

COMBINED AXIAL LOAD - BENDING MOMENT
BEHAVIOUR OF REINFORCED CONCRETE SHORT COLUMNS

by

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ABSTRACT

Extensive and sometimes comprehensive research on the ultimate strength of reinforced concrete columns has been carried out almost since the first use of this structural material. This thesis reviews briefly some of the more significant experimental and analytical work in this field.

An experimental investigation has been carried out on 6" by 6" square section concrete columns, with symmetrical reinforcement. Loads were applied at eccentricities e/t equal to 0.00 ± 0.01 ; 0.10 ± 0.01 ; 0.30 ± 0.01 ; 0.50 ± 0.01 ; 0.75 ± 0.01 and ∞ . A total of 100 column specimens were tested, divided into 4 groups, each having 25 specimens. The percentage of reinforcement in each group was: 1.24; 2.27; 5.14 and 7.14 respectively. The variations of concrete strength considered in the project were 4,000 and 6,000 psi.

The scope of this thesis was purposely divided into three groups depending on the predominant type of stress to which the columns were subjected, namely: (1) pure flexure; (2) axial compression; (3) combined axial compression and flexure.

From the comparison of the results, it was found that the experimentally determined mean strength of the column specimens was in good agreement with a mean value of 1.04 and a standard deviation of 0.09 in comparison with the loads calculated from the formulae in the American Concrete Institute Building Code Requirement ACI 318-63 and a mean value of 0.99 and a standard deviation of 0.02 for the strain gradient method.

A computer program using the strain gradient method is presented. The input requires only the geometry of the member, stress-strain characteristics of the concrete, the longitudinal reinforcement and the magnitude and location of the applied loading. Final results of the calculated maximum axial loads and maximum external moments are obtained and printed out.

This thesis analysed and discussed the test results, reviewed the ultimate strength theories, thus gave an overall picture of the developement and scope of the research on ultimate strength of reinforced concrete short columns.

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NOTATION

A_s	=	area of tension steel, sq.in.
A_s'	=	area of compression steel, sq.in.
a	=	depth of equivalent rectangular stress block = $K_1 c$, in.
a_b	=	depth of equivalent rectangular stress block = $K_1 c_b$, in.
b	=	width of rectangular member, in.
c	=	distance from extreme compression fiber to neutral axis at ultimate strength, in.
C_b	=	distance from extreme compression fibre to neutral axis for balanced condition = $d(87,000)/(87,000+f_y)$
d	=	distance from extreme compression fiber to centroid of tension reinforcement
d'	=	distance from extreme compression fibre to centroid of compression reinforcement
d''	=	distance from plastic centroid to centroid of tension reinforcement
E_s	=	modulus of elasticity of reinforcement
E_c	=	modulus of elasticity of concrete
e'	=	eccentricity of load with respect to centroid of tension reinforcement
e	=	eccentricity of load with respect to plastic centroid of section
e_b	=	eccentricity of load P_b measured from plastic centroid of section
f_c	=	concrete strength

- f_c' = compression strength of 6" by 12" concrete cylinders, psi.
- f_c'' = flexural compressive strength of concrete member
- f_y = yield strength of reinforcement, psi.
- f_{s1} = compressive steel stress, psi.
- f_{s2} = tensile steel stress, psi.
- K_1 = the ratio of average stress to maximum stress
- K_d = distance from extreme compression fibre to neutral axis at ultimate strength
- L = span length, ft.
- M_u' = ultimate moment, kip-in.
- M_u^T = ultimate moment of column under pure bending, kip-in.
- $m = \frac{f_y}{0.85f_c'}$
- P_u' = ultimate load, kips.
- P_u^T = ultimate lateral load of column under pure bending, kips.
- $p = \frac{A_s}{bd}$
- $p' = \frac{A_s'}{bd}$
- P_b = axial load capacity at simultaneous crushing of concrete and yielding of tension steel, kips. (balanced conditions)
- P_o = ultimate load of concentrically loaded column, kips.
- $q = \frac{A_s}{bd} \cdot \frac{f_y}{f_c'}$

- t = over-all depth of a rectangular section in the direction of bending
- V = coefficient of variation
- δ = deflection
- ϵ = strain
- $\epsilon_{s1}, \epsilon_{s2}$ = steel strains
- ϵ_c = compressive strain in concrete
- ϵ_o = compressive strain in concrete corresponding to maximum stress (Fig. 4.1.a)
- ϵ_s = steel strain
- ϵ_x = strain in the X-direction
- ϵ_u = useful limit of compressive strain in concrete (Fig. 4.1.a)
- ρ = radius of curvature
- ϕ = angle change per unit length, curvature
- σ = standard deviation

CHAPTER I

INTRODUCTION

The basic function of columns in most normal frameworks is to carry the vertical load down to some lower level. In ordinary construction, there are many cases in which members are subjected to a combination of bending moments and axial loads. Generally, in reinforced concrete members, the direct axial load in such a combination is a compressive force. Lateral earth pressures which act upon foundation walls, columns connected to beams eccentrically, together with the effect of the rigidity of the connections, will cause bending moments to be applied to the columns. Wind loads acting upon a building also force the columns to bend sideways. In other cases, internal columns in the framework of a symmetrical structure, are subjected to out of balance bending moments, which also cause the column to bend. All of these, are ordinary causes of combined compressive force and flexure in the members that are affected by the above conditions.

Generally, problems involving compression and bending come into one of the two classes: the first includes those members under compression failure, with the crushing of concrete before yielding of the tension steel; the second covers those under tension failure, with the tension steel yield first.

In the past forty years, the strength of reinforced concrete columns, their behaviour under various conditions of loading, and proper procedures for their design have been of intense interest among Structural and Civil Engineers.

In 1638, Galilei's work regarding flexure of beams, was exclusively devoted to ultimate strength. Hooke's Law was formulated 40 years later, and Navier developed the fundamental theorems of the theory of elasticity in 1821. By 1930, the phenomena of failure and ultimate strength of columns in axial compression were fairly well covered, and theories for ultimate strength had also been developed. Tests and theories for eccentrically loaded columns were developed sometime later.

All present knowledge regarding ultimate strength of reinforced concrete columns is based on the results of tests. Consequently, ultimate strength methods give an indication of being both accurate and useful, while the straight-line method does not produce either consistent or satisfactory results.

The investigation described in this thesis was carried out to determine the effects of concrete quality, percentage and yield strength of reinforcement, and eccentricity on the actual failure load of the reinforced concrete tied columns and to the predicted ultimate strength, while the length and the nominal dimensions of the cross-section remained unchanged. The validity of the ACI Design Formulae and the strain gradient

method were also checked.

The experimental results were analysed and compared with the theoretical results obtained from the equations of the American Concrete Institute Building Code, and the strain gradient method.

CHAPTER II

REVIEW OF LITERATURE

The subject of 'Mechanics of Materials' has been a science of empirical character from the earliest developments due to the nature of the problem involved. This is particularly the case with concrete and reinforced concrete. The first attempts to establish a mathematical design procedure for reinforced concrete members were characterised by loading to failure a number of specimens and trying to establish an equation which correlated with the test results. In current terminology, these early theories are referred to as inelastic or ultimate theories.

In 1638, the work done by Galilei, attempted to describe the behaviour of beams in flexure following ultimate load tests on masonry cantilever beams.

In 1821, Navier's theory of bending was developed, based on the earlier's work of Bernoulli and Hooke's Law.

In 1894, the Straight-Line theory, was presented by Coignet and tédesco (3)*, to explain the behavior of concrete beams and columns. The theory was mathematically simple and was accurate enough for design purpose at that time. This method assumed a linear distribution of strain and that the stresses of both the concrete and steel

* Numerals in parentheses refer to corresponding items in the List of References.

were directly proportional to strain (the concrete was assumed to have no tension capability). No account was taken of the plastic behavior of the material nor was there any specific determination of the masimum load that could be carried.

From 1920 to 1930, another important development was taking place in the philosophy of column design. Before that time, bending stresses were generally neglected in the design of concrete building columns, and such stresses together with other effects, were assumed to be allowed for in an overall factor of safety. The Slope-Deflection and the Moment-Distribution Methods developed to analyse monolithic structures, enabled the bending moments as well as the axial loads on columns in frames to be determined easily. Consequently, current design codes required columns to be designed for both the axial load and bending moment obtained from a rational analysis.

Members subjected to axial compression have been represented in tests by axially-loaded columns. Although this type of load may never encountered in actual construction, it represents a limiting case for the columns under combined axial load and bending.

About 1900, Considère (4) proposed the following equation for the allowable load of concentrically loaded reinforced concrete columns, using appropriate safety factors to determine the allowable unit stresses for concrete and steel. Strain compatability was not considered.

$$P = A_c f_c + A_{st} f_s$$

where - f_c is allowable stress in extreme fiber of concrete, psi.

- f_s is allowable tensile stress of steel, psi.

- A_c and A_{st} are areas of concrete and steel respectively.

The Straight-Line Theory (Standard Theory) was established later, by using the transformed area formula, and the strain compatability was taken into account.

$$P = A_c f_c \left[1 + (n - 1) p_{st} \right]$$

where - n is the modular ratio (E_s/E_c)

- p_{st} is the ratio of total longitudinal reinforcement area to the gross area of concrete.

In 1921, McMillan (5) studied column test data, and found that building columns under load may develop steel stresses, due to plastic action, considerably higher than those predicted by the Straight-Line theory. This important study led to a number of ACI column research investigations,

carried out by Slater & Lyse (66), Slater (7), and Richart (8), and others in the 1930's. From their work, rational equations for the ultimate load of concentrically loaded reinforced concrete columns were developed. The ultimate load for tied columns is simply the sum of the strengths of the longitudinal reinforcement and the concrete. The ultimate strength of tied columns were shown to be as follows:

$$P = 0.85A_c f'_c + A_{st} f_y$$

where - f'_c is the ultimate concrete cylinder strength

- f_y is the yield point stress of longitudinal reinforcement.

In 1938, Richart and Olsen (11), carried out an investigation of 82 eccentrically-loaded columns, and presented an expression relating the ultimate load on an eccentrically-loaded column, to that of an axially-loaded column. In which,

$$\frac{P_{ecc.}}{P_{axial}} = \frac{1}{1 + C \frac{ec}{k^2}}$$

where - e is the eccentricity of the load with respect to the centroid of the column cross-section,

- c is the distance from the centroid to the extreme fiber in compression,

- C is an empirical constant less than unity,

- k is the radius of gyration of the column section.

This empirical equation, based on experimental tests, was capable of predicting the ultimate strengths for failures in compression, but was not capable of predicting either the ultimate strengths for failures by yielding of the reinforcement nor if a member would fail in tension or by crushing of the concrete.

In 1937 and 1943, Whitney (12) (37) made an important contribution on eccentrically loaded columns. He presented ultimate theories for the compression and tension failures of members subjected to axial compression and bending. Whitney assumed that the non-linear concrete compressive stress distribution could be approximated to a rectangular stress block with a uniform stress intensity of $0.85f_c'$ over some depth 'a' such that equilibrium is satisfied. The following equation for compression failures in combined bending and axial load was developed.

$$P = \frac{2A_s' f_y}{\frac{2e}{d'} + 1} + \frac{b t f_c'}{\frac{3te}{d^2} + 1.178}$$

where - A_s' is the area of compression reinforcement,

- d' is the distance from extreme compression fibre to centroid of compression reinforcement,

- d is the distance from centroid of tension reinforcement to compression face of member,

- b is the width of a rectangular member,
- t is the total depth of section.

For compression failures, this equation reduces to the ultimate load for a centrically-loaded column, with $e = 0$.

In 1944, Jensen (13) made an attempt to use the in-elastic properties of the stress-strain curve for concrete. He assumed a trapezoidal stress distribution and derived the properties of this trapezoid as a function of cylinder strength by analysis of the observed ultimate strength of reinforced concrete beams.

Followed by a number of extensive studies of Jensen's theory, the ultimate strength theories for eccentrically loaded columns were developed.

Many additional tests have been reported between the years 1900 to 1950, from both European and North American investigators. Among these, are included those of Von Thullie (14), 1896; Bach and Graf (15), 1913; Baumann (16), 1934; Bittner (17), 1935; Brandzaeg (18), 1935; Thomas (19), 1938; The Russian Specifications (20), 1938; Anderson (21), 1941; and Richart (22), 1947.

In 1951, Hognestad (25) published a more refined ultimate strength theory, based on more realistic, and more complex assumptions regarding the stress-strain relation of concrete, derived empirically from his tests. His investigation not only predicted the types of failure and the

ultimate strength of the columns, but also the behavior of the column throughout the entire loading range. In his paper, Hognestad also introduced the dimensionless form of interaction diagram; with the application of this diagram; with the application of this diagram, one can determine the behaviour in which the strength varied between the limits of axial compression to pure flexure of the column, simply and accurately.

Both the ACI (American Concrete Institute) Building Code (ACI - 318 - 71) (23) and the NBC (National Building Code of Canada, 1965) (24) permit the ultimate strength method to be used in the design of reinforced concrete columns, using a ϕ factor to reduce the predicted actual strength capacity of the column, to obtain the design ultimate strength. These ϕ factors were intended to take into account the importance of the member and the accuracy of the predicted equation and allow for any errors in construction techniques. The ACI code specifies $\phi = 0.7$ for tied columns and $\phi = 0.75$ for spirally hooped columns; while the NBC code specifies $\phi = 0.75$ for both.

CHAPTER III

EXPERIMENTAL INVESTIGATION

3.1 Object and Scope of Investigation

The object of this investigation was to experimentally study the basic behavior of reinforced concrete short columns subject to combined bending and axial load.

The major variables studied in this program were the percentage of reinforcement, steel yield stress, concrete strength and eccentricity of the applied loading, while the length and nominal dimensions of the columns remained unchanged.

Details of the nominal dimension of the reinforced concrete columns is given in Figure (3.2.a).

3.2 Material

3.2.1 Concrete Mixtures

The concrete used for all specimens was supplied by Dominion Building Materials Ltd. The concrete was transit mixed during the transportation from the batch plant to the laboratory of the University of Ottawa. Two design compression strengths, of 4000 psi. and 6000 psi. at 28 days, were used in the project as being typical of concrete strengths presently used in construction. The mix proportions of the concrete are shown in Table (3.2.1.a) and Table (3.2.1.b) below. The grading curves of coarse and fine aggregates are shown in Fig. (3.2.1).

TABLE (3.2.1.a) Mix Proportion by Weight with Compression Strength of 4000 psi. (Slump = 3")

MATERIAL	WEIGHT PER CUBIC YARD OF CONCRETE
Standard Portland Cement	500 lbs.
Sand	1,530 lbs.
3/4" Coarse Aggregate	1,910 lbs.
Water	294 lbs.

$$W/C = 294/500 = 0.59$$

TABLE (3.2.1.b) Mix Proportion by Weight with Compression Strength of 6000 psi. (Slump = 3")

MATERIAL	WEIGHT PER CUBIC YARD OF CONCRETE
Standard Portland Cement	700 lbs.
Sand	1,140 lbs.
3/4" Coarse Aggregate	1,960 lbs.
Water	286 lbs.

$$W/C = 286/700 = 0.41$$

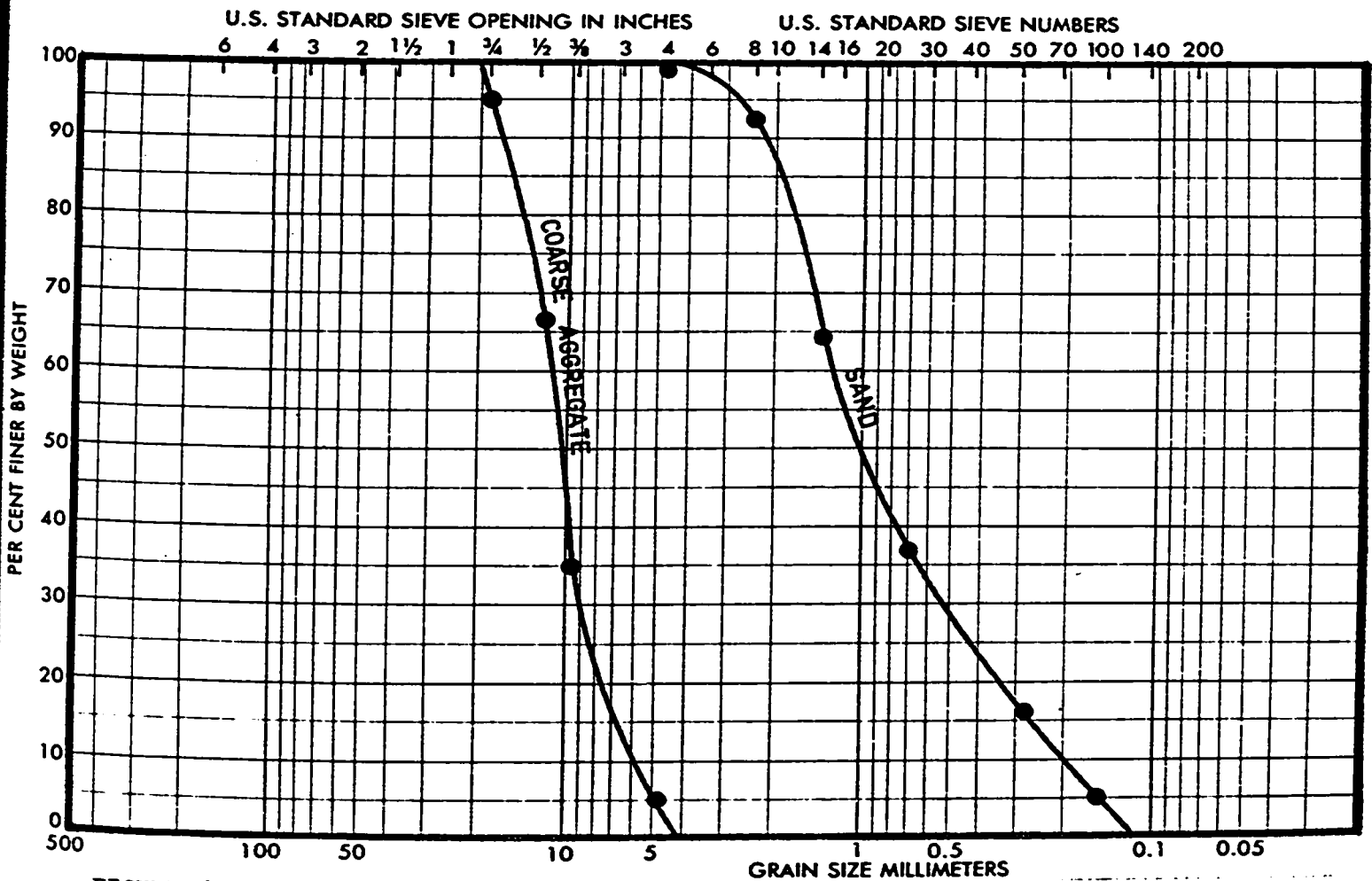


FIGURE (3.2.1) GRADING CURVES OF COARSE AGGREGATE & SAND

3.2.2 Reinforcing Steel

Three sizes of deformed bars of Nos. 7, 6, and 4, and one size of plain bar of No. 3, were used in the tests. The ties for all the specimens were manufactured by hand in the laboratory from #2, 1/4 in. plain bars. Average properties of the various lots of reinforcement as determined from tension tests in the laboratory are given in Table (3.2.2.a), and used for the calculations in both the ACI Design Formulae and the Strain Gradient Method.

All longitudinal reinforcement was specified to be Grade 60 steel.

TABLE (3.2.2.a)
AVERAGE TENSILE PROPERTIES OF
REINFORCING STEEL

BAR SIZE (in.)	NO. OF TESTS	AVERAGE YIELD POINT (ksi.)		AVERAGE ULTIMATE STRENGTH (ksi.)	USED IN SERIES NO.
		MEAN	C.of V.		
3/8	25	54.40	1.6%	76.00	4
4/8	25	65.50	2.9%	92.80	3
6/8	25	52.60	1.4%	110.00	2
7/8	25	67.40	0.5%	112.10	1

* The yield point is defined by a residual strain of 0.2% instead of 0.5% of the total strain under load; the yielding stress was found to be very close in both cases.

Typical stress-strain curves for various size of reinforcement are shown in Appendix A, from Fig. (A.1) page 107 to Fig. (A.4) page 110.

3.3 Types of Specimens

The 100 test specimens was divided into four groups, all of them being tied columns. Since the investigation was confined to combined axial and bending problems, the specimens were kept fairly short, ($\frac{L}{r} = 30$) so that the results would not be affected by the occurrence of buckling failures. An outline of the test program is given in Table (3.3.a), which indicates that the major variables were: size and yield stress of reinforcement, concrete strength and the eccentricity of applied loading.

Columns were completely identified by a numeral, a capital letter and followed by another numeral. The first numeral -- 1 through 4 -- indicates the size of the reinforcement; the capital letter -- A, B, C, D, DE or E -- indicates the eccentricity of load; the final numeral -- 1 to 5 -- indicates one of five companion specimens.

For example:-

1B - 1 is a column in group one, with 7/8 in. longitudinal reinforcement, loaded with a eccentricity of $e/t = 0.1$, and the first of this group of specimens.

The four groups of specimens with the different sizes of longitudinal reinforcement are shown in Fig. (3.2.a). Concrete cylinders, 6 by 12 in. were made as control specimens.

TABLE (3.3.a)
OUTLINE OF TESTS

GROUP NO.	ECCENTRICITY e/t	SIZE OF REINFORCEMENT (in.)	CONCRETE STRENGTH (psi.)	$P \frac{f_y}{0.85f_c'}$	TOTAL NO. OF COLUMNS	TESTED*
1	A (0.0)	7/8	4000	1.26	5	V
	B (0.1)				5	V
	C (0.3)				5	V
	D (0.5)				5	V
	E (∞)				5	H
2	B (0.1)	6/8	6000	0.404	5	V
	C (0.3)				5	V
	D (0.5)				5	V
	DE (0.75)				5	V
	E (∞)				5	H
3	A (0.0)	4/8	4000	0.479	5	V
	B (0.1)				5	V
	C (0.3)				5	H
	D (0.5)				5	H
	E (∞)				5	H
4	B (0.1)	3/8	6000	0.097	5	V
	C (0.3)				5	H
	D (0.5)				5	H
	DE (0.75)				5	V
	E (∞)				5	H

* H - Horizontally tested specimens
V - Vertically tested specimens

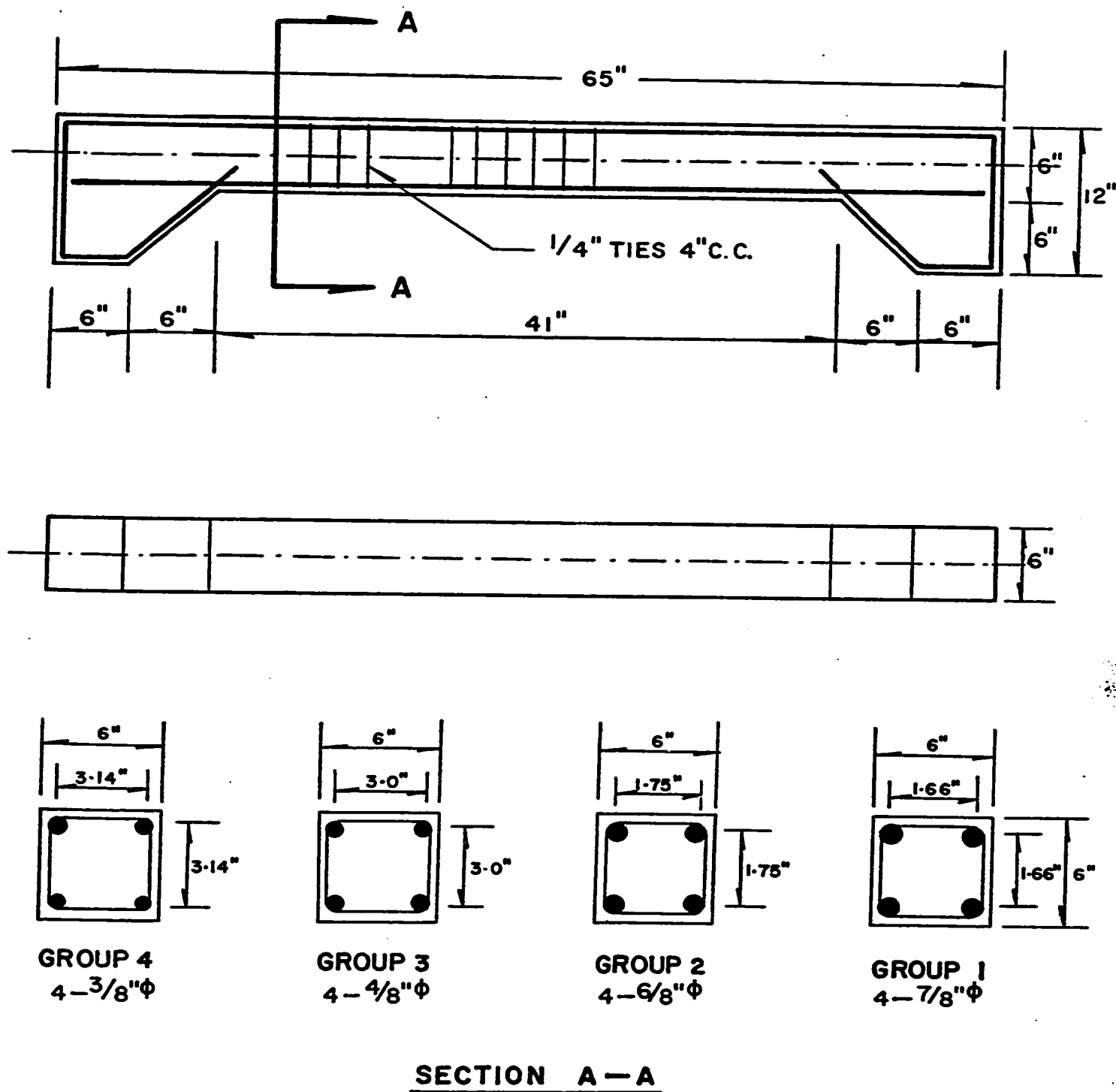
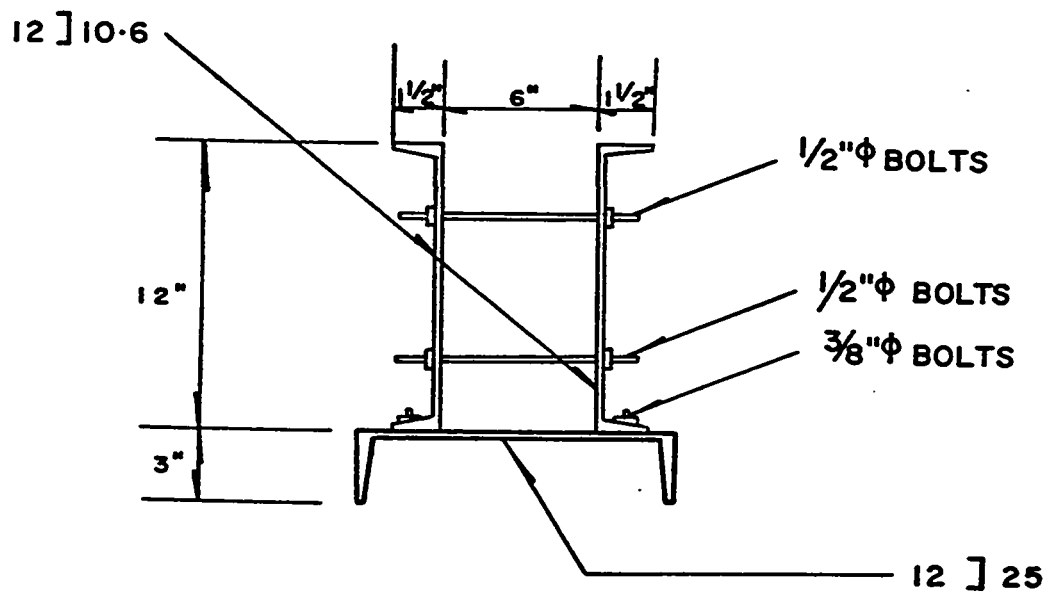
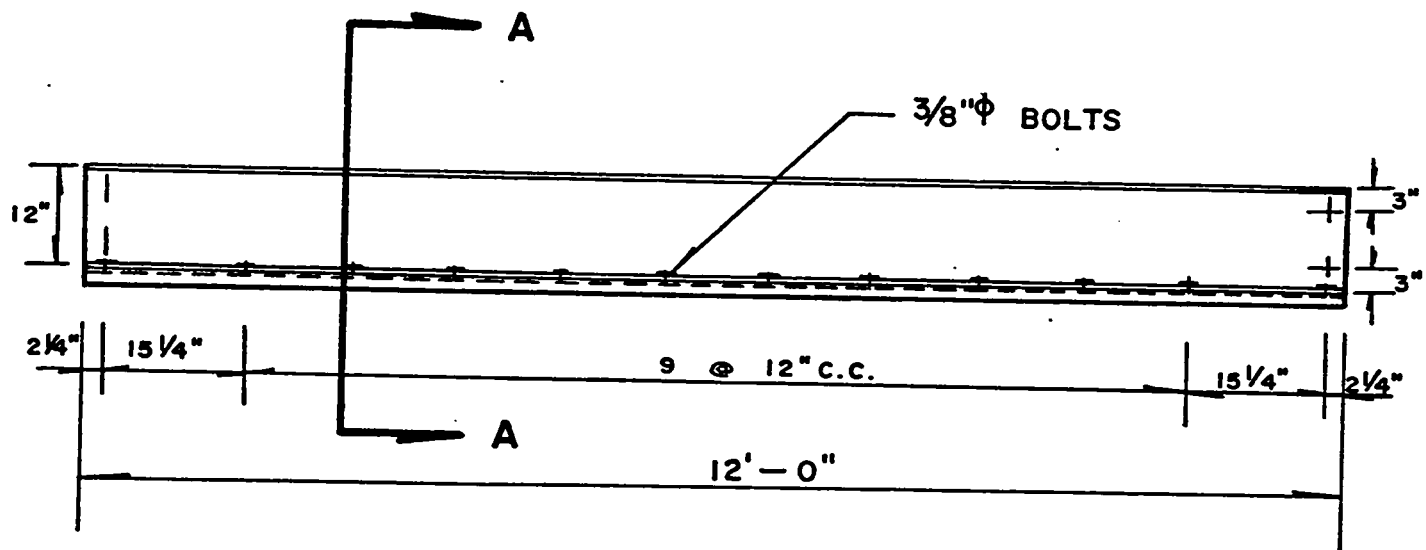


FIG. (3.2.a) DETAIL OF COLUMN SPECIMENS

3.4 Fabrication and Curing

The concrete was mixed and supplied by the Dominion Building Material Limited, and the specimens were cast in the structures laboratory of the University of Ottawa throughout the tests. The total time from mixing of concrete to the final placing and compacting operations, was kept within one hour and thirty minutes.

The reinforcement was assembled into a cage before it was placed in the forms for casting. The longitudinal bars were placed inside the ties and welded to them. Spacing blocks were used to provide accurate spacing and protecting coverings of the longitudinal reinforcement. Wooden blocks were inserted in the center portion of the steel forms before casting. Steel forms, as shown in Fig. (3.4.a), were used for each casting for the entire project.



SECTION A - A

FIG. (3.4.a) TYPICAL DRAWING FOR STEEL FORM

3.5 Testing Methods and Procedure

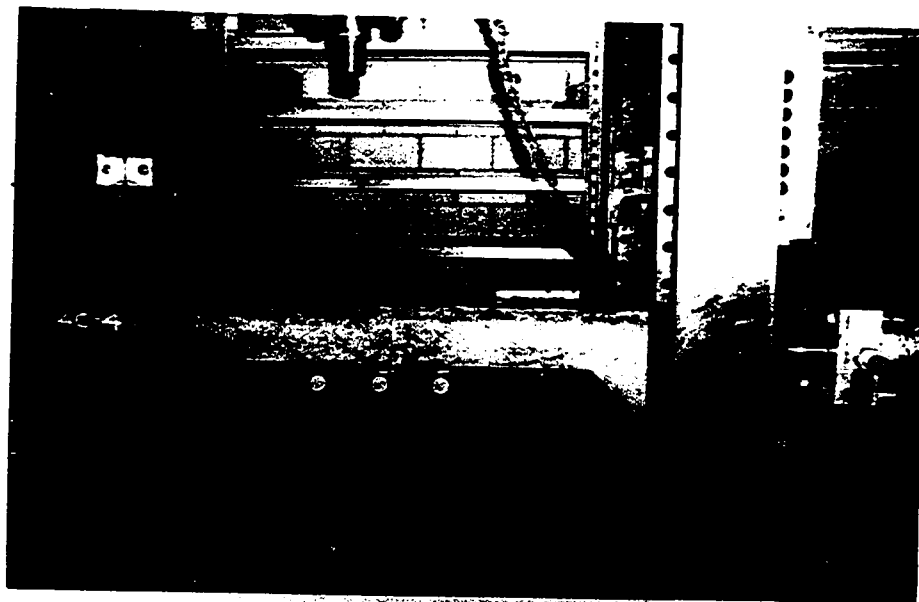
All five test specimens of a group of reinforced concrete columns and the twenty associated 6 by 12 in. concrete cylinders, were cast at the same time, and were tested within a 24 hour period, at the age of 28 days. The columns were subjected to short term loading, the time between the first and last increments of the applied load was approximately 30 minutes.

3.5.1 Reinforced Concrete Columns

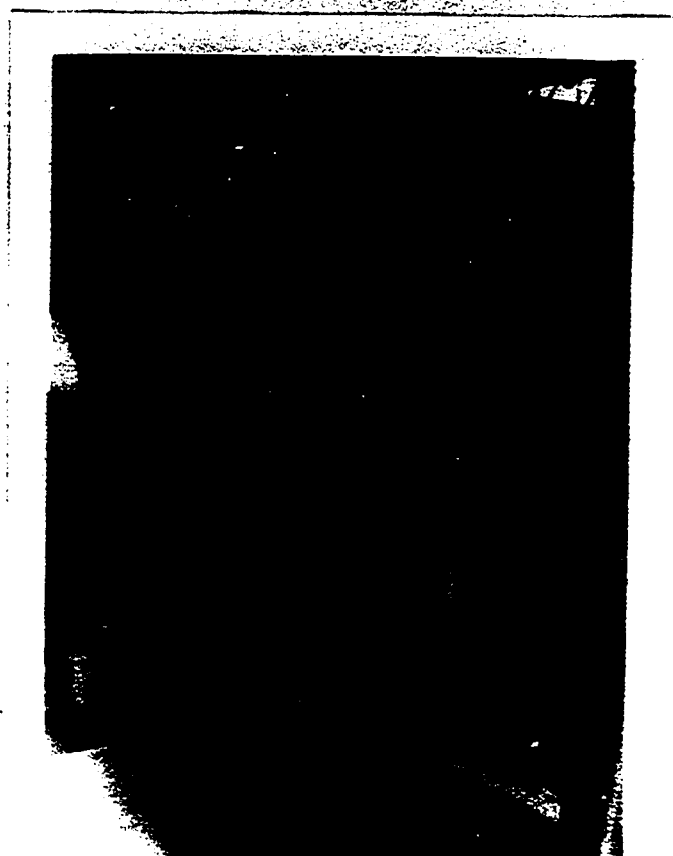
3.5.1.a Columns under Combined Axial and Eccentric Loadings

A total of 75 column specimens were assigned to be tested under axial and eccentric loadings, with eccentricities e/t equal to 0.00 ± 0.01 ; 0.10 ± 0.01 ; 0.30 ± 0.01 ; 0.50 ± 0.01 and 0.75 ± 0.01 etc.; of which 25 were tested horizontally in a special testing frame, with a 100 kips capacity. The other 50 specimens were tested vertically in a 600,000-lb. capacity Baldwin Southwark Tate-Emery hydraulic testing machine. A view of both the test set ups during a column test are shown in Fig. (3.5.1.a) and (b), and a sketch of the testing equipment for each is given in Fig. (3.5.2.a) and (b), respectively. The test method is indicated by an horizontal and vertical tested specimen, respectively in Figs. (3.5.2.a) and (b).

The columns were cast in a horizontal position in order to avoid a differential in concrete quality along the column length, but, on the other hand, this would cause a strength differential through the cross-section of the column.

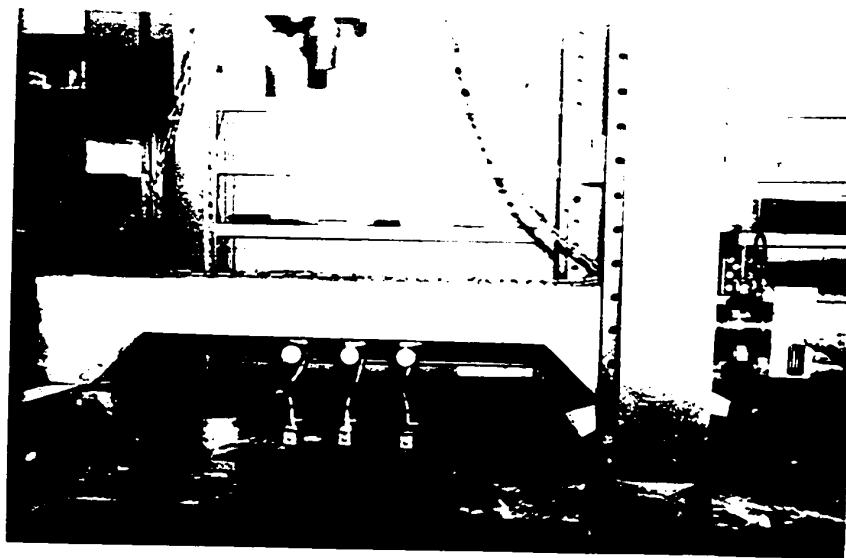


(a)

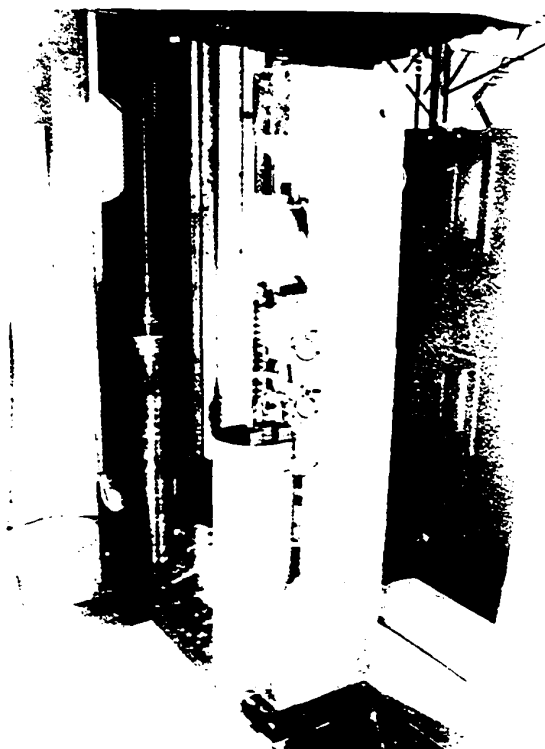


(b)

FIG.(3.5.1) GENERAL VIEW OF A TESTED SPECIMEN UNDER ECCENTRIC LOADING



(a)



(b)

FIG.(3.5.1) GENERAL VIEW OF A TESTED SPECIMEN UNDER ECCENTRIC LOADING

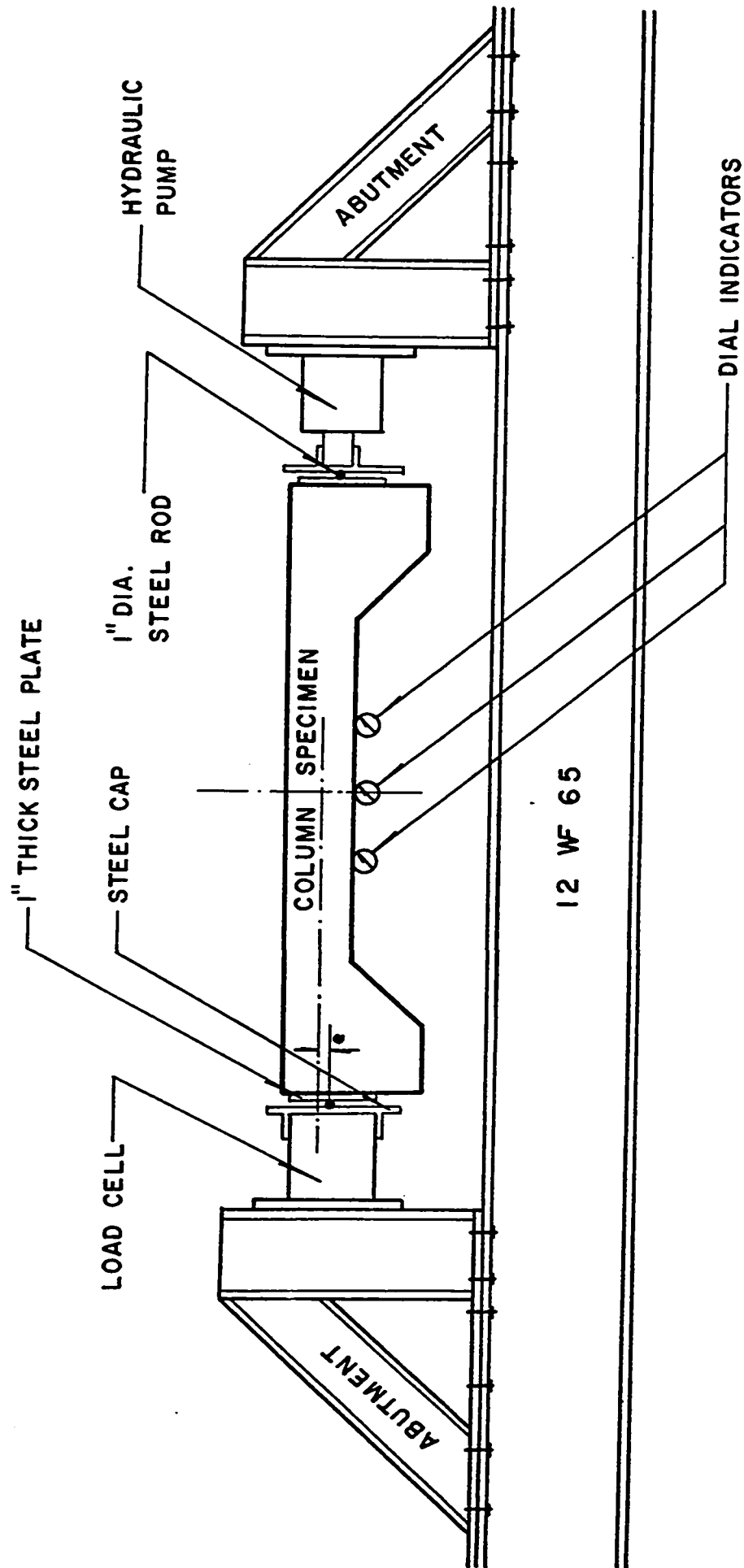


FIG.(3.5.2.a) ARRANGEMENT OF TESTING

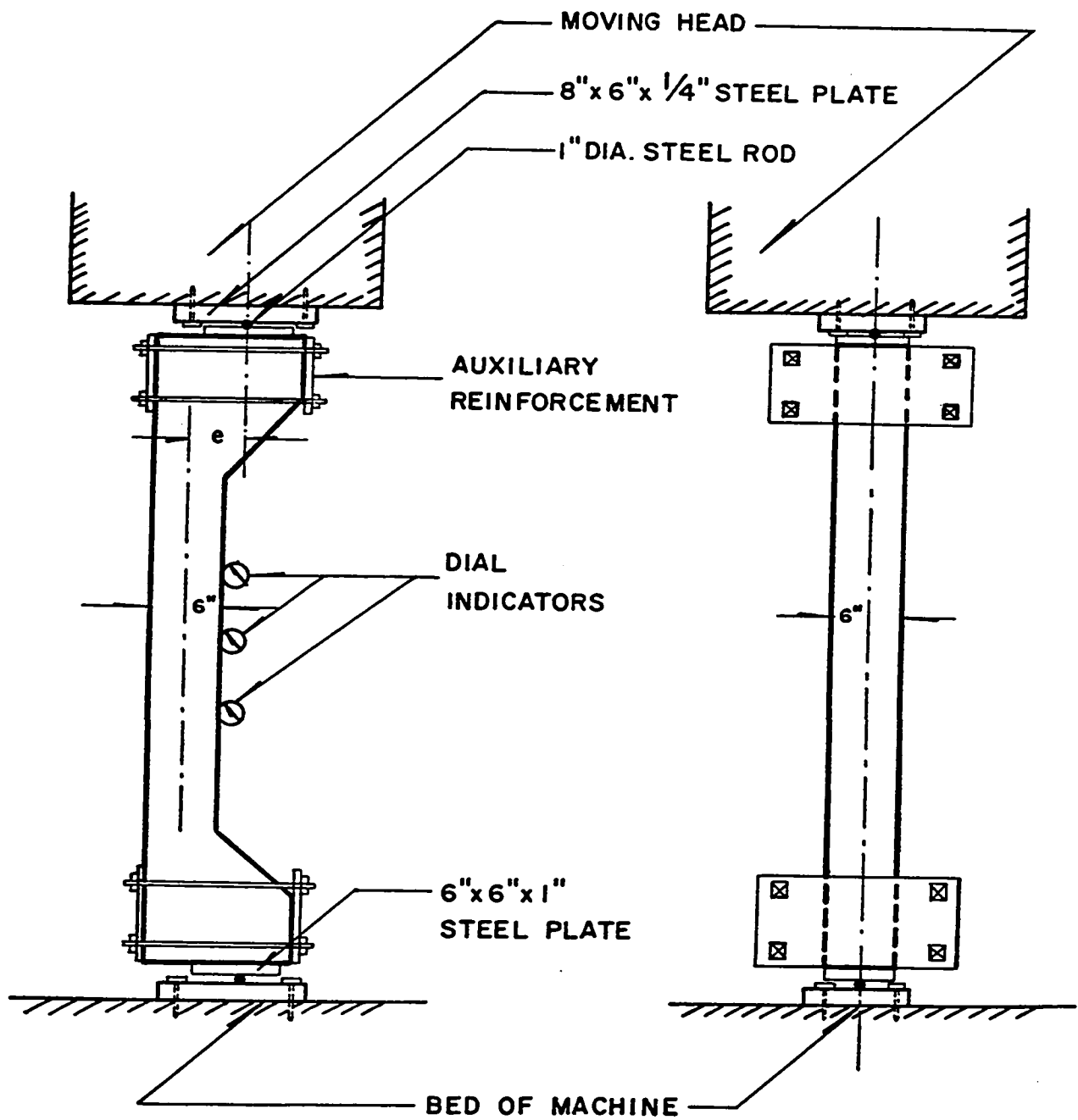


FIG.(3.5.2.b) ARRANGEMENT OF TESTING

Five or ten test specimens were cast at one casting, and the concrete was compacted in the forms by means of a small electrically driven pencil vibrator.

The day after the columns were cast, the forms were removed, and the columns and the control cylinders (20 for each casting), were cured under wet sacks in the laboratory for 28 days before testing.

The method and procedure used with both testing apparatuses were the same. In order to avoid the crushing of the concrete at both ends of the specimen, two 1" thick steel bearing plates were used, one at each end face of the specimen.

For the specimens tested horizontally in the testing frame, a steel cap was used for both the ram and the load cell. To obtain single axis hinged ends at both ends of the specimen, two 1" diameter steel rollers were put in between the end bearing plates and the steel caps, both of the which were provided with semi-circular grooves to keep the steel rollers in the desired positions.

For the specimens tested vertically in the 600,000-lb. Baldwin Southwark Tate-Emery hydraulic machine, two $1\frac{1}{4}$ " steel bearing plates, screwed to the movable head and the bed of the machine respectively, were provided with semi-circular grooves, to keep the 1" roller in place. Two - 12" x 6" x $1\frac{1}{2}$ " steel plates were clamped to both capitals of the columns by 4 - $1\frac{1}{2}$ " thread steel bars (which can be

seen in Fig. (3.5.2.b). These steel bars were placed on each capital of the column and prestressed before the test, by tightening the nuts, to reduce the possibility of diagonal tension failure in the capitals, because it was found impracticable to reinforce the capitals sufficiently with only embedded bar reinforcement.

The lateral deflection of the column was measured with three 0.001 in. dial indicators; one located at the midspan of the specimen, and the other two at 6" to the left and right of the midspan of the specimen respectively, in the plane of eccentricity.

All columns were carefully centered and levelled or plumbed in the testing machine. Initial readings of all gages were taken immediately before application of any loading. Loading proceeded to failure in 15 to 20 increments for every specimen.

3.5.1.b Columns under Pure Flexure

Twenty five specimens were tested under flexure, ie, with the eccentricity e/t equal to infinity. The method of testing was as for a simply-supported beam; loaded at two points located symmetrically about midspan. A view of the testing frame during the test of a specimen is shown in Fig. (3.5.3.a) and a sketch of the testing equipment is given in Fig. (3.5.3.b).

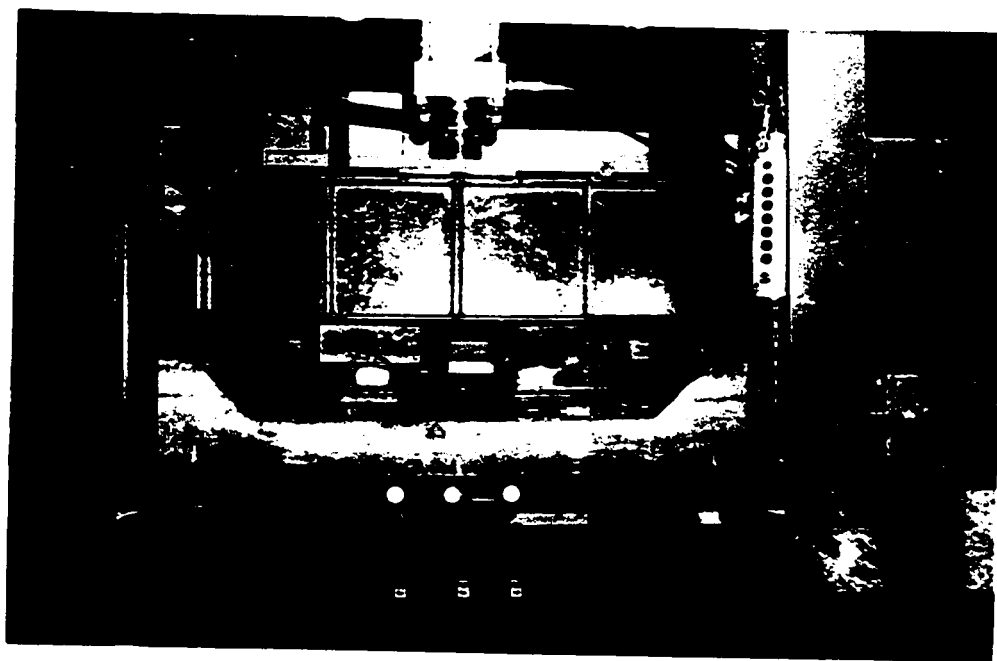


FIG.(3.5.3.a) GENERAL VIEW OF TESTED SPECIMEN UNDER PURE BENDING



FIG.(3.5.4.a) 6" BY 12" CONCRETE CYLINDER UNDER TEST

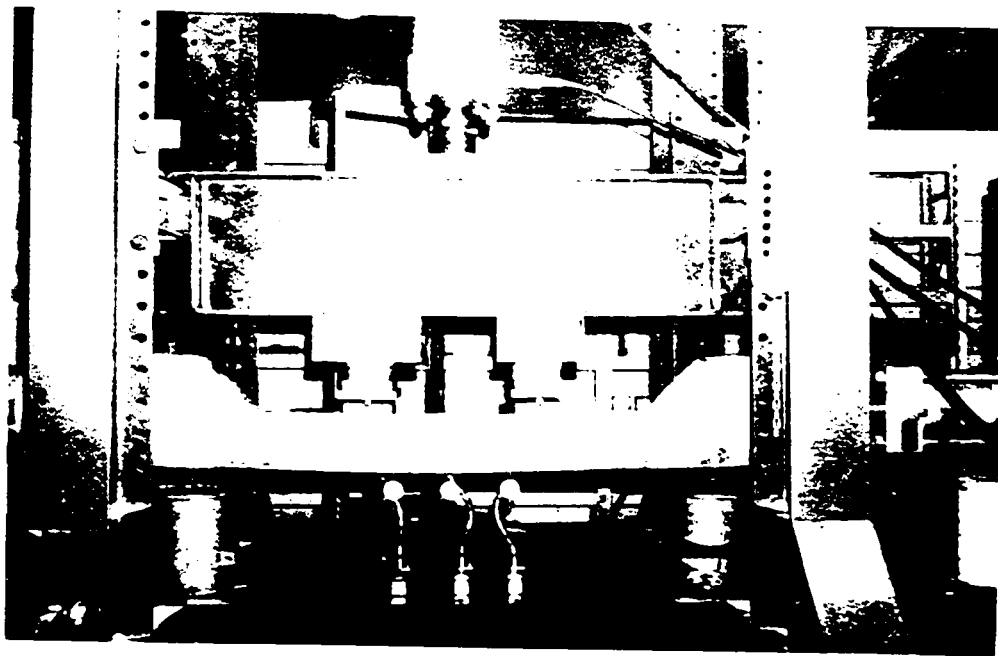


FIG.(3.5.3.a) GENERAL VIEW OF TESTED SPECIMEN UNDER PURE BENDING



FIG.(3.5.4.a) 6" BY 12" CONCRETE CYLINDER UNDER TEST

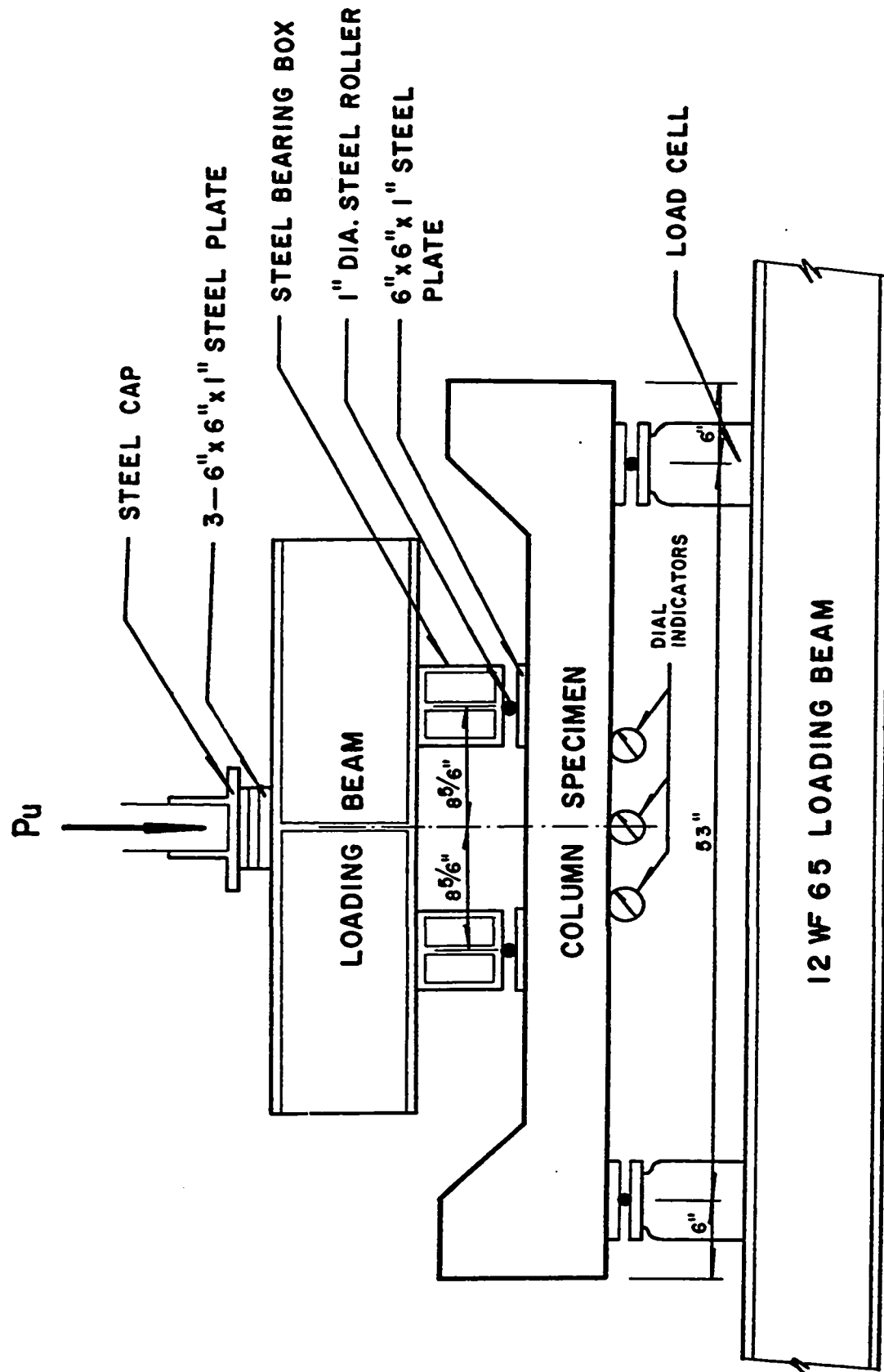


FIG (3.5.3.a) SKETCH OF TEST SETUP FOR BEAM COLUMN

The steel loading beam, located under the ram, was supported symmetrically by two load transfer blocks, both with 1" thick steel bearing plates resting directly on the upper surface of the specimen. Two 1" diameter steel rollers were put in between the steel blocks and the bearing plates. The load was applied by the ram, and transferred to the test specimen through the steel beam. The concrete beam-column was supported by two load cells each with a capacity of 50 kips, 6" from each end of the column. The load was measured from the reactions of the load cells.

The lateral deflections of the specimen, and the increments of the applied loading, were measured and applied the same way as for a column tested under combined axial and bending.

3.5.2 Concrete Cylinders

The compression strength of the concrete cylinders were tested on the same day as the specimen and thus gave the actual concrete cylinder compression strength at that age. A total of 20 test cylinders were cast at the same time as the corresponding columns, hence no differential time effects would be considered between the testing specimens and the cylinders.

Both end faces of the concrete cylinder were capped with standard sulphur capping compound and tested to failure in a 300,000-lb. capacity Forney hydraulic concrete testing machine. A view of a 6" by 12" concrete cylinder under test is shown in Fig. (3.5.4.a).

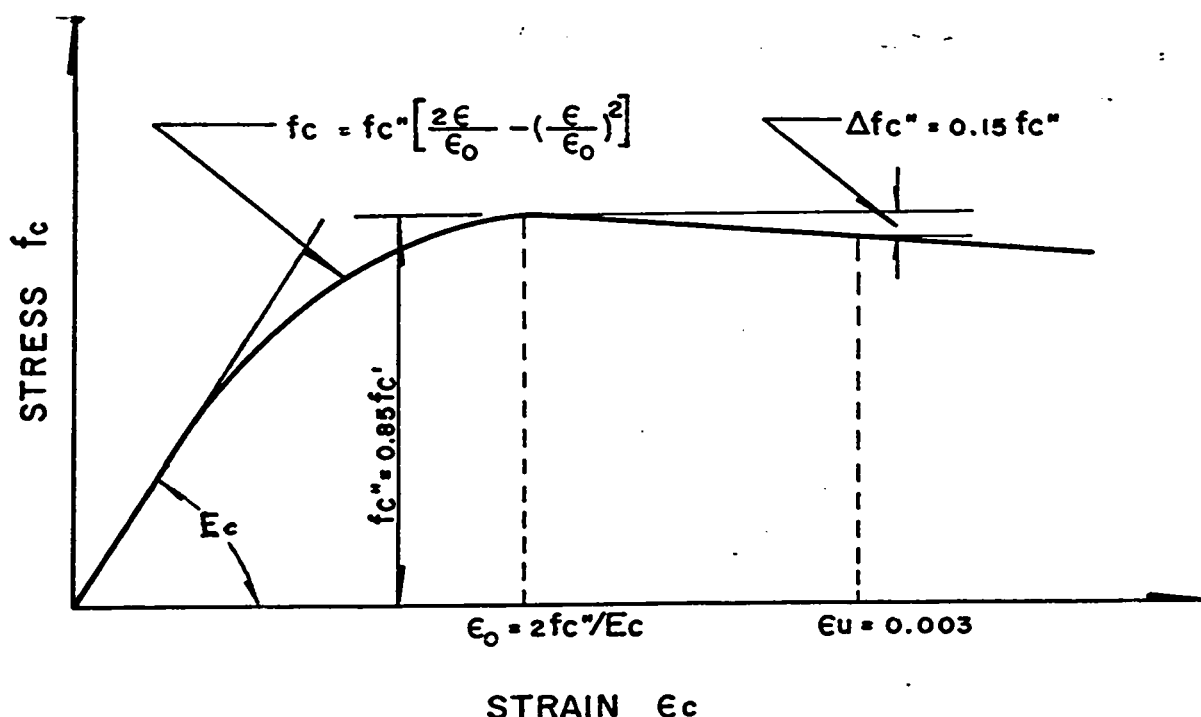
CHAPTER IV

THEORETICAL INVESTIGATION

4.1 Ultimate Strength Analysis of Reinforced Concrete Columns as for ACI 318 - 56 or NBC 1965 or CSA - A23.3 - 1970

The ultimate strength analysis of reinforced concrete member presented herein is based on the following assumptions:-

- (1) A linear distribution of strain over the cross section is assumed at all load levels.
- (2) The stress-strain relationship assumed for concrete in compression used in the strain-gradient method is that derived by Hognestad as shown in Fig. (4.1.a).



**FIG.(4.1.a) ASSUMED STRESS — STRAIN RELATIONSHIPS
FOR CONCRETE IN COMPRESSION**

The initial modulus of elasticity, E_c is assumed in accord with Inge Lyse's Equation.

$$E_c = 1,800,000 + 460f_c' \quad \dots\dots\dots(4.1.1)$$

- (3) It is assumed in the present analysis that no tension stresses can be taken by the concrete.
- (4) It is assumed that no general slip occurs between concrete and reinforcing steel, although local slip must be present at tension cracks in the concrete.
- (5) In the ACI Design Formulae, the maximum strain at the extreme compression fiber at ultimate strength shall be assumed equal to 0.003.
- (6) At ultimate strength the concrete stress is not proportional to strain. The variation of compressive concrete stress can be assumed to be uniform at an intensity of $0.85f_c'$ over an equivalent compression zone bounded by the edge of the cross section and a straight line located parallel to the neutral axis at a distance $a = K_1c$ from the fiber of maximum compressive strain. The distance "c" from the fiber of maximum strain to the neutral axis is measured in a direction perpendicular to that axis. The fraction " K_1 " shall be taken as 0.85 for $f_c' \leq 4000$ psi., and decreased by 0.05 for every 1000 psi. above 4000.

In the above assumptions, Nos. 1, 3 and 4 are common for both the ACI and strain gradient method.

All the theoretical calculations presented use the final eccentricity of the columns taken from the experimental results.

4.1.1 Reinforced Concrete Columns under Balanced Failure

The balanced condition is the loading condition which produces at ultimate strength, simultaneously, a strain of 0.003 in the extreme fiber of concrete and yield strain in the tension steel, and assume that the compression steel reaches its yield point stress, from the strain relationship is shown in Fig. (4.1.1.a) :-

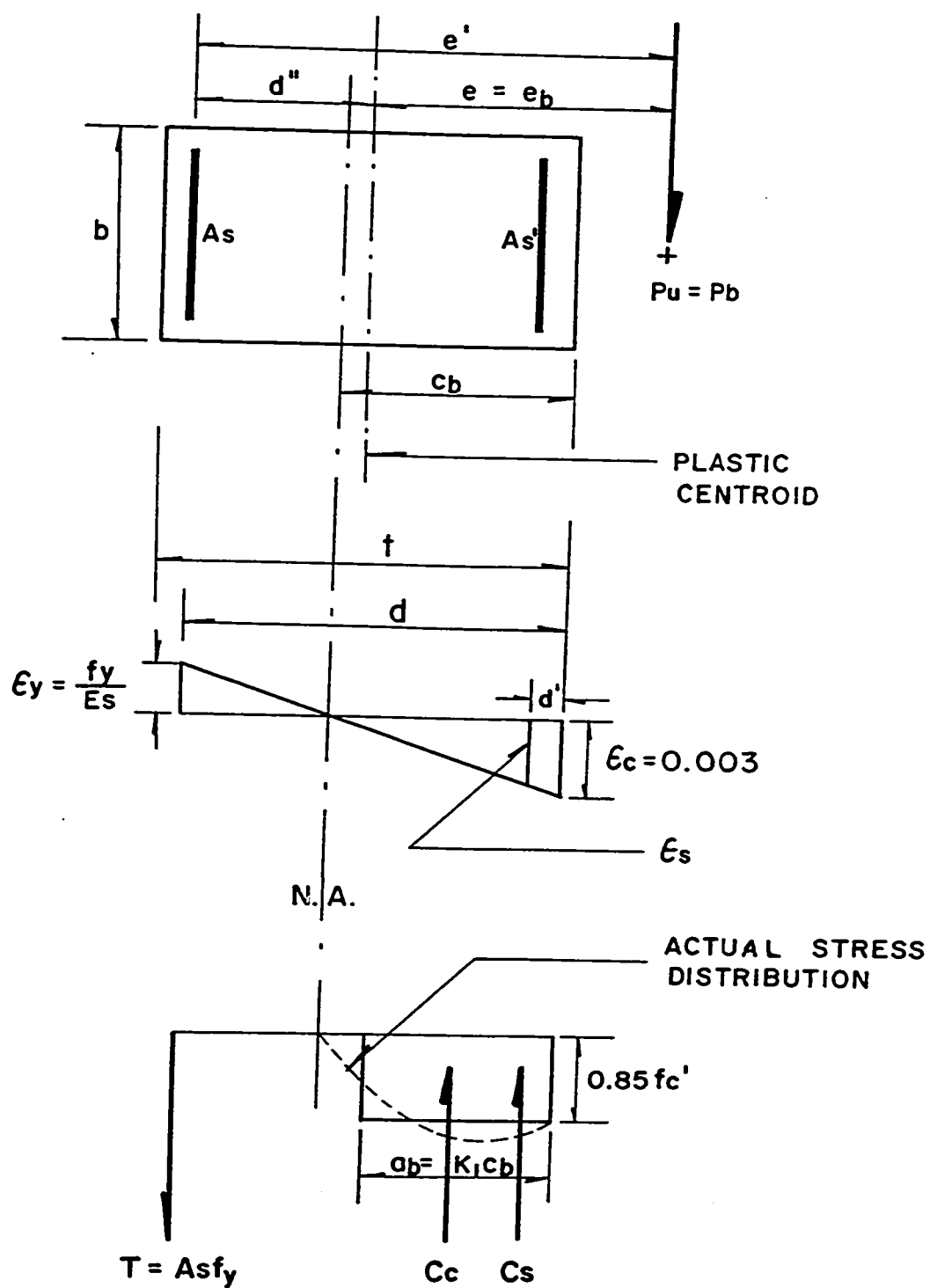


FIG. (4.1.1.a) BALANCED CONDITION—
RECTANGULAR SECTION

$$\frac{c_b}{d} = \frac{\epsilon_u}{f_y / E_s + 0.003}$$

$$c_b = \frac{0.003}{f_y / (29 \times 10^6) + 0.003} \cdot d$$

$$= \frac{87,000d}{f_y + 87,000} \dots\dots\dots(4.1.1.a)$$

$$\begin{aligned} C_c &= 0.85f_c' K_1 c_b b \\ &= 0.85f_c' a_b b \dots\dots\dots(4.1.1.b) \end{aligned}$$

$$C_s = A_s' (f_y - 0.85f_c') \dots\dots\dots(4.1.1.c)$$

$$T = A_s f_y \dots\dots\dots(4.1.1.d)$$

Force equilibrium requires.

$$\begin{aligned} P_b &= C_c + C_s - T \\ &= 0.85f_c' K_1 b c_b + A_s' (f_y - 0.85f_c') - A_s f_y \dots\dots\dots(4.1.1.e) \end{aligned}$$

Taking moments about the plastic centroid,

$$P_e e_b = C_c (d - a_b/2 - d'') + C_s (d - d' - d'') + T d''$$

$$\begin{aligned} P_e e_b = M_b &= 0.85f_c' K_1 c_b \cdot b \cdot (d - 0.5K_1 c_b - d'') + A_s' (f_y - 0.85f_c') \cdot \\ &\quad (d - d' - d'') + (A_s f_y) d'' \dots\dots\dots(4.1.1.f) \end{aligned}$$

4.1.2 Reinforced Concrete Columns under Compression Failure

When the ultimate eccentric load P_u' exceeds the balanced value P_b or when the eccentricity e is less than the balanced values e_b , the capacity of the section is controlled by compression failure of the concrete. In this case, the tensile force T will be based on a stress less than the yield strength and may actually be compressive.

Using the principles of statics, considering the actual strain variation as the unknown, the ultimate eccentric load can be found. However, two approximate procedures are provided in the ACI Code to simplify the calculations.

In one approach, P_u' is assumed to decrease linearly from P_o to P_b as the moment increases from zero to $M_b = P_b \cdot e_b$, as shown in an interaction diagram of Fig. (4.1.2.a).

$$\frac{P_o - P_u'}{P_o - P_b} = \frac{P_u' e}{P_b e_b} \dots\dots\dots(4.1.2.a)$$

Solving Equation (4.1.2.a) for P_u' gives,

$$P_u' = \frac{P_o}{1 + \left[\left(P_o / P_b \right) - 1 \right] e / e_b} \dots\dots\dots(4.1.2.b)$$

or

$$P_u' = P_o - (P_o - P_b) \frac{M_u'}{M_b} \dots\dots\dots(4.1.2.c)$$

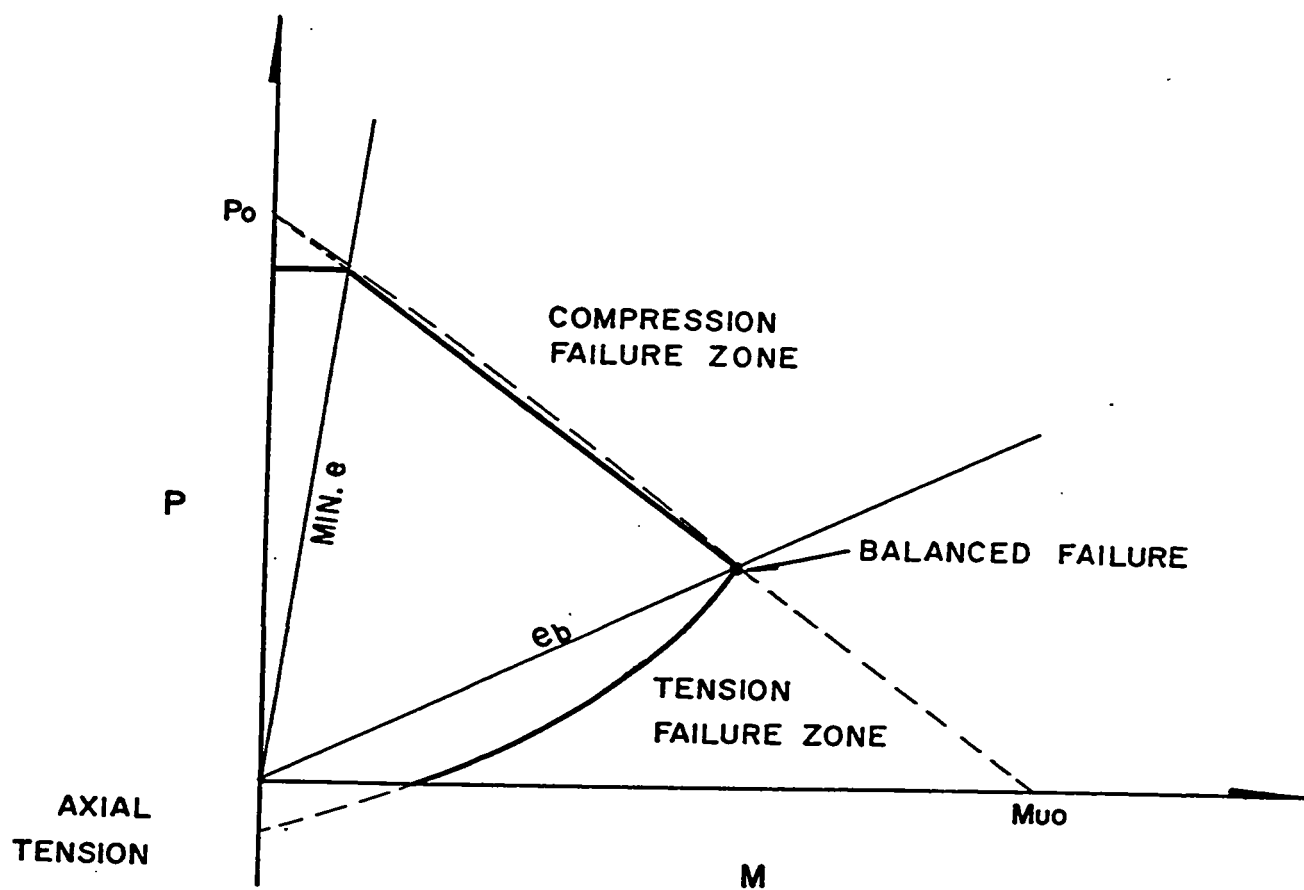


FIG.(4.1.2.a) INTERACTION DIAGRAM, COMPRESSION PLUS BENDING, ULTIMATE STRENGTH P_0 AND M_{uo}

A second approximate procedure may be applied for symmetrical reinforcement in terms of the column dimensions. Considering the moment equilibrium with respect to the tension steel, in Fig. (4.1.2.b) gives,

$$P_u' \left(e + \frac{d - d'}{2} \right) = C_c \left(d - \frac{a}{2} \right) + C_s (d - d') \quad \dots\dots\dots(4.1.2.a)$$

where

$$C_c = 0.85 f_c' b a \quad \dots\dots\dots(4.1.2.b)$$

When compression controls, compression steel yields at ultimate strength, thus

$$C_s = A_s' f_y \quad \dots\dots\dots(4.1.2.c)$$

Substituting Equations (4.1.2.b) and (4.1.2.c) in Equation (4.1.2.a) gives,

$$P_u' = \frac{\frac{3te}{d^2} \frac{f_c' b t}{3 \left(\frac{d-d'}{2} \right) t}}{\frac{3te}{d^2} \frac{f_c' b t}{3 \left(\frac{d-d'}{2} \right) t}} + \frac{A_s' f_y}{\left(\frac{e}{d-d'} + \frac{1}{2} \right)} \quad \dots\dots\dots(4.1.2.d)$$

With the boundry condition of $P_u' = P_o$ and $e = 0$, then

$$P_o = 0.85 f_c' b t + 2 A_s' f_y \quad \dots\dots\dots(4.1.2.e)$$

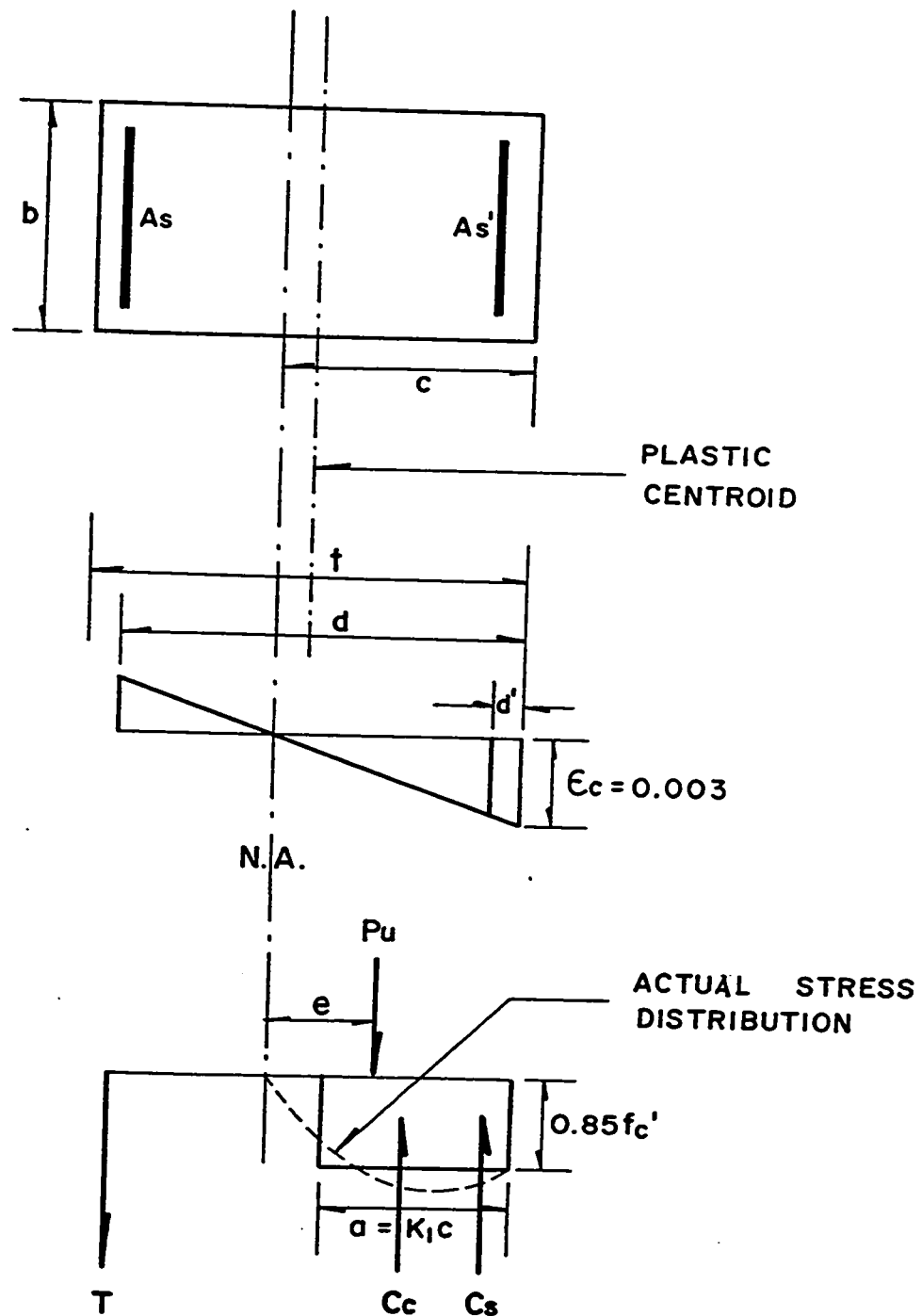


FIG.(4.1.2.b) ULTIMATE CAPACITY CONTROLLED BY COMPRESSION—RECTANGULAR SECTION

Substituting the above boundary condition in Equation (4.1.2.d), and neglecting the displacement of concrete by compression steel, it becomes,

$$P_u' = \frac{b t f_c'}{\left(\frac{3te}{d^2} + 1.18 \right)} + \frac{A_s' f_y}{\left(\frac{e}{d - d'} + 0.5 \right)}$$

.....(4.1.2.f)

It is to be observed that the direct use of statics gives probably the more accurate answer; and the result will be the same if the reduction for concrete displaced by compression steel is being applied. The first approach of straight line interaction gives the most conservative value.

