

INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI

**A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor MI 48106-1346 USA
313/761-4700 800/521-0600**



Université d'Ottawa • University of Ottawa

The Hydraulics of Flow on Stepped Ogee-Profile Spillways

Mehdi Azhdary Moghaddam

A Dissertation
Submitted to School of Graduate Studies
under the supervision of
Dr. R.D. Townsend

in partial fulfilment of the requirements for the degree of
Doctor of Philosophy in Civil Engineering

Department of Civil Engineering
Faculty of Engineering
University of Ottawa
Ottawa, Canada, K1N 6N5

The Doctor of Philosophy in Civil Engineering is a joint program between
Carleton University and the University of Ottawa, which is administrated
by the Ottawa-Carleton Institute for Civil Engineering

©Mehdi Azhdary Moghaddam, Ottawa, Canada, 1997



**National Library
of Canada**

**Acquisitions and
Bibliographic Services**

**395 Wellington Street
Ottawa ON K1A 0N4
Canada**

**Bibliothèque nationale
du Canada**

**Acquisitions et
services bibliographiques**

**395, rue Wellington
Ottawa ON K1A 0N4
Canada**

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-26104-2

Abstract

Because of its simple form, its effectiveness as an energy dissipator, and its contribution to dam stability, the *stepped* spillway is one of the oldest *drop* structure used in the construction of small dams. With the relatively recent introduction of Roller Compacted Concrete (RCC) and particularly (rockfill) gabions as dam construction materials, the *stepped* spillways continues to be an attractive option among the various traditional forms of drop structure for small dams.

The *steps* on the spillway's downstream face significantly increase the energy dissipation thereon over that would occur on the equivalent *smooth* (ogee-profile) version. As a consequence stilling basin structures, usually introduced at the toe of spillways (to reduce excess energy in the spillway floor) can be either reduced in size or eliminated entirely .

Although *stepped* spillways were first introduced around 694 B.C., in most previous studies dealing with the hydraulic performance of *stepped* spillways are *site-specific*. This research, which includes both mathematical and physical elements, investigates the effect of different *step* configurations, in contribution with different spillway slopes, on the hydraulic performance of (ogee-profile) *stepped* spillways. The main goal of the research was to develop improved criteria for the design of *stepped* spillways.

The numerical study, a 2-D finite element model was developed to estimate, based on a trial-error procedure, the *stepped* spillway water surface for a given design head. In the supporting experimental programme three different spillway slopes

(45°, 50°, and 60°) were investigated and in each case the experiments were repeated for four different *step* configurations. This provides data set of 12 combinations of spillway slope and *step* arrangement.

Improved criteria were developed to (i) distinguish between the possible flow regimes on *stepped* spillways, (ii) estimate both *stepped* spillway friction factors and energy dissipation rates, and (iii) determine optimum *step* configurations for a given set of boundary conditions.

In the name of God

Acknowledgements

The author wishes to express his sincere appreciation to his supervisor, Dr. R.D. Townsend, for his guidance, advice, and encouragement during the course of this research.

The scholarship provided by the Ministry of Culture and Higher Education of the I.R. of Iran is gratefully acknowledged.

The author is grateful to Mr. R. Moore, the Hydraulics Laboratory technician, and Mr. B. Cotter, of the workshop staff, for constructing the experimental facility.

The author also wishes to thank his parents, parents-in-law, brothers and sisters for their continuous encouragement. The patience of the author's daughter, Yeganeh, has been appreciated. This study would not have been possible without the encouragement, patience, and overwhelming support of the authors wife, **Touran.**

Table of Content

Abstract	ii
Acknowledgements	iv
Table of Content	v
List of Symbols	ix
List of Figures	xiii
List of Plates	xviii
List of Tables	xix

Chapter 1

Introduction	1
1.1. Preamble	1
1.2. Statement of the problem	3
1.3. Study Objectives	4
1.4. Scope of the Research	5
1.5. Thesis Description	6

Chapter 2

The "Ogee"-Profile Spillway	7
2.1. Introduction	7
2.2. Spillway Types	8
2.3. The Ogee Spillway	9
2.3.1. Crest Detail	9
2.3.2. Downstream Face	11
2.3.3. Energy Dissipators	15

2.4. Discharge Coefficient	18
----------------------------------	----

Chapter 3

Literature Review	30
3.1. Historical Review	30
3.2. Gabion Stepped Spillways	31
3.3. RCC and Concrete Stepped Spillways	32
3.3.1. Block Stepped Spillway	32
3.4. Hydraulic Characteristics	35

Chapter 4

Experimental Studies	52
4.1. Introduction	52
4.2. Experimental Setup and Measurements	53
4.3. Spillway Models	54
4.3.1. Smooth Ogee Spillway Models	55
4.3.2. Stepped Ogee-Profile Spillway Models	56

Chapter 5

Hydraulics of Stepped Spillway	75
5.1. Drop Structure	75
5.2. Flow over a Stepped Spillway	82
5.2.1. Nappe Flow	82
5.2.2. Skimming Flow	85
5.3 Boundary Layer Growth	86
5.4. Flow Aeration	87
5.5. Flow Resistance	89
5.6. Energy Dissipation	93
5.6.1. Energy Dissipation for Nappe Flow	94

Chapter 6

Experimental Results	115
6.1. Introduction	115
6.2. Discharge Coefficient	115
6.3. Friction Factor for Stepped Spillways	116
6.4. Energy Dissipation for Skimming Flow	118

Chapter 7

Numerical Modelling of Spillway Flow	157
7.1. Introduction	157
7.2. Literature Review	158
7.3. Proposed Numerical Model	162
7.3.1. Boundary Conditions	163
7.4. Finite Element Method	165
7.4.1. Finite Element Discretization	166
7.4.2. Weighted-Residual Form	167
7.4.3. Finite Element Model	170
7.4.4. Interpolation Functions	171
7.4.5. Coordinate Transformation	174
7.4.6. Numerical Integration	176
7.4.7. Assembly of Element Equations	176
7.4.8. Imposition of Boundary Conditions	177
7.4.9. Solution of Equations	178
7.4.10. Postprocessing	178
7.5. Discussion	179
7.6. Conclusion	180

Chapter 8

Summary, Conclusions, and Recommendations for Future Research	194
--	-----

8.1. Summary	194
8.2. Main Conclusions	196
8.3. Future Research	198

Chapter 9

References	200
-------------------------	------------

Appendix	215
-----------------------	------------

List of Symbols

a	= coefficient
a_i	= constant coefficient
a'_i	= constant coefficient
A	= area of cross section
b	= coefficient
c_f	= coefficient of fluid friction
C	= discharge coefficient
C_a	= local air concentration
C_c	= Chezy coefficient
C_D	= drag coefficient
C_g	= centrifugal acceleration factor
C_s	= discharge coefficient for stepped spillway
d	= location of water surface profile
ds	= arclength of an infinitesimal line element along the boundary
D_H	= hydraulic depth
E_0	= energy at upstream
E_1	= energy at section 1
E_2	= energy at section 2
$E_{\max}$	= maximum energy
f	= Darcy-Weisbach friction factor
F_{r1}	= Froude number at section 1
g	= gravitational acceleration
h	= height of step
h_s	= static head at the inception point
H	= operating head
H_d	= design head
H_e	= theoretical head on the crest
J	= Jacobian matrix

k_s = equivalent sand roughness
 k'_s = roughness of spillway surface
 K = constant coefficient
 l = length of step
 L = effective length of the crest
 L_d = distance from the wall to the position y_1
 L_T = the distance from the start of the growth of the boundary layer
 L_j = length of the jump
 L_s = roughness spacing
 m = number of Gauss points
 n = manning's constant
 n_1 = number of nodes
 n_i = normal outward vector on the boundary
 n_x = component of unit vector in x-direction
 n_y = component of unit vector in y-direction
 N = constant coefficient
 N_s = number of steps
 p_1 = the proportion of the energy loss per step
 p = hydrostatic pressure
 p_n = order of triangle
 q = discharge per unit width
 Q = discharge
 r = local coordinate
 r^2 = coefficient of determination
 r_i = location of Gauss points
 R = radius of bucket
 R_c = radius of curvature
 s = local coordinate
 t_n = secondary variable
 u = velocity in the x-direction

v = velocity in y-direction
 V = velocity
 V_0 = approach velocity
 V_c = critical velocity
 $V_{\max}$ = maximum velocity
 x = horizontal coordinate
 x_s = distance along the slope which the boundary layer grows
 X_l = the horizontal length from the crest to the saturation point
 y = vertical coordinate
 y_1 = depth at section 1
 y_2 = depth at section 2
 y_{99} = flow depth corresponding to 99 percent air concentration
 y_c = critical depth
 y_d = depth of water
 y_I = the depth of flow at the inception point
 y_n = normal depth
 y_p = pool depth
 y_w = depth corresponding to a pure water column
 w = weight function
 z = elevation above datum
 α_i = weight factor
 α_j = weight factor
 β = the angle of the jet where it hits the bed
 Γ_e = boundary of the element
 γ = specific weight of the fluid
 ΔE = energy lost
 δ = boundary layer thickness
 δ_{ij} = Kronecker-Delta
 θ = downstream slope
 κ = bed curvature

λ_j^e = Lagrange interpolation function

ρ = mass density of the water

Ψ = stream function

Ω = flow domain

Ω^e = element

ω = vorticity

List of Figures

Fig.2.1. Examples of spillway types: (a) Free-overfall, (b) Ogee, (c) Side-channel	22
Fig.2.2. Further examples of spillway types: (a) Open-channel, (b) Shaft, (c) Siphon (after Novak, et. al, 1990)	23
Fig.2.3. Cavitation prevention criterion for design head - ogee spillways (after Chaudhry, 1993)	24
Fig.2.4. Spillway crest shape (after U.S. Army Corps of Engrs., 1977) ...	24
Fig.2.5. Principal zones of aerated flow on spillway face	24
Fig.2.6. Self-aeration mechanism on spillway face (after Chanson, 1993) ..	25
Fig.2.7. Recommended radius of bucket (after Creager et al. , 1964)	25
Fig.2.8. Examples of standard stilling basins: (a) Type II, (b) Type III (after Peterka, 1958)	26
Fig.2.9. Flip bucket and Ski jump spillways (USBR, 1977)	27
Fig.2.10. Roller bucket energy dissipators (USBR, 1977)	27
Fig.2.11. Examples of a stepped-spillway	28
Fig.2.12. Various types of step geometry	28
Fig. 2.13. H-Q relationship for standard ogee spillway (after Baban, 1995)	29
Fig.3.1. Examples of stepped spillway structures: (a) Almansa dam (Spain, 1576), (b) Tibi-Alicante dam (Spain, 1594), (c) Pabellon dam (Mexico, 1730), (d) Gabion stepped spillway profile	46
Fig.3.2. Unit head loss over plain gabion steps (after Peyras, et al., 1992) .	47
Fig.3.3. Brownwood Country Club dam (after Hansen, 1986) (USA)	47
Fig.3.4. Spillway of the De Mist Kraal weir (after Hollingworth and Druyts, 1986) (S. Africa)	48
Fig.3.5. Spillway of the Zaaihoek dam (after Hollingworth and Druyts, 1986) (S. Africa)	48
Fig.3.6. Stepped spillway energy dissipation for Upper Stillwater,	

Monksville, and Stagecoach Dam (after Hensen and Reinhardt, 1991)	49
Fig.3.7. Energy dissipation characteristics at the toe of stepped spillways (after Frizell, 1992)	49
Fig.3.8. $\Delta E/E_0$ vs $y_c/N_s h$ relationship (after Christodoulou, 1993)	50
Fig.3.9. Flow velocity at the toe of a stepped spillway (after Bindo, et al., 1993)	50
Fig.3.10. Velocity profile on a spillway: (a) smooth, (b) Stepped (after Rice, and Kadavy, 1996)	51
Fig. 4.1. General arrangement of the experimental flume	61
Fig.4.2. General profile(s) of smooth spillway model(s) (45°, 50°, 60°) ..	62
Fig.4.3. Water surface profiles for smooth 45° spillway model	62
Fig.4.4. Water surface profiles for smooth 50° spillway model	63
Fig.4.5. Water surface profiles for smooth 60° spillway model	63
Fig.4.6. Observed water surface profiles for (a) 45-CL-BL model, (b) 45-CS-BL model	64
Fig.4.7. Observed water surface profiles for (a) 45-CL-BS model, (b) 45-CS-BS model	65
Fig.4.8. Observed water surface profiles for (a) 50-CL-BL model, (b) 50-CS-BL model	66
Fig.4.9. Observed water surface profiles for (a) 50-CL-BS model, (b) 50-CS-BS model	67
Fig.4.10. Observed water surface profiles for (a) 60-CL-BL model, (b) 60-CS-BL model	68
Fig.4.11. Observed water surface profiles for (a) 60-CL-BS model, (b) 60-CS-BS model	69
Fig.5.1. Simple drop structures (a) vertical, (b) inclined	98
Fig.5.2. Comparison of eq.(5.26) with experimental data from various sources	99
Fig.5.3. Comparison of eq.(5.27) with experimental data from various	

sources	100
Fig.5.4. Comparison of eq.(5.28) with Rajaratnam, and Chamani's (1995)	
data	101
Fig.5.5. Comparison of eq.(5.29) with experimental data from various	
sources	102
Fig.5.6. Observed y_1/h versus computed y_1/h	103
Fig.5.7. Observed y_p/h versus computed y_p/h	104
Fig.5.8. Observed $\cos\beta$ versus computed $\cos\beta$	105
Fig.5.9. Observed $\Delta E/E_0$ versus computed $\Delta E/E_0$	106
Fig.5.10. Characteristic flows over a stepped spillway: (a) nappe flow with	
fully developed hydraulic jump, (b) nappe flow with partially	
developed hydraulic jump, (c) skimming flow	107
Fig.5.11. Nappe flow profile with fully-developed hydraulic jump on a	
single step	108
Fig.5.12. Comparison of eq.(5.38) with various criteria developed by	
others	109
Fig.5.13. Comparison of eq.(5.38) with experimental data	110
Fig.5.14. Comparison of eq.(5.45),and (5.43), with experimental data from	
various sources	111
Fig.5.15. Comparison of eq.(5.46), and (5.44), with experimental data from	
various sources	111
Fig.5.16. Friction factor for stepped channels on flat slopes ($\theta < 12^\circ$)	
(after Chansen, 1994)	112
Fig.5.17. Friction factor of stepped channels on steep slopes ($\theta > 27^\circ$)	
(after Chansen, 1994)	112
Fig.5.18. Friction factor for stepped surfaces (after Tozzi, 1994)	113
Fig.5.19. Comparison of eq.(5.71) with experimental data from various	
sources	114
Fig.6.1. H-Q relationships for 45° models	124
Fig.6.2. H-Q relationships for 50° models	125

Fig.6.3. H-Q relationships for 60° models	126
Fig.6.4. Comparison of eq.(6.4) with CLBL model experimental data ...	127
Fig.6.5. Comparison of eq.(6.4) with CLBS model experimental data ...	128
Fig.6.6. Comparison of eq.(6.4) with CSBL model experimental data ...	129
Fig.6.7. Comparison of eq.(6.4) with CSBS model experimental data ...	130
Fig.6.8. Comparison of eq.(6.4) with 45° stepped model experimental data	131
Fig.6.9. Comparison of eq.(6.4) with 50° stepped model experimental data	132
Fig.6.10. Comparison of eq.(6.4) with 60° stepped model experimental data	133
Fig.6.11. Observed f versus computed f	134
Fig.6.12. $\Delta E/E_0$ versus x/H_d ; 45-CL-BL model	135
Fig.6.13. $\Delta E/E_0$ versus x/H_d ; 45-CL-BS model	136
Fig.6.14. $\Delta E/E_0$ versus x/H_d ; 45-CS-BL model	137
Fig.6.15. $\Delta E/E_0$ versus x/H_d ; 45-CS-BS model	138
Fig.6.16. $\Delta E/E_0$ versus x/H_d ; 50-CL-BL model	139
Fig.6.17. $\Delta E/E_0$ versus x/H_d ; 50-CL-BS model	140
Fig.6.18. $\Delta E/E_0$ versus x/H_d ; 50-CS-BL model	141
Fig.6.19. $\Delta E/E_0$ versus x/H_d ; 50-CS-BS model.	142
Fig.6.20. $\Delta E/E_0$ versus x/H_d ; 60-CL-BL model	143
Fig.6.21. $\Delta E/E_0$ versus x/H_d ; 60-CL-BS model	144
Fig.6.22. $\Delta E/E_0$ versus x/H_d ; 60-CS-BL model	145
Fig.6.23. $\Delta E/E_0$ versus x/H_d ; 60-CS-BS model	146
Fig.6.24. $\Delta E/E_0$ versus x/H_d ; 45° smooth model	147
Fig.6.25. $\Delta E/E_0$ versus x/H_d ; 50° smooth model	148
Fig.6.26. $\Delta E/E_0$ versus x/H_d ; 60° smooth model	149
Fig.6.27. $\Delta E/E_0$ versus y_c/h ; 45° stepped models	150
Fig.6.28. $\Delta E/E_0$ versus y_c/h ; 50° stepped models	151
Fig.6.29. $\Delta E/E_0$ versus y_c/h ; 60° stepped models	152

Fig.6.30. Comparison of eq.(6.14) with experimental data 45° stepped models	153
Fig.6.31. Comparison of eq.(6.14) with experimental data 50° stepped models	154
Fig.6.32. Comparison of eq.(6.14) with experimental data 60° stepped models	155
Fig.6.33. Observed $\Delta E/E_0$ versus computed $\Delta E/E_0$	156
Fig.7.1. Flow over a spillway	182
Fig.7.2. Finite element mesh	183
Fig.7.3. Construction of the interpolation functions for a typical quadratural triangular elements	183
Fig.7.4. Comparison of depth of water between the numerical simulation output and experimental data for 45° smooth model	184
Fig.7.5. Comparison of depth of water between the numerical simulation output and experimental data for 50° smooth model	185
Fig.7.6. Comparison of depth of water between the numerical simulation output and experimental data for 60° smooth model	186
Fig.7.7. Comparison of depth of water between the numerical simulation output and experimental data for 45-CL-BL model	187
Fig.7.8. Comparison of depth of water between the numerical simulation output and experimental data for 45-CS-BS model	188
Fig.7.9. Comparison of depth of water between the numerical simulation output and experimental data for 50-CL-BL model	189
Fig.7.10. Comparison of depth of water between the numerical simulation output and experimental data for 50-CS-BS model	190
Fig.7.11. Comparison of depth of water between the numerical simulation output and experimental data for 60-CL-BL model	191
Fig.7.12. Comparison of depth of water between the numerical simulation output and experimental data for 60-CS-BS model	192
Fig.7.13. Observed y_d versus Simulated y_d	193

List of Plates

Plate 4.1. Model 45-CL-BL	70
Plate 4.2. Model 45-CS-BL	71
Plate 4.3. Model 50-CL-BL	72
Plate 4.4. Model 50-CS-BL	73
Plate 4.5. Model 60-CL-BL	74

List of Tables

Table 2.1. K- and N- values deduced by different researchers (Senturk, 1994)	21
Table 2.2. USBR- recommended values of K and N	21
Table 3.1. Examples of Gabion stepped spillways	42
Table 3.2. RCC dams containing stepped spillways	43
Table 3.3. Embankment dam with stepped spillway protection in the UK, and Russia (Baker, 1992)	44
Table 3.4. Stepped spillways for protection of the downstream slope of embankment dams in the USA (Frizell, 1992)	45
Table 5.1. Statistical parameters of the developed equations for a single vertical drop	97
Table 6.1. Values of H and q for all models	123
Table 7.1. Sampling points and weights in Gauss quadrature (Bathe, 1996)	181
Table A.1. Location of water surface profile for 45° smooth model	216
Table A.2. Location of water surface profile for 45-CL-BL model	217
Table A.3. Location of water surface profile for 45-CL-BS model	218
Table A.4. Location of water surface profile for 45-CS-BL model	219
Table A.5. Location of water surface profile for 45-CS-BS model	220
Table A.6. Location of water surface profile for 50° smooth model	221
Table A.7. Location of water surface profile for 50-CL-BL model	222
Table A.8. Location of water surface profile for 50-CL-BS model	223
Table A.9. Location of water surface profile for 50-CS-BL model	224
Table A.10. Location of water surface profile for 50-CS-BS model	225
Table A.11. Location of water surface profile for 60° smooth model	226
Table A.12. Location of water surface profile for 60-CL-BL model	227
Table A.13. Location of water surface profile for 60-CL-BS model	228
Table A.14. Location of water surface profile for 60-CS-BL model	229

Table A.15. Location of water surface profile for 60-CS-BS model	230
---	------------

Chapter 1

Introduction

1.1. Preamble

A spillway is usually the most important appurtenant facility to a dam. The function of the spillway is to provide an efficient and safe means of conveying flood discharges to the downstream channel. The spillway design primarily depends on the design flood, dam type and location, and reservoir size and operation. Spillways can be classified based on (i) their function (main, emergency, and auxiliary), (ii) their hydraulic type (*free overfall*, *overflow*, *chute*, *siphon*, etc.), and (iii) their mode of control (*ungated*, and *gated*).

While the number of new dams under construction in the world is declining, the number of existing dams that have had to be upgraded to meet current hydraulic and seismic criteria is increasing. This is especially true for smaller embankment dams that have been judged to have inadequate spillway capacity and are unable to store or pass safely floods in accordance with the current criteria of at least one half the theoretical *Probable Maximum Flood* (PMF). Many failures of dams have been caused by improperly designed spillways or by spillways of insufficient capacity. Several embankment dams have been identified as unable to pass their design flows without failure due to overtopping (Frizell, 1991).

In addition to providing sufficient capacity, the spillway must be hydraulically and structurally adequate. For a typical overflow spillway the flow accelerates along the downstream face and high velocities are attained at the spillway's toe. The spillway's outlet structure must be designed to ensure that the spillway discharges will not erode the downstream channel bed, or undermine the toe of the dam itself. Therefore, the spillway's surface must be erosion resistant to withstand the high scouring velocities created by the drop from the reservoir surface to the tailwater.

In dam design the traditional approach to handling the excessively high kinetic energies associated with large flows is through some form of *stilling basin* structure located at the foot of the spillway. Depending on the expected Froude Number range of the incoming flow, the form of the stilling basin can range from a simple concrete apron to a complex structure that may include rows of chute blocks, baffle piers and a plain or dentated end sill (Peterka 1958, USBR 1977). If the basin requires all three features this can add substantially to the overall costs, consequently alternative solutions to the problem merit investigation. One possible solution is to consider a *stepped-spillway* instead of the traditional smooth ogee-profile spillway.

The *stepped spillway* is a spillway whose face is provided with a series of *steps* from near the crest to the toe. The *steps* significantly increase the rate of energy dissipation taking place on the spillway face. They also reduce the size of the required downstream energy dissipation basin. A *stepped spillway* offers a number of advantages over an equivalent smooth spillway:

1. compatibility with *Roller Compacted Concrete* (RCC) and *gabion* hydraulic structures;
2. reduction of dissipation basin size;
3. reduction of the risk of cavitation;
4. in the case of an auxiliary spillway the construction programme will be much shortened;

5. an increase in the discharge capacity; and
6. low cost.

For these reasons, there has been renewed interest in this form of spillway (Young, 1982, Sorensen, 1985, Frizell ,1991, Diez-Cascon, et al., 1991).

1.2. Statement of the problem

To the author's knowledge, only one practical guideline for the design of *stepped spillways* has been produced (CIRIA, 1978). In this guideline, however, there is no indication of the degree to which the spillway *steps* may impact on energy dissipation. In practice, therefore, one has to rely on a physical model study of a particular spillway design to estimate the energy dissipated. This kind of model has been investigated for the six following cases:

1. The Upper Stillwater Dam, (Young, 1982);
2. Monksville Dam, (Sorensen, 1985);
3. The Upper Stillwater Dam, (Huston, 1987);
4. Model built in the Hydraulic Lab. of Iberduero, SA, (Diez-Cascon, et al. 1991);
5. M'Bali dam, (Bindo, et al., 1993); and
6. Salado Creek Site 10 (Rice, and Kadavy, 1996).

In general, the results obtained for each of the above cases are valid only for that case, i.e. they are site-specific. In light of these limitations, Sorensen (1985) points the need for further research on *stepped spillway* structures. Degoutte et al. (1992) state that "*... until now, the size requirements for stilling basins downstream of stepped spillways were not well known*". Frizell (1992) identifies the lack of general design criteria that quantifies the energy dissipation on stepped spillways, and Christodoulou (1993) states that "*... the performance of stepped spillways is not*

well known". Although several researchers have studied *stepped spillways* in the past, there remain considerable gaps in knowledge in the design of these structures. For example, we are still unable to predict the energy dissipated on the structure, or the spillway water surface profile without building a physical model. The water surface profile is an important consideration when establishing the height of the spillway side-walls. **Given these remaining unanswered aspects of *stepped spillway* design and operation, there is clearly a need for further research to address these important issues.**

The analysis of water flow over a spillway is an important engineering problem, however the analysis presents several difficulties in view of the following considerations:

1. it is a non-linear problem, therefore, it might have non-unique solutions in certain cases;
2. the free-water surface is unknown beforehand;
3. the flow is *subcritical* upstream of the spillway crest and is *supercritical* downstream of the spillway crest; and
4. there is a control section near the crest.

Furthermore, any numerical simulation of the spillway flow must account for possible significant departure from the hydrostatic of fluid-induced pressures on the spillway face, produced by centripetal fluid acceleration at the crest and toe sections.

1.3. Study Objectives

This study examines the hydraulics of flow on various *stepped* ogee-profile spillway configurations, both from *theoretical* and *physical-modelling* viewpoints. The main objectives of the research were:

- (i) to investigate the hydraulic performance of several physical models having different downstream slopes and different step configurations, and
- (ii) to develop a numerical model for predicting the stepped spillway water surface profile corresponding to the *design-discharge* condition.

The study requirements included:

1. a review of the hydraulics of flow on stepped spillways and the *criteria* for *flow classification*;
2. in the physical-modelling portion of the study, measure *energy dissipation* for different spillway *downstream slopes* and *different step sizes*;
3. investigate the *performances* of different stepped spillway models for a wide range of discharge; and
4. develop a two-dimensional Finite Element (FE) model to predict the spillway water surface profile and related *energy dissipation* for the *design-discharge* condition.

1.4. Scope of the Research

From the literature review it was found that, so far, *stepped spillways* studies have been largely *site-specific* and that the results of these studies have been used purely as a *guide* for other similar cases. This study examined and compared the hydraulic performance of various *Ogee-profile stepped spillways*, having different *downstream slopes* and *step configurations*. The effectiveness of the various designs was judged by comparing the hydraulic performance of each *stepped* profile with its equivalent *smooth* profile over a wide range of discharges. Although Sorensen (1985) was, the first to initiate a combination of small and large steps on the spillway profile, this application was not related to considerations of energy dissipation. The present study is the first to examine the use of a combination of step sizes to optimize energy dissipation occurring on stepped spillways. Based on

the results of this study and previous studies several new criteria for the design of stepped spillways were developed.

A 2-D Finite Element (FE) model was developed which determines: (i) the free water surface of the spillway flow, and (ii) estimates the amount of energy dissipated by the stepped profile, based on the *design-discharge* condition. The data obtained from the laboratory investigations were employed to validate the numerical model. This study is *unique* in two respects:

- (1) unlike earlier studies concerning *stepped* spillways (which are largely site-specific), this study investigates and compares the hydraulic performance of different *stepped* spillway configurations, consequently it is more general in nature, and
- (2) the numerical model developed in this study represents a useful design aid in regards to estimating: (i) the spillway water surface profile, and (ii) the energy dissipated by the *stepped* spillway configuration.

1.5. Thesis Description

Chapter 2 briefly reviews the hydraulics of *Ogee-Profile* spillways and their outlet *energy dissipator* structures. Chapter 3 is a literature review on the subject of *stepped spillways*. In Chapter 4 the experimental programme is described and in Chapter 5 the hydraulics of *stepped spillways* is explained. In Chapter 6 the experimental results are analyzed and several new design criteria are proposed. Chapter 7 deals with the development of the numerical model. Finally, Chapter 8 combines the summary, conclusions and recommendations for future research.

Chapter 2

The "Ogee"-Profile Spillway

2.1. Introduction

Spillways are hydraulic structures provided at storage and diversion dams. At storage dams, they release surplus or flood water which can not be contained in the allotted reservoir storage space. At diversion dams, they are designed to bypass flows exceeding those which are directed into the diversion system.

To propose a composite design for a spillway, it is necessary to consider the various factors influencing spillway size and type, and to correlate alternatively selected components. The components of a spillway are: *(i)* the control structure; *(ii)* the discharge channel; *(iii)* the terminal structure; and *(iv)* the entrance and/ or outlet channels.

After the hydraulic size and outflow characteristics of the spillway are determined by routing of the design flood, the general dimensions of the control structure can be selected. A specific spillway layout can be developed by considering *(i)* the type of dam; *(ii)* the topography; *(iii)* the foundation conditions; *(iv)* the hydraulic condition of the control structure; and *(v)* the overall economy. Site conditions greatly influence the selection of location, type and components of a spillway. In the design of a spillway the following considerations are also necessary:

1. the steepness of the terrain traversed by the spillway control and discharge

- channel;
2. the class and amount of excavation material and the possibility for its use as embankment material;
 3. the potential for scour of the spillway and downstream channel surfaces and the need for lining; and
 4. the stability of the excavated slopes.

2.2. Spillway Types

A spillway can be constructed in several locations: within the body of the dam, at the one end of it, or independently in a saddle. Spillways are ordinarily classified according to their most prominent features, either as they relate to the control, to the discharge channel, or to some other component. The following are common types:

1. A *free-overfall (straight drop)* spillway is one in which the flow drops freely from the crest (Fig.2.1.a).
2. An *Ogee (overflow)* spillway has an ogee- or S-shaped profile. The upper curve of the ogee is generally made to conform closely to the profile of the lower nappe of a ventilated sheet falling from an *equivalent* sharp-crested weir (Fig.2.1.b).
3. A *side-channel* spillway is one in which the control weir is placed along the side of and approximately parallel to the upper portion of the spillway discharge channel (Fig.2.1.c).
4. An *open-channel (trough or chute)* spillway is a spillway whose discharge is conveyed from the reservoir to the downstream river level through an open channel, placed either along a dam abutment or through a saddle (Fig.2.2.a).
5. A *conduit or tunnel* spillway is a closed channel used to convey the discharge around or under a dam.
6. A *shaft (morning glory or drop inlet)* spillway is one in which the water enters over

a horizontal-positioned lip, drops through a vertical or sloping shaft, and then flows to the downstream river channel through a horizontal or near horizontal conduit or tunnel (Fig.2.2.b).

7. A *siphon spillway* is a closed conduit system formed in the shape of a U, and positioned so that the inside of the bend of the upper passageway is at normal reservoir storage level. Siphonic action takes place after the air in the bend over the crest has been expelled from the conduit (Fig.2.2.c).

2.3. The Ogee Spillway

An ogee spillway can be used on concrete-gravity, arch, and buttress dams, where part of the dam's length may be used as a spillway. It is probably the most extensively-used spillway to safely pass the flood flow out of a reservoir. An ogee spillway has three main parts: (i) the crest, (ii) the sloping downstream face, and (iii) the energy dissipator at the toe.

2.3.1. Crest Detail

The crest profile is so chosen as to provide a high discharge coefficient without causing dangerous cavitation conditions and vibrations. The profile is usually made to conform to the lower nappe flow from a well-ventilated sharp-crested equivalent rectangular weir with an approach velocity $V_0 \approx 0$. Therefore, for the *design discharge* condition, theoretically the pressures along the crest section are atmospheric and a high discharge coefficient is assured. This idea is believed to have been proposed by Muller (1908) (Subramanya, 1989). Operating heads smaller than the design head (H_d) cause smaller nappe trajectories and hence result in positive pressures (above atmospheric) on the spillway face and consequently lower discharge coefficients.

Similarly, for operating heads greater than H_d , the lower nappe trajectory tends to pull away from the spillway surface thereby producing negative pressures (sub-atmospheric) and higher discharge coefficients.

The shape of the crest is based on H_d , which is selected for a given site such that the minimum pressure head at the crest is higher than -6 m in order to prevent cavitation. Fig.2.3 may be used to select H_d so that this condition for the minimum pressure is satisfied. For operating heads (H) greater than H_d , the nappe is projected beyond the control structure profile and hence a negative pressure is developed in the space between the structure and the water column. Usually, the design head is selected such that $1.3 < H/H_d < 1.5$. In this range, acceptable levels of sub-atmospheric pressures are produced on the spillway face. This results in an increase in the discharge capacity of the spillway and at the same time does not result in cavitation damage of the spillway surface.

Applying the principle of projectile motion, it is found that the equation of the lower nappe profile is a second-degree parabola. The general form of this equation is written as:

$$\frac{y}{H_d} = K \left(\frac{x}{H_d} \right)^N \quad (2.1)$$

in which x and y are the coordinates of the downstream curve of the spillway with the origin of coordinates being located on the apex. K and N are constants, whose values depend upon the inclination of the spillway's upstream face and the approach velocity of the fluid. Many attempts have been made by various researchers to determine the values of K and N . Table 2.1. presents some examples of these attempts. The crest shapes of uncontrolled ogee spillways have been extensively used by the US Bureau

of Reclamation (USBR). For low approach velocities, typical values of K and N can be found in table 2.2.

The crest profile upstream of the apex is usually given by a series of compound curves. Cassidy (1970) reported the equation for the upstream portion of a vertical faced spillway as:

$$\frac{y}{H_d} = 0.724 \left(\frac{x}{H_d} + 0.270 \right)^{1.85} - 0.432 \left(\frac{x}{H_d} + 0.27 \right)^{0.625} + 0.126 \quad (2.2)$$

which is valid for the following regions:

$$-0.27 \leq \frac{x}{H_d} \leq 0 \quad ; \quad 0 \leq \frac{y}{H_d} \leq 0.126 \quad (2.2.a)$$

Based on extensive laboratory investigations the U.S. Army Corps of Engineers (1977) developed several empirical relationships for the crest shape. These relationships are shown in Fig.2.4.

2.3.2. Downstream Face

The downstream face of an ogee spillway usually has a very steep gradient and consequently the flow on the face quickly becomes supercritical. Because of this steepness some difficulties arise in the analysis of the flow conditions. The development of a boundary layer starts near the crest. Once this boundary layer extends to the water surface, air is entrained in the high velocity flow. Air entrainment or *insufflation* is the process by which air enters the body of water as a result of turbulence of the *free* water surface (Falvey, 1980, Wood 1991). The intersection

point of the boundary layer and the water surface is called the *point of inception*.

Based on Keller and Rostagi's (1975) study, Wood (1991) presented the following equation for development of the boundary layer in the spillway flow field:

$$\frac{\delta}{x_s} = 0.021 \left(\frac{x_s}{h_s} \right)^{0.11} \left(\frac{k_s}{x_s} \right)^{0.10} \quad (2.3)$$

in which

δ = boundary layer thickness;

k_s = equivalent sand roughness;

x_s = distance along the slope which the boundary layer grows; and

h_s = static head at the inception point.

This equation is applicable in the range $5^\circ \leq \theta \leq 70^\circ$, where θ is the slope of the spillway face.

The designer needs to know the amount of air entrainment in order to select the height of the spillway side walls. If the walls are designed to insufficient heights, overtopping may occur and this may lead to dangerous erosion on the outsides of the walls. Knowing the amount of air entrainment is also helpful in assessing the *cavitation* potential, since air near the channel bottom reduces the possibility of *cavitation*. Air entrainment decreases the density of the air-water mixture by increasing the velocity of flow. At the same time, the excess velocity of flow increases the air friction, which decreases the velocity of flow.

Flow over a spillway that includes air entrainment may be divided into the following four vertical zones (Fig.2.5).

1. **Upper zone:** comprises a small mass of flying water particles ejected from the mixing zone. This zone is not important for engineering applications.
2. **Mixing zone:** has surface waves of random amplitude and frequency. To prevent overtopping of the side walls, it is necessary to take the height of these waves into consideration.
3. **Underlying zone:** the surface waves do not penetrate the underlying zone, and the air concentration at any depth in this zone is determined by the number and the size of air bubbles.
4. **Air-free zone:** exists only in that part of the channel where aeration is still developing.

Wood (1991) divided the development of self-aerated flows in a wide channel into the following four regions (Fig.2.6):

1. **Nonaerated flow region:** in this region the turbulent boundary layer does not reach the water surface and hence no air is entrained.
2. **Partially-aerated flow region:** air concentration profiles vary with distance, however, the air does not reach the channel bottom in the partially-aerated region.
3. **Fully-aerated flow region:** air concentration profiles vary with distance, however, the air reaches the channel bottom in the partially-aerated region.
4. **Uniform (fully-developed) aerated flow region:** the flow conditions reach an equilibrium state and are constant with distance. A large amount of air is entrained, and *bulking* of the flow occurs.

In the uniform aerated region mean-flow properties, such as flow depth (y) and depth-averaged air concentration, (C_w) are functions of discharge, channel bottom slope, bottom roughness, and the fluid properties. The U.S. Army Corps of Engineers

recommends increasing the flow depth by 20 percent to account for bulking (U.S. Army Corps of Engrs., 1965).

By using Straub and Anderson's data (1958) from an artificially-roughened flume, Hager (1991) developed the following empirical expression in the SI system for a wide rectangular channel in the uniform aerated region :

$$y_{99} = y_w + 1.35y_w \left(\frac{\sin^3 \theta y_w}{n^2 g^3} \right)^{0.25} \quad (2.4)$$

in which

y_{99} = flow depth corresponding to 99 percent air concentration;

y_w = depth corresponding to a *pure* water column;

n = Manning's constant;

θ = the slope; and

g = gravitational acceleration.

At the toe of the spillway a smooth transition is needed. This smooth transition avoids excessive vibration that could induce structural damage and possible failure. It also prevents the toe of the spillway from scouring the foundation due to the impact of the falling water. Therefore, the surface at the spillway toe is usually designed as a circular curve or *bucket*. To be thoroughly effective the bucket should tangent to the foundation as well as to the spillway's terminal downstream slope. The radius R of the bucket, measured in feet, may be estimated approximately by the following empirical formula (Chow, 1959):

$$R = 10^{(V+6.4H_{dam}+16)/(3.6H_{dam}+64)} \quad (2.5)$$

where

V = the velocity, (ft/sec) of the flow at the spillway toe, and
 H_{dam} = the dam height, (ft).

To find the radius of bucket, Creager et al. (1964) suggested the dimensions shown in Fig.2.7.

2.3.3. Energy Dissipators

To prevent scour and erosion of the toe of a dam, as well as the downstream channel, energy dissipators are provided to dissipate a sufficient amount of energy before water enters the downstream channel. The flow velocity at the toe of a high-head spillway is usually around 32-35 m/sec. This high velocity may cause serious scour and erosion of the downstream channel if proper precautions are not taken. The common types of energy dissipators which are used as *outlet* structures in spillway design, are:

(a) **Straight drop:** This kind of energy dissipator is commonly used in small drainage structures, and low head spillways. The aerated free-falling nappe in the straight drop usually produces supercritical flow on the apron. Therefore, a hydraulic jump may be formed downstream.

(b) **Stilling basin:** The stilling basin structure comes in several forms and is designed to dissipate the energy of the flow by utilizing the development and enhancement of the hydraulic jump within the basin. This is a solution for dams up to 60-70 m high. The location of the hydraulic jump is important for determining the length of channel requiring protection. Energy loss in a hydraulic jump is expressed as:

$$\Delta E = E_1 - E_2 = \frac{(y_2 - y_1)^2}{4y_1 y_2} \quad (2.6)$$

in which

E_1 = energy at section 1;

y_1 = depth at section 1;

E_2 = energy at section 2;

y_2 = depth at section 2; and

ΔE = energy lost in the jump.

Depending upon the range of Froude number of the upstream flow, the following devices may be incorporated, either singularly or in combination, in stilling basin design:

(i) **Paved apron:** A simple paved apron may be used in the case of *low-head* structures. The length of paved apron depends on the maximum length of the anticipated range of hydraulic jumps.

(ii) **Chute blocks:** These devices are placed at the entrance of the stilling basin to *corrugate* the jet, and lift a portion of it from the basin floor. This creates a greater number of energy dissipating eddies, which results in a shorter length of jump than would be possible without them. These blocks also reduce the tendency of the jump to sweep off the apron at tail water elevations below conjugate depth.

(iii) **Baffle blocks:** These structures are located in the stilling basin a certain distance downstream from the chute blocks. Their function is to dissipate energy mostly by impact action. In certain circumstances, they must be designed to withstand impact from ice or floating debris and/ or damage due to cavitation.

(iv) **End sill:** This device is located at the exit of the stilling basin. It can be either

solid or dentated. Its function is to reduce further the length of the jump and to control scour immediately downstream from the structure. For large basins that are designed for high incoming velocities, the sill is usually dentated to perform the additional function of diffusing the residual portion of the high-velocity jets that may reach the end of the basin.

Fig.2.8 shows the basic types of stilling basins commonly used as outlet structures.

(c) **Flip buckets:** When the downstream channel bed is relatively stable, the *flip-bucket* or *ski-jump* spillway can be considered, Fig.2.9. This form of dissipator structure projects the high velocity jet away from the base of the dam into the downstream channel. This is achieved via a concave section at the bottom of the chute section. The spillway overflow is discharged into atmosphere completely above the tailwater level. Under ideal conditions, these kinds of energy dissipators have been found to be very efficient in dissipating the excess energy.

(d) **Roller buckets:** When the discharge channel has an appreciable natural erosion resistance, the *roller bucket* can be used. This kind of dissipator requires substantially higher tailwater levels than conventional hydraulic jump basins. Two types of roller buckets have been developed, Fig.2.10. Scour will occur in the streambed at the point of impingement when the jet dives, but will be filled in by the ground roller when the jet rides.

(e) **Stepped spillways:** Another possible energy dissipator option is to dissipate part of the kinetic energy of the spillway flow on the spillway face itself. An ogee stepped spillway is an ogee spillway containing a series of drops or *steps* in the invert. At a point just downstream of the spillway crest, steps are introduced into the profile so that

the envelope of their tips follows the standard profile down to the toe of the spillway, Fig.2.11. Therefore, the total fall is divided into a number of smaller falls. At each fall, retarding forces are derived from the reaction of the steps to the descending flow. The steps significantly increase the rate of energy dissipation taking place on the spillway face and eliminate or greatly reduce the need for a large energy dissipation basin at the toe of the spillway. Step geometries are either horizontal, inclined, or pooled, Fig.2.12. Frizell (1992) examined the hydraulic performance of different step slopes and showed that horizontal steps would have the greatest energy dissipation.

2.4. Discharge Coefficient

A curve relating the upstream reservoir level and the spillway discharge is called the *spillway rating curve*. The relationship takes the form:

$$Q = CLH_e^{1.5} \quad (2.7)$$

where

C = discharge coefficient,

L = effective length of the crest,

H_e = theoretical head on the crest $\left(= H + \frac{V^2}{2g} \right)$, and

$\frac{V^2}{2g}$ = velocity head component.

An overflow spillway can be considered as a short-crested weir. The basic difference between a broad-crested weir and a short-crested weir is that nowhere above the short-crested structure can the curvature of the streamlines be neglected; i.e. the pressure

distribution is other than hydrostatic.

From an economical point of view, spillways must safely discharge a peak flow under the smallest possible head, while on the other hand the negative pressures on the crest must be limited to avoid the danger of cavitation. Theoretically, there should be atmospheric pressure on the crest. In practice, however, friction between the surface of the spillway and the lower nappe will introduce some negative pressures. If the spillway is operating under a head lower than its design head, positive pressures occur throughout the crest region and the discharge coefficient is reduced.

An operating head (H) greater than the design head (H_d) will cause negative pressure at all points of the crest profile and will increase the discharge coefficient. The avoidance of severe negative pressure on the crest, which may cause cavitation on the crest or vibration of the structure, should be considered an important design criterion on high-head spillways. Model experiments indicate, however, that the design head may be safely exceeded by up to 50%, but beyond this harmful cavitation may develop (Chow, 1959).

The discharge coefficient, C , is influenced by a number of factors, such as (i) the depth of the approach flow, (ii) relation of the actual crest shape to the ideal lower-nappe shape, (iii) upstream spillway slope, (iv) downstream apron interference, and (v) downstream submergence (USBR, 1977). Through extensive laboratory tests, the U.S. Bureau of Reclamation has compiled information on the coefficient of discharge for smooth spillways. There are two conditions for which the value of C is determined (Baban, 1995):

(i) when $H/H_d > 1.33$ the velocity head is negligible, then $C = 2.225$ and

(ii) when $H/H_d < 1.33$ the velocity head should not be neglected. The value of C calculated from the experimental curves is presented in Fig. 2.13.

Researchers	K	N
Creager-Scimemi (1937)	0.5	1.85
Creager-Scimemi (1945)	0.47	1.80
Smetana (1948)	0.461	1.85
de Marchi (1928)	0.556	2.0

Table 2.1. K- and N- values deduced by different researchers (Senturk, 1994)

Upstream Face Slope	K	N
Vertical	0.500	1.850
1 (H) : 0.33 (V)	0.517	1.836
1 (H) : 1 (V)	0.534	1.776

Table 2.2 USBR- recommended values of K and N

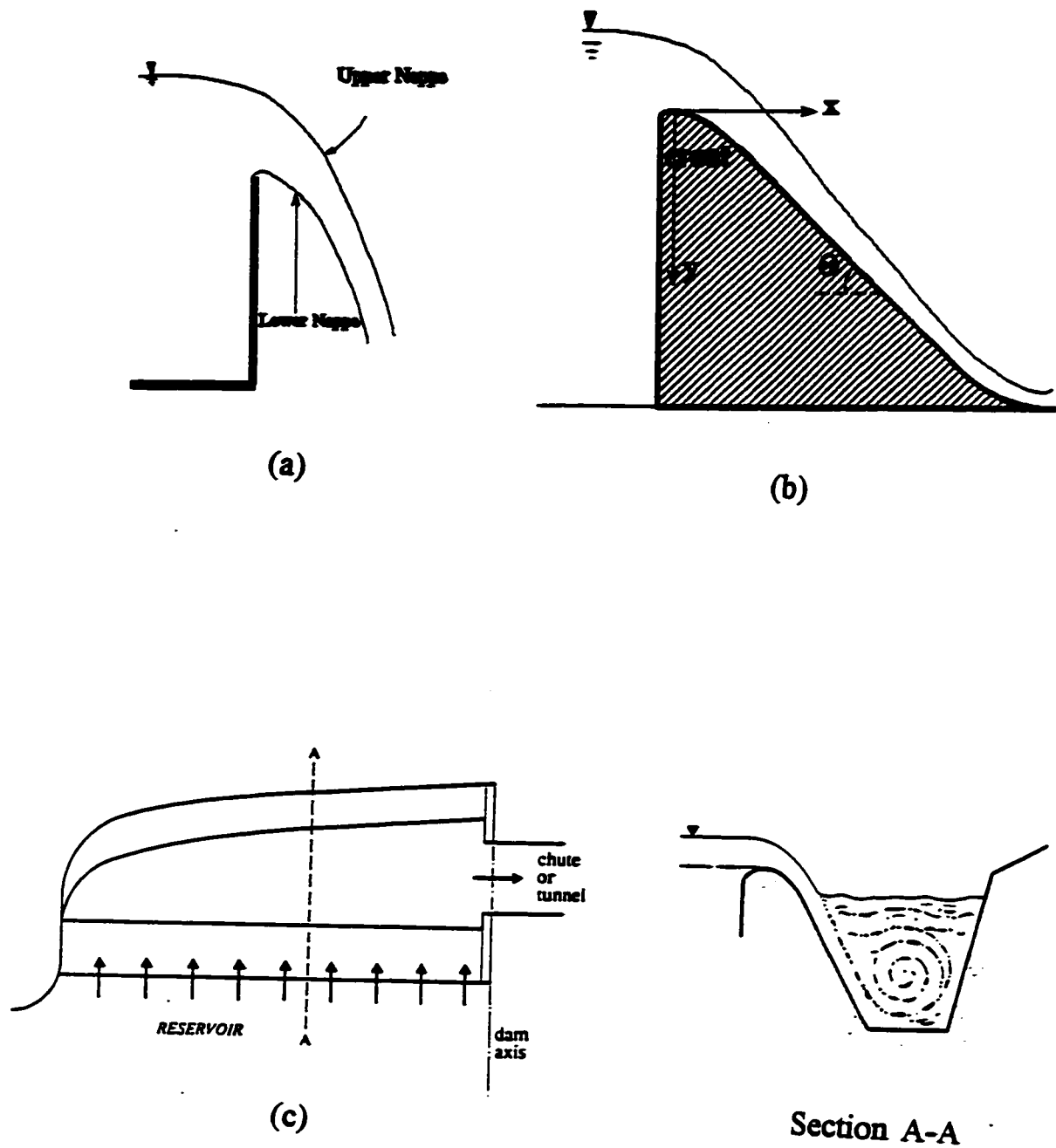


Fig.2.1. Examples of spillway types: (a) Free-overfall, (b) Ogee, (c) Side-channel

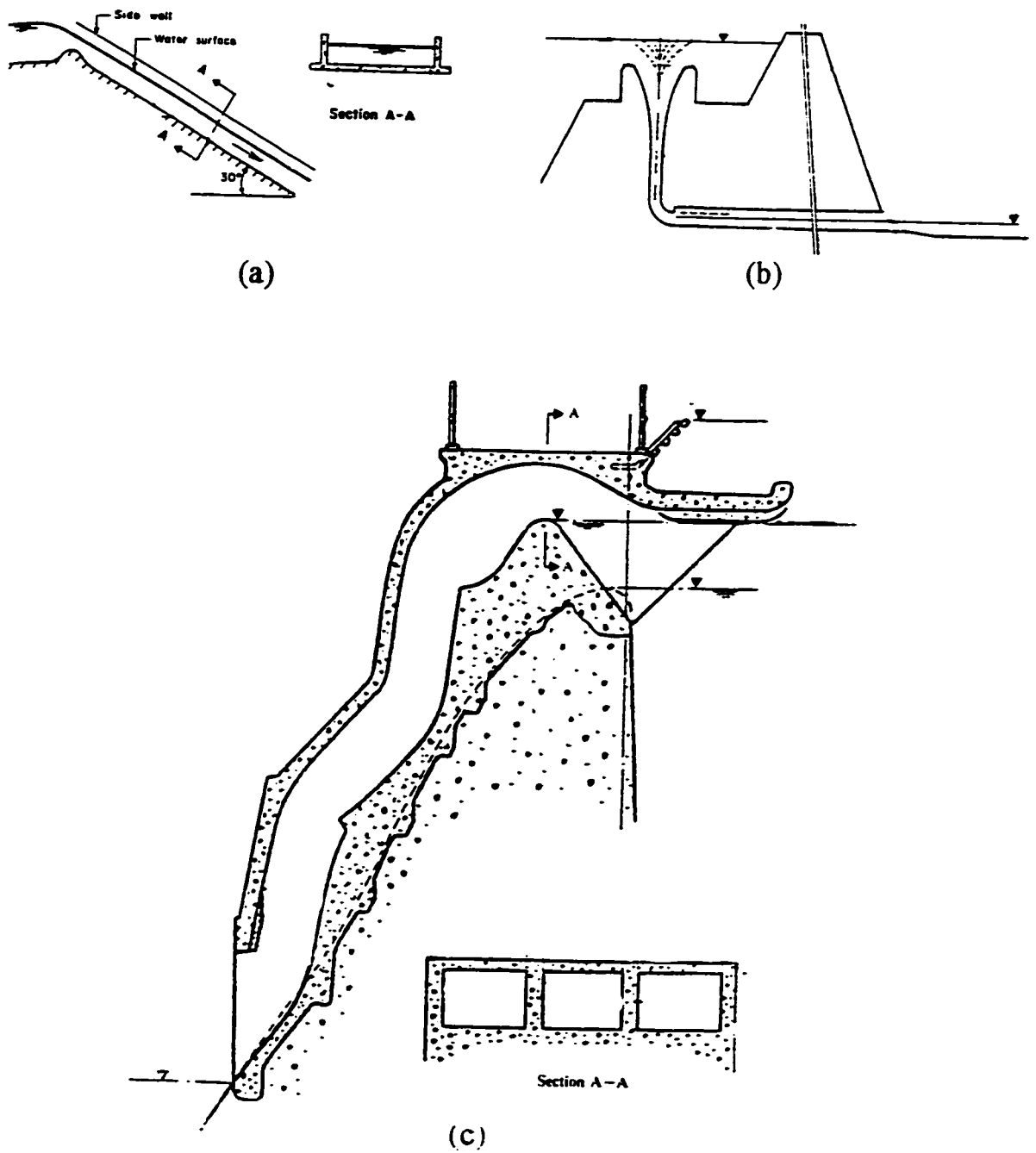


Fig.2.2. Further examples of spillway types: (a) Open-channel, (b) Shaft, (c) Siphon (after Novak et al., 1990)

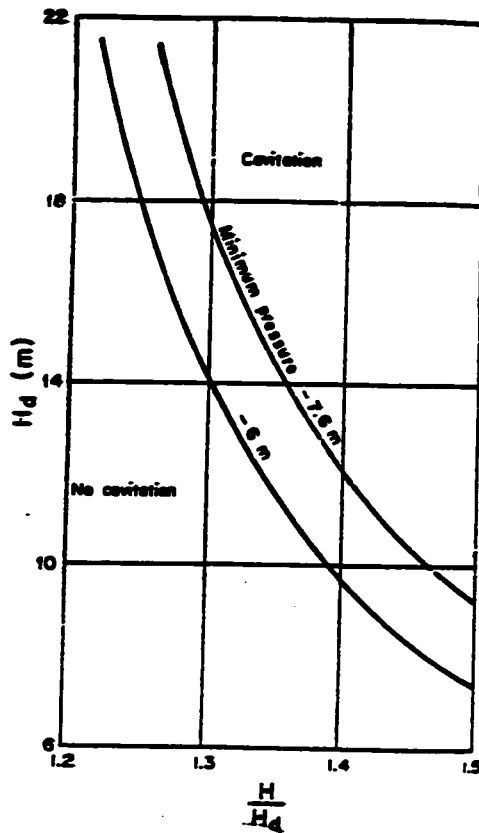


Fig.2.3. Cavitation prevention criterion for design head - ogee spillways (after Chaudhry, 1993)

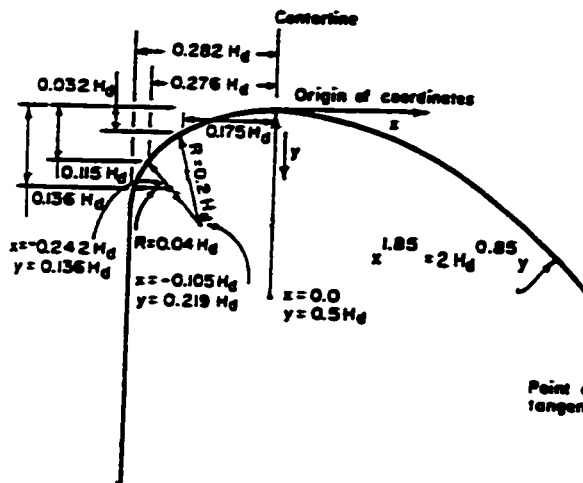


Fig.2.4. Spillway crest shape (after U.S. Army Corps of Engrs., 197)

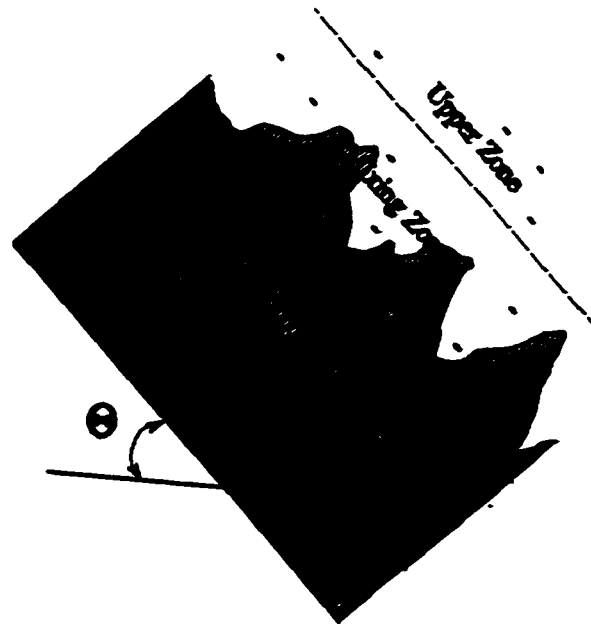


Fig.2.5. Principal zones of aerated flow on spillway face

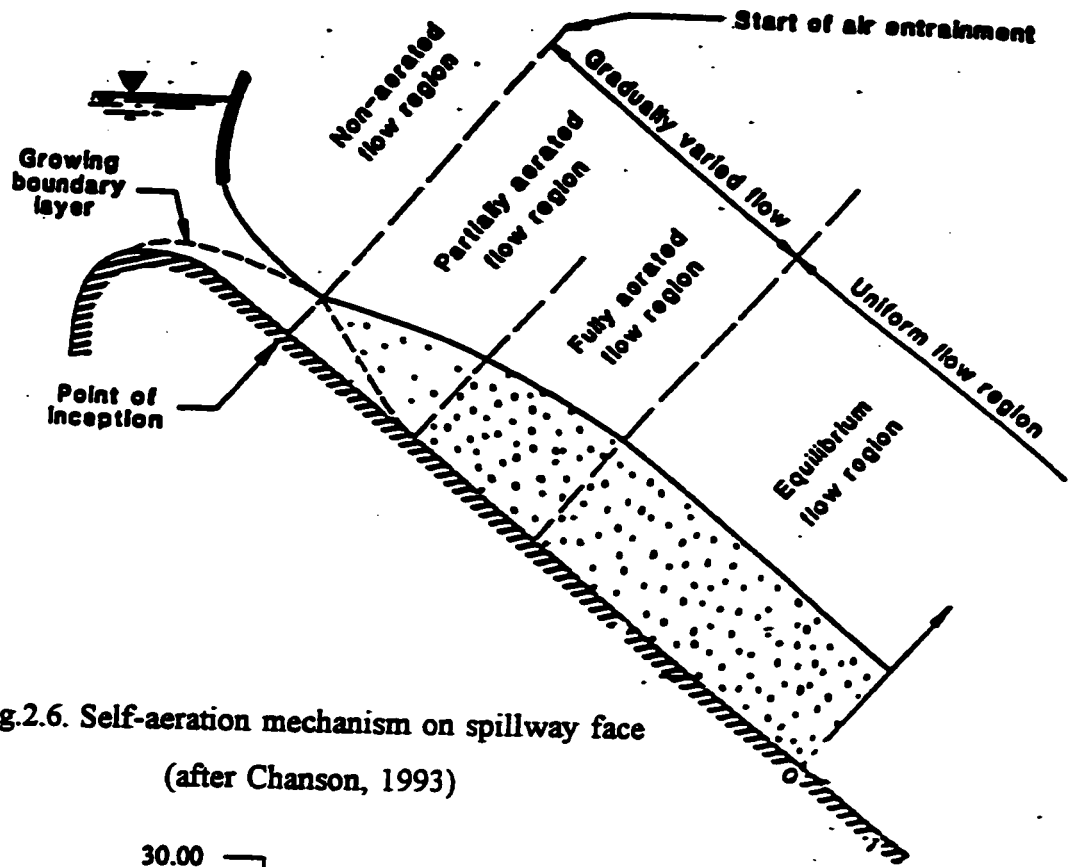


Fig.2.6. Self-aeration mechanism on spillway face
(after Chanson, 1993)

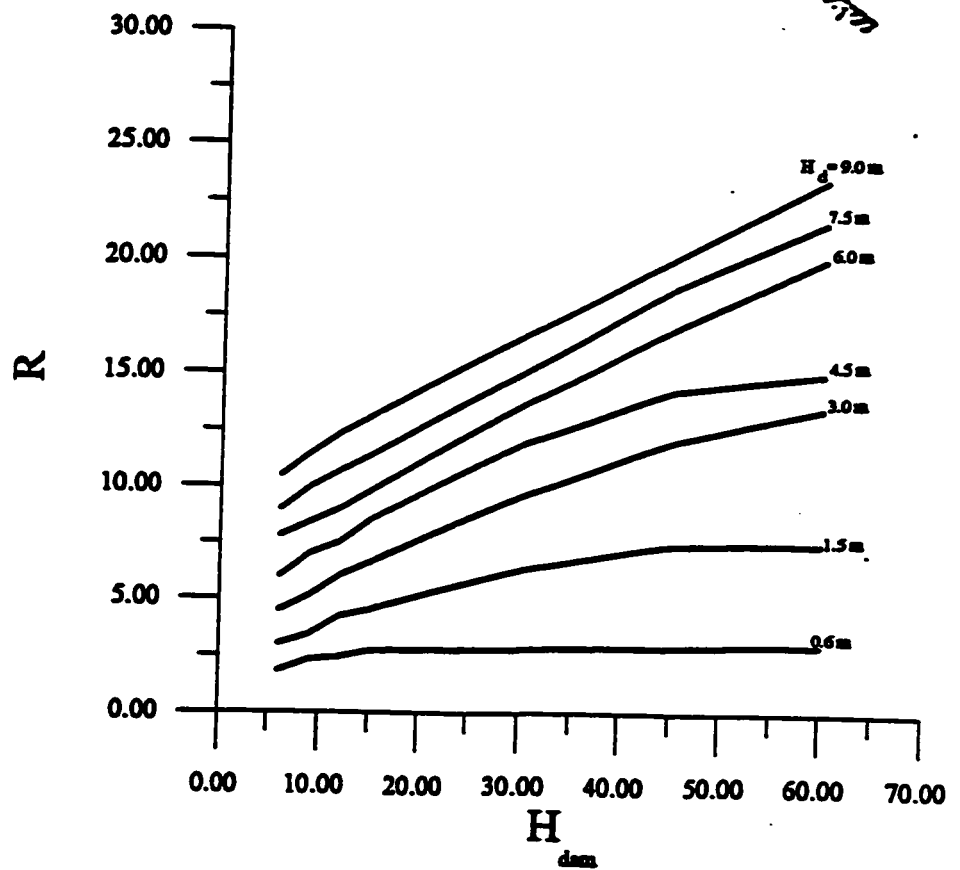
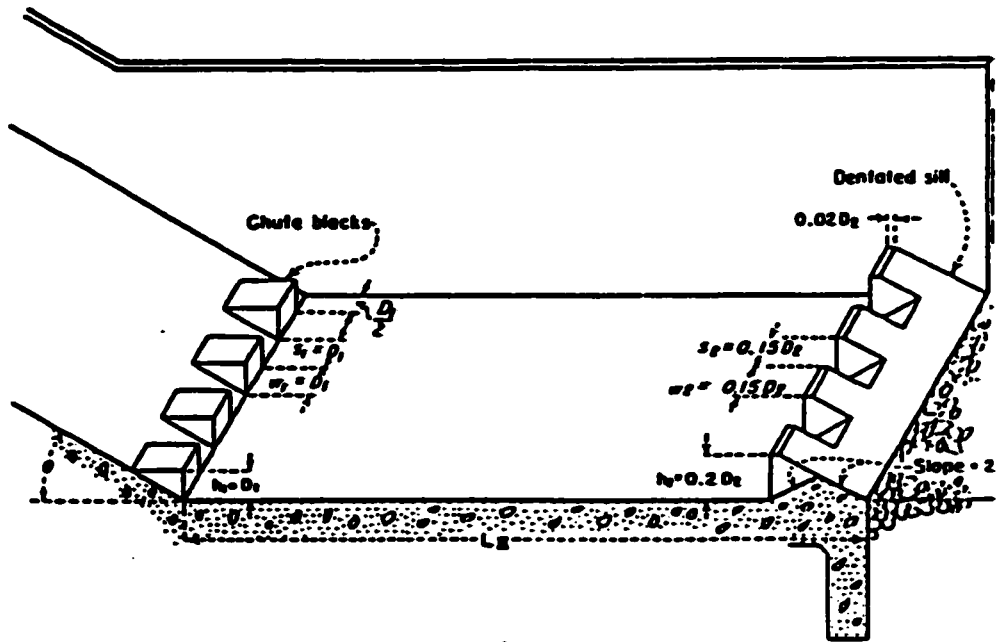
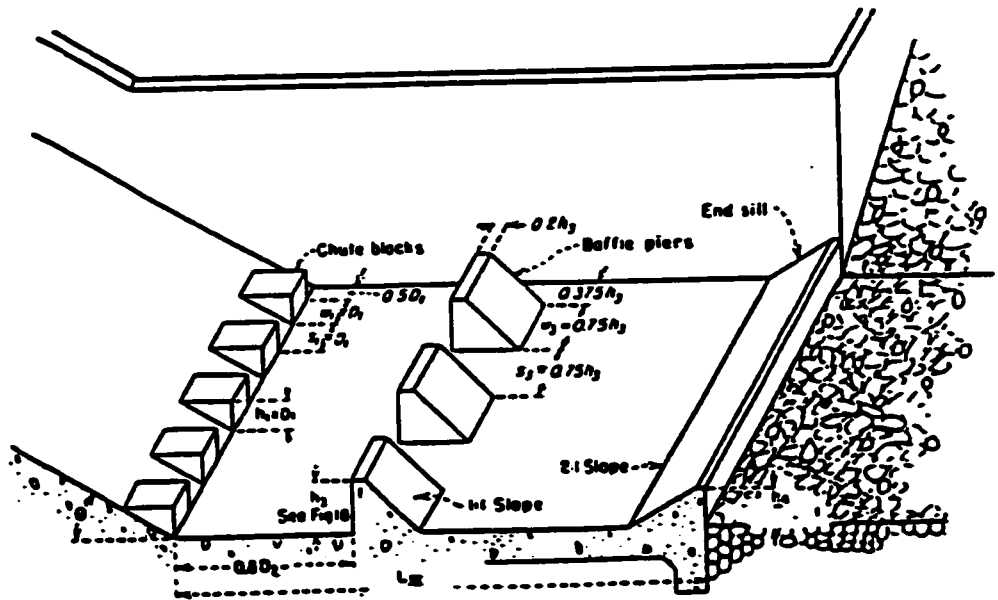


Fig.2.7. Recommended radius of bucket (after Creager et al. , 1964)



(a)



(b)

Fig.2.8. Examples of standard stilling basins: (a) Type II, (b) Type III (after Peterka, 1958)

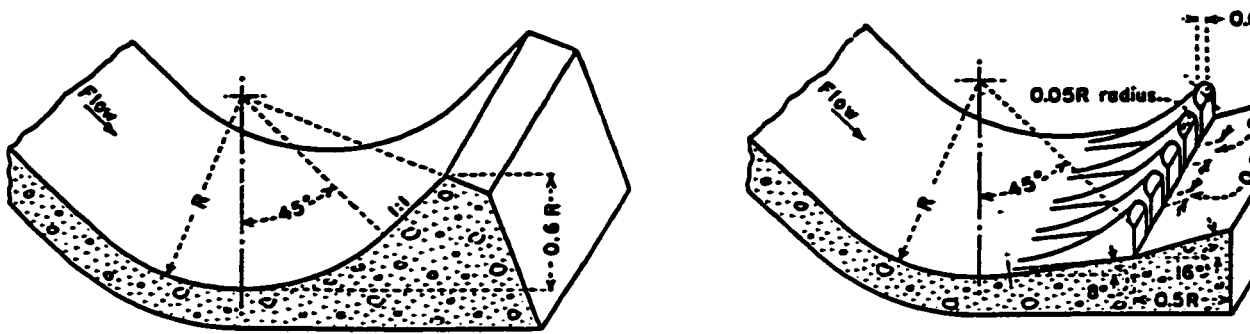


Fig.2.9. Flip bucket and Ski jump spillways (USBR, 1977)

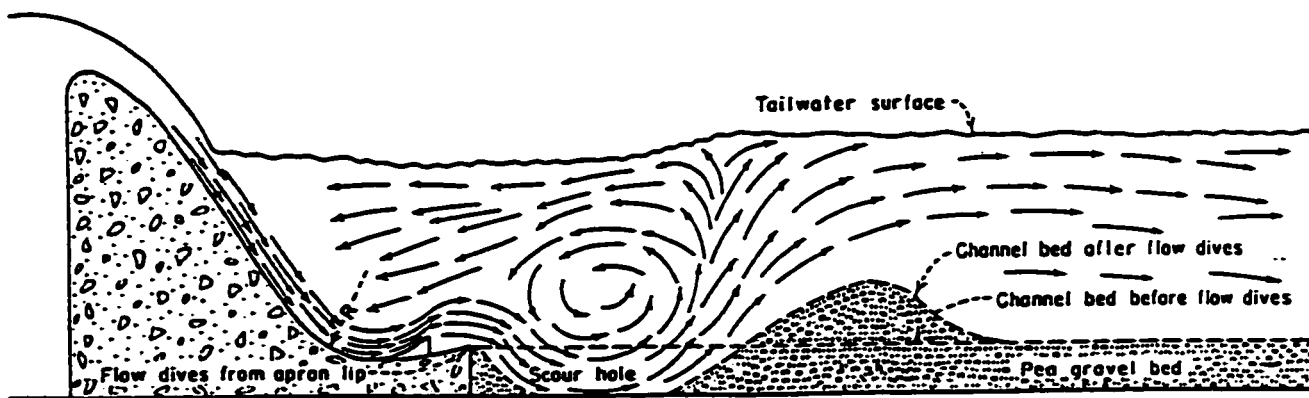


Fig.2.10. Roller bucket energy dissipators (USBR, 1977)

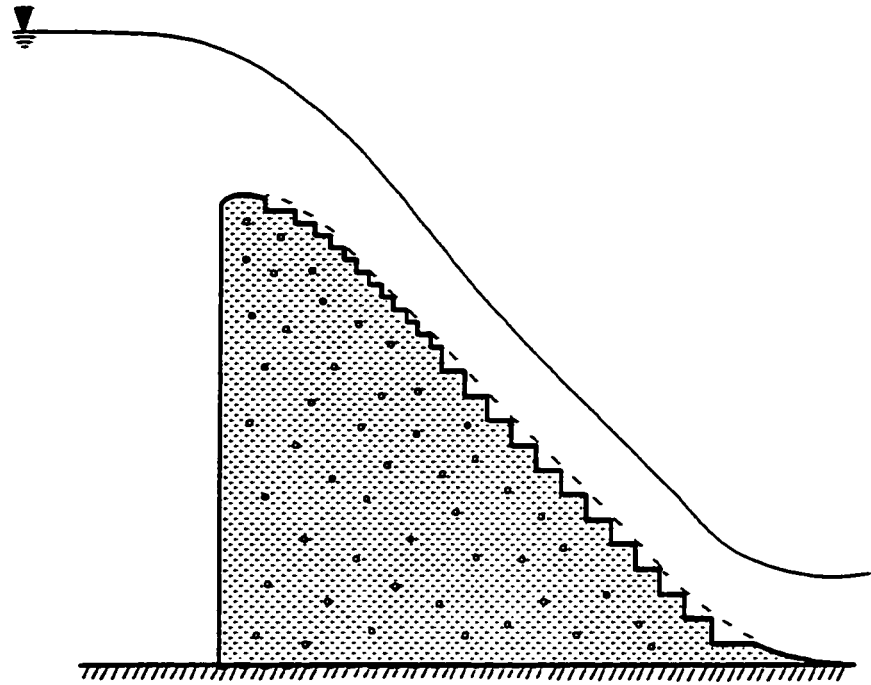


Fig.2.11. Examples of a stepped-spillway

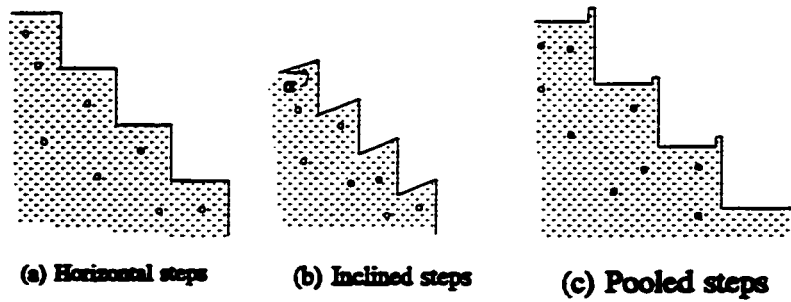


Fig.2.12. Various types of step geometry

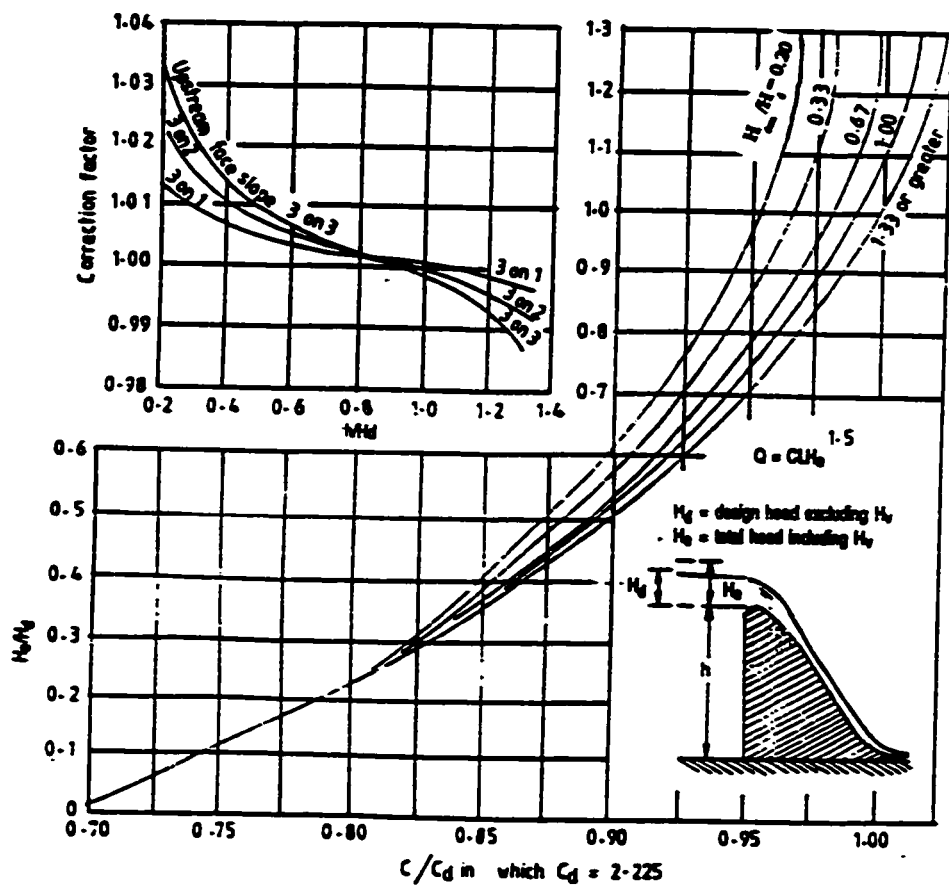


Fig. 2.13. H-Q relationship for standard ogee spillway (after Baban, 1995)

Chapter 3

Literature Review

3.1. Historical Review

Drop structures and *stepped* spillway profiles have been used for more than 2500 years. The oldest *stepped* spillways are the spillways of the two Khosr dams, in Iraq, built by the Assyrian King Sennacherib in 694 B.C. These dams were designed to supply water to the Assyrian capital city Nineveh (near the present city of Mosul). Both dams feature a *stepped* downstream face and were intended to discharge the entire river flow over their crests. Much later, the Romans built *stepped* overflow dams in Syria and Tunisia. After the fall of the Roman empire, several *stepped* spillways in Iraq and Spain were designed by Moslem civil engineers. Spanish engineers, who benefitted from Roman and Moslem precedents, were the most exceptional engineers in dam expertise before 1850. Towards the end of the 14th century they built the Almansa dam, in Spain, which in 1576 was raised while keeping the lower original part intact (Fig.3.1.a). A similar structure is the Tibi-Alicante Dam built between 1579 and 1594 on the river Monegre, in Spain (Fig.3.1.b). The largest dam with a *stepped* spillway is the Puentes dam, in Spain, which was built in 1791. This dam was washed out in 1802 after a foundation failure. In central Mexico, several *stepped* overflow dams, like the Pabellon Dam, 1730 (Fig.3.1.c) were built by Spanish Engineers during the 18th-19th centuries.

