

HIGH-TEMPERATURE FIBRE SENSOR POTENTIAL AND LOW NOISE FIBRE LASER

BY

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STATEMENT OF ORIGINAL CONTRIBUTIONS

This thesis is submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics at University of Ottawa. I hereby declare that this thesis is my own work and has not been submitted in substantially the same form for the award of a degree at this or any other academic institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except where due reference is made in the text. Any research-related collaboration or assistance is fully acknowledged.

The research presented in this thesis is my own original work, except where otherwise acknowledged. The theoretical framework for wave propagation in non-uniform media, as detailed in Chapter 3 and 4, is built upon a foundational mathematical derivation provided by my co-supervisor, Dr. Liang Chen. This derivation is presented in its entirety in Appendix A. My original contribution was to apply this framework to the specific problem of OFDR sensing in dynamic environments, to solve the resulting equations using the WKB approximation as shown in Appendix B, and to interpret the physical meaning of the complex phase terms for multi-parameter sensing. All experimental work, data analysis, and interpretation of results presented herein are my own.

ABSTRACT

This thesis presents a comprehensive investigation into the physical phenomena that define the operational limits of two distinct classes of fibre optic systems: distributed sensors and random lasers. The research addresses fundamental challenges in both domains, focusing first on the characterization of material stability in sensors deployed in extreme thermal environments, and second on the development of all-optical, passive methodologies for controlling the intrinsic dynamic instabilities of disordered laser systems. The unifying theme is the progression from identifying performance-limiting mechanisms to harnessing a quantitative understanding of these mechanisms for enhanced characterization and control.

The first part of the thesis investigates the performance of silica optical fibre as a distributed sensor using Optical Frequency Domain Reflectometry (OFDR) under intense thermal loads where material degradation is the primary constraint. Measurements in an electrical arc plasma, combined with non-linear calibration, enabled spatially resolved temperature sensing up to approximately 1300 °C. This established OFDR as a multi-parameter diagnostic tool, yielding simultaneous measurements of temperature, thermally-induced residual stress, and dynamic optical path length changes. The study further demonstrates that the fundamental limit of cross-correlation sensing is governed by the fibre's fictive and glass transition temperatures, beyond which structural relaxation induces waveguide breakdown and signal decorrelation. To enable continuous operation, a study in a hydrogen flame was conducted to formalize the mechanism of sensor failure. The irreversible, thermally-driven rearrangement of the fibre's glass network was quantified, leading to the establishment of decorrelation time as a new, physically-grounded metric for sensor durability. Measurements indicating a 50% loss of signal correlation in approximately 20 seconds at 1000 °C provide a quantitative framework for assessing the measurement validity window of fibre sensors at their material limits.

The second part of the thesis addresses the control of chaotic modal behaviour in Erbium-doped Random Fibre Lasers (EDRFLs). Two distinct, passive, all-optical strategies are developed and demonstrated. The first is a feedback-based approach, where Self-Injection Locking (SIL) to a novel Random-Fibre-Grating Ring (RFGR) resonator is used to discipline the laser's dynamics by slaving its frequency to the stable resonances of the external passive system. This method was shown to be effective, suppressing mode

competition to achieve stable, single-frequency operation with a 1.23 kHz linewidth and a 70 dB side-mode suppression ratio. The second is a nonlinearity-based approach that harnesses the laser’s internal dynamics. By engineering a two-section cavity, a robust Optical Bistability (OB) was established and exploited as a mechanism for passive, self-stabilization, resulting in a measured order-of-magnitude reduction in intensity noise.

This thesis establishes a framework for sensor durability assessment, demonstrates two paradigms for the all-optical control of disordered lasers, and introduces a methodology for the internal characterization of active photonic devices. A central element of this work is the application of OFDR, which served both as the sensing technology in the first part of the research and as the *in-situ* diagnostic tool in the second. This diagnostic application provided the first spatially resolved map of internal refractive index dynamics within an active Random Fibre Laser (RFL), which identified the origin of the observed bistability as a hybrid mechanism of population-driven (Kramers–Kronig) and localized, intensity-driven (Kerr) nonlinearities.

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LIST OF ACRONYMS

- AC** Alternating Current 19, 20, 214
- ADC** Analog-to-Digital Converter 17, 20
- ASE** Amplified Spontaneous Emission xx, 119, 124, 126, 127, 139, 140, 170, 172, 175, 180, 181, 183, 186, 225–230
- AUI** Auxiliary Interferometer xiii, 16, 17, 55, 94, 96, 173
- BPD** Balanced Photodetector xvi, 95
- CARS** Coherent Anti-Stokes Raman 91
- CCD** Charge-Coupled Device 91
- CW** Continuous-wave 188
- DAQ** Data Acquisition 59, 96, 108, 173
- DC** Direct Current xiv, 19, 40, 214
- DFB** Distributed Feedback 160
- DFT** Discrete Fourier Transform 20, 21
- DSHI** Delayed Self-Heterodyne Interferometry 150
- EDF** Erbium-Doped Fibre ix, xx, xxi, 122–128, 142, 145–148, 154, 156, 159–162, 166, 168–179, 183, 185–187, 189, 221, 223, 225, 226, 230
- EDFA** Erbium-Doped Fibre Amplifier 122
- EDFL** Erbium-Doped Fibre Laser 124–126, 133, 168

EDRFL Erbium-Doped Random Fibre Laser iii, x, xix, xx, 7, 116, 122, 123, 126, 137, 138, 141–143, 145–149, 154–156, 158, 159, 161, 163, 166, 167, 169–171, 174, 177, 179, 180, 188, 189, 225, 230

ESA Excited-State Absorption 125, 229

FBG Fibre Bragg Grating xix, 55, 92, 97, 103, 130–132, 196, 213

FFT Fast Fourier Transform xiii, 23, 25, 60, 61, 80, 99, 149

FRM Faraday Rotating Mirror xiii, xvi, 16, 95

FSR Free Spectral Range 158

FUT Fibre Under Test 16, 18, 56

FWHM Full Width at Half Maximum xx, 158, 178

FWM Four-Wave Mixing 126, 128, 181

GODC Germanium Oxygen Deficiency Centre 31, 36, 39, 51

IFFT Inverse Fast Fourier Transform 23, 60, 87, 99

IR Infrared xiv, 36–38, 41, 42, 51, 91, 92, 97, 103, 106, 110

KK Kramers–Kronig 175, 176

LK Lang-Kobayashi 151, 155, 164, 220, 222, 223

LTI Linear Time-Invariant 220

MFD Mode Field Diameter 125, 126

MI Main Interferometer 55, 56, 94

OB Optical Bistability iv, xx, 4, 116, 137, 143, 168, 180, 225

ODE Ordinary Differential Equation 206

OFDR Optical Frequency-Domain Reflectometry ii–iv, viii–xi, xiii, xiv, xvi, xvii, xx, 3, 4, 8–11, 13, 15–22, 28, 29, 35, 40, 42, 43, 45–47, 50, 51, 53–56, 62, 66, 69, 70, 72, 75, 76, 79, 80, 86, 88–90, 92–94, 101, 107, 108, 110, 113, 141, 167, 169, 170, 172–177, 182, 189, 193, 195, 196, 205, 208–212

OIL Optical Injection Locking 132

OPL Optical Path Length viii, xiii, xv, xvi, 14, 23–25, 27, 57–61, 69–71, 73–76, 78, 80–85, 88, 89, 100, 210, 211

OSA Optical Spectrum Analyzer xix, 149, 156

OSNR Optical Signal-to-Noise Ratio 148, 149

OTDR Optical Time-Domain Reflectometry 80, 89

PBS Polarization Beam Splitter 17, 57

PC Polarization Controller 57, 148, 149

PD Photodetector xiii, xvi, 16, 95

PM Polarization-Maintaining 171, 177, 225

PSF Point Spread Function 21

RBS Rayleigh Backward Scattering xiii, 13, 14, 23, 24, 26, 32, 35, 48, 55, 58–61, 64, 67, 68, 80, 84, 87

RFG Random Fibre Grating ix, x, xviii–xx, 4, 116, 118, 122, 128–132, 136, 137, 142, 145–148, 152, 154, 155, 157–164, 168–173, 177–179, 181, 190, 215–217, 225, 229

RFGR Random Fibre Grating Ring iii, x, xix, xx, 7, 135–137, 143, 144, 146, 148–165, 220, 222, 223

RFL Random Fibre Laser iv, 4, 10, 116–122, 128, 133–136, 144–146, 158, 166, 167, 169, 189

RIN Relative Intensity Noise xx, 117, 120, 125, 127, 135, 144, 149, 159, 160

RS Rayleigh Scattering 118, 128, 145, 146

SA Saturable Absorber xix, 138, 139

SBS Stimulated Brillouin Scattering 126, 145

SIL Self-Injection Locking iii, ix, x, xix, 4, 116, 132–137, 142, 144, 146, 147, 149, 150, 154, 155, 157–164

SM Single-Mode 225

SMF Single Mode Fibre 57, 92, 106, 112, 176

SMSR Side-Mode Suppression Ratio xx, 163, 164

SRS Stimulated Raman Scattering 145

SVEA Slowly Varying Envelope Approximation 203

TLS Tunable Laser Source xiii, xvi, 16, 17, 55, 94, 95, 173

UV Ultraviolet xiv, 31, 35, 36, 38–40, 42, 51, 66, 82, 83, 91, 106, 110

WDM Wavelength Division Multiplexer 148

WKB Wentzel–Kramers–Brillouin ii, xi, 11, 205–210

ZNCC Zero-Mean Normalized Cross-Correlation viii, 26, 27, 48, 60, 61, 64, 67, 70, 75, 76, 87, 88, 100

INTRODUCTION

Chapter 1

INTRODUCTION TO DISTRIBUTED FIBRE SENSING AND RANDOM FIBRE LASERS

1.1 THE ROLE OF PHOTONICS IN ADVANCED TECHNOLOGICAL SYSTEMS

The advancement of modern technological systems is inseparably tied to our ability to generate, manipulate, and measure information and energy with increasing precision. At the core of many such systems is photonics, where light serves as a medium for communication, computation, metrology, and control. Progress in these technologies relies on two interdependent capabilities. The first is the acquisition of detailed knowledge of the physical world via optical sensing, requiring sensors that transduce physical parameters into optical signals with high fidelity. The second is the generation of highly stable and spectrally pure optical radiation, which provides the reference against which subtle physical changes are measured. High-fidelity optical systems therefore demand concurrent advances in both

1.1. THE ROLE OF PHOTONICS IN ADVANCED TECHNOLOGICAL SYSTEMS

measurement and light generation. This thesis contributes to this progress by addressing critical challenges at the operational limits of distributed fibre optic sensing and advanced fibre laser systems.

Fibre optic sensors offer distinct advantages over electronic counterparts, including immunity to electromagnetic interference, compactness, and suitability for harsh environments. Among these, Optical Frequency Domain Reflectometry (OFDR) enables high-resolution, spatially distributed measurements of temperature and strain via the intrinsic Rayleigh backscattering signature of standard optical fibres [1]. Extending this capability to extreme thermal environments, such as electrical arc plasma and flame combustion, presents a significant challenge [2, 3]. In these conditions, the limitation is no longer the instrumentation but the fundamental thermo-mechanical stability of the silica fibre itself. As temperatures exceed 1100 °C and approach the fibre’s glass transition (T_g) and fictive temperatures (T_f), reduced viscosity [4] facilitates rapid structural relaxation and irreversible material rearrangements [5]. This is accompanied by highly nonlinear material properties, such as a temperature-dependent thermo-optic coefficient [6, 7]. This structural degradation manifests as a severe decorrelation of the Rayleigh scattering signature, ultimately establishing a hard physical boundary for continuous optical measurement near 1300 °C. Therefore, the first part of this thesis develops a quantitative, non-linear framework to characterize these material limitations and establish the sensor’s operational reliability up to its absolute physical threshold.

Complementary to advancements in sensing, fibre lasers provide efficient, high-quality, and robust coherent light sources. This part of the research focuses on Random Fibre Lasers

1.1. THE ROLE OF PHOTONICS IN ADVANCED TECHNOLOGICAL SYSTEMS

(RFLs), whose operation relies on disordered feedback mechanisms such as scattering particles [8], intrinsic Rayleigh scattering [9], or engineered Random Fibre Gratings (RFGs) [10, 11]. While enabling simple coherent sources, these mechanisms also seed intrinsic dynamic instabilities. The coherent superposition of multiply scattered light produces a dense spectrum of potential lasing modes, with competition for gain generating chaotic intensity fluctuations and frequency noise. The main challenge is therefore not generating random laser light but controlling its complex dynamics to achieve high spectral purity.

This part of thesis addresses this control challenge by investigating two distinct, all-optical, and passive paradigms. The first is a feedback-based strategy that seeks to impose an external loop upon the chaotic system. We propose Self-Injection Locking (SIL), a technique established for laser stabilization [12, 13], using a novel random-fibre-grating-based ring resonator without reliance on complex electronic feedback loops. This method aims to suppress the intrinsic mode competition by forcing the laser to lock its frequency and phase to a single, highly stable external reference. The second methodology explores the harnessing of the system's internal nonlinear dynamics. We study the Optical Bistability (OB), a phenomenon well-investigated in conventional resonant cavities [14], as a mechanism for passive self-stabilization in optical systems. This approach does not seek to suppress nonlinearity but to structure it to create a robust equilibrium state within the disordered system itself. Validating this paradigm requires a direct link between the macroscopic laser behaviour and its microscopic internal physics. To this end, the investigation is supported by a novel diagnostic application of OFDR. This technique is employed to spatially resolve the internal refractive index dynamics, thereby providing direct experimental access to the

1.2. RESEARCH OBJECTIVES AND MOTIVATIONS

underlying Kramers-Kronig and Kerr-effect nonlinearities [15, 16] that govern the bistable regime.

1.2 RESEARCH OBJECTIVES AND MOTIVATIONS

This thesis investigates the physical phenomena at the material and dynamic limits of fibre optic systems to develop frameworks for quantifying sensor durability and passively controlling complex laser dynamics. The work is motivated by challenges in distributed sensing and random laser engineering.

The first part of this thesis tackles high-resolution, distributed thermal sensing in extreme environments characterized by sub-millimetre spatial scales and large temperature gradients (*e.g.*, air plasma, flame combustion). Under such conditions, silica fibre sensors are pushed to their absolute thermo-mechanical limits. As temperatures cross the glass transition boundary, the silica network enters a supercooled liquid state, causing the frozen-in density fluctuations responsible for Rayleigh backscattering to undergo irreversible structural relaxation. This invalidates the standard assumption of a static sensing element and creates an ambiguity between the sensor’s environmental response and its own physical evolution. Thus, moving beyond mere environmental measurements, a quantitative framework is needed to accurately map non-linear thermal responses, characterize the sensor’s *in-situ* condition, and rigorously define its operational limits.

The second part of this thesis focuses on controlling the intrinsic dynamic instabilities

1.3. CONTRIBUTIONS

that limit Random Fibre Laser performance. We pursue all-optical, passive control through two complementary strategies. The first disciplines the laser dynamics by locking its frequency to the stable resonances of an external passive cavity, enforcing coherent single-mode operation. The second exploits intrinsic nonlinearities to create a self-stabilizing equilibrium, turning instability into a means of control. Advancing this approach requires diagnostics capable of directly resolving the active physics within the laser cavity.

1.3 CONTRIBUTIONS

The major contributions of this thesis to the fields of fibre optic sensing and laser physics include:

1. Methodology for Quantitative Diagnostics in Extreme Environments

We introduce a novel methodology that uses Rayleigh spectral decorrelation as a quantitative diagnostic tool in extreme thermal environments. The residual spectral shift variance indicates induced mechanical stress and successfully pinpoints the exact onset of the fibre's glass phase transition (T_g), while the correlation coefficient quantifies temperature measurement reliability under high thermal gradients. This approach converts spectral decorrelation from a limitation into a source of physical insight, providing complementary mechanical and structural data to conventional spectral shift analysis.

2. Quantitative Metric for Sensor Durability Assessment

1.3. CONTRIBUTIONS

We introduce and experimentally validate decorrelation time as a quantitative metric for evaluating fibre sensor performance in high-temperature environments. Defined as the time for the Rayleigh backscattering signature to lose correlation with its reference, it reflects irreversible structural changes in silica. This framework transforms high-temperature sensing from qualitative testing to quantitative assessment, establishing fundamental physical thresholds (*e.g.*, instantaneous signal loss at the ~ 1300 °C fictive temperature limit) and temporal confidence metrics (*e.g.*, 50% decorrelation in ~ 20 s at 1000 °C) to define measurement validity windows and guide robust sensor design.

3. All-Optically Stabilized Low-Noise Random Laser

We demonstrate an Erbium-doped Random Fibre Laser (EDRFL) stabilized via self-injection locking to a Random-Fibre-Grating Ring (RFGR) resonator. Adaptive multi-path interference in the RFGR enables single-frequency operation with a 1.23 kHz linewidth, 70 dB side-mode suppression, and a $1 \text{ Hz}/\sqrt{\text{Hz}}$ frequency noise floor. This fully passive, all-optical feedback effectively suppresses random-laser chaos, achieving performance comparable to electronically stabilized phase-locked lasers and offering a low-cost, compact path to high-coherence light sources for communications, metrology, and sensing.

4. Passive Stabilization of Random Lasers via Nonlinearity

We demonstrate the first use of optical bistability in an Erbium-doped Random Fibre Laser (EDRFL) for passive noise reduction. Within the bistable regime, a

1.4. THESIS STRUCTURE

stable operating point clamps gain and loss, suppresses mode competition, and lowers intensity noise by nearly an order of magnitude. This reveals a new framework for controlling complex lasers through intrinsic nonlinearity, showing that disordered systems can achieve self-stabilizing equilibrium and enabling pathways toward passively stable, all-optical switching and memory devices.

5. *In-Situ* Diagnostic Technique for Active Fibre Devices

We employ Optical Frequency Domain Reflectometry (OFDR) as a high-resolution, *in-situ* diagnostic for active fibre devices. Using this method, I achieve the first spatially resolved mapping of refractive index dynamics in an operating random fibre laser, revealing its bistability as a hybrid of population-driven and Kerr-effect nonlinearities. This approach enables direct probing of spatial and temporal dynamics within active fibres rather than inferring them solely from output characteristics, providing a powerful tool for validating models and optimizing nonlinear fibre components.

1.4 THESIS STRUCTURE

This thesis is structured as follows:

Introduction:

- **Chapter 1** introduces the research field, establishes the scientific context for the work, and defines the specific challenges in distributed sensing and random fibre lasers

1.4. THESIS STRUCTURE

that motivate this thesis. It concludes by outlining the primary research objectives and the principal contributions of the work.

Part I: Distributed Sensing in Extreme Thermal Environments:

- **Chapter 2** establishes a necessary theoretical foundation. It details the principles of Optical Frequency Domain Reflectometry (OFDR), introducing the dual-domain cross-correlation methodology. It further examines the complex thermo-mechanical phase behaviours of silica fibre, specifically structural relaxation and fictive temperature (T_f) memory, and the relevant physics of radiative heat transfer that govern the sensor's interaction with its environment.
- **Chapter 3** presents the first experimental investigation, applying the OFDR technique to the transient, high-gradient thermal environment of an electrical arc plasma. This chapter focuses on characterizing the fibre's thermo-mechanical response. It establishes the fundamental continuous measurement limit of OFDR at approximately 1300 °C, demonstrating how exceeding the fibre's glass transition (T_g) and fictive (T_f) temperatures triggers structural relaxation, localized waveguide breakdown, and irreversible internal stress.
- **Chapter 4** builds upon the findings of the previous chapter by addressing the challenge of continuous thermal exposure in a hydrogen flame. This chapter formalizes sensor degradation into a quantitative framework for evaluating durability and measurement validity over time.

1.4. THESIS STRUCTURE

Part II: All-Optical Control of Random Fibre Lasers

- **Chapter 5** provides the theoretical foundation for this section. It covers the noise physics of Random Fibre Lasers (RFLs), the properties of erbium-doped gain media, and the fundamental principles of the two distinct control methodologies investigated: self-injection locking and optical bistability.
- **Chapter 6** details the design and characterization of the feedback-based control approach. It presents the experimental demonstration of an RFL stabilized by self-injection locking to a Random-Fibre-Grating Ring resonator and quantifies its performance in achieving low-noise, single-frequency operation.
- **Chapter 7** explores the nonlinearity-based control approach. It presents the experimental demonstration of harnessing optical bistability for passive stabilization and introduces the novel use of OFDR as an *in-situ* diagnostic tool to resolve the underlying physical mechanisms governing this behaviour.

Conclusion and Appendices

- **Chapter 8** provides the concluding synthesis of the research. It consolidates the key findings from both parts, explicitly addresses the research objectives, discusses the broader scientific and technological implications of the work, and proposes specific directions for future investigation.

1.4. THESIS STRUCTURE

- **Appendices** contain the detailed mathematical derivations that underpin the theoretical and analytical frameworks used throughout the thesis. Appendix A and B provide the formal electromagnetic theory for Rayleigh backscattering, deriving the wave propagation equation from first principles and its WKB solution to establish the basis for the OFDR sensing principle. Appendix C, D, and E provide the detailed models for the laser systems: Appendix C details the transfer-matrix model of the passive Random-Fibre-Grating Ring resonator; Appendix D presents the Lang-Kobayashi rate-equation model for the self-injection locked laser; and Appendix E develops the spatio-temporal propagation and rate-equation model for the bistable laser system.

PART I

DISTRIBUTED FIBRE SENSING IN EXTREME THERMAL ENVIRONMENTS

Chapter 2

PRINCIPLES OF DISTRIBUTED SENSING IN EXTREME THERMAL ENVIRONMENTS

2.1 INTRODUCTION

This chapter establishes the theoretical, methodological, and materials-science foundations for the distributed sensing experiments in this thesis. To comprehensively interpret physical phenomena in extreme thermal regimes, the discussion is structured around four core pillars. First, it details the optical principles of Rayleigh Backscattering (RBS) and Optical Frequency Domain Reflectometry (OFDR), introducing a dual-domain cross-correlation methodology. Second, it examines the thermo-mechanical response of vitreous silica, focusing on non-linear optical properties, structural relaxation, and fictive temperature (T_f) evolution. Third, it analyzes the distinct heat transfer and radiative coupling dynamics of electrical arc plasmas and hydrogen flames. Finally, these pillars are synthesized into a tailored measurement strategy designed to satisfy quasi-static assumptions while cap-

2.2. *DISTRIBUTED FIBRE SENSING VIA RAYLEIGH BACKSCATTERING*

turing permanent structural memory. Together, these frameworks enable the quantitative assessment of sensor performance and material durability explored in Chapter 3 and 4.

2.2 DISTRIBUTED FIBRE SENSING VIA RAYLEIGH BACKSCATTERING

2.2.1 THE ORIGIN AND UTILITY OF THE RAYLEIGH SIGNATURE

Rayleigh backscattering (RBS) in optical fibres originates from microscopic, non-propagating fluctuations in the refractive index of the glass core. During manufacturing, the rapid quenching of the viscous silica preform permanently arrests sub-wavelength variations in material density and dopant concentration within the amorphous matrix (see Figure 2.1, bottom right). These frozen-in structural inhomogeneities act as static scattering centres, providing a unique and repeatable intrinsic fingerprint that functions as a continuous, weak fibre Bragg grating with a random period [1, 17].

Thermal (ΔT) or mechanical ($\Delta \epsilon$) perturbations modify the local effective refractive index (n_{eff}) and physical length (L) (see Figure 2.1, top). These thermo-optic and thermal expansion effects alter the optical path length (OPL) between scattering centres, manifesting as a proportional spectral shift ($\Delta \lambda$) of the complex RBS interference spectrum (see Figure 2.1, Bottom left). By calculating this shift at discrete spatial intervals, the continuous RBS signature enables high-resolution, fully distributed mapping of temperature

2.3. OPTICAL FREQUENCY DOMAIN REFLECTOMETRY (OFDR)

and strain profiles.

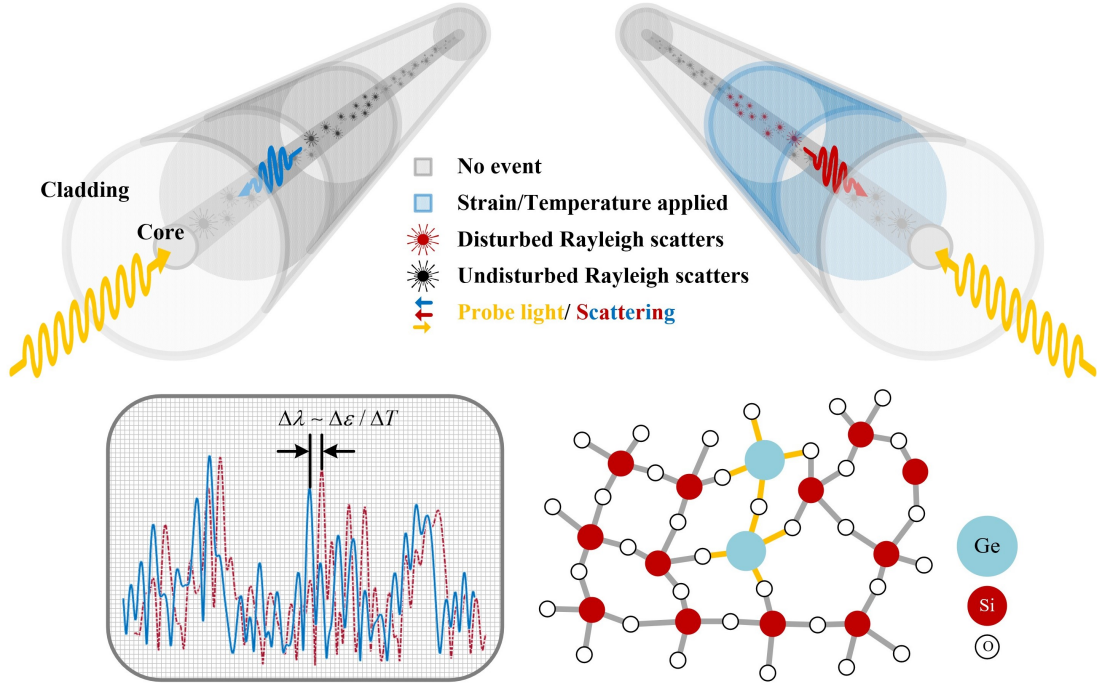


Figure 2.1: Conceptual illustration of distributed sensing via Rayleigh backscattering. Top: intrinsic glass inhomogeneities act as static scattering centres; temperature/strain alters the local refractive index and length. Bottom left: the resulting proportional spectral shift ($\Delta\lambda$). Bottom right: microscopic origins of scattering in the amorphous network.

2.3 OPTICAL FREQUENCY DOMAIN REFLECTOMETRY (OFDR)

OFDR is an interferometric technique used to measure the spatially resolved Rayleigh backscattering profile of an optical fibre. By interrogating the fibre in the optical frequency domain and applying a Fourier transform, the system achieves high spatial resolution and sensitivity.

2.3. OPTICAL FREQUENCY DOMAIN REFLECTOMETRY (OFDR)

2.3.1 OFDR SYSTEM

The OFDR architecture is composed of two primary interferometric networks: a main measurement interferometer and an auxiliary interferometer (AUI), as illustrated in Figure 2.2.

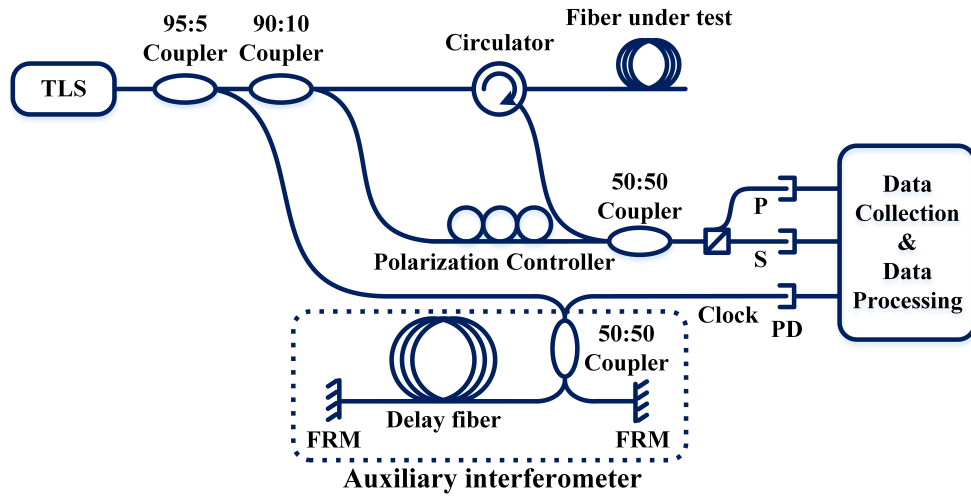


Figure 2.2: Schematic diagram of the Optical Frequency Domain Reflectometry (OFDR) experimental setup. The system consists of a main measurement interferometer and an auxiliary interferometer (AUI). Key components include a tunable laser source (TLS), a cascaded optical coupler network (95:5, 90:10, 50:50), an optical circulator, Faraday Rotator Mirrors (FRM), a polarization beam splitter for polarization diversity detection (P and S channels), and photodetectors (PD).

Light from a Tunable Laser Source (TLS) is split into the AUI and the main interferometer. Within the main network, the light is further divided into measurement and reference arms. The probe light is injected into the Fibre Under Test (FUT) through an optical circulator. Coherent Rayleigh backscattered light, originating from microscopic inhomogeneities in the FUT, returns through the circulator and is mixed with the reference

2.3. OPTICAL FREQUENCY DOMAIN REFLECTOMETRY (OFDR)

light at a 50:50 coupler to produce an interference signal.

Since standard single-mode fibres do not maintain polarization, the backscattered signal undergoes random polarization evolution. This can lead to signal fading if the returning probe field becomes orthogonal to the reference field. To mitigate this, polarization diversity detection is employed: a Polarization Beam Splitter (PBS) separates the mixed signal into two orthogonal polarization states (P and S). These are captured by independent photodetectors, ensuring that the total scattering amplitude is recovered regardless of the local polarization state [18]. Simultaneously, the AUI generates an external clock signal to trigger the analog-to-digital (ADC) converters. This compensates for the inherent tuning non-linearity of the TLS, ensuring that the data is sampled at uniform intervals in the optical frequency domain [19, 20].

2.3.2 BASIC PRINCIPLE OF OFDR

The fundamental principle of Optical Frequency-Domain Reflectometry (OFDR) lies in the linear mapping of the spatial position of scattering centres along an optical fibre into discrete, measurable beat frequencies. This is achieved by interrogating the fibre with a laser source whose optical frequency $\nu(t)$ is swept linearly in time. For a tuning rate γ (Hz/s) and an initial frequency ν_0 , the instantaneous frequency is expressed as:

$$\nu(t) = \gamma t + \nu_0. \quad (2.1)$$

2.3. OPTICAL FREQUENCY DOMAIN REFLECTOMETRY (OFDR)

The instantaneous phase $\phi(t)$, defined as the time integral of the optical frequency, evolves quadratically with time:

$$\phi(t) = \int_0^t 2\pi\nu(t')dt' = 2\pi\nu_0 t + \pi\gamma t^2 + \phi_0, \quad (2.2)$$

where ϕ_0 represents the initial phase constant. The electric field of the reference light reaching the detector, $E_r(t)$, is characterized by a slowly varying amplitude A_{r0} and this time-dependent phase, expressed in complex notation as:

$$E_r(t) = \frac{1}{2}A_{r0}e^{j\phi(t)} + c.c. = A_{r0} \cos[\phi(t)], \quad (2.3)$$

where *c.c.* denotes the complex conjugate.

The probe light propagates through the measurement arm into the FUT. Under a quasi-static assumption ($T_{\text{scan}} \approx 1$ s), the scattering distribution remains stationary during the laser sweep. For a discrete scattering element at a round-trip delay τ , the backscattered field $E_p(t)$ is a delayed and scaled version of the source field. The interaction is governed by the complex reflectivity $r(\nu, \tau) = |r(\nu, \tau)|e^{j\psi(\nu, \tau)}$, which accounts for the magnitude and phase of the Rayleigh speckle. The backscattered field is expressed as:

$$E_p(t) = \frac{1}{2}A_{p0}r(\nu, \tau)e^{j\phi(t-\tau)} + c.c., \quad (2.4)$$

where A_{p0} is the incident amplitude. The photodetector measures the optical intensity $I(t)$, proportional to the time-averaged square of the total electric field $\langle (E_r + E_p)^2 \rangle$. After

2.3. OPTICAL FREQUENCY DOMAIN REFLECTOMETRY (OFDR)

expanding the square and averaging the rapidly oscillating optical cycles, the DC terms are filtered, leaving the difference-frequency interference term. The resulting AC signal is:

$$I_{\text{AC}}(t) = \frac{1}{2}\eta A_{\text{r0}}A_{\text{p0}}r(\nu, \tau)e^{j[\phi(t)-\phi(t-\tau)]} + c.c., \quad (2.5)$$

where η represents the detector sensitivity.

Beat frequency: Using the quadratic phase evolution previously established, the phase difference between the reference and delayed probe light, $\Phi_{\text{diff}}(t)$, is given by:

$$\begin{aligned} \Phi_{\text{diff}}(t) &= \phi(t) - \phi(t - \tau) \\ &= (2\pi\nu_0 t + \pi\gamma t^2) - (2\pi\nu_0(t - \tau) + \pi\gamma(t - \tau)^2) \\ &= 2\pi\gamma\tau t + 2\pi\nu_0\tau - \pi\gamma\tau^2. \end{aligned} \quad (2.6)$$

Given that the round-trip delay τ is typically on the order of nanoseconds for laboratory-scale fibres, the quadratic term $\pi\gamma\tau^2$ is negligible compared to the linear term $2\pi\gamma\tau t$ over the duration of the sweep. Consequently, the phase difference is approximately linear in time:

$$\Phi_{\text{diff}}(t) \approx 2\pi(\gamma\tau)t + 2\pi\nu_0\tau. \quad (2.7)$$

The frequency of the resulting interference signal, defined as the beat frequency ν_b , is the time derivative of this phase difference. It is directly proportional to the time delay τ ,

2.3. OPTICAL FREQUENCY DOMAIN REFLECTOMETRY (OFDR)

and thus to the spatial position of the scattering centre:

$$\nu_b = \frac{1}{2\pi} \frac{d\Phi_{\text{diff}}}{dt} = \gamma\tau. \quad (2.8)$$

Substituting this back into the interference expression, the detected AC signal for a single scattering centre is a sinusoidal wave in the frequency domain:

$$I_{\text{AC}}(\nu) = \frac{1}{2} \eta A_{\text{r0}} A_{\text{p0}} r(\nu, \tau) e^{j2\pi\nu\tau} + c.c.. \quad (2.9)$$

Discrete sampling and Fourier transform: In experimental implementation, the interference signal is uniformly sampled by an analog-to-digital converter (ADC) as the laser scans over a total bandwidth $\Delta\nu$. The discrete frequency samples are defined as $\nu_n = \nu_0 + n\delta\nu$ for $n = 0, \dots, N - 1$. The sampled intensity $I[\nu_n]$ for a scattering event at delay τ is:

$$I[\nu_n] = \frac{1}{2} \eta A_{\text{r0}} A_{\text{p0}} r(\nu_n, \tau) e^{j2\pi\nu_n\tau} + c.c.. \quad (2.10)$$

To resolve the spatial distribution, a DFT is applied to convert the signal from the optical frequency domain index n to the delay index k . According to the Discrete Convolution Theorem, the DFT of the product of the reflectivity and the phase term is the circular convolution of their individual transforms:

$$\tilde{I}[\tau_k] = \mathcal{DFT}\{I[\nu_n]\} = \frac{1}{2N} \eta A_{\text{r0}} A_{\text{p0}} \mathcal{DFT}\{r(\nu_n, \tau)\} \otimes \mathcal{DFT}\{e^{j2\pi\nu_n\tau}\} + c.c., \quad (2.11)$$

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where the discrete delay grid is defined by $\tau_k = k/\Delta\nu$. The term $\mathcal{DFT}\{r(\nu_n, \tau)\}$ represents the intrinsic spatial impulse response of the scattering centres. The term $\mathcal{DFT}\{e^{j2\pi\nu_n\tau}\}$, resulting from the finite sweep range $\Delta\nu$, yields the system's Point Spread Function (PSF). For large N , this approximates a *sinc* Dirichlet kernel centered at τ :

$$\mathcal{DFT}\{e^{j2\pi\nu_n\tau}\} \approx e^{j2\pi\nu_0\tau} e^{j\pi\Delta\nu(\tau-\tau_k)} \frac{\sin[\pi\Delta\nu(\tau-\tau_k)]}{\pi\Delta\nu(\tau-\tau_k)}. \quad (2.12)$$

The final measured signal $\tilde{I}[\tau_k]$ is thus the intrinsic scattering fingerprint of the fibre blurred by the system's PSF.

Delay-time and spatial resolution: For an ideal point reflector, such as a fibre end-face Fresnel reflection, the reflectivity is frequency-independent: $r(\nu_n, \tau) = r_0 e^{j\psi_0}$. Its DFT is a Kronecker delta, $Nr_0 e^{j\psi_0} \delta(\tau_k)$, which reduces the measured signal in eq. (2.11) to the system's *sinc*-shaped PSF. According to the Rayleigh criterion, two adjacent scattering features are just resolved when the peak of one *sinc* function coincides with the first zero of the other, requiring:

$$\pi\Delta\nu(\tau_k - \tau) = \pi. \quad (2.13)$$

This defines the minimum resolvable delay as $\Delta\tau = 1/\Delta\nu$. This temporal resolution is converted to spatial resolution Δz by accounting for the group refractive index n_g and the round-trip propagation:

$$\Delta z = \frac{c}{2n_g} \Delta\tau = \frac{c}{2n_g \Delta\nu}. \quad (2.14)$$

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In the wavelength domain, this fundamental limit is expressed as:

$$\Delta z = \frac{\lambda_i \lambda_f}{2n_g \Delta \lambda}, \quad (2.15)$$

where λ_i and λ_f are the start and end wavelengths of the scan, and $\Delta \lambda$ is the total sweep range. Consequently, the spatial resolution of an OFDR system is dictated by the spectral breadth of the interrogation, with wider scans enabling the resolution of finer structural details.

2.4 METHODOLOGY: DUAL-DOMAIN CORRELATION ANALYSIS

To extract physical measurements from raw optical signals, this study applies a dual-domain analysis methodology. The distinction between these domains lies in how they resolve the propagation of backscattered light:

- **Spectral Domain:** By isolating a physical volume of the fibre, this method evaluates the collective interference pattern of all scattering centres within a spatial window. Cross-correlation yields a single, averaged scalar shift representing the bulk thermodynamic state (temperature or strain). It fundamentally assumes the maintenance of a single-mode waveguide.
- **Temporal Domain:** By resolving the absolute time-of-flight (Optical Path Length,

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OPL), this method tracks independent photon arrival times. If localized heating disrupts the core's refractive index or violates the single-mode condition, the temporal domain can resolve multiple simultaneous delay shifts, providing insight into waveguide integrity.

2.4.1 DATA PROCESSING PIPELINES

Mathematically, the distinction between the two analysis methods depends on the domain in which a windowing function (*e.g.*, a rectangular window) is applied to the data.

Spectral Domain Pipeline (Spatial Window): To measure localized temperature or strain, the processing isolates a specific physical length of the fibre, as illustrated in Figure 2.3. The sequence is as follows:

1. **FFT to Distance Domain:** The raw wavelength-domain interference fringes undergo a Fast Fourier Transform (FFT) to generate a global spatial reflectivity profile, mapping the Rayleigh scattering along the entire fibre.
2. **Spatial Windowing (Δz):** A sliding window is applied in the distance domain to isolate a discrete physical segment of the fibre (*e.g.*, a 2 mm gauge length).
3. **IFFT to Wavelength Domain:** An Inverse Fast Fourier Transform (IFFT) is performed on this isolated segment, recovering the local Rayleigh backscattering (RBS) spectrum.

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4. **Spectral Shift Calculation:** Cross-correlation determines the wavelength shift ($\delta\lambda$) between the measurement and reference spectra.

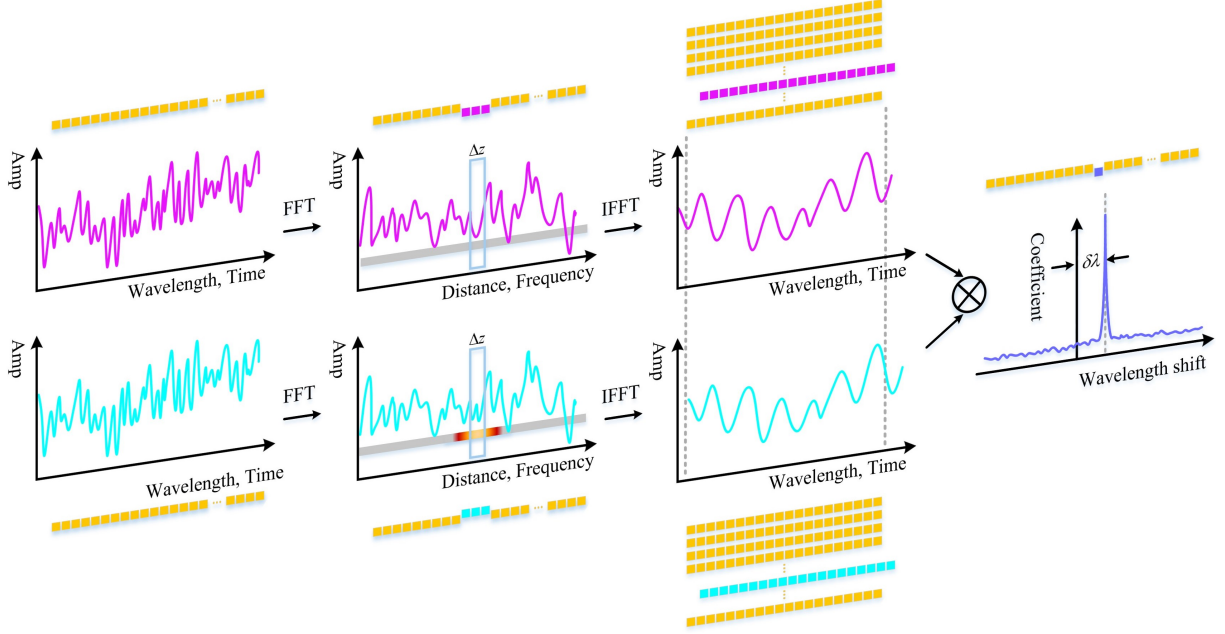


Figure 2.3: Data processing pipeline for spectral domain analysis: Raw fringes are transformed to the distance domain, windowed spatially (Δz), and inverse-transformed to recover the local RBS spectrum. Cross-correlation against a reference yields the local spectral shift ($\delta\lambda$).

Temporal Domain Pipeline (Wavelength Window): To monitor high-sensitivity, integrated changes in optical path length (OPL) and resolve potential multipath interference during transient structural events, the processing path is inverted (Figure 2.4).

1. **Wavelength Windowing ($\Delta\lambda$):** A sliding window is applied directly to the raw interference fringes in the wavelength domain. This isolates a specific spectral band of the backscattered signal.

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2. **FFT to Distance Domain:** An FFT is applied to this windowed spectral data, transforming it into a local spatial reflectivity profile.
3. **Spatial Shift Calculation:** Cross-correlation is performed directly on these spatial profiles. Operating in the time-delay domain allows the algorithm to identify local maxima representing multiple discrete time-delay shifts ($\delta\tau$) if the light propagates through multiple optical paths or modes within the same spatial location.

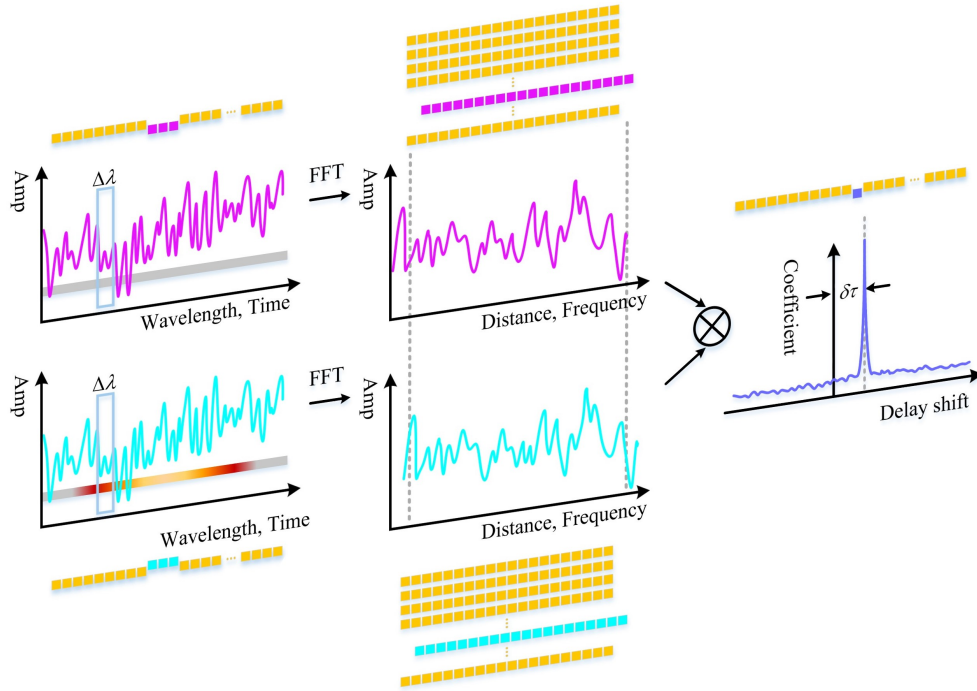


Figure 2.4: Data processing pipeline for temporal domain analysis: A spectral window ($\Delta\lambda$) is applied to raw fringes before an FFT transforms the data into the distance domain. Cross-correlation of spatial profiles yields the time-delay shift ($\delta\tau$), enabling high-resolution OPL tracking.

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2.4.2 ZERO-MEAN NORMALIZED CROSS-CORRELATION (ZNCC)

Since Rayleigh backscattering produces a stochastic interference pattern, comparing measurement and reference traces requires a correlation metric immune to uniform amplitude and intensity fluctuations. This study employs ZNCC. For a discrete window of N points, the correlation function Q_{ZNCC} at shift index k is defined as:

$$Q_{ZNCC}(k) = \frac{\sum_{i=1}^N [I_m(i) - \bar{I}_m] [I_r(i - k) - \bar{I}_r]}{\sqrt{\sum_{i=1}^N [I_m(i) - \bar{I}_m]^2} \sqrt{\sum_{i=1}^N [I_r(i - k) - \bar{I}_r]^2}}, \quad (2.16)$$

where $I_m(i)$ and $I_r(i)$ are the intensities of the measurement and reference traces, and $\bar{I}_m, \bar{I}_r$ are their respective means. This normalized approach ensures that the correlation peak depends solely on the similarity of the scattering fingerprints. To simplify notation, we define the mean-subtracted signals as $\Delta I_{m,r}$ and use the $\otimes$ operator to denote this process.

Spectral Domain Analysis (Temperature and Stress): In the spectral domain, ZNCC is applied to the recovered RBS spectrum $\Delta I(\nu)$. The spectral shift $\delta\nu$ and the correlation quality Q_ν are extracted as:

$$\begin{aligned} \delta\nu &= \arg \max \{ \Delta I_m(\nu) \otimes \Delta I_r(\nu) \}, \\ Q_\nu &= \max \{ \Delta I_m(\nu) \otimes \Delta I_r(\nu) \}. \end{aligned} \quad (2.17)$$

The shift $\delta\nu$ is linearly related to changes in temperature ΔT and strain $\Delta\epsilon$ by the

2.4. METHODOLOGY: DUAL-DOMAIN CORRELATION ANALYSIS

sensing equation [21]:

$$\frac{\delta\lambda}{\lambda} = -\frac{\delta\nu}{\nu} = \left(\frac{1}{n_{eff}} \frac{\partial n_{eff}}{\partial T} + \frac{1}{L} \frac{\partial L}{\partial T} \right) \Delta T + (1 - p_e) \Delta\epsilon = K_T \Delta T + K_\epsilon \Delta\epsilon, \quad (2.18)$$

where K_T and K_ϵ are the temperature and strain sensitivity coefficients. For standard silica fibre at 1550 nm, $K_T \approx 6.7 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ ($\sim 10 \text{ pm}/^\circ\text{C}$) and $K_\epsilon \approx 0.78 \times 10^{-6} \text{ } \mu\epsilon^{-1}$ ($\sim 1.2 \text{ pm}/\mu\epsilon$). While the thermo-optic effect contributes approximately 95–98% of K_T , it is critical to note that K_T becomes non-linear at elevated temperatures. The correlation quality Q_ν serves as a diagnostic: values approaching unity indicate high measurement fidelity, while significant drops indicate irreversible structural decorrelation of the glass network.

