# THE FILTRATION OF DRAG REDUCING FLUIDS

by

CHENG TZE FU

A THESIS SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR

THE DRGREE OF MASTER OF APPLIED SCIENCE

in the

DEPARTMENT OF CHEMICAL ENGINEERING

UNIVERSITY OF OTTAWA

AUGUST, 1977

ProQuest LLC
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106-1346

## ABSTRACT

The filtration of drag reducing fluids has been studied in order to determine the variations in the resistance of filter cake and filter septum at different pressures.

At the beginning of operation, both average specific cake resistance and filter medium resistance of filtration are important at any particular pressure and constant temperature; however, the filter cake resistance plays a dominant role in the remainder of a filtration run after a certain time interval. The average specific filtration resistance and filter medium resistance were computed from the relationships of filtrate volume versus effective time; the effective time of commencement of filtration was obtained by least-squares fitting of a developed equation. The results obtained from the data collected during constant pressure filtration indicated that average specific filtration resistance was decreased by increasing the concentration of Polyox WSR Coagulant in the aqueous solution slurry. However, the filter medium resistance was found to increase under the same conditions.

# ACKNOWLEDGEMENT

The author gratefully acknowledges the guidance and advice of his research director Dr. William Kozicki, Professor and Associate Dean of Engineering and Science, without whose idea and patience this work could not have been carried out.

Also, the author is indebted to Dr. G. Neale and Mr. Jack Son for their helpful discussions and innumerable suggestions. Thanks are also due to Mr. G. Gasperetti and his assistants for technical assistance and for constructing the apparatus.

Finally, the author is grateful to Union Carbide Canada Ltd. who supplied Polyox WSR Coagulant for this work.

# NOMENCLATURE

$A$ = cross-sectional area of the filter bed, $ft^2$.

$C_i$ = the concentration of the polymer solution in the aqueous slurry, dimensionless.

$e_x$ = local value of voidage, $\varepsilon_x/(1-\varepsilon_x)$, dimensionless.

$F(v)$ = function of filtrate volume serving as a correction term, defined by Kozicki, Rao and Tiu.

$g_c$ = conversion factor, $lb_m$-ft/$lb_f$-$sec^2$.

$J_R$ = correction factor for the conventional filtration resistance $\alpha_R$, dimensionless.

$K$ = function defined by equation (II-37).

$k_i$ = a dimensionless constant.

$L$ = total cake thickness, ft.

$m$ = moisture ratio, ratio of mass of wet to dry cake, dimensionless.

$p$ = applied filtration pressure, $lb_f/ft^2$.

$p_1$ = hydraulic pressure at the interface of medium and cake, $lb_f/ft^2$.

$p_s$ = solids compressive pressure at distance x from the medium, $lb_f/ft^2$.

$p_x$ = pressure, $lb_f/ft^2$.

$q_1$ = superficial velocity of the filtrate at the exit of the cake, ft/sec.

$q_x$ = local superficial velocity of the liquid at distance x from the medium, ft/sec.

$R_H$ = hydraulic radius, ft.
$R_m$ = filter medium resistance, $ft^{-1}$.
$r_x$ = local superficial velocity of solids at distance x from the medium, ft/sec.
s = mass fraction of solids in the slurry, dimensionless.
$s_o$ = specific surface per unit volume of solid particle, $ft^2/ft^3$.
T = temperature, $^oC$.
$\bar{t}$ = average time, sec.
u = linear velocity, ft/sec.
$u_{l,x}$ = true average velocity of liquid, ft/sec.
$u_{s,x}$ = true average velocity of solids, ft/sec.
$u_x$ = true average relative velocity of liquid to solids, ft/sec.
v = cumulative filtrate volume per unit area, $ft^3/ft^2$.
$v_o$ = function defined by equation (II-38).
$v_i$ = filtrate volume interval of initial stage, $ft^3/ft^2$.
w = mass of cake solids per unit area, $lb_m/ft^2$.
$w_x$ = mass of cake solids per unit area at distance x from the medium, $lb_m/ft^2$.
x = distance from the medium, ft.

## GREEK LETTERS

$\mu$ = viscosity, $lb_m$/ft-sec.
$\varepsilon$ = local value of porosity of the filter cake, dimensionless.

$\varepsilon_1$ = local value of porosity at filter medium, dimensionless.

$\varepsilon_i$ = porosity in infinitesimal surface layer of cake, dimensionless.

$\varepsilon_x$ = local value or porosity at distance x from the medium, dimensionless.

$\varepsilon_{avg}$ = average porosity for entire cake, dimensionless.

$\varepsilon_{avgx}$ = average porosity for the portion of cake between medium and distance x, dimensionless.

$\Theta$ = time, sec.

$\Theta_o$ = effective time of commencement of filtration defined by equation (II-41), sec.

$\Theta_e$ = effective time of filtration as defined in equation (II-42).

$\rho$ = the density of liquid, $lb_m/ft^3$.

$\rho_s$ = the density of solids, $lb_m/ft^3$.

$\alpha$ = average specific filtration resistance, $ft/lb_m$.

$\alpha_i$ = the average specific filtration resistance in the polymer solution slurry, $ft/lb_m$.

$\alpha_R$ = average psecific filtration resistance defined by Carman (4) and Ruth (15).

$\alpha_{H_2O}$ = the average specific filtration resistance in the aqueous slurry, $ft/lb_m$.

$\alpha_x$ = local specific cake resistance defined by equation (II-21), $ft/lb_m$.

# TABLE OF CONTENTS

## LIST OF FIGURES

## LIST OF TABLES

# CHAPTER I

## INTRODUCTION

Many studies on drag reducing fluids have been done in the past thirty years, and, recently, such fluids have been used in various applications, particularly in studies involving transport phenomena. The addition of polymer solutions to fluids changes some of their behaviour and consequently affects the models for representing these fluids. Drag reduction (27), usually occurring in turbulent flow, takes place in the laminar sublayer region near the wall (3,17,31). However, in the filtration of such a polymer solution, the effective velocity of slip, and the suitability of a model for representing the process, must be experimentally determined because the flow is laminar (5,19,21,29).

Time or cumulative filtrate volume can be used as a measure of the filtration efficiency at constant temperature and constant pressure. If, at a given time, the accumulated filtrate volume increases with the addition of polymer solution, this is an indication that drag reducing fluids can improve filtration properties through, for example, the enlargement of the porosity of filter cake or through the reduction of the average specific resistance of filtration.

The cumulative filtrate volume (9) is determined by filter medium resistance and filter cake resistance in the early stage of filtration. After the filter cake has been built up, the filter medium resistance can be neglected and filter cake resistance plays a dominant role in the later stages of filtration. Therefore, the effective time of filtration (9) or cumulative filtrate volume depends on the average specific filtration resistance.

The purpose of this laboratory work is to observe what the effect of drag reducing fluids will be on filtration processes, and how much drag reducing fluid is required to increase the true average relative velocity of liquid to solid particles (18), or the porosity of filter cake (9). It is hoped that the results of this research will lead to increased efficiency and reduced cost of filtration.

# CHAPTER II

# LITERATURE SURVEY AND DEVELOPMENT OF EQUATIONS

## II-A. Theory

Filtration has been studied and applied in many fields. The theory of filtration has been developed along many lines, each using different mechanisms to explain the phenomena involved. However, the results of those approaches are consistent with each other. For example, McCabe and Smith (11), and Bennett and Myers(2) derived different equations using different symbols, but the physical meaning and final results are the same. The cumulative volume of filtrate and filtration time are the two most important factors in those equations for constant pressure filtration. They all explained that filter medium resistance and filter cake resistance determine the properties of filtration and the velocity of filtrate which exits from the cake.

For purposes of easy mathematical treatment and convenient industrial practice,filtration operations are classified according to the variation of pressure and flow rate with time. The different categories into which the filtration operations are classified are as follows :

(A) For a given aqueous slurry, the major variable under the control of the operator is an overall pressure drop; e.g., when outlet pressure is constant, the pressure drop is controlled by the variation of inlet pressure. If the inlet pressure remains constant during operation, the velocity of filtrate is a maximum at the start of filtration and decreases continuously to a minimum at the end of filtration. This method of operation is called "Constant Pressure Filtration".

(B) If the pressure drop is to be varied during a filtration run, the run is usually started at zero inlet gauge pressure which is then gradually increased, either continuously or in steps, to reach the desired maximum pressure at the end of filtration. By increasing the inlet pressure continuously and smoothly, the velocity of filtrate can be maintained constant after a certain initial time period of filtration. This method has been named "Constant-Rate Filtration."

(C) Various combinations of constant rate and constant pressure operation are used in industry. A common one is to use constant rate until the inlet pressure reaches a specified maximum and then to maintain at expected constant pressure until the end of the filtration. This minimizes loss of solids through a filter septum when filter medium resistance is low, and avoids packing solid particles into the septum.

Of the above mentioned classifications, constant pressure and constant rate filtration are very commonly and widely used in laboratories as well as in industry. A combination of these two types of filtration is employed by many industries whereby the filtration is conducted at constant rate until the maximum pressure is attained and then continued at constant pressure to the extent desired. Hence, the investigations reported here are confined to filtration at constant pressure and constant rate.

Early filtration theory assumed that the liquid moves past a stationary solid (filter cake) during the operation (11,2,22,25). It is, however, false to assume that the solid particle velocity is zero. For highly concentrated slurries, it is important to take into account the velocity of the solids, as shown by Shirato and his coworkers in 1969 (18).

## II-B. Development of Equations

Basic filtration equation can be derived (13) from the modified form of the Poiseuille equation (22) for the rate of flow of a Newtonian fluid through a solid bed :

$$g_c\left(\frac{dp_x}{dx}\right) = -\frac{k_i \mu}{R_H^2}\,u \qquad \text{(II-1)}$$

where, $g_c$= conversion factor, $lb_m$-ft/$lb_f$-$sec^2$

$k_i$= a dimensionless constant

$p_x$= pressure, $lb_f/ft^2$

$R_H$= hydraulic radius, ft

u = linear velocity, ft/sec

x = distance from the medium in the direction of the flow, ft

$\mu$ = viscosity, $lb_m$/ft-sec

The hydraulic radius is defined as :

$$R_H = \left( \frac{\text{flow area}}{\text{wetted perimeter}} \right) \left( \frac{\text{length of path}}{\text{length of path}} \right)$$

$$= \left( \frac{\text{void volume}}{\text{surface area of solids}} \right) \qquad \text{(II-2)}$$

since,

$$\left( \frac{\text{void volume}}{\text{volume of solids}} \right) = \frac{\varepsilon}{1-\varepsilon} \qquad \text{(II-3)}$$

then,

$$\text{void volume} = \frac{\varepsilon}{1-\varepsilon} \text{(volume of solids)} \qquad \text{(II-4)}$$

Substituting (II-4) in (II-2) yields

$$R_H = \frac{\varepsilon \text{ (volume of solids)}}{(1-\varepsilon)\text{(surface area of solids)}} \qquad \text{(II-5)}$$

The surface area of solids per unit volume of solids (exclusive of voids) is termed the specific surface, $S_o$. Now, we obtain :

$$R_H = \frac{\varepsilon}{(1-\varepsilon)S_o} \qquad \text{(II-6)}$$

In Figure II-1, a schematic diagram of a filter cake is shown. As the cake is compressed, the porosity decreases with time at a given distance, x, from the filter medium. Decrease in porosity is caused by solid flowing into the voids as the cake is compressed. The compressive, squeezing action causes the flow rate of the liquid to increase as the medium is approached (25,26). At a distance, x, from the medium, the hydraulic radius is :

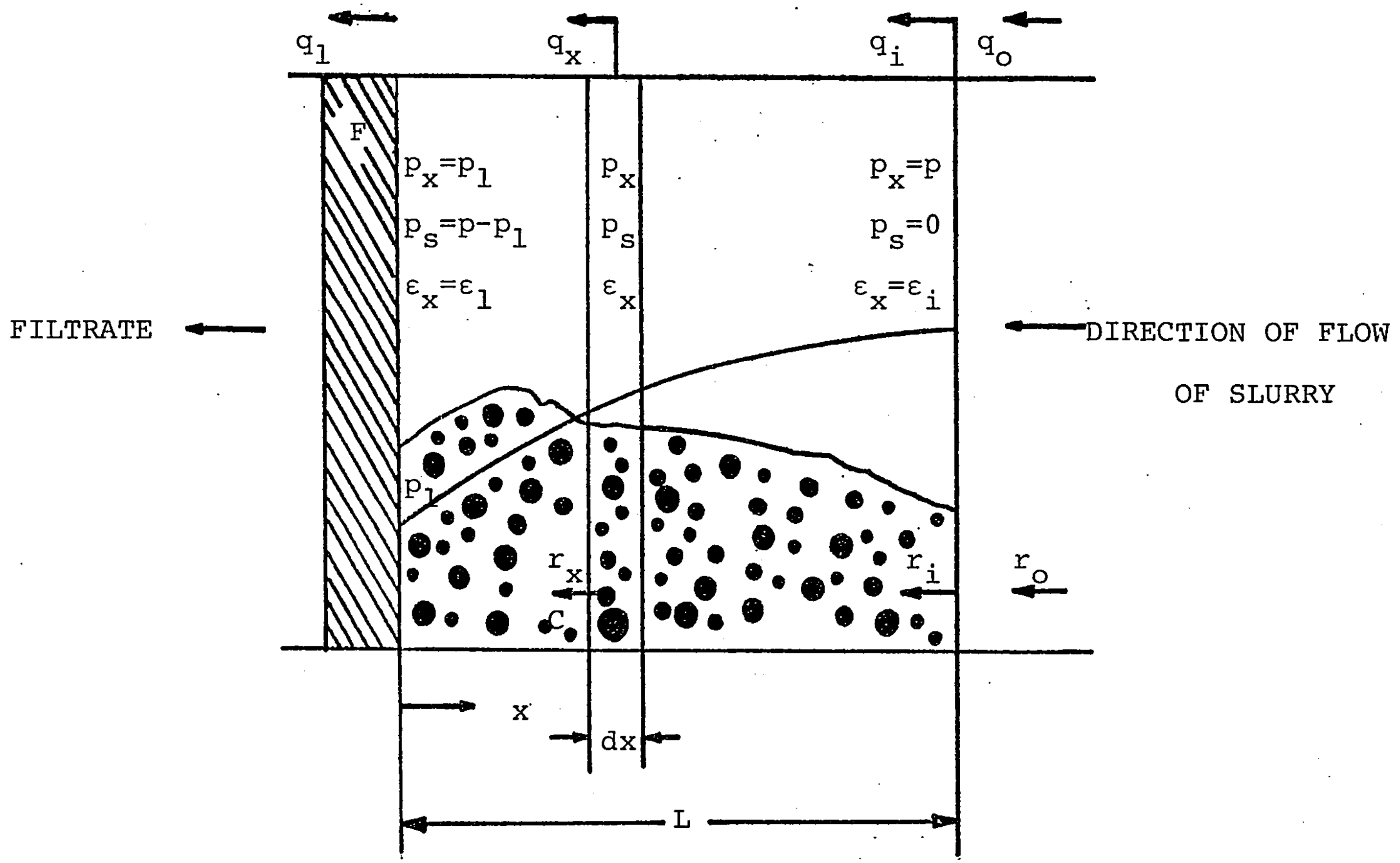
$$R_H = \frac{\varepsilon_x}{(1-\varepsilon_x)S_o} \qquad \text{(II-7)}$$

It should be recognized that the internal flow rates of liquid, $q_x$, and solid, $r_x$, are not constant due to the continuous compression of the cake.

The true average velocity of the liquid, $u_{l,x}$, $ft^3/(ft^2)(sec)$ or ft /sec, is given by the superficial velocity divided by the per cent voids at x, that is :

$$u_{l,x} = \frac{q_x}{\varepsilon_x} \qquad \text{(II-8)}$$

Here x is a distance from the filter medium as shown in Figure II-1, and the true average velocity of the solids is defined as :



| | | | |
|---|---|---|---|
| C | CAKE | q | FLOW RATE OF LIQUID |
| F | FILTER MEDIUM | $p_x$ | HYDRAULIC PRESSURE |
| $\varepsilon$ | POROSITY | $p_s$ | COMPRESSIVE PRESSURE |

FIGURE II-1 SCHEMATIC DIAGRAM OF FILTER CAKE AND MEDIUM

$$u_{s,x} = \frac{r_x}{1-\varepsilon_x} \qquad \text{(II-9)}$$

Therefore, the true average relative velocity, $u_x$ of liquid to solids is represented by :

$$u_x = \frac{q_x}{\varepsilon_x} - \frac{r_x}{1-\varepsilon_x} \qquad \text{(II-10)}$$

The apparent relative flow rate of liquid to solids based upon unit cross-sectional area is :

$$\varepsilon_x u_x = q_x - \frac{\varepsilon_x}{1-\varepsilon_x} r_x = q_x - e_x r_x \qquad \text{(II-11)}$$

where,

$$e_x = \frac{\varepsilon_x}{1-\varepsilon_x} = \text{the local void ratio}$$

Substituting equation (II-7) and (II-10) into the modified Poiseuille equation, we obtain :

$$g_c\left(\frac{dp_x}{dx}\right) = \frac{k_i \mu}{\left(\frac{\varepsilon_x}{(1-\varepsilon_x) S_o}\right)^2} \left(\frac{q_x}{\varepsilon_x} - \frac{r_x}{1-\varepsilon_x}\right) \qquad \text{(II-12)}$$

or,

$$g_c\left(\frac{dp_x}{dx}\right) = \frac{k_i \mu (1-\varepsilon_x)^2 S_o^2}{\varepsilon_x^2} \left(\frac{q_x}{\varepsilon_x} - \frac{r_x}{1-\varepsilon_x}\right) \qquad \text{(II-13)}$$

Or,

$$g_c\left(\frac{dp_x}{dx}\right) = \frac{k_i\mu(1-\varepsilon_x)^2 S_o^{\ 2}}{\varepsilon_x^{\ 3}}(q_x - e_x r_x) \qquad \text{(II-14)}$$

The minus sign usually appearing in the pressure gradient term has been dropped since x is measured from the filter medium, i.e., in the direction opposite to the direction of flow, as indicated in Figure II-1. Tiller and Shirato (25) clearly indicated the necessity to measure distance from the fixed medium rather than from the moving cake surface.

Suppose the total mass of dry solid is $w$ $lb_m/ft^2$. Then the mass of solids in the filter cake per square foot in the distance dx is given by :

$$\begin{aligned} dw_x &= \text{(density) (volume of solids)} \\ &= \rho_s(1-\varepsilon_x)\ Adx \qquad \text{(II-15)} \end{aligned}$$

or, per unit area :

$$dw_x = \rho_s(1-\varepsilon_x)\ dx \qquad \text{(II-16)}$$

Therefore,

$$dx = \frac{dw_x}{\rho_s(1-\varepsilon_x)} \qquad \text{(II-17)}$$

As liquid passes along the particles in the filter bed (16), the drag on each particle adds to the drag on the downstream particle; consequently, the net compressive pressure builds up as the filter medium is approached which accounts for the decrease in porosity. Tiller mentioned (22,23) that Grace and Ruth (22,7) have assumed that there is point contact between the particles in a filter cake. The build-up of compressive pressure throughout the cake will be in accord with the relation :

$$dp_s = - dp_x \tag{II-18}$$

which is equivalent to :

$$p_s + p_x = p \tag{II-19}$$

where p is applied pressure (23), $p_x$ is the hydraulic pressure at a distance x from the surface of the cake and $p_s$ is the solid compressive pressure. Equation (II-18) is predicated on the basis of point contact and would have to be modified if there were area contact between particles. Substituting equation (II-17) and (II-18) in equation (II-14) we obtain :

$$\frac{dp_s}{dw_x} = - \frac{\mu k_i S_o^2 (1-\varepsilon_x)}{g_c \varepsilon_x^3 \rho_s} (q_x - e_x r_x) \tag{II-20}$$

In classical filtration theory, it is convenient to lump those parameters which are functions only of the pressure drop across the filter bed together in a single quantity, $\alpha_x$, called the local specific cake resistance. Mathematically, $\alpha_x$ is defined by :

$$\alpha_x = \frac{k_i S_o^2 (1-\varepsilon_x)}{\varepsilon_x^3 \rho_s} = \frac{k_i}{R_H^2 \rho_s \varepsilon_x (1-\varepsilon_x)} \tag{II-21}$$

with the aid of equation (II-21), equation (II-20) can be rearranged as follows :

$$g_c \frac{dp_s}{dw_x} = -\mu\alpha_x(q_x - e_x r_x) \qquad \text{(II-22)}$$

When the flow takes place through a fixed, compressible bed in which the solids are not moving (14,24), $r_x$ is zero and $q_x$ is constant(although the average liquid velocity $q_x/\varepsilon_x$ may vary). Equation (II-22) then becomes:

$$g_c \frac{dp_s}{dw_x} = -\mu\alpha_x q_x \qquad \text{(II-23)}$$

This is the basic differential equation for filtration with dilute concentration slurries. Shirato et al (18) indicated it is probable that the effect of the solid movement can be neglected when the ratio of the fraction of solid in the slurry to the fraction in the surface layer of the cake is less than 0.5. In this case equation (II-23) can be used. Otherwise, equation (II-22) must be used.

The basic differential equation relating the change in flow rate to the time rate of change of porosity according to Tiller and Cooper (26) is :

$$\left( \frac{\partial q_x}{\partial x} \right)_\Theta = \left( \frac{\partial \varepsilon_x}{\partial \Theta} \right)_x \qquad \text{(II-24)}$$

and, according to Shirato (18), the relationship between the change of solid rate and the time rate of change of porosity is :

$$\left( \frac{\partial r_x}{\partial x} \right)_\Theta = - \left( \frac{\partial \varepsilon_x}{\partial \Theta} \right)_x \qquad \text{(II-25)}$$

Combining equation (II-24) and (II-25) leads to :

$$\left( \frac{\partial q_x}{\partial x} \right)_\Theta = - \left( \frac{\partial r_x}{\partial x} \right)_\Theta \qquad \text{(II-26)}$$

or,

$$dq_x + dr_x = 0 \qquad \text{(II-27)}$$

Integration of equation (II-27) yields $q_x + r_x = \text{constant}$. Evaluation of the constant at $x=0$, where $r_x=0$ and $q_x=q_1$, produces :

$$q_x + r_x = q_1 \qquad \text{(II-28)}$$

When equation (II-22) is integrated across the entire thickness of the cake, the following equation is obtained (18) :

$$q_1 = \frac{dv}{d\Theta} = \frac{g_c(p-p_1)}{\mu J_R \alpha_R W} = \frac{g_c p}{\mu (J_R \alpha_R W + R_m)} \qquad \text{(II-29)}$$

which is an improvement over the previous result of Tiller and Shirato (25) in which only the variation of the superficial velocity $q_x$ with respect to the distance was considered. The conventional filtration resistance, $\alpha_R$, is defined by Carman (4) and Ruth (15) as :

$$\alpha_R = \frac{p - p_1}{\int_o^{p-p_1} \frac{dp_s}{\alpha_x}} \tag{II-30}$$

The coefficient, $\alpha_R$, ft/lb$_m$, is also known as the average specific cake resistance. As the name suggests, it is an average quantity for the entire cake and as indicated by McCabe and Smith (11), this coefficient must be measured experimentally for each sludge. For a particular sludge, it is only a function of the pressure drop across the cake.

