Effects of Videotaped Solutions on the Transfer of Problem Solving Skills in Mathematics at the Grade Seven Level

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ABSTRACT

Mathematics is a broad field, mastered at a functional level by many preschoolers, and taught in its different forms to students from four years of age to adults in school. A review of the literature revealed a large number of instructional interventions in mathematics. Bruner (1960) has long advocated discovery learning. Collaborative learning, working with peers, was effective in developing strategies (Collins, 1992). Worked examples have been used from kindergarten (Villasenor and Kepner, 1993), to university (Schoenfeld, 1985). Allowing a student to watch a peer solving a problem on videotape combines all three types of intervention, and offers many advantages to the learner. A review of the literature did not find evidence that a videotaped intervention had previously been used in mathematics problem solving at the grade seven level.

The current study was exploratory in nature, with both a quantitative and qualitative component. Three isomorphic problems were developed which were in areas of interest suggested by students at the same grade level. These problems were considered to be "real world" problems because they involved construction projects similar to ones which the students might have encountered in school or at home. The videotape of a grade seven student solving one of these problems was obtained and used as an instructional

intervention.

Thirty-four grade seven students were divided into six groups, three experimental and three control groups. Each of the three experimental groups received a different sequence of problems to solve, but each group saw the videotaped problem solution before solving a common final problem. One control group received the same sequence of problems as each of the experimental groups, but they did not see the video before solving the final problem.

The students who viewed the video reported that watching the student on the video solve the problem did have an impact on their work. After watching the videotaped problem solution these students spent less time analyzing the next problem, suggesting that the video had resulted in the development of a mental model for this type of problem. Support for this conclusion was also found in the increased amount of time spent on Global Monitoring. These students appeared to be more aware of their global plan, and in many cases to have a better global plan. The improved global plan was demonstrated in their selection of a higher level strategy to solve the second problem.

The performance of both the experimental and control groups might have been influenced by the effects of having practised on the first problem. This study found that having one problem to solve did not have a positive effect on the second problem solution for either group. Practise, combined with seeing a videotaped problem solution which was different from the one just attempted, was also not beneficial.

The intervention which was most effective was the one in which the students saw the solution of the problem which they had just attempted. All of the students receiving this treatment developed a more complete global plan, and used a more advanced strategy on their second problem. Two of the six subjects solved the problem correctly when they had not been able to solve their first problem.

Table of Contents

APPENDICES

List of Tables

List of Figures

CHAPTER 1

INTRODUCTION

Problem solving has always been an integral part of mathematics. In the broad sense of the term, problem solving occurs whenever obstacles prevent the movement from an initial state to a goal state. The success of students problem solving in mathematics has long been of interest to educators. One way to encourage and develop problem solving skills is through innovations in the design and delivery of curriculum.

The study of problem solving was enhanced by the advent of computer technology and information processing theory. Being able to look at the constituent parts of a problem solution led to a shift in emphasis from product to process. Within the information processing framework breakdowns in processing and retrieval can be isolated and dealt with. A student may be experiencing problems because information cannot gain access to short term memory because of problems related to the sensory register (Broadbent, 1954). It may be necessary to teach rehearsal strategies to get information from short term memory to long term memory, or storage allocation mechanisms to facilitate storage and retrieval of information brought to long term memory. Difficulties may also be a result of poor executive control mechanisms which allocate time and resources during problem solving.

To examine human problem solving we need to take a broader perspective. Problem solving is a process which takes the problem solver from the initial state to the goal state (Voss, 1991). But there are many other factors which impact on the human problem solver: resources which have been internalized including the knowledge base, belief systems, and affect variables related to mathematics and problem solving. Control mechanisms determine how to allocate time, which resources to use and how much effort to expend in analyzing the problem and creating a global plan. Other control decisions need to be made during the execution of the solution. The student must decide if and when to monitor his* activity, how to correct errors, and where to go when an impasse is reached.

Knowledge Acquisition

To design an effective instructional intervention it is necessary to know what types of knowledge the student will encounter. Current research centres on three types of knowledge. Procedural Knowledge relates to how to perform a task (Ohlsson and Rees, 1991), Conceptual Knowledge is the network of relationships between pieces of information (Eisenhart et al, 1993) and Strategic Knowledge, knowing when to apply knowledge (Borkowski et al, 1986).

* All references to subjects will use the male forms: he, him, his.

How the different types of knowledge develop has been widely studied by Hiebert and Lefevre (1986) and others. According to Hiebert and Lefevre (1986), Procedural and Conceptual Knowledge are closely linked but there is an over emphasis on Procedural Knowledge during the school years. Without the link neither of these types of knowledge can be used effectively.

The acquisition of knowledge by children has been extensively studied by Piaget and his colleagues. Their work has been extended by Sternberg (1985) and others using an information processing framework. Sternberg views knowledge acquisition as a threefold process. The first component is Selective Encoding, picking out relevant information, second is Selective Combination, combining information into a plausible whole, and Selective Comparison, comparing and integrating knowledge with existing knowledge.

Instructional Interventions

Educators must select the proper instructional mode for the subject area and age level of their students. Being aware of the current thinking on information processing, or learning theory does not ensure the proper delivery of material. There may not be one framework which can be used to develop curriculum for all age levels and subject matter.

Research into instructional interventions has not been conclusive. But some interesting trends have emerged. Seeing examples of problems being solved has proven to be an effective device with children of different ages. From grade one (Villasenor and Kepner, 1993) through to university, (Schoenfeld, 1985), research has shown that seeing worked examples and working through the solution have been effective in developing all three types of knowledge.

Worked examples are a form of discovery learning, an instruction intervention which Bruner (1960) has advocated for many years. With worked examples ownership of learning is shifted from the teacher to the student. By working through completed examples, or correcting faulty ones, students are able develop their own strategies, and link the three types of knowledge. In some cases this technique has proven to be more effective than a teacher taught lesson (Villasenor and Kepner, 1993).

Chronological age is a crucial factor for mathematics teachers for several reasons. But research shows that students who are experiencing difficulty in mathematics exhibit patterns of thinking characteristic of younger children. Mathematical ability may have its own developmental time line. What are the implications of this? If, for example, a child in grade six is experiencing difficulty in mathematics, he may be viewing his work from the perspective of a child in grade four, then the instruction which he is receiving at the grade six level may be inappropriate.

Delivering a program to children which was designed by adults, may create a similar mismatch.

The work on expert/novice differences supports this position. Novices respond to the surface features of a problem and represent it based on a classification of these features. Experts extract the deep structure of the problem. Experts and novices perceive problems differently. Perhaps there is a similar difference in the perception of adults and children. If a child perceives a problem differently from the adult who designs or delivers the curriculum then poor performance may result from this mismatch.

Given these constraints, seeing worked examples performed by a student of the same age might be a promising avenue of research. The present project represents an exploratory study of this intervention. By focusing on the process, the impact of this intervention can be examined and discussed with respect to the individual parts of the problem solution as well as the correctness of the solution.

Theory

To study problem solving, a theory which can account for the problem solving process in mathematics, as well as the learning which occurs, is needed. The Induction framework of Holland et al (1985) was chosen because it provides an explanatory framework on several levels. Induction assumes a problem solving approach to learning. It accounts for learning through the concept of rule clusters at a micro level, and mental models at a macro level. Feedback from the problem solving situation affects the strength of the rules contributing to the solution. A change in rule strength will affect whether or not this rule is invoked for some future problem. The Induction framework can also account for the use of analogy in problem solving. Through spreading activation (see page 48), the problem solver is led to the appropriate rule which is associated with the mental model used for the analogy.

In order to examine the processes involved in mathematical problem solving an explanatory framework used in the field of mathematics was needed. The work of two authors, Schoenfeld (1985), and Rowe (1985), provided the basis for this framework.

Schoenfeld (1985) has developed a framework for examining problem solving at a macroscopic level. He argues that there are four categories of knowledge and behaviour which must be dealt with when examining problem solving behaviour. The four categories are: Resources, Heuristics, Control, and Belief Systems. Schoenfeld used these four categories to examine problem solving protocols. From this work a group of six episodes emerged which may or may not be present in problem solving. These episodes are: Reading, Analysis, Exploration, Local Assessment and New Information, Planning and

Implementation, and Verification.

Rowe (1985) adopted a different perspective in developing her explanatory framework. She proposes eighteen variables which can be collapsed into seven groupings. These seven are: Directions/Stimulus Passage Directed Activity, Solution Directed Activity, Reasoning, Critical Evaluation/Judgement, Pause, Memory Related Activities, and Changing the Conditions.

Based on a preliminary analysis of the protocols in the current study, eight categories from the work of Schoenfeld and Rowe emerged which accounted for the problem solving behaviour of the students. These categories were used to develop the coding grid employed in the analysis of the protocols in the current study.

The Research Problem

Many different instructional interventions are discussed in the research literature. The majority of these interventions are founded on some form of adult input, either at the level of development or delivery. There is evidence that novices and experts perceive problems differently, and that young children develop their own methods of problem solving which are different from those they receive through instruction at school. The question of whether adults perceive mathematical problem solving from a perspective incompatible with or foreign to the school age child has not been adequately explored. The present study was designed to examine how effective viewing a student at the same grade level solving a problem is in teaching a student to solve a quasimorphic problem.

A focused review of the literature and an articulation of the specific research questions are presented in chapter 2. The methodology is described in chapter three. Chapters 4 and 5 present a quantitative and qualitative analysis and interpretation of the results, followed by a brief discussion of the two types of analysis in chapter six.

CHAPTER 2

FOCUSED REVIEW OF THE LITERATURE

Mathematics education is a broad field, and over the years there has been extensive work done on educational interventions which will produce the best results in the classroom. With the shift in interest from product to process has come the need to revise old methods of delivery and to explore new avenues. The impact of computer technology was first felt in the classroom with programmes designed to make remedial drill of concepts more entertaining. Teaching programmes were developed which made good use of the novelty and entertainment value of the computer.

This review of the literature has been focused on several themes. The first theme centres on a conceptual framework for mathematical problem solving which could be used to develop an appropriate methodology to assess the processes underlying the problem solving process. The second theme relates to the acquisition of knowledge in mathematics and the variables which impact on that knowledge. Third, instructional interventions in mathematics are reviewed. The chapter concludes with a statement of the research problem.

Information Processing Theory

The emergence of information processing theory was facilitated by two events: the lack of viability of Behaviourism to explain mental processes, and the advent of computer technology. The paradigm pioneered by Donald Broadbent (1954) became the model for research for over a decade. It was based on the assumptions that behaviour consisted of events that could be decomposed into pieces of information that could be operated on to produce an outcome. Complex behaviours could be decomposed into component processes which could in turn be temporally ordered to produce a model of the dynamic of the behaviour. Information processing theory provides a methodology to access the processes used by students while problem solving in mathematics, permitting a change in focus from the product to the process.

A Model of Human Information Processing

The advent of computer technology facilitated a move away from the behaviourists' explanations of problem solving. Researchers began to look at how humans attended to input of stimuli through the various senses. Since humans are bombarded with multiple stimuli during the course of their daily activities, the concept of **sensory registers** gained popularity. Initially it was believed that the sensory registers allowed people to focus on a single stimulus while excluding all others (Broadbent, 1954). It was later shown that people did process information to which they were not consciously attending (Moray, 1969), and they could process information from more than one source at the same time (Treisman, 1964). The sensory registers are not simply an "all or nothing" switch, but rather a means of allocating attentional resources. When an individual processes information he attempts to match the signal with an internal representation (Neisser, 1967), and he does this by performing a serial and exhaustive search (Sternberg, 1969).

This work on the initial stages of the processing of information came to be integrated with existing elements of the memory system. Information passes into the "short-term store" (which came to be known as short term memory [STM]), (Atkinson and Shiffrin, 1968). This is a working system where a finite amount of information, approximately seven units (Miller, 1956), stays briefly, fifteen to thirty seconds, before decaying. Information can be placed in a "rehearsal buffer" and remain there for a longer time. The capacity of STM can be increased by chunking information (Newell and Simon, 1972) but the number of psychologically meaningful chunks remains at approximately seven. The longer information is held in short term memory, the greater the probability that it will pass into the "long-term store" (long term memory [LTM]).

Storage in LTM is viewed as relatively permanent. LTM has a practically limitless capacity, although retrieval may be hampered by the addition of new information. Information can be stored as individual units or chunks, as well as episodes or productions. Episodes may enter LTM and be stored according to their content or context as spatio-temporal, or semantic memory (Tulving, 1972). Atkinson and Shiffrin (1968), discuss several storage mechanisms which allocate space to information according to its context, and mechanisms of retrieval including search procedures. Retrieval mechanisms soon became the focus of attention as executive control processes.

Several researchers have examined the processes of executive control from a theoretical perspective. Anderson (1987) examined the nature and structure of the information processing paradigm, Sternberg (1985) worked on the function of the system while others looked at the regulation and coordination of processing (Pressley et al, 1985, Anderson, 1987). The efficiency of the executive control processes is a determinant of academic success and may be the source of individual developmental differences (Baron, 1978; Lawson and Chinnappan, 1994; Sternberg, 1985).

Problem Solving Defined

Within the more narrow context of the information processing paradigm, problem solving can be defined as a search for a solution within a problem space (Newell and Simon, 1972). The problem solver attempts to go from the initial state of the problem to the goal state. He constructs a problem space which contains these two states, and any possible obstacles or actions which might be taken to reach the goal (Holyoak, 1990). Each action taken adds more problem states to the problem space. Problem solving is a process, the process which takes the problem solver from the initial problem state, through the actions required and new problem states, to the goal state (Holyoak, 1990; Newell and Simon, 1972; Voss, 1991).

The computer brings to the problem solving session only those resources which pertain to the problem being solved. Because people are affected by past experiences, their beliefs, and other variables, human problem solving must be examined from a broader perspective. Learning has been viewed as a problem solving experience, whether it is within the context of interacting with the environment (Ginsburg and Opper, 1979; Vygotsky, 1978), or in the classroom with problems presented by the teacher. Figure 1 presents a model of the problem space and the factors which impact on the human problem solver.

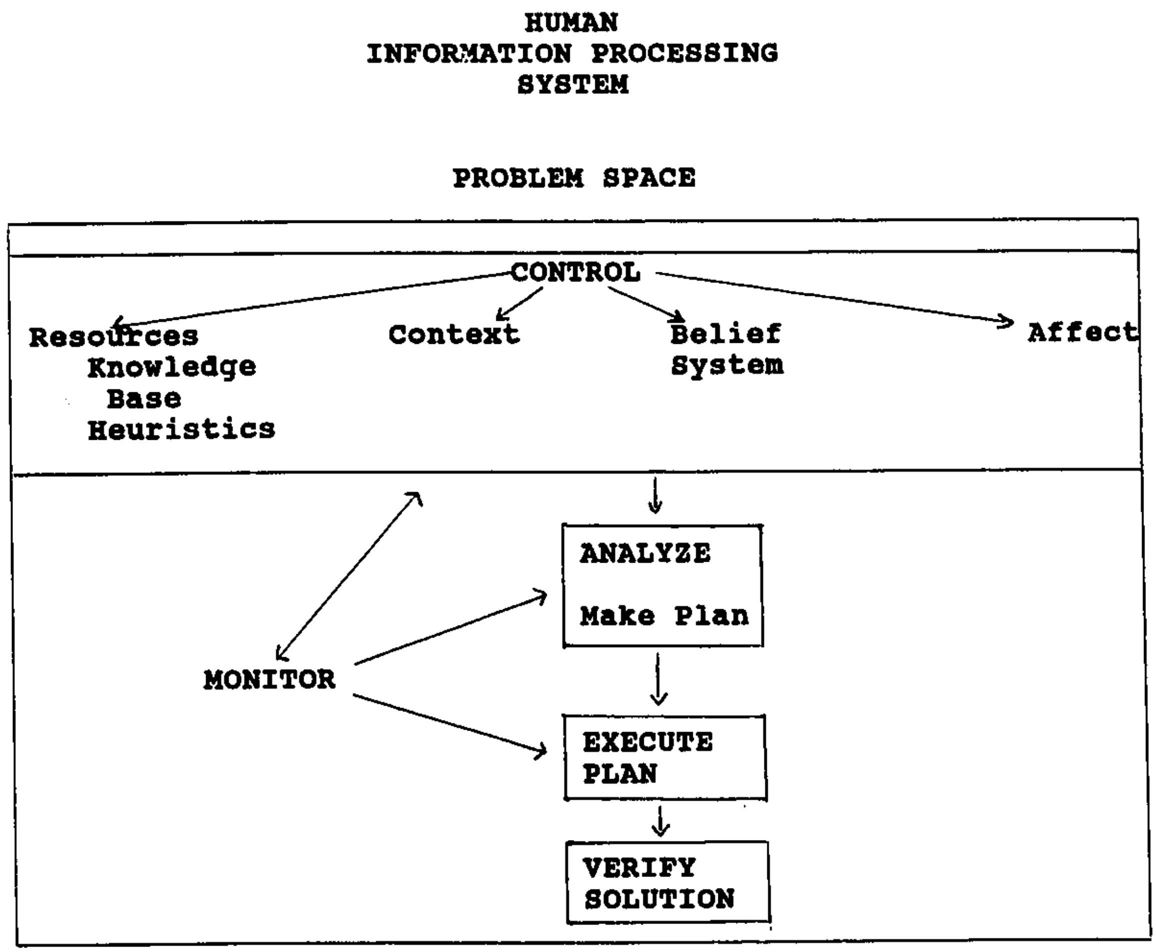

Figure 1. A Model of Human Information Processing
(Adapted from Schoenfeld, 1985)

Within the human information processing system, the individual's resources, the context of the problem, beliefs and affective variables are being regulated by control mechanisms. The question is analyzed and a plan is made. During analysis and the development of the plan, the individual may monitor his progress. If the plan is considered to meet the requirements of the problem, it is executed. During execution of the plan each step may be monitored at both a global and local level. At a global level, as each step in the plan is about to be executed, it is matched to the plan. At the local level, the execution of the steps are scrutinized for errors. Once the answer is obtained the problem solver may choose to verify the solution. Once the solution has been verified, the problem solver may attempt to match it with existing knowledge, and retain it for future reference.

Types of Knowledge

There are three types of knowledge or information, with which people work when problem solving, which are discussed in the literature. The first, **Procedural Knowledge**, is the knowledge of how to perform the task at hand (Eisenhart et al, 1993; Ohlsson and Rees, 1991), the "step by step procedures executed in a specific sequence" (Hiebert and Lefevre, 1986 pg. 8). It may be the formal language or symbol representation system of the domain, eg. knowing that 6(X+1)=48 is syntactically correct even though the answer may not be known. Or it may be the algorithms, heuristics, or rules for completing a task, eg. the heuristics of Polya (1954).

The second type of knowledge is **Conceptual Knowledge**, (referred to as Declarative Knowledge by some researchers). As with procedural knowledge, conceptual knowledge includes the facts in a specific domain needed to perform a task (Anderson, 1987), but within the context of the network of relationships between these pieces of information (Eisenhart et al, 1993), it is goal independent (Ohlsson and Rees, 1991). Knowledge of how units of knowledge are related permits flexibility in accessing and using the information (Carpenter, 1986). A unit of conceptual knowledge cannot be an isolated piece of information, it is a network where the linking relationships are as prominent as the discrete pieces of information (Hiebert and Lefevre, 1986). Teaching for conceptual knowledge is critical for increasing understanding (Eisenhart et al, 1993).

The final type of knowledge, **Strategic Knowledge**, provides the knowledge of the goals and objectives of a procedure, and when to apply those procedures (Borkowski, Johnston and Reid, 1986). Given an area problem in geometry, strategic knowledge will allow the problem solver to correctly select the formula A = l x w, or A = 1/2 (l x w), based on the type of figure involved.

Conceptual knowledge develops from an early age. Its development is an ongoing process which results from an individual's interaction with his environment (Vygotsky, 1978). As conceptual knowledge is gained, related procedural knowledge develops. Concepts and procedures are two types of knowledge which are inextricably linked (Silver, 1986), but when the child begins to attend school the emphasis begins to shift from the conceptual to the procedural (Hiebert and Lefevre, 1986). According to Carpenter (1986), emphasizing procedural knowledge in mathematics education, may result in an impoverished conceptual knowledge, tenuous links between the conceptual knowledge learned and related procedural knowledge, and children acquiring flawed procedural knowledge. Procedures can be learned by rote but unless they are linked to conceptual knowledge their usefulness is limited (Carpenter, 1986). The links between procedural and conceptual knowledge are achieved through the development of strategic knowledge (Chi, 1985).

Being able to link procedural, conceptual and strategic knowledge in a meaningful way is critical for successful problem solving. In addition executive control strategies must be taught to enable the student to effectively implement his knowledge. Teaching executive control strategies is a critical role for teachers (Marzano et al, 1988; Perkins and Swartz, 1989; Nandi, 1990). But if adults develop these strategies from their own perspective, and impose a structure upon them for delivery, they may not reflect the knowledge structure of a young student. Sinclair and Sinclair (1986) demonstrated that young children at home develop a numerical world incompatible with the one they encounter at school. This may well be the case in later years as well, and must be considered when examining metacognitive performance in problem solving situations.

Development of a Knowledge Base in Mathematics

Sternberg (1985) describes knowledge acquisition as a threefold process. One component is **Selective Encoding.** The student must be able to differentiate what is relevant from what is irrelevant. In mathematics, as in life, frequently there are distracters imbedded in a problem with which the student must deal. Successful problem solving, and accurate knowledge acquisition, is dependent upon the student's ability to selectively encode the information used to solve the problem.

The second component is **Selective Combination,** the ability to combine information to form an integrated, plausible whole. The student must be able to take the information from a problem and combine it into a meaningful whole which can be used in the future to solve a similar problem. By taking the relevant information from the problem, and combining it with the operations used in the solution, the student may be able to formulate a single piece, or chunk, of information for storage in LTM. This new chunk will facilitate the solution of a new problem.

Finally Sternberg discusses **Selective Comparison** as a means of knowledge acquisition. In order to make maximum use of newly acquired information, the student must be able to compare it to, and integrate it with, existing information. This must be done during, as well as after, problem solving. During problem solving the student needs to constantly be searching memory for existing information which might share some similarities with the problem being solved. If each problem is completely novel, problem solving will take a long time. Being able to draw on existing knowledge greatly reduces solution latency. Once the new information has been encoded an attempt must be made to combine it with existing information to which it might be related in some way. This establishes a larger, integrated knowledge base which leads to greater success.

Piaget has also stated quite clearly that intellectual development results from an interplay between internal and external factors (Ginsburg and Opper, 1979). Children from a very young age are interacting with, and interpreting, their environment. Through the processes of assimilation and accommodation they learn about their world and begin to develop the knowledge base and many of the skills required at school. As Piaget discovered, many mathematical milestones are developmental. Concrete and abstract reasoning develop at different stages of childhood. An understanding of part-whole relationships doesn't manifest itself until the child is five to six years of age (Sophian and McCorgray, 1994).

Children bring to school some sophisticated mathematical concepts (Baroody and Ginsburg, 1986; Gelman and Meck, 1986; Sinclair and Sinclair, 1986). They can count as well as do addition. There is a sequence through which addition develops beginning with a Count All from the First Number (CAF) strategy up to a Count On from the Largest (COL) strategy (Baroody and Ginsburg, 1986). Children can perform CAF addition before they receive formal instruction in mathematics. Hiebert and Lefevre (1986) have found that at the preschool stage conceptual and procedural knowledge are closely related. The strategies used are meaningful to the child because they have been discovered, tested and developed by the child himself.

Children in the home learn by doing. They are active investigators of their environment not passive recipients of information. They manipulate objects from a very early age, learn their characteristics and uses (Vygotsky, 1978). Once the utility of an object is established, the child will experiment with that object, attempting to find new uses for it. The inquisitiveness of the preschooler and his natural lack of inhibition are important factors in the rate at which young children develop intellectually.

At daycare and nursery schools children interact, developing social skills, but also learning from each other how to use their toys. Children will imitate each other in play and extend the actions of their peers. Children will informally instruct each other, argue, and problem solve. Adult intercession quite often has a negative impact upon learning. Once it is perceived that there is a right way and a wrong way to use an object experimentation and learning ceases (Piaget, 1972; Vygotsky, 1978).

Seeing examples of problems being solved can be an important instructional device through the school years as well (Cooper and Sweller, 1987; Schoenfeld, 1985; Silver, 1986; Sweller and Cooper, 1985; Zhu and Simon, 1987). As early as grade one problem solving can be used effectively to assist in the recall of number facts, and the use of advanced strategies (Villasenor and Kepner, 1993). With worked examples the responsibility for learning is shifted from the teacher to the student who now actively reads and problem solves (Zhu and Simon, 1987). Worked examples as an instructional device led to shorter time for acquisition of concepts (Sweller and Cooper, 1985), and facilitated automation of problem solving operators particularly with low ability students (Cooper and Sweller, 1987).

There are many aspects of the knowledge base which a student brings to a problem solving session. They include a memory of facts (Russell and Ginsburg, 1984), and an assortment of algorithms and heuristics. These known facts must be learned or become automatic (Fuson, 1982). Once they become automatic they are incorporated into simple algorithms, when these become automatic they in turn are incorporated into more complex algorithms (Case, 1978). The student must automatize as many facts and procedures as possible to facilitate problem solving (Rabinowitz 1990). Computational algorithms with "bugs" can cause errors (Brown and Burton, 1978). Mayer (1982) found that students used algorithms and heuristics to categorize problems prior to their solution. Selecting the wrong algorithm or not knowing how to correctly apply the proper algorithm (Schoenfeld, 1985) can have equally adverse effects upon the problem solution.

Mathematical Knowledge as Applied to Problem Solving

From the discussion above it is apparent that there are a number of elements to the knowledge base. Problem solving cannot occur without a sound knowledge base, but there is more involved in successful problem solving. There are a number of contextual variables which come into play; the problem context, the individual's belief systems, feelings, and culture, all impact on the student and the problem solving session. The nature of the instructional intervention also impacts on the overall process. The role of contextual variables will be discussed in a later section.

The Knowledge base is itself composed of a number of elements. Schoenfeld (1985) groups most of the elements discussed into two categories: Resources, and Heuristics. Each will be discussed in turn as it relates to the context of problem solving.