Temporal Domain Analysis (OPL and Dynamics): In the temporal domain, ZNCC is applied directly to the spatial reflectivity profile $\Delta I(\tau)$. Perturbations to the fibre alter the group refractive index n_g and physical length L , manifesting as a time-delay shift $\delta\tau$ in the backscattered signal. This domain enables femtosecond-level resolution of changes in the optical path length (OPL).

When extreme thermal gradients or structural breakdown disrupt the single-mode waveguide condition, the temporal ZNCC algorithm is configured to resolve multiple propagating modes or discrete scattering paths. Rather than extracting a single scalar shift, the algorithm evaluates local maxima across the correlation vector to identify multiple

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simultaneous delay shifts ($\delta\tau$) and their corresponding quality metrics (Q_τ):

$$\begin{aligned}\delta\tau &= \arg \max\{\Delta I_m(\tau) \otimes \Delta I_r(\tau)\}, \\ Q_\tau &= \Delta I_m(\tau) \otimes \Delta I_r(\tau).\end{aligned}\tag{2.19}$$

Physically, the spatial shift $\delta\tau(z)$ represents the integrated time-of-flight of the backscattered light. The cumulative delay at position z reflects the integrated thermal and mechanical modifications from the proximal end of the fibre up to that point:

$$\delta\tau(z) = \frac{2}{c} \int_0^z [\Delta n_g(\zeta) + n_g(\zeta)\alpha_L \Delta T(\zeta)] d\zeta,\tag{2.20}$$

where Δn_g is the dominant term, representing the thermal shift of electronic polarizability, and α_L is the coefficient of thermal expansion. This approach tracks absolute signal propagation, providing a robust diagnostic for dynamic processes where spectral correlation may fail due to rapid structural reconfiguration or severe waveguide damage. By utilizing the wide 100 nm OFDR sweep, this dual-domain methodology bridges macroscopic thermal mapping with microscopic analysis of light propagation in non-uniform dielectric media.

2.4.3 PHYSICAL BASIS FOR DUAL-DOMAIN SENSING

The dual-domain analysis methodology is grounded in the theory of wave propagation within non-uniform dielectric media. The Rayleigh backscattering signature arises from the coherent superposition of waves reflected by stochastic microscopic index fluctuations. The

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round-trip phase φ_{RT} of light backscattered from position z is defined by the integral of the local wavenumber $k(z)$:

$$\varphi_{RT}(z) = 2 \int_0^z k(\zeta) d\zeta. \quad (2.21)$$

By decomposing the complex wavenumber into its real and imaginary components, $k(z) = k'(z) + ik''(z)$, the round-trip phase separates into two physically distinct terms:

$$\varphi_{RT}(z) = \underbrace{2 \int_0^z k'(\zeta) d\zeta}_{\text{Real Part: Phase/Delay}} + \underbrace{2i \int_0^z k''(\zeta) d\zeta}_{\text{Imaginary Part: Amplitude/Signature}}. \quad (2.22)$$

This decomposition provides the formal justification for the dual-domain method:

- **Real Part:** Determined by the real component of the refractive index $n'(z)$. Perturbations in temperature or strain modify n' , shifting the accumulated phase. This manifests as a time delay $\delta\tau$ in the temporal domain or a spectral shift $\delta\lambda$ in the spectral domain.
- **Imaginary Part:** Determined by the imaginary component of the refractive index $n''(z)$, which governs local absorption and the statistical properties (strength and phase) of the Rayleigh scattering. Irreversible modifications to the fibre's microscopic network fundamentally alter this random refractive index profile, leading to the spectral and spatial decorrelation observed during annealing or structural degradation.

Consequently, OFDR functions as a probe for both the real and imaginary components of the fibre's complex refractive index, providing a comprehensive assessment of its thermodynamic

2.5. THERMO-MECHANICAL RESPONSE OF SILICA FIBRE

and structural state.

2.5 THERMO-MECHANICAL RESPONSE OF SILICA FIBRE

The response of a silica fibre sensor to temperature is approximately linear only within a limited range. At the extreme temperatures investigated in this work, the material's thermo-mechanical behaviour is governed by the structural state of its amorphous network. This state is characterized by the fictive temperature (T_f), defined as the temperature at which the glass's non-equilibrium structure would be frozen in thermodynamic equilibrium [22]. In the Ge-doped core, the evolution of T_f introduces non-linearities, non-reproducible drift, and permanent structural memory.

2.5.1 LINEAR REGIME AND THERMO-OPTIC DOMINANCE

At temperatures well below the glass transition range (typically < 500 °C), the relationship between temperature change and the induced spectral shift ($\delta\lambda$) or optical path length change ($\delta\tau$) is linear and predictable. As established in eq. (2.18), this thermal response is governed by the interplay of the thermo-optic effect and thermal expansion.

These two mechanisms are not of equal magnitude. The first-order thermo-optic coefficient ($\partial n_{eff}/\partial T$) of silica fibre is approximately 1.09×10^{-5} °C⁻¹, whereas the linear thermal expansion coefficient (α_L) is significantly smaller, approximately 5.36×10^{-7} °C⁻¹ [23]. Thermal expansion contributes only about 2% to the total thermal response in silica glass,

2.5. THERMO-MECHANICAL RESPONSE OF SILICA FIBRE

with the dominant 98% arising from the thermo-optic effect [24, 25]. Physically, this temperature-induced refractive index change originates primarily from the thermal shift of the material’s excitonic UV absorption peak (near 10.4 eV), a phenomenon strongly associated with Germanium Oxygen-Deficient Centres (GODCs) in the doped core [26]. In this regime, the intrinsic Rayleigh scattering signature is structurally static and highly correlated with the reference state after thermal cycling; the glass is effectively frozen at a constant T_f established during manufacture [27, 28].

2.5.2 HIGH-TEMPERATURE NONLINEAR REGIME AND PHASE TRANSITIONS

As temperatures exceed 1000 °C, the linear thermal response model fails. The refractive index follows a quadratic dependence, $n(T) = n_0 + \alpha_n T + \beta_n T^2$, where the second-order thermo-optic coefficient ($\beta_n \approx 1.61 \times 10^{-9} \text{ °C}^{-2}$) drives a continuous increase in thermal sensitivity [23].

The subsequent structural evolution is caused by the Ge-doped core’s atomic network. The Ge-O bond ($\approx 1.72 \text{ Å}$) is longer and more labile than the Si-O bond ($\approx 1.61 \text{ Å}$) [29], and germanium incorporation promotes the formation of strained 3- and 4-membered ring structures at elevated temperature [5]. These factors reduce the local viscosity and depress the glass transition temperature (T_g) relative to the pure silica cladding [27].

As vitreous silica is an amorphous solid, its phase transition is governed by an exponentially decreasing viscosity. Pure silica is traditionally mapped using mechanical thresholds:

2.5. THERMO-MECHANICAL RESPONSE OF SILICA FIBRE

the strain point (~ 1108 °C, $\sim 10^{14.5}$ Poise), the annealing point (~ 1190 °C, $\sim 10^{13.4}$ Poise), and the softening point (~ 1670 °C, $\sim 10^{7.65}$ Poise) [4]. However, for distributed optical sensing, it is more instructive to categorize the Ge-doped core into three thermodynamic regimes based on structural mobility:

1. **Glassy State (< 1100 °C):** The network acts as a rigid solid. Density and concentration fluctuations are arrested, meaning the intrinsic RBS profile is static and governed by the original fictive temperature (T_f). Spectral shifts are reversible and correspond to pure thermo-optic and expansion effects without network rearrangement.
2. **Glass Transition Region (1100 °C to 1200 °C):** Spanning the strain and annealing points, this regime marks the onset of macroscopic structural mobility. The network undergoes non-exponential relaxation, causing the RBS signature to evolve over time. This manifests as non-reproducible sensor drift and irreversible spectral decorrelation.
3. **Supercooled Liquid State (> 1200 °C):** The glass behaves as a high-viscosity fluid, maintaining continuous local thermal equilibrium ($T_f = T$). Macroscopic memory of the factory-frozen structure is erased. Because density fluctuations are governed by thermodynamic statistical mechanics rather than being frozen-in, the Rayleigh scattering intensity becomes proportional to the absolute temperature [30, 31]. Rapid quenching from this state locks in an exceptionally high T_f , resulting in permanently increased scattering losses.

2.5. THERMO-MECHANICAL RESPONSE OF SILICA FIBRE

2.5.3 FICTIVE TEMPERATURE AND STRUCTURAL MEMORY OF GE-DOPED SILICA FIBRE

The structural state of vitreous silica is governed not by its current ambient temperature, but by its thermal history. To describe this non-equilibrium state, the fictive temperature (T_f) represents the temperature at which the liquid-like molecular network was arrested during cooling [22].

As a glass-forming liquid is cooled through its transition range, its viscosity increases exponentially, causing the structural relaxation time to eventually exceed the experimental cooling time. Narayanaswamy demonstrated that this relaxation is highly non-exponential, governed by a broad spectrum of relaxation times rather than a single mechanism [32]. Due to this kinetic slowdown, the glass retains a memory of its thermal history, where the final room-temperature structure encodes the specific cooling trajectory.

The structural state of the glass is characterized by the fictive temperature (T_f), defined as the temperature at which the instantaneous glass structure would be in equilibrium. And its behaviour transitions across the cooling range: in the liquid state ($T \gg T_g$), the structure relaxes instantaneously, such that $T_f = T$, whereas in the glassy state ($T \ll T_g$), the structural relaxation effectively ceases, and the fictive temperature T_f reaches a limiting value, T_f' . This dependency is quantified by the Moynihan relationship [33]:

$$\frac{d \ln |q|}{d(1/T_f')} = -\frac{\Delta h^*}{R}, \quad (2.23)$$

2.5. THERMO-MECHANICAL RESPONSE OF SILICA FIBRE

where q is the cooling rate, Δh^* is the activation enthalpy for structural relaxation and R is the ideal gas constant.

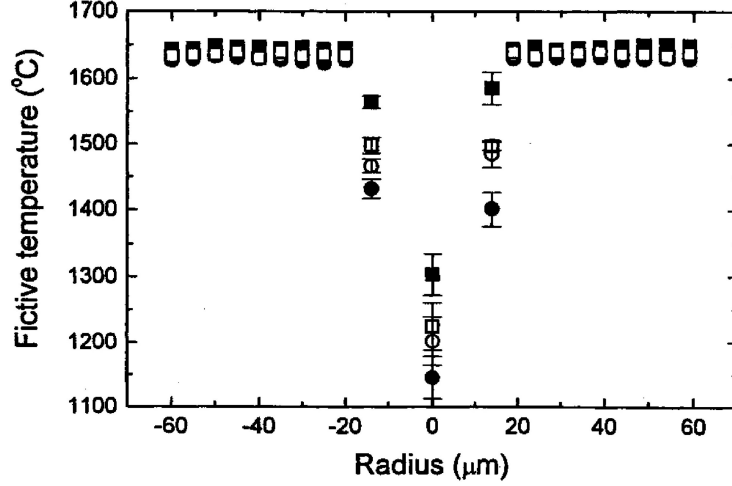


Figure 2.5: Radial profile of the fictive temperature (T_f) across a standard Ge-doped single-mode optical fibre. The pure silica cladding solidifies rapidly, locking in a high baseline T_f , whereas the lower-viscosity core undergoes extended structural relaxation, resulting in a pronounced T_f depression. The discrete markers denote varying cooling rates during the drawing process: ■ fast cooling at 30000 °C/s; □, ○ intermediate cooling at 9000 °C/s; ● slow cooling at 3000 °C/s (adapted from Kim *et al.* [34]).

Standard single-mode optical fibres possess a non-uniform initial structure due to the extreme cooling rates ($> 6.5 \times 10^3$ °C/s) used during drawing. Because the fibre is a composite structure, the pure silica outer cladding, which possesses higher viscosity, solidifies first and locks in an extremely high T_f (1620–1660 °C). In contrast, the Ge-doped core remains a supercooled liquid longer, allowing more structural relaxation and resulting in a lower baseline T_f (1150–1300 °C) (see Figure 2.5) [34]. This reflects that a faster cooling rate (q) provides less time for the atomic network to relax, locking the glass into a

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higher-energy state with a higher T_f .

Furthermore, as the core and inner cladding continue to cool, they are mechanically constrained by the already-rigid outer cladding. This constrained volume relaxation locks severe residual stresses and distinct micro-density variations into the core's RBS signature. In high-temperature OFDR sensing, exceeding these baseline T_f values effectively erases the factory-frozen memory, replacing it with a new structural signature upon subsequent cooling.

2.6 HEAT TRANSFER DYNAMICS IN EXTREME ENVIRONMENTS

In high-temperature open environments, energy transfer to the optical fibre is governed by the interplay of convective heat transfer and spectrally coupled radiation. The efficiency of this energy deposition depends on the emission spectrum of the source, gas-phase dynamics, and the temperature-dependent absorption coefficients of Ge-doped silica.

2.6.1 SPECTRALLY SELECTIVE ABSORPTION IN GE-DOPED SILICA

Vitreous silica is a spectrally selective medium. Its initial coupling with external heat sources is dictated by two primary absorption regions [35]:

Ultraviolet (UV) Absorption: Ge-doped silica fibres exhibit a very strong UV band

2.6. HEAT TRANSFER DYNAMICS IN EXTREME ENVIRONMENTS

around 240–243 nm (5.1 eV) due to germanium oxygen deficiency centres (GODC). For a typical 3–4 mol% Ge fibre, the absorption coefficient at 240 nm is on the order of 10–30 cm^{-1} (equivalent to thousands of dB/m, see Figure 2.6). A much weaker shoulder or second peak appears near 330–340 nm (~ 3.7 – 3.8 eV) (typically $\sim 10^{-2}$ cm^{-1}). No other UV bands are seen in pristine Ge fibres under normal conditions. Vacuum UV studies show the 5.15 eV band in all Ge-doped samples and a higher-energy structure near 165–180 nm (7.0–7.5 eV) [36–38]. This allows for highly efficient, localized energy deposition directly into the fibre core when exposed to UV-rich environments.

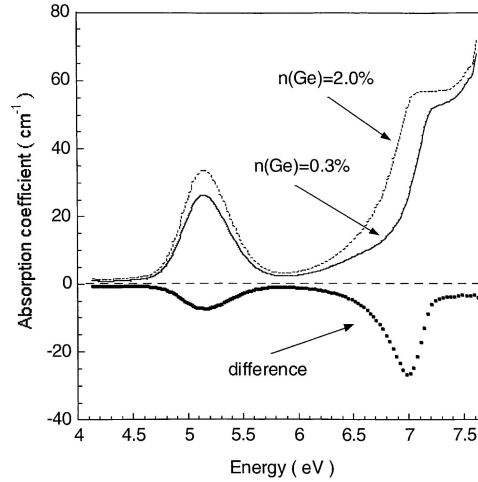


Figure 2.6: Vacuum UV absorption spectra of homogeneous Ge-doped silica (0.3 and 2 mol%) and difference spectrum (0.3–2%) (adapted from Anedda *et al.* [38]).

Infrared (IR) Absorption: Absorption in this region closely follows that of pure silica, with attenuation governed by the vibrational resonances of Si-O-Si and Ge-O-Si bonds. As shown in Figure 2.7, the near-IR region (800–1600 nm) is characterized by very low loss, forming the well-known telecom windows around 850, 1310, and

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1550 nm, where attenuation reaches its minimum and is primarily limited by the Rayleigh-scattering background. The glass becomes effectively opaque for wavelengths longer than $\sim 3.5\text{--}4.0\text{ }\mu\text{m}$ due to multi-phonon absorption. Weak absorption features may arise from OH-related impurities, including the overtone near $\sim 1380\text{ nm}$ and the fundamental vibration at $\sim 2.7\text{--}2.85\text{ }\mu\text{m}$ [39]. In the mid-IR, strong absorption bands appear at $9\text{--}9.5$, 12.5 , and $21\text{--}23\text{ }\mu\text{m}$, corresponding to fundamental Si-O-Si vibrational modes [35]. This broad absorption band is responsible for the dominant thermal radiation coupling in optical fibres.

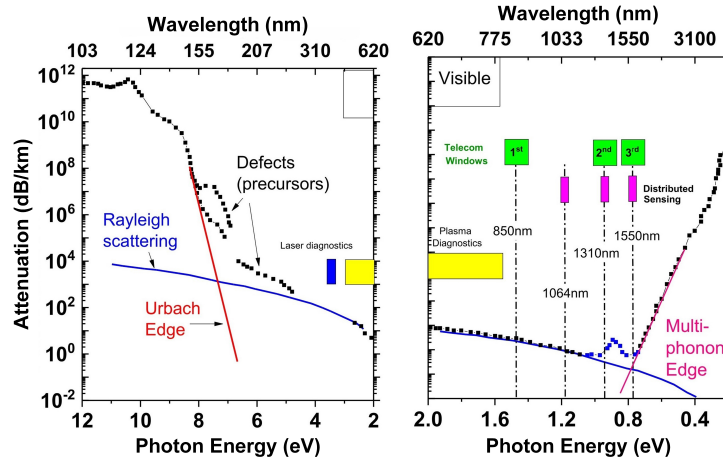


Figure 2.7: Illustration of the attenuation in a silica fibre and of the main mechanisms explaining its spectral dependence (adapted from Girard *et al.* [40]).

2.6.2 DYNAMIC ABSORPTION AT ELEVATED TEMPERATURES

The absorption profile of Ge-doped silica is highly temperature-dependent. In the extreme thermal environments generated by plasmas and flames, optical absorption coefficients

2.6. HEAT TRANSFER DYNAMICS IN EXTREME ENVIRONMENTS

evolve dynamically through two primary mechanisms:

- **Urbach Absorption Edge:** The fundamental ultraviolet electronic absorption edge extends into the optical bandgap as an exponential curve known as the Urbach tail [41]. While negligible at room temperature, heating toward 1000 °C and beyond intensifies thermal lattice vibrations (phonons) and increases structural disorder. This causes the Urbach edge to broaden and shift toward smaller photon energies (longer wavelengths) [42, 43]. Consequently, the overall UV absorption cross-section of the silica network increases significantly at high temperatures, accelerating the coupling efficiency of external radiation into the fibre core.
- **Multiphonon Absorption Edge:** At extreme temperatures, the population of high-energy phonon states increases significantly. This thermal excitation induces a pronounced shift of the IR absorption edge toward shorter wavelengths, effectively narrowing the fibre’s transparency window from the long-wavelength side [44]. Compounded by thermal expansion and intensified atomic motion which broaden the fundamental vibrational bands, this encroaching multiphonon edge drastically increases the absorption coefficient in the near- to mid-IR. At temperatures approaching 2000 °C, the silica network becomes highly opaque to broadband thermal radiation, driving the rapid bulk heating observed in flame environments.

Collectively, these dynamic effects ensure that silica fibre at elevated temperatures couples with external energy fundamentally differently than at room temperature, driving the extreme internal thermal gradients and waveguide breakdown investigated in this work.

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2.6.3 DIFFERENTIATED HEATING MECHANISMS: ARC PLASMA VS. HYDROGEN FLAME

The two experimental environments investigated in this thesis present distinct thermodynamic and radiative profiles. To accurately capture their thermal dynamics, convective heat transfer from the gas phase must be treated alongside optical radiative absorption.

Electrical Arc Plasma (Chapter 3): An electrical glow discharge is a stable, luminous, non-thermal plasma generated by a high electric field in air. Within this environment, the fibre is subjected to a dual-mechanism, bi-directional heating process:

- **Convective Heat Transfer (Surface Heating):** In an atmospheric-pressure discharge, high-kinetic-energy electrons undergo frequent inelastic collisions with nitrogen and oxygen molecules, rapidly transferring energy to the heavy particles. This generates a high-temperature fluid flow (with temperatures ~ 2200 K) that creates massive convective heat transfer at the fibre surface. This forced convection acts as the dominant bulk heating mechanism, driving thermal energy to conduct radially inward from the cladding.
- **Radiative/Optical Absorption (Volumetric Heating):** Simultaneously, as excited nitrogen and oxygen species relax, they emit intense, discrete photons in the ultraviolet (UV) band [45] (Figure 2.8). Unlike convective heat, which must diffuse through the cladding, this UV radiation penetrates the transparent silica and couples directly to germanium oxygen deficiency centres (GODC) in

2.6. HEAT TRANSFER DYNAMICS IN EXTREME ENVIRONMENTS

the core. This radiation initiates electronic and vibrational excitation within the core volume on femtosecond to picosecond timescales.

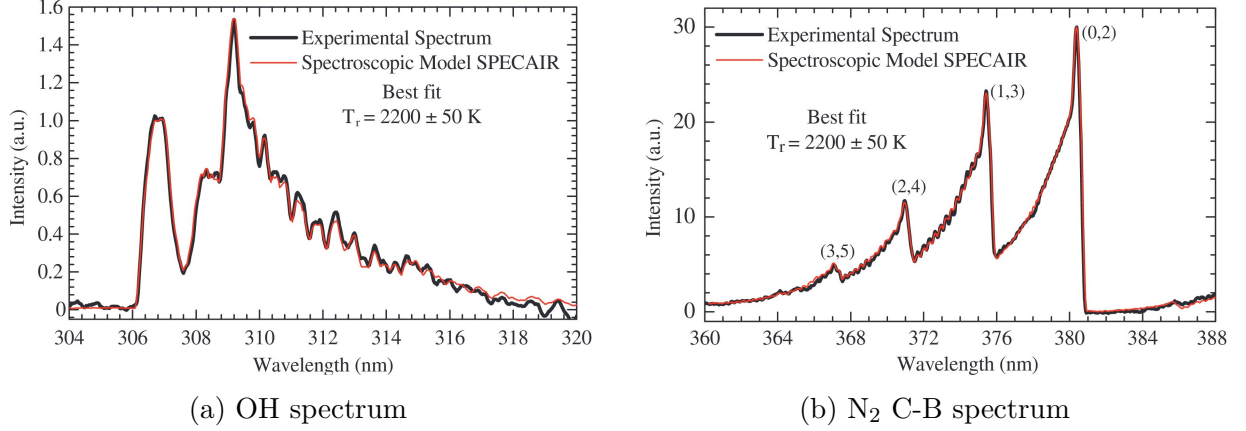


Figure 2.8: Measured (a) OH and (b) N₂ C-B emission spectra from a DC air glow discharge. SPECAIR fitting yields a rotational temperature of 2200 ± 50 K (adapted from Laux *et al.* [45]).

The resulting extreme combined heating rates (potentially $> 10^6$ K/s) generate profound radial thermal gradients: convection heats the cladding from the outside, while UV absorption heats the core from the inside. This loading induces permanent network reconfiguration and frozen-in stresses that modify the intrinsic Rayleigh scattering signature. In OFDR, this manifests as a quantifiable drop in spectral correlation, providing a diagnostic tool to map plasma-induced modification zones.

Hydrogen Flame (Chapter 4): In contrast to the plasma, a hydrogen flame is a combustion source operating near local thermodynamic equilibrium. Energy release is driven by chemical reactions that produce molecular radicals in electronically and vibrationally excited states. Within this environment, structural degradation is gov-

2.6. HEAT TRANSFER DYNAMICS IN EXTREME ENVIRONMENTS

erned by a tripartite mechanism of convective transfer, radiative bulk absorption, and thermo-chemical interaction:

- **Convective Heat Transfer (Surface Heating):** High-velocity, high-temperature combustion gases (predominantly water vapor and unreacted hydrogen) envelop the fibre. This generates massive forced convection at the silica boundary layer, initiating inward radial thermal diffusion. The fluid dynamics of the flame typically result in a more uniform and broadly distributed heating envelope compared to the localized arc discharge.
- **Radiative/Optical Absorption (Bulk Lattice Heating):** The hydrogen flame radiates a continuous thermal spectrum across the visible and infrared (IR) regions, peaking in the IR due to Planck's law and the vibrational emissions of hot water vapor and OH radicals (Figure 2.9) [46]. As established in section 2.4.3, the extreme ambient temperature causes the multi-phonon absorption edge of the silica network to shift toward shorter wavelengths. Consequently, this intense IR radiation strongly couples with the fundamental vibrational modes of the bulk Si-O-Si and Ge-O-Si lattice [44], driving delocalized bulk heating of the entire fibre cross-section.

2.6. HEAT TRANSFER DYNAMICS IN EXTREME ENVIRONMENTS

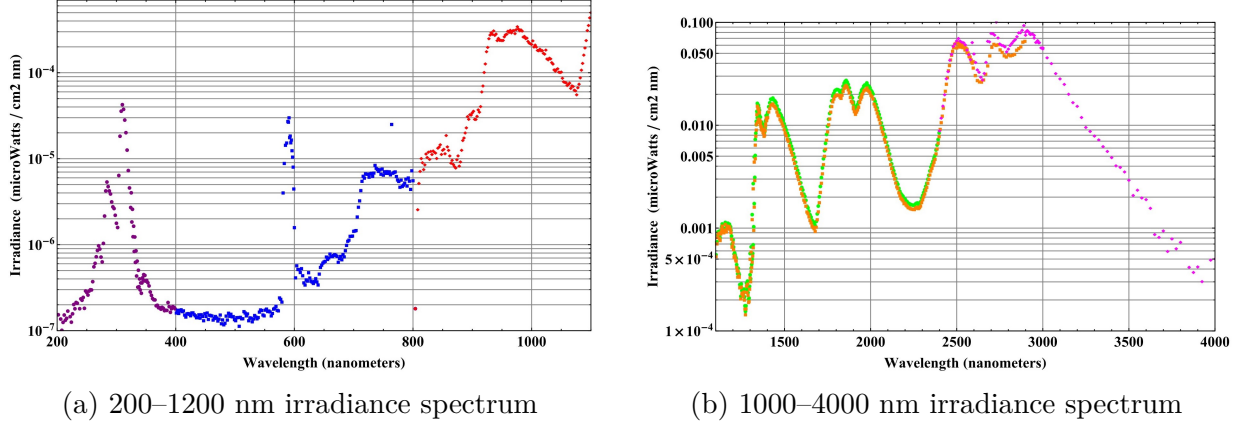


Figure 2.9: Irradiance spectra of a standard hydrogen flame: (a) UV to near-IR and (b) Near- to mid-IR measured using calibrated PMT and PbS/MCT detectors (adapted from Arens *et al.* [46]).

- **Thermo-Chemical Interaction (Hydrogen Diffusion):** Beyond thermal effects, the hydrogen-rich environment introduces a severe chemical dynamic. At elevated temperatures, the expanded silica lattice exhibits significantly increased permeability, allowing molecular hydrogen to diffuse rapidly into the glass matrix [47]. These species chemically react with strained bridging bonds to form terminal hydroxyl (-OH) groups and oxygen-deficient defect centres, which accelerate viscous flow and irreversible network modification [48].

The combined action of convective surface heating, broadband radiative phonon excitation, and chemical bond breaking permanently alters the microscopic density fluctuations of the core. In OFDR measurements, this translates to permanent shifts in the Rayleigh scattering signature and an accelerating drop in spectral correlation as the fibre degrades.

2.6. HEAT TRANSFER DYNAMICS IN EXTREME ENVIRONMENTS

2.6.4 TEMPERATURE DEFINITIONS AND THERMAL TIME SCALES

The fibre's state in extreme environments is governed by a dynamic balance of convective transfer, radiative absorption, and thermal diffusion. To interpret the sensing data, a clear distinction must be made between the intrinsic source temperature, the actual core temperature (T_{core}), and the structural fictive temperature (T_f).

- 1. Intrinsic Source Temperature:** This defines the maximum thermal potential of the external environment. An electrical glow discharge is a non-thermal plasma with kinetic temperatures typically ranging from 5000 K to over 10000 K. In contrast, a hydrogen flame operates near local thermodynamic equilibrium with gas temperatures between 1000–2500 K. The silica fibre never reaches the peak plasma temperature, as such conditions would result in immediate material vaporization.
- 2. Actual Core Temperature (T_{core}):** The OFDR spectral shift measures the instantaneous thermal state of the Ge-doped fibre core (T_{core}). Heat deposited at the cladding surface diffuses radially inward, a process governed by the thermal diffusivity of silica ($\alpha \approx 1.2 \text{ mm}^2/\text{s}$ [49]). This induces a core temperature lag on the order of milliseconds [50, 51]. For the multi-second exposures considered in this work, this delay is negligible, and the fibre reaches a quasi-steady macroscopic thermal equilibrium. Consequently, a 1-second OFDR scan acquired during the thermal plateau satisfies the snapshot criterion, ensuring that T_{core} is temporally stationary during measurement.
- 3. Structural Fictive Temperature (T_f):** While T_{core} governs the reversible thermo-

2.6. HEAT TRANSFER DYNAMICS IN EXTREME ENVIRONMENTS

optic response, the long-term material evolution is dictated by the fictive temperature (T_f), which encodes the frozen-in structural state. T_f is determined by the cooling rate; rapid quenching during drawing locks in high baseline values (1620–1660 °C for cladding, 1150–1300 °C for the core) [33, 34]. When T_{core} approaches or exceeds this range, the network becomes viscous and the factory-frozen memory is erased. Subsequent cooling establishes a new T_f constrained by the rigid cladding. The permanent spectral shifts observed post-exposure directly reflect this structural reconfiguration.

The structural response of vitreous silica to this thermal loading spans multiple temporal regimes. Electronic and phonon excitations occur on femtosecond to picosecond scales as radiation couples to covalent bonds. Localized bond-angle rearrangements and short-range structural relaxation follow on nanosecond to microsecond scales. Finally, on millisecond to second scales, viscous flow and network reconfiguration establish the frozen-in stresses and micro-density variations that modify the Rayleigh scattering signature. These modifications provide a persistent diagnostic of the fibre’s thermal history, even after the source is removed.

2.7. SYNTHESIZED MEASUREMENT STRATEGY FOR EXTREME ENVIRONMENTS

2.7 SYNTHESIZED MEASUREMENT STRATEGY FOR EXTREME ENVIRONMENTS

2.7.1 QUASI-STATIC ASSUMPTION AND MEASUREMENT TIME

The fundamental requirement for accurate spatial mapping in OFDR is the quasi-static assumption: the physical and structural state of the fibre must remain stationary during the laser frequency sweep. For an acquisition time $T_{\text{sweep}} \approx 1$ s, the condition $\Delta T(t) \approx \text{constant}$ must hold to ensure that the interference fringes are recorded sequentially without temporal evolution.

If the fibre is subjected to a highly transient thermal field where the temperature or strain changes significantly during T_{sweep} , the Rayleigh phase and amplitude evolve continuously. This violation of the frozen-state assumption introduces several measurement artifacts:

- **Phase-smearing:** Non-linear phase evolution during the sweep smears the spatial peaks after the Fourier transform and broadens the local Rayleigh spectrum.
- **Loss of Correlation:** The evolving scattering signature reduces the cross-correlation coefficient Q between the reference and measurement traces.
- **Spatial and Spectral Distortion:** The reconstructed thermal distribution becomes

2.7. SYNTHESIZED MEASUREMENT STRATEGY FOR EXTREME ENVIRONMENTS

distorted; peak temperatures are systematically underestimated, yielding a time-averaged profile rather than an instantaneous thermal snapshot.

These artifacts are inherent to all swept-frequency interrogation systems. To extract valid physical data, the measurement strategy must be synchronized with the thermal dynamics of the source and the structural relaxation time of the silica network.

2.7.2 MEASUREMENT STRATEGY AND STRUCTURAL MEMORY

The extreme temperatures of plasmas and flames induce permanent structural rearrangements in the silica network. If a high-temperature measurement were correlated against a pre-heating room-temperature reference, the irreversible micro-density changes would lead to complete decorrelation, preventing temperature extraction.

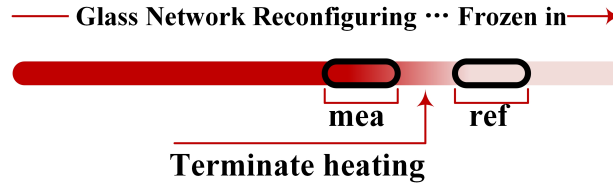


Figure 2.10: OFDR data acquisition timing strategy. To mitigate irreversible structural decorrelation, the measurement trace (mea) is acquired during a quasi-steady thermal state, and the reference trace (ref) is recorded subsequently at room temperature after structural modifications have been permanently frozen.

To resolve the conflict between the quasi-static requirement and the structural memory effect, a specific data acquisition strategy is employed (Figure 2.10). This approach leverages

2.7. SYNTHESIZED MEASUREMENT STRATEGY FOR EXTREME ENVIRONMENTS

the kinetic relationship between cooling rate and fictive temperature (T_f) to intentionally freeze the altered glass structure before acquiring a stable reference.

The experimental sequence is structured as follows:

- **Heating and Reconfiguration:** As the heat source is applied, the glass network drives toward a new equilibrium, effectively erasing its factory-established T_f . Once the macroscopic thermal distribution reaches a quasi-steady plateau, the system satisfies the stationary requirement.
- **Measurement Acquisition (Snapshot Criterion):** The OFDR scan is triggered during this plateau. The sweep must be sufficiently fast to capture the network state before further structural relaxation or thermal drift smears the interference fringes.
- **Termination and Rapid Air Cooling:** Upon removing the heat source, the fibre undergoes rapid convective cooling. This outpaces the network's ability to relax back to a low-temperature equilibrium, effectively freezing-in the high-temperature structural state (T_f). This process is mechanically heterogeneous: the high-viscosity cladding solidifies first, mechanically constraining the lower-viscosity core and locking in severe residual stresses [33, 34].
- **Reference Acquisition:** Because the structure is now frozen in this new state, a room-temperature reference scan securely captures the optical memory of the new thermal history.

2.7. SYNTHESIZED MEASUREMENT STRATEGY FOR EXTREME ENVIRONMENTS

By acquiring the reference trace post-exposure, the methodology decodes the high-temperature measurement against its newly modified structural state. It is critical to recognize that the resulting spectral shift ($\delta\lambda$) is a convolution: it is dominated by the reversible thermo-optic effect but intrinsically contains the irreversible structural mismatch between the hot-mobile state and the cold-constrained state. Consequently, a drop in the correlation coefficient (Q) is inevitable. Rather than being discarded as noise, this correlation loss is utilized as a vital diagnostic metric for the localized severity of network reconfiguration.

2.7.3 SPECTRAL DECORRELATION AS A DIAGNOSTIC METRIC

Irreversible structural changes induced by transient extreme temperatures permanently modify the fibre's stochastic Rayleigh backscattering signature. Consequently, the RBS spectrum recorded after a thermal event no longer correlates perfectly with its initial reference state. This phenomenon, defined as spectral decorrelation, serves as a direct macroscopic indicator of the microscopic rearrangement of the glass network.

The degree of similarity is quantified by the peak value of the ZNCC function, the correlation coefficient Q . A value of $Q = 1$ represents perfect structural fidelity, while $Q \rightarrow 0$ indicates a complete loss of the original scattering fingerprint. The rate and magnitude of this decorrelation are fundamentally dependent on the peak temperature reached and the duration of exposure. This metric provides two critical insights for high-temperature sensing:

2.7. SYNTHESIZED MEASUREMENT STRATEGY FOR EXTREME ENVIRONMENTS

Measurement Fidelity: A low correlation coefficient indicates that the spectral shift $\delta\lambda$ is derived from two fundamentally dissimilar patterns. This establishes a localized diagnostic threshold: once Q drops below a critical value, the calculated temperature or strain becomes statistically unreliable.

Sensor Durability: The progression of decorrelation provides a quantitative measure of material degradation and the operational lifetime of the sensor under sustained thermal load. By monitoring the time required for Q to reach a specific degradation limit (*e.g.*, 0.5), the operational envelope and durability of the fibre can be mapped as a function of temperature.

This diagnostic framework is central to the investigations in Chapter 3 and 4, where spectral decorrelation is used to quantify the localized impact of arc discharges and the long-term degradation kinetics in hydrogen combustion environments.

2.7.4 CONSIDERATIONS FOR LONG-TERM SENSOR VIABILITY

The experimental data in Chapter 3 and 4 focus on highly transient thermal events, monitoring the fibre over timescales from seconds to minutes. This regime is chosen to capture dynamic processes such as arc fusion splicing, plasma interactions, and fibre deformation in hydrogen flames. However, extending standard single-mode fibre sensing to continuous, long-term operation (weeks to months) in ultra-high-temperature environments introduces significant material viability challenges that must be addressed.

2.8. CHAPTER SUMMARY

Prolonged exposure of standard telecom fibre to temperatures exceeding 1000 °C leads to severe degradation. Protective coatings (acrylate and polyimide) decompose above 250 °C and 400 °C, respectively, leaving the bare glass vulnerable to micro-cracking and moisture-induced damage upon cooling. At elevated temperatures, devitrification of the amorphous silica network increases optical loss and embrittlement. Over longer durations (weeks to months), thermal diffusion of Ge dopants into the cladding alters the refractive index profile and numerical aperture, ultimately degrading single-mode guidance.

For practical long-term deployment of OFDR distributed sensing in extreme environments (*e.g.*, industrial furnaces, aerospace engines, or nuclear reactors), significant modifications to the sensing fibre are required. Standard telecom fibre must be replaced with high-temperature waveguides, such as pure-silica-core fibres to mitigate dopant diffusion, or sapphire fibres for operation up to ~ 1600 °C. Mechanical stability must be ensured through metallic coatings (*e.g.*, gold or copper) or ceramic capillary packaging. In addition, pre-annealing is necessary to relax residual stress and stabilize the fictive temperature, thereby minimizing long-term drift in the Rayleigh scattering signature.

2.8 CHAPTER SUMMARY

This chapter has synthesized the optical, thermodynamic, and material-science frameworks necessary to resolve fibre behaviour in extreme environments. The following points represent the fundamental physical pillars of this work:

2.8. CHAPTER SUMMARY

Wavenumber Decomposition and Dual-Domain Diagnostics: By treating backscattered light through the lens of a complex wavenumber ($k = k' + ik''$), we decouple the fibre's response. The real component (phase/delay) enables high-precision tracking of reversible thermodynamic variables like temperature and strain. The imaginary component (amplitude/correlation) provides a direct window into the irreversible structural integrity of the waveguide, allowing us to resolve microscopic multipath dynamics and waveguide breakdown.

Non-Equilibrium Structural Memory (T_f): The structural state of the Ge-doped core is governed by its fictive temperature (T_f), a non-equilibrium parameter that encodes the fibre's thermal history. At extreme temperatures, the glass transitions from a rigid solid to a viscous fluid, erases its factory-frozen memory, and establishes a new structural state. The measurement strategy presented here leverages the kinetics of rapid quenching to intentionally freeze this high-temperature state, allowing the OFDR signature to function as a permanent record of the thermal event.

Radiative Spectral Coupling and Energy Deposition: Heat transfer in extreme environments is a competition between surface-limited convection and volumetric radiative absorption. The air plasma environment induces extreme radial gradients via targeted UV coupling with core defect states (GODCs), whereas the hydrogen flame drives more uniform bulk heating through broadband IR coupling with the fundamental vibrational modes of the silica lattice. This distinction fundamentally dictates the rate and mechanism of material degradation.

2.8. CHAPTER SUMMARY

Correlation Loss as a Material Diagnostic: In extreme sensing, the drop in spectral correlation (Q) is not merely a signal processing artifact but a measure of structural transform within the glass network. By monitoring the progression of decorrelation, we transition from simple temperature mapping to an active assessment of sensor durability and the onset of irreversible structural reconfiguration.

These principles provide the analytical framework required for the experimental investigations in the subsequent chapters, bridging the gap between raw interferometric data and the complex physical reality of fibre sensors in plasmas and flames.

Chapter 3

DISTRIBUTED HIGH-TEMPERATURE MONITORING IN AN ELECTRICAL ARC PLASMA ENVIRONMENT

Building upon the theoretical principles of Optical Frequency Domain Reflectometry (OFDR) established in Chapter 2, this chapter presents the first experimental application of the technique within one of the most demanding thermal environments: the electrical arc plasma. Accurate, high-resolution temperature mapping in such conditions is a critical yet persistent challenge for applications ranging from specialty fibre splicing to plasma diagnostics and materials science.

This study confronts this challenge by demonstrating the feasibility of employing standard telecommunications fibre as a distributed sensor for ultra-high temperature measurements. The central aim is to push the fibre beyond its conventional operating limits, investigating its complete thermo-mechanical response up to and through its glass transition region

3.1. INTRODUCTION: MOTIVATION FOR EXTREME TEMPERATURE SENSING

(1100 to 1200 °C). Consequently, the analysis extends beyond simple temperature profiling to provide a comprehensive characterization of the thermally-induced stress, optical path length variations, and dynamic material phase transitions that occur within the intense, localized heating zone.

3.1 INTRODUCTION: MOTIVATION FOR EXTREME TEMPERATURE SENSING

High-temperature measurement in electric arc discharges has been investigated by several groups. Hatakeyama *et al.* employed a microradiation thermometer to measure fibre core temperatures up to 2000 °C during single-mode fibre fusion splicing [2]. Mohanna estimated the arc temperature at about 1450 °C, assuming a Gaussian distribution, by relating internal stress relaxation to fibre temperature and annealing time [52]. Rego *et al.* used blackbody radiation to determine 1450 °C for fibre exposed to an electric arc [53], and later measured 1400 ± 50 °C using an electrically insulated Type S thermocouple [3].

Despite these efforts, accurately estimating extreme temperatures within the sub-millimetre heating zone of a splicing arc remains challenging. Fibre temperature during an electric arc discharge critically influences splicing loss, grating fabrication, and splicer optimization. This makes OFDR a strong candidate for distributed high-temperature sensing in such open thermal equilibrium systems. Leveraging OFDR's high spatial and temporal resolution would enable a detailed mapping of thermal radiation induced by the

3.1. INTRODUCTION: MOTIVATION FOR EXTREME TEMPERATURE SENSING

arc.

Optical frequency-domain reflectometry (OFDR) offers the high sensitivity and spatial resolution required to capture localized Rayleigh scatter patterns. The principle relies on the static, random amplitude and phase distribution of local Rayleigh spectra along the fibre, which can be modelled as a long, weak fibre Bragg grating (FBG) with a random period [1]. It is widely applied in distributed temperature and strain measurements [54, 55].

In this work, I demonstrate the feasibility of using OFDR within a fusion splicer to perform distributed ultrahigh-temperature measurements on standard telecom fibre. The system achieves an 8 μm spatial resolution over a fibre segment shorter than 10 mm by monitoring the thermally induced wavelength shift in the Rayleigh backscattering (RBS) spectrum [1]. The method enables temperature detection up to approximately 1300 °C generated by an electrical arc. The conversion from spectral wavelength shift to absolute temperature relies on a non-linear high-temperature calibration model, which is thoroughly established and discussed in Chapter 4.

3.1.1 OFDR SYSTEM FOR DISTRIBUTED SENSING

The experimental setup, designed for distributed high-temperature measurement in a commercial fusion splicer (Ericsson, Sweden), is shown in Figure 3.1. The system is based on a Mach-Zehnder interferometer architecture. A tunable laser source (TLS, New-Focus Venturi-TLB-6600) injects light that is split into two parallel branches: an auxiliary interferometer (AUI) and a main interferometer (MI). The AUI output serves as an external

3.1. INTRODUCTION: MOTIVATION FOR EXTREME TEMPERATURE SENSING

clock for data acquisition, a widely adopted frequency-sampling method used to compensate for the nonlinearity of the laser tuning [20].

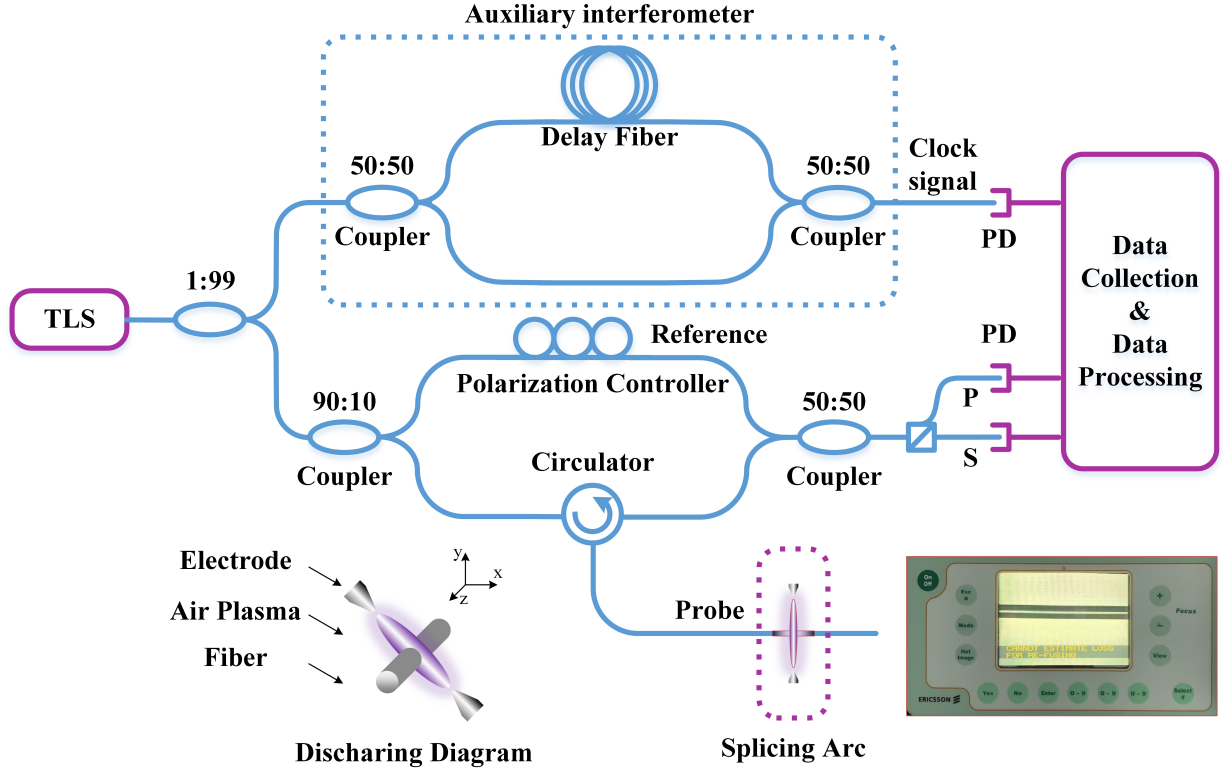


Figure 3.1: Schematic of the optical frequency-domain reflectometry (OFDR) system for electrical arc discharge measurement applied with different currents.

The MI is used for the interrogation of the fibre under test (FUT) by splitting the injected light into reference and probe arms. In the probe arm, light is guided by a circulator into the FUT and returns as backward Rayleigh scattering, which is delayed and encoded with information about the dynamic discharge action. These reflected probe fields then recombine with the reference fields to create detectable beat patterns. To mitigate signal

3.1. INTRODUCTION: MOTIVATION FOR EXTREME TEMPERATURE SENSING

fading from polarization misalignment, a polarization diversity technique is employed: a polarization beam splitter (PBS) and a polarization controller (PC) balance the reference light evenly across two orthogonal polarization states (S and P) [18]. The final interference signal is recorded by photodetectors and digitized for post-processing.

