{*2})$$

$$\sigma = 0.968 \, \alpha^{0.045} \, y_1^{+0.031}$$

$$\left(\frac{\epsilon_h}{\nu}\right)_1 = \frac{0.963 \, K_1 \alpha^{0.045} \, \delta_1^+}{6} \, y_1^{+0.031} (1 + \eta_1^{*2} - 2 \eta_1^{*4})$$

(V.9a)

By differentiating Equation (V.9a) with respect to y_1^+,

$$\frac{d\left(\frac{\epsilon_h}{\nu}\right)_1}{dy_1^+} = B\left[0.031 y_1^{+\,-0.969}(1+\eta_1^{*2}-2\,\eta_1^{*4})+\frac{2y_1^{+0.031}}{\delta_1^+}(4\,\eta_1^{*3}-\eta_1^{*})\right]$$

where, $B = 0.968\ K_1\,\alpha^{0.045}\,\delta_1^+ / 6$. (V.9b)

Therefore , Z is given by

$$Z = \frac{r_1^+\,(\alpha-1)}{y_1^+ + r_1^+} + \frac{0.968\ K_1\,\alpha^{0.045}\,\delta_1^+\,(\,E+F\,)H}{6\,(\,G+1/Pr\,)} \qquad (V.10)$$

where,

$$E = 0.031\ y_1^{+\,-0.969}(1+\eta_1^{*2}-2\,\eta_1^{*4})$$

$$F = 2\ y_1^{+0.031}(\,4\,\eta_1^{*3}-\eta_1^{*})/\,\delta_1^+$$

$$G = B\ y_1^{+0.031}(\,1+\eta_1^{*2}-2\,\eta_1^{*4}\,)$$

$$H = r_1^+\,(\alpha-1\,)$$

3. POTENTIAL FLOW REGION $\delta_1^+ \leqslant y_1^+ \leqslant (r_m^+ - r_1^+)$

Since eddy diffusivity of heat in the potential flow region is invariant with the radial distance , the second term in Equation (V.8) vanishes and hence Z is given by,

$$Z = r_1^+\,(\alpha-1)/(\,y_1^+ + r_1^+\,) \qquad (V.11)$$

OUTER WALL REGION

4. POTENTIAL FLOW REGION $\delta_2^+ \leq y_2^+ < \delta_{2max}^+$

As in the case of the inner wall region, eddy diffusivity within this potential flow region is constant and hence the second term in the expression for Z drops out. Thus Z is given by,

$$Z = \left[(\alpha - 1) r_2^+\right] / \left[\alpha (r_2^+ - y_2^+)\right] \qquad (V.12)$$

5. TURBULENT CORE REGION $26 < y_2^+ < \delta_2^+$

Eddy diffusivity within the turbulent core region is given by,

$$\left.\frac{\epsilon_m}{\nu}\right|_2 = \frac{0.968\ K_2\ \delta_1^{+0.031}\ \delta_2^+\ \alpha^{0.045}}{6} (1 + \eta_2^{*2} - 2\eta_2^{*4}) \qquad (V.13)$$

From Equation (V.13),

$$\frac{d\left(\frac{\epsilon_m}{\nu}\right)_2}{dy_2^+} = \frac{0.968\ K_2\ \delta_1^{+0.031}\ \delta_2^+ \alpha^{0.045}}{6} (8\eta_2^{*3} - 2\eta_2^*) / \delta_2^+ \qquad (V.14)$$

Thus,

$$Z = \frac{(\alpha - 1)\ r_2^+}{(r_2^+ - y_2^+)} + \frac{2(\eta_2^* - 4\eta_2^{*3})\ r_2^+\ (1 - 1/\alpha)\ M}{\delta_2^+ (1 + \eta_2^{*2} - 2\eta_2^{*4}) M + 1/Pr} \qquad (V.15)$$

where, $M = 0.968\ K_2\ \delta_1^{+0.031}\ \alpha^{0.045} / 6$

6. SUBLAYER REGION $0 \leqslant y_2^+ \leqslant 26$

Velocity and eddy diffusivity relationships within this sublayer region is given by,

$$\left(\frac{\epsilon_m}{\nu}\right)_2 = n^2 u_2^+ y_2^+ \left[1 - \exp(-n^2 u_2^+ y_2^+)\right], \quad n^2 = 0.0154 \tag{V.16a}$$

$$\sigma = 0.968\, \delta_1^{+0.031} \alpha^{0.045} \tag{V.16b}$$

$$u_2^+ = \int_0^{y_2^+} \frac{dy_2^+}{1 + n^2 u_2^+ y_2^+ \left[1 - \exp(-n^2 u_2^+ y_2^+)\right]} \tag{V.16c}$$

The same procedure as in the case of the inner wall sublayer region has to be used in this case to obtain the value of the function Z.

APPENDIX VI

ON THE NUMERICAL METHOD FOR THE EIGEN-VALUE PROBLEM

The choice of a particular numerical method for solving a differential equation is rather difficult as each method both advantages as well as disadvantages. Basically, the three important factors to be considered before making any final decision are

i) the accuracy required,

ii) the ease with which the method can be expressed in computer language and then executed,

iii) the flexibility as to its usability as a forward as well as backward integration method.

When all these three factors are considered together, Runge-Kutta method seems to be a wise choice. The accuracy of a step by step solution of a differential equation is often difficult to determine. The Euler's method of solving differential equation is simple and self starting, however, it was abandoned because the accuracy obtained is poor and the truncation error is much larger than that in any other method. The predictor-corrector method (e.g. Milne's method) was discarded as it is generally unstable and not self starting.

The Runge-Kutta method , on the other hand , is simple and self starting. This means that once the boundary values are known , the solution for the succeeding points is straightforward. It does not require evaluation of derivatives beyond the first derivative and is also stable. The accuracy in the Runge-Kutta method can be increased by increasing the number of steps and this can be done at any stage of integration. However , a decrease of step size adds to the computation time and also increases the possible round-off error. The latter can be minimised by using double precision. The Runge-Kutta method has fewer formulas and it is not complicated. In addition to all these , the method can be used for backward integration just by changing the sign of the increment in the independent variable.

Any differential equation of higher degree can be reduced to a system of first order equations. Therefore , to solve a second order differential equation one can solve for the first derivative and then it can be solved as a first order differential equation. Instead of doing this , Collaz(21) has suggested a Runge-Kutta scheme for solving differential equation of second order. He has also mentioned a rule of thumb for finding a reasonably good step size. According to him , step size should be such that the the second and the third coefficients in the Runge-Kutta scheme are approximately equal. In the present work, however, the effect of step has been investigated for every possible case. It is obvious that the accuracy of computation can be increased by decreasing step size. But this can not be done indefinitely for the following reasons:

i) with a decrease of step size, the storage requirement for the computer increases,

ii) the computation time also increases with decreasing step size,

iii) with an increase number of steps, the computation suffers from truncation error. These errors are propagated and accumulated in the step by step computation.

Unlike predictor-corrector formula, Runge-Kutta method has a disadvantage. It is not possible in this method to estimate the error at each stage of calculation. However, this can be overcome by varying the number of steps in a wide range until the magnitude of the required result converges, where ever possible, to a satisfactory value.

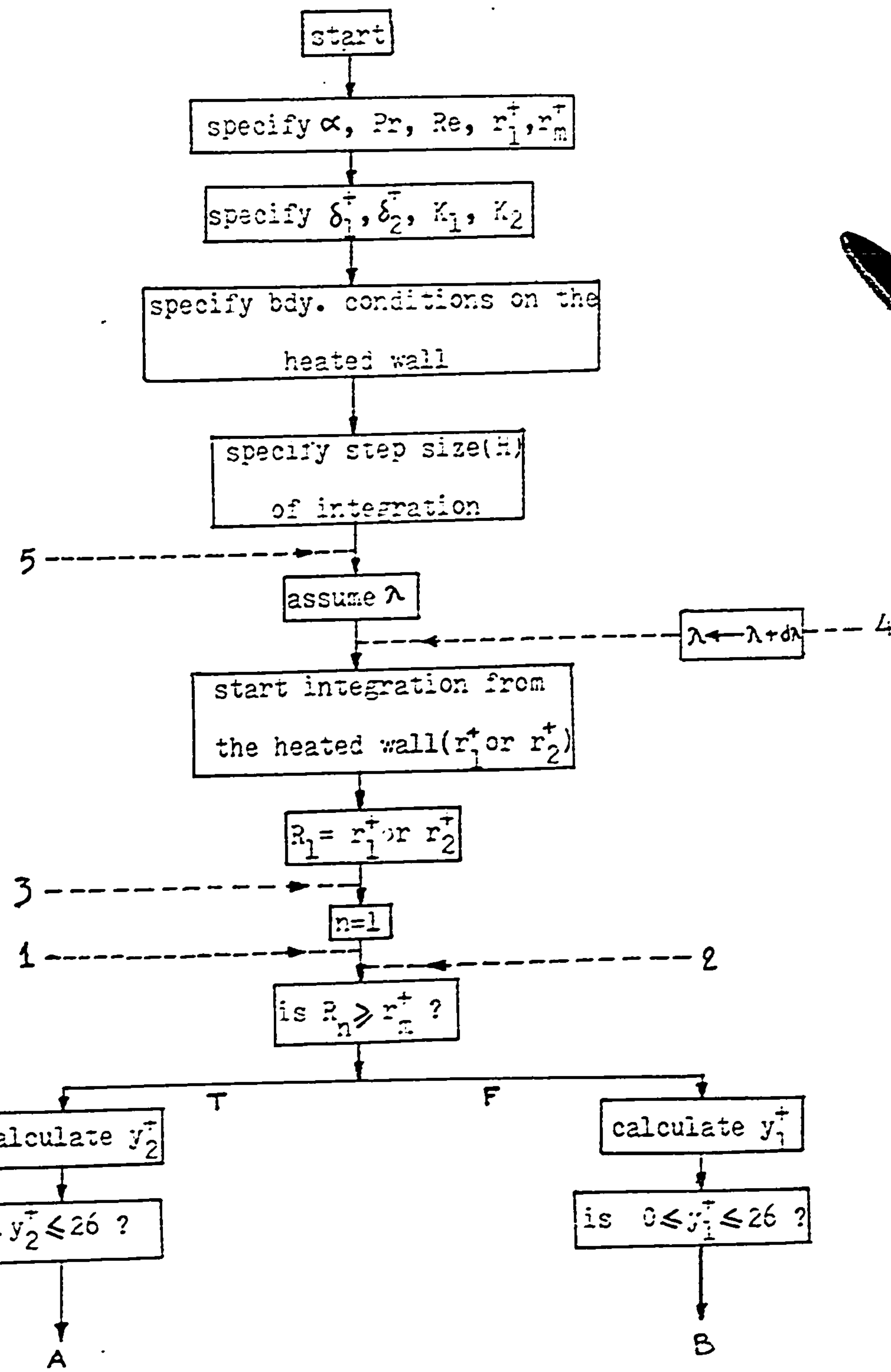
It has been observed that the effect of step size is very important in the evaluation of the eigen-constants. The calculation of the eigen-constants requires a correct estimate of the eigen-values. Therefore, special care has also to be taken to estimate the eigen-values.

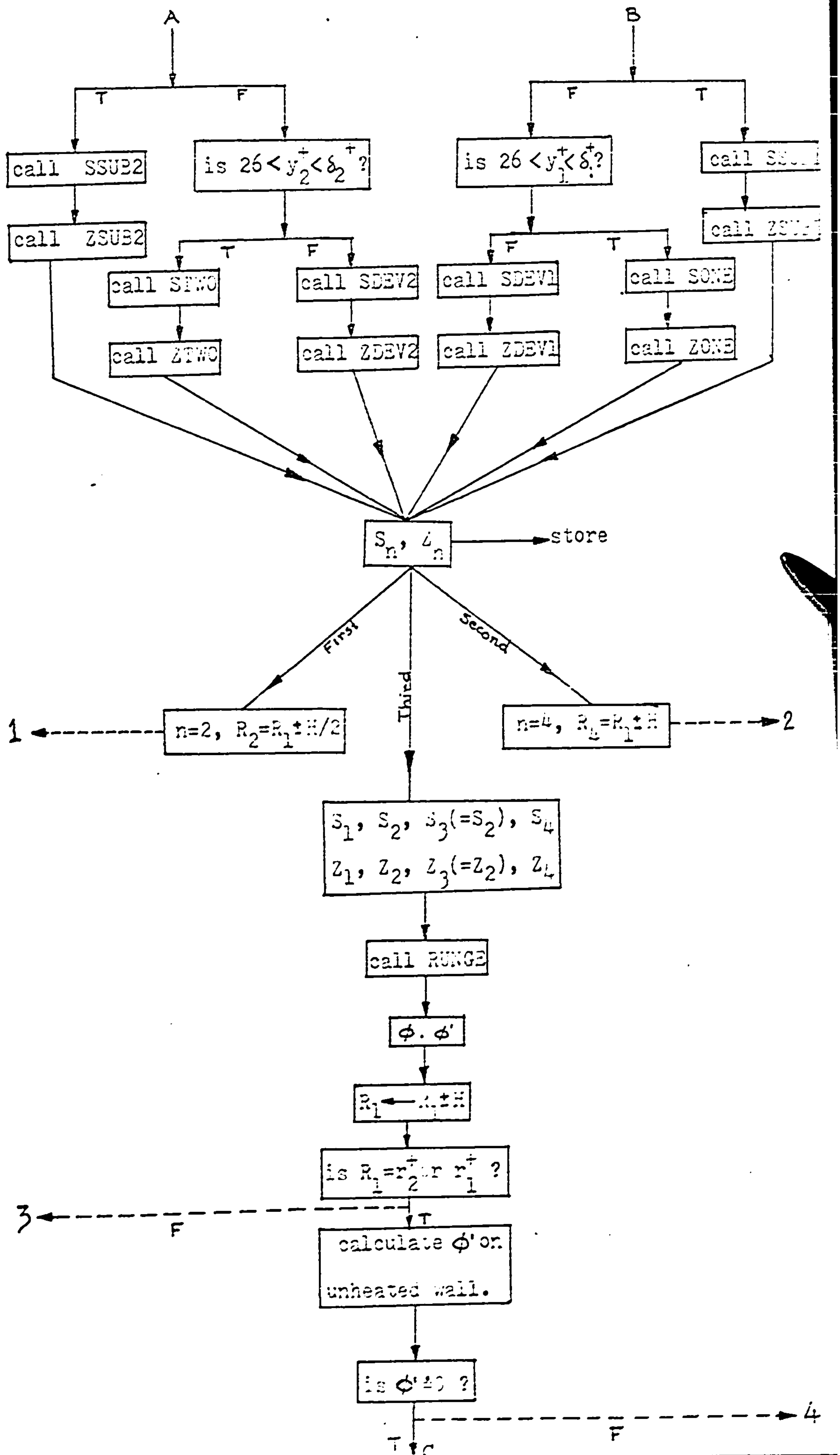
The calculation of the eigen-values involves a trial and error procedure. First, the eigen-value is assumed and the integration is started from the heated wall of the annulus. If the magnitude of the eigen-function on the insulated wall satisfies the required boundary condition, the assumed eigen-value is a correct one. It is apparent that depending on the number of steps in the integration these eigen-values will be different. Thus, for the estimation of a single eigen-value, one has to investigate the effect of step size very carefully. As a matter of fact this is the most lengthy operation in the whole work. Once the eigen-values are estimated by the above way, these are fed into the computer to calculate the corresponding eigen-constants. The step size in each case is again varied widely until a satisfactorily steady character of the required result is observed.

