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Multitone Signalling on Telephone Lines

by

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A thesis submitted to the
School of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of

Master of Applied Science

Ottawa-Carleton Institute for Electrical Engineering

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James A. Cargill, Ottawa, Canada, 1988

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ABSTRACT

The objective of this thesis is to evaluate the use of multitone signalling in an ISDN environment. The transmission rate limits of frequency division multiplexing over a telephone loop are determined. Channel noise, attenuation and the dispersive nature of the telephone line are the factors limiting the baud rate. The optimum transmission rate is determined for QAM and PAM carrier systems using either linear or decision feedback equalization. The effects of transmitted power, subchannel bandwidth and the subchannel filter are presented. The optimum transmission rates are compared to the theoretical channel limit as well as the rates which can be achieved using baseband and orthogonal signalling techniques.

It is shown that in terms of transmission rate the optimum system depends upon the type of equalization which is used. For linear equalization the parallel transmission schemes are significantly better than baseband transmission. With decision feedback equalization baseband transmission provides the highest baud rate. Orthogonal signalling is superior to multitone transmission with either type of equalization.

Transmission over a loaded telephone loop is also considered. In this type of channel it is shown that the parallel transmission schemes are superior to baseband regardless of the type of equalization which is used.

Table of Contents

Table of Contents	i
Table of Illustrations	iii
Table of Acronyms	v
CHAPTER 1	1
1.1 INTRODUCTION	1
1.2 ORGANIZATION OF THIS THESIS	2
1.3 REVIEW OF RELATED WORK	4
CHAPTER 2 TRANSMISSION CAPACITY	8
2.1 CHANNEL CHARACTERISTICS	8
2.2 NOISE	10
2.3 CHANNEL CAPACITY	13
2.3.1 SHANNON THEOREM	13
2.3.2 CAPACITY CALCULATION	17
2.3.3 LOADED VOICEBAND LOOP	20
CHAPTER 3 METHOD OF ANALYSIS	22
3.1 MULTITONE SIGNALLING	22
3.2 CALCULATION OF ERROR RATE	25
3.2.1 MSE CALCULATIONS	26
3.2.2 CHERNOFF BOUND	28
3.3 BASEBAND TRANSMISSION	29
3.4 CARRIER SYSTEMS	30
3.4.1 QUADRATURE AMPLITUDE MODULATION	31
3.4.2 DOUBLE SIDE-BAND MODULATION	32
3.4.3 SINGLE SIDE-BAND MODULATION	33
3.5 MULTITONE SYSTEM DESIGN	34
CHAPTER 4	36
4.1 BIT RATE COMPARISON	36
4.1.1 LINEAR EQUALIZATION	36
4.1.2 DECISION FEEDBACK EQUALIZATION	41
INFLUENCE OF DECISION FEEDBACK	46

FREQUENCY DEPENDENCE OF MSE	49
4.2 POWER DISTRIBUTION	52
4.3 FILTER EFFECTS	56
4.4 P_e vs MSE-CHERNOFF BOUND CALCULATIONS	56
4.5 VOICEBAND CHANNEL CALCULATIONS	62
CHAPTER 5 SUMMARY AND RESEARCH SUGGESTIONS	64
5.1 SUMMARY	64
5.2 SUGGESTIONS FOR FURTHER RESEARCH	66
APPENDIX A MSE CALCULATION	68
APPENDIX B BASEBAND EQUIVALENT EQUATIONS	77
PAM SIGNALLING	78
QAM SIGNALLING	81
APPENDIX C CHERNOFF BOUND	85
APPENDIX D SIGNAL CONSTELLATIONS	89
APPENDIX E PROGRAM LISTING	92
REFERENCES	115

Table of Illustrations

FIGURES

FIGURE 1.1 Telephone Cable	3
FIGURE 1.2 Multitone System	4
FIGURE 2.1 26 AWG Twisted Pair Cable Characteristics	9
FIGURE 2.2 Crosstalk Noise Sources	12
FIGURE 2.3 Shannon Capacity	15
FIGURE 2.4 Signal and Noise Power Distribution	16
FIGURE 2.5 Water-Pouring Analogy	17
FIGURE 2.6 Accumulated Channel Throughput	18
FIGURE 3.1 One Tone	23
FIGURE 3.2 PAM Signal Constellation	30
FIGURE 3.3 QAM System	32
FIGURE 4.1 8 L PAM Transmission	38
FIGURE 4.2 Linearly Equalized Systems	40
FIGURE 4.3 OQAM Signal Spectrum	41
FIGURE 4.4 QAM Subchannels	44
FIGURE 4.5 PAM Subchannels	45
FIGURE 4.6 Decision Feedback Equalized Systems	48
FIGURE 4.7 MSE vs T for a QAM System	50
FIGURE 4.8 Feedback Influence	51
FIGURE 4.9 Frequency Dependence of MSE	53
FIGURE 4.10 Folded Noise for 4L Baseband	54
FIGURE 4.11 Bit Rate vs Frequency for QAM system	57
FIGURE 4.12 Signal and Noise Power Distribution	58
FIGURE 4.13 Effect of filter on bit rate	59
FIGURE 4.14 Kalet and MSE comparison	61

TABLES

TABLE 2.1 Capacity vs Bandwidth	19
TABLE 2.2 Capacity vs Power	19
TABLE 2.3 Capacity vs Cable Length	20
TABLE 2.4 Voiceband Channel Capacity	21
TABLE 3.1 Baseband Equations	31
TABLE 3.2 QAM Equations	33
TABLE 3.3 PAM Equations	34
TABLE 4.1 Multitone Linear Equalization	36
TABLE 4.2 Linear Equalization	39
TABLE 4.3 Multitone Decision Feedback Equalization	42
TABLE 4.4 Comparison of Feedback Effect	43
TABLE 4.5 Decision Feedback Equalization	47
TABLE 4.6 Channel Filters	60
TABLE 4.7 Voiceband Channel	63

Table of Acronyms

a	- one-half the distance between adjacent symbols
B	- channel bandwidth
BER	- bit error rate
$B(f)$	- feedback response
$C(f)$	- telephone loop response
DSB	- double side-band
E'	- energy in a QAM constellation (V^2)
$E(f)$	- equalizer response
e_k	- error
$F(f)$	- channel filter
$h(t)$	- channel response
h_0	- impulse response at time 0
K_c	- crosstalk coupling factor
K_e	- bit error rate factor
K_p	- water-pouring constant
L	- channel attenuation (Kalet)
M	- bits per symbol
MSE	- mean square error
$M(f)$	- message spectrum
N_0	- white noise spectral density (W/Hz)
$N(f)$	- total noise power spectral density (b/s/Hz)
N_x	- crosstalk noise spectral density (W/Hz)
$n(t)$	- noise
OQAM	- orthogonal quadrature amplitude modulation
PAM	- pulse amplitude modulation
P_k	- Kalet's transmitted power (W)
P_e	- probability of a symbol error
$P(f)$	- pulse transform

QAM - quadrature amplitude modulation
 $R_s(f)$ - spectral efficiency (b/s/Hz)
 $R(f)$ - channel response without $E(f)$
 $r(t)$ - received signal
 $S(f)$ - transmitted power spectral density (W/Hz)
 SSB - single side-band
 $U(e_{+/-}^{-j2\pi fT})$ - spectral factorization products
 W - bandwidth (Kalet) (Hz)
 X_n - input data symbol

CHAPTER 1

INTRODUCTION

This chapter describes the objectives and incentives for the research into multitone signalling. A review of published material related to this subject is also presented.

1.1 INTRODUCTION

The need for digital communications continues to grow as digital equipment makes more incursions into our daily lives. Computers are found in most modern offices and other automated equipment such as smart terminals and facsimile machines are becoming part of the standard issue for an office in the '80's. The spread of these digital tools of modern commerce creates an ever increasing need for digital communications[1]. It is this demand, which is providing a great part of the impetus to the current interest in integrated services digital networks, ISDN, and integrated digital networks, IDN[2]. The ISDN networks, although a common topic in technical journals, are not yet commonly available and the level of acceptance they will achieve is yet to be determined[3,4]. In addition, the 144 kb/s data rate which the most common ISDN configuration offers may prove to be insufficient for some users[5]. With this considerable uncertainty surrounding ISDN an investigation of different means of providing access is still appropriate. This thesis is such an investigation.

Providing digital service to the more isolated or unusual subscriber is another problem which is compounded by the fact that the transmission lines to these users generally have loading coils which severely limit the bandwidth. Modems are now available which permit digital transmission over telephone channels with significant attenuation variation over the bandwidth of interest. There seems to have been little work however regarding access to ISDN over a loaded loop which runs between a subscriber and a central office. Such a loop would have attenuation and phase

characteristics which are not as severe as those which modems are generally designed to cope with.

This study is a theoretical investigation into the transmission rate which can be achieved over a telephone loop using parallel frequency transmission and optimum equalization in each subchannel. Frequency division multiplexing in the form of multitone signalling has been considered before as a means of compensating for variations in the phase and amplitude response of a telephone line[6-11,13,19,26]. The idea of dividing the available bandwidth into a number of smaller subchannels is applied in this thesis as a means of attempting to improve the transmission rate. A modulation method more appropriate to the transmission characteristics of each frequency range can be chosen once the channel is divided into subchannels. In this manner the frequency dependence of the channel and the noise can be dealt with. This multitone technique has already been analyzed for linearly equalized quadrature amplitude modulation systems, QAM, on a generalized channel with Gaussian noise[13] and for orthogonal quadrature amplitude modulation systems, OQAM, with overlapping signal spectrums[6]. The study reported in this thesis considers a telephone loop in the presence of crosstalk noise, with non-overlapping signal spectrums and with either linear or decision feedback equalization. QAM as well as other carrier modulation methods are used. The achieved transmission rate is compared to that which results from OQAM and baseband modulation. The telephone loop is a twisted copper pair and transmission is considered with and without any loading coils. Figures 1.1 and 1.2 show block diagrams of the multitone transmission system considered in this thesis. The approach studied here differs from other published studies in that it is an investigation of ISDN access over telephone loops without repeaters, no restrictions are placed upon the subchannel bandwidths and optimum equalizers are used for each of the tones. Implementation difficulties are set aside in order to determine the limit of this approach. The objective of this thesis is to determine if multitone transmission over a telephone loop offers any benefit which would justify further investigation.

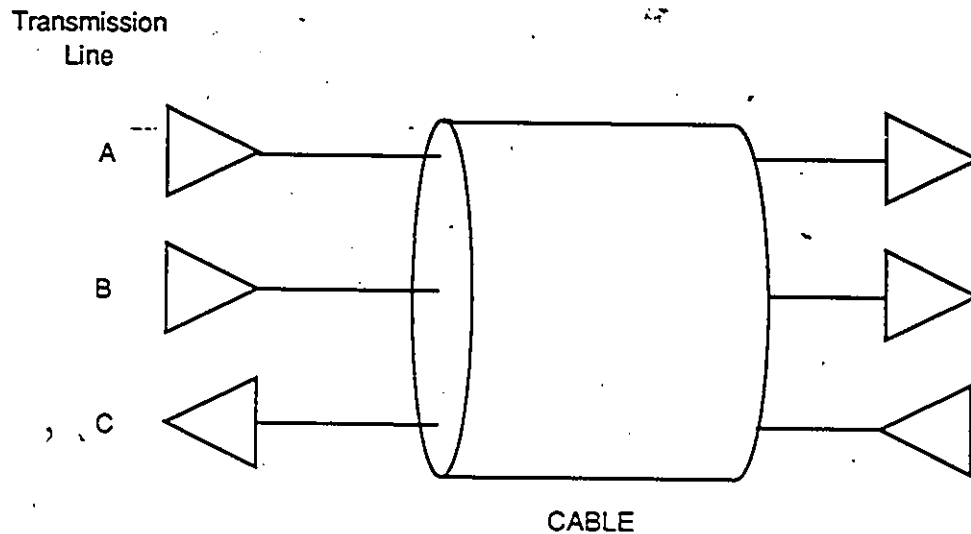


FIGURE 1. 1 Telephone Cable. It shows the bidirectional nature of transmission. The transmission systems on lines A,B and C could represent switched voiceband loops, digital systems or the multitone system considered in this thesis.

1.2 ORGANIZATION OF THIS THESIS

Chapter 2 introduces the telephone cable and the sources of noise in the system. The theoretical capacity of the loop is determined. In Chapter 3 the proposed transmission system and the analysis method is discussed. This includes the error rate calculation and the conditions set out for acceptable transmission quality. Chapter 4 presents the results of the analysis including a discussion of the effects of factors such as transmitting power, subchannel filters and subchannel bandwidths. A comparison is made between the mean squared error analysis method used in this study and an analysis method based upon direct bit error rate calculations[13]. Finally, the summary and conclusions are provided in Chapter 5. The derivation of the equations in Chapter 3 is included in the appendices.

In order to prevent confusion regarding the type of transmission medium being referred to the following convention is used. The copper pair transmission line between a subscriber and the first switching point is referred to as a *loop* and is

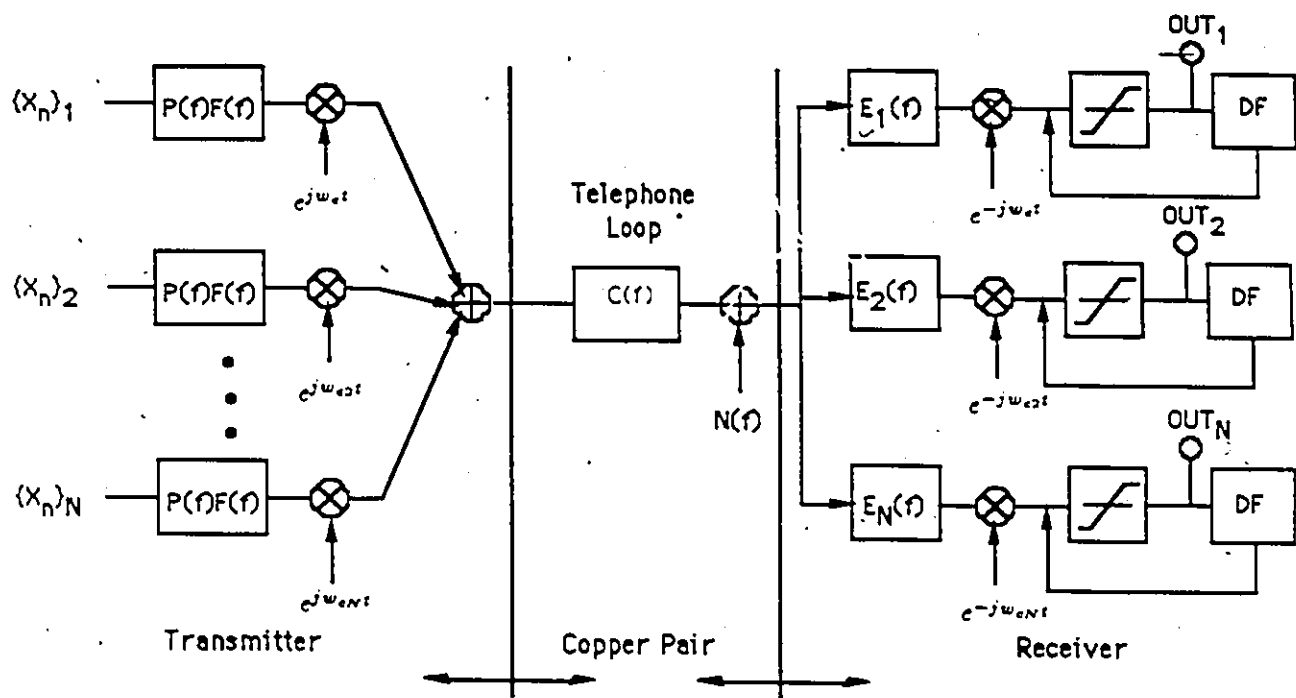


FIGURE 1. 2 Multitone System. This block diagram shows the basic components of the multitone system. The telephone loop is a copper pair. Each of the tones represents a digital modulation system centered about a carrier frequency ω_{ci} . Also shown are the shaping filters and the equalization blocks that make up the transmitter and receiver of each tone. The input data sequence to each tone is represented by $\{X_n\}$.

generally less than 3 miles long. A telephone loop is the medium over which the transmission systems in this thesis will operate. A telephone line can be any length and can pass through a number of switches. A telephone channel is a connection between subscribers and makes no reference to length or the type of link which exists between subscribers. A loaded loop is a transmission loop with loading coils inserted.

1.3 REVIEW OF RELATED WORK

The multitone approach to digital signalling in a loaded loop is not new[6,14,17]. The advantages of this approach are well-known and include reduced impulse noise sensitivity, increased capacity and less stringent equalization requirements[10]. Frequency division multiplexing is used by the telephone industry as a means of providing increased service to a subscriber without having to install additional cable[14]. Single channel systems use carrier techniques to transmit two analog voiceband signals on the same loop. AT&T's SLC-1 system[14] is an example of this application. Multi-channel techniques are used with double side-band amplitude modulation to provide up to 8 voice channels on a single pair.

One of the two main stream approaches that have evolved in high bit-rate voice band modem design is that of parallel frequency transmission[15]. In 1964 Holsinger [16] showed that an optimal signal set is time limited orthogonal sinusoidal signals. Later, in 1966, Chang [9,10] proposed an orthogonally modulated parallel scheme which eliminated the need for high tolerance subchannel filters and guard bands between subchannels. The idea was of interest to a number of researchers although until recently the difficult phase and timing requirements have limited the practical application of the idea[6]. In the 1980's Hirosaki combined Chang's proposal with a discrete Fourier transform approach to produce a realizable orthogonal QAM system[7,8,19]. The parallel signals are formed by using QAM modulation with the two input data lines staggered by $T/2$. T is the period of the input data symbol. The adjacent channels are both modulated with the staggered data lines. For example, if there are three QAM channels, the I channel of QAM systems 1 and 3 would be modulated by data streams that are in phase and the I channel of QAM system 2 would be modulated by the data stream that is staggered by $T/2$. This results in a system similar to the multitone approach in terms of the advantages derived by using parallel subchannels for transmission. The question still remains however as to whether this multitone approach provides a greater transmission throughput than a baseband system occupying the same bandwidth. The discrete Fourier transform approach provides a convenient means of transmitting over a series of N channels

each with a bandwidth B . Hirotsaki's OQAM technique requires that the input data sequences be synchronized. The transmission rate achieved using OQAM is included in this analysis and comparison to other multitone systems and baseband modulation is made.

From a wider perspective multitone signalling in the form of frequency division multiplexing has found use in most areas of communications. From Bell mastergroup systems which transmit 600 analog voiceband channels, to broadcast radio, to television, frequency division multiplexing is an essential element in the efficient utilization of various transmission media[14].

In order to analyze the transmission systems in this thesis mean squared error analysis is used. The minimum mean squared error is determined for each tone and a bound on the probability of error is determined using this MSE value. A more direct approach is presented in a paper by Kalet[13]. He analyzes QAM transmission in an additive white Gaussian environment with the transmission channel being approximated by step functions. Under these conditions the bit error rate equation for QAM reduces to a simple closed form. Equations (1) and (2) are from Kalet's paper and they show respectively, the approximation which is used for the energy in a QAM constellation and the bit error rate formula. The bit error rate formula cannot however be reduced to this simple form after introducing continuous functions for channel response and frequency dependant crosstalk noise. Equation (3) shows the 'Kalet type' expression which is necessary for the transmission conditions studied in this thesis. Equation (3) is suitable for analyzing multitone QAM systems which use linear equalization. The calculation is based upon the received signal-to-noise ratio and does not lend itself to analysis of systems using decision feedback equalization. In Chapter 4 the results of a Kalet type analysis are compared to those obtained using MSE analysis for linearly equalized multitone systems.

The energy E' in a QAM signal constellation can be determined using equation (1).

$$E' = 2^{M+1} a^2 / 3, \quad (1)$$

where E' is the energy in a QAM signal constellation in which each symbol represents M bits of information. The term ' a ' is equal to one-half the distance between adjacent symbols. A more comprehensive discussion of signal constellations is provided in Appendix D. Kalet's equation for probability of error is given in equation (2).

$$P_e \simeq \exp \left[- \left(\frac{1.5 P_k L}{N_0 W 2^M} \right) \right], \quad (2)$$

where P_e is the probability of an error occurring in any one direction, (As will be explained in a later section this probability must be modified to take into consideration the possible ways an error may occur within a given constellation.), P_k represents the transmitted power as defined in Kalet's paper, N_0 is the noise spectral density in the channel, M is the number of bits represented by each symbol, L is the channel attenuation and W is the channel bandwidth.

If the frequency dependant nature of the channel and the crosstalk noise are considered and the message spectrum of the transmitted data is flat with respect to frequency the probability of error for a channel with attenuation $L(f)$ is given by equation (3).

$$P_e \simeq \frac{1}{(f_2 - f_1)} \int_{f_1}^{f_2} \exp - \left(\frac{3L(f)P_k(f)}{2^{M+1}W(N_o + N_x(f))} \right) df, \quad (3)$$

where $N_x(f)$ is the frequency dependant crosstalk noise, N_o is the white noise in the noise model used in this study, and $P_k(f)$ is the frequency dependant transmitter power. It is equation (3) which is used for comparison purposes in Chapter 4.

CHAPTER

TRANSMISSION CAPACITY

It is possible to determine the theoretical capacity of a channel given knowledge of the channel's transmission characteristics and the noise which is present[21-23].

This chapter describes the noise and channel characteristics of the telephone loop which is to be studied.

2.1 CHANNEL CHARACTERISTICS

The telephone loop is a 26-AWG twisted copper pair cable with a large number of pairs. This permits the use of average values in computing the crosstalk noise[24]. The loop is unconditioned, which means no loading coils are used for voiceband equalization. All the systems considered will work on cables possessing these average characteristics. The average phase and attenuation characteristic of a 26-AWG copper pair is shown in Figure 2.1, this characteristic was obtained from a Bell System Specification[27]. The low frequency effect of the terminating hybrid transformers is modelled by a steep high-pass filter which has a rolloff matching that shown in the characteristic curve for C-message weighting[28]. The length of the cable is 3 miles or 4.8 km., a length chosen to represent the maximum distance likely to be encountered in a subscriber loop[30] without loading coils. The line characteristics shown in Figure 2.1 are used in all calculations. Calculations for a different wire gauge with the same cable construction would result in transmission rates which are larger in magnitude but the conclusions regarding the ranking of systems would remain the same. The 26-AWG cable represents the most common size, other wire sizes which may be encountered would be larger and the attenuation would be lower.

An assessment of multitone performance over a loaded loop is also made. For this analysis the channel loss characteristic is assumed to be the same as that of a 26-AWG cable with H88 loading[28] and is shown in Figure 2.2.

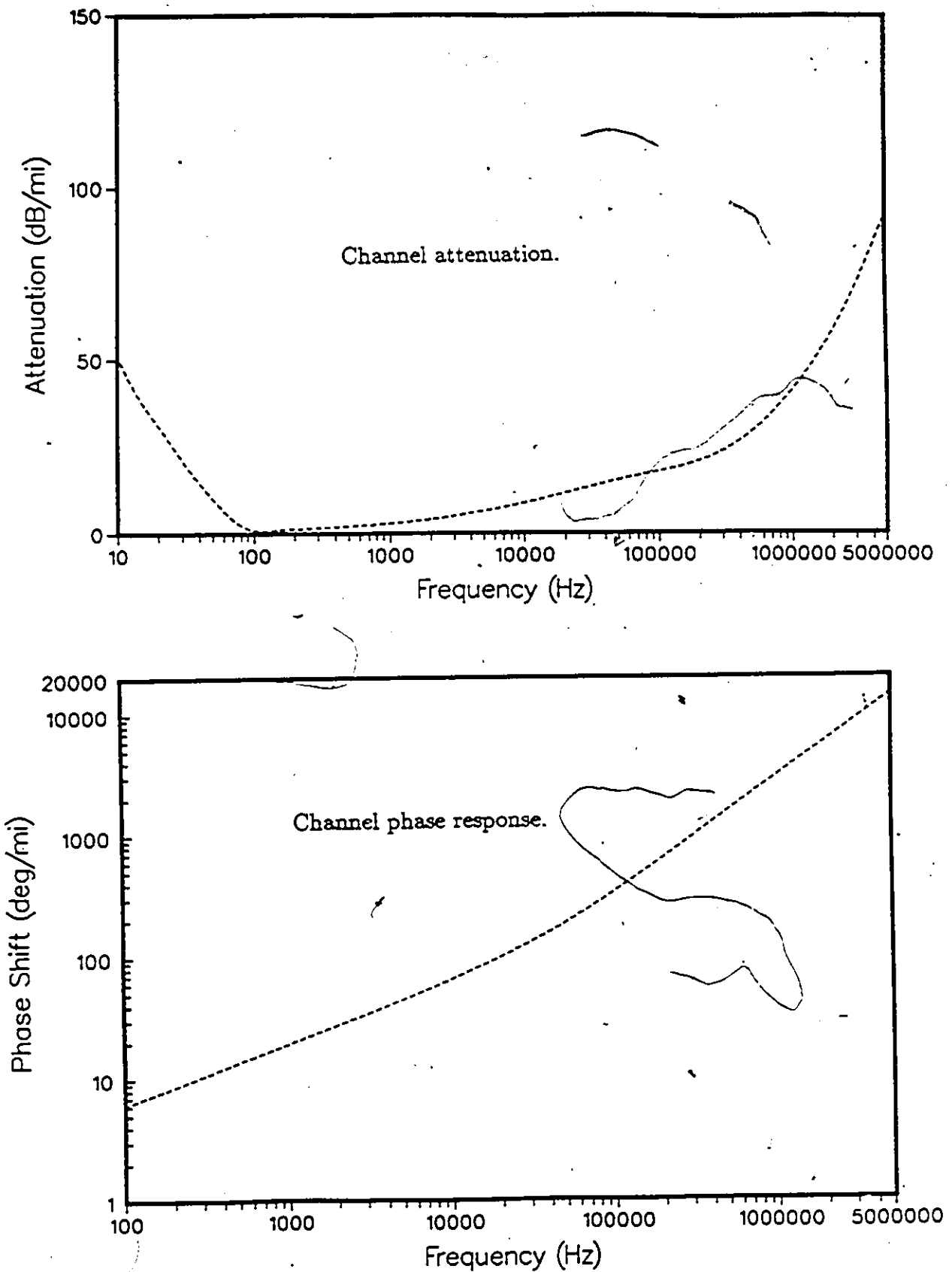


FIGURE 2.1 Plots of the channel characteristics for a 26-AWG twisted pair cable.

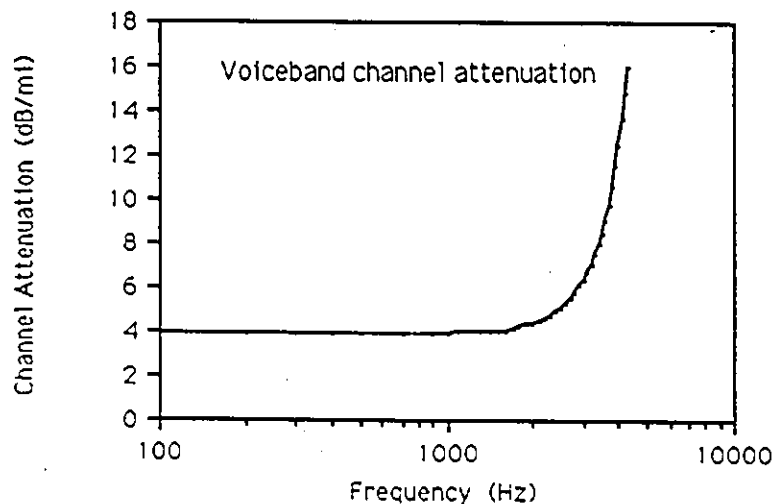


FIGURE 2. 2 Voiceband channel characteristic.

2.2 NOISE

The transmission systems in this study and most ISDN systems are designed for use on a loop intended originally for analog signals[2]. This results in a number of the noise sources which in the analog system were not a concern becoming serious factors which limit the transmission rate and the cable length which can be used. The noise sources which should be considered include impulse, shot, thermal and crosstalk noise[31,32]. Echo impairments due to impedance discontinuities or other dispersive effects result in intersymbol interference which is treated separately in this analysis.

Impulse noise, which is characterized by large, infrequent amplitude peaks, is due to sources such as lightning, switching transients and dial pulses on voice pairs sharing the cable with the digital system. Switching transients tend to be concentrated at central offices and die out quickly with distance. The problem that impulse noise represents is often discussed in literature but due to its wide statistical variation it is difficult to model[31]. Previous work in the area of impulse noise has shown

that if systems are ordered by relative performance in the presence of Gaussian noise the same ordering will be valid for impulse noise[12]. It is noted however that frequency division multiplexing used with narrow subchannel bandwidths is effective in combatting impulse noise effects[10]. For this reason, in the presence of impulse noise, the multitone systems considered here have an inherent advantage over single channel systems. Due to the difficulty of modelling the noise however the level of this advantage over the reference baseband systems is not determined. The comparison of systems in this study is based upon the achieved transmission rate. The infrequent nature of the impulse noise means that it will not provide the ultimate limit to the transmission rate of the system. That limit will result from the crosstalk and white noise.

