GEOSTATISTICAL MODELLING OF MICROFOSSIL ABUNDANCE DATA IN UPPER JURASSIC SHALE, TOJEIRA SECTIONS, CENTRAL PORTUGAL

by

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ABSTRACT

Every count can be seen as a statistical sample from a statistical population in paleontology. It can be defined as the collection of individuals of a specific group of microfossils that is present in the geological sample from which the count comes. In general, when sampling from a stratigraphic section one should consider only the vicinity within the layer from which the sample comes.

The sampled population proportions (percents) can be considered to be random variables.

Along a stratigraphical column the proportions of a certain taxon may show an overall increase or an overall decrease; these phenomena are called trends. There may also exist more local increases alternating with decreases. These systematic changes (fluctuations) may be expressed by the autocorrelation coefficient value between successive proportions for a number of the samples ordered from bottom to top.

Care is required in evaluating such autocorrelation coefficient values, because one of the series of variables, the sample numbers, has a fixed, logical (stratigraphic) order. This logical order of the samples may be correlated to the other series of variables, the proportions. There may be an interdependence in the sense that some taxon property (e.g., its proportion) in a certain sample is strongly related to the one in the sample immediately below. Series with interdependence because of an inherent logical order in time are called time series.
The analysis of the time series is helped by geostatistical modelling

based on probability theory. Probability models give a good description of reality, when uncertainty is a factor.

This method of treating autocorrelated proportions used for this study was originally developed by F.P. Agterberg. The geological applications of the geostatistical model used for this project have previously been restricted to ore minerals in ore deposits and occurrence of volcanic rocks in large regions (Agterberg, 1974 and 1984).

It is useful in spatial pattern analysis to assume that a feature can be significantly correlated with features that occur at some distance from it. A basic tool of spatial pattern analysis is the autocorrelation function of a random variable. The method to be discussed in this research can be applied to phenomena that occur in either two - dimensional or in three dimensional space.

When the geostatistical model was applied, it was required to estimate R values representing correlation coefficients between proportion variables for volumes and a binary variable defined for presence or absence of the feature for which the proportions were determined. These R values can be derived from the mean and variance of taxon proportion distributions by using the FF FORTRAN program that solves Hermite polynomials. Alternatively, the R - values can be determined from graphs or tables based on output from the FF FORTRAN program. New graphs and tables, especially for small mean proportions and their variances, have been prepared for this thesis project.

Several models in one or two dimensions produce exponential autocorrelation functions (Matern , 1960 and Switzer, 1965) which are frequently used in one - dimensional time series analysis. When the autocorrelation function is known, it is possible to estimate the variance of the distrbution of a taxon measured as a proportion in a sample derived from a volume of any size and shape.

A model that can often be used is the signal - plus - noise model of the statistical theory of communication. The signal may satisfy an autoregressive process model. The noise (random shock) consists of uncorrelated values and is independent of the signal.

Theoretical models for frequency distributions in spatial analysis have previously been proposed by several authors. These include the the Neyman - Scott clustering model, the de Wijsian model, the Prokhorov model and a model based on the transfer function theory of Matheron which is the basis for the model used in this project.

When a random variable has both a stationary mean and a covariance function, it may be assumed that the discrete Gaussian model of Matheron can provide useful methods to obtain frequency distribution models in spatial analysis.

Before the start of the geostatistical modelling of the frequency distribution of the microfossils, the time series were tested as to whether or not, they could be described by various autoregressive process models that would explain the data set.

Newly prepared FORTRAN as well as existing SAS package programs (ARIMA, SPECTRA and CORR procedures) were used for statistical calculations and SAS graphic software was used for the plotting of graphs.

For application of the geostatistical modelling, two stratigraphic sections were studied. They are called Tojeira 1 and Tojeira 2 and are located in the Montejunto area of central Portugal. Data were taken from a 1986 Ph. D. thesis by B.Stam and also provided by F.M.Gradstein who identified and counted species of foraminifers from samples newly collected during the summer of 1986. With Stam's data, 3 of 14 species in the Tojeira 1 section and 5 of 14 species in the Tojeira 2 section were shown to have exponential autocorrelation functions. They are Eoguttulina sp., Epistomina mosquensis and Ophthalmidium strumosum in Tojeira 1 and Eoguttulina sp., Epistomina mosquensis, Ophthalmidium strumosum, Spirillina tenuissima and agglutinants in the Tojeira 2 section.

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CHAPTER 1: Introduction

1.1 Purpose of study

The objective of this new research project is the development of techniques to establish the reliablity of microfossil abundance data in micropaleontology.

Statistical models are applied for estimating the precision of counts of species of different types. These methods will allow estimation of the precision of averages computed from values for a single taxon in samples of different size. It also has a bearing on the correlation of data on abundance of many taxa with one another and in relation with lithological information.

This methodology can be used in several other fields of geoscience. These include chemical analysis of rock samples, petrographic modal analysis, geostatistical ore - reserve estimation, geochemical exploration, line - spacing problems in exploration geophysics, drilling for hidden targets, and regional resource estimation.

The methods used are applicable to discrete or continuous variables defined for samples located in two dimensional or three - dimensional space. Spatial pattern analysis is an important topic of study in geomathematics. Generally, it is useful in spatial pattern analysis to assume that a feature can be significantly correlated vith features which occur at some distance from it. For this reason, geological pattern analysis may require a more elaborate mathematical apparatus than is available in current methods of pattern recognition, to perform the multivariate analysis of attributes observed simultaneously at the same points in space.

Application of this methodology to the bivariate normal distribution leads to the " prob - normal." distribution which has turned out to be very useful. A similar type of distribution can be obtained by applying the method to a bivariate exponential distribution (Agterberg 1984).

1.2 acknowledgements

Sincere thanks are due to Dr. F. P. Agterberg for suggesting the research topic, for his advice, continued support and constructive criticism.

I wish to thank Dr. F. M. Gradstein, who co - supervised the project.

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1.3 Geology

1.3.1 Introduction

The data used in this thesis were derived from published reports and from information collected by the author following field investigation in 1986 of several geological sections.

The relevant sections were chosen carefully. The first consideration was that they had to be well exposed for detailed sampling. Secondly, the sections had to fit into a well known micropaleontologic and biostratigraphic framework (Age - control). Thirdly, some aspects of the paleogeographic setting of the areas had to be known. This was necessary to enable recognition of trends in basin tectonics and basin development. The Montejunto area of central Portugal, in the Lusitanian basin, was found to fit these criteria.

Much of the literature concerning the stratigraphy of Portugal has been written in Frnch and Portuguese, and published in journals and memoirs that are not readily accessible.

The biostratigraphic framework, based on ammonites, is well known and discussed by Choffat, (1880 - 1893), Ruget - Perrot (1961), Mouterde et al. (1972), and Poulton (1983).

The paleogeographic setting of the areas has been documented by Ruget Perrot (1961), Mouterde and Ruget (1975), Wilson (1975a), Wilson and Exton (1979) and Exton and Gradstein (1984). Stam (1986) has made detailed comparisons between Middle and Late Jurassic foraminifera from Portugal and the Grand Banks of Newfoundland. He has used quantitative methods, to examine some paleoecological parameters controlling the distribution and abundance of Jurassic benthonic foraminifera. He also evaluated the biostratigraphic usefulness of Jurassic planktonic and benthonic foraminifera.

1.3.2 Paleogeography of the Lusitanian basin

A useful summary of the stratigraphy of the Lusitanian basin can be found in the guidebook of the field trips accompanying the 27^{e} I.G.C. (Paris), Livret Guide G - 14 (1980). Data on Triassic exposures were presented by Palain (1976) and the Lower Jurassic has been summarized by Hallam (1971) and Mouterde and Ruget (1975). The Triassic and Jurassic rocks of Portugal have been studied since the end of the nineteenth century (Choffat 1880). The Lusitanian basin originated in the Late Triassic - Early Jurassic as a result of movements along Hercynian (Paleozoic) normal basement faults; one of which is the Nazare strike - slip fault. Triassic sediments in the Lusitanian basin rest unconformably on igneous, metamorphic and sedimentary rocks of Precambrian and Paleozoic ages (Da Costa, 1950; Howie and Barss, 1975). The Late Triassic through Middle Jurassic development of the basin was generally uniform. It started with the deposition of nonmarine and marginal marine to shallow marine sands, shales and evaporites. Dolomites and dolomitic limestone were deposited throughout the Sinemurian (Doubinger et al. 1970 ; Mouterde et al. 1973; Mouterde and Ruget, 1975). This was followed in the late Sinemurian and throughout the Pliensbachian by normal marine sedimentation of shales. alternating with limestone. These late Sinemurian sediments contained the first occurrence of ammonites. At the beginning of the Toarcian. the shale facies became predominant. with an upward trend towards more calcareous sedimentation. This trend continued throughout the Aalenian and Bajocian until the Bathonian. At this time most of the basin was covered by shallow carbonate deposits. During the Callovian, also characterized by shallow water deposits, reactivation of basement faults and halokinesis changed the basin configuration to such an extent that a northern and a southern sub - basin can be recognized, separated along the Nazare fault line (Stam, 1986).

In the northern sub - basin, the deposition of oolitic limestone and sandstone over shaly and " reefal " limestone (Mouterde et al., 1973), indicates a (middle) Callovian regression. lower Oxfordian sediments appear absent, because of uplift (Wilson and Exton, 1979). Sedimentation resumed in the late Oxfordian with the deposition of marginal marine carbonates and fresh water sediments forming algal marsh facies (Wilson, 1975). This was followed in the Kimmeridgian by fluvial clastics. In late early Oxfordian times, more rapid subsidence in the southern sub - basin resulted in normal marine sedimentation with abundant ammonites. Sediments include high energy carbonates (Wilson, 1975). In late Oxfordian time this high energy facies was gradually replaced by low - energy shale sedimentation and by the end of the Oxfordian, and at the beginning of the Kimmeridgian. shale facies were predominant. Reactivation of basement faults and salt movements in the early Kimmeridgian resulted in a complex facies distribution, with marine clastics in the Montejunto area, and shelf carbonates to the northeast of the Montejunto area.

1.3.3 Montejunto area

The Montejunto area is located some 50 km north of Lisbon (Fig.1.1). and is dominated by the 664 m. high "Montejunto" mountain some 7 km. northeast of Vila Verda.

An almost complete, but tilted sequence of Bathonian through Kimmeridgian marine sediments, rest on the top and flanks of Montejunto.

The geology and macropaleontology of the area has been described by Choffat (1880 - 1893), Ruget - Perrot (1961), and Mouterde et al. (1972, 1973).

The formations found in this sequence are the Cabacos, Montejunto, Tojeira, Cabrito, Abadia, and Amaral Formations (Figs. 1.2 - 1.3).

1.4 Stratigraphy

The stratigraphy of the area has been explained by Stam [illegible]986) as follows:

1.4.1 **Middle Jurassic**

Rocks of the middle Jurassic age are exposed near the top of the Montejunto mountain. The oldest strata consist of thick bedded (0.5 -

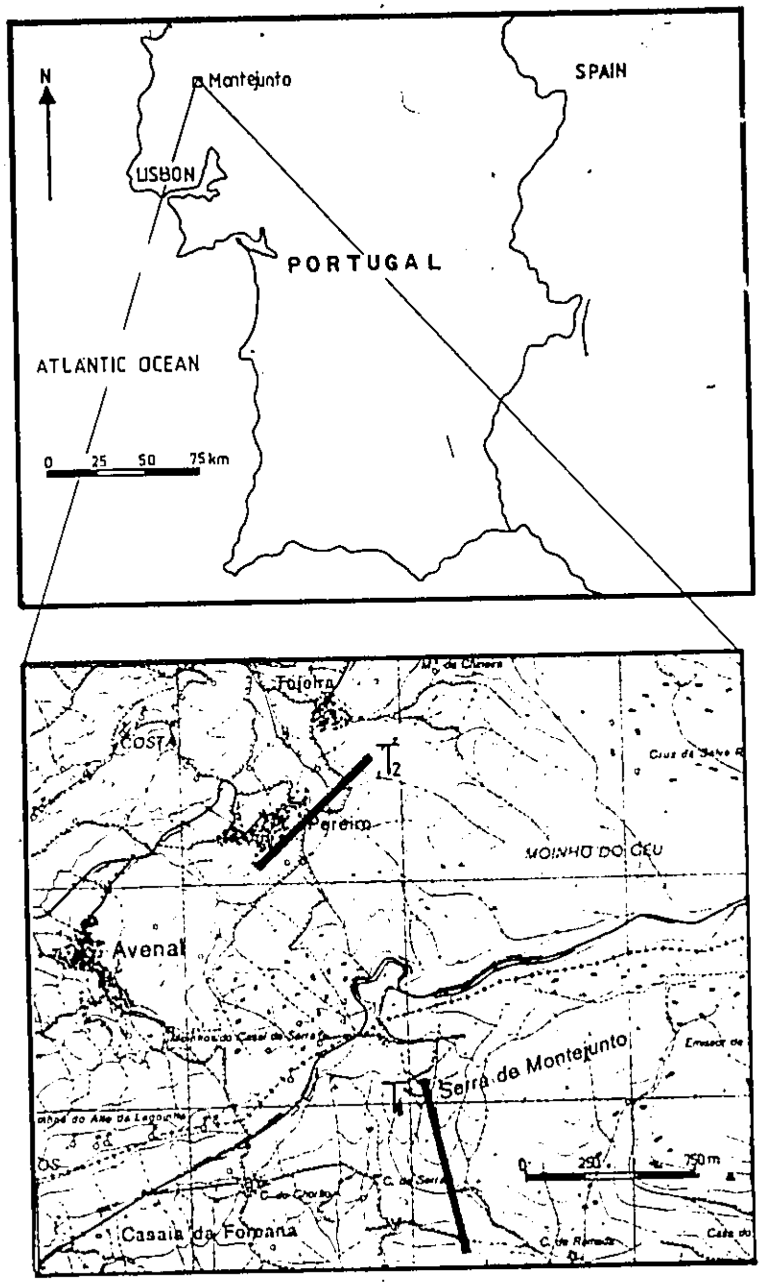


Figure 1.1 Location map of the Montejunto area Tojeira 1 and Tojeira 2 sections

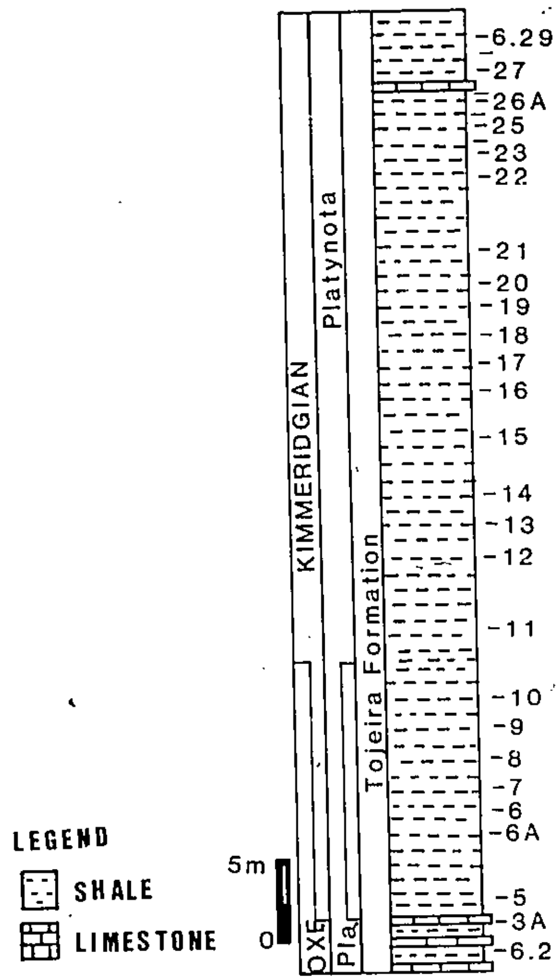


Figure 1.2 The Tojeira 1 section with sample numbers 6.2 - 6.29 (after Stam,1986)

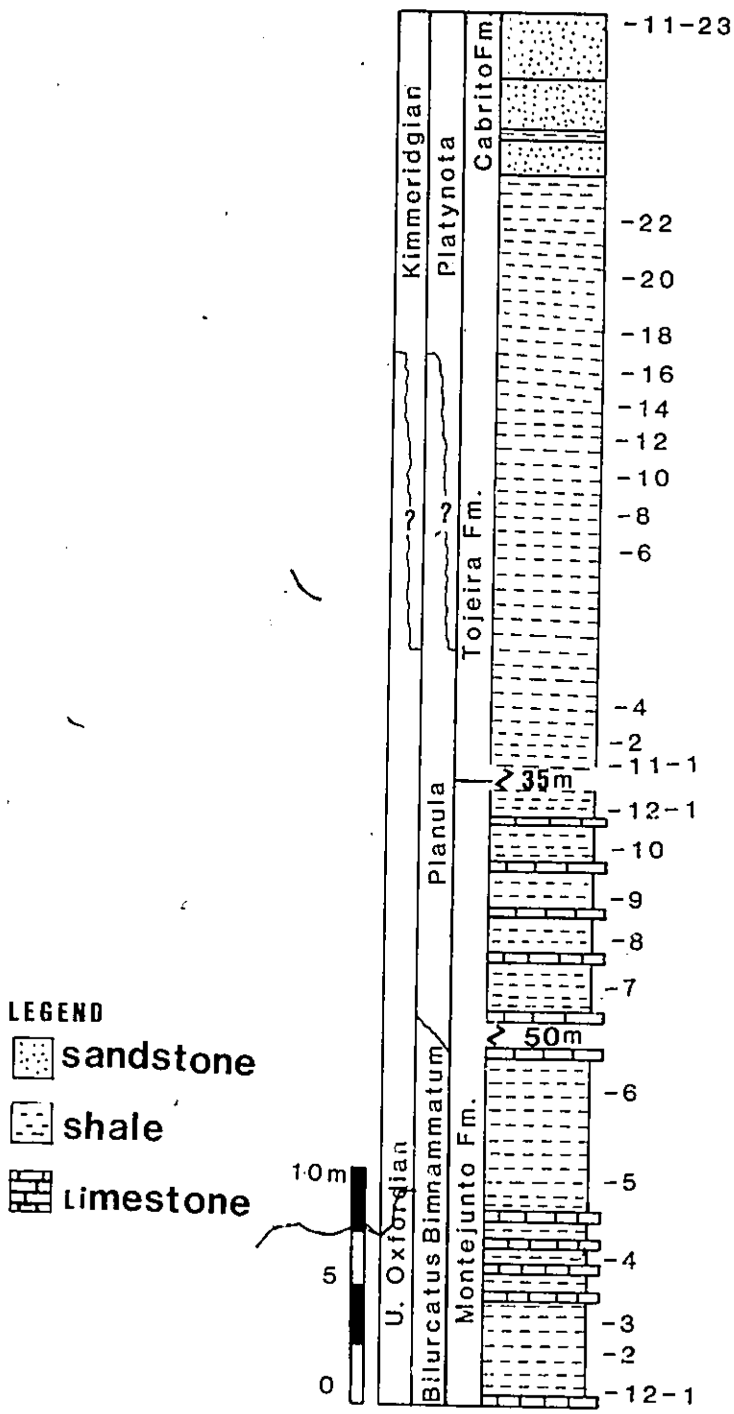


Figure 1.3 The Tojeira 2 section with sample numbers 12.1 - 12.11 and 11.1 - 11.23 (after Stam 1986)

2m) beige to grey, mainly micritic, sometimes oolitic/pisolitic limestones with rare crinoids and *Ostrea*. No ammonites were observed nor are any reported in the literature. Based on occurrences of the larger foraminifer *Meyendorffina bathonica* Bizon, Zbyszwski et al., (1966) believe these strata belong in the upper Bathonian.

The Upper Jurassic formations are separated from these Middle Jurassic sediments by a hiatus. The upper Callovian Lamberti zone and the lower Oxfordian Mariae zone are missing.

1.4.2 Upper Jurassic

1.4.2.1 *Cabacos Formation* (Cordatum - Transversarum zones)

The basal 30 m of this formation consists of highly bituminous platy, thinly bedded, grey and shaly limestone. This is overlain by at least 200m of thickly bedded (0.5 - 2m) yellow to dark brown bituminous limestone, which is fine - grained, micritic and locally oolitic. Ammonites occur throughout the formation but are rare in the lower part.

1.4.2.2 *Montejunto Formation* (Transversarium - Planula zones)

The lower part of this 150 to 200 m thick formation consists of thick bedded (up to 2m) white to grey micritic limestone, changing to a thickly bedded alternation of bluish - grey limestone and hard grey shale. Ammonites are common throughout the formation. At the top near the (covered) contact with the overlying Tojeira Formation. several shale and limestone beds are very rich in ammonites.

1.4.2.3 *Tojeira Formation* (Planula - Platynota zones)

This formation is at least 70 m thick and consists almost entirely of dark grey shales, in which limonite concretions are common. Some calcareous shale intercalations occur in the lower part of the formation. Locally, in the middle part of the formation, the shale is brownish - red. Close to the boundary with the overlying Cabrito Formation, there is a marked increase in silt content. Pyritized ammonites are common to abundant in the lower part, but are relatively rare in the upper part of formation.

1.4.2.4 *Cabrito Formation* (? Platynota zone)

This formation consists of a massive, coarse - grained, micaceous sandstone unit and a conglomeratic unit containing pebbles of quartz and pink feldspar. The sandstone is dark yellow to brown in colour, near - massive, and is generally poorly sorted. Crinoids, shell debris and large (up to 40 cm) fragments of coral colonies are common to abundant. In the Montejunto area, the Cabrito formation is about 40 - 50 m thick, thickening rapidly towards the west. This was interpreted by Mouterde et al. (1973). as indicative of a westerly source for the influx of these clastics. A few badly preserved ammonites have been reported

(Mempel. 1955 : Mouterde et al.. 1973).

1.4.2.5 *Abadia Formation* (? Kimmeridgian)

Characteristic of this formation are the blue colour of its sediments and the presence of abundant black plant remains. Ostracods and thin - shelled bivalves are common, but no ammonites were observed. It consists of very thickly bedded (up to 4 m), coarse - grained blue coloured sandstone that shows ripple marks and channelling. The sandstone alternates with bluish sandy siltstone/shale.

1.4.2.6 *Amaral Formation* (? Kimmeridgian)

This formation is up to 70 m thick, and consists of well sorted brownish quartz sandstone containing plant remains, and sometimes showing cross bedding. A few *Ostrea* and pelecypods are present, but no ammonites were observed.