4.1.3 Reinforced Concrete Columns under Tension Failure

When the ultimate capacity P_u' is less than the balanced value P_b or the eccentricity e is greater than the balanced value e_b , the capacity of the section is controlled by tension in the steel. A tension failure is initiated by yielding of the tension steel, this produces a movement of the neutral axis towards the compression zone. In this situation the ultimate strain in the tension steel will be greater than the yield strain. Referring to Fig. (4.1.3.a) and assuming that the strain in the compression steel is greater than the yield strain, one can obtain,

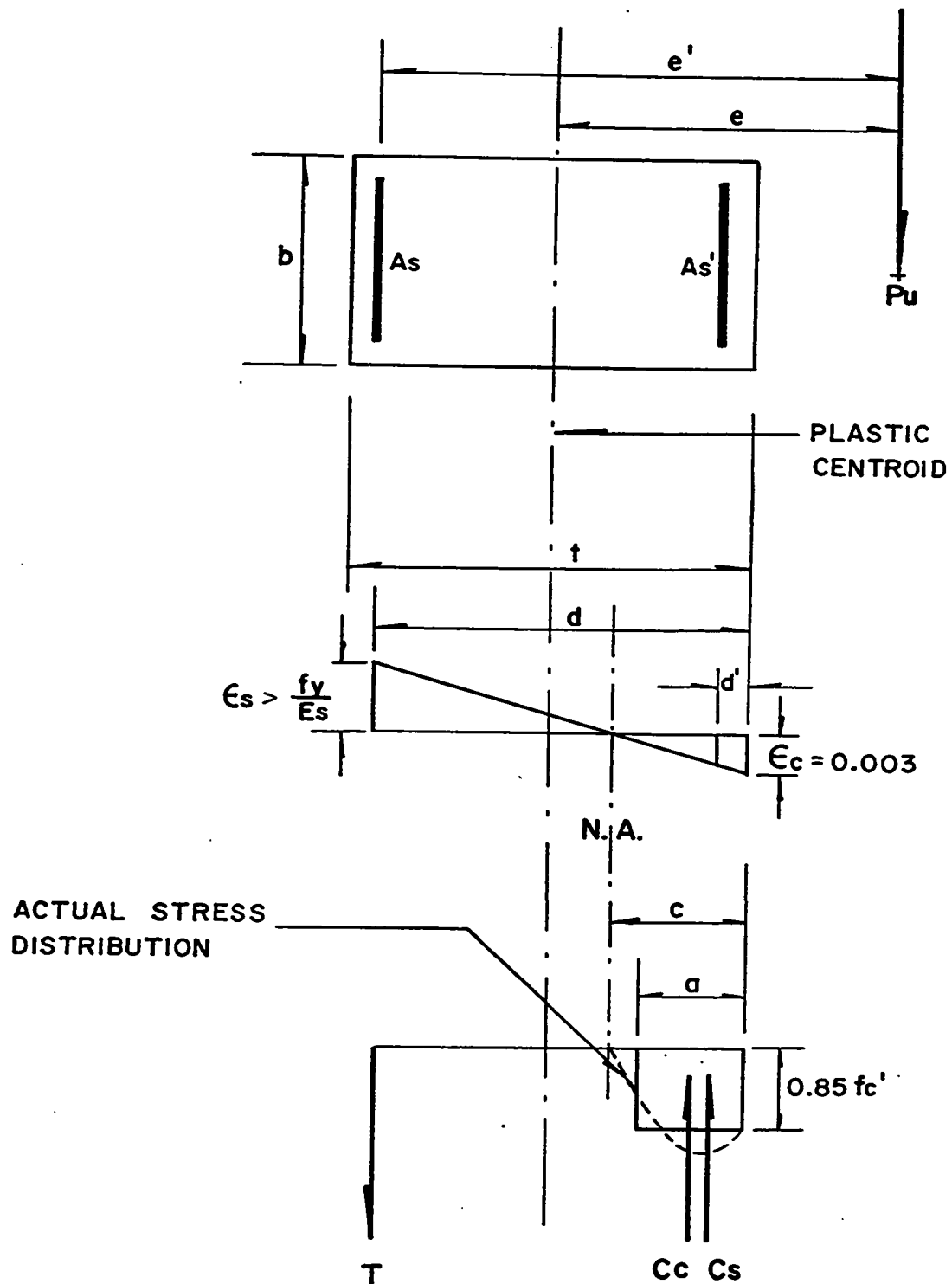


FIG. (4.1.3.a) ULTIMATE CAPACITY CONTROLLED
BY TENSION — RECTANGULAR SECTION

$$T = A_s f_y$$

$$C_s = A_s' (f_y - 0.85f_c')$$

$$C_c = 0.85f_c' K_1 c b$$

From force equilibrium,

$$P_u' = 0.85f_c' K_1 c b + A_s' (f_y - 0.85f_c') - A_s f_y \quad \dots\dots\dots (4.1.3.a)$$

Let

$$\left. \begin{aligned} p &= \frac{A_s}{bd} \\ p' &= \frac{A_s'}{bd} \\ m &= \frac{f_y}{0.85f_c'} \end{aligned} \right\} \dots\dots\dots (4.1.3.b)$$

From moment equilibrium with respect to the tension steel,

$$P_u' e' = 0.85f_c' (K_1 c) b \left(d - \frac{K_1 c}{2} \right) + A_s' (f_y - 0.85f_c') (d - d') \quad \dots\dots\dots (4.1.3.c)$$

Substituting Equations (4.1.3.a) and (4.1.3.b) in Equation (4.1.3.c), and rearrange,

$$P'_u = 0.85f_c'bd \left\{ p'(m-1) - pm + (1 - e'/d) + \sqrt{(1 - e'/d)^2 + 2 \left[(e'/d)(pm - p'm + p') + p'(m-1)(1 - d'/d) \right]} \right\}$$

.....(4.1.3.d)

In our case, $A_s = A_s'$ and $p = p'$, Equation (4.1.3.d) reduced to

$$P'_u = 0.85f_c'bd \left\{ -p + 1 - e'/d + \sqrt{(1 - e'/d)^2 + 2p \left[(m-1)(1 - d'/d) + e'/d \right]} \right\}$$

.....(4.1.3.e)

4.1.4 Reinforced Concrete Beams with Equal Tension and Compression Steel

When eccentricity 'e' becomes very large so that bending is predominant, the effect of the direct compression caused by P'_u is small. The member acts as a beam column and can be analyzed as a flexural member when the tensile steel limits the strength. When e becomes infinite, it is approximately the case of pure moment resistance, with the moment capacity M_u^T in simple bending, and $P_u^T = 0$.

From force equilibrium as shown in Fig. (4.1.4.a) requires that

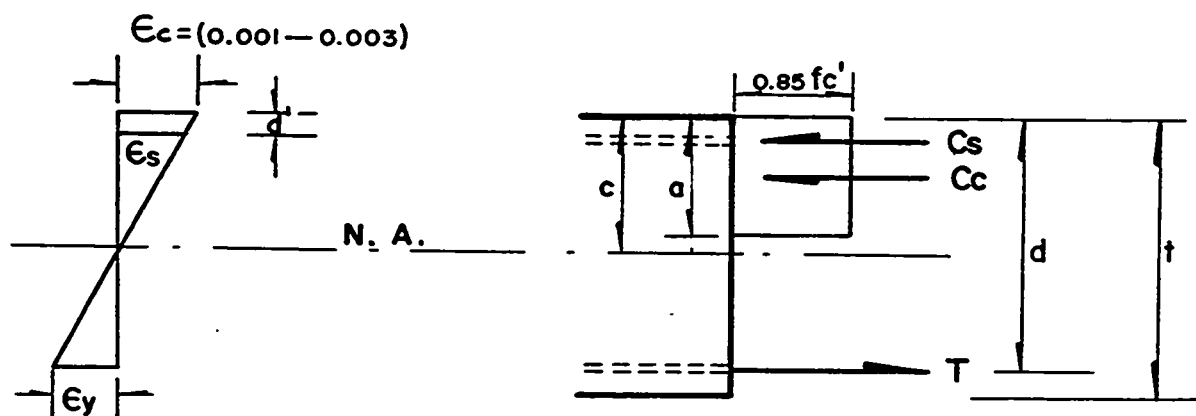


FIG. (4.1.4.a) ULTIMATE STRENGTH, FLEXURE
— RECTANGULAR SECTION

$$A_s f_y = A'_s f_s + 0.85 f'_c b a \quad \dots\dots\dots (4.1.4.a)$$

where

$$a = K_1 \cdot c$$

From strain diagram

$$\epsilon_s = \frac{\epsilon_c \times (c - d')}{c} \quad \dots\dots\dots (4.1.4.b)$$

where ϵ_c varied from 0.001 to 0.003

From stress-strain relationship of reinforcement, and a assigned value of ϵ_c (varied from 0.001 to 0.003), f_s can be determined.

Using Equations (4.1.4.a) and (b), and considering force equilibrium, c can be obtained by trial and error.

From moment equilibrium about the centroid of the tension steel, the moment is found to be :-

$$M_u^T = A_s' f_s (d - d') + 0.85 f_c' \times b \times K_1 c (d - K_1 c / 2) \dots\dots\dots(4.1.4.c)$$

The maximum load capacity of a structural member is attained when a mechanism forms. A two point loaded beam column collapses by forming a plastic hinge in the centre of the span, as shown in the Fig. (4.1.4.b).

Using either the equilibrium method or the mechanism method, the ultimate load P_u^T can be found,

$$P_u^T = \frac{6M_u^T}{L} \dots\dots\dots(4.1.4.h)$$

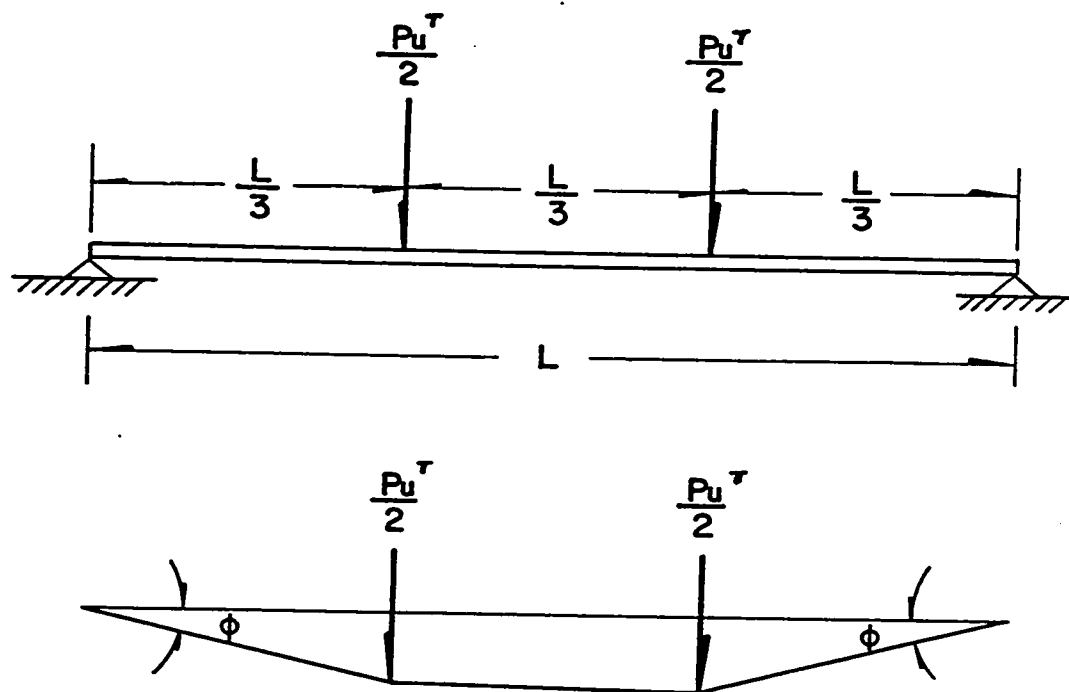


FIG. (4.1.4.b) COLLAPSE MECHANISM OF
A BEAM COLUMN

4.2 Strain Gradient Method

For cross sections consisting of linear materials, it is generally possible, and usually most convenient to write a single continuous expression relating axial load, moment and curvature. Unfortunately, the response of reinforced concrete to external effects is very difficult to evaluate, due to the nonlinear behavior of the concrete. Instead, it is more convenient to define the relationship between P , M and ϕ by a number of discrete points—The strain-gradient method.

The determination of the relation between P , M and ϕ evolves directly from the application of ordinary principles of mechanics of materials. If a linear distribution of strain across the section is assumed and the stress-strain relationships known or assumed for both concrete and steel, the elemental force on each element of material can be determined. By algebraically summing these elemental forces, the external moment and force necessary are found.

The method assumes a linear strain distribution over the depth of the section is as shown in Fig. (4.2). A generalized stress-strain characteristic for concrete, was derived by Hognestad as given in Sec. 4.1. The characteristics for the steel were taken directly from the experimental results. From the stress-strain relationships for the concrete and the reinforcement, the distribution of stresses over the cross-section can be determined as shown in Fig. (4.2.c) and (d).

From these stresses, the external force and moment necessary to keep the member in equilibrium is found. For instance, the external moment is the algebraic sum of the external applied moment plus the axial load deformation moment. The moment curvature relationship will be obtained by varying the strain gradient and keeping the axial load equal to the desired value by moving the position of the neutral axis.

It is obvious that, the moment capacity which is available to carry external applied moment is equal to the member moment capacity less the moment capacity required by the axial load-lateral deformation moment.

Strain-Gradient Method :-

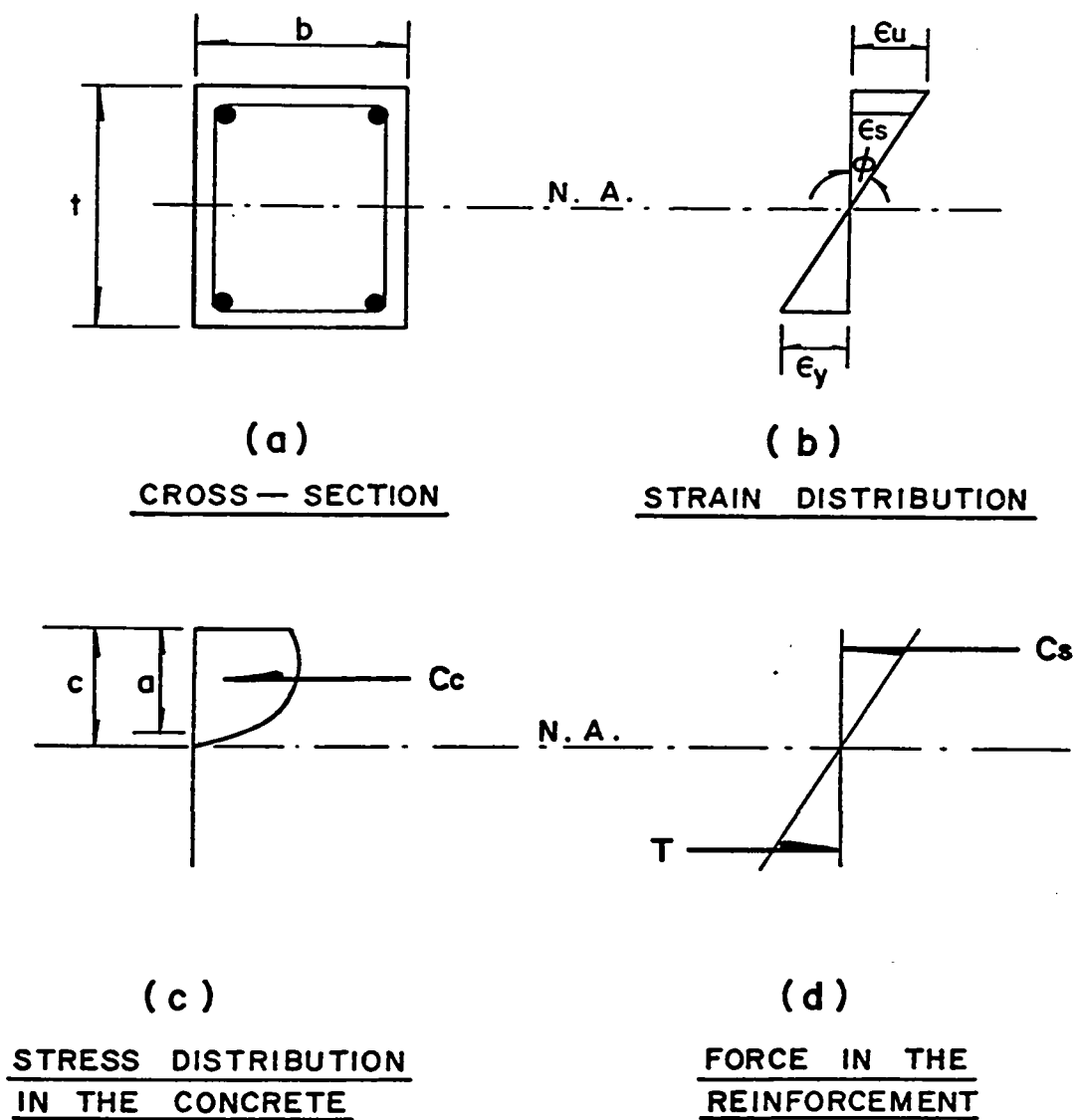


FIG.(4.2.) METHOD OF ANALYSIS

The method is iterative, it is convenient to list the steps as follows:-

- (1) Specify an axial load.
- (2) Select a strain gradient.
- (3) Select the position of the neutral axis.
- (4) Determine the curvature by computing the angle between the line representing the strain distribution and the assumed position of the neutral axis.
- (5) Sum algebraically the stress times the area over which they act to obtain the calculated axial load under equilibrium. If the calculated axial load and the chosen load agree with 1% — go to next step. If not, repeat step (3).
- (6) Sum the bending moment caused by the normal stress to obtain the resulting bending moment.
- (7) Increase strain gradient and obtain the maximum external moment until the maximum strain in the concrete reaches 0.003.
- (8) Write out the calculated results of maximum axial load and maximum external moment for that curvature.
- (9) Assign a new value of axial load until all experimental results have been considered.

If a sufficient number of discrete points relating P , M and ϕ are determined, the load-moment-curvature relationship for the section will be described.

4.2.1 Reinforced Concrete Columns under Balanced Failure

As shown in Fig. (4.2.1.1), the balance point 'b' is a common point to both the failure and yield interaction diagram, simultaneously, a strain of 0.003 in the extreme fibre of concrete and the yield strain in the tension steel. From the stress-strain diagram in Fig. (4.2.1.2.a) gives,

$$\begin{array}{lcl} T = A_s f_{s1} & \text{---} & \text{See notation} \\ C_c = 0.85 f_c' a b & \text{---} & \\ C_s = A_s f_{s2} & \text{---} & \end{array}$$

where

$$a = K_1 \cdot c$$

$$P_u' = C_c - T + C_s \dots\dots\dots(4.2.1.1)$$

$$\begin{aligned} M_u' &= C_c (t/2 - a/2) + T (t/2 - d') \\ &+ C_s (t/2 - d') \dots\dots\dots(4.2.1.2) \end{aligned}$$

Solve Equations (4.2.1.1) and (4.2.1.2) by trial and error method.

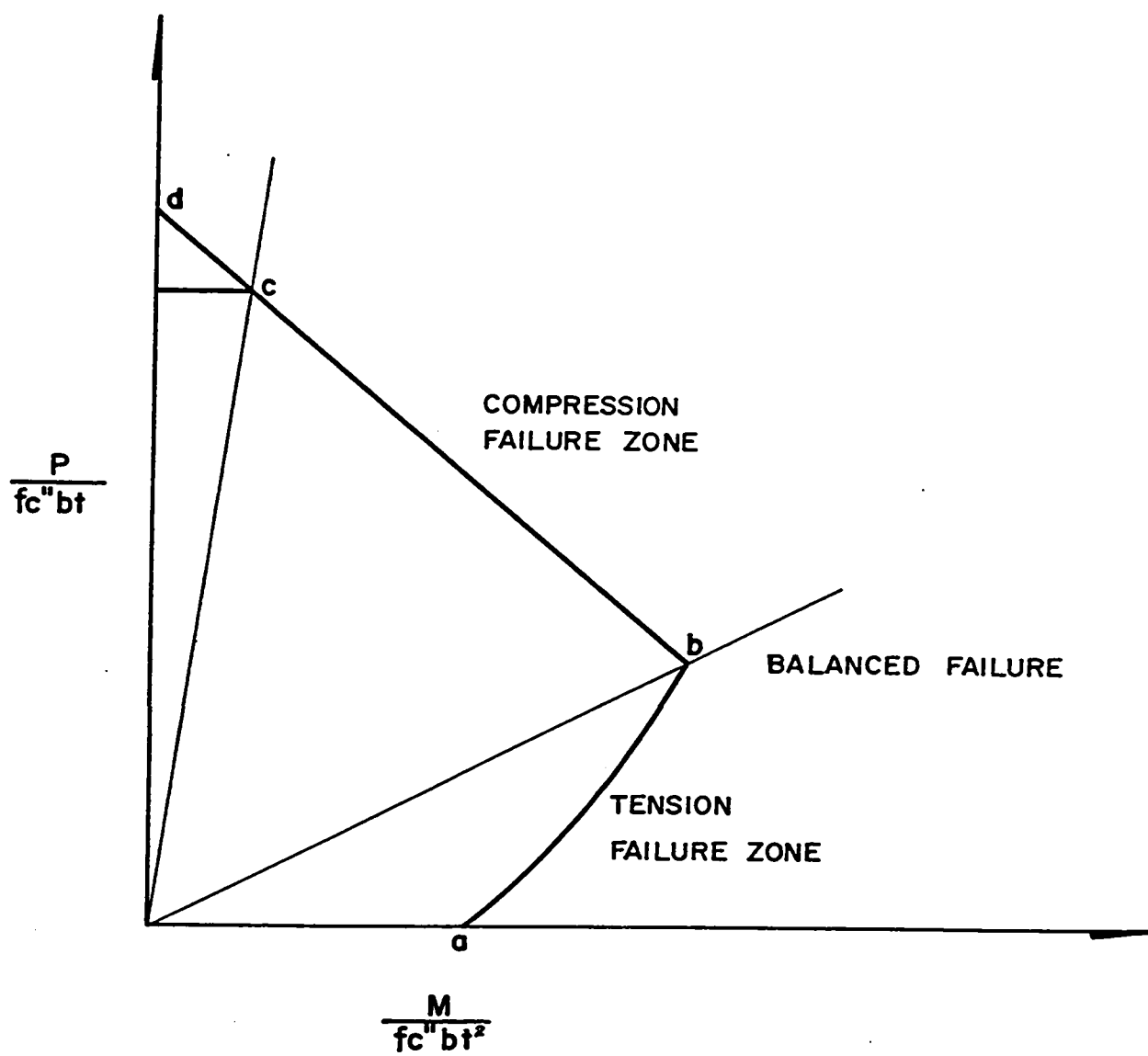


FIG.(4.2.1.1) INTERACTION DIAGRAM

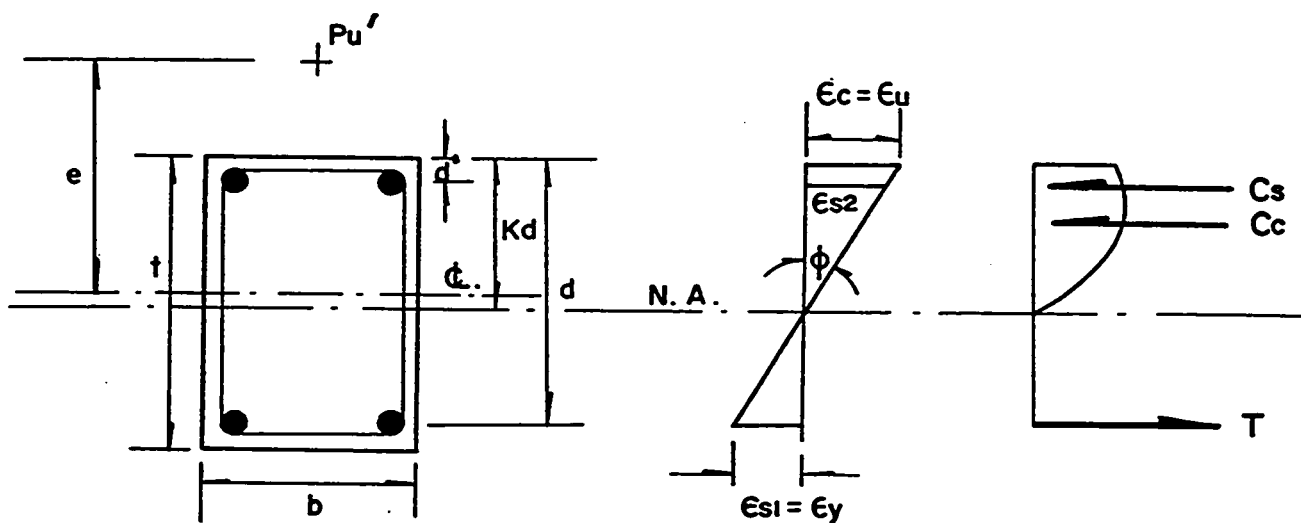


FIG. (4.2.1.2.a) POINT b (FIG. 4.2.1.1.)

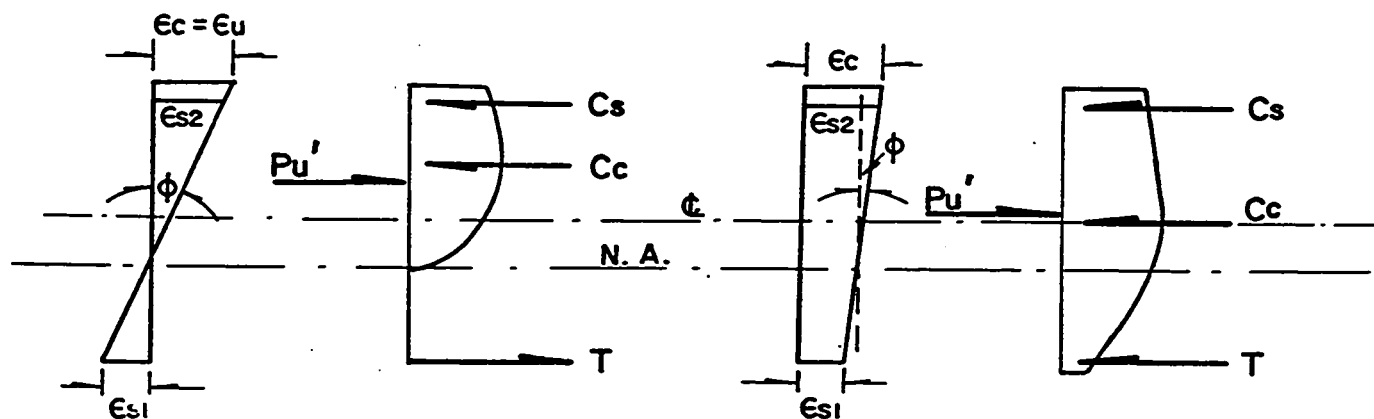


FIG. (4.2.1.2.b) LINE b—c

FIG.(4.2.1.2.c) LINE c—d

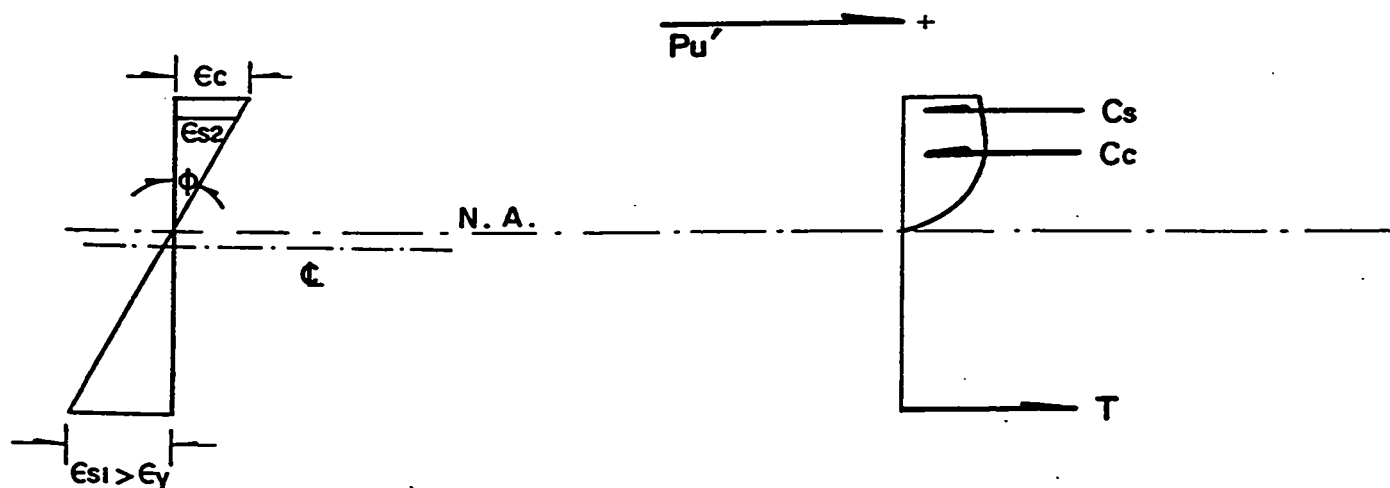


FIG.(4.2.1.2.d) LINE a—b

FIG.(4.2.1.2) STRESS — STRAIN RELATIONSHIP

4.2.2 Reinforced Concrete Columns under Compression Failure

For moderate eccentricities, when compression governs, the behavior of the column is represented by the line b-c in the interaction diagram in Fig. (4.2.1.1).

From the stress-strain diagram in Fig. (4.2.1.2.b) the equilibrium equation applies

$$P_u' = C_c - T + C_s \dots\dots\dots(4.2.1.3)$$

$$M_u' = C_c (t/2 - a/2) + C_s (t/2 - d') + T (t/2 - d') \dots\dots\dots(4.2.1.4)$$

For zero or small eccentricities, as the interval line d-c, in the interaction diagram, and considering equilibrium as in Fig. (4.2.1.2.c), gives

$$P_u' = C_c + C_s + T \dots\dots\dots(4.2.1.5)$$

$$M_u' = C_c (t/2 - c/2) - T (t/2 - d') + C_s (t/2 - d') \dots\dots\dots(4.2.1.6)$$

4.2.3 Reinforced Concrete Beam-columns under Tension Failure

For large eccentricities, when tension governs, as represented by the curve line a-b in the interaction diagram, and from the Fig. (4.2.1.2.d), we have,

$$P_u' = C_c + C_s - T \quad \dots\dots\dots(4.2.1.7)$$

$$M_u' = C_c (t/2 - a/2) + T (t/2 - d') + C_s (t/2 - d') \quad \dots\dots\dots(4.2.1.8)$$

When eccentricity become infinity, as the point a in the interaction diagram, we have,

$$T = C_c + C_s \quad \dots\dots\dots(4.2.1.9)$$

$$M_u^r = C_c (d - a/2) + C_s (d - d') \quad \dots\dots\dots(4.2.1.1.0)$$

and

$$P_u^r = \frac{6M_u^r}{L}$$

where L = span length (53")

CHAPTER V

DISCUSSION AND SUMMARY OF TEST RESULTS

5.1 General Behaviour and Modes of Failure of Specimens

All specimens with the eccentricity equal to 0.00 ± 0.01 ; 0.10 ± 0.01 ; 0.30 ± 0.01 ; 0.50 ± 0.01 and 0.75 ± 0.01 , failed in two modes: compression failure and tension failure. Only a few columns had the combination of the variables that the mode of failure was close enough to the balanced conditions.

All those specimens, which were designed as beam-columns, with the eccentricity equal to infinity failed by tension failure.

The observed modes of failure of the individual test columns are listed in Table (5.1), page 53, and the various typical failure phenomena are shown in Figs. (5.1.1) to (5.1.4), on page 56 to page 59. The load-deflection curves for each member are presented in Appendix B, from Fig. (B.1) page 112 to Fig. (B.85) page 196, the maximum load in each curve indicates the failure load for the member.

FIG. (5.1) TEST RESULT OF GROUP I, II, III, & IV

SPECIMEN NO.	f'_c (psi.)	REINFORCEMENT		e/t	$P_{exp.}$ (Kips)	MODE OF FAILURE*
		TYPE	f_y (psi.)			
1A-1 1A-2 1A-3 1A-4 1A-5	_____	Deformed	_____	0.0	_____	_____
1B-1 1B-2 1B-3 1B-4 1B-5	_____	Deformed	_____	0.1	_____	_____
1C-1 1C-2 1C-3 1C-4 1C-5	3,500 c.of v. (3.5%)	Deformed	67,400	0.3	112.00 110.00 110.50 112.12 115.00	C C C C C
1D-1 1D-2 1D-3 1D-4 1D-5	3,500 c.of v. (3.5%)	Deformed	67,400	0.5	74.50 72.50 68.00 69.00 70.00	C C C C C
1E-1 1E-2 1E-3 1E-4 1E-5	4,300 c.of v. (7.5%)	Deformed	67,400	0	25.00 24.94 25.00 22.00 24.98	T T T T T
2B-1 2B-2 2B-3 2B-4 2B-5	_____	_____	_____	0.1	_____	_____
2C-1 2C-2 2C-3 2C-4 2C-5	4,600 c.of v. (5.0%)	Deformed	52,600	0.3	100.00 104.00 112.50 105.50 106.50	C C C C C

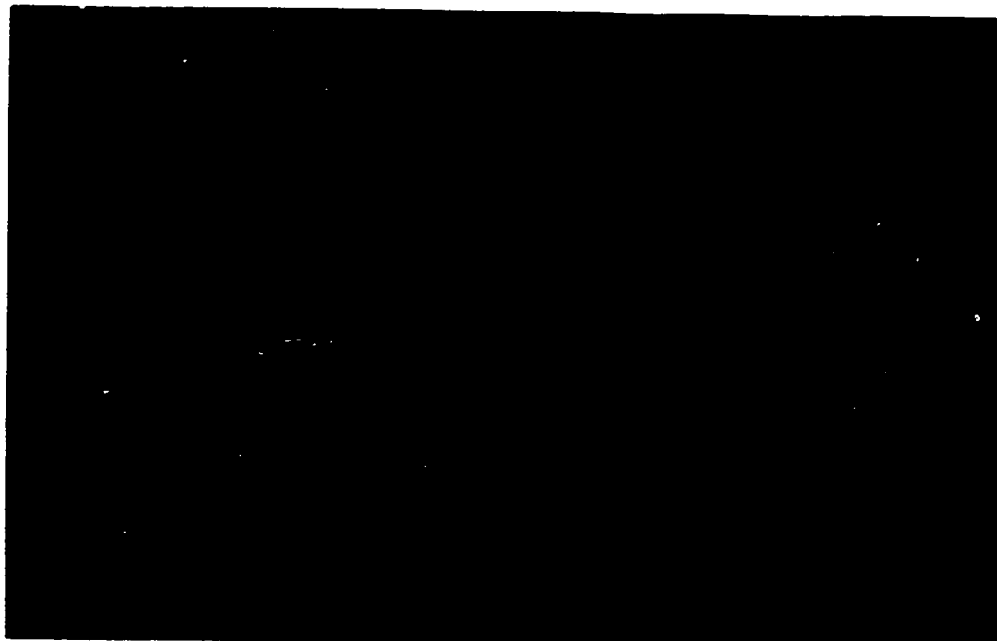
Continued

SPECIMEN NO.	f'_c (psi.)	REINFORCEMENT		e/t	$P_{exp.}$ (Kips)	MODE OF FAILURE*
		TYPE	f_y (psi.)			
2D-1 2D-2 2D-3 2D-4 2D-5	4,000 c.of v. (5.1%)	Deformed	52,600	0.5	59.85 61.77 57.43 61.90 61.10	C C C C C
2DE-1 2DE-2 2DE-3 2DE-4 2DE-5	4,600 c.of v. (5.0%)	Deformed	52,600	0.75	45.10 47.70 45.45 45.50 43.50	T T T T T
2E-1 2E-2 2E-3 2E-4 2E-5	4,000 c.of v. (5.1%)	Deformed	52,600	∞	18.00 19.60 19.96 19.30 19.70	T T T T T
3A-1 3A-2 3A-3 3A-4 3A-5	4,000 c.of v. (4.6%)	Deformed	65,500	0.0	165.00 156.50 164.60 162.00 171.00	C C C C C
3B-1 3B-2 3B-3 3B-4 3B-5	3,700 c.of v. (5.3%)	Deformed	65,500	0.1	103.10 103.90 104.00 113.00 111.00	C C C C C
3C-1 3C-2 3C-3 3C-4 3C-5	3,700 c.of v. (5.3%)	Deformed	65,500	0.3	71.33 69.00 75.82 67.50 67.50	C C C C C
3D-1 3D-2 3D-3 3D-4 3D-5	3,500 c.of v. (4.3%)	Deformed	65,500	0.5	42.66 41.71 43.98 43.15 41.55	C & T C & T C & T C & T C & T
3E-1 3E-2 3E-3 3E-4 3E-5	3,500 c.of v. (4.3%)	Deformed	65,500	∞	11.50 11.74 12.39 11.92 11.37	T T T T T

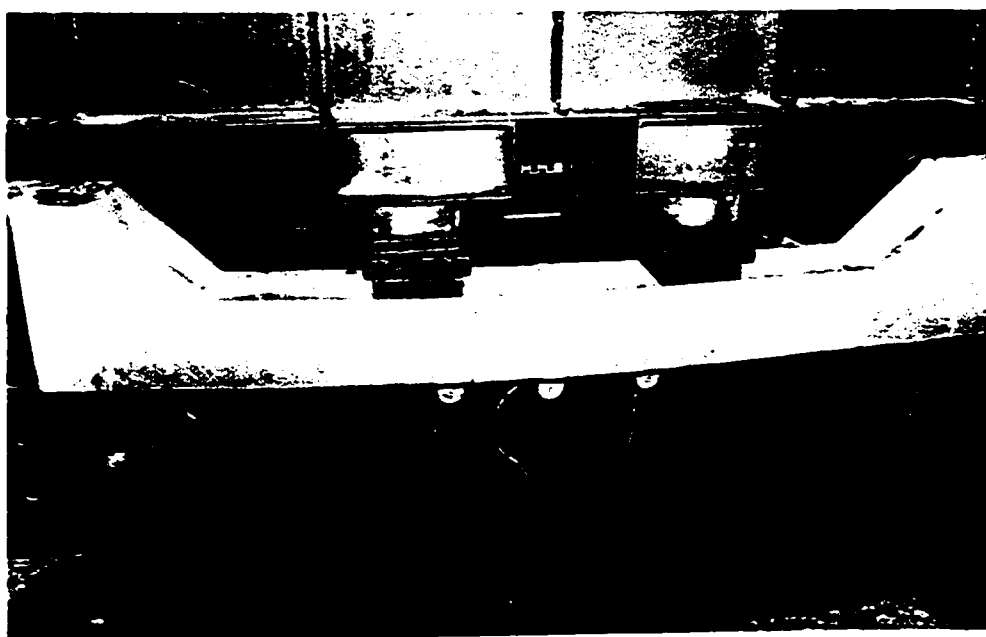
Continued

SPECIMEN NO.	f'_c (psi.)	REINFORCEMENT		e/t	$P_{exp.}$ (Kips)	MODE OF FAILURE*
		TYPE	f_y (psi.)			
4B-1 4B-2 4B-3 4B-4 4B-5	3,200 c.of v. (7.3%)	Plain	54,400	0.1	88.25 80.92 82.11 88.51 82.01	C C C C C
4C-1 4C-2 4C-3 4C-4 4C-5	4,600 c.of v. (7.9%)	Plain	54,400	0.3	61.00 65.30 55.00 60.00 67.00	C & T C & T C & T C & T C & T
4D-1 4D-2 4D-3 4D-4 4D-5	4,900 c.of v. (6.0%)	Plain	54,400	0.5	30.50 30.74 30.50 30.57 31.77	T T T T T
4DE-1 4DE-2 4DE-3 4DE-4 4DE-5	4,100 c.of v. (4.9%)	Plain	54,400	0.75	18.63 16.30 18.35 17.85 21.95	T T T T T
4E-1 4E-2 4E-3 4E-4 4E-5	4,900 c.of v. (6.0%)	Plain	74,500	∞	8.32 7.85 8.55 8.15 8.54	T T T T T

* C, T and C & T indicate compression, tension and near balance failure, respectively.

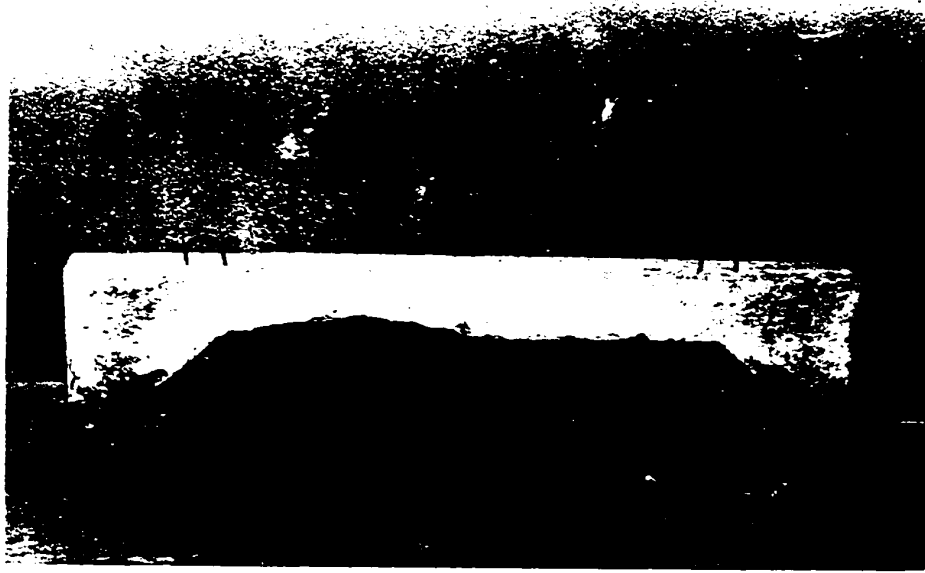


COMPRESSION FAILURE

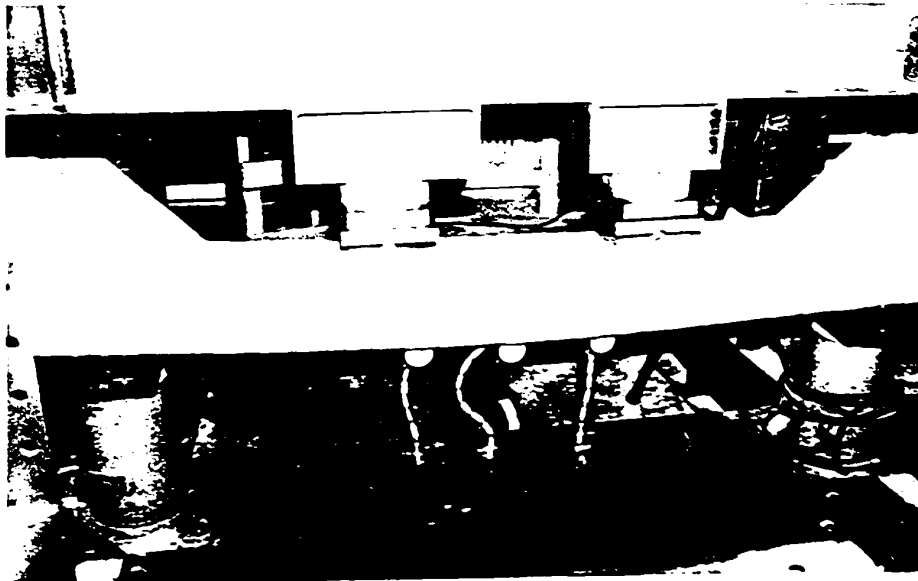


TENSION FAILURE

**FIG.(5.1.1) TYPICAL FAILURE PHENOMENA
OF GROUP I**

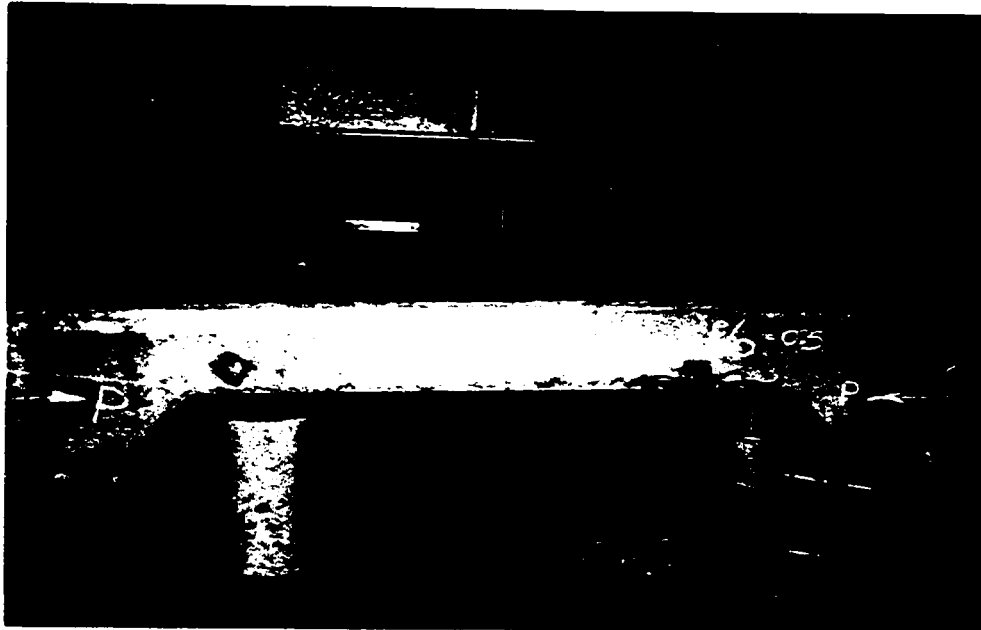


COMPRESSION FAILURE

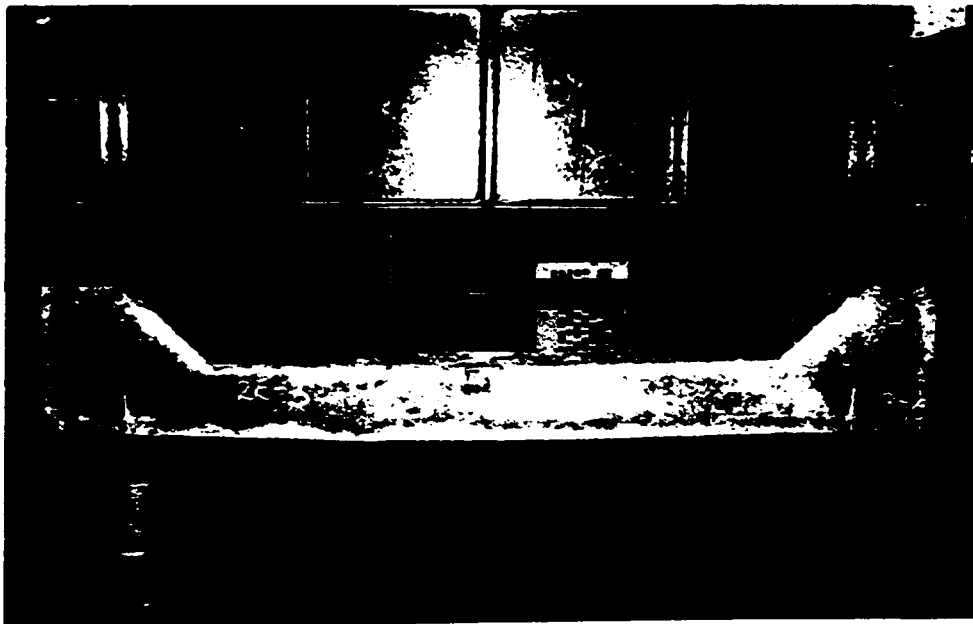


TENSION FAILURE

FIG.(5.1.1) TYPICAL FAILURE PHENOMENA
OF GROUP I

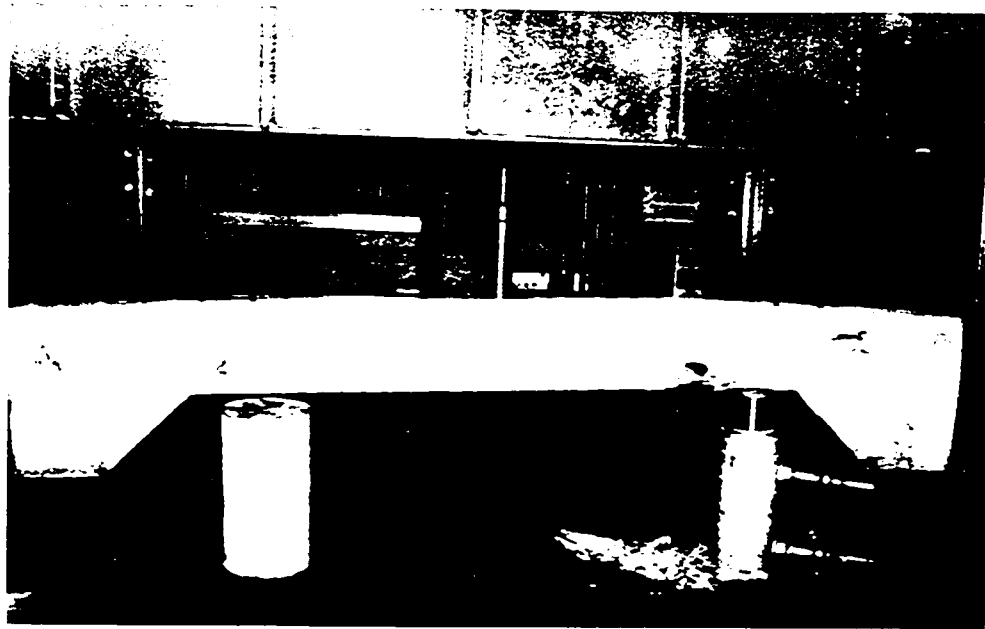


COMPRESSION FAILURE

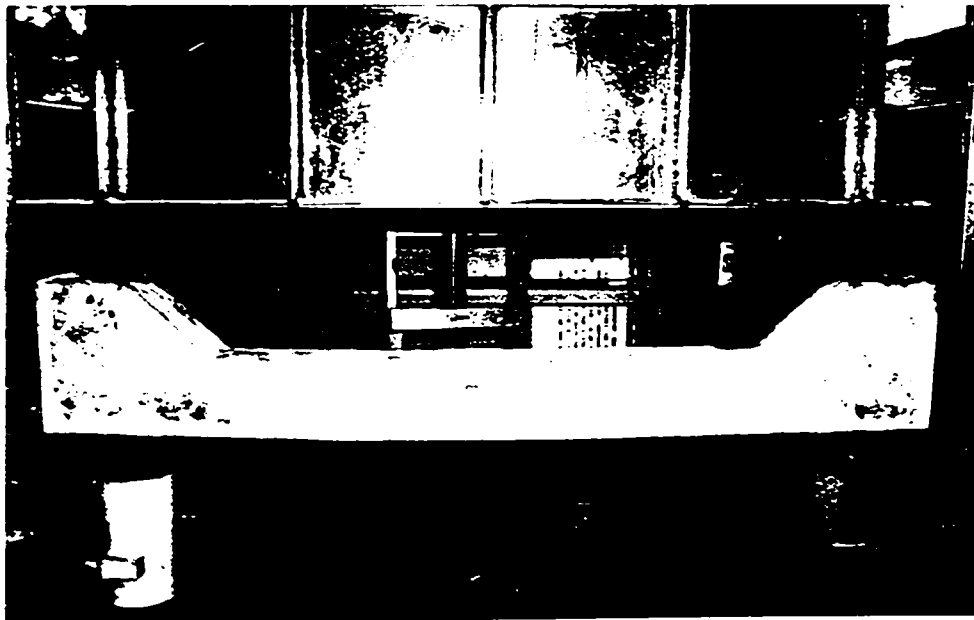


TENSION FAILURE

**FIG.(5.1.2) TYPICAL FAILURE PHENOMENA
OF GROUP 2**



COMPRESSION FAILURE

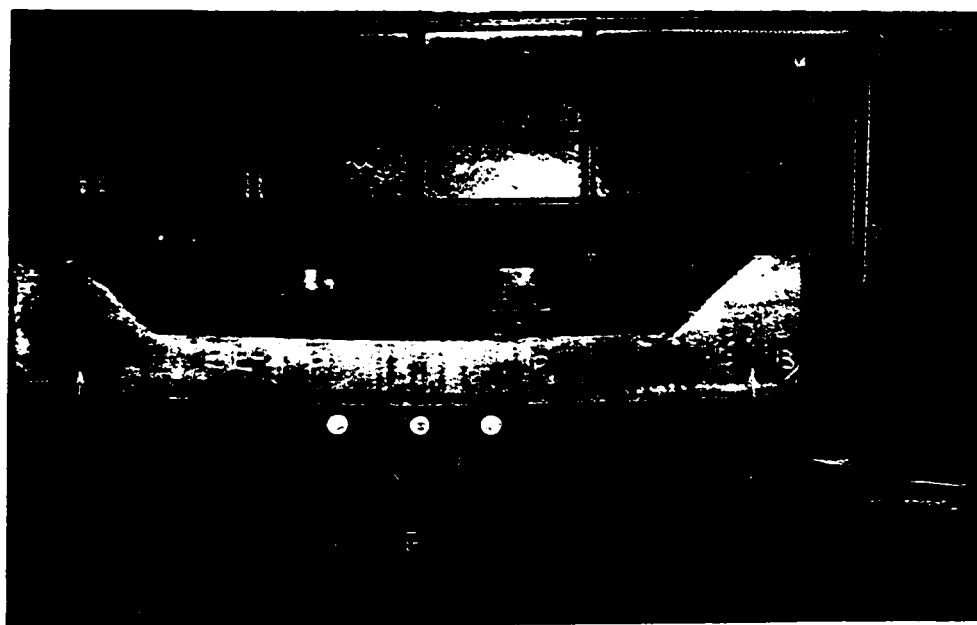


TENSION FAILURE

FIG.(5.1.2) TYPICAL FAILURE PHENOMENA
OF GROUP 2

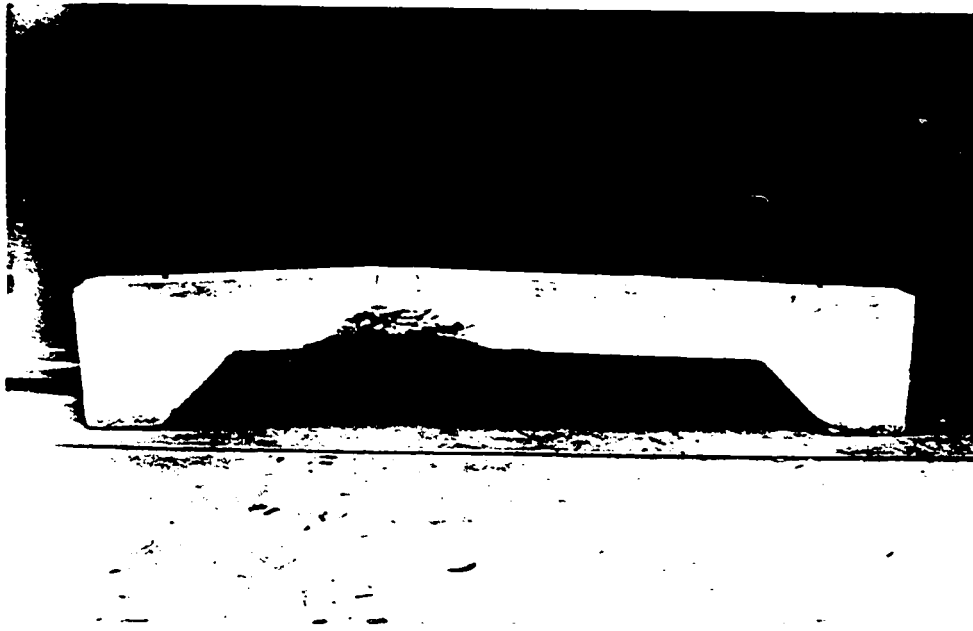


COMPRESSION FAILURE

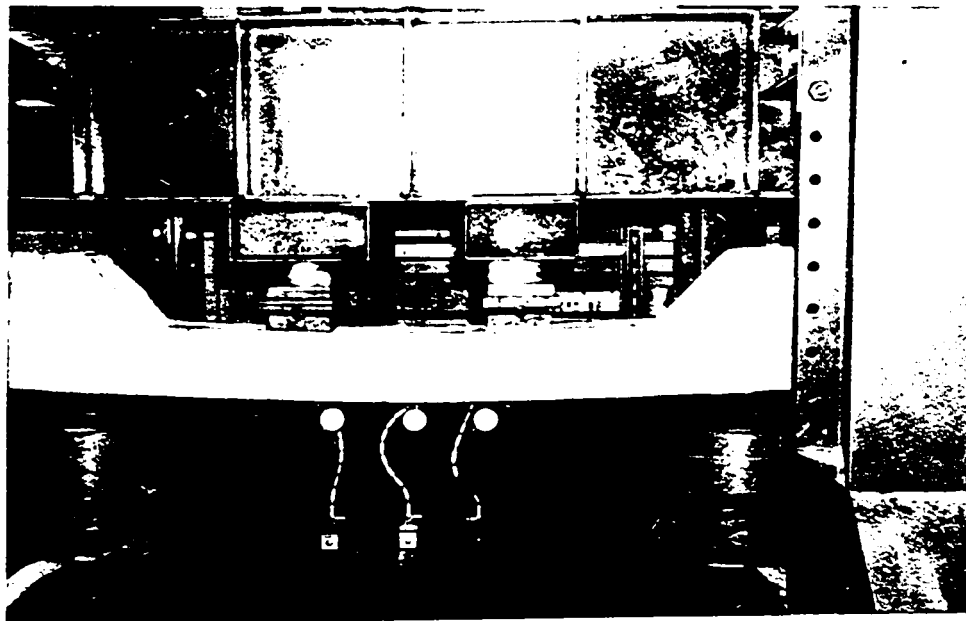


TENSION FAILURE

**FIG.(5.1.3) TYPICAL FAILURE PHENOMENA
OF GROUP 3**



COMPRESSION FAILURE

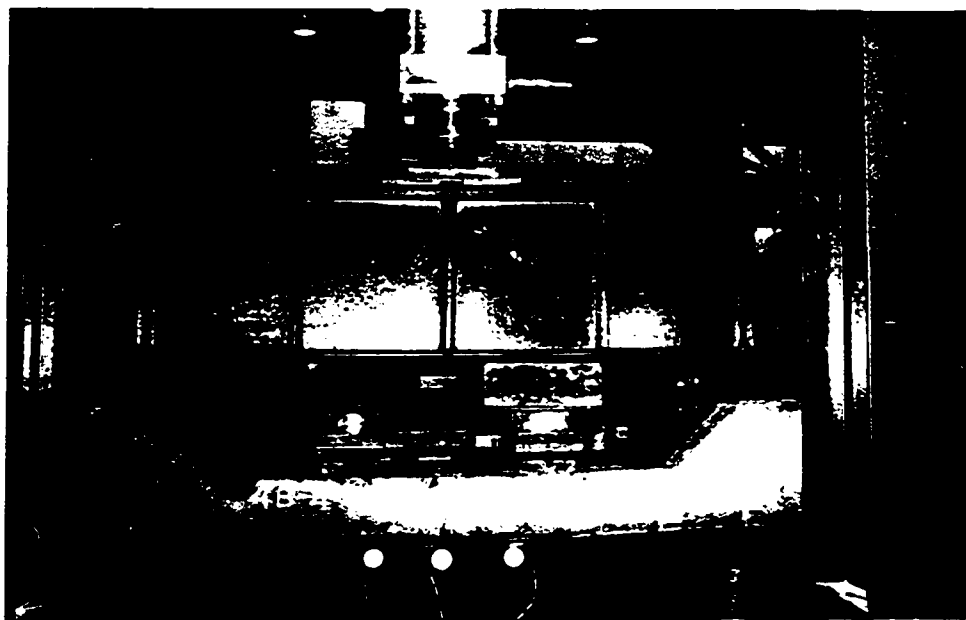


TENSION FAILURE

FIG.(5.1.3) TYPICAL FAILURE PHENOMENA
OF GROUP 3

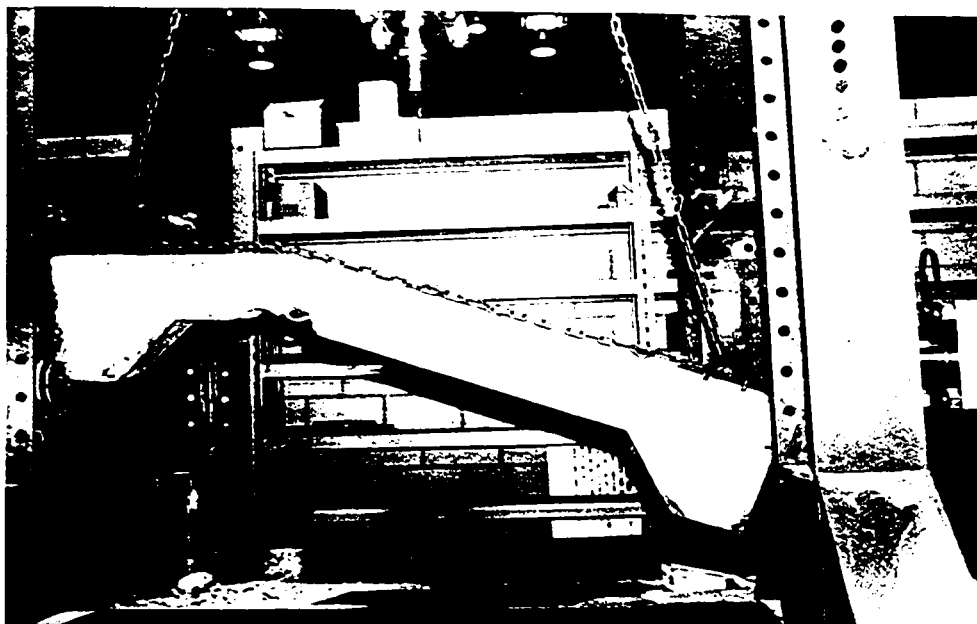


COMPRESSION FAILURE

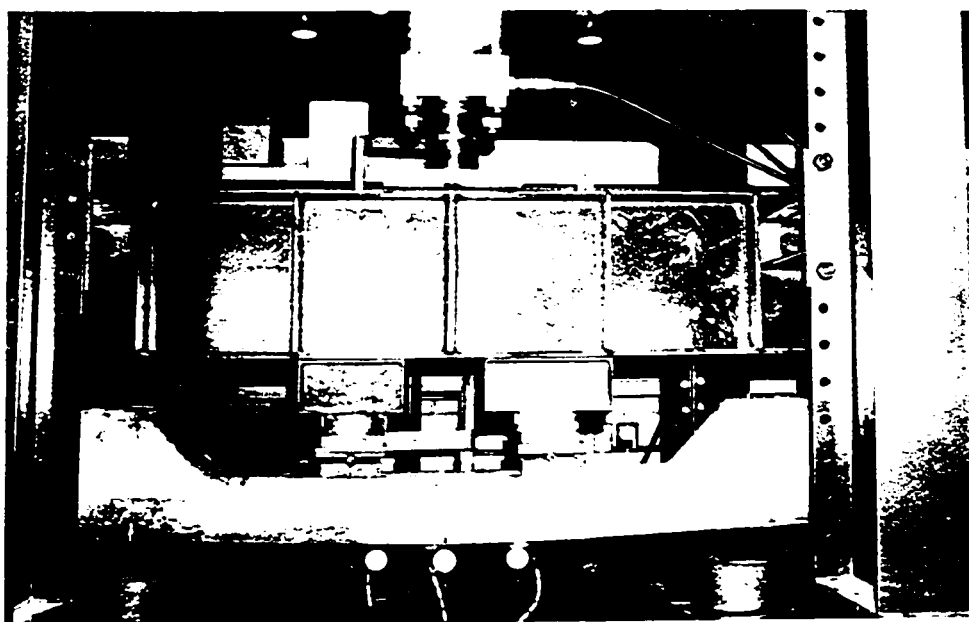


TENSION FAILURE

**FIG.(5.1.4) TYPICAL FAILURE PHENOMENA
OF GROUP 4**



COMPRESSION FAILURE



TENSION FAILURE

FIG.(5.1.4) TYPICAL FAILURE PHENOMENA
OF GROUP 4

5.2 Analysis of Experimental Results

Among the 100 specimens tested, 15 failed in shear through the capitals of the specimens. The reason was insufficient embedded bar reinforcement to reinforce the capitals against shear failure. To solve this problem, four auxiliary tension rods were used to clamp plates around each capital through the rest of the tests.

The results of the remaining 85 tests are believed to be significant, and yet are subjected to various errors as indicated below.

- (1) Small initial eccentricities due to errors in centering and adjusting the specimens in the testing machines, and the nonhomogenous properties of concrete along the transverse direction.
- (2) The position of the reinforcing bars in the cross-section of the member may vary $\pm 1/8$ in. from the nominal values. This error is not important for compression failure members, but for tension failure members, it changes the internal moment arm and thus affects the ultimate loads.
- (3) The rate of applying load increments, and the total time that used for testing each column specimen can not be controlled to be identical.

Hence, it is possible but not too significant that the ultimate loads, may have been influenced by the uncontrolled time factors.

5.3 Analysis of Theoretical Results

Both the results of the American Concrete Institute design equations and the strain gradient method were obtained by

computer programs. Table (5.3) on page 62 shows the results obtained from the ACI Design Formulae, and from the strain gradient method.

5.4 Comparision of Test Results with the ACI (American Concrete Institute) Design Formulae

The test results show general agreement with the design formulae for the full range of conditions from axially-loaded columns to beam-columns under pure bending. The actual strengths for all specimens under all cases are slightly larger than the calculated values as shown in Table (5.4) on page 64. The arithmetic mean of the ratios of measured to computed ultimate load is 1.04, the overall standard deviation is 0.09, and the overall coefficient of variation is 8.94 per cent.

Due to the known variations of concrete strength, steel yield stress and dimension, the measured results will not necessarily be equal to the theoretical results but still may be significant.

From the measured variabilities of the concrete cylinder stress (8%) and the steel yield stress (3%) and assuming a dimensional variability of 3% it can be shown that only deviations of measured to calculated loads greater than 6% are significant.

Hence within the accuracy of the experimental results the ACI design formulae accurately predicts the failure load of columns under combined axial load and moment.

TABLE (5.3) CALCULATED VALUES P_{ult} OF ACI DESIGNED FORMULARS
AND STRAIN GRADIENT METHOD (KIPS)

SPECIMEN NO.	e/t	P_{exp}	P_{ACI}	P_{SGM}^*	SPECIMEN NO.	e/t	P_{exp}	P_{ACI}	P_{SGM}^*
4E-1	∞	8.32	8.83	8.15	3D-1	0.5	42.66	44.11	4.164
4E-2		7.85	8.83	8.15	3D-2		41.71	38.79	41.64
4E-3		8.55	8.83	8.92	3D-3		43.98	43.49	43.95
4E-4		8.15	8.83	8.15	3D-4		43.15	43.92	43.38
4E-5		8.54	8.83	8.92	3D-5		41.55	43.83	41.64
4D-1	0.5	30.50	28.09	30.39	3C-1	0.3	71.33	59.32	71.44
4D-2		30.75	28.18	30.83	3C-2		69.00	58.24	68.75
4D-3		30.50	25.90	30.39	3C-3		75.82	63.82	75.13
4D-4		30.57	28.73	30.83	3C-4		67.50	60.79	67.08
4D-5		31.77	26.95	31.94	3C-5		67.50	61.05	66.93
4C-1	0.3	61.00	55.68	61.37	3B-1	0.1	103.10	98.56	102.92
4C-2		65.38	55.68	64.73	3B-2		103.90	98.45	102.92
4C-3		55.00	54.47	54.76	3B-3		104.00	98.64	103.53
4C-4		60.00	54.11	59.74	3B-4		113.00	98.79	112.33
4C-5		67.00	58.47	67.03	3B-5		111.00	98.84	110.34
4DE-1	0.75	18.63	15.84	19.50	3A-1	0.0	165.00	159.60	164.52
4DE-2		16.30	15.90	16.40	3A-2		165.00	159.60	156.44
4DE-3		18.35	15.90	18.34	3A-3		164.60	159.40	163.29
4DE-4		17.85	16.00	17.74	3A-4		162.00	159.70	159.31
4DE-5		21.95	16.16	22.03	3A-5		171.00	159.50	169.84
4B-1	0.1	88.25	69.54	87.96	2E-1	∞	18.00	19.47	17.11
4B-2		80.92	73.18	80.69	2E-2		19.60	19.59	19.61
4B-3		82.11	69.50	82.46	2E-3		19.96	19.59	19.59
4B-4		83.51	68.99	88.04	2E-4		19.30	19.59	19.92
4B-5		82.01	71.49	82.07	2E-5		19.69	20.15	19.57
3E-1	∞	11.50	11.77	12.01	2DE-1	0.75	45.10	46.23	45.21
3E-2		11.74	11.77	12.20	2DE-2		47.70	46.43	47.54
3E-3		12.39	12.11	12.78	2DE-3		45.45	45.25	45.79
3E-4		11.92	12.11	12.51	2DE-4		45.30	43.38	45.67
3E-5		11.37	11.77	11.89	2DE-5		43.50	44.88	45.18

Continued

SPECIMEN NO.	e/t	P _{exp}	P _{ACI}	P _{SGM} *	SPECIMEN NO.	e/t	P _{exp}	P _{ACI}	P _{SGM} *
2D-1	0.5	59.85	59.13	60.10	1D-1	0.5	74.50	74.07	73.94
2D-2		61.77	58.85	62.36	1D-2		72.50	72.96	72.02
2D-3		57.43	55.85	57.83	1D-3		68.00	71.98	68.10
2D-4		61.90	59.22	62.36	1D-4		69.00	73.42	68.59
2D-5		61.10	60.59	60.90	1D-5		70.00	72.61	69.51
2C-1	0.3	100.00	93.51	99.81	1C-1	0.3	112.00	110.67	110.98
2C-2		104.00	94.51	102.99	1C-2		111.00	114.20	110.83
2C-3		112.50	95.62	112.82	1C-3		110.50	106.93	110.83
2C-4		105.50	97.20	105.58	1C-4		112.12	106.69	115.65
2C-5		106.50	96.36	106.73	1C-5		115.00	104.91	114.44
1E-1	∞	25.00	27.62	25.63					
1E-2		24.94	27.62	25.45					
1E-3		25.00	27.62	26.12					
1E-4		22.00	27.28	21.71					
1E-5		24.98	27.62	25.76					

NOTE: Results obtained from ACI (American Concrete Institute) Design Formulae.

* Results obtained from Strain Gradient Method.

TABLE (5.4) COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS
BY ACI FORMULAE (KIPS)

SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$
4E-1	∞	8.32	8.83	0.94	3D-1	0.5	42.66	44.11	0.97
4E-2		7.85	8.83	0.89	3D-2		41.71	38.79	1.04
4E-3		8.55	8.83	0.97	3D-3		43.98	43.49	1.01
4E-4		8.15	8.83	0.92	3D-4		43.15	43.92	0.98
4E-5		8.54	8.83	0.97	3D-5		41.55	43.83	0.94
4DE-1	0.75	18.63	15.84	1.17	3C-1	0.3	71.33	59.32	1.20
4DE-2		16.30	15.90	1.02	3C-2		69.00	58.24	1.18
4DE-3		18.35	15.90	1.15	3C-3		75.82	63.82	1.18
4DE-4		17.85	16.00	1.11	3C-4		67.50	60.79	1.10
4DE-5		21.95	16.16	1.35	3C-5		67.50	61.05	1.10
4D-1	0.5	30.50	28.09	1.08	3B-1	0.1	103.10	98.56	1.04
4D-2		30.75	28.18	1.09	3B-2		103.90	98.45	1.05
4D-3		30.50	25.90	1.17	3B-3		104.00	98.64	1.05
4D-4		30.57	28.73	1.06	3B-4		113.00	107.60	1.05
4D-5		31.77	26.95	1.17	3B-5		111.00	104.30	1.05
4C-1	0.3	61.00	55.68	1.09	3A-1	0.0	165.00	159.60	1.04
4C-2		65.30	55.68	1.17	3A-2		156.60	159.60	0.99
4C-3		55.00	54.47	1.01	3A-3		164.60	159.40	1.04
4C-4		60.00	54.11	1.09	3A-4		162.00	159.70	1.02
4C-5		67.00	58.47	1.14	3A-5		171.00	159.50	1.00
4B-1	0.1	88.25	69.54	1.26	2E-1	∞	18.00	19.47	0.93
4B-2		80.92	73.18	1.10	2E-2		19.60	19.59	1.00
4B-3		82.11	69.50	1.18	2E-3		19.96	19.59	1.02
4B-4		88.51	68.99	1.28	2E-4		19.30	19.59	0.99
4B-5		82.01	71.49	1.14	2E-5		19.69	20.15	0.97
3E-1	∞	11.50	11.77	0.98	2DE-1	0.75	45.10	46.23	0.97
3E-2		11.74	11.77	1.00	2DE-2		47.70	46.43	1.02
3E-3		12.39	12.11	1.02	2DE-3		45.45	45.25	1.02
3E-4		11.92	12.11	0.98	2DE-4		45.30	43.38	1.04
3E-5		11.37	11.77	0.97	2DE-5		43.50	44.88	0.96

Continued

SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$
2D-1	0.5	59.85	59.13	1.01	1D-1	0.5	74.50	74.07	1.00
2D-2		61.77	58.85	1.04	1D-2		72.50	72.96	0.99
2D-3		57.43	55.85	1.03	1D-3		68.00	71.98	0.95
2D-4		61.90	59.22	1.04	1D-4		69.00	73.42	0.93
2D-5		61.10	60.59	1.01	1D-5		70.00	72.61	0.96
2C-1	0.3	100.00	93.51	1.06	1C-1	0.3	112.00	110.67	1.01
2C-2		104.00	94.51	1.10	1C-2		111.00	114.20	0.97
2C-3		112.50	95.62	1.08	1C-3		110.50	106.93	1.03
2C-4		105.50	97.20	1.08	1C-4		112.12	106.69	1.05
2C-5		106.50	96.36	1.10	1C-5		115.00	104.91	1.09
1E-1	∞	25.00	27.62	0.90					
1E-2		24.94	27.62	0.90					
1E-3		25.00	27.62	0.90					
1E-4		22.00	27.28	0.81					
1E-5		24.98	27.62	0.90					

Mean value of $P_{exp} / P_{cal} = 1.04$

Standard deviation = 0.09

Coefficient of variation = 8.94%

5.4.1 Effects of Concrete Quality; Strength of Reinforcement and Eccentricity

Figs. (5.4.1) to (5.4.3) show the curves of ratio of measured* to computed ultimate load against concrete strength, reinforcement ratio and eccentricity ratio, respectively. It may be observed that the ratio of measured to computed ultimate load varied about ± 10 percent from unity.

5.4.2 Interaction Diagram

Interaction diagrams are presented in Figs. (5.4.2.1) to (5.4.2.4), from page 68, to page 71; comparing the maximum measured load capacity of the specimens with the theoretical ultimate strengths. The material properties of the cross-section with a variation of concrete strength from 3400 to 4600 psi.; percentage of reinforcement varied from 1.27 (plain bar) to 7.14 percent; and steel yield strength from 43,000 to 67,400 psi.

For concentrically-loaded columns, small initial eccentricities due to errors in centering the specimens, as well as the inhomogeneous properties of the concrete along the lateral direction, will generally cause lateral deflections, and thus reduce the column strength. For eccentrically-loaded columns, the errors in entering may increase or reduce the column strength, depending on the direction of the error. The ultimate loads of concentrically-loaded columns are far more sensitive due to such centering errors, than those of eccentrically-loaded columns.

* each point represent the average ratio P_{exp}/P_{ACI} of each group.