Resources

In his work on problem solving Schoenfeld (1985) describes two elements of the student's mathematical knowledge. The first, Resources, encompasses all the facts learned, and the procedures and skills acquired by the individual which can be brought to bear on a problem. This is similar to the knowledge base discussed above, but transcends the constituent elements to examine how the knowledge is stored and subsequently accessed, and how it is organized. An information processing approach must be taken to examine this mathematical knowledge. Two students with similar mathematical backgrounds may not approach a problem and attempt to solve it in the same way. Elements of the knowledge base are resources only in so far as they can be accessed and used appropriately.

Schoenfeld (1985) suggests that there are four classes of Resources. The first is a set of relevant facts. Facts acquired are "known" to various degrees and they are consequently indexed according to the extent to which they are known. Does a student recognize a line as being perpendicular to another? Does he know what parallel lines are? Being able to access these concepts as facts on a test is quite different from recognizing them as important elements for a problem solution. The degree to which they are known determines whether they will be used correctly. Proper use of these facts leads to the awareness of when to access other facts during the solution. This network of facts, known to varying degrees, is potentially different for each student.

A second class of Resources in Schoenfeld's work are the algorithmic procedures known by the individual. Algorithmic procedures may include all standard geometric constructions, algebraic rules and arithmetic procedures. Can the student construct parallel lines? Can he perform the four steps repeated in a division question? Knowledge of these algorithms is essential for performance beyond the elementary level in mathematics. But again what distinguishes problem solvers is the ability to know when and where to use these algorithms in a particular problem.

Routine procedures compose the third class of Resources. By selecting the appropriate variables from a problem the student is able to derive a formula to obtain the desired quantity. Implicit in this class is the belief that there is more discretion in selecting variables and deriving a formula than there is in choosing an algorithmic procedure to solve the problem.

Finally Schoenfeld discusses relevant competencies, the ability to produce mathematical arguments to verify procedures as they are used. By examining and evaluating potential paths for a solution the student draws upon more than simply the algorithms and procedures which he is using in the problem itself. Can the student bisect an angle in order to locate a point midway between both the line segments which make up the angle? While this may not be one of the main components in the solution of the larger problem, it may be necessary to ultimately derive the answer.

These four classes of Resources are inter-related. The greater the Resources and the competency with them, the better the student will be in problem solving. The procedural knowledge associated with these Resources will help on measures of crystallized intelligence (Sternberg, 1985). but the associated conceptual knowledge is also a factor in problem solving situations (Hiebert and Lefevre, 1986).

Heuristics

The second area of the knowledge base involved in problem solving which Schoenfeld (1985) examines is Heuristics. They are rules of thumb for successful problem solving. For an experienced mathematician, an heuristic can provide a strategy or technique for making progress on an unfamiliar or non-standard problem. These strategies may include drawing a picture, treating the problem as solved, exploiting related problems and many others. When Polya (1954) reintroduced heuristics to mathematics, he prefaced his work by advocating that the student acquire as much independent problem solving experience as possible. While teachers need to assist students, it is important that the student do a reasonable share of the work.

The new enthusiasm for heuristics led to teachers taking the strategies which mathematicians had found useful and teaching them to novices as aides for problem solving. Regrettably without the knowledge base built through experience which mathematicians possess, these tools were of little value to the novice. They knew neither when, where, nor why to apply these strategies with any consistency. Polya (1954) advocates modelling by the teacher as the most effective way of teaching heuristics. If the student witnesses the teacher successfully completing a problem after asking certain questions, he will note the success and associate it with the questions asked by the teacher, and be induced to use these questions himself in similar situations (Polya, 1954). This optimism is not shared by Schoenfeld (1985) who cites several studies which show that there is little transfer of the knowledge gained in the use of strategies to related and non-related problems.

Schoenfeld suggests that too much emphasis has been placed upon heuristics in recent years. Heuristic strategies are used by good problem solvers when they experience difficulties in a problem solution. Their ability to successfully employ heuristics stems from the fact that they can extract the deep structure from a problem more efficiently than a novice, and subsequently know which heuristic applies to that particular problem. Poor problem solvers have not developed the ability to extract the deep structure of a problem, or learned the questions necessary to engage in the inner dialogue which Polya (1954) recommends. This inner dialogue suggests avenues as yet unexplored or new perspectives on the problem, but are based on experience.

Heuristics are popular in mathematics education but Schoenfeld (1985) condemns them as too broad, and containing few clues for their unambiguous interpretation. The explanations which appear in the short dictionary of heuristics which Polya (1954) presents in his work confirm Schoenfeld's assessment.

Contextual Variables

Problem solving is not simply a matter of retrieving and manipulating data as the Information Processing framework might suggest. A computer will take the question input, access the necessary knowledge and operations from memory, perform the calculation and provide the answer. The human problem solver must deal with other variables which do not always relate to the problem being solved.

The problem solver is constantly trying to create a fit between the problem context, and the context of previously solved problems (Rabinowitz, 1990). Problem solving is a continuous process of adapting and shaping the problem context (Romberg, 1992; Sternberg, 1985).

There are several contextual variables which may have an impact on the solution: the context of the problem, the student's belief system, feelings and culture may be influential. Each of these will be dealt with in turn.

The Role of Problem Context

All problem solving is based on knowledge (Greeno, 1980). Two ways in which students are asked to demonstrate their understanding of the mathematics which they are studying is with arithmetic problems, and word or story problems. They may solve arithmetic problems of the variety 1+2=__, or they may solve what are called word or story problems, a question in prose from which the arithmetic problem must be developed by the student. The difference in the format of the presentation, numbers or words, is the problem context.

Newell and Simon (1972) found that the method of presenting problems, i.e. words versus numbers, affects the strategy employed and the outcome. Different types of processing occur for story and arithmetic problems (Bilsky et al, 1986). Arithmetic problems can be solved simply by manipulating the numerals in the appropriate fashion. The rules or procedures will have been covered in class and practised by the student. A problem occurs when there is no obvious way to go from the given state outlined in the question, to the goal state (Mayer, 1985). The addition example given above would be considered a problem only if the student did not know how to add to obtain the answer. In the literature, resolving this dilemma is studied from the perspective of learning theory rather than problem solving. There are, of course, examples of questions of a more complex nature presented in this or more formal algebraic notation (Hall et al, 1989; Schoenfeld, 1985). These may provide the opportunity to study problem solving if it is not readily apparent to the student how to derive the answer.

In mathematics, story problems form a fundamental sort of "ill-structured problem" (Newell, 1969; Simon, 1973). Ill-structured problems have several defining characteristics: the initial or goal state may not be clearly stated (Reitman, 1965), the solution paths may not be clearly delimited (Frederiksen, 1984), and there may be no single correct solution (Simon, 1973). Ill-structured problems are the primary context for applying mathematical knowledge (Briars and Larkin, 1984). The student must read the problem, and attempt to derive some sort of understanding of what is being asked (Mayer, 1982, 1985). Once there is an understanding this must be translated into an internal representation (Briars and Larkin, 1984; Mayer, 1985; Schoenfeld, 1982). An accurate internal representation is dependent on a correct reading of the text and the ability to understand what is meant by the question. These are linguistic skills quite independent of mathematics ability. Nesher (1986) found that the complexity of the text, not the mathematical operations, influences the processing of the word-problem text.

The text gives the information regarding the initial and goal states, but the state transition operators are not always defined (Briars and Larkin, 1984; Hall et al, 1989). One reason students have difficulty representing the problem is that they rely too heavily on the key word method (Carey, 1991; Schoenfeld, 1982). The word "left" is a key word used for the subtraction operation and students will react to the word "left" in a question and subtract even when it is used in the context "Mr. Left ..." (Schoenfeld, 1982).

In order to construct an accurate representation of the word problem, the student must deal with linguistic factors, as well as select factual knowledge from the question, and attempt to recognize problem types in the question and in memory (Mayer, 1982). Problem solving proceeds from this base as an elaborative, interdependent exploration of two problem spaces: the situational context of the story problem, and the quantitative constraints, (the arithmetic relationship), given explicitly or implicitly (Hall et al, 1989). Conceptual errors are more prevalent than computational errors because students tend to leave the mathematical formalism and reason within the situational context (Hall et al, 1989).

The Role of Beliefs

The role which the student's belief system plays in problem solving is receiving more attention (Kaput, 1989; Mandler, 1989; Marshall, 1989; McLeod, 1985, 1989; Schoenfeld, 1985; Simon, 1976; Sowder, 1989). Because real world learning shapes learning in a formal context, and vice versa, Schoenfeld (1985) suggests that this issue be addressed within the classroom at an early age.

One's mathematical world view shapes the way in which one does mathematics. Schoenfeld (1985) has developed a framework for examining problem solving at a macroscopic level. One of the four categories of knowledge and behaviour which must be dealt with is Belief Systems. Individuals develop models of the real world by abstracting from their experiences (Schoenfeld, 1985). This world view affects how they perceive mathematical problems.

Most of a student's experience with mathematics occurs in the classroom, as a result, when problems of a mathematical nature are encountered outside of the classroom, they do not always see any association with mathematics. The meaninglessness which is associated with school mathematics reduces the motivation of students when they encounter new problems in school (Kaput, 1989). Formal knowledge and procedures are supplanted by a "makes sense epistemology" (Schoenfeld, 1985). The problem is not put into its proper mathematical context and the student uses real world contexts to try to solve them. Schoenfeld labels people who ignore their mathematical background as naive empiricists. He cites several examples where students have demonstrated their proficiency with a mathematical procedure in a mathematics class and subsequently ignored this knowledge in solving a real world problem.

Mathematical experiences in the classroom, both positive and negative, are critical to the development of a mathematical belief system (Borasi, 1994). If mathematical argumentation is used only to verify established knowledge, then its utility in the processes of discovery is unknown to the students. There may be a strong relationship between proof problems and discovery problems if the student has been taught to look for them.

It is critical to develop a mathematical belief system in students at an early age. Students who enjoy mathematics are confident of their ability (Kloosterman and Cougan, 1994). The student's belief system must allow them to see the inter-relationships as well as the intra-relationships in the various branches of mathematics. Polya (1954) advises that having the students check their own work consolidates their belief that they can problem solve. Of equal importance is the ability to see mathematical applications in real world problems (Schoenfeld, 1985).

The Role of Affect

The belief that a student has about his proficiency in problem solving is important (Nickerson, 1990), but of equal importance is how the student feels about the problem. Positive or negative feelings about a problem will influence whether the student will continue with the solution or give up (Silver and Metzger, 1989). The student's evaluation of his answer will influence his belief in his problem solving proficiency, and determine how he feels about the next problem which he must solve. Silver and Metzger advocate getting the student more actively involved in the process of refining his work to develop positive feelings toward problem solving.

Problem solving involves cycles of tension and relaxation (McLeod, 1989). Tension occurs when an impasse is reached, and relaxation occurs when the blockage is overcome. Tension can be a positive element in problem solving when it leads to an increase in committment to the solution of the problem. Good problem solvers use stress constructively (Mandler, 1989; McLeod, 1989), while poor problem solvers do not realize that frustration is an integral part of problem solving and quit prematurely.

Marshall (1989) found a link between schema acquisition and emotion. A successful solution led to a positive emotion which carried over into the next problem solving session. The strength of the feelings towards problem solving varies with each successive problem solving session and the positive or negative emotion associated with it (Mandler, 1989; Marshall, 1989). Negative affect had an impact at the executive control level creating stress which may lead to lower motivational levels, decreased attention to what is considered important, and developing superficial learning strategies (Kaput, 1989; Mandler, 1989; McLeod, 1989; Shaw, 1990). Creating a positive affect is critical to successful problem solving.

Control

In a problem solving situation decisions must be made about information to heed or ignore, paths to take or abandon, procedures to invoke, and knowledge to access. Success in problem solving is quite often determined by the quality of the control decisions made. The importance which researchers place on control strategies is evidenced by the number of attempts to teach such strategies (Polya, 1954; Schoenfeld, 1985). A brief discussion of control, metacognition and how they relate to problem solving in mathematics follows.

Control and Metacognition

The term metacognition as used in the psychological literature refers to control processes similar to those discussed by Schoenfeld (1985). Our understanding of metacognition has evolved from simply the knowledge of ones own cognitive processes (Flavell, 1976), to the current twofold definition which includes both knowledge, and self monitoring of cognitive resources (Brown, 1987; Paris and Winograd, 1990). Self monitoring may include executive decision making which focuses on deliberate processes such as classifying and planning, and the more automatized processes of executive regulation which regulates attention, and the intensity and speed of processing (Kluwe, 1987; Yussen, 1985). The change in focus from the traditional information processing approach which emphasizes the control of processing through executive decisions, to the current conceptualization which emphasizes the knowledge which enables the control of processing, is more amenable to an educator's perspective.

Control represents the active decision making in problem solving. This may include making plans, selecting goals and subgoals, monitoring and assessing solutions as they evolve, and revising and/or abandoning plans when such action should be taken (Schoenfeld, 1985; Sternberg, 1985). The proficiency with which Control is exercised in problem solving frequently determines the success of the problem solving session.

Within the context of problem solving in mathematics attempts have been made to teach control strategies. Polya (1954) outlines twelve categories of problem solving strategies, Schoenfeld (1985) suggests that there are at least as many more. Polya suggests four phases of problem solving. The first is understand the problem, the second make a plan, third carry out the plan, fourth look back. The questions which he recommends for each phase provide the control decisions for the student. Within this decision making framework the student can attack most problems. Frequently students apply this and other control heuristics in a rote fashion with no regard for how appropriate it may be, or without monitoring its effectiveness (Schoenfeld, 1985).

Current research suggests that there is more involved in problem solving than the Polya framework implies. The ability to make a plan depends upon the conditional knowledge of when and where a particular heuristic is appropriate (O'Sullivan and Pressley, 1984), and beliefs about the self as a problem solver (Borkowsky et al, 1990). Knowledge of heuristics is not sufficient to insure their appropriate implementation (Paris et al, 1983).

Control in Mathematical Problem Solving

Control in a problem solving session can take many forms. Schoenfeld (1985) discusses the ways in which problem solvers exercise control. Control decisions which are made before the student actually begins his solution are often critical to his success. The initial control decisions are at a global level. Into which category of problem does this question fall? Which type of heuristic might be used? An incorrect decision at this point potentially could result in the student spending the rest of his time in a fruitless attempt to solve the problem.

Next, the student must select a specific strategy to pursue. The student must also allocate his various resources including time. Problem solutions frequently have multiple steps. A global plan allocating time to the various stages of a problem solving strategy can help to ensure that a dead end is discovered before the solution is irrevocably lost. Pre-planning the correct sequence of a solution is also important. Most of the above Control decisions can and should occur at the outset of the problem solving session. Many other Control decisions occur during the solution attempt.

Each stage of the solution must be monitored to evaluate its progress and appropriateness. If it is not yielding any progress, or if new information becomes available which suggests a change in direction, a Control decision must be made. A good knowledge base will enable the student to recognize these points and make the appropriate decisions. Many poor problem solvers will pursue a plan of attack long after it has ceased to yield any useful information.

In a problem solving session a student must not only be efficient in terms of resource allocation, but resourceful as well. The manner in which he navigates through the solution, abandoning useless avenues, and selecting promising ones will determine his success. Successful problem solvers are perceptive in selecting and pursuing approaches, quick in recovering from inappropriate choices, and alert in monitoring the entire problem-solving session.

Instructional Interventions

Bruner (1960) has long been an advocate of discovery learning. The act of discovery is a matter of re-arranging evidence in such a way that one is able to go beyond the evidence to new insights. The discovery can be self initiated, or the result of studying a previously solved problem. With worked examples the responsibility for learning is shifted from the teacher to the student who now actively reads and problem solves (Zhu and Simon, 1987), sees strategies being applied, and consolidates the belief in his ability to solve problems (Polya, 1954; Simon, 1980). In attempting to apply strategies the student will develop procedures which help to generate novel solution strategies (McDaniel and Schlager, 1990). Studying worked examples and solving questions lead to acquisition, and life long retention, of algorithms (Bruner, 1960; Simon, 1980).

Solving problems has other advantages. Generating and solving problems leads to invented strategies which are used more effectively (Christensen and Cooper, 1991). As few as one example may be all that is necessary for prototype extraction (Dijkstra, 1988) provided all defining and highly correlated variables are included. Worked examples as an instructional device led to shorter time for acquisition of concepts (Sweller and Cooper, 1985), and facilitated automation of problem solving operators particularly with low ability students (Cooper and Sweller, 1987). Collaborative problem solving was also found to be highly effective in developing strategies and belief systems (Christensen and Cooper, 1991; Collins, 1992; Greeno, 1989).

Errors made by a student can be used by the teacher for diagnostic purposes, but they can also be used by the student as a springboard for inquiry (Borasi, 1994). This inquiry can help to consolidate learned content, reinforce correct solution paths, and enhance confidence in their problem solving ability.

In a formal instructional setting it is important to avoid ambiguities in problem solving and ensure that instruction is well structured (Derry, 1990). Instruction of specific strategies narrows the range of strategies selected during problem solving (Lembke and Reys, 1994). But once the student has extracted the prototype, irrelevant problem attributes which correlate highly need to be introduced to help consolidate the prototype (Dijkstra, 1988). Critically examining questions in the textbook also helps to define the problem prototype, and develop self-instruction strategies which can be applied when the problem solving becomes more generalized (Collins, 1992; Dijkstra, 1988; Simon, 1980). Teaching the strategies to students proved to be less effective than allowing the students to discover them, and often led to inappropriate application (Christensen and Cooper, 1991). But allowing the students to see a number of different strategies applied to a single problem caused them to be more likely to take multiple perspectives to new problems (Bransford et al, 1990).

For some children, especially those with reading difficulties, converting the external language of the problem into an internal representation is difficult (Rabinowitz, 1990). Pictures can substantially improve a student's ability to make this conversion from the external to the internal (Hembree, 1992; Levin, 1983) by aiding in the recall of relevant information (Shaw, 1990). Organizational pictures which accompany a mathematics problem are effective in deriving a problem representation and solution.

The role of the teacher is critical. If the goals of schools are directed at student learning, then the role of the teacher should complement that of the student. Teachers must facilitate the creation of knowledge and thinking by students (Romberg, 1992). It is important to have consistency between the way the material is presented and the student's preferred mode of learning (Verschaffel, 1994).

The teacher must use formal instruction to complement discovery learning. Teaching number sense, i.e. mental computation, estimation etc, leads to more effective strategy use and knowledge organization (Markovits and Sowder, 1994). The instructional component is critical. Certain aspects of number sense, e.g. numerosity, can cause students to ignore other cues (Pelham et al, 1994). Causal discourse linking superordinate principles to the underlying concepts is an effective means of helping the student to organize material (Woodward, 1994).

Regardless of the intervention employed, a sound knowledge base is critical to combine with newly introduced material (Anzai and Simon 1979; Derry, 1990). Learning results from the selective search through memory for knowledge to combine with new information to form schema which are then retained in memory (Anzai and Simon, 1979, Dijkstra, 1988; Romberg, 1992). Finally the cognitive strategies to access and manipulate this material must be developed (Dijkstra, 1988).

Critique of Existing Literature

Research in mathematics education is rich with examples of factors affecting performance. Student characteristics, personality type, learning style, and level of intelligence have been examined as possible influences. School curriculum, teacher style, classroom and social environment have also been studied. What emerges is that mathematics education is a diverse field, and there are many influences on performance.

One factor frequently mentioned in research on problem solving is age. There are limits upon what problems a student can be expected to solve. A first grader cannot be expected to derive a proof in Euclidean geometry. Likewise a university mathematics major might not be challenged by a problem involving a simple addition question. The problem must be appropriate to the level of development of the student. Frequently in the research age is an independent variable, less often is it a dependent variable in the study of problem solving.

One notable exception to this is the work by Vygotsky (1978). His concept of the "zone of proximal development" has given new insights into the developmental aspects of problem solving. Vygotsky suggests that a student may know all of a given concept but cannot demonstrate mastery of the next concept in the sequence. But lying between these two concepts is a zone in which the student may know some of the prerequisite knowledge for the next concept. He is able to show mastery of the second concept when he has someone who can fill in the holes in his knowledge base. Vygotsky's work suggests that chronological age and independent performance may not accurately reflect the student's ability unless the zone of proximal development is examined.

It may be more appropriate to examine mathematics skills and problem solving ability in terms of their relationship to the student's overall mathematical development, than to his chronological age. Russell and Ginsberg (1984) found that students who were experiencing difficulties in math exhibited patterns of thinking characteristic of younger children. Their knowledge base and execution of strategies were at an earlier developmental level. Higher order algorithms in mathematics are made up of simpler, lower order algorithms. Case (1978) and Fuson (1982) suggest that an algorithm must become automatic before it can be incorporated into a more complex one. Students who take longer to automatize algorithms will lag behind their peers.

The ability of the student impacts on problem solving in a number of ways. Higher performing students access more information and use it more effectively especially with difficult problems (Lawson and Chinnappan, 1994). More able students think proceptually, that is they can more easily resolve the process/concept ambiguity. High achieving students are also more willing to give and receive help in group situations (Nattiv, 1994).

There is a long noted difference between male and female students in performance in mathematics, and representation in university mathematics courses. A series of studies reported in Johnson (1984), which were done in the 1950's, found that male students outperformed female students in mathematics (Casey, 1958; Milton, 1958; Nakamura, 1955; Sweeney, 1953). Johnson (1984) attempted to replicate some of these studies and obtained similar results. Johnson suggests that the difference was related to math aptitude, with SAT mathematics scores being the best predictor of performance for both male and female students.

Studies conducted by Hanna (1986) and Wolleat et al

(1980) offer some insights into this disparity in performance between boys and girls. Hanna (1986) found that boys did better in only two of five math subtests administered, Geometry and Measurement. There was no significant difference in scores on the Arithmetic, Algebra and Probability, and Statistics subtests. She further found that girls omitted more items in all areas than did boys. Wolleat et al (1980) found that boys attribute their success in math to ability and their failure to lack of effort. Conversely the girls attribute their success to effort and their failure to lack of ability. These findings suggest an interesting question. Do girls respond to initial difficulty by quitting because they feel that they do not possess the ability?

The problem context and format seem to have differing effects upon the outcome. But rewording the problem into a female context, or changing the gender of the actors in the problem, did little to alter the sex based difference in performance on math problems (Johnson, 1984).

One of the areas of interest in problem solving that has come from the work on Information Processing Theory is the expert/novice paradigm. It was believed that to improve a student's problem solving ability one need only teach them to problem solve as the experts do. Attempts were made to identify the characteristics of the expert problem solver and then incorporate these into the curriculum for novices. Eventually this approach fell from favour because it was found that experts and novices perceive things differently (Novick, 1988).

In studies comparing the problem solving of university professors and advanced PhD students (experts), with undergraduate students (novices) (Bowden, 1985; Chi et al, 1981; Novick, 1988), the main difference between the expert and novice problem solvers is in the way that they represent the problem. Novices tend to look at the surface features of the problem, with an over reliance on key words (Chi et al, 1981; Novick, 1988). Experts use surface as well as structural features. Novices also tend to do more poorly when there were time constraints, when there was no time limit there was no difference in the number of correct answers for experts and novices. The only difference was in the solution latency, novices taking longer (Bowden, 1985).

Skill in problem solving is dependent on schema acquisition (Owen and Sweller, 1985). Because analogy plays an important role in the expert's problem solving (Novick, 1988), it is important that the schemas acquired be complete. When the source and target problems share surface and structural similarities, positive transfer occurs regardless of expertise, but when there are only structural similarities only experts can make the positive transfer (Novick, 1988).

In problem solving the initial categorization is based on surface features, but the problem representation is completed with schemas from the knowledge base (Chi et al, 1981). Since the schemas which a novice possesses are based on surface features they are unable to complete the problem representation correctly (Chi et al, 1981).

Frameworks of Problem Solving

Learning has been viewed as a problem solving experience, whether it is within the context of interacting with the environment (Ginsburg and Opper, 1979; Vygotsky, 1978), or in the classroom with problems presented by the teacher. Several frameworks have been developed by cognitive scientists to account for the processes involved in problem solving. The four most prominent are ACT* (Anderson, 1983), SOAR (Laird, Newell and Rosenbloom, 1987), PDP (Rumelhart, McClelland and the PDP Research Group, 1986), and Induction (Holland, Holyoak, Nisbett and Thagard, 1986). Table 1 provides a comparative overview of each of these frameworks.

ACT*, SOAR and Induction each use productions as their main processing unit. A production is a condition-action pair, for example IF condition A exists, THEN do action B. The PDP framework uses rules only as its processing unit. Early computer simulations could only accommodate the firing of a single rule or production at one time. This was a critical limiting factor in simulating human problem solving since a large amount of information could be processed by humans apparently simultaneously. But all four models allow

Table 1

An overview of ACT*, SOAR, PDP and Induction.

	ACT*	SOAR	PDP	INDUCTION
PROCESSING	Productions Spread of Activation Parallel	Productions All rules Firing Parallel	Processing Units Levels of Activation Parallel	Rules Spread of Activation Limited Parallelism
PROBLEM SOLVING				
1. Novice	General Procedures	Universal Subgoaling		Random Search of Mental Models
2. Inter-mediate		General Methods	Common Elements	Systematic Search of Mental Models
3. Expert	Better Path Choices	Expert Choices	Associate Patterns	Induction
LEARNING	Knowledge Compilation Fluid	Chunking Fluid	Revise Strength Parameters Fluid	Rule Clusters Revise Strength Fluid
TYPES OF KNOWLEDGE	Procedural Conceptual	Procedural Conceptual	Procedural Conceptual	Procedural Conceptual
TYPICAL APPLICATION	Science Language Math	All Areas	Science Language	All Areas

parallel processing, at different levels. ACT*, PDP and Induction explain the search through a problem space by reference to different levels of spreading activation. Spreading activation is a process wherein one rule or production will activate another one closely related to it until a rule or production relevant to the problem is found. SOAR, in contrast, allows all rules to fire simultaneously, the ones most closely related to the problem will have a greater strength and carry more weight in the bidding process and will be most likely to be executed. Induction has rules firing and competing with each other based upon their current strength which is determined by their past successes. In ACT*, Induction and PDP learning does not involve static knowledge units, it is fluid, the result of revising strengths of rules (productions) and rule clusters. In SOAR knowledge accumulates through a process called chunking (the grouping of clusters of related productions).