In all of the above examples, *stepped* spillways were selected (i) to contribute to the stability of the dam, (ii) for their simplicity of shape, or (iii) for a combination of the two. The spillway of the New Croton dam, USA, (1906) is probably the first *stepped* spillway designed specifically to dissipate energy (Wegman, 1907).

3.2. Gabion Stepped Spillways

Gabions are a common construction material for *stepped* spillways (Peyras, et al., 1992). Their simple geometric form and easy positioning make the construction of gabion spillway structures easy and cheap. *Stepped* gabion spillways are structurally stable, resistant to water loads, and efficient energy dissipators, Fig.3.1.d. The flexibility that they provide to dams and their drainage capacity make them a highly suitable material for spillway construction. *Stepped* gabion spillways are frequently used in dams less than 5 m high. From a technical point of view, *stepped* gabion spillways can withstand unit-width flowrates q up to $3 \text{ m}^3/\text{s}/\text{m}$ without great damage, if the setting of the gabions complies faithfully with the code of practice (Peyras, et al., 1992). Table 3.1 includes two examples of gabion *stepped* spillways.

Peyras, et al. (1992) studied the hydraulics of gabion *stepped* spillways. They recommended *stepped* spillways as a good option for small dams. They found about 10-30% saving on stilling basin length by using a *stepped* spillway. Their research focussed on describing flows over small, *stepped*, homogeneous gabion spillways to quantify accurately the energy dissipation. Based on their experiments they developed a graph (Fig.3.2) for estimating the amount of dissipated energy. With increasing $q^2/(gH^3)$, the head loss per unit height drops sharply, due to the change from *nappe* to *skimming* flow.

3.3. RCC and Concrete Stepped Spillways

Roller Compacted Concrete (RCC) is regarded as a powerful new material by dam builders around the world. In addition to its use for gravity dams, RCC has found increasing use for spillways as well as for foundation improvements for dams. *Stepped* spillways are natural extensions of RCC placement techniques. Table 3.2 lists some examples of RCC dams with *stepped* spillways.

In Norway, *steps* were introduced in a tunnel spillway with a free surface flow to enhance the energy dissipation (Chanson, 1994). This design was selected to allow detrainment upstream of a vertical shaft and to prevent entrainment of air in the shaft. In the UK a *stepped* intake of a morning-glory type spillway was selected for Ladybower reservoir, following model tests which showed that this arrangement provided a larger discharge capacity than a smooth intake (Chanson, 1994).

Recently the use of RCC for existing small earth embankments has received a great amount of interest. RCC *block-steps* are more frequently being placed on the downstream slopes of dams to allow for overtopping during flood conditions (Hansen, 1986).

3.3.1. Block Stepped Spillway

Dam Safety inspections have concluded that a large number of both small and large embankment dams are unsafe due to predicted overtopping during extreme flood events (Frizell, 1992). In addition, the capacity of a spillway often needs to be increased, either because of changes in methods of hydrological assessment, or requirements for higher standards of safety. In some cases it is even necessary to replace the existing

spillway completely. However, often the preferred solution is to augment capacity with an auxiliary spillway, which generally comes into operation at a higher reservoir level than the existing spillway (Pravdivets, and Bramley, 1989).

Recently there has been considerable interest in the use of *stepped-block* spillways for the construction of such auxiliary spillways, because of: (i) the ease of construction, and (ii) the low cost per installed unit discharge capacity (Pravdivets, and Bramley, 1989). *Stepped* block protection has considerable potential as a low-cost method for the construction of chute spillways and the protection of embankments from erosion by overtopping flow. By using *stepped* blocks, the overall construction programme is shorter because of the reduction in preparatory works required before dam construction.

The use of RCC to solve an inadequate spillway capacity condition was developed by Freese and Nicholas (Hansen, 1986) for the Brownwood Country Club dam, Texas (Fig. 3.3). After several possible solutions were investigated, it was decided to place *stair-stepped* RCC on the dam's downstream slope. The entire project was completed in two weeks at a cost of about one-third of the other options studied. This solution has already produced similar designs for modifying two existing embankments in Colorado and two more in North Carolina (Hansen, 1986).

Wedge-block spillways differ from conventional *stepped* spillways in a number of respects. Apart from the drainage holes, an important consideration is that wedge-block spillways are considerably flatter than traditional *stepped* spillways. In addition, an under-drain is an important feature with adequate water egress at the toe. The wedge block choice for the spillway at Brushes Clough, in the UK, proved to be an economical solution and straight forward to construct. The loose blocks have not moved or shown any sign of distress due to hydraulic forces and have been tested up

to $q=0.75 \text{ m}^2/\text{sec}$ (Baker, and Gardiner,1995).

Research in Russia, the UK and the USA has shown that spillways constructed from small, lightweight, wedge-shaped concrete blocks, can utilize flow hydrodynamics to achieve a high degree of stability. Table 3.3 lists several examples of *stepped* block protection in the UK and Russia, and Table 3.4 lists similar examples in the USA.

Pravdivets, and Bramley (1989) studied the hydraulics of *stepped* block spillways. They found that the optimum block stability is given by:

$$4 < \frac{l}{h} < 6 \quad (3.1)$$

where

l = length of *steps*, and

h = height of *steps*.

Frizell (1992) studied the hydraulics of block *stepped* spillways through a physical model test programme. The emphasis of his research was on producing a stable *stepped* spillway overlay that still provides energy dissipation on 2(H):1(V) embankment slopes. Frizell's results showed that the *steps* provide a good energy dissipator, and they are also very stable.

The Construction Industry Research and Information Association (CIRIA) design guide (Baker, and Gardiner, 1995) recommends a minimum block thickness of 100 mm for a discharge intensity $q = 1.525 \text{ m}^2/\text{sec}$ on a 1(H):3(V) slope. However, North West Water Engineering (NWW) opted to increase the average block thickness to 212 mm, giving a block weight of 120 kg and hence a greater resistance to removal or damage

(Baker, 1995).

3.4. Hydraulic Characteristics

Poggi (1949) (Essery and Horner, 1978) performed extensive experimental research on the flow behaviour of a scale model of a *stepped* spillway operating as a series of *pooled steps*. The objective of his investigations was to provide subcritical approach flow to every drop by creating a series of *hydraulic jumps*. Thus, almost complete dissipation of the energy attained by the change in level was achieved on all *steps*. The resulting design consists of large pools and small drops, inherently a costly design.

When a flow enters a spillway having *steps* of constant geometry, it accelerates over the initial section of the structure. This section is termed the *transition zone*. On *steps* lower down the spillway, equilibrium is established and the flow geometry is the same at each *step*, although the depth varies across each *step*. This section is referred to as the *uniform zone*. The length of the transition zone is influenced by the physical characteristics of the spillway and by the discharge per unit width. On a prototype spillway the length of the transition zone is generally small in comparison with the length of the spillway. Therefore, in order to provide design information, it is only necessary to make energy measurements on flows in the uniform zone.

In the transition zone containing the first series of *steps*, when the flow accelerates to a maximum velocity (V_{max}) and air starts to be entrained, the flow is smooth with slight undulations reflecting the *steps* below. Then at V_{max} , air is drawn into the flow and intense turbulence starts and grows toward the bottom of the spillway. The water cushion on the *steps* gives way to bottom rollers in which air bubbles can be seen to gyrate. The current skims over these rollers and the *step* lips. The air entrainment

velocity is solely governed by spillway features, like *step* height and length. Peyras, et al. (1992) recorded values of the order of 5.5-6 m/s depending on spillway slope. Once $V_{\max}$ is reached, flow velocity drops slightly. In addition to a loss of h m potential energy per *step*, the flow also dissipates some of its kinetic energy.

It must be noted that *cavitation* damage may occur on *stepped* spillways, but the risk of cavitation damage is reduced by the flow aeration phenomenon. Peterka (1953) and Russell and Sheehan (1974) showed that 5-8% of air concentration next to the spillway bottom may prevent cavitation damage on concrete surfaces. Further, the high rate of energy dissipation along *stepped* spillways reduces the flow momentum. Essery and Horner (1978) performed a physical model study of the complex flow behaviour down *stepped* spillways. To the author's knowledge their report is the only available guideline for design of *stepped* spillways. Unfortunately the results obtained from this study apply only to a restricted range of designs.

Young (1982) studied the feasibility of a *stepped* spillway for the Upper Stillwater Dam, Utah. He estimated a 75 percent reduction of energy through the use of a *stepped* spillway. He noted that "*the stepped spillway can be used with dams where the unit discharge is low and stilling basin construction is to be limited*". He found that at small discharges ($0.25 \leq q \leq 0.75 \text{ m}^2/\text{s}$), the flow strikes each *step* and loses nearly all of its velocity before accelerating to the next *step*. The results of his study confirmed an enormous saving in concrete and excavation along with reducing the area over which uplift pressures can develop.

Sorenson (1985) performed a physical model investigation of *stepped* spillways. He found that the best way to eliminate the deflecting jet of water was to add a few *steps* further up the face of the spillway. For a model spillway, the scaled *steps* form the

dominant *surface roughness*. He stated that "*the model surface roughness and resulting turbulent boundary layer growth and air entrainment were not sufficient to adequately simulate the prototypical conditions*". So, if the Reynolds numbers are in the order of 10^5 , the scale effects will be minimal and predicted prototypical toe velocities should be closer to true prototypical values.

The spillway sections adopted for De Mist Krall weir on the Little Fish river, and Zaaihoek dam on the Slang river, both in South Africa, are shown in Fig.3.4 and Fig.3.5 respectively (Hollingworth and Druyts, 1986). Choosing *stepped* spillways for these dams was for two purposes:

1. to help to dissipate the energy; and
2. to facilitate the placing of rollcrete and facing concrete.

The upstream faces of these two structures are vertical and their downstream faces are *stepped*. The De Mist Kraal spillway was hydraulically tested on a small-scale model (1:75). A *step* size 1 m high by 0.6 m wide was found to be optimum, for the required discharge range. The same general configuration was adopted for the Zaaihoek dam.

Houston (1987) studied the hydraulic model performance of the Upper Stillwater *stepped* spillway. She stated that "*the crest section must be appropriately designed to produce the desired flow conditions down the remainder of the stepped face*." In her studies, she found a reduction of 85 percent for the required stilling basin length. She emphasized the needs for further research, regarding the design data that relates the flow depths to energy dissipation on *stepped* spillways.

Rajaratnam (1990), in his work on *stepped* spillway flow, defined a coefficient of

fluid friction, C_f , as:

$$C_f = \frac{2y_n^3 g \sin \theta}{q^2} \quad (3.2)$$

where

y_n = normal depth ,and

$$\sin \theta = \frac{h}{\sqrt{l^2 + h^2}} \quad (3.3)$$

In his experiments C_f was found to vary between $0.11 \leq C_f \leq 0.2$, with an average of 0.18 and for small flow rates the range was found to be $0.25 \leq C_f \leq 0.28$.

Stephenson (1991) found that the energy loss over *stepped* spillways is proportional to the length of the face once uniform depth is reached. It means the incremental energy loss equals the incremental height of the dam. Therefore, the energy loss ratio increases with dam height and is not only a function of *step* configuration. He also noted that "*... a recessed top step kept the jet against the dam face, giving a more uniform loss gradient down the dam face.*" His results show that energy dissipation increases up to a certain limit as the *step* sizes are increased, beyond which there is limited advantage in increasing the *step* height.

Diez-Cascon, et al. (1991) studied the hydraulic behaviour of a prototype *stepped* spillway, based on tests on a scale model. The 1/10 scale model represented a *stepped* spillway containing *steps* of 30 cm length and 60 cm height. They observed that the *stepped* spillway behaved in the same way as a smooth one, with some unknown roughness, and they identified the trapped vortices in the *steps* as the main reason for

energy loss. They presented a theoretical-experimental approach to the hydraulic behaviour of *skimming* flow on the maximum air entrainment and swelling of flow. They also observed a 91% reduction in upstream fluid energy as a result of using a *stepped* spillway.

Hensen and Reinhardt's (1991) results are reproduced in Fig.3.6. It shows that with increase in discharge the rate of energy dissipation decreases. At Monksville dam, New Jersey, no terminal energy dissipator structure was required because of the dense foundation rock at the end of the spillway. A 0.61 m high *step* appears to produce a good combination of hydraulic efficiency and simple constructibility. Also, for Monksville Dam the designers determined that the training wall height should be twice that required by model similitude at maximum discharge to account for freeboard, accommodate bulking of the flow, and reduce spray. For Upper Stillwater Dam, laboratory studies indicated that, for the design discharge condition, the *stepped* spillway produced a 70 % energy reduction over a conventional smooth spillway. This, in turn, led to an 85 percent reduction in the necessary length of stilling basin.

Frizell (1992) mentioned that many embankment dams have been identified as unable to pass their design flow without failure due to overtopping. She discussed the use of *stepped* spillways, for providing erosion protection as well as dissipating energy. Energy dissipation characteristics of several *site-specific* model tests are shown in Fig.3.7. These data are plotted in prototype values and show the ratio of the kinetic energy at the dam toe to the total available head versus the unit discharge. The kinetic energy was calculated using measured velocities on the *steps* near the dam toe. The ratio reduces to the average *stepped* spillway velocity over the theoretical maximum velocity for a given dam height. Most of the information is for dams with 0.6 m *step* height.

Peyras, et al. (1992) studied the hydraulics of gabion *stepped* spillways, based on 1:5 scale model tests. They examined gabion deformations and factors affecting resistance to floods. The tests revealed some deformation of gabions, due to movement of the stone filling. They recommended, for expected floods exceeding $q = 1.5 \text{ m}^2/\text{s}$, that wire mesh and lacing must be strengthened. It was also advised to stiffen the zinc-coated box with a third row of extra binding wire and increase the number of diaphragms in the gabions. In general, from an economic point of view, they found a saving of 5-10% on project cost, using *stepped* spillways.

Chanson (1993) stated that up to 99% of the total head developed at a dam site can be dissipated by using a *stepped* spillway. In his study he considered the effects of air entrainment in *stepped* spillways. He noticed better efficiency for a *stepped* spillway with slopes steeper than 30° .

Christodoulou (1993) found that energy loss due to the *steps* depend primarily on the ratio of y_c/h , where y_c = critical depth, as well as on the number of *steps* N_s . He found that for higher values of y_c/h , the effect of N_s becomes appreciable at a certain y_c/h , and that the relative energy loss increases with N_s . In his experiments 10 and 13 *step* models were used and discharges ranged between 10 and 45 l/s. He indicated that the most important parameters governing energy dissipation are y_c/h , and the number of *steps* N_s . Using Sorensen's (1985) data, he developed a graph to find the amount of energy dissipated (see Fig.3.8). It has been established that energy dissipation decreases with an increase in $y_c/N_s h$.

Bindo, et al.'s (1993) tests for the case of the M'Bali dam showed that:

1. the discharge coefficient was not affected by the presence of *steps*;
2. the *step* heights progressively increase toward the toe of the spillway, so that

- the flow would not *jump* for low discharges;
3. the energy dissipation along the chute varied from 90 % (for low discharges) to 50 % (design discharge);
 4. the hydraulic jump was located within the basin; and,
 5. downstream scouring was limited.

They also recommended that "*the height of the steps be chosen in such a way that, for the maximum flow, air enters the flow some 5 to 10 m higher than the elevation of the stilling basin.*" They developed Fig.3.9 for determining the flow velocity.

Chanson (1994) indicated that energy dissipation in a *stepped* spillway increases with dam height. Also, for a given dam height and using a *stepped* spillway, the rate of energy dissipation decreases when the discharge increases. Chamani and Rajaratnam (1994) presented a method to estimate the energy loss on a *stepped* spillway with nappe flow. By analyzing the data of Horner (1969), and Moore (1943), they found that the relative energy loss could be up to 97% for $y_c/h < 0.8$, which is a very low discharge.

Rice and Kadavy (1996) studied the physical models for the Saldo Creek Site 10, San Antonio, Texas. It is a RCC *stepped* spillway with slope 2.5(H) : 1 (V). This study was limited to only three discharges. They found the energy dissipation for *stepped* models was 71 %, whereas for a *smooth* model it was 25 %. Figs. 3.10.a and 3.10.b present typical velocity distributions for *stepped* and *smooth* models respectively. From these figures it can be concluded that with the use of *steps* the momentum transfer caused by the roughness of *steps* penetrates vertically into the flow and results in lower velocities compared to the smooth model.

Name	Slope (°)	Step height (m)	No. of steps	Remarks
Rietspruit outfall, South Africa			2 to 4	3 successive <i>stepped</i> weirs
San Paolo weirs, Brazil 1986	63	1	9,11 and 8	Solid transport and flood flow controls. 3 successive weirs

Table 3.1. Examples of gabion *stepped* spillways

Dam Name	Slope ($^{\circ}$)	Dam Height H (m)	Max. Dis. q (m^2/s)	Step Height h (m)	No. of Steps N_s	Type of Steps
Clywedog dam, UK, 1968	60	72	2.8	0.76		Precast concrete
De Mist Kraal weir, South Africa 1986	59	30	29	1	19	Horizontal
Zaaihoek dam, South Africa 1986	58.2	45	15.6	1	40	Horizontal
Monksville, USA 1987	52	36.6	9.3	0.61		Horizontal
Olivettes dam, France 1987	53.1	36	6.6	0.6	47	Horizontal
Upper Stillwater, USA 1987	72 & 59	61	11.6	0.61		Horizontal
M'Bali dam, CAR 1990	51.3	24.5	16	0.8	36	Horizontal
New Victoria dam, Australia 1993	72 & 51.3	52	5.4	0.6	85	Horizontal
Petit-Saut dam Guyana 1994	51.3	37	4	0.6		Horizontal

Table 3.2. RCC dams containing *stepped* spillways

Dam Name	Slope (°)	Dam height (m)	Max. Dis. (m ³ /s)	Type of Steps
Brushes Clough dam, UK, 1859/1991	18.43	26	3.66	Wedge concrete blocks inclined downward $\theta = -5.6$
Dneiper hydro plant, Ukraine, 1976	8.75		60	Concrete block system. Horizontal steps
Sonsnovski dam, Russia, 1978		11	3.3	<i>Stepped</i> block system
Lukhovitsky dams, Russia 1978, 1980, and 1981		11, 11.5, 7.5	3, 3.3, 2.9	<i>Stepped</i> block system
Transbaikal region dam, Russia, 1986	14	9.4	20	<i>Stepped</i> block system

Table 3.3 Embankment dams with *stepped* spillway protection in the UK, and Russia; (Baker, 1989)

Dam and location	Unit Dis. m ² /s	Hydrau- lic height (m)	Head (m)	Downst- ream Slope (H:V)	Downstream concrete facing and Placement technique
Lahontan, NV	6.32	33.53	1.83	2:1	Conventional formed
Brownwood Country Club, TX	2.29	5.79	1.68	2:1	RCC unformed
Kerrville, TX	31.12	6.40	7.32	0.8:1	RCC unformed
McClure, NM	9.04	36.27	3.13	2.18:1	RCC unformed
Spring Creek, CO	2.62	15.24	1.36	2.3:1	RCC unformed
Goose Lake, CO	0.85	10.67	0.73	1:1	RCC unformed
Upper Las Vegas WASH	21.37	11.28	5.49	2.5:1	RCC
Ringtown No. 5, PA	4.46	18.29	1.83	2.75:1	RCC unformed

Table 3.4. *Stepped* spillways for protection of the downstream slope of embankment dams in the USA (Frizell, 1992)

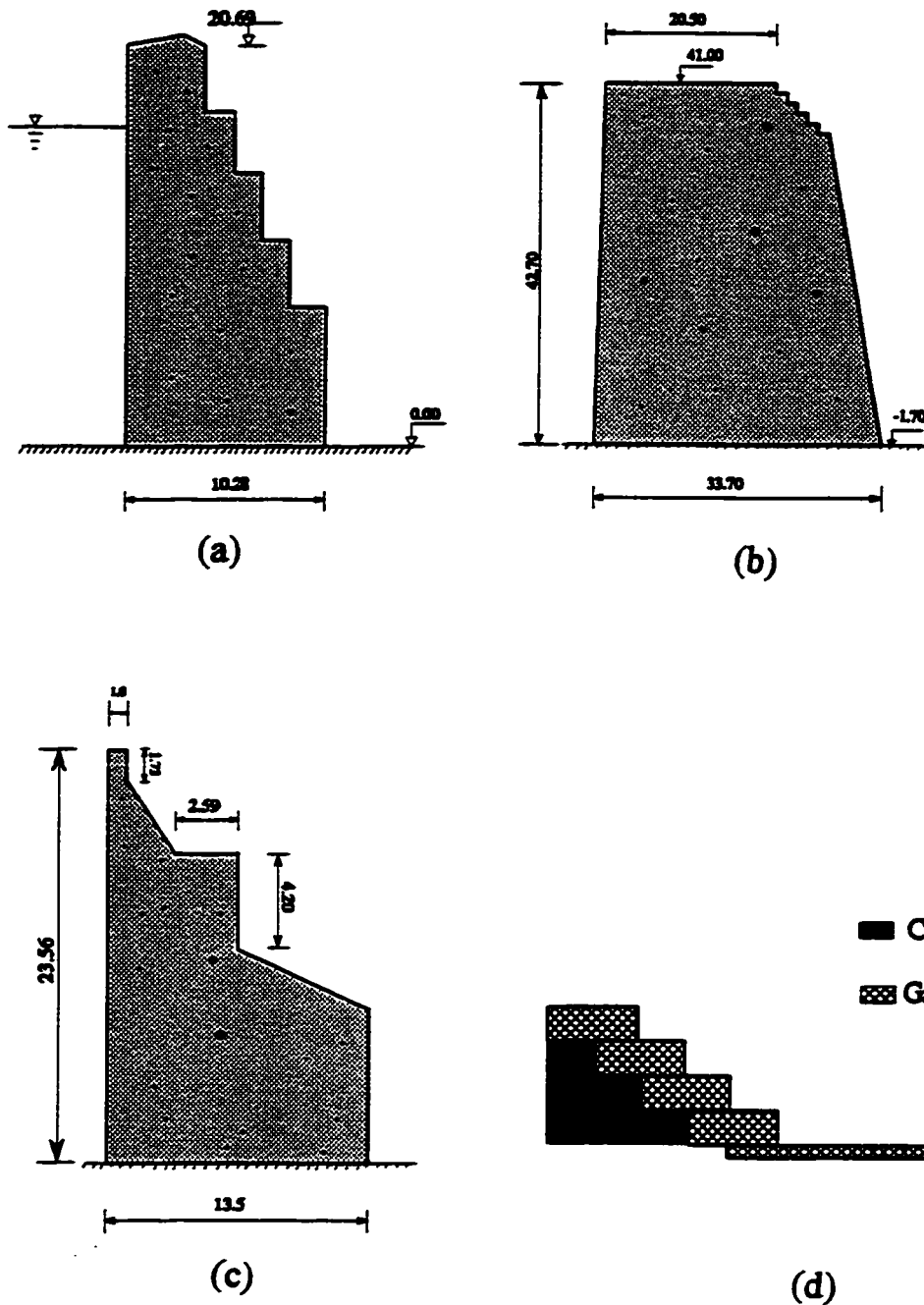


Fig.3.1. Examples of stepped spillway structures: (a) Almansa dam (Spain, 1576), (b) Tibi-Alicante dam (Spain, 1594), (c) Pabellon dam (Mexico, 1730)
(d) Gabion stepped spillway profile

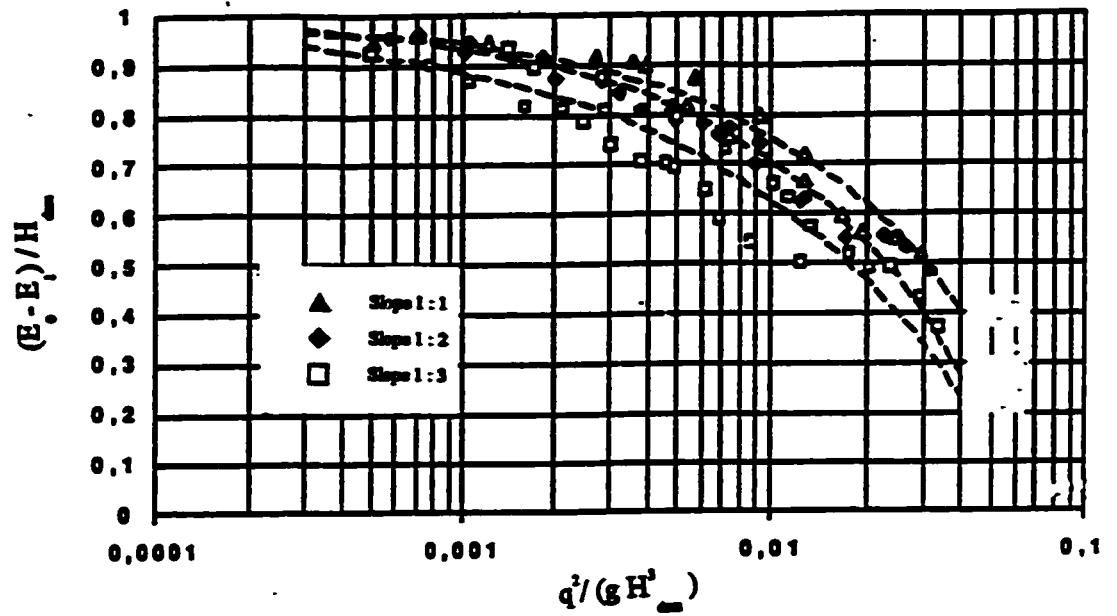


Fig. 3.2. Unit head loss over plain gabion steps (after Peyras, et al., 1992)

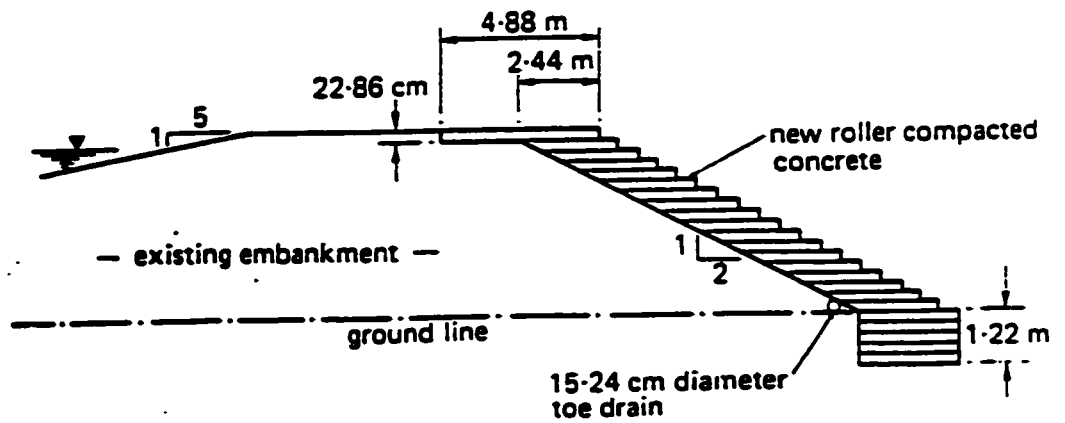


Fig.3.3. Brownwood Country Club dam (after Hansen, 1986) (USA)

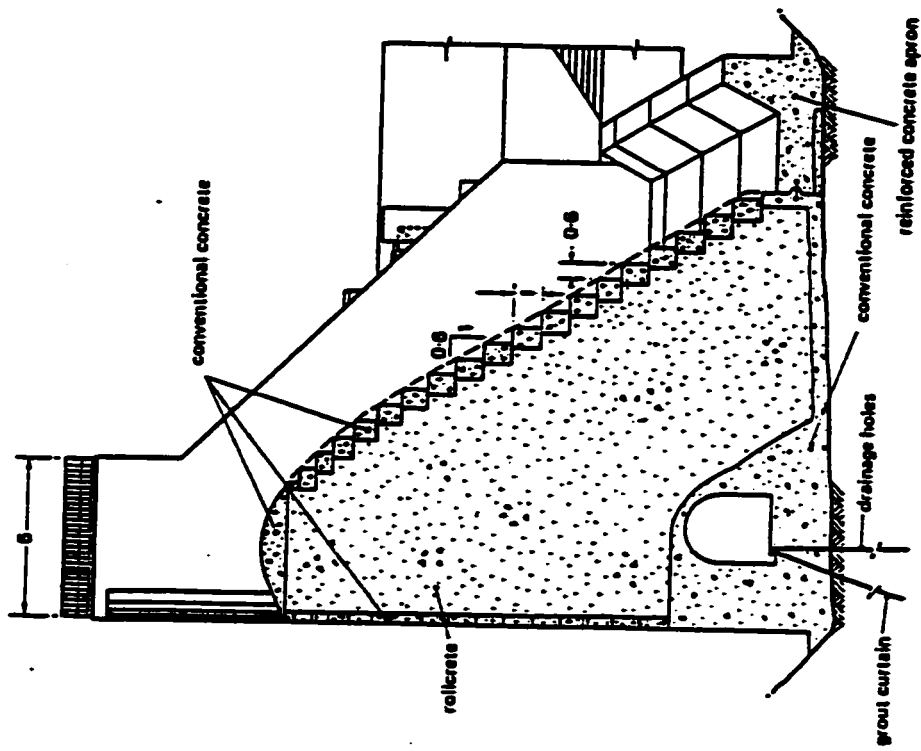


Fig.3.5. Spillway of the Zaaikoek dam
(after Hollingworth and Druyts, 1986) (S. Africa)

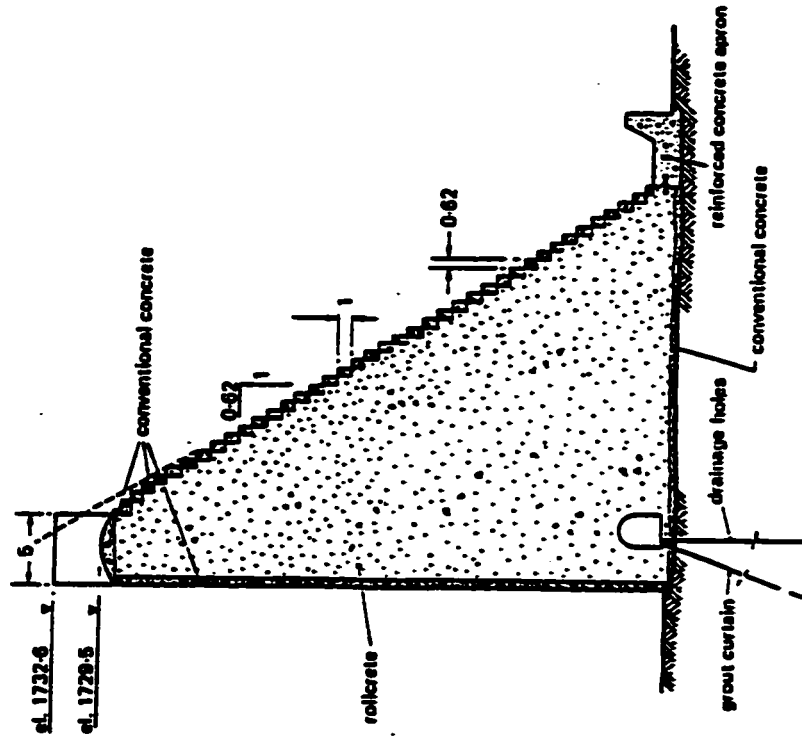


Fig.3.4. Spillway of the De Mist Kraal weir
(after Hollingworth and Druyts, 1986) (S. Africa)

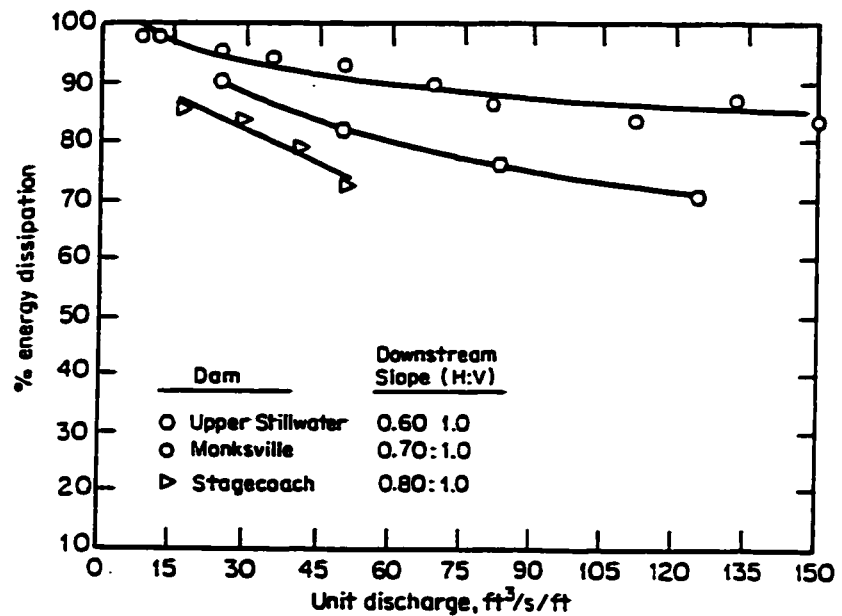


Fig.3.6. Stepped spillway energy dissipation for Upper Stillwater, Monksville, and Stagecoach Dam (after Hensen and Reinhardt, 1991)

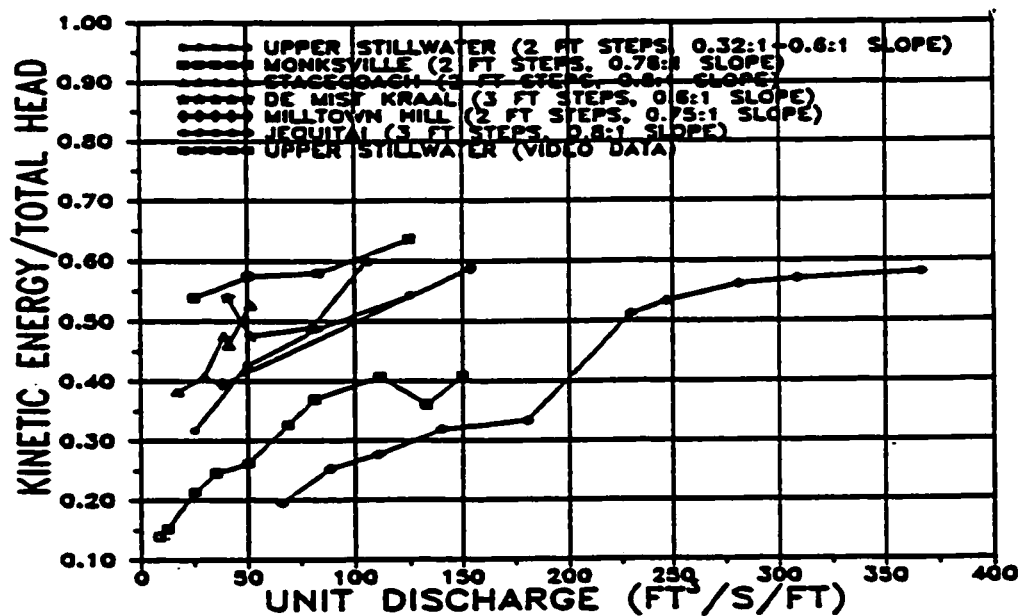


Fig.3.7. Energy dissipation characteristics at the toe of stepped spillways (after Frizell, 1992)

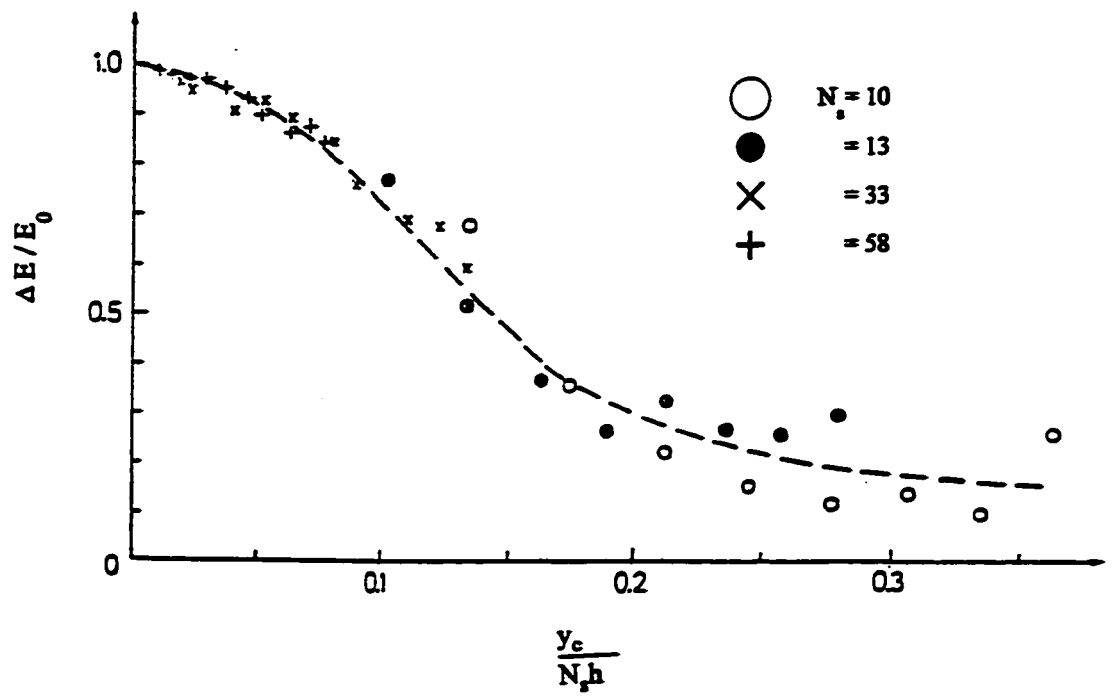


Fig.3.8. $\Delta E/E_0$ vs $y_c/N_s h$ relationship (after Christodoulou, 1993)

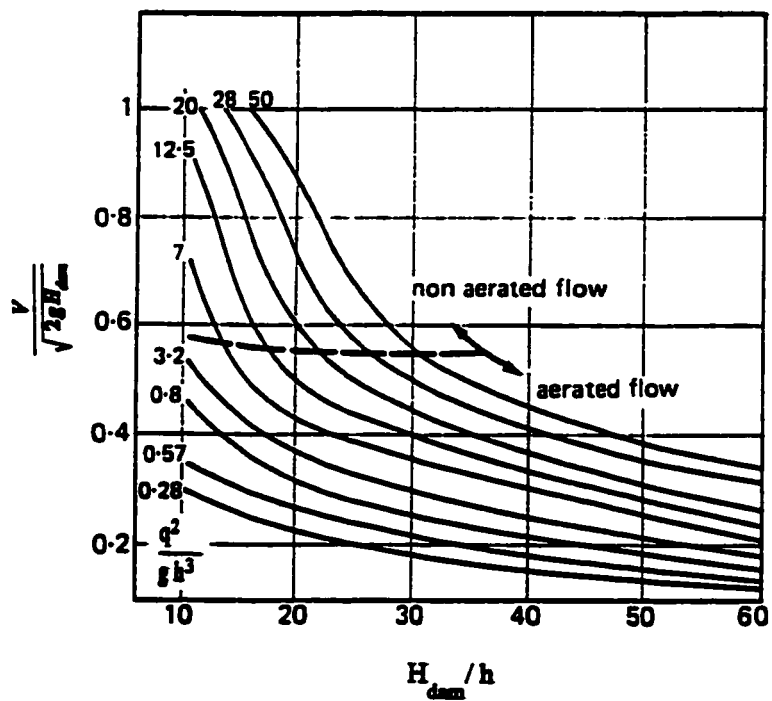
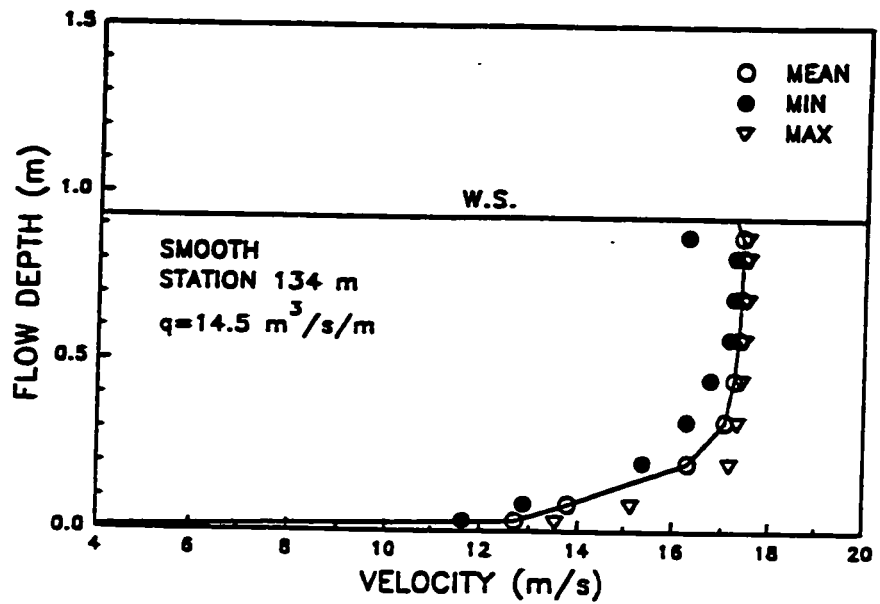
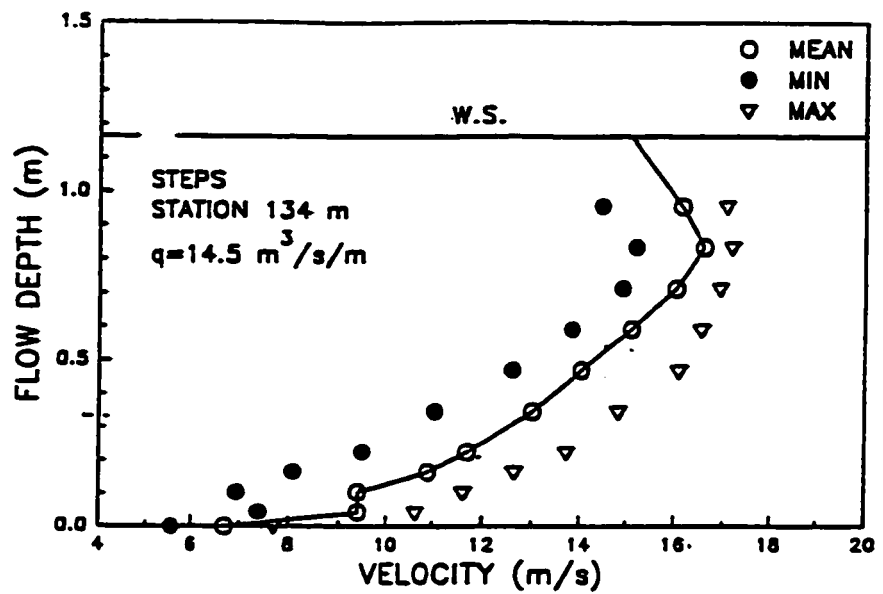


Fig.3.9. Flow velocity at the toe of a stepped spillway (after Bindo, et al., 1993)



(a)



(b)

Fig.3.10. Velocity profile on a spillway: (a) smooth, (b) Stepped (after Rice, and Kadavy, 1996)

Chapter 4

Experimental Studies

4.1. Introduction

Physical models of hydraulic structures are probably the most common type of hydraulic model to be studied. They are cheap, generally easy to operate, and easy to interpret. Physical model studies are used to investigate the anticipated performance of hydraulic structures. The model is usually a reduced-size representation of the proposed hydraulic structure and is designed to investigate specific areas of concern related to the hydrodynamic performance of the waterways. Modelling laws based to the theoretical analysis and laboratory experience have been developed which permit correct simulation of the particular hydrodynamic phenomena under investigation. Once the model laws are understood and proper consideration is given to their limiting range of application, successful laboratory investigations can be completed which will in fact simulate the performance of the full scale structure.

The hydraulic model plays a key role in the development of final design concepts for hydraulic structures. Although its most important role is to verify the adequacy of the design concept, it is also used to *fine tune* details such as length of a stilling basin, or

the height of a spillway's training walls, etc., which normally produce construction cost saving that more than offset the cost of the hydraulic model investigation.

The principal goal of this study was to experimentally investigate the *energy dissipation* and the *location of the free surface water* on *ogee stepped spillway* models installed in a rectangular channel. This chapter provides details on the experimental setup and apparatus used as well as the procedures followed during the course of the laboratory investigations.

4.2. Experimental Setup and Measurements

The experimental programme was performed in the large adjustable-slope flume located in the Hydraulics Laboratory of the Department of Civil Engineering, University of Ottawa. The glass-wall flume is 12 m long and has a cross section 0.387 m wide $\times$ 0.61 m deep. A head tank with a sluice gate control section is located at the upstream end of the flume. Water supply to the tank is via a 203 mm diameter pipe with a *perforated-T* end section that ensure as even flow distribution to the tank. Screens are installed in the tank to eliminate any residual large-scale turbulence in the flows exiting the tank. Discharge to the flume is adjusted by a butterfly control valve in the 203 mm diameter supply line. A variable-height weir is located at the flume's downstream end and flows exiting the flume discharge to a collection tank situated directly under the flume's outlet. Various types of sharp-crested weirs can be installed in the collection tank to measure flow rates in the flume.

The flume is supported on two (wide flange) steel beams, which are pivoted close to the upstream end of the flume and supported on two manually-driven screw-type jacks close to the downstream end of the flume. Longitudinal slope adjustment of the flume

is achieved through vertical movement of the jacks. A drawing of the general arrangement of the flume is shown in Fig.4.1.

Water surface profiles within the test section of the flume were recorded as follows:

- (i) a series of piezometer tapping points, located at intervals along the centreline of the flume floor and connected to a manometer board by polyethylene tubing, provided depth readings upstream and downstream of the various model spillways tested.
- (ii) a vernier point-gauge, having an accuracy of ± 0.1 mm, were used to record water surface profiles along the downstream faces of the model spillways. [Note : due to the inherently unstable water surface associated with a model spillway flow, the actual measurement accuracy when recording our model spillway flow profiles was closer to $\pm 2-3$ mm].

4.3. Spillway Models

Flow over hydraulic structures involves significant vertical velocity components and it is therefore necessary to construct the model to a natural (i.e. undistorted) scale. Any exaggeration of the vertical scale would cause an unacceptable exaggeration of vertical flow components and for this reason must be avoided .

Because the flow is dominated by gravitational effects, models of free surface flow around hydraulic structures must be scaled according to the Froude criterion. To ensure freedom from scale according to the fairly large and scales ranging from about 1:15 to 1:60 are common (Sharp, 1981). U.S. Bureau of Reclamation (1980) spillway models for large dams have typically been constructed to scale ratios between 1:30 and 1:100. The Bureau recommends that medium-size spillway models not be smaller than

a 1:60 scale.

The scale chosen will generally depend on the facilities available. Facilities like flume sizes, discharge and so on. The model scale must satisfy two requirements:

- (i) it must allow the whole model to be incorporated in the channel available, and
- (ii) it must be such that the model discharge is within the capability of the existing pump.

Because of *cavitation* considerations in spillway design, *stepped* spillways are practical only for *small-* to *moderate-* height dams. Accordingly, based on the dimensions of the University of Ottawa tilting flume and also on the capacity of the laboratory pump, a hypothetical spillway (height $H_{\text{dam}} = 19.0$ m and design head, $H_d = 2.5$ m) was selected as the prototype for this model study. The model to prototype scale ratio adopted for the study was 1:50 which, given the dimension of the flume, meant that the model spillways were 38.0 cm high and the design head was 5 cm. The model spillways, which were constructed from plexiglass blocks, included (i) *smooth* ogee-profile spillway models, and (ii) various combinations of *stepped* ogee-profile spillway models.

4.3.1. Smooth Ogee Spillway Models

All ogee-profile spillways investigated in this study had vertical upstream faces. The spillway crest geometry used was that proposed by the Waterways Experiment Station (WES) (1977). It includes the three-arc curve upstream from the crest section. Downstream from the crest the following power function applies (see sec. 2.3.1 for more detail):

$$\left(\frac{y}{H_d}\right) = 0.5 \left(\frac{x}{H_d}\right)^{1.85} \quad (4.1)$$

This shape continues down to the point where the slope of the curve meets the terminal spillway slope. In this study 3 different terminal slopes were investigated, namely: 45°, 50°, 60°.

Based on Creager, et al's recommendation (1964) a 14 cm radius of curvature was chosen for the spillway toe transition curve for all models. This ensured a smooth transition of the flow from the spillway to the downstream channel. In this study the profile of these models were produced using the computer program GEMLSPS. Figs.4.2 shows the smooth 45°, 50°, 60° models. The experimental outputs were plotted in Figs. 4.3, 4.4, and 4.5.

4.3.2. Stepped Ogee-Profile Spillway Models

In this study the effectiveness of different *ogee-profile stepped* spillway configurations were examined by considering : (i) different downstream slopes; (ii) different *step* sizes; and (iii) different arrangement of the *steps*.

For each spillway slope considered, the various *stepped spillway profiles* were arranged so that the tips of the *steps* conformed to the corresponding smooth profile. The *steps* themselves were introduced to the profile just below the crest section. Each of the 3 (*base*) *stepped* models constructed in two parts: (i) the *crest* part, which was 1/3 of the total spillway height (H_{dam}), and (ii) the *bottom* part, which had a height = 2/3 H_{dam} . While each parts of the models had a uniform *step* size throughout its profile, the *step* sizes of the base models were different from each other.

Based on the literature review (Sorensen, 1985, Christodoulou, 1993, Bindo, 1993, Rice and Kadavy, 1996) two different *step* sizes were chosen:

1. $1/20 H_{dam}$ which considered as *large-step* sizes;
2. $1/40 H_{dam}$ which considered as *small-step* sizes.

Since the *crest* and *bottom* parts of the different base models could be interchanged, this permitted testing a wide variety of *step* arrangements, as follows:

Model 45-CL-BL

This model has a (1H: 1V) downstream slope. The crest part has 10 large-size *steps* and the bottom part has 16 large-size *steps*. Except for a few *steps* on the upper portion of the crest section the height of *steps* are identical. The few *steps* on the upper portion of the crest have a smaller height to provide the jet (Sorensen, 1985). Among all 45° *stepped* models this model has the largest *step* size and the least number of *steps*. Plate 4.1 shows this model. The water surface profiles are plotted in Fig. 4.6.a.

Model 45-CS-BL

This model has a (1H : 1V) downstream slope. The crest part has 20 small-size *steps* and the bottom part has 16 large-size *steps*. Therefore, this model provides smoother face on the upper part of the model. Plate 4.2 shows this model. The water surface profiles are plotted in Fig. 4.6.b.

Model 45-CL-BS

This model has a (1H: 1V) downstream slope. The crest part has 10 large-size *steps* and the bottom part has 32 small-size *steps*. In this model the upper part is rougher than the base part. Thus more energy should dissipate on the crest part and there will be a smooth transition to the stilling basin at the toe of spillway. The water surface profiles are plotted in Fig. 4.7.a.

Model 45-CS-BS

This model has a (1H: 1V) downstream slope. The crest part has 20 small-size *steps*, and the bottom part has 32 small-size *steps*. This model has the highest number of *steps*, and also the smallest *step* size. The water surface profiles are plotted in Fig. 4.7.b.

Model 50-CL-BL

This model has a (1H: 1.192V) downstream slope. On the crest part this model has 7 large-size *steps*, and on the bottom part it has 12 large-size *steps*. Except the few *steps* on the upper part of the crest the height of *steps* are identical. Those few *steps* on the upper part of the crest have smaller height to provide the jet. Plate 4.3 shows this model. The water surface profiles are plotted in Fig. 4.8.a.

Model 50-CS-BL

This model has a (1H: 1.192V) downstream slope. On the crest part this model has

12 small-size *steps*, and on the bottom part it has 12 large-size *steps*. The smaller *steps* on the upper part provides a smooth transient to the base part. Plate 4.4 shows this model. The water surface profile are plotted in Fig. 4.8.b.

Model 50-CL-BS

This model has a (1H: 1.192V) downstream slope. On the crest part this model has 7 large-size *steps*, and on the bottom part it has 24 small-size *steps*. In this model the crest part is rougher than the base part. Thus more energy should dissipate on the crest part and there will be a relative smooth transition to the stilling basin at the toe of spillway. The water surface profiles are plotted in Fig. 4.9.a.

Model 50-CS-BS

This model has a (1:H 1.192V) downstream slope. On the crest part, this model has 12 small-size *steps*, and on the bottom part it has 24 small-size *steps*. Among all the 50° models, this model has the largest number of *steps*, and the smallest *step* size. The water surface profiles are plotted in Fig. 4.9.b.

Model 60-CL-BL

This model has a (1H: 1.732V) downstream slope. On the crest part this model has 7 large-size *steps*, and on the bottom part it has 12 large-size *steps*. Except the few *steps* on the upper part of spillway the height of *steps* are identical. This model has the largest *step* size and the least amount of *step* among all 60° models. Plate 4.5 shows this model. The water surface profiles are plotted in Fig. 4.10.a.

Model 60-CS-BL

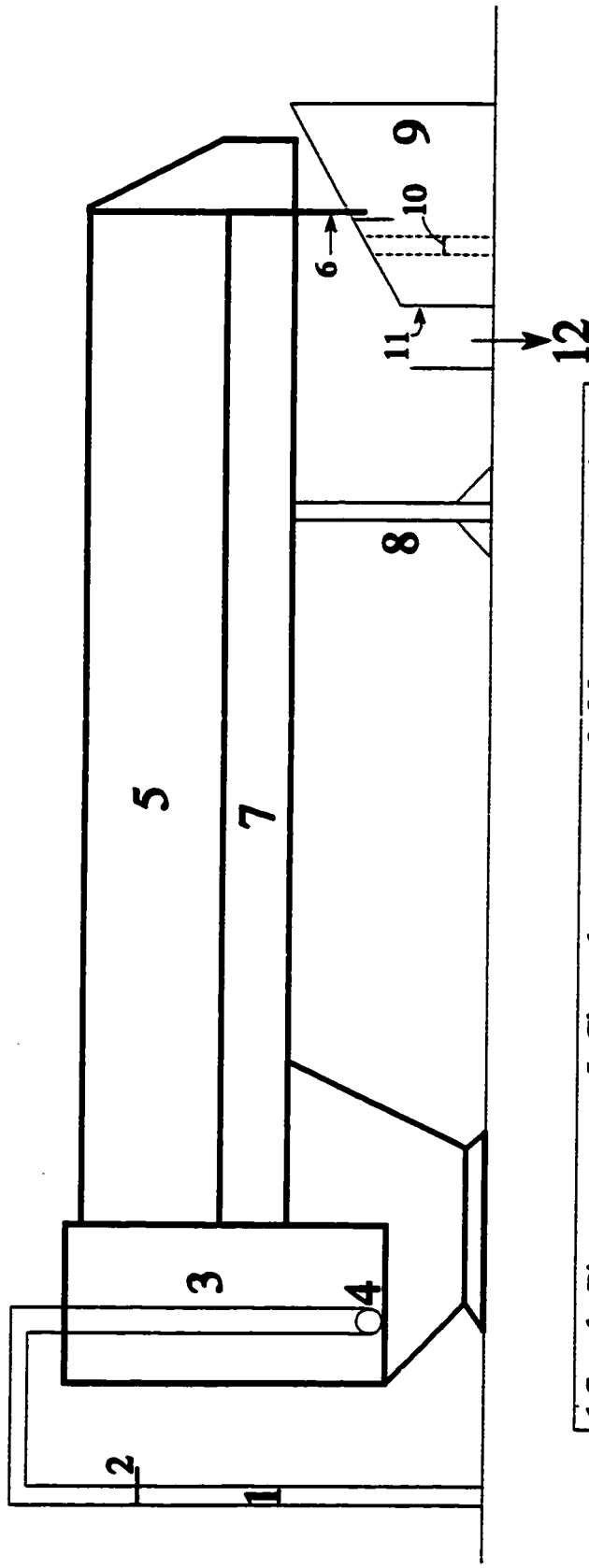
This model has a (1H: 1.732V) downstream slope. On the crest part it has 12 small-size *steps* and on the bottom part it has also 12 large-size *steps*. The *steps* on the crest part have lower height than the bottom part. It provides a smoother transition to the rough surface at the downstream. The water surface profiles are plotted in Fig. 4.10.b.

Model 60-CL-BS

This model has a (1H: 1.732V) downstream slope. On the crest part it has 7 large-size *steps*, and on the bottom part it has 24 small-size *steps*. In this model the crest part is rougher than the bottom part. Thus more energy should dissipate on the crest part and there will be a relative smooth transition to the stilling basin at the toe of spillway. The water surface profiles are plotted in Fig. 4.11.a.

Model 60-CS-BS

This model has a (1H: 1.732V) downstream slope. This model has 12 small-size *steps* on the crest part, and 24 small-size *steps* on the bottom part. The *steps* are identical. This model provides smoother model than the rest of 60° models. The water surface profiles are plotted in Fig. 4.11.b.