Essentially, for a single wavelength, the time delay of light propagating in a single-mode fibre (SMF) is a function of its physical length and local refractive index. As the laser wavelength is tuned, the light in the reference arm arrives at the photodetector before the delayed, scattered light from the probe arm. This generates a beat frequency corresponding to the time delay. By performing a Fourier transform on this interference pattern, a spatially resolved profile of the Rayleigh scattering is obtained.

The spatial resolution of this measurement is determined by the spectral bandwidth of the laser scan according to the equation:

$$\Delta z = \frac{\lambda_i \lambda_f}{2n_g \Delta \lambda}, \quad (3.1)$$

where λ_i and λ_f are the start and end wavelengths of the scan, $\Delta \lambda$ is the total scanned wavelength range, and n_g is the group index of the fibre (assumed to be 1.47). In this setup, the laser scanned from 1520 nm to 1620 nm, corresponding to a spatial resolution (Δz) of approximately 8 μm . A delay shift in the measured Rayleigh pattern occurs under the arc condition due to the increased OPL relative to the fibre at room temperature.

However, the high temperature and steep thermal gradients from the arc induce non-uniform changes in both the refractive index and physical length of the fibre, invalidating

3.1. INTRODUCTION: MOTIVATION FOR EXTREME TEMPERATURE SENSING

the simple approximation of a constant OPL. The propagation of the electric field in such an inhomogeneous and time-varied medium can be described by the 1D Maxwell equation:

$$\frac{\partial^2 E_z}{\partial z^2} + \frac{\partial}{\partial z} \left(\frac{1}{\epsilon_r} E_z \frac{\partial \epsilon_r}{\partial z} \right) = \frac{1}{c^2} \left(\epsilon_r \frac{\partial^2 E_z}{\partial t^2} + 2 \frac{\partial \epsilon_r}{\partial t} \frac{\partial E_z}{\partial t} + E_z \frac{\partial^2 \epsilon_r}{\partial t^2} \right). \quad (3.2)$$

By assuming the system is in an open thermal equilibrium state over a small range, we can consider the waveguide to be static ($\partial \epsilon / \partial t = 0$) and simplify the refractive index as a function of position. The thermally induced optical path length, $\tilde{\Gamma}(l)$ affecting the Rayleigh scattering response at any location l can then be modelled as:

$$\tilde{\Gamma}(l) = \int_0^l \sqrt{\epsilon_r(z)} dz, \quad (3.3)$$

where $\epsilon_r(z)$ is spatial varying relative permittivity. This model allows for the quantification of localized temperature changes by measuring the resulting delay shift in the RBS.

3.1.2 EXPERIMENTAL PROCEDURE AND DATA PROCESSING

EXPERIMENTAL PROCEDURE

The measurement process followed a controlled sequence: fibre preparation and installation, arc discharge, and finally data acquisition and processing.

A stripped segment of standard single-mode fibre was held in embossed grooves, aligned and clamped on the stage of the fusion splicer. Care was taken to ensure the fibre remained

3.1. INTRODUCTION: MOTIVATION FOR EXTREME TEMPERATURE SENSING

immobile during the discharges, with no external pulling or pushing forces applied. The splicer's electrode gap and fusion time were fixed, while the arc current was varied between 4.0 and 6.0 mA to control the glow heating temperature. A total discharge duration of 7 seconds was set for each measurement to ensure the heating area reached an open thermal equilibrium state. The Rayleigh backscattering signature was first measured just before the end of the 7-second discharge. Subsequently, a reference RBS signature was recorded at room temperature after the fibre had cooled.

An external stimulus, such as temperature, changes the fibre's local OPL, producing both a spectral shift in the RBS spectrum and a corresponding change in the backscattering delay response. Correlating heated and reference measurements in spectral and temporal domains enables precise determination of these shifts and quantification of the stimulus.

DATA ACQUISITION

The tunable laser source was set to a tuning rate of 100 nm/s. The auxiliary interferometer, equipped with a 24.3 m delay fibre, generated a stable sampling clock at a rate of 1.55 MHz for the Data Acquisition (DAQ) system. For each measurement, the interference signals for both the S and P polarization states were detected by a photodetector (Thorlabs-PDB130C). The acquisition for a single trace took 1.03 seconds and was digitized by a DAQ card (NI-PCI6115), generating 1.6 mega-samples of raw data.

DATA PROCESSING METHODOLOGY

The 1.6 mega-samples of raw interference data acquired during each measurement were pro-

3.1. INTRODUCTION: MOTIVATION FOR EXTREME TEMPERATURE SENSING

cessed using customized digital signal processing algorithms. Building upon the theoretical framework established in Chapter 2, the raw data was transformed to extract three distinct physical metrics: spectral correlation (for temperature and stress), temporal correlation (for OPL changes), and spatial-temporal mapping (for dynamic process monitoring).

Spectral Correlation: To obtain the distributed temperature profiles, the raw optical frequency-domain data was first transformed into the spatial (time-delay) domain via a Fast Fourier Transform (FFT). A moving window was then applied to isolate the local Rayleigh backscattering (RBS) signal at specific positions along the fibre.

For this analysis, a moving window containing 31 points was used to retrieve the RBS from the heated region, which corresponded to a localized spatial resolution of approximately 250 μm . The analysed heating area spanned up to 6 mm, extracted from a total of 10 mm of converted fibre length. Fundamentally, the length of the window used before applying the Inverse Fast Fourier Transform (IFFT) to retrieve the local Rayleigh spectra determines the weight of the contribution from the local Rayleigh pattern. When a window of small length was applied, the limited local data points led to insufficient spectral resolution, making it difficult to confidently determine the wavelength shift. Conversely, applying a larger window would excessively average out the steep temperature gradients, blurring the detailed information derived from the localized spectrum. Once the optimized local spectra were retrieved via IFFT, a Zero-Mean Normalized Cross-Correlation (ZNCC) operation was performed against the room-temperature reference spectra to calculate the local wavelength shift.

3.1. INTRODUCTION: MOTIVATION FOR EXTREME TEMPERATURE SENSING

Temporal Correlation: To analyze the absolute OPL changes and permanent elongation, the data was processed purely in the time-delay domain. While the 100 nm laser sweep bandwidth provides an intrinsic sub-picosecond temporal resolution, this was further enhanced digitally. Prior to executing the primary FFT on the raw data, a heavy zero-padding algorithm was applied. This mathematically interpolated the points in the resulting time-domain Rayleigh pattern. By applying the ZNCC algorithm directly to these zero-padded time-domain traces (comparing the heated state to the reference state), the system could precisely track the accumulated delay shift with an enhanced digital resolution better than ± 20 fs.

Spatial-Temporal Mapping: To resolve the highly transient physical transitions occurring during the plasma arc discharge, a spatial-temporal mapping method was employed. Instead of processing the entire 1.6 mega-sample array as a single static measurement, a sliding window was applied directly across the raw interference fringes in the optical frequency domain. By performing a localized FFT on each successive window, the spatial/delay response of both the RBS and the fibre end-face Fresnel reflection was mapped as a function of time throughout the duration of the laser scan. Unlike the correlation methods, which require a reference trace, this mapping directly tracks the instantaneous absolute position and intensity of the scatterers, allowing for the continuous monitoring of the waveguide's dynamic physical state.

3.2. RESULTS AND DISCUSSION

3.2 RESULTS AND DISCUSSION

This section presents and analyzes a set of experimental results obtained from the OFDR system. The analysis is structured to cover five key aspects: the primary temperature measurements, the mechanical stress induced by the thermal gradients, the impact on the optical path length, the reliability of the measurements, and finally, the dynamic monitoring of the process itself.

3.2.1 DISTRIBUTED TEMPERATURE PROFILE MEASUREMENT

The main result of this investigation is the measurement of the distributed temperature profile along the optical fibre during an electric arc discharge. The distributed temperature profile was estimated by converting the measured spectral shifts into absolute temperature using the comprehensive high-temperature calibration model developed in Chapter 4. Unlike standard room-temperature approximations that assume a linear sensitivity of 10 pm/°C [56], this calibration accounts for the highly non-linear thermo-optic and thermal expansion behaviours of silica at extreme temperatures. With the reference trace recorded at a room temperature of 22.0 °C, the estimated distributed high-temperature profiles are presented in Figure 3.2 as a function of fibre position for five different arc currents.

The profiles exhibit a Gaussian-like distribution, with the peak temperature and the width of the heated zone increasing as the applied current rises. This demonstrates that the high-density energy release from the discharge creates a steep temperature gradient

3.2. RESULTS AND DISCUSSION

on a sub-millimetre scale. The heat transferred to the fibre is primarily dissipated to the surroundings via radiation and thermal conduction along the fibre's length.

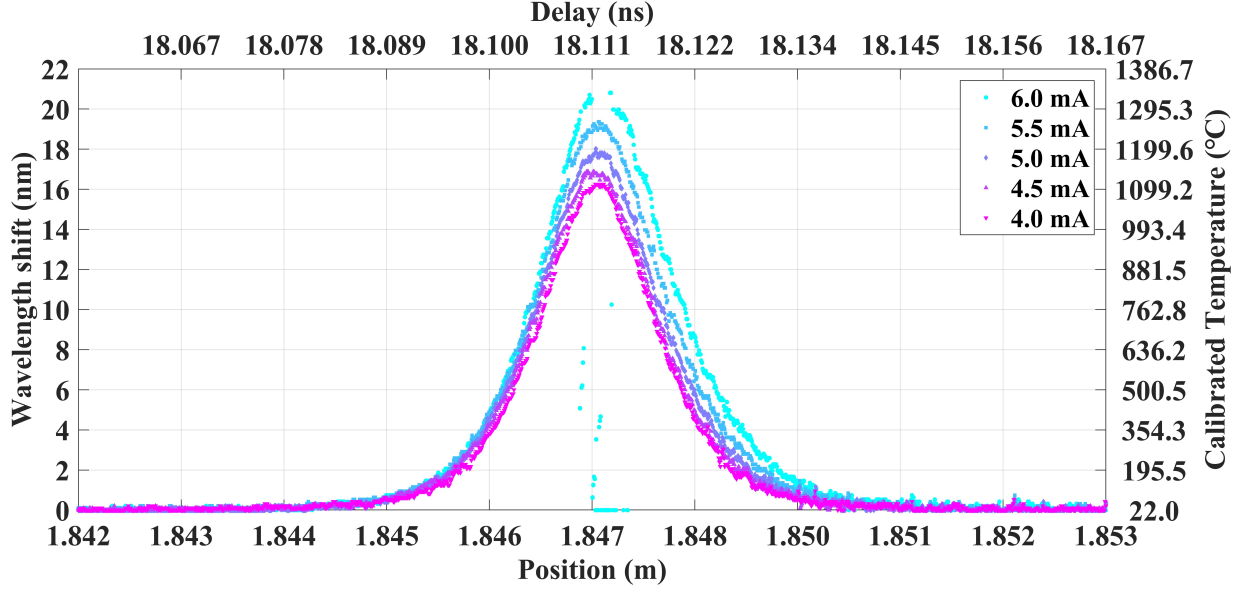


Figure 3.2: Distributed spatial profiles of the measured wavelength shift (left axis) and the corresponding calibrated absolute temperature (right axis) for five applied arc currents. The temperature conversion utilizes the non-linear high-temperature calibration model. At 6.0 mA, the peak temperature approaches 1300 °C, and the appearance of stray data points (data drop-out) indicates a loss of local spectral correlation as the silica network approaches its theoretical fictive temperature (T_f). Reported temperatures carry an estimated uncertainty of ± 20 °C, the 250 μm spatial averaging window, and the calibration mapping.

Based on this accurate non-linear conversion, the maximum detected wavelength shifts (approximately 20 nm to 22 nm) correspond to an estimated peak temperature approaching 1300 °C to 1350 °C. Crucially, this upper measurable limit coincides closely with the fictive temperature (T_f) of the silica fibre, as discussed in Chapter 2. Because the fibre experiences rapid quenching during manufacturing (and subsequent cooling from the arc), its silica

3.2. RESULTS AND DISCUSSION

network is frozen at this high T_f . When the applied arc heating approaches or exceeds this fictive temperature, the glass transitions into a highly mobile, supercooled liquid state. This rapid structural reconfiguration intrinsically decorrelates the Rayleigh backscattering pattern from its room-temperature reference, establishing a fundamental physical limit for continuous cross-correlation temperature tracking.

3.2.2 ANALYSIS OF MEASUREMENT LIMITATIONS AT HIGH TEMPERATURES

At the highest current of 6.0 mA, significant data scatter is observed in the temperature profile (Figure 3.2). These stray points result from a degradation of the ZNCC calculation. As the fibre approaches such high temperatures, processes like material viscosity changes begin to alter the fibre's physical micro-structure. This decorrelates the RBS spectrum from its room-temperature reference, reducing the reliability of the ZNCC method. In essence, the stray dots indicate that a significant structural change, such as the glass transition has occurred.

The relationship between the maximum measured wavelength shift, the calibrated absolute temperature (derived from the Chapter 4 model), and the applied arc discharge current is shown in Figure 3.3. Each black dot represents an individual maximum measurement extracted from the tests, while the red dot with error bars indicates the mean value and standard error for a given current. As the applied current increases from 4.0 mA to 6.0 mA, the peak calibrated temperature rises continuously from approximately 1100 to over 1300 °C.

3.2. RESULTS AND DISCUSSION

Importantly, to ensure data reliability, only values derived from a correlation coefficient greater than 50% were included in this plot. The increasing data scatter and drop-out observed as temperatures approach and exceed 1300 °C further substantiate this threshold as the physical measurement limit. This boundary directly corresponds to the high fictive temperature (T_f) frozen into the fibre by fast cooling rates during test; surpassing it triggers rapid structural relaxation that destroys the local Rayleigh spectral signature.

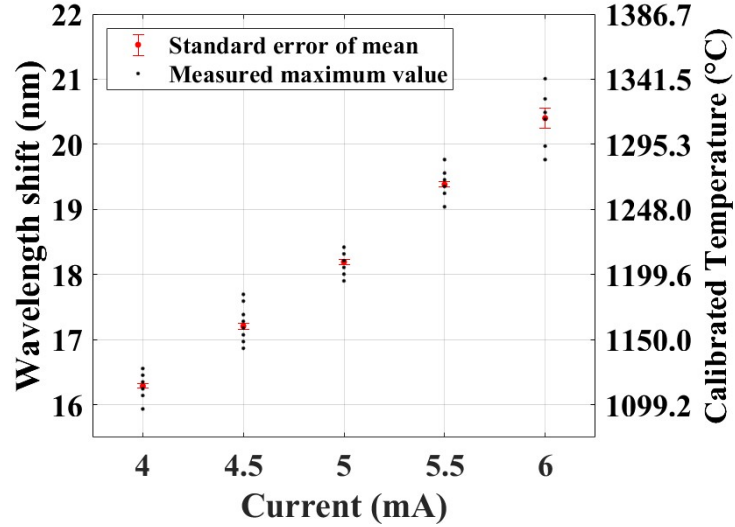


Figure 3.3: Relationship between the maximum measured wavelength shift, the calibrated peak temperature, and the applied discharge current. Black dots represent individual maximum measurements extracted from the tests, while red markers indicate the mean and standard error of the mean. The trend demonstrates a physical measurement threshold near 1300 °C, beyond which structural relaxation triggers severe signal decorrelation. The absolute temperature estimations carry an uncertainty of ± 20 °C arising from the 50% correlation threshold, spatial averaging limitations and calibration tolerances.

The system detected maximum spectral shifts exceeding 20 nm, equivalent to an estimated peak temperature approaching 1300 °C within a near-equilibrium thermal system.

3.2. RESULTS AND DISCUSSION

To our knowledge, this is the first demonstration of OFDR’s capability to continuously track spatially-resolved temperature profiles in standard single-mode fibre right up to the material’s glass transition and fictive temperature boundaries.

3.2.3 STATISTICAL ANALYSIS OF THERMALLY INDUCED INTERNAL STRESS

Beyond a simple temperature measurement, the extreme thermal gradients imposed by the arc are expected to induce significant mechanical stress and permanent structural changes in the fibre. The physical origin of this stress is complex, arising from the high-density energy transfer from the arc to the vitreous silica. This process likely involves localized changes in the refractive index due to optical absorption from the ultraviolet (UV) band [35], thermal diffusion of dopants, and, most critically, viscosity changes as the material approaches its glass transition phase [57, 58].

When the fibre is heated above its glass transition temperature (T_g) and then undergoes rapid natural cooling, the non-uniform energy distribution and dissipation result in irreversible structural deformation. Because the structural relaxation time of the glass drops exponentially above T_g , the silica flows and reconfigures, subsequently freezing this deformation into the network upon quenching. This frozen-in deformation manifests as a permanent internal stress. The following analysis presents a statistical method to quantify this effect.

3.2. RESULTS AND DISCUSSION

METHODOLOGY FOR STATISTICAL STRESS ANALYSIS

To investigate the cumulative internal stress arising from repeated heating and cooling cycles, a series of multiple discharge tests were performed at different currents. The measurement sequence involved capturing the RBS spectrum traces both during the discharge ($\mathbf{M}_i$) and after the fibre had cooled to room temperature ($\mathbf{R}_i$).

The analysis focused on quantifying the permanent change between cycles by comparing the reference spectra from consecutive traces (*e.g.*, $\mathbf{R}_{i+1}$ vs. $\mathbf{R}_i$). This was accomplished by performing a ZNCC operation to determine the residual spectral shift between two cooled-down states. This spectral shift is a direct measure of the residual strain, which corresponds to the internal stress formed during the preceding heating/cooling cycle. The measured shifts were then converted to an equivalent strain using a sensitivity coefficient of 1 pm/ $\mu\epsilon$, a standard value for single-mode fibre at 1550 nm [56, 59].

The results of the statistical analysis are shown in Figure 3.4. Figure 3.4 (a) plots the standard deviation of the residual spectral shifts as a function of the applied arc current. This standard deviation represents the variability, and thus the magnitude, of the internal stress locked into the fibre during each cycle. The data reveal a clear behavioural dichotomy. At lower currents (4.0 mA to 5.0 mA), the induced stress remains negligible, indicating that the fibre acts largely elastically. However, as the current increases from 5.0 mA to 5.5 mA, the standard deviation rises dramatically by over two orders of magnitude. Referring back to the calibrated temperature profiles (Figure 3.3), the 5.0 mA to 5.5 mA range corresponds precisely to peak temperatures crossing the 1100 °C to 1200 °C threshold.

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This provides a direct, macroscopic signature that the silica network has reached its glass transition temperature (T_g). Crossing this transition point allows the glass structure to locally reconfigure, trapping massive, irreversible residual stress upon cooling.

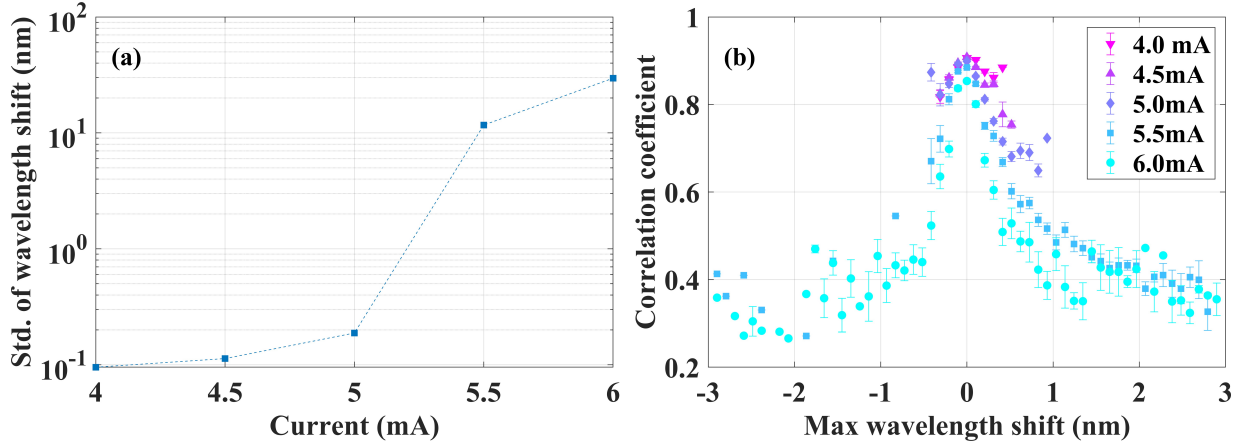


Figure 3.4: (a) Standard deviation of the residual spectral shift in the fused region as a function of applied arc current, showing a massive increase as the fibre crosses the glass transition threshold between 5.0 and 5.5 mA. (b) Distribution of correlation coefficients, showing the severe degradation of spectral similarity and increased residual stress at currents exceeding 5.0 mA.

Figure 3.4 (b) provides further evidence by plotting the correlation coefficient between the RBS spectra before and after a discharge cycle against the maximum wavelength shift. The correlation coefficient serves as a reliable metric for structural similarity. As shown, the correlation remains tightly clustered and highly reliable for currents up to 5.0 mA. However, as the thermal load crosses the T_g threshold into the 5.5 mA and 6.0 mA regimes, the distribution scatters widely across a ± 3 nm shift range, and the correlation drops sharply. This loss of spectral correlation definitively verifies that the fused silica has crossed its glass transition boundary, permanently altering the frozen-in density fluctuations that generate

3.2. RESULTS AND DISCUSSION

the local Rayleigh scattering signature.

In summary, a correlation-based method is proposed and demonstrated to statistically analyze the internal stress induced by repeated arc discharges. By comparing the fibre's state after successive heating and cooling cycles, we observed that negligible permanent deformation occurred at currents up to 5.0 mA. Conversely, massive irreversible changes were triggered as the current exceeded 5.0 mA (crossing the 1100 °C to 1200 °C boundary). This method successfully isolates and quantifies the exact onset of the material's glass phase transition purely through OFDR.

These permanent changes also manifest as an irreversible expansion of the fibre's physical length, which can be measured as a delay shift in the time domain. This effect is discussed next.

3.2.4 THERMAL IMPACT ON OPL AND TIME DELAY

The intense heating from the arc discharge alters not only the fibre's temperature but also its physical properties, mainly the refractive index and physical length. These two factors collectively determine the OPL, which can be directly measured in the time domain as a delay of the Rayleigh scattering response [60]. This subsection investigates the thermal impact on OPL by analyzing the time-delay shift of the Rayleigh scattering pattern.

This delay shift arises from the combined effects of thermal-induced refractive index change and material expansion across the heated region. For this analysis, the influence of chromatic dispersion arising from the extreme temperature gradient is considered negligible.

3.2. RESULTS AND DISCUSSION

The goal is to measure the accumulated delay shift by correlating the time-domain Rayleigh pattern of the heated fibre against a non-heated reference measurement, a technique established in distributed sensing [59, 61].

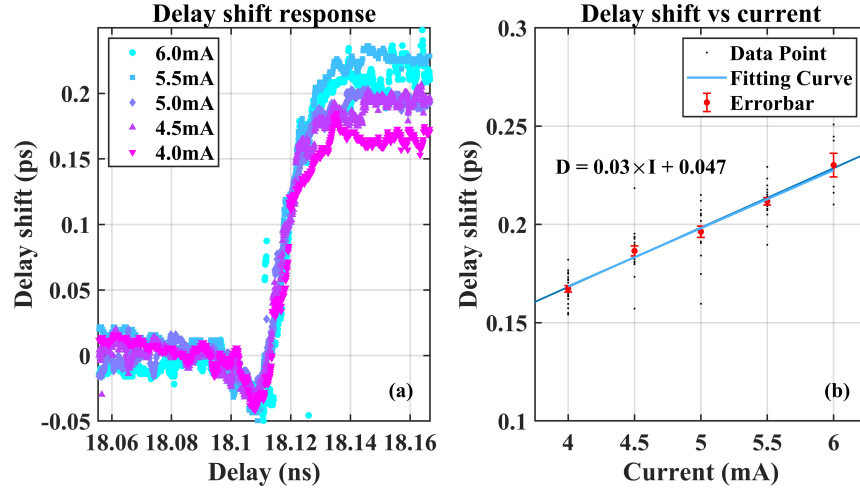


Figure 3.5: (a) Profile of the delay shift across the heated fibre region for different currents, showing competing physical effects. (b) The net OPL change (delay shift) for a single discharge as a function of applied current.

By converting the OFDR signal into the time domain, we can analyze the thermally induced time delay shift. While the spectral bandwidth of 100 nm provides an intrinsic delay accuracy in the sub-picosecond range, this resolution was enhanced by applying a zero-padding algorithm to the raw data prior to the Fourier transformation. A ZNCC operation was then performed on the Rayleigh patterns in the time domain to precisely resolve the delay shift with a resolution better than ± 20 fs. The detailed results of this high-resolution analysis are shown in Figure 3.5.

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DYNAMIC OPL CHANGES DURING A SINGLE DISCHARGE

The variation of the OPL during a single, high-power discharge reveals a complex interplay of physical effects, as shown in Figure 3.5 (a). The delay shift profile exhibits two competing contributions:

- i. **A negative shift:** A reduction in OPL is observed in the hottest central region (~ 18.11 ns).

This is attributed to a glass transition-induced reduction of the refractive index in the silica.

- ii. **A positive shift:** A dominant positive shift in OPL surrounds the central dip. This is caused by the combined effects of thermal expansion and the positive thermo-optic coefficient of the material.

Interestingly, the location of the maximum temperature (from Figure 3.2) does not perfectly align with the minimum delay shift. This suggests that the electronic transitions affecting the refractive index may occur more rapidly or at a slightly different location than the bulk thermal expansion of the material. When averaged across the entire heating range, the net delay shift for a single discharge appears to be directly proportional to the applied current, as seen in Figure 3.5 (b).

OBSERVATION OF MULTIPLE DELAY MODES

Under conditions of this case, a weaker discharge (4.5 mA) applied to a fibre intentionally positioned offset from the heating centre, a multiple delay shift phenomenon was observed

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(Figure 3.6 (a)). A distinct delay shift around 0.1 ps was obtained between the first and second lines. This temporal separation corresponds to a physical path length difference of approximately 10 μm . Fundamentally, this phenomenon arises from the extreme, localized heating, which temporarily disrupts the fibre's waveguide structure. The corresponding calibrated temperature profile (Figure 3.6 (b)) reveals a peak temperature of approximately 960 $^{\circ}\text{C}$. While the majority of the profile maintains a high correlation coefficient, the colour bar overlay reveals specific locations near the heating centre where the correlation coefficient drops below 50% (indicated by cyan markers). Although 960 $^{\circ}\text{C}$ is well below the softening point of the pure silica cladding, it approaches the glass transition temperature of the Ge-doped core, where the glass network begins to undergo structural relaxation. This process modifies the frozen-in density fluctuations responsible for Rayleigh scattering, manifesting as a localized drop in the correlation coefficient.

Consequently, the multiple delay shifts are likely caused by a transient breakdown of the waveguide condition driven by this core-specific phase transition and the resulting thermo-mechanical stress. As the Ge-doped core structurally relaxes while the surrounding silica cladding remains rigid, massive internal stress is generated due to the mismatch in their thermal expansion properties. This stress, combined with the core's transition, severely disrupts the local refractive index profile, violating the strict requirements for single-mode light confinement. This failure allows light to couple into higher-order cladding modes or reflect within micro-cavities formed by the transient stress fields. Each optical path results in a distinct arrival time, which is resolved as a discrete, separated delay shift in the high-resolution time-domain OFDR measurement [36].

3.2. RESULTS AND DISCUSSION

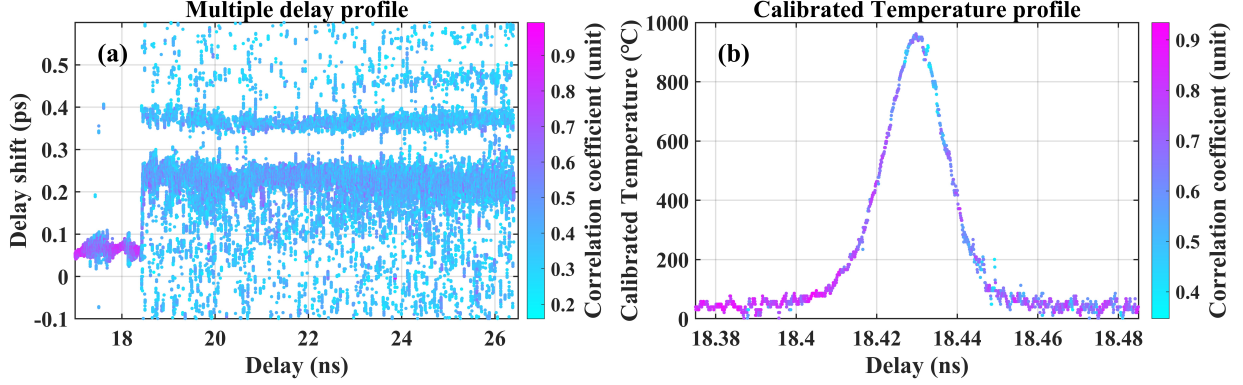


Figure 3.6: Analysis of transient waveguide breakdown under moderate, offset heating (4.5 mA). (a) The high-resolution time-domain response reveals multiple, discrete delay shifts corresponding to split optical paths. (b) The calibrated temperature profile confirms a peak localized temperature of approximately 960 °C (estimated uncertainty ± 20 °C). The colour bar reveals localized drops in the correlation coefficient (cyan markers) near the heating centre, indicating that the Germanium-doped core has approached its specific glass transition temperature (T_g), triggering structural relaxation and localized waveguide failure.

This finding demonstrates the ability of time-domain analysis to probe not just thermal changes, but the fundamental breakdown and modification of the fibre waveguide condition at a micro-scale.

CUMULATIVE OPL CHANGES FROM REPEATED DISCHARGES

Finally, by fixing the initial reference trace and tracking the residual delay shift over repeated discharge cycles, we can quantify the cumulative and permanent changes to the fibre's OPL. As shown in Figure 3.7, at lower currents (4.0 mA, 4.5 mA, and 5.0 mA), the fibre behaves purely elastically. Across 20 consecutive measurement cycles, the delay shift fluctuates closely around zero. This indicates that the thermal expansion and refractive

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index changes induced during the arc are completely reversible upon cooling, leaving no permanent elongation or structural modification.

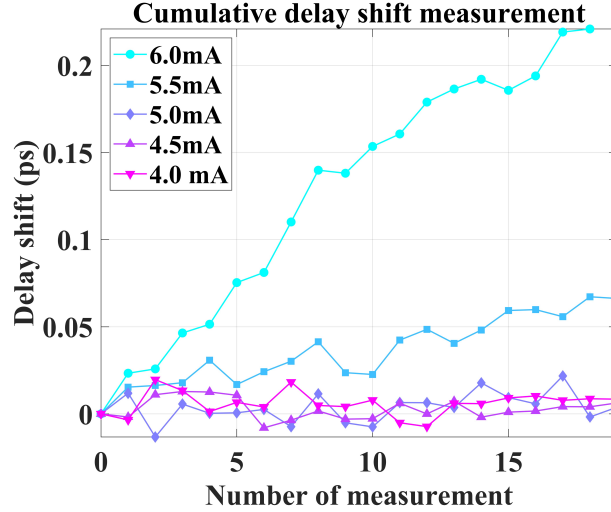


Figure 3.7: Cumulative delay shift as a function of the number of discharges. As the applied current crosses the glass transition threshold (> 5.0 mA), the OPL permanently increases with each cycle, indicating irreversible elongation and structural relaxation.

However, a distinct shift occurs for currents of 5.5 mA and 6.0 mA. At these levels, a clear cumulative increase in the delay shift is observed with each successive discharge. As established by the calibrated temperature profile (Figure 3.3), the transition from 5.0 mA to 5.5 mA corresponds to the silica network crossing its glass transition temperature (T_g) in the 1100 °C to 1200 °C regime. Entering this phase allows the glass structure to relax. Consequently, a permanent elongation and deformation of the fibre occurs, likely driven by the fibre's own weight and minute pulling forces from the discharge convection currents while the material is in this structurally mobile state.

Crucially, this temporal measurement directly corroborates the spectral stress analysis

3.2. RESULTS AND DISCUSSION

from the previous section (Figure 3.4). Both the frequency-domain (residual wavelength shift) and time-domain (cumulative delay shift) measurements independently identify the exact same 5.0 mA to 5.5 mA boundary as the physical threshold for irreversible structural phase transition.

In summary, the temporal correlation analysis provides critical insights into the macroscopic physical state of the fibre. It reveals the dynamic competition between physical effects during a discharge, allows for the detection of subtle localized waveguide breakdown (such as coexisting optical modes), and definitively quantifies the onset of permanent material elongation triggered by repeated thermal stress crossing the glass transition boundary.

3.2.5 ANALYSIS OF MEASUREMENT RELIABILITY VIA CORRELATION COEFFICIENT

One critical aspect of any sensing technique is understanding its reliability, particularly when operating at the limits of the sensor’s material properties. For OFDR, the correlation coefficient between the reference and measurement spectra provides a direct quantitative measure of data quality. This subsection analyzes the correlation coefficient Q to assess the reliability of the temperature and OPL measurements.

The correlation coefficient Q indicates the similarity between two spectra in comparison, and the reliability of the spectral shift ($\delta\nu$) or delay shift ($\delta\tau$) values obtained by the ZNCC method. A coefficient approaching 1.0 represents high similarity, whereas lower values indicate deformation of the spectral signature due to severe thermal gradients or irreversible

3.2. RESULTS AND DISCUSSION

phase transitions in the fused silica.

To quantify the operational limit, a reliability threshold must be defined. Although thresholds such as 50% or 60% may appear empirical, they are fundamentally grounded in the statistical nature of cross-correlation. The Rayleigh backscattering spectrum is a complex pseudo-random pattern, and ZNCC produces a noise floor of spurious peaks from random spectral alignments. Measurement failure occurs when the true correlation peak (representing the physical shift) degrades to the level of the strongest spurious peak. Below this threshold, the algorithm locks onto false peaks, yielding erroneous shift values that appear as random outliers in the data.

Statistical analysis of OFDR Rayleigh spectra indicates that as the correlation coefficient drops below approximately 0.5, the probability of false peak detection increases sharply, rendering the measurement statistically unreliable. Therefore, in this analysis, a threshold of $Q \geq 50\%$ is adopted as a conservative boundary to separate physically meaningful measurements from noise-dominated errors. To illustrate the sensitivity of the system to this boundary, calculations based on a stricter 60% threshold are also presented. The percentages of the coefficient above 50% or 60% generally gave the reliability of the temperature and OPL measurement at different currents, as shown in Figure 3.8 (c, d) for temperature measurement and Figure 3.9 (b) for delay shift measurement.

RELIABILITY OF SPECTRAL CORRELATION MEASUREMENTS (TEMPERATURE)

First, I analyze the reliability of the temperature measurements derived from spectral shifts.

3.2. RESULTS AND DISCUSSION

The correlation coefficients from the multiple tests were grouped by applied current and plotted as a function of the measured spectral shift, as shown in Figure 3.8 (a, b).

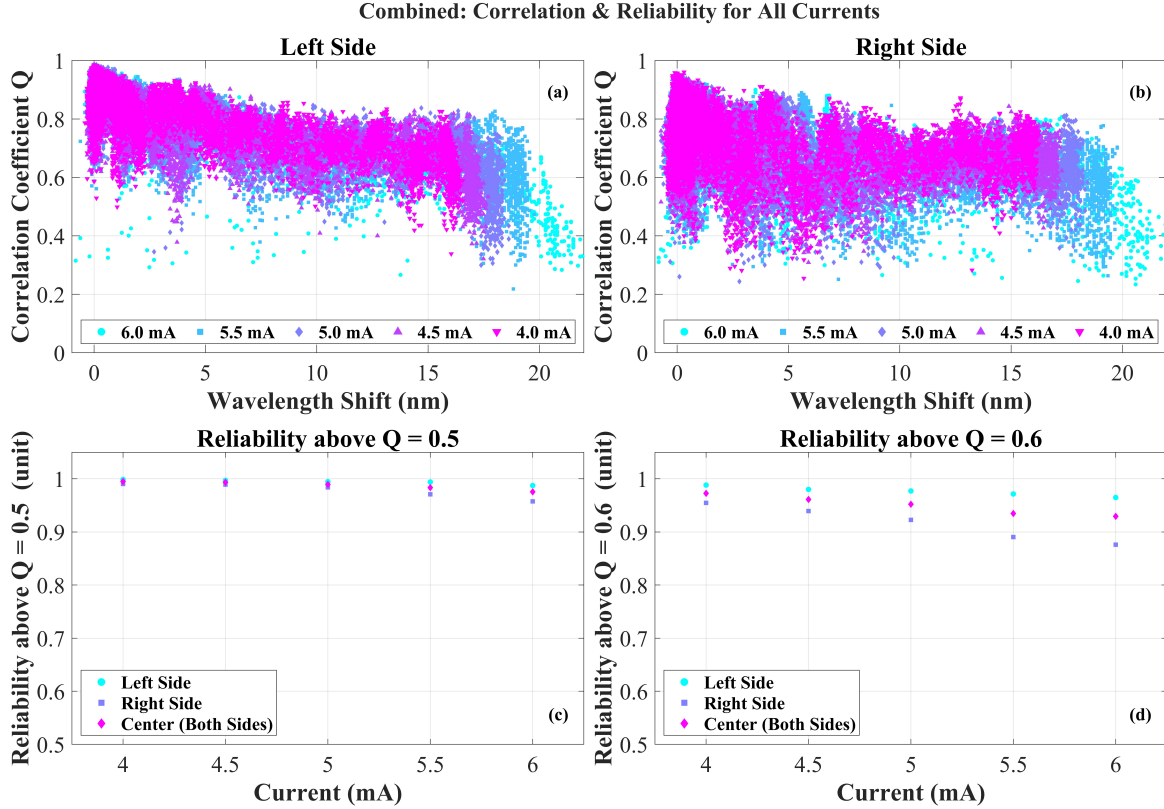


Figure 3.8: Statistical analysis of spectral correlation. (a, b) Correlation coefficient versus spectral shift for the left and right sides of the heated zone. (c, d) Percentage of measurements with a correlation coefficient above 50% and 60% for different currents.

Two key trends emerge. Firstly, for both sides of the heated zone, the correlation coefficient consistently decreases as the spectral shift (and thus temperature) increases, reducing its spectral similarity to the room-temperature reference. Secondly, a notable difference appears when comparing the left-hand side (Figure 3.8 (a)) and right-hand side

3.2. RESULTS AND DISCUSSION

(Figure 3.8 (b)) of the heating zone. The correlation on the right-hand side is lower since the light returning from the right side has propagated twice through the entire turbulent, high-temperature region, accumulating more phase distortion and OPL variation than light scattered from the left side.

To provide a clear operational metric, I calculated the percentage of measurements that maintained a correlation coefficient above a set threshold. Figure 3.8 (c, d) show the percentage of reliable measurements (coefficient $> 50\%$ and $> 60\%$, respectively) as a function of current. This analysis provides a clear quantitative assessment of the temperature sensing reliability, showing a predictable decline as the thermal load approaches the fibre's deformation point. Robustness checks confirm that key temperature profiles remain unchanged within the 40–70% threshold range, validating the chosen 50–60% cut-off as a conservative yet non-arbitrary fidelity criterion.

RELIABILITY OF TEMPORAL CORRELATION MEASUREMENTS (OPL)

A similar reliability analysis was performed for the time-domain measurements. Figure 3.9 (a) shows the resulting correlation coefficients. The analysis reveals that the decorrelation of the time-domain Rayleigh patterns is not directly caused by the magnitude of the delay shift itself. Instead, the degradation in correlation is driven by the intense temperature distribution of the arc. The physical alterations to the fibre are most severe at the point of peak temperature, under which the lowest temporal correlation coefficients are observed.

3.2. RESULTS AND DISCUSSION

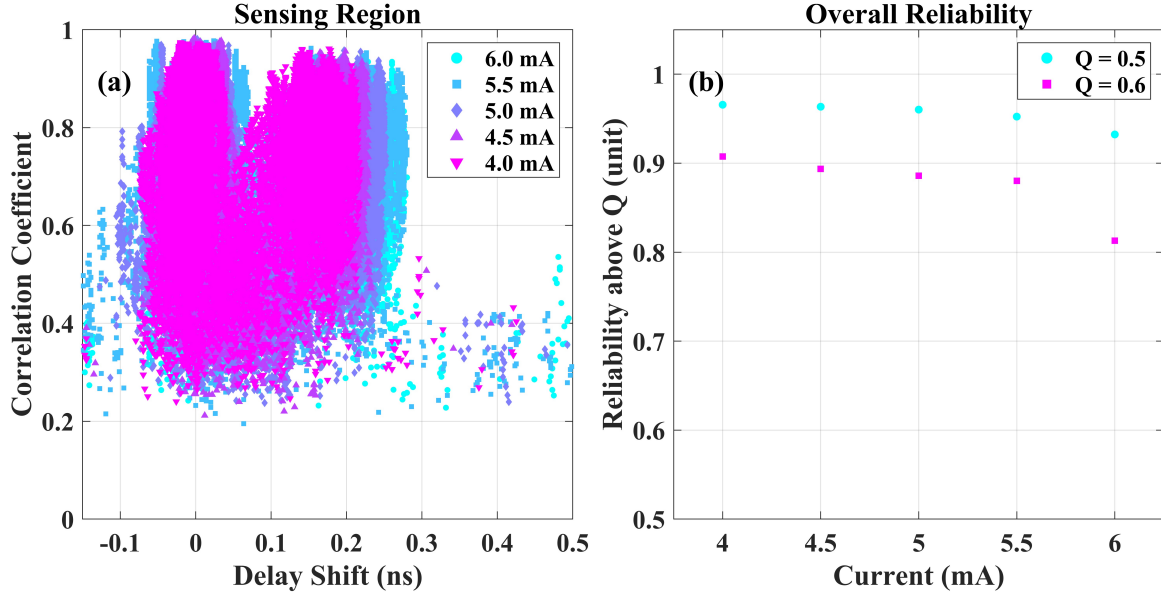


Figure 3.9: Statistical analysis of temporal correlation. (a) Correlation coefficient as a function of the maximum delay shift. (b) Percentage of measurements with a correlation coefficient above 50%.

Figure 3.9 (b) shows the percentage of temporal measurements with a correlation coefficient above 50%. This provides a direct analogue to the spectral reliability analysis, confirming that the reliability of the time-delay measurement also degrades predictably as the applied current and resulting physical changes increase.

In summary, the correlation coefficient serves as an essential tool for quantitatively evaluating the reliability of both spectral and temporal OFDR measurements. The analysis confirms that measurement reliability degrades as the thermal load increases, a direct consequence of the physical decorrelation of the Rayleigh pattern due to structural changes in the fibre. This method allows for the establishing confidence and operational boundaries for

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the sensing system, particularly when measuring extreme temperatures near the material's phase transition point.

3.2.6 METHODOLOGY FOR DYNAMIC SPATIAL-TEMPORAL MAPPING

To resolve the rapid physical transitions occurring during the plasma arc discharge, a spatial-temporal mapping method was developed. Standard OFDR provides a static spatial profile by processing the full frequency-sweep data. However, by applying a sliding window to the raw interference fringes (frequency domain) and performing an FFT operation, we can map the spatial/delay response from RBS as a function of time within a single 100 nm laser scan. Unlike OTDR, which is limited by the pulse repetition rate, this OFDR-based approach allows for continuous monitoring within a single scan.

In this approach, each window represents a discrete time segment of the discharge event. By tracking the intensity and spatial position of the backscattered signal in each window, it generate a continuous response mapping of the fibre state. Crucially, this method does not rely on cross-correlation against a reference; instead, it directly maps the raw spatial/delay information, specifically the Fresnel reflection from the fibre end-face and the local Rayleigh scattering, to observe the instantaneous and integrated response of the waveguide.

The fibre end-face reflection acts as a high-contrast absolute marker. Any change in OPL due to localized heating or electronic excitation causes this reflection to shift in the time-delay domain. This shift provides an integrated measure of the cumulative changes in the refractive index (Δn) and physical length (ΔL) along the discharge zone.

3.2. RESULTS AND DISCUSSION

DYNAMIC EVOLUTION OF OPL DURING DISCHARGE

The dynamic spatial-temporal mapping of the fibre end reflection reveals a complex, multi-stage evolution of the OPL, as detailed in Figure 3.10. This evolution demonstrates a clear competition between instantaneous electronic effects and diffusion-limited thermal responses, governed by the high-temperature thermodynamics of the silica network.

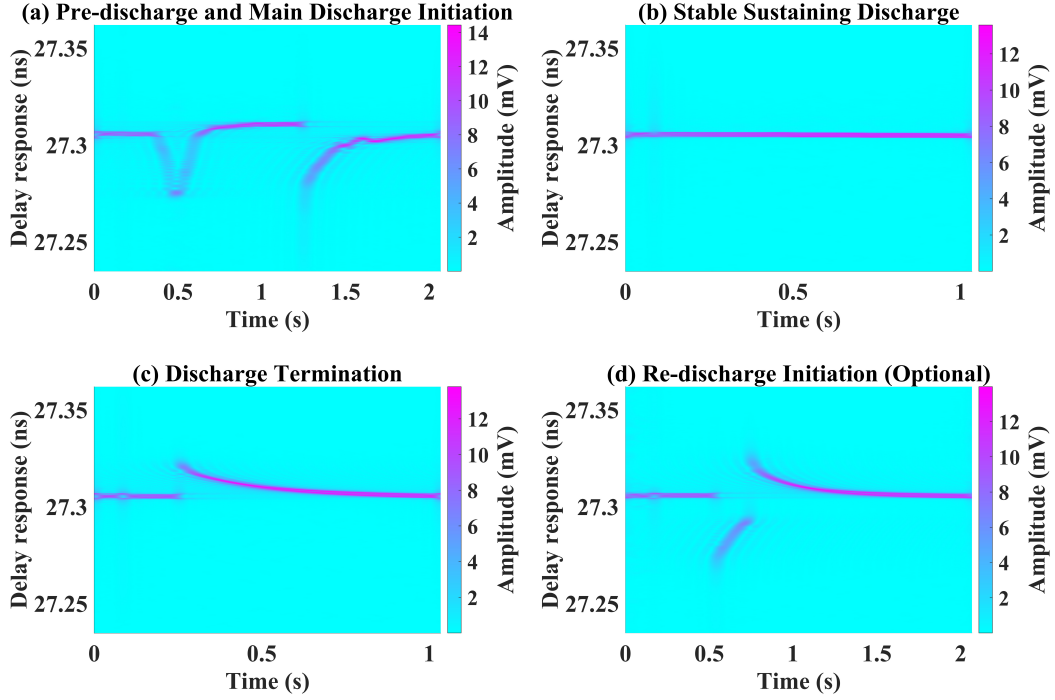


Figure 3.10: Dynamic OPL change monitored via the spatial-temporal mapping of the fibre end reflection: (a) initiation of pre-discharge and main discharge; (b) during a stable discharge; (c) at the termination of the discharge; (d) initiation of a re-discharge (optional).

Figure 3.10 (a) Pre-discharge and Main Discharge Initiation: The process begins with a low-current pre-discharge. At main discharge, a near-instantaneous decrease

3.2. RESULTS AND DISCUSSION

in OPL is observed. This represents the fast electronic response: UV radiation from the plasma initiates electronic transitions in the Ge-O and Si-O bonds, momentarily suppressing the refractive index via a change in local polarizability. This is followed by a slower upward rise in OPL, representing the thermal diffusion phase. Because of the low thermal diffusivity ($\alpha \approx 0.12 \text{ mm}^2/\text{s}$) of the silica fibre, there is a characteristic thermal lag of several hundred milliseconds before the heat front reaches the fibre core. Once reached, the rising temperature drives a positive thermo-optic effect and thermal expansion, causing the OPL to rise. This dip-then-rise cycle repeats with greater magnitude upon the ignition of the high-current main discharge.

Figure 3.10 (b) Stable Sustaining Discharge: During a continuous discharge, the OPL reaches a quasi-steady-state plateau. At this stage, the fibre core temperature may approach the glass transition region ($T \approx T_g$). The silica network transitions into a metastable, supercooled liquid state with increased mobility. The stable OPL observed is a balance between the high-temperature thermo-optic coefficient and the thermal diffusion of the glass. While the fibre remains below the softening point, the increased mobility allows for the structural relaxation and reconfiguration.