<u>Induction Framework</u>

Each of these models, ACT*, SOAR, PDP and Induction assume a problem solving approach to learning, but SOAR and Induction more closely approximate a unified theory of cognition (Holland et al, 1986; Newell, 1991). The theoretical framework embodied in Induction is ideally suited for use in the area of mathematics. Holland et al (1986) define Induction as the modification of knowledge through its use, emphasizing the role of experience in processing. Learning is a result of experience in any domain. Learning in mathematics at a micro level may include the development of new rule clusters, at a macro level new mental models may be formed which more efficiently handle a problem situation. Induction is further defined as a problem-directed activity based on feedback regarding the success or failure of predictions generated by the system. This approach closely parallels activity in the mathematics classroom. The cumulative effects of learning can be accounted for in the Induction framework with the concept of rule strength. Students are continually involved in problem solving, and learning can be measured by their ability to respond to the experiences from previous problem solving situations. As rules are strengthened, they are more likely to be used in future problems which are similar to them, thus reducing solution latency.

Analogy plays an important role in mathematics problem solving. In the Induction framework the development of conceptual knowledge can be observed through the mental models created during, or as a result of, problem solving. Analysis of the productions embodied in these mental models will reveal the student's procedural knowledge. Knowledge is stored hierarchically and problem solving involves a search for appropriate rules at a micro level, or mental models at a macro level, to aid in the problem solution. The hierarchical structure facilitates spreading activation, the mechanism by which rules are found which most closely match the information in virtual memory. Students attempt to find mental models which may contain analogous problem solutions or resort to individual rules.

Everyday living, as well as mathematics, presents many examples of problem situations which cannot be solved by some previously learned behaviour. The Induction framework is well suited to explain these problems. When faced with a novel problem situation a student will search through his memory for similar situations for which he has a solution. If he is unable to find something closely related he will search for an experience which may share some commonalities with the new situation. When a solution is derived, the rules which contributed to the solution will gain strength. If the problem was truly unique the solution may be saved in its entirety for future reference.

The Holland et al model provides an ideal framework to examine the research problem in this study. Because the current research study is exploratory in nature, the data will be examined from both a quantitative and qualitative perspective where appropriate. The nature of the data, and the proposed method of analysis, require a fluid model which has the potential for a broad application. The Induction model meets these requirements.

Research Problem

The Induction model of Holland et al (1986) has been chosen as the theoretical framework for this study for a number of reasons. The role of experience in learning and problem solving can be explained by the increase in rule strength. Elaborate proofs can be retained as mental models which students acquire and bring to new problem situations. In novel situations or with ill structured problems the concept of spreading activation can be invoked to explain the discovery of new solution paths.

Within the context of the Holland et al (1986) framework of Induction, and the foregoing review of the literature, the research problem for the present study can be stated as:

> -to determine the effectiveness of a videotape of a grade 7 student problem solving in assisting students to develop the rules, mental models and strategies necessary for the solution of isomorphic mathematics problems.

The global conceptualization of human problem solving presented in Figure 1 emphasizes that there are a number of factors which impact on the problem solving of students. Factors other than the actual mathematics can play a prominent role, but students may not learn of their importance in mathematics class (Nickerson, 1990; Silver and Metzger, 1989). The modelling of the teacher may not be sufficient to convince the student to incorporate the strategies into his problem solving repertoire.

The research on discovery learning (Bruner, 1960), and collaborative learning (Collins, 1992), shows that students can learn on their own and from each other. This work suggests that a videotape of a same grade level peer problem solving might be an effective means of providing worked examples for a student to study. Students studying worked examples have been able to develop the required skills for problem solving (Schoenfeld, 1985; Vallasenor and Kepner, 1993). Seeing a peer use a strategy might have a greater impact on a student and encourage him to use the strategy in his own problem solving.

From the research problem stated above, a global hypothesis and six subhypotheses were developed. Each hypothesis will be presented with a brief discussion following.

The global hypothesis was stated as follows:

> A videotaped problem solving protocol will increase the efficiency of a grade 7 student when problem solving.

It was hypothesized that the videotaped problem solving protocol would act as an instructional intervention for the students and increase their efficiency. Problem solving efficiency is measured on two dimensions: the percentage of solution time spent on the different episodes, and the correctness of the answer. Watching a student at their grade level solve a problem may help them to go beyond the surface features of a similar problem. This will increase the probability of them correctly solving the similar problem, and reduce the solution latency. The way in which the time will be reduced is the subject of the six sub-hypotheses, the discussion of which follows.

Each of the sub-hypotheses was constructed to explain the time changes in the global hypothesis as it related to each episode in the protocol. The first sub-hypothesis pertained to the Analysis Episode.

> A videotaped problem solving protocol will reduce the amount of time spent on Analysis.

Seeing the videotaped problem solution will help the student to see the underlying structure of the problem. They may be able to form a mental model which they will be able to draw on for their own solution. Having this mental model will reduce the search time in the Analysis Episode because of its recency in long term memory, and the isomorphic nature of the problem will facilitate transfer. A more complete global plan will result.

> A videotaped problem solving protocol will increase the amount of time spent on Problem Representation.

More time will be spent on Problem Representation if the student is able to transfer the problem solving strategies of the student in the video. In the video, time was spent making a pictorial representation of the project in the problem. The diagram was labelled and used to formulate the global plan.

If the student saw the value of the diagram for both planning and monitoring the solution, he will subsequently employ it in his solution, resulting in more time being spent on this episode.

> A videotaped problem solving protocol will reduce the amount of time spent on Solution Directed Activity.

As a result of the more complete global plan, the time spent on the Solution Directed Activity Episode will be reduced. The global plan will be more complete and there will be less uncertainty associated with the execution of the plan. The student will be able to state briefly and with confidence what he is going to do and then do it.

> A videotaped problem solving protocol will increase the amount of time spent on Global Monitoring.

Having a more complete global plan, and having seen the student in the video modelling how to refer to the plan as a check on the progress of the solution, will result in an increase in time spent on Global Monitoring. The rationale given by the student in the video for his global monitoring should reinforce the value of this activity for the overall solution.

> A videotaped problem solving protocol will increase the amount of time spent on Local Monitoring.

Students check their work infrequently. They get the information, perform the arithmetic and move on to the next question. The video demonstrated how to check the numbers which you write, the labels used, and the arithmetic performed. The student in the video made an error, but caught his mistake through local monitoring. Time spent of this activity, Local Monitoring, will increase as a result.

> A videotaped problem solving protocol will increase the amount of time spent on Verification.

Another area which is frequently ignored is the verification of work once it has been completed. When an answer is obtained the student considers the problem finished. The Verification Episode is an integral part of most expert's problem solutions and was included in the video taped problem solution. Seeing the student in the video include a verification of his solution will increase the time spent by the student on his next solution.

CHAPTER 3

METHODOLOGY

To be successful a study must be founded on a sound theoretical framework and a strong methodology. The theoretical framework for this study was discussed in the previous chapter. In addition a rigorous approach to the methodology is imperative when the data source involves verbal reports.

The data analysis is based in part on verbal reports from students problem solving. Because of the controversy which has surrounded verbal reports since their early use, it is necessary to give a brief account of how verbal reports have evolved.

The Development of Verbal Reports

A verbal report is an account given by the subject in his own words. There are a number of different types of verbal reports which have been discussed in the literature. Introspection, the introspective observation and analysis of thinking (Ericsson and Crutcher, 1991), was popular at the turn of the century but these reports required skilled observers. The Gestalt psychologists used verbal reports to study problem solving (Duncker, 1945; Wertheimer, 1945). In studying the development of children's thinking Piaget and his followers used verbal reports as well (Inhelder and Piaget, 1958).

A controversy arose over the nature of thought and what could be obtained through introspection. Questions of whether problem solutions were derived from individual parts or a complete whole, and whether the reasoner is conscious of the factors which aid in bringing about the solution, were difficult to resolve (Rowe, 1985). The danger of over rationalizing is great when the subject must report on his inferences about the psychological and physiological processes.

Retrospective accounts, an introspective report of what had occurred after the problem solving was completed, shared many of the same problems. The subject must partially or completely finish a task before he gives an account of what he can remember having occurred. The subjective nature of the account, the possibility of forgetting, the dangers of rationalization and the threat of mixing current and past knowledge rendered retrospection no more attractive than introspection as a source of valid and reliable data.

The current use of verbal reports is based on an information processing framework (Newell and Simon, 1972). The most recent debate over verbal reports revolves around the criticisms by Nisbett and Wilson (1977), and the response from Ericsson and Simon (1980). Nisbett and Wilson question whether there can in fact be any introspective access to higher order thought processes. They argue that only the product of thinking is accessible. Their failure to adequately specify the type of verbal report which they were examining, or the information to be obtained, led to the rebuttal by Ericsson and Simon.

Ericsson and Simon (1980; 1993) argue from an information processing framework that only information currently, or previously heeded in short term memory, or fixed and retrievable in long term memory, can be reliably reported. The failure to report in the Nisbett and Wilson study is more likely attributable to the failure to inform the subjects that they would be required to report processes when they were finished the task.

Self reports are forms of introspection, and in the narrower sense used by Ericsson and Simon (1984) do not rely on specific recall and are more susceptible to distortion from inference than other forms of reports. Ericsson and Simon base their arguments for reliability and validity on two types of verbal reports: concurrent verbal protocols, and retrospective reports. Nisbett and Wilson did not distinguish these two from introspection. Their work was based primarily on retrospective report data.

A concurrent verbal protocol is the subject's oral account of his activity while in the act of problem solving (Ericsson and Simon, 1993). These verbalizations are descriptions of the activity in short term memory. Because there is limited space in short term memory, what is verbalized will reflect only what the problem solver deems to be of great enough importance to be brought to bear on the problem at that time (Ericsson and Simon, 1993; Garner, 1988). Direct reporting of what is in short term memory does not create excessive demands on resources nor does it lead the problem solver away from his solution (Ericsson and Simon, 1993; Russo et al, 1989). Some information will of course be lost, and the final protocol will not reflect the complete thinking process, but it will provide an accurate enough account to permit examination of what occurred (Ericsson and Simon, 1993).

In spite of the criticism of concurrent verbal protocols, (Nisbett and Wilson, 1977), there is growing theoretical and empirical support for this type of data when used appropriately (Ericsson and Simon, 1993; Garner, 1988).

Retrospective Reports

Retrospective data is obtained in a post problem solving interview. It is the subject's account of what he thought and did during the problem solving session. The researcher asks questions about the problem solution which affords the student the opportunity to elaborate on strategies used (Kuiper and Kassirer, 1984). Because of the time delay between the actual events and the retrospective interview, the information obtained is derived from the subject's long term memory. Not all of what was done will be stored in long term memory, particularly events which may not have contributed in a positive way to the solution (Ericsson and Simon, 1993; Garner, 1988). Consequently the researcher must be sensitive to this constraint and probe the subject's memory with a view to identifying what was done during the problem solving session. When analyzing the data the information obtained in the retrospective interview must be cross referenced with the concurrent verbal protocol.

When taken in isolation both concurrent verbal protocols and retrospective data have shortcomings. But based on the work of Ericsson and Simon (1993), concurrent verbal protocols and retrospective reports, when used together, can be a source of reliable data (Ericsson and Crutcher, 1991).

With the three sources of information: concurrent verbal protocols, retrospective data, and the solution itself, the researcher has the opportunity to view the problem solving session from a number of perspectives. Each one serves as a check on the researcher's observations, and provides the opportunity to go beyond the solution itself to examine the underlying cognitive processes.

Because the processes involved in problem analysis and representation are complex, not readily observable, nor often obvious to the problem solver himself, the written solution to the problem is not a source of sufficient information. What is needed is a methodology which can access the processes which the problem solver uses to analyze and represent the problem.

A twofold approach has proven to be effective in accessing strategies and related knowledge (Haastrup, 1987; Lundeberg, 1987). Concurrent verbal protocols followed by a retrospective debriefing has proven to be a good source of this type of information (Kuipers and Kassirer, 1984). The information obtained from the retrospective debriefing enriches the concurrent verbal protocol and reduces the level of inference when analyzing the data (Grotjahn, 1987).

Population and Sample

The population chosen was grade seven students. In grade seven, students do not have a choice of whether or not to study mathematics, but they do for the first time have a teacher who has a certain amount of specialized training in the field of mathematics. Problem solving is an integral part of the mathematics curriculum. Even if problem solving was taught as a separate unit, by the final month of the academic year problem solving should have been covered in class.

The sample consisted of thirty-five grade seven students from five classes in an intermediate school in the Ottawa area. All students in the five classes were invited to participate, but only those for whom signed parental permission forms (Appendix A) were obtained were included in the study.

Thirty-six students initially volunteered for the study and returned permission forms, one subsequently changed his mind and withdrew from the study. The students were placed into one of six groups. The groups were formed so that they were as well matched as possible with respect to gender and ability. Ability was determined by asking the classroom teacher to evaluate each student on their ability to solve word problems. Each teacher was asked whether the student was above average, average or below average. In forming each group an attempt was made to have a representation of boys and girls from each ability group.

Once the six groups were formed they were randomly assigned to either the experimental group, or the control group.

The Experimental Tasks

Three word problems were developed and each subject was asked to solve one or two of the three problems. Each problem was a "construction" problem, that is, each problem involved building something.

Prior to the study a group of sixty grade seven students was asked to make a list of things which they had made either in school as project or at home with a parent or friend. Their teachers were also surveyed to find the type of projects which were assigned in Design and Technology and Family Living courses. A list was compiled and the students were asked to check five of the projects which they had already done or were interested in doing. From this survey three of the most frequently listed projects were selected and a problem for each context was developed.

Once each question was designed they were given to a panel of three grade seven mathematics teachers to rate as to whether they were representative of questions which might appear in their mathematics programme. Criticisms were made relating to the wording and language used, these were addressed and the questions were returned to the teachers for a second evaluation. The three teachers agreed that the questions and the mathematics involved were appropriate for students at the grade seven level.

A second criterion for the problems was that they be as isomorphic as possible. It was important that problem difficulty not be a factor in the study. In constructing the problems each one was laid out in a similar fashion. They began with a paragraph naming three students and making a general statement about what project they wanted to make. This was followed by a chart with information related to the projects. Each chart consisted of four rows and six columns of information related to the question. Next there was a paragraph which outlined in detail what each of the three students wanted. Finally there was a single sentence asking how much material one of the students would require for their project.

Two graduate students in education examined the three questions to assess whether they were indeed isomorphic. These judges agreed that the questions were similar in form and content. While the solution paths were similar, they were not identical and consequently the problems were considered to be quasimorphic.

The three problems are presented in Tables 2 to 4. Each question was then given to a group of four grade seven students to determine whether there was any readily apparent difficulties in solving the problems. Four of the twelve solutions were complete and correct, one for each of the Fort and Suit problems, and two correct for the Birdhouse problem. The questions as seen by these students were used in the study.

The Videotaped Instructional Intervention

The instructional intervention used in the present study was a video tape of a grade seven student solving one of the above construction problems. To obtain this video tape, a class of grade seven students was asked to take part in a morning workshop on problem solving in mathematics. Four students volunteered and returned a signed parental permission form (Appendix A). They were excused from class and taken to

Table 2

Statement for the Fort Problem.

Pat, Jan and Kelly have decided to build their own play fort from cardboard boxes left over from new appliances their parents purchased. Pat will build a fort with one tower. Jan will build a fort with a tower and two small rooms. Kelly will build a fort with a tunnel and one small room. The chart below shows how much cardboard is required for each size play fort.

	FRONT/ BACK	FRONT/ BACK	SIDES	SIDES	ROOF	FLOOR
	NO WINDOWS	WITH WINDOWS	NO WINDOWS	WITH WINDOWS		
SMALL ROOM	550 sq cm	500 sq cm	550 sq cm	500 sq cm	850 sq cm	300 sq cm
LARGE ROOM	750 sq cm	700 sq cm	750 sq cm	700 sq cm	950 sq cm	300 sq cm
TOWER	850 sq cm	800 sq cm	850 sq cm	800 sq cm	950 sq cm	300 sq cm
TUNNEL	950 sq cm	900 sq cm	950 sq cm	900 sq cm	950 sq cm	900 sq cm

Pat decided to build a fort with one tower with windows at the front but not at the back. Jan will build a fort with one tower with one window and two small rooms with one window in each room. Kelly wants a fort with a large room with windows on two sides and a tunnel with no windows.

1. How much cardboard will Pat need altogether?

____

Table 3

Statement for the Suit Problem.

Sue, Jim and Wendy have decided to sew their own suits. Sue is a size 8, Jim a size 10, and Wendy a size 6. They have each chosen the same pattern but will make different style suits. The chart below shows how much material is required for each size suit. Remember that both styles of jacket require a lining.

SIZE	JACKET	JACKET	PANTS	PANTS	LINING	VEST
	DOUBLE BREASTED	SINGLE BREASTED	NO CUFF	CUFFED		
6	200 sq cm	170 sq cm	100 sq cm	120 sq cm	120 sq cm	150 sq cm
8	225 sq cm	195 sq cm	125 sq cm	150 sq cm	135 sq cm	175 sq cm
10	250 sq cm	220 sq cm	150 sq cm	175 sq cm	150 sq cm	200 sq cm
12	275 sq cm	245 sq cm	175 sq cm	200 sq cm	165 sq cm	225 sq cm

Sue will sew a suit with a double breasted jacket, cuffed pants and a vest. Jim wants a suit with a single breasted jacket, cuffed pants and no vest. Wendy wants a single breasted jacket, pants with no cuffs and a vest.

1. How much material will Sue need altogether?

Table 4

Statement for the Birdhouse Problem.

Bob, Jack and Wally have decided to build their own birdhouses for the grade 7 woodwork project. Bob will build a six bird birdhouse, Jack a twelve bird birdhouse, and Wally a ten bird birdhouse. They have each chosen the same pattern but will make different style houses. The chart below shows how much wood is required for each size birdhouse.

	FRONT/ BACK	FRONT/ BACK	ROOF	ROOF	SIDE	FLOOR
SIZE	FANCY	PLAIN	COTTAGE	A FRAME		
6 BIRD	550 sq cm	450 sq cm	500 sq cm	700 sq cm	450 sq cm	300 sq cm
8 BIRD	700 sq cm	600 sq cm	500 sq cm	700 sq cm	450 sq cm	300 sq cm
10 BIRD	850 sq cm	750 sq cm	500 sq cm	700 sq cm	750 sq cm	300 sq cm
12 BIRD	950 sq cm	850 sq cm	500 sq cm	700 sq cm	900 sq cm	300 sq cm

Bob decided to build a six bird birdhouse with a fancy front, plain back and an A Frame roof. Jack will build a twelve bird birdhouse with a fancy front and back and a Cottage roof. Wally wants a ten bird birdhouse with a plain front and back and an A Frame roof.

1. How much wood will Bob need altogether?

____

a seminar room where the workshop took place.

The four students were asked to work together to solve a group of math problems taken from their mathematics text (Appendix B). They were given one problem at a time. After each question they were asked to explain what they had done and why they had done it. When they had solved four problems, a group discussion was held. The researcher probed more deeply behaviours which the students had exhibited which related to global planning, graphically representing the problem, monitoring, and verification of the solution. The students were then asked to brainstorm a list of criteria which they felt were important to remember when problem solving.

The students were placed in pairs and took turns solving two more questions, this time being videotaped at the Table of Observation. As the students worked the researcher reinforced behaviours which related to the list which the students had discussed.

The following morning the students returned to the seminar room individually and solved each of the three problems to be used in the study. They were videotaped as they worked. The four students produced eight correct solutions.

The researcher reviewed the eight correct solutions. Three were eliminated because of poor audio or video quality. The video which best exemplified good global planning, problem representation, monitoring, and verification was selected for the study. Permission was obtained from the parents of the student to use the video in the research study (Appendix C).

Data Collection and Preparation

Data collection was conducted on an individual basis. Each student was withdrawn from class for a single session which lasted approximately twenty to thirty minutes. The school had provided a seminar room for the exclusive use of the researcher for the duration of the data collection. Each session followed the format as outlined below.

Procedure

Each student was brought into the seminar room which had been allocated by the school administration for the research project. Before the data collection began, the student was seated at the Table of Observation (Figure 2), and introduced to the data collection procedure.

The Table of Observation is designed to allow the experimenter to record the subject's written work, facial expressions and gestures. This provides a much richer protocol. The top of the table is made of plexiglass and is tilted like a drafting table. A mirror is located under the table and a video camera is placed behind the table. The camera is focused on the subject's reflection in the mirror,

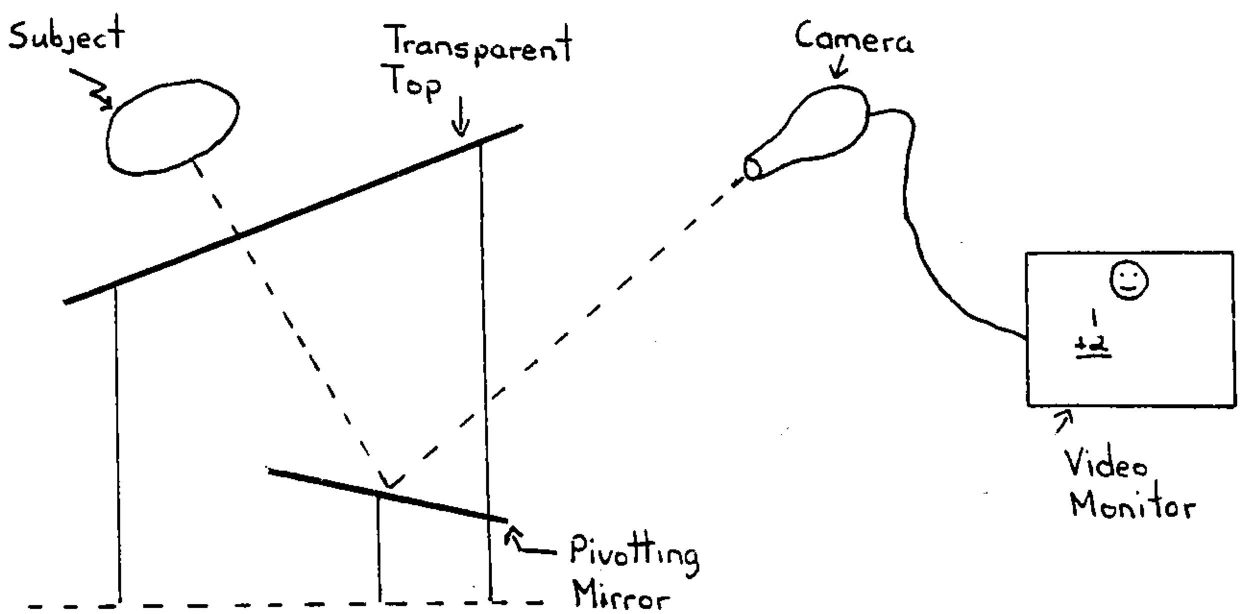

Figure 2. Table of Observation

the mirror reverses the image to allow the written work to appear on the screen as the subject would see it.

On the counter beside the table was a television, two video cassette recorders, and an audio cassette recorder. The concurrent verbal protocols and retrospective interviews were both recorded by the video camera. The audio cassette recorder was used as a backup. Each student was recorded on a mini cassette in the video camera. The camera was also connected to a video cassette recorder which provided a video backup. The second video cassette recorder was connected to a monitor and the students in the treatment group were shown the video of the student solving a problem on this television.

Once the procedure was explained, any questions which the student had were answered before beginning the data collection.

Instructions

Each student received a standard set of instructions prior to the beginning of the taping which related to the use of the Table of Observations and the process of providing concurrent verbal protocols (Appendix D). Next they received instructions related to the expectations for them in the study (Appendix E). These were based on which group they were in.

Warm Up Problems

Each student was given the opportunity to practise using the Table of Observation and accustoming themselves to thinking aloud as they problem solved. The warm up problems (Appendix F) allowed the students to perform arithmetic which was not difficult for them as a way of reducing any anxiety they might have while they became comfortable with working at the Table of Observation. When the student finished they were asked if they felt comfortable with the think aloud technique. If they said they weren't, they were given more time to practise. If they were comfortable the instructions for the problems were given and the first problem was given.

The Experimental Design

Each of the six groups in the present study had a different treatment, an overview of the sequence of tasks is given in Table 5 below.

Students in the first group, P1VP3, were given the Fort Problem to solve. When they had completed their solution or 10 minutes had passed they were asked to stop, put down their marker and watch the student on television solve the problem which they had just attempted. After they had finished watching the video they were given the Birdhouse Problem to solve. They were again allowed a maximum of 10 minutes to solve the problem. While the students were problem solving, the researcher made note of any aspects of the solution which were not adequately explained by the student. These issues were addressed in the retrospective interview.

Table 5

The sequence of tasks for each group.

GROUP	FIRST PROBLEM	VIDEO	SECOND PROBLEM	INTERVIEW
P1VP3	FORT PROBLEM	FORT PROBLEM VIDEO	BIRDHOUSE PROBLEM	INTERVIEW
P2VP3	SUIT PROBLEM	FORT PROBLEM VIDEO	BIRDHOUSE PROBLEM	INTERVIEW
VP3	NO PROBLEM	FORT PROBLEM VIDEO	BIRDHOUSE PROBLEM	INTERVIEW
P1P3	FORT PROBLEM	NO VIDEO	BIRDHOUSE PROBLEM	INTERVIEW
P2P3	SUIT PROBLEM	NO VIDEO	BIRDHOUSE PROBLEM	INTERVIEW
P3	NO PROBLEM	NO VIDEO	BIRDHOUSE PROBLEM	INTERVIEW

The final part of the session was devoted to an interview in which the researcher questioned them about the differences in their approach to solving the first and second problem. These questions focused on the aspects of problem solving discussed previously. Any questions which the student had were answered and then they were allowed to return to class.

Students in the second group, P2VP3, were asked to solve the Suit Problem first. They were then shown the same video of the Fort Problem as P1VP3. The Birdhouse Problem was presented next, followed by the interview. The time constraints were the same as for P1VP3.

After receiving their instructions, students in group VP3 were shown the video of the Fort Problem. They were then given the Birdhouse Problem to solve, and they were subsequently interviewed.

Group P1P3 was given the Fort Problem to solve. They did not watch video following their solution attempt. They were given the Birdhouse Problem to solve immediately after they finished their first problem. They were interviewed after they completed this problem.

Students in P2P3 were presented with the Suit Problem. They did not watch the video. The Birdhouse Problem was given to them and following completion of the problem they were interviewed.

The final group, P3, were simply asked to solve the Birdhouse Problem and then they were interviewed.

Retrospective Interviews

The questions used in the retrospective interview are listed in Appendix G. The same format was employed to allow comparison of the data from the six groups. The questions in Appendix G formed the nucleus of the interview, but were supplemented with questions related to issues which were noted in the problem solving session. Questions related to the video were omitted when interviewing the three control groups.