Thermal and shot noise sources are well documented and can be modelled successfully as white Gaussian noise. Low frequency noise such as 60 Hz hum and the effects of transmission line transformers at very low frequencies, less than 100 Hz, are not well represented by this model. No attempt is made to consider low frequency noise as for every system studied a low frequency cut-off filter is included to eliminate DC and low frequency signals. This filter introduces little in terms of additional signal distortion as the transformers generally found at the transmission line terminations severely distort signals at these low frequencies. The noise model used in this study is Gaussian white noise which represents all noise sources except for crosstalk. The level of the white noise is set at the worst case permissible level in a voice channel[28]. This level was extended over all frequencies of interest. The value of $2.5 \times 10^{-14} \text{ W/Hz}$ compares well with values used by others in similar studies[33].

$$N_0 = 2.5 \times 10^{-14} \text{ W/Hz} \quad (4)$$

The remaining noise source of concern is crosstalk. The published studies on the subject of multitone signals in a telephone loop have often neglected to deal with the subject of crosstalk[13]. The two types of crosstalk of concern are far-end crosstalk or FEXT, and near-end crosstalk or NEXT. Models for these noise sources have long been established[31,32,34]. Both types of crosstalk depend upon the transmitted

signal strength and both are frequency dependant. This means the significance of crosstalk noise will increase as the frequency increases. The frequency dependence of the channel attenuation coupled with the frequency dependence of crosstalk noise are the major factors constraining the designer of a system intended for use on telephone loops. Figure 2.3 is a pictorial representation of the sources of crosstalk in a cable.

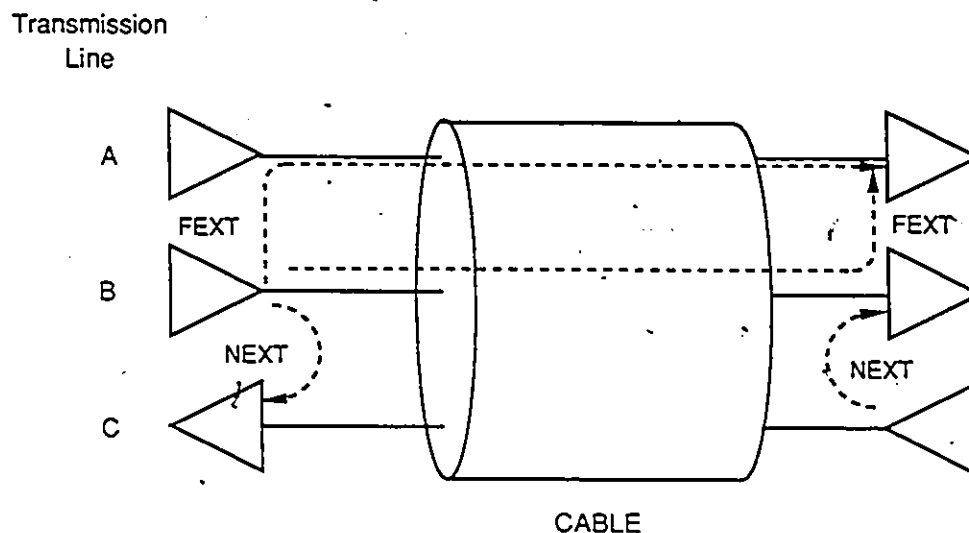


FIGURE 2. 3 A representation of the crosstalk noise sources in a multipair telephone cable. [page 302, ref. 28]

⁴FEXT is proportional to frequency raised to the power 2, that is, Kf^2 . FEXT results from the coupling of signals at the far end of the cable, 'far end' is defined as the end furthest from the receiver. This coupled signal is subject to losses during the coupling process and in transmission from the point of coupling to the receiver. The actual FEXT is therefore dependant upon the loss in the transmission line. Due to the length of the cable in this study and the fact that NEXT is generally the limiting factor in digital transmission on this type of cable [24,33] the effect of FEXT is not included.

Near-end crosstalk, as the name implies, results from crosstalk between the transmitter end of one loop and the receiver of another loop at the same end of the cable. It is independent of cable length when the total cable loss exceeds 10 dB [24]. The coupling is a random process dependant upon frequency raised to the power 1.5, $f^{1.5}$ [24,25]. When a large number of interferers exist average statistics may be used [35] noting that the cyclostationary character of the noise could result in a slightly lower margin or eye opening [36]. Near-end crosstalk noise power modelled using the average value approach can be represented at any frequency as in equation (5).

$$N_x = S(f)K_c f^{1.5} \text{ W/Hz} \quad (5)$$

K_c is a measure of the crosstalk coupling between copper pairs and is generally in the range of 10^{-11} to 10^{-15} [31]. The value used in this study is 0.632×10^{-13} [33]. The magnitude of the coupled noise is directly proportional to the transmitted power represented in equation (5) as $S(f)$.

Throughout this analysis it is assumed that the noise is uncorrelated and stationary. The thermal noise is by its nature uncorrelated however crosstalk noise may be cyclostationary. If all the transmitted signals use the same clock source the noise will be cyclostationary. While the coupling process is random the number of interferers which actually contribute to the crosstalk is small. A cyclostationary noise source could affect the error rate if the signal were sampled at an instant when the noise was at a peak. The effect of the noise peak will of course depend upon the equalizer response. The assumption of stationary noise is therefore based upon the introduction of random phasing to the clock signals on different transmission loops.

The noise model used in this study can be summarized in equation (6) below.

$$N(f) = N_o + S(f)0.632 \times 10^{-13} f^{1.5} \text{ W/Hz}, \quad (6)$$

where N_o is the white noise spectral density, $S(f)$ is the transmitted power, f is the frequency in Hz and the constant is a function of the crosstalk coupling.

2.3 CHANNEL CAPACITY

As a reference for comparison with the transmission rates achieved by the systems considered here we will determine the channel capacity as defined by Shannon[21,22].

2.3.1 SHANNON THEOREM

The error free capacity of the telephone loop is determined using the Shannon theorem. The spectral efficiency, R_s , at any frequency, f , can be determined if the signal-to-noise ratio is known.

$$R_s(f) = \log_2(1 + S(f)/N(f)) \quad \text{b/s/Hz} \quad (7)$$

$R_s(f)$ is the spectral efficiency in b/s/Hz that the channel can support in error free transmission at a frequency f . The word efficiency when used in this manner is actually a measure of the data rate capability of the channel. Figure 2.4 is the plot of this equation for the telephone loop. $S(f)/N(f)$ is the signal-to-noise ratio at the frequency f .

$$\text{Channel Capacity} = \int_0^B \log_2(1 + S(f)/N(f)) df \quad \text{b/s} \quad (8)$$

The channel capacity is the maximum error free transmission rate which is possible over a channel bandwidth B with the given signal-to-noise ratio. Equation (8) provides the theoretical digital channel capacity given knowledge of the signal-to-noise ratio with respect to frequency. It has been shown that the optimum power distribution is one in which the signal power and noise power add to a constant value over the channel bandwidth[12,21]. This power distribution has been described using an analogy to water-pouring because it fits the image of pouring a fixed amount of water (a limited amount of power) into a vessel so that the water finds its own level. Figure 2.5 shows the signal and power distribution with a water-poured power distribution over a telephone loop. The signal power drops off quickly at frequencies

beyond 100 kHz. The signal and noise powers add to a constant which is chosen so that the total transmitted power is equal to a desired value.

The constant to which the noise and signal power add in the optimum distribution above, is represented by the variable K_p in the formula below. In equation (9) the transmitted power and the noise reflected through the channel to the transmitter add to a constant K_p .

$$K_p = M(f)|P(f)|^2 + \frac{N(f)}{|C(f)|^2} \text{ W/Hz} \quad (9)$$

$M(f)|P(f)|^2$ represents the transmitted power spectrum and $N(f)/|C(f)|^2$ is the noise that would appear at the transmitter. The term $N(f)$ is a function of the power spectrum because of the presence of crosstalk noise. The signal-to-noise ratio at the receiver is given by equation (10).

$$S(f)/N(f) = \frac{M(f)|P(f)|^2|C(f)|^2}{N(f)} \quad (10)$$

2.3.2 CAPACITY CALCULATION

The equations discussed in the previous section can be used to generate a plot which shows the capacity in terms of b/s/Hz vs frequency, Figure 2.4. This plot shows that for the telephone line the theoretical capacity of the channel decreases rapidly as frequency increases. This is due to both the increasing channel attenuation and the high crosstalk levels resulting from the $f^{1.5}$ frequency dependence. Figure 2.6 shows that as the frequency increases, the transmitted power should be reduced which in effect will lower the near-end crosstalk. Figure 2.7 shows the accumulation of the channel throughput with respect to frequency. Given the optimum power distribution, the majority of the capacity is accumulated at the very low frequencies where a high signal-to-noise ratio exists. This is evident from Table 2.1 which shows that the channel capacity is little affected by a 50% reduction in bandwidth.

SHANNON CAPACITY 26 AWG copper pair

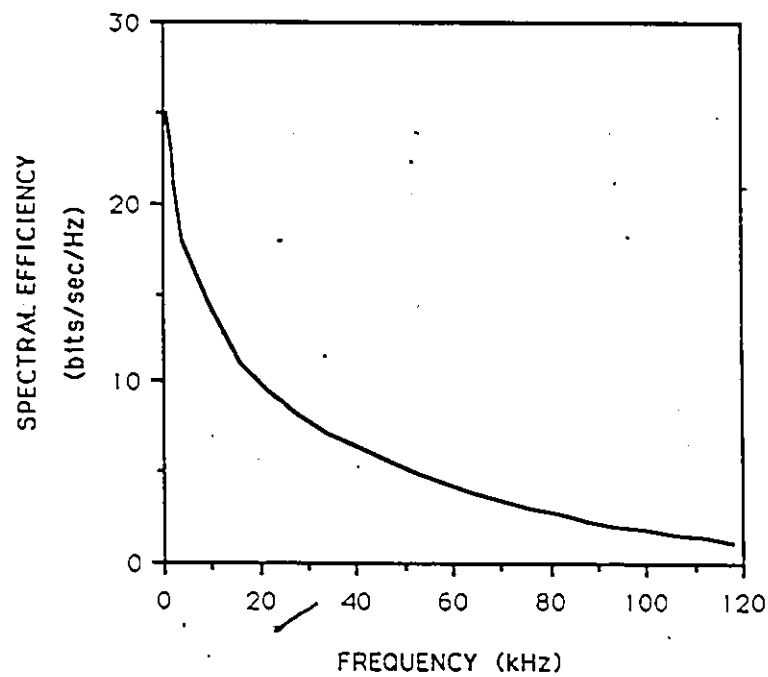


FIGURE 2. 4 A plot showing the spectral efficiency variation with frequency.
[$P=0.25$ W, $L=4.8$ km., Noise = White noise + Crosstalk]

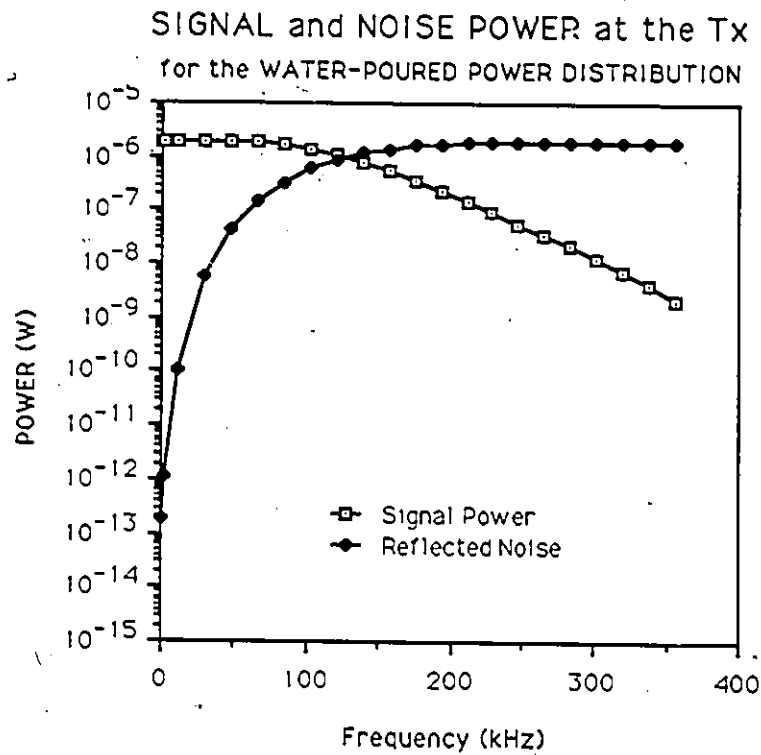


FIGURE 2.5 A plot showing the signal and noise power distribution which exists if a water-pouring distribution is used. The noise power has been 'reflected' back through the channel so it is the noise as it would appear from the perspective of the transmitter. [$P=0.25$ W, $L=4.8$ km., Noise = White noise + Crosstalk]

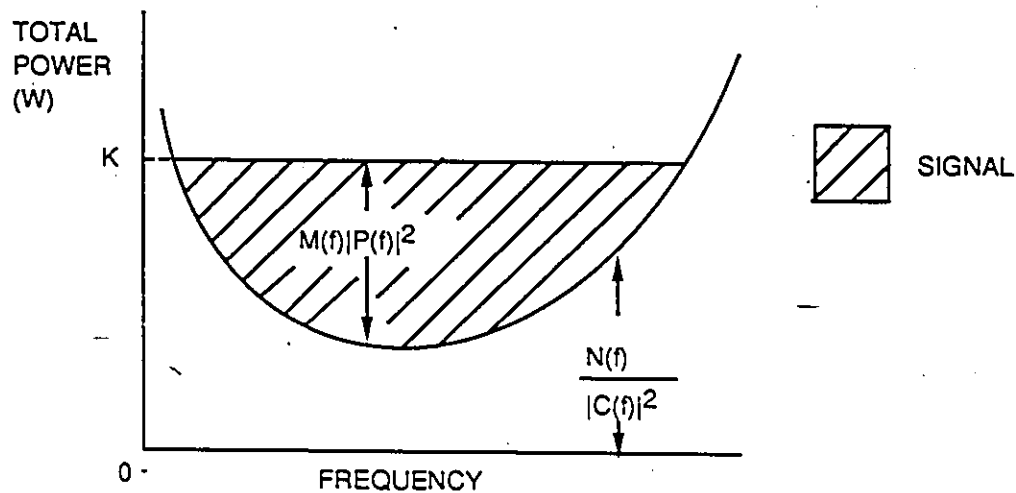


FIGURE 2. 6 A diagram of the water-pouring analogy for optimum power distribution. The noise and signal power add to a constant over the frequencies of interest. K is chosen so that the total signal power is at the desired level.

BANDWIDTH (kHz)	30	60	100	110	120
CAPACITY (kbits/s)	421	564	675	690	702

TABLE 2. 1 Shannon Capacity vs Bandwidth [$P=0.25$ W, $L=4.8$ km., Noise = White noise + Crosstalk, a large number of interfering pairs is assumed]

It is worthy of note that the capacity is little altered by changes to the total transmitted power. As can be seen in the table below reducing the power by a factor of 10 has little effect. The absence of a strong dependence upon transmitted power is a result of the dominance of the crosstalk noise. The importance of the crosstalk noise increases with frequency. As crosstalk becomes dominant the signal-to-noise ratio becomes less dependant upon signal power levels and more dependant upon the relationship between the channel attenuation and the crosstalk coupling factor. A

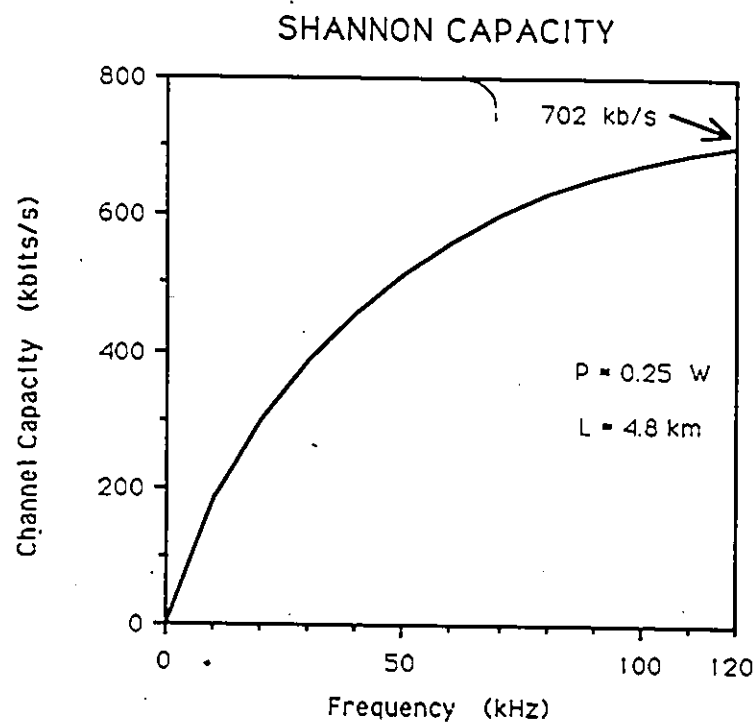


FIGURE 2.7 A plot showing the accumulation of throughput with frequency. The majority of the total capacity is accumulated at low frequencies. [P=0.25 W, L=4.8 km., 26-AWG., Noise = White noise + Crosstalk]

higher white noise level would result in a closer relationship between channel capacity and transmitted power. If white noise is the only limiting factor the Shannon capacity is 2.6 Mb/s. This rate may prove to be of interest if recent advances in methods of coping with crosstalk noise are successful[37].

POWER (W)	0.01	0.10	0.25	1.0	50.0
CAPACITY (kbits/s)	621	690	702	744	749

TABLE 2. 2 Capacity vs Power [B=120 kHz, L=4.8 km., Noise = White noise + Crosstalk]

There is a significant link between length and Shannon capacity. This link is to be expected from equations (8) and (10) which both include the channel attenuation in the form of $|C(f)|^2$, a factor dependant upon the cable length.

CABLE LENGTH		CAPACITY (kbits/s)
(km.)	(miles)	
3.2	2.0	1227
4.8	3.0	702
6.4	4.0	368

TABLE 2. 3 Capacity vs Cable length [P=0.25 W, B=120 kHz, Noise = White noise + Crosstalk]

2.3.3 LOADED VOICEBAND LOOP

The capacity of a loaded voiceband loop is considered using the compensated channel characteristic provided in [28] (reproduced in Figure 2.2) as a model. As the table below shows this channel is incapable of supporting the 144 kb/s transmission rate often quoted as the ISDN rate. The capacity is however considerably higher than the 23.5 kbit/s value used as a benchmark in many studies[15]. This is due to the fact that the channel considered in this study has not suffered any degradation due to multiple regeneration and retransmission. The loaded loop is capable of supporting the data rates of the component parts of ISDN transmission. That is, either a combination of a B channel (68 kb/s) and a number of D (16 kb/s) channels or a number of D channels can be transmitted over the loaded loop. Multitone and baseband transmission rates on this channel are compared in Chapter 4.

CABLE LENGTH		CAPACITY (kbits/s)
(km.)	(miles)	
10.0	6.2	83
7.3	4.5	87
4.8	3.0	91

TABLE 2. 4 Voiceband Channel Capacity [$P=0.25$ W, $B=0.1$ to 3.5 kHz,
Noise = White noise + Crosstalk]

CHAPTER 3

METHOD OF ANALYSIS

When using multitone transmission the available channel bandwidth is divided into a number of subchannels which because of their smaller bandwidths have transmission characteristics that exhibit less frequency dependence. The modulation technique is chosen to match the transmission characteristic of each subchannel. This is multitone signalling and it has been considered for a number of applications[6-11,13]. The difference of note in this study is the presence of crosstalk noise and the general approach to the investigation is intended to prevent current technology limits from influencing the research. This chapter describes the multitone system, the modulation methods and the calculations used to design the multitone systems. The MSE calculations were first presented in the format used here in a paper by McGee [38] and the Chernoff bound development is parallel to that in a paper by Saltzberg[39]. The development of these equations is included in the Appendices for reference. The equivalent baseband channel equations used for this study are detailed in Appendix B.

3.1 MULTITONE SIGNALLING

A multitone system can be broken down into the block form shown in Figure 1.1. This shows the basic components of the transmission system. A more detailed view of a single tone in Figure 3.1 reveals the transmission components which must be dealt with in the analysis.

The input data is represented by the sequence of real or complex-valued data symbols $\{X_n\}$. The input modulates the pulse function $P(f)$ and is filtered by the subchannel filter $F(f)$. For carrier systems this baseband signal is modulated by a carrier signal f_c . The channel $C(f)$ and the noise $N(f)$ distort the signal before the equalizer prefilter $E(f)$ is encountered. The demodulator restores the baseband

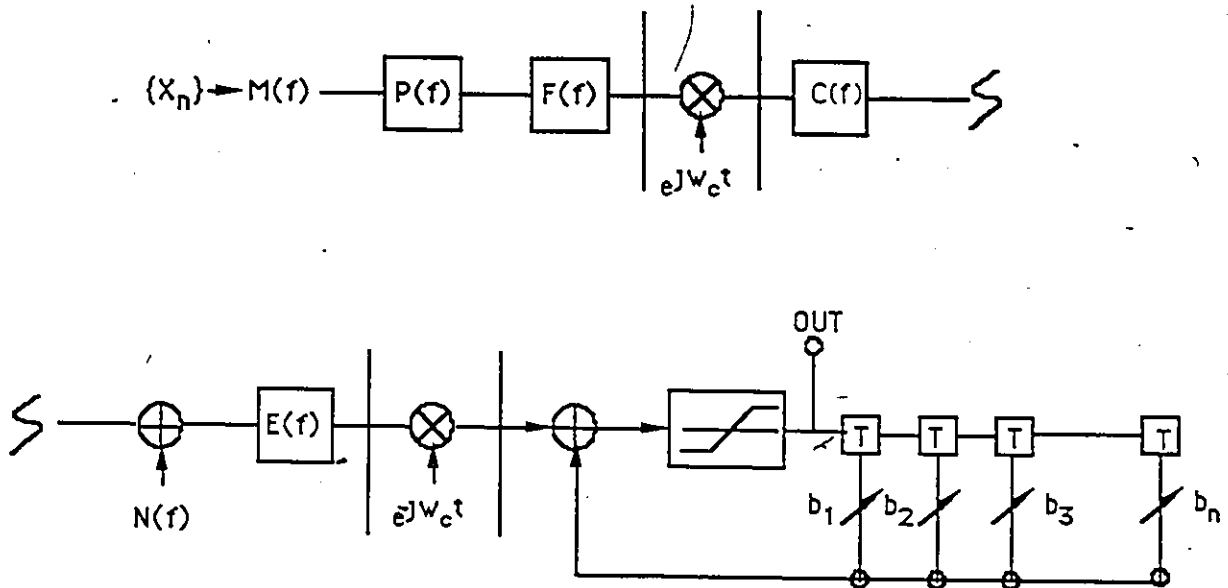


FIGURE 3. 1 One tone of a multitone system.

signal and decision feedback removes the effects of post-cursors before the decision stage is reached.

The input data sequence $\{X_n\}$ in this study consists of independent symbols. Each of these symbols is equi-probable and represents a point in the signal constellation of the modulation system being used. For the carrier QAM system the input data symbols are complex valued to accommodate the two dimensional type of transmission QAM represents. The data streams which modulate adjacent subchannels are asynchronous.

$M(f)$ is the message spectrum and is a measure of the correlation between data symbols and the spectral distribution of the data sequence. As shown in equations (18) and (19) correlations between symbols are represented by the summation of expected values. This thesis considers only uncorrelated data and as a result the message spectrum is flat with respect to frequency for all forms of modulation.

The pulse function $P(f)$ is the frequency representation of the pulse. An impulse is used so the spectrum is flat.

The transmitted power for the subchannel is the integral of the three factors above; the message spectrum, $M(f)$, the pulse function squared, $|P(f)|^2$, and the channel filter squared, $|F(f)|^2$. For the purpose of comparison between systems the power level for each subchannel is set to 0.25 W. When considering multitone systems this rule is relaxed and instead a limit is placed on the overall system power level. These limits on transmitted power are introduced to permit comparison between systems on a relatively equal footing. It can be anticipated from Chapter 2 that power levels are not a major concern and indeed results in Chapter 4 will show this to be true.

The channel filter in each subchannel is positioned before the modulator. A variety of filters have been analyzed. In some systems a 20% raised-cosine filter are used, in others full-cosine or brick-wall filters are used. The 20% raised-cosine filter is a compromise between the desire to maximize the use of available bandwidth and the need to use realizable filter specifications. The other channel filters are considered in determining the limits of the multitone approach, in evaluating equalizer effects and in calculating the performance of other approaches to multitone signalling. The brick-wall filter has a flat passband characteristic and vertical cut-off edges. The full-cosine rolloff filter has a bandwidth of $2/T$ and has been referred to as the Nyquist filter[6]. T is the period of one symbol. This filter is used in evaluating the orthogonal QAM approach in which the signal spectrum of one tone is allowed to overlap into another tone[10].

Transmission is studied with and without channel filters for the baseband systems. A cut-off filter is included for the systems where carrier modulation is used in subchannels above the reach of the baseband system. As a result of the presence of the equalizer prefilter the introduction of this cut-off filter has little impact on the baseband baud rate. The optimum equalizer prefilter tends to attenuate the channel above the frequency $1/2T$ to reduce noise, as a result introducing a high frequency cut-off filter is of little consequence. The equalizer prefilter alone is not however sufficient to ensure that the subchannels above the baseband frequencies may be used for carrier transmission.

The channel response, $C(f)$, is known for all frequencies[27] and is plotted in Figure 2.1.

The optimum equalizer prefilter is placed before the demodulator. This placement has been suggested before [40] and it is convenient for the purpose of this analysis. The feedback stage eliminates the effect of post-cursors from previously transmitted symbols. The equalizer prefilter and the feedback coefficients are determined using mean squared error analysis. For linear equalization the prefilter, in effect the entire equalizer, assumes the task of eliminating both the pre- and post-cursors.

These are the components of the subchannel transmission system. The following section will describe the approach used in calculating the optimum equalizer and determining the MSE for each subchannel.

3.2 CALCULATION OF ERROR RATE

The allowable error rate for each multitone system is 10^{-6} . When analyzing the multitone systems the error rate within each subchannel may exceed this nominal value slightly however the system error rate can never exceed 10^{-6} . Tight restrictions are placed on the error rate within each subchannel in order to ensure that no single tone will have an excessive number of errors while adjacent channels have error rates significantly lower than the nominal value. Due to these restrictions there is little difference in performance between a multitone system which requires only that the overall system error rate does not exceed 10^{-6} and a multitone system in which the error rate of each subchannel is less than 10^{-6} . Gray coding is used so that one symbol error will usually result in one bit error. The symbol error rate is determined using a bound to convert the calculated MSE to an upper limit on error rate. The MSE for the subchannel is calculated after determining the optimum equalizer. Each subchannel is analyzed in this manner for each of the modulation options. The MSE and bit error rate calculations are detailed below.

3.2.1 MSE CALCULATIONS

The calculation of mean square error is similar for both carrier and baseband systems. The development of the MSE equation and the equalizer equation is detailed in Appendix A with the differences between carrier and baseband systems explained in Appendix B.

If the received signal is represented by

$$r(t) = \sum_{n=-\infty}^{+\infty} X_n h(t - nT) + n(t) \quad (11)$$

- then at the sampling instant $kT + t_0$ the desired voltage is X_k . The delay encountered by the transmitted signals is represented by t_0 assuming the equalizer prefilter is compensating for the phase distortion introduced by the channel.

$$r(kT + t_0) = \sum_{n=-\infty}^{+\infty} X_n h(t_0 - nT + kT) + n(kT + t_0) \quad (12)$$

If we drop reference to the delay t_0 we can write the received signal as

$$r_k = X_k h_0 + \sum_{\substack{n=-\infty \\ n \neq k}}^{+\infty} X_n h_{k-n} + n_k \quad (13)$$

The desired signal is $X_k h_0$. The error in the signal is

$$\sum_{\substack{n=-\infty \\ n \neq k}}^{+\infty} X_n h_{k-n} + n_k \quad (14)$$

and an error in decoding may occur if the magnitude of this term exceeds one-half the distance between levels. Thus the mean squared error in the received signal can be written

$$|e_k|^2 = \left| \sum_{n=-\infty}^{+\infty} X_n h_{k-n} + n_k - X_k h_0 \right|^2 \quad (15)$$

In Appendix A it is shown that this term can be written as in equation (16) below.

$$\text{MSE} = \int_{-\infty}^{+\infty} N(f) |E(f)|^2 df + \int_{-1/2T}^{1/2T} M(f) \left| \sum_{m=-\infty}^{+\infty} H(f+m/T) - T - TB(f) \right|^2 df \quad (16)$$

The term $H(f + m/T)$ is the folded overall channel response which is the product of the channel $C(f)$, the equalizer prefilter $E(f)$, the subchannel filter $F(f)$ and the pulse function $P(f)$. $B(f)$ is the feedback

$$B(f) = \sum_{k=1}^{+\infty} b_k e^{-j2\pi k f T}, \quad (17)$$

where the term b_k represents the gain of each feedback tap.

The feedback acts to eliminate the effect of post-cursors by subtracting the residual channel response of previously decoded signals from the current signal. The message spectrum $M(f)$ is a function of the input data and the modulation being used.

$$M(f) = \sum_{k=-\infty}^{+\infty} \frac{m_k}{T} e^{-j2\pi k f T}, \quad (18)$$

where m_k is the autocorrelation of the input data sequence.

$$m_k = E(X_k X_0^*) \quad (19)$$

For uncorrelated input data m_k is equal to zero for all k except $k = 0$. Equation (18) is the message spectrum used in the baseband calculations.

The first half of equation (16) is the noise contribution to the error. The second half results from intersymbol interference. It is possible to eliminate all intersymbol interference through an appropriate equalizer design however allowing some intersymbol interference results in a MSE which is lower or equal to the MSE of a zero-forcing equalizer[42].