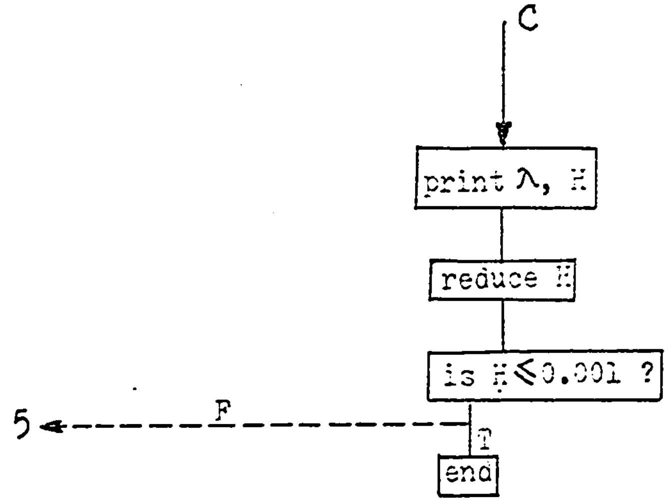
APPENDIX VII

1. A simplified Flow Diagram for a single Eigen-value

Main Program




A
B
T
F
call SSUB2
call ZSUB2
is 26 < y2+ < δ2+ ?
call STWO
call ZTWO
call SDEV2
call ZDEV2
is 26 < y1+ < δ1+ ?
call SDEV1
call ZDEV1
call SONE
call ZONE
Sn, Zn
store
First
Second
Third
n=2, R2=R1±H/2
1
n=4, R4=R1±H
2
S1, S2, S3(=S2), S4
Z1, Z2, Z3(=Z2), Z4
call RUNGE
φ. φ'
is R1=r2+ or r1+ ?
3
calculate φ' on unheated wall.
4
C



Formulae for different calling programs are given below: (details appear in appendix V)

Name of Calling Program	Name of Function	Radial location
	Outer Wall Region	
SSUB2	'S'	Sublayer region
ZSUB2	'Z'	,,
STWO	'S'	Turbulent core
ZTWO	'Z'	,,
SDEV2	'S'	Potential Flow
ZDEV2	'Z'	,,

contd.

continued from last page

Inner Wall Region

SDEV1	'S'	Potential Flow Regi
ZDEV1	'Z'	"
SONE	'S'	Turbulent core "
ZONE	'Z'	"
SSUB1	'S'	Sublayer Region
ZSUB1	'Z'	"

FUNCTION 'RUNGE'

Input:-

i) Boundary conditions on the heated wall (ϕ and ϕ')

ii) Step size of Integration (H)

iii) Eigen value (λ)

iv) Functions 'S' and 'Z' (i.e. S_1 to 4 and Z_1 to 4)

v) The second order differential equation

Output:-

i) Weighted coefficients in the Runge-Kutta scheme(k_1 & k'_1

ii) ϕ and ϕ' for the following step of integration.

The Runge-Kutta scheme of integration is given below:

$$\phi_1 = \phi_1 + H.\phi_1' + k_1$$

$$\phi_1' = \phi_1' + k_1'/H$$

$$\phi_1'' = -Z_1.\phi_1' - S_1.\lambda^2.\phi_1$$

$$k_1 = H^2.\phi_1''/2$$

$$\phi_2 = \phi_1 + H.\phi_1'/2 + k_1/4$$

$$\phi_2' = \phi_1' + k_1/H$$

$$\phi_2'' = -Z_2.\phi_2' - S_2.\lambda^2.\phi_2$$

$$k_2 = H^2.\phi_2''/2$$

$$\phi_3 = \phi_2$$

$$\phi_3' = \phi_1' + k_2/H$$

$$\phi_3'' = -Z_2.\phi_3' - S_2.\lambda^2.\phi_3$$

$$k_3 = H^2.\phi_3''/2$$

$$\phi_4 = \phi_1 + H.\phi_1' + k_3$$

$$\phi_4' = \phi_1' + 2.k_3/H$$

$$\phi_4'' = -Z_4.\phi_4' - S_4.\lambda^2.\phi_4$$

$$k_4 = H^2 \phi''_4/2$$

$$k_1 = (k_1 + k_2 + k_3)/3$$

$$k'_1 = (k_1 + 2k_2 + 2k_3 + k_4)/3$$

Communication with the main program and other subprogram:-

Suppose it is necessary to evaluate the eigen-function and its slope at a certain radial location given by R=0.078 and suppose it is located at the 79th. step of calculation where step size is 0.001 (=H). Before calling the subprogram 'RUNGE' , the main program and other subprograms must supply the followings:

i) the second order differential equation to be integrated and the value of λ ,

ii) ϕ and ϕ' at the 78th. step where R=0.077

iii) coefficients in the Runge-Kutta scheme (k_1 & k'_1) at the 78th. step,

iv) the functional value of S and Z at R=0.077 , 0.0775 and 0.078 (these are designated by S_1 , S_2 , S_4 and Z_1 , Z_2 , Z_4 respectively).

Once these are known the Runge-Kutta program is called to evaluate the followings:

i) ϕ , ϕ' , and ϕ'' for all the four coefficients in the Runge-Kutta scheme,

ii) coefficients k_1, k_2, k_3 and k_4,

iii) k_1 and k_1' at the 79th. step.

When k_1 and k_1' at the 79th. step are known the eigen-function and its slope at that step (where R=0.078) are given by,

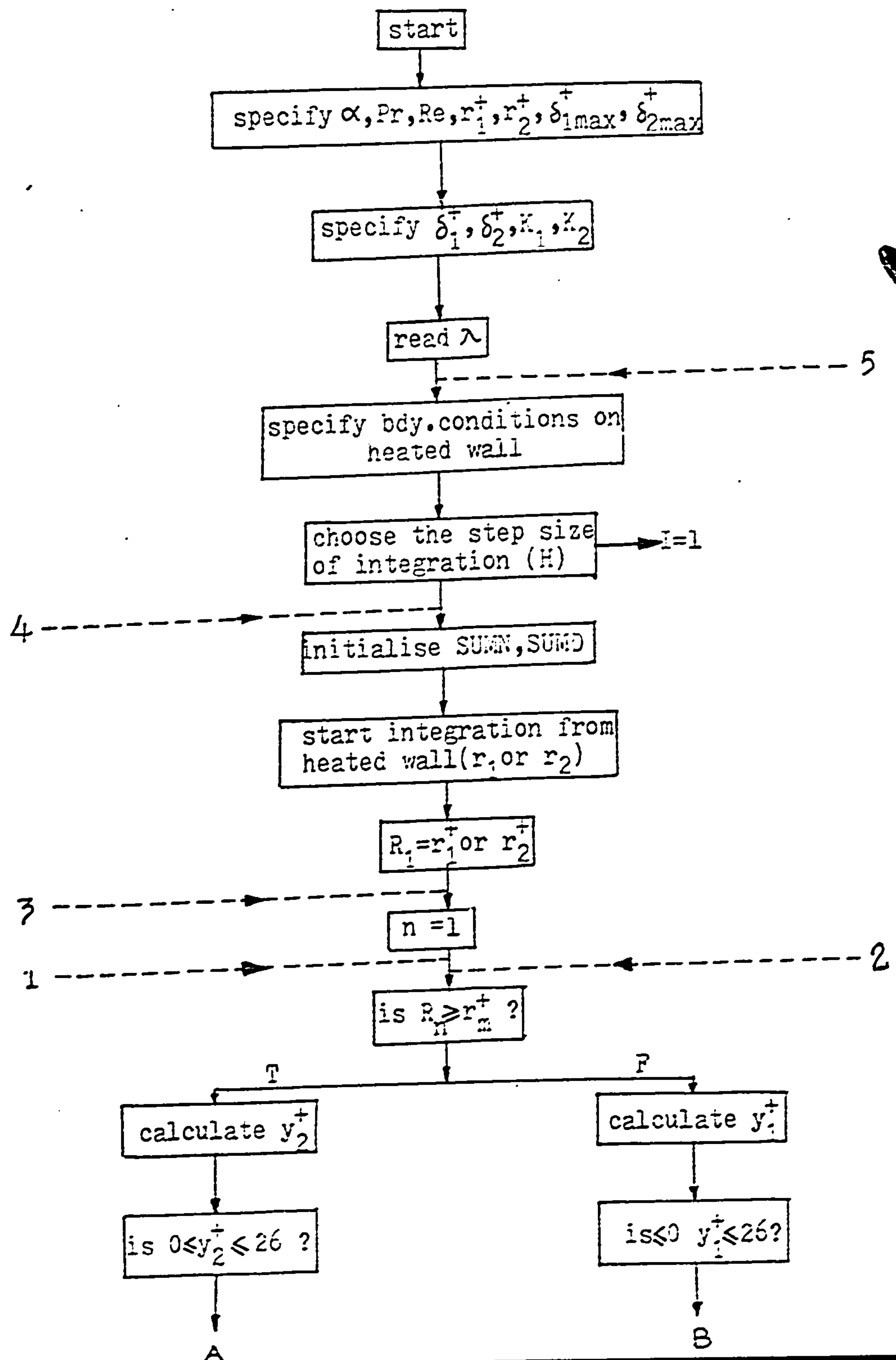
$$\phi(79) = \phi(78) + 0.001\,\phi_1'(78) + k_1(79)$$

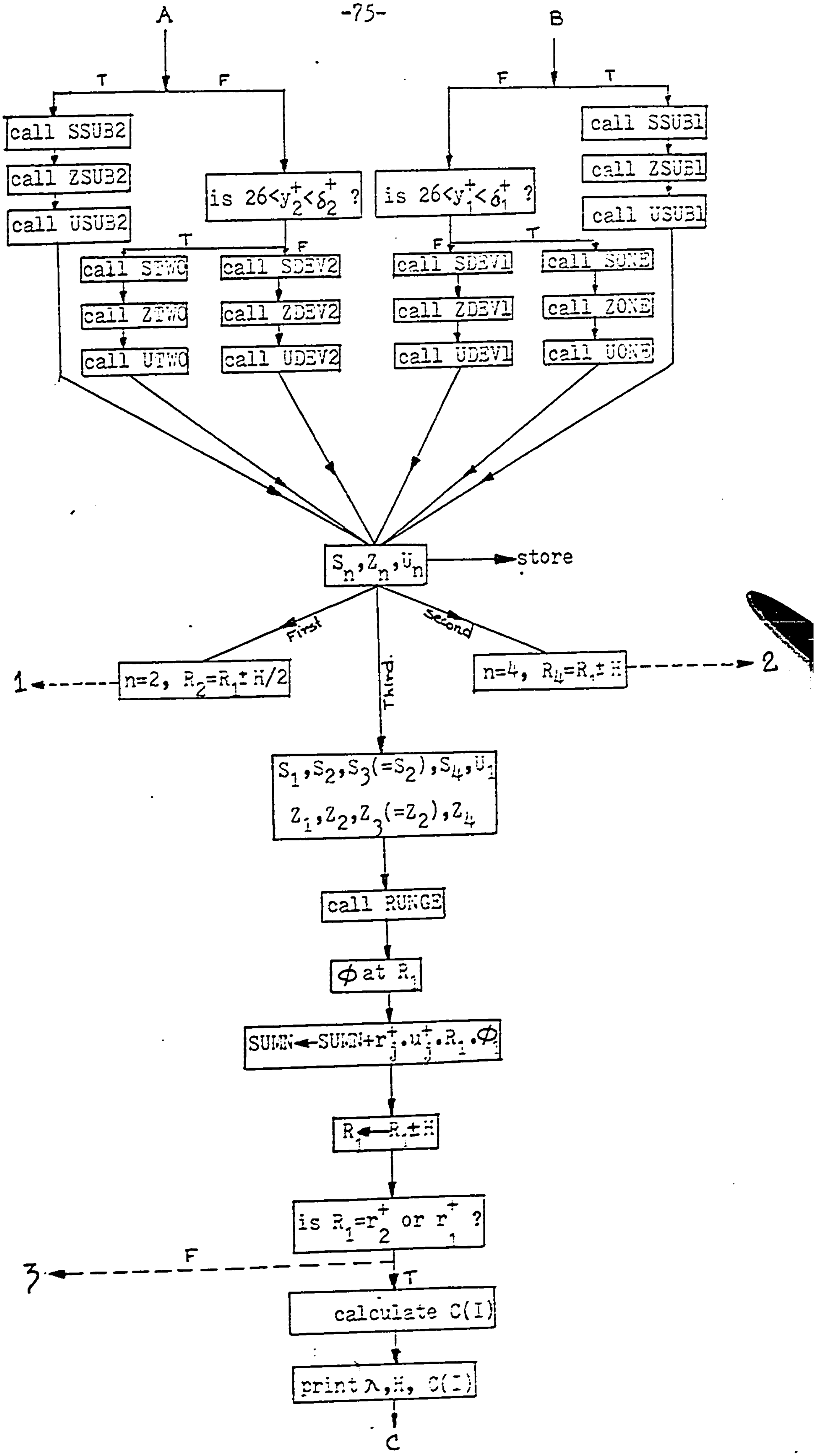
and

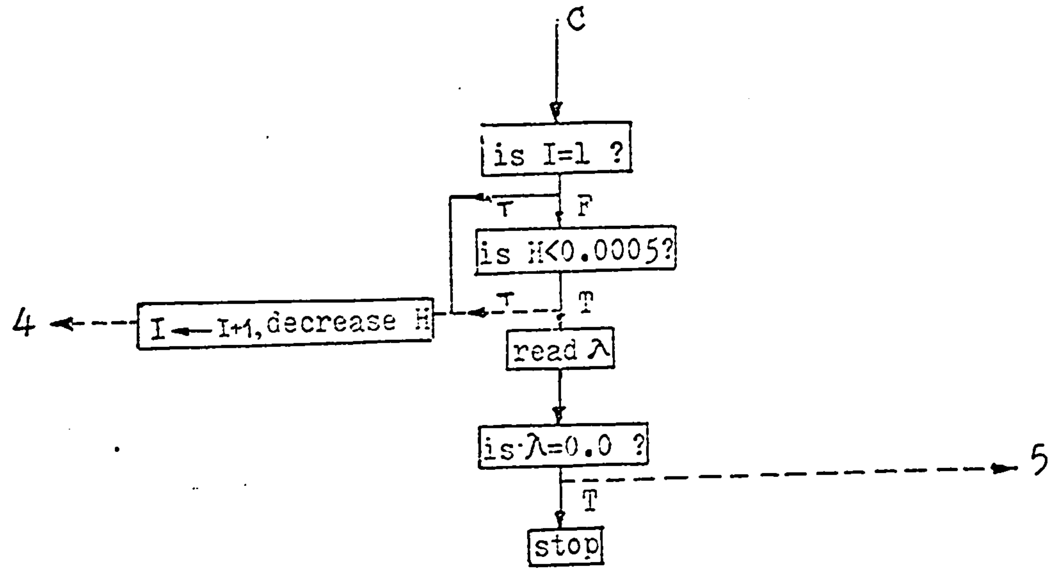
$$\phi'(79) = \phi'(78) + k_1'(79)/0.001.$$

2. A Simplified Flow Diagram For Eigen-Constant

Main Program




A
B
T
F
call SSUB2
call ZSUB2
call USUB2
is 26<y2+<δ2+ ?
is 26<y1+<δ1+ ?
call SSUB1
call ZSUB1
call USUB1
call STWO
call ZTWO
call UTWO
call SDEV2
call ZDEV2
call UDEV2
call SDEV1
call ZDEV1
call UDEV1
call SONE
call ZONE
call UONE
Sn, Zn, Un
store
First
Second
Third
n=2, R2=R1± H/2
n=4, R4=R1± H
1
2
S1, S2, S3(=S2), S4, U1
Z1, Z2, Z3(=Z2), Z4
call RUNGE
Φ at R1
SUMN←SUMN+rj+·uj+·R1·Φ
R1←R1±H
is R1=r2+ or r1+ ?
3
calculate C(I)
print λ, H, C(I)
C



Notes:

1. A description of the Calling Programs has been given with the previous flow chart. U stands for velocity.
2. The upper sign corresponds to the case where the inner wall is heated and the lower sign corresponds to the case where the outer wall is heated.
3. The flow diagram for Temperature distribution and Nusselt Number is a slight extension of the above flow diagram.