1.5 Section descriptions

Two sections have been sampled in detail by the author during the summer of 1986. The first, called Tojeira 1, is located near the hamlet of Ramada, and covers the top 65m of the Tojeira Formation. The second, Tojeira 2, is located near the villages of Pereiro and Tojeiro and covers the top of the Montejunto Formation and the upper part of the Tojeira Formation. Stam (1986) has made a quantitative analysis of the foraminiferal assemblages in 137 samples from five Portuguese sections of Middle and Late Jurassic age, two of which were sections Tojeira 1 and Tojeira 2 (Figs.1.4 - 1.5). Sampling strategy involved trying to match as many levels as possible to those previously sampled by Stam (1986) and achieving regular spacing of the samples. Although there is no certainty that the sedimentation rate was constant, the monotonous Tojeira shale lithology suggests so. Less continuous outcrops in the Tojeira 2 section did not allow regular sample intervals. Samples T_2/1 to T_2/3 are from the Tojeira 2 section and were taken from a small outcrop of the Montejunto Formation in the down faulted block west of Montejunto Mountain. Sample T_2/4 was taken along a dirt road halfway between T_2/3 and T_2/5. These samples are separated by a gap in the outcrop of approximately 50 m. Samples T_2/5 and T_2/6 were taken along the asphalt road that runs from Pereiro to Tojeira and correspond to the top of section 12 of Stam (1986). Samples T_2/1 - 5 could be matched in the field to the approximate lithostratigraphic position (± 1/2m) of samples 12/2 - 7 studied by Stam (1986) (Fig.1.3). T_2/7 is a new sample taken from a ditch halfway between T_2/6 and T_2/8. Stam (1986), had previously indicated that there was no exposure at this point. This sample location was probably estimated too low in the section, and should be estimated at an 80 m. thickness. Samples T_2/8 and T_2/9 are on the W side of the asphalt road just before the first house entry from Pereiro to Tojeira. Samples T_2/10 to T_2/13 were

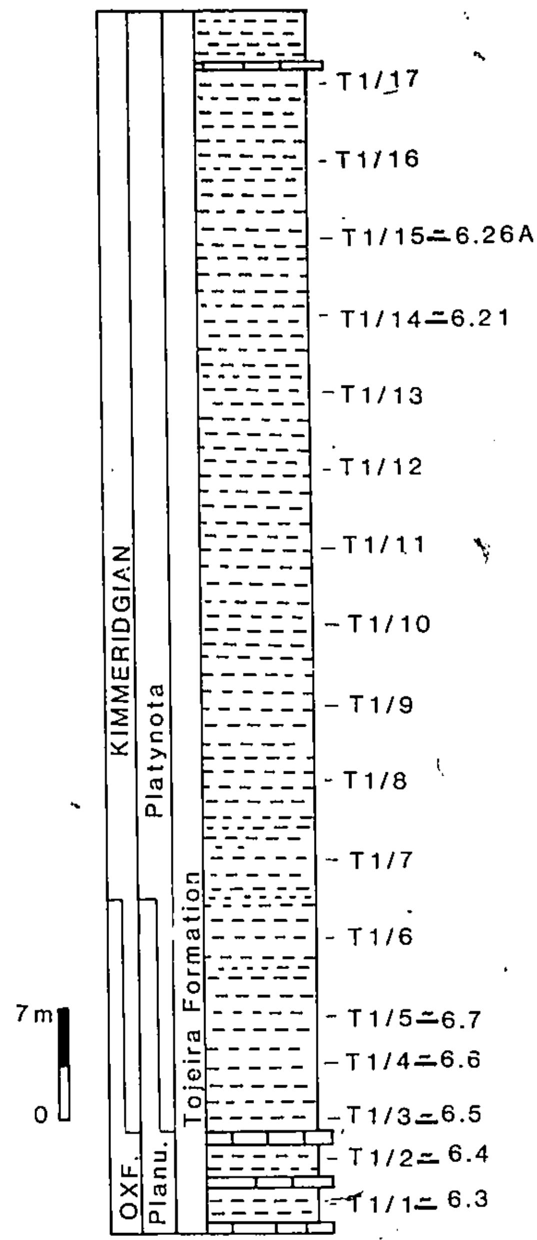


Figure 1.4 The Tojeira 1 section with F.M. Gradstein's sample numbers T1/1 - T1/17 as they correspond to Stam's samples

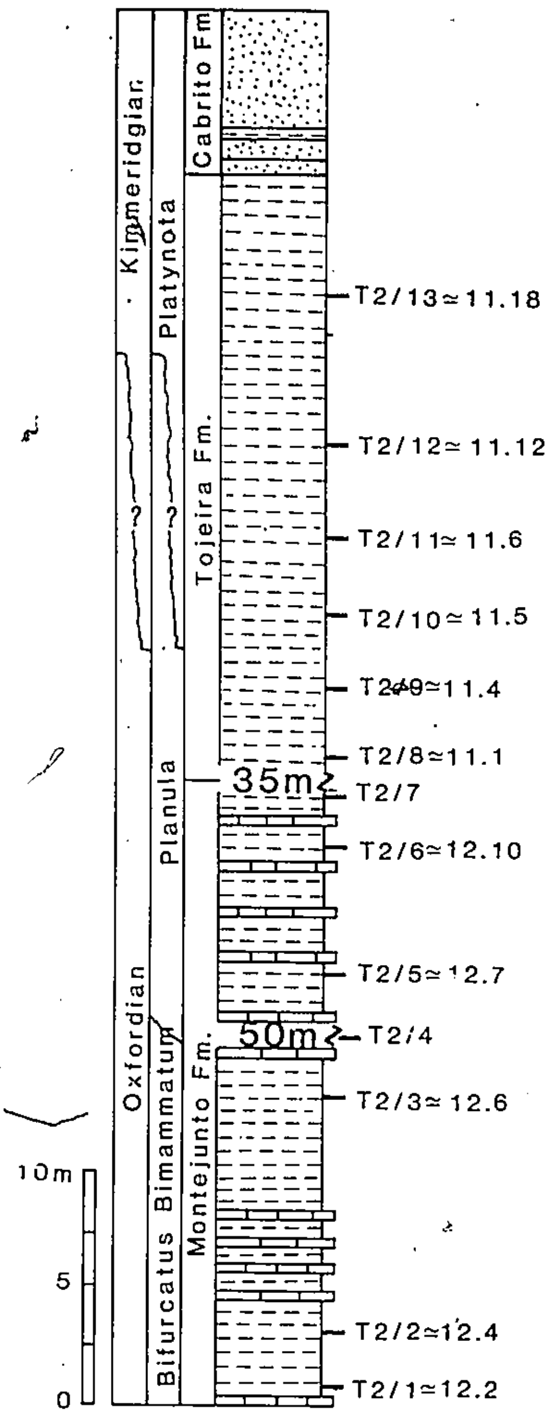


Figure 1.5 The Tojeira 2 section with F.M. Gradstein's sample numbers T2/1 - T2/13 as they correspond to Stam's samples

taken from a steep outcrop above the dirt road just West of the asphalt road. The thickness of this outcrop mentioned by Stam (1986), was probably underestimated by 30 % or more. Because of easily recognizable landmarks in the section, all but a few of our samples could be matched to Stam's (1986) sampling levels (see Fig.1.5). Sample T_1/1, from the Tojeira 1 section east of Montejunto, was taken as a first sample from under outcrop 17 of Stam (1986). Samples T_1/2, T_1/2A, 2B, 2C, and 2D were collected, at 5m. intervals. in the same layer located above outcrop 17. Samples T_1/3 and T_1/17 were taken in order at 5 m intervals. Several of the newly collected Tojeira 1 samples correspond with Stam's (1986) samples, as shown in Fig. 1.4.

1.6 Preparation of Data

The samples were prepared in the laboratories of the Geological Survey of Canada, Bedford Institute of Oceanography, Dartmouth, Nova Scotia. Samples were processed according to the " Industrial Soap Method " of Thomas (1985. 1986). This included washing the samples over a set of sieves sized 500, 125, and 63 microns. The 63 - 125 micron size fraction was used neither for this study nor by Stam(1986). All residues were split such that several hundred foraminiferal shells remained in one part. Each split was spread over one or more picking trays and picked clean of foraminifers. Next, using a microsplitter the benthic taxa were identified and counted by F. Gradstein (in 1987).

Percentages of taxa were calculated and presented as shown in Tables 1.3 - 1.4. Stam (1986) previously used the same method and his results are shown in Tables 1.1 - 1.2. A planktonic/ benthonic (P/B) ratio was also calculated and shown for the data sets in both tables. Faunal analysis of the Tojeira 1 and Tojeira 2 sections are discussed below. Stam (1986) took thirty one samples for foraminiferal analysis. The frequency patterns of seventeen taxonomic categories were examined, fifteen of which concern proportion counts on species or species groups. These taxonomic units are: Eoguttulina sp. (E. oolithica and E. metensis), Epistomina mosquensis, E. uhligi, all other Epistomina sp., Lenticulina muensteri , all other Lenticulina sp., Nodosaria/Dentalina sp., Pseudolamarckina rjasanensis , Spirillina elongata, Spirillina infima, Spirillina tenuissima, Ophthalmidium carinatum, Ophthalmidium strumosum, agglutinants (Ammobaculites sp. Reophax sp., Trochammina sp., Textularia sp., Bigenerina sp.). All other species are grouped in a Restgroup, usually comprising less than 15% of the benthonic microfauna.

For this study, samples used to create a geostatistical model included the thirty one samples reported by Stam (1986) plus fifteen samples collected by the author from the Tojeira 1 and thirteen new samples collected by the author and the thirty samples reported by Stam (1986) for the Tojeira 2 section.

TABLE 1.1 DATA SET OF TOJEIRA 1 SECTION

Sample number	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	15	16	17	18
6.2	5.0	4.1	.0	.0	1.5	7.9	6.6	4.8	10.4	.0	.0	1.8	.9	45.3	11.7	-	36	558
6.3	2.5	14.8	.0	.0	.0	24.7	5.7	4.9	.2	.0	.0	.9	18.8	17.9	9.6	.01	24	453
6.3A	.5	19.0	.0	.5	.0	22.5	3.8	4.6	1.1	.0	.0	.0	26.8	13.6	7.6	-	22	369
6.4	5.0	5.4	.2	1.2	.0	19.8	10.6	3.4	4.2	.0	.0	2.4	.4	26.1	21.3	-	34	501
6.5	2.6	1.8	15.1	.0	13.3	4.7	4.9	.0	.0	.0	.0	2.9	7.2	34.8	11.9	.01	32	511
6.6A	1.2	13.5	84	1.5	16.2	5.4	.3	.6	14.4	.0	.0	13.8	6.0	11.7	7.0	.09	21	333
6.6	2.3	.22	24.2	11.8	.0	9.9	4.8	2.5	3.8	.0	.0	10.8	3.8	9.1	14.8	.02	30	372
6.7	5.4	3.4	18.1	3.4	.0	11.1	3.1	3.8	3.8	.2	.0	6.9	.6	29.2	11.0	.09	32	524
6.8	4.5	.3	12.4	1.0	.0	5.6	2.1	2.0	35.4	.0	5.1	5.3	.0	15.4	10.9	.13	26	396
6.9	6.4	2.6	26.1	4.3	.3	4.6	4.1	2.6	3.7	.0	10.9	5.7	.0	23.2	5.5	.40	27	349
6.10	5.1	1.1	27.6	2.5	2.5	1.4	.6	2.2	1.1	.3	29.6	1.7	.3	20.7	3.3	1.19	27	362
6.11	3.7	1.1	13.2	17.6	.0	23.0	6.1	2.0	.5	.2	10.9	1.1	.2	11.1	9.3	.08	31	614
6.12	21.6	1.3	23.6	3.0	3.0	9.3	3.3	.0	.0	.0	12.3	1.3	.0	15.0	6.3	.17	27	301
6.13	3.9	.6	23.3	5.7	4.8	8.7	2.7	.3	1.2	.0	14.0	1.5	.0	23.0	10.3	.60	26	335
6.14	7.6	1.5	15.5	.7	.9	4.8	3.8	1.8	2.4	2.3	4.8	5.0	.7	31.3	6.9	.44	37	459
6.15	2.4	.3	35.2	7.8	6.0	8.1	7.2	1.5	.6	.3	10.5	1.5	.0	15.1	3.5	.41	30	332
6.16	7.8	.5	21.4	3.1	7.7	15.2	5.5	1.4	2.1	2.9	15.0	1.7	.0	11.2	4.5	.28	37	420
6.17	2.9	1.1	46.3	2.0	1.9	5.1	2.8	1.1	1.4	8.3	6.4	4.1	.0	13.2	3.4	.16	30	916
6.18	14.0	.3	18.0	1.7	5.3	3.9	7.2	2.8	.8	6.6	8.0	4.7	.0	20.8	5.9	.44	34	361
6.19	8.5	.8	39.7	.6	3.5	3.5	4.4	2.5	1.0	1.9	11.6	4.1	.0	8.3	9.6	.11	30	484
6.20	6.0	.8	30.4	6.0	1.4	9.1	7.4	1.5	1.9	.4	12.6	4.4	1.0	9.1	8.0	.06	35	517
6.21	11.4	.8	13.5	1.7	1.2	15.0	11.3	1.9	1.9	7.2	9.3	4.2	.0	13.5	7.1	.06	37	527
6.22	4.6	3.5	23.0	.3	.8	11.8	6.6	1.6	.5	2.4	11.8	7.6	.0	22.3	3.2	.30	35	382
6.23	3.3	1.5	33.5	.8	6.2	11.4	9.3	2.1	1.6	1.9	10.9	4.8	.0	7.7	5.0	.16	25	376
6.24	7.1	.6	37.1	3.2	3.1	6.6	3.6	1.5	1.2	5.8	1.0	3.4	.0	9.0	16.8	.11	35	412
6.25	7.4	.6	36.6	7.4	8.4	9.1	1.2	3.9	.6	1.9	.0	1.9	.0	13.6	7.4	.31	33	309
6.26	17.1	.9	38.8	4.9	.6	6.7	2.7	1.7	.6	2.9	3.2	.6	.0	13.3	6.0	.08	27	345
6.26A	2.2	1.0	34.8	3.7	3.4	16.2	3.3	2.5	.3	5.2	9.1	4.6	.0	7.0	6.7	.02	43	767
6.27	10.6	.3	36.5	1.2	1.8	11.9	8.8	.3	1.2	2.7	6.4	5.5	.0	8.8	4.0	.34	28	329
6.28	11.8	.0	27.4	3.4	2.3	14.0	2.4	2.0	.3	1.1	17.1	2.3	.0	9.4	6.5	.70	28	351
6.29	6.2	.2	16.7	3.2	2.5	6.5	.6	2.5	.0	5.3	21.6	4.9	.4	16.0	13.4	.64	29	474

A1 = Agglutinants
A2 = *Eoguttulina* spp.
A3 = *E. mosquensis*
A4 = *E. ubligi*
A5 = *Epistomina* spp.
A6 = *L. muensteri*
A7 = *Lenticulina* spp.
A8 = *Nodosaria/Dentalina* spp.
A9 = *O. carinatum*
A10 = *O. strumosum*
A11 = *P. riasanensis*
A12 = *S. elongata*
A13 = *S. infima*
A14 = *S. tenuissima*
15 = Restgroup
16 = P/B-ratio
17 = number of species
18 = total number

TABLE 1.2 DATA SET OF TOJEIRA 2 SECTION

Sample number	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	15	16	17	18
12.1	4.4	.0	2.8	1.2	2.3	3.7	4.6	.0	1.2	1.6	57.7	2.3	.2	7.5	10.5	.02	31	433
12.2	2.4	1.9	34.6	6.2	1.9	5.4	3.3	.0	2.9	1.9	24.0	1.4	.5	3.9	9.7	.01	27	208
12.3	3.8	1.0	4.8	1.0	1.9	3.4	5.3	.0	7.1	13.3	44.8	.0	.0	3.5	10.1	.01	28	210
12.4	2.3	.0	.0	.5	10.3	3.3	.9	.0	2.8	2.3	61.0	3.3	.0	3.3	10.0	.00	21	213
12.5	4.2	1.1	3.2	2.7	10.1	7.0	7.4	.0	2.7	3.2	42.0	1.1	1.1	4.2	9.4	.05	25	188
12.6	4.8	1.6	.0	.4	1.2	3.2	1.6	.0	4.8	3.2	64.1	6.5	.0	2.4	6.2	.00	21	248
12.7	3.8	2.4	3.8	.5	4.3	8.7	4.3	.5	1.0	1.0	38.6	5.7	1.0	12.2	14.2	.04	32	210
12.8	3.1	5.2	5.2	1.3	6.5	1.3	.4	.4	4.3	3.9	30.9	3.9	.9	7.3	25.4	.00	28	230
12.9	4.2	.5	.0	.5	4.6	4.7	.5	.0	4.1	1.8	46.8	8.3	2.8	7.9	13.3	.01	27	218
12.10	3.0	.0	.0	.4	4.6	6.8	2.1	.0	3.4	3.8	52.7	3.8	.4	7.1	11.9	.03	26	237
12.11	4.1	1.8	5.4	1.4	5.0	1.9	3.2	.5	2.3	3.2	46.8	8.3	1.8	6.6	10.1	.03	28	222
11.1	2.8	19.0	8.3	.8	5.6	5.2	.4	7.9	2.0	1.6	19.4	2.0	5.2	11.2	8.6	.76	27	252
11.2	2.7	17.9	5.0	.4	2.7	3.0	1.1	2.7	2.3	1.9	14.9	3.4	14.9	14.9	12.2	.66	27	262
11.3	4.2	26.3	2.5	.4	5.0	2.5	1.3	.8	2.9	1.3	15.0	2.1	16.3	13.4	6.0	.84	25	240
11.4	6.9	16.6	3.7	.4	2.4	2.8	2.0	7.3	4.1	.4	13.8	3.7	22.4	5.6	7.9	.24	28	246
11.5	3.4	13.8	3.8	.0	1.9	4.8	3.8	2.9	3.8	1.9	16.2	1.9	16.7	14.8	10.3	.34	30	210
11.6	2.5	19.0	5.0	.0	3.0	3.0	3.0	6.5	2.5	1.0	17.0	1.5	12.0	13.5	10.5	.93	23	200
11.7	1.1	18.6	3.8	.4	4.6	.3	2.3	4.2	1.9	.8	16.7	4.2	19.4	10.3	8.7	.40	26	263
11.8	3.6	16.4	2.5	.4	4.3	2.5	2.9	2.5	1.4	1.1	12.5	3.6	27.5	9.3	9.5	.44	26	280
11.9	3.7	6.7	5.2	1.5	5.2	3.7	4.4	8.9	3.7	.0	11.9	5.2	14.8	7.4	17.7	.11	21	135
11.10	2.2	31.1	1.9	.0	3.0	2.7	1.5	.7	3.4	.4	17.6	1.5	16.5	11.0	6.5	.75	26	267
11.11	1.9	16.7	2.7	.0	3.8	1.2	4.2	.8	3.0	.8	11.4	6.1	27.4	16.7	3.3	.29	24	263
11.12	1.7	10.9	1.1	.0	5.7	5.2	2.3	2.6	1.7	.3	15.2	4.0	29.8	11.8	7.7	.16	29	375
11.13	.4	23.2	1.1	.0	6.4	9.7	2.2	2.5	4.6	.7	16.4	.4	16.1	10.5	5.8	.78	26	280
11.14	2.4	22.8	1.6	.0	5.3	3.2	2.0	.8	6.1	.0	13.4	1.2	18.3	17.8	5.1	.81	26	246
11.15	.7	21.5	1.7	.0	4.3	2.3	3.6	6.3	3.0	.3	7.3	1.3	20.9	20.8	6.0	.80	25	302
11.16	.4	37.6	1.4	.0	2.2	3.6	2.5	9.0	3.2	.4	10.4	.7	11.5	12.3	4.8	.47	21	279
11.17	1.4	24.9	2.8	.5	5.1	3.7	4.6	8.8	3.2	.0	8.8	.5	8.3	20.9	6.5	.61	25	217
11.18	1.4	31.7	1.6	1.3	5.2	5.2	2.3	2.3	1.6	.0	11.8	2.6	15.4	13.7	3.9	.90	27	306
11.19	1.9	25.0	2.3	1.2	6.5	3.1	2.7	3.1	5.8	.4	13.5	1.5	15.4	10.7	7.2	.51	28	260

B1 = *Eoguttulina* spp.
B2 = *E. mosquensis*
B3 = *E. ubligi*
B4 = *Epistomina* spp.
B5 = *L. muensteri*
B6 = *Lenticulina* spp.
B7 = *Nodosaria/Dentalina* spp.
B8 = *P. ηasanensis*
B9 = *S. elongata*
B10 = *S. intima*
B11 = *S. tenuissima*
B12 = *O. carinatum*
B13 = *O. strumosum*
B14 = Agglutinants
15 = Restgroup
16 = P/B-ratio
17 = number of species
18 = total number

Table 1. 3 Data set of new samples of Tojeira 1 section

(F.M. Gradstein's data)

	1	2	3	4	5	6	7	8	9
T_1/1	6.07	0.00	0.92	0.00	13.79	3.68	1.29	0.37	1.47
T_1/2	1.97	0.00	2.96	0.00	20.20	6.90	1.97	0.00	0.49
T_1/2A	1.99	0.28	1.42	0.28	23.01	7.95	3.41	0.85	0.85
T_1/2B	5.68	0.44	0.00	0.00	18.34	1.75	2.62	0.00	1.31
T_1/2C	2.49	0.83	5.81	0.00	7.88	7.05	3.73	0.00	2.07
T_1/2D	1.80	0.36	0.72	0.00	19.42	3.24	1.44	0.00	3.24
T_1/3	1.89	28.77	6.60	0.00	8.96	3.07	1.89	3.77	0.00
T_1/4	4.40	11.00	3.77	0.00	12.26	1.89	10.06	7.86	15.09
T_1/5	0.36	15.07	7.72	0.00	7.35	0.73	1.84	12.50	1.47
T_1/6	0.49	27.36	4.97	0.00	13.93	1.24	0.99	14.42	4.98
T_1/7	0.73	38.63	23.23	0.00	15.16	2.20	0.49	6.36	0.98
T_1/8	0.59	25.09	4.72	0.00	24.19	2.36	2.94	9.43	3.24
T_1/9	0.00	22.39	8.95	2.98	8.21	2.24	2.24	11.19	1.49
T_1/10	0.39	18.82	15.29	0.39	11.76	1.57	2.74	7.84	4.70
T_1/11	1.17	32.42	1.95	0.00	14.84	1.56	3.90	5.86	3.51
T_1/12	0.99	25.74	1.98	0.00	7.42	1.73	2.97	3.46	2.47
T_1/13	0.00	9.16	3.05	0.76	19.84	2.67	3.82	3.05	4.20
T_1/14	0.00	18.94	3.28	0.00	18.43	2.27	4.80	10.10	3.03
T1/15	0.51	16.58	4.85	0.00	9.18	1.78	2.55	6.89	3.57
T1/16	0.00	39.05	1.90	0.00	25.71	2.40	0.95	1.90	2.40
T1/17	0.00	38.94	12.87	0.66	16.50	2.64	0.99	15.51	0.33

Table 1.3 (concluded)

	10	11	12	13	14	15	16	17	18
T_1/1	9.19	30.83	0.73	1.47	25.56	4.60	0.00	26	544
T_1/2	5.41	13.79	1.48	0.00	39.47	5.43	0.00	20	203
T_1/2A	12.50	7.95	6.82	1.14	23.01	8.52	0.00	25	352
T_1/2B	20.09	5.24	16.16	2.18	17.47	8.73	0.00	18	225
T_1/2C	6.22	28.63	8.71	1.24	16.60	8.71	0.00	25	241
T_1/2D	3.24	39.21	6.11	0.00	15.11	7.91	0.00	20	278
T_1/3	11.38	17.69	3.07	0.00	10.61	2.36	0.03	22	424
T_1/4	7.86	15.09	5.34	0.63	12.58	5.03	0.01	22	318
T_1/5	0.00	18.75	18.01	4.04	9.93	2.20	0.01	18	272
T_1/6	3.48	14.18	3.98	1.24	7.71	0.99	0.18	21	402
T_1/7	0.49	2.20	2.69	2.93	2.44	1.47	0.18	22	409
T_1/8	2.65	11.80	4.13	1.18	5.01	0.00	0.02	18	339
T_1/9	1.49	6.72	0.00	0.00	32.09	0.00	0.08	15	134
T_1/10	5.10	15.68	7.84	0.00	7.84	0.00	0.16	20	255
T_1/11	1.95	9.37	4.69	5.86	10.55	2.34	0.31	24	256
T_1/12	3.22	26.48	6.43	5.69	9.40	1.98	0.18	32	404
T_1/13	0.00	19.08	4.96	10.30	17.94	1.14	0.06	21	262
T_1/14	0.50	9.85	0.50	18.18	8.58	1.51	0.20	22	396
T_1/15	2.29	17.86	26.02	0.26	4.33	3.32	0.08	22	392
T1/16	2.85	10.47	4.76	2.85	3.33	1.42	0.77	19	210
T1/17	0.00	5.28	0.66	1.32	3.30	0.99	0.78	22	303

Table 1.4 Data set of the new samples of Tojeira 2 section (F.M. Gradstein's data).

	1	2	3	4	5	6	7	8	9
T_2/1	0.40	0.00	69.70	17.82	2.37	0.00	0.40	0.00	1.78
T_2/2	0.90	0.00	0.00	0.00	1.81	6.04	0.90	0.00	2.72
T_2/3	0.59	0.00	3.55	1.77	2.96	11.24	1.77	0.00	2.96
T_2/4	0.00	0.00	3.09	0.00	1.03	13.40	2.06	0.00	2.06
T_2/5	0.35	1.05	28.67	28.67	1.40	15.73	1.08	0.00	0.35
T_2/6	0.49	0.00	4.90	0.00	11.27	16.67	1.96	0.00	0.00
T_2/7	0.00	47.59	11.44	13.85	4.22	3.67	0.60	1.80	0.60
T_2/8	0.46	51.15	3.69	8.29	6.91	3.69	1.38	0.00	0.00
T_2/9	0.00	25.22	10.62	6.64	1.33	3.10	0.00	5.31	2.65
T_2/10	0.00	0.43	0.87	0.00	2.17	12.17	0.87	0.00	0.87
T_2/11	0.00	23.7	5.18	7.41	16.3	0.00	1.48	0.00	1.48
T_2/12	1.28	35.26	3.20	5.77	5.13	7.69	0.00	0,20	0.00
T_2/13	0.00	54.06	1.63	0.00	9.35	2.44	2.03	3.66	2.44

1 = Eoguttulina sp.

2 = Epistomina mosquensis

3 = Epistomina uhligi

4 = Epistomina sp.

5 = Lenticulina muensteri

6 = Lenticulina sp.

7 = Nodoseria/Dentalina sp.