Figure 3.10 (c) Discharge Termination: Upon termination, the UV radiation and electronic excitation cease instantly, resulting in an immediate discontinuity in OPL. This is followed by a slow, exponential decay governed by thermal contraction and cooling. Crucially, the final OPL baseline after cooling often differs from the initial reference. This permanent shift indicates that the fibre has re-frozen at a new fictive temperature

3.2. RESULTS AND DISCUSSION

(T_f). The rapid quenching from the discharge temperature to room temperature traps the silica in a higher-energy structural state with a permanently modified refractive index, a change that correlates with the observed permanent spectral decorrelation in the Rayleigh signal.

Figure 3.10 (d) Re-discharge Initiation (optional): Re-discharge is optionally applied to an already fused joint to reduce splice loss and exhibits a distinct dynamic signature compared to the pre- and main discharge processes. The initial rise resembles the main discharge (Figure 3.10 (a)), while the subsequent rise suggests modulation of the arc current. A transient current drop between the pre- and main re-discharge induces an instantaneous electronic index recovery (as shown in Figure 3.10 (c)), accompanied by localized temperature fluctuations that may briefly drive the glass back across the glass transition temperature T_g before stable heating is re-established.

This dynamic behaviour is categorized by two distinct physical regimes:

- i. Fast electronic/structural response:** A near-instantaneous decrease in index due to UV polarizability shifts and the transition from a frozen glass to an active supercooled liquid state as the core crosses T_g .
- ii. Slow thermal response:** A diffusion-limited increase in OPL occurring over milliseconds ($t > \tau$), governed by the bulk heating of the fibre. The final state of the fibre is a function of the cooling rate, which determines the permanent frozen structure in the material's fictive temperature.

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DYNAMIC IMPACT ON LOCAL RAYLEIGH SCATTERING

Beyond overall OPL changes, spatial-temporal mapping reveals fundamental modifications to the local Rayleigh scattering signature. Figure 3.11 compares Fresnel reflection and RBS intensity during the dynamic plasma event with the initial room-temperature trace, evidencing rapid structural reconfiguration of the glass.

In contrast to the OPL changes in the Fresnel reflection (Figure 3.11 (a)), a highly localized transient surge in RBS intensity, appearing as a spark, is observed at the onset of re-discharge (Figure 3.11 (b)). This spike serves as a thermodynamic signature of the silica network crossing the glass transition threshold ($T > T_g$).

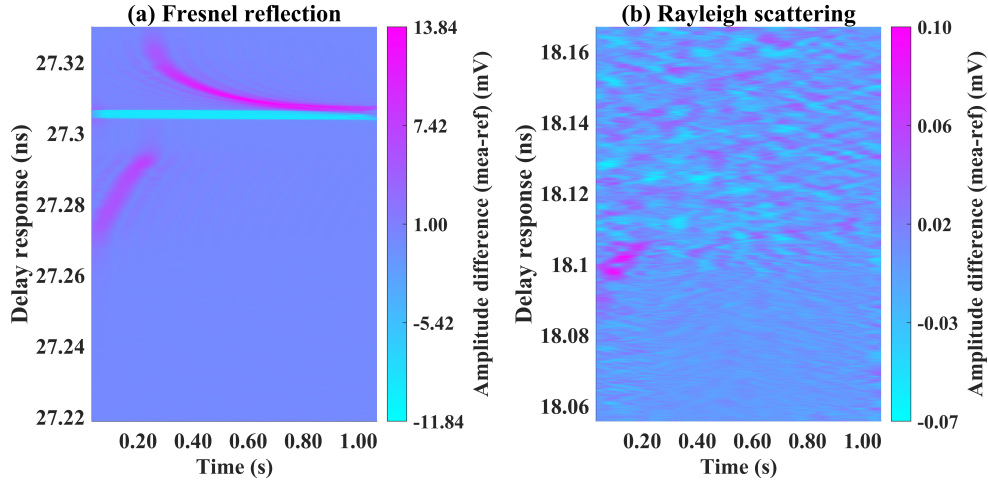


Figure 3.11: (a) OPL change during a re-discharge event. (b) The corresponding local Rayleigh spectrum at the discharge centre, showing a transient increase in intensity (a spark indicated at delay of 18.1 ns) at the moment of ignition.

In the solid state, Rayleigh scattering originates from frozen-in density fluctuations dictated by the fibre initial fictive temperature (T_f). As the plasma rapidly heats the core

3.2. RESULTS AND DISCUSSION

past T_g , the silica network transitions into a metastable, supercooled liquid. In this regime, density fluctuations are no longer statically locked but become active thermodynamic variables. Because the isothermal compressibility of this liquid state exceeds that of the quenched glass, the local scattering cross-section increases dramatically, manifesting as the observed optical spark.

The extreme transience of this signal is dictated by the fibre's low thermal diffusivity ($\alpha \approx 0.12 \text{ mm}^2/\text{s}$). Radial and longitudinal heat diffusion dissipates the localized thermal gradient, dropping the core temperature back below T_g and abruptly re-freezing the silica network. Although glass phase transitions could in principle contribute to slow dynamics, the fact that the Rayleigh signal largely recovers its baseline intensity implies that the peak core temperature remained below the softening point ($\sim 1670 \text{ }^\circ\text{C}$), and the observed reversibility (upon cooling) is consistent with refractive-index and strain effects rather than irreversible structural reorganisation.

In summary, spatial-temporal mapping successfully decouples the competing high-temperature physics within the fibre: the instantaneous, radiation-induced electronic polarizability drop versus the slower, diffusion-limited thermo-optic response governing the OPL. Furthermore, the detection of the transient Rayleigh spark provides an *in-situ* optical indicator of the phase transition. This method offers a powerful, real-time diagnostic for probing the thermodynamic state and structural mobility of optical waveguides under extreme thermal shock.

3.2. RESULTS AND DISCUSSION

3.2.7 MODELLING ASSUMPTIONS AND MEASUREMENT LIMITATIONS

To interpret the OFDR spectral and temporal shifts as distributed temperature profiles, several physical modelling assumptions and simplifications were necessary. Grouping these assumptions clarifies the operational limits and expected accuracy of the reported measurements.

First, no full transient thermal diffusion modelling was performed in this work. Instead, the system was assumed to be in an open thermal equilibrium, forming a quasi-static state where the dielectric properties of the fibre do not change over the measurement window ($\partial\epsilon_r/\partial t \approx 0$). This is justified by time-scale reasoning: the OFDR trace acquisition takes 1.03 seconds and was triggered 6 seconds into a 7-second continuous arc discharge. Because the radial thermal diffusion time of standard silica fibre ($\alpha \approx 0.12 \text{ mm}^2/\text{s}$ for $r = 62.5 \text{ }\mu\text{m}$) is on the order of tens to hundreds of milliseconds [50, 51], the sub-millimetre heated zone reaches a quasi-steady state well before the measurement begins, allowing the static OFDR spatial profiling to map the thermal distribution without solving complex transient heat equations.

Second, the system is inherently subject to spatial-averaging effects. The OFDR setup provides a sensing window of $250 \text{ }\mu\text{m}$ with a spatial resolution of $\Delta z \approx 8 \text{ }\mu\text{m}$. Consequently, the measured spectral shift at any given data point represents the integrated, average optical path length change over that $250 \text{ }\mu\text{m}$ window. Because the localized electric arc plasma induces incredibly steep temperature gradients, any microscopic thermal peaks narrower than this resolution limit are subjected to a spatial low-pass filtering effect. As a

3.2. RESULTS AND DISCUSSION

result, the true maximum peak temperatures at the dead-centre of the arc may be slightly underestimated due to this spatial averaging.

Third, absolute temperature estimation relies on the non-linear calibration model developed in Chapter 4, which incorporates the temperature-dependent thermo-optic and thermal expansion coefficients of fused silica, thereby eliminating the overestimation associated with room-temperature linear sensitivities (10 pm/°C). The reported temperatures carry an overall uncertainty of approximately ± 20 °C, arising from three primary sources: (1) the 50% correlation threshold, below which spectral shift estimation becomes unstable; (2) the 250 μm spatial averaging window in the IFFT processing, which smooths and slightly underestimates sharp thermal gradients; and (3) inherent calibration limits at extreme temperatures. Despite this uncertainty, the steady-state calibration is assumed to remain valid under transient heating conditions. The close agreement between the maximum measured temperature (~ 1300 °C) and the theoretical fictive temperature (T_f) of the fibre supports this assumption and confirms the physical limits of the sensing approach.

Finally, physical limitations arise at extreme thermal loads. As the fibre approaches transient temperatures, processes like material viscosity changes begin to alter the fibre's physical micro-structure. This decorrelates the RBS spectrum from its room-temperature reference, reducing the reliability of the ZNCC method.

3.3. CHAPTER CONCLUSION

3.3 CHAPTER CONCLUSION

In summary, this chapter demonstrates a novel approach for distributed high-temperature measurement within a commercial fusion splicer using OFDR. To our knowledge, this is the first demonstration of the capability of OFDR for spatially resolved temperature sensing in a standard single-mode fibre at temperatures approaching 1300 °C (corresponding to wavelength shifts over 20 nm). By applying a Zero-Mean Normalized Cross-Correlation (ZNCC) analysis to the Rayleigh backscattering spectra, rapid and extreme temperature changes were successfully mapped across an approximately 6 mm heating area with sub-millimetre spatial resolution.

Building upon this foundation, the key findings of the study are threefold. First, the spectral correlation analysis yielded distributed thermal profiles with estimated peak temperatures approaching 1300 °C, based on the rigorous non-linear calibration established in Chapter 4. Crucially, this maximum measurable threshold was found to be intrinsically linked to the fibre’s fictive temperature (T_f), demonstrating the fundamental physical limit of cross-correlation sensing as the glass network begins to undergo a phase transition. This spectral analysis was also extended to introduce a novel statistical method for quantifying the permanent internal stress induced by repeated heating cycles. Second, in the time domain, the analysis of femtosecond-level delay shifts revealed the complex dynamics of the optical path length (OPL), successfully decoupling the competing effects of the fibre’s instantaneous electronic and diffusion-limited thermal responses. This temporal

3.3. CHAPTER CONCLUSION

approach uncovered unique phenomena, such as the formation of multiple delay modes, providing direct evidence of the fibre’s waveguide structure being disrupted during the core’s glass transition. Finally, the reliability of all measurements was quantitatively assessed by analyzing the correlation coefficient, establishing clear operational boundaries for the technique at extreme temperatures near the material’s structural relaxation point.

In conclusion, this work establishes OFDR not merely as a high-temperature thermometer, but as a multi-parameter diagnostic tool capable of simultaneously probing thermal, mechanical, and optical phenomena in real time. The dynamic spatial-temporal mapping method, unconstrained by the pulse repetition rates that limit OTDR, offers distinct advantages for monitoring rapid thermodynamic processes. The insights gained have direct implications for optimizing industrial applications, such as fusion splicing and specialty fibre fabrication. Moreover, the demonstrated sensitivity to OPL changes and optical mode evolution opens promising new avenues for research, including the study of group birefringence in dynamically stressed materials and the *in-situ* optical analysis of glass phase transitions.

Chapter 4

DISTRIBUTED TEMPERATURE SENSING OF A HYDROGEN FLAME AND FIBRE DURABILITY ANALYSIS BY OFDR

Following the investigation of extreme temperatures generated by an electrical arc in Chapter 3, this chapter transitions to a different but equally challenging high-temperature environment: a hydrogen combustion flame. While the previous chapter focused on short-duration, high-energy plasma, this work addresses the need for detailed, spatially resolved temperature mapping in continuous combustion processes. Such capabilities are critical for applications like micro-combustor design and flame diagnostics. Furthermore, this study extends beyond simple temperature measurement to investigate the crucial aspects of fibre sensor performance and durability under prolonged thermal load.

4.1 INTRODUCTION: CHALLENGES AND MOTIVATION FOR FLAME SENSING

Unlike conduction or convection heating, radiant heat released from plasma or γ -ray can be faster and store a higher density of energy. It is important to investigate fibre durability under this type of heating for sensing application, such as micro-combustion system, mining and nuclear industries. In contrast to the nitrogen ions emission induced by high-voltage splicing arc [62] and γ -ray dose [63] arising from the radioactive decay of atomic nuclei, a hydrogen flame exhibits a broad black-body spectrum from UV to IR [46], which enables sufficient energy exchange with the vitreous silica fibre under test. The resulting temperature rises up from 1500 to 2100 °C [64, 65] and a significant thermal gradient appears from the inner core to the outer shell.

The design of micro power systems as a stable energy source with decent combustion efficiency has received considerable attention [66], and giving a general characterization of the thermal distribution becomes essential for further optimization. In terms of micro-fibre fabrication, establishing a uniform flame zone is key to tapering with a small core diameter [67]. The characterization of the temperature gradient surrounding the fibre in situ could make the fabrication repeatable and reliable. Conventionally, flame temperature is measured with the CARS (Coherent Anti-Stokes Raman Scattering) technique [68] or computed tomography with a CCD camera [64]. However, estimation of the temperature distribution with an optical lens, especially for the inner flame core, could inevitably

4.1. INTRODUCTION: CHALLENGES AND MOTIVATION FOR FLAME SENSING

introduce large uncertainty due to image distortions. Long exposure times and lower spatial resolution may also lead to a compromise on detailed dynamic features.

Optical frequency-domain reflectometry (OFDR) provides both the high sensitivity and spatial resolution necessary by measuring the localized Rayleigh scatter pattern, which varies with temperature and applied strain [1]. However, distributed high-temperature measurement based on OFDR has been a challenge in extreme conditions. A modified FBG (*Type II fs-IR*) inscribed by a femtosecond laser can endure over 1130 °C with a relative 2000 GHz frequency shift (~ 16 nm wavelength shift), and single-mode fibre up to 950 °C has also been measured in a furnace with a 1600 GHz frequency shift (~ 12.8 nm wavelength shift) [69]. It is noted that single-mode fibre subjected to the plasma radiation of an electric arc can be measured with a maximum spectral shift of 21 nm during a short arc excitation time of a few seconds [62]. The thermal sensitivities of SMF and silica-based FBGs have been studied across different ranges from room temperature to 1000 °C [6, 7, 70]. It has also been found that silica-based fibre sensors show varying thermal sensitivities depending on their packaging and coating materials [71, 72]. Improvements in the temperature range of telecom fibres have also been achieved using adaptive algorithms [69].

However, it is still unclear how long the commonly used temperature and wavelength shift relationship (*i.e.*, 10 pm/°C) is maintained by silica fibre under high temperatures, especially near the glass softening range where a gradual rearrangement of the glass network occurs [5]. The sustained linearity of SMF under high temperatures could be critical for reliable measurement.

4.2. EXPERIMENTAL SETUP AND METHODOLOGY

In this study, we propose a novel method to measure the distributed temperature of a hydrogen flame using telecom fibre based on OFDR, capable of making spatially distributed measurements ranging from micrometers to meters. Spatial temperature profiles are obtained during combustion with varied H_2 gas flow rates. The temperature gradient over the flame is monitored by telecom fibre, and its durability under high temperature is studied using a Rayleigh spectrum correlation method. A 16 nm spectral shift, equivalent to 1100 °C, is measured over a duration of up to a few seconds following numerical calibration. The softening condition essentially determines the upper exposure time limit for placing telecom fibre under extreme heat for sensing; prolonged exposure permanently changes the silicate structure and eventually alters the spectral shape of the Rayleigh scatter pattern. This allows us to quantitatively estimate the fibre durability time (operation time) under the radiation heating of a hydrogen flame via a decorrelation time. This is achieved by monitoring the spectral shift in the OFDR system and recording when the correlation coefficient drops to specific percentage levels. To the best of our knowledge, above 1050 °C, the time taken for the correlation to drop to 50% is only a few seconds, which demonstrates the limits and capabilities of Rayleigh scattering in telecom fibre for high-temperature measurement.

4.2 EXPERIMENTAL SETUP AND METHODOLOGY

To achieve the objectives outlined, a specialized experimental system was developed. This section details the OFDR setup used for distributed sensing, the design of the hydrogen

4.2. EXPERIMENTAL SETUP AND METHODOLOGY

combustion environment, and the specific procedures for data acquisition and the numerical calibration required to convert measured spectral shifts into temperature values.

4.2.1 OFDR SYSTEM AND COMBUSTION SETUP

The OFDR system designed for distributed high-temperature measurement is shown in Figure 4.1. A tunable laser source (TLS, New-Focus-Venturi-TLB-6600) is coupled into a Mach-Zehnder interferometer. An auxiliary interferometer (AUI) branch, circled by a dashed square, functions as an external clock to ensure accurate and linear frequency sampling during the laser scan [20]. The main interferometer (MI) branch splits the light into reference and probe arms to interrogate the fibre under test. Light propagating into the probe arm is backscattered from the fibre, and this signal is then combined with the reference arm light to generate detectable beat patterns at the photodetector. The spatial resolution Δz of the Rayleigh scattering measurement is determined by the spectral bandwidth of the tuning range:

$$\Delta z = \frac{\lambda_i \lambda_f}{2n_g \Delta \lambda}, \quad (4.1)$$

where n_g is the group index of the silica fibre assumed as a constant value of 1.47, λ_i and λ_f are the initial and final wavelengths, and $\Delta \lambda$ is the total scanned wavelength range for the measurement. In this setup, the laser scanned from 1520 to 1620 nm, which corresponds to a spatial resolution Δz of approximately 7.6 μm .

The combustion setup, partially shown in the inset of Figure 4.1, consists of a cylindrical stainless tube with an inner diameter of ~ 0.87 mm, which serves as a vertical gas channel.

4.2. EXPERIMENTAL SETUP AND METHODOLOGY

A bare optical fibre, with its coating mechanically stripped, is positioned directly above the tube. Non-premixed hydrogen fuel was generated continuously by a commercial water electrolysis device (QL-300, Hydrogen Generator) and regulated by a gas mass flow controller (FMA5506, OMEGA). The flame was considered a typical laminar flow in quiescent air, and the presence of the fibre was assumed to be negligible to the overall temperature distribution of the flame.

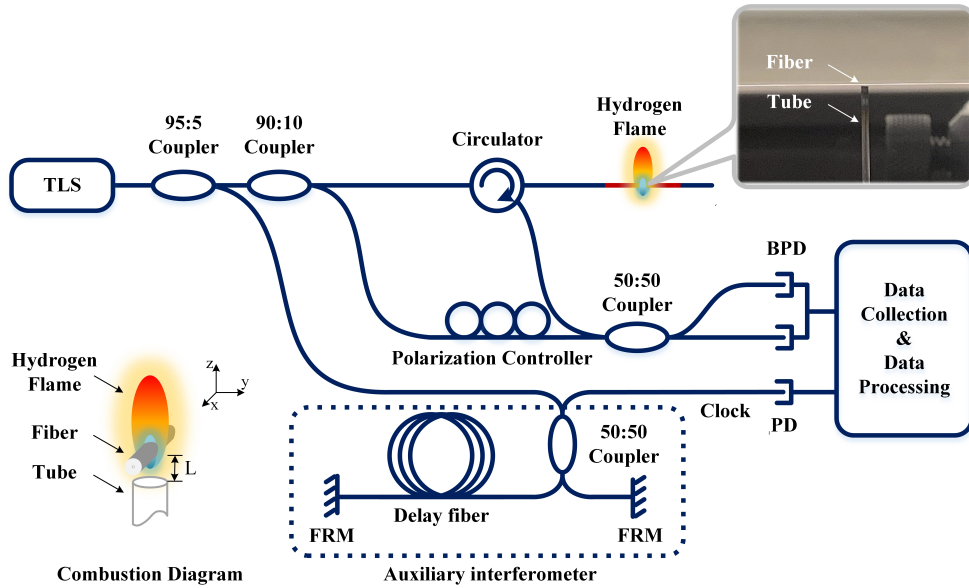


Figure 4.1: Schematic of the optical frequency-domain reflectometry system for hydrogen flame measurement. TLS: tunable light source; FRM: Faraday rotating mirror; PD: photodetector; BPD: balanced photodetector. Circled by a dashed square, an Auxiliary Michelson interferometer was configured as a clock generator for digital acquisition, equipped with a 25-meter length of delay fibre and two Faraday rotating mirrors. An inset at top right shows a front view of solid tube located closely underneath a tested fibre, which were indicated respectively in a white arrow. Combustion diagram at bottom left showed a basic layout with a telecom fibre placed in the flame core for temperature measurement along the x axis. Capital L represents the vertical distance between fibre and nozzle along the z axis.

4.2. EXPERIMENTAL SETUP AND METHODOLOGY

4.2.2 DATA ACQUISITION PROCEDURE

The light source's tuning rate was set to 100 nm/s. The AUI, equipped with a 25.0 m delay fibre, generated a sampling rate of 3.2 MHz for the Data Acquisition (DAQ) card (NI-PCI6115). Each complete scan took 1.13 seconds, generating a 3.4 mega-sample raw data file for post-processing.

Two sets of experiments were performed. First, to measure the temperature distribution as a function of fuel supply, the H₂ flow rate was varied from 2.0 to 7.5 mL/min in steps of 0.5 mL/min. Second, to map the vertical temperature profile of the flame, the fibre's vertical distance from the nozzle was adjusted from 0 to 200 μm in steps of 25 μm using a high-precision stage, while the flow rate was held constant at 4.5 mL/min.

For each measurement, the fuel was ignited and the fibre was allowed to heat up for several seconds before data acquisition began. Immediately after the measurement was complete, the gas supply was terminated. To ensure measurement consistency, several minutes were allowed for the system and the heated fibre to cool down to room temperature before a new reference trace was acquired. This procedure avoids introducing artificial errors by maintaining the same referential conditions for each relative measurement.

4.2.3 NUMERICAL TEMPERATURE CALIBRATION

While the spectral shift in telecom fibre is approximately linear with temperature over a small range, this relationship becomes highly nonlinear at the transition temperatures

4.2. EXPERIMENTAL SETUP AND METHODOLOGY

relevant to this study. Therefore, a numerical calibration method was required. As the true temperature within the flame is unknown, an assumption was made by referencing the well-characterized thermal sensitivity of a *Type II fs-IR* FBG, which was experimentally tested in a furnace [70].

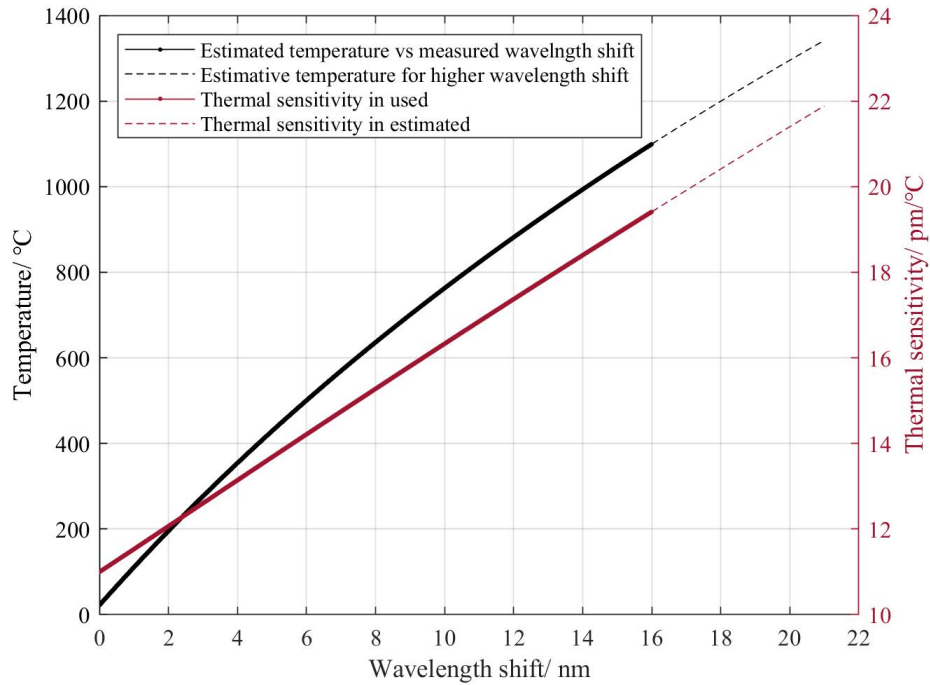


Figure 4.2: Evaluation of temperature and thermal sensitivity in terms of a given spectral shift, neglecting the coating or packaging effects on the interrogated fibre. The wavelength shift was initially obtained by a cross-correlation method, and the conversion relationship was constructed using a third-order polynomial. (i) The black, solid-dot line gives a general estimation that was adopted for the remaining temperature conversions. The black dashed line forecasts the temperature changes associated with wavelength shifts; (ii) The red, solid line is the thermal sensitivity associated with the converted temperature for the wavelength shift, and the red dashed line is the estimated thermal sensitivity intended for higher wavelength shifts.

4.2. EXPERIMENTAL SETUP AND METHODOLOGY

A third-order polynomial was used to model the relationship between the measured wavelength shift ($\delta\nu$) and the estimated temperature (T), fitted using a least squares method with known slope rates at 50, 500, and 1000 °C, plus the room temperature point (22 °C). The resulting cubic equation used for temperature conversion is:

$$\delta\nu = \left(6.0819 \times 10^{-10}\right) T^3 + \left(2.8827 \times 10^{-6}\right) T^2 + \left(1.0872 \times 10^{-2}\right) T - 0.2406, \quad (4.2)$$

where $\delta\nu$ is the wavelength shift in units of nm compared to the Rayleigh spectrum at 22 °C.

This relationship, shown in Figure 4.2, fits reasonably well with other published data from room temperature to 1000 °C [6, 7]. The model shows that the thermal sensitivity is not constant, starting at ~ 11 pm/°C, increasing to ~ 14 pm/°C at 500 °C, and rising above ~ 18 pm/°C at 1000 °C. As indicated by the red dashed line, this trend is projected to continue, reaching an estimated sensitivity of approximately 22 pm/°C at 1300 °C.

The physical origin of this non-linearity is the competing effects of thermal expansion and refractive index changes at high temperatures. While thermal expansion, which is proportional to temperature, tends to dominate at lower temperatures, the significant changes in the refractive index at higher temperatures introduce the non-linear terms, causing the observed bending at 500–600 °C in the response curve seen in Figure 4.2.

4.3. RESULTS AND DISCUSSION

4.3 RESULTS AND DISCUSSION

This section presents the core experimental findings of the chapter. We begin by detailing the data processing steps and validating a key measurement assumption. We then present three key analyses: first, the detailed spatial characterization of the hydrogen flame itself; second, an investigation into how high temperatures affect the fibre’s physical state; and third, a quantitative analysis of the fibre’s durability over time in the combustion environment.

4.3.1 DATA PROCESSING AND MEASUREMENT VALIDATION

The raw interferometric signal was digitized and processed using a three-step routine to extract the distributed temperature information.

- i. Time-domain conversion:** A Fast Fourier Transform (FFT) was performed on both the measurement trace $i_m(\nu)$ and the reference $i_r(\nu)$ to reconstruct the Rayleigh scattering intensity as a function of time-of-flight along the fibre $\tilde{I}_m(\tau)$ and $\tilde{I}_r(\tau)$.
- ii. Local spectrum recovery:** A moving window was applied to the time-domain data to isolate a specific region of interest. An Inverse Fast Fourier Transform (IFFT) was then used to recover the local optical spectrum corresponding to that fibre segment. In this analysis, a spatial sensing resolution of 346 μm was used. The window was moved in steps of 7.6 μm .

4.3. RESULTS AND DISCUSSION

iii. Spectral shift calculation: The wavelength shift between the measurement spectrum $\tilde{I}_m(\nu)$ and the reference spectrum $\tilde{I}_r(\nu)$ was determined using the zero-mean normalized cross-correlation (ZNCC) method [62, 69]. A spectral shift $\delta\nu$ in cross-correlated spectra is accompanied by a quality metric, the correlation coefficient Q , defined as:

$$\begin{aligned}\delta\nu &= \arg \max \{ \Delta I_m(\nu) \otimes \Delta I_r(\nu) \} \\ Q &= \max \{ \Delta I_m(\nu) \otimes \Delta I_r(\nu) \},\end{aligned}\tag{4.3}$$

where $\Delta I_m(\nu) = \tilde{I}_m(\nu) - \bar{I}_m(\nu)$ and $\Delta I_r(\nu) = \tilde{I}_r(\nu) - \bar{I}_r(\nu)$, representing the spectra with their respective mean values $\bar{I}_m(\nu)$ or $\bar{I}_r(\nu)$ subtracted.

Before presenting the temperature profiles, a key assumption must be validated: that the change in the fibre's optical path length (OPL) due to thermal expansion and refractive index change is negligible compared to the spatial sensing resolution. To verify this, we calculated the extra time-delay (retardation) over the heated section by correlating the temporal Rayleigh patterns. As shown in Figure 4.3, the maximum extra retardation induced by the flame was ~ 0.4 ps. This corresponds to a total heated interaction length of ~ 80 ps. Since the induced retardation is less than 0.5% of the total length of the processed signal segment, its effect on the determined spatial location is minor compared to the 346 μm spatial sensing resolution, thus validating our approach.

4.3. RESULTS AND DISCUSSION

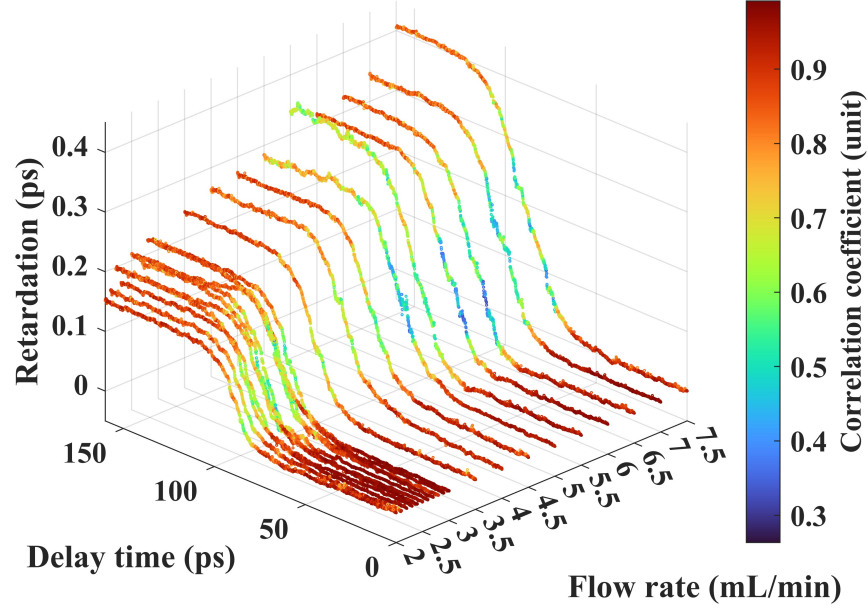


Figure 4.3: Extra retardation (subtraction to room temperature delay) versus time delay of light travelling in fibre over radiation heating at different flow rate from 2.0 mL/ min to 7.5 mL/min. The thermal induced extra retardation amounts to 0.15 ps at 2.0 mL/min and 0.4 ps at 7.5 mL/min. Note that colour bar on right gives the reliability of OFDR on retardation measurement by associated correlation coefficient.

4.3.2 SPATIAL TEMPERATURE PROFILE MAPPING OF H₂ FLAME

Using the established methodology, we performed a detailed investigation of the hydrogen flame's thermal structure with respect to two key combustion parameters: the fuel flow velocity ($\mathbf{F}$) and the vertical distance ($\mathbf{L}$) of the interrogated fibre relative to the tube top. The 2D temperature distributions are shown in Figure 4.4 (i, ii), respectively. It should be noted that the reported temperatures carry an estimated uncertainty of ± 20 °C arising from the 50% correlation threshold, 346 μm spatial averaging, and the wavelength-to-temperature

4.3. RESULTS AND DISCUSSION

calibration.

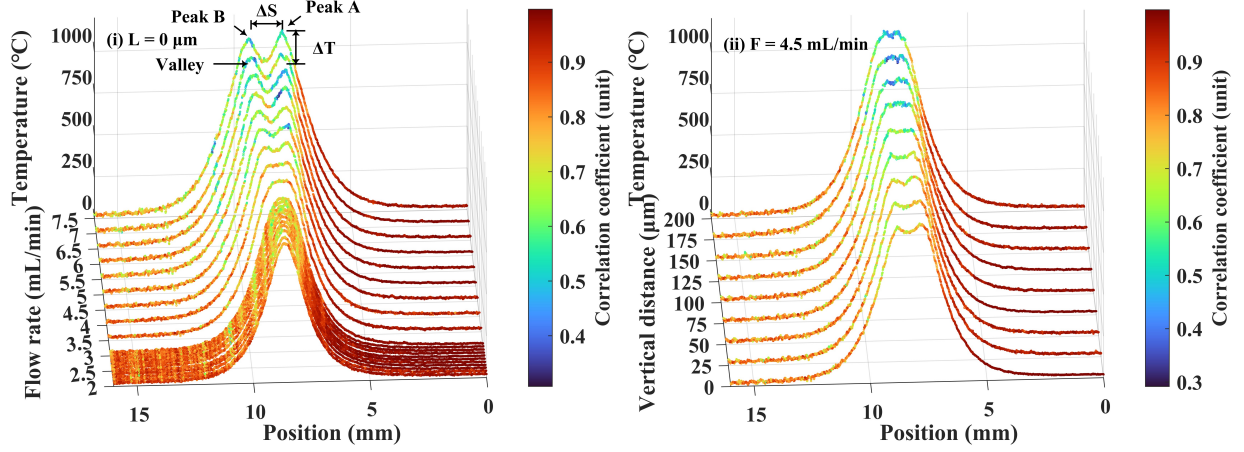


Figure 4.4: (i) Distributed temperature profile of a hydrogen flame at different flow rates (F parameter) in units of mL/min. With a fixed spacing $L = 0 \mu\text{m}$ between the tube and fibre, a 2D flame structure was obtained below 1100°C with a slightly asymmetrical shape at high flow rates. It showed a characteristic combustion of a micro-jet flame where thermal radiation grows higher and broader as the fed fuel increases, while the cooler temperature is kept stationary in the core above a certain flow rate. Peaks A and B, and the valley indicated by the arrow, are the extreme points at both sides and the centre of the temperature field. ΔT represents the gradient between the peak (A or B) and the valley, while ΔS is the spatial separation between Peaks A and B, which grows with increasing flow rate. (ii) With an incremented spacing (L parameter), temperature growth was observed by adjusting the displacement scale while keeping the flow rate constant at $F = 4.5 \text{ mL/min}$. Note that the color bar on the right side of each figure indicates the reliability of the distributed temperature measurement: red dots show a high correlation coefficient, and inversely for blue. Estimated temperature uncertainty is $\pm 20^\circ\text{C}$ (arising from spatial averaging $\approx 346 \mu\text{m}$, and calibration).

First, the effect of the fuel flow rate on the flame's thermal structure was investigated. As shown in Figure 4.4 (i), the temperature due to radiation heat near the tube lip grew with the fuel velocity. The core temperature, however, stayed constant at around 900°C as the flow rate was raised above 3.5 mL/min . At the highest flow rate of 7.5 mL/min , the

4.3. RESULTS AND DISCUSSION

hottest regions of the flame are on the sides of the centre, separated by 1.4 mm, with a temperature difference of 140–190 °C over a dimension of 2.5 mm above 900 °C.

This temperature distribution is characteristic of a micro-jet flame where the reaction zone is lifted off from the tube as the flow rate increases. The inner temperature appears cooler than the outer because it is an oxygen-deficient region with less direct contact with ambient air, causing the chemical reaction to reach a dynamic balance. The temperature distribution was essentially affected by the shape of the micro-flame, which also determined the details of the thermal interactions between the flame and the solid tube [73, 74]. The observed asymmetry in the flame shape is likely due to an unbalanced thermal interaction with the tube walls, caused by an imperfectly flattened nozzle. It should also be noted that repeated thermal reactions gave rise to the uneven growth of oxides on the tube surface.

Next, the impact of the vertical distance (L) between the fibre and the nozzle was studied while keeping the flow rate constant at 4.5 mL/min. The temperature distribution was measured by precisely adjusting the vertical position of the tube entrance relative to the fibre, ensuring the spacing was equally incremented from 0 to 200 μm in steps of 25 μm using a high-precision stage. As shown in Figure 4.4 (ii), increasing the spacing caused the temperature at the flame’s centre to grow significantly, while the sides expanded only slightly. In both sets of experiments, the maximum measured temperature reached up to 1100 °C in the hottest regions, a value comparable to that achieved with specialized *Type II fs-IR* FBGs. It is noteworthy that the standard telecom fibre survived these repeated thermal annealing treatments with no measurable increase in optical loss.

4.3. RESULTS AND DISCUSSION

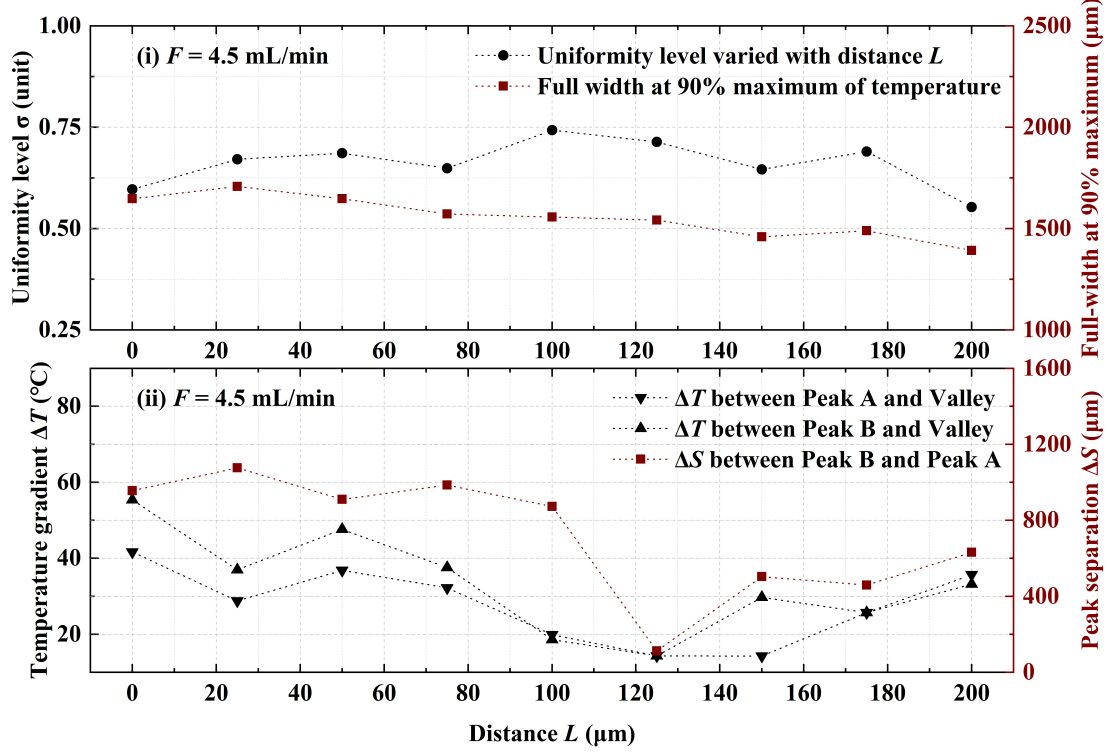


Figure 4.5: Characterization of a hydrogen flame at a flow rate of 4.5 mL/min regarding (i) the uniformity level with the full-width at 90% of the maximum temperature, and (ii) the local thermal gradient varying with the distance L between the fibre and the tube entrance.

Temperature uniformity is essential for applications like fabricating small-diameter tapered fibres, as non-uniform heating can release internal strain unevenly and introduce leaky modes that limit the final taper diameter. To quantify the flame's suitability for such applications, we characterize its uniformity with respect to the distance L by referring to the full-width at 90% of the maximum temperature field, which is denoted as length W .

4.3. RESULTS AND DISCUSSION

The uniformity coefficient, σ , is defined as:

$$\sigma = \frac{\int_0^W [T(x) - T_{90\%max}] dx}{(T_{max} - T_{90\%max}) \times W}, \quad (4.4)$$

where a value of $\sigma = 1$ signifies a perfectly uniform (flat-top) temperature field.

Generally, as shown in Figure 4.5 (i), the full width at 90% maximum of the temperature field decreases slowly as the distance increases, from $\sim 1650 \mu\text{m}$ at the initial position to $\sim 1400 \mu\text{m}$ at $L = 200 \mu\text{m}$. The analysis reveals that the position at $L = 100 \mu\text{m}$ provides the largest uniformity level. The shape of the temperature field changes significantly with distance, as detailed in Figure 4.5 (ii); the temperature gradient, ΔT , experiences a fast fall as the fibre moves towards $L = 100 \mu\text{m}$ and rises again above $L = 125 \mu\text{m}$, with the peak separation, ΔS , showing a similar behaviour. This quantitative finding is visually confirmed in Figure 4.4 (ii), where the temperature profile at $L = 100 \mu\text{m}$, shows a distinct flat top at approximately 1000°C .

The hydrogen flame's behaviour, governed by the flow rate F and distance L , is critical for achieving a uniform temperature profile. The spatial temperature distribution of the hydrogen flame characterizes temperature uniformity under different F and L settings, providing a guideline for designing a reliable tapering process with consistent diameter and taper length. The glass transition plays a major role in this process and can be further inferred from the temperature-dependent correlation shown in Figure 4.6, which will be discussed in the following section.

4.3. RESULTS AND DISCUSSION

4.3.3 ANALYSIS OF TEMPERATURE-DEPENDENT FIBRE DEFORMATION

Beyond measuring the external flame temperature, it is crucial to understand how such extreme heat affects the sensing fibre itself. The occurrence of deformation limits the fibre's applicability for higher-temperature and long-term measurements. This is because the glass transition in fused silica is not a single point but a complex series of softening and melting processes, particularly in a network randomly mixed with GeO_2 and SiO_2 species. Understanding this boundary is essential for ensuring quantitative and reliable measurements.

As the fibre is exposed to the broad radiation spectrum of the hydrogen flame [46], a temperature field is formed along its length. The heating mechanism is primarily driven by the absorption of this radiation by the fused silica, which has known absorption bands in both the ultraviolet (UV) and infrared (IR) regions [35]. In this section, we analyze the resulting fibre deformation by relating the degradation of the spectral correlation coefficient to the measured temperature.

The statistical results, gathered from tests at various flow rates, are exhibited in Figure 4.6. The correlation coefficient, which quantifies the similarity between the heated and reference spectra, shows a distinct, temperature-dependent decay. It remains high (above 90%) until approximately 400 °C, then begins to spread out, and finally drops significantly above 900 °C. This behaviour is consistent with measurements of SMF in a furnace, which also show a rapid decorrelation at similar spectral shifts (12 nm)[69]. At an estimated temperature of 1100 °C, corresponding to a 16 nm spectral shift, the correlation

4.3. RESULTS AND DISCUSSION

falls to approximately 50%.

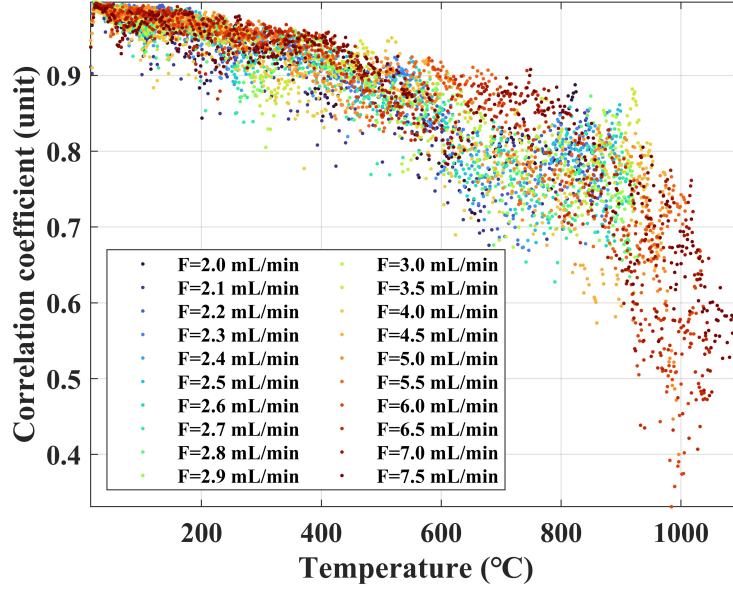


Figure 4.6: Temperature-dependent correlation between Rayleigh spectra from the measurement and reference. Statistically, dots represent results with various flow rates in different colors, illustrating a general tendency of the correlation varying with increasing thermal levels. It decreases slowly to 90% as the temperature rises to 400 °C. The correlation starts to spread out as it enters the range from 400 to 800 °C, for which nonlinear thermal coefficients are expected for temperatures higher than 400 °C with OFDR measurement. A significant drop is shown here when the temperature exceeds 900 °C, which could be related to the rearrangement process of the glass structure. It shows a correlation of 50% around 1100 °C as the maximum temperature limited by fast glass transition.

The physical interpretation of this trend is directly linked to the structural rearrangement of the glass network at high temperatures. The rapid decorrelation above 900–1000 °C, well above the annealing temperature, is likely caused by the breaking and rebuilding of mixed $\text{SiO}_2\text{-GeO}_2$ rings within the fibre core, along with the relaxation of densification and compressed stress [5]. The measurement is ultimately limited by this fast glass transition,

4.3. RESULTS AND DISCUSSION

which becomes highly active as the temperature approaches the melting point of GeO_2 (1115 °C). It is noteworthy that no evident optical loss was observed in the Rayleigh scattering, suggesting that the fibre's cladding acts as an effective barrier against the invasion of hydroxyl (OH) ions into the core, a factor that enhances the fibre's sustainability for these high-temperature measurements [75].

4.3.4 CHARACTERIZATION OF FIBRE DURABILITY IN A COMBUSTION ENVIRONMENT

Building upon the analysis of temperature-dependent deformation, a final experiment was conducted to quantify the fibre sensor's durability over time, a critical parameter for practical applications. This was achieved by continuously monitoring the fibre in the flame and analyzing the decorrelation time as a function of temperature.

To study the time-dependent durability, a single-mode fibre was placed in the flame at a constant flow rate. The OFDR system was programmed to continuously acquire a series of 20 traces over a total period of approximately 140 seconds. For each measurement trace, a corresponding reference trace was taken after the interrogated fibre had naturally cooled down to room temperature. This procedure ensures that each measurement is compared against the most recent state of the permanently altered fibre, allowing us to track the progressive degradation.

A timestamp was automatically embedded in each trace through a data channel with the DAQ function. The interval time between the start of two consecutive traces was

4.3. RESULTS AND DISCUSSION

approximately 7.32 seconds on average, which includes 1.13 seconds for the measurement scan and additional time for the system to restart. Since the reference for each measurement was updated after the heating was complete, all timestamps can be viewed as being temporally reversed relative to the end of each heating cycle.