Equation (16) requires calculation of the equalizer prefilter and the feedback response before the error can be determined. As shown in Appendix A the equalizer prefilter response is given by equation (20).

$$E(f) = \frac{TR^*(f)}{N(f)U_-(e^{j2\pi fT})U_0} \quad (20)$$

$U_-(e^{j2\pi fT})$ is a factor which results from spectral factorization of equation (21).

$$U_+(e^{-j2\pi fT})U_-(e^{j2\pi fT}) = 1/M(f) + \sum_{m=-\infty}^{+\infty} \frac{|R(f + m/T)|^2}{N(f + m/T)} \quad (21)$$

U_0 is the DC value of the spectral factors. $R(f)$ is the overall channel response excluding the equalizer prefilter.

$$R(f) = C(f)F(f)P(f) \quad (22)$$

The equalizer is designed to minimize the mean square error for the subchannel.

3.2.2 CHERNOFF BOUND

Since the MSE cannot be directly related to the error rate and designing an equalizer with minimum error rate as the main criteria is considerably more difficult than the MSE approach[34], a suitable conversion between the two is required. The method chosen to convert MSE to bit error rate is the Chernoff bound[39] which provides an upper bound on bit error rate. The development of the bound is detailed in Appendix C. The use of the bound requires calculation of the overall channel impulse response at sampling time zero, h_0 . Sampling time zero refers to the sampling instant used in the receiver to decode a given symbol. The overall channel impulse response must take into account all factors from the pulse response, $P(f)$, to the feedback, $B(f)$. As shown in Appendix C once the MSE is known an approximation to the symbol error rate can be calculated.

$$P_e \simeq K_e Q\left(\frac{h_0}{\sqrt{\text{MSE}}}\right) \quad (23)$$

This Q function approximation is good for small MSE levels. h_0 is the overall channel impulse response at time $t = 0$. Since multilevel coding is used the symbol error rate must be adjusted by a factor relative to the number of bits represented by each symbol. That is, since 2^M symbols describe M bits and with Gray coding a symbol error results in an error of one bit, the bit error rate must be less than the symbol error rate by a factor M .

$$\text{bit error rate} = BER \simeq \frac{P_e}{M} \simeq \frac{K_e Q(\frac{h_0}{\sqrt{MSE}})}{M} \quad (24)$$

The constant K_e is dependant upon the signal constellation and is explained below.

3.3 BASEBAND TRANSMISSION

As explained previously in this chapter, if baseband transmission is used in a lower subchannel, and carrier modulation in an upper subchannel, then a high frequency cut-off filter is included in the baseband subchannel. When the transmission rates of baseband systems with and without the high frequency cut-off filters are compared little difference is noted. All the baseband systems include a filter to eliminate noise from very low frequencies thus removing the influence of 60 Hz pickup, $1/f$ noise and other low frequency sources.

The factor K_e in the bit error rate calculation is given by equation (25).

$$K_e = \frac{2(2^M - 1)}{2^M} \quad (25)$$

K_e results from consideration of the signal constellation and the fact that the 'Q' function provides the probability of an error occurring in one direction only. That is to say, if 'Z' represents the uncertainty surrounding a symbol then the probability that Z exceeds one-half the distance between symbols, $P(|Z| > h_0)$, is the probability an error will occur in decoding the symbol. As shown in Figure 2.2 if $Q(x) = P(Z > h_0)$ and $2Q(x) = P(|Z| > h_0)$ then the overall probability of a symbol error must take into consideration the arrangement of symbols and the probability of each symbol being transmitted.

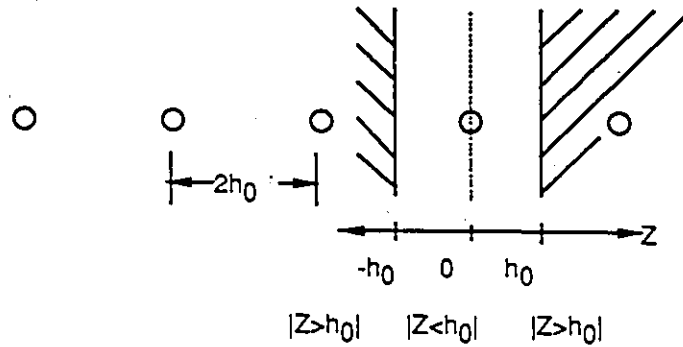


FIGURE 3. 2 Signal constellation for baseband signalling. Z represents the uncertainty surrounding the decoded symbol 'd'

For all but two symbols transmitted using the baseband signal constellation there are two possible decoding errors. For the outside symbols a decoding error can only occur in one direction. Therefore 2^M symbols can be decoded incorrectly in $2(2^M - 1)$ possible ways. Finally this factor is adjusted to account for the fact that each symbol occurs with probability $1/2^M$ and equation(25) is the result.

3.4 CARRIER SYSTEMS

Three carrier systems are considered : quadrature amplitude modulation, QAM, double side-band pulse amplitude modulation, DSB and single side-band pulse amplitude modulation, SSB. The analysis is performed using the same equations detailed for baseband transmission in Appendix A. The differences between the carrier and the baseband system calculations are briefly discussed below and are reflected in the baseband equivalent channels which are determined for each carrier system. The main differences exist in the channel filtering and the effects of modulation and demodulation on the signal and noise. These differences and the development of the equivalent baseband equations are detailed in Appendix B.

$\text{MSE} = \int_{-\infty}^{+\infty} N(f) E(f) ^2 df + \int_{-1/2T}^{1/2T} M(f) \left \sum_{m=-\infty}^{+\infty} H(f + m/T) - T - TB(f) \right ^2 df \quad (16)$	
$E(f) = \frac{TR^*(f)}{N(f)U_-(e^{j2\pi fT})U_0} \quad (20)$	$M(f) = \sum_{k=-\infty}^{+\infty} \frac{E(X_k X_0^*)}{T} e^{-j2\pi k f T} \quad (18)$
$B(f) = \sum_{k=1}^{+\infty} b_k e^{-j2\pi k f T} = \frac{U_+(e^{-j2\pi f T})}{U_0} - 1 \quad (26)$	$\text{BER} \sim \left(\frac{2(2^M - 1)}{M 2^M} \right) Q\left(\frac{h_0}{\sqrt{\text{MSE}}} \right) \quad (27)$
$U_+(e^{-j2\pi f T})U_-(e^{j2\pi f T}) = 1/M(f) + \sum_{m=-\infty}^{+\infty} \frac{ R(f + m/T) ^2}{N(f + m/T)} \quad (21)$	
$H(f) = P(f)F(f)C(f)E(f) \quad (28)$	$N(f) = N_0 + M(f)P^2(f)0.632 \times 10^{-13} f^{1.5} \quad (6)$

TABLE 3. 1 Equations for baseband transmission.

3.4.1 QUADRATURE AMPLITUDE MODULATION

QAM introduces the use of complex data symbols. The input data sequence $\{X_n\}$ consists of complex numbers. The modulation and demodulation of QAM signals can be depicted as in Figure 3.3. The I and the Q channels form what is in effect two DSB pulse amplitude signals which have been modulated by carriers at the same frequency but offset by 90 degrees in phase. This fact is used in the analysis of the QAM signals and in the development of the equivalent baseband channel in Appendix B. The presence of two signals on the same line results in an additional source of intersymbol interference which must be accounted for in the baseband equivalent channel.

The demodulation of the received signal results in a reduction in the noise con-

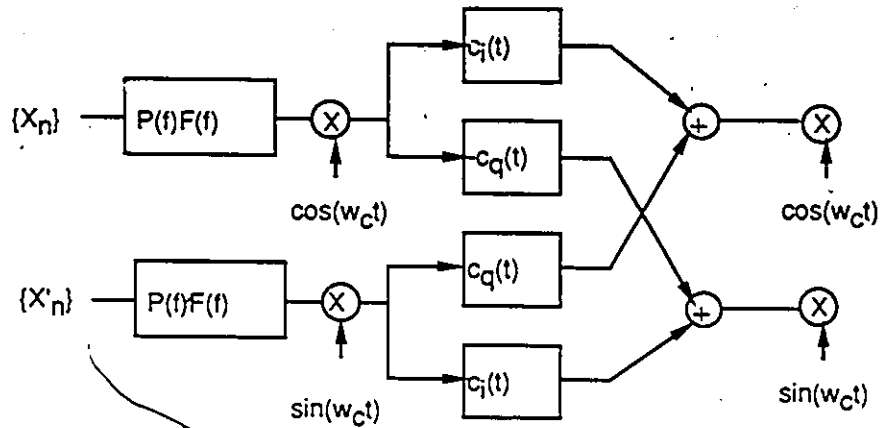


FIGURE 3. 3 Representation of a QAM modulation system.

tribution to the MSE by a factor of one-half.

The factor K_e in the bit error rate calculation is again dependant upon the signal constellation. The QAM constellation is more complicated than that of the baseband or PAM cases in that the constellation is drawn on two axes instead of one. It is therefore impossible to develop a closed form expression for K_e without first determining the signal constellation. A further discussion regarding K_e is given in Appendix D.

3.4.2 DOUBLE SIDE-BAND MODULATION

DSB signals result from the modulation of a real input data sequence $\{X_n\}$. The channel filters have a bandwidth in excess of $1/T$. The baseband equivalent channel shown in equation (32) is developed in Appendix B. The factor K_e in the bit error rate calculation is the same as that developed for baseband transmission, equation(25). This results from the fact that signal constellation is the same for baseband as for PAM transmission. As was the case for QAM, demodulation results in a reduction in the MSE noise contribution by a factor of two.

$\text{MSE} = \frac{1}{2} \int_{-\infty}^{+\infty} N(f) E(f) ^2 df + \int_{-1/2T}^{1/2T} M(f) \left \sum_{m=-\infty}^{+\infty} H_{eq}(f + m/T) - T - TB(f) \right ^2 df \quad (29)$	
$H_{eq}(f) = \frac{1}{2} P(f) F(f) [H(f + f_c)] \quad (30)$	$H(f) = E(f) C(f) \quad (22)$
$M(f) = \sum_{k=-\infty}^{+\infty} \frac{m_k}{2T} e^{-j2\pi k f T} \quad (31)$	$\text{BER} = \frac{P_e}{M} \sim \frac{1}{M} K_e Q\left(\frac{h_0}{\sqrt{\text{MSE}}}\right) \quad (24)$
$K_e = \text{dependant upon QAM constellation}$	

TABLE 3. 2 Equations for QAM transmission.

3.4.3 SINGLE SIDE-BAND MODULATION

SSB transmission offers the advantage of lower bandwidth requirements when compared to DSB transmission while providing the same baud rate. The difficulty which is well recognized is that of generating the SSB signal[44]. This implementation problem is not addressed in the analysis and the SSB system is evaluated on the basis of the transmission rate which it achieves. As can be seen from Table 2.3 the analysis of the SSB system is the same as that for DSB. The only difference lies in the choice of subchannel filters. The SSB signal is generated through the expedient of filtering the upper side-band of the DSB signal. The SSB signal carries the same amount of information as the DSB signal but with slightly more than half the bandwidth requirement. The form of the baseband equivalent channel equation is the same for SSB as for DSB however the SSB subchannel filter $F(f)$ removes most of the signal above the subchannel center frequency f_c . The single side-band filter maintains a 20% raised-cosine rolloff so the actual bandwidth of the SSB system is $1.4/2T$. In some literature this is referred to as vestigial single side-band modulation. As a result

of the overlap into the upper side-band the SSB scheme considered in this study is not as efficient as the QAM scheme.

$\text{MSE} = \frac{1}{2} \int_{-\infty}^{+\infty} N(f) E(f) ^2 df + \int_{-1/2T}^{1/2T} M(f) \left \sum_{m=-\infty}^{+\infty} H_{eq}(f + m/T) - T - TB(f) \right ^2 df \quad (29)$	
$H_{eq}(f) = \frac{1}{4} P(f) F(f) \left[H(f + f_c) + H^*(f_c - f) \right] \quad (32)$	$H(f) = E(f) C(f) \quad (22)$
$M(f) = \sum_{k=-\infty}^{+\infty} \frac{E(X_k X_k^*)}{T} e^{-j2\pi k f T} \quad (18)$	$\text{BER} \approx \left(\frac{2(2^M - 1)}{M 2^M} \right) Q\left(\frac{h_0}{\sqrt{\text{MSE}}} \right) \quad (27)$

TABLE 3. 3 Equations for PAM transmission.

3.5 MULTITONE SYSTEM DESIGN

The multitone systems are designed using the MSE-Chernoff bound equations described in the previous section. The bit error rate for each system is set to 10^{-6} . Two design approaches are used; the first is to fix the subchannel bandwidth and to then determine the optimum transmission rate which can be achieved in that subchannel and the second is to place no restriction on the subchannel bandwidth expanding the frequency range until the desired error rate is reached. In both approaches maximizing the total system transmission rate is the objective. The carrier modulation best suited for use at each frequency is determined based solely upon the achieved bit rate. At low frequencies where baseband modulation is feasible baseband transmission rates are weighed against those achieved using carrier systems occupying the same range of frequencies. The optimum multitone systems thus determined are

compared to the transmission rates achieved by the approaches discussed in Chapter 1 and to the rates achieved by baseband modulation alone. The results of this analysis are presented in the next chapter.

CHAPTER 4

COMPARISON OF SYSTEMS

In this chapter the application of the previously developed design calculations is presented. Optimum multitone systems using either linear or decision feedback equalization are determined. Comparisons are made to the performance of other parallel transmission schemes already in use.

4.1 BIT RATE COMPARISON

4.1.1 LINEAR EQUALIZATION

The analysis of multitone systems with linearly equalized subchannels shows that an improvement of almost 20% in bit rate can be achieved. This is the improvement relative to the highest bit rate achieved using a single baseband channel. In every type of carrier modulation the multitone system is superior to the the bit rate offered by a single channel using that type of modulation. As shown in Table 4.1 below the improvement offered by multitone signalling is dependant upon the modulation chosen.

Table 4.1 shows the highest transmission rate which can be achieved with each type of modulation. The systems defined as single channel consist of a single carrier channel extending from 100 Hz. to an upper frequency bound determined by the number of levels and the bit error rate. The number of levels and the bandwidth for the maximum rate system is included in brackets. The multitone results are again the highest rates which can be achieved. The overall system bandwidth is reported in brackets beneath the bit rate.

It was found that the multitone systems are relatively insensitive to subchannel bandwidth as long as the number of levels was optimized for each subchannel. Thus a large number of combinations can be found which provide transmission rates close to those identified as the maximum. This table does not include any results for

orthogonal signalling. The multitone systems therefore do not have overlapping signal spectrums. Figure 4.1 provides a representative sample of the frequency spectra of the systems in Tables 4.1 and 4.2.

		DSB-PAM	SSB-PAM	QAM	BASEBAND
TRANS. RATE	SINGLE CHANNEL	184 kbits/s (8L, 64 kHz)	180 kbits/s (8L, 42 kHz)	215 kbits/s (32Q, 51 kHz)	260 kbits/s (8L, 52 kHz)
	MULTITONE	187 kbits/s (64 kHz)	277 kbits/s (50 kHz)	310 kbits/s (64 kHz)	
% IMPROVEMENT (over single channel)		2 %	54 %	44 %	

TABLE 4. 1 20% raised-cosine filters are used as channel filters in all systems. The results are the maximum transmission rate which can be achieved. The terms in brackets are the number of levels, or points in the QAM constellation and the system bandwidth. The multitone systems are insensitive to the choice of subchannel bandwidth so a typical result is reported.

These results are for systems with subchannel filters that have 20% raised-cosine characteristics.

As Table 4.1 shows the multitone approach offers an improvement over baseband signalling when SSB or QAM carrier modulation is used. DSB is not better than baseband in terms of bit rate. When single-channel 8-level SSB-PAM and DSB-PAM modulation are compared on the basis of the transmission rate DSB modulation is slightly better. This is shown in Figure 4.2 where the bit rate achieved using 8 level PAM transmission is plotted against the frequency at which the subchannel begins. The advantage of DSB transmission increases as the start frequency is increased. This is of significance since due to the superiority of QAM in terms of spectral density PAM transmission is only considered for use at frequencies where QAM cannot be used. At these frequencies the advantage of DSB transmission over SSB makes it

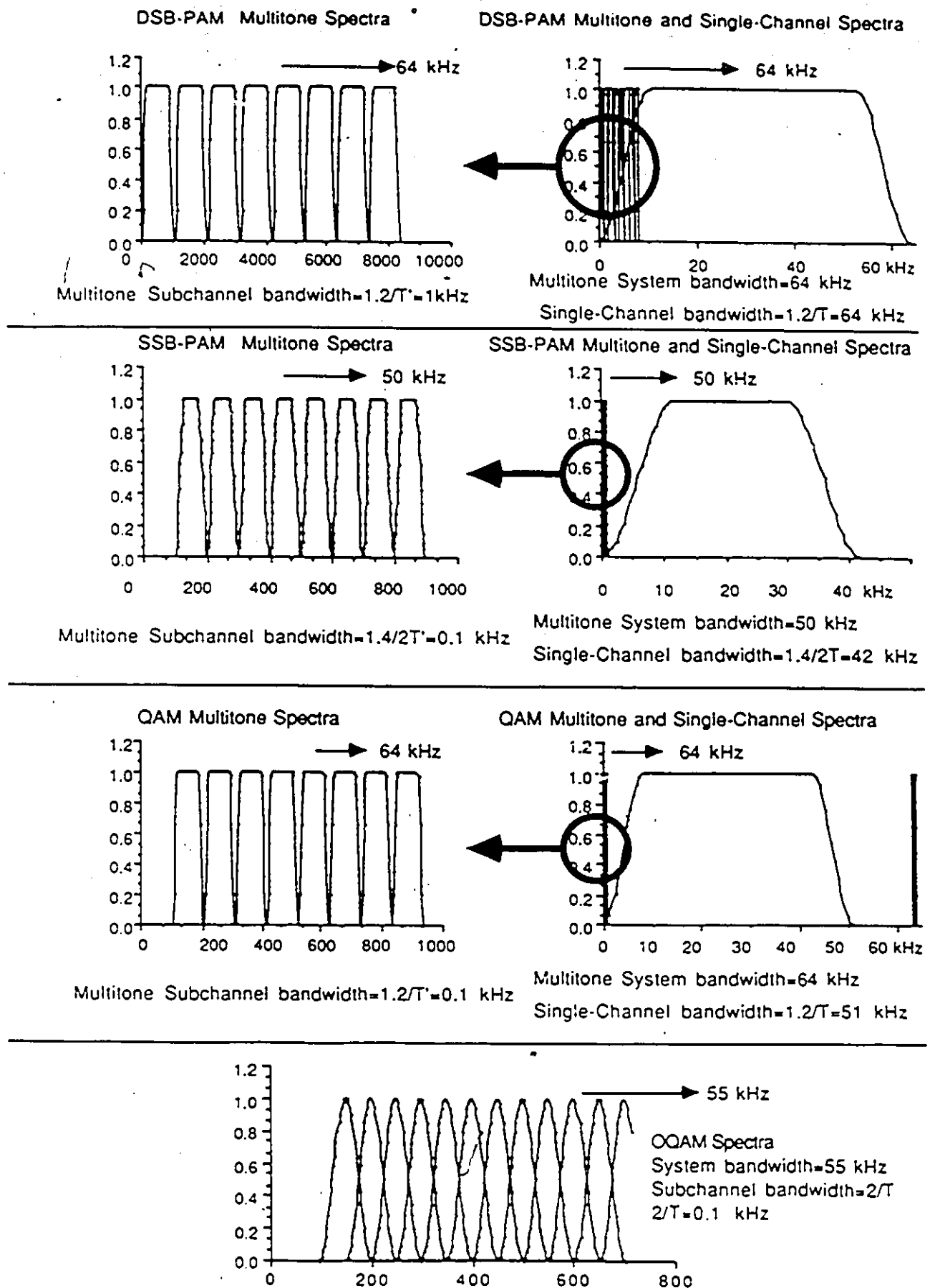


FIGURE 4. 1 A plot of the frequency spectra for the optimum linearly equalized single-channel and multitone systems. The choice of multitone subchannel bandwidth proved not to be critical. The vertical scale is arbitrary.

viable. If the subchannel bandwidth is restricted to some value at a frequency where either DSB or SSB transmission is possible the SSB system is superior by virtue of its higher ratio of bits/Hz. This observation is confirmed by the relative performance of the multitone systems using PAM modulation. The SSB system is much better than the system that relies upon DSB modulation.

The result for baseband transmission includes the effect of the low frequency distortion which results from the channel's inability to transmit signals at DC. The linearly equalized baseband system suffers from the presence of signal at this frequency. The effect of this distortion is limited however as when the performance of a 2-level baseband system was compared to that which is achieved with a baseband system using bipolar coding the coded system is only 5% better.

The choice of subchannel bandwidths is not very significant. The multitone systems designed with fixed subchannel bandwidths generally performed at the same level regardless of the subchannel bandwidth. As Figure 4.3 and Table 4.2 show it is possible to establish systems with relatively few tones, that is to say wide subchannel bandwidths, that performed as well or better than the narrow bandwidth multitone systems.

Orthogonal QAM signalling offers an alternative to the need to ensure the physical separation of subchannels in the frequency spectrum. Due to the orthogonal nature of the signals the signal spectrums may overlap in the manner illustrated in Figure 4.3. The center frequencies of each subchannel are separated by $1/T$ where T is one symbol interval. As can be seen in Table 4.2 the OQAM system provides a significantly better transmission rate than any of the multitone systems. The transmission rate achieved using OQAM is slightly better than that achieved with a multitone system using brick-wall filters even though the theoretical efficiency in terms of symbols/Hz is the same. This is due to a higher level of intersymbol interference and higher crosstalk noise levels at frequencies approaching the edge of the subchannel. If the transmission rates of the QAM multitone systems are compared to the OQAM transmission rate it is noted that the differences do not directly correspond to the improvement in efficiency, with efficiency still defined as symbols/Hz. The increase

CARRIER 8 LEVEL PAM MODULATION LINEAR and DECISION FEEDBACK EQUALIZATION

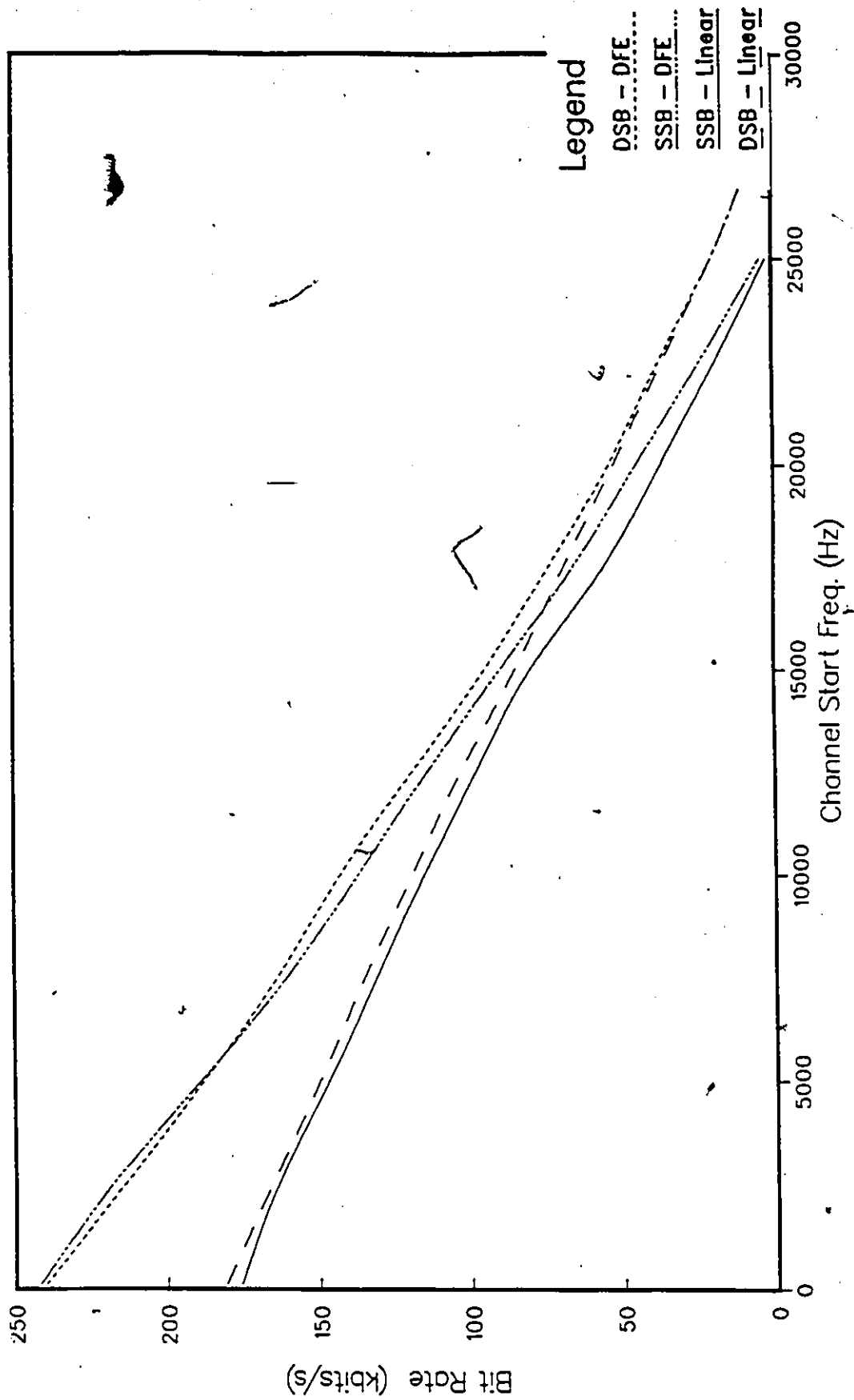


FIGURE 4. 2 A plot showing the effect of subchannel start frequency on bit rate for 8-level PAM systems with 20% raised-cosine filters.

MODULATION		FILTER	OPTIMAL SINGLE CHANNEL (kbits/s)	MULTITONE	
				BANDWIDTH B (kHz)	BIT RATE (kbits/s)
QAM	20 % RAISED COSINE $B=1.2/T$	215	0.001	297	
			0.1	319	
			1.0	312	
			10.0	308	
	BRICK WALL $B=1/T$	236	0.1	350	
			1.0	360	
			10.0	355	
	FULL COSINE $B=2/T$	204	0.1	231	
			1.0	213	
10.0			225		
DSB - PAM	20 % RAISED COSINE	184	0.1	171	
			1.0	187	
			10.0	183	
SSB - PAM	20 % RAISED COSINE	176	0.1	277	
			1.0	276	
			10.0	268	
QAM / DSB COMBINATION 20 % RAISED COSINE (6 TONES)			Syst. B = 55.0	320	
BASEBAND			Syst. B = 47.0	260	
OQAM	FULL COSINE f_c spacing = $1/T$ $B=2/T$	0.1	377		
		1.0	371		
		5.0	352		
		10.0	330		

TABLE 4. 2, A table of bit rates for linearly equalized systems. The optimum single channel and multitone result for each type of modulation is listed. The results for different filter types are included. [Single channel $P=0.25$ W, Multitone $P=1$ W]

LINEARLY EQUALIZED SYSTEMS

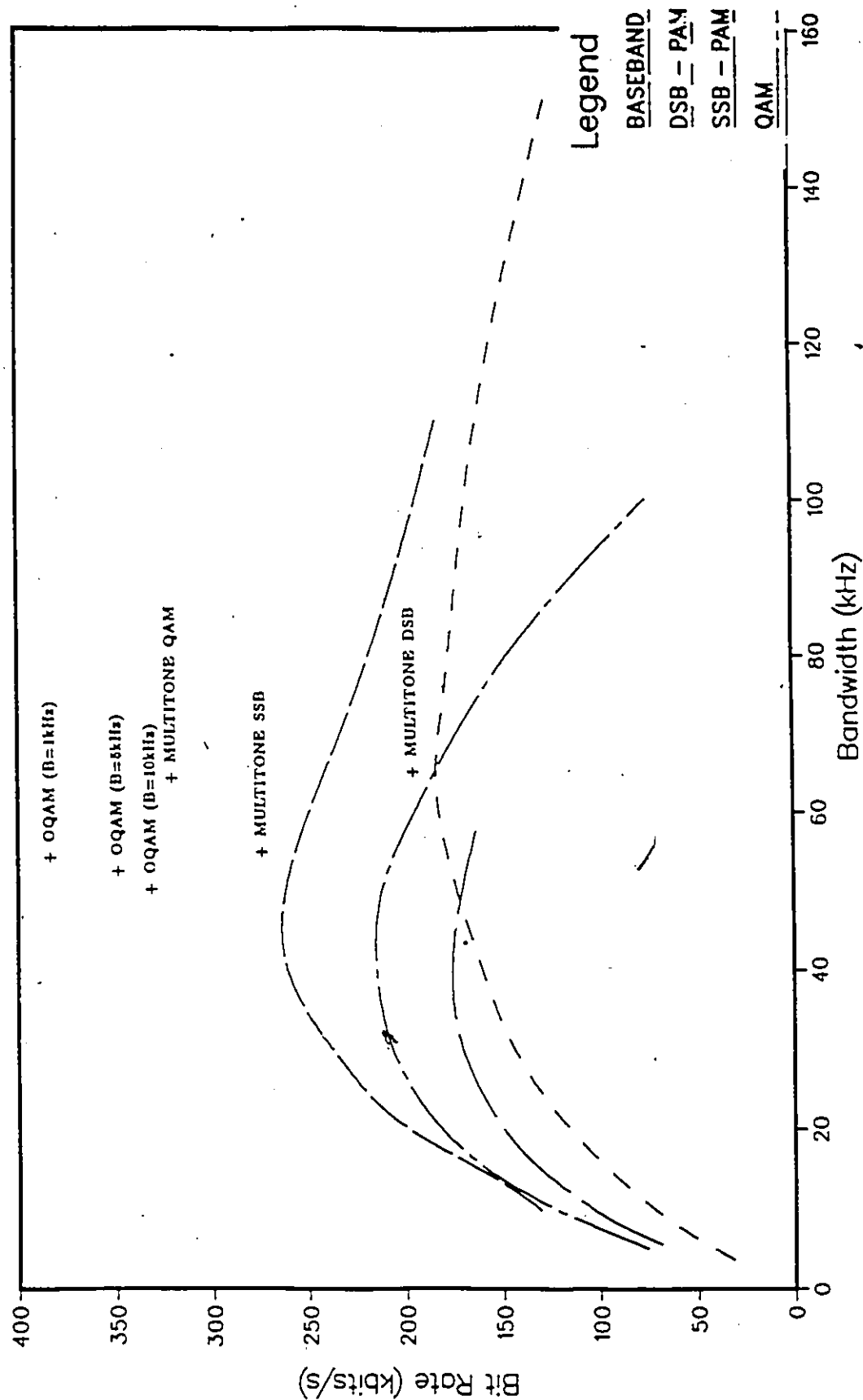


FIGURE 4.3 A plot of the bit rate and system bandwidth for linearly equalized systems. The continuous lines represent the single channel results. Individual points represent multitone systems. 20% raised-cosine channel filters are used in all systems except the OQAM systems which use full-cosine filters.

in transmission rate achieved in using OQAM is lower than the improvement in efficiency. This difference is explained by the increase in crosstalk which occurs in the OQAM system as a result of the overlapping signal spectrums.