TABLE: 1

Relationship between Reynolds Number and r_2^+ for different radius-ratio

α =1.010	r_2^+	4000	90000	250000	660000	1240000
	Re	12276.4	31689.9	101401.3	300152.1	603040.9
α =1.614	r_2^+	800	2000	7000	20000	40000
	Re	9084.6	26703.0	111086.0	357995.2	769188.8
α =2.310	r_2^+	600	2000	5000	10000	30000
	Re	10463.1	42482.4	119736.2	259423.5	872398.8
α =3.834	r_2^+	600	1000	3000	9000	20000
	Re	14416.3	26120.0	91237.3	310958.4	750309.1
α =5.625	r_2^+	400	1000	3000	8000	20000
	Re	10144.8	29601.6	102965.4	307486.2	844065.0
α =9.370	r_2^+	400	1000	3000	6000	20000
	Re	11222.1	32592.7	113125.8	245114.0	925227.2
α =15.32	r_2^+	400	800	3000	7000	40000
	Re	11843.7	26570.7	119002.7	305863.4	2074510.6
α =1000	r_2^+	400	2000	20000		
	Re	12776.2	81008.7	1042779.2		

TABLE 2

CORELATION AMONG HYDRODYNAMIC BOUNDARY LAYER THICKNESSES, MIXING LENGTH CONSTANT AND AXIAL DISTANCE FROM ENTRANCE.

RADIUS RATIO=5.625, REYNOLDS NUMBER=29601.6

x^+	0.4157	0.4916	0.6228	0.7818	4.8600	7.2589
K_1	0.78561	0.71199	0.67782	0.66136	0.60106	0.59453
δ_1^+	30.3	43.7	57.2	70.6	272.5	353.1
δ_2^+	45.0	65.0	85.0	105.0	405.0	524.9

RADIUS RATIO=5.625, REYNOLDS NUMBER=10144.8

x^+	1.1376	1.3403	1.5258	2.2419	5.8428	6.1453
K_1	0.78561	0.70042	0.68340	0.65061	0.61939	0.61939
δ_1^+	29.1	42.0	48.4	67.8	132.4	136.9
δ_2^+	45.0	65.0	75.0	105.0	205.0	211.9

RADIUS RATIO=5.625, REYNOLDS NUMBER=102965.4

x^+	0.1431	0.2221	0.5364	1.7151	4.4435	8.5729
K_1	0.74179	0.67229	0.62963	0.59779	0.58487	0.57852
δ_1^+	38.0	65.7	141.7	349.2	694.9	1081.9
δ_2^+	55.0	95.0	205.0	505.0	1005.0	1564.8

RADIUS RATIO=5.625, REYNOLDS NUMBER=307486.2

x^+	0.0926	0.3171	0.5879	1.2209	6.0880	9.8220
K_1	0.67229	0.61434	0.60106	0.58807	0.57067	0.56602
δ_1^+	74.2	215.5	356.9	639.6	2123.6	2933.9
δ_2^+	105.0	305.0	505.0	905.0	3005.0	4151.6

RADIUS RATIO=5.625, REYNOLDS NUMBER=844065.0

x^+	0.1251	0.2283	0.4597	2.0018	7.4556	10.830
K_1	0.61602	0.60106	0.58807	0.57223	0.56140	0.55987
δ_1^+	217.9	360.8	646.6	2146.9	5719.2	7396.4
δ_2^+	305.0	505.0	905.0	3005.0	8005.0	10352.5

RADIUS RATIO=1.01 , REYNOLDS NUMBER=29601.6

x^+	0.5465	1.0797	1.7163	4.2599	7.2548	11.0117
K_1	0.40104	0.40104	0.40104	0.40104	0.40104	0.40104
δ_1^+	44.9	74.8	104.7	204.5	304.2	415.5
δ_2^+	45.0	75.0	105.0	205.0	305.0	416.5

RADIUS RATIO=2.31 , REYNOLDS NUMBER=29601.6

x^+	0.5458	1.0274	2.8292	4.8740	7.2475	8.9042
K_1	0.51979	0.50030	0.48772	0.48278	0.48032	0.47910
δ_1^+	45.9	79.3	171.2	254.6	338.1	390.7
δ_2^+	55.0	95.0	205.0	305.0	405.0	467.9

TABLE 3

LIST OF EIGEN-VALUES AND EIGEN-CONSTANTS: INNER WALL IS AT CONSTANT TEMPERATURE AND OUTER WALL IS INSULATED.

PRANDTL NUMBER=0.001, REYNOLDS NUMBER=29601.6 , RADIUS RATIO=5.625

x^+=0.4157

n	λ_n	C_n
1	31.376	3.93555
2	115.45	2.38212
3	200.07	1.28816

x^+=0.4916

n	λ_n	C_n
1	30.788	3.93141
2	113.31	2.38165
3	197.09	1.41019

x^+=0.6228

n	λ_n	C_n
1	30.220	3.92936
2	111.26	2.38 556
3	193.58	1.39091

x^+=0.7818

n	λ_n	C_n
1	29.765	3.92842
2	109.66	2.38923
3	191.01	1.39376

x^+=4.8600

n	λ_n	C_n
1	26.610	3.97106
2	103.48	2.21541
3	176.62	1.24119

x^+=7.2589

n	λ_n	C_n
1	24.9827	3.99159
2	102.92	1.74251
3	175.54	1.18038

PRANDTL NUMBER=0.01 , REYNOLDS NUMBER=29601.6, RADIUS RATIO=5.625

$x^+=0.4157$

n	λ_n	C_n
1	10.086	4.14303
2	37.170	2.47474
3	64.623	1.58432
4	91.114	1.85073
5	116.891	2.49943
6	143.846	2.23024

$x^+=0.4916$

n	λ_n	C_n
1	9.9741	4.12457
2	36.716	2.47598
3	63.851	1.55322
4	90.067	1.85900
5	115.59	2.45742
6	142.30	2.24702

$x^+=0.6228$

n	λ_n	C_n
1	9.8552	4.11884
2	36.286	2.48880
3	63.142	1.52863
4	89.126	1.87450
5	114.44	2.42001
6	140.94	2.24801

$x^+=0.7818$

n	λ_n	C_n
1	9.7701	4.12762
2	35.997	2.50733
3	62.701	1.51812
4	88.576	1.88305
5	113.78	2.39395
6	140.11	2.23376

$x^+=4.8600$

n	λ_n	c_n
1	9.4621	4.69982
2	37.057	2.58415
3	63.326	1.39240
4	89.992	1.73639
5	114.73	2.14953
6	141.72	2.24901

$x^+=7.2589$

n	λ_n	c_n
1	9.2802	4.94421
2	37.984	2.05333
3	64.640	1.29798
4	91.101	1.52626
5	117.45	2.18093
6	143.73	2.39501

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=29601.6 , RADIUS RATIO=5.625

$x^+=0.4157$

n	λ_n	C_n
1	2.0698	17.91473
2	8.1462	9.96249
3	14.173	10.06092
4	19.935	5.59362
5	25.883	7.81812
6	31.713	2.55269
7	37.575	6.53503

$x^+=0.4916$

n	λ_n	C_n
1	2.3365	17.12285
2	9.0671	9.74762
3	15.794	8.02318
4	22.236	6.14870
5	28.752	6.79555
6	35.343	4.30245
7	41.853	5.09944

$x^+=0.6228$

n	λ_n	C_n
1	2.5501	17.13636
2	9.8402	9.95417
3	17.168	6.96053
4	24.194	6.74111
5	31.273	6.38394
6	38.486	5.31577
7	45.548	3.95442

$x^+=0.7818$

n	λ_n	C_n
1	2.7320	17.66935
2	10.533	10.35299
3	18.417	6.43423
4	25.983	7.20197
5	33.587	6.00262
6	41.356	5.93549
7	48.893	2.87950

$x^+=4.8600$

n	λ_n	c_n
1	3.9084	30.00272
2	17.146	11.74804
3	30.225	1.19883
4	42.530	8.10315
5	54.554	1.73964
6	67.074	9.84548
7	78.629	-5.05835

$x^+=7.2589$

n	λ_n	c_n
1	3.9010	33.03728
2	19.169	7.45515
3	33.081	-1.17679
4	46.613	7.27346
5	59.752	0.96103
6	72.797	11.26537
7.	85.776	-6.87283

PRANDTL NUMBER=50 , REYNOLDS NUMBER=29601.6 , RADIUS RATIO=5.625

$x^+=0.4157$

n	λ_n	c_n
1	2.8401	329.98416
2	8.9622	-147.84287
3	14.581	95.26684
4	19.965	-47.22767
5	25.128	29.01965
6	30.406	-12.19549
7	35.573	20.54384

$x^+=0.4916$

n	λ_n	c_n
1	3.2411	326.65410
2	10.291	-177.81426
3	16.727	107.98287
4	22.956	-56.77501
5	28.970	43.36272
6	35.011	-28.67032
7	41.007	23.69306

$x^+=0.6228$

n	λ_n	c_n
1	3.6431	330.77239
2	11.470	-192.24418
3	18.577	112.37282
4	25.527	-60.81300
5	32.226	48.42334
6	38.918	-37.64582
7	45.603	22.87518

$x^+=0.7818$

n	λ_n	c_n
1	4.0595	339.13800
2	12.587	-199.44587
3	20.294	112.76760
4	27.893	-62.24423
5	35.185	49.41712
6	42.431	-42.69485
7	49.653	21.97648

$x^+=4.8600$

n	λ_n	C_n
1	5.2513	345.14325
2	14.632	-197.35241
3	30.253	117.65334
4	38.273	-67.35872
5	46.358	58.25673
6	58.369	-49.15273
7	70.222	27.53981

$x^+=7.2589$

n	λ_n	C_n
1	5.3527	346.23541
2	16.353	-217.62431
3	35.243	125.21430
4	41.127	-75.24315
5	49.892	68.35142
6	63.246	-54.12584
7	88.614	39.25386

PRANDTL NUMBER=1000, REYNOLDS NUMBER=29601.6, RADIUS RATIO=5.625

$x^+=0.4157$

n	λ_n	c_n
1	0.3783	602.19363
2	5.6364	50.86339
3	10.614	1.60131
4	15.642	-8.46632
5	20.845	17.65699
6	25.881	-30.23979
7	31.053	38.88579

$x^+=0.4916$

n	λ_n	c_n
1	0.4589	601.53298
2	6.4531	17.73356
3	12.272	17.37602
4	18.051	-0.32677
5	24.002	7.50642
6	29.905	-8.94879
7	35.836	7.85481

$x^+=0.6228$

n	λ_n	c_n
1	0.5124	601.64088
2	7.1332	1.08759
3	13.624	-11.34612
4	20.031	7.82613
5	26.677	-0.79084
6	33.278	0.13299
7	39.844	-16.81225

$x^+=0.7818$

n	λ_n	c_n
1	0.5476	602.21537
2	7.7452	-7.56602
3	14.852	-27.58262
4	21.826	13.37468
5	29.114	-8.12631
6	36.320	5.03573
7	43.442	-35.47308

$x^+=4.8600$

n	λ_n	c_n
1	0.6315	608.55354
2	14.059	-36.33760
3	27.098	-95.71186
4	38.452	48.34883
5	50.113	-96.80440
6	61.288	60.18626
7	72.407	-293.12602

$x^+=7.2589$

n	λ_n	c_n
1	0.6112	609.30042
2	16.232	-73.00855
3	29.832	-137.24559
4	42.385	47.01091
5	54.457	-140.20930
6	66.583	82.85696
7	76.231	-248.67241

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=10144.8, RADIUS RATIO=5.625

$x^+=1.1376$

n	λ_n	C_n
1	1.7581	15.06981
2	7.3592	5.87969
3	13.027	6.58392
4	18.564	1.13964
5	24.285	5.28829
6	29.981	-0.83713
7	35.605	5.58531

$x^+=1.3403$

n	λ_n	C_n
1	1.9713	14.54538
2	8.0921	6.60144
3	14.376	4.14772
4	20.507	2.50500
5	26.736	3.62162
6	32.970	0.94412
7	38.976	2.62233

$x^+=1.5258$

n	λ_n	C_n
1	2.0720	14.52143
2	8.4361	6.93618
3	15.026	3.48489
4	21.387	2.94321
5	27.794	2.83439
6	34.193	1.58404
7	40.467	1.41834

$x^+=2.2419$

n	λ_n	C_n
1	2.2893	14.83689
2	9.3605	7.67506
3	16.756	1.74959
4	23.597	3.61130
5	30.467	1.00832
6	37.697	2.67434
7	44.583	-1.11682

$x^+=5.8428$

n	λ_n	c_n
1	2.5931	17.13022
2	12.007	5.85338
3	20.626	-2.27173
4	29.120	4.54416
5	37.409	-2.58421
6	45.920	3.90524
7	54.551	-4.78637

$x^+=6.1453$

n	λ_n	c_n
1	2.5967	17.27477
2	12.176	5.54629
3	20.866	-2.51875
4	29.388	4.58063
5	37.850	-2.68521
6	46.377	3.98041
7	55.086	-4.75422

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=102965.4, RADIUS RATIO=5.625

$x^+=0.1431$

n	λ_n	C_n
1	2.4293	19.05384
2	9.3968	13.24124
3	16.313	9.61837
4	22.913	11.31176
5	29.592	7.52290
6	36.273	10.35020
7	42.884	6.60705

$x^+=0.2221$

n	λ_n	C_n
1	2.9247	19.03040
2	11.176	13.02984
3	19.427	7.99801
4	27.319	11.19641
5	35.214	7.59661
6	43.226	11.67087
7	51.071	5.70134

$x^+=0.5364$

n	λ_n	C_n
1	3.8949	25.89132
2	14.674	17.50212
3	25.617	8.07708
4	36.074	15.41168
5	46.556	8.83582
6	57.308	16.19291
7	67.735	4.17054

$x^+=1.7151$

n	λ_n	C_n
1	5.1407	45.18590
2	20.423	28.50220
3	36.390	6.24158
4	51.597	20.86912
5	66.977	10.10010
6	82.314	15.58118
7	97.014	1.19177

$x^+=4.4435$

n	λ_n	c_n
1	5.9820	67.11977
2	26.664	32.25751
3	48.576	3.23750
4	67.960	16.13337
5	88.639	9.42160
6	108.60	15.47375
7	127.96	-1.95281

$x^+=8.5729$

n	λ_n	c_n
1	6.1116	81.24749
2	32.577	16.45611
3	57.226	-2.09387
4	81.016	11.75562
5	104.34	9.63409
6	127.18	20.87081
7	149.85	-3.38353

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=307486.2, RADIUS RATIO=5.625

$x^+=0.0920$

n	λ_n	C_n
1	3.2845	19.31329
2	12.600	11.37659
3	21.841	11.27563
4	30.706	7.81866
5	39.547	12.77791
6	48.503	6.78512
7	57.287	13.25442

$x^+=0.3171$

n	λ_n	C_n
1	4.9748	34.98043
2	18.733	20.18371
3	32.599	15.70918
4	45.846	15.07574
5	58.998	20.99492
6	72.504	15.55650
7	85.538	17.22412