A correction factor, $J_R$, to account for the variation of $q_x$ with x and the movement of solids as a result of compaction of the cake has been given by Tiller and Shirato (25) as follows :

$$J_R = \int_o^1 \left(\frac{q_x}{q_1}\right) d \left(\frac{w_x}{w}\right) \tag{II-31}$$

For computational purpose, $J_R$ can be placed in the form(18):

$$J_R = \int_o^1 \left( 1 - \frac{(\varepsilon_x - \varepsilon_{avgx})(m-1)}{(1-\varepsilon_x)\varepsilon_{avg}(1-ms)} \left(\frac{x}{L}\right) s \right) d \left(\frac{w_x}{w}\right) \tag{II-32}$$

Tiller and Shirato (18) also showed the average specific filtration resistance, $\alpha$, ft/lb$_m$, is given by :

$$\alpha = J_R \alpha_R \qquad \text{(II-33)}$$

In practice, $\alpha_R$ is a good approximation of the average filtration resistance and $J_R$ approaches unity when the slurry is dilute.

The pressure at the interface of the filter medium and the filter cake, $p_1$, in equation (II-29) is related to the flow rate,$q_1$, at the exit of the cake by :

$$g_c p_1 = \mu q_1 R_m \qquad \text{(II-34)}$$

The filter medium resistance, $R_m$, in 1/ft, can be defined by analogy with equation (II-30). It has been generally observed (11) that the magnitude of the filter medium resistance depends on the pressure drop and on the properties of the flow. Also an old, used filter medium offers more resistance to flow than a new, clear one. Since the medium resistance plays an important role only during the early stages of filtration, it can be effectively assumed to be constant and is usually determined experimentally (13). It has been reported that the magnitude of $R_m$ varies from experiment to experiment even for the same sludge and filter under the same filtration conditions.

Making use of the following relationship (13,10):

$$w = \frac{s}{(1-ms)} v \qquad \text{(II-35)}$$

equation (II-29) can be rewritten in terms of the volume of filtrate per unit area in the following form (9,10) :

$$q_1 = \frac{dv}{d\Theta} = \frac{K}{2(v+v_o)} \qquad \text{(II-36)}$$

where,

$$K = \frac{2g_c p(1-ms)}{\mu\rho s\alpha} \qquad \text{(II-37)}$$

and

$$v_o = \frac{R_m(1-ms)}{\alpha s\rho} \qquad \text{(II-38)}$$

### II-B.1. Constant Pressure Filtration

After the filter cake has been built up, the pressure, $p_1$, at the interface of the filter medium and cake becomes small in comparison with the pressure, p, at the cake surface, and $\alpha_R$ and $\alpha$, a function of its pressure, are assumed to remain constant. Experimentally it is also observed that the moisture ratio, m, ratio of mass of wet to dry cake, is effectively constant after a relatively short interval for a particular run. Hence K and $v_o$ in equation (II-36) will take on constant values in constant pressure filtration after some initial period of filtration.

During the initial stages of filtration, when the cake thickness is small and $p_1$ is not insignificant compared to p, $\alpha_R$ becomes a function of $(p-p_1)$ and varies continuously with an increase in the amount of filtrate v. Variations in the pressure in the early part of the initial stage and the variation of m ascribable to the initial stage are additional possible sources of variation superimposed on K and $v_o$ in the initial stage.

Using the previous equation (II-36), a new form can be obtained, as follows :

$$\frac{d\Theta}{dv} = \frac{2}{K}(v+v_o) \tag{II-39}$$

This differential equation forms the basis for analyzing the experimental data. However, $\frac{d\Theta}{dv}$ and v can not be evaluated accurately.

To improve the accuracy a parabolic relationship involving the volume per unit area of filtrate v and time $\Theta$ at constant pressure, was devised by Kozicki, Rao, and Tiu, (9) in 1970, by integrating equation (II-39) and applying the limit v=0 at $\Theta$=0 and $v=v>v_i$ at $\Theta=\Theta$, to obtain:

$$\Theta = \frac{1}{K}(v^2 + 2vv_o) + \Theta_o \tag{II-40}$$

where,

$$\Theta_o = \int_o^{v_i} F(v)\, dv \tag{II-41}$$

$\Theta_o$ is the effective time of commencement of filtration (9,10) and is determined by the length of the initial period of filtration. If the initial period is short, the $\Theta_o$ value will be small. If the initial period is long, the value of $\Theta_o$ becomes large. It is very difficult to ensure that the initial period of filtration is the same in different runs. The initial time period of filtration is the time taken to attain the final desired pressure head from zero pressure to the expected range on the pressure gauge. The initial period is determined by the adjustment of the valve located on the circulation line. If the valve is closed quickly the initial time period of filtration will be short. If closed slowly, the initial time period will be long. F(v) is defined by Kozicki, Rao and Tiu, as a function of filtrate volume serving as a correction term yielding the correct $d\Theta/dv$ in the initial period (9). The correction is zero beyond the initial stage where $v>v_i$.

Equation (II-40) can be fitted by the method of least-squares to the experimental data collected during the main filtration stage. In the least-squares analysis, the constants in equation (II-40) are evaluated such that the variance is a minimum.

As mentioned earlier, it is difficult to control the initial time period of filtration. Consequently, the results are not comparable under different conditions. In order to solve this problem, Kozicki and his coworkers also developed a method (10) to analyse the experimental data by the method of least-squares using filtration volume v and effective time of filtration $(\Theta-\Theta_o)$. Since the runs conducted at the same pressure were subjected to different initial periods, the corresponding $\Theta_o$ values were obtained by least-squares fitting of equation (II-40) separately to each set of data corresponding to a particular run. With appropriate $\Theta_o$ values, equation (II-40) can be changed as follows :

$$\Theta_e = \Theta - \Theta_o = \frac{1}{K} (v^2 + 2 \; v \; v_o) \qquad \text{(II-42)}$$

where, $\Theta_e$ is the effective time of filtration as defined in equation (II-42). The least-squares plot of effective time, $\Theta_e$, versus filtration volume, v, will pass through the origin. This method allows the experimental data to be analyzed easily and accurately and makes the filtration results more comparable between runs. The method is illustrated in Figure (II-2) and (II-3).

From equation (II-42), K and $v_o$ can be obtained by the method of least-squares. The average specific filtration resistance and filter medium resistance can be computed by making use of equation (II-37) equation (II-38) and other basic filtration properties. Both resistances and other properties suggest the characterization of the filtration; e.g., the average specific filtration resistance will show the role of porosity and imply the mechanism of construction in filter cake. All properties of filtration-porosity, moisture ratio, resistances of both filter medium and cake-are discussed in Chapter IV and V.

The empirical equation of average specific filtration resistance was found as follows (11) :

$$\alpha=\alpha_o p^s \qquad \text{(II-43)}$$

where, $\alpha_o$ and s are empirical constants. This equation is restricted to higher pressure drops (above 50 PSI) (7). It is not suitable for low pressure drops because it indicates a zero resistance at zero p.

It was quoted by McCabe and Smith (7) that s is a quantitative measure of compressibility. It is zero for incompressible slurries and is positive for compressible ones. Constant s usually falls between 0.1 and 1.0, the large values applying to the more compressible slurries. This constant is commonly called the compressibility coefficient.

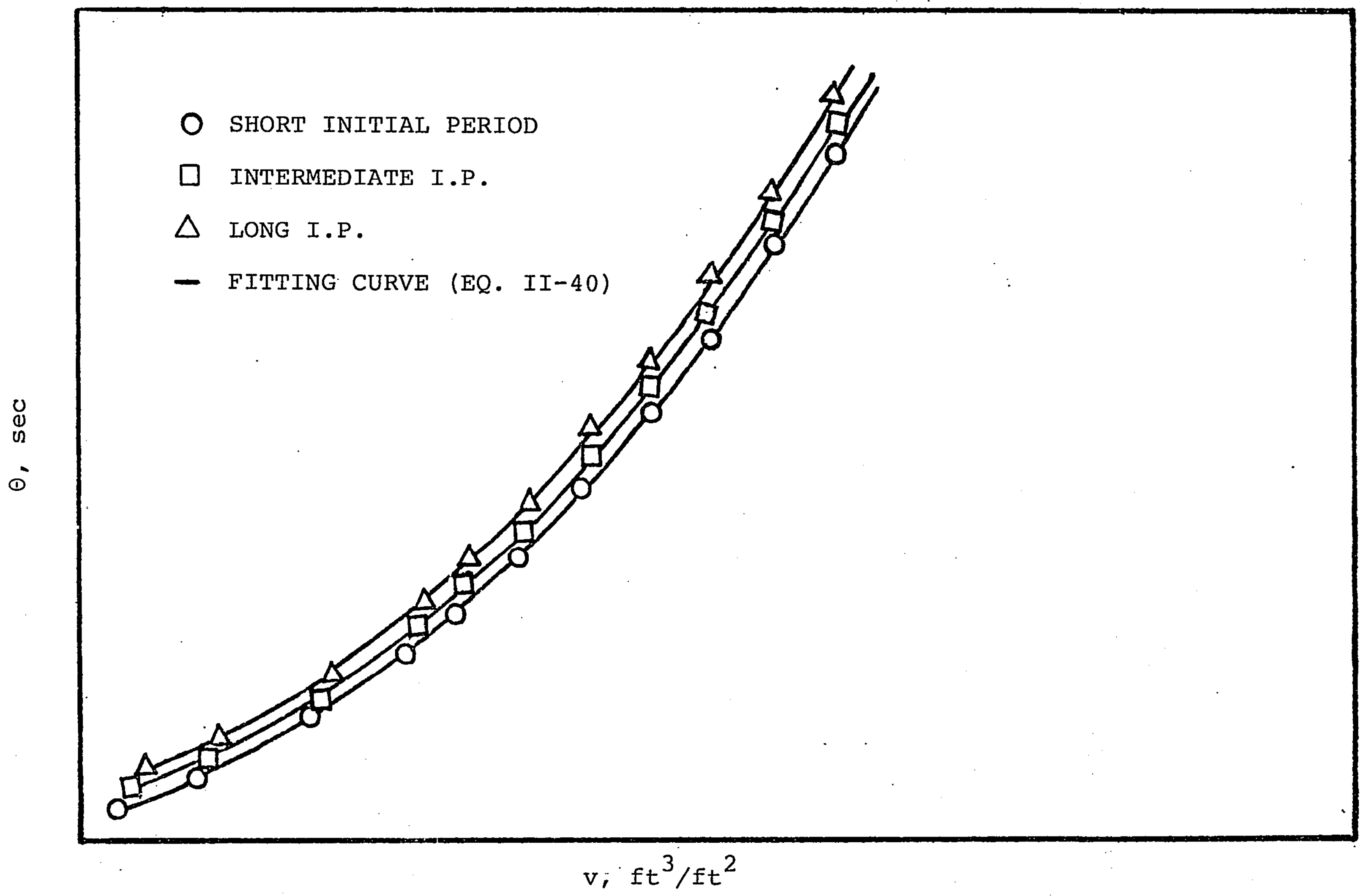


FIGURE II-2 SCHEMATIC PLOT OF TIME $\theta$ VERSUS CUMULATIVE FILTRATE VOLUME PER UNIT AREA v FOR CONSTANT PRESSURE RUN WITH DIFFERENT INITIAL PERIODS

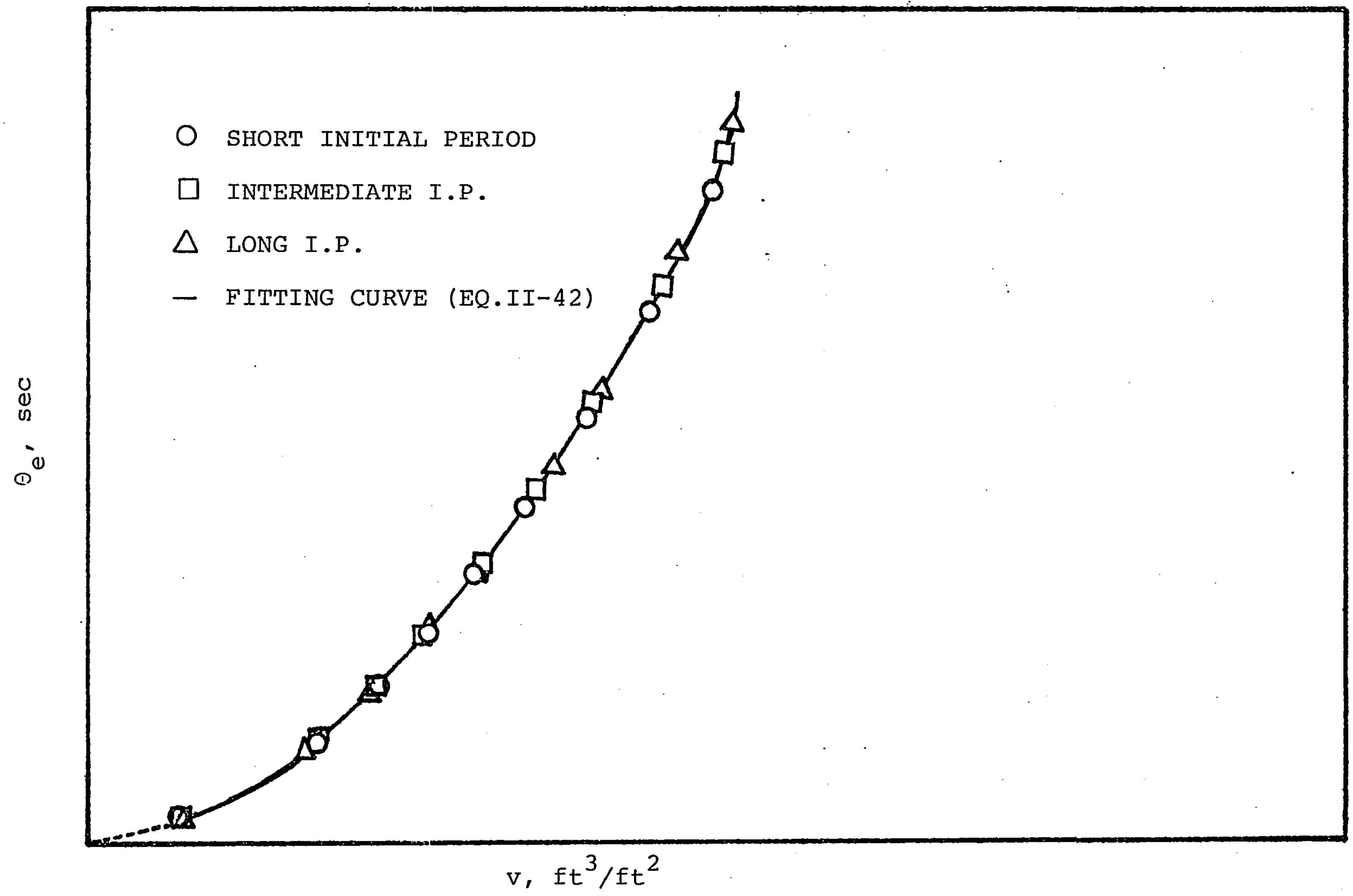


FIGURE II-3 SCHEMATIC PLOT OF v VERSUS $\Theta_e$ FOR A PARTICULAR CONSTANT PRESSURE RUN WITH DIFFERENT INITIAL PERIODS

# CHAPTER III

# EXPERIMENTAL ASPECTS

In this chapter, the preparation of Polyox WSR Coagulant aqueous solution, the measurement of the density and viscosity of the filtrate, the experimental equipment and procedure will be described in detail.

## III-A. Preparation of Polyox WSR Coagulant Aqueous Solutions

Filtration runs were conducted at constant pressure with Polyox WSR Coagulant aqueous solutions. The concentration of Polyox WSR Coagulant was initially 1% by weight and was diluted to the appropriate concentration for each run. Because the polymer solution concentrations were low (less than 100 PPM), the fluid could be assumed to be Newtonian. Polyox WSR Coagulant is soluble in either cold or hot water. However, as with most water soluble thickeners, the particles have a tendency to agglomerate when the Polyox WSR Coagulant particles are first wetted with water. It was found by experience that a uniformly smooth solution could be obtained with 1% Polyox WSR Coagulant by weight, provided care was taken in its preparation. The recipe for the preparation of 1% polymer solutions by wt. is as follows:

(1) Begin with a 2gm sample of Polyox WSR Coagulant powder.

(2) Pour 200 ml of distilled water into a 600 ml beaker, as shown in Figure III-1.A.

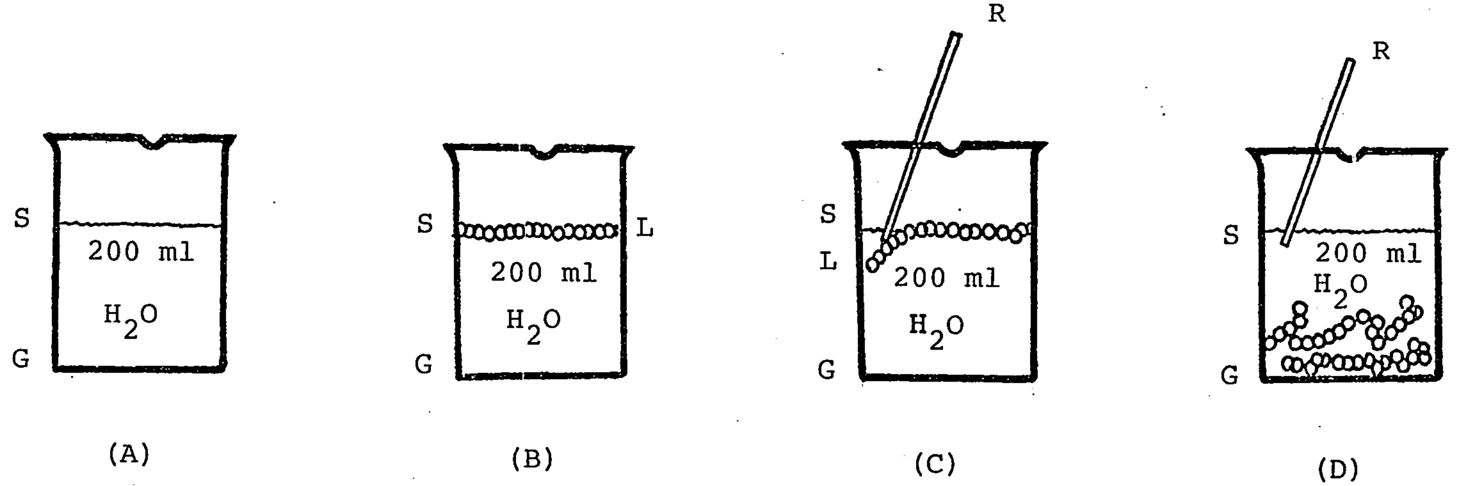
(3) Spread some Polyox WSR Coagulant powder uniformly on the surface of the water to form a thin layer, as shown in Figure III-1.B.

(4) Using the glass rod, push the thin layer of polymer into the water from side to side very slowly and gently, as shown in Figure III-1.C.

(5) After the whole thin layer has sunk into the water, push it to the bottom of the beaker, shown in Figure III-1.D. Then spread more Polyox WSR Coagulant powder as described in procedure (2) and repeat the above procedures until the Coagulant powder has been exhausted.

(6) Agitate the solution with the glass rod for a little while, leave the glass bar in the beaker, cover the solution in the beaker with thick paper and close the cover by rubber bands to avoid oxidation with air. Finally store the solution in the dark for about 24 hours before using.

Like most polymers which are very difficult to dissolve in water in a short time, Polyox WSR Coagulant must be allowed a sufficiently long time to dissolve com-



G 600 ml GLASS BEAKER

S DISTILLED WATER SURFACE

L THIN LAYER FORMED BY POLYOX WSR COAGULANT POWDER

R GLASS ROD

FIGURE III-1 PREPARATION OF POLYOX WSR COAGULANT AQUEOUS SOLUTION pletely. The time interval selected was 24 hours. The preparation of one per cent by weight Polyox WSR Coagulant aqueous solutions requires patience and care.

Three concentrations of Polyox WSR Coagulant with Calcium Carbonate slurry were prepared in aqueous solution. The concentrations for Polyox WSR Coagulant were 20 PPM, 60 PPM, and 100 PPM by weight and 2.5% Calcium Carbonate by wt.

The structure of Polyox WSR Coagulant is shown in Figure III-1.E, and other properties of Polyox WSR Coagulant are given by Union Carbide of Canada Ltd..

## III-B. Apparatus

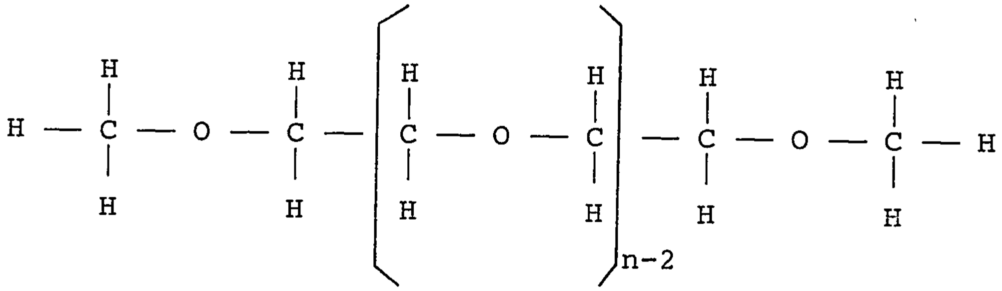
The filtration equipment, as shown schematically in Figure III-2, consisted of a feed tank, A, a Boston rotary gear pump, B, an oscillation unit, C, a filtration unit, D and a graduated cylinder, E. Each part of the experimental apparatus will be sketched and explained in detail.