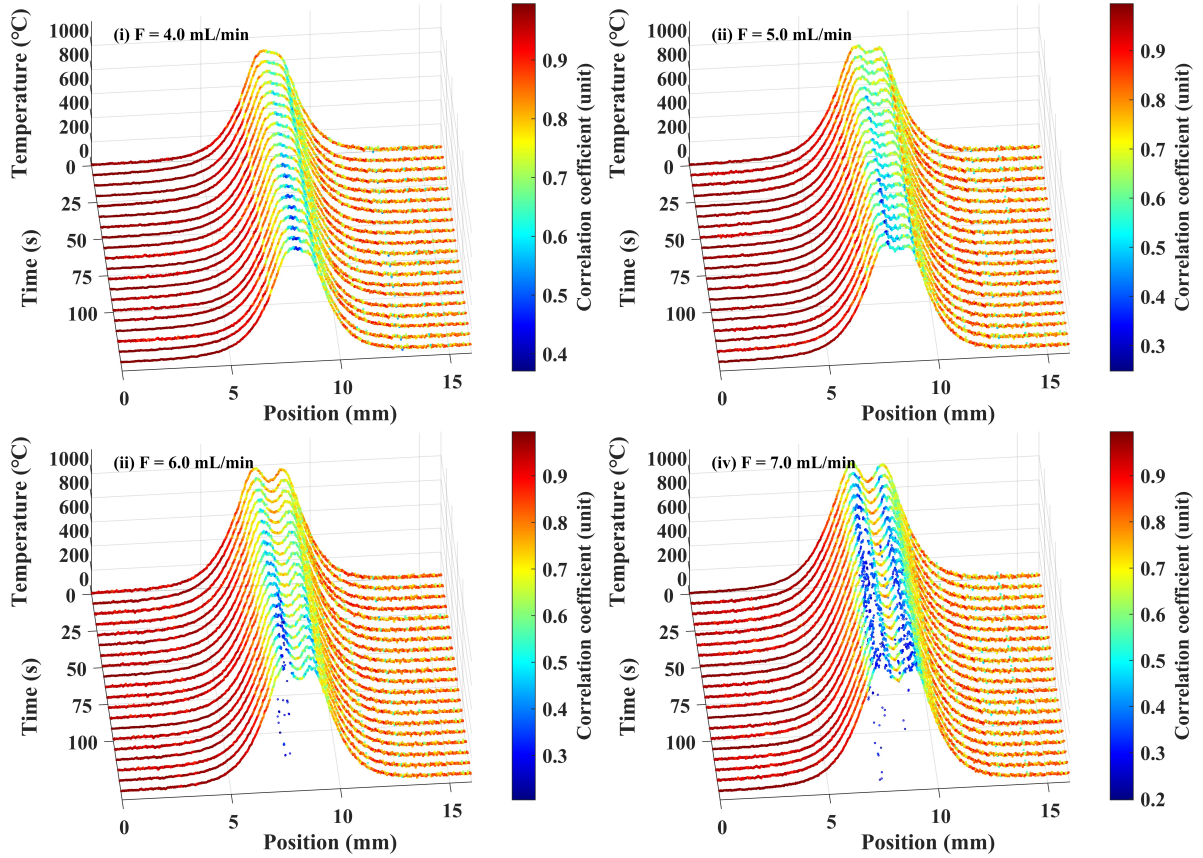


Figure 4.7: Time-varying distributed temperatures obtained from flow rates of (i) 4.0, (ii) 5.0, (iii) 6.0, and (iv) 7.0 mL/min. The associated correlation coefficient in the temperature profile, as indicated by the color bar on the right side of the figure, progressively degrades over time, mainly at the top region where the temperature reaches its highest point. It shows different degradation rates in terms of flow rate. With a 140-second free burn of a hydrogen flame, the resolved temperature in the innermost zone of the flame ultimately degrades down to 900 °C.

4.3. RESULTS AND DISCUSSION

The results of this continuous monitoring are shown in Figure 4.7 for four different flow rates. The time-varying distributed temperature profiles clearly demonstrate the degradation of the measurement over the 140-second burn. As the silica fibre continuously absorbs radiant heat, it progressively deforms. This is visually indicated by the degradation of the correlation coefficient (color bar), which is most pronounced in the hottest regions of the flame.

This degradation is a direct result of the physical changes occurring within the fibre. The fibre undergoes progressive physical alteration due to thermal expansion along with the processes of softening and rearranging within the glass structure. Unlike the discrete nitrogen ion emissions in the UV band from an electric arc [62], the broad black-body radiation of the hydrogen flame plays the major role here. A permanent, irreversible change occurs in the structure due to abundant energy absorption by the $\text{SiO}_2\text{-GeO}_2$ glass matrix, particularly in the IR band. This absorption permanently affects the Rayleigh pattern and hence the local spectrum. It is this decorrelation that gives rise to the apparent degradation of the measured spectral shift and ultimately restricts the measurable temperature range for the OFDR system over time.

The analysis confirms that the time evolution of the hydrogen flame under thermal dynamic equilibrium is slower than the OFDR response time, validating the spatial measurements. However, it also highlights that higher temperature measurements beyond 1100 °C would only be feasible with a much faster system response time (<1 s) to capture the profile before significant decorrelation occurs.

4.3. RESULTS AND DISCUSSION

To characterize the fibre's durability quantitatively, we introduce the metric of decorrelation time: the time required for the correlation coefficient to drop to a specific threshold at a given temperature. To analyze this across a wide temperature gradient (from room temperature to beyond 1100 °C), a high flow rate of $F = 10$ mL/min was used, and 12 arrays were collected for averaging. The decorrelation time was calculated by poly-fitting the time-dependent correlation coefficient at different temperature ranges and then solving for the time at which the coefficient dropped to specific levels (50%, 55%, 60%, 65%, and 68%).

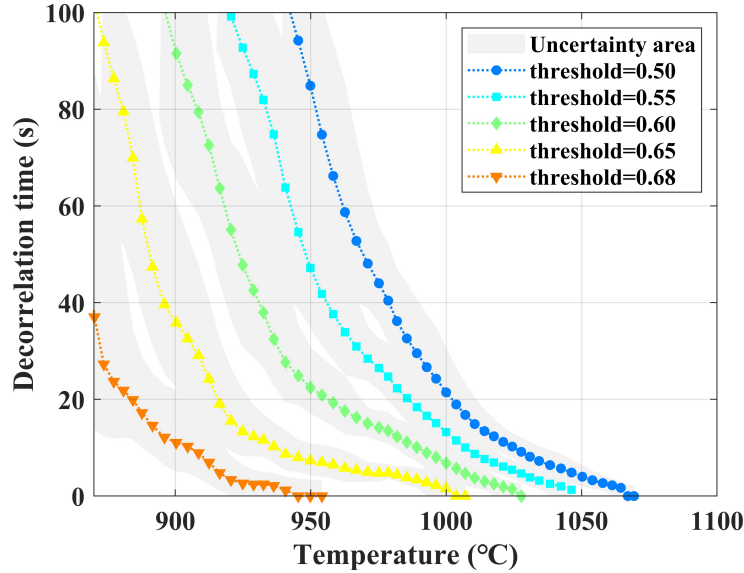


Figure 4.8: Decorrelation time evaluated at certain ranges of temperature when the coefficient separately drops to 50% (blue circle), 55% (cyan square), 60% (green square), 65% (yellow upper-triangle), and 67.9% (orange lower-triangle) of correlation. The shaded grey area for each marked line represents the 95% confidence interval. The correlation drops to 50% at 1000 °C in approximately 20 s, while taking the same time to reach 55% at 990 °C, 60% at 960 °C, 65% at 920 °C, and only 68% at 880 °C. This indicates rapidly changing thermal coefficients in the small temperature range above 880 °C.

4.3. RESULTS AND DISCUSSION

The results are presented in Figure 4.8. The analysis clearly shows the trade-off between operating temperature and sensor lifetime. For example, at 1000 °C, the signal decorrelates to 50% in approximately 20 seconds. At lower temperatures, the fibre is more durable; a 65% correlation is maintained for 20 seconds at 920 °C. The grey areas represent the 95% confidence intervals, calculated over a small spatial region of 11 points (88 μm). This shows that the decorrelation time is also associated with the spatial sensing resolution, as the selected size of the fibre segment determines the total amount of absorbed radiation energy and thus the degree of deformation.

The concept of decorrelation time provides a physical explanation for the upper limits of using telecom fibre for high-temperature measurements, especially in furnaces where long heating and thermal equilibrium processes are required. During these long exposures, the correlation is progressively erased by glass structural changes. The faster heat release of a hydrogen flame, however, enables the characterization of these high-temperature limits on a shorter timescale. It implies the feasibility of using standard SMF for measuring fast combustion processes, where the high-density thermal energy exposure is time-limited. Provided the applied temperature remains below the point where significant deformation occurs, the fibre is reusable. Ultimately, a trade-off between the sustainable measurement time and the upper limit of temperature must be considered in any practical scenario, and the decorrelation time provides a critical metric for making this assessment.

4.4. CHAPTER CONCLUSION

4.4 CHAPTER CONCLUSION

This chapter transitioned the application of OFDR from the short-duration, high-energy environment of an electrical arc to the continuous, high-temperature domain of a hydrogen flame. The study provided a spatially-resolved characterization of the combustion process with detailed analysis of the optical fibre sensor's performance and durability under prolonged thermal load. The findings confirm OFDR's capability as a useful diagnostic tool for extreme combustion environments and establish a new framework for understanding the operational limits of silica-based fibre sensors.

The investigation yielded two primary sets of results. Firstly, by implementing a non-linear calibration based on a third-order polynomial, we achieved distributed temperature mapping of the hydrogen flame, resolving peak temperatures up to 1100 °C (corresponding to a 16 nm spectral shift). This high-resolution mapping enabled a detailed characterization of the flame's thermal structure with respect to fuel flow rate and position, culminating in a quantitative analysis of thermal uniformity with direct applications for processes like fibre tapering. Secondly, this work investigated the fibre sensor itself as a subject of study. By analyzing the temperature-dependent degradation of the Rayleigh spectral correlation, we quantified the onset of permanent structural deformation above 900 °C. A decorrelation time is introduced to assess the fibre's operational lifetime, demonstrating that the sensor's signal integrity drops by 50% in approximately 20 seconds at 1000 °C.

The significance of this work lies in its dual contribution to both combustion diagnostics

4.4. CHAPTER CONCLUSION

and sensor science. The concept of a decorrelation time provides a physical explanation for the known limitations of using standard fibres in long-duration, high-temperature applications (*e.g.*, furnaces) and offers a practical methodology for determining sensor reusability. It demonstrates that standard, low-cost telecom fibre can be effectively deployed for transient measurements in extreme environments like micro-combustors, provided that its durability limits are understood. Ultimately, this study establishes that the trade-off between the maximum measurable temperature and the sustainable measurement time is a critical, quantifiable factor for reliable high-temperature sensing applications.

PART II

RANDOM FIBRE LASERS WITH
ULTRA-LOW NOISE

Chapter 5

BACKGROUND AND THEORETICAL PRINCIPLES OF RANDOM FIBRE LASERS

5.1 INTRODUCTION

In this chapter, we establish the theoretical foundation for Random Fibre Lasers (RFLs), which form the basis of the experimental investigations discussed in Chapter 6 and Chapter 7. The chapter begins by examining the noise characteristics inherent to RFLs, and then explores the operational principles of Erbium-Doped Random Fibre Lasers (EDRFLs). We then examine the unique role of Random Fibre Gratings (RFGs) in inducing distributed feedback and their contribution to laser performance. Next, the chapter covers the principles of self-injection locking (SIL) for laser stabilization and noise reduction, and is followed by a discussion of Optical Bistability (OB) and its relevance to nonlinear dynamics in random fibre lasers. This chapter provides the essential theoretical framework for understanding how the unique properties of random feedback can be harnessed and controlled, ultimately enabling

5.2. NOISE PROPERTIES OF RANDOM FIBRE LASERS

the low-noise and stable laser performance demonstrated in the subsequent experimental work.

5.2 NOISE PROPERTIES OF RANDOM FIBRE LASERS

Low-noise laser operation is essential for high-precision metrology, coherent communications, fibre optical sensing, and quantum technologies. In fibre lasers, ultra-low noise typically refers to two key metrics: Relative Intensity Noise (RIN), with levels below -160 dBc/Hz, and spectral purity, indicated by a Lorentzian linewidth of approximately 100 Hz or less. In this section, I focus on the noise characteristics inherent to Random Fibre Lasers (RFLs) and provide motivation for the noise reduction techniques explored in this thesis.

5.2.1 COHERENT VS. INCOHERENT FEEDBACK AND ITS IMPACT ON LASER NOISE

The feedback mechanism in a RFL governs its spectral properties and noise performance. This feedback, generated by light scattering back to the gain medium, can be broadly categorized as either coherent (resonant) [10, 11] or incoherent (non-resonant) [8, 9, 76]. Incoherent feedback is governed by intensity fluctuations, whereas coherent feedback involves amplitude or field interference. Depending on whether the optical phase is preserved between scattering events, this distinction leads to markedly different noise characteristics [77–79].

The RFLs examined in this thesis operate in the coherent feedback regime, which

5.2. NOISE PROPERTIES OF RANDOM FIBRE LASERS

dominates in one-dimensional waveguide systems such as optical fibres. In this regime, the elastic nature of scattering and the transverse confinement of light preserve phase relationships over long paths, allowing scattered light from different points to interfere constructively or destructively. Phase coherence enables phase-sensitive interference of multiply scattered light, giving rise to randomly distributed, spectrally narrow resonant modes whose feedback strength depends critically on accumulated phase matching [78, 80]. Coherent feedback can arise from intrinsic Rayleigh scattering (RS) in standard fibres or from engineered Random Fibre Gratings (RFGs). RS relies on weak, naturally occurring inhomogeneities, requiring kilometre-scale fibre lengths to accumulate sufficient feedback and yielding a dense spectrum of random modes with MHz or sub-MHz spacing [9, 81]. In contrast, RFGs introduce enhanced scattering centres through index modulations, enabling much shorter cavities ranging from centimetres to metres, yet still supporting random modes with complex, spectrally overlapping envelopes on nanometre or sub-nanometre scales [10, 11]. Both RS- and RFG-based systems can incorporate Raman [9], Brillouin [82], or erbium-doped gain [83, 84], with scattering from random density fluctuations or engineered index variations producing the high-coherence, spectrally narrow resonant modes that underpin RFL operation.

While this coherent mechanism enables lasing, its reliance on phase-sensitive interference is the direct origin of the dominant noise sources in RFLs:

Mode Competition and Hopping: The distributed feedback from local scattering supports an extremely high density of potential lasing modes. Unlike the regularly spaced

5.2. NOISE PROPERTIES OF RANDOM FIBRE LASERS

modes of a conventional cavity, these random modes are often closely spaced and spectrally overlapped, leading them to compete intensely for the available gain. This intense competition causes the laser to switch unpredictably between modes, resulting in large fluctuations in both output intensity and instantaneous frequency. In basic RFL configurations, mode competition is the primary source of intensity noise and a major contributor to linewidth broadening and frequency jitter.

Environmental Phase Sensitivity: The resonant modes are defined by the interference of light traversing countless paths within the fibre. Consequently, their relative phases are highly sensitive to environmental perturbations. Thermal fluctuations, acoustic vibrations, and mechanical stress alter the fibre's local refractive index and physical path lengths, which in turn modifies the interference conditions. This leads to fluctuations in the feedback strength and phase for any given mode, causing variations in output power (intensity noise) and shifts in the lasing frequency (frequency noise and drift).

Amplified Spontaneous Emission (ASE): As a fundamental source of quantum noise, ASE arises from the spontaneous decay of excited states in the gain medium. In the distributed-feedback environment of an RFL, spontaneously emitted photons are scattered and amplified along multiple paths. This not only adds a broad noise background but also simultaneously seed numerous lasing modes, thereby exacerbating the modal competition and contributing to the overall noise profile.

5.2. NOISE PROPERTIES OF RANDOM FIBRE LASERS

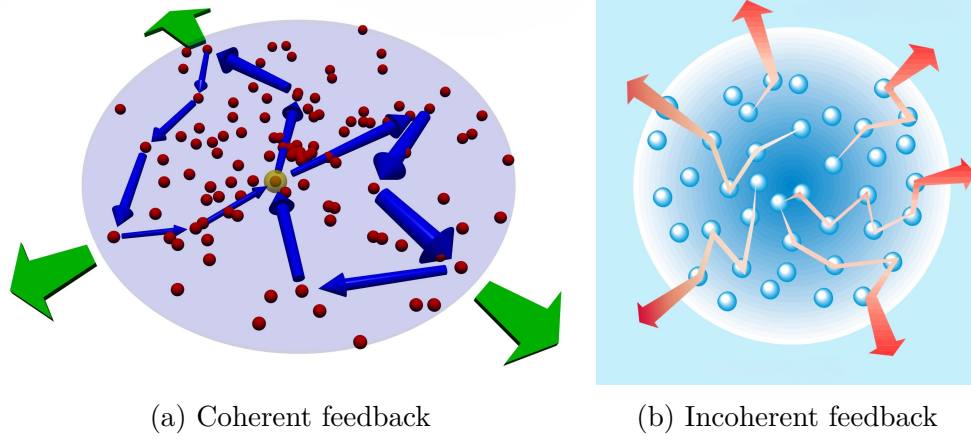


Figure 5.1: Comparison of emission spectra from random lasers operating under (a) coherent and (b) incoherent feedback regimes. (a) Coherent feedback preserves optical phase, producing numerous sharp, randomly spaced resonant modes and leading to mode competition and noise (adapted from <https://www.eng.yale.edu/stonegroup/science.html>). (b) Incoherent feedback randomizes phase, yielding a single smooth, broad emission peak governed by gain narrowing and lacking fine modal structure (adapted from Wiersma [85]).

As shown in Figure 5.1 (a), the coherent nature of the feedback is the primary source of the intrinsic noise challenges in RFLs. Firstly, the phase-sensitive interference makes the laser highly susceptible to environmental perturbations. Thermal fluctuations, acoustic vibrations, or mechanical stress alter the fibre’s refractive index and physical path lengths, changing the interference conditions. This causes the resonant frequencies of the random modes to drift, leading to frequency noise. Secondly, the disordered structure supports a very high density of potential lasing modes within the gain bandwidth. These modes compete intensely for the available gain, leading to mode hopping and large fluctuations in the output power, which manifest as high Relative Intensity Noise (RIN), particularly at low frequencies [86]. While a single, stable coherent mode can exhibit a narrow linewidth [82,

5.2. NOISE PROPERTIES OF RANDOM FIBRE LASERS

87], the complex interplay of these modes often leads to stochastic intensity dynamics and chaotic behaviour [88], which have been studied extensively through light-wave kinetics [89, 90] and analyses of transient dynamics [91].

In contrast, incoherent feedback occurs in strongly scattering, typically three-dimensional media where the optical phase is randomized over very short distances, eliminating constructive interference [77]. As shown in Figure 5.1 (b), feedback in this regime arises from the total intensity of scattered light returned to the gain medium, with phase information lost or averaged out. Lasing occurs when the average backscattered intensity, amplified by the gain medium, exceeds system losses, often producing broad emission spectra without sharply defined resonant modes. Such systems are commonly realized with laser dyes [92], semiconductors [93], or rare-earth-doped materials [94] combined with scattering elements such as dielectric [95] or semiconductor nanoparticles [92], powders [93, 96], or other disordered structures [97], with quasi-one-dimensional implementations demonstrated in photonic crystal fibres filled with dye and TiO_2 particles [8]. Incoherent feedback requires either sufficiently strong scattering to approach the Anderson localization threshold [98] or conditions where coherence is degraded by nonlinear effects or extremely long cavity lengths [76]. Because no closed scattering loops preserve optical phase, phase coherence is not maintained, and lasing depends solely on the average intensity of backscattering exceeding losses rather than strict phase matching. The resulting emission spectrum is broad and smooth, shaped primarily by the gain profile and narrowed only by gain saturation, lacking the sharp resonant modes and phase-sensitive interference of coherent random lasers. Consequently, incoherent RFLs avoid mode hopping and are less sensitive to environmental

5.3. THE GAIN MEDIUM: ERBIUM-DOPED FIBRE

phase fluctuations, but they also sacrifice the spectral purity achievable in coherent systems.

The RFLs investigated in this thesis (Chapter 6 and Chapter 7) utilize low-doped erbium-doped fibre (EDF) as the gain medium in conjunction with random fibre gratings (RFGs), operating under a coherent feedback regime owing to strong scattering from microscale refractive index modulation within the fibre core. The objective of this part is to mitigate noise intrinsic to coherent feedback, such as intensity noise and mode hopping due to mode competition, as well as frequency noise driven by phase sensitivity. To mitigate these challenges, we implemented self-injection locking and saturable absorption, which significantly reduced intensity noise, suppressed mode hopping, and narrowed the output linewidth.

5.3 THE GAIN MEDIUM: ERBIUM-DOPED FIBRE

The gain medium employed in the RFLs is erbium-doped fibre (EDF), widely used in modern photonics due to its amplification in the 1530–1565 nm range [99, 100], which aligns with the C-band and corresponds to a low-loss transmission window in optical fibre communications [101]. The high efficiency of EDFs, along with their compatibility with standard fibre components and suitability for semiconductor pumping schemes, has led to their widespread adoption in telecommunications amplifiers (EDFA) [102] as well as in versatile laser applications. Achieving ultra-low noise and narrow linewidth in EDRFLs requires careful optimization of both the spectroscopic properties of the Er^{3+} ions and the laser cavity design. This section reviews the physical mechanisms that enable gain in

5.3. THE GAIN MEDIUM: ERBIUM-DOPED FIBRE

EDF and discusses the principal noise and stability challenges that must be addressed in designing low-noise EDRFLs.

5.3.1 GAIN MECHANISM AND PROPERTIES OF EDF

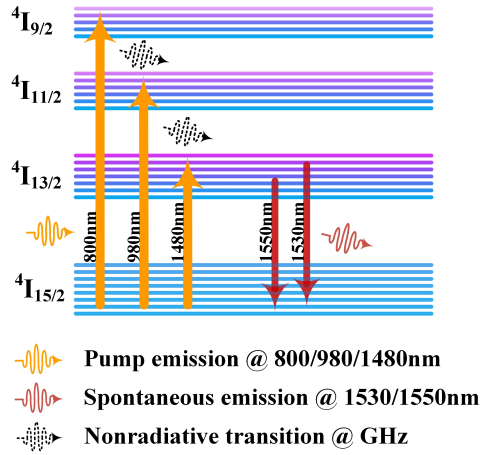


Figure 5.2: Energy level diagram of Er^{3+} ions in silica glass host with four lowest manifolds: $4I_{15/2}$, $4I_{13/2}$, $4I_{11/2}$. The diagram illustrates the Stark-split energy levels of Er^{3+} ions in an erbium-doped fibre, showing the 800 nm, 980 nm and 1480 nm pump absorption pathways to the metastable $4I_{13/2}$ level and the subsequent 1550 nm emission transition to the ground state $4I_{15/2}$.

Erbium ions in a silica host exhibit several energy manifolds, and the spectroscopic properties of trivalent erbium (Er^{3+}) are determined by their electronic configurations within the silica glass matrix of the optical fibre. Figure 5.2 illustrates the four lowest energy levels of Er^{3+} associated with the $4f$ inner electronic configuration, namely $4I_{15/2}$, $4I_{13/2}$, $4I_{11/2}$, and $4I_{9/2}$. The local environment surrounding the Er^{3+} ions modulates and splits these energy levels through local electric fields, with the magnitude of the splitting depending on the field

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strength [103].

External energy is supplied via pump photons from semiconductor laser diodes operating at wavelengths corresponding to strong Er^{3+} absorption bands. Population inversion is achieved either by optical pumping near 980 nm through the $^4I_{15/2} \rightarrow ^4I_{11/2}$ transition, which exhibits a peak absorption cross-section on the order of 1×10^{-25} to 10^{-24} m^2 in silica fibres [15], followed by rapid non-radiative relaxation to the metastable $^4I_{13/2}$ level, or by direct pumping at 1480 nm via the $^4I_{15/2} \rightarrow ^4I_{13/2}$ transition, which has a comparable cross-section but broader absorption bandwidth [104]. The fluorescence lifetime τ of the metastable level $^4I_{13/2}$ in EDF is around 10 ms [105], which corresponds to the radiative decay from $^4I_{13/2}$ to $^4I_{15/2}$ and has 99.93% quantum efficiency [102]. In essence, 980 nm pumping establishes a three-level laser system, whereas 1480 nm pumping operates as a quasi-two-level system.

Spontaneous decay from the metastable $^4I_{13/2}$ state generates ASE, which acts as a fundamental source of quantum noise and can initiate laser oscillation. The pumping process increases the population density in the $^4I_{13/2}$ level relative to the ground state $^4I_{15/2}$, thereby establishing population inversion. Once the pump intensity exceeds the lasing threshold, stimulated emission dominates at the $^4I_{13/2} \rightarrow ^4I_{15/2}$ transition, providing coherent optical amplification.

In Erbium-doped Fibre Laser EDFL systems, 980 nm pumping provides a lower noise figure and high gain per meter, making it attractive for compact amplifiers and laser designs; however, the larger quantum defect at this wavelength, arising from non-radiative

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relaxation from the $^4I_{11/2}$ to $^4I_{13/2}$ level, induces thermal loading within the fibre [102]. This heating alters the refractive index and physical length of the fibre, shifting the cavity phase and degrading stability, thereby necessitating enhanced thermal management for reliable operation. In contrast, 1480 nm pumping benefits from a smaller quantum defect, yielding higher pump-to-signal conversion efficiency and reduced heat generation per unit of optical gain [102]. The negligible excited-state absorption (ESA) at 1480 nm further enables distributed pump absorption along the fibre length, producing a flatter population inversion, higher saturation power, and more efficient energy extraction compared to 980 nm-pumped systems. Consequently, 1480 nm-pumped EDFL exhibit lower thermo-optic drift, improved linewidth stability, and reduced relative intensity noise (RIN) over multiple round trips, with overall gain dynamics and noise performance governed by the interplay between absorption efficiency, emission bandwidth, and lifetime.

5.3.2 HOST GLASS AND FIBRE GEOMETRY

The material properties of the silica host not only define the thermal and mechanical stability of the fibre but also critically shape the spectroscopic characteristics of EDFLs. The small-signal gain, bandwidth, saturation power, nonlinear susceptibility, and lasing threshold of an erbium-doped fibre (EDF) are determined by the Er^{3+} concentration, core composition, and mode-field diameter (MFD).

Typical Er^{3+} concentrations in EDF range from 500 to 2500 ppm. High concentrations increase small-signal gain but also risk inducing concentration quenching due to ion-ion

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interactions [106] and clustering effects [107]. Co-dopants such as aluminum (Al) broaden the gain bandwidth and help suppress clustering [107], while germanium (Ge) increases the core refractive index, enhancing pump absorption and optical gain efficiency [106]. Reducing the MFD further strengthens pump–signal overlap and boosts gain per unit length, but it also elevates intracavity intensities, exacerbating nonlinear effects such as four-wave mixing (FWM), stimulated Brillouin and Raman scattering (SBS) [108]. Therefore, in selecting EDFs for EDRFL design, it is crucial to balance small-signal gain, low ASE background, efficient pump absorption, and control of nonlinear effects.

5.3.3 NOISE AND INSTABILITY ARISING FROM THE GAIN MEDIUM

Erbium-doped fibre (EDF) provides efficient optical gain in the telecom band but its intrinsic spectroscopic and dynamic properties also give rise to fundamental noise and stability limits. In an EDFL, the pumping conditions, population inversion, and gain saturation dynamics govern intensity and frequency stability, with fluctuations in the pump transferring directly to the signal. These processes are described by the laser rate equations [109], which account for field–population interactions and spontaneous emission. As a result, EDFLs are limited by ASE [105], relative intensity and frequency noise, mode competition, spatial hole burning, and other nonlinear effects. Characterizing these mechanisms is essential for designing low-noise random fibre lasers. The primary noise mechanisms are as follows:

Amplified spontaneous emission (ASE): The process of stimulated emission is inevitably accompanied by spontaneous emission. These spontaneously emitted photons,

5.3. THE GAIN MEDIUM: ERBIUM-DOPED FIBRE

originating from the decay of excited Er^{3+} ions within the fibre, are amplified as they propagate through the gain medium [105]. In the laser system, this ASE constitutes a broadband quantum noise floor that degrades the optical signal-to-noise ratio and competes with the lasing mode for available gain. The power of this noise is described by [110]:

$$P_{\text{ASE}} = 2 n_{\text{sp}} h\nu (G - 1) \Delta\nu, \quad (5.1)$$

where n_{sp} is the spontaneous emission factor, $h\nu$ the photon energy, G the small-signal gain in linear scale, and $\Delta\nu$ the optical noise bandwidth [99]. Since both G and $\Delta\nu$ increase with the active fibre length and Er^{3+} concentration, using shorter fibres with lower doping concentrations can help suppress ASE.

Pump noise transduction: The population inversion in the EDF acts as a low-pass filter that transduces fluctuations from the pump source into Relative Intensity Noise (RIN) on the laser output. The dynamics of this process are governed by the upper-state lifetime (τ_g) of the Er^{3+} ions, a fundamental property of the doped fibre. The transfer function for this noise is given by:

$$\text{RIN}_{\text{sig}}(f) = \text{RIN}_{\text{pump}}(f) |1 + j 2\pi f \tau_g|^{-2}. \quad (5.2)$$

This relationship illustrates how a microscopic property of the EDF dictates how extrinsic pump noise manifests as intensity noise in the final laser system.

Nonlinear effects: The geometry of the EDF whose tight confinement of light within

5.4. *RANDOM FIBRE GRATINGS (RFGS)*

its small core, leads to high intracavity optical intensities. These high intensities can induce nonlinear optical effects, such as Four-Wave Mixing (FWM), which can generate unwanted frequency sidebands and degrade the spectral purity of the laser output. Therefore, the selection of an EDF with specific geometric and doping properties is a critical strategy for managing nonlinear impairments within the laser cavity. In particular, employing a low-concentration and long-length EDF plays an essential role in suppressing these nonlinear phenomena by controlling the mode confinement and doping levels.

5.4 RANDOM FIBRE GRATINGS (RFGs)

Similar to disordered electronic systems, where both the degree of disorder and electron–electron interactions govern localization [111], the study of dimensionality and photon–photon interactions in disordered optical media is central to realizing photon localization [77, 80] and its promising applications [78]. Random fibre lasers (RFL) represent a non-conventional class of lasers in which disorder-induced scattering provides feedback, either through intrinsic Rayleigh scattering (RS) in long fibres [112] or through random fibre gratings (RFG) with artificially enhanced and spectrally engineered feedback [113]. While RS-based RFL require kilometre-scale fibres and offer limited control, RFG combine strong scattering and diffraction due to index modulation, enabling compact architectures with enhanced coherent feedback. Such RFG-based RFL can achieve low thresholds and narrow linewidths [11], and their stability against drift and noise can be further improved

5.4. RANDOM FIBRE GRATINGS (RFGS)

by employing diverse gain mechanisms such as stimulated Raman or Brillouin scattering [9, 114]. These systems thus provide not only a promising platform for high-performance lasers but also a unique setting to explore photon localization in one-dimensional disordered media.

5.4.1 FABRICATION VIA FEMTOSECOND-LASER INSCRIPTION

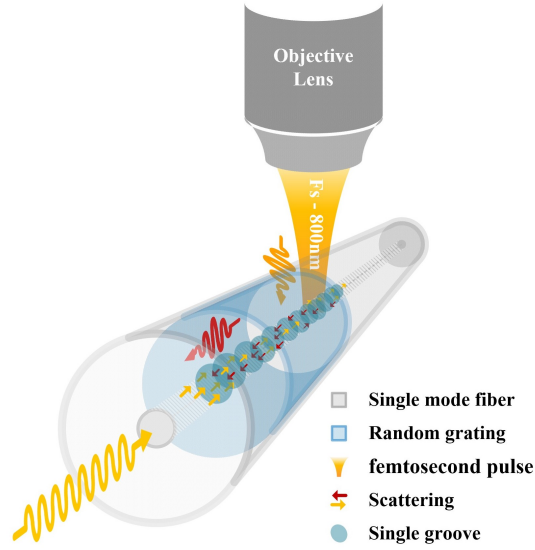


Figure 5.3: Schematic of Random Fibre Grating (RFG) fabrication using focused femtosecond (800 nm) laser pulses. The laser is focused through an objective lens onto a single-mode fibre, inducing localized refractive index modifications that form individual grating grooves. The resulting random grating structure enhances reflection by providing coherent feedback.

The RFGs employed in this work are fabricated using a point-by-point inscription technique with a femtosecond (fs) laser [115, 116], as shown in Figure 5.3. This direct-write method

5.4. RANDOM FIBRE GRATINGS (RFGS)

involves focusing high-intensity fs-laser pulses into the core of an optical fibre. Each pulse creates a highly localized, permanent modification of the refractive index. By precisely translating the fibre relative to the fixed laser focus, a series of these index-modulation points can be inscribed along the core. To create the disordered structure of an RFG, a controlled random variation is introduced into the spacing between adjacent inscription points. This technique offers excellent control over the grating's length, average index contrast, and the statistical properties of the disorder, allowing the feedback characteristics to be tailored for specific laser designs.

5.4.2 CHARACTERISTICS AND ROLE IN RFLS

The RFG serves as the primary feedback element in the laser cavity, acting as a highly efficient, spectrally complex distributed mirror. Its two primary roles are providing distributed feedback and defining the potential lasing modes.

Distributed feedback: In contrast to a conventional Fibre Bragg Grating (FBG), which relies on periodic index modulation and exhibits a single, well-defined reflection peak, an RFG consists of a randomized sequence of scattering planes. As light propagates through the structure, reflections from these planes interfere coherently, producing a complex, pseudo-random reflection spectrum. Although rich in spectral detail, this spectrum is static and deterministic for a given grating, thereby offering strong wavelength-selective feedback within a spectral band determined by the average spacing of the scattering sites.

5.4. RANDOM FIBRE GRATINGS (RFGS)

Mode selection: The reflection spectrum of an RFG comprises a dense set of narrow, high-Q resonant peaks. Each peak corresponds to a wavelength at which multiple scattered waves interfere constructively, forming a localized high-reflectivity state. These sharp resonances define the potential longitudinal modes of the random laser. When coupled with a gain medium, the laser preferentially oscillates on those RFG modes that experience the highest net gain, resulting in emission with a narrow spectral profile.

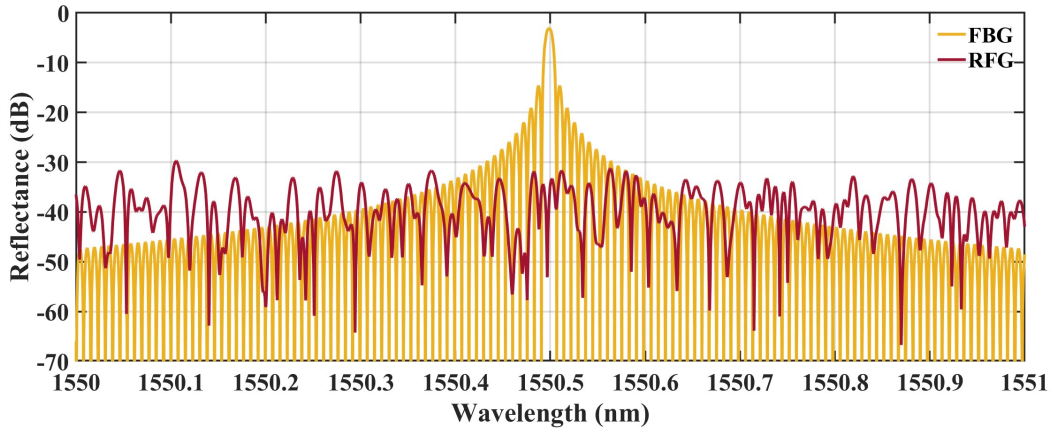


Figure 5.4: Comparison of simulated reflection spectra for a periodic Fibre Bragg Grating (FBG) and a Random Fibre Grating (RFG). The FBG (yellow line) displays a localized, deterministic peak governed by the periodic Bragg condition. In contrast, the RFG (red line) exhibits a stochastic distribution of resonant modes across a broad spectral range, resulting from multiple coherent scattering in the disordered refractive index profile.

Figure 5.4 illustrates the contrast between periodic and disordered feedback. A conventional FBG provides a deterministic, narrowband reflection at the Bragg wavelength, whereas an RFG produces a stochastic spectrum arising from random phase accumulation

5.5. SELF-INJECTION LOCKING (SIL)

along the fibre. Each spectral spike corresponds to a localized resonance within the disordered medium. In contrast to the single-mode selection of an FBG, the RFG supports a dense manifold of modes, where lasing occurs on those experiencing the highest net gain. The spectral overlap of these modes leads to strong mode competition and stochastic intensity dynamics, as discussed in Section 5.2.

5.5 SELF-INJECTION LOCKING (SIL)

Optical injection locking (OIL) is an optical frequency and phase synchronization technique based on the photon-photon interaction when an external light is injected into a laser cavity [109]. Injection locking was first studied for electronic oscillators by Adler in 1946 [117]. Following the invention of the laser in 1960 [118], Pantell extended Adler's oscillator locking theory to optics, showing that an external signal could pull and lock a laser's frequency [119]. This was experimentally confirmed by Stover and Steier in 1966, who demonstrated OIL by injecting a weak signal into a He-Ne laser stabilized its frequency and narrowed its linewidth [12]. Injection locking has played a critical role in advancing laser frequency stabilization and linewidth narrowing across multiple platforms. Early demonstrations by Byer's group showed that a low-power single-frequency Nd:YAG could injection-lock a high-power, multimode Nd:YAG oscillator, enabling stable single-mode operation for precision spectroscopy and metrology [120]. Around the same time, Kobayashi and Kimura demonstrated injection locking in AlGaAs diode lasers, achieving significant linewidth reduction and noise suppression, a breakthrough for optical communications [121]. Lang

5.5. SELF-INJECTION LOCKING (SIL)

then developed the theoretical foundation by incorporating the linewidth enhancement factor into rate equations, providing the standard model for predicting locking range and dynamics in semiconductor lasers [13]. The technique extended to fibre systems when Jones and Urquhart injection-locked an EDFL with a diode seed, enabling narrow-linewidth, telecom-band fibre sources [122]. More recently, chip-scale self-injection-locked lasers have demonstrated sub-hertz instantaneous linewidth [123], achieving ultra-low noise performance comparable to bulk cavity-stabilized systems, paving the way for compact sources in atomic clocks, precision spectroscopy, and coherent communication.

5.5.1 CONCEPT OF SELF-INJECTION LOCKING

Self-injection locking (SIL) is a particularly elegant variant of this technique. Instead of using a separate master laser, a fraction of the laser’s own output is passed through a narrowband optical filter, typically a high-Q resonator, and then fed back into the main laser cavity. This time-delayed, spectrally purified feedback signal acts as the master signal, forcing the laser to lock to one of the narrow resonant modes of the external filter. SIL has proven highly effective for the spectral purification of both semiconductor and fibre lasers and is the key technique used in this work to overcome the intrinsic instabilities of RFLs. The complex dynamics of this locking process are described by a multi-delay variant of the Lang-Kobayashi equations, the full model for which is detailed in Appendix D.

5.5. SELF-INJECTION LOCKING (SIL)

5.5.2 PRINCIPLE OF INJECTION LOCKING

The fundamental physics of injection locking can be understood by considering the laser as an oscillator with a natural, free-running frequency. When an external, stable signal is injected, its electromagnetic field interacts with the field inside the laser cavity. This interaction effectively applies a torque to the phase of the slave laser's field, pulling its frequency towards that of the injected signal. If the frequency difference between the two is small enough (within the locking range) and the injected power is sufficient, the slave laser's frequency and phase will become completely synchronized, or locked, to the master signal. The locking range is proportional to the injection strength and the bandwidth of the laser resonator. Within this range, the slave laser inherits the frequency stability and spectral purity of the master signal.

5.5.3 NOISE REDUCTION MECHANISM IN SIL

When applied to a noisy laser like an RFL, SIL is exceptionally effective at suppressing both frequency and intensity noise.

Frequency noise and linewidth narrowing: The origin of a laser's linewidth lies in spontaneous emission, which introduces phase noise. Every spontaneous emission event instantaneously adds a phasor of unit amplitude and random phase to the laser field. The resulting intensity perturbation decays via relaxation oscillations; during that recovery the population inversion changes, altering both the imaginary (gain)

5.5. SELF-INJECTION LOCKING (SIL)

and real (index) parts of the refractive index. Because the phase shift induced by this index change is delayed relative to the original event, the phase and amplitude noises become correlated, which significantly broadens the laser line beyond the basic Schawlow-Townes limit. Self-injection locking directly counteracts this intrinsic phase noise. The high-Q external resonator acts as a short-term phase memory. It filters the rapid phase fluctuations (*i.e.*, frequency noise) of the free-running RFL and returns a phase-stabilized optical signal. This feedback continuously corrects the laser's intracavity phase, forcing it to follow the highly stable resonance of the passive external cavity. This results in a transfer of stability: the laser's linewidth is narrowed by orders of magnitude, inheriting the spectral purity of the high-Q resonator.

Intensity noise suppression: As established in Section 5.2 the main source of intensity noise in RFLs is the intense competition between a dense spectrum of random modes. By providing highly selective, resonant feedback at a single frequency, the locking mechanism gives one specific mode a significant gain advantage over all others. This chosen mode is amplified preferentially, suppressing the oscillation of its competitors and reducing mode hopping. This leads to a stable, single-mode output with a dramatic reduction in low-frequency relative intensity noise (RIN).

5.5.4 ROLE OF HIGH-Q RESONATORS (RFGR RESONATOR)

In this work, the high-Q resonator for SIL is an all-fibre Random-Fibre-Grating Ring (RFGR), the basic diagram is shown in Figure 5.5. This compact device is formed by

5.5. SELF-INJECTION LOCKING (SIL)

splicing an RFG into a closed loop of optical fibre. The RFGR provides the sharp, stable spectral filtering required for robust locking.

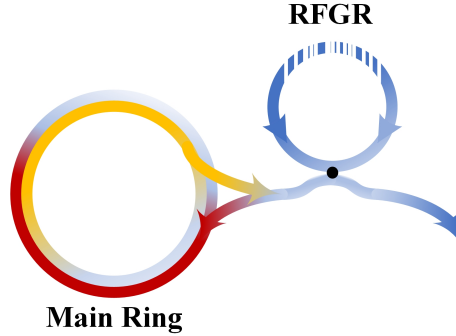


Figure 5.5: Schematic of the self-injection locking (SIL) architecture. The main ring contains the gain medium, while the Random-Fibre-Grating Ring (RFGR) acts as a high-Q, frequency-selective feedback element, enforcing single-mode operation.

Achieving single-mode oscillation with a stand-alone RFG is challenging because the erbium gain bandwidth is orders of magnitude broader than the mode spacing. By embedding the RFG within a fibre ring, the filtering effect is greatly enhanced. The ring structure itself imposes a periodic set of resonant frequencies (similar to a Fabry-Pérot cavity), while the RFG within the ring introduces its own complex, high-resolution spectral signature due to multiple scattering interference from its random structure. The combination of these two effects creates an extremely sharp and selective transmission filter.

During SIL, a fraction of the radiation from the main RFL cavity is sent into the RFGR. Only the light whose frequency precisely matches a resonant mode of this composite RFGR cavity can circulate efficiently and be fed back to the main laser. This selectively amplified, phase-correlated feedback is what locks the main laser and ensures robust, stable operation.

5.6. OPTICAL BISTABILITY

The random nature of the RFG, comprising thousands of scattering planes, creates a diffuse resonance cloud, allowing the locking condition to be robustly maintained even under small environmental perturbations like thermal drift. This enhances the environmental stability of the locked laser system. This complex filtering action is captured in our theoretical model (Appendix D) by treating the RFGR as a linear time-invariant filter whose effect is described by its complex impulse response, $h_{\text{RFGR}}(t)$. While SIL provides an active method for noise suppression through external feedback, the following section explores how internal nonlinearities can be utilized for passive stabilization.

5.6 OPTICAL BISTABILITY

Optical Bistability (OB) is a nonlinear phenomenon where a system can exhibit two stable output states for a single input, characterized by a hysteretic response, as typically shown in Figure 5.6 (c). First proposed by Szöke *et al.* in 1969 [124] and experimentally demonstrated in sodium vapour by Gibbs *et al.* in 1976 [14], the effect forms the foundation for all-optical control applications such as switching and memory elements [125]. The emergence of bistability fundamentally relies on two ingredients: a strong optical nonlinearity and a feedback mechanism. While OB was well-studied in conventional Fabry-Pérot cavities, its implementation and utility within a disordered random laser cavity are a main focus of this thesis. This section outlines the theoretical principles of the specific bistable mechanism engineered in our Erbium-Doped Random Fibre Laser (EDRFL).

5.6. OPTICAL BISTABILITY

5.6.1 PRINCIPLE OF OPTICAL BISTABILITY IN EDRFLs

Optical bistability in the EDRFL is fundamentally a consequence of intensity-dependent absorption and the distributed memory of the system's excitation history. Unlike bulk systems, the saturable absorber (SA) in a fibre laser is an extended medium where the population inversion, $\Delta N(z)$, is non-uniformly distributed along the fibre coordinate z . This spatial distribution, and its dependence on the direction of pump power evolution, gives rise to the hysteretic response.

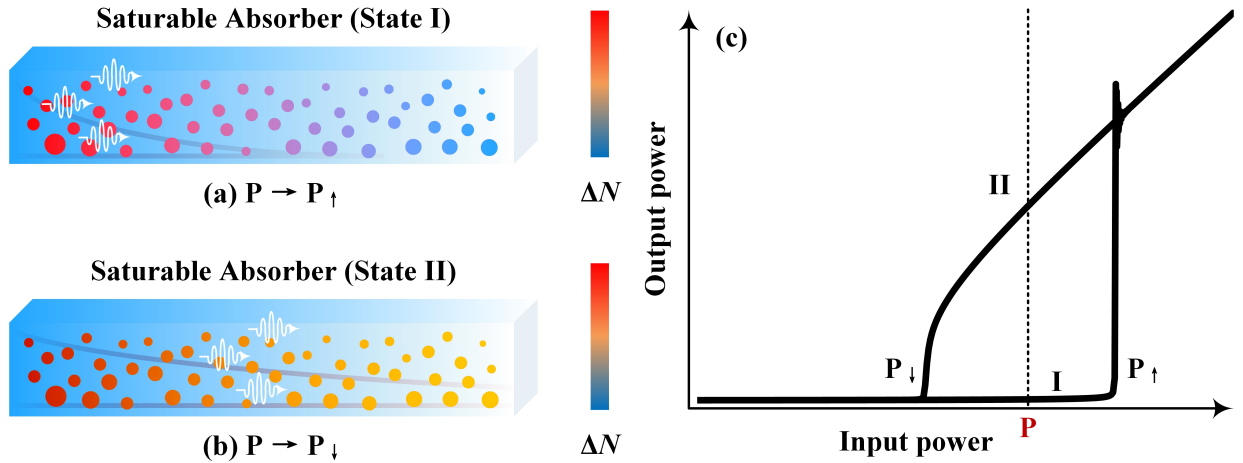


Figure 5.6: Mechanism of distributed absorptive bistability in an EDRFL. (a) State I (the upper branch): At low input powers, a steep longitudinal gradient of ΔN exists; Erbium ions near the input are partially excited, while the remainder of the fibre stays in the ground state, maintaining high total attenuation. (b) State II (the lower branch): Once bleached, the intense intracavity laser field reinforces a high population inversion across the entire length of the SA, maintaining a low-loss state. (c) Hysteresis curve (simulation): The macroscopic output power as a function of input power, illustrating the coexistence of two stable states (I and II) at a single input value P , determined by the system's memory of its excitation history.

5.6. OPTICAL BISTABILITY

The bistable transition can be understood through the evolution of this longitudinal population profile:

State I (the upper branch): As the input power increases from zero (Figure 5.6 (a)), the optical field is strongly attenuated as it propagates. Consequently, while the ions near the input end may reach a state of inversion, the power is insufficient to bleach the downstream sections of the fibre. These regions remain in the ground state, providing the dominant loss that prevents laser oscillation. When the input power reaches the switch-up threshold ($P_{\uparrow}$), the accumulated ASE power finally saturates the entire length of the SA. The resulting jump in net gain initiates lasing. Once lasing begins, the high intracavity photon flux acts to self-reinforce the saturation of the absorber.

State II (the lower branch): On this branch (Figure 5.6 (b)), the system exhibits a radically different population distribution. As the external pump power is reduced toward the switch-down threshold ($P_{\downarrow}$), the internal lasing field keeps the population inversion $\Delta N(z)$ high across the entire SA length. This memory of the lasing state keeps the system locked in a low-loss configuration, allowing stimulated emission to persist at power levels where it previously could not start. Consequently, for a single input power P , the system can support two distinct, stable longitudinal population profiles. This distributed inversion is the physical basis for the bistable operation, providing a mechanism to clamp the laser dynamics and passively mitigate the intensity noise characteristic of random feedback.