○ — 3/8" ϕ , PLANE BAR
 ● — 4/8" ϕ , 6/8" ϕ AND
 7/8" ϕ DEFORMED BAR

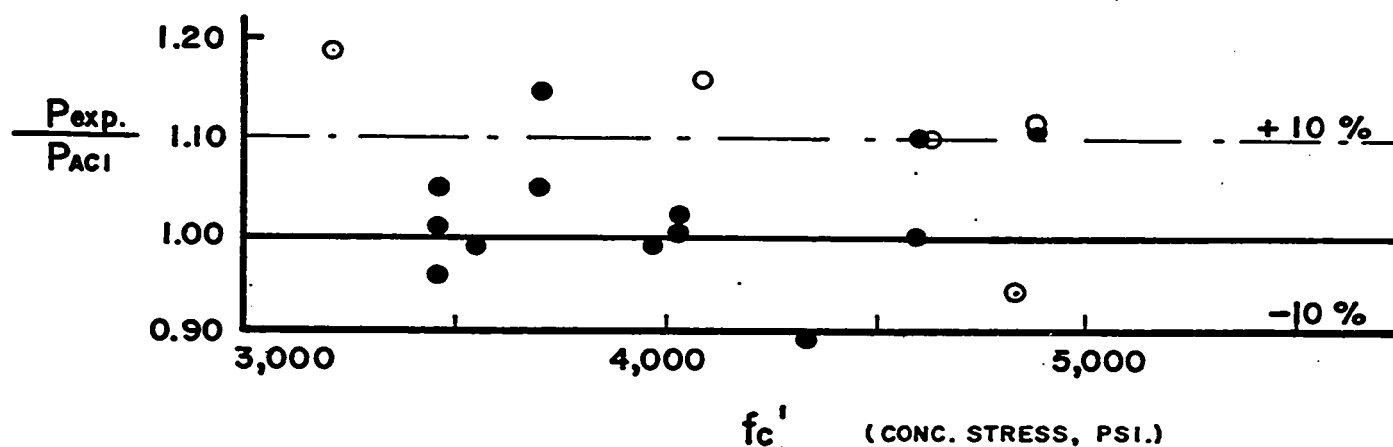


FIG.(5.4.1) $P_{exp.}/P_{cal.}$ VERSUS ULTIMATE CONCRETE STRESS

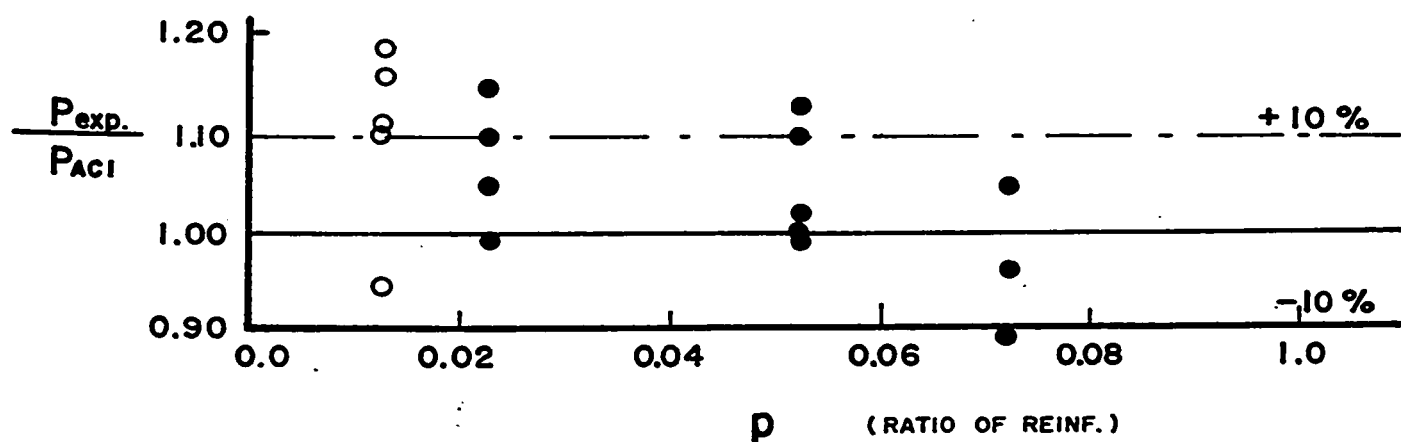


FIG.(5.4.2) $P_{exp.}/P_{cal.}$ VERSUS RATIO OF REINFORCEMENT

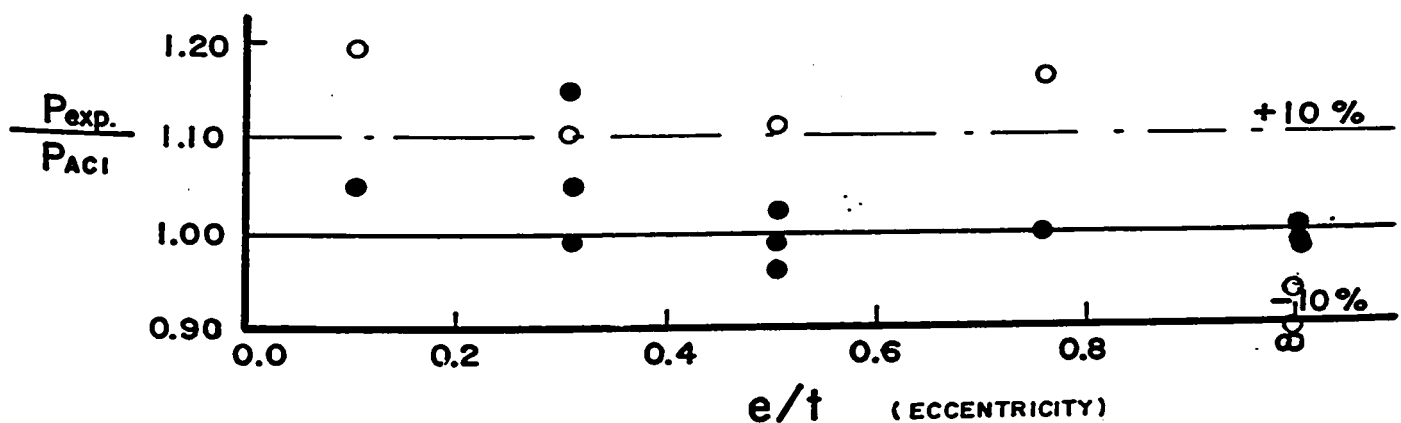


FIG.(5.4.3) $P_{exp.}/P_{cal.}$ VERSUS ECCENTRICITY RATIO OF LOAD (ACI DESIGN FORMULAE)

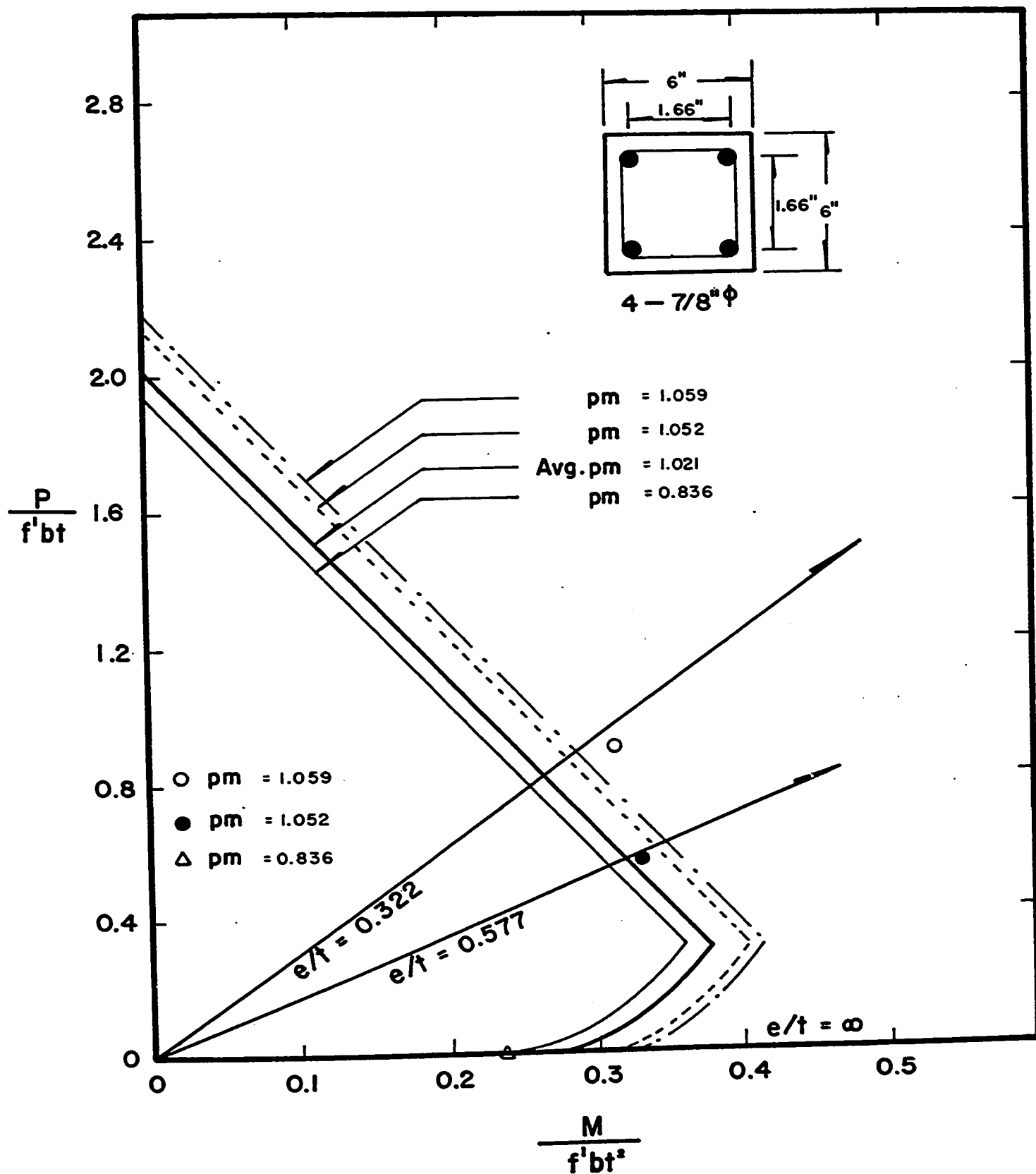
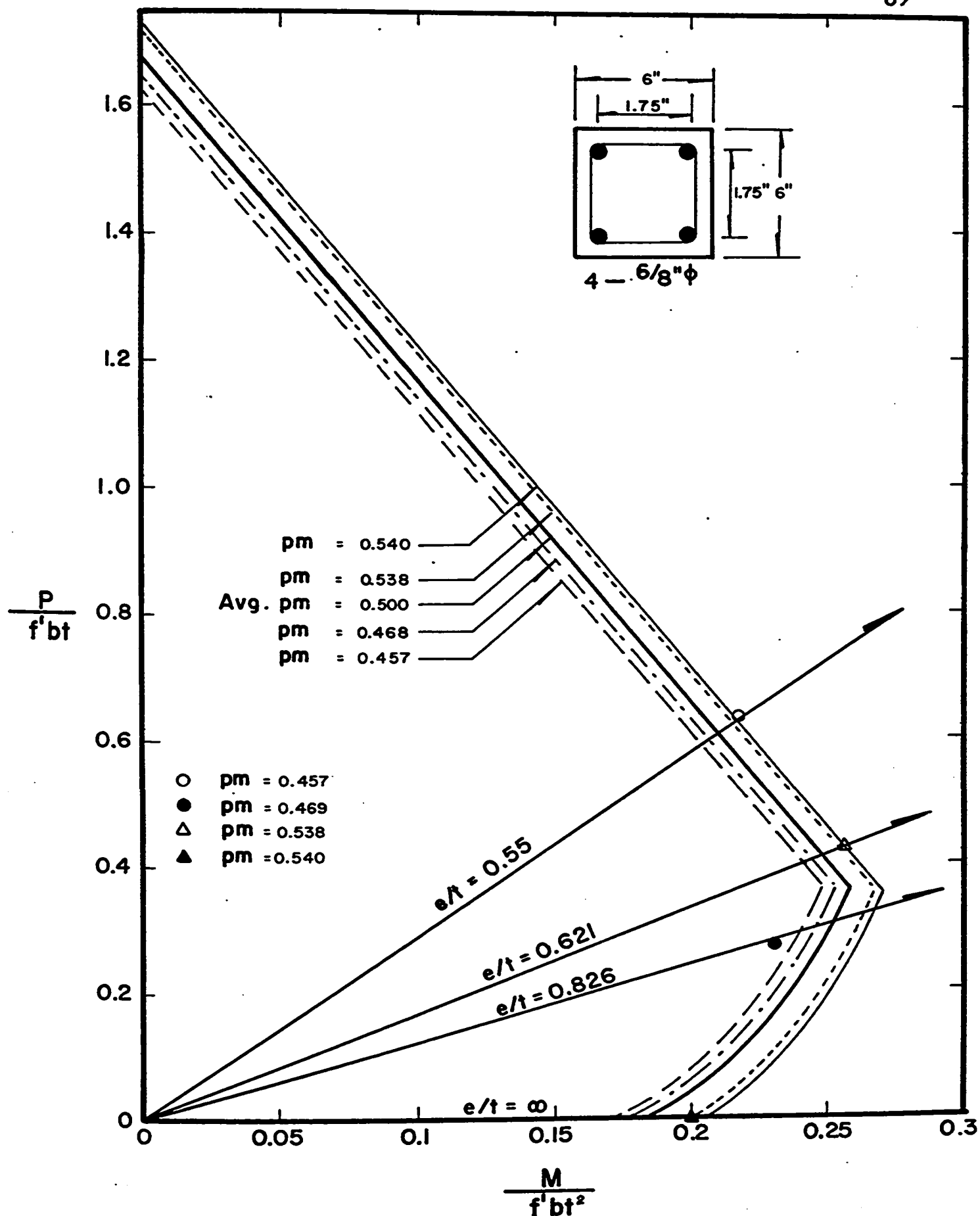


FIG.(5.4.2.1) INTERACTION DIAGRAM FOR GROUP I
(ACI DESIGN FORMULAE)



**FIG.(5.4.2.2) INTERACTION DIAGRAM
FOR GROUP 2
(ACI DESIGN FORMULAE)**

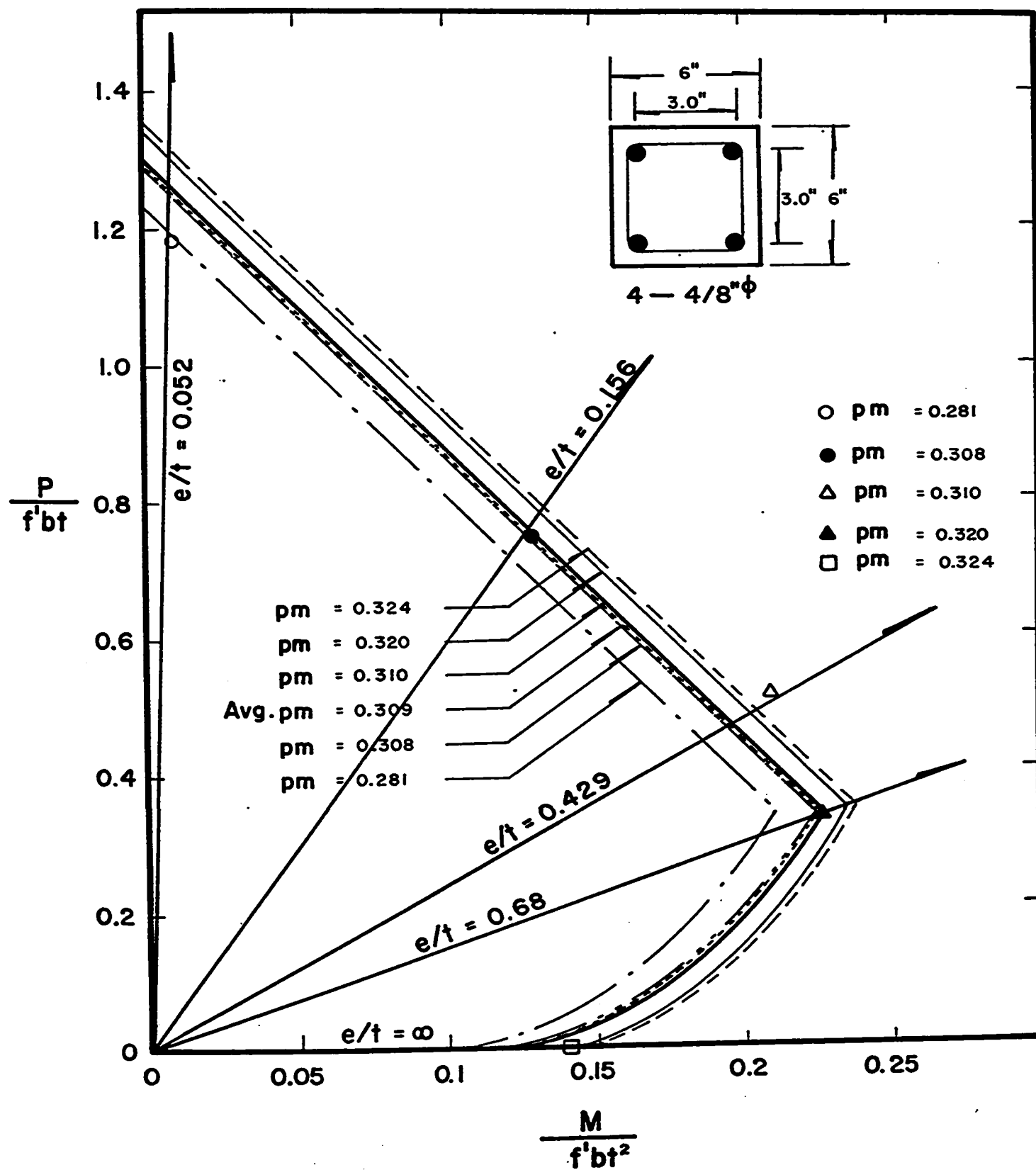
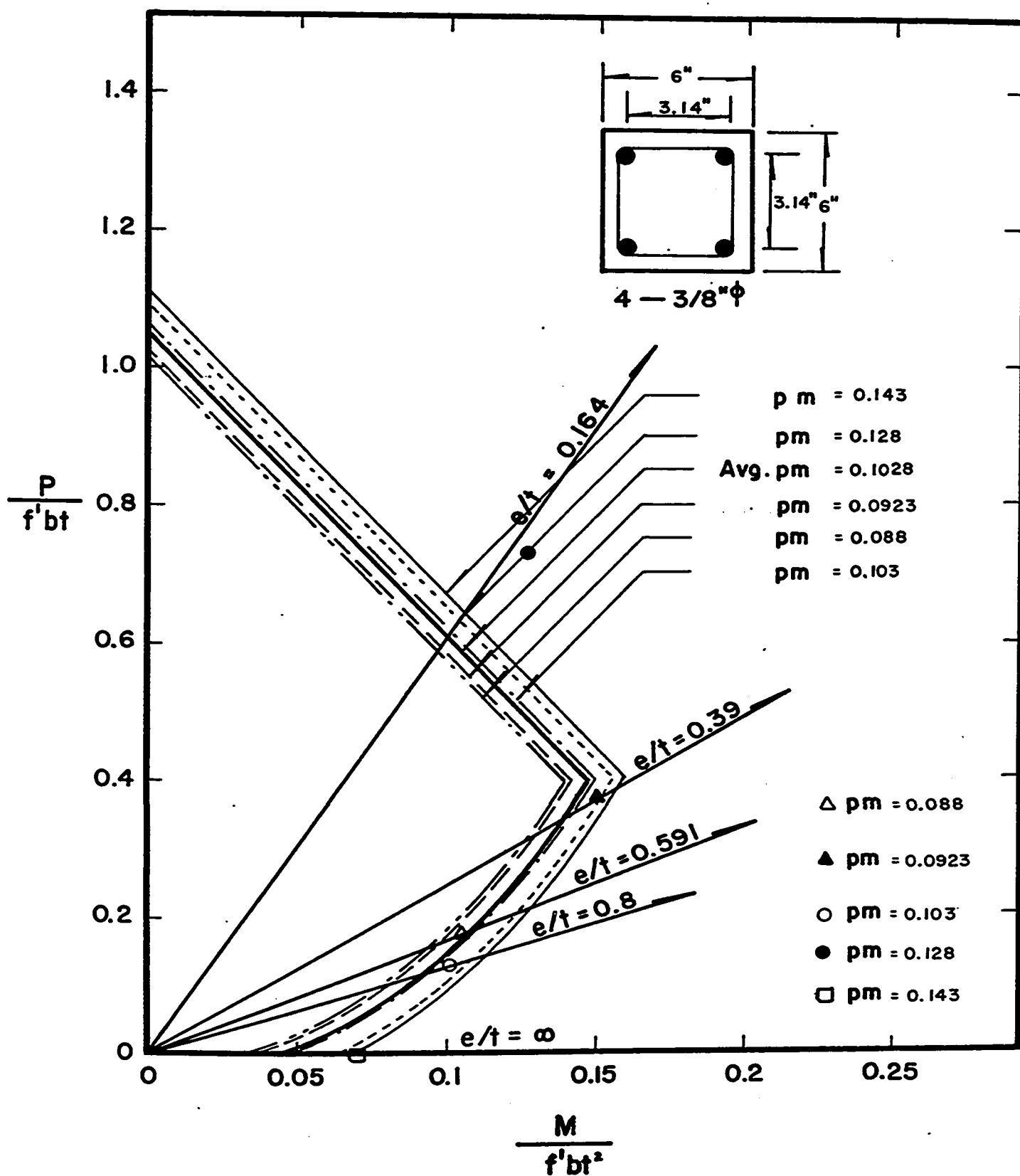


FIG. (5.4.2.3) INTERACTION DIAGRAM FOR GROUP 3
(ACI DESIGN FORMULAE)



**FIG.(5.4.2.4) INTERACTION DIAGRAM
FOR GROUP 4
(ACI DESIGN FORMULAE)**

From Figs. (5.4.2.1) to (5.4.2.4), it can be seen that the accuracy of the ACI Design formulae will generally be within ± 10 percent.

5.4.3 Variation of $P_{exp.}/P_{cal.}$

A histogram and the fitted normal curve of the ratios of the experimental results to the theoretical results using the ACI formulae are shown in Fig. (5.4.3.1) of page The total population of 85 ratios was subjected to a statistical analysis.

The arithmetic mean of the total population is 1.04, the standard deviation of the ratios between the measured and calculated ultimate loads is 0.09 and the coefficient of variation is 8.94 percent. The probability of failure, calculated from the theoretical distribution of the ultimate load capacity, when using a ϕ factor of 0.70, under the ACI Building Code was found to be 0.03% and was 0.19% when using a ϕ factor of 0.75 under the NBC Building Code.

The test results appear to be distributed at random and no systematic trends of variation were found. The column design formulae, using the ϕ reduction factor were so conservative that there is a very low probability of the maximum column capacity would be less than the ultimate design strength.

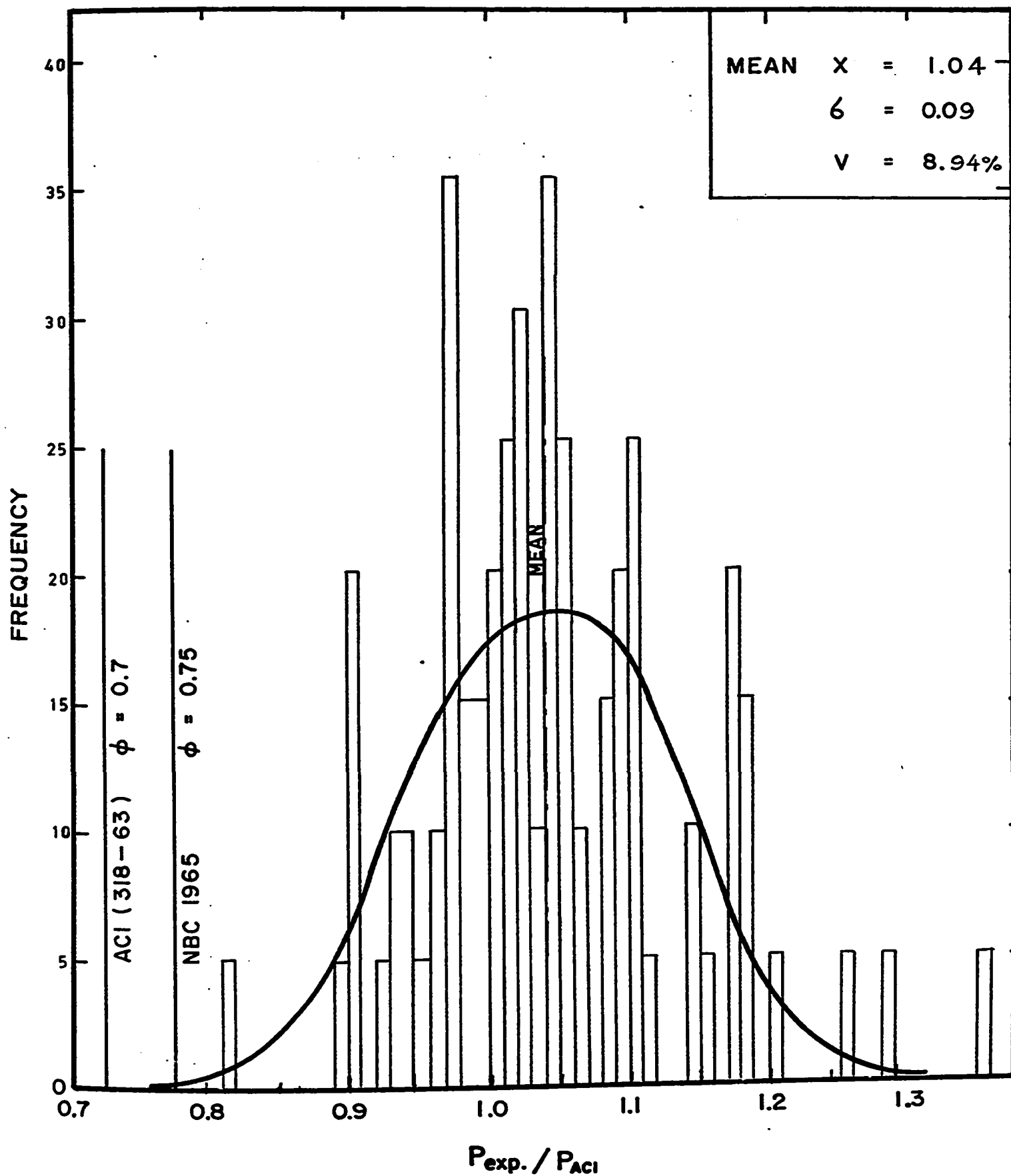


FIG.(5.4.3.1) HISTOGRAM AND FITTED
NORMAL CURVE

5.5 Comparison of Test Results with the Strain Gradient Method

The test results showed close agreement with the results obtained from the Strain-Gradient Method for the full range of conditions from axially-loaded columns to beam columns under pure flexure. The actual strengths for all specimens under all cases are slightly less than the calculated values as shown in Table (5.5) on page 75. The arithmetic mean of the ratios of measured to computed ultimate load is 0.99, the overall standard deviation is 0.02, and the overall coefficient of variation is 1.94 percent.

**TABLE (5.5) COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS
BY STRAIN GRADIENT METHOD (KIPS)**

SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$
4E-1	∞	8.32	8.15	1.02	3D-1	0.5	42.66	41.64	1.02
4E-2		7.85	8.15	0.95	3D-2		41.71	41.64	1.00
4E-3		8.55	8.92	0.96	3D-3		43.98	43.95	1.00
4E-4		8.15	8.15	1.00	3D-4		43.15	43.38	1.00
4E-5		8.54	8.92	0.96	3D-5		41.55	41.64	1.00
4DE-1	0.75	18.63	19.50	0.96	3C-1	0.3	71.33	71.44	1.00
4DE-2		16.30	16.40	0.99	3C-2		69.00	68.75	1.00
4DE-3		18.35	18.34	1.00	3C-3		75.82	75.13	1.01
4DE-4		17.85	17.74	1.01	3C-4		67.50	67.08	1.00
4DE-5		21.95	22.03	0.99	3C-5		67.50	66.93	1.01
4D-1	0.5	30.50	30.39	1.00	3B-1	0.1	103.37	102.92	1.00
4D-2		30.75	30.83	1.00	3B-2		105.56	102.92	1.03
4D-3		30.50	30.39	1.00	3B-3		106.53	103.53	1.03
4D-4		30.57	30.83	0.99	3B-4		117.08	112.33	1.04
4D-5		31.77	31.94	0.99	3B-5		106.53	110.34	0.97
4C-1	0.3	61.00	61.37	0.99	3A-1	0.0	165.00	164.52	1.00
4C-2		65.30	64.73	1.01	3A-2		156.00	156.44	1.00
4C-3		55.00	54.76	1.00	3A-3		166.00	163.29	1.02
4C-4		60.00	59.74	1.00	3A-4		162.00	159.31	1.02
4C-5		67.00	67.03	1.00	3A-5		171.00	169.84	1.00
4B-1	0.1	88.25	87.96	1.00	2E-1	∞	18.00	17.11	1.05
4B-2		80.92	80.69	1.00	2E-2		19.60	19.61	1.00
4B-3		82.11	82.46	0.97	2E-3		19.96	19.59	1.00
4B-4		88.51	88.04	1.00	2E-4		19.30	19.92	0.97
4B-5		82.01	82.07	1.00	2E-5		19.69	19.57	1.00
3E-1	∞	11.50	12.01	0.96	2DE-1	0.75	45.10	45.21	1.00
3E-2		11.74	12.20	0.96	2DE-2		47.70	47.54	1.00
3E-3		12.39	12.78	0.97	2DE-3		45.45	45.79	1.00
3E-4		11.92	12.51	0.95	2DE-4		45.30	45.67	0.99
3E-5		11.37	11.89	0.96	2DE-5		43.50	45.18	0.96

Continued

**TABLE (5.5) COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS
BY STRAIN GRADIENT METHOD (KIPS)**

SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$
4E-1	∞	8.32	8.15	1.02	3D-1	0.5	42.66	41.64	1.02
4E-2		7.85	8.15	0.95	3D-2		41.71	41.64	1.00
4E-3		8.55	8.92	0.96	3D-3		43.98	43.95	1.00
4E-4		8.15	8.15	1.00	3D-4		43.15	43.38	1.00
4E-5		8.54	8.92	0.96	3D-5		41.55	41.64	1.00
4DE-1	0.75	18.63	19.50	0.96	3C-1	0.3	71.33	71.44	1.00
4DE-2		16.30	16.40	0.99	3C-2		69.00	68.75	1.00
4DE-3		18.35	18.34	1.00	3C-3		75.82	75.13	1.01
4DE-4		17.85	17.74	1.01	3C-4		67.50	67.08	1.00
4DE-5		21.95	22.03	0.99	3C-5		67.50	66.93	1.01
4D-1	0.5	30.50	30.39	1.00	3B-1	0.1	103.37	102.92	1.00
4D-2		30.75	30.83	1.00	3B-2		105.56	102.92	1.03
4D-3		30.50	30.39	1.00	3B-3		106.53	103.53	1.03
4D-4		30.57	30.83	0.99	3B-4		117.08	112.33	1.04
4D-5		31.77	31.94	0.99	3B-5		106.53	110.34	0.97
4C-1	0.3	61.00	61.37	0.99	3A-1	0.0	165.00	164.52	1.00
4C-2		65.30	64.73	1.01	3A-2		156.00	156.44	1.00
4C-3		55.00	54.76	1.00	3A-3		166.00	163.29	1.02
4C-4		60.00	59.74	1.00	3A-4		162.00	159.31	1.02
4C-5		67.00	67.03	1.00	3A-5		171.00	169.84	1.00
4B-1	0.1	88.25	87.96	1.00	2E-1	∞	18.00	17.11	1.05
4B-2		80.92	80.69	1.00	2E-2		19.60	19.61	1.00
4B-3		82.11	82.46	0.97	2E-3		19.96	19.59	1.00
4B-4		88.51	88.04	1.00	2E-4		19.30	19.92	0.97
4B-5		82.01	82.07	1.00	2E-5		19.69	19.57	1.00
3E-1	∞	11.50	12.01	0.96	2DE-1	0.75	45.10	45.21	1.00
3E-2		11.74	12.20	0.96	2DE-2		47.70	47.54	1.00
3E-3		12.39	12.78	0.97	2DE-3		45.45	45.79	1.00
3E-4		11.92	12.51	0.95	2DE-4		45.30	45.67	0.99
3E-5		11.37	11.89	0.96	2DE-5		43.50	45.18	0.96

Continued

SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$	SPECIMEN NO.	e/t	P _{exp}	P _{cal}	$\frac{P_{exp}}{P_{cal}}$
2D-1	0.5	59.85	60.10	1.00	1D-1	0.5	74.50	73.94	1.01
2D-2		61.77	62.36	0.99	1D-2		72.50	72.02	1.01
2D-3		57.43	57.83	0.99	1D-3		68.00	68.10	1.00
2D-4		61.90	62.36	0.99	1D-4		69.00	68.59	1.00
2D-5		61.10	60.90	1.00	1D-5		70.00	69.51	1.00
2C-1	0.3	100.00	99.81	1.00	1C-1	0.3	112.00	110.98	1.01
2C-2		104.00	102.99	1.01	1C-2		111.00	110.83	1.00
2C-3		112.50	112.82	1.00	1C-3		110.50	110.83	1.00
2C-4		105.50	105.58	1.00	1C-4		112.12	115.65	0.98
2C-5		106.50	106.73	1.00	1C-5		115.00	114.44	1.00
1E-1	∞	25.00	25.63	0.97					
1E-2		24.94	25.45	0.98					
1E-3		25.00	26.12	0.96					
1E-4		22.00	21.71	1.01					
1E-5		24.98	25.76	0.97					

Mean value of $P_{exp} / P_{cal} = 0.99$

Standard deviation = 0.02

Coefficient of variation = 1.94%

5.5.1 Effects of Concrete Quality; Strength of Reinforcement and Eccentricity

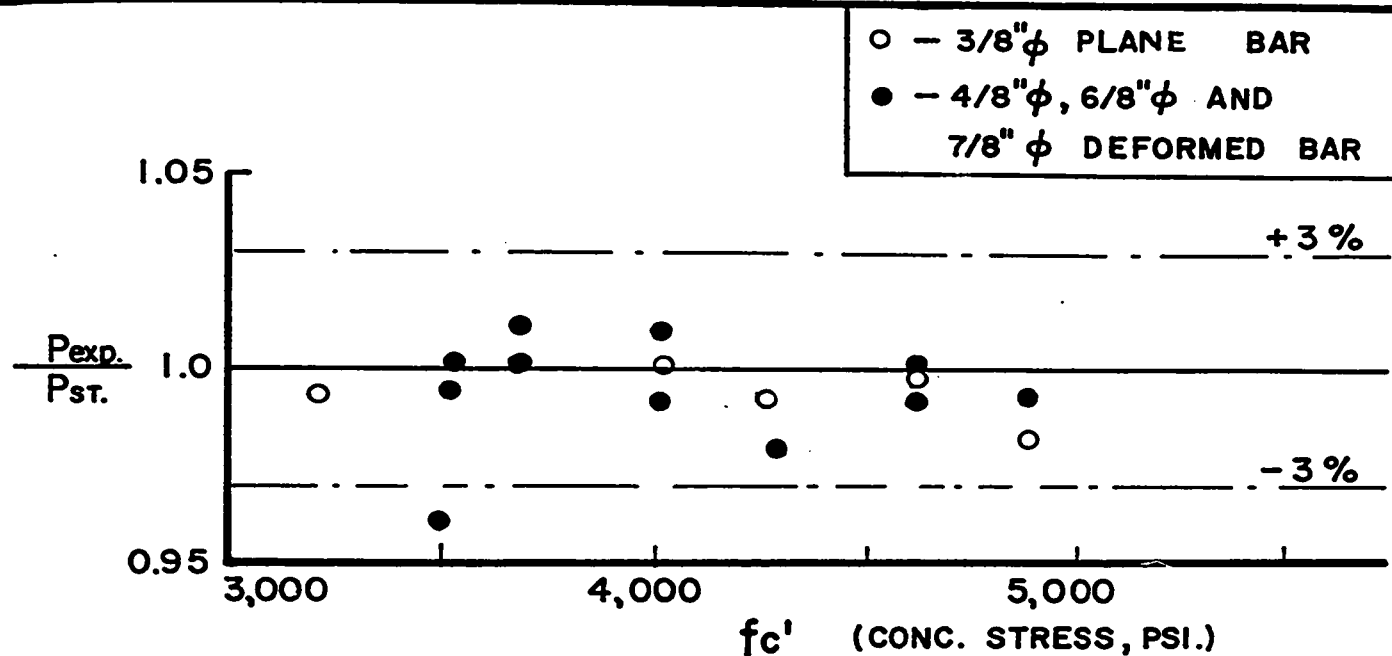
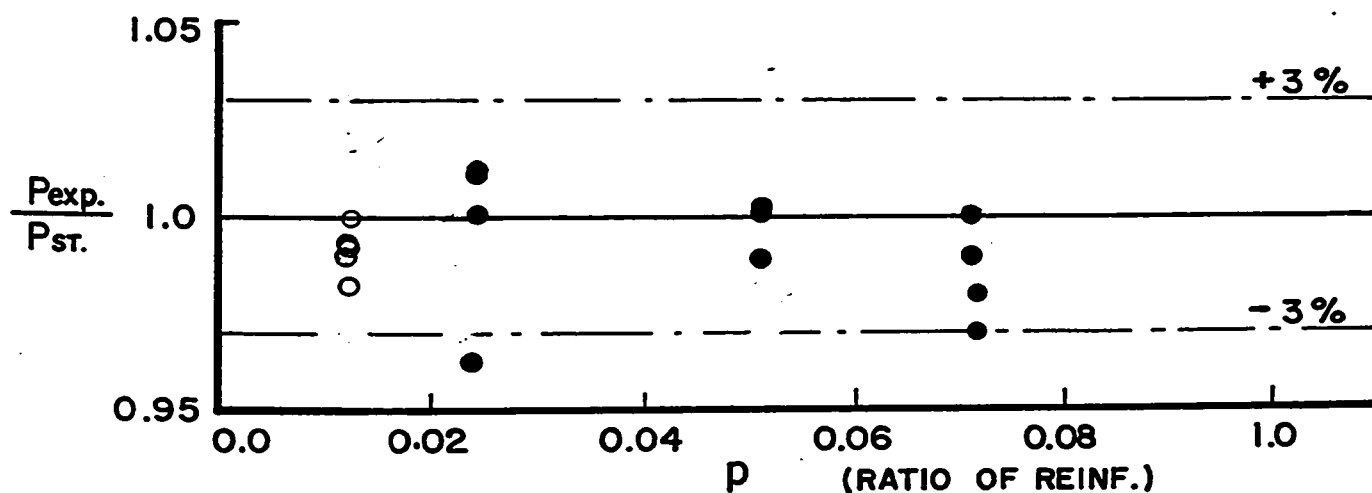
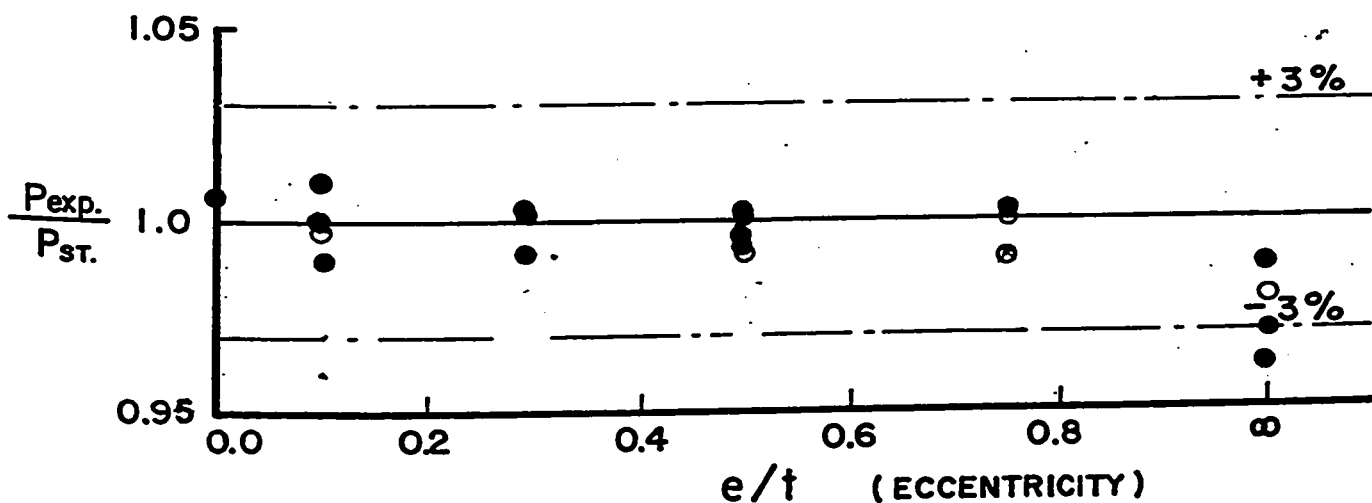
Figs. (5.5.1) to (5.5.3) show the curves of ratio of measured* to computed ultimate load against concrete strength, reinforcement ratio and eccentricity ratio, respectively. It can be seen from these curves that the ratio of measured to computed ultimate load varied within the limit of ± 3 percent from unity.

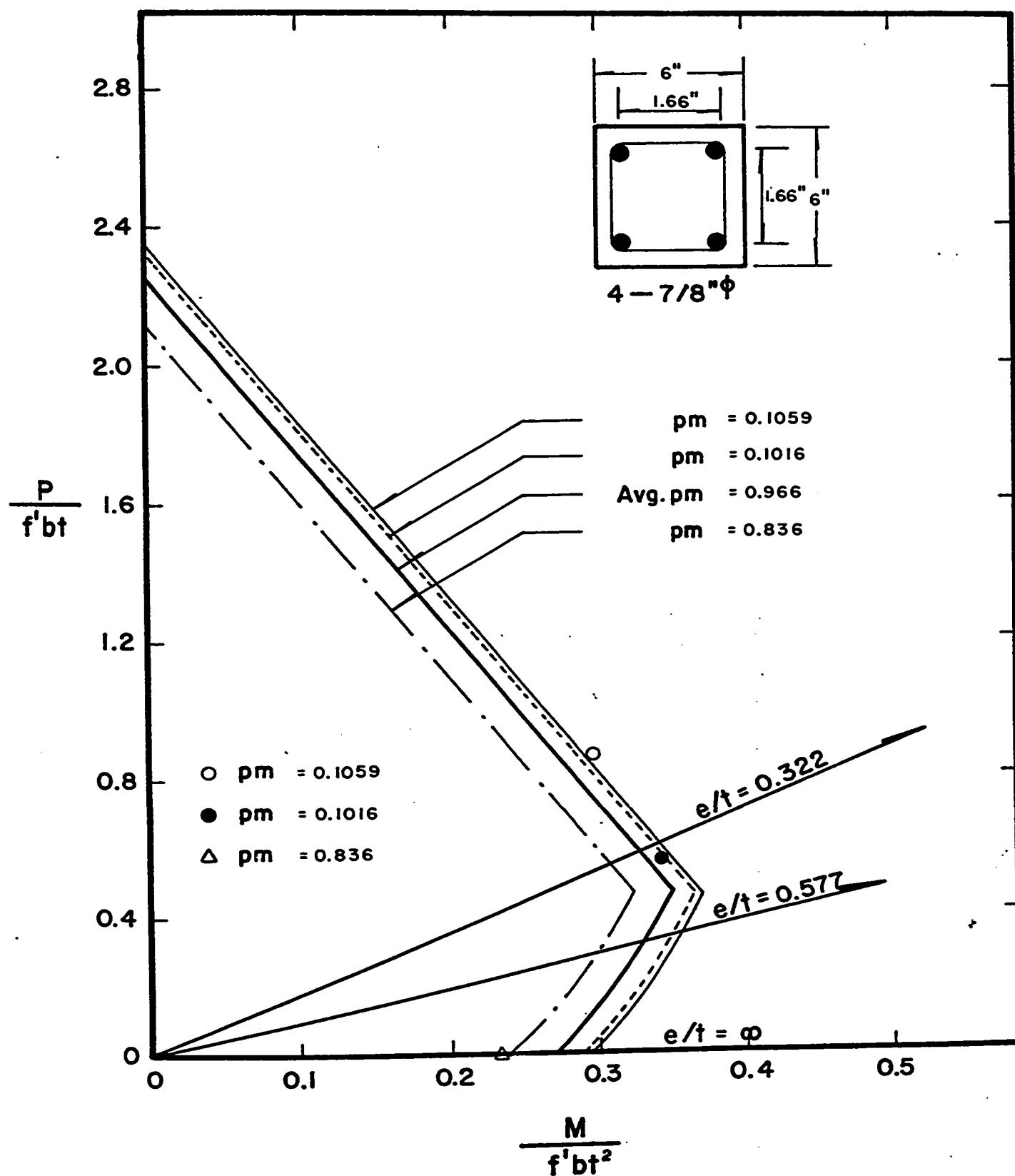
5.5.2 Interaction Diagram

From Figs. (5.5.2.1) to (5.5.2.4), a series of interaction curves, compare the maximum load capacity of the columns to the theoretical ultimate strengths. The material properties of the cross-section are: concrete strength from 3400 to 4600 psi.; percentage of reinforcement varied from 1.27 (plain bar) to 7.14 percent; and steel yield strength from 43,000 to 67,400 psi.

It can be seen from Figs. (5.5.2.1) to (5.5.2.4) that the experimental values lie well in between ± 10 percent of the theoretical curves.

* each point represent the average ratio P_{exp}/P_{ACI} of each group.

FIG. (5.5.1) $P_{exp.}/P_{cal.}$ VERSUS ULTIMATE CONCRETE STRESSFIG. (5.5.2) $P_{exp.}/P_{cal.}$ VERSUS RATIO OF REINFORCEMENTFIG. (5.5.3) $P_{exp.}/P_{cal.}$ VERSUS ECCENTRICITY RATIO OF LOAD (STRAIN GRADIENT METHOD)



**FIG.(5.5.2.1) INTERACTION DIAGRAM
FOR GROUP I
(STRAIN GRADIENT METHOD)**

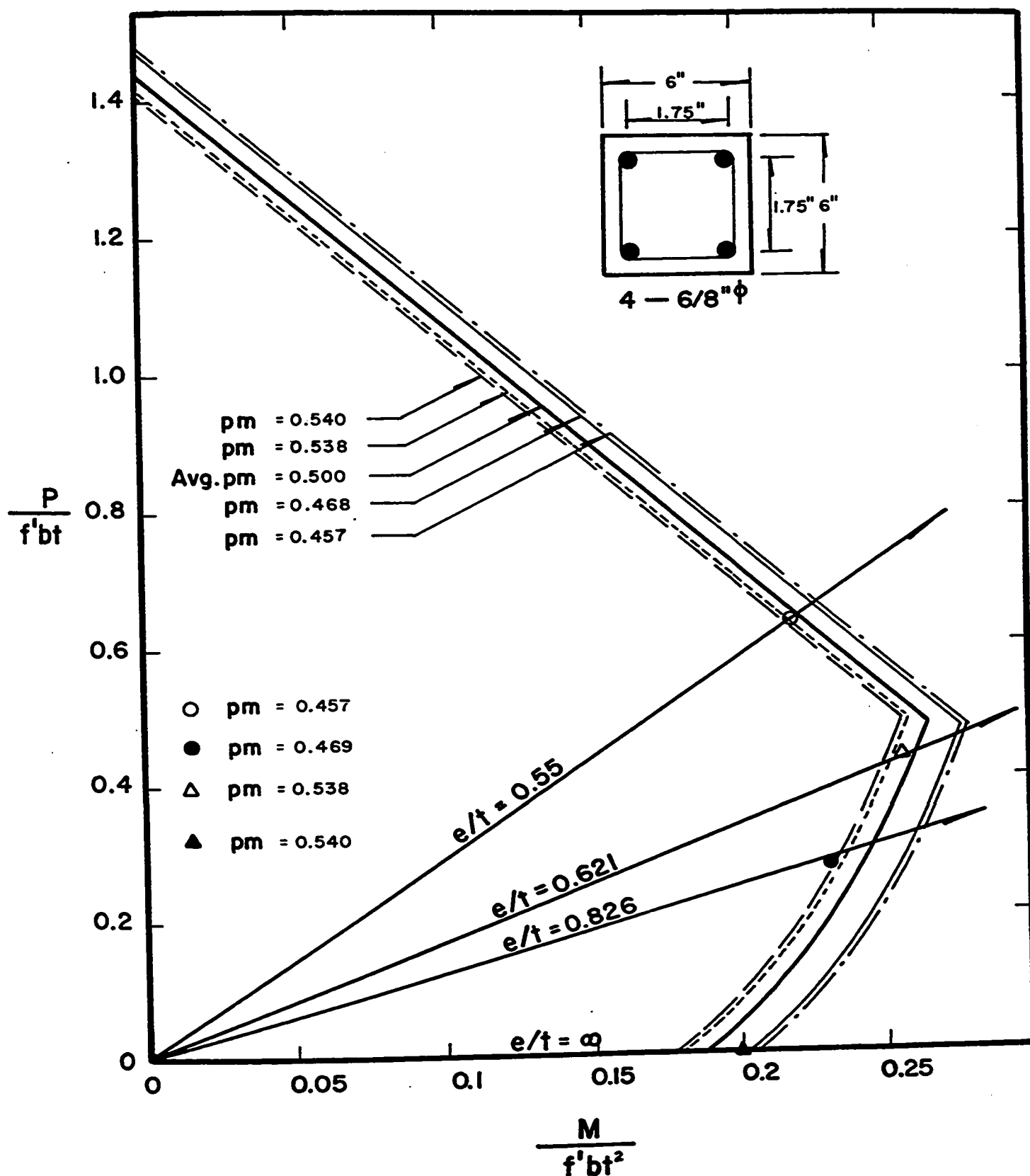


FIG.(5.5.2.2) INTERACTION DIAGRAM FOR GROUP 2
(STRAIN GRADIENT METHOD)

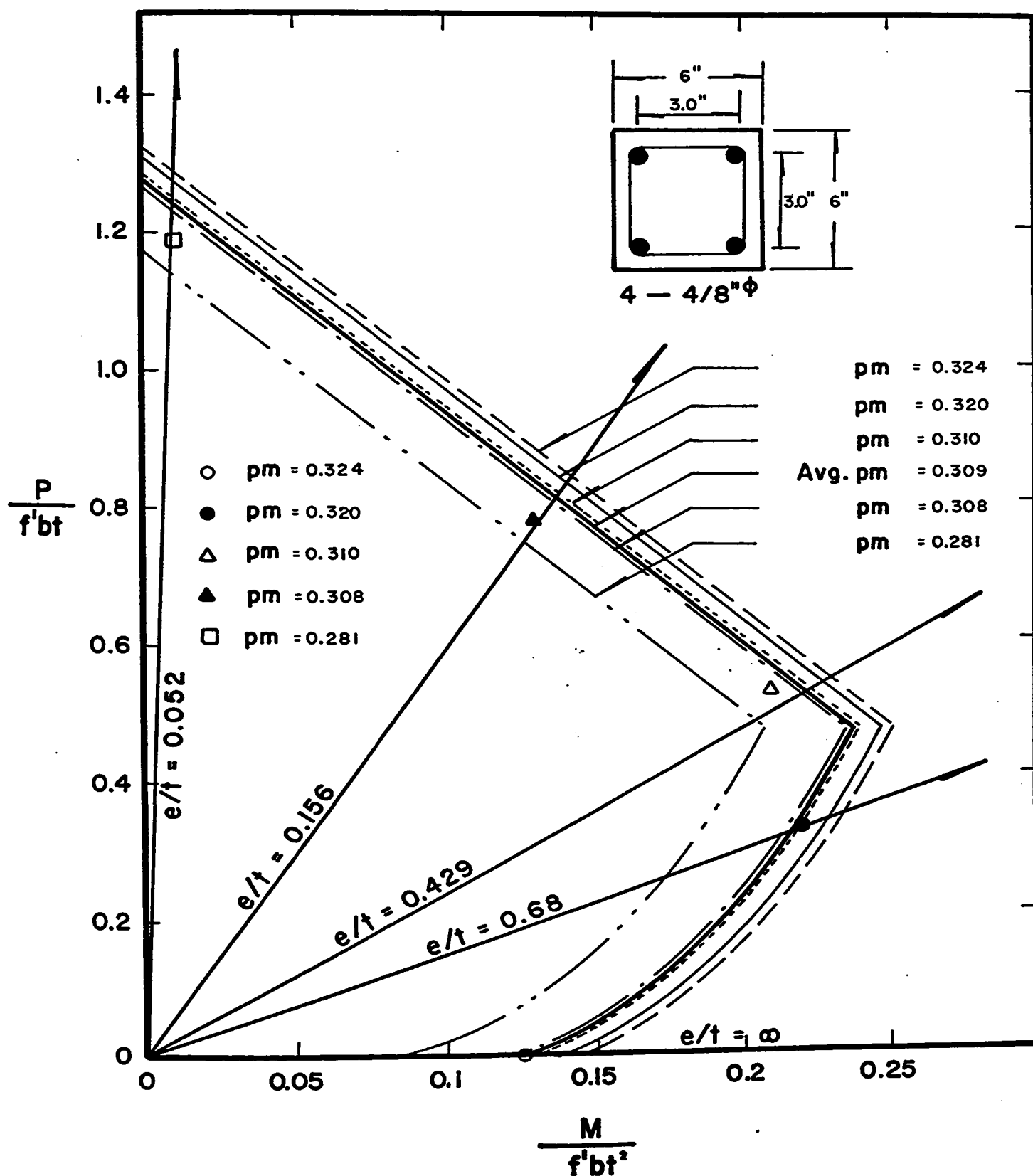


FIG. (5.5.2.3) INTERACTION DIAGRAM FOR GROUP 3
(STRAIN GRADIENT METHOD)

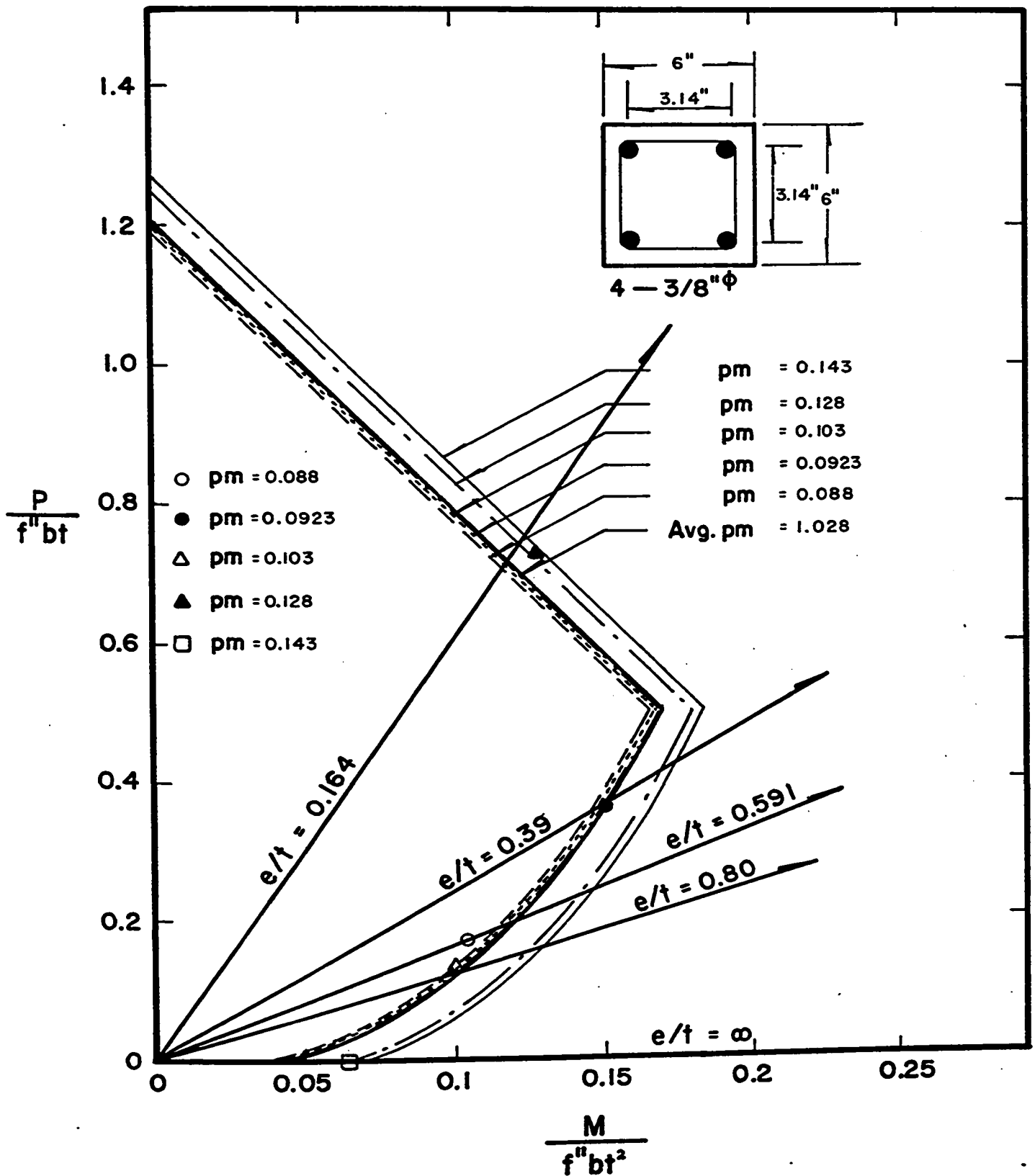


FIG. (5.5.2.4) INTERACTION DIAGRAM FOR GROUP 4
(STRAIN GRADIENT METHOD)

5.5.3 Variation of P_{exp}/P_{cal}

In Fig. (5.5.3.1) of page 84, shows the histogram and fitted normal curve of the ratios of the experimental results to the theoretical results given by the Strain Gradient Method. The total population of 85 ratios was studied and subjected to a statistical analysis.

The arithmetic mean of the total population is 0.99, the standard deviation of the ratios between the measured and calculated ultimate loads is 0.02, and the coefficient of variation is 1.94 percent.

Within the scope of the tests reported herein, the ultimate loads computed from the Strain Gradient Method slightly overestimate the column capacities, within one percent on the average.

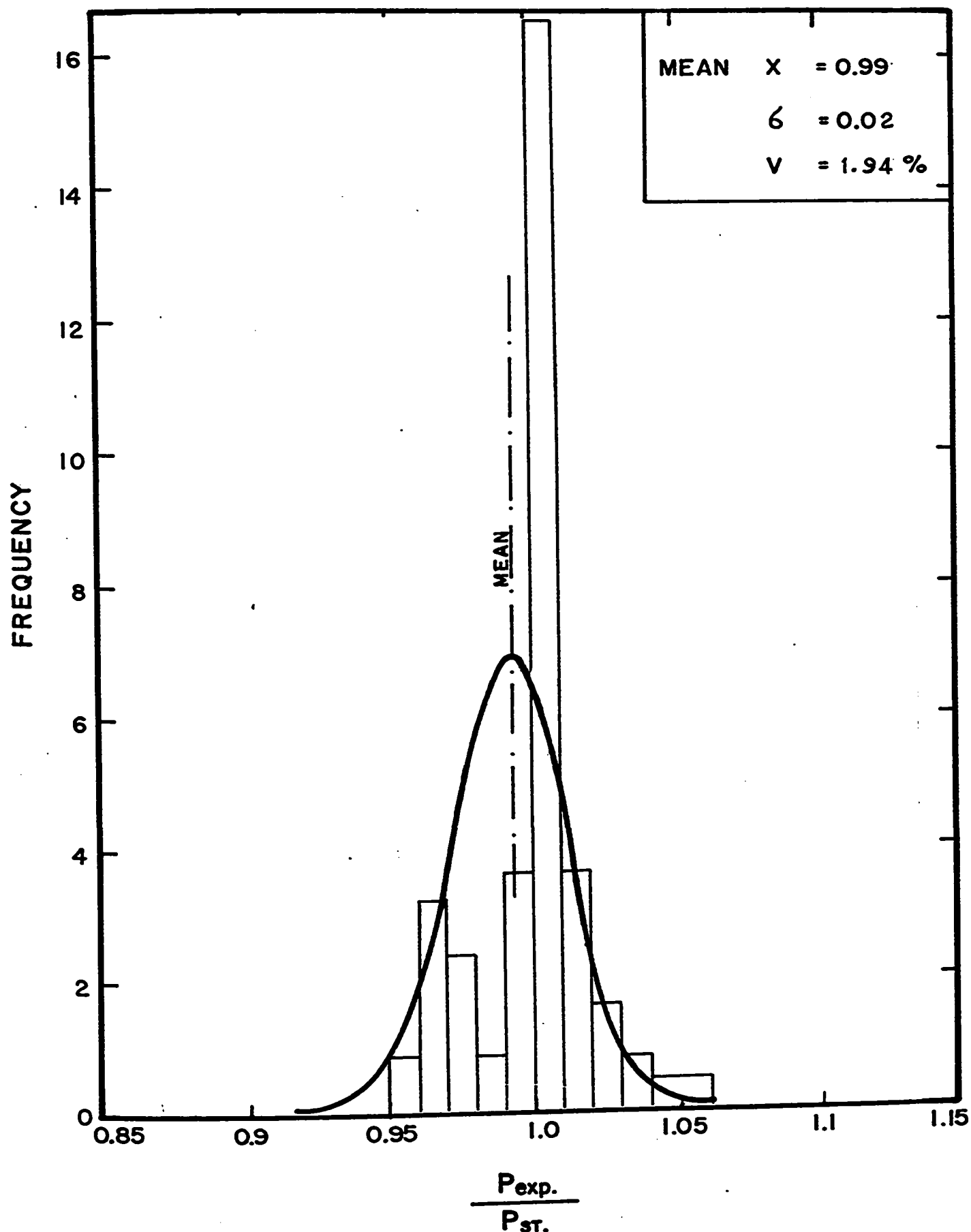


FIG.(5.5.3.1) HISTOGRAM AND FITTED
NORMAL CURVE

5.6 Comparisions with Previous Investigations

The ratios of the computed values from the ACI Design Formulas and the test results are compared with the ratios presented by Chow (26), Hognestad (27), The ACI Column Investigation (27), and Bach and Graf (15).

In Table (5.4), on page 64, shows the ratios of the measured to the calculated results obtained in the project, with the mean value of 1.04 , the standard deviation of 0.09 , and the coefficient of variation of 8.94 percent.

In Table (5.6.2), on page 86, shows the ratios of the measured to the calculated results presented by Chow, with the mean value of 1.15, the standard deviation of 0.0931 and coefficient of variation of 8.3 percent.

Table (5.6.3) on page 87, shows the ratios of the measured to the calculated results presented by Hognestad, with a mean value of 0.97, standard deviation of 0.059, and coefficient of variation of 6.08 percent.

Table (5.6.4) on page 90, gives the ratios of the measured to the calculated results presented by The ACI Column Investigation, with a mean value of 1.00, standard deviation of 0.074, and the coefficient of variation of 7.40 percent.

Table (5.6.5), on page 91, gives the ratios of the measured to the calculated results by Bach and Graf, with a mean value of 1.035, standard deviation of 0.0703, and coefficient of variation of 6.79 percent.

Similiar results were found with the specimens reported by Hognestad, The ACI Column Investigation and Bach and Graf, and are in close agreement with results presented in Table (5.4).

The ratios of the computed values from the Strain Gradient Method and the test results are also compared with the ratios presented by Chow. The ratios of the measured to the calculated results presented by Chow, with the mean value of 0.97, the standard deviation of 0.076, and coefficient of variation of 7.9%, were compared to the results obtained in Table (5.5), with the mean value of 0.99, standard deviation of 0.02 and coefficient of variation of 1.94%.

Since the agreement between Chow's results and the ACI design formulae is much worse than that between Hognestad, The ACI Column Investigation, the Bach and Graf and the ACI design formulae, it is concluded that Chow's results must contain some experimental errors.

TABLE (5.6.3) COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS
UNDER ECCENTRIC LOADS — BY HOGNESTAD

SPECIMEN	f'_c (psi.)	f_y (ksi.)	% OF REINF.	P_{exp} (Kips)	P_{cal} (Kips)	$\frac{P_{exp}}{P_{cal}}$
B = 10 in. T = 10 in.	5280.00	60.00	2.48	388.00	452.00	0.86
	5660.00			441.00	481.00	0.92
	4250.00			343.00	372.00	0.92
	4070.00			352.00	356.00	0.99
	2270.00			222.00	212.00	1.05
	2020.00			191.00	192.00	1.00
	5280.00			239.00	240.00	1.00
	5830.00			253.00	260.00	0.97
	4250.00			213.00	206.00	1.03
	4070.00			190.00	197.00	0.96
	2270.00			118.50	116.00	1.02
	1970.00			100.00	103.00	0.97
	5660.00			133.50	154.00	0.87
	4830.00			140.00	158.00	0.89
	4630.00			125.90	134.30	0.94
	4290.00			116.00	129.40	0.90
	1880.00			60.50	66.00	0.92
	1690.00			64.00	52.00	1.03
	4810.00			84.50	85.70	0.99
	5600.00			81.00	92.10	0.88
	3800.00			80.00	77.70	1.03
	4290.00			81.00	81.10	1.00
	1690.00			50.50	48.00	1.05
	1730.00			52.00	49.00	1.06
	4810.00			48.20	45.60	1.06
	5600.00			42.80	46.90	0.91
	4290.00			46.10	44.50	1.04
	4590.00			45.50	45.00	1.01
	2310.00			39.00	37.70	1.04
	1770.00			32.80	34.00	0.96
	4080.00			456.00	437.00	1.04
	4040.00			420.00	436.00	0.96
	2020.00			225.00	268.00	0.84
	1520.00			202.00	222.00	0.91

Continued

TABLE (5.6.3) Continued

SPECIMEN	f'_c (psi.)	f_y (ksi.)	% OF REINF.	P_{exp} (Kips)	P_{cal} (Kips)	$\frac{P_{exp}}{P_{cal}}$
B = 10 in. T = 10 in.	5240.00	60.00	2.48	274.00	254.00	1.08
	5810.00			284.00	312.00	0.91
	4080.00			256.00	245.00	1.04
	4040.00			248.00	244.00	1.02
	1970.00			141.00	147.00	0.96
	1520.00			126.80	135.00	0.94
	5520.00			162.00	177.90	0.91
	5810.00			152.00	179.50	0.85
	4700.00			156.00	164.90	0.95
	4260.00			146.00	158.16	0.92
	1820.00			99.00	103.00	0.96
	1820.00			99.00	102.00	0.97
	5100.00			89.00	99.10	0.90
	5170.00			91.20	99.10	0.92
	4700.00			94.00	97.70	0.96
	4370.00			89.50	96.60	0.93
	1880.00			73.00	75.80	0.96
	1730.00			65.50	74.10	0.88
	5100.00			46.10	47.80	0.97
	5170.00			44.00	48.00	0.92
	4260.00			43.50	47.20	0.92
	4370.00			44.00	47.20	0.93
	2300.00			44.50	44.30	1.00
	1770.00			45.00	42.60	1.06
	3870.00			500.00	513.00	0.98
	4070.00			485.00	517.00	0.94
	2070.00			353.00	376.00	0.94
	4150.00			315.00	306.00	1.03
	5050.00			326.00	343.00	0.95
	4300.00			303.00	318.00	0.95
	4010.00			284.00	298.00	0.95
	2300.00			252.00	224.00	1.12

Continued

5.7 Studies of Deflection

The deflections of all columns tested were observed by means of three dial indicators: at centre span and 6" to the left and right of the centre of the column along the longitudinal direction. These dial indicators were attached to the compression face of the columns as shown in Figs. (3.5.2.a) and (b). The load-deflection curves for all columns presented in Figs.(B.1) to (B.85), on page 112 to page 196.

The deflections at mid-height with respect to the hinged ends were used to calculate the actual eccentricities at failure. All the ACI Design Equations for ultimate loads of eccentrically-loaded members reported in this thesis were referred to the actual eccentricities at failure.

The deflections at mid-height of the columns were also considered theoretically. They can be obtained easily by introducing the simple assumption that the deflected shape was a cosine wave, as shown in Fig. (5.7.1).

$$Y = \delta \cos \frac{\pi x}{L} \dots\dots\dots(1)$$

$$\frac{d^2 y}{dx^2} = -\delta \left(\frac{\pi}{L} \right)^2 \cos \frac{\pi x}{L}$$

but

$$\frac{d^2 y}{dx^2} = \frac{-1}{\rho}$$

$$\therefore \frac{1}{\rho} = \delta \left(\frac{\pi}{L} \right)^2 \cos \frac{\pi x}{L} \dots\dots\dots(2)$$

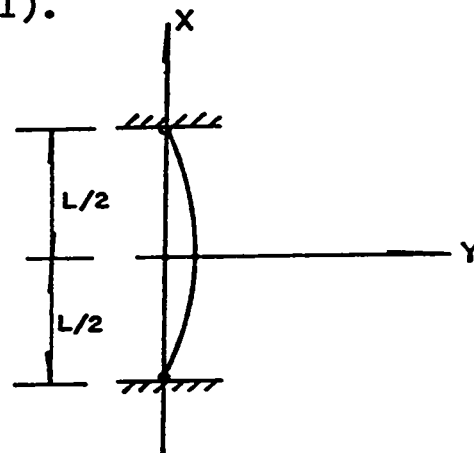


FIG. (5.7.1) DEFLECTED
SHAPE OF A COLUMN

Apply boundary condition, when $X = 0$, $Y = \delta$

Thus

$$\frac{1}{\rho} = \delta \left(\frac{\pi}{L} \right)^2 \dots\dots(3)$$

From Fig. (5.7.2), consider column under deflection.

$$\frac{1}{\rho} = \frac{\epsilon_x}{Y} = \frac{\epsilon_c}{Kd} = \phi \dots\dots(4)$$

Substitute Eq. (4) in Eq. (3), we have,

$$\delta \left(\frac{\pi}{L} \right)^2 = \phi$$

$$\therefore \delta = \phi \cdot \left(\frac{L}{\pi} \right)^2 \dots\dots(5)$$

In our case, as shown in Fig. (5.7.3) gives,

$$(\epsilon_c + \epsilon_{sl}) = d \cdot \phi$$

$$\therefore \phi = \frac{\epsilon_c + \epsilon_{sl}}{d} \dots\dots(6)$$

Substitute Eq. (6) in Eq. (5), we obtain,

$$\delta = \left(\frac{\pi}{L} \right)^2 \left(\frac{\epsilon_c + \epsilon_{sl}}{d} \right)$$

$$= K \cdot (\epsilon_c + \epsilon_{sl})$$

where $K = \pi^2 / L^2 d$, and ϵ_c , and ϵ_{sl} can be obtained for each column, from the Strain Gradient Method.

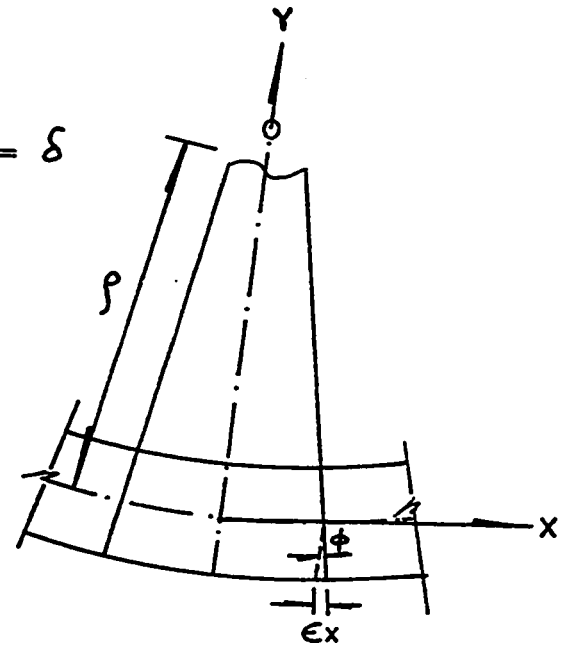


FIG.(5.7.2) COLUMN UNDER DEFLECTION

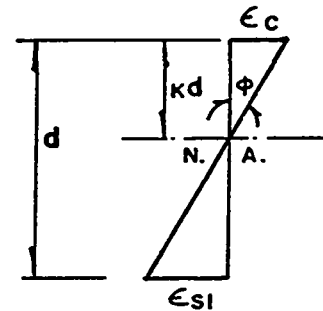


FIG.(5.7.3) STRAIN RELATIONSHIP

The test results of the deflections at mid-height were compared with the theoretical results as shown in Table (5.7.1), and were found to be in a satisfactory agreement. The errors are probably due to the centering of the specimen; the beam-column effect under various eccentricities; the assumptions of the maximum concrete strain equal to 0.003 and the assumption that the deflected shape of the specimens is a cosine curve.

**TABLE (5.7.1) COMPARISON BETWEEN OBSERVED AND
CALCULATE CENTRE DEFLECTIONS**

SPECIMEN *	INITIAL ECCENTRICITY e/t	DEFLECTION AT FAILURE		
		δ exp.		δ cal.
		MEAN	C.of V.	
4E (1 - 5)		0.481	11.1%	0.437
4DE (1 - 5)	0.75	0.261	7.7%	0.320
4D (1 - 5)	0.5	0.601	12.6%	0.642
4C (1 - 5)	0.3	0.537	9.6%	0.420
4B (1 - 5)	0.1	0.431	14.4%	0.448
2E (1 - 5)		0.363	16.5%	0.324
2DE (1 - 5)	0.75	0.463	12.9%	0.418
2D (1 - 5)	0.5	0.614	9.1%	0.550
2C (1 - 5)	0.3	0.380	5.5%	0.305
3E (1 - 5)		0.359	1.4%	0.384
3D (1 - 5)	0.5	1.050	6.3%	0.964
3C (1 - 5)	0.3	0.725	15.7%	0.734
3B (1 - 5)	0.1	0.413	2.0%	0.470
3A (1 - 5)	0.0	0.033	6.4%	0.034
1E (1 - 5)		0.460	4.8%	0.390
1D (1 - 5)	0.5	0.449	11.4%	0.415
1C (1 - 5)	0.3	0.312	10.3%	0.330

* Average Values of 5 Specimens in a Same Group.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

From this investigation, some main conclusions may be summarised as follows:-

- 1) The short column design formulae given in the ACI 318-56, NBC 1965 and CSA.A23.3-1970 Building Codes predicts the measured failure loads of columns within $\pm 10\%$, provided that the actual eccentricity of the load at failure is used in the calculations. The average ratio of measured load / predicted load is 1.04 with a standard deviation of 0.09.
- 2) The ACI - CSA Design Formulae together with the recommended " ϕ " factor of 0.70 implies a probability of the ultimate section capacity of a laboratory specimen being less than the design ultimate load of 0.03%.
- 3) The probability of the actual load capacity being less than the design ultimate load is 0.19%, using a " ϕ " factor of 0.75 as recommended by the Canadian Standard (CSA.A23-1970), together with the ACI - NBC - CSA prediction formulae.
- 4) The strain gradient method using the measured steel stress-strain curve and Hognestad stress-strain curve for the concrete predicts the experimental values within $\pm 3\%$. The average measured load / predicted load ratio is 0.99 with a standard deviation of 0.02.

- 5) Calculations of the lateral deflection from the strain gradient method assuming that the deflected shape of an eccentrically loaded long column deflects laterally in the form of a cosine agree reasonably well with the measured lateral deflections of the laboratorial specimens.
- 6) The accuracy of the results obtained from the ACI Design Formulae, increases as the percentage of the longitudinal reinforcement increases.
- 7) The ratio of the measured to computed ultimate load using the ACI Design Formulae, becomes close to unity as the ratio e/t approaching infinity.
- 8) The ratio of the measured to computed ultimate load using the strain gradient method, approaches close to unity as the ratio e/t increases.
- 9) The moment with respect to the balanced condition is increased with an increase of longitudinal reinforcing steel.
- 10) The ductility of the column section is decreased with the increase of the longitudinal reinforcing steel.
- 11) The experimental results using deformed bars rather than plain bars, give more consistant agreement with the predicted results from both the strain gradient method and ACI - NBC - CSA methods.

6.2 Recommendations

- 1) Longitudinal stirrups are recommended in the capitals to transmit the eccentric load to the prismatic section of the column.
- 2) Test the Strain Gradient Method experimentally, by installing strain gages in the specimen to check the correspondance of strain / strain gradient and load.
- 3) To develop a general computer program to represent the behavior of long column under uni-axial and bi-axial bending, by the Strain Gradient Method, for the analysis of the reinforced concrete columns.