8 = Psedolamarckina rjasanensis

9 = Spirillina elongata

Table 1.4. (concluded)

	10	11	12	13	14	15	16	17	18
T_2/1	1.78	1.19	0.00	0.00	1.39	1.58	0. 00	20	520
T_2/2	13.29	62.84	0.30	0.00	3.32	7.85	0.00	15	331
T_2/3	5.92	37.28	0.59	0.59	14.20	16.57	0.00	19	169
T_2/4	0.00	24.74	0.00	2.06	51.55	0.00	0.00	15	097
T_2/5	1.05	3.15	0.00	0.00	6.99	11.54	0.00	19	286
T_2/6	2.45	42.65	0.98	0.00	12.25	6.37	0.00	16	204
T_2/7	1.20	3.01	1.20	0.00	3.01	7.83	0.48	17	166
T_2/8	0.00	5.99	2.30	5.53	9.22	1.38	0.40	20	217
T_2/9	0.00	12.83	0.44	2.65	25.67	3.54	0.23	20	226
T_2/10	6.52	33.04	3.45	3.04	18.26	8.26	0.01	20	230
T_2/11	7.41	11.85	1.48	3.70	12.6	7.41	1.21	26	135
T_2/12	1.28	8.33	2.56	1.92	8.97	15.38	0.83	20	156
T_2/13	0.41	9.76	1.63	1.21	8.94	2.44	1.06	20	246

10 = Spirillina infima

11 = Spirillina tenuissima

12 = Ophthalmidium carinatum

13 = Ophthalmidium strumosum

14 = agglutinants

15 = Restgroup

16 = P/B - ratio

17 = Number of species

18 = Total number

CHAPTER 2: Statistical time series analysis

2.1 Introduction

Before applying the geostatistical modelling, time series models were applied as a possible strategy for explaining the available data structure from a statistical perspective.
Time series analysis does not have to fit the available data exactly. They can provide a model they only approximates the time process as long as the model explains the behavior of the available realization in a statistically adequate manner. Statistical tests are avaliable to establish whether time series explain the data structure or not.

It is noted that most statistical calculations for this project were done using the Statistical Analysis System (SAS). For this reason, a brief overview of the SAS system is given before the actual statistical tests are described later.

2.2 SAS package programs

The SAS system is a software package for data analysis. The SAS language has its own vocabulary and syntax - words and the rules for putting them together.

The statements in a SAS program are divided into two kinds of steps; DATA steps and PROC steps (or procedure steps). These are the building blocks of all SAS programs. Usually DATA steps create SAS data sets and PROC steps process SAS data sets: DATA and PROC steps can appear in any order and any number of data sets can be used. The DATA step begins with a data statement and can include any number of program statements. The PROC step asks SAS to call a procedure from its library, and execute that procedure. The PROC step begins with a PROC statement.

SAS output consists of two main parts:

1 - The SAS log which contains the SAS statements used in the job, notes, and error messages.

2 - The SAS procedure output (File name , listing).

During this research, several SAS software systems were used to assist in the geostatistical modelling.

SAS/Basics software is for the structure of the SAS language and its procedures are fundamental to the (SAS) system.

SAS/Statistics software is for the statistical analysis procedures in the SAS system. They range from simple descriptive statistics to complex multivariate techniques. Essentially, the user communicates with the SAS system by giving it a data file and the necessary PROC statements.

SAS/Graph software is a device - intelligent system for producing information and presenting it graphically. The software can be combined with other software in the SAS system. Version 5 SAS/Graph software runs on the main frame with the IBM 3179G, eight colour graphics terminal at the University of Ottawa.

SAS/ETS software is a package for econometrics and time series analysis. Procedures include;

- Time series extraction and plotting,
- Forecasting and time series analysis,
- Modeling

2 .2.1 SAS package program ARIMA

The ARIMA model and techniques are described by Box and Jenkins (1976). This model is an algebraic statement showing how a time series variable (X_t) is related to its own past values (X_{t-1}, X_{t-2}, X_{t-3}). Consider the algebraic expression,

$$X_t = C + \phi_1 X_{t-1} + a_t \quad (1)$$

In Eq.1, X_t is related to its own immediately past value (X_{t-1}). where C is a constant term, ϕ_1 is a fixed coefficient whose value determines the relationship between X_t and X_{t-1} , and the a_t term is a probabilistic " shock" (random error, white noise) element. The generating mechanisms for two common ARIMA processes are written as follows:

$$X_t = C + \phi_1 X_{t-1} + a_t \quad (2)$$
$$X_t = C - \theta_1 a_{t-1} + a_t \quad (3)$$

The term $\phi_1 X_{t-1}$ is called an *autoregressive* (abbreviated AR) process. A first order autoregressive process is abbreviated AR(1). In general, the label AR(p), means that the longest time lag attached to an

AR term in that process, is p time periods. The term $\theta_1 a_{t-1}$ is called a *moving - average* (abbreviated MA) process. Again , a first order process is abbreviated MA(1). The label MA(q) means that the longest time lag attached to a MA term in that process is q time periods. The following additional processes can be considered:

$$X_t = C + \phi_1 X_{t-1} + \phi_2 X_{t-2} + a_t \qquad (4)$$

$$X_t = C - \theta_1 a_{t-1} - \theta_2 a_{t-2} + a_t \qquad (5)$$

$$X_t = C + \phi_1 X_{t-1} - \theta_1 a_{t-1} + a_t \qquad (6)$$

Eq.4 is an AR(2) process because it contains only AR terms and the maximum time lag on the AR terms is two. Eq.5 is an MA(2) process since it has only MA terms, with a maximum time lag on the MA terms of two. Eq.6 is an example of a mixed process which contains both AR and MA terms. It is an ARMA(1,1) process.

Let the AR order of a process be designated p, where p is some non - negative integer. Let q, also a non - negative integer, be the MA order of the process. The value d, another non - negative integer stands for the number of times a realization must be differenced to achieve a stationary mean. After a differenced series has been modeled, it is integrated d times. The letter " I " in the ARIMA refers to this integration step, and it corresponds to the number of times (d) the original series has been differenced. ARIMA processes are characterized by the values of p, d, and q in this manner;
ARIMA (p, d, q) (AutoRegresive Integrated Moving Average).

It is convenient to write ARIMA models in backshift notation using the multiplicative backshift operator B. The operator B is defined such that any variable which it multiplies has its time subscript shifted back by the power of B. This is shown in Eq.7.

$$B^k X_t = X_{t-k} \qquad (7)$$

A constant is unaffected when multiplied by B, since, a constant has no time subscript.

$$B^k C = C \qquad (8)$$

This model has two advantages over many other traditional single - series methods. The first advantage is that ARIMA is derived from a solid foundation of classical probability theory and mathematical statistics. Many other historically popular univariate methods are derived by intuitive methods The second advantage is that ARIMA models are a family of models, not just a single model. Box and Jenkins (1964) have developed a strategy that guides the analyst in choosing one or more appropriate models out of this larger family of models. However, the construction of proper ARIMA models may require more experience and computer time than some of the other univariate methods.

The ARIMA method applies only to stationary time series. Although many non - stationary series arise in practice, most can be transformed into stationary series.

In ARIMA analysis, the observations in a single time series (one variable - univariate) are assumed to be statistically dependent, that is, sequentially or serially correlated.

The term X_t is a numerical value of an observation. The subscript t refers to the time period. A sequence of N observations can be represented as: $X_1 , X_2 , X_3, \ldots , X_N$. The ARIMA model describes how any given observation (X_t) is related to previous observations ($X_{t-1} , X_{t-2} , \ldots ,$). The mean of a stationary series indicates the overall level of this series. The sample mean is calculated as,

$$\bar{x} = \frac{1}{N} \sum_{t=1}^{N} X_t \qquad t = 1 , 2 , \ldots , N \quad (9)$$

The sample variance $s^2(x)$, is calculated as

$$s^2(x) = \frac{1}{N} \sum_{t=1}^{N} (X_t - \bar{x})^2 \qquad (10)$$

When the mean of a series is stationary, a new series W_t is created. The series W_t is calculated as X_t minus sample mean. Where the sample mean ($\bar{x}$) is calculated as before.

$$W_t = X_t - \bar{x} \qquad (11)$$

The new series (W_t) is indistinguishable from the series X_t , except that it has a mean of zero ($W_t = 0$). In fact, the two series X_t and W_t have all the same statistical properties except for their means. For example, the variances of the two series are identical.

2.2.2 Autocorrelation

The objective in autocorrelation analysis is to calculate a correlation coefficient (r_k), for each set of ordered pairs (W_t , W_{t+k}). The k subscript represents the number of lags or time periods within the series. After calculating estimated autocorrelation coefficients, they are plotted graphically using an estimated autocorrelation function.

The autocorrelation coefficient (r_k) measures the direction and strength of the statistical relationship between ordered pairs of observations on two random variables. It is a dimensionless number that can take on values only between - 1 and +1. A value of - 1 means perfect negative correlation and a value of +1 means perfect positive correlation. If $r_k = 0$ then X_{t+k} and X_t are not correlated at all in the available data. The formula for calculating the autocorrelation coefficients is,

$$r_k = \frac{\sum_{t=1}^{N-k} (X_t - \bar{x}) (X_{t+k} - \bar{x})}{\sum_{t=1}^{N} (X_t - \bar{x})^2} \qquad (12)$$

Eq.(12) can be written more compactly, because W_t is defined as $X_t - x$ Eq.(12) becomes,

$$r_k' = \frac{\sum_{t=1}^{N} W_t \, W_{t+k}}{\sum_{t=1}^{N} (W_t)^2} \qquad (13)$$

Many authors suggest that the maximum number of useful estimated autocorrelations is roughly N/4 where N is the number of observations.

2.3 Partial autocorrelation

An estimated partial autocorrelation function is broadly similar to an estimated autocorrelation coefficient function (a.c.f.). The idea of partial autocorrelation analysis is to see how W_t and W_{t+k} are related. The estimated partial autocorrelation coefficient measuring this relationship between W_t and W_{t+k} is designated as $\hat{\phi}_{kk}$, which is an estimation of the correspondence parameters ϕ_{kk}. The most accurate way of calculating partial autocorrelation coefficients (p.a.c.f.) is to estimate a series of least squares regression coefficients. Estimated partial autocorrelation coefficients are also found by applying regression techniques. The first step is to consider the true-regression relationship between W_{t+1} and the preceding value W_t. This is shown in Eq.(14).

$$W_{t+1} = \phi_{11} W_t + u_{t+1} \qquad (14)$$

The term ϕ_{11} is the partial autocorrelation coefficient to be estimated for k = 1, u_{t+1}, which in turn, is the error term representing all the factors affecting W_{t+1} that do not appear elsewhere in the regression equation. This is because there are no other W's between W_k and W_{t+1}. However, there is a slightly less accurate, though easily computed, way to estimate the ϕ_{kk} coefficients. It involves using the previously calculated autocorrelation coefficients (r_k). As long as the data series is stationary, the following set of recursive equations gives fairly good estimates of the partial autocorrelations (Yule - Walker equations).

$$\hat{\phi}_{11} = r_1$$

$$\phi_{kk} = \frac{\sum_{j=1}^{k-1} \hat{\phi}_{k-1,j} r_{k-j}}{1 - \sum_{j=1}^{k-1} \hat{\phi}_{k-1,j} r_j} \qquad (k = 2, 3, \ldots)$$

where $\hat{\phi}_{kj} = \hat{\phi}_{k-1,j} - \hat{\phi}_{kk} \hat{\phi}_{k-1,k-j}$
($k = 3, 4, \ldots, \; j = 1, 2, \ldots, k - 1$)

If X_t is a stationary normal distribution, then it has a constant expected mean $\mu = E(X_t)$ for all X's where E (. . .) denotes mathematical expectation.

$$\mu = E(X_t) = E(X_{t+m}) \qquad (16)$$

The estimated constant variance, $\sigma^2(x) = \nu_o(x) = E(X_t - \mu)^2$ for all X's satisfies

$$\sigma^2(x = E(X_{t+m} - \mu)^2 \qquad (17)$$

and the constant autocovariance is

$$\nu_k(x) = E[(X_t - \mu)(X_{t+m} - \mu)]^2.$$

For any two X's separated by k time periods (LAG), the constant autovariance is,

$$\nu_k = E[(X_k - \mu)(X_{t+k} - \mu)] = E[(X_{t+m} - \mu)(X_{t+k+m} - \mu)]^2$$

When k = 0, ($X_t - \mu$) times ($X_{t+k} - \mu$) is simply ($X_t - \mu$)^2 in which case $\nu_k(x) = 0 = \nu_o(x) = \sigma^2(x)$. The estimated autocorrelation coefficients are derived as shown below

$$\rho_k = \frac{\nu_k}{\nu_o} \qquad (18)$$

The estimation of auto and partial correlation coefficient values could be chosen according to different criterian. Box and Jenkins (1976) favor choosing according to the maximum likelihood (ML) criterion. However ML estimates of ARIMA procedure may require relatively large amounts of computer time. For this reason, Box and Jenkins (1976) suggest using the least - squares (LS) criterion. LS estimates, are either exactly or very nearly ML estimates.

2.2.3.1 Testing autocorrelation coefficients

An estimated autocorrelation coefficient (r_k) and theoretical autocorrelation coefficient (ρ_k) are not exactly equal to each other due to sampling error. Bartlett (1946) has derived an approximate expression for the standard error of the sampling distribution of the r_k values. This estimated standard error s(r_k) is calculated as follows,

$$s(r_k) = (1 + 2\sum r_j^2)^{1/2} N^{-1/2} \qquad (19)$$

using the null hypothesis $H_o : \rho_k = 0$ for $k = 1, 2, \ldots, N$. It is common when using the estimated $s(r_k)$ in place of the true standard error $s(r_k)$ to refer to the t - distribution in this way,

$$t_{r_k} = \frac{r_k - \rho_k}{s(r_k)} \tag{20}$$

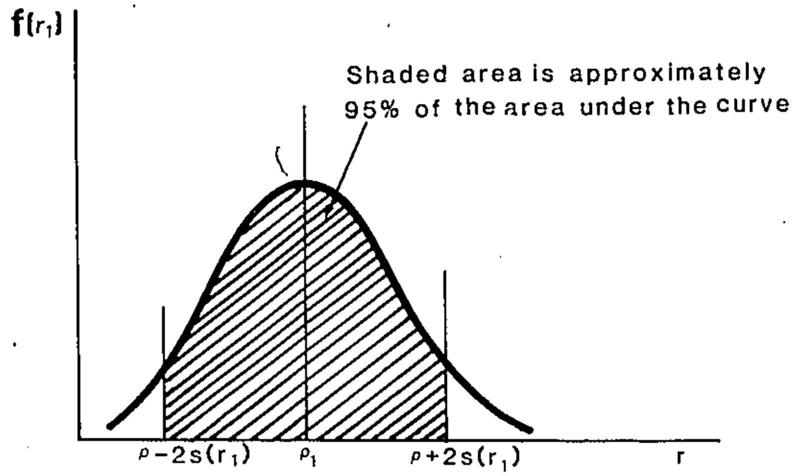
Fig. 2.1 illustrates these ideas. The label on the horizontal axis shows that this is a distribution of all possible values of r_1 for a certain sample size N. This distribution is centered on the parameter ρ_1 which is unknown. Since this is approximately a normal (or t) distribution with an estimated standard errror, $s(r_1)$ given by Eq. 19, the interval $\rho_1 \pm 2s(r_1)$ contains about 95 % of all possible r_1 values. This is represented by the shaded area under the curve. If MA is white noise, there would be no autocorrelation within the process and the MA order would be zero. In that case Eq. 19 would be

$$s(r_k) = N^{-1/2} \tag{21}$$

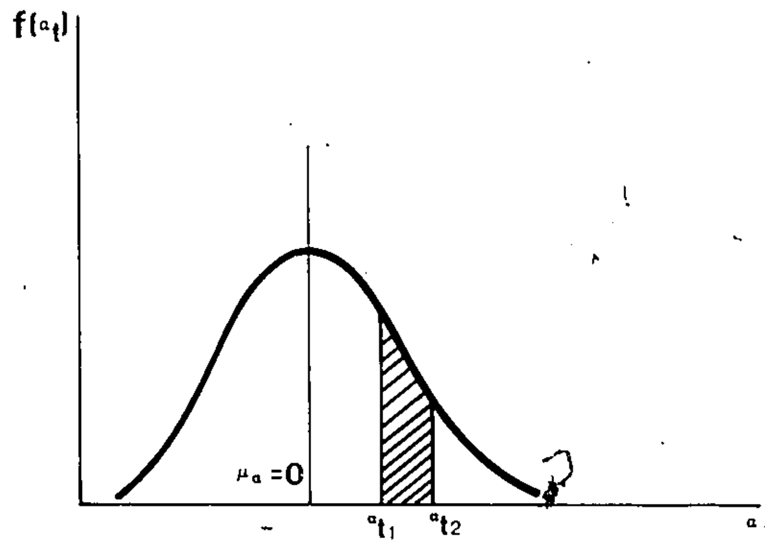
for all k.

2.2.3.2 Testing partial autocorrelation coefficients

Partial autocorrelation coefficients can also be tested and their statistical significance estimated. The required estimated standard error is



2.1



2.2

Figure 2.1 Example of approximately normal (or t) sample distribution for r with estimated standard error $s(r_1)$ given by Eq. 19

Figure 2.2 Normal distirbution of white noise (random shock) a_t (after Pankratz, 1983)

$$s(\hat{\phi}_{kk}) = N^{-1/2} \quad (22)$$

Testing the null hypothesis $H_o : \phi_{11} = 0$ and t - statistic:

$$t_{\hat{\phi}_{11}} = \frac{\hat{\phi}_{11} - \phi_{11}}{s(\hat{\phi}_{11})} \quad (23)$$

2.2.3.3 Stationarity

Stationarity implies that the AR coefficients must satisfy certain conditions. If p = 0 , the series are either a pure MA model or a white - noise series. All pure MA models and white noise are stationary, so there are no stationary conditions to check. For an AR(1) or ARMA(1,q) process, the stationarity requirement is that the absolute value of ϕ_1 must be less than one ($|\phi_1| < 1$). The ϕ_1 is not known, instead of it $\hat{\phi}_1$ can be used. Table 2.1 shows a summary of stationary conditions for the ARMA procedure.

TABLE 2.1 Stationary conditions of time series (after Pankratz, 1983)

Model type	Stationarity conditions
ARMA (0,q)	Always stationary
AR(1) or ARMA(1,q)	$\lvert\phi_1\rvert < 1$
AR(2) or ARMA(2,q)	$\lvert\phi_2\rvert < 1$
	$\phi_2 + \phi_1 < 1$
	$\phi_2 - \phi_1 < 1$

2.3 The random shock (white noise, random error) model

The a_t terms in an ARIMA process are usually assumed to be normally indentical and independently distributed random variables with a mean of zero and a constant variance.Those variables are often called " white noise ". Fig. 2.2 illustrates this idea with a normal distribution centered on zero. The horizontal axis shows the values are independently distributed and therefore not autocorrelated. That is, knowing the set of past random shocks (a_{t-1}, a_{t-2}, a_{t-3}, . . . ,). would not help to predict the current shock a_t. Random shocks (a_t) are statistically independent, meaning that they are not autocorrelated. They can not be observed in practice but, they can be estimated. The residuals ($\hat{a}_t$) are calculated from the estimated model. The residuals can be tested.

If the random shocks are serially correlated, then there is an autocorrelation pattern in X_t that has not been accounted for by the AR and MA terms in that model. If the residuals are autocorrelated, they are not white noise, so that the other model should be searched for residuals.

A residual autocorrelation coefficient is basically the same as any other estimated autocorrelation coefficient. The only difference is that the residuals ($\hat{a}_t$) from an estimated model instead of observations in a realization (X_t), are used to calculate the autocorrelation coefficients. Eq. 11 is used for this calculation,

$$r_k(\hat{a}) = \frac{\sum_{t=1}^{N-k} (\hat{a}_t - \bar{a})(\hat{a}_{t+k} - \bar{a})}{\sum_{t=1}^{N} (\hat{a}_t - \bar{a})^2} \qquad (24)$$

where $r_k(\hat{a})$ is the residual autocorrelation coefficient.

If the random shocks are uncorrelated, estimation of $(\hat{a})$ should also be uncorrelated. Therefore, the residual autocorrelation coefficients for an ARIMA model will ideally have autocorrelation coefficients that are all statistically zero. But all residual autocorrelations cannot be expected to be zero. The reason is, that the residuals are calculated from a calculation using only estimates of the ARIMA coefficients (not their true values). Therefore, the sampling error will cause some residual autocorrelations to be nonzero.

The AR (1) procedure can be written as $(1 - \phi B)W_t = a_t$ or

$$X_t = \mu(1 - \phi_1) + \phi_1 X_{t-1} + a_t \qquad (25)$$

The random shock a_t can not be observed during the period $t - 1$ but μ, ϕ_1 and X_{t-1} are known parameters. Using μ, ϕ_1 and X_{t-1} to find the calculated value of X_t designated $\hat{X}_t$:

$$\hat{X}_t = \mu(1 - \phi_1) + \phi_1 X_{t-1} \qquad (26)$$

at time t, X_t can be observed, then the random shock a_t can be found by subtracting the calculated value $\hat{X}_t$ from the observed value X_t in Eq 25

$$X_t - \hat{X}_t = a_t \qquad (27)$$

μ and ϕ_1 are known, but in practice, parameters of the ARIMA procedure are not known. They must be estimated from the data. These estimates in the present case, are $\hat{\mu}$ and $\hat{\phi}_1$. Modifying (26) accordingly, the calculated value $\hat{X}_t$ becomes

$$\hat{X}_t = \hat{\mu} (1 - \hat{\phi}_1) + \hat{\phi}_1 X_{t-1} \qquad (28)$$

When $\hat{X}_t$ is calculated from estimates of parameters rather than known parameters, (27) does not give the exact value of the random shock a_t. An estimate of the random shock a_t is denoted $\hat{a}_t$ and called a residual:

$$X_t - \hat{X}_t = \hat{a}_t \qquad (29)$$

Eq. 29 is the definition of a residual for any ARIMA model.
AR(1), $\hat{X}_t$ depends on $\hat{\mu}$ and $\hat{\phi}_1$ along with X_{t-1}, as shown Eq. 28
MA(1), $\hat{X}_t$ depends on $\hat{\mu}$ and $\hat{\theta}_1$ along with $\hat{a}_{t-1}$.

$$\hat{X}_t = \hat{\mu} - \hat{\theta}_1 \hat{a}_{t-1} \qquad (30)$$

ARIMA(1,1), $\hat{X}_t$ depends on $\hat{\mu}$, $\hat{\phi}_1$, $\hat{\theta}_1$ along with X_{t-1} and $\hat{a}_{t-1}$;

$$\hat{X}_t = \hat{\mu} (1 - \hat{\phi}_1) + \hat{\phi}_1 X_{t-1} - \hat{\theta}_1 \hat{a}_{t-1} \qquad (31)$$

$$\hat{C} = \hat{\mu} (1 - \hat{\phi}_1) \qquad (32)$$

2.3.1 t_b - test

Approximate t values are calculated for residual autocorrelation coefficients using Bartlett's approximation for standard error of estimated autocorrelations. The null hypothesis; $H_o: \rho_k \ (\hat{a}) = 0$ can be tested for each residual autocorrelation coefficient and the number of standard errors (t) away from zero each residual autocorrelation coefficient falls can be calculated.

$$t_b = \frac{r_k (\hat{a}) - 0}{s \left[r_k(\hat{a}) \right]} \qquad (33)$$

In practice, if the absolute value of a residual autocorrelation coefficient t - value is less than 1.25 at lags 1, 2, and 3, and less than about 1.6 at larger lags, it means that the random shocks at that lag are independent. If any residual autocorrelation coefficient t - value is larger than the critical values suggested above. The null hypothesis, is rejected and it is assumed that the random shocks from the estimated model are correlated.

2.3.2 Chi - squared test

There is another way of estimating residual autocorrelation coefficient t - values. Ljung and Box (1978) suggest a test statistic based on all the residual autocorrelations as a set. K is a number of residual autocorrelations. The null hypothesis is tested about the correlations among the random shocks,

$$H_o: \rho_1(a_t) = \rho_2(a) = \ldots = \rho_k(a) = 0$$

with this test statistic

$$Q = N(N+2)\sum_{k=1}^{K}(N-k)^{-1} r_k^2(\hat{a}) \qquad (34)$$

where N is the number of observations using the estimated model. The statistic Q approximately follows a chi - squared distribution with (K - m) degrees of freedom, where m is the number of parameters estimated in the ARIMA model. If Q is large (significantly different from zero) then the residual autocorrelations, as a set, are significantly different from zero, and the random shocks of the estimated model are probably autocorrelated.

2.4 Periodogram

A time series with N observations can be represented by a trigonometric polynomial,

$$X_t = a_o/2 + \sum (a_k \cos w_k t + a_b \sin w_k t) \qquad (35)$$

where $\omega = \frac{2\pi k}{N}$ $\qquad k = 0, 1, 2, \ldots, m$

$$a_k = \frac{\sum_{t=1}^{N} x_t \cos\omega_k t}{N} \qquad (36)$$

$$b_k = \frac{2\sum_{t=1}^{N} x_t \sin\omega_k t}{N} \qquad (37)$$

$$a_o = x/2$$

$m = N/2$ (if N is even) and $m = (N - 1)/2$ (if N is odd)

The periodogram is defined as

$$I_k (\omega_k) = N (a^2_k + b^2_k) /2 \qquad (38)$$

for the decomposition of the process into two - degrees - of - freedom components for each of the m frequencies. When N is even, the last periodogram value is a one - degree - of - freedom component.

