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**FACULTY OF GRADUATE AND
POSTDOCTORAL STUDIES**

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**Identification and Characterization of a Cost-Effective Combination of a System for Arctic
Surveillance : The Northern Watch Project**

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M.Sc. Systems Science

Thesis

**Identification and characterization of a cost-effective
combination of systems for Arctic surveillance: The
Northern Watch project**

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Abstract

This thesis discusses a new stream of research and analysis which will form part of the Northern Watch Technology Demonstration project.

The objective of the thesis is to develop and illustrate a procedure for identifying and characterizing combinations of sensors and systems that will provide cost-effective options for Arctic maritime surveillance. A ship detection simulation is constructed, the results of which are used to produce a set of ranked options for combinations of sensors used to conduct maritime surveillance at a strategic choke point located at the Barrow Strait, in Canada's Northwest Passage. The overall objective of the surveillance is to improve on the detection, classification, and identification of maritime vessels. The modeled performance and effectiveness of each sensor is evaluated in relation to the multiple objectives using the Analytical Hierarchy Process (AHP) to rank alternative sensor effectiveness and performance. The results indicate that the most cost-effective solution is to install an Automatic Identification System (AIS) sensor at the Northern Watch station. However, due to practical concerns, alternatives to this approach are presented and discussed.

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“The important thing is not to stop questioning.” --Albert Einstein

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1 Introduction

This thesis identifies and characterizes combinations of sensors and systems toward providing cost-effective options for Arctic maritime surveillance. Surveillance of the maritime approaches and narrows of the Canadian Arctic is a subject of substantial and increasing interest to the Government of Canada.

The research surveys different methods of performing multi-criteria decision making (MCDM) and multi-objective optimization (MOO) using evolutionary algorithms for evaluating surveillance systems. Geographical information systems are used to identify candidate sensor sites. A simulation model is constructed using MATLAB to simulate the passage of vessels through the Barrow Strait and determine analytically the effectiveness of different sensor combinations for detecting them.

Separately, a number of expert decision-makers were asked to rate the relative importance of a set of performance measures for candidate sensors in a series of pairwise comparisons. This application of the Analytical Hierarchy Process, combined with the output of the simulation, yields a preferred ranking of sensor combinations. That ranking is the final output of the paper.

This research is part of a larger project undertaken by Defence Research and Development Canada's (DRDC), called the Northern Watch Technology Demonstration Project. The following section explains the broader context within which the project is situated.

1.1 *Project context*

The Northwest Passage is a route through the waters of the Canadian Arctic Archipelago which connects the Atlantic and Pacific Oceans. Until recent years, Canada's Arctic waterways have been largely inaccessible due to the presence of sea ice. However, climate change has caused the ice to melt rapidly and the Passage was completely open for the first time in September 2007 since satellite measurement began in 1979 (European Space Agency, 2007). The Passage is normally more accessible from May to September. However, with the rapid changes in the summer ice, the Passage is expected to be more accessible to more ships over longer periods of time each year (ACIA, 2005: 934-935). Even without a complete opening of the Northwest Passage, climate change and melting sea ice has resulted in greater interest in accessing natural resources in the Arctic Archipelago. As a result, monitoring maritime traffic in the waters of the Arctic Archipelago has become one of the essential elements for Canada to assert its sovereignty over those waters.

In order to achieve this goal, four Defence R&D Canada centres are participating in the Northern Watch Technology Demonstration Project (TDP). The ultimate goal of the Project is to develop a common operating picture and recognized maritime picture for the Arctic. Note that the term "picture" is used in the military sense, signifying an overall, integrated view of all information pertaining to the geographical area of interest.

The Government of Canada is interested in protecting our sovereignty and natural resources in the North by performing surveillance of the Canadian waterways and tracking shipping routes (cf. McRae, 1986, Birchall, 2006, and Beauchamp and Huebert, 2008). However, there are many different requirements and constraints for surveillance of the North. The cost of performing surveillance in this remote area is very high. There are many criteria which the decision maker must consider: different measures of sensor performance, cost, power usage, etc. As a result, it is in the Government of Canada's interest to invest in a cost-effective solution for this task that takes into account the multi-criteria elements of the decision making problem. Accordingly, multi-objective methods will be needed to determine the preferred surveillance options.

The important question is what would be a cost-effective placement of sensors in the Canadian Arctic archipelago to afford satisfactory maritime surveillance coverage of the Northwest Passage. The favoured solution will be one which will obtain the maximum amount of information for surveillance while keeping the cost of installation and operation at lower relative levels. This paper seeks, through the use of simulation and MCDM, to find such a solution for Northern surveillance.

1.2 Northern Watch background

Northern Watch (NW) is a Technology Demonstration Project. It has a number of aims related to Arctic surveillance including the following as described in the Northern Watch Technology Demonstration Project document (Northern Watch, 2007a: 2-3):

- 8.2 “Demonstrate the effective use of surveillance at a selected “choke point” on Barrow Strait and prove that this is a first step, scalable option toward providing greater Arctic surveillance capabilities;
- 8.3 Investigate the effective use of surveillance at strategic approaches like Hudson Strait or Amundsen Gulf, through a choke point results extrapolation study; and participation in northern exercises;
- 8.4 Investigate and demonstrate the effectiveness of surveillance from platforms of opportunity (civilian or CF vessels, satellite-based sensors, etc.);
- 8.5 Develop and demonstrate concepts for employment of sensor integration (local and remote) and picture compilation at a simulated operations centre including the conducting of trade offs for various configurations of sensor systems, communication systems and common operating picture (COP) compilation systems;
- 8.6 Investigate options for the communication of relevant data to fusion centres located in Southern Canada;”

From these statements of intent, it is apparent that some element of decision analysis will be needed. Paragraph 8.5 indicates this directly, as it refers to tradeoffs which naturally imply some process of decision making with respect to a particular goal or multiple goals. The goal of the NW TDP is stated as follows:

“The objective of the NW TD [technology demonstration] project is to identify and characterize combinations of sensors and systems that will provide a cost-effective RMP [recognized maritime picture] for the Canadian Arctic.”
(Northern Watch, 2007a: 3).

To do this, the NW TDP will make use of “a mixture of passive and active as well as local and wide-area surveillance assets” (Northern Watch, 2007a: 2).

Within this context, this research will analyze the relative performance of a number of candidate sensor combinations for the NW TDP, and yield a combined expert rating of those combinations that are judged to be most effective.

1.3 Thesis objectives

The Arctic maritime surveillance situation clearly calls for a cost-effective approach to balance the constraints imposed by the environment, geography and project parameters such as cost and sensor maintenance, against the stated objective of building a coherent, cost-effective and usable RMP for the Canadian Arctic.

The fields of optimization and decision analysis already contain ongoing research on tools that are useful in designing the kind of surveillance system needed under NW. Multicriteria methods, e.g., Multi-Criteria Decision Making (MCDM) and evolutionary Multi-Objective Optimization (MOO) methods, in particular, will be of interest. For the geographical aspects of sensor placement and effectiveness, geographical information systems (GIS) will be used to determine appropriate choke points and sensor placements.

Therefore, the objective of the thesis will be to use MCDM algorithms to develop a set of ranked options for different configurations of sensors in the Arctic for the Northern Watch TDP. This will be done through the development of a shipping detection model and a model of the detection capabilities of different types of sensors and their effectiveness in a simulated environment. It will also include practical factors such as cost, weather, communication, and sensor maintenance considerations.

The research question to be answered by this thesis project is:

How can simulation, MCDM algorithms and GIS methods be used to generate a ranked set of alternatives for configurations of sensors in an Arctic maritime surveillance context to best achieve Canada’s stated objectives?

In addition to the simple output of the set of ranked alternatives, the main contribution to knowledge of this thesis will be twofold: first, it contains a solid literature review on MCDM and MOO. Second, the particular combination of GIS, simulation and MCDM tools used to determine the ranked set of alternatives represents a new and untried approach.

1.4 Outline of the Thesis

The thesis consists of five main parts:

- i. Introduction: in this introduction chapter background has been provided from the Northern Watch technology demonstration project underway as well as the statement of the objective of this thesis within this context.
- ii. Literature review: extensive search and discussion of advantages and disadvantages of widely used multi-criteria decision making and multi-objective optimization methods, as well as the relevant literature on geographical information systems, and surveillance methods are presented in this chapter.
- iii. Methodology: in this chapter, discussion of the precise methodology adopted for the Northern Watch project and applied in this research is presented.
- iv. Analysis and Results: analysis of the thesis applied methodology results are presented in Chapter 4. In this chapter a set of ranked options for the Northern Watch project is provided.
- v. Conclusion and Recommendation: finally in this chapter, conclusion of the research and future research recommendations are presented.

2 Literature Review

The following literature review provides background on a number of suitable decision analysis tools that are useful for the selection of sensor combinations. It provides background from the literature on the Northern Watch project, considers the use of GIS for this particular application, and examines a number of multicriteria algorithms for MCDM and MOO analyses.

2.1 *GIS in NW context*

The term “Geographical Information System” refers to any computerized tool to link geographic information with other, non-direct geographic data of interest. The data of interest could be sea ice concentrations, as in the present example, or it could be information about disease prevalence, average temperatures, or deposition of pollutants, for example. These are types of data that could be stored in any normal database. What makes GIS distinct from a regular database is the inclusion of geographical or positional information along with the data of interest. In other words, the database contains information not only about types of pollutants and their rates of deposition, but also about the location of their deposition. This in turn allows pollution deposition to be represented on a map, and for database query operations to be conducted on the data based on their spatial characteristics. Databases are useful in knowing what, how, and even when, but GIS is a tool specially adapted to illustrating also where phenomena occur. For example, de la Torre et al. (2009: 10) use GIS to combine economic data and temperature predictions from global climate models to predict the effect of climate change on agricultural land prices in Latin America on a region-by-region basis.

In the context of the Northern Watch project, GIS is useful in visualizing the “big picture”: the view of the system of sensors as a whole. Its geographical orientation is also helpful because it shows the relative positions of the possible sensor sites, as well as the relative sizes of the detection radii of the different types of sensors. GIS allows planners and users to put the system under development into perspective. For a system as dependent on positional information as NW, this could be a critical element of the planning process. GIS also has the additional flexibility of being able to represent environmental conditions such as ice cover, bathymetric data, etc., and manipulating those data as required by project needs. For example, risk of natural disasters is highly dependent on spatial considerations (e.g. position of populations relative to volcanoes, fault lines, etc.) and on environmental considerations (frequency, intensity and trajectory of hurricanes, precipitation profiles and likelihood of drought, etc.). Dilley et al. (2005) use GIS to combine demographic and economic data with analysis of the location and characteristics of “hotspots” for six types of natural hazards to derive natural disaster risk profiles at the sub-national level and identify regions at particular risk.

GIS technology can be applied to maritime surveillance problems. For instance, Moutray and Ponsford (2003: 385) allude to the value of using GIS-based information to deploy

surface and air assets more efficiently for maritime surveillance tasks. The authors do not specifically mention the possibility that GIS could also be used to deploy fixed sensors such as radar sites, however. Sevgi et al. (2001), examine the use of shore-based radar for maritime surveillance using GIS.

Moutray and Ponsford (2003) further indicate that the integration of maritime surveillance assets and information is a problem whose solution can benefit from the use of optimization techniques combined with GIS analysis. They illustrate the need for optimization with an example of coordination between land-based radar, patrol vessels and surveillance aircraft:

“The important issue is how efficiently [aircraft and patrol vessels] can be deployed. One aircraft flying a reconnaissance pattern and recording the identity and position of targets of opportunity can spend a significant portion of its air-time seeing little or nothing to report. Contrast that with the same aircraft flying directly from target to target based on continuing real-time display of target locations. The increase in effectiveness is dramatic. Given that it cannot “see” very far, vectoring a patrol vessel from target to target results in an even greater increase in effectiveness. This increased effectiveness is the only way to achieve the enhanced levels of surveillance necessary to address today’s EEZ requirements and still operate within available budgets.” (Moutray and Ponsford, 2003, p. 388)

Ou and Zhu (2008) describe how GIS can be used in combination with airborne surveillance using the International Maritime Organization’s Automatic Identification System (AIS). In their example, GIS is used not to optimize the detection or tracking of targets, or to optimize the placement of sensors. Instead, they use it as a tool to assist in processing information collected by airborne AIS receivers. In particular, GIS can be used with geocoded information to perform queries to isolate desired subsets of data. Ou and Zhu describe the benefits of using GIS in the marine surveillance context as follows: “The main advantage of using a GIS system for queries is that users achieve a great visual presentation of their query results – a map. The GIS system also provides a spatio-temporal query interface for interactive query by drawing the Area-Of-Interest (AOI) directly on the map. The query results for the vessel targets are automatically displayed as a layer and could be labelled by any attribute or a combination of attributes.” (Ou and Zhu, 2008: 660).

The literature shows that the application of GIS to maritime surveillance problems may be a worthwhile inquiry to pursue. In the following sections, we turn our attention from spatial analysis to decision making and optimization techniques that may be useful in the specific context of the Northern Watch problem.

2.2 Multicriteria Algorithms - MCDM and MOO: Main differences

MCDM is used when various alternatives are compared against each other based on multiple criteria. MOO investigates a solution space and is not limited to selecting a specific alternative. In MOO, different objectives are determined and a solution space is defined in

order to find optimal solutions by comparing a series of non-dominated solution sets. The solution space is defined by the constraints, and each constraint provides a limit. It is important to recall that MCDM is a form of assisted decision-making – or decision analysis – and MOO is a process of optimization. The two can be related but are not the same. These sections investigate these different multicriteria algorithms toward determining the more appropriate model for use in the NW problem context.

To illustrate the differences between these two types of tools, we use a simple analogy. Suppose that a country needs to make a decision on the purchase of helicopters for a number of purposes. These helicopters must meet a number of decision-making criteria, including cost, multi-role capability, fuel efficiency, carrying capacity, cold-weather operations, etc. We can use both MCDM and MOO tools to assist the decision-making process.

In MCDM, the alternatives and criteria under consideration are pre-determined. Using different algorithms, the available pre-determined alternatives are compared on the basis of each criterion. In the case of the helicopters, for example, the decision maker compares different available models sold by manufacturers over which the decision maker does not have control, such as helicopter-A, helicopter-B, helicopter-C, etc. The decision maker grades the available models based on the stated criteria of cost, fuel efficiency, parts availability, maintenance, etc. Other decision makers may also supply similar grades. An algorithm is applied to combine the input of all decision makers on all the criteria, and a winning alternative is selected based on the best results.

Multi-objective optimization is not limited to pre-determined alternatives. Instead, a MOO process will seek to optimize the value of the defined variables. In the case of a helicopter, different explicit objective functions such as overall cost, minimum amount of maintenance, etc., are defined. The constraints will define the allowed values that these variables can take, and the objective functions will define what we want to achieve in terms of each. For example, a cost constraint will determine the maximum money available for purchasing a helicopter, and an objective function will be created to determine what is fuel “efficient”, etc. MOO, in other words, customizes a helicopter based on these parameters, by running an optimization program that will satisfy the constraints and minimize or maximize the objective functions. It would then be up to a manufacturer to design and build a helicopter that met the design parameters corresponding to the optimal point as determined by MOO.

In the following sections, we examine a number of MCDM and MOO tools with the intent of evaluating their usefulness in the context of Northern Watch.

2.3 Multi-objective optimization (MOO)

MOO investigates a solution space and is not limited to selecting a specific alternative. In MOO, different objectives are determined and a solution space is defined in order to find optimal solutions by comparing a series of non-dominated solution sets. The solution space is defined by the constraints. Each constraint provides a limit.

It is important to recall that unlike MCDM, MOO is a process of optimization. A multi-objective optimization problem (2.1) has a number of objective functions which are to be minimized or maximized.

$$\begin{array}{lll} \text{Maximize / Minimize} & f_m(x) & m = 1, 2, 3, \dots, M \\ \text{Subject to} & g_i(x) \geq 0 & j = 1, 2, 3, \dots, J \end{array} \quad (2.1)$$

The solution x is a vector of n_i decision variables as shown below (2.2):

$$x = (x_1, x_2, \dots, x_n)^T \quad (2.2)$$

The MOO methodologies considered in this report mainly use evolutionary algorithms to arrive at their optimal solutions (also see section 2.3.1, below). Generally, the evolutionary methodology begins with an arbitrary set of feasible solution points, and generates a second set (a “child” generation) according to specific rules. It then compares the fitness of each individual solution in the child generation with the parents and keeps any child whose overall fitness is higher than its parents. This way, at each generation the algorithm retains a set of non-dominated solutions. The generational process is reiterated until the non-dominated set arrives at an optimal point, with no better feasible outcomes available (Deb, 2001).

In the following sections, we examine a number of different approaches to MOO, each one implementing a variation of an evolutionary algorithm, which is the current state of the art for optimization of this kind.

2.3.1 Evolutionary algorithms

The evolutionary algorithm imitates nature’s evolutionary principle to determine an optimal solution (Deb, 2001). The primary reason for using evolutionary algorithms is their ability to find multiple Pareto-optimal solutions in a single simulation run. In classical optimization methods, including MCDM, the general approach is to convert the multiple objective problem into a set of single objective problems and emphasize one Pareto-optimal solution at a time. When that method is used, it requires many iterations to obtain a number of different solutions. In addition, there is no guarantee that every run will not return the same solution. The evolutionary algorithms not only produce a set of solution points in each run, but also the algorithm runs on multiple objectives simultaneously where classical optimization can only approximate that result by focusing on one objective at a time. Thus, in general, evolutionary algorithms are more time efficient and each iteration yields a more useful result.

2.3.2 Strength Pareto Evolutionary Algorithm – SPEA

In this method, there are two populations considered and compared; the regular population and the archived (or external) population. At the initial step, both sets are empty. The first step is placing all non-dominated solutions from the current population into the archive group. At the same time, any dominated members or duplicates are removed from the

archive. This will result in a newly populated archive which will respect a predefined limit on the number of entries. (Nedjah and Mourelle, 2005: 536-538). If the archive exceeds this limit, more members are discarded by using a clustering technique. The clustering technique is based on the nearness of the solutions to one another and seeks to preserve the maximum diversity within the population size limit. The next step is assigning fitness values to both archive and regular population members. The fitness value F is determined for each member by assigning a strength value, $S \in [0,1)$. S_i , the strength value for an archive member i , is calculated by dividing the number of population members that are dominated by or equal to i , by the population size plus one as shown in (2.3).

$$S_i = \frac{n_i}{N + 1} \quad (2.3)$$

The $(N + 1)$ ensures that the strength of any archive member is never one or more (Deb, 2001:250).

In the next step the fitness value for a population member j , F_j , is determined as the summation of the strength of all members of the archive population not dominated by j , plus one. The addition of one ensures that the fitness of any population member is more than the fitness of any archive member.

This is followed by the mating selection phase where the combined regular and archive population are available as parents. A binary tournament selection is made wherein two random groups are selected and the individual with the highest fitness is taken from each. Those two individuals are then paired for mating. Those individuals are then returned to the pool and the process is iterated again to produce the next offspring and so on. This is repeated N times, where N is the size of the population. (Nedjah and Mourelle, 2005: 537).

The SPEA algorithm has some disadvantages. One of these disadvantages is related to fitness assignment. Because fitness is determined by the strength of the archive members that dominate the given member of the population, members that are dominated by the same archive members will have identical fitness values. As a result, this leads to a situation where, if there is only one member in the archive then all population members will have the same fitness value. At this point, the selection pressure is decreased since there is no way to distinguish members based on their fitness and this algorithm will behave like a random search. Another issue arises when the clustering technique is applied. During this procedure, some members of the non-dominated set are discarded to keep within the limit of the size of the archive. As a result, some outer solutions may be lost and that reduces the diversity of the non-dominated set, and increases the risk of the algorithm getting trapped in a local maximum. The last potential weakness of this algorithm is that if many members do not dominate each other, this will provide very little information with which to select the fittest. When this occurs, density information must be used but SPEA's clustering mechanism only uses density information about the archive and not the population. (Zitzler et al., 2001: 4-5).

The complexity of this algorithm at every step is no larger than:

$$O(MN^2) \quad (2.4)$$

where M is the number of objectives and N is the size of the population. (Deb, 2001:256)
This is the same order of complexity as many of the other evolutionary algorithms considered below.

2.3.3 Strength Pareto Evolutionary Algorithm - SPEA2

This algorithm was designed to improve SPEA and overcome the issues mentioned above (Zitzler et al., 2005: 5). Thus, this method uses similar steps as SPEA. The first step is creating two empty population sets: general and archive. Then the fitness value of each member is calculated. In order to avoid the situation where members would have an identical fitness value, mentioned as a disadvantage above, with SPEA2 both dominating and non-dominating members are taken into account when calculating the strength and fitness. Strength values, $S(i)$, are calculated for members of both the population and the archive. For member i , strength is equal to the cardinality of the set of individuals in the archive and population that are dominated by i , as follows:

$$S(i) = |\{j | j \in P_t + \bar{P}_t, i \geq j\}| \quad (2.5)$$

where P_t is the population and $\bar{P}_t$ is the archive.

The raw fitness value is determined as the sum of the strength values of all members of the population and the archive that dominate member i :

$$R(i) = \sum_{j \in P_t + \bar{P}_t, j \geq i} S(j) \quad (2.6)$$

As with SPEA, lower fitness values are more desirable, indicating better fitness. If most of the members do not dominate one another, the fitness method mentioned above will only provide a little information to determine fitness. Here determining the density will improve the situation. Also, as discussed in the previous section, introducing the density value D_i for a member i , will improve one of the weaknesses of SPEA. The density function $D(i)$ is determined by measuring the distance between the member i and all the other members of the population and the archive. Summing the density and raw fitness results in the fitness value $F(i)$ for the member i as shown in (2.7) (Zitzler et al., 2001: 7):

$$F(i) = R(i) + D(i) \quad (2.7)$$

Defining the fitness in SPEA 2 is followed by the environmental selection process. The first step in this selection process is to copy all non-dominated members from both archive and population to the archive for the next generation in order to keep the best solutions. If the size of the future generation of the archive is the same as the previous one, meaning $|\bar{P}_{t+1}| = \bar{N}$, the environmental selection is completed. However, if this condition is not satisfied, there are two other possibilities. The first possibility is that there are too few members in the next

generation of the archive. In this situation, the remaining spaces are filled with the dominated solutions with the best fitness. The second possibility is when there are too many members in $\bar{P}_{t+1}$ and an archive truncation procedure is applied in order to reduce the size of the group. This procedure is an iterative process where the member with the smallest distance to another member is removed. If there is more than one member with the smallest distance, then the tie is broken by comparing the second smallest distance. The next step checks to see if the simulation is concluded. If the current generation counter, t is greater than or equal to the maximum number of the generations T , then the simulation is complete and the set of the non-dominated solutions is returned as an output. Otherwise, the mating selection is performed by binary tournament selection as discussed in SPEA.

The disadvantage of this algorithm, like other evolutionary multi-objective optimization algorithms, is that its ability to find optimal solutions is greatly degraded when number of objectives is increased (Ishibuchi et al., 2006:1148).

The complexity of computation in this algorithm is related to the size of the archive and the population. This complexity is as follows:

$$O((N + \bar{N})^2) \quad (2.8)$$

Also the complexity of the environmental selection is as follows:

$$O((N + \bar{N})^3) \quad (2.9)$$

One of SPEA2's advantages is that its complexity is not dependent on the number of objectives, as it is for other algorithms. This might make it more computationally practical in cases with larger numbers of objectives. As noted above, however, increasing numbers of objectives do impair SPEA2's ability to find optimal solutions. In addition, its complexity does vary as the cube of the size of the combined population and archive, rather than the square as is the case for most of the other algorithms considered here (Ishibuchi et al., 2006).

2.3.4 Non-dominated Sorting Genetic Algorithm – NSGA

The first part of this algorithm sorts the population into different fronts. The sorting is done by calculating the value for objective functions of each member of the population. Then each member's objective function value is compared with those of each other member. Once the comparison is completed, each member that is not dominated by at least one other member is placed in the first non-dominated front. The members of the first front are assigned a large dummy fitness value. To decide which remaining member will be placed in the second front, all the members in the first front are eliminated and the sorting is performed on the remaining members. This process continues until all members of the populations are sorted into fronts (Srinivas and Deb, 1994:9). Therefore, each front includes members that are mutually non-dominated. As a result, the population is now represented by number of mutually exclusive equivalent fronts, P_j . The first front is the one nearest to the true Pareto-optimal front and each front after that is farther away. The next step is the fitness assignment procedure which starts with the first front and proceeds in succession into further fronts

(Coello Coello, 2003:124-125). All the solutions in the first front are assigned a fitness equal to N or the population size since these members are equally important in terms of their closeness to the Pareto-optimal solution.

$$F_j = N \quad (2.10)$$

To maintain diversity in this algorithm, NSGA uses a sharing function. The sharing function is the measure of the proximity of other members on the same front, and is governed by a parameter σ_{share} that is set by the user and remains constant throughout the simulation. σ_{share} has a value between 0 and 1 representing the distance between two members. The sharing function is used to calculate a niche count which serves as a measure of the number of other solutions nearby. The final fitness is then calculated by dividing the initial fitness F_j by the niche count in order to degrade the fitness of the solutions which are clustered together: the higher the niche count in the denominator, the more clustered are the solutions, the more their fitness is degraded. This is repeated for all the solutions in the first front, and the subsequent fronts are each assigned a maximum fitness that is slightly lower than the minimum fitness of the previous front, which ensures that the fronts are kept separate in terms of their fitness. NSGA uses a stochastic random process to conduct mating selection according to final fitness. This will ensure that the members in the first front have a higher chance of surviving than those in the other fronts. (Deb, 2001:199-208)

The NSGA's main advantage is in its assignment of fitness using non-dominated fronts. This sorting into fronts allows us to easily assess any new point in the solution space. Is the new point dominated by or does it dominate the first front? This allows us to maintain the elitist selection process and ensures that the algorithm proceeds towards the Pareto-optimal solution more quickly than non-elitist algorithms. NSGA also has a sharing function which assures diversity, unlike some other algorithms such as SPEA.

There are three main criticisms of the NSGA approach: (Deb et al., 2002:182)

1 – High computational complexity of non-dominated sorting: the computational complexity of this method is $O(MN^3)$ where M is the number of objectives and N is the population size. This complexity is due to the non-dominated sorting procedure in every generation, and it makes this method computationally expensive for large population sizes.

2- Lack of elitism: Elitism is a technique which preserves the best individuals from each generation. NSGA algorithm does not have elitism on an individual basis. It uses dummy fitness values based on the front on which each solution is found but the solutions own fitness is not calculated as explained above. This can lead to the loss of good solutions as the fitness of each solution is not individually recognized or stored.

3- Need for specifying the sharing parameter: To ensure diversity in the population in order to get a wide variety of solutions, this algorithm uses a sharing operator. Without it, a lack of diversity will result in getting caught in local optima (also known as premature convergence). The sharing requires defining a sharing parameter (σ_{share}). Use of this parameter introduces dependency of the algorithm on a fixed parameter, while a parameter-

less mechanism is most often desired. The performance of NSGA is sensitive to the choice of parameter σ_{share} .

2.3.5 Non-dominated Sorting Genetic Algorithm - NSGA II

NSGA II is an improved version of the NSGA algorithm. This algorithm has a similar structure, with a few significant differences. It starts by sorting all non-dominated solutions into fronts using a faster sorting algorithm where the complexity is defined, similar to other algorithms considered here, as

$$O(MN^2) \quad (2.11)$$

This sorting algorithm considers two characteristics of each member. The first characteristic is the domination count which is the number of other members that dominate the member. The second one is the set of the members that this member dominates. Defining these characteristics will assist with the determination of position for each member in regards to the fronts, resulting in less complexity and therefore better processing times.

As mentioned in the previous section, introducing σ_{share} in the fitness determination process to ensure diversity is one of the disadvantages of NSGA. To improve this situation, NSGA II has a parameter-less mechanism to ensure diversity. Therefore, NSGA II is no longer sensitive to the choice of parameter value. The diversity is ensured by performing density estimation. To do this, each member's crowding distance is calculated by averaging the distance to the two nearest neighbours. The crowding distance is calculated for each individual member of the population under each separate objective function. For each objective function, the boundary members (those with the highest and lowest objective values) are assigned infinite distances to avoid discarding them and losing diversity in the solution set. The results for each objective function are normalized with respect to the maximum and minimum values of that objective function, in order to eliminate dimension mismatch. The overall crowding distance is then determined by summing all of the individual distance values for each objective. In general, the greater the crowding distance the more isolated that member is.

The fitness is determined by the number of the non-domination front in which the member is found: if the member is in the first front the fitness value is 1, where a member in the second front has a fitness of 2, and so on.

The population consisting of parents and offspring is sorted into non-domination fronts. Those members in the first front (who therefore have the best fitness) are automatically selected to be part of the next new base population. If there are fewer than N members in the first front, then all the first front members will be included and the difference will be made up by taking members from the second front, then the third and so on until there are N members in the population. In order to determine which member of each front will be included with the first front members in the population, a crowded comparison operator is used. The crowded comparison operator prefers member in the lower number fronts to be

included. In cases where both members are in the same front, the one with greater crowding distance or which occupies a less crowded region is preferred (Sareni et al., 2004: 116). The new population with size N is used for mating selection to create the next offspring generation through crossover and mutation. This is done by binary tournament selection but the selection is done using the results of the crowded comparison operator, ensuring that an elite selection will be made. This process is repeated until the simulation reaches the predetermined number of generations.

As discussed above, NSGA II is preferred to NSGA as it addresses most of the main difficulties with the older algorithm. However, like other multiobjective evolutionary algorithms, NSGA II suffers a loss of performance as the number of objectives increases. (Deb, 2002:182-186)

2.3.6 Pareto Archived Evolution Strategy – PAES

In this algorithm, “instead of using real parameters, binary strings were used and bitwise mutations were employed to create offspring.” (Deb et al., 2002:183) PAES begins with single solution (i.e. one member). A copy of that member is made and mutation operator is applied to the copy. Now, the mutant is compared with the original member. At each step of this algorithm, the mutant or offspring is compared with the parent. If the offspring dominates the parent, it is accepted as the next parent and the iteration continues. However, if the parent dominates, the offspring is discarded and new offspring is found. On the other hand, if neither the parent nor offspring dominate each other, they are compared with the archive of best solutions. In this case, if the offspring dominates the existing members of the archive then it is accepted as a new parent and the dominated solutions are discarded. However, if the offspring does not dominate any of the existing members of the archive, both parent and offspring are checked for their nearness with the solutions of the archive. If the offspring resides in a less crowded area, then it is accepted as a new parent and added to the archive. The reason for measuring nearness is to avoid crowding and ensure diversity (Knowles and Corne, 2000).

The complexity of this algorithm for N evaluations has been determined to be $O(aMN)$ where a is the length of the archive. The archive size is usually chosen proportional to the population size, N , and the overall complexity for PAES is $O(MN^2)$. This is comparable to most other evolutionary MOO algorithms discussed here, so is of no particular advantage or disadvantage.

2.3.7 Hybrid algorithm

Neither single- nor multi-objective genetic algorithms are ideal for all situations. According to Ishibuchi et al. (2006), multiple coordinated runs of single objective genetic algorithms can outperform multi-objective ones under some circumstances and vice versa according to the following cases:

- 1- NSGA II can outperform single objective genetic algorithms in their application to single-objective problems by avoiding becoming trapped in local optima.
- 2- Single objective algorithms outperform NSGA II in multi-objective problems by conforming more closely to the Pareto front. This is because NSGA II does not perform well where there are many objectives.
- 3- Multiple runs of single objective algorithms can provide a more diverse set of solutions than a single run of NSGA II.

It is possible to combine the advantages of both single and multi-objective algorithms to obtain results closer to the Pareto optimal solutions. For that reason, Ishibuchi et al. (2006) propose a simple probabilistic hybrid that applies either the single objective method or the multi-objective method to each iteration. This can be applied at the stage of parent selection or at the generation update stage. The decision maker assigns the probability of using the single objective method vs. the multi objective method.

In testing, Ishibuchi et al. (2006), found that the hybrid is more likely to avoid being caught by local optima than the single objective algorithm was and had better convergence with the Pareto front than NSGA II in a four objective problem. The use of a hybrid algorithm preserves the best features of each of the model types.

The algorithms presented in Section 2.3 will be considered as possible approaches for the Northern Watch problem. See Section 2.5, below, for discussion of algorithm selection. The following section lays out some possible options for multi-criteria decision making approaches.

2.4 Multi-criteria decision making (MCDM)

MCDM is a discipline whose goal is to support decision makers who must choose between numerous and conflicting existing alternatives. MCDM aims to understand the conflicts between alternatives and derive ways of trading off between them to choose an alternative in a logical, analytical, transparent and rigorous manner (cf. Triantaphyllou, 2000).

MCDM problems involve ranking pre-determined alternatives in terms of a number of decision criteria where the number of alternatives and criteria are finite. The alternatives are pre-determined in that they are not editable by the decision maker, who must take them or leave them as they are.

The following are the steps involved in using numerical analysis to support MCDM in a group decision making model such as the ones considered for use in this thesis:

- 1- Determining alternatives $A_1 \dots A_m$ and criteria $C_1 \dots C_n$
- 2- Weighting each criterion by assigning a relative numerical value (w_j) to each. Usually the weights are normalized to add up to one (1.0) (Triantaphyllou, 2000).

- 3- Collecting individual preference rankings or scorings of the involved decision making “committee”. Each score – called “performance values” a_{ij} – indicates the performance of alternative A_i in terms of criterion C_j .
- 4- Combining the input of all decision makers into an aggregate score using a specific algorithm
- 5- Ranking of each alternative based on the previous step

Note that not all MCDM models involve input from more than one decision maker.

Dominance in MCDM is defined as follows: alternative A_1 dominates A_2 if the performance value $a_{1j} \geq a_{2j}$ for all j .

2.4.1 Condorcet’s Method

Condorcet’s Method is a “performance voting scheme where each voter ranks all the candidates from most preferred to least preferred. The Condorcet winner is that candidate which is preferred by a majority of the voters in a pair wise comparison with every other candidate.” (Emond, 2006:3)

In this method, the decision makers are provided with a list of comparisons and the list includes all the possible binary combinations of the alternatives. For each pair, the decision makers provide their preference. The number of votes is counted and the majority determine the winner in each pair wise comparison. If the majority of the voters select candidate A over candidate B, we can write this as a proposition $A > B$. This is repeated for the other pairs and the result is a set of propositions. From this set of propositions, one of the alternatives is preferred to all the others. The preferred alternative is the dominant winner since it is preferred by the majority over any other candidate.

The Condorcet method can also be used to rank candidates. The same procedure is followed to determine the dominant candidate. Then this candidate is eliminated and the next winner becomes the second ranked, and so on.

The method is relatively easy to understand and simple to implement. However, one of the disadvantages of this method is the possible existence of cycles in the propositions. Given three candidates A, B and C, the set of relationships (2.12) below is an example of this disadvantage.

$$\begin{aligned} A &> B \\ B &> C \\ C &> A \end{aligned} \tag{2.12}$$

As shown in (2.12), no conclusion can be drawn as to which candidate is the dominant winner and in fact there might not always be a winner. If this happens, it is difficult to resolve the outcome.

2.4.2 Borda's Method

In Borda's method, voters rank alternatives by assigning points in the order of their preference. This "procedure involves the summation of rank scores which are assigned according to the position of an option or candidate within the input rankings." (Emond, 2006:3) If compared with the method above, it is clear that this method is complementary to that of Condorcet. Again, it is fairly simple to explain to the voter, and the results are relatively easy to determine. However, there are many problems that prevent Borda's method from being of use in many situations. A winner by the Condorcet method is not necessarily a Borda's method winner and vice versa. It can also produce results that are seemingly perverse: if option B is persistently ranked second but the summation of its scores is the highest compared to other options, B is the winner according to Borda's method even though B was not the preferred choice over all others by the majority. A further problem with Borda's method is that it is open to manipulation by participants assigning low rank to "dangerous" rival options in order to influence their score totals, or assigning high rank to spurious choices. Also, by nominating a large number of candidates who are very similar to one another the probability of one of these candidates being elected is greater. In order to avoid this situation, diversity among candidates is required.

Borda's method also implies that the greater the separation of two options in the rankings, the greater the difference in preference between them though this is not always true – and the voters can not assign the same score between two options. Note that the Borda's ranking system cannot "measure the degree of intensity of preference held by the rankers. For example, if option A is two or more places ahead of B in one ranking and B is one place ahead of A in another ranking, this is not evidence that somehow A is globally preferred to B." (Emond, 2006:4).

2.4.3 Arrow's Theorem and Independence of Irrelevant Alternatives

The Arrow's Theorem or Impossibility Theorem states that in a situation with at least two decision makers and at least three alternatives, it is impossible to satisfy all of the following axioms (Arrow, 1950):

(Note: The "social welfare function" is the system that converts individual preferences into a single social ordering.)

- 1- The social welfare function should take into account all of the voters' preferences. Also the outcome should include the ranking of all alternatives meaning none of the alternatives is left out. It also has to yield a unique result meaning that each time the same set of voters' preferences has been added the same outcome is expected (i.e., it is deterministic, and cannot be probabilistic).
- 2- If an individual changes his or her input to rank an alternative higher, this should also be reflected in the social welfare function meaning the alternative should be ranked higher in the revised outcome.

- 3- Independence of Irrelevant Alternatives: if one of the alternatives is removed from the list, the result of the ranking system should not change.
- 4- The social welfare function is not to be imposed. The individuals have to be free to choose from all of the alternatives.
- 5- The social welfare function is not to be dictatorial, meaning its outcome cannot be determined by one person.

The third axiom, also known as Independence of Irrelevant Alternatives, does not consider the complexity of voting systems. In fact, it has been shown that there are no ranking systems which would satisfy both Condorcet and the independence of irrelevant alternatives.

According to Arrow's theorem, by changing the alternatives which are unavailable (meaning that they are clearly dominated and will not win), the outcome can be affected as well.

2.4.4 The Weighted Sum Model (WSM)

In this method we consider m alternatives and n decision criteria. The best alternative in the case of maximization is the one which satisfies (2.13) (Triantaphyllou, 2000: 6-7);

$$A_{WSM} = \max \sum_{j=1}^n a_{ij} w_j \quad \text{for } i=1, 2, 3, \dots, m \quad (2.13)$$

where A_{WSM} is the weighted sum model of the best alternative, a_{ij} as mentioned above is the performance value of the i -th alternative of the j -th criterion and w_j is the importance weight of that criterion.

Therefore as shown in (2.13), the total value of each alternative is equivalent to the sum of the products. The summation method used reveals one of the difficulties with the model: this method cannot be applied directly to multi-dimensional problems (i.e. problems with different units of measure between criteria). In this case, in adding different dimensions and different units, the additive utility assumption which is the assumption that the overall desirability is equal to the sum of all the products as given in the above equation, is violated. As a result, comparison becomes impossible. It is imperative that all criteria have the same dimension (or be dimensionless) for this method to be useful.

2.4.5 The Weighted Product Model (WPM)

In this method, each alternative is compared with the other alternative(s) by multiplying together a number of ratios, one for each criterion. Then the result is raised to the power equivalent to the weight of that criterion. The following is the method to find the ratio between two alternatives A_k and A_l :

$$R(A_k|A_l) = \prod_{j=1}^n (a_{kj}/a_{lj})^{w_j} \quad (2.14)$$

a_{ij} as mentioned above is the performance value of the i -th alternative in terms of the j -th criterion and w_j is the importance weight of that criterion.

If equation (2.14) is equal to or greater than one, it is concluded that A_k is more desirable to A_l in the case of maximization. The best alternative in the case of maximization is the one that is better than or equal to all other alternatives. It is important to keep in mind that “the weighted product method tends to penalize poor performance on one attribute more heavily” than does the WSM (Norris and Marshall, 1995: 12)

One of the advantages of this model is that any units of measurement are eliminated due to the use of ratios for comparison. Due to this property, this method is also called *dimensionless analysis* (Triantaphyllou, 2000). Therefore, this method can be used for both single- and multi-dimensional decision making unlike the WSM method described above.

One of the disadvantages of this method is the weight of the criteria being used as a power factor. In the case of heterogeneous criteria, high values for different criteria can cancel each other. As a result, the obtained ranking may be false (Mogharreban, 2006). Another related disadvantage is that the values of the weights must each be greater than one, since they are being used as exponents. This may seem trivial, but will require the conversion of more traditional normalized weights whose value is set between 0 and 1, summing to 1 overall.

2.4.6 Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process is a very well-known MCDM technique introduced by Saaty in the early 1970s, and widely used in practical applications since then (Saaty, 1980; Warren, 2004: 1). It is called an “analytic hierarchy” because it decomposes the overall decision making problem of choosing between n alternatives on the basis of m criteria into a number of sub-problems. These sub-problems are hierarchically structured and the decomposition makes each step computationally and cognitively easier to manage.

AHP provides a framework for structuring the processes both of assigning weights to the decision criteria and of comparing the alternatives on the basis of those criteria. It relies mainly on subjective pairwise comparisons between criteria or between alternatives to create a set of square reciprocal comparison matrices, which are then used to assign weights to the criteria, and to rank the alternatives in terms of each criterion. The rankings in terms of individual criteria are then aggregated using the weights assigned to the criteria themselves, giving a final relative score to each alternative.

AHP also includes a mechanism to check the fitness – called “consistency” in AHP – of its results at each step. The rankings of criteria or alternatives are considered to be consistent if they are found to uphold the transitivity property; i.e. if $A > B$, and $B > C$, then $A > C$. To do

this, we take the largest eigenvalue λ_{max} of each comparison matrix. Saaty proved that, theoretically, the largest eigenvalue of a consistent reciprocal matrix is equal to the matrix size, n (Teknomo, 2006). Since real applications of AHP do not conform to the theoretical ideal, λ_{max} will generally be close to, but not exactly equal to n . The consistency of the matrix can therefore be determined by taking the difference between λ_{max} and n . Saaty formulated this difference as a consistency index, CI :

$$CI = \frac{\lambda_{max} - n}{n - 1} \quad (2.15)$$

CI is then compared to a random consistency index, RI , representing the average consistency index of a randomly generated sample of 500 matrices with size n , according to the following table (Teknomo, 2006):

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Comparing CI to RI as a ratio, we derive the consistency ratio, CR :

$$CR = \frac{CI}{RI} \quad (2.16)$$

If CR is found to be less than 10%, we declare the evaluation represented by the comparison matrix to be consistent.

In aggregating the results of each comparison matrix to arrive at a set of final scores for each alternative, AHP is very similar to WSM except that AHP uses relative values instead of actual ones, and makes extensive use of normalization. Therefore, unlike WSM, it can be used for both single- or multi-dimensional decision making problems.

The overall matrix built in the AHP method is constructed by using the relative importance of the alternatives in terms of each criterion. According to AHP, the best alternative in the case of maximization is indicated as shown in (2.17):

$$A_{AHP} = \max \sum_{j=1}^n a_{ij} w_j \quad \text{for } i=1, 2, 3, \dots, m \quad (2.17)$$

Where a_{ij} (for $i=1, 2, 3, \dots, n$) is determined by pair wise comparison of the impact of m alternatives on the i -th criterion. Clearly this is similar to the form of the WSM. AHP's main distinction lies in its use of decomposition and pairwise comparison to make the process more intuitive and accessible to decision-makers. This is possibly because the pairwise comparisons closely resemble the methods of decision-making that most people use in their daily lives, based on simple preferences between two options.

Two other main advantages of AHP include its ability to handle multiple criteria easily, and its ability to incorporate both quantitative analytical data and subjective or qualitative judgements within the same framework. The latter, in particular, makes AHP useful in more complex situations in which a diverse range of input information types is expected, as would likely be the case for NW.

Despite its advantages in terms of accessibility, usability, and flexibility, however, there are serious concerns about AHP in the literature. Warren (2004), for example, raises concerns about the assumptions upon which the AHP is based. Warren (2004: 9-12) devotes considerable attention to Saaty's apparent misunderstanding of the rating scale type in use in AHP: is it a ratio, ordinal or interval scale, or some combination of these? The importance of this question relates to the mathematical operations that can be performed on the matrix entries. For example, because the numerical scale presented to the decision maker in making pairwise comparisons ranges from $\frac{1}{9}$ to 9, its range does not include an absolute zero – one characteristic that is necessary to the definition of a ratio scale, which “must possess an absolute zero which then enables division and multiplication, as well as addition and subtraction.” (Warren, 2004: 11).

Another problem with the numerical scale used is that its chosen scale necessarily implies that if option A is mildly preferred to option B, then A receives a score of 3, and B receives a score of 1/3. Because of the later multiplication and division operations performed using these scores, it becomes necessary to interpret this as meaning that A is three times as important as B, despite the fact that this is not how the numerical scale is explained to the decision maker completing each pairwise comparison (Warren, 2001: 11). As a result, it is also impossible to assign magnitudes of preference in excess of the limitations of the maximum and minimum values of the AHP scale: ranging from $\frac{1}{9}$ to 9.