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APPENDIX A

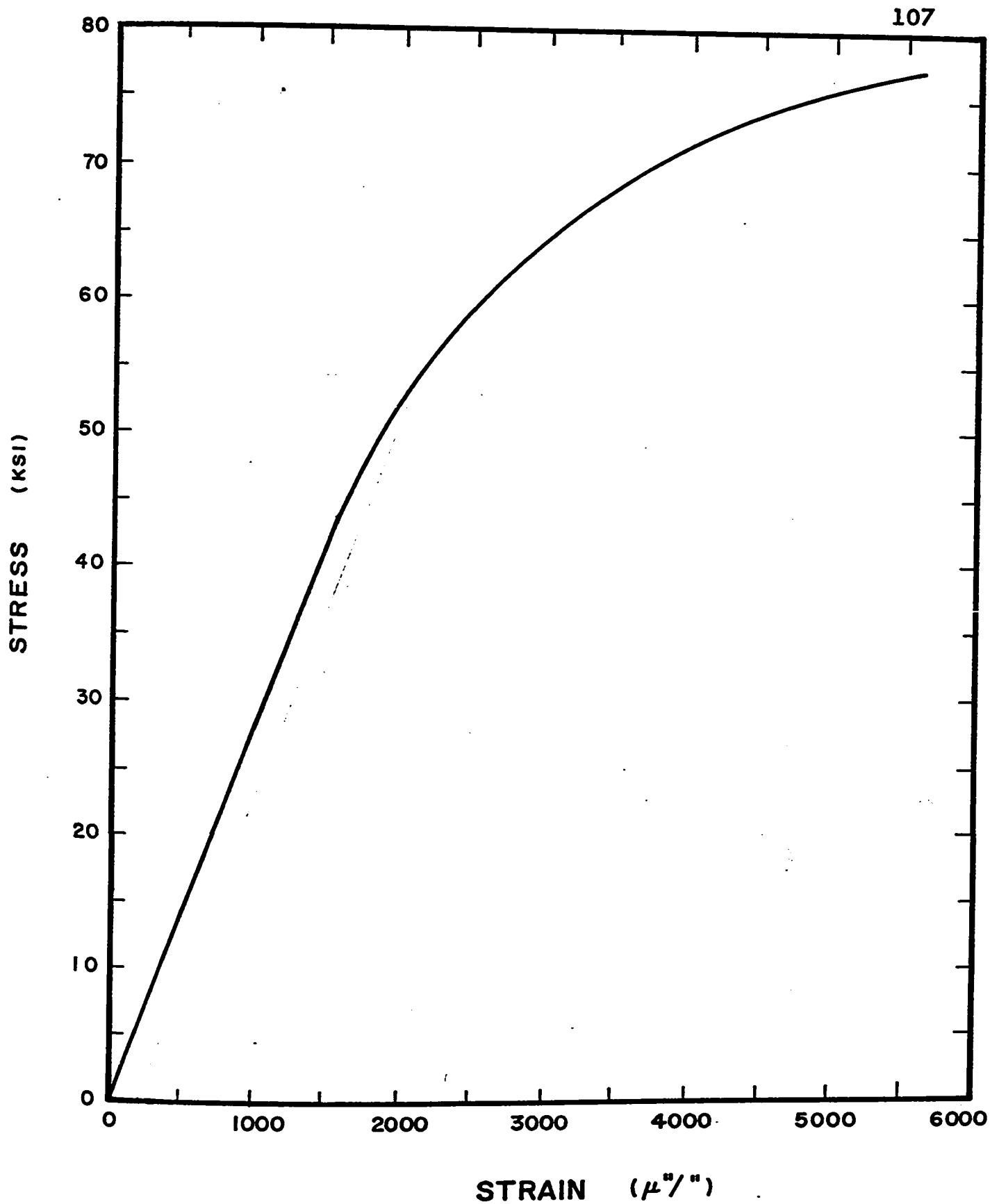
STEEL PROPERTIES

FIGURE (A.1)

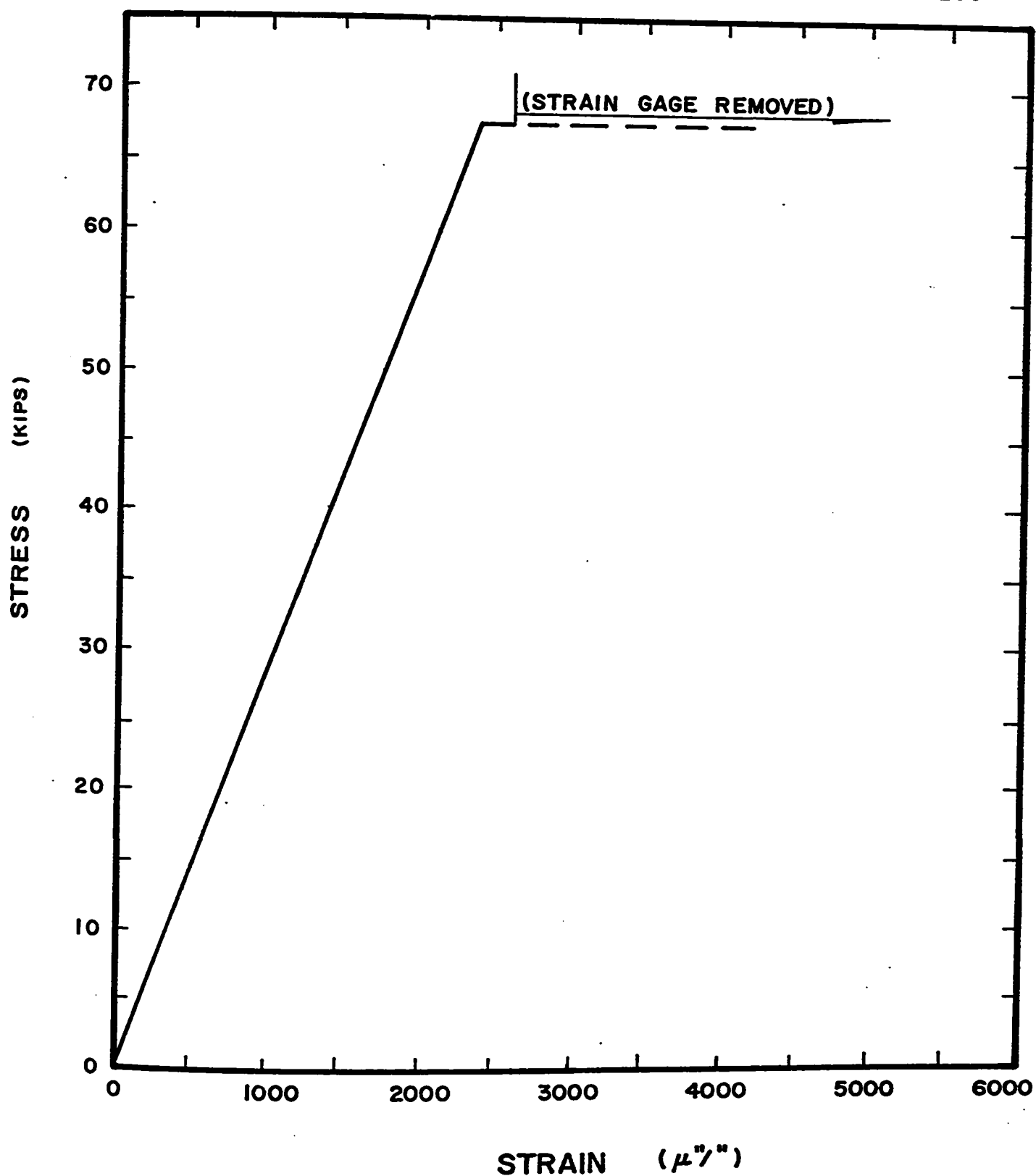
to

FIGURE (A.4)

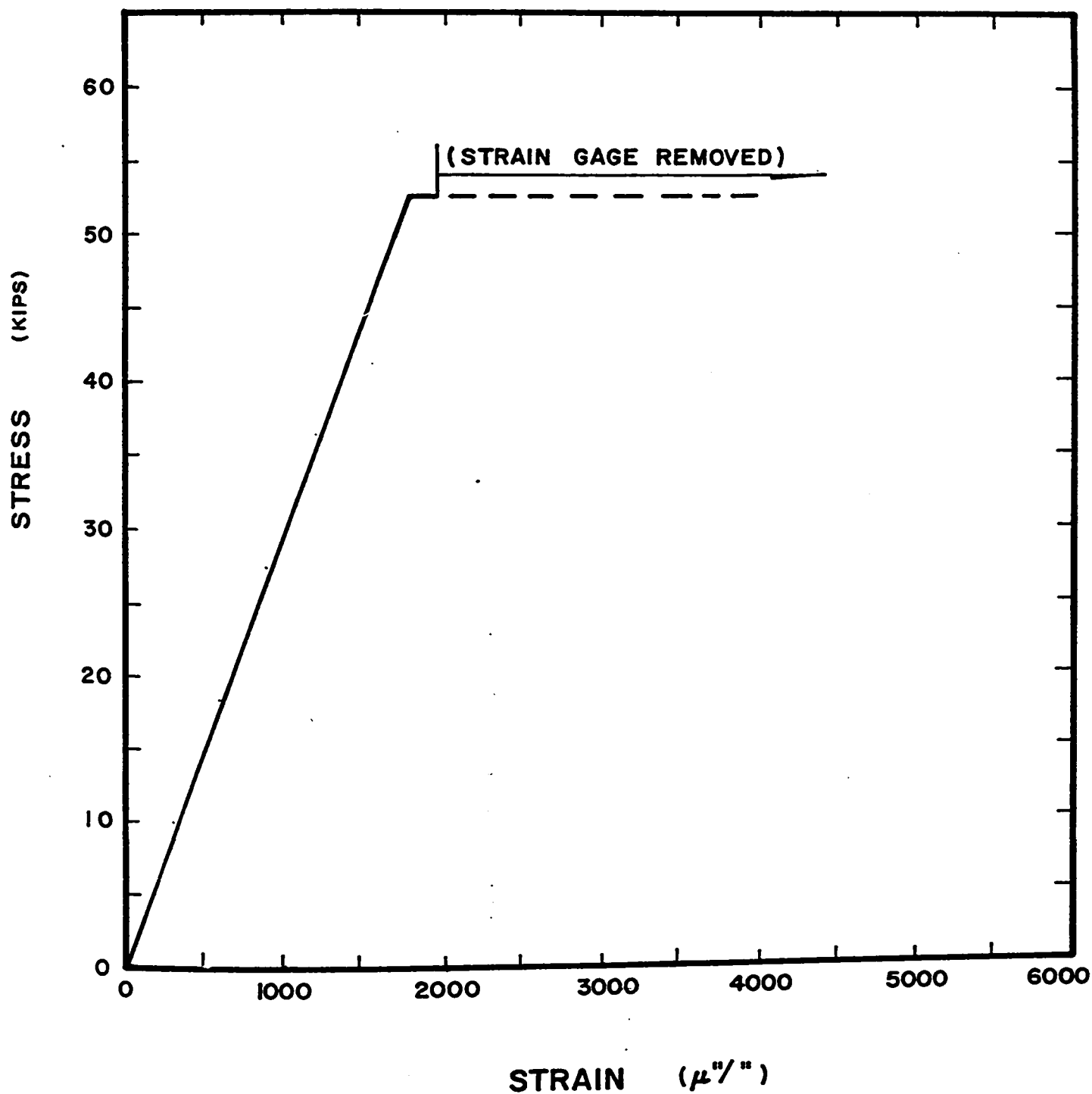
STRESS-STRAIN CURVES OF
LONGITUDINAL REINFORCEMENTS



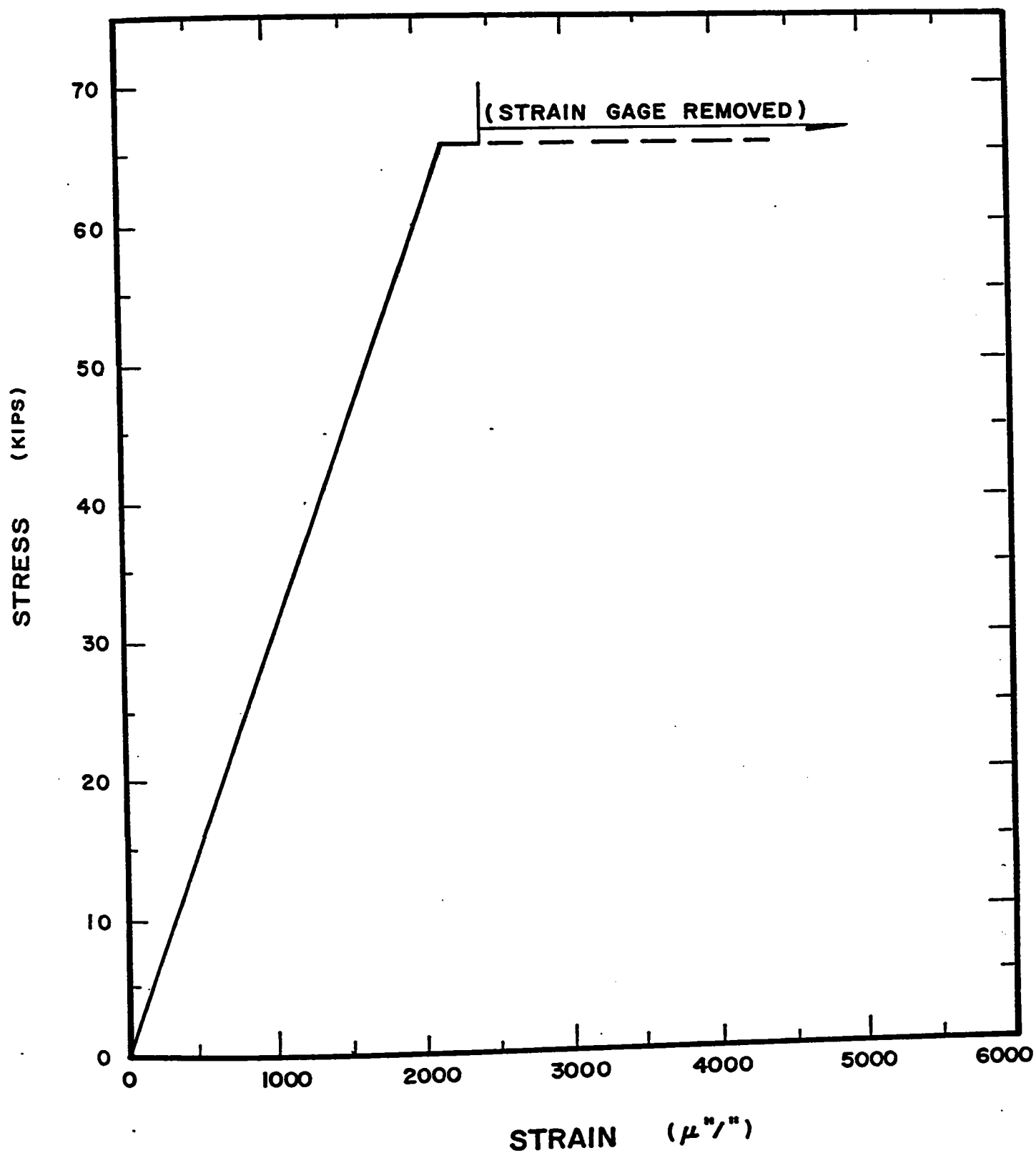
**FIG.(A.1) STRESS — STRAIN CURVE OF #3
LONGITUDINAL REINFORCEMENT (PLAIN BAR)**



**FIG.(A.2) STRESS — STRAIN CURVE OF #4
LONGITUDINAL REINFORCEMENT
(DEFORM BAR)**



**FIG.(A.3) STRESS — STRAIN CURVE OF #6
LONGITUDINAL REINFORCEMENT
(DEFORM BAR)**



**FIG.(A.4) STRESS — STRAIN CURVE OF #7
LONGITUDINAL REINFORCEMENT
(DEFORM BAR)**

APPENDIX B

INDIVIDUAL GRAPHS OF SPECIMENS

FIGURE (B.1)

to

FIGURE (B.85)

LOAD-DEFLECTIONS CURVES

OF SPECIMENS

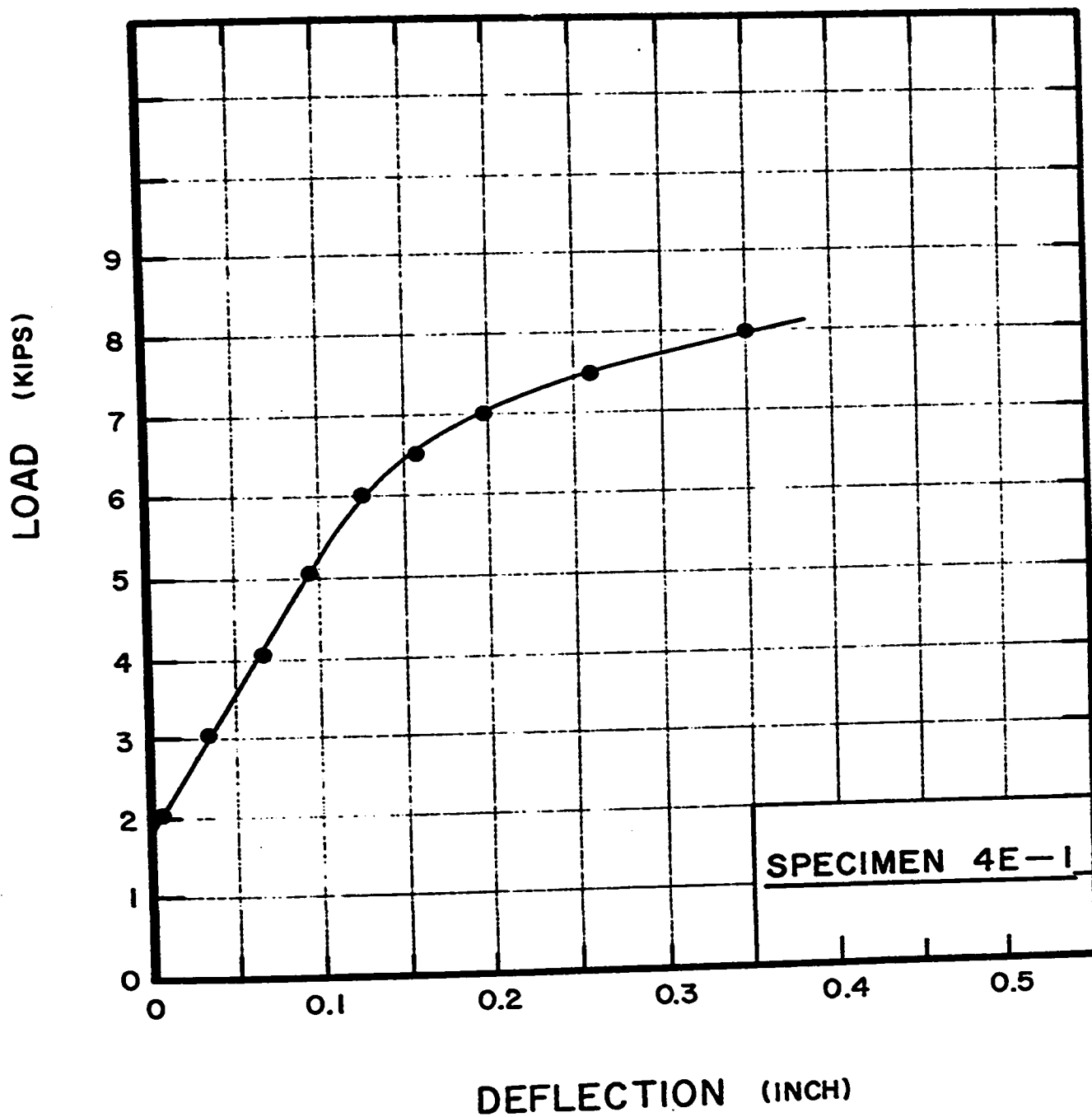


FIG. (B. 1) LOAD—DEFLECTION CURVE
OF SPECIMEN 4E-1

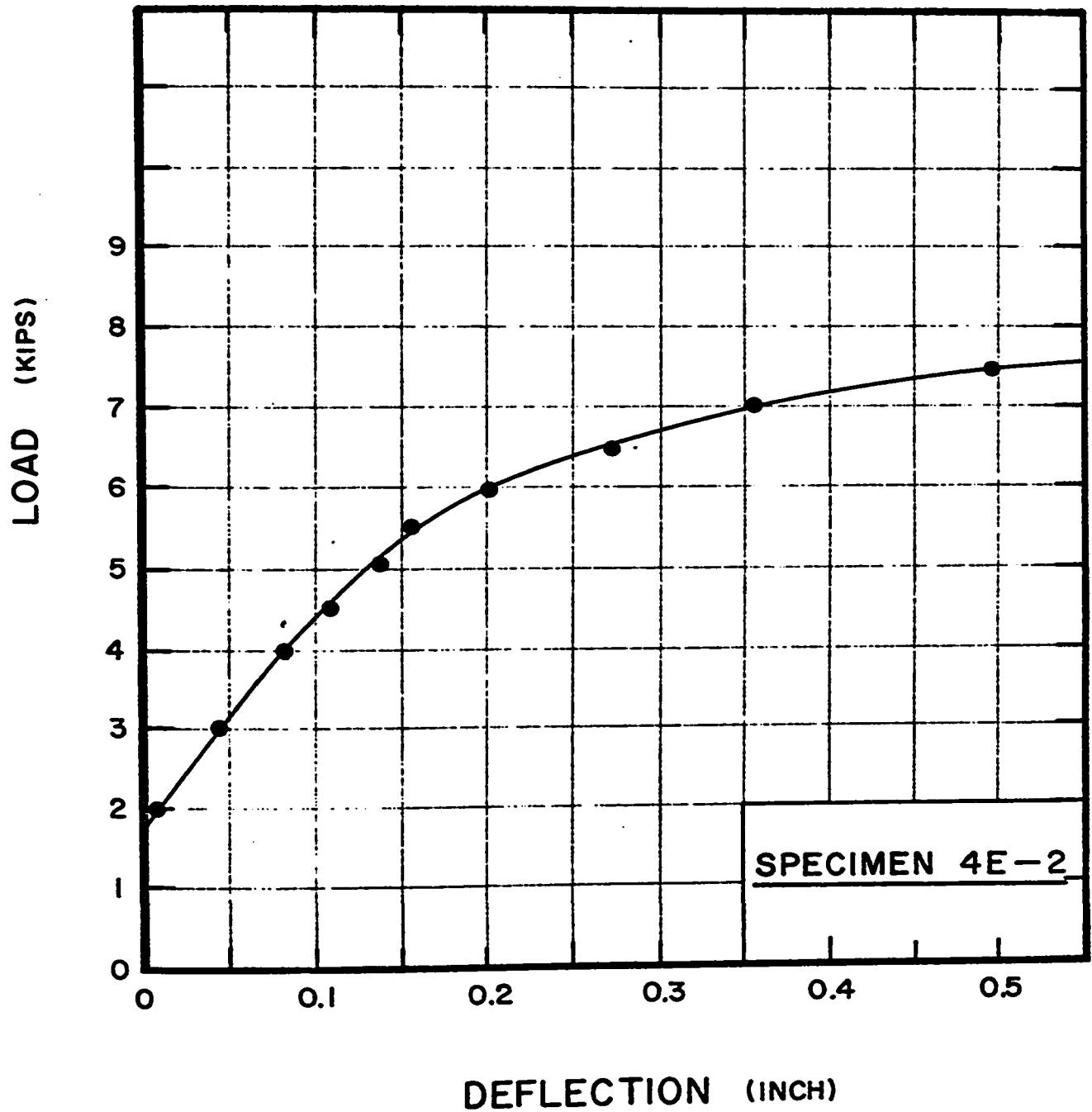


FIG. (B. 2) LOAD—DEFLECTION CURVE
OF SPECIMEN 4E-2

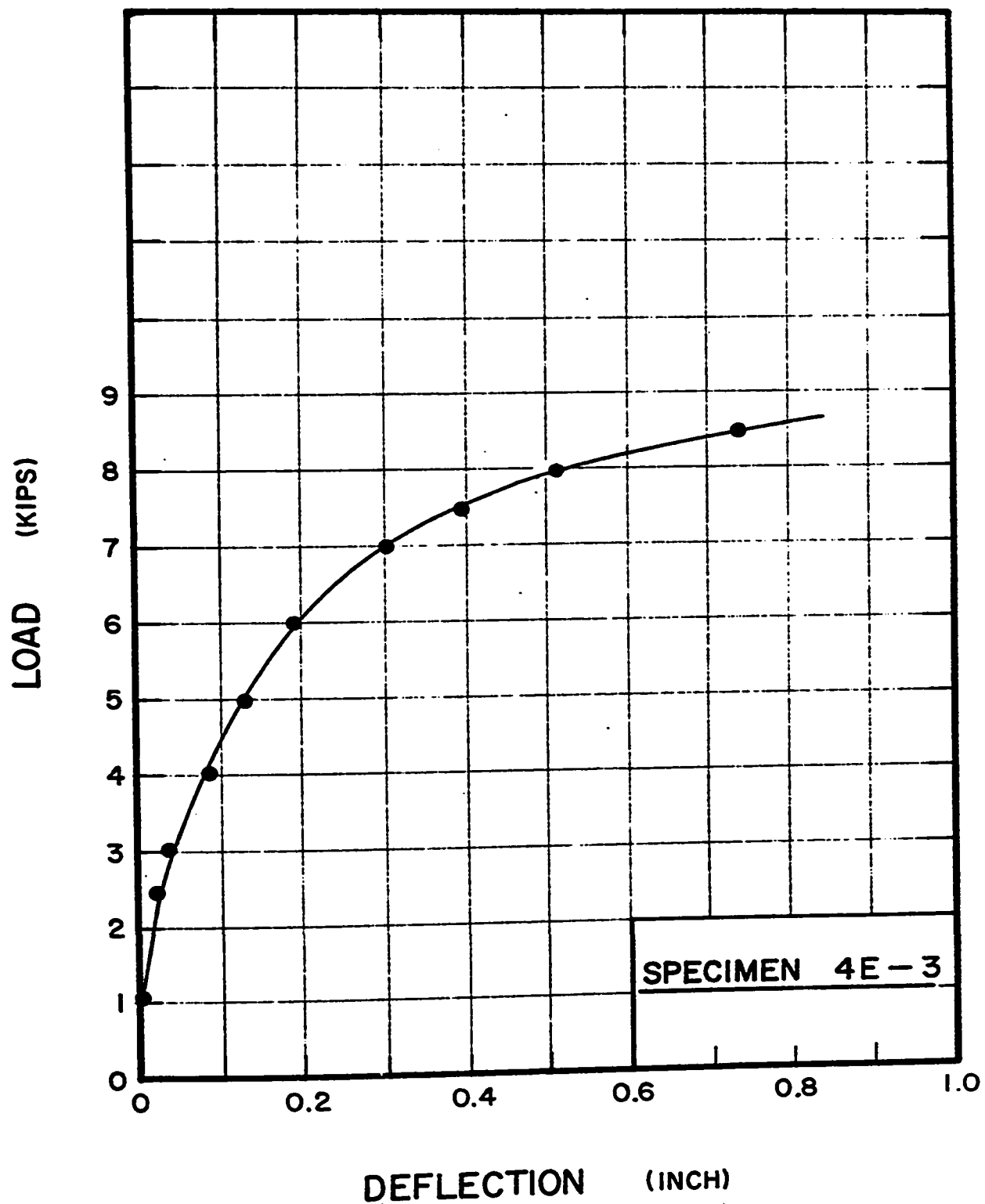


FIG.(B. 3) LOAD—DEFLECTION CURVE
OF SPECIMEN 4E-3

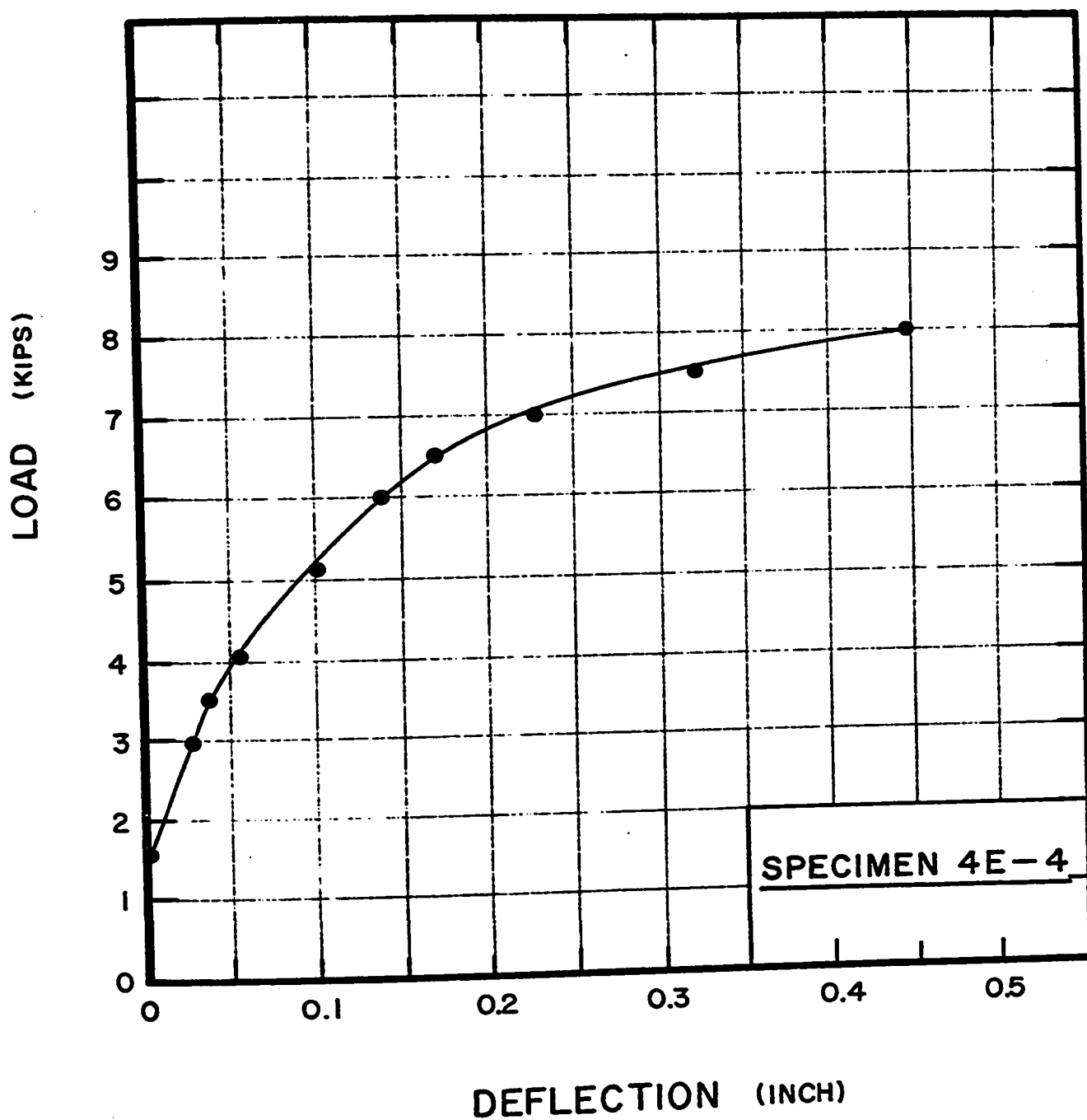


FIG. (B. 4) LOAD—DEFLECTION CURVE
OF SPECIMEN 4E-4

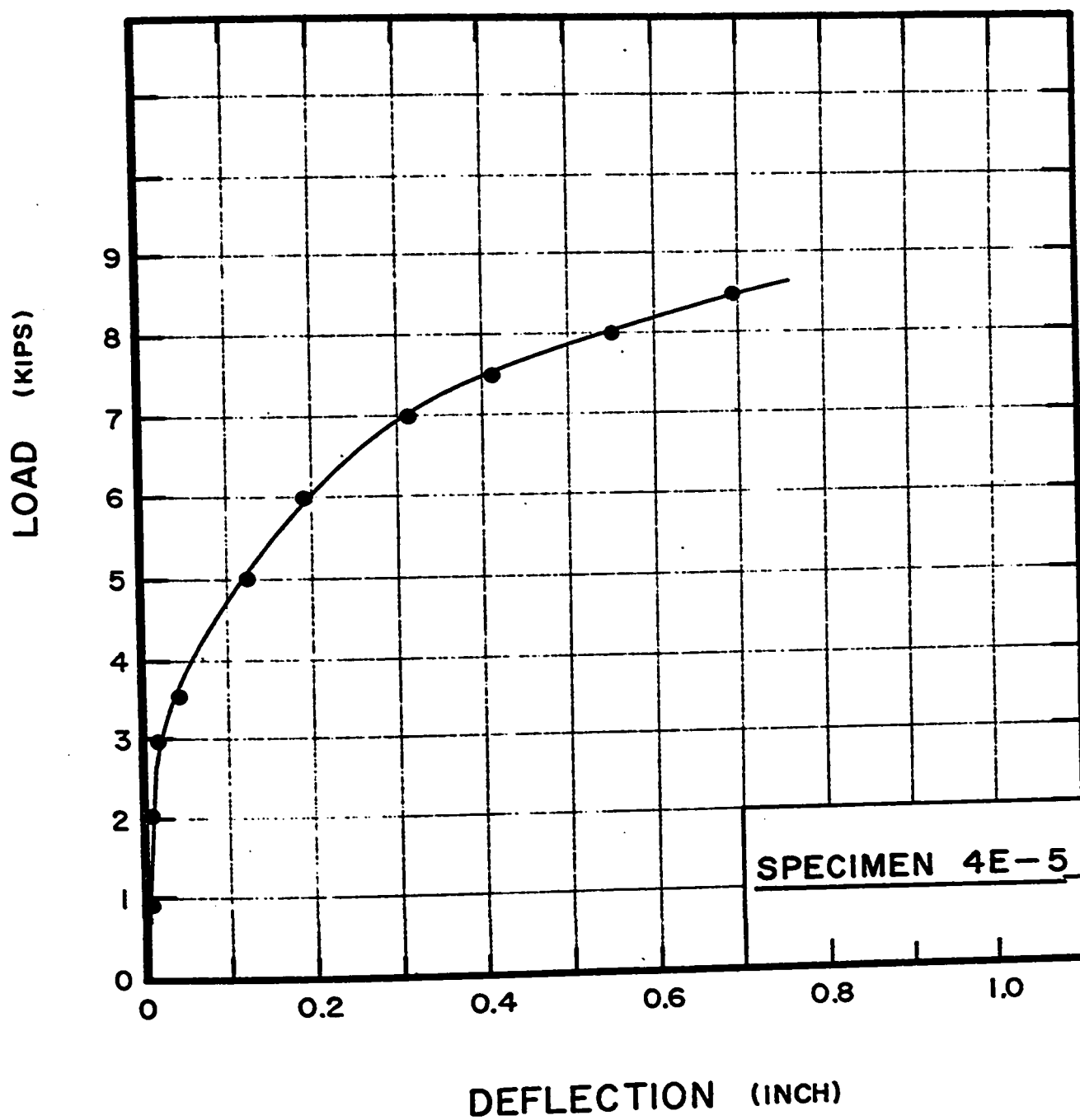
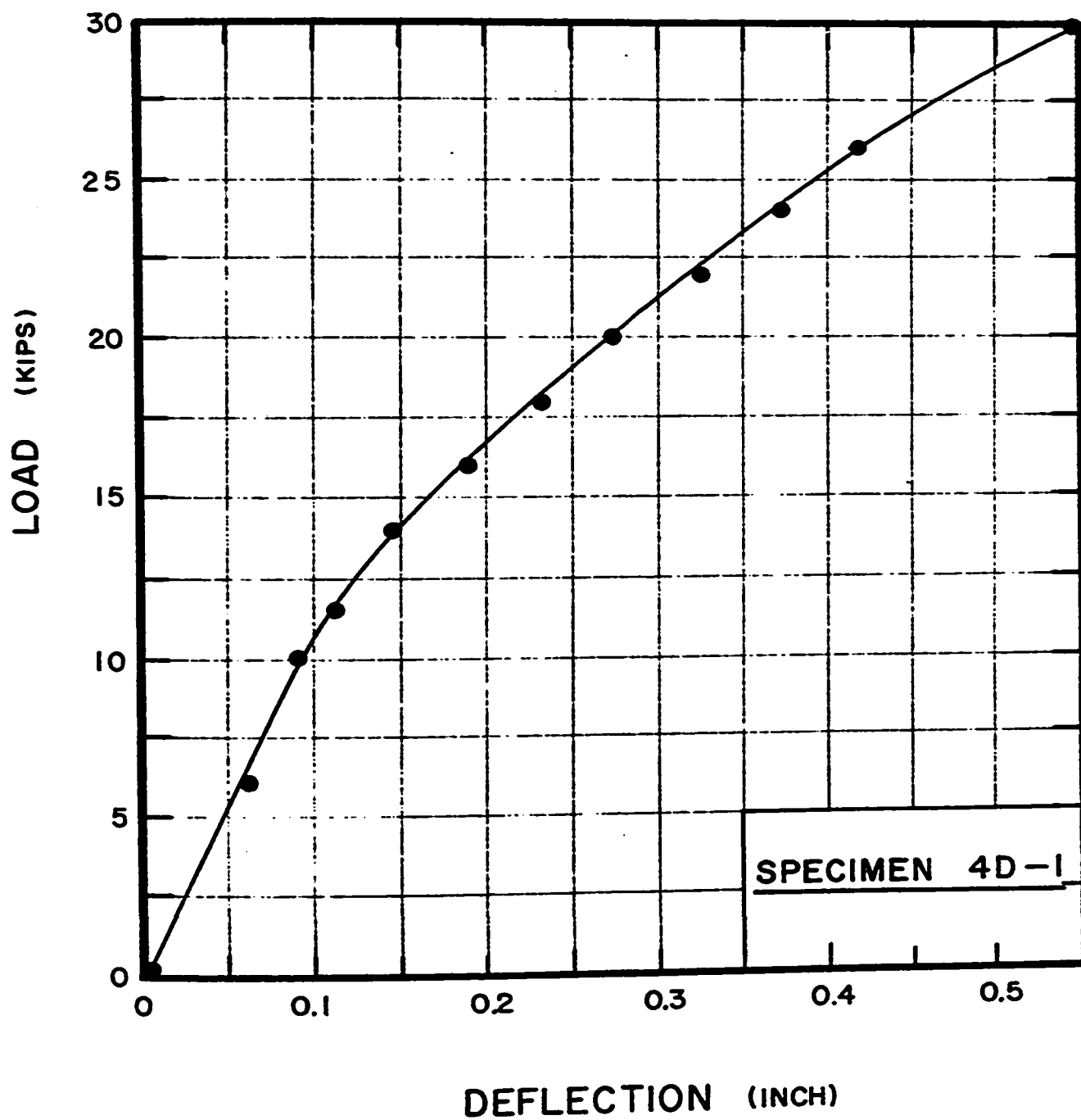
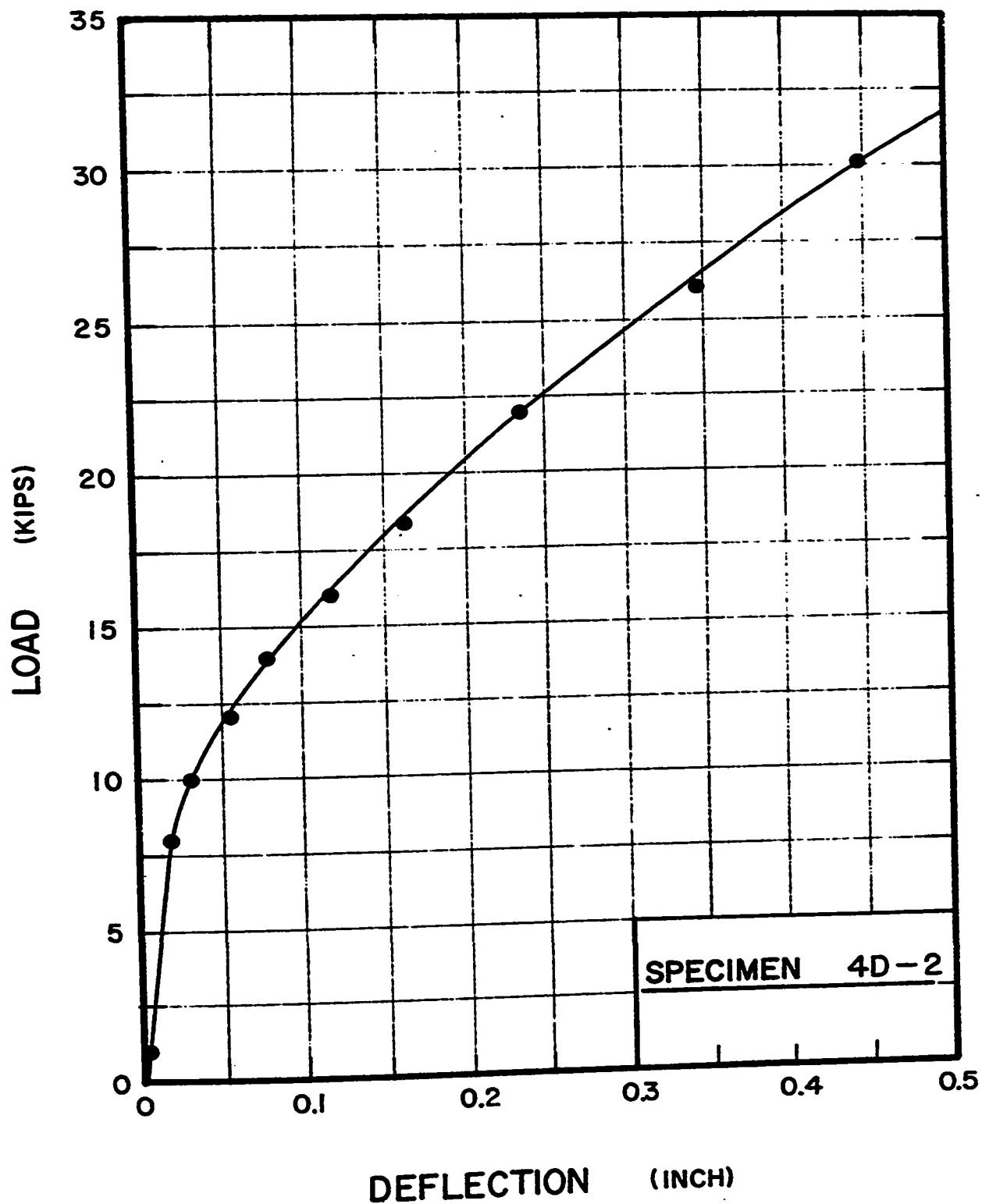


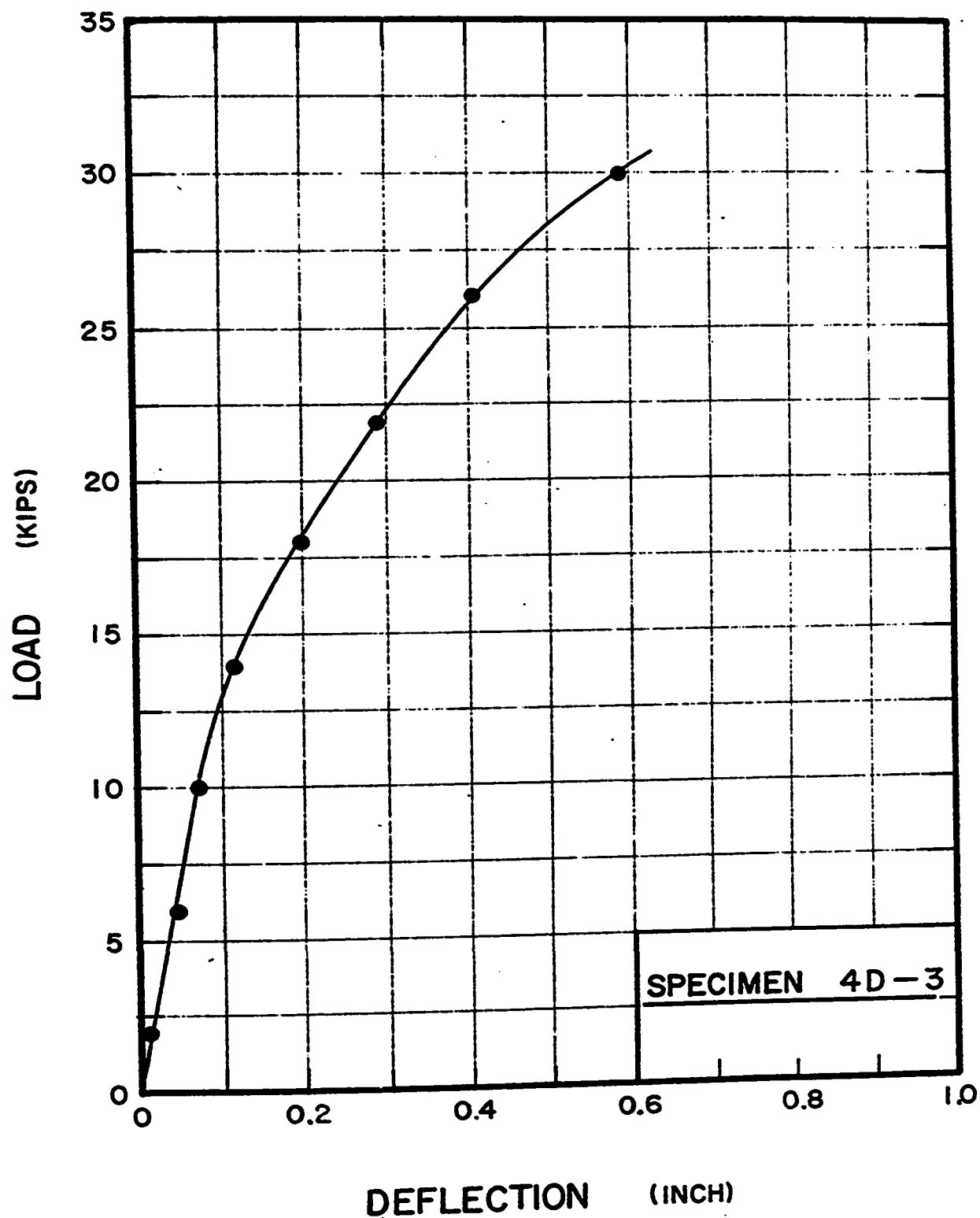
FIG. (B. 5) LOAD—DEFLECTION CURVE
OF SPECIMEN 4E-5



**FIG. (B. 6) LOAD—DEFLECTION CURVE
OF SPECIMEN 4D—1**



**FIG.(B. 7) LOAD—DEFLECTION CURVE
OF SPECIMEN 4D—2**



**FIG.(B. 8) LOAD—DEFLECTION CURVE
OF SPECIMEN 4D—3**

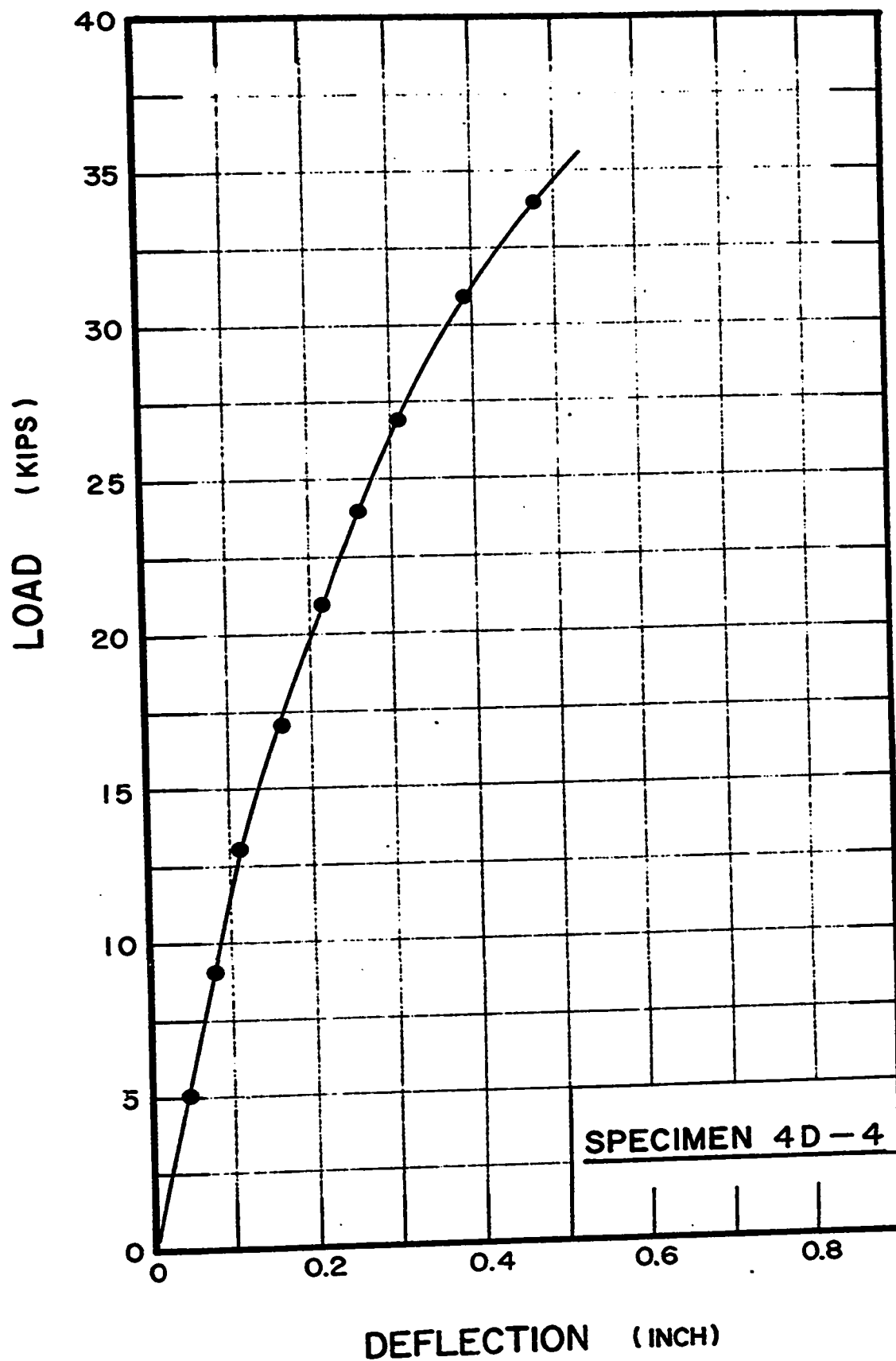


FIG.(B. 9) LOAD-DEFLECTION
CURVE OF SPECIMEN 4D-4

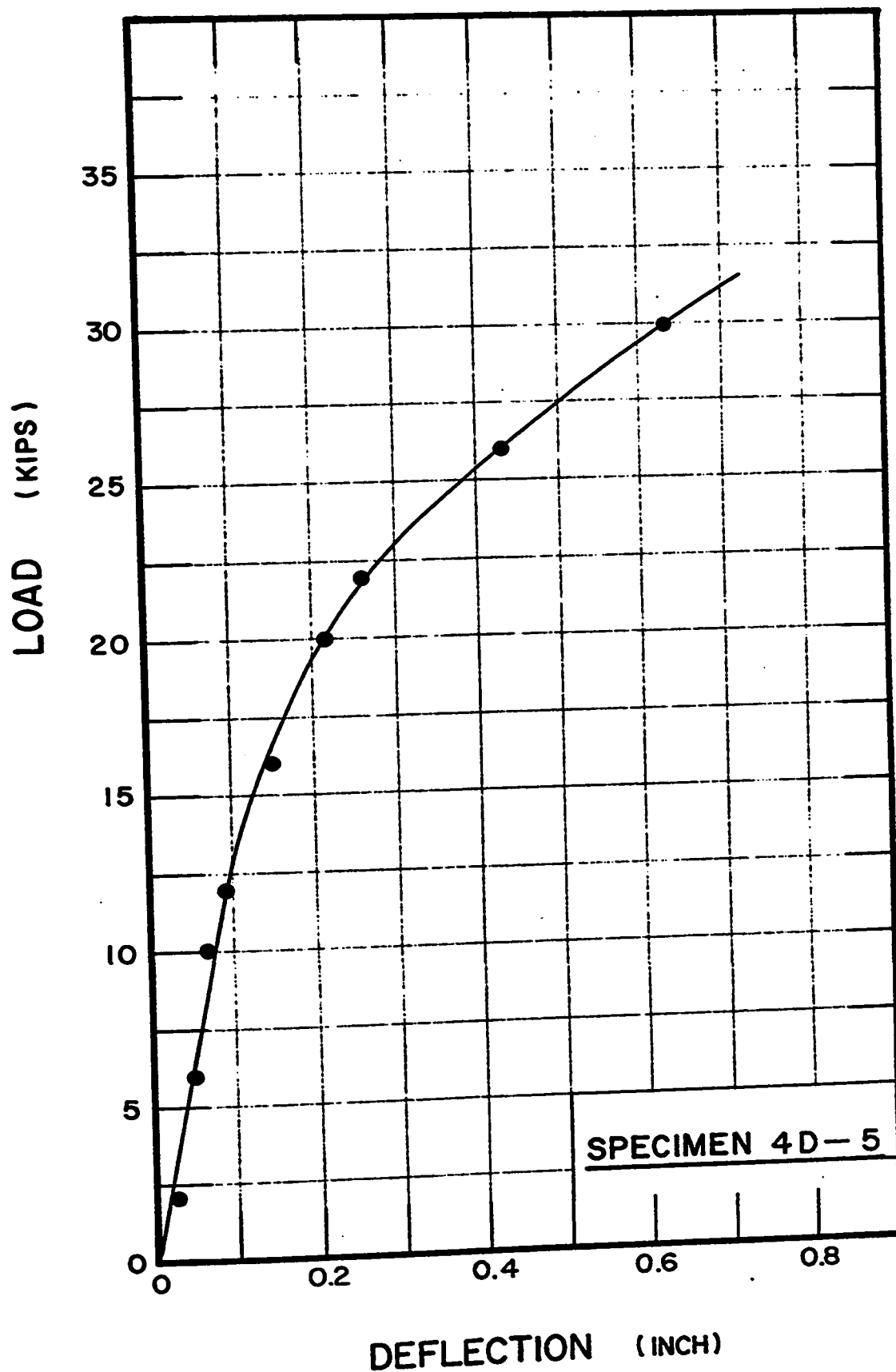
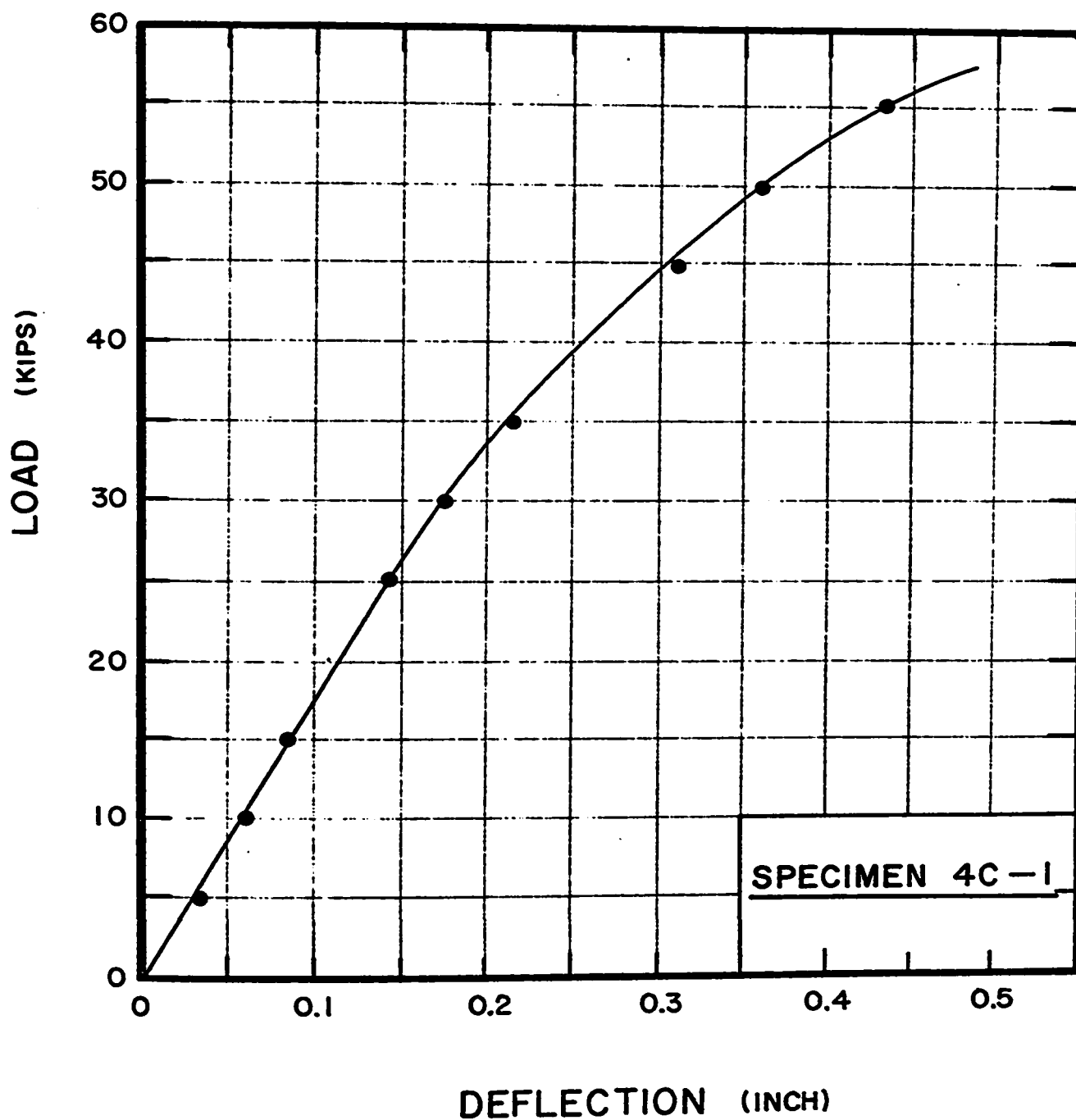
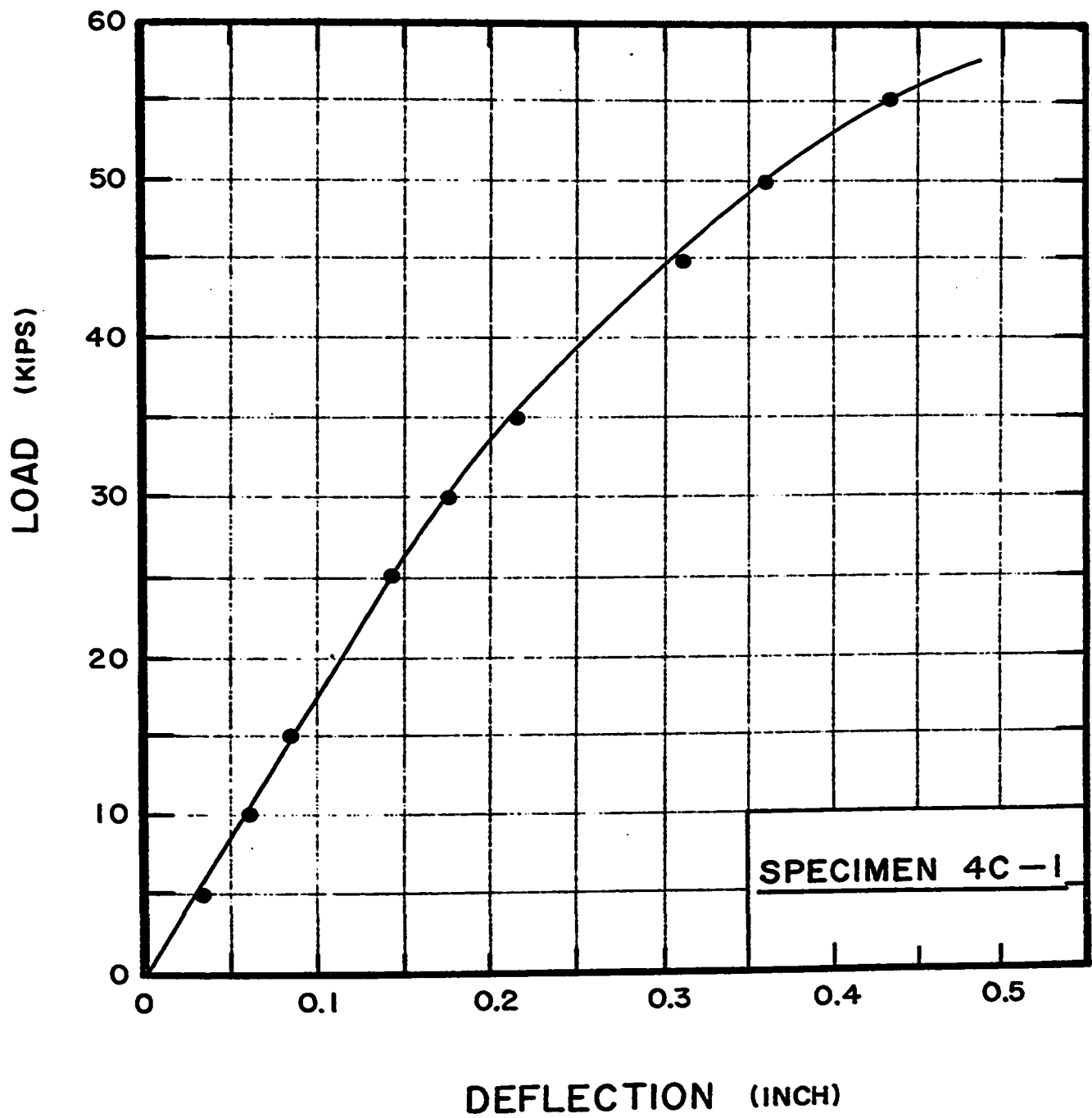


FIG.(B. 10) LOAD-DEFLECTION
CURVE OF SPECIMEN 4D-5



**FIG. (B. 11) LOAD—DEFLECTION CURVE
OF SPECIMEN 4C-1**



**FIG. (B. 11) LOAD—DEFLECTION CURVE
OF SPECIMEN 4C-1**

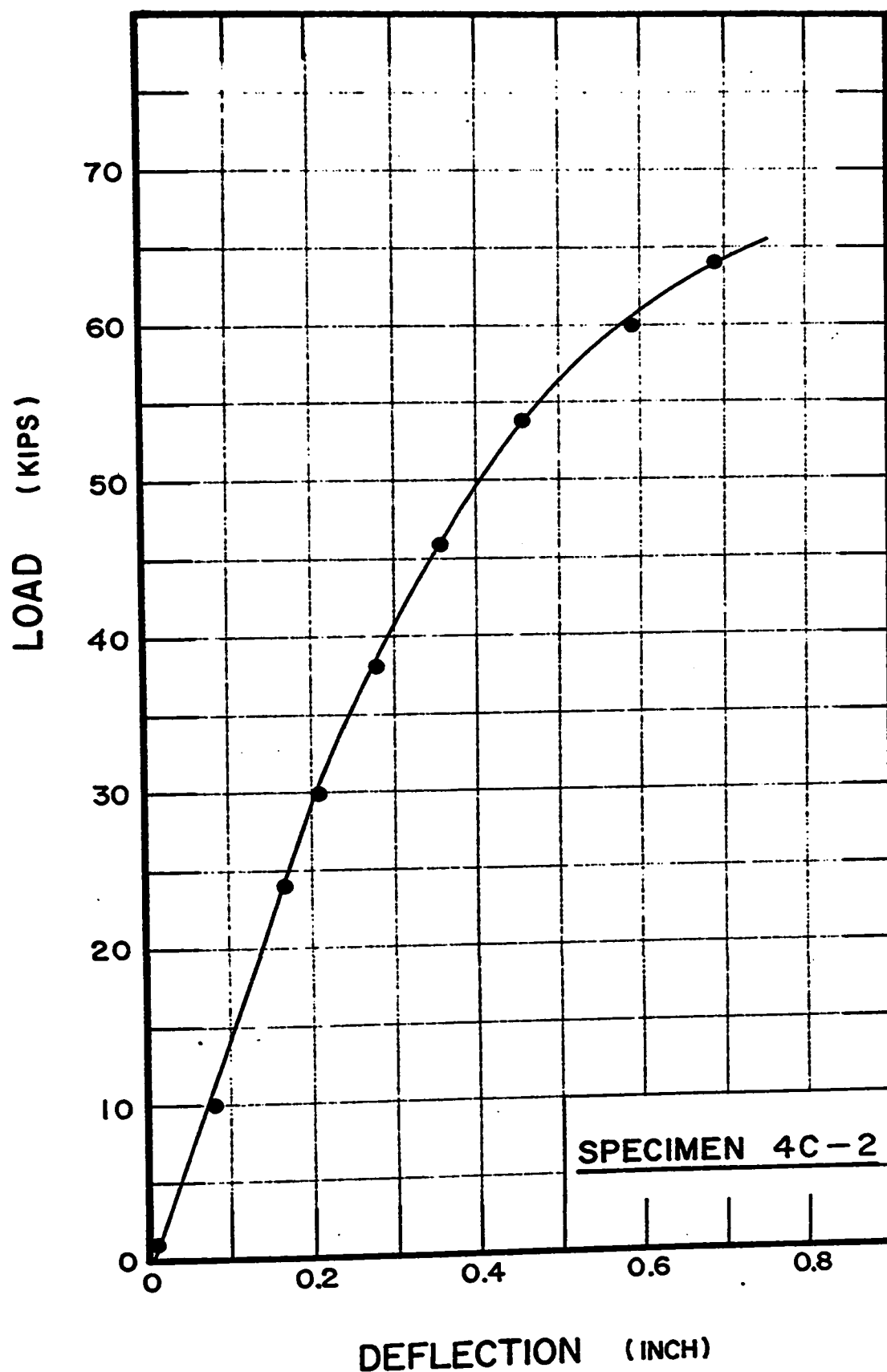
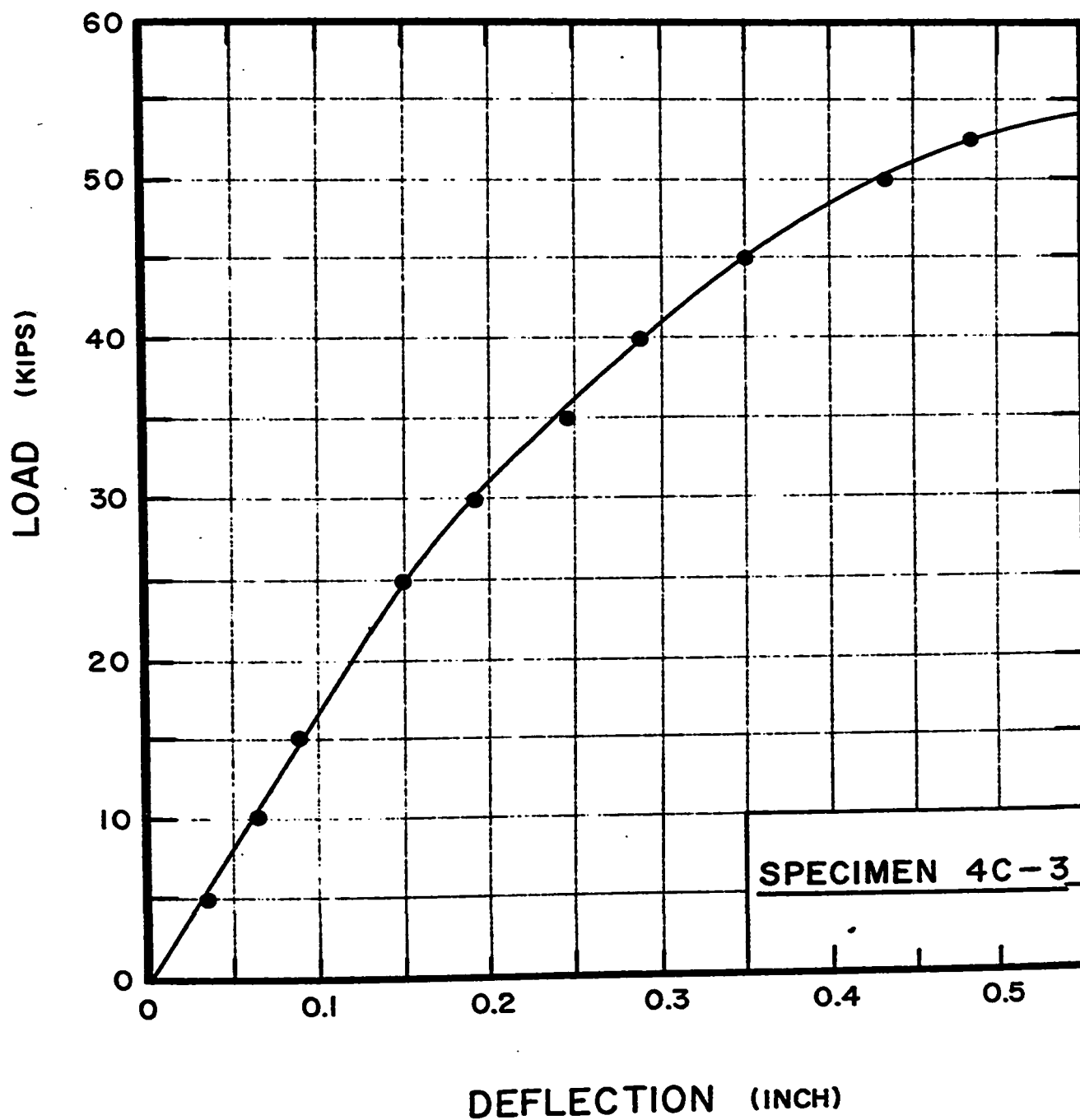


FIG.(B. 12) LOAD-DEFLECTION
CURVE OF SPECIMEN 4C-2



**FIG. (B. 13) LOAD—DEFLECTION CURVE
OF SPECIMEN 4C-3**

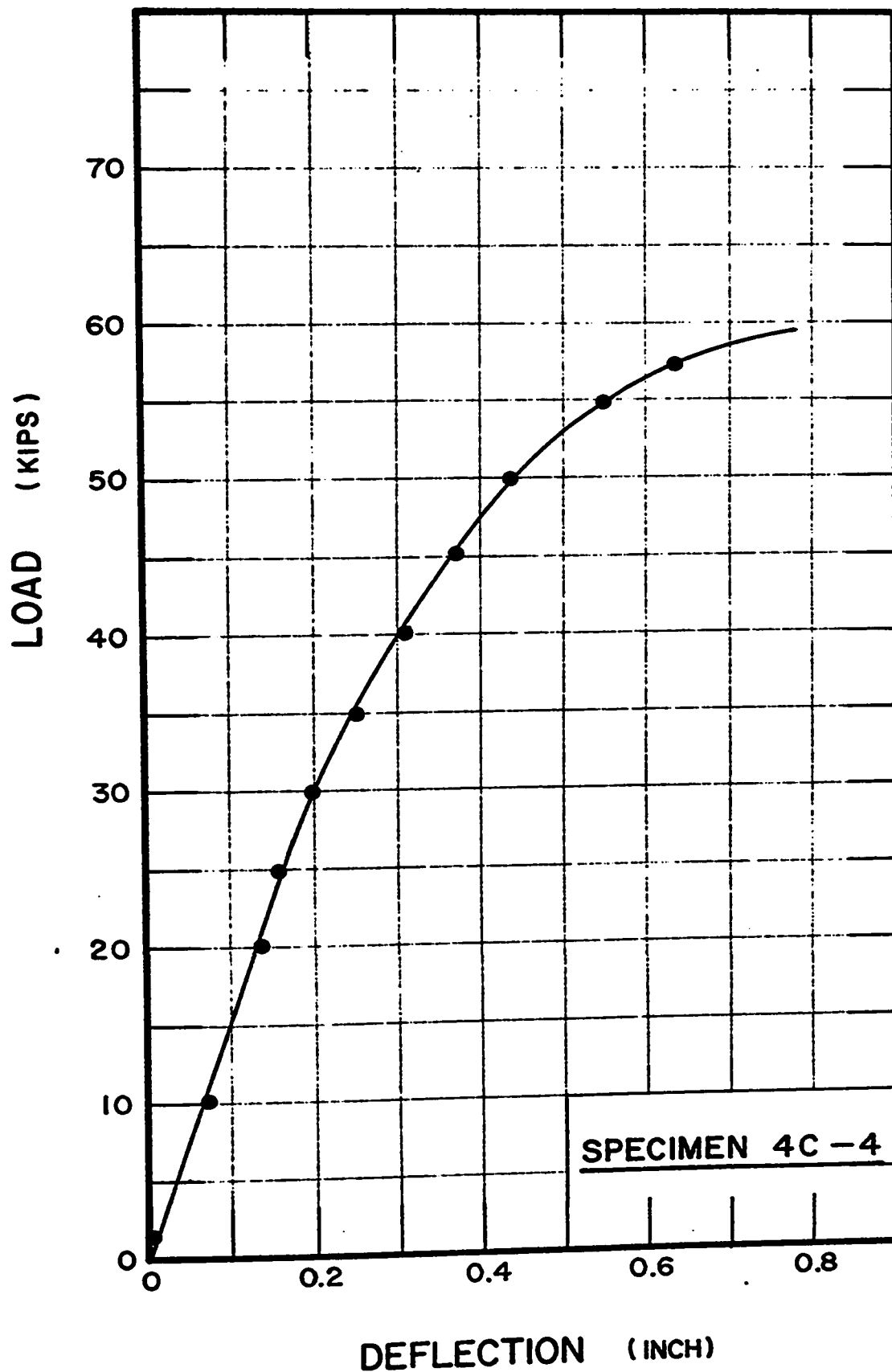


FIG.(B.14) LOAD-DEFLECTION
CURVE OF SPECIMEN 4C-4

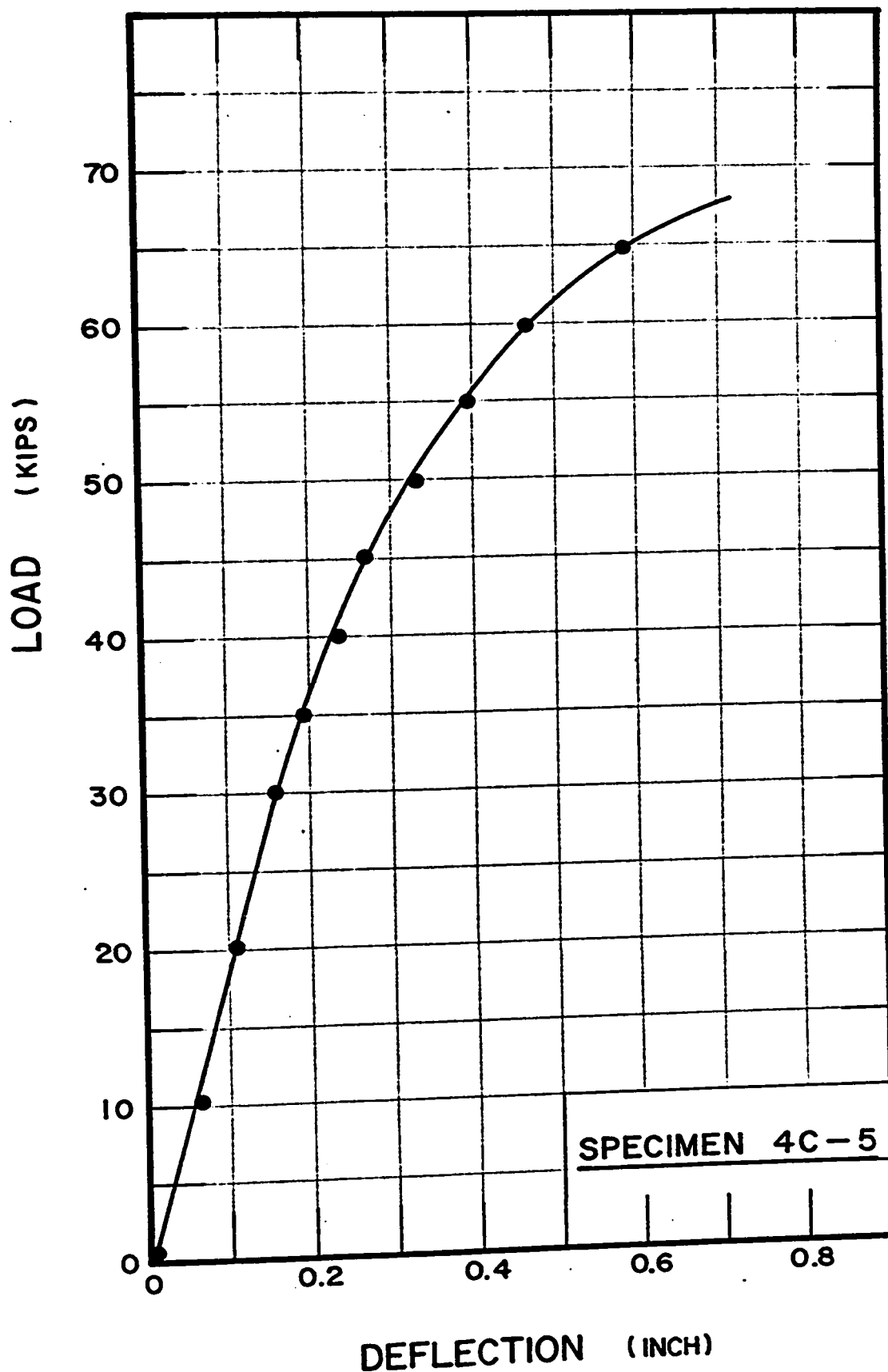
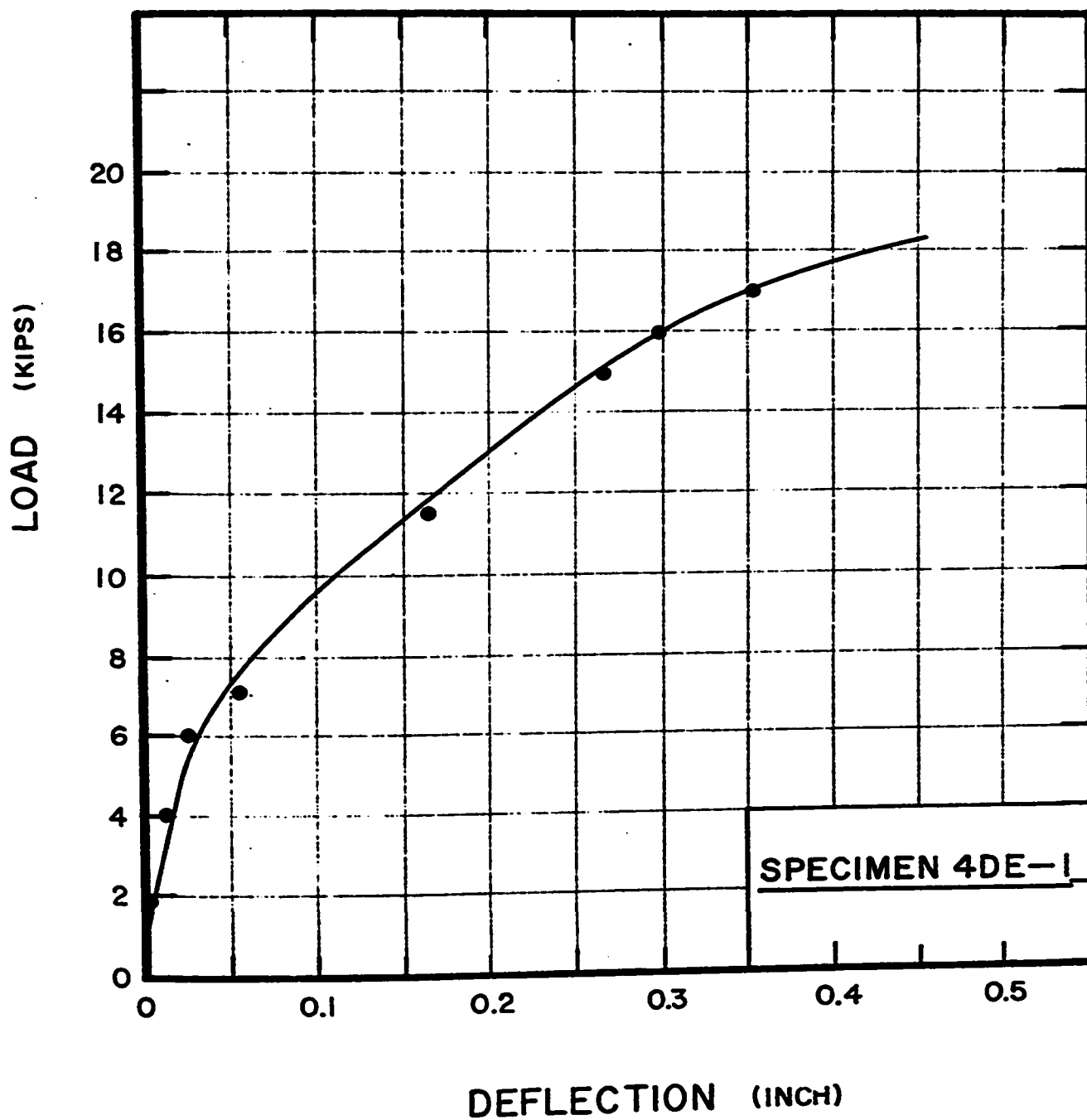