### III-B.1. Feed Tank

As shown in Figure III-3, the feed tank was fitted with a cooling coil, C, a mixer, A, an inlet water valve, B and a thermometer, T. The mixer, A, agitated the slurry completely before each filtration run and kept the slurry uniformly smooth during the whole operation. After every

$$-\left[ CH_2 - O - CH_2 \right]-$$

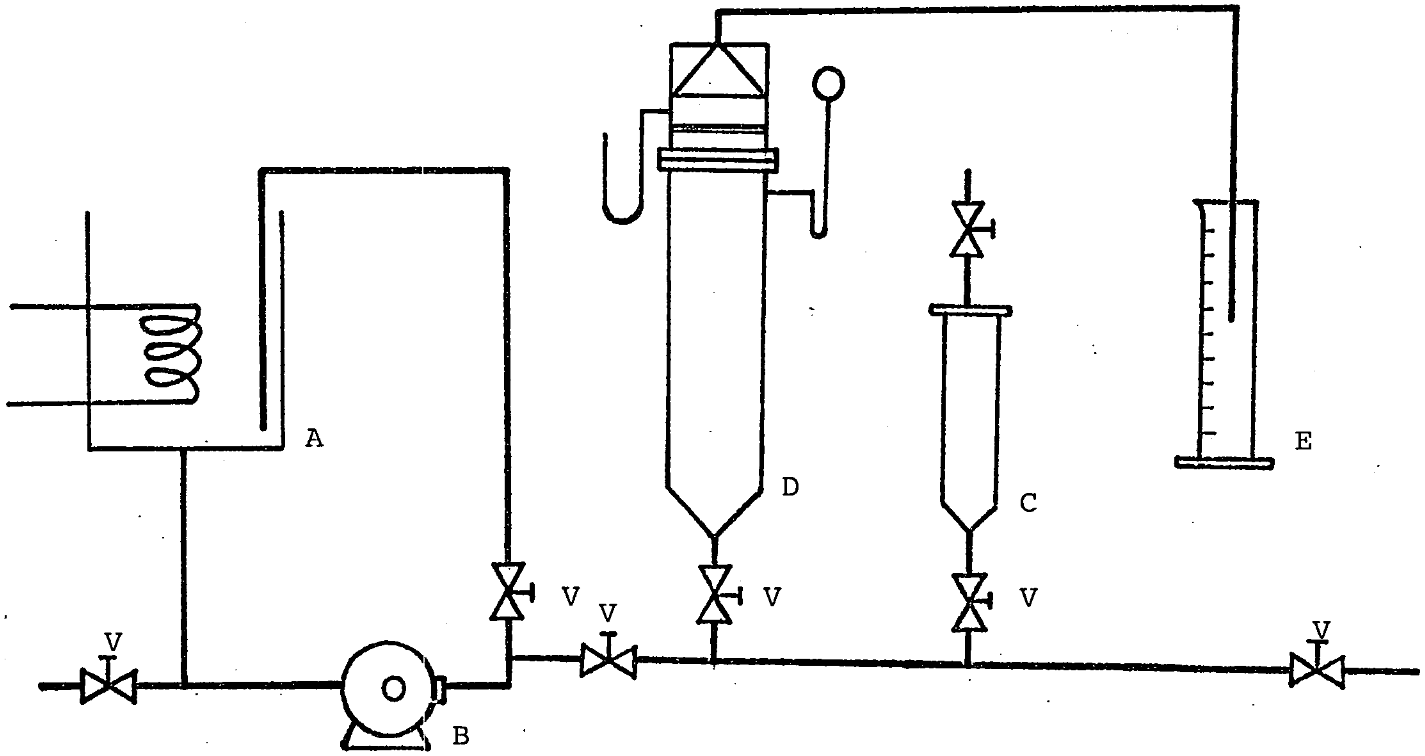
MONOMOLECULAR STRUCTURE OF POLYOX WSR COAGULANT



$$H - \underset{H}{\overset{H}{C}} - O - \underset{H}{\overset{H}{C}} - \left[ \underset{H}{\overset{H}{C}} - O - \underset{H}{\overset{H}{C}} \right]_{n-2} - \underset{H}{\overset{H}{C}} - O - \underset{H}{\overset{H}{C}} - H$$

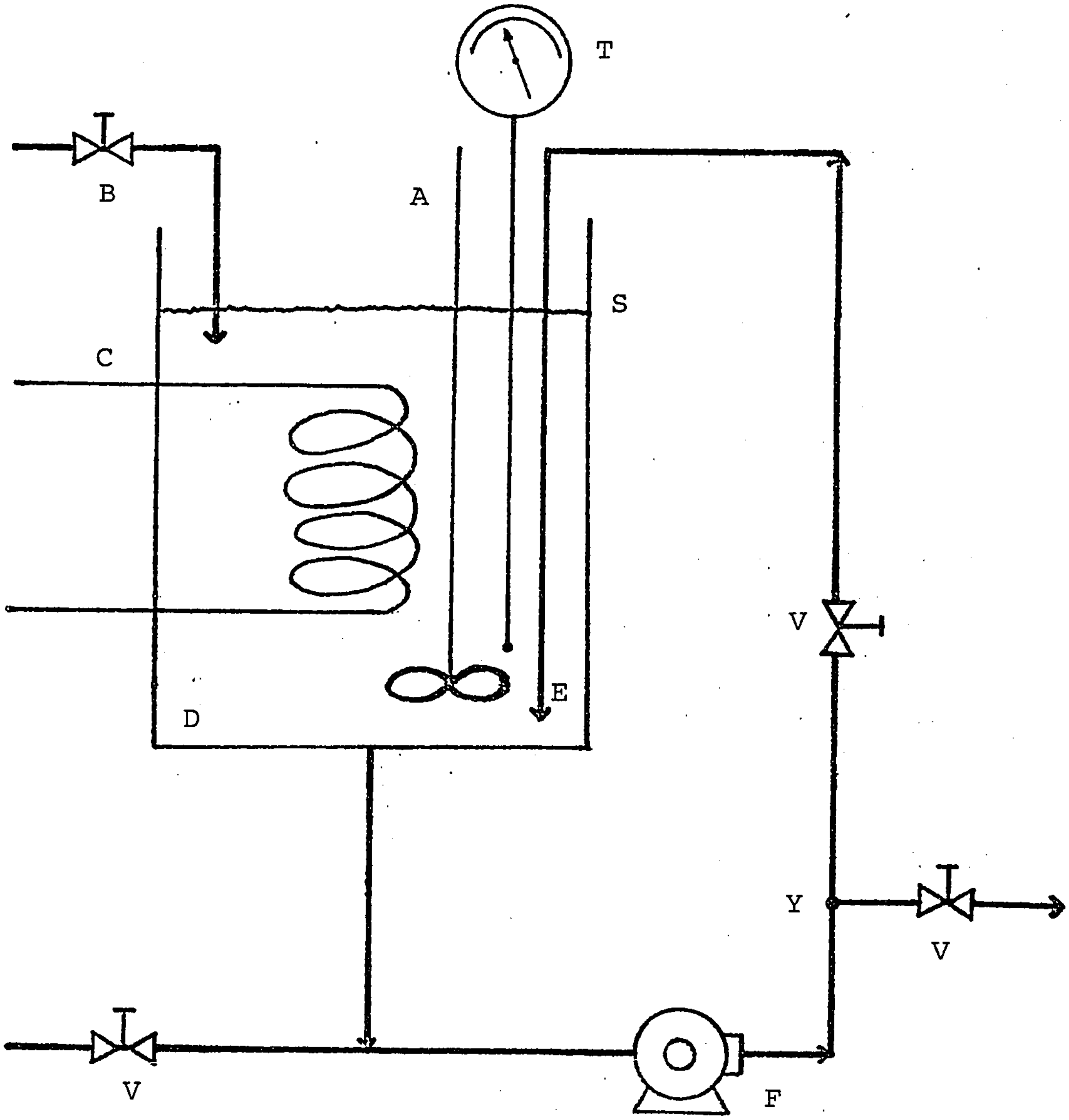
POLYOX WSR COAGULANT IN POLYMER STATE

FIGURE III-1.E IDEALIZED STRUCTURE OF POLYOX WSR COAGULANT



|   |   |   |   |
|---|---|---|---|
| A | FEED TANK | D | FILTRATION UNIT |
| B | PUMP | E | GRADUATED CYLINDER |
| C | OSCILLATION UNIT | V | VALVE |

FIGURE III-2 SCHEMATIC DIAGRAM OF FILTRATION EQUIPMENT



| | | | |
|---|---|---|---|
| A | AGITATOR | F | PUMP |
| B | INLET WATER VALVE | S | LIQUID SURFACE |
| C | COOLING COIL | T | THERMOMETER |
| D | FEED TANK | V | VALVE |
| E | CIRCULATION PIPE | Y | Y-JOINT 1 |

FIGURE III-3 FEED TANK

run, the system was washed by water from inlet water valve, B, in order to remove traces of reducing fluid and Calcium Carbonate slurry. The flow circulated through the tube, E, and a cooling coil, C, situated in the feed tank, removed heat generated by the pump and by the circulation to maintain a constant feed temperature. The temperature was monitored by the thermometer, T. The cooling coil was adjusted manually. To prevent air from entering the slurry (20), the end of the recirculation line, E was installed near the bottom of the feed tank.

### III-B.2. Pump

A 3 horsepower electric variable speed motor (SCR Precision Motor, Model E300), manufactured by the Boston Gear Works of Quincy, Mass. U. S. A., drove a gear pump to transport the slurry from the feed tank to the filtration unit and maintained the necessary pressure head. The gear pump was cooled by the circulating slurry; the heat generated by the pump was removed by a cooling coil in the tank, as described above.

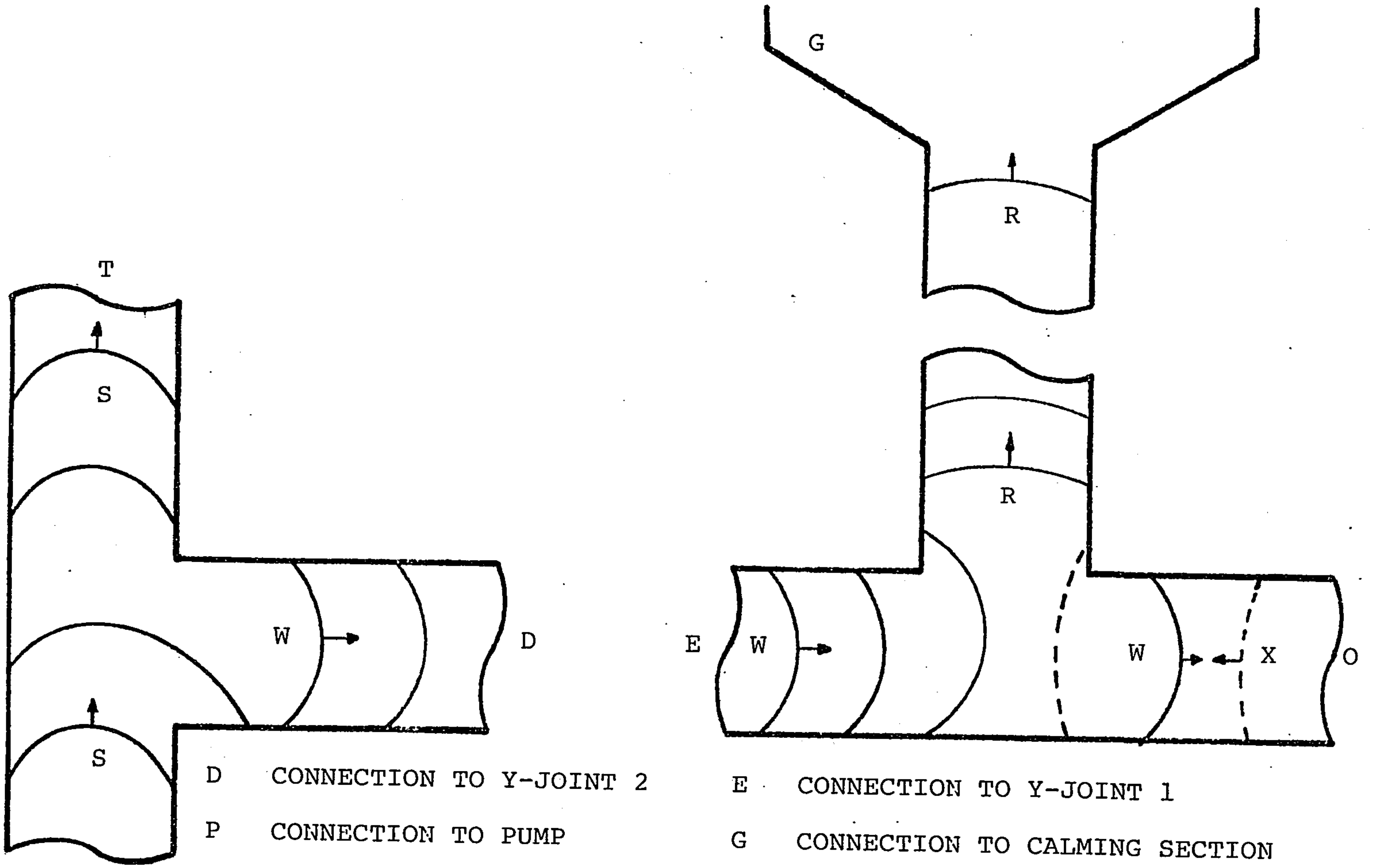
The gear pump (model 525), manufactured by the Brown & Sharpe Company of North Kingstown, R.I., U.S.A., generated a vibrating pressure wave from the rotary gears during operation. This wave made the needle of the Diaphragm Pressure Gauge unstable and causing it to oscillate wildly. The installation of Y-Joints points, a vibration cell and the oscillation unit helped reduce needle oscillations, as shown schematically in Figure III-4, Figure III-7 and Figure III-6. The details of these devices will be discussed later.

### III-B.3. Oscillation Unit and other Wave Reducing Systems

These devices were set up to stabilize the surge of fluid flow in the pipes, and to decrease the wave of vibration in the pressure gauge needle. Two Y-Joints, shown in Figure III-4, an oscillation unit, shown in Figure III-6, and a vibrating cell, shown in Figure III-7 and III-8 were effective in reducing the surge generated by the rotary gears of the pump. The vibrating cell was packed with a stainless steel mesh and with various sizes of stainless steel balls. The flow through the vibrating cell was thereby stabilized through the action of the mesh and the balls such that a given uniform reading could be maintained.

### III-B.4. Filtration Unit

As shown in Figure III-2, the filtration equipment included a filtration unit which played an important role during each run. The detailed installation of this part is sketched in Figure III-5. From Y-Joint 2, the slurry



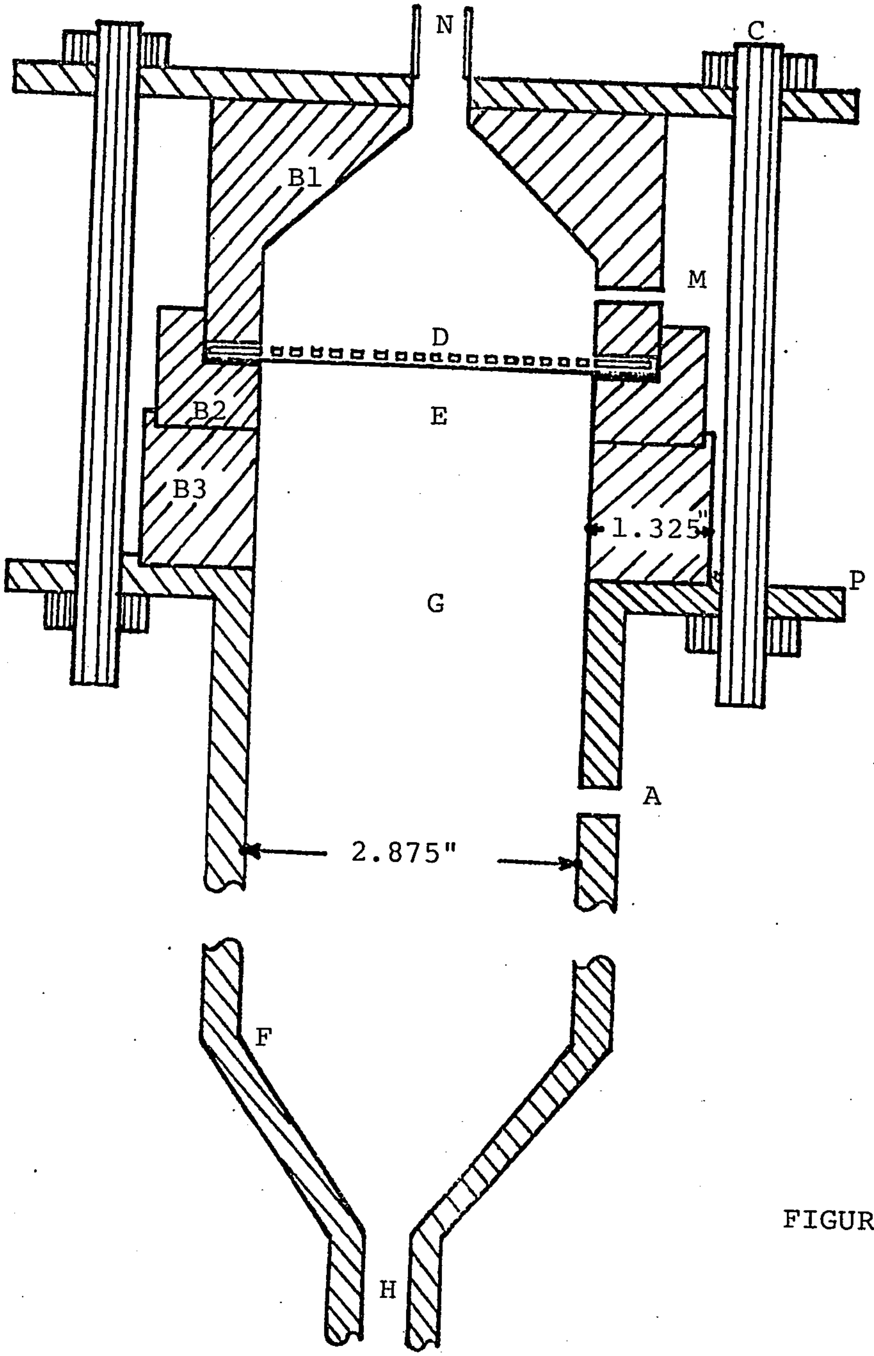
| | |
|---|---|
| D | CONNECTION TO Y-JOINT 2 |
| P | CONNECTION TO PUMP |
| S | SURGE |
| T | CONNECTION TO FEED TANK |
| W | WEAKENED WAVE |

(A) Y-JOINT 1

| | |
|---|---|
| E | CONNECTION TO Y-JOINT 1 |
| G | CONNECTION TO CALMING SECTION |
| O | CONNECTION TO OSCILLATION UNIT |
| R | RIPPLE |
| W | WEAKENED WAVE |
| X | REFLECTIVE WAVE FROM CLOSED VALVE |

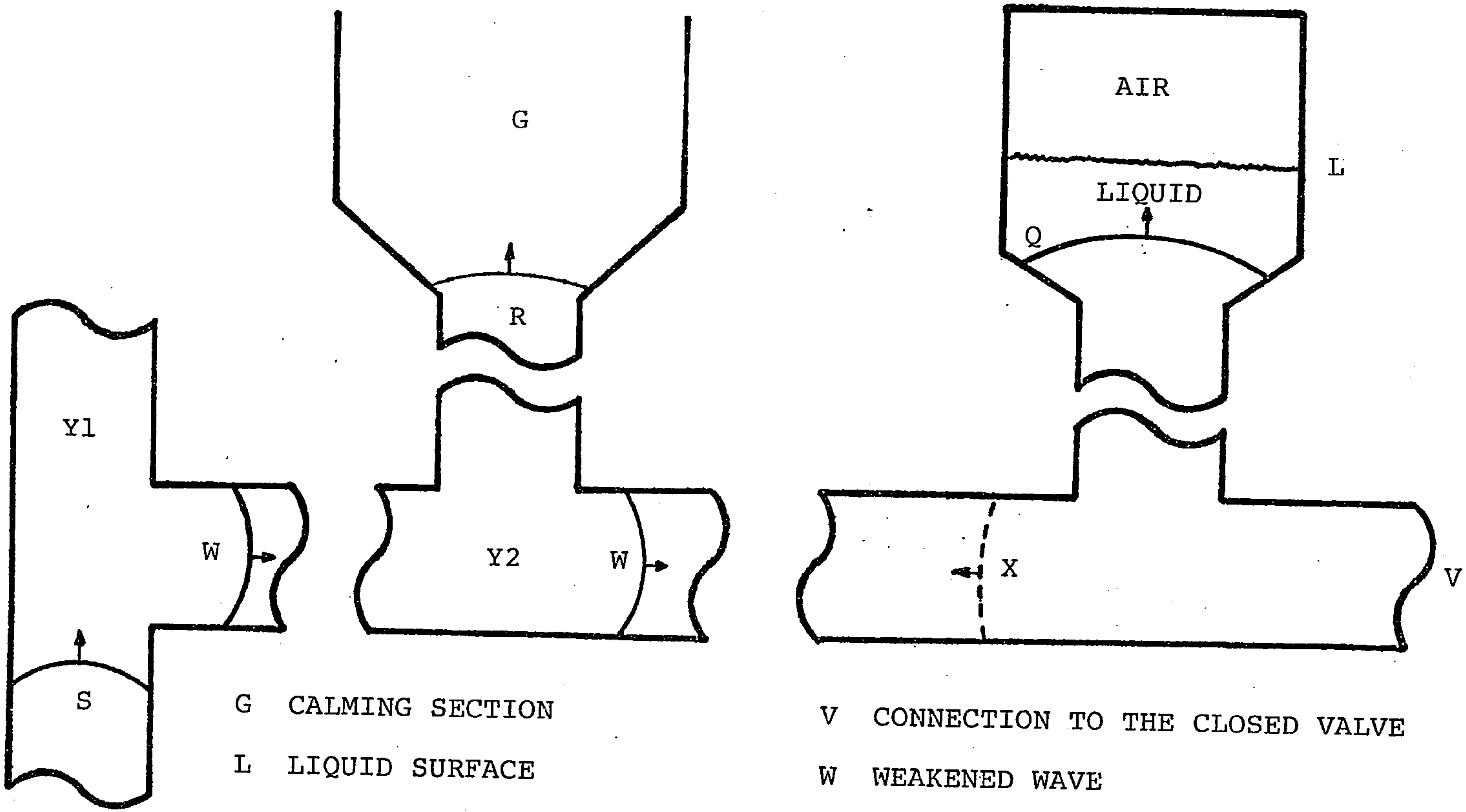
(B) Y-JOINT 2

FIGURE III-4 THE SKETCH OF THE FUNCTION OF Y-JOINTS



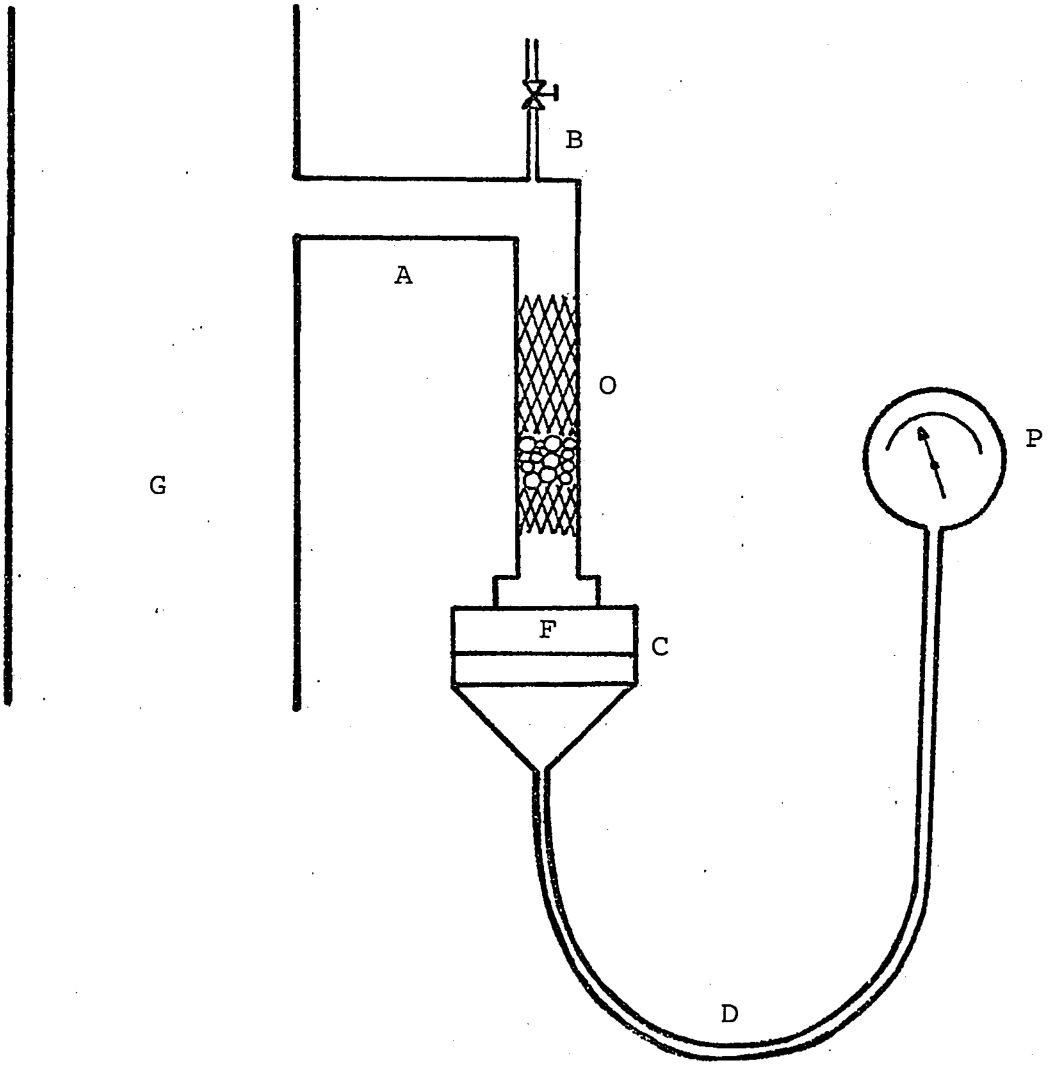
| | |
|---|---|
| A | CONNECTION TO DIAPHRAGM PRESSURE GAUGE |
| B | PLEXIGLASS PIECES |
| C | THREADED STEEL RODS |
| D | PERFORATED BRASS PLATE |
| E | FILTER MEDIUM |
| F | FILTRATION COLUMN |
| G | CALMING SECTION |
| H | CONNECTION TO Y-JOINT 2 |
| M | CONNECTION TO MANOMETER |
| N | CONNECTION TO GRADUATE CYLINDER |
| P | FLANGE PLATES |