4.1.2 DECISION FEEDBACK EQUALIZATION

Introducing decision feedback equalization to the subchannels results in a reversal of the conclusions regarding the performance of the multitone approach. Baseband signalling now provides the highest transmission rate of all the systems considered. Multitone signalling with DSB-PAM modulation is no longer an improvement over the best rate achieved using a single DSB channel. Multitone QAM is little better than the best single channel QAM system and the improvement in using multitone transmission with SSB modulation is significantly lower than that which is achieved when only linear equalization is used.

		DSB-PAM	SSB-PAM	QAM	BASEBAND
TRANS. RATE	SINGLE CHANNEL	246 kbits/s (8L, 97 kHz)	241 kbits/s (8L, 57 kHz)	340 kbits/s (320, 80 kHz)	434 kbits/s (8L, 87 kHz)
	MULTITONE	190 kbits/s (60 kHz)	279 kbits/s (54 kHz)	350 kbits/s (60 kHz)	
% IMPROVEMENT (over single channel)		-23%	15%	2.9%	

TABLE 4. 3 A table showing the improvement achieved using multitone signalling with decision feedback equalization. 20% raised-cosine filters are used in all systems. The results are the maximum transmission rate which can be achieved. The terms in brackets are the number of levels and the bandwidth for the single channel systems and the system bandwidth for the multitone systems. The multitone systems are insensitive to the choice of subchannel bandwidth so a typical result is reported. [Single channel $P=0.25$ W, Multitone $P=1$ W]

The frequency spectra of the systems in Table 4.3 and 4.5 are very similar to those shown in Figure 4.1. The only difference of note is the larger bandwidth of the single-channel or one tone systems. The choice of subchannel bandwidth in the multitone systems again proved to be of little importance (see Table 4.5). The reason for this shift in performance is the differential improvement offered by decision feedback. The introduction of decision feedback improves the performance of subchannels with wide bandwidths far more than it improves the performance of narrow bandwidth subchannels. This effect, which is explained in a later section, effectively eliminates one of the main advantages offered by the multitone approach. Since the decision feedback equalizer is better able to cope with large changes in channel attenuation and noise levels, being able to reduce the size of these changes through the use of multitones is no longer a significant gain. This fact is aptly demonstrated by the difference in improvements observed in the change from linear equalization to decision feedback equalization for multitone systems and baseband transmission. The baseband bit rate is dramatically higher when decision feedback equalization is used. This results from the ability of the decision feedback equalizer to deal with the channel null at the low frequencies as well as its enhanced ability to compensate for the frequency dependant nature of the channel attenuation and noise. The multitone systems which already have narrow subchannels and small variations in attenuation and noise benefit little from the introduction of decision feedback equalization.

The carrier PAM subchannels exhibit the same performance characteristics as was described for linearly equalized subchannels. That is, DSB modulation is slightly better than SSB modulation if no restriction is placed upon the channel bandwidth. A sample case for DSB and SSB modulation with DFE is plotted in Figure 4.1.

The carrier system subchannel bandwidths exhibit a characteristic behavior as the spectral density M (bits/symbol) is changed. Figures 4.4 and 4.5 plot the start and end frequencies for QAM and DSB subchannels using 20% raised-cosine filters, decision feedback equalization and different modulation levels. Design plots of this sort can allow a designer to quickly determine both the bandwidth of a subchannel and the baud rate which can be achieved in that subchannel with a bit error rate of

		MULTITONE SYSTEMS			
		DSB-PAM	SSB-PAM	QAM	BASEBAND
BIT RATE	LINEAR EQ.	187 kbits/s	277 kbits/s	319 kbits/s	260 kbits/s
	DFE	190 kbits/s	279 kbits/s	350 kbits/s	434 kbits/s
% INCREASE (relative to linear eq.)		1.6 %	0.7 %	9.7 %	67 %

TABLE 4. 4 A table comparing the bit rates for optimum systems with linear and decision feedback equalization. 20% raised-cosine channel filters were used on all systems. The system bandwidths are found in Table 4.1 and 4.3. [Single channel $P=0.25 W$, Multitone $P=1 W$]

10^{-6} . Similar curves can be generated for other combinations of channel filters, error rates and types of modulation.

OQAM is no longer much better than multitone signalling. Due to the enhanced ability of the decision feedback equalizer to cope with intersymbol interference there is no longer any difference in the transmission rates of the multitone system which uses brick-wall filters and the OQAM system. It must be noted when making this comparison that OQAM is feasible while the brick-wall filter multitone system is not. It is worthy of note that the only multitone systems which derived significant benefit from the introduction decision feedback equalization are the systems which have wide subchannel bandwidths. The narrow subchannel systems show little change.

The optimum baseband system provides a transmission rate which is 61% of the Shannon limit while the optimum combined baseband/carrier system provides 56%. The baseband/carrier combination is of doubtful value as it requires a baseband bandwidth which is almost as wide as that of the optimum baseband channel. This means there is little advantage being gained in terms of a simpler equalizer design at the expense of a lower information rate. A more conventional multitone system

QAM SUBCHANNEL START and END FREQUENCIES DECISION FEEDBACK EQUALIZATION

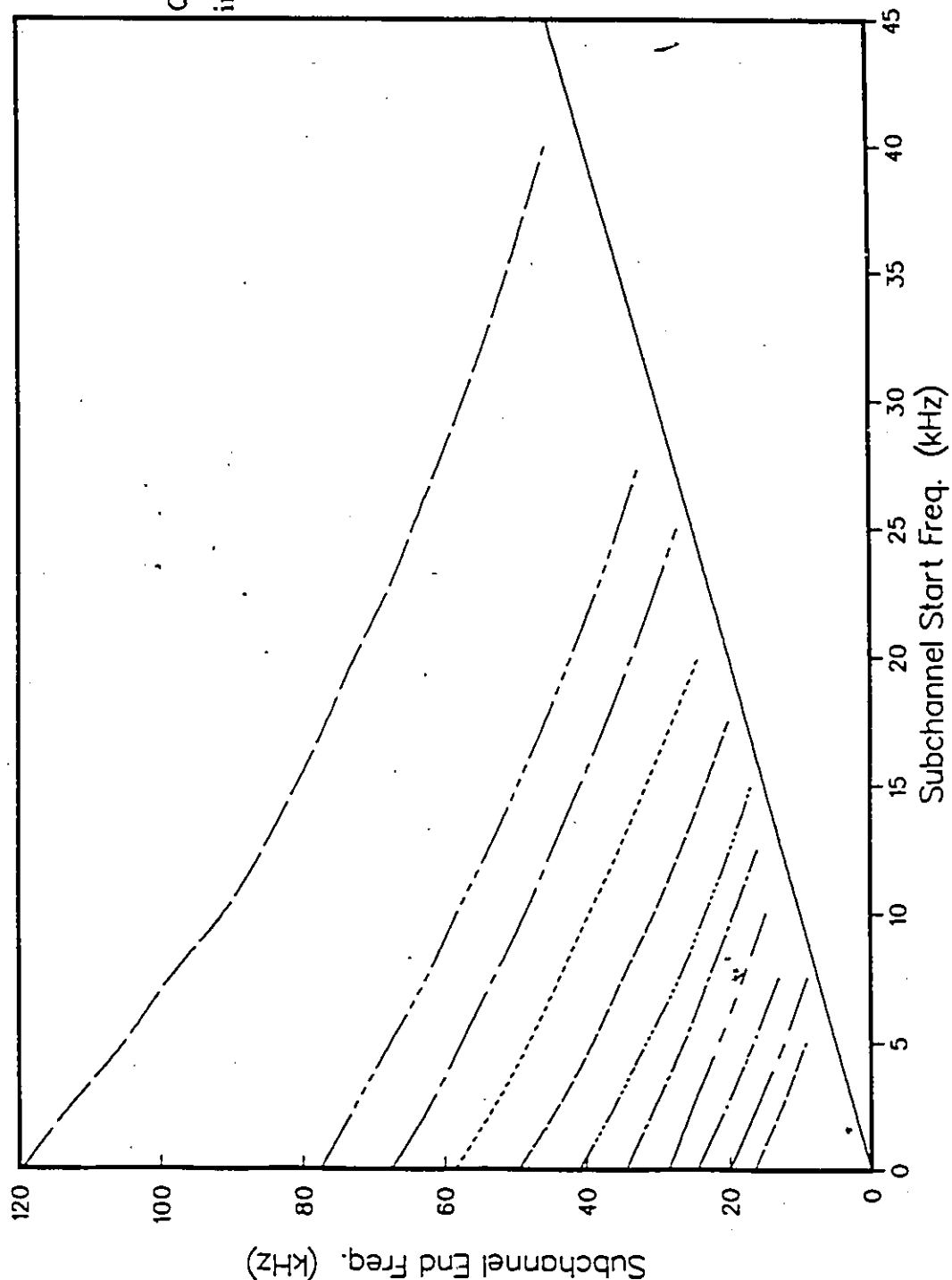


FIGURE 4. 4 A plot of subchannel start and end frequencies for QAM subchannels with decision feedback equalization. The frequencies plotted are the filter cut-offs. That is, the start frequency $= f_c - 1.2/2t$ and the end frequency $= f_c + 1.2/2T$. 20 % raised-cosine filters are used.

DSB-PAM SUBCHANNEL START and END FREQUENCIES DECISION FEEDBACK EQUALIZATION

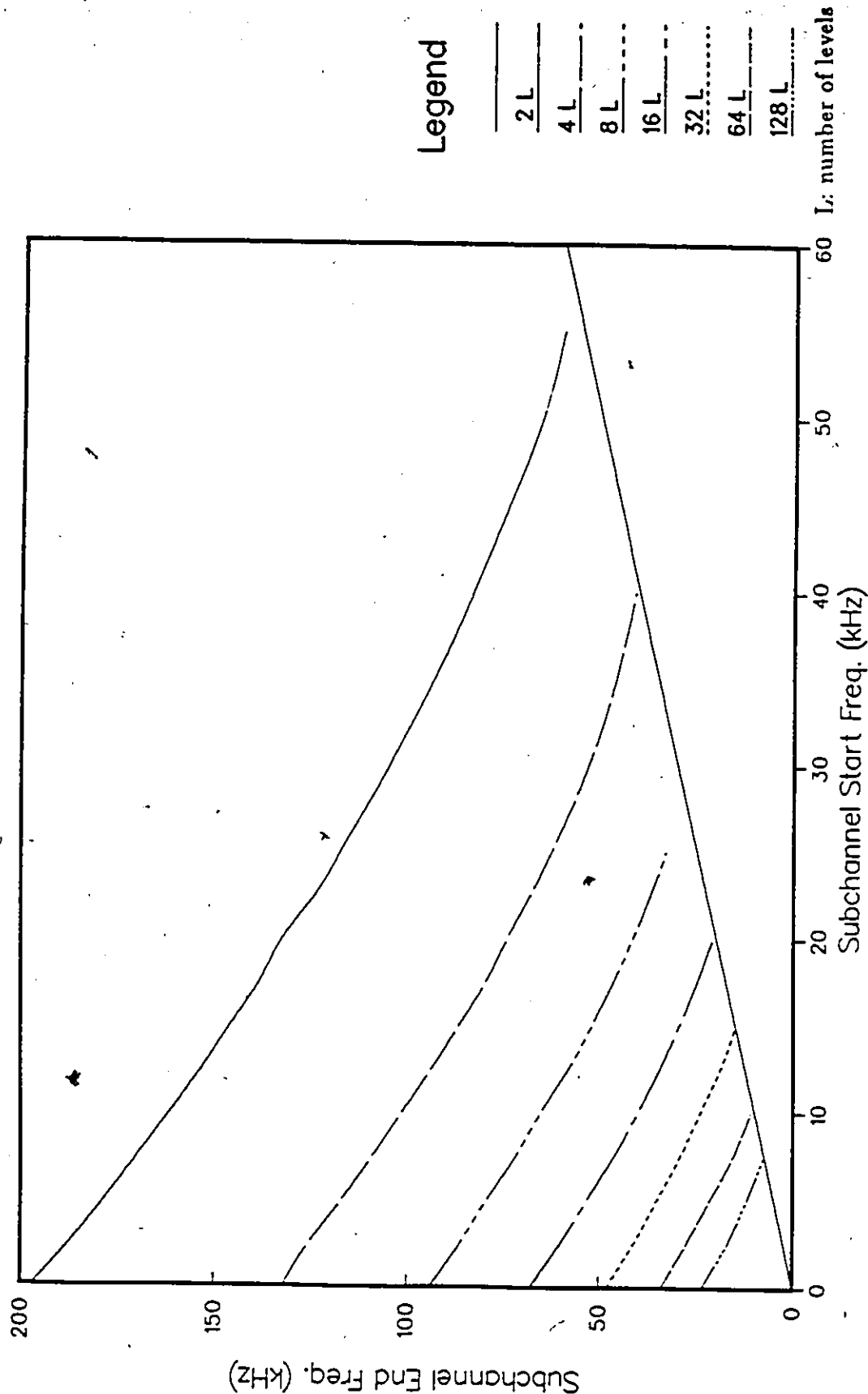


FIGURE 4. 5 A plot of subchannel start and end frequencies for DSB-PAM subchannels with decision feedback equalization. The frequencies plotted are the filter cut-offs. That is, the start frequency $= f_c - 1.2/2T$ and the end frequency $= f_c + 1.2/2T$. 20 % raised-cosine filters are used.

which combines QAM and DSB modulation provides a bit rate which is 54% of the Shannon limit. The OQAM system achieves 57%. These values are listed in Table 4.5 and plotted in Figure 4.6.

INFLUENCE OF DECISION FEEDBACK

One of the main advantages of multitone signalling is that it permits the designer to divide the available bandwidth into narrow subchannels which have consistent transmission characteristics over the frequency range. If decision feedback can negate the effects of large changes in attenuation and noise multitone signalling becomes less attractive. The reason for the differential improvement of large channel bandwidths over small when decision feedback is introduced is demonstrated by Figure 4.7. This plot shows, relative to the symbol rate, the influence of decision feedback for a QAM system with 10 bits/symbol. The MSE is plotted for both linear and decision feedback equalization. As the plot shows the linear and the decision feedback equalizers perform equally well at low symbol rates, that is, with narrow subchannel bandwidths. As the symbol rate increases however the performance diverges. There is actually very little difference noted until bandwidths greater than 1 kHz are reached.

Figure 4.8 shows the influence of decision feedback across the channel bandwidth for an 8 level baseband channel using a 20% raised-cosine filter. The decision feedback equalizer is more effective than the linear equalizer at the higher frequency end of the subchannel. As the frequency is brought down to the center of the channel, down to DC, the roles are reversed and the MSE contribution of the linearly equalized system is lower than that of the decision feedback equalized system. At DC there is a dramatic increase in the linear systems MSE which is a result of the channel null at very low frequencies. As the section below describes the linear equalizer actually enhances the noise, thereby increasing the MSE, in attempting to compensate for a steep increase in channel attenuation.

It is the linear equalizer's inability to perform well in wide subchannel bandwidths that makes linearly equalized multitone systems attractive. Decision feedback

MODULATION		FILTER	OPTIMAL SINGLE CHANNEL (kbits/s)	MULTITONE	
				BANDWIDTH B (kHz)	BIT RATE (kbits/s)
QAM	20 % RAISED COSINE B=1.2/T	340	0.1	319	
			1.0	328	
			10.0	350	
	BRICK WALL B=1/T	359	0.1	350	
			1.0	370	
			10.0	380	
	FULL COSINE B=2/T	264	0.1	231	
			1.0	213	
			10.0	225	
DSB - PAM	20 % RAISED COSINE	246	0.1	171	
			1.0	187	
			10.0	190	
SSB - PAM	20 % RAISED COSINE	241	0.1	279	
			1.0	275	
			10.0	268	
QAM / DSB COMBINATION 20 % RAISED COSINE			Syst. B = 60.0	360	
BASEBAND			Syst. B. = 87.0	434	
BASEBAND \ DSB COMBINATION			Syst. B = 77.0	400	
OQAM	FULL COSINE f_c spacing = 1/T B = 2/T	0.1	377		
		1.0	371		
		5.0	352		
		10.0	330		
		20.0	313		

TABLE 4. 5 A table of bit rates for decision feedback equalized systems. The optimum single channel and multitone result for each type of modulation is listed. The results for different filter types are included. [Single channel $P=0.25$ W, Multitone $P=1$ W]

DECISION FEEDBACK EQUALIZED SYSTEMS

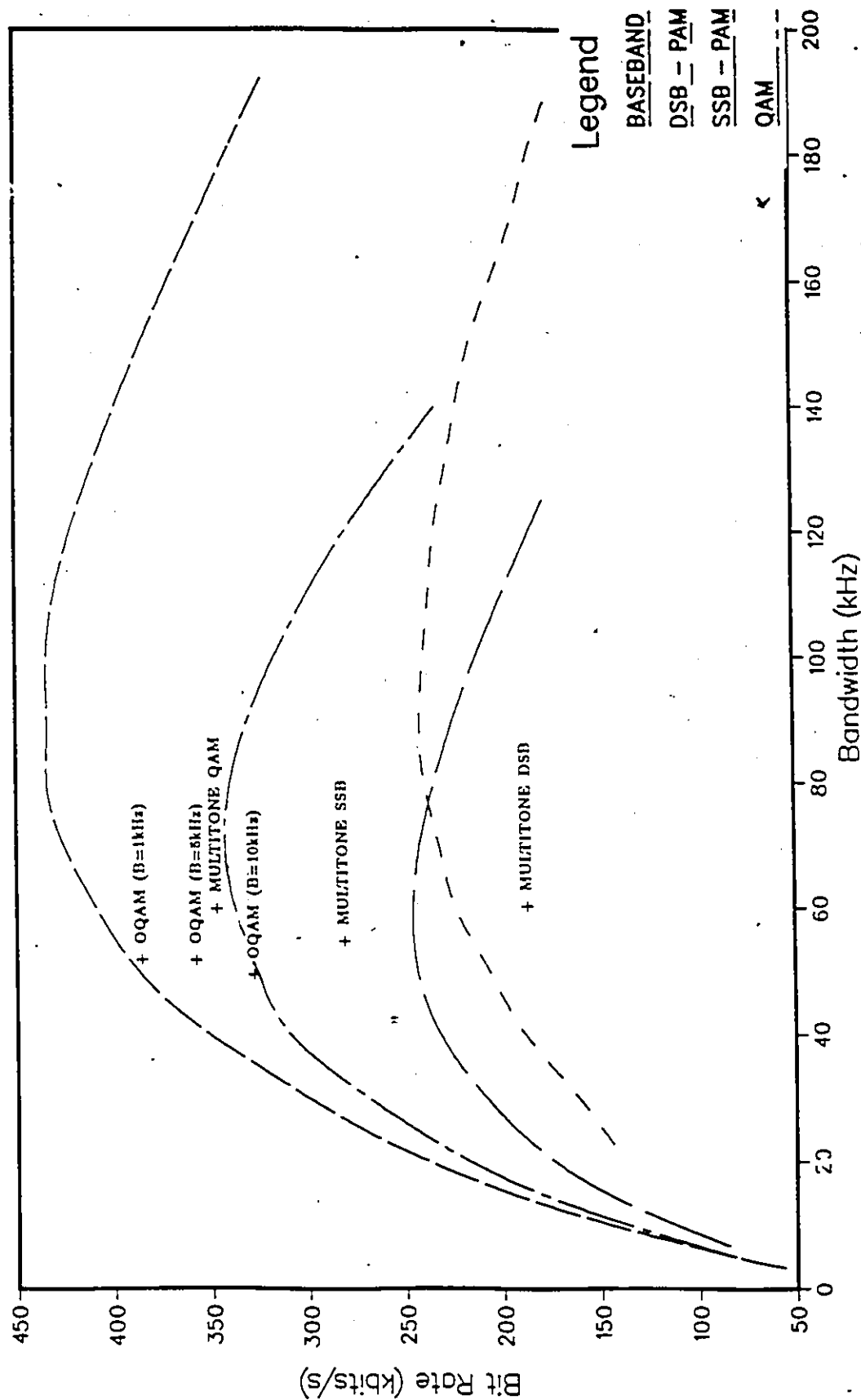


FIGURE 4. 6 A plot of the bit rate and system bandwidth for decision feedback equalized systems. The continuous lines represent the single channel results. Individual points represent multitone systems. 20% raised-cosine channel filters are used on all systems except OQAM systems which use full-cosine filters.

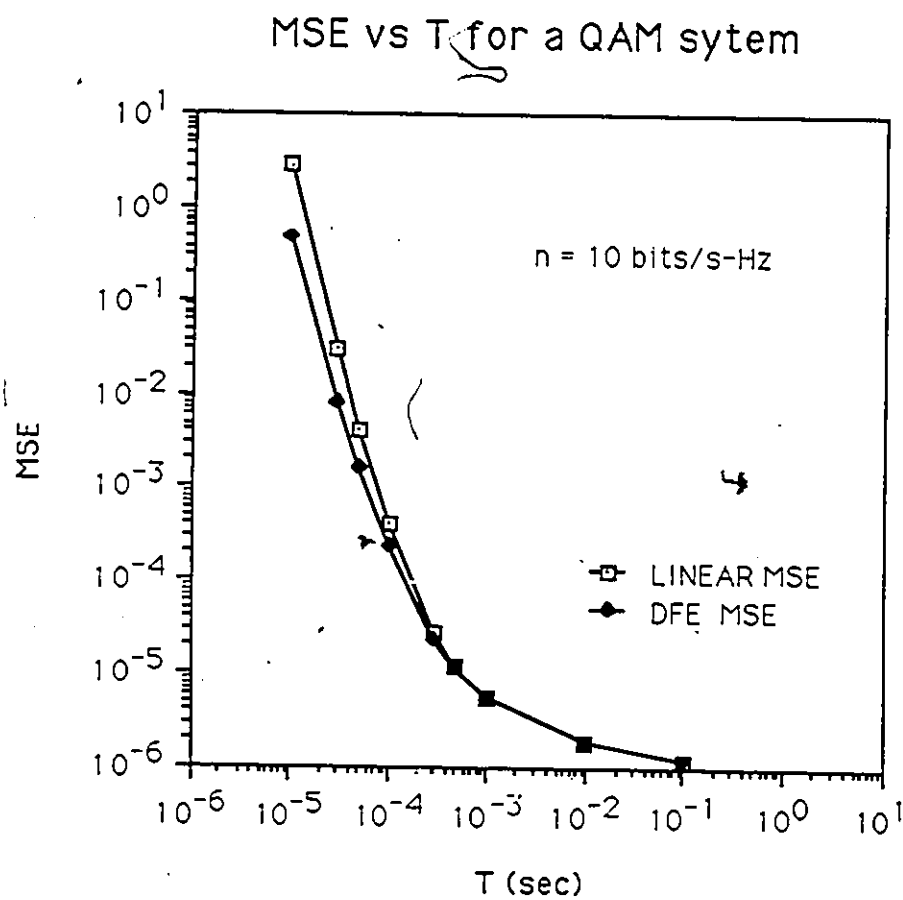


FIGURE 4. 7 A plot showing the influence of feedback on MSE for a QAM system. The wider the channel bandwidth the greater the influence of feedback.

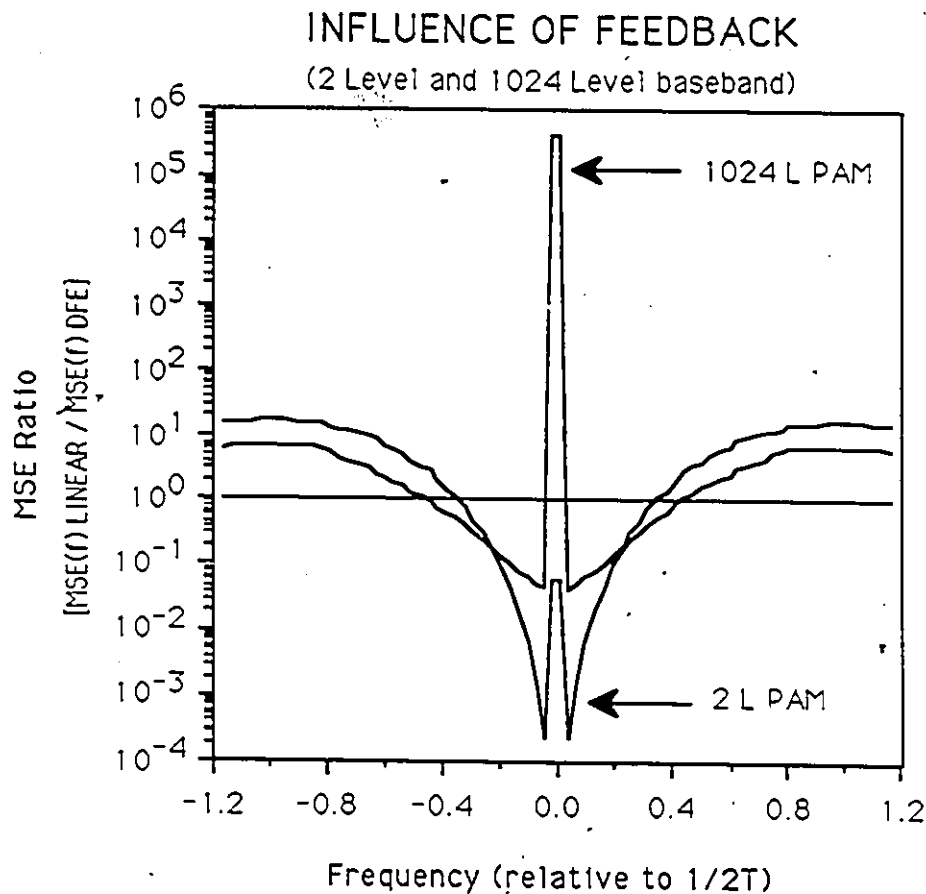


FIGURE 4. 8 A plot showing the influence of decision feedback on baseband channels. The greater the variance from 1.0 the greater the influence of feedback. The wide channel 2-level system benefits more from the introduction of feedback than the narrow bandwidth 1024-level system. The impact of the channel null at $f=0$ is obvious

is better able to cope with wide bandwidths and thus the multitone approach can offer less of an advantage in terms of transmission rate.

FREQUENCY DEPENDENCE OF MSE

The two types of equalizers in this study minimize the MSE in very different ways. As noted above the linear equalizer is unable to perform as well as the decision feedback equalizer in wide subchannels. An examination of the frequency dependence of the error contribution to the total MSE helps demonstrate why this is true. As Figure 4.9 shows, the error contribution to the MSE is linear over frequency for the decision feedback equalizer. For the linear equalizer the contribution is much greater at the upper frequency end of the channel. If one remembers that the crosstalk noise is frequency dependant it is reasonable to expect that increasing the channel bandwidth of a linearly equalized system will make the impact of the high frequency error contribution more significant. That is, due to the tendency of the linear equalizer to enhance noise at high frequencies, higher noise levels at the upper frequency end will degrade a linearly equalized channel's performance more rapidly than a channel with decision feedback equalization. The folded noise spectrum for these two channels after the equalizer prefilter is plotted in Figure 4.10. It clearly demonstrates the tendency of the linear equalizer to enhance high frequency noise while the decision feedback system results in a flat noise spectrum.