The fact that Saaty assumed his scale to be a ratio scale does more than bring simple row operations into question. It also raises the question of whether or not it is possible to justify using the dominant eigenvector, which contains the maximum eigenvalue λ_{max} , in the process of determining consistency. Though using the dominant eigenvector in this way appears to work well, strictly speaking, a number of authors find its use questionable, and have suggested using the geometric mean instead (Warren, 2004: 13). Some of this debate is of a highly academic nature and is likely of limited practical importance to the problem of designing a system for Arctic surveillance, if AHP were to be used. This perception is reinforced by Warren's own statement that several attempts to simulate the relative merits of using different measures, including the dominant eigenvector and the geometric mean, have not yielded conclusive results favouring one or the other (2001: 13).

The AHP also suffers from what is commonly known as the “rank-reversal” problem, which arises from the fact that the relative weights assigned to the criteria are not independent of the normalization that is performed on the weights of the options or alternatives under each criteria. AHP implicitly and incorrectly assumes this independence, leading to situations in which, when an option is added to the problem, the ranking of preferred options is often reversed. (Warren, 2004: 14) No satisfactory solution to this problem has yet been devised.

Finally, Warren (2004: 15-16) also cites a number of concerns with AHP's heavy use of normalization to make it possible to relate numbers measured in different units. AHP relies on additive normalizing constants, but those constants are not consistent between levels. This means that one can normalize without difficulty within a given level, but applying criteria weights from a different level can lead to the normalized numbers effectively being ranked on different scales. This is not intuitively obvious in following the AHP algorithm, but has been shown analytically and, as above, no convincing solution has yet been found.

In summary, AHP is appealing because it is easy to implement and easy for decision makers who are not experts in MCDM to understand. On the other hand, the literature has identified a number of flaws in the theoretical basis of AHP that will require careful consideration.

2.4.7 ELECTRE

The Elimination and Choice Translating Reality, or ELECTRE, method deals with outranking relations through a pair-wise comparison of alternatives under each of the criteria (Triantaphyllou, 2000). In this method, an alternative is dominated by other alternatives if it is not equal to or better than the rest under all criteria. The decision maker can, however, still select an alternative that is not quantitatively dominant, as sometimes happens in real-world decision-making.

The ELECTRE method starts with the decision matrix. The first step is normalizing the decision matrix followed by the second step of applying the weights for each criterion to the normalized matrix. The normalization procedure ensures dimensionless comparable entries while bringing the magnitude of the values into the same range across different criteria or alternatives. The decision maker normalizes the decision matrix by creating a relation between the performance value a_{ij} and the other performance values under the same criterion according to equation (2.18), below:

$$x_{ij} = \frac{a_{ij}}{\sqrt{\sum_{k=1}^m a_{kj}^2}} \quad (2.18)$$

where the summation is the sum of the other performance values for the criterion in question.

As mentioned in equation (2.18), the weight is applied to the normalized matrix by multiplying the weight vector containing the weight for each criterion by the matrix itself.

In the third step of this method, the concordance set is determined. This set includes all criteria for which a given alternative is preferred in pair-wise comparison. A discordance set is constructed as well, which is simply the complement of the concordance set. The fourth step of the method constructs a concordance matrix by summing the weights of the criteria found in the concordance set. The result is the concordance index, C_{ij} which indicates the relative importance of alternative A_i over A_j . The discordance index complements the

concordance index and a discordance matrix is also constructed. At this point, a threshold is introduced in order to quantitatively define dominance between two alternatives. The threshold value, $C_{threshold}$, can be determined by taking the average concordance index, or can be set to any value desired. Once this value is defined, it is compared to each entry in the concordance matrix. If C_{ij} is greater than or equal to $C_{threshold}$ then the criterion has a value of 1 in the concordance dominance matrix F . Otherwise it will have a value of zero. As before, the discordance dominance matrix, G , is the complement of F . As a result, the concordance and discordance dominance matrices are binary matrices containing values of 1 and 0. The next step is to determine the aggregate dominance matrix by taking the cross product of the concordance dominance matrix and its complement as shown below:

$$e_{kl} = f_{kl} \times g_{kj} \quad (2.19)$$

In the last step, if $e_{kl} = 1$, then alternative A_k is preferred to A_l by using both concordance and discordance criteria. The columns in the aggregate dominance matrix which contain at least one element equal to 1, are dominated by the corresponding row. Therefore any alternative or column that contains a 1 can be eliminated.

This method may result in a final set containing one or more alternatives for the decision maker. It does not assume that a unique ranking always exists in practice. A number of different candidate rankings can be derived from the remaining alternatives in the final set.

A clear disadvantage of ELECTRE is that it does not necessarily yield a final solution. It may help to eliminate some alternatives from consideration, but for the purposes of NW, it will be necessary to make a final decision on a single alternative to pursue. For this reason, while ELECTRE may be interesting to consider for comparison purposes, it will not likely be one of the algorithms used to obtain a final ranking.

2.4.8 TOPSIS

The Technique for Order Preference by Similarity to Ideal Solution – TOPSIS – is a refined method derived from ELECTRE. This method uses the same steps as ELECTRE to normalize and weight the decision matrix. The difference between the two is that TOPSIS then determines ideal and negative-ideal solutions which represent the theoretical most preferred and least preferred alternatives. The ideal solution denoted as A^* and negative ideal solution denoted as A^- , are defined as follows (Triantaphyllou,2000:19):

$$\begin{aligned} A^* &= \{(max v_{ij} | j \in J), (min v_{ij} | j \in \bar{J}), i = 1, 2, \dots, m\} \\ A^- &= \{(min v_{ij} | j \in J), (max v_{ij} | j \in \bar{J}), i = 1, 2, \dots, m\} \end{aligned} \quad (2.20)$$

where v_{ij} is the weighted normalized performance measure,

$J = \{j=1,2,3,\dots,n \text{ and } j \text{ is associated with benefit criteria}\}$, and
 $\bar{J} = \{j=1,2,3,\dots,n \text{ and } j \text{ is associated with cost/loss criteria}\}$

This is followed by using the n-dimensional Euclidean distance method to measure the distance, S_i^* , between the ideal solution and the actual alternatives. Similarly the distance from the negative-ideal solution is measured. The relative closeness to the ideal-solution, C_i^* , is calculated based on these distances. Based on these closeness values, the alternatives are ranked from highest to lowest in order to determine the best solution. Clearly, the alternative with the highest rank is closest to the ideal solution and therefore the best alternative.

For the purposes of NW, TOPSIS may be more useful than ELECTRE, in that it will yield a ranking of alternatives that can be used to direct the project. The use of the distance measure relative to the ideal solution is also useful in that it can serve as a “yardstick” to show how close each ranking is to the ideal, and therefore as a way of comparing rankings with one another.

2.4.9 MARCUS

Multicriteria Analysis and Ranking Consensus Unified System or *MARCUS* is based on a branch and bound algorithm which finds solution(s) to a problem by using a solution concept described by Emond (2006). The purpose of MARCUS is to assist decision makers in reaching a group consensus. MARCUS was developed at DRDC’s Centre for Operational Research and Analysis (CORA) using FORTRAN which uses only integer arithmetic. The input to MARCUS is integer rank values by decision maker. For example, given four alternatives A, B, C and D, can be ranked by a voter as follows:

<A C B D>

The input to MARCUS would be as shown below;

(1 3 2 4) indicating A has been ranked as first, B (the second alternative) is ranked third, C as second, and finally D has been ranked fourth.

If an alternative has not been ranked, the input to MARCUS will be 0.

If two alternatives are tied, then both will have a same rank assigned to them as shown below:

<B A—C D> → (2 1 2 3)

However, if a weak preference is indicated, then there will be two inputs with each carrying half of the weight assigned. For example:

<A B ≥ C D> → (1 2 2 4) and (1 2 3 4)

The output of MARCUS is a single ranking of alternatives which represents a consensus of the group. This “consensus” is determined by finding a “centroid” ranking – that is, a

ranking which minimizes the total distance (determined according to a defined metric) between itself and the input rankings provided by the decision-makers.

Even though MARCUS allows decision makers to rank alternatives as a tie or incomplete ranking in which not all options are ranked, it does not determine the strength of preference by the voters. For example, if A is preferred to B and B is preferred to C, MARCUS will give a consensus ranking:

<A B C>

But the above does not indicate the strength of agreement among the decision makers. Did all decision makers strongly agree that A is a better alternative compared to B? There are methods of sensitivity analysis involving tying any two adjacent ranked options that can be used to provide some measure of the level of consensus between decision-makers on the final consensus ranking (Eles, 2006).

The greatest advantage of MARCUS is that it is able to incorporate ties and weak preferences as input in addition to simple strong-preference rankings, and is able to calculate a consensus ranking among all decision-makers using a rigorous process involving the distance measure. On the other hand, MARCUS is computationally demanding (Emond, 2006), which puts a practical limit on the number of input rankings and alternatives that can be considered.

The algorithms described in this section may be used to conduct the analysis needed to identify feasible and cost-effective options for the NW TDP. The following section details the selection of the algorithm to be used from among the possible options.

2.5 MCDM Algorithm selection

As discussed in the literature review above, there are different algorithms which could be used to determine the best combination of sensors at the choke points, to perform surveillance. Each algorithm has advantages and disadvantages. For this project, AHP was chosen because it is logical and relatively simple to implement. However, AHP also has a number of flaws of which we will need to be aware (see section 2.2.6, above).

Neither the weighted sum method nor the weighted product method is suited to analysis of the problem at hand in NW. WSM is not able to address the kind of multi-dimensional analysis needed in such a complex situation. The WPM requires a level of certainty in the values assigned to the weights of each criterion that is very difficult to achieve in a situation involving multiple decision-makers. Achieving agreement on the w_j exponents would be a very difficult task without some kind of analytical framework to assist decision-makers.

The ELECTRE method might be a more suitable candidate than these because of its ability to eliminate dominated alternatives, but it was not chosen because it suffers from other drawbacks. Foremost among these is the fact that it does not necessarily yield a usable

ranking at the end, but only eliminates dominated alternatives. Second, it has the same disadvantage as WPM in that it requires that the weights assigned to each criteria be precisely known, without specifying a means of determining those weights. TOPSIS would be an improvement as it can yield a final ranking, but since it is based on ELECTRE it still depends on a very precise knowledge of the weights and therefore is not practical for the situation of NW, where there are multiple decision makers, each with a different view of the relative importance of the criteria.

MARCUS was also a possible candidate. However, the following disadvantages were sufficient to favour AHP over MARCUS for this application. For MARCUS, it is not possible to quantify relative preference; it is not possible, for example, to indicate that option A is preferred twice as much as option B. Also, the number of alternatives can greatly affect MARCUS' performance. As Emond (2006 : 20) states: "The problem of searching the set of all weak orderings of n objects for optimal solutions quickly becomes impractical when n reaches the 15 to 20 range. Even to handle problems with n as large as 15 require very efficient code on the fastest computers available." This scenario with 9 alternatives (see Table 8, below) is not within the range in which computation becomes unmanageable, but it is a possible concern.

The main reason that MARCUS is not the ideal tool to use for the NW application is that, though it was designed for group decision-making, it involves decision-makers ranking the alternatives in order of preference. Since for NW, a simulation model will be used to generate predictions of sensor performance, the MARCUS method is not appropriate. What is needed is a system that will allow decision-makers to rank the importance of the criteria, and then apply the aggregated criteria weights to the results of the simulation.

In order to accomplish the modeling for ships and sensors, software packages such as MATLAB will be sufficient. MATLAB's Statistics package provides functions which will be adequate to determine the probabilities mentioned in the previous sections. As a result, MATLAB can be used for the purpose of determining MOPs of the Northern Watch Station (NWS) based on the simulation. (MathWorks, 2009)

Under the circumstances, it is judged that the most appropriate, applicable, and simplest MCDM tool for the NW problem is selected as AHP. Given the circumstances of the NW TDP and the simulated environment that is part of this research, AHP provides the most flexible option that meets the needs of the situation. Due to the hierarchical nature of AHP it is easy to break down the main problem into sub-problems to make it easier for decision makers to comprehend each choice or comparison they are being asked to make. Furthermore, the intuitive pairwise-comparison approach taken in AHP makes it easier for decision makers to grasp the full range of tradeoffs involved in each comparison without any need for complex mathematical techniques. Also, AHP is capable of handling multiple criteria and multiple different dimensions, which will be necessary in the NW context. A further advantage of using AHP in the computerized environment of the Expert Choice software tool is that it immediately informs the decision maker of the consistency index rating of his or her rankings, allowing corrections to be made by reassigning weights to the criteria if the CI is found to be greater than 0.1 (Expert Choice, 2000).

Given its advantages in the specific context of NW, this paper will employ AHP to conduct the MCDM analysis, using the Expert Choice software package and develop a ranking for the Arctic surveillance alternatives.

It must be noted that, at this stage of the NW TDP, some sensors alternatives have already been selected based on their performance and availability. As a result, it will not be necessary to use any MOO algorithm to determine additional alternatives. Accordingly, only AHP will be used to choose among these determined alternatives. Using these tools, we should be able to construct a model that will give us usable results to test in the field, and ultimately to make up the new northern surveillance system under Northern Watch.

3 Methodology

The particular nature of the Arctic surveillance problem of interest to this thesis makes it difficult to conduct direct experiments or even obtain direct observations to aid the research. The geographic location of the area is remote and difficult to access. The types of equipment and vessels needed to test surveillance systems are expensive to procure, install and maintain. In addition, the nature of the research is speculative: even if the area were not so remote and access were easy, the reduction in ice cover and increases in traffic that make maritime surveillance of the Northwest Passage acutely important have yet to occur.

Given all of these conditions, the research proposed here is well suited to an approach using a simulation of the maritime surveillance system to provide input to the MCDM model. This chapter contains a discussion of the precise methodology chosen.

3.1 Methodology Overview

The thesis is focused on developing a ranked set of options for different configurations of sensors to perform surveillance of the Northwest Passage. The first stage in doing this will be to obtain information about the effectiveness of different sensors and combinations of sensors in the Northwest Passage. Because of the practical difficulties, described above, that prevent direct observation of the system of interest, it is more directly feasible in this thesis to build a simulation model of the capabilities of the Northern Watch station.

From these models, it will be possible to determine measures of performance (MOP) and measures of effectiveness (MOE) for the types of sensors available. These measures can then be used as input parameters for a multicriteria AHP model (see section 2.5, above) which will also take into account other variables such as the cost of installing and maintaining sensors, the practicality of providing power to the sensors, the difficulties of communicating with the sensor, etc. The AHP model will yield a number of possible options, as well as a relative ranking of the selected surveillance alternatives.

Figure 1, below, presents a flowchart view of the process to be used in developing the model and conducting the ranking of the options presented. Note that “MOP” in the diagram refers to Measures of Performance, discussed in greater detail in section 3.4, below.

The process flow described in the diagram shows that the results of the GIS analysis will be used to inform the shipping simulation model, which will then produce estimates of sensor performance. Those estimates will then be used as inputs to the AHP model, along with information on cost and power consumption of the different sensors considered. Finally, the AHP model will yield the necessary set of ranked options for Northern Watch.

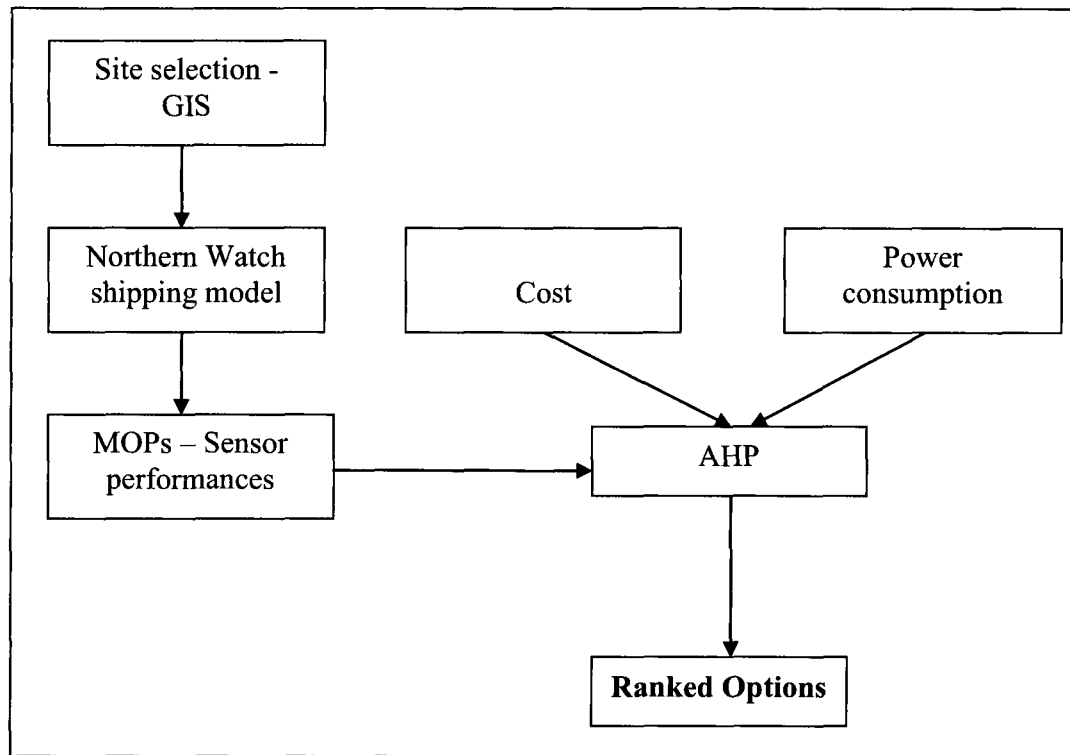


Figure 1 - Option ranking problem – flow chart

The main portion of the work on the project will come in two areas:

- 1) develop the model to simulate the sensors used in Arctic marine surveillance and to measure the effectiveness of those sensors under simulated conditions; and
- 2) adapt the results of the simulation model to produce input for the data requirements of the AHP multicriteria ranking of surveillance options.

The following sections discuss in greater detail each subcomponent of the thesis methodology as described in Figure 1.

3.2 Use of GIS for site selection

In order for the simulated detection of ships by sensors to be representative of the specific context of Northern Watch, it is important that the simulation accurately reflect the geography of the Barrow Strait. For this purpose, the GIS description of the area is instructive.

For this project, ArcGIS software version 9.3 was used to generate all of the analysis and maps presented in this paper. Layers containing the outlines of the land areas of Canada, Greenland and Alaska were downloaded from the Natural Resources Canada GEOGratis

library. These data are provided in the Clarke 1866 Lambert Conformal Conic projection (Natural Resources Canada, 2008).

For the ice coverage and concentration, the U.S. National Ice Center's Arctic Sea Ice Charts and Climatologies in Gridded Format were used. These Arctic sea ice charts from the U.S. National Snow and Ice Data Center (NSIDC) were provided in the WGS 1984 Stereographic North Pole projection (National Ice Center, 2006).

This dataset contains Arctic sea ice concentration climatology derived from the National Ice Center's weekly or biweekly operational ice chart time series. The charts used in the climatology are from 1972 through 2007 and the monthly climatology products include the median, maximum, minimum, first quartile, and third quartile concentrations. The dataset also contains information about the frequency of occurrence of ice. All of these data are available on 33 year, 10 year, and 5 year time scales.

Though the data are available from 1972 to 2007, the 5-year interval files, from 2003-2007 show the most current changes in sea ice. After analysis of these data, it becomes clear that the recent files would provide a better result for this research. Therefore, this project used the 2003-2007 data because these are most likely to represent the recent effects of climate change on Arctic sea ice. This dataset also contains median, maximum, minimum, and information about the frequency of occurrence of ice on a monthly basis. However, after analysis, only the median scenarios were selected for further analysis.

The dataset is a 361 by 361 unsigned-byte array with values as described in Table 1.

Value	Description
0 - 100	Ice concentration (percent) (in multiples of 5)
108	Fast ice
157	Undigitized (appears in some early charts)
253	Areas not covered
254	Land

Table 1 - EASE-Grid Climatology Data File Values (Source: National Ice Center, 2006)

Parameter values are given as percentages. That is, each grid cell has a value between 0 and 100 representing the percentage of ice concentration. By definition, fast ice is 100 percent concentration because it is physically connected to the shore.

There was an issue of matching the coordinate systems between the maps from the U.S. NSIDC and Natural Resources Canada (NRCan). This issue was resolved using a transformation and changing the data frame's projection and exporting the data. As a result, all layers are in Clarke 1866 Lambert Conformal Conic projection.

3.2.1 GIS results

Northwest Passage Routes:

Though it is usually called “the Northwest Passage”, there are actually a number of different routes that can be taken to lead westward from the Davis Strait to the Beaufort Sea through the waterways of the Arctic archipelago. In this project, we will consider three major routes for simplicity. The routes are described below as they are traversed from east to west:

Route 1: Davis Strait – Lancaster Sound – Barrow Strait – Viscount Melville Sound – McClure Strait – Beaufort Sea

Route 2: Davis Strait – Lancaster Sound – Barrow Strait – Viscount Melville Sound – Prince of Wales Strait – Amundsen Gulf – Beaufort Sea

Route 3: Davis Strait – Lancaster Sound – Barrow Strait – Peel Sound – Victoria Strait – Queen Maud Gulf – Dease Strait – Coronation Gulf – Dolphin and Union Strait – Amundsen Gulf – Beaufort Sea

Route 1 is the least practical of these, as the McClure Strait usually hosts a relatively high concentration of ice (see Figures 2 and 4, below). Routes 2 and 3 are generally more easily navigable and are therefore more likely to be appealing for surface transits even though they are longer than Route 1. Submarine transits may not be subject to the same restrictions as they can simply sail under the ice.

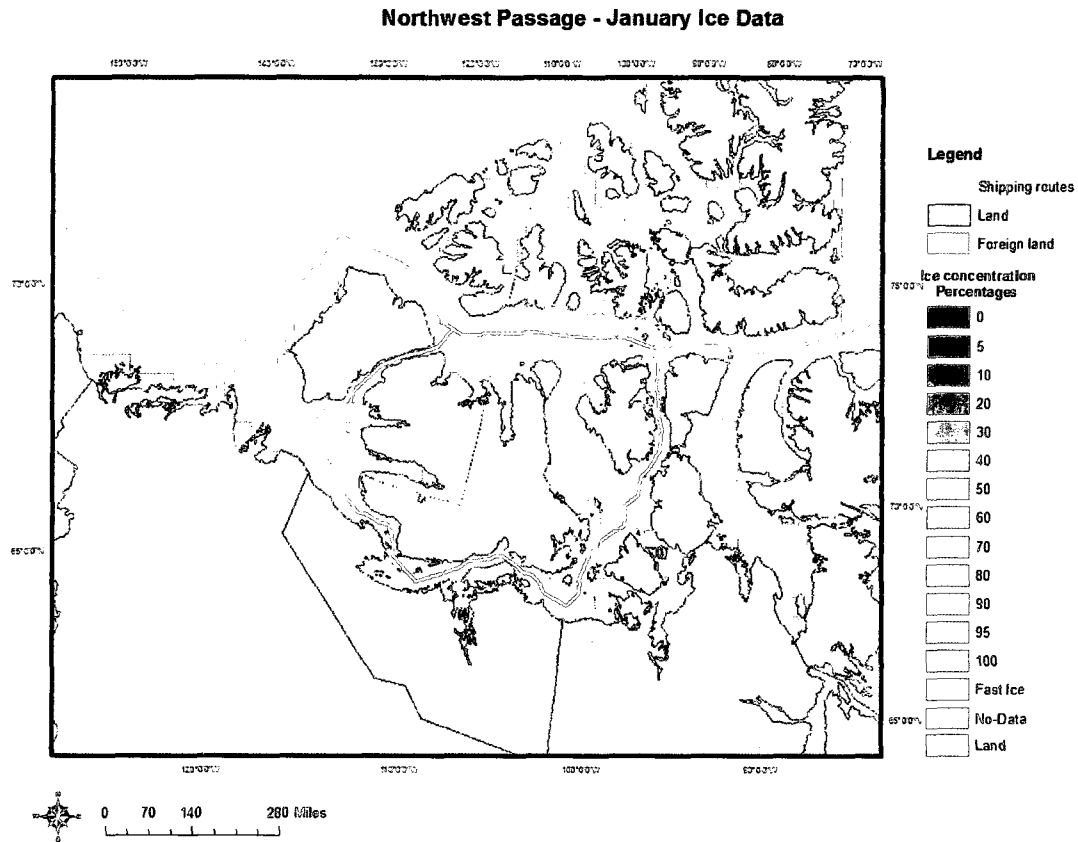


Figure 2 - Northwest Passage showing January median ice conditions 2003-2007

From this analysis of possible route, it is clear that Barrow Strait and Lancaster Sound are the prime strategic narrows (“choke points”) that allow commanding surveillance of the entire Passage. Barrow Strait has a slight advantage in that it is somewhat narrower than Lancaster Sound, which is why it was chosen as the main choke point for the trials of the Northern Watch project (MacLeod et al., 2008: 1, and Waller et al., 2008: 3).

In order to evaluate the navigability of the Northwest Passage under any given ice conditions, a polygon with an approximate area of 1,845,400 square kilometres was created which would enclose all three routes. The intersection of this polygon and the ice data for all 12 months was then created. Since these new intersect layers include land as well as ice, only ice concentration percentages and fast ice were selected for the analysis from the tables using a SQL query ($\text{GRIDCODE} \leq 108$, Table 1). The statistical tool was then used to obtain the mean. The intersection of that polygon and the ice map for the month of November is shown in Figure 3.

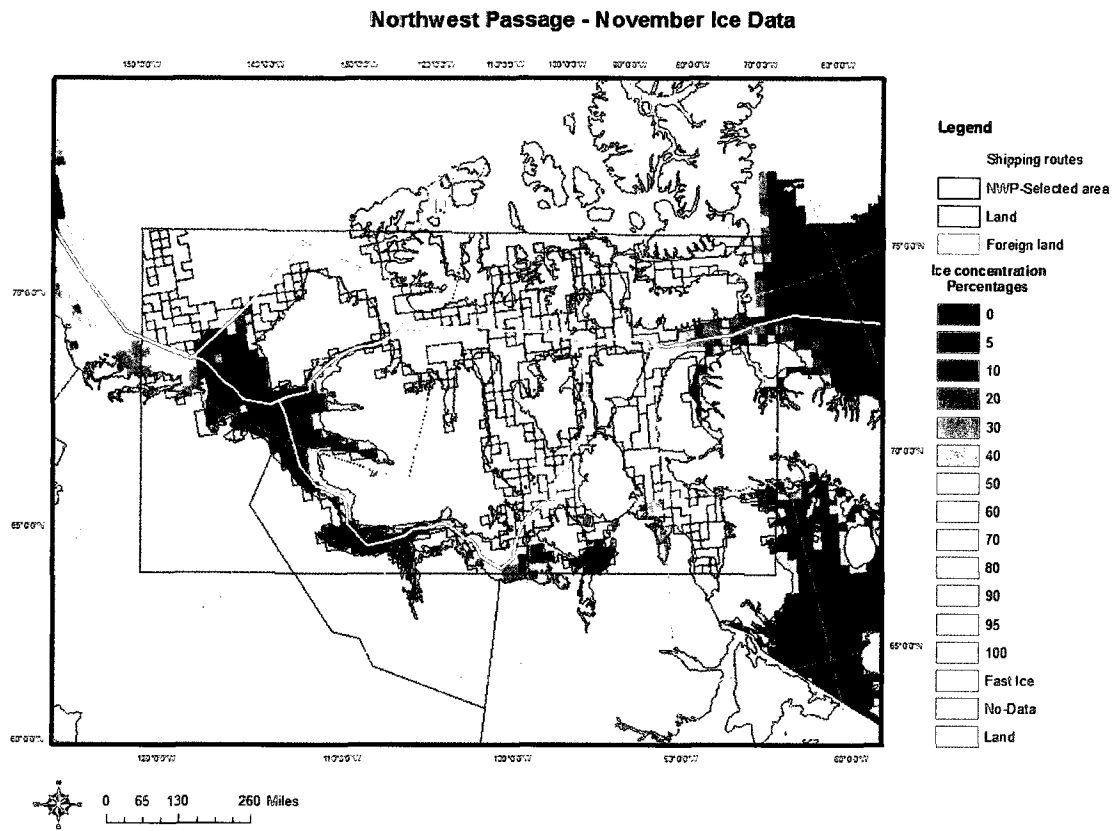


Figure 3 - Polygon and ice data used for statistical analysis of ice concentrations

Using the statistics tool, graphs of ice concentration for the selected area of the Northwest Passage were created. See Figure 4 for a representative sample of the results (complete results are found in Appendix A). These graphs excluded land and the area within this selection that did not contain any values. According to these plots, it is clear that between the month of May and October, the ice index mean value is lowest with August having the lowest mean value of ~48%. As a result, this is the best natural time-frame for the highest traffic in the passage (refer to Figure 4).

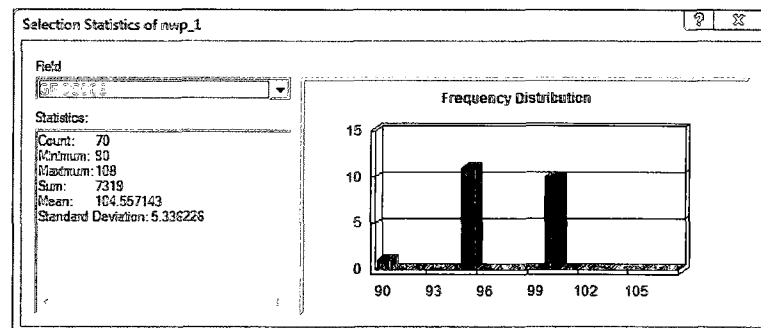


Figure 4 - Ice concentration percentage statistics for the month of January 2003-2007

A report summary was created for January and August to determine what the highest quantity of high ice concentrations were. In August, as shown in Figure 5, the area of the Northwest Passage had between 5-30% ice based on the numbers while January had mostly fast ice. The fast ice in month of January covered about 69% of the passage.

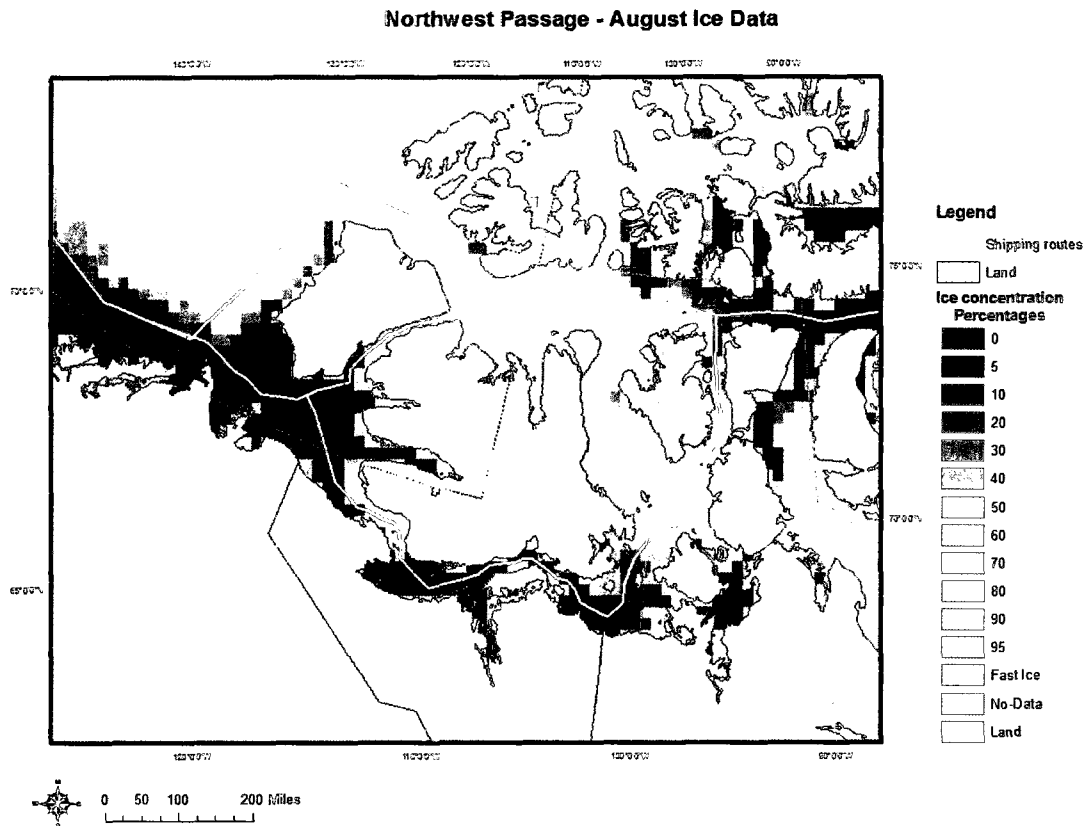


Figure 5 - Northwest Passage showing August median ice conditions 2003-2007

The map of northern communities and airports was analyzed in GIS in order to add value to the decision-making processes of placing the sensor site. This shape file was available at Natural Resources Canada. The file is based on 1996 census data (Natural Resources Canada, 2008).

3.2.2 GIS Conclusion

This section presents the results of the GIS analysis discussed in the previous section. It discusses the possible chokepoints to be considered by the NW TDP.

Placement of sensors:

Figure 6 shows the position of the potential sensor sites and types along the Passage.

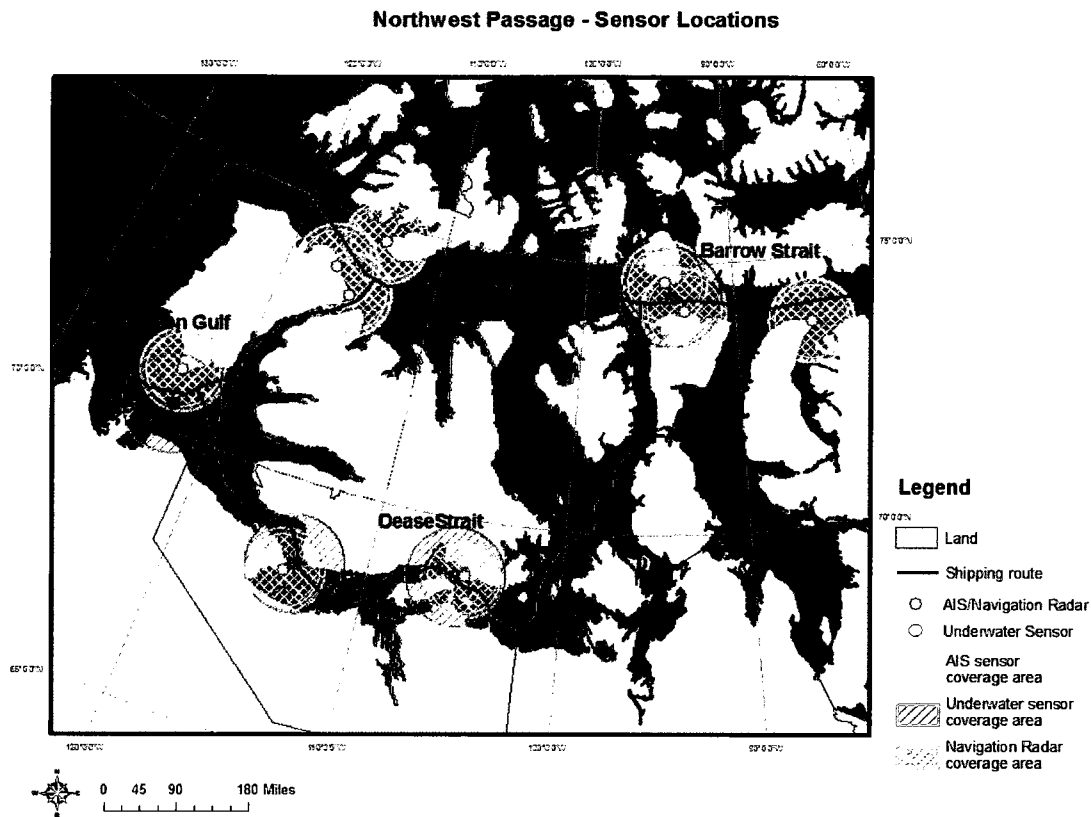


Figure 6 - Possible placement of sensors

The different types of sensors available for use in the NW TDP are discussed in detail in section 3.3.2, below. However, for the purposes of GIS, there are four main sensor types to consider: navigation radar, Automatic Identification System (AIS), underwater sensors, and electro-optical/infrared (EO/IR) sensors. In Figure 6 above, three of these sensors and their coverage areas are shown.

Nine combined radar and AIS stations and four underwater sensors were considered for the location analysis. AIS and navigation radars were located approximately 5km inland from the shore line while underwater sensors are located exactly in the middle of the channels.

The ArcGIS ArcToolBox and shape files consisting of lines were used to determine these distances. The location of each sensor was dependant on the following factors:

- 1- Width of the passage: It is important to place the sensors at the narrowest choke point to increase the probability of detection. A shape file consisting of lines was created to measure the distance between various chokepoints along the shipping

routes as shown in Figure 7. These distances were determined from the ArcGIS software.

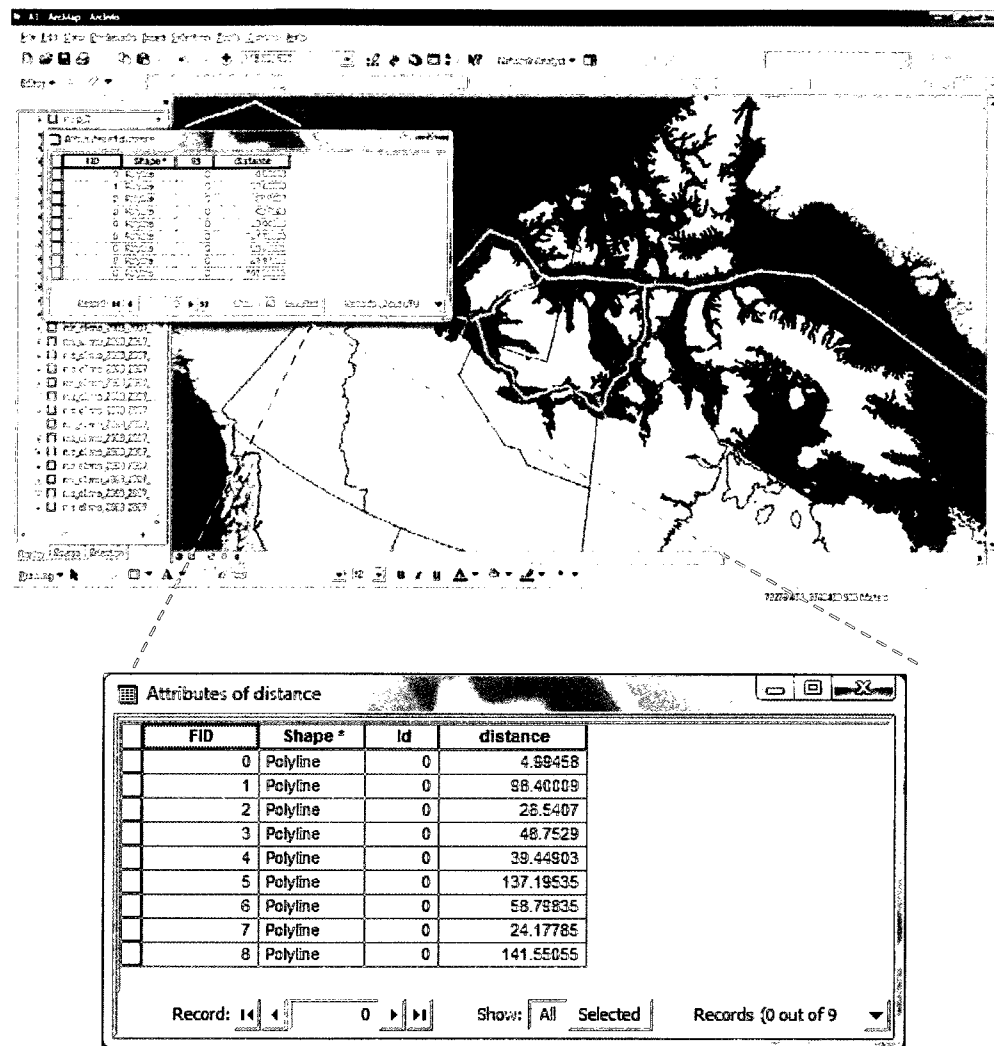


Figure 7 - Chokepoint distances as determined from ArcGIS

- 2- Ice concentration percentage: as mentioned above, some areas along the passage are covered with ice or mostly fast ice during most months of the year (such as Route 1). Even though this area is the shortest distance between Pacific and Atlantic oceans, the ice conditions make it very difficult for vessels to go through. Figure 8 shows the ice index for the month of September. According to this map, Routes 2 and 3 are partially clear of ice during this month.
- 3- Accessibility by land and air: one factor affecting sensor placement is site accessibility and closeness to an existing community to reduce cost. If the sensor system is near a community, it will be more cost-effective in the long run for

maintenance and data collection purposes. Figure 9 shows the northern communities in the Arctic region.

It is clearly most advantageous to try to cover choke points with sensors to achieve the highest possible overall control over the traffic inside the Northwest Passage. For this reason, Barrow Strait and the Amundsen Gulf (refer to Figure 6) are very important to cover as a first effort, to “close the gates” at the eastern and western ends of the Passage, respectively.

It must also be noted that as shown in Figure 3, the created polygon is only an approximation and it contains data from some areas that are not part of the shipping routes.

As a secondary checkpoint, it will be important to control one of two strategic narrows along Route 3, the southernmost of the routes. The September ice charts (Figure 8) show that this will be the most ice-free of the routes and therefore the easiest for ships to transit. The Dease Strait offers an excellent natural choke point at which to install a further sensor site (see Figure 6). This site is also good because of its proximity to Cambridge Bay, which could serve as a logistical base.