FIG.(B. 15) LOAD—DEFLECTION
CURVE OF SPECIMEN 4C-5



**FIG. (B. 16) LOAD—DEFLECTION CURVE
OF SPECIMEN 4DE-1**

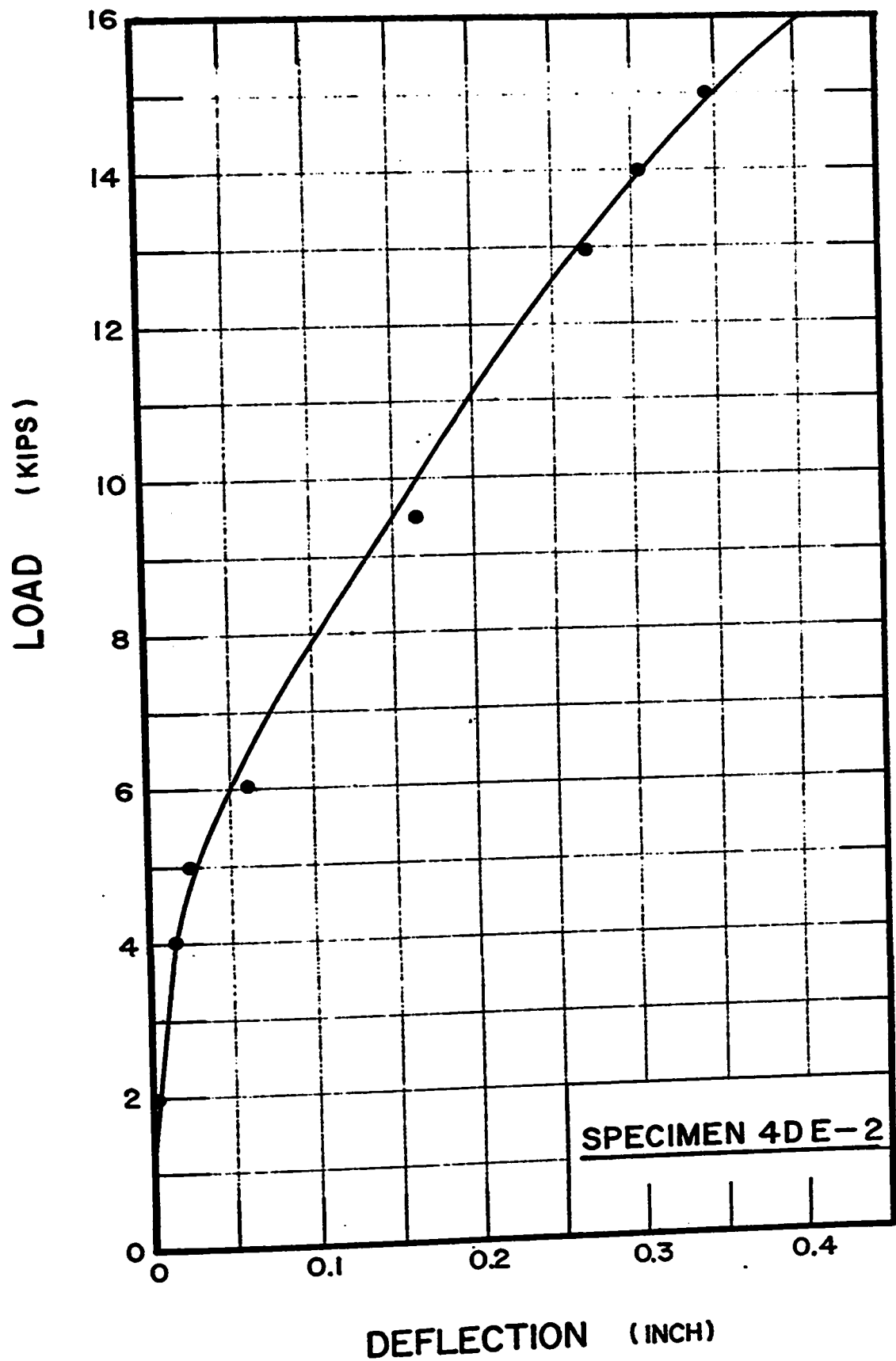
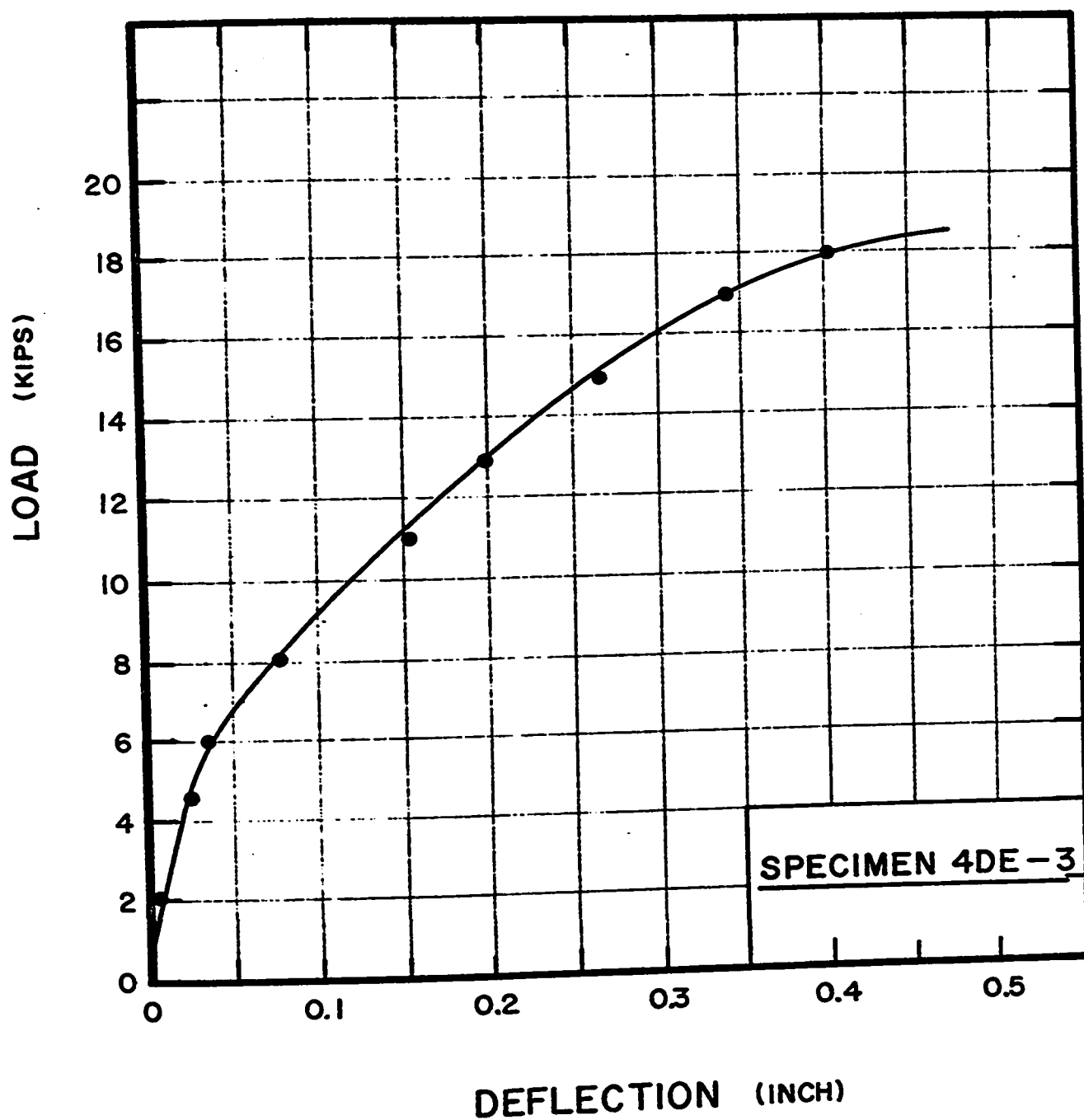
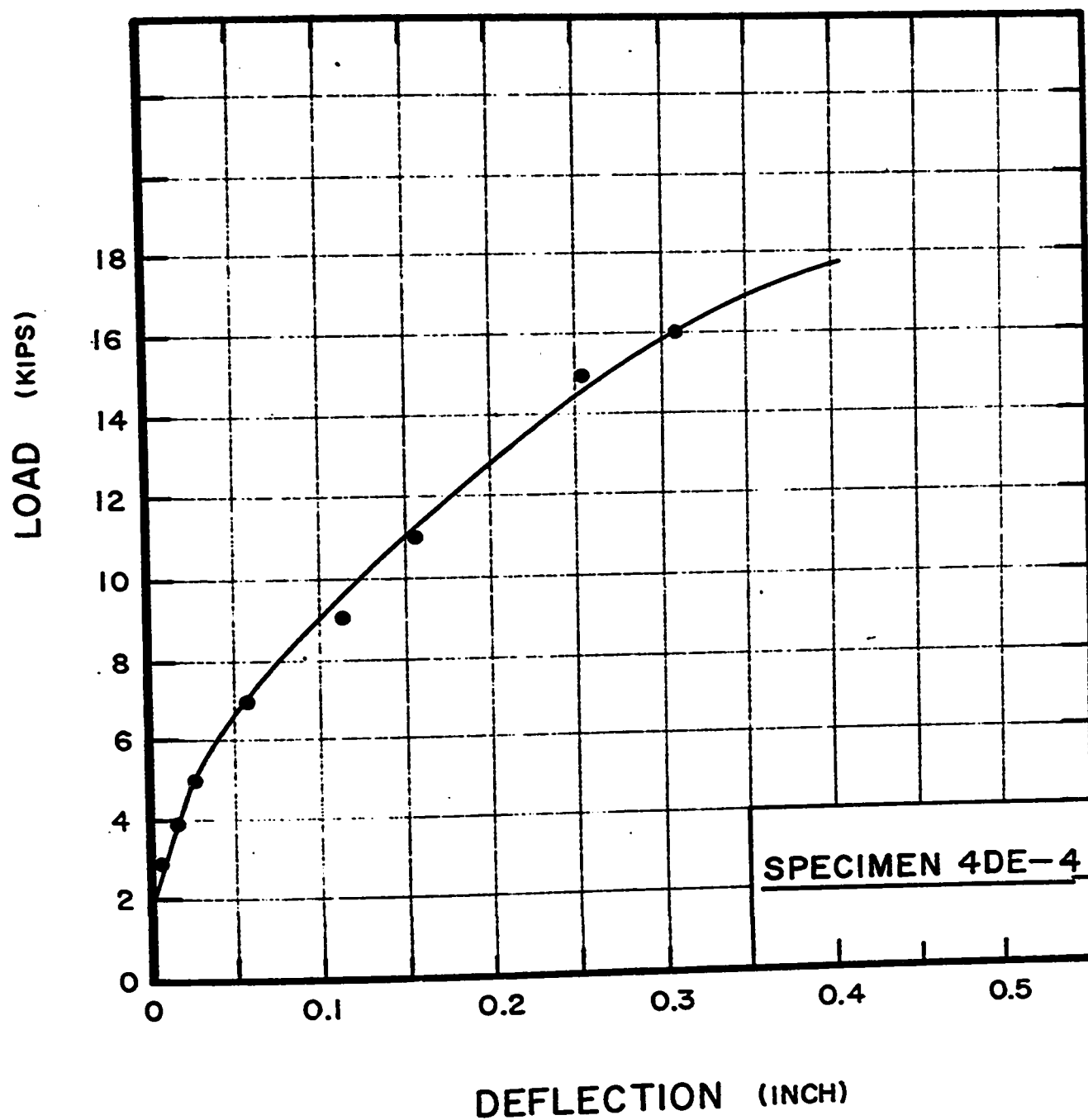


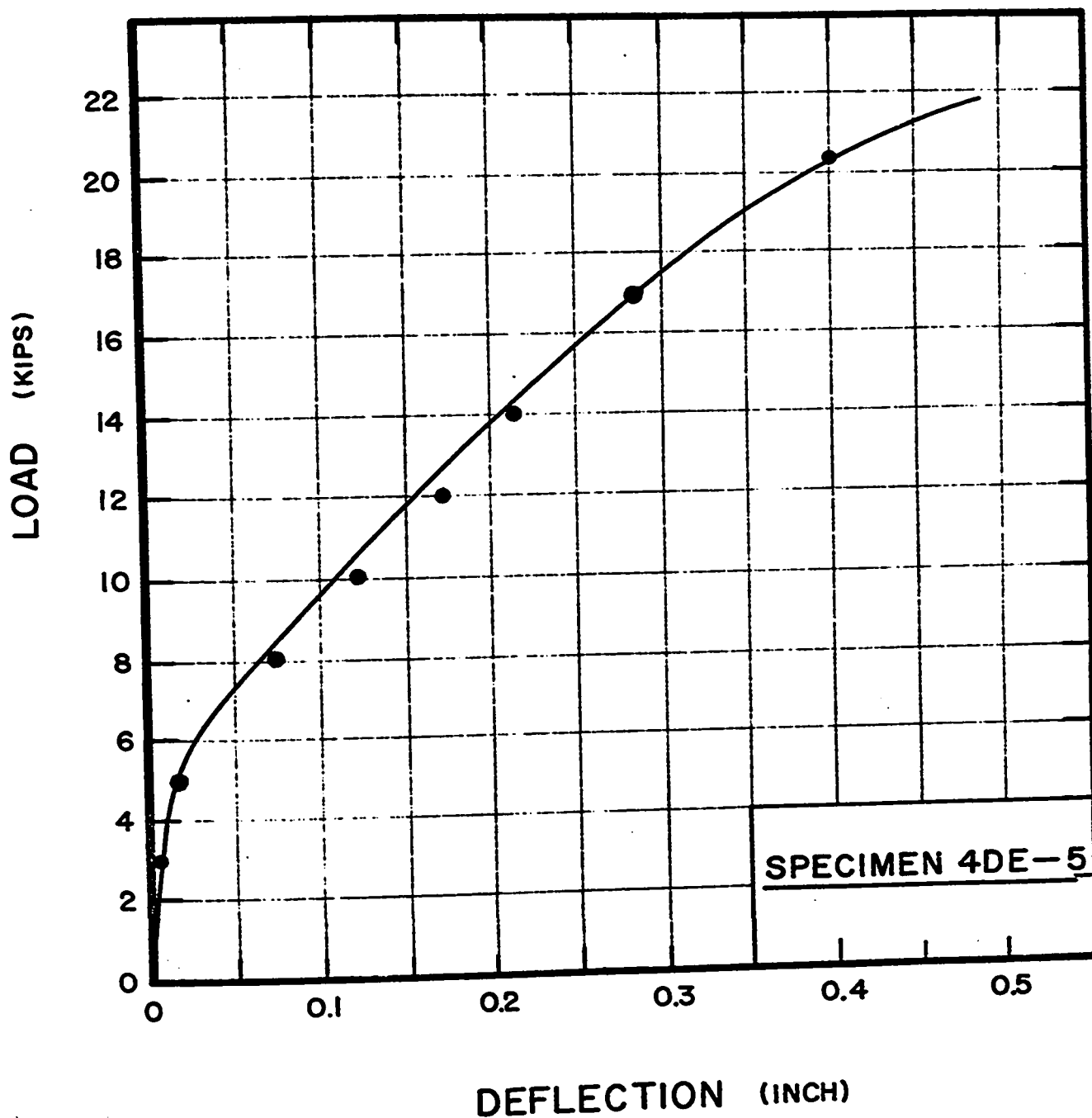
FIG.(B. 17) LOAD-DEFLECTION
CURVE OF SPECIMEN 4DE-2



**FIG. (B. 18) LOAD—DEFLECTION CURVE
OF SPECIMEN 4DE-3**



**FIG. (B. 19) LOAD—DEFLECTION CURVE
OF SPECIMEN 4DE-4**



**FIG. (B. 20) LOAD—DEFLECTION CURVE
OF SPECIMEN 4DE-5**

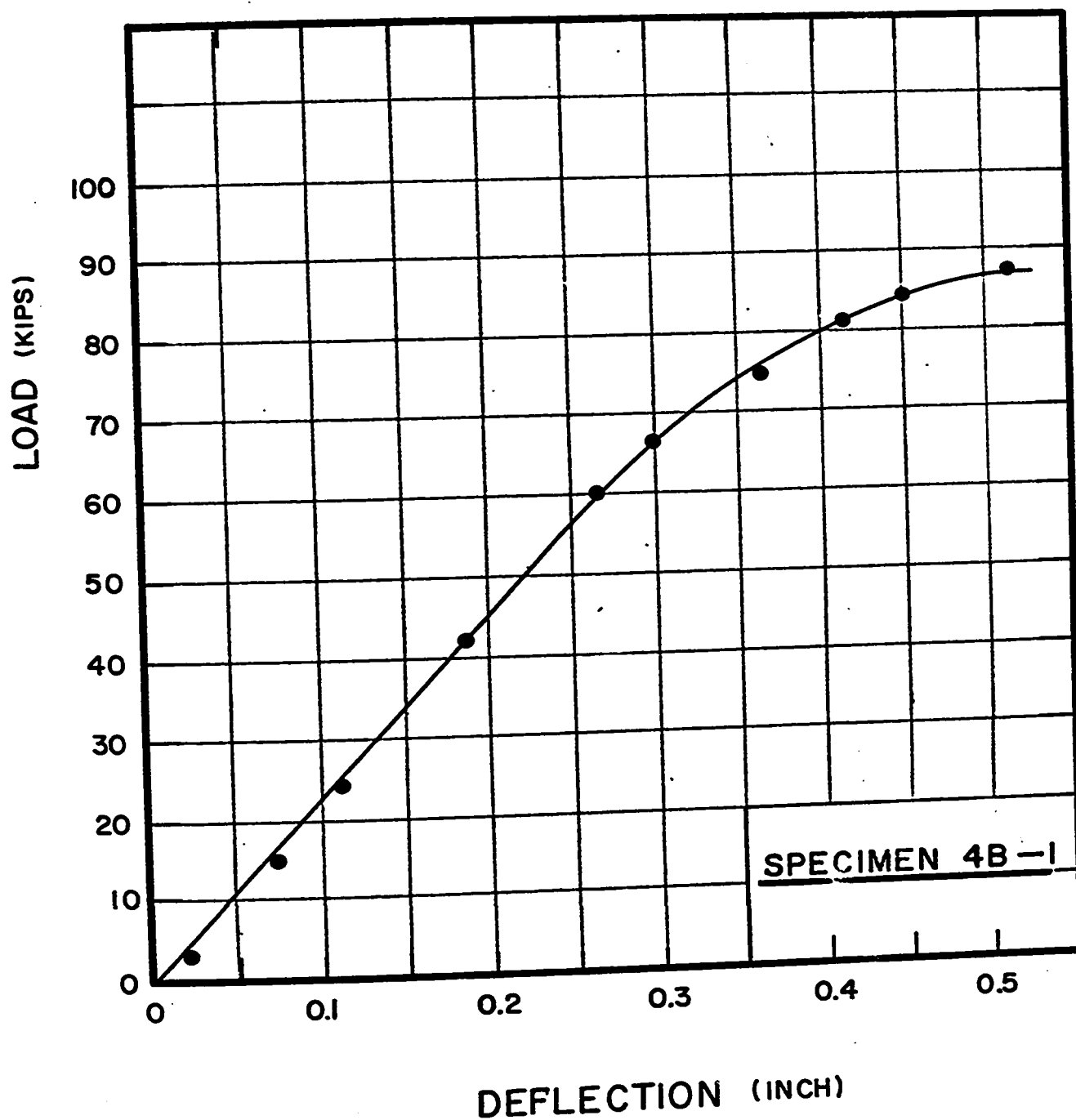
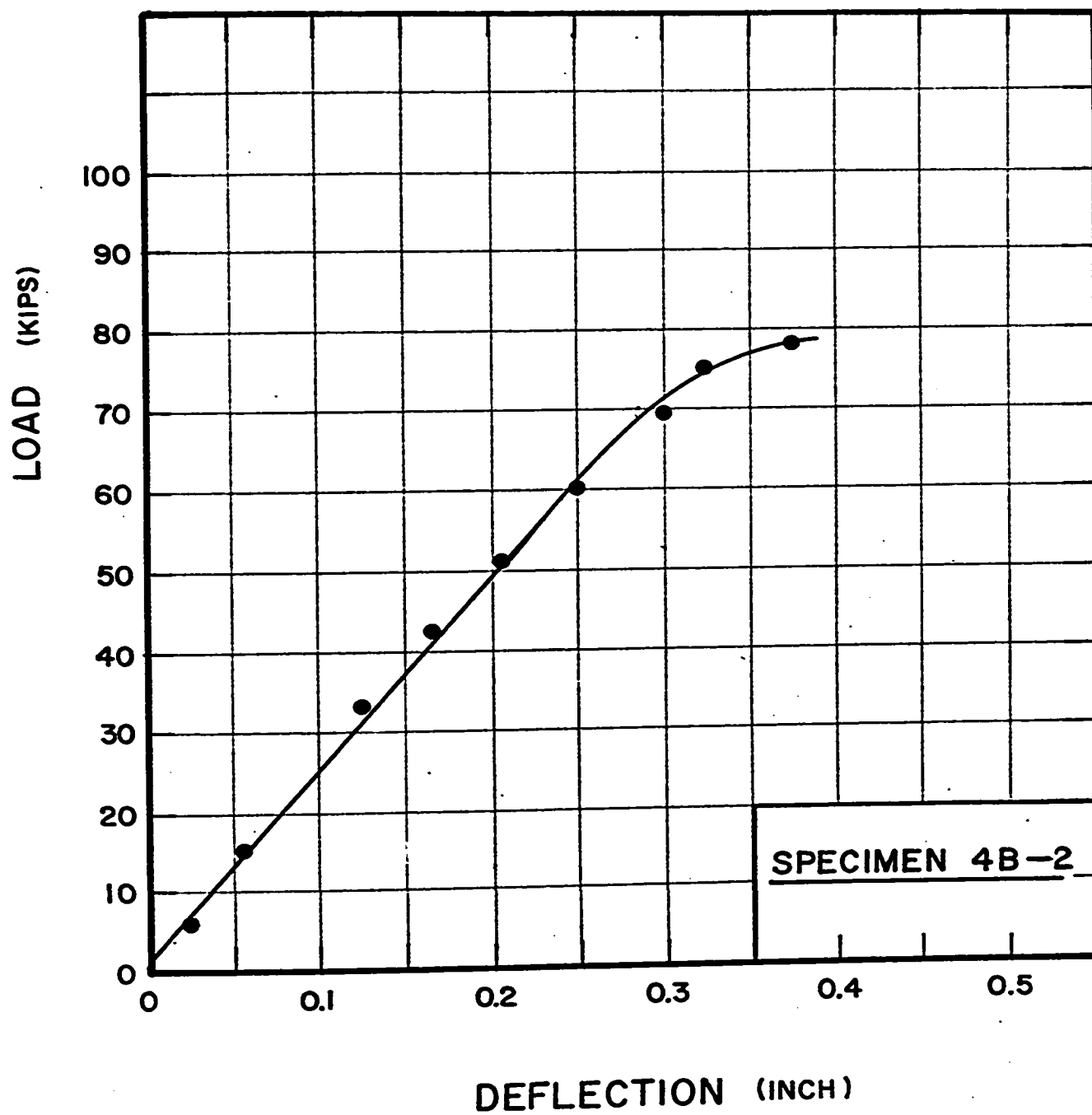
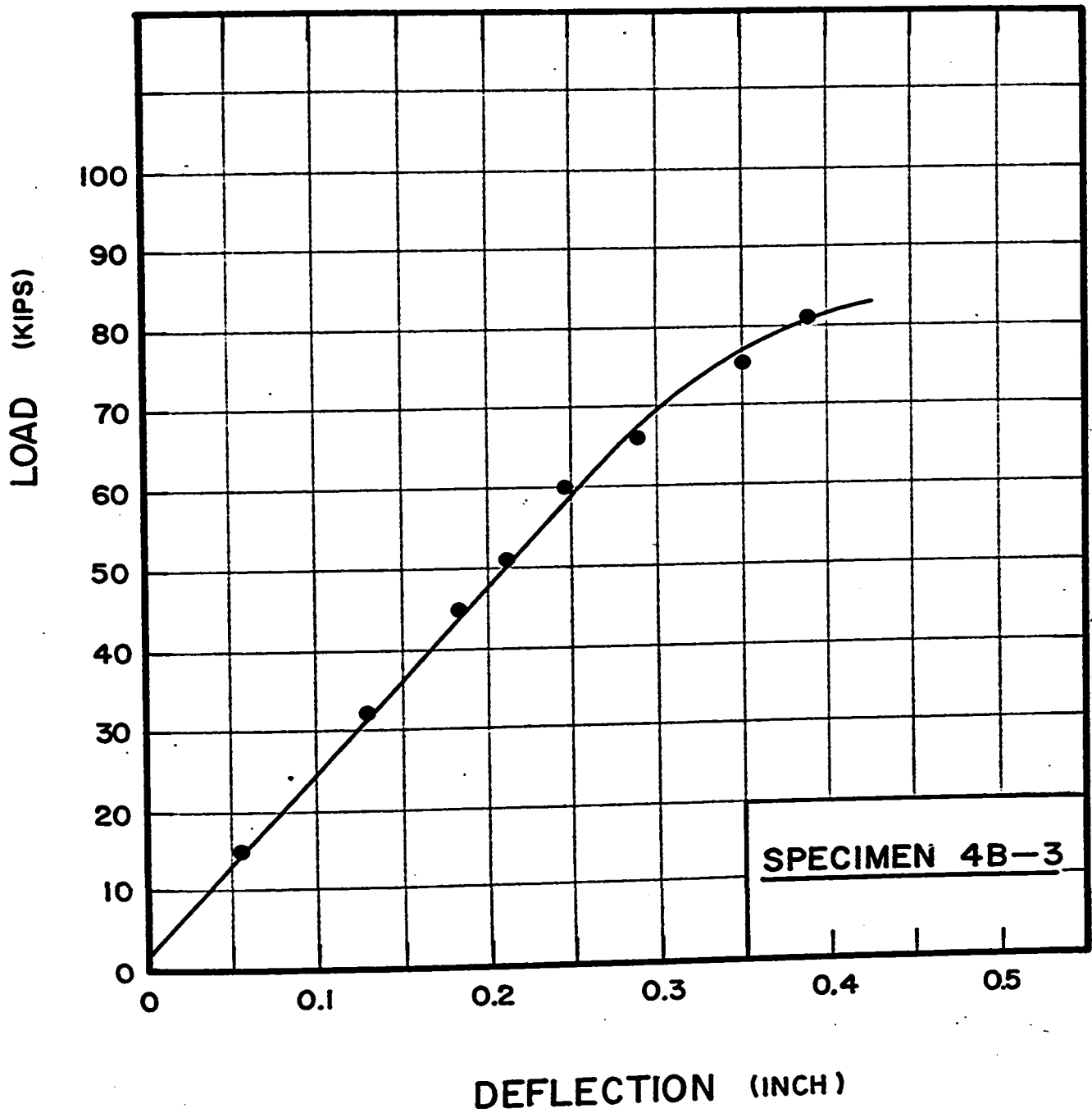


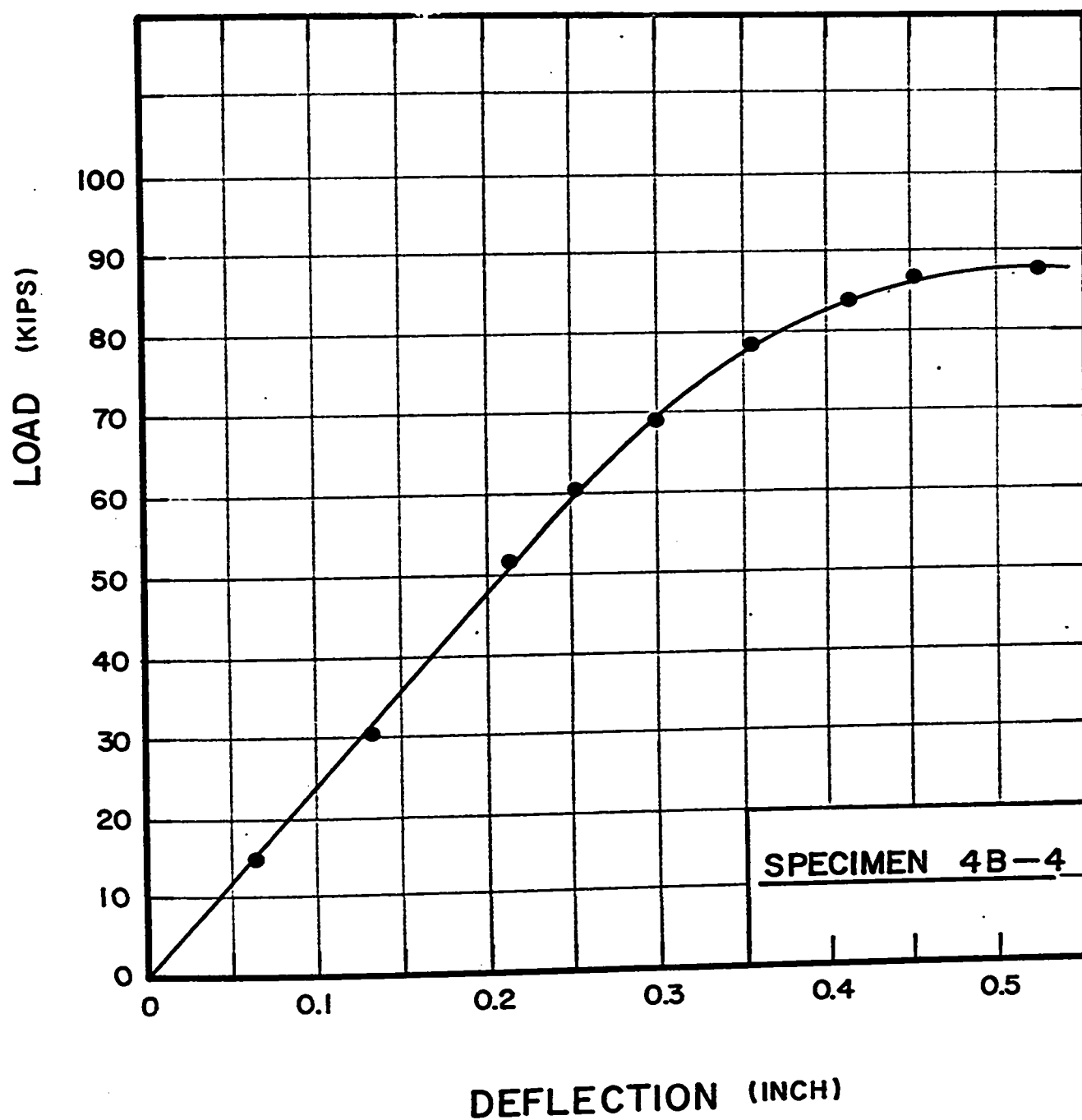
FIG.(B. 21) LOAD — DEFLECTION CURVE
OF SPECIMEN 4B-1



**FIG. (B. 22) LOAD — DEFLECTION CURVE
OF SPECIMEN 4B-2**



**FIG. (B. 23) LOAD — DEFLECTION CURVE
OF SPECIMEN 4B-3**



**FIG.(B.24) LOAD — DEFLECTION CURVE
OF SPECIMEN 4B — 4**

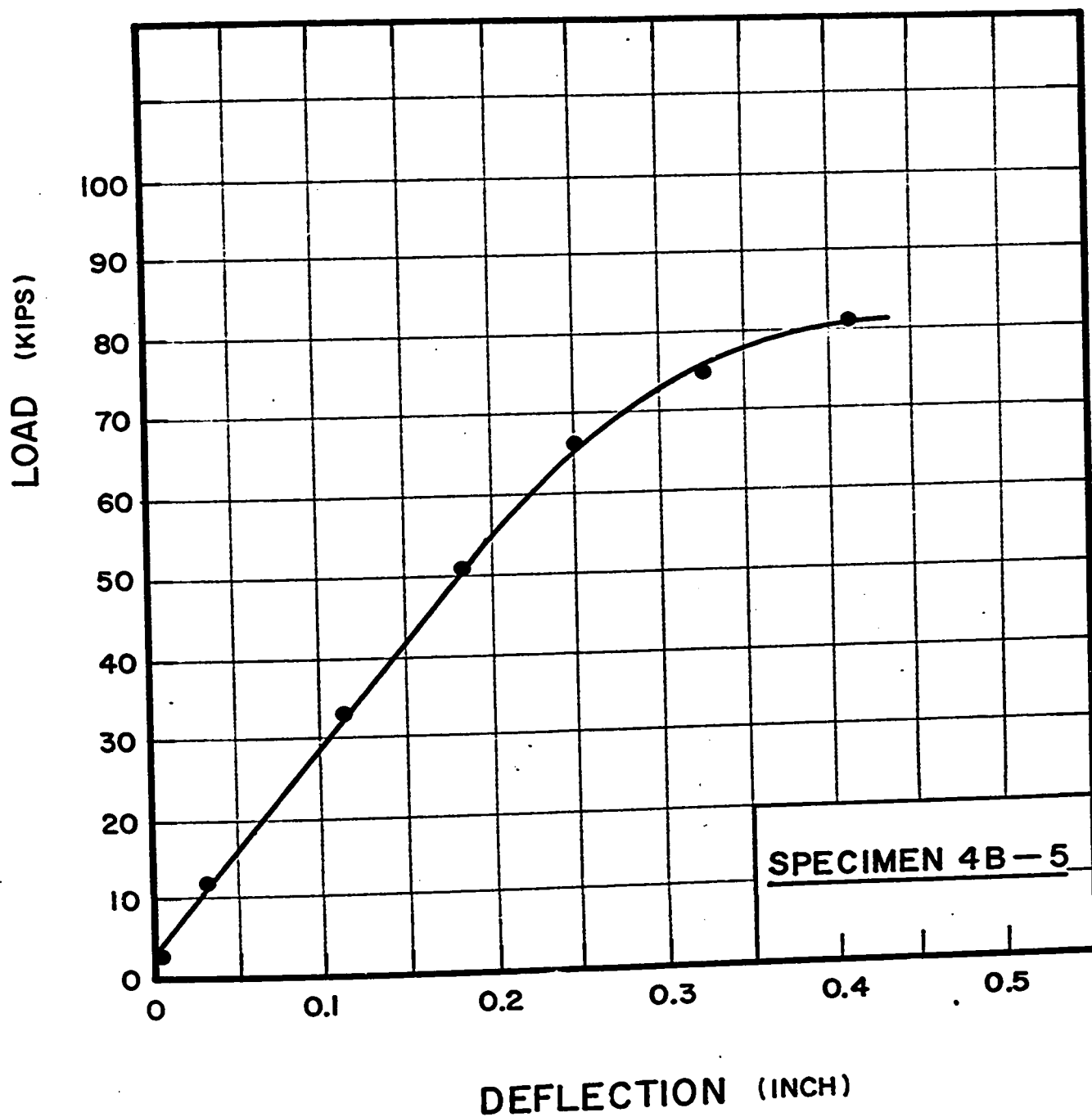


FIG.(B. 25) LOAD — DEFLECTION CURVE
OF SPECIMEN 4B-5

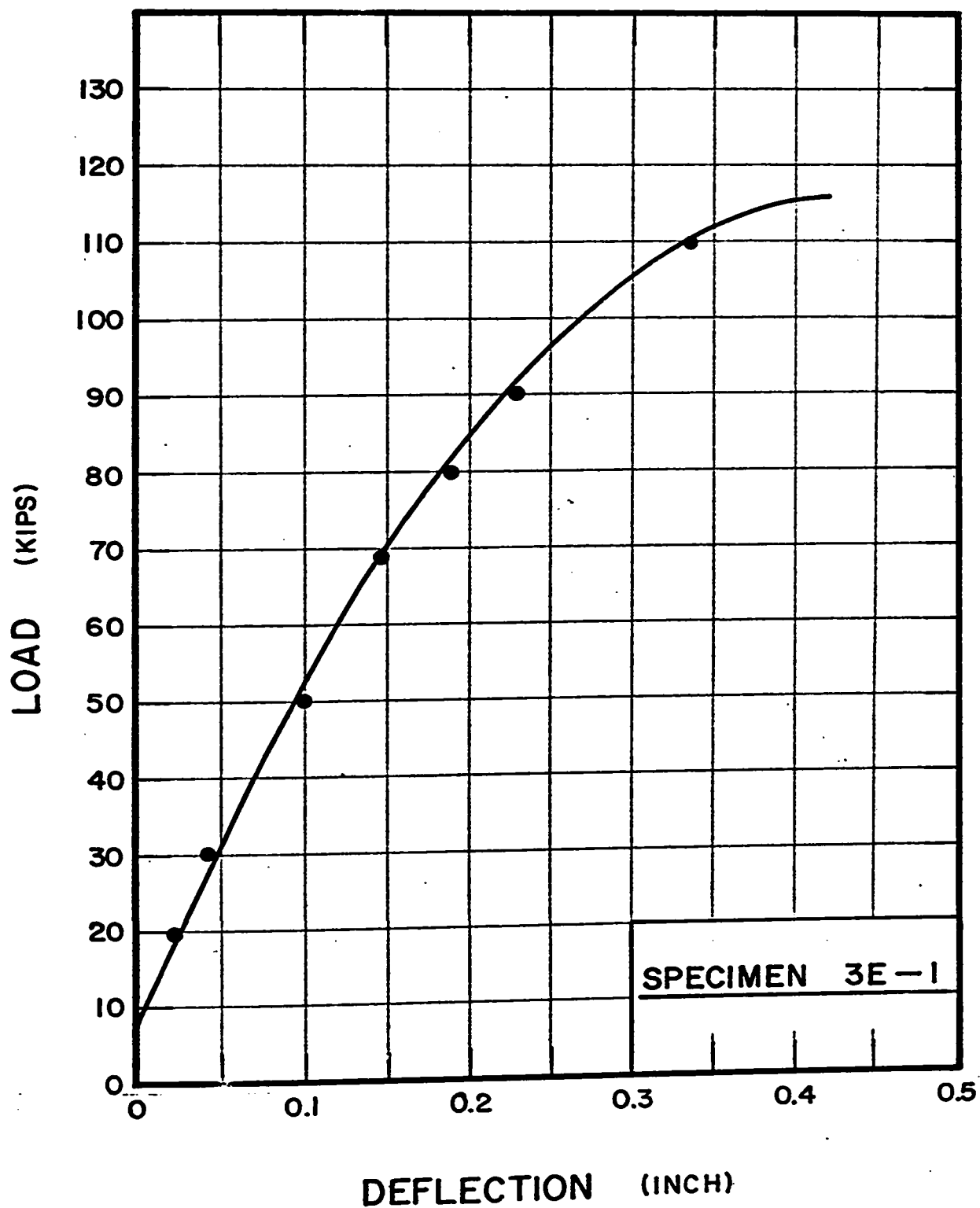


FIG.(B.26) LOAD-DEFLECTION CURVE
OF SPECIMEN 3E-1

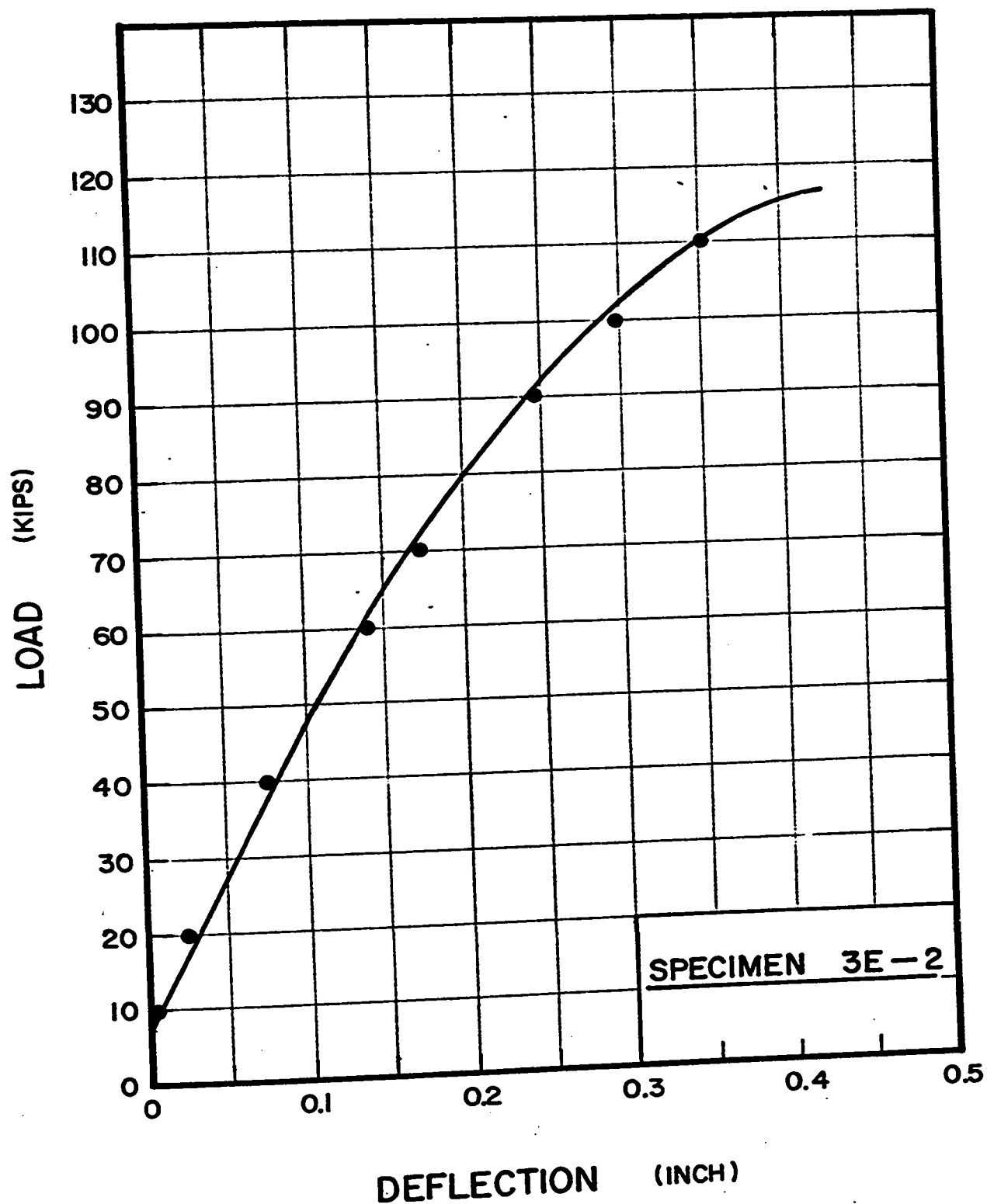


FIG.(B.27) LOAD-DEFLECTION CURVE
OF SPECIMEN 3E-2

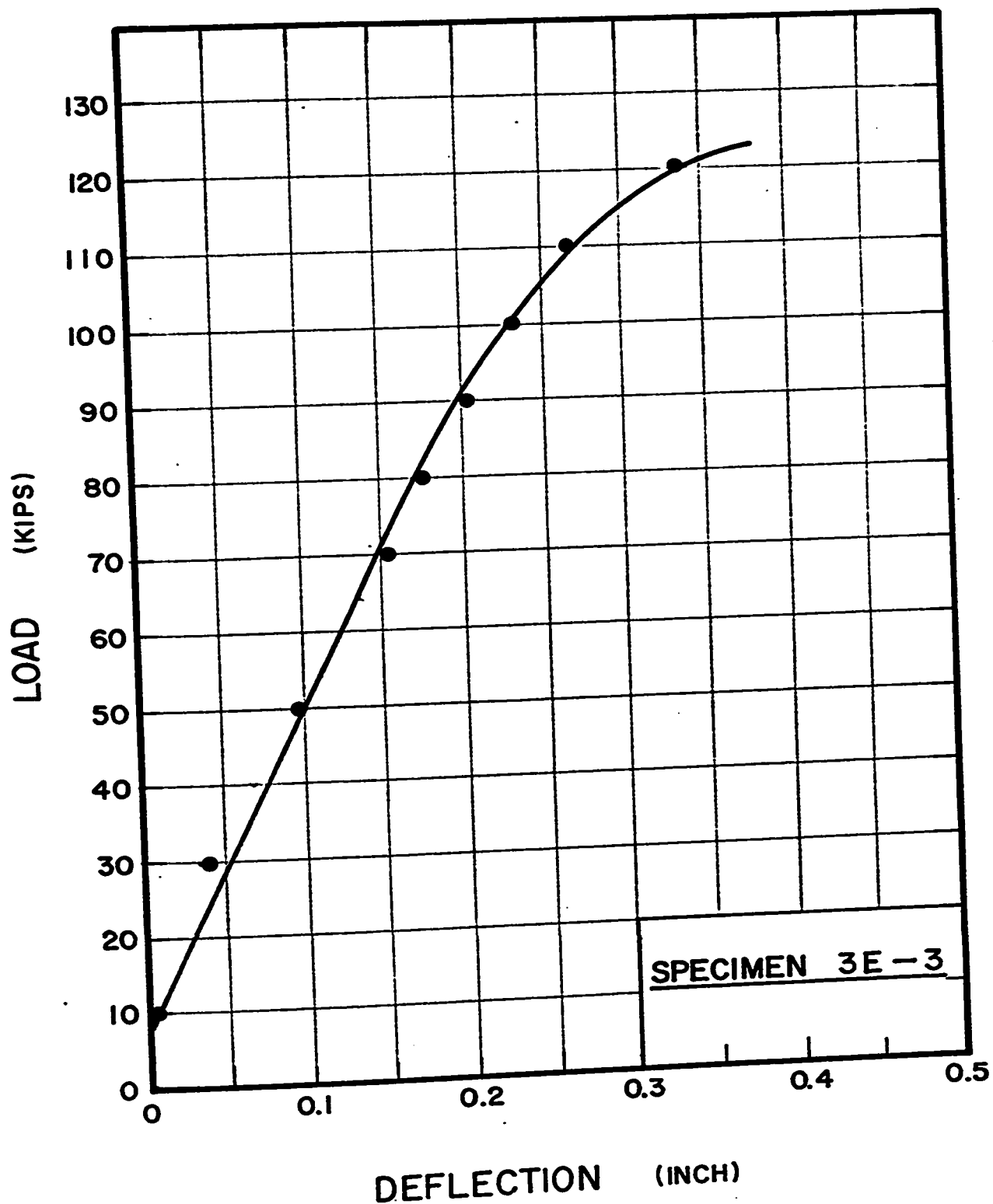


FIG.(B.28) LOAD—DEFLECTION CURVE
OF SPECIMEN 3E-3

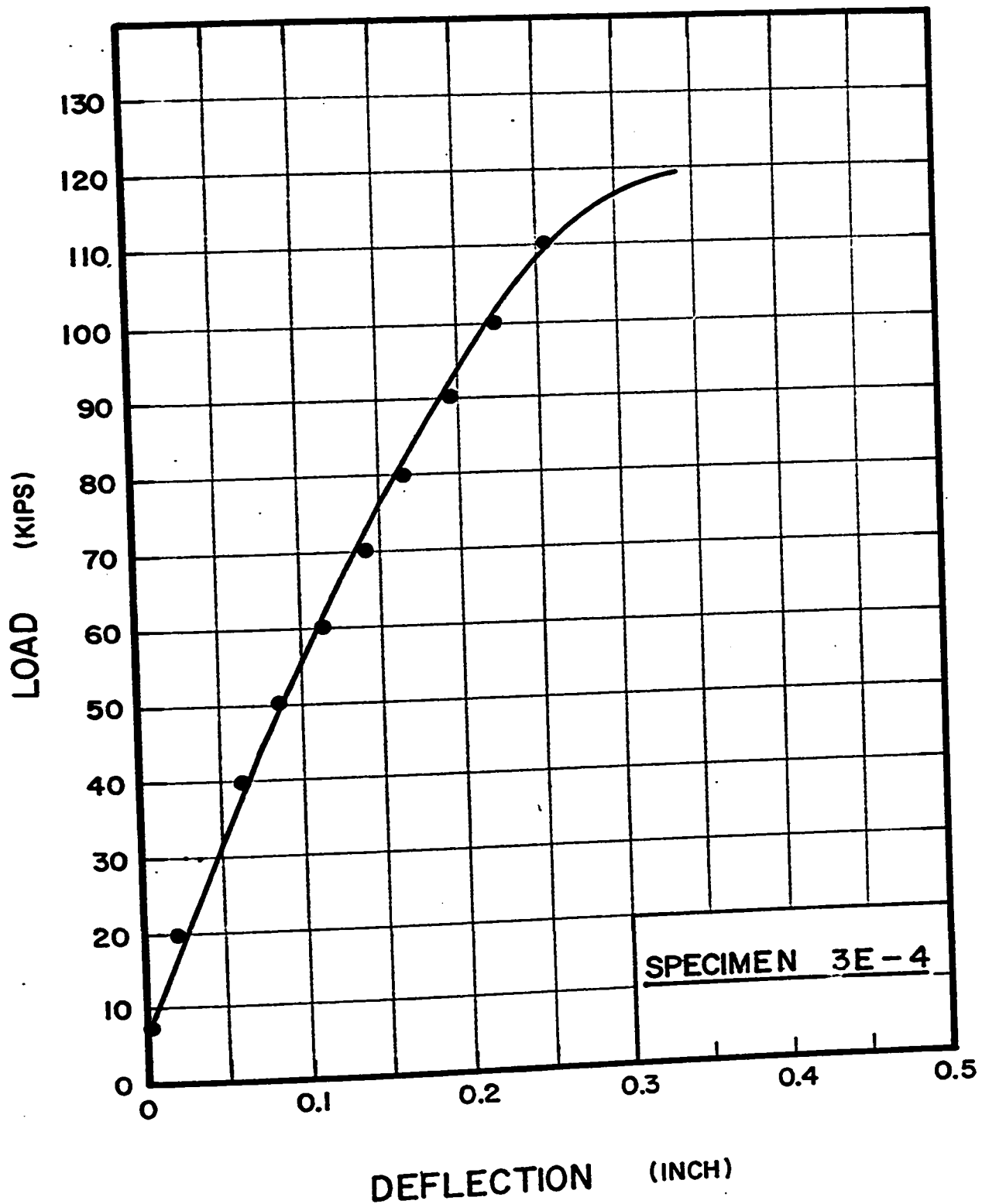


FIG.(B.29) LOAD — DEFLECTION CURVE
OF SPECIMEN 3E — 4

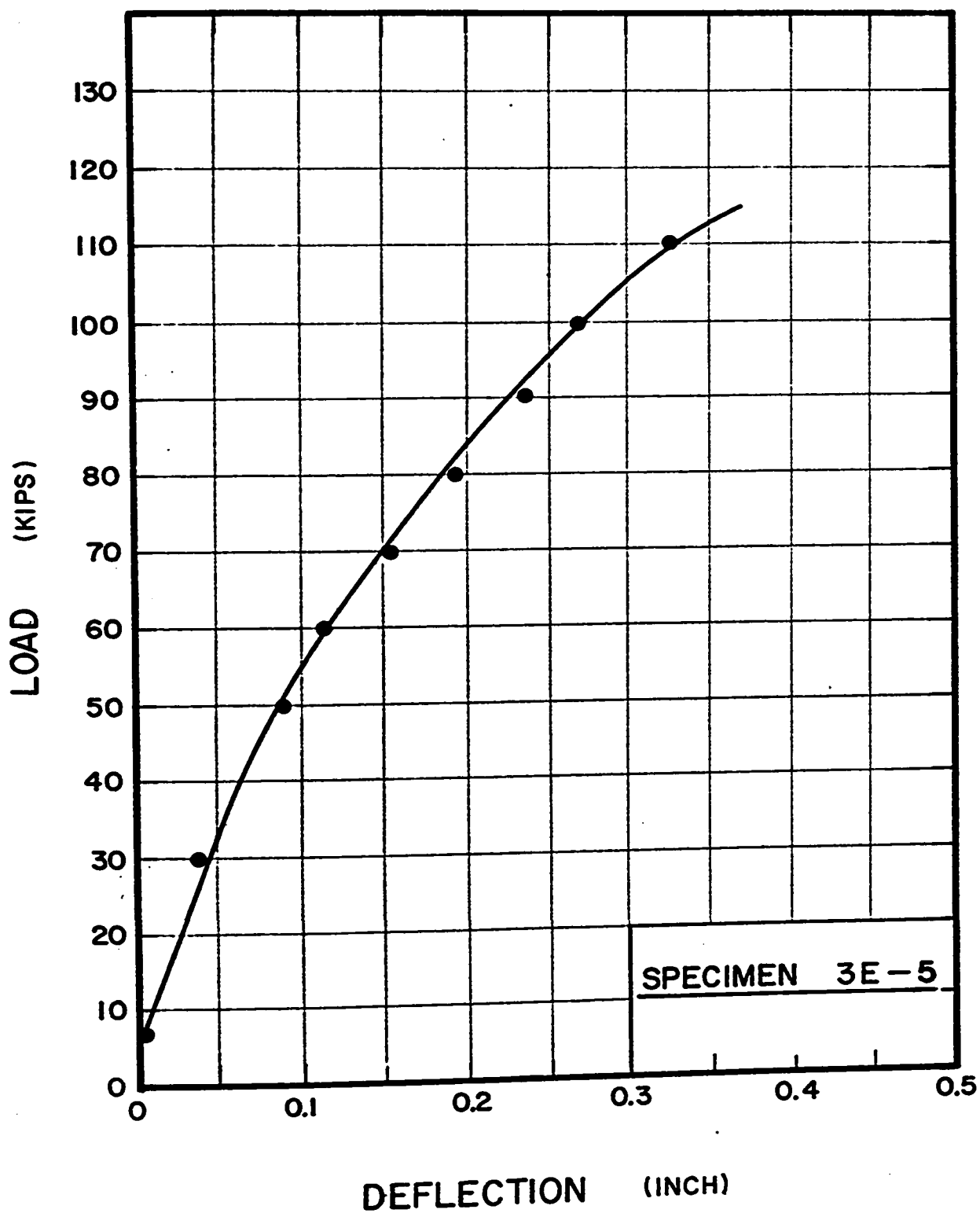


FIG.(B.30) LOAD—DEFLECTION CURVE
OF SPECIMEN 3E—5

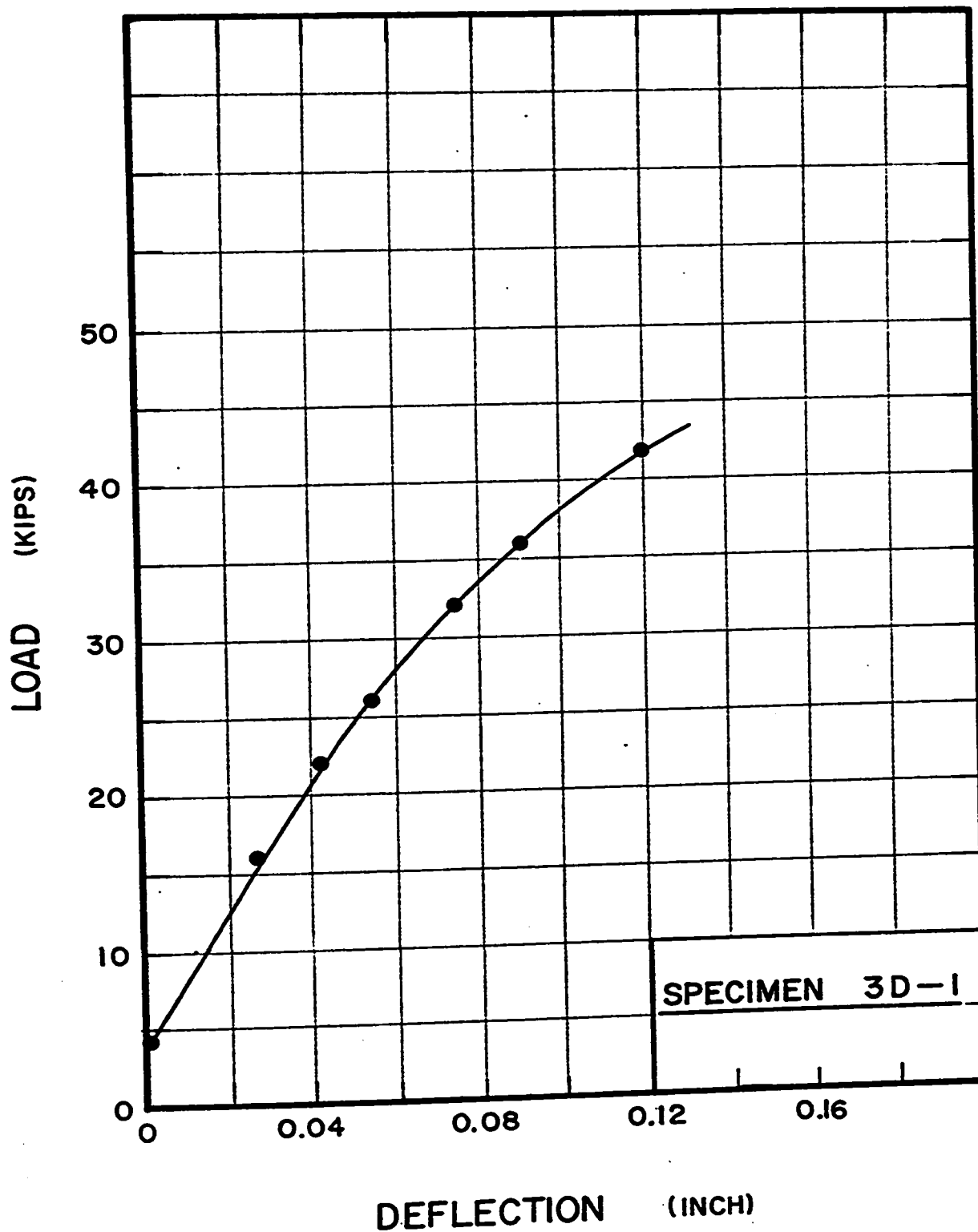
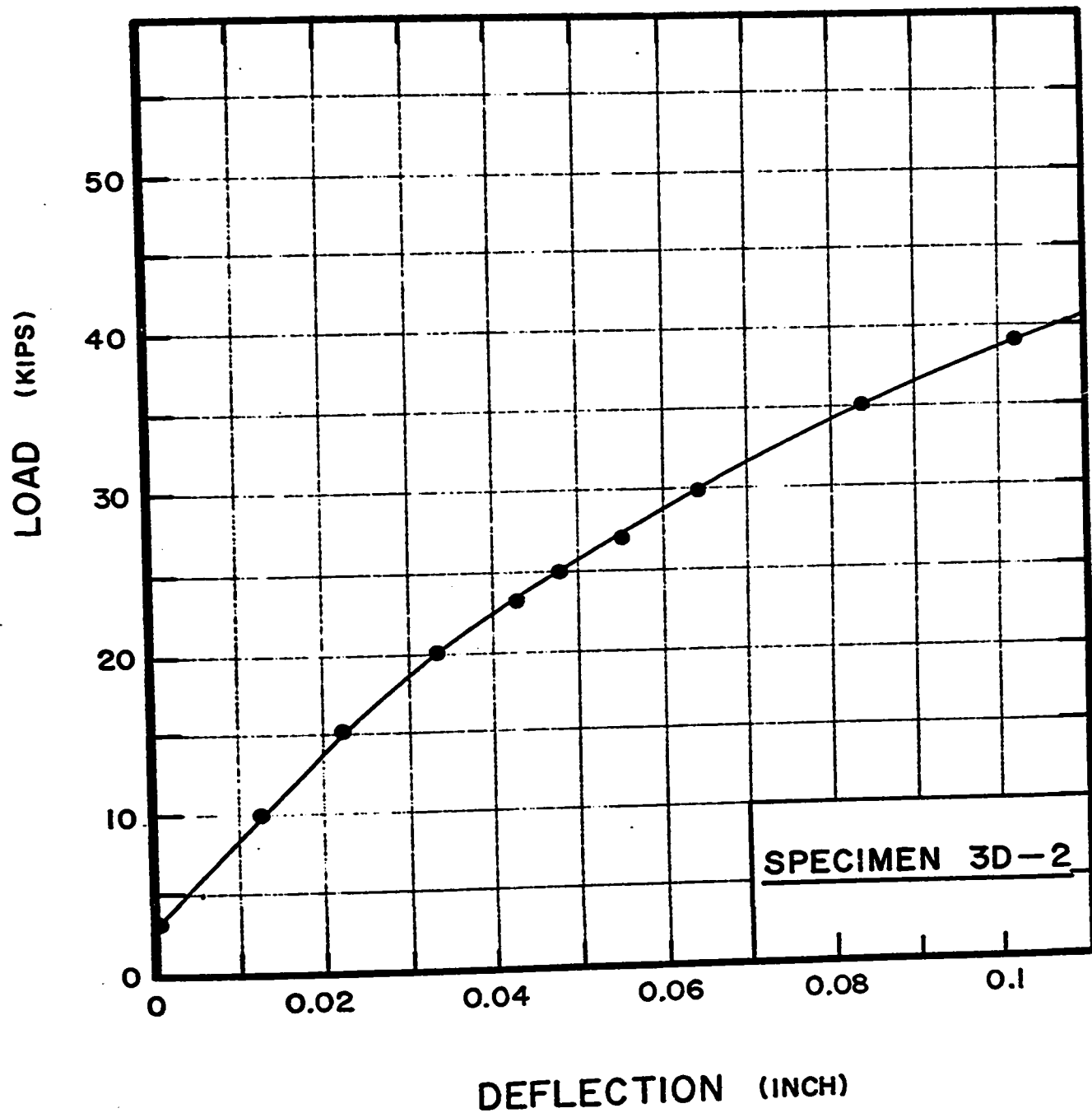
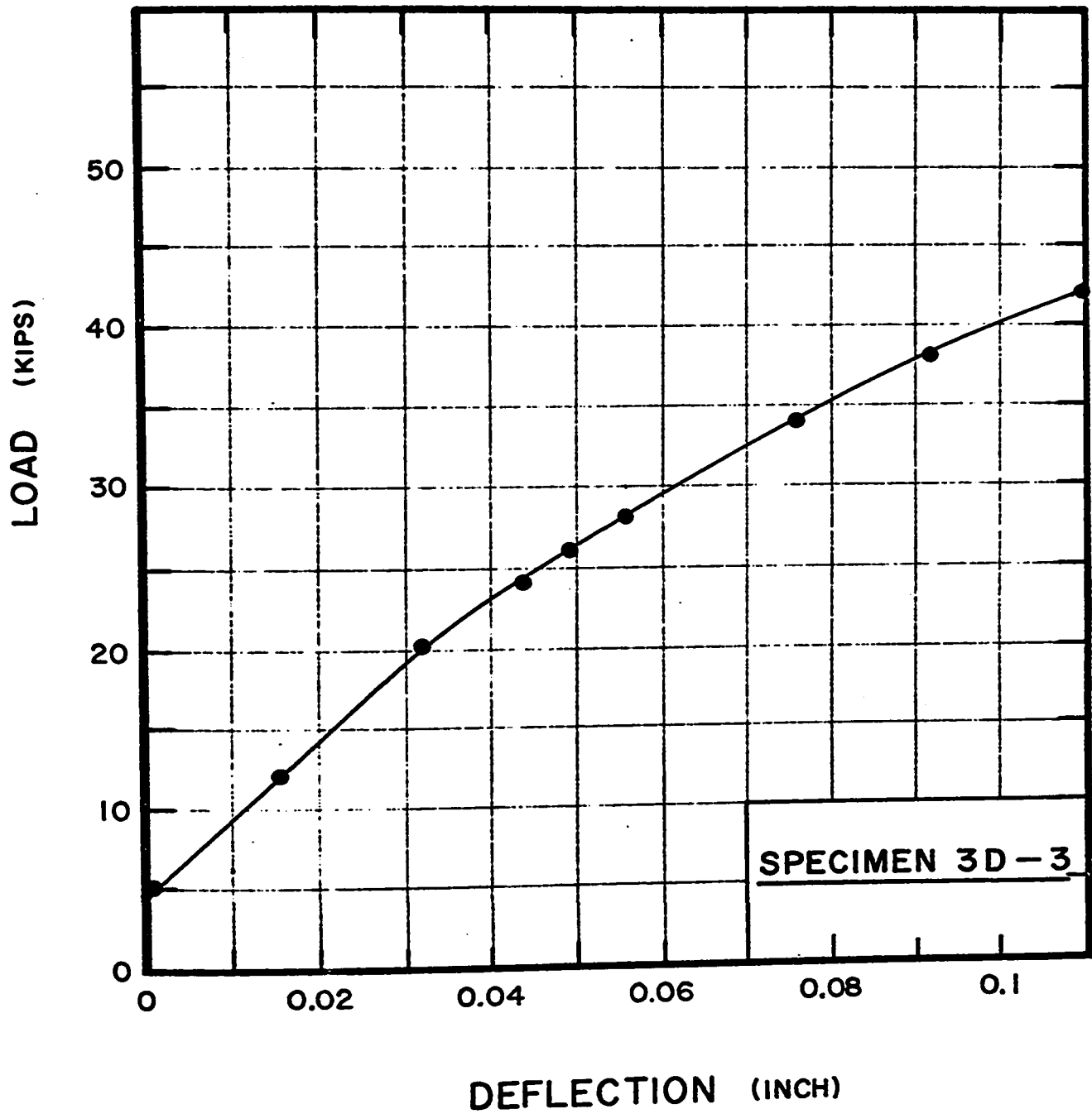


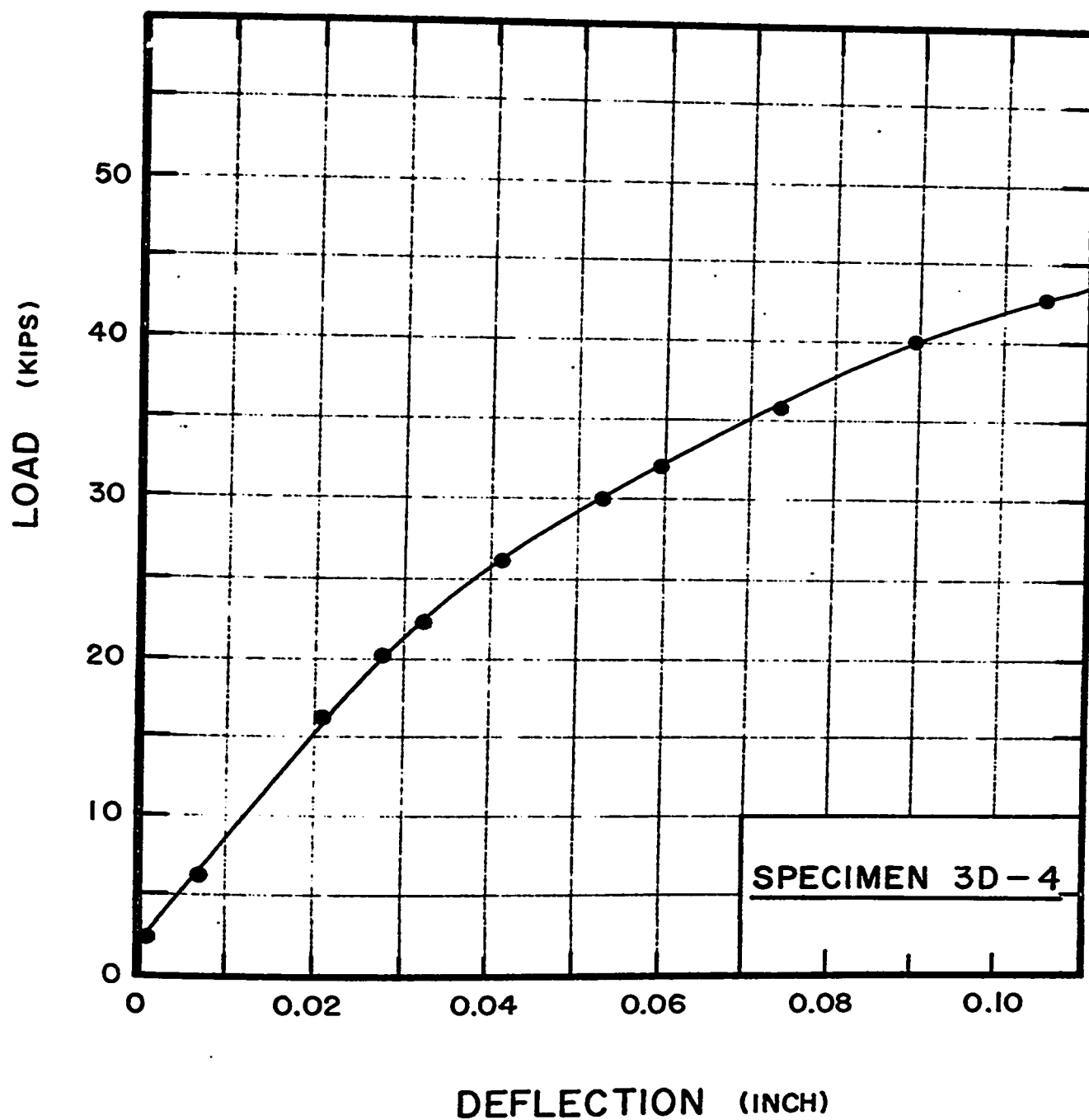
FIG.(B. 31) LOAD—DEFLECTION CURVE
OF SPECIMEN 3D—1



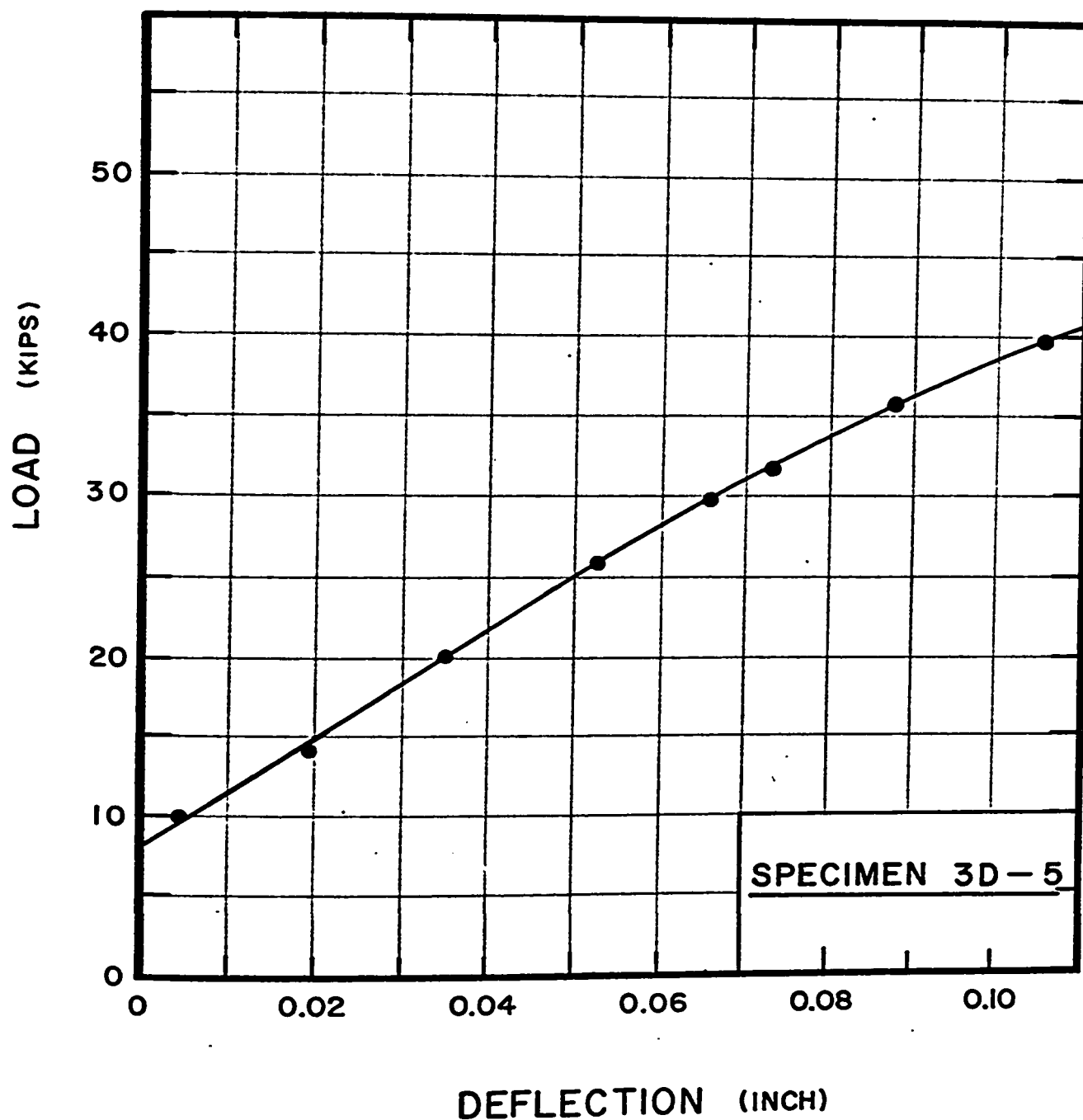
**FIG. (B. 32) LOAD—DEFLECTION CURVE
OF SPECIMEN 3D-2**



**FIG. (B. 3 3) LOAD—DEFLECTION CURVE
OF SPECIMEN 3D-3**



**FIG. (B. 34) LOAD—DEFLECTION CURVE
OF SPECIMEN 3D-4**



**FIG. (B. 35) LOAD—DEFLECTION CURVE
OF SPECIMEN 3D-5**

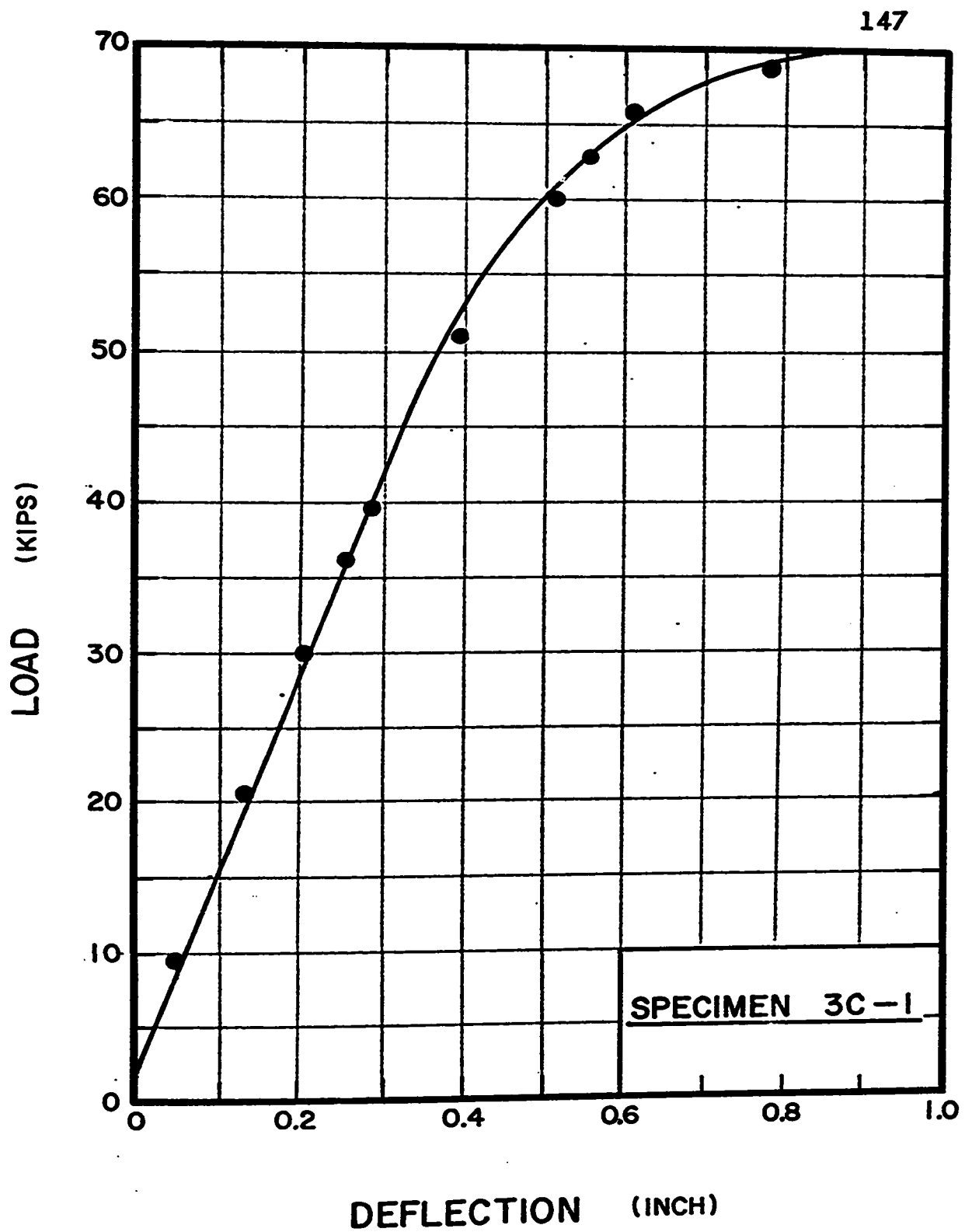
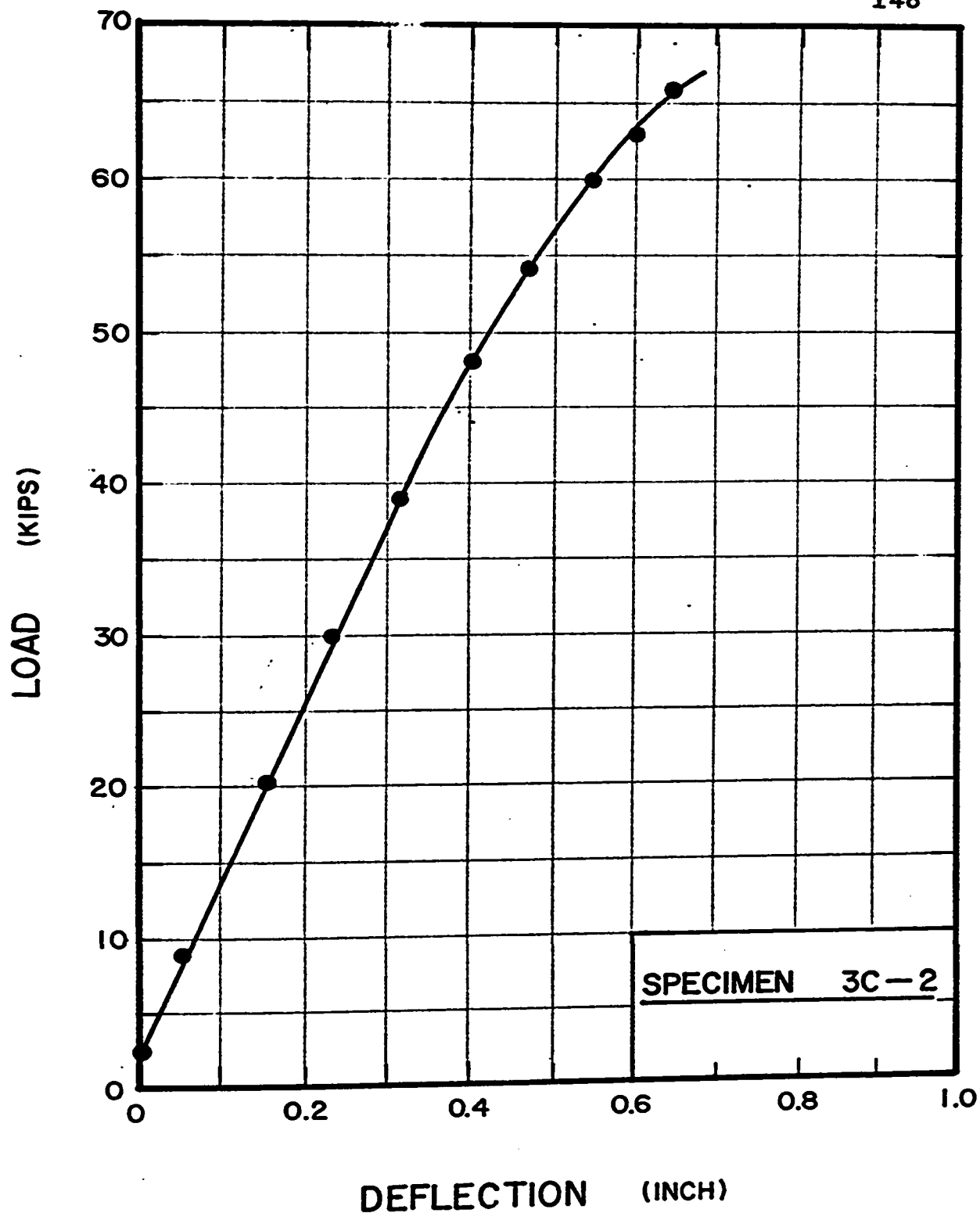