$x^+=0.5879$

n	λ_n	C_n
1	5.9951	51.14651
2	22.790	30.23928
3	39.816	18.40888
4	56.010	24.88690
5	72.256	24.70368
6	88.951	26.09713
7	105.03	15.02117

$x^+=1.2209$

n	λ_n	C_n
1	7.3639	80.19110
2	28.403	48.36350
3	50.188	17.13179
4	70.917	38.73753
5	92.226	26.33816
6	113.88	29.69797
7	134.65	10.29693

$x^+=6.0880$

n	λ_n	c_n
1	9.9345	168.80308
2	46.170	44.17077
3	83.445	7.35633
4	117.48	26.95573

$x^+=9.8220$

n	λ_n	c_n
1	9.9916	191.19220
2	53.617	16.63962

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=844065.0, RADIUS RATIO=5.625

$x^+=0.1251$

n	λ_n	c_n
1	5.1733	86.76370
2	19.546	52.70368
3	34.033	36.99347
4	47.971	38.47690
5	61.838	52.95235
6	75.689	42.84421
7	89.241	36.50007

$x^+=0.2283$

n	λ_n	c_n
1	6.3101	128.67784
2	23.935	75.02549
3	41:877	51.66747
4	58.425	68.54442
5	75.548	62.84629
6	92.978	62.03154
7	109.62	40.87307

$x^+=0.4597$

n	λ_n	c_n
1	7.8633	205.87642
2	30.230	110.26783
3	52.978	61.76612
4	74.688	79.14521
5	96.308	85.15772
6	118.69	73.47097
7	144.083	142.69058

$x^+=2.0018$

n	λ_n	c_n
1	11.190	474.70079
2	47.926	155.79393
3	86.987	44.55886
4	124.58	83.13304

$x^+=7.4556$

n	λ_n	c_n
1	12.480	704.70225
2	72.053	68.53913
3	137.67	90.88201

$x^+=10.8300$

n	λ_n	c_n
1	12.230	747.52795
2	85.560	-149.32374

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=29601.6 , RADIUS RATIO=1.01

$x^{+}=0.5465$

n	λ_n	C_n
1	2.6560	9.15766
2	7.9973	8.15053
3	13.373	7.93675
4	18.794	5.86090
5	24.263	6.16005
6	29.770	3.93222
7	35.312	4.86959

$x^{+}=1.0797$

n	λ_n	C_n
1	3.0606	9.55516
2	9.2148	8.75879
3	15.411	7.79264
4	21.663	6.39941
5	27.972	5.52001
6	34.327	4.35408
7	40.722	3.90278

$x^{+}=1.7163$

n	λ_n	C_n
1	3.3889	10.69899
2	10.215	9.70181
3	17.118	7.90660
4	24.110	6.60743
5	31.183	4.93192
6	38.314	4.32432
7	45.482	3.12440

$x^{+}=4.2599$

n	λ_n	C_n
1	4.1798	15.11852
2	12.762	11.81992
3	21.712	7.26715
4	30.897	6.04525
5	40.112	3.30504
6	49.230	4.27513
7	58.248	1.73633

$x^+=7.2548$

n	λ_n	c_n
1	4.6956	18.89755
2	14.780	11.85107
3	25.566	6.05371
4	36.396	5.82593
5	46.920	2.64028
6	57.326	4.79573
7	67.739	0.84240

$x^+=11.0117$

n	λ_n	c_n
1	5.0947	22.10141
2	16.750	11.34517
3	29.249	5.17784
4	41.163	6.22824
5	52.881	2.10532
6	64.610	5.04692
7	76.161	-0.10296

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=29601.6 , RADIUS RATIO=2.31

$x^+=0.5458$

n	λ_n	C_n
1	2.4706	11.68695
2	8.5405	8.39349
3	14.501	7.85391
4	20.388	6.30467
5	26.372	6.56835
6	32.340	4.22323
7	38.375	4.92705

$x^+=1.0274$

n	λ_n	C_n
1	2.9187	12.41409
2	10.053	9.05548
3	17.116	7.26260
4	24.061	7.29812
5	31.170	5.75079
6	38.230	4.90634
7	45.386	3.13297

$x^+=2.8292$

n	λ_n	C_n
1	3.7142	14.19715
2	12.996	11.25874
3	22.448	6.50406
4	31.756	7.52215
5	41.190	3.59255
6	50.396	4.86524
7	59.778	0.26715

$x^+=4.8740$

n	λ_n	C_n
1	4.1501	21.13070
2	14.999	11.65151
3	26.257	17.59260
4	36.978	7.20864
5	47.968	2.65959
6	58.590	5.43399
7	69.135	-1.39525

$x^+=7.2475$

n	λ_n	c_n
1	4.2315	24.25538
2	16.356	15.34510
3	29.332	11.74770
4	41.289	2.06210
5	53.289	1.24871
6	64.950	5.97871
7	76.663	-2.85397

$x^+=8.9042$

n	λ_n	c_n
1	4.5541	25.85117
2	17.780	10.36739
3	31.018	3.18835
4	43.680	7.87392
5	56.053	1.81201
6	68.348	6.17885
7	80.632	-3.71579

TABLE 4

LIST OF EIGEN-VALUES AND EIGEN-CONSTANTS: OUTER WALL IS AT CONSTANT TEMPERATURE AND INNER WALL IS INSULATED.

PRANDTL NUMBER=0.001, REYNOLDS NUMBER=29601.6, RADIUS RATIO=5.625

$x^+=0.4157$

n	λ_n	C_n
1	52.995	-1.77397
2	129.25	-2.29572
3	208.81	-1.93883

$x^+=0.4916$

n	λ_n	C_n
1	52.041	-1.77809
2	127.03	-2.30864
3	205.18	-1.95980

$x^+=0.6228$

n	λ_n	C_n
1	51.151	-1.78304
2	124.89	-2.32405
3	201.62	-1.98202

$x^+=0.7818$

n	λ_n	C_n
1	50.460	-1.78952
2	123.13	-2.34595
3	198.71	-1.99979

$x^+=4.8600$

n	λ_n	C_n
1	46.601	-1.90376
2	112.78	-2.16750
3	185.97	-1.73599

$x^+=7.2589$

n	λ_n	C_n
1	45.723	-1.90116
2	113.04	-2.11795
3	182.29	-1.71482

$x^+=4.8600$

n	λ_n	C_n
1	16.606	-2.11324
2	40.094	-3.66995
3	66.571	-3.04430
4	90.525	-3.03636
5	117.21	-0.14575
6	141.95	-2.30687

$x^+=7.2589$

n	λ_n	C_n
1	16.700	-2.15534
2	41.280	-4.35403
3	67.043	-1.58102
4	92.865	-2.13297
5	118.75	-0.89652
6	144.71	-2.53925

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=29601.6 , RADIUS RATIO=5.625

$x^+=0.4157$

n	λ_n	C_n
1	3.7575	-9.05665
2	9.0381	-8.39269
3	14.546	-8.15298
4	20.226	-6.74536
5	25.954	-5.87330
6	31.740	-4.90462
7	37.616	-4.65287

$x^+=0.4916$

n	λ_n	C_n
1	4.1778	-9.05357
2	10.053	-9.34463
3	16.206	-8.14658
4	22.440	-7.46849
5	28.803	-6.28431
6	35.335	-5.77857
7	41.700	-4.42618

$x^+=0.6228$

n	λ_n	C_n
1	4.5193	-9.59729
2	10.875	-10.42771
3	17.539	-8. 39117
4	24.241	-8.31928
5	31.166	-6.57827
6	38.180	-6.63941
7	45.224	-4.31123

$x^+=0.7818$

n	λ_n	C_n
1	4.8209	-10.38085
2	11.592	-11.55178
3	18.699	-8.71298
4	25.852	-9.24014
5	33.333	-6.89174
6	40.910	-7.30943
7	48.540	-4.10886

$x^+=4.8600$

n	λ_n	c_n
1	6.9516	-20.96836
2	17.616	-27.16050
3	30.517	-7.51787
4	42.417	-5.93909
5	54.761	10.30503
6	66.383	-4.98553
7	78.290	-4.41688

$x^+=7.2589$

n	λ_n	c_n
1	7.2388	-23.25206
2	19.622	-28.65982
3	33.511	2.78088
4	46.661	-2.59353
5	59.435	1.88707
6	72.250	-5.04994
7	85.198	-0.55379

PRANDTL NUMBER=50 , REYNOLDS NUMBER=29601.6, RADIUS RATIO=5.625

$x^+=0.4157$

n	λ_n	C_n
1	1.7248	-189.13449
2	6.0431	-18.51212
3	10.801	-17.37995
4	15.866	-7.41080
5	20.986	-6.34039
6	26.028	-6.17757
7	31.220	-8.25153

$x^+=0.4916$

n	λ_n	C_n
1	1.9918	-190.16427
2	6.8649	-30.60835
3	12.466	-11.39691
4	18.267	-8.74698
5	24.144	-4.93477
6	30.055	-5.76317
7	35.998	-3.90587

$x^+=0.6228$

n	λ_n	C_n
1	2.1473	-193.69626
2	7.5543	-36.56992
3.	13.825	-7.98837
4	20.250	-10.12643
5	26.799	-4.01486
6	33.424	-5.85571
7	40.002	-0.29608

$x^+=0.7818$

n	λ_n	C_n
1	2.2502	-197.53628
2	8.1637	-40.71647
3	15.041	-6.24523
4	22.048	-11.42741
5	29.222	-3.25529
6	36.470	-5.23060
7	43.597	3.88670

$x^+=4.8600$

n	λ_n	c_n
1	2.5378	-220.57233
2	14.473	-78.04402
3	27.190	35.16009
4	38.797	27.54323
5	50.270	83.45620
6	61.580	-7.34090
7	72.741	-3.64753

$x^+=7.2589$

n	λ_n	c_n
1	2.5478	-223.28466
2	16.478	-70.34801
3	30.064	86.99508
4	42.610	30.19313
5	54.700	34.43628
6	66.885	4.23737
7	79.316	19.43060

PRANDTL NUMBER=1000, REYNOLDS NUMBER=29601.6, RADIUS RATIO=5.625

$x^+=0.4157$

n	λ_n	C_n
1	0.68897	-508.99504
2	5.6883	1.55725
3	10.600	-23.70110
4	15.700	-8.74228
5	20.836	-9.92238
6	25.880	-11.50036
7	31.066	-17.49061

$x^+=0.4916$

n	λ_n	C_n
1	0.79665	-510.56868
2	6.5004	-17.52445
3	12.242	-8.49794
4	18.093	-10.98668
5	23.990	-6.20215
6	29.906	-9.75842
7	35.847	-6.8550

$x^+=0.6228$

n	λ_n	C_n
1	0.85432	-512.66934
2	7.1773	-31.21008
3	13.601	-0.97285
4	20.078	-13.92632
5	26.646	-3.88914
6	33.277	-9.57514
7	39.850	2.61363

$x^+=0.7818$

n	λ_n	C_n
1	0.88973	-514.66360
2	7.7902	-42.34300
3	14.820	2.43531
4	21.881	-16.82424
5	29.070	-1.91785
6	36.320	-7.77925
7	43.434	14.23701

$x^{+}=4.8600$

n	λ_n	C_n
1	0.96088	-526.53498
2	14.213	-141.95634
3	26.955	93.43100
4	38.567	71.99080
5	49.973	181.80904
6	61.223	-11.78445
7	72.146	-14.71155

$x^{+}=7.2589$

n	λ_n	C_n
1	0.97051	-528.31601
2	16.215	-126.86034
3	29.813	192.40194
4	42.336	71.22086
5	54.366	76.10057
6	66.404	20.76219
7	78.214	64.79201

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=10144.8, RADIUS RATIO=5.625

$x^+=1.1376$

n	λ_n	C_n
1	3.2729	-8.45861
2	8.0501	-6.14474
3	13.265	-4.80096
4	18.765	-3.14329
5	24.385	-3.10680
6	30.048	-2.18726
7	35.736	-2.89096

$x^+=1.3403$

n	λ_n	C_n
1	3.6261	-8.46931
2	8.8301	-7.50614
3	14.564	-4.66859
4	20.566	-3.69455
5	26.806	-2.81498
6	33.078	-2.00492
7	39.186	-1.34660

$x^+=1.5258$

n	λ_n	C_n
1	3.7671	-8.64850
2	9.1452	-8.11295
3	15.128	-4.64757
4	21.403	-3.84873
5	27.913	-2.42425
6	34.374	-1.76245
7	40.658	-0.30454

$x^+=2.2419$

n	λ_n	C_n
1	4.0971	-9.39135
2	9.9321	-9.89474
3	16.730	-4.59505
4	23.694	-3.46712
5	30.692	-0.09932
6	37.793	-0.90539
7	44.809	2.09828

$x^+=5.8428$

n	λ_n	c_n
1	4.6901	-11.58540
2	12.297	-12.77769
3	20.864	1.54122
4	29.185	-1.05991
5	37.450	1.63235
6	46.100	-2.58363
7	54.721	1.14061

$x^+=6.1453$

n	λ_n	c_n
1	4.7154	-11.69694
2	12.460	-12.71888
3	21.092	2.15387
4	29.460	-1.22599
5	37.865	1.03759
6	46.513	-2.05134
7	55.329	1.56277

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=102965.4, RADIUS RATIO=5.625

$x^+=0.1431$

n	λ_n	C_n
1	4.4081	-8.85709
2	10.588	-9.17856
3	17.015	-9.36758
4	23.551	-9.20105
5	30.116	-8.96571
6	36.714	-8.70621
7	43.334	-8.48986

$x^+=0.2221$

n	λ_n	C_n
1	5.2133	-10.00938
2	12.560	-11.28980
3	20.226	-10.45776
4	27.912	-10.63976
5	35.761	-10.37770
6	43.565	-10.34973
7	51.370	-8.88858

$x^+=0.5364$

n	λ_n	C_n
1	6.8167	-15.60359
2	16.450	-18.41116
3	26.506	-15.02378
4	36.476	-16.21031
5	46.820	-14.03364
6	57.093	-16.48770
7	67.429	-11.17802

$x^+=1.7151$

n	λ_n	C_n
1	9.3162	-30.89897
2	22.377	-36.98908
3	36.590	-23.91498
4	51.253	-32.67896
5	67.002	-23.09250
6	82.380	-22.50639
7	97.270	-2.41115

$x^+=4.4435$

n	λ_n	c_n
1	11.180	-48.34658
2	27.977	-63.25689
3	49.006	-33.70499
4	68.570	-26.72429
5	89.136	16.27675
6	108.87	-4.06957
7	127.56	9.41609

$x^+=8.5729$

n	λ_n	c_n
1	11.950	-58.78267
2	33.713	-76.67212
3	58.363	3.96115
4	81.800	-4.36079
5	104.49	3.56794
6	126.99	-6.77089

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=307486.2, RADIUS RATIO=5.625

$x^+=0.0926$

n	λ_n	C_n
1	5.7918	-10.55573
2	13.950	-11.77938
3	22.451	-11.32465
4	31.006	-11.56111
5	39.714	-11.69226
6	48.373	-11.70948
7	57.060	-10.98217

$x^+=0.3171$

n	λ_n	C_n
1	8.5981	-21.30033
2	20.805	-25.61004
3	33.534	-22.01780
4	46.170	-22.70996
5	59.271	-22.16162
6	72.088	-23.22970
7	85.006	-17.80606

$x^+=0.5879$

n	λ_n	C_n
1	10.488	-32.66741
2	25.372	-39.26778
3	40.910	-31.10112
4	56.248	-33.14745
5	72.286	-28.93385
6	88.079	-34.62074
7	104.05	-23.01539