5.6. OPTICAL BISTABILITY

The complex interplay between the propagating optical powers (pump, ASE, and laser signal) and the population dynamics in both the gain and absorber sections is described quantitatively by the detailed propagation and rate equation model presented in Appendix E.

5.6.2 CONCURRENT DISPERSIVE EFFECTS AND HYBRID BISTABILITY

While saturable absorption is the driver of bistability, the transition is inherently hybrid, involving a concurrent dispersive nonlinearity. The population dynamics (ΔN) governing gain and absorption are fundamentally linked to the fibre's complex refractive index. Any intensity-dependent change in the gain spectrum, $\delta g(\nu)$, induces a corresponding change in the real part of the refractive index, $\delta n_{\text{kk}}(\nu)$, according to the Kramers-Kronig relations:

$$\delta n_{\text{kk}}(\nu_c) = -\frac{c}{\pi^2} \mathcal{P} \int_0^\infty \frac{\delta g(\nu')}{\nu'^2 - \nu_c^2} d\nu'. \quad (5.3)$$

In addition to this resonant contribution, the high intracavity intensities, particularly at fibre interfaces and splice points, induce a non-resonant electronic nonlinearity via the optical Kerr effect:

$$\delta n_{\text{k}}(z) = n_2 I(z), \quad (5.4)$$

where n_2 is the nonlinear refractive index and $I(z)$ is the local optical intensity. As demonstrated in Chapter 7, these power-dependent index variations modify the phase conditions of the random cavity, creating a hybrid bistable regime that effectively clamps the laser dynamics and suppresses mode competition. The clamping of both gain and loss

5.6. OPTICAL BISTABILITY

establishes a robust equilibrium, leading to a significant reduction in intensity noise and demonstrating a novel pathway for creating passively stabilized random lasers.

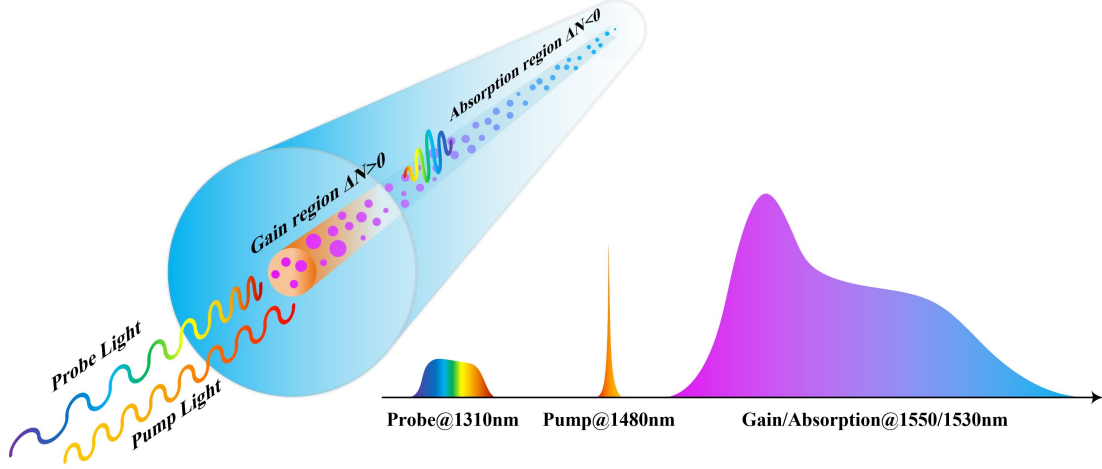


Figure 5.7: Principle of *in situ* diagnosis for population dynamics in a bistable EDRFL. An O-band probe light (1310 nm) interrogates the longitudinal state of the fibre. The pump-induced transition between gain ($\Delta N > 0$) and absorption ($\Delta N < 0$) regions modifies the local refractive index via the Kramers-Kronig relation. These index variations, along with localized Kerr nonlinearities, are detected as wavelength shifts in the Rayleigh scattering pattern, providing a spatial map of the laser's internal state.

To resolve the spatial evolution of these nonlinearities, an *in situ* diagnostic based on phase-sensitive Optical Frequency Domain Reflectometry (OFDR) is employed (Figure 5.7). By utilizing a probe signal in the O-band (1310 nm), the internal state of the C-band (1550 nm) laser is interrogated without spectral interference. The longitudinal distribution of gain ($\Delta N > 0$) and absorption ($\Delta N < 0$) manifests as local variations in the effective refractive index, causing measurable wavelength shifts in the Rayleigh scattering spectrum. This spectro-temporal mapping provides direct observation of the spatial memory within

5.7. CHAPTER SUMMARY

the saturable absorber, confirming the existence of the two distinct population distribution modes that underpin the system's stable, low-noise operation.

5.7 CHAPTER SUMMARY

This chapter has established the essential theoretical groundwork for the design and analysis of the erbium-doped random fibre lasers (EDRFLs) investigated in this thesis. We began by identifying the core challenge inherent to this class of laser: the intrinsic intensity and frequency noise that arises from its reliance on coherent random feedback. This mechanism, while enabling lasing in a disordered medium, leads to severe mode competition and a high sensitivity to environmental perturbations, which are the primary obstacles to stable, low-noise operation.

We then examined the key physical components that constitute the laser systems. The principles of the erbium-doped fibre (EDF) were detailed, establishing it as both the gain medium and, through its population dynamics and nonlinear properties, a main source of system noise and instability. The random fibre grating (RFG) was introduced as the engineered feedback element, providing strong, spectrally complex, and localized feedback that enables the construction of compact and versatile random lasers.

Finally, the chapter detailed the two advanced physical principles harnessed in this work to control the laser's behaviour. First, self-injection locking (SIL) was presented as a powerful technique for active stabilization. By using a high-Q random-fibre-Grating Ring

5.7. CHAPTER SUMMARY

(RFGR) to provide spectrally pure optical feedback, SIL is capable of suppressing mode competition and dramatically narrowing the laser's linewidth, enforcing coherence and stability. Second, Optical Bistability (OB) was introduced as a fundamentally different approach: harnessing nonlinearity for passive control. The mechanism, driven by saturable absorption in a two-section EDRFL design, creates a hysteretic switching behaviour that contains a robustly stable operating regime, offering a pathway to passively mitigate intensity fluctuations.

Collectively, these principles, from the origins of noise in random systems to the advanced techniques for its control, provide the complete theoretical framework necessary to understand the experimental designs, interpret the results, and appreciate the significance of the low-noise, stabilized random fibre lasers demonstrated in the subsequent chapters.

Chapter 6

SELF-INJECTION LOCKING OF A LOW-NOISE ERBIUM-DOPED RANDOM FIBRE LASER

The preceding chapter discussed the principles of random fibre lasers (RFLs) and their instability caused by chaotic mode competition. To address this, we extend basic RFL designs by implementing an all-optical stabilization technique. This chapter presents an erbium-doped RFL stabilized by self-injection locking (SIL) with a high-Q random fibre grating ring (RFGR) resonator.

The RFGR supplies narrowband distributed feedback from random scattering within a ring, enabling SIL to suppress noise and narrow the linewidth. In this work, we demonstrate and characterize this RFGR-SIL laser configuration. We investigate its spectral properties, efficiency, linewidth, long-term stability, and its relative intensity noise (RIN) and frequency noise. The results show performance comparable to that of electronically stabilized

6.1. INTRODUCTION

commercial lasers, achieved through a fully passive, all-optical locking mechanism.

6.1 INTRODUCTION

Laser sources with high spectral purity, characterized by narrow linewidth and low intensity and frequency noise, are essential for applications requiring high coherence, such as precision metrology, optical communications, fibre sensing, and quantum technologies. Conventional lasers achieve this through well-defined cavities formed by discrete reflectors. In contrast, random lasers rely on feedback from disordered media, where interference and disorder can lead to Anderson localization [111]. Studying light in such systems is key to understanding and controlling photon localization [77, 80], with implications for both fundamental physics and emerging applications [78].

Random fibre lasers (RFLs) use disordered scattering as feedback, typically via intrinsic Rayleigh scattering (RS) from fibre inhomogeneities [112] or engineered random fibre gratings (RFGs) with enhanced index modulation [113]. Compared to RS-based feedback, RFGs introduce both scattering and diffraction, strengthening coherent feedback. To reduce low-frequency drift and intensity noise, various gain mechanisms have been explored, including stimulated Brillouin scattering (SBS) [114] and stimulated Raman scattering (SRS) [9]. Erbium-doped fibre (EDF) offers broader gain, shorter build-up time, and longer photon lifetime. RS-based erbium-doped random fibre lasers (EDRFLs) exhibit reduced thermal frequency noise at high frequencies due to their long cavity lengths [86].

6.1. INTRODUCTION

However, despite EDF being a common gain medium, EDRFLs often suffer from multi-mode operation, mode hopping, broad linewidth, and noise due to the dense random mode structure within EDF's broad gain bandwidth. While early demonstrations showed the potential for single-mode operation with RS and EDF gain [126], long fibre lengths yield high mode density and intensity instability. RFGs mitigate this by shortening the feedback length and increasing scattering density via deep index modulation using CO₂ lasers [113] or femtosecond pulses [127], offering greater environmental stability. Notably, RFLs based on artificial Rayleigh fibres [83] and randomly distributed grating arrays [127–130] have demonstrated linewidth narrowing and improved stability.

Single-mode operation can be achieved using an optical filter to select one narrow-linewidth mode, but any drift in the filter's peak frequency leads to corresponding laser frequency drift. This limitation can be mitigated by incorporating a random fibre grating in a ring (RFGR), which forms a highly adaptive, high-Q interferometric structure. Thousands of random modes establish interference within nanoseconds due to the short travel time in the RFGR, minimizing frequency drift. Under environmental perturbations, a new, stable resonance can rapidly form in the RFGR, providing feedback into the main EDF gain ring. This self-injection locking (SIL) mechanism ensures fast laser build-up, frequency locking, and narrow linewidth. Mode alignment between the RFGR and the main ring is maintained via polarization control.

In this chapter, we demonstrate a compact erbium-doped random fibre laser EDRFL based on SIL with RFGR. The configuration achieves single-mode operation with ultra-low intensity and frequency noise. The RFGR is fabricated by multi-photon absorption using

6.2. LASER DESIGN AND CHARACTERIZATION METHOD

800-nm femtosecond laser irradiation over a 5-cm fibre section, forming ~ 2000 randomly spaced sub-gratings, each $\sim 25\text{ }\mu\text{m}$ long, with random periods uniformly distributed from 0 to $2.5\text{ }\mu\text{m}$ [116]. The combined effects of distributed resonant feedback and SIL suppress noise and enhance mode stability. The laser's performance is systematically evaluated and compared to reference lasers.

6.2 LASER DESIGN AND CHARACTERIZATION METHOD

This section details the physical design of the stabilized EDRFL, explains the theoretical principles that govern its operation, and discusses the mechanisms through which this design enhances laser performance.

6.2.1 THE TWO-RING SELF-INJECTION LOCKING CONFIGURATION

The laser's architecture based on the SIL mechanism is realized through a compact two-ring topology, as illustrated schematically in Figure 6.1 (a). The design consists of a main ring that provides the optical gain, which is then critically coupled to a passive secondary ring containing the RFG. This secondary ring functions as the high-Q resonant reflector that stabilizes the entire system.

The main ring incorporates a 10-m EDF (Er-407, 9.3 dB/m @ 1533 nm, INO) with low erbium ion density, which serves as the gain medium. The EDF is backward-pumped by a 1480 nm semiconductor laser (FITEC Inc.) via a 1480/1550 nm wavelength division

6.2. LASER DESIGN AND CHARACTERIZATION METHOD

multiplexer (WDM, Micro-Optics Inc.). The use of a long, low-doping EDF helps suppress the $1/f$ noise of spontaneous emission [131], while acting as a distributed high-pass filter to reduce phase and amplitude fluctuations. Backward pumping further enhances saturation gain, enabling operation with high optical signal-to-noise ratio (OSNR). The RFGR is formed by inserting a 5-cm RFG into a 1.74-m fibre loop, coupled to the main ring via a 95:5 coupler. The RFG consists of a dense array of randomly spaced sub-gratings. The main ring and the RFGR are connected via two critical control components: a 50:50 optical coupler and a polarization controller (PC).

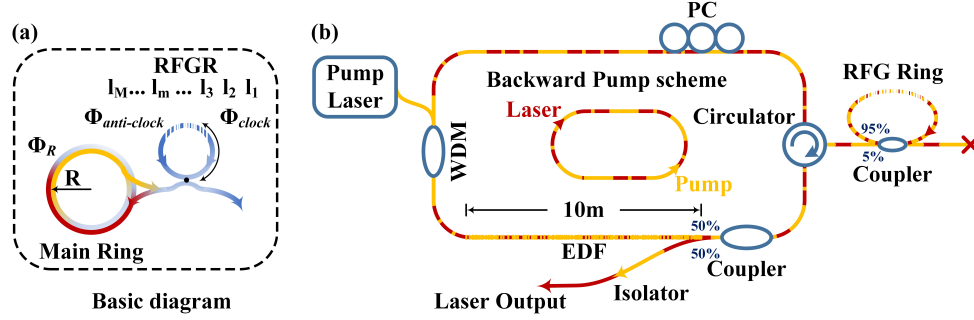


Figure 6.1: Schematic of EDRFL with (a) basic diagram of two-ring configuration; (b) set up in backward pumping scheme.

The 50:50 coupler determines the feedback coupling strength, κ , a key parameter in the Lang-Kobayashi model that governs the locking dynamics (see eq. (D.4) in Appendix D). The choice of a 50:50 ratio provides the optimal power exchange between the two cavities. This strong coupling is essential to ensure the feedback signal from the RFGR is sufficiently powerful to overcome the main laser's intrinsic noise, thereby establishing a wide and robust locking range [132].

6.2. LASER DESIGN AND CHARACTERIZATION METHOD

The PC acts as an adjustable optical element that counteracts the random birefringence of the fibre. By carefully adjusting the controller, a special, stable state of polarization, known as a polarization eigenstate, can be established. This eigenstate is a specific polarization that has the unique property of reproducing itself perfectly after one full circulation around the loop. Only when such a stable, self-consistent state is found can a powerful and coherent resonance form within the main cavity. This adjustment also provides fine control over the total round-trip phase delay, which allows the resonance frequency of the main ring to be precisely tuned to a target mode of the RFGR, satisfying the final condition for SIL.

6.2.2 CHARACTERIZATION METHODS

To evaluate the performance of the stabilized EDRFL, a suite of characterization techniques was employed to measure its spectral purity, intensity noise, and frequency stability.

First, the fundamental lasing characteristics were determined. The output power and slope efficiency were measured using an optical power meter while varying the pump power. The laser spectrum and OSNR were characterized using an optical spectrum analyzer (OSA).

Next, the laser's stability was quantified. The intensity noise was analyzed by directing the output onto a fast photodetector; the resulting time-domain signal was captured by a digital oscilloscope and then post-processed using a Fast Fourier Transform (FFT) to calculate the RIN spectrum. Frequency noise was measured using an unbalanced Michelson

6.3. *PRINCIPLE OF RFGR-BASED SELF-INJECTION LOCKING*

interferometer, which acts as a frequency discriminator to convert frequency fluctuations into measurable intensity fluctuations. The interferometer was constructed with a symmetric 3×3 optical coupler and included a 100-m fibre delay line in one arm, enabling phase noise characterization up to a 100 kHz bandwidth [133].

Finally, the laser’s high-coherence properties were measured. The laser linewidth and long-term stability were determined using a delayed self-heterodyne interferometry (DSHI) setup. This interferometer incorporated a 200-km delay line to ensure phase decorrelation and an acousto-optic modulator to create a 200-MHz frequency shift for the heterodyne beat note, which was then analyzed to extract the linewidth and track frequency jitter over time.

6.3 PRINCIPLE OF RFGR-BASED SELF-INJECTION LOCKING

The laser operates on the principle of SIL, where the RFGR acts as a high-Q, frequency-selective filter. Light generated in the main gain ring circulates, and a portion is coupled into the RFGR. Only light at frequencies that are resonant within the RFGR is re-injected into the main ring with phase coherence. This spectrally pure feedback forces the laser to oscillate on a single longitudinal mode, simultaneously stabilizing its frequency and suppressing competing modes.

This interaction is described by a set of theoretical models that connect the physical

6.3. PRINCIPLE OF RFGR-BASED SELF-INJECTION LOCKING

structure of the RFGR to the dynamic behaviour of self-injection locking.

6.3.1 THE DYNAMICS OF SELF-INJECTION LOCKING (LANG-KOBAYASHI MODEL)

The temporal dynamics of the locking process are described by a modified Lang-Kobayashi (LK) model, which is detailed in Appendix D. This model captures the evolution of the laser's complex electric field $E(t)$, which is under the influence of the spectrally filtered feedback from the RFGR. The core equation of this model is an integro-differential equation:

$$\frac{dE(t)}{dt} = \frac{1}{2}(1 + i\alpha_H)g(t)E(t) - \frac{E(t)}{\tau_{ph}} + \kappa \int_0^\infty h_{\text{RFGR}}(\tau)E(t - \tau)d\tau, \quad (6.1)$$

where the critical third term is the delayed feedback integral, which represents the influence of the RFGR. Unlike a simple delayed-feedback model, it treats the feedback as a convolution of the laser's entire electric field history, $E(t - \tau)$ with the RFGR time-domain impulse response, $h_{\text{RFGR}}(t)$.

6.3.2 MODELLING THE RFGR FEEDBACK (TRANSFER-MATRIX METHOD)

The impulse response, $h_{\text{RFGR}}(t)$, provides the link between the laser's dynamics and the physical structure of the feedback element. The impulse response is the inverse Fourier transform of the transfer function of the RFGR in the frequency domain, $H_{\text{RFGR}}(\omega)$. This transfer function, which describes the amplitude and phase response of the filter, is

6.3. PRINCIPLE OF RFGR-BASED SELF-INJECTION LOCKING

calculated from its physical structure using the detailed transfer-matrix model presented in Appendix C. This model treats the RFG as a concatenation of individual grating segments and calculates the total reflection and transmission coefficients for the entire ring structure (see eq. (C.18)). This bridges the physical grating design (Appendix C) with the temporal dynamics of self-injection locking, determined by the impulse response of the RFGR, which in turn governs the laser's dynamic behaviour (Appendix D).

6.3.3 THE STEADY-STATE RESONANCE CONDITION

For stable, steady-state locking to occur, the dynamic system described by eq. (6.1) must settle into a state that satisfies a phase resonance condition. This condition can be understood by analyzing the optical paths and phase accumulation within the coupled-ring system at the level of individual scattering elements. Let m denote the index of a sub-grating in the RFG. Consider a photon interacting with the m th grating, either reflected or transmitted in opposite directions. From this point, light can follow many possible trajectories through the RFG, including multiple reflections and returns to the same direction at the original point. Such a complete loop is treated as a closed optical path.

To describe how these closed-loop trajectories contribute to mode formation and resonance, we consider their role in defining the system's eigenmodes and associated optical phases. Physically, the system supports eigenmodes determined by the propagation direction (clockwise or counter-clockwise). A mode starting clockwise within the RFGR is referred

6.3. PRINCIPLE OF RFGR-BASED SELF-INJECTION LOCKING

to as the reflection mode, with its characteristic phase denoted $\Phi_{\text{clock}_m}(\beta)$. Conversely, a mode propagating anti-clockwise, termed the transmission mode, has a characteristic phase $\Phi_{\text{anti-clock}_m}(\beta)$. After undergoing multiple reflective and transmissive interactions involving both amplitude and phase modulation within the RFGR, both modes return to the initial m th grating from opposite directions, each having accumulated distinct phase shifts. For a single round trip, the total accumulated phase must be an integer multiple of 2π . For clockwise propagation, two types of closed-loop paths exist: (1) those confined within the RFGR, and (2) those traversing the main ring. The total optical phase for a characteristic closed loop involving the m th grating that couples the main ring and the RFGR can be expressed as eq. (6.2):

$$\Phi_{R_m}(\beta) = (2\pi R + l_m) \cdot \beta + \Phi_g(\beta) + \pi, \quad l_m \in \{l_1, l_2, \dots, l_M\}, \quad (6.2)$$

where β denotes the propagation constant of a random mode, $2\pi R$ is the effective length of the main ring, l_m is an additional effective length from multiple scattering within the RFGR, and Φ_g is the gain-induced phase shift and an additional phase shift of π accounts for the two crossings of the coupler. The final constructive interference condition requires that the sum of all phase contributions from the main ring $\Phi_{R_m}(\beta)$ and the bidirectional paths within the RFGR $\Phi_{\text{clock}_m}(\beta)$ and $\Phi_{\text{anti-clock}_m}(\beta)$ satisfies:

$$\Phi_{R_m}(\beta) + \Phi_{\text{clock}_m}(\beta) + \Phi_{\text{anti-clock}_m}(\beta) = 2n\pi, \quad n \in \mathbb{Z}. \quad (6.3)$$

6.3. PRINCIPLE OF RFGR-BASED SELF-INJECTION LOCKING

The unique nature of the RFG, comprising thousands of random scattering planes, creates a diffuse resonance cloud that allows this locking condition to be robustly maintained even under minor environmental perturbations, enhancing the laser’s stability.

6.3.4 ROLE OF RFGR AND SIL IN PERFORMANCE ENHANCEMENT

Self-injection locking in the EDRFL is thus facilitated by the multiple scattering in the RFGR, which selectively feeds back radiation from the main ring that matches its resonant condition. The performance of the locked laser is dependent on two unique properties of the RFGR that are not present in a simple stand-alone RFG: its enhanced spectral filtering and its environmental robustness.

First, achieving single-mode oscillation using a stand-alone RFG is challenging. The gain bandwidth of EDF is orders of magnitude broader than the mode spacing in the RFG, and the round-trip gain difference between adjacent modes is minimal. As a result, the RFG’s weak wavelength selectivity is insufficient to suppress competing modes. By embedding the RFG in a fibre ring (RFGR), the filtering effect is enhanced. The ring structure imposes a periodic set of resonances, and the multiple scattering interference within the RFG modulates the phase of these resonances. This imposes stricter phase conditions across a periodic spectral range, which suppresses coupling between random modes and strongly favours oscillation in a single, stabilized mode. This enhanced spectral selectivity is physically represented in our model (Appendix D) by the RFGR’s time-domain impulse response, $h_{\text{RFGR}}(t)$. For a high-Q filter like the RFGR, this impulse response is

6.4. RESULTS AND DISCUSSION

a long, ringing signal that decays slowly over time. This long memory means that the feedback term in the LK equation (eq. (D.4)) is a weighted average of the laser's field over a long history, which is a much more effective filter than the short-memory, fast-decaying impulse response of a low-Q stand-alone RFG.

Second, the RFG comprises thousands of randomly spaced scattering planes and acts as a diffuse resonance cloud. This structure allows the laser's resonant condition to adapt dynamically and smoothly to small, time-dependent perturbations, such as thermal or acoustic fluctuations. This adaptive behaviour enables stable optical feedback to be maintained over many round trips without a long re-establishment process, which enhances the environmental robustness of the random laser. The effectiveness of this robust feedback is captured in the phase rate equation (eq. (D.10)) of our model. The sine term, $\sin[\phi_\tau - \phi + \theta(\tau)]$, describes how the feedback signal continuously applies a phase torque to correct the laser's phase. The long, complex impulse response of the RFGR ensures that this corrective torque is averaged over many scattering paths and delays, making the lock less sensitive to fluctuations in any single path and thus more robust against environmental drift.

6.4 RESULTS AND DISCUSSION

The experimental characterization of the EDRFL produced a set of results that demonstrate the profound impact of the RFGR-based SIL technique. This section systematically analyzes the laser's output characteristics, starting with its basic performance and the properties of the feedback element, then progressing to detailed noise analyses and the final verification

6.4. RESULTS AND DISCUSSION

of its high-coherence, single-mode operation.

6.4.1 LASING CHARACTERISTICS AND RFGR SPECTRAL PROPERTIES

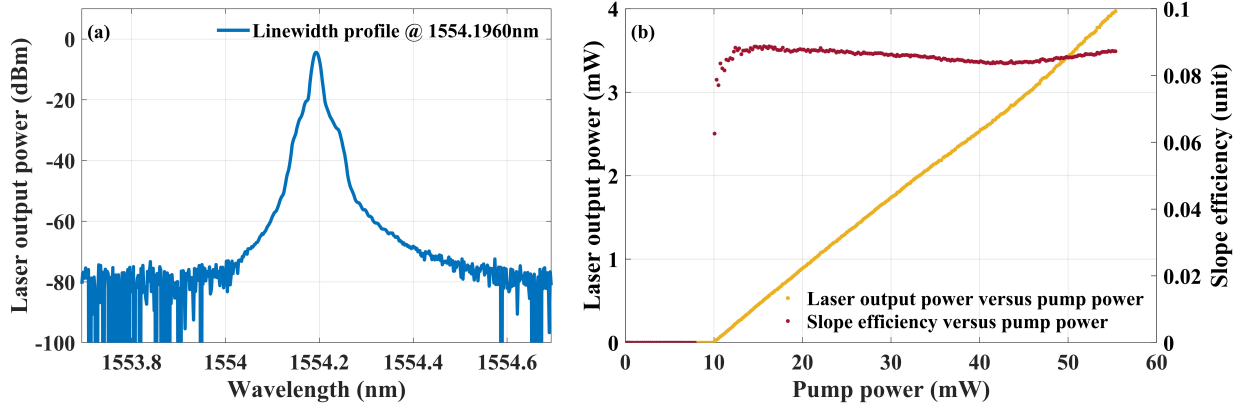


Figure 6.2: (a) laser output spectrum from OSA; (b) laser output power growth and slope efficiency against pump power.

The fundamental lasing characteristics were established. The optical spectrum, characterized at 1554.1960 nm with 70 dB of EDRFL by OSA as shown in Figure 6.2 (a), reveals a single-mode lasing at 1554.1960 nm. Figure 6.2 (b) shows the output power as a function of pump power, along with the extracted slope efficiency. A lasing threshold is observed near 10 mW. Beyond this threshold, the output power increases nearly linearly with pump power, yielding a slope efficiency of approximately 8.8 % at 55 mW of pump input. This consistent efficiency across the pump range indicates efficient pump-to-signal conversion and minimal parasitic losses. These results verify the effectiveness of the long, low-doped EDF for gain extraction and the robust optical feedback provided by the RFGR.

6.4. RESULTS AND DISCUSSION

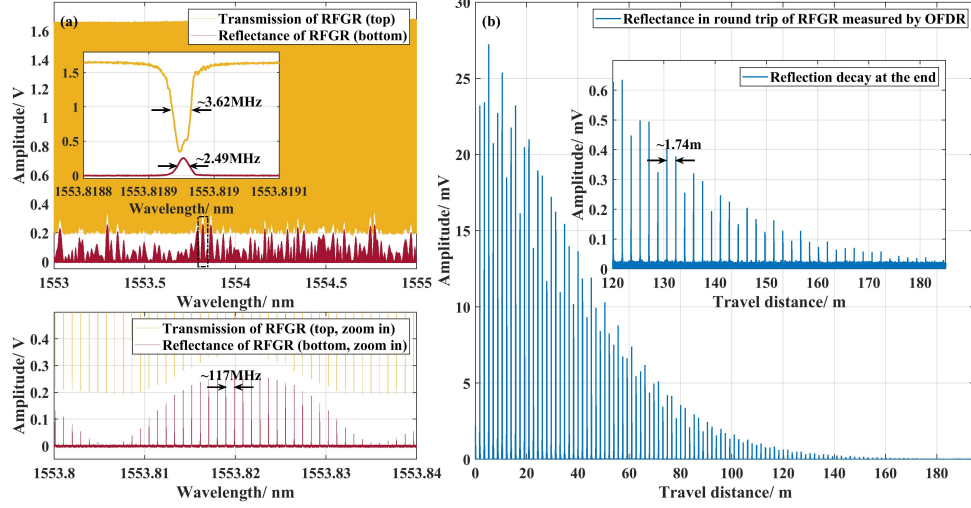


Figure 6.3: (a) Transmission and reflection spectrum of RFGR and (b) a time delayed sequence of light circulating in the RFGR.

To understand the feedback mechanism enabling the SIL, the spectral properties of the high-Q RFGR resonator were independently characterized. To investigate the optical characteristics of the RFGR as feedback, linear swept-wavelength interferometry was carried out over a 10 nm spectral range. Reflection and transmission spectra were recorded separately. As shown in Figure 6.3 (a, top left), the red-shaded region marks the random modes corresponding to a resonant wavelength range, while the yellow-shaded region indicates the inverse spectral response in transmission. These experimentally measured spectral features are predicted by the transfer-matrix model detailed in Appendix C. In this model, RFGR reproduces the resonant peaks in reflection and the corresponding dips in transmission that arise from the input/output of the ring structure. These features arise from multiple scattering at randomly spaced Fabry-Pérot cavities in RFG, which

6.4. RESULTS AND DISCUSSION

collectively form a low-finesse but dense distribution of local resonators. The resulting structure produces broad, overlapping random modes that can couple to the eigenmodes of the fibre ring.

The reflection spectrum exhibits high finesse, with a Q factor of 7.75×10^7 , as shown in the inset. Notably, the full width at half maximum (FWHM) of the transmission spectrum is broader than that of the reflection by a factor of 1.45. Figure 6.3 (a, bottom left) shows a free spectral range (FSR) of 117 MHz.

To further interpret this FSR, a Fourier transform of the reflection spectrum was performed, as shown in Figure 6.3 (b). A dominant peak separation of 1.74 m corresponds to the physical round-trip length of the RFGR. The presence of additional peaks at longer delays, with a maximum effective path length of approximately 184 m shown in the inset, confirms that resonant photons are trapped by multiple scattering events within the RFGR. This significantly extends their residence time inside the ring. This distributed ensemble of scattering-induced resonant delays is the physical origin of high Q-factor in RFGR; it acts as a phase modulator, stabilizing the random laser by averaging out phase fluctuations.

6.4.2 INTENSITY NOISE SUPPRESSION

With the feedback element characterized, the laser's intensity noise performance was evaluated to quantify the effectiveness of SIL in suppressing mode competition. The relative intensity noise of the RFGR-based EDRFL was measured and compared against a basic RFL using only a single RFG for feedback, and a commercial, electronically stabilized laser.

6.4. RESULTS AND DISCUSSION

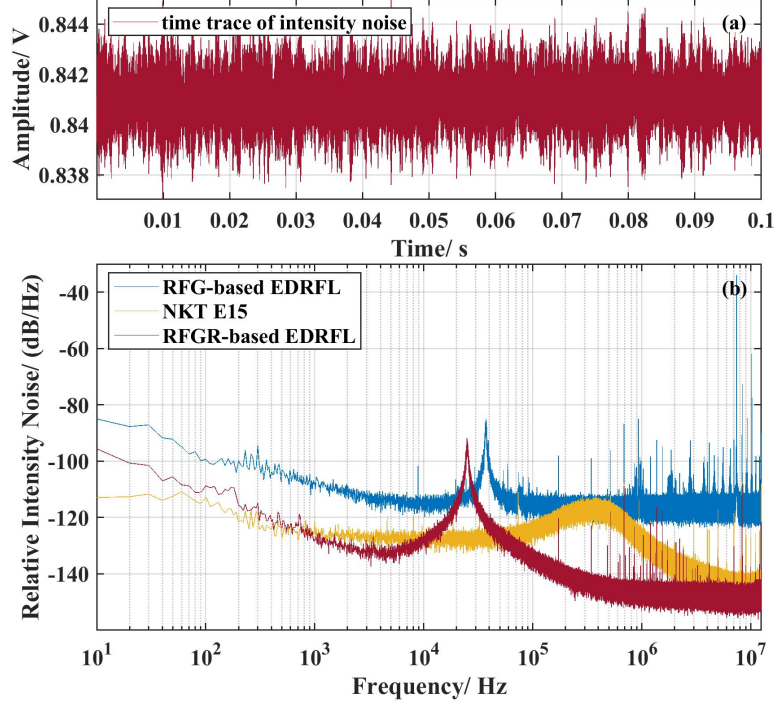


Figure 6.4: (a) Time trace of intensity noise and (b) RIN properties of the proposed EDRL compared with RFG (blue) feedback and phase-locked EDF laser (NKT, yellow).

The measured RIN spectra averaged over ten measurements are shown in Figure 6.4 (b), with a representative time-domain trace in Figure 6.4 (a). The single-RFG laser (blue trace) exhibits multiple discrete noise peaks in the megahertz range. In contrast, the RFG-based SIL laser (red trace) shows a clean spectrum with these peaks suppressed. The SIL laser's RIN is reduced to a floor of -140 dB/Hz for frequencies above 100 kHz, a reduction of over 10 dB compared to the single-RFG laser and comparable to the performance of the commercial phase-locked laser. A single peak corresponding to the intrinsic relaxation oscillation of the EDF remains visible in the kilohertz range. The multiple noise peaks in

6.4. RESULTS AND DISCUSSION

the single-RFG laser are a direct signature of mode competition and hopping within the EDF gain bandwidth [77].

The suppression of these peaks in the RFGR-based laser shows that SIL has effectively quenched this mode competition. This stabilization is described in our model by the feedback term in the power rate equation (eq. (D.9)). The term $2\kappa \int_0^\infty |h_{\text{RFGR}}(\tau)| \sqrt{PP_\tau} \cos[\phi_\tau - \phi + \theta(\tau)]$, acts to selectively reinforce the power $P(t)$ of the single mode that satisfies the phase-matching condition, while other out-of-phase modes are suppressed through destructive interference. This stabilizes the laser in a single-mode state, eliminating mode-hopping as a source of noise and resulting in the observed low-RIN performance. The residual low-frequency $1/f$ intensity noise below 3 kHz arises from several interconnected factors. Primarily, it is attributed to the slow relaxation time of Erbium ions in the fibre (approximately 8–10 ms), which keeps the laser system in a non-equilibrium state. Furthermore, this low-frequency noise is driven by non-equilibrium thermal fluctuations in the gain cavity, pump source instabilities and dispersion (such as DFB laser noise), and long-term frequency drift. Together, these elements cause thermally induced refractive index fluctuations that manifest as noise [131].

6.4.3 FREQUENCY NOISE REDUCTION AND LONG-TERM STABILITY

To assess the laser's spectral stability, the frequency noise was measured using an unbalanced Michelson interferometer. The measured frequency noise spectrum of the RFGR-based SIL laser is shown in Figure 6.5 (b) and compared to the reference lasers.

6.4. RESULTS AND DISCUSSION

The RFGR-based EDRFL shows a noise reduction of 10 dB at 1 Hz and up to 25 dB at 100 kHz relative to the single-RFG laser. The system achieves a frequency noise floor of $1 \text{ Hz/Hz}^{1/2}$ above 10 kHz, a suppression of 30 dB. The frequency noise of the SIL laser is over 5 dB lower than that of the commercial electronically stabilized laser for frequencies above 2 kHz. The corresponding phase jitter over 1 second for the proposed EDRFL is shown in Figure 6.5 (a).

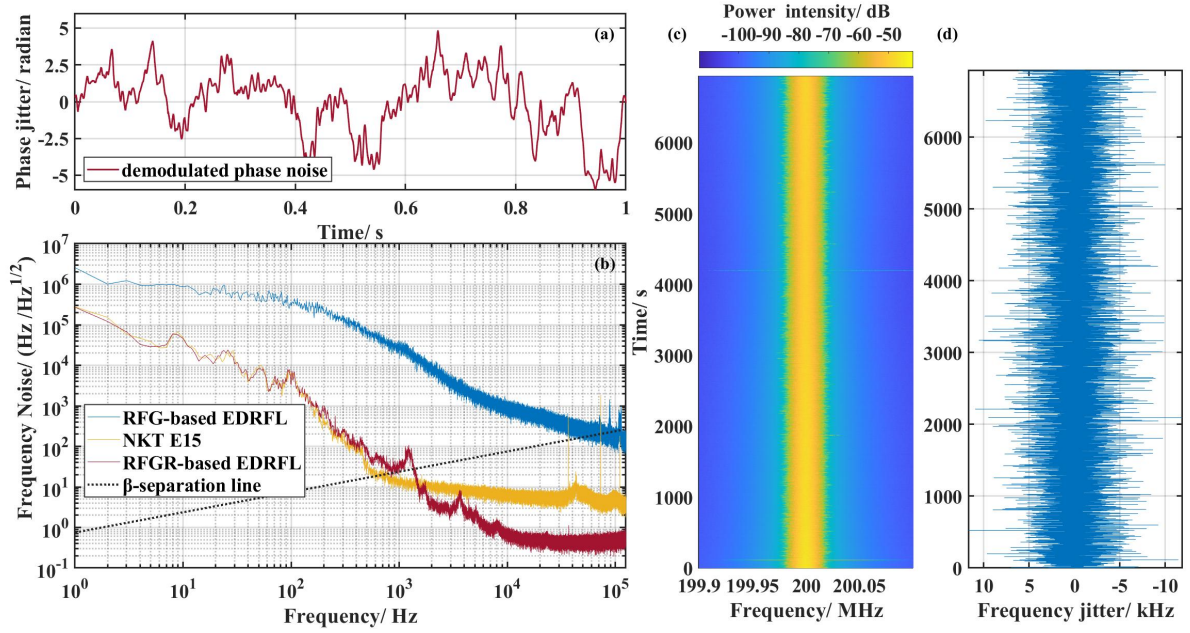


Figure 6.5: (a) Phase jitter of the proposed EDRFL over one second. (b) Frequency noise spectral density of the RFGR-based EDRFL (red) compared with a single-RFG laser (blue) and a commercial phase-locked EDF laser (E15, NKT, yellow). (c) and (d) Long-term stability of the laser's linewidth and frequency over a 7000-second period.

EDF lasers typically suffer from equilibrium thermal noise in passive components and non-equilibrium thermal noise in the gain medium, both inversely related to their respective

6.4. RESULTS AND DISCUSSION

effective lengths [131, 134]. Low-frequency noise below 100 Hz is mainly due to the intrinsic relaxation time of EDF and environmental and pump-induced fluctuations, while shot noise above 10 kHz is effectively suppressed by SIL from the high-Q RFGR. Compared to short-length RFG feedback, the longer effective cavity in the RFGR configuration reduces both intensity and frequency noise.

This suppression of phase noise is a consequence of the self-injection locking mechanism. This effect is described by the phase rate equation in our theoretical model (see eq. (D.10) in Appendix D). The second integral term $\kappa \int_0^\infty |h_{\text{RFGR}}(\tau)| \sqrt{\frac{P}{P}} \sin[\phi_\tau - \phi + \theta(\tau)]$ represents the corrective feedback from the RFGR. The sine function acts as a phase torque. When the laser's current phase $\phi(t)$ drifts away from the phase of the spectrally pure feedback signal, the sine term becomes non-zero and pushes the phase back towards the stable locking point. This continuous phase correction is the physical mechanism that counteracts the intrinsic noise and is responsible for the observed low frequency noise floor.

The long-term stability of the laser was characterized using delayed self-heterodyne interferometry. The results are shown in Figure 6.5 (c, d). The laser maintains stable operation over a 7000-second period, with a peak-to-peak frequency jitter standard deviation of 3.0 kHz. These results confirm the effectiveness of the SIL mechanism against slow environmental drifts.

6.5. SINGLE-MODE OPERATION AND LINEWIDTH MEASUREMENT

6.5 SINGLE-MODE OPERATION AND LINEWIDTH MEASUREMENT

Lastly, we verified the effectiveness of SIL in achieving single-mode operation by measuring the resultant laser linewidth and quantifying the capability of the noise-suppression and mode-selection mechanisms.

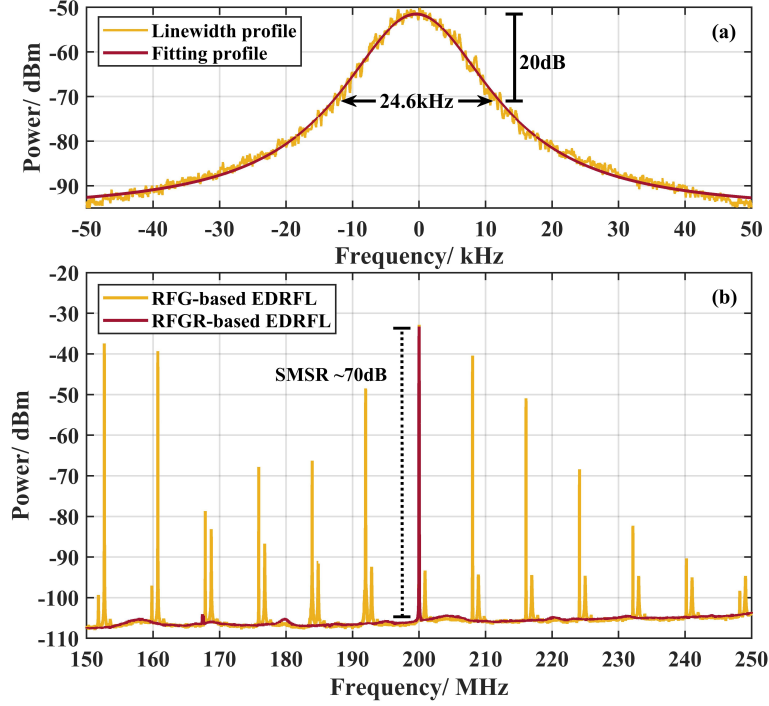


Figure 6.6: (a) Measured laser beat note spectrum with a Lorentzian fit, yielding a 20-dB linewidth of 24.6 kHz. (b) Comparison of the SMSR for the RFG-based EDRFL (red, 70 dB SMSR) and a multi-mode single-RFG laser (blue).

As shown in Figure 6.6 (b), the RFG-based EDRFL exhibits single-mode lasing with

6.6. CHAPTER CONCLUSION

a side-mode suppression ratio (SMSR) of 70 dB. This is in contrast to the multi-mode oscillation observed in the RFG-based configuration. An SMSR of this magnitude provides evidence that the SIL mechanism has suppressed mode competition. It confirms that the laser operates in a stable, single-mode regime, which validates the use of a single-mode theoretical framework, such as the Lang-Kobayashi (LK) model (Appendix D), to describe its dynamics.

The measured beat note from the delayed self-heterodyne interferometer is shown in Figure 6.6 (a). A Lorentzian fit to the spectrum yields a 20-dB linewidth of 24.6 kHz, corresponding to a 3-dB linewidth of 1.23 kHz. The narrow Lorentzian profile is characteristic of a laser whose phase is stabilized by self-injection locking through the RFGR. This demonstrates that the RFGR feedback effectively suppresses phase noise as described in our model. Combined with the 70-dB SMSR, the 1.23-kHz linewidth confirms that SIL via the RFGR converts the multi-mode random laser into a stable single-frequency source.

6.6 CHAPTER CONCLUSION

In this chapter, an Erbium-Doped Random Fibre Laser stabilized by self-injection locking to a Random Fibre Grating Ring resonator was demonstrated. The all-optical stabilization mechanism was shown to suppress the intrinsic mode competition and phase noise characteristic of free-running random lasers.

The laser's performance is consistent with a theoretical framework. The dynamic

6.6. CHAPTER CONCLUSION

evolution of the system is described by the Lang-Kobayashi model (Appendix D), which shows how the filtered feedback forces the laser to settle into a stable, single-mode state. This final state is one that satisfies the steady-state phase-matching condition, where the sum of the phase contributions from the main ring and the multiple scattering paths within the RFGR is an integer multiple of 2π for a single resonant mode. This condition is derived from the physical structure of the RFGR, which is modelled via the transfer-matrix method of Appendix C. The model quantitatively explains the observed performance: the feedback term in the power rate equation (eq. (D.9)) leads to the high side-mode suppression ratio, while the phase-correcting influence described in the phase rate equation (eq. (D.10)) results in the measured linewidth narrowing.

The experimental results validate this theoretical framework. The laser achieves a 1.23 kHz linewidth, a 70 dB side-mode suppression ratio, and a frequency jitter standard deviation of 3.0 kHz over 7000 seconds. This work establishes that RFGR-based self-injection locking is a method for transforming unstable random fibre lasers into single-frequency sources with noise performance comparable to that of commercial, electronically-stabilized systems.

Chapter 7

OPTICAL BISTABILITY AND STABILITY CONTROL IN AN ERBIUM-DOPED RANDOM FIBRE LASER

Following the exploration of techniques for noise reduction via self-injection locking in 6, this chapter investigates a fundamentally different approach to laser stabilization. Instead of suppressing nonlinearities, I explore the possibility of harnessing a specific nonlinear phenomenon, optical bistability, as a mechanism for control and noise reduction in a random fibre laser.

Harnessing the complex nonlinear dynamics of inherently unstable random fibre lasers (RFLs) allows us to move beyond studying instability to actively controlling it, unlocking novel optical functionalities. I demonstrate this principle by characterizing optical bistability in an Erbium-doped RFL (EDRFL) built with an unpumped Erbium-doped fibre (EDF) as a saturable absorber and an inscribed random fibre grating for distributed feedback. The laser

7.1. INTRODUCTION: HARNESSING NONLINEARITY FOR LASER STABILITY

exhibited a distinct hysteresis loop wherein the dominant and suppressed modes displayed opposing cycles, a clear signature of strong inter-modal gain competition. High-resolution optical frequency domain reflectometry (OFDR) spatially resolves the refractive-index bistability, revealing a hybrid mechanism driven by both Er^{3+} population dynamics and localized Kerr nonlinearity. Crucially, operating within the bistable regime where the intracavity power clamps both the gain and saturable loss establishes a robust equilibrium that actively suppresses mode competition, reducing intensity noise by nearly an order of magnitude. These results establish a practical strategy for converting instability into a tool for control, paving the way for passively stabilized random lasers and novel all-optical switching applications.

7.1 INTRODUCTION: HARNESSING NONLINEARITY FOR LASER STABILITY

Precise control of laser emission is fundamental to photonics from telecommunications to precision metrology. Random lasers (RFLs), which replace conventional mirrors with distributed feedback from disordered media, form a distinct class of coherent sources offering structural simplicity and spectral versatility [8]. However, the complex interplay of multiple scattering paths often fuels chaotic mode competition and severe intensity fluctuations, limiting their use in stability-critical applications. Since the first demonstrations of Erbium-doped random fibre lasers (EDRFLs) [10, 135], significant efforts have focused on mitigating

7.1. INTRODUCTION: HARNESSING NONLINEARITY FOR LASER STABILITY

laser instability through engineered random fibre gratings (RFGs) [87, 113] or refined cavity design [129, 136–139]. These approaches typically aim to suppress nonlinear effects or favor single-mode operation to achieve stability. A less-explored but promising alternative is to harness nonlinearity as a mechanism for control rather than suppression.

Optical bistability (OB), characterized by two steady-state outputs for a single input, represents such a powerful nonlinear phenomenon. First proposed by Szöke *et al.* in 1969 [124] and experimentally demonstrated in sodium vapour in 1976 [14], OB provides a foundation for all-optical control functionalities like switching and memory [125]. Bistability arising from nonlinear dispersion or absorption has been extensively studied in conventional cavities, including Fabry-Pérot resonators [14, 140] and fibre ring cavities [141]. In erbium-doped fibre lasers (EDFLs), in particular, OB has been achieved by leveraging the strong resonant nonlinearity of Er^{3+} ions. For instance, Horowitz *et al.* demonstrated absorption bistability and linewidth narrowing in an EDF-based Fabry-Pérot cavity [142], while others explored its role in single-frequency stabilization [143] and managing multi-wavelength operation [144]. Further studies have demonstrated active control over the bistable region's width and thresholds by injecting an external control laser into the cavity [145]. These studies established that an unpumped section of EDF can act as an effective saturable absorber, creating the intensity-dependent loss required for bistable operation.

Despite this extensive research in conventional lasers, the role of OB in random fibre laser systems remains largely unexplored, especially if a stable bistable regime can be reliably established within a disordered cavity, which lacks well-defined mirrors and supports a multitude of competing modes. Furthermore, it is an open question whether such a regime

7.2. EXPERIMENTAL DESIGN AND METHODOLOGY

could be exploited to enhance, rather than degrade, laser stability. Recent work has shown that using an unpumped EDF as a saturable absorber in a ring laser can lead to a transition from stable single-mode operation into regular mode-hopping instabilities as pump power is increased [146]. Depending on the operating conditions and cavity configuration, the use of a saturable absorber can either stabilize or destabilize a laser. The primary objective is to demonstrate and characterize optical bistability in an EDRFL and to investigate its direct impact on mode competition and intensity noise.