If (X_t) is a sequence of normal independent (0, σ^2) random variables, then the a_k and b_k , being linear combinations of the X_t , will be normally distributed. Since the sine and cosine functions are orthogonal, (Fuller, 1969), the a_k and b_k are independent.

Fuller (1969) suggested that the distributional properties of the periodogram ordinates are easily obtained when the time series is normal white noise.

2.4.1 Fisher's kappa test

A statistic that can be used to test the white noise hypothesis is

$$\xi = \left[(1/m) \sum I_n(\omega_k)\right]^{-1} I_n(L) \qquad (39)$$

where $I_n(L)$ is the largest periodogram ordinate in a sample of m periodogram ordinates each with two degrees of freedom. A table of the distribution of ξ is given by Davis (1941).

Under the null hypothesis that a time series is normal white noise, the periodogram ordinates are multiples of independent chi - squares, each with two degrees of freedom.

If the largest ordinate is $P_{max.}$, the sum of all periodogram values is P and sample size is m - 1. Then Fisher's kappa test has

$$\text{Test Statistic} = (m - 1) P_{max} / P \qquad (40)$$

2.4.2 Kolmogorov - Smirnov test

Bartlett (1946) suggested a test based on the normalized cumulative periodogram which is a uniform (0, 1) distribution. Therefore, the normalized periodogram as a sample distribution function is plotted and the Kolmogorov - Smirnov test of the hypothesis function is plotted and the Kolmogorov - Smirnov test of the hypothesis is applied to tell whether the original time series is white noise or not.

The cumulative periodogram is plotted against k. The upper and lower bounds are drawn according to the Kolmogorov - Smirnov test. If the normalized cumulative periodogram passes the 5% or 95% boundary lines, the data rejects the hypothesis of independence.

Birnbaum (1952) indicated that for $(m - 1) > 30$, the 95% point for the Kolmogorov - Smirnov statistic is approximately $1.36\ (m1)^{1/2}$ and the 99% point is approximately $1.63\ (m - 1)^{1/2}$.

The periodogram was calculated with a SAS package program called the SPECTRA procedure. SPECTRA creates an output whose variables can contain values of the periodograms and estimates of spectral densities. The periodogram ordinates can be smoothed by a moving average to produce estimated spectral densities. SPECTRA can also test whether or not the data are white noise, according to Fisher's kappa and Bartlett's Kolmogorov - Smirnov test.

CHAPTER 3: Geostatistical model:

3.1 Correlogram

A set of autocorrelation coefficients is a graph called a correlogram in which r_k is plotted against the lag k. The correlogram is helpful in identifying that type of ARIMA model gives the best representation of observed values.

For stationary series, the correlogram is compared with the theoretical autocorrelation functions of different ARMA processes in order to choose the one that is most appropriate. The autocorrelation coefficient function of the MA(q) process is easy to recognize as it will ' cut off ' at lag q, whereas the autocorrelation coefficient function (a.c.f) of an AR(p) process is a mixture of damped exponentials and sinusoids and dies out slowly.

If a non - stationary series contains a trend, then the values of r_k may not come down to zero quickly. This indicates a non - stationary process.

3.2 Trend

Agterberg (1974a) suggested that the systematic variations called " trends " or the spatial variablity of measurable features can be divided into regional variations of a systematic nature and more local, unpredictable, fluctuations.

AR processes have theoretical a.c.f'.s which tail off toward zero. This tailing off might follow a simple exponential decay pattern, a damped sine wave, or a more complicated decay or wave pattern. Fig. 3.1 shows the theoretical a.c.f.s and p.a.c.f.s for two types of stationary AR(1) processes. Any stationary AR(1) process has a theoretical a.c.f. showing exponential decay and a p.a.c.f. with an asterisk at lag 1. If ϕ_1 is positive the a.c.f. decays on the positive side and the p.a.c.f asterisk is positive. This is illustrated by Fig. 3.1.

If ϕ_1 is negative, the AR(1) a.c.f decays with alternating signs, starting from the negative side while the p.a.c.f asterisk is negative. This is illustrated by Fig. 3.2.

MA processes have a theoretical p.a.c.f which tails off to zero after lag q. This tailing off may be either some kind of exponential decay or some type of damped wave pattern. Fig 3.3 shows two MA(1) theoretical a.c.f's and p.a.c.f's. Any MA(1) process has a theoretical a.c.f with an asterisk at lag 1 followed by a cut-off to zero, and a theoretical p.a.c.f. which tails off toward zero.

If θ_1 is negative, the asterisk in the a.c.f is positive whereas the p.a.c.f. decays exponentially, with an alternating sign starting on the positive side. This is illustrated by Fig. 3.3.

If θ_1 is positive, the a.c.f. asterisk is negative, while the p.a.c.f. decays exponentially on the negative side. This is illustrated by Fig. 3.4. ARMA processes or mixed processes have theoretical a.c.f'.s with both AR and MA characteristics. The a.c.f. tails off toward zero after the first q - p lags with either exponential decay or a damped

Figure 3.1 Examples of theoretical a.c.f.'s and p.a.c.f.'s for stationary AR(1) processes with $\phi_1>0$

```
                                 SAS                                            3
                                          14:27 WEDNESDAY, APRIL 29, 1987
                            ARIMA PROCEDURE

                  NAME OF VARIABLE         =          B1
                  MEAN OF WORKING SERIES=    2.84667
                  STANDARD DEVIATION      =  1.43079
                  NUMBER OF OBSERVATIONS=         30
                            AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   2.04716    1.00000   |                    |********************|
  1  0.952839    0.46545   |                    |*********           |
  2  0.638966    0.31212   |                    |******              |
  3  0.440271    0.21506   |                    |****                |
  4  0.728687    0.35595   |                    |*******             |
  5  0.721859    0.35262   |                    |*******             |
  6  0.564276    0.27564   |                    |******              |
  7 -0.148975   -0.07277   |                   *|                    |
  8  0.178041    0.08697   |                    |**                  |
  9   0.01608    0.00785   |                    |                    |
 10 0.0867185    0.04236   |                    |*                   |
                           '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1   -0.35597   |             *******|                    |
           -0.01553   |                    |                    |
            0.05279   |                    |*                   |
           -0.05504   |                   *|                    |
       5   -0.12524   |                ****|                    |
       6   -0.20078   |                ****|                    |
       7    0.33761   |                    |*******             |
       8   -0.19931   |                ****|                    |
       9    0.07789   |                    |**                  |
      10    0.03349   |                    |*                   |

                        PARTIAL AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1    0.46545   |                    |*********           |
       2    0.12189   |                    |**                  |
       3    0.03986   |                    |*                   |
       4    0.28239   |                    |******              |
       5    0.12452   |                    |**                  |
       6    0.00929   |                    |                    |
       7   -0.37665   |            ********|                    |
       8    0.17039   |                    |***                 |
       9   -0.16726   |                 ***|                    |
      10   -0.05192   |                   *|                    |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   24.65  6  0.000  0.465  0.312  0.215  0.356  0.353  0.276
```

```
                                 SAS                                            4
                                          14:27 WEDNESDAY, APRIL 29, 1987
                  ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                       APPROX.
          PARAMETER      ESTIMATE    STD ERROR    T RATIO  LAG
          MU               2.7608     0.844737       3.27    0
          MA1,1          0.606397     0.331335       1.83    1
          AR1,1          0.888313       0.2109       4.21    1

          CONSTANT ESTIMATE  =  0.308345

          VARIANCE  ESTIMATE =   1.66459
          STD ERROR ESTIMATE =   1.29019
          AIC                =    103.73
          SBC                =   107.934
          NUMBER OF RESIDUALS=        30

                     CORRELATIONS OF THE ESTIMATES

                            MU      MA1,1      AR1,1
               MU         1.000    -0.488     -0.574
               MA1,1     -0.488     1.000      0.898
               AR1,1     -0.574     0.898      1.000

                  AUTOCORRELATION CHECK OF RESIDUALS

   TO   CHI                          AUTOCORRELATIONS
  LAG  SQUARE DF   PROB
    6    7.00  3  0.072  0.060 -0.103 -0.192  0.164  0.258  0.203
   12   16.98  9  0.049 -0.297  0.098  0.030  0.181 -0.162 -0.236
   18   24.22 15  0.061 -0.133  0.245 -0.091 -0.018 -0.140 -0.090
   24   27.22 21  0.164  0.026 -0.023 -0.134  0.011 -0.077  0.031

               MODEL FOR VARIABLE B1
               ESTIMATED MEAN      =    2.7608

                AUTOREGRESSIVE FACTORS
                                 FACTOR 1
               1-.888313B**(1)

                MOVING AVERAGE FACTORS
                                 FACTOR 1
               1-.606397B**(1)
```

Figure 3.2 Examples of theoretical a.c.f.'s and p.a.c.f.'s for stationary AR(1) processes with $\phi_1 < 0$

```
                              SAS                                    7
                    14:27 WEDNESDAY, APRIL 29, 1987
                 ARIMA PROCEDURE

           NAME OF VARIABLE            =       B6
           MEAN OF WORKING SERIES      =  3.90333
           STANDARD DEVIATION          =  2.06292
           NUMBER OF OBSERVATIONS      =       30
                   AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    4.25566     1.00000 |                    |********************|
  1  -0.552789    -0.12990 |             .   ***|        .           |
  2  -0.406812    -0.09559 |             .    **|        .           |
  3   0.437443     0.10279 |             .      |**      .           |
  4   0.085343     0.02005 |             .      |        .           |
  5   0.817843     0.19218 |             .      |****    .           |
  6   -1.27698    -0.30007 |             . ******|       .           |
  7   0.236197     0.05550 |             .      |*       .           |
  8  0.0830526     0.01952 |             .      |        .           |
  9  -0.385826    -0.09066 |             .    **|        .           |
 10  -0.292304    -0.06869 |             .     *|        .           |
                            '.' MARKS TWO STANDARD ERRORS

                 INVERSE AUTOCORRELATIONS

      LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
        1     0.01110 |             .      |        .           |
        2     0.17511 |             .      |****    .           |
        3    -0.18919 |             .  ****|        .           |
        4     0.03787 |             .      |*       .           |
        5    -0.21713 |             .  ****|        .           |
        6     0.22299 |             .      |****    .           |
        7    -0.03498 |             .     *|        .           |
        8     0.10951 |             .      |**      .           |
        9     0.04786 |             .      |*       .           |
       10     0.11592 |             .      |**      .           |

                 PARTIAL AUTOCORRELATIONS

      LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
        1    -0.12990 |             .   ***|        .           |
        2    -0.11440 |             .    **|        .           |
        3     0.07636 |             .      |**      .           |
        4     0.03533 |             .      |*       .           |
        5     0.22582 |             .      |*****   .           |
        6    -0.26738 |             . *****|        .           |
        7     0.03030 |             .      |*       .           |
        8    -0.08778 |             .    **|        .           |
        9    -0.03499 |             .     *|        .           |
       10    -0.13952 |             .   ***|        .           |

          AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                       AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    6.28  6  0.392 -0.130 -0.096  0.103  0.020  0.192 -0.300
```

```
                              SAS                                    8
                    14:27 WEDNESDAY, APRIL 29, 1987
           ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                     APPROX.
   PARAMETER      ESTIMATE        STD ERROR    T RATIO  LAG
   MU              3.90487         0.329428      11.85    0
   MA1,1          0.286939          1.23055       0.23    1
   AR1,1          0.143605          1.27118       0.11    1

   CONSTANT ESTIMATE    =   3.34411

   VARIANCE ESTIMATE    =   4.62506
   STD ERROR ESTIMATE   =   2.15059
   AIC                  =   133.943
   SBC                  =   138.146
   NUMBER OF RESIDUALS=          30

          CORRELATIONS OF THE ESTIMATES

                   MU      MA1,1      AR1,1
   MU           1.000     -0.011     -0.013
   MA1,1       -0.011      1.000      0.989
   AR1,1       -0.013      0.989      1.000

        AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                      AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    4.66  3  0.198  0.001 -0.059  0.107  0.051  0.163 -0.274
 12    7.20  9  0.617  0.022  0.001 -0.109 -0.105 -0.136 -0.097
 18   12.28 15  0.658 -0.135  0.096 -0.088 -0.148  0.142  0.018
 24   16.51 21  0.740  0.115 -0.080 -0.068  0.120 -0.037  0.017


   MODEL FOR VARIABLE B6
   ESTIMATED MEAN        =   3.90487

    AUTOREGRESSIVE FACTORS
                   FACTOR 1
   1-.1436050**(1)

    MOVING AVERAGE FACTORS
                   FACTOR 1
   1-.2869398**(1)
```

Figure 3.3 Examples of theoretical a.c.f.'s and p.a.c.f.'s for stationary AM(1) processes with $\theta_1<0$

```
                                   SAS                                          1
                                        14:27 WEDNESDAY, APRIL 29, 1987
                            ARIMA PROCEDURE

                 NAME OF VARIABLE          =         A7
                 MEAN OF WORKING SERIES=     4.73548
                 STANDARD DEVIATION      =   2.81477
                 NUMBER OF OBSERVATIONS=          31
                                AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    7.92293     1.00000  |                    |********************|
  1    1.10392     0.13933  |                 .  |***   .             |
  2 -0.0276046    -0.00348  |                 .  |      .             |
  3    1.22344     0.15442  |                 .  |***   .             |
  4   -1.09178    -0.13780  |                 ***|      .             |
  5   0.754833     0.09527  |                .   |**     .            |
  6  -0.727153    -0.09178  |                .  **|      .            |
  7  -0.841242    -0.10618  |                .  **|      .            |
  8  -0.607848    -0.07672  |                .  **|      .            |
  9   -1.96482    -0.24799  |                .*****|     .            |
 10   -1.67154    -0.21097  |                . ****|      .           |
                            '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.38413  |          ********|                    |
            2     0.40548  |               .  |********            |
            3    -0.37965  |          ********|                    |
            4     0.37481  |               .  |*******             |
            5    -0.30921  |               .******|    .           |
            6     0.25903  |               .  |*****  .            |
            7    -0.14434  |               .   ***|    .           |
            8     0.15599  |               .  |***    .            |
            9     0.00262  |               .  |       .            |
           10     0.16411  |               .  |***    .            |

                          PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.13933  |               .  |***    .            |
            2    -0.02335  |               .  |       .            |
            3     0.16139  |               .  |***    .            |
            4    -0.19177  |               . ****|    .            |
            5     0.16925  |               .  |***    .            |
            6    -0.20026  |               . ****|    .            |
            7     0.03173  |               .  |*      .            |
            8    -0.18538  |               . ****|    .            |
            9    -0.11470  |               .   **|    .            |
           10    -0.25130  |               .*****|    .            |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                            AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    2.95  6  0.814  0.139 -0.003  0.154 -0.138  0.095 -0.092
```

```
                     SAS                                          2
                              14:27 WEDNESDAY, APRIL 29, 1987
            ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                     APPROX.
  PARAMETER     ESTIMATE          STD ERROR    T RATIO  LAG
  MU             4.72516          0.598224        7.90    0
  MA1,1        -0.163017          0.187074       -0.87    1

  CONSTANT ESTIMATE  =    4.72516

  VARIANCE  ESTIMATE =      8.275
  STD ERROR ESTIMATE =    2.87663
  AIC                =    155.444
  SBC                =    158.312
  NUMBER OF RESIDUALS=         31

            CORRELATIONS OF THE ESTIMATES

                         MU     MA1,1
            MU        1.000     0.030
            MA1,1     0.030     1.000

          AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                         AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    3.64  4  0.457 -0.006 -0.025  0.177 -0.185  0.14  -0.093
 12   10.47 10  0.400 -0.082 -0.039 -0.221 -0.156 -0.219  0.105
 18   16.63 16  0.410 -0.126 -0.119  0.013 -0.146  0.196  0.041
 24   22.68 22  0.420  0.044  0.238  0.020  0.043 -0.050 -0.019

      MODEL FOR VARIABLE A7
      ESTIMATED MEAN        =   4.72516

       MOVING AVERAGE FACTORS
                        FACTOR 1
      1+0.163017B**(1)
```

Figure 3.4 Examples of theoretical a.c.f.'s and p.a.c.f.'s for stationary AM(1) processes with $\theta_1 > 0$

SAS 9
14:27 WEDNESDAY, APRIL 29, 1987
ARIMA PROCEDURE

NAME OF VARIABLE = B7
MEAN OF WORKING SERIES= 2.75667
STANDARD DEVIATION = 1.57452
NUMBER OF OBSERVATIONS= 30

AUTOCORRELATIONS

LAG	COVARIANCE	CORRELATION
0	2.47912	1.00000
1	-0.511729	-0.20642
2	1.02654	0.41407
3	-0.437066	-0.17630
4	0.537716	0.21690
5	-0.248813	-0.10036
6	0.31268	0.12613
7	-0.870349	-0.35107
8	-0.33489	-0.13508
9	-0.209986	-0.08470
10	-0.288159	-0.11623

'.' MARKS TWO STANDARD ERRORS

INVERSE AUTOCORRELATIONS

LAG	CORRELATION
1	0.10318
2	-0.37825
3	0.02025
4	0.03499
5	-0.21618
6	-0.11661
7	0.36096
8	0.19961
9	-0.09582
10	-0.00428

PARTIAL AUTOCORRELATIONS

LAG	CORRELATION
1	-0.20642
2	0.38800
3	-0.05398
4	0.03618
5	0.01862
6	0.01705
7	-0.35606
8	-0.37136
9	0.14940
10	0.00678

AUTOCORRELATION CHECK FOR WHITE NOISE

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	11.15	6	0.084	-0.206	0.414	-0.176	0.217	-0.100	0.126

SAS 10
14:27 WEDNESDAY, APRIL 29, 1987
ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

ESTIMATES DID NOT IMPROVE AFTER HIGH RIDGE.
ESTIMATES MIGHT NOT HAVE CONVERGED.

PARAMETER	ESTIMATE	APPROX. STD ERROR	T RATIO	LAG
MU	2.74923	0.259869	10.58	0
MA1,1	0.119394	0.187791	0.64	1

CONSTANT ESTIMATE = 2.74923

VARIANCE ESTIMATE = 2.59019
STD ERROR ESTIMATE = 1.60941
AIC = 115.633
SBC = 118.435
NUMBER OF RESIDUALS= 30

CORRELATIONS OF THE ESTIMATES

	MU	MA1,1
MU	1.000	0.002
MA1,1	0.002	1.000

AUTOCORRELATION CHECK OF RESIDUALS

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	7.81	4	0.099	-0.044	0.400	-0.108	0.197	-0.070	0.075
12	17.44	10	0.065	-0.370	-0.195	-0.125	-0.141	-0.072	-0.077
18	23.91	16	0.092	-0.049	-0.107	0.155	-0.124	0.164	-0.115
24	30.51	22	0.107	0.016	-0.117	0.121	-0.087	0.121	-0.071

MODEL FOR VARIABLE B7
ESTIMATED MEAN = 2.74923

MOVING AVERAGE FACTORS
FACTOR 1
1-.119394B**(1)

sine wave. The theoretical p.a.c.f. tails off to zero after the first p - q lags. Fig. 3.5 shows theoretical a.c.f's and p.a.c.f's for some types of ARMA (1, 1) processes.

Non - stationary series show a trend, which may be either in the mean , the variance or both. The series can be decomposed into two parts. The first one, representing the mean of the series, accounts for the nonstationary trend by a deterministic function, which depends on the origin. The second one is a stochastic part with zero mean. Pandit et al. 1983 has shown that many of the nonstationary data can be modelled by explicitly including polynomial, exponential or sinusoidal functions, dependent on the choice of origin, to represent the mean of the series. Furthermore, the series with a nonstationary variance may be stationary in the natural logarithm.

3.2.1 Exponential trend

Exponential trends very often arise in the data obtained from scientific applications. Many of these systems can be well represented by differential equations with constant coefficients, and the nature of noise is generally unknown. If the constant of proportionality is denoted by c, then the equation of the system is

$$\frac{dX(t)}{dt} = -cX(t) \qquad (41)$$

Figure 3.5 Examples of theoretical a.c.f.'s and p.a.c.f.'s two stationary ARMA(1,1) processes

```
                              SAS                                   5
                                   14:27 WEDNESDAY, APRIL 29, 1987
                          ARIMA PROCEDURE

              NAME OF VARIABLE        =        B5
              MEAN OF WORKING SERIES=     4.49667
              STANDARD DEVIATION     =    2.12783
              NUMBER OF OBSERVATIONS=          30
                          AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    4.52766     1.00000  |                    |********************|
  1   0.709677     0.15674  |                .   |***  .              |
  2   -1.67178    -0.36924  |                *******|   .             |
  3  -0.614223    -0.13566  |               .  ***|     .            |
  4   0.292387     0.06458  |              .      |*     .            |
  5   0.139454     0.03080  |              .      |*     .            |
  6   0.213731     0.04721  |              .      |*     .            |
  7 -0.0601248    -0.01328  |              .      |      .            |
  8  -0.643481    -0.14212  |              .   ***|.     .            |
  9  -0.522414    -0.11538  |              .    **|      .            |
 10  0.0617407     0.01364  |              .      |      .            |
                              '.' MARKS TWO STANDARD ERRORS

                    INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    -0.21865  |               .  ****|     .          |
           2     0.36645  |               .      |*******         |
           3     0.02426  |               .      |     .          |
           4     0.02148  |               .      |     .          |
           5     0.08662  |               .      |**   .          |
           6     0.01198  |               .      |     .          |
           7     0.05845  |               .      |*    .          |
           8     0.10023  |               .      |**   .          |
           9     0.05235  |               .      |*    .          |
          10     0.04799  |               .      |*    .          |

                    PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1     0.15674  |               .      |***  .          |
           2    -0.40372  |               ********|    .          |
           3     0.01085  |               .      |     .          |
           4    -0.06937  |               .     *|     .          |
           5    -0.02711  |               .     *|     .          |
           6     0.06307  |               .      |*    .          |
           7    -0.05135  |               .     *|     .          |
           8    -0.11133  |               .    **|     .          |
           9    -0.09157  |               .    **|     .          |
          10    -0.06065  |               .     *|     .          |

              AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                        AUTOCORRELATIONS
LAG  SQUARE DF    PROB
  6    6.42  6   0.378  0.157 -0.369 -0.136  0.065  0.031  0.047
```

```
                              SAS                                   6
                                   14:27 WEDNESDAY, APRIL 29, 1987
                 ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                      APPROX.
           PARAMETER    ESTIMATE    STD ERROR   T RATIO  LAG
           MU            4.50075     0.483908      9.30    0
           MA1,1       -0.647902     0.376026     -1.72    1
           AR1,1       -0.298897     0.446744     -0.67    1

           CONSTANT ESTIMATE  =   5.84601

           VARIANCE  ESTIMATE =   4.44094
           STD ERROR ESTIMATE =   2.10735
           AIC                =   132.909
           SBC                =   137.112
           NUMBER OF RESIDUALS=        30

                     CORRELATIONS OF THE ESTIMATES

                              MU     MA1,1    AR1,1
                  MU       1.000    -0.003    0.009
                  MA1,1   -0.003     1.000    0.904
                  AR1,1    0.009     0.904    1.000

                    AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                          AUTOCORRELATIONS
LAG  SQUARE DF    PROB
  6    3.00  3   0.392 -0.054 -0.234 -0.122  0.089 -0.019  0.037
 12    5.92  9   0.748 -0.017 -0.077 -0.140  0.086 -0.166 -0.002
 18    9.15 15   0.870 -0.073  0.092  0.010  0.074 -0.164  0.028
 24   14.82 21   0.832  0.081  0.109  0.048 -0.139 -0.091  0.041


            MODEL FOR VARIABLE B5
            ESTIMATED MEAN        =   4.50075

             AUTOREGRESSIVE FACTORS
                             FACTOR 1
            1+0.298897B**(1)

              MOVING AVERAGE FACTORS
                             FACTOR 1
            1+0.647902B**(1)
```

```
                              SAS                                    5
                                      21:31 TUESDAY, JULY 14, 1987
                        ARIMA PROCEDURE

                NAME OF VARIABLE         =        A2
                MEAN OF WORKING SERIES=    2.77097
                STANDARD DEVIATION    =    4.48713
                NUMBER OF OBSERVATIONS=         31
                        AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    20.1343    1.00000  |                    |********************|
  1    9.83615    0.48853  |              .     |**********          |
  2    4.60299    0.22861  |            .       |*****               |
  3    6.48433    0.32205  |            .       |******              |
  4    5.80549    0.28834  |          .         |******   .          |
  5    1.74721    0.08678  |          .         |**       .          |
  6 -0.0175513   -0.00087  |          .         |         .          |
  7  -0.106569   -0.00529  |          .         |         .          |
  8  -0.377554   -0.01875  |          .         |         .          |
  9  -0.502992   -0.02498  |          .         |         .          |
 10  -0.975797   -0.04846  |                   *|         .          |
                            '.' MARKS TWO STANDARD ERRORS

                   INVERSE AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1    -0.39887  |             ********|                    |
       2     0.15046  |                     |***   .             |
       3    -0.17302  |              .   ***|      .             |
       4    -0.12338  |              .    **|      .             |
       5     0.05685  |              .      |*     .             |
       6     0.03231  |              .      |*     .             |
       7     0.02655  |              .      |*     .             |
       8     0.03444  |              .      |*     .             |
       9    -0.04236  |              .     *|      .             |
      10     0.01083  |              .      |      .             |

                   PARTIAL AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1     0.48853  |              .      |**********          |
       2    -0.01319  |              .      |      .             |
       3     0.28289  |              .      |******.             |
       4     0.03506  |              .      |*     .             |
       5    -0.12060  |              .    **|      .             |
       6    -0.07626  |              .    **|      .             |
       7    -0.06344  |              .     *|      .             |
       8     0.00149  |              .      |      .             |
       9     0.04306  |              .      |*     .             |
      10    -0.01458  |              .      |      .             |

             AUTOCORRELATION CHECK FOR WHITE NOISE

 TO   CHI                       AUTOCORRELATIONS
LAG SQUARE DF  PROB
  6  17.22  6 0.009  0.489  0.229  0.322  0.288  0.087 -0.001
```

```
                              SAS                                    6
                                      21:31 TUESDAY, JULY 14, 1987
              ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                   APPROX.
PARAMETER      ESTIMATE     STD ERROR    T RATIO  LAG
MU              2.70841       1.23847       2.19    0
MA1,1         -0.278989      0.366777      -0.76    1
AR1,1          0.254803      0.365735       0.70    1

CONSTANT ESTIMATE   =    2.0183

VARIANCE  ESTIMATE =    16.8526
STD ERROR ESTIMATE =    4.10519
AIC                =    178.664
SBC                =    182.966
NUMBER OF RESIDUALS=         31

              CORRELATIONS OF THE ESTIMATES

                       MU      MA1,1    AR1,1
         MU         1.000     -0.022   -0.035
         MA1,1     -0.022      1.000    0.867
         AR1,1     -0.035      0.867    1.000

              AUTOCORRELATION CHECK OF RESIDUALS

 TO   CHI                            AUTOCORRELATIONS
LAG SQUARE DF   PROB
 6.   3.22  3  0.358 -0.003 -0.009  0.213  0.200 -0.025 -0.021
 12   3.36  9  0.948  0.010 -0.014 -0.007 -0.020 -0.044  0.001
 18   4.56 15  0.995 -0.041 -0.066 -0.005 -0.088  0.024 -0.059
 24   6.75 21  0.999 -0.075  0.014 -0.002 -0.056 -0.078 -0.062

        MODEL FOR VARIABLE A2
        ESTIMATED MEAN        =    2.70841

         AUTOREGRESSIVE FACTORS
                     FACTOR 1
        1-.254803B**(1)

         MOVING AVERAGE FACTORS
                     FACTOR 1
        1+0.278989B**(1)
```