$x^+=1.2209$

n	λ_n	C_n
1	13.068	-54.32915
2	31.510	-63.94877
3	51.007	-43.04094
4	70.543	-55.14178
5	91.634	-43.14128
6	112.765	-60.43790
7	133.92	-37.44346

$x^+=6.0880$

n	λ_n	c_n
1	18.069	-123.42943
2	47.437	-163.19616
3	84.381	-76.80610
4	118.11	-60.81118

$x^+=9.8220$

n	λ_n	c_n
1	18.763	-140.19657
2	54.770	-187.78088
3	95.651	2.28267
4	134.52	-6.65900

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=844065.0, RADIUS RATIO=5.625

$x^+=0.1251$

n	λ_n	c_n
1	8.9884	-56.18737
2	21.727	-66.38129
3	35.022	-57.92350
4	48.263	-57.56785
5	61.974	-57.71183
6	75.437	-54.92471
7	89.003	-44.86027

$x^+=0.2283$

n	λ_n	c_n
1	10.944	-88.98246
2	26.470	-105.00462
3	42.730	-84.74852
4	58.853	-81.61190
5	75.693	-76.18286
6	92.235	-72.78115
7	108.90	-50.75099

$x^+=0.4597$

n	λ_n	c_n
1	13.646	-154.68160
2	33.004	-173.19671
3	53.544	-117.39307
4	73.948	-112.70638
5	95.568	-89.46959
6	117.04	-98.31119
7	139.41	-58.79324

$x^+=2.0018$

n	λ_n	c_n
1	19.388	-411.93292
2	49.228	-372.72781
3	85.333	-168.85722
4	122.47	-237.94921

$x^{+}=7.4556$

n	λ_n	c_n
1	21.269	-616.82859
2	72.103	-609.56484
3	133.22	-149.88458

$x^{+}=10.8300$

n	λ_n	c_n
1	21.270	-653.55937
2	81.748	-636.72467
3	147.45	93.61113

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=29601.6 , RADIUS RATIO=1.01

$x^+=0.5465$

n	λ_n	C_n
1	2.6644	-9.25543
2	7.9936	-8.32396
3	13.364	-7.97561
4	18.783	-6.04683
5	24.250	-6.14267
6	29.758	-4.18533
7	35.298	-4.88035

$x^+=1.0797$

n	λ_n	C_n
1	3.0729	-9.61303
2	9.2197	-8.94126
3	15.410	-7.67210
4	21.657	-7.08309
5	27.957	-5.54336
6	34.305	-5.74884
7	40.690	-4.32250

$x^+=1.7163$

n	λ_n	C_n
1	3.3925	-10.69992
2	10.230	-9.99195
3	17.124	-7.64963
4	24.104	-8.52006
5	31.159	-5.42831
6	38.274	-7.84350
7	45.425	-3.72676

$x^+=4.2599$

n	λ_n	C_n
1	4.2205	-14.96000
2	12.801	-14.30294
3	21.709	-8.05958
4	30.848	-17.29115
5	40.028	1.34032
6	49.115	-12.97610
7	58.101	8.79139

$x^+=7.2548$

n	λ_n	c_n
1	4.7620	-18.42311
2	14.784	-20.66272
3	25.525	-6.00523
4	36.301	-21.55918
5	46.768	11.67791
6	57.136	1.20197
7	67.518	2.56410

$x^+=11.0117$

n	λ_n	c_n
1	5.18795	-21.19694
2	16.750	-31.35804
3	29.169	9.43334
4	41.008	-11.64275
5	52.668	3.26501
6	64.278	-1.49666
7	75.895	-1.16189

PRANDTL NUMBER=0.7 , REYNOLDS NUMBER=29601.6 , RADIUS RADIUS=2.31

$x^+=0.5458$

n	λ_n	C_n
1	3.3831	-8.63935
2	8.9049	-8.90166
3	17.648	-12.95152
4	20.483	-6.81791
5	26.395	-6.63812
6	32.331	-5.25298
7	38.344	-4.49566

$x^+=1.0274$

n	λ_n	C_n
1	3.9651	-9.66642
2	10.460	-10.49267
3	17.187	-8.24931
4	24.039	-8.49994
5	31.030	-5.93425
6	38.043	-7.48387
7	45.221	-4.25040

$x^+=2.8292$

n	λ_n	C_n
1	5.0239	-14.16543
2	13.309	-16.25794
3	22.182	-9.95841
4	31.499	-14.09400
5	41.014	-3.58355
6	50.206	-8.73877
7	59.504	6.21956

$x^+=4.8740$

n	λ_n	C_n
1	5.6534	-17.69464
2	15.194	-21.89303
3	26.071	-11.72833
4	37.043	-13.86444
5	47.795	8.83375
6	58.350	-4.25597
7	68.952	5.85914

$x^+=7.2475$

n	λ_n	c_n
1	6.0672	-20.32972
2	16.880	-27.23410
3	29.385	-4.69406
4	41.232	-11.77765
5	53.100	11.80694
6	64.496	0.76300
7	76.451	-5.83454

$x^+=8.9042$

n	λ_n	c_n
1	6.2600	-21.62122
2	17.929	-29.86652
3	31.091	4.09438
4	43.528	-7.15820
5	55.758	4.34841
6	67.989	-2.59587
7	80.251	-3.02353

TABLE 5

ENTRANCE REGION NUSSELT NUMBERS AND ENTRANCE LENGTHS : INNER WALL AT CONSTANT TEMPERATURE AND OUTER WALL INSULATED.

RADIUS RATIO = 5.625 , REYNOLDS NUMBER = 29601.6

Pr = 0.001		Pr = 0.01		Pr = 0.7		Pr = 50		Pr = 1000	
x^+	Nu_x	x^+	Nu_x	x^+	Nu_x	x^+	Nu_x	x^+	Nu_x
0.4157	9.5696	0.4157	17.9997	0.4157	114.9102	0.4157	523.8417	0.4157	1415.0354
0.4916	9.3383	0.4916	17.1236	0.4916	107.5565	0.4916	446.9726	0.4916	1248.5638
0.6228	9.0678	0.6228	15.9945	0.6228	103.7194	0.6228	415.8953	0.6228	1140.2256
0.7818	8.7871	0.7818	[illegible]0269	0.7818	101.0783	0.7818	412.4361	0.7818	1078.4984
4.8600	8.3804	4.8600	10.8528	4.8600	79.4474	4.8600	409.5783	4.8600	1074.2642
7.2789	8.3790	7.2589	10.6649	7.2589	74.5685	7.2589	400.0429	7.2589	1065.8433
8.00	8.3581	8.00	10.5961	9.00	73.3108	9.00	400.0121	9.00	1061.7325
17.00	8.2143	11.00	10.4498	14.00	71.0207	12.00	399.8723	15.00	1059.6231
25.00	8.1756	16.00	10.3885	19.00	69.5807	15.00	399.5231	20.60	1051.5837
34.00	8.1281	21.00	10.3787	22.00	68.9877	20.00	399.4183		
		25.00	10.3772	28.00	68.2027	23.80	399.3107		
		33.00	10.3768	32.30	67.8564				

TABLE 5

ENTRANCE REGION NUSSELT NUMBERS AND ENTRANCE LENGTHS : INNER WALL IS AT CONSTANT TEMPERATURE AND OUTER WALL IS INSULATED.

RADIUS RATIO = 5.625 , PRANDTL NUMBER = 0.7

Re=10144.8		Re=102965.4		Re=307486.2		Re=844065.0	
x^+	Nu_x	x^+	Nu_x	x^+	Nu_x	x^+	Nu_x
1.1376	63.1463	0.1431	1205.7426	0.0926	1864.3287	0.1251	8351.6425
1.3403	56.5351	0.2221	615.7851	0.3171	776.4521	0.2283	3016.3825
1.5258	54.0313	0.5364	301.4516	0.5879	521.3472	0.4597	2063.4371
2.2419	48.6081	1.7151	207.8216	1.2209	455.8211	2.0018	1375.5231
5.8428	40.5135	4.4435	197.0659	6.0880	426.3312	7.4556	1304.7313
6.1453	40.1907	8.5729	181.2379	9.8220	412.6188	10.8300	1128.3862
11.00	38.2508	12.00	177.1171	14.00	408.2625	14.00	1119.4835
15.00	37.1992	16.00	173.8078	19.00	404.5124	29.00	1108.5152
23.00	36.0303	29.00	168.0264	29.00	400.0750	38.00	1101.9261
30.30	35.5690	37.05	166.4473	41.00	397.6459	45.10	1098.3571

TABLE 5

ENTRANCE REGION NUSSELT NUMBERS AND ENTRANCE LENGTHS : INNER WALL IS AT CONSTANT TEMPERATURE AND OUTER WALL IS INSULATED.

REYNOLDS NUMBER = 29601.6 , PRANDTL NUMBER = 0.7

$\alpha = 1.01$		$\alpha = 2.31$	
x^+	Nu_x	x^+	Nu_x
0.5465	88.6293	0.5458	94.9020
1.0797	83.0898	1.0274	88.2901
1.7163	78.4959	2.8292	82.7764
4.2599	66.4131	4.8740	75.2901
7.2548	60.5241	7.2475	71.7802
11.0117	57.9002	8.9042	64.2853
19.00	52.9093	11.00	62.3428
28.00	50.1099	15.00	59.7826
34.00	49.0340	19.00	58.0366
43.00	48.0667	23.00	56.8009
48.00	47.7385	33.00	55.0209
55.76	47.4080	45.30	54.1152

TABLE 6

ENTRANCE REGION NUSSELT NUMBERS AND ENTRANCE LENGTHS : OUTER WALL IS AT CONSTANT TEMPERATURE AND INNER WALL IS INSULATED.

RADIUS RATIO = 5.625 , REYNOLDS NUMBER = 29601.6

Pr = 0.001		Pr = 0.01		Pr = 0.7		Pr = 50		Pr = 1000	
x^+	Nu_x	x^+	Nu_x	x^+	Nu_x	x^+	Nu_x	x^+	Nu_x
0.4157	6.0731	0.4157	14.6172	0.4157	103.5634	0.4157	503.0112	0.4157	1147.4321
0.4916	5.8662	0.4916	13.8954	0.4916	99.8442	0.4916	431.4241	0.4916	1133.6672
0.6228	5.5943	0.6228	12.8753	0.6228	96.8571	0.6228	405.7382	0.6228	1128.0173
0.7818	5.3894	0.7818	11.9662	0.7818	93.9142	0.7818	403.1945	0.7818	1062.5872
4.8600	5.1993	4.8600	7.6812	4.8600	78.6552	4.8600	401.4523	4.8600	1059.3885
7.2589	5.1443	7.2589	7.0803	7.2589	73.4553	7.2589	399.5352	7.2589	1057.4943
8.00	5.1154	8.00	6.9372	9.00	72.9535	9.00	399.3243	9.00	1053.2452
15.00	5.0786	11.00	6.6045	14.00	70.1754	12.00	399.1575	15.00	1045.7813
25.00	5.0134	16.00	6.4762	19.00	65.0132	15.00	399.0123	20.80	1038.5634
35.00	5.0112	21.00	6.4574	22.00	62.7792	18.00	398.9243		
		25.00	6.4541	28.00	59.6944	21.00	398.8471		
		34.00	6.4512	33.76	57.9132	24.10	398.7612		

TABLE 6

ENTRANCE REGION NUSSELT NUMBERS AND ENTRANCE LENGTHS : OUTER WALL IS AT CONSTANT TEMPERATURE AND INNER WALL IS INSULATED.

RADIUS RATIO = 5.625 , PRANDTL NUMBER = 0.7

Re=10144.8		Re=102965.4		Re=307486.2		Re=844065.0	
x^+	Nu_x	x^+	Nu_x	x^+	Nu_x	x^+	Nu_x
1.1376	50.0421	0.1431	1179.3472	0.0926	1753.4283	0.1251	7982.3042
1.3403	48.3038	0.2221	597.5784	0.3171	759.4872	0.2283	2897.2215
1.5258	47.1382	0.5364	292.8953	0.5879	506.0446	0.4597	2031.2738
2.2419	44.4443	1.7151	201.4982	1.2209	441.8402	2.0018	1352.9594
5.8428	42.5426	4.4435	191.8524	6.0880	418.8134	7.4556	1293.2765
6.1453	38.1724	8.5729	175.5196	9.8220	406.2853	10.8300	1119.9812
11.00	34.2534	21.00	169.0113	15.00	403.3726	14.00	1112.4753
15.00	31.9785	26.00	160.0516	20.00	401.5312	28.00	1103.3524
19.00	30.3956	31.00	153.8125	29.00	399.8725	33.00	1101.1426
25.00	28.9253	35.00	150.2397	35.00	398.5825	38.00	1098.7434
31.45	28.0813	39.75	147.1753	42.30	398.4536	46.70	1093.4685

TABLE 6

ENTRANCE REGION NUSSELT NUMBERS AND ENTRANCE LENGTHS : OUTER WALL IS AT CONSTANT TEMPERATURE AND INNER WALL IS INSULATED.

REYNOLDS NUMBER = 29601.6 , PRANDTL NUMBER = 0.7

$\alpha = 1.01$		$\alpha = 2.31$	
x^+	Nu_x	x^+	Nu_x
0.5465	87.9853	0.5458	91.4391
1.0797	82.1156	1.0274	86.1305
1.7163	78.4024	2.8292	81.2225
4.2599	65.5732	4.8740	73.9624
7.2548	59.7456	7.2475	70.5657
11.0117	57.3372	8.9042	63.7903
19.00	52.1571	11.00	61.4781
28.00	49.3891	15.00	58.2762
34.00	48.1658	19.00	57.1893
43.00	47.1897	23.00	56.0321
48.00	46.9762	33.00	54.8235
56.81	46.5681	46.12	53.9233

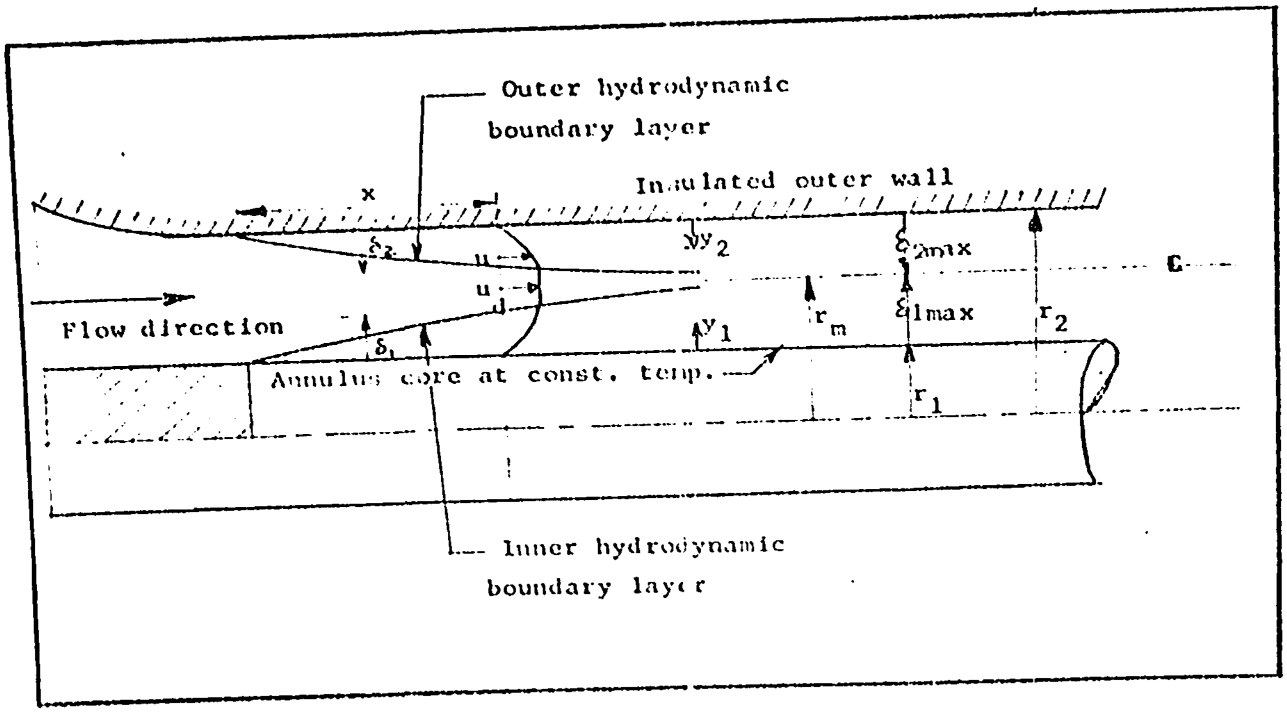


Fig. 1 Idealized model showing the development of hydrodynamic boundary layers in an Annulus.