In this work, I demonstrate and characterize optical bistability in an EDRFL using an unpumped EDF section as a saturable absorber and an RFG for distributed feedback. High-resolution Optical Frequency Domain Reflectometry (OFDR) is applied to spatially resolve pump-induced refractive index changes along the RFL cavity, revealing a hybrid bistable behaviour governed by both Er^{3+} population dynamics and localized Kerr nonlinearity. Crucially, I show that operating within the bistable regime actively suppresses mode competition and reduces intensity noise, establishing a direct link between microscopic nonlinearities and macroscopic laser stability. These results introduce a practical framework for turning instability into optical functionalities, enabling passively stabilized RFLs and advancing all-optical control applications.

7.2 EXPERIMENTAL DESIGN AND METHODOLOGY

To investigate the proposed bistable dynamics, a specialized experimental setup was designed and constructed. This section details the principle of operation and physical configuration of

7.2. EXPERIMENTAL DESIGN AND METHODOLOGY

the laser, the OFDR system used for internal state analysis, and the underlying theoretical framework connecting the measured quantities.

7.2.1 LASER CONFIGURATION AND PRINCIPLE OF BISTABILITY

The optical bistability observed in EDRFL arises from the interplay between nonlinear gain saturation in the pumped Erbium-doped fibre (EDF) and saturable absorption in the unpumped EDF. The primary mechanism is the intensity-dependent absorption of the unpumped EDF, which acts as a nonlinear loss element. Feedback is provided by a random fibre grating (RFG), which comprises a permanently inscribed, statistically varying index modulation that supports a dense ensemble of weakly resonant modes across a broad spectral range, acting as a distributed mirror. This distributed feedback forms the laser cavity, enabling the intracavity power to build sufficiently to saturate the absorber. The bistable operation results from the combination of this nonlinearity and distributed feedback, and proceeds through the following sequence:

1. **Off State:** At low pump powers, amplified spontaneous emission (ASE) from the gain section is strongly attenuated by the highly absorptive unpumped EDF.
2. **Approaching Threshold U_{th} :** As pump power increases, ASE builds up to bleach the absorber, and partially inverts its population, reducing absorption.
3. **Switch-Up (U_{th}):** Once the net gain exceeds cavity loss, and lasing initiates. The resulting strong intra-cavity field rapidly saturates the absorber, sharply reducing

7.2. EXPERIMENTAL DESIGN AND METHODOLOGY

cavity loss and causing a transition to a high-power state.

4. **On State:** As the pump decreases from above U_{th} , the strong laser field maintains the absorber saturated and clamps the gain and loss, sustaining lasing below U_{th} .
5. **Approaching Threshold L_{th} :** Continued pump reduction weakens the intra-cavity field. The balance of gain and absorption becomes delicate, modulated by population dynamics in both EDFs.
6. **Switch-Down (L_{th}):** Below L_{th} , the intra-cavity intensity is insufficient to maintain absorber saturation. The absorber reverts to a high-loss state, and lasing ceases.

This hysteresis between U_{th} and L_{th} defines the bistable response of the system, governed by the coupled nonlinearities and distributed feedback of the random cavity.

The experimental setup of EDRFL is shown in (Figure 7.1 (a)). A ring resonator contains two separate EDFs: one is pumped as a gain medium while the other is unpumped as an absorber. The gain medium is a 10-metre polarization-maintaining (PM) EDF (ER-407, INO), a specific design choice to stabilize the laser's gain characteristics by minimizing polarization-dependent gain fluctuations and isolating the bistability dynamics. The saturable absorber is a 12-metre single-mode EDF (M5-980-125, Thorlabs). This length was determined empirically to provide sufficient ground-state absorption for a clear hysteresis loop without introducing excessive cavity loss that would prevent lasing at accessible pump powers. A 0.1-metre RFG was inscribed point-by-point near one end of

7.2. EXPERIMENTAL DESIGN AND METHODOLOGY

the unpumped EDF using a femtosecond laser, creating enhanced Rayleigh scattering of approximately -60 dBm/mm at 1550 nm.

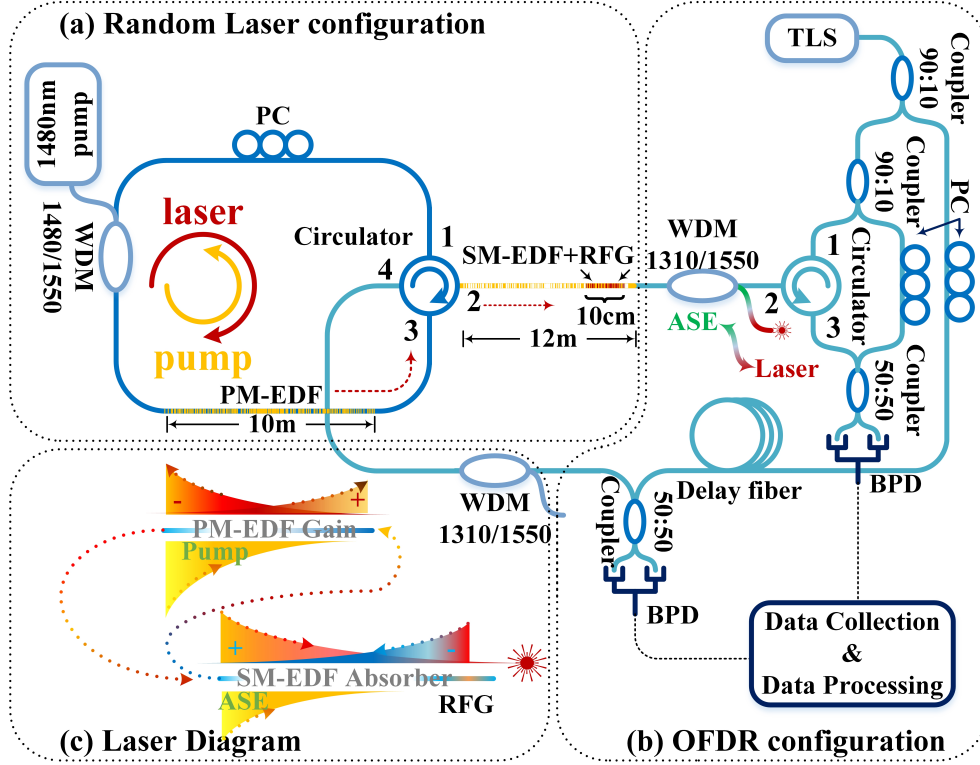


Figure 7.1: (a) Diagram of random laser configuration with backward pumping; (b) OFDR setup using two reference arms for simultaneous Rayleigh scattering measurements in EDF gain and absorber; (c) Diagram show light propagation direction and power growth in pump, ASE, and laser within the gain medium, along with power reduction in the absorber.

As depicted in Figure 7.1 (c), a backward-pumping scheme is employed. The backward-propagating ASE from the gain section seeds the unpumped EDF and decays toward the RFG. Multiple reflections from the RFG generate standing-wave patterns with counter-propagating waves, which saturate the unpumped EDF, suppress off-resonance modes, and

7.2. EXPERIMENTAL DESIGN AND METHODOLOGY

prolong the resonant mode's effective relaxation time due to the metastable state of Er^{3+} ions. In contrast, the gain medium supports a unidirectional travelling wave, producing a spatially uniform gain that reduces intensity fluctuations and enhances random laser stability.

7.2.2 OFDR SYSTEM FOR INTERNAL STATE CHARACTERIZATION

While measuring the laser's output power reveals the macroscopic bistable behaviour, understanding its microscopic origin requires probing the internal state of the laser cavity. To this end, a high-resolution Optical Frequency Domain Reflectometry (OFDR) system was implemented, as illustrated in Figure 7.1 (b).

An O-band tunable laser source (TLS, T100S-HP-O+, EXFO) serves as the interrogator. The light is split into three paths: one probe arm and two reference arms. In the probe arm, light travels from port 2 to port 3 of the circulator, enters RFG and unpumped EDF absorber, and is partially reflected via Rayleigh scattering. The transmitted wave then interrogates the EDF gain section from port 3 to port 4. Both EDFs act as the probe arm, enabling simultaneous interferometric detection of refractive index changes in the gain and absorber regions. The Rayleigh-scattered signals interfere with reference beams, generating fringes recorded by two balanced photodetectors (PDB435C, Thorlabs) and digitized using a Data Acquisition DAQ card (PXI-6115, National Instruments). A delay fibre compensates the optical path difference between the probe and reference arms. An auxiliary interferometer (AUI), not shown in the diagram, provides an external clock using

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frequency-sampling method to correct for laser tuning nonlinearity [20]. The reference signal reaches photodetector earlier than the probe signal, introducing a time delay that translates into the beat frequency.

7.2.3 THEORETICAL FRAMEWORK FOR REFRACTIVE INDEX MEASUREMENT

The ability of OFDR to measure refractive index changes relies on a well-established theoretical framework that connects the observed spectral shifts in the Rayleigh back-scattering to the physical properties of the fibre. As introduced in Chapter 5, the two primary mechanisms contributing to refractive index modulation in this EDRFL are resonant effects tied to the Er^{3+} population and non-resonant Kerr effects.

Resonant transitions in both EDFs induce broadband optical dispersion, enabling nonlinear effects even at off-resonance wavelengths. This nonlinearity, resembling a slow Kerr response [147], combines with saturable absorption to produce hybrid bistability involving both dispersion and absorption. Pump-induced population inversion alters the local susceptibility, causing refractive index changes on the order of 10^{-7} to 10^{-6} . The corresponding nonlinear index ($n_2 \sim 10^{-15} \text{ m}^2/\text{W}$) is 10^5 to 10^6 times greater than that from the Kerr effect in silica fibre [15, 16]. Radiation and non-radiation processes [148], combined with spatial variations in waveguide confinement and pump/laser overlap, further modulate the local refractive index and affect mode stability under optical bistability.

Optical Frequency-Domain Reflectometry (OFDR) [149] offers high spatial resolution

7.2. EXPERIMENTAL DESIGN AND METHODOLOGY

and sensitivity for distributed fibre strain and temperature sensing [1]. In Rayleigh scattering measurements, the spatial resolution Δz depends on the frequency sweep range $\Delta\nu$ and is given by:

$$\Delta z = \frac{c}{2n_g\Delta\nu}, \quad (7.1)$$

where n_g is the group index of the EDF. For a laser sweep from 1300 nm to 1324.3 nm, Δz is approximately 24.3 μm .

In the experiment, the EDF gain is pumped by 1480 nm laser diode, while the unpumped EDF absorber is indirectly driven by ASE from the gain fibre. The complex susceptibility of EDF, which includes absorption and dispersion, varies with pump rate and scales with the nonlinear polarization arising from population inversion [15]. Both EDFs are interrogated by OFDR at wavelengths outside the Er^{3+} absorption bands to measure the pump-induced refractive index change $\delta n_{kk}(\nu)$, which results from the redistribution of Er^{3+} ion populations and is fundamentally related to the gain spectrum $g(\nu)$ via the Kramers–Kronig (KK) relation:

$$\delta n_{kk}(\nu_c) = -\frac{c}{\pi^2} \mathcal{P} \int_0^\infty \frac{\delta g(\nu')}{\nu'^2 - \nu_c^2} d\nu', \quad (7.2)$$

where c is the speed of light in vacuum and $\mathcal{P}$ denotes the Cauchy principal value. The gain spectrum $\delta g(\nu')$ is determined by the fractional population inversion of Er^{3+} ions, which is itself governed by the standard Er^{3+} rate equations as a function of local pump and signal intensities. This provides a direct theoretical pathway from pump power to refractive index change. Pumping enhances gain near Er^{3+} resonances (1480 nm and 1530 nm), contributing positively to $\delta n_{kk}(\nu_c)$, while lasing depletes the gain, yielding a negative contribution. These

7.2. EXPERIMENTAL DESIGN AND METHODOLOGY

refractive index changes, governed by population differences between energy levels, are encoded in the real part of the susceptibility.

Additionally, strong pump and laser fields induce refractive index modulation via optical Kerr effect:

$$\delta n_k = n_2 \delta I, \quad (7.3)$$

where δn_k scales with the local intensity variation δI , with n_2 denoting the Kerr nonlinear coefficient. At fibre boundaries such as SMF-EDF splices, this yields an enhanced and linear δn_k response and intensity-driven hysteresis loops.

OFDR quantifies both KK- and Kerr-type δn by detecting local spectral shifts in Rayleigh scattering, where the relative shift $\delta \nu$ at probe frequency ν_c corresponds to the refractive index change via:

$$\frac{\delta n}{n_g} = -\frac{\delta \nu}{\nu_c}, \quad (7.4)$$

where n_g is group index of the EDF and δn is the local refractive index change caused by δn_{kk} and δn_k . By cross-correlating Rayleigh signals under pumped and unpumped conditions, spatially resolved refractive index variations can be extracted. The next section discusses how these effects manifest in the observed spectral and spatial behaviour.

7.3. RESULTS AND DISCUSSION

7.3 RESULTS AND DISCUSSION

With the experimental apparatus and theoretical principles established, this section presents and analyzes the results obtained from the EDRFL system. I begin by characterizing the feedback element, then explore the macroscopic bistable behaviour, and finally delve into the microscopic dynamics revealed by OFDR.

7.3.1 CHARACTERIZATION OF THE RFG DISTRIBUTED FEEDBACK ELEMENT

Before analyzing the laser's dynamic behaviour, it is essential to first characterize the key component responsible for the distributed feedback: the femtosecond-laser-inscribed random fibre grating (RFG).

To elucidate the role of the random fibre grating (RFG) in enabling distributed feedback and mode selection, I first characterize its structural and spectral properties. The polarization-maintaining (PM) EDF exhibits an absorption coefficient of 9.3 dB/m at 1533 nm, while the single-mode (SM) EDF shows 6.3 dB/m at 1531 nm. The RFG is inscribed at one end of the SM-EDF using 100 fs pulses at 0.6 μ J, with repetition rates randomly varied between 237 and 262 Hz [116]. The resulting grating, with spatial periods ranging from 0.450 to 0.547 μ m, enhances Rayleigh scattering to -60 dBm/mm at 1550 nm, providing the broadband distributed feedback necessary for random lasing.

7.3. RESULTS AND DISCUSSION

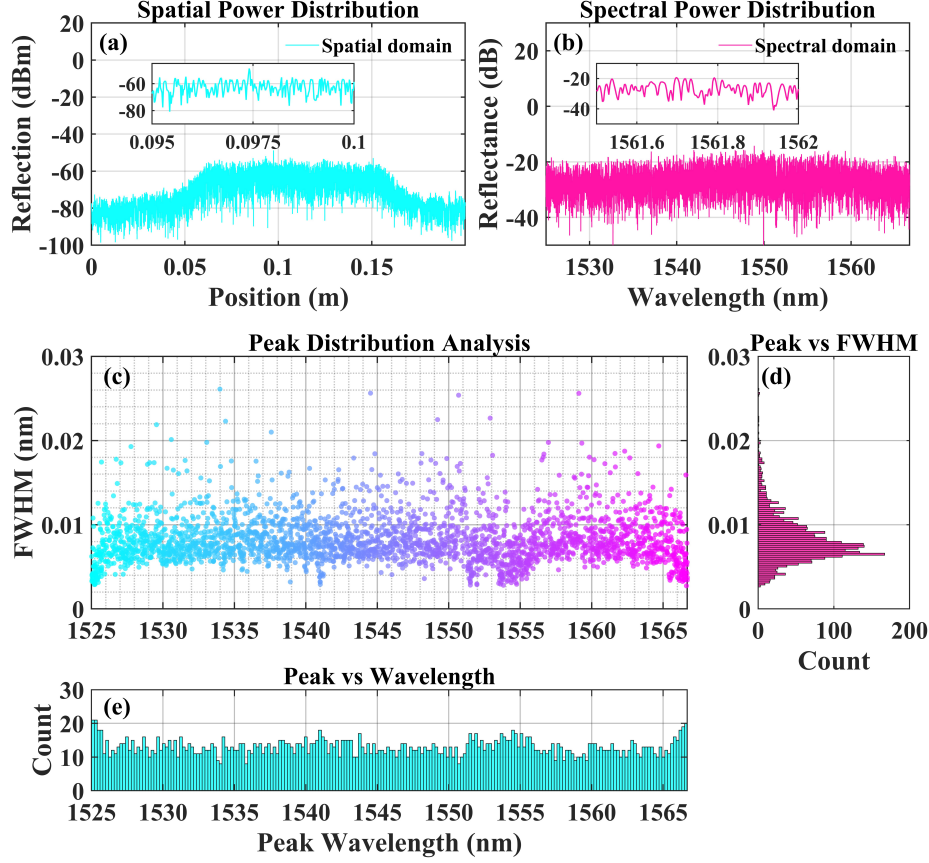


Figure 7.2: (a) Spatial distribution of the 10-mm RFG in the unpumped EDF; (b) reflectance spectrum measured via optical backscatter reflectometry; (c) reflection peak distribution vs. wavelength and linewidth; (d) peak count vs. FWHM; (e) peak count vs. wavelength.

Figure 7.2 (a, b) presents the spatial and spectral characterization of a 10-mm RFG segment inscribed in the unpumped EDF. The reflection spectrum reveals densely distributed peaks across 1525–1565 nm, with an average reflectance of -20 dB. Statistical analysis in Figure 7.2 (c-e) shows over 2500 discrete peaks within this 40-nm window, exhibiting full-width at half-maximum (FWHM) values between 3 and 25 pm. This high mode density

7.3. RESULTS AND DISCUSSION

indicates a strongly disordered scattering medium supporting a multitude of quasi-localized modes.

Within the laser cavity, each wavelength experiences a combination of local gain from the pumped EDF and loss from the RFG and unpumped EDF absorption. The spectrally non-uniform reflectance of the RFG modulates this gain-loss balance, rendering the cavity both spatially and spectrally disordered. This interplay enables selective feedback and saturable absorption, facilitating the emergence of bistable dynamics in the EDRFL system.

7.3.2 OBSERVATION OF OPTICAL BISTABILITY AND SPECTRAL HYSTERESIS

Having confirmed the properties of the distributed feedback element, the investigation now turns to the primary macroscopic phenomenon of interest: the bistable switching of the laser output and its spectral manifestation.

The interplay between distributed feedback and saturable absorption is critical for establishing optical bistability, I investigate the manifestation of optical bistability and spectral hysteresis in the EDRFL. Through controlled variation of pump power and polarization, I experimentally measure optical bistability and spectral hysteresis, revealing threshold-dependent mode selection, polarization-sensitive feedback, and mode competition.

7.3. RESULTS AND DISCUSSION

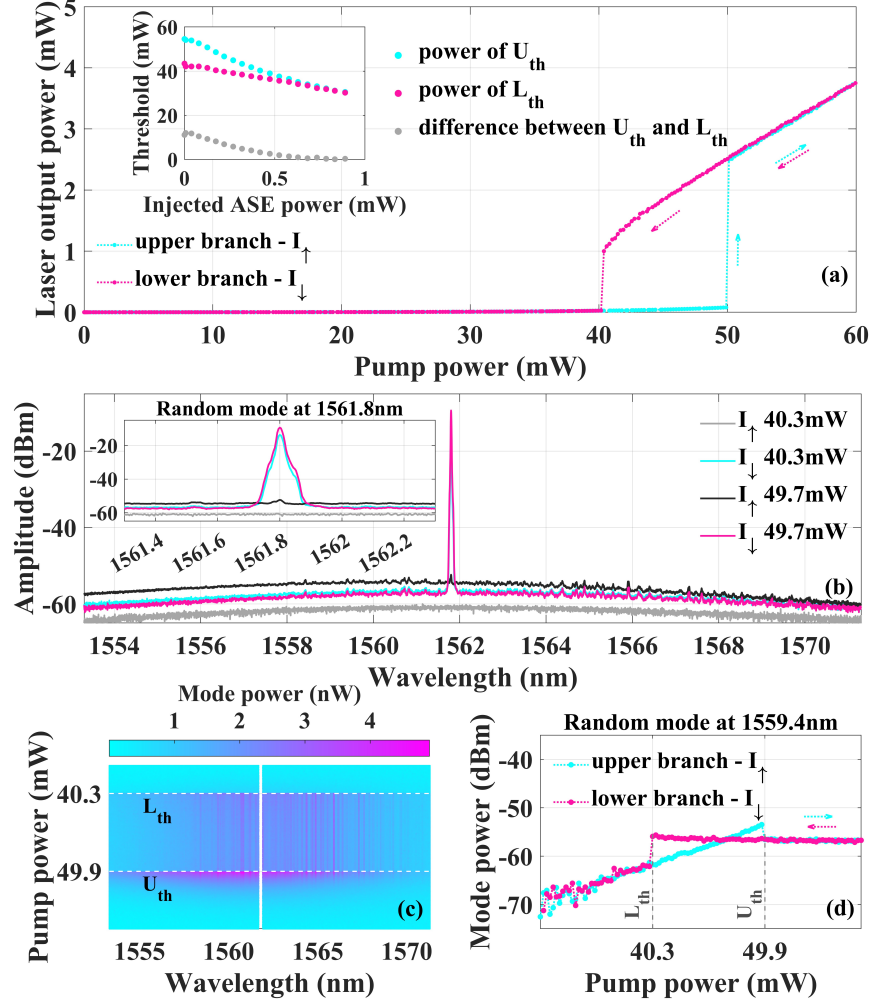


Figure 7.3: (a) OB measurement in the EDRFL showing dual thresholds under step-wise pump variation (counterclockwise hysteresis). Inset: injection of 1 mW ASE collapses the dual thresholds; (b) Random modes above the ASE baseline (1553.3–1571.3 nm) under identical pump conditions, with or without crossing the bistable region. Inset: dominant mode at 1561.8 nm; (c) Spectral evolution with step-wise pump changes; dashed lines mark threshold transitions; (d) Hysteresis loop of random mode at 1559.4 nm extracted from (c).

Figure 7.3 (a) shows a counterclockwise hysteresis loop in the EDRFL output as pump

7.3. RESULTS AND DISCUSSION

power is varied, with lasing turn-on at 49.9 mW and turn-off at 40.3 mW. This bistability is suppressed by injecting ~ 1 mW of ASE, which pre-saturates the absorber and collapses the dual thresholds into a single point (inset). Figure 7.3 (b) compares random modes with and without crossing the thresholds, revealing a dominant mode at 1561.8 nm and suppressed modes above the ASE level (1553.3–1571.3 nm). Polarization adjustment shifts the lasing mode and modifies the bistable region, confirming the role of polarization-dependent gain and feedback.

The bistable operation was tested under mild fibre movement and laboratory vibration; no spontaneous switching or linewidth broadening was observed. Cavity physics indicates low sensitivity to external perturbations because the high-Q RFG resonances dominate over environmental phase noise. The primary limitation remains experimental (fibre handling) rather than fundamental.

Figure 7.3 (c) tracks spectral evolution with step-wise pump changes, with threshold transitions marked by dashed lines. Figure 7.3 (d) shows a representative hysteresis loop at 1559.4 nm, where suppressed modes exhibit inverted hysteresis, gaining power as the pump decreases along the lower branch. Due to dense mode spacing and shared gain, strong inter-modal competition dynamically redistributes gain as the dominant mode weakens. This competition contributes to the refractive index increase near the lower threshold. While polarization mode competition acts as a modulating factor on the bistability, the fundamental mechanism remains saturable absorption. Other nonlinear effects such as four-wave mixing (FWM) are considered negligible due to the low power levels and lack of phase matching in the disordered cavity.

7.3. RESULTS AND DISCUSSION

7.3.3 SPATIALLY RESOLVED MAPPING OF REFRACTIVE INDEX BISTABILITY

The observation of a hysteresis loop in the laser's output power leads to the question of its physical origin within the laser cavity. By using OFDR system described previously, I can directly map the refractive index changes spatially to correlate them with the macroscopic bistable states.

To probe the microscopic origin of the bistability, I use OFDR to map the pump-induced refractive index change (δn) along the entire laser cavity. An off-resonant 1310 nm probe is insensitive to direct absorption, and captures δn arising from changes in the Er^{3+} population inversion, linked to the gain spectrum via the Kramers-Kronig relations (eq. (7.2)).

As pump power increases (laser-off), population inversion builds up, increasing δn_{kk} at probe wavelength range. Upon reaching the upper threshold, lasing initiates and rapidly depletes the inversion, and δn_{kk} drops. As pump power decreases (laser-on), inversion continues falling, further reducing δn_{kk} . When the pump falls below the lower threshold, lasing ceases, inversion recovers, and δn_{kk} rises again. This sequence forms a clockwise hysteresis loop, driven by population-inversion dynamics and its impact on refractive index.

7.3. RESULTS AND DISCUSSION

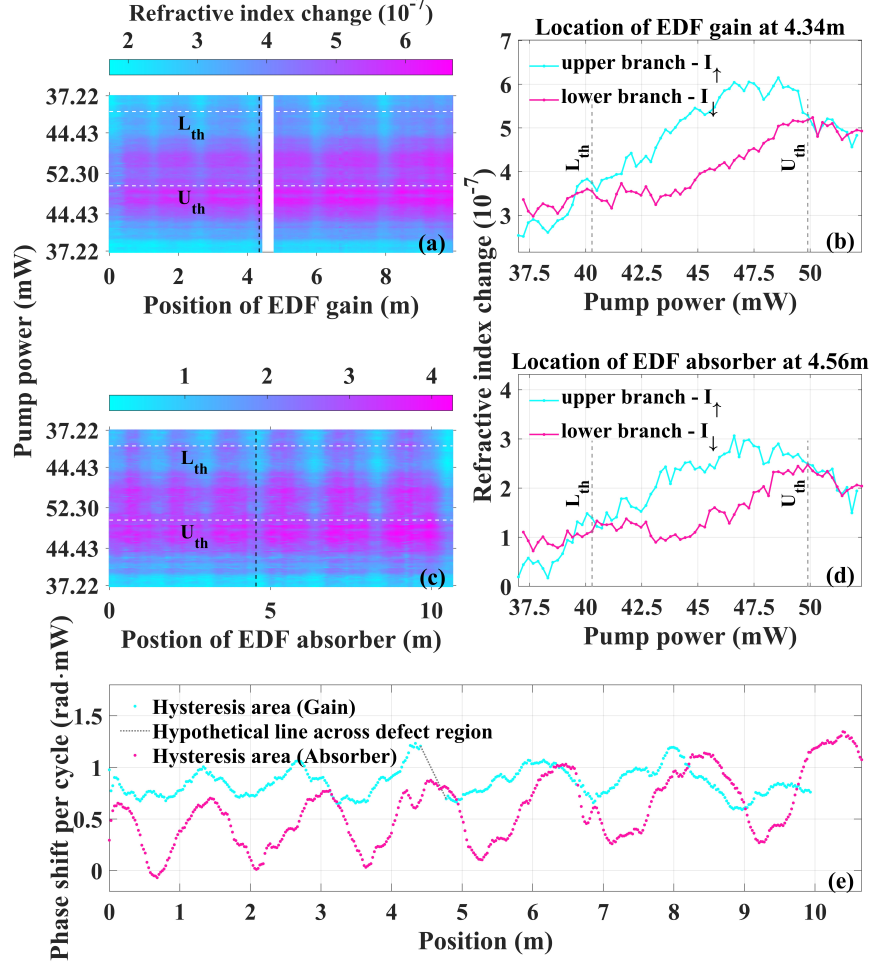


Figure 7.4: Spatial mapping of refractive index bistability in the EDF gain (a) and absorber (c); hysteresis loops of δn at selected locations in the EDF gain (b) and absorber (d); (e) hysteresis area (rad · mW) quantifying phase shift across the bistable regime, as defined in eq. (7.5).

This bistability arises from the coupling among pump, laser, and bidirectional ASE, which modulate δn_{kk} via spatially varying inversion redistribution in both gain and absorber regions. As shown in Figure 7.4 (a, c), the refractive index bistability exhibits periodic spatial variation. The extracted hysteresis loops in Figure 7.4 (b, d) are consistently clockwise,

7.3. RESULTS AND DISCUSSION

indicating absorption/gain-driven nonlinearity that results in dispersion-based bistability at the probe wavelength.

To quantify the spatially distributed hysteresis in both the gain and absorber, I define the hysteresis area $A(\tau)$ as the accumulated phase shift per pump cycle ($\text{rad} \cdot \text{mW}$):

$$A(\tau) = \frac{2\pi\Delta l}{\lambda_c} \int_{L_{\text{th}}}^{U_{\text{th}}} [\delta n_{\uparrow}(p, \tau) - \delta n_{\downarrow}(p, \tau)] dp, \quad (7.5)$$

where $\delta n_{\uparrow}$ and $\delta n_{\downarrow}$ represent the refractive index on the up- and down-steps of the pump at position τ .

Spatial variations in $A(\tau)$ reflect the influence of fibre geometry on refractive index modulation and bistable dynamics. As shown in Figure 7.4 (e), the hysteresis area follows the six-arc fibre layout, where geometric factors, such as bend radius, waveguide boundary conditions, and pump/laser mode overlap, affect the spatial redistribution of population inversion. These geometric dependencies lead to nonuniform δn , unveiling the role of structural layout in shaping local nonlinear interactions and gain-loss balance.

7.3.4 ANALYSIS OF LOCALIZED NONLINEARITY AT FIBRE INTERFACES

While the overall mapping reveals a cavity-wide phenomenon, the theory suggests that nonlinear effects may be enhanced at material or geometric discontinuities. Therefore, I focus analysis on specific regions, such as the splices between fibres, to investigate this localized nonlinearity.

7.3. RESULTS AND DISCUSSION

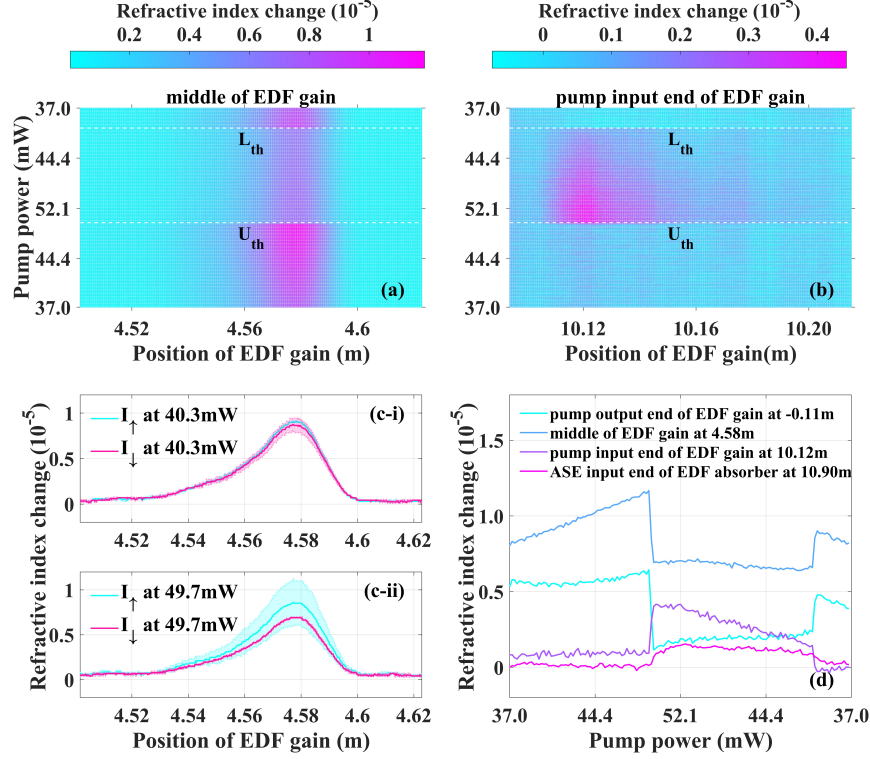


Figure 7.5: Spatially-resolved bistability in refractive index near the central region of EDF gain (a) and at the input end of the EDF absorber (b). Fluctuations of the distributed refractive index near the EDF gain centre as pump crosses the upper (c-i) and lower (c-ii) thresholds. (d) Hysteresis cycles of the refractive index change at specific locations, showing both clockwise and counterclockwise cycles.

I further examine δn near material discontinuities, such as splice interfaces. Figure 7.5 shows spatially resolved refractive index measurements within the central gain region and at the splices, where enhanced nonlinearity is expected. As shown in Figure 7.5 (a, b), these regions exhibit larger δn than uniform fibre sections. Figure 7.5 (c-i, c-ii) compares δn variations with and without threshold crossing, a clear transitions occurs at the bistability thresholds on a timescale of seconds. Hysteresis in Figure 7.5 (d) exhibit both clockwise

7.3. RESULTS AND DISCUSSION

and counterclockwise cycles, depending on Kerr-type nonlinearity driven by local intensity distribution (eq. (7.3)).

As pump power increases, forward ASE depletes inversion, gradually raising δn_k via the Kerr effect (*e.g.*, at -0.11 m, 4.58 m). At splices, δn_k scales with local intensity change. Once lasing starts, backward ASE drives population redistribution, enhancing δn_k at the pump and absorber inputs. Simultaneously, depletion suppresses forward ASE, reducing δn_k near the output and defect regions. As the pump decreases, the cycle reverses, completing the hysteresis loop. Elevated δn at the EDF centre reflects stronger local-intensity-driven Kerr response.

7.3.5 DYNAMIC STABILITY AND NOISE REDUCTION IN THE BISTABLE REGIME

The implication of these findings relates to laser stability. While nonlinearities and mode competition are often sources of instability, the establishment of a bistable equilibrium suggests the possibility of a stable operating regime. This subsection investigates the temporal dynamics and intensity noise of the laser to test this hypothesis.

Temporal and statistical variations in refractive index were evaluated in both the gain and absorber regions. Figure 7.6 (a) shows the averaged δn over a 24.3 cm window, plotted in cyan (gain) and magenta (absorber). In the gain region, the standard deviation of δn increases from $\pm 2.1 \times 10^{-8}$ to $\pm 5.6 \times 10^{-8}$, while in the absorber it reaches $\pm 6.2 \times 10^{-8}$ above the upper threshold. This suggests that the polarization-maintaining EDF (gain)

7.3. RESULTS AND DISCUSSION

exhibits reduced power-induced fluctuations compared to the single-mode EDF (absorber). Both curves follow similar trends, offset by approximately 2×10^{-7} .

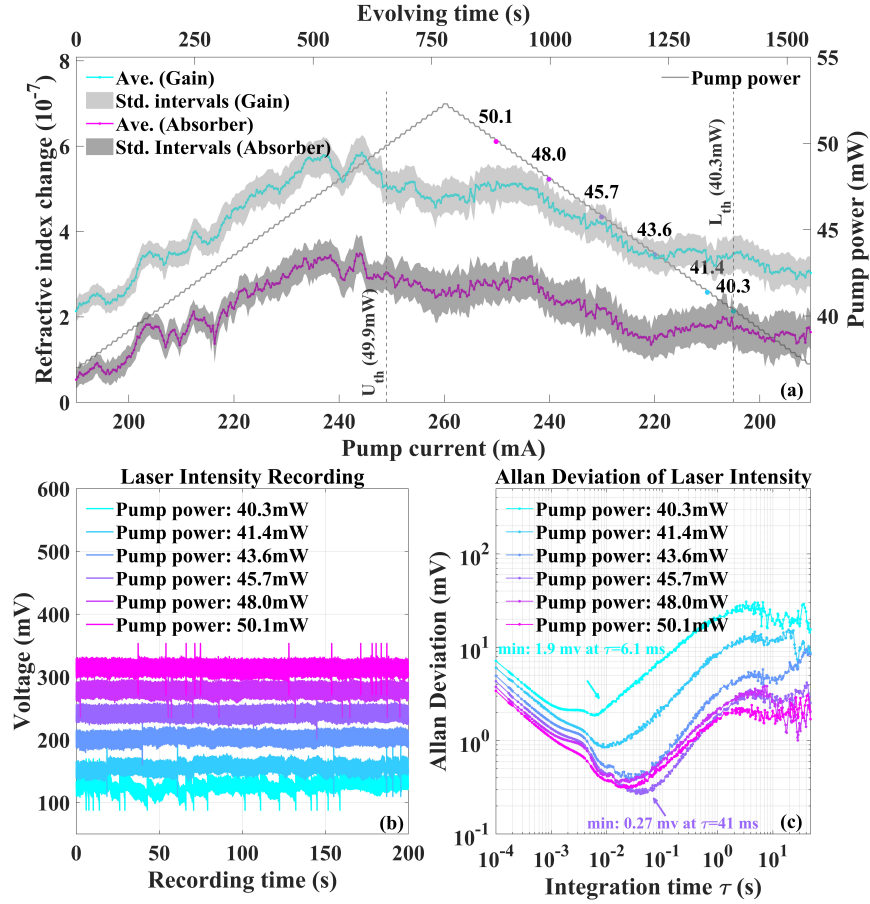


Figure 7.6: Temporal and statistical analysis of refractive index and laser intensity stability. (a) Averaged δn in the gain (cyan) and absorber (magenta) regions versus pump power; shaded areas indicate standard deviation. (b) Real-time intensity traces show mode-hopping at 40.3/41.4 mW and enhanced stability at 43.6/45.7 mW. (c) Allan deviation shows minimum noise (0.27 mV at 41 ms) at 45.7 mW, while higher noise at 40.3 mW (1.9 mV at 6.1 ms) reflects critical slowing down and increased mode competition.

Figure 7.6 (b) presents real-time laser intensity traces under different pump powers.

7.4. CHAPTER CONCLUSION

Near the lower threshold (40.3 and 41.4 mW), mode-hopping and instability dominate. As the pump increases to 43.6 and 45.7 mW, intensity becomes significantly more stable with minimal δn modulation, extending steady lasing beyond 160 seconds. Throughout measurements, the laser consistently operated in a continuous-wave (CW) mode, and no self-pulsing behaviour (such as Q-switching) was observed. This transition is further quantified in Figure 7.6 (c), where Allan deviation analysis shows improved intensity stability at 45.7 mW, indicating optimal stability. In contrast, larger deviations at lower pump powers (1.9 mV at 6.1 ms for 40.3 mW) reflect critical slowing down, where perturbation recovery time increases and the system becomes more sensitive to minor fluctuations [150].

The system stabilizes when gain and absorption reach dynamic equilibrium. As shown in Figure 7.3 (a, d), suppressed random modes are progressively amplified near the lower threshold, while the dominant mode weakens. This disrupts the gain-loss balance and intensifies mode competition.

7.4 CHAPTER CONCLUSION

In this chapter, a comprehensive investigation into optical bistability within a novel EDRFL configuration was presented. The results confirmed the existence of robust bistability and, through high-resolution internal probing, elucidated its physical origins and, most importantly, demonstrated its utility as a mechanism for passive laser stabilization.

In this work, optical bistability in an Erbium-doped random fibre laser has been

7.4. CHAPTER CONCLUSION

demonstrated and comprehensively characterized. By employing an unpumped EDF section as a saturable absorber alongside a random fibre grating for distributed feedback, a robust hysteresis loop was established, where gain competition drives opposing cycles between dominant and suppressed modes. High-resolution OFDR enabled the first spatially resolved mapping of the associated refractive index bistability, revealing a hybrid hysteresis driven by both population-level (Kramers–Kronig) dynamics and localized, intensity-dependent (Kerr) effects. Crucially, the bistable regime was shown to contain a window of exceptional stability. Operating the laser where intracavity power clamps both gain and saturable loss actively suppresses mode competition and reduces intensity noise by nearly an order of magnitude.

This work provides direct experimental validation that a stable equilibrium can be achieved in a complex disordered system without active electronic feedback, offering a passive approach for engineering ultra-stable random lasers. The demonstrated principles could lead to passively stabilized, narrow-band RFL sources for high-precision sensing and metrology. The bistable nature of the EDRFL establishes it as a promising platform for all-optical applications such as optical memory elements, logic gates, and tunable switches, where the state can be flipped using an external optical field.

Several limitations in this study suggest directions for future work. First, the observed bistability is governed by millisecond-scale Er^{3+} population dynamics, limiting its suitability for high-speed switching applications. Exploring similar mechanisms in systems with faster nonlinearities, such as semiconductor-based random lasers, would be valuable. Second, a detailed investigation of the spectral linewidth of individual random modes within the

7.4. *CHAPTER CONCLUSION*

hysteresis loop could provide deeper insights into the laser's coherence and ultimate stability limits. Finally, while the stable operating regime was empirically identified, developing a comprehensive theoretical model could enable predictive design by optimizing fibre parameters and RFG properties to engineer the stability window.

CONCLUSIONS

Chapter 8

CONCLUSION

8.1 SYNTHESIS OF RESEARCH

This thesis has undertaken a fundamental investigation of the physical phenomena that define the material and dynamic performance limits of advanced fibre optic systems. The research was driven by challenges in two key areas: the characterization of distributed sensors under extreme thermal conditions, and the passive, all-optical control of complex random fibre lasers. The work progressed from identifying core scientific ambiguities and control challenges in these domains to developing new quantitative frameworks and experimental methodologies to address them. This concluding chapter consolidates the key findings, evaluates the objectives outlined in the introduction, reflects on the broader implications of the research, and proposes directions for future study.

8.2. SUMMARY OF FINDINGS AND FULFILLED OBJECTIVES

8.2 SUMMARY OF FINDINGS AND FULFILLED OBJECTIVES

The experimental and analytical work conducted has successfully met the research objectives, providing new insights and capabilities in both sensing and laser physics.

8.2.1 CHARACTERIZATION OF SENSOR PERFORMANCE AT MATERIAL LIMITS

The first part of this thesis was motivated by the challenge of performing reliable distributed sensing in extreme thermal environments where the sensor's own material instability introduces a critical ambiguity into the measurement. The objective was to establish a quantitative framework for characterizing the sensor's condition *in-situ*.

This objective was fulfilled through a two-stage investigation. The first, presented in Chapter 3, utilized the high-gradient environment of an electrical arc plasma to establish a methodology for using the Rayleigh spectral and temporal correlations as quantitative diagnostic signals. By applying a rigorous non-linear calibration, this study demonstrated the capability of OFDR to continuously track spatially resolved temperature profiles up to approximately 1300 °C. Crucially, the investigation identified that the fundamental physical limit of cross-correlation sensing is intrinsically tied to the fibre's fictive temperature (T_f) and glass transition temperature (T_g). As the extreme thermal load forces the silica network across this boundary, rapid structural relaxation triggers severe signal decorrelation. Furthermore, it was demonstrated that the statistical properties of the residual spectral

8.2. SUMMARY OF FINDINGS AND FULFILLED OBJECTIVES

and temporal shifts provide direct, macroscopic measures of permanent internal stress, transient waveguide breakdown, and irreversible material elongation. This work effectively transformed a known limitation—spectral decorrelation—into an *in-situ* thermodynamic signature of the glass phase transition.

Building on this, the investigation in Chapter 4 addressed the temporal evolution of this degradation under the sustained thermal load of a hydrogen flame. This led to the formulation and validation of decorrelation time as a new, physically grounded metric for sensor durability. By quantifying the reliable operational duration (*e.g.*, 50% decorrelation in ~ 20 seconds at $1000\text{ }^{\circ}\text{C}$), this work provides the quantitative framework necessary for defining a sensor’s measurement validity window and assessing its reusability, thereby resolving the initial ambiguity between the sensor’s response and its changing state.

8.2.2 ALL-OPTICAL PASSIVE CONTROL OF RANDOM FIBRE LASERS

The second part of the thesis was motivated by the challenge of controlling the intrinsic dynamic instabilities of random fibre lasers using all-optical, passive methodologies. The investigation pursued two distinct conceptual strategies for control.

The first strategy—disciplining the laser’s dynamics by slaving its frequency to an external, passive reference—was successfully demonstrated in Chapter 6. The use of a novel random-fibre-grating ring resonator for self-injection locking was shown to be highly effective at suppressing chaotic mode competition. This all-optical feedback mechanism enforced single-frequency operation, achieving a 1.23 kHz linewidth and a 70 dB side-mode

8.3. SIGNIFICANCE AND IMPLICATIONS

suppression ratio, demonstrating that performance comparable to electronically stabilized systems can be achieved through a structurally simple, passive design.

The second strategy—harnessing the system’s own internal nonlinearities to create a self-regulating state—was validated in Chapter 7. A robust optical bistability was engineered in an erbium-doped random fibre laser, and a stable operational regime was identified within its hysteresis loop that resulted in an order-of-magnitude reduction in intensity noise. This confirmed that instability can be transformed into a mechanism for control. This investigation was made possible by a novel diagnostic application of OFDR, which provided the first spatially resolved map of the internal refractive index dynamics of an active laser. This revealed the physical origin of the bistability as a hybrid of cavity-wide, population-driven (Kramers–Kronig) and localized, intensity-driven (Kerr) nonlinearities, fulfilling the final research objective.

8.3 SIGNIFICANCE AND IMPLICATIONS

The contributions of this thesis carry implications that extend beyond the specific systems studied. The framework for sensor durability provides a new paradigm for the engineering of sensing systems intended for any application near material limits, shifting the focus from simple measurement to the quantitative characterization of reliability. The two successful laser stabilization schemes offer a new toolkit for the design of coherent light sources, presenting distinct physical philosophies for achieving control in complex photonic systems. Finally, the methodological innovation of using OFDR for *in-situ* diagnostics of active

8.4. *LIMITATIONS AND FUTURE OUTLOOK*

devices provides a powerful new capability for the experimental validation of theoretical models and the optimization of a wide range of active and nonlinear fibre components.

8.4 LIMITATIONS AND FUTURE OUTLOOK

This research also defines boundaries that point to future inquiry. The thermal analysis of the sensor could be refined by incorporating non-linear thermo-optic coefficients for greater accuracy at extreme temperatures. The application of the durability metric to novel high-temperature fibres, such as sapphire FBGs and metal-coated fibres, would be a valuable extension. In the laser systems, the millisecond-scale switching time of the demonstrated bistability, limited by erbium population dynamics, motivates the exploration of these control concepts in media with faster nonlinearities, such as semiconductor-based random lasers, to pursue high-speed applications. The theoretical models in Appendices A–E provide a framework for designing systems with tailored properties. Future work could extend the use of OFDR as an internal diagnostic to probe complex fibre phenomena such as soliton dynamics and spatial hole burning.

APPENDICES

A

WAVE PROPAGATION IN A SPATIALLY NON-UNIFORM DIELECTRIC MEDIUM

This appendix derives the one-dimensional wave equation governing an electromagnetic field propagating in a linear, non-uniform, and potentially lossy dielectric medium, forming the theoretical basis for analyzing Rayleigh backscattering in optical fibres.