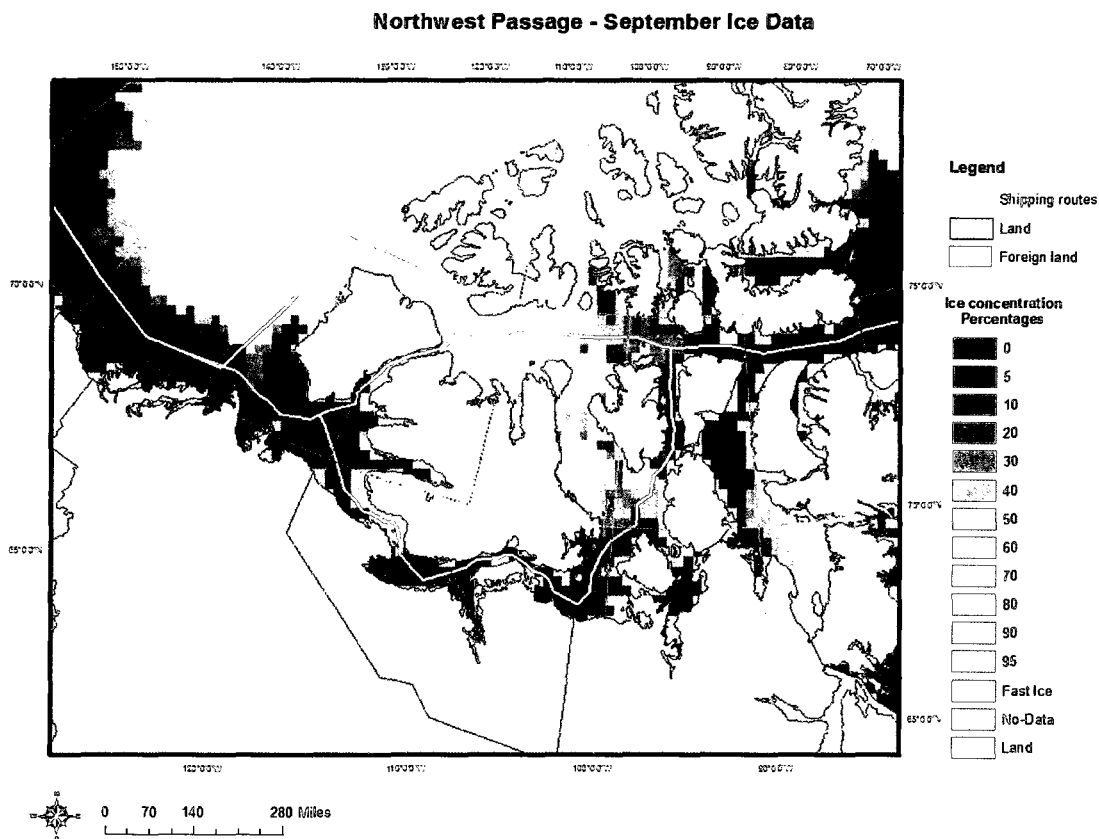


Figure 8 - Northwest Passage showing September ice conditions

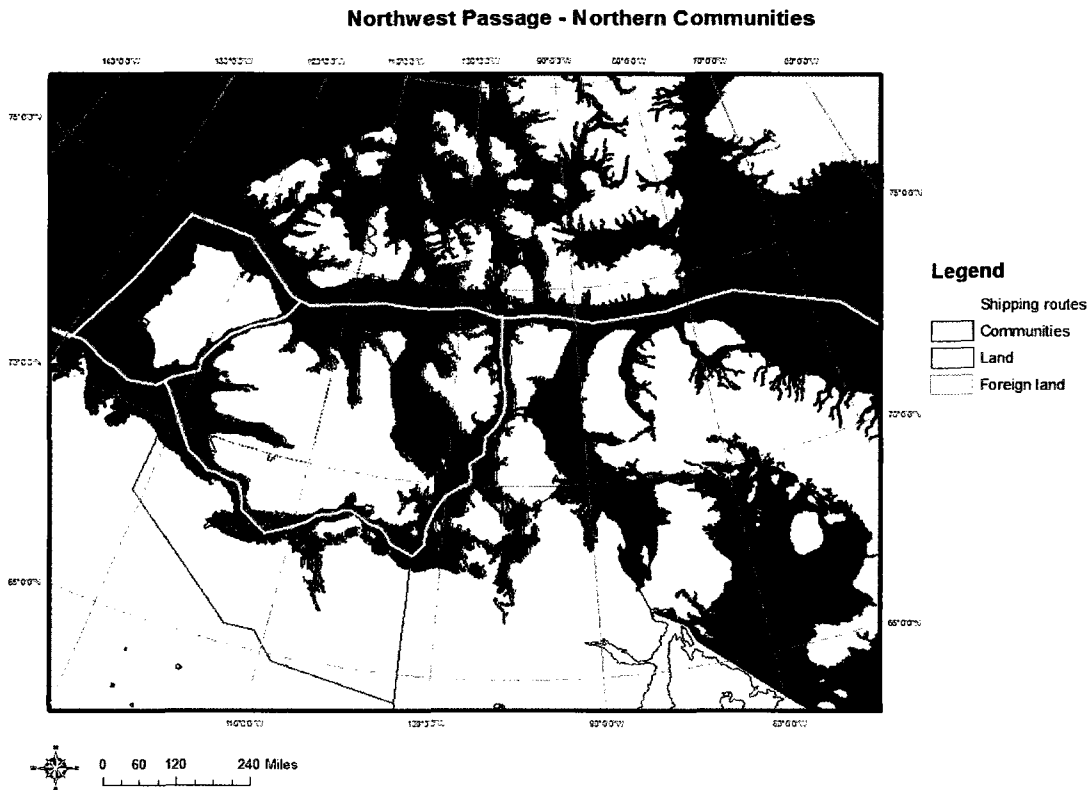


Figure 9 - Northern communities

From an examination of any of the maps of the Northwest Passage it is clear that the Barrow Strait chokepoint makes an excellent point for the Northern Watch sensor trials since all possible routes must pass through the Strait. However, the GIS analysis presented here shows that other potential chokepoints could be considered as part of Northern Watch, though this information is not sufficient to make an informed decision on sensor placement. A shipping model is needed to simulate the performance of these sensors in detecting vessels passing through their area of coverage. The following section outlines the development of that model.

3.3 Simulation model

A simulation model was constructed to evaluate the relative performance of the four different types of sensors (see Table 2) in detecting vessels transiting through the Northwest Passage. The sensors were considered both alone and in combination. The purpose of the simulation model is to generate statistical measures of performance that can be used to complete the data requirements for the AHP model to inform decision makers' ranking of the selected sensor combinations (see Section 3.7, below).

3.3.1 Ship generation

The yellow lines in Figures 2, 4, 5, 7, 8 and 9 show that there are three different shipping routes which could be taken by large commercial ships through the Northwest Passage. There are two ship sizes considered by the shipping model: small and large. Small ships would be approximately 15 m in length, and include vessel such as a recreational sail boat or commercial fishing boat, while large ships would be approximately 140 m long, and include large ocean-going freighters or ice breakers being used as a cruise ship (Waller et al., 2008: 14). A small ship is assumed to travel at a speed of 7 NM/hour while a large ship travels at a higher speed of 12 NM/hour. In the model we estimate the probability of a vessel appearing in the passage as considered to be 50% small or 50% large.

As mentioned above, there are 3 routes considered for the traffic through the Northwest Passage. However, since the Barrow Strait is common to all of them, only the Barrow Strait is considered for this study. Accordingly, an area of interest (AOI) has been constructed, as a box containing a point A, which is the entry point for any given vessel into the passage, and a point B where the vessel exits the AOI (see also Figure 10 below). Points A and B are selected at a random within the box for each run. Point A's latitude is between 74.075N and 74.481N and its longitude is 90.919W. Point B's latitude is between 74.239N and 74.561N and its longitude is 99.913W. Vessel V is assumed to travel from point A to point B as shown in Figure 10, below. The coordinates of the AOI correspond to a representative section of the navigable channel through the Barrow Strait, sufficiently close to a good candidate sensor site that we can adequately study the sensor types and combinations required by the research.

The speed of the vessel determines how long the target is within detection range of each sensor, and because the simulation operates on a discrete timescale of one minute per "tick", the faster the vessel is moving, the fewer the number of ticks (or opportunities for the sensors to detect the vessels) will occur between point A and point B. This means that if the vessel is travelling at a higher speed, the probability of detection is lower overall due to the number of opportunities the sensor is given to detect. However, the probability of detection for each sensor considers the size (width and length) and source level (in dB) for each vessel type.

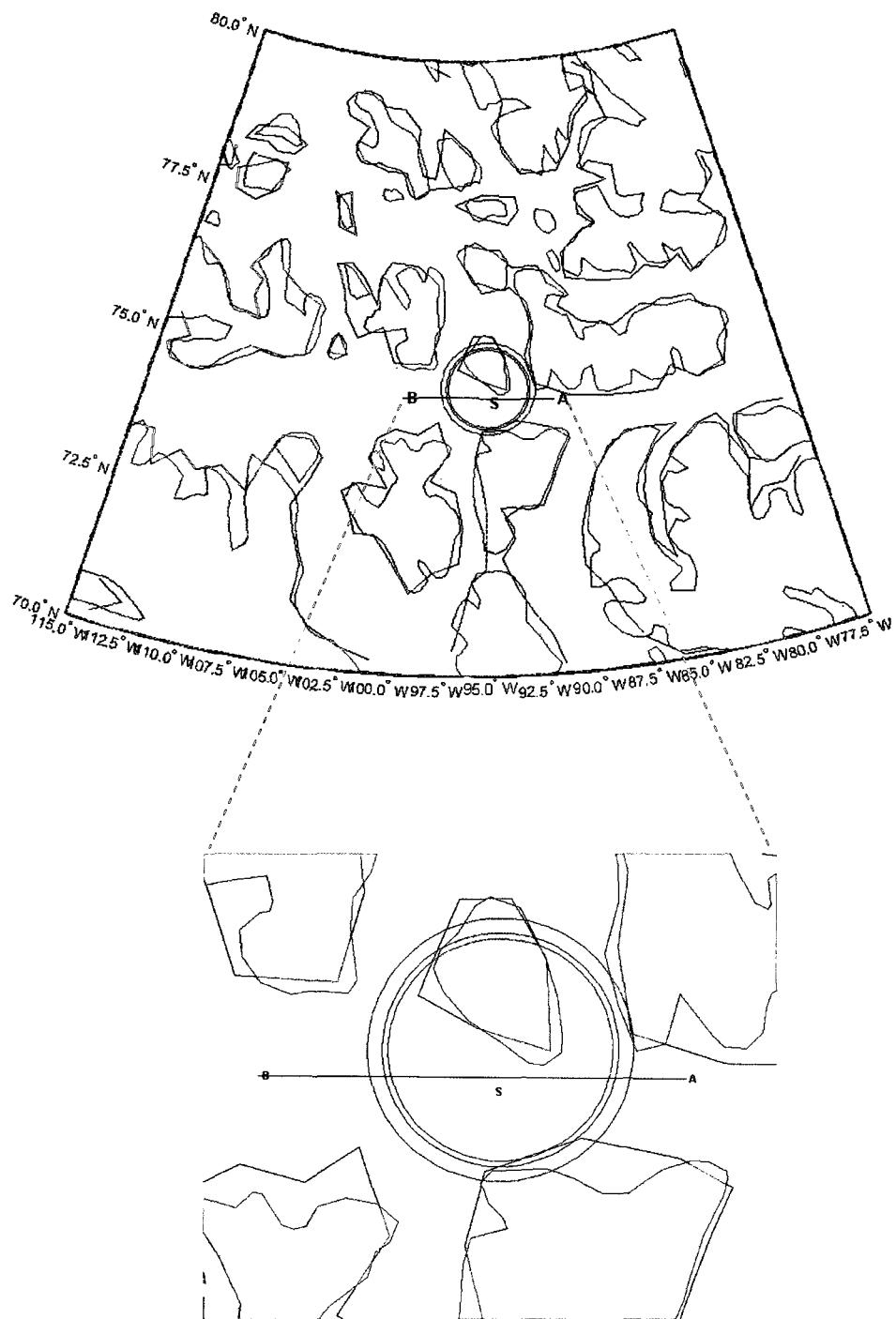


Figure 10 - MATLAB map output showing a sample route from point A to point B

Because of the random latitude of points A and B, each vessel arrives at the passage on a random path. Depending on the vessel type, the sensor ranges are set up for detection,

classification and identification as noted in Tables 3 and 4 below. The intersection of the vessel path with the various sensor contours provides the total distance over which the vessel is within each range bracket (i.e., region in which the probability of detection is constant with respect to a given sensor).

The speed of the vessel and this distance determine the length of time the vessel is within a certain range bracket. From this is determined the number of detection attempts the sensor will have for the vessel in that range bracket. The number of true detections relative to the total number of possible attempts/chances determines the overall detection rate for the given sensor. Generally, if the vessel is closer to the sensor, the chances of detection is higher. The sensors are set to record detection (or not) every minute.

The distance and the time stamp of the first and last detection are noted. The total simulation run is based on the length of the passage which is predetermined for this study as discussed above. This information is used to calculate the measures of effectiveness, as described above.

3.3.2 Sensor model

The general purpose of NW is to monitor the Northwest Passage by detecting, classifying and identifying vessels passing through it. The model of the sensors depends mainly on the effectiveness of the different types of sensors which may be installed to monitor the Barrow Strait choke point. The sensor types available for the NW TDP are as follows:

Abbreviation	Full name	Sensor type
S1	Sensor 1	Conventional search radars
S2	Sensor 2	Automatic Identification System (AIS)
S3	Sensor 3	Electro-optical/Infrared (EO/IR)
S4	Sensor 4	Underwater (UW)

Table 2 - Simulation model sensor nomenclature

These sensors are deemed to be practical for the Northern Watch application (McLeod et al., 2008). The simulation model makes it possible to derive specific quantitative measures of the effectiveness and performance of each of these sensors, as described below.

3.3.2.1 Measures of sensor performance

For each sensor, it is essential to know the level of their capabilities, the most important of which is generally the range at which each can detect, classify and identify a vessel. The tables below summarize the ranges at which each sensor detects, identifies and classifies a small and a large vessel, respectively. Note that 1 international nautical mile (NM) is 1853 meters.

Sensor	Range (NM)		
	Detect	Classify	Identify
S1. Navigation Radar	21	N/A	N/A
S2. AIS	41	41	41
S3. EO/IR	34	1.6	0.65
S4. Underwater Sensor	54+	N/A	N/A

Table 3 - Sensor predicted performances – small vessel (adapted from McLeod et al., 2008: 45)

Sensor	Range (NM)		
	Detect	Classify	Identify
S1. Navigation Radar	40	N/A	N/A
S2. AIS	46	46	46
S3. EO/IR	40	5.5	3.6
S4. Underwater Sensor	54+	N/A	N/A

Table 4 - Sensor predicted performances – large vessel (adapted from McLeod et al., 2008: 46)

For each of the sensors considered here, a contour plot with various probabilities for detection, classification and identification is determined based on Tables 3 and 4 specifications. (These values are estimated by the NW team due to lack of real empirical data from Arctic trials that are yet to be undertaken)

Sensor performances are directly related to range. As the vessel range from the sensor increases, the probabilities of detection, classification, and identification by each sensor decrease. As a result, performances are functions of range.

Figure 11 demonstrates an ideal relation between probability of detection by Navigation Radar and range:

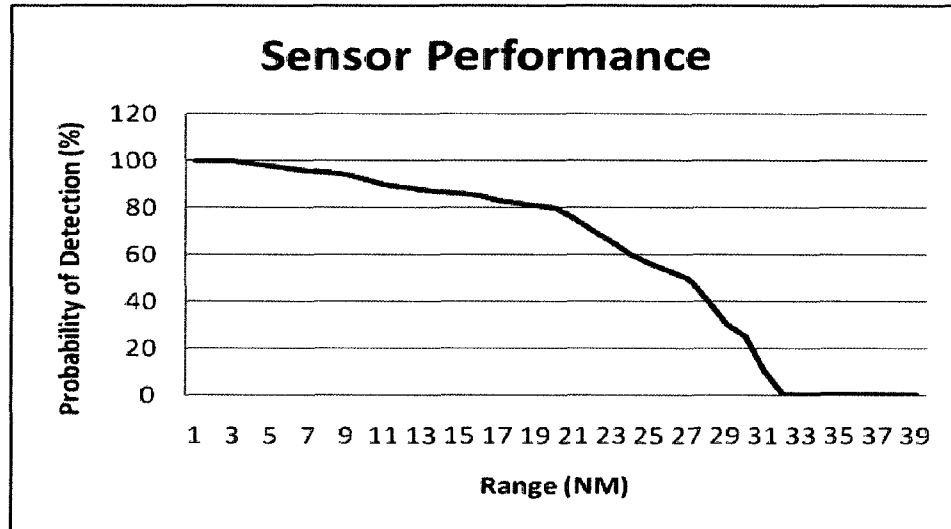


Figure 11 – Navigation Radar probability of detection as a function of range – ideal conditions

The above figure shows that as the range between a vessel and a sensor increases, the probability of detection decreases. At 32 NM the probability of detection is zero for Navigation radar sensor. As mentioned earlier, with the available data from each sensor, a different curve is developed for this project based on sensor specifications. This estimated curve is for Navigation radar developed for the model is shown in Figure 12. In this figure, the relationship between detection probabilities and range is a jack-knifed step function rather than a smooth curve.

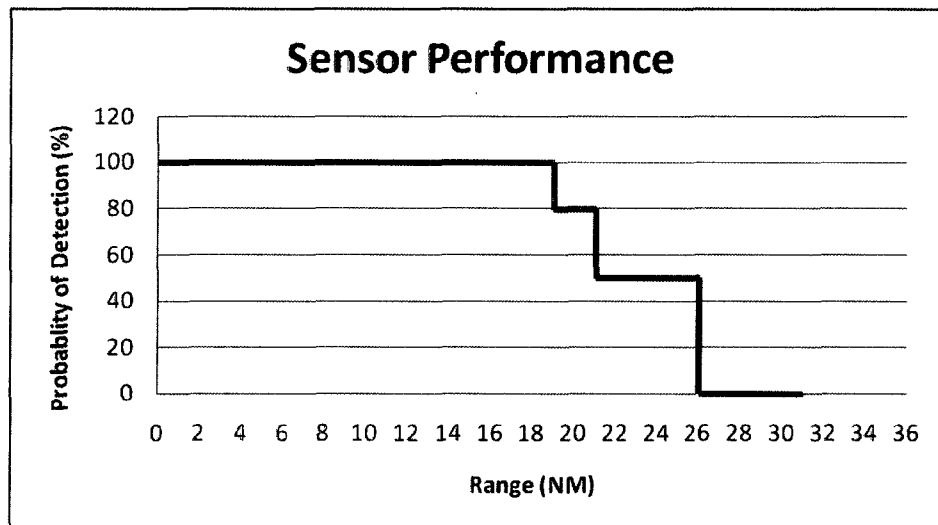


Figure 12 – Navigation Radar probability of detection as a function of range – as modelled

For sensors' measures of performance, a contour plot based on step-function detection has been generated. This plot shows the ranges at which the detection, classification and identification are achieved with different jack-knifed confidence levels of 100%, 80%, and 50%. Figure 13 illustrates the contour plot:

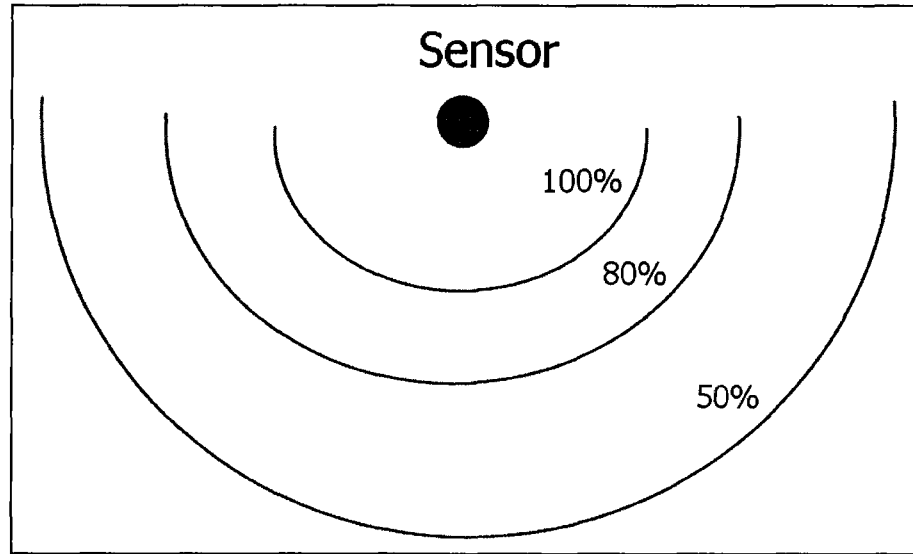


Figure 13 - Example of a contour plot for probabilities of detection of vessels

Weather has a significant role in detection, classification and identification probabilities. In this model, we apply two types of conditions considered for the weather: “ideal” and “poor”. Ideal is considered to be full daylight on a clear day with no ice. Poor conditions refers to darkness and snowfall with moderate ice conditions (Waller et al., 2008: 14). In ideal weather, which occurs 80% of the time, the probabilities are 100%, 80% and 50%. However, if the weather is poor, the probabilities will drop to 85%, 65% and 25%. The probability of poor weather occurring is set at 20% of the time.

As mentioned earlier, range is an important factor in determining the probabilities of detection, classification and identification. To obtain a value for each contour, minimum, median, and maximum range (R_{min} , R_{med} , and R_{max}) for detection/classification/identification must be considered. However, due to lack of data, it is not possible to measure these ranges accurately as expected by the sensor options (this field data is anticipated, but not yet available). As a result, for the purpose of this research these ranges were estimated.

Estimated ranges and the associated probabilities used in the model are provided in Table 5 below for the measures of performance (MOPs) of each sensor type for small and large vessels.

	R_{min} (Nm)		R_{med} (Nm)		R_{max} (Nm)	
	small	large	small	large	small	large
<i>Navigation Radar</i>	0-18	0-38	19-20	39-40	21-25	41-45
<i>AIS</i>	0-39	0-44	40-41	45-46	42-46	47-51
<i>EO/IR</i>	0-32	0-38	33-34	39-40	35-39	41-45
<i>Under Water Sensor</i>	0-52	0-52	53-54	53-54	55-59	55-59

Table 5 - Detection ranges for a NWS for small and large vessels

R_{min} is associated with the range at which a sensor is able to detect a vessel at a 100% probability in good weather. R_{med} and R_{max} refer to the probabilities of detection at 80% and 50% respectively for good weather conditions.

As shown in Tables 3 and 4 above, not all the sensors are capable of classifying and identifying a vessel. For example, Navigation radar is only capable of detection; AIS' classification and identification is only limited to a prior detection within the same range as detection; and UW sensors' ability to classify and identify is not discussed in this paper as the data are not available. However, for an EO/IR sensor, the classification and identification ranges vary from those for detection. For the purpose of classification and identification, no probability contour plot has been considered. If the vessel is within classification and identification range of the EO/IR sensor, it is assumed that the vessel is, at the same time, both classified and identified. It must be noted that this assumption is only valid for this report and in a real scenario even if the target is within range, it still might not be classified or identified especially in poor weather conditions. Field data are anticipated but not yet available for Arctic conditions.

As mentioned earlier, the goals of the NWS are to detect, classify, and identify vessels passing through while they are in range. As stated by Waller (2008), detection, classification, and identification are range-dependant. The goals must be accomplished considering different weather conditions.

3.3.2.2 Other variables such as cost, power, etc

This section will consider variables which will affect the ranking of the options. Cost is one of the important variables. Total cost includes the cost of purchase, and the power consumption requirements of each sensor. The performance of each sensor is not only limited to its ability to detect, classify and identify targets. Other factors such as volume and frequency of data which it can provide and also its power usage should also be considered. In this thesis, only data for installation cost and power consumption are available. It is recommended to consider other factors such as operation cost and communication usage in future studies as this information comes available.

3.4 *MOPs – Measures of Performance and MOEs – Measures of Effectiveness*

The general purpose of the NW TDP is to detect, classify and identify vessels passing through the Northwest Passage. The measure of effectiveness of a combination of sensors is a measurement of the probability that the sensor combination will detect an undeclared vessel under all weather conditions, while passing by a choke point.

According to Gauthier et al. (2004), "MOEs measure the ability of a given architecture to meet well defined objectives within the operational context." (as cited in Waller et al., 2008:

13). In other words, they are a quantification of the ability of a given object of study to meet specified objectives under a given set of conditions.

In the case of Northern Watch, the most important MOE is the probability that a sensor will detect a given ship and at what range. The probability of a vessel being detected is defined as P_{det} while probabilities of identification and classification are P_{id} and P_{cl} respectively.

The P_{det} is dependent on a few factors. The size of a vessel could have an impact on the probability of detecting it. Another factor is the weather as discussed above. In poor weather conditions, the probability of detection decreases. Also the weather conditions have an effect on classification and identification as some sensors used for classification and identification might not be able to perform well in such conditions (e.g., cameras).

One other factor is range. At a higher range, it is expected that the probability of detection would be lower than detecting an object closer to the sensor. Also at higher range the signal-to-noise ratio (SNR) is expected to be higher, resulting in false alarms.

When a vessel is travelling at a closer range to a sensor, the probability of detection is higher. For example, the probability of detecting a small vessel by the Navigation radar at 15 NM is predicted to be at 100%. However, if the same vessel is travelling 20 NM away from the same sensor, then the probability of detection will be reduced to 80%.

The first measure of performance (MOP) is therefore the detection rate. This rate is calculated based on the results of the simulation trial and probability of detection as a function of range.

The geographical location of the intersection of route AB with each sensor contour is noted (Figure 10). The distances between the points of intersection are then calculated based on the distance, the speed of the vessel and the direction of the track from A to B which determines the azimuth, the time which the vessel spends in any given range bracket, is measured. Based on this time and the corresponding coverage area, the probability of detection is measured every minute. For example, if the time spent in the 50% area is 200 minutes, the sensor has 200 chances to detect the vessel but due to the distance it will only be able to detect (within some non-zero detection area) 48% of the time. After all the detection rates associated with each range bracket have been calculated, they are summed up and averaged over the total time spent within detection range. This detection range is the distance between the point at which AB first intersects with the sensor contours and the point at which it last intersects.

Another performance measure is the time and the location where the vessel is first detected by a given sensor relative to the length of the passage. That determines the amount of time (t_{det}) before a vessel is detected while within range. It is important to minimize this time and allow detection as early as possible and continued detection and tracking through as much of the vessel's transit as possible. This MOP is expressed as a ratio of time of first detection to the total time the vessel spends in the Barrow Strait choke point Passage. As a result, smaller fractions represent better performance and higher effectiveness than large fractions do.

Tracking of a vessel is the final measure of performance for any sensor. The length of the time a sensor can track a given ship is determined as (t_{track}). This measurement determines the ability of the sensor to first detect and then track a target. It is important to maximize the length of this time. This MOP is expressed as a fraction, with 1.00 representing full tracking coverage of the vessel from its entry into the study area in question to its exit, and 0.00 representing a vessel transiting completely undetected and untracked, i.e., an AB path that is completely outside the non-zero detection contours.

For each sensor it is important to determine the first detection time and the relative distance to be able to measure the performance of the sensor. The first detection time relative to the ship route through the passage determines the ability of the sensor to track. Also tracking could be measured by the amount of time the vessel is within detection range from $t_{det(1)}$ to $t_{det(n)}$ where n represents the last detection time. It is desired to maximize the length of tracking. Once a vessel is detected, it is only tracked for the rest of its route until it is out of range of the sensor.

There are a number of MOPs that can be defined to quantify the performance of the tracking provided by the NW TDP (Waller et al., 2008). Tracking in this project is considered as a one of the measures of performance for each sensor. The tracking occurs when the vessel is detected by the sensor within the sensor's R_{max} . The period of time from the first detection to the last, relative to the length of the passage, determines the amount of time a given sensor is able to track a vessel as shown in the equation (3.1).

$$track = (t_{det(n)} - t_{det(1)}) / t_{total} \quad (3.1)$$

t_{total} is the total amount of time the vessel is traveling in the passage from point A to point B.

3.5 Other variables: cost and power

The performance of each sensor is not only limited to its ability to detect, classify and identify targets. Beyond strict performance, there are a number of other variables which will affect the ranking of the alternative surveillance options. Cost is one of the important variables. Other factors such as power usage are also important. It must be noted that there is a base station designated "second site" in the tables below located at the NWS. This site also consumes power. Only one base station is required for any number of sensors at a given location.

Sensor	Cost
S1. Navigation Radar	\$250,000
S2. AIS	\$25,000
S3. EO/IR	\$1,500,000
S4. Underwater sensor	\$2,000,000

Table 6 - Installation costs of individual sensors

Sensor	Average (kWh/day)	Peak(kWh/day)
S1. Navigation Radar	9360	12720
S2. AIS	36	48
S3. EO/IR	53040	58320
S4. UW sensor	15600	15600
Second Site	6000	6000

Table 7 - Power consumption of individual sensors and “second site” base station

3.6 Simulation scenarios, trials, and illustrative example

To summarize the content of the previous section: there are 4 different types of sensors: Navigation Radar, AIS, EO/IR and Underwater sensors. There are two different vessel types: small vessel with a speed of 7 Nm/hr and large vessel with a speed of 12 Nm/hr. There are 2 different weather types: ideal and poor.

In this simulation, a single trial is defined as a given vessel type under given weather conditions travelling from the same point A to the same point B for each of a number of possible sensor combinations defined below. Therefore, at each trial, sensors perform detection, classification and identification on an identical simulated targets under identical conditions.

The probability of each type of vessel entering the coverage area of the simulation is 50%, meaning that at every individual trial there is an equal chance of either type of vessel travelling in the passage. The vessel type is determined at the beginning of each trial.

The probability of ideal weather is 80% and of poor weather is 20%. These values have been chosen since the simulation only considers summer months that the passage is open and clear of ice, during which time good weather is more likely to prevail. Under ideal weather conditions, the probability associated with each contour is at 100%, 80% and 50% (as in Figure 14). However, in the case of poor weather these probabilities will be reduced to 85%, 65% and 25% but the ranges remain the same as under ideal conditions. As a result, it must be noted that only the probabilities change and R_{max} , R_{mid} or R_{min} remain the same.

Since there are 4 different sensors available for this study as mentioned in Section 3 above, there will be 15 different combinations of all sensors for this study using the equation (3.2) below:

$$N_{config} = \sum_{i=1}^m \frac{m!}{i! (m-i)!} \quad (3.2)$$

where m is the number of alternatives, which in this case is 4.

However, after some initial analysis about actual sensor deployment, it became clear that combining some sensors did not yield better results than the individual sensors themselves.

Therefore, some combinations were considered to be dominated and eliminated from this study. For example, as discussed earlier only two of these sensors have classification and identification capabilities. Therefore, it will be beneficial to combine these two sensors to add extra capabilities. This factor was considered when selecting the actual combinations to be included in the study presented here. On the other hand, since some sensors such as those underwater have a large area of coverage, if it is combined with a navigation radar with a small area of coverage, the overall detection rate and first time of detection is simply the same as the underwater sensor and there is no improvement in any of the three detection measures. As a result, such combinations where one sensor is clearly dominated by another have been eliminated. Table 8 contains a list of all feasible combinations of sensors considered herein:

Scenario	Combination
1	S1
2	S2
3	S3
4	S4
5	S1 & S2
6	S2 & S3
7	S2 & S4
8	S1 & S3
9	S1, S2 & S4

Table 8 - Combination of sensors considered for this study

From this table one can see that only 9 combinations have been considered in this study.

Detection measures

Overall detection rate:

There are 3 different scenarios considered for this measurement. In the first scenario, the vessel travels from A to B passing through all the rings as shown in Figure 14.

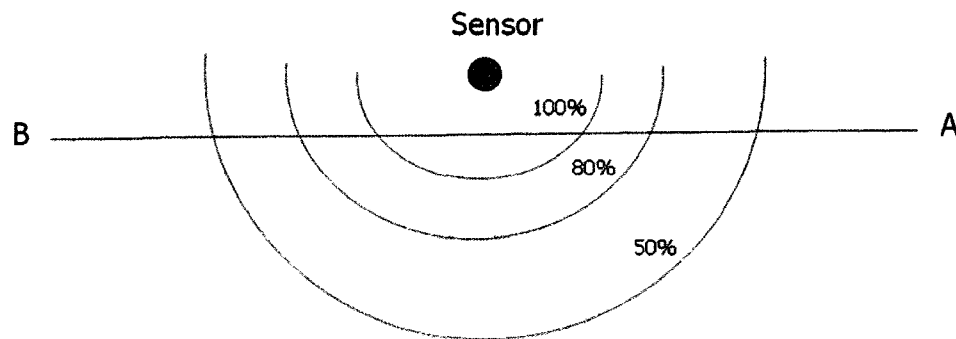


Figure 14 - Vessel V traveling from point A to point B through all probability detection rings

In the second scenario, the vessel only passes through the 80% and 50% rings as shown in Figure 15. As a result, the overall detection rate is decreased only leaving a maximum detection rate of 80%. In the last scenario, the vessel is far away from the sensor and only crosses the 50% contour. These scenarios are shown in the figure below:

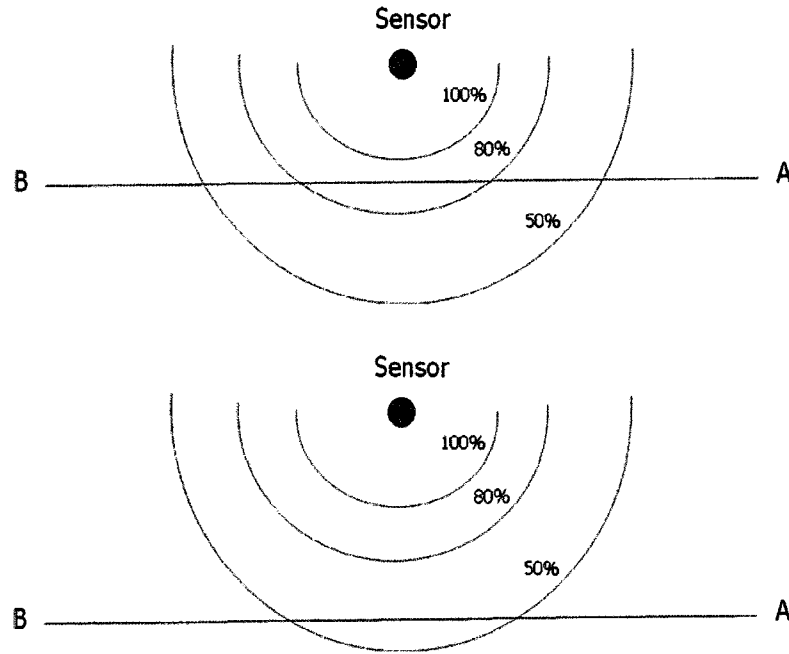


Figure 15 - Vessel V traveling from point A to point B through some probability detection rings

To determine the detection rate, the intersection of AB and each ring has been measured. Using the MATLAB function *polyxpoly* given the points of intersection, the latitude and longitude of the point(s) of intersection are returned. Using these intersection points and the speed of the vessel in that simulation trial, the total time spent in a specific range bracket is calculated. Based on that, each sensor has a certain number of minutes in each range bracket in which to actually detect the vessel. These results are stored in an array of true and false values. If the vessel is detected, then the result is true. Otherwise, the result of detection is false. These values are stored in the array element corresponding to the minute in which the detection attempt was made. As soon as the vessel is out of detection range, which is the last intersection point of AB and the outermost contour (50%), the number of true detections are added up. This number is then divided by the total of possible times the vessel was within the detection range in minutes. This new value is the probability detection rate.

The first detection time is the first time the vessel is detected. As soon as the vessel travel line crosses the 50% detection line, the counter starts. As soon as the value of detection is true, the counter stops. At this point, the counter value is added to the time from A to the

intersection line to calculate the first detection time. The first detection position is then measured based on the first detection time ($t_{det(1)}$), the first intersection of AB and the 50% contour and the azimuth which is calculated based on the route AB. Then the distance is measured from this first detection location and the sensor location to confirm the results.

Tracking is the proportion of the passage the vessel spent detected. As mentioned above, the time of first detection is noted. Then the last detection time is recorded as $t_{det(n)}$. At this point, the length of the time from $t_{det(1)}$ to $t_{det(n)}$ is calculated, which in combination with vessel speed determines the total time t_{total} for the vessel to travel from point A to point B. As a result, the period of a time a vessel is tracked is the ratio of the total amount of time spent detected ($t_{det(n)} - t_{det(1)}$) to t_{total} .

It has been noted elsewhere that only two of the four sensors have classification and identification capabilities. The AIS sensor classifies and identifies at the time of detection. Therefore, in the model, as soon as the first detection occurs, classification and identification take place as well. The other sensor with these capabilities is EO/IR. However classification and identification of a target take place at a very close range to the sensor. These ranges for small and large vessels are shown in Table 9, below:

	Classification (NM)	Identification (NM)
Fishing boat	1.6	0.65
Icebreaker	5.5	3.6

Table 9 - Classification and identification ranges of EO/IR sensor for fishing boats and icebreakers (in NM distance from sensor)

By looking at this table, and comparing with Table 5, it becomes clear that classification and identification are only possible when the vessel is in the 100% detection range. In the model, in order to configure these capabilities, once the vessel is within the 100% range, its distance to the sensor is calculated at every minute. As soon as this distance is less than the given classification and identification ranges in the table above, the values of classification and identification are set to “true”.

Having a combination of different sensors promises to improve the probably of detection as well as adding some other performance measures such as classification and identification. For example, combining underwater sensors and AIS will improve the detection rate but also because of capabilities of AIS sensors with classification and identification, we will have a better performance overall.

In order to measure effectiveness of these five scenarios (Scenarios 5-9, Table 8) involving combinations of more than one sensor, it is important that we keep the same AB track, vessel type, and weather conditions as used for the individual sensor performances. This will simplify the comparison between different combinations. Accordingly, results from the individual sensors are stored for this comparison. As mentioned above, to measure the detection rate for each sensor, a binary array is created. In this array true values or “1” refer to successful detection of the target. In a case of combining S1 and S2, we are provided with

one of these binary arrays for each sensor, for a total of two. But it must be noted that these arrays are not necessarily the same size, since depending on the sensor's coverage area, the vessel can spend more or less time within range and therefore a sensor might have less time to perform detection. In order to avoid problems with regard to potential differences in the size of the arrays, the two resulting arrays are compared in size. If the time of first intersection of line AB and the outermost contour for S1 is less than that of S2, that means S1 has larger areas of coverage and therefore a larger detection array. So in this case, S2's detection array is padded with zeroes at the beginning and the end. The size of padding is determined by difference in time of intersection as shown below in the code for MATLAB:

```
(t_A_max_S1 < t_A_max_S2)
    A_diff_s1_s2 = (fix(t_A_max_S2 - t_A_max_S1));
    padding_A = zeros(1,(A_diff_s1_s2));
    s2_padded_track = cat(2,padding_A,s2_padded_track);
(t_max_B_S1 < t_max_B_S2)
    B_diff_s1_s2 = (fix(t_max_B_S2 - t_max_B_S1));
    padding_B = zeros(1,(B_diff_s1_s2));
    s2_padded_track = cat(2,s2_padded_track,padding_B);
```

The first detection time in the case of combinations of sensors, is obtained simply by comparing each individual sensor's first detection time. The combination's t_{det1} is the smallest $t_{det(1)}$ of the two or three sensors. This rule also applies to the last detection time. In some cases, $t_{det(1)}$ is determined by one sensor while $t_{det(n)}$ is determined by another sensor. In that case, the overall tracking percentage is higher since the vessel spent more time being tracked by multiple sensors rather than one. However, in most scenarios, tracking percentage is as high as or better than the sensor with the largest area of coverage.

In the case of combined sensors where one of them is able to classify and identify, it is assumed that if that sensor has already classified and identified a vessel individually, that information can be re-used for the combinations as well.

The following is an outcome of one simulation trial where the results of individual sensors and combination sensors are shown. Each trial consists of 9 individual passages between the same point A, the same point B, with the same vessel type and the same weather conditions. The only difference between the individual passages is the sensor combination in use. Note that for the individual sensors only (S1, S2, S3 and S4 when not in combination with one another), the "Scenario 1" designation indicates that the track AB passes through all three range brackets of that sensor. "Scenario 2" would indicate that it passes only through the two outermost brackets, and "Scenario 3" only the one outermost bracket.

```
----- vessel: icebreaker -----
Weather condition is ideal
***** S1 *****
Scenario 1: First detection was at t=138.278 minutes
The overall detection by S1-Navigation Radar is %92.6316
The vessel was detected and tracked %54.0816 of its trip across the passage.

***** S2 *****
Scenario 1: First detection was at t=97.0503 minutes
```

The overall probability of detection rate by S2-AIS Radar is %93.5252
The vessel was detected and tracked %63.4351 of its trip through the passage.
The vessel was classified by S2
The vessel was identified by S2

***** S3 *****

Scenario 1: First detection was at t=140.278 minutes
The overall probability of detection rate by S3-EO/IR Radar is %92.4211
The vessel is detected and tracked %53.7389 of its trip through the passage.

***** S4 *****

Scenario 1: First detection was at t=44.5005 minutes
The overall probability of detection rate by S4-UnderWater Radar is %95.4481
The vessel is detected and tracked %80.243 of its trip through the passage.

***** S1 & S2 *****

The overall probability rate of detection by S1 and S2 is %94.0647
First detection was at t=97.0503 minutes by S2
Last detection was at t=652.497 minutes by S2
The vessel is detected and tracked %63.4351 of its trip through the passage by S1 and S2.

***** S2 & S3 *****

The overall probability rate of detection by S2 and S3 is %93.705
First detection was at t=97.0503 minutes by S2
Last detection was at t=652.497 minutes by S2
The vessel is detected and tracked %63.4351 of its trip through the passage by S2 and S3.
The vessel was classified by S2.
The vessel was identified by S2.

***** S2 & S4 *****

The overall probability rate of detection by S2 and S4 is %95.3125
First detection was at t=44.5005 minutes by S4
Last detection was at t=747.119 minutes by S4
The vessel is detected and tracked %80.243 of its trip through the passage by S2 and S4.
The vessel was classified by S2.
The vessel was identified by S2.

***** S1 & S3 *****

The overall probability rate of detection by S1 and S3 is %97.0526
First detection was at t=138.278 minutes by S1
Last detection was at t=611.823 minutes by S1
The vessel is detected and tracked %54.0816 of its trip through the passage by S1 and S3.

***** S1 & S2 & S4 *****

The overall probability rate of detection by S1, S2 and S3 is %95.3125
First detection was at t=44.5005 minutes by S4
Last detection was at t=652.497 minutes by S4
The vessel is detected and tracked %69.4366 of its trip through the passage by S1, S2, and S4.
The vessel was classified by S2.
The vessel was identified by S2.

These results are presented here simply to illustrate the type of statistics produced by the shipping model for a typical simulation trial. Discussion of the results of a full simulation run (consisting of 100 trials) will be presented in chapter 4, below. The raw data for a full simulation run is presented in Appendix B.

3.7 Expert Choice (AHP) setup

Figure 16, below, shows how the hierarchy of factors for consideration using AHP as configured. This section also explains in greater detail the formulation of the AHP model for this application.

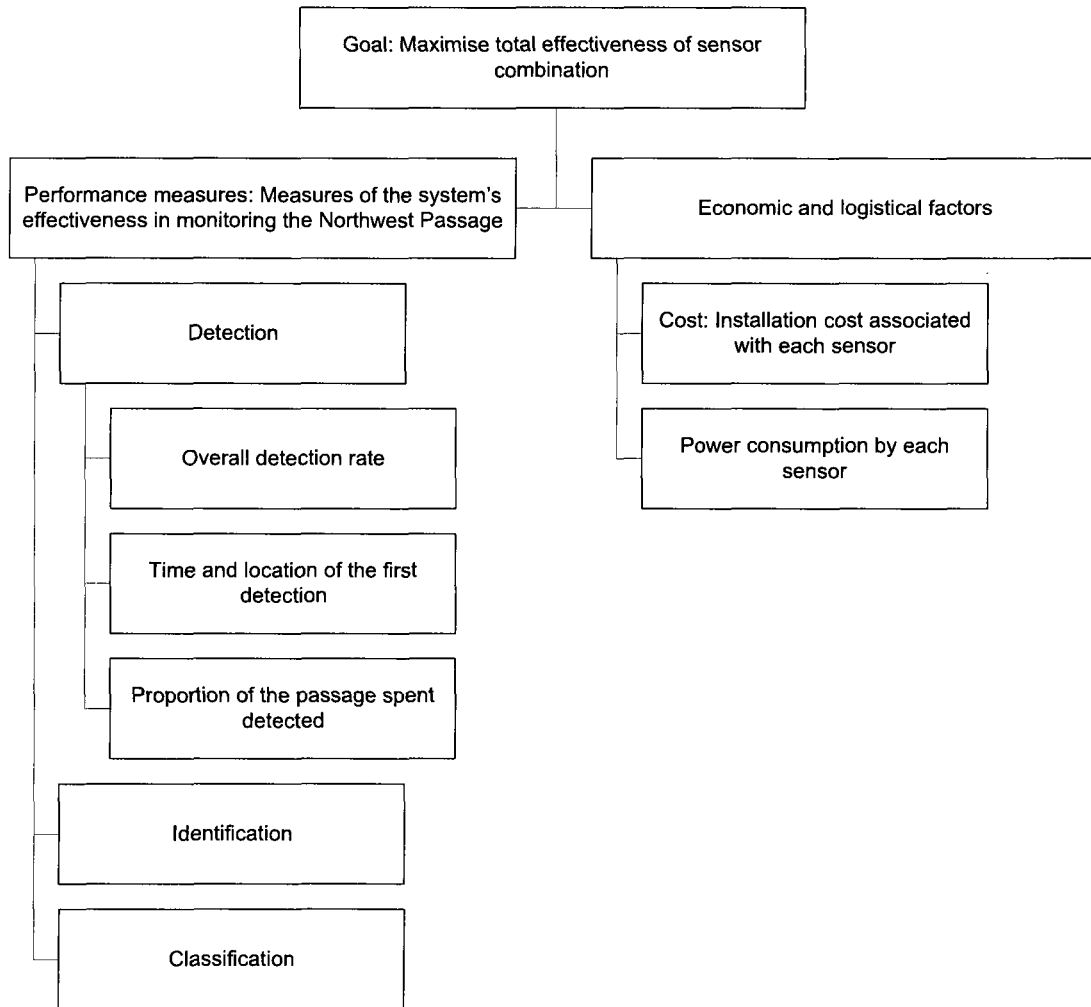


Figure 16 - Conceptual diagram of AHP hierarchy for Northern Watch

The hierarchy depicted in Figure 16, above, was set up using the Expert Choice 2000 software tool. Figure 17, below, presents the model as constructed in the software. This is a direct representation of the same hierarchy as in Figure 16 in terms of the Expert Choice 2000 software.