1. Supply Pipe	5. Channel	9. Measurement Tank
2. Control Valve	6. Control Gate	10. Baffled Plates
3. Tank	7. W-Beam	11. Rectangular S.C. Weir
4. Perforated T Unit	8. Jack	12. Underground Sump

Fig. 4.1. General arrangement of the experimental flume

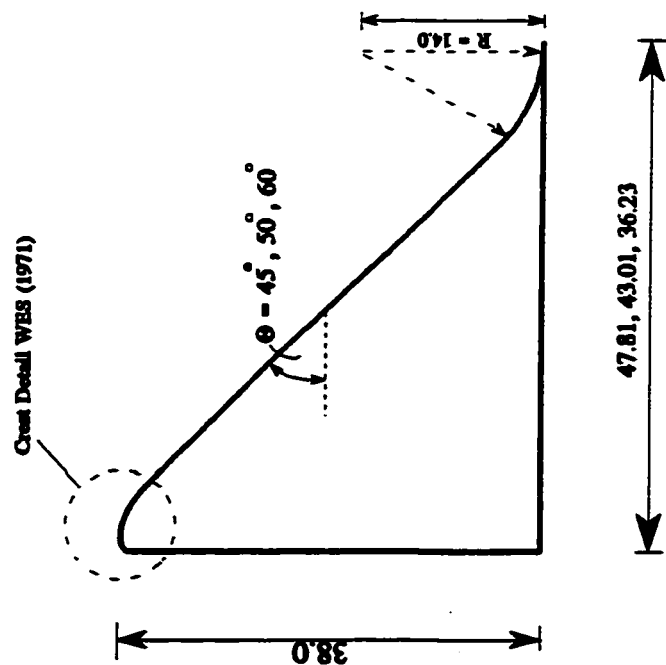


Fig.4.2. General profile(s) of smooth spillway model(s) ($45^\circ, 50^\circ, 60^\circ$)

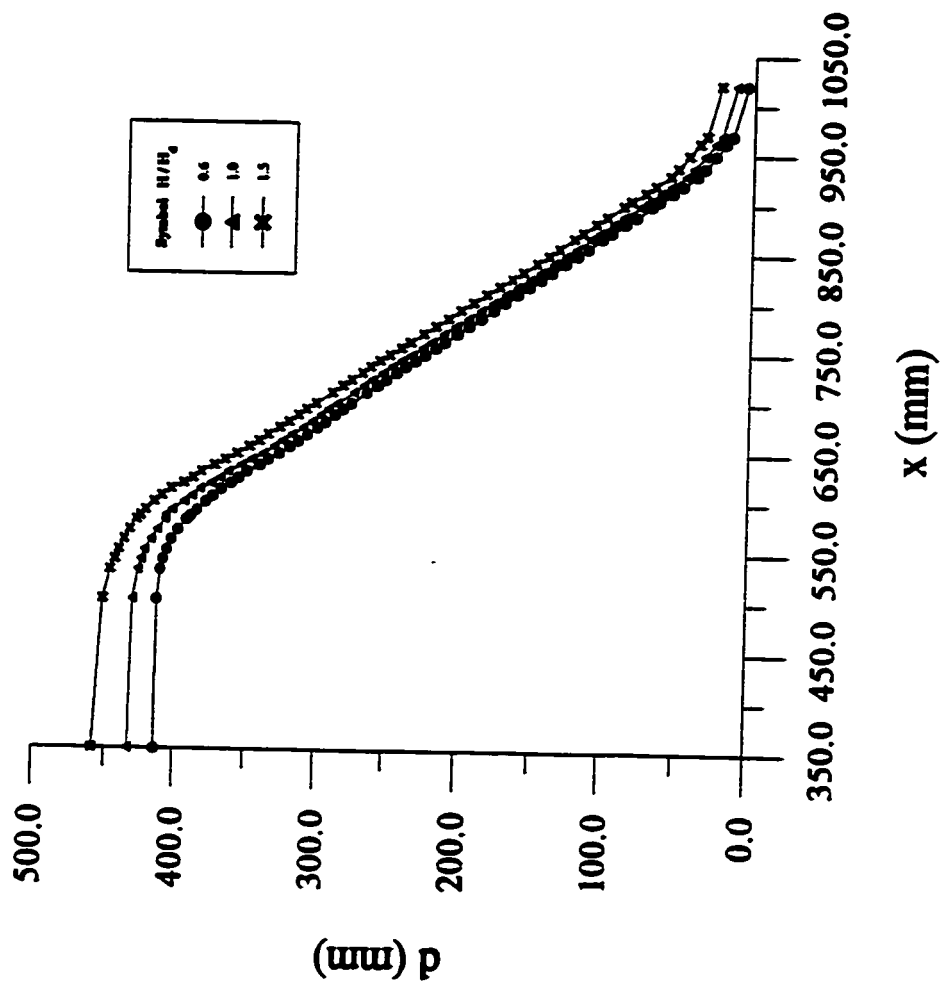


Fig.4.3. Water surface profiles for smooth 45° spillway model

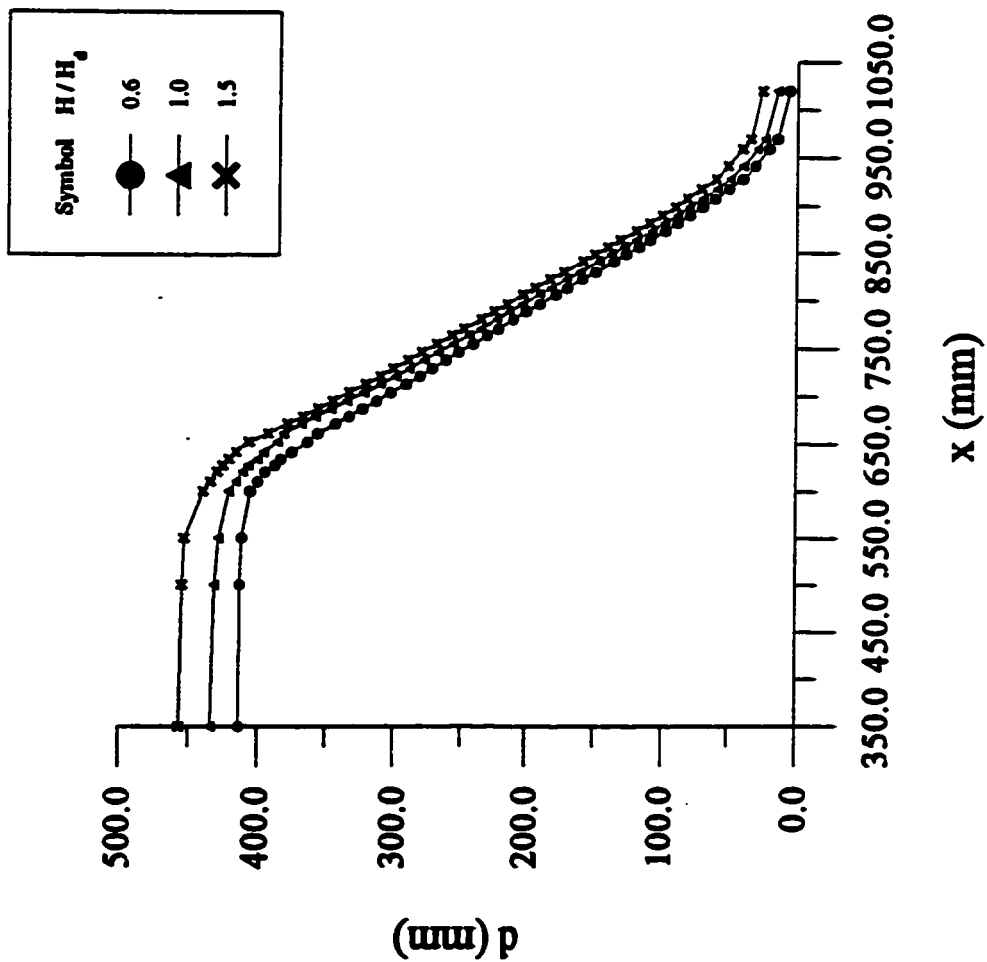


Fig.4.4. Water surface profiles for smooth 50° spillway model

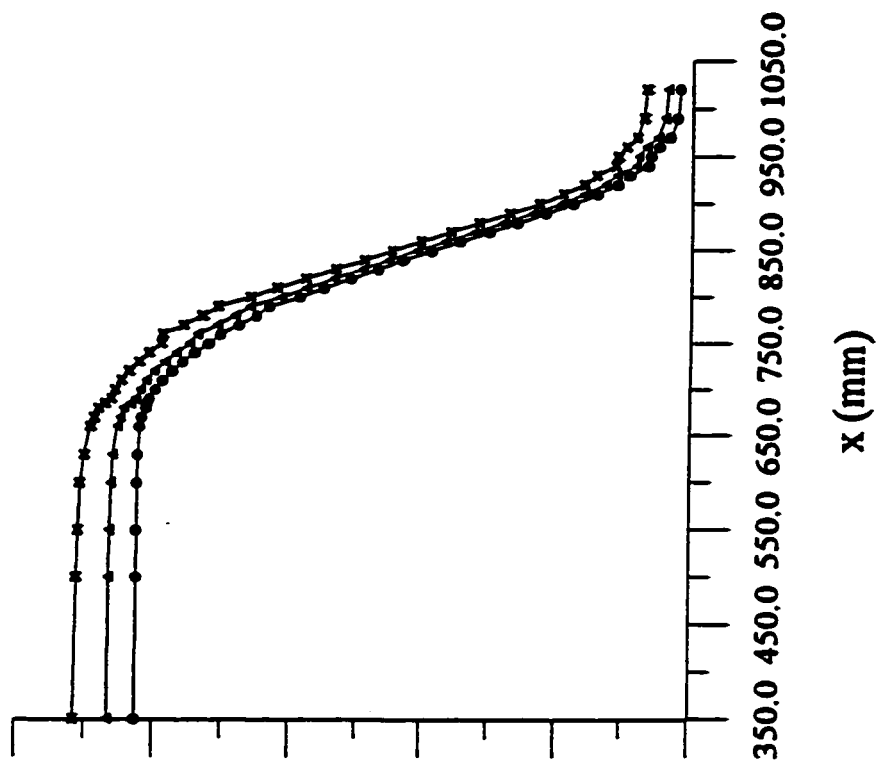


Fig.4.5. Water surface profiles for smooth 60° spillway model

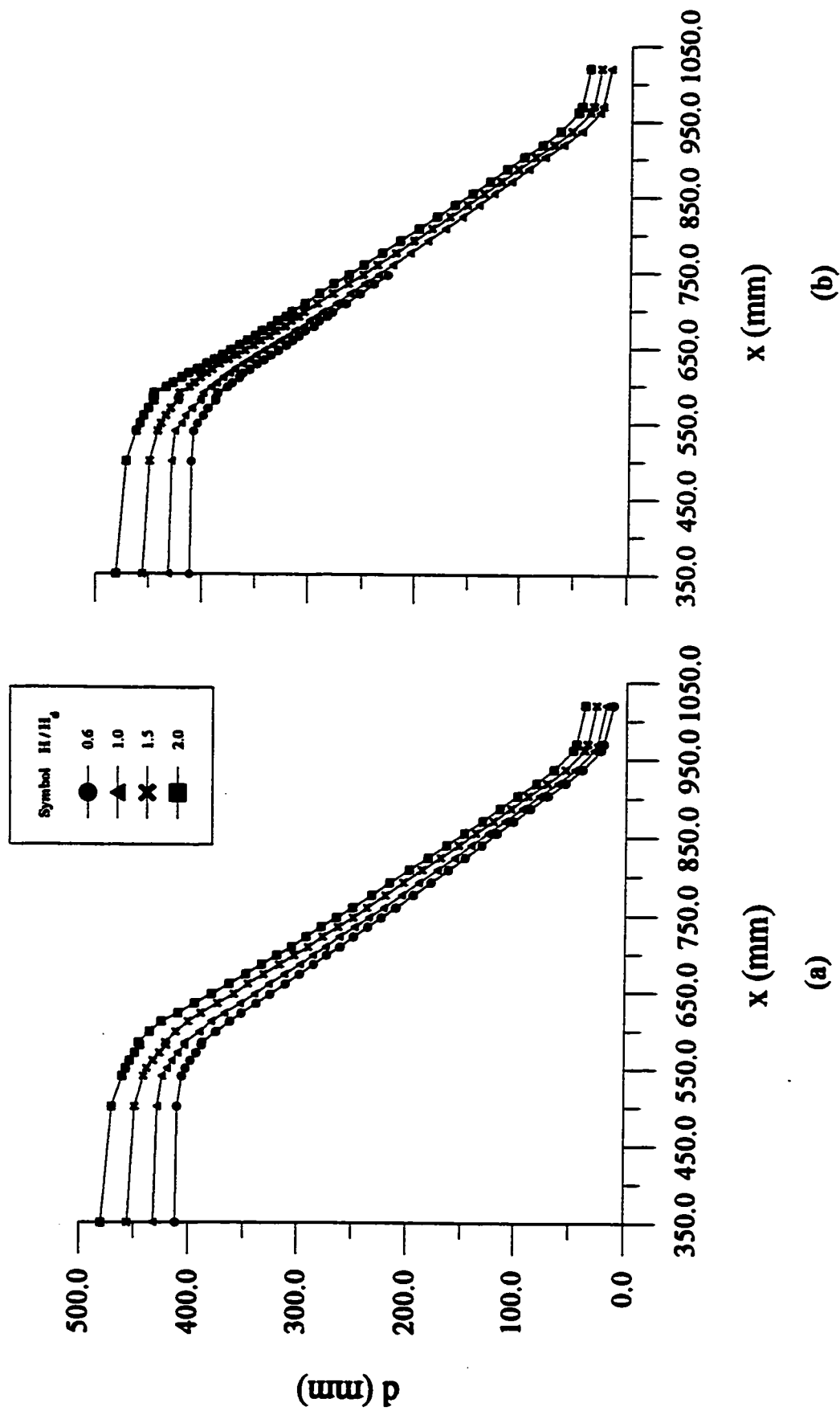


Fig.4.6. Observed water surface profiles for (a) 45-CL-BL model, (b) 45-CS-BL model

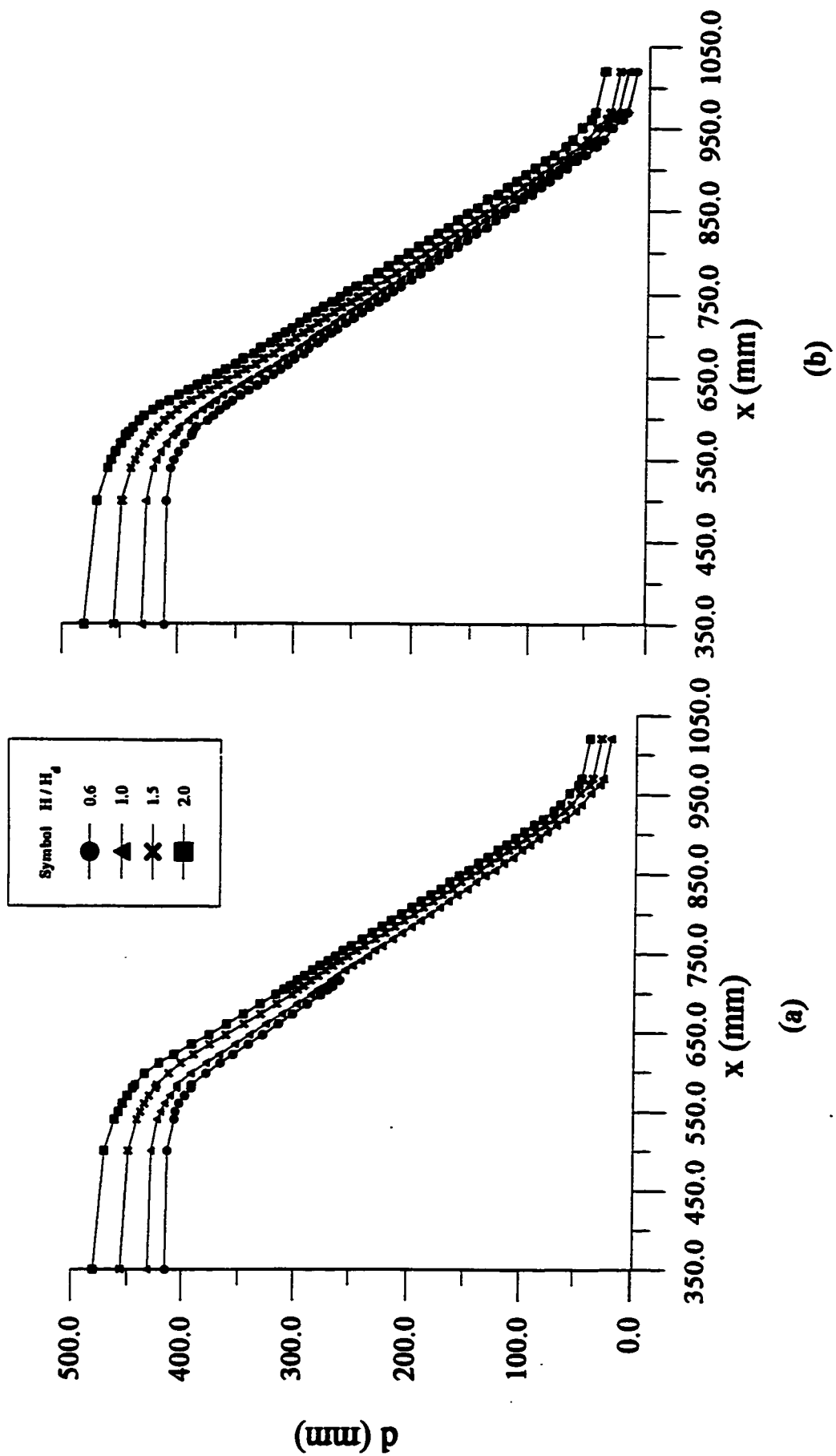


Fig.4.7. Observed water surface profiles for (a) 45-CL-BS model, (b) 45-CS-BS model

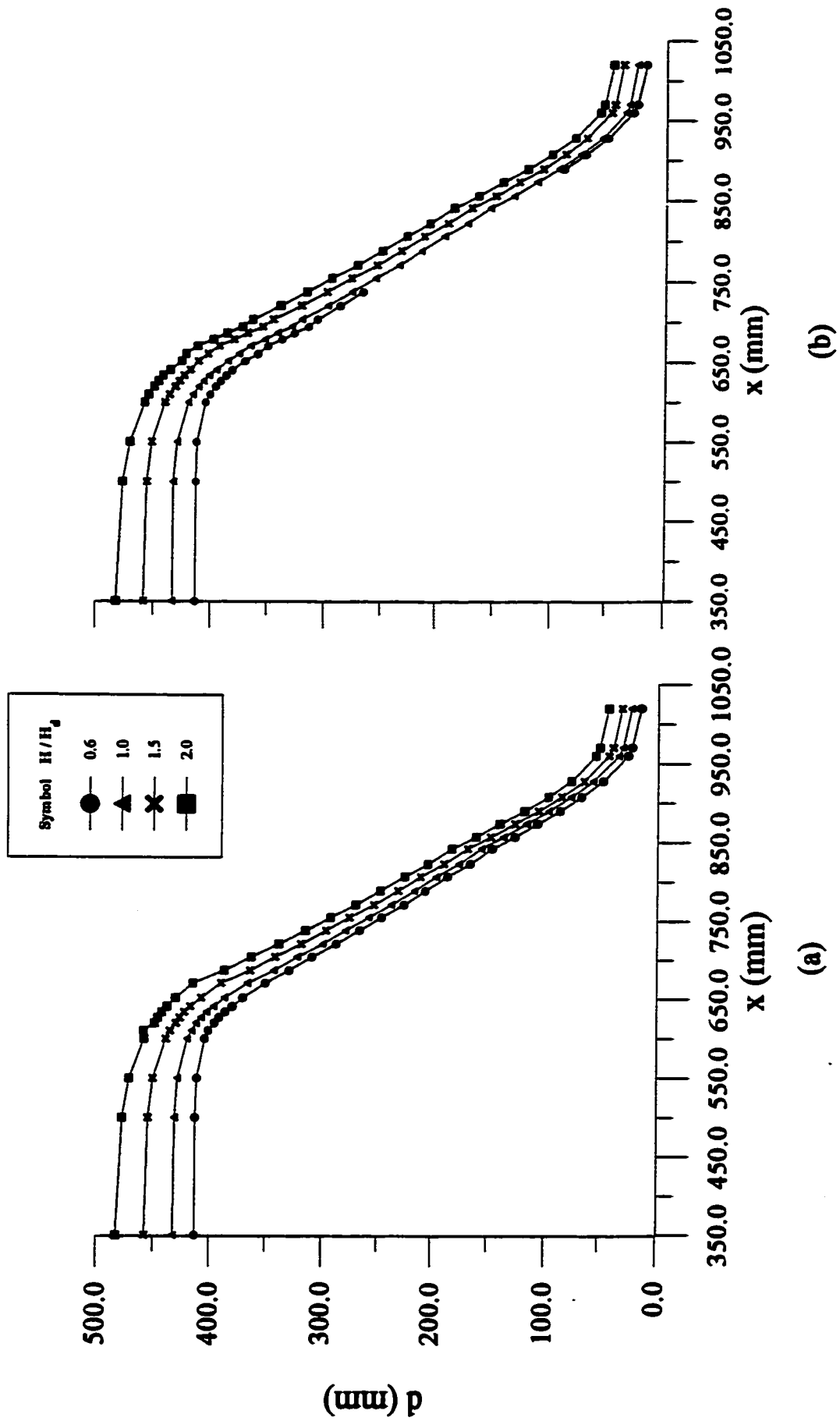


Fig.4.8. Observed water surface profiles for (a) 50-CL-BL model, (b) 50-CS-BL model

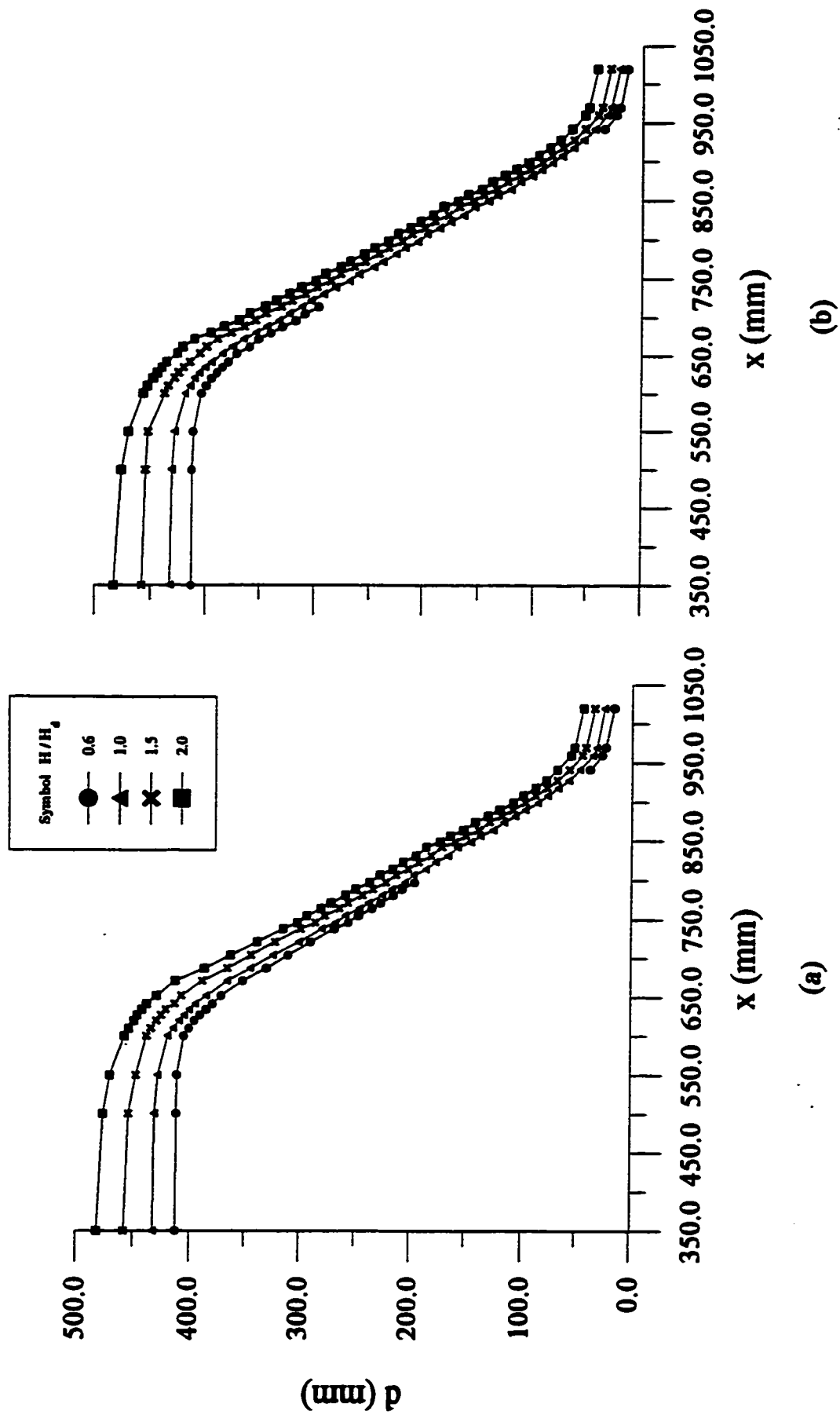


Fig.4.9. Observed water surface profiles for (a) 50-CL-BS model, (b) 50-CS-BS model

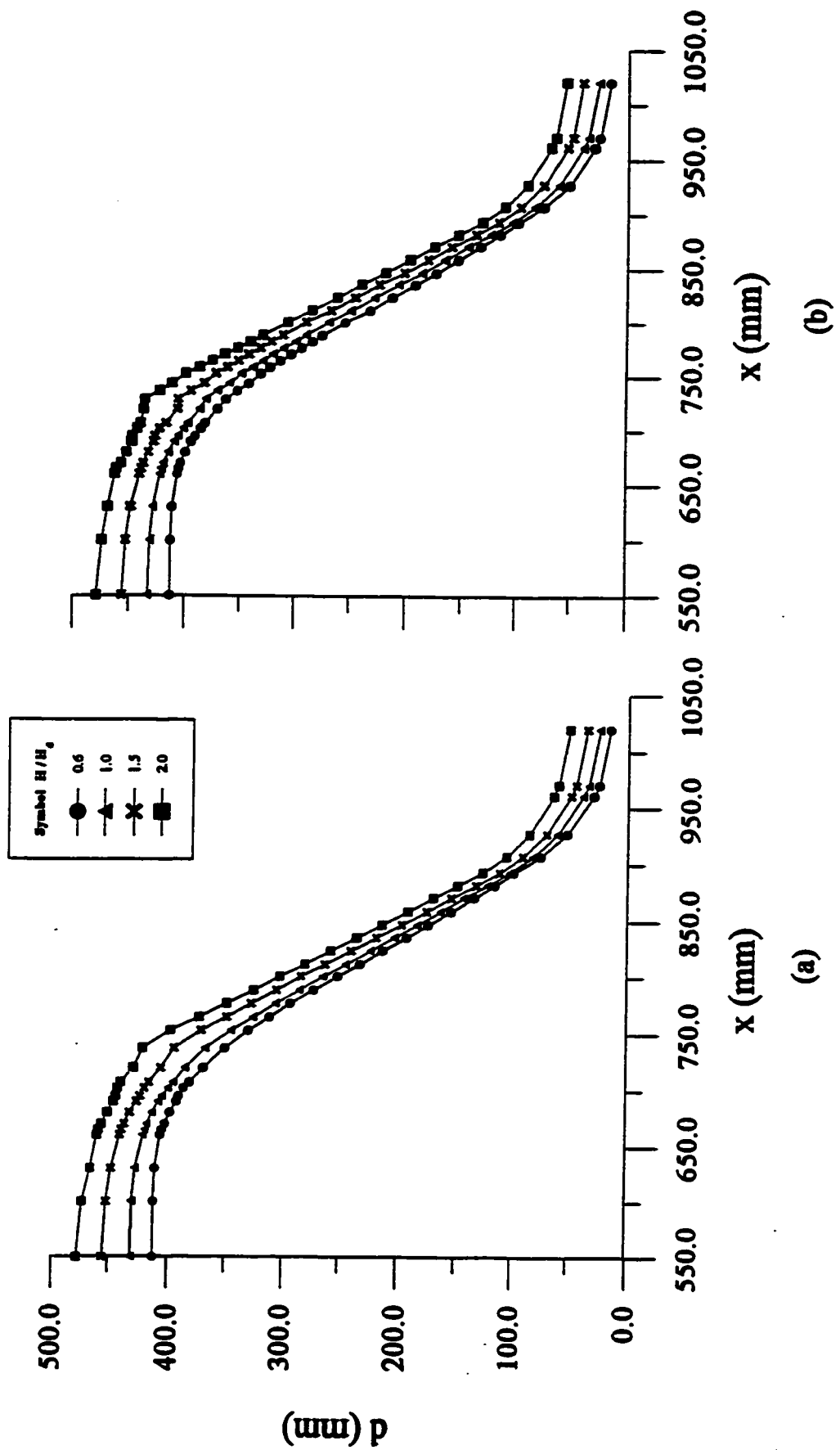


Fig.4.10. Observed water surface profiles for (a) 60-CL-BL model, (b) 60-CS-BL model

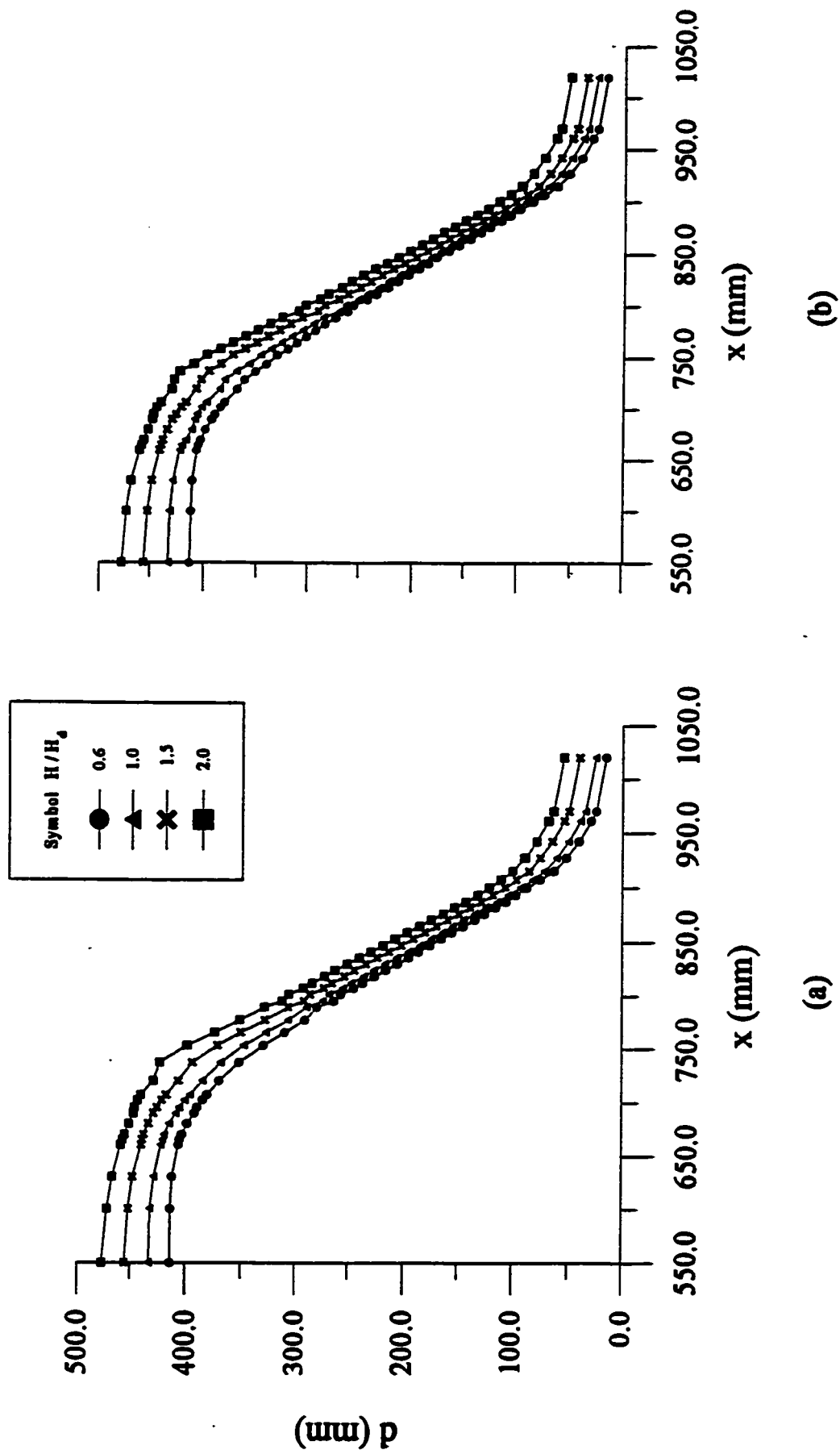


Fig.4.11. Observed water surface profiles for (a) 60-CL-BS model, (b) 60-CS-BS model

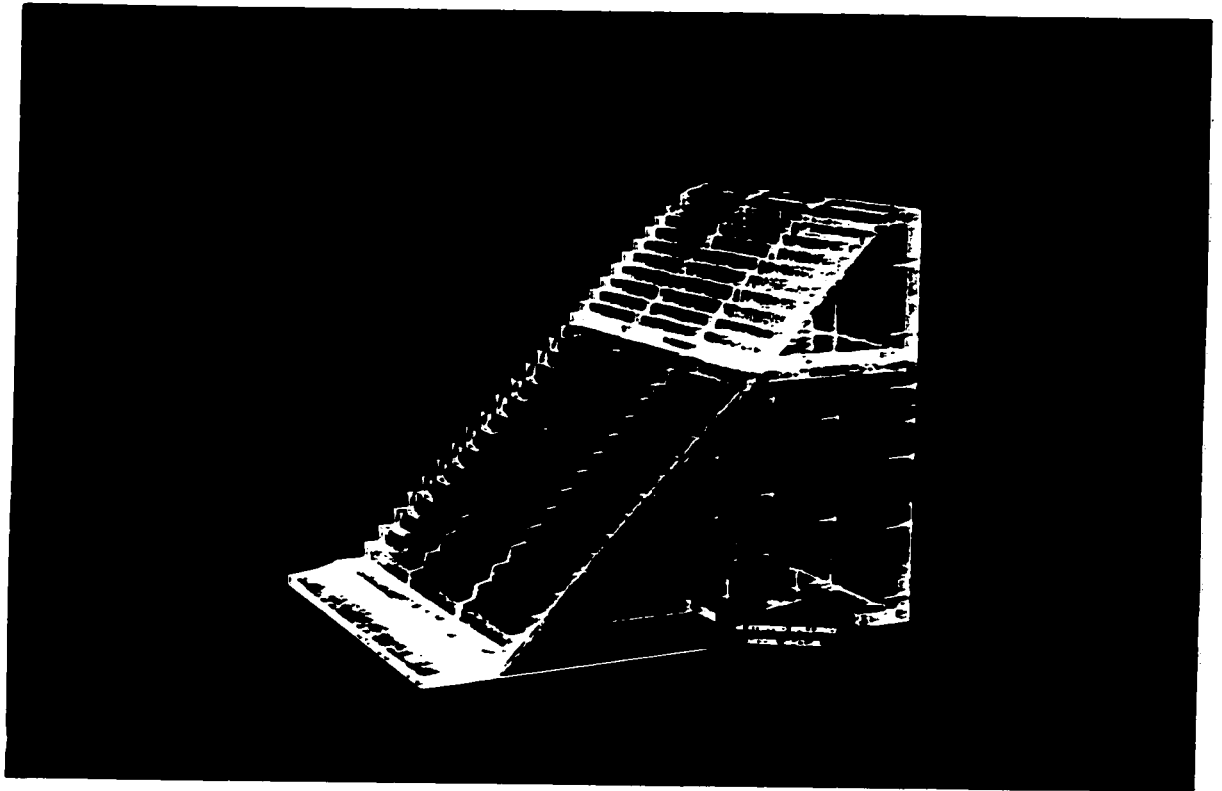


Plate 4.1. Model 45-CL-BL

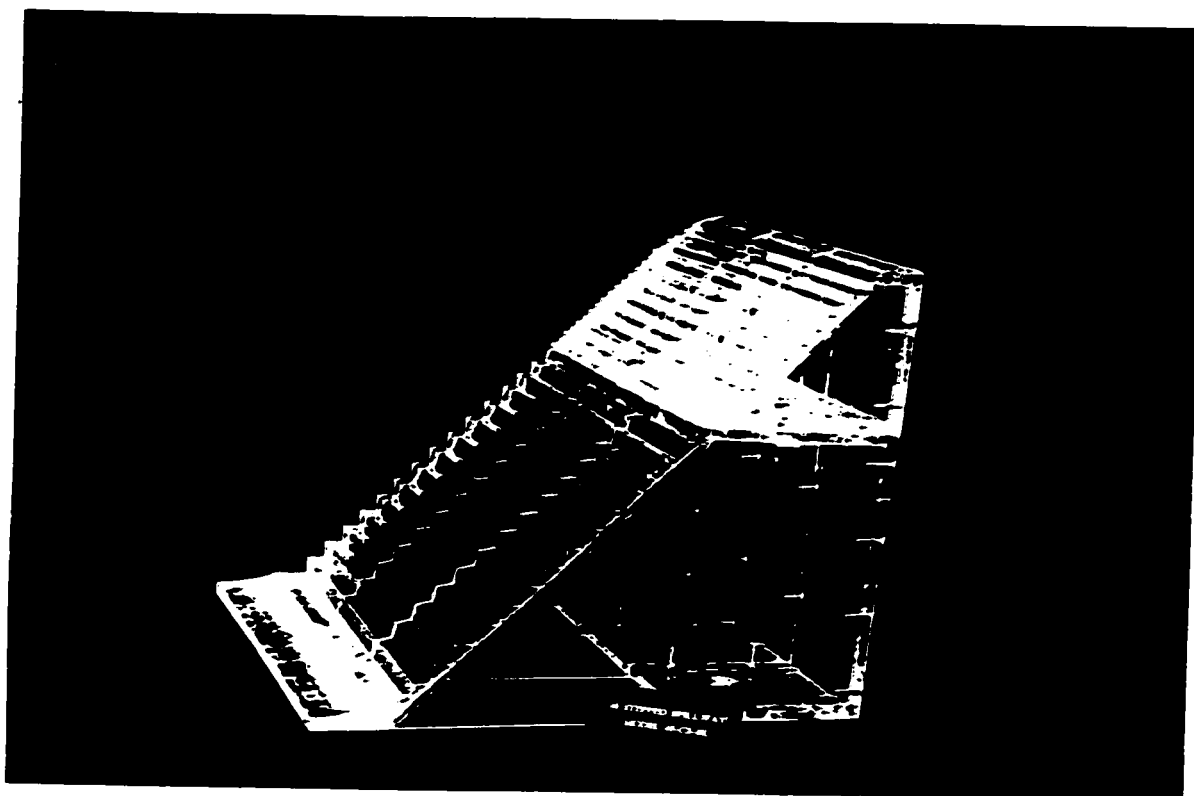


Plate 4.2. Model 45-CS-BL

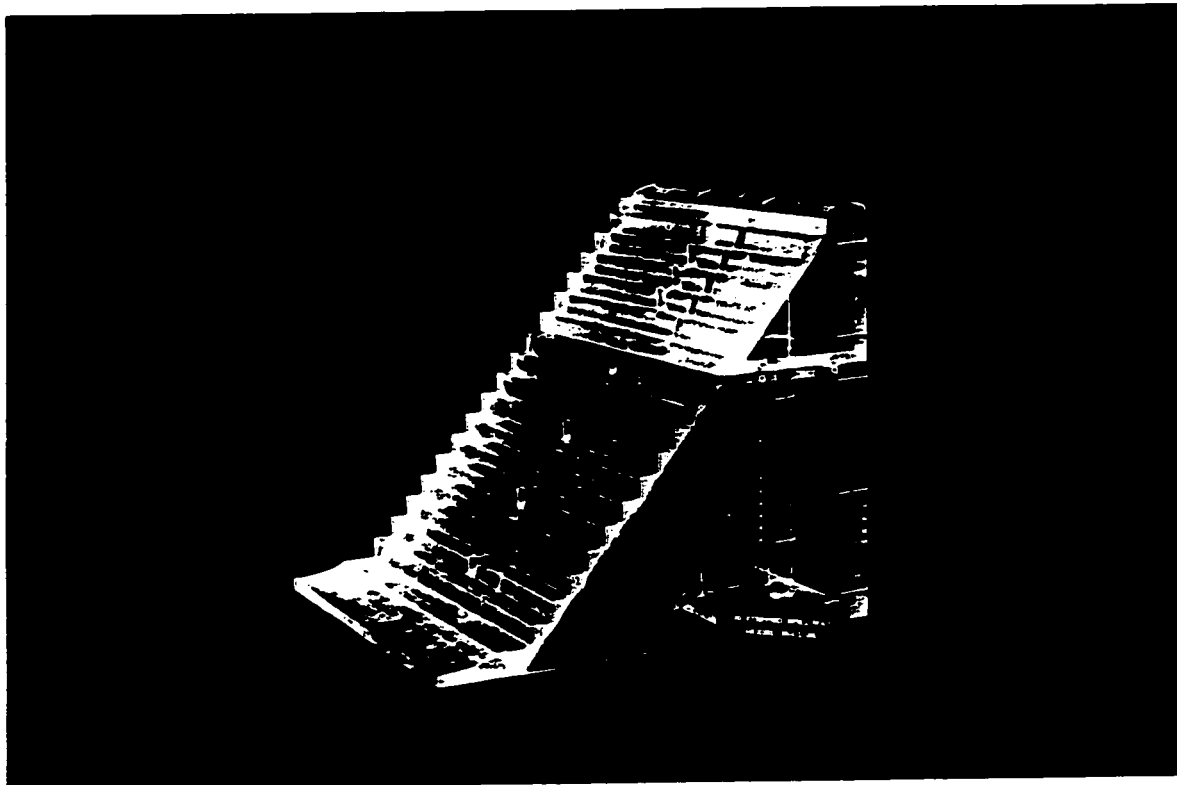


Plate 4.3. Model 50-CL-BL

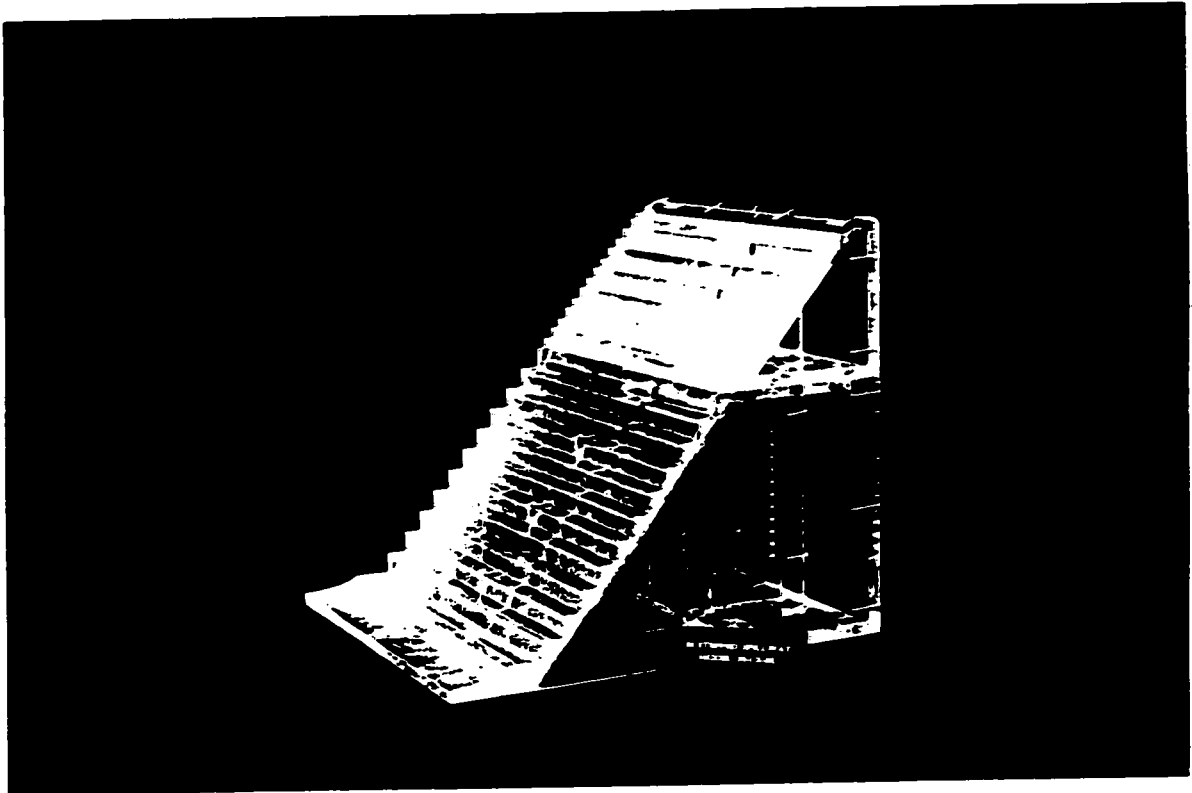


Plate 4.4. Model 50-CS-BL

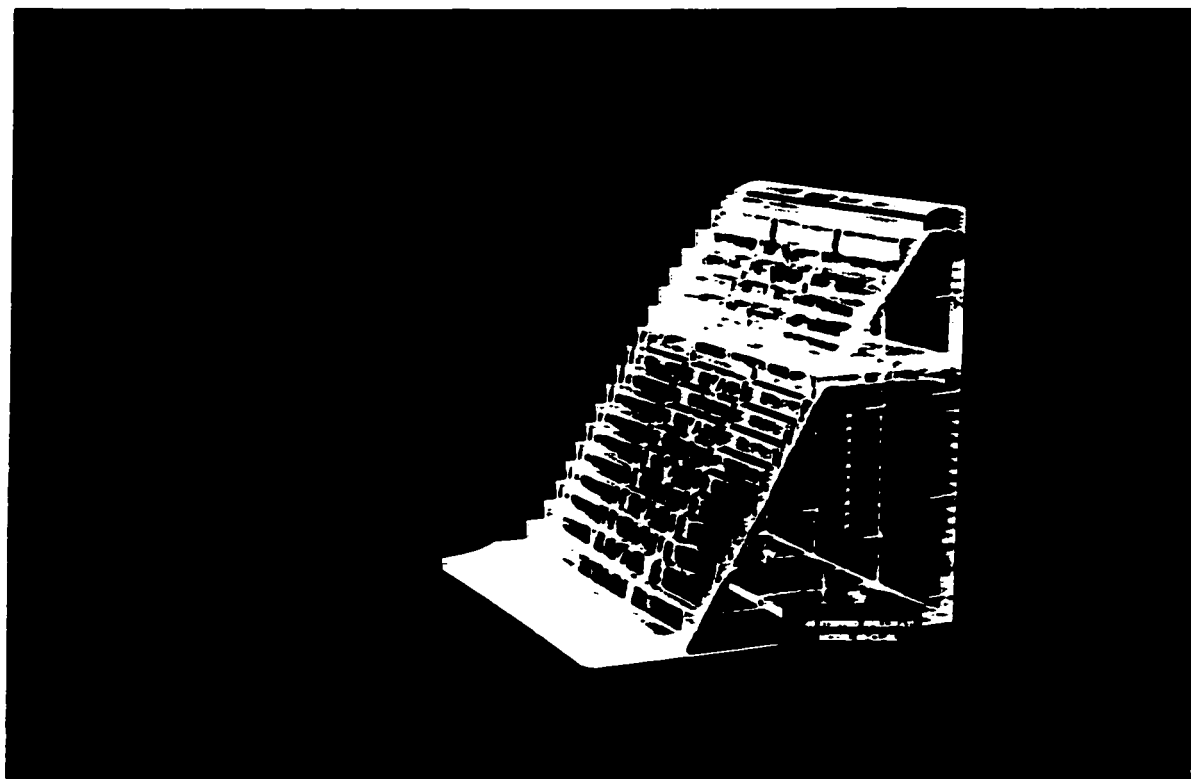


Plate 4.5. Model 60-CL-BL

Chapter 5

Hydraulics of Stepped Spillway

5.1. Drop Structure

Drop structures or free overfalls are usually used in irrigation canal systems, flood conveyance channels, and degrading streams. These structures in the bed level are provided at suitable places to economize the cost of the earthfill. They may be vertical, or inclined, Fig.5.1. Rectangular vertical drop structures are probably as frequently constructed in practice as the inclined ones.

Moore (1943) studied two physical models of drop structures having 0.15 m, and 0.45 m height. His study is very useful from a practical viewpoint. He found that the energy loss at the base of the fall is a function of the relative fall (h/y_c). He also noted that the depth of water standing behind the fall is greater than the depth of water shooting from the base of the fall. Using the momentum equation he found that the pool depth, y_p , can be predicted by :

$$\left(\frac{y_p}{y_c}\right)^2 = \left(\frac{y_1}{y_c}\right)^2 + 2\left(\frac{y_c}{y_1}\right) - 3 \quad (5.1)$$

White (1943), in the discussion of Moore's paper, developed a method to predict the energy loss at the base of the drop. He used the following assumptions:

1. the recirculating flow in the pool at the bottom of the drop was the same as the backwater flow;
2. the velocity of the supercritical stream immediately downstream of the drop was the same as the uniform velocity in a thicker stream at the side of the pool;
3. the angle of the falling jet was not affected by the presence of the pool;
4. the energy loss at the drop was due to the mixing with the pool; and
5. the effect of pool in the calculating the velocity of jet, V , was neglected.

These assumptions were questioned by many researchers, including Moore, in particular the velocity is not uniform in the thicker stream (2nd assumption), and the pool would effect the flow. White (1943) found

$$\frac{E_1}{y_c} = \frac{\sqrt{2}}{1.06 + \sqrt{\frac{h}{y_c} + 1.5}} + \frac{\left(1.06 + \sqrt{\frac{h}{y_c} + 1.5}\right)^2}{4} \quad (5.2)$$

Rand (1955) used his own sets of experimental data and Moore's data to develop the following correlations:

$$\frac{y_1}{h} = 0.54 \left(\frac{y_c}{h} \right)^{1.275} \quad (5.3)$$

$$\frac{y_2}{h} = 1.66 \left(\frac{y_c}{h} \right)^{0.81} \quad (5.4)$$

$$\frac{y_p}{h} = \left(\frac{y_c}{h} \right)^{0.66} \quad (5.5)$$

$$\frac{L_d}{h} = 4.30 \left(\frac{y_c}{h} \right)^{0.81} \quad (5.6)$$

where

L_d = distance from the wall to the position y_1 .

Gill (1979) modified White (1943) assumptions. He assumed that:

1. the uniform velocity occurs in the thinner layer below the pool;
2. the velocity is not constant in the mixing zone;
3. the effect of the pool in the calculation of velocity is considered as

$$V = \sqrt{2g(h + 1.5y_c - y_p)} \quad (5.7)$$

4. the effect of the pool in regards to energy loss is neglected.

Gill's assumptions modified some of the weakness of White's assumption. But assuming the pool would not effect the energy loss was incorrect. Gill's relationships are:

$$\frac{y_1}{h} = 0.524 \frac{y_c}{h} \left(\frac{y_c}{h} - 0.0053 \right)^{0.283} \quad (5.8)$$

$$\frac{y_p}{h} = 1.067 \left(\frac{y_c}{h} - 0.0016 \right)^{0.697} \quad (5.9)$$

$$\cos \beta = 0.968 \left(\frac{y_c}{h} - 0.0058 \right)^{0.305} \quad (5.10)$$

where

β = the angle of the jet where it hits the bed.

El-Khashab, et al. (1987) developed theoretical model for a trapezoidal weir drop structure, which was operating in a trapezoidal channel. A good agreement was reported between the theoretical and observed data.

Chanson (1994), using the trajectory equations of the nappe centreline, developed the following expressions:

$$\frac{y_1}{h} = 0.625 \left(\frac{y_c}{h} \right)^{1.326} \quad (5.11)$$

$$\frac{y_p}{h} = 0.998 \left(\frac{y_c}{h} \right)^{0.675} \quad (5.12)$$

$$L_d = 2.171 \left(\frac{y_c}{h} \right)^{0.525} \quad (5.13)$$

$$\tan \beta = 0.855 \left(\frac{y_c}{h} \right)^{-0.582} \quad (5.14)$$

Rajaratnam and Chamani (1995) examined experimentally the energy loss at the base of a drop structure. They developed the following relationships:

$$\frac{\Delta E}{E_0} = 0.896 \left(\frac{y_c}{h} \right)^{-0.766} \quad (5.15)$$

$$\frac{y_p}{h} = 1.107 \left(\frac{y_c}{h} \right)^{0.719} \quad (5.16)$$

where

$$\Delta E = E_0 - E_1 \quad (5.17)$$

The energy upstream of the drop E_0 can be written as:

$$E_0 = h + 1.5y_c \quad (5.18)$$

and

$$E_1 = y_1 + \frac{V_1^2}{2g} \quad (5.19)$$

Fig.5.1.a shows a vertical rectangular drop structure. The following assumptions apply to Fig.5.1.a and the analysis performed in the present study:

1. the shear force at the bed is negligible; and
2. the angle of the jet where it impacts the pool is the same when it impacts the bed.

Applying the momentum principle to sections 1-1 and a-a , gives:

$$F_1 - F_a = M_a - M_1 \quad (5.20)$$

where

$$F_1 = \frac{1}{2} \gamma y_1^2 \quad ; \quad F_a = \frac{1}{2} \gamma y_p^2 \quad ; \quad M_1 = \rho q V_1 \quad ; \quad M_a = \rho q V \cos \beta$$

where

γ = specific weight of the fluid, and

ρ = mass density of the water.

Therefore the eq.(5.20) can be written as:

$$\rho q V \cos \beta + \frac{1}{2} \gamma y_p^2 = \rho q V_1 + \frac{1}{2} \gamma y_1^2 \quad (5.21)$$

The critical depth, y_c , occurs some distance upstream of the brink. Applying the momentum equation for sections c-c and a-a gives:

$$\frac{1}{2} \gamma y_c^2 = \rho q V \cos \beta - \rho q V_c \quad (5.22)$$

Assuming no energy loss between sections c-c and a-a, then

$$h + \frac{V_c^2}{2g} = y_p + \frac{V^2}{2g} \quad (5.23)$$

Also

$$V_c = \sqrt{g y_c} \quad (5.24)$$

where

V_c = critical velocity.

Therefore eq. (5.23) can be written as:

$$h + 1.5 y_c = y_p + \frac{V^2}{2g} \quad (5.25)$$

Based on the data obtained by Moore (1943), Rand (1955), and Rajaratnam and Chamani (1995), one can write:

$$\frac{y_1}{h} = 0.505 \left(\frac{y_c}{h} \right)^{1.222} \quad (5.26)$$

Therefore, solving eqs. (5.21), (5.22), (5.25), and (5.26) gives:

$$\frac{y_p}{h} = 1.026 \left(\frac{y_c}{h} - 0.03 \right)^{0.598} \quad (5.27)$$

$$\cos \beta = 0.99 \left(\frac{y_c}{h} \right)^{0.466} \quad (5.28)$$

and

$$\frac{\Delta E}{E_0} = 0.077 \left(\frac{y_c}{h} \right)^{-0.813} \quad (5.29)$$

Figs.5.2, 5.3, 5.4, and 5.5 compare the above equations with the experimental data obtained in this study. Fig.5.3 shows that with increasing discharge the depth of pool formed on the drop structure would increase. When $y_c = h$, the depth of pool would be $y_p = h$, which means the whole space is filled.

The coefficient of determination (r^2), the *statistical* F-Observed for eqs. (5.26), (5.27), (5.28), and (5.29), and F-Critical considering a 1 % *level of significance* and the corresponding *degrees of freedom* have are presented in Table 5.1. Since for the above equations F-Observed $\gg$ F-Critical, it can be concluded that good agreement was achieved between the semi-empirical proposed equation and Moore's (1943), Rand's (1955), and Rajaratnam and Chamani's (1995) data. Figs. 5.6, 5.7, 5.8, and 5.9 show the observed parameters versus the predicted parameters.

Fig.5.4 presents the y_c/h vs $\cos \beta$. It can be concluded that the angle of the jet where it hits the bed is decreased when the discharge is increased. The value predicted by eq.(5.28) is fairly close to the experimental data observed. Fig.5.5 presents the energy loss at the base of a drop structure. It can be seen with increasing discharge the energy loss will decrease. When $y_c = h$, the energy loss rate would be the minimum (0.1). Applying eq. (5.29) shows a reliable agreement

with the available experimental data.

In general, in this research some of the weakest assumptions for the theory of hydraulics of rectangular vertical drop structures are replaced with better assumptions. The resulting semi-empirical equations found show very reliable agreement with the available experimental data.

5.2. Flow over a Stepped Spillway

The flow over a *stepped* spillway can be divided into *nappe* flow and *skimming* flow regimes (Fig.5.10). Peyras, et al. (1992) described two types of *nappe* flow:

- (i) *nappe* flow with a fully-developed hydraulic jump for low discharge and small flow depth, and
- (ii) *nappe* flow with partially-developed hydraulic jump.

In the *nappe* flow regime, the flow from each *step* hits the *step* below as a falling jet, with the energy dissipation occurring by jet break-up in the air, and jet mixing on the *step*, with or without the formation of a partial hydraulic jump on the *step*. In the *skimming* flow regime, the water flows down the *stepped* face as a coherent stream, skimming over the *steps* and is cushioned by the recirculating fluid trapped between them.

5.2.1. Nappe Flow

Consider a *nappe* flow with the unit discharge q , Fig.5.11. When the flow hits the *step* below, part of the flow is directed upstream (against the flow direction) and part of the flow continues downstream. When the upstream-directed flow meets the vertical *step* wall, it is forced to reverse direction and a pool is formed. This flow is recirculated in the pool and joins the downstream-directed flow with a lesser

velocity.

Based on the experimental data of Essery and Horner (1978) and Peyras et al. (1991) for gabion *stepped* spillways, the onset flow condition for chute *stepped* spillways may be estimated as:

$$\left(\frac{y_c}{h}\right)_{\text{onset}} = 1.01 - 0.37 \frac{h}{l} \quad (5.30)$$

Skimming flow will occur when $y_c/h > (y_c/h)_{\text{onset}}$.

Rajaratnam (1990) suggested that nappe flow occurs when $y_c/h < 0.8$. Based on his experiences with South African dams, Stephenson (1991) suggested the following conditions for nappe flow:

$$\tan\Theta = \frac{h}{l} < 0.2 \quad (5.31)$$

and

$$\frac{y_c}{h} < \frac{1}{3} \quad (5.32)$$

A recent re-analysis (Chanson, 1994) showed that skimming flow regime on a chute spillway occurs for discharges larger than a critical value defined as:

$$\left(\frac{y_c}{h}\right)_{\text{onset}} = 1.057 - 0.465 \frac{h}{l} \quad (5.33)$$

This relation was obtained using a linear regression of the data of Essery and Horner (1978), Peyras, et al. (1991), and Beitz and Lawless (1992) with a regression coefficient of 0.79, which indicated a relatively poor correlation. This was mostly due to infiltration through the gabion material of Peyras, et al. (1991).

Theoretically, to have a fully nappe flow, the length of *steps* should be larger than the length of a fully-developed hydraulic jump, L_j , plus the length of drop (L_d), or

$$l > L_d + L_j \quad (5.34)$$

For the value of L_d in eq.(5.34) , eq.(5.13) which was developed by Chanson(1994) can be used. Chaudhry (1993) suggested:

$$L_j = 6 y_2 \quad (5.35)$$

One can find y_2 from the classical hydraulic jump relationship:

$$\frac{y_2}{y_1} = 0.5 \left(\sqrt{1 + 8 Fr_1^2} - 1 \right) \quad (5.36)$$

where $Fr_1 = \text{Froude No.} \left(= \frac{q}{\sqrt{g y_1^3}} \right)$

From eqs.(5.26) and (5.36) we can write:

$$\frac{y_2}{h} = 0.548 \left(\frac{y_c}{h} \right)^{2.508} \quad (5.37)$$

By applying eq.(5.13) and eq.(5.37) in eq.(5.34) the following criteria for the *nappe* flow can be developed:

$$\frac{l}{h} > 5.24 \left(\frac{y_c}{h} \right)^{2.15} \quad (5.38)$$

Fig.5.12 presents eq.(5.38) and the criteria developed for the data of Essery and Horner (1978) and Peyras et al. (1991), Rajaratnam (1990), and Stephenson (1991). It can be seen a *nappe* flow regime for higher discharges is possible, but this high discharge needs very flat slope. Also for steeper slopes, it shows that the discharge must be very low to have a *nappe* flow regime. Because of the implied large

increase in concrete volume in the case of concrete gravity dam these conditions make such a structure unattractive from an economic viewpoint. The *step* height can also be a problem if the concrete thickness is limited. This is because the forces caused by impinging water can crack the concrete and cause internal stresses, which add to the rapid deterioration of the concrete.

Fig.5.13 verifies the eq.(5.38) with the experimental data found in this study. It can be seen that these criteria are highly applicable to the results of measured values. This figure indicates that a nappe flow is also possible for very high downstream slopes, but it needs very low discharges. At these discharges most of energy can be dissipated by using the *stepped* spillway.

5.2.2. Skimming Flow

Skimming flow occurs at moderate to high discharges. No nappe is visible and the spillway is submerged beneath a strong, relatively smooth current. In the *skimming* flow regime, the external edges of the *steps* form a pseudo-bottom over which the flows pass. Beneath this, horizontal-axis vortices develop, filling the zone between the main flow and the *step*. These recirculating vortices are maintained through the transmission of shear stress from the fluid flowing past the edges of the *steps*. *Skimming* flows are characterized by large friction losses and the air-entrainment process.

Downstream of the point of inception, a layer containing a mixture of air and water extends through the fluid. Far downstream, the flow will become uniform and for a given discharge any measurement of flow depth, air concentration, and velocity distribution will not vary along the spillway. Chanson (1994) states that, when the *skimming* flow becomes fully-developed, a *stepped* spillway behaves in the same way as an *unstepped* one.

5.3 Boundary Layer Growth

From approximately the crest section of the spillway, a turbulent boundary layer will develop on the concrete surface. A short distance downstream of the point of inception of this boundary layer, where the boundary layer extends to the water surface, the turbulence next to the free surface become large enough to initiate natural free-surface aeration. On a smooth spillway, Keller and Rostagi (1977) suggested that:

$$\frac{L_I}{k'_s} = f_1(F_*, \sin\theta) \quad (5.39)$$

and

$$\frac{y_I}{k'_s} = f_2(F_*, \sin\theta) \quad (5.40)$$

where

L_I = the distance from the start of the growth of the boundary layer,

$$F_* = \frac{q}{\sqrt{g \sin\theta} (k'_s)^3} \quad (5.41)$$

y_I = the depth of flow at the inception point, and

k'_s = roughness of spillway surface

$$k'_s = h \cos\theta \quad (5.42)$$

For a smooth concrete spillway Cain and Wood (1981) found that:

$$\frac{L_I}{k'_s} = 13.6 (\sin\theta)^{0.0796} (F_*^{0.713}) \quad (5.43)$$

$$\frac{y_I}{k'_s} = \frac{0.223}{(\sin\theta)^{0.04}} (F_*^{0.643}) \quad (5.44)$$

Using the data of Bindo (1993), Frizell (1991.), Sorensen (1985), and Tozzi (1994), Chanson (1994) found that, for a *stepped* spillway with a downstream slope $27^\circ < \theta < 53^\circ$:

$$\frac{L_I}{k'_s} = 9.719(\sin\theta)^{0.0796} (F_*^{0.713}) \quad (5.45)$$

and

$$\frac{y_I}{k'_s} = \frac{0.4034}{(\sin\theta)^{0.04}} (F_*^{0.592}) \quad (5.46)$$

Eq.(5.45) and (5.46) are plotted in Figs.5.14 and 5.15. Applying eq.(5.43) for a *stepped* spillway is inaccurate, as it overestimates the location of the inception point. Eq. (5.45) implies that, with increasing discharge the point of inception moves downstream.

5.4. Flow Aeration

Flow aeration on the spillway, also called *self-aeration*, was initially studied because of the effects of entrained air on the thickness of the flow. Air entrainment increases the bulk flow depth, and this is used as a design parameter for the height of spillway sidewalls (Falvey, 1980). Also the presence of air within the boundary layer reduces the shear stress between the flow layers and hence the shear force. Further, the presence of air within high-velocity flows may prevent or reduce the damage caused by cavitation (May, 1987). Although the flow conditions between smooth and *stepped* spillways are different , it is assumed that the

mechanisms of air entrainment are similar. Once the flow becomes fully-developed then a *stepped* spillway can act as a smooth spillway with high roughness.

In nappe flows, air entrainment occurs near the impact point of the falling jet with the horizontal *step* and the hydraulic jump. On the other hand large de-aeration also occurs downstream of the falling jet and downstream of jump. Therefore, the net aeration is small and it is believed that the effect of air entrainment on nappe flows can be neglected.

In skimming flows, the air entrainment increases the flow depth after the point of inception. When the flow becomes uniform the flow depth stays constant. It has been observed that at low discharges the air entrainment is higher than high discharges. In general to find the energy dissipation, if one applies the bulking effect the result is highly overestimated. The local air concentration, C_a , is defined as the volume of air per unit volume, which can be expressed as:

$$C_a = \frac{\text{Air Volume}}{\text{Air Volume} + \text{Water Volume}} \quad (5.47)$$

It is generally accepted that air concentration in the flow on a wide chute (width-depth ratio greater than 5) spillway can be estimated by U.S. Army Corps. of Engg. formula (1977) in fps units as follows:

$$C_a = 0.701 \log_{10} \left(\frac{\sin \theta}{q^{1/5}} \right) + 0.971 \quad (5.48)$$

The results of Knauss (1979) indicate a comparable rate of air entrainment for both smooth and rough flows. His relationship takes the form:

$$C_a = 1.44 \sin \theta - 0.08 \quad (5.49)$$

Chanson (1993) states that the uniform air concentration on a *stepped* spillway can

be expected to be similar to that for a smooth spillway, where the air concentration is a function of slope only. Senturk (1994) suggested the following empirical equation for the air entrainment on an ogee-smooth spillway:

$$C_a = 10 \sqrt{F_r - 1} \quad (5.50)$$

5.5. Flow Resistance

In any real fluid flow, energy is continuously being dissipated. This occurs because the fluid has to do work against resistance originating in the fluid's viscosity. Whether the flow is laminar or turbulent the basic resistance mechanism is the shear stress by which a slow-moving layer of fluid exerts a retarding force on an adjacent layer of faster moving fluid. Between the adjacent layers moving with different velocity there will be a retarding force exerted on the faster-moving layer, because it is continuously receiving low-velocity fluid and losing high-velocity fluid. This retardation will be in the form of a shear force, since it acts parallel to the interface between the two layers. It is well established by observation that fluid actually in contact with a solid surface has no motion along the surface. The created transverse velocity gradient enables the solid surface to exert on the moving fluid a drag force, which is transmitted outwards through successive faster-moving fluid layers. Flow resistance therefore depends on: *(i)* the presence of solid surfaces; and *(ii)* the strength of the viscosity or turbulence.

Studies of skimming flow over large elements in open channels and pipes indicate that the classical flow resistance must be modified to take into account the shape of the roughness elements. Indeed, the flow resistance is the sum of the *skin* resistance and the *form* resistance of the *steps*. Neglecting the effect of the flow aeration, Chanson (1994) showed that the friction factor was a function of the *step* roughness height and channel slope.

$$f = f \left(\frac{k'_s}{D_H}; \theta \right) \quad (5.51)$$

where

f = Darcy-Weisbach friction factor; and

D_H = hydraulic depth, $\left(= 4 \frac{A}{P} \right)$; where P = wetted perimeter.

Noori (1984) experimentally studied the form drag resistance of two-dimensional *flat-stepped* channels. He expressed the drag coefficient as:

$$C_D = \frac{2.02}{\left(\frac{y_n}{h} \right)^{0.067} F_r^{1.96}} \quad (5.52)$$

Based on Grinchuk et al.'s (1977) data and Noori's (1984) data for the flat slope ($\theta < 12^\circ$) and $0.02 < k'_s/D_H < 0.3$ the following equation is obtained:

$$\frac{1}{\sqrt{f}} = 1.42 L_n \left(\frac{D_H}{k'_s} \right) - 1.25 \quad (5.53)$$

Fig.5.16 shows good agreement between eq.(5.53) and the data obtained by Grinchuk et al. (1977) and Noori (1984).

Fig.5.17 shows the results obtained from several authors for steep slopes ($\theta > 27^\circ$). A comparison between Figs.5.16 and 5.17 shows substantial increase of the friction factor. For flat channels and $0.02 < k'_s/D_H < 0.3$ the space of recirculating fluid beneath the pseudo-bottom formed by the *step* edges is relatively thin. The recirculating vortices do not fill the entire cavity between the edges of adjacent *steps*. Therefore stable recirculating eddies cannot develop. For steep slopes, the

space between the edges of the *steps* was filled completely by the recirculating vortices. The energy dissipation and the flow resistance are functions of the energy required to maintain the circulation of vortices. Chanson (1994) indicated that the stable recirculation occurs for slopes larger than 27° on a *stepped* chute.

Vittal et al. (1977) performed experiments in open channel with two dimensional triangular roughness elements. For a roughness height to length ratio of 1/5, their results satisfy:

$$f = 1.44 \frac{k'_s}{L_s} \left(\frac{y_n}{k'_s} + 0.5 \right)^{-3/8} \quad (5.54)$$

where

L_s = the roughness spacing.

Rajaratnam and Katopodis (1991) studied the hydraulics of *steep*pass fishways. Using theoretical considerations and experimental observations, they developed an expression that relates the flow rate, the slope of the fishways, and the depth of flow. They also obtained friction factors for *steep*pass fishways in the range of 0.4 to 4.

Stephenson (1991) by assuming the *skimming* flow reaches uniform depth found:

$$f = \frac{1}{\left(1.14 + 2 \log_{10} 4 \frac{y_n}{h} \right)^2} \quad (5.55)$$

This equation has a singularity point when $y_n/h = 0.067$. This equation was recommended for $y_n/h > 0.2$.

Gevorkyan and Kalantarova (1992) studied the flow past *stepped teeth* opposed to

the flow with a roughness height to length ratio of 1/4. They reported the following equation:

$$f = 8g \left(\frac{1000}{52 - 5.1 \frac{y_n}{k'_s}} \right)^{-2} \quad (5.56)$$

Chanson (1994) for the experiments of Hartung and Scheuerlein (1970) and Scheuerlein (1973) developed the following empirical equation:

$$\frac{1}{\sqrt{f}} = -3.2 \log_{10} \left((1.7 + 8.1 \sin \theta) \frac{k'_s}{D_H} \right) \quad (5.57)$$

These data were obtained for mountain rivers and rockfill channels. This equation is a very rough estimation of the friction factor. The results obtained shows very high friction factor. Therefore applying this equation estimates very high friction factor.

Tozzi (1994) developed the following expressions:

(i) for 6.69 (H) : 1 (V)

$$\frac{1}{\sqrt{f}} = 3.68 + 0.28 \log_{10} \frac{y_n}{k'_s} \quad (5.58)$$

(ii) for 2 (H) : 1 (V)

$$\frac{1}{\sqrt{f}} = 3.25 + 0.39 \log_{10} \frac{y_n}{k'_s} \quad (5.59)$$

(iii) for 0.75 (H) : 1 (V), when $y_n/k'_s > 1.80$

$$\frac{1}{\sqrt{f}} = 2.16 + 1.24 \log_{10} \frac{y_n}{k'_s} \quad (5.60)$$

otherwise $f = 0.163$.

Fig.5.18 presents these equations. It can be seen that the y_n/k'_s decreases, the friction factor increases. Applying eqs.(5.58),(5.59), and (5.60) obtained by Tozzi (1994) gives very low friction factor.

Qibei and Baker (1995) studied the resistance forces on wedge-shaped blocks used for spillway protection. They assumed that the flow passing over the *steps* is similar to that passing through an abrupt enlargement. For a low-slope downstream surface, they developed the following expression

$$C_c = \sqrt{2 \left(\frac{l}{h} \right) \left(\frac{y_n}{h} \right)} \quad (5.61)$$

where

C_c = the Chezy coefficient.

In the next chapter the friction factor based on the experimental data of this research will be presented.

5.6. Energy Dissipation

Although the mechanisms of energy loss are quite different between the *nappe* flow and *skimming* flow regimes, but both flows can dissipate a major proportion of the flow energy. A *stepped* spillway with *nappe* flow can be considered as a succession of *drop* structures. The energy dissipation occurs by jet breakup in the air, jet impact and jet mixing on the *steps*, with the formation of fully-developed or partially-developed hydraulic jumps on the *steps*. In a *skimming* flow regime,

the *steps* act as large roughness elements. Most of energy is dissipated to maintain stable horizontal vortices beneath the pseudo-bottom formed by the external edges of the *steps*. The vortices are maintained through the transmission of turbulent shear stress between the skimming stream and the recirculating fluid underneath.

5.6.1. Energy Dissipation for Nappe Flow

Chanson (1993) assumed that the head loss at any intermediary *step* equals the *step* height. He expressed the total head loss for fully-developed hydraulic jump as:

$$\frac{\Delta E}{E_0} = 1 - \frac{0.54 \left(\frac{y_c}{h} \right)^{0.275} + 1.72 \left(\frac{y_c}{h} \right)^{-0.55}}{1.5 + \frac{H_{dam}}{y_c}} \quad (5.62)$$

In this equation all assumptions of Rand (1955) were applied. Eq.(5.62) is in good agreement with Moore's (1943) data, but when the number of *steps* increase it varies significantly.

Chamani and Rajaratnam (1994) presented a method to estimate the energy loss on a *stepped* spillway with nappe flow. Analyzing the data obtained by Horner (1969), they introduced the concept of *average relative energy loss per step*. They found the following equation:

$$\frac{\Delta E}{E_0} = 1 - \frac{\left((1-p_1)^{N_s} \left[1 + 1.5 \left(\frac{y_c}{h} \right) \right] + \sum_{i=1}^{N_s-1} -(1-p_1)^i \right)}{N_s + 1.5 \left(\frac{y_c}{h} \right)} \quad (5.63)$$

where

p_1 = the proportion of the energy loss per *step*;

$$p_1 = a - b \log \left(\frac{y_c}{h} \right) \quad (5.64)$$

a, b = coefficients described by the following equations:

$$a = 0.30 - 0.35 \left(\frac{h}{l} \right) \quad (5.65)$$

$$b = 0.54 + 0.27 \left(\frac{h}{l} \right) \quad (5.66)$$

All these equations are valid for $y_c/h < 0.8$. They found that the relative energy loss could be up to 0.97. All these correlations were based on the Horner's (1969) data.

The maximum energy can be stated as:

$$E_{\max} = H_{\text{dam}} + 1.5y_c \quad (5.67)$$

Assuming the vertical distance between the crest and the first *step* as the height of one *step*, then

$$H_{\text{dam}} = (N_s + 1)h \quad (5.68)$$

Energy at the base of a spillway can be expressed as :

$$E_1 = y_1 + \frac{V_1^2}{2g} \quad (5.69)$$

Referring to eq.(5.26), eq.(5.69) can be expressed as:

$$\frac{E_1}{h} = 0.505 \left(\frac{y_c}{h} \right)^{1.222} \left(1 + 3.882 \left(\frac{y_c}{h} \right)^{-0.666} \right) \quad (5.70)$$

Therefore the energy dissipation can be written as:

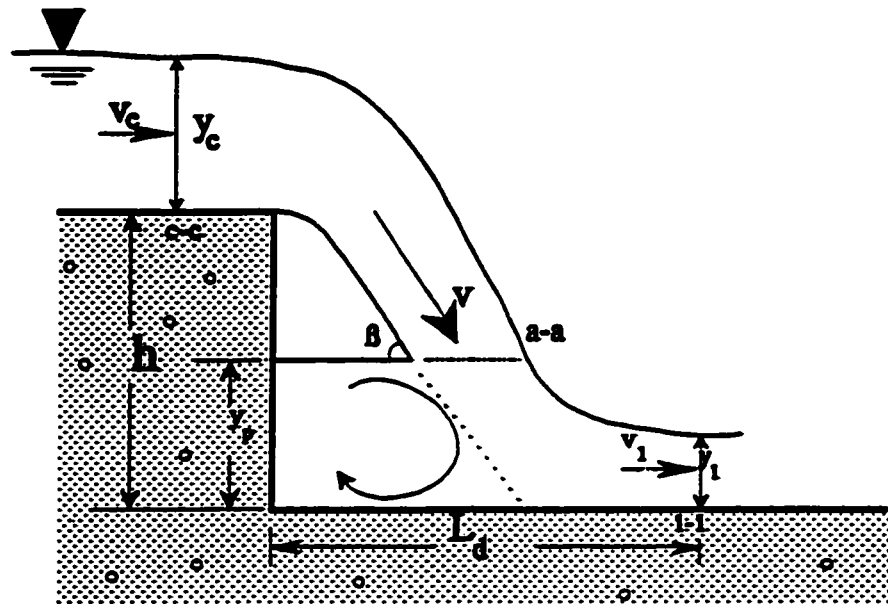
$$\frac{\Delta E}{E_0} = 1 - \frac{0.505 \left(\frac{y_c}{h} \right)^{1.222} \left(1 + 3.882 \left(\frac{y_c}{h} \right)^{-0.666} \right)}{N_s + 1 + 1.5 \left(\frac{y_c}{h} \right)} \quad (5.71)$$

Fig.5.19 the data obtained by Moore(1943) and Essery and Horner (1978) superimposed on eq. (5.71). This equation does not have the limitations applied to the Rand's (1955) equations which was used by Chanson (1993), or the correlation of Horner(1969)'s data which was used by Chamani and Rajaratnam (1994).

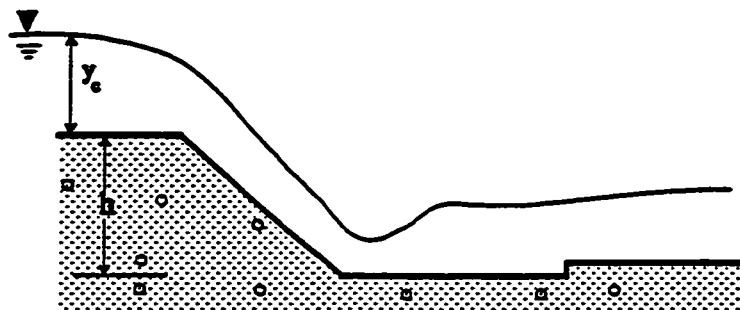
From their experimental data, Peyras et al. (1991) found that the rate of energy dissipation of nappe flows, with partially-developed hydraulic jumps, was within 10% of the values obtained for nappe flows with fully-developed hydraulic jumps for same flow conditions.