FIGURE III-5 FILTRATION UNIT



| | |
|---|---|
| G | CALMING SECTION |
| L | LIQUID SURFACE |
| Q | OSCILLATION UNIT |
| R | RIPPLE |
| S | SURGE |
| V | CONNECTION TO THE CLOSED VALVE |
| W | WEAKENED WAVE |
| X | REFLECTIVE WAVE |
| Y1 | Y JOINT 1 |
| Y2 | Y JOINT 2 |

FIGURE III-6 OSCILLATION UNIT



A CONNECTING TUBE FROM CALMING SECTION OF FILTRATION UNIT

B AIR INLET AND OUTLET VALVE

C DIAPHRAGM UNIT

D CAPILLARY TUBE CONNECTING TO THE PRESSURE GAUGE DIAL

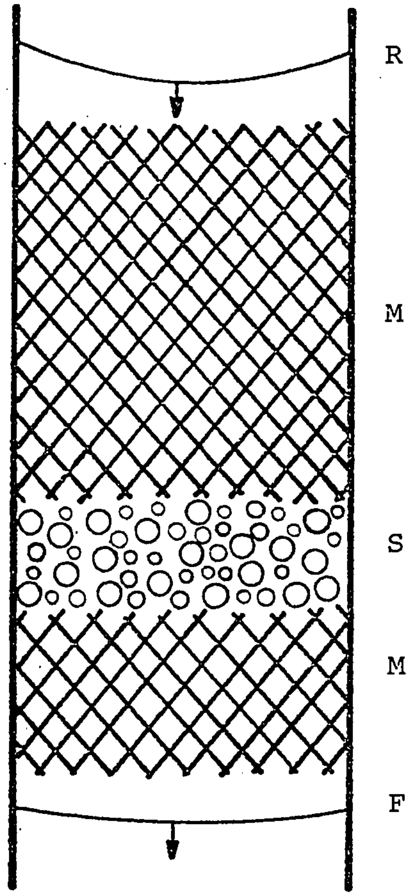
F CLEAN OUT RING

G CALMING SECTION

O VIBRATING CELL

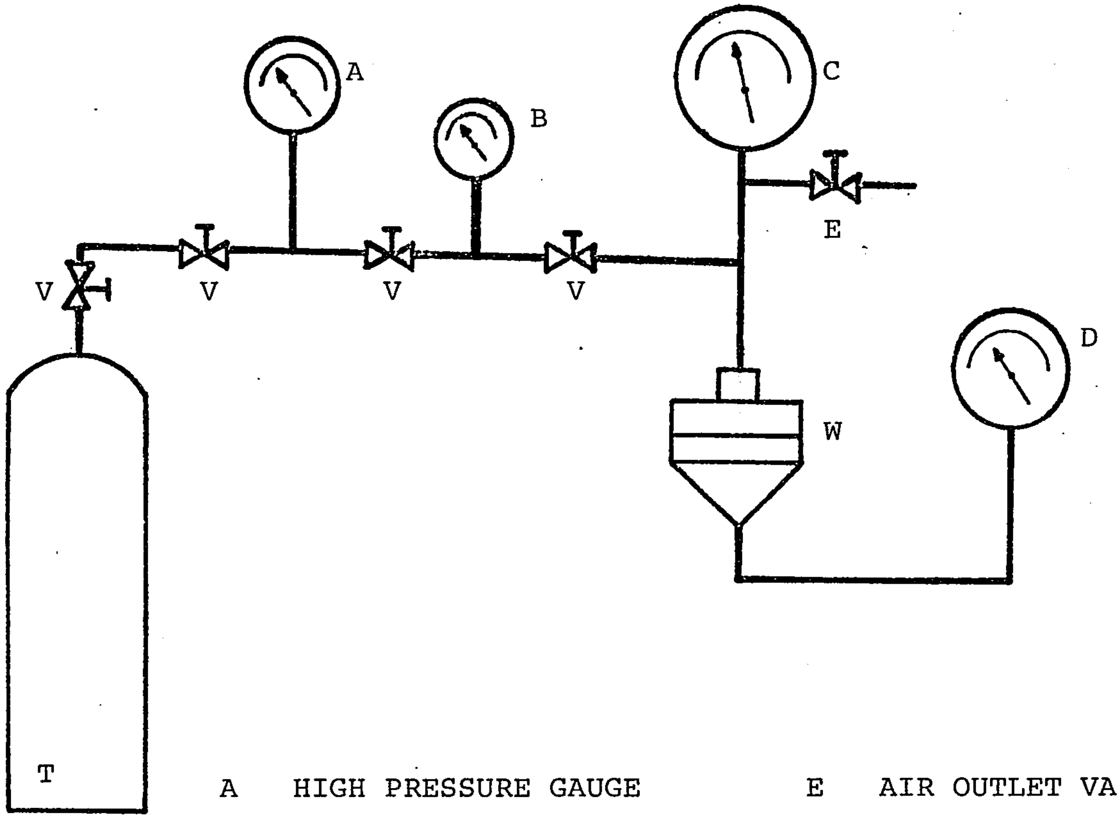
P PRESSURE GAUGE DIAL

FIGURE III-7 DIAPHRAGM PRESSURE GAUGE



| | |
|---|---|
| F | STABLE FLOW TO DIAPHRAGM UNIT |
| M | STAINLESS STEEL MESH |
| R | RIPPLE FROM CALMING SECTION OF FILTRATION UNIT |
| S | RANDOM SIZES OF STAINLESS STEEL BALL |

FIGURE III-8 DETAILED SKETCH OF VIBRATING CELL was transported to a circular calming section, 11 inches long, made of plexiglass and a stainless steel filtration column, both with an inside diameter of 2.875 inches. The length of the calming section was determined by the resisdence time of the slurry during a filtration run (13). The length was calculated to make sure no appreciable amount of change in the concentration of slurry in the calming would seriously affect the values of the quantities calculated from the filtration data. The length of the calming section was also set up to keep the settling of solid particles in the calming section to a minimum and to obtain a uniform velocity of slurry at the test section. A diaphragm pressure gauge was connected to the stainless steel column to measure the different upstream side pressure ranges in the filtration unit under the requirement of each filtration run. Shown in Figure III-7, the diaphragm pressure gauge consisted of a vibrating cell, a diaphragm unit, capillary tube and gauge dial. The function of the vibrating cell was described in the last paragraph; the diaphragm unit included a clean-out ring, F, to trap impurities which were deposited from the slurry during the filtration runs. Between the diaphragm unit and the indicating pressure dial, a two foot long capillary tube, made of flexible material, was connected. The air inlet and outlet valve was designed to facilitate the expulsion of the air present in the calming section connecting to the filtration unit. The pressure gauge had to be unmounted and cleansed of impurities from time to time. For each different pressure range run, the diaphragm pressure was calibrated against the standard pressure gauges as shown in Figure III-9. Bases on the experimental data listed in Table 25 of Appendix A, the calibrated curve is shown in Figure A-1 of Appendix A. Between the upper piece and the lower piece of the plexiglass unit, the filter medium was supported by uniformly perforated brass plate, whose perforation diameter was 0.1 inches, to maintain the surface of the filter medium perpendicular to the direction of flow of the slurry under different pressure ranges. The plexiglass unit permitted observation of the formation of cake during a filtration run and the transportation of slurry in the calming section. The upper portion of the plexiglass unit received the filtrate from the filter medium. A cone was selected for the shape of this upper portion to minimize the volume. A manometer was connected near the perforated brass plate to measure the downstream pressure of filtration. The lower parts of the plexiglass unit, being a part of the calming section, was designed to withstand the high filtration pressures during operation. Therefore, the wall thickness of these parts was chosen thick enough not to fail. The flange for joining the plexiglass unit and the stainless steel column by means of



| | | | |
|---|---|---|---|
| A | HIGH PRESSURE GAUGE | E | AIR OUTLET VALVE |
| B | LOW PRESSURE GAUGE | T | $N_2$ GAS CONTAINER |
| C | STANDARD PRESSURE GAUGE | V | VALVES |
| D | DIAPHRAGM PRESSURE GAUGE | W | WATER IN DIAPHRAGM UNIT |

FIGURE III-9 CALIBRATION EQUIPMENT FOR DIAPHRAGM PRESSURE GAUGE eight threaded steel rods and the flange plates as shown in Figure III-5 was designed for easy dismantling and reassembly. Leak-tightness was achieved by using Teflon gaskets. A $2000^{ml}$ graduated cylinder was used to measure and collect the cumulative volume of filtrate from the upper cone plexiglass at different time intervals during each filtration run.

## III-C. Materials

### III-C.1. Filter Medium

In this work, twill weave cloth was selected for the filter septum with the following requirements (1,11) :

(1) It must retain the solids to be filtered, giving a reasonably clear filtrate.

(2) It must have adequate porosity and a minimum tendency to plug or blind.

(3) It must have strength and mechanical resistance to withstand the process conditions.

(4) It must permit the cake formed to discharge cleanly and completely.

(5) It must not be expensive.

Style 3630, twill weave filter cloth, manufactured by the Shriver Company, was used. Other specifications of this twill weave were given in that company's literature. The resistance of a filter medium varies for each experimental run even for the same sludge and filter. In ordinary condition, a used filter medium offers more resistance than a fresh one. A filter paper, Cat. No. 9-790-2D from the Fisher Scientific Company., was used as the prime filter on the top of a twill cloth in order to prevent suspension particles from plugging the filter septum. Adding some filter paper can also prevent sagging of the filter under the process condition (6). The filter medium, consisting of twill cloth and filter paper, was changed at the beginning of each filtration run. To maintain the consistency, a clean filter surface was provided for each new run.

### III-C.2. Chemicals

The Polyox WSR Coagulant was supplied by Union Carbide Canada Ltd. To date, testing has shown no evidence of toxicity or irritation due to handling Polyox WSR Coagulant or exposure to Coagulant dust. Polyox WSR Coagulant can absorb moisture from the atmosphere. Therefore, to maintain the original moisture content, Polyox WSR Coagulant must be stored in a tightly closed container and in a dry atmosphere as recommended by the manufacturer. Other properties of Polyox WSR Coagulant aqueous solution will be discussed in the next section.

Ordered from the Fisher Scientific Company, C-62, a

25 lbs package of Calcium Carbonate was used in this research work.

### III-D. Procedure

500 miligrams of Calcium Carbonate was put into a stainless steel feed tank with 19500 milliliters of distilled water. After 40 minutes of agitation the Polyox WSR Coagulant aqueous solution was added to the tank, and the slurry was mixed until a uniform slurry composition at the controlled temperature was obtained. For this step, a mixing time of 35 minutes was chosen. Before each filtration run, the entire equipment was washed with distilled water. The filter cloth was immersed in water for about 90 minutes to prevent suspension particles and small solids from passing through the filter medium at the start of filtration. The cleaning valve of the oscillation unit was closed to entrap air for damping pressure surges. Circulated by the gear pump, the slurry was gradually introduced into the calming section and the oscillation unit by careful manual adjustment of the valves. Before the slurry could reach the filter septum which was placed between the plexiglass and the perforated brass plate, the calming section and the connecting tube to the diaphragm unit were filled with the slurry. At the same time, the air present in these regions was expelled from the air

inlet and outlet valves. The complete expulsion of air was signalled by overflow of the slurry from the valve situated on the top of the vibrating cell, whereupon this valve was closed for the rest of filtration operation. The filtration pressure, as indicated by the diaphragm pressure gauge, was approached gradually by carefully closing the valve located in the by-pass line. The valve was controlled by careful manipulation taking care not to repture the filter septum. When the first droplet of filtrate appeared on the surface of the perforated brass plate at the downstream side of the filter medium, a stop watch was started. The pressure of the downstream side was recorded by a manometer. The filtrate was discharged into a 2000 milliliters graduated cylinder manufactured by the Fisher Scientific Company. 500 milliliters of filtrate sample was kept to measure the density and viscosity; this sample was chosed from an cumulative filtrate range between 2000-4000 millimeter. In the meantime, the valve at the circulation line was adjusted carefully to increase the pressure at the upstream side. When the desired pressure head was reached, the time interval was recorded. Subsequently, the cumulative volume of filtrate in the graduated cylinder and the corresponding time interval were recorded. Fifteen intervals were usually measured during each filtration run. The above procedure was repeated for each run with different grades of concentration of Polyox WSR Coagulant aqueous solution. The filtration run with pure Calcium Carbonate 2.5% concentration by weight without polymer solution followed same procedure but with a mixing time of about 75 minutes.

The temperature of the slurry was kept at 25±1°C as measured by two thermometers throughout every filtration run. The temperature of the filtrate was also monitored in the graduated cylinder during the filtration operation. It was verified there was no appreciable change in temperature to affect the flow characteristics of the fluids. In order to avoid influencing the flow behavior due to temperature differences, the temperature was fixed at 25°C in the feed tank for each filtration run. Filtrate temperature was monitored continually.

Before closing the valve to the filtration unit, the upstream pressure was reduced by manipulating the valve on the by-pass circulation line after sufficient experimental data had been obtained. The variable speed motor and agitator were then turned off. The filtrate, accumulated in the upper plexiglass section above the perforated brass plate, and in the connecting tube to the graduated cylinder, was siphoned and the initial volume $v_i$ of filtrate, was measured. The plexiglass assembly was then dismantled for washing. Next, the cake was carefully transfered into a beaker and weighed. The weight of dry cake was also measured after a period of drying in an oven at 110°C. After every filtration run, the complete system was washed with water to remove traces of polymer solution and accumulated solid particles absorbed on the inside surfaces.

The data collected during the constant pressure filtration have been tabulated in Appendix A.

## III-E. Measurement of Density and Viscosity of Filtrate

The densities and viscosities of different grades of Polyox WSR Coagulant aqueous solution were measured at a constant temperature of 25°C.

Density was measured with the Digital Densimeter DMA 02C(NO. 199), manufactured by the Anton Paar Company (Graz, Austria). All data were obtained with the fluid temperature constant at $25 \pm 0.01^{\circ}C$. Viscosity was measured with the Cannon-Fenske Viscometer with the temperature maintained bath constant at $25 \pm 0.01^{\circ}C$. The experimental results and calculations will be discussed in Chapter IV. The procedures for measuring density and viscosity followed those outlined in the instruction manuals.

## CHAPTER IV

## RESULTS AND DATA CORRELATION

The cumulative filtrate volume per unit area in filtration is influenced by the following factors :

(1) System temperature.

(2) Applied pressure on the surface of the cake.

(3) Concentration of drag reducing fluids and concentration of Calcium Carbonate in aqueous slurry.

(4) The properties of filter cake, e.g., porosity.

(5) The properties of filter medium.

(6) The density and viscosity of filtrate.

For each constant pressure filtration run, the temperature was maintained constant at $25 \pm 1^{\circ}C$, the filter medium in the filtration unit was replaced and the concentration of Calcium Carbonate added to the aqueous slurry was maintained constant. In constant pressure filtration, the applied pressure p is constant, therefore the cumulative filtrate volume in filtration depends only on the concentration of drag reducing fluids in the slurry. In each experimental filtration run, the concentration of aqueous Polyox WSR Coagulant solution added to the slurry was less than 100 PPM. Thus the slurries in the filtration of drag reducing fluids can be assumed as Newtonian.

Using the method of least-squares as mentioned in Chapter II, and equation (II-40) and equation (II-42), as an example, a plot of the data (see Part B-1 of Appendix B) collected with different initial periods in three filtration runs using the same aqueous slurries of 2.5% Calcium Carbonate by weight and 20 PPM concentration of Polyox WSR Coagulant solution at a constant pressure of 50 PSI in each case, is given in Figure IV-1. Then, a fitted curve was obtained using of equation (II-42) as shown in Part B-2 of Appendix B. Similarly, at 150 PSI, a fitted curve was also obtained as shown in Figure IV-2. The curve, shown in Figure IV-1 and Figure IV-2, demonstrates the considerable success of the parabolic form of the relationship.

The initial periods, the varying time intervals used to attain the final pressure, are classified as short, intermediate, or long, depending on the relative rates of the variation of pressure with time from zero gauge pressure at the commencement of filtration to the final desired pressure at the main stage of filtration. It is noted that $\Theta_o$ always increases with the time taken to attain the final pressure. Least-squares fitting of equation (II-40) separately to the set of data corresponding to each run was performed using a digital computer (IBM 360).

With the necessary time corrections for the appropriate $\Theta_o$ values for the three different initial periods

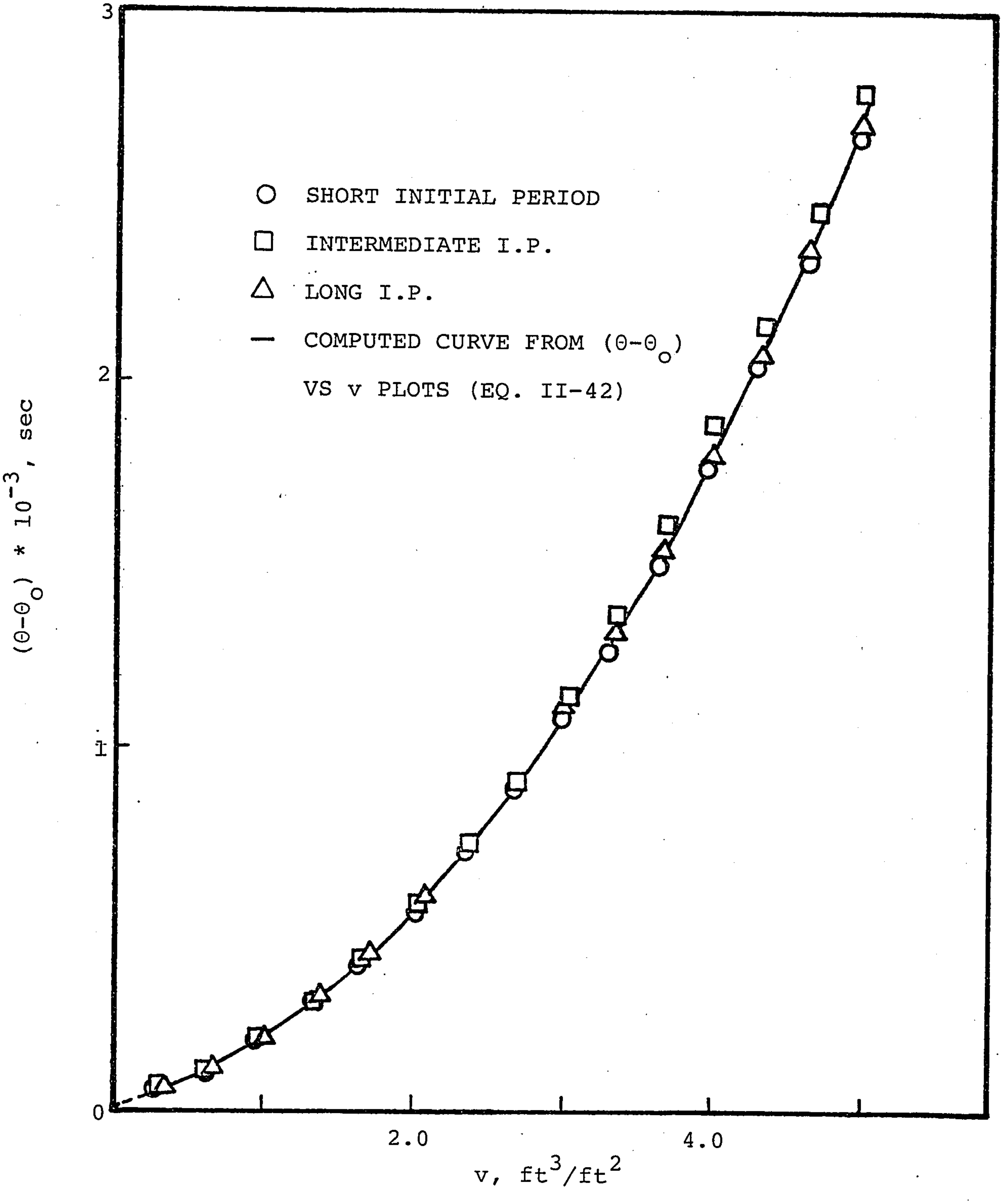


FIGURE IV-1 EFFECTIVE TIME $(\Theta-\Theta_o)$ VS. CUMULATIVE FILTRATE VOLUME v FOR 20 PPM POLYMER SOLUTION IN 2.5% $CaCo_3$ SLURRY, 50 PSI

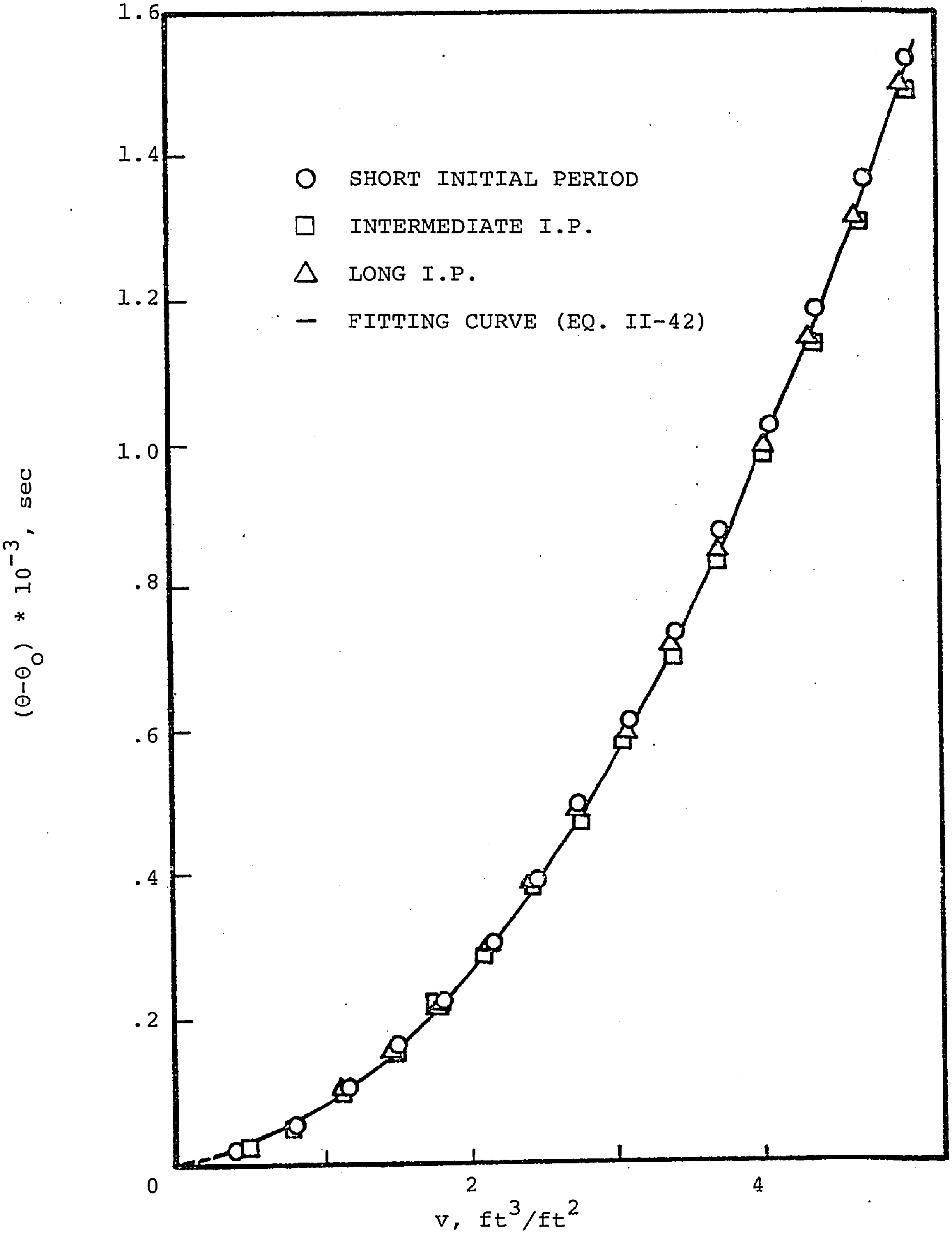


FIGURE IV-2 EFFECTIVE TIME ($\Theta-\Theta_o$) VS. CUMULATIVE FILTRATE VOLUME v FOR 20 PPM POLYMER SOLUTION IN 2.5% $CaCO_3$ SLURRY, 150 PSI of filtration, the fitting constants, K and $v_o$ were computed from equation (II-42) for each particular concentration in the slurry at each constant pressure. In Figure IV-3, data are plotted as effective time $(\Theta-\Theta_o)$ versus cumulative filtrate volume per unit area v using the aqueous slurries of different concentrations of drag reducing fluid containing 2.5% $CaCO_3$ by weight at the constant pressure of 50 PSI; a similar plot at a constant pressure of 150 PSI is shown in Figure IV-4. The computer programs used for least-squares fitting both of equation (II-40) and equation (II-42) are given in Appendix B along with the sample calculation of constant pressure filtration data.