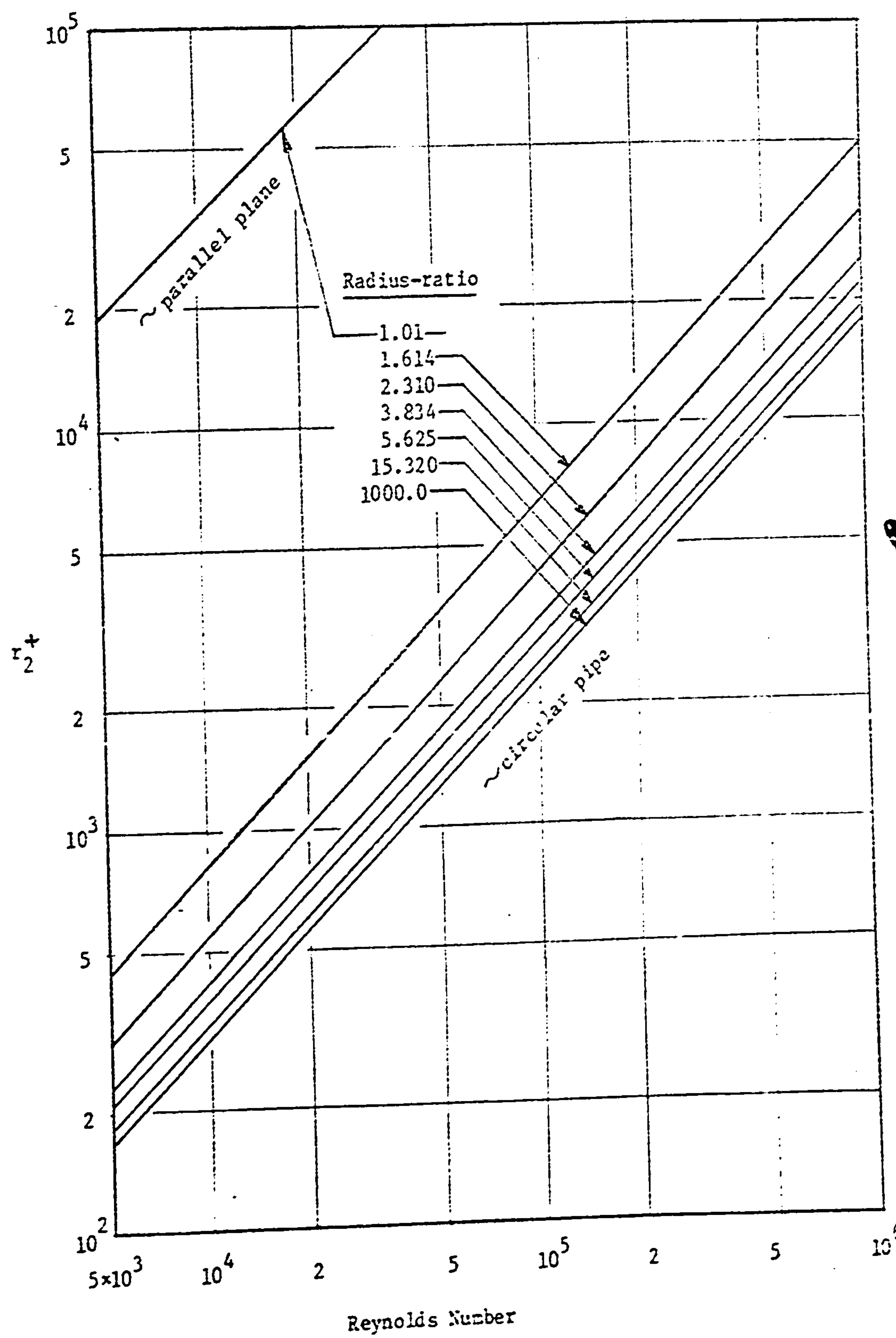


Fig.2 Relationship between Reynolds Number and r_2^+ for different values of α

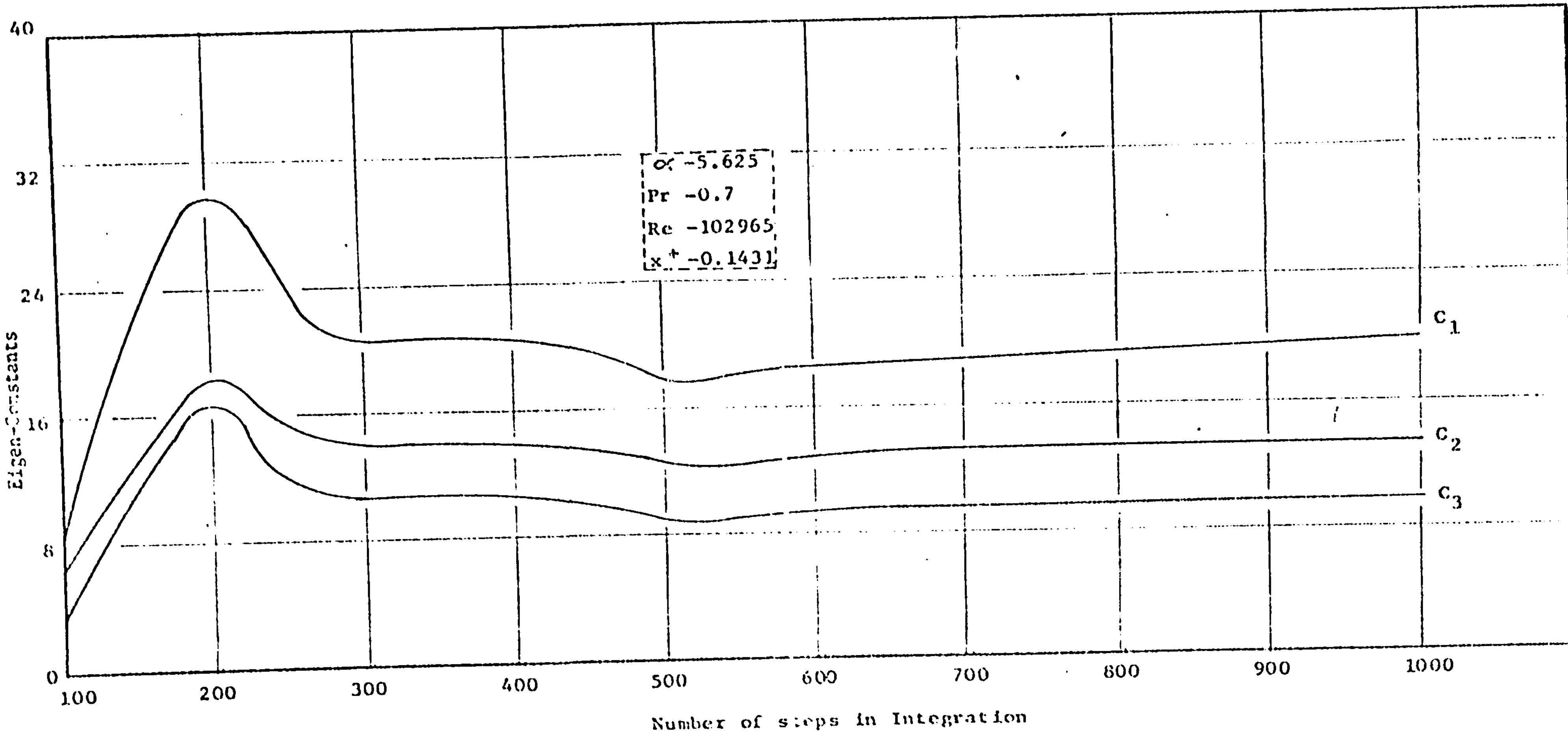


Fig. 3 Typical variation of Eigen-Constants with the number of steps in the Runge-Kutta method.

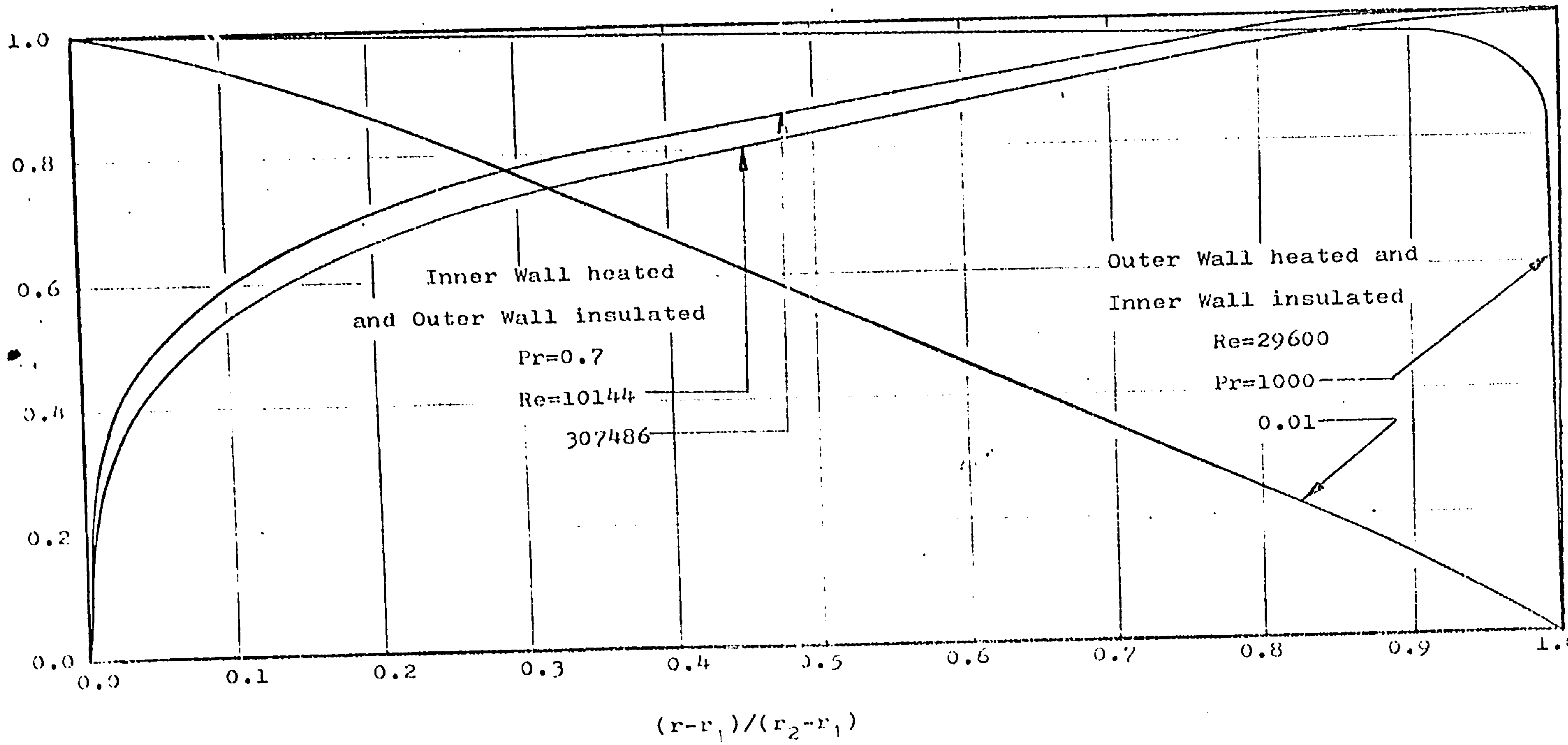


Fig.-4 Typical variation of Fully Developed Temperature Profile

(Radius ratio=5.625)

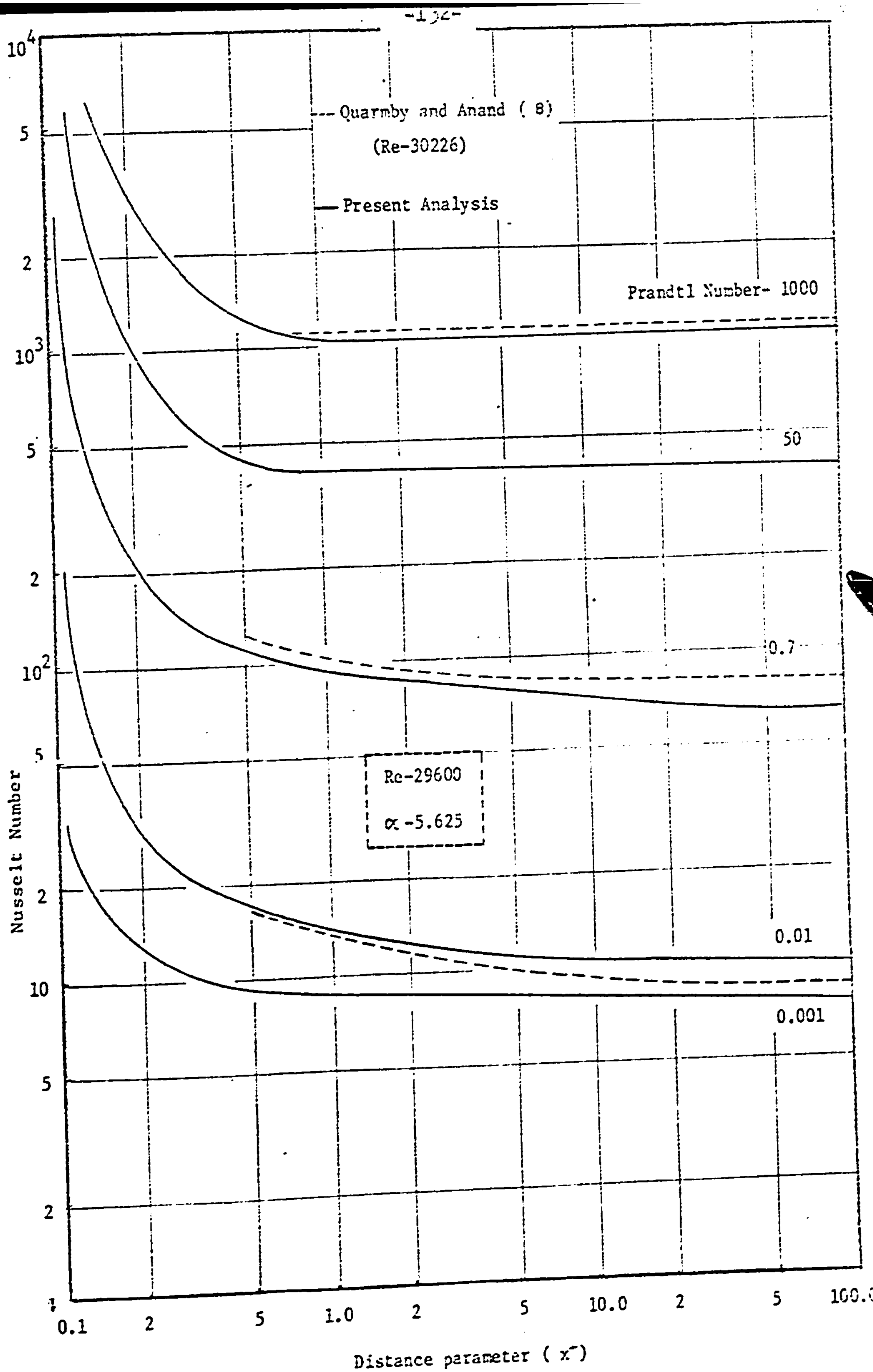


Fig.5a Effect of Prandtl Number on heat transfer.