4.2 POWER DISTRIBUTION

As discussed in the section regarding the Shannon limit, crosstalk noise is the dominant noise source in this study. Figure 4.11 confirms this in a plot showing the limit of transmission rate in b/s/Hz for a QAM system with 1 Hz subchannels. There are three lines in the plot, the first shows the bit rate which is possible if only white noise is present, the second shows the limit imposed if crosstalk were the only noise source, and the third is the result of the combined noise sources. It is noted that this third line cannot be distinguished from the line resulting from crosstalk noise

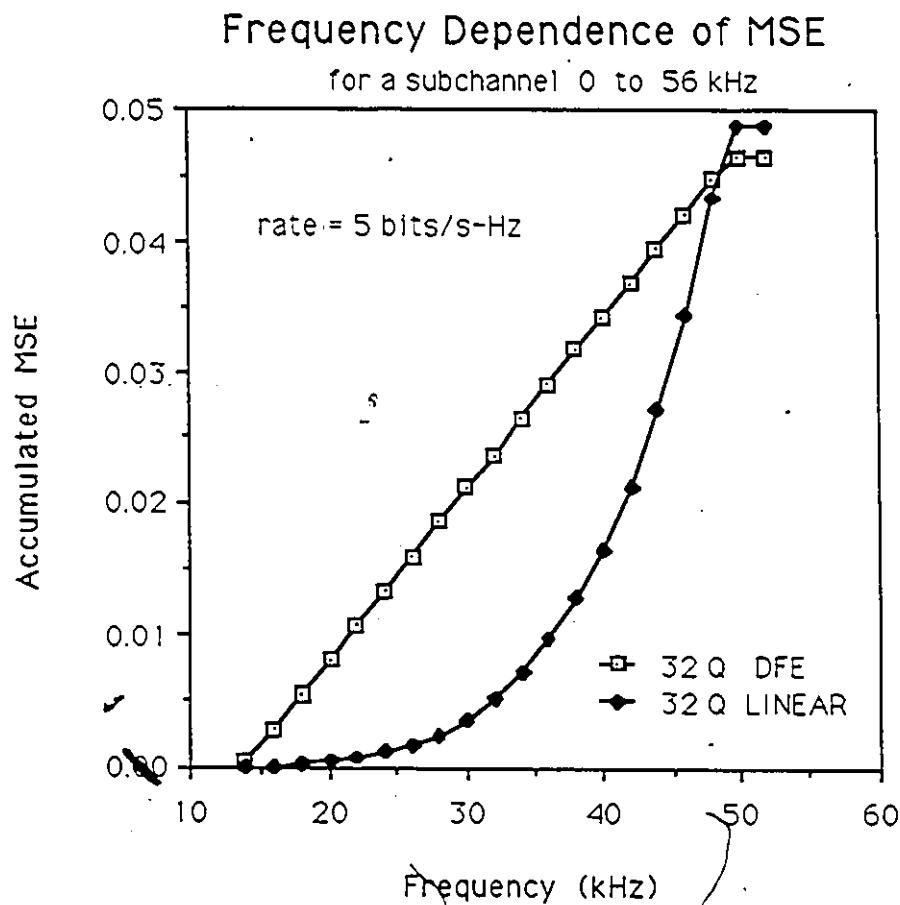


FIGURE 4. 9 A plot showing the linear accumulation of error over a decision feedback equalized channel and the dominance of the error contribution from the high frequency end of a linearly equalized channel.

FOLDED NOISE FOR 4L BASEBAND SYSTEM

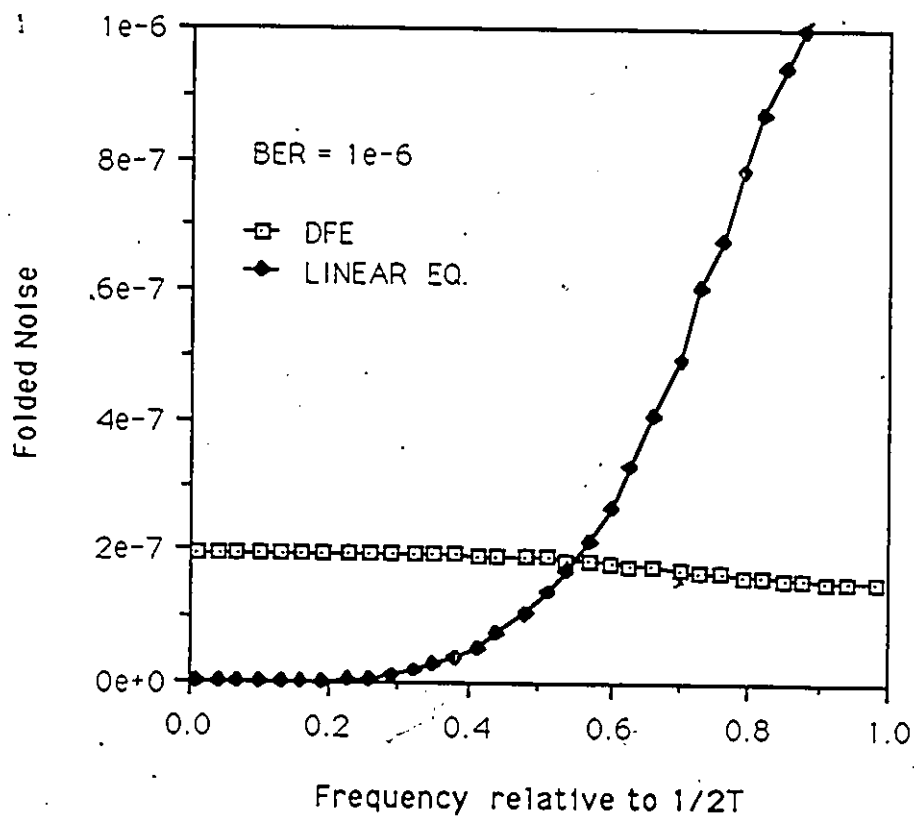


FIGURE 4. 10 A plot showing the folded noise after the equalizer prefilter for a baseband system with linear equalization and with decision feedback equalization. The folded noise over the feedback equalized channel is flat while the linearly equalized channel's noise is significantly skewed with frequency.

alone. It is evident from this plot that the white noise can only play a role in limiting the bit rate at very low frequencies. The relationship between increases in signal strength and increases in crosstalk noise make concerns about power distribution pointless. The achieved bit rate becomes a function of the channel attenuation and not the signal power distribution. This observation is confirmed in this simulation since concerns about matching the transmitted powers of the multitone and single tone systems have proven unnecessary. As Table 2.2 implies as long as the total power is above a threshold the actual power level and consequently the power distribution are not important.

The performance of baseband modulation relative to the multitone systems, which can match spectral efficiency to the channel's capability, is also explained by consideration of power distribution. Figure 4.12 shows the optimum signal and noise power distribution for the telephone loop and transmitted power level in this study. As can be seen from the plot of signal power the optimum distribution is relatively flat with respect to frequency. Baseband modulation provides just such a distribution. Thus even though baseband modulation in no way matches the spectral efficiency in b/s/Hz to the channel capability it still is close to the optimum approach.

The Shannon capacity for a channel which is limited only by white noise is 2.6 Mb/s. A calculation using OQAM signalling and 1 Hz subchannels suggests that a bit rate of approximately 1.6 Mb/s can be achieved. The possibility of completely eliminating crosstalk was raised in a recent paper by Steiglitz, Honig and Gopinath[37]. Their proposal is to use a multichannel adaptive finite-impulse-response filter to cancel near-end crosstalk. If a design of this sort were successful and far-end crosstalk could still be neglected rates in excess of T1 rates (1.544 Mb/s) could be achieved. This multichannel approach is outside the scope of this thesis and is mentioned only to emphasize the considerable impact of near-end crosstalk on the capacity of telephone wires. Multilevel baseband transmission under this condition can maintain a bit rate of 1.45 Mb/s. Thus in a white noise environment OQAM transmission provides a higher transmission rate than baseband. Multilevel QAM provides approximately the same bit rate as baseband transmission.

The dominance of crosstalk noise can be more fully described by considering the contribution of each error component to the total MSE. Equation (16) permits us to separate the intersymbol interference from the noise contribution. It is also possible to separate the noise contribution into the white noise and crosstalk components. This process clearly demonstrates the dominance of crosstalk as well as the frequency dependant nature of the noise. In wide bandwidth QAM channels with decision feedback equalization and 2 bits/symbol over 70% of the error is due to crosstalk, white noise contributes approximately 10% and the rest is due to intersymbol interference. The same QAM system operating in a narrow bandwidth channel at high frequencies has a MSE which is 84% due to crosstalk and 16% due to white noise, the intersymbol interference contribution is insignificant. A higher level QAM system with 16 bits/symbol operating in a narrow bandwidth channel at low frequencies has the same distribution, 84% of the MSE is due to crosstalk and 16% due to white noise. Baseband systems breakdown in the same manner. The 2 level baseband has a MSE which is 58% due to crosstalk and 31% due to white noise. The intersymbol interference contribution is only significant in wide channel bandwidths where large variations in attenuation exist across the bandwidth.

4.3 FILTER EFFECTS

There was considerable concern over the choice of subchannel filters. While the 20% raised-cosine filter is a reasonable choice there still remains some question regarding the effect of the subchannel filter on the baud rate. As Figure 4.13 shows, the transmission rate which can be achieved decreases as the excess bandwidth of the filter is increased. The changes in transmission rate are not however directly related to the changes in excess bandwidth. Excess bandwidth refers to the bandwidth in excess of $1/T$. For the 20% raised-cosine filters the excess is 20%. The full-cosine filter has an excess of 100%. The baud rates achieved while using the 20% raised-cosine filters are not 20% lower than the baud rates achieved using brick-wall filters. Table 4.6 lists the bit rates for different filter types and it is noted that the rates are not directly proportional to the changes in excess bandwidth. This is due to the

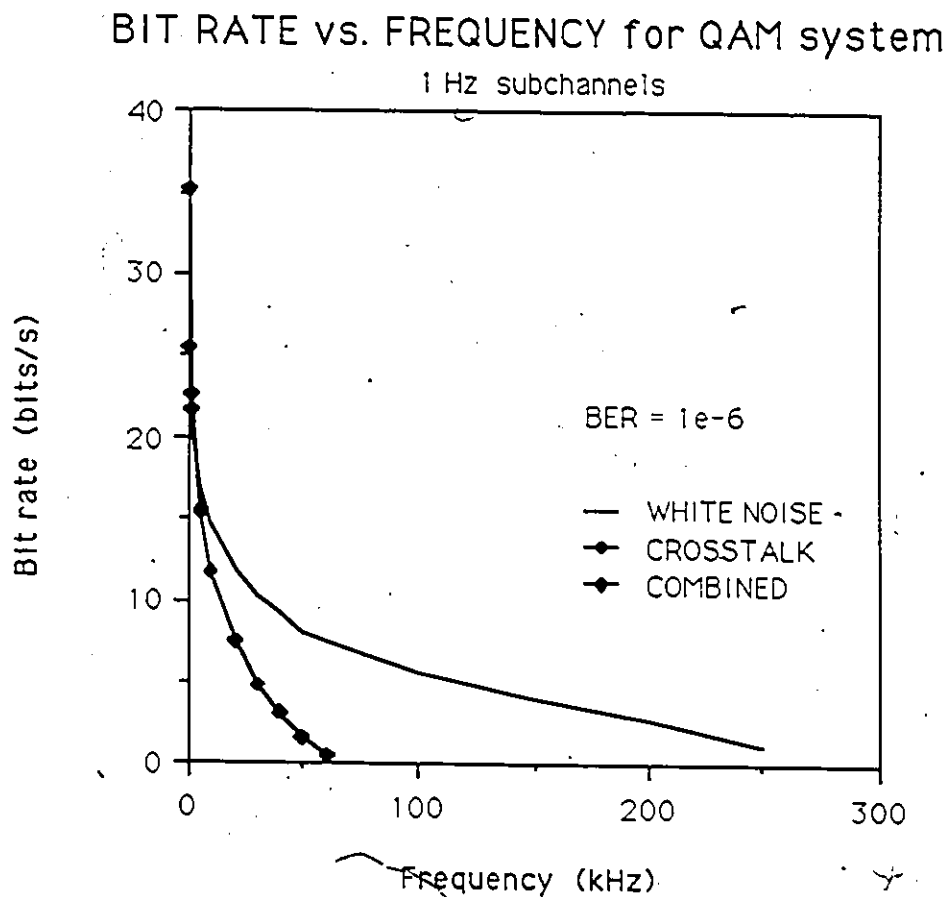


FIGURE 4. 11 A plot showing the bit rate which can be achieved in 1 Hz subchannels at an error rate of 10^{-6} . The smooth line shows the limit imposed by white noise. The second is the limit imposed by crosstalk and the third, which overlays the crosstalk-only line, shows the bit rate for combined white and crosstalk noise systems.

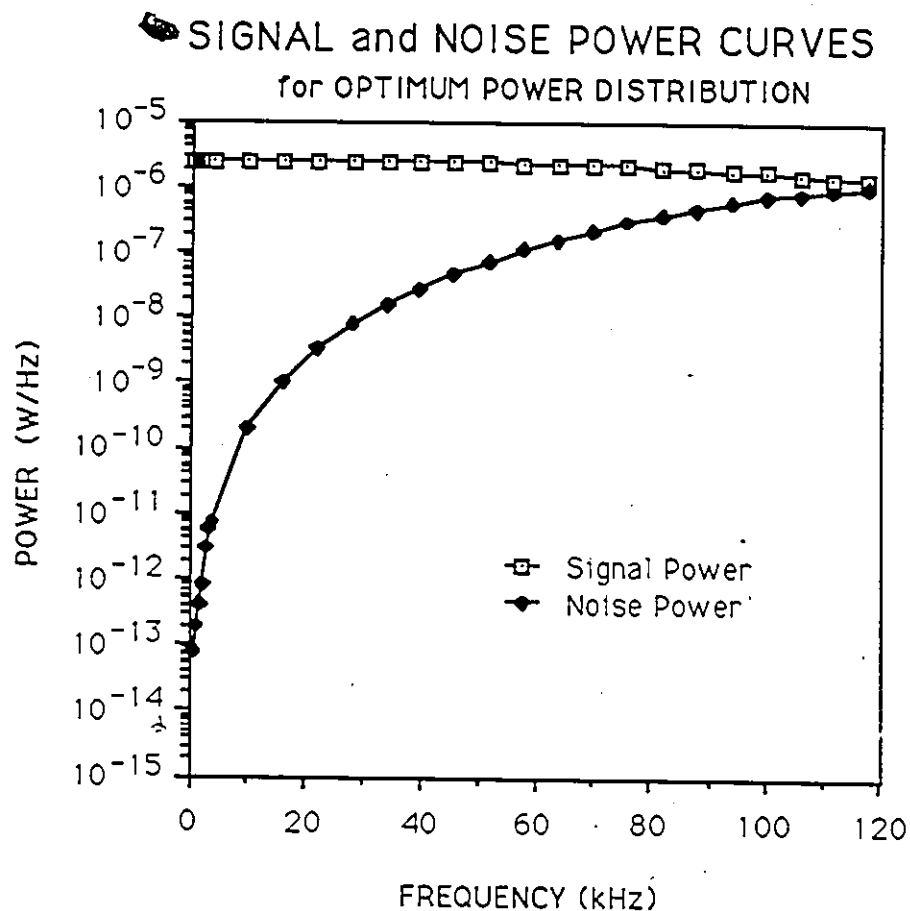


FIGURE 4. 12 A plot showing the optimum signal and power distribution for the telephone loop in this study. The noise has been 'reflected' through the channel response so that the noise power is as it would appear from the perspective of the transmitter. [P=0.25 W, L=4.8 km.]

IMPACT OF FILTER ON BIT RATE

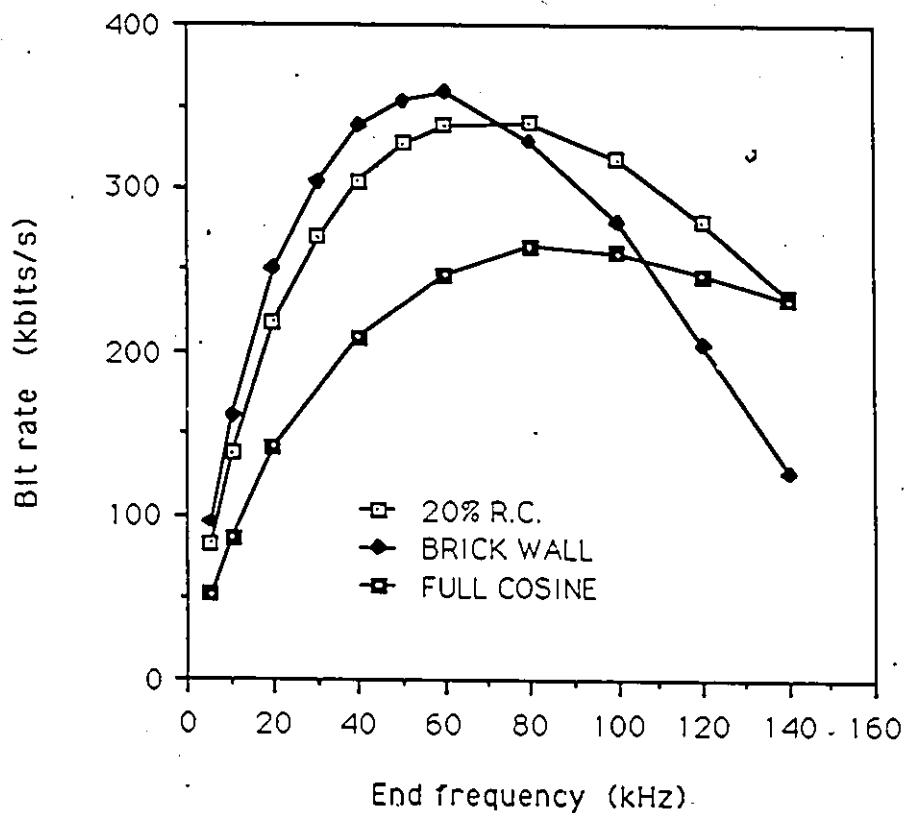


FIGURE 4. 13 A plot showing the bit rate for a single QAM tone with three different filter types. The more severe filter types provide a higher maximum baud rate but perform poorly in wide channel bandwidths. The start frequency of each channel is 100 Hz.

higher intersymbol interference which results from the more stringent subchannel filters. As Figure 4.13 shows, at higher frequencies the subchannels with the less severe filters can maintain higher bit rates. The observations regarding the effect of excess bandwidth on system baud rate permit one to conclude that for decision feedback equalized channels a multitone system cannot achieve a better transmission rate than baseband transmission simply through a different choice of channel filters.

	EXCESS BANDWIDTH	TRANSMISSION RATE	
		SINGLE CHANNEL	MULTITONE
BRICK WALL FILTER	0 %	359 kbits/s	380 kbits/s
20 % RAISED COSINE	20 %	340 kbits/s	350 kbits/s
		95 %	92 %
FULL COSINE	100 %	264 kbits/s	231 kbits/s
		74 %	61 %

TABLE 4. 6 A table comparing the bit rates of systems with different filter types. The bit rate percentages represent the bit rate relative to the rate achieved by the 0 % excess bandwidth system.

4.4 P_e vs MSE-CHERNOFF BOUND CALCULATIONS

As discussed in Chapter 1 another method of calculating bit rates is available. For the purpose of clarity it is referred to in this discussion as the 'Kalet calculation'.

This approach permits direct calculation of the bit rate which can be achieved in a subchannel with linear equalization. The calculation is complicated when frequency dependant crosstalk noise and channel attenuations are present however it can be used as an approximation if the appropriate integrals are used as in equation (4). This approximation is compared to the results obtained using the MSE-Chernoff bound approach of this study. As Figure 4.14 shows, the calculation of bit rate using these two methods is very close. The slight differences can be attributed to the approximations made in developing the Kalet calculation and the fact that the Chernoff bound is an upper limit on the error probability and not an exact calculation. Thus the Kalet calculation offers another approach to calculating the error rate which can be achieved using linear equalization. It does not however determine the equalizer function which is necessary to achieve this transmission rate and is not necessarily a more simple calculation.

4.5 VOICEBAND CHANNEL CALCULATIONS

The performance of multitone signalling on a voiceband channel with loading coils is considered in this section. Decision feedback equalization is used in all cases. This channel is not as severe in terms of channel distortion as is usually encountered when discussing voiceband channels[15]. The channel used in this analysis is a 4.5 mile pair with no repeaters or bridged taps. With this channel the OQAM approach is clearly superior to baseband transmission. As can be seen in Table 4.7 a single QAM system is almost as good as the best baseband system. When OQAM is introduced the achievable transmission rate is 17% better than that which can be achieved using baseband modulation. This result, which appears to be in contradiction with the conclusion made in the analysis above, is explained by noting that the voiceband channel places a severe limitation on the bandwidth. In the analysis above no such restrictions were placed on the system bandwidth. Indeed, the optimum baseband system above has a bandwidth which is 45% greater than the best OQAM system. If the bandwidth of the baseband systems was limited to the bandwidth occupied by the multitone systems the conclusions regarding transmission rate would be less

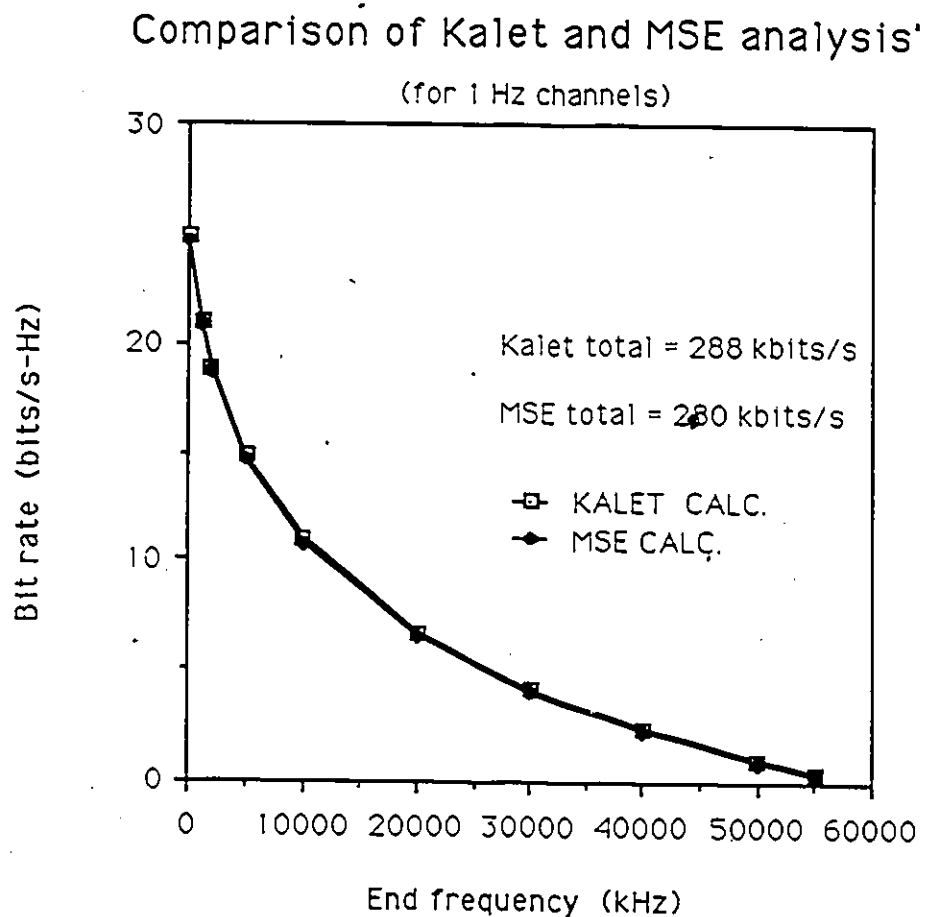


FIGURE 4.14 A plot showing the difference in results between Kalet and MSE performance calculations. The bit rate is the rate achieved by a multitone QAM system with 1 Hz subchannels and linear equalization. There is almost no difference in the two plots.

clear. For the channel considered in Section 4.2.1 it can be seen from Figure 4.5 that a baseband system occupying the same bandwidth as the OQAM system has approximately the same bit rate.

For the voiceband channel the problems associated with transmission at frequencies approaching DC are more significant than is the case when transmission is possible over a wider bandwidth. That is, as the low frequency null occupies a greater proportion of the channel bandwidth the influence of the null is increased. The advantage of multitone signalling in a channel of this sort will increase if the low frequency performance of the channel degrades, thus for systems operating in voiceband channels over long distances and through many repeaters the advantage of multitone signalling will be even greater.

	BASEBAND	QAM (20 % r.c.)	MULTITONE QAM	OQAM
BIT RATE (kbits/s)	60.1	58.9	63.2	71.0

TABLE 4. 7 A table comparing the bit rates of systems operating in a voiceband channel. All system channel filters are 20% raised-cosines except for the OQAM which uses a full-cosine filter. [P = 0.25 W, L = 4.5 km., 26-AWG cable with loading coils]

CHAPTER 5

SUMMARY AND SUGGESTIONS FOR FURTHER RESEARCH

5.1 SUMMARY

This thesis has considered the performance of multitone signalling over a telephone loop. The conclusions are summarized below.

The transmission rate achieved using multitone signalling is lower than that which can be achieved with decision feedback equalized baseband modulation. OQAM provides the highest transmission rate of all the carrier schemes. It is possible to design two tone systems consisting of baseband modulation and DSB-PAM which have a higher transmission rate than OQAM however the improvement is small and the rate remains lower than the optimum baseband rate.

Multitone QAM transmission rates can approach those of OQAM if the filter excess bandwidth is small. Practical limits on subchannel filter design will however result in large differences in the rates which can actually be achieved by these two approaches. Hirosaki has already demonstrated that an OQAM system of the type used in this analysis is practical[19]. This is despite the stringent requirements placed upon the phase and clock recovery circuitry. Only the requirement that the data in adjacent channels be synchronized may prevent the use of OQAM when choosing between carrier systems. If frequency bands are to be allocated to specific tasks or if the data sources are in physically separated locations then neither OQAM or baseband transmission can be used.

The performance of the frequency division multiplexed systems is in general little affected by the type of equalization which is used. Only in the case of the multitone system using a brick-wall filter with its severe spectral rolloff characteristic is a significant improvement gained by introducing decision feedback equalization. The

multitone systems are unaffected by equalization due to the fact that the tones are narrow enough that there is no significant change in the noise and channel characteristics across the bandwidth. In this situation it has been shown that the linear equalizer performs almost as well as a decision feedback equalizer. The use of linear equalization and the fact that relatively few tones are required to provide good performance simplifies the design of the multitone transmission system. In order to maximize the baseband transmission rate decision feedback equalization is required and the large channel bandwidth will result in the need for significant phase compensation at the equalizer. If linear equalization is used the performance of the baseband systems falls below that of all the OQAM and multitone schemes with the exception of one using double side-band modulation.

In a crosstalk noise environment spectral allocation of transmitted power is unimportant. Due to the one-to-one correlation between crosstalk noise power and signal power the channel response plays the major role in determining the channel capacity once the crosstalk level is established. For the specific noise levels in this study a small relationship did exist between power allocation and channel capacity. This relationship resulted from the presence of white noise as well as the crosstalk. The crosstalk noise is dominant however and the distribution of transmitted power with respect to frequency is relatively unimportant.

The bandwidth required by the optimum multitone systems is generally lower than the bandwidth of the baseband systems. This points to the possibility that if the channel bandwidth were restricted to a value less than 60 kHz then multitone signalling may provide a higher transmission rate than baseband. In addition, no coding techniques were considered. It is noted that if power-limited orthogonal techniques[45] were used in subchannels above the multitone bandwidth the combined transmission rate could approach that of the optimum baseband system.

The low frequency channel distortion had little effect on the baseband transmission rate. The carrier systems avoided the problem entirely by not placing any signal at the low frequencies. The baseband system will always have some signal content at frequencies approaching DC. For this reason any degradation in the DC

characteristic of the channel will impact upon the baseband transmission rate while the carrier systems will be unaffected. The null will also increase the complexity of both the DFE and the echo canceller in the baseband system. This factor must be considered when applying the conclusions made here to other channels.

When transmission in a voiceband channel is considered, frequency division signalling provides the highest transmission rate. This is due to the restricted channel bandwidth and the DC distortion discussed above. The OQAM transmission rate is significantly better than baseband and again this difference will increase when other voiceband channels with more severe DC distortion are encountered.

A number of papers in areas related to this research have shown that coded multitone systems outperform baseband modulation.[6-8,11] It must be noted however that in many cases these authors are using channel characteristics which include the effects of bridged taps. The presence of bridged taps introduces nulls into the frequency characteristic. Nulls in the characteristic make the use of multitone techniques a logical choice. It is also noted that in comparing baseband to multitone these authors restricted themselves to low level (2-4) baseband modulation. It was found that if higher levels are used the baseband system performs on a par with multitone. It is expected that this will be the case if coding is introduced to the system.

Multitone and OQAM signalling both offer alternatives which may be considered to baseband transmission on telephone lines. The complexity of the design and the nature of the data source are the major factors which would dictate such consideration. Depending upon the channel limitations, the bandwidth as well as the availability of receiver equalization technology, baseband modulation and OQAM will both provide a higher transmission rate than multitone transmission. Only in the case of distributed sources for the transmitted data is multitone signalling an obvious choice.

5.2 SUGGESTIONS FOR FURTHER RESEARCH

The calculations in this thesis assumed that an infinite tapped delay line was available for the equalization. An analysis of a receiver with a limited number of equalizer taps should be made to determine if practical limits to the technology affect the ranking of the systems considered here. Such an analysis could be performed in the time domain which would lend itself more easily to consideration of phase and timing jitter. Phase and timing jitter were not considered in this thesis since when transmitting over telephone lines both these problems can be practically eliminated as the changes are slow relative to the symbol rate and hence the tap coefficient refresh rate[46]. The actual effect of phase and timing jitter as well as random tap fluctuations cannot however be fully considered until the configuration of the equalizer and the number of taps is determined.

The effect of impulse noise was not considered although it was noted that multitone and OQAM systems would be less sensitive than baseband systems. A study could be performed to determine the level of this advantage over baseband transmission.

Coding is another area which remains largely unaddressed. It was assumed that the impact on each of the systems of error correction coding would be proportional so that no difference would occur in the ranking. There remains however the possibility that power-limited techniques would permit the use of bandwidth which cannot presently be used. As the multitone and OQAM systems both occupy less bandwidth than baseband transmission it could prove that the relative ranking of the systems is changed when such techniques are introduced.

APPENDIX A

MSE CALCULATION

The received signal is represented as:

$$r(t) = \sum_{k=-\infty}^{+\infty} X_k h(t - kT) + n(t) \quad (33)$$

X_k is the transmitted digit sequence ,

$h(t)$ is the channel impulse response

$n(t)$ is the noise after equalization

It is assumed that the received and decoded digital sequence is the same as that which is transmitted. That is to say, few errors are made in the decoding. The error signal can be represented as

$$\text{error} = e_{k'} = r(t + kT) - \sum_{i=1}^{+\infty} X_{k'-i} b_i - X_{k'} \quad (34)$$

The mean square error can then be determined. The MSE equations developed below were first published by McGee[38]. They are included here only for reference.

$$E\{|e_0|^2\} = E\left\{\left(\sum_{k=-\infty}^{+\infty} X_k h(t - kT) + n(t) - \sum_{i=1}^{+\infty} X_{-i} b_i - X_0\right) \left(\sum_{k'=-\infty}^{+\infty} X_{k'}^* h^*(t - k'T) + n^*(t) - \sum_{i'=1}^{+\infty} X_{-i'}^* b_{i'}^* - X_0^*\right)\right\} \quad (35)$$

Those cross products involving the noise term and any other term are equal to zero due to the lack of correlation between the factors. The other cross products are dealt with as shown below.