Eq. No.	r^2	F-Observed	F-Critical
(5.26)	0.97	226.3	5.85
(5.27)	0.97	226.3	5.85
(5.28)	0.87	45	9.5
(5.29)	0.93	86.33	5.93

Table 5.1. Statistical parameters of the developed equations for a single vertical drop



(a)



(b)

Fig.5.1. Simple drop structures (a) vertical, (b) inclined

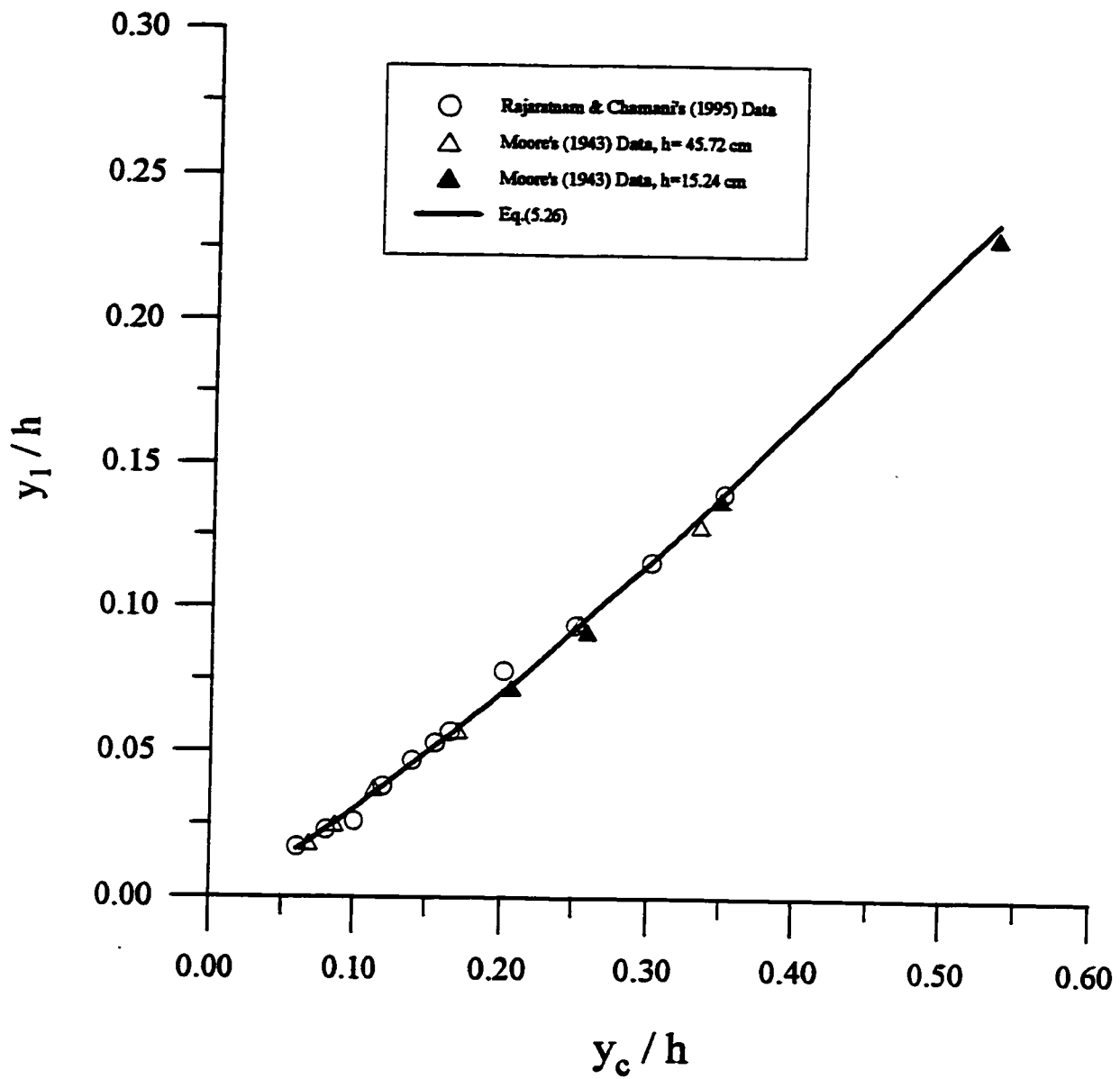


Fig. 5.2 Comparison of eq. (5.26) with experimental data from various sources

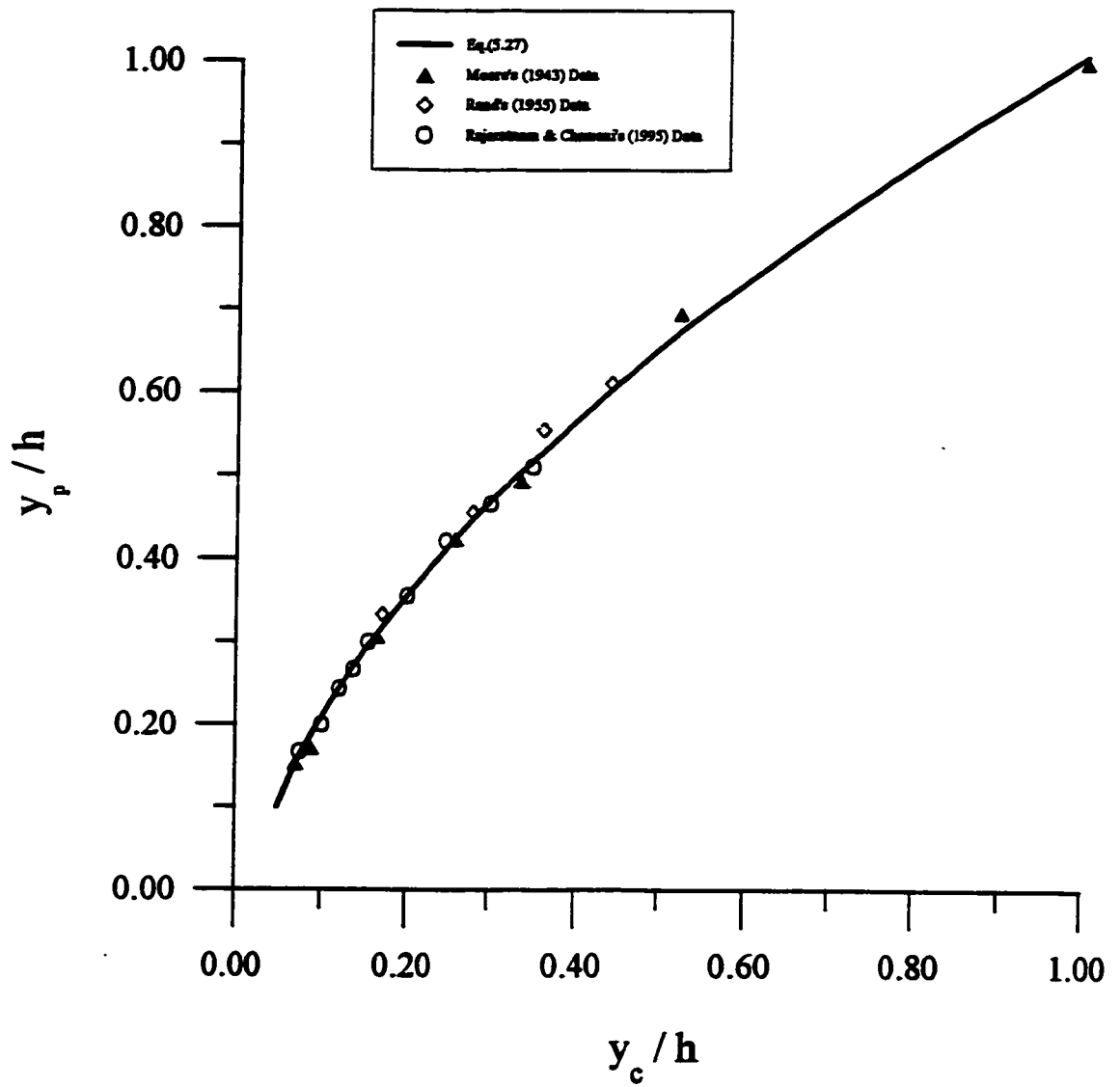


Fig. 5.3. Comparison of eq. (5.27) with experimental data from various sources

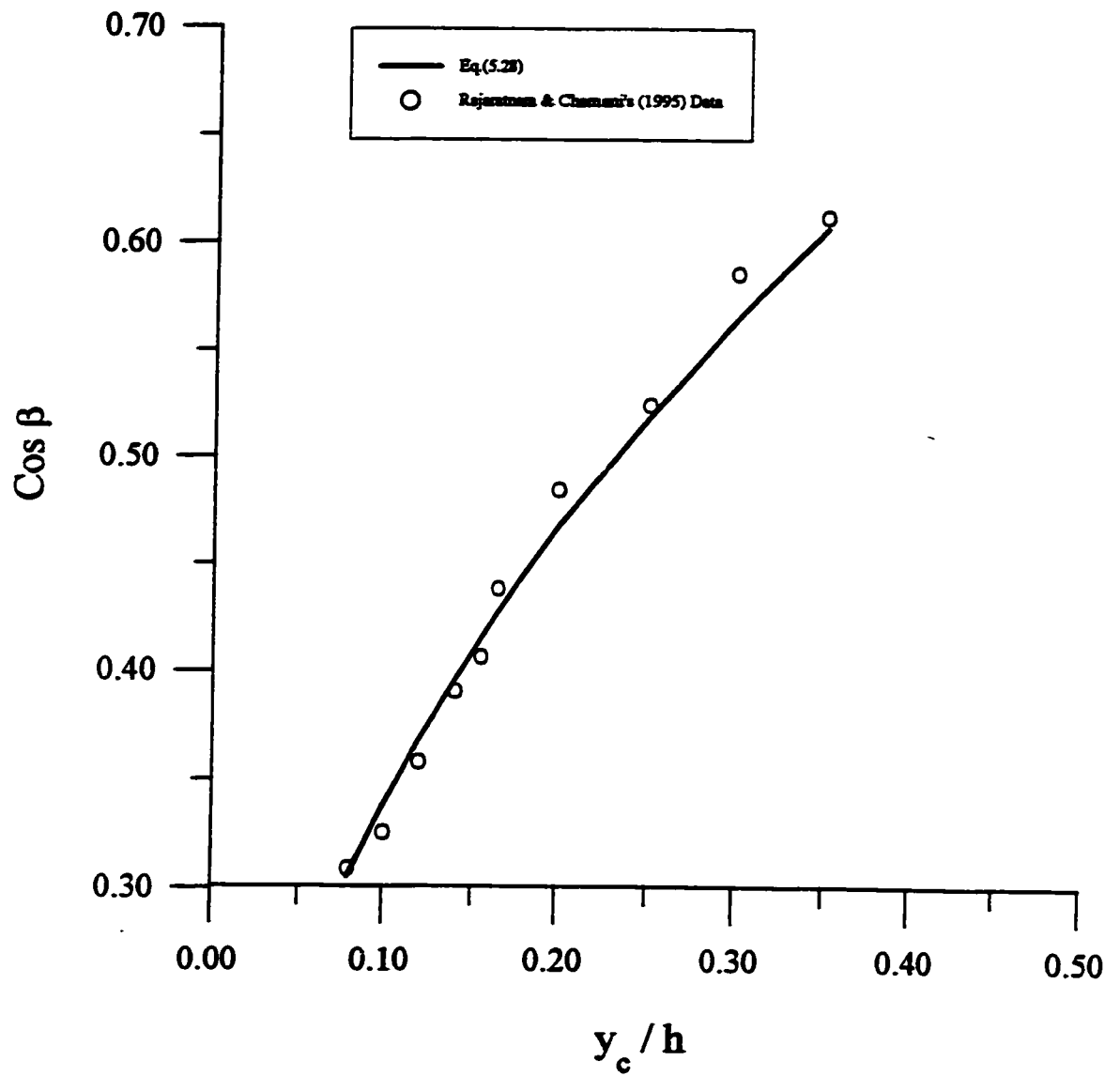


Fig.5.4. Comparison of eq. (5.28) with Rajaratnam and Chamani's (1995) data

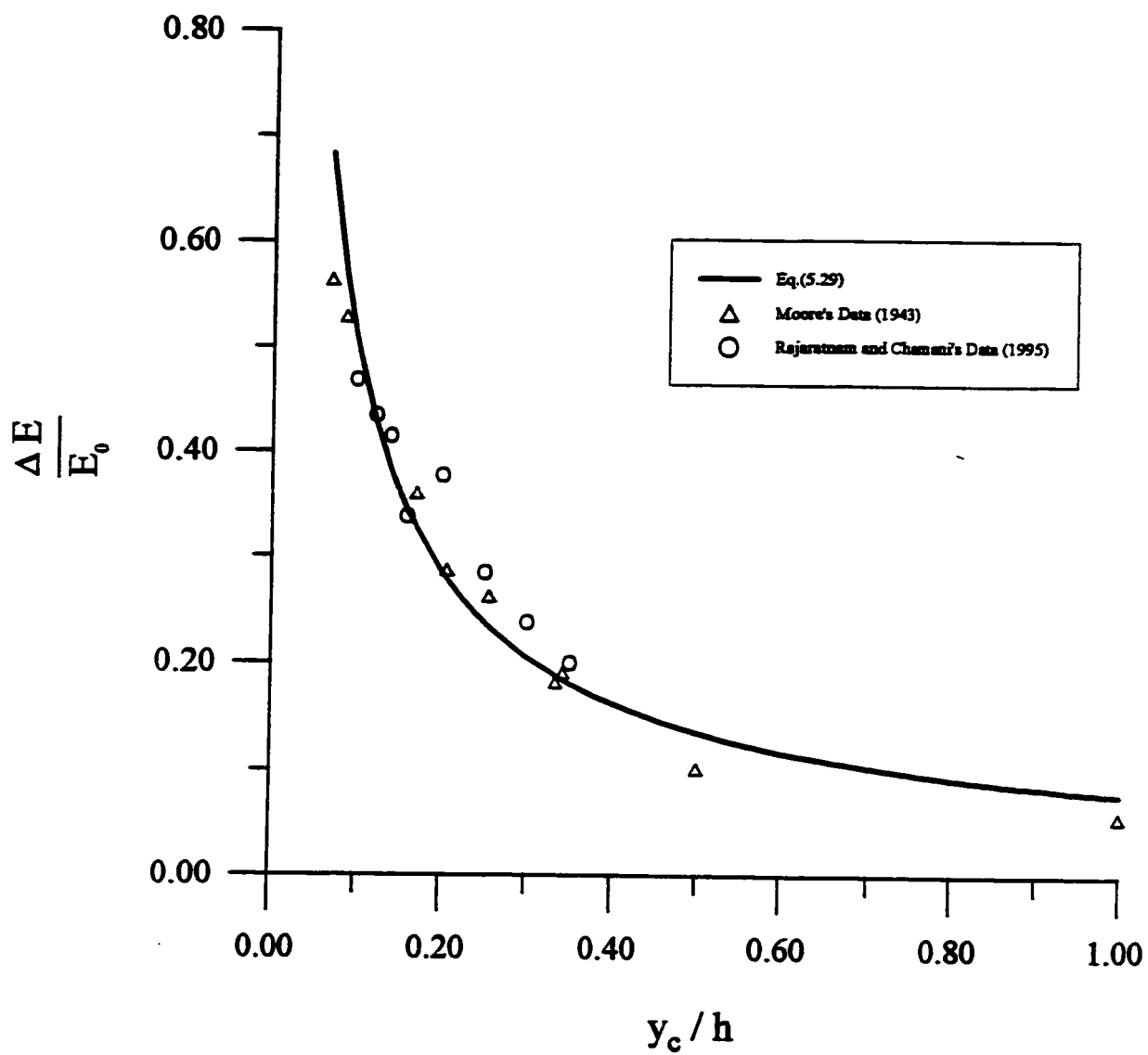


Fig. 5.5. Comparison of eq. (5.29) with experimental data from various sources

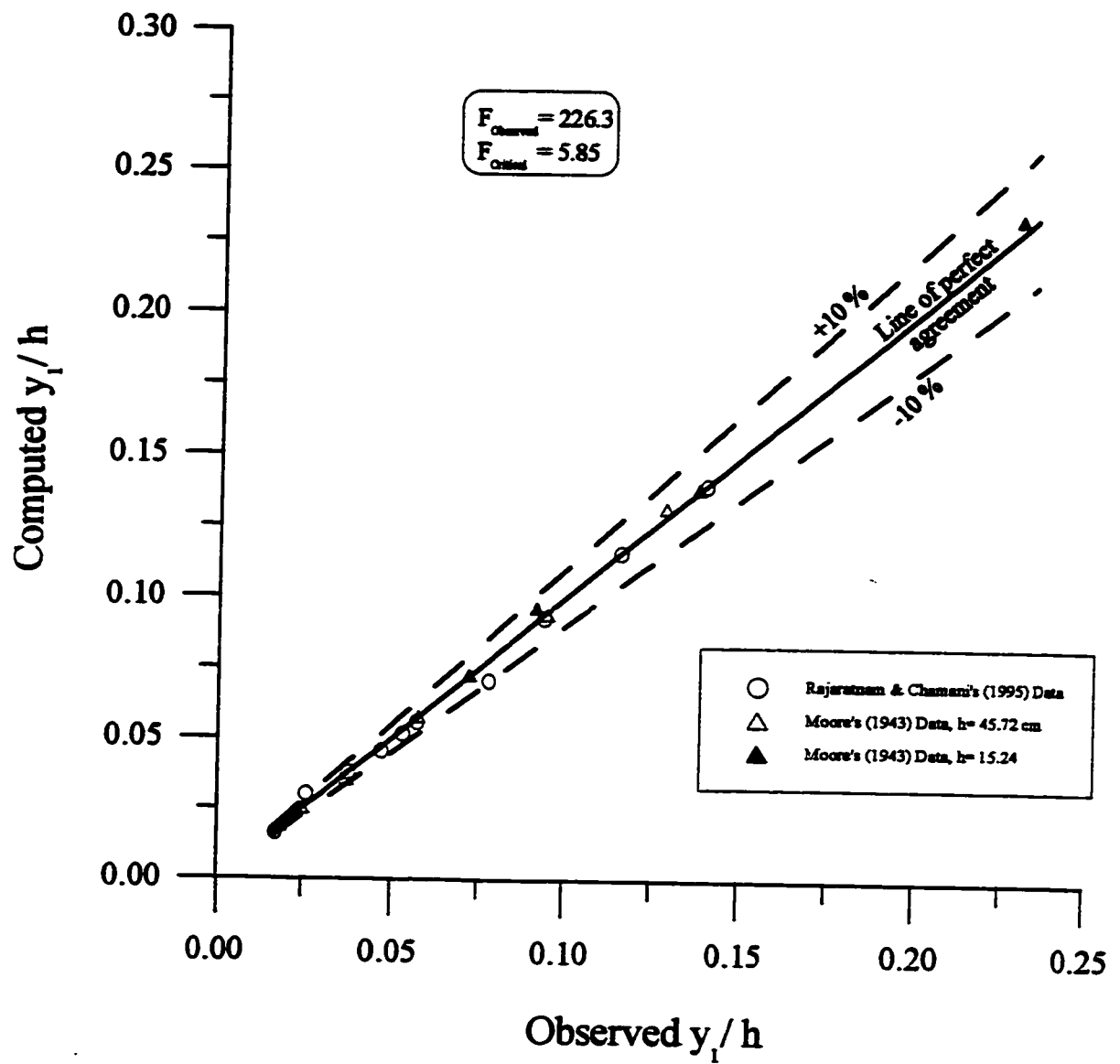


Fig. 5.6. Observed y_i/h versus computed y_i/h

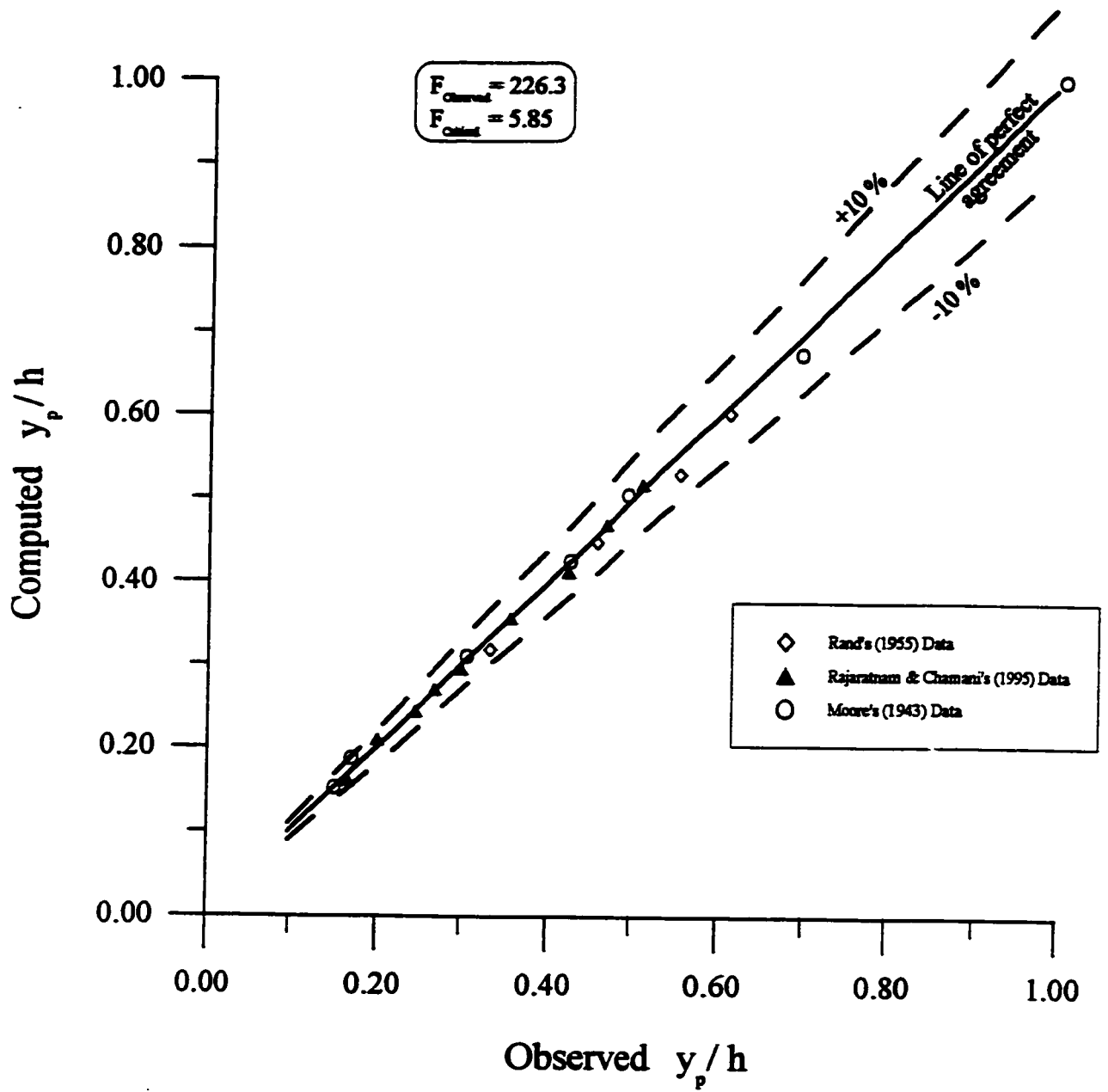


Fig. 5.7. Observed y_p/h versus computed y_p/h

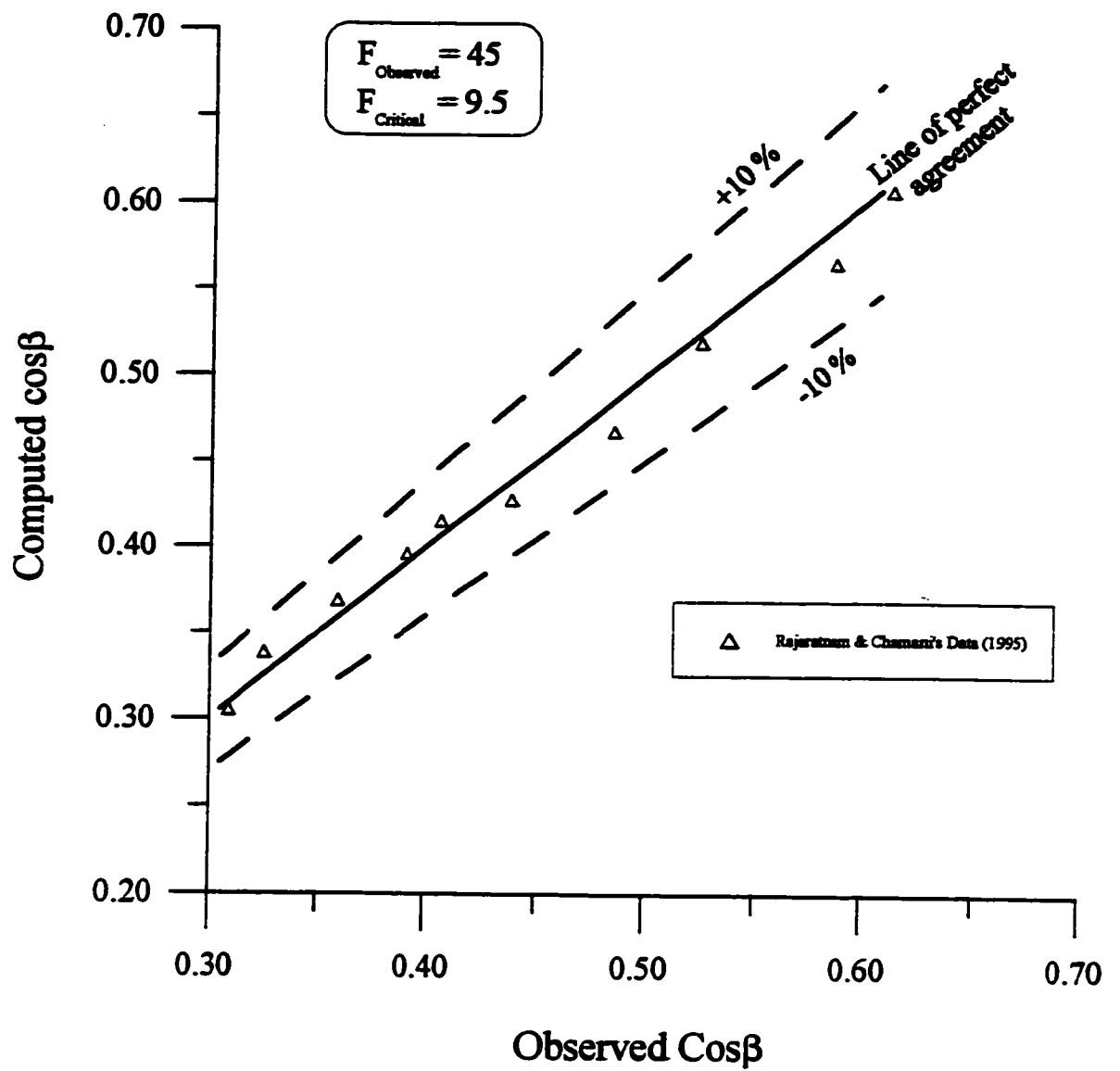


Fig. 5.8. Observed $\cos\beta$ versus computed $\cos\beta$

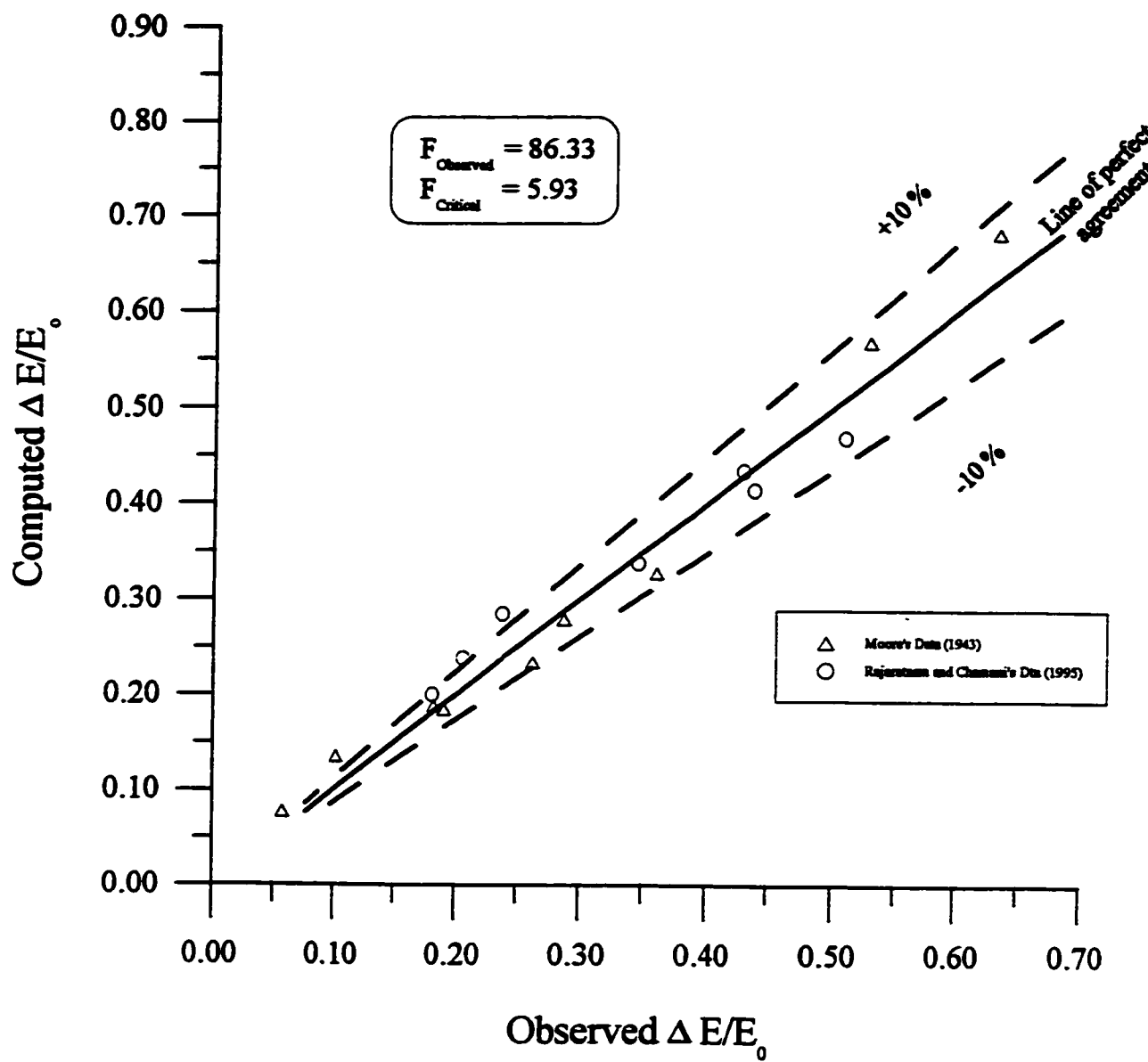
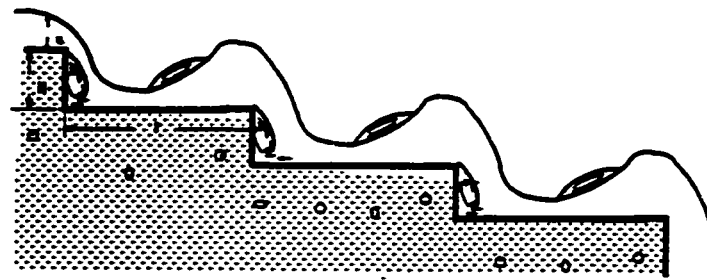
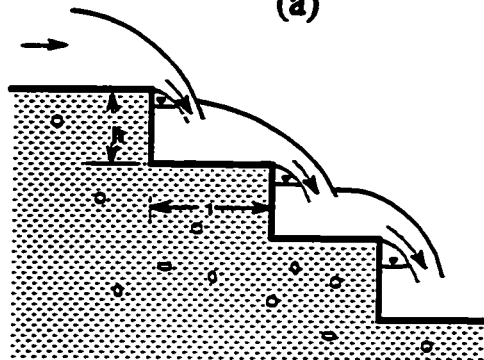


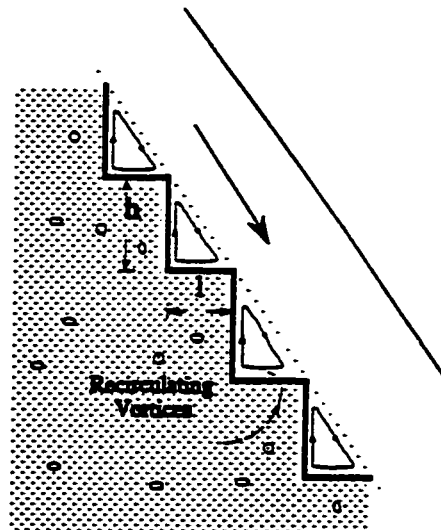
Fig. 5.9. Observed $\Delta E / E_0$ versus computed $\Delta E / E_0$



(a)



(b)



(c)

Fig.5.10. Characteristic flows over a stepped spillway: (a) nappe flow with fully developed hydraulic jump, (b) nappe flow with partially developed hydraulic jump, (c) skimming flow

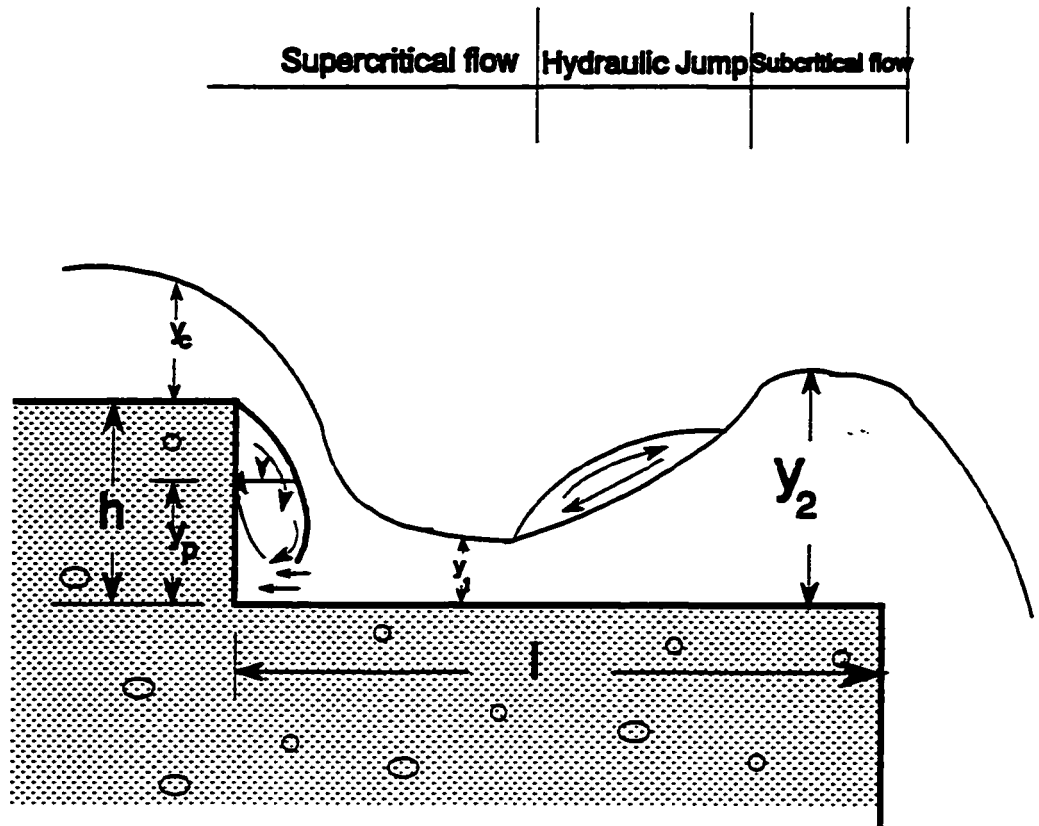


Fig.5.11. Nappe flow profile with fully-developed hydraulic jump on a single step

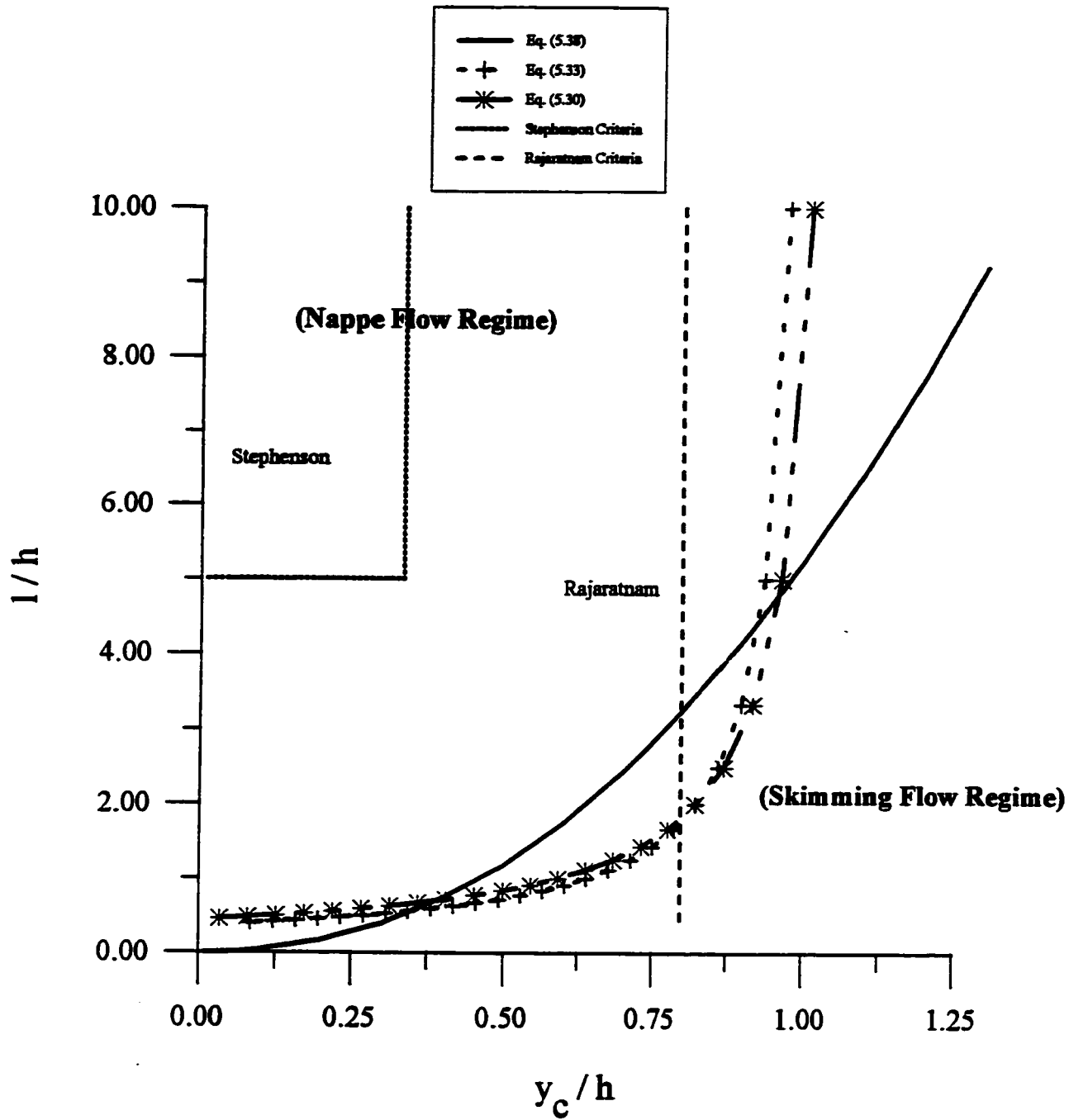


Fig. 5.12. Comparison of eq. (5.38) with various criteria developed by others

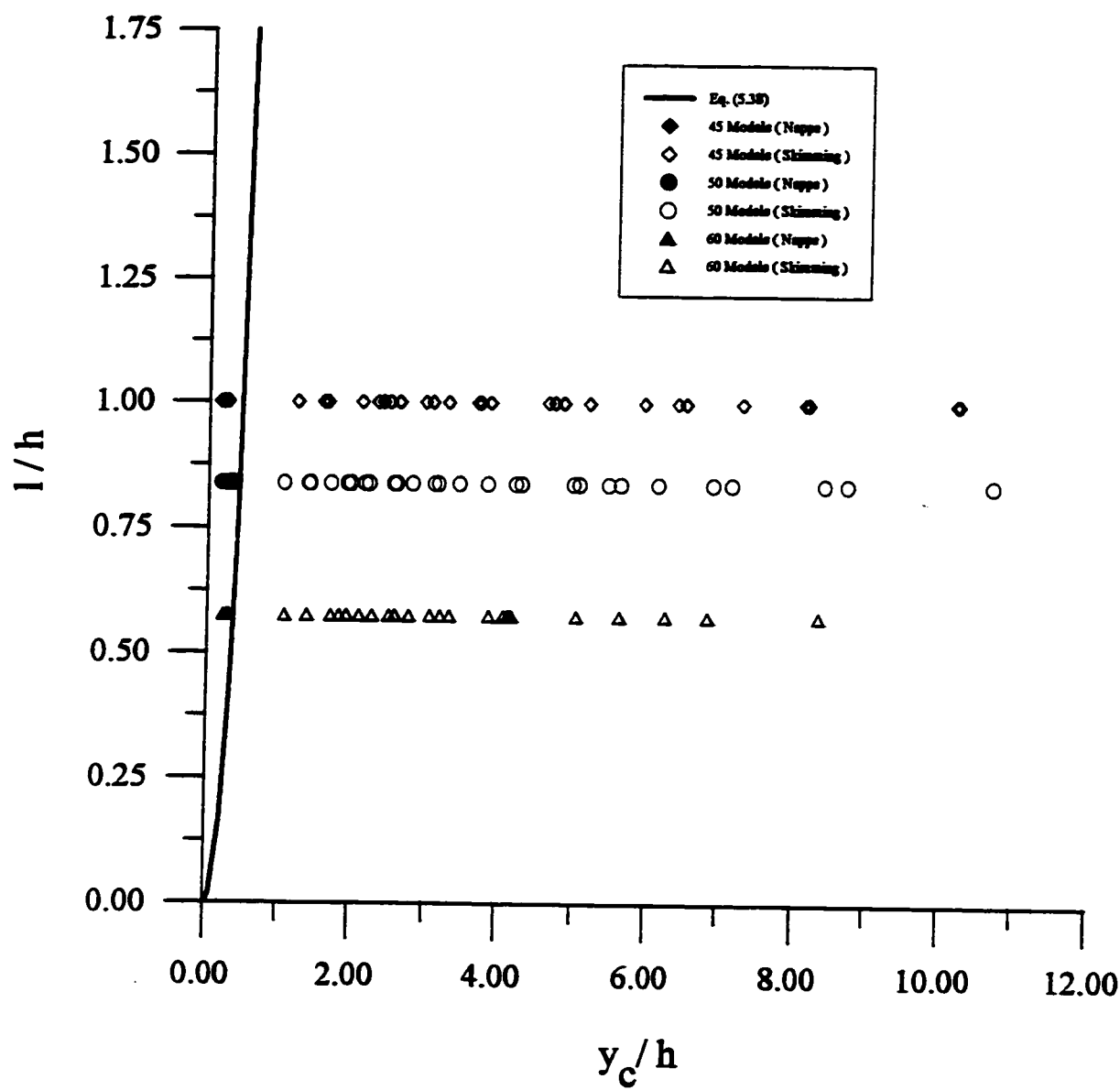


Fig. 5.13. Comparison of eq. (5.38) with experimental data

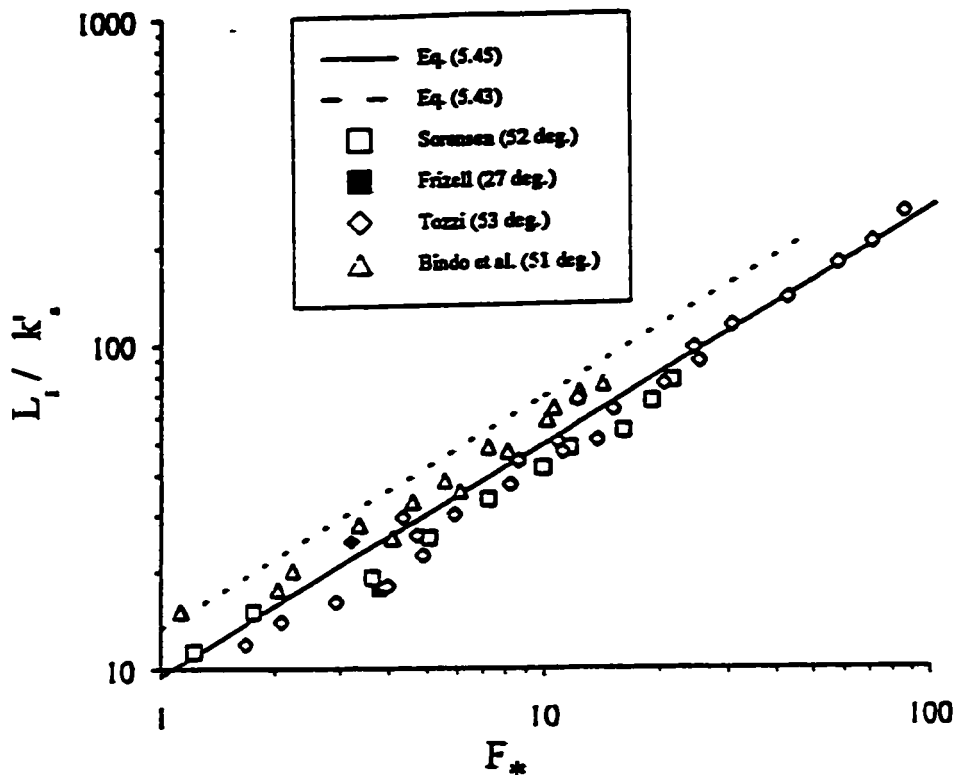


Fig.5.14. Comparison of eq.(5.45),and (5.43), with experimental data from various sources

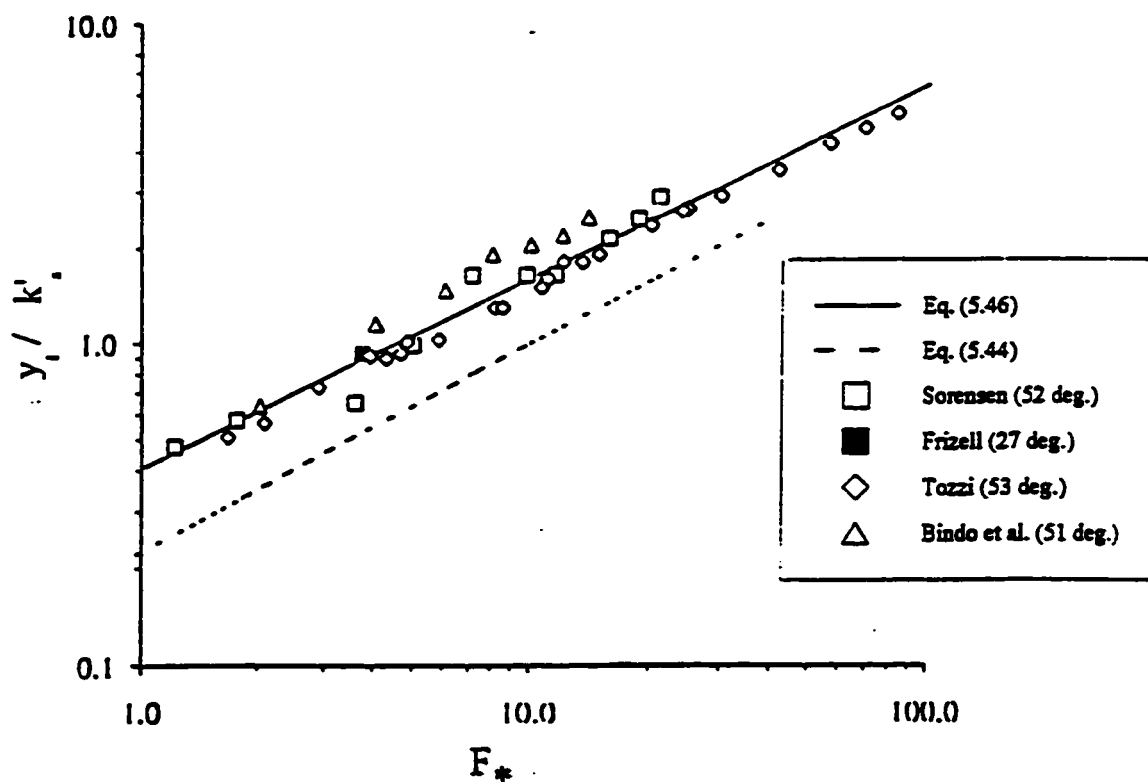


Fig.5.15. Comparison of eq.(5.46), and (5.44), with experimental data from various sources

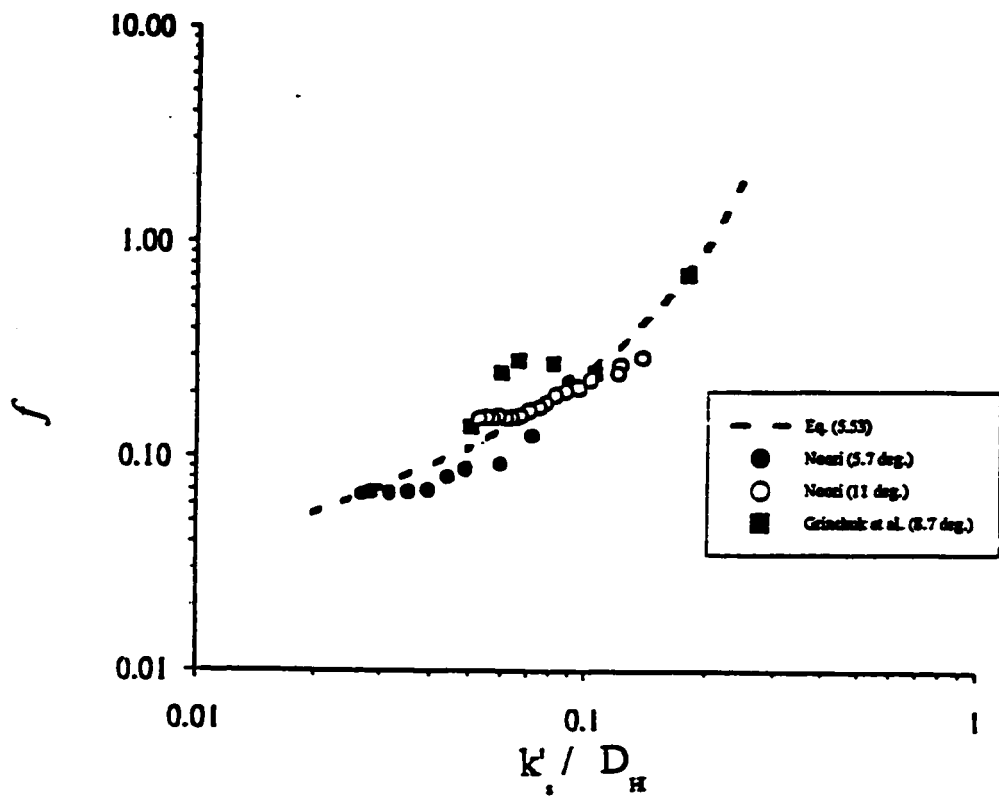


Fig.5.16. Friction factor for stepped channels on flat slopes ($\theta < 12^\circ$)
(after Chansen, 1994)

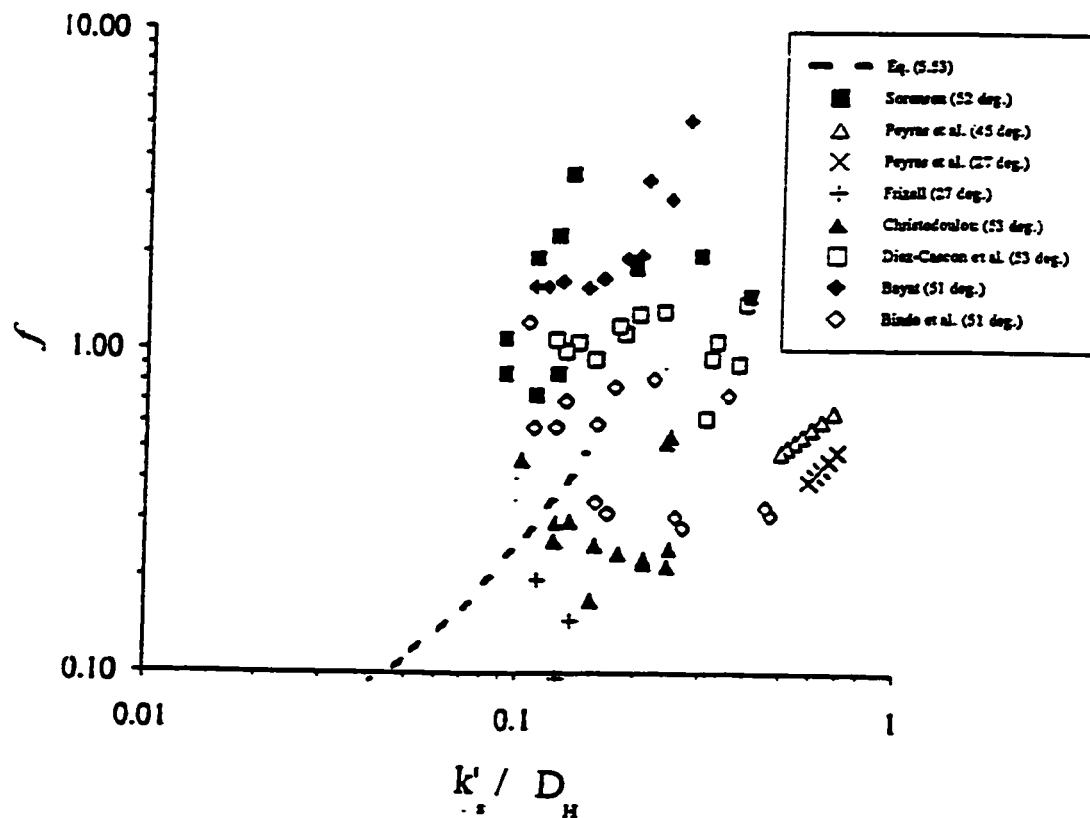


Fig.5.17. Friction factor of stepped channels on steep slopes ($\theta > 27^\circ$)
(after Chansen, 1994)

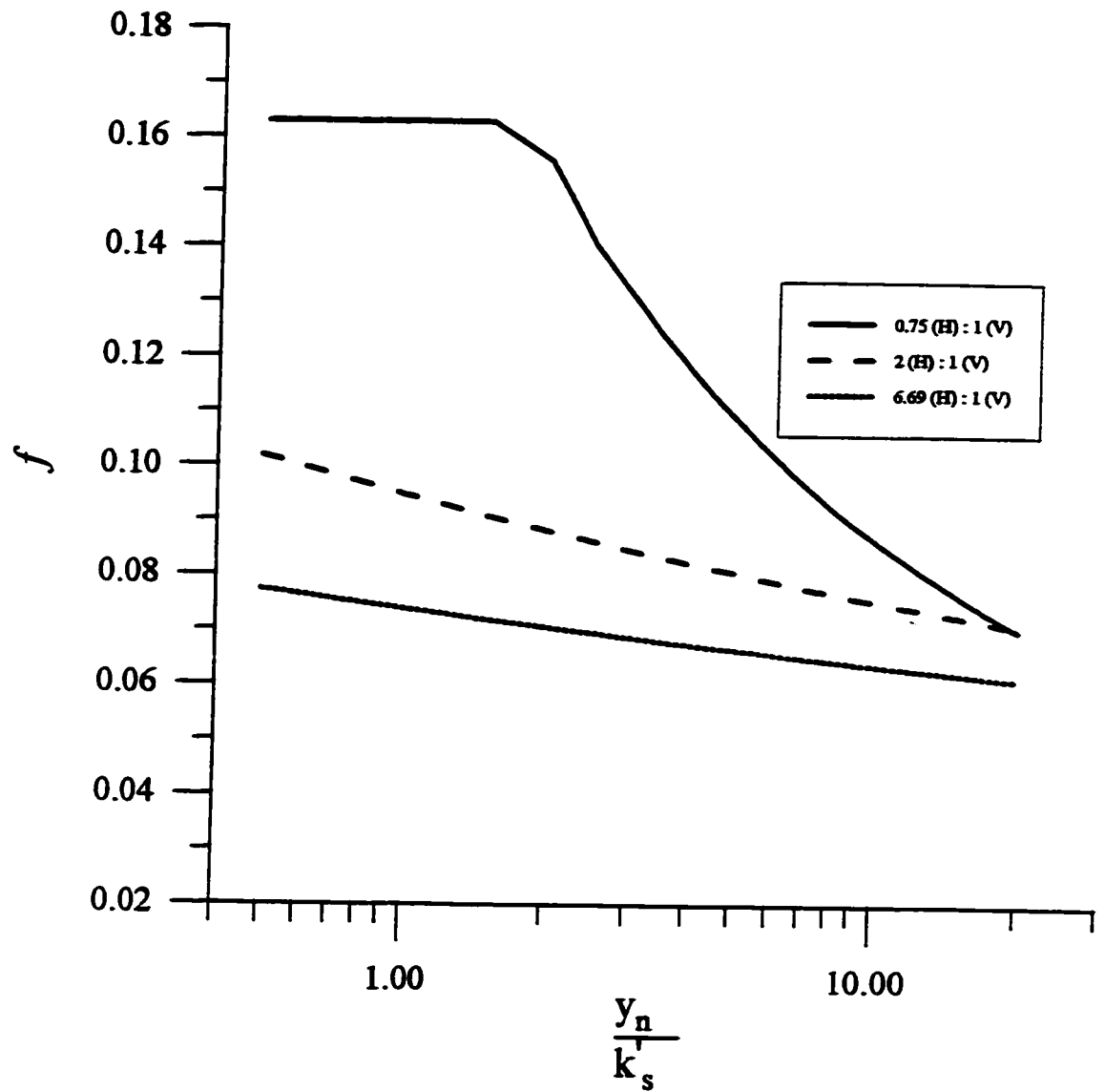


Fig. 5.18. Friction Factor for stepped surfaces (after Tozzi, 1994)

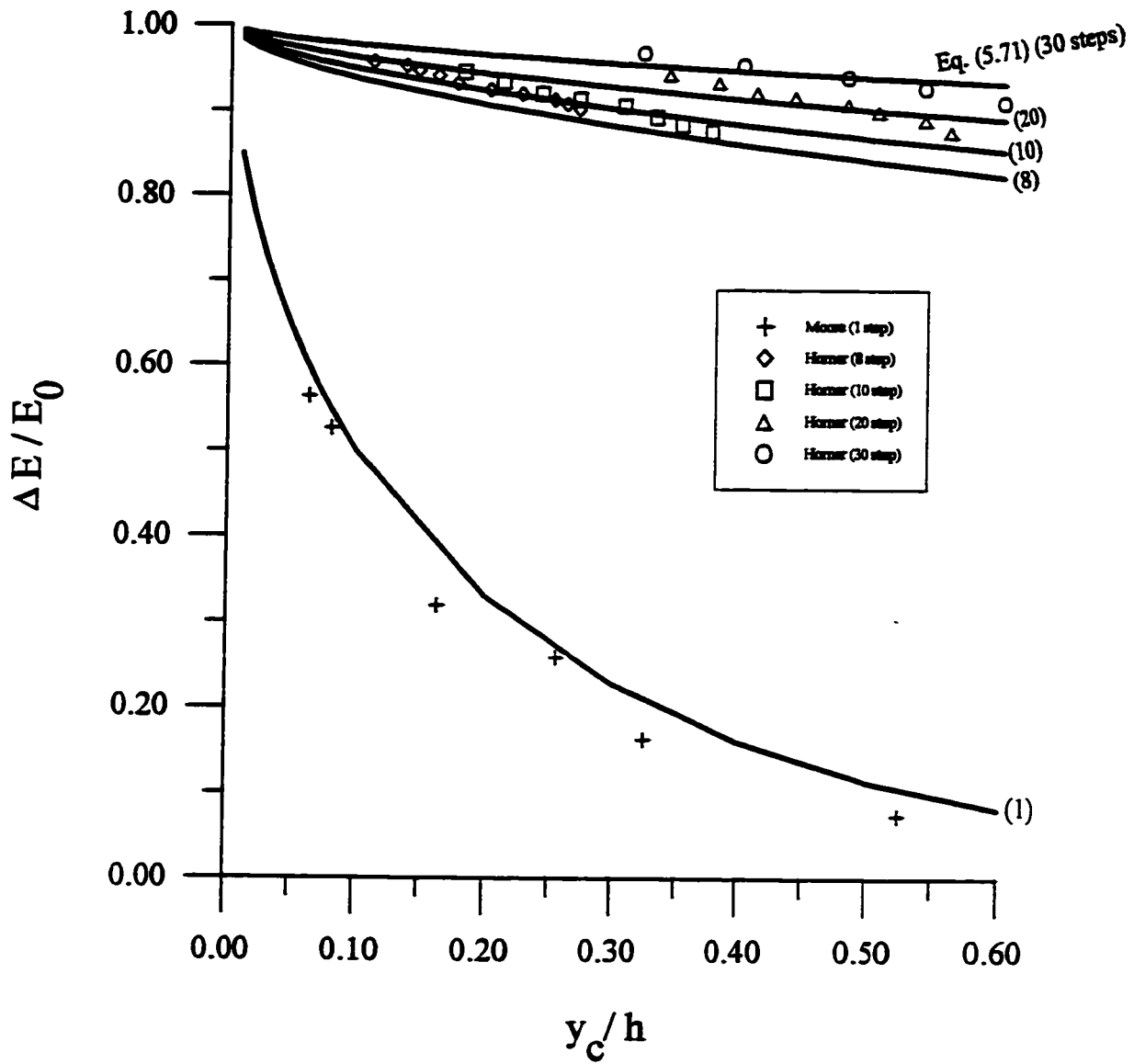


Fig. 5.19. Comparison of Eq. (5.71) with Experimental data from various sources

Chapter 6

Experimental Results

6.1. Introduction

The most common types of hydraulic model studied so far were physical-type models, which are used to study the performance of hydraulic structures. These models have an important role in the design and development stages. The literature review carried out in Chapter 3 showed that the *stepped* spillways studied, so far, were site-specific. The experimental models and setup were explained thoroughly in Chapter 4. In this chapter the results obtained from the experimental programme would be discussed.

6.2. Discharge Coefficient

In Chapter 2, the rational for a *coefficient of discharge* for a smooth spillway was presented. In this section, the *coefficient of discharge* for 12 *stepped* spillway models is determined on the basis of experimental observations for the different models. In order to determine the effect of the *steps* on the coefficient of discharge, the upstream head above the crest, and the corresponding discharges were measured. At low heads it was noted that the water surface upstream of the crest is essentially horizontal a short distance upstream of the crest. By increasing the discharge this distance also increases. To ensure that the effect of the curvature of water surface upstream of the crest would not effect the measurements, the head

measuring station was located about $12 H_d$ upstream from the spillway crest.

Table 6.1 lists the discharges and the corresponding upstream heads, H , for all models. The H - Q relationships are also plotted in Figs. 6.1, 6.2, and 6.3. The rating curve for these models takes the form:

$$Q = C_s L H^{1.5} \quad (6.1)$$

Comparing with the smooth models, it was noted that the coefficient of discharge increased for *stepped* spillways (by approximately 8%). Also it was observed that the downstream slope of the models does not effect the *discharge coefficient*. Therefore it can be concluded that:

$$C_s = 1.08 C \quad (6.2)$$

6.3. Friction Factor for Stepped Spillways

Since the fluid has to do work against resistance, the energy is being continuously dissipated. In the *skimming-flow regime* on *stepped* spillways, the resistance mostly depends on: *i)* the shape of the surface, and *ii)* the vortices on the *steps*. In Chapter 5, different methods to determine the friction factors for rough boundaries were reviewed. It was found that the friction factor found by these methods could not be applied to *stepped* spillways with large downstream slope. In this section the main effort was to develop a general equation for the friction factor for the different *stepped* spillway models studied.

For non-uniform gradually-varied flows, the friction factor can be found from the energy equation:

$$f = \frac{8gy_n^2}{q^2} \frac{D_H}{4} \frac{\Delta H}{\Delta s} \quad (6.3)$$

where

ΔH = the total head loss over a distance Δs ; and

$\Delta H/\Delta s$ = the friction slope.

Based on the results of our experimental studies and using the Grid Search Method, the following equation develops:

$$f = 72.59 \left(\frac{8700 \frac{y_n}{k'_s}}{\left(\left(449 \frac{y_n}{k'_s} + 18 \right) \frac{k'_s}{D_k} - 67.5 (\ln \sqrt{f})^{1.23} \right)} \right)^{-1.25} \quad (6.4)$$

Figs. 6.4, 6.5, 6.6, and 6.7 show the comparison of the experimental data obtained in this research with eq.(6.4). Based on a *goodness of fit* test ($r^2 = 0.968$, $F_{\text{Observed}} = 536 \gg F_{\text{Critical}} = 3.6$), a good agreement between the experimental data and eq. (6.4) is observed. Fig. 6.8 shows the observed f versus the computed f from eq. (6.4).

Figs. 6.4, 6.5, 6.6, and 6.7 plotted y_n / k'_s vs f for different spillway slopes. It can be seen that, as y_n / k'_s decreases the f increases. Eq. (6.4) is a general equation to find the f . This equation shows that the higher the Q the smaller the f will be. This consequence is mostly due to the effect of the trapped vortices on the main spillway flow. With increasing discharge the effect of the trapped vortices, (in the spaces between the *steps*) on the main stream will be reduced.

Figs 6.9, 6.10, and 6.11 compare y_n / k'_s vs f for the different *stepped* models. It can be seen that the CSBS and CLBS models provided the highest f , whereas the

CLBL model has the smallest friction factor. This results mostly because of the larger number of *steps* in the CSBS and CLBS models. Concerning the f , it would appear that the number of *steps* is more important than the size of the *steps*.

6.4. Energy Dissipation for Skimming Flow

In the *skimming-flow regime*, most of the energy is dissipated in the maintenance of stable depression vortices between the *steps*. Chanson (1993) proposed that, for an open channel *stepped* spillway, if uniform flow conditions are reached at the downstream end of the spillway, the total head loss can be expressed as:

$$\frac{\Delta E}{E_0} = 1 - \frac{\left(\frac{f}{8\sin\theta}\right)^{1/3} \cos\theta + 0.5\left(\frac{f}{8\sin\theta}\right)^{-2/3}}{1.5 + H_{dam}/y_c} \quad (6.5)$$

To find the energy at any point along the spillway Bernoulli's equation is used,

$$\text{i.e.: } \left(E = z + \frac{p}{\gamma} + \frac{v^2}{2g} \right) ; \text{ where}$$

z = elevation above datum, and

p = hydrostatic pressure.

A hydrostatic pressure distribution is a good assumption for an open-channel flow on mild slopes. However, as the slope increases the pressure distribution no longer remains hydrostatic. Therefore, a pressure distribution correction must be made on steep slopes, which is:

$$p = \gamma y_n \cos\theta \quad (6.7)$$

where $y_n = y \cos\theta$. Therefore

$$p = \gamma y_n \cos^2\theta \quad (6.8)$$

Near the crest and the toe of the spillway the streamlines have a substantial curvature in the vertical plane. The piezometric pressure at the boundary must be corrected by an appropriate factor. This factor is equal to the force produced per unit area of a mass of water undergoing centrifugal acceleration. For the channel invert, the factor is determined from the following relationship:

$$C_g = \frac{y_n V^2}{g R_c} \quad (6.9)$$

where

C_g = centrifugal acceleration factor,

R_c = radius of curvature.

Therefore, for the *crest* region:

$$p = \gamma y_n - C_g \quad (6.10)$$

and for the *toe* region:

$$p = \gamma y_n + C_g \quad (6.11)$$

Substituting the values found by eq.(6.9) in eqs.(6.10) and (6.11), the corrected pressure for the required locations can be found. Applying eqs. (6.8), (6.10), and (6.11) one can find the energy at any intermediate *step*.

In *skimming-flow regime*, when the discharge is low ($0.25 < H/H_d < 0.5$), the lower nappe hits the *step* tips. The water will be projected into the air and reattached a few *steps* downstream. Also, some of the water upon reattachment, will be splashed into the air. Because of this effect, measuring the water surface profile at

$H/H_d < 0.5$ would be very difficult.

For different slopes the energy dissipation rate ($\Delta E/E_0$) along the spillway bottom axis (horizontal axis) were plotted in Figs. 6.12 to 6.23. In Figs. 6.24, 6.25, and 6.26 the energy dissipation rate $\Delta E/E_0$ vs x/H_d are also plotted for the smooth models. These figures graphically illustrate the energy dissipation from the crest to the toe of the spillway. Comparing Figs. 6.12 to 6.23 with Figs. 6.24 to 6.26 it can be concluded that the *stepped* models dissipate about 4 times of their corresponding *smooth* models. From Figs. 6.12 to 6.23 it can be seen that the energy dissipation in the spillway section immediately downstream from the crest beginning ($x/H_d < 1.5$) have almost the same dissipation rate for all discharges. For lower discharges, when $H/H_d \leq 1.25$, and $x/H_d > 1.5$, the energy dissipation rate is significantly higher. Upon increasing the discharge, the rate of energy dissipation in the middle part of the spillway would decrease. At a point near the toe of the spillway, this rate would sharply decrease and become almost zero, i.e. the energy dissipation becomes constant after this point, which we will hereafter refer to as the *saturation point*. It was observed that, upon increasing the downstream slope the saturation point moved upstream from the toe toward the middle of the spillway. Since the horizontal length of the spillway would shorten for steeper downstream slope, the *saturation point* would stay the same for each of these models. Therefore, the dissipation would be constant after a certain length. The horizontal length of spillway has to be long enough to reach to the *saturation point*. The *saturation point* can be found from the following relationship:

$$X_1 \geq \frac{8H_d}{\tan\theta} \quad (6.12)$$

where

X_1 = the horizontal length from the crest to the *saturation point*.

In high dams normally this criteria is satisfied. This is an important criterion for low to moderate dams. It was also observed that for very high discharges (H/H_d

> 2), the energy dissipation rate associated with a *stepped* spillway would be insignificant.

Figs. 6.12 to 6.23 show that the energy dissipation rate will decrease with increasing discharge. It appears that, at high discharges ($H/H_d > 2$), even if the length of spillway is large enough to have a fully-developed flow, the effect of the *steps* will be very small. In general, a *stepped* spillway will act the same as a *smooth* spillway after $H/H_d > 1.5$.

Figs. 6.27, 6.28, and 6.29 show the energy dissipation at the toe of the different *stepped* models studied, for $\theta = 45^\circ$, 50° and 60° . It can be seen that overall the CLBS models provide the highest energy dissipation rate among all the models studied. The model tests indicated that larger *steps* at the upper portion of the spillway, (where the depth of flow was lower), would have more effect on the spillway flow. At the toe of the spillway the small *steps* will have a large number of smaller vortices. Therefore, at the toe of the spillway the transition of flow to the stilling basin would be very smooth.

In spite of high f associated in the CSBS models, the vortices created in the *step* spaces were not large. Therefore the energy dissipation in these models is not the highest. The CLBL models produced the strongest vortices but the number of vortices is small. Both the CLBS models and CSBL models provided a combination of strong vortices and a large number of *steps*. In the case of *skimming-flow*, the water shoots down the face in a regular stream and *steps* act as a form of *intensive-friction* in retarding the flow. The total energy dissipation above the spillway and the residual energy at the bottom of the spillway are functions of H_{dam} , θ , q , N_s , and f .

$$\frac{\Delta E}{E_0} = f(H_{dam}, \theta, q, N_s, f) \quad (6.13)$$

For the different models studied the friction factor can be found from eq.(6.4). Among the 5 principal factors influencing energy dissipation, H_{dam} , θ , q , are determined (i) by the purpose of structure, (ii) by site topography, and (iii) the design flood. Moreover, once N_s is selected f is determined. Using the data obtained for different θ , q , and N_s and using the Grid Search Method the following equation is obtained:

$$\frac{\Delta E}{E_0} = 1 - 0.268 \left(\frac{9.773 \left(\frac{y_c}{N_s h} \operatorname{cosec} \theta \right)^{0.461}}{(6.652 f \sec \theta)^{0.056} + 0.966} \right)^{0.809} \quad (6.14)$$

Figs. 6.30, 6.31, and 6.32 compare this equation with the data obtained for each set of models. A good agreement between the experimental data and eq.(6.14) was achieved ($r^2 = 0.9$, $F\text{-Observed} = 152 \gg F\text{-Critical} = 3.62$). Fig. 6.33 shows the observed $\Delta E/E_0$ versus the computed $\Delta E/E_0$ from eq. (6.14). Accordingly, this equation enables one to estimate the amount of energy dissipated on a *stepped* spillway structure.

In most of the literature the energy dissipation rate reported was for low discharges. This study shows that this rate is in the range of 50-60 percent for the design head. When the head over the crest (H) is more than twice the design discharge the *stepped* spillway will not be effective anymore.