or
$$\frac{dX(t)}{d(t)} + c\,X(t) = 0 \qquad (42)$$

The solution of an n^{th} order linear differential equation has the form of a sum of n exponentials. The homogenous differential equation is,

$$\frac{df}{dt} = f \qquad \text{or} \qquad \frac{df}{dt} - f = 0 \qquad (43)$$

To solve this, we want a function that gives back the same function after differentiation. Other than the trivial solution which is f(t) = 0, the simplest function that can be-tried is f(t) = 1. If f(t) is differentiated, we must get 1 whereas f(t) = 1 gives 0. In order to obtain 1, we must add t to the function Let us try:

$$f(t) = 1+t \qquad (43a)$$

If 43a is differentiated we must get 1 + t. To do this we would have to add $t^2/2$ to the function. Thus,

$$f(t) = 1 + t + t^2/2 \qquad (43b)$$

It is clear that the desired function is the infinite sum

$1 + t + t^2/2! + t^3/3! +,\ldots+ t^n/t!$ with $n \to \infty$

where

$n! = n\,(n-1)\,(n-2)\,\ldots 3.2.1$

This is the exponential function

$$e^{t} = \sum_{n=0}^{\infty} \frac{t^{n}}{n!} \qquad (44)$$

which is a solution of $\frac{df}{dt} - f = 0$. This being a homogenous differential equation, the general solution is, $f(t) = c\,e^{t}$. By similar reasoning, the solution of $\frac{df}{dt} = \beta f$ becomes

$$e^{\beta t} = 1 + \beta t + \frac{(\beta t)^{2}}{2} + \ldots + \frac{(\beta t)^{n}}{n!} + \cdots \qquad (45)$$

The general solution is $f(t) = c\,e^{\beta t}$. Integration of Eq. 42 gives

$$(\beta + a)\,c\,e^{\beta t} = 0 \qquad (46)$$

This gives the characteristic equation

$\beta + a = 0$ and $\beta = -a$. The solution of Eq. 42 takes the form $f(t) = c\,e^{-at}$ or for any time period (h) the correlogram shows

$$r(h) = c\,e^{-ah} \qquad (47)$$

When a logarithmic scale is used for vertical axis, the exponential curve becomes a straight line.

3.3 Probit transformation

So - called tolerance values in response to a stimulus vary from one sample to another in the population. If tolerance is measured by λ, Finney (1952) suggested that the distribution of tolerances may be expressed as,

$$dP = f(\lambda)\, d\lambda \qquad (48)$$

If a λ_o is assigned to the whole population, all individuals will respond whose tolerances are less than λ_o, and their proportion P in the population satisfies

$$P = \int_0^{\lambda_o} f(\lambda)\, d\lambda \qquad (49)$$

If this equation is extended from 0 to ∞, it becomes,

$$\int_0^{\infty} f(\lambda)\, d\lambda = 1 \qquad (50)$$

Usually, the distribution of the tolerance can be made approximately normal by an appropriate transformation.

Bliss (1934) first proposed the name " probit " for his modification of the normal equivalent deviate, which he increased by 5 so as to simplify the arithmetical procedure by avoiding negative values. P corresponds to a probability in a normal distribution with mean equal to 1. The probit of P is Y. 5 and a variance equal to 1.

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Y-5} e^{(-1/2)u^2} du \qquad (51)$$

A table giving probits for specified values of P was prepared by Bliss (1935a). It was reproduced by Fisher and Yates (1963) as table IX of their Statistical tables for Biological, Agricultural and Medical Research. The values shown in their table IX are equal to fractiles ($\cdot$u) of the normal distribution in standard form augmented by 5.
For example: $u_{0.98} + 5 = 7.054$ and $u_{0.98} = 2.054$

The effect of a transformation from sample percentages to probits is illustrated in Fig. 3.6. The normal curve becomes a straight line when the probit transformed values are plotted on probability paper.

3.4 Frequency Distributions

The average of N observations is defined as

$$y = \bar{x}_N = (1/N) (x_1 + x_2 + . . . + x_N) \qquad (52)$$

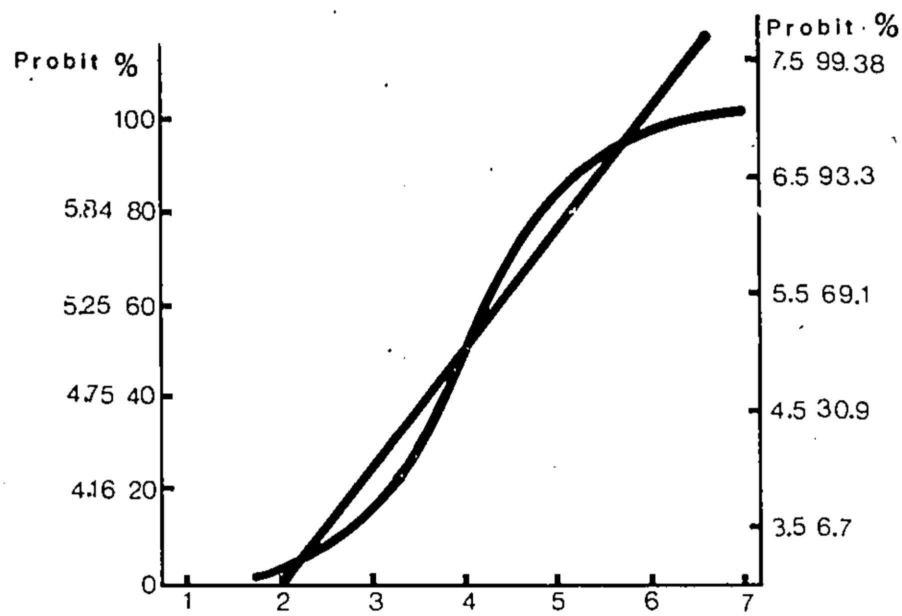
Suppose that the standard error is s(x), the variance can be written as

$$s^2(y) = (1/N) s^2(x) \qquad (53)$$

Eq. 52 represents one of the more important theorems of statistical theory. Eq. 53 can also be written as,

$$F_N = N s^2(\bar{x}) /s^2(x) = 1 \text{ or } F_N = \frac{Ns^2 (x_N)}{s^2 (x)} = 1 \qquad (54)$$

The factor F_N is equal to the relative variance of the mean, multiplied by the number of observations (N). For uncorrelated data, it



3.6

Figure 3.6 Effect of probit transformation (The normal curve is transformed to a straight line when the ordinates are measured on a linear scale of probits instead of percentages after Finney, 1952) is equal to 1. Eq. 54 is not satisfied for averages of autocorrelated data. For averages of two values

$$s^2(\bar{x}_2) = \frac{s^2(x_t + x_{t+1})}{2} = (1/4)\ [s^2(x_t) + s(x_t, x_{t+1}) + s(x_{t+1}, x_t) + s^2(x_{t+1})] \quad (55)$$

where $s(x_t, x_{t+1})$ and $s(x_{t+1}, x_t)$ are covariances. If the series is stationary,

$$s^2(\bar{x}_2) = (1/2)\ s^2(x)[\,1+r_1] \quad (56)$$

or

$$F_2 = (1+r_1) \quad (57)$$

where r_1 is the first autocorrelation coefficient (Agterberg, 1974b) A similar expression can be developed for $s^2(\bar{x})$. If $y = \bar{x}$ represents the mean of N values then,

$$s^2(y) = s^2(x)[\,1/N + (2/N^2)\sum_{j=1}^{N}(N-j)\,r_j\,] \quad (58)$$

These variances for the average values of successive samples were calculated by a FORTRAN program which was prepared by the author. (F FORTRAN, Appendix I).

Agterberg (1974b) has shown that Eq. 58 becomes approximately

$$F_1 = 1 - c + (2c/a)\ \{\,1 + (e^{-al} - 1)\ /\ al\,\} \quad (59)$$

where the autocorrelation coefficient r (l) or r_l changes with distance or " lag " l . With $\mathrm{Lim}_{l \to \infty} = 1 - c + 2c/a$ for large values of l.
The a and c estimated from the correlogram can be used for Eq. 59.

A theoretical curve based on the exponential model can be constructed using the variance values plotted against lag. This allows a comparison between experimental and theoretical values.

In an early application of the so called discrete Gaussian model, Matheron (1974) has considered the following distribution model for binary patterns.

$$X = \Phi \left[(RZ - b) (1 - R^2)^{-1/2} \right] \qquad (60)$$

In Eq. 60, X is a random variable generated from a binary variable X_o. The variable Z has a standard normal distribution with zero mean and unit variance, Φ (u) represents the fractile of the cumulative normal distribution function with,

$$\Phi (u) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{u} \exp (x^2/2) \, dx \qquad (61)$$

If m represents the mean value of X for the study region, the constant b in Eq. 60 is defined as,

$$\Phi(b) = 1 - m \qquad (62)$$

The constant R in Eq. 60 represents the correlation coefficient between the Gaussian random variable Z of Eq. 60 and a Gaussian variable Z_o which corresponds to the binary variable X_o which assume a value equal to one for presence and a value equal to zero for absence of the feature which proportion values are measured. Agterberg (1981) demonstrated that:

$$X = \Phi \left[\frac{RZ - b}{(1 - R^2)^{-1/2}}\right] = 1 - \Phi(b) - \phi(b) \sum_{j=1}^{\infty} R^j H_j(Z) H_{j-1}(b)/(j!) \qquad (63)$$

This is equivalent to assuming that the probit of X satisfies a normal distribution. The variance of X,

$$s^2 = \phi^2(b) \sum_{j=1}^{\infty} R^{2j} \left[H_{j-1}(b) \right]^2 \qquad (64)$$

where,

$$\phi(u) = (2\pi)^{-1/2} \exp(-u^2/2) \qquad (65)$$

and $H_j(u)$ denotes a standarized Hermite polynomial.

3.4.1 Hermite polynomials

These satisfy,

$$H_j(u) = \left(u^j - \frac{j^{(2)}}{2.1!} u^{j-2} + \frac{j^{(4)}}{2^2.2!} u^{j-4} - \frac{j^{(6)}}{2^3.3!} u^{j-6} + \ldots - \right) / (j!)^{1/2} \qquad (66)$$

where

$$j^{(t)} = j(j-1)(j-2)\ldots(j-t+1)$$

The first twelve standardized Hermite polynomials are:

$H_o(u) = 1,$

$H_1(u) = u$

$H_2(u) = (u^2 - 1)(2)^{-1/2}$

$H_3(u) = (u^3 - 3u)(6)^{-1/2}$

$H_4(u) = (u^4 - 6u^2 + 3)(24)^{-1/2}$

$H_5(u) = (u^5 - 10u^3 + 15u)(120)^{-1/2},$

$H_6(u) = (u^6 - 15u^4 + 45u^2 - 15)(720)^{-1/2},$

$H_7(u) = (u^7 - 21u^5 + 105u^3 - 105u)(5040)^{-1/2},$

$H_8(u) = (u^8 - 28u^6 + 210u^4 - 420u^2 + 105)(40320)^{-1/2},$

$H_9(u) = (u^9 - 36u^7 + 378u^5 - 1260u^3 + 945u)(362880)^{-1/2},$

$H_{10}(u) = (u^{10} - 45u^8 + 630u^6 - 3150u^4 + 4725u^2 - 945)(3628800)^{-1/2},$

$H_{11}(u) = (u^{11} - 55u^9 + 990u^7 - 6930u^5 + 17325u^3 - 10395u)(39916800)^{-1/2}.$

$H_{12}(u) = (u^{12} - 66u^{10} + 1485u^8 - 13860u^6 + 51975u^4 - 62370u^2 + 10395)(47900160)^{-1/2}.$

The variances were calculated from these twelve Hermite polynomials with the FF FORTRAN computer program which is listed in Appendix I. For preparing the FF program use was made of an unpublished FORTRAN computer program by F.P. Agterberg called the HERMITE program in which Hermite polynomials until the 11th degree only were used.

3.4.2 Mean and variance relationships

The values of means and their corresponding fractile (= probit - 5) values were taken from Table IX of Fisher and Yates (1963) for probits of a normal distribution and are shown in Table 3.1

TABLE 3.1 Input data for the FF FORTRAN program

m = mean	b = probit - 5	Φ (b) = probability
0.000001	4.75350	0.00000490
0.000010	4.26510	0.00004480
0.000100	3.71900	0.00039590
0.001000	3.09020	0.00336950
0.010000	2.32600	0.02667380
0.020000	2.05400	0.04839320
0.030000	1.88100	0.06801560
0.040000	1.75100	0.08612650
0.050000	1.64500	0.10311090
0.060000	1.55500	0.11908110
0.070000	1.47600	0.13422660
0.080000	1.40500	0.14868130
0.090000	1.34100	0.16233740
0.100000	1.28200	0.17540180

Probit values (u) can be used in Eq. 65 which is the prob - normal distribution for calculating the probability values as shown table 3.1 Table 3.1 was used as input for the FF FORTRAN program.

The value of R can be derived from the mean (x) and variance (s^2) by using Eq. 64 or it can be estimated from mean and variance graphs or tables as shown in Appendix II and III.

First, the means and variance relationships are plotted as shown in Appendix II. Next the vertical axis was changed to represent variance/mean (1.0 - mean) in accordance with a suggestion made by Tukey (1984) in a discussion of a paper by Agterberg (1984b). This quantity was plotted against the mean in Appendix II. Finally, the variance and mean relationships are examined in detail in Appendix II by plotting variances against logarithmic means. Appendix II. All graphs in Appendix II were plotted by the author using SAS graphic software.

The R values are tabulated with respect to their means and variances, their mean and [variance/ (mean (1.0 - mean)] and their logarithmic means and variances in Appendix III.

The frequency distribution of X in Eq. 60 becomes a straight line, if the function is plotted on normal probability paper. The ordinate represents the cumulative frequency percentage. A straight line is also obtained approximately by plotting the probit of the percentage value on normal probability paper and connecting the probits with one another.

3.4.3 Sampling errors and binomial distribution

If a variable X can assume a discrete set of values $x_1, x_2, \ldots$ x_k with respective probabilities $p_1, p_2, \ldots, p_k$ when $p_1 + p_2, \ldots,$ $p_k = 1$, then this distribution is a discrete probability distribution. This is because X has been defined as the function P(x) which has the respective values $p_1, p_2, \ldots, p_k$ for $X = x_1, x_2, \ldots, x_k$ is called the probability function or frequency function of X. It is binomial if it is represented by

$$P(x) = n\,C_x p^x q^{n-x} = \frac{n!}{x!\,(n-x)!} p^x q^{n-x} \qquad (67)$$

where $q = 1 - p$

If the distribution of microfossil species was a binomial distribution model, the binomial mean is

$$\mu = np \qquad (68)$$

where n the total of counts and p is the mean of proportion. The binomial standard deviation is,

$$\sigma = (n.p.q)^{1/2} \qquad (69)$$

The binomial error can be defined as $\hat{\sigma}_B = \frac{100}{n}\sigma$. The white noise (see next section) has variance given by,

$$\sigma^2_N = (1 - c)\, s^2 \qquad (70)$$

and standard deviation

$$\hat{\sigma}_N = (\sigma^2_N)^{1/2} \qquad (71)$$

The binomial error and white noise error can be compared with each other. They would be equal to one another if all white noise is due to sampling.

3.4.4 Signal - plus - noise model

If a time series is weakly stationary and consists of two components; a systematic component (signal) and white noise component, a signal-plus- noise model can be applied.

The concept for models of this type is that a series of observed values x_k is the sum of the series signal (s_t) and white noise (a_t) with $x_t = s_t + a_t$. In geochemical applications, the signals are systematic variations in element concentration of a limited spatial extent. The signal may satisfy an autoregression process model (AR). The noise consists of uncorrelated values.

The corresponding three autocovariance functions are $C_{xx}(l)$, $C_{ss}(l)$ and $C_{aa}(l)$ respectively, with

$$C_{aa} (l) = 0 \text{ if } l = 0 \text{ and } C_{xx} (l) = C_{ss} (l) + C_{aa}(l) \qquad (72)$$

Because the noise is independent of the signal,

$C_{sa}(l) = C_{as}(l) = 0$

Also $C_{xs} (l) = C_{sx}(l) = C_{ss} (l)$

If the data X have variance or $C_{xx} (0) = 1$, then

$\rho_{xx} (l) = C_{xx}(l)$

However in general; $\rho_{ss}(l)$ and $\rho_{aa} (0) = 1 > C_{aa}(0)$. The value c can be defined as $C_{ss}(0) = c$ where c is a constant with $0 \le c \le 1$. Suppose that

$$C_{ss}(l) = c\, e^{-al} \qquad (73)$$

Agterberg (1978) has shown that the signal then satisfies a Markov process of the first order ($X_t = \phi_1 X_{t-1} + a_t$) with

$$\rho_{xx}(l) = c\, e^{-al} \quad \text{if } l = 0 \qquad (74)$$

$$\rho_{xx}(0) = 1$$

If the noise is absent, $c = 1$, or if the series consists of noise only, then $c = 0$.

CHAPTER 4 : Application of statistical and geostatistical models to middle - upper Jurassic foraminifers in Tojeira-sections, Montejunto area, Portugal.

4.1- Introduction

The following application studies are designed to show how to construct a geostatistical model. Tables 1.1,1.2,1.3 and 1.4, which show the data sets of Tojeira 1 and Tojeira 2 section, are used to calculate the autocorrelation coefficients with the SAS package called ARIMA.

Ideally, the number of observations should be a minimum of about 30 in order to proceed with the ARIMA procedure. This usually allows sufficient degrees of freedom for adequate identification and estimation. Occasionally, some analyses may use fewer than 30 observations, but the results then must be interpreted cautiously (see SAS Time Series User Guide Version 5).

Box and Jenkins (1976) suggest that about 50 observations are required for an ARIMA procedure. However, Jenkins (1979,p.64) presents an example of a (multiveriate) ARIMA procedure based on only 14 observations. The key is not necessarily the total number of observations but rather the amount of " statistical noise " in the data. If the noise factor (the variance of the random shocks or white noise) is small, it may be possible to extract enough information from relatively few observations in order to develop a useful ARIMA procedure.

Table 1.1 is the data set of Tojeira 1 that was used with the ARIMA procedure to calculate the autocorrelation coefficients and residuals. These autocorrelation coefficients were plotted against lags. Printout 1 (see appendix I) shows that 3 species of the 31 samples have autocorrelation coefficents which are significantly different from zero. These three species (Eoguttulina sp., Epistomina mosquensis and Ophthalmidium strumosum) were examined more closely.

The SAS variable name should be less than 8 characters so A2 was used in place of Eoguttulina sp. The A indicates it is from Tojeira 1 and the 2 indicates it is from the second row of the Table 1.1 The B indicates it is from Tojeira 2 section (Table 1.2)

4.2 - Tojeira 1 section

4.2.1 - EOGUTTULINA SP.

The number of observations is 31 (N = 31) as shown in Table 1.1 so that there are approximately 31/4 = 8 autocorrelations and also 8 approximately partial autocorrelations. These are useful primarily for identifying the AR order of the ARIMA procedure.

4.2.1.1 Time series analysis - searching for a good model to fit the data set

Figure 4.1 Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the AR(1) time series model to A2(<u>Eoguttulina</u> sp.)

```
                                SAS
                                     21:31 TUESDAY, JULY 14, 1987  1
                          ARIMA PROCEDURE

                NAME OF VARIABLE        =        A2
                MEAN OF WORKING SERIES=     2.77097
                STANDARD DEVIATION     =    4.48713
                NUMBER OF OBSERVATIONS=          31
                          AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    20.1343     1.00000 |                    |********************|
  1    9.83615     0.48853 |                .   |**********          |
  2    4.60299     0.22861 |               .    |*****  .            |
  3    6.48433     0.32205 |               .    |******  .           |
  4    5.80549     0.28834 |              .     |******   .          |
  5    1.74721     0.08678 |              .     |**       .          |
  6 -0.0175513    -0.00087 |              .     |         .          |
  7  -0.106569    -0.00529 |              .     |         .          |
  8  -0.377554    -0.01875 |              .     |         .          |
  9  -0.502992    -0.02498 |              .     |         .          |
 10  -0.975797    -0.04846 |              .    *|         .          |
                          '.' MARKS TWO STANDARD ERRORS

                  INVERSE AUTOCORRELATIONS

        LAG  CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1     -0.39887 |          ********|    .               |
          2      0.15046 |             .    |***  .              |
          3     -0.17302 |             .     ***|  .              |
          4     -0.12338 |             .      **|  .              |
          5      0.05685 |             .    |*    .              |
          6      0.03231 |             .    |*    .              |
          7      0.02655 |             .    |*    .              |
          8      0.03444 |             .    |*    .              |
          9     -0.04236 |             .   *|    .               |
         10      0.01083 |             .    |    .               |

                  PARTIAL AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1     0.48853 |             .    |**********          |
          2    -0.01319 |             .    |    .               |
          3     0.28289 |             .    |******.             |
          4     0.03506 |             .    |*   .               |
          5    -0.12060 |             .  **|    .               |
          6    -0.07626 |             .  **|    .               |
          7    -0.06344 |             .   *|    .               |
          8     0.00149 |             .    |    .               |
          9     0.04306 |             .    |*   .               |
         10    -0.01458 |             .    |    .               |

           AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                       AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   17.22  6  0.009   0.489  0.229  0.322  0.288  0.087 -0.001
```

```
                                SAS
                                     21:31 TUESDAY, JULY 14, 1987  2
               ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                     APPROX.
        PARAMETER     ESTIMATE     STD ERROR    T RATIO   LAG
        MU             2.73605        1.3501       2.03     0
        AR1,1         0.479748        0.1619       2.96     1

        CONSTANT ESTIMATE   =    1.42344

        VARIANCE  ESTIMATE  =    16.3211
        STD ERROR ESTIMATE  =    4.03994
        AIC                 =    176.735
        SBC                 =    179.602
        NUMBER OF RESIDUALS=          31

               CORRELATIONS OF THE ESTIMATES

                          MU      AR1,1
               MU        1.000    -0.034
               AR1,1    -0.034     1.000

               AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                       AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    4.05  4  0.399   0.015 -0.142  0.200  0.212 -0.043 -0.049
 12    4.14 10  0.941   0.007 -0.007 -0.003 -0.020 -0.036  0.009
 18    4.99 16  0.995  -0.034 -0.056 -0.007 -0.069  0.030 -0.051
 24    7.00 22  0.999  -0.075  0.027  0.019 -0.050 -0.075 -0.054

          MODEL FOR VARIABLE A2
          ESTIMATED MEAN      =    2.73605

           AUTOREGRESSIVE FACTORS
                       FACTOR 1
          1-.479748B**(1)
```

The estimated a.c.f and p.a.c.f are shown in Fig. 4.1. The estimated autocorrelation function decays toward zero at the positive side, while the theoretical p.a.c.f has a single positive asterisk at lag 1 when q = 1, p = 0, d = 0 for ARIMA (q d p) or ARIMA (1 0 0) or AR (1). The AR (1) model can be selected, tentatively, for processing the data,

$$X_t = C + \hat{\phi}_1 X_{t-1} + a_t \qquad (A)$$

Model A has two parameters requiring estimation. Fig. 4.1 results from fitting model (A) where C = 1.423 and $\hat{\phi}_1 = 0.479$
The absolute value of $\hat{\phi}_1$ is less than 1. This model is stationary. The absolute t - value (from Fig. 4.1) is greater than 2.0 and this estimated coefficient value is different from zero at the 5% significance level.