FIG. (B. 36) LOAD-DEFLECTION CURVE
OF SPECIMEN 3C-1



**FIG.(B.37) LOAD—DEFLECTION CURVE
OF SPECIMEN 3C—2**

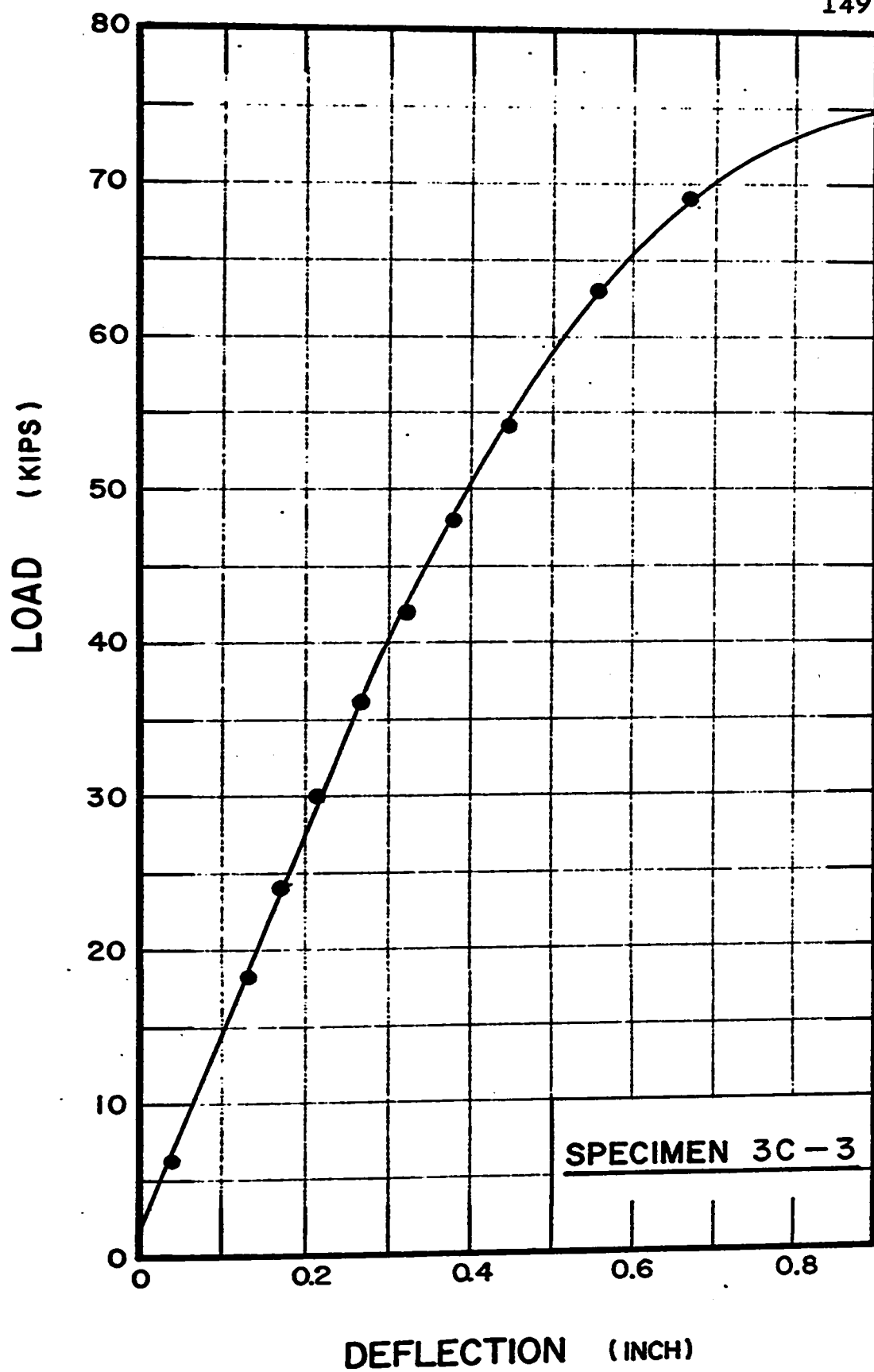
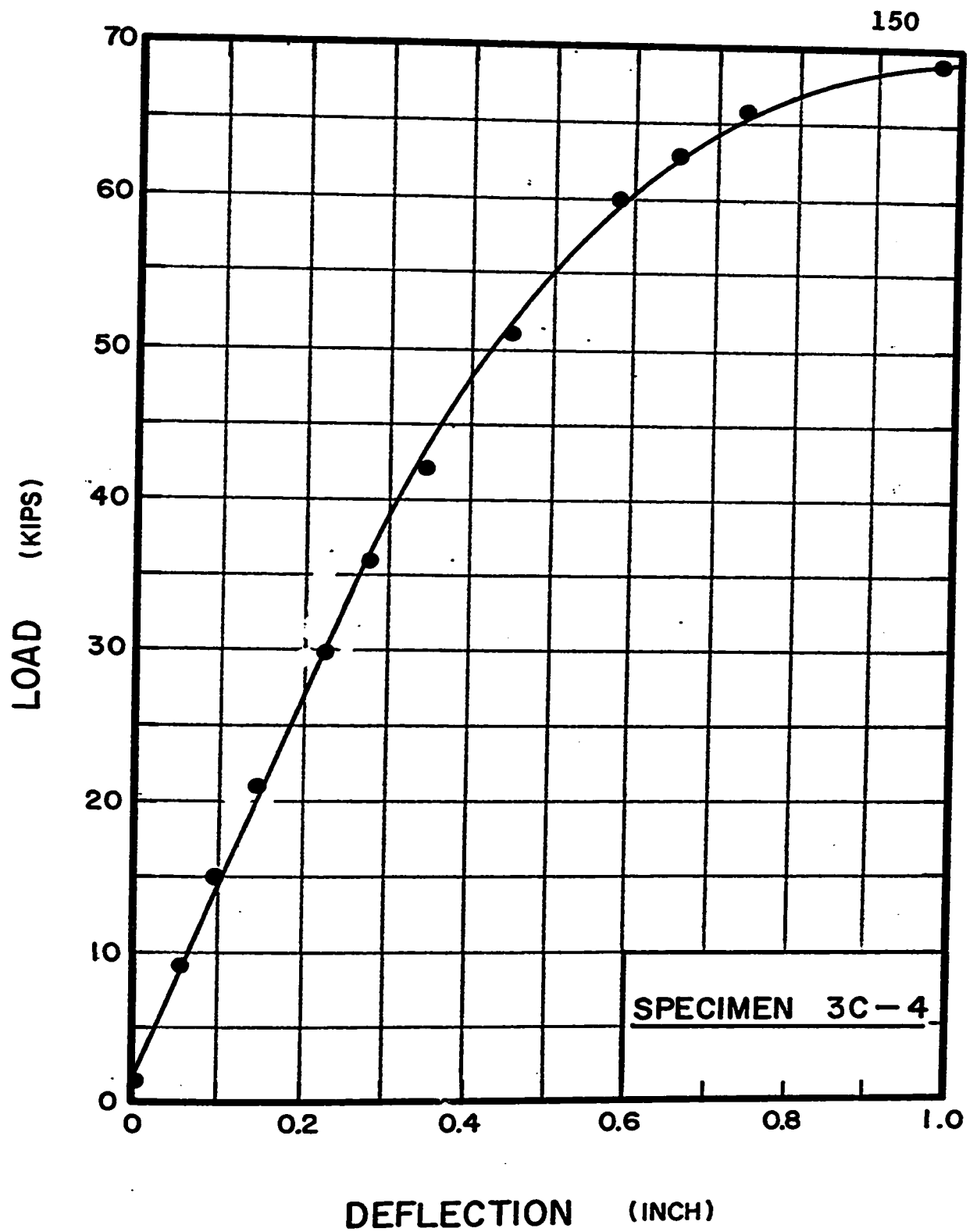
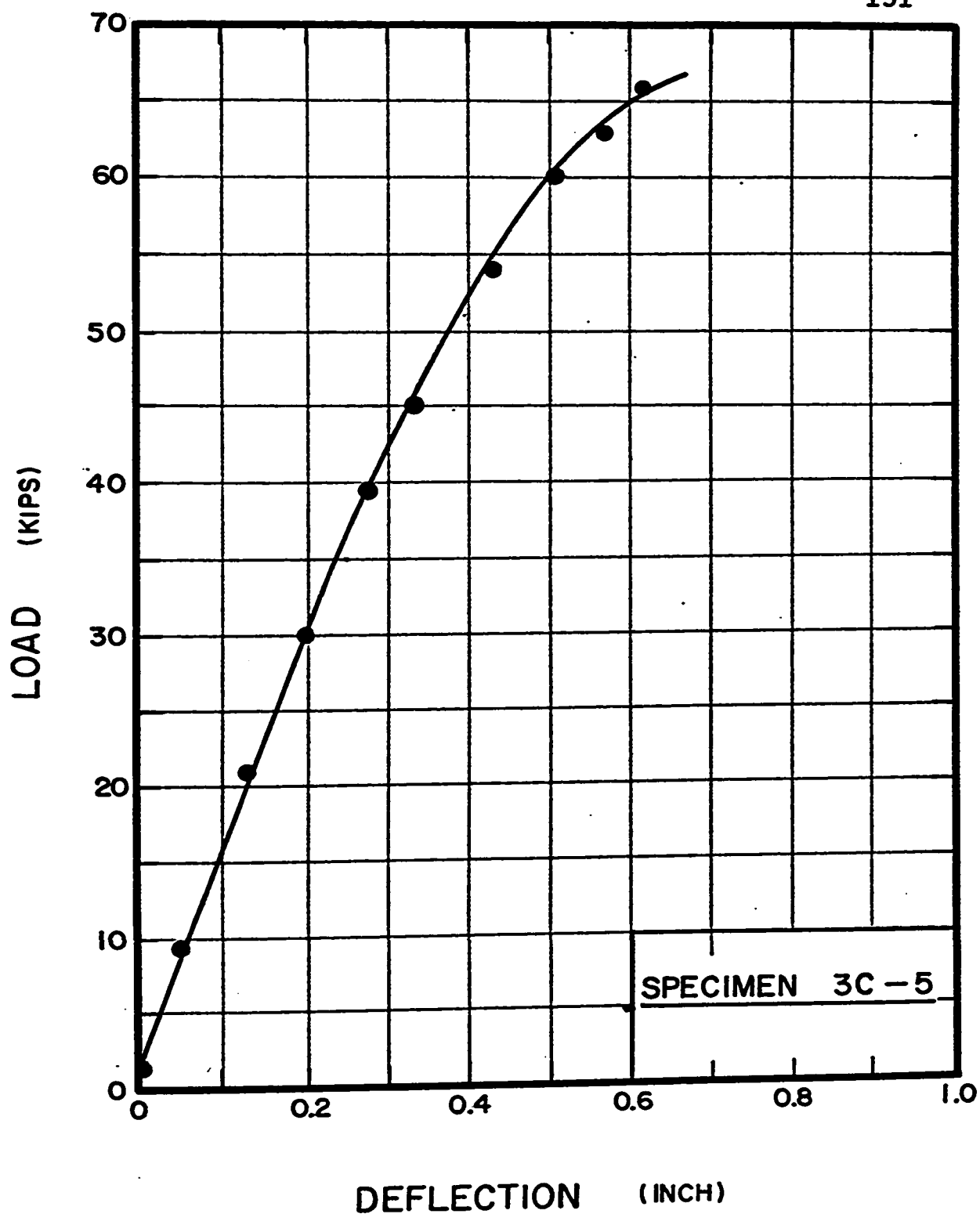


FIG.(B. 38) LOAD—DEFLECTION
CURVE OF SPECIMEN 3C-3



**FIG. (B.39) LOAD — DEFLECTION CURVE
OF SPECIMEN 3C — 4**



**FIG. (B.40) LOAD — DEFLECTION CURVE
OF SPECIMEN 3C-5**

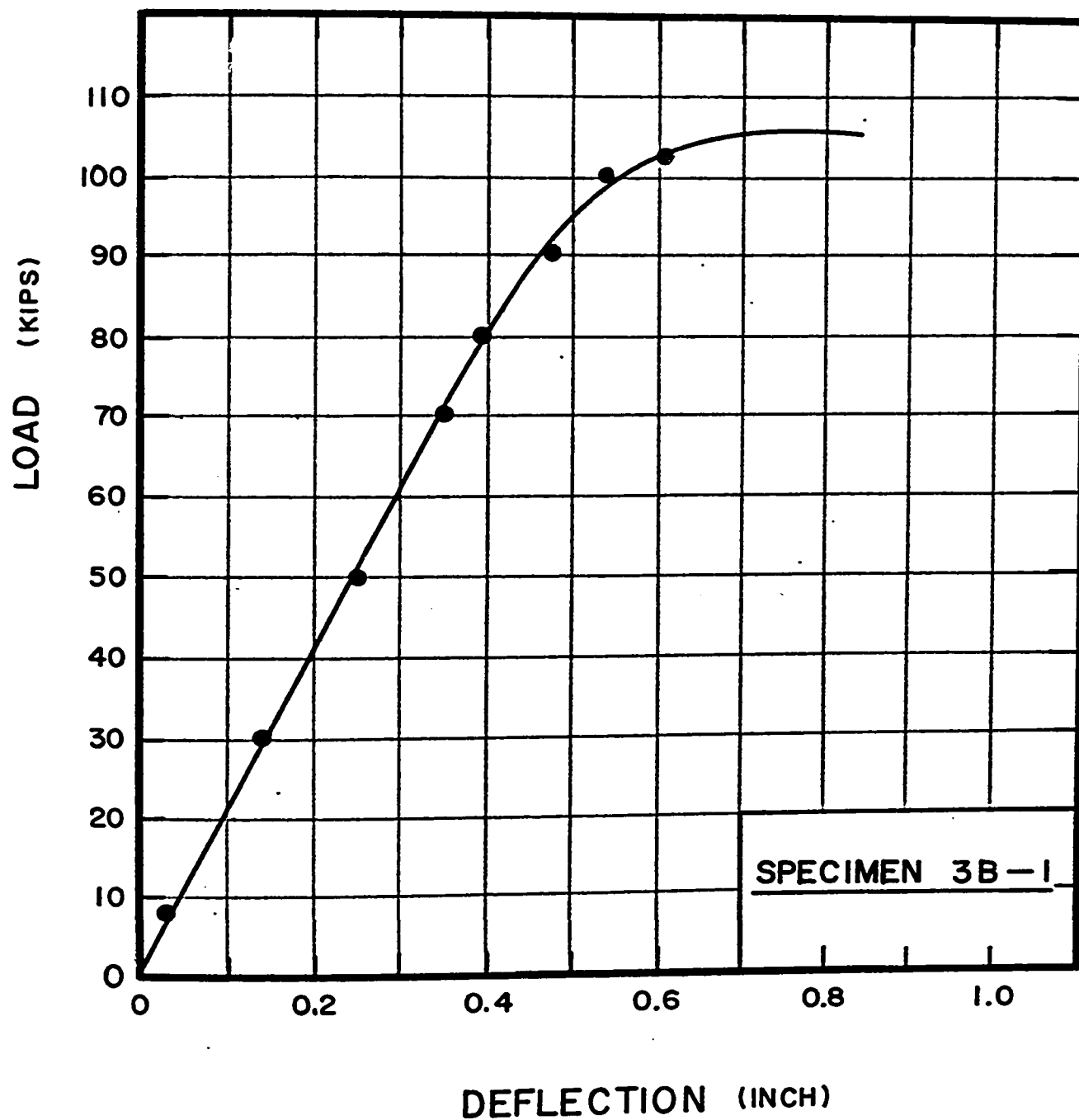
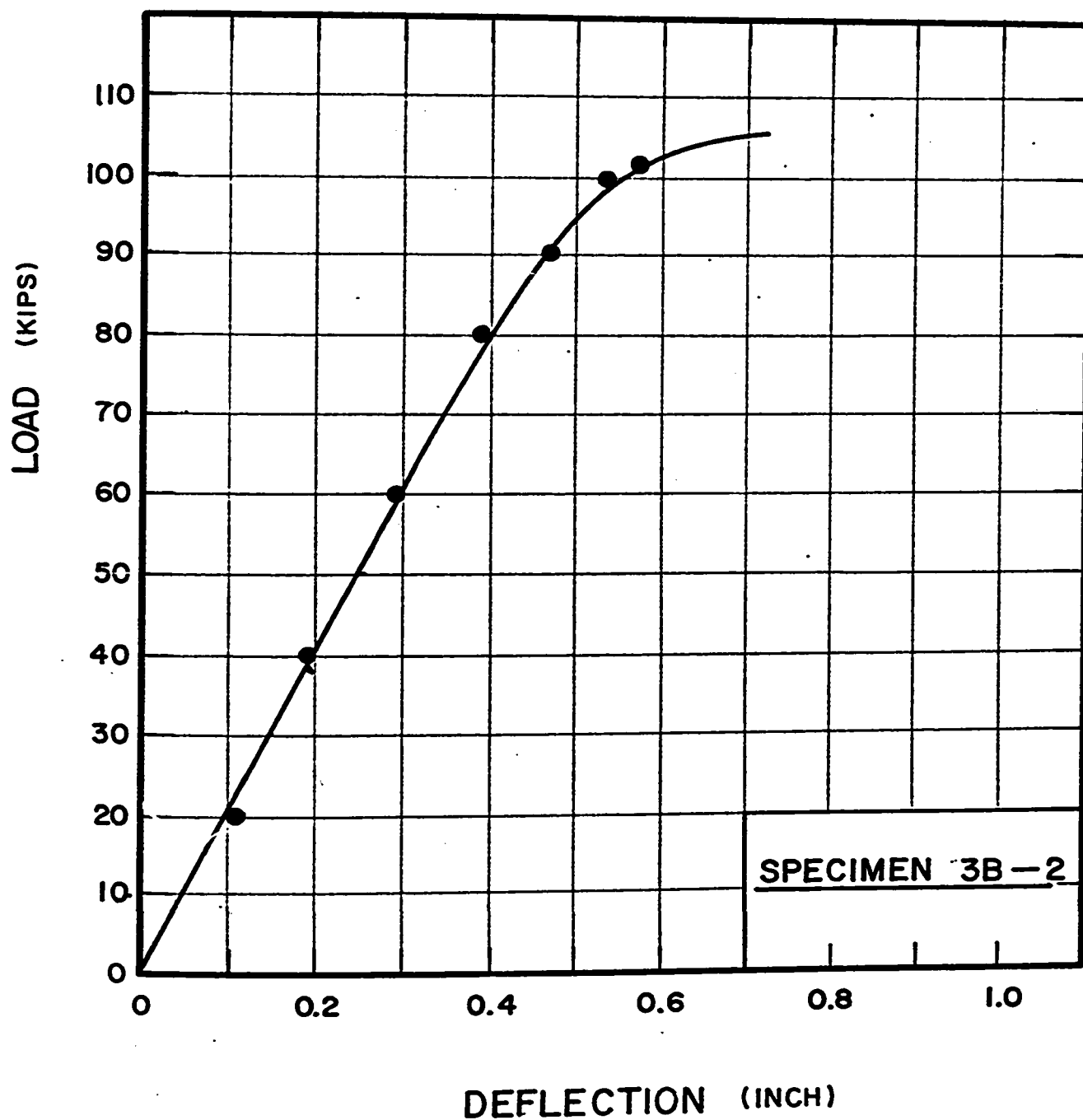
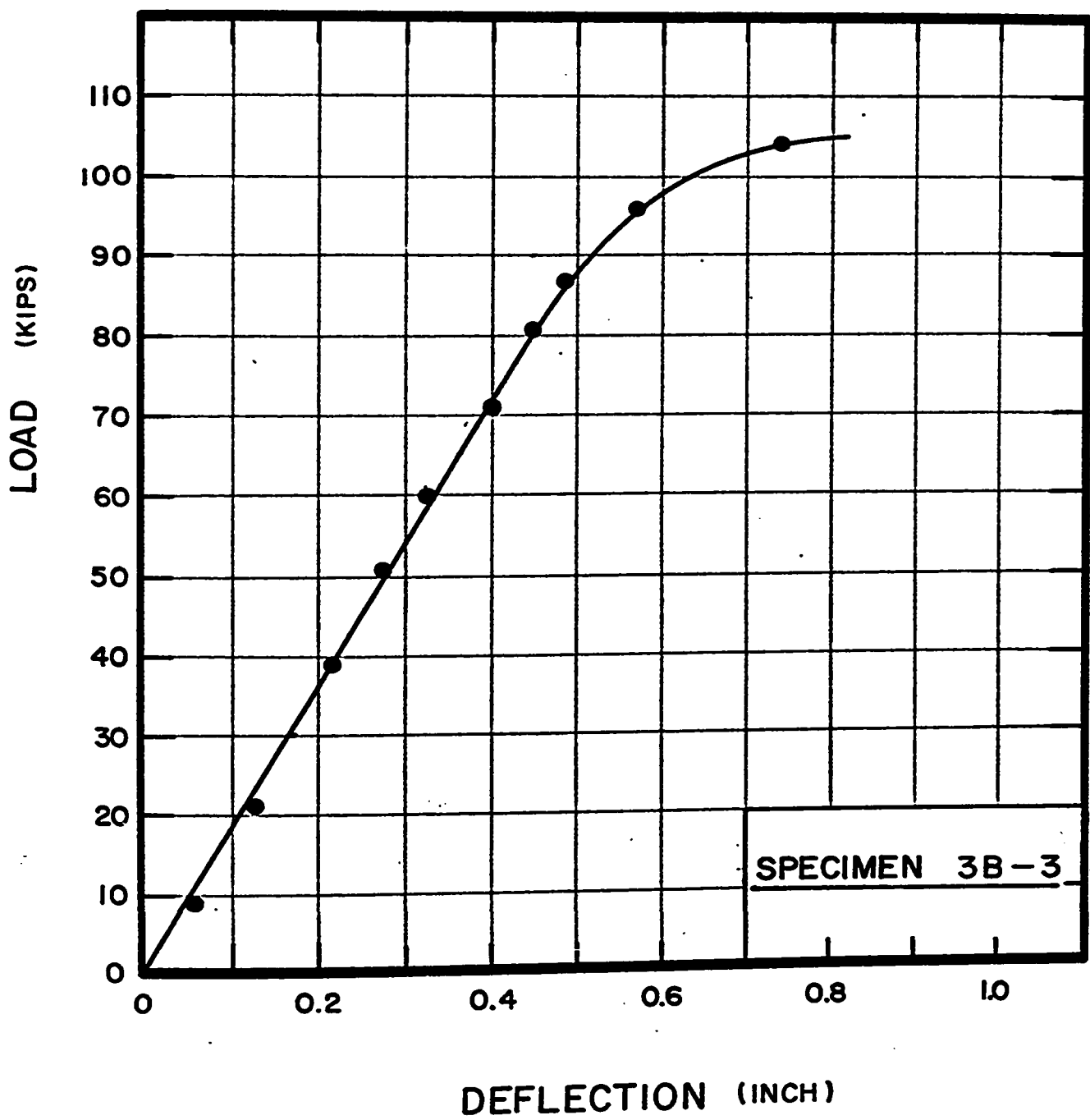


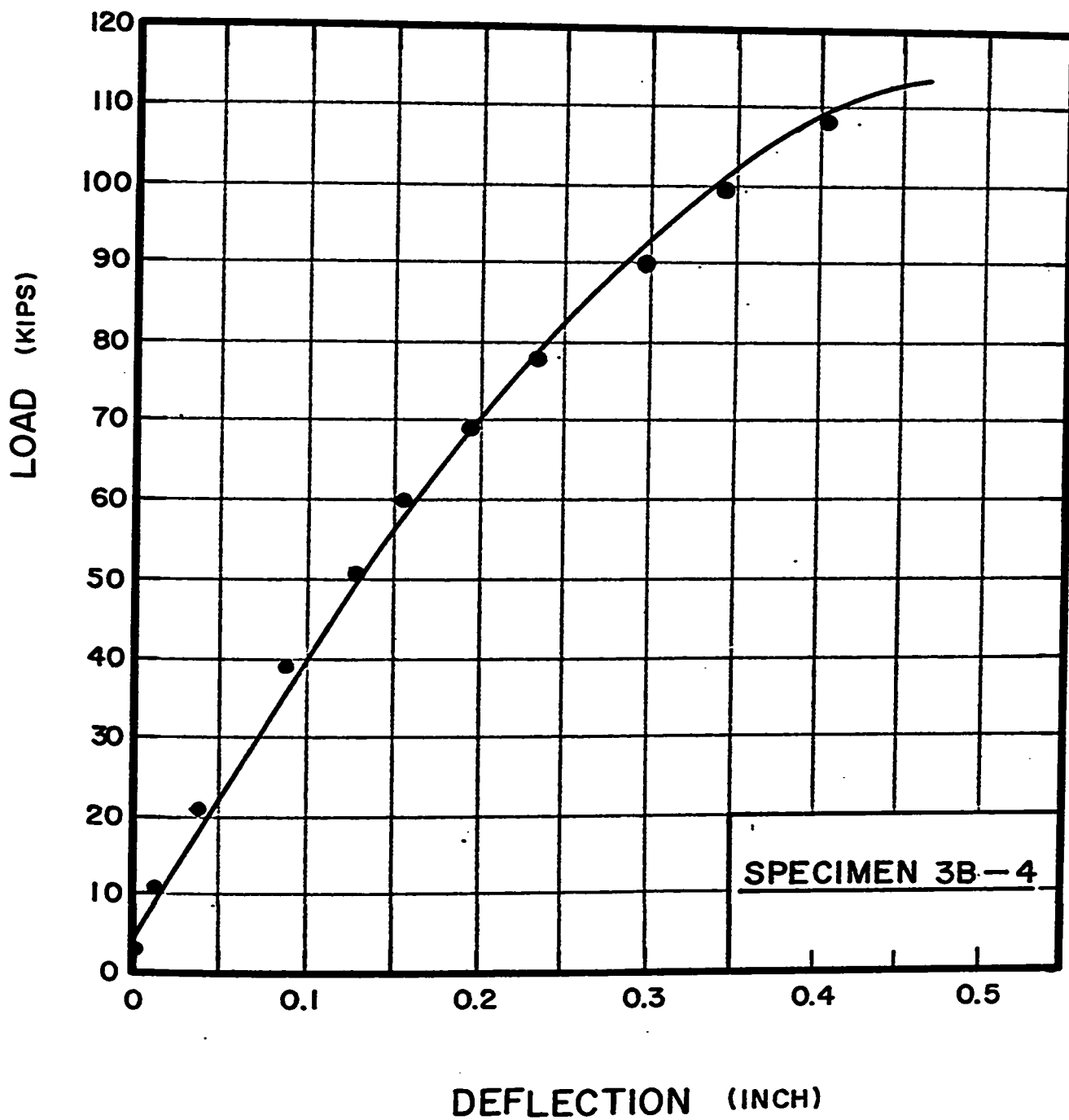
FIG.(B. 41) LOAD — DEFLECTION CURVE
OF SPECIMEN 3B-1



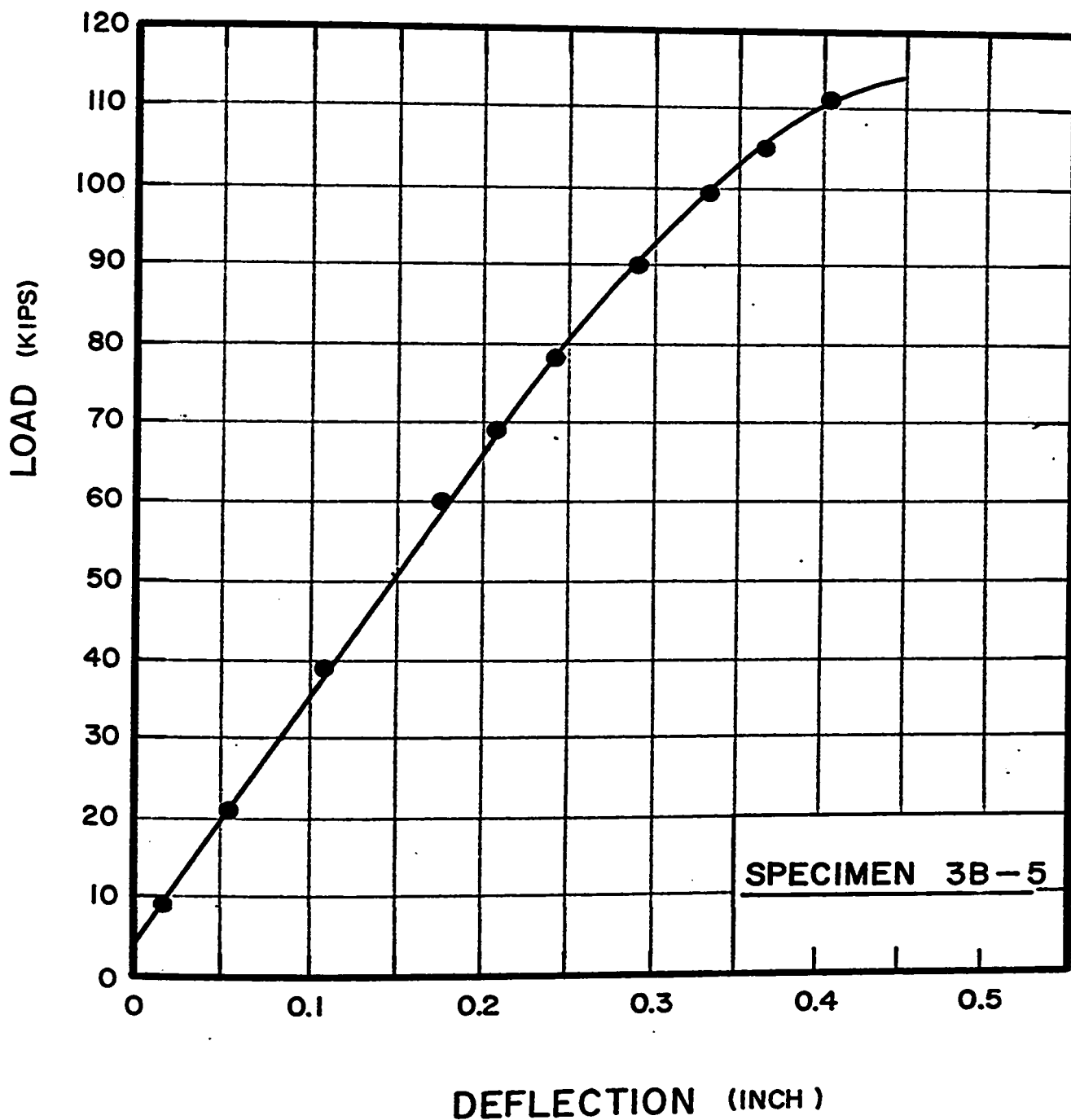
**FIG. (B. 42) LOAD — DEFLECTION CURVE
OF SPECIMEN 3B-2**



**FIG. (B. 43) LOAD — DEFLECTION CURVE
OF SPECIMEN 3B-3**



**FIG. (B. 44) LOAD — DEFLECTION CURVE
OF SPECIMEN 3B — 4**



**FIG.(B. 45) LOAD—DEFLECTION CURVE
OF SPECIMEN 3B—5**

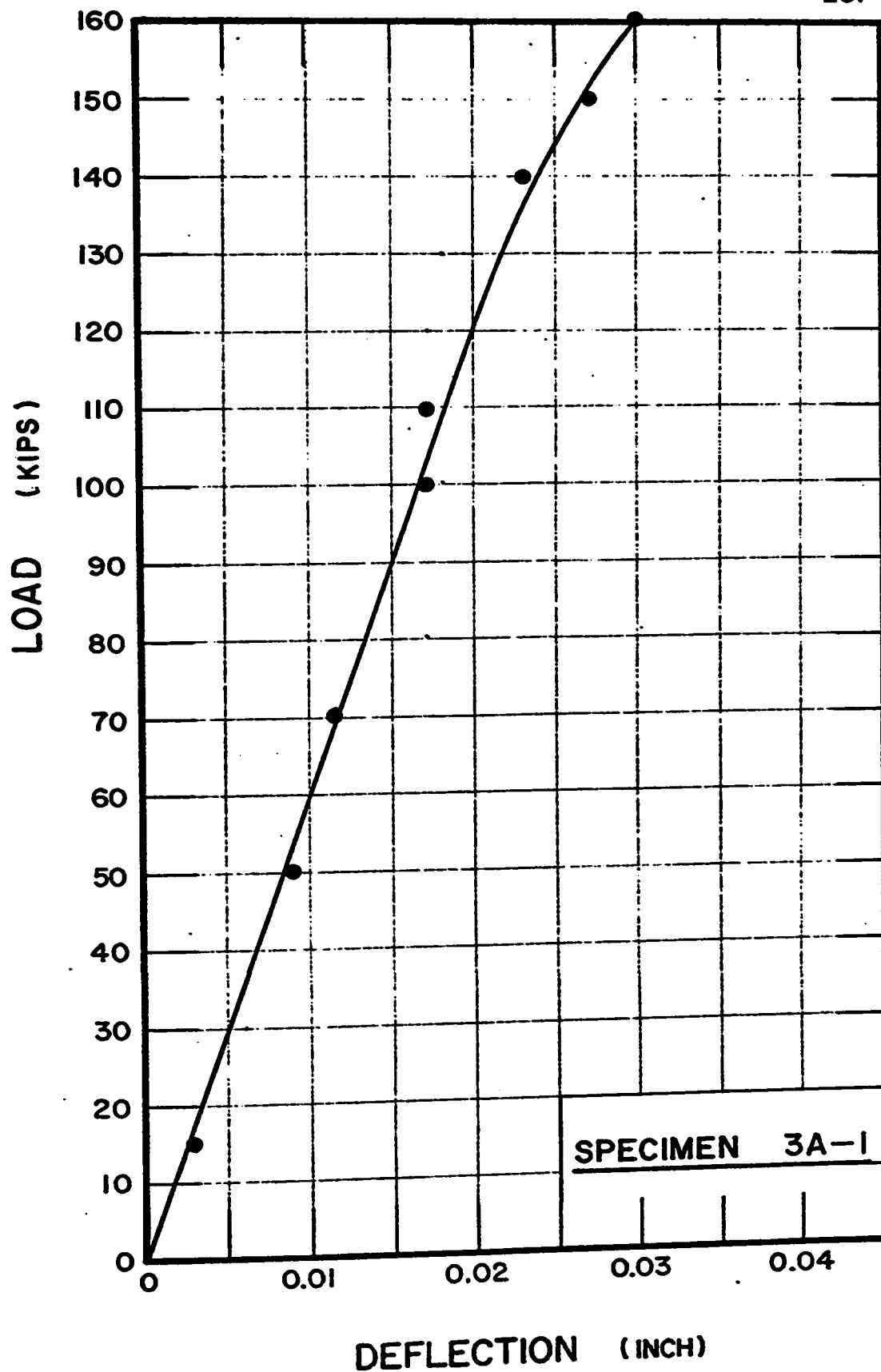


FIG.(B.46) LOAD-DEFLECTION
CURVE OF SPECIMEN 3A-1

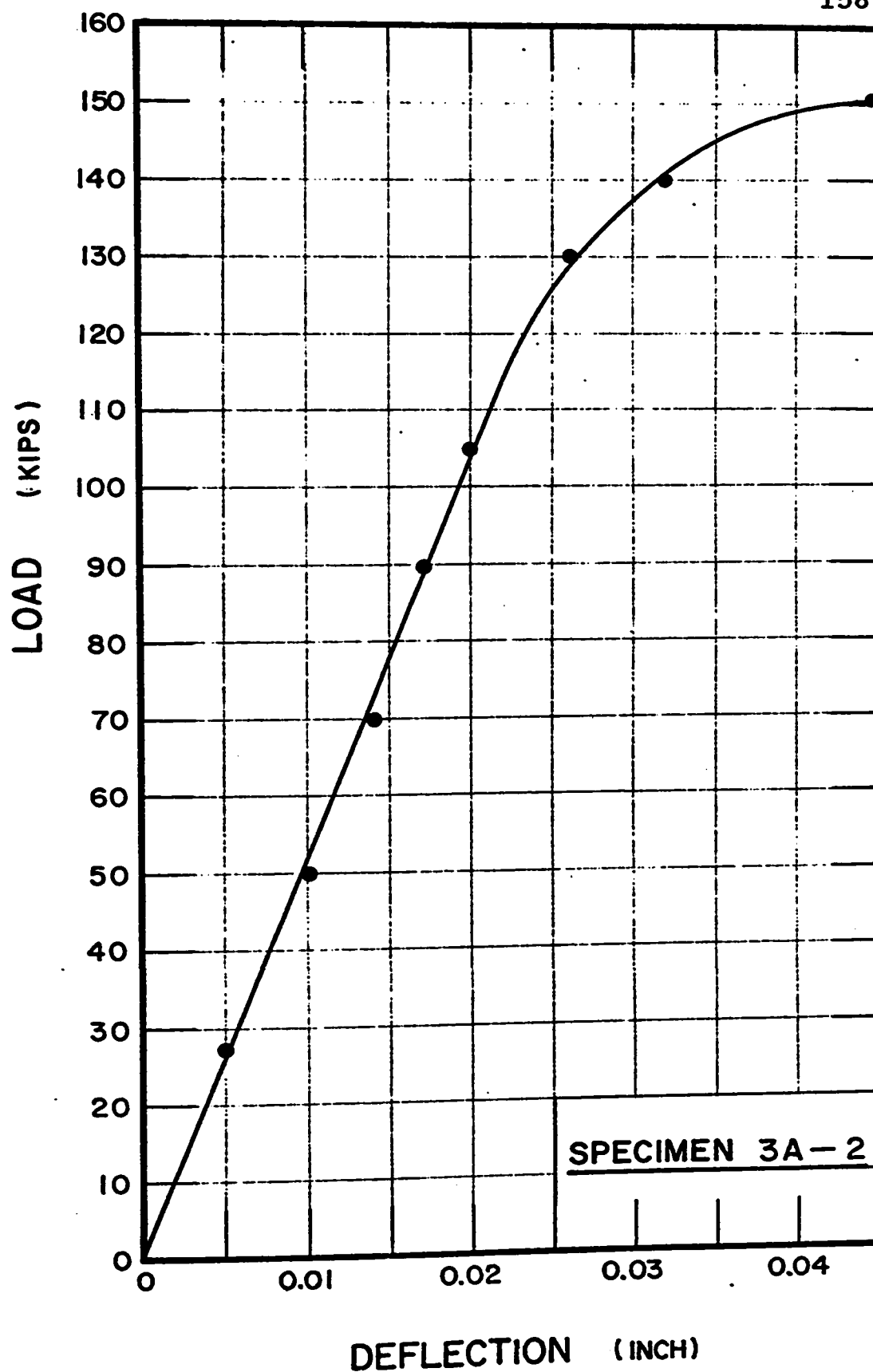
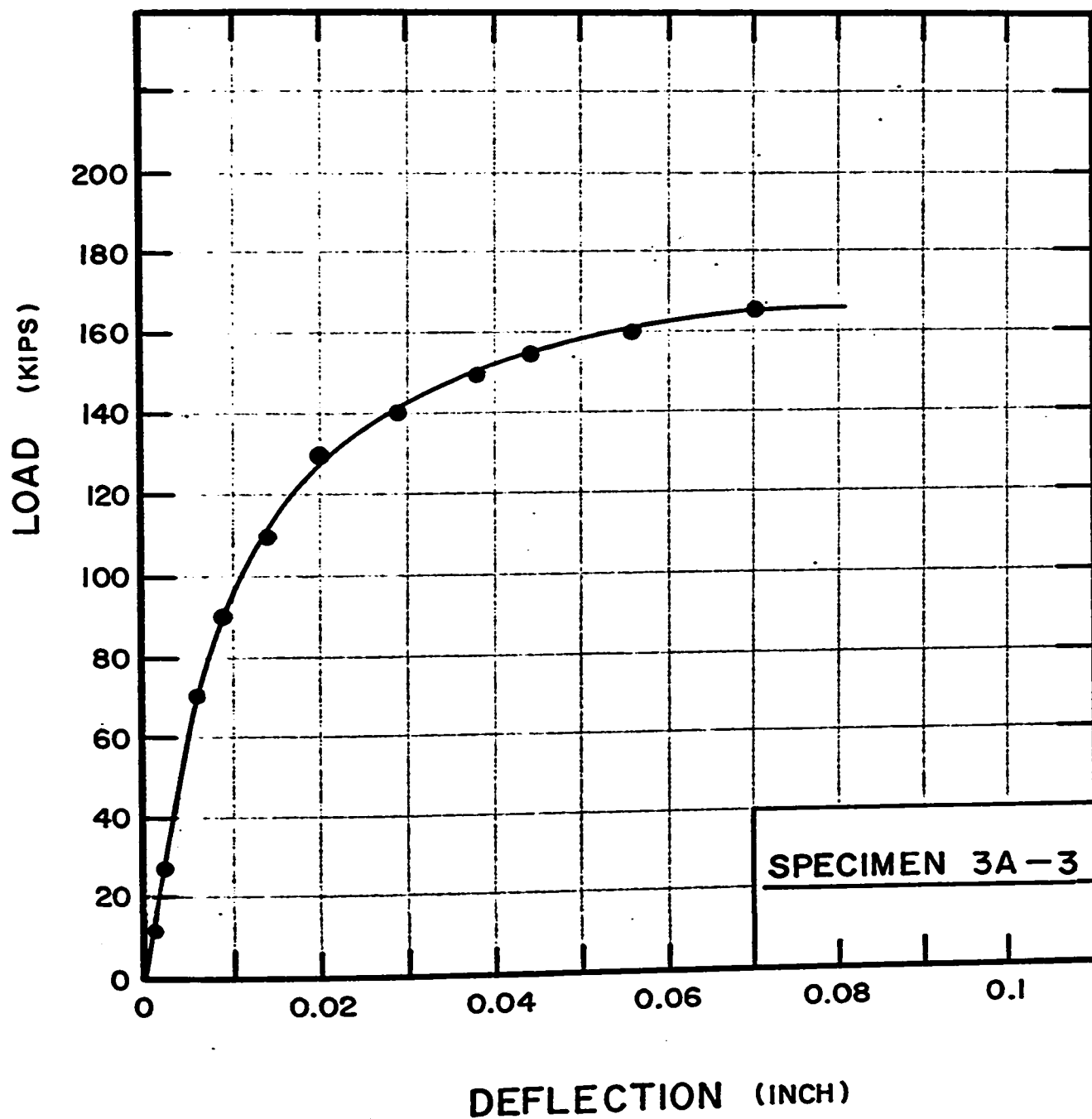
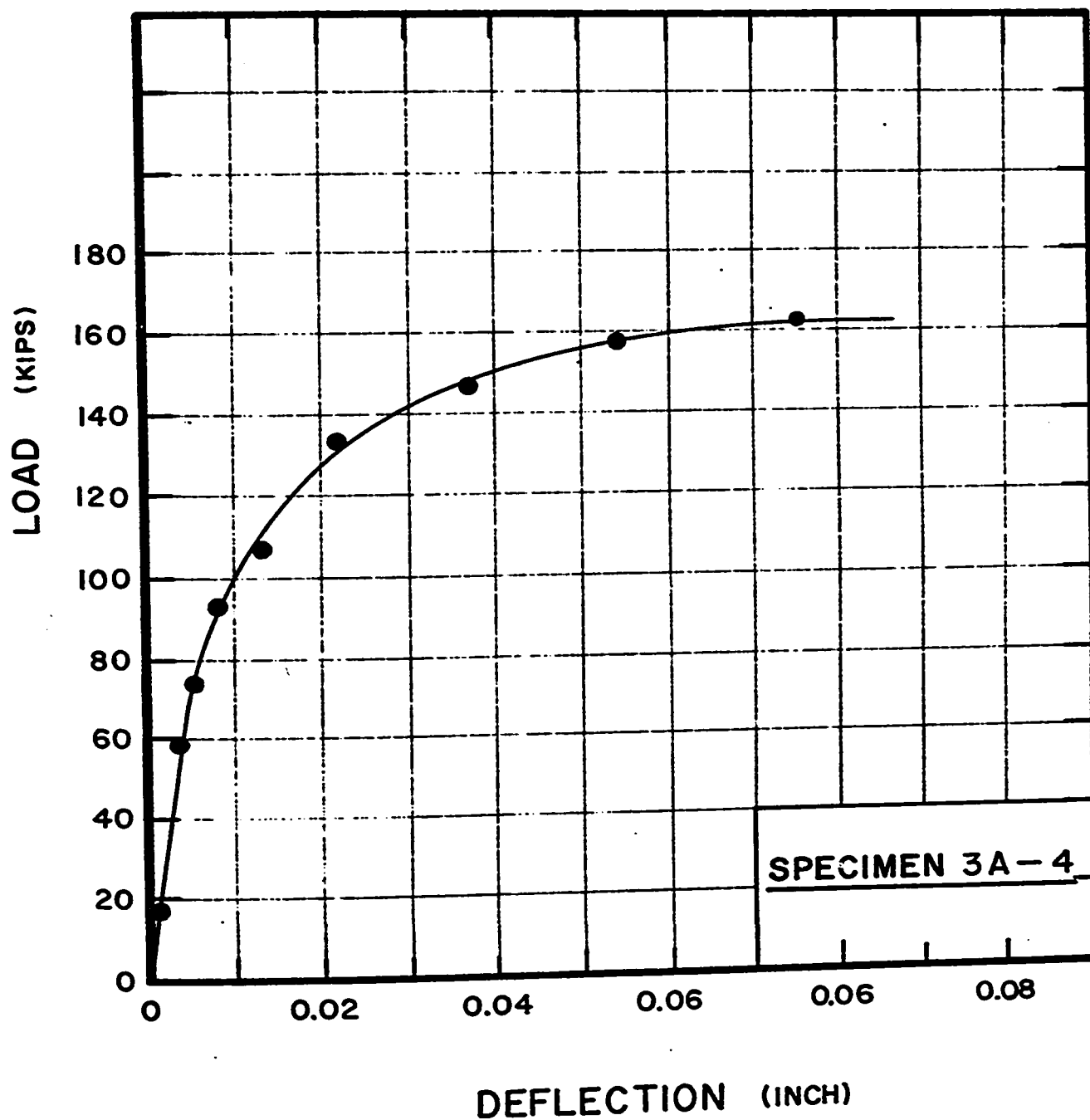


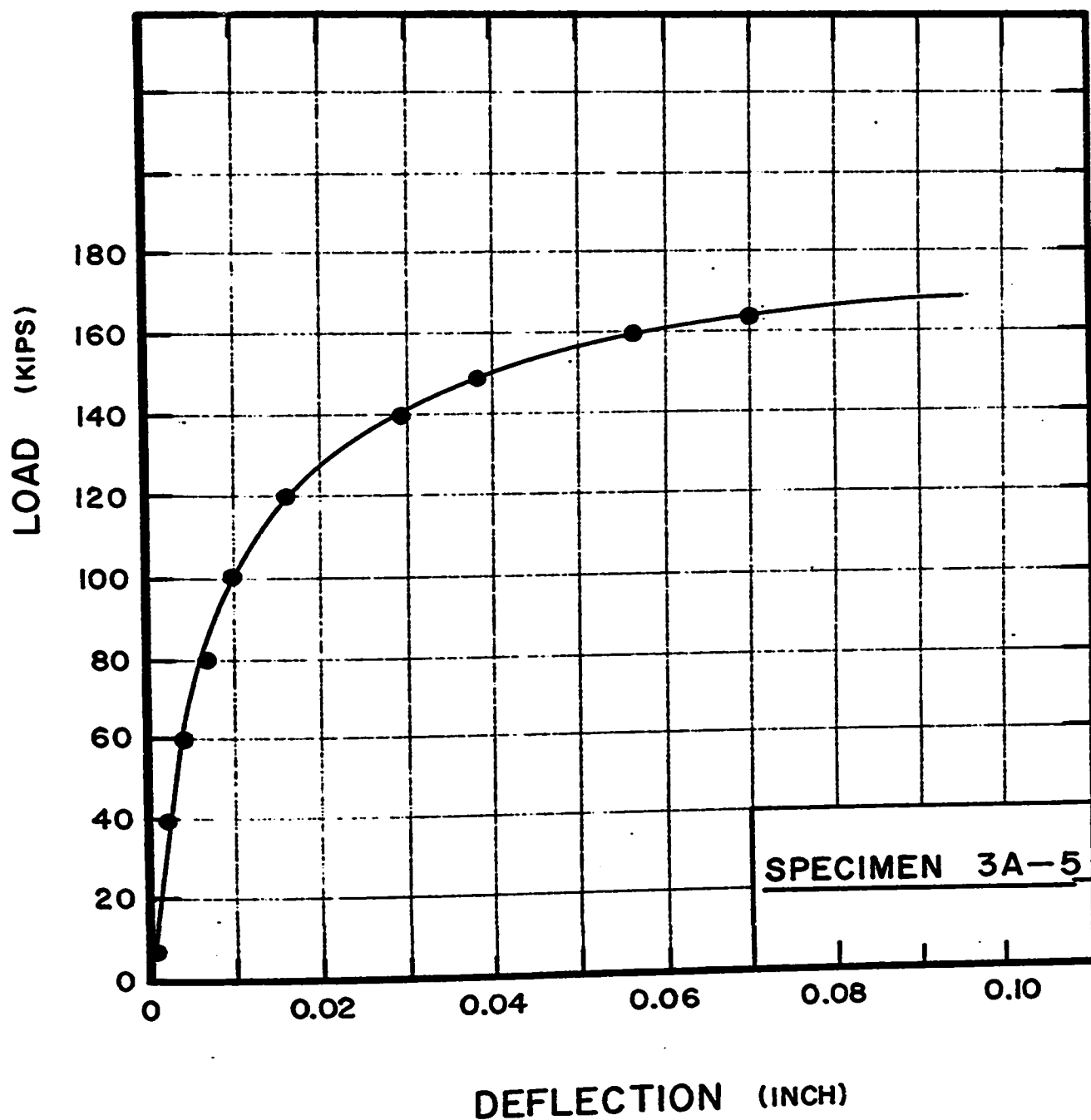
FIG.(B. 47) LOAD—DEFLECTION
CURVE OF SPECIMEN 3A—2



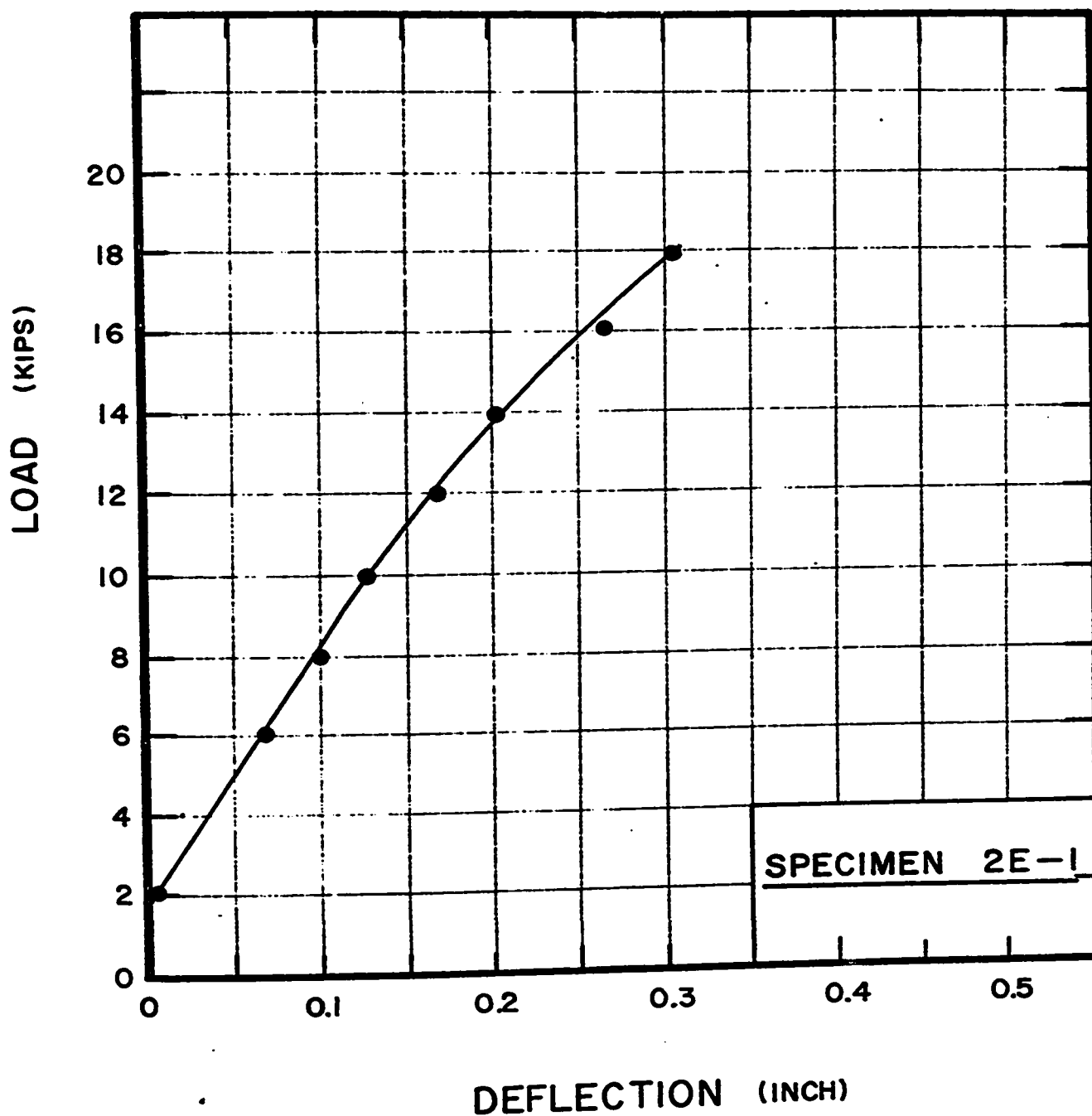
**FIG. (B. 4 8) LOAD — DEFLECTION CURVE
OF SPECIMEN 3A — 3**



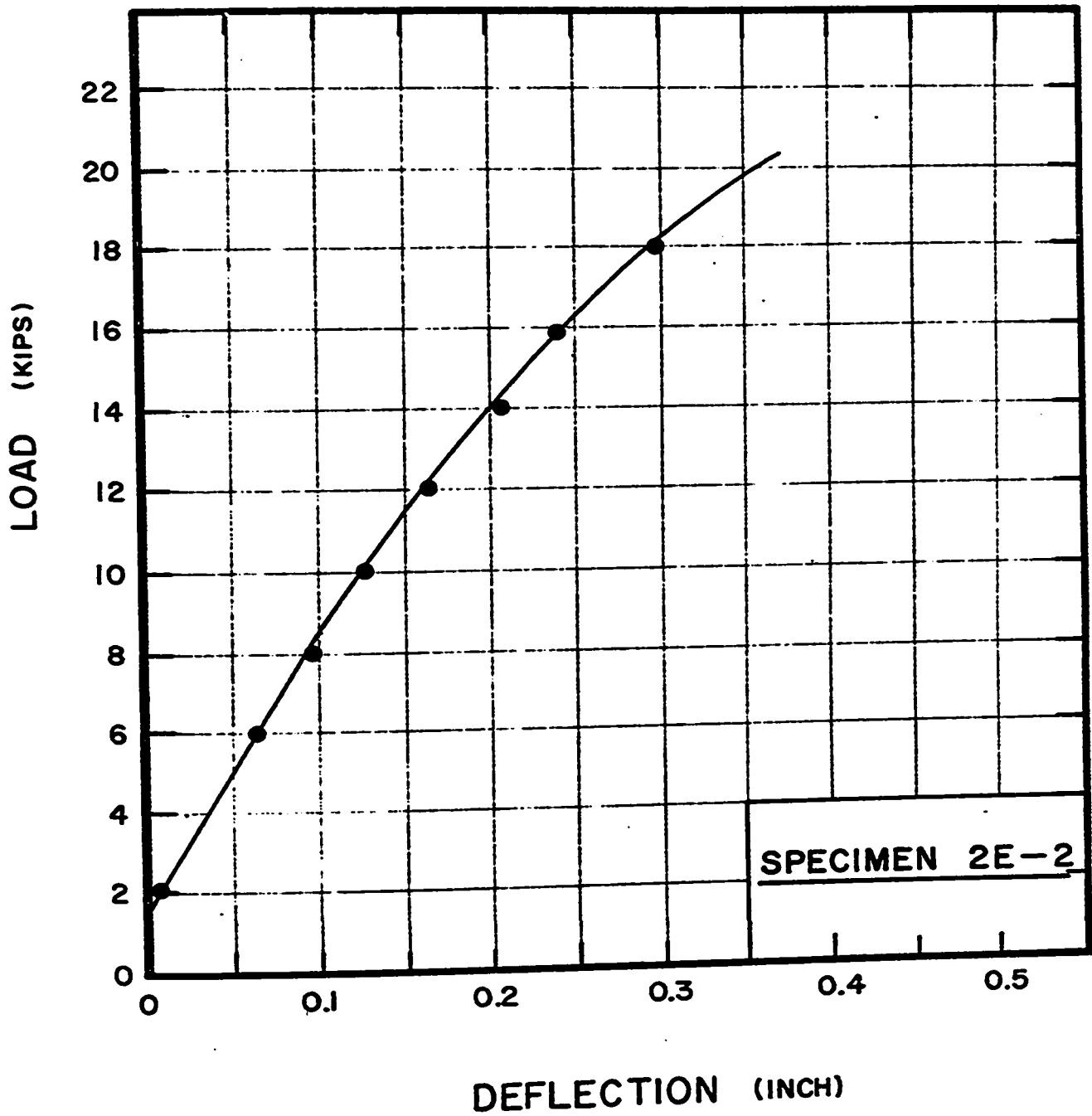
**FIG. (B. 49) LOAD—DEFLECTION CURVE
OF SPECIMEN 3A-4**



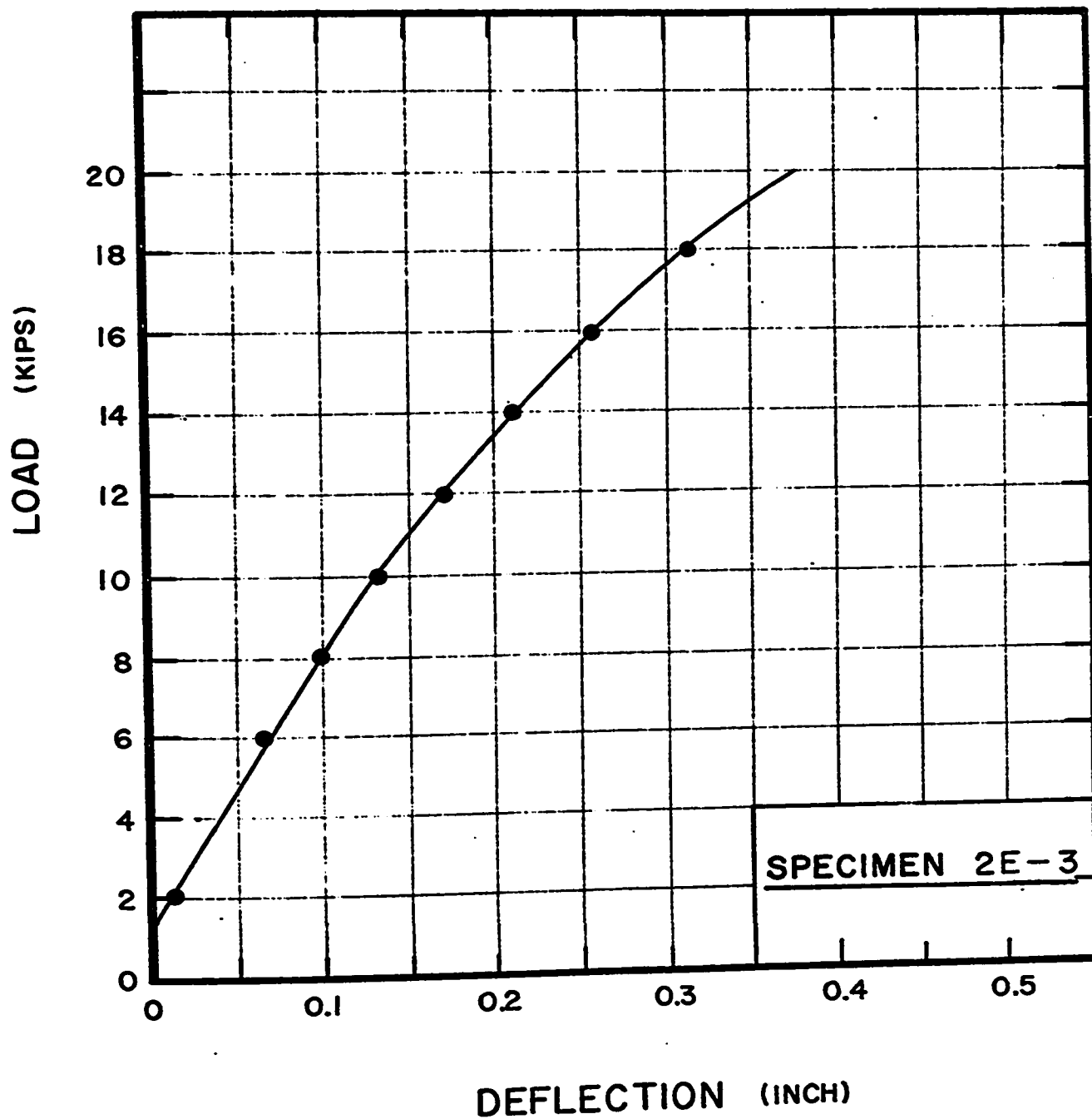
**FIG. (B. 50) LOAD—DEFLECTION CURVE
OF SPECIMEN 3A—5**



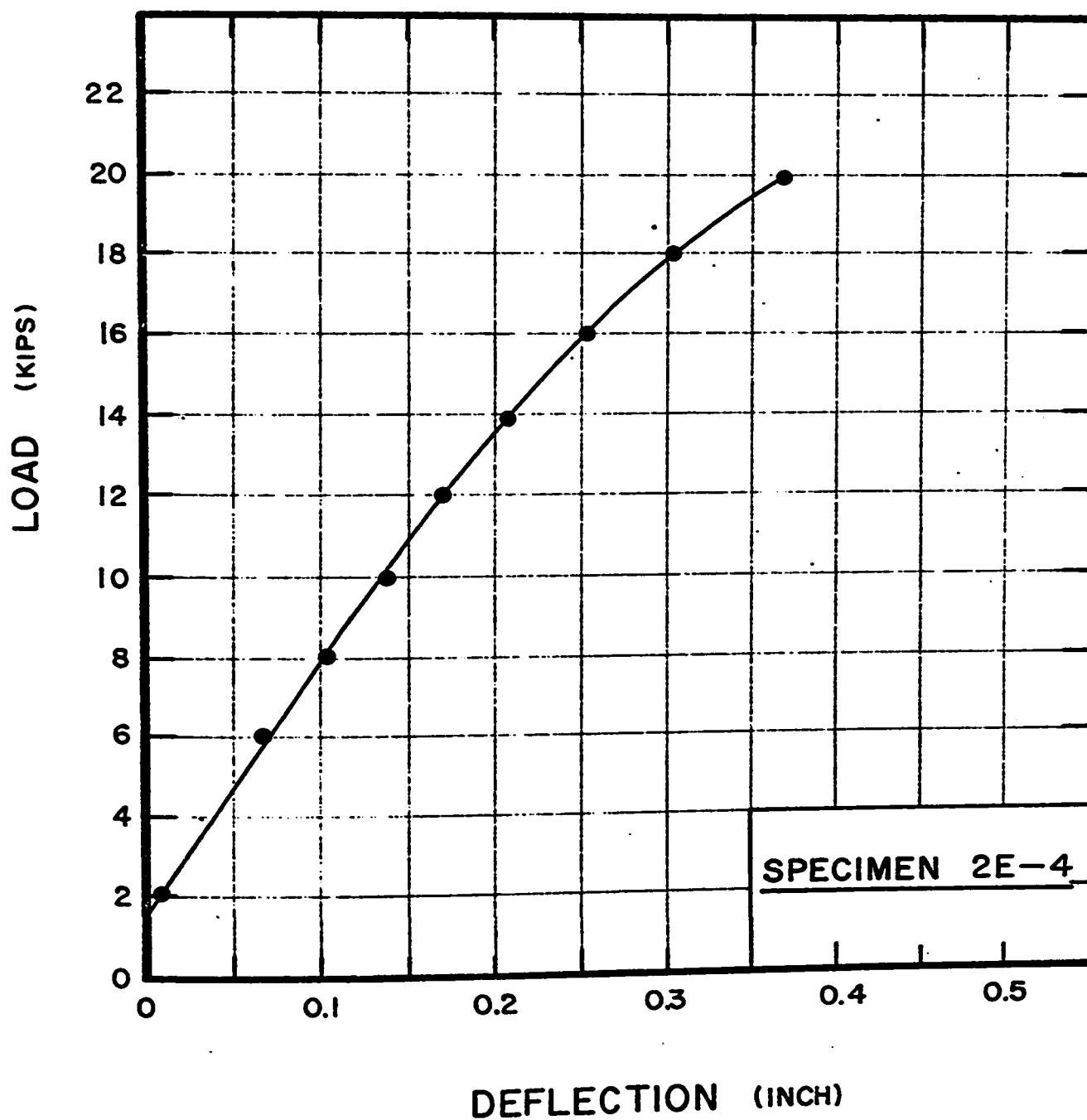
**FIG. (B. 51) LOAD—DEFLECTION CURVE
OF SPECIMEN 2E—1**



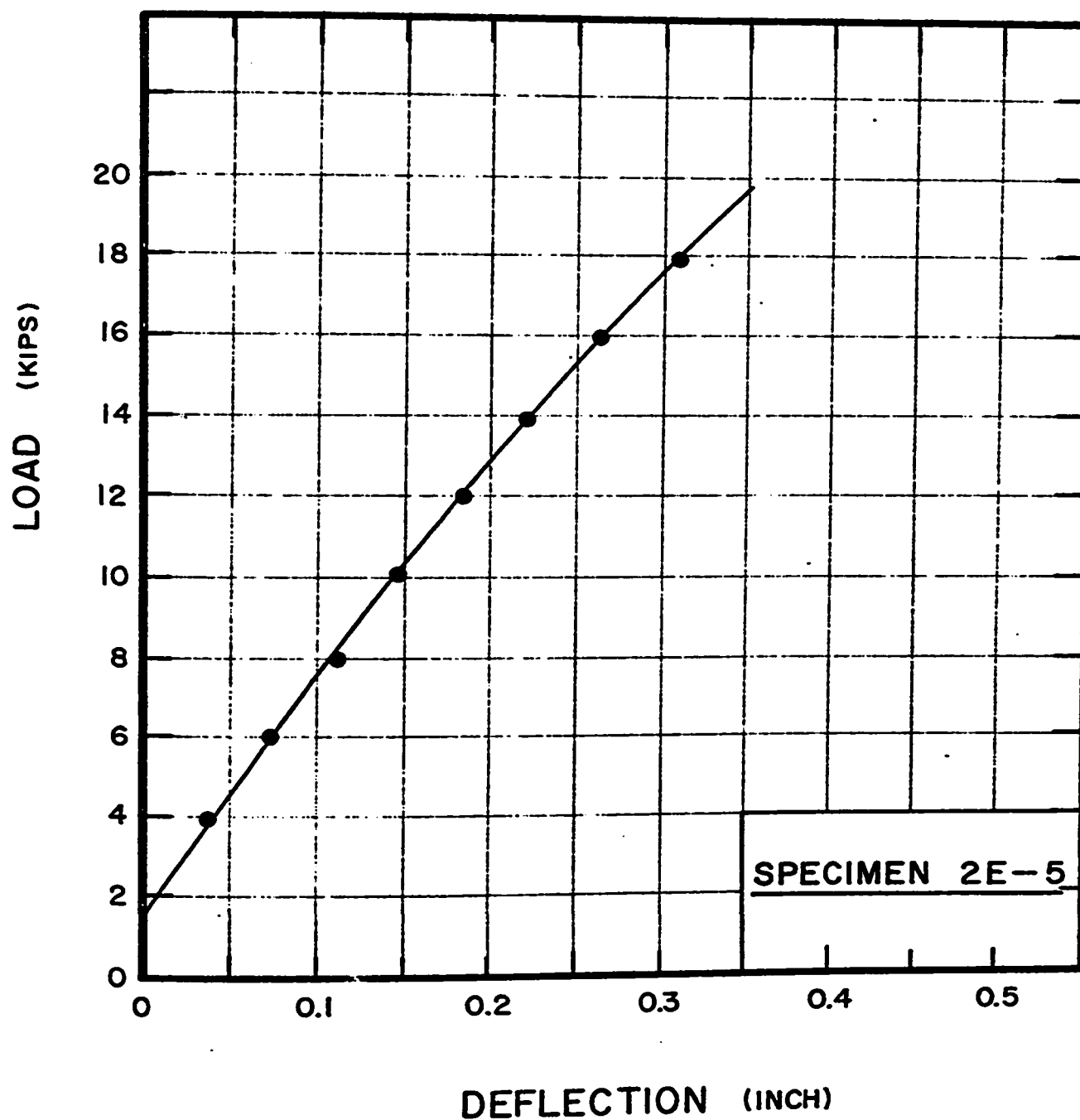
**FIG. (B. 52) LOAD—DEFLECTION CURVE
OF SPECIMEN 2E-2**



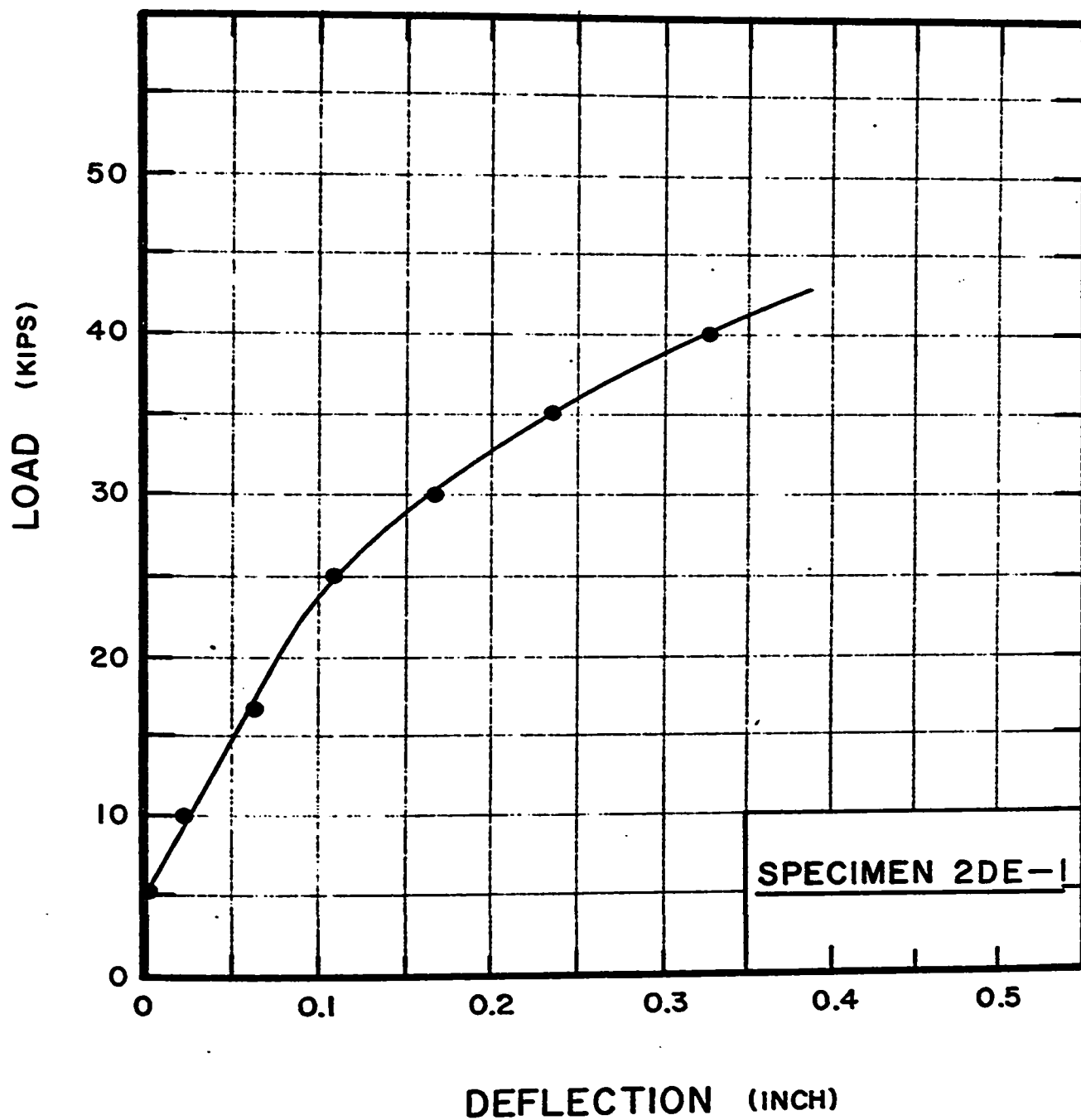
**FIG. (B. 53) LOAD—DEFLECTION CURVE
OF SPECIMEN 2E-3**



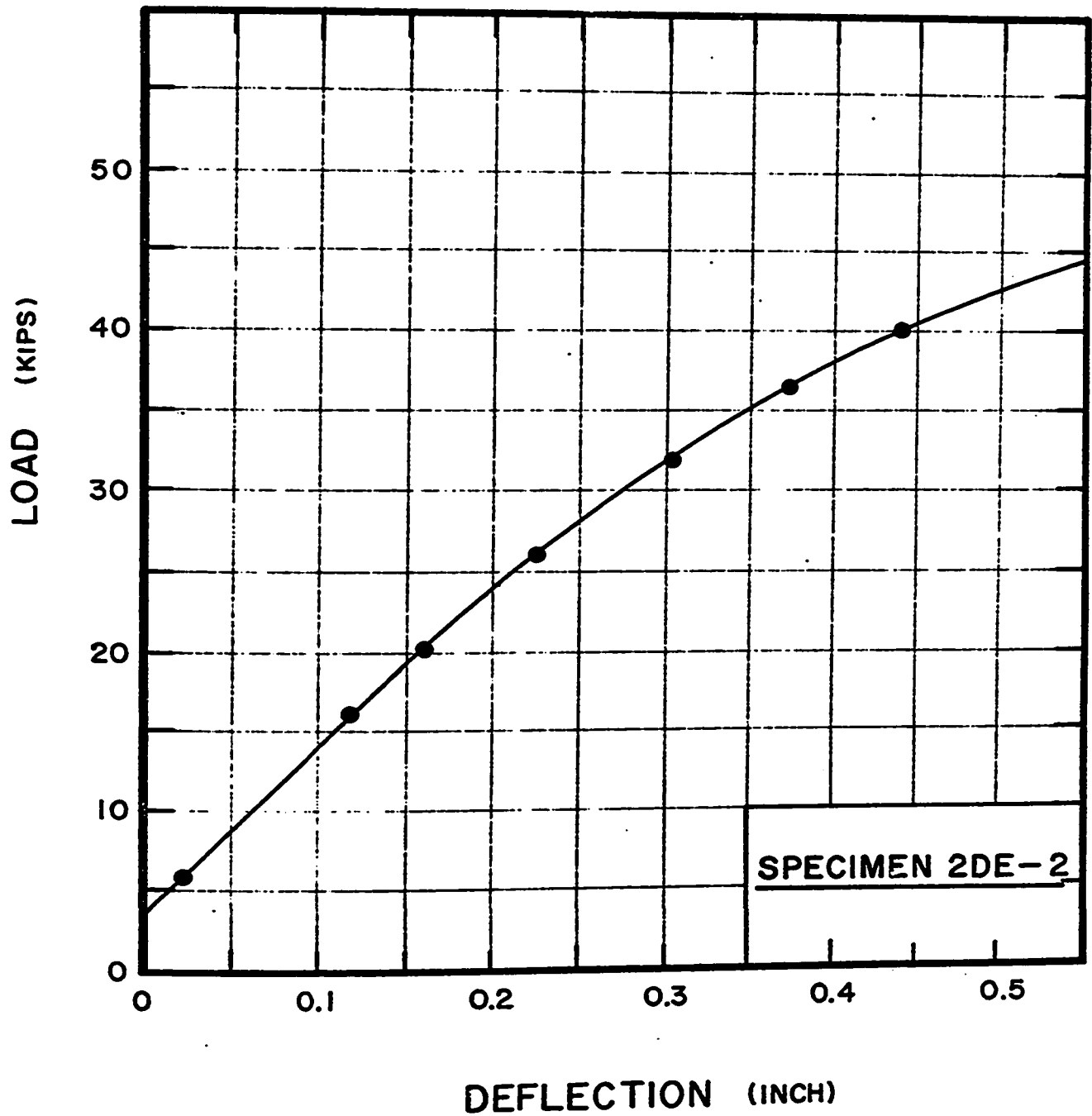
**FIG. (B. 5 4) LOAD—DEFLECTION CURVE
OF SPECIMEN 2E—4**