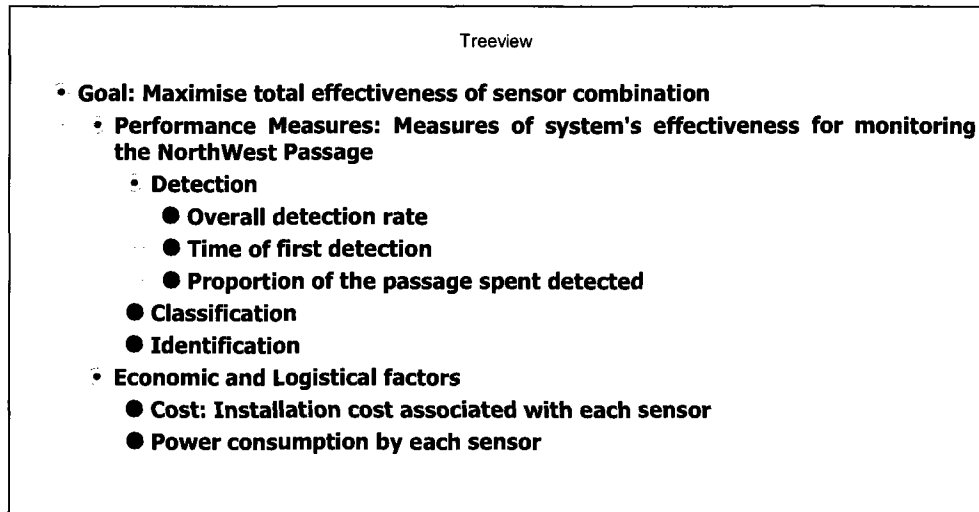


Figure 17 - Tree view of the AHP model in Expert Choice

This model was presented to a panel of five experts to collect their views on assigning weights to the criteria without presenting them with the different alternatives. The experts were selected for their expertise either on the Northern Watch project or in MCDM (see Table 10, below).

The alternatives in this case are the various sensors, both individually and in combination. Note that the alternatives are not subject to direct ranking by the participants: they are to be ranked according to their measures of performance evaluated in the context of the structure of relative weights determined by the combined pairwise judgements of the participants.

The participants were given the option of using the numerical, verbal, or graphical comparison tools available in Expert Choice to set the relative priorities of the performance measures in the hierarchy, and each did so according to their preference (see Appendix E). Expert Choice allows the decision maker to revisit the decision if the priorities are inconsistent by displaying the consistency index for each of the comparison matrices. Any index value higher than 0.1 is considered inconsistent. Expert Choice points out the inconsistent decisions from highest to lowest. At this stage the decision maker can readjust its judgement to achieve a better consistency.

The following table presents the priorities assigned to the first level of the hierarchy (i.e. performance measures versus economic and logistical factors) by each expert as well as some profile information about the experts themselves.

Decision Maker	Name	Affiliation	Expertise	Weight assigned to Performance Measures	Weight assigned to Economic and Logistical factors
1	Matthew MacLeod	DRDC CORA	Northern Watch	0.583	0.417
2	David Waller	DRDC Ottawa	Northern Watch	0.600	0.400
3	Kendall Wheaton	DRDC CORA	Northern Watch	0.655	0.345

4	Daniel Lane	University of Ottawa	AHP	0.615	0.385
5	Noosha Tayebi	University of Ottawa	MCDM	0.750	0.250

Table 10 - Decision makers' expertise and results

All participants were deemed to have equal weight in the collaboration scheme, thus, it was possible to consolidate all expert's results by using a spreadsheet (see Appendix E) to calculate average priorities for each of the weights in the pairwise comparison matrices of Expert Choice, and then to re-enter the calculated averages into a "combined" iteration of the AHP model. This model yields a combined set of weights or "priorities" for the objectives and sub-objectives to be used in the model.

The combined model was then used as the basis for determining the favoured alternative. This was done using the data grid view in Expert Choice (see Appendix F – AHP data grids, below). In the data grid, the objectives are listed in the columns and the alternatives are listed in the rows. For each objective, a utility function is defined to govern the assignment of scores to the performance of each alternative with respect to that objective. For example, for the installation cost of each possible sensor combination, a decreasing linear formula was created which assigned maximum utility to alternatives having a cost near zero, and minimum utility to alternatives having a cost around the maximum for this scenario, which was \$2.3 million (see Figure 18, below).

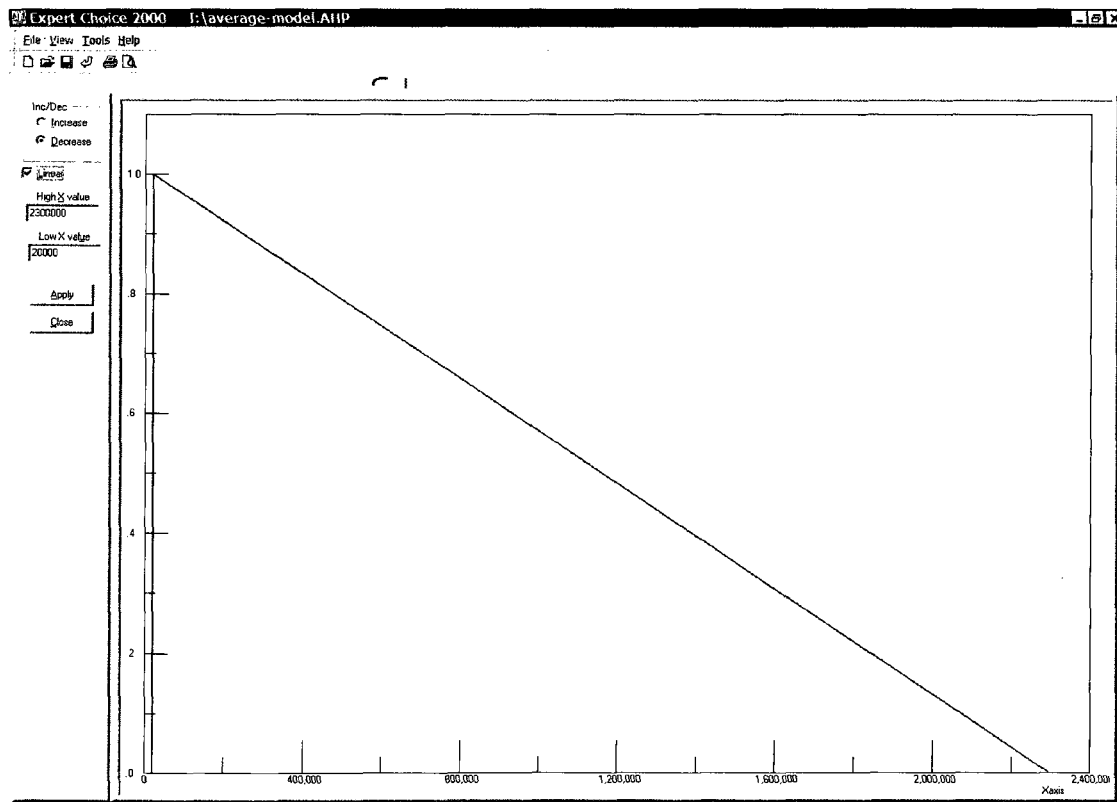


Figure 18 - Linear utility function assigned to the installation cost objective

In this way, alternatives with high installation costs will receive a lower utility score on this objective than those that have lower costs. This is consistent with the overall management objective of having a cost-effective sensor solution. Other objectives were handled in a similar way:

The objective of overall detection rate is a percentage scale, so the utility function was bounded at 0 and 100. It was an increasing function, but was not assumed to be linear, having instead a strongly concave shape that reflected the fact that sensor combinations which detect and track contacts for less than half of the time in which detection and tracking are possible are effectively useless to the managers of the NW TDP (see Figure 19, below). Performance ratings in the range of 75-100% are strongly preferred, and the deeply concave shape of the utility function is a way of implementing this preference in the model.

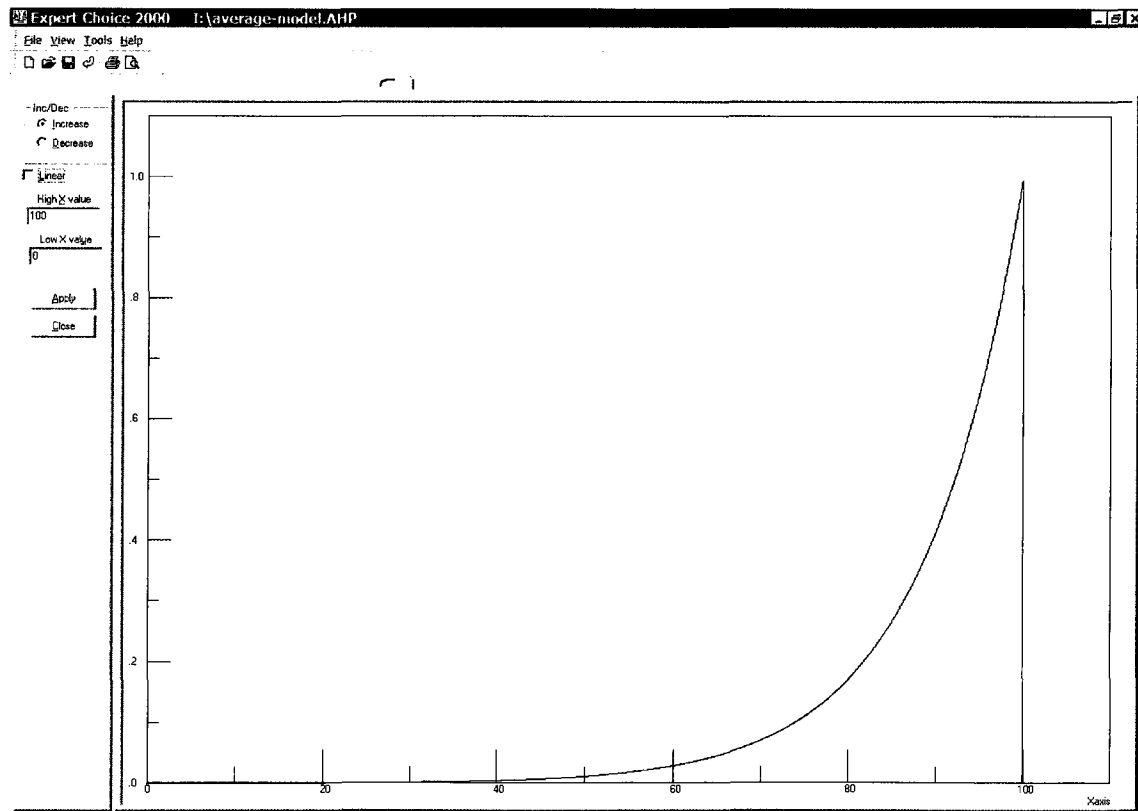


Figure 19 - Concave utility function assigned to the overall detection rate objective

The time of first detection objective was placed on a decreasing convex utility curve, with the minimum value set at 30 minutes and the maximum at 600 minutes. Earlier times of first detection – corresponding to lower numerical values on this scale – are moderately preferred

over late ones. To reflect this, the curve was given a convex configuration with a more moderate curvature than in the previous example (see Figure 20, below).

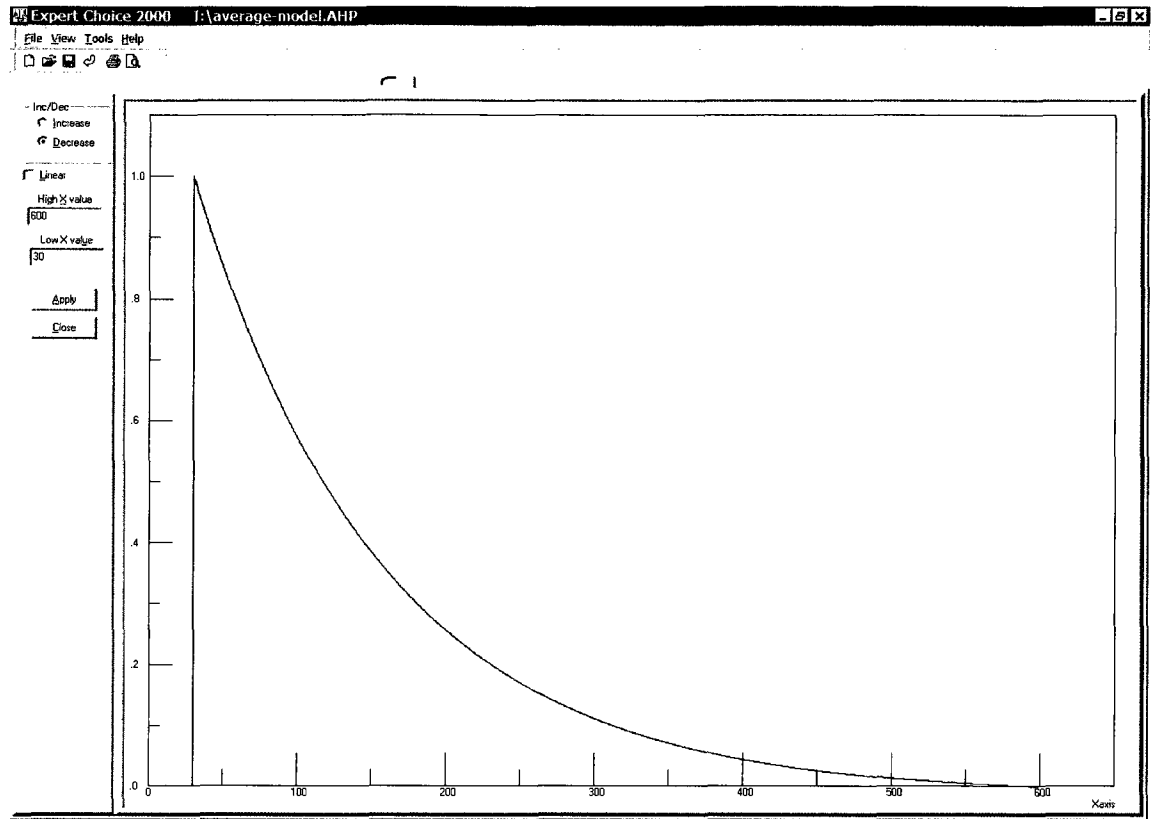


Figure 20 - Concave decreasing utility function assigned to the time of first detection objective

The tracking objective again uses a percentage scale. In this case, however, the preference for higher proportions over lower ones is a weaker preference and so, while the utility function for this objective is non-linear, its concavity is much shallower than in the earlier examples (see Figure 21, below).

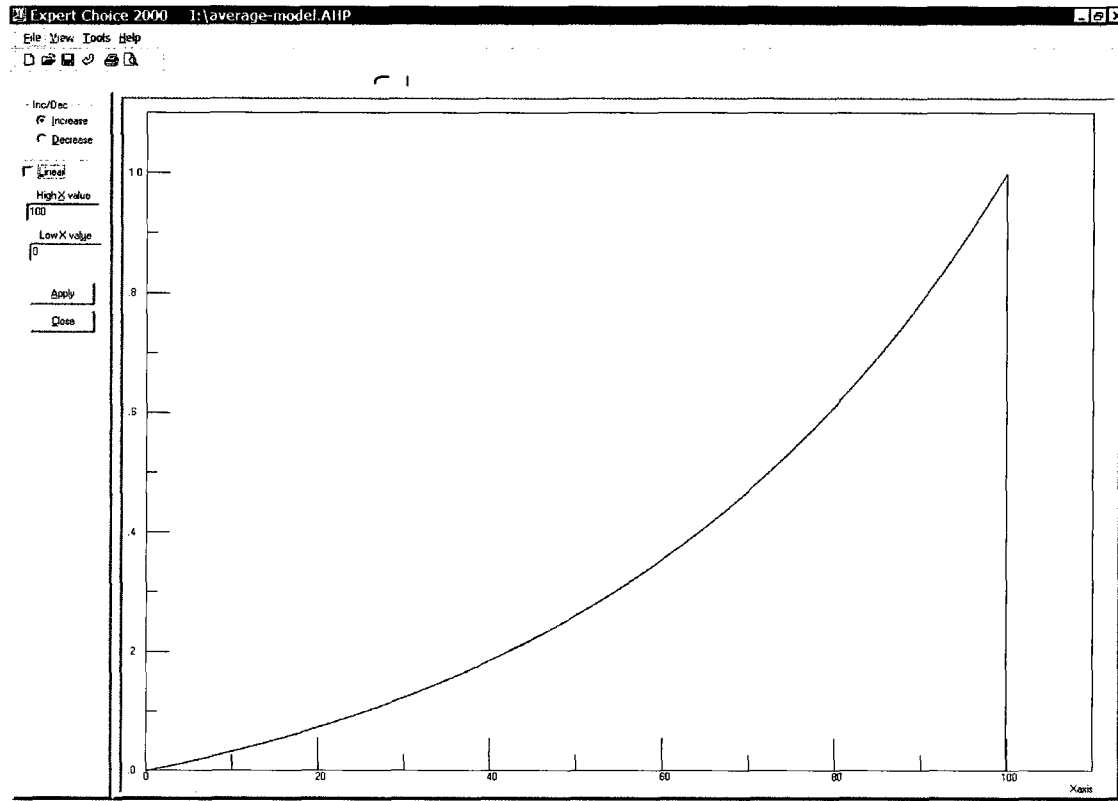


Figure 21 - Concave increasing utility function assigned to the tracking objective

The classification objective, because of the way the simulation is constructed, is essentially a binary choice: during any given trial of the simulation, the vessel is classified or it is not. Therefore, the utility function chosen for this objective is equally simple: it is a linear, increasing function bounded by 0 (signifying unclassified) and 1 (signifying classified), with lowest utility (0.0) assigned to the low end of the scale and highest utility (1.0) assigned to the high end of the scale.

The identification objective is handled in an identical fashion to classification, as it is modelled in the same way in the simulation.

The installation cost objective is described in the example given above (see Figure 18 and accompanying text). It should be noted that the values chosen to mark the upper and lower bounds of the utility scale are significant. In this case, the bounds were chosen to approximate the extremes of the cost characteristics of the available alternatives. As a result, the assignment of utility becomes relative to the alternatives presented in the model, not to any external benchmark. For example, if the NW TDP had a known budget for installation costs, we could use it to define the bounds of the utility function's domain. This information is not available in this context, however, so the relative measure will suffice. This issue is discussed further in section 4.2 below.

Finally, the power consumption objective follows the same pattern as installation cost, though with different bounds on the domain as the scale of this objective is measured in kWh/day rather than dollars.

The following table summarizes the seven utility functions created for this AHP model:

Objective	Function type	Lower bound	Upper bound	Curvature
Overall detection rate	Increasing	0	100	-1.13E-01
Time of first detection	Decreasing	30	600	-2.25E-01
Proportion of passage spent detected	Increasing	0	100	-4.81E-01
Classification	Increasing	0	1	Linear
Identification	Increasing	0	1	Linear
Installation cost	Decreasing	20000	2300000	Linear
Power consumption	Decreasing	600	80000	Linear

Table 11 - Summary of utility functions assigned to objectives in Expert Choice AHP model

3.7.1 Transfer of simulation results into Expert Choice (AHP)

Once the results of the simulation are available for the seven objectives listed in the rows of the data grid, we enter their values into the cells of the data grid. The value used in each case was the overall global mean achieved for the alternative in question over 100 trials of the simulation, regardless of vessel type or weather conditions. These values, evaluated by Expert Choice using the utility functions already defined, yielded a score for each alternative in terms of that objective.

The results of the AHP model are discussed in detail in section 4.2, below.

This chapter presented the methodology applied to the NW TDP analysis. It consisted of the development of a model for simulating and measuring, via repeated simulation trials, the alternative sensor performance. Data on performance was provided as input to the multicriteria AHP evaluation model that enabled ranking of the sensor alternatives including sensor combinations.

The following chapter will lay out and discuss the results of the analysis that flows from methodology described in this chapter.

4 Analysis and Results

In this chapter the results of simulation model and decision analysis methods are presented and analyzed based on the methodology presented in the previous chapter.

The final outcome of the research will be presented as a robust order of ranked options consisting of different combinations of sensors. The list of sensor alternatives has been provided in the previous section.

4.1 *Simulation results*

As mentioned in the previous sections, there are 9 different combinations of sensors considered for this study. In order to ensure statistical significance of the result, a simulation run was represented by 100 executed simulations of the AB passage trials. In this run, to have an overview of each different scenario, some variables were selected randomly such as vessel type, weather condition and route AB. In order to ensure consistency across all of the sensor combinations, each combination used the same values for these variables. In other words, the same vessel type, weather conditions and route from A to B were used to test all 9 different combinations for a given trial.

The following figure shows one of the outputs of a simulation run of 100 vessel passage trials in MATLAB. In this figure, detection rate for each of the four individual sensor sets is shown in the vertical axis against the run number in the horizontal axis. It is clear from this figure that the detection rate of the Navigation radar alternative fluctuates from 0% to 94.85%. On the other hand, other sensors such as underwater sensors have a more consistent detection rate. The minimum value for this sensor type is 75.88% and the maximum value was 95.94%. Similar consistency was also noticeable in AIS and EO/IR.

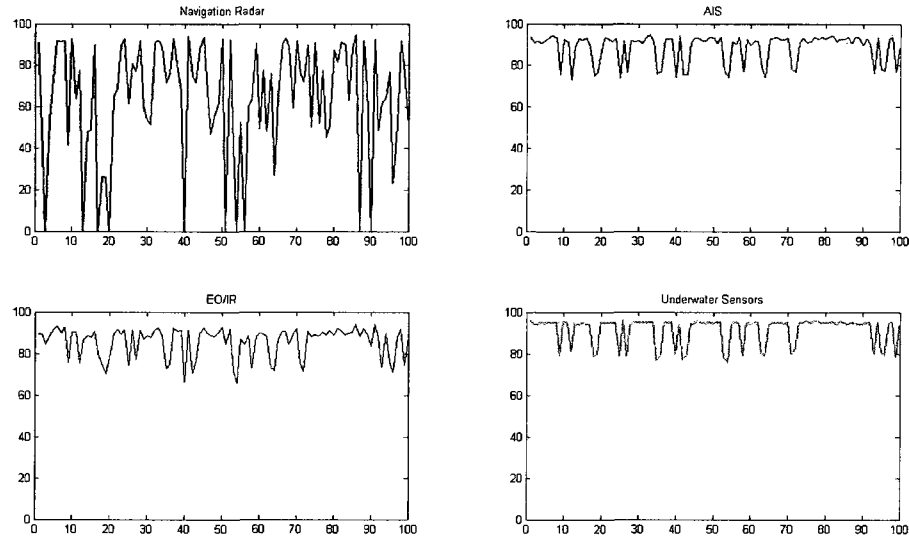


Figure 22 - Detection rate output plot from MATLAB

The high variability in detection rate of the Navigation radar is due to its detection ranges (R_{min} , R_{mid} , and R_{max}). In a case of a fishing boat, these ranges as shown in Table 5 are much smaller than any other sensor. As a result, if the vessel is traveling in the middle of the passage or farther away from the sensor, the probabilities of detection drop significantly. In some cases, Navigation radar (S1) was not able to detect the target at all, resulting in a trials with 0% detection rate. In other similar cases, where the vessel was farther away from the sensor, the route AB only crossed into the 50% probability range bracket. If the vessel travels exclusively or largely within this area, the detection rate will decrease significantly. Also, a low detection rate could result in increased false alarm detections.

From the 100 trials in each simulation run, MATLAB was used to create Excel spreadsheet files, each containing tables of one of the performance measures, enumerated by trial. For example, in the spreadsheet for overall detection rate, the nine sensor combinations are in the rows, and the 100 runs are in the columns, with each cell showing the value of the overall detection rate calculated by MATLAB for that sensor combination for that run. Those spreadsheets were used to calculate averages for each of the performance measures for each sensor combination across all runs. Appendix C contains sample results of the simulation for single run of 100 trials. These averages were used as performance measure inputs to the data grid for AHP in Expert Choice.

The simulation set of 100 trial results was divided to 4 different categories depending on the vessel type and the weather conditions as follows:

Fishing boat – ideal weather
Icebreaker – ideal weather
Fishing boat – poor weather
Icebreaker – poor weather

These categories were created to understand the different sensor performances depending on the vessel and weather type. The following figure shows the box plot created for all 100 runs by the sensor percentage detection performance measure:

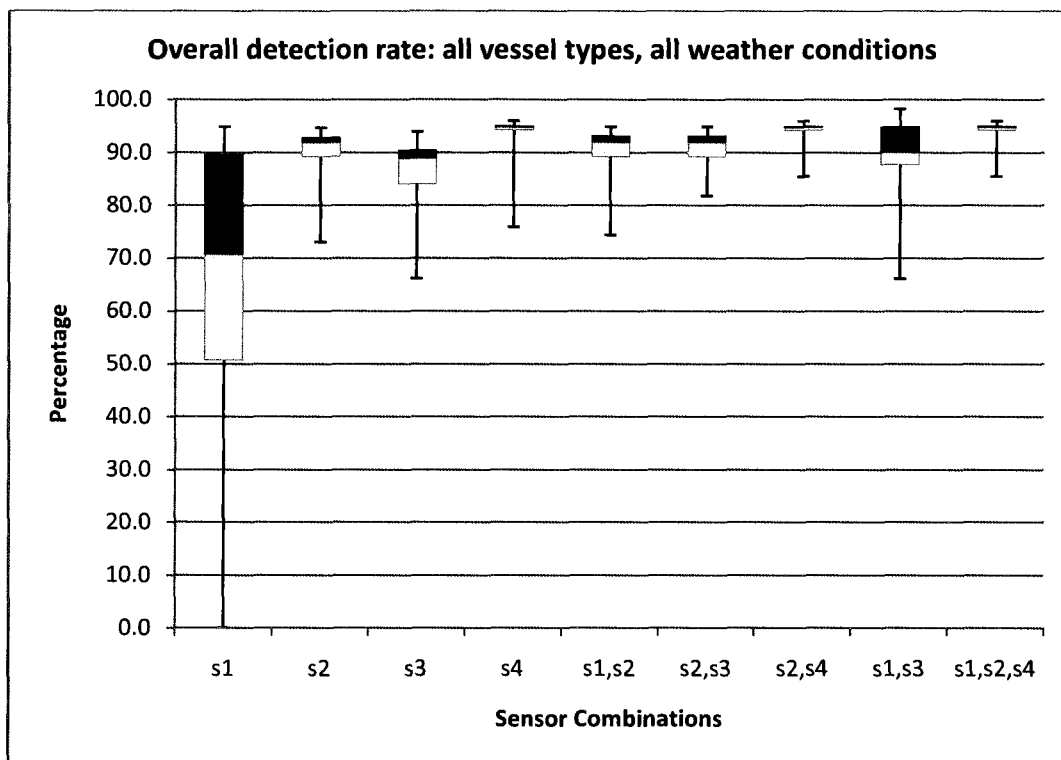


Figure 23 - Box plot of detection rate for all vessel types in all weather conditions

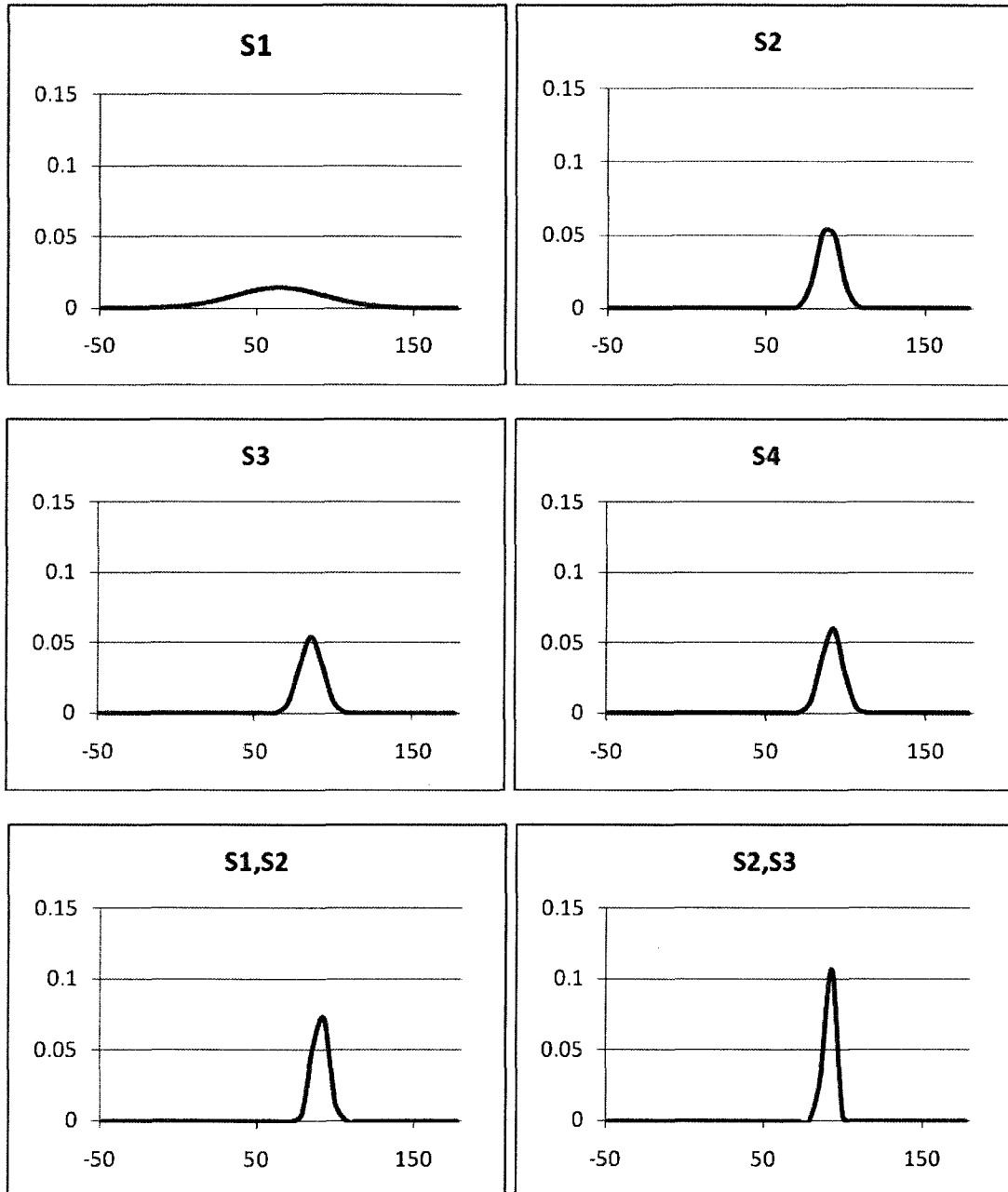
The figure above is a box plot of 100 trials created in Excel for a single simulation run. In order to create this plot, Min, Max, and Quartile values were calculated. Min refers to minimum value among 100 runs for each combination while Max refers to the maximum value. For this plot, 1st quartile and 3rd quartile referring to 25% and 75% respectively are calculated.

As shown in Figures 22 and 23, above, Navigation radar, S1, has a poor overall detection rate. The minimum value is 0% while the maximum only reaches 94.84% as discussed above. The median detection rate hovers just over 70%.

The underwater sensor, S4, holds one of the best performances with low variability in the detection rate. Most importantly, Figure 23 clearly shows that combinations of sensors

including the underwater sensor tend to have better detection rate or at least as good as the top performing sensor especially in the case of S1,S2, and S4.

Below are graphs showing the standard normal distributions of detection rate for each of the nine sensor combinations. The x-axis is the detection rate percentage, while the y-axis represents the Z-value. These graphs were plotted in Excel using median, and standard deviation.



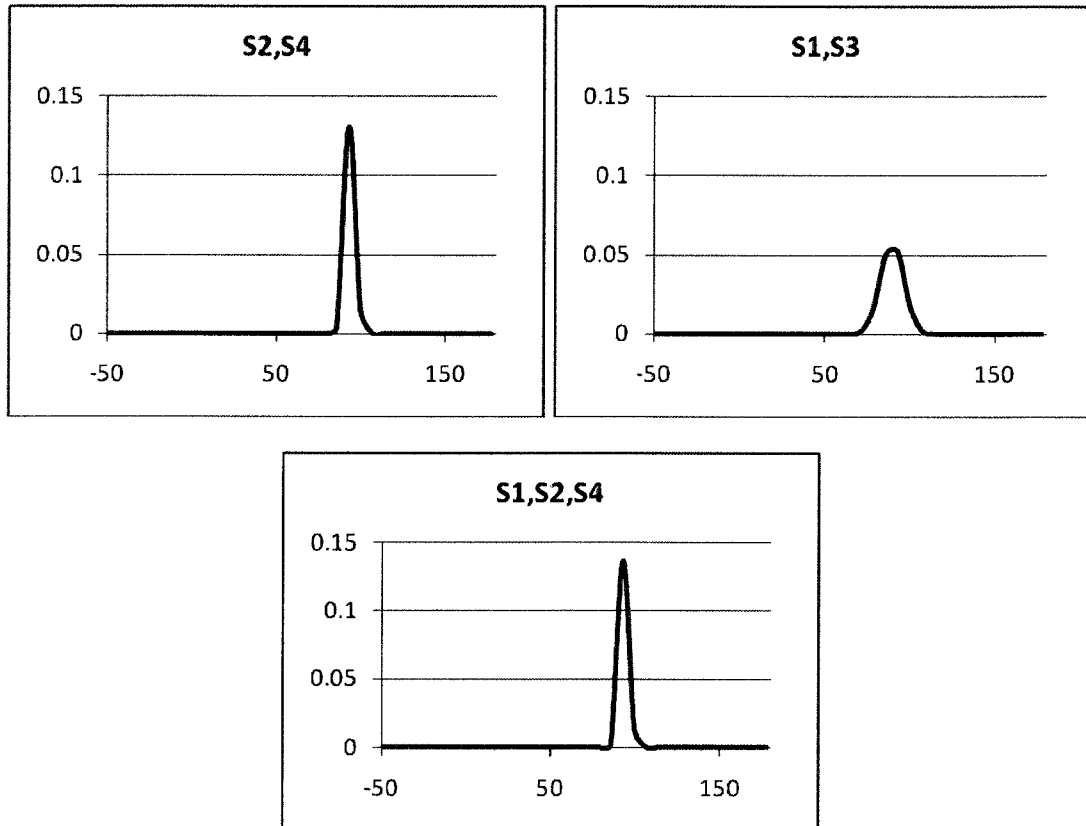


Figure 24 - Normal distributions of simulated detection rate of sensors and combinations

According to the plots in Figure 24, the combinations of sensors have better performances compared to single sensor performance. The normal distributions show that the median for the combinations, especially S1 S2 S4 is very high. Also it must be noted that the variances are very low resulting in tightness in the normal distribution curves. The only individual sensor with good detection rates is S4. As a result, all the combinations that include S4, yield better and higher peaks in these curves.

The following subsections report on the findings of the simulation runs for the various model definitions by vessel type, and weather conditions.

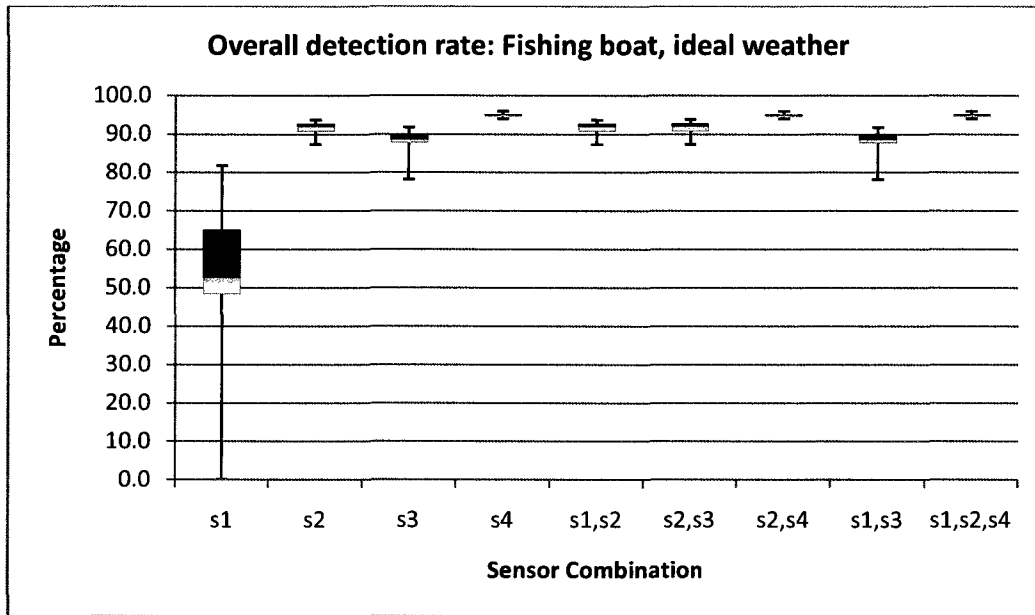


Figure 25 - Box plot of detection rate for a fishing boat under ideal weather conditions

4.1.1 Model Results for Small Vessels and Ideal Weather Conditions

Figure 25 shows similar characteristics to the plot of overall detection rate presented in Figure 23. S1 continues to have the poorest performance, with wide variability, low maximum and a median in the 50-55% range. When the vessel type is a fishing boat, S1 has poor overall performance because the size of its detection range brackets is small in comparison to the other sensors (see Table 5, above). S4, by contrast, has the best performance profile in this box plot, with a very low variability (ranging between 94.0 and 95.9%). Indeed, S4's performance is so good this plot also shows that those combinations which include S4 (S2 S4 and S1 S2 S4) have the same performance profile on detection rate that S4 has. This means that S4's performance is so strong that the other sensors in combination are dominated by S4; if they were to be taken out of the combination, the overall performance would be the same since S4's performance is so strongly dominant.

Similarly, the weak performance of S1 is itself dominated by that of S3 when the two are placed in combination (S1 S3): the combination has the same performance profile as S3 alone, meaning that S1 does not make an appreciable contribution to improving the detection rate when the two are used in combination. This same pattern is repeated in the S1 S2 combination, whose profile is identical to that of S2 alone.

It is also possible that the relative lack of improvement in performance from combinations when compared with individual sensors may result from the fact that some of the individual sensors already offer a very high level of performance; S4 is consistently in the 94-95% range. This is because the innermost range bracket for any sensor has a 100% chance of detection. Accordingly, the larger this innermost bracket

becomes, the greater will be the overall detection rate for the same vessel track from A to B, relative to other sensors. In such cases, where a given sensor already has a large 100% range bracket, there is not much room for improvement. By contrast, as we shall see below, cases where weather is not ideal – and therefore the probability of detection in the innermost range bracket is less than 100% – lead to situations in which individual sensor performance is substantially lower and the improvement gained from combinations is more pronounced.

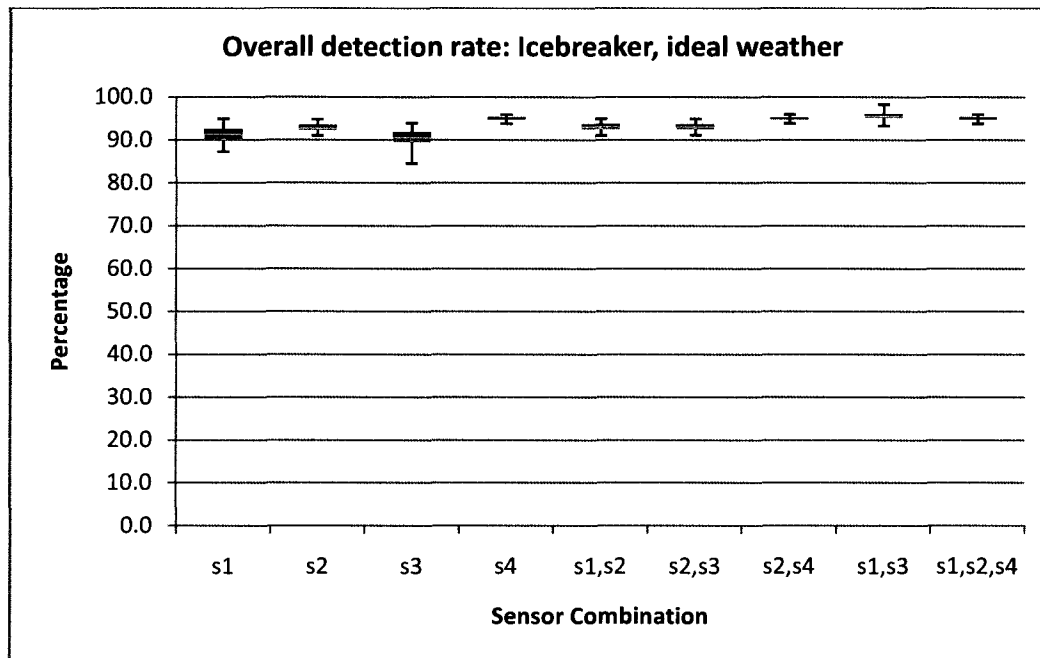


Figure 26 - Box plot of detection rate for an icebreaker under ideal weather conditions

4.1.2 Model Results for Large Vessels and Ideal Weather Conditions

Consistent with Figures 25 and 23, Figure 26 show that S2 and S4 tend to have a slightly better performance than S1 and S3, generally. However, of interest in this particular case is the fact that S1 performs better than S3 does, with higher values for all quantiles. This is because of the different profile of larger range brackets for radar when considering a larger target (see Table 5, above). Because of the nature of radar, the size of the target is an important factor in determining the range at which that target can be detected. This is not true of other types of sensors such as AIS or the underwater sensor, whose performance does not depend on target size. Also consistent with the plots above, S4 continues to be the top performer with less variability and higher values achieved for all quantiles than other sensors. It is also notable that, again, combinations of sensors tend to yield better performance profiles than individual sensors alone.

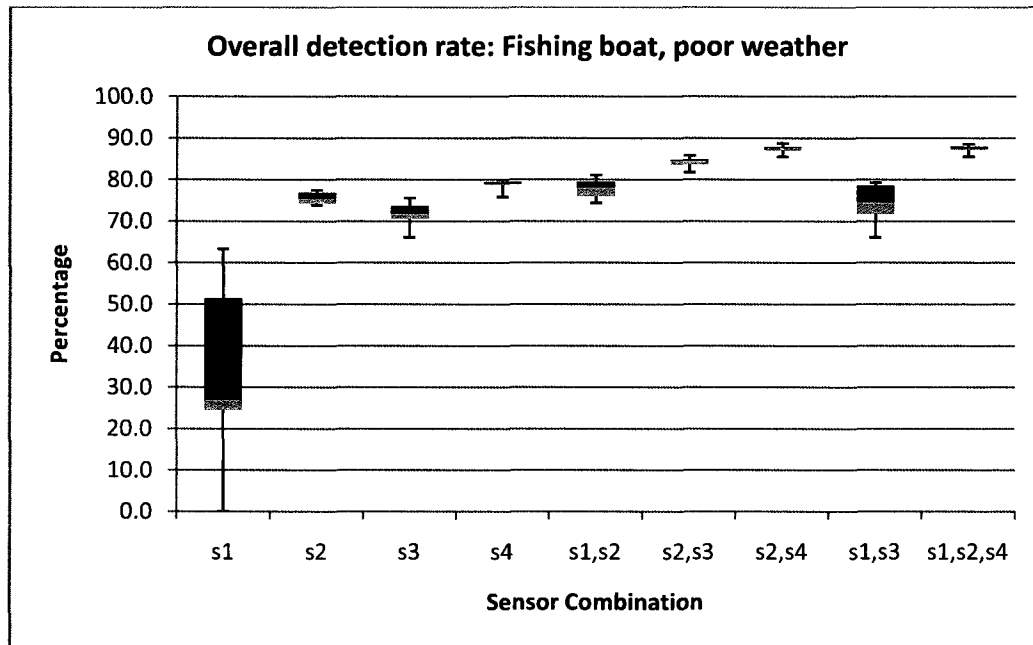


Figure 27 - Box plot of detection rate for a fishing boat under poor weather conditions

4.1.3 Model Results for Small Vessels and Poor Weather Conditions

While the relative performance of the various sensors and combinations under poor weather conditions is consistent with that under ideal conditions, there are two notable differences: first, the quantiles for all sensors and all combinations score significantly lower under poor conditions. This is to be expected, given that the model assigns lower detection probabilities to each range bracket under poor weather than under ideal weather. Second, the variability of each sensor or combination appears to be relatively less than for the same vessel type under ideal conditions.

As noted above, this figure shows that under poor weather conditions the performance advantage of sensor combinations over individual sensors in isolation becomes much more apparent. Consider for example, S2, S3 and combination S2 S3: S2 has a median of 75.4%, and S3 has a median of 71.9%, but the combination of S2 S3 has a median of 84.6%. Furthermore, from Figure 27 we can see that combinations under poor weather are not subject to domination by the highest performing sensor they contained in the same way that was evident for ideal weather (see above). Indeed, under these weather conditions and this vessel type, all combinations outperform the individual sensors that compose them for all quintiles.