The random shocks (a_t) are independent. If the shocks in a given model are correlated, the model must be reformulated, because it does not fully capture the statistical relationship among the X's. Fig.4.1 shows the residual a.c.f for model (A). The random shock t - value is less than 1.25 for lag 1. The random shock at this lag is independent. The absolute t - values and the chi - squared statistic are all relatively small. Consequently, it may be concluded that all random shocks are uncorrelated and model (A) is statistically adequate as a model with $X_t = 1.423 + 0.479X_{t-1} + a_t$. This result is shown on Fig. 4.1 with B notation.

Figure 4.2 Printout of estimated a.c.f.'s and P.a.c.f.'s after fitting the MA(1) time series model to A2(<u>Eoguttulina</u> sp.)

```
                         SAS                                   3
                                   21:31 TUESDAY, JULY 14, 1987
                   ARIMA PROCEDURE

            NAME OF VARIABLE         =        A2
            MEAN OF WORKING SERIES=    2.77097
            STANDARD DEVIATION       =    4.48713
            NUMBER OF OBSERVATIONS=         31
                          AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    20.1343     1.00000  |                    |********************|
  1    9.83615     0.48853  |                .   |**********          |
  2    4.60299     0.22861  |                .   |*****               |
  3    6.48433     0.32205  |                .   |******  .           |
  4    5.80549     0.28834  |                .   |******  .           |
  5    1.74721     0.08678  |                .   |**      .           |
  6 -0.0175513    -0.00087  |                .   |        .           |
  7  -0.106569    -0.00529  |                .   |        .           |
  8  -0.377554    -0.01875  |                .   |        .           |
  9  -0.502992    -0.02498  |                .   |        .           |
 10  -0.975797    -0.04846  |                .  *|        .           |
                      '.' MARKS TWO STANDARD ERRORS

              INVERSE AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1   -0.39887  |          ********|                    |
  2    0.15046  |             .    |***  .              |
  3   -0.17302  |             . ***|     .              |
  4   -0.12338  |             .  **|     .              |
  5    0.05685  |             .    |*    .              |
  6    0.03231  |             .    |*    .              |
  7    0.02655  |             .    |*    .              |
  8    0.03444  |             .    |*    .              |
  9   -0.04236  |             .   *|     .              |
 10    0.01083  |             .    |     .              |

              PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    0.48853  |             .    |**********          |
  2   -0.01319  |             .    |     .              |
  3    0.28289  |             .    |******              |
  4    0.03506  |             .    |*    .              |
  5   -0.12060  |             .  **|     .              |
  6   -0.07626  |             .  **|     .              |
  7   -0.06344  |             .   *|     .              |
  8    0.00149  |             .    |     .              |
  9    0.04306  |             .    |*    .              |
 10   -0.01458  |             .    |     .              |

           AUTOCORRELATION CHECK FOR WHITE NOISE

 TO   CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   17.22  6  0.009   0.489  0.229  0.322  0.288  0.087 -0.001
```

```
                         SAS                                   4
                                   21:31 TUESDAY, JULY 14, 1987
             ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                     APPROX.
PARAMETER      ESTIMATE           STD ERROR    T RATIO  LAG
MU              2.72391             1.07226       2.54    0
MA1,1         -0.493009            0.164662      -2.99    1

CONSTANT ESTIMATE     =    2.72391

VARIANCE  ESTIMATE    =    16.4746
STD ERROR ESTIMATE    =    4.05889
AIC                   =    177.042
SBC                   =     179.91
NUMBER OF RESIDUALS=            31

           CORRELATIONS OF THE ESTIMATES

                        MU      MA1,1
           MU        1.000      0.014
           MA1,1     0.014      1.000

          AUTOCORRELATION CHECK OF RESIDUALS

 TO   CHI                          AUTOCORRELATIONS
LAG  SQUARE DF  PROB
  6    4.06  4  0.397  0.060  0.137  0.200  0.219  0.004 -0.004
 12    4.25 10  0.935  0.004 -0.016 -0.012 -0.025 -0.049 -0.015
 18    5.89 16  0.989 -0.048 -0.079 -0.015 -0.101  0.009 -0.072
 24    8.91 22  0.994 -0.074 -0.005 -0.022 -0.064 -0.093 -0.079

      MODEL FOR VARIABLE A2
      ESTIMATED MEAN     =     2.72391

       MOVING AVERAGE FACTORS
                    FACTOR 1
      1+0.493009B**(1)
```

Fig. 4.2 shows the estimated a.c.f and p.a.c.f for the MA (1) process where $\theta_1 < 0$ (θ_1 is negative). The asterisks in the a.c.f indicate positive values. The a.c.f decays exponentially, starting on the positive side, and the p.a.c.f also damps out exponentially starting on the positive side. The estimated a.c.f drops to nearly zero after lag 1. This model is written as,

$$X_t = C - \theta_1 a_{t-1} + a_t \qquad (B)$$

where q = 0, p = 1 and d = 0 for ARIMA (p d q) or MA(1). The B model is a pure MA process or white noise series, and no further checks are required (Fig. 4.2).

Consider the estimated a.c.f in Fig. 4.3. Only the first autocorrelation is significantly different from zero at the 5% significance level. Only the first asterisk in the a.c.f falls outside the 95% confidence belt on the printout. The position of this belt is based on Bartlett's approximation for the standard error (Eq. 19)

$$s(r_1) = (1 + 2.0)^{1/2} (31)^{-1/2} = 0.179$$

used in the 95% confidence interval = $\rho_1 \pm 2\, s (r_1)$. For the null hypothesis $H_0 : \rho_1 = 0$, this interval is $\pm 2\, s (r_1) = \pm 0.358$. Because $r_1 > 2s (r_1)$ the asterisk in the a.c.f falls outside the belt on the printout.

Figure 4.3 Printout of the ARMA(1,1) time series model for A2(*Eoguttulina* sp.)

```
                                  SAS                                         3
                                          9:28 THURSDAY, APRIL 23, 1987
                             ARIMA PROCEDURE

                  NAME OF VARIABLE          =        A2
                  MEAN OF WORKING SERIES=     2.77097
                  STANDARD DEVIATION        =     4.48713
                  NUMBER OF OBSERVATIONS=          31
                             AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    20.1343     1.00000  |                    |********************|
  1    9.83615     0.48853  |                    |**********          |
  2    4.60299     0.22861  |              .     |*****  .            |
  3    6.48433     0.32205  |              .     |******  .           |
  4    5.80549     0.28834  |             .      |******   .          |
  5    1.74721     0.08678  |             .      |**       .          |
  6 -0.0175513    -0.00087  |             .      |         .          |
  7  -0.106569    -0.00529  |             .      |         .          |
  8  -0.377554    -0.01875  |             .      |         .          |
  9  -0.502992    -0.02498  |             .      |         .          |
 10  -0.975797    -0.04846  |             .     *|         .          |
                             '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    -0.39887  |           ********|                    |
          2     0.15046  |              .    |***  .              |
          3    -0.17302  |              .  ***|     .             |
          4    -0.12338  |              .   **|     .             |
          5     0.05685  |              .     |*    .             |
          6     0.03231  |              .     |*    .             |
          7     0.02655  |              .     |*    .             |
          8     0.03444  |              .     |*    .             |
          9    -0.04236  |              .    *|     .             |
         10     0.01083  |              .     |     .             |

                        PARTIAL AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1     0.48853  |              .     |**********         |
          2    -0.01319  |              .     |     .             |
          3     0.28289  |              .     |******.            |
          4     0.03506  |              .     |*    .             |
          5    -0.12060  |              .   **|     .             |
          6    -0.07626  |              .   **|     .             |
          7    -0.06344  |              .    *|     .             |
          8     0.00149  |              .     |     .             |
          9     0.04306  |              .     |*    .             |
         10    -0.01458  |              .     |     .             |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                              AUTOCORRELATIONS
LAG   SQUARE DF    PROB
  6    17.22  6   0.009   0.489   0.229   0.322   0.288   0.087 -0.001
```

```
                                  SAS                                         4
                                          9:28 THURSDAY, APRIL 23, 1987
                      ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                           APPROX.
             PARAMETER     ESTIMATE      STD ERROR    T RATIO  LAG
             MU             2.70841        1.23847       2.19    0
             MA1,1        -0.278989       0.366777      -0.76    1
             AR1,1         0.254803       0.365735       0.70    1

             CONSTANT ESTIMATE   =     2.0183

             VARIANCE  ESTIMATE =     16.8526
             STD ERROR ESTIMATE =     4.10519
             AIC                =     178.664
             SBC                =     182.966
             NUMBER OF RESIDUALS=          31

                      CORRELATIONS OF THE ESTIMATES

                            MU      MA1,1      AR1,1
                  MU      1.000    -0.022     -0.035
                  MA1,1  -0.022     1.000      0.867
                  AR1,1  -0.035     0.867      1.000

                  AUTOCORRELATION CHECK OF RESIDUALS

 TO     CHI                              AUTOCORRELATIONS
LAG   SQUARE DF    PROB
  6     3.22  3   0.358 -0.003 -0.009  0.213  0.200 -0.025 -0.021
 12     3.36  9   0.948  0.010 -0.014 -0.007 -0.020 -0.044  0.001
 18     4.56 15   0.995 -0.041 -0.066 -0.005 -0.088  0.024 -0.059
 24     6.75 21   0.999 -0.075  0.014 -0.002 -0.056 -0.078 -0.062

               MODEL FOR VARIABLE A2
               ESTIMATED MEAN        =    2.70841

                AUTOREGRESSIVE FACTORS
                                FACTOR 1
               1-.2548038**(1)

                MOVING AVERAGE FACTORS
                                FACTOR 1
               +0.2789898**(1)
```

For r_2 , $s(r_2) = (1 + 2 r_1^2)^{1/2} 31^{-1/2} = 0.218$
The 95% interval for r_2 is $\rho_2 \pm 2 s(r_2)$ Setting $\rho_2 = 0$ gives $\pm 2 s(r_2) = 0.436$ and $r_2 < 0.436$

The autocorrelations decrease gradually to zero so that the mean of the series is probably stationary. Since $\hat{\phi}_1 = 0.259$ and $|\hat{\phi}_1| < 1$ then according to Table 2.1 this series is stationary. An AR procedure seems appropriate because the a.c.f decays toward zero rather then cutting off sharply to zero. A decaying a.c.f is also consistent with a mixed (ARMA) procedure. A model of this working series is an ARMA(1, 1) (see Fig. 4.3). ARMA (1, 1) can be written as

$$X_t = C + \phi_1 X_{t-1} - \theta_1 a_{t-1} + a_t \qquad (AB)$$

For model (AB), estimated coefficients from Fig. 4.3 are;

$\hat{\phi}_1 = 0.255$

$\hat{\theta}_1 = -0.279$

$\hat{C} = \hat{\mu}(1 - \hat{\phi}_1) = 2.018$

If the p.a.c.f was tested, the standard error according to Eq.22 would be

$$s(\hat{\phi}_{11}) = 31^{-1/2} = 0.197$$

The t - statistic is calculated using Eq. 23 used to test the null hypothesis $H_o : \phi_{11} = 0$

$$t_{\hat{\phi}_{11}} = \frac{0.49 - 0}{0.197} = 2.487$$

The absolute value of the t - statistic is greater than 2.0. The $\hat{\phi}_{11}$ is different from zero at the 5% significance level so the null hypothesis ϕ_{11} is rejected.

Let k = 2 and test the null hypothesis $H_o : \phi_{22} = 0$

$$t_{\hat{\phi}_{22}} \text{ - statistic } = \frac{-0.01319 - 0}{0.179} = -0.07$$

The absolute value of the t value is less than 2.0 so the null hypothesis is acceptable and $\phi_{22} = 0$.

Fig. 4.3 shows the result of estimating the ARIMA procedure of Eoguttulina sp. The ϕ and μ were estimated using the likelihood criterion. In Fig.4.3 $\hat{\phi}_1 = 0.255$ and $\hat{\mu} = 2.708$ and the estimated constant is found to be C = 2.018. This model satisfies the stationarity requirement $| \hat{\phi}_1 | < 1.0$. The t - statistic for $H_o : \phi_1 = 0$ is as follows,

$$t = \frac{0.2548 - 0}{0.365} = 0.698 = 0.70 \text{ (as in ~~Fig.4.3~~)}$$

The t value (0.70) is less than 2.0, so the null hypothesis is acceptable for $\phi_1 = 0$. According to the result of this statistical significance testing, the coefficient quality is not good or, as shown in Fig. 4.3, $\hat{\phi}_1$ and $\hat{\theta}_1$ are not statistically significant.

If Fig. 4. 3 was statistically adequate, the random shocks a_t could be tested for independence using the residuals $\hat{a}_t$ from the estimated equation. The residuals are estimates of the random shocks and these shocks are assumed to be statistically independent. The estimated a.c.f of the residuals are used to test whether the shocks are independent. The 8 of the 31 residual autocorrelations are examined using the t_b - test. Using Eq. 33, we obtain

$$s\,[\,r_1\,(\hat{a}_1)\,] = (\,1 + 2.0\,)^{1/2}\,31^{-1/2} = 0.179$$

$$t_{b_1} = -\,\frac{-\,0.003 - 0}{0.179} = -\,0.015$$

$$s\,[\,r_2\,(\hat{a}_2)\,] = (\,1 + 2\,(\,0.01)^2\,)^{1/2}\,31^{-1/2} = 0.1796$$

$$t_{b_2} = \frac{0.01 - 0}{0.1796} = 0.0556$$

The absolute value of the residual a.c.f t_b - value is less than 1.25 at lags 1 and 2. Furthermore, according to the X^2 (chi - squared) test (Eq. 34), if 6 residual autocorrelations are assumed, then K = 6 and three parameters will be estimated (ϕ, θ, and μ) so that m = 3. Therefore, there are K - m = 3 degrees of freedom. From Fig. 4.3 this value is 3.22, and according to the chi - squared table the critical value with degrees of freedom equal to 3 at the 5% significance level is 7.81. For 6 degrees of freedom, the value determined from Fig. 4.3

is 3.36 and the corresponding table value is 16.9. The calculated values are less than the critical values. The shocks appear to be independent according to the t_b - test as well as the chi - squared test.

The model (AB) may be a good model for processing the data set. However, the estimated coefficients are not statistically significant.

Model ARMA (1, 2) (see the Fig. 4.4) and Model ARMA (2, 1) (see Fig. 4.5) have estimated coefficients with absolute t - values less than 2.0. They are not significantly different from zero at the 5% significance level. The selected time series model is AR (1) or model (A). The working data set has a good fit with the AR (1) model. If the AR (1) model and ARMA (1, 1) model are essentially the same in all other respects, the AR (1) model would be selected because it has one less coefficient to estimate.

4.2.1.2 SPECTRA procedure

In Fig. 4.6, illustrating the SPECTRA process, it is shown that the maximum periodogram ordinate is 168.265 and the periodogram ordinate total is 624.164, for M-1=15 (31 samples have 16 periodogram ordinates).

$$\text{Kappa test statistic : } \frac{15 * 168.265}{624.164} = 4.0438$$

The table value is 5.019 for the 5% significance level . The calculated value is less than the table value. Consequently, the null hypothesis is

Figure 4.4 Printout of the ARMA(1,2) time series model for A2(*Eoguttulina* sp.)

```
SAS                                              7
                    21:31 TUESDAY, JULY 14, 1987
          ARIMA PROCEDURE

      NAME OF VARIABLE        =      A2
      MEAN OF WORKING SERIES=   2.77097
      STANDARD DEVIATION    =   4.48713
      NUMBER OF OBSERVATIONS=        31
                AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    20.1343     1.00000  |                    |********************|
 1    9.83615     0.48853  |                    |**********          |
 2    4.60299     0.22861  |             .      |*****  .            |
 3    6.48433     0.32205  |             .      |******  .           |
 4    5.80549     0.28834  |          .        .|******    .         |
 5    1.74721     0.08678  |          .         |**        .         |
 6 -0.0175513    -0.00087  |          .         |          .         |
 7  -0.106569    -0.00529  |          .         |          .         |
 8  -0.377554    -0.01875  |          .         |          .         |
 9  -0.502992    -0.02498  |          .        *|          .         |
10  -0.975797    -0.04846  |          .         |          .         |
                            '.' MARKS TWO STANDARD ERRORS

          INVERSE AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1   -0.39887  |          ********|                    |
  2    0.15046  |           .      |***  .              |
  3   -0.17302  |           .   ***|     .              |
  4   -0.12338  |           .    **|     .              |
  5    0.05685  |           .      |*    .              |
  6    0.03231  |           .      |*    .              |
  7    0.02655  |           .      |*    .              |
  8    0.03444  |           .      |*    .              |
  9   -0.04236  |           .     *|     .              |
 10    0.01083  |           .      |     .              |

          PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    0.48853  |           .      |**********          |
  2   -0.01319  |           .      |     .              |
  3    0.28289  |           .      |******.             |
  4    0.03506  |           .      |*    .              |
  5   -0.12060  |           .    **|     .              |
  6   -0.07626  |           .    **|     .              |
  7   -0.06344  |           .     *|     .              |
  8    0.00149  |           .      |     .              |
  9    0.04306  |           .      |*    .              |
 10   -0.01458  |           .      |     .              |

       AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                        AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   17.22  6  0.009   0.489  0.229  0.322  0.288  0.087 -0.001
```

```
SAS                                              8
                    21:31 TUESDAY, JULY 14, 1987
       ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

DID NOT CONVERGE AFTER  15 ITERATIONS.
```

Figure 4.5 Printout of the ARMA(2,1) time series model for A2(*Eoguttulina* sp.)

```
                              SAS                                        9
                                     21:31 TUESDAY, JULY 14, 1987
                      ARIMA PROCEDURE

             NAME OF VARIABLE         =       A2
             MEAN OF WORKING SERIES=   2.77097
             STANDARD DEVIATION    =   4.48713
             NUMBER OF OBSERVATIONS=        31
                      AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    20.1343    1.00000  |                    |********************|
  1    9.83615    0.48853  |                    |**********          |
  2    4.60299    0.22861  |                    |*****               |
  3    6.48433    0.32205  |                    |******              |
  4    5.80549    0.28834  |                    |******              |
  5    1.74721    0.08678  |                    |**                  |
  6 -0.0175513   -0.00087  |                    |                    |
  7  -0.106569   -0.00529  |                    |                    |
  8  -0.377554   -0.01875  |                    |                    |
  9  -0.502992   -0.02498  |                    |                    |
 10  -0.975797   -0.04846  |                   *|                    |
                          '.' MARKS TWO STANDARD ERRORS

                   INVERSE AUTOCORRELATIONS

    LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
      1    -0.39887  |            ********|                    |
      2     0.15046  |                    |***                 |
      3    -0.17302  |                 ***|                    |
      4    -0.12338  |                  **|                    |
      5     0.05685  |                    |*                   |
      6     0.03231  |                    |*                   |
      7     0.02655  |                    |*                   |
      8     0.03444  |                    |*                   |
      9    -0.04236  |                   *|                    |
     10     0.01083  |                    |                    |

                   PARTIAL AUTOCORRELATIONS

    LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
      1     0.48853  |                    |**********          |
      2    -0.01319  |                    |                    |
      3     0.28289  |                    |******              |
      4     0.03506  |                    |*                   |
      5    -0.12060  |                  **|                    |
      6    -0.07626  |                  **|                    |
      7    -0.06344  |                   *|                    |
      8     0.00149  |                    |                    |
      9     0.04306  |                    |*                   |
     10    -0.01458  |                    |                    |

              AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                         AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   17.22  6  0.009   0.489  0.229  0.322  0.288  0.087 -0.001
```

```
                              SAS                                       10
                                     21:31 TUESDAY, JULY 14, 1987
               ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                         APPROX.
          PARAMETER      ESTIMATE      STD ERROR    T RATIO   LAG
          MU               2.7248        1.33062       2.05     0
          MA1,1         -0.618452        1.05121      -0.59     1
          AR1,1        -0.0861893        1.09536      -0.08     1
          AR1,2          0.202429       0.618922       0.33     2

          CONSTANT ESTIMATE    =   2.40807

          VARIANCE  ESTIMATE  =   17.2844
          STD ERROR ESTIMATE  =   4.15745
          AIC                 =   180.333
          SBC                 =   186.069
          NUMBER OF RESIDUALS=         31

                  CORRELATIONS OF THE ESTIMATES

                        MU     MA1,1     AR1,1     AR1,2
          MU         1.000     0.017     0.012    -0.026
          MA1,1      0.017     1.000     0.985    -0.952
          AR1,1      0.012     0.985     1.000    -0.934
          AR1,2     -0.026    -0.952    -0.934     1.000

                AUTOCORRELATION CHECK OF RESIDUALS

  TO    CHI                          AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
   6    3.03  2  0.220 -0.015 -0.035  0.171  0.219 -0.041 -0.027
  12    3.13  8  0.926  0.001 -0.006 -0.007 -0.019 -0.039  0.002
  18    4.19 14  0.994 -0.034 -0.063 -0.003 -0.080  0.029 -0.057
  24    6.16 20  0.999 -0.070  0.017  0.008 -0.051 -0.076 -0.058

          MODEL FOR VARIABLE A2
          ESTIMATED MEAN        =    2.7248

           AUTOREGRESSIVE FACTORS
                           FACTOR 1
          1+.0861893B**(1)-.202429B**(2)

           MOVING AVERAGE FACTORS
                           FACTOR 1
          1+0.618452B**(1)
```

```
                                          10:59 THURSDAY, JANUARY 29, 1987
                  S P E C T R A   P R O C E D U R E

-----  TEST FOR WHITE NOISE FOR VARIABLE A2 -----

FISHER'S KAPPA: (M-1)*MAX(P(*))/SUM(P(*))
PARAMETERS:     M-1         =        15
                MAX(P(*))   =   168.265
                SUM(P(*))   =   624.154
TEST STATISTIC: KAPPA       =    4.0438

BARTLETT'S KOLMOGOROV-SMIRNOV STATISTIC:
        MAX ABS DIFFERENCE OF THE STANDARDIZED PARTIAL SUMS
        OF THE PERIODOGRAM AND THE CDF OF A UNIFORM(0,1).
```

SAS   10:59 THURSDAY, JANUARY 29, 1987   14

OBS	FREQ	PERIOD	P_01	S_01
1	0.00000	.	0.000	11.1863
2	0.20268	31.0000	168.265	10.4613
3	0.40537	15.5000	105.681	8.5197
4	0.60805	10.3333	71.399	6.2609
5	0.81073	7.7500	22.491	4.3251
6	1.01342	6.2000	20.024	3.1938
7	1.21610	5.1667	41.391	2.8915
8	1.41878	4.4286	40.494	2.7985
9	1.62147	3.8750	38.464	2.7398
10	1.82415	3.4444	33.453	2.4574
11	2.02683	3.1000	20.190	1.9559
12	2.22952	2.8182	15.978	1.5114
13	2.43220	2.5833	9.490	1.1496
14	2.63488	2.3846	12.804	0.9343
15	2.83757	2.2143	9.957	0.8465
16	3.04025	2.0667	8.093	0.8072

Figure 4.6 Printout of the SAS SPECTRA procedure for A2(Eoguttulina sp.)

rejected (the hypothesis of independence). The series is not pure white noise.

The SAS package program calculated the Kolmogorov - Smirnov test for 1% probablity. The 5% test was calculated by the author using the equation:

$$\text{Test} : \frac{1.36}{(15)^{1/2}} = 0.351$$

Here 1.36 is a constant and 15 represents the number of the periodogram ordinates (Fig. 4.6). Furthermore, investigation of the white noise hypothesis was done by plotting the cumulative periodogram values against the lag values as shown in Fig. 4.7. The $k/(15)^{1/2} \pm 0.351$ values were plotted against lag values in the same figure. Fig. 4.7 shows that the series of Eoguttulina sp. is not white noise.

Statistical properties of the Eoguttulina sp. series are well known now so that the geostatistical model can be applied to it. All the required statistics could be calculated using the SAS package program.