Assume the phasing is adjusted so the sample is taken at $t=0$. The first cross product is then equal to equation (36).

(i)

$$E\left\{\sum_{k=-\infty}^{+\infty} X_k h(-kT) \sum_{k'=-\infty}^{+\infty} X_{k'}^* h^*(-k'T)\right\} \quad (36)$$

define the term m_k as

$$m_k = E\{X_k X_0^*\} \quad (37)$$

since

$$m_k = E\{X_{k'+k} X_{k'}^*\} \quad (38)$$

then

$$m_{k-k'} = E\{X_k X_{k'}^*\} \quad (39)$$

therefore, since $h(-kT)$ is a deterministic function:

$$E\left\{\sum_{k=-\infty}^{+\infty} X_k h(-kT) \sum_{k'=-\infty}^{+\infty} X_{k'}^* h^*(-k'T)\right\} = \sum_{k=-\infty}^{+\infty} \sum_{k'=-\infty}^{+\infty} m_{k-k'} h(-kT) h^*(-k'T) \quad (40)$$

the Fourier transform of this term is

$$= \sum_{k=-\infty}^{+\infty} \sum_{k'=-\infty}^{+\infty} m_{k-k'} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H(f_1) H^*(f_2) e^{-j2\pi f_1 kT} e^{j2\pi f_2 k'T} df_1 df_2$$

let $k - k' = k''$ then $k = k'' + k'$

define the message spectrum $M(f)$ as

$$M(f) = \sum_{k''=-\infty}^{+\infty} \frac{m_{k''}}{T} e^{-j2\pi f k''T} \quad (41)$$

then the expected value in equation (36) may be written as

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H(f_1) H^*(f_2) T M(f_1) \sum_{k'=-\infty}^{+\infty} e^{-j2\pi(f_1 - f_2)k'T} df_1 df_2 \quad (42)$$

with the Poisson Summation formula:

$$\sum_{k=-\infty}^{+\infty} p(kT) e^{-j2\pi f kT} = \frac{1}{T} \sum_{m=-\infty}^{+\infty} P(f - \frac{m}{T}) \quad (43)$$

it is possible to write (42) as

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H(f_1) H^*(f_2) T M(f_1) \left(\frac{1}{T}\right) \sum_{m=-\infty}^{+\infty} \delta(f_1 - f_2 - \frac{m}{T}) \quad (44)$$

integrate over f_2

$$= \int_{-\infty}^{+\infty} H(f_1) M(f_1) \sum_{m=-\infty}^{+\infty} H^*(f_1 - \frac{m}{T}) df_1 \quad (45)$$

$M(f_1)$ is periodic in $1/T$ and $H^*(f - \frac{m}{T})$ is periodic in $1/T$

Therefore the integral can be written as:

$$E\left\{ \sum_{k=-\infty}^{+\infty} X_k h(-kT) \sum_{k'=-\infty}^{+\infty} X_{k'}^* h^*(-k'T) \right\} = \int_{\frac{-1}{2T}}^{\frac{1}{2T}} M(f) \sum_{m=-\infty}^{+\infty} H(f + \frac{m}{T}) H^*(f + \frac{m}{T}) df \quad (46)$$

(ii)

$E\{X_0 X_0^*\} = m_0$ as per the definition in equation (37) above.

$$M(f) = \sum_{k=-\infty}^{+\infty} \frac{m_k}{T} e^{-j2\pi f kT}$$

$$T M(f) = \sum_{k=-\infty}^{+\infty} m_k e^{-j2\pi f kT}$$

$$T \int_{\frac{-1}{2T}}^{\frac{1}{2T}} M(f) df = \int_{\frac{-1}{2T}}^{\frac{1}{2T}} \sum_{k=-\infty}^{+\infty} m_k e^{-j2\pi f kT} df$$

$$T \int_{\frac{-1}{2T}}^{\frac{1}{2T}} M(f) df = \int_{\frac{-1}{2T}}^{\frac{1}{2T}} m_0 df$$

$$T^2 \int_{\frac{-1}{2T}}^{\frac{1}{2T}} M(f) df = m_0$$

therefore

$$E\{X_0 X_0^*\} = T^2 \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) df \quad (47)$$

(iii)

$$E\left\{ \sum_{k=-\infty}^{+\infty} X_k h(-kT) X_0^* \right\}$$

Using the same arguments as were presented in (i) above it is possible to show this expectation is equal to:

$$= \int_{-\infty}^{+\infty} T M(f) H(f) df$$

Once again $M(f)$ is periodic in $1/T$ as is $H(f)$ we can write:

$$= \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T M(f) \sum_{m=-\infty}^{+\infty} H\left(f + \frac{m}{T}\right) df \quad (48)$$

(iv)

$$E\left\{ \sum_{k'=-\infty}^{+\infty} X_{k'}^* h^*(-k'T) X_0 \right\}$$

In the same way as in (iii) above it can be shown that this expectation is equal to:

$$= \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T M(f) \sum_{m=-\infty}^{+\infty} H^*\left(f + \frac{m}{T}\right) df \quad (49)$$

(v)

$$E\left\{\sum_{k=-\infty}^{+\infty}\sum_{i'=1}^{+\infty}X_kX_{-i'}^*h(-kT)b_{i'}^*\right\}$$

Write the sum for i' as $-\infty$ to $+\infty$ noting that $b_{i'}^*$ is equal to zero for $i' < 1$.

$$E\left\{\sum_{k=-\infty}^{+\infty}\sum_{i'=-\infty}^{+\infty}X_kX_{i'}^*h(-kT)b_{-i'}^*\right\}$$

$$B(f) = \sum_{k=1}^{+\infty} b_k e^{-j2\pi k f T}$$

The expectation $E\{X_kX_{i'}^*\}$ is $m_{k-i'}$ it can therefore be written:

$$\sum_{k=-\infty}^{+\infty}\sum_{i'=-\infty}^{+\infty}m_{k-i'}\int_{-\infty}^{+\infty}H(f_1)e^{-j2\pi f_1 k T}df_1\int_{-\frac{1}{2T}}^{\frac{1}{2T}}TB^*(f_2)e^{j2\pi f_2 i' T}df_2$$

let $k'' = k - i'$

$$= \sum_{k=-\infty}^{+\infty}\sum_{k''=-\infty}^{+\infty}m_{k''}\int_{-\infty}^{+\infty}H(f_1)e^{-j2\pi f_1 k T}df_1\int_{-\frac{1}{2T}}^{\frac{1}{2T}}T^2B^*(f_2)e^{j2\pi f_2(k-k'')T}df_2$$

$$= \sum_{k=-\infty}^{+\infty}\int_{-\infty}^{+\infty}H(f_1)e^{-j2\pi f_1 k T}df_1\int_{-\frac{1}{2T}}^{\frac{1}{2T}}TB^*(f_2)M(f_2)e^{j2\pi f_2 k T}df_2$$

$$= \int_{-\infty}^{+\infty}H(f_1)\sum_{m=-\infty}^{+\infty}\delta(f_1 - f_2 - \frac{m}{T})df_1\int_{-\frac{1}{2T}}^{\frac{1}{2T}}TB^*(f_2)M(f_2)df_2$$

integrate over f_1

$$= \int_{-\frac{1}{2T}}^{\frac{1}{2T}}T\sum_{m=-\infty}^{+\infty}H(f_2 + \frac{m}{T})B^*(f_2)M(f_2)df_2 \quad (50)$$

(vi)

In the same manner it can be shown that the expectation

$$E\left\{\sum_{k=-\infty}^{+\infty}\sum_{i'=-\infty}^{+\infty}X_k^*X_{i'}h^*(-kT)b_{-i'}\right\}=\int_{-\frac{1}{2T}}^{\frac{1}{2T}}T\sum_{m=-\infty}^{+\infty}H^*(f_2+\frac{m}{T})B(f_2)M(f_2)df_2 \quad (51)$$

(vii)

The noise terms ~~can be dealt~~ with by noting that the expected value of the product of the complex conjugates is the autocorrelation function at time 0.

$$E\{n(t)n^*(t)\}=R(0)$$

$$\mathcal{F}\{R(0)\}=N(f)$$

$$R(0)=\mathcal{F}^{-1}\{N(f)\}$$

therefore

$$E\{n(t)n^*(t)\}=\int_{-\infty}^{+\infty}N(f)df \quad (52)$$

It should be noted that the $N(f)$ term above is the noise spectrum after passing through the equalizer prefilter and in order to be consistent with the variables used in this study the integral should actually be written as in (53).

$$E\{n(t)n^*(t)\}=\int_{-\infty}^{+\infty}N(f)|E(f)|^2df \quad (53)$$

(viii)

$$E\left\{\sum_{k=-\infty}^{+\infty}\sum_{i'=1}^{+\infty}X_{-i}X_{-i'}^*b_ib_{i'}^*\right\}$$

$$= \sum_{i=-\infty}^{+\infty} \sum_{i'=-\infty}^{+\infty} m_{i-i'} \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T^2 B(f_1) B^*(f_2) e^{-j2\pi f_1 i T} e^{+j2\pi f_2 i' T} df_1 df_2$$

$$k'' = i - i'$$

$$= \sum_{i=-\infty}^{+\infty} \sum_{k''=-\infty}^{+\infty} m_k'' \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T^2 B(f_1) B^*(f_2) e^{-j2\pi f_1 i T} e^{+j2\pi f_2 (k''-i) T} df_1 df_2$$

$$= \sum_{i=-\infty}^{+\infty} \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T^2 B(f_1) B^*(f_2) e^{-j2\pi (f_1 - f_2) i T} df_1 df_2$$

This integral will equal zero unless $i = 0$ we can therefore write:

$$E\left\{ \sum_{k=-\infty}^{+\infty} \sum_{i'=1}^{+\infty} X_{-i} X_{-i'}^* b_i b_{i'}^* \right\} = T^2 \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) B(f) B^*(f) df \quad (54)$$

(ix)

$$\begin{aligned} & E\left\{ \sum_{i=1}^{+\infty} X_i X_0^* b_i \right\} \\ &= \sum_{i=-\infty}^{+\infty} m_i \int_{-\frac{1}{2T}}^{\frac{1}{2T}} B(f) e^{-j2\pi f_1 i T} df \\ &= T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) B(f) df \end{aligned} \quad (55)$$

(x)

$$E\left\{ \sum_{i'=1}^{+\infty} X_i^* X_0 b_{i'} \right\} = T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) B^*(f) df \quad (56)$$

By combining the above expectations into equation (35) we have :

$$\begin{aligned}
 \text{MSE} = & \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) \sum_{m=-\infty}^{+\infty} H(f + \frac{m}{T}) H^*(f + \frac{m}{T}) df - \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T \sum_{m=-\infty}^{+\infty} H(f + \frac{m}{T}) B^*(f) M(f) df \\
 & - \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T M(f) \sum_{m=-\infty}^{+\infty} H(f + \frac{m}{T}) df + \int_{-\infty}^{+\infty} N(f) |E(f)|^2 df \\
 & - \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T \sum_{m=-\infty}^{+\infty} H^*(f + \frac{m}{T}) B(f) M(f) df + T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) B(f) df \\
 & - T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) B^*(f) df + T^2 \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) df \\
 & + T^2 \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) B(f) B^*(f) df - \int_{-\frac{1}{2T}}^{\frac{1}{2T}} T M(f) \sum_{m=-\infty}^{+\infty} H^*(f + \frac{m}{T}) df
 \end{aligned}$$

which can be written as :

$$\text{MSE} = \int_{-\infty}^{+\infty} N(f) |E(f)|^2 df + \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) \left| \sum_{m=-\infty}^{+\infty} H(f + \frac{m}{T}) - T - T B(f) \right|^2 df \quad (57)$$

As shown in [38] the equalizer which ensures the minimum mean squared error can be determined from this formula. The MSE is minimized when the differential equation (58) is equal to zero.

$$\frac{\partial}{\partial E^*} 0 = |E(f)| N(f) + R^*(f) M(f) \left(\sum_{m=-\infty}^{+\infty} E(f + \frac{m}{T}) H(f + \frac{m}{T}) - T - T B(f) \right) \quad (58)$$

with $R(f) = P(f)C(f)$ the feedback coefficients are minimized by setting (59) equal to zero

$$\frac{\partial}{\partial b_k^*} \int_{-\frac{1}{2T}}^{\frac{1}{2T}} M(f) e^{j2\pi k' f T} \left(\sum_{m=-\infty}^{+\infty} E(f + \frac{m}{T}) H(f + \frac{m}{T}) - T - T \sum_{k=1}^{+\infty} b_k e^{-j2\pi k f T} \right) df \quad (59)$$

$k' = 1, 2, 3, \dots$ These formulae can be solved so that the optimal equation can be written.

$$E(f) = \frac{TR^*(f)}{U_{+0}U_{-}(e^{j2\pi fT})N(f)} \quad (60)$$

$R(f)$ is the overall channel response excluding the equalizer prefilter $U_{-}(e^{j2\pi fT})$ and $U_{+}(e^{-j2\pi fT})$ are the products of the spectral factorization process where:

$$\frac{1}{M(f)} + \sum_{m=-\infty}^{+\infty} \frac{|R(f + \frac{m}{T})|^2}{N(f + \frac{m}{T})} = U_{+}(e^{-j2\pi fT})U_{-}(e^{j2\pi fT}) \quad (61)$$

$$R(f) = P(f)C(f) \quad (62)$$

The feedback coefficients are determined from

$$\begin{aligned} B(f) &= \frac{U_{+}(e^{-j2\pi fT})}{U_{+0}} - 1 \\ &= \sum_{k=1}^{+\infty} b_k e^{-j2\pi k fT} \end{aligned} \quad (63)$$

A short form equation for the minimum mean squared error is:

$$\begin{aligned} \text{MMSE} &= \frac{T}{U_{+0}U_{-0}} \\ &= T \exp \left(-T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \ln \left(\frac{1}{M(f)} + \sum_{m=-\infty}^{+\infty} \frac{|R(f + \frac{m}{T})|^2}{N(f + \frac{m}{T})} \right) df \right) \end{aligned} \quad (64)$$

For the case of linear equalization the prefilter equation is:

$$E(f) = \frac{TR(f)/N(f)}{\frac{1}{M(f)} + \sum_{m=-\infty}^{+\infty} \frac{|R(f + \frac{m}{T})|^2}{N(f + \frac{m}{T})}} \quad (65)$$

These are the equations which are used for baseband transmission.

APPENDIX B

CARRIER SYSTEM'S BASEBAND EQUIVALENT EQUATIONS

The MSE and related formulae which have been developed are for baseband transmission. It is necessary to develop a new set of equations for carrier transmission. These equations will reflect the effects of modulation and demodulation on the signal and the noise as well as considering the differences in terms of noise for the PAM and QAM transmission systems.

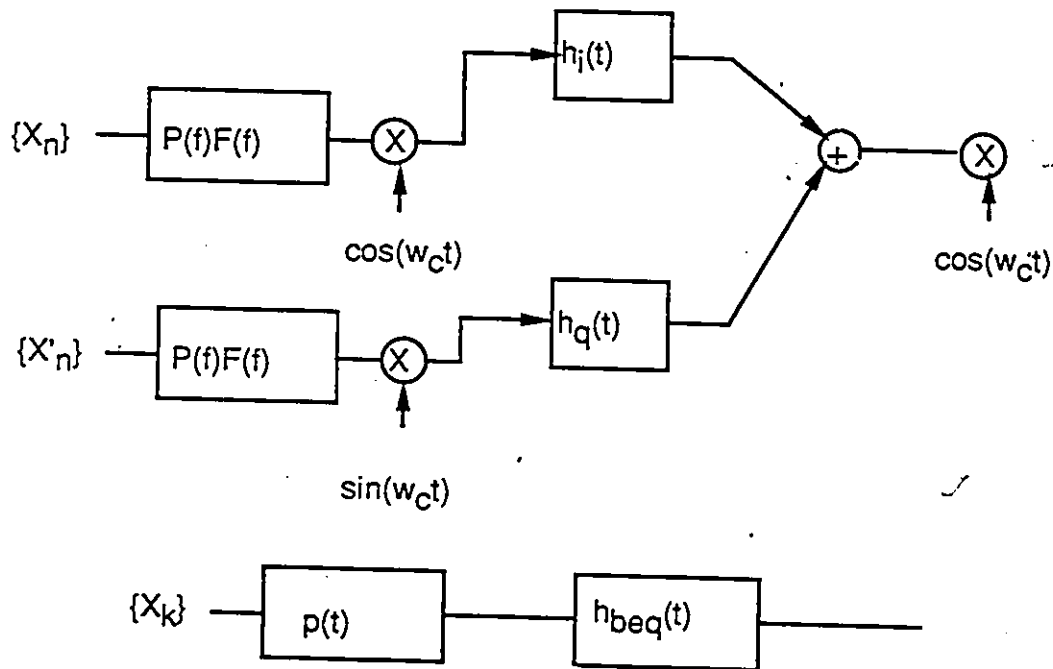


FIGURE B. 1 Carrier System Diagram. $\{X_k\}$ is a set of complex input data points

In the development below the following convention will be used:

LPF - denotes a low pass filter

$\Re$ - Real Part

* - indicates convolution

$\mathcal{F}$ - Fourier Transform

$\Im$ - Imaginary Part

PAM SIGNALLING

The received carrier signal can be written as in equation (66) below.

$$\text{Demodulated signal} = \text{LPF} \left[\Re \left\{ \sum_{k=-\infty}^{+\infty} X_k p(t - kT) e^{j2\pi f_c t} \right\} * h(t) \cos(\omega_c t) \right], \quad (66)$$

where $p(t - kT)$ is the pulse function.

The baseband representation of this signal is (67).

$$\Re \left\{ \sum_{k=-\infty}^{+\infty} X_k p(t - kT) h_b(t - kT) \right\} \quad (67)$$

If we consider equation (66) and dropping reference to the common factor "t" we can write (68).

$$\text{Demodulated signal} = \text{LPF} \left[\left[\frac{\sum_{k=-\infty}^{+\infty} X_k p(-kT) e^{j2\pi f_c t} + \sum_{k=-\infty}^{+\infty} X_k^* p^*(-kT) e^{-j2\pi f_c t}}{2} \right] * h(t) \left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right] \right] \quad (68)$$

$$\begin{aligned} &= \frac{1}{4} \left[\sum_{k=-\infty}^{+\infty} X_k p(-kT) e^{j2\pi f_c t} * h(t) e^{-j2\pi f_c t} \right. \\ &\quad \left. + \sum_{k=-\infty}^{+\infty} X_k^* p^*(-kT) e^{-j2\pi f_c t} * h^*(t) e^{j2\pi f_c t} \right] \\ &= \frac{1}{2} \Re \left\{ \sum_{k=-\infty}^{+\infty} X_k p(-kT) e^{j2\pi f_c t} * h(t) e^{-j2\pi f_c t} \right\} \quad (69) \end{aligned}$$

Take Fourier transform of (69).

$$\frac{1}{2} \int_{-\infty}^{+\infty} \Re \left\{ \left\{ \sum_{k=-\infty}^{+\infty} X_k p(-kT) e^{j2\pi f_c t} * h(t) e^{-j2\pi f_c t} \right\} e^{-j2\pi f t} dt \right\}$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} \Re \left\{ \sum_{k=-\infty}^{+\infty} X_k p(-kT) e^{j2\pi f_c t} * h(t) e^{-j2\pi(f_c+f)t} dt \right\} \quad (70)$$

Consider part of (70).

$$\begin{aligned} p(t-kT) e^{j2\pi f_c t} * h(t) &= \int_{-\infty}^{+\infty} \mathcal{F}\{p(t-kT) e^{j\pi f_c t}\} \mathcal{F}\{h(t)\} e^{j2\pi f' t} df' \\ &= \int_{-\infty}^{+\infty} H(f') \int_{-\infty}^{+\infty} p(t'-kT) e^{j\pi f_c t'} dt' e^{j2\pi f' t} df' \\ &= \int_{-\infty}^{+\infty} H(f') \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P(f'') e^{j\pi f''(t'-kT)} df'' e^{j\pi f_c t'} e^{j2\pi f' t} dt' e^{j2\pi f' t} df' \\ &= \int_{-\infty}^{+\infty} H(f') \int_{-\infty}^{+\infty} P(f'') e^{j\pi f'' kT} df'' \delta(f'' + f_c - f') e^{j2\pi f' t} df' \\ &= \int_{-\infty}^{+\infty} H(f') P(f' - f_c) e^{j\pi(f' - f_c)kT} e^{j2\pi f' t} df' \end{aligned} \quad (71)$$

(71) is substituted back into (70)

$$\begin{aligned} &\frac{1}{2} \int_{-\infty}^{+\infty} \Re \left\{ \sum_{k=-\infty}^{+\infty} X_k \int_{-\infty}^{+\infty} H(f') P(f' - f_c) e^{j\pi(f' - f_c)kT} e^{j2\pi(f' - f_c - f)t} df' dt \right\} \\ &= \frac{1}{2} \Re \left\{ \sum_{k=-\infty}^{+\infty} X_k \int_{-\infty}^{+\infty} H(f') P(f' - f_c) e^{j\pi(f' - f_c)kT} \delta(f' - f_c - f) df' \right\} \\ &= \frac{1}{2} \Re \left\{ \sum_{k=-\infty}^{+\infty} X_k H(f_c + f) P(f) e^{-j2\pi f kT} \right\} \end{aligned} \quad (72)$$

Take the inverse Fourier transform.

$$\int_{-\infty}^{+\infty} \frac{1}{2} \Re \left\{ \sum_{k=-\infty}^{+\infty} X_k H(f_c + f) P(f) e^{-j2\pi f kT} \right\} e^{j2\pi f t} df \quad (73)$$

This equation must equal (2).

$$\Re\left\{\sum_{k=-\infty}^{+\infty} X_k p(t-kT) h_b(t-kT)\right\}$$

Therefore

$$p(t)h_b(t) = \frac{1}{2} \int_{-\infty}^{+\infty} H(f_c + f) P(f) e^{j2\pi f T} df \quad (74)$$

Thus

$$H_b(f) = \frac{1}{2} H(f_c + f) \quad (75)$$

If we continue to ignore the influence of noise at the receiver the MSE can be written as in (76)

$$\text{MSE} = E\left\{\sum_{k \neq 0} X_k^2 [\Re\{h_b(-kT)\}]^2\right\} \quad (76)$$

The pulse function $p(t)$ has been dropped as it has the same influence on the carrier and the baseband systems and therefore is secondary to the development undertaken here.

Since $M(f) = \sum_{k=-\infty}^{+\infty} E\{X_k X_0^*\} e^{-j2\pi k f T} / T$ and since the data is uncorrelated we can write equation (77).

$$\text{MSE} = TM(f) \sum_{k \neq 0} X_k^2 [\Re\{h_b(-kT)\}]^2 \quad (77)$$

$$= TM(f) \left[\frac{1}{4} \sum_{k=-\infty}^{+\infty} [h_b^2(-kT) + 2|h_b(-kT)|^2 + h_b^{*2}(-kT)] - |\Re\{h_0\}|^2 \right]$$

$$= TM(f) \left[\frac{1}{4} \sum_{m=-\infty}^{+\infty} \frac{1}{T} \left[\int_{-\infty}^{+\infty} \left(H_b(f) H_b(-f + \frac{m}{T}) + 2H_b(f + \frac{m}{T}) H_b^*(f) \right. \right. \right. \\ \left. \left. \left. + H_b^*(f) H_b(-f - \frac{m}{T}) \right) df \right] - |\Re\{h_0\}|^2 \right]$$

$$= M(f) \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \left| \sum_{m=-\infty}^{+\infty} H_{eq}(f + \frac{m}{T}) - T \right|^2 df \quad (78)$$

with

$$H_{eq}(f) = \frac{1}{2} [H_b(f) + H_b^*(-f)] \quad (79)$$

This formula is valid for both SSB and DSB transmission. The channel filter for the SSB transmission appears in the transfer function $h(t)$ and therefore there is no need to consider the analytic representation of a baseband SSB signal.

QAM SIGNALLING

For QAM transmission the calculation is complicated by the possible influence of the quadrature channel. Due to the symmetry of the QAM constellations used in this study it is possible to isolate the I and the Q channels when calculating the probability of error of the overall system. This point is discussed again in Appendix C while developing the Chernoff bound equations. It is reasonable to treat the I and Q channels as separate PAM channels as long as any influence from the quadrature channel on the channel under consideration is accounted for. Since no restriction was placed upon $\{X_n\}$ during the previous development we can still write h_b as:

$$h_b(t) = \frac{1}{2} \int_{-\infty}^{+\infty} H(f + f_c) P(f) e^{j2\pi w_c t} df$$

and

$$H_b(f) = \frac{1}{2} H(f + f_c)$$

If we again ignore the influence of noise the MSE for the I channel in a QAM system is:

$$\text{MSE} = E \left\{ \left(\Re \left\{ \sum_{k=-\infty}^{+\infty} X_k h_b(t - kT) \right\} - E_L \left\{ \Re \{ X_0 h_b(0) \} \right\} \right)^2 \right\} \quad (80)$$

The expectation represented by E_L takes into account the possibility of $h_b(t)$ being complex by providing a measure of the expectation of the possible real valued

components of all the symbols at each I channel level.

$$\text{MSE} = \frac{1}{4} E \left\{ \left(\left[\sum_{k=-\infty}^{+\infty} X_k h_b(-kT) - E_L \{X_0 h_b(0)\} \right] + \left[\sum_{k=-\infty}^{+\infty} X_k^* h_b^*(-kT) - E_L \{X_0^* h_b^*(0)\} \right] \right) \right. \\ \left. \left(\left[\sum_{k'=-\infty}^{+\infty} X_{k'} h_b(-k'T) - E_L \{X_0 h_b(0)\} \right] + \left[\sum_{k'=-\infty}^{+\infty} X_{k'}^* h_b^*(-k'T) - E_L \{X_0^* h_b^*(0)\} \right] \right) \right\}$$

For independent and uncorrelated input data X_k this formula may be written as:

$$\text{MSE} = \frac{1}{4} E \left\{ \left(\sum_{k \neq 0} X_k h_b(-kT) - E_L \{X_0 h_b(0)\} + \sum_{k \neq 0} X_k^* h_b^*(-kT) - E_L \{X_0^* h_b^*(0)\} + X_0 h_b(0) + X_0^* h_b^*(0) \right) \right. \\ \left. \left(\sum_{k \neq 0} X_k h_b(-kT) - E_L \{X_0 h_b(0)\} + \sum_{k \neq 0} X_k^* h_b^*(-kT) - E_L \{X_0^* h_b^*(0)\} + X_0 h_b(0) + X_0^* h_b^*(0) \right) \right\}$$

This is due to the fact that under these conditions for input data the following is true:

$$E \left\{ \sum_{k \neq 0} X_k h_b(-kT) E_L \{X_0 h_b(0)\} \right\} = 0$$

$$p(X_0 X_k) = 0 \text{ for } k \neq 0$$

$$\text{MSE} = \frac{1}{4} E \left\{ \sum_{k \neq 0} X_k^2 h_b^2(-kT) + 2 \sum_{k \neq 0} |X_k|^2 |h_b(-kT)|^2 + \sum_{k \neq 0} X_k^{*2} h_b^{*2}(-kT) - 2X_0 h_b(0) E_L \{X_0 h_b(0)\} - 2X_0^* h_b^*(0) E_L \{X_0^* h_b^*(0)\} \right. \\ \left. - 2X_0^* h_b^*(0) E_L \{X_0 h_b(0)\} + X_0^2 h_b^2(0) + 2|X_0|^2 |h_b(0)|^2 + X_0^{*2} h_b^{*2}(0) + (E_L \{X_0 h_b(0)\})^2 \right. \\ \left. + 2|E_L \{X_0 h_b(0)\}|^2 + (E_L \{X_0^* h_b^*(0)\})^2 \right\}$$

$$\text{MSE} = E \left\{ \frac{1}{4} \sum_{k \neq 0} |X_k h_b(-kT) + X_k^* h_b^*(-kT)|^2 + \left(\Re \{X_0 h_b(0) - E_L \{X_0 h_b(0)\}\} \right)^2 \right\}$$

For symmetric QAM constellations the expected values $E\{\sum_{k \neq 0} X_k^2\}$ and $E\{\sum_{k \neq 0} X_k^2\}$ are both equal to zero. Therefore the formula reduces to :

$$\text{MSE} = \frac{1}{4} E \left\{ 2 \sum_{k \neq 0} |X_k|^2 |h_b(-kT)|^2 \right\} + E \left\{ \left| \Re \{ X_0 h_b(0) - E_L \{ X_0 h_b(0) \} \} \right|^2 \right\}$$

consider the second part of the equation

$$\begin{aligned} & E \left\{ \left(\Re \{ X_0 h_b(0) - E_L \{ X_0 h_b(0) \} \} \right)^2 \right\} \\ &= \frac{1}{4} E \left\{ (X_0 h_b(0) - E_L \{ X_0 h_b(0) \} + X_0^* h_b(0) - E_L \{ X_0^* h_b(0) \}) \right. \\ &\quad \left. (X_0 h_b(0) - E_L \{ X_0 h_b(0) \} + X_0^* h_b(0) - E_L \{ X_0^* h_b(0) \}) \right\} \\ &= \frac{1}{4} \left[E_L \{ (X_0 h_b(0))^2 \} - (E_L \{ X_0 h_b(0) \})^2 + 2 E_L |X_0 h_b(0)|^2 - 2 |E_L \{ X_0 h_b(0) \}|^2 \right. \\ &\quad \left. + E_L \{ (X_0^* h_b(0))^2 \} - (E_L \{ X_0^* h_b(0) \})^2 \right] \\ &= \frac{1}{4} \left[E_L |X_0 h_b(0) + X_0^* h_b(0)|^2 - |E_L \{ X_0 h_b(0) + X_0^* h_b(0) \}|^2 \right] \\ &= \frac{1}{4} \left[E_L \left\{ \left| \left(\frac{X_0 + X_0^*}{2} + \frac{X_0 - X_0^*}{2} \right) h_b(0) + \left(\frac{X_0 + X_0^*}{2} - \frac{X_0 - X_0^*}{2} \right) h_b^*(0) \right|^2 \right\} \right. \\ &\quad \left. - \frac{1}{4} \left[E_L \{ X_0 h_b(0) \} + E_L \{ X_0^* h_b(0) \} \right] \left[E_L \{ X_0^* h_b(0) \} + E_L \{ X_0 h_b(0) \} \right] \right] \\ &= E_L \left| \left(\frac{X_0 + X_0^*}{2} \right) \left(\frac{h_b(0) + h_b^*(0)}{2} \right) + \left(\frac{X_0 - X_0^*}{2} \right) \left(\frac{h_b(0) - h_b^*(0)}{2} \right) \right|^2 \\ &\quad - \left| E_L \left\{ \left(\frac{X_0 + X_0^*}{2} \right) \left(\frac{h_b(0) + h_b^*(0)}{2} \right) \right\} + E_L \left\{ \left(\frac{X_0 - X_0^*}{2} \right) \left(\frac{h_b(0) - h_b^*(0)}{2} \right) \right\} \right|^2 \end{aligned}$$

The expectation $E_L \{ (X_0 - X_0^*) (h_b(0) - h_b^*(0)) \}$ is equal to zero because as explained in Appendix D the constellation is symmetric and the E_L expectation is done over all possible values at this I channel level.