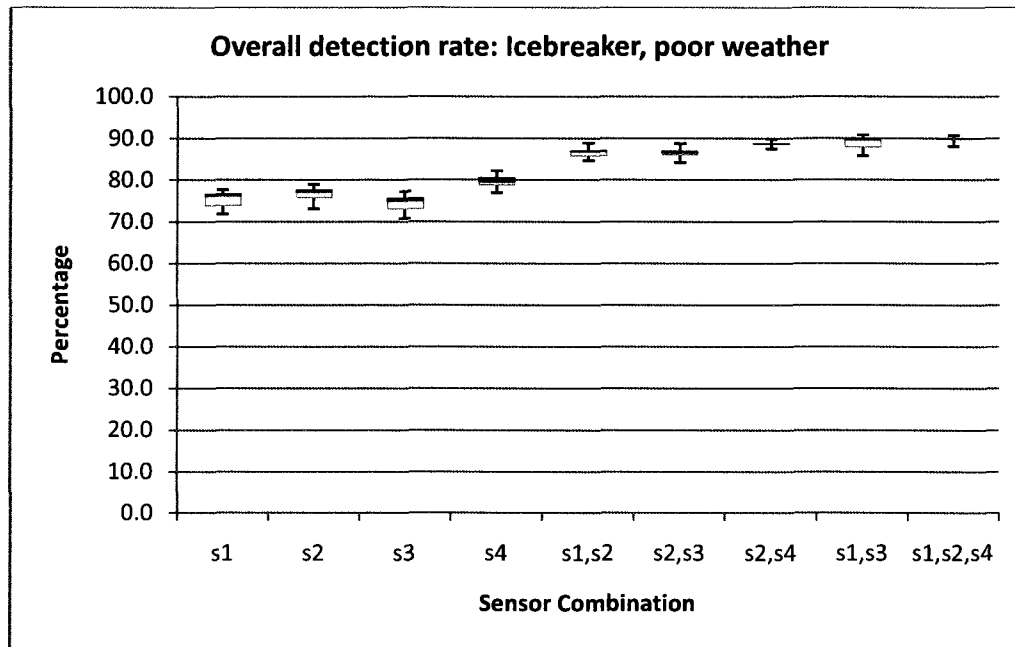


Figure 28 - Box plot of detection rate for an icebreaker under poor weather conditions

4.1.4 Model Results for Large Vessels and Poor Weather Conditions

The general pattern visible in this figure is consistent with that seen in Figure 27, above. The performance levels for all sensors and combinations are lower than under ideal weather conditions. The combinations all show higher performance for all quintiles than the individual sensors that compose them. The main distinction between these results (Figure 28) and those shown in Figure 27 is the performance of S1, which is much more competitive with the other sensor types for the icebreaker vessel type than it is for fishing boats.

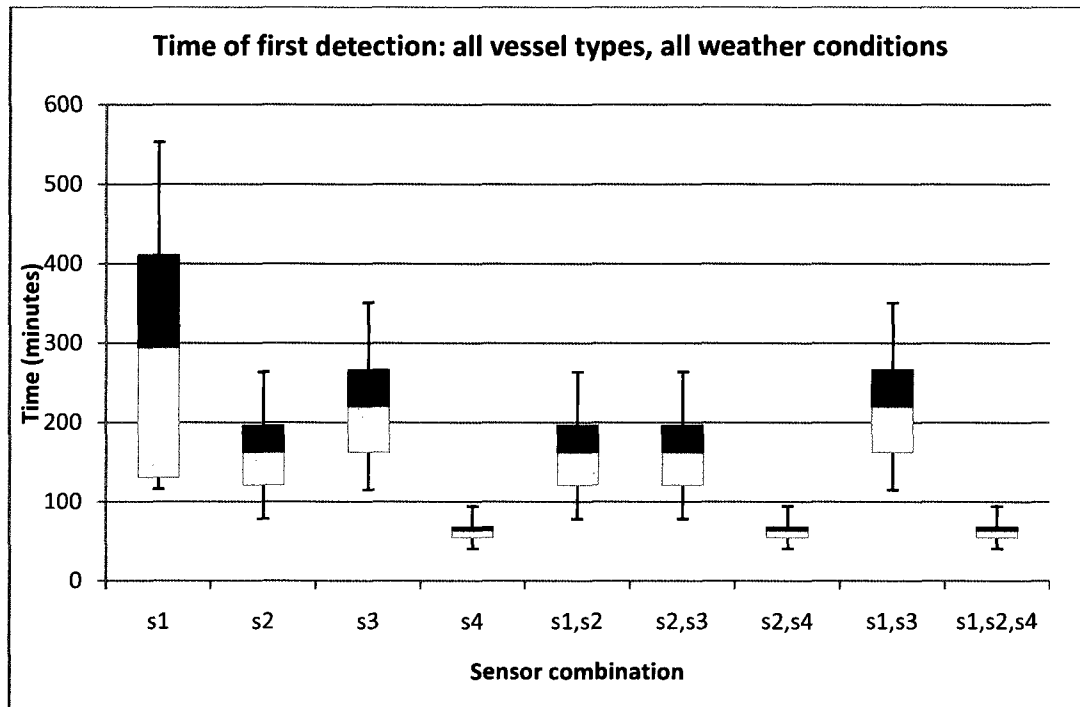


Figure 29 - Box plot of time of first detection for all vessel types under all weather conditions

4.1.5 Model Results for All Vessels and All Weather Conditions

Figure 29 shows the same familiar pattern of performance between the various combinations. It also clearly shows the dominance of some sensors over others. Specifically, the fact that S1 S2 has the same performance profile as S2 indicates the dominance of S2 over S1; likewise with the S2 S3. Most clearly dominant is S4, which consistently has the lowest (i.e., best) times of first detection as well as the least variability in its results. In combination with other sensors, S4's profile dominates that of others, even others such as S2 which are dominant over S1 or S3 in turn.

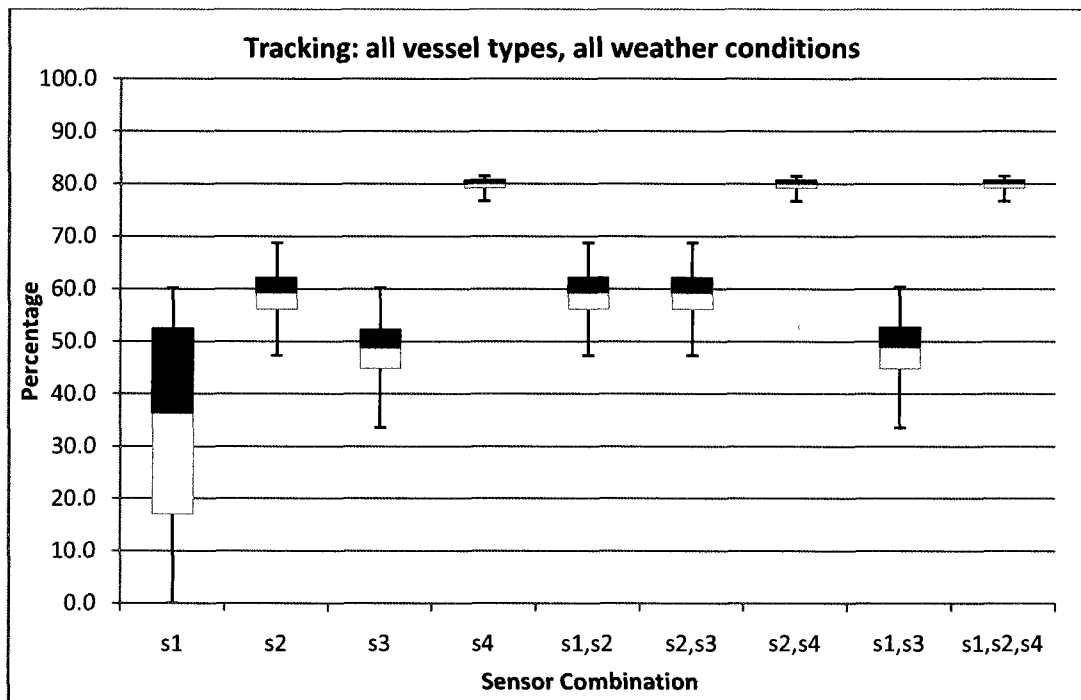


Figure 30 - Box plot of tracking performance for all vessel types under all weather conditions

Again in Figure 30 we see the clear advantage of S4, which dominates all of the other individual sensors and the combinations as well. S2 gives the second-strongest performance, but yields both substantially lower average tracking values and greater variability than S4. Overall, this plot repeats the same pattern of relative performance between the different combinations.

In summary, for some MOPs, it is possible to see that certain individual sensor types are clearly top performers, dominant over others to the point that it becomes moot to combine them with other, lower-performing sensors as described above. However, where no sensor is clearly dominant in this way, there are some scenarios in which it is advantageous to have a combination of sensors. As discussed above, this is most evident under poor weather conditions, where combinations yield consistently better results than individual sensors on their own.

The advantage of multiple sensors in combination is most evident because it doubles the number of detection opportunities for each tick of the clock. If one sensor does not detect the vessel in any given minute, there is still a chance that the second one may. This significantly improves the overall probability of detection in any given clock tick, and results in higher performance for virtually all of the measures considered.

4.2 AHP results

The statistical results of the simulation were entered into the data grid of the AHP model as discussed in section 3.7.1, above. (See Appendix F for the full results of the AHP analysis. Some information contained in Appendix F is repeated here, for clarity.)

Sensor combination	Normalized Priority
S1	0.072
S2	0.171
S3	0.054
S4	0.077
S1, S2	0.167
S2, S3	0.128
S2, S4	0.142
S1, S3	0.055
S1, S2, S4	0.134

Table 12 - Final “priority” scores for all sensor combinations in the combined AHP model

Expert Choice 2000 refers to the overall final score of any given alternative as its “priority” with respect to the overall goal, which is to maximize the total effectiveness of the sensor combination. Note that the distributive mode of synthesis was used, because this mode divides the priority of each objective and assigns it to alternatives relative to their performance under that objective (Expert Choice, 2000).

As shown in Table 12, S2 is the combination which receives the highest score, at 0.171. The reason for this is not difficult to determine when one considers the strong advantage of S2 (i.e., AIS-type sensors) in terms of the economic and logistical factors included in the model. It has the lowest installation cost by a factor of 10 from the next least expensive combination, in this case S1 at \$250,000. This leads it to have a very high utility, indeed, the highest of all the alternatives in terms of the installation cost objective. This advantage is even higher when one considers the fact that the power consumption of S2 is less than that of its nearest competitor by a factor of 2.82, again yielding a high utility score for that objective to add to this alternative’s overall score.

When combined with its strong performance on the performance measures such as detection rate, S2’s clearly dominant economic and logistical score is enough to put it in first place in terms of relative ranking, despite the fact that the priority of economic and logistical factors is only 0.347 compared to the complementary 0.653 for performance measures.

It is not difficult to see why, S2 would be the best choice to install in the Arctic to monitor marine traffic. First of all, AIS is a lightweight system composed mainly of a VHF transceiver connected to a small computer which is used to identify vessels based on their AIS transponder codes. The low installation and operating costs of such a simple system are undoubtedly appealing from the perspective of the NW TDP. However, there are some practical considerations that have not been modeled here that would need to be taken into account. First and most importantly, AIS is technically mandatory since it is

required by international maritime regulations, but it can easily be disabled by any vessels not concerned with compliance with international law and interested in deceiving a shore-based monitoring station. Most notably, warships are not required to activate their AIS transponder at all times. If a vessel evading detection can completely circumvent the surveillance offered by S2 simply by deactivating its AIS transponder, then S2 does not offer any effective surveillance at all. Practically speaking, it will be necessary to pair the AIS sensor with at least one other sensor which is capable of detecting vessels whether they participate in AIS or not.

The model for the financial cost of each sensor combination introduces a potential source of bias into the ranking process. This is because, as discussed below, the utility function is too closely bounded to the actual range of costs for the available options (see Table 6 and Table 11, above). Because the range minimum is \$20,000, S2 is the lowest cost option and therefore S2 obtains very nearly the maximum utility in this decreasing linear utility function. This is perhaps logical: if a sensor combination has a very low cost of installation – making it as close to “free” as possible – then this may make it highly attractive to the Department of National Defence. It must be noted that this problem is solely due to the way in which the AHP function was programmed, not due to any intrinsic characteristic of the NW project or any decision of the NW staff.

The real problem occurs at the opposite end of the scale. The range maximum has been set at \$2.3 million, where the utility falls arbitrarily to zero. Therefore, the most costly sensor combination, S1 S2 S4, falls very near to this and is assigned a utility rating (for this objective) very near to zero.

The problem with this setup is that the lowest cost option receives full points under the installation cost objective while the highest cost option receives almost no points. This might make sense if one assumes that the government’s budget is no larger than the cost of the most expensive sensor combination, but is not necessarily realistic in the context of a budget that might be much higher than the cost of any of these combinations. The AHP utility function should have been programmed differently, a possibility which is discussed further below.

The overall effect is for this portion of the model to be strongly biased against the sensor combinations containing more than one sensor. This is because a combination of two sensors will necessarily be more expensive than either of the component sensors alone, by definition. Therefore, though the combination of multiple sensors may have higher performance in detection measures, the more expensive ones will be unfairly penalized because they fall towards the higher end of the feasible range. From Appendix E, we see that the weight assigned to installation cost by the polled experts under the “economic and logistical factors” node of the AHP hierarchy is:

$$0.347 \times 0.737 = 0.256$$

This is a significant portion of the overall priority weighting of the whole model.

The problem is that there is no way to correct this without being somewhat arbitrary. A better approach might be to set the upper cost limit to some higher number. With more information about the government budget for the installation of the sensors, it would be possible to model this more effectively. This issue must be left for further study.

As discussed in section 2.4.6, above, AHP is recognized in the literature as having a number of advantages and disadvantages inherent in its structure and methodology. These bear re-examination here in light of the experience gained through the use of AHP in this project.

4.3 Discussion of AHP methodology

The first advantage of AHP is that it makes the process of assigning relative importance to alternatives and criteria much simpler for participating decision-makers to understand, and the experience of this particular example bears this out. The use of pairwise comparisons keeps the number of factors under consideration at any given time low, so it is much more intuitive for the decision-maker to indicate his or her level of preference with some precision. He or she does not have to balance multiple dimensions against each other simultaneously. This was an advantage in the case of this project because it made it possible for decision-makers to get right into making pairwise comparisons immediately, with a minimum of explanation of the system. It allowed the decision-makers to remain experts in the subject matter at hand (i.e. sensors and their characteristics relative to NW) rather than forcing them to become experts in the MCDM support system (i.e., AHP). This saved time and made it much easier for the project team to obtain the input of a larger number of subject matter experts in a shorter amount of time.

The other two advantages of AHP listed in section 2.4.6 refer to the fact that AHP is able to handle multiple criteria easily, and that it is able to incorporate both quantitative and qualitative input data. The research conducted here has reinforced the former conclusion, as we were able to use AHP to evaluate nine alternatives in terms of seven criteria without creating an unmanageably complex computation. As for the ability to include both qualitative and quantitative inputs, this was one of the most appealing features of AHP when compared to the other MCDM methods considered in Chapter 2. It is in its flexibility that AHP truly shines. The expert decision-makers can first rank the criteria relative to one another and then go on to rank the alternatives, or they can simply rank the criteria and then input external data to permit the ranking of the alternatives – the approach actually taken in this research project. If the shipping simulation had not been undertaken, it would have been possible for the decision-makers to rank the alternatives relative to one another and to arrive at a conclusion. However, since the simulation was prepared and its result was not known before the decision-makers were asked to rank the criteria, for this research the approach of combining decision-makers' rankings of criteria with quantitative output from the simulation makes sense. Incorporating these quantitative inputs to the model also helps to increase the objectivity of the MCDM exercise.

The disadvantages of AHP as argued by some in the relevant literature are outlined in Section 2.4.6 as well. In particular, the questions raised by Warren (2004) concern the analytical basis of AHP. Warren (2004: 9-12) points out that the type of rating scale used in AHP certainly underlines very valid concerns about the lack of strict mathematical rigor in AHP. However, they do not actually address the next logical extension of the question, which is whether this lack of strict rigor will have a significant effect on the decision-making result.

The lack of an absolute zero in the rating scale, the difficulties associated with the assignment of inverse scores through each pairwise comparison, the difficulties implied for the evaluation of consistency, and the absolute limit on the maximum and minimum scores are all legitimate criticisms of the AHP system. Over time, new methods would be devised that would resolve the inconsistencies and contradictions that these identified flaws imply. However, for the purposes of decision making in a context involving many criteria, it is not required that the method be analytically perfect. It is sufficient that it be functional and yield a coherent result, which AHP does in the case observed here. The exact sensitivity of the AHP system to the flaws identified by Warren and others has yet to be determined, but this could be a rewarding area of further study.

It should be noted that some of the main concerns raised in section 2.4.6 do not apply to the problem considered in this paper. For example, rank reversal is not an issue in this case, as alternatives were not added in stages: all were considered simultaneously.

This concludes the discussion of the results of the simulation and AHP, and of the analysis of the same. In the following chapter, we draw conclusions based on this analysis and provide recommendations for the NW TDP and for future research.

5 Conclusion and Recommendations

This section summarizes the main conclusion of the AHP results. Limitations and suggestions about possible areas of future research are presented.

5.1 Conclusion

From the analysis detailed above, one may conclude that AIS is the preferred sensor combination for the NW application. The AIS sensor had a high detection rate in the simulation runs, the first detection time is short compared to other sensor combinations if the vessel is within range, and AIS has the capability to track a vessel for a longer period of time. However, it may be more advisable to combine AIS with at least one other sensor in order to account for practical considerations not included in the model. For example, one might consider the second-best score in Table 11 as discussed in section 5.2, below.

As noted in section 4.2, above, the decision makers strongly emphasized the initial cost of a sensor: they assigned a relatively high weight to the installation cost associated with each combination. As a result, since AIS's cost is the least by a large margin compared to other combinations, it was ranked first in terms of cost.

The AIS sensor also consumes the least amount of energy in a day. As a result, AIS (Sensor 2) received a high utility score for the power consumption criterion as well. This was another factor that helped to put AIS at the top of the ranked options.

However, for practical purposes, it is important to remember that there are some gaps in the AHP model that may limit the usefulness of this result (especially the issue about the ease with which AIS can be deceived). This is because the model treats each type of sensor as if they had the same capabilities aside from the distribution of their range brackets. Any given sensor's ability to resist malicious attempts to deceive it is not one of the characteristics that is scored in the model, though it probably should be as discussed in section 4.2. Indeed, it is a well-known, fundamental military principle never to rely on any one source of information or sensor but always to confirm any given reading by at least one other method.

Also, in a realistic situation, not all sensors will be detecting, classifying and identifying as the vessels are within range. Usually, these sensors work best in a layered configuration where one sensor might be cued or triggered by another sensor or an external command. In a layered configuration any given sensor can be used to confirm or to complement the results of another sensor.

Under these circumstances, it seems that some combination of sensors that can cover one another's weaknesses may be most appropriate. The model should be updated to take these considerations into account.

5.2 Recommendation

Based on the analysis conducted here, NW should consider installing AIS as its primary sensor type, with another sensor as backup. As Table 11 shows, the second-best option using the weights determined by AHP would be the combination of S1 and S2. This would certainly be a cost-effective way of having at least one level of redundancy in the system in case either sensor failed or was deceived. A third option that is probably more robust in that it offers higher performance characteristics overall than the S1 S2 combination would be S2 S4. This would, however, come at a higher cost and budget constraints might play a role in deciding the feasibility of this option.

Ultimately, this analysis has yielded a small number of feasible options for the NW TDP's decision makers to consider. With some further refinements to the model, as discussed in the next section, that small number could be reduced even further to arrive at a single conclusion that would meet all of the project's stated needs.

5.3 Future Study

The ice data used for the GIS analysis were the most recent data available. However, it is recommended to consider more ice data for a better understanding of the rate of change in the ice in the Northwest Passage.

As discussed above, it would be helpful to improve the AHP model to include tactical considerations such as vulnerability of sensors to electronic warfare techniques or deception by a malicious or hostile party. This is important considering the military/sovereignty-oriented purpose of the system.

Future study should also include a more rigorous analytical method to determine the utility functions for each of the objectives in AHP, which were done by estimation in this case. As discussed in Section 3.7, the utility functions in Expert Choice were set using a qualitative evaluation of each criterion. For greater rigor a more analytical procedure could be used, possibly through a collaborative process between the experts.

Another improvement would be to undertake improved modeling of the equipment installation budget to make a smoother gradient of utility assignments between the highest and lowest cost options. This would help to reduce some of the apparent bias due to the setting of the lower and upper bounds of the utility range, as discussed in section 4.2, above.

The simulation model can be improved. Having access to real maritime traffic would help with the modeling of the shipping. Currently, the shipping model is automated, but incorporating real data would allow the creation of a real time model. As the model is currently, the sensors' performances and probability detection ranges are estimated values based on theoretical specification values. Each sensor will perform differently in

the real environment and their performance can be represented as a step function. The performance of each sensor could vary depending on its unique characteristics. For example, EO/IR will not perform well relative to other sensor types under poor weather conditions. In another scenario, Navigation radar would perform differently depending on the angle and the height at which it is installed. False alarm rates affect the performance of each sensor. For example, Navigation radar has a higher false detection rate than AIS. Therefore, it is recommended to use each sensor combination in a real time experiment to understand their real performance. Some sensors – such as underwater sensors – have been used for experiments and trials in other locations. Using the results of these experiments, the sensor team could provide more details on the performance of such sensor which would improve the realism of the simulation results.

Future study could include the use of genetic algorithms in MOO to optimize design of the system. This could be used in combination with GIS for sensor placement, for example, to yield a much more sophisticated analysis of the whole NW TDP.

Expert Choice 2000 is limited in group decision analysis. However, the later versions of Expert Choice provide this capability. It would be instructive to update the AHP model to a more recent software platform with improved capabilities that would allow weighting of experts' inputs for example.

In this report, only data for installation cost and power consumption are available. It is recommended to consider other factors such as operation cost and communication usage in future study.

It is also recommended that investigation be done on detailed performance of each individual sensor. It would be helpful to have a higher-resolution breakdown of range brackets, for example, which would add some depth and subtlety to the simulation model.

Arctic weather conditions can affect the performance of each sensor. Some sensors such as EO/IR might not perform as well as considered for this study in poor weather conditions. Therefore, it is recommended to investigate each performance in similar conditions to the NW station.

Real data from northern traffic could be collected to better understand the traffic in the Northwest Passage. Such data could help with the modeling and simulation.

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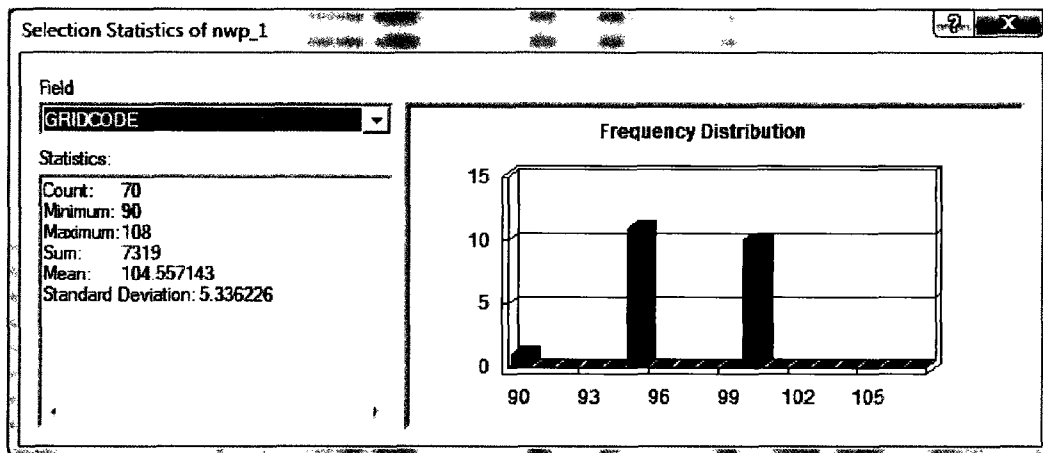
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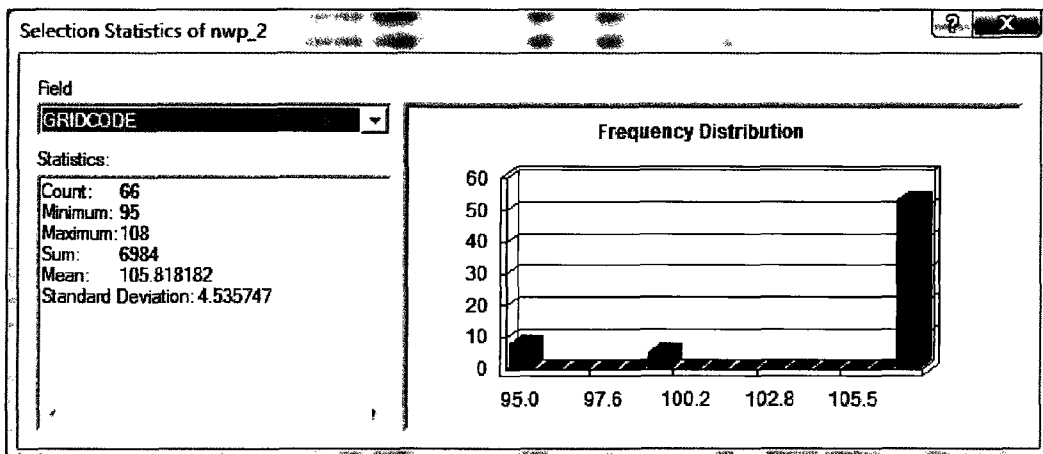
Appendix A – Ice concentration percentage data

The 12 graphs shown in this appendix are outputs from ArcGIS. They each display statistical information on ice concentration in a given month within the area of interest contained within the polygon (see Figure 7, above) and a histogram showing the frequency distribution of ice cells within the area of interest. Ice concentration is expressed as a percentage (i.e. 90 means the cell is 90% ice-covered). Percentages of 108 were used to indicate fast ice (ice which is attached to the shore, and is deemed to be 100% covered). These graphs excluded land and the area within the selection that did not contain any values.

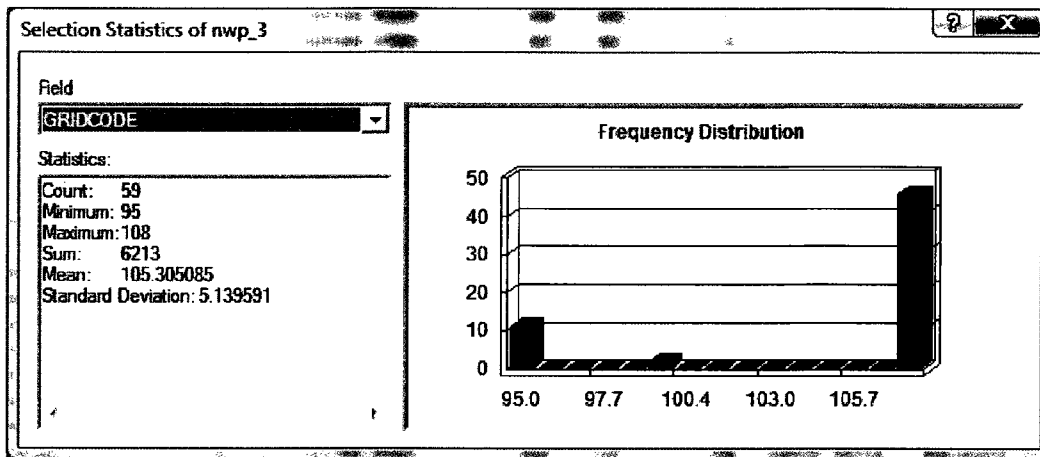
According to these plots, it is clear that between the month of May and October, the ice index mean value is to be its lowest with August having the lowest mean value of ~48%. As a result, this is the best natural time-frame for the highest traffic in the passage.



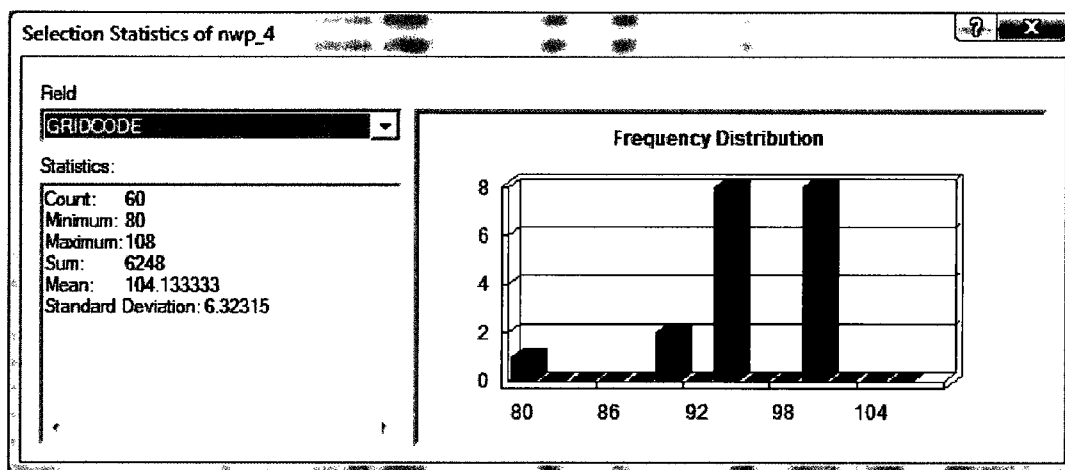
January



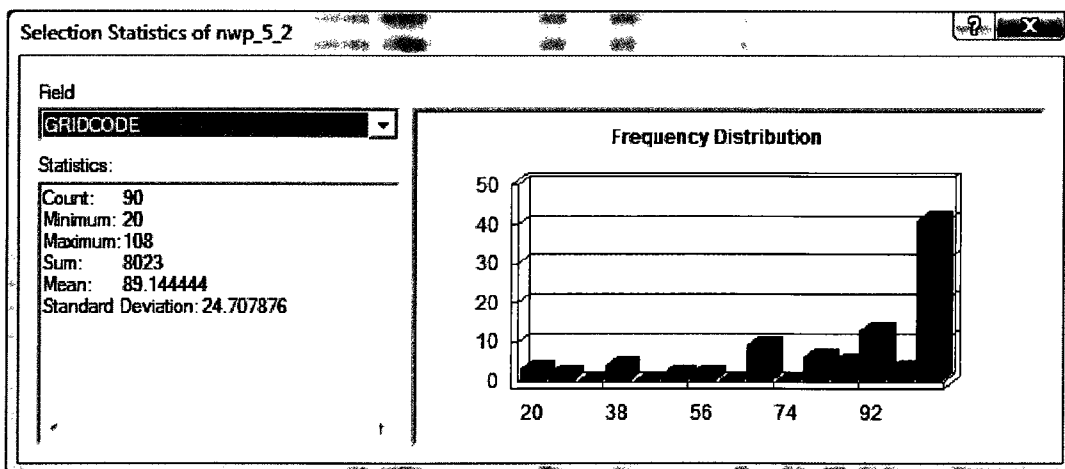
February



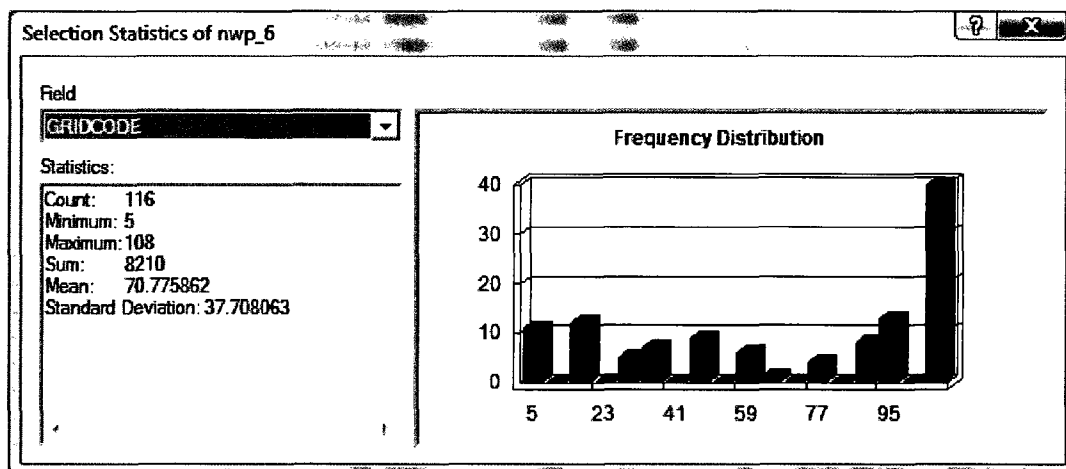
March



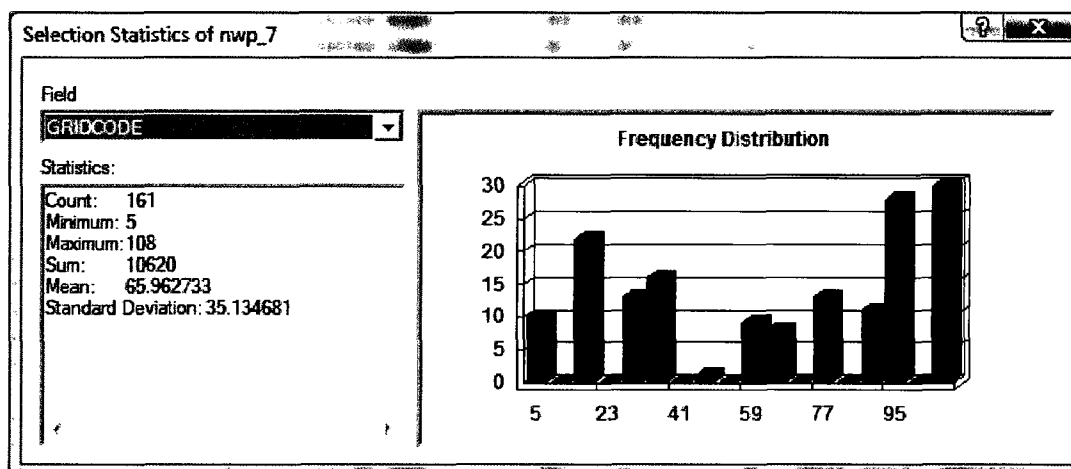
April



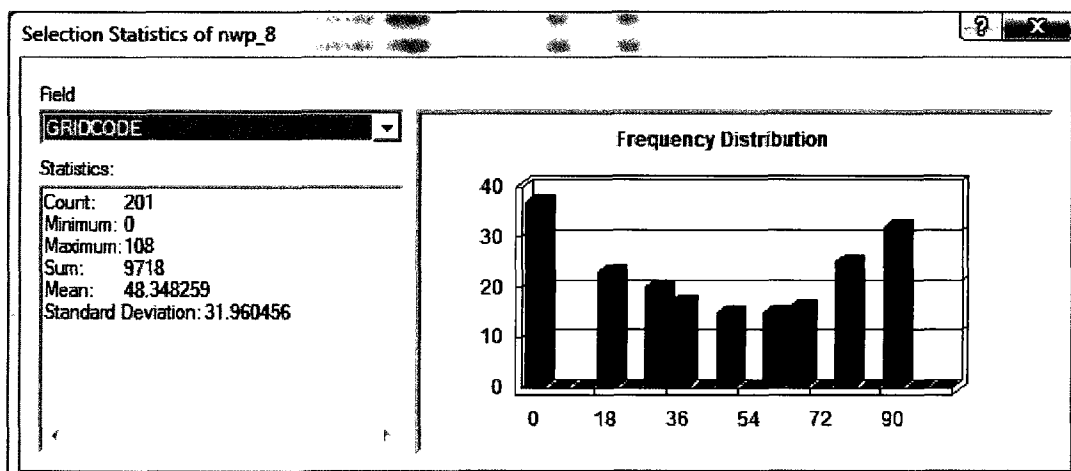
May



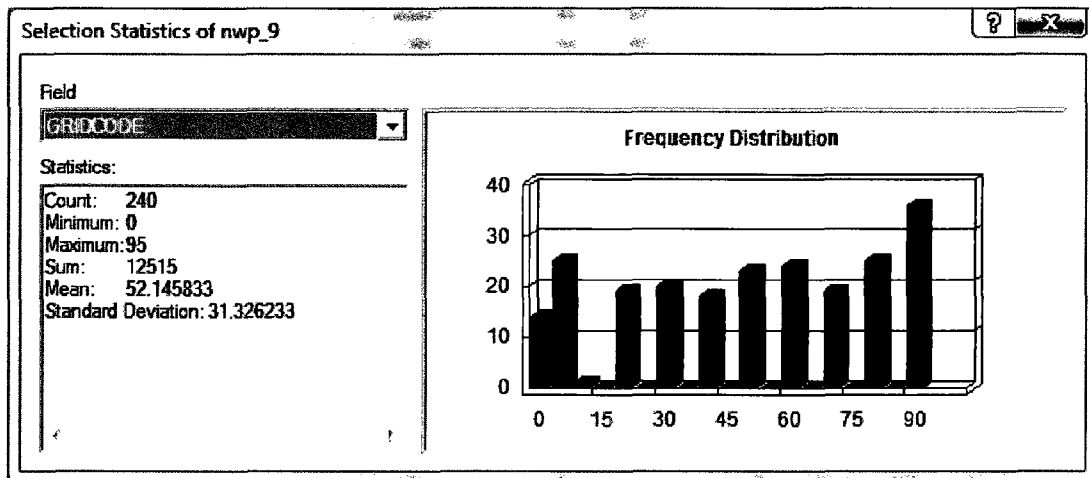
June



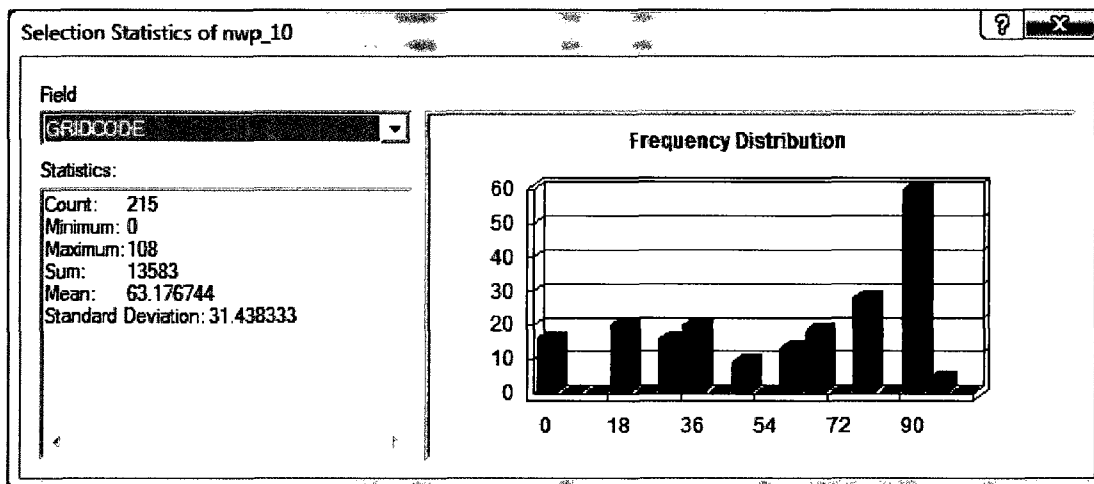
July



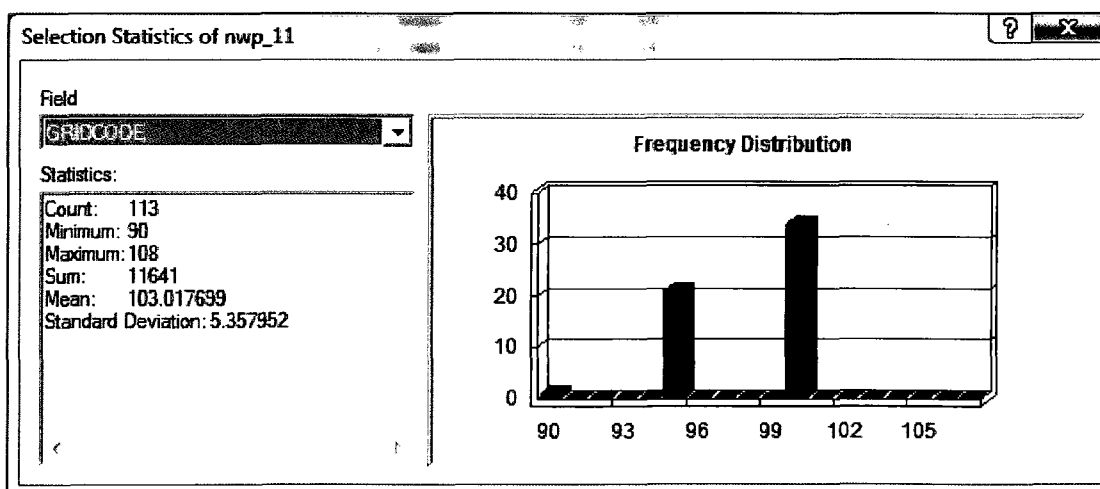
August



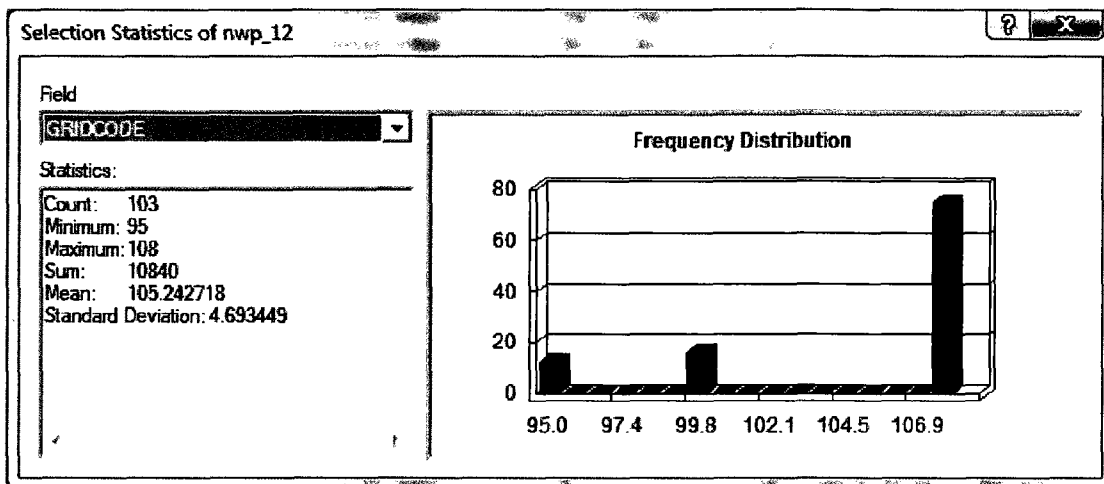
September



October



November



December

Appendix B – Shipping model code

This appendix contains the code of the shipping model as programmed in MATLAB.