The pattern of autocorrelation coefficients plotted against lag or correlogram shows that the trend is approximately exponential. If the vertical axis was given a logarithmic scale, the exponential curve becomes a straight line. The best fitting line with the constants a and c can be constructed. When estimated from the correlogram in Fig. 4.8, these values are, a = 0.24 (a is the slope of best fitting line) and c = 0.76.

Eq. 47 becomes

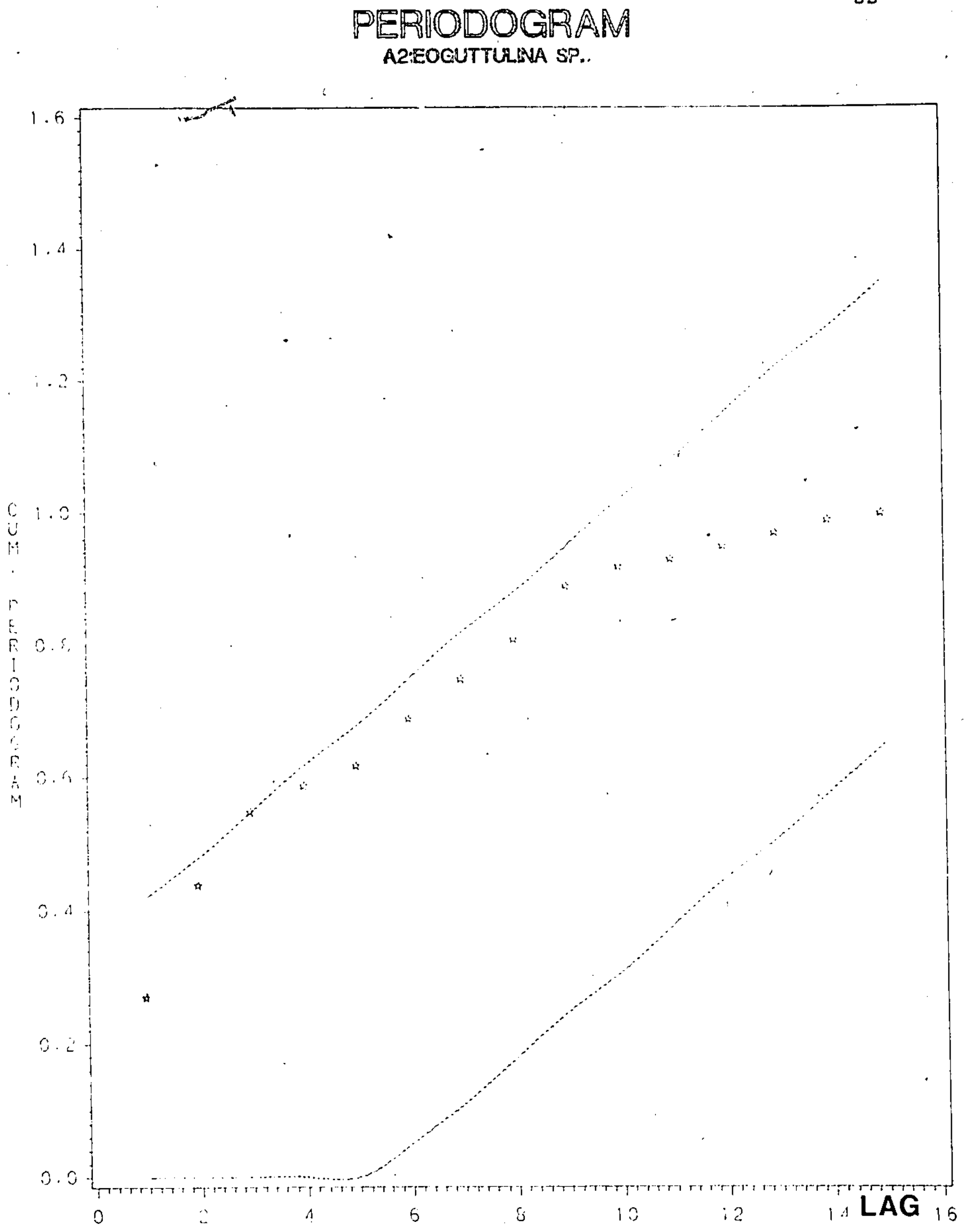
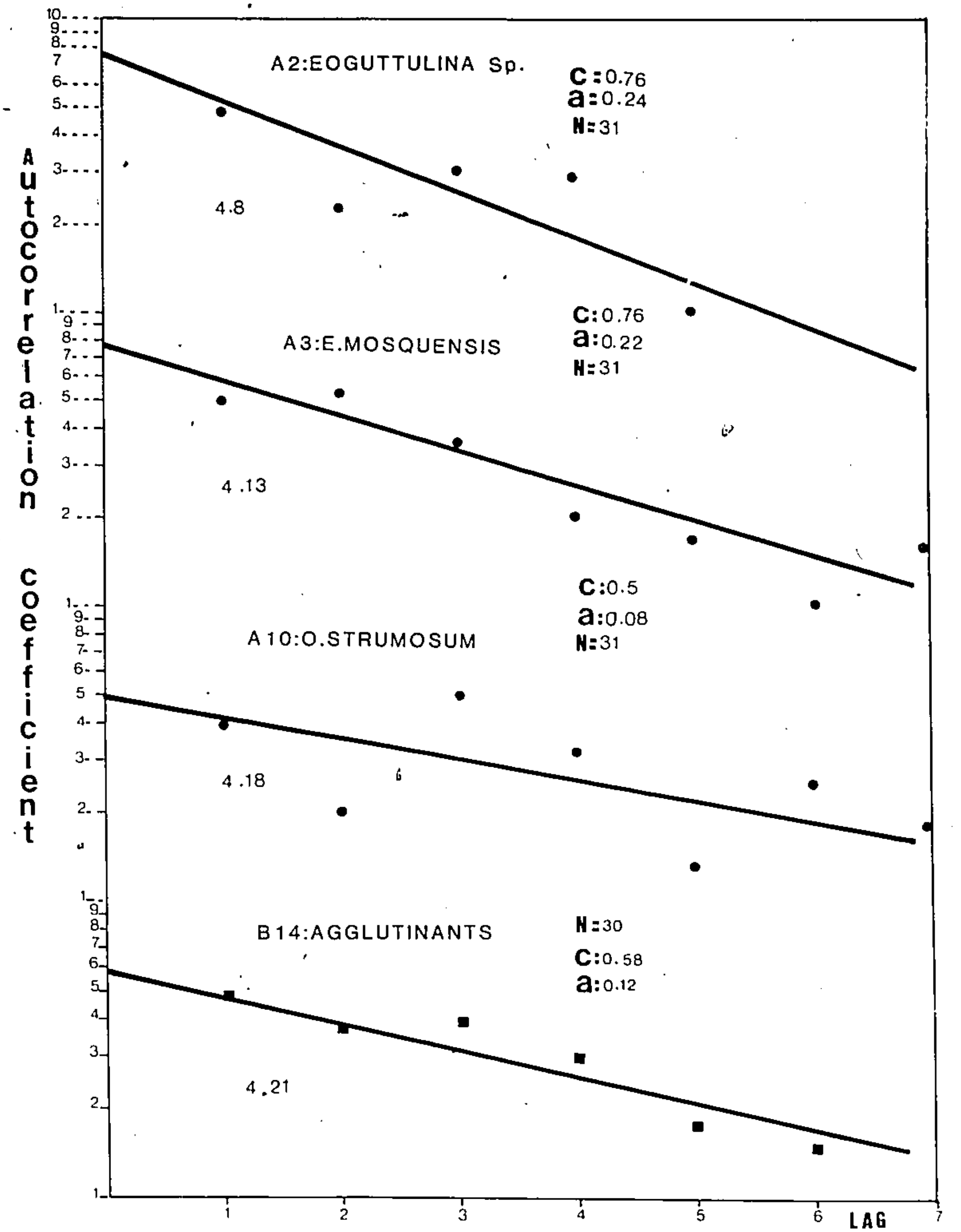


Figure 4.7 The periodogram for A2(Eoguttulina sp.).

Figure 4.8 The correlogram for A2(<u>Eoguttulina</u> sp.) and the best-fitting exponential curve with a = 0.24 and c = 0.76

Figure 4.13 The correlogram for A3(<u>E. mosquensis</u>) with best - fitting exponential curve. From this correlogram, a = 0.22 and c = 0.76 were estimated

Figure 4.18 The correlogram for A10(<u>O. strumosum</u>) and the best-fitting exponential curve with a = 0.08 and c = 0.50


Autocorrelation coefficient
A2:EOGUTTULINA Sp.
C:0.76
a:0.24
N:31
4.8
A3:E.MOSQUENSIS
C:0.76
a:0.22
N:31
4.13
A10:O.STRUMOSUM
C:0.5
a:0.08
N:31
4.18
B14:AGGLUTINANTS
N:30
C:0.58
a:0.12
4.21
LAG
1
2
3
4
5
6
7

$$r(k) = 0.76e^{-0.24k}$$

This result can be used in Eq. 54. $S^2(x_N)$ values which are variances for N adjacent samples, are calculated by the F FORTRAN program (see Appendix I). For a series of 31 values, there are 30 pairs of adjacent values, 29 sets of three adjacent values, etc.. Observed relative variances (F_N) were calculated using values of $S^2(x_N)$ with the results shown in Table 4.1.

TABLE 4.1 Theoretical and observed values of the relative variance of Eoguttulina sp.

k	$S^2(X_N)$	F_N	F_L
0	0	0	0.24
1	20.85	1	0.94
2	15.95	1.53	1.53
3	12.54	1.80	2.06
4	9.53	1.83	2.50
5	8.00	1.92	2.88
6	7.10	2.04	3.21
7	5.63	1.89	3.50

The value F_N represents the relative variance calculated according to Eq. 54. We have for example,

$$F_1 = \frac{1*20.85}{20.85} = 1$$

$$F_2 = \frac{2 * 15.95}{20.85} = 1.53$$

If the data were uncorrelated (white noise model), all F_N values would approximately be equal to 1. The F_N factors for $N > 1$ are considerably larger than unity as predicted by Eq. 54 which is not valid for autocorrelated data. The standard deviation of the mean of 31 uncorrelated data would be

$$s(x_{31}) = \frac{4.48}{\sqrt{31}} = 0.804$$

The population mean (μ) would be in the interval $x \pm 0.804 * t_{0.975}$ with a probability of 95% or because $t_{0.975}$ (30) = 2.06

$P (1.114 < \mu < 4.426) = 95\%$

Because the data are autocorrelated, Eq.54 must be used instead with

$s(x_{31}) = \sqrt{31}.\ s (x)/\sqrt{F_{31}}$

It means that the above estimate of $s (x_{31}) = 0.804$ must be corrected by the factor $\sqrt{F_{31}}$. But $\sqrt{F_{31}}$ can not be estimated from the data by the direct method used, for example, to estimate F_2. However, by assuming that the series is stationary and consists of two components characterized by a and c, by approximation, we can use Eq. 59 to calculate F_{31}. This gives

$F_{31} = 1 - 0.76 + (2* 0.76/0.24) [1 + (e-0.24*31 - 1)/0.24*31]$

or $F_{31} = 5.72$, and

$s(x_{31}) \simeq \sqrt{5.72 * 0.804} = 1.923$

The above confidence interval for the mean must be replaced by

$P(0.00 < \mu < 6.76) = 95\%$

In fact, the lower limit of the confidence interval is negative, however the mean can never be a negative. According to Eq. 59,

$F_0 = 1 - 0.76 = 0.24$

$F_1 = 0.24 + 6.33 [1 + (e^{-0.24*1} - 1)/0.24*1] = 0.94$

$F_2 = 0.24 + 6.33 [1+ (e^{-0.24*2} - 1)/0.24*2] = 1.53$

(as above in Table 4.1).

The F_N and F_L variance values are plotted against lag as shown in Fig. 4.9. The correspondence between the experimental and theoretical values is good for this example.

The Eoguttulina data, pairs of Eoguttulina data and the sets of 4 adjacent Eoguttulina data were transformed with the probit transformation (Table 4.2) using the table value. Frequency distribution tables for these data were constructed as in Table 4.3.

Calculated cumulative frequency distributions were plotted on normal probability paper using upper class limits. They fall approximately on the straight lines (Chapter 2). Eq. 60 provides a theoretical straight line on the probability paper for percentage values of this type. The distribution for the 31 values of A2 (Eoguttulina sp.) has mean ($\bar{x}$) = 0.0277 and variance $s^2(x) = 0.002013$. By using

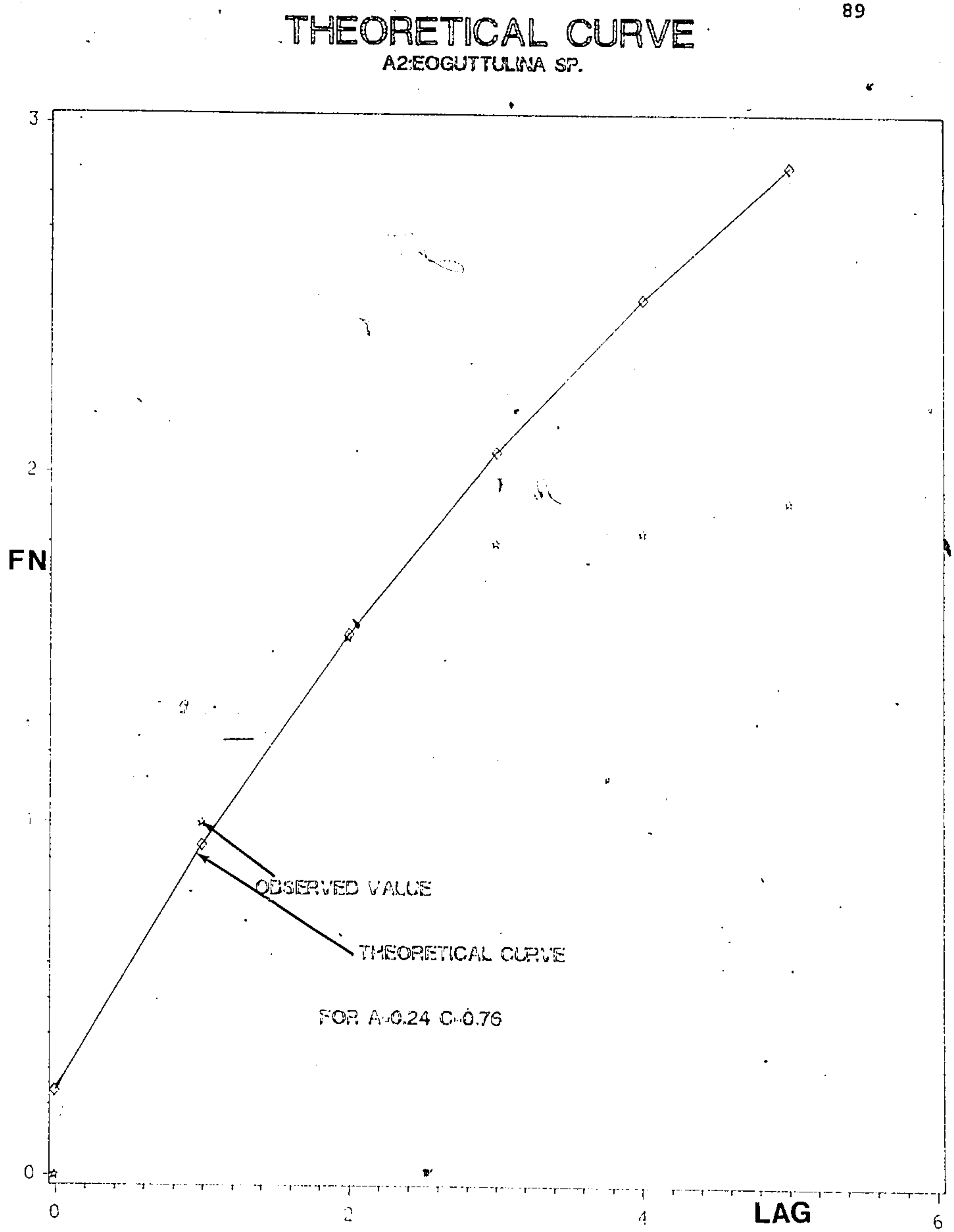


Figure 4.9 Theoretical curve for a = 0.24 and c = 0.76 in comparison with observed values of Eoguttulina sp.

Table 4.2 Probit transformation of A2 (Eoguttulina sp.) from the Tojeira 1 section.

Sample single %	probit	sample pairs %	probit	sample set of 4 %	probit
4.1	3.261				
14.8	3.955	9.45	2.686		
19.0	4.122				
5.4	3.393	12.0	3.835	10.825	3.764
1.8	2.903				
13.5	3.897	7.65	3.570		
2.2	2.986				
3.4	3.175	2.80	3.080	5.225	3.376
0.3	2.252				
2.6	3.057	1.45	2.816		
1.1	2.710				
1.1	2.710	1.1	2.710	1.275	2.766
1.3	2.774				
0.6	2.488	0.95	2.654		
1.5	2.830				
0.3	2.252	0.90	2.634	0.925	2.644
0.5	2.424				
1.1	2.710	0.80	2.591		

(continued)

Table 4.2 (concluded)

0.3	2.252				
0.8	2.591	0.6	2.488	0.70	2.543
0.8	2.591				
0.8	2.591	0.8	2.591		
3.5	3.188				
1.5	2.830	2.5	2.424	1.65	2.510
0.6	2.488				
0.6	2.488	0.6	2.488		
0.9	2.634				
1.0	2.674	0.95	2.654	0.775	2.578
0.3	2.252				
0.0	2.187	0.15	1.916		
0.2	2.182				

Table 4.3 Cumulative frequency distribution of A2(Eoguttulina sp.) for 31, 15 and 7 samples.

N = 31 (single observed values)

class interval	mid point	Freq.	Cum. freq	Freq %	Cumfreq %	plotting value
2.2 - 2.6	2.4	13	13	41.96	41.96	40.4
2.6 - 3.0	2.8	10	23	32.26	74.19	72.3
3.0 - 3.4	3.2	5	28	16.13	90.32	88.3
3.4 - 3.8	3.6	0	28	0.00	90.32	88.3
3.8 - 4.2	4.0	3	31	9.68	100.00	97.9

N = 15 (pairs of observed values)

1.7 - 2.3	2.0	1	1	6.67	6.67	4.3
2.3 - 2.8	2.55	9	10	60.0	66.67	63.0
2.8 - 3.3	3.05	2	12	13.33	80.00	76.1
3.3 -3.8	3.55	3	15	20.00	100.00	97.9

N = 7 (4 of set of observed values)

2.6 - 3.0	2.8	5	5	71.43	71.43	63.6
3.0 - 3.4	3.4	1	6	14.29	85.71	77.73
3.4 - 3.8	3.6	1	1	14.29	100.00	90.90

NOTE :

$$\text{Plotting value} = \frac{\text{cum. freq.}*3 - 1}{(N*3 + 1)/100}$$

Appendix II or III, R = 0.34 was found. From Eq.62, it follows that

$\Phi (b) = 1 - 0.0277 = 0.97229$

The probit of 0.97229 is 6.916 (see Table III) and

$b = 6.916 - 5 = 1.916$

Eq. 60 becomes $Y_1 = 0.3615z - 2.037$ using the above value for N = 31. This function is plotted on normal probability paper which is the same paper used for the probit transformed data (Fig. 4.10 a).

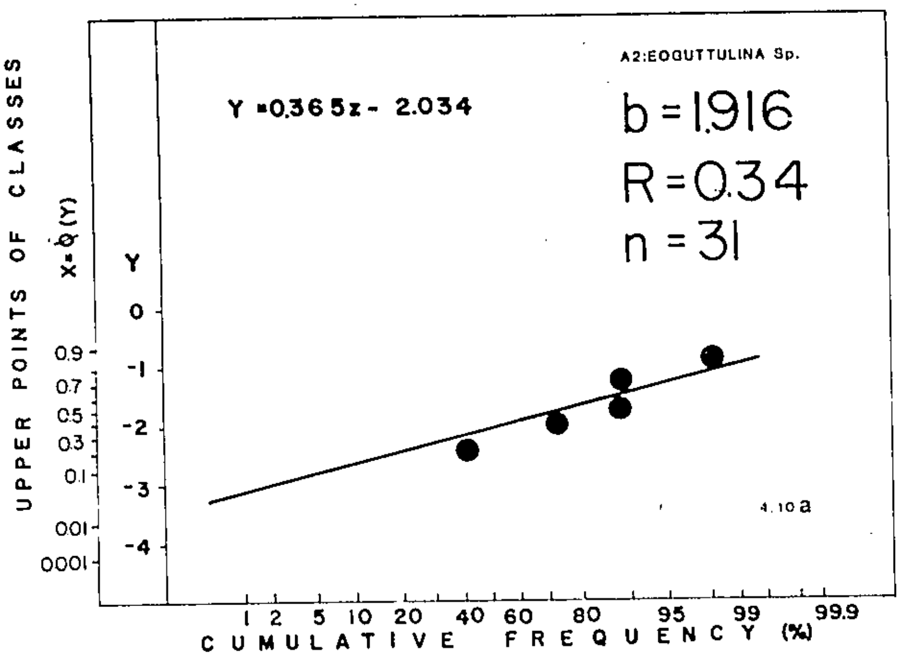
For the distribution of 15 successive values (pairs of adjacent values) of A2(Eoguttulina sp.), the mean is the same as that of 31 single values of A2(Eoguttulina sp.) and its variance. Using Eq. 54,

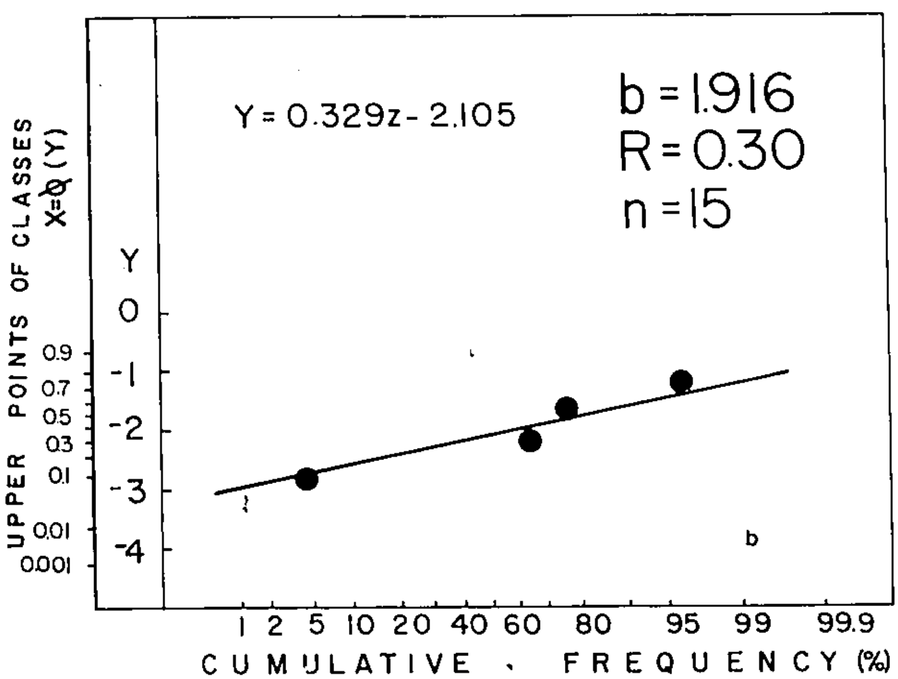
$$s^2 (x_2) = \frac{F_2 s^2 (x)}{2} = 0.001540$$

From Appendix II or III, it follows that R = 0.30 with b = 1.916 (b is dependent on the mean). Eq. 60 becomes $Y_2 = 0.329z - 2.105$. This Y function is plotted on normal probability paper which is the same paper used before for the probit transformed data of the paired values of A2 (Eoguttulina sp.) Fig. 4.10b.

The distribution of the 7 successive relative values of Eoguttulina sp.(sets of 4 adjacent measured values) also has the same mean and b coefficient. Its variance is,

Figure 4.10a,b,c Experimental frequency distributions for 31, 15, and 7 values of A2(*Eoguttulina* sp.) plotted on two normal probability scales. The values of b and R were obtained from the means and variances They determine the positions and slopes of the lines for probnormal distributions satisfying Eq.60


A2:EOGUTTULINA Sp.
Y =0.365z - 2.034
b = 1.916
R = 0.34
n = 31
UPPER POINTS OF CLASSES
X=Φ(Y)
Y
0
-1
-2
-3
-4
0.9
0.7
0.5
0.3
0.1
0.01
0.001
4.10a
1 2 5 10 20 40 60 80 95 99 99.9
CUMULATIVE FREQUENCY (%)


Y = 0.329z - 2.105
b = 1.916
R = 0.30
n = 15
UPPER POINTS OF CLASSES
X=Φ(Y)
Y
0
-1
-2
-3
-4
0.9
0.7
0.5
0.3
0.1
0.01
0.001
b
1 2 5 10 20 40 60 80 95 99 99.9
CUMULATIVE FREQUENCY (%)

$$s^2 (x_4) = \frac{F_4 s^2 (x)}{4} = 0.000916$$

By using Appendix II or III, it follows that R = 0.25. Eq. 60 becomes Y = 0.26z - 1.98. For Y_4, this function is shown graphically in the same manner as the other Y functions. The vertical scales are $X = \phi (Yn)$ in Fig.4.10c. There is good agreement between the observed values and the theoretical distributions as can be seen in Fig. 4.10 a, b, c.,which means that Eoguttulina sp.is a unique species. The distribution of this taxon could be of biostratigraphic value.It could be used for correlation between sections using the abundance data for this species as well as for correlation with other species within the same samples.