The retrospective interview allowed the subject to elaborate on the concurrent verbal protocol, enabled the researcher to clarify issues which arose from the protocol, and provided the opportunity to subsequently group the data in standard categories. The interview was conducted in an informal manner to allow the students an opportunity to reduce any anxiety associated with the session and when the researcher finished his questions, the students were encouraged to seek feedback on their work. Positive feedback was given and some informal teaching done related to the problems which they had worked before they were asked to return to class.

Data Preparation

The data obtained was in the form of the video taped concurrent verbal protocol and retrospective interview. For the initial transcription of the protocols the audio tapes were used because they were easier to work with. The final transcription of the problem solving sessions followed as closely as possible the guidelines of Ericsson and Simon (1993). The transcripts were a verbatim account of the session, pauses and incomplete thoughts were included (Appendix H).

When transcribing each protocol, an attempt was made to include only one thought on a each line. This was done to facilitate the coding process. Each line was numbered, with the second problem solution following the numbering of the first session.

Following transcription the video tapes were reviewed and long hand notes were made of what was happening that was not captured in the verbal portion of the protocol. Diagrams were reproduced on the protocol, and any gestures were noted. From this data a one paragraph summary was made for each problem answered (Appendix I).

The transcriptions were then timed and divided into five second intervals. At this point it was possible to begin the formal coding of the protocols. The retrospective interviews were part of the transcription, although they were not timed. They were available for a later stage of the analysis.

Development of the Coding Grid

It was necessary to develop a coding grid which could be used to describe the problem solving session, and allow the isolation of the students' attempts at analysis, problem representation and other aspects of problem solving. The development of the coding grid drew on the procedures outlined by Ericsson and Simon (1993), and Rowe (1985). The theoretical basis is derived from the work of Rowe (1985), and Schoenfeld (1985).

Schoenfeld (1985) has developed a framework for examining problem solving at a macroscopic level. He argues that there are four categories of knowledge and behaviour which must be dealt with when examining problem solving behaviour. The four categories are: Resources, Heuristics, Control and Belief Systems. These have been discussed earlier in Chapter 2 but a brief review at this point is in order.

Resources are the student's mathematical knowledge. Heuristics, the second category, are rules of thumb for successful problem solving. The category Control, as used by Schoenfeld, is comparable to the term metacognition as used in the psychological literature. Control can be exercised at a macro or micro level in the problem solving session. Belief Systems are the person's mathematical world view, and influence the way they perceive mathematical problems. These four categories form the dimensions of an explanatory framework which deals with mathematical problem solving.

Based on this work Schoenfeld suggests that a problem solving protocol can be broken down into episodes. He outlines six possible episodes: Reading, Analysis, Exploration, Local Assessment and New Information, Planning and Implementation, and Verification. Not all episodes need be present in any given protocol. These episodes are connected by Transition periods which mark a change of orientation and do not fit into the episode preceding or following it.

A different perspective is presented by Rowe (1985). She presents her taxonomy as a template only. There are eighteen variables which can be collapsed into seven groupings. The first is Directions/Stimulus Passage Directed Activity which deals with the initial reading of the problem, derivation of information, and the question being asked. Solution Directed Activity is problem solving behaviour directed at producing a solution to the problem. Reasoning, a third grouping, involves the complex cognitive processes necessary to derive a solution path. Rowe includes Person Related Activity which is comparable to the world view discussed by Schoenfeld (1985), including an affective component. Critical Evaluation/Judgement involves an assessment by the problem solver of the accuracy of the problem solution. Pause is a category which accounts for the parts of a solution in which there may be no activity, or an activity not related to the problem solution. A sixth grouping is Memory Related Activities. These are attempts to link up previously experienced situations with the current problem. The final category is Changing the Conditions of the Problem, which may involve attempting some illicit method of reaching a solution.

From the work of Rowe and Schoenfeld eight categories were selected from which to develop a coding grid for the protocols (Table 6). It is important that the data reflect the categories used in the coding grid (Newell and Simon, 1972). An initial examination of six protocols led to the dropping of three categories, and the splitting of one category into two separate categories (Table 7). The Reading category was eliminated because the subjects in the present study were instructed to read the entire question before beginning to solve the problem, any errors in reading which had an impact on the solution were noted on the protocol. Any re-reading was placed in the Analysis category because it was in the context of creating a plan. Memory Related Activity was also eliminated because the subjects did not refer back to past experience. The videotape of the protocol made the Pause category unnecessary because in most cases it was possible to ascertain why the student was pausing from his actions. The two aspects of Monitoring, local and global, were included as separate categories to help distinguish the impact of each on the overall solution.

For each of the six categories, subcategories were developed which helped to elaborate each category. The resulting grid was then applied to the analysis of the protocols to determine whether it was sensitive to the aspects of the problem solving process being examined in this study.

Table 6

Broad categories for the analysis of the protocols.

Category	Source	Description
Reading	Schoenfeld (1985)	Initial reading of question, re-reading during solution
Analysis	Schoenfeld (1985)	Understanding the problem
Memory Related Activities	Rowe (1985)	Reference to protocol or past experience
Exploration	Schoenfeld (1985)	A search beyond the parameters of the question
Solution Directed Activity	Rowe (1985)	Solution generating process
Monitoring	Schoenfeld (1985)	Local - of arithmetic, written work Global - of implementation of plan
Pause	Rowe (1985)	Stop in solution, no overt behaviour
Verification	Schoenfeld (1985)	Checking of solution when complete

Table 7

Coding grid for each episode.

Category	Subcategory
ANALYSIS	A1 Examine question
	A2 Select what is necessary
	A3 Re read question
	A4 Re read chart
PROBLEM REPRESENTATION	A5 Make a mental representation
	A6 Draw a diagram
	A7 Ask for assistance or clarification
SOLUTION DIRECTED ACTIVITY	S1 Write part of problem
	S2 State arithmetic problem
	S3 Perform computation
	S4 Label
	S5 State answer
GLOBAL MONITORING	M1 Refer back to part of plan
	M2 Refer to procedure
	M3 Re read question
	M4 Refer to plan and re read chart
	M8 Reach an impasse
LOCAL MONITORING	M5 Correct written work
	M6 Correct computation
	M7 Correct labels
VERIFICATION	V1 Review calculation
	V2 Assess appropriateness of solution

As a further check three graduate students volunteered to test the grid. The coding grid was explained to them and the way in which it was to be applied was discussed. Three protocols were randomly selected and each graduate student analyzed the protocols. The researcher compared their work. The percentages of agreement for the three coders were: Analysis .92, Problem Representation .84, Solution Directed Activity .95, Global Monitoring .75, Local Monitoring .91, Verification .93. The researcher met with the graduate students and discussed their work. There were a couple of categories which they felt were ambiguous. These were discussed and redefined. The revised grid was applied to three new protocols by the researcher and graduate students, and the overall percentage of agreement was almost .92. Correlations for individual codes exceeded .80. This final grid was adopted for the analysis of all the protocols and is the one presented in Table 7.

Coding Procedure

Each protocol in the present study will be analyzed using the six episodes discussed above. Within each episode there were a number of category codes. Each category code was prefaced by the first letter of the episode and a number e.g. A2: Analysis Episode .

The coding procedure for each protocol was as follows.

1. The Protocol was read entirely without any coding.

2. Each line was re-read and given a code signifying which episode it fell into and the nature of the activity (Appendix H).
3. The video tape of the protocol was reviewed to insure that the coding was representative of what had occurred.
4. When a group had been completely coded, the first protocol was coded a second time as a check on the consistency of the coding procedure.

The retrospective interviews were not coded. They were designed to elicit information of an exploratory nature comparing the two problem solutions. They were used in the qualitative analysis of the protocols, and as a supplement to the concurrent verbal protocols.

To facilitate a comparison of protocols, a graphical representation was created which took as its base the five second time intervals of each protocol (Appendix J). On this matrix was mapped the different episodes which constituted the protocol. A different colour was assigned to each episode. When at least three seconds of the five second interval was spent on a given episode, the full five second portion of the matrix was coloured. This produced a striking graphic display of the protocol on a time line which could be compared easily with the representations of other protocols.

When all of the protocols were coded analysis began. The analysis was conducted on two levels. First a quantitative analysis was performed and this is reported in Chapter 4. A qualitative analysis was conducted next and the results appear in Chapter 5.

CHAPTER 4

QUANTITATIVE ANALYSIS OF THE RESULTS

This chapter is devoted to the presentation of a quantitative analysis of the problem solutions. In the Methodology chapter two assumptions were made: the first with respect to the sampling procedure, and the second about the quasimorphism of the problems. The first section of this chapter addresses these assumptions. The remainder of the chapter reports the quantitative analysis of the global hypothesis and each of the subhypotheses.

Confirmation of Assumptions

Before dealing with the hypotheses, it is necessary to confirm the assumptions discussed in the Methodology section regarding the sampling procedure and questions used in the study.

While every effort was made to randomly assign students to treatment groups, a brief analysis was undertaken to verify that the groups were equivalent. The solution times for the two groups of students who solved the Fort Problem (P1VP3 and P1P3), as their initial problem were compared. A Mann-Whitney U Test was conducted to compare the times for groups P1VP3 and P1P3. As can be seen in Table 8, there was no significant

Table 8

Summary statistics comparing the solution times for the initial problem solved by each group.

Problem	Group	N	M	Md	Statistic	p
Fort	P1VP3	6	3.18	2.91	U=16	>0.05
	P1P3	6	3.38	3.00		
Suit	P2VP3	5	2.95	2.75	U=14	>0.05
	P2P3	6	2.97	2.26		
Fort	P1VP3 P1P3	12	3.27	2.91	U=53	>0.05
Suit	P2VP3 P2P3	11	2.96	2.43		
Fort	P1VP3 P1P3	12	3.27	2.91	U=46	>0.05
Birdhouse	P3	5	2.09	1.67		
Suit	P2VP3 P2P3	11	2.96	2.43	U=23	>0.05
Birdhouse	P3	5	2.09	1.67		

U- Mann-Whitney U Statistic

difference in the times spent by the two groups. When the times of the two groups who solved the Suit Problem first, P2VP3 and P2P3, were compared there was also no difference. Because groups VP3 and P3 only solved one problem, and VP3 saw the video before solving the problem, these two groups could not be compared.

To ensure that the Fort Problem, Suit Problem and Birdhouse Problem were of equal difficulty, a Mann-Whitney U Test was conducted on the three possible comparisons using the times spent by each group solving the first problem. The following comparisons are reported in Table 8: Fort Problem and Suit Problem, Fort Problem and Birdhouse Problem, Suit Problem and Birdhouse Problem. The Birdhouse Problem scores were obtained from group P3 which solved only the Birdhouse Problem. Group VP3 also solved only the Birdhouse Problem but was not included because they had viewed the video before attempting this problem, and all other comparisons were conducted on the subjects first solution attempt.

None of the three comparisons produced significant differences in scores (Table 8). The means for the groups P1VP3 and P2VP3 solving the Fort and Suit Problems respectively, were very close, the sample sizes were 12 and 11. Because one group could not be included in the Birdhouse Problem analysis, (VP3), the sample size for this group was only 5. This may have had some impact on the mean, 2.09, which was lower than the other groups.

These results were taken as confirmation of the assumption that the groups are comparable and the problems of equal difficulty as discussed in the Methodology chapter.

Research Hypotheses

The global research hypothesis was stated as follows:

> A videotaped problem solving protocol increases the efficiency of a grade 7 student when problem solving.

There were six sub-hypotheses related to the research hypothesis. They were stated as follows:

> A videotaped problem solving protocol reduces the amount of time spent on Analysis.
>
> A videotaped problem solving protocol increases the amount of time spent on Problem Representation.
>
> A videotaped problem solving protocol reduces the amount of time spent of Solution Directed Activity.
>
> A videotaped problem solving protocol increases the amount of time spent on Global Monitoring.
>
> A videotaped problem solving protocol increases the amount of time spent on Local Monitoring.
>
> A videotaped problem solving protocol increases the amount of time spent on Verification.

Discussion of the Global Hypothesis

As discussed earlier, there were six groups in the present study. For the purpose of testing the global hypothesis, these six groups were combined into two: groups P1VP3, P2VP3, and VP3 which saw the video formed the experimental group, and groups P1P3, P2P3, and P3 which did not see the video were the control group. All groups solved the Birdhouse Problem, which will be referred to as Problem 3. Groups P1VP3, P2VP3, P1P3, and P2P3 solved an initial problem, either the Fort or Suit Problem. These two initial problems will be referred to as Problem 1 throughout the discussion of the global hypothesis and sub-hypotheses.

To test the global hypothesis, subjects in the experimental groups P1VP3 and P2VP3 were grouped together, as were the subjects in the control groups P1P3 and P2P3. The two groups which solved only the final problem, VP3 and P3, were not included. A Wilcoxon Signed-Rank Test was applied to the differences in time spent on the two problems. All time scores represent the time in minutes spent on the solution of the problem excluding the time spent on the initial reading of the problem. The results of this analysis are reported in Table 9. The statistics reported include the mean amount of time spent on each problem, the median score, the Wilcoxon T Statistic and the p value. The alpha = 0.05 level of significance was adopted throughout.

Table 9 shows that, as the hypothesis suggests,

Table 9

Summary statistics for the solution times of subjects who solved Problem 1 and Problem 3.

Group	Problem	N	Solution Cor	Solution Inc	M	Md	Statistic	p
Experimental	1	11	0	11	3.08	2.75	T=8.5	<.05
	3	11	2	9	2.24	1.67		
Control	1	12	3	9	3.18	2.50	T=0	<.05
	3	12	1	11	1.82	1.725		

T=Wilcoxon Signed-Rank T Statistic
Problem 1 refers to both the Fort Problem and the Suit Problem
Problem 3 refers to the Birdhouse Problem
N - Number in group
Cor - Correct
Inc - Incorrect
M - Mean
Md - Median

experimental group did spend significantly less time on Problem 3 than they did on their first problem. The observed Wilcoxon T statistic was 8.5, the critical value was 13. The subjects in the control group also spent less time on Problem 3, (T=0, critical value=17). The mean for both of the samples for the experimental group were larger than the median, indicating that there were a number of subjects who spent longer on their solution thus raising the mean. The same was true for the contol group's scores for the first problem.

The experimental group also showed an improvement in their performance with two subjects solving the final problem correctly compared to no correct answers for the first problem. The control group did more poorly on Problem 3. One subject solved Problem 3 correctly compared with three who obtained the correct solution to the first problem. In order to ascertain if there was a difference between the two treatments, these two groups were compared and a Mann-Whitney U Test was conducted.

Because of the small sample size, the Mann-Whitney U Test was employed as it is the nonparametric counterpart of the independent groups t-test. Significance of the U statistic was determined by reference to the Table of Critical Values for the Mann-Whitney U Statistic (Hayes and Winkler, 1971). Since zero was a possible choice, (the subject could choose to spend no time on a given grid category), zero values were included in computing rankings for the Mann-Whitney U

Statistic. Where non-zero values did not meet the criteria for the test statistic, that is N>4, no analysis was conducted.

Table 10 shows the mean time spent by subjects in each group on their solution to Problem 3, the median score, the U Statistic and the p value. Once again only the subjects who solved Problem 1 and Problem 3 were included in this analysis. The mean score for the experimental group was larger than the control group, but the difference between the two groups was not statistically significant. The U Score was 61.5 for N=11 and 12, the critical value was 38. Once again the sample for the experimental group was positively skewed.

The Mann-Whitney U statistic for the difference in times spent on Problem 3 by male and female students within each treatment group is presented in Table 10. In the experimental group the male subjects spent significantly more time on Problem 3 than the female students, ($U = 2$, $p<.05$, critical value = 5). In the control group there was no significant difference between the male and female subjects. As can be seen there is very little difference between the means or medians for the male and female subjects. Of interest is the direction of change for the subjects in the experimental group. For the male subjects, only four of the six showed a positive difference in P1 - P3, spending more time on their first problem (P1) than their second problem (P3). Two of the subjects spent more time on Problem 3 than Problem 1. This

Table 10

Summary statistics for the solution times in minutes, for Problem 3 by gender and treatment.

Treatment	Gender	N	Direction of Change +	-	NC	M	Md	Statistic	p
Experimental		11				2.24	1.67	U=61.5	>.05
Control		12				1.82	1.725		
Experimental	Male	6	4	2	0	2.95	2.54	U=2	<.05
	Female	5	5	0	0	1.39	1.30		
Control	Male	6	6	0	0	1.77	1.725	U=17	>.05
	Female	6	6	0	0	1.88	1.82		
Experimental	Male	6	4	2	0	2.95	2.54	T=8.5	>.05
Control	Male	6	6	0	0	1.77	1.725		
Experimental	Female	5	5	0	0	1.39	1.30	T=11.0	>.05
Control	Female	6	6	0	0	1.88	1.82		

NC - No change
U=Mann-Whitney U Statistic
T=Wilcoxon Signed-Rank T Statistic

may account for the higher mean score for the male subjects. For the female subjects in the experimental group and the male and female subjects in the control group, the difference was positive, they spent less time on Problem 3 than on Problem 1.

In Table 10 the times for the male and female students are compared across treatments. The mean amount of time spent by the male subjects in the experimental group is greater than the control group, but the difference is not statistically significant, (T = 8.5, critical value = 7). The spread between the median scores is smaller than the mean scores with the median score for the experimental group being lower than the mean indicating that a few extreme scores positively skewed the distribution of the scores for the male subjects.

The relationship between the means for the two treatment groups is reversed for the female subjects. The mean time spent by the females in the experimental group is less than the mean for the control group, but the difference is not statistically significant, (T = 11, critical value = 5). Both median scores are very close in value to the means.

As discussed earlier the teachers were asked to rate their students who had volunteered for the study on their ability to solve word problems in mathematics. When the subjects were divided into treatment groups there were enough high ability subjects, but not enough low and middle ability subjects in the two treatment groups to permit a statistical analysis. The summary statistics for all three groups are reported in Table 11, and the Wilcoxon Signed-Rank T statistic for the high ability group is presented.

In the experimental group there were only two subjects in each of the low and middle ability groups who solved Problem 1 and Problem 3. The mean time spent on Problem 3 by the low ability group was less than half that spent on Problem 1. The mean time dropped in the middle ability group as well. While the drop was not as dramatic, it was of the same magnitude as the high ability group. The high ability group did have a statistically significant drop in time spent on Problem 3 ($T=1$, critical value 3, $p < .05$). The drop in the median score was even more dramatic with the distribution of times for Problem 3 being positively skewed.

There were three subjects in each of the low and middle ability groups in the control group. There was very little change in the mean times for the low ability group. The time spent by the middle ability group on Problem 3 was nearly half that spent on Problem 1. But again there were too few subjects to permit a statistical analysis. The drop in mean times in the high ability group was not as large as the middle ability group, but there were enough subjects to permit an analysis and the change in time did prove to be statistically significant ($T=0$, critical value 2, $p<.05$).

Next the experimental and control groups were compared on each of the six main episodes. Each of the six episodes on the grid was broken down into two or more subcategories as

Table 11

Summary statistics for the percentage of solution time spent on Problem 1 and Problem 3 for ability and treatment variables.

Group	Ability	Problem	N	M	Md	Statistic	p
Experimental	Low	1	2	4.18	-	-	-
		3	2	2.07	-		
	Middle	1	2	2.94	-	-	-
		3	2	2.34	-		
	High	1	7	2.80	2.60	T=1	<.05
		3	7	2.26	1.67		
Control	Low	1	3	1.75	-	-	-
		3	3	1.57	-		
	Middle	1	3	3.37	-	-	-
		3	3	1.52	-		
	High	1	6	3.79	3.06	T=0	<.05
		3	6	2.10	2.17		

T=Wilcoxon Signed-Rank T Statistic

discussed previously. For this analysis the percentage of the protocol time spent on the actual solution, (ie. the total protocol time minus the time for the initial reading of the problem) was used to compare the impact of the treatments on the individual episodes. The percentage of the protocol time spent on each subcategory for Problem 3 was computed. For the purpose of analysis, the time percentages of each subcategory were added and consequently produced one score for that episode. The time percentages for each of the seventeen subjects in the video group were compared with the seventeen subjects in the non video group. Each of the six groups solved the same final problem which permitted a between groups analysis.

Table 12 shows the mean, median and U statistic for each episode. The dependent variable for each table was the percentage of protocol time spent on that episode, the independent variables were the video or non-video intervention.

The global hypothesis stated that watching a videotaped problem solving protocol would increase the efficiency of a problem solver. Support for this hypothesis was found in two of the six episodes. Both the Analysis and the Global Monitoring Episodes produced significant differences. The mean percentage of time spent on the Problem Representation Episode was almost identical for the experimental and control groups, but there were too few subjects who spent time on this

Table 12

Summary statistics for the percentage of time spent on each of the major categories.

Category	Group	N	M	Md	A=0	Statistic	p
Analysis	Experimental	17	17.35	16	0	U=96	<.05
	Control	17	28.94	21	0		
Problem Representation	Experimental	17	2.23	0	14	-	-
	Control	17	2.18	0	15		
Solution Directed Activity	Experimental	17	47.70	47	0	U=146.5	>.05
	Control	17	48.29	48	0		
Global Monitoring	Experimental	17	29.23	25	0	U=92	<.05
	Control	17	20.82	17	1		
Local Monitoring	Experimental	17	1.41	0	10	-	-
	Control	17	0.41	0	16		
Verification	Experimental	17	1.47	0	14	-	-
	Control	17	0.53	0	15		

U=Mann-Whitney U Statistic
A=0 refers to the number of subjects spending no time on that episode.
M refers to the mean of the percentage of protocol time spent on that episode.

episode, the experimental group (N=3), and the control group (N=2), to permit testing. The means for the two groups in the Solution Directed Activity Episode, were very close and no significant difference was found. The Local Monitoring and Verification Episodes had too few subjects spending time on these episodes to permit analysis.

Discussion of the First Sub-Hypothesis

The first sub-hypothesis states that the videotaped problem solving protocol reduces the percentage of time spent on Analysis. As was seen in Table 12, the control group did spend significantly more time on Analysis than the experimental group. The experimental group would need to spend less time on analysis due to an increased understanding of the mental model required to solve these problems.

When the experimental and control groups were examined individually, it was found that both groups spent less time on the second problem (Table 13). Subjects in the experimental group spent ten percent less of their protocol time on the Analysis Episode in the second problem than on the first. This produced a T score of 10.5, critical value of 13, $p<.05$. The drop in time spent by the control group was less dramatic, but was also statistically significant at the .05 level.

Within the experimental group there was no statistically significant difference in the means for the male and female subjects (Table 14). The means were close, less than 2

Table 13

Summary statistics for the percentage of solution time spent on Problem 1 and Problem 3 in the Analysis Episode by treatment group.

Treatment	Problem	N	M	Md	Statistic	p
Experimental	P1	11	26.18	17.0	T=10.5	<.05
	P3	11	16.18	14.0		
Control	P1	12	32.17	27.0	T=12.5	<.05
	P3	12	25.92	20.5		

T=Wilcoxon Signed-Rank T Statistic

Table 14

Summary statistics for the percentage of solution time spent on the Analysis Episode of Problem 3 by gender and treatment.

Treatment	Gender	N	Direction of Change +	-	NC	M	Md	Statistic	p
Experimental	Male	6	4	2	0	15.33	10.50	U=8.5	>.05
	Female	5	4	1	0	17.20	15.00		
Control	Male	6	3	3	0	30.17	22.50	U=14.0	>.05
	Female	6	4	2	0	21.67	17.00		
Experimental	Male	6	4	2	0	15.33	10.50	U=9.0	>.05
Control	Male	6	3	3	0	30.17	22.50		
Experimental	Female	5	4	1	0	17.20	15.00	U=14.5	>.05
Control	Female	6	4	2	0	21.67	17.00		

U=Mann-Whitney U Statistic

percentage points apart. The means for the male and female subjects in the control group were more spread out but the difference was not statistically significant. The male subjects spent on average a larger percentage of their time on the Analysis Episode, but this was in part due to one subject who spent much more time than the others. More than two thirds of the experimental group spent less time on Analysis in Problem 3, four of six male, and four of five female subjects. Seven of twelve subjects in the control group spent less time on Analysis.

The results of the comparisons of the treatment groups for the male and female subjects are presented in Table 14. While the mean score for the male subjects in the experimental group was just over half that of subjects in the control group, the difference was not statistically significant. This may be due in part to one score that was much higher than the others in the group. The female subjects in the experimental group had a lower mean score than the control but the difference was not statistically significant.

The Analysis Episode consisted of four subcategories. Table 15 shows the summary statistics for the four subcategories of the Analysis Episode. Only the first subcategory, A1 Examine the Question, produced a significant difference. The control group spent more time examining the question (A1), trying to ascertain what must be done to solve the problem. This finding is in keeping with the first

Table 15

Summary statistics for the percentage of time spent on the four subcategories of the Analysis Episode.

Subcategory	Group	N	M	Md	A=0	Statistic	p
A1	Experimental	17	4.9	3	7	U=87.5	<.05
	Control	17	13.2	10	4		
A2	Experimental	17	4.6	5	4	U=125.5	>.05
	Control	17	3.8	0	9		
A3	Experimental	17	5.1	3	7	U=132	>.05
	Control	17	6.0	0	9		
A4	Experimental	17	2.7	0	9	U=108	>.05
	Control	17	5.9	4	6		

U=Mann-Whitney U Statistic
A=0 refers to the number of subjects who spent no time on that episode.
M refers to the mean of the percentage of protocol time spent on that episode.

A1 Examine Question
A2 Select What Is Necessary
A3 Re-read Question
A4 Re-read Chart

subhypothesis. The experimental group was better prepared to analyze the problem and make their plan. The control group also spent more time, on average, re-reading the question (A3) and the chart (A4) and less time selecting what is necessary (A2) but there is not enough evidence of a difference on these three subcategories.

As can be seen from Table 15, the percentage of time which the control group spent re-reading the chart (A4), was more than twice that of the experimental group. Because of the large number of zero values, 15, these tied scores contributed to a non-significant U statistic. Clearly, of those who spent time re-reading the chart, subjects in the experimental group had to spend less time on this activity. The subjects in the control group were not able to develop a plan as quickly, hence the need to check the chart as well as the question in an effort to find a meaning for the problem.

The final area of interest is how each individual group within the two treatments performed in the Analysis Episode.