At the beginning of each trial, a vessel type and weather type are determined, along with the position of points A and B (the starting and ending points of the vessel's track, respectively). The characteristics of each sensor are defined. The points of intersection of the line AB with each range bracket of each sensor are calculated. Based on all of the above information, the MOPs (detection rate, time of first detection, proportion of passage spent detected, classification and identification) are recorded and statistics on the MOPs are compiled for each run of 100 trials.

```
=====

clc;
clear all;

for J=1:1:100

%*****%
%                               V                               %
%*****%
%creating a fishing boatvessels at random
rand_v = rand(1);
if (rand_v <= 0.5)
    v_speed = nm2km(7); %fishing boat travels at about 7Nm/hr
    v_rng = km2deg(v_speed); %convert km to arclength for rng
    v = 'fishing boat';
    v_type = 1;
else
    v_speed = nm2km(10); %Ice breaker travels at about 7Nm/hr
    %dist2 = v_speed/2;
    v_rng = km2deg(v_speed);
    v = 'icebreaker';
    v_type = 2;
end
%fprintf('vessel type is %s\n',v);
fprintf('\n----- vessel: %s ----- \n',v);

%*****%
%                               weather                               %
%*****%
rand_weather = rand(1);
if (rand_v <=80)
    w = 'ideal';
    p_min = 1;
    p_mid = 0.80;
    p_max = 0.50;
else
    w = 'poor';
```

```

    p_min = 0.85;
    p_mid = 0.65;
    p_max = 0.25;
end
fprintf('Weather condition is %s\n',w);

%*****%
%                               S1                               %
%*****%
%Navigation Radar - s1
%Depending on the type of vessel, the detection contours change.
if (v_type == 1)
    s1_detect_min = 18; %80 detection
    s1_detect_mid = 20; %100 detection
    s1_detect_max = 25;
    s1_class = 0;
    s1_ident = 0;
else
    s1_detect_min = 38; %80 detection
    s1_detect_mid = 40; %100 detection
    s1_detect_max = 45;
    s1_class = 0;
    s1_ident = 0;
end
s1_lat = 74.69;
s1_lon = -94.81;
s1_location = [s1_lat, s1_lon];
%convert distances from nautical miles to kilometers
s1_r_min = nm2km(s1_detect_min);
s1_r_mid = nm2km(s1_detect_mid);
s1_r_max = nm2km(s1_detect_max);

t_max_max_S1 = 0;
t_A_max_S1 = 0;
t_max_mid_S1 = 0;
t_mid_min_S1 = 0;
t_min_min_S1 = 0;
t_min_mid_S1 = 0;
t_mid_max_S1 = 0;
t_mid_mid_S1 = 0;
t_max_B_S1 = 0;

%*****%
%                               S2                               %
%*****%
%AIS - s2
if (v_type == 1)
    s2_detect_min = 39; %80 detection
    s2_detect_mid = 41; %100 detection
    s2_detect_max = 46; %50 detection
    s2_class = 0;
    s2_ident = 0;
else
    s2_detect_min = 44; %80 detection
    s2_detect_mid = 46; %100 detection
    s2_detect_max = 51; %50 detection

```

```

    s2_class = 0;
    s2_ident = 0;
end
s2_lat = 74.69;
s2_lon = -94.81;
s2_location = [s2_lat, s2_lon];
%convert distances from nautical miles to kilometers
s2_r_min = nm2km(s2_detect_min);
s2_r_mid = nm2km(s2_detect_mid);
s2_r_max = nm2km(s2_detect_max);

t_max_max_S2 = 0;
t_A_max_S2 = 0;
t_max_mid_S2 = 0;
t_mid_min_S2 = 0;
t_min_min_S2 = 0;
t_min_mid_S2 = 0;
t_mid_max_S2 = 0;
t_mid_mid_S2 = 0;
t_max_B_S2 = 0;

%*****0%
%                               S3                               %
%*****0%
%EO/IR
if (v_type == 1)
    s3_detect_min = 32; %80 detection
    s3_detect_mid = 34; %100 detection
    s3_detect_max = 39;
    s3_class = 1.6;
    s3_ident = 0.65;
else
    s3_detect_min = 38; %80 detection
    s3_detect_mid = 40; %100 detection
    s3_detect_max = 45;
    s3_class = 5.5;
    s3_ident = 3.6;
end
s3_lat = 74.69;
s3_lon = -94.81;
s3_location = [s3_lat, s3_lon];
%convert distances from nautical miles to kilometers
s3_r_min = nm2km(s3_detect_min);
s3_r_mid = nm2km(s3_detect_mid);
s3_r_max = nm2km(s3_detect_max);

t_max_max_S3 = 0;
t_A_max_S3 = 0;
t_max_mid_S3 = 0;
t_mid_min_S3 = 0;
t_min_min_S3 = 0;
t_min_mid_S3 = 0;
t_mid_max_S3 = 0;
t_mid_mid_S3 = 0;
t_max_B_S3 = 0;

```

```

%*****%
%                               S4                               %
%*****%
%UnderWater - S4
if (v_type == 1)
    s4_detect_min = 52; %80 detection
    s4_detect_mid = 54; %100 detection
    s4_detect_max = 59;
else
    s4_detect_min = 52; %80 detection
    s4_detect_mid = 54; %100 detection
    s4_detect_max = 59;
end
s4_lat = 74.45;
s4_lon = -95.00;
s4_location = [s4_lat, s4_lon];
%convert distances from nautical miles to kilometers
s4_r_min = nm2km(s4_detect_min);
s4_r_mid = nm2km(s4_detect_mid);
s4_r_max = nm2km(s4_detect_max);

t_max_max_S4 = 0;
t_A_max_S4 = 0;
t_max_mid_S4 = 0;
t_mid_min_S4 = 0;
t_min_min_S4 = 0;
t_min_mid_S4 = 0;
t_mid_max_S4 = 0;
t_mid_mid_S4 = 0;
t_max_B_S4 = 0;

%creating a box
latA_min = 74.075;
latA_max = 74.481;
latA_rand = (latA_max - latA_min) * rand(1);
latA = latA_min + latA_rand;
lonA = -90.919;
v_start = [latA lonA];

latB_min = 74.239;
latB_max = 74.561;
latB_rand = (latB_max - latB_min) * rand(1);
latB = latB_min + latB_rand;
lonB = -99.913;
v_end = [latB lonB];

%calculating the time of travel between start and end point (the box)
d_total = distance(latA,lonA,latB,lonB);
d_total_km = deg2km(d_total);
t_total = (d_total/v_rng)*60;

%geographic track from starting and ending points
v_line = track2('rh',latA,lonA,latB,lonB);

```

```

v_az = azimuth (latA,lonA,latB,lonB);

%*****%
%          S1          %
%*****%

%creating a the contour circles for the area of coverage and intersection of
%each circle with the vessel route.

s1_cnt_min = scircle1(s1_lat,s1_lon,km2deg(s1_r_min));
[v_s1_min_lat,v_s1_min_lon] =
polyxpoly(s1_cnt_min(:,1),s1_cnt_min(:,2),v_line(:,1),v_line(:,2),'unique');
v_s1_min = [v_s1_min_lat,v_s1_min_lon];

s1_cnt_mid = scircle1(s1_lat,s1_lon,km2deg(s1_r_mid));
[v_s1_mid_lat,v_s1_mid_lon] =
polyxpoly(s1_cnt_mid(:,1),s1_cnt_mid(:,2),v_line(:,1),v_line(:,2),'unique');
v_s1_mid = [v_s1_mid_lat,v_s1_mid_lon];

s1_cnt_max = scircle1(s1_lat,s1_lon,km2deg(s1_r_max));
[v_s1_max_lat,v_s1_max_lon] =
polyxpoly(s1_cnt_max(:,1),s1_cnt_max(:,2),v_line(:,1),v_line(:,2),'unique');
v_s1_max = [v_s1_max_lat,v_s1_max_lon];

fprintf('***** S1 *****\n');

if (v_s1_max ~= '')
    %scenario 3 (crossing only the max contour)
    d_max_max_S1 = distance (v_s1_max_lat(1),v_s1_max_lon(1),v_s1_max_lat(2),v_s1_max_lon(2));
    t_max_max_S1 = (d_max_max_S1 / v_rng)*60;
    d_A_max_S1 = distance (latA,lonA,v_s1_max_lat(1),v_s1_max_lon(1));
    t_A_max_S1 = (d_A_max_S1 / v_rng)*60;
    d_max_B_S1 = distance (v_s1_max_lat(2),v_s1_max_lon(2),latB,lonB);
    t_max_B_S1 = (d_max_B_S1 / v_rng)*60;
    if (v_s1_mid ~= '')
        %scenario 2 (crossing the max and mid contours)
        d_max_mid_S1 = distance (v_s1_max_lat(1),v_s1_max_lon(1),v_s1_mid_lat(1),v_s1_mid_lon(1));
        t_max_mid_S1 = (d_max_mid_S1 / v_rng)*60;
        d_mid_max_S1 = distance (v_s1_mid_lat(2),v_s1_mid_lon(2),v_s1_max_lat(2),v_s1_max_lon(2));
        t_mid_max_S1 = (d_mid_max_S1 / v_rng)*60;
        d_mid_mid_S1 = distance (v_s1_mid_lat(1),v_s1_mid_lon(1),v_s1_mid_lat(2),v_s1_mid_lon(2));
        t_mid_mid_S1 = (d_mid_mid_S1 / v_rng)*60;
    if (v_s1_min ~= '')
        %scenario 1 (crossing all the contours)
        d_mid_min_S1 = distance (v_s1_mid_lat(1),v_s1_mid_lon(1),v_s1_min_lat(1),v_s1_min_lon(1));
        t_mid_min_S1 = (d_mid_min_S1 / v_rng)*60;
        d_min_min_S1 = distance (v_s1_min_lat(1),v_s1_min_lon(1),v_s1_min_lat(2),v_s1_min_lon(2));
        t_min_min_S1 = (d_min_min_S1 / v_rng)*60;
        d_min_mid_S1 = distance (v_s1_min_lat(2),v_s1_min_lon(2),v_s1_mid_lat(2),v_s1_mid_lon(2));
        t_min_mid_S1 = (d_min_mid_S1 / v_rng)*60;
    else
        fprintf('The vessel is out of detection range (no intersection with s1_min).\n');
    end
else

```

```

        fprintf('The vessel is out of detection range (no intersection with s1_mid).\n');
    end
else
    fprintf('The vessel is out of detection range (no intersection with s1_max).\n');
end

s1_t_det_50 = 0;
s1_t_det_last = 0;
t_last = 0;
c1_s1 = 0;
c2_s1 = 0;
c3_s1 = 0;
c4_s1 = 0;
c5_s1 = 0;
sz1_s1 = 0;
sz2_s1 = 0;
sz3_s1 = 0;
sz4_s1 = 0;
sz5_s1 = 0;
pdet_s1 = 0;

s1_v_max_max = 0;
s1_v_mid_mid = 0;
s1_v_min_min = 0;
s1_v_max_mid = 0;
s1_v_mid_min = 0;
s1_v_min_mid = 0;
s1_v_mid_max = 0;
s1_v_A_max = 0;
s1_v_max_B = 0;
s1_v_max_mid_min = 0;
s1_v_A_B = 0;

%calculating the probabilities
%scenario 1 (crossing all contours)
if((t_max_mid_S1 ~=0) && (t_mid_min_S1 ~=0) && (t_min_min_S1 ~=0))
    for i=1:(round(t_max_mid_S1))
        detect_num = rand(1);
        if(detect_num <= p_max)
            s1_v_max_mid(i) = true;
            c1_s1 = c1_s1 + 1;
            if(c1_s1==1)
                s1_t_det_50 = i + t_A_max_S1;
                fprintf('Scenario 1: First detection was at t=%g minutes\n',s1_t_det_50);
                [p1_s1_lat,p1_s1_lon] = reckon(v_s1_max_lat(1),v_s1_max_lon(1),((i/60)*v_rng),v_az);
                dist_v_s1 = deg2nm(distance(s1_lat,s1_lon,p1_s1_lat,p1_s1_lon));
            end
        end
        else
            s1_v_max_mid(i) = false;
        end
        sz1_s1 = numel(s1_v_max_mid);
    end
    %sz1_s1=length(s1_v_max_mid);

    for i=1:(round(t_mid_min_S1))
        detect_num = rand(1);

```

```

        if (detect_num <= p_mid)
            s1_v_mid_min(i) = true;
            c2_s1 = c2_s1 + 1;
        else
            s1_v_mid_min(i) = false;
        end
        sz2_s1=numel(s1_v_mid_min);
    end
    %sz2_s1=length(s1_v_mid_min);

    for i=1:1:(round(t_min_min_S1))
        detect_num = rand(1);
        if (detect_num <= p_min)
            s1_v_min_min(i) = true;
            c3_s1 = c3_s1 + 1;
        else
            s1_v_min_min(i) = false;
        end
        sz3_s1=numel(s1_v_min_min);
    end
    % sz3_s1=length(s1_v_min_min);

    for i=1:1:(round(t_min_mid_S1))
        detect_num = rand(1);
        if (detect_num <= p_mid)
            s1_v_min_mid(i) = true;
            c4_s1 = c4_s1 + 1;
        else
            s1_v_min_mid(i) = false;
        end
        sz4_s1=numel(s1_v_min_mid);
    end
    %sz4_s1=length(s1_v_min_mid);

    for i=1:1:(round(t_mid_max_S1))
        detect_num = rand(1);
        if (detect_num <= p_max)
            s1_v_mid_max(i) = true;
            c5_s1 = c5_s1 + 1;
            t_last = i;
        else
            s1_v_mid_max(i) = false;
        end
        sz5_s1=numel(s1_v_mid_max);
    end
    s1_t_det_last = t_last+t_min_mid_S1+t_min_min_S1+t_mid_min_S1+t_max_mid_S1+ t_A_max_S1;
    %sz5_s1=length(s1_v_mid_max);

    pdet_s1 = (c1_s1+c2_s1+c3_s1+c4_s1+c5_s1)/(sz1_s1+sz2_s1+sz3_s1+sz4_s1+sz5_s1);
    %fprintf('The detection probability rate by S1-Navigation Radar is %g\n',pdet_s1*100);
    s1_v_max_mid_min = cat(2, s1_v_max_mid,s1_v_mid_min,s1_v_min_min,s1_v_min_mid,
s1_v_mid_max);
end

%calculating the probabilities
%scenario 2 (crossing max and mid contours)

```

```

if((t_max_mid_S1 ~=0) && (t_mid_mid_S1 ~=0) && (t_mid_min_S1 ==0))
    for i=1:1:(round(t_max_mid_S1))
        detect_num = rand(1);
        if (detect_num <= p_max)
            s1_v_max_mid(i) = true;
            c1_s1 = c1_s1 + 1;
            if (c1_s1==1)
                s1_t_det_50 = i + t_A_max_S1;
                fprintf('Scenario 2: First detection was at t=%g minutes\n',s1_t_det_50);
                [p1_s1_lat,p1_s1_lon] = reckon(v_s1_max_lat(1),v_s1_max_lon(1),((i/60)*v_rng),v_az);
                dist_v_s1 = deg2nm(distance(s1_lat,s1_lon,p1_s1_lat,p1_s1_lon));
            end
        else
            s1_v_max_mid(i)=false;
        end
        sz1_s1=numel(s1_v_max_mid);
    end
    %sz1_s1=length(s1_v_max_mid);

    for i=1:1:(round(t_mid_mid_S1))
        detect_num = rand(1);
        if (detect_num <= p_mid)
            s1_v_mid_mid(i) = true;
            c2_s1 = c2_s1 + 1;
        else
            s1_v_mid_mid(i)=false;
        end
        sz2_s1=numel(s1_v_mid_mid);
    end
    %sz2_s1=length(s1_v_mid_mid);

    for i=1:1:(round(t_mid_max_S1))
        detect_num = rand(1);
        if (detect_num <= p_max)
            s1_v_mid_max(i) = true;
            c3_s1 = c3_s1 + 1;
            t_last = i;
        else
            s1_v_mid_max(i)=false;
        end
        sz3_s1=numel(s1_v_mid_max);
    end
    s1_t_det_last = t_last + t_max_mid_S1 + t_mid_mid_S1 + t_A_max_S1;
    %sz3_s1=length(s1_v_mid_max);

    pdet_s1 = (c1_s1+c2_s1+c3_s1)/(sz1_s1+sz2_s1+sz3_s1);
    %fprintf('The detection probability rate by S1-Navigation Radar is %g\n',pdet_s1*100);
    s1_v_max_mid_min = cat(2, s1_v_max_mid,s1_v_mid_mid,s1_v_mid_max);
end

%calculating the probabilities
%scenario 3 (crossing max only)
if((t_max_max_S1) ~=0 && (t_max_mid_S1 ==0) && (t_mid_min_S1 ==0))
    for i=1:1:(round(t_max_max_S1))
        detect_num = rand(1);
        if (detect_num <= p_max)

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        s1_v_max_max(i) = true;
        cl_s1 = cl_s1 + 1;
        if (cl_s1==1)
            s1_t_det_50 = i + t_A_max_S1;
            fprintf('Scenario 3: First detection was at t=%g minutes\n',s1_t_det_50);
            [p1_s1_lat,p1_s1_lon] = reckon(v_s1_max_lat(1),v_s1_max_lon(1),((i/60)*v_rng),v_az);
            dist_v_s1 = deg2nm(distance(s1_lat,s1_lon,p1_s1_lat,p1_s1_lon));
        end
        t_last = i;
    else
        s1_v_max_max(i) = false;
    end
    end
    sz1_s1=numel(s1_v_max_max);
end
s1_t_det_last = t_last + t_A_max_S1;
%sz1_s1=length(s1_v_max_max);
pdet_s1 = (cl_s1)/(sz1_s1);
%fprintf('The detection probability rate by S1-Navigation Radar is %g\n',pdet_s1*100);
s1_v_max_mid_min = cat(2, s1_v_max_max);
end

%position of last detection:
%p2_s1_lat,p2_s1_lon] = reckon (latA,lonA,((s1_t_det_last/60)*v_rng),v_az);

fprintf('The overall detection by S1-Navigation Radar is %g\n',pdet_s1*100);
%fprintf('The last detection was at t=%g minutes\n',s1_t_det_last);
%fprintf('The vessel is detected at the first %g %g of its trip through the
passage.\n',(s1_t_det_50/t_total)*100);
fprintf('The vessel was detected and tracked %g of its trip across the passage.\n\n',((s1_t_det_last-
s1_t_det_50)/t_total)*100);
pdet_s1_J(J) = pdet_s1*100;
s1_t_det_50_J(J) = s1_t_det_50;
%s1_t_det_total_J(J) = (s1_t_det_50/t_total)*100;
s1_t_det_total_J(J) = ((s1_t_det_last-s1_t_det_50)/t_total)*100;
s1_v_class_J(J) = 0;
s1_v_ident_J(J) = 0;

% s1_v_max_max = 0;
% s1_v_mid_mid = 0;
% s1_v_max_mid = 0;
% s1_v_min_min = 0;
% s1_v_mid_min = 0;
% s1_v_max_mid = 0;

%*****0%
%                S2                %
%*****0%

%creating a the contour circles for the area of coverage and intersection of
%each circle with the vessel route.

s2_cnt_min = scircle1(s2_lat,s2_lon,km2deg(s2_r_min));
[v_s2_min_lat,v_s2_min_lon] =
polyxpoly(s2_cnt_min(:,1),s2_cnt_min(:,2),v_line(:,1),v_line(:,2),'unique');
v_s2_min = [v_s2_min_lat,v_s2_min_lon];

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s2_cnt_mid = scircle1(s2_lat,s2_lon,km2deg(s2_r_mid));
[v_s2_mid_lat,v_s2_mid_lon] =
polyxpoly(s2_cnt_mid(:,1),s2_cnt_mid(:,2),v_line(:,1),v_line(:,2),'unique');
v_s2_mid = [v_s2_mid_lat,v_s2_mid_lon];

s2_cnt_max = scircle1(s2_lat,s2_lon,km2deg(s2_r_max));
[v_s2_max_lat,v_s2_max_lon] =
polyxpoly(s2_cnt_max(:,1),s2_cnt_max(:,2),v_line(:,1),v_line(:,2),'unique');
v_s2_max = [v_s2_max_lat,v_s2_max_lon];

fprintf('***** S2 *****\n');

if (v_s2_max ~= '')
    %scenario 3 (crossing only the max contour)
    d_max_max_S2 = distance (v_s2_max_lat(1),v_s2_max_lon(1),v_s2_max_lat(2),v_s2_max_lon(2));
    t_max_max_S2 = (d_max_max_S2 / v_rng)*60;
    d_A_max_S2 = distance (latA,lonA,v_s2_max_lat(1),v_s2_max_lon(1));
    t_A_max_S2 = (d_A_max_S2 / v_rng)*60;
    d_max_B_S2 = distance (v_s2_max_lat(2),v_s2_max_lon(2),latB,lonB);
    t_max_B_S2 = (d_max_B_S2 / v_rng)*60;
    if (v_s2_mid ~= '')
        %scenario 2 (crossing the max and mid contours)
        d_max_mid_S2 = distance (v_s2_max_lat(1),v_s2_max_lon(1),v_s2_mid_lat(1),v_s2_mid_lon(1));
        t_max_mid_S2 = (d_max_mid_S2 / v_rng)*60;
        d_mid_max_S2 = distance (v_s2_mid_lat(2),v_s2_mid_lon(2),v_s2_max_lat(2),v_s2_max_lon(2));
        t_mid_max_S2 = (d_mid_max_S2 / v_rng)*60;
        d_mid_mid_S2 = distance (v_s2_mid_lat(1),v_s2_mid_lon(1),v_s2_mid_lat(2),v_s2_mid_lon(2));
        t_mid_mid_S2 = (d_mid_mid_S2 / v_rng)*60;
        if (v_s2_min ~= '')
            %scenario 1 (crossing all the contours)
            d_mid_min_S2 = distance (v_s2_mid_lat(1),v_s2_mid_lon(1),v_s2_min_lat(1),v_s2_min_lon(1));
            t_mid_min_S2 = (d_mid_min_S2 / v_rng)*60;
            d_min_min_S2 = distance (v_s2_min_lat(1),v_s2_min_lon(1),v_s2_min_lat(2),v_s2_min_lon(2));
            t_min_min_S2 = (d_min_min_S2 / v_rng)*60;
            d_min_mid_S2 = distance (v_s2_min_lat(2),v_s2_min_lon(2),v_s2_mid_lat(2),v_s2_mid_lon(2));
            t_min_mid_S2 = (d_min_mid_S2 / v_rng)*60;
        else
            fprintf ('The vessel is out of detection range (no intersection with s2_min).\n');
        end
    else
        fprintf ('The vessel is out of detection range (no intersection with s2_mid).\n');
    end
else
    fprintf ('The vessel is out of detection range (no intersection with s2_max).\n');
end

%calculating the probabilities
%scenario 1 (crossing all contours)
s2_t_det_50 = 0;
s2_t_det_last = 0;
t_last = 0;
c1_s2 = 0;
c2_s2 = 0;
c3_s2 = 0;
c4_s2 = 0;

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```

c5_s2 = 0;
sz1_s2 = 0;
sz2_s2 = 0;
sz3_s2 = 0;
sz4_s2 = 0;
sz5_s2 = 0;
pdet_s2 = 0;

s2_v_max_max = 0;
s2_v_mid_mid = 0;
s2_v_min_min = 0;
s2_v_mid_min = 0;
s2_v_max_mid = 0;
s2_v_mid_max = 0;
s2_v_min_mid = 0;
s2_v_A_max = 0;
s2_v_max_B = 0;
s2_v_A_B = 0;
s2_v_max_mid_min = 0;

if((t_max_mid_S2 ~=0) && (t_mid_min_S2 ~=0) && (t_min_min_S2 ~=0) && (t_min_min_S2 ~=0))
    for i=1:(round(t_max_mid_S2))
        detect_num = rand(1);
        if(detect_num <= p_max)
            s2_v_max_mid(i) = true;
            c1_s2 = c1_s2 + 1;
            if(c1_s2==1)
                s2_t_det_50 = i + t_A_max_S2;
                fprintf('Scenario 1: First detection was at t=%g minutes\n',s2_t_det_50);
                [p1_s2_lat,p1_s2_lon] = reckon(v_s2_max_lat(1),v_s2_max_lon(1),((i/60)*v_rng),v_az);
                dist_v_s2 = deg2nm(distance(s2_lat,s2_lon,p1_s2_lat,p1_s2_lon));
                s2_v_class = 1;
                s2_v_ident = 1;
            end
        else
            s2_v_max_mid(i) = false;
        end
        %sz1_s2=sz1_s2+1;
    end
    %sz1_s2=length(s2_v_max_mid);
    sz1_s2 = numel(s2_v_max_mid);

    for i=1:(round(t_mid_min_S2))
        detect_num = rand(1);
        if(detect_num <= p_mid)
            s2_v_mid_min(i) = true;
            c2_s2 = c2_s2 + 1;
        else
            s2_v_mid_min(i) = false;
        end
        %sz2_s2=sz2_s2+1;
    end
    %sz2_s2=length(s2_v_mid_min);
    sz2_s2 = numel(s2_v_mid_min);

    for i=1:(round(t_min_min_S2))

```

```

detect_num = rand(1);
if (detect_num <= p_min)
    s2_v_min_min(i) = true;
    c3_s2 = c3_s2 + 1;
else
    s2_v_min_min(i) = false;
end
%sz3_s2=sz3_s2+1;
end
sz3_s2 = numel(s2_v_min_min);
%sz3_s2=length(s2_v_min_min);

for i=1:1:(round(t_min_mid_S2))
    detect_num = rand(1);
    if (detect_num <= p_mid)
        s2_v_min_mid(i) = true;
        c4_s2 = c4_s2 + 1;
    else
        s2_v_min_mid(i) = false;
    end
    %sz4_s2=sz4_s2+1;
end
sz4_s2=numel(s2_v_min_mid);
%sz4_s2=length(s2_v_min_mid);

for i=1:1:(round(t_mid_max_S2))
    detect_num = rand(1);
    if (detect_num <= p_max)
        s2_v_mid_max(i) = true;
        c5_s2 = c5_s2 + 1;
        t_last = i;
    else
        s2_v_mid_max(i) = false;
    end
    % sz5_s2=sz5_s2+1;
end
%sz5_s2=length(s2_v_mid_max);
s2_t_det_last = t_last + t_min_mid_S2 + t_min_min_S2 + t_mid_min_S2 + t_max_mid_S2 +
t_A_max_S2;
sz5_s2=numel(s2_v_mid_max);
pdet_s2 = (c1_s2+c2_s2+c3_s2+c4_s2+c5_s2)/(sz1_s2+sz2_s2+sz3_s2+sz4_s2+sz5_s2);
s2_v_max_mid_min = cat(2,
s2_v_max_mid,s2_v_mid_min,s2_v_min_min,s2_v_min_mid,s2_v_mid_max);
end

%calculating the probabilities
%scenario 2 (crossing max and mid contours)
if((t_max_mid_S2 ~=0) && (t_mid_mid_S2 ~=0) && (t_mid_min_S2 ==0))
    for i=1:1:(round(t_max_mid_S2))
        detect_num = rand(1);
        if (detect_num <= p_max)
            s2_v_max_mid(i) = true;
            c1_s2 = c1_s2 + 1;
            if (c1_s2==1)
                s2_t_det_50 = i + t_A_max_S2;
                fprintf('Scenario 2: First detection was at t =%g minutes\n',s2_t_det_50);
            end
        end
    end
end

```

```

        [p1_s2_lat,p1_s2_lon] = reckon (v_s2_max_lat(1),v_s2_max_lon(1),((i/60)*v_rng),v_az);
        dist_v_s2 = deg2nm(distance(s2_lat,s2_lon,p1_s2_lat,p1_s2_lon));
        s2_v_class = 1;
        s2_v_ident = 1;
    end
    else
        s2_v_max_mid(i) = false;
    end
    end
    %sz1_s2=sz1_s2+1;
end
%sz1_s2=length(s2_v_max_mid);
sz1_s2=numel(s2_v_max_mid);

for i=1:(round(t_mid_mid_S2))
    detect_num = rand(1);
    if (detect_num <= p_mid)
        s2_v_mid_mid(i) = true;
        c2_s2 = c2_s2 + 1;
    else
        s2_v_mid_mid(i) = false;
    end
    %sz2_s2=sz2_s2+1;
end
%sz2_s2=length(s2_v_mid_mid);
sz2_s2=numel(s2_v_mid_mid);

for i=1:(round(t_mid_max_S2))
    detect_num = rand(1);
    if (detect_num <= p_max)
        s2_v_mid_max(i) = true;
        c3_s2 = c3_s2 + 1;
        t_last = i;
    else
        s2_v_mid_max(i) = false;
    end
    s2_t_det_last = t_last + t_max_mid_S2 + t_mid_mid_S2 + t_A_max_S2;
    %sz3_s2=sz3_s2+1;
end
%sz3_s2=length(s2_v_mid_max);
sz3_s2=numel(s2_v_mid_max);

pdet_s2 = (c1_s2+c2_s2+c3_s2)/(sz1_s2+sz2_s2+sz3_s2);
s2_v_max_mid_min = cat(2, s2_v_max_mid,s2_v_mid_mid,s2_v_mid_max);
end

%calculating the probabilities
%scenario 3 (crossing max only)
if((t_max_max_S2) ~=0 && (t_max_mid_S2 ==0) && (t_mid_min_S2 ==0))
    for i=1:(round(t_max_max_S2))
        detect_num = rand(1);
        if (detect_num <= p_max)
            s2_v_max_max(i) = true;
            c1_s2 = c1_s2 + 1;
            if (c1_s2==1)
                s2_t_det_50 = i + t_A_max_S2;
                fprintf('Scenario 3: First detection was at t =%g minutes\n',s2_t_det_50);
            end
        end
    end
end

```

```

        [p1_s2_lat,p1_s2_lon] = reckon (v_s2_max_lat(1),v_s2_max_lon(1),((i/60)*v_rng),v_az);
        dist_v_s2 = deg2nm(distance(s2_lat,s2_lon,p1_s2_lat,p1_s2_lon));
        s2_v_class = 1;
        s2_v_ident = 1;
    end
    t_last = i;
else
    s2_v_max_max(i) = false;
end
% sz3_s2=sz3_s2+1;
end
s2_t_det_last = t_last + t_A_max_S2;
%sz1_s2=length(s2_v_max_max);
sz1_s2=numel(s2_v_max_max);
pdet_s2 = (c1_s2)/(sz1_s2);
s2_v_max_mid_min = cat(2, s2_v_max_max);
end

%position of the last detection:
%p2_s2_lat,p2_s2_lon] = reckon (latA,lonA,((s2_t_det_last/60)*v_rng),_az);
fprintf('The overall probability of detection rate by S2-AIS Radar is %%%g\n',pdet_s2*100);
fprintf('The vessel was detected and tracked %%%g of its trip through the passage.\n',((s2_t_det_last-
s2_t_det_50)/t_total)*100);
pdet_s2_J(J) = pdet_s2*100;
s2_t_det_50_J(J) = s2_t_det_50;
s2_t_det_total_J(J) = ((s2_t_det_last-s2_t_det_50)/t_total)*100;
s2_v_class_J(J) = (s2_v_class);
s2_v_ident_J(J) = (s2_v_ident);
if (s2_v_class_J(J)==1)
    fprintf('The vessel was classified by S2 \n')
end
if (s2_v_ident_J(J)==1)
    fprintf('The vessel was identified by S2 \n')
end
fprintf ('\n');

%*****%
%          S3          %
%*****%

%creaing a the contour circles for the area of coverage and intersection of
%each circle with the vessel route.

s3_cnt_min = scircle1(s3_lat,s3_lon,km2deg(s3_r_min));
[v_s3_min_lat,v_s3_min_lon] =
polyxpoly(s3_cnt_min(:,1),s3_cnt_min(:,2),v_line(:,1),v_line(:,2),'unique');
v_s3_min = [v_s3_min_lat,v_s3_min_lon];

s3_cnt_mid = scircle1(s3_lat,s3_lon,km2deg(s3_r_mid));
[v_s3_mid_lat,v_s3_mid_lon] =
polyxpoly(s3_cnt_mid(:,1),s3_cnt_mid(:,2),v_line(:,1),v_line(:,2),'unique');
v_s3_mid = [v_s3_mid_lat,v_s3_mid_lon];

s3_cnt_max = scircle1(s3_lat,s3_lon,km2deg(s3_r_max));

```

```

[v_s3_max_lat,v_s3_max_lon] =
polyxpoly(s3_cnt_max(:,1),s3_cnt_max(:,2),v_line(:,1),v_line(:,2),'unique');
v_s3_max = [v_s3_max_lat,v_s3_max_lon];

fprintf('***** S3 *****\n');

if (v_s3_max ~= '')
    %scenario 3 (crossing only the max contour)
    d_max_max_S3 = distance(v_s3_max_lat(1),v_s3_max_lon(1),v_s3_max_lat(2),v_s3_max_lon(2));
    t_max_max_S3 = (d_max_max_S3 / v_rng)*60;
    d_A_max_S3 = distance(latA,lonA,v_s3_max_lat(1),v_s3_max_lon(1));
    t_A_max_S3 = (d_A_max_S3 / v_rng)*60;
    d_max_B_S3 = distance(v_s3_max_lat(2),v_s3_max_lon(2),latB,lonB);
    t_max_B_S3 = (d_max_B_S3 / v_rng)*60;
    if (v_s3_mid ~= '')
        %scenario 2 (crossing the max and mid contours)
        d_max_mid_S3 = distance(v_s3_max_lat(1),v_s3_max_lon(1),v_s3_mid_lat(1),v_s3_mid_lon(1));
        t_max_mid_S3 = (d_max_mid_S3 / v_rng)*60;
        d_mid_max_S3 = distance(v_s3_mid_lat(2),v_s3_mid_lon(2),v_s3_max_lat(2),v_s3_max_lon(2));
        t_mid_max_S3 = (d_mid_max_S3 / v_rng)*60;
        d_mid_mid_S3 = distance(v_s3_mid_lat(1),v_s3_mid_lon(1),v_s3_mid_lat(2),v_s3_mid_lon(2));
        t_mid_mid_S3 = (d_mid_mid_S3 / v_rng)*60;
        if (v_s3_min ~= '')
            %scenario 1 (crossing all the contours)
            d_mid_min_S3 = distance(v_s3_mid_lat(1),v_s3_mid_lon(1),v_s3_min_lat(1),v_s3_min_lon(1));
            t_mid_min_S3 = (d_mid_min_S3 / v_rng)*60;
            d_min_min_S3 = distance(v_s3_min_lat(1),v_s3_min_lon(1),v_s3_min_lat(2),v_s3_min_lon(2));
            t_min_min_S3 = (d_min_min_S3 / v_rng)*60;
            d_min_mid_S3 = distance(v_s3_min_lat(2),v_s3_min_lon(2),v_s3_mid_lat(2),v_s3_mid_lon(2));
            t_min_mid_S3 = (d_min_mid_S3 / v_rng)*60;
        else
            fprintf('The vessel is out of detection range (no intersection with s3_min).\n');
        end
    else
        fprintf('The vessel is out of detection range (no intersection with s3_mid).\n');
    end
end

else
    fprintf('The vessel is out of detection range (no intersection with s3_max).\n');
end

%calculating the probabilities
%scenario 1 (crossing all contours)
s3_t_det_50 = 0;
s3_t_det_last = 0;
t_last = 0;
c1_s3 = 0;
c2_s3 = 0;
c3_s3 = 0;
c4_s3 = 0;
c5_s3 = 0;
sz1_s3 = 0;
sz2_s3 = 0;
sz3_s3 = 0;
sz4_s3 = 0;
sz5_s3 = 0;

```

```

s3_v_max_max = 0;
s3_v_mid_mid = 0;
s3_v_min_min = 0;
s3_v_max_mid = 0;
s3_v_mid_min = 0;
s3_v_min_mid = 0;
s3_v_mid_max = 0;
s3_v_max_mid_min = 0;

if((t_max_mid_S3 ~=0) && (t_mid_min_S3 ~=0) && (t_min_min_S3 ~=0))
    for i=1:round(t_max_mid_S3)
        detect_num = rand(1);
        if(detect_num <= p_max)
            s3_v_max_mid(i) = true;
            c1_s3 = c1_s3 + 1;
            if(c1_s3==1)
                s3_t_det_50 = i + t_A_max_S3;
                fprintf('Scenario 1: First detection was at t=%g minutes\n',s3_t_det_50);
                [p1_s3_lat,p1_s3_lon] = reckon(v_s3_max_lat(1),v_s3_max_lon(1),((i/60)*v_rng),v_az);
                dist_v_s3 = deg2nm(distance(s3_lat,s3_lon,p1_s3_lat,p1_s3_lon));
            end
        else
            s3_v_max_mid(i) = false;
        end
    end
    sz1_s3=numel(s3_v_max_mid);

    for i=1:round(t_mid_min_S3)
        detect_num = rand(1);
        if(detect_num <= p_mid)
            s3_v_mid_min(i) = true;
            c2_s3 = c2_s3 + 1;
        else
            s3_v_mid_min(i) = false;
        end
    end
    sz2_s3=numel(s3_v_mid_min);

    s3_v_class = 0;
    s3_v_ident = 0;
    for i=1:t_min_min_S3
        detect_num = rand(1);
        if(detect_num <=p_min)
            s3_v_min_min(i) = true;
            c3_s3 = c3_s3 + 1;
            if((s3_v_class == 0) || (s3_v_ident == 0))
                %This is a test to know at what distance the class and ident
                %actually start happening.
                if(s3_v_class ==1)
                    fprintf('The distance from s3 and v for class is %g \n', etc_dist_v_s3);
                end
                if(s3_v_ident ==1)
                    fprintf('The distance from s3 and v for ident is %g \n', etc_dist_v_s3);
                end
            end
        end
    end

```

```

[etc_s3_lat,etc_s3_lon] = reckon
(v_s3_max_lat(1),v_s3_max_lon(1),(((i+t_max_mid_S3+t_mid_min_S3)/60)*v_rng),v_az);
etc_dist_v_s3 = deg2nm(distance(s3_lat,s3_lon,etc_s3_lat,etc_s3_lon));
% fprintf('The distance between sensor 3 and vessel is %g \n', etc_dist_v_s3);
if ((etc_dist_v_s3 <= s3_class) && (s3_v_class == 0))
    s3_v_class = 1;
end
if ((etc_dist_v_s3 <= s3_ident) && (s3_v_ident == 0))
    s3_v_ident = 1;
end
end
else
    s3_v_min_min(i) = false;
end
end
sz3_s3=numel(s3_v_min_min);

for i=1:round(t_min_mid_S3)
    detect_num = rand(1);
    if (detect_num <= p_mid)
        s3_v_min_mid(i) = true;
        c4_s3 = c4_s3 + 1;
    else
        s3_v_min_mid(i) = false;
    end
end
sz4_s3 = numel(s3_v_min_mid);

for i=1:round(t_mid_max_S3)
    detect_num = rand(1);
    if (detect_num <= p_max)
        s3_v_mid_max(i) = true;
        c5_s3 = c5_s3 + 1;
        t_last = i;
    else
        s3_v_mid_max(i) = false;
    end
end
sz5_s3 = numel(s3_v_mid_max);
end
s3_t_det_last = t_last + t_min_mid_S3 + t_min_min_S3 + t_mid_min_S3 + t_max_mid_S3 +
t_A_max_S3;
pdet_s3 = (c1_s3+c2_s3+c3_s3+c4_s3+c5_s3)/(sz1_s3+sz2_s3+sz3_s3+sz4_s3+sz5_s3);
s3_v_max_mid_min =
cat(2,s3_v_max_mid,s3_v_mid_min,s3_v_min_min,s3_v_min_mid,s3_v_mid_max);

%calculating the probabilities
%scenario 2 (crossing max and mid contours)
if((t_max_mid_S3 ~=0) && (t_mid_mid_S3 ~=0) && (t_mid_min_S3 ==0))
    for i=1:round(t_max_mid_S3)
        detect_num = rand(1);
        if (detect_num <= p_max)
            s3_v_max_mid(i) = true;
            c1_s3 = c1_s3 + 1;
            if (c1_s3==1)
                s3_t_det_50 = i + t_A_max_S3;
            end
        end
    end
end

```

```

        fprintf('Scenario 2: First detection was at t=%g minutes\n',s3_t_det_50);
        [p1_s3_lat,p1_s3_lon] = reckon (v_s3_max_lat(1),v_s3_max_lon(1),((i/60)*v_rng),v_az);
        dist_v_s3 = deg2nm(distance(s3_lat,s3_lon,p1_s3_lat,p1_s3_lon));
    end
    else
        s3_v_max_mid(i) = false;
    end
end
sz1_s3=numel(s3_v_max_mid);

for i=1:round(t_mid_mid_S3)
    detect_num = rand(1);
    if (detect_num <= p_mid)
        s3_v_mid_mid(i) = true;
        c2_s3 = c2_s3 + 1;
    else
        s3_v_mid_mid(i) = false;
    end
end
sz2_s3=numel(s3_v_mid_mid);

for i=1:round(t_mid_max_S3)
    if (detect_num <= p_max)
        s3_v_mid_max(i) = true;
        c3_s3 = c3_s3 + 1;
        t_last = i;
    else
        s3_v_mid_max(i) = false;
    end
end
s3_t_det_last = t_last + t_max_mid_S3 + t_mid_mid_S3 + t_A_max_S3;
sz3_s3=numel(s3_v_mid_max);
pdet_s3 = (c1_s3+c2_s3+c3_s3)/(sz1_s3+sz2_s3+sz3_s3);
s3_v_max_mid_min = cat(2,s3_v_max_mid,s3_v_mid_mid,s3_v_mid_max);
end

%calculating the probabilities
%scenario 3 (crossing max only)
if((t_max_max_S3)~=0 && (t_max_mid_S3==0) && (t_mid_min_S3==0))
    for i=1:round(t_max_max_S3)
        detect_num = rand(1);
        if (detect_num <= p_max)
            s3_v_max_max(i) = true;
            c1_s3 = c1_s3 + 1;
            if (c1_s3==1)
                s3_t_det_50 = i + t_A_max_S3;
                fprintf('Scenario 3: First detection was at t=%g minutes\n',s3_t_det_50);
                [p1_s3_lat,p1_s3_lon] = reckon (v_s3_max_lat(1),v_s3_max_lon(1),((i/60)*v_rng),v_az);
                dist_v_s3 = deg2nm(distance(s3_lat,s3_lon,p1_s3_lat,p1_s3_lon));
            end
            t_last = i;
        else
            s3_v_max_max(i) = false;
        end
    end
end
s3_t_det_last = t_last+t_A_max_S3;

```

```

    sz1_s3=numel(s3_v_max_max);
    pdet_s3 = (c1_s3)/(sz1_s3);
    s3_v_max_mid_min = cat(2,s3_v_max_max);
end

%position of the last detection:
%[p2_s3_last,p2_s3_lon] = reckon (latA, lonA,
%((s3_t_det_last/60)*v_rng),v_az);

fprintf('The overall probability of detection rate by S3-EO/IR Radar is %%%g\n',pdet_s3*100);
fprintf('The vessel is detected and tracked %%%g of its trip through the passage.\n',((s3_t_det_last-
s3_t_det_50)/t_total)*100);
pdet_s3_J(J) = pdet_s3*100;
s3_t_det_50_J(J) = s3_t_det_50;
s3_t_det_total_J(J) = ((s3_t_det_last-s3_t_det_50)/t_total)*100;
s3_v_class_J(J) = (s3_v_class);
s3_v_ident_J(J) = (s3_v_ident);
if (s3_v_class_J(J)==1)
    fprintf('The vessel was classified by S3 \n');
end
if (s3_v_ident_J(J)==1)
    fprintf('The vessel was identified by S3 \n');
end
fprintf ('\n');

%*****%
%                               %
%                               %
%*****%

%creating a the contour circles for the area of coverage and intersection of
%each circle with the vessel route.

s4_cnt_min = scircle1(s4_lat,s4_lon,km2deg(s4_r_min));
[v_s4_min_lat,v_s4_min_lon] =
polyxpoly(s4_cnt_min(:,1),s4_cnt_min(:,2),v_line(:,1),v_line(:,2),'unique');
v_s4_min = [v_s4_min_lat,v_s4_min_lon];

s4_cnt_mid = scircle1(s4_lat,s4_lon,km2deg(s4_r_mid));
[v_s4_mid_lat,v_s4_mid_lon] =
polyxpoly(s4_cnt_mid(:,1),s4_cnt_mid(:,2),v_line(:,1),v_line(:,2),'unique');
v_s4_mid = [v_s4_mid_lat,v_s4_mid_lon];

s4_cnt_max = scircle1(s4_lat,s4_lon,km2deg(s4_r_max));
[v_s4_max_lat,v_s4_max_lon] =
polyxpoly(s4_cnt_max(:,1),s4_cnt_max(:,2),v_line(:,1),v_line(:,2),'unique');
v_s4_max = [v_s4_max_lat,v_s4_max_lon];

fprintf('***** S4 *****\n');

if (v_s4_max ~= '')
    %scenario 3 (crossing only the max contour)
    d_max_max_S4 = distance (v_s4_max_lat(1),v_s4_max_lon(1),v_s4_max_lat(2),v_s4_max_lon(2));
    t_max_max_S4 = (d_max_max_S4 / v_rng)*60;
    d_A_max_S4 = distance (latA,lonA,v_s4_max_lat(1),v_s4_max_lon(1));

```

```

t_A_max_S4 = (d_A_max_S4 / v_rng)*60;
d_max_B_S4 = distance (v_s4_max_lat(2),v_s4_max_lon(2),latB,lonB);
t_max_B_S4 = (d_max_B_S4/v_rng)*60;
if (v_s4_mid ~= '')
    %scenario 2 (crossing the max and mid contours)
    d_max_mid_S4 = distance (v_s4_max_lat(1),v_s4_max_lon(1),v_s4_mid_lat(1),v_s4_mid_lon(1));
    t_max_mid_S4 = (d_max_mid_S4 / v_rng)*60;
    d_mid_max_S4 = distance (v_s4_mid_lat(2),v_s4_mid_lon(2),v_s4_max_lat(2),v_s4_max_lon(2));
    t_mid_max_S4 = (d_mid_max_S4 / v_rng)*60;
    d_mid_mid_S4 = distance (v_s4_mid_lat(1),v_s4_mid_lon(1),v_s4_mid_lat(2),v_s4_mid_lon(2));
    t_mid_mid_S4 = (d_mid_mid_S4 / v_rng)*60;
    if (v_s4_min ~= '')
        %scenario 1 (crossing all the contours)
        d_mid_min_S4 = distance (v_s4_mid_lat(1),v_s4_mid_lon(1),v_s4_min_lat(1),v_s4_min_lon(1));
        t_mid_min_S4 = (d_mid_min_S4 / v_rng)*60;
        d_min_min_S4 = distance (v_s4_min_lat(1),v_s4_min_lon(1),v_s4_min_lat(2),v_s4_min_lon(2));
        t_min_min_S4 = (d_min_min_S4 / v_rng)*60;
        d_min_mid_S4 = distance (v_s4_min_lat(2),v_s4_min_lon(2),v_s4_mid_lat(2),v_s4_mid_lon(2));
        t_min_mid_S4 = (d_min_mid_S4 / v_rng)*60;
    else
        fprintf('The vessel is out of detection range (no intersection with s4_min).\n');
    end
else
    fprintf('The vessel is out of detection range (no intersection with s4_mid).\n');
end
else
    fprintf('The vessel is out of detection range (no intersection with s4_max).\n');
end

%calculating the probabilities
%scenario 1 (crossing all contours)
s4_t_det_50 = 0;
s4_t_det_last = 0;
t_last = 0;
c1_s4 = 0;
c2_s4 = 0;
c3_s4 = 0;
c4_s4 = 0;
c5_s4 = 0;
sz1_s4 = 0;
sz2_s4 = 0;
sz3_s4 = 0;
sz4_s4 = 0;
sz5_s4 = 0;
pdet_s4 = 0;

s4_v_max_max = 0;
s4_v_mid_mid = 0;
s4_v_min_min = 0;
s4_v_max_mid = 0;
s4_v_mid_min = 0;
s4_v_min_mid = 0;
s4_v_mid_max = 0;
s4_v_max_A = 0;
s4_v_B_max = 0;
s4_v_max_mid_min = 0;