$\Re(h_b(0))$ has been normalized to 1 so that two of the remaining terms cancel. The final term $E_L \{ \Re(X_0) \Im(X_0) \Re(h_b(0)) \Im(h_b(0)) \}$ is equal to zero again due to the symmetry of the constellation. This leaves the following equation for MSE.

$$\text{MSE} = \frac{1}{2} E \left\{ \sum_{k \neq 0} |X_k|^2 |h_b(-kT)|^2 \right\} + E \left\{ \left| \frac{X_0 - X_0^*}{2} \right|^2 \left| \frac{h_b(0) - h_b^*(0)}{2} \right|^2 \right\}. \quad (81)$$

This can be shown to be the same as writing

$$= M(f) \int_{-\infty}^{+\infty} \left| \sum_{m=-\infty}^{+\infty} H_{eq}(f + \frac{m}{T}) - T \right|^2 df$$

with $M(f)$ defined as $M(f) = E\{X_k X_k^*\}/2T$ for uncorrelated input data and

$$H_{eq}(f) = H_b(f) \quad (82)$$

The MSE determined using these equations is the MSE associated with one 'axis' of the QAM constellation.

APPENDIX C

CHERNOFF BOUND

The received signal is represented as:

$$r(t) = \sum_{k=-\infty}^{+\infty} X_k h(t - kT) + n(t)$$

X_k is the transmitted digit sequence

$h(t)$ is the channel impulse response

$n(t)$ is the noise after equalization

$$r(t) = X_m h_0 + \sum_{k \neq 0} Z_k + n(t)$$

$$Z_k = X_{m-k} h_k$$

If we define $Z = \sum_{k \neq 0} Z_k + n(t)$ then as shown in reference [39] and in the diagram below an error occurs if $(Z < h_0)$ for the symbol $\{X_m = N - 1\}$, and if $(Z > h_0)$ for the symbol $\{X_m = -N + 1\}$. An error occurs for either of these Z conditions for symbols other than $\{X_m = -N + 1, N - 1\}$. Thus if we note that $P(Z > h) = P(Z < h)$ due to symmetry the error probability per symbol is :

$$P = \text{error probability per symbol} = \frac{2(N-1)}{N} P(z > h(0)) \quad (83)$$

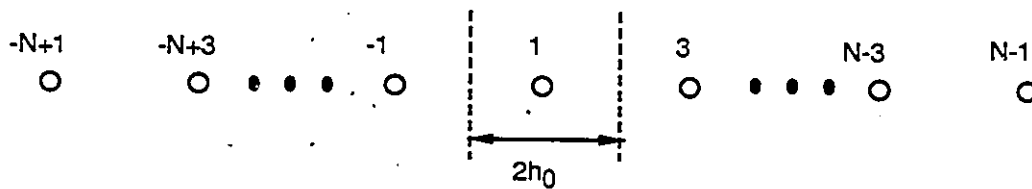


FIGURE C. 1 Simple PAM Constellation. The spacing between symbols is '2' and only real valued symbols are transmitted.

Saltzberg shows that the probability of error can be written as:

$$P(Z > h(0)) \leq E\{e^{sn}\} E\{e^{-sh(0)}\} \prod_{k \neq 0} E\{e^{sZ(-kT)}\} \quad (84)$$

Each of these expectations can be found which results in the equation:

$$P(Z > h(0)) \leq \exp\left(-sh(0) + \frac{1}{2}s^2\sigma_x^2 \sum_{k \neq 0} h_k + \frac{1}{2}s^2\sigma_n^2 + \frac{1}{2}s^2n_0\right)$$

If this formula is optimized with respect to s the resulting approximation is :

$$P(Z > h(0)) \leq \exp\left(\frac{-\frac{1}{2}h_0^2}{\sigma_x^2 \sum_{k \neq 0} h_k + \sigma_n^2 + n_0}\right) \quad (85)$$

which can be approximated with a Q function

$$\simeq Q\left(\frac{h(0)}{\sqrt{\sigma_x^2 \sum_{k \neq 0} h_k + \sigma_n^2 + n_0}}\right) \quad (86)$$

n_0 in this case represents the thermal noise contribution

σ_n^2 represents the noise contribution

σ_x^2 represents the input data

h_k is the channel impulse response (including the effect of feedback in the case of decision feedback)

h_0 is the overall channel response at time $t=0$

For real, uncorrelated input data:

$$\sigma_x^2 = E\left\{\sum_{k=-\infty}^{+\infty} X_k X_0 e^{-j2\pi k f T}\right\} \quad (87)$$

The noise term is simple the autocorrelation at time $t=0$ as discussed previously in Appendix A:

$$\sigma_n^2 + n_0 = E\{n^2(t)\} = R(0) = \int_{-\infty}^{+\infty} \tilde{N}(f) |E(f)|^2 df \quad (88)$$

Thus the denominator of the Q function operand is equal to the square root of the MSE error formula developed earlier.

For the simple constellation above the probability of a symbol error is written:

$$P \simeq \frac{2(N-1)}{N} Q\left(\frac{h_0}{\sqrt{\text{MSE}}}\right) \quad (89)$$

QAM

In the case of QAM transmission the input data X_k is complex. We are able to treat the I and the Q channels separately as long as the mean squared error is calculated with consideration given to the possible influence of the quadrature channel. We are considering only symmetric constellations and independent, uncorrelated data points so the factor by which the probability of error $P(Z > h(0))$ is multiplied to determine the symbol error is the same in both the 'I' and the 'Q' directions. Thus if the probability of error is calculated using the bound just developed it is necessary to consider the 'one dimensional' characteristic of the calculation. That is, when the modulation index $M(f)$ is calculated, a factor of 2 must be introduced to ensure the $M(f)$ value is the same as that which would be achieved if only one dimension in the constellation were considered. For QAM systems the $M(f)$ term which appears in the MSE calculation is :

$$M(f) = \left(\frac{\sum_{k=-\infty}^{+\infty} E(X_k X_k^*) e^{-j2\pi k f T}}{2T} \right) \quad (90)$$

The probability of a symbol error can be calculated for the I or the Q channel of a QAM system. Since the probability of error $P(Z > h(0))$ is the same for both the I and Q 'directions' the probability of a symbol error for the QAM system can be determined by multiplying $P(Z > h(0))$ by a constant K_e which is determined for each constellation.

PAM SYSTEMS

For both carrier PAM and QAM systems the noise contribution in the mean squared error calculation is multiplied by a factor of 1/2 due to the effect of demodulation. Thus instead of equation (88) we have equation (91).

$$\sigma_n^2 + n_0 = \frac{1}{2} E\{n^2(t)\} = R(0) = \frac{1}{2} \int_{-\infty}^{+\infty} N(f) |E(f)|^2 df \quad (91)$$

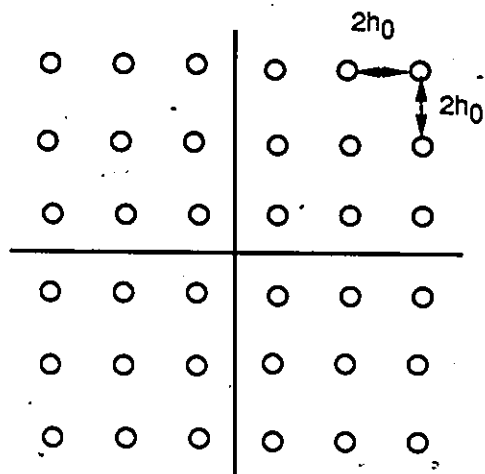


FIGURE C. 2 Symmetric Constellation. Either rectangular or 'cross' constellations may be used.

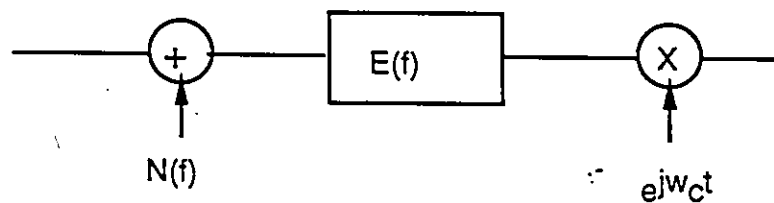


FIGURE C. 3 Noise/Receiver effect. The noise contribution to the MSE is calculated after demodulation has taken place.

APPENDIX D

SIGNAL CONSTELLATIONS

The constellation of a signal set provides two pieces of information which are necessary in order to calculate the bit error rate. A factor related to the probability of a symbol error and the average energy in a signal must be determined from the constellation. The process for determining this information is briefly described below.

BASEBAND AND PAM MODULATION

For baseband and PAM modulation the input data symbols $\{X_k\}$ are real valued. They are centered about 0 and for this analysis the distance between symbols is normalized to 2. This distance is represented as $2a$ or $2h_0$ in the formula used in this thesis. A general representation of the real symbol set is given below.

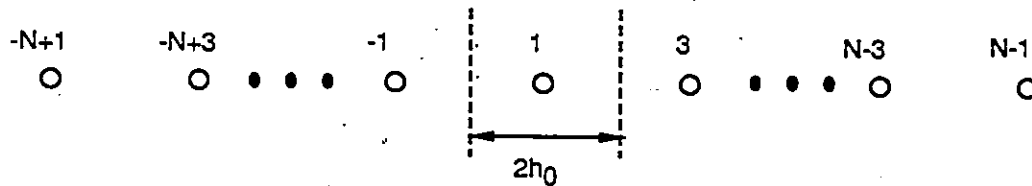


FIGURE D. 1 General 'Real' Symbol Representation.

The value N is the number of symbols and M is the number of bits each symbol represents.

$$N = 2^M$$

A decoding error occurs if the demodulator decides on a symbol which was not transmitted. This occurs if the error in the signal exceeds h_0 . The probability of this occurring is determined using the equations developed previously. In the general model, for symbols other than the two on either end, an error can occur in two

directions. For the symbols on the end an error can only occur in one direction. Therefore if the transmission of each symbol is equally likely the overall probability of a decoding error occurring is given by (92).

$$P = 2(N - 1)P(\text{error} > h_0)/N \quad (92)$$

A measure of the average energy in the constellation is required in order to determine the MSE. If the symbols are represented by X_n where X_n can have any value in the set $\{N - 1, N - 3, \dots, -N + 1\}$ then the average energy is given by (93).

$$\text{Energy} = \sum_{k=-N+1}^{N-1} \{X_k^2\}/N \quad (93)$$

assuming each symbol is equally probable. This value is also equal to the m_k value in the $M(f)$ equation if the symbols are uncorrelated. The energy of this symbol set can be approximated by (94).

$$\text{Energy} \simeq 4^M/3 \quad (94)$$

QAM MODULATION

QAM modulation requires symbols represented by complex values. The I and the Q channels provide parallel transmission and for each symbol interval T two pieces of information are sent. All QAM constellations in this thesis were symmetric about the I and the Q axis.

The spacing between symbols remains equal to 2 and is represented by $2a$ or $2h_0$. Due to the symmetry of the constellation the MSE is the same in both the I and Q channels. It is therefore possible to determine the probability of a symbol error by simply studying one channel. Except for symbols on the outside edge of the constellation an error can occur in any of four directions. If $P(\text{error} > h_0)$ represents the probability of an error occurring in one direction the probability of one of these inside symbols being in error is equal to $4P(\text{error} > h_0)$. This must be tempered by

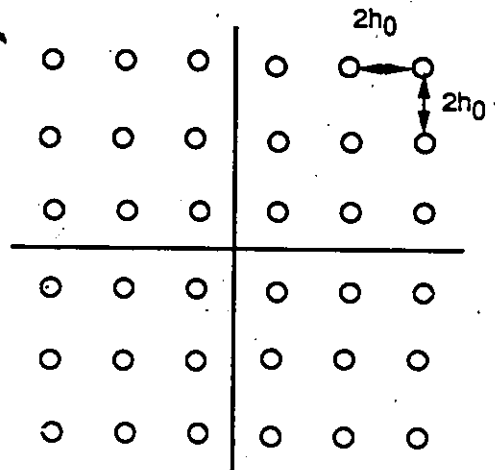


FIGURE D. 2 General QAM Constellation. The transmitted symbols are represented by complex numbers in order to account for both the I and the Q channels.

the probability of that symbol being transmitted. Depending upon the positioning of the outside symbols their probability of error can be multiplied by two or three. It is necessary to consider each constellation separately when determining the K_e factor. The K_e factor ranges from a low of 2 for 4 symbol QAM to a maximum of 4 as the number of symbols increases.

The energy of the constellation must also be determined for each system. It can be approximated by (95).

$$\text{Energy}_{(\text{QAM})} = 2 \sum_{i=1}^{N/2} \frac{(2i-1)^2}{N} \quad (95)$$

Once again for uncorrelated data points this value is the m_k which is required for the $M(f)$ calculation.

APPENDIX E

PROGRAM LISTING

Two program listings are included. The first is the program used to calculate the performance of a baseband transmission system. The required input is the message spectrum $E\{X_0 X_k^*\}$ and the estimated channel end frequency. The K_e factor is calculated based upon the M value (equation (25)). The probability of error is calculated based upon the Chernoff bound. The bandwidth which is entered is the subchannel end frequency including the excess bandwidth. That is, the $1.2/T$ bandwidth is required for the 20% raised-cosine system. The filter characteristic is determined in the subprogram SNRBASE.PAS. The output as it appears in this listing provides the MSE and probability of error for linear and decision feedback equalized systems occupying the specified bandwidth. An iterative process is used to determine the bandwidth for that choice of M which results in a probability of error of 10^{-6} . Also listed is the breakdown of the ISI, white noise and crosstalk contributions to the MSE.

The second listing is for QAM systems. The difference from the first program exists in the baseband equivalent channel and in the routine which retrieves the channel attenuation values. This program has the option of estimating $E\{X_0 X_k^*\}$ using equation (1). This would result in non-integer values for M, a fact which must be considered in determining system throughput. Also included is the SNR program for the carrier systems using the 20% raised-cosine filter.

The subprograms are:

SNRBASE.PAS - determines the channel attenuation and crosstalk characteristic at f , where f is some frequency in Hz.

COMPLEX.PAS - complex number handling routines

POWER.PAS - returns a variable raised to the power i

FFTPART.PAS - calculates the Fast Fourier Transform and the inverse FFT


```
{program msebascl.pas;}
```

```
{ This program calculates the mean square error of a baseband PAM system  
  based on the signalling frequency, the background noise and the crosstalk.
```

```
{The MSE is calculated assuming optimal equalization with an infinite delay  
  line for the case of decision feedback.
```

```
  The probability of error is calculated using a Chernoff bound.}
```

```
{updated april 17 88}
```

```
type grump = array[0..200]of real;
```

```
const      WhiteNoise = 2.5e-14;
```

```
var
```

```
  justaninteger, NumOfSamples, n, il, iLast, m, i, iNew, iStart, marker: integer;  
  FoldedFreq, AbsFreq, DFMSE, Xtalk, T, Pulse, sum, MessageSpectrum: real;  
  Freq, U0, sum1, ProbOfErrorFactor, NumberOfLevels, Power: real;  
  ChanEnd, Qfactor, ProbOfError, FilterRes, df, PrevChanEnd, ak2: real;  
  XtalkNoise, LinEq, OverallChanResponse, DFEq, ChanResponse, Noise, g, gl: grump;  
  DfISISum, LinISISum, LinNoiseSum, WhiteNoisePowerLim, ImpulseResAtZero: real;  
  justanumber, BitsPerLevel, PowerLimitingFactor: real;  
  LinXtalkNoiseSum, LinWhiteNoiseSum, DFXTalkNoiseSum: real;  
  DFWhiteNoiseSum, fd, DFMSE1, DFMSEshortFormSum, LinMSEshortFormSum: real;  
  DFNoiseSum, FreqLimit, sumd, sumh, dNd, cd: real;  
  ComplexMatrix: vector;  
  ComplexZero: complex;
```

```
{*****}
```

```
{Obtain the system parameters}
```

```
{Output: channel frequency limits, data sequence information}
```

```
begin
```

```
  writeln(' what is the autocorrelation of the input data?');  
  readln(ak2);  
  writeln(' what is the number of levels?');  
  readln(NumberOfLevels);  
  BitsPerLevel:=ln(NumberOfLevels)/ln(2);  
  ProbOfErrorFactor:=2*(NumberOfLevels-1)/NumberOfLevels;  
  writeln(' what is the estimated channel end?');  
  readln(ChanEnd);  
  PrevChanEnd:=0.0;
```

```
{*****}
```

```
{Obtain the channel parameters}
```

```
{Input: channel limits, data sequence information}
```

```
{Output: channel response, noise}
```

```
while (ChanEnd<>PrevChanEnd) do begin
```

```
  ProbOfError:=0.0;
```

```
  marker:=0;
```

```

PrevChanEnd:=ChanEnd;;
justanumber:=0;
ImpulseResAtZero:=0.0;
T:=(1+FilterRolloff)/(2*(ChanEnd));
MessageSpectrum:=ak2/T;
PowerLimitingFactor:=MessageSpectrum/(T*0.25);
WhiteNoisePowerLim:=WhiteNoise*PowerLimitingFactor;
df:=(1/T)/32;
Power:=0.0;
if (odd(trunc(32*(1+FilterRolloff)))) then
  NumOfSamples:=trunc(32*(1+FilterRolloff))-1 else
  NumOfSamples:=trunc(32*(1+FilterRolloff));
FreqLimit:=df*((NumOfSamples div 2)-0.5);
NumOfSamples:=NumOfSamples-1;
writeln(NumOfSamples);
Freq:=-FreqLimit;
for i:=0 to (NumOfSamples) do begin
  AbsFreq:=abs(Freq);
  snr(AbsFreq,Xtalk,ChanResponse[i],FilterRes,T);
  Pulse:=1.0;
  Noise[i]:=WhiteNoisePowerLim+MessageSpectrum*Xtalk*sqr(Pulse);
  XtalkNoise[i]:=MessageSpectrum*Xtalk*sqr(Pulse);
  Power:=Power+sqr(Pulse*FilterRes);
  Freq:=Freq+df;
  delay(300);
end;
writeln(' finished with snr');
Power:=Power*MessageSpectrum*df/PowerLimitingFactor;

{*****}
{Calculate the folded sum}
{Input: channel response, noise}
{Output: folded sum}

il:=0;
i:=0;
DFMSEshortFormSum:=0.0;
sumh:=0.0;
sumd:=0.0;
LinMSEshortFormSum:=0.0;
suml:=0.0;
Freq:=-FreqLimit;

while (Freq<= FreqLimit) do begin
  sum:=0.0;

  {*****}
  {If 'f' is close to zero average in the null}

  if (Freq< df) and (Freq>-df) then begin
    sumd:=0.0;
    dfd:=0.2;
    sumh:=0.0;
    iNew:=round((Freq+FreqLimit)/dfd);

```

```

if (df<=100.0)then begin
  fd:=dfd/2.0;
  while(fd<df) do begin
    cd:=1e-30+exp(15*ln(fd/(fd+12)));
    if(fd=dfd/2.0)then begin
      sumd:=sumd+ln(1/MessageSpectrum);
      sumh:=sumh+Noise[iNew]/(Noise[iNew]/MessageSpectrum);
    end;
    if(fd<>dfd/2.0) then begin
      sumh:=sumh+Noise[iNew]/(Noise[iNew]/MessageSpectrum+cd);
      sumd:=sumd+ln(1/MessageSpectrum+cd/Noise[iNew]);
    end;
    fd:=dfd+fd;
  end;
  sumh:=sumh*dfd;
  sumd:=sumd*dfd;
end;

if(df>100.0) then begin
  fd:=dfd/2.0;
  while (fd<100.0) do begin
    cd:=1e-30+exp(15*ln(fd/(fd+12)));
    sumd:=sumd+ln(1/MessageSpectrum+cd/Noise[iNew]);
    sumh:=sumh+Noise[iNew]/(Noise[iNew]/MessageSpectrum+cd);
    fd:=fd+dfd;
  end;
  sumh:=sumh*dfd;
  sumd:=sumd*dfd;
  sumh:=sumh+(df-100.0)/(1/MessageSpectrum+
    ChanResponse[iNew]/Noise[iNew]);
  sumd:=sumd+(df-100.0)*ln(1/MessageSpectrum+
    ChanResponse[iNew]/Noise[iNew]);
end;

sum:=sumh/df;
sum:=1/sum;
gl[i]:=exp(sumd/df);
gl[i]:=sum;
LinEq[i]:=T*(sqrt(gl[i]-1/MessageSpectrum))/
  (gl[i]*sqrt(Noise[iNew]));
ComplexMatrix[i].re:=(sumd/df);ComplexMatrix[i].im:=0.0;
i:=i+1;
i:=i+1;
DFMSEshortFormSum:=DFMSEshortFormSum+2*sumd/df;
LinMSEshortFormSum:=LinMSEshortFormSum+2/sum;
Freq:=Freq+2*df;
gl[i]:=exp(sumd/df);
gl[i]:=sum;
LinEq[i]:=T*sqrt(gl[i]-1/MessageSpectrum)/
  (gl[i]*sqrt(Noise[iNew]));
ComplexMatrix[i].re:=(sumd/df);ComplexMatrix[i].im:=0.0;
i:=i+1;
i:=i+1;
end;

```

```

{*****}
{If 'f' is not close to zero}

sum:=0.0;
for m:=-1 to 1 do begin
  FoldedFreq:=Freq+m/T;
  iNew:=round((FoldedFreq+FreqLimit)/df);
  if(iNew <= NumOfSamples)and (iNew >= 0)and(Noise[iNew]>0.0)then begin
    sum:=sum+sqr(Pulse)*ChanResponse[iNew]/Noise[iNew];
  end;
end;
sum:=sum+1/(MessageSpectrum);
gl[i]:=sum;
gl[i]:=sum;
if (Freq>=(-1/(2*T))) and (Freq<=(1/(2*T))) then begin
  if (marker<1) then iStart:=i;
  iLast:=i;
  ComplexMatrix[i].re:=ln(sum);ComplexMatrix[i].im:=0.0;
  i:=i+1;
  marker:=1;
  DFMSEshortFormSum:=DFMSEshortFormSum+ln(sum);
  LinMSEshortFormSum:=LinMSEshortFormSum+1/sum;
end;
LinEq[i]:=T*sqrt(ChanResponse[i]/(Noise[i]*gl[i]));
Freq:=Freq+df;
i:=i+1;
end;

{*****}
{Spectral Factorization}
{Input: folded sum}
{Output: spectral factor}

m:=5;
n:=32;
writeln('first twiddle');
twiddle(ComplexMatrix,m);
writeln('fft');
fft(ComplexMatrix,m);
ComplexZero.re:=0.0;ComplexZero.im:=0.0;
addc(ComplexMatrix[0],ComplexMatrix[(n div 2)+1],ComplexMatrix[0]);
ComplexMatrix[0].re:=ComplexMatrix[0].re/2.0;
ComplexMatrix[0].im:=ComplexMatrix[0].im/2.0;
U0:=exp(ComplexMatrix[0].re/32);
for i:=(n div 2) to n-1 do ComplexMatrix[i]:=ComplexZero;
writeln('inverse twiddle');
twiddle(ComplexMatrix,m);
writeln('inverse fft');
ifft(ComplexMatrix,m);

for i:=0 to n-1 do ComplexMatrix[i].re:=exp(ComplexMatrix[i].re);

```

```

{*****}
{Calculate the decision feedback equalizer values}
{Input: spectral factor, channel response}
{Output: equalizer response}

for i:=0 to (iStart-1) do begin
  DFEq[i]:=T*sqrt(ChanResponse[i])*Pulse/
    (Noise[i]*U0*ComplexMatrix[(31-(iStart-1-i))].re);
  OverallChanResponse[i]:=sqrt(ChanResponse[i])*Pulse*DFEq[i];
  ImpulseResAtZero:=ImpulseResAtZero+OverallChanResponse[i];
end;

for i:=iStart to iLast do begin
  DFEq[i]:=T*sqrt(ChanResponse[i])*Pulse/
    (Noise[i]*U0*ComplexMatrix[(1-iStart)].re);
  OverallChanResponse[i]:=sqrt(ChanResponse[i])*Pulse*DFEq[i]-
    T*(ComplexMatrix[(1-iStart)].re/U0-1);
  justaninteger:=(iLast-iStart) div 2;
  if (i>(iStart+trunc(justaninteger-1)))and
    (i<=(iStart+trunc(justaninteger+1))) then begin
    DFEq[i]:=T*(sqrt(g[i]-1/MessageSpectrum)*sqrt(Noise[i]))/
      (Noise[i]*U0*ComplexMatrix[(1-iStart)].re);
    OverallChanResponse[i]:=(sqrt(g[i]-1/MessageSpectrum)*
      sqrt(Noise[i]))*DFEq[i]-T*(ComplexMatrix[(1-iStart)].re/U0-1);
  end;
  ImpulseResAtZero:=ImpulseResAtZero+OverallChanResponse[i];
end;

for i:=iLast+1 to NumOfSamples do begin
  DFEq[i]:=T*sqrt(ChanResponse[i])*Pulse/
    (Noise[i]*U0*ComplexMatrix[(1-(iLast+1))].re);
  OverallChanResponse[i]:=sqrt(ChanResponse[i])*DFEq[i]*Pulse;
  ImpulseResAtZero:=ImpulseResAtZero+OverallChanResponse[i];
end;

{*****}
{Calculate the components of the MSE contribution}

DFNoiseSum:=0.0;
DfISISum:=0.0;
LinISISum:=0.0;
DFWhiteNoiseSum:=0.0;
DFXtalkNoiseSum:=0.0;
LinNoiseSum:=0.0;
LinXtalkNoiseSum:=0.0;
LinWhiteNoiseSum:=0.0;

Freq:=-FreqLimit;
justaninteger:=NumOfSamples+1;
justaninteger:=justaninteger div 2;
for i:=0 to NumOfSamples do begin
  {*****}

```

```

{Noise factors}
{Input: equalizer response, noise}
{Output: noise integral}

DFWhiteNoiseSum:=DFWhiteNoiseSum+
  WhiteNoisePowerLim*sqr(abs(DFEq[i]));
DFXtalkNoiseSum:=DFXtalkNoiseSum+
  XtalkNoise[i]*sqr(DFEq[i]);
LinXtalkNoiseSum:=LinXtalkNoiseSum+
  XtalkNoise[i]*sqr(LinEq[i]);
LinWhiteNoiseSum:=LinWhiteNoiseSum+
  WhiteNoisePowerLim*sqr(LinEq[i]);
LinNoiseSum:=LinNoiseSum+Noise[i]*sqr(abs(LinEq[i]));
DFNoiseSum:=DFNoiseSum+Noise[i]*sqr(abs(DFEq[i]));

{*****}
{ISI contribution}
{Input: equalizer, channel response}
{Output: ISI integral}

sum:=0.0;
sum1:=0.0;
if (Freq<(1/(2*T))) and (Freq>(-1/(2*T))) then begin
  for m:=-1 to 1 do begin
    FoldedFreq:=Freq+m/T;
    iNew:=round((FoldedFreq+FreqLimit)/df);
    if (iNew <= NumOfSamples) and (iNew >= 0) then begin
      sum:=sum+DFEq[iNew]*sqr(ChanResponse[iNew])*Pulse;
      sum1:=sum1+LinEq[iNew]*sqr(ChanResponse[iNew])*Pulse;
    end;
  end;
  if (Freq<df)and (Freq>-df) then begin
    iNew:=round((Freq+FreqLimit)/df);
    sum:=DFEq[iNew]*(sqr(g[iNew]-1/MessageSpectrum)*
      sqr(Noise[iNew]));
    sum1:=LinEq[iNew]*(sqr(g[iNew]-1/MessageSpectrum)*
      sqr(Noise[iNew]));
  end;
  sum:=sqr(sum-T*ComplexMatrix[(1-iStart)].re/U0);
  sum1:=sqr(sum1-T);
end;

Freq:=Freq+df;
LinISISum:=LinISISum+sum1;
DfISISum:=DfISISum+sum;
end;

{*****}
{MSE total}

DFNoiseSum:=DFNoiseSum*df;
LinNoiseSum:=LinNoiseSum*df;
DFWhiteNoiseSum:=DFWhiteNoiseSum*df;
DFXtalkNoiseSum:=DFXtalkNoiseSum*df;
LinXtalkNoiseSum:=LinXtalkNoiseSum*df;

```

```

LinWhiteNoiseSum:=LinWhiteNoiseSum*df;
LinISISum:=LinISISum*df*MessageSpectrum;
DfISISum:=DfISISum*df*MessageSpectrum;
DFMSE:=DfISISum+DFNoiseSum;
DFMSE1:=T*exp(-T*df*DFMSEshortFormSum);

```

```

{*****}

```

```

{Output routine}

```

```

writeln(' The decision feedback MSE is ',DFMSE1);
writeln(' noise = ',DFNoiseSum,' ISI = ',DfISISum,
' long form calculation MSE = ',DFMSE);
writeln(' The white noise contribution is ',DFWhiteNoiseSum);
writeln(' The crosstalk contribution is ',DFXtalkNoiseSum);
writeln(' Short form mse = ',T/sqr(U0));
writeln(' Impulse Response at f=0 = ',ImpulseResAtZero*df);
writeln(' ');
Qfactor:=ImpulseResAtZero*df/sqrt(DFMSE);
q(Qfactor,ProbOfError);
ProbOfError:=ProbOfErrorFactor*ProbOfError/BitsPerLevel;
writeln(' Channel End = ',ChanEnd:9,' 1/T = ',(1/T):9,
' Pe = ',ProbOfError:10);
writeln(' ');writeln(' ');

writeln(' Linear MSE = ',(LinISISum+LinNoiseSum));
writeln(' Short form MSE = ',(sqr(T)*LinMSEshortFormSum*df));
writeln(' The ISI contribution is ',LinISISum,
' the noise is ',LinNoiseSum);
writeln(' The white noise contribution is ',LinWhiteNoiseSum);
writeln(' The crosstalk contribution is ',LinXtalkNoiseSum);
writeln(' ');
Qfactor:=1/sqrt(LinISISum+LinNoiseSum);
q(Qfactor,ProbOfError);
ProbOfError:=ProbOfErrorFactor*ProbOfError/BitsPerLevel;
writeln(' Channel End= ',ChanEnd:9,
' 1/T = ',(1/T):9,' Pe= ',ProbOfError);
writeln(' ');writeln(' ');
sound(1200);
delay(50);
nosound;
writeln(' Signalling Rate = ',(BitsPerLevel*(1/T)):10,' bits/s');
writeln(' ');
writeln('do you want to change the channel end frequency? ',ChanEnd);
readln(justanumber);
if justanumber>100.0 then ChanEnd:=justanumber;
end;
writeln(' ');
writeln(' Power = ',Power:10);
writeln(' ');
writeln(' ');

```

```

end.