(inner wall heated)

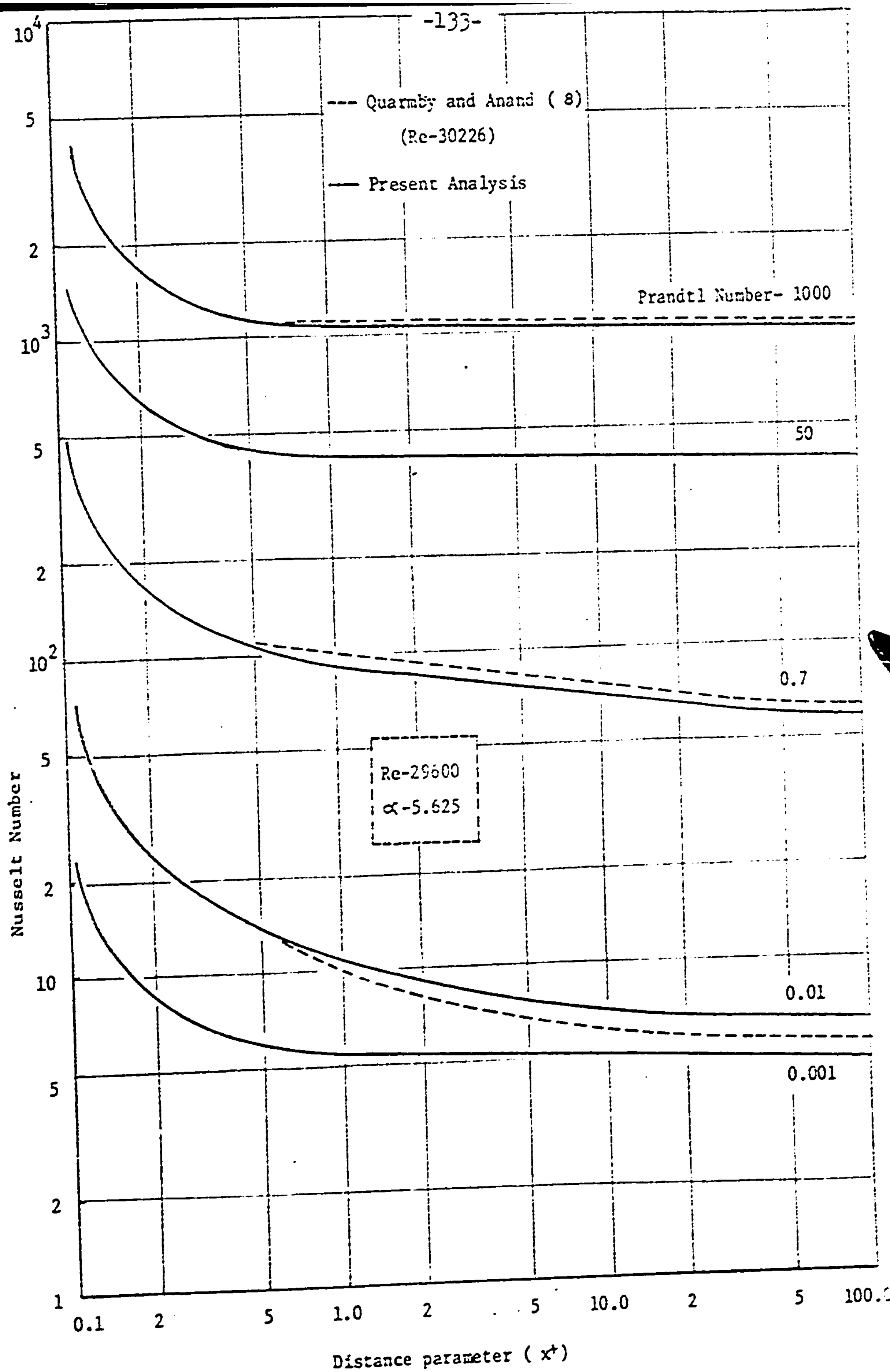


Fig.-5b Effect of Prandtl Number on heat transfer.

(Outer wall heated)

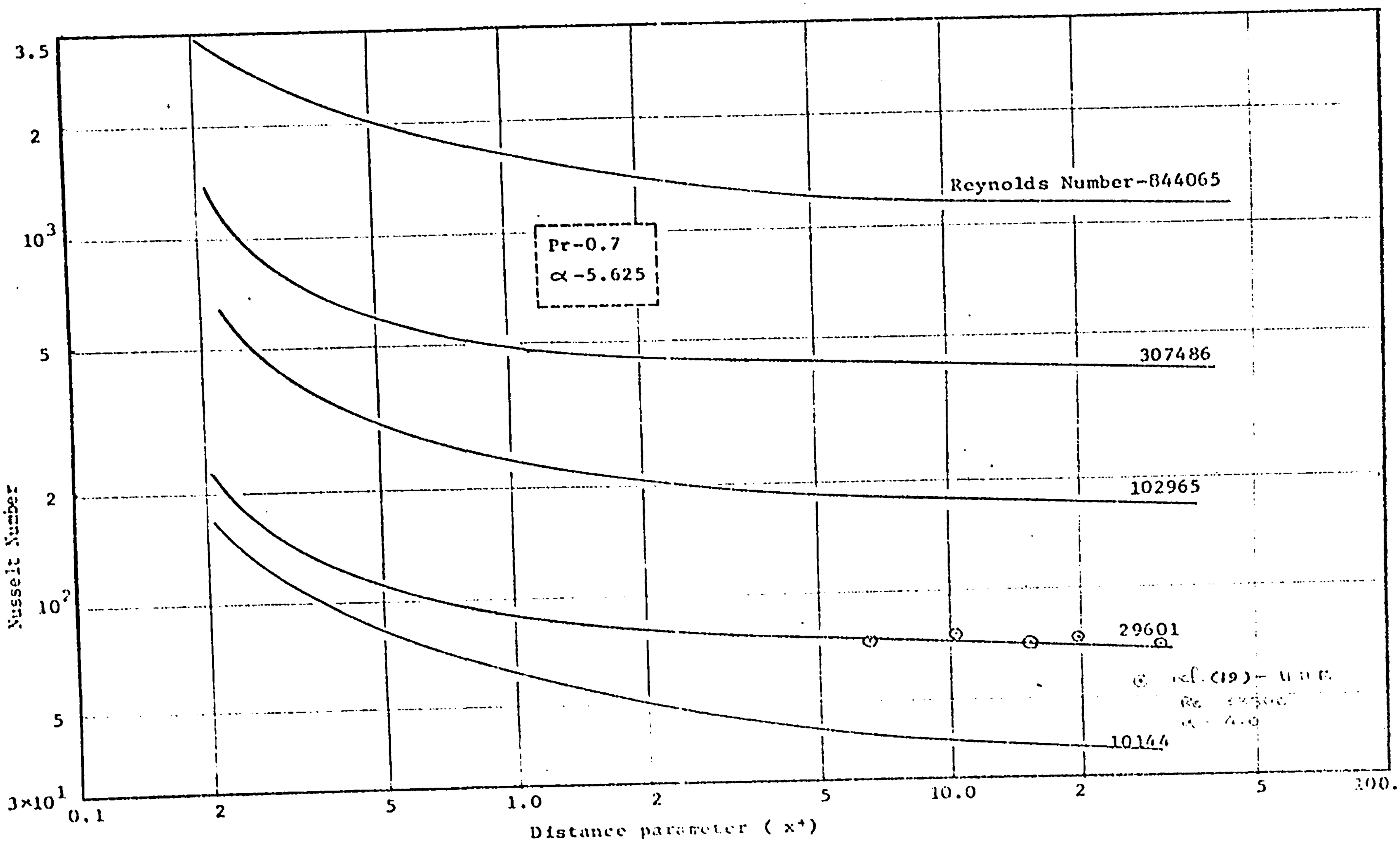


Fig.-6a Effect of Reynolds Number on heat transfer (Inner wall heated)

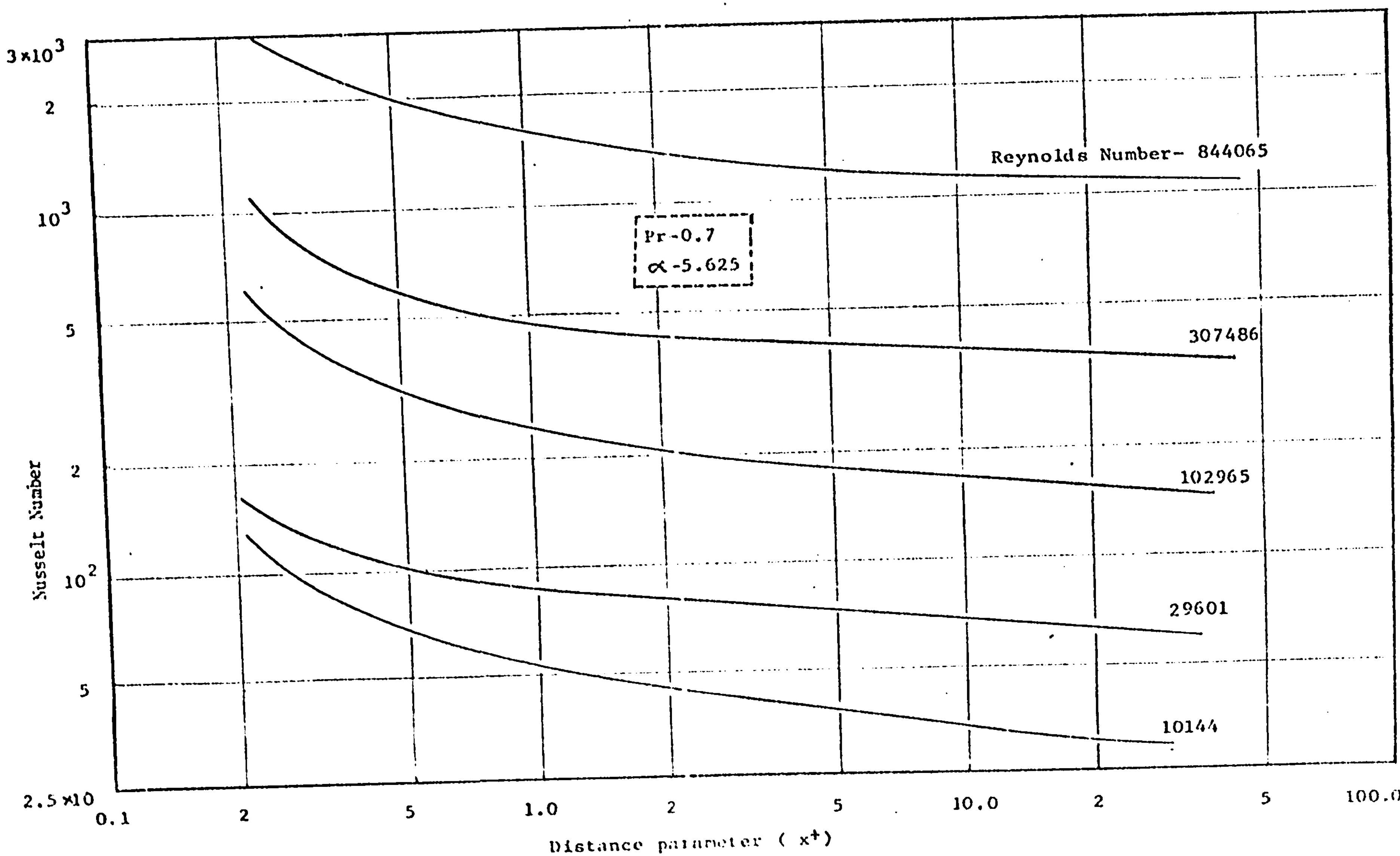


Fig.-6b Effect of Reynolds Number on heat transfer (Outer wall heated)

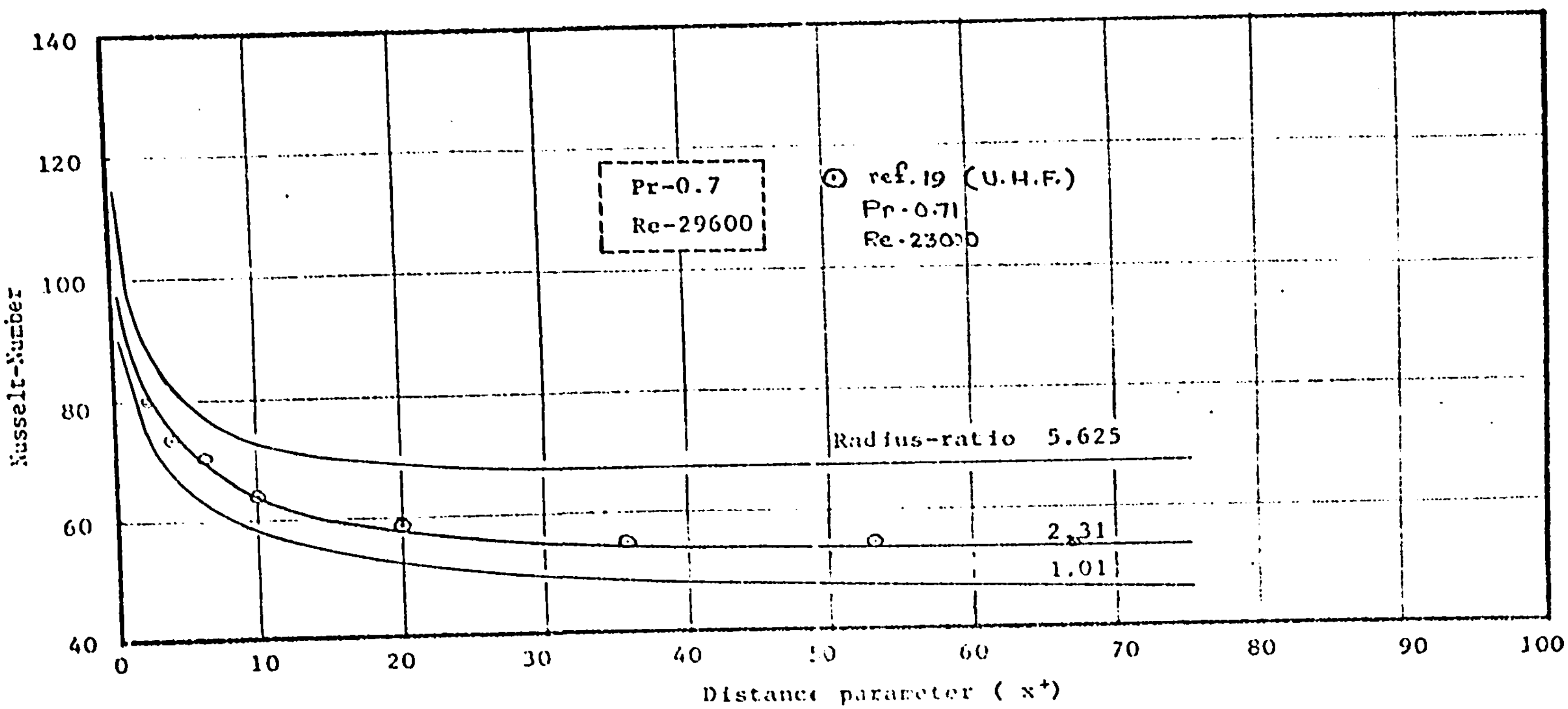


Fig.7a Effect of Radius-ratio on heat transfer.
(Inner wall heated)