Smooth 45		CLBL 45		CLBS 45		CSBL 45		CSBS 45	
H (m)	q (m2/sec)	H (m)	q (m2/sec)	H (m)	q (m2/sec)	H (m)	q (m2/sec)	H (m)	q (m2/sec)
0.076	0.046	0.127	0.109	0.127	0.107	0.127	0.108	0.102	0.075
0.064	0.035	0.102	0.075	0.102	0.075	0.102	0.077	0.076	0.048
0.051	0.024	0.076	0.050	0.076	0.049	0.076	0.048	0.064	0.037
0.038	0.015	0.064	0.035	0.064	0.037	0.064	0.037	0.051	0.026
0.030	0.010	0.051	0.026	0.051	0.025	0.051	0.025	0.038	0.016
		0.038	0.016	0.038	0.015	0.038	0.017	0.030	0.011
		0.030	0.009	0.034	0.014	0.030	0.010		

Smooth 50		CLBL 50		CLBS 50		CSBL 50		CSBS 50	
H (m)	q (m2/sec)	H (m)	q (m2/sec)	H (m)	q (m2/sec)	H (m)	q (m2/sec)	H (m)	q (m2/sec)
0.076	0.048	0.127	0.113	0.127	0.110	0.127	0.110	0.127	0.110
0.064	0.035	0.102	0.079	0.102	0.077	0.102	0.077	0.102	0.077
0.051	0.024	0.076	0.049	0.076	0.049	0.076	0.051	0.076	0.048
0.038	0.015	0.064	0.037	0.064	0.037	0.064	0.037	0.064	0.035
0.030	0.011	0.051	0.025	0.051	0.025	0.051	0.025	0.051	0.025
		0.038	0.016	0.038	0.015	0.038	0.016	0.038	0.015
		0.031	0.011	0.030	0.010	0.030	0.010	0.030	0.010

Smooth 60		CLBL 60		CLBS 60		CSBL 60		CSBS 60	
H (m)	q (m2/sec)	H (m)	q (m2/sec)	H (m)	q (m2/sec)	H (m)	q (m2/sec)	H (m)	q (m2/sec)
0.076	0.047	0.102	0.077	0.102	0.077	0.102	0.077	0.102	0.078
0.064	0.036	0.076	0.049	0.076	0.049	0.076	0.050	0.076	0.051
0.051	0.024	0.064	0.037	0.064	0.037	0.064	0.037	0.064	0.037
0.038	0.015	0.051	0.026	0.051	0.027	0.051	0.025	0.051	0.026
0.030	0.010	0.038	0.016	0.038	0.016	0.038	0.016	0.038	0.016
		0.030	0.011	0.030	0.011	0.030	0.011	0.030	0.011