There were six groups in the study. The groups were divided so that three received the video intervention, and three similar groups did not. The first experimental group solved the Fort Problem (P1), watched a video of the Fort Problem being solved by a student (V), and then solved the Birdhouse Problem (P3). The first group is referred to as P1VP3. The second experimental group, P2VP3, solved the Suit Problem (P2), watched the Fort Problem video (V), and solved the Birdhouse Problem (P3). The third experimental group, VP3, watched the Fort Problem video (V), and then solved the Birdhouse Problem (P3).

The three control groups were similar to the ones discussed above but they did not watch the video. The first control group, P1P3, solved the Fort Problem (P1) and then solved the Birdhouse Problem (P3). The second control group, P2P3, solved the Suit Problem (P2), and then solved the Birdhouse Problem (P3). The final control group, P3, solved the Birdhouse Problem (P3) only.

A Wilcoxon Signed Rank test was conducted on the change in percentage of time, P1 - P3, for each episode by the four groups which solved two problems. A similar analysis was done for the subcategories of each episode. The results of each will be reported with the appropriate subhypothesis.

The results of the Wilcoxon Test and the direction of the change in time percentage for the first subhypothesis is reported in Table 16. The first experimental group, P1VP3, showed a positive change in direction in the Analysis Episode. Five of the six subjects had a positive difference, indicating less time spent on Analysis in the second problem than the first. The difference was not statistically significant. None of the subcategories produced a statistically significant difference. Five of the six subjects in this group spent less time in the second problem on subcategory A2, Select What is Necessary. They may have been better prepared to find what

Table 16

Summary statistics for the change in percentage of time spent on the Analysis Episode by subjects who solved both Problem 1 and Problem 3.

Group	Subcategory	N	Direction of Change			Statistic	p
			Pos	Neg	NC		
P1VP3		6	5	1	0	T=4	>.05
	A1	6	2	0	4	-	-
	A2	6	5	1	0	T=4	>.05
	A3	6	3	2	1	T=5	>.05
	A4	6	1	1	4	-	-
P2VP3		5	4	1	0	T=1.5	>.05
	A1	5	1	2	2	-	-
	A2	5	2	2	1	-	-
	A3	5	0	3	2	-	-
	A4	5	3	2	0	T=4.5	>.05
P1P3		6	5	1	0	T=1	<.05
	A1	6	3	3	0	T=10	
	A2	6	2	2	2	-	-
	A3	6	4	1	0	T=4	>.05
	A4	6	1	0	5	-	-
P2P3		6	2	4	0	T=8.5	>.05
	A1	6	2	3	1	T=6	>.05
	A2	6	1	4	1	T=3	>.05
	A3	6	1	4	1	T=5	>.05
	A4	6	2	4	0	T=7.5	>.05

T=Wilcoxon Signed-Rank T Statistic
NC - no change

A1 Examine Question
A2 Select What is Necessary
A3 Re-read Question
A4 Re-read Chart
NB The critical T value for N=5 is 1. If the score of the one subject which was in the opposite direction is not the lowest in absolute value, the T score will be too high to produce a statistically significant difference.

was necessary to solve the problem, but the difference was not significantly different.

Four of five subjects in P2VP3 spent more time on Analysis in their first problem, but this difference was not statistically significant. None of the subcategories showed a significant difference.

The statistically significant difference in times spent on the Analysis Episode by the experimental group, P1VP3 and P2VP3 combined, which was reported in Table 13 could not be attributed to any of the subcategories.

The difference which was reported in the time spent by the control group on Analysis (Table 13), could be attributed to the first control group, P1P3, which spent significantly less time on their second problem, ($T=1$, critical value = 2, $p<.05$). None of the subcategories produced a statistically significant difference.

Four of the six subjects in the second control group, P2P3, spent more time on the second problem than on the first. None of the subcategories produced statistically significant differences.

Both the experimental and control groups spent less time on the Analysis Episode when solving their second problem. The video resulted in the experimental group spending significantly less time on the Analysis Episode as the first subhypothesis stated. It appears that learning may have occurred as a result of viewing the video.

Discussion of the Second Sub-Hypothesis

The second sub-hypothesis states that a videotaped problem solving protocol increases the amount of time spent on Problem Representation. There was very little difference in the means of the percentage of protocol time spent on Problem Representation by the experimental and control groups (Table 12). Only three of the subjects in the experimental group, and two in the control group spent any time at all on Problem Representation in Problem 3.

Problem Representation was an integral part of the videotape solution. The student who solved the problem on the videotape, did spend time on problem representation, he drew a diagram, labelled it, and explained why he was doing it. But the merits of problem representation as such were not discussed by the student in the video, or the experimenter.

Within the experimental group there were some interesting changes. The difference between the time spent on Problem Representation by subjects in the experimental group for Problem 1 and Problem 3 was not statistically significant (Table 17). As can be seen the mean did drop for the second problem, and the number of subjects who spent no time at all increased from five to eight. No one in the control group spent time on Problem Representation in the second problem, this was down from seven who did in the first problem. This did produce a statistically significant difference (T=0, critical value=3).

Table 17

Summary statistics for the percentage of solution time spent on the Problem Representation Episode for Problem 1 and Problem 3 by treatment.

Treatment	Problem	N	Direction of Change +	-	NC	M	Md	A=0	Statistic	p
Experimental	1	11				4.64	2.0	5	T=6.5	>.05
	3	11				3.45	0.0	8		
	P1-P3	11	4	2	5					
Control	1	12				6.67	4.5	5	T=0	<.05
	3	12				0.0	0.0	12		
	P1-P3	12	7	0	5					

T=Wilcoxon Signed-Rank T Statistic

A=0 represents the number of subjects spending no time on Problem Representation

The final part of Table 17 shows the direction of change from Problem 1 to Problem 3. Almost half of each group, (5 of 11, and 5 of 12), showed no change. This was because none of these subjects spent any time on Problem Representation for either problem. Of those who did spend time on Problem Representation, in the experimental group four of the six had a positive change, spending more time on Problem 1 than Problem 3. As discussed above, all seven of the control group subjects had a positive change. This may be a result of a practise effect, the subjects did not see a need to represent the problem graphically because they felt they knew the structure of the problem. The fact that two of the subjects in the experimental group did have a negative change may be because they saw the value of problem representation in the video, or had their initial attempts at problem representation reinforced by the video.

Due to the small number of subjects in the two groups who spent time on Problem Representation, no further analysis was conducted.

Representation of the problem could facilitate the solution of these construction problems. When the subject was able to see what he was to construct, he was better able to pick out the errors in his global plan. For the subjects in the present study problem representation formed a small percentage of their solution time. They may not have been taught to make a representation of the problem, or made aware of the value of making a representation when doing problems of this nature. In either case a valuable problem solving technique was not used by many subjects.

Discussion of the Third Sub-Hypothesis

Solution Directed Activity was the episode on which the most time was spent by the experimental and control groups. The third subhypothesis states that the videotaped problem solving protocol increases the amount of time spent on this episode. The percentage of protocol time spent by the two groups on Solution Directed Activity was very close, 47.70% for the experimental group, and 48.29% for the control group (Table 12). This difference was not statistically significant, and as a result did not support the third subhypothesis.

Within each group there was a shift in the percentage of time spent on the Solution Directed Activity Episode from the first problem to the second. Both groups spent about 35% of their solution time for the first problem on this episode. The experimental group increased the amount of time spent by over fourteen percent to 49.91% of their solution time on the second problem (Table 18). But this was not a statistically significant shift, $T=13.5$, critical value = 13.

The shift in the time percentage by the control group was statistically significant, $T=7$, $p<0.05$. They spent over 51% of their time on the Solution Directed Activity Episode in the

Table 18

Summary statistics for the percentage of solution time spent on Problem 1 and Problem 3 in the Solution Directed Activity Episode by treatment group.

Treatment	Problem	N	M	Md	Statistic	p
Experimental	P1	11	35.18	40.0	T=13.5	>.05
	P3	11	49.91	50.0		
Control	P1	12	35.33	37.0	T= 7.0	<.05
	P3	12	51.17	52.0		

T=Wilcoxon Signed-Rank T Statistic

second problem.

The time spent actually solving the problem, writing numbers or labels, and doing the arithmetic, was most important to the subjects in this study. The increase in the amount of time from the first to the second problem may be a result of having had one problem from which to gain experience. Perhaps the subjects had a better idea of what was required to solve a problem of this nature and so spent less time on other aspects of the problem, as was seen in the discussion of the Analysis Episode. This would result in a larger percentage of their time being devoted to the Solution Directed Activity Episode.

The male subjects in the experimental group spent less time on the Solution Directed Activity Episode than the female subjects (Table 19). The ten percentage point difference was not statistically significant, (U=6, critical value = 5). Interestingly all of the female subjects spent a greater percentage of their time on this episode in their second problem than their first. This may be a result of less time being spent on the Analysis Episode by four of the five female subjects (Table 14) which produced a higher time percentage for this episode. Only two male subjects devoted more of their protocol time to the Solution Directed Activity Episode in their second problem, this is the same shift noted for the Analysis Episode (Table 14). The video intervention appears to have a greater impact on the female subjects.

Table 19

Summary statistics for the percentage of solution time spent on the Solution Directed Activity Episode of Problem 3 by gender and treatment.

Treatment	Gender	N	Direction of Change +	-	NC	M	Md	Statistic	p
Experimental	Male	6	4	2	0	45.17	43.50	U=6.0	>.05
	Female	5	0	5	0	55.60	50.00		
Control	Male	6	1	5	0	45.83	45.50	U=14.0	>.05
	Female	6	2	4	0	56.50	56.50		
Experimental	Male	6	4	2	0	45.17	43.50	U=18.0	>.05
Control	Male	6	1	5	0	45.83	45.50		
Experimental	Female	5	0	5	0	55.60	50.00	U=14.0	>.05
Control	Female	6	2	4	0	56.50	56.50		

U=Mann-Whitney U Statistic

In the control group the female subjects also devoted a greater percentage of their time to the Solution Directed Activity Episode than did their male counterparts. The approximately eleven percentage point difference noted in Table 19 was not statistically significant (U=14, critical value = 7). This was the inverse of the 8.5% difference favouring the male subjects, in time spent on the Analysis Episode (Table 14). The male subjects in the control group spent more time developing their plan, while the female subjects spent more time on Solution Directed Activities.

When the male subjects in the experimental and control groups were compared, there was virtually no difference in the mean percentage of time spent on the Solution Directed Activity Episode. Five of the six male subjects in the control group devoted more of their second problem to this episode than the first. Only two males in the experimental group had a negative change. The videotaped problem solution may have had a positive impact reducing the percentage of protocol time that the male subjects required to solve their second problem.

The times spent by the female subjects in the experimental and control groups were also very close and did not produce a significant difference (Table 19). All of the female subjects in the experimental group and four of six in the control group had a negative change in the percentage of time spent on this episode.

When the five subcategories of the Solution Directed Activity Episode were examined, only one of them produced a statistically significant difference between the experimental and control groups. Both experimental and control subjects spent more time on the first subcategory, S1: Write part of the problem, than the other four (Table 20). There was no significant difference between the two groups, the means were only 0.9% apart. One of the control group subjects spent no time at all writing part of the problem, he simply wrote a single numeral from the chart for an answer.

Very little time was spent on the second subcategory, S2: State arithmetic problem. Only one subject in the experimental group, and three in the control group, actually verbalized the arithmetic problem which they had written before they solved it.

Subcategory S3: Perform computation, consumed the second greatest amount of time in this episode. The means once again were very close and did not result in a significant difference between treatment groups. One subject from each group did not spend any time on this subcategory, both of them selecting a single value for an answer.

There was a large difference in the means for the experimental and control groups on subcategory S4: Label, 4.8% and 0.59% respectively. A statistical analysis could not be performed because only one subject in the control group spent any time labelling his work. Because seven subjects in the

Table 20

Summary statistics for the five subcategories of the Solution Directed Activity Episode.

Subcategory	Group	N	M	Md	A=0	Statistic	p
S1	Experimental	17	22.9	25	0	U=130.5	>.05
	Control	17	22.0	19	1		
S2	Experimental	17	0.41	0	16	-	-
	Control	17	1.8	0	14		
S3	Experimental	17	14.29	14	1	U=112.0	>.05
	Control	17	14.7	14	1		
S4	Experimental	17	4.8	0	10	-	-
	Control	17	0.59	0	16		
S5	Experimental	17	5.64	5	4	U=88.5	<.05
	Control	17	9.17	6	0		

U=Mann-Whitney U Statistic
A=0 refers to the number of subjects who spent no time on that episode.
M refers to the mean of the percentage of protocol time spent on that episode.

S1 Write part of problem
S2 State arithmetic problem
S3 Perform computation
S4 Label
S5 State Answer

experimental group did label, watching the video may have had a positive impact.

The final subcategory S5: State Answer, was the one which did produce a statistically significant difference. The control group spent 9.17% of their time stating the answer compared to 5.64% for the experimental group ($U=88.5$, critical value 97, $p<.05$). All of the subjects in the control group took time to state their answer, four of the experimental subjects simply did their calculations, obtained an answer, and stated that they were finished. Whether this was a result of the videotaped problem solution is difficult to ascertain.

Table 21 shows the change in the percentage of time spent on each subcategory for the four groups which solved two problems.

The first experimental group, P1VP3, viewed the problem which they had just solved, on the videotape. For that group there was a statistically significant shift in the amount of time spent on their second problem, five of the six subjects spent more time on P3 than on P1. The subjects in this group spent significantly less time writing the problem, subcategory S1, ($T=2$, critical value = 2, $p<.05$) in P3. None of the other subcategories produced a statistically significant value for T, subcategories S2 and S4 each had 3 subjects who did not have a change in time which did not permit analysis. Once again, seeing a solution to the problem just attempted may have resulted in a greater percentage of time spent on

Table 21

Summary statistics for the change in percentage of time spent on the Solution Directed Activity Episode by subjects who solved both Problem 1 and Problem 3.

Group	Subcategory	N	Direction of Change Pos	Neg	NC	Statistic	p
P1VP3		6	1	5	0	T=2	<.05
	S1	6	1	5	0	T=2	<.05
	S2	6	2	1	3	-	-
	S3	6	2	4	0	T=7	>.05
	S4	6	1	2	3	-	-
	S5	6	3	3	0	T=8	>.05
P2VP3		5	3	2	0	T=5	>.05
	S1	5	2	3	0	T=6	>.05
	S2	5	0	0	5	-	-
	S3	5	2	3	0	T=6	>.05
	S4	5	1	2	2	-	-
	S5	5	1	3	1	-	-
P1P3		6	0	6	0	T=0	<.05
	S1	6	1	5	0	T=3	>.05
	S2	6	0	2	4	-	-
	S3	6	0	5	1	T=0	<.05
	S4	6	2	0	4	-	-
	S5	6	1	5	0	T=1	<.05
P2P3		6	3	3	0	T=6.5	>.05
	S1	6	3	3	0	T=7	>.05
	S2	6	2	0	4	-	-
	S3	6	4	2	0	T=4	>.05
	S4	6	0	0	6	-	-
	S5	6	1	5	0	T=2	<.05

T=Wilcoxon Signed-Rank T Statistic
NC indicates no change
S1 Write Part of Problem
S2 State Arithmetic Problem
S3 Perform Computation
S4 Label
S5 State Answer
P1VP3 Problem 1, Video, Problem 3
P2VP3 Problem 2, Video, Problem 3
P1P3 Problem 1, Problem 3
P2P3 Problem 2, Problem 3

Solution Directed Activity because a smaller percentage of time was devoted to Analysis.

The second experimental group, P2VP3, had only five subjects. They saw the same videotaped problem solution as the first group, which was the solution for a problem different from the first one they attempted. Their time change was split with three having a positive change, more time spent on their first problem, and two spending more time on the second problem. There was also no significant change in any of the subcategories for this group. Seeing a problem solution different from the one they had attempted was not as effective in changing the amount of time spent on the Solution Directed Activity Episode.

The first control group, P1P3, had a statistically significant change in time (T=0, critical value = 2, p<.05). All of the subjects in group P1P3 spent more time on the Solution Directed Activity Episode in Problem 3. They had solved the same first problem as group P1VP3, which also had a significant change. In two of the subcategories the negative change in time for P1P3 was significant. Subcategory S3 Perform Computation, required more time in the second problem, (T=0, critical value = 2, p<.05), as did S5 State Answer, (T=1, critical value = 2, p<.05).

The final control group, P2P3, did not have a significant change in time on their second problem, with three subjects recording a positive change, and three a negative change.

Only one subcategory, S5 State Answer, did show a significant change with more time spent on the final problem, (T=2, critical value = 2, p<.05). The finding that the control group spent more time on S5 State Answer, has been discussed earlier, the same distribution of scores was evident in the two control groups, one positive change, five negative. Neither of the groups which solved the Suit Problem first, spent significantly less time on Solution Directed Activity in their second problem.

Both the experimental and control groups spent most of their protocol time on the Solution Directed Activity Episode, the majority of this time being spent on two aspects of this episode: writing part of the problem, and performing the computation. The subjects in the present study either did not realize, or believe in, the importance of the other episodes in contributing to a correct solution. Writing the problem and performing the arithmetic was deserving of the greatest percentage of their solution time.

Discussion of the Fourth Sub-Hypothesis

The fourth subhypothesis examined the effects of the treatment on the Global Monitoring of the subjects. The Global Monitoring Episode produced a U score of 92, p<0.05 (Table 12). This hypothesis states that the experimental group spends more time on Global Monitoring which involved referring back to the global plan and checking the progress of the solution. This hypothesis was supported.

While the experimental group did spend more time on Global Monitoring than the control group, (Table 22), the percentage of protocol time spent dropped from the first to the second problem. The drop was over four percentage points but was not statistically significant.

The control group spent less time on the second problem as well, but the drop was less than one percentage point, and was not statistically significant.

The male subjects in the experimental group spent a greater percentage of their protocol time on global monitoring than the female subjects (Table 23). The difference was almost nine percentage points but did not result in a statistically significant U statistic (U=7, critical value=5). Four of the six male subjects spent a greater percentage of their protocol time on global monitoring in their first problem than their second, four of the five female subjects did as well.

In the control group, the male subjects did spend significantly more time on the Global Monitoring Episode than the female subjects, (U=7, critical value = 7, $p<.05$). An examination of the Direction of Change column shows that four of the six male subjects spent a greater percentage of their time on the second problem on this episode. The female subjects had a much different change, five of the six subjects had a positive change, they spent a greater percentage of

Table 22

Summary statistics for the percentage of solution time spent on Problem 1 and Problem 3 in the Global Monitoring Episode by treatment group.

Treatment	Problem	N	M	Md	Statistic	p
Experimental	P1	11	32.36	37.0	T=24.5	>.05
	P3	11	28.27	25.0		
Control	P1	12	23.17	24.0	T=36.0	>.05
	P3	12	22.42	21.0		

T=Wilcoxon Signed-Rank T Statistic

Table 23

Summary statistics for the percentage of solution time spent on the Global Monitoring Episode of Problem 3 by gender and treatment.

Treatment	Gender	N	Direction of Change +	Direction of Change -	Direction of Change NC	M	Md	Statistic	p
Experimental	Male	6	4	2	0	32.33	31.0	U=7	>.05
	Female	5	4	1	0	23.40	21.0		
Control	Male	6	2	4	0	25.50	30.5	U=7	<.05
	Female	6	5	1	0	19.33	14.0		
Experimental	Male	6	4	2	0	32.33	31.0	U=6	<.05
Control	Male	6	2	4	0	25.50	30.5		
Experimental	Female	5	4	1	0	23.40	21.0	U=9	>.05
Control	Female	6	5	1	0	19.33	14.0		

U=Mann-Whitney U Statistic

their time on global monitoring while doing their first problem.

When the male subjects in the two groups were compared, the males in the experimental group spent a significantly greater percentage of their time on global monitoring, ($U=6$, critical value = 7, $p<.05$). There was an inverse relationship in the Direction of Change for the two male groups. Four male subjects in the experimental group showed a positive change, more time on the first problem, and two a negative change. The inverse occurred in the control group.

Four of the six male subjects in the experimental group spent less time on global monitoring after watching the video, but this was still a significantly greater percentage than the males in the control group. Four of six males in the control group devoted more time to global monitoring in the second problem, but did not place the same emphasis on it as the males in the experimental group.

The females in the experimental group also devoted a greater percentage of their time to the Global Monitoring Episode than the females in the control group. The four percentage point difference was not statistically significant. Both groups of females showed a positive direction of change, with four of five experimental group females spending more time on global monitoring in the first problem, and five of six females in the control group.

There were five subcategories in the Global Monitoring

Episode. The experimental and control groups had identical means for the first subcategory M1 Refer Back to Part of Plan (Table 24). Only six of the seventeen subjects in either group spent time on this subcategory. No statistical analysis was conducted.

The experimental group spent a greater percentage of their protocol time on the second subcategory, M2 Refer to Procedure. Fifteen of the seventeen subjects in the experimental group referred back to their procedure compared to only eight of seventeen in the control group. The difference in the means resulted in a U Statistic of 96.5 which was not statistically significant (critical value=96). This may have been a result of the experimental group having a more highly developed global plan to which they were able to refer while solving the problem.

The experimental group spent nearly twice as much time on the third subcategory, M3 Re-read Question, than the control group. The difference was not statistically significant (U=115, critical value = 96). Nine of the seventeen experimental, and six of seventeen control group subjects spent time on this subcategory. Perhaps subjects in the control group did not refer back to the question as often because they did not have as clear a global plan, or their plan was incomplete.

The experimental group seems to have developed a global plan which they have referred to frequently. They spent a

Table 24

Summary statistics for the five subcategories of the Global Monitoring Episode.

Subcategory	Group	N	M	Md	A=0	Statistic	p
M1	Experimental	17	2.47	0	11	-	-
	Control	17	2.47	0	11		
M2	Experimental	17	5.35	5	2	U=96.5	>.05
	Control	17	3.47	0	9		
M3	Experimental	17	7.29	4	8	U=115	>.05
	Control	17	3.88	0	11		
M4	Experimental	17	14.76	15	0	U= 94.5	<.05
	Control	17	10.17	13	4		
M8	Experimental	17	0.0	0	17	-	-
	Control	17	0.18	0	16		

U=Mann-Whitney U Statistic
A=0 refers to the number of subjects who spent no time on that episode.
M refers to the mean of the percentage of protocol time spent on that episode.

M1 Refer Back to Part of Plan
M2 Refer to Procedure
M3 Re-read Question
M4 Refer to Plan and Re-read Chart
M8 Reach an Impasse

significantly larger percentage of time on subcategory M4 Refer to Plan and Re-read Chart, than the control group (U=94.5, critical value = 96, p<.05). All of the subjects in the experimental group spent some of their solution time on this subcategory, four of the seventeen control group subjects spent no time on M4. The final subcategory was M8 Reach an Impasse. Only one subject was unable to solve the second problem. This subject was in the control group. Either practising, or the video, enabled most of the subjects to at least complete the second problem.

The testing for subcategory effects within each of the treatment groups is reported in Table 25. The first experimental group, P1VP3, had five of six subjects showing a positive change in direction, spending a greater percentage of their time on global monitoring in their first problem. But for only one of the subcategories was there a testable positive change in direction. Five of six subjects had a statistically significant positive change of direction for M3 Re-read Question, (T=0, critical value=0, p<.05). Subcategory M8 Reach an Impasse, also had a positive change in direction. One subject had reached an impasse in their first problem, the other five showed no change. The majority of the subjects in the other three subcategories showed a negative change in direction, but none of these changes were statistically significant.

Group P2VP3 showed a non significant positive change in

Table 25

Summary statistics for the change in percentage of time spent on the Global Monitoring Episode by subjects who solved both Problem 1 and Problem 3.

Group	Subcategory	N	Direction of Change			Statistic	p
			Pos	Neg	NC		
P1VP3		6	5	1	0	T=5	>.05
	M1	6	1	3	2	-	-
	M2	6	1	3	2	-	-
	M3	6	5	0	1	T=0	<.05
	M4	6	2	3	1	T=7	>.05
	M8	6	1	0	5	-	-
P2VP3		5	3	2	0	T=7	>.05
	M1	5	1	2	2	-	-
	M2	5	1	4	0	T=1	>.05
	M3	5	2	2	1	-	-
	M4	5	2	3	0	T=6	>.05
	M8	5	1	0	4	-	-
P1P3		6	3	3	0	T=10.5	>.05
	M1	6	2	2	2	-	-
	M2	6	1	2	3	-	-
	M3	6	4	1	1	T=2.5	>.05
2	M4	6		3	1	T=3.5	>.05
	M8	6	0	1	5	-	-
P2P3		6	4	2	0	T=9	>.05
	M1	6	0	3	3	-	-
	M2	6	2	1	3	-	-
	M3	6	3	1	2	-	-
	M4	6	3	2	1	T=3.5	>.05
	M8	6	1	0	5	-	-

T=Wilcoxon Signed-Rank T Statistic
NC indicates no change
M1 Refer Back to Part of Plan
M2 Refer to Procedure
M3 Re-read Question
M4 Refer to Plan and Re-read Chart
M8 Reach an Impasse

direction, (three of five subjects). Testing of the subcategories resulted in no statistically significant changes. Once again M3 and M8 were the two subcategories which did not have a negative change. For subcategory M3 Re-read Question, the change for two of the subjects was positive, two negative, and one showed no change. One subject reached an impasse in his first problem but was able to obtain an answer for his second, the other four subjects in subcategory M8, were able to obtain an answer for both of their questions.

The first control group, P1P3, was equally split with three subjects having a positive and three having a negative change. It was not possible to test three of the five subcategories because of the number of students showing no change. M1, M2 and M8 had too many subjects showing no change. This was the group in which one subject reached an impasse in their second problem. There was a positive change in direction for subcategory M3 Re-read Question, less time was spent on this activity in the second problem, but the difference was not statistically significant (T=2.5, critical value=0). Three subjects spent more time on M4 Refer to Plan and Re-read Chart, in their second problem solution, two did not and one showed no change. This change was not statistically significant (T=3.5, critical value=0).

Group P2P3 had a non-significant positive change in direction for the Global Monitoring Episode, (four of six subjects). Because of the larger number of subjects with no change, only one subcategory was tested. There was a positive change in direction for subcategory M4 Refer to Plan and Re-read Chart. The change was not statistically significant (T=3.5, critical value=0).