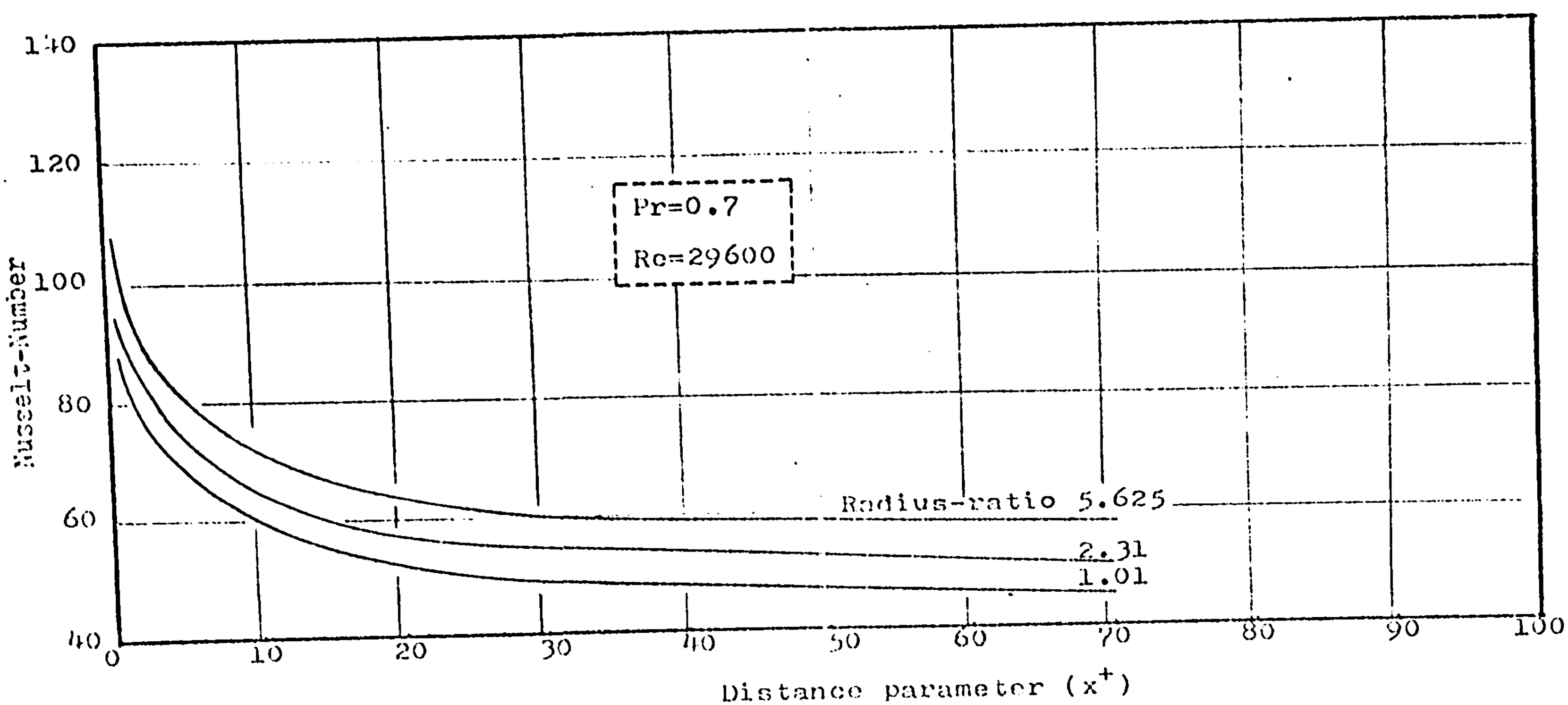


Fig.-7b Effect of Radius-ratio on heat transfer.

(Outer wall heated)

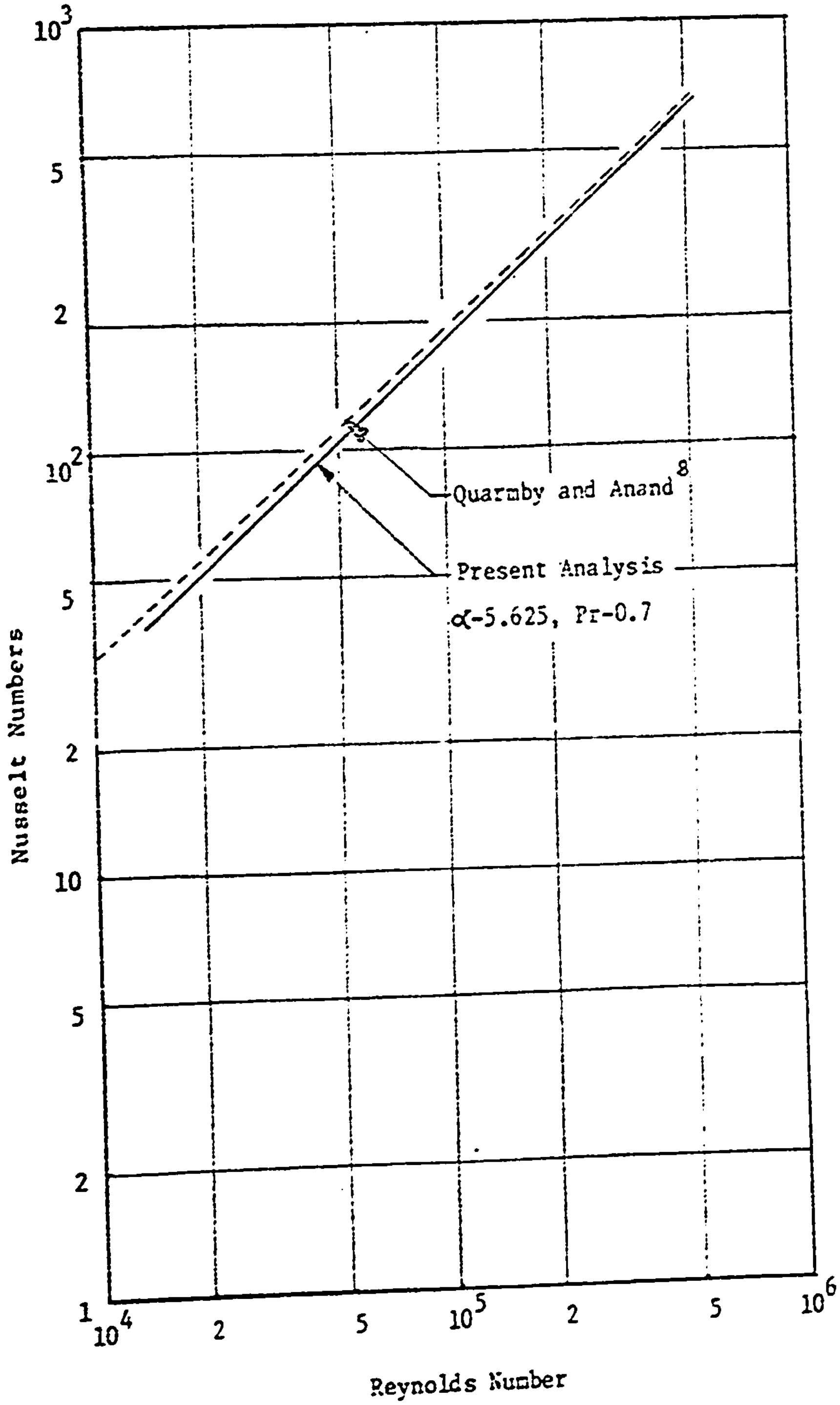


Fig.-8a Fully Developed Nusselt Numbers

(Inner wall heated)

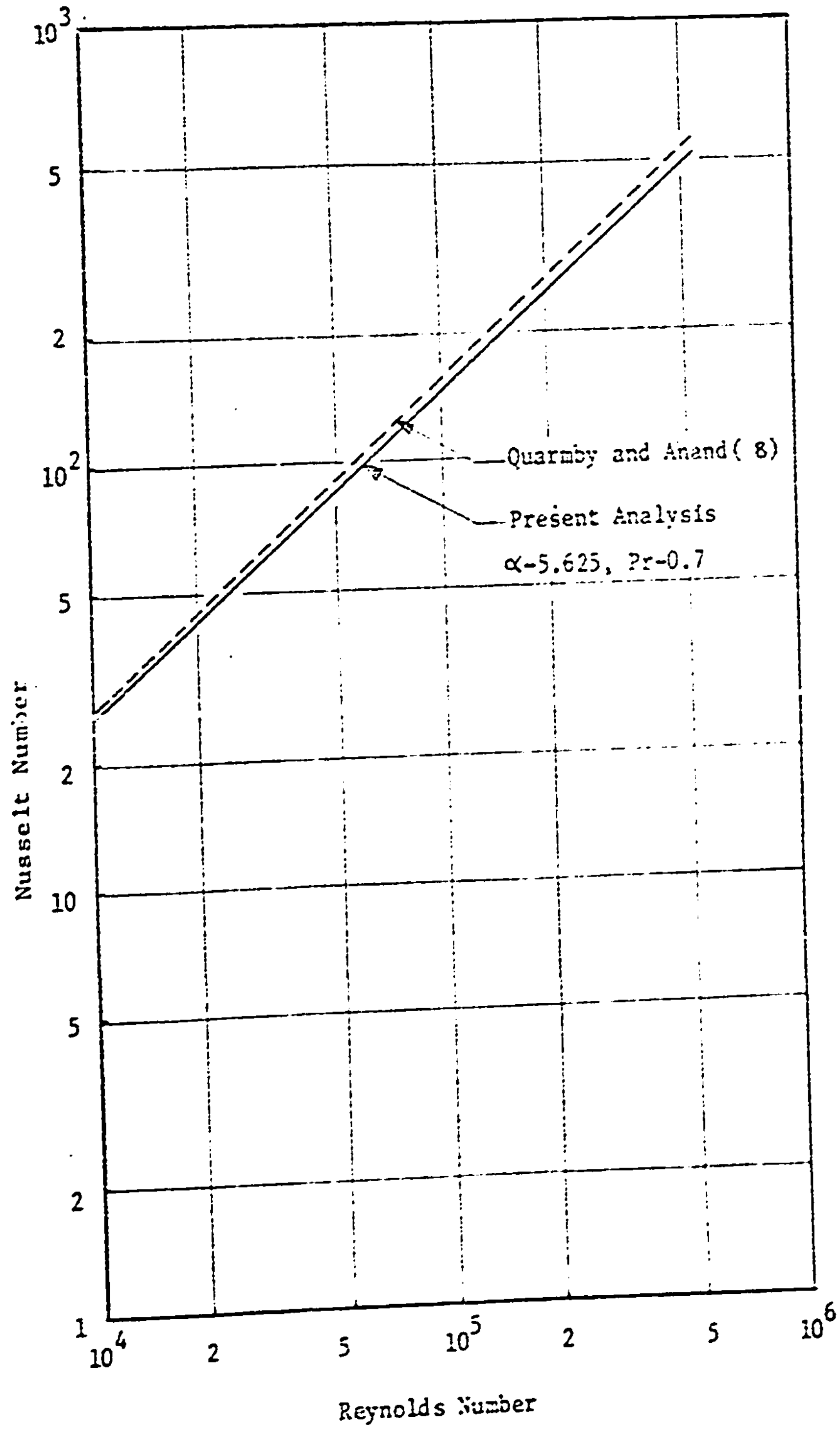


Fig.8b Fully Developed Nusselt Numbers

(Outer wall heated)

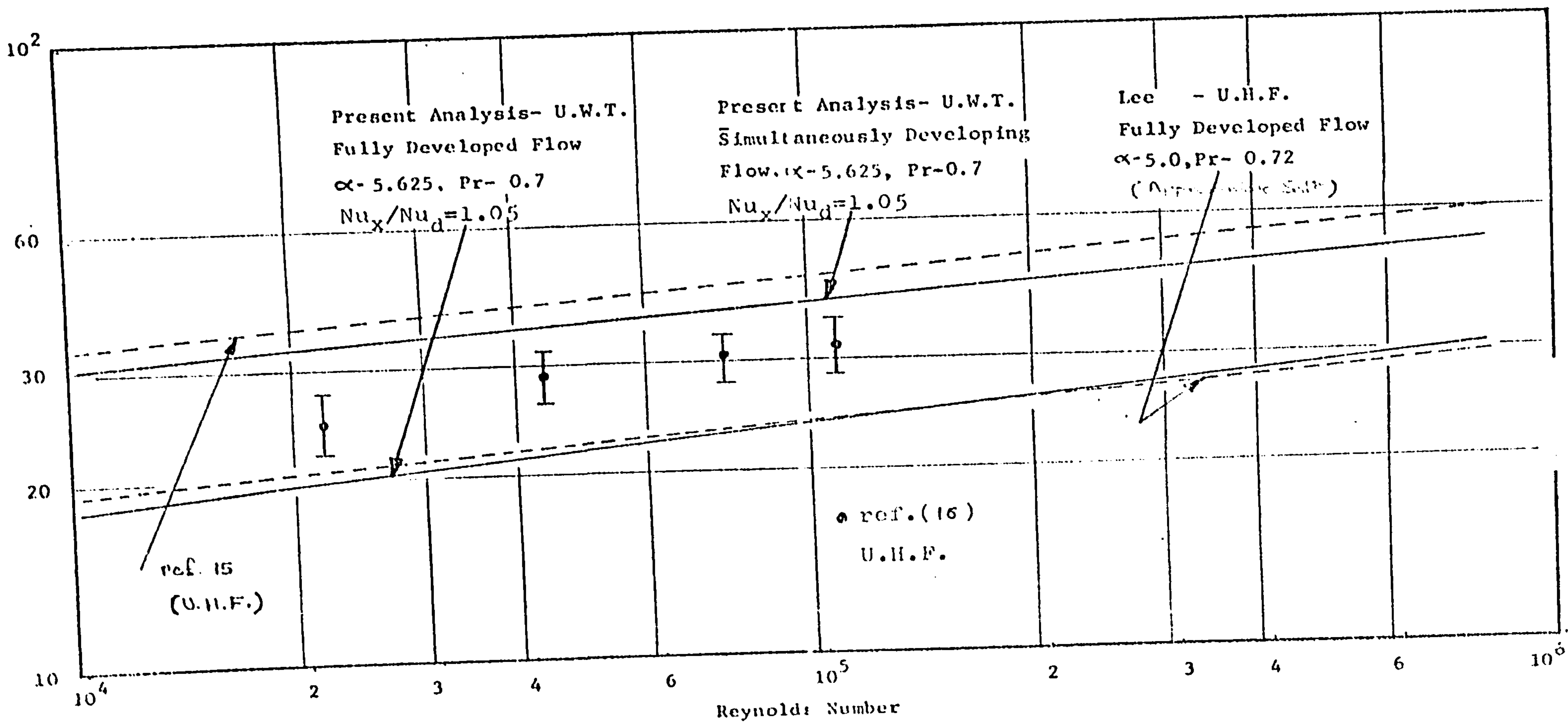


Fig.- 9 Entrance Length, Concentric Annulus with heating on the core wall.

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