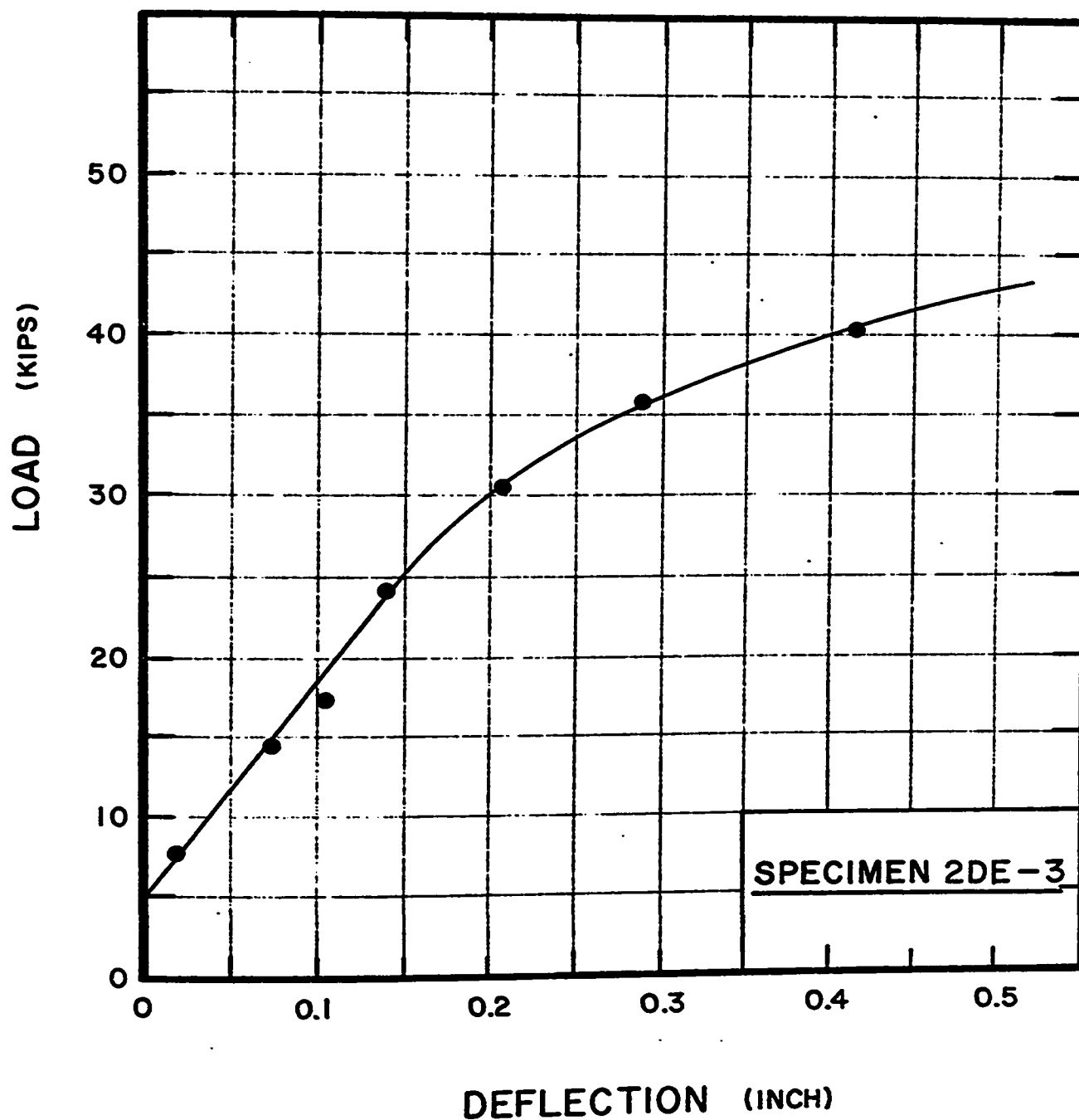
**FIG. (B. 5 5) LOAD—DEFLECTION CURVE
OF SPECIMEN 2E-5**



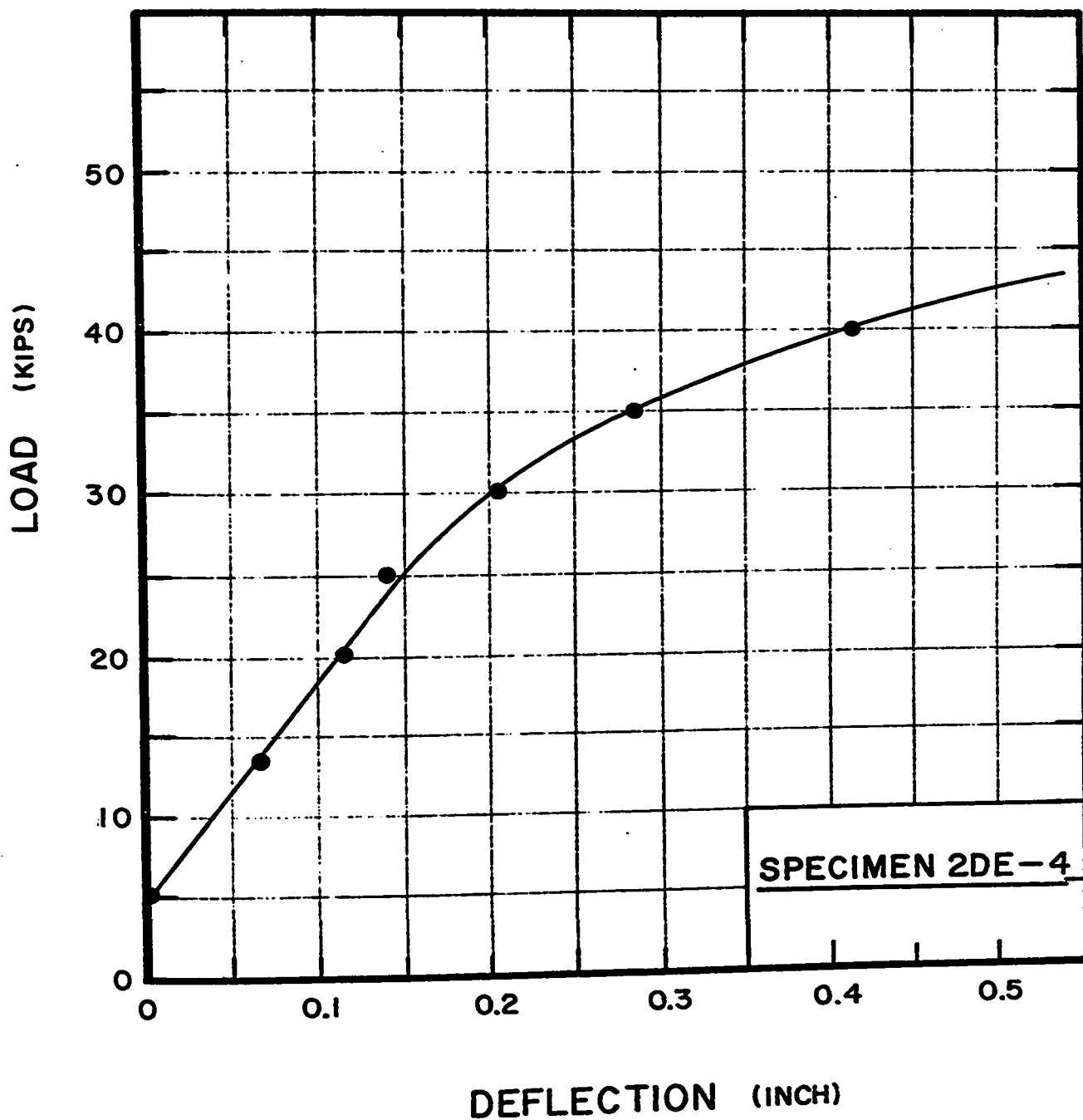
**FIG. (B. 56) LOAD—DEFLECTION CURVE
OF SPECIMEN 2DE—1**



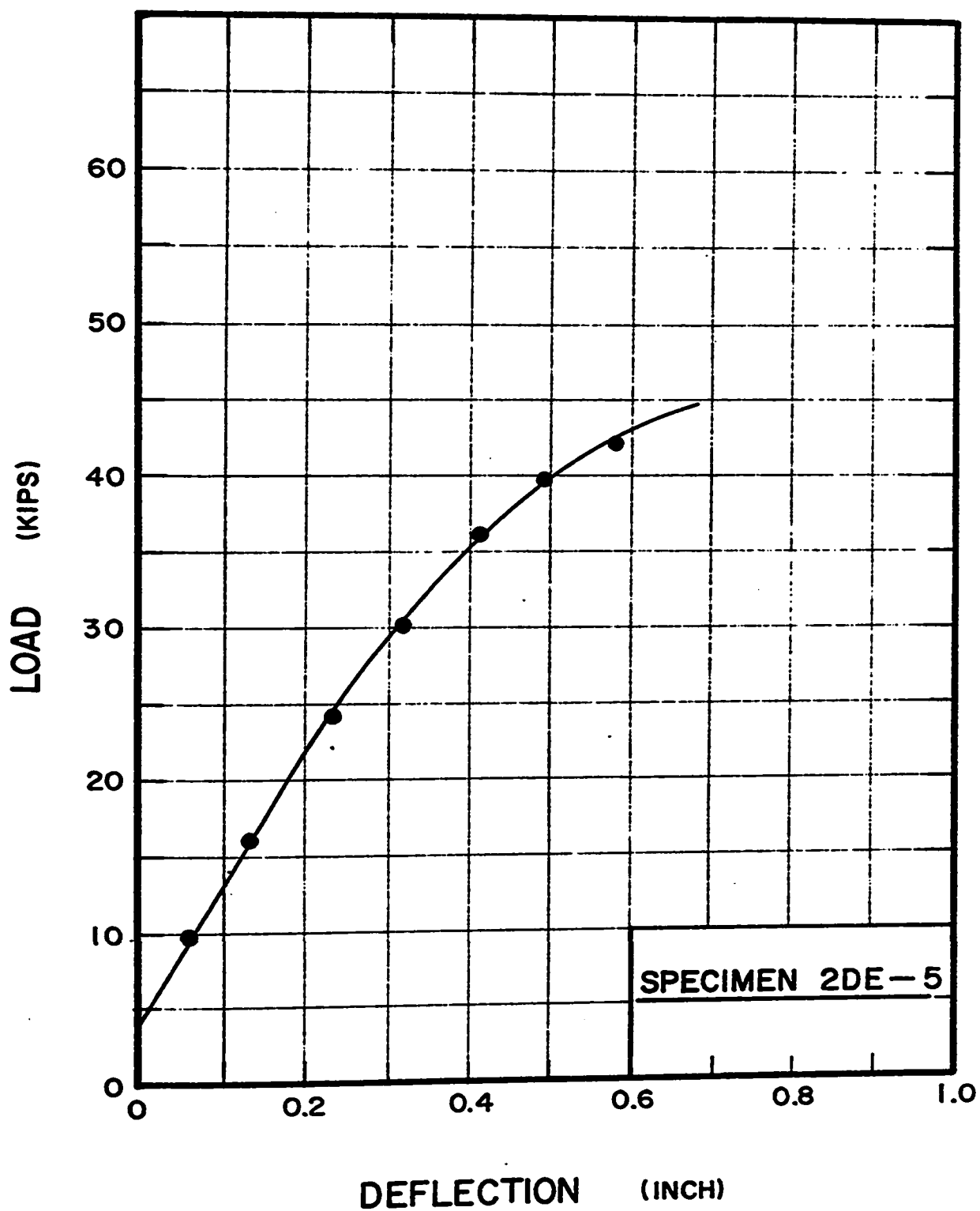
**FIG. (B. 57) LOAD—DEFLECTION CURVE
OF SPECIMEN 2DE-2**



**FIG. (B. 58) LOAD—DEFLECTION CURVE
OF SPECIMEN 2DE-3**



**FIG. (B. 59) LOAD—DEFLECTION CURVE
OF SPECIMEN 2DE-4**



**FIG.(B.60) LOAD—DEFLECTION CURVE
OF SPECIMEN 2DE-5**

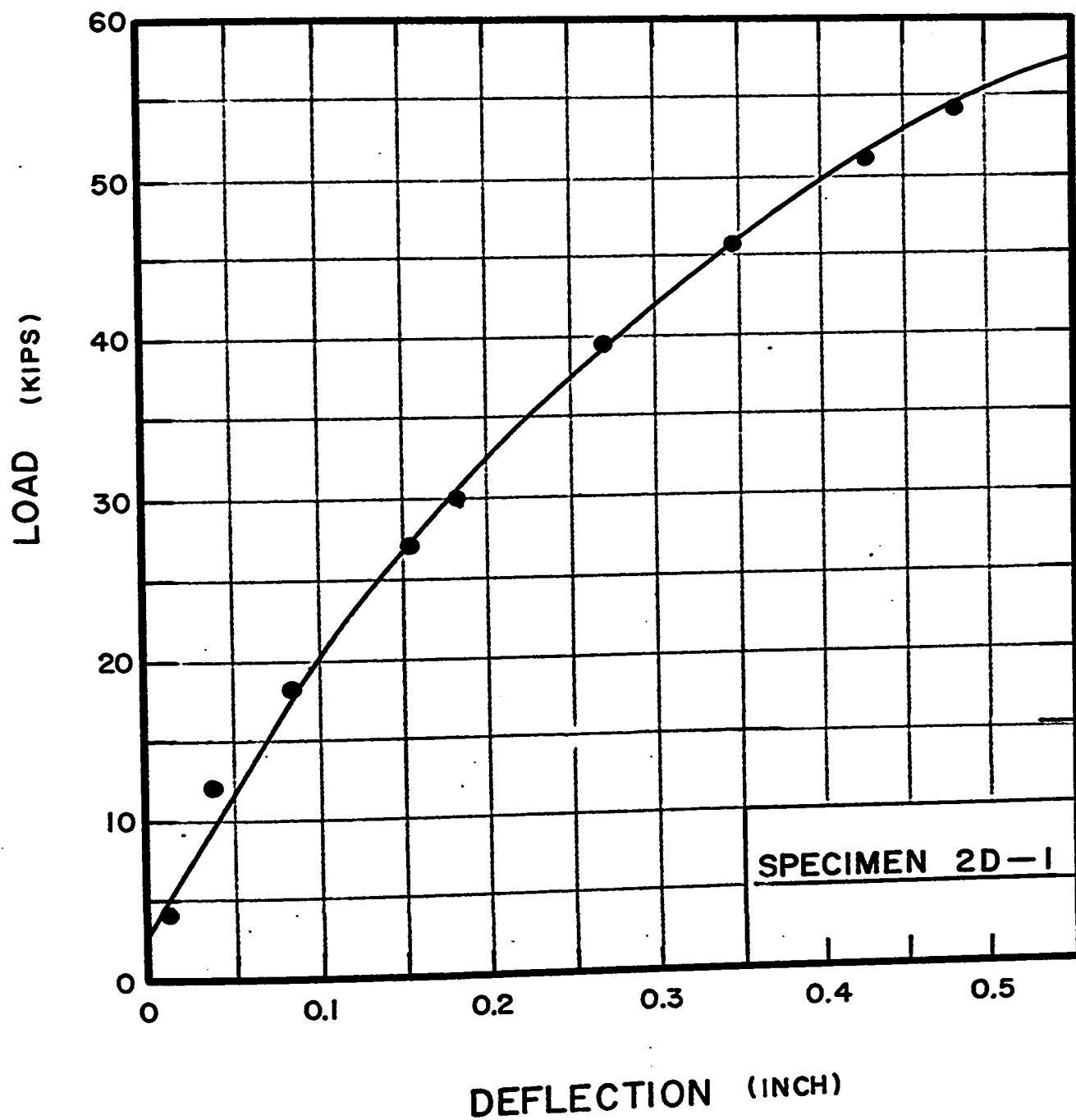


FIG.(B. 61) LOAD — DEFLECTION CURVE
OF SPECIMEN 2D-1

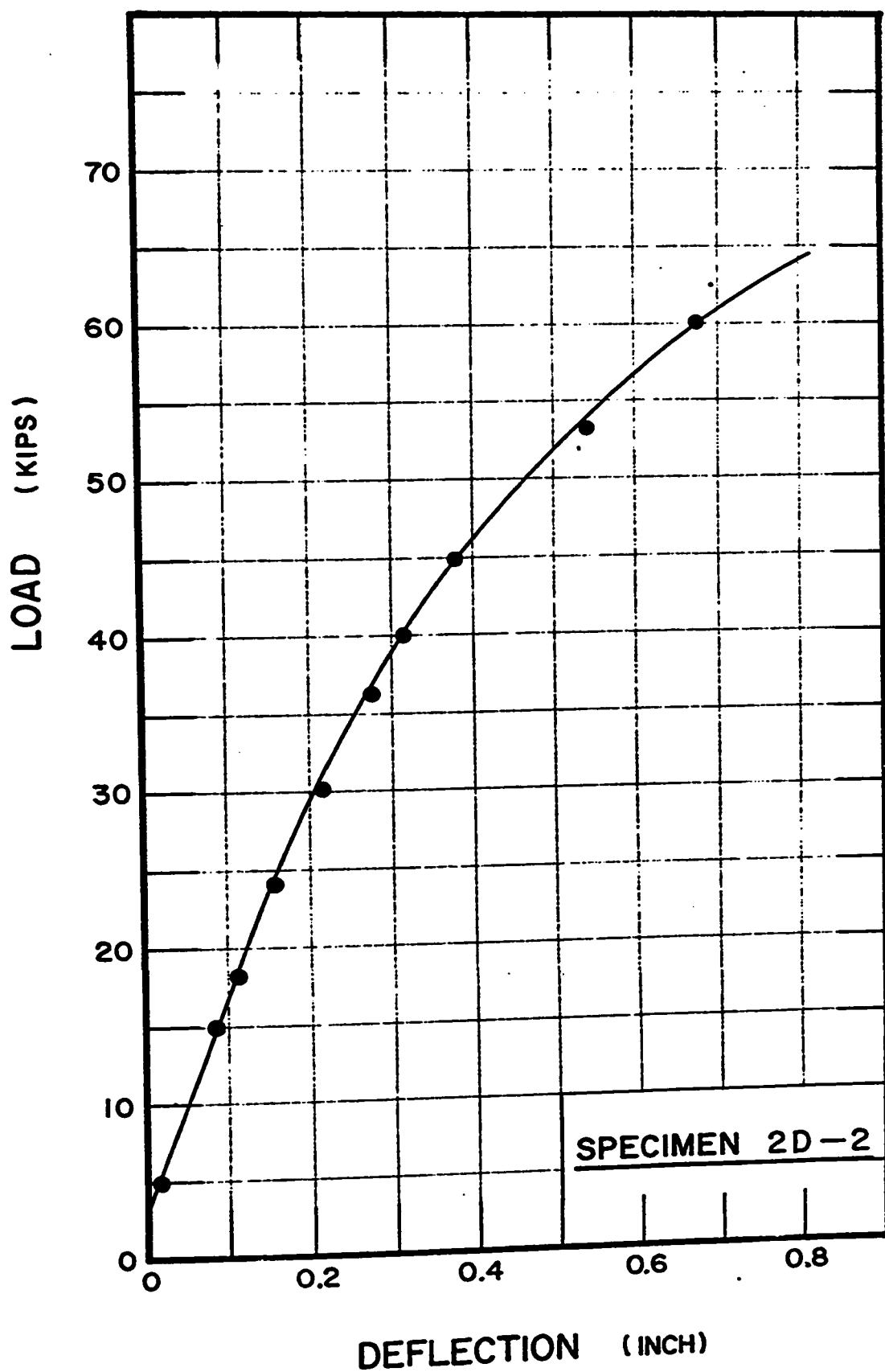
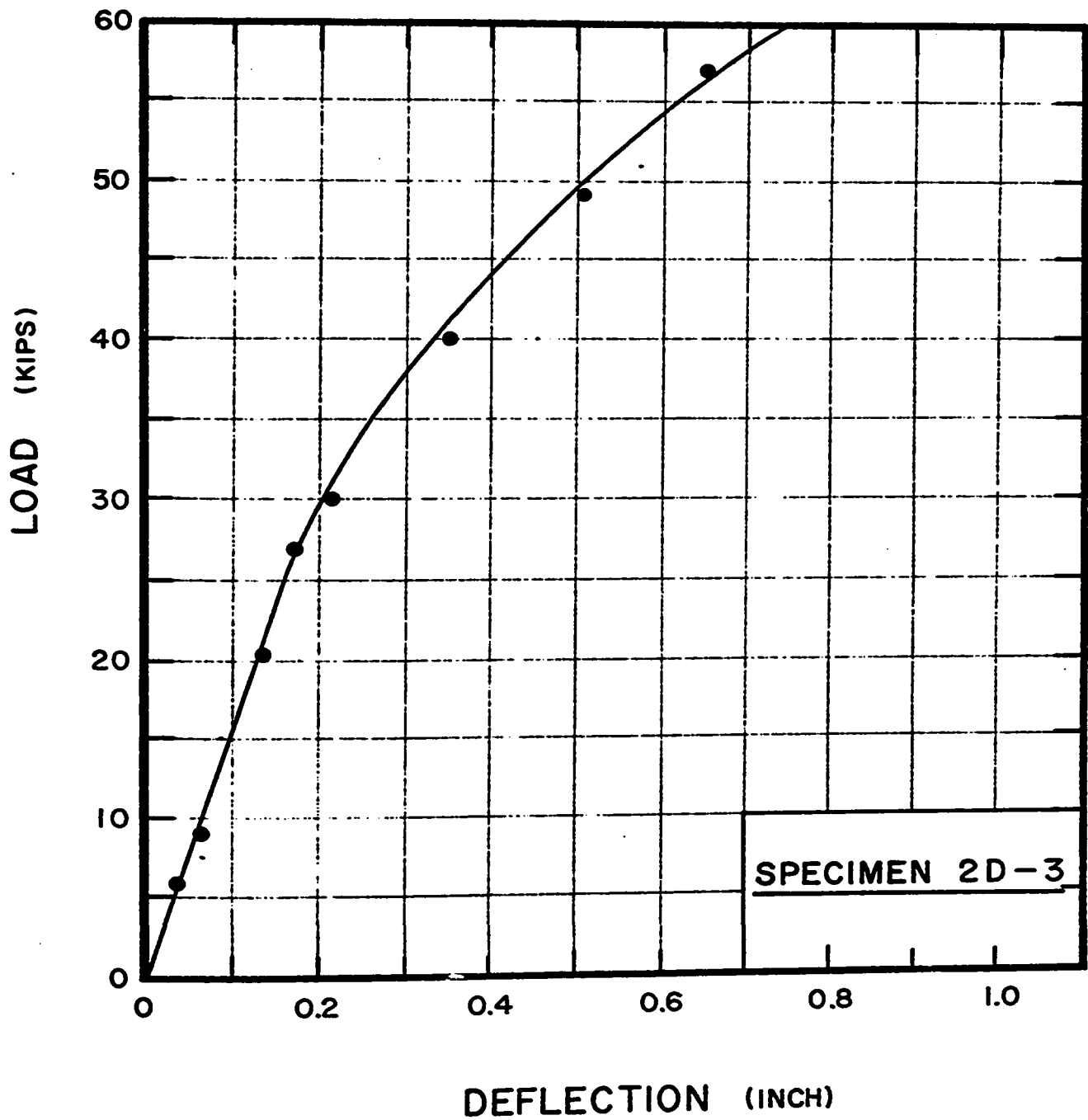
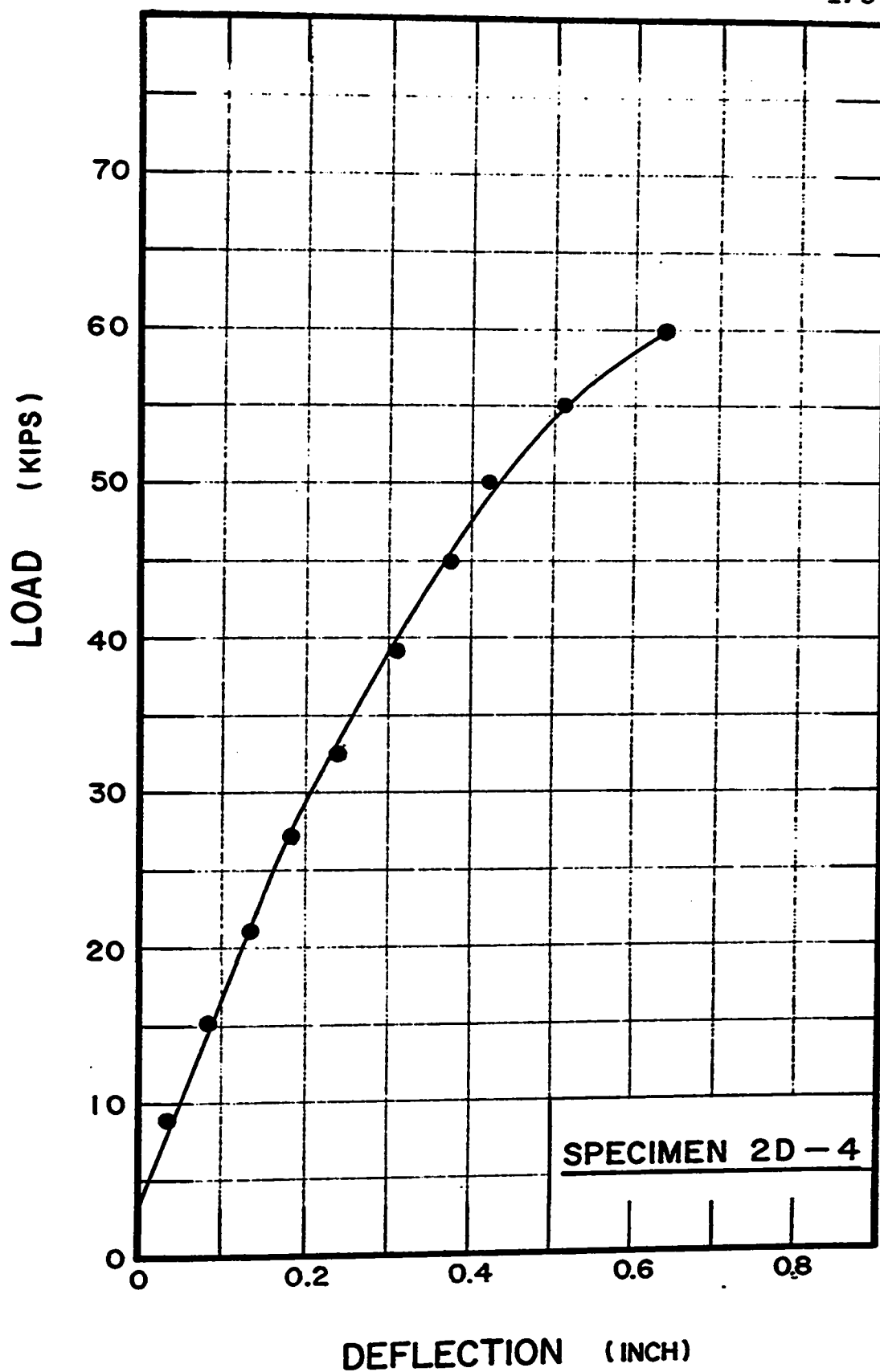


FIG.(B. 62) LOAD—DEFLECTION
CURVE OF SPECIMEN 2D-2



**FIG. (B. 63) LOAD—DEFLECTION CURVE
OF SPECIMEN 2D-3**



**FIG.(B.64) LOAD-DEFLECTION
CURVE OF SPECIMEN 2D-4**

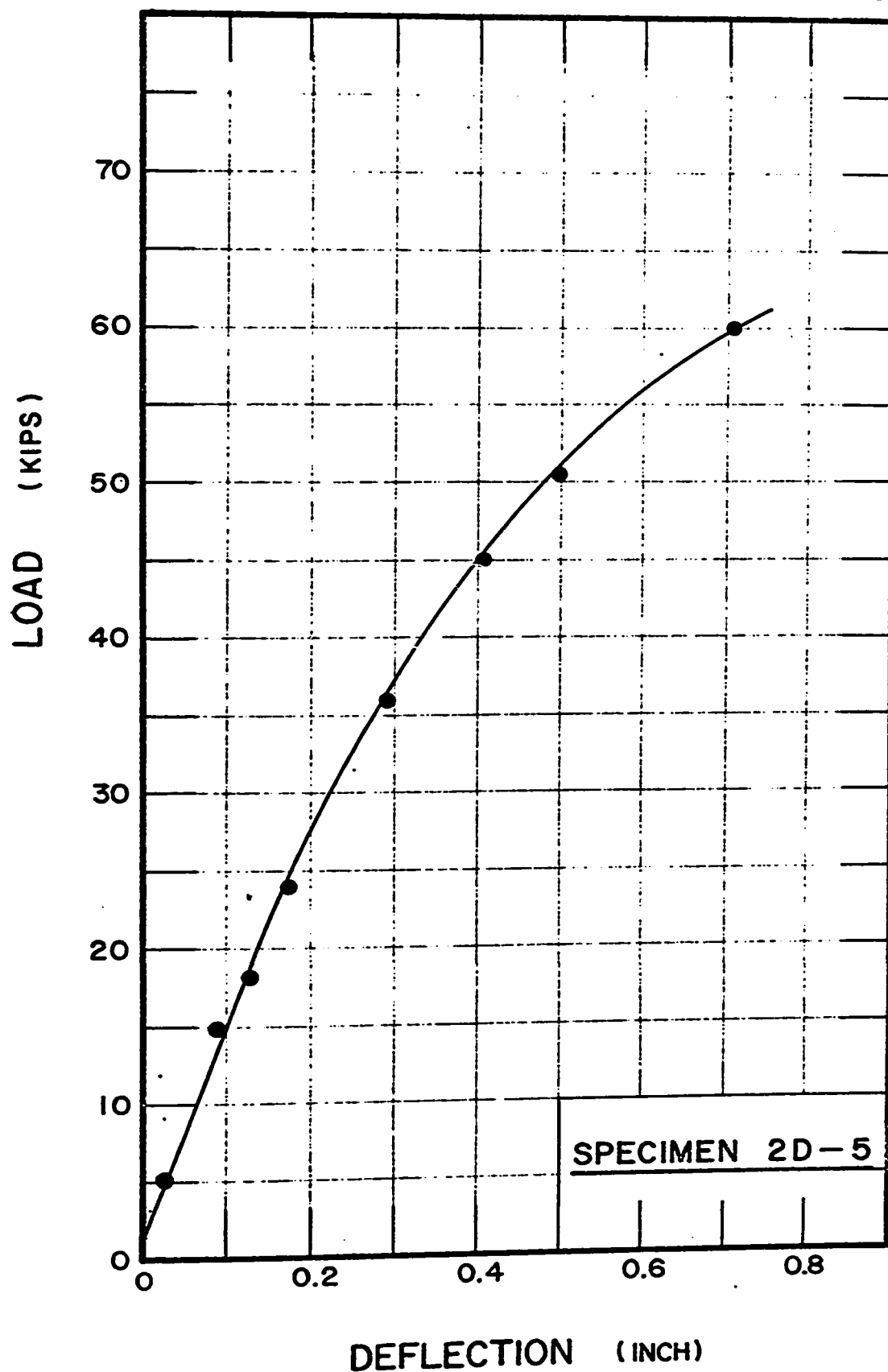


FIG.(B. 65) LOAD—DEFLECTION
CURVE OF SPECIMEN 2D—5

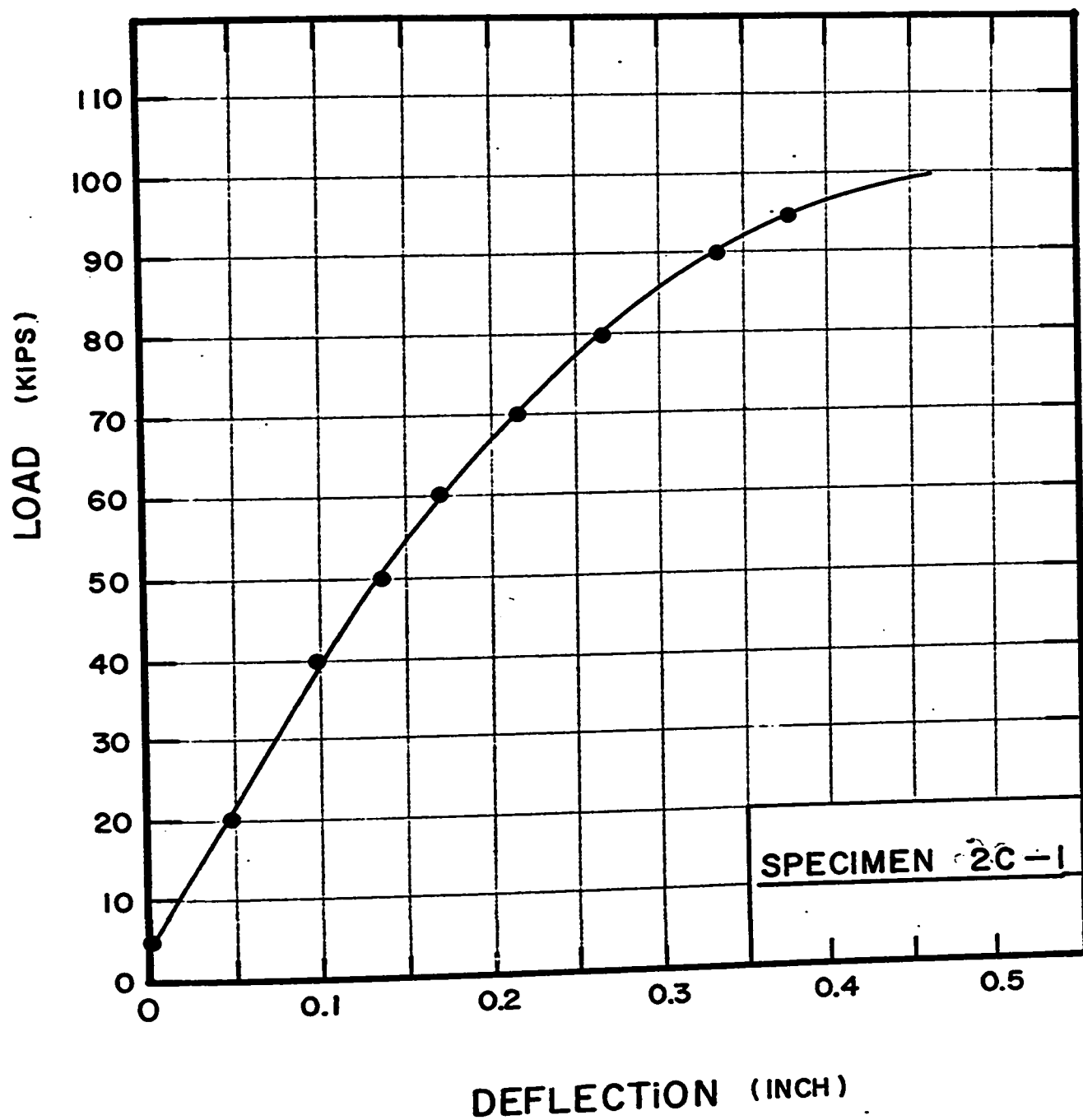


FIG. (B. 6 6) LOAD—DEFLECTION CURVE
OF SPECIMEN 2C-1

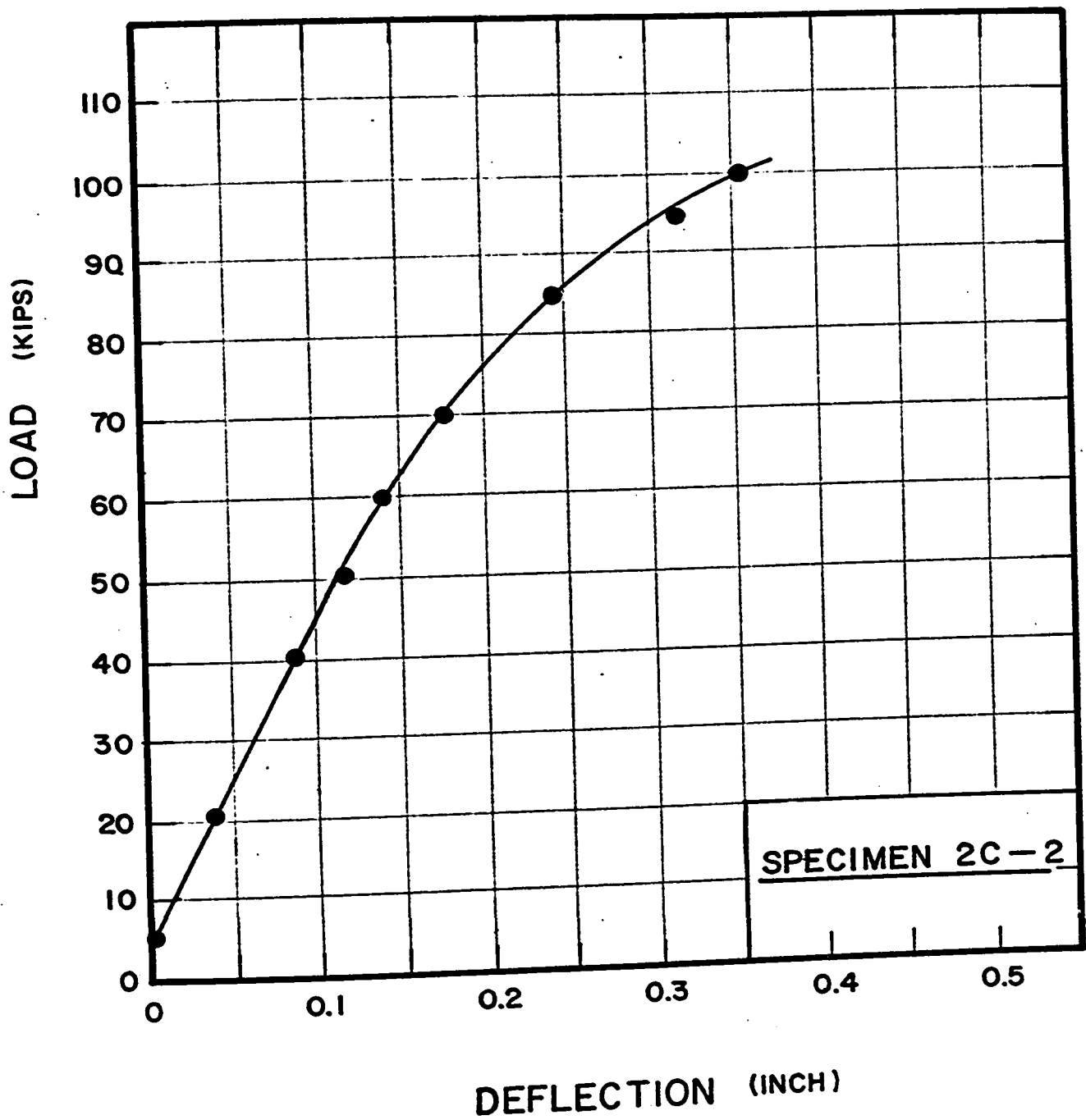
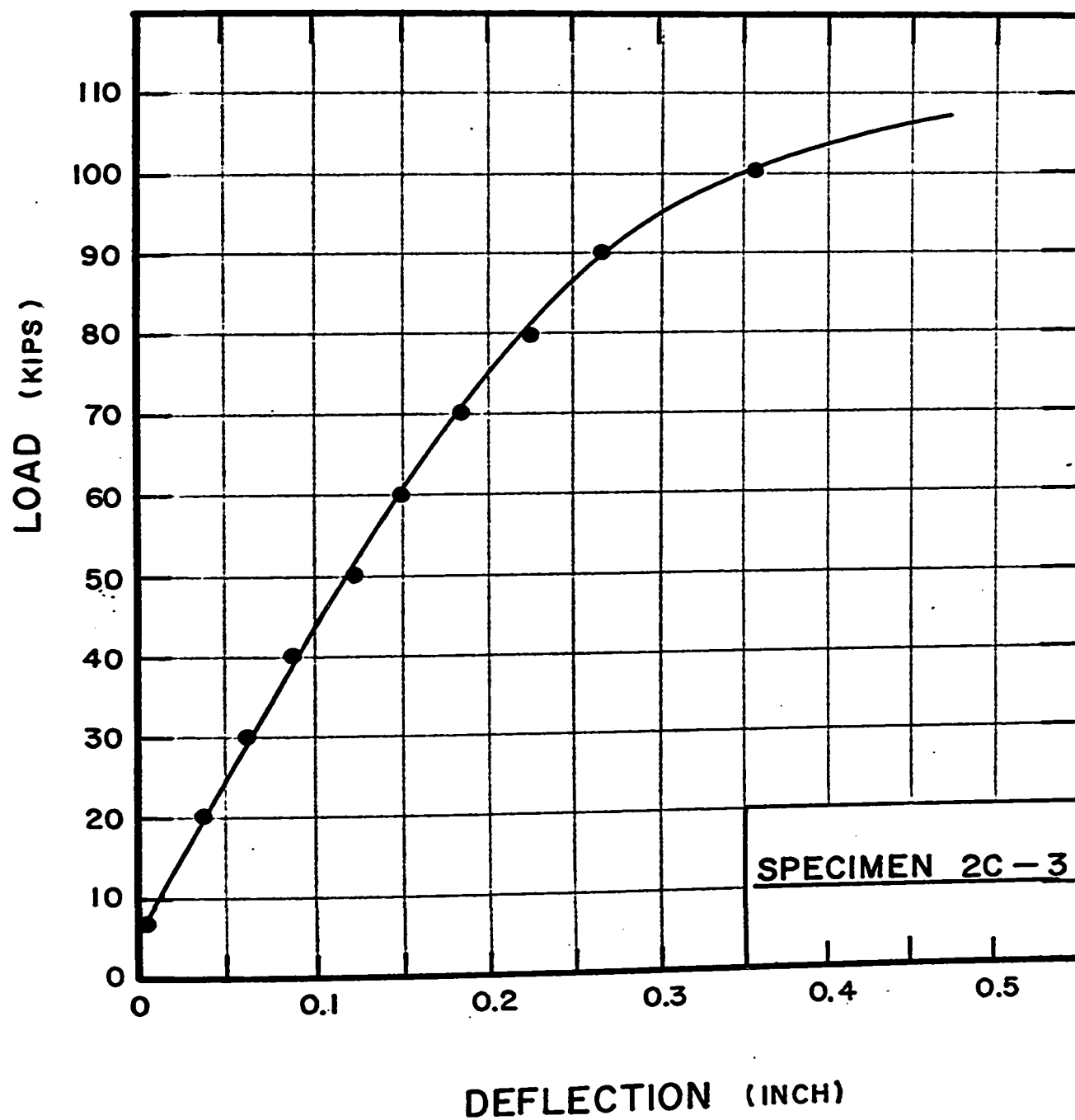
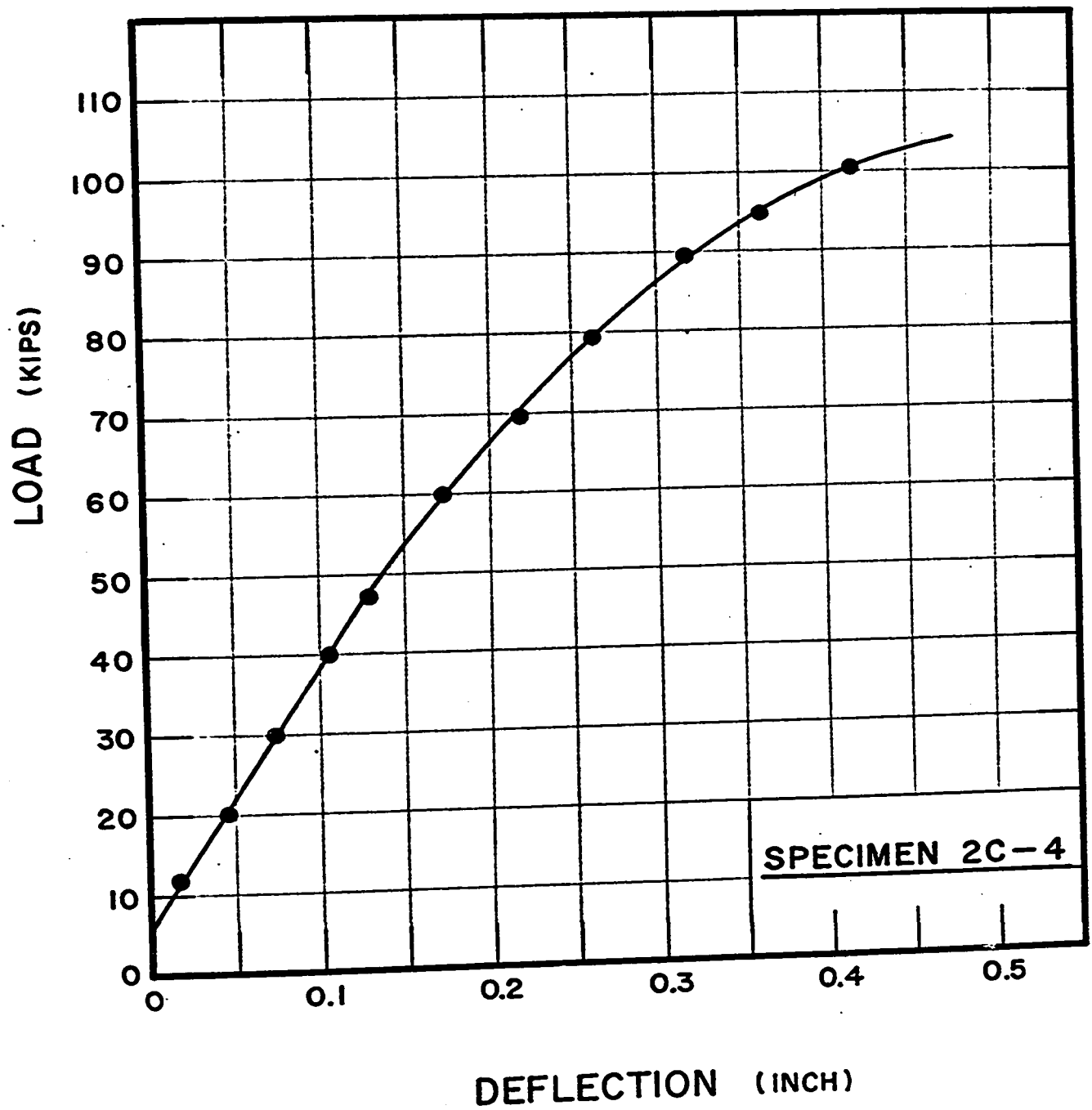


FIG. (B. 67) LOAD — DEFLECTION CURVE
OF SPECIMEN 2C-2



**FIG.(B.68) LOAD-DEFLECTION CURVE
OF SPECIMEN 2C-3**



**FIG. (B.69) LOAD—DEFLECTION CURVE
OF SPECIMEN 2C-4**

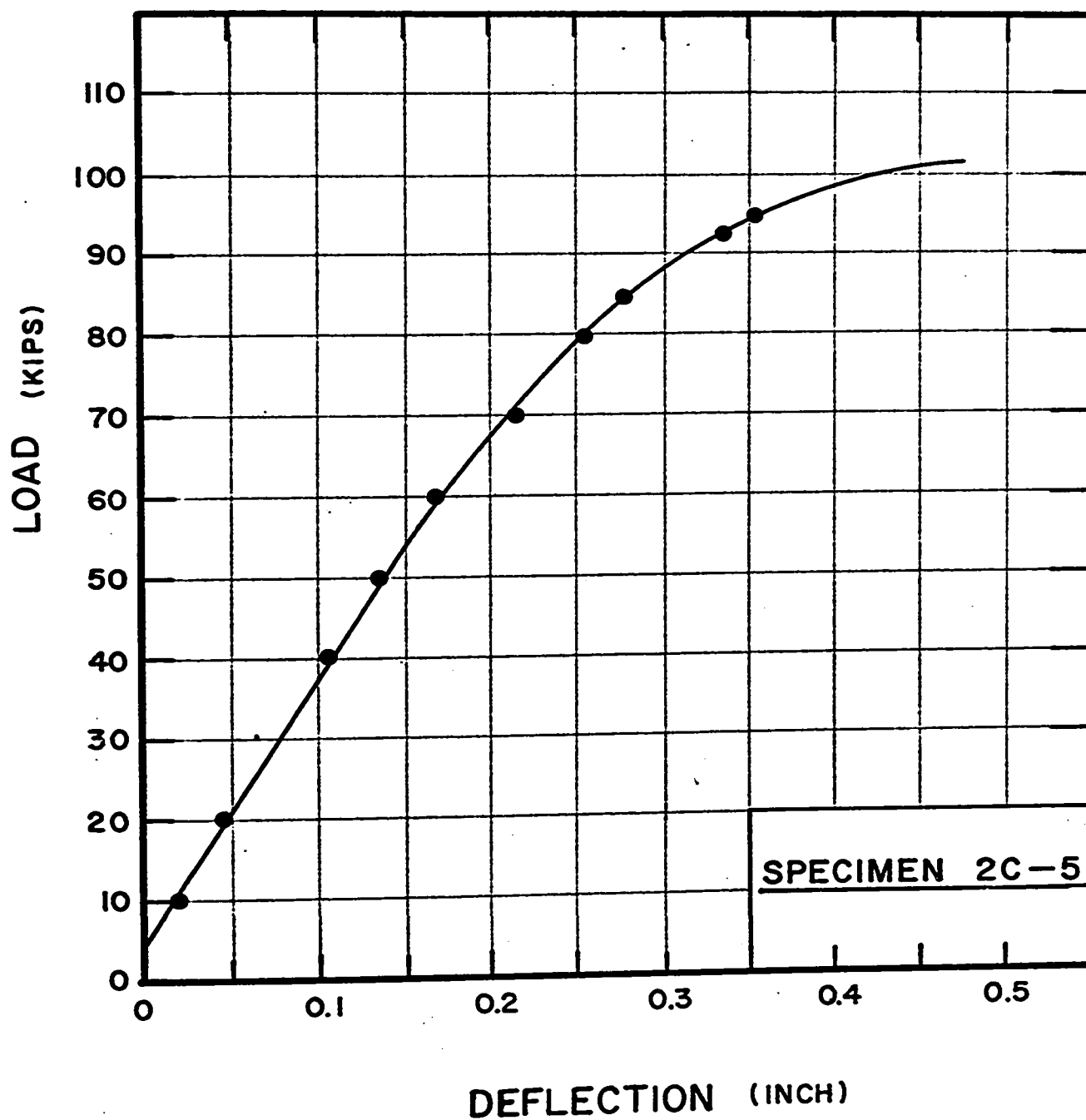
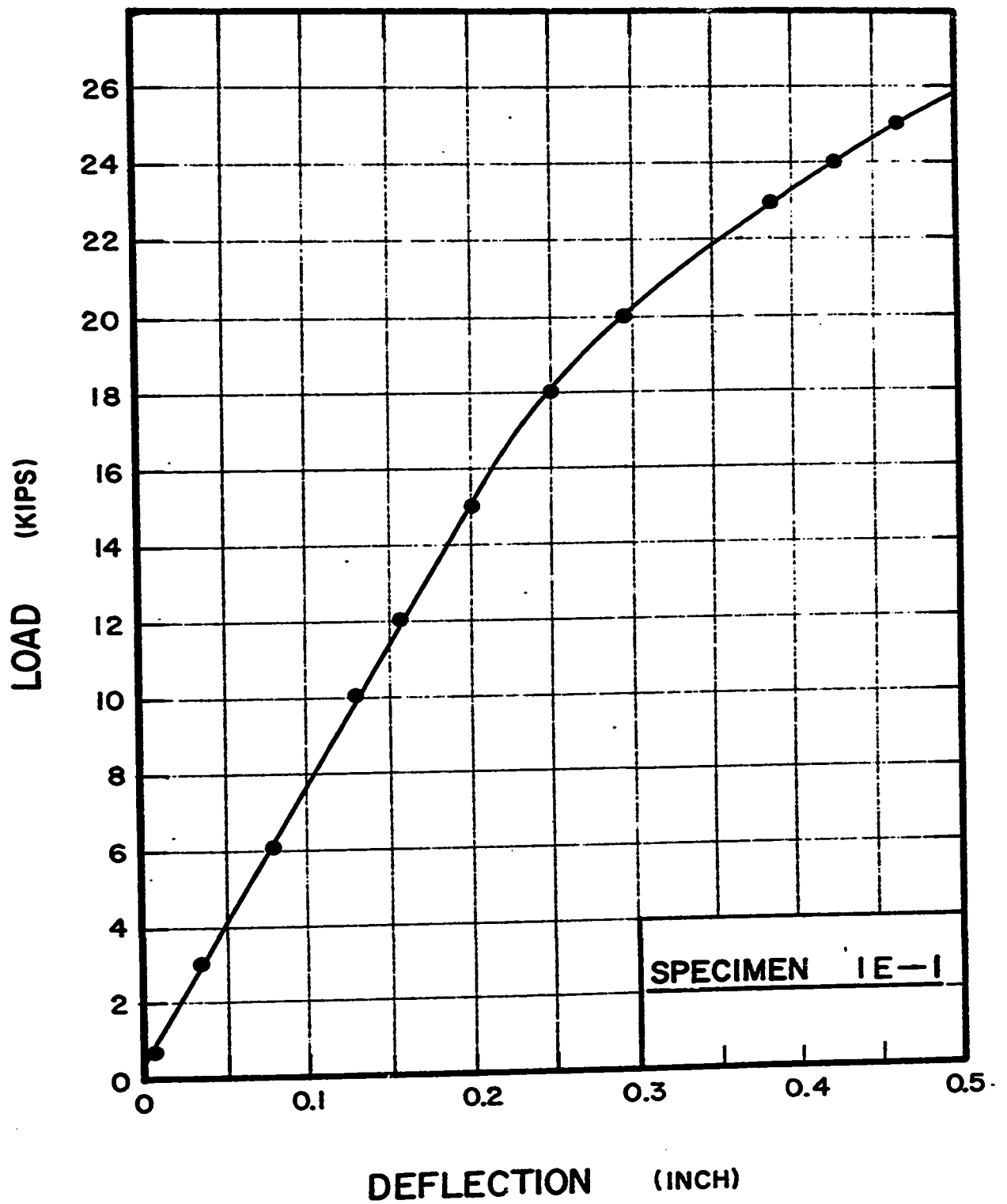


FIG.(B. 70) LOAD—DEFLECTION CURVE
OF SPECIMEN 2C-5



**FIG.(B. 71) LOAD—DEFLECTION CURVE
OF SPECIMEN IE—1**

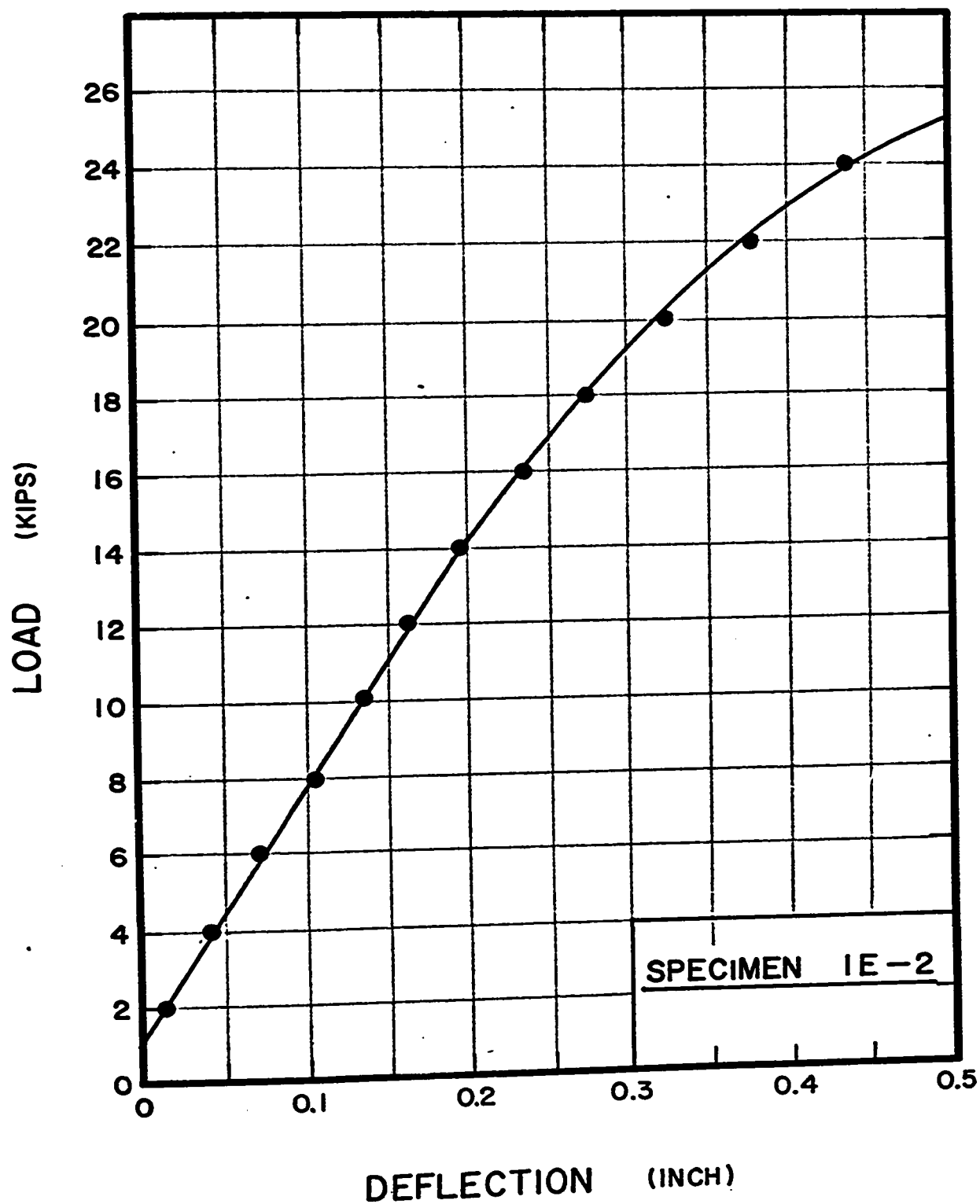
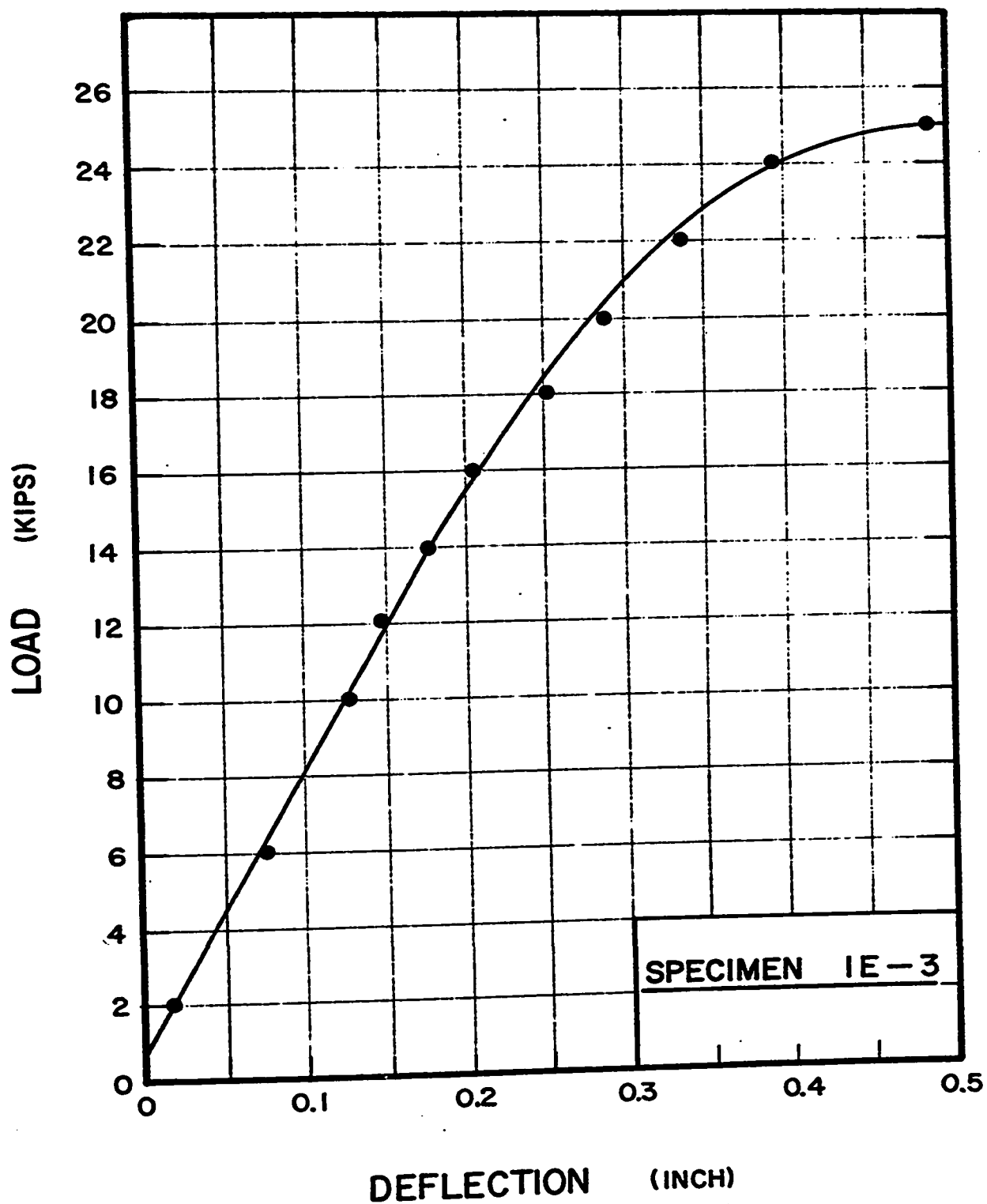


FIG.(B.72) LOAD — DEFLECTION CURVE
OF SPECIMEN 1E — 2



**FIG.(B.73) LOAD—DEFLECTION CURVE
OF SPECIMEN IE—3**

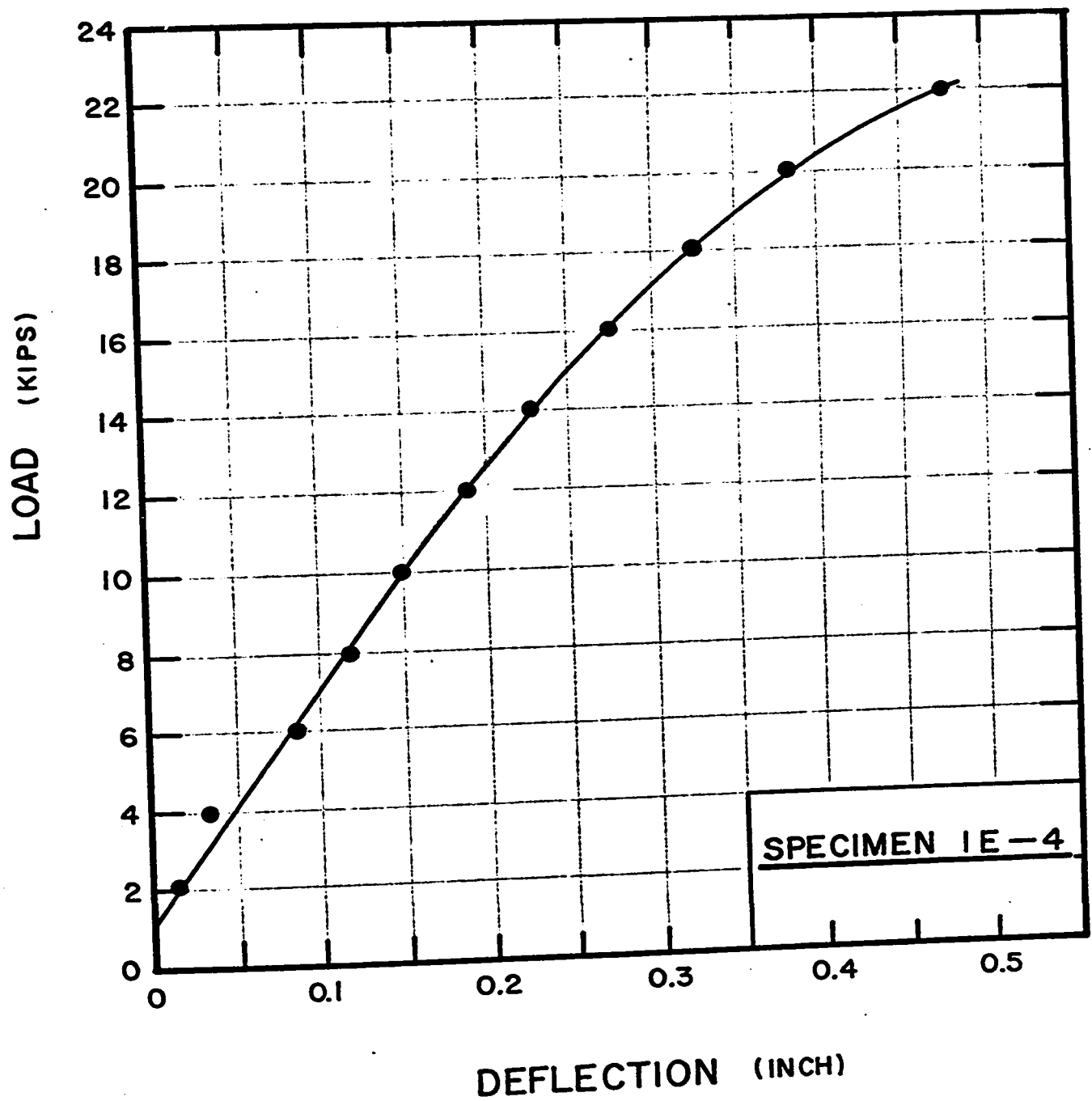
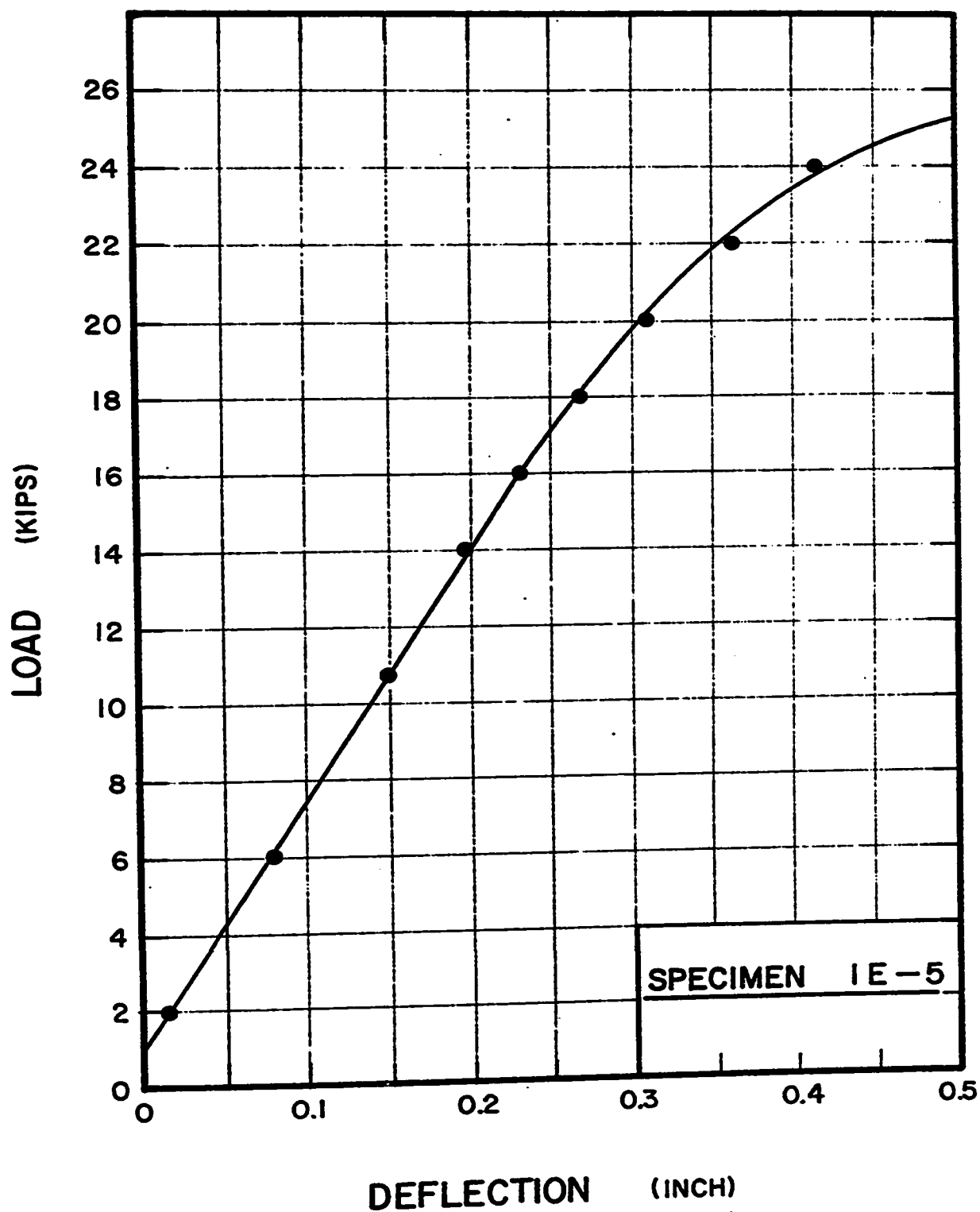
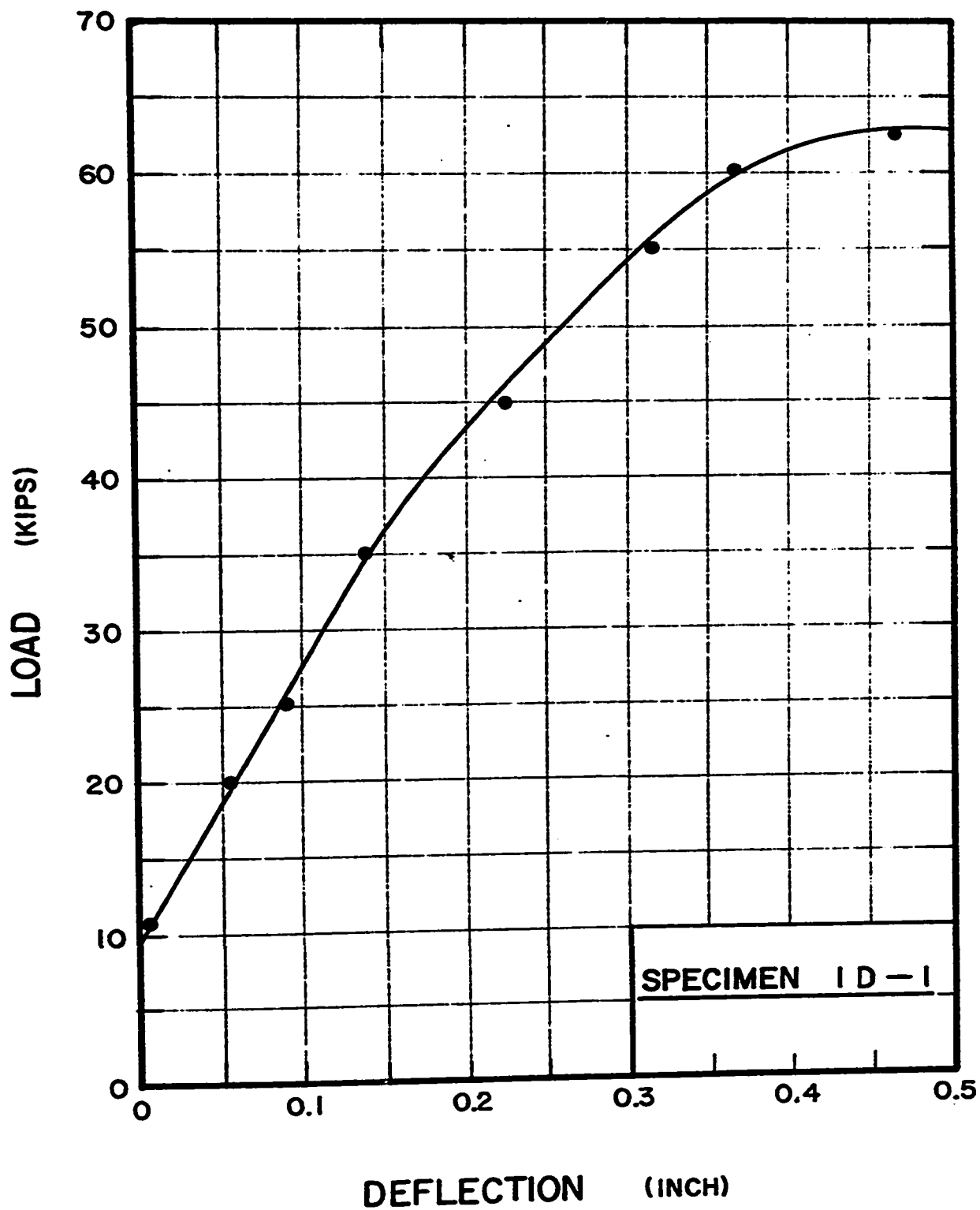