Global Monitoring was an important part of the problem solving protocols of the subjects in this study. The male subjects took greater advantage of this episode with the experimental group males spending significantly more time on global monitoring than the control group males. The chart was used extensively by subjects in the experimental group to check their global plan. The video may have been instrumental in showing the subjects in the experimental group the value of the chart as a check on their progress.

Discussion of the Fifth Sub-Hypothesis

The fifth subhypothesis dealt with the Local Monitoring Episode. It was hypothesized that the videotaped protocol would increase the amount of time spent on local monitoring. The percentage of time spent by subjects in the experimental group on Local Monitoring was more than three times as great as that spent by the control group, 1.41% compared to 0.41% (Table 12). The median score for both groups was 0 (Table 26), with 4 subjects in the experimental group doing some local monitoring of the work on their second problem, but only one subject in the control group. Because of the small numbers it was not possible to test this difference. This sub-hypothesis was not supported.

Local monitoring was not done very often by subjects in either group. There was a decrease in time spent by subjects in the experimental group on this episode. As can be seen in Table 26

Table 26

Summary statistics for the percentage of solution time spent on the Local Monitoring Episode for Problem 1 and Problem 3 by treatment.

Treatment	Problem	N	Direction of Change +	-	NC	M	Md	A=0	Statistic	p
Experimental	1	11				1.91	0	8	-	-
	3	11				1.36	0	7		
	P1-P3	11	2	3	6					
Control	1	12				1.67	0	7	-	-
	3	12				0.58	0	11		
	P1-P3	12	4	1	7					

T=Wilcoxon Signed-Rank T Statistic
A=0 represents the number of subjects spending no time on Problem Representation

there were too few subjects to permit testing. Three subjects in the experimental group did some local monitoring in the first problem, four in the second. While there was an increase of one in the number of subjects doing some local monitoring in the second problem, these four subjects spent a smaller percentage of the protocol time on this episode.

The control group also had few subjects who spent time on local monitoring. Five subjects did some local monitoring in the first problem, but only one did in the second. It was not possible to test the difference in times because of the small numbers.

The bottom part of Table 26 shows that the majority of the subjects in both groups did not spend time on local monitoring. In the control group there was a definite shift away from local monitoring, four subjects spending part of their time on local monitoring in the first problem and only one in the second. Seven subjects spent no time in either problem on local monitoring.

In the experimental group two subjects showed a positive change in direction, three a negative change. Six subjects did not spend any time in either the first or second problem doing local monitoring.

There were too few subjects who spent time on the Local Monitoring Episode to permit further testing.

Local Monitoring is an important part of a problem solution. Errors of omission or substitution, can be caught before they have an impact on the problem solution. The subjects in the present study did not avail themselves of the opportunity to pick up the errors they might have made. Perhaps they felt constrained by time, or they may have been confident of their work and did not see any need to monitor it. Whatever the reason this valuable aspect of problem solving was ignored by most of the subjects in both the experimental and control groups.

Discussion of the Sixth Sub-Hypothesis

The final subhypothesis states that the videotaped problem solving protocol increases the amount of time spent on the Verification Episode.

The number of subjects conducting a verification of their work was small, one from the experimental group, and three from the control group (Table 27). Because of the small numbers a statistical analysis was not possible. This final hypothesis could not be confirmed.

In the experimental group only one subject performed a verification of their solution, this was on the second problem.

Three subjects in the control group spent time on the Verification Episode. One subject attempted to verify both of his problem solutions, spending a greater percentage of time on the second problem. The other subject spent time verifying his second problem only.

Table 27

Summary statistics for the percentage of solution time spent on the Verification Episode for Problem 1 and Problem 3 by treatment.

Treatment	Problem	N	Direction of Change +	-	NC	M	Md	A=0	Statistic	p
Experimental	1	11				0	0	11	-	-
	3	11				.27	0	10		
	P1-P3	11	0	1	10					
Control	1	12				.73	0	10	-	-
	3	12				.75	0	10		
	P1-P3	12	1	2	9					

T=Wilcoxon Signed-Rank T Statistic
A=0 represents the number of subjects spending no time on Problem Representation

No further testing was conducted because of the small number of subjects attempting to verify their solutions. Verification did not appear to be a very important part of the problem solving repertoire of these subjects. Perhaps verifying work completed isn't stressed in school, or possibly these subjects did not feel there was a need to verify this type of problem solution. Whatever the reason, a valuable opportunity to check the work completed to insure its accuracy was missed.

CHAPTER 5

QUALITATIVE ANALYSIS OF THE RESULTS

The quantitative analysis in the previous chapter presents one aspect of what occurs while problem solving. To complement this information the protocols will also be examined from a qualitative perspective. In the first section the solution paths employed by the subjects to solve the problems will be discussed. Examining the strategies used by the subjects, and determining which strategies led to success and which led to failure, shall lead to pertinent insights. The solution path selected by the subject is related to the quality of the global planning occurring to solve a problem. This issue will be examined in the second section. In the final section, the treatment effects will be discussed. It was hypothesized that the videotaped problem would have an impact on the problem solving processes adopted by these subjects. Through the data collected in the retrospective interviews, it was possible to isolate aspects of the videotaped problem solution which had a critical effect on the solution process, in particular the global planning process.

Solution Paths

In order to better understand the solution paths chosen by the subjects in the present study, it is important to be aware of the possible choices open to them. Figure 3 shows

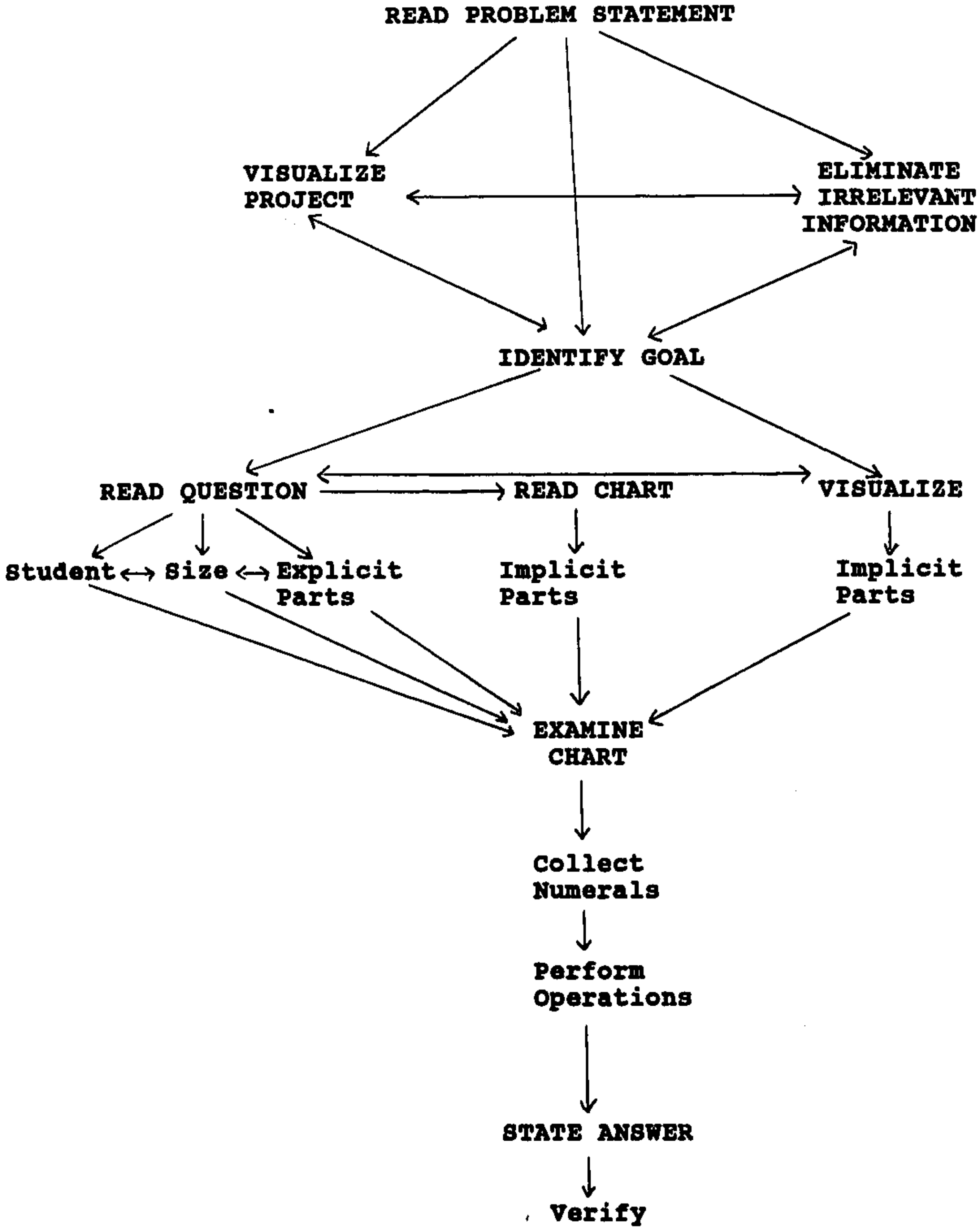

Figure 3. Template to Identify Potential Solution Paths

the potential routes to generate a correct solution to any of the three problems.

All the subjects read the problem statement out loud. Each problem had a written part and a chart. After reading the problem statement, the goal needed to be identified. This could be done by visualizing the project, or by re-examining the question and locating the relevant, and eliminating the irrelevant, information.

Once the goal was identified, the subject needed to develop a global plan to solve the problem. There were several sources of information available to the problem solver. There was information in the problem statement and in the chart accompanying the problem statement. By accessing the information from either of these two sources, or past experience, the subject could develop a global plan for the problem. The quality of the global plan depended upon the ability to use these information sources.

After the global plan had been developed, the subject would refer to the chart to extract the numerals to be used to obtain the answer. Once all the numerals had been collected, and the necessary arithmetic operation performed, the subject would state the answer obtained. This could be followed by a verification of the work done to ensure that the global plan had been correctly implemented.

The actual solution paths chosen by the students are discussed in the next section.

Strategies

The subjects in the present study rarely displayed a complete solution path as outlined in the preceding section. They selected certain solution elements, but ignored others. The resulting combination of elements by the subjects, produced four distinct categories of solution paths. These solution paths resulted from the choices made by the subjects while solving the problems, and are referred to as strategies in the following discussion.

The four strategies used by the subjects were: Random Search Strategy, Explicitly Stated Parts Strategy, Chart Monitoring Strategy, and Project Visualization Strategy. An overview of each follows.

Random Search Strategy

Solutions grouped into this category were characterized by a lack of organization and no clear idea of what was being asked in the problem statement. Subjects using this strategy did not visualize the project or identify the goal (Figure 4). A comprehensive plan for the solution of the problem was not developed, instead a random search was undertaken, re-reading the problem statement or examining the chart, in an apparent attempt to discover what was being asked in the question.

One or more of the three students named in the problem statement might be selected, and a search for a value

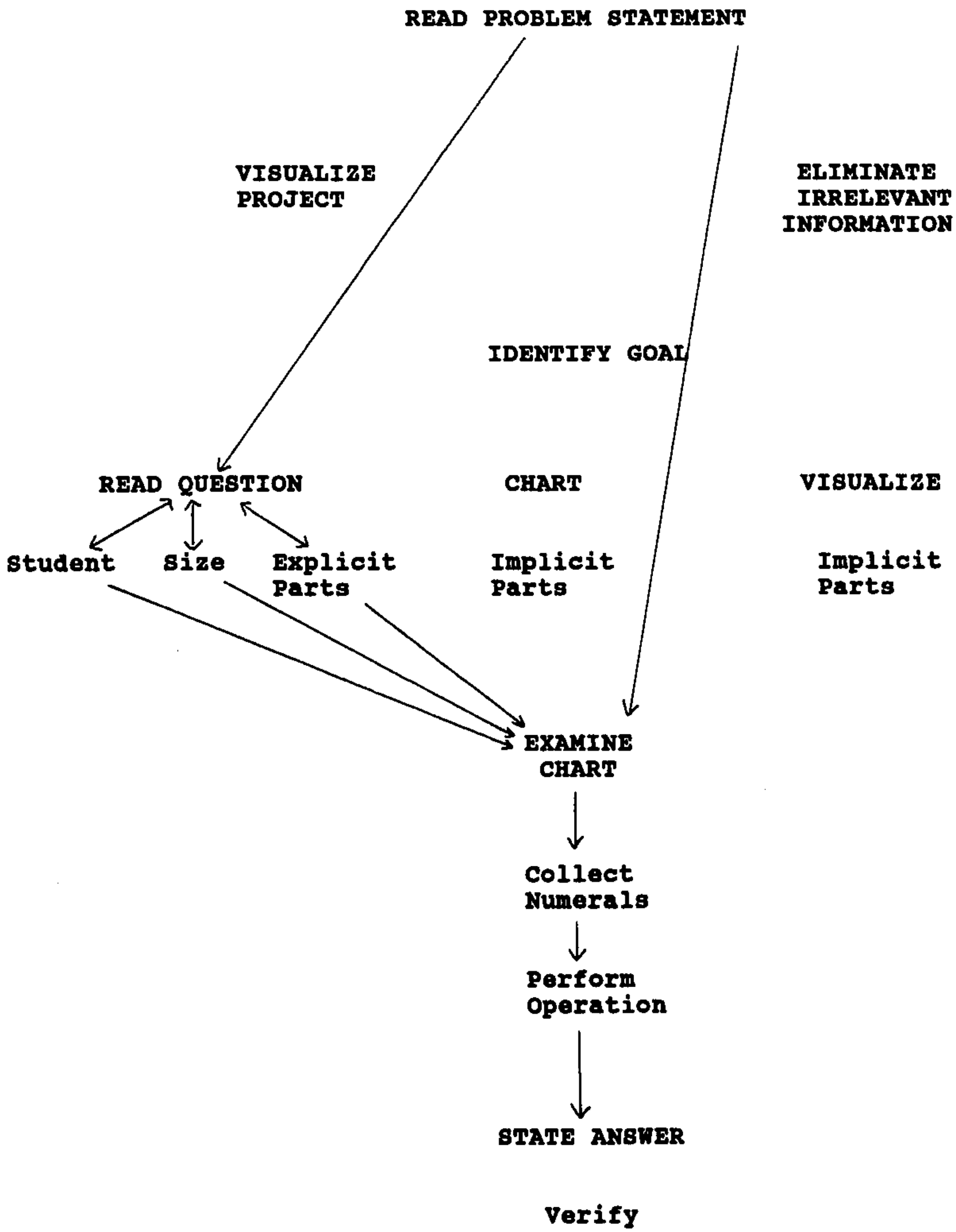

Figure 4. Template to Identify Potential Solution Paths for the Random Search Strategy

associated with that student or students conducted. Frequently a single value from the chart would be selected to represent the answer to the entire project. The choice might be based on the size, or one of the explicitly stated parts e.g. "a fort with one tower with windows at the front but not at the back", (excerpt from the Fort Problem). The chart would be examined for something with "a window" and that numeral written as the answer. A numeral adjacent to the size of the project, eg. suit size 8, or a numeral for each size mentioned in the problem statement might be selected. This strategy produced a faulty or incomplete global plan, and resulted in an incorrect answer.

Explicitly Stated Parts Strategy

The Explicitly Stated Parts Strategy followed a different path from the Random Search Strategy (Figure 5). After the initial reading of the problem statement a goal was identified. Unfortunately the goal identified was incorrect. It included only the parts which were explicitly mentioned in the problem statement.

The Explicitly Stated Parts Strategy was characterized, as the name suggests, by the selection of the parts explicitly mentioned in the problem statement. The strategy path alternated between the problem statement, where the explicitly stated parts were mentioned, and the chart where the appropriate values were located. Since these were real world

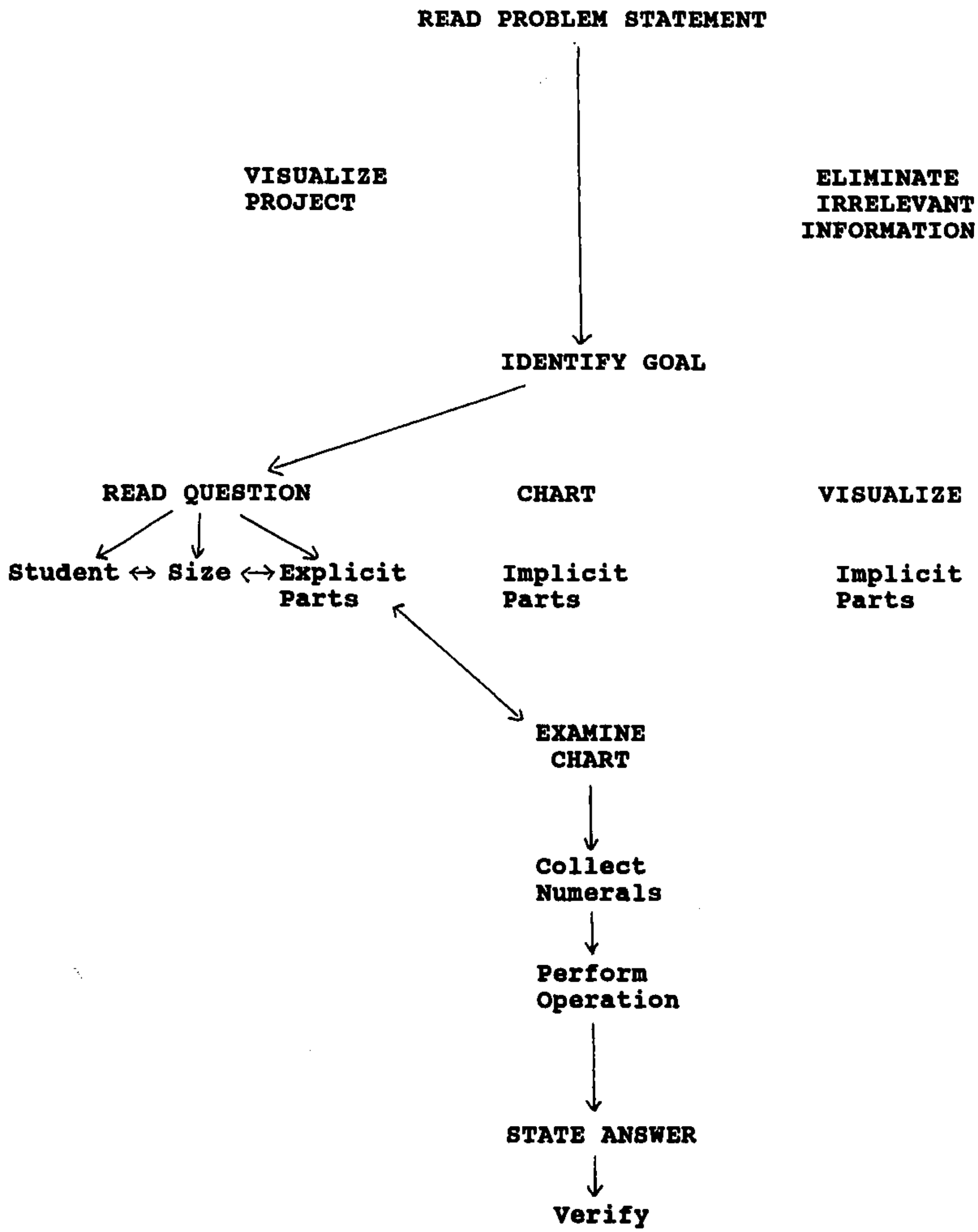

Figure 5. Template to Identify Potential Solution Paths for the Explicitly Stated Parts Strategy

construction projects, there were elements which varied e.g. the style of the front for the birdhouse, and others which remained constant, e.g. the birdhouse floor. The parts which varied were explicitly mentioned in the problem statement, those which did not vary were not mentioned, but values for these parts were included in the chart.

Once the numerals had been selected, the appropriate arithmetic operation was performed, and the answer stated. It was possible to verify the solution but this seldom occurred. A more complete analysis of the problem statement was required to realize that there was more required than what was explicitly stated. This strategy would lead to a birdhouse with a front, back and roof. The remaining parts could be found in the chart i.e. sides and floor, but this information was not included in the solution, perhaps because it was not mentioned in the question. Without these other parts the project was incomplete, and the solution incorrect. This strategy resulted in a more highly developed global plan than the Random Search Strategy, but was still not adequate by itself, to obtain a correct solution.

Chart Monitoring Strategy

The Chart Monitoring Strategy began as the Explicitly Stated Parts Strategy had, with the reading of the problem statement and the identification of the goal (Figure 6). Once the goal had been identified the problem statement and chart

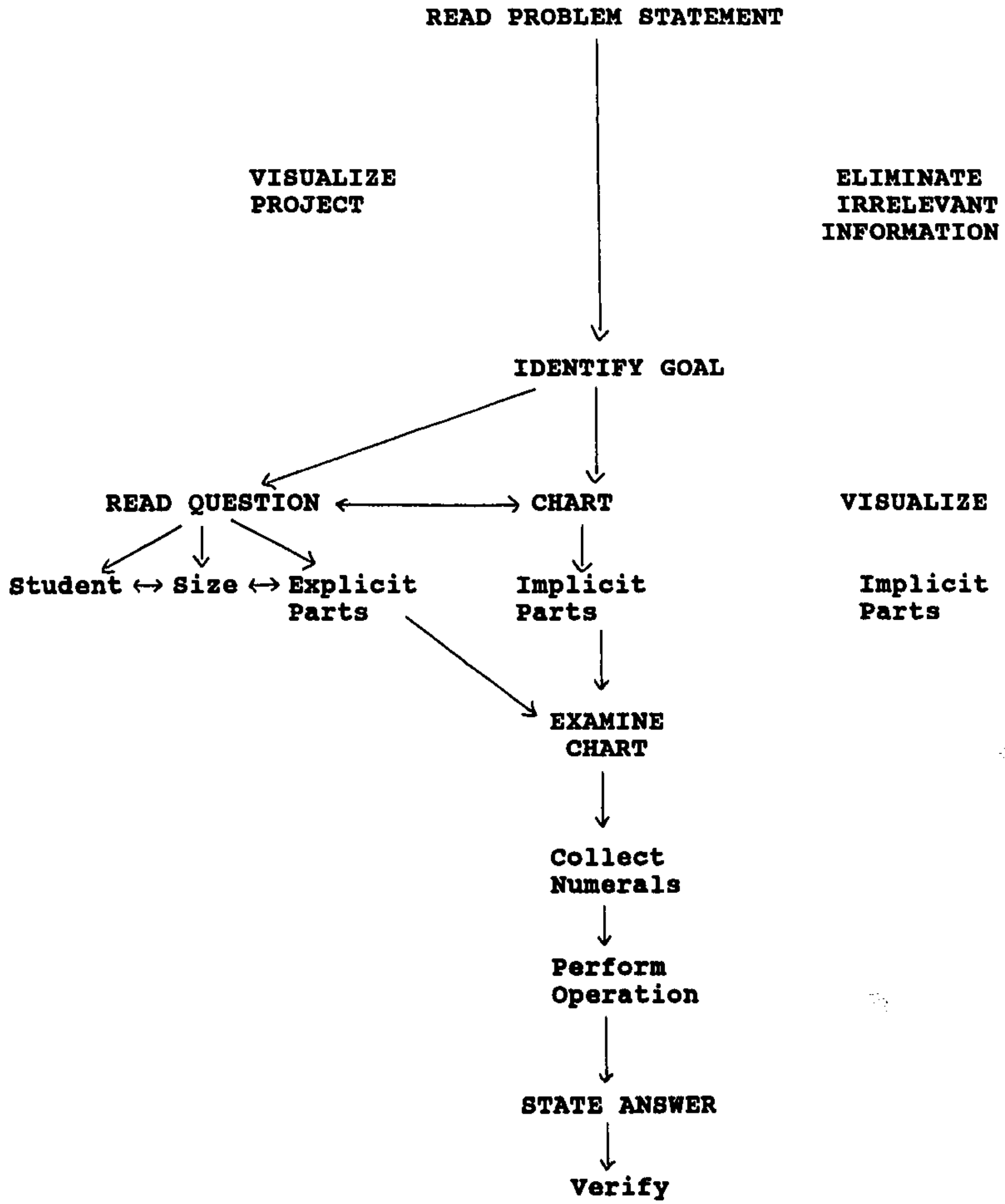

Figure 6. Template to Identify Potential Solution Paths for the Chart Monitoring Strategy

would be re-examined. Examination of the chart would frequently make the additional information apparent. In the Explicitly Stated Parts Strategy if the additional information was noticed, it was ignored.

Noting the additional information, might lead to the realization of why these parts were on the chart. The global plan could then be modified before it was implemented. But noting the additional information did not necessarily lead to an awareness of its relevance. Some or all of the additional parts could be included simply because they were on the chart. If the correct goal was not identified, a correct solution would not be reached.

The Chart Monitoring Strategy could be an effective strategy when accompanied by an accurate identification of the goal. While this strategy did not always produce an efficient solution path, it was more effective than the previous two strategies because it could generate a correct solution.