To distinguish the curves in Figure IV-3 and Figure IV-4, triangles refer to filtration runs on water slurries without the Polyox WSR Coagulant. Squares refer to 20 PPM, circles to 60 PPM and crosses to 100 PPM concentration of aqueous Polyox WSR Coagulant in slurries at each particular constant pressure filtration run. The variation of these curves can be classified into three stages; the early stage, transition stage and main stage of filtration. It is evident that the curve drawn with triangles is always located above the others during the main stage of filtration. When the concentration of polymer solution in the slurry is increased, the curve

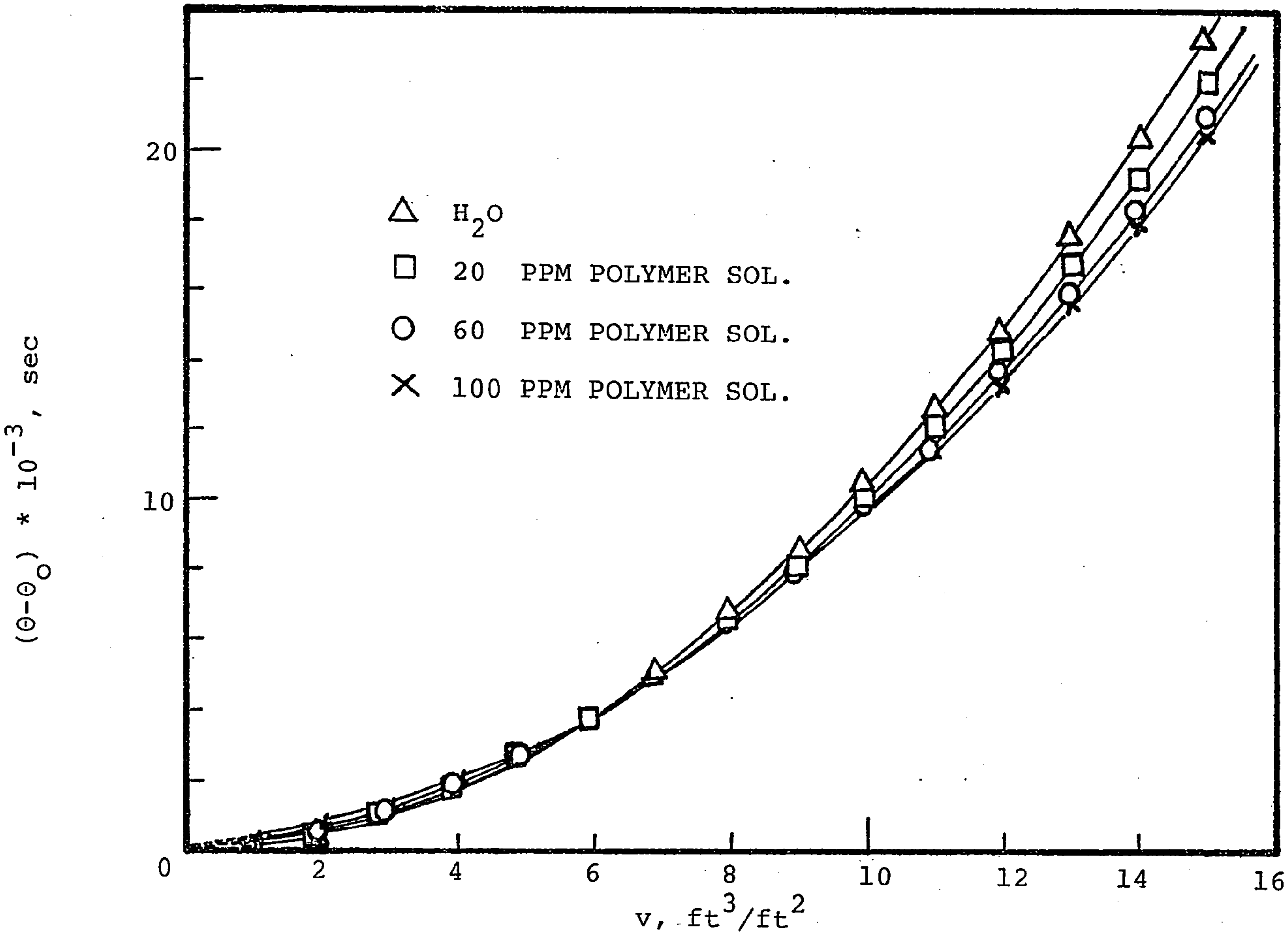


FIGURE IV-3 PLOT OF CUMULATIVE FILTRATE VOLUME v VS. EFFECTIVE TIME $(\Theta-\Theta_o)$ FOR VARIOUS POLYMER CONCENTRATIONS AT CONSTANT PRESSURE OF 50 PSI

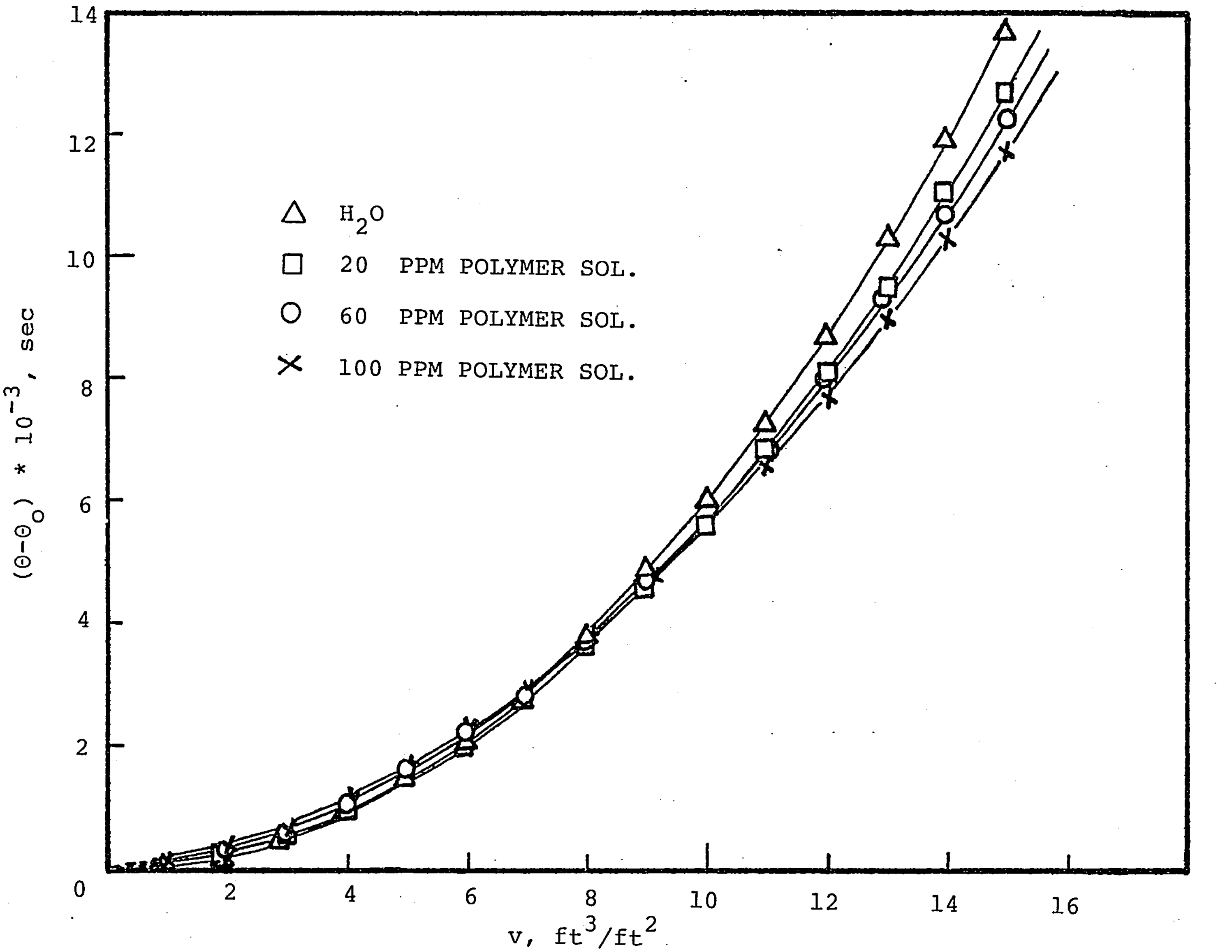


FIGURE IV-4 PLOT OF CUMULATIVE FILTRATE VOLUME v VS. EFFECTIVE TIME $(\Theta-\Theta_o)$ FOR VARIOUS POLYMER CONCENTRATIONS AT CONSTANT PRESSURE OF 150 PSI moves down. However, during the early stage the relative positions of the curves are reversed. The curves are shown in Figure IV-5 and Figure IV-6 to larger scale.

The variation of the curves can be explained as follows. Filter medium resistance influences the cumulative volume v only in the early stage of filtration. At the beginning of filtration, both resistance of filter medium and filter cake are important. But, after a certain time interval, the filter medium resistance can be neglected compared to filter cake resistance which becomes dominant for the remainder of the time. Therefore, the increasing filter medium resistance and the decreasing filter cake resistance are the relevant factors in the early stage and the main stage regions, respectively. The variation of resistance in filter medium $R_m$ and cake $\alpha$ can be computed using equation (IV-37) and (IV-38) through the fitting constants, K and $v_o$ and other properties of filtration (Table 1 and Table 2).

The effective time difference at constant v values between aqueous slurries with polymer solution and without polymer solution is clearly demonstrated in Table 3, although these relationships are already plotted in Figure IV-3 and Figure IV-4.

As an example :

When $v = 15\ ft^3/ft^2$, the time difference between

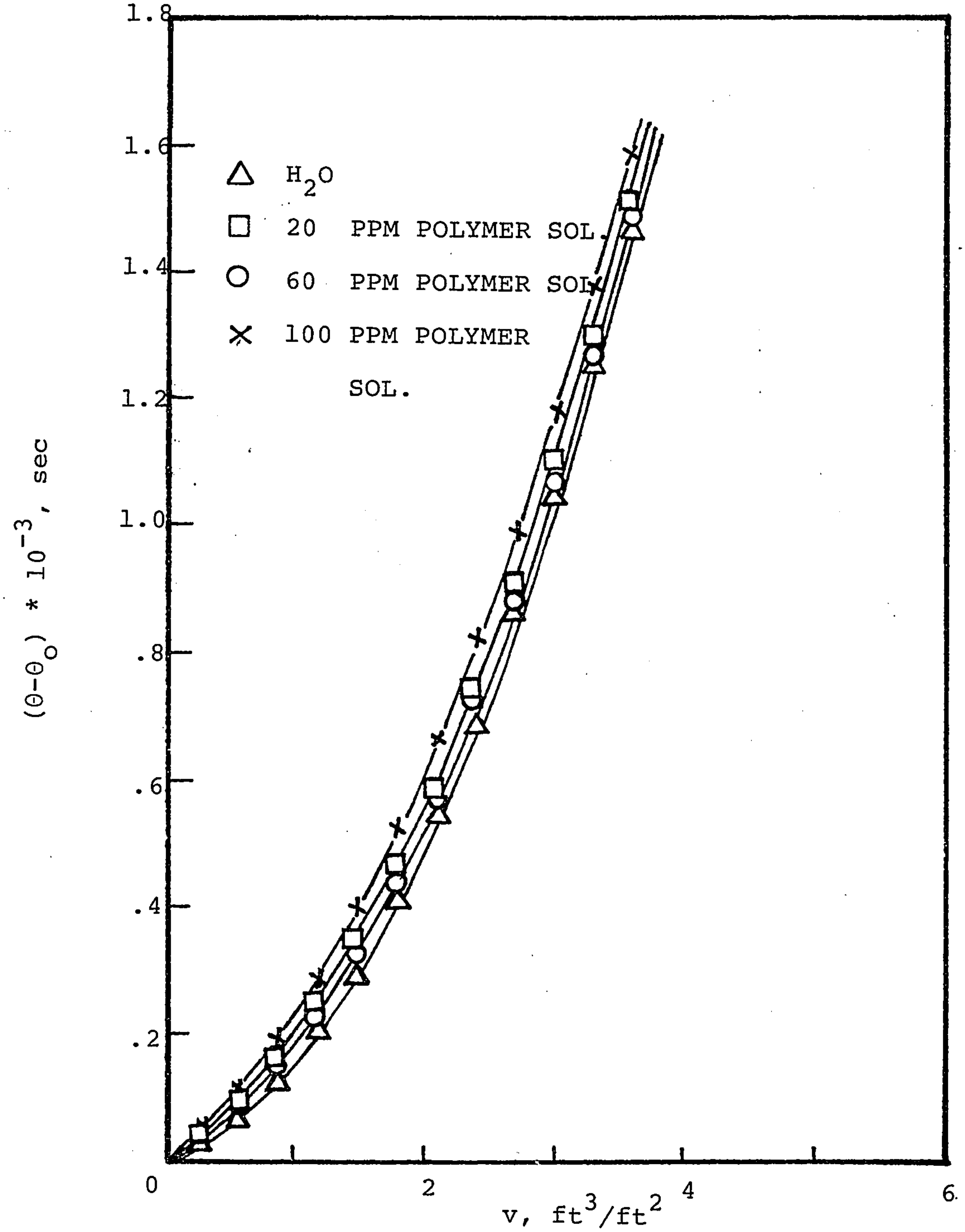


FIGURE IV-5 PLOT OF v VS. $(\Theta-\Theta_o)$ FOR VARIOUS POLYMER CONCENTRATIONS IN AN ENLARGED SCALE AT CONSTANT PRESSURE OF 50 PSI

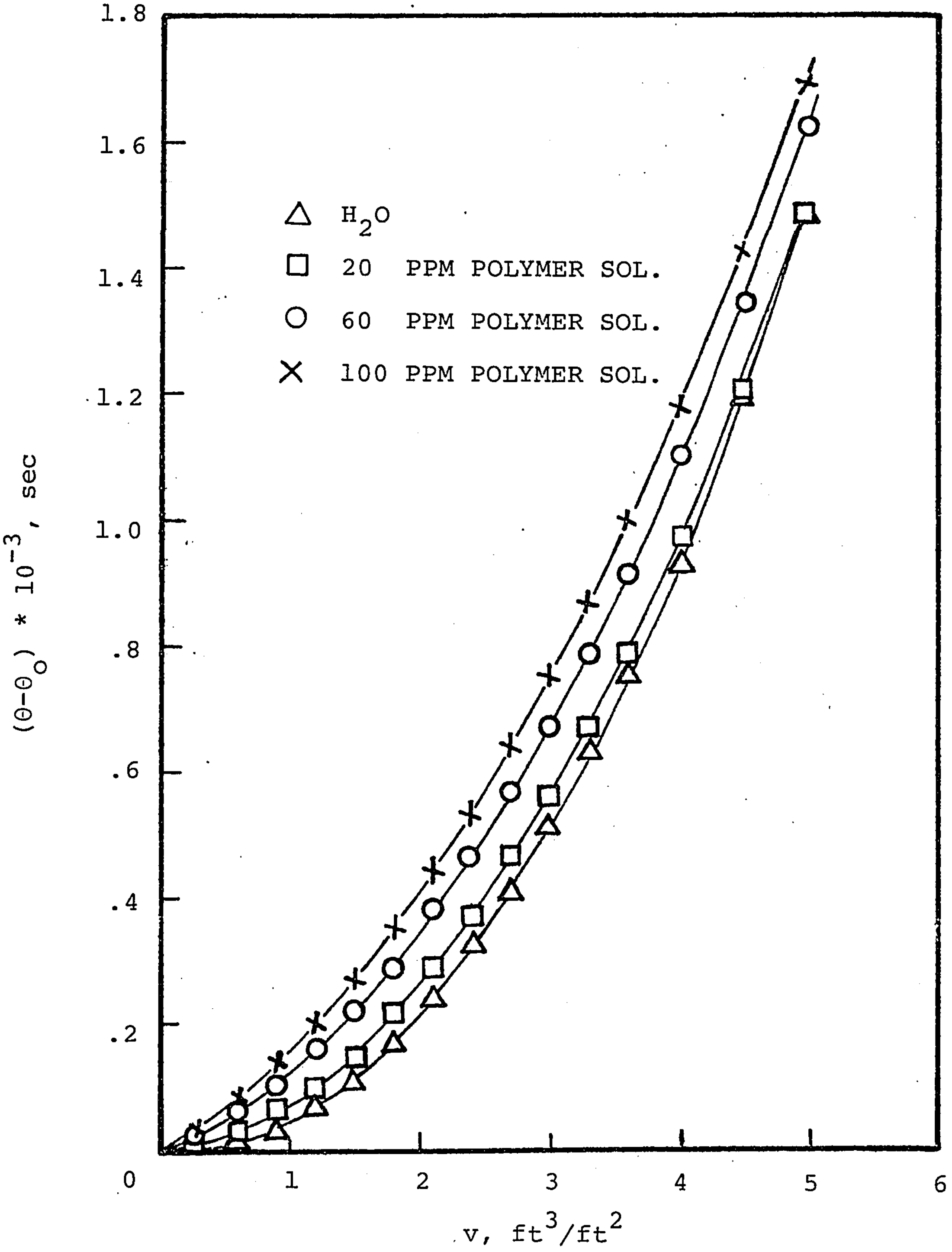


FIGURE IV-6 PLOT OF v VS. $(\Theta-\Theta_o)$ FOR VARIOUS POLYMER CONCENTRATIONS IN AN ENLARGED SCALE AT CONSTANT PRESSURE OF 150 PSI

## TABLE 1

### THE FILTER BED CHARACTERISTICS COMPUTED FROM $(\Theta-\Theta_o)$ VERSUS v DATA FOR EACH PARTICULAR CONCENTRATION AT CONSTANT PRESSURE OF 50 PSI

| CONC. OF POLYMER | 0 PPM | 20 PPM | 60 PPM | 100 PPM |
|---|---|---|---|---|
| $K*10^2$ | 0.99 | 1.07 | 1.17 | 1.21 |
| $v_o*10^2$ | 23.02 | 41.21 | 66.13 | 88.84 |
| M | 2.28 | 2.29 | 2.29 | 2.29 |
| $R_m*10^{-10}$ 1/ft | 1.79 | 3.04 | 4.35 | 5.53 |
| $\alpha*10^{-10}$ ft/$lb_m$ | 4.72 | 4.47 | 3.98 | 3.77 |
| $\Delta\alpha=\alpha_{H_2O}-\alpha_i$ ft/$lb_m$ | 0 | 0.25 | 0.74 | 0.95 |
| $\frac{\Delta\alpha}{\text{CONC.}}$ % |  | 1.26 | 1.23 | 0.95 |

# TABLE 2

## THE FILTER BED CHARACTERISTICS COMPUTED FROM $(\Theta-\Theta_o)$ VERSUS v DATA FOR EACH PARTICULAR CONCENTRATION AT CONSTANT PRESSURE OF 150 PSI

| CONC. OF POLYMER | 0 PPM | 20 PPM | 60 PPM | 100 PPM |
|---|---|---|---|---|
| $K*10^2$ | 1.61 | 1.84 | 2.03 | 2.26 |
| $v_o*10^2$ | 10.93 | 22.63 | 78.52 | 132.64 |
| M | 2.05 | 2.09 | 2.09 | 2.13 |
| $R_m*10^{-10}$ 1/ft | 1.57 | 2.91 | 8.98 | 13.42 |
| $\alpha*10^{-10}$ ft/$lb_m$ | 8.74 | 7.83 | 6.97 | 6.16 |
| $\Delta\alpha=\alpha_{H_2O}-\alpha_i$ ft/$lb_m$ | 0 | 0.91 | 1.77 | 2.59 |
| $\frac{\Delta\alpha}{\text{CONC.}}\%$ |  | 4.56 | 2.96 | 2.59 |

## TABLE 3

### THE VALUES OF EFFECTIVE TIME BETWEEN POLYMER SLURRY AND AQUEOUS SLURRY AT A FIXED CUMULATIVE FILTRATE VOLUME FOR 50 PSI FILTRATION RUNS

| V $ft^3/ft^2$ | $\Theta_e$(0) sec | $\Theta_e$(20 PPM) sec | $\Theta_e$(60 PPM) sec | $\Theta_e$(100 PPM) sec |
|---|---|---|---|---|
| 1 | 147 | 171 | 199 | 228 |
| 3 | 1048 | 1073 | 1109 | 1181 |
| 5 | 2756 | 2722 | 2704 | 2792 |
| 7 | 5272 | 5120 | 4984 | 5062 |
| 9 | 8595 | 8266 | 7948 | 7992 |
| 11 | 12726 | 12160 | 11596 | 11580 |
| 13 | 17664 | 16801 | 15928 | 15828 |
| 15 | 23410 | 22190 | 20945 | 20735 |

the aqueous slurry and the 100 PPM polymer concentration slurry runs at constant pressure 50 PSI is calculated below as

$$\Delta\Theta_e = \Theta_{e,H_2O} - \Theta_{e,100\ \mathrm{PPM}}$$
$$= 23410-20735$$
$$= 2675\ \mathrm{sec}$$

This example indicates that filtration time can be reduced at a particular cumulative volume v, after the addition of drag reducing fluids. In other words, the filtration of drag reducing fluids can provide a more effective result in practice.

The filter bed characteristics of filtration computed from the $(\Theta - \Theta_o)$ versus v relationship and equation (II-37) and (II-38), are presented in Table 1 and Table 2 for different constant pressure runs of 50 and 150 PSI, respectively. It will be noted that the moisture ratios determined experimentally are almost identical when the concentration of the Polyox WSR Coagulant solution in the slurry is augmented.

As mentioned before, the filter medium resistance is influenced not only by the initial period of filtration, but also by the concentration of the polymer solution in aqueous slurry. It is also evident that average specific filtration resistance varies inversely with the concentration of Polyox WSR Coagulant solution. This implies that the flow rate of filtrate can be increased after the addition of drag reducing fluids during filtration because the resistance during the main stage of filtration is reduced. A detailed mechanism for the action of filter cake will be discussed in the Chapter V.