```

```

s4_v_A_B = 0;

if((t_max_mid_S4 ~=0) && (t_mid_min_S4 ~=0) && (t_min_min_S4 ~=0))
    for i=1:round(t_max_mid_S4)
        detect_num = rand(1);
        if (detect_num <= p_max)
            s4_v_max_mid(i) = true;
            c1_s4 = c1_s4 + 1;
            if (c1_s4==1)
                s4_t_det_50 = i + t_A_max_S4;
                fprintf('Scenario 1: First detection was at t=%g minutes\n',s4_t_det_50);
                [p1_s4_lat,p1_s4_lon] = reckon(v_s4_max_lat(1),v_s4_max_lon(1),((i/60)*v_rng),v_az);
                dist_v_s4 = deg2nm(distance(s4_lat,s4_lon,p1_s4_lat,p1_s4_lon));
            end
        else
            s4_v_max_mid(i) = false;
        end
    end
    sz1_s4=numel(s4_v_max_mid);

    for i=1:round(t_mid_min_S4)
        detect_num = rand(1);
        if (detect_num <= p_mid)
            s4_v_mid_min(i) = true;
            c2_s4 = c2_s4 + 1;
        else
            s4_v_mid_min(i) = false;
        end
    end
    sz2_s4=numel(s4_v_mid_min);

    for i=1:round(t_min_min_S4)
        detect_num = rand(1);
        if (detect_num <= p_min)
            s4_v_min_min(i) = true;
            c3_s4 = c3_s4 + 1;
        else
            s4_v_min_min(i) = false;
        end
    end
    sz3_s4=numel(s4_v_min_min);

    for i=1:round(t_min_mid_S4)
        detect_num = rand(1);
        if (detect_num <= p_mid)
            s4_v_min_mid(i) = true;
            c4_s4 = c4_s4+1;
        else
            s1_v_min_mid(i) = false;
        end
    end
    sz4_s4 = numel(s4_v_min_mid);
end

for i=1:round(t_mid_max_S4)
    detect_num = rand(1);
    if (detect_num <= p_max)

```

```

        s4_v_mid_max(i) = true;
        c5_s4 = c5_s4+1;
        t_last = i;
    else
        s4_v_mid_max(i) = false;
    end
    sz5_s4 = numel(s4_v_mid_max);
end
s4_t_det_last = t_last + t_min_mid_S4 + t_min_min_S4 + t_mid_min_S4 + t_max_mid_S4 +
t_A_max_S4;
pdet_s4 = (c1_s4+c2_s4+c3_s4+c4_s4+c5_s4)/(sz1_s4+sz2_s4+sz3_s4+sz4_s4+sz5_s4);
s4_v_max_mid_min = cat (2, s4_v_max_mid, s4_v_mid_min, s4_v_min_min, s4_v_min_mid,
s4_v_mid_max);
fprintf('the overall probability of detection by S4-UnderWater Radar is %g\n',pdet_s4*100);
end

%calculating the probabilities
%scenario 2 (crossing max and mid contours)
if((t_max_mid_S4 ~=0) && (t_mid_mid_S4 ~=0) && (t_mid_min_S4 ==0))
    for i=1:round(t_max_mid_S4)
        detect_num = rand(1);
        if (detect_num <= p_max)
            s4_v_max_mid(i) = true;
            c1_s4 = c1_s4 + 1;
            if (c1_s4==1)
                s4_t_det_50 = i + t_A_max_S4;
                fprintf('Scenario 2: First detection was at t=%g minutes\n',s4_t_det_50);
                [p1_s4_lat,p1_s4_lon] = reckon (v_s4_max_lat(1),v_s4_max_lon(1),((i/60)*v_rng),v_az);
                dist_v_s4 = deg2nm(distance(s4_lat,s4_lon,p1_s4_lat,p1_s4_lon));
            end
        else
            s4_v_max_mid(i) = false;
        end
    end
    sz1_s4=numel(s4_v_max_mid);

    for i=1:round(t_mid_mid_S4)
        detect_num = rand(1);
        if (detect_num <= p_mid)
            s4_v_mid_mid(i) = true;
            c2_s4 = c2_s4 + 1;
        else
            s4_v_mid_mid(i) = false;
        end
    end
    sz2_s4=numel(s4_v_mid_mid);

    for i=1:round(t_mid_max_S4)
        detect_num = rand(1);
        if (detect_num <= p_max)
            s4_v_mid_max(i) = true;
            c3_s4 = c3_s4 + 1;
            t_last = i;
        else
            s4_v_mid_max(i) = false;
        end
    end
end

```

```

        sz3_s4 = numel(s1_v_mid_max);
    end
    s4_t_det_last = t_last + t_max_mid_S4 + t_mid_mid_S4 + t_A_max_S4;
    pdet_s4 = (c1_s4+c2_s4+c3_s4)/(sz1_s4+sz2_s4+sz3_s4);
    s4_v_max_mid_min = cat(2,s4_v_max_mid,s4_v_mid_mid,s4_v_mid_max);
    %fprintf('the overall probability of detection by S4-UnderWater Radar is %g\n',pdet_s4*100);
end

%calculating the probabilities
%scenario 3 (crossing max only)
if((t_max_max_S4) ~=0 && (t_max_mid_S4 ==0) && (t_mid_min_S4 ==0))
    for i=1:t_max_max_S4
        detect_num = rand(1);
        if (detect_num <= p_max)
            s4_v_max_max(i) = true;
            c1_s4 = c1_s4 + 1;
            if (c1_s4==1)
                s4_t_det_50 = i + t_A_max_S4;
                fprintf('Scenario 3: First detection was at t=%g minutes\n',s4_t_det_50);
                [p1_s4_lat,p1_s4_lon] = reckon(v_s4_max_lat(1),v_s4_max_lon(1),((i/60)*v_rng),v_az);
                dist_v_s4 = deg2nm(distance(s4_lat,s4_lon,p1_s4_lat,p1_s4_lon));
            end
            t_last = i;
        else
            s4_v_max_max(i) = false;
        end
    end
    sz1_s4=numel(s4_v_max_max);
    s4_t_det_last = t_last + t_A_max_S4;
    pdet_s4 = (c1_s4)/(sz1_s4);
    %fprintf('the overall probability of detection by S4-UnderWater Radar is %g\n',pdet_s4*100);
end

%position of last detection
%[p2_s4_last,p2_s4_lon] =
%reckon(latA,latB,((s4_t_det_last/60)*v_rng),v_az);

fprintf('The overall probability of detection rate by S4-UnderWater Radar is %%%g\n',pdet_s4*100);
fprintf('The vessel is detected and tracked %%%g of its trip through the passage.\n\n',((s4_t_det_last-
s4_t_det_50)/t_total)*100);
pdet_s4_J(J) = pdet_s4*100;
s4_t_det_50_J(J) = s4_t_det_50;
s4_t_det_total_J(J) = ((s4_t_det_last-s4_t_det_50)/t_total)*100;
s4_v_class_J(J) = 0;
s4_v_ident_J(J) = 0;

%*****%
%                               S1 & S2                               %
%*****%

t_det_s1_s2 = 0;
s1_s2_t_det = 0;
s1_s2_t_det_last = 0;
A_diff_s1_s2 = 0;
B_diff_s1_s2 = 0;
diff_s1s2 = 0;

```

```

%compare t_A_max for S1 and S2
fprintf('***** S1 & S2 *****\n');
if ((t_A_max_S1 < t_A_max_S2) && (t_A_max_S1 ~=0)) %S1 has bigger rings and S2 is not zero
    A_diff_s1_s2 = abs(fix(t_A_max_S2 - t_A_max_S1));
    padding_A = zeros(1,(A_diff_s1_s2-1));
    s2_v_max_mid_min = cat(2,padding_A,s2_v_max_mid_min);
elseif ((t_A_max_S2 < t_A_max_S1) && (t_A_max_S2 ~=0))
    A_diff_s1_s2 = abs(fix(t_A_max_S1 - t_A_max_S2));
    padding_A = zeros(1,(A_diff_s1_s2-1));
    s1_v_max_mid_min = cat(2,padding_A,s1_v_max_mid_min);
end

if ((t_max_B_S1 < t_max_B_S2) && (t_max_B_S1 ~=0)) %S2 has bigger rings pad S1 with zeros
    B_diff_s1_s2 = abs(fix(t_max_B_S2 - t_max_B_S1));
    padding_B = zeros(1,(B_diff_s1_s2-1));
    s2_v_max_mid_min = cat(2,s2_v_max_mid_min,padding_B);
elseif ((t_max_B_S2 < t_max_B_S1) && (t_max_B_S2 ~=0))
    B_diff_s1_s2 = abs(fix(t_max_B_S1 - t_max_B_S2));
    padding_B = zeros(1,(B_diff_s1_s2-1));
    s1_v_max_mid_min = cat(2,s1_v_max_mid_min,padding_B);
end

%if there is no intersection of the v with a sensor, then that sensors has
%an array of 0 and 1 (size of the array is the size of the array of the
%other sensor with intersection)
if (t_A_max_S1 == 0)
    s1_v_max_mid_min = zeros(1,length(s2_v_max_mid_min));
end
if (t_A_max_S2 == 0)
    s2_v_max_mid_min = zeros(1,length(s1_v_max_mid_min));
end
%compare the size of arrays from S1 and S2
diff_s1s2 = length(s1_v_max_mid_min) - length(s2_v_max_mid_min);

%if one is bigger than the other, pad the smaller one with zeros
padding = zeros(1,abs(diff_s1s2));
if (diff_s1s2 > 0) %s1 array is larger than s2
    s2_v_max_mid_min = cat(2,s2_v_max_mid_min,padding);
else
    s1_v_max_mid_min = cat(2,s1_v_max_mid_min,padding);
end

%use OR function
s1_s2 = s1_v_max_mid_min | s2_v_max_mid_min ;

%calculate detection rate, first detection time (compare det_t for S1 and
%S2, and percentage rate
c_s1_s2 = numel(find(s1_s2));
pdet_s1_s2 = c_s1_s2/(numel(s1_s2));
fprintf('The overall probability rate of detection by S1 and S2 is %g\n',pdet_s1_s2*100);

%first detection time is whichever sensor detects frist (check for 0
%detection)
if ((s1_t_det_50 < s2_t_det_50) && (s1_t_det_50 ~= 0))
    s1_s2_t_det = s1_t_det_50;

```

```

    fprintf('First detection was at t=%g minutes by S1\n',s1_s2_t_det);
else
    s1_s2_t_det = s2_t_det_50;
    fprintf('First detection was at t=%g minutes by S2\n',s1_s2_t_det);
end

%last detection time is whichever sensor detects last (check for 0
%detection)
if((s1_t_det_last > s2_t_det_last))
    s1_s2_det_last = s1_t_det_last;
    fprintf('Last detection was at t=%g minutes by S1\n',s1_s2_det_last);
else
    s1_s2_det_last = s2_t_det_last;
    fprintf('Last detection was at t=%g minutes by S2\n',s1_s2_det_last);
end
t_det_s1_s2 = (s1_s2_det_last - s1_s2_t_det)/t_total;
fprintf('The vessel is detected and tracked %%%g of its trip through the passage by S1 and
S2.\n',t_det_s1_s2*100);
%store results in J
pdet_s1_s2_J(J) = pdet_s1_s2 * 100;
s1_s2_t_det_J(J) = s1_s2_t_det;
s1_s2_t_det_total_J(J) = t_det_s1_s2*100;
s1_s2_v_class_J(J) = (s2_v_class);
s1_s2_v_ident_J(J) = (s2_v_ident);

%*****%
%                S2 & S3                %
%*****%
t_det_s2_s3 = 0;
s2_s3_t_det = 0;
s2_s3_t_det_last = 0;
A_diff_s2_s3 = 0;
B_diff_s2_s3 = 0;
diff_s2s3 = 0;

%compare t_A_max for S2 and S3
fprintf('***** S2 & S3 *****\n');
if((t_A_max_S2 < t_A_max_S3) && (t_A_max_S2 ~=0)) %S2 has bigger rings and S2 is not zero
    A_diff_s2_s3 = (fix(t_A_max_S3 - t_A_max_S2));
    padding_A = zeros(1,(A_diff_s2_s3));
    s3_v_max_mid_min = cat(2,padding_A,s3_v_max_mid_min);
elseif((t_A_max_S3 < t_A_max_S2) && (t_A_max_S3 ~=0))
    A_diff_s2_s3 = (fix(t_A_max_S2 - t_A_max_S3));
    padding_A = zeros(1,(A_diff_s2_s3));
    s2_v_max_mid_min = cat(2,padding_A,s2_v_max_mid_min);
end

if((t_max_B_S2 < t_max_B_S3) && (t_max_B_S2 ~=0)) %S2 has bigger rings pad S1 with zeros
    B_diff_s2_s3 = abs(fix(t_max_B_S3 - t_max_B_S2));
    padding_B = zeros(1,(B_diff_s2_s3-1));
    s3_v_max_mid_min = cat(2,s3_v_max_mid_min,padding_B);
elseif((t_max_B_S3 < t_max_B_S2) && (t_max_B_S3 ~=0))
    B_diff_s2_s3 = abs(fix(t_max_B_S2 - t_max_B_S3));
    padding_B = zeros(1,(B_diff_s2_s3-1));
    s2_v_max_mid_min = cat(2,s2_v_max_mid_min,padding_B);

```

```

end

%if there is no intersection of the v with a sensor, then that sensor has
%an array of 0 (size of the array is the size of the array of the
%other sensor with intersection)
if (t_A_max_S2 == 0)
    s2_v_max_mid_min = zeros(1,length(s3_v_max_mid_min));
end
if (t_A_max_S3 == 0)
    s3_v_max_mid_min = zeros (1,length (s2_v_max_mid_min));
end
%compare the size of arrays from S2 and S3
diff_s2s3 = length (s2_v_max_mid_min) - length (s3_v_max_mid_min);

%if one is bigger than the other, pad the smaller one with zeros
padding = zeros(1,abs(diff_s2s3));
if (diff_s2s3 > 0) %s1 array is larger than s2
    s3_v_max_mid_min = cat(2,s3_v_max_mid_min,padding);
else
    s2_v_max_mid_min = cat(2,s2_v_max_mid_min,padding);
end

%use OR function
s2_s3 = s2_v_max_mid_min | s3_v_max_mid_min ;

%calculate detection rate, first detection time (compare det_t for S2 and
%S3, and percentage rate
c_s2_s3 = numel(find(s2_s3));
pdet_s2_s3 = c_s2_s3/(numel(s2_s3));
fprintf('The overall probability rate of detection by S2 and S3 is %%%g\n',pdet_s2_s3*100);

%first detection time is whichever sensor detects first (check for 0
%detection)
if ((s2_t_det_50 < s3_t_det_50) && (s2_t_det_50 ~= 0))
    s2_s3_t_det = s2_t_det_50;
    fprintf('First detection was at t=%g minutes by S2\n',s2_s3_t_det);
else
    s2_s3_t_det = s3_t_det_50;
    fprintf('First detection was at t=%g minutes by S3\n',s2_s3_t_det);
end

%last detection time is whichever sensor detects last (check for 0
%detection)
if ((s2_t_det_last > s3_t_det_last))
    s2_s3_det_last = s2_t_det_last;
    fprintf('Last detection was at t=%g minutes by S2\n',s2_s3_det_last);
else
    s2_s3_det_last = s3_t_det_last;
    fprintf('Last detection was at t=%g minutes by S3\n',s2_s3_det_last);
end
t_det_s2_s3 = (s2_s3_det_last - s2_s3_t_det)/t_total;
fprintf('The vessel is detected and tracked %%%g of its trip through the passage by S2 and
S3.\n',t_det_s2_s3*100);
%store results in J
pdet_s2_s3_J(J)=pdet_s2_s3 *100;

```

```

s2_s3_t_det_J(J) = s2_s3_t_det;
s2_s3_t_det_total_J(J) = t_det_s2_s3*100;
s2_s3_v_class_J(J) = (s2_v_class);
if (s2_s3_v_class_J(J) == 1)
    fprintf('The vessel was classified by S2.\n');
end
s2_s3_v_ident_J(J) = (s2_v_ident);
if (s2_s3_v_ident_J(J) == 1)
    fprintf('The vessel was identified by S2.\n');
end

%*****%
%                S2 & S4                %
%*****%

t_det_s2_s4 = 0;
s2_s4_t_det = 0;
s2_s4_t_det_last = 0;
A_diff_s2_s4 = 0;
B_diff_s2_s4 = 0;
diff_s2s4 = 0;

%compare t_A_max for S2 and S4
fprintf('***** S2 & S4 *****\n');
if ((t_A_max_S2 < t_A_max_S4) && (t_A_max_S2 ~= 0)) %S2 has bigger rings and S4 is not zero
    A_diff_s2_s4 = (fix(t_A_max_S4 - t_A_max_S2));
    padding_A = zeros(1,(A_diff_s2_s4));
    s4_v_max_mid_min = cat(2,padding_A,s4_v_max_mid_min);
elseif ((t_A_max_S4 < t_A_max_S2) && (t_A_max_S4 ~= 0))
    A_diff_s2_s4 = (fix(t_A_max_S2 - t_A_max_S4));
    padding_A = zeros(1,(A_diff_s2_s4));
    s2_v_max_mid_min = cat(2,padding_A,s2_v_max_mid_min);
end

if ((t_max_B_S2 < t_max_B_S4) && (t_max_B_S2 ~= 0)) %S4 has bigger rings pad S2 with zeros
    B_diff_s2_s4 = (fix(t_max_B_S4 - t_max_B_S2));
    padding_B = zeros(1,(B_diff_s2_s4));
    s4_v_max_mid_min = cat(2,s4_v_max_mid_min,padding_B);
elseif ((t_max_B_S4 < t_max_B_S2) && (t_max_B_S4 ~= 0))
    B_diff_s2_s4 = (fix(t_max_B_S2 - t_max_B_S4));
    padding_B = zeros(1,(B_diff_s2_s4));
    s2_v_max_mid_min = cat(2,s2_v_max_mid_min,padding_B);
end

%if there is no intersection of the v with a sensor, then that sensor has
%an array of 0 and 1 (size of the array is the size of the array of the
%other sensor with intersection)
if (t_A_max_S2 == 0)
    s2_v_max_mid_min = zeros(1,length(s4_v_max_mid_min));
end
if (t_A_max_S4 == 0)
    s4_v_max_mid_min = zeros(1,length(s2_v_max_mid_min));
end
%compare the size of arrays from S2 and S4
diff_s2s4 = length(s2_v_max_mid_min) - length(s4_v_max_mid_min);

%if one is bigger than the other, pad the smaller one with zeros

```

```

padding = zeros(1,abs(diff_s2s4));
if (diff_s2s4 > 0) %s1 array is larger than s2
    s4_v_max_mid_min = cat(2,s4_v_max_mid_min,padding);
else
    s2_v_max_mid_min = cat(2,s2_v_max_mid_min,padding);
end

%use OR function
s2_s4 = s2_v_max_mid_min | s4_v_max_mid_min ;

%calculate detection rate, first detection time (compare det_t for S2 and
%S4, and percentage rate
c_s2_s4 = numel(find(s2_s4));
pdet_s2_s4 = c_s2_s4/(numel(s2_s4));
fprintf('The overall probability rate of detection by S2 and S4 is %%%g\n',pdet_s2_s4*100);

%first detection time is whichever sensor detects first (check for 0
%detection)
if ((s2_t_det_50 < s4_t_det_50) && (s2_t_det_50 ~= 0))
    s2_s4_t_det = s2_t_det_50;
    fprintf('First detection was at t=%g minutes by S2\n',s2_s4_t_det);
else
    s2_s4_t_det = s4_t_det_50;
    fprintf('First detection was at t=%g minutes by S4\n',s2_s4_t_det);
end

%last detection time is whichever sensor detects last (check for 0
%detection)
if ((s2_t_det_last > s4_t_det_last))
    s2_s4_det_last = s2_t_det_last;
    fprintf('Last detection was at t=%g minutes by S2\n',s2_s4_det_last);
else
    s2_s4_det_last = s4_t_det_last;
    fprintf('Last detection was at t=%g minutes by S4\n',s2_s4_det_last);
end
t_det_s2_s4 = (s2_s4_det_last - s2_s4_t_det)/t_total;
fprintf('The vessel is detected and tracked %%%g of its trip through the passage by S2 and
S4.\n',t_det_s2_s4*100);
%store results in J
pdet_s2_s4_J(J)=pdet_s2_s4 *100;
s2_s4_t_det_J(J) = s2_s4_t_det;
s2_s4_t_det_total_J(J) = t_det_s2_s4*100;
s2_s4_v_class_J(J) = (s2_v_class);
if (s2_s4_v_class_J(J)==1)
    fprintf('The vessel was classified by S2. \n');
end
s2_s4_v_ident_J(J) = (s2_v_ident);
if (s2_s4_v_ident_J(J)==1)
    fprintf('The vessel was identified by S2. \n');
end

%*****%
%                S1 & S3                %
%*****%
t_det_s1_s3 = 0;
s1_s3_t_det = 0;

```

```

s1_s3_t_det_last = 0;
A_diff_s1_s3 = 0;
B_diff_s1_s3 = 0;
diff_s1s3 = 0;

%compare t_A_max for S1 and S3
fprintf('***** S1 & S3 *****\n');
%from A to max
if ((t_A_max_S1 < t_A_max_S3) && (t_A_max_S1 ~= 0)) %S1 has bigger rings and S1 is not zero
    A_diff_s1_s3 = (fix(t_A_max_S3 - t_A_max_S1));
    padding_A = zeros(1,(A_diff_s1_s3));
    s3_v_max_mid_min = cat(2,padding_A,s3_v_max_mid_min);
elseif ((t_A_max_S3 < t_A_max_S1) && (t_A_max_S3 ~= 0))
    A_diff_s1_s3 = (fix(t_A_max_S1 - t_A_max_S3));
    padding_A = zeros(1,(A_diff_s1_s3));
    s1_v_max_mid_min = cat(2,padding_A,s1_v_max_mid_min);
end

%from max to B
if ((t_max_B_S1 < t_max_B_S3) && (t_max_B_S1 ~= 0))
    B_diff_s1_s3 = (fix(t_max_B_S3 - t_max_B_S1));
    padding_B = zeros(1,(B_diff_s1_s3));
    s3_v_max_mid_min = cat(2,s3_v_max_mid_min,padding_B);
elseif ((t_max_B_S3 < t_max_B_S1) && (t_max_B_S3 ~= 0))
    B_diff_s1_s3 = (fix(t_max_B_S1 - t_max_B_S3));
    padding_B = zeros(1,(B_diff_s1_s3));
    s1_v_max_mid_min = cat(2,s1_v_max_mid_min,padding_B);
end

%if there is no intersection of the v with a sensor, then that sensor has
%an array of 0 and 1 (size of the array is the size of the array of the
%other sensor with intersection)
if (t_A_max_S1 == 0)
    s1_v_max_mid_min = zeros(1,length(s3_v_max_mid_min));
end
if (t_A_max_S3 == 0)
    s3_v_max_mid_min = zeros(1,length(s1_v_max_mid_min));
end
%compare the size of arrays from S1 and S3
diff_s1s3 = length(s1_v_max_mid_min) - length(s3_v_max_mid_min);

%if one is bigger than the other, pad the smaller one with zeros
padding = zeros(1,abs(diff_s1s3));
if (diff_s1s3 > 0) %s1 array is larger than s3
    s3_v_max_mid_min = cat(2,s3_v_max_mid_min,padding);
else
    s1_v_max_mid_min = cat(2,s1_v_max_mid_min,padding);
end

%use OR function
s1_s3 = s1_v_max_mid_min | s3_v_max_mid_min ;

%calculate detection rate, first detection time (compare det_t for S1 and
%S3, and percentage rate
c_s1_s3 = numel(find(s1_s3));

```

```

pdet_s1_s3 = c_s1_s3/(numel(s1_s3));
fprintf('The overall probability rate of detection by S1 and S3 is %%%g\n',pdet_s1_s3*100);

%first detection time is whichever sensor detects first (check for 0
%detection)
if ((s1_t_det_50 < s3_t_det_50) && (s1_t_det_50 ~= 0))
    s1_s3_t_det = s1_t_det_50;
    fprintf('First detection was at t=%g minutes by S1\n',s1_s3_t_det);
else
    s1_s3_t_det = s3_t_det_50;
    fprintf('First detection was at t=%g minutes by S3\n',s1_s3_t_det);
end

%last detection time is whichever sensor detects last (check for 0
%detection)
if ((s1_t_det_last > s3_t_det_last))
    s1_s3_det_last = s1_t_det_last;
    fprintf('Last detection was at t=%g minutes by S1\n',s1_s3_det_last);
else
    s1_s3_det_last = s3_t_det_last;
    fprintf('Last detection was at t=%g minutes by S3\n',s1_s3_det_last);
end
t_det_s1_s3 = (s1_s3_det_last - s1_s3_t_det)/t_total;
fprintf('The vessel is detected and tracked %%%g of its trip through the passage by S1 and
S3.\n',t_det_s1_s3*100);

%store results in J
pdet_s1_s3_J(J) = pdet_s1_s3 * 100;
s1_s3_t_det_J(J) = s1_s3_t_det;
s1_s3_t_det_total_J(J) = t_det_s1_s3*100;
s1_s3_v_class_J(J) = (s3_v_class);
if (s1_s3_v_class_J(J) == 1)
    fprintf('The vessel was classified by S3. \n');
end
s1_s3_v_ident_J(J) = (s3_v_ident);
if (s1_s3_v_ident_J(J) == 1)
    fprintf('The vessel was identified by S3. \n');
end

%*****%
%          S1 & S2 & S4          %
%*****%

t_det_s1_s2_s4 = 0;
s1_s2_s4_t_det = 0;
s1_s2_s4_t_det_last = 0;
A_diff_s1_s2 = 0;
A_diff_s1_s4 = 0;
A_diff_s2_s4 = 0;
B_diff_s1_s2 = 0;
B_diff_s1_s4 = 0;
B_diff_s2_s4 = 0;
c = 0;
d = 0;

%compare t_A_max for S1, S2 and S4
fprintf('***** S1 & S2 & S4 *****\n');

```

```

%from A to max
%S1 has the largest rings
%S2 is zero
%S4 is zero
%both S2 and S4 are zero
if((t_A_max_S1 < t_A_max_S2) && (t_A_max_S1 < t_A_max_S4) && (t_A_max_S1 ~=0)) ||
((t_A_max_S2 == 0) && (t_A_max_S1 < t_A_max_S2)) || ((t_A_max_S4 == 0) && (t_A_max_S1 <
t_A_max_S2)) || ((t_A_max_S2 ==0) && (t_A_max_S4 ==0))
    A_diff_s1_s2 = (fix(t_A_max_S2 - t_A_max_S1));
    A_diff_s1_s4 = (fix(t_A_max_S4 - t_A_max_S1));
    padding_A_s2 = zeros(1,abs(A_diff_s1_s2));
    padding_A_s4 = zeros(1,abs(A_diff_s1_s4));
    s2_v_max_mid_min = cat(2,padding_A_s2,s2_v_max_mid_min);
    s4_v_max_mid_min = cat(2,padding_A_s4,s4_v_max_mid_min);
elseif((t_A_max_S1 < t_A_max_S2) && (t_A_max_S4 < t_A_max_S1) && (t_A_max_S4 ~=0)) ||
((t_A_max_S2 < t_A_max_S1) && (t_A_max_S4 < t_A_max_S2) && (t_A_max_S4 ~=0)) ||
((t_A_max_S1 == 0) && (t_A_max_S4 < t_A_max_S2)) || ((t_A_max_S2 == 0) && (t_A_max_S4 <
t_A_max_S1)) || ((t_A_max_S1 == 0) && (t_A_max_S2 == 0))
    A_diff_s1_s4 = (fix(t_A_max_S1 - t_A_max_S4));
    A_diff_s2_s4 = (fix(t_A_max_S2 - t_A_max_S4));
    padding_A_s1 = zeros(1,abs(A_diff_s1_s4));
    padding_A_s2 = zeros(1,abs(A_diff_s2_s4));
    s1_v_max_mid_min = cat(2,padding_A_s1,s1_v_max_mid_min);
    s2_v_max_mid_min = cat(2,padding_A_s2,s2_v_max_mid_min);
elseif((t_A_max_S2 < t_A_max_S1) && (t_A_max_S2 < t_A_max_S4) && (t_A_max_S2 ~=0)) ||
((t_A_max_S1 ==0) && (t_A_max_S2 < t_A_max_S4)) || ((t_A_max_S4 ==0) && (t_A_max_S2 <
t_A_max_S1)) || ((t_A_max_S1 ==0) && (t_A_max_S4 ==0))
    A_diff_s1_s2 = (fix(t_A_max_S1 - t_A_max_S2));
    A_diff_s2_s4 = (fix(t_A_max_S4 - t_A_max_S2));
    padding_A_s1 = zeros(1,abs(A_diff_s1_s2));
    padding_A_s4 = zeros(1,abs(A_diff_s2_s4));
    s1_v_max_mid_min = cat(2,padding_A_s1,s1_v_max_mid_min);
    s4_v_max_mid_min = cat(2,padding_A_s4,s4_v_max_mid_min);
end

%from max to B
if((t_max_B_S1 < t_max_B_S2) && (t_max_B_S1 < t_max_B_S4) && (t_max_B_S1 ~=0)) ||
((t_max_B_S2 == 0) && (t_max_B_S1 < t_max_B_S2)) || ((t_max_B_S4 == 0) && (t_max_B_S1 <
t_max_B_S2)) || ((t_max_B_S2 ==0) && (t_max_B_S4 ==0))
    B_diff_s1_s2 = (fix(t_max_B_S2 - t_max_B_S1));
    B_diff_s1_s4 = (fix(t_max_B_S4 - t_max_B_S1));
    padding_B_s2 = zeros(1,abs(B_diff_s1_s2));
    padding_B_s4 = zeros(1,abs(B_diff_s1_s4));
    s2_v_max_mid_min = cat(2,s2_v_max_mid_min,padding_B_s2);
    s4_v_max_mid_min = cat(2,s4_v_max_mid_min,padding_B_s4);
elseif((t_max_B_S1 < t_max_B_S2) && (t_max_B_S4 < t_max_B_S1)) || ((t_max_B_S2 < t_max_B_S1)
&& (t_max_B_S4 < t_max_B_S2)) || ((t_max_B_S1 == 0) && (t_max_B_S4 < t_max_B_S2)) ||
((t_max_B_S2 == 0) && (t_max_B_S4 < t_max_B_S1)) || ((t_max_B_S1 == 0) && (t_max_B_S2 == 0))
    B_diff_s1_s4 = (fix(t_max_B_S1 - t_max_B_S4));
    B_diff_s2_s4 = (fix(t_max_B_S2 - t_max_B_S4));
    padding_B_s1 = zeros(1,abs(B_diff_s1_s4));
    padding_B_s4 = zeros(1,abs(B_diff_s2_s4));
    s1_v_max_mid_min = cat(2,s1_v_max_mid_min,padding_B_s1);
    s2_v_max_mid_min = cat(2,s2_v_max_mid_min,padding_B_s4);

```

```

elseif((t_max_B_S2 < t_max_B_S1) && (t_max_B_S2 < t_max_B_S4)) || ((t_max_B_S1 == 0) &&
(t_max_B_S2 < t_max_B_S4)) || ((t_max_B_S4 == 0) && (t_max_B_S2 < t_max_B_S1)) || ((t_max_B_S1
== 0) && (t_max_B_S4 == 0))
    B_diff_s1_s2 = (fix(t_max_B_S1 - t_max_B_S2));
    B_diff_s2_s4 = (fix(t_max_B_S4 - t_max_B_S2));
    padding_B_s1 = zeros(1,abs(B_diff_s1_s2));
    padding_B_s4 = zeros(1,abs(B_diff_s2_s4));
    s1_v_max_mid_min = cat(2,s1_v_max_mid_min,padding_B_s1);
    s4_v_max_mid_min = cat(2,s4_v_max_mid_min,padding_B_s4);
end

%compare the size of arrays from S1 and S2 and S4
%find the largest array and add the difference to the smaller one
c = max (length(s1_v_max_mid_min),length(s2_v_max_mid_min));
d = max (c, length(s4_v_max_mid_min));

if (length (s1_v_max_mid_min)< d )
    padding = zeros(1,(d - length (s1_v_max_mid_min)));
    s1_v_max_mid_min = cat(2,s1_v_max_mid_min,padding);
end
if (length (s2_v_max_mid_min)< d )
    padding = zeros(1,(d - length (s2_v_max_mid_min)));
    s2_v_max_mid_min = cat(2,s2_v_max_mid_min,padding);
end
if (length (s4_v_max_mid_min)< d )
    padding = zeros(1,(d - length (s4_v_max_mid_min)));
    s4_v_max_mid_min = cat(2,s4_v_max_mid_min,padding);
end

%use OR function
s1_s2_s4 = s1_v_max_mid_min | s2_v_max_mid_min | s4_v_max_mid_min;

%calculate detection rate, first detection time
c_s1_s2_s4 = numel(find(s1_s2_s4));
pdet_s1_s2_s4 = c_s1_s2_s4/(numel(s1_s2_s4));
fprintf('The overall probability rate of detection by S1, S2 and S3 is %%%g\n',pdet_s1_s2_s4*100);

%first detection time is whichever sensor detects first (check for 0
%detection)
if ((s1_t_det_50 < s2_t_det_50) && (s1_t_det_50 < s4_t_det_50) && (s1_t_det_50 ~= 0))
    s1_s2_s4_t_det = s1_t_det_50;
    fprintf('First detection was at t=%g minutes by S1\n',s1_s2_s4_t_det);
elseif ((s2_t_det_50 < s1_t_det_50) && (s2_t_det_50 < s4_t_det_50) && (s2_t_det_50 ~= 0))
    s1_s2_s4_t_det = s2_t_det_50;
    fprintf('First detection was at t=%g minutes by S2\n',s1_s2_s4_t_det);
elseif ((s4_t_det_50 < s1_t_det_50) && (s4_t_det_50 < s2_t_det_50) && (s4_t_det_50 ~= 0))
    s1_s2_s4_t_det = s4_t_det_50;
    fprintf('First detection was at t=%g minutes by S4\n',s1_s2_s4_t_det);
elseif (s1_t_det_50 == 0)
    if (s2_t_det_50 < s4_t_det_50)
        s1_s2_s4_t_det = s2_t_det_50;
        fprintf('First detection was at t=%g minutes by S2\n',s1_s2_s4_t_det);
    else
        s1_s2_s4_t_det = s4_t_det_50;
        fprintf('First detection was at t=%g minutes by S4\n',s1_s2_s4_t_det);
    end
end

```

```

elseif (s2_t_det_50 == 0)
    if (s1_t_det_50 < s4_t_det_50)
        s1_s2_s4_t_det = s1_t_det_50;
        fprintf('First detection was at t=%g minutes by S1\n',s1_s2_s4_t_det);
    else
        s1_s2_s4_t_det = s4_t_det_50;
        fprintf('First detection was at t=%g minutes by S4\n',s1_s2_s4_t_det);
    end
elseif (s4_t_det_50 == 0)
    if (s1_t_det_50 < s2_t_det_50)
        s1_s2_s4_t_det = s1_t_det_50;
        fprintf('First detection was at t=%g minutes by S1\n',s1_s2_s4_t_det);
    else
        s1_s2_s4_t_det = s4_t_det_50;
        fprintf('First detection was at t=%g minutes by S4\n',s1_s2_s4_t_det);
    end
end

%last detection time is whichever sensor detects last
if ((s1_t_det_last > s2_t_det_last) && (s1_t_det_last > s4_t_det_last))
    s1_s2_s4_det_last = s1_t_det_last;
    fprintf('Last detection was at t=%g minutes by S1\n',s1_s2_s4_det_last);
elseif ((s2_t_det_last > s1_t_det_last) && (s2_t_det_last > s4_t_det_last))
    s1_s2_s4_det_last = s2_t_det_last;
    fprintf('Last detection was at t=%g minutes by S2\n',s1_s2_s4_det_last);
elseif ((s4_t_det_last > s1_t_det_last) && (s4_t_det_last > s2_t_det_last))
    s1_s2_s4_det_last = s4_t_det_last;
    fprintf('Last detection was at t=%g minutes by S4\n',s1_s2_s4_det_last);
end
t_det_s1_s2_s4 = (s1_s2_s4_det_last - s1_s2_s4_t_det)/t_total;
fprintf('The vessel is detected and tracked %%%g of its trip through the passage by S1, S2, and
S4.\n',t_det_s1_s2_s4*100);

%store results in J
pdet_s1_s2_s4_J(J) = pdet_s1_s2_s4 * 100;
s1_s2_s4_t_det_J(J) = s1_s2_s4_t_det;
s1_s2_s4_t_det_total_J(J) = t_det_s1_s2_s4 * 100;
s1_s2_s4_v_class_J(J) = (s2_v_class);
if (s1_s2_s4_v_class_J(J) == 1)
    fprintf('The vessel was classified by S2. \n');
end
s1_s2_s4_v_ident_J(J) = (s2_v_ident);
if (s1_s2_s4_v_ident_J(J) == 1)
    fprintf('The vessel was identified by S2. \n');
end

end %End of simulation (J)

%write the results in excel
pdet =
[pdet_s1_J;pdet_s2_J;pdet_s3_J;pdet_s4_J;pdet_s1_s2_J;pdet_s2_s3_J;pdet_s2_s4_J;pdet_s1_s3_J;pdet_s
1_s2_s4_J];
t_det =
[s1_t_det_50_J;s2_t_det_50_J;s3_t_det_50_J;s4_t_det_50_J;s1_s2_t_det_J;s2_s3_t_det_J;s2_s4_t_det_J;s
1_s3_t_det_J;s1_s2_s4_t_det_J];

```

```

t_total =
[s1_t_det_total_J;s2_t_det_total_J;s3_t_det_total_J;s4_t_det_total_J;s1_s2_t_det_total_J;s2_s3_t_det_total
_J;s2_s4_t_det_total_J;s1_s3_t_det_total_J;s1_s2_s4_t_det_total_J];
%xlswrite('pdet.xls', pdet);
%xlswrite('t_det.xls', t_det);
%xlswrite('t_total.xls', t_total);

%%statistical analysis on the data

%*****mean*****%

%detection rate
mean_pdet_s1 = mean (pdet_s1_J);
mean_pdet_s2 = mean (pdet_s2_J);
mean_pdet_s3 = mean (pdet_s3_J);
mean_pdet_s4 = mean (pdet_s4_J);
mean_pdet_s1_s2 = mean (pdet_s1_s2_J);
mean_pdet_s2_s3 = mean (pdet_s2_s3_J);
mean_pdet_s2_s4 = mean (pdet_s2_s4_J);
mean_pdet_s1_s3 = mean (pdet_s1_s3_J);
mean_pdet_s1_s2_s4 = mean (pdet_s1_s2_s4_J);
mean_pdet =
[mean_pdet_s1;mean_pdet_s2;mean_pdet_s3;mean_pdet_s4;mean_pdet_s1_s2;mean_pdet_s2_s3;mean_pdet_s
et_s2_s4;mean_pdet_s1_s3;mean_pdet_s1_s2_s4];

%first detection time
mean_t_det_s1 = mean(s1_t_det_50_J);
mean_t_det_s2 = mean(s2_t_det_50_J);
mean_t_det_s3 = mean(s3_t_det_50_J);
mean_t_det_s4 = mean(s4_t_det_50_J);
mean_t_det_s1_s2 = mean(s1_s2_t_det_J);
mean_t_det_s2_s3 = mean(s2_s3_t_det_J);
mean_t_det_s2_s4 = mean(s2_s4_t_det_J);
mean_t_det_s1_s3 = mean(s1_s3_t_det_J);
mean_t_det_s1_s2_s4 = mean(s1_s2_s4_t_det_J);
mean_t_det =
[mean_t_det_s1;mean_t_det_s2;mean_t_det_s3;mean_t_det_s4;mean_t_det_s1_s2;mean_t_det_s2_s3;mea
n_t_det_s2_s4;mean_t_det_s1_s3;mean_t_det_s1_s2_s4];

%boxplot
%h=boxplot
([pdet_s1_J;pdet_s2_J;pdet_s3_J;pdet_s4_J;pdet_s1_s2_J;pdet_s2_s3_J;pdet_s2_s4_J;pdet_s1_s3_J;pdet_s
1_s2_s4_J], 'notch', 'on');
%h=subplot(2,1,1);
%subplot(2,1,1);
%p= prctile (pdet_s1_J,[5 25 75 90]);
%plot (p)
subplot(2,2,1);
plot(pdet_s1_J,'--bs');title('pdet S1');
subplot(2,2,2);
plot(pdet_s2_J,'--rs');title('pdet S2');
subplot(2,2,3);
plot(pdet_s3_J,'--blacks');title('pdet S3');
subplot(2,2,4);
plot(pdet_s4_J,'--ys');title('pdet S4');

```

```

%circles for the map - S1
s1_location_min= scircle1(s1_lat,s1_lon,km2deg(s1_r_min));
s1_location_mid= scircle1(s1_lat,s1_lon,km2deg(s1_r_mid));
s1_location_max= scircle1(s1_lat,s1_lon,km2deg(s1_r_max));

%circles for the map - S2
s2_location_min= scircle1(s2_lat,s2_lon,km2deg(s2_r_min));
s2_location_mid= scircle1(s2_lat,s2_lon,km2deg(s2_r_mid));
s2_location_max= scircle1(s2_lat,s2_lon,km2deg(s2_r_max));

%circles for the map - S3
s3_location_min= scircle1(s3_lat,s3_lon,km2deg(s3_r_min));
s3_location_mid= scircle1(s3_lat,s3_lon,km2deg(s3_r_mid));
s3_location_max= scircle1(s3_lat,s3_lon,km2deg(s3_r_max));

%circles for the map - S4
s4_location_min= scircle1(s4_lat,s4_lon,km2deg(s4_r_min));
s4_location_mid= scircle1(s4_lat,s4_lon,km2deg(s4_r_mid));
s4_location_max= scircle1(s4_lat,s4_lon,km2deg(s4_r_max));

%*****MAP*****%
%return map specific data
latlim = [70 80];
lonlim = [-115 -77];
%h=worldmap('Canada');
h=worldmap(latlim,lonlim);

load coast;
land = shaperead('landareas', 'UseGeoCoords', true);
geoshow(h,land,'FaceColor',[0.5 0.7 0.5]);
plotm(lat, long);
sum (isnan(lat));

linem([latA latB],[lonA lonB]);
textm(latA,lonA,' A ','fontweight','bold', 'color','blue');
textm(latB,lonB,' B ','fontweight','bold', 'color','blue');
%textm(s3_lat,s3_lon,' S ','fontweight','bold', 'color','black');
%textm(p1_s3_lat,p1_s3_lon,'X','fontweight','bold', 'color','black');

% %*****%
% % S1 %
% %*****%
% plotm(s1_location_mid,'yellow');
% plotm(s1_location_min,'yellow');
% plotm(s1_location_max,'yellow');
% % if (v_s1_max~=')
% % plotm(v_s1_max(:,1),v_s1_max(:,2),'blue');
% % end
% % if (v_s1_mid~=')
% % plotm(v_s1_mid(:,1),v_s1_mid(:,2),'b');
% % end
% % if (v_s1_min~=')
% % plotm(v_s1_min(:,1),v_s1_min(:,2),'b');