FIG. (B. 74) LOAD — DEFLECTION CURVE
OF SPECIMEN IE — 4



**FIG.(B.75) LOAD—DEFLECTION CURVE
OF SPECIMEN 1E-5**



**FIG.(B. 76) LOAD—DEFLECTION CURVE
OF SPECIMEN ID—1**

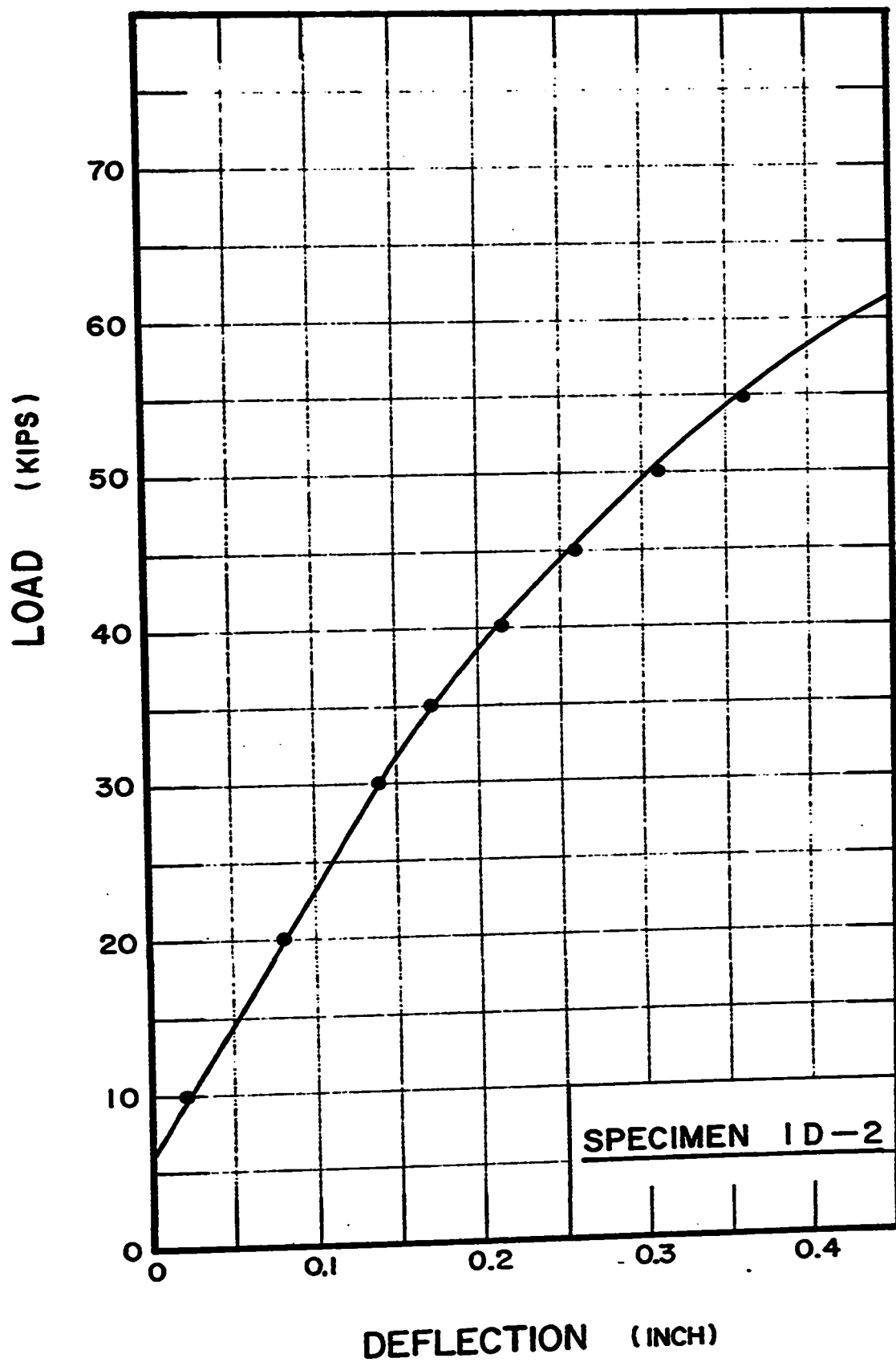
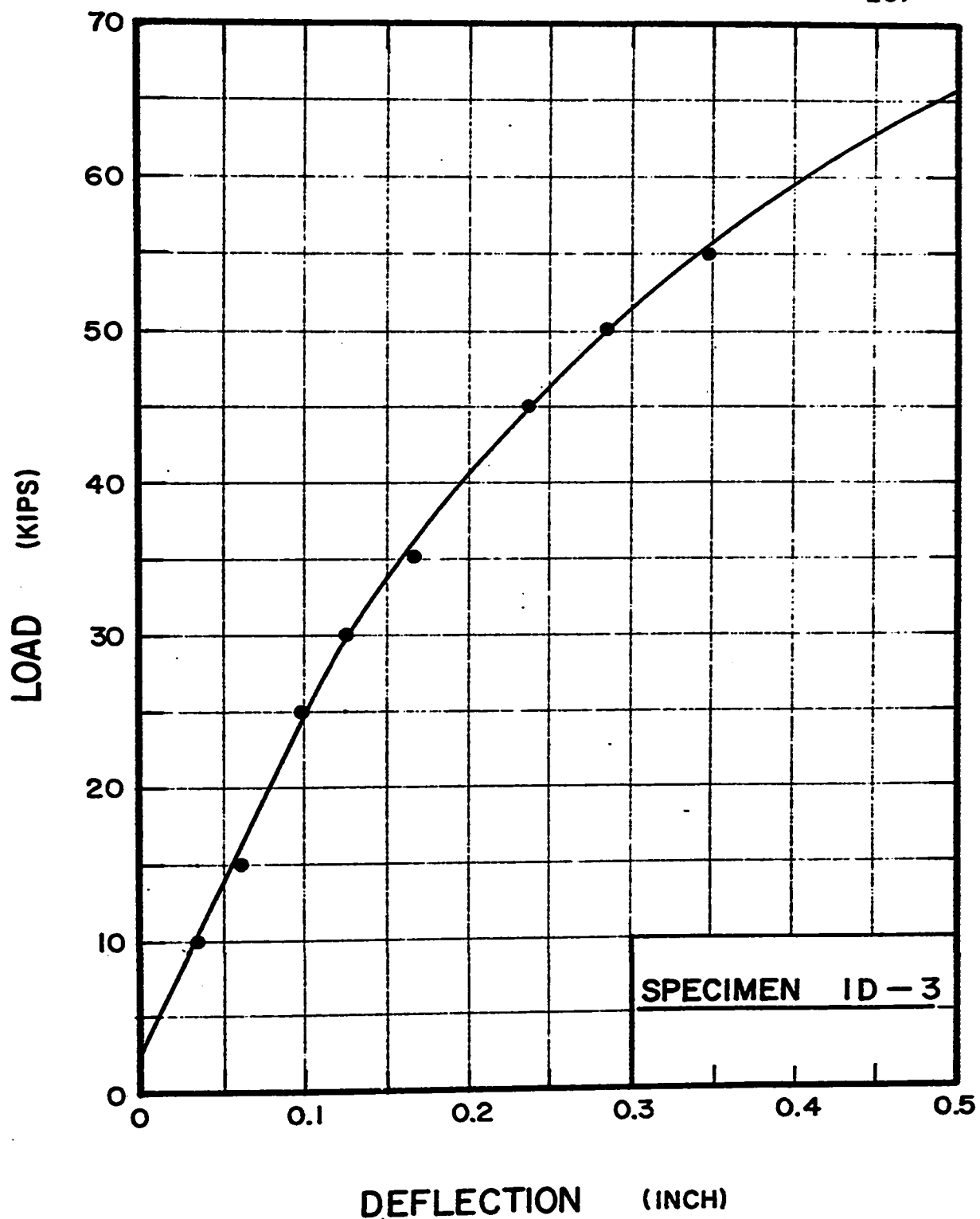
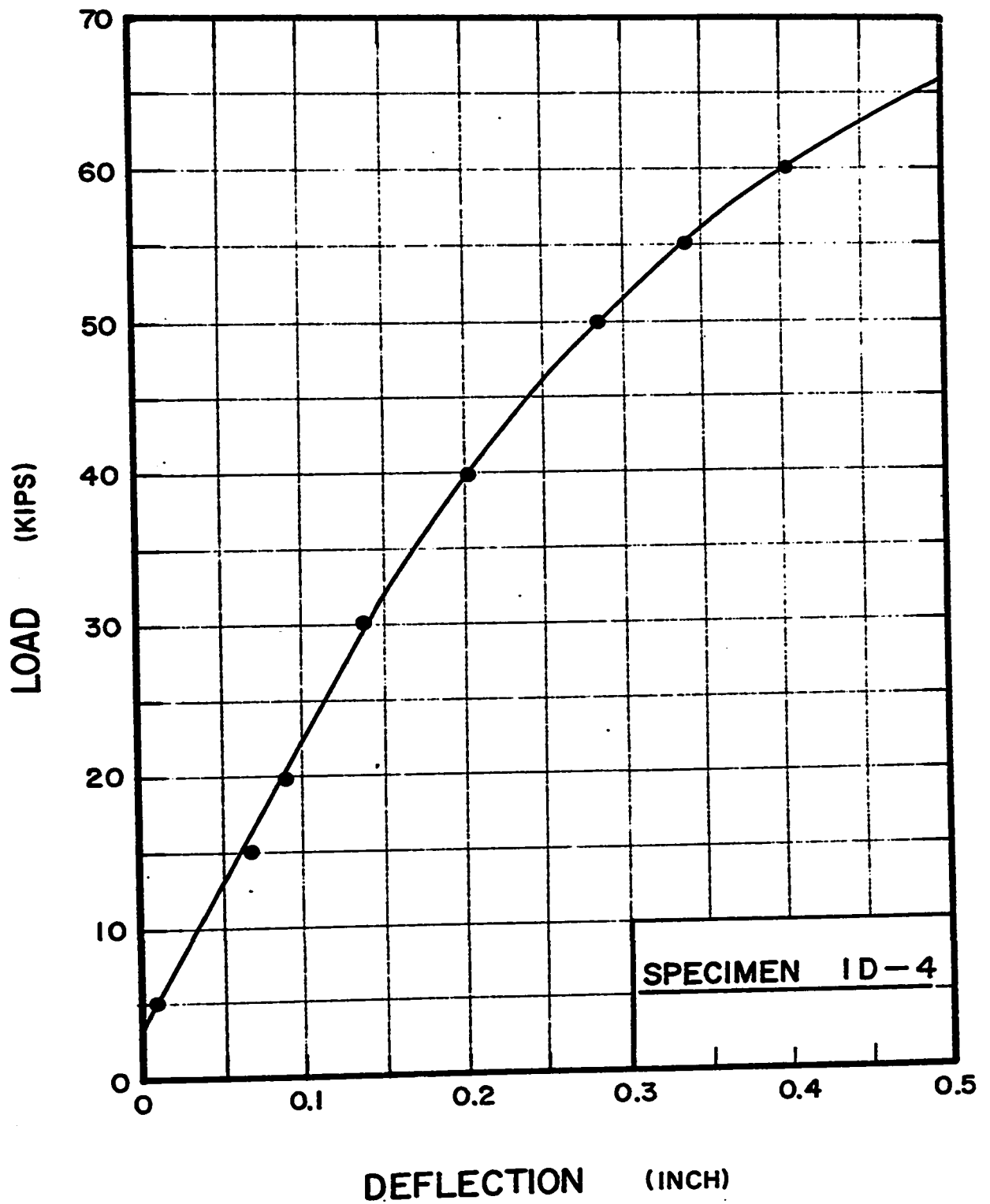


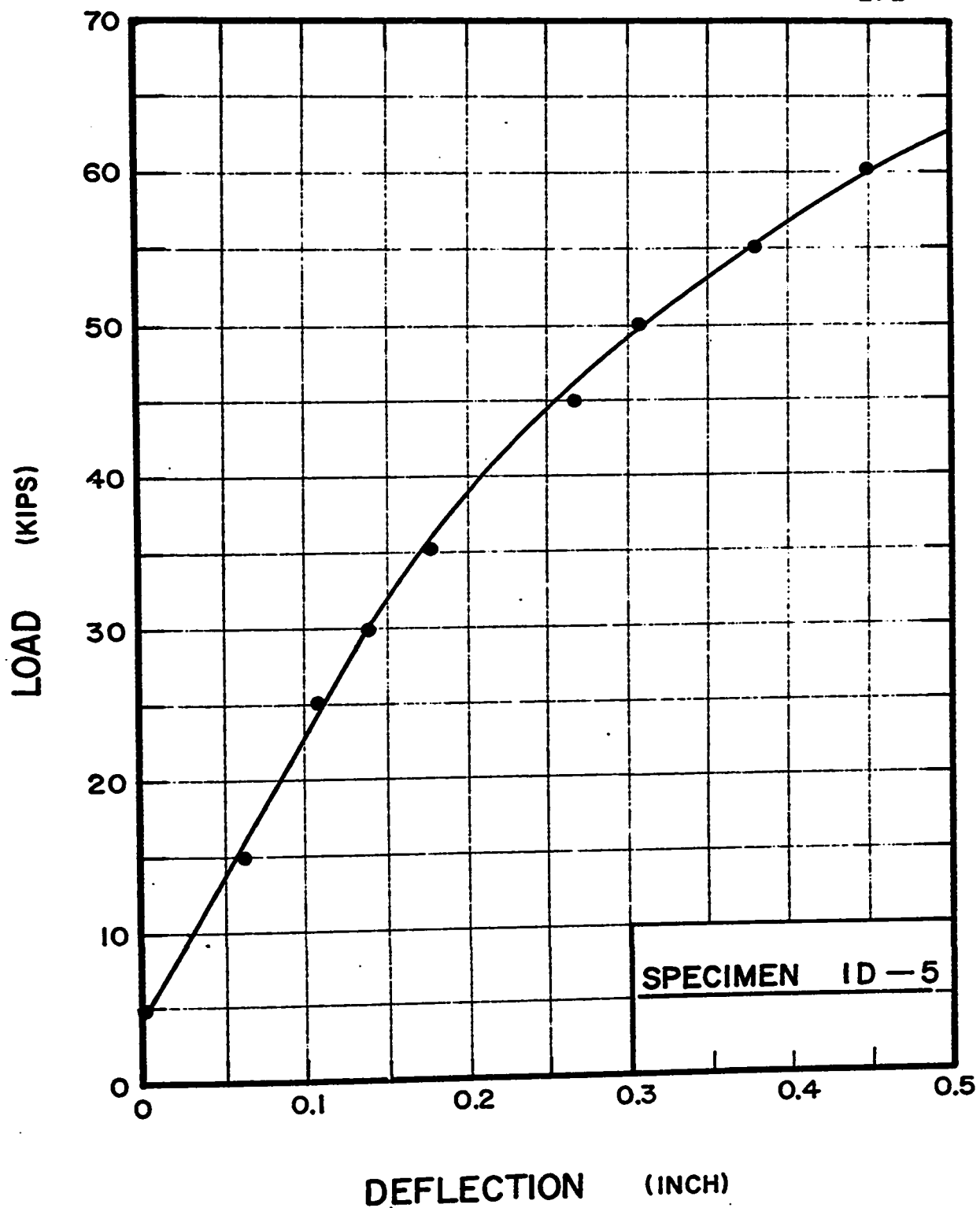
FIG.(B.77) LOAD-DEFLECTION
CURVE OF SPECIMEN ID-2



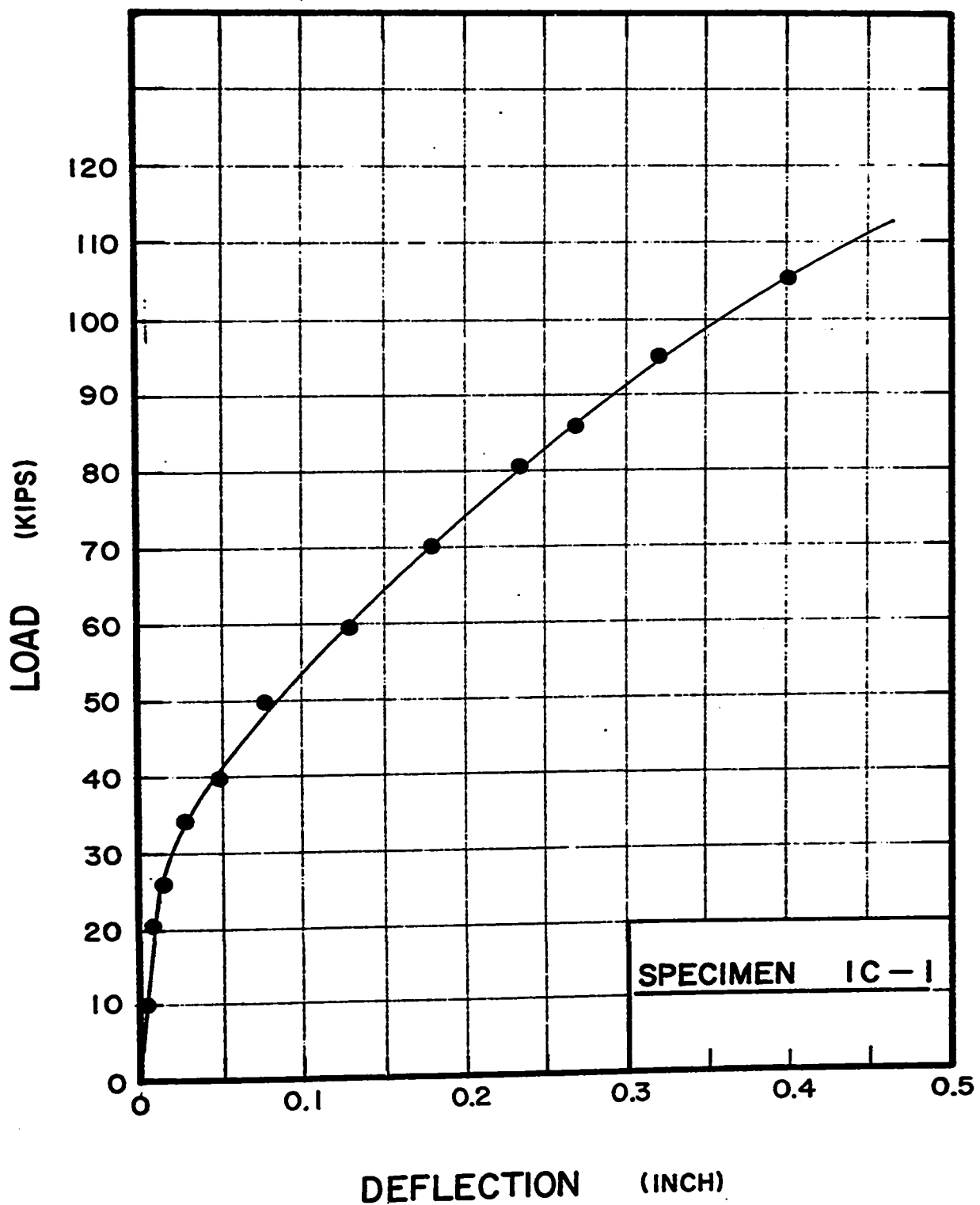
**FIG.(B.78) LOAD—DEFLECTION CURVE
OF SPECIMEN ID-3**



**FIG.(B.79) LOAD—DEFLECTION CURVE
OF SPECIMEN ID—4**



**FIG.(B.80) LOAD—DEFLECTION CURVE
OF SPECIMEN ID—1**



**FIG.(B. 81) LOAD—DEFLECTION CURVE
OF SPECIMEN IC—1**

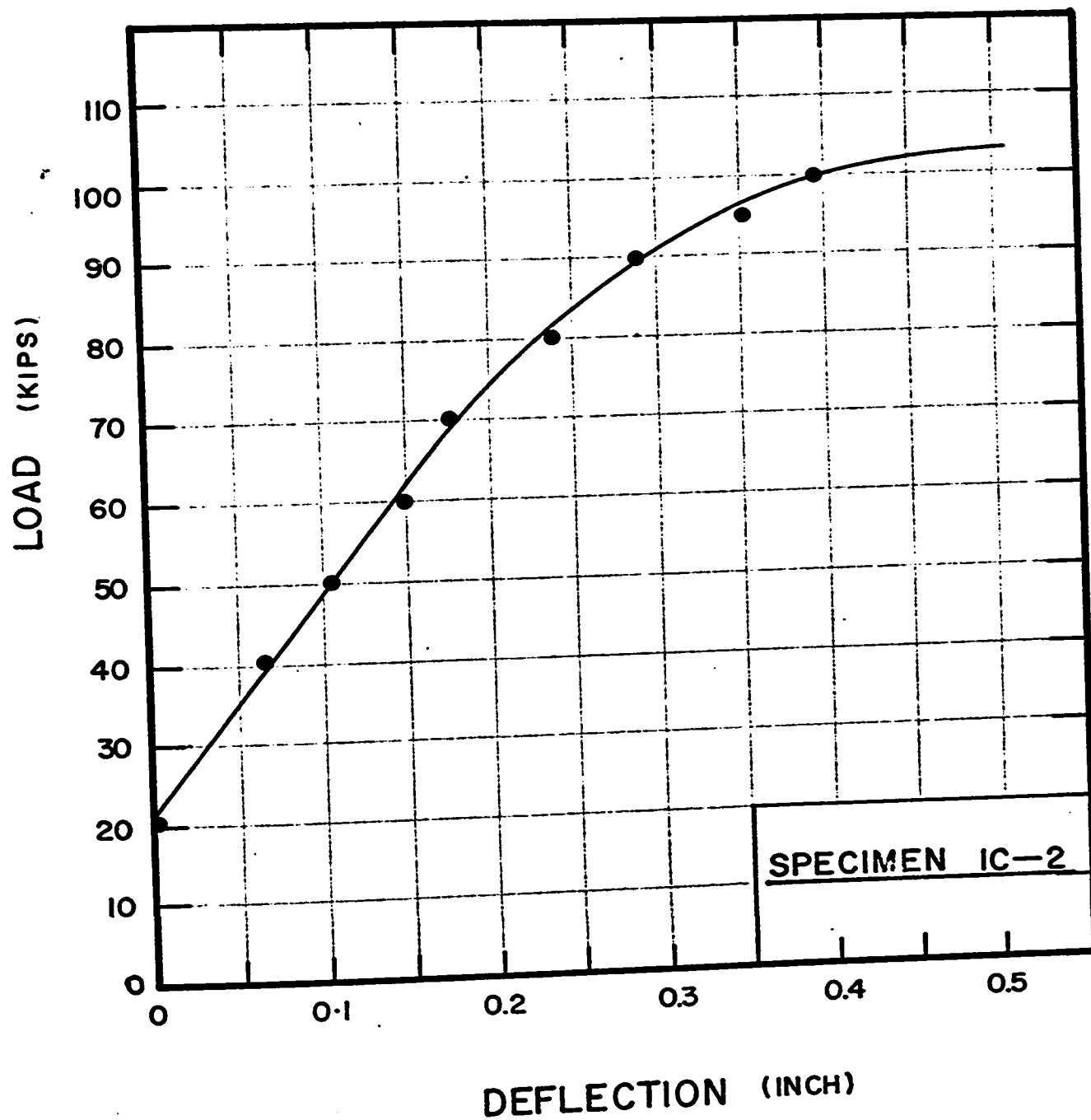
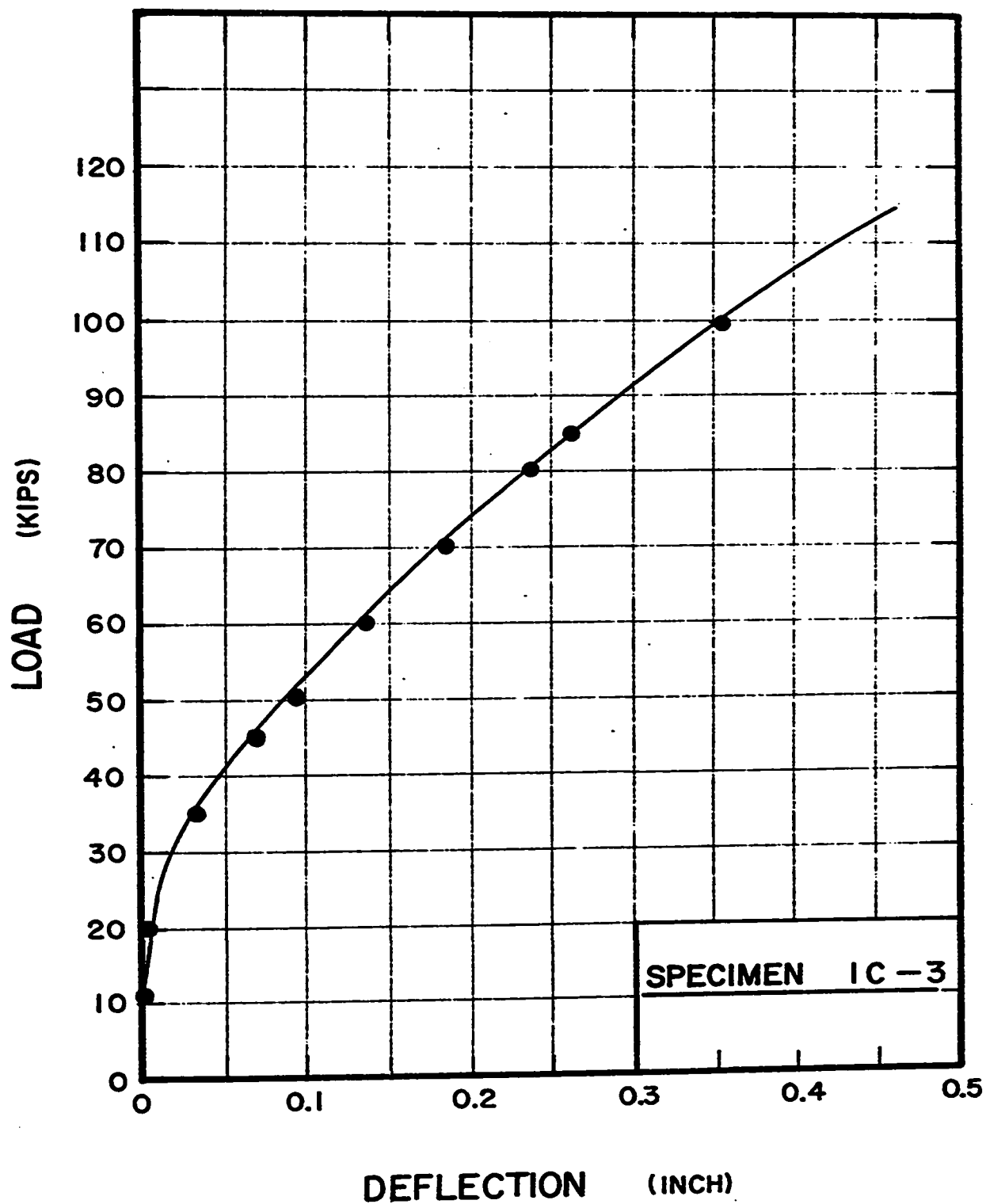
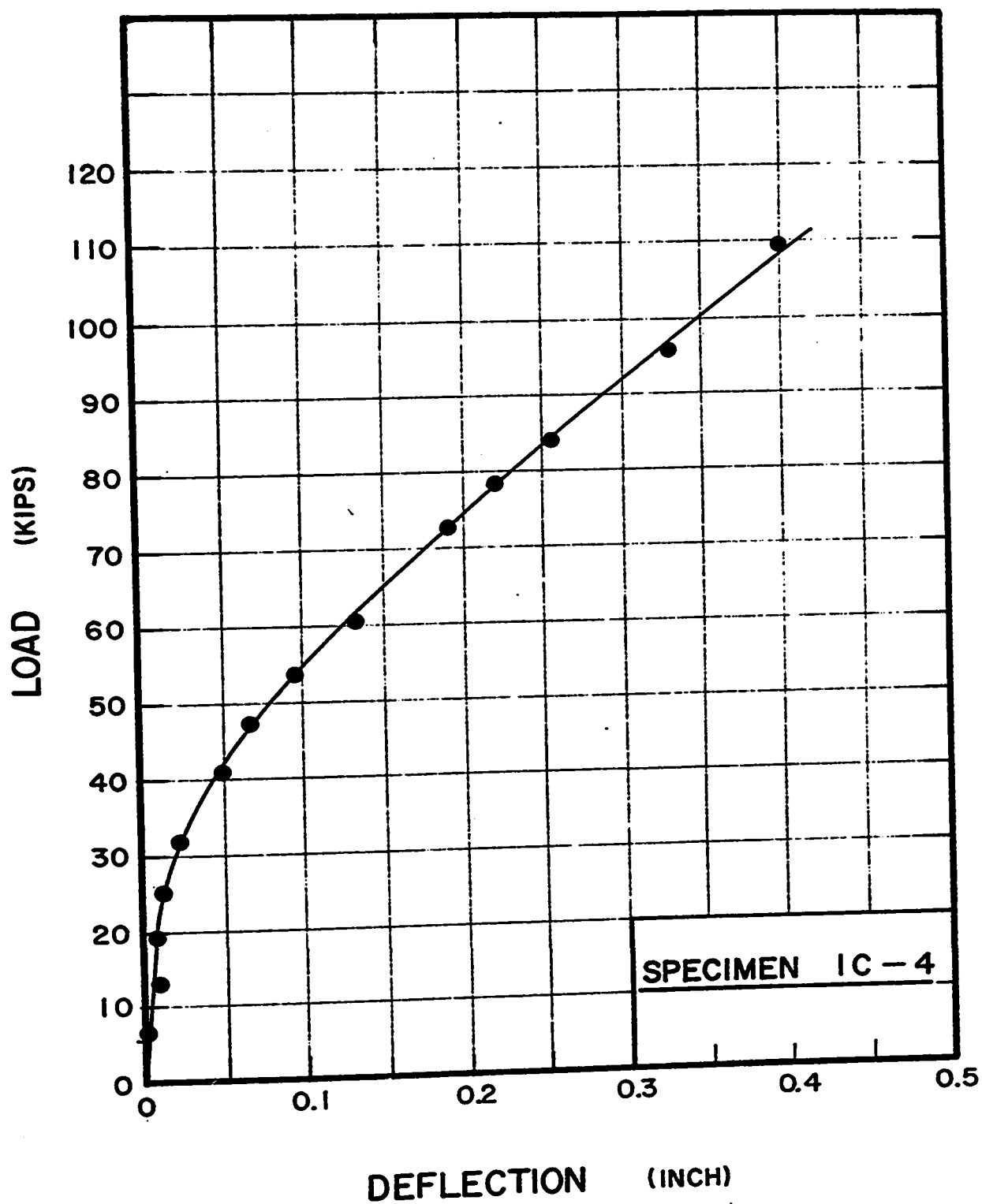


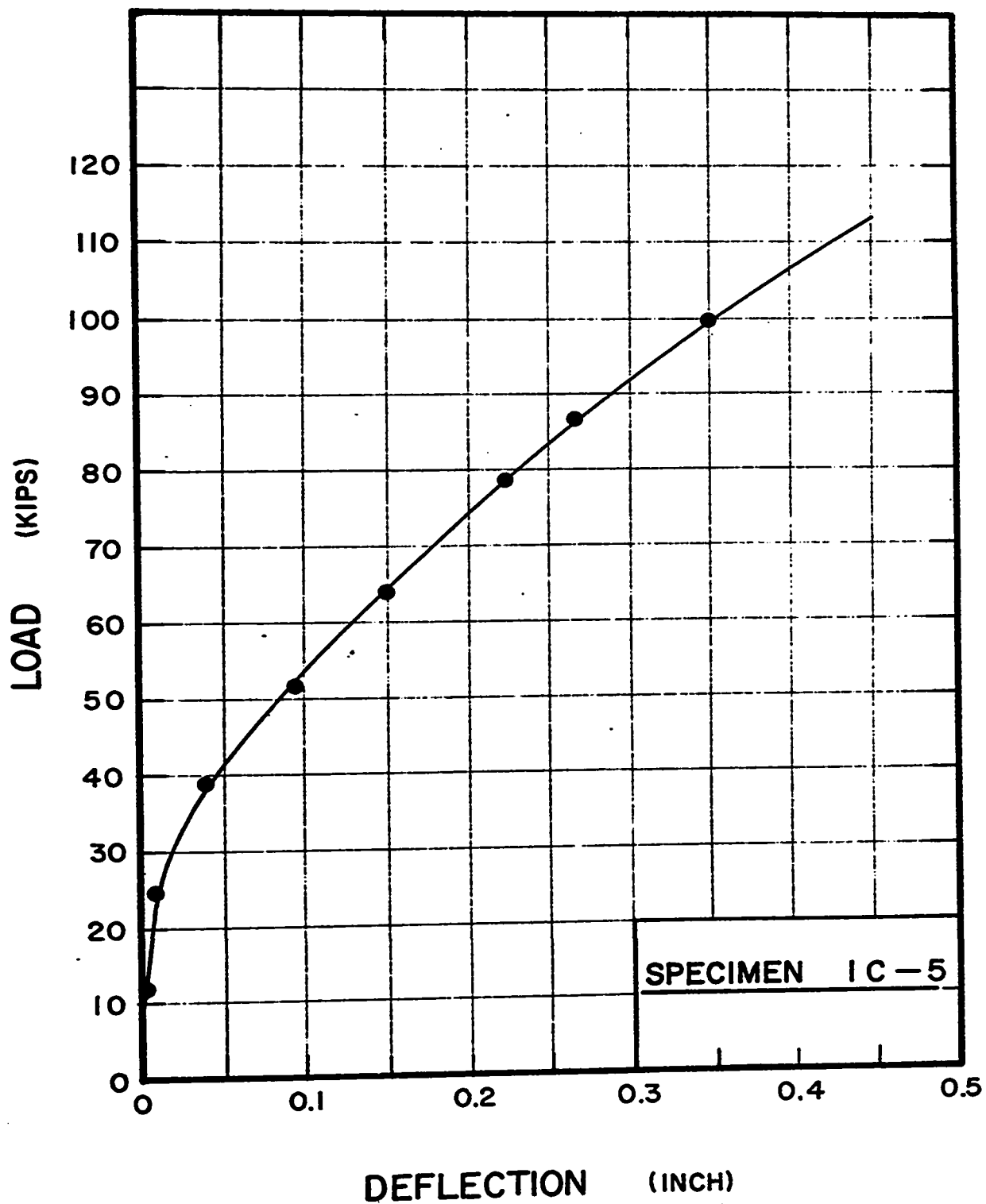
FIG.(B.82) LOAD—DEFLECTION CURVE
OF SPECIMEN IC-2



**FIG.(B.83) LOAD—DEFLECTION CURVE
OF SPECIMEN IC—3**



**FIG.(B.84) LOAD—DEFLECTION CURVE
OF SPECIMEN IC-4**



**FIG.(B.85) LOAD—DEFLECTION CURVE
OF SPECIMEN IC-5**

APPENDIX C

E X A M P L E S

EXAMPLE A

COLUMN DUE TO ECCENTRIC LOADING
BY
ACI DESIGN FORMULAE

EXAMPLE B

COLUMN DUE TO PURE BENDING
BY
STRAIN-GRADIENT METHOD

(A) Reinforced Concrete Columns due to Eccentric Loading

Example: Column 4D-4

$$\text{Given: } e/t = 0.3$$

$$f_c' = 48,708 \text{ psi.}$$

$$f_y = 45,386 \text{ psi.}$$

$$d = 4.57''$$

$$d' = 1.43''$$

$$d'' = 1.57''$$

$$A_s = A_s' = 0.11 \text{ sq.in.}$$

$$e = 3.38''$$

Refer to Figs. (4.1.1.a) and (4.1.3.a)

$$\begin{aligned} C_b &= \frac{87d}{87 + f_y} = \frac{87 \times 4.57}{87 + 45.386} \\ &= \frac{87 \times 4.57}{132.386} = 3.0'' \end{aligned}$$

$$a = K_1 C_b = 0.85 \times 3 = 2.55''$$

$$\begin{aligned} C_c &= 0.85 f_c' a b \\ &= 0.85 \times 4.871 \times 2.55 \times 6 \\ &= 63.4 \text{ kips.} \end{aligned}$$

$$\begin{aligned}
 C_s &= A_s (f_y - 0.85f_c') \\
 &= 0.22 (45.39 - 0.85 \times 4.871) \\
 &= 9.08 \text{ kips.}
 \end{aligned}$$

$$T = A_s f_y = 0.22 \times 45.386 = 9.99 \text{ kips.}$$

$$\begin{aligned}
 P_b &= C_s + C_c - T = 9.08 + 63.4 - 9.99 \\
 &= 62.49 \text{ kips.}
 \end{aligned}$$

Substitute in Eq. (4.1.3.c)

$$\begin{aligned}
 P_b e_b &= C_c \left(d - \frac{a}{2}\right) + C_s (d - d') \\
 &= 63.4 \left(4.57 - \frac{2.55}{2}\right) + 9.08 (4.57 - 1.43) \\
 &= 173.5 \text{ k-in.}
 \end{aligned}$$

$$e_b = \frac{173.5}{62.49} = 2.78" < e = 3.38"$$

Therefore tension controls.

$$p = \frac{AS}{bd} = \frac{0.22}{6 \times 4.95} = 0.0074$$

$$m = \frac{f_y}{0.85f_c'} = \frac{45.386}{0.85 \times 4.871} = 1.097$$

Substitute into Eq. (4.1.3.e)

$$\begin{aligned}
 P_u &= 0.85f_c'bd \left\{ \left(-p + 1 - e'/d \right) \right. \\
 &\quad \left. + \sqrt{\left(1 - \frac{e'}{d} \right)^2 + 2p \left[(m - 1)\left(1 - \frac{d'}{d} \right) + \frac{e'}{d} \right]} \right\} \\
 &= 0.85 \times 4.87 \times 6 \times 4.57 \\
 &\quad \left\{ -0.0074 + 1 - 4.95/4.57 \right. \\
 &\quad \left. + \sqrt{\left(1 - 4.95/4.57 \right)^2 + 2 \times 0.0074 \left[(9.97)\left(1 - \frac{1.43}{4.57} \right) + \left(\frac{4.95}{4.57} \right) \right]} \right\} \\
 &= 28.8 \text{ kips.}
 \end{aligned}$$

For NBC design load, $\phi = 0.75$

$$\begin{aligned}
 P_{\text{design}} &= 0.75 \times 28.8 \\
 &= 21.6 \text{ kips.}
 \end{aligned}$$

For ACI design load, $\phi = 0.7$

$$\begin{aligned}
 P_{\text{design}} &= 0.7 \times 28.8 \\
 &= 20.2 \text{ kips.}
 \end{aligned}$$

(B) Reinforced Concrete Columns due to Pure BendingExample: Column 3E-5

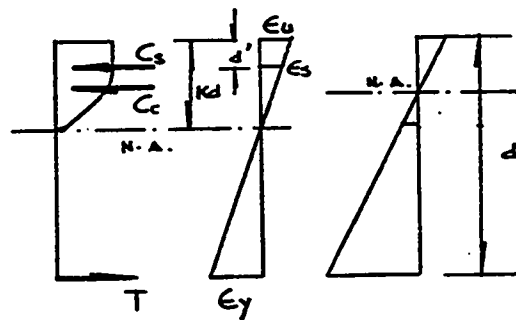
From Strain Diagram as Shown below.

$$\epsilon_y = \epsilon_u \left(\frac{1}{K} - 1 \right)$$

$$\epsilon_s = \left(\frac{Kd - d'}{Kd} \right) \epsilon_u$$

if compression steel below N.A. then

$$\epsilon_s = \left(\frac{d' - Kd}{Kd} \right) \epsilon_u$$

Given: $f_c' = 3.461$ ksi. $f_y = 65.525$ ksi. $d = 4.69$ " $d' = 1.31$ " $A_s = A_s' = 0.2$ sq.in. $e/t = \infty$ 

1st TRIAL :

$$\text{Assume: } \begin{cases} \epsilon_u &= 0.003 \\ K &= 0.4 \end{cases}$$

$$\therefore Kd = 0.4 (4.69) = 1.876"$$

$$\begin{aligned} \epsilon_y &= \left(\frac{4.69}{1.876} - 1 \right) 0.003 \\ &= 0.003 (1.5) \\ &= 0.0045 \end{aligned}$$

From Stress-Strain diagram of steel

$$f_y = 65,525 \text{ psi.}$$

$$\begin{aligned} T &= 2 \times 0.2 \times 65.525 \\ &= 26.20 \text{ kips.} \end{aligned}$$

$$\begin{aligned} \epsilon_s &= 0.003 \left(\frac{1.876 - 1.31}{1.876} \right) \\ &= 0.003 (0.218) \\ &= 0.000646 \end{aligned}$$

$$C_s = 20 \times 0.2 \times 2 = 8.0 \text{ k}$$

From Stress-Strain diagram of concrete

$$f_c' = 2.492 \text{ ksi.}$$

$$C_c = 0.85 f_c' a \times B$$

$$= 0.85 \times 0.85 \times 2.492 \times 1.876 \times 6$$

$$= 28.05 \text{ k}$$

$$C_c + C_s = 28.05 + 8.0 = 34.05 \text{ k} > T = 26.20$$

(NO GOOD)

2nd TRIAL :

$$\text{Assume: } \begin{cases} \epsilon_u = 0.0026 \\ K = 0.34 \end{cases}$$

$$\therefore Kd = 1.60$$

From Stress-Strain diagram of concrete, obtains,

$$f_c' = 2.720 \text{ psi.}$$

$$\epsilon_y = 0.0026 \left(\frac{1}{0.34} - 1 \right) = 0.0026 \times 1.94 \\ = 0.00505$$

$$T = 2 \times 0.2 \times 65.525 = 26.20 \text{ k.}$$

$$C_c = 0.72 \times 2.72 \times 1.60 \times 6 = 18.80 \text{ k.}$$

$$\begin{aligned} \epsilon_s &= 0.0026 \left(\frac{1.60 - 1.31}{1.31} \right) = 0.0026 (0.221) \\ &= 0.000575 \end{aligned}$$

From Stress-Strain diagram of steel, gives,

$$f_s = 17,000 \text{ psi.}$$

$$C_s = 3.4 \times 2 = 6.8 \text{ k.}$$

$$\therefore C_c + C_s = 18.80 + 6.8 = 25.60 \doteq T = 26.20 \text{ k.}$$

For

$$P_u = \frac{6M_u}{L}$$

Take moment about the centroid of Rectangular concrete block,

$$M_u = M_1 - M_2$$

$$M_1 = 26.20 (d - a/2) = 26.20 (4.69 - \frac{1.36}{2}) = 105 \text{ k'}$$

$$M_2 = 6.8 (1.31 - 0.68) = 4.28 \text{ k'}$$

$$\therefore M_u = -4.28 + 105.0 = 100.72 \text{ k'}$$

Therefore,

$$P_u = 6M/L = \frac{6 \times 100.72}{53} = 11.41 \text{ k' } \doteq 11.37 \text{ k'}$$

(O.K.)

APPENDIX D

COMPUTER PROGRAMS

PROGRAM A

PROGRAM FOR ACI DESIGN

FORMULAE

PROGRAM B

PROGRAM FOR STRAIN-GRADIENT

METHOD

A). PROGRAM FOR ACI DESIGN FORMULAE

The arguments used in the program are defined as follows:-

$$TM = m$$

$$PUA = \text{Ultimate load of column under concentric loading}$$

$$PUB = \text{Ultimate load of column under pure bending}$$

$$PUBB = \text{Ultimate load of column under balanced condition}$$

$$PUC = \text{Ultimate load of column under eccentric loading (compression failure)}$$

$$PUT = \text{Ultimate load of column under eccentric loading (tension failure)}$$

$$D = d$$

$$DP = d'$$

$$E = \text{Initial eccentricity}$$

$$EI = \delta \text{ (deflection)}$$

$$FY = f_y$$

$$FC = f_c'$$

$$AS = A_s$$

$$EC = e = E + \delta$$

$$EB = e_b$$

$$AB = a_b$$

$$EP = e'$$

$$MUB = \text{Balanced moment, kip-in.}$$

$$p = \rho = \frac{A_s}{bd}$$

 GENERAL SOLUTIONS FOR SHORT REINFORCED CONCRETE COLUMNS
 SUBJECTED TO AXIAL AND BENDING---BY ACI DESIGN FORMULAE

 COMMON AND DIMENSION STATEMENT

DIMENSION TM(100),PUA(100), PUC(100),PUBB(100),PUT(100),
 D(100),DP(100),E(100),EI(100),FY(20),FC(20),AS(20),
 SEC(100),EB(100),AB(100),EP(100),CK(10), P(100),PP(20),MUB(1000),
 ETT(100),CC(100),CS(100),EBB(100),ABB(100),AK(100),AKM(100),
 EAP(100),AM(100),BP(100), CP(100),CM(100),DDP(100),BM(1000),
 ECDM(100),EEP(100),EM(100), C(1000),DK(1000),PUB(1000),
 EFORCE(100),AXP(100),AXM(100),BXP(100),BXM(100),CXP(100),CXM(100),
 EAO(20),A1(20),A2(20),A3(20),A4(20),STC(5),STA(1000),FD(1000),
 EXP(100),DXM(100),EXP(100),EXM(100),XBM(100),FS(1000)

 INPUT DATA

N=100

N=20

READ (1,100) (E(1),I=1,M)
 READ (1,100) (D(1),I=1,M)
 READ (1,100) (DP(1),I=1,M)
 READ (1,100) (EI(1),I=1,M)
 READ (1,100) (FC(J),J=1,N)
 READ (1,100) (FY(J),J=1,N)
 READ (1,100) (AS(J),J=1,N)
 READ (1,100) (FORCE(1),I=1,M)
 READ (1,209) (STC(K),K=1,5)
 READ (1,209) (CK(KK),KK=1,10)
 READ (1,200) (A0(J),J=1,N)
 READ (1,200) (A1(J),J=1,N)
 READ (1,200) (A2(J),J=1,N)
 READ (1,200) (A3(J),J=1,N)

```

CC19 100 FCRMAT (8F10.5)
CC20 209 FCRMAT (5F10.5)
CC21 200 FCRMAT (5E13.6)
CC22 WRITE (3,909) (E(I),I=1,M)
CC23 909 FCRMAT (//,5X,10F12.5//)
CC24 WRITE (3,908) (D(I),I=1,M)
CC25 908 FCRMAT (//,5X,10F12.5//)
CC26 WRITE (3,908) (DP(I),I=1,M)
CC27 WRITE (3,908) (EI(I),I=1,M)
CC28 WRITE (3,908) (FC(J),J=1,N)
CC29 WRITE (3,908) (FY(J),J=1,N)
CC30 WRITE (3,908) (AS(J),J=1,N)
CC31 WRITE (3,908) (FCRC(I),I=1,M)
CC32 B=6.0
CC33 T=6.0
CC34 I=1
CC35 DO 110 J=1,N
CC36 CC 900 JJ=1,5
CC37 WRITE (3,11) I
CC38 11 FCRMAT (13)
CC39 R=E(I)/T
CC40 WRITE (3,505)
CC41 505 FCRMAT (1H1,15X,'E/D'//)
CC42 PP(J)=0.
CC43 PP(J)=AS(J)/(B*T)
CC44 WRITE (3,506) R
CC45 506 FCRMAT (12X,F6.2)
CC46 P(I)=0.
CC47 P(I)=AS(J)/(E*D(I))
CC48 WRITE (3,710)
CC49 710 FCRMAT (//,5X,'THE VALUE OF P'//)
CC50 WRITE (3,720) P(I)
CC51 720 FCRMAT (12X,F17.6)
CC52 AB(I)=C.
CC53 AB(I)=(87.0/(87.0+FY(J)))*0.85*D(I)
CC54 EC(I)=C.
CC55 EC(I)=E(I)+EI(I)
CC56 WRITE (3,410)
CC57 410 FCRMAT (//,5X,'THE VALUES OF EC')
CC58 WRITE (3,500) EC(I)
CC59 TM(J)=0.
CC60 TM(J)=FY(J)/(0.85*FC(J))

```

```

0061 AK(I)=C.
0062 AK(I)=(P(I)*FY(J))/FC(J)
0063 AKM(I)=0.
0064 AKM(I)=P(I)*TM(J)
0065 WRITE (3,810)
0066 810 FORMAT (///,5X,'THE VALUES FO P*FY
0067 WRITE (3,500) AK(I)
0068 WRITE (3,820)
0069 820 FORMAT (///,5X,'THE VALUE OF P*M')
0070 WRITE (3,500) AKM(I)
0071 IF (E(I).LE.0.) GO TO 120
0072 IF (E(I).GE. 1000.) GO TO 220
0073 EB(I)=C.
0074 TT(J)=0.
0075 CC(I)=C.
0076 CS(J)=0.
0077 PUBB(I)=0.
0078 TT(J)=AS(I)*FY(J)
0079 CS(J)=AS(I)*(FY(J)-0.85*FC(J))
0080 CC(I)=0.85*FC(J)*AB(I)*B
0081 PUBB(I)=CC(I)+CS(J)-TT(J)
0082 EB(I)=
      E      TT(J)*(T/2-DP(I))
      E+CC(I)*(T/2-AB(I)/2)
      E+CS(J)*(T/2-DP(I))/PUBB(I)
      WRITE (3,430)
0083 430 FORMAT (///,5X,'THE VALVES OF EB')
0084 WRITE (3,500) EB(I)
0085

```

[illegible]

IF (EC(I)-EB(I)) 301,302,303

PLA(J)=0.
AP(J)=0.
AM(J)=0.

[illegible]
$$PUA(J) = 0.85 * FC(J) * (B * T - 2 * AS(J)) + 2 * AS(J) * FY(J)$$

```

0091      AP(J)=PUA(J)/(FC(J)*H*T)
0092      WRITE (3,510)
0093      510 FORMAT (//,5X,'THE VALUES OF PUA')
0094      WRITE (3,500) PUA(J)
0095      500 FORMAT (//,5X,F17.6)
0096      WRITE (3,560)
0097      560 FCRMAT (//,3X,'THE VALUE OF P/PU',2X,'THE VALVE FO M/MU')
0098      WRITE (3,601) AP(J),AM(J)
0099      601 FCRMAT (//,10X,2F10.2)
0100      AXP(I)=0.
0101      AXM(I)=0.
0102      AXP(I)=FCRCE(I)/(FC(J)*B*T)
0103      WRITE (3,599)
0104      599 FORMAT (//,3X,'THE VALUE OF PEXP./PU',2X,'THE VALVE OF MEXP/PU')
0105      WRITE (3,601) AXP(I),AXM(I)
0106      GO TO 400
0107      220 BF(I)=0.
0108      DC 221 K=1,5
0109      KK=1
0110      281 C(KK)=CK(KK)*D(I)
0111      CK(KK)=0.85*C(KK)
0112      STA(KK)= ABS(STC(K)*(C(KK)-DP(I))/C(KK))
0113      FS(KK)= A0(J)+A1(J)*STA(KK)+A2(J)*STA(KK)**2
0114      &+A3(J)*STA(KK)**3+A4(J)*STA(KK)**4
0115      FC(KK)= AS(J)*FS(KK)+0.85*FC(J)*B*DK(KK)-AS(J)*FY(J)
0116      IF (ABS(FO(KK)).GT.(0.01*AS(J)*FY(J))) GO TO 227
0117      MUB(KK)=AS(J)*FS(KK)*(D(I)-DP(I))
0118      &+C.85*FC(J)*B*DK(KK)*(D(I)-DK(KK)/2.)
0119      PUB(KK)= 6.*MUB(KK)/53.
0120      BP(KK)=MUB(KK)/(FC(J)*B*T)
0121      WRITE (3,520)
0122      520 FORMAT (//,5X,'THE VALUES OF PUB')
0123      WRITE (3,500) PUB(KK)
0124      WRITE (3,560)
0125      WRITE (3,601) BP(I),BM(KK)
0126      227 KK=KK+1
0127      IF (KK.LE.10) GO TO 281
0128      221 CONTINUE

```

```

0127 BXP(I)=0.
0128 BXM(I)=0.
0129 XBM(I)=0.
0130 XBM(I)=FORCE(I)*53./6.
0131 BXP(I)=XBM(I)/(FC(J)*8*T)
0132 WRITE (3,599)
0133 WRITE (3,601) BXP(I),BXM(I)
0134 GC TO 400
0135 301 PUC(I)=0.
0136 CP(I)=0.
0137 CN(I)=C.
C
C *****
C CCNPUTE THE ULTIMATE LOAD DUE TO ECCENTRIC LOADING(COMP.FAILURE)
C *****
C
0138 PUC(I)=AS(J)*FY(J)/((EC(I)/(D(I)-DP(I))+0.5)
&+8*T*FC(J)/((3*8*EC(I)/(D(I)*2))+1.18)
0139 CP(I)=
& PUC(I)/(FC(J)*B*T)
0140 CM(I)=CP(I)*EC(I)/T
0141 WRITE (3,530)
0142 530 FCRMAT (/,5X,'THE VALUES OF PUC')
0143 WRITE (3,500) PUC(I)
0144 WRITE (3,560)
0145 WRITE (3,601) CP(I),CM(I)
0146 CXP(I)=0.
0147 CXP(I)=0.
0148 CXP(I)=FCRCE(I)/(FC(J)*8*T)
0149 CXM(I)=CXP(I)*EC(I)/T
0150 WRITE (3,599)
0151 WRITE (3,601) CXP(I),CXM(I)
0152 GC TO 400
0153 302 CCP(I)=C.
0154 CCM(I)=0.
C
C *****
C CCNPUTE THE ULTIMATE LOAD DUE TO BALANCED CONDITION
C *****
C
0155 DOP(I)=
& PUBB(I)/(FC(J)*B*T)
0156 CCM(I)=CCP(I)*EC(I)/T

```

```

C158 540 FORMAT (//,5X,'THE VALVES OF PUBB.')
C159 WRITE (3,500) PUBB(I)
C160 WRITE (3,560)
C161 WRITE (3,601) DDP(I),DDM(I)
C162 EXP(I)=0.
C163 DDM(I)=0.
C164 EXP(I)=FORCE(I)/(FC(J)*B*T)
C165 CXM(I)=DXP(I)*EC(I)/T
C166 WRITE (3,595)
C167 WRITE (3,601) DXP(I),DXM(I)
C168 GC TO 400
C169 303 PUT(I)=0.
C170 EP(I)=0.
C171 EEP(I)=0.
C172 EM(I)=0.
C173 EP(I)=EC(I)+3.-DP(I)

```

C *****

C COMPUTE THE ULTIMATE LOAD DUE TO ECCENTRIC LOADING
(TENSION FAILURE)

C *****

```

C174 PUT(I)=C.85*FC(J)*B*D(I)*(-P(I)+1.0-EP(I)/D(I)
&+SQRT((1.0-EP(I)/D(I))**2+
&2.0*P(I)*((TM(J)-1.0)*(1.0-DP(I)/D(I)+EP(I)/D(I))))
C175 EEP(I)=PUT(I)/(FC(J)*B*T)
C176 EM(I)=EEP(I)*EC(I)/T
C177 WRITE (3,550)
C178 550 FCRMAT (//,5X,'THE VALVES OF PUT.')
C179 WRITE (3,500) PUT(I)
C180 WRITE (3,560)
C181 WRITE (3,601) EEP(I),EM(I)
C182 EXP(I)=0.
C183 CXM(I)=0.
C184 EXP(I)=FORCE(I)/(FC(J)*B*T)
C185 CXM(I)=EXP(I)*EC(I)/T
C186 WRITE (3,595)
C187 WRITE (3,601) EXP(I),EXM(I)
C188 GC TO 400
C189 400 I=I+1
C190 900 CONTINUE
C191 110 CONTINUE

```

STOP
ENDC192
C193

B). PROGRAM FOR STRAIN-GRADIENT METHOD

The arguments used in the program are defined as follows:-

STC = Strain of Concrete

AK = Factor

AS = Area of tension steel, sq.in.

EC = Eccentricity of load with respect to plastic centroid of section

DP = Distance from extreme compression fibre to centroid of tension reinforcement

D = Distance from extreme compression fibre to centroid of tension reinforcement

DK = Distance from extreme compression fibre to neutral axis at ultimate strength

STA = Strain in tension reinforcement

STB = Strain in compression reinforcement

FY = Yield strength of tension reinforcement, psi.

FC = Compressive strength of concrete cylinders, psi.

FS = Tensile steel stress, psi.

AB = Depth of equivalent rectangular stress block = $K_1 c$

A0	—	A4	}	Coefficients of equation
B1	—	B4		
C1	—	C4		

BM0 = Internal resisting moment, kip-in.

BM1 = Internal resisting moment in terms of
the strength of steel, kip-in.

BM2 = Internal resisting moment in terms of
the strength of compression steel, kip-in.

PU = P_u , Ultimate load, kips.

C CALCULATIONS OF ULTIMATE AXIAL LOAD NAD ULTIMATE MOMENT
C FROM THE STRAIN GRACIENT METHOD FOR REINFORCING CONCRETE
C COLUMNS SUBJECTED TO AXIAL AND BENDING
C
C

0001 DIMENSION FORCE(50),STC(6),AK(45),AS(50),EC(50),DP(50),
& D(50),DK(3000),STA(3000),STB(3000),FCAL(3000),
& FCA(3000),FSB(3000),FSC(3000),MCAL(3000),
& MMAX(50),PEPS(50),MEPS(50), FC1(3000),FC2(3000),
& FY(3000),FC(3000),FS(3000),AB(3000),FC3(3000),FC4(3000),
& A0(50),A1(50),A2(50),FC5(3000),FC6(3000),A3(50),FC10(3000),
& C0(50),C1(50),C2(50),C3(50),R(50),FC7(3000),FC8(3000),FC9(3000)
& A4(50), C4(50),F0(300),BM0(300),BM1(300),RM2(300),PJ(300)
REAL MCAL,MMAX,MEPS

0002

0003 READ(1,100) (FORCE(I),I=1,35)

0004 100 FORMAT(8F10.5)

0005 READ(1,100) (D(I),I=1,35)

0006 READ (1,100) (EC(I),I=1,35)

0007 READ (1,100) (AS(I),I=1,35)

0008 READ (1,100) (DP(I),I=1,35)

0009 READ (1,100) (AK(J),J=1,42)

0010 READ (1,100) (STC(K),K=1,6)

0011 READ (1,100) (R(I),I=1,35)

0012 READ (1,299) (A0(I),I=1,35)

0013 299 FORMAT (5E13.6)

0014 READ (1,299) (A1(I),I=1,35)

0015 READ (1,299) (A2(I),I=1,35)

0016 READ (1,299) (A3(I),I=1,35)

0017 READ (1,299) (A4(I),I=1,35)

0018 READ (1,200) (C0(I),I=1,35)

0019 READ (1,200) (C1(I),I=1,35)

0020 READ (1,200) (C2(I),I=1,35)

0021 READ (1,200) (C3(I),I=1,35)

0022 READ (1,200) (C4(I),I=1,35)

0023 200 FORMAT (6E13.6)

0024 WRITE (3,208)

0025 208 FORMAT(///,20X,'FORCE')

0026 WRITE (3,202) (FORCE(I),I=1,35)

0027 202 FORMAT (///,2X,8F16.6)

0028 WRITE(3,201)

0029 201 FORMAT (///,2X,'THEVALVES OF D(I)'/)

0030	WRITE (3,202) (D(I),I=1,35)
0031	WRITE (3,204)
0032	204 FORMAT (///,2X,'THE VALVES OF EC')
0033	WRITE (3,202) (EC(I),I=1,35)
0034	WRITE (3,205)
0035	205 FORMAT (///,20X,'AS')
0036	WRITE (3,202) (AS(I),I=1,35)
0037	WRITE (3,601)
0038	601 FORMAT (///,20X,'DP')
0039	WRITE (3,202) (DP(I),I=1,35)
0040	WRITE (3,206)
0041	206 FORMAT (///,20X,'AK')
0042	WRITE (3,202) (AK(I),I=1,42)
0043	WRITE (3,207)
0044	207 FORMAT (///,20X,'STC')
0045	WRITE (3,202) (STC(K),K=1,6)
0046	WRITE (3,302)
0047	302 FORMAT (///,20X,'AO')
0048	WRITE (3,509) (AO(I),I=1,35)
0049	WRITE (3,303)
0050	303 FORMAT (///,'A1')
0051	WRITE (3,509) (A1(I),I=1,35)
0052	WRITE (3,304)
0053	304 FORMAT (///,'A2')
0054	WRITE (3,509) (A2(I),I=1,35)
0055	WRITE (3,305)
0056	305 FORMAT (///,'A3')
0057	WRITE (3,509) (A3(I),I=1,35)
0058	WRITE (3,306)
0059	306 FORMAT (///,'A4')
0060	WRITE (3,509) (A4(I),I=1,35)
0061	WRITE (3,406)
0062	406 FORMAT (///,'C0')
0063	WRITE (3,509) (C0(I),I=1,35)
0064	WRITE (3,407)
0065	407 FORMAT (///,'C1')
0066	WRITE (3,509) (C1(I),I=1,35)
0067	WRITE (3,408)
0068	408 FORMAT (///,'C2')
0069	WRITE (3,509) (C2(I),I=1,35)
0070	WRITE (3,409)
0071	409 FORMAT (///,'C3')
0072	WRITE (3,509) (C3(I),I=1,35)

```

0073 WRITE(3,501)
0074 501 FORMAT (///,'C4')
0075 WRITE(3,509) (C4(I),I=1,35)
0076 509 FORMAT (///,2X,8E13.6/)
C
C
0077 B=6.
0078 T=6.
C
C
0079 DO 1000 I=26,35
0080 PEPs(I)=0.
0081 PEPs(I)=PEPs(I) +0.01*FORCE(I)
0082 MMAX(I)=0.
0083 MMAX(I)=MMAX(I)+FORCE(I)*EC(I)
0084 MEPS(I)=0.
CC85 MEPS(I)=MEPS(I) +0.01*MMAX(I)
C
C
0086 WRITE (3,19)
0087 19 FORMAT (///,5X,'E/D')
0088 WRITE (3,29) R(I)
0089 29 FORMAT (///,F8.4)
0090 DO 999 K=1,6
0091 J=1
0092 61 IF (STC(K).LT.0.0001) GO TO 999
0093 IF (STC(K).GT.0.003) GO TO 999
0094 26 IF (J.GT.42) GO TO 999
0095 DK(J)=AK(J)*D(I)
0096 STA(J)=0.
0097 STB(J)=0.
0098 FCA(J)=0.
0099 FSB(J)=0.
0100 FSC(J)=0.
0101 FCAL(J)=0.
0102 MCAL(J)=0.
0103 FO(J)=0.
0104 BMO(J)=0.
0105 BM1(J)=0.
0106 BM2(J)=0.
0107 PU(J)=0.
0108 AB(J)=0.
0109 FY(J)=0.

```

0110

FS(J)=0.

0111

FC(J)=0.

0112

AB(J)=0.85*DK(J)

0113

IF (OK(J)-D(I)) 50,150,250

C

0114

50 IF (DK(J)-DP(I)) 10,20,30

0115

10 STA(J)=STA(J)+STC(K)*L./AK(J)-L.)

0116

STB(J)=STB(J)+STC(K)*(DP(I)-DK(J))/DK(J)

0117

IF (STA(J).GT.0.1 .OR. STB(J).GT.0.1) GO TO 101

C

0118

FY(J)=FY(J)+A0(I)+A1(I)*STA(J)+A2(I)*STA(J)**2

0119

GA3(I)*STA(J)**3+A4(I)*STA(J)**4

0120

FS(J)=FS(J)+A0(I)+A1(I)*STB(J)+A2(I)*STB(J)**2

0121

GA3(I)*STB(J)**3+A4(I)*STB(J)**4

0122

FC1(J)= CO(I)+C1(I)*19/20*STC(K)+C2(I)*(19/20*STC(K))**2

0123

GA3(I)*19/20*STC(K)**3+C4(I)*(19/20*STC(K))**4

0124

FC2(J)= CO(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2

0125

GA3(I)*17/20*STC(K)**3+C4(I)*(17/20*STC(K))**4

0126

FC3(J)= CO(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2

0127

GA3(I)*15/20*STC(K)**3+C4(I)*(15/20*STC(K))**4

0128

FC4(J)= CO(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2

0129

GA3(I)*13/20*STC(K)**3+C4(I)*(13/20*STC(K))**4

0130

FC5(J)= CO(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2

0131

GA3(I)*11/20*STC(K)**3+C4(I)*(11/20*STC(K))**4

0132

FC6(J)= CO(I)+C1(I)*9/20*STC(K)+C2(I)*(9/20*STC(K))**2

0133

GA3(I)*9/20*STC(K)**3+C4(I)*(9/20*STC(K))**4

0134

FC7(J)= CO(I)+C1(I)*7/20*STC(K)+C2(I)*(7/20*STC(K))**2

0135

GA3(I)*7/20*STC(K)**3+C4(I)*(7/20*STC(K))**4

0136

FC8(J)= CO(I)+C1(I)*5/20*STC(K)+C2(I)*(5/20*STC(K))**2

0137

GA3(I)*5/20*STC(K)**3+C4(I)*(5/20*STC(K))**4

0138

FC9(J)= CO(I)+C1(I)*3/20*STC(K)+C2(I)*(3/20*STC(K))**2

0139

GA3(I)*3/20*STC(K)**3+C4(I)*(3/20*STC(K))**4

0140

FC10(J)=CO(I)+C1(I)*1/20*STC(K)+C2(I)*(1/20*STC(K))**2

0141

GA3(I)*1/20*STC(K)**3+C4(I)*(1/20*STC(K))**4

0142

FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)+FC8(J)+FC9(J)+FC10(J))

0143

FCA(J)=FC(J)*B*DK(J)

0144

FSB(J)=FSB(J)+AS(I)*FY(J)

0145

FSC(J)=FSC(J)+AS(I)*FS(J)

0146

WRITE (3,7)

0147

7 FORMAT (//,10X,'FSB',10X,'FSC',10X,'FCA')

0148

WRITE (3,9) FSB(J),FSC(J),FCA(J)

0149

WRITE (3,9) FSB(J),FSC(J),FCA(J)

0150

WRITE (3,9) FSB(J),FSC(J),FCA(J)

```

0137 5 FORMAT ( 3F17.6)
0138 IF (EC(I).LT.1000.) GO TO 310
0139 FO(J)=FO(J)+FSC(J)+FSB(J)-FCA(J)
0140 IF (ABS(FO(J)).LE.( 0.01*FCA(J))) GO TO 110
0141 J=J+1
0142 GO TO 26
C
0143 110 BM1(J)=BM1(J)+FSB(J)*(D(I)-AB(J)/2)
0144 BM2(J)=BM2(J)+FSC(J)*(DP(I)-AB(J)/2)
0145 BMQ(J)=BMQ(J)+BM1(J)+BM2(J)
0146 PU(J)=PU(J)+6.*BMQ(J)/53.
0147 IF (ABS(PU(J))-FORCE(I)).LE.PEPS(I) GO TO 210
0148 J=J+1
0149 GO TO 26
C
0150 210 WRITE (3,39)
0151 39 FORMAT (///,2X,'I',5X,'J',5X,'K',
&X,'FORCE',5X,'CAL.PU',5X,'PURE BENDING MU')
0152 WRITE (3,500) I,J,K,FORCE(I),PU(J),BMQ(J)
0153 GO TO 999
0154 310 FCAL(J)=FCAL(J)+FCA(J)-FSC(J)-FSB(J)
0155 MCAL(J)=MCAL(J)+FSB(J)*(T/2-DP(I))
&-FSC(J)*(T/2-DP(I))
&+FCA(J)*(T/2-AB(J)/2)
GO TO 300
0156
C
C
0157 20 STA(J)=STA(J)+STC(K)*(1./AK(J)-1.)
0158 STB(J)=STB(J)
0159 IF (STA(J).GT.0.1 .OR. STB(J).GT.0.1) GO TO 101
C
0160 FY(J)=FY(J)+A0(I)+A1(I)*STA(J)+A2(I)*STA(J)**2
&+A3(I)*STA(J)**3+A4(I)*STA(J)**4
0161 FS(J)=FS(J)+A0(I)+A1(I)*STB(J)+A2(I)*STB(J)**2
&+A3(I)*STB(J)**3+A4(I)*STB(J)**4
0162 FC1(J)= CO(I)+C1(I)*19/20*STC(K)+C2(I)*(19/20*STC(K))**2
&+C3(I)*(19/20*STC(K))**3+C4(I)*(19/20*STC(K))**4
0163 FC2(J)= CO(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2
&+C3(I)*(17/20*STC(K))**3+C4(I)*(17/20*STC(K))**4
0164 FC3(J)= CO(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2
&+C3(I)*(15/20*STC(K))**3+C4(I)*(15/20*STC(K))**4
0165 FC4(J)= CO(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2
&+C3(I)*(13/20*STC(K))**3+C4(I)*(13/20*STC(K))**4

```

0166	FC5(J)= C0(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2	
	&+C3(I)*(11/20*STC(K))**3+C4(I)*(11/20*STC(K))**4	
0167	FC6(J)= C0(I)+C1(I)* 9/20*STC(K)+C2(I)*(9/20*STC(K))**2	
	&+C3(I)*(9/20*STC(K))**3+C4(I)*(9/20*STC(K))**4	
0168	FC7(J)= C0(I)+C1(I)* 7/20*STC(K)+C2(I)*(7/20*STC(K))**2	
	&+C3(I)*(7/20*STC(K))**3+C4(I)*(7/20*STC(K))**4	
0169	FC8(J)= C0(I)+C1(I)* 5/20*STC(K)+C2(I)*(5/20*STC(K))**2	
	&+C3(I)*(5/20*STC(K))**3+C4(I)*(5/20*STC(K))**4	
0170	FC9(J)= C0(I)+C1(I)* 3/20*STC(K)+C2(I)*(3/20*STC(K))**2	
	&+C3(I)*(3/20*STC(K))**3+C4(I)*(3/20*STC(K))**4	
0171	FC10(J)=C0(I)+C1(I)* 1/20*STC(K)+C2(I)*(1/20*STC(K))**2	
	&+C3(I)*(1/20*STC(K))**3+C4(I)*(1/20*STC(K))**4	
0172	FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)	
	&+FC8(J)+FC9(J)+FC10(J))	
	C	
0173	FCA(J)=FC(J)*B*DK(J)	
0174	FSB(J)=FSB(J)+AS(I)*FY(J)	
0175	FSC(J)=0.	
0176	IF (EC(I).LT.1000.) GO TO 330	
0177	F0(J)=F0(J)+FSB(J)-FCA(J)+FSC(J)	
0178	IF (ABS(F0(J)).LE.(0.01*FCA(J))) GO TO 130	
0179	J=J+1	
0180	GO TO 26	
	C	
	C	
0181	130 BM1(J)=BM1(J)+FSB(J)*(D(I)-AB(J)/2)	
0182	BM2(J)=BM2(J)+FSC(J)*(DP(I)-AB(J)/2)	
0183	BM0(J)=BM0(J)+BM1(J)+BM2(J)	
0184	PU(J)=PU(J)+6.*BM0(J)/53.	
0185	IF (ABS(PU(J))-FORCE(I)).LE.PEPS(I)) GO TO 210	
0186	J=J+1	
0187	GO TO 26	
	C	
	C	
0188	330 FCAL(J)=FCAL(J)+FCA(J)-FSC(J)-FSB(J)	
0189	MCAL(J)=MCAL(J)+FSB(J)*(T/2-DP(I))	
	&-FSC(J)*(T/2-DP(I))	
0190	&+FCA(J)*(T/2-AB(J)/2)	
	GO TO 300	
	C	
	C	
0191	30 STA(J)=STA(J)+STC(K)*D(I)/DK(J)-STC(K)	
0192	STB(J)=STB(J)+((DK(J)-DP(I))*STC(K))/DK(J)	

0193 IF (STA(J),GT,0.1)OR, SIB(J),GT,0.1) GO TO 101

C

0154 FY(J)=FY(J)+AO(I)+A1(I)*STA(J)+A2(I)*STA(J)**2

Σ+A3(I)*STA(J)**3+A4(I)*STA(J)**4

0155 FS(J)=FS(J)+AO(I)+A1(I)*STB(J)+A2(I)*STB(J)**2

Σ+A3(I)*STB(J)**3+A4(I)*STB(J)**4

0196 FC1(J)= CO(I)+C1(I)*19/20*STC(K)+C2(I)*(19/20*STC(K))**2

Σ+C3(I)*(19/20*STC(K))**3+C4(I)*(19/20*STC(K))**4

0197 FC2(J)= CO(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2

Σ+C3(I)*(17/20*STC(K))**3+C4(I)*(17/20*STC(K))**4

0198 FC3(J)= CO(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2

Σ+C3(I)*(15/20*STC(K))**3+C4(I)*(15/20*STC(K))**4

0199 FC4(J)= CO(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2

Σ+C3(I)*(13/20*STC(K))**3+C4(I)*(13/20*STC(K))**4

0200 FC5(J)= CO(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2

Σ+C3(I)*(11/20*STC(K))**3+C4(I)*(11/20*STC(K))**4

0201 FC6(J)= CO(I)+C1(I)* 9/20*STC(K)+C2(I)*(9/20*STC(K))**2

Σ+C3(I)*(9/20*STC(K))**3+C4(I)*(9/20*STC(K))**4

0202 FC7(J)= CO(I)+C1(I)* 7/20*STC(K)+C2(I)*(7/20*STC(K))**2

Σ+C3(I)*(7/20*STC(K))**3+C4(I)*(7/20*STC(K))**4

0203 FC8(J)= CO(I)+C1(I)* 5/20*STC(K)+C2(I)*(5/20*STC(K))**2

Σ+C3(I)*(5/20*STC(K))**3+C4(I)*(5/20*STC(K))**4

0204 FC9(J)= CO(I)+C1(I)* 3/20*STC(K)+C2(I)*(3/20*STC(K))**2

Σ+C3(I)*(3/20*STC(K))**3+C4(I)*(3/20*STC(K))**4

0205 FC10(J)=CO(I)+C1(I)* 1/20*STC(K)+C2(I)*(1/20*STC(K))**2

Σ+C3(I)*(1/20*STC(K))**3+C4(I)*(1/20*STC(K))**4

0206 FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)

Σ+FC8(J)+FC9(J)+FC10(J))

C

0207 FCA(J)=FC(J)*B*DK(J)

0208 FSB(J)=FSB(J)+FY(J)*AS(I)

0209 FSC(J)=FSC(J)+FS(J)*AS(I)

C

0210 IF (EC(I),LT,1000.) GO TO 630

0211 FO(J)=FO(J)+FSB(J)-FCA(J)-FSC(J)

0212 IF (ABS(FO(J)).LE.(0.01*FSB(J))) GO TO 430

0213 J=J+1

0214 GO TO 26

C

0215 430 BM1(J)=BM1(J)+(FSB(J)-FSC(J))*(D(I)-AB(I)/2)

0216 BM2(J)=BM2(J)+FSC(J)*(D(I)-DP(I))

0217 BM0(J)=BM0(J)+BM1(J)+BM2(J)

0218	PU(J)=PU(J)+6.*BM0(J)/53.	
0219	IF (ABS(PU(J)-FORCE(I)).LE.PEPS(I)) GO TO 210	
0220	J=J+1	
0221	GO TO 26	
0222	630 FCAL(J)=FCAL(J)+FCA(J)-FSB(J)+FSC(J)	
0223	MCAL(J)=MCAL(J)+FCA(J)*(T/2-AB(J)/2)	
	8 +FSC(J)*(T/2-DP(I))	
	8+FSB(J)*(T/2-DP(I))	
0224	GO TO 300	
0225	150 STA(J)=STA(J)	
0226	STB(J)=STB(J)+(STC(K)/DK(J))*(DK(J)-DP(I))	
0227	IF (STA(J).GT.0.1 .OR. STB(J).GT.0.1) GO TO 101	
0228	FY(J)=FY(J)+A0(I)+A1(I)*STA(J)+A2(I)*STA(J)**2	
	8+A3(I)*STA(J)**3+A4(I)*STA(J)**4	
0229	FS(J)=FS(J)+A0(I)+A1(I)*STB(J)+A2(I)*STB(J)**2	
	8+A3(I)*STB(J)**3+A4(I)*STB(J)**4	
0230	FC1(J)= CO(I)+C1(I)*19/20*STC(K)+C2(I)*(19/20*STC(K))**2	
	8+C3(I)*(19/20*STC(K))**3+C4(I)*(19/20*STC(K))**4	
0231	FC2(J)= CO(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2	
	8+C3(I)*(17/20*STC(K))**3+C4(I)*(17/20*STC(K))**4	
0232	FC3(J)= CO(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2	
	8+C3(I)*(15/20*STC(K))**3+C4(I)*(15/20*STC(K))**4	
0233	FC4(J)= CO(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2	
	8+C3(I)*(13/20*STC(K))**3+C4(I)*(13/20*STC(K))**4	
0234	FC5(J)= CO(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2	
	8+C3(I)*(11/20*STC(K))**3+C4(I)*(11/20*STC(K))**4	
0235	FC6(J)= CO(I)+C1(I)* 9/20*STC(K)+C2(I)*(9/20*STC(K))**2	
	8+C3(I)*(9/20*STC(K))**3+C4(I)*(9/20*STC(K))**4	
0236	FC7(J)= CO(I)+C1(I)* 7/20*STC(K)+C2(I)*(7/20*STC(K))**2	
	8+C3(I)*(7/20*STC(K))**3+C4(I)*(7/20*STC(K))**4	
0237	FC8(J)= CO(I)+C1(I)* 5/20*STC(K)+C2(I)*(5/20*STC(K))**2	
	8+C3(I)*(5/20*STC(K))**3+C4(I)*(5/20*STC(K))**4	
0238	FC9(J)= CO(I)+C1(I)* 3/20*STC(K)+C2(I)*(3/20*STC(K))**2	
	8+C3(I)*(3/20*STC(K))**3+C4(I)*(3/20*STC(K))**4	
0239	FC10(J)=CO(I)+C1(I)* 1/20*STC(K)+C2(I)*(1/20*STC(K))**2	
	8+C3(I)*(1/20*STC(K))**3+C4(I)*(1/20*STC(K))**4	
0240	FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)	
	8+FC8(J)+FC9(J)+FC10(J)	223

C241

FSB(J)=FSB(J)

C242

FSC(J)=FSC(J)+FS(J)*AS(I)

C243

FCA(J)=FC(J)*B*DK(J)

C244

FCAL(J)=FCAL(J)+FCA(J)+FSC(J)

C245

MCAL(J)=MCAL(J)+FCA(J)*(T/2-AB(J)/2)

C246

E+FSC(J)*(T/2-DP(I))

GO TO 300

C247

250 STA(J)=STA(J)+(STC(K)/DK(J))*(DK(J)-D(I))

C248

STB(J)=STB(J)+(STC(K)/DK(J))*(DK(J)-DP(I))

C249

IF (STA(J).GT.0.1 .CR. STB(J).GT.0.1) GO TO 101

C250

FY(J)=FY(J)+AO(I)+A1(I)*STA(J)+A2(I)*STA(J)**2

C251

E+A3(I)*STA(J)**3+A4(I)*STA(J)**4

C252

FS(J)=FS(J)+AO(I)+A1(I)*STB(J)+A2(I)*STB(J)**2

C253

E+A3(I)*STB(J)**3+A4(I)*STB(J)**4

C254

FC1(J)= CO(I)+C1(I)*19/20*STC(K)**3+C4(I)*(19/20*STC(K))**4

C255

E+C3(I)*(19/20*STC(K))**3+C4(I)*(19/20*STC(K))**4

C256

FC2(J)= CO(I)+C1(I)*17/20*STC(K)+C2(I)*(17/20*STC(K))**2

C257

E+C3(I)*(17/20*STC(K))**3+C4(I)*(17/20*STC(K))**4

C258

FC3(J)= CO(I)+C1(I)*15/20*STC(K)+C2(I)*(15/20*STC(K))**2

C259

E+C3(I)*(15/20*STC(K))**3+C4(I)*(15/20*STC(K))**4

C260

FC4(J)= CO(I)+C1(I)*13/20*STC(K)+C2(I)*(13/20*STC(K))**2

C261

E+C3(I)*(13/20*STC(K))**3+C4(I)*(13/20*STC(K))**4

C262

FC5(J)= CO(I)+C1(I)*11/20*STC(K)+C2(I)*(11/20*STC(K))**2

C263

E+C3(I)*(11/20*STC(K))**3+C4(I)*(11/20*STC(K))**4

C264

FC6(J)= CO(I)+C1(I)* 9/20*STC(K)+C2(I)*(9/20*STC(K))**2

C265

E+C3(I)*(9/20*STC(K))**3+C4(I)*(9/20*STC(K))**4

C266

FC7(J)= CO(I)+C1(I)* 7/20*STC(K)+C2(I)*(7/20*STC(K))**2

C267

E+C3(I)*(7/20*STC(K))**3+C4(I)*(7/20*STC(K))**4

C268

FC8(J)= CO(I)+C1(I)* 5/20*STC(K)+C2(I)*(5/20*STC(K))**2

C269

E+C3(I)*(5/20*STC(K))**3+C4(I)*(5/20*STC(K))**4

C270

FC9(J)= CO(I)+C1(I)* 3/20*STC(K)+C2(I)*(3/20*STC(K))**2

C271

E+C3(I)*(3/20*STC(K))**3+C4(I)*(3/20*STC(K))**4

C272

FC10(J)=CO(I)+C1(I)* 1/20*STC(K)+C2(I)*(1/20*STC(K))**2

C273

E+C3(I)*(1/20*STC(K))**3+C4(I)*(1/20*STC(K))**4

C274

FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)

C275

E+FC8(J)+FC9(J)+FC10(J)

```

0263       IF (T,LE,AB(J)) AB(J)=T
0264       FSB(J)=FSB(J)+FY(J)*AS(I)
0265       FSC(J)=FSC(J)+FS(J)*AS(I)
0266       FCA(J)=FC(J)*8*DK(J)
          C
0267       FCAL(J)=FCAL(J)+FCA(J)+FSC(J)+FSB(J)
0268       MCAL(J)=MCAL(J)+FCA(J)*(T/2-AB(J)/2)
          C-FSB(J)*(T/2-DP(I))
          C+FSC(J)*(T/2-DP(I))
          GO TO 300
          C
0269
          C
          C
0270       300 IF (ABS(FCAL(J)-FORCE(I)),LE,PEPS(I)) GO TO 60
          C
0271       101 J=J+1
0272       GO TO 26
          C
          C
0273       60 WRITE (3,59)
0274       59 FORMAT (///,2X,'I',5X,'J',5X,'K',5X,'FORCE',
          5X,'CAL.PU',5X,'CAL.MU')
0275       WRITE (3,500) I,J,K,FORCE(I),FCAL(J),MCAL(J)
0276       500 FORMAT (///,2X,3(I3,5X),3(F17.6))
          C
0277       WRITE (3,69)
0278       69 FORMAT (///,2X,'I',5X,'J',5X,'K',5X,'CAL.MU',5X,'EXTERNAL MU',
          5X,'STC')
0279       WRITE (3,500) I,J,K,MCAL(J),MMAX(I),STC(K)
          C
0280       WRITE (3,79)
0281       79 FORMAT (///,2X,'DK',5X,'D',5X,'STA',5X,
          5X,'STB',5X,'FCA',5X,'FSB',5X,'FSC')
0282       WRITE (3,600) DK(J),D(I),STA(J),STB(J),FCA(J),
          FSB(J),FSC(J)
0283       600 FORMAT (///,5X,7F17.6)
          C
0284       999 CONTINUE
0285       1000 CONTINUE
0286       RETURN
0287       END

```