In Figure IV-7, the points indicated by triangles, squares, circles and crosses represent the average values of $\alpha$ given in Table 1 and Table 2. These values were computed from equation (II-37) after fitting at each concentration and each pressure to equation (II-42) which relates $\Theta_e$ to v. It was found that almost identical values (represented by filled circles in Figure IV-7) of average cake resistance could be also obtained by least-squares fitting equation (II-43) to values of $\alpha$ corresponding to the three different initial time periods at each pressure and each concentration. Equation (II-40) and equation (II-37) were used to calculate K and $\alpha$, respectively. The computer program for this calculation is listed in Part C of Appendix B. The dashed-line plot in Figure IV-7 corresponds to a power function of the form $\alpha = \alpha_o p^s$. The empirical equations for each concentration are also shown on this figure. The compressibility coefficient, s, a quantitative measure of compressibility, varies inversely with concentration of drag reducing fluid. The results of this study indicate

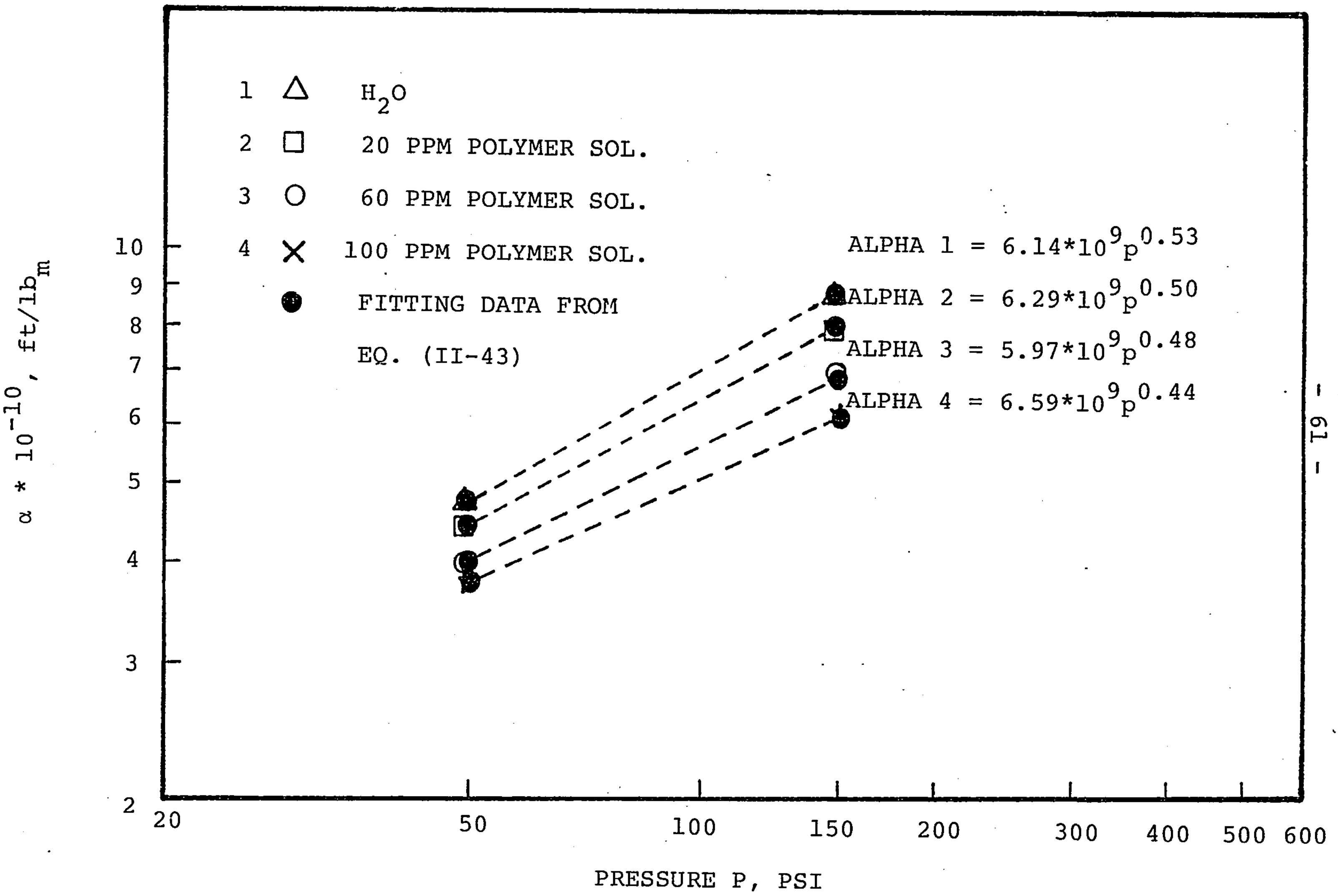


FIGURE IV-7 PLOT OF AVERAGE SPECIFIC FILTRATION RESISTANCE VS. APPLIED PRESSURE that the compressibility coefficient of the drag reducing fluid slurry is less than unity, and the slurry is therefore slightly compressible (7,8).

The ratio of reducing resistance in filtration after the addition of polymer solution to the slurry is defined as :

$$\text{ratio of reducing resistance} = \frac{\alpha_{H_2O} - \alpha_i}{C_i} \qquad \text{(IV-1)}$$

where,

$\alpha_i$ = the average specific filtration resistance in the polymer solution slurry,

$\alpha_{H_2O}$ = the average specific filtration resistance in the aqueous slurry,

$C_i$ = the concentration of the polymer solution in the aqueous slurry.

This ratio can be used to select an optimum concentration of polymer solution.

Sample calculations for the determination of the density and viscosity of filtrate are reported in Appendix C. The experimental data show that the density of filtrate does not change with the addition of drag reducing fluid. This can be explained by noting that the concentration of Polyox WSR Coagulant in all cases is below 100 PPM. The average viscosity of polymer solution for different pressures is also listed in Appendix C. The viscosity of filtrate can be assumed constant for a particular concentration of Polyox WSR Coagulant in the aqueous slurry as shown by the analytical values of viscosity which are quite close. Therefore, it can be assumed that the fluid is Newtonian in these filtration runs. The variation of filtrate viscosity with concentration of Polyox WSR Coagulant in $H_2O$ is plotted in Figure IV-8. Curve A expresses the viscosity of Polyox WSR Coagulant solution before filtration; curve B represents the viscosity of filtrate collected after filtration. Two curves are identical as shown in Figure IV-8.

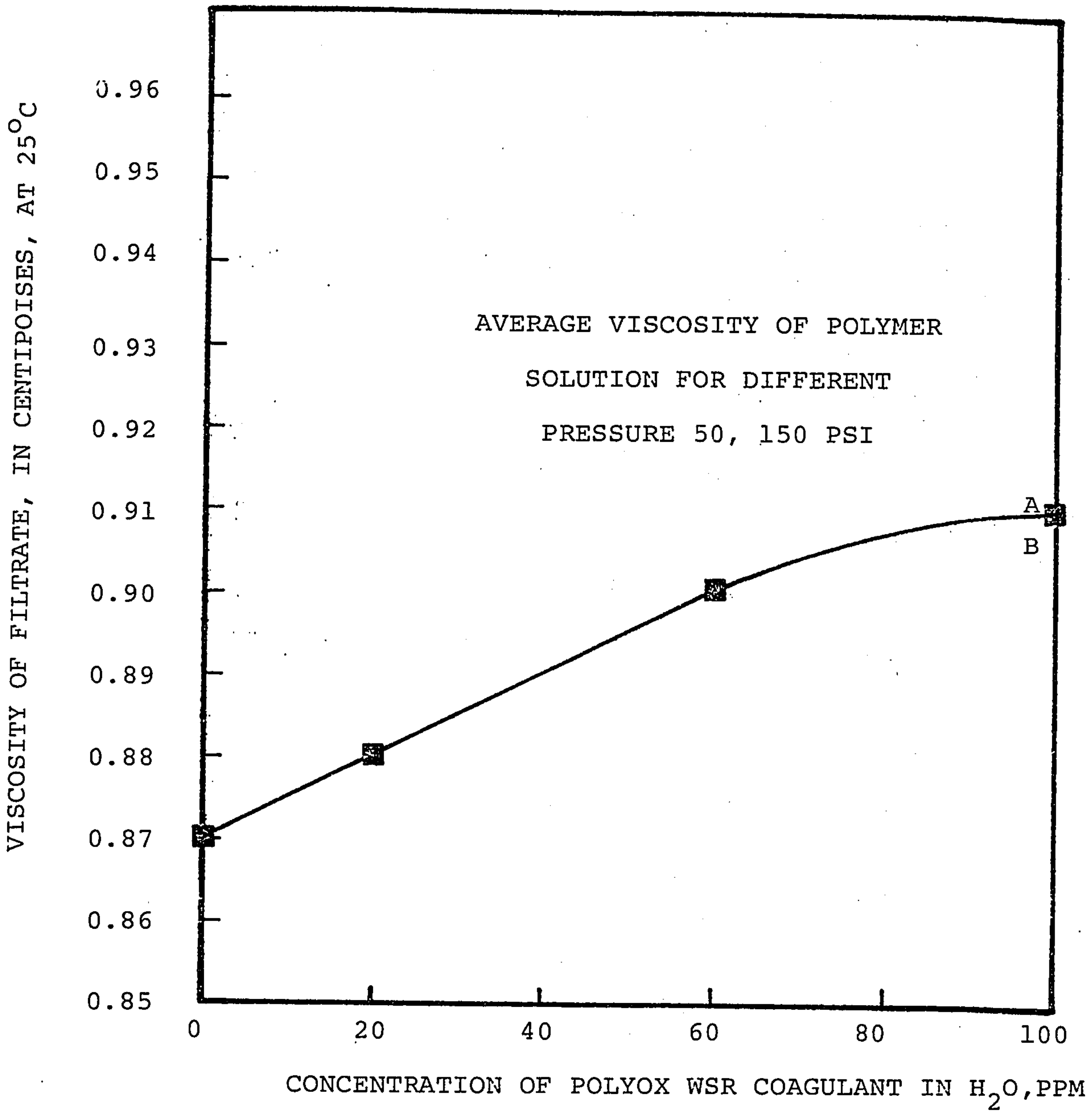


FIGURE IV-8 THE VARIATION OF VISCOSITY IN POLYOX WSR COAGULANT SOLUTION VERSUS CONC. OF POLYOX WSR COAGULANT IN $H_2O$

# CHAPTER V

## DISCUSSION AND CONCLUSION

The results indicate that filter medium resistance increases with increasing concentration of polymer solution in the slurry under the constant temperature and constant pressure filtration runs.

To reduce the filter medium resistance, a higher porosity filter septum could be used. At the start of filtration, small solid particles may pass through the filter medium. In industrial practice, however, the initial filtrate can be returned to the feed tank for reprocessing. After a short time, when the filter cake has been built up, the filtrate becomes clear and can be collected. However, in the laboratory, it is not convenient to reflux the filtrate at the beginning of filtration. The reflux of filtrate will change the concentration of slurry in the feed tank, vary the quantity of cumulative filtrate volume in the graduated cylinder, and make the calculation of the result more complicated. Fortunately, it is possible to select different substances for the filter septum to solve this problem.

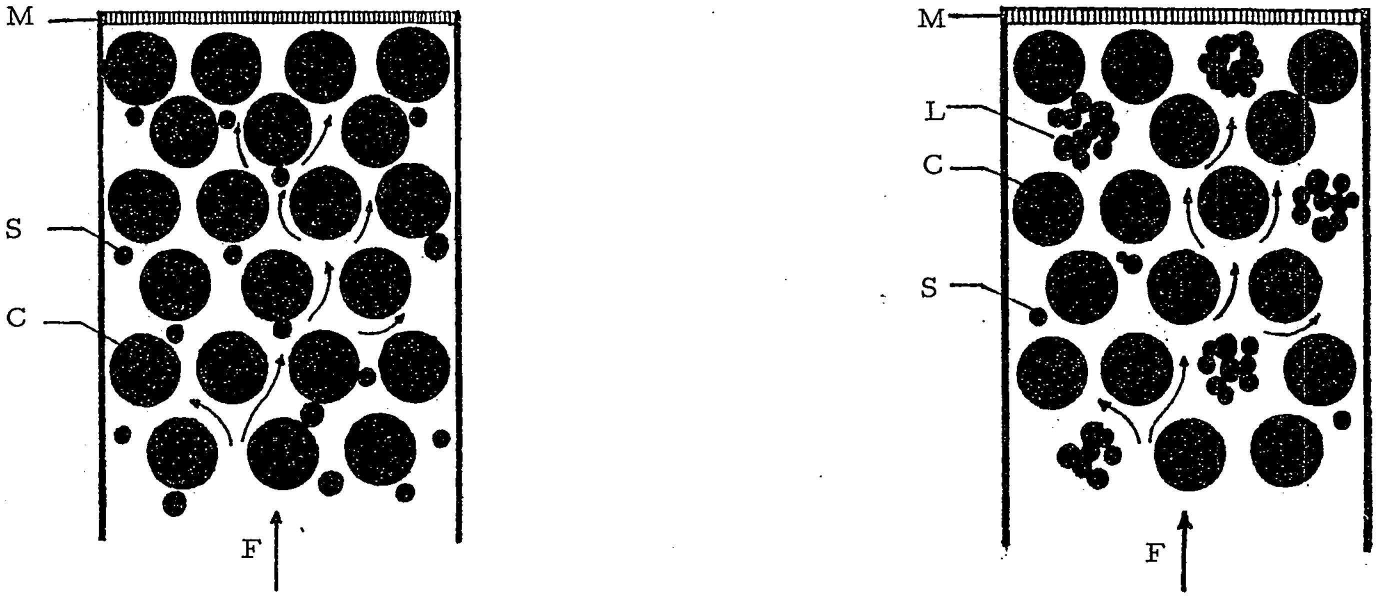
The increase in the resistance of the filter medium upon the addition of drag reducing fluid may be explained by noting that the variation of filter medium resistance depends on the amount of polymer solvent absorbed by the filter septum. The higher concentration of polymer solution will result in greater absorption by the filter medium.

The aqueous polymer solution can aggregate the suspension or gel to reform a new shape of solid particle in the slurry. These new solids are known as "Aggregated Particles" (30), and are formed by the mechanisms known as "Flocculation and Coagulation" (12). The diameter and the total size of aggregated particle are larger than those of the suspension in the aqueous slurry in the absence of polymer solvent. During filtration, these aggregated particles may form a filter cake with higher porosity than that without any flocculation and coagulation, as shown schematically in Figure V-1.

The reduction of the average friction in the pores during passage of the filtrate may be explained by the aggregation of particles and their subsequent behaviour as a normal large solid particle.

This result is evident from the following relationship :

$$\alpha_x = \frac{k_i}{R_H^2 \rho_s \varepsilon (1-\varepsilon)} \qquad \text{(V-1)}$$



(A) FILTER CAKE WITHOUT FLOCCULATION

C CALCIUM CARBONATE

SOLID PARTICLE

F FLOW

(B) FILTER CALE WITH FLOCCULATION

L AGGREGATED PARTICLE

M FILTER MEDIUM

S SUSPENSION

FIGURE V-I THE SCHEMATIC STRUCTURE OF FILTER CAKE

When the porosity, ε, and hydraulic radius, $R_H$, increase due to aggregation, the local specific cake resistance will decrease, and the average specific filtration resistance must then be reduced.

Figure V-1 is a schematic showing the difference between filter cake with and without aggregated particles. The aggregated particles are formed from suspensions in the slurry. The concentration of the polymer solution in the slurry is a very important aspect of aggregation. If the concentration of polymer solution in the slurry is increased, the polymer solution will make the aggregated particle much stronger and larger in the slurry (12). These stronger and larger particles will establish a much higher porosity and lower resistance in the filter cake. However, there is a limitation to the agglomerates. After a certain degree of flocculation, the particles can not be enlarged further and the effect of a change in the polymer concentration is therefore much reduced. However, There are several methods available for avoiding this limitation, such as using a different type of polymer (12).

It was quoted by Purchas (12) that the optimum polymer concentration for the maximum flocculating effect varies with the size of the solid particles, the molecular weight of the polymer, the intensity of agitation and the time of constant agitation. In this research, all these factors are fixed. The experimental results show that the average specific filtration resistance decreases when the concentration of the Polyox WSR Coagulant solution in the slurry increases. The results also provide information about the variation of average specific filtration resistance for each PPM polymer concentration with different concentration of Polyox WSR Coagulant solution in the slurry. This variation was defined as the ratio of reducing resistance in equation (IV-1). From Table 1 and 2, it can be seen that the ratio of reducing resistance varies with the concentration of polymer solution in the slurry. From these data, we can select the optimum concentration for the lowest specific filtration resistance. The optimum concentration of polymer solution to give maximum flocculating effect is not the same as the economic optimum; the latter often occurs at a much lower concentration level,since it depends upon many other factors such as the variation in equipment cost and in running costs.

In conclusion, it is indicated that the drag reducing fluids in filtration can increase the porosity of filter cake and decrease the average specific filtration resistance. The superficial velocity of the filtrate at the exit of the cake can therefore be increased.

In practice, the benefits of increasing the efficiency of filtration resulting from the addition of a polymer must be weighed against the cost of the polymer. The results of this work suggest that the required concentration of polymer can be optimized.

In some processes, drag reducing fluids are used in the transportation of slurries. The findings of this research indicate that the removal of such fluids may not be necessary.

## CHAPTER VI

## REFERENCES

1. A.I.Ch.E. Journal. "Equipment Testing Procedure Batch Pressure Filters." Amer. Inst. of Chem. Eng. (1967).
2. Bennett, C.O. and Myers, J.E., "Momentum, Heat, and Mass Transfer", McGraw-Hill, New York (1974).
3. Berman, N.S., A.I.Ch.E. Journal, 15, 137 (1961).
4. Carman, P.C., Trans. Inst. Chem. Engrs. (London), 16, 168 (1938).
5. Cox, L.A. and Mohney, W.K., Ind. and Eng. Chem., 45, 1798 (1953).
6. Gieger, H.J.A,5, "1st World Filtration Congress", Paries, (1974).
7. Grace, H.P., Chem. Eng. Progr., 49, 303 (1953).
8. Grace, H.P., Chem. Eng. Progr., 49, 367 (1953).
9. Kozicki, W., Rao, A.R.K., and Tiu, C., Candian Journal Chem. Eng. 48, 463 (1970).
10. Kozicki, W., Rao, A.R.K., and Tiu, C., Ind. Eng. Chem. Fundam., 9, 261 (1970).
11. McCabe and Smith, "Unit Operations of Chemical Engrg.", McGraw-Hill Inc., New York (1967).
12. Purchas, D.B., "Industrial Filtration of Liquids", Press, Ohio (1971).

13. Rao, A.R.K, Ph.D. Thesis, University of Ottawa (1971).

14. Ruth, B.F., Ind. Eng. Chem., 25, 153 (1933).

15. Ruth, B.F., Ind. Eng. Chem., 38, 564 (1946).

16. Scheidegger, A.E, "The Physis of Flow through Porous Media", Toronto (1974).

17. Seyer, F.A., J. Fluid Mech., 40, 807 (1970).

18. Shirato, M., Sambuichi, M., Kato, H., and Aragaki, T., A.I.Ch.E. Journal. 15, 405 (1969).

19. Shirato, M., Aragaki T. and Fujiyoshi S., A.8 "First World Filtration Congress", Paris (1974).

20. Shook C.A. and Liebe, J.O., Can. J. Chem. Eng., 54, 118 (1976).

21. Sutterby, J.L., A.I.Ch.E. Journal, 12, 63 (1966).

22. Tiller, F.M., Chem. Eng. Progr., 49, 467 (1953).

23. Tiller, F.M., Chem. Eng. Progr., 51, 282 (1955).

24. Tiller, F.M., and Shirato, M., A.I.Ch.E. Journal, 10, 1, 62, (1963).

25. Tiller, F.M., and Shirato, M., A.I.Ch.E. Journal, 10, 61 (1964).

26. Tiller, F.M., and H.R. Cooper, A.I.Ch.E. Journal, 6, 595 (1960).

27. Toms, B.A., Proc. Int. Cong. Rheol. II, P. 135, North Holland P.Co., Amsterdam (1949).

28. Virk, P.S. et al., J. Fluid Mech., 30, 305 (1967).

29. Vossoughi, S. and Seyer, F.A., Can. J. Chem. Eng., 52, 666 (1974).

30. Wakeman R.J., "Filtration Post-Treatment Processes". Elsevier, New York (1975).

31. Wells, P.S. et al., J. Fluid Mech., 30, 305 (1967).

# VII APPENDICES

# APPENDIX A

## FILTRATION DATA COLLECTED AT CONSTANT TEMPERATURE

## $25^{\circ}C$ AND CONSTANT PRESSURE 50 PSI OR 150 PSI

# TABLE A (FROM TABLE A-1 TO TABLE A-24)

# CONSTANT PRESSURE FILTRATION

# DATA

## TABLE A-1

Constant pressure filtration data using
2.5% $CaCO_3$ aqueous slurry

| | |
|---|---|
| Filtration run no. | : S1F1 |
| Pressure | : 50 PSI |
| Mass of Calcium Carbonate | : 500.003 gm |
| Mixing time for preparation of slurry | : 79 min 54 sec |
| Density of filtrate | : 0.99710 $gm/cm^3$ |
| Viscosity of filtrate | : 0.8731 cp |
| Moisture ratio | : 2.2953 |

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 47 | 400 |
| 2 | 92 | 820 |
| 3 | 169 | 1260 |
| 4 | 263 | 1660 |
| 5 | 383 | 2080 |
| 6 | 526 | 2500 |
| 7 | 690 | 2920 |
| 8 | 884 | 3360 |
| 9 | 1108 | 3820 |
| 10 | 1302 | 4180 |
| 11 | 1551 | 4600 |
| 12 | 1808 | 5000 |
| 13 | 2113 | 5440 |
| 14 | 2414 | 5840 |
| 15 | 2746 | 6260 |

## TABLE A-2

Constant pressure filtration data using
2.5% $CaCO_3$ aqueous slurry

Filtration run no. : S1F2
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.000gm
Mixing time for preparation of slurry : 75 min 14 sec
Density of filtrate : 0.99710 $gm/cm^3$
Viscosity of filtrate : 0.8731 cp
Moisture ratio : 2.2850

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 57 | 400 |
| 2 | 102 | 820 |
| 3 | 180 | 1240 |
| 4 | 290 | 1680 |
| 5 | 418 | 2100 |
| 6 | 570 | 2520 |
| 7 | 727 | 2900 |
| 8 | 929 | 3340 |
| 9 | 1146 | 3760 |
| 10 | 1373 | 4160 |
| 11 | 1633 | 4580 |
| 12 | 1926 | 5020 |
| 13 | 2214 | 5420 |
| 14 | 2533 | 5840 |
| 15 | 2896 | 6280 |

## TABLE A-3

Constant pressure filtration data using
2.5% $CaCO_3$ aqueous slurry

Filtration run no. : S1F3
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.003 gm
Mixing time for preparation of slurry : 75 min 30 sec
Density of filtrate : 0.99710 $gm/cm^3$
Viscosity of filtrate : 0.8731 cp
Moisture ratio : 2.2712

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 60 | 400 |
| 2 | 101 | 820 |
| 3 | 164 | 1220 |
| 4 | 252 | 1620 |
| 5 | 378 | 2080 |
| 6 | 507 | 2480 |
| 7 | 662 | 2900 |
| 8 | 827 | 3300 |
| 9 | 1023 | 3720 |
| 10 | 1239 | 4140 |
| 11 | 1470 | 4540 |
| 12 | 1733 | 4960 |
| 13 | 2019 | 5380 |
| 14 | 2315 | 5780 |
| 15 | 2662 | 6220 |

## TABLE A-4

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 20 PPM Polyox WSR Coagulant solution

Filtration run no. : S1F4
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.003 gm
Mixing time for preparation of slurry : 75 min 15 sec
Density of filtrate : 0.99711 gm/$cm^3$
Viscosity of filtrate : 0.8749 cp
Moisture ratio : 2.2967