```

```
{*****}
```

```
{subprogram snrbase.pas}
```

```
{Input: frequency}
```

```
{Output: channel attenuation}
```

```
{update Oct. 23}
```

```
const pi = 3.1415926;
```

```
Length= 4.8;
```

```
FilterRolloff = 0.2;
```

```
procedure loss(var Freq,Attenuation:real); {find the attenuation at a certain}
```

```
type b=array[1..40]of real;
```

```
var i,ip,inew,p:integer;
```

```
CA,FR:b;
```

```
begin
```

```
CA[1]:=2.45 ;FR[ 1]:= 1.0 ;CA[ 2]:= 2.45 ;FR[ 2]:= 5.0 ;
CA[ 3]:=2.45 ;FR[ 3]:=10.0 ;CA[ 4]:= 2.45 ;FR[ 4]:= 15.0 ;
CA[ 5]:=2.45 ;FR[ 5]:=20.0 ;CA[ 6]:= 2.45 ;FR[ 6]:= 30.0 ;
CA[ 7]:=2.45 ;FR[ 7]:=50.0 ;CA[ 8]:= 2.45 ;FR[ 8]:= 70.0 ;
CA[ 9]:=2.45 ;FR[ 9]:=100.0 ;CA[10]:=2.45 ;FR[10]:= 150.0 ;
CA[11]:=2.45 ;FR[11]:=200.0 ;CA[12]:=2.45 ;FR[12]:= 300.0 ;
CA[13]:=2.45 ;FR[13]:=500.0 ;CA[14]:=2.45 ;FR[14]:= 700.0 ;
CA[15]:=2.92 ;FR[15]:=1000.0 ;CA[16]:=3.57 ;FR[16]:=1500.0 ;
CA[17]:=4.1 ;FR[17]:=2000.0 ;CA[18]:=4.995 ;FR[18]:=3000.0 ;
CA[19]:=6.5 ;FR[19]:=5000.0 ;CA[20]:=7.422 ;FR[20]:=7000.0 ;
CA[21]:=8.692 ;FR[21]:=10000.0 ;CA[22]:=10.294 ;FR[22]:=15000.0 ;
CA[23]:=11.503 ;FR[23]:=20000.0 ;CA[24]:=13.232 ;FR[24]:=30000.0 ;
CA[25]:=15.29 ;FR[25]:=50000.0 ;CA[26]:=16.5 ;FR[26]:=70000.0 ;
CA[27]:=17.73 ;FR[27]:=100000.0 ;CA[28]:=19.25 ;FR[28]:=150000.0 ;
CA[29]:=20.67 ;FR[29]:=200000.0 ;CA[30]:=23.59 ;FR[30]:=300000.0 ;
CA[31]:=29.26 ;FR[31]:=500000.0 ;CA[32]:=34.48 ;FR[32]:=700000.0 ;
CA[33]:=41.2 ;FR[33]:=1000000.0 ;CA[34]:=50.49 ;FR[34]:=1500000.0 ;
CA[35]:=58.35 ;FR[35]:=2000000.0 ;CA[36]:=71.55 ;FR[36]:=3000000.0 ;
CA[37]:=92.53 ;FR[37]:=5000000.0 ;
```

```
p:=0;
```

```
i:=1;
```

```
while (p < 1) do begin
```

```
if (Freq >= FR[i]) then ip:=i;
```

```
if (Freq <= FR[(i+1)]) then inew:=i+1;
```

```
if (Freq <= FR[(i+1)]) then p:=1;
```

```
i:=i+1;
```

```
end;
```

```
Attenuation:=(Length/1.6)*((((CA[inew]-CA[ip])/
```

```
(FR[inew]-FR[ip]))*(Freq-FR[ip]))+CA[ip];
```

```
end;
```



```

{*****}
{SNR procedure}
{Input: frequency, center frequency}
{Output: crosstalk factor, channel response, filter response}

procedure snr(var Freq,xtalk,ChanRes,FilterAtf,T:real);
var tpf,Attenuation:real;
begin
  loss(Freq,Attenuation);
  tpf:=sin(pi*T*(Freq-1/(2*T))/FilterRolloff);
  if(Freq>((1-FilterRolloff)/(2*T)))and
    (Freq<((1+FilterRolloff)/(2*T)))then FilterAtf:=(1-tpf)/2 else
    if(Freq<((1-FilterRolloff)/(2*T)))then FilterAtf:=1.0 else
      FilterAtf:=0.0;
  ChanRes:=sqr(FilterAtf)*exp((-Attenuation/10)*ln(10));
  if (Freq<100) then ChanRes:=1e-30+exp(15*ln(Freq/(Freq+12)));
  xtalk:=sqr(FilterAtf)*0.632e-13*exp(1.5*ln(Freq));
end;

```

```

{*****}
{subprogram complex.pas}
{W.F. McGee}

const  dbcon = 4.3429448190;
        phasecon=57.295779513;
        twopi= 6.2831853072;

type complex=record re,im:real end;

procedure addc(a,b:complex;var c:complex);
{c:=a+b}
begin
    c.re:=a.re+b.re;
    c.im:=a.im+b.im
end;

procedure subc(a,b:complex;var c:complex);
{c:=a-b}
var temp:complex;
begin
    c.re:=a.re-b.re;
    c.im:=a.im-b.im
end;

procedure equalc(a:complex;var b:complex);
{b:=a}
begin
    b.re:=a.re;
    b.im:=a.im
end;

procedure makec(x,y:real;var c:complex);
{z:=x+jy}
begin
    c.re:=x;
    c.im:=y
end;

procedure conjugate(a:complex; var b:complex);
{b:=a*; if a=x+jy then z=x-jy}
begin
    makec(a.re,-a.im,b)
end;

procedure multc(a,b:complex;var c:complex);
{c:=a*b}
var t1,t2:real;
begin
    t1:=a.re; t2:=b.re;
    c.re:=t1*t2-a.im*b.im;
    c.im:=t1*b.im+t2*a.im
end;

procedure divc(a,b:complex;var c:complex);

```

```

{c:=a/b}
var den:real;temp:complex;
begin
  den:=b.re*b.re+b.im*b.im;
  conjugate(b,temp);
  multc(a,temp,c);
  c.re:=c.re/den;c.im:=c.im/den
end;

function angle(z:complex):real;
{angle(z) in radians}
begin
  with z do begin
    if (re=0.0) and (im=0.0) then angle:=0.0
    else if (re=0.0) and (im>0.0) then angle:=TWOPI/4.0
    else if (re=0.0) and (im<0.0) then angle:=-TWOPI/4.0
    else if (re<0.0) and (im>=0.0) then angle:=arctan(im/re)+TWOPI/2.0
    else if (re<0.0) and (im<0.0) then angle:=arctan(im/re)-TWOPI/2.0
    else angle:=arctan(im/re)
  end
end;

function amag(z:complex):real;
begin
  amag:=sqrt(sqr(z.re)+sqr(z.im))
end;

procedure polarc(z:complex;var dB,deg:real);
{magnitude (in dB) and phase (in degrees) of z}
var temp:real;
begin
  temp:=sqrt(z.re)+sqr(z.im);
  db :=ln(temp)*DBCON;
  deg:=angle(z)*PHASECON
end;

```

```

{*****}
{subprogram power.pas}

```

```

function power2(i:integer):integer;
begin
  if i<=0 then power2:=1
  else power2:=2*power2(i-1)
end;

```

```

{*****}
{subprogram fftpart.pas}
{based on Oppenheim and Schaefer}

type vector=array[0..512]of complex;
procedure twiddle(var a:vector;var m:integer);
{twiddles the vector elements}
var n,i,nhalf,j,k:integer;
    temp:complex;
begin
    n:=power2(m);
    nhalf:=n div 2;
    j:=0;
    for i:=0 to n-2 do begin
        if (i<j) then begin
            temp:=a[j];a[j]:=a[i];a[i]:=temp;
        end;
        k:=nhalf;
        while (k<(j+1)) do begin
            j:=j-k;k:=k div 2
        end;
        j:=j+k
    end
end;

procedure fft(var a:vector;m:integer);
var n,i,j,l,le,le1,ip,nl:integer;
    u,w,t:complex;
begin
    n:=power2(m);
    nl:=power2(m)-1;
    for l:=1 to m do begin
        le:=power2(l);
        le1:=le div 2;
        u.re:=1.0;u.im:=0.0;
        w.re:=cos(pi/le1);w.im:=-sin(pi/le1);
        for j:=0 to le1-1 do begin
            i:=j;
            while (i<=nl) do begin
                ip:=i+le1;
                multc(a[ip],u,t);
                subc(a[i],t,a[ip]);
                addc(a[i],t,a[ip]);
                i:=i+le
            end;
            multc(u,w,u)
        end
    end
end;

procedure ifft(var a:vector;m:integer);
var n,i,j,l,le,le1,ip,nl:integer;
    u,w,t:complex;
begin

```

```

n:=power2(m);
nl:=power2(m)-1;
for l:=1 to m do begin
  le:=power2(l);
  le1:=le div 2;
  u.re:=1.0;u.im:=0.0;
  w.re:=cos(pi/le1);w.im:=sin(pi/le1);
  for j:=0 to le1-1 do begin
    i:=j;
    while (i<=nl) do begin
      ip:=i+le1;
      multc(a[ip],u,t);
      subc(a[i],t,a[ip]);
      addc(a[i],t,a[i]);
      i:=i+le
    end;
    multc(u,w,u)
  end;
end;
for i:=0 to n-1 do begin
  a[i].re:=a[i].re/n;a[i].im:=a[i].im/n;
end;
end;

```

```
{program MSEqcl.pas;}
```

```
{This program calculates the mean square error of a passband QAM system  
based upon the signalling frequency, the background noise and the crosstalk
```

```
The MSE is calculated assuming optimal equalization with an infinite delay  
line for the case of decision feedback.
```

```
The probability of error is calculated using a Chernoff bound.}
```

```
{updated Oct. 26 }
```

```
const      WhiteNoise = 2.5e-14;  
var NumOfSamples,n,justaninteger,iLast,m,i,iNew,iFirst,marker:integer;  
    DFISISum,DFNoiseSum,FoldFreq,FreqPlusCarrier,MSE,ChanStartFreq:real;  
    Freq,Xtalk,T,Pulse,sum,MessageSpectrum:real;  
    justanumber,U0,sum1,ProbErrorFac,Power,ChanEndFreq:real;  
    Qfactor,ProbOfError,CenterFreq,FilterAtf,df,PrevChanEnd,ak2:real;  
    LinMSE,Linsum,DFWhiteNoiseSum,DFXtalkNoiseSum,OverallChanResponse:real;  
    WhiteNoisePowerLim,NumOfSymbols,PowerFactor,BitsperSymbol:real;  
    LinEqRes,XtalkNoiseRes,DFEqRes,ChanRes,Noise,PeriodicFn:grump;  
    BasebandFreqLimit,LinXtalkNoiseSum,LinWhiteNoiseSum,LinISISum:real;  
    ComplexMatrix:vector;  
    ComplexZero:complex;
```

```
{*****}  
{Obtain the system parameters}  
{Output: channel frequency limits, data sequence information}
```

```
begin  
    ak2:=0.0;  
    writeln(' ');  
    writeln(' what is the autocorrelation of the input data?');  
    readln(ak2);  
    writeln(' what is the probability of error factor?');  
    readln(ProbErrorFac);  
    writeln('how many symbols?');  
    readln(NumOfSymbols);  
    BitsperSymbol:=ln(NumOfSymbols)/ln(2);  
    writeln(' what is the frequency starting point?');  
    readln(ChanStartFreq);  
    writeln(' what is the estimated channel end?');  
    readln(ChanEndFreq);
```

```
{*****}  
{Obtain the channel parameters}  
{Input: channel frequency limits, data sequence information}  
{Output: channel response, noise}
```

```
ProbOfError:=0.0;
```

```

PrevChanEnd:=0.0;
marker:=0;
while (ChanEndFreq<>PrevChanEnd) do begin
  PrevChanEnd:=ChanEndFreq;
  justanumber:=0.0;
  Power:=0.0;
  CenterFreq:=(ChanEndFreq+ChanStartFreq)/2;
  T:=(1+FilterRolloff)/(2*(CenterFreq-ChanStartFreq));
  MessageSpectrum:=ak2/(2*T);
  PowerFactor:=MessageSpectrum/(0.25*T);
  df:=(1/T)/128;
  Power:=0.0;
  if (odd(trunc(128*(1+FilterRolloff)))) then
    NumOfSamples:=trunc(128*(1+FilterRolloff))-1 else
    NumOfSamples:=trunc(128*(1+FilterRolloff));
  justaninteger:=NumOfSamples div 2;
  BasebandFreqLimit:=df*(justaninteger-0.5);
  NumOfSamples:=NumOfSamples-1;
  writeln(NumOfSamples);
  Freq:=-BasebandFreqLimit;
  WhiteNoisePowerLim:=WhiteNoise*PowerFactor;
  for i:=0 to (NumOfSamples) do begin
    FreqPlusCarrier:=Freq+CenterFreq;
    snr(FreqPlusCarrier,Xtalk,ChanRes[i],FilterAtf,CenterFreq,T);
    Pulse:=1.0;
    XtalkNoiseRes[i]:=MessageSpectrum*Xtalk*sqr(Pulse);
    Noise[i]:=WhiteNoisePowerLim+XtalkNoiseRes[i];
    Power:=Power+sqr(Pulse*FilterAtf);
    delay(300);
    Freq:=Freq+df;
  end;
  writeln(' finished with snr');
  Power:=Power*df*MessageSpectrum/PowerFactor;
  writeln('Power = ',Power);

```

```

{*****}
{Calculate the folded sum}
{input: channel response and noise}
{output: folded sum}

i:=0;
Freq:=-BasebandFreqLimit;
while (Freq<= BasebandFreqLimit) do begin
  sum:=0.0;
  for m:=-1 to 1 do begin
    FoldFreq:=Freq+m/T;
    iNew:=round((FoldFreq+BasebandFreqLimit)/df);
    if (iNew <= NumOfSamples) and (iNew >= 0) then
      sum:=sum+sqr(Pulse)*ChanRes[iNew]/Noise[iNew];
  end;
  sum:=0.5*sum+1/(MessageSpectrum);
  PeriodicFn[i]:=sum;
  if (Freq>=(-1/(2*T))) and (Freq<=(1/(2*T))) then begin

```

```

    if (marker<1) then iFirst:=1;
    iLast:=i;
    ComplexMatrix[i-iFirst].re:=ln(sum);ComplexMatrix[i-iFirst].im:=0.0;
    marker:=1;
  end;
  Freq:=Freq+df;
  i:=i+1;
end;

```

```

{*****}
{Spectral Factorization}
{input: folded sum}
{output: spectral factor}

```

```

m:=7;
n:=trunc(exp(m*ln(2)));

OverallChanResponse:=0.0;
writeln('twiddle');
twiddle(ComplexMatrix,m);
writeln('fft');
fft(ComplexMatrix,m);
ComplexZero.re:=0.0;ComplexZero.im:=0.0;
ComplexMatrix[0].re:=ComplexMatrix[0].re/2.0;
ComplexMatrix[0].im:=ComplexMatrix[0].im/2.0;
ComplexMatrix[n div 2].re:=ComplexMatrix[n div 2].re/2.0;
ComplexMatrix[n div 2].im:=ComplexMatrix[n div 2].im/2.0;
U0:=exp(ComplexMatrix[0].re/128);
for i:=((n div 2)+1) to n-1 do ComplexMatrix[i]:=ComplexZero;
writeln(' twiddling again');
twiddle(ComplexMatrix,m);
writeln('inverse fft');
ifft(ComplexMatrix,m);
sum:=0.0;
suml:=0.0;
for i:=0 to n-1 do begin
  ComplexMatrix[i].re:=exp(ComplexMatrix[i].re);
  writeln(i,' ',sqr(ComplexMatrix[i].re)-PeriodicFn[i+iFirst]);
  suml:=suml+ln(ComplexMatrix[i].re);
end;
writeln('(U0) ',ln(U0),' integral ',(T*df*suml));

```

```

{*****}
{Calculate the equalizer values}
{input: spectral factor, channel response}
{output: equalizer response}

```

```

Linsum:=0.0;
for i:=0 to (iFirst-1) do begin
  DFEqRes[i]:=T*sqr(ChanRes[i])*Pulse/(Noise[i]*U0*

```



```

        ComplexMatrix[(127-(iFirst-1-1))].re);
    LinEqRes[i]:=T*Pulse*sqrt(ChanRes[i])/(PeriodicFn[i]*Noise[i]);
    OverallChanResponse:=OverallChanResponse+
        0.5*Pulse*DfEqRes[i]*sqrt(ChanRes[i]);
end;
writeln('middle bit');
for i:=iFirst to iLast do begin
    DfEqRes[i]:=T*sqrt(ChanRes[i])*Pulse/(Noise[i]*U0*
        ComplexMatrix[(i-iFirst)].re);
    LinEqRes[i]:=T*Pulse*sqrt(ChanRes[i])/(PeriodicFn[i]*Noise[i]);
    OverallChanResponse:=OverallChanResponse+
        0.5*sqrt(ChanRes[i])*Pulse*DfEqRes[i]-
        T*(ComplexMatrix[(i-iFirst)].re/U0-1);
    Linsum:=Linsum+1/PeriodicFn[i];
end;
writeln('last bit');
for i:=iLast+1 to NumOfSamples do begin
    DfEqRes[i]:=T*sqrt(ChanRes[i])*Pulse/(Noise[i]*U0*
        ComplexMatrix[(i-(iLast+1))].re);
    LinEqRes[i]:=T*Pulse*sqrt(ChanRes[i])/(PeriodicFn[i]*Noise[i]);
    OverallChanResponse:=OverallChanResponse+
        0.5*Pulse*DfEqRes[i]*sqrt(ChanRes[i]);
end;

```

```

{*****}
{Calculate the contributions to MSE}

```

```

DFWhiteNoiseSum:=0.0;
DFXtalkNoiseSum:=0.0;
DFNoiseSum:=0.0;
LinXtalkNoiseSum:=0.0;
LinWhiteNoiseSum:=0.0;
LinISISum:=0.0;
LinMSE:=0.0;
Freq:=CenterFreq-BasebandFreqLimit;

```

```

{*****}

```

```

{Noise}
{input: equalizer response, noise}
{ouput: noise integral}

```

```

for i:=0 to NumOfSamples do begin
    DFWhiteNoiseSum:=DFWhiteNoiseSum+WhiteNoisePowerLim*sqr(abs(DfEqRes[i]
    LinWhiteNoiseSum:=LinWhiteNoiseSum+
        WhiteNoisePowerLim*sqr(abs(LinEqRes[i]));
    DFXtalkNoiseSum:=DFXtalkNoiseSum+XtalkNoiseRes[i]*sqr(abs(DfEqRes[i]))
    LinXtalkNoiseSum:=LinXtalkNoiseSum+
        XtalkNoiseRes[i]*sqr(abs(LinEqRes[i]));
end;

```

```

DFNoiseSum:=0.5*(DFWhiteNoiseSum+DFXtalkNoiseSum)*df;
DFWhiteNoiseSum:=0.5*DFWhiteNoiseSum*df;
DFXtalkNoiseSum:=0.5*DFXtalkNoiseSum*df;
LinXtalkNoiseSum:=LinXtalkNoiseSum*df*0.5;
LinWhiteNoiseSum:=LinWhiteNoiseSum*df*0.5;

```

```

{*****}

```

```

{ISI}
{input: equalizer, channel response}
{output: ISI integral}

```

```

DFISISum:=0.0;
Freq:=-1/(2*T)+0.5*df;
i:=0;
while Freq <= (1/(2*T)-0.5*df) do begin
  sum:=0.0;
  sum1:=0.0;
  for m:=-1 to 1 do begin
    FoldFreq:=Freq+m/T;
    iNew:=round((FoldFreq+BasebandFreqLimit)/df);
    if (iNew <= NumOfSamples) and (iNew >= 0) then begin
      sum:=sum+0.5*DFEqRes[iNew]*Pulse*sqrt(ChanRes[iNew]);
      sum1:=sum1+0.5*LinEqRes[iNew]*Pulse*sqrt(ChanRes[iNew]);
    end;
  end;
  DFISISum:=DFISISum+sqr(sum-T*ComplexMatrix[i].re/U0);
  LinISISum:=LinISISum+sqr(sum1-T);
  Freq:=Freq+df;
  i:=i+1;
end;
DFISISum:=DFISISum*df*MessageSpectrum;
LinISISum:=LinISISum*df*MessageSpectrum;

```

```

{*****}

```

```

{MSE total}

```

```

MSE:=DFISISum+DFNoiseSum;
LinMSE:=LinISISum+LinXtalkNoiseSum+LinWhiteNoiseSum;

```

```

{*****}

```

```

{Output routine}

```

```

writeln(' Impulse response at t=0 ',OverallChanResponse*df);
writeln(' Noise = ',DFNoiseSum,' ISI = ',DFISISum,
  ' Long form DFE MSE = ',MSE);
writeln(' white noise = ',DFWhiteNoiseSum,
  ' crosstalk = ',DFXtalkNoiseSum);
writeln(' short form MSE = ',T/sqr(U0));

```

```

Qfactor:=abs(OverallChanResponse*df)/sqrt(T/sqr(U0));
q(Qfactor,ProbOfError);
ProbOfError:=ProbErrorFac*ProbOfError;
ProbOfError:=ProbOfError/BitsperSymbol;
writeln(' Start frequency = ',ChanStartFreq,
' End frequency = ',ChanEndFreq);
writeln(' 1/T = ',(1/T):9,' Pe= ',ProbOfError:10);
writeln(' ');
writeln(' Linear noise = ',(LinXtalkNoiseSum+LinWhiteNoiseSum),
' ISI = ',LinISISum,' linear MSE = ',LinMSE);
writeln(' white noise = ',LinWhiteNoiseSum,
' crosstalk = ',LinXtalkNoiseSum);
writeln(' short form MSE = ',sqr(T)*LinMSE*df);

Qfactor:=1.0/sqrt(LinMSE);
q(Qfactor,ProbOfError);
ProbOfError:=ProbErrorFac*ProbOfError;
ProbOfError:=ProbOfError/BitsperSymbol;
writeln(' Start frequency = ',ChanStartFreq,
' End frequency = ',ChanEndFreq);
writeln(' 1/T = ',(1/T):9,' Pe= ',ProbOfError:10);

for i:=1 to 3 do writeln(' ');
  writeln(' do you want to change the Channel End Frequency? ');
  writeln(' ',ProbOfError,' ',ChanEndFreq);
  readln(justanumber);
  if justanumber>100.0 then ChanEndFreq:=justanumber;
end;

end.

```

{*****}

{subprogram snrbase.pas}

{Input: frequency}

{Output: channel attenuation}

{update Oct. 23}

const pi = 3.1415926;
Length= 4.8;
FilterRolloff = 0.2;

procedure loss(var Freq,Attenuation:real); {find the attenuation at a certain

type b=array[1..40]of real;

var i,ip,inew,p:integer;

CA,FR:b;

begin

CA[1]:=2.45 ;FR[1]:= 1.0	;CA[2]:= 2.45 ;FR[2]:= 5.0 ;
CA[3]:=2.45 ;FR[3]:=10.0	;CA[4]:= 2.45 ;FR[4]:= 15.0 ;
CA[5]:=2.45 ;FR[5]:=20.0	;CA[6]:= 2.45 ;FR[6]:= 30.0 ;
CA[7]:=2.45 ;FR[7]:=50.0	;CA[8]:= 2.45 ;FR[8]:= 70.0 ;
CA[9]:=2.45 ;FR[9]:=100.0	;CA[10]:=2.45 ;FR[10]:= 150.0 ;
CA[11]:=2.45 ;FR[11]:=200.0	;CA[12]:=2.45 ;FR[12]:= 300.0 ;
CA[13]:=2.45 ;FR[13]:=500.0	;CA[14]:=2.45 ;FR[14]:= 700.0 ;
CA[15]:=2.92 ;FR[15]:=1000.0	;CA[16]:=3.57 ;FR[16]:=1500.0 ;
CA[17]:=4.1 ;FR[17]:=2000.0	;CA[18]:=4.995 ;FR[18]:=3000.0 ;
CA[19]:=6.5 ;FR[19]:=5000.0	;CA[20]:=7.422 ;FR[20]:=7000.0 ;
CA[21]:=8.692 ;FR[21]:=10000.0	;CA[22]:=10.294 ;FR[22]:=15000.0 ;
CA[23]:=11.503 ;FR[23]:=20000.0	;CA[24]:=13.232 ;FR[24]:=30000.0 ;
CA[25]:=15.29 ;FR[25]:=50000.0	;CA[26]:=16.5 ;FR[26]:=70000.0 ;
CA[27]:=17.73 ;FR[27]:=100000.0	;CA[28]:=19.25 ;FR[28]:=150000.0 ;
CA[29]:=20.67 ;FR[29]:=200000.0	;CA[30]:=23.59 ;FR[30]:=300000.0 ;
CA[31]:=29.26 ;FR[31]:=500000.0	;CA[32]:=34.48 ;FR[32]:=700000.0 ;
CA[33]:=41.2 ;FR[33]:=1000000.0	;CA[34]:=50.49 ;FR[34]:=1500000.0 ;
CA[35]:=58.35 ;FR[35]:=2000000.0	;CA[36]:=71.55 ;FR[36]:=3000000.0 ;
CA[37]:=92.53 ;FR[37]:=5000000.0;	

p:=0;

i:=1;

while (p < 1) do begin

if (Freq >= FR[i]) then ip:=i;

if (Freq <= FR[(i+1)]) then inew:=i+1;

if (Freq <= FR[(i+1)]) then p:=1;

i:=i+1;

end;

Attenuation:=(Length/1.6)*((((CA[inew]-CA[ip])/
(FR[inew]-FR[ip]))*(Freq-FR[ip]))+CA[ip]);

end;

```

{*****}
{SNR procedure}
{Input: f=equency, center frequency}
{Output: crosstalk factor, channel response, filter response}

procedure snr(var Freq,xtalk,ChanRes,FilterAtf,T:real);
var tpf,Attenuation:real;
begin
  loss(Freq,Attenuation);
  tpf:=sin(pi*T*(Freq-1/(2*T))/FilterRolloff);
  if(Freq>((1-FilterRolloff)/(2*T)))and
    (Freq<((1+FilterRolloff)/(2*T)))then FilterAtf:=(1-tpf)/2 else
    if(Freq<((1-FilterRolloff)/(2*T)))then FilterAtf:=1.0 else
      FilterAtf:=0.0;
  ChanRes:=sqr(FilterAtf)*exp((-Attenuation/10)*ln(10));
  if (Freq<100) then ChanRes:=1e-30+exp(15*ln(Freq/(Freq+12)));
  xtalk:=sqr(FilterAtf)*0.632e-13*exp(1.5*ln(Freq));
end;

```

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