Project Visualization Strategy

Project Visualization was the strategy which most closely approximated the solution path presented in Figure 3. After the initial reading of the problem statement, time was devoted to the visualization of the project (Figure 7). This visualization could be a diagram actually drawn, or a mental representation of the project. Part of the process of identifying the goal involved the selection of the relevant

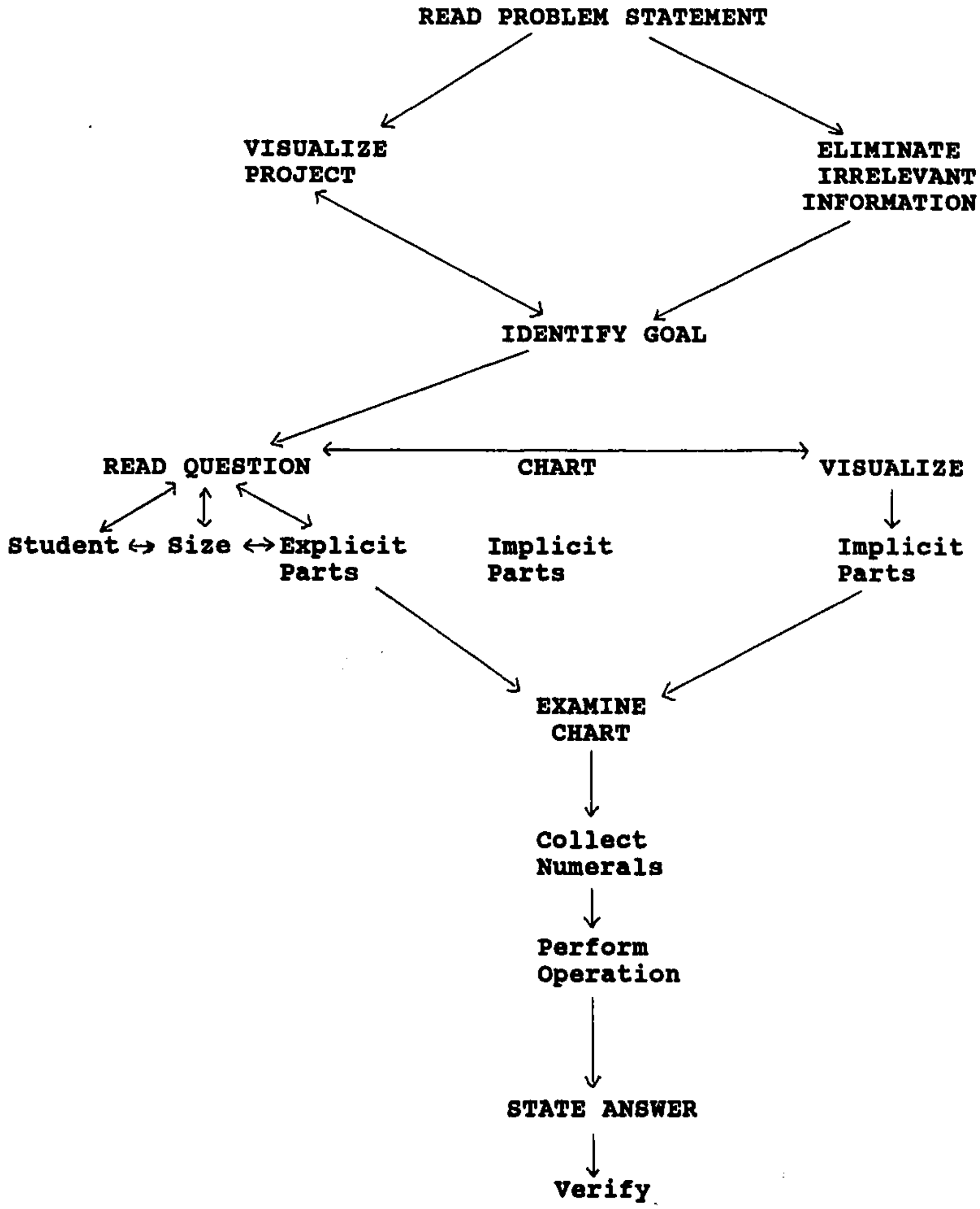

Figure 7. Template to Identify Potential Solution Paths for the Project Visualization Strategy

information, and the elimination of the irrelevant information. "Project visualization", and "eliminate irrelevant information", were not a part of the other three strategies. This early work on goal identification was important for the formation of a complete global plan.

Once the goal had been identified the question was re-read and the specific components for the project, e.g. student and size, were selected. The values for each of these components were obtained from the chart, and then the visualization was referred to and the implicit parts were identified. The chart was consulted once again and the values were found for these parts. Unlike the Chart Monitoring Strategy, the chart was not used for the purposes of monitoring an established global plan. The chart was referred to during the development of the global plan, and subsequently used to get the values for the different project parts.

When all the numerals were collected the arithmetic was performed, the answer obtained, and stated. Finally a verification of the solution could be done with the visualization as a reference if needed. This strategy was most likely to lead to an efficient solution path and a correct answer.

Impact of Strategy Choice on Global Planning

The quality of the global planning done by the subjects in the present study was related to their choice of solution strategy. The four strategies discussed in the preceding section required varying degrees of analysis prior to the actual development of the global plan. As the amount of analysis increased, so too did the likelihood of a good quality global plan, and a correct solution.

Global Planning Within The Random Search Strategy

The quality of the global plans associated with each solution strategy is reported in Table 28. The Random Search Strategy was used in thirteen protocols, and in twelve of them it resulted in a faulty global plan. In the most common example of this strategy subjects would select a single numeral as the answer to the question. In the time spent re-reading the problem, it did not appear that they had any understanding of what the question was asking them to find. They might pick out a few key words, e.g. a fort with one tower with windows (Fort Problem), and attempt to find a value on the chart for that phrase.

Another example of this strategy involved the addition of all the numbers on the chart for a particular attribute, e.g. size. Other subjects selected a numeral for a particular attribute for each person mentioned in the question.

The Random Search Strategy was characterized by the lack of, or ineffective, analysis of the problem, which resulted in a faulty or incomplete global plan. One subject simply added

Table 28

Summary of the quality of the global plan for each strategy choice.

		Group									
		Experimental					Control				
Strategy	Global	P1VP3		P2VP3		VP3	P1P3		P2P3		P3
	Plan	P1	P3	P2	P3	P3	P1	P3	P2	P3	P3
Random Search	Faulty	5	0	1	2	0	3	1	0	0	0
	Incomp.	0	0	0	0	1	0	0	0	0	0
	Complete	0	0	0	0	0	0	0	0	0	0
Explicitly Stated Parts	Faulty	0	0	0	0	0	0	0	0	0	0
	Incomp.	1	3	3	2	4	2	4	3	5	3
	Complete	0	0	0	0	0	0	0	3	0	0
Chart Monitoring	Faulty	0	0	1	1	0	0	0	0	0	0
	Incomp.	0	0	0	0	0	0	0	0	0	0
	Complete	0	2	0	0	1	0	0	0	0	2
Project Visual.	Faulty	0	0	0	0	0	0	0	0	0	0
	Incomp.	0	1	0	0	0	0	1	0	0	0
	Complete	0	0	0	0	0	1	0	0	1	0

Incomp.- Incomplete
Visual.- Visualization

everything up (2-1, Table 29), he felt that when there was a chart present you simply selected the numerals associated with the size which the question specified. Another subject realized that there was irrelevant information (5-1, Table 29), but he still said that he just added up the numbers, not making any plan. The one subject who was able to develop an incomplete global plan using this strategy, performed extensive global monitoring and revised his initial strategy.

Global Planning Within the Explicitly Stated Parts Strategy

The Explicitly Stated Parts Strategy was used thirty-three times (Table 28). Use of this strategy resulted in an incomplete global plan thirty times, three subjects were able to develop a complete global plan with this strategy.

The construction problems given to the subjects contained explicitly stated information to help solve the question. There was also information which was contained in the chart but not explicitly discussed which was required to obtain a correct solution. The Explicitly Stated Parts Strategy, when used on its own, could not lead to a correct solution because it did not take into account the information not explicitly discussed. The three subjects who did have complete global plan used global monitoring extensively to modify their initial plan.

Some subjects, when reading the problem, made note of the explicitly stated parts and went to the chart and picked out

Table 29

Excerpts from the followup interviews with subjects.

1-1	"When I had a diagram ... it was easier cause I just looked at it and you could picture it better."
2-1	"I just add everything up and that will be the answer" "You figure that by the chart you have to add everything because there's so many numbers you wouldn't multiply."
5-1	"I saw that he wanted to make a 6 bird house so um I just went down the columns to find what he wanted."
5-2	"with just a roof and front and back it'd look kinda stupid." "... things that are put in there extra to just to like screw you up and I make sure to avoid those."
5-3	"I just find the information and write it down and then do whatever multiplication I have to do or whatever." "Nope, same format."
5-5	"I just do whatever is there and never really think about what I'm doing." (Did you do anything different?) "Not really."

the appropriate numerals associated with these parts. They realized that they must add the numerals, did so, and stated their answer. There did not appear to be any attempt to give thought to the construction project in the problem statement. Most of the subjects who used this strategy, treated these questions as mathematics questions, rather than real world construction problems, and finding the explicitly stated parts may have seemed a reasonable method of obtaining an answer.

A number of subjects used this strategy for both of their problems and acknowledged that they had done little analysis of the problem, or global planning for their solution. One subject said that his plan was to find the information and then just do whatever operation was required (5-3, Table 29). Another said that he just did what was asked and didn't even think about it (5-5, Table 29). Some relied on the chart to help them pick out the explicitly stated parts in the question (5-1, Table 29).

The Explicitly Stated Parts Strategy was the most frequently used strategy by subjects in this study. Use of this strategy suggests a separation of mathematics problems in school, from problems in the real world which require mathematics. By attempting to obtain information from the question alone, students could not make a complete global plan, nor obtain a correct answer for the problem.

Global Planning Within the Chart Monitoring Strategy

The Chart Monitoring Strategy was used in seven protocols, and resulted in a complete global plan in five of those protocols (Table 28). Subjects using this strategy, as discussed earlier, took advantage of the chart to develop a complete global plan. After reading the question and noting the relevant information, the subject would examine the chart and note that there was additional information. This additional information led the subjects to reflect upon the difference between what was stated in the question and what was in the chart, and to conduct a more critical analysis of the question before completing their global plan. This frequently led to the development of a complete global plan.

The success of this strategy was in part dependent upon a careful reading of the question and chart. Noting that there was extra information contained in the chart was not sufficient to develop a complete global plan. The two faulty plans which were developed using this strategy were a result of the subjects inability to distinguish what was relevant. Some of the relevant extra information in the chart was not included in their global plan, leaving it incomplete.

Global Planning Within The Project Visualization Strategy

Project Visualization, the fourth strategy, was the least frequently used strategy. The defining characteristic of this strategy was the production of some form of visual representation of the project. In two of the four protocols which contained this strategy, the global plan was complete (Table 28).

Very few subjects visualized the project at any point in their solution, but those who did were quick to realize the benefits of doing so. When a diagram was drawn by the subject, it gave him something concrete to which he could refer as he was developing his global plan (1-1, Table 29). It also enabled him to distinguish what information was relevant and needed to be a part of the solution (5-2, Table 29).

Making a visual representation required the construction of the project before the solution began. In doing so the subject was able to note the inadequacy of the explicitly stated information when they looked at their diagram (5-2, Table 29). The subjects with a visual representation referred to it as they progressed through their solution. The impact of the visual representation was twofold: it improved the quality of the global plan, and it facilitated global monitoring.

Strategy Choice and Problem Solving Success

When the subjects were problem solving, the likelihood of obtaining a correct solution to the problem depended, in part, on the solution strategy which they employed. Choosing the Random Search Strategy was least likely to result in a correct solution being obtained (Table 30). None of the thirteen protocols in which this strategy was used produced a correct answer. Without some form of systematic analysis and global planning it was unlikely that a correct answer would be found.

Subjects using the Explicitly Stated Parts Strategy were also unlikely to obtain a correct answer. In Table 30 it can be seen that of the thirty-three protocols in which this strategy was used, only three contained correct answers. These correct answers were the result of effective global monitoring during the solution directed activity episode. The subjects in these three protocols referred back to the question and realized that their global plan was incomplete. They made the necessary corrections to their plan and obtained the correct answer.

Of the four solution strategies, the one which resulted in the highest percentage of correct answers was the Chart Monitoring Strategy (Table 30). Five of the seven protocols in which this strategy was used resulted in a correct solution. Subjects using this strategy made a global plan, and then used the chart to monitor their work. This led to a more complete global plan, and may have had a carryover effect leading to monitoring of the solution as it unfolded.

The Project Visualization Strategy was used in four protocols, two of which resulted in a correct solution (Table 30). Three of these four protocols were for the second problem which the subject was asked to solve, and twice it led

Table 30

Summary of the relationship between strategy choice and solution.

		Group									
		Experimental					Control				
Strategy	Answer	P1VP3		P2VP3		VP3	P1P3		P2P3		P3
		P1	P3	P2	P3	P3	P1	P3	P2	P3	P3
Random Search	Correct	0	0	0	0	0	0	0	0	0	0
	Incorrect	5	0	1	2	1	3	1	0	0	0
Explicitly Stated Parts	Correct	0	0	0	0	0	0	0	3	0	0
	Incorrect	1	3	3	2	4	2	4	3	5	3
Chart Monitoring	Correct	0	2	0	0	1	0	0	0	0	2
	Incorrect	0	0	1	1	0	0	0	0	0	0
Project Visual.	Correct	0	0	0	0	0	0	1	0	1	0
	Incorrect	0	1	0	0	0	1	0	0	0	0

Visual.- Visualization

to a correct solution. In the case of one of the correct solutions, the subject had used this strategy in his first protocol and obtained an incorrect answer. The other two subjects had not used this strategy for their first problem.

Changes in Strategy Choice and Global Planning

It was hypothesized that the videotaped problem solution would be an effective instructional intervention. If the video was effective, the two experimental groups P1VP3 and P2VP3 should select a more efficient strategy for the solution of their second problem, the Birdhouse Problem. The video should also lead to an improved global plan. The next section examines the change in strategy choice for the experimental and control groups. The following section examines the impact of the video on the global planning of the experimental and control groups.

Impact of The Video on Strategy Choice

The subjects in the experimental group P1VP3 first solved the Fort Problem, watched a video of a student solving the Fort Problem, and then solved the Birdhouse Problem. Five of the six subjects used the Random Search Strategy to solve the Fort Problem (Table 28). The other subject in this group used the Explicitly Stated Parts Strategy. Neither of these strategies was adequate to obtain a correct solution.

For each of the six subjects there was an improvement in the quality of the strategy chosen after watching the video (Table 31). No one obtained the correct answer to the Fort Problem, but two of the six subjects obtained the correct solution for the Birdhouse Problem. Both of the correct solutions came from the subjects who had used the Explicitly Stated Parts Strategy to solve the Fort Problem. These subjects used the Chart Monitoring Strategy for the Birdhouse Problem. One of the subjects attributed his success to the video showing that there were also implicit elements to the problem (1-4, Table 32).

The four remaining subjects had used the Random Search Strategy to solve the Fort Problem. When they were doing the Birdhouse Problem they used the Explicitly Stated Parts Strategy. Their global plans were incomplete, containing only the explicitly stated information, but more closely approximated a correct solution path than their plan for the Fort Problem.

One subject felt more confident about his answer to the Birdhouse Problem because of his use of a diagram (1-1, Table 32). Another learned the need to look at all the information before beginning the solution (1-5, Table 32). The video helped another to realize that there was more than a single numeral required (1-6, Table 32).

The video had a positive impact on all of the subjects in

Table 31

Summary of Strategies Used by Subjects for their Two Problems.

		Subject 1		2		3		4		5		6	
		Problem											
Group	Strategy	F	B	F	B	F	B	F	B	F	B	F	B
P1VP3	Random Search	*				*		*		*		*	
	Exp Stat Part			*			*				*		*
	Chart Monitor				*				*				
	Project Visual		*										
P1P3	Random Search	*						*	*	*			
	Exp Stat Part		*	*	*						*	*	*
	Chart Monitor												
	Project Visual					*	*						
VP3	Random Search						*						
	Exp Stat Part		*						*		*		*
	Chart Monitor				*								
	Project Visual												

F - Fort Problem
B - Birdhouse Problem
Exp Stat Part - Explicitly Stated Parts
Chart Monitor - Chart Monitoring
Project Visual - Project Visualization

Table 32

Excerpts from the interviews with subjects in group P1VP3 and group P1P3.

1-1	"When I had a diagram ... it was easier cause I just looked at it and you could picture it better."
1-4	"It (video) showed me where I went wrong ... like don't forget the sides and the floor and the roof."
1-5	"I read it (problem) more. He showed me to look at all the information before you write anything down."
1-6	"I add up all the sides instead of just getting one answer."
3-2	"We're supposed to do a four step problem but I find it easier to just add it up."
4-3	"Ya I checked for which uh kind of roof he had and I didn't check for what kinda thing they had there cause I didn't notice it."
4-5	"On the second problem I added three numbers ... the first one I only had to find one number."
5-2	"with just a roof and front and back it'd look kinda stupid." "... things are put in there extra to just to like screw you up and I make sure to avoid those."
6-1	"I just looked at all the things that ... he was gonna use and wrote them all down and...just added them altogether."
6-2	"My teacher teaches us like X = the problem, X = what we're doing and X = the answer. (Do you use those steps?) "Not usually. In class he says we have to, so I use those steps."
6-5	"I always start with addition then go to subtraction then go to multiplication then go to division."

Group P1VP3. They had a better idea of what was expected, and because of their strategy choice, were able to make a more effective global plan to solve the Birdhouse Problem.

The second group which saw the video, P2VP3, had only five subjects. They first solved the Suit Problem, saw the video of the Fort Problem, and then solved the Birdhouse Problem. While the sequence for Group P2VP3 was similar to Group P1VP3, the results of the strategy choice was quite different for the two groups.

When solving the Suit Problem, one subject used the Random Search Strategy, three the Explicitly Stated Parts Strategy, and one the Chart Monitoring Strategy (Table 33). Each of these strategies led to a faulty or incomplete global plan, and an incorrect answer.

None of the five subjects in Group P2VP3 improved their global planning after watching the video. Four of the subjects used the same strategy for the Birdhouse Problem as they had used for the Suit Problem.

The video had a positive impact on one of the subjects who realized that there were other parts needed for the project. This subject included the implicitly stated parts, but did so for each person named in the problem. The strategy chosen by this subject could best be described as a Random Search Strategy because of the lack of a clear idea of what the question asked. The plan which he developed was clearly faulty and led to an incorrect solution.

Table 33

Summary of Strategies Used by Subjects for Their Two Problems.

		Subject 1		2		3		4		5		6	
		Problem											
Group	Strategy	S	B	S	B	S	B	S	B	S	B	S	B
P2VP3	Random Search			*	*				*				
	Exp Stat Part					*	*	*		*	*		
	Chart Monitor	*	*										
	Project Visual												
P2P3	Random Search												
	Exp Stat Part	*	*	*		*	*	*	*	*	*	*	*
	Chart Monitor												
	Project Visual				*								
P3	Random Search												
	Exp Stat Part				*		*		*				
	Chart Monitor		*								*		
	Project Visual												

S - Suit Problem
B - Birdhouse Problem
Exp Stat Part - Explicitly Stated Parts
Chart Monitor - Chart Monitoring
Project Visual - Project Visualization

The difference in the effectiveness of the video with the two groups may be related to the first problem solved. The initial problem for group P1VP3 was the Fort Problem, group P2VP3 solved the Suit Problem. The Fort Problem was the one which both groups watched on the video. Seeing a problem being worked out was not sufficient to insure an improvement in strategy choice. It may be more appropriate to attribute the improvement in the strategy choice of group P1VP3 to the fact that these subjects saw the problem which they had just completed being solved in the video.

Groups P1P3 and P2P3 were the two control groups which received the same sequence of problems as P1VP3 and P2VP3 respectively. Because practise might have had an impact on the choice of solution strategy for subjects in each of these four groups, it was necessary to examine the changes in solution strategy choices for subjects not viewing the video.

Group P1P3 solved the Fort and Birdhouse Problems as did group P1VP3. Three of the subjects in P1P3 used the Random Search Strategy to solve the Fort Problem, two the Explicitly Stated Parts Strategy, and one the Project Visualization Strategy (Table 31). Unlike group P1VP3 in which all subjects used a better strategy and developed an improved global plan, only two subjects in group P1P3 changed strategies. Two of the subjects who had used the Random Search Strategy switched to the Explicitly Stated Parts Strategy. As a result of this change their global plan improved, but not enough to allow them to obtain a correct solution. The other four subjects used the same strategy and did not improve their global plan. The subject who had used the Project Visualization Strategy had developed a complete plan for the Fort Problem but had made an incorrect assumption about one of the sides and as a result obtained the wrong answer. For the Birdhouse Problem his initial plan was incomplete, but with effective global monitoring he was able to correct the plan and obtain a correct solution (4-3, Table 32).

The two subjects who did improve their global plan had selected a single value to answer the Fort Problem. In the Birdhouse Problem they added the explicitly stated parts, but still did not obtain the correct solution. The improvement of the global plans for these two subjects reflects the change in strategy employed which may have been a result of practise, or it could also be past experience suggesting that they needed to do more to find the answer (4-5, Table 32).

The greater improvement noted for group P1VP3 cannot be attributed to practise alone, it appears that the video did have an impact on the strategy choice of subjects in this group.

Group P2P3 solved the Suit Problem before attempting the Birdhouse Problem but did not see the video which group P2VP3 watched between problems. All of the subjects in P2P3 used the Explicitly Stated Parts Strategy to solve the Suit Problem (Table 33). Three of the subjects used global monitoring to alter their global plan as they solved the problem. This led to the development of a complete global plan and a correct solution.

Five of the subjects who used the Explicitly Stated Parts Strategy for the Suit Problem used the same strategy to solve the Birdhouse Problem. Two of the subjects who had used global monitoring failed to do so for the Birdhouse Problem and consequently developed an incomplete global plan.

One subject used the Project Visualization Strategy to solve the Birdhouse Problem and developed a complete global plan and obtained the correct answer (5-2, Table 32).

There was little difference in the impact of practise and the video on groups P2VP3 and P2P3. There was little improvement in strategy choice or global planning. In some cases practise seemed to make the subject less willing to monitor his work.

Group VP3 watched the video of the Fort Problem and then solved the Birdhouse Problem. The choice of strategy by the subjects in group VP3 was not unlike that of the subjects in P2VP3 (Table 31). One subject used the Random Search Strategy, four the Explicitly Stated Parts Strategy, and one the Chart Monitoring Strategy.

Only one subject in group VP3 used a strategy which was better than the Explicitly Stated Parts Strategy. This subject was able to develop a complete global plan using the Chart Monitoring Strategy. He realized the need to eliminate the irrelevant information, did some local monitoring of his written work, and obtained a correct answer. This subject attributed his success to ignoring the method advocated by his teacher (3-2, Table 32).

The strategy choice of the other five subjects did not lead to an effective global plan and solution. The subject who used the Random Search Strategy developed a faulty global plan, the other four subjects used the Explicitly Stated Parts Strategy and developed incomplete global plans.

Group P3 solved the Birdhouse Problem only. There was little difference in the strategy choice between groups VP3 and P3. In group P3 three subjects used the Explicitly Stated Parts Strategy, two the Chart Monitoring Strategy (Table 33). Both of the subjects using the Chart Monitoring Strategy developed complete global plans and produced a correct solution. These two subjects had been rated as good problem solvers by their teachers. They had the skills to solve the problem and monitor their work to find errors. One subject was able to select the relevant information and perform the appropriate operation (6-1, Table 32). The other subject was not able to articulate as well how he had solved the problem. He referred only to the order in which he performed the four operations (6-5, Table 32).

The plans for the other three subjects were incomplete. One subject echoed the concerns of other students regarding the relevance of, and need to, follow the method taught in class (6-2, Table 32). A number of subjects felt that there was one method for problem solving which could be used in all instances. This might account for the difficulty which many subjects experienced in selecting a strategy to solve the problem.

Impact of the Video on Global Planning

In this section, the quality of the global planning done by each subject in the experimental and control groups will be examined. A summary of these results is presented in Table 34.

When we compare the two experimental groups which solved two problems, P1VP3 and P2VP3, with the two control group which also solved the same two problems, P1P3 and P2P3, we do see certain similarities and differences. In the Solution column of Table 34 we see that two of the eleven subjects in the experimental group obtained the correct solution for the Birdhouse Problem, as did two of the twelve subjects in the control group. This represents very little difference. What is of interest is that both of the subjects in the experimental group who answered the Birdhouse Problem correctly had improved their global planning from the first problem which they had solved incorrectly. The two subjects in the control group who had solved this problem correctly had also solved their first problem correctly. What we saw in the control group was continued good problem solving, in the

Table 34

Summary of the relationship between global planning and solution correctness, and the change in global planning for individual subjects in each group.

Group	Solution		Global Plan			Change in Global Plan		
	Cor	Incor	Comp	Incom	Faulty	Impr	No Change	Worse
P1VP3	2	4	1	5	0	6	0	0
P2VP3	0	5	0	2	3	0	4	1
VP3	1	5	1	4	1			
P1P3	1	5	1	4	1	2	4	0
P2P3	1	5	1	5	0	0	4	2
P3	2	3	1	4	0			

Cor = Correct, Incor = Incorrect
Comp = Complete, Incom = Incomplete
Impr = Improved

experimental group there was an improvement in the problem solving ability of these subjects.

The real difference between the experimental and control groups was the change in their choice of strategy and ability to develop an appropriate global plan. The global plans of seven of the eleven experimental group subjects improved after the videotaped problem had been viewed (Table 34). Only two of the twelve control group subjects improved their global plan on their second problem. Clearly, practise alone was not enough to lead to an improved global plan, the videotaped problem solution did have a positive impact. The six subjects in the experimental groups who improved their global plan were in the group which viewed the solution to the problem which they had just attempted to solve. Seeing the solution to a problem within the immediate experience of the subject, was an effective intervention.

One of the subjects in the experimental groups developed a less effective global plan for his second problem, four showed no change. Two subjects in the control groups had a worse global plan, and eight showed no change (Table 34).

When the two groups which only solved one problem (VP3 and P3) were included, there was little difference in the number of correct solutions between the experimental and control groups. Three of seventeen subjects in the experimental groups P1VP3, P2VP3 and VP3, solved the Birdhouse Problem correctly, compared to four of seventeen in the control groups P1P3, P2P3 and P3 (Table 34). One subject in each group did not develop a complete global plan before they began their solution, they obtained the correct answer through careful monitoring of their progress, and changing their plan during the solution. Eleven of the seventeen subjects in the experimental group developed an incomplete global plan, and four developed a faulty global plan (Table 34). In the control group thirteen subjects attempted to solve their problem with an incomplete global plan, and one had a faulty plan.

The difference in performance of the experimental and control groups may be attributed to the change in their choice of strategy, and ability to develop an effective global plan. Seeing the videotaped solution of a problem just attempted led to a greater improvement in both of these areas.

CHAPTER 6

CONCLUSIONS

The objectives of the present study have been twofold. First to determine how effective a videotape of a grade seven student problem solving would be as an instructional intervention, and second to determine what aspects of mathematical problem solving the video influenced. In order to meet these objectives, the interpretation of the results was conducted on two different levels. In Chapter 4 the results of a quantitative analysis of the data testing the global hypothesis and each of the subhypotheses were presented. In Chapter 5 the protocols were analyzed from a qualitative perspective. While the former was directed at confirming or rejecting the hypotheses, the latter sought to provide insights into the actual problem solving process.

The following sections will attempt to reconcile the results of both analyses, and discuss the impact of this instructional intervention on problem solving in mathematics. This chapter will conclude with recommendations for future research.

Mathematical Problem Solving

From the literature a conceptualization of human problem solving was derived and presented in chapter 2, Figure 1.

From this conceptualization a model solution path for the construction problems used in the present study was developed and presented in chapter 5, Figure 3. The model did accurately reflect what occurred in the problem solving of these students.