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 56 | 400 |
| 2 | 114 | 820 |
| 3 | 188 | 1220 |
| 4 | 292 | 1660 |
| 5 | 410 | 2080 |
| 6 | 543 | 2480 |
| 7 | 700 | 2900 |
| 8 | 879 | 3320 |
| 9 | 1067 | 3720 |
| 10 | 1267 | 4120 |
| 11 | 1506 | 4540 |
| 12 | 1751 | 4940 |
| 13 | 2030 | 5360 |
| 14 | 2328 | 5780 |
| 15 | 2661 | 6220 |

## TABLE A-5

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 20 PPM Polyox WSR Coagulant solution

| | |
|---|---|
| Filtration run no. | : S1F5 |
| Pressure | : 50 PSI |
| Mass of Calcium Carbonate | : 500.007 gm |
| Mixing time for preparation of slurry | : 75 min 28 sec |
| Density of filtrate | : 0.99711 gm/$cm^3$ |
| Viscosity of filtrate | : 0.8740 cp |
| Moisture ratio | : 2.2983 |

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 74 | 400 |
| 2 | 135 | 820 |
| 3 | 222 | 1260 |
| 4 | 326 | 1680 |
| 5 | 448 | 2100 |
| 6 | 605 | 2560 |
| 7 | 748 | 2920 |
| 8 | 939 | 3360 |
| 9 | 1117 | 3760 |
| 10 | 1324 | 4160 |
| 11 | 1562 | 4580 |
| 12 | 1807 | 4980 |
| 13 | 2082 | 5400 |
| 14 | 2369 | 5800 |
| 15 | 2706 | 6240 |

## TABLE A-6

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 20 PPM Polyox WSR Coagulant solution

Filtration run no. : S1F6
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.005 gm
Mixing time for preparation of slurry : 75 min 43 sec
Density of filtrate : 0.99711 $gm/cm^3$
Viscosity of filtrate : 0.8740 cp
Moisture ratio : 2.2727

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 72 | 400 |
| 2 | 130 | 800 |
| 3 | 213 | 1220 |
| 4 | 324 | 1660 |
| 5 | 450 | 2080 |
| 6 | 589 | 2480 |
| 7 | 753 | 2900 |
| 8 | 938 | 3320 |
| 9 | 1157 | 3760 |
| 10 | 1383 | 4180 |
| 11 | 1625 | 4580 |
| 12 | 1897 | 5000 |
| 13 | 2180 | 5400 |
| 14 | 2484 | 5820 |
| 15 | 2812 | 6240 |

## TABLE A-7

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 60 PPM Polyox WSR Coagulant solution

Filtration run no. : S1F7
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.000 gm
Mixing time for preparation of slurry : 76 min 55 sec
Density of filtrate : 0.99711 $gm/cm^3$
Viscosity of filtrate : 0.8973 cp
Moisture ratio : 2.2965

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 61 | 400 |
| 2 | 129 | 820 |
| 3 | 220 | 1240 |
| 4 | 318 | 1620 |
| 5 | 473 | 1940 |
| 6 | 538 | 2320 |
| 7 | 725 | 2820 |
| 8 | 911 | 3260 |
| 9 | 1108 | 3680 |
| 10 | 1318 | 4100 |
| 11 | 1552 | 4520 |
| 12 | 1800 | 4940 |
| 13 | 2068 | 5360 |
| 14 | 2354 | 5780 |
| 15 | 2671 | 6220 |

## TABLE A-8

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 60 PPM Polyox WSR Coagulant solution

Filtration run no. : S1F8
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.004 gm
Mixing time for preparation of slurry : 75 min 05 sec
Density of filtrate : 0.99711 $gm/cm^3$
Viscosity of filtrate : 0.8984 cp
Moisture ratio : 2.2971

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 61 | 400 |
| 2 | 126 | 820 |
| 3 | 214 | 1240 |
| 4 | 330 | 1680 |
| 5 | 460 | 2100 |
| 6 | 608 | 2520 |
| 7 | 766 | 2920 |
| 8 | 958 | 3360 |
| 9 | 1150 | 3760 |
| 10 | 1369 | 4180 |
| 11 | 1614 | 4600 |
| 12 | 1881 | 5020 |
| 13 | 2151 | 5420 |
| 14 | 2440 | 5820 |
| 15 | 2760 | 6240 |

## TABLE A-9

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 60 PPM Polyox WSR Coagulant solution

Filtration run no. : S1F9
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.004 gm
Mixing time for preparation of slurry : 75 min 24 sec
Density of filtrate : 0.99711 gm/cm$^3$
Viscosity of filtrate : 0.8964 cp
Moisture ratio : 2.2882

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 82 | 400 |
| 2 | 142 | 800 |
| 3 | 227 | 1220 |
| 4 | 331 | 1640 |
| 5 | 458 | 2080 |
| 6 | 599 | 2500 |
| 7 | 759 | 2920 |
| 8 | 935 | 3340 |
| 9 | 1132 | 3760 |
| 10 | 1343 | 4180 |
| 11 | 1597 | 4640 |
| 12 | 1836 | 5040 |
| 13 | 2088 | 5440 |
| 14 | 2360 | 5820 |
| 15 | 2651 | 6240 |

## TABLE A-10

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 100 PPM Polyox WSR Coagulant solution

Filtration run no. : S1F10
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.000 gm
Mixing time for preparation of slurry : 75 min 19 sec
Density of filtrate : 0.99716 gm/cm$^3$
Viscosity of filtrate : 0.9117 cp
Moisture ratio : 2.2828

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 96 | 400 |
| 2 | 185 | 800 |
| 3 | 298 | 1220 |
| 4 | 430 | 1660 |
| 5 | 566 | 2080 |
| 6 | 711 | 2480 |
| 7 | 875 | 2900 |
| 8 | 1060 | 3320 |
| 9 | 1262 | 3740 |
| 10 | 1473 | 4140 |
| 11 | 1703 | 4540 |
| 12 | 1967 | 4960 |
| 13 | 2249 | 5380 |
| 14 | 2555 | 5800 |
| 15 | 2895 | 6240 |

## TABLE A-11

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 100 PPM Polyox WSR Coagulant solution

Filtration run no. : S1F11
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.002 gm
Mixing time for preparation of slurry : 75 min 27 sec
Density of filtrate : 0.99711 gm/$cm^3$
Viscosity of filtrate : 0.9125 cp
Moisture ratio : 2.2872

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 81 | 400 |
| 2 | 156 | 800 |
| 3 | 254 | 1220 |
| 4 | 380 | 1680 |
| 5 | 502 | 2080 |
| 6 | 640 | 2480 |
| 7 | 801 | 2900 |
| 8 | 980 | 3320 |
| 9 | 1175 | 3740 |
| 10 | 1391 | 4160 |
| 11 | 1622 | 4580 |
| 12 | 1874 | 5000 |
| 13 | 2143 | 5420 |
| 14 | 2430 | 5840 |
| 15 | 2708 | 6240 |

## TABLE A-12

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 100 PPM Polyox WSR Coagulant solution

Filtration run no. : S1F12
Pressure : 50 PSI
Mass of Calcium Carbonate : 500.001 gm
Mixing time for preparation of slurry : 75 min 30 sec
Density of filtrate : 0.99716 $gm/cm^3$
Viscosity of filtrate : 0.9137 cp
Moisture ratio : 2.2884

| sample no. | time, sec | volume,ml |
|---|---|---|
| 1 | 98 | 400 |
| 2 | 188 | 800 |
| 3 | 300 | 1220 |
| 4 | 432 | 1680 |
| 5 | 565 | 2100 |
| 6 | 711 | 2520 |
| 7 | 874 | 2940 |
| 8 | 1045 | 3360 |
| 9 | 1255 | 3780 |
| 10 | 1473 | 4200 |
| 11 | 1731 | 4660 |
| 12 | 1961 | 5040 |
| 13 | 2209 | 5420 |
| 14 | 2493 | 5820 |
| 15 | 2814 | 6240 |

## TABLE A-13

Constant pressure filtration data using
2.5% $CaCO_3$ aqueous slurry

| | |
|---|---|
| Filtration run no. | : S2F1 |
| Pressure | : 150 PSI |
| Mass of Calcium Carbonate | : 500.002 gm |
| Mixing time for preparation of slurry | : 75 min 13 sec |
| Density of filtrate | : 0.99708 $gm/cm^3$ |
| Viscosity of filtrate | : 0.8937 cp |
| Moisture ratio | : 2.0201 |

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 30 | 580 |
| 2 | 55 | 980 |
| 3 | 90 | 1360 |
| 4 | 142 | 1780 |
| 5 | 202 | 2180 |
| 6 | 269 | 2560 |
| 7 | 351 | 2960 |
| 8 | 449 | 3380 |
| 9 | 555 | 3780 |
| 10 | 680 | 4200 |
| 11 | 817 | 4620 |
| 12 | 977 | 5060 |
| 13 | 1137 | 5460 |
| 14 | 1322 | 5880 |
| 15 | 1522 | 6300 |

## TABLE A-14

Constant pressure filtration data using
2.5% $CaCO_3$ aqueous slurry

Filtration run no. : S2F2
Pressure : 150 PSI
Mass of Calcium Carbonate : 500.002 gm
Mixing time for preparation of slurry : 77 min 46 sec
Density of filtrate : 0.99708 $gm/cm^3$
Viscosity of filtrate : 0.8937 cp
Moisture ratio : 2.0719

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 52 | 400 |
| 2 | 77 | 860 |
| 3 | 120 | 1340 |
| 4 | 169 | 1740 |
| 5 | 236 | 2180 |
| 6 | 310 | 2580 |
| 7 | 399 | 3000 |
| 8 | 491 | 3380 |
| 9 | 604 | 3800 |
| 10 | 730 | 4220 |
| 11 | 862 | 4620 |
| 12 | 1015 | 5040 |
| 13 | 1174 | 5440 |
| 14 | 1357 | 5860 |
| 15 | 1554 | 6280 |

## TABLE A-15

Constant pressure filtration data using

2.5% $CaCO_3$ aqueous slurry

Filtration run no. : S2F3
Pressure : 150 PSI
Mass of Calcium Carbonate : 500.006 gm
Mixing time for preparation of slurry : 75 min 28 sec
Density of filtrate : 0.99708 $gm/cm^3$
Viscosity of filtrate : 0.8937 cp
Moisture ratio : 2.0719

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 59 | 600 |
| 2 | 84 | 980 |
| 3 | 114 | 1320 |
| 4 | 161 | 1720 |
| 5 | 230 | 2180 |
| 6 | 303 | 2580 |
| 7 | 390 | 3000 |
| 8 | 485 | 3400 |
| 9 | 597 | 3820 |
| 10 | 728 | 4260 |
| 11 | 860 | 4660 |
| 12 | 1014 | 5080 |
| 13 | 1168 | 5460 |
| 14 | 1343 | 5860 |
| 15 | 1554 | 6300 |

## TABLE A-16

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 20 PPM Polyox WSR Coagulant solution

Filtration run no. : S2F4
Pressure : 150 PSI
Mass of Calcium Carbonate : 500.001 gm
Mixing time for preparation of slurry : 75 min 35 sec
Density of filtrate : 0.99711 gm/cm$^3$
Viscosity of filtrate : 0.8775 cp
Moisture ratio : 2.0879

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 36 | 400 |
| 2 | 75 | 880 |
| 3 | 128 | 1360 |
| 4 | 183 | 1760 |
| 5 | 249 | 2160 |
| 6 | 328 | 2580 |
| 7 | 412 | 2980 |
| 8 | 514 | 3400 |
| 9 | 632 | 3840 |
| 10 | 752 | 4240 |
| 11 | 896 | 4680 |
| 12 | 1047 | 5100 |
| 13 | 1203 | 5500 |
| 14 | 1383 | 5920 |
| 15 | 1553 | 6300 |

## TABLE A-17

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 20 PPM Polyox WSR Coagulant solution

| | |
|---|---|
| Filtration run no. | : S2F5 |
| Pressure | : 150 PSI |
| Mass of Calcium Carbonate | : 500.003 gm |
| Mixing time for preparation of slurry | : 78 min 59 sec |
| Density of filtrate | : 0.99711 $gm/cm^3$ |
| Viscosity of filtrate | : 0.8767 cp |
| Moisture ratio | : 2.0885 |

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 46 | 500 |
| 2 | 78 | 900 |
| 3 | 124 | 1340 |
| 4 | 175 | 1740 |
| 5 | 241 | 2160 |
| 6 | 314 | 2560 |
| 7 | 400 | 2980 |
| 8 | 494 | 3380 |
| 9 | 603 | 3800 |
| 10 | 729 | 4240 |
| 11 | 855 | 4640 |
| 12 | 1000 | 5060 |
| 13 | 1158 | 5480 |
| 14 | 1329 | 5900 |
| 15 | 1510 | 6320 |

## TABLE A-18

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 20 PPM Polyox WSR Coagulant solution

| | |
|---|---|
| Filtration run no. | : S2F6 |
| Pressure | : 150 PSI |
| Mass of Calcium Carbonate | : 500.003 gm |
| Mixing time for preparation of slurry | : 75 min 20 sec |
| Density of filtrate | : 0.99713 gm/$cm^3$ |
| Viscosity of filtrate | : 0.8756 cp |
| Moisture ratio | : 2.0874 |

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 59 | 420 |
| 2 | 96 | 860 |
| 3 | 146 | 1320 |
| 4 | 199 | 1720 |
| 5 | 269 | 2160 |
| 6 | 344 | 2560 |
| 7 | 428 | 2960 |
| 8 | 526 | 3380 |
| 9 | 636 | 3800 |
| 10 | 762 | 4240 |
| 11 | 889 | 4640 |
| 12 | 1035 | 5060 |
| 13 | 1188 | 5460 |
| 14 | 1353 | 5860 |
| 15 | 1538 | 6280 |

## TABLE A-19

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 60 PPM Polyox WSR Coagulant solution

| | |
|---|---|
| Filtration run no. | : S2F7 |
| Pressure | : 150 PSI |
| Mass of Calcium Carbonate | : 500.005 gm |
| Mixing time for preparation of slurry | : 75 min 27 sec |
| Density of filtrate | : 0.99711 $gm/cm^3$ |
| Viscosity of filtrate | : 0.8935 cp |
| Moisture ratio | : 2.0874 |

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 47 | 400 |
| 2 | 109 | 900 |
| 3 | 179 | 1340 |
| 4 | 260 | 1780 |
| 5 | 365 | 2300 |
| 6 | 457 | 2720 |
| 7 | 556 | 3120 |
| 8 | 674 | 3560 |
| 9 | 792 | 3960 |
| 10 | 923 | 4380 |
| 11 | 1075 | 4820 |
| 12 | 1231 | 5220 |
| 13 | 1384 | 5620 |
| 14 | 1578 | 6080 |
| 15 | 1764 | 6480 |

## TABLE A-20

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 60 PPM Polyox WSR Coagulant solution

Filtration run no. : S2F8
Pressure : 150 PSI
Mass of Calcium Carbonate : 500.010 gm
Mixing time for preparation of slurry : 76 min 06 sec
Density of filtrate : 0.99714 gm/$cm^3$
Viscosity of filtrate : 0.8902 cp
Moisture ratio : 2.0929

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 55 | 400 |
| 2 | 113 | 860 |
| 3 | 183 | 1320 |
| 4 | 256 | 1740 |
| 5 | 339 | 2180 |
| 6 | 423 | 2580 |
| 7 | 519 | 3000 |
| 8 | 619 | 3400 |
| 9 | 735 | 3820 |
| 10 | 867 | 4260 |
| 11 | 1015 | 4700 |
| 12 | 1174 | 5140 |
| 13 | 1368 | 5620 |
| 14 | 1508 | 5940 |
| 15 | 1692 | 6340 |

## TABLE A-21

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 60 PPM Polyox WSR Coagulant solution

Filtration run no. : S2F9
Pressure : 150 PSI
Mass of Calcium Carbonate : 500.000 gm
Mixing time for preparation of slurry : 75 min 25 sec
Density of filtrate : 0.99707 gm/$cm^3$
Viscosity of filtrate : 0.8950 cp
Moisture ratio : 2.1002

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 59 | 400 |
| 2 | 106 | 840 |
| 3 | 171 | 1320 |
| 4 | 236 | 1720 |
| 5 | 316 | 2160 |
| 6 | 399 | 2580 |
| 7 | 493 | 3000 |
| 8 | 596 | 3420 |
| 9 | 710 | 3840 |
| 10 | 828 | 4240 |
| 11 | 966 | 4660 |
| 12 | 1108 | 5060 |
| 13 | 1270 | 5480 |
| 14 | 1454 | 5920 |
| 15 | 1642 | 6340 |

## TABLE A-22

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 100 PPM Polyox WSR Coagulant solution

Filtration run no. : S2F10
Pressure : 150 PSI
Mass of Calcium Carbonate : 500.003 gm
Mixing time for preparation of slurry : 75 min 17 sec
Density of filtrate : 0.99714 gm/$cm^3$
Viscosity of filtrate : 0.9064 cp
Moisture ratio : 2.1265

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 56 | 400 |
| 2 | 113 | 820 |
| 3 | 189 | 1260 |
| 4 | 265 | 1660 |
| 5 | 356 | 2100 |
| 6 | 448 | 2520 |
| 7 | 544 | 2920 |
| 8 | 651 | 3340 |
| 9 | 763 | 3740 |
| 10 | 888 | 4160 |
| 11 | 1029 | 4580 |
| 12 | 1178 | 5000 |
| 13 | 1340 | 5420 |
| 14 | 1516 | 5840 |
| 15 | 1702 | 6260 |

## TABLE A-23

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 100 PPM Polyox WSR Coagulant solution

Filtration run no. : S2F11
Pressure : 150 PSI
Mass of Calcium Carbonate : 500.000 gm
Mixing time for preparation of slurry : 77 min 37 sec
Density of filtrate : 0.99714 gm/$cm^3$
Viscosity of filtrate : 0.9051 cp
Moisture ratio : 2.1285

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 53 | 400 |
| 2 | 106 | 840 |
| 3 | 168 | 1240 |
| 4 | 243 | 1660 |
| 5 | 326 | 2080 |
| 6 | 421 | 2520 |
| 7 | 520 | 2940 |
| 8 | 623 | 3340 |
| 9 | 741 | 3760 |
| 10 | 875 | 4200 |
| 11 | 1007 | 4600 |
| 12 | 1155 | 5020 |
| 13 | 1322 | 5460 |
| 14 | 1485 | 5860 |
| 15 | 1665 | 6280 |

## TABLE A-24

Constant pressure filtration data using 2.5% $CaCO_3$ by weight aqueous slurry with 100 PPM Polyox WSR Coagulant solution

Filtration run no. : S2F12
Pressure : 150 PSI
Mass of Calcium Carbonate : 500.006 gm
Mixing time for preparation of slurry : 75 min 33 sec
Density of filtrate : 0.99713 gm/cm$^3$
Viscosity of filtrate : 0.9062 cp
Moisture ratio : 2.1267

| sample no. | time, sec | volume, ml |
|---|---|---|
| 1 | 84 | 400 |
| 2 | 154 | 840 |
| 3 | 233 | 1260 |
| 4 | 320 | 1700 |
| 5 | 406 | 2100 |
| 6 | 501 | 2520 |
| 7 | 609 | 2960 |
| 8 | 715 | 3360 |
| 9 | 836 | 3780 |
| 10 | 960 | 4180 |
| 11 | 1101 | 4600 |
| 12 | 1254 | 5020 |
| 13 | 1411 | 5420 |
| 14 | 1595 | 5860 |
| 15 | 1783 | 6280 |

## TABLE A-25

### Calibrated data for diaphragm pressure gauge

(A) First Calibration

| Species | Pressure, PSI | | | |
|---|---|---|---|---|
| Standard Pressure Gauge | 50 | 100 | 150 | 180 |
| Diaphragm Pressure Gauge | 42 | 92 | 142 | 172 |

(B) Second Calibration

| Species | Pressure, PSI | | | |
|---|---|---|---|---|
| Standard Pressure Gauge | 50 | 100 | 150 | 180 |
| Diaphragm Pressure Gauge | 42.5 | 92 | 143 | 172.5 |

(C) Third Calibration

| Species | Pressure, PSI | | | |
|---|---|---|---|---|
| Standard Pressure Gauge | 50 | 100 | 150 | 180 |
| Diaphragm Pressure Gauge | 42.5 | 92.5 | 142.5 | 172.5 |

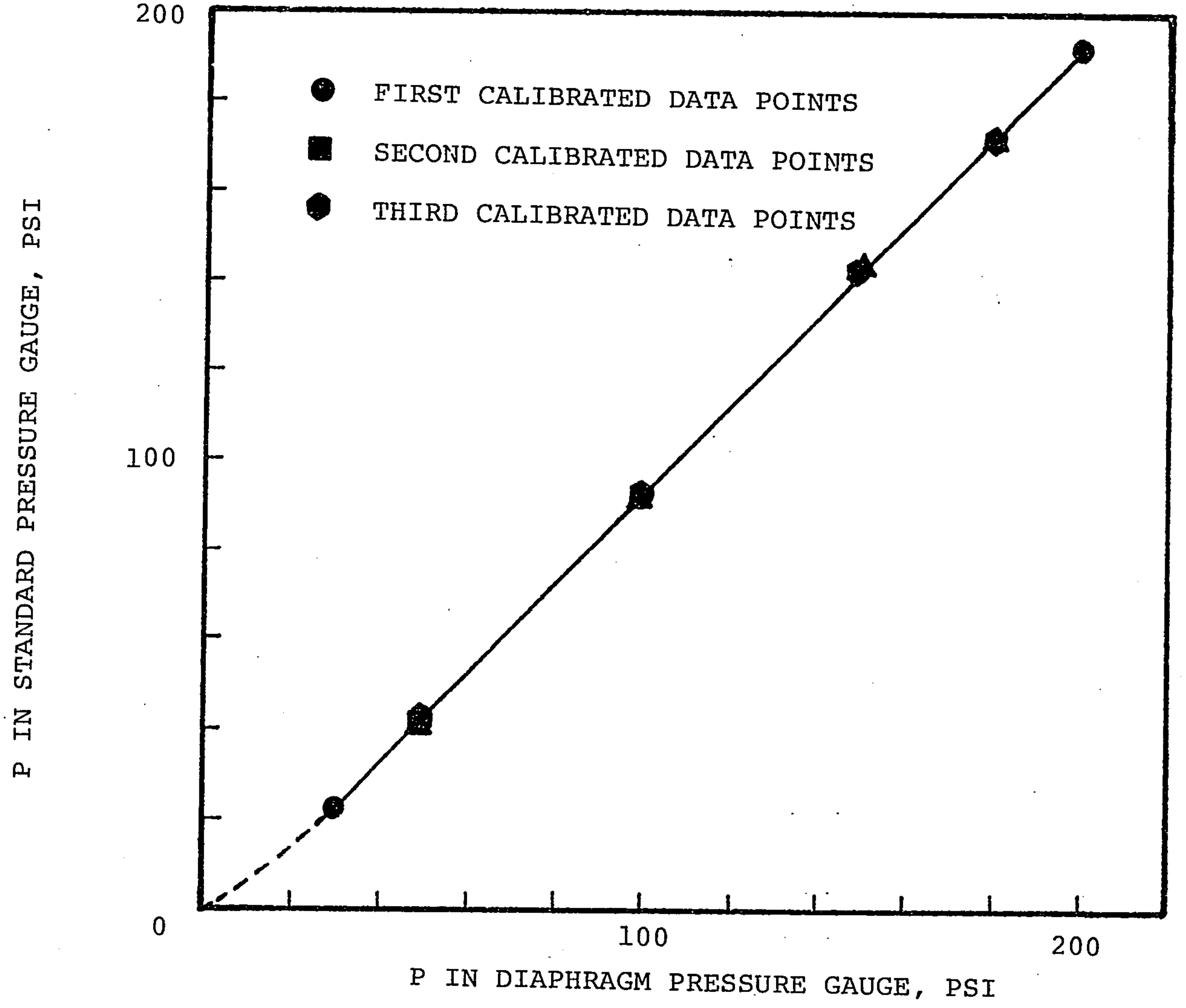


FIGURE A-1 CALIBRATION CURVE OF DIAPHRAGM PRESSURE GAUGE

# APPENDIX B

## COMPUTER PROGRAMS AND SAMPLE CALCULATIONS OF $R_m$ AND $\alpha$

## PART B-1

## Basic calculation of experimental data and least-squares fitting of equation (II-40)

### SAMPLE CALCULATION

Using filtration time and filtrate volume of experimental data listed in Table 4 to Table 6 in Appendix A, the constants of equation (II-40) can be obtained by using the method of least-squares as shown in Program 1.