```

```

%% %% end
% textm(s1_lat,s1_lon,' S1 ','fontweight','bold', 'color','black');
% %textm(p1_s1_lat,p1_s1_lon,'X','fontweight','bold', 'color','red');
% %textm(p2_s1_lat,p2_s1_lon,'Y','fontweight','bold', 'color','blue');

%*****%
%                               S2                               %
%*****%
plotm(s2_location_mid,'red');
plotm(s2_location_min,'red');
plotm(s2_location_max,'red');
% if (v_s2_max~=')
%   plotm(v_s2_max(:,1),v_s2_max(:,2),'blue');
% end
% if (v_s2_mid~=')
%   plotm(v_s2_mid(:,1),v_s2_mid(:,2),'b');
% end
% if (v_s2_min~=')
%   plotm(v_s2_min(:,1),v_s2_min(:,2),'b');
% end
textm(s2_lat,s2_lon,' S2 ','fontweight','bold', 'color','black');
%textm(p1_s2_lat,p1_s2_lon,'X','fontweight','bold', 'color','red');

% %*****%
% %                               S3                               %
% %*****%
% plotm(s3_location_mid,'black');
% plotm(s3_location_min,'black');
% plotm(s3_location_max,'black');
% % if (v_s3_max~=')
% %   plotm(v_s3_max(:,1),v_s3_max(:,2),'blue');
% % end
% % if (v_s3_mid~=')
% %   plotm(v_s3_mid(:,1),v_s3_mid(:,2),'b');
% % end
% % if (v_s3_min~=')
% %   plotm(v_s3_min(:,1),v_s3_min(:,2),'b');
% % end
% textm(s3_lat,s3_lon,' S3 ','fontweight','bold', 'color','black');
% %textm(p1_s3_lat,p1_s3_lon,'X','fontweight','bold', 'color','red');

%*****%
%                               S4                               %
%*****%
plotm(s4_location_mid,'black');
plotm(s4_location_min,'black');
plotm(s4_location_max,'black');
% if (v_s4_max~=')
%   plotm(v_s4_max(:,1),v_s4_max(:,2),'blue');
% end
% if (v_s4_mid~=')
%   plotm(v_s4_mid(:,1),v_s4_mid(:,2),'b');
% end
% if (v_s4_min~=')
%   plotm(v_s4_min(:,1),v_s4_min(:,2),'b');
% end

```

```
textm(s4_lat,s4_lon,' S4 ','fontweight','bold', 'color','black');  
%textm(p1_s4_lat,p1_s4_lon,'X','fontweight','bold', 'color','black');  
hold off;
```

Appendix C – Simulation results

This appendix contains the results of the run of 100 simulation trials upon which all of the analysis in this report are based. The information is presented in the form of tables containing the relevant MOP statistics for each trial. Each column represents one of the hundred trials. Each row represents the value of that statistic using the sensor indicated by the row (i.e. a detection rate of 93.67 for S2) for the vessel type and weather conditions indicated in the column heading (e.g. “ideal” weather and “icebreaker” vessel type).

weather	'ideal'	'ideal'	'ideal'	'poor'	'ideal'	'poor'	'ideal'	'poor'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'
vessel	'icebreaker'	'fishing boø'	'icebreaker'	'fishing boø'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'
s1	93.02	77.78	67.79	0.00	93.87	75.16	71.91	88.24	93.60	67.55	47.11	55.56		
s2	92.75	92.87	92.89	74.36	93.79	75.37	75.61	90.94	92.94	92.91	91.97	91.89		
s3	92.43	90.49	90.97	66.40	91.17	70.70	76.18	89.82	92.20	90.00	88.64	87.95		
s4	95.06	94.66	95.34	79.82	94.77	77.14	79.23	94.40	94.48	95.65	94.93	94.32		
s1,s2	92.92	92.87	92.89	74.36	93.96	85.74	86.87	91.13	93.29	92.91	91.97	91.89		
s2,s3	93.09	92.87	92.89	81.79	93.96	84.26	86.30	91.32	93.29	92.91	91.97	91.89		
s2,s4	95.06	94.66	95.34	87.95	94.91	87.57	89.11	94.40	94.48	95.65	94.93	94.32		
s1,s3	97.09	90.49	90.97	66.40	95.59	87.25	89.66	95.25	96.60	90.00	88.64	87.95		
s1,s2,s4	95.06	94.66	95.34	87.95	94.91	88.14	89.97	94.40	94.48	95.65	94.93	94.32		
weather	'ideal'	'ideal'	'ideal'	'poor'	'ideal'	'poor'	'ideal'	'ideal'	'ideal'	'poor'	'ideal'	'ideal'	'ideal'	'ideal'
vessel	'fishing boø'	'icebreaker'	'fishing boø'	'icebreaker'	'fishing boø'	'icebreaker'	'fishing boø'	'icebreaker'	'fishing boø'	'icebreaker'	'fishing boø'	'icebreaker'	'fishing boø'	'icebreaker'
s1	61.75	92.67	0.00	92.34	41.22	0.00	52.65	0.00	60.43	63.37	90.51	49.79		
s2	93.68	93.29	90.73	93.52	77.07	74.44	91.56	90.06	93.37	77.23	92.15	89.83		
s3	90.02	92.66	84.39	91.94	71.43	66.14	86.93	84.51	88.42	73.18	88.19	89.96		
s4	95.14	94.34	95.07	95.06	78.00	75.88	94.54	95.29	94.75	78.83	94.81	94.33		
s1,s2	93.68	93.63	90.73	93.69	79.42	74.44	91.56	90.06	93.37	81.09	92.15	89.83		
s2,s3	93.82	93.46	90.73	93.69	84.53	82.71	91.56	90.06	93.37	84.55	92.15	89.97		
s2,s4	95.05	94.34	95.07	95.06	87.51	85.49	94.54	95.29	94.75	88.13	94.81	94.33		
s1,s3	90.02	96.44	84.39	95.87	74.49	66.14	86.93	84.51	88.42	78.87	95.83	89.96		
s1,s2,s4	95.14	94.34	95.07	95.06	87.61	85.49	94.54	95.29	94.75	87.44	94.81	94.33		
weather	'ideal'	'ideal'	'poor'	'poor'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'poor'	'poor'	'poor'	'poor'
vessel	'fishing boø'	'fishing boø'	'icebreaker'	'fishing boø'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'	'icebreaker'
s1	77.81	48.36	75.94	26.86	65.22	90.69	93.19	88.89	59.51	91.94	76.86	72.37		
s2	91.95	91.35	78.97	74.23	92.24	93.22	93.35	92.29	92.42	92.93	78.57	76.62		
s3	89.54	87.99	72.98	71.85	86.76	90.48	90.75	84.50	89.49	91.74	75.64	71.43		
s4	95.05	95.22	80.23	79.10	94.01	94.73	94.86	94.78	94.55	95.32	79.66	82.14		
s1,s2	91.95	91.35	87.35	75.92	92.24	93.22	93.53	92.49	92.42	93.29	88.95	85.34		
s2,s3	91.95	91.50	86.67	83.66	92.24	93.59	93.53	92.49	92.42	92.93	88.78	86.46		
s2,s4	95.05	95.22	89.83	87.36	94.01	94.73	94.86	94.78	94.55	95.32	88.70	88.57		
s1,s3	89.40	87.99	87.77	74.30	86.76	95.24	97.80	94.93	89.49	95.66	89.61	85.75		
s1,s2,s4	95.05	95.22	90.68	86.98	94.01	94.73	94.86	94.78	94.55	95.32	89.83	90.00		

weather	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'
vessel	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'
s1	89.60	50.49	90.95	51.93	77.81	45.50	50.78	87.19	81.84	91.43	89.74	63.50	91.51	92.80
s2	90.91	92.99	90.93	91.54	93.56	92.59	93.58	92.86	93.48	94.29	91.51	92.80	89.90	95.44
s3	90.69	88.11	88.96	88.73	90.05	88.91	91.18	88.99	91.72	91.02	89.04	95.44	93.80	92.80
s4	94.86	94.93	95.43	94.84	95.45	95.43	94.74	94.99	94.75	95.47	93.80	95.44	91.70	92.80
s1,s2	90.91	92.99	91.11	91.54	93.56	92.59	93.58	92.86	93.48	94.46	91.70	92.80	92.08	92.80
s2,s3	91.09	92.99	91.11	91.54	93.56	92.59	93.58	92.86	93.61	94.46	92.08	92.80	95.44	95.44
s2,s4	94.86	94.93	95.43	94.84	95.45	95.43	94.74	94.99	94.75	95.47	93.80	95.44	95.80	95.44
s1,s3	95.80	87.95	94.92	88.73	90.05	88.91	91.18	93.26	91.72	94.62	95.80	89.59	93.80	95.44
s1,s2,s4	94.86	94.93	95.43	94.84	95.45	95.43	94.74	94.99	94.75	95.47	93.80	95.44		

weather	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'
vessel	'icebreaker'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'
s1	88.58	94.85	0.00	91.68	69.64	0.00	92.56	48.59	61.65	64.67	76.70	23.03	77.44	71.03
s2	92.78	93.65	90.39	93.47	93.58	89.95	92.86	90.66	76.47	93.32	78.04	77.44	75.88	79.46
s3	90.16	93.89	88.13	91.87	90.00	83.40	93.93	88.29	73.51	89.98	75.88	71.03	81.21	79.46
s4	94.67	94.33	95.00	94.77	94.46	94.47	94.92	95.12	79.62	95.15	81.21	79.46	88.85	78.17
s1,s2	93.16	94.15	90.39	93.81	93.58	89.95	93.20	90.66	79.55	93.32	88.85	78.17	88.18	84.72
s2,s3	93.35	93.98	90.39	93.81	93.58	89.95	92.86	90.66	85.70	93.32	88.18	84.72	88.84	87.74
s2,s4	94.67	94.48	95.00	94.77	94.46	94.47	94.92	95.12	87.83	95.15	88.84	87.74	90.68	71.96
s1,s3	95.21	98.09	88.13	95.64	90.00	83.40	98.24	88.29	79.32	89.98	90.68	71.96	90.54	87.84
s1,s2,s4	94.67	94.48	95.00	94.77	94.46	94.47	94.92	95.12	87.93	95.15	90.54	87.84		

weather	'ideal'	'ideal'	'poor'	'ideal'
vessel	'fishing boz'	'icebreaker'	'icebreaker'	'fishing boat'
s1	51.16	92.00	76.15	51.78
s2	92.76	93.61	77.30	89.24
s3	87.52	91.78	74.55	87.73
s4	94.93	95.33	78.44	94.82
s1,s2	92.76	93.78	86.83	89.24
s2,s3	92.76	93.78	87.35	89.24
s2,s4	94.84	95.19	88.67	94.82
s1,s3	87.52	95.80	90.78	87.73

weather	'ideal'	'ideal'	'ideal'	'poor'	'poor'	'ideal'	'poor'	'poor'	'poor'	'ideal'	'ideal'	'ideal'	'poor'	'poor'	'ideal'	'ideal'	'ideal'	'poor'	'poor'
vessel	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'
s1	120.38	381.29	391.82	0.00	118.96	179.51	173.74	177.44	127.77	411.55	429.59	527.45							
s2	79.45	175.74	174.92	217.70	81.24	134.74	130.17	136.74	87.69	195.32	195.92	244.27							
s3	117.38	238.36	243.86	297.12	119.96	176.51	174.74	177.44	130.77	263.07	266.30	316.32							
s4	40.91	60.67	65.56	73.58	41.31	82.54	59.72	60.39	43.22	68.64	67.01	88.28							
s1,s2	79.45	175.74	174.92	217.70	81.24	134.74	130.17	136.74	87.69	195.32	195.92	244.27							
s2,s3	79.45	175.74	174.92	217.70	81.24	134.74	130.17	136.74	87.69	195.32	195.92	244.27							
s2,s4	40.91	60.67	65.56	73.58	41.31	82.54	59.72	60.39	43.22	68.64	67.01	88.28							
s1,s3	117.38	238.36	243.86	297.12	118.96	176.51	173.74	177.44	127.77	263.07	266.30	316.32							
s1,s2,s4	40.91	60.67	65.56	73.58	41.31	82.54	59.72	60.39	43.22	68.64	67.01	88.28							
weather	'ideal'	'ideal'	'ideal'	'poor'	'poor'	'ideal'	'ideal'	'ideal'	'ideal'	'poor'	'poor'	'ideal'	'ideal'	'ideal'	'poor'	'poor'	'ideal'	'ideal'	'ideal'
vessel	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'
s1	406.75	122.75	0.00	124.77	413.82	0.00	424.65	0.00	422.74	377.94	182.24	422.35							
s2	188.22	85.99	230.27	87.28	184.12	218.06	191.88	236.67	196.96	174.59	137.60	184.64							
s3	253.79	122.75	309.44	125.77	251.89	298.52	261.36	313.68	265.73	241.32	186.24	254.57							
s4	64.39	41.59	78.68	42.00	64.03	74.10	66.45	84.28	68.66	61.66	64.99	63.83							
s1,s2	188.22	85.99	230.27	87.28	184.12	218.06	191.88	236.67	196.96	174.59	137.60	184.64							
s2,s3	188.22	85.99	230.27	87.28	184.12	218.06	191.88	236.67	196.96	174.59	137.60	184.64							
s2,s4	64.39	41.59	78.68	42.00	64.03	74.10	66.45	84.28	68.66	61.66	64.99	63.83							
s1,s3	253.79	122.75	309.44	124.77	251.89	298.52	261.36	313.68	265.73	241.32	182.24	254.57							
s1,s2,s4	64.39	41.59	78.68	42.00	64.03	74.10	66.45	84.28	68.66	61.66	64.99	63.83							
weather	'ideal'	'ideal'	'poor'	'poor'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'poor'	'poor'	'ideal'	'ideal'	'ideal'	'poor'	'poor'	'ideal'	'ideal'	'ideal'
vessel	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'	'icebreaker'	'fishing boe'
s1	385.78	515.82	130.25	409.07	553.01	176.82	185.39	198.15	400.48	162.70	126.12	187.08							
s2	179.17	235.32	84.64	173.47	229.86	137.71	142.46	151.68	178.22	122.36	92.72	135.37							
s3	243.74	309.08	121.25	252.01	306.17	178.82	187.39	200.15	246.02	162.70	129.12	178.08							
s4	62.51	84.74	41.40	61.71	80.73	61.53	68.40	69.01	63.68	58.50	42.45	71.02							
s1,s2	179.17	235.32	84.64	173.47	229.86	137.71	142.46	151.68	178.22	122.36	92.72	135.37							
s2,s3	179.17	235.32	84.64	173.47	229.86	137.71	142.46	151.68	178.22	122.36	92.72	135.37							
s2,s4	62.51	84.74	41.40	61.71	80.73	61.53	68.40	69.01	63.68	58.50	42.45	71.02							
s1,s3	243.74	309.08	121.25	252.01	306.17	176.82	185.39	198.15	246.02	162.70	126.12	178.08							
s1,s2,s4	62.51	84.74	41.40	61.71	80.73	61.53	68.40	69.01	63.68	58.50	42.45	71.02							

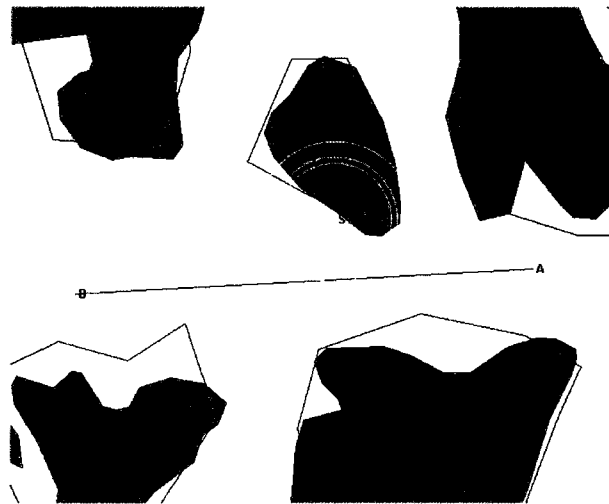
weather	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'
vessel	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'
s1	50.63	16.08	50.81	18.50	26.34	16.52	20.41	49.85	29.86	57.16	48.24	21.50					
s2	59.57	55.89	60.64	56.61	59.61	55.14	57.25	59.48	61.45	65.84	58.44	57.51					
s3	50.74	44.08	50.70	45.45	49.10	44.02	46.21	50.07	51.78	56.81	48.35	46.36					
s4	78.73	80.42	78.23	80.13	81.24	79.34	80.43	78.11	81.10	80.57	77.68	80.85					
s1,s2	59.57	55.89	60.64	56.61	59.61	55.14	57.25	59.48	61.45	65.84	58.44	57.51					
s2,s3	59.57	55.89	60.64	56.61	59.61	55.14	57.25	59.48	61.45	65.84	58.44	57.51					
s2,s4	78.73	80.42	78.23	80.13	81.24	79.34	80.43	78.11	81.10	80.57	77.68	80.85					
s1,s3	50.74	44.08	50.81	45.45	49.10	44.02	46.21	50.07	51.78	57.16	48.35	46.36					
s1,s2,s4	78.73	80.42	78.23	80.13	81.24	79.34	80.43	78.11	81.10	80.57	77.68	80.85					
weather	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'	'ideal'
vessel	'icebreaker'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'	'icebreaker'	'fishing boz'
s1	49.11	60.24	0.00	57.45	24.25	0.00	58.23	11.19	27.08	24.00	58.05	11.11					
s2	59.23	68.78	52.92	66.68	58.58	51.04	67.36	54.67	59.95	58.79	67.69	54.69					
s3	49.45	60.24	40.95	57.68	48.16	38.27	58.69	42.53	49.70	47.93	58.40	41.73					
s4	78.48	81.46	79.52	81.13	81.17	78.48	80.88	80.12	81.16	81.08	81.07	80.00					
s1,s2	59.23	68.78	52.92	66.68	58.58	51.04	67.36	54.67	59.95	58.79	67.69	54.69					
s2,s3	59.23	68.78	52.92	66.68	58.58	51.04	67.36	54.67	59.95	58.79	67.69	54.69					
s2,s4	78.48	81.46	79.52	81.13	81.17	78.48	80.88	80.12	81.16	81.08	81.07	80.00					
s1,s3	49.45	60.36	40.95	57.68	48.16	38.27	58.69	42.53	49.70	47.93	59.09	41.73					
s1,s2,s4	78.48	81.46	79.52	81.13	81.17	78.48	80.88	80.12	81.16	81.08	81.07	80.00					
weather	'ideal'	'ideal'	'poor'	'ideal'	'icebreaker'	'poor'	'ideal'	'icebreaker'	'poor'	'ideal'	'icebreaker'	'poor'	'ideal'	'icebreaker'	'poor'	'ideal'	'icebreaker'
vessel	'fishing boz'	'icebreaker'	'icebreaker'	'fishing boat'	'icebreaker'	'fishing boat'	'icebreaker'	'fishing boat'	'icebreaker'	'fishing boat'	'icebreaker'	'fishing boat'	'icebreaker'	'fishing boat'	'icebreaker'	'fishing boat'	'icebreaker'
s1	17.05	56.83	56.34	15.60													
s2	56.04	65.89	65.42	55.72													
s3	44.80	56.83	56.68	44.33													
s4	80.49	80.87	79.99	80.14													
s1,s2	56.04	65.89	65.42	55.72													
s2,s3	56.04	65.89	65.42	55.72													
s2,s4	80.49	80.87	79.99	80.14													
s1,s3	44.80	56.94	56.68	44.33													
s1,s2,s4	80.49	80.87	79.99	80.14													

Appendix D – MATLAB map results

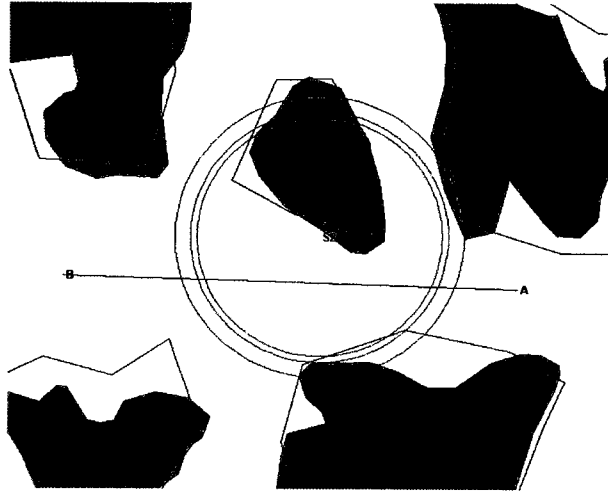
The following maps are the output of the MATLAB simulation runs. These maps indicate the three detection rings for each sensor or sensor combination. The following nine figures show a snapshot of the Barrow Strait map used in MATLAB for the shipping model.

The line AB shows the route taken by the vessel through the passage in each trial. The vessel enters at point A and exits at point B. S1, S2 and S3 are located on land, and S4 is located underwater in the geographic centre of the passage, approximately 35km from either shore.

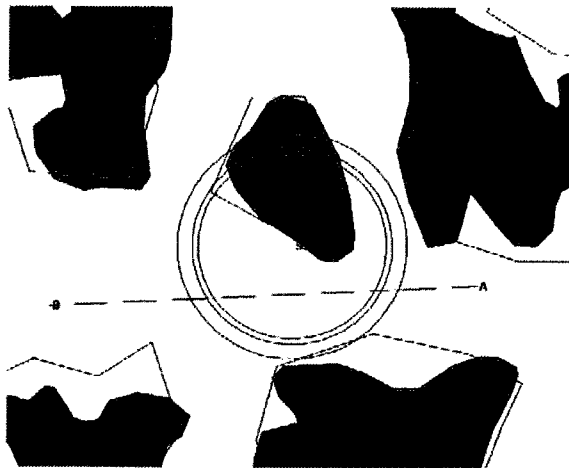
The rings around the sensors represent the range brackets for that sensor. Note that the maps shown in this appendix show only the correct geometry of range brackets for a vessel of the type “fishing boat”: for an icebreaker, the rings would be larger in each case. Note also that weather conditions do not affect the size or geometry of the range brackets themselves, but only the probability of detection in each bracket.



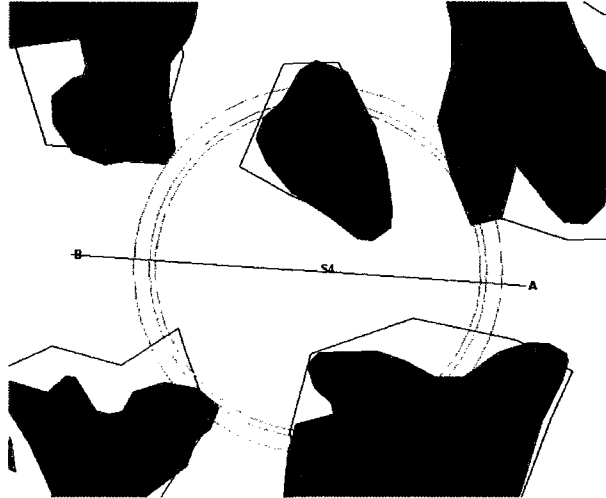
Navigation Radar, fishing boat, ideal weather – Scenario 2 where the route AB intersects only the two outer most probability contours.



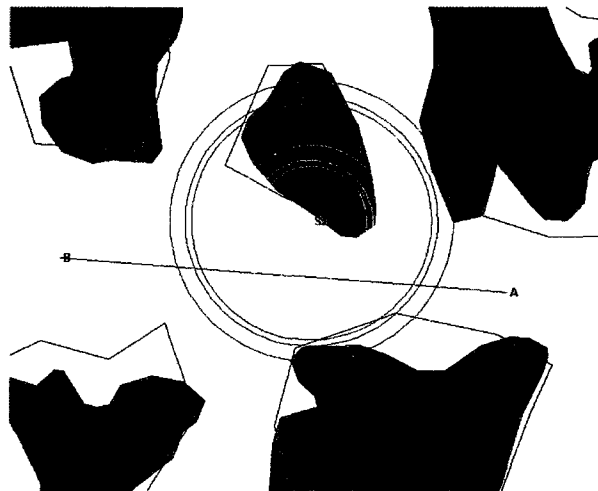
AIS, fishing boat, ideal weather – Scenario 1 where the route AB intersects all of the probability contours.



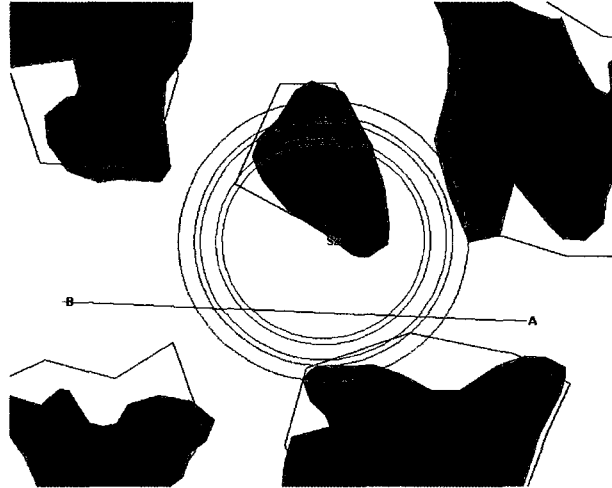
EO/IR, fishing boat, ideal weather – Scenario 1 where the route AB intersects all of the probability contours.



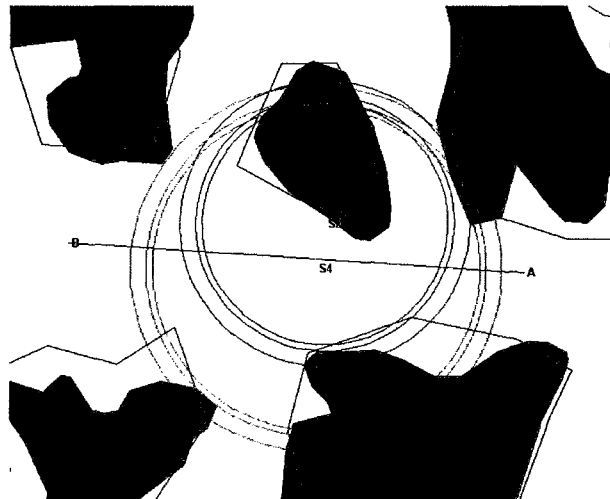
Underwater Sensor, fishing boat, ideal weather – Scenario 1 where the route AB intersects all of the probability contours. Note that the Underwater Sensor is located in the middle of the passage at Barrow Strait.



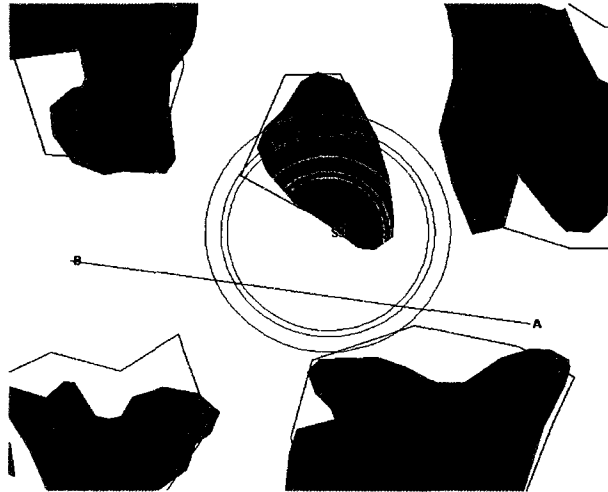
Navigation Radar and AIS, fishing boat, ideal weather – Scenario 1 for the AIS and Scenario 2 for the Navigation Radar.



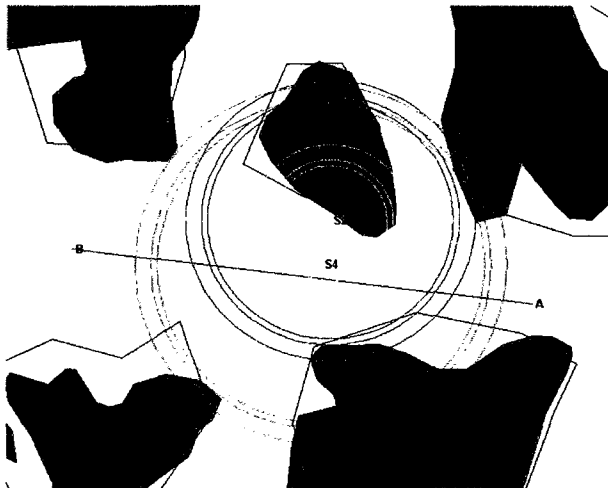
AIS and EO/IR, fishing boat, ideal weather – Scenario 1 for both sensors. Note that in this example, the lowest probability contour (50%) for EO/IR is the same as the highest probability contour (100%) for AIS.



AIS and Underwater Sensor, fishing boat, ideal weather – Scenario 1 for both sensors where route AB intersects all the probability contours.



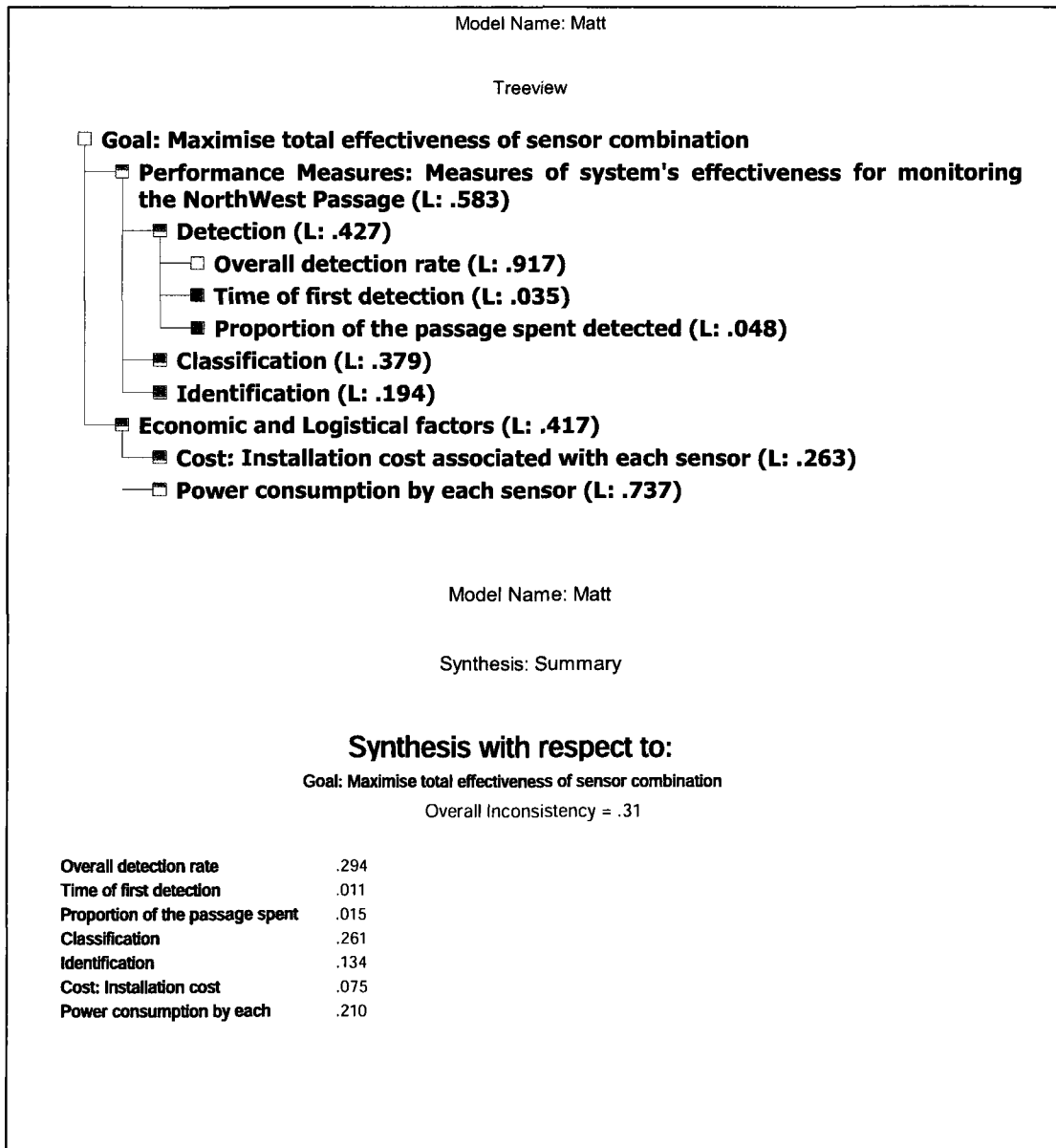
Navigation Radar and EO/IR, fishing boat, ideal weather – Scenario 1 for the EO/IR sensor and Scenario 3 for the Navigation Radar. In this example, route AB only intersects only the lowest probability contour (50%) of the Navigation Radar.



Navigation Radar, AIS and Underwater Sensor, fishing boat, ideal weather – Scenario 1 for both AIS and Underwater Sensor and Scenario 3 for the Navigation Radar. In this example, route AB only intersects the lowest probability contour (50%) of the Navigation Radar while it intersects all the probability contours for the other two sensors.

Appendix E – AHP data, and results

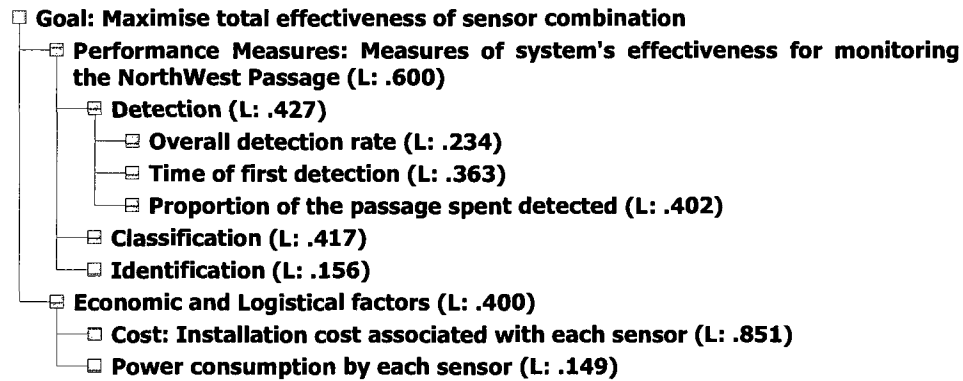
This appendix presents six panels of AHP results – one per page. The first five are summaries based on the pairwise comparison choices made by each decision maker. They show first a tree view of the weights calculated by Expert Choice for each criterion, then a synthesis summary which calculates the cumulative weight of each criterion when multiplied through the tree structure. The final panel on the last page shows the same result views, but for the cumulative analysis representing the combination of all decision makers' input.



Decision maker #1

Model Name: David

Treeview



Model Name: David

Synthesis: Summary

Synthesis with respect to:

Goal: Maximise total effectiveness of sensor combination

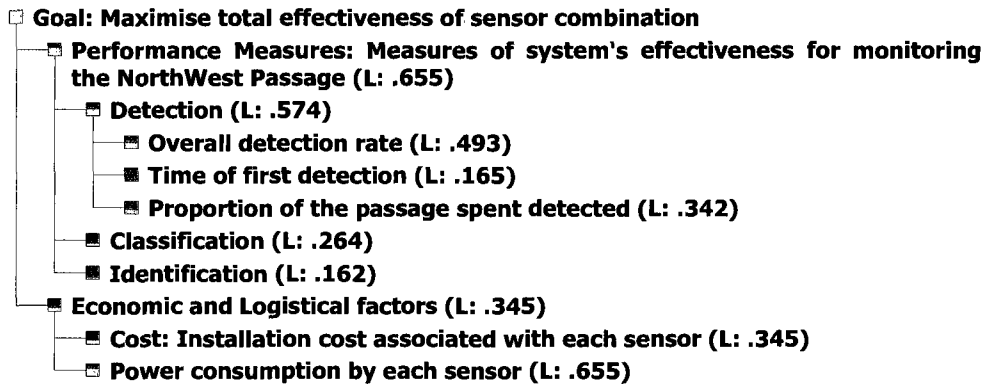
Overall Inconsistency = .00

Overall detection rate	.126
Time of first detection	.196
Proportion of the passage spent	.217
Classification	.212
Identification	.079
Cost: Installation cost	.145
Power consumption by each	.025

Decision maker #2

Model Name: kendall

Treeview



Model Name: kendall

Synthesis: Summary

Synthesis with respect to:

Goal: Maximise total effectiveness of sensor combination

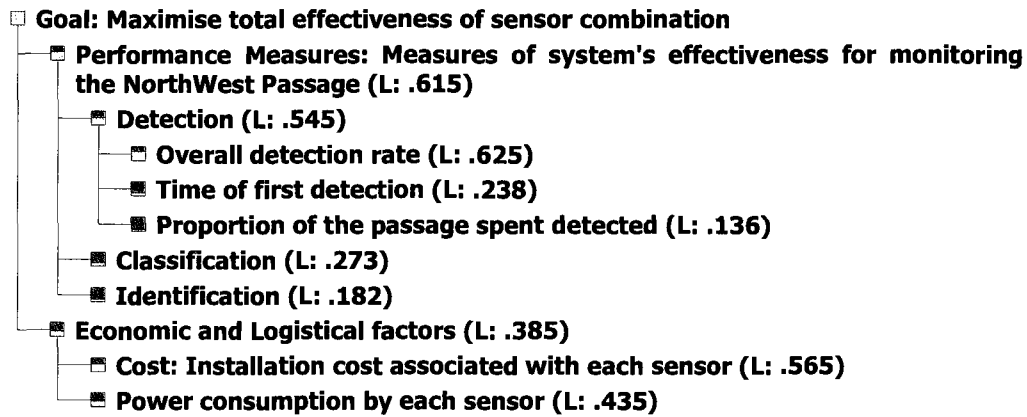
Overall Inconsistency = .01

Overall detection rate	.280
Time of first detection	.093
Proportion of the passage spent	.194
Classification	.129
Identification	.079
Cost: Installation cost	.078
Power consumption by each	.147

Decision maker #3

Model Name: Lane

Treeview



Model Name: Lane

Synthesis: Summary

Synthesis with respect to:

Goal: Maximise total effectiveness of sensor combination

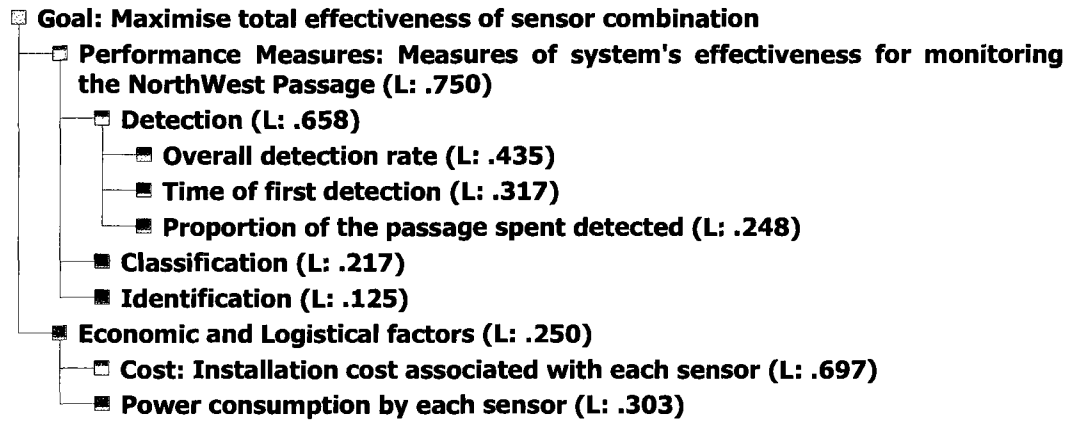
Overall Inconsistency = .01

Overall detection rate	.283
Time of first detection	.108
Proportion of the passage spent	.062
Classification	.141
Identification	.094
Cost: Installation cost	.177
Power consumption by each	.136

Decision maker #4

Model Name: Noosha

Treeview



Model Name: Noosha

Synthesis: Summary

Synthesis with respect to:

Goal: Maximise total effectiveness of sensor combination

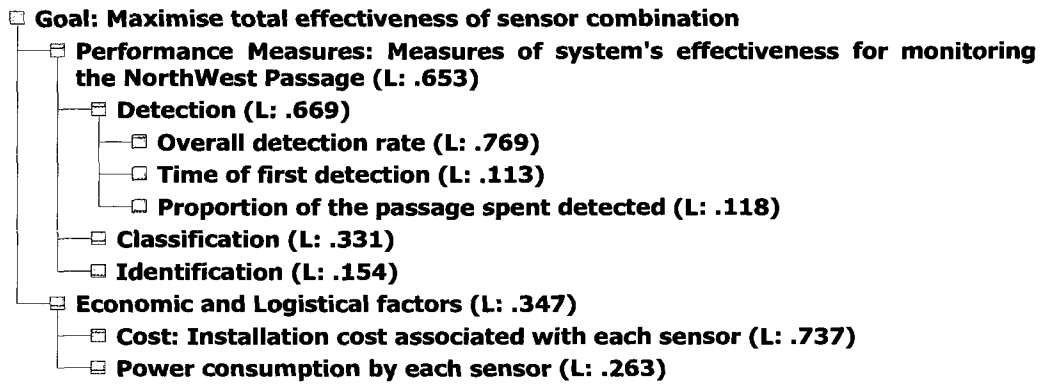
Overall Inconsistency = .01

Overall detection rate	.303
Time of first detection	.221
Proportion of the passage spent	.173
Classification	.100
Identification	.058
Cost: Installation cost	.101
Power consumption by each	.044

Decision maker #5

Model Name: average-model - v2

Treeview



Model Name: average-model - v2

Synthesis: Summary

Synthesis with respect to:

Goal: Maximise total effectiveness of sensor combination

S1	.061
S2	.162
S3	.055
S4	.084
S1, S2	.160
S2, S3	.129
S2, S4	.148
S1, S3	.058
S1, S2, S4	.142

Overall results

Appendix F – AHP Data grids

This appendix presents the “data grid” output of Expert Choice, which contains the performance of each alternative (sensor combination) relative to each objective or decision criterion of the AHP model.

Distributive mode	Priority	INCR
Alternative	Total	Performance Measures: Measures Detection Overall detection rate (L: .769)
☒ S1	.072	64.29
☒ S2	.171	88.67
☒ S3	.054	85.68
☒ S4	.077	91.44
☒ S1, S2	.167	90.08
☒ S2, S3	.128	90.78
☒ S2, S4	.142	93.38
☒ S1, S3	.055	89.24
☒ S1, S2, S4	.134	93.52

Distributive mode	Priority	DECR
Alternative	Total	Performance Measures: Measures Detection Time of first detection (L: .113)
☒ S1	.072	294.28
☒ S2	.171	161.96
☒ S3	.054	220.18
☒ S4	.077	63.23
☒ S1, S2	.167	161.96
☒ S2, S3	.128	161.96
☒ S2, S4	.142	63.23
☒ S1, S3	.055	219.82
☒ S1, S2, S4	.134	63.23

Distributive mode	Priority	INCR
Alternative	Total	Performance Measures: Measures Detection Proportion of the passage spent detected (L: .118)
☒ S1	.072	32.63
☒ S2	.171	59.21
☒ S3	.054	48.72
☒ S4	.077	79.92
☒ S1, S2	.167	59.21
☒ S2, S3	.128	59.21
☒ S2, S4	.142	79.92
☒ S1, S3	.055	48.82
☒ S1, S2, S4	.134	79.92

Distributive mode	Priority	INCR
Alternative	Total	Performance Measures: Measures Classification (L: .331)
☒ S1	.072	0
☒ S2	.171	1
☒ S3	.054	0
☒ S4	.077	0
☒ S1, S2	.167	1
☒ S2, S3	.128	1
☒ S2, S4	.142	1
☒ S1, S3	.055	0
☒ S1, S2, S4	.134	1

Distributive mode	Priority	INCR
Alternative	Total	Performance Measures: Measures Identification (L: .154)
☒ S1	.072	0
☒ S2	.171	1
☒ S3	.054	0
☒ S4	.077	0
☒ S1, S2	.167	1
☒ S2, S3	.128	1
☒ S2, S4	.142	1
☒ S1, S3	.055	0
☒ S1, S2, S4	.134	1

Distributive mode	Priority	DECOR
Alternative	Total	Economic and Logistical factor Cost: Installation cost associated with each sensor (L: .737)
☒ S1	.072	250000
☒ S2	.171	25000
☒ S3	.054	1500000
☒ S4	.077	2000000
☒ S1, S2	.167	275000
☒ S2, S3	.128	1525000
☒ S2, S4	.142	2025000
☒ S1, S3	.055	1750000
☒ S1, S2, S4	.134	2275000

Distributive mode	Priority	DECOR
Alternative	Total	Economic and Logistical factor Power consumption by each sensor (L: .263)
☒ S1	.072	17040
☒ S2	.171	6042
☒ S3	.054	61680
☒ S4	.077	21600
☒ S1, S2	.167	17082
☒ S2, S3	.128	61722
☒ S2, S4	.142	21642
☒ S1, S3	.055	72720
☒ S1, S2, S4	.134	32682

Appendix G – Expert Choice alternative ranking

This appendix shows a summary of the final output of Expert Choice. Each alternative is shown with its normalized priority score, and the histograms are a visual representation of the relative magnitude of those scores.

S1	.072
S2	.171
S3	.054
S4	.077
S1, S2	.167
S2, S3	.128
S2, S4	.142
S1,S3	.055
S1, S2, S4	.134