A.1 MAXWELL'S EQUATIONS IN A DIELECTRIC MEDIUM

We begin with Maxwell's equations in a region with no free charges ($\rho = 0$) or free currents ($\mathbf{J} = 0$). The constitutive relations for a linear, isotropic, non-magnetic dielectric are:

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E} = \epsilon \mathbf{E}, \tag{A.1a}$$

$$\mathbf{B} = \mu_0 \mathbf{H}, \tag{A.1b}$$

A.2. DERIVATION OF THE VECTOR WAVE EQUATION

where ϵ is the spatially varying electric permittivity and ϵ_r is the relative permittivity, which is directly related to the complex refractive index n by $n^2 = \epsilon_r$. Maxwell's equations then take the form:

$$\nabla \cdot (\epsilon \mathbf{E}) = 0, \quad (\text{A.2a})$$

$$\nabla \cdot \mathbf{B} = 0, \quad (\text{A.2b})$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (\text{A.2c})$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial(\epsilon \mathbf{E})}{\partial t}. \quad (\text{A.2d})$$

A.2 DERIVATION OF THE VECTOR WAVE EQUATION

To derive the wave equation for the electric field $\mathbf{E}$, we take the curl of Faraday's Law (A.2c) and substitute Ampère-Maxwell Law (A.2d). Using the vector identity $\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$, we obtain:

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 \frac{\partial^2 \epsilon \mathbf{E}}{\partial t^2}. \quad (\text{A.3})$$

In a non-uniform medium where ϵ is a function of position, the divergence term $\nabla \cdot \mathbf{E}$ is generally non-zero. From Gauss's Law (A.2a), we have $\nabla \cdot (\epsilon \mathbf{E}) = \epsilon(\nabla \cdot \mathbf{E}) + \mathbf{E} \cdot \nabla \epsilon = 0$. This allows us to express the divergence term as:

A.3. THE SCALAR HELMHOLTZ EQUATION

$$\nabla \cdot \mathbf{E} = -\frac{1}{\epsilon} \mathbf{E} \cdot \nabla \epsilon = -\frac{1}{\epsilon_r} \mathbf{E} \cdot \nabla \epsilon_r. \quad (\text{A.4})$$

And we simplified the (A.3) into expression:

$$-\nabla^2 \mathbf{E} - \nabla \left(\frac{1}{\epsilon} \mathbf{E} \cdot \nabla \epsilon \right) = -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} (\epsilon \mathbf{E}). \quad (\text{A.5})$$

A.3 THE SCALAR HELMHOLTZ EQUATION

For a single-mode fibre with a cylindrically symmetric waveguide, we focus on the propagation component $E(z, t)$ along the z-axis, neglecting transverse derivative terms in the curl and divergence operations on the field. Under the slowly varying envelope approximation, this is a valid simplification, leading to the one-dimensional scalar wave equation:

$$\frac{\partial^2 E_z}{\partial z^2} + \frac{\partial}{\partial z} \left(\frac{1}{\epsilon_r} E_z \frac{\partial \epsilon_r}{\partial z} \right) = \frac{1}{c^2} \left(\epsilon_r \frac{\partial^2 E_z}{\partial t^2} + 2 \frac{\partial \epsilon_r}{\partial t} \frac{\partial E_z}{\partial t} + E_z \frac{\partial^2 \epsilon_r}{\partial t^2} \right), \quad (\text{A.6})$$

where $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ is the speed of light in vacuum. For further simplification, we assume small spatial fluctuations, *i.e.*,

$$\epsilon_r(z) = \bar{\epsilon}_r + \delta \epsilon_r(z). \quad (\text{A.7})$$

Considering a spatially non-uniform but temporally static waveguide in the fibre, *i.e.*, $\partial \epsilon_r / \partial t = 0$, and assuming that the spatial fluctuations are small enough to retain only

A.3. THE SCALAR HELMHOLTZ EQUATION

first-order terms, *i.e.*, $\frac{\partial^2 \epsilon_r}{\partial z^2} = 0$, we obtain:

$$\frac{\partial^2 E}{\partial z^2} = \frac{\epsilon_r}{c^2} \frac{\partial^2 E}{\partial t^2}. \quad (\text{A.8})$$

By applying a Fourier transform to move into the frequency domain:

$$E(z, t) = \frac{1}{\sqrt{2\pi}} \int \tilde{E}(z, \omega) e^{-i\omega t} d\omega, \quad (\text{A.9})$$

we obtain the time-independent Helmholtz equation:

$$\frac{\partial^2 \tilde{E}(z, \omega)}{\partial z^2} + \frac{\epsilon_r(z)}{c^2} \omega^2 \tilde{E}(z, \omega) = 0. \quad (\text{A.10})$$

Define the spatially varying wavenumber $k(z) = \frac{\omega}{c} \sqrt{\epsilon_r(z)} = \frac{\omega}{c} \sqrt{\bar{\epsilon}_r + \delta\epsilon_r(z)}$, we obtain:

$$\frac{\partial^2 \tilde{E}}{\partial z^2} + k^2(z) \tilde{E} = 0. \quad (\text{A.11})$$

A.4. THE NONLINEAR EQUATION FOR THE COMPLEX PROPAGATION PHASE

A.4 THE NONLINEAR EQUATION FOR THE COMPLEX PROPAGATION PHASE

To analyze the phase and amplitude evolution of the wave, we postulate a solution of the form representing a combination of forward-probing wave and backward-scattering wave:

$$\tilde{E} = \tilde{E}_f + \tilde{E}_b. \quad (\text{A.12})$$

By substituting (A.12) into the 1D Helmholtz equation (A.11), we have:

$$\left[\frac{\partial^2 E_b}{dz^2} + k^2(z)E_b \right] = - \left[\frac{\partial^2 E_f}{dz^2} + k^2(z)E_f \right]. \quad (\text{A.13})$$

This indicates that the backward wave is governed by a wave equation where the source term on the right-hand side is determined by how forward wave satisfies the local wave equation.

To evaluate the source term, we firstly introduce the forward wave as $\tilde{E}_f = A_f(z)e^{i\varphi_f(z)}$ where $\varphi_f(z)$ is a complex propagation phase, and its first and second derivatives are:

$$\frac{\partial \tilde{E}_f}{\partial z} = \left(\frac{\partial A_f}{\partial z} \right) e^{i\varphi_f} + i \frac{\partial \varphi_f}{\partial z} \tilde{E}_f, \quad (\text{A.14a})$$

$$\frac{\partial^2 \tilde{E}_f}{\partial z^2} = \left[i \frac{\partial^2 \varphi_f}{\partial z^2} - \left(\frac{\partial \varphi_f}{\partial z} \right)^2 \right] \tilde{E}_f + \left[2i \frac{\partial \varphi_f}{\partial z} \frac{\partial A_f}{\partial z} + \frac{\partial^2 A_f}{\partial z^2} \right] e^{i\varphi_f}. \quad (\text{A.14b})$$

A.4. THE NONLINEAR EQUATION FOR THE COMPLEX PROPAGATION PHASE

And similarly, we introduce the backward wave as $\tilde{E}_b = A_b(z)e^{i\varphi_b(z)}$, and obtain its first and second derivatives as:

$$\frac{\partial \tilde{E}_b}{\partial z} = \left(\frac{\partial A_b}{\partial z} \right) e^{-i\varphi_b} - i \frac{\partial \varphi_b}{\partial z} \tilde{E}_b, \quad (\text{A.15a})$$

$$\frac{\partial^2 \tilde{E}_b}{\partial z^2} = \left[-i \frac{\partial^2 \varphi_b}{\partial z^2} - \left(\frac{\partial \varphi_b}{\partial z} \right)^2 \right] \tilde{E}_b + \left[-2i \frac{\partial \varphi_b}{\partial z} \frac{\partial A_b}{\partial z} + \frac{\partial^2 A_b}{\partial z^2} \right] e^{-i\varphi_b}. \quad (\text{A.15b})$$

By substituting (A.14b) and (A.15b) into (A.13), we find that the local solutions to the wave equation satisfy the nonlinear Riccati equations:

$$k^2(z) + i \frac{\partial^2 \varphi_f}{\partial z^2} - \left(\frac{\partial \varphi_f}{\partial z} \right)^2 = 0, \quad (\text{A.16a})$$

$$k^2(z) - i \frac{\partial^2 \varphi_b}{\partial z^2} - \left(\frac{\partial \varphi_b}{\partial z} \right)^2 = 0, \quad (\text{A.16b})$$

which in turn allows us to obtain the exact coupled-wave equation:

$$\left[\frac{\partial^2 A_b}{\partial z^2} - 2i \frac{\partial \varphi_b}{\partial z} \frac{\partial A_b}{\partial z} \right] e^{-i\varphi_b} = - \left[\frac{\partial^2 A_f}{\partial z^2} + 2i \frac{\partial \varphi_f}{\partial z} \frac{\partial A_f}{\partial z} \right] e^{i\varphi_f}. \quad (\text{A.17})$$

To solve this equation, we apply the Slowly Varying Envelope Approximation (SVEA), which assumes that the amplitudes A_f and A_b vary much more slowly along z than the optical phase. Under this approximation, the second-derivative terms $\frac{\partial^2 A_f}{\partial z^2}$ and $\frac{\partial^2 A_b}{\partial z^2}$ can

A.4. THE NONLINEAR EQUATION FOR THE COMPLEX PROPAGATION PHASE

be neglected, reducing the coupled-wave equation to:

$$\left[-2i \frac{\partial \varphi_b}{\partial z} \frac{\partial A_b}{\partial z} \right] e^{-i\varphi_b} \approx - \left[2i \frac{\partial \varphi_f}{\partial z} \frac{\partial A_f}{\partial z} \right] e^{i\varphi_f}. \quad (\text{A.18})$$

Solving for the derivative of the backward-wave amplitude yields:

$$\frac{\partial A_b(z)}{\partial z} = \frac{\partial \varphi_f(z)/\partial z}{\partial \varphi_b(z)/\partial z} \frac{\partial A_f(z)}{\partial z} e^{i[\varphi_f(z)+\varphi_b(z)]}, \quad (\text{A.19})$$

which shows that a small variation in the forward amplitude A_f at position z produces a corresponding contribution to the backward amplitude A_b , weighted by the relative phase evolution of the forward and backward waves. By integrating over the fibre end point L back to the origin, We obtain the total backscattered amplitude at input ($z = 0$):

$$A_b(0) = \int_L^0 \frac{\partial \varphi_f(\zeta)/\partial \zeta}{\partial \varphi_b(\zeta)/\partial \zeta} \frac{\partial A_f(\zeta)}{\partial \zeta} e^{i[\varphi_f(\zeta)+\varphi_b(\zeta)]} d\zeta. \quad (\text{A.20})$$

Therefore, given static random dielectric profile $\epsilon_r(z)$ and small fluctuations $\delta\epsilon_r(z)$, the Rayleigh backscattering field deep inside fibre at a given location z is found for given spectral component ω by:

$$\tilde{E}_b(z) = A_b(z)e^{-i\varphi_b(z)} = e^{-i\varphi_b(z)} \int_L^z \frac{\partial \varphi_f(\zeta)/\partial \zeta}{\partial \varphi_b(\zeta)/\partial \zeta} \frac{\partial A_f(\zeta)}{\partial \zeta} e^{i[\varphi_f(\zeta)+\varphi_b(\zeta)]} d\zeta. \quad (\text{A.21})$$

B

WKB SOLUTION FOR THE COMPLEX PROPAGATION PHASE

This appendix solves the nonlinear phase equations derived in Appendix A for forward and backward propagating waves. Using the Wentzel-Kramers-Brillouin (WKB) approximation, we analyze wave propagation in media with slowly varying properties and establish the principles enabling OFDR systems to detect variations in complex refractive index

B.1 THE NONLINEAR PHASE EQUATIONS

As shown in Appendix A, the complex propagation phases of the forward wave $\varphi_f(z)$, and the backward wave $\varphi_b(z)$, satisfy a pair of conjugate nonlinear Riccati equations:

$$\left(\frac{d\varphi_f}{dz}\right)^2 - i\frac{d^2\varphi_f}{dz^2} = k^2(z), \quad (\text{B.1a})$$

$$\left(\frac{d\varphi_b}{dz}\right)^2 + i\frac{d^2\varphi_b}{dz^2} = k^2(z), \quad (\text{B.1b})$$

B.2. THE WKB APPROXIMATION SOLUTION

where $k^2(z) = (\omega/c)^2 \epsilon_r(z)$. In high-temperature or dynamic sensing applications, the relative permittivity $\epsilon_r(z)$ and the wavenumber $k(z)$ are complex. Solving these equations are essential for modelling the round-trip phase evolution of the backscattered signal.

B.2 THE WKB APPROXIMATION SOLUTION

The WKB method provides an approximate solution to (B.1) by transforming the nonlinear problem into a linear one. This approach is physically justified in optical fibres where refractive index variations occur over length scales much larger than the optical wavelength. To reduce (B.1) to a second-order linear ordinary differential equation ODE, we define $p_+(z) \equiv \left(\frac{d\varphi_f}{dz} \right)$ and $p_-(z) \equiv \left(\frac{d\varphi_b}{dz} \right)$. After rearrangement, the governing equations become:

$$p'_+(z) = -i \left(p_+^2(z) - k^2(z) \right), \quad (\text{B.2a})$$

$$p'_-(z) = i \left(p_-^2(z) - k^2(z) \right). \quad (\text{B.2b})$$

Both (B.2a) and (B.2b) have the Riccati form $y' = a + by + cy^2$. Applying the standard substitution $y(z) = -\frac{1}{c} \frac{u'(z)}{u(z)}$, and taking $c = -i$ for the forward wave and $c = i$ for the

B.2. THE WKB APPROXIMATION SOLUTION

backward wave, we obtain:

$$p_+(z) = -i \frac{u'_+(z)}{u_+(z)}, \quad (\text{B.3a})$$

$$p_-(z) = i \frac{u'_-(z)}{u_-(z)}. \quad (\text{B.3b})$$

Substituting each expression into the corresponding Riccati equation (B.2) yields the same linear Helmholtz equation for both the forward and backward waves:

$$u''(z) + k^2(z) u(z) = 0, \quad (\text{B.4})$$

which has two fundamental WKB solutions to this equation represent forward (+) and backward (−) propagating waves with slowly varying amplitudes:

$$u_+(z) \approx \frac{C_+}{\sqrt{k(z)}} \exp \left(+i \int_0^z k(\zeta) d\zeta \right), \quad (\text{B.5a})$$

$$u_-(z) \approx \frac{C_-}{\sqrt{k(z)}} \exp \left(-i \int_0^z k(\zeta) d\zeta \right). \quad (\text{B.5b})$$

The factor $\frac{C_{\pm}}{\sqrt{k(z)}}$ ensure that the energy flux of the wave is conserved as it propagates. The exponential term describes the primary phase accumulation. By inverting the substitution in (B.3) and integrating the phase derivative, we obtain the complex phase functions. The forward phase φ_f is linked with the forward solution u_+ , while the backward phase φ_b

B.3. ANALYSIS OF THE BACKSCATTERED FIELD FOR OFDR SENSING

is derived from the same formalism from backward solution u_- :

$$\varphi_f(z) = -i \ln u_+(z) = \int_0^z k(\zeta) d\zeta + \frac{i}{2} \ln \frac{k(z)}{k(0)}, \quad (\text{B.6a})$$

$$\varphi_b(z) = i \ln u_-(z) = \int_0^z k(\zeta) d\zeta - \frac{i}{2} \ln \frac{k(z)}{k(0)}. \quad (\text{B.6b})$$

B.3 ANALYSIS OF THE BACKSCATTERED FIELD FOR OFDR SENSING

The WKB solution simplifies the expression for the backscattered signal measured by the OFDR. The total signal at the detector is the coherent sum of all backscattered waves, expressed by (A.20) from Appendix A:

$$A_b(0) = \int_L^0 \frac{\partial \varphi_f(\zeta)/\partial \zeta}{\partial \varphi_b(\zeta)/\partial \zeta} \frac{\partial A_f(\zeta)}{\partial \zeta} e^{i[\varphi_f(\zeta) + \varphi_b(\zeta)]} d\zeta,$$

where the term $\frac{dA_f(\zeta)}{d\zeta}$ acts as the local scattering strength, representing the portion of the forward wave's amplitude that is scattered backwards at position ζ . The exponential factor accounts for the full round-trip phase accumulated by the light as it propagates from the input end to the scattering point ζ and back.

B.3. ANALYSIS OF THE BACKSCATTERED FIELD FOR OFDR SENSING

B.3.1 ANALYSIS OF THE PHASE DERIVATIVE RATIO

The factor in the integral involves the ratio of the derivatives of the forward and backward phase. Differentiating the WKB solutions gives:

$$\frac{\partial \varphi_f(z)}{\partial z} = k(z) + \frac{i}{2} \frac{\partial k(z)/\partial z}{k(z)}, \quad (\text{B.7a})$$

$$\frac{\partial \varphi_b(z)}{\partial z} = k(z) - \frac{i}{2} \frac{\partial k(z)/\partial z}{k(z)}. \quad (\text{B.7b})$$

The ratio is therefore:

$$\frac{\partial \varphi_f(z)/\partial z}{\partial \varphi_b(z)/\partial z} = \frac{k(z) + \frac{i}{2} \frac{\partial k(z)/\partial z}{k(z)}}{k(z) - \frac{i}{2} \frac{\partial k(z)/\partial z}{k(z)}} = \frac{1 + \frac{i}{2} \frac{\partial k(z)/\partial z}{k^2(z)}}{1 - \frac{i}{2} \frac{\partial k(z)/\partial z}{k^2(z)}}. \quad (\text{B.8})$$

Within the WKB approximation, the term $\frac{\partial k(z)/\partial z}{k^2(z)}$ is considered small. Using the approximation $(1+x)/(1-x) \approx 1+2x$ for small x , the ratio is approximately $1 + i \frac{\partial k(z)/\partial z}{k^2(z)}$. For most OFDR applications, it is reasonable to approximate the ratio as unity. However, in media with large refractive-index gradients, this term can contribute to both the amplitude and phase of the Rayleigh backscattering, such as a crack or break in the fibre core, the end-face of the fibre.

B.3. ANALYSIS OF THE BACKSCATTERED FIELD FOR OFDR SENSING

B.3.2 ANALYSIS OF THE ROUND-TRIP PHASE TERM

The critical term is the round-trip phase in the exponential, $\varphi_{RT}(z) = \varphi_f(z) + \varphi_b(z)$. Summing the WKB solutions (B.6a) and (B.6b) yields:

$$\varphi_{RT}(z) = \left[\int_0^z k(\zeta) d\zeta + \frac{i}{2} \ln \frac{k(z)}{k(0)} \right] + \left[\int_0^z k(\zeta) d\zeta - \frac{i}{2} \ln \frac{k(z)}{k(0)} \right] = 2 \int_0^z k(\zeta) d\zeta. \quad (\text{B.9})$$

The second logarithmic correction terms, which determined by the spatial fluctuations of refractive index, have opposite signs for the forward and backward paths and therefore cancel out. This means that the total complex phase of a backscattered wave is simply twice the integral of the local wavenumber from the input to the scattering point.

By decomposing the wavenumber $k(z) = k'(z) + ik''(z) = (\omega/c) [n'(z) + in''(z)]$, we can separate the round-trip phase into its physically distinct real and imaginary parts:

$$\varphi_{RT}(z) \approx \underbrace{2 \int_0^z k'(\zeta) d\zeta}_{\text{Real Part: Determines Phase}} + \underbrace{2i \int_0^z k''(\zeta) d\zeta}_{\text{Imaginary Part: Determines Attenuation}}. \quad (\text{B.10})$$

This separation provides a direct link between the physical properties of the fibre on local complex refractive index profile which creates the Rayleigh scattering signature and signal measured by OFDR. The real part of this phase is directly related to the round-trip Optical Path Length (OPL), which we denote as $\tilde{\Gamma}(z)$. Therefore, the thermally or strain-induced

B.3. ANALYSIS OF THE BACKSCATTERED FIELD FOR OFDR SENSING

OPL affecting the Rayleigh scattering response at any location z can be modelled as:

$$\tilde{\Gamma}(z) = 2 \int_0^z n'(\zeta) d\zeta. \quad (\text{B.11})$$

B.3.3 SENSING CHANGES IN THE REAL REFRACTIVE INDEX (k')

The phase of the backscattering is determined by the real part of the round-trip phase.

The total accumulated optical phase for a signal scattered at position z is:

$$\varphi_{\text{total}}(z) = \text{Re}[\varphi_{RT}(z)] \approx 2 \int_0^z k'(\zeta) d\zeta = \frac{2\omega}{c} \int_0^z n'(\zeta) d\zeta. \quad (\text{B.12})$$

A change in the real part of the refractive index, $\Delta n'$, induced by temperature or strain, modifies k' and produces a measurable total phase shift $\Delta\varphi_{\text{total}}(z)$. In OFDR, this manifests as a delay shift of the Rayleigh backscattering signature at position z , or equivalently, as a spectral shift in the frequency domain.

B.3.4 SENSING CHANGES IN THE IMAGINARY REFRACTIVE INDEX (k'')

The amplitude of the backscattered signal is determined by the imaginary part of the round-trip phase, which not only governs attenuation but also the Rayleigh scattering strength and overall signature. The total round-trip factor for the electric field scattered at

B.3. ANALYSIS OF THE BACKSCATTERED FIELD FOR OFDR SENSING

position z is $e^{-\text{Im}[\varphi_{RT}(z)]}$, where

$$\text{Im}[\varphi_{RT}(z)] \approx 2 \int_0^z k''(\zeta) d\zeta = \frac{2\omega}{c} \int_0^z n''(\zeta) d\zeta. \quad (\text{B.13})$$

1. **Change in Attenuation:** The imaginary part of the round-trip phase governs the signal attenuation. A localized increase in absorption ($\Delta n'' > 0$) at a specific position will therefore alter the measured intensity of the Rayleigh backscattering signal originating from all points at or beyond that position. This often appears as a step-like drop in the signal's amplitude profile.
2. **Transformation of the Scattering Signature:** The physical processes that alter n'' often correspond to a change in the material's microscopic structure. This change fundamentally transforms the random dielectric profile, $\delta\epsilon_r(z)$, which is the very source of the Rayleigh scattering signature. In essence, the intrinsic fingerprint of the fibre at that location is permanently altered.

This transformation of the scattering signature leads to a rapid decorrelation of the measured Rayleigh pattern over time. As a result, traditional cross-correlation methods used for phase-based sensing become unreliable or fail completely, as they depend on a stable scattering signature. This necessitates a direct analysis of the both signal's phase and amplitude to quantify the change in n'' , making it a distinct and complementary sensing modality to phase-based measurements.

C

RANDOM FIBRE GRATING RING RESONATOR MODEL

C.1 NORMAL FIBRE BRAGG GRATING

For a uniform, reflective fibre Bragg grating (FBG) with constant period Λ and uniform index modulation (i.e., zero longitudinal phase variation $d\phi/dz = 0$), the slowly varying envelopes of the forward-propagating field $R(z)$ and the backward-propagating field $S(z)$ satisfy the linear coupled-mode equations:

$$\begin{aligned}\frac{dR}{dz} + i\left(\kappa_{dc} + \frac{1}{2}\Delta\beta\right)R &= -i\kappa_{ac}^*S, \\ \frac{dS}{dz} - i\left(\kappa_{dc} + \frac{1}{2}\Delta\beta\right)S &= i\kappa_{ac}R,\end{aligned}\tag{C.1}$$

where

$$\kappa_{dc} = \frac{4\pi}{\lambda}\eta\Delta\bar{n}, \quad \kappa_{ac} = \kappa_{dc}/2.$$

C.2. RANDOM FIBRE GRATING

The self-coupling (DC) coefficient κ_{dc} represents the phase shift per unit length imposed by the average index change $\Delta\bar{n}$ of the grating. Here, λ is the free-space wavelength, Λ is the grating period, and η is the mode-overlap integral between forward and backward fields. The alternating-index (AC) coupling coefficient κ_{ac} quantifies the power transfer strength from the forward to backward propagating mode.

The detuning parameter $\Delta\beta$ measures the phase mismatch between the guided forward/backward modes and the Bragg condition, which controls the mode coupling efficiency:

$$\Delta\beta = \beta_f + \beta_b - \frac{2\pi}{\Lambda}, \quad (\text{C.2})$$

with the propagation constants $\beta_{f,b} = 2\pi n_{\text{eff}}/\lambda$. Because the grating is uniform, $d\phi/dz = 0$ and all coefficients are constant along z .

C.2 RANDOM FIBRE GRATING

The solution for a single grating segment of length L_0 can be expressed in the transfer-matrix form:

$$M_j = \begin{bmatrix} A - \frac{i\delta}{\alpha}B & -\frac{i\kappa_{ac}}{\alpha}B \\ \frac{i\kappa_{ac}}{\alpha}B & A + \frac{i\delta}{\alpha}B \end{bmatrix}, \quad (\text{C.3})$$

where the parameters are defined as

$$\alpha^2 = \kappa_{ac}^2 - \delta^2, \quad \delta = \kappa_{dc} + \frac{1}{2}\Delta\beta,$$

C.3. RANDOM FIBRE GRATING RING RESONATOR

and

$$A = \cosh(\alpha L_0), \quad B = \sinh(\alpha L_0).$$

The overall transfer matrix for a random fibre grating (RFG) formed by concatenating N segments is the product:

$$\mathbf{M}_{\text{RFG}} = M_N M_{N-1} \cdots M_1 = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}. \quad (\text{C.4})$$

From the transfer matrix, the scattering matrix $\mathbf{S}_{\text{RFG}}$ is derived by:

$$\mathbf{S}_{\text{RFG}} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}, \quad (\text{C.5})$$

with elements

$$s_{11} = -\frac{m_{21}}{m_{22}}, \quad s_{12} = \frac{1}{m_{22}}, \quad s_{21} = \frac{m_{11}m_{22} - m_{12}m_{21}}{m_{22}}, \quad s_{22} = \frac{m_{12}}{m_{22}}. \quad (\text{C.6})$$

C.3 RANDOM FIBRE GRATING RING RESONATOR

For a lossless, reciprocal optical coupler characterized by coupling ratio ρ , the transmission and cross-coupling coefficients τ and κ satisfy $\tau^2 + \kappa^2 = 1$ and are real-valued. The 4×4

C.3. RANDOM FIBRE GRATING RING RESONATOR

coupler matrix describing clockwise and counter-clockwise propagation is:

$$\mathbf{M}_{\text{cpl}} = \begin{bmatrix} \tau & -j\kappa & 0 & 0 \\ -j\kappa & \tau & 0 & 0 \\ 0 & 0 & \tau & -j\kappa \\ 0 & 0 & -j\kappa & \tau \end{bmatrix}. \quad (\text{C.7})$$

The upper 2×2 block describes the clockwise pair $(E_1, E_2) \rightarrow (E_3, E_4)$, and the lower block treats the counter-clockwise pair $(E_5, E_6) \rightarrow (E_7, E_8)$.

Assuming the RFG is placed at the midpoint of the ring, the propagation distance from the coupler to the grating is $\ell/2$. The complex propagation factor for a half-arc is:

$$p(\omega) = \exp \left[-\frac{1}{2}\alpha\ell + j\beta(\omega)\frac{\ell}{2} \right], \quad (\text{C.8})$$

where α represents loss, and $\beta(\omega)$ is the propagation constant.

The field propagation between inputs and outputs of the coupler and grating is described by the diagonal propagation matrix:

$$\mathbf{P} = p \mathbf{I}_4, \quad (\text{C.9})$$

C.3. RANDOM FIBRE GRATING RING RESONATOR

which relates the fields as

$$\begin{bmatrix} E'_4 \\ E_2 \\ E_6 \\ E'_8 \end{bmatrix} = \mathbf{P} \begin{bmatrix} E_4 \\ E'_2 \\ E'_6 \\ E_8 \end{bmatrix}. \quad (\text{C.10})$$

FIELD RELATIONS

The RFG acts as an inhomogeneous Bragg reflector with complex scattering coefficients $s_{ij}(\omega)$. Using the scattering matrix (C.5), the fields inside the ring satisfy:

$$\begin{aligned} E'_2 &= s_{11}E'_4 + s_{12}E'_8, \\ E'_6 &= s_{21}E'_4 + s_{22}E'_8, \end{aligned} \quad (\text{C.11})$$

while the coupler relates inputs and outputs as:

$$\begin{aligned} E_3 &= \tau E_1 - j\kappa E_2, \\ E_4 &= -j\kappa E_1 + \tau E_2, \\ E_7 &= \tau E_5 - j\kappa E_6, \\ E_8 &= -j\kappa E_5 + \tau E_6. \end{aligned} \quad (\text{C.12})$$

Letting the normalized input be $\gamma \equiv E_5/E_1$, and inserting the propagation relations

$$E'_4 = pE_4, \quad E'_8 = pE_8,$$

C.3. RANDOM FIBRE GRATING RING RESONATOR

we rewrite E_2, E_6 in terms of E_4, E_8 as:

$$\begin{aligned} E_2 &= p^2(s_{11}E_4 + s_{12}E_8), \\ E_6 &= p^2(s_{21}E_4 + s_{22}E_8). \end{aligned} \tag{C.13}$$

Define auxiliary variables:

$$A = p^2 s_{11}, \quad B = p^2 s_{12}, \quad C = p^2 s_{21}, \quad D = p^2 s_{22}. \tag{C.14}$$

The coupled fields E_4, E_8 satisfy the linear system with determinant

$$\Delta = (1 - \tau A)(1 - \tau D) - \tau^2 BC. \tag{C.15}$$

Solving for E_4/E_1 and E_8/E_1 yields:

$$\begin{aligned} \frac{E_4}{E_1} &= -j\kappa \frac{(1 - \tau D) + \tau B\gamma}{\Delta}, \\ \frac{E_8}{E_1} &= -j\kappa \frac{\gamma(1 - \tau A) + \tau C}{\Delta}. \end{aligned} \tag{C.16}$$

Subsequently,

$$\begin{aligned} \frac{E_2}{E_1} &= A \frac{E_4}{E_1} + B \frac{E_8}{E_1}, \\ \frac{E_6}{E_1} &= C \frac{E_4}{E_1} + D \frac{E_8}{E_1}, \end{aligned} \tag{C.17}$$

C.3. RANDOM FIBRE GRATING RING RESONATOR

and

$$\begin{aligned}\frac{E_3}{E_1} &= \tau - j\kappa \frac{E_2}{E_1} = \tau + \kappa^2 \frac{A[(1 - \tau D) + \tau B\gamma] + B[\gamma(1 - \tau A) + \tau C]}{\Delta}, \\ \frac{E_7}{E_1} &= \gamma\tau - j\kappa \frac{E_6}{E_1} = \gamma\tau + \kappa^2 \frac{C[(1 - \tau D) + \tau B\gamma] + D[\gamma(1 - \tau A) + \tau C]}{\Delta}.\end{aligned}\tag{C.18}$$

Accordingly, E_3/E_1 and E_7/E_1 represent the transmission and reflection coefficients, respectively.

SPECIAL CASE: NO GRATING

When no grating is present, the scattering matrix reduces to the identity:

$$s_{11} = s_{22} = 1, \quad s_{12} = s_{21} = 0,$$

which gives $A = D = p^2$, $B = C = 0$, and

$$\Delta = 1 - \tau^2 p^2.$$

In this limit, the expressions collapse to the standard ring-resonator results.

D

SELF-INJECTION LOCKING MODEL

This appendix details the self-injection locking model developed to simulate the dynamics of a random fibre grating ring (RFGR) resonator. The model is based on multi-delay variant of the Lang-Kobayashi (LK) equations, which are standard for describing semiconductor lasers with external optical feedback. The system is described by two coupled delay-differential equations: one governing the population inversion of the gain medium, and the other describing the evolution of the complex electric field under the influence of feedback from RFGR.

A key innovation of this model lies in its treatment of the complex feedback from the RFGR. The model treats the RFGR as a linear, time-invariant (LTI) filter whose effect on the laser's electric field is fully captured by its complex impulse response, denoted as $h_{\text{RFGR}}(t)$. The feedback term in the LK equation is therefore not a simple delayed copy of the field, but rather a convolution of laser's electric field history with impulse response.

According to principle of Fourier duality: the time domain impulse response $h_{\text{RFGR}}(t)$ is the inverse Fourier transform of the system's frequency-domain transfer function $H(\omega)$. The

D.1. RATE EQUATION FOR POPULATION INVERSION

transfer function can be readily calculated using a transfer-matrix model (see eq. (C.18)).

The impulse response $h_{\text{RFGR}}(t)$ is given by:

$$h_{\text{RFGR}}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H_{\text{RFGR}}(\omega) e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{E_7}{E_1}(\omega) e^{i\omega t} d\omega. \quad (\text{D.1})$$

The following sections will present the two core rate equations of the model.

D.1 RATE EQUATION FOR POPULATION INVERSION

The first component of the model describes the gain medium, an erbium-doped fibre (EDF), which is treated as a two-level atomic system. The dynamics of the fractional population of the excited state, N_2 , are governed by the following rate equations:

$$\frac{dN_2(t)}{dt} = W_{12}(N_{\text{tot}} - N_2) - \left(W_{21} + \frac{1}{\tau_{21}}\right)N_2 - \Gamma[\sigma_{e,s}N_2 - \sigma_{a,s}(N_{\text{tot}} - N_2)] \frac{|E(t)|^2}{h\nu_s A_{\text{core}}}, \quad (\text{D.2})$$

where N_{tot} is the total erbium ion density and τ_{21} is the spontaneous lifetime of the upper laser level. The pump absorption rate W_{12} and stimulated emission rate W_{21} are functions of the pump power P_p :

$$W_{12} = \frac{\sigma_{a,p} P_p}{h\nu_p A_{\text{core}}}, \quad W_{21} = \frac{\sigma_{e,p} P_p}{h\nu_p A_{\text{core}}}. \quad (\text{D.3})$$

The parameters used in this equation are defined in Table D.1.

D.2. RATE EQUATION FOR THE COMPLEX ELECTRIC FIELD

Symbol	Meaning	Value / Origin
h	Planck constant	6.626^{-34} J·s
$\nu_{s,p}$	frequency of laser, pump fields	$1.935, 2.027 \times 10^{14}$ Hz
N_2	Erbium ion density in $^4I_{13/2}$ manifold	ions·m $^{-3}$
N_{tot}	total Er $^{3+}$ density	1.2×10^{25} m $^{-3}$
τ_{21}	spontaneous lifetime	10 ms
$\sigma_{e/a,s}$	emission and absorption cross-sections at 1550 nm (laser)	$6.0, 4.0 \times 10^{-25}$ m 2
$\sigma_{e/a,p}$	emission and absorption cross-sections at 1480 nm (pump)	$2.0, 3.0 \times 10^{-25}$ m 2
Γ	overlap factor between optical mode and doped core	0.80
β_{sp}	fraction of spontaneous-emission into lasing mode	10^{-5}
A_{core}	Cross-sectional area of the fibre core	1.25^{-10} m 2

Table D.1: Key symbols, meanings, and values/origins used in population rate equation

D.2 RATE EQUATION FOR THE COMPLEX ELECTRIC FIELD

The second component of the model governs the evolution of the complex electric field. It is described by an integro-differential LK equation that includes terms for gain, cavity loss, and the distributed feedback from the RFGR. The full dynamic equation is:

$$\underbrace{\frac{dE(t)}{dt}}_{\text{Rate of Change}} = \underbrace{\frac{1}{2}(1 + i\alpha_H)g(t)E(t)}_{\text{1. Complex Gain}} - \underbrace{\frac{E(t)}{\tau_{\text{ph}}}}_{\text{2. Cavity Loss}} + \underbrace{\kappa \int_0^\infty h_{\text{RFGR}}(\tau)E(t - \tau) d\tau}_{\text{3. Delayed Feedback}}, \quad (\text{D.4})$$

where α_H is the Henry linewidth enhancement factor, τ_{ph} is the photon lifetime in the ring cavity, and κ is the feedback coupling strength. The temporal growth rate of the laser

D.3. DECOMPOSITION INTO POWER AND PHASE EQUATIONS

power gt is proportional to the net gain $G(T)$:

$$G(t) = \Gamma[\sigma_{\text{e,s}}N_2 - \sigma_{\text{a,s}}(N_{\text{tot}} - N_2)], \quad g(t) = \frac{c}{n_{\text{eff}}}G(t). \quad (\text{D.5})$$

In the LK equation, the first term represents EDF gain and the accompanying phase noise, which broadens the linewidth through the Henry factor α_H . The second term accounts for cavity losses, characterized by the photon lifetime τ_{ph} , acting as a damping mechanism that must be balanced by the gain. The third convolution term describes the delayed feedback from the RFGR, which provides a spectrally purified, phase-stabilized signal that pulls the laser frequency toward its resonance, thereby suppressing noise.

D.3 DECOMPOSITION INTO POWER AND PHASE EQUATIONS

For numerical simulation, it is advantageous to decompose the single complex equation eq. (D.4) into two coupled real-valued equations for the instantaneous optical power, $P(t) = |E(t)|^2$, and phase, $\phi(t)$. Begin with the electric field in the polar form, $E(t) = \sqrt{P(t)} e^{i\phi(t)}$, whose time derivative is

$$\dot{E} = \frac{1}{2} \left(\frac{\dot{P}}{P} + 2i\dot{\phi} \right) E. \quad (\text{D.6})$$

Substituting this into the LK equation (eq. (D.4)) and dividing by $E(t)$ gives:

$$\frac{1}{2} \left(\frac{\dot{P}}{P} + 2i\dot{\phi} \right) = \frac{1}{2} (1 + i\alpha_H) g - \frac{1}{\tau_{\text{ph}}} + \frac{\kappa}{E(t)} \int_0^\infty h_{\text{RFGR}}(\tau) E(t - \tau) d\tau. \quad (\text{D.7})$$

D.3. DECOMPOSITION INTO POWER AND PHASE EQUATIONS

The feedback integral can be rewritten by expressing the fields and the impulse response in polar coordinates. Let $P_\tau = P(t - \tau)$, $\phi_\tau = \phi(t - \tau)$, and $h_{\text{RFGR}}(\tau) = |h_{\text{RFGR}}(\tau)|e^{i\theta(\tau)}$. The feedback term becomes:

$$\kappa \int_0^\infty h_{\text{RFGR}}(\tau) \frac{E(t - \tau)}{E(t)} d\tau = \kappa \int_0^\infty |h_{\text{RFGR}}(\tau)| \sqrt{\frac{P_\tau}{P}} e^{i(\phi_\tau - \phi + \theta(\tau))} d\tau. \quad (\text{D.8})$$

By equating the real and imaginary parts of eq. (D.7) separately, we arrive at the final rate equations for power and phase. The power rate equation is obtained from the real part:

$$\dot{P}(t) = gP - \frac{2P}{\tau_{\text{ph}}} + 2\kappa \int_0^\infty |h_{\text{RFGR}}(\tau)| \sqrt{PP_\tau} \cos[\phi_\tau - \phi + \theta(\tau)] d\tau. \quad (\text{D.9})$$

The phase rate equation is obtained from the imaginary part:

$$\dot{\phi}(t) = \frac{\alpha_{\text{H}}}{2} g + \kappa \int_0^\infty |h_{\text{RFGR}}(\tau)| \sqrt{\frac{P_\tau}{P}} \sin[\phi_\tau - \phi + \theta(\tau)] d\tau. \quad (\text{D.10})$$

The parameters for these equations are defined in Table D.2.

Symbol	Meaning	Value / Origin
α_{H}	Henry linewidth-enhancement factor	0.8
n_{eff}	Effective refractive index of the fibre mode	1.4567
τ_{ph}	photon lifetime in ring cavities	2.1 μs
κ	coupling factor	$1 \times 10^7 \text{ s}^{-1}$

Table D.2: Key symbols, meanings, and values/origins used in complex field rate equation

E

OPTICAL BISTABILITY IN ERBIUM-DOPED RANDOM FIBRE LASER

E.1 MODEL ESTABLISHMENT

This section presents the theoretical model used to simulate the dynamics of the Erbium-Doped Random Fibre Laser (EDRFL). The model describes a system composed of two distinct sections: a gain medium and a saturable absorber with optical feedback. As shown in diagram Figure 7.1 (c), the gain section consists of a polarization-maintained (PM) Erbium-Doped Fibre (EDF) pumped at 1480 nm. The absorber section is an unpumped single-mode (SM) EDF, terminated by a Random Fibre Grating (RFG). In this configuration, backward-propagating Amplified Spontaneous Emission (ASE) from the gain medium is injected into the absorber and subsequently reflected by the RFG, establishing a feedback loop. The combination of gain saturation, saturable absorption, and delayed feedback provides the necessary conditions for optical bistability (OB).

E.1. MODEL ESTABLISHMENT

Following the approach applied in ring-cavity erbium-doped fibre lasers [151], the model is formulated using coupled propagation and rate equations for a two-level atomic system. It describes the spatio-temporal evolution of the optical powers and the population inversion of Er^{3+} ions. The index i denotes the two EDF sections, with $i = \alpha$ representing the gain medium and $i = \beta$ representing the absorber.

E.1.1 PROPAGATION AND RATE EQUATIONS

The propagation of optical powers along the fibre axis z at time t is described by the following equations. The evolution of the pump power P_p^α , which exists only in the gain medium, is given by:

$$\frac{dP_p^\alpha(z, t)}{dz} = \{[\gamma_a^\alpha(v_p) + \gamma_e^\alpha(v_p)] \eta^\alpha(v_p) N^\alpha(z, t) - \gamma_a^\alpha(v_p)\} \times P_p^\alpha(z, t). \quad (\text{E.1})$$

The external control signal power P_s^i , propagates according to:

$$\frac{dP_s^i(z, t)}{dz} = - \left\{ [\gamma_a^i(v_s) + \gamma_e^i(v_s)] \eta^i(v_s) N^i(z, t) - \gamma_a^i(v_s) \right\} \times P_s^i(z, t). \quad (\text{E.2})$$

The forward (+) and backward propagating (−) ASE powers $P_f^{i,\pm}$ evolve as:

$$\frac{dP_f^{i,\pm}(z, v, t)}{dz} = \pm \left\{ [\gamma_a^i(v) + \gamma_e^i(v)] \eta^i(v) N^i(z, t) - \gamma_a^i(v) \right\} \times P_f^{i,\pm}(z, v, t) \pm 2hv\Delta v \gamma_e^i(v) N^i(z, t). \quad (\text{E.3})$$

E.1. MODEL ESTABLISHMENT

In these equations: P_p^α , P_s^i , and $P_f^{i,\pm}$ are the powers for the pump, control signal, and ASE, respectively. The pump propagates in the forward (+) direction, while the control signal propagates backward (-). v_p and v_s are the central frequencies of the pump and control signals. The fluorescence spectrum is discretized into frequency bins v of width Δv . h is the Planck constant. $\gamma_a^i(v)$ and $\gamma_e^i(v)$ are the experimentally measured absorption and emission coefficients. $N^i(z, t)$ is the normalized population of the upper laser level ($^4I_{13/2}$), representing the fractional population inversion.

The nonlinearity in the system arise originates from the interaction between the light field and the Er^{3+} ions. The dipole transition between the ground state ($^4I_{15/2}$) and the excited state ($^4I_{13/2}$) modifies the local polarization and hence the refractive index. In the bistable regime, this process can be interpreted as a light-induced first-order phase transition, in which the spontaneous ordering of atomic dipoles gives rise to a macroscopic effect [152]. The coupling between dipole and electromagnetic field is mediated by the population inversion, N^i , whose dynamics are governed by the rate equation:

$$\frac{dN^i(z, t)}{dt} = W_{a,0}^i(z, t) - \left[W_{a,2}^i(z, t) + W_{e,2}^i(z, t) + \frac{1}{\tau^i} \right] N^i(z, t), \quad (\text{E.4})$$

where $W_{a,0}^i$, $W_{a,2}^i$, and $W_{e,2}^i$ are the transition rates for absorption from the ground state, absorption from the excited state, and stimulated emission, respectively. These rates are functions of the optical powers present in the fibre:

$$W_{a,0}^i(z, t) = \frac{P_p^i(z, t) \gamma_a^i(v_p)}{h v_p N_T^i} + \frac{P_s^i(z, t) \gamma_a^i(v_s)}{h v_s N_T^i} + \sum_v \frac{P_f^{i,\pm}(z, v, t) \gamma_a^i(v)}{h v N_T^i}, \quad (\text{E.5})$$

E.1. MODEL ESTABLISHMENT

$$W_{a,2}^i(z, t) = \frac{P_p^i(z, t) \gamma_a^i(v_p) \eta_2^i(v_p, t)}{h v_p N_T^i \eta_0^i(v_p)} + \frac{P_s^i(z, t) \gamma_a^i(v_s) \eta_2^i(v_s, t)}{h v_s N_T^i \eta_0^i(v_s)} + \sum_v \frac{P_f^{i,\pm}(z, v, t) \gamma_a^i(v) \eta_2^i(v, t)}{h v N_T^i \eta_0^i(v)}, \quad (\text{E.6})$$

$$W_{e,2}^i(z, t) = \frac{P_p^i(z, t) \gamma_e^i(v_p) \eta_2^i(v_p, t)}{h v_p N_T^i \eta_0^i(v_p)} + \frac{P_s^i(z, t) \gamma_e^i(v_s) \eta_2^i(v_s, t)}{h v_s N_T^i \eta_0^i(v_s)} + \sum_v \frac{P_f^{i,\pm}(z, v, t) \gamma_e^i(v) \eta_2^i(v, t)}{h v N_T^i \eta_0^i(v)}. \quad (\text{E.7})$$

In these rate equations, τ^i is the metastable state lifetime, N_T^i is the density of Er^{3+} ions per unit length, and η_2^i is the spatial overlap factor between the optical fields and the excited ions, with a maximum value of η_0^i [153]. For the absorber section ($i = \beta$), the pump power is zero ($P_p^\beta = 0$), so its population dynamics are driven solely by the external control signal and the ASE from the gain section. In this model, we use $N_T^\alpha = 9.2 \times 10^{14} \text{ m}^{-1}$, $N_T^\beta = 7.3 \times 10^{14} \text{ m}^{-1}$, and $\eta_2^\alpha = \eta_2^\beta = 0.77$.

E.2. BOUNDARY CONDITIONS AND FEEDBACK

E.2 BOUNDARY CONDITIONS AND FEEDBACK

To complete the model, boundary conditions are defined to couple the ASE powers between the gain and absorber sections, thereby closing the feedback loop:

$$\begin{aligned} P_{\text{f}}^{\beta,+}(0, v, t) &= T P_{\text{f}}^{\alpha,-}(0, v, t), \\ P_{\text{f}}^{\beta,-}(L^{\beta}, v, t) &= R_{\text{RFG}}(v) P_{\text{f}}^{\beta,+}(0, v, t), \\ P_{\text{f}}^{\alpha,-}(L^{\alpha}, v, t + \Delta t) &= T P_{\text{f}}^{\beta,-}(0, v, t), \end{aligned} \tag{E.8}$$

where L^{α} and L^{β} are the lengths of the gain and absorber fibres, T is the transmission coefficient of the component coupling the two sections (e.g., a circulator), and $R_{\text{RFG}}(v)$ is the spectral reflectivity of the RFG. The time delay Δt represents the round-trip propagation time through the absorber section and coupling components, which introduces delayed feedback into the system. The values for all model parameters are listed in Table E.1.

E.3 MODEL ASSUMPTIONS AND PARAMETERS

The model is based on the following simplifying assumptions: (1) Excited-state absorption (ESA) and cooperative up-conversion processes are neglected, as their contribution is smaller in the power regime considered. (2) The pump level is assumed to deplete rapidly, as the non-radiative decay rate from the pump level to the upper laser level is much larger than the pumping rate. This justifies the use of a two-level system approximation.

E.3. MODEL ASSUMPTIONS AND PARAMETERS

This set of coupled differential equations, along with the specified boundary conditions and assumptions, forms a complete mathematical description of the EDRFL. Numerical solution of this system allows for the simulation of its dynamic behaviour, including the emergence of optical bistability and the influence of an external control signal on its hysteresis loop.

Table E.1: Designated Model Parameters

Parameter	Annotation	Parameter	Annotation
h	Planck constant	v_p	Centre frequency of pump
z	Spatial coordinate along the fibre axis	v_s	Centre frequency of control signal
t	Temporal coordinate	v	Single frequency of fluorescence
α	Label for EDF gain	Δv	Bandwidth of fluorescence
β	Label for EDF absorber	γ_a^i	Absorption coefficient
$P_p^i, i \in \{\alpha, \beta\}$	Pump power in EDFs ($P_p^\beta = 0$)	γ_e^i	Emission coefficient
$P_s^i, i \in \{\alpha, \beta\}$	Power of control signal	$\eta_0^i, i \in \{\alpha, \beta\}$	Overlapping factor (ground)
$P_f^{i,+}$	Power of forward ASE noise	$\eta_2^i, i \in \{\alpha, \beta\}$	Overlapping factor (upper)
$P_f^{i,-}$	Power of backward ASE noise	$W_{a,0}^i$	Absorption rate (ground)
$N^i, i \in \{\alpha, \beta\}$	Population inversion ratio	$W_{e,2}^i$	Emission rate (upper)
$N_T^i, i \in \{\alpha, \beta\}$	Population density per unit length	$W_{a,2}^i$	Absorption rate (upper)
$L^i, i \in \{\alpha, \beta\}$	Total length of EDF		

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