The effective time of commencement of filtration and other values calculated in Program 1, are listed in Table B-1.

TABLE B-1

Resistances computed from experimental data (Table 4-6 of Appendix A) in the form of $\Theta$ versus v

| Initial time period | $\Theta_o$ sec | $K \times 10^2$ | $v_o \times 10^2$ | $\alpha \times 10^{-10}$ ft/lb$_m$ | $R_m \times 10^{-10}$ 1/ft |
|---|---|---|---|---|---|
| Short | 13.6 | 1.07 | 36.11 | 4.46 | 2.66 |
| Intermediate | 22.5 | 1.04 | 41.62 | 4.60 | 3.16 |
| Long | 25.4 | 1.10 | 46.25 | 4.33 | 3.31 |

# COMPUTER PROGRAM 1

The following is the program (Program 1) which was used to fit the constant pressure filtration data to equation (II-40) by the method of least-squares, utilizing an IBM digital computer. The form of the equation used in the computation is :

$$Y = C\ (1) + C\ (2)X + C\ (3)X^2$$

The notation used in the computation is explained below in terms of the notation used in equation (II-40).

$$C\ (1) = \theta_o \qquad Y = \theta$$

$$C\ (2) = \frac{2v_o}{K} \qquad X = v$$

$$C\ (3) = \frac{1}{K}$$

```
C       ***********************************************
C
C       ANALYSIS OF FILTRATION PROGRAM NO.1
C
C       ***********************************************
        IMPLICIT REAL *4 (A-H,O-Z)
        REAL *4 AA(47)
        REAL *8 P,T,KO,MT
        LOGICAL *1 IMAG1 (5152)/5152*' '/
        INTEGER IMAG4 (1288)/1288*0/
        DIMENSION X(45),Y(45),YY(15,10),P(90),C(5),S(5),
       *A(2),B(2),T(45),XA(45),YA(45),YB(45),AC(45),AB(45),
       *VA(45),VAA(45)
        REAL *4 Z(15)/0.0,.3,.6,.9,1.2,1.5,1.8,2.1,2.4,
       *2.7,3.0,3.3,3.6,4.0,4.5/
        RSQ=100
        MD=2
```

```
      M=6
      DO 10 I=1,M
C
C
C     CALCULATION OF TIME=Y(K) AND ACCUMULATED VOLUME=X(K)
C
C
      N=15
      READ (5,1) NP
    1 FORMAT (I3)
      READ (5,2) WM,DM,UA,DA
    2 FORMAT (2F8.3,F5.4,F7.5)
      READ (5,4) (AC(K),K=1,N)
    4 FORMAT (15F3.0)
      READ (5,4) (AB(K),K=1,N)
      READ (5,7) (VA(K),K=1,N)
    7 FORMAT (15F5.0)
      VI=125
      VIA=1000*28.316
      VIB=VI/VIA
      VIV=VIB/0.045082
      DO 5 K=1,N
      VAA(K)=VA(K)/1276.541912
      Y(K)=AC(K)*60+AB(K)
      X(K)=VIV+VAA(K)
      XA(K)=X(K)
    5 YA(K)=Y(K)
      WRITE (6,12) NP
   12 FORMAT ('1',  ///,5X,'CONSTANT PRESSURE',5X,I3,
     *4X,'PSI',//,5X,'TEMPERATURE DEGREES OF F',//,5X,
     *'CONCENTRATION',//,5X,'FILTER MEDIUM',//,5X,'NO.',
     *5X,'SYMBOL:',//,5X,'INITIAL PERIOD TIME',//,5X,
     *'MIXING TIME',//,5X,'ROTARY SPEED:',//,5X,'CONDI-
     *TIONS:',//)
```

```
C
C
C       CALCULATE THE CONSTANTS C1,C2,C3,
C
C
        CALL RLFOTH(X,Y,N,RSQ,MD,ID,P,C,S,A,B,IER)
        CALL RLDOPM(C,ID,A,B,T)
        T0=C(1)
        K0=1/C(3)
        V0=C(2)*K0/2
C
C       DENSITY=D
C       MASS RATIO=R
C       VISCOSITY=U
C       AVERAGE SPECIFIC RESISTANCE=ALFA
C       FILTER MEDIUM RESISTANCE=RM
C       MASS OF WET CAKE=WM
C       MASS OF DRY CAKE=DM
C
        PD=NP*144.
        GC=32.174
        SCC=0.025
        D=DA*2.2046*28.316
        U=UA*0.00067197
        R=WM/DM
        MT=1.0-R*SCC
        ALFAA=2*GC*PD*MT*C(3)
        ALFAB=U*D*SCC
        ALFA=ALFAA/ALFAB
        RMA=V0*ALFA*D*SCC
        RM=RMA/MT
        WRITE(6,100)(C(II),II=1,3)
  100   FORMAT (' ',4X,'C1=',F9.4,8X,'C2=',F9.4,12X,'C3=',
       *F10.4)
```

```
      WRITE (6,41) T0,K0,V0
  41  FORMAT (/,5X,'T0=',F9.4,8X,'K=',F9.4,13X,'V0=',
     *F9.4)
      WRITE (6,42) R,ALFA,RM
  42  FORMAT (/,5X,'R=',F6.4,12X,'ALFA=',F14.1,5X,
     *'RM=',F14.1)
      N=15
      DO 52 J=1,N
      IF (J.GT.15) GO TO 50
      YB(J)=YA(J)-C(1)
      YY(J,I)=C(2)*Z(J)+C(3)*Z(J)**2
      WRITE (6,51) J,XA(J),J,YB(J),J,Z(J),J,I,YY(J,I)
  51  FORMAT (/,5X,'XA(',I2,')=',F4.2,9X,'YB(',I2,')=',
     *F7.2,10X,'Z(',I2,')=',F4.2,9X,'YY(',I2,',',I2,')=',
     *F9.2)
      GO TO 52
  50  YB(J)=YA(J)-C(1)
      WRITE (6,53) J,XA(J),J,YB(J)
  53  FORMAT (/,5X,'XA('I2,')=',F4.2,9X,'YB(',I2,')=',
     *F7.2)
  52  CONTINUE
  10  CONTINUE
C
C     PLOTTING STATEMENTS
C
      IA=15.0
      INC=1
      READ (5,70) (AA(K),K=1,40)
  70  FORMAT (20A4/10A4/10A4)
      AA(41)=0.0
      AA(42)=5.
      AA(43)=0.0
      AA(44)=1500.0
      AA(45)=0.0
```

```
      N=15
      CALL USPLX (Z,YY,N,M,INC,IA,AA,IMAG1,IMAG4,IER)
      WRITE (6,72) IER
72    FORMAT ('1',5X,'IER=',I3)
      STOP
      END
```

## PART B-2

## Sample calculation of $R_m$ and $\alpha$, and fitting constants of equation (II-42) by method of least-squares

With appropriate $\Theta_o$ values, a plot of effective time versus filtrate volume per unit area can be obtained using equation (II-42), as shown in Figure IV-3 and Figure IV-4. The calculation of the fitting constants, K and $v_o$ and of resistance, $\alpha$ and $R_m$, were computed on an IBM 360 digital computer; the program is listed at the end of this part.

The sample calculation of average specific filtration resistance and filter medium resistance will be given below. This calculation is based on the experimental data collected by three different initial time period filtration runs conducted at a constant pressure of 50 PSI, using 20 PPM Polyox WSR Coagulant with 2.5% $CaCO_3$ by weight aqueous slurry, as shown in Table 4 to Table 6 of Appendix A.

(1) Average specific filtration resistance

Given data :

Mass fraction of solid in the slurry, $s=0.025$
Viscosity of filtrate at 25°C, $\mu=0.8743$ cp
Density of filtrate at 25°C, $\rho=0.9971$ gm/cm$^3$
Average moisture ratio, $m=2.2892$
Applied pressure, $p=50$ PSI

Conversion factor, $g_c$=32.174 $lb_m$-ft/$lb_f$-$sec^2$

Fitting constant of equation (II-42) from Program 2 are :

K = 0.0107

$v_o$ = 0.4121

Solution :

Substituting above data into equation (II-37), average psecific filtration resistance can be obtained as :

$$\alpha = \frac{2g_c p\ (1-ms)}{\mu s \rho K}$$

$$= \frac{2(32.174)(50 \times 144)(1-2.2892 \times 0.025)}{(0.0005875)(0.025)(62.2444)(0.0107)}$$

$$= 4.47 \times 10^{10}\ ft/lb_m$$

(2) Filter medium resistance

Making use of equation (II-38) and known values, filter medium resistance can be obtained as follows :

$$R_m = \frac{v_o \alpha s \rho}{(1-ms)}$$

$$= \frac{(0.4121)(4.47 \times 10^{10})(0.025)(62.2444)}{(1-2.2892 \times 0.025)}$$

$$= 3.04 \times 10^{10}\ 1/ft$$

# COMPUTER PROGRAM 2

The following is the program (Program 2) which was used to fit the constant pressure filtration data to equation (II-42) by the method of least-squares, utilizing an IBM 360 digital computer. The form of equation used in the computation is :

$$Y = C\ (2) + C\ (3)X$$

The notation used in the computation is explained below in terms of the notation used in equation (II-42).

$$C\ (1) = \frac{2v_o}{K} \qquad Y = \frac{\theta_e}{v}$$

$$C\ (2) = \frac{1}{K} \qquad X = v$$

```
C         *********************************************
C
C         ANALYSIS OF FILTRATION PROGRAM NO.2
C
C         *********************************************
          IMPLICIT REAL *4 (A-H,O-Z)
          REAL *8 P(90),T(24),MT,ALFAA,ALFAB,ALFA,RMA,RM,
         *K0,V0
          REAL *4 Z(15)/1.0,2.0,3.0,4.0,5.0,6.0,7.0,
         *8.0,9.0,10.0,11.0,12.0,13.0,14.0,15.0/
          REAL X(45),Y(45),YY(15,10),C(8),A(5),B(5),AA(47),
         *DB(3),XA(45),YA(45),YB(45),AC(45),AB(45),VA(45),
         *VAA(45),W(3),UB(3),S(8)
          INTEGER IMAG4 (1288)
          LOGICAL*1 IMAG1(5152)/5152*' '/
          EQUIVALENCE (IMAG4(1),IMAG1(1))
C
C         CALCULATION OF TIME=Y(K) AND ACCUMULATED VOLUME=X(K)
C
```

```
      RSQ=100
      MD=1
      M=4
      DO 10 I=1,M
      N=45
      READ (5,1) NP
    1 FORMAT (I3)
      READ (5,2) (W(J),J=1,3)
    2 FORMAT (3F7.4)
      READ (5,8) (UB(J),J=1,3)
    8 FORMAT (3F5,4)
      READ (5,9) (DB(J),J=1,3)
    9 FORMAT (3F6,5)
      READ (5,4) (X(K),K=1,N)
      READ (5,7) (Y(K),K=1,N)
    4 FORMAT (15F5.2/15F5.2/15F5.2)
    7 FORMAT (10F8.2/10F8.2/10F8.2/10F8.2/5F8.2)
      DO 5 K=1,N
      XA(K)=X(K)
      Y(K)=Y(K)/X(K)
    5 YA(K)=Y(K)
      WRITE (6,12) NP
      WRITE ('1', ///,5X,'CONSTANT PRESSURE',4X,I3,4X,
     *'PSI',//,5X,'TEMPERATURE 25 DEGREES OF C',//,5X,
     *'CONCENTRATION',//,5X,'FILTER MEDIUM',//,5X,'NO.',
     *//,5X,'SYMBOL:',//,5X,'INITIAL PERIOD TIME',//,
     *5X,'MIXING TIME',//,5X,'ROTARY SPEED:',//,5X,
     *'CONDITIONS:',//)
C
C     ***********************************************
C
C     CALCULATE THE CONSTANTS C1,C2,C3,
C
C     ***********************************************
C
```

```
      CALL RLFOTH (X,Y,N,RSQ,MD,ID,P,C,S,A,B,IER)
      WRITE (6,15) ID,IER
   15 FORMAT (/,5X,'ID=',I3, 14X,'IER=',I2)
      CALL RLDOPM (C,ID,A,B,T)
      T0=0.0
      K0=1.0/C(2)
      V0=C(1)*K0/2.
C
C     ***********************************************
C
C     MASS RATIO=W(I)
C     VISCOSITY=UB(I)
C     DENSITY=DB(I)
C     AVERAGE MASS RATIO=R
C     AVERAGE VISCOSITY=U
C     AVERAGE DENSITY=D
C     AVERAGE SPECIFIC RESISTANCE=ALFA
C     FILTER MEDIUM RESISTANCE=RM
C
C     ***********************************************
C
      PD=NP*144.
      GC=32.174
      SCC=0.025
      DT=DB(1)+DB(2)+DB(3)
      DA=DT/3
      D=DA*2.2046*28.316
      UT=UB(1)+UB(2)+UB(3)
      UA=UT/3
      U=UA*0.00067197
      WT=W(1)+W(2)+W(3)
      R=WT/3
      MT=1.0-R*SCC
```

```
      ALFAA=2*GC*PD*MT
      ALFAB=U*D*SCC*K0
      ALFA=ALFAA/ALFAB
      RMA=V0*ALFA*D*SCC
      RM=RMA/MT
      WRITE (6,100)(C(II),II=1,2)
  100 FORMAT (' ',4X,'C1=',F9.4,8X,'C2=',F9.4)
      WRITE (6,41)T0,K0,V0
  41  FORMAT (/,5X,'T0=',F9.4,8X,'K=',F9.4,13X,'V0=',
     *F9.4)
      WRITE (6,42) R,ALFA,RM
  42  FORMAT (/,5X,'R=',F6.4,12X,'ALFA=',F14.1,5X,
     *'RM=',F14.1)
      WRITE (6,46) UA,DA
  46  FORMAT (/,5X,'UA=',F5.4,12X,'DA=',F10.6)
      N=45
      DO 52 J=1,N
      IF (J.GT.15) GO TO 50
      YY(J,I)=C(1)*Z(J)+C(2)*Z(J)**2
      WRITE (6,51) J,XA(J),J,YA(J),J,Z(J),J,I,YY(J,I)
  51  FORMAT (/,5X,'XA(',I2,')=',F4.2,9X,'YA(',I2,')=',
     *F7.2,10X,'Z(',I2,')=',F5.2,9X,'YY(',I2,',',I2,')=',
     *F9.2)
      GO TO 52
  50  WRITE (6,53) J,XA(J),J,YA(J)
  53  FORMAT (/,5X,'XA(',I2,')=',F4.2,9X,'YA(',I2,')=',
     *F7.2)
  52  CONTINUE
  10  CONTINUE
C
C     ***********************************************
C
C     PLOTTING STATEMENTS
C
```

```
C       ************************************************
C
        IA=15.0
        INC=1
        READ (5,70)(AA(K),K=1,40)
   70   FORMAT (20A4/10A4/10A4)
        AA(41)=0.0
        AA(42)=15.0
        AA(43)=0.0
        AA(44)=23000.0
        AA(45)=0.0
        N=15
        CALL USPLX (Z,YY,N,M,INC,IA,AA,IMAG1,IMAG4,IER)
        WRITE (6,71)
   71   FORMAT ('1',9X,'1: 2.5% WEIGHT OF CACO3 IN AQUEOUS
       *SOLUTION',//,10X,'2: 2.5% WEIGHT OF CACO3 WITH 20
       *PPM POLYOX WSR COAGULANT AQUEOUS SOLUTION',//,
       *10X,'3: 2.5% WEIGHT OF CACO3 WITH 60 PPM POLYOX
       *WSR COAGULANT AQUEOUS SOLUTION',//,10X,'4: 2.5%
       *WEIGHT OF CACO3 WITH 100 PPM POLYOX WSR COAGULANT
       *AQUEOUS SOLUTION')
        WRITE (6,72) IER
   72   FORMAT ('1',5X,'IER=',I3)
        STOP
        END
```

## PART B-3

## PROGRAM USED TO FIT THE AVERAGE SPECIFIC FILTRATION RESISTANCE TO EQUATION (II-43). (COMPUTER PROGRAM 3)

The following is the program (Program 3) which was used to fit the average specific filtration resistance to equation (II-43) by method of least-squares.

The form of equation used in the computation is

$$\log \alpha = \log \alpha_o + s \log p$$

```
C
C     ************************************************
C     ANALYSIS OF EQUATION
C     α=α_o p^s
C     AL=ALPHA
C     SP=SLOPE*LN(PRESSURE)
C     TAL=LN(ALPHA)
C     ************************************************
C
      REAL X(40),Y(40),SP(10),TAL(10),AL(10),A(40),B(40)
      M=4
      DO 10 I=1,M
      NUM=6
      READ (5,4)(B(J),J=1,NUM)
      READ (5,7)(A(J),J=1,NUM)
    4 FORMAT (3F14.1/3F14.1)
    7 FORMAT (6F4.0)
      DO 15 J=1,NUM
      AX=A(J)
      BY=B(J)
      X(J)=ALOG(AX)
      Y(J)=ALOG(BY)
      WRITE (6,55) J,X(J),J,Y(J),J,A(J),J,B(J)
   55 FORMAT (/,5X,'X(',I2,')=',F8.5,9X,'Y(',I2,')=',
     *F10.6,5X,'A(',I2,')=',F8.2,9X,'B(',I2,')=',F15.2)
   15 CONTINUE
      CALL LEAST(Y,X,NUM,SLOPE,RCEP)
```

```
      X=EXP(RCEP)
      WRITE (6,81) SLOPE,RCEP,R,SLOPE
81    FORMAT (/,5X,'LN(ALPHA)=',F8.4,'*LN(P)=',F8.4,
     *//,5X,'ALPHA=',F16.2,'*P**',F8.4)
      SP(1)=SLOPE*3.912023005
      SP(2)=SLOPE*5.010635294
      DO 25 L=1,2
      TAL(L)=RCEP+SP(L)
      TALA=TAL(L)
      AL(L)=EXP(TALA)
      WRITE(6,45) L,TAL(L),L,AL(L)
45    FORMAT (/,5X,'LN(AVERAGE ALPHA)',I2,2X,':',2X,
     *F8.4,5X,'AVERAGE ALPHA',I2,2X,':',2X,F15.2)
25    CONTINUE
10    CONTINUE
      STOP
      END


*******************************************************
      SUBROUTINE LEAST (Y,X,NUM,SLOPE,RCEP)
      DIMENSION X(50),Y(50)
      SUMX=0.0
      SUMY=0.0
      SUMXX=0.0
      SUMXY=0.0
      DO 20 I=1,NUM
      XX=X(I)
      YY=Y(I)
      SUMX=SUMX+XX
      SUMY=SUMY+YY
      SUMXX=SUMXX+XX**2
      SUMXY=SUMXY+XX*YY
20    CONTINUE
```

```
      ANUM=NUM
      DET=SUMX*SUMX-ANUM*SUMXX
      RCEP=(SUMX*SUMXY-SUMY*SUMXX)/DET
      SLOPE=(SUMX*SUMY-ANUM*SUMXY)/DET
30    WRITE (6,31) SLOPE,RCEP
31    FORMAT (/,5X,'SLOPE BY LEAST=',F8.4,5X,'RCEP=',
     *F8.4,/)
      RETURN
      END
```

# APPENDIX C

## SAMPLE CALCULATION OF VISCOSITY AND DENSITY OF FILTRATE

## PART C-1

### The measurement of the density of filtrate

All measurements were taken at a constant temperature of 25°C using the "Digital Densimeter DMA 02C". The density of filtrate can be calculated using the equation given in the "Instruction Manual of Digital Densimeter" as follows :

$$\rho_1-\rho_2=K(T_1^2-T_2^2) \qquad \text{(C-1)}$$

where,

$\rho$=density

T=freqency

K=constant

### SAMPLE CALCULATION

The filtration run was conducted at a constant pressure of 50 PSI, using 20 PPM Polyox WSR Coagulant solution with 2.5% $CaCO_3$ by weight aqueous slurry as shown in Table 4 of Appendix A. Then, the density of filtrate in this filtration run was calculated using equation (C-1) and known data.

Given data :

Density of n-Octane at 25°C, $\rho_{n\text{-}C_8H_{18}}=0.6988 gm/cm^3$

Density of water at 25°C, $\rho_{H_2O}=0.9971$ gm/cm$^3$

Measured data :

$T_{H_2O}=239147$

$T_{n-C_8H_{18}}=222298$

$T_{sample}=239148$

Solution :

$\rho_{H_2O}-\rho_{n-C_8H_{18}}=0.2983$ gm/cm$^3$

Substituting to equation (C-1), obtains

$$K=\frac{\rho_{H_2O}-\rho_{n-C_8H_{18}}}{T_{H_2O}{}^2-T_{n-C_8H_{18}}{}^2}$$

$$=\frac{0.2983}{(239147^2-222298^2)}$$

$$=3.8366 \times 10^{-11}$$

Then

$$\rho_{sample}=\rho_{H_2O}+K(T_{sample}{}^2+T_{H_2O}{}^2)$$

$$=0.9971+(3.8366 \times 10^{-11})(239148^2-239147^2)$$

$$=0.9971 \text{ gm/cm}^3$$

## PART C-2

### The measurement of the viscosity of filtrate

All the measurements were taken at constant temperature of 25°C by using the Cannon-Fenske Routine Viscometer No. 50, M795 (Precision Scientific Co.).

### SAMPLE CALCULATION

Filtration runs were conducted at a constant pressure of 50 PSI, using 2.5% $CaCO_3$ by weight aqueous slurry with 20 PPM Polyox WSR Coagulant solution. The viscosity of filtrate in this filtration run was measured using the Cannon-Fenske Routine Viscometer.

Given data :

Viscometer constant, $k_v$ =0.004026 centistokes/sec

Density of filtrate, $\rho$=0.7791 $gm/cm^3$

Measured data :

Average efflux time, $\bar{t}$=217.9 sec

Solution :

From "Instructions for the use of the Cannon-Fenske Routine Viscometer", Kinematic viscosity is

$$\upsilon=k_v\bar{t} \qquad \text{(C-2)}$$

Then

$$\mu=k_v\rho\bar{t} \qquad (C\text{-}3)$$

And

$$\mu= (0.004026)(0.7791)(217.9)$$
$$= 0.8749 \text{ cp}$$

The average viscosity of filtrate for each particular concentration of polymer solution in the slurry was computed as shown in Table C-1.

TABLE C-1

Average viscosity of filtrate at 25°C

| P PSI | average μ ($H_2O$), cp | average μ (20 PPM), cp | average μ (60 PPM), cp | average μ (100 PPM), cp |
|---|---|---|---|---|
| 50 | 0.873 | 0.874 | 0.897 | 0.912 |
| 150 | 0.873 | 0.877 | 0.893 | 0.906 |

The viscosity of polymer solution without filtration was measured at a constant temperature of 25°C, as shown in Table C-2.

## TABLE C-2

Viscosity of polymer solutions at 25°C

| viscosity ($H_2O$),cp | viscosity (20 PPM),cp | viscosity (60 PPM),cp | viscosity (100 PPM),cp |
|---|---|---|---|
| 0.870 | 0.884 | 0.894 | 0.905 |