Table 6.1. Values of H and q for all models

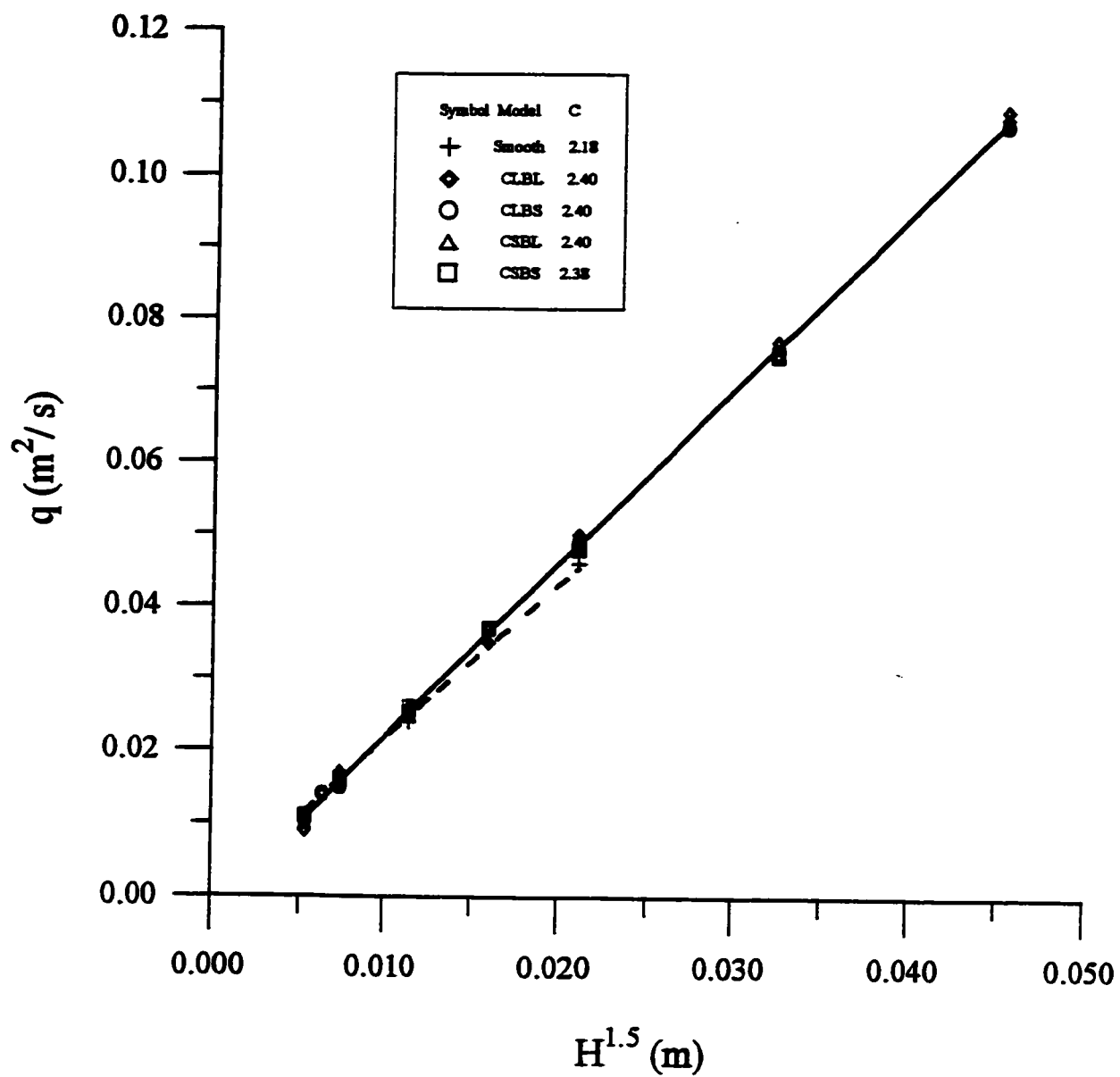


Fig. 6.1. H-Q relationship for 45° models

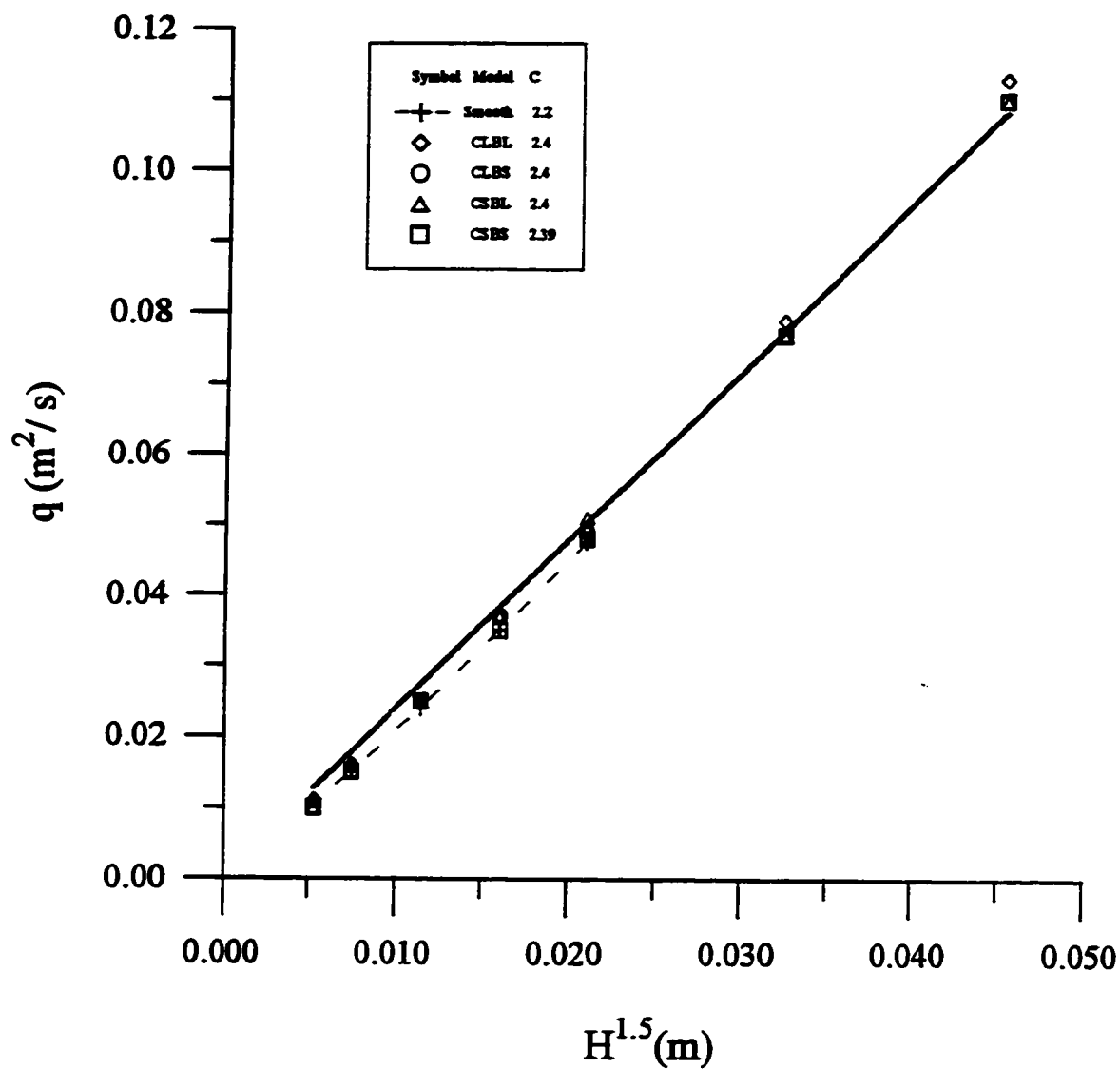


Fig. 6.2. H-Q relationship for 50°models

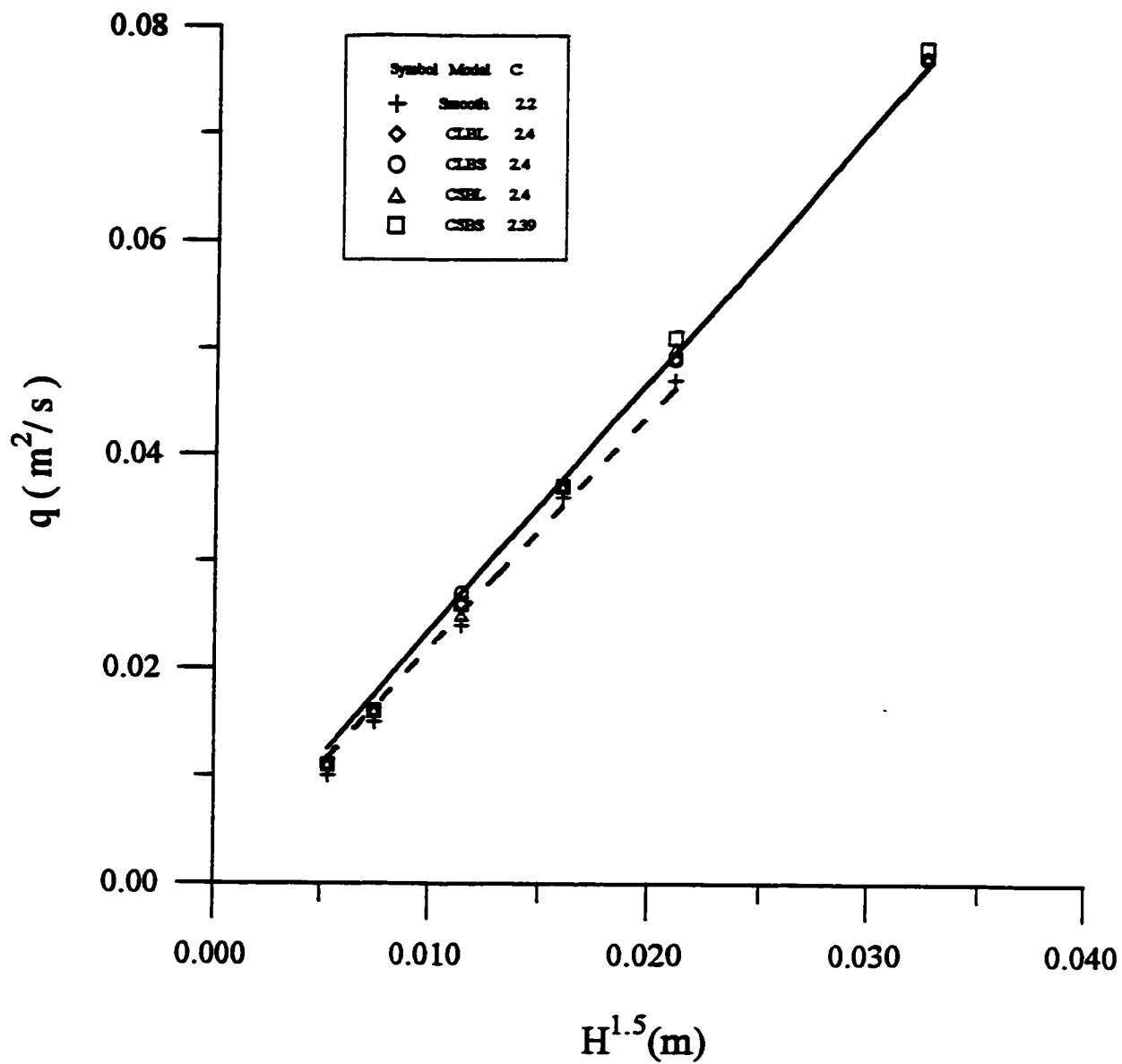


Fig. 6.3. H-Q relationships for 60° models

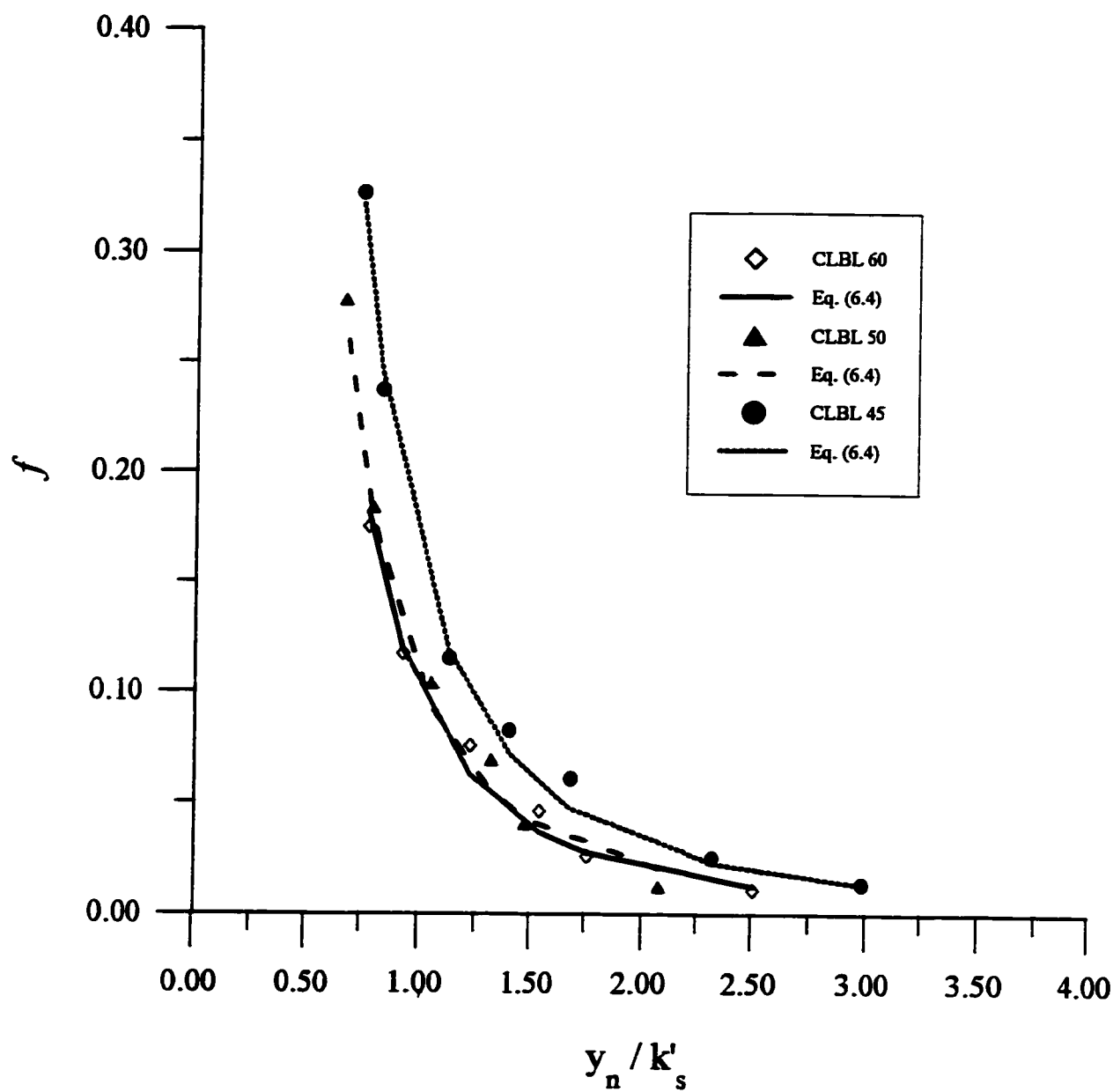


Fig. 6.4. Comparison of eq. (6.4) with CLBL model experimental data

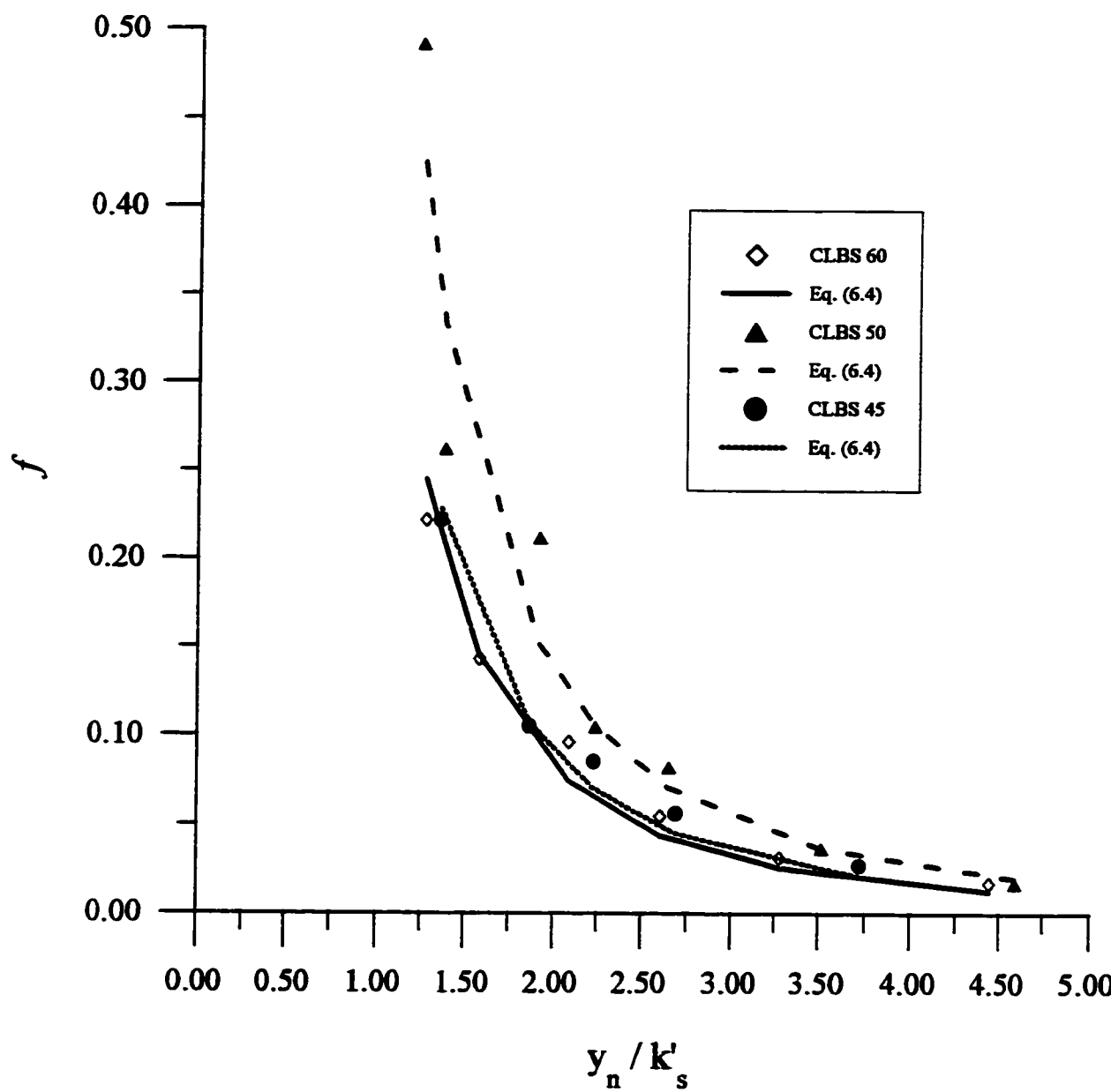


Fig. 6.5. Comparison of eq. (6.4) with CLBS model experimental data

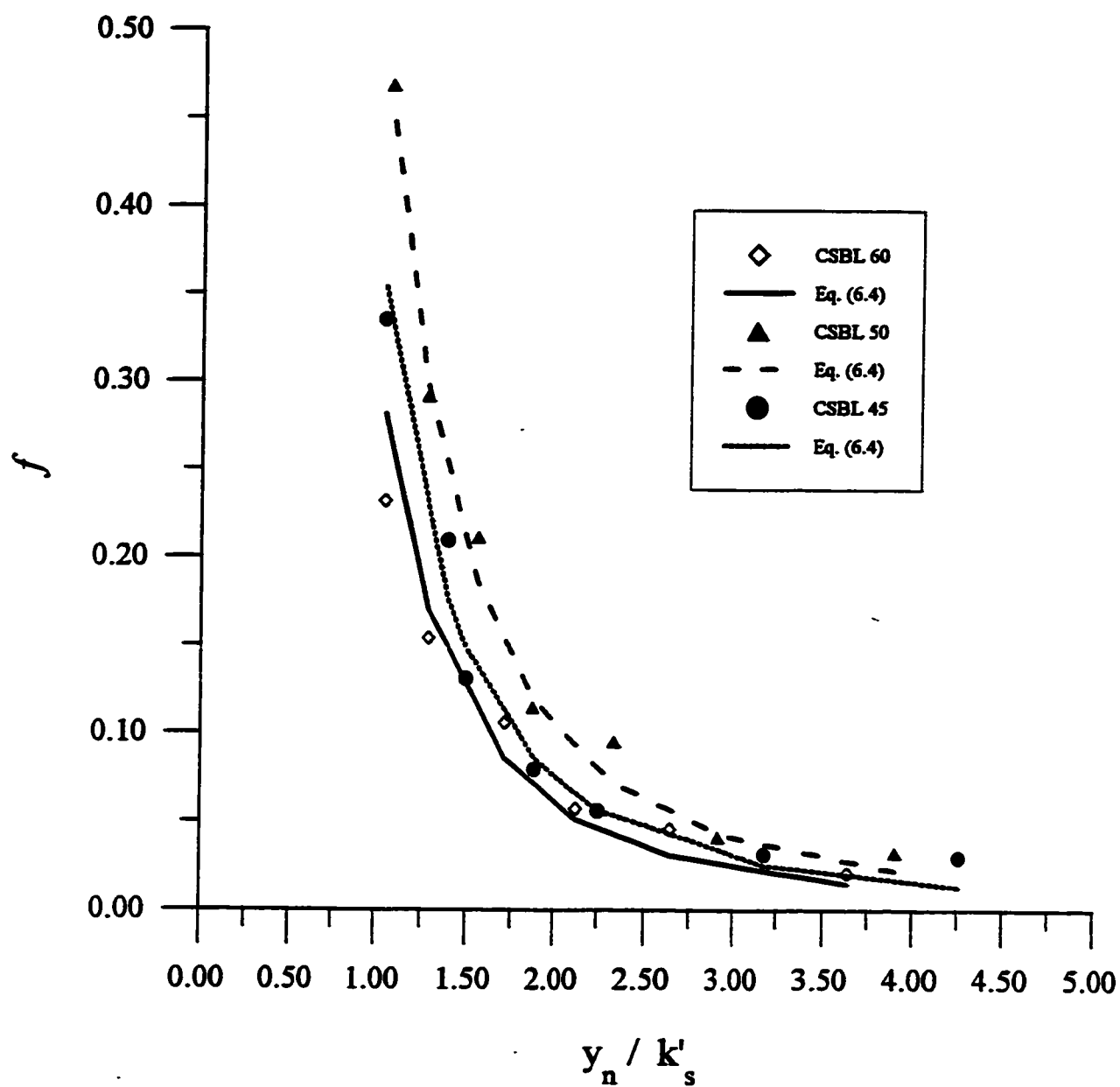


Fig. 6.6. Comparison of eq. (6.4) with CSBL model experimental data

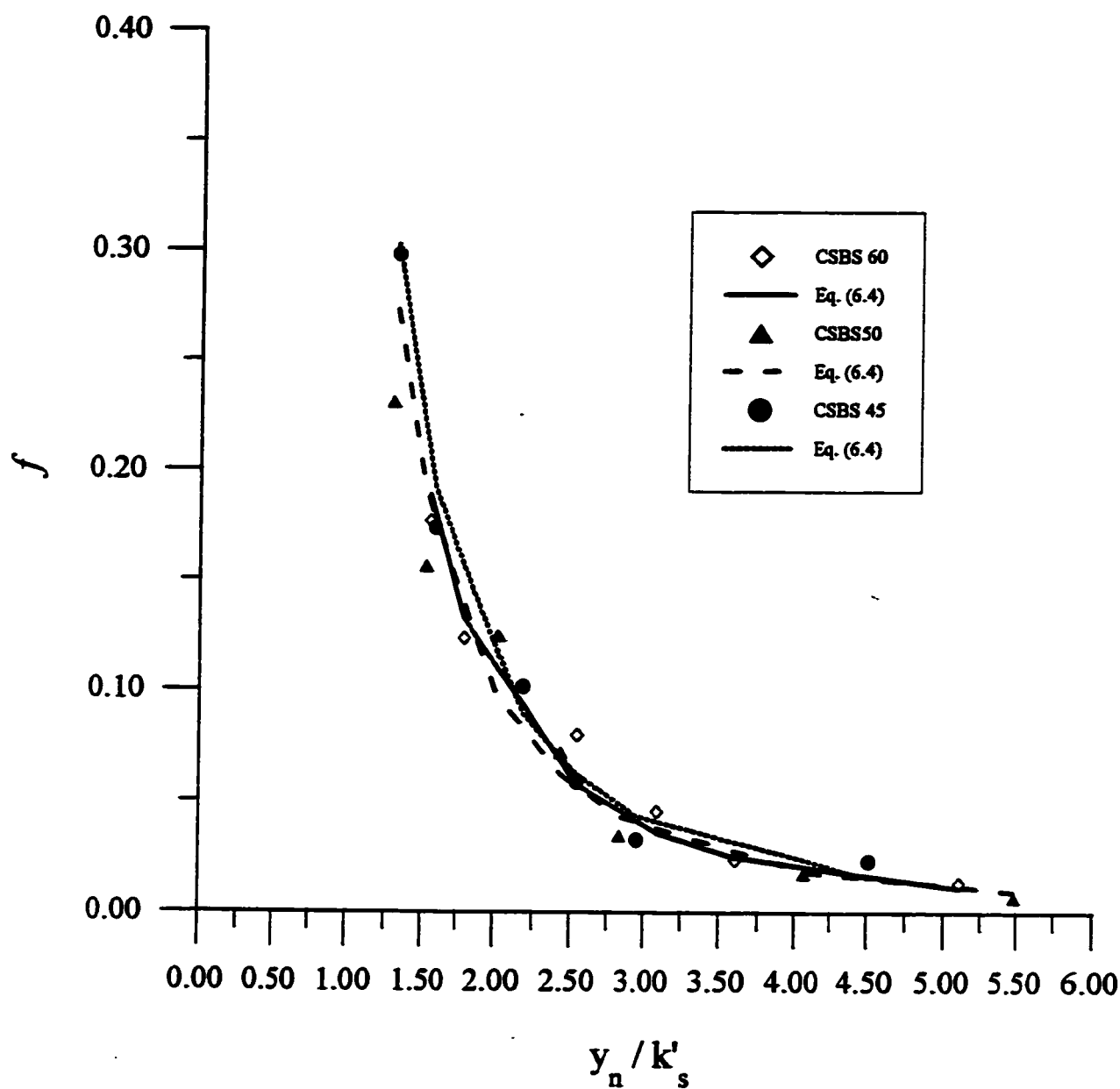


Fig. 6.7. Comparison of eq. (6.4) with CSBS model experimental data

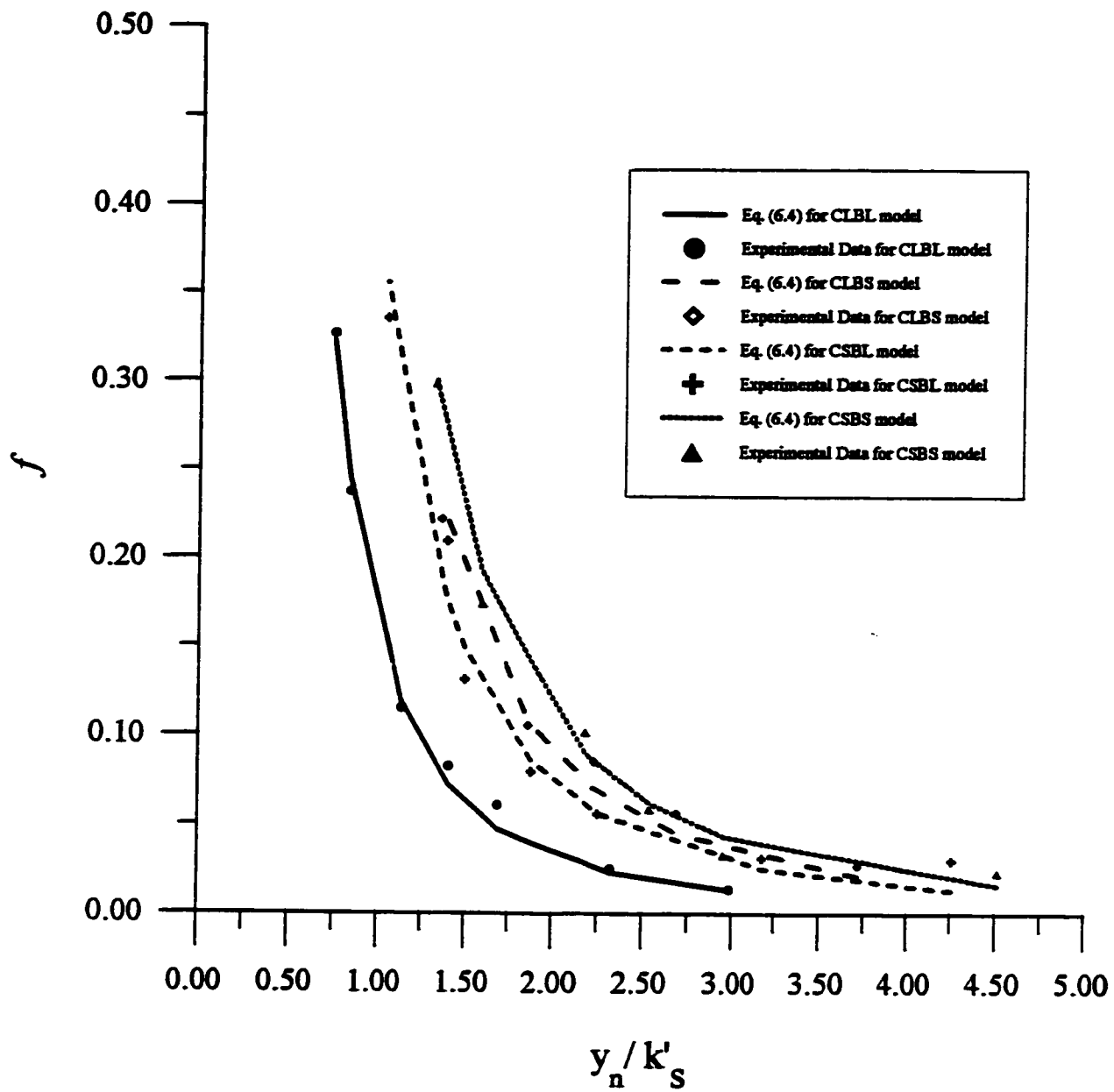


Fig. 6.8. Comparison of eq. (6.4) with 45° stepped model experimental data

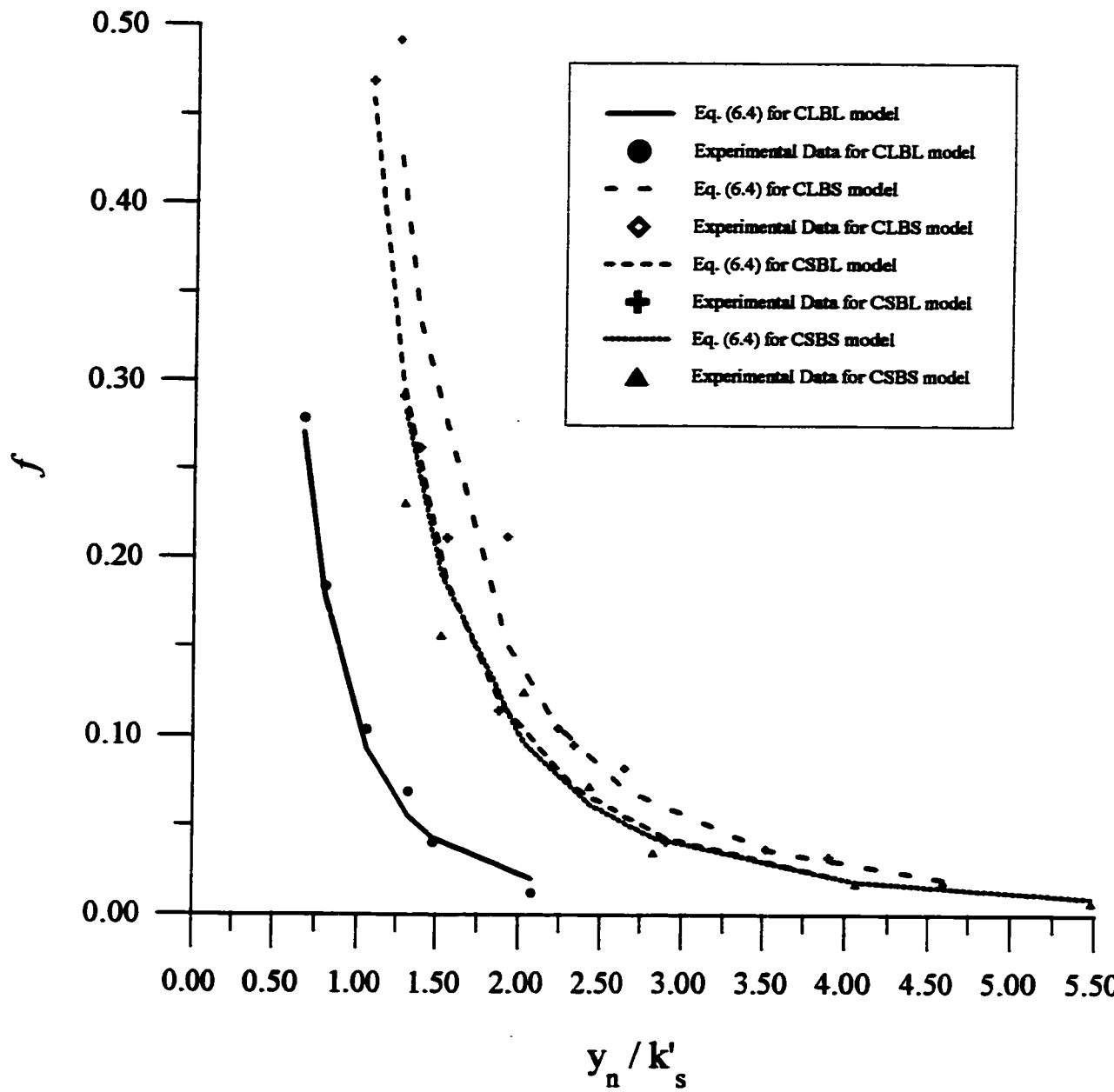


Fig. 6.9. Comparison of eq. (6.4) with 50°stepped model experimental data

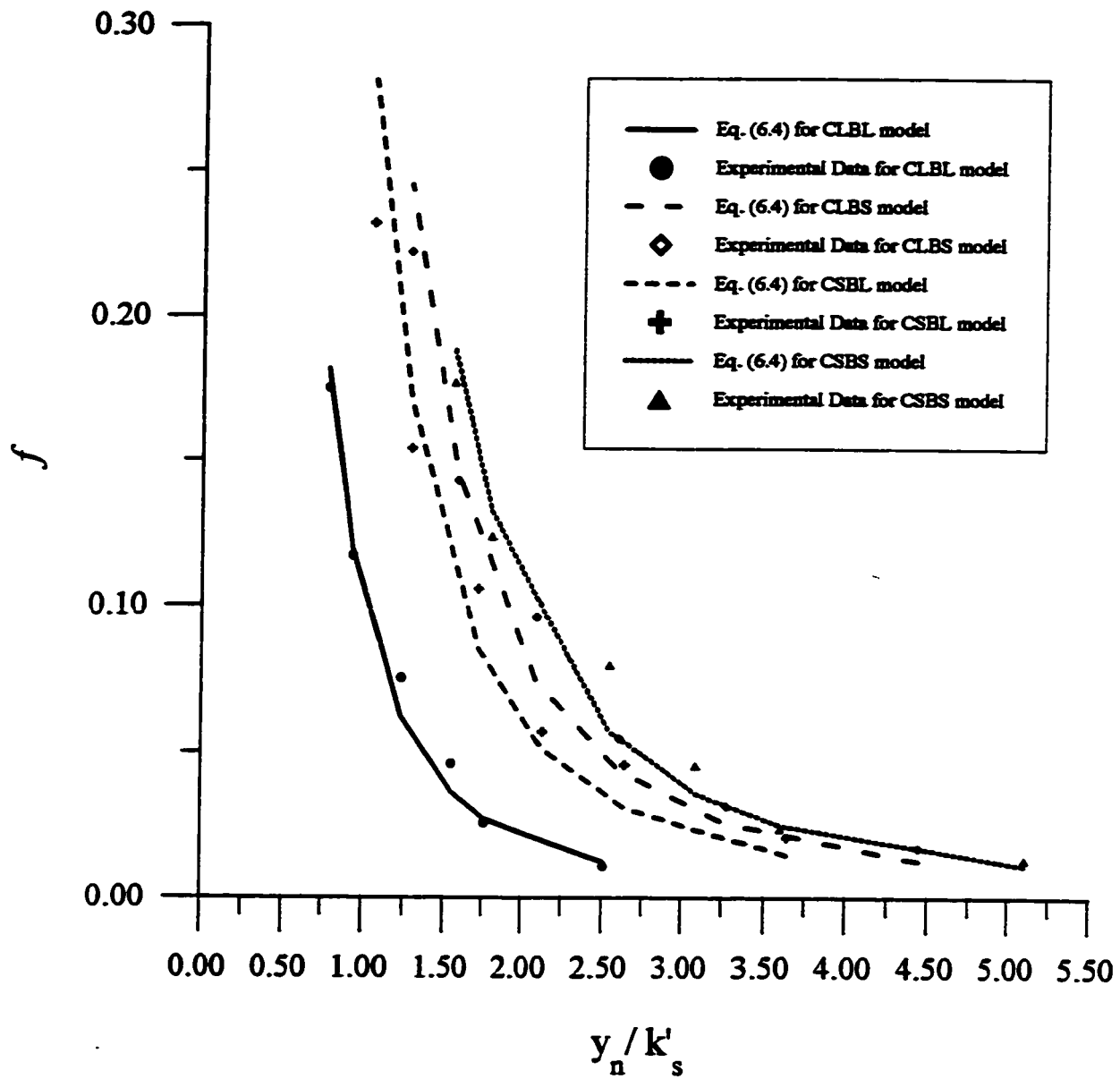


Fig. 6.10. Comparison of eq.(6.4) with 60° stepped model experimental data

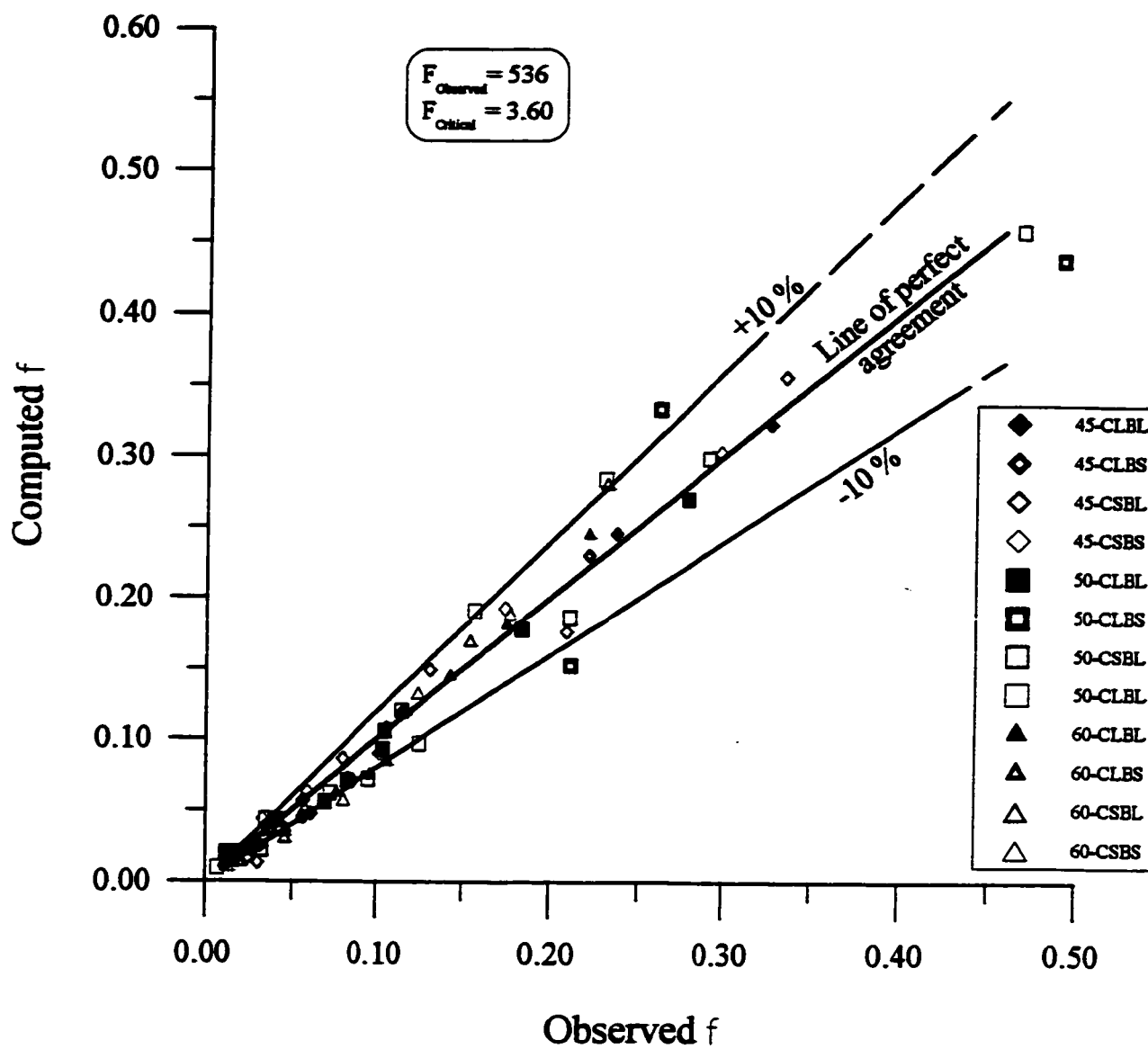


Fig. 6.11. Observed f versus computed f

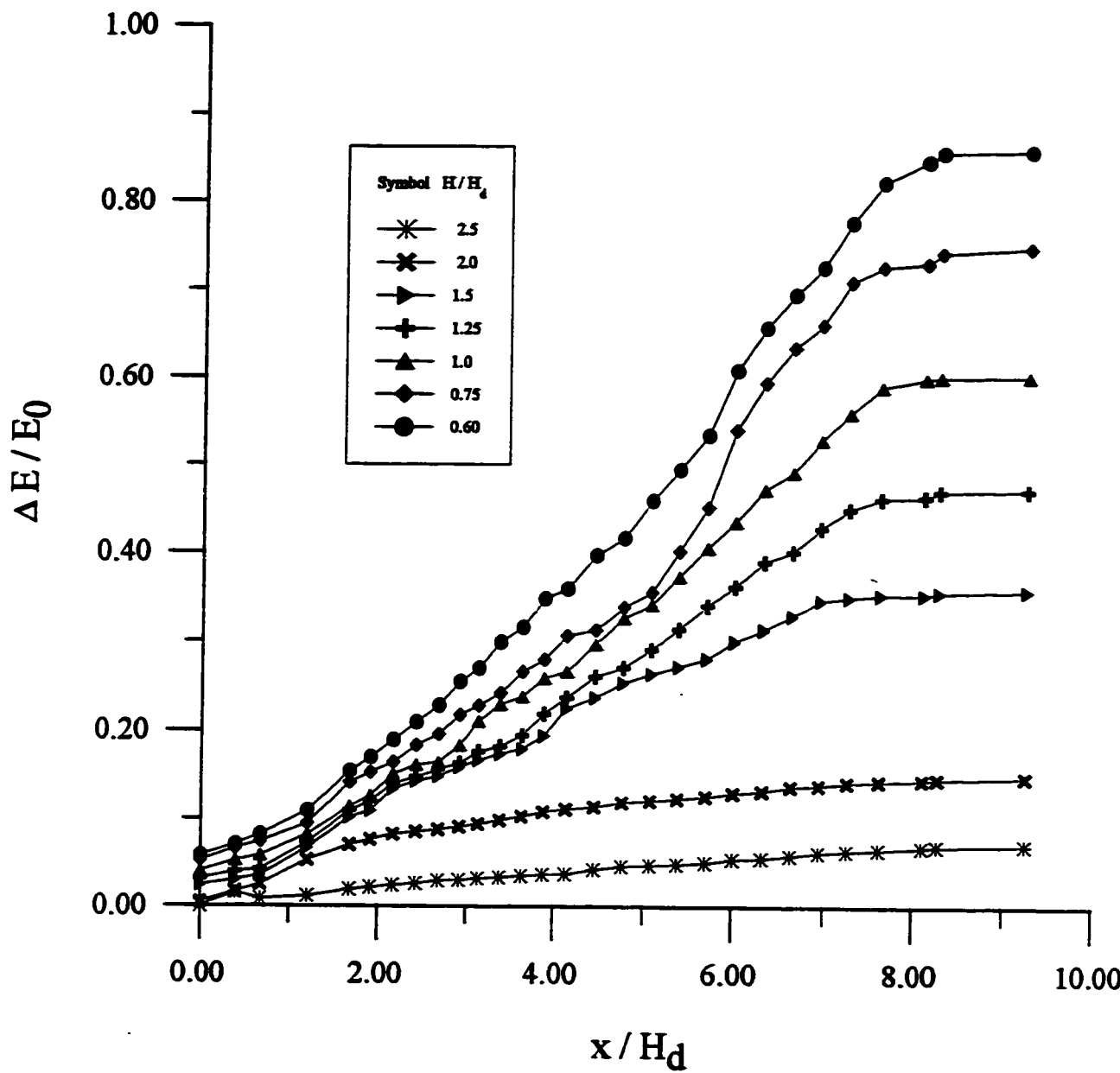


Fig. 6.12. $\Delta E/E_0$ versus x/H_d ; 45-CL-BL model

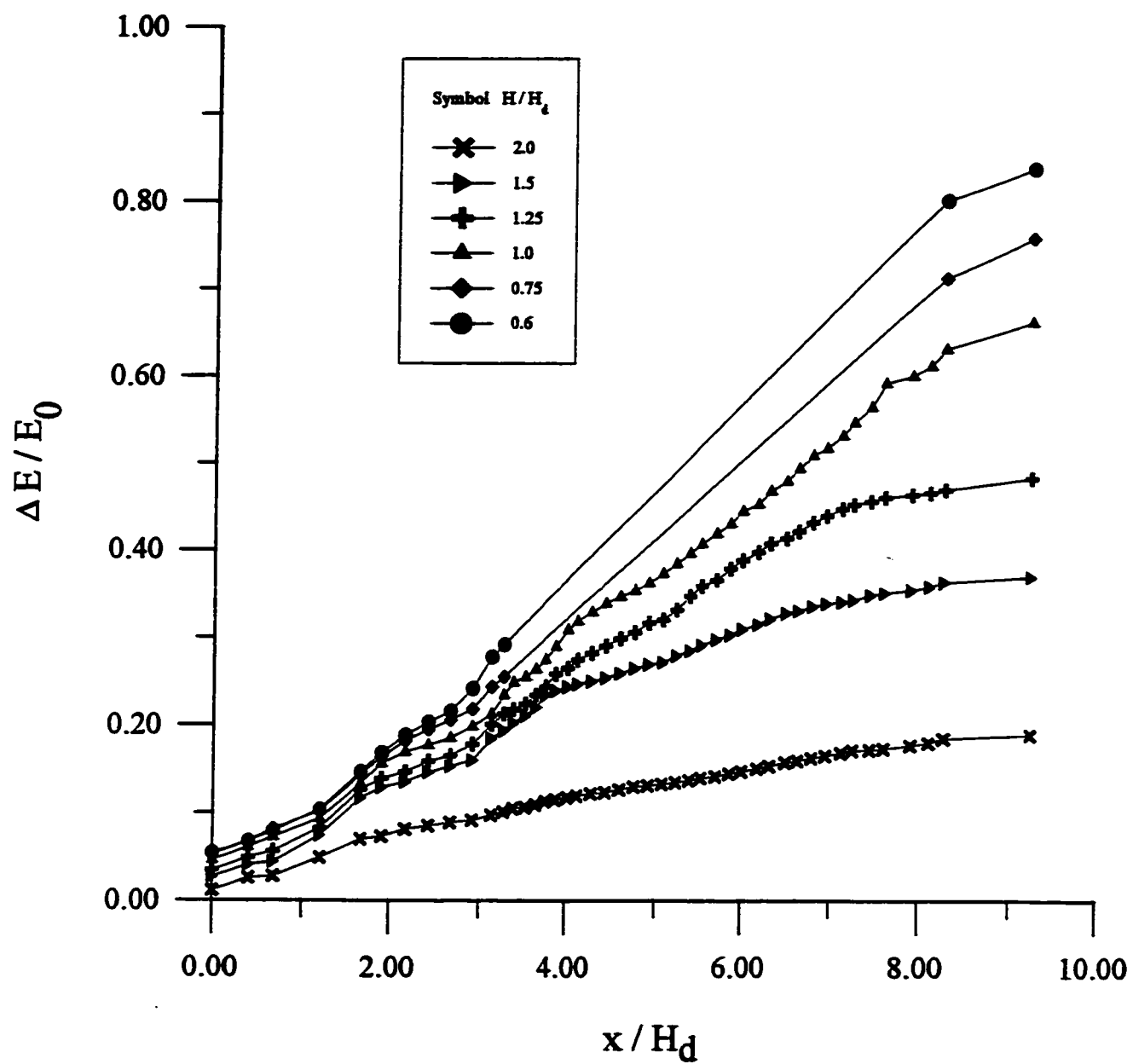


Fig. 6.13. $\Delta E/E_0$ versus x/H_d ; 45-CL-BS model

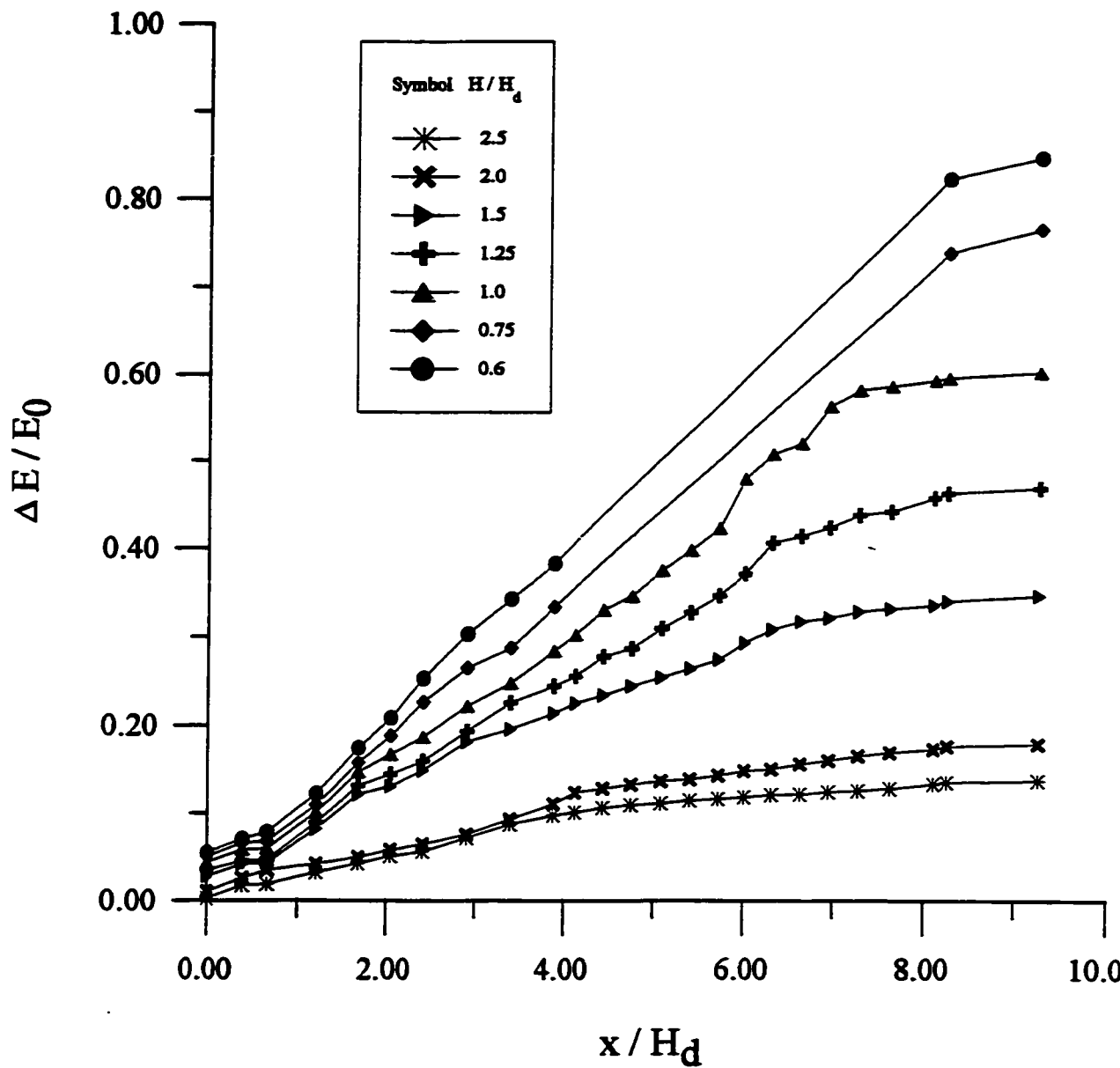


Fig. 6.14. $\Delta E/E_0$ versus x/H_d ; 45-CS-BL model

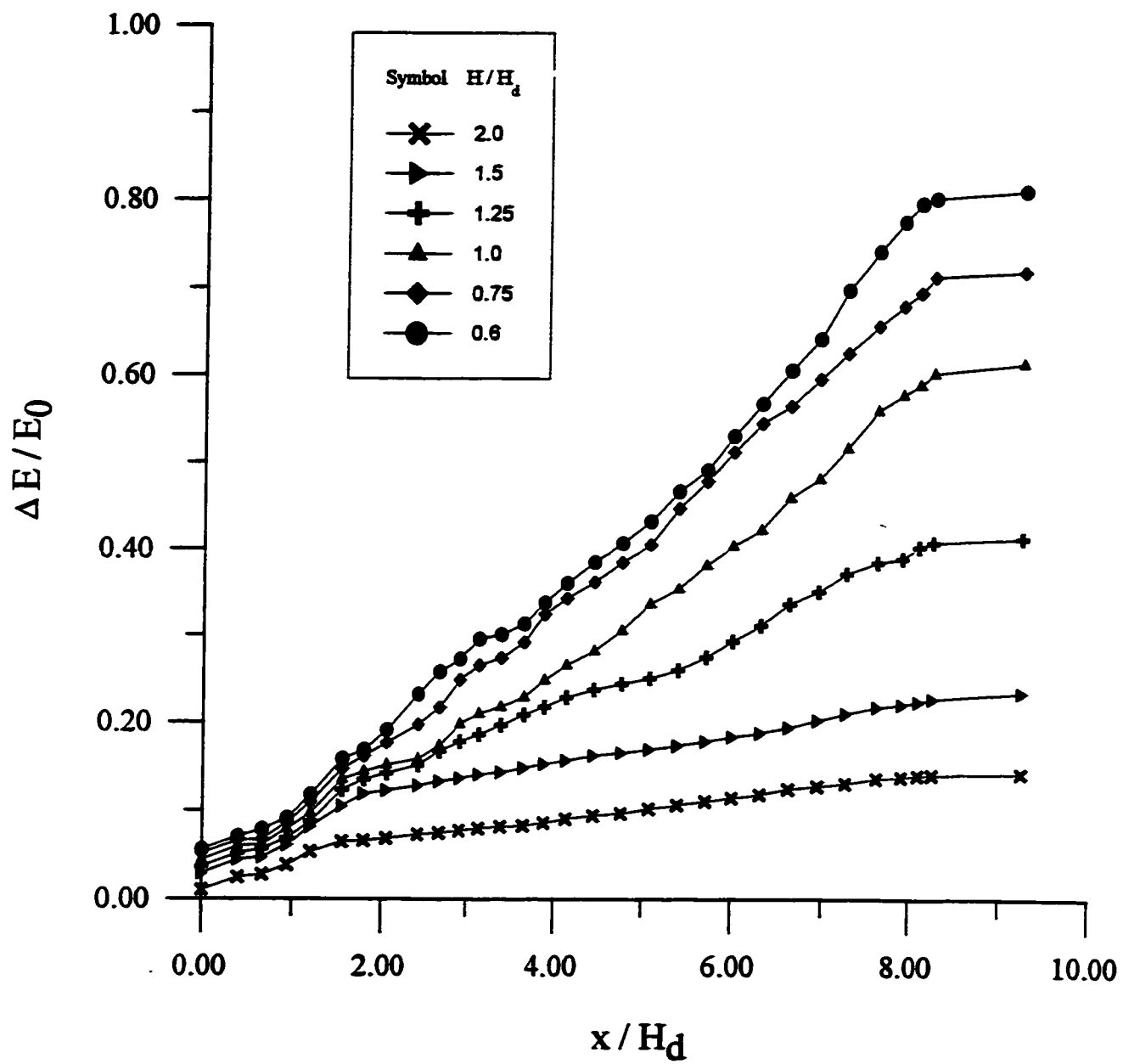


Fig. 6.15. $\Delta E/E_0$ versus x/H_d ; 45-CS-BS model

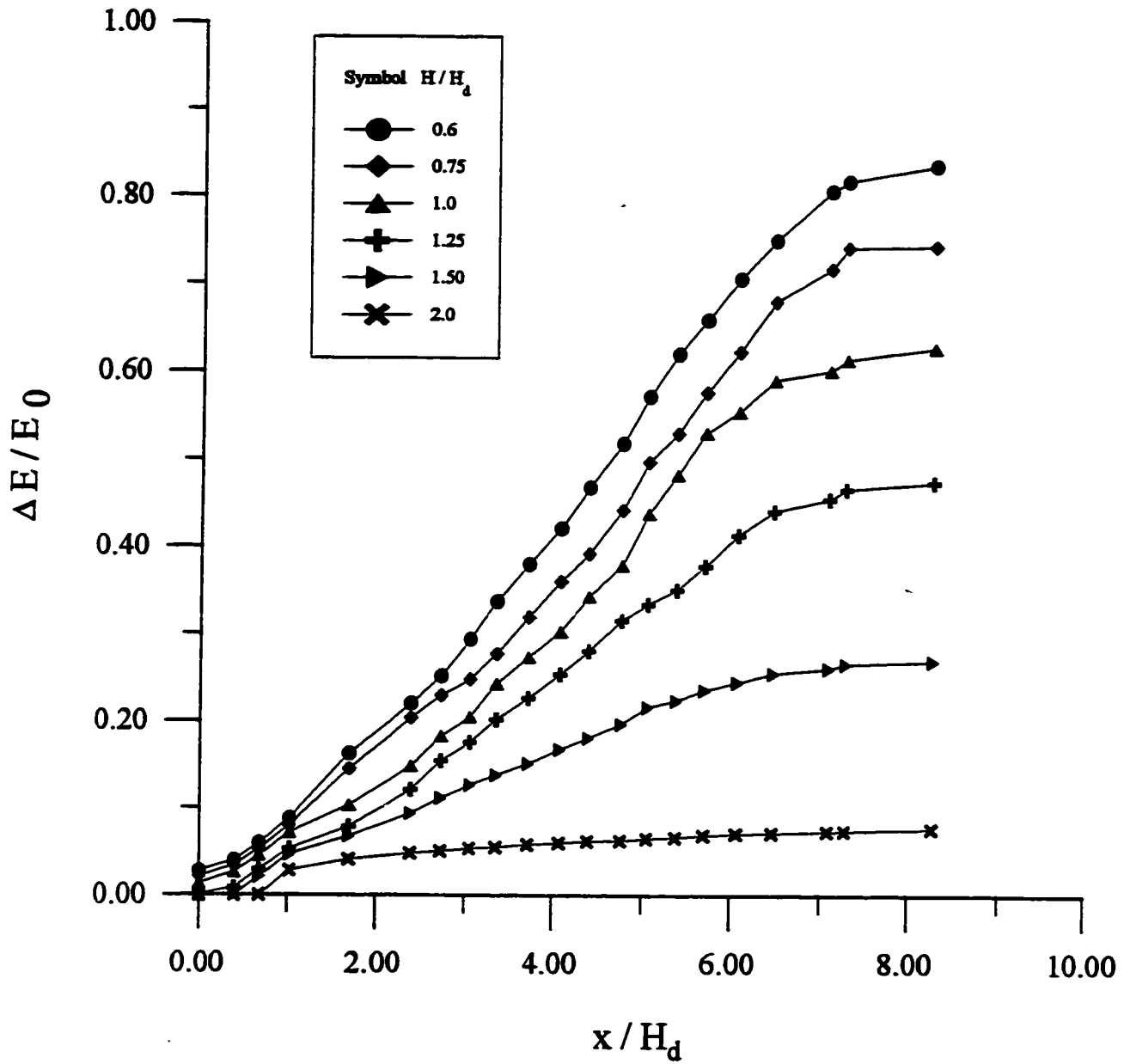


Fig. 6.16. $\Delta E/E_0$ versus x/H_d ; 50-CL-BL model

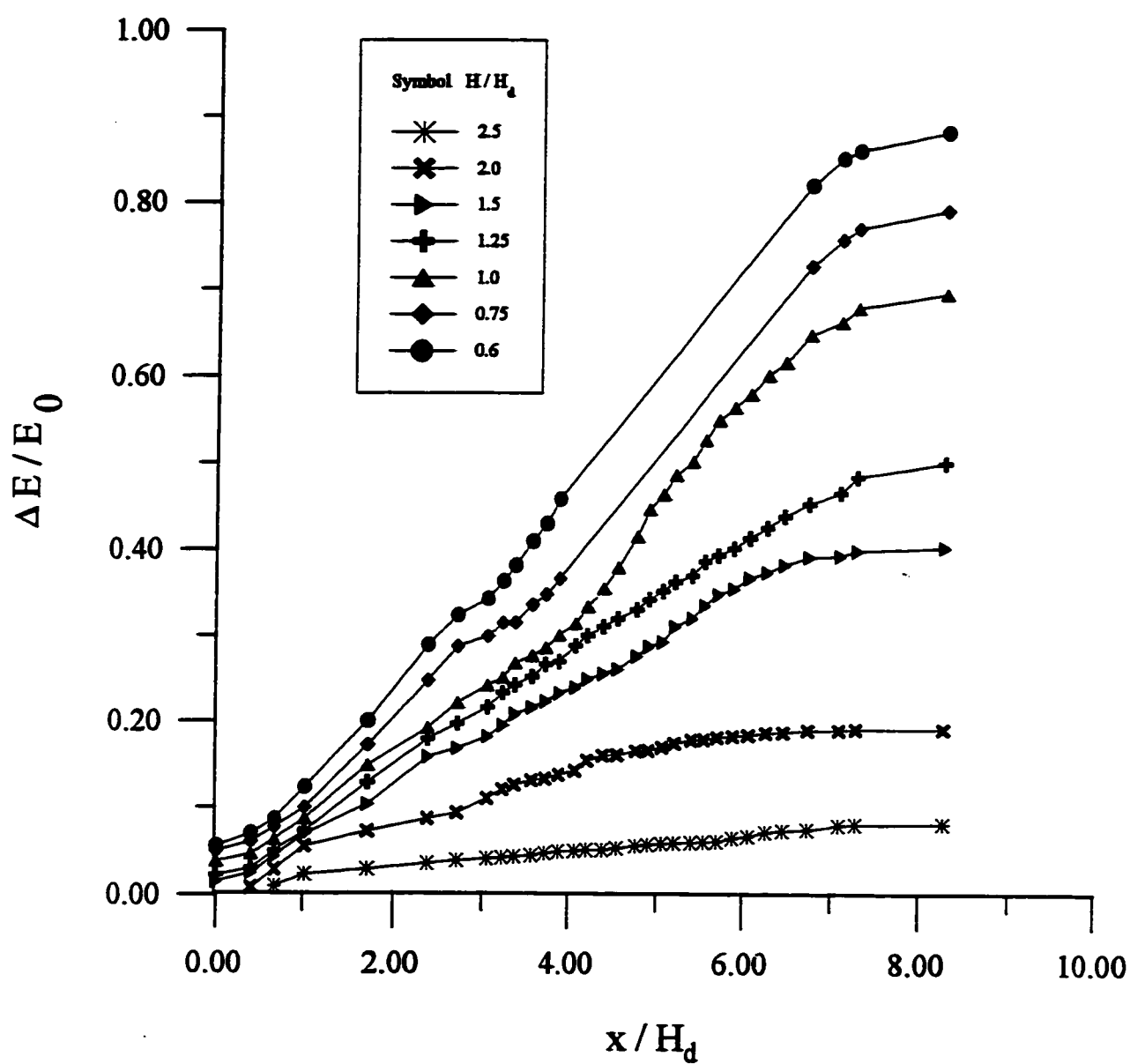


Fig. 6.17. $\Delta E/E_0$ versus x/H_d ; 50-CL-BS model

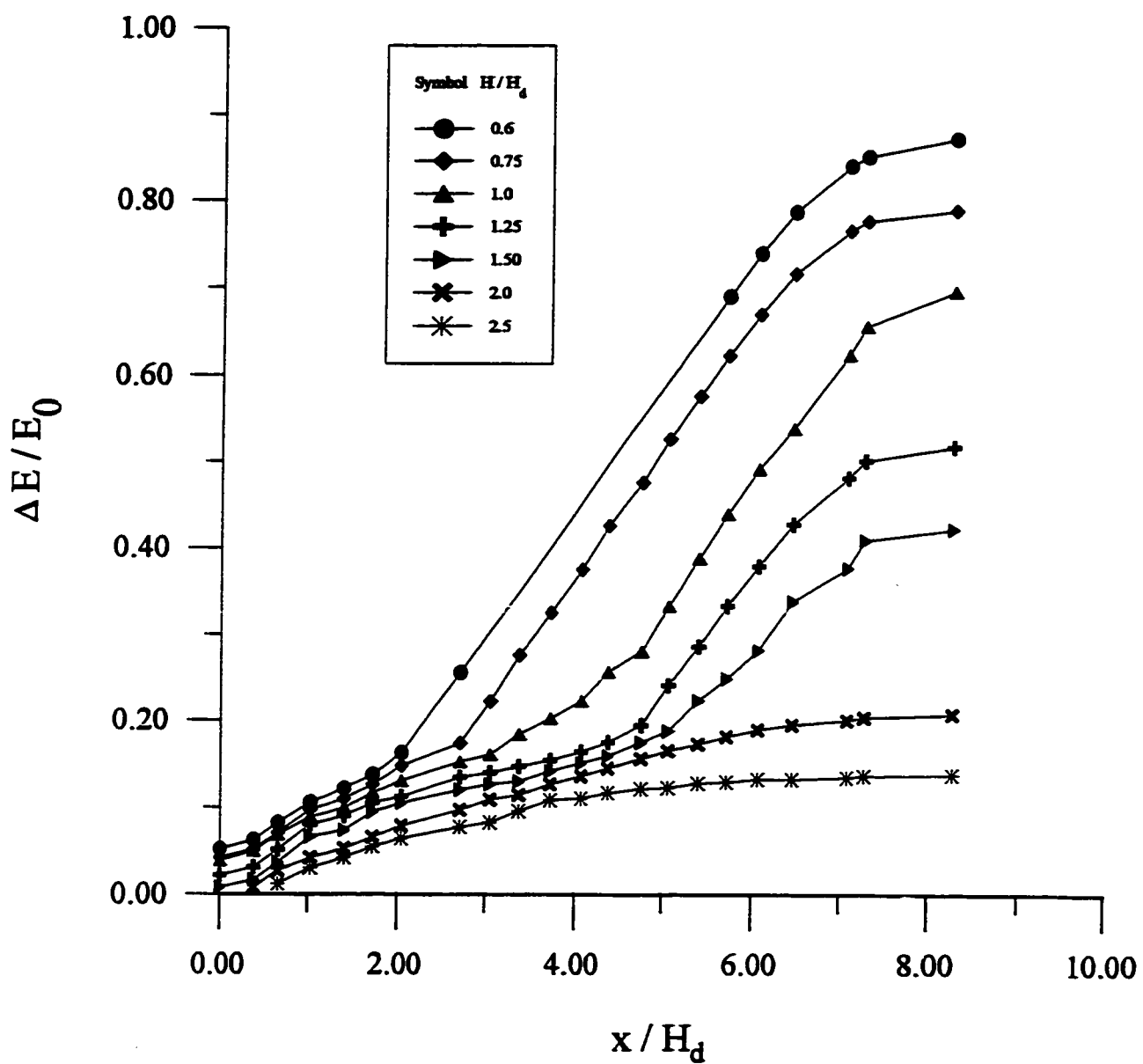


Fig. 6.18. $\Delta E/E_0$ versus x/H_d ; 50-CS-BL

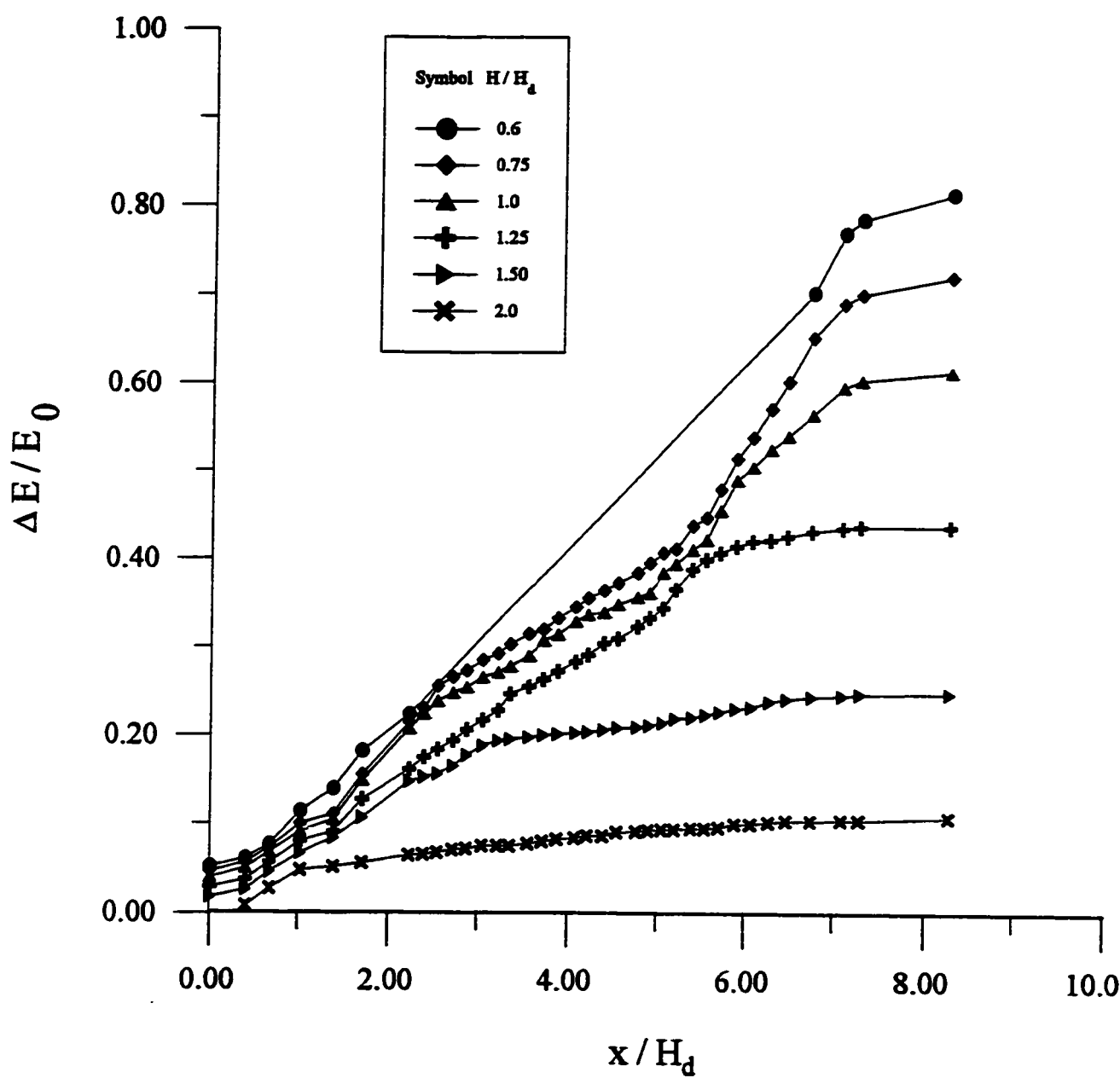


Fig. 6.19. $\Delta E/E_0$ versus x/H_d ; 50-CS-BS

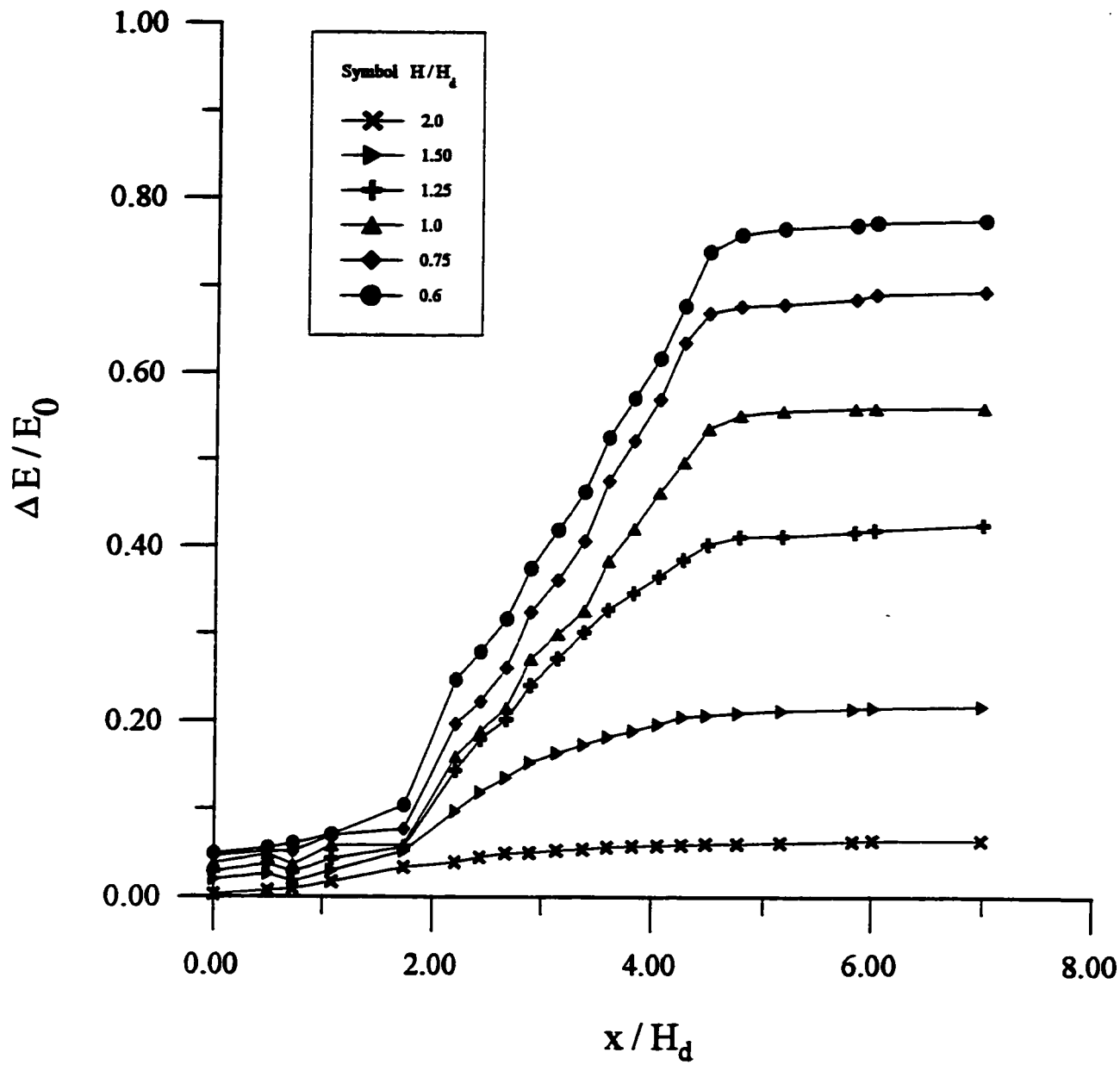


Fig. 6.20. $\Delta E/E_0$ versus x/H_d ; 60-CL-BL model

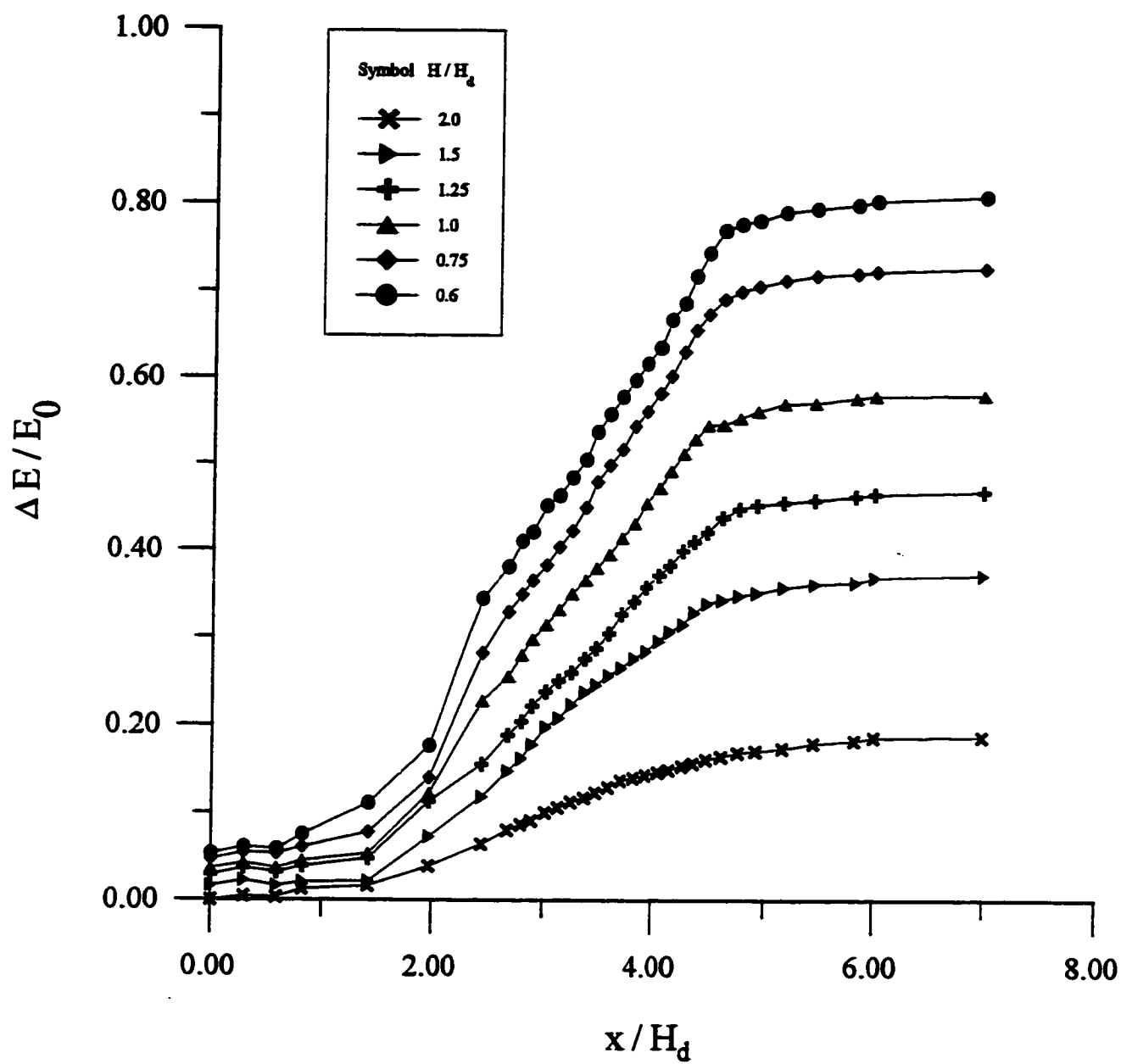


Fig. 6.21. $\Delta E/E_0$ versus x/H_d ; 60-CL-BS model

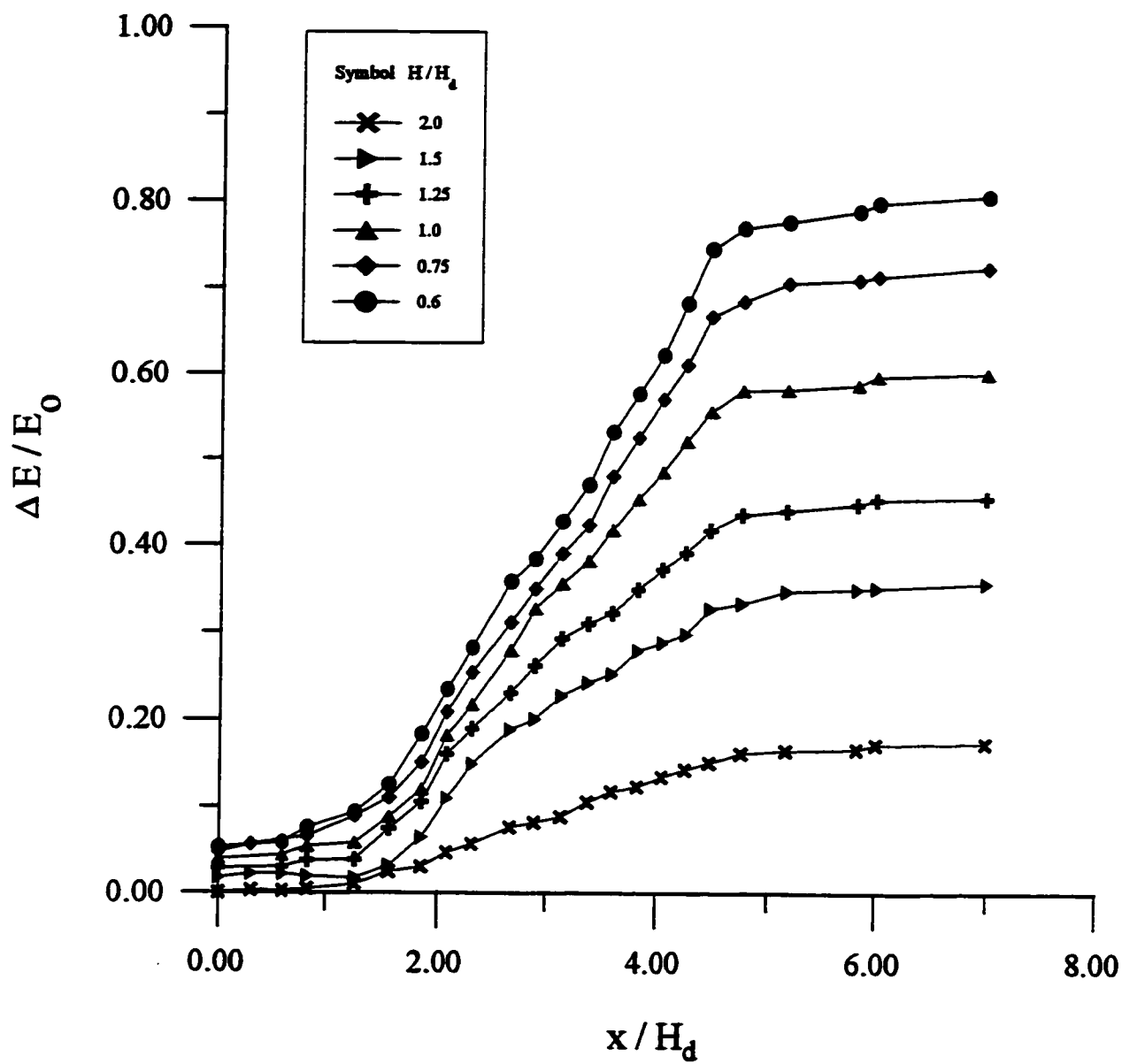


Fig. 6.22. $\Delta E/E_0$ versus x/H_d ; 60-CS-BL

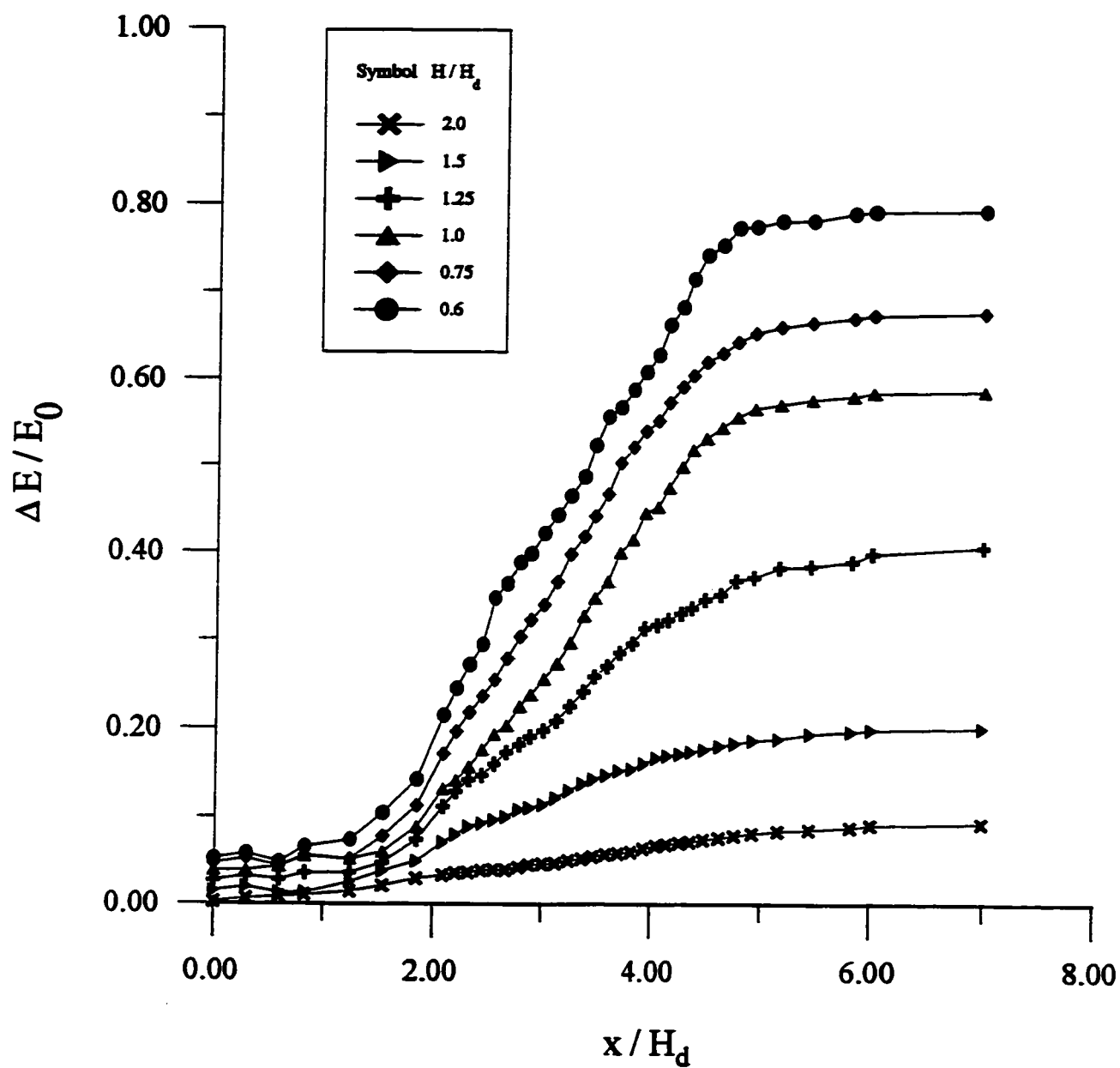


Fig. 6.23. $\Delta E/E_0$ versus x/H_d ; 60-CS-BS model

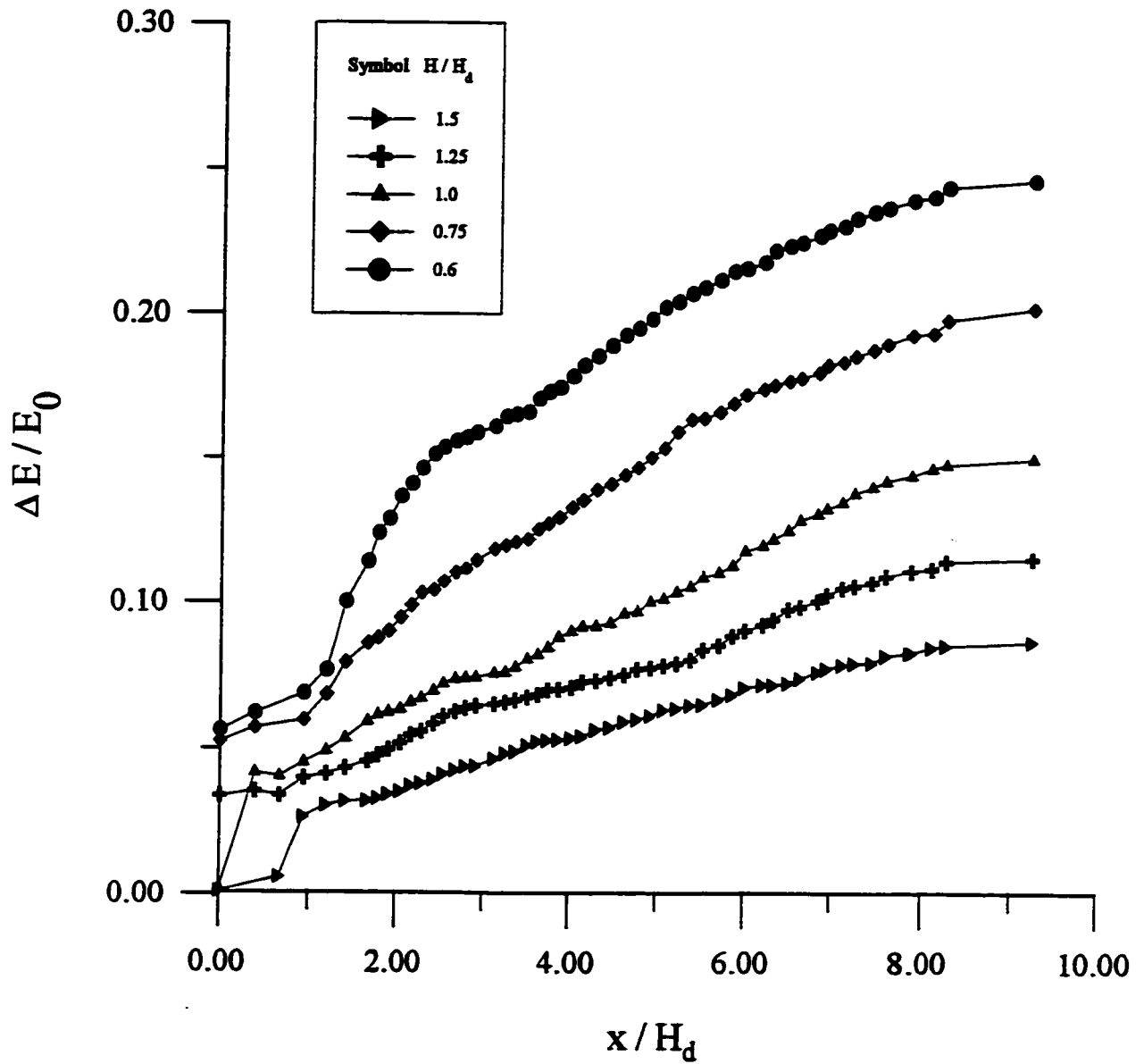


Fig. 6.24. $\Delta E/E_0$ versus x/H_d ; 45° smooth model

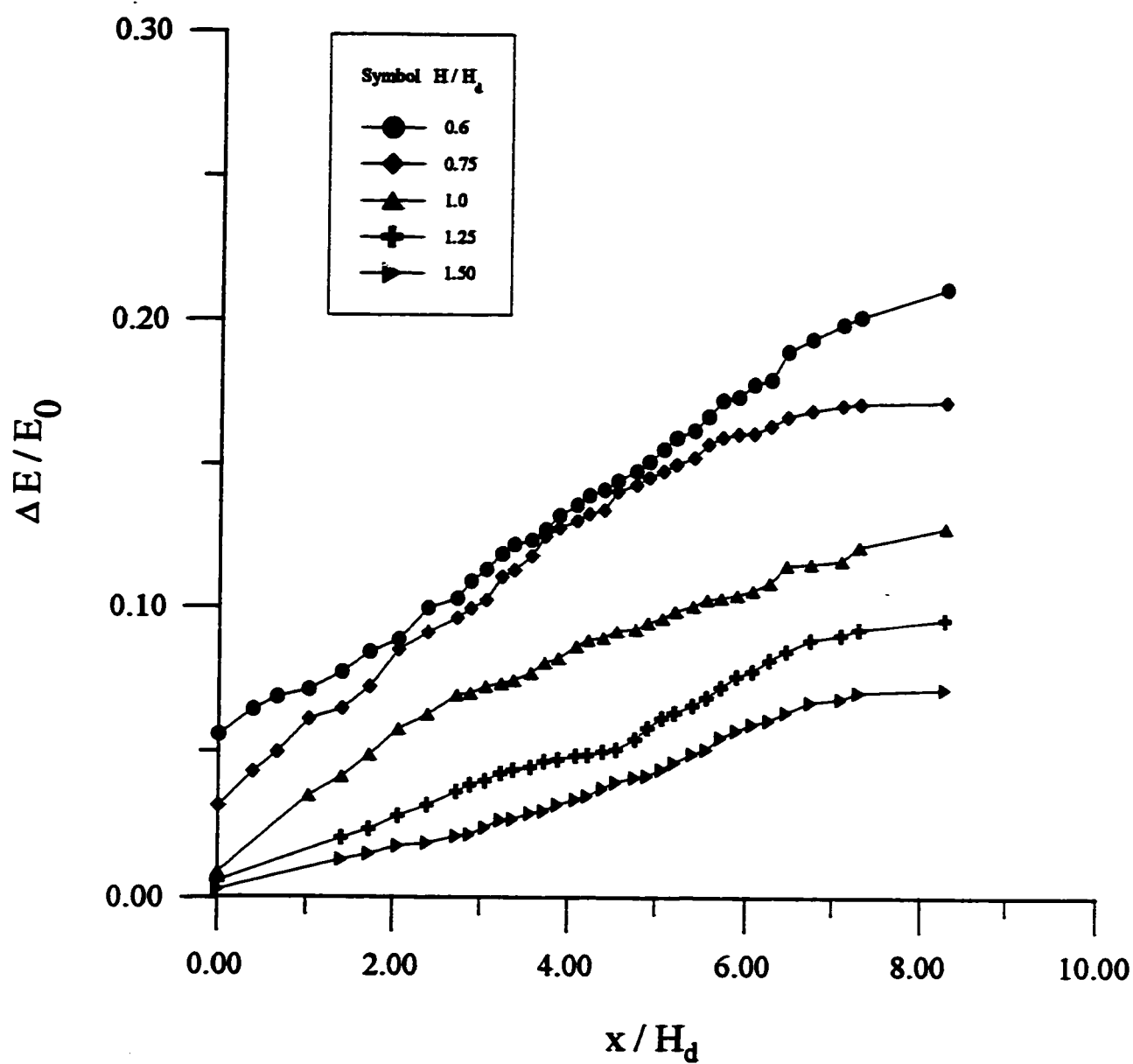


Fig. 6.25. $\Delta E/E_0$ versus x/H_d ; 50° smooth model

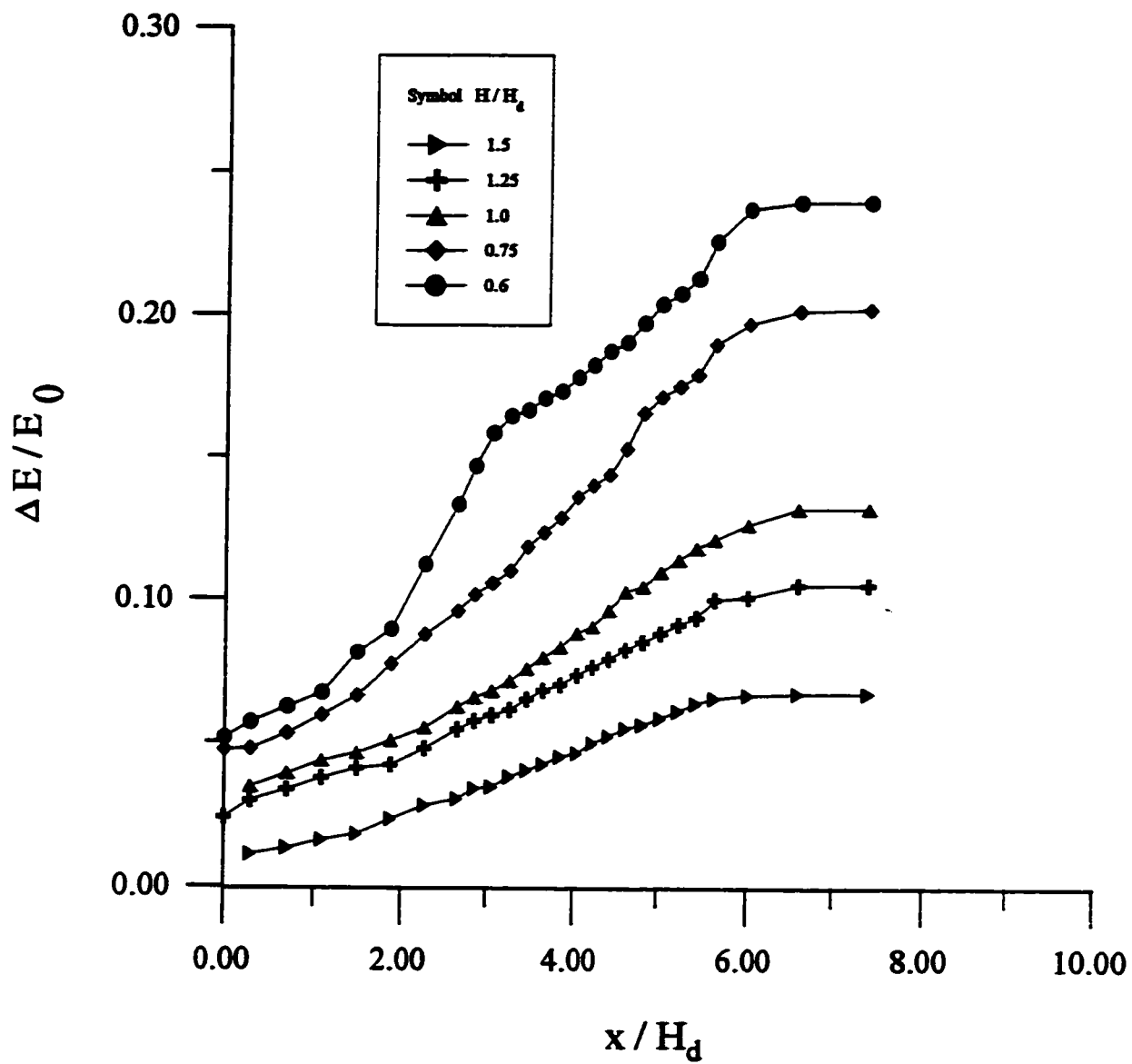


Fig. 6.26. $\Delta E/E_0$ versus x/H_d ; 60° smooth model

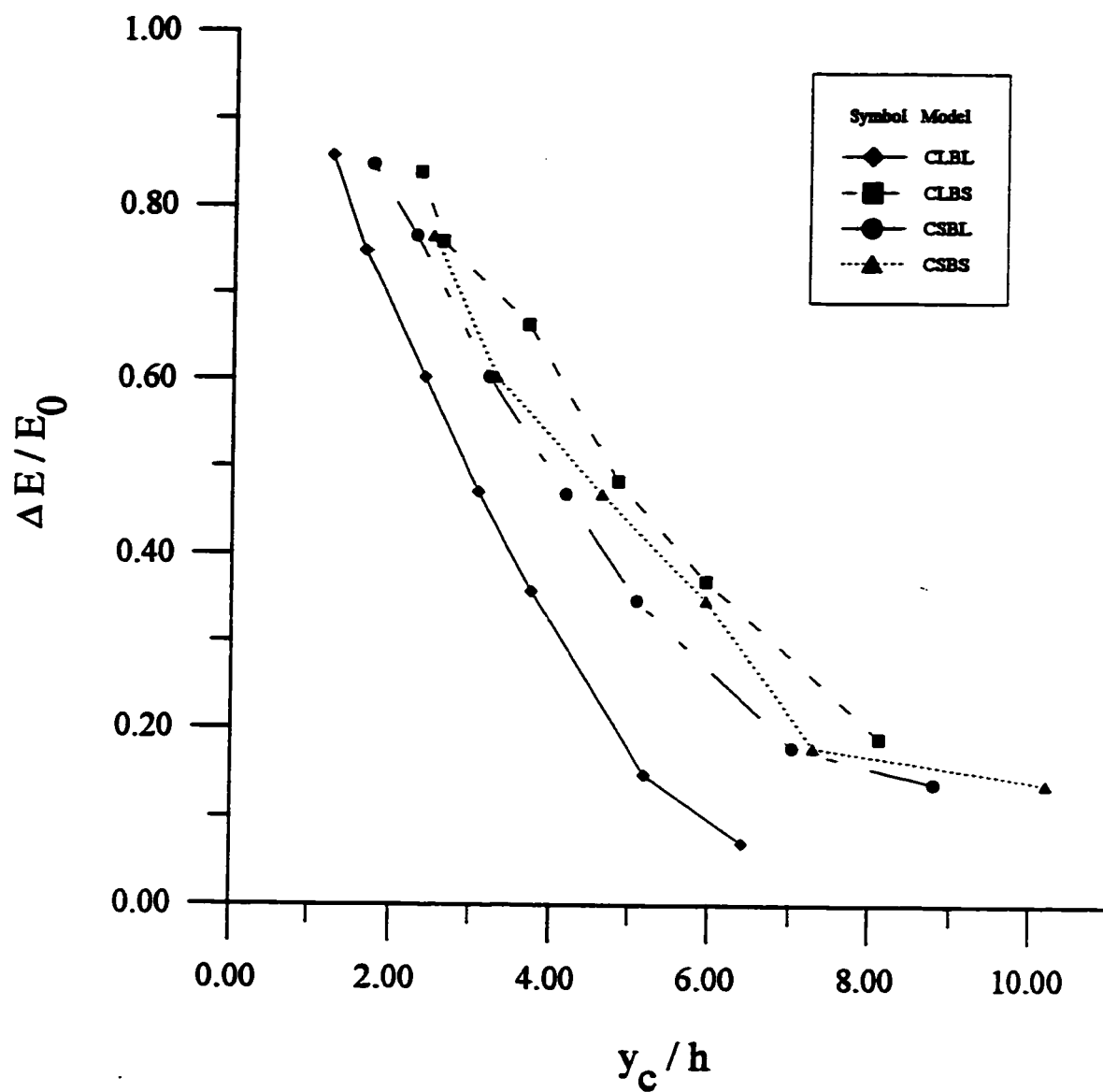


Fig. 6.27. $\Delta E/E_0$ versus y_c/h ; 45°stepped models

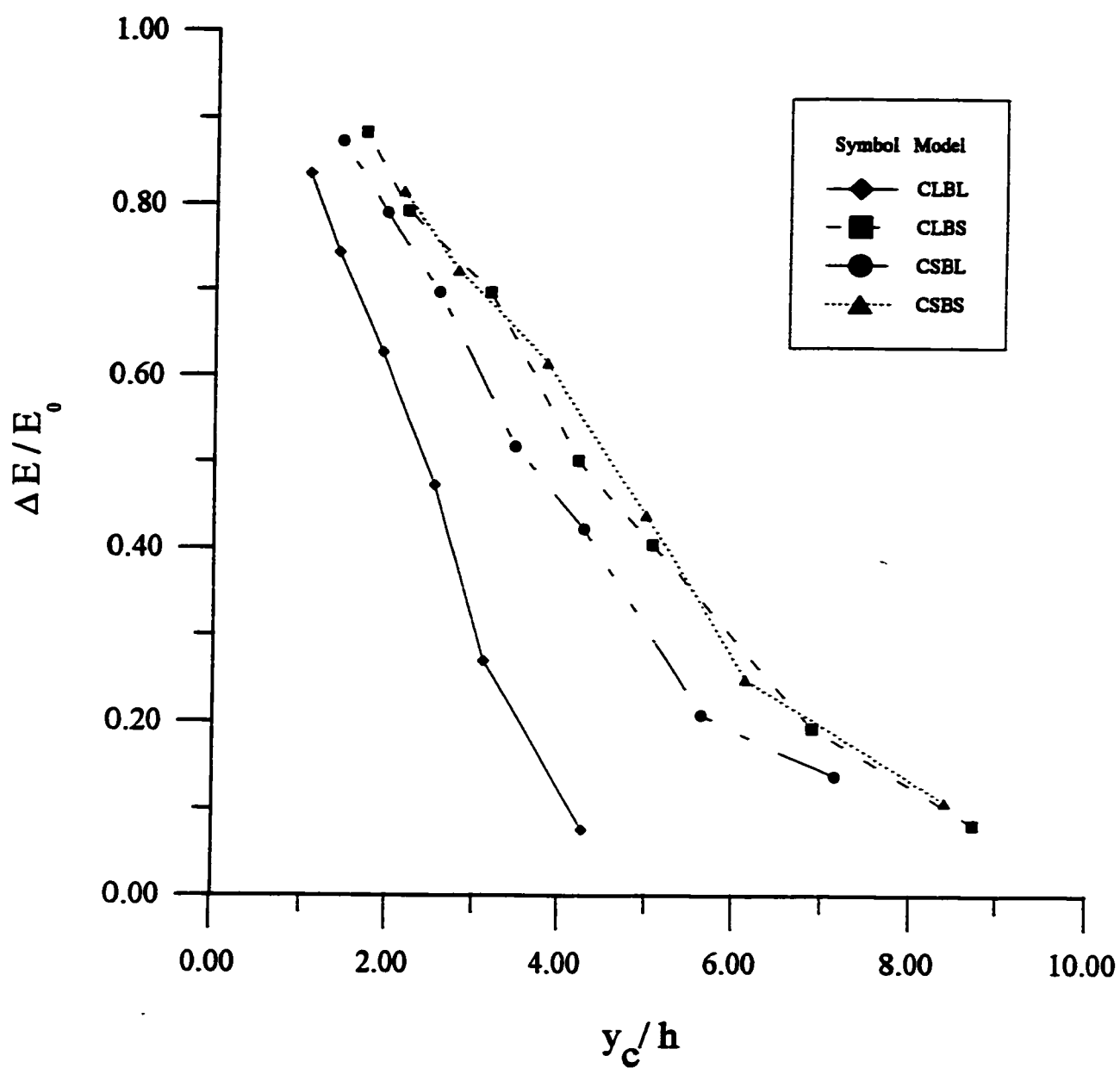


Fig. 6.28. $\Delta E/E_0$ versus y_c/h ; 50° stepped models

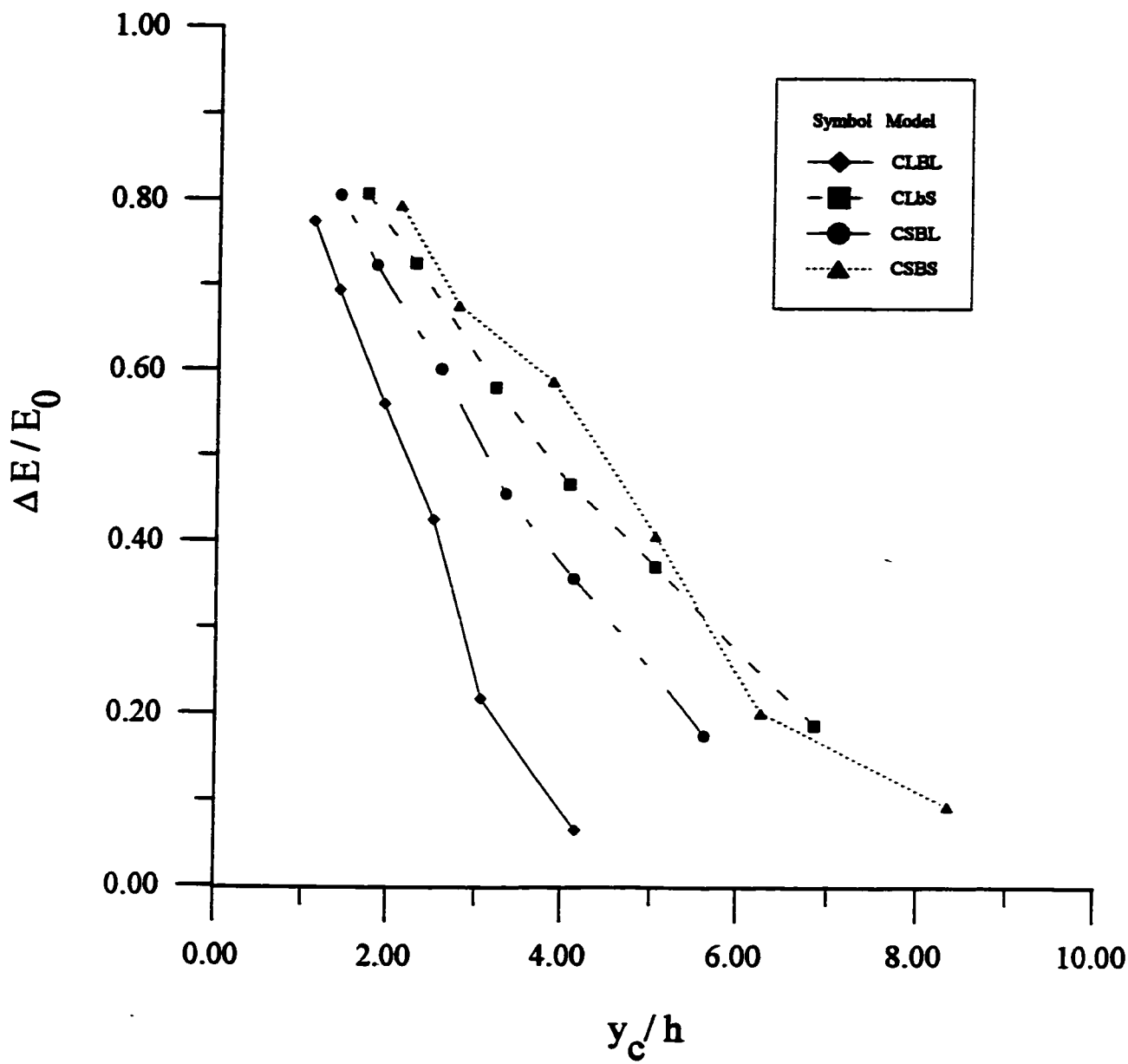


Fig. 6.29. $\Delta E/E_0$ versus y_c/h ; 60° stepped models

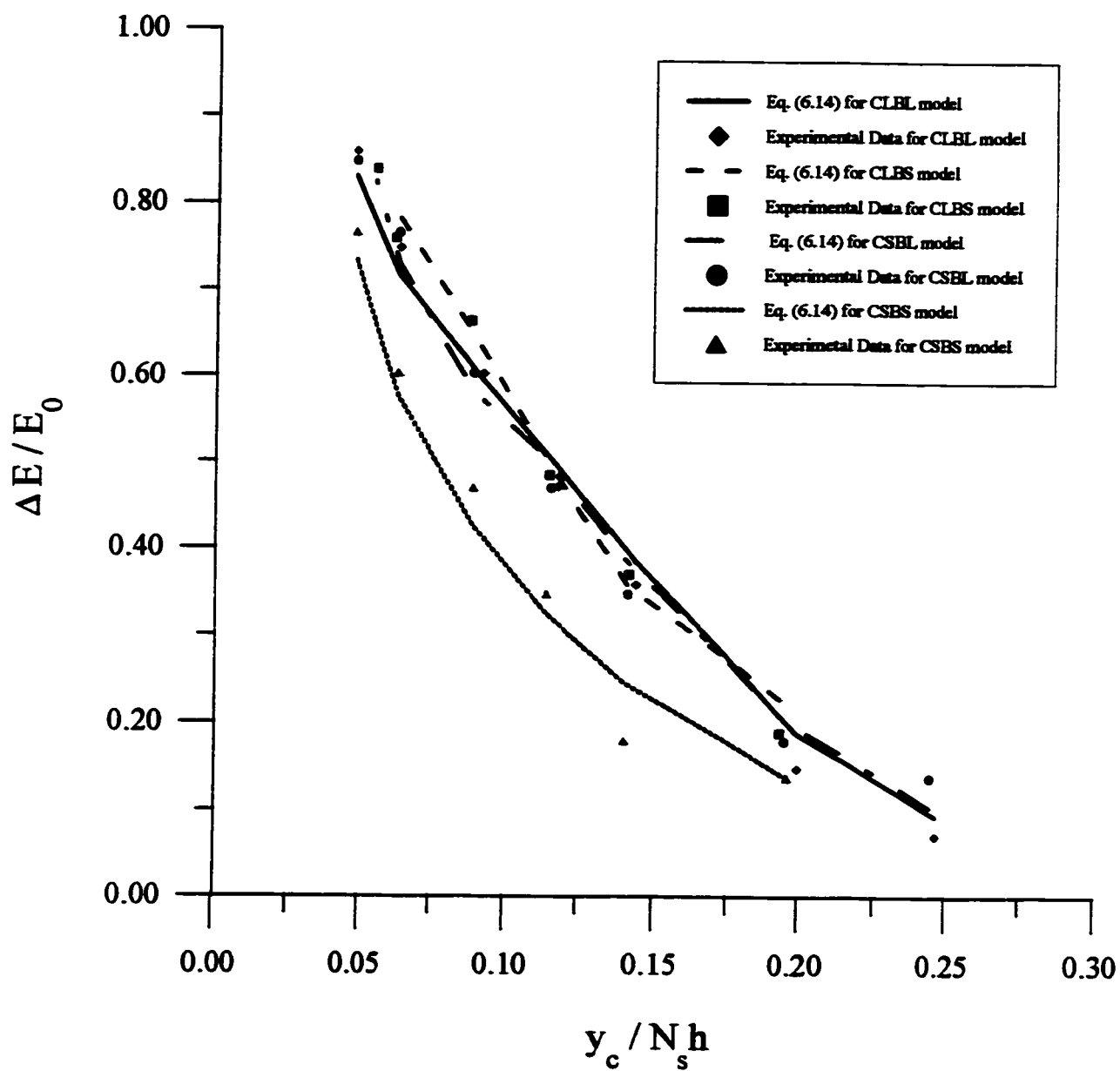


Fig. 6.30. Comparison of eq. (6.14) with experimental data 45° stepped models

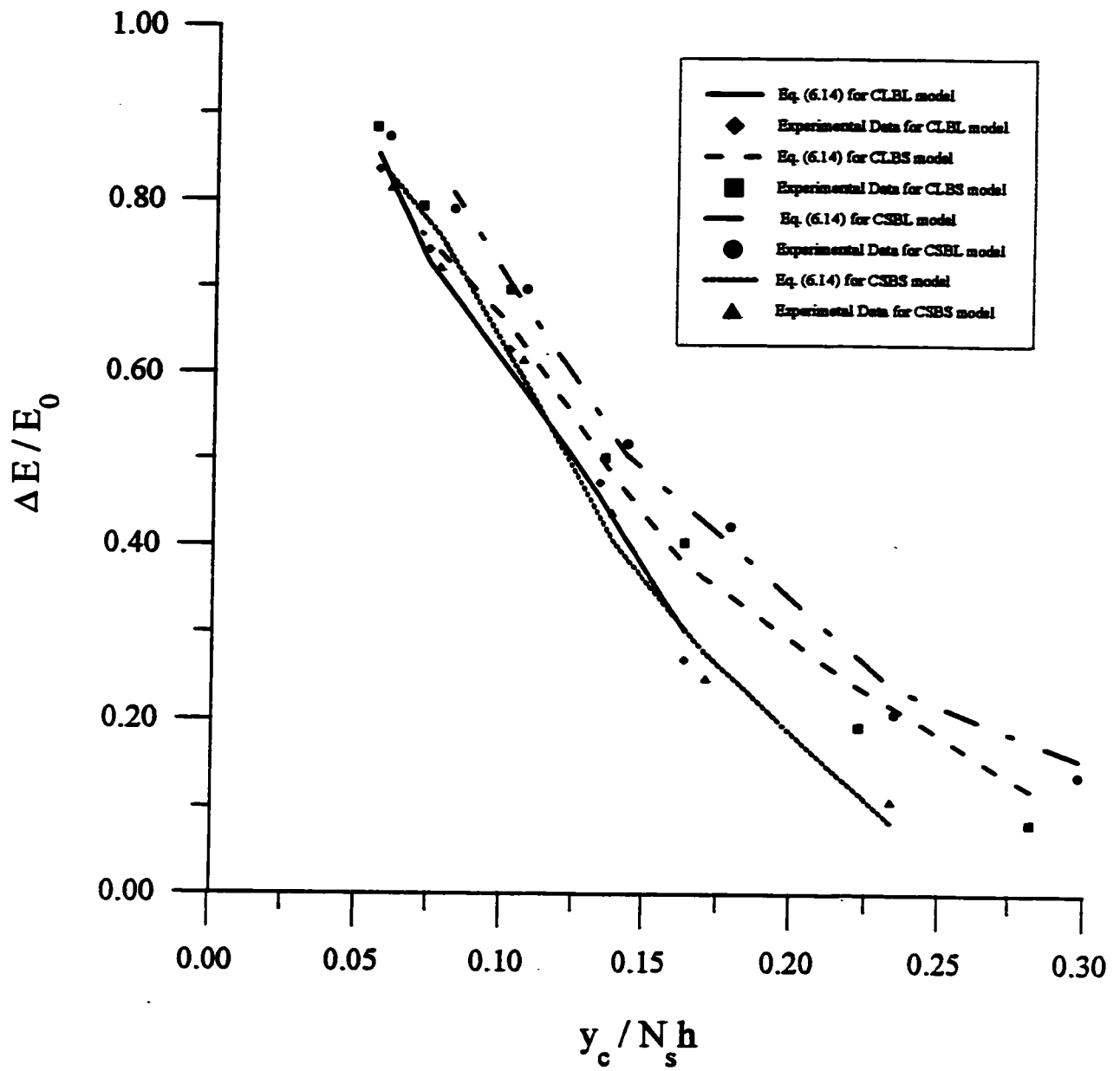


Fig. 6.31. Comparison of eq.(6.14) with experimental data 50° stepped models

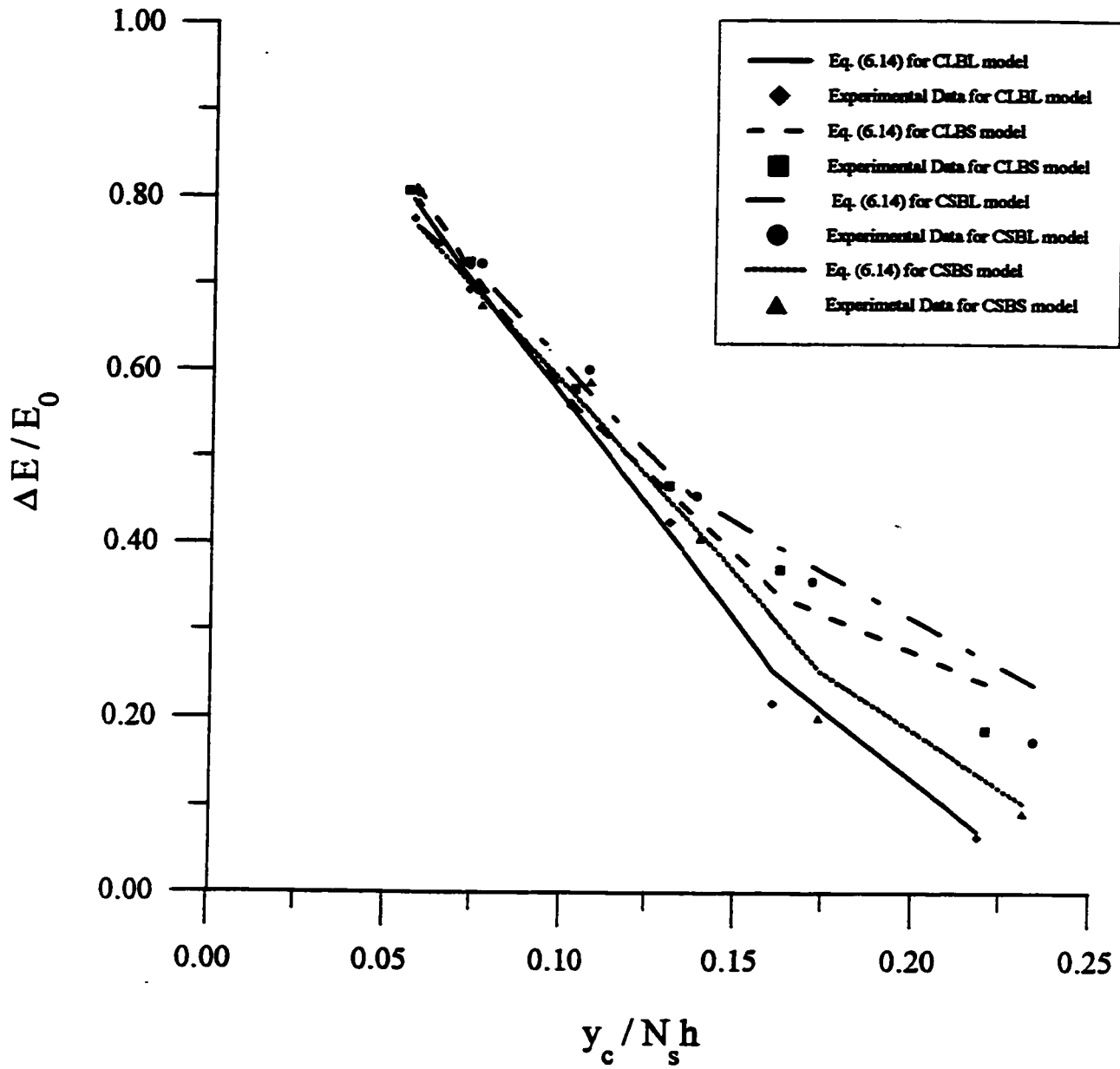


Fig. 6.32. Comparison of eq. (6.14) with experimental data 60° stepped models

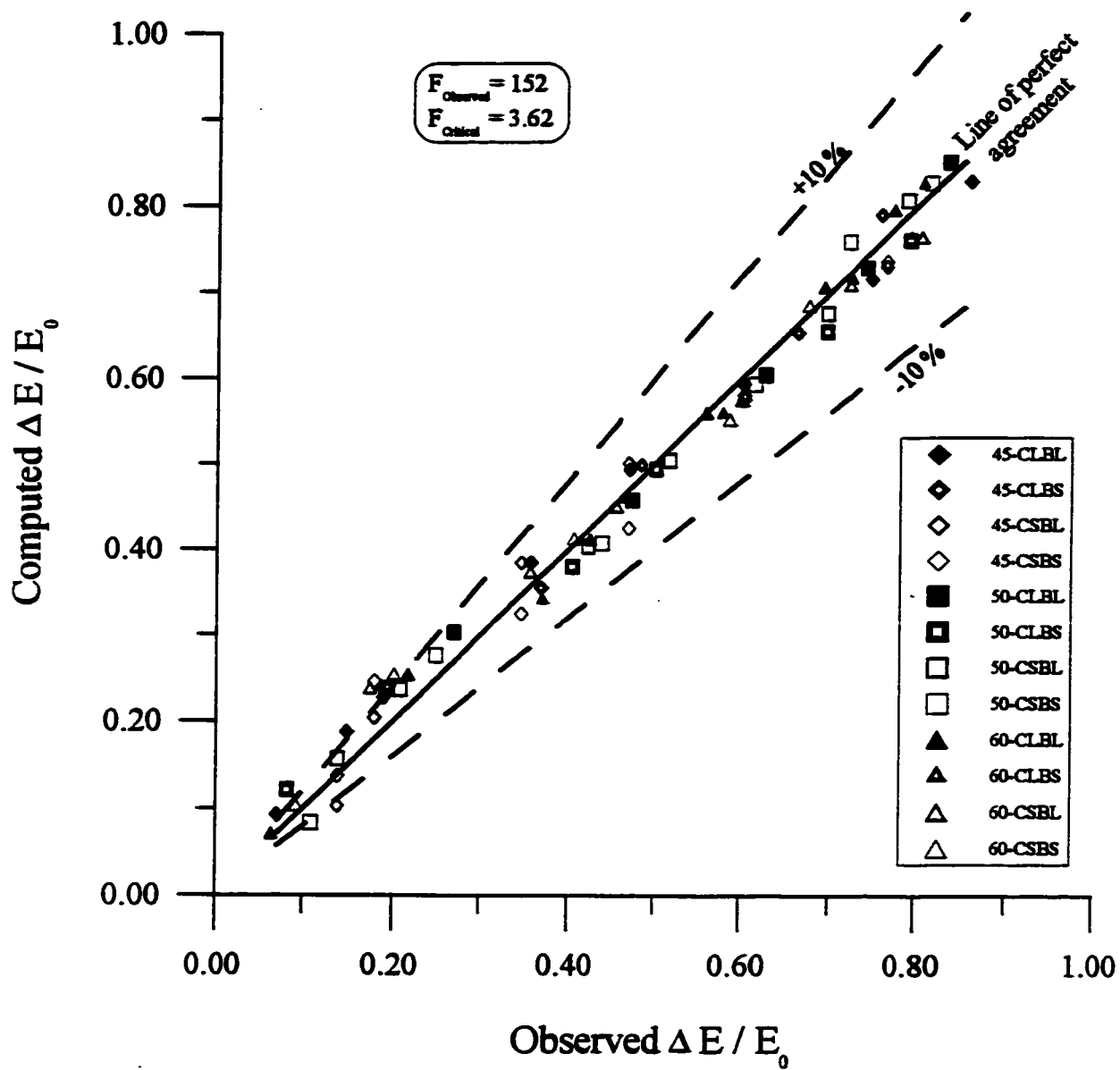


Fig. 6.33. Observed $\Delta E / E_0$ versus Computed $\Delta E / E_0$

Chapter 7

Numerical Modelling of Spillway Flow

7.1. Introduction

Virtually every phenomenon in nature, whether biological, geological, or mechanical, can be described with the aid of the law of physics, in terms of algebraic, differential, or integral equations relating various quantities of interest. Most engineers and scientists studying physical phenomena are involved with two major tasks:

1. Mathematical formulation of the physical process;
2. Numerical analysis of the mathematical model.

Development of the mathematical model of a process is achieved through assumptions concerning how the process works. In a numerical simulation, one uses a numerical method and a computer to evaluate the mathematical model and estimate the characteristics of the process.

While the derivation of the governing equations for most problems is not excessively difficult, their solution by exact methods of analysis is a formidable task. In such

cases, approximate methods of analysis provide alternative means of finding solutions. Among these, the finite difference method (FDM) and the variational methods, such as the Rayleigh-Ritz and Galerkin methods, are most frequently used in the literature. In the finite difference approximation of a differential equation, the derivatives in the latter are replaced by difference quotients (or the function is expanded in a Taylor series) that involve the values of the solution at discrete mesh points of the domain. The resulting algebraic equations are solved, after imposing the boundary conditions, for the values of the solution at the mesh points. This method suffers from the disadvantage that the approximation functions for problems with arbitrary domains are difficult to construct.

The finite element method (FEM) overcomes the disadvantage of the traditional methods, such as FDM, by providing a systematic procedure for derivation of the approximation functions over subregions of the domain. In general, in the FEM, a given domain is divided into subdomains, called finite elements, and an approximate solution to the problem is developed over each of these. The subdivision of a whole into parts has two advantages:

1. It allows accurate representation of complex geometries and inclusion of dissimilar materials.
2. It enables accurate representation of the solution within each element, to bring out local effects (e.g. large gradients of the solution).

7.2. Literature Review

The analysis of water flow over a spillway presents a difficult mathematical problem because of the following :

1. the flow is non-linear;
2. the free water surface is unknown beforehand;
3. the flow is subcritical upstream of the crest of the spillway;
4. the flow is supercritical downstream of the crest of the spillway; and
5. there is a control section near the crest.

Depending on whether the flow is supercritical or subcritical, Southwell and Varisey (1946) introduced two different iterative methods. Although they apply a relaxation method to fluid streamlines, they found difficulty in determination of the free stream boundary. Ikegawa and Washizu (1973) introduced a new iteration procedure by combining the use of the variational principle (VP) and the FEM. In this procedure the stream function and the free surface profile are independent quantities. The result obtained by their model shows good agreement with those obtained using an empirical formula.

Chan et al. (1973) using VP and FEM, studied free surface ideal fluid flows. Their study was focussed on a steady two-dimensional, inviscid fluid with a free surface boundary. To find the location of the free surface, they introduced an iterative algorithm. Depending on the number of elements introduced, 10-20 iterations were normally required for finding the exact location of the free surface. In their study the entire domain had only 72 elements.

Taylor et al. (1978) used FEM to study models of turbulent flow. Dao-Yang and Man-Ling (1979) developed a two dimensional mathematical model for flow over a spillway. They developed a synchronous iteration to find the discharge and location of the free surface. Bathe and Khoshgoftar (1979) presented an iterative procedure for finite element analysis of free seepage problems. The procedure does not need

changing in the finite element mesh, but instead uses a non-linear pressure dependent permeability.

Salonen et al. (1981) used FEM to analyze a two-dimensional incompressible irrotational inviscid fluid flow with the velocity components as basic unknowns. By using a modified least squares method, i.e. a combination of the conventional least squares and subdomain collocation method, they found a remarkable increase in accuracy compared with the conventional method. Bettess and Bettess (1983) used variable geometry isoparametric elements and a variational method to solve free surface problems. Since this method allows the free surface to be curved rather than be piecewise straight, the representation is more accurate.

O'Carroll and Toro (1984) presented an algorithm for flow over a weir. In their algorithm they tried to solve the stream function and free surface location simultaneously. This is a rapid convergent iterative procedure for computing the correct value of critical discharge. Rasanen and Salonen (1984) using FEM suggested the importance of curvature of the free surface on velocity. Rasanen and Salonen (1985) used the velocity components (u,v-formulation) as basic unknowns and FEM to develop an algorithm for the solution of free-surface problems. The basis of the method is the modified least squares function. They also used an iteration method to find the location of free surface.

Heng et al. (1986), by using a different iterative approach and FEM, tried to solve the flow under a sluice gate, i.e basically a free surface flow. Their study showed that the results are very sensitive to the finite element grid used. Aitchison and Karageorghis (1988) used the boundary element method (BEM) to study the conventional flow of an incompressible inviscid fluid past a plate in a channel of finite width and infinite

length. The results show more accuracy because of the property of BEM to deal only with the boundary of the region under consideration. Berger and Winant (1991) developed a one-dimensional numerical model for shallow water, termed CURVE1D. This model however has a significant bed curvature limitation. The limit is $-2 < \kappa y \leq 0.54$, where κ is the bed curvature and y is the depth.

Henderson et al. (1991), by a direct mathematical approach instead of trial-and-error methods, presented a two-dimensional model for free surface flow over a weir. The model mostly was developed for studying cavitation damage. They used BEM for the numerical approach. Ratmanan and Engelman (1993), using FEM and the Galerkin approach, presented an algorithm to simulate viscous free-surface flows. This algorithm is valid only for a continuous boundary condition, and for fluid with high viscosity. Bettess et al. (1994) used FEM to find the flow profile along a crump weir. They adopted a rectangular element with variable geometry. By using the Newton-Raphson iteration method, the location of the free-surface was found. Khan and Steffler (1996) verified the applicability of a 1-D model velocity average and momentum (VAM) equations on flow over curved beds. This method can be used even for steep (near vertical) slopes. Using Montes (1994) experimental data, they found the VAM equations model extremely well.

In spite of many researchers having studied fluid problems using FEM and BEM, there is no specific procedure for solving all free-surface problems. This is because there is a moving boundary at the free-surface. The location of the free-surface is very important in the study of flows over spillways.

7.3. Proposed Numerical Model

In order to establish a stable upstream boundary condition, a large-volume reservoir is assumed. It is also assumed that flow in the domain can be described by velocities, by location of the free surface, and by discharge. It is necessary to impose certain conditions upon the flow to derive the governing equations. First, it can be assumed that the flow out of the reservoir is time-independent. This assumption means that the water level of the reservoir (the stagnation level) does not change with time and the potential energy of the reservoir is therefore time-independent. Furthermore, it is assumed the spillway flow is (i) two-dimensional; (ii) steady; (iii) incompressible; (iv) non-viscous; and (v) irrotational on smooth and rotational on stepped models.

For flow downstream of the ogee-profile stepped spillway, the acceleration and boundary layer development are taking place on the flow goes down the spillway face. When the boundary layer meets the free surface, the flow becomes fully developed. At this point, the point of inception, the phenomenon of air entrainment or *insufflation* occurs. Due to the mixture of air and water after the point of inception, defining a precise free surface is very difficult in practice.

The analysis of fluid flow problems, principally two-dimensional ones, is facilitated by the concomitant consideration of a scalar. The stream function Ψ is a scalar field function. This parameter was introduced by d'Alembert (1752) and was extensively used by Lagrange (1781). The intimate relation between the stream function and the continuity relation in two-dimensional fluid motion was demonstrated by Rankine (1864). The stream function for two-dimensional flow of an incompressible fluid is defined by:

$$u = \frac{\partial \Psi}{\partial y} \quad (7.1)$$

and

$$v = -\frac{\partial \Psi}{\partial x} \quad (7.2)$$

in which

u = velocity in the x-direction; and

v = velocity in the y-direction.

The stream functions automatically satisfy continuity. Also in a two-dimensional flow

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \omega \quad (7.3)$$

in which

ω = vorticity.

By substituting eq.(7.1) and eq.(7.2) in eq.(7.3), one gets:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\omega \quad (7.4)$$

This is the governing equation for flow over a stepped spillway. When the flow is irrotational $\omega = 0$, and eq.(7.4) becomes the Laplace equation. For rotational flow, $\omega \neq 0$, and eq.(7.4) is termed the Poisson equation. The singular vortex elements in the form of elements of circulation, are generated on solid walls in order to destroy slip velocities.

7.3.1. Boundary Conditions

The boundary conditions are as follows:

1. At the free surface, it is necessary to satisfy:

(i)

$$\Psi = 0 \quad (7.5)$$

and

(ii) An additional boundary condition on the free surface must be satisfied. The pressure is prescribed there and is taken as atmospheric. Bernoulli's law then gives the following boundary condition:

$$\left[\left(\frac{\partial \Psi}{\partial x} \right)^2 + \left(\frac{\partial \Psi}{\partial y} \right)^2 \right]^{1/2} = \frac{\partial \Psi}{\partial n_i} = -\sqrt{2gf(x)} \quad (7.6)$$

in which

n_i = the normal outward vector on the boundary;

$f(x)$ = the distance of the free surface measured from the stagnation level (Fig.7.1); and

2. At the bottom of the channel:

$$\Psi = q \quad (7.7)$$

3. It is also assumed the flow is uniform on the far downstream.

4. The final assumption is that Ψ varies linearly along the boundaries.

The goal of this numerical study is to predict the location of the free surface for a given discharge q and flow field. The following procedure is suggested:

1. for an assumed q , estimate the corresponding water surface profile;
2. find the function $f(x)$;
3. by using FEM find the value of Ψ at nodal points in the flow region;
4. calculate $\partial \Psi / \partial n_i$ using Ψ 's obtained in step 3;

5. substitute step 4 in eq. (7.6) find a new value for $f(x)$;
6. repeat steps 3, 4, and 5 until the profile of free boundary surface is found to have converged.

The **SPIL-FLOW** programme has been developed for the above algorithm.

7.4. Finite Element Method

The finite element method (FEM) is a simple two-dimensional geometric shape that can be used to approximate a given two dimensional domain as well as the solution over it. The FEM is endowed with three basic features that account for its superiority over other competing methods:

- (i) a geometrically complex domain of the problem is represented as a collection of geometrically simple subdomains, called *finite elements*.
- (ii) over each finite element, the approximation functions are derived using the basic idea that any continuous function can be represented by a linear combination of algebraic polynomials; and
- (iii) algebraic relations among the undetermined coefficients (i.e., nodal values) are obtained by satisfying the governing equations, often in a weighted-integral sense, over each element.

Therefore, the ability to represent domains with irregular geometries makes the finite element method a valuable practical tool for the solution of boundary value problems. In FEM the approximation functions are often taken to be algebraic polynomials, and the undetermined parameters represent the values of the solution at a finite number of preselected points, called *nodes*, on the boundary and in the interior of the element. The approximation functions are derived using concepts from interpolation theory, and therefore called *interpolation functions*. One finds the degree of the interpolation

functions depends on the number of nodes in the element and the order of the differential equation being solved.

In a two-dimensional problem, not only one seeks an approximate solution to a given problem on a domain but also one approximates the domain by a suitable finite element mesh. Consequently, in the finite element analysis of two dimensional problems, the approximation errors are due to the approximation of the solution as well as discretization errors due to approximation of the domain. The finite element mesh consists of simple two dimensional elements that allow unique derivation of interpolation functions. The elements are connected to each other at nodal points on the boundaries of the elements.

The major steps are as follows:

1. discretization of the domain into a set of finite elements;
2. weighted-integral formulation of the governing differential equation (7.4);
3. derivation of finite element interpolation functions;
4. development of the finite element model using the weighted-residual form;
5. assembly of finite elements to obtain the global system of algebraic equations;
6. imposition of boundary conditions;
7. solution of equations; and
8. post-computation of solution and quantities of interest.

These steps will be explained in detail in the following.

7.4.1. Finite Element Discretization

In a two-dimensional problem, the interpolation functions depend on:

- (i) the number of nodes in the element; and
- (ii) the shape of the element.

The shape of the element must be such that its geometry is uniquely defined by a set of points, which serve as the element nodes in the development of the interpolation functions.

The representation of a given region by a set of elements is an important step in finite element analysis. The choice of element type, number of elements, the density of elements depends on:

- (i) the geometry of the domain;
- (ii) the problem to be analyzed; and
- (iii) the degree of accuracy desired.

The general rules of mesh generation for element formulation include the following:

1. select elements that characterize the governing equations of the problem;
2. the number, shape, and type (i.e., linear or quadratic) of elements should be such that the geometry of the domain is represented as accurately as desired;
3. the density of elements should be such that regions of large gradients of the solution are adequately modeled (i.e., more or higher-order elements should be used in regions of large gradients); and
4. mesh refinements should be gradually from high-density regions to low-density regions.

7.4.2. Weighted-Residual Form

In the development of the weighted form, one needs only consider an arbitrary typical

element. Assuming that Ω^e is such an element (Fig.7.2), one can develop the finite element model of eq.(7.4) over Ω^e .