The global hypothesis stated that a videotaped problem solution reduces the amount of time spent solving a subsequent problem. While the present study was exploratory, and the sample size small, there was support for this hypothesis. When the experimental and control groups were compared, there was not a significant difference in the amount of time spent on the second problem. The experimental group increased the number of correct solutions from zero to two, while the control group fell from three correct to zero on their second problem. The video intervention appears to have improved the efficiency and quality of their problem solving.

The Induction framework of Holland et al (1986) discusses the role of rules, mental models and strategies in learning. In mathematics how can these best be developed? Bruner (1961) and others have discussed discovery learning, peer teaching and related instructional interventions as effective methods for learning. Following confirmation of the global hypothesis, that the videotaped problem solution was an effective instructional intervention, the processes involved in each of the problem solving sessions were examined to determine what impact the peer videotaped intervention had on each episode.

If the video intervention was to facilitate learning, this might be reflected in the development by the subjects of a mental model and the related strategies for solving this type of problem. In the next section we will examine the evidence for the development of a mental model as a result of the intervention.

The Development of a Mental Model

Before solving a problem a student must analyze the question and develop a global plan for its solution. This global plan will, if successful, form the basis of a mental model which the student can invoke when a similar problem is encountered. Because the video intervention resulted in a reduction in time spent on the analysis of the problem, it might be argued that this was because a better mental model was now available. The design of the study allowed the researcher to examine this possibility while controlling for the chance that the mental model was developed as a result of practise alone.

With the appropriate mental model it is expected that less time will be needed to analyze the problem before solving it. The first subhypothesis stated that the amount of time spent on Analysis would be lower for the group receiving the video intervention. There was empirical evidence to support this hypothesis. The experimental group not only spent less time on Analysis in their second problem, but they spent significantly less time than the control group. The main reason for this decrease in time spent on Analysis was due to less time being spent examining the question. It is possible that less time was needed to examine the question because the mental model was available to be invoked following the initial reading of the problem.

The control group also spent less time on Analysis in their second problem. This raises the question of whether the video intervention was the reason for the drop in time spent on Analysis, or whether it was a practise effect from solving the initial problem. Was a mental model developed? If so, was it the result of the video, practise, or a combination of the two? This question will be addressed in the next section.

The second source of support for the efficacy of the video intervention came from the examination of the third subhypothesis. Contrary to the subhypothesis which states that the video reduces time spent on Solution Directed Activity, the experimental group spent a greater percentage of their solution time on Solution Directed Activity in their second problem although the increase was not statistically significant. One possible explanation for this finding is that the students were more efficient with the other aspects of their solution after the video intervention and consequently the percentage of time spent on Solution Directed Activity increased, (the time spent on Analysis had dropped as discussed earlier). Another possible explanation is that because this episode involved writing down the numbers and adding them, there may not have been as much of an opportunity for a decrease in the amount of time needed to do this.

The control group also spent a greater percentage of their solution time on Solution Directed Activity, this increase was statistically significant. The difference may be accounted for in one of the sub categories of this episode. The control group spent significantly more time stating the answer. If the students in the experimental group were working with a more complete global plan they may have felt more confident of their answers and either did not bother to state them, or did so very briefly. The control group was more tentative and tried to confirm their answer while stating it, or expressed their answer in a questioning manner.

The experimental group spent significantly more time on Global Monitoring as hypothesized. If their global plan was based on a mental model developed in the first problem and consolidated by the video intervention, these subjects might feel more confident referring to the global plan which they had developed, to monitor their progress. The experimental group spent more time referring to the plan and re-reading the chart. The control group may not have been as confident of their global plan and would be less likely to refer to it to check their progress.

The following section will present a discussion of the strategies employed by the problem solvers as evidence of the impact of the video intervention on this aspect of learning.

The Impact on Strategy Selection

The video intervention seemed to have an impact on the problem solving process, as measured by the percentage of time spent on the different episodes in the solution. To determine if learning was occurring, the verbal protocols were examined for some change in the problem solving process. Were the students becoming faster problem solvers, or were they becoming better problem solvers? To answer this question, the problem solving strategies employed by the students were examined.

As discussed in Chapter 5, the subjects in the present study used one of four strategies to problem solve. The four strategies were ranked according to their adequacy for deriving a correct solution to this type of problem. The first two strategies: Random Search and Explicitly Stated Parts, were not sufficient by themselves to obtain a correct answer. The final two strategies: Chart Monitoring, and Project Visualization, when properly applied could lead to a correct solution. The strategies were progressively more sophisticated. Change to a higher order strategy to solve the second problem, was regarded as evidence of learning having occurred.

Group P1VP3 saw a videotaped solution of the problem which they had attempted. The student in the video used the highest level strategy, Project Visualization, to solve the problem. When the solutions to the first and second problem were examined, each student in this group had used a higher order strategy to solve the second problem. One of the students actually jumped from the lowest strategy to the highest after watching the video. None of the six subjects in this group solved the first problem correctly, two solved the second one correctly. Both of these students attributed their success to the video, they knew what they had to do to solve the problem and consequently spent more time developing their plan. They were less tentative finding and stating their answer.

The second group, P2VP3, attempted as their first problem the Suit Problem, which was different from the problem in the video. All but one student used the same strategy to solve the second problem. The student who changed strategies, used a lower one for his second problem. None of these students obtained a correct solution to the second problem. They claimed that the video had reassured them that they were solving the problem correctly. The video did have a positive impact on one of the students who realized that there were other parts necessary for the project. Unfortunately he still did not identify the question correctly before beginning his analysis and he developed a faulty global plan in which he added the parts required by all the people mentioned in the question.

Seeing the video did have an impact on the strategy choice of the subjects in groups P1VP3 and P2VP3. Seeing the problem which they had just attempted being solved on the video was more effective for improving their strategy choice.

The final experimental group, VP3, did not solve a problem before watching the video and solving the Birdhouse Problem. As a group their choice of strategy was somewhat higher than P2VP3, but not as good as P1VP3. Seeing the solution to the problem just attempted was the most effective instructional intervention.

The control group P1P3 solved the same sequence of problems as group P1VP3 but they did not view the video. Only two of the students in this group improved their choice of strategy compared with all six students in the corresponding experimental group. One student used the Project Visualization Strategy to solve both problems, and solved the second one correctly, as a result of more careful use of the chart to select values for his project. For this group as a whole, the improvement was not as dramatic without the video intervention.

The control group P2P3, solved the same two problems which group P2VP3 solved. As with the second experimental group, only one student improved his strategy choice. He employed the Project Visualization Strategy for his second problem and obtained a correct answer. This student attributed his success to using this strategy, the other students in P2P3 had no clear idea of what strategy they had used. Practise was of little value to the students in this group.

The final control group, P3, solved the final problem only. Their strategy choice was similar to the choices of the other two control groups which had solved two problems. Practise alone was not influential in improving strategy choice.

SUMMARY

The present study was designed to examine the impact of a videotaped problem solution on the problem solving of grade seven mathematics students. A review of the literature indicated that peer tutoring and discovery learning were in and of themselves effective teaching techniques. It was also reported that as few as one example was necessary for prototype extraction (Dykstra, 1988) and that when the source and target problems share surface and structural similarities transfer is much more successful (Novick, 1988). Peer tutoring and discovery learning were combined with the aid of videotape technology to form an educational intervention which was not found in the literature.

One of the six groups, group P1VP3, worked a problem, saw a correct solution to that problem, and then solved a second problem which shared surface and structural similarities. The protocols of the subjects in this group were compared with subjects in other groups. Three groups had a practise problem, one viewed the same problem as group P1VP3. Two groups did not have a practise problem, one of these groups saw the video.

Transfer did occur for the experimental groups, these subjects spent less time on the problem they attempted after watching the video than the control groups. The drop in time spent by the students described as above average problem solvers by their teachers, was statistically significant. There were too few average and below average problem solvers to permit analysis, but the magnitude of the time change for these two groups was equal to or greater than the high performing group.

Subjects in the experimental group P1VP3 spent less time on analysis of the second problem. When they watched the videotape of the problem which they had just attempted, they seem able to develop an appropriate mental model which aided the solution of the isomorphic second problem. They picked out the deep structure of the problem quickly and were able to make an effective global plan and execute it.

There was no significant difference between the experimental and control groups on the percentage of time spent actually solving the problem, the Solution Directed

Activity Episode, but the experimental group did spend more time global monitoring. Because they may have had a better understanding of the problem, the experimental group may have been more aware of the necessity of a global plan, and been better able to make an effective global plan. They could then refer to this plan while executing it. Groups with a poor global plan did not see the need to refer to their plan as a check on their progress. Both male and female subjects in the experimental group spent more time global monitoring, for the male subjects the difference was statistically significant.

Too few subjects performed local monitoring or a verification of their solutions to permit analysis. Both of these activities were present in the videotaped solution, but were not stressed and did not appear to be an important aspect of problem solving for these subjects.

Practise on one problem did not appear to be adequate to develop the mental model necessary for a successful problem solution. The two control groups which did have a practise problem did not perform significantly better on their second problem. They did not receive any feedback from the researcher on their first problem, and by not verifying or monitoring their problems, they did not give themselves any feedback.

All of the subjects in the experimental group who saw the solution to the problem which they had attempted, improved their strategy selection and global planning. The modelling of the correct solution for a problem with which they had experience was effective in developing the mental model necessary to solve a quasimorphic problem. These subjects knew whether or not their answer was correct, and were able to learn on what aspects of the problem they had erred. The other experimental group which did have a problem on which to practise, was no more effective on their second problem than the control groups. Seeing a videotaped solution of a new problem did not provide feedback based on their personal experience, and consequently learning may not have occurred to the same extent that it did for the other experimental group.

DIRECTIONS FOR FUTURE RESEARCH

Viewing the videotape of a peer's problem solution within the context of a student's immediate experience, was an effective intervention and supports the research on peer teaching and discovery learning. Some empirical support has been found for the efficacy of a videotaped problem solution in lieu of actual contact with a teacher or peer. It remains for future research to determine whether the videotape of an adult's solution would have been as effective.

The video intervention was administered once and produced positive results. This approach was taken because of the literature on prototype extraction and development of mental models. The impact of repeated use, of a videotaped problem solution remains to be explored. Perhaps repeated use outside of the context of personal experience would be as effective as use within context. The long term impact of this intervention was also not explored. Whether these students would continue to perform this well after a week or a month requires further study.

It would also be interesting to see whether this intervention was more effective than seeing a written solution to the problem. The video input and explanation might be superfluous if the positive results were simply a result of the feedback on the problem just solved.

The skills learned by the group which saw the videotaped problem within the context of their immediate experience, were transferred to a quasimorphic problem. How often this process must be repeated before the skills involved are mastered and can subsequently be transferred to related problems needs to be explored.

Having demonstrated the effectiveness of this intervention in one area opens up the possibility of its use with other age and grade levels, as well as other subject areas.

CONCLUSIONS AND EDUCATIONAL IMPLICATIONS

The results of the present study indicate that allowing a student to attempt to solve a problem and then providing a videotaped demonstration by a peer of a correct solution to that problem will enhance the student's performance on a quasimorphic problem. That peers are effective teachers has been well established in the literature. The impact of this work hopefully will be to encourage the extension, through the use of current technology, of the role of students in helping their peers.

In mathematics the link between conceptual and procedural knowledge is critical. Because this intervention was successful in developing the mental model necessary to solve a quasimorphic problem, perhaps it can be used by educators to establish or reinforce the link between these two types of knowledge.

The individualized attention of a tutor, be it a peer or an adult, is beneficial to the student. A peer teaching video such as the one used in this study is inexpensive to obtain, and can be used for many students and many years. The results of the present study indicate that this intervention is effective with good problem solvers, and may also work well with low and average students. As a result it may be useful in both a remedial and enrichment context.

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APPENDIX A: PARENTAL PERMISSION FORM

Dear Parents

Your child has expressed an interest in participating in a research project being conducted by Wayne Sheldrick, a PhD student from the Faculty of Education at the University of Ottawa.

The study will be examining the effects watching a peer solve a mathematics word problem on video has on the strategies which the student will use. Students will be asked to solve either one or two word problems. The students will work at a table which enables us to video tape their written work as well as what they say as they are solving the problem. The problem solution, including what they say as they work will be transcribed in a written form. The written solution will be examined to determine what strategies were used and how effective the video of the peer problem solving was in teaching effective strategies. The video of your child will not be viewed by anyone other than the researcher.

Would you please sign the consent forms at the bottom if you agree to allow your child to participate in the study and be videotaped, and to have excerpts of the session quoted anonymously. If you have any questions please contact me at the University of Ottawa at 564-2249. If you would like a summary of the results when they become available please include your mailing address at the bottom of the page.

Yours truly

Wayne Sheldrick

CONSENT FOR AUDIO-VISUAL RECORDING

I, (parent or guardian) ______________________________
agree to the video-taping of my son/daughter's problem solving session. I agree that such material will be used by the researcher only to examine strategies used by the student while problem solving. All reasonable precautions will be taken to ensure confidentiality.

______________	______________	________
Signature	Witness	Date

CONSENT TO ALLOW EXCERPTS TO BE QUOTED

I, (parent or guardian) ______________________________
agree to allow excerpts of the problem solving session to be quoted in written form. I understand that my child will not be identified by name.

______________	______________	________
Signature	Witness	Date

APPENDIX B: GRADE SEVEN MATHEMATICS PROBLEMS

1. In a 200 m race Jan finished 7 m behind Pat. Kelly was 8 m in behind Jan. Willi was 14 m in front of Kelly. Who won the race and by how much? Who came last?

2. There are 8 chairs in a rectangular classroom. How can the chairs be placed along the walls so that there are three chairs along each wall?

3. A square garden with a perimeter of 48 m is enclosed by a fence with posts 2 m apart. How many posts are there?

4. Al, Beth, Cindy and Don are seated around a card table. Partners are seated on opposite sides of the table. Al is seated on Don's right. Beth is not seated on Don's left. Who is Cindy's partner?

APPENDIX C: PERMISSION TO USE VIDEO FOR TEACHING

Dear Parents

Your child has expressed an interest in participating in a research project being conducted by a PhD student from the Faculty of Education at the University of Ottawa.

Students will practise solving word problems in small groups. The following day they will be taken from class individually to solve three mathematics word problems. Students will work at a special table which will enable us to videotape their written work as well as what they say while they are solving the problem. One problem solution from all those obtained will be selected and used as a teaching tool. It will be viewed by other grade 7 students in the next phase of the study to demonstrate how a grade 7 student problem solves.

Would you please sign the consent form at the bottom if you agree to allow your child to participate in the study, be videotaped, and have a portion of the videotape used as a teaching tool in the next phase of the study. If you have any questions please contact me at the University of Ottawa at 564-2249. If you would like a summary of the results when they become available please include your mailing address at the bottom of the page.

Yours truly

Wayne Sheldrick

CONSENT FOR AUDIO-VISUAL RECORDING AND USE

I, (parent or guardian) ______________________________
agree to the video-taping of my son/daughter's problem solving session. I agree that such material will be used by the researcher to examine strategies used by the student while problem solving and may be used as a teaching tool in the next phase of the study. All reasonable precautions will be taken to ensure confidentiality.

______________________	______________________	__________
Signature	Witness	Date

APPENDIX D: GENERAL INSTRUCTIONS

The table at which you are sitting was specially designed to allow us to video tape what you are writing and saying as you are solving some problems. The mirror under the table turns the image around for the camera so that the final picture will show what you are writing just as you see it. It is important as you are solving each problem that you speak clearly and say what you are thinking and doing at the time. This takes a little practise so you will be doing a few warmup questions. Everything which you think and do is important so try to say out loud what you are doing even if it does not seem very important to you.

During the warmup problems you can ask any questions which you may about what you are doing and whether you are doing it properly. During the experiment you will have to work on your own, I will not be able to help you. I will speak only to remind you to continue to talk if you have stopped.

Do you have any questions before you start the warmup questions?

APPENDIX E: GROUP RELATED INSTRUCTIONS

Group P1VP3: I am going to give you a problem to solve. I would like you to talk aloud as you solve the problem and write your work on this sheet of acetate. Tell me when you are finished. After you have finished the first question I am going to show you a video of a student solving a problem. When the video is finished I will give you a second problem to solve. Please remember to talk out loud as you are solving the problems. Do you have any questions?

Group P2VP3: I am going to give you a problem to solve. I would like you to talk aloud as you solve the problem and write your work on this sheet of acetate. Tell me when you are finished. After you have finished the first question I am going to show you a video of a student solving a problem. When the video is finished I will give you a second problem to solve. Please remember to talk out loud as you are solving the problems. Do you have any questions?

Group VP3: I am going to ask you to watch a video of a student solving a problem. When the video is finished I will give you a problem to solve. I would like you to talk aloud as you solve the problem and write your work on this sheet of acetate. Tell me when you are finished. Do you have any questions?

Group P1P3: I am going to give you a problem to solve. I would like you to talk aloud as you solve the problem and write your work on this sheet of acetate. Tell me when you are finished. I will give you a second problem to solve. Please remember to talk out loud as you are solving the problems. Do you have any questions?

Group P2P3: I am going to give you a problem to solve. I would like you to talk aloud as you solve the problem and write your work on this sheet of acetate. Tell me when you are finished. I will give you a second problem to solve. Please remember to talk out loud as you are solving the problems. Do you have any questions?

Group P3: I am going to give you a problem to solve. I would like you to talk aloud as you solve the problem and write your work on this sheet of acetate. Tell me when you are finished. Do you have any questions?

APPENDIX F: WARMUP QUESTIONS

ADD	275 +634	SUBTRACT	706 -384
MULTIPLY	423 X 5		

APPENDIX G: RETROSPECTIVE INTERVIEW QUESTIONS

1. Did you do anything different when you were doing the second problem?

(Experimental Group)

2. Did the video help you to solve the second problem?

3. How did the video help you?

4. The boy in the video used a diagram. Do you think it would have been helpful if there had been a diagram provided on the sheet?

(Control Group)

4. Do you think it would have been helpful if there had been a diagram provided on the sheet?

6. Would it have helped if you had drawn a diagram?

7. Do you ever draw diagrams to help you solve problems in math?

(Experimental Group)

8. The boy in the video checked his work. Do you ever check your work?

(Control Group)

8. Do you ever check your work?

9. Would it be helpful to check your work?

APPENDIX H: A TRANSCRIBED PROTOCOL

GROUP 1 - 4 H M

Pat, Jan and Kelly have decided to build their own play fort from cardboard boxes left over from new appliances their parents purchased. Pat will build a fort with one tower. Jan will build a fort with a tower and two small rooms. Kelly will build a fort with a tunnel and one small room. The chart below shows how much cardboard is required for each size play fort.

Pat decided to build a fort with one tower with windows at the front but not at the back. Jan will build a fort with one tower with one window and two small rooms with one window in each room. Kelly wants a fort with a large room with windows on two sides and a tunnel with no windows.

How much cardboard will Pat need altogether?

A1 1 Um ...

A3 2 Oh so Pat wanted to buy a tower with windows

A2 3 Which is ...

A2 4 With windows ...

A2 5 With windows is

S1 6 800 sq cm

M3 7 Pause

M4 8 The front should be

M4 9 800 sq cm

M3 10 Pause

A7 11 Is it just for Pat?

A7 12 How much cardboard Pat needs?

M Is that what the question asks?

A7 13 Would this be right? This answer?

M Solve the way you think is right.

M3 14 Pause

M3 15 Okay

M3 16 Pat will build a fort with one tower and

M3 17 Pat decided to build a fort with one tower

M4 18 Which ... is 800 cm

M4 19 800 sq cm

M4 20 And with 1 window on the front would be 800 sq cm

S5 21 I think it's this answer I think.

VIDEO

READS

Bob, Jack and Wally have decided to build their own birdhouses for the grade 7 woodwork project. Bob will build a six bird birdhouse, Jack a twelve bird birdhouse, and Wally a ten bird birdhouse. They have each chosen the same pattern but will make different style houses. The chart below shows how much wood is required for each size birdhouse.

Bob decided to build a six bird birdhouse with a fancy front, plain back and an A Frame roof. Jack will build a twelve bird birdhouse with a fancy front and back, and a Cottage roof. Wally wants a ten bird birdhouse with a plain front and back and an A Frame roof.

How much wood will Bob need altogether?

A1 22 Okay so Bob decided to build a 6 bird birdhouse with a fancy front

A2 23 So a fancy front would ...

S1 24 Would be 550 sq cm

M3 25 And and a plain back

M4 26 A plain back is

S1 27 450 sq cm

M3 28 And an A frame roof...

M4 29 So an A frame roof S1 30 700

M4 31 And a plain roof ...

M2 32 No ...

M1 33 Oh then you would add the two sides

M4 34 Which would be

S1 35 5, 450

M4 36 And the other side

S1 37 450

M4 38 And then the floor

S1 39 300

M2 40 Then you add them up

S3 41 0

S3 42 5 + 5 + 5 + 5 is 20 carry the 2

S3 43 And then 2 + 3 is 5 + 4 is 9 + 5 is 14 + 4 is 18 + 7 is 25 and + 4 is 29

V1 44 Ya

INTERVIEW

M Did you do anything different from your first problem to your second problem?

4 Um

I added the sides and the floor.

M Anything else.

4 I think there were

M After watching the video, did it help you to solve the second problem?

4 Ya it did

M Can you tell me how?

4 Because it showed me where I went wrong ... and it showed me that ... like that you had to add the sides like don't forget the sides and the floor ... and the roof

M The boy in the video used a diagram. Do you think it would have been helpful if there had have been a diagram provided on the sheet?

4 Ya I think so.

M How about if you had drawn a diagram?

4 Um I think it would have helped me but like I'm not used to drawing diagrams so ...

M When you're solving a problem do you have a plan in your head that you go through to solve the problem?

4 Um ... not ... I don't think so.

M The boy in the video checked his work. Do you ever check your work or do you think that you should check your work?

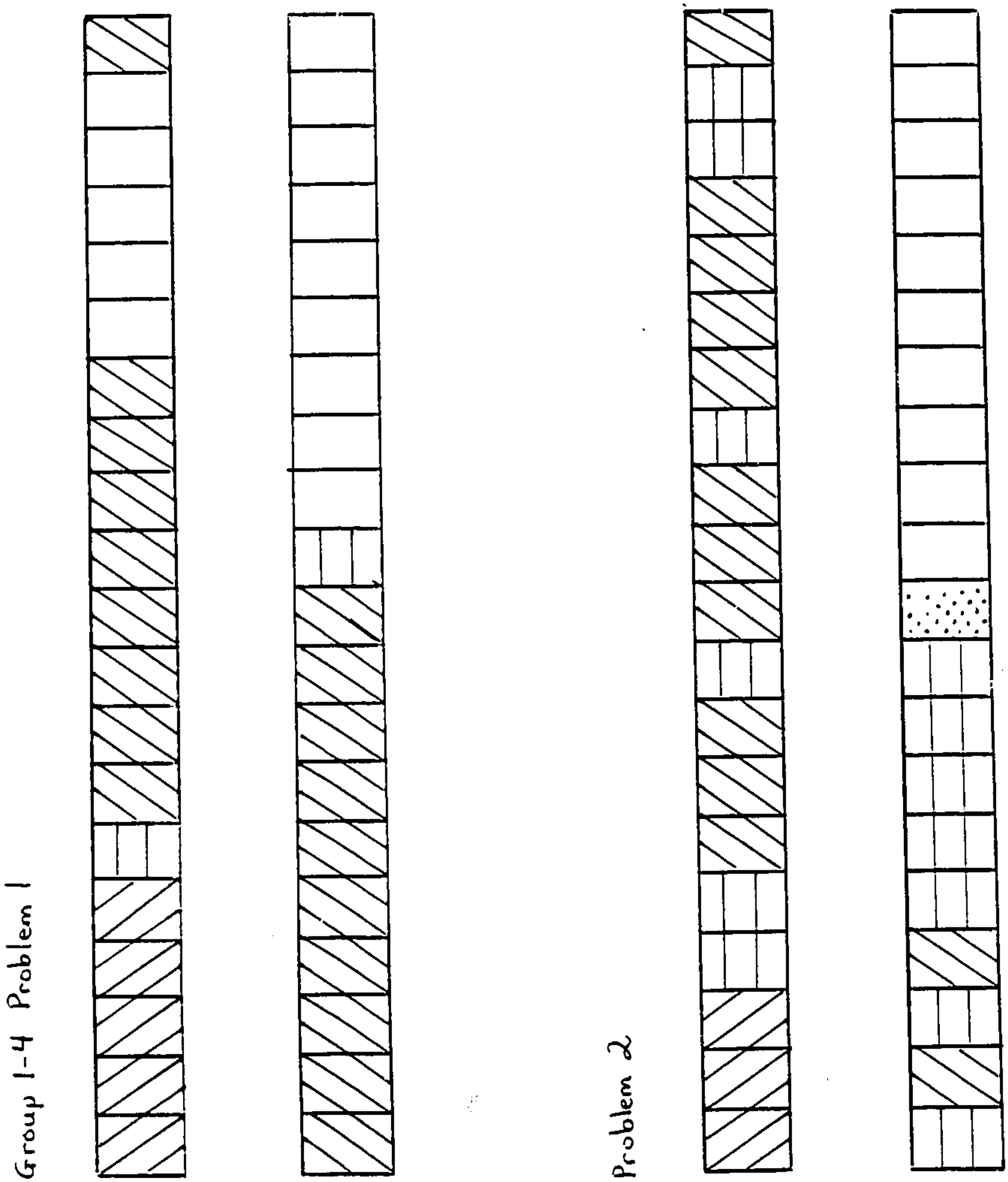
4 Um ... sometimes I check my work

APPENDIX I: SUMMARY FOR A PROTOCOL

GROUP 1 - 4 H M
PROBLEM 2

The subject searched the question for the necessary information. He wrote the value for the front and then re-read the question, checked the chart for the back and roof. He started to add a second roof, realized his mistake and moved on to the implicit parts. He checked the chart and wrote the value for each side and the floor. He stated his intention to add the numbers up. In lines 41-43 he performs the addition. In line 44 he verifies orally that the answer is correct (2900). There was no apparent plan, he seemed to simply work across the columns of the chart but he did realize the answer was not a single value as in Problem 1 and that the parts he saw as he crossed the chart were necessary.

APPENDIX J: Coded Protocol Timeline



Analysis
Problem Representation
Solution Directed Activity
Global Monitoring
Local Monitoring
Verification