To develop the weighted-residual form for eq.(7.4) one may proceed as follows:

i) Multiply eq.(7.4) with a weight function w , which is assumed to be once-differentiable with respect to x and y , and then integrate the resulting equation over the element domain Ω^e :

$$\int_{\Omega^e} w \left[\frac{\partial}{\partial x} \left(w \frac{\partial \Psi}{\partial x} \right) + \frac{\partial}{\partial y} \left(w \frac{\partial \Psi}{\partial y} \right) \right] dx dy = 0 \quad (7.8)$$

ii) Distribute the differentiation equally between Ψ and w . To achieve this, one can integrate the first two terms in eq.(7.8) by parts. First note the identities:

$$\frac{\partial}{\partial x} \left(w \frac{\partial \Psi}{\partial x} \right) = \frac{\partial w}{\partial x} \frac{\partial \Psi}{\partial x} + w \frac{\partial}{\partial x} \left(\frac{\partial \Psi}{\partial x} \right) \quad (7.9)$$

or

$$-w \frac{\partial}{\partial x} \left(\frac{\partial \Psi}{\partial x} \right) = \frac{\partial w}{\partial x} \frac{\partial \Psi}{\partial x} - \frac{\partial}{\partial x} \left(w \frac{\partial \Psi}{\partial x} \right) \quad (7.10)$$

and

$$\frac{\partial}{\partial y} \left(w \frac{\partial \Psi}{\partial y} \right) = \frac{\partial w}{\partial y} \frac{\partial \Psi}{\partial y} + w \frac{\partial}{\partial y} \left(\frac{\partial \Psi}{\partial y} \right) \quad (7.11)$$

or

$$-w \frac{\partial}{\partial y} \left(\frac{\partial \Psi}{\partial y} \right) = \frac{\partial w}{\partial y} \frac{\partial \Psi}{\partial y} - \frac{\partial}{\partial y} \left(w \frac{\partial \Psi}{\partial y} \right) \quad (7.12)$$

Then, recall the component form of the gradient (or divergence) theorem, i.e.,

$$\int_{\Omega^e} \frac{\partial}{\partial x} \left(w \frac{\partial \Psi}{\partial x} \right) dx dy = \oint_{\Gamma^e} w \left(\frac{\partial \Psi}{\partial x} \right) n_x ds \quad (7.13)$$

and

$$\int_{\Omega^e} \frac{\partial}{\partial y} \left(w \frac{\partial \Psi}{\partial y} \right) dx dy = \oint_{\Gamma^e} w \left(\frac{\partial \Psi}{\partial y} \right) n_y ds \quad (7.14)$$

where n_x and n_y are the components of the unit normal vector on the boundary of the element, Γ^e , and ds is the arclength of an infinitesimal line element along the boundary.

$$\bar{n} = n_x \bar{i} + n_y \bar{j} \quad (7.15)$$

Using the above equations one obtains:

$$\int_{\Omega^e} \left[\frac{\partial w}{\partial x} \frac{\partial \Psi}{\partial x} + \frac{\partial w}{\partial y} \frac{\partial \Psi}{\partial y} \right] dx dy - \oint_{\Gamma^e} w \left[n_x \frac{\partial \Psi}{\partial x} + n_y \frac{\partial \Psi}{\partial y} \right] = 0 \quad (7.16)$$

The specification of Ψ constitutes the essential boundary condition, and hence Ψ is the primary variable. The specification of the coefficient of the weight function in the boundary expression

$$t_n \equiv n_x \frac{\partial \Psi}{\partial x} + n_y \frac{\partial \Psi}{\partial y} \quad (7.17)$$

constitutes the natural boundary conditions; thus, t_n is the secondary variable of the formulation. The function $t_n = t_n(s)$ denotes the projection of the vector $\mathbf{n} \cdot \nabla \Psi$ along the unit normal $\mathbf{n}$. By definition, t_n is positive outward from the surface as one moves counter-clockwise along the boundary Γ^e .

iii) Use the definition eq.(7.17) in eq.(7.16) and write the weighted-residual form of eq.(7.4) as:

$$\int_{\Omega^e} \left[\frac{\partial w}{\partial x} \frac{\partial \Psi}{\partial x} + \frac{\partial w}{\partial y} \frac{\partial \Psi}{\partial y} \right] dx dy - \oint_{\Gamma^e} w t_n ds = 0 \quad (7.18)$$

The weighted-residual form in eq.(7.18) forms the basis of the finite element model of eq.(7.4).

7.4.3. Finite Element Model

The weighted-residual form in eq.(7.18) requires that the approximation chosen for Ψ should be at least linear in both x and y , so that there are no terms in eq.(7.4) that are identically zero. Since the primary variable is simply the function itself, the Lagrange family of interpolation functions is admissible. Suppose that Ψ is approximated over a typical finite element Ω^e by the expression:

$$\psi(x,y) \approx \Psi^e(x,y) = \sum_{j=1}^n \psi_j^e \lambda_j^e(x,y) \quad (7.19)$$

where ψ_j^e is the value of Ψ^e at the j th node (x_j, y_j) of the element, and λ_j^e are the Lagrange interpolation functions, with the property

$$\lambda_i^e(x_j, y_j) = \delta_{ij} \quad (7.20)$$

where δ_{ij} = Kronecker-Delta ,i.e.

$\delta_{ij} = 1$ if $i=j$, and

$\delta_{ij} = 0$ if $i \neq j$.

Substituting the finite element approximation eq.(7.19) for ψ into the weighted-residual form eq.(7.18), one obtains:

$$\int_{\Omega^e} \left[\frac{\partial w}{\partial x} \left(\sum_{j=1}^n \psi_j^e \frac{\partial \lambda_j^e}{\partial x} \right) + \frac{\partial w}{\partial y} \left(\sum_{j=1}^n \psi_j^e \frac{\partial \lambda_j^e}{\partial y} \right) \right] dx dy - \oint_{\Gamma^e} w t_n ds = 0 \quad (7.21)$$

This equation must hold for any weight function w . Since n independent algebraic equations are required to solve for n unknowns $\psi_1, \psi_2, \dots, \psi_n$, then n independent functions for w : ($w=\lambda_1, \lambda_2, \dots, \lambda_n$) must be chosen. This particular choice is a natural one when the weight function is viewed as a virtual variation of the dependent unknown (i.e., $w = \delta\psi = \sum_{i=1}^n \delta\psi_i \lambda_i$). One can label the algebraic equation resulting from substitution of λ_i for w into above eq.(7.21) as the first algebraic equation. The i th algebraic equation is obtained substituting $w = \lambda_i$ into eq.(7.21):

$$\sum_{j=1}^n \left[\int_{\Omega^e} \left(\frac{\partial \lambda_i}{\partial x} \left(\frac{\partial \lambda_j}{\partial x} \right) + \frac{\partial \lambda_i}{\partial y} \left(\frac{\partial \lambda_j}{\partial y} \right) \right) dx dy \right] \psi_j - \oint_{\Gamma^e} \lambda_i t_n ds = 0 \quad (i=1,2,\dots,n) \quad (7.22)$$

or

$$\sum_{j=1}^n K_{ij}^e \psi_j^e = Q_i^e \quad (7.23)$$

where

$$K_{ij}^e = \int_{\Omega^e} \left(\frac{\partial \lambda_i}{\partial x} \frac{\partial \lambda_j}{\partial x} + \frac{\partial \lambda_i}{\partial y} \frac{\partial \lambda_j}{\partial y} \right) dx dy \quad (7.24)$$

and

$$Q_i^e = \oint_{\Gamma^e} t_n \lambda_i ds \quad (7.25)$$

In matrix notation, eq.(7.23) takes the following form:

$$[K^e] (\Psi) = (Q^e) \quad (7.26)$$

This completes the development of finite element model.

7.4.4. Interpolation Functions

The finite element approximation $\Psi^e(x,y)$, of $\psi(x,y)$ over an element Ω^e , must satisfy the following conditions in order for the approximate solution to be convergent to the true one:

1. Ψ^e must be differentiable, as required in the weighted-residual form of the problem;
2. the polynomials used to represent Ψ^e must be complete; and
3. all terms in the polynomial should be linearly independent.

The number of linearly independent terms in the representation of Ψ^e dictates the shape and number of degrees of freedom of the element. In general, by using Pascal's triangle, which contains the terms of polynomials of various degrees in the two coordinates x and y , p th-order triangular element has n_1 nodes, with

$$n_1 = \frac{1}{2}(p+1)(p+2) \quad (7.27)$$

and a complete polynomial of the p_n th degree is given by

$$\psi(x,y) = \sum_{i=1}^{n_1} a_i x^{m_1} y^{m_2} = \sum_{j=1}^{n_1} \psi_j \lambda_j, \quad m_1 + m_2 \leq p_n \quad (7.28)$$

For this study, a second-order triangular element ($p_n=2$) was developed, which contains 6 nodes. The positions of the six nodes in the triangle are at the vertices and at the midpoints of the three sides (Fig.7.3). The polynomial involves six constants which can be expressed in terms of the nodal values of the variable being interpolated:

$$\psi = \sum_{i=1}^6 \psi_i \lambda_i(x,y) \quad (7.29)$$

where λ_i are the quadratic interpolation functions.

The quadratic polynomial associated with the quadratic (six-node) triangular element is:

$$\psi^e(x,y) = a_1 + a_2x + a_3y + a_4xy + a_5x^2 + a_6y^2 \quad (7.30)$$

The derivatives of ψ^e are:

$$\frac{\partial \Psi^e}{\partial x} = a_2 + a_4 y + 2a_5 x, \quad \frac{\partial \Psi^e}{\partial y} = a_3 + a_4 x + 2a_6 y \quad (7.31)$$

Since the transformation from the global coordinate, (x,y), system to the local coordinate, (r,s), system involves only rotation and translation, the second order polynomial in the global coordinate system has still the second order in the local coordinate system. Therefore,

$$\Psi^e(s,r) = a'_1 + a'_2 r + a'_3 s + a'_4 rs + a'_5 r^2 + a'_6 s^2 = \sum_{j=1}^6 \Psi_j^e(r,s) \lambda_j \quad (7.32)$$

where a'_i ($i=1,2,\dots,6$) are constants depending on the a_i and the angle of rotation.

The interpolation functions of a 6 number-node element are:

$$\lambda_1 = 1 - r - s - \frac{1}{2}\lambda_4 - \frac{1}{2}\lambda_6 \quad (7.33)$$

$$\lambda_2 = r - \frac{1}{2}\lambda_4 - \frac{1}{2}\lambda_5 \quad (7.34)$$

$$\lambda_3 = s - \frac{1}{2}\lambda_5 - \frac{1}{2}\lambda_6 \quad (7.35)$$

$$\lambda_4 = 4r(1 - r - s) \quad (7.36)$$

$$\lambda_5 = 4rs \quad (7.37)$$

and

$$\lambda_6 = 4s(1-r-s) \quad (7.38)$$

These functions are constructed in the usual way, namely, Ω_i must be unity at node i and zero at all other nodes.

7.4.5. Coordinate Transformation

The transformation of a quadrilateral element of a finite element mesh from a local coordinate (r,s) system to a global coordinate (x,y) system is for the purpose of numerically evaluating the integrals. Consider the element coefficients

$$K_{ij}^e = \int_{\Omega^e} \left[\frac{\partial \lambda_i^e}{\partial x} \frac{\partial \lambda_j^e}{\partial x} + \frac{\partial \lambda_i^e}{\partial y} \frac{\partial \lambda_j^e}{\partial y} \right] dx dy \quad (7.39)$$

The integrand is a function of the global coordinate. One has to rewrite in terms of r and s using the following transformations:

$$x = \sum_{j=1}^m x_j^e \lambda_j^e(r,s) \quad (7.40)$$

$$y = \sum_{j=1}^m y_j^e \lambda_j^e(r,s) \quad (7.41)$$

This integrand contains not only functions but also derivatives with the respect to the global coordinates (x,y) . Therefore, one has to relate $\partial \Psi_i^e / \partial x$ and $\partial \Psi_i^e / \partial y$ to $\partial \Psi_i^e / \partial r$ and $\partial \Psi_i^e / \partial s$ using the above transformations. Hence, by the chain rule of partial differentiation, one can write:

$$\frac{\partial \lambda_i^e}{\partial r} = \frac{\partial \lambda_i^e}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial \lambda_i^e}{\partial y} \frac{\partial y}{\partial r} \quad (7.42)$$

$$\frac{\partial \lambda_i^e}{\partial s} = \frac{\partial \lambda_i^e}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial \lambda_i^e}{\partial y} \frac{\partial y}{\partial s} \quad (7.43)$$

or, in matrix notation,

$$\begin{Bmatrix} \frac{\partial \lambda_i^e}{\partial r} \\ \frac{\partial \lambda_i^e}{\partial s} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix}^e \begin{Bmatrix} \frac{\partial \lambda_i^e}{\partial x} \\ \frac{\partial \lambda_i^e}{\partial y} \end{Bmatrix} \quad (7.44)$$

which gives the relation between the derivatives of λ_i^e with respect to the global and local coordinate.

The Jacobian matrix is defined as:

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix}^e \quad (7.45)$$

Therefore

$$\begin{Bmatrix} \frac{\partial \lambda_i^e}{\partial x} \\ \frac{\partial \lambda_i^e}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial \lambda_i^e}{\partial r} \\ \frac{\partial \lambda_i^e}{\partial s} \end{Bmatrix} \quad (7.46)$$

where $[J]^{-1}$ is the invert of the Jacobian matrix.

7.4.6. Numerical Integration

A very important numerical integration procedure, in which the positions of the sampling points and the weights have been optimized, is the Gauss quadrature (Bathe, 1996). It can be written as follows:

$$\int_{-1}^1 g(\xi) d\xi = \sum_{i=1}^m \alpha_i g(\xi_i) \quad (7.47)$$

or, when extended to a two-dimensional case:

$$\int_{-1}^1 \int_{-1}^1 g(r,s) dr ds = \int_{-1}^1 \sum_{i=1}^m \alpha_i g(r_i, s) ds = \sum_{j=1}^m \sum_{i=1}^m \alpha_j \alpha_i g(r_i, s_j) \quad (7.48)$$

where

α_i and α_j = the weighting factors,

r_i and r_j = the locations, and

m = the number of Gauss points.

Table (7.1) lists the value of α_i and r_i for one-dimensional $g(r)$. In this study a 4-point Gauss quadrature was chosen.

7.4.7. Assembly of Element Equations

The assembly of finite element equations is based on two principles:

1. *Continuity* of primary variables, and
2. *Equilibrium* of secondary variables.

The continuity of the primary variables at the interelement nodes guarantees the continuity of the primary variable along the entire interelement boundary. For the

secondary variables, at the interface between the two elements, the fluxes from them should be equal in magnitude and opposite in sign.

In general, when several elements are connected, the assembly of the elements is carried out by putting element coefficients K_{ij}^e and Q_i^e into proper locations of the global coefficient matrix and the right hand vector. This is done by means of the connectivity relations, i.e. the correspondence of the local node number to the global node number.

7.4.8. Imposition of Boundary Conditions

There are two types of boundary conditions for this study:

1. **Essential (Dirichlet) boundary conditions**, i.e. boundary conditions on primary variables;
2. **Natural (Neumann) boundary conditions** , i.e. conditions on secondary variables.

The procedure for implementing the boundary conditions on the primary variables involves modifying the assembled coefficient matrix and the right-hand column vector by the following three operations:

1. moving the known products to the right-hand column of the matrix equation;
2. replacing the columns and rows of the assembled coefficient matrix corresponding to the known primary variable by zeros, and setting the coefficient on the main diagonal to unity;
3. replacing the corresponding component of the right-hand column by the

specified value of the variable.

Finally the full matrix form is:

$$\begin{bmatrix} K_{11} & K_{12} & . & . & . & K_{1N} \\ K_{21} & K_{22} & . & . & . & K_{2N} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ K_{N1} & K_{N2} & . & . & . & K_{NN} \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \\ . \\ . \\ . \\ \psi_N \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ . \\ . \\ . \\ Q_N \end{Bmatrix} \quad (7.49)$$

7.4.9. Solution of Equations

Gaussian elimination and back-substitution was used to solve the equation system eq.(7.49). This is the most effective direct solution technique (Bathe, 1996, Hoffman, 1992). Scaling and pivoting are essential elements of Gauss elimination.

7.4.10. Postprocessing

The finite element solution at any point (x,y) in an element Ω^e is given by:

$$\Psi^e(x,y) = \sum_{j=1}^n \psi_j^e \lambda_j^e(x,y) \quad (7.50)$$

and its derivatives are computed as:

$$\frac{\partial \Psi^e}{\partial x} = \sum_{j=1}^n \psi_j^e \frac{\partial \lambda_j^e}{\partial x} \quad , \quad \frac{\partial \Psi^e}{\partial y} = \sum_{j=1}^n \psi_j^e \frac{\partial \lambda_j^e}{\partial y} \quad (7.51)$$

The above equations can be used to compute the solution and its derivatives at any

point (x,y) in the element.

7.5. Discussion

The effect of air entrainment in the model was studied by applying the U.S. Army Corps. of Engg. formula (eq.(5.48)). Based on this equation it can be concluded that, with increase in discharge for a specific downstream slope, the air entrainment will decrease. After the point of inception eq.(5.48) was applied to the water surface profile found using SPIL-FLOW. It was found that, for $H/H_d > 1.25$, the required length to establish uniform flow could not be achieved. Therefore our numerical model can not be applied in situations where uniform flow conditions on the spillway face cannot be achieved.

In order to apply our model and verify it, detailed information regarding the coordinates of the spillway surface, discharge, and location of water surface profile are necessary. Therefore, for the purpose of comparison, the model was applied to several series of test data in which $H/H_d < 1.25$ and where uniform flow condition on the spillway face were achieved. Figs. 7.4 to 7.12 show the results of these verification tests.

The maximum total number of elements used was 576 with 1305 nodes (the SPIL-FLOW model can handle 1000 elements). The maximum number of iterations was 30. As indicated in Figs. 7.4. to 7.12, the model predicts water surface profiles better for low range of H/H_d ($H/H_d < 1.0$). The maximum difference between the numerical simulation and the experimental results is about 10 % (Fig. 7.13), with $r^2 = 0.947$.

In spite of applying the air entrainment coefficient it is found that the flow-depth

results of the numerical modelling are always less than the experimental data. In regards to predicting the spillway water surface elevation profiles the model underpredicts, by a maximum 10%, the observed water surface profiles. The reason for this may be that the actual air entrainment coefficient is higher than the one estimated by eq.(5.48). This will lead to underestimating the energy dissipation as well as the height of the necessary retaining walls.

7.6. Summary

In this chapter, an algorithm that applies a 2-D finite element method to find the location of the free water surface for flow over a spillway has been proposed. The main conclusions are:

- (i) Based on the nature of the governing equation and the boundary conditions, the variational method (with quadratic triangular elements) was applied to develop the FEM procedures.
- (ii) In order to find the location of the free water surface for flow over a spillway, a *trial and error* procedure based on Bernoulli's equation has been used.
- (iii) The effect of air entrainment was studied by applying the well-known and generally accepted U.S. Army Corps. of Engg. formula (eq.(5.48)).
- (iv) The model has been applied to the test data corresponding to $H/H_d < 1.25$, which is the data for which uniform flow conditions were achieved on the model spillways. For this range of data the model predicts a good comparison.

m	r_i	α_i
1	0.0	2.0
2	± 0.57735	1.0
3	± 0.77460 0.0	0.55556 0.88889
4	± 0.86114 ± 0.33998	0.34785 0.65215
5	± 0.90618 ± 0.53847 0.0	0.23693 0.47863 0.56889
6	± 0.93247 ± 0.66121 ± 0.23862	0.17132 0.36076 0.46791

Table 7.1. Sampling points and the weights in Gauss quadrature (after Bathe, 1996)

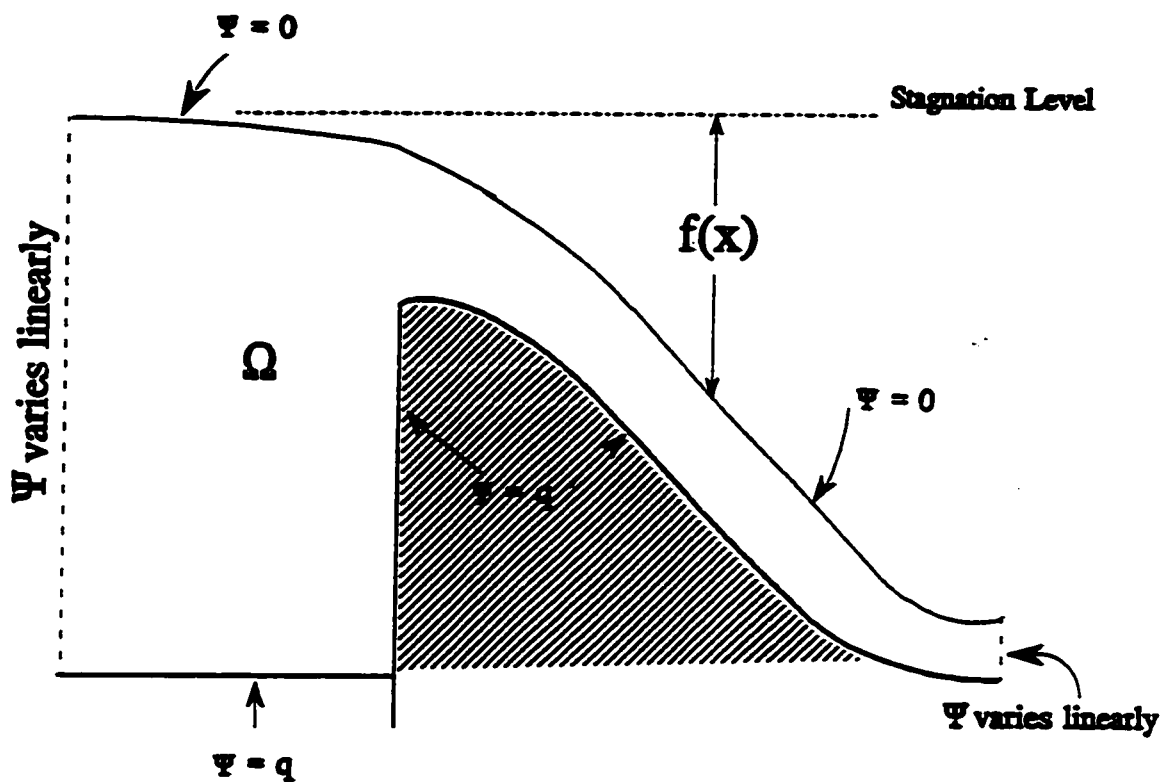


Fig.7.1. Flow over a spillway

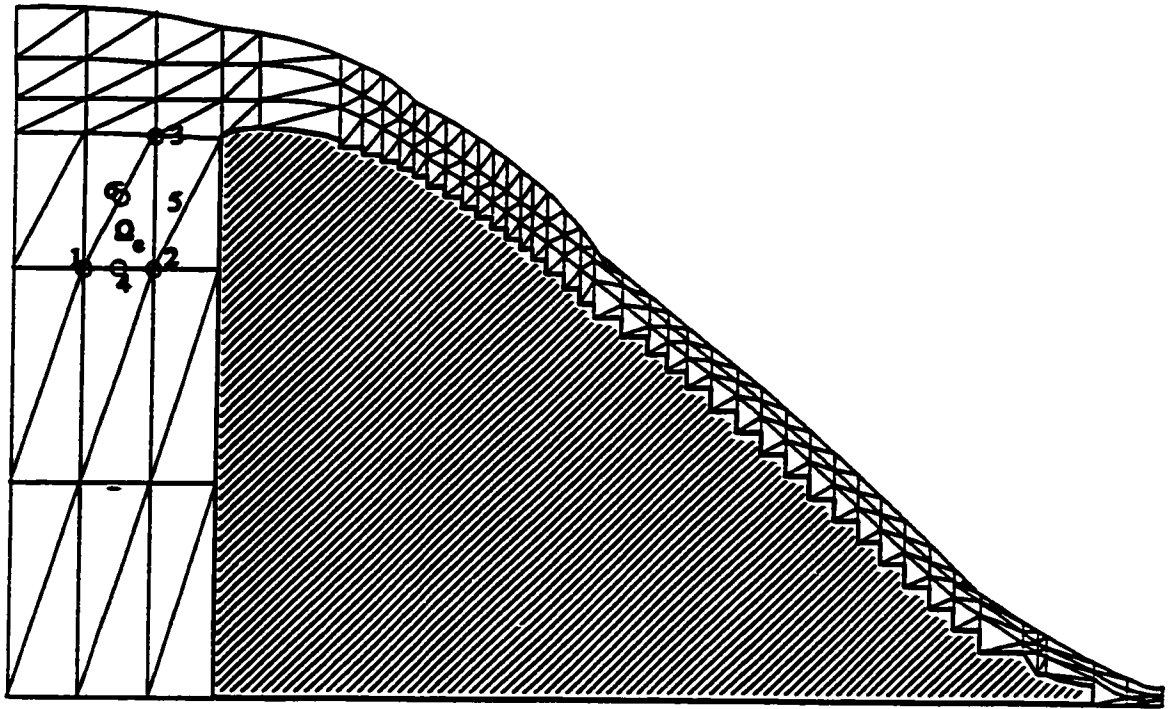


Fig.7.2. Finite element mesh

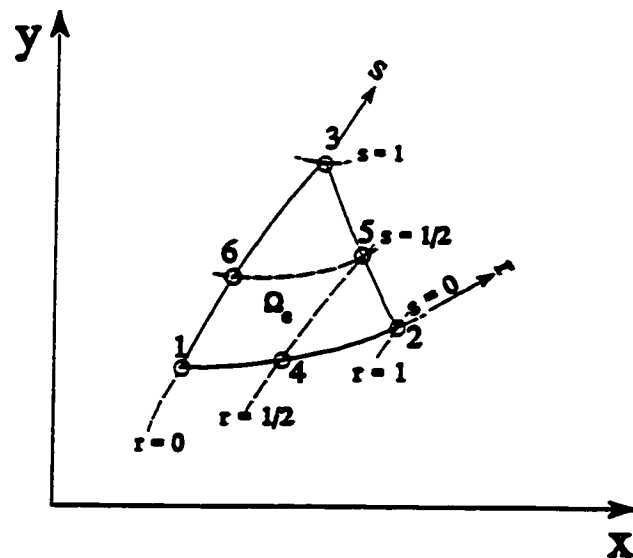


Fig.7.3. Construction of the interpolation functions
for a typical quadratural triangular elements

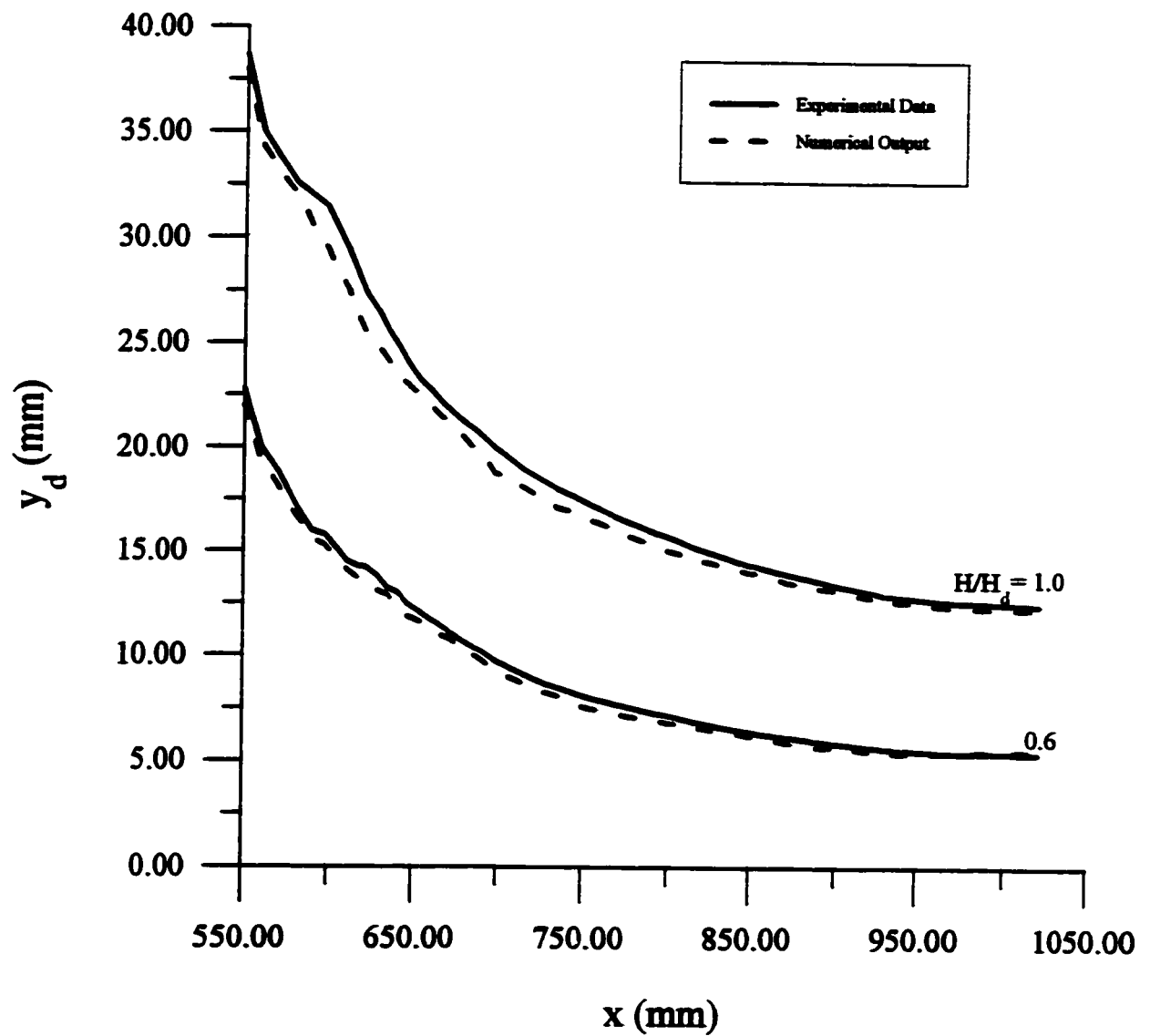


Fig.7.4. Comparison of depth of water between the numerical simulation output and experimental data for 45° smooth model

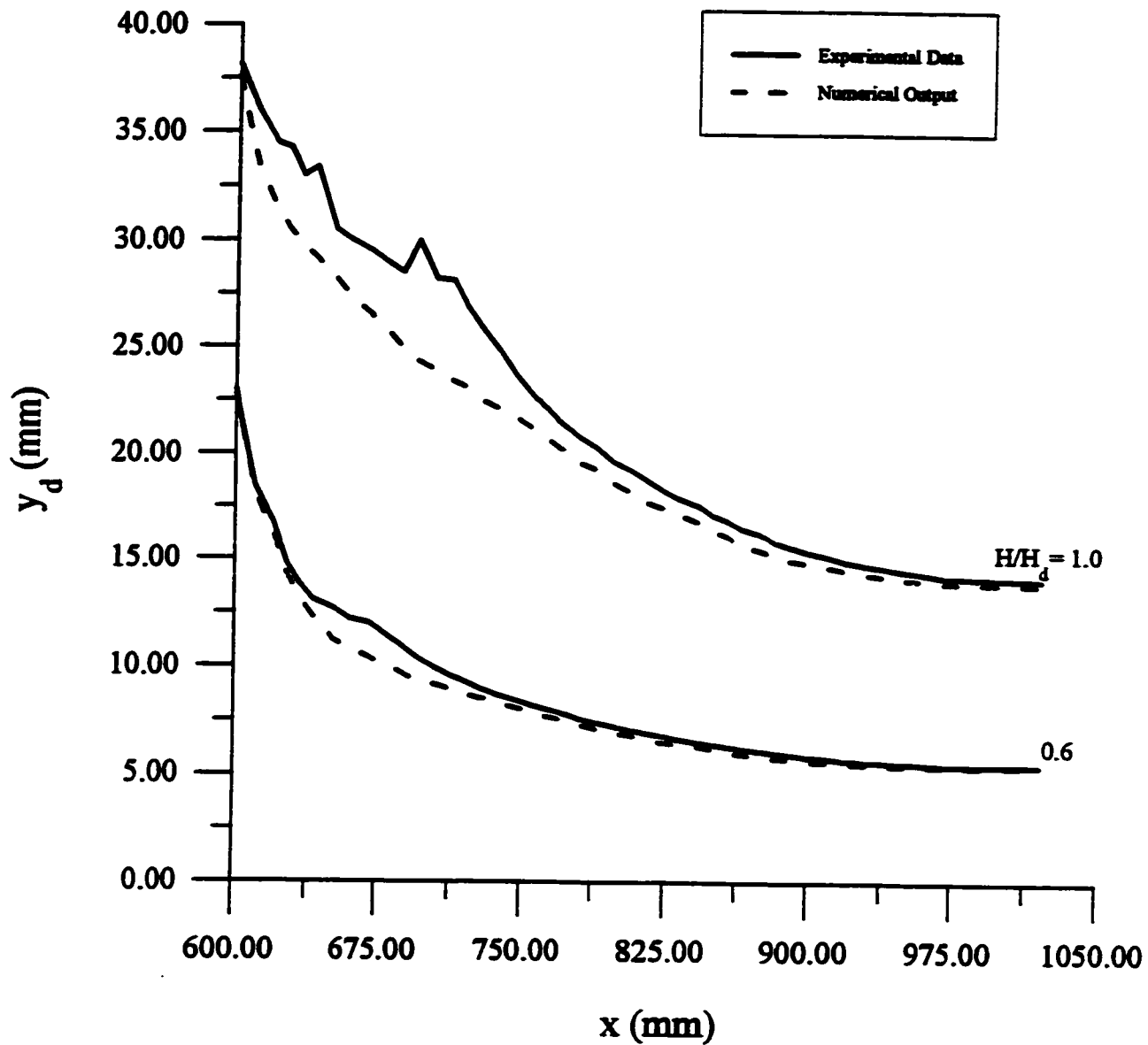


Fig. 7.5. Comparison of depth of water between the numerical simulation output and experimental data for 50° smooth model

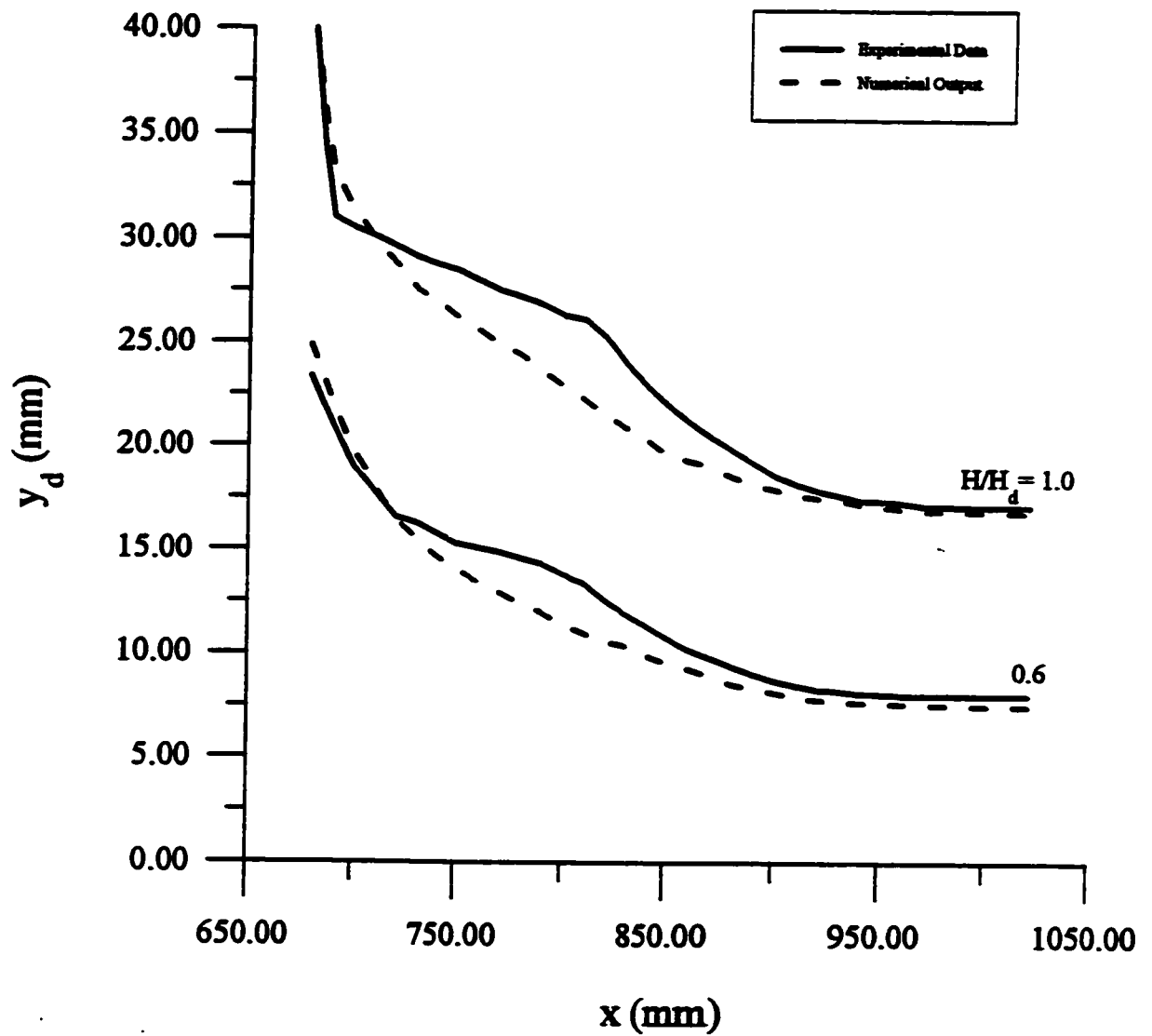


Fig. 7.6. Comparison of depth of water between the numerical simulation output and experimental data for 60° smooth model

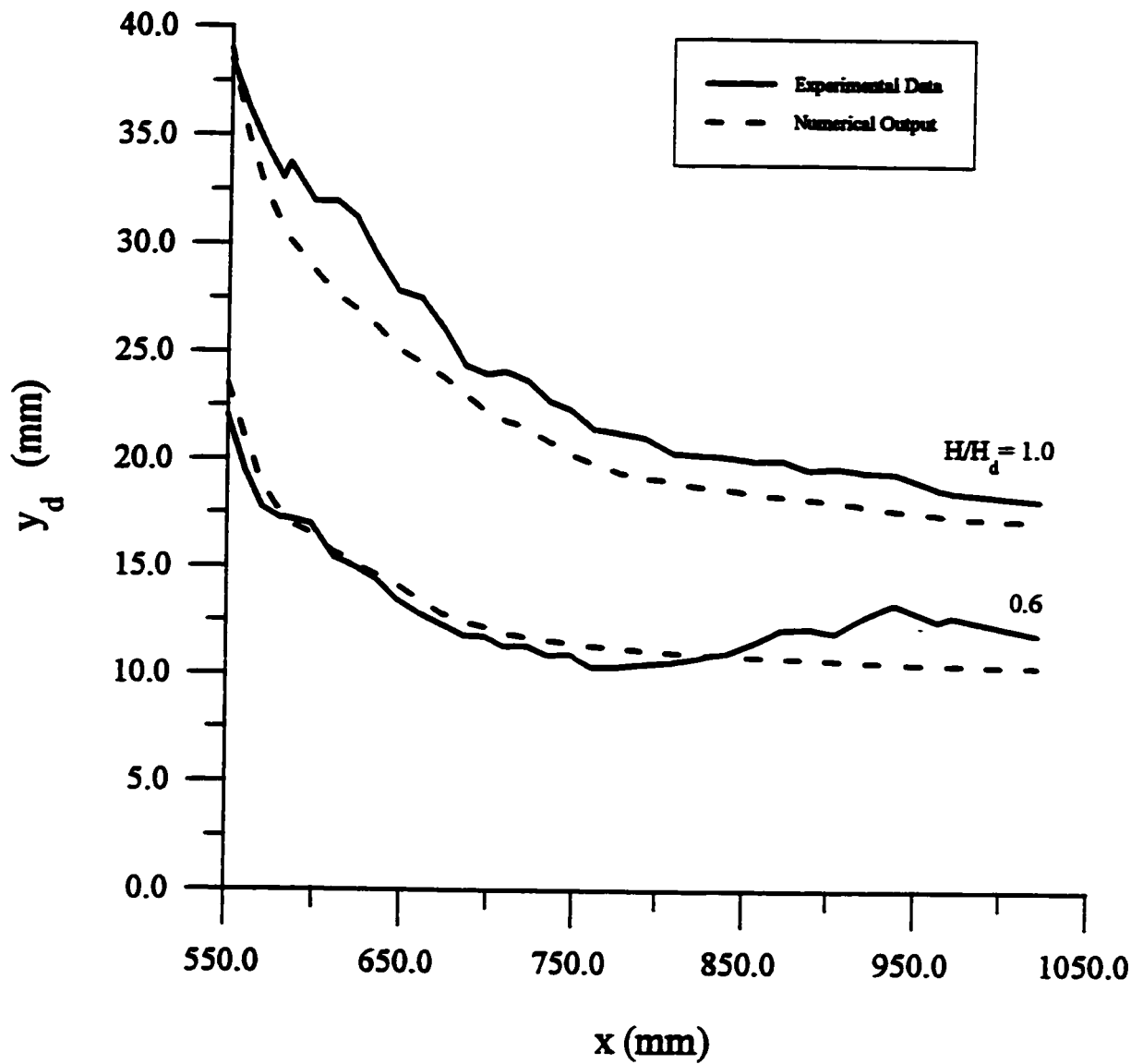


Fig. 7.7. Comparison of depth of water between the numerical simulation output and experimental data for 45-CL-BL model

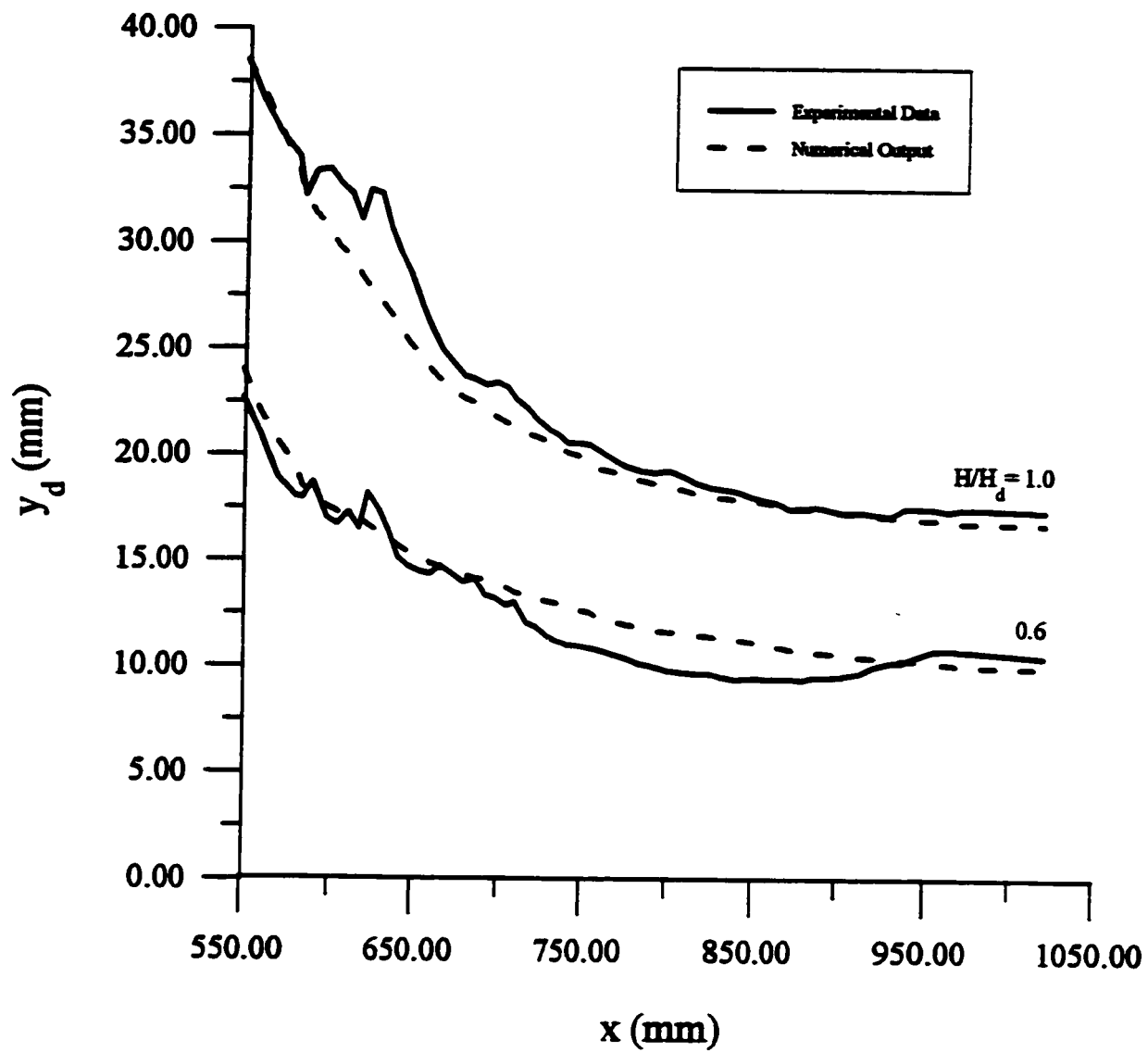


Fig. 7.8. Comparison of depth of water between the numerical simulation output and experimental data for 45-CS-BS model

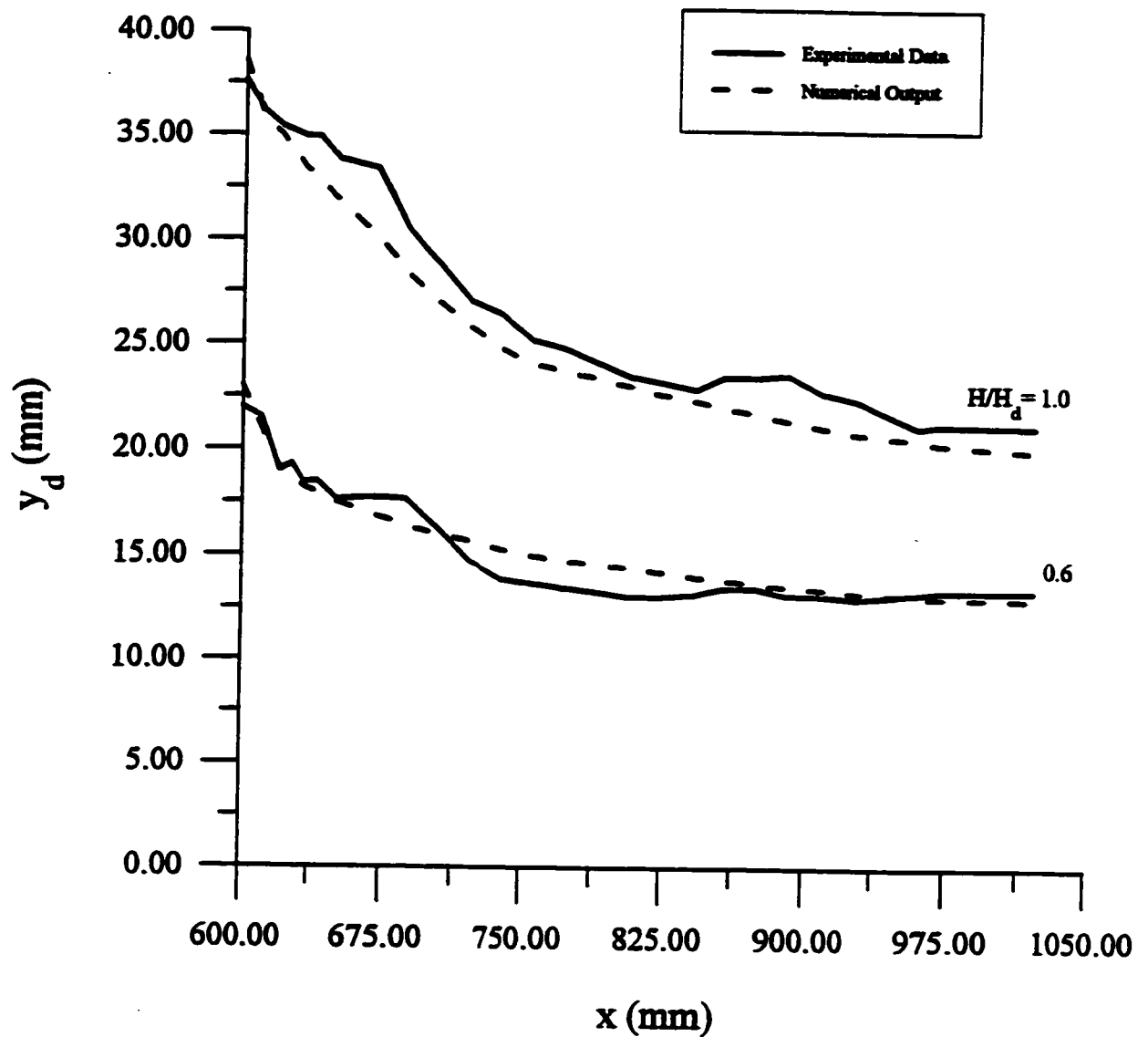


Fig. 7.9. Comparison of depth of water between the numerical simulation output and experimental data for 50-CL-BL model

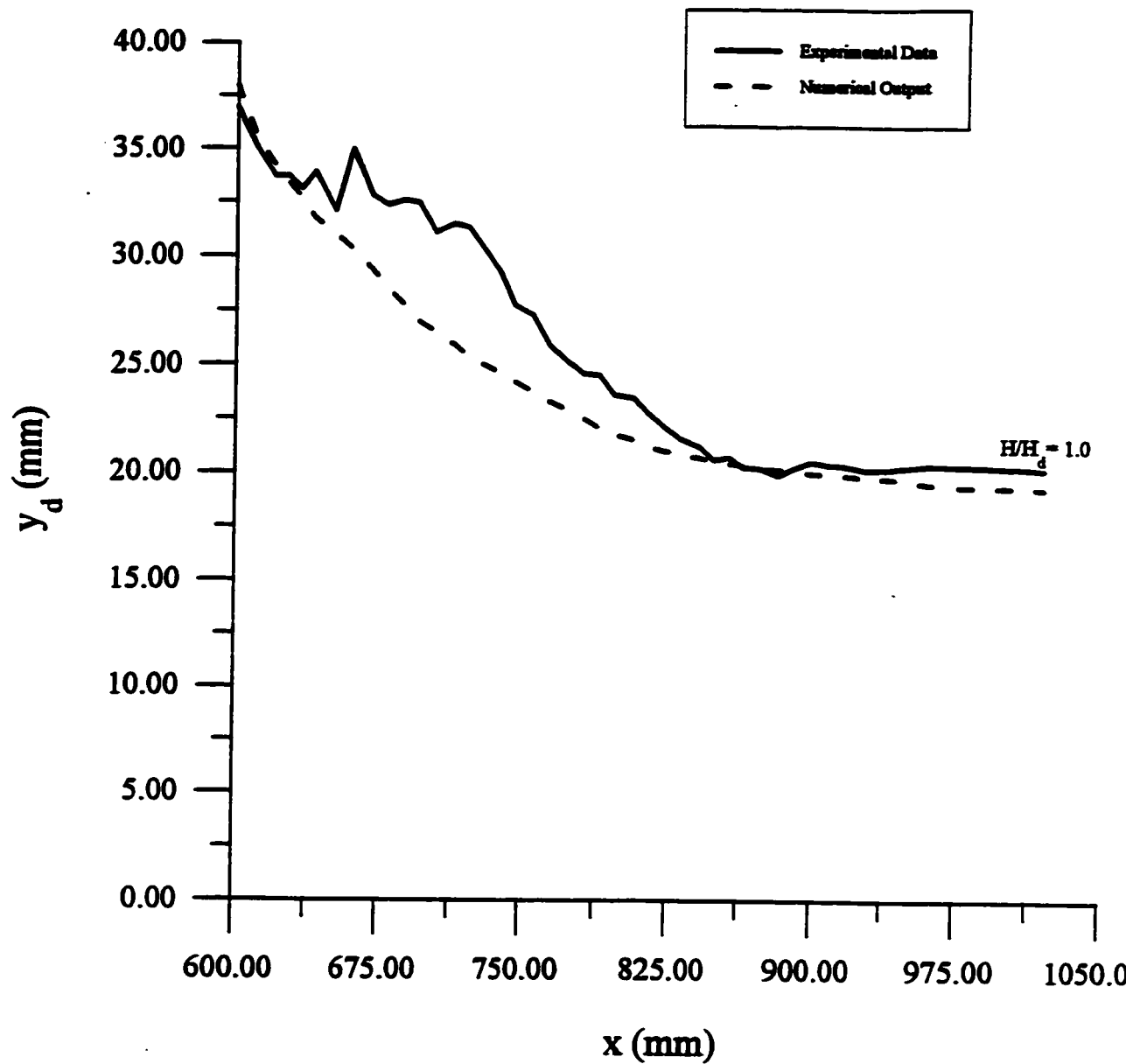


Fig. 7.10. Comparison of depth of water between the numerical simulation output and experimental data for 50-CS-BS model

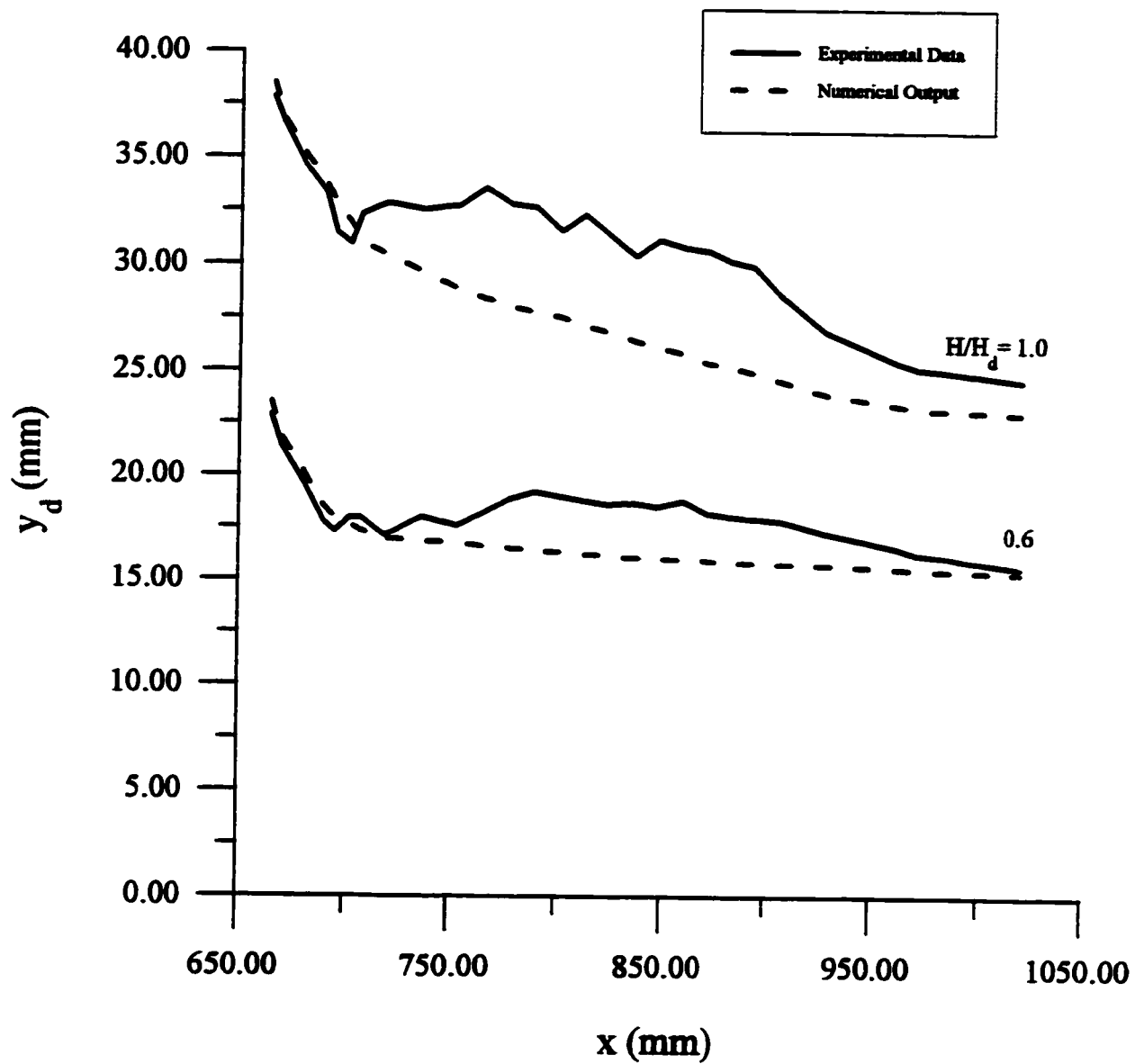


Fig. 7.11. Comparison of depth of water between the numerical simulation output and experimental data for 60-CL-BL model

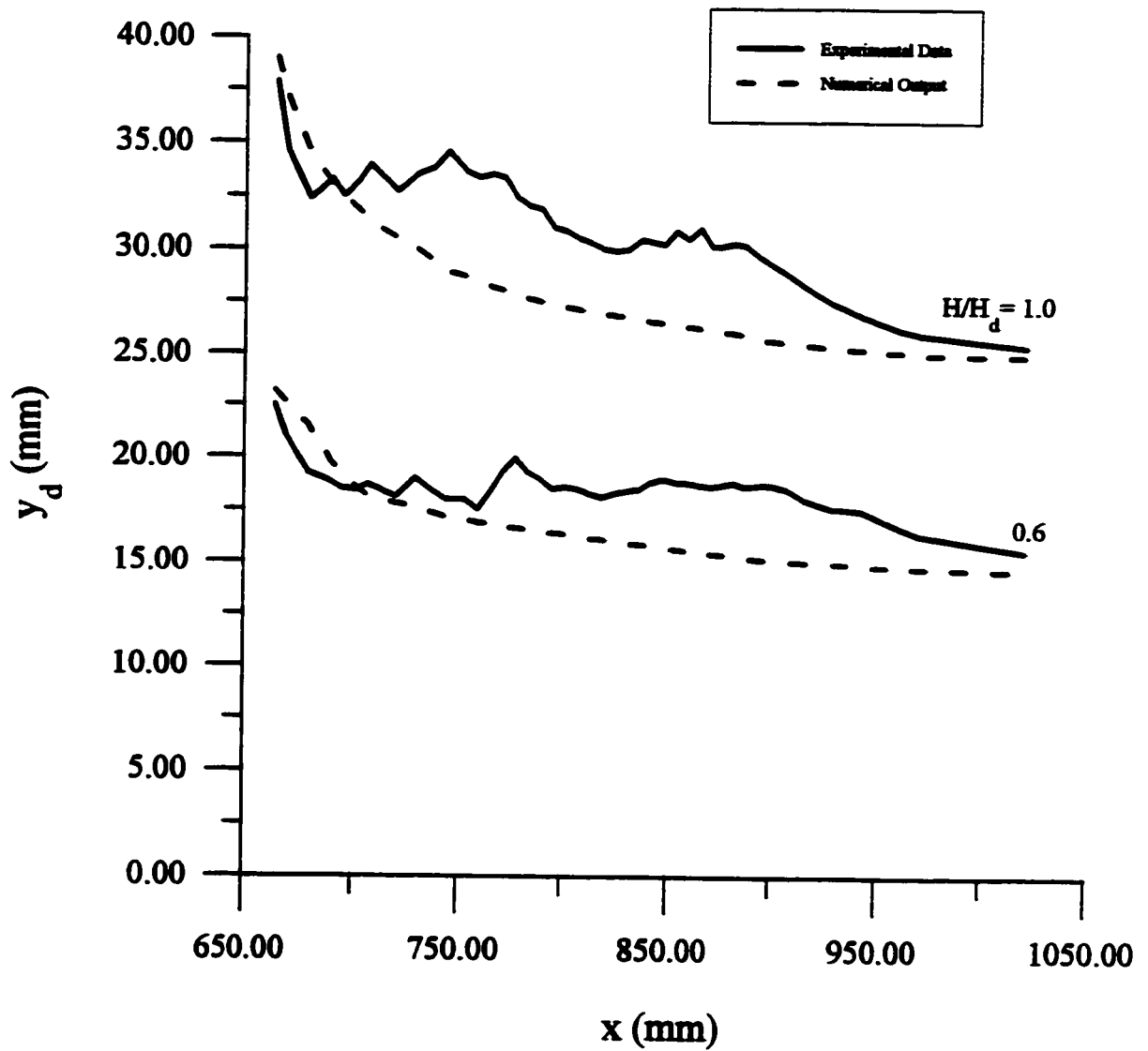


Fig. 7.12. Comparison of depth of water between the numeric simulation output and experimental data for 60-CS-BS model

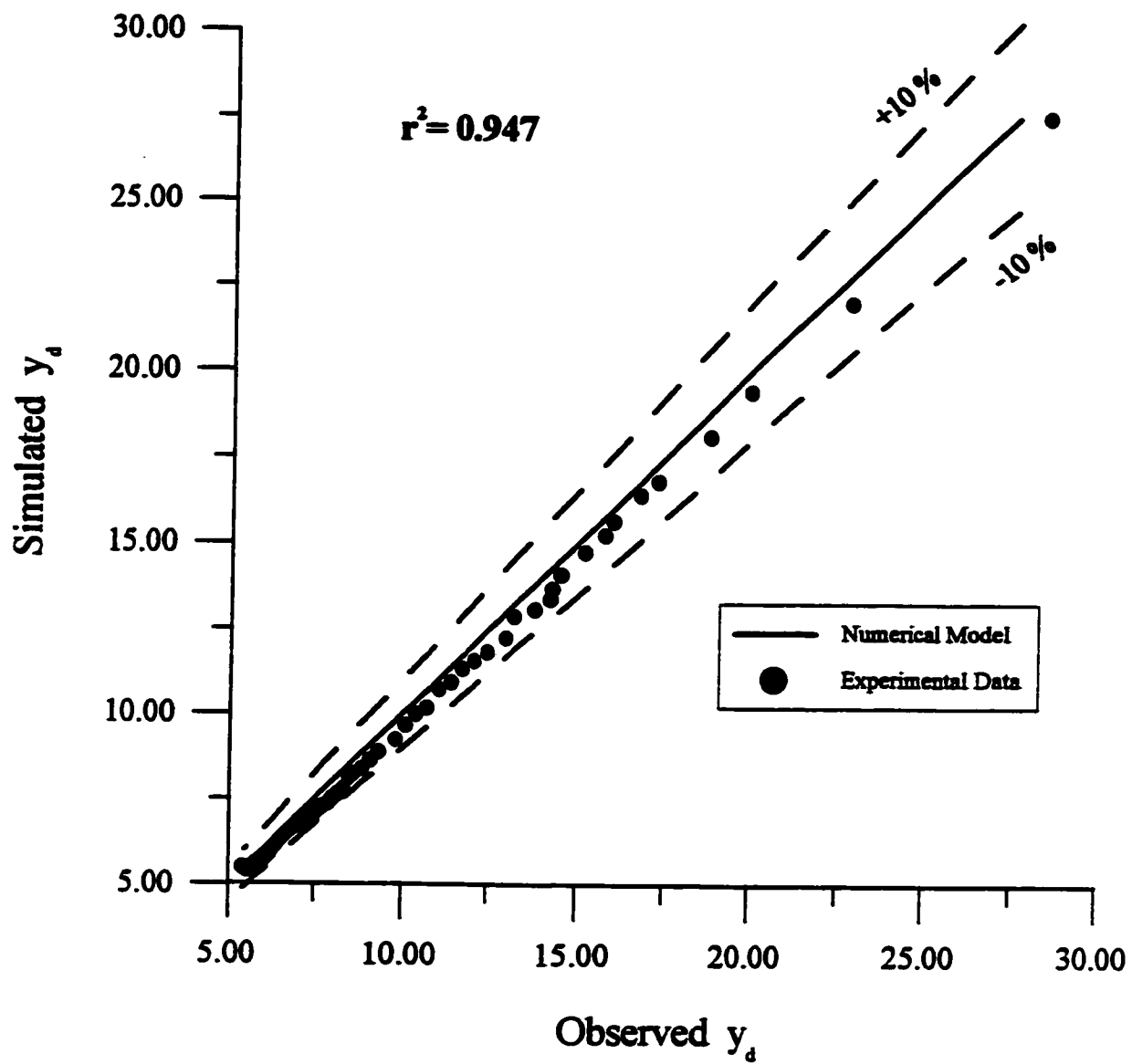


Fig. 7.13. Observed y_d versus Simulated y_d

Chapter 8

Summary, Conclusions, and Recommendations for Future Research

8.1. Summary

The *stepped* ogee-profile spillway is a spillway whose downstream face is provided with a series of *steps* from near the crest to the toe. The *steps*, which are arranged so that their tips conform to the corresponding smooth ogee-profile, significantly increase the rate of energy dissipation taking place on the spillway face. Accordingly, if a stilling basin is also required, its size is significantly reduced by the action of the *steps*. In this research the hydraulic characteristics of various combination of ogee-profile *stepped* spillway were investigated experimentally and numerically. The performance of the various *stepped* models were compared to those for the corresponding *smooth* profile.

Twelve *stepped* ogee-profile models and three smooth ogee-profile models were tested in this study. All models had vertical upstream faces and the spillway crest geometry used was that proposed by the U.S. Waterways Experiment Station (WES) (1977). The models, which included downstream slopes of 45° , 50° , and 60° had an assumed model to prototype scale ratio of 1:50. The computer program GEMLSPS

was used to generate the geometry of the models. Model performance was examined for a wide range of operational head, H (i.e. from $0.5 H_d$ to $2.5 H_d$). In the case of the *stepped* models, at the lower discharges the water jet upon impacting the first *step* was projected into the air and became reattached a few *steps* downstream. Upon reattachment, some water was splashed into the air, and measuring this particular water surface profile was virtually impossible. It was also found that the effect of *steps* was insignificant for $H/H_d > 2.5$, and *stepped* models acted as the *smooth* models.

The flow over a *stepped* spillway can be divided into *nappe* and *skimming* flow regimes. *Nappe* flow regime can be described as (i) *nappe* flow with fully developed hydraulic jump, and (ii) *nappe* flow with partially-developed hydraulic jump. In the case of *nappe* flow with fully developed hydraulic jump, each *nappe* would perform the same as a drop structure. In this research the drop structures were studied extensively (see sec. 5.1). Based on the results of this research, new criteria were developed for the drop structure. Applying these criteria, a general equation was derived to distinguish between a fully-developed *nappe* flow regime and a *skimming* flow regime (eq. (5.38)). For a fully-developed *nappe* flow regime, a general equation based on the momentum principle was developed to predict the amount of dissipated energy (eq. (5.71)). This equation was found to be more applicable to the available data from other sources than similar equations found in the literature (Chanson 1993, Chamani, and Rajaratnam, 1994). Although the *nappe* flow regime was studied, the main emphasis of this research was the *skimming* flow regime. From an economical and practical viewpoint, in order to have a *nappe* flow regime one needs flat slopes. In this study, the models studied had relatively steep downstream slopes.

It was found that compared to an equivalent smooth spillway the discharge coefficient increased for the case of a *stepped* spillway (see eq. (6.2)). Increased discharge capacity is one of the advantages of *stepped* spillways over equivalent smooth spillways. This increase provides more deployment of flood to the downstream channel in a given time period. Another important hydraulic characteristic of a

stepped spillway is its friction factor. This aspect was also investigated in this research and a general relationship was developed (eq. (6.4)). This equation was also verified based on the experimental data. It is shown that, in general, the models with the larger numbers of *steps* (CSBS- type models) provide higher friction factors.

It was found that energy dissipation on *stepped* spillways is a function of the height of dam (H_{dam}), slope (θ), discharge (q), number of *steps* (N_s), and the friction factor (f). Based on the experimental results, a general equation for the rate of energy dissipation for flow over a *stepped* ogee-profile spillway was developed (eq. (6.14)). Also, based on the different models tested, it was found that a certain horizontal length was required for the energy dissipation to reach an *equilibrium* rate (eq. (6. 12)).

A 2-D non-linear Finite Element (FE) model was developed for estimating the spillway water surface profile. Because of the problems such as non-linearity of the governing equation, and unknown boundary conditions beforehand, the analysis of flow over a spillway is a difficult mathematical problem. Using Bernoulli's law, an algorithm (SPIL-FLOW) was introduced to find the location of the free water surface over a spillway. This numerical model was verified using the experimental data.

8.2. Main Conclusions

Based on the results obtained in this study, the main conclusions are:

1. Applying the momentum principle, semi-empirical equations for y_1/h , y_2/h , $\cos\beta$, and $\Delta E/E_0$, namely eqs. (5.26), (5.27), (5.28) and (5.29), for a drop structure were developed in this study. As noted in Table 5.1. the results show good agreement with the data published by Moore (1943), Gill (1978), and Chanson (1993).

2. Equation (5.38) was developed to describe fully-developed *nappe* flow. From this equation, the possibility of fully-developed *nappe* flow with high discharge can be predicted. Also the equation shows that fully-developed *nappe* flow is possible even for steep slopes.
3. Equation (5.71) for the energy dissipation rate for fully-developed *nappe* flow over an ogee-profile *stepped* spillway was developed. This equation does not have the limitations of similar ones found in the literature.
4. Based on the results of the experiments, the discharge coefficient for a *stepped* spillway was determined (see eq. (6.2)). It was found that for the same head condition on the crest this coefficient is higher for *stepped* spillways than for *smooth* spillways.
5. Based on the experimental data and the Grid Search Method, a general implicit equation (eq. (6.4)) was developed for the friction factor (f). It was found that the number of *steps* is more important than the size of the *steps*.
6. It was found that energy dissipation mainly occurs in the middle part of a *stepped* spillway (see section 6.4). To reach the maximum possible dissipation rate (saturation point), a certain minimum horizontal length (expressed in eq. (6.12)) is required. Based on the experimental results this length criterion was found.
7. It was found that the energy dissipation rate is a function of H_{sum} , θ , q , N_s , and f . A general equation (eq. (6.14)) was developed for the energy dissipation rate for a *skimming* flow regime.
8. Among the 12 models tested, in general stepped spillway configurations having large steps near the crest followed by small steps near the toe, (e.g. CLBS-type

models) gave the highest energy dissipation rate.

9. A 2-D non-linear Finite Element (FE) model (SPIL-FLOW) was developed to estimate the water surface profile on an ogee-profile *stepped* spillway. A good agreement ($r^2 = 0.947$) was obtained between the results generated by the numerical model and the experimental (physical model) results.

8.3. Future Research

Future research investigations should be carried out to provide information in the following areas:

1. The mechanism of flow recirculation in *skimming* flow regime needs to be studied in detail. The flow recirculation is the major source of energy dissipation in *skimming* flow regime. To study this mechanism one needs to build large scale physical models. A ratio of 1:10 is recommended.
2. One of the most important water quality parameters in rivers and streams is the *dissolved oxygen content*. Deep and slow pools of water upstream of a dam reduce the air-water gas transfer process. *Stepped* spillways can enhance the oxygen transfer during the water release. Also *stepped* spillways are used in water treatment plants to enhance the air-water transfer of atmospheric gases. This process needs to be studied in detail for both *nappe* flow and *skimming* flow regimes.
3. On a *stepped* spillway, the high rate of energy dissipation reduces the flow momentum in comparison with a smooth spillway. The presence of air within the flow will increase the flow depth, which results in reduction of flow velocity. The reduction of flow velocity also reduces the risk of cavitation as

the cavitation index increases. Prevention of cavitation damage requires the knowledge of the air concentration in the layers close to the spillway bottom. Further study is required to examine the efficiency of *stepped* spillways in reducing cavitation.

4. Although Kells (1993) among the others have made preliminary studies of the hydraulics of gabion *stepped* spillways, there is still a need for further studies of fundamental nature on this important topic. The modelling of flow through and over the *stepped* spillway needs to be studied in detail. It is very important and essential for gabion *stepped* spillways. In order to investigate the flow through and over the *stepped* spillway, one needs to build several large scale physical models.

Chapter 9

References

Akin, J.E., (1987), "Finite Element Formulation of Flows with Singularities," Proc. of the Second Int. Conf. on Finite Elements in Water Resources, Imperial College, London, July 9-12 , pp. 99-112.

Atchison, J.M., Karagerorghis, A., (1988), "Numerical Solution of a Boundary Element Method," Int. J. for Numerical Methods in Fluids, Vol. 8, pp. 91-96.

Baban, R., (1995), " Design of Diversion Weirs", John Wiely & Sons Ltd, UK.

Baker, R., Gardiner, K.D., (1995), "Building Blocks," Int. Water Power & Dam Construction, Jan., Vol. 47, No. 11, pp. 16-18.

Baker, R., (1989), "Pre-Cast Concrete Blocks for Dam Spillways," Int. Water Power & Dam Construction, July, Vol. 41, No. 7, pp. 60-66.

Bathe, K.J., (1996), "Finite Element Procedure," Prentice Hall, Englewood Cliffs, N.J.

Bathe, K.J., Khoshgoftaar, M.R., (1979), "Finite Element Free Surface Seepage Analysis without Mesh Iteration," Int. J. for Numerical and Analytical Methods in Geomechanics, Vol. 3, pp. 13-22.

Bathe, K.J., Oden, J.T., Wunderlich, W., (1977), "Formulations and Computational Algorithms in Finite Element Analysis, Massachusetts Institute of Technology, Boston, MA.

Bathe, K.J., Wilson, E.L., (1976), "Numerical Methods in Finite Element Analysis," Prentice Hall, Englewood Cliffs, N.J.

Bayat, H.O., (1991), "Stepped Spillway Feasibility Investigation," Proc. 17th ICOLD Congress, Vienna, Austria, pp. 1803-1817.

Beitz, E., Lawless, M., (1992), "Hydraulic Model Study for Dam on GHFL 3791 Isaac River at Burton Gorge," Water Resources Commission Report, Brisbane, Australia.

Berger, R.C., (1994), "Strength and Weakness of Shallow Water Equations in Steep Open Channel Flow," Proc. Nat. Conf. on Hydraulic Engrg., ASCE, Buffalo, N.Y., Aug 1-5, pp. 1257-1262 .

Berger, R.C., Stockstill, R.L., (1994), "Considerations in 2-D Modelling of Hydraulically Steep Flow," Proc. Nat. Conf. on Hydraulic Engrg., ASCE, Buffalo, N.Y., Aug 1-5, pp. 1285-1289.

Berger, R.C., Winant, E.H., (1991), "One Dimensional Finite Element Model for Spillway Flow," Proc. Nat. Conf. on Hydraulic Engrg., ASCE, Nashville, Tennessee, July 29-Aug. 2., pp. 388-391.

Bettess, O., White, W.R., Bettess, R., (1994), "Recirculation in Flow over Crump Weirs," Proc. Nat. Conf. on Hydraulic Engrg., ASCE, Buffalo, N.Y., Aug 1-5, pp. 103-117.

Bettess, P., Bettess, J.A., (1983), "Analysis of Free Surface Flows using Isoparametric Finite Elements," Int. J. for Numerical Methods in Engineering, Vol. 19, pp. 1675-1689.

Bindo, M., Gautier, J., Lacroix, F., (1993), "The Stepped Spillway of M'Bali Dam," Int. Water Power & Dam Construction, Jan., Vol. 45, No. 1, pp. 35-36.

Cain, P., Wood, I.R., (1981), "Measurements of Self-Aerated Flow on a Spillway," J. Hydraulic Div., ASCE, Vol. 107, No. HY11, pp. 1425-1444.

Cassidy, J.J., (1970), "Design Spillway Crest for High-Head Operation," J. Hydraulic Div., Proc. ASCE, Vol. 96, No. HY3, pp. 745-753.

Cassidy, J.J., (1965), "Irrotational Flow over Spillways of Finite Height," J. Mech. Div., ASCE, Vol. 91, No. EM6, pp. 155-173.

Chamani, M.R., Rajaratnam, N., (1994), "Jet Flow on Stepped Spillways," J. Hydraulic Engrg., ASCE, Vol. 120, No. 2, pp. 254-259.

Chan, S.T.K., Larock, B.E., Herrmann, L.R., (1973), "Free Surface Ideal Fluid by Finite Elements," J. Hydraulic Engrg., ASCE, Vol. 99, No. 6, pp. 959-974.

Chanson, H., (1996) : Discussion appearing in Rajaratnam, N., Chamani, M.R., (1995).

Chanson, H., (1994), "Comparison of Energy Dissipation between Nappe and Skimming Flow Regimes on Stepped Chutes," J. of Hydraulic Res., IAHR, Vol. 32, No. 2, pp. 213-218.

Chanson, H., (1993), "Self-Aerated Flows on Chutes and Spillways," J. Hydraulic

Engrg., ASCE, Vol. 119, No. 2, pp. 220-243.

Chanson, H., (1993), "Stepped Spillway Flows and Air Entrainment," Can. J. Civ. Eng. CSCE, Vol. 20, pp. 422-435.

Chanson, H.,(1992), "Air Entrainment in Chutes and Spillways," Research Report No. CE 133, Dept. of Civil Engineering, Univ. of Queensland, Australia.

Chaudhry, M.H., (1993), "Open-Channel Flow," Prentice-Hall, Inc., New Jersey, N.Y.

Chow, V.T., (1959), "Open Channel Hydraulics," McGraw- Hill, New York, N.Y.

Christodoulou, G.C., (1993), "Energy Dissipation on Stepped Spillway," J. Hydraulic Engrg., ASCE, Vol. 119, No. 5, pp. 473-482.

Creager, W.P., Justin, J.D., Hinds, J., (1964), "Engineering for Dams," John Wiley & Sons, Inc., New York, N.Y.

Dao-Yang, D., Man-Ling, L., (1979), "Mathematical Model of Flow over a Spillway Dam," Proc. 8th ICOLD Congress, New Delhi, pp. 959-976.

Degoutte, G., Peyras, L., and Royet, P., (1992): Discussion appearing in Rajaratnam, N., (1990).

Diersch, H.J., Schirmer, A., Busch, K.F., (1977), "Analysis of Flows with Initially Unknown Discharge," J. Hydraulic Engrg., ASCE, Vol. 103, No. 3, pp. 213-232.

Diez-Cascon, J., Blanco, J.L., Rivilla, J., Garcia, R., (1991), "Studies on Hydraulic Behaviour of Stepped Spillway," Int. Water Power & Dam Construction, Sept., Vol.

43, No. 9, pp. 22-26.

Dressler, R.F., (1978), "New Nonlinear Shallow-Flow Equations with Curvature," J. Hydraulic Res., IAHR, Vol. 16, No. 3, pp. 205-222.

El-Khashab, A.M., Helweg, O., Alajaji, A., (1987), "Theoretical Flow Model for Drop Structures," Proc. of Nat. Conf. on Hydraulic Engrg., ASCE, Aug. 3-7, Williamsburg, Virginia, pp. 1112-1117.

Ellis, J., (1985), "Numerical Modelling of Spillway Flows," 2nd International Conference on the Hydraulic of Floods & Flood Control, Cambridge-England, Sept. 24-26, pp. 73-90.

Essery, I.T.S., Horner, M.W., (1978), "The Hydraulic Design of Stepped Spillways," CIRIA Report No. 33, 2nd ed., London, U.K.

Essery, I.T.S., Tebbutt, T.H.Y., Rasaratnam, S.K., (1978), "Design of Spillway for Re-aeration of Polluted Waters," CIRIA, London, U.K.

Falvey, H.T., (1980), "Air-Water in Hydraulic Structures," USBR, Engrg. Monograph No. 41, Denver, Co.

Frind, E.O., Matanga, G.B., (1985), "The Dual Formulation of Flow for Contaminant Transport Modelling," Water Resources Research, Vol. 21, No.2, pp.159-169.

Frizell, K.H., (1991), "Stepped Spillway Design for Flow over Embankment," Proc. Nat. Conf. Hydraulic Engrg., ASCE, Nashville, Tennessee, July 29, Aug. 2, pp.118-123.

Frizell, K.H., (1992), "Hydraulics of Stepped Spillways for RCC Dams and Dam Rehabilitations," Proc. 3rd Specialty Conf. on Roller Compacted Concrete, ASCE, San Diego, CA, pp.423-439.

Garg, S.K., (1983), "Irrigation Engineering and Hydraulic Structures," Khana Publishers, Delhi, India.

Gevorkyan, S.G., Kalantrova, Z.K., (1992), "Design of Tsunami-Protective Embankment with a Complex front Surface," Hydrotechnical Construction, pp.286-290.

Gill. M.A., (1980), "Effect of Boundary Roughness on Hydraulic Jump," Int. Water Power & Dam Construction, Jan., Vol. 32, No. 1, pp. 22-24.

Gill, M.A., (1979), "Hydraulic of Rectangular Vertical Drop Structures," J. Hydraulic Res., IAHR, Vol. 17, No.4, pp. 289-302.

Golze, A.R., (1977), "Handbook of Dam Engineering," Van Nostrand Reinhold Co., New York, N.Y.

Grinchuck, A.S., Pravdivets, Y.P., Shektman, N.V., (1977), "Tests of Earth Slope Revetments Permitting Flow of Water at Large Specific Discharge," Hydrotechnical Construction, pp. 367-373.

Guymon, G.L., (1973), "Finite Element Solution for General Fluid Motion," J. Hydraulic Div., ASCE, Vol. 99, No. HY6, pp. 913-919.

Hager, W.H., (1992), "Energy Dissipators and Hydraulic Jump," Kluwer Academic Publishers, The Netherlands.

Hager, W.H., (1991), "Experiments on Standard Spillway Flow," Proc. Instn. Civ. Engrs., Part 2, Sept., pp. 399-416.

Hansen, K.D., Reinhardt, W.G., (1991), "Roller Compacted Concrete Dams," McGraw-Hill, New York, N.Y.

Hansen, K.D., (1986), "Roller Compacted Concrete Developments in the USA," Int. Water Power & Dam Construction, Jan., Vol. 38, No. 1, pp. 9-12.

Hartung, F., Scheuerlein, H., (1970), "Design of Overflow Rockfill Dams," Proc. 10th ICOLD Congress, Montreal, Canada, pp. 587-598.

Henderson, H.C., Kok, M., Dekoning, W.L., (1991), "Computer Aided Spillway Design Using the Boundary Element Method and Non-linear Programming," Int. J. of Numerical Methods in Fluids, Vol. 13, pp. 625- 641.

Henderson, F.M., (1966), "Open Channel Flow," The McMillan Company, New York, N.Y.

Heng, F.L., Mitsoulis, E., Prinos, P., (1986), "Modelling of Flow under Sluice Gates with FEM," VI Int. Conference on Finite Elements in Water Resources, Lisbon, pp. 483-493.

Hoffman, J.D., (1992), "Numerical Methods for Engineers and Scientists," McGraw- Hill, New York, N.Y.

Hollingworth, F., Druyts, F.H.W.M., (1986), "Rollcrete: Some Applications to Dams in South Africa," Int. Water Power & Dam Construction, Jan., Vol. 38, No.1, pp. 13-16.

Horner, M.W., (1969), " An Analysis of Flow on Cascade of Steps," Ph.D. Thesis, Univ. of Birmingham, UK

Huston, K.L., (1987), "Stepped Spillway design with a RCC Dam," Proc. Nat. Conf. on Hydraulic Engrg., ASCE, Williamsburg, Virginia, Aug. 3-7, pp. 122-127.

Ikegawa, M., Washizu, K., (1973), "Finite Element Method Applied to Analysis of Flow over a Spillway Crest," Int. J. for Numerical Methods in Eng. Vol. 6, pp. 179-189.

Keller, R.J., Rostagi, A.K., (1977), "Design Chart for Predicting Critical Point on Spillways," J. Hydraulic Div., ASCE, Vol. 103, No. HY12, pp. 1417-1429.

Kells, J.A., (1993), "Spatially Varied Flow over Rockfill Embankment," Can. J. Civ. Eng., Vol 20, pp. 820-827.

Kennedy, J.B., Neville, A.M., (1986), "Basic Statistical Methods for Engineers and Scientists," Harper & Row Publ., New York, N.Y.

Khan, A.A., Steffler, P.M., (1996), "Vertically Averaged and Momentum Equations Model for Flow over Curved Beds," J. Hydraulic Engrg., ASCE, Vol. 122, No. 1, pp. 3-9.

Knauss, J., (1979), "Computation of Maximum Discharge at Overflow Rockfill Dams," Proc. 13th ICOLS Congress, New Delhi, India, pp. 143-159.

Knight, D.W., MacDonald, J.A., (1979), "Hydraulic Resistance of Artificial Strip Roughness," J. Hydraulic Div., ASCE, Vol. 105, No. HY6, pp. 675-690.

- Liggett, J.A., (1991), "Forty Year of Computational Hydraulics," Proc. Nat. Conf. on Hydraulic Engrg., ASCE, Nashville, Tennessee, July 29-Aug. 2., pp. 1124-1133.**
- Logie, C.V., Rahe, J.H., (1987), "Hydraulic Design Pamo Spillway," Proc. Nat. Conf. on Hydraulic Engrg., ASCE, Williamsburg, Virginia, Aug. 3-7, pp. 408-413.**
- Luke, J.C., (1967), "A Variational Principle for a Fluid with a Free Surface," J. of Fluid Mech., U.K., Vol. 27, part 2, pp. 395- 397.**
- Marchi, E., (1993), "On the Free Overfall," J. Hydraulic Res., Vol. 31, No. 6, pp. 777-790.**
- May, R.W.P., (1989), "Cavitation in Hydraulic Structures: Occurrence and Prevention," Hydraulic Research Report, Wallingford, UK.**
- McCorodale, J.A., Li, C.Y., (1971), "Finite Element Analysis of Sluice Gate Flow," Trans. Eng. Inst. Can., Vol. 14, pp. 189-202.**
- Mohamed Ali, H.S., (1994), "Bed Roughened Stilling Basins," Proc. Instn. Civ. Engrs., Wat. Merit. & Energy, Vol. 106, Dec., pp. 331-341.**
- Montes, J.S., (1994), "Potential-Flow Solution to 2D Transition from Mild to Steep Slope," J. Hydraulic Engrg., ASCE, Vol. 120, No. 5, pp. 601-621.**
- Moore, W.L., (1943), "Energy Loss at the Base of a free Overfall," Trans. ASCE, Vol. 108, pp. 1343-1360, Discussions: (1) White, M.P., (1943), Tans. ASCE, Vol. 108, pp. 1361-1364.**
- Neuman, S.P., Witherspoon, P.A., (1971), "Analysis of Non-steady Flow with a Free Surface Using the Finite Element Method," Water Resources Research, Vol. 7, pp.**

611-623.

Noori, B.M.A., (1984), **Form Drag Resistance of Two Dimensional Stepped Steep Open Channel,** Proc. 1st Int. Conf. on Hydraulic Design in Water Resources Eng., Southampton, UK, pp. 1.133-1.147.

Novak, P., Moffat, A.I.B., Nalluri, C., (1990), **"Hydraulic Structures,"** Unwin Hyman Ltd, London, UK.

O'Carroll, M.J., Toro, E.F., (1984), **"Numerical Computations of Critical Flow over a Weir,"** Int. J. for Numerical Methods in Fluids, Vol. 4, pp. 499- 509.

O'Carroll, M.J., Harison, H.T., (1976), **"Variation Techniques for Free Streamline Problems,"** Proc. 2nd Int. Symp. Finite Element in Flow Problems, Genoa, pp. 485-495.

Patankar, S.V., (1980), **"Numerical Heat Transfer and Fluid Flow,"** Hemisphere Publishing Co., U.S.A.

Peterka, A.J., (1958), **"Hydraulic Design of Stilling Basins and Energy Dissipators,"** Engineering Monograph No. 25, USBR, Denver, Co.

Peterka, A.J., (1953), **"The Effect of Entrained Air on Cavitation Pitting,"** Joint Meeting Paper, IAHR/ASCE, Minneapolis, Minnesota, Aug., pp. 507-518.

Peyras, L., Royet, P., Degoutte, G., (1992), **"Flow and Energy Dissipation over Stepped Gabion Weirs,"** J. Hydraulic Engrg., ASCE, Vol. 118, No. 5, pp. 707-717.

Peyras, L., Royet, P., Degoutte, G., (1991), **"Flows and Dissipation of Energy on Gabion Weirs,"** J. Houille Blanche, No.1, pp. 37-47.

Pravdivets, Y.P., Bramley, M.E., (1989), "Stepped Protection Blocks for Dam Spillways," *Int. Water Power & Dam Construction*, July , Vol. 41, No. 7, pp. 49-56.

Qibei, L., Baker, R., (1995), "Dam Spillways: Theoretical Analysis of Flow Parameters for Wedge-Shaped Spillway Protection Block," *J. Dam Engineering*, Vol. VI, Issue I, pp. 65-76.

Rajaratnam, N., Chamani, M.R., (1995), "Energy Loss at Drop," *J. of Hydraulic Res., IAHR*, Vol. 33, No. 3, pp.373-384. Discussions: (1) Chanson, H., (1996), *J. of Hydraulic Res., IAHR*, Vol. 34, No. 2, pp. 273-278.

Rajaratnam, N., Katopodis, C. (1991), "Hydraulic of Steeppass Fishways," *Can. J. Civ. Eng.*, Vol 18, pp. 1024-1032.

Rajaratnam, N., (1990), "Skimming Flow in Stepped Spillways," *J. Hydraulic Engrg., ASCE*, Aug. 3-7, Vol. 116, No. 4, pp. 587-591. Discussions: (1) Degoutte, G., Peyras, L., and Royet, P., (1992), *J. Hydraulic Engrg., ASCE*, Vol. 118, No. 1, pp. 111-114.

Rajaratnam, N., Subramanya, K., Muralidhar, D., (1968), "Flow Profiles over Sharp-Crested Weir," *J. Hydraulic Div., ASCE*, Vol. 94, No. HY3, pp. 843-847.

Ramanan, N. Engelman, M., (1993), "An Algorithm for Simulation of Steady Free Surface Flows," *Proc. of the 8th Int. Conf. in Numerical Methods in Laminar and Turbulent Flow*, 18th-23rd July, Swansea, pp. 435-447.

Rand, W., (1955), "Flow Geometry at Straight Drop Spillways," *Proceedings, ASCE*, 81E, Paper 791, pp. 1-13.

Rasanen, S.M., Salonen, E-M., (1985), "A u,v-formulation for Free Surface Flow," Proc. of the 4th Int. Conf. in Numerical Methods in Laminar and Turbulent Flow, 9th-12th July, Swansea, pp. 1393-1401.

Rasanen, S.M., Salonen, E-M., (1984), "A Model for Understanding Free Surface Flow," Numerical Methods for Non-linear Problems, Vol. 2, pp. 849-860.

Rice, C.E., Kadavy, K.C., (1996), "Model Study of a Roller Compacted Concrete Stepped Spillway," J. Hydraulic Engrg., ASCE, Vol. 122, No. 6, pp. 292-297.

Rice, C.E., Kadavy, K.C., (1994), "Energy Dissipation for a RCC Stepped Spillway," Proc. Nat. Conf. on Hydraulic Engrg., ASCE, Buffalo, N.Y., Aug 1-5, pp. 548-552.

Russel, S.O., Sheehan, G.J., (1974), "Effect of Entrained Air on Cavitation Damage," Can. J. Civ. Eng., Vol. 1, pp. 97-107.

Salonen, E-M, Lehtonen, P., Pramils, A., (1981), "Further u,v-Formulation for Ideal Fluid Flow," Proc. of the Laminar and Turbulent Flow, Pineridge Press, pp. 1185-1196.

Scheuerlein, H., (1973), "The Mechanics of Flow in Steep, Rough Open Channels," Proc. 15th IAHR Congress, Istanbul, Turkey, pp. 1-9.

Senturk, F., (1994), "Hydraulics of Dams and Reservoirs," Water Resources Publications, USA.

Sharp, J.J., (1981), "Hydraulic Modelling," Butterworths, London, UK.

Smith, I.M., Griffiths, D.V., (1988), "Programming the Finite Element Method,"

John Wiely & Sons Ltd, UK.

Smith, N., (1971), "A History of Dams," The Chaucer Press, Peter Davies, London, UK.

Sorensen,R.M., (1985), "Stepped Spillway Hydraulic Model Investigation," J. Hydraulic Engrg., ASCE, Vol. 111, No. 12, pp. 1461-1472.

Southwell, R.V., Vaisey, G., (1943), "Relaxation Methods Applied to Engineering Problems. VIII. Plane-Potential Problems Involving Specified Normal Gradients," Proc. Roy. Soc. A. pp. 129-151.

Sternberg, R.L., Kalinowski, A.J., Papadakis, J.S., (1979), " Non-linear Partial Differential Equations in Engineering and Applied Sciences," Marcel Pakker Inc., New York, N.Y.

Stephenson, D., (1991), "Energy Dissipation down Stepped Spillways," Int. Water Power & Dam Construction, Sept.,Vol. 43, No. 9, pp. 27-30.

Straub, L.G., Anderson, A.G., (1958), "Experiments on Self-Aerated Flow in Open Channels," J. Hydraulic Div., ASCE, Vol. 84, No. HY7, pp. 1890-1,1890-35.

Subramanya, K., (1989), "Flow in Open Channel," Tata McGraw Hill Publishing Co. Ltd., New Delhi, India.

Taylor, C., Hughes, T.G., Morgan, K., (1978), "Finite Elements solution of One Equation Models of Turbulent Flow," J. of Computational Physics, Vol. 29, pp. 163-172.

Tozzi, M.J., (1994), "Residual Energy in Stepped Spillways," Int. Water Power &

Dam Construction, May, Vol. 46, No. 5, pp. 32-34.

U.S. Army Corps of Engineers, (1995), "Hydraulic Design of Spillway," ASCE Press, New York, N.Y.

U.S. Army Corps of Engineers Waterways Experimenta Station, (1977), "Hydraulic Design Criteria," Vicksburg, Miss.

U.S. Army Corps of Engineers Waterways Experiment Station, (1961), "Prototype Tests of Spillway Crest," Vicksburg, Miss.

U.S.B.R., (1977), "Design of Small Dams," Bureau of Reclamation, Washington, D.C.

Vermeyen, T.B., Merritt, D.H., Taylor, M., (1991), "Design of High Capacity Spillways for Ritschard Dam," Proc. Nat. Conf. on Hydraulic Engrg., ASCE, Nashville, Tennessee, July 29, Aug. 2, pp.394-399.

Vittal, N., Ranga Raju, K.G., Garde, R.J., (1977), "Resistance of Two Dimensional Triangular Roughness," J. of Hydraulic Res., IAHR, Vol. 15, No. 1, pp.19-35.

Washizu, K., (1985), "Some Application of Finite Element Techniques to Nonlinear Free Surface Fluid Flow Problems," Proc. of the 4th Int. Conf. in Numerical Methods in Laminar and Turbulent Flow, 9th-12th July, Swansea, pp.3-15.

Wegman, E., (1907), "The Design of the New Croton Dam," Trans. ASCE, Paper 1047, June, pp.398-457.

White, M.P., (1943): Discussion appearing in Moore, W.L., (1943).

Wood, I.R., (1991), "Air Entrainment in Free-Surface Flows," IAHR Structures Design Manual No. 4, Hydraulic Design Considerations, Balema Publ., Rotterdam, The Netherlands.

Wortman, R.T., (1989), "Spillway Discharge Calculations in NWSDAMBRK," J. Hydraulic Engrg., Vol. 115, No. 7, pp.976-981.

Young, M.F., (1982), "Feasibility Study of a Stepped Spillway," Proc. Hydraulic Div. Spec. Conf., ASCE, New York, N.Y., pp.96-106.

Zienkiewicz, O.C., Szmelter, J., Perire, J., (1990), "Compressible and Incompressible Flow; An Algorithm for All Seasons," Computer Methods in Applied Mechanics and Engineering, Vol. 78, pp.105-121.

Zienkiewicz, O.C., Taylor, R.L., (1989), "Finite element method," McGraw- Hill, New York, N.Y.

Appendix

The following tables are the location of water surface profile measured for all the experimental models.

x	H/Hd				
	0.60	0.75	1.00	1.25	1.50
	d				
0	414.5	422.1	434.8	447.5	460.2
210	414.3	421.7	434.4	446.6	459.5
350	414.1	421.4	433.4	446.0	458.5
500	413.5	420.3	431.0	442.1	452.1
530	411.4	417.4	427.2	437.5	447.6
540	409.4	415.4	425.0	435.2	444.6
550	406.8	412.6	422.7	432.3	441.6
560	403.3	408.9	418.3	428.2	438.1
570	398.8	404.2	413.7	423.7	433.6
580	392.8	398.2	408.1	418.9	428.6
584	389.9	396.3	407.3	417.9	426.0
590	385.5	391.9	402.8	413.4	422.1
598	378.9	384.0	394.6	405.0	416.1
604	373.9	379.0	389.2	399.2	410.6
611	367.9	373.0	382.8	392.5	403.7
617	361.1	365.9	375.0	384.2	395.2
622	355.7	359.8	368.8	377.9	388.6
629	350.3	354.1	363.0	371.9	382.4
635	341.7	345.4	354.1	362.6	372.7
641	335.3	338.6	347.2	355.6	365.4
647	327.9	331.1	339.5	347.7	357.4
654	320.3	323.3	331.4	339.6	349.0
660	314.2	317.3	325.2	333.3	342.6
666	307.9	311.0	318.7	326.7	335.8
673	300.5	303.5	311.2	319.0	328.0
679	294.2	297.2	304.7	312.5	321.3
686	287.8	290.8	298.2	305.9	314.6
692	282.1	285.1	292.4	300.0	308.6
698	275.8	278.9	286.0	293.5	302.0
709	264.3	267.4	274.3	281.6	290.0
716	257.1	260.1	266.9	274.1	282.4
722	250.9	253.9	260.6	267.7	275.9

x	H/Hd				
	0.60	0.75	1.00	1.25	1.50
	d				
729	243.6	246.6	253.3	260.2	268.4
735	237.5	240.4	247.0	253.8	262.0
741	231.3	234.2	240.7	247.5	255.5
747	224.8	227.7	234.2	240.8	248.7
754	217.5	220.4	226.8	233.2	241.0
760	211.4	214.2	220.5	227.0	234.7
768	203.2	206.0	212.2	218.5	226.2
776	195.1	197.8	203.9	210.2	217.7
784	186.9	189.7	195.7	201.8	209.3
792	178.8	181.5	187.4	193.5	200.9
800	170.7	173.3	179.2	185.2	192.5
808	162.1	164.8	170.5	176.4	183.7
816	153.6	156.2	161.9	167.7	174.9
824	145.1	147.7	153.3	159.0	166.1
831	137.6	140.2	145.8	151.5	158.4
840	128.0	130.6	136.1	141.7	148.6
848	119.5	122.1	127.5	133.1	139.9
855	112.4	114.9	120.3	125.8	132.6
865	102.4	104.9	110.3	115.7	122.3
871	96.4	98.9	104.2	109.6	116.2
880	87.5	89.9	95.2	100.5	107.0
887	80.5	82.9	88.1	93.4	99.8
898	69.5	71.9	77.1	82.3	88.7
903	64.6	66.9	72.1	77.3	83.6
912	55.7	58.0	63.1	68.2	74.5
919	48.7	51.0	56.1	61.2	67.4
929	38.8	41.1	46.1	51.2	57.3
937	33.8	36.1	41.1	46.1	52.2
950	26.4	28.6	33.6	38.6	44.6
962	19.5	21.8	26.7	31.6	37.6
970	14.9	17.2	22.1	27.0	33.0
1020	5.4	7.6	12.5	17.3	23.2

Table A.1. Location of water surface profile for 45° smooth model

		H/Hd					
		d					
		0.6	0.75	1	1.25	1.5	2
x							
0.0	411.5	419.1	431.8	444.5	457.2	482.6	508.0
210.0	411.5	419.1	431.8	444.5	456.6	481.5	506.7
350.0	411.6	418.2	431.8	443.8	455.6	480.0	504.3
500.0	410.6	416.6	429.0	439.0	449.7	470.6	491.6
540.0	406.1	412.3	424.3	430.9	441.4	461.0	482.2
550.0	403.1	409.4	419.5	428.8	438.4	457.4	481.0
560.0	398.5	405.8	415.3	424.8	432.5	454.1	475.8
570.0	393.5	400.3	410.4	418.3	426.8	449.7	470.9
580.0	388.3	393.3	404.0	411.6	420.6	444.8	465.6
584.0	387.9	393.5	403.7	411.9	421.9	446.0	460.6
598.0	375.5	380.6	390.5	401.1	411.9	435.8	451.6
611.0	363.2	368.5	379.8	390.1	400.9	425.4	436.6
622.0	352.5	357.9	367.7	379.0	389.5	410.3	422.0
635.0	338.4	343.9	353.3	364.1	374.5	395.0	406.2
647.0	325.5	330.5	339.8	350.2	359.9	380.0	391.3
660.0	311.8	316.2	326.5	336.3	346.7	364.1	375.4
673.0	298.8	303.1	312.6	321.8	331.9	348.8	360.4
686.0	285.8	289.8	298.4	307.7	317.2	333.8	345.8
698.0	273.8	277.5	285.9	294.3	303.8	319.8	331.7

		H/Hd					
		d					
		0.6	0.75	1	1.25	1.5	2
x							
709.0	261.8	265.4	274.6	282.0	291.0	306.6	318.5
722.0	249.8	252.9	262.1	268.8	277.6	293.0	304.8
735.0	236.9	240.2	248.7	255.5	263.8	279.1	290.7
747.0	224.1	226.9	235.5	242.6	250.2	264.9	276.3
760.0	210.8	214.2	221.9	229.4	237.8	250.8	262.2
776.0	194.9	197.3	205.7	212.9	220.6	233.2	244.7
792.0	178.9	181.6	190.1	196.4	204.3	216.4	227.8
808.0	163.1	165.2	173.3	179.8	187.1	199.0	210.3
824.0	147.4	149.7	157.5	163.7	170.3	182.0	193.3
840.0	131.6	134.2	141.7	147.6	153.6	165.1	176.3
855.0	117.0	119.8	125.5	131.2	137.0	148.1	159.2
871.0	101.6	104.5	109.5	115.0	120.4	131.1	142.1
887.0	86.2	89.2	93.6	98.8	104.3	114.8	125.7
903.0	70.5	73.4	78.2	83.1	88.3	98.4	109.3
919.0	54.7	57.4	61.5	66.2	70.9	81.1	91.9
937.0	39.8	41.1	45.9	50.1	54.7	64.9	75.5
962.0	23.0	24.0	29.2	33.3	37.9	48.1	58.7
970.0	20.2	21.2	26.0	30.3	34.8	45.0	55.6
1020.0	11.9	13.4	18.1	22.5	27.0	37.2	47.8

Table A.2. Location of water surface profile for 45-CL-BL model

H/Hd								
		0.6	0.75	1	1.25	1.5	2	2.5
x	d							
760.0				222.7	230.4	238.3	249.4	281.5
768.0				213.7	221.3	228.9	240.1	272.1
776.0				205.9	213.4	220.6	231.9	262.8
784.0				197.5	204.9	212.0	223.2	253.5
792.0				189.6	197.0	204.0	215.2	244.1
800.0				181.2	188.7	195.3	206.5	234.8
808.0				172.9	180.1	186.7	197.8	225.5
816.0				164.7	171.9	178.3	189.2	216.2
824.0				156.8	164.1	170.1	181.0	206.9
831.0				148.3	155.6	161.3	172.1	198.8
840.0				140.6	147.7	153.5	164.1	188.3
848.0				132.5	139.7	145.1	155.7	179.0
855.0				124.3	131.3	136.7	147.2	171.5
864.0				116.0	123.0	128.3	138.7	162.2
871.0				108.0	114.7	119.9	130.3	154.9
880.0				99.8	106.4	111.6	121.9	145.6
887.0				92.2	98.6	103.7	114.0	138.3
895.0				84.7	90.9	95.8	106.1	130.2
903.0				76.4	82.6	87.4	97.7	122.2
912.0				67.8	73.8	78.5	88.8	113.1
919.0				59.7	65.5	70.1	80.5	106.1
929.0				49.8	55.2	59.9	70.2	96.0
937.0				44.5	49.3	54.0	64.2	89.7
952.0				36.3	41.0	45.6	55.9	78.2
962.0				28.1	32.6	37.4	47.6	70.5
970.0	21.0	20.1	25.5	29.6	34.3	44.5	64.4	
1020.0	15.0	14.0	18.9	22.5	27.1	37.3	45.2	

H/Hd								
		0.6	0.75	1	1.25	1.5	2	2.5
x	d							
0.0	415.4	419.1	431.8	444.5	457.2	482.6	508.0	
210.0	414.5	417.9	431.5	444.0	456.5	481.6	506.6	
350.0	414.3	417.8	430.8	443.8	455.8	479.8	504.3	
500.0	413.3	416.6	428.2	438.9	449.6	470.4	491.3	
540.0	407.1	412.3	422.4	432.0	441.6	461.7	482.6	
550.0	406.2	408.9	419.2	428.6	438.4	458.2	479.3	
560.0	402.7	405.3	415.4	425.0	434.6	454.8	475.5	
570.0	397.7	400.3	410.7	420.5	430.3	450.5	471.0	
580.0	391.9	394.4	404.4	414.5	424.2	445.8	466.6	
584.0	391.9	395.5	404.9	414.0	422.9	443.3	464.3	
598.0	378.9	381.8	392.5	402.2	412.3	434.6	457.0	
611.0	366.5	368.9	378.9	390.5	401.2	421.1	448.0	
622.0	355.1	357.4	367.2	379.3	390.1	407.4	439.4	
635.0	341.5	343.7	353.2	365.3	376.0	392.1	428.7	
647.0	328.9	331.0	340.8	351.2	361.6	376.8	418.2	
660.0	315.2	317.1	326.0	335.6	345.5	361.4	403.2	
673.0	301.8	303.5	311.8	321.6	331.1	346.3	389.6	
686.0	288.3	289.9	297.8	307.2	316.6	331.7	373.5	
698.0	276.3	277.3	285.0	294.3	303.0	317.7	358.9	
704.0	270.3	271.4	278.9	287.9	297.1	310.9	351.4	
709.0	265.3	265.7	272.8	282.6	291.5	304.7	345.2	
716.0	259.3	259.6	267.3	276.5	285.2	298.1	336.4	
722.0			261.4	270.0	278.9	291.4	328.8	
729.0			254.5	263.0	272.0	284.1	319.6	
735.0			248.2	256.8	265.8	277.5	311.8	
741.0			241.0	249.5	258.7	269.8	304.0	
747.0			235.3	243.8	252.3	263.4	296.8	
754.0			229.4	237.2	245.5	256.6	288.5	

Table A.3. Location of water surface profile for 45-CL-BS model

x	H/Hd						
	0.6	0.75	1	1.25	1.5	2	2.5
	d						
0.0	411.5	419.1	431.8	444.5	457.2	482.6	508.0
210.0	411.0	418.7	431.3	443.6	456.5	481.5	506.9
330.0	411.0	418.7	431.3	443.3	455.4	480.5	504.7
500.0	410.1	417.5	428.6	439.1	449.6	471.3	491.2
540.0	408.0	413.9	425.9	434.4	442.1	462.3	481.6
550.0	404.9	410.6	419.7	429.3	438.9	458.6	481.7
560.0	399.6	405.9	416.2	425.5	434.8	455.2	475.5
570.0	395.1	401.2	410.4	418.7	430.5	451.0	470.0
580.0	388.1	393.2	402.4	412.1	423.5	445.8	461.6
584.0	388.2	392.6	402.0	411.4	422.4	446.5	463.1
590.0	386.0	390.4	399.8	410.5	421.7	445.9	465.3
598.0	377.0	382.0	392.3	403.0	412.8	434.5	453.3
604.0	372.8	377.0	387.3	397.4	408.3	428.3	448.2
611.0	366.0	370.9	381.1	391.9	402.7	420.0	440.3
617.0	362.2	365.2	375.2	385.4	396.7	413.7	434.0
622.0	355.3	360.3	370.1	380.2	391.9	405.0	425.5
629.0	348.8	353.0	362.8	373.1	385.0	396.9	417.0
635.0	341.8	346.2	356.2	366.4	376.7	389.2	409.2
641.0	335.3	339.4	349.5	359.1	368.5	382.1	402.0
647.0	328.4	333.4	342.3	351.6	360.9	374.7	394.2
654.0	321.3	325.7	334.0	343.1	352.5	366.3	385.7
660.0	315.2	319.2	327.0	336.0	345.6	359.2	378.2
666.0	309.3	312.1	319.3	328.0	337.3	351.0	369.9
673.0	303.2	306.8	313.3	321.8	331.3	344.5	363.2

x	H/Hd						
	0.6	0.75	1	1.25	1.5	2	2.5
	d						
679.0	294.9	298.3	305.0	313.2	322.4	335.7	354.1
686.0	290.3	293.0	299.7	307.9	317.1	330.0	348.8
692.0	282.9	285.9	292.6	300.8	310.0	322.1	340.7
698.0	278.0	280.7	287.2	295.6	304.7	316.6	335.1
709.0	265.8	267.9	274.6	283.6	292.5	304.2	322.3
722.0	253.0	255.2	261.9	270.6	278.2	290.9	308.7
735.0	239.7	242.5	249.0	256.9	264.2	277.3	294.6
747.0	226.4	229.3	235.6	243.1	250.4	263.4	280.1
760.0			222.4	229.4	236.8	250.0	265.8
776.0			206.1	212.8	219.4	232.4	248.0
792.0			189.7	196.2	202.8	215.5	230.8
808.0			173.5	179.7	185.7	198.3	213.1
824.0			157.4	163.4	169.1	181.3	196.0
840.0			141.2	147.0	152.4	164.4	178.8
855.0			126.0	130.7	135.9	147.4	161.4
871.0			109.8	114.8	119.3	130.4	144.2
887.0			93.5	98.4	103.1	114.1	127.5
903.0			78.3	82.2	86.8	97.8	111.0
919.0			61.2	65.2	69.6	80.5	93.4
937.0			44.8	48.9	53.4	64.3	77.0
962.0			28.0	32.5	36.7	47.6	60.2
970.0	19.1	21.3	25.0	29.5	33.7	44.5	57.1
1020.0	12.5	14.6	17.8	22.3	26.5	37.3	49.7

Table A.4. Location of water surface profile for 45-CS-BL model

x	H/Hd					
	0.6	0.75	1	1.25	1.5	2
0.0	411.5	419.1	431.8	444.5	457.2	482.6
210.0	411.3	419.1	431.4	444.0	456.2	481.9
350.0	410.9	418.5	431.1	443.7	455.4	481.0
500.0	410.0	416.9	428.4	439.4	449.7	470.9
540.0	406.4	412.6	422.6	432.6	442.0	461.9
550.0	403.7	409.5	419.5	429.1	438.4	458.9
560.0	399.9	405.8	415.6	425.4	434.9	455.4
570.0	394.7	400.9	410.9	420.9	430.4	450.9
580.0	389.0	393.0	405.0	414.9	425.0	446.9
584.0	387.9	392.2	402.2	413.4	422.5	444.3
590.0	385.2	390.0	399.8	410.8	418.7	440.4
598.0	375.5	381.5	392.0	402.2	412.8	435.4
604.0	370.7	376.6	386.8	397.0	407.2	431.1
611.0	365.1	370.7	380.1	390.6	401.7	423.2
617.0	359.5	365.0	374.1	385.0	396.4	417.0
622.0	354.7	360.1	369.0	380.1	389.6	408.6
629.0	347.6	352.9	362.5	372.8	381.4	400.3
635.0	340.4	345.9	354.7	365.4	374.0	392.0
641.0	333.1	338.9	347.6	358.1	367.0	384.3
647.0	326.7	331.9	340.6	350.6	359.4	376.7
654.0	319.5	324.3	332.1	342.1	350.6	368.0
660.0	313.4	317.9	325.0	335.0	343.5	360.5
666.0	306.8	310.0	317.0	326.8	334.9	352.0
673.0	300.9	304.2	310.8	320.5	328.4	345.7
679.0	293.0	296.3	302.7	312.1	319.7	336.7
686.0	288.2	291.1	297.6	307.0	314.0	330.9
692.0	280.4	283.9	290.3	299.2	306.0	322.7
698.0	275.2	279.4	285.4	293.8	300.3	317.0
704.0	268.9	273.0	279.2	287.2	293.6	310.1
709.0	263.6	267.3	273.2	281.2	287.5	303.9
716.0	256.6	260.8	266.7	274.7	280.7	297.1
722.0	250.4	254.3	260.2	268.2	274.1	290.3

x	H/Hd					
	0.6	0.75	1	1.25	1.5	2
729.0	243.4	247.8	253.2	261.3	266.9	283.0
735.0	237.2	241.4	247.0	254.9	260.5	276.3
741.0	230.1	234.3	239.6	247.3	252.8	268.6
747.0	224.3	228.7	233.8	241.1	246.5	262.2
754.0	217.9	222.2	227.5	234.5	239.8	255.4
760.0	211.3	215.6	220.7	227.7	232.8	248.3
768.0	202.6	206.7	211.8	218.6	223.7	239.1
776.0	194.9	198.9	204.0	210.6	215.7	230.9
784.0	186.7	190.7	195.8	202.1	207.2	222.2
792.0	179.1	183.0	188.2	194.1	199.2	214.1
800.0	170.9	174.8	180.3	185.6	190.7	205.6
808.0	162.8	166.5	172.0	177.2	182.2	197.0
816.0	154.7	158.7	163.7	168.8	173.8	188.4
824.0	147.0	151.0	155.8	160.8	165.7	180.2
831.0	138.6	142.6	147.4	152.2	157.0	171.4
840.0	131.0	135.1	139.8	144.5	149.1	163.4
848.0	123.1	127.1	131.7	136.5	140.9	155.0
855.0	115.0	119.0	123.5	128.1	132.4	146.5
865.0	107.0	110.9	115.3	119.9	124.1	138.0
871.0	99.0	102.9	107.0	111.7	115.7	129.6
880.0	90.9	94.7	99.0	103.5	107.4	121.2
887.0	83.6	87.0	91.6	96.0	99.7	113.3
895.0	76.0	79.5	84.0	88.3	91.9	105.5
903.0	68.1	71.4	75.8	80.1	83.7	97.1
912.0	59.8	62.8	67.4	71.4	75.0	88.1
919.0	52.0	54.7	59.3	63.3	66.7	79.8
929.0	42.5	44.6	49.4	53.2	56.6	69.6
937.0	36.8	39.1	44.0	47.3	50.7	63.7
952.0	29.3	31.3	36.0	39.1	42.5	55.3
962.0	21.4	23.2	27.9	31.0	34.3	47.0
970.0	18.3	20.5	25.0	28.0	31.2	43.9
1020.0	10.5	12.7	17.4	20.3	23.6	36.1

Table A.5. Location of water surface profile for 45-CS-BS model

		H/Hd				
		0.6	0.75	1	1.25	1.5
x	d					
	0.0	0.6	0.75	1	1.25	1.5
0.0	413.0	420.6	433.3	446.0	458.7	
210.0	413.0	420.6	433.3	445.7	457.7	
350.0	412.9	419.8	433.3	444.9	456.6	
500.0	412.8	420.3	430.8	443.2	454.3	
550.0	411.3	419.0	428.2	439.8	453.0	
600.0	405.3	400.0	420.7	430.1	439.4	
610.0	399.9	400.6	415.6	424.4	433.8	
620.0	394.3	396.1	410.4	419.6	428.9	
627.0	387.3	390.1	406.8	416.1	425.2	
634.0	382.6	386.1	399.9	411.1	420.6	
641.0	375.0	379.2	395.6	405.0	415.3	
652.0	363.2	367.8	385.2	388.6	406.4	
661.0	356.2	361.5	380.9	379.9	392.1	
671.0	343.4	348.4	367.8	366.2	377.9	
679.0	333.2	338.1	357.4	355.4	366.8	
687.0	323.1	327.8	346.3	344.6	355.7	
695.0	313.3	318.1	334.6	334.4	345.2	
704.0	302.3	307.0	322.5	322.8	333.3	
713.0	291.0	295.5	309.5	310.9	321.0	
721.0	281.2	285.4	298.7	300.6	310.5	
729.0	271.8	276.0	288.6	290.9	300.5	
738.0	261.4	265.5	277.5	280.1	289.5	
746.0	252.2	256.2	267.5	270.6	279.8	
755.0	241.6	245.4	256.1	259.5	268.6	

		H/Hd				
		0.6	0.75	1	1.25	1.5
x	d					
	764.0	230.6	234.4	244.6	248.2	257.1
771.0	222.1	225.8	235.6	239.4	248.2	
781.0	209.9	213.6	223.1	226.8	235.5	
789.0	200.2	203.8	213.1	216.9	225.3	
797.0	190.5	194.1	203.0	206.9	215.3	
807.0	178.3	181.8	190.5	194.4	202.6	
814.0	169.8	173.2	181.7	185.6	193.8	
823.0	158.8	162.1	170.3	174.3	182.4	
831.0	149.1	152.3	160.3	164.3	172.4	
842.0	135.6	138.8	146.7	150.7	158.5	
849.0	127.1	130.3	137.9	142.0	149.7	
857.0	117.7	120.8	128.2	132.4	140.0	
864.0	109.5	112.5	119.8	124.0	131.6	
874.0	97.7	100.7	107.9	112.1	119.5	
882.0	88.3	91.3	98.2	102.5	109.9	
890.0	79.0	81.9	88.7	93.0	100.3	
899.0	69.5	72.4	79.1	83.5	90.7	
908.0	60.1	62.9	69.5	73.9	81.1	
918.0	49.7	52.4	58.9	63.3	70.4	
928.0	39.2	41.9	48.4	52.7	59.7	
942.0	30.2	32.9	39.2	43.5	50.5	
960.0	19.8	22.4	28.7	33.0	39.8	
970.0	14.0	16.6	22.8	27.1	33.9	
1020.0	5.5	8.0	14.1	18.5	25.2	

Table A.6. Location of water surface profile for 50° smooth model

x	H/Hd							
	0.6	0.75	1	1.25	1.5	2	2.5	
	d							
0.0	413.0	420.6	433.3	446.0	458.7	484.1	509.5	
210.0	413.0	420.4	433.0	445.7	458.2	483.8	508.7	
350.0	413.0	420.3	432.4	445.2	457.7	482.9	507.4	
500.0	413.0	419.9	431.4	443.2	454.5	477.7	498.9	
550.0	411.5	418.7	429.1	439.8	450.4	471.6	491.1	
600.0	404.5	411.2	420.2	429.5	439.3	458.6	477.4	
610.0	401.5	407.3	416.2	425.5	435.3	458.6	473.9	
620.0	396.0	402.5	412.4	420.8	430.6	449.5	470.0	
627.0	391.8	398.1	407.7	417.2	427.1	446.5	467.3	
634.0	386.4	392.8	402.9	412.5	422.8	442.7	463.6	
641.0	380.0	387.0	396.4	406.9	417.5	438.3	459.1	
652.0	370.6	376.7	386.8	397.2	408.1	430.9	451.8	
671.0	350.7	357.6	366.4	378.2	389.8	415.1	437.3	
687.0	329.6	335.4	342.5	353.5	364.3	387.4	423.2	
704.0	309.3	315.4	321.9	332.0	342.0	363.5	405.7	
721.0	287.3	293.0	299.6	309.9	318.7	338.7	384.9	
738.0	266.4	271.6	278.9	289.1	296.8	315.2	362.9	
755.0	246.7	250.7	258.2	268.2	275.4	292.8	336.6	
771.0	226.5	230.0	237.8	247.1	253.5	270.1	314.9	
789.0	206.3	209.8	217.2	226.0	232.0	247.9	290.5	
807.0	186.1	189.6	196.5	205.2	210.8	225.9	265.8	
823.0	166.0	169.0	176.3	184.4	189.5	204.1	243.5	
842.0	146.2	149.2	155.9	164.0	168.5	182.4	217.0	
857.0	126.5	129.5	136.5	142.9	147.7	160.9	197.1	
874.0	106.5	109.0	116.5	121.9	126.5	139.5	175.9	
890.0	86.2	89.0	96.6	101.4	105.6	118.3	156.0	
908.0	66.2	69.0	75.8	81.1	84.7	97.2	137.7	
928.0	46.0	49.4	55.4	60.6	63.9	76.1	114.5	
960.0	24.2	27.0	32.1	37.6	40.9	53.0	86.5	
970.0	20.9	24.0	28.8	34.2	37.4	49.3	77.9	
1020.0	13.4	16.0	21.1	26.4	29.6	41.5	57.3	

Table A.7. Location of water surface profile for 50-CL-BL model

x	H/Hd					
	0.6	0.75	1	1.25	1.5	2
0.0	413.0	420.6	433.3	446.0	458.7	484.1
210.0	413.0	420.6	433.3	446.0	458.5	483.6
350.0	412.6	420.0	432.5	445.3	458.0	482.1
500.0	412.1	419.5	431.5	442.9	454.3	476.8
550.0	411.5	418.2	428.8	439.8	448.0	470.5
600.0	405.4	410.6	419.8	429.6	438.6	458.0
610.0	400.6	406.0	415.1	425.4	435.0	454.4
620.0	395.7	400.8	409.8	420.5	430.1	449.9
627.0	390.6	395.8	405.2	416.7	426.2	446.8
634.0	385.1	390.5	400.3	412.2	421.9	442.9
641.0	379.3	384.9	394.9	405.0	414.0	438.5
652.0	371.8	374.5	385.0	397.1	407.8	430.3
671.0	351.8	355.8	366.2	378.8	388.9	413.5
687.0	330.4	334.8	345.3	358.0	366.1	387.1
704.0	310.9	315.3	324.5	335.3	344.5	363.4
721.0	290.7	294.0	301.4	314.2	323.0	339.0
738.0	269.3	273.4	280.3	291.7	299.7	315.7
746.0	256.7	260.6	268.2	278.7	286.5	302.8
755.0	247.5	251.5	259.3	270.3	277.9	294.2
764.0	235.1	238.9	246.3	257.7	265.0	281.2
771.0	227.1	230.0	238.3	249.1	256.8	272.3
781.0	215.7	218.3	225.9	236.6	244.2	259.5
						274.3

x	H/Hd					
	0.6	0.75	1	1.25	1.5	2
789.0	207.2	209.5	217.0	227.9	234.9	250.0
797.0	195.9	197.7	205.0	215.2	222.6	237.3
807.0			196.3	206.7	213.4	228.1
814.0			185.1	195.1	201.7	216.5
823.0			176.2	185.6	192.1	206.8
831.0			165.2	173.9	180.2	194.7
842.0			156.9	164.5	171.0	185.0
849.0			145.7	152.5	159.0	172.4
857.0			137.1	143.6	149.9	163.3
864.0			126.1	132.1	138.7	151.5
874.0			116.9	122.7	129.3	142.0
882.0			106.5	111.9	118.6	130.7
890.0			97.1	102.0	108.8	120.5
899.0			85.7	90.5	97.2	108.8
908.0			76.5	81.2	87.9	99.2
918.0			65.9	70.4	76.9	88.0
928.0			56.2	60.7	67.0	78.0
942.0	36.8	38.8	46.1	49.9	56.1	66.9
960.0	25.7	27.5	34.1	38.1	43.9	54.7
970.0	22.3	24.3	31.0	34.9	40.4	51.0
1020.0	15.2	17.0	23.5	27.4	32.5	43.1
						56.3

Table A.8. Location of water surface profile for 50-CL-BS model

x	H/Hd					
	0.6	0.75	1	1.25	1.5	2
0.0	413.0	420.6	433.3	446.0	458.7	484.1
210.0	413.0	420.6	433.3	446.0	458.5	483.4
350.0	413.0	420.6	433.0	445.5	457.9	482.0
500.0	412.8	420.1	432.6	444.3	455.6	477.1
550.0	412.2	419.4	429.6	441.3	451.6	470.7
600.0	404.7	410.8	419.9	430.3	440.1	457.8
610.0	400.6	407.2	415.9	426.3	436.3	454.5
620.0	395.4	401.8	410.9	421.3	431.4	449.7
627.0	391.0	397.5	406.4	417.4	427.8	446.0
634.0	386.3	392.8	401.7	413.2	423.8	442.0
641.0	381.2	387.4	395.2	408.7	418.4	435.8
652.0	370.1	377.6	385.0	399.5	411.3	426.1
661.0	358.8	366.9	375.5	391.2	402.8	422.3
671.0	349.8	357.2	365.5	380.2	392.8	412.0
679.0	337.4	344.3	352.7	366.7	380.0	398.1
687.0	326.5	333.4	341.5	355.0	368.0	386.0
695.0	314.5	321.1	328.8	341.8	354.7	372.7

x	H/Hd					
	0.6	0.75	1	1.25	1.5	2
704.0	306.6	313.0	320.3	332.5	345.3	363.5
721.0	286.1	290.9	297.4	309.1	320.9	339.0
738.0	266.1	269.5	276.1	287.2	298.3	316.1
755.0		250.2	255.2	265.7	276.4	294.2
771.0		230.5	234.5	244.1	254.2	271.5
789.0		210.7	213.7	222.7	232.7	249.6
807.0		190.9	193.1	201.5	211.2	227.7
823.0		171.2	172.8	180.5	189.9	206.0
842.0		151.4	152.3	159.9	168.9	184.6
857.0		131.6	132.5	140.0	148.0	163.2
874.0		111.9	112.8	120.1	127.8	141.8
890.0	87.9	92.0	93.0	100.2	107.3	120.6
908.0	68.1	72.1	73.2	80.3	87.0	99.4
928.0	48.3	52.2	53.4	60.5	67.5	78.3
960.0	26.5	30.2	32.5	38.7	45.3	55.1
970.0	23.2	26.9	29.9	35.6	42.6	51.5
1020.0	16.0	19.2	23.3	28.1	35.0	43.6

Table A.9. Location of water surface profile for 50-CS-BL model

		H/Hd						
		0.6	0.75	1	1.25	1.5	2	2.5
		d						
x								
0.0	413.0	420.6	433.3	446.0	458.7	484.1	509.5	
210.0	413.0	420.6	433.3	445.9	458.2	483.4	508.4	
350.0	413.0	420.4	432.5	444.8	457.7	482.8	506.8	
500.0	412.9	420.1	431.5	442.1	455.0	476.7	498.6	
550.0	411.6	418.3	429.1	440.2	453.0	470.5	491.6	
600.0	404.2	409.9	419.5	429.0	438.4	457.5	477.1	
610.0	400.1	405.7	415.0	425.1	435.3	453.9	473.2	
620.0	395.1	401.0	410.7	419.6	429.4	448.9	468.7	
627.0	390.2	396.2	406.2	415.8	425.9	445.2	465.4	
634.0	385.1	391.0	401.1	411.1	421.4	441.2	461.0	
641.0	379.6	385.4	395.4	405.7	415.6	436.9	453.1	
652.0	371.9	375.9	385.1	395.9	406.1	427.0	443.3	
661.0	360.3	365.0	376.0	387.0	400.1	421.9	442.2	
671.0	351.7	355.5	365.8	377.2	390.0	411.1	431.1	
679.0	340.8	344.6	354.4	365.5	377.3	396.5	416.4	
687.0	330.1	334.2	344.6	355.0	365.5	383.9	403.4	
695.0	318.0	323.0	333.5	344.3	355.1	370.3	389.4	
704.0	309.4	313.9	324.1	335.0	346.0	360.7	379.5	
713.0	296.6	302.3	312.5	320.2	331.5	346.4	364.8	
721.0		293.5	303.8	311.3	321.6	336.3	354.5	
729.0		283.2	292.4	299.5	309.2	324.1	342.0	
738.0		272.8	281.8	289.1	298.6	313.2	330.8	
746.0		260.1	268.7	276.7	286.2	300.0	317.4	
755.0		251.7	260.3	268.3	277.6	291.0	308.2	

		H/Hd						
		0.6	0.75	1	1.25	1.5	2	2.5
		d						
x								
764.0		238.7	246.9	255.3	264.1	277.4	294.4	
771.0		230.4	238.3	247.4	255.1	268.4	285.3	
781.0		218.3	226.1	234.8	242.2	255.7	272.3	
789.0		209.3	217.5	225.8	232.8	246.4	262.8	
797.0		197.3	205.1	213.4	220.1	233.8	250.0	
807.0		188.6	196.5	204.5	210.8	224.5	240.6	
814.0		177.2	184.8	192.8	198.8	212.5	228.5	
823.0		167.9	175.1	183.5	189.1	202.7	218.6	
831.0		156.4	163.6	171.7	177.3	191.0	206.6	
842.0		147.2	154.2	162.5	167.6	181.3	196.8	
849.0		135.4	142.1	150.4	155.3	168.9	184.4	
857.0		126.7	133.7	141.6	146.4	159.8	175.3	
864.0		115.3	122.3	130.6	134.8	148.0	163.4	
874.0		106.4	113.2	121.7	125.3	138.5	153.8	
882.0		95.6	102.4	110.8	114.2	127.3	142.5	
890.0		86.3	93.2	100.8	104.3	117.2	132.3	
899.0		75.6	82.5	89.3	92.8	105.7	120.5	
908.0		66.6	73.4	79.9	83.4	96.2	110.9	
918.0		56.3	62.8	68.7	72.5	85.1	99.7	
928.0		47.0	53.2	58.8	62.6	75.2	89.6	
942.0	33.6	37.2	42.7	47.8	51.7	64.1	78.5	
960.0	23.3	26.1	31.4	35.8	39.7	52.1	66.3	
970.0	20.0	22.6	27.9	32.2	36.1	48.4	62.7	
1020.0	12.9	15.2	20.2	24.3	28.3	40.6	54.8	

Table A.10. Location of water surface profile for 50-CS-BS model

H/Hd					
x	0.6	0.75	1	1.25	1.5
	d				
0.0	413.0	420.6	433.3	446.0	458.7
210.0	412.8	420.6	433.3	445.9	458.4
350.0	412.7	420.6	432.9	445.7	457.9
500.0	412.2	420.2	432.3	445.5	456.1
550.0	412.0	420.1	432.1	445.0	455.1
600.0	411.8	419.6	430.9	444.0	453.7
630.0	411.4	419.1	429.8	442.5	450.3
660.0	410.1	416.7	426.3	438.5	445.9
670.0	408.7	415.2	424.1	435.0	443.3
680.0	405.2	412.9	421.7	431.0	440.2
685.0	404.6	411.3	416.9	429.5	435.7
690.0	403.1	408.1	413.0	428.5	431.1
700.0	398.6	402.2	410.1	424.5	428.2
710.0	393.3	397.1	405.6	418.9	423.6
720.0	386.6	391.1	399.6	412.5	418.2
730.0	379.1	383.2	391.9	404.9	411.2
740.0	370.1	375.1	384.0	396.8	403.8
750.0	360.0	365.1	374.2	387.0	394.8
760.0	351.7	357.4	368.4	384.0	393.6
770.0	337.8	343.5	354.0	368.7	378.6
780.0	325.3	330.8	340.8	355.0	364.9

H/Hd					
x	0.6	0.75	1	1.25	1.5
	d				
790.0	315.2	320.4	330.1	344.0	353.8
800.0	293.1	297.9	307.0	320.2	329.6
810.0	274.9	279.3	288.1	300.7	309.8
820.0	255.1	259.3	267.8	279.8	288.6
830.0	233.9	237.9	246.0	257.6	266.2
840.0	213.9	217.6	225.5	236.6	245.0
850.0	192.8	196.4	204.0	214.7	222.9
860.0	171.8	175.4	182.7	193.1	201.0
870.0	150.3	153.8	161.0	171.0	178.7
880.0	129.9	133.4	140.3	150.0	157.6
890.0	108.6	112.0	118.7	128.1	135.5
900.0	87.2	90.6	97.1	106.3	113.6
910.0	69.5	72.8	79.2	88.2	95.3
920.0	53.8	57.1	63.4	72.2	79.2
930.0	44.9	48.2	54.4	63.1	70.1
940.0	30.7	33.9	40.0	48.6	55.4
950.0	28.9	32.1	38.2	46.8	53.6
960.0	23.0	26.2	32.3	40.8	47.6
970.0	15.2	18.4	24.4	32.9	39.6
990.0	10.0	13.2	19.1	27.5	34.2
1020.0	8.0	11.2	17.1	25.5	32.1

Table A.11. Location of water surface profile for 60° smooth model

		H/Hd				
		0.6	0.75	1	1.25	1.5
x		d				
0.0	413.0	420.6	433.3	446.0	458.7	484.1
210.0	413.0	420.6	433.3	446.0	458.6	483.2
550.0	412.2	419.2	431.2	443.3	455.2	477.7
600.0	412.1	419.1	430.6	441.5	452.0	473.3
630.0	411.2	417.5	428.1	438.3	448.2	466.3
660.0	406.9	412.5	422.1	431.7	441.3	460.6
665.0	405.4	411.0	420.4	429.8	439.7	459.5
670.0	403.4	409.0	418.6	428.2	437.4	456.7
680.0	399.2	404.3	414.1	423.8	433.3	451.9
690.0	393.3	398.1	408.8	418.7	427.8	446.8
695.0	391.3	396.7	405.5	415.6	425.0	445.1
702.0	387.5	391.9	400.4	410.9	420.9	443.8
707.0	382.0	387.3	396.3	406.9	417.2	440.6
720.0	370.1	375.9	385.8	396.2	406.7	430.3
737.0	351.6	357.9	368.5	382.1	395.5	423.1
753.0	330.6	335.2	345.7	358.8	372.2	399.0
765.0	312.7	316.7	326.6	339.1	350.0	374.2

		H/Hd				
		0.6	0.75	1	1.25	1.5
x		d				
777.0	294.2	297.1	306.7	319.0	328.3	350.0
789.0	273.0	275.9	285.6	298.4	306.6	326.7
801.0	252.4	255.5	264.5	276.7	284.6	303.7
812.0	233.3	236.6	245.3	256.3	262.9	280.8
824.0	213.2	216.1	224.3	235.4	240.9	258.4
836.0	193.1	196.1	203.4	214.5	219.1	236.2
847.0	174.7	177.8	184.1	193.5	197.5	214.2
859.0	154.6	157.9	163.8	172.0	176.0	192.3
871.0	134.6	138.1	143.6	150.8	154.6	170.6
882.0	116.3	120.0	123.1	129.6	133.4	149.1
893.0	98.7	98.9	102.9	108.5	112.1	127.6
907.0	75.1	76.2	81.5	87.1	90.9	106.3
927.0	51.8	53.9	59.9	65.7	69.8	85.0
961.0	29.6	32.3	38.4	44.5	48.8	63.9
970.0	25.0	27.9	33.8	40.0	44.3	59.4
1020.0	15.6	18.6	24.6	30.9	35.2	50.2

Table A.12. Location of water surface profile for 60-CL-BL model

x	H/Hd					
	0.6	0.75	1	1.25	1.5	2
	d					
0.0	413.0	420.6	433.3	446.0	458.7	484.1
210.0	413.0	420.6	433.3	446.0	458.7	483.3
550.0	413.0	420.5	432.8	444.4	455.5	477.3
600.0	413.0	420.3	431.6	442.0	452.1	472.5
630.0	411.8	418.2	428.2	438.5	448.2	467.6
660.0	406.7	412.6	422.1	431.6	440.6	459.7
665.0	405.3	411.1	420.7	430.2	439.3	457.9
670.0	403.8	409.6	418.8	428.2	437.7	456.0
680.0	399.0	404.7	414.5	424.4	433.8	452.0
690.0	393.0	399.2	409.0	419.1	429.1	447.7
695.0	390.6	396.7	405.7	415.8	426.0	446.6
702.0	385.8	392.8	400.9	411.1	421.4	443.8
707.0	381.6	386.9	396.9	407.3	417.8	441.1
720.0	371.0	374.8	385.7	395.9	407.1	429.1
737.0	352.6	357.9	369.5	382.0	394.6	423.2
753.0	330.0	335.9	348.4	360.9	372.8	399.3
765.0	310.7	315.9	327.9	339.8	351.1	375.0
777.0	291.5	296.2	307.4	317.6	328.8	352.1
789.0	279.7	280.1	289.5	296.0	307.5	329.1
795.0	263.9	265.1	274.1	281.0	292.3	313.4
801.0	257.7	260.3	268.1	275.3	286.5	307.0
807.0	245.4	247.0	255.3	262.1	273.1	292.9
812.0	236.8	239.4	247.9	254.7	265.6	284.5

x	H/Hd					
	0.6	0.75	1	1.25	1.5	2
	d					
818.0	226.5	227.5	236.1	243.0	254.2	272.3
824.0	216.2	218.8	227.1	233.7	244.8	262.7
830.0	205.9	208.5	216.9	223.0	234.4	251.7
836.0	195.6	198.9	206.4	212.6	224.0	240.6
841.0	187.0	189.6	195.9	202.0	213.1	229.7
847.0	176.7	179.3	185.5	191.8	202.5	218.8
853.0	166.3	168.9	175.3	181.9	191.8	208.1
859.0	155.8	159.3	164.9	171.5	181.1	197.1
865.0	145.2	148.8	155.0	161.2	170.4	186.2
871.0	134.7	138.7	144.8	150.8	159.9	175.3
876.0	125.9	128.5	134.7	140.3	149.4	164.4
882.0	115.4	118.9	124.5	130.1	138.8	153.7
887.0	106.6	109.2	114.3	119.5	128.4	142.9
893.0	97.2	98.8	103.9	109.1	117.9	132.1
900.0	87.8	88.3	92.8	98.9	107.1	121.4
907.0	75.3	77.2	82.1	88.4	96.5	110.8
915.0	63.2	66.1	71.4	77.7	85.7	100.0
927.0	52.0	55.2	60.9	67.0	75.2	89.4
942.0	40.6	44.2	50.0	56.3	64.5	78.8
961.0	29.6	33.3	39.5	45.7	53.9	68.2
970.0	25.0	28.7	34.9	41.2	49.4	63.8
1020.0	15.6	19.4	25.6	32.0	40.2	54.5

Table A.13. Location of water surface profile for 60-CL-BS model

H/Hd						
x	0.6	0.75	1	1.25	1.5	2
	d					
0	413.0	420.6	433.3	446.0	458.7	484.1
210	413.0	420.6	433.3	446.0	458.7	484.0
550	412.9	420.6	433.0	444.6	455.6	478.0
600	412.6	419.9	431.3	442.0	452.6	473.3
630	411.3	417.9	428.5	438.5	448.5	468.3
660	406.8	412.8	422.5	431.9	441.0	462.6
665	405.4	411.4	421.2	430.4	439.5	461.1
670	403.9	410.0	419.5	428.4	437.5	456.9
680	399.7	405.6	414.9	424.1	433.1	452.6
690	394.2	400.1	409.7	419.1	428.5	448.0
695	390.7	396.6	406.3	415.9	427.0	447.2
702	385.7	391.5	401.3	411.2	423.1	443.3
707	381.9	387.6	397.5	407.6	417.3	440.4
720	370.5	376.7	386.9	396.9	407.1	437.6
729	363.2	371.1	381.4	394.0	406.6	436.6
737	352.7	359.4	370.7	383.8	395.0	423.0
744	342.1	348.7	359.9	373.3	383.1	412.3
753	331.4	338.9	349.1	362.8	372.2	400.0
759	323.1	328.4	338.8	351.9	362.2	387.4

H/Hd						
x	0.6	0.75	1	1.25	1.5	2
	d					
765	313.5	319.0	329.8	342.4	352.1	375.9
771	303.6	309.5	320.7	333.4	342.6	364.5
777	294.1	300.1	310.8	322.9	332.3	352.6
783	283.9	289.6	299.7	311.7	322.4	341.2
789	274.8	279.8	289.9	300.9	311.7	330.2
801	254.7	258.6	269.5	279.4	290.0	307.4
812	232.7	238.0	249.8	258.3	267.3	284.6
824	212.4	217.7	228.4	237.3	246.1	262.1
836	192.2	196.8	206.9	215.3	224.1	240.6
847	173.6	177.7	186.3	193.2	202.1	218.8
859	153.5	157.7	165.7	172.3	181.2	196.8
871	133.5	137.7	144.9	151.3	159.4	175.3
882	115.2	117.4	124.4	130.1	137.8	153.8
893	97.7	98.3	103.8	109.3	117.2	132.4
907	74.5	76.2	82.8	88.2	95.7	111.1
927	51.2	54.8	60.7	66.8	74.7	89.7
961	29.3	32.9	39.2	45.6	53.3	68.3
970	24.9	28.4	35.0	41.1	48.7	63.8
1020	15.7	19.3	25.7	31.9	39.6	54.6

Table A.14. Location of water surface profile for 60-CS-BL model

H/Hd						
x	0.6	0.75	1	1.25	1.5	2
	d					
0.0	413.0	420.6	433.3	446.0	458.7	484.1
210.0	412.9	420.5	433.2	446.0	458.7	483.4
550.0	412.7	420.0	432.3	444.3	455.6	477.4
600.0	411.9	419.4	431.0	441.6	452.1	472.9
630.0	410.8	417.3	428.1	438.5	448.1	468.2
660.0	406.6	412.3	422.0	431.7	441.0	460.0
665.0	405.0	410.6	420.4	430.2	439.5	458.3
670.0	403.0	408.6	416.6	428.6	437.7	456.3
680.0	398.7	404.4	411.9	424.0	434.0	451.9
690.0	393.2	398.6	408.8	419.5	429.5	448.1
695.0	389.8	395.3	406.5	416.1	426.0	447.3
702.0	384.9	390.4	402.7	411.4	421.1	444.0
707.0	381.1	386.5	397.9	407.8	417.3	439.9
720.0	368.8	374.7	385.7	397.1	407.6	429.7
729.0	362.0	367.9	381.0	394.4	403.1	427.7
737.0	351.4	357.5	369.5	382.2	395.8	422.6
744.0	341.0	346.5	357.6	370.7	385.1	410.1
753.0	331.0	335.7	346.6	358.9	373.3	398.1
759.0	320.5	325.9	336.3	349.2	361.6	385.7
765.0	311.4	316.1	326.5	340.3	350.5	373.4
771.0	302.3	306.7	316.3	328.7	339.7	361.4
777.0	293.0	296.9	305.4	318.4	328.6	349.6
783.0	283.3	286.9	295.0	307.7	317.6	337.7
789.0	273.3	276.6	284.9	296.4	306.3	326.2
795.0	261.6	261.8	270.0	281.1	290.4	309.9

H/Hd						
x	0.6	0.75	1	1.25	1.5	2
	d					
801.0	256.0	256.5	263.9	275.2	283.8	303.0
807.0	242.9	243.7	251.0	261.5	270.2	288.9
812.0	234.5	236.4	243.4	253.5	261.9	280.5
818.0	223.1	224.7	231.8	241.3	249.5	267.9
824.0	214.4	216.3	222.9	232.1	240.2	258.1
830.0	204.4	206.9	213.0	221.8	229.5	247.1
836.0	194.3	196.8	203.4	211.6	218.9	236.1
841.0	185.9	186.8	193.4	201.4	208.1	225.1
847.0	177.5	177.0	183.2	190.8	197.4	214.2
853.0	165.7	168.1	173.9	180.5	186.7	203.3
859.0	155.4	157.8	163.5	170.0	176.0	192.5
865.0	145.0	147.5	154.0	159.9	165.4	181.7
871.0	134.7	136.7	143.1	149.1	154.9	171.0
876.0	126.1	126.6	133.2	138.4	144.2	160.3
882.0	115.8	116.3	123.3	127.8	133.6	149.5
887.0	107.2	105.7	113.2	117.3	123.0	138.8
893.0	97.9	95.3	102.7	106.8	112.4	128.2
900.0	86.1	84.7	92.3	96.3	101.9	117.5
907.0	75.7	74.2	81.8	86.1	91.4	106.9
915.0	63.3	63.6	71.2	75.6	80.9	96.3
927.0	52.0	53.0	60.5	65.2	70.4	85.8
942.0	40.5	42.3	49.8	54.6	60.0	75.2
961.0	29.7	31.7	39.2	44.2	49.6	64.7
970.0	25.0	27.2	34.7	39.9	45.1	60.2
1020.0	15.5	18.0	25.4	30.9	36.0	51.0

Table A.15. Location of water surface profile for 60-CS-BS model