4.2.2 EPISTOMINA MOSQUENSIS

As in section 4.2.1 the ARIMA procedure is used to calculate autocorrelation coefficients. The estimated a.c.f's for the AR (1) model is seen in Fig. 4.11 and the model shown in Fig. 4.12. ARMA (1, 2) may be good time series models for the data set of Epistomina mosquensis The AR (1) was tested and the estimated a.c.f for this model is shown in Fig. 4.11. The first two autocorrelations are significantly different from zero, at the 5% level of significance. The first and second asterisks in the a.c.f extend beyond the 95% confidence belt of two standard errors. Since $\hat{\phi}_1 > 0$ and the absolute value of $\phi_1 <$ 1 then, AR (1) is stationary. The AR process has theoretical a.c.f's

Figure 4.11 Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the AR(1) time series model to A3(*E. mosquensis*)

```
                              SAS                                    11
                                         21:31 TUESDAY, JULY 14, 1987
                       ARIMA PROCEDURE

           NAME OF VARIABLE          =         A3
           MEAN OF WORKING SERIES=        22.471
           STANDARD DEVIATION        =   12.6522
           NUMBER OF OBSERVATIONS=            31
                          AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    160.079     1.00000 |                    |********************|
  1    79.9485     0.49943 |                    |**********          |
  2    85.2347     0.53245 |                    |***********         |
  3    55.3794     0.36469 |                    |*******             |
  4    32.2471     0.20145 |                    |****                |
  5    27.9955     0.17489 |                    |***                 |
  6    14.9058     0.09312 |                    |**                  |
  7    25.9934     0.16238 |                    |***                 |
  8    23.4033     0.14620 |                    |***                 |
  9    19.8307     0.12388 |                    |**                  |
 10    12.4919     0.07804 |                    |**                  |
                          '.' MARKS TWO STANDARD ERRORS

                    INVERSE AUTOCORRELATIONS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
         1    -0.05880 |                   *|                    |
         2    -0.39872 |            ********|                    |
         3    -0.21208 |                ****|                    |
         4     0.10481 |                    |**                  |
         5     0.14240 |                    |***                 |
         6     0.13767 |                    |***                 |
         7    -0.12063 |                  **|                    |
         8    -0.13749 |                 ***|                    |
         9    -0.00195 |                    |                    |
        10     0.08213 |                    |**                  |

                    PARTIAL AUTOCORRELATIONS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
         1     0.49943 |                    |**********          |
         2     0.37708 |                    |********            |
         3     0.01664 |                    |                    |
         4    -0.18030 |                ****|                    |
         5     0.00757 |                    |                    |
         6     0.03097 |                    |*                   |
         7     0.16038 |                    |***                 |
         8     0.06029 |                    |*                   |
         9    -0.06668 |                   *|                    |
        10    -0.11801 |                  **|                    |

              AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   26.46  6  0.000  0.499  0.532  0.365  0.201  0.175  0.093
```

```
                              SAS                                    12
                                         21:31 TUESDAY, JULY 14, 1987
              ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                        APPROX.
         PARAMETER      ESTIMATE      STD ERROR    T RATIO   LAG
         MU              21.4662        4.17757       5.14     0
         AR1,1          0.542912       0.153571       3.54     1

         CONSTANT ESTIMATE   =    9.81194

         VARIANCE  ESTIMATE  =    123.046
         STD ERROR ESTIMATE  =    11.0926
         AIC                 =    239.445
         SBC                 =    242.313
         NUMBER OF RESIDUALS=          31

                 CORRELATIONS OF THE ESTIMATES

                              MU       AR1,1
                 MU         1.000     -0.025
                 AR1,1     -0.025      1.000

               AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                              AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    6.35  4  0.175 -0.256  0.304  0.092 -0.034  0.080 -0.075
 12   12.15 10  0.275  0.103  0.033  0.063 -0.000  0.183 -0.252
 18   17.59 16  0.349  0.144 -0.107 -0.145  0.022 -0.164  0.054
 24   22.29 22  0.443 -0.045 -0.021 -0.038 -0.060 -0.135 -0.105

         MODEL FOR VARIABLE A3
         ESTIMATED MEAN       =    21.4662

         AUTOREGRESSIVE FACTORS
                           FACTOR 1
         1-.542912B**(1)
```

Figure 4.12 Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the ARMA(1,2) time series model to A3(<u>E. mosquensis</u>)

```
                              SAS                                 13
                                   21:31 TUESDAY, JULY 14, 1987
                       ARIMA PROCEDURE

            NAME OF VARIABLE          =      A3
            MEAN OF WORKING SERIES=      22.471
            STANDARD DEVIATION    =     12.6522
            NUMBER OF OBSERVATIONS=          31
                       AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    160.079     1.00000  |                    |********************|
  1    79.9485     0.49943  |                    |**********          |
  2    85.2347     0.53245  |                    |***********         |
  3    58.3794     0.36469  |                    |*******             |
  4    32.2471     0.20145  |                    |****                |
  5    27.9955     0.17489  |                    |***                 |
  6    14.9058     0.09312  |                    |**                  |
  7    25.9934     0.16238  |                    |***                 |
  8    23.4033     0.14620  |                    |***                 |
  9    19.8307     0.12388  |                    |**                  |
 10    12.4919     0.07804  |                    |**                  |
                             '.' MARKS TWO STANDARD ERRORS

                 INVERSE AUTOCORRELATIONS

      LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
        1    -0.05880  |                   *|                    |
        2    -0.39872  |            ********|                    |
        3    -0.21208  |                ****|                    |
        4     0.10481  |                    |**                  |
        5     0.14240  |                    |***                 |
        6     0.13767  |                    |***                 |
        7    -0.12063  |                  **|                    |
        8    -0.13749  |                 ***|                    |
        9    -0.00195  |                    |                    |
       10     0.08213  |                    |**                  |

                 PARTIAL AUTOCORRELATIONS

      LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
        1     0.49943  |                    |**********          |
        2     0.37708  |                    |********            |
        3     0.01664  |                    |                    |
        4    -0.18030  |                ****|                    |
        5     0.00757  |                    |                    |
        6     0.03097  |                    |*                   |
        7     0.16038  |                    |***                 |
        8     0.06029  |                    |*                   |
        9    -0.06668  |                   *|                    |
       10    -0.11801  |                  **|                    |

            AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                           AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   25.46  6  0.000  0.499  0.532  0.365  0.201  0.175  0.093
```

```
                              SAS                                 14
                                   21:31 TUESDAY, JULY 14, 1987
             ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                     APPROX.
     PARAMETER      ESTIMATE      STD ERROR   T RATIO  LAG
     MU              19.7269        6.28075      3.14    0
     MA1,1          0.552066       0.204949      2.69    1
     MA1,2         -0.473937       0.187166     -2.53    2
     AR1,1          0.772596       0.153447      5.03    1

     CONSTANT ESTIMATE    =  4.48598

     VARIANCE  ESTIMATE =    96.3968
     STD ERROR ESTIMATE =    9.81819
     AIC                =    234.573
     SBC                =    240.309
     NUMBER OF RESIDUALS=         31

               CORRELATIONS OF THE ESTIMATES

                    MU     MA1,1     MA1,2     AR1,1
     MU          1.000     0.015     0.006    -0.020
     MA1,1       0.015     1.000    -0.144     0.534
     MA1,2       0.006    -0.144     1.000     0.321
     AR1,1      -0.020     0.534     0.321     1.000

             AUTOCORRELATION CHECK OF RESIDUALS

 TO     CHI                         AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    1.10  2  0.578 -0.006 -0.058 -0.017 -0.088 -0.046 -0.120
 12    4.37  8  0.823  0.052  0.109  0.086  0.052  0.122 -0.160
 18    9.57 14  0.793  0.062  0.012 -0.173 -0.063 -0.142  0.130
 24   16.29 20  0.699  0.114 -0.006  0.025 -0.008 -0.159 -0.125

       MODEL FOR VARIABLE A3
       ESTIMATED MEAN         =  19.7269

        AUTOREGRESSIVE FACTORS
                      FACTOR 1
       1-.772596B**(1)

        MOVING AVERAGE FACTORS
                      FACTOR 1
       1-.552066B**(1)+0.473937B**(2)
```

that decay gradually to zero. Fig. 4.11 shows a simple exponential decay pattern with $\hat{\phi}_1 = 0.543$. The constant (C) = μ (1 - $\hat{\phi}_1$) gives C = 9.812. Other ARIMA estimation routines automatically test the hypothesis that the true coefficient is zero. A t - value to test each coefficient for being different from zero is 3.54, which is larger than the 95% confidence limit of 2.0. The coefficient is therefore statistically significant. The p.a.c.f supports this as a good model for the time series. The t - value for the p.a.c.f at lag 1 is 2.78 and this value is also greater then 2.0. Consequently $\hat{\phi}_{11}$ is also different from zero at the 5% significance level.

If the random shock process was tested as the null hypothesis by calculating how many standard errors (t_b) away from zero each residual autocorrelation coefficient falls, it follows that $\hat{a}_1 = -0.256$ with standard error 0.179 and t_b - value of - 1.43. The absolute value of this t_b value is larger then 1.25 for lag 1, so the residuals are significantly autocorrelated. If the residuals are autocorrelated, they are not white noise. Another model must be found with residuals that are consistent with the independence assumption. Finally, AR (1) is not a good model for this situation.

For the model ARMA (1 ,2), the test of p.a.c.f gives (φ_{kk}) = 0.197 . The null hypothesis $H_o : \rho_1 = 0$ results in

$$t_{\phi_{11}} = \frac{0.499 - 0}{0.197} = 2.53$$

The absolute value of the t - statistic greater than 2.0 and $\hat{\phi}_{11}$ is different from zero at the 5% significance level. We also have

$\hat{\phi}_1 = 0.77$

$\hat{\theta}_1 = 0.55$

$\hat{\theta}_2 = - 0.47$ (from Fig. 4.12)

The stationarity requirement for ARIMA (1, 2) with p = 1 , q = 2 is that $|\hat{\phi}_1| < 1$ for the AR coefficient. Further,

$\theta_2 + \theta_1 < 1$, $0.076 < 1$

and

$\theta_2 - \theta_1 < 1$, $- 1.023 < 1$

It may be concluded that the A3 (Epistomina mosquensis) variable is stationary. Its autocorrelation coefficients decrease gradually to zero.

The random shock (white noise) was tested with ARIMA (1,2). The standard error is 0.179 and the first residual value is - 0.355. Its absolute value is less than 1.25 for lag 1. For lag 2, the t_b - value is 0.18 and is less than 1.25 too. The random shocks (a_t) in model ARMA (1, 2) are assumed to be statistically independent. The chi - squared test for random shocks shows that the calculated value is 1.10 for df = 2. The critical value with df = 2 at the 5% significance level is 5.99 (chi - squared table taken from statistical tables). The calculated chi - squared value is less than the critical value. The conclusion is that the residual autocorrelations are not significantly different from zero. The hypothesis is accepted that the random shocks are independent. The model ARMA (1,2) provides a good fit for working with the data set. This model becomes:

$$X_t = 4.48 + 0.77X_{t-1} - 0.55a_{t-1} + 0.474a_{t-2} + a_t$$

Now that a model which provides a good fit to the data set of Epistomina mosquensis has been selected, geostatistical moddeling can be applied. As shown in Fig. 4.12, the autocorrelation plotted against the lag has an exponential trend (signal - plus - noise model). If the vertical axis has a logarithmic scale (as in Fig. 4.13 see page 83), the exponential curve becomes a straight line. The stationary series has constant values c and a which are estimated from the correlogram. We have

$a = 0.22$ and

$c = 0.76$

Consequently, Eq. 47 becomes

$$r(k) = 0.76\, e^{-0.22k}$$

$S^2(x)$ are two or more than two adjacent variances which are calculated with the F FORTRAN program (Appendix I). F_N values are relative variances which are constant. They are calculated according to Eq. 54 as shown in table 4.4. The F_L values are theoretical relative variances which are calculated using Eq. 59 as seen in Table 4.4.

TABLE 4.4 Theoretical and observed relative variance of Epistomina Mosquensis

k	$S^2(x)$	F_N	F_L

0	0	0	0.24
1	165.42	1	0.95
2	123.42	1.49	1.56
3	108.48	1.97	2.09
4	95.01	2.29	2.55
5	80.60	2.44	2.96
6	69.77	2.53	3.31

The theoretical and observed relative variance values were plotted against lag as shown in Fig. 4.14. The correspondence between experimental and theoretical values is a good fit.

The mean standard deviation is $s(\overline{x}_{31}) = \dfrac{0.1286}{(31)^{1/2}} = 0.023$

The population mean (μ) lies in the interval $x \pm 0.023t_{0.975}$ (two tails) with a probability of 95% or because $t_{0.975}$ (30) = 2.06 $P(0.177 < \mu < 0.272) = 95\%$. Using Eq. 54 we obtain, $s(\bar{x}_{31}) = (31)^{1/2} s(x)(F_{31})^{1/2}$. This means that estimated values above $(x_{31}) = 0.023$ must be corrected by the factor $(F_{31})^{1/2}$The value F_{31} can be calculated as 6.14.

$F_{31} = 6.14$ and $(F_{31})^{1/2} * 0.023 = 0.057$

The above confidence interval for the mean must be replaced by,

$P(0.168 < \mu < 0.282) = 95\%$

From Fig. 4.12 the estimated mean is 0.01972 which remains

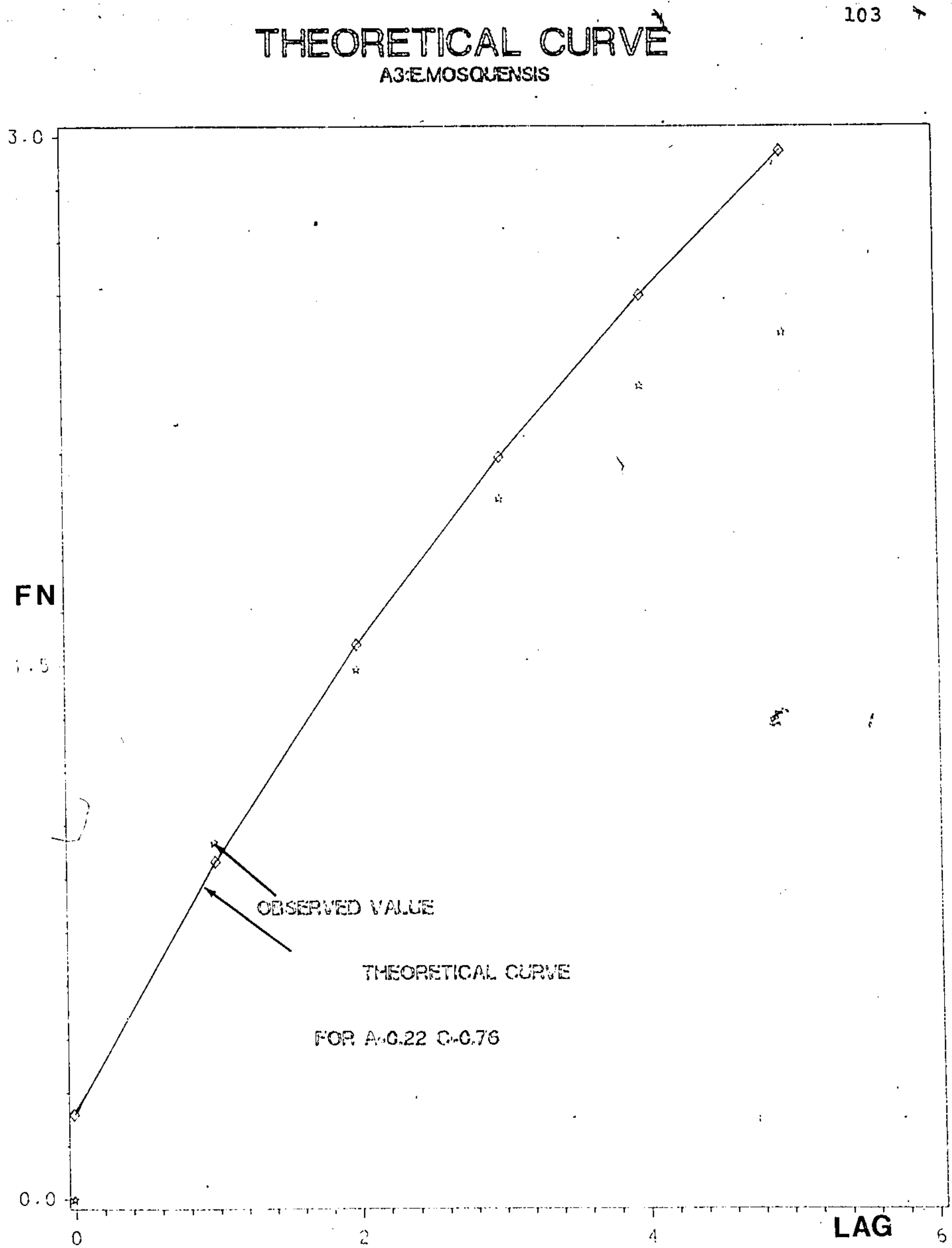


Figure 4.14 Theoretical curve for a = 0.22 and c = 0.76 in comparison with observed values of A3(E. mosquensis)

Table 4.5 Probit transformation of A3 (Epistomina mosquensis) from the Tojeira 1 section.

Samples single %	probit	sample pairs %	probit	sample set of 4 %	probit
0.0	2.187				
0.0	2.187	0.0	2.187		
0.0	2.187				
0.2	2.187	0.1	2.187	0.0	2.187
15.1	3.968				
8.4	3.621	11.75	3.812		
24.2	4.300				
18.1	4.081	21.15	4.198	16.45	4.024
12.4	3.845				
26.1	4.360	19.25	4.131		
27.6	4.405				
13.2	3.883	20.4	4.173	19.8	4.151
23.6	4.281				
23.3	4.271	23.45	4.276		
15.5	3.985				
35.2	4.620	25.35	4.336	24.4	4.304
21.4	4.207				
46.3	4.907	33.85	4.583		
18.0	4.085				

(continued)

Table 4.5 (concluded)

39.7	4.739	28.85	4.442	31.35	4.514
30.4	4.487				
13.5	3.897	21.95	4.226		
23.0	4.261				
33.5	4.574	28.25	4.424	25.1	4.329
37.1	4.671				
36.6	4.658	36.81	4.664		
38.8	4.715				
34.8	4.609	36.8	4.663	36.8	4.663
36.5	4.655				
27.4	4.399	31.95	4.531		
16.7	4.034				

Table 4.6 Cumulative frequency distributions of the <u>Epistomina</u> <u>mosquensis</u> for 31, 15 and 7 samples.

N = 31

Class interval	Freq.	Cum freq	Cum %	Plott. value
2.1 - 2.7	4	4	12.90	11.70
2.7 - 3.3	0	4	0.000	11.70
3.3 - 3.9	4	8	12.90	11.70
3.9 - 4.5	14	22	70.97	69.15
4.5 - 5.1	9	31	100.0	97.88

N = 15

2.0 - 2.8	2	2	13.13	10.9
2.8 - 3.6	0	2	13.13	10.9
3.6 - 4.4	7	9	60.0	56.5
4.4 - 5.2	6	15	100.0	95.7

N = 7

2.0 - 2.8	1	1	14.29	9.1
2.8 - 3.6	0	1	14.29	9.1
3.6 - 4.4	6	7	100.0	90.1

within the confidence interval.

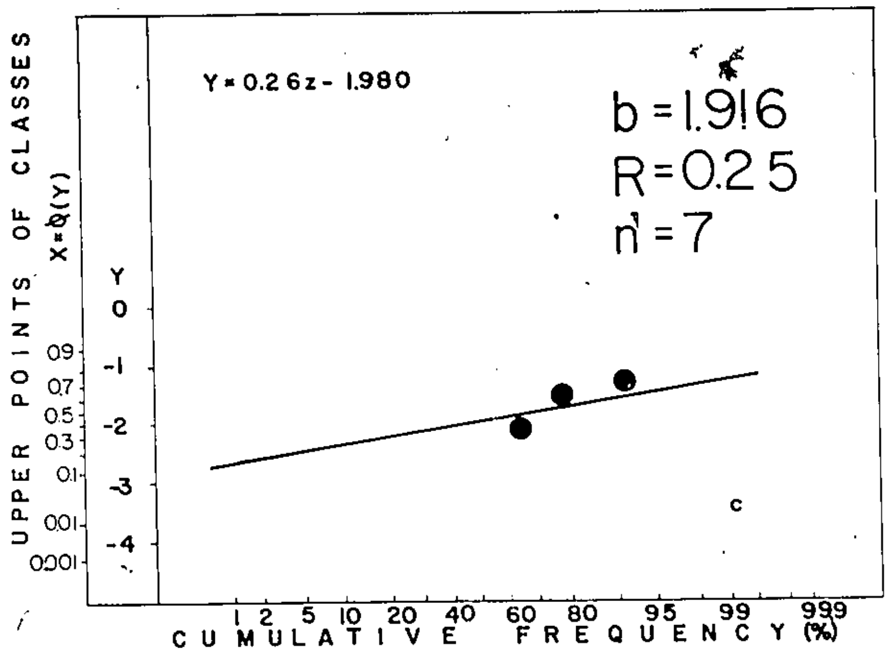
The raw data set for single, paired and sets of 4 of adjacent values of Epistomina mosquensis series were transformed with the probit transformation (Table 4.5) using table. The frequency distribution for these data is shown in Table 4.6. Calculated cumulative frequency distributions were plotted on normal probability paper using upper class limits. They are approximately normal distributions which plot as straight lines. This distribution has a mean $\bar{x} = 0.2247$ and a variance $s^2(x) = 0.016538$ for 31 values of A3. The R can be obtained from the mean and variance relationships. However, the mean and variance were too large to be found on standard prepared tables and graphs. Because of this, the R values were estimated from a different graph which has been plotted by Agterberg (1984).
The scale of this graph is so small that R values estimated from this graph are not very precise. The estimated R value is 0.39.

For Eq. 62 the coefficient $\Phi(b) = 1 - m = 1 - 0.2247 = 0.7753$. The probit value in Table is 5.756 for $\Phi = 0.775$. Calculating $b = 5.756 - 5 = 0.756$.Eq. 60 becomes, $Y = 0.4235Z - 0.821$.

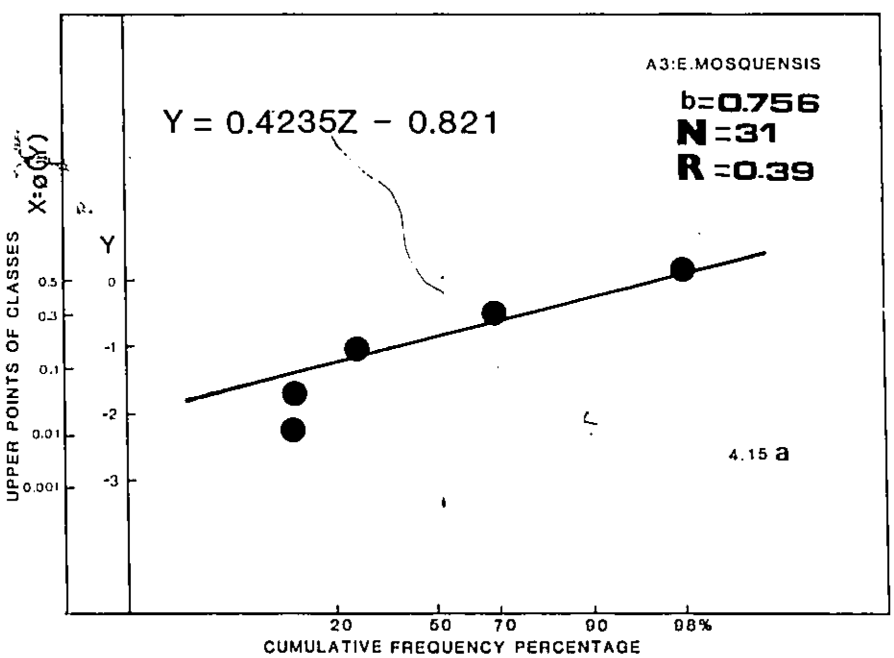
This function is plotted on normal probability paper (Fig 4.15a), which is the same paper used for the probit transformed single data of Epistomina mosquensis, using the values shown below.

$Z = 0$	$y = -0.821$
$Z = 1$	$y = -0.397$
$Z = 2$	$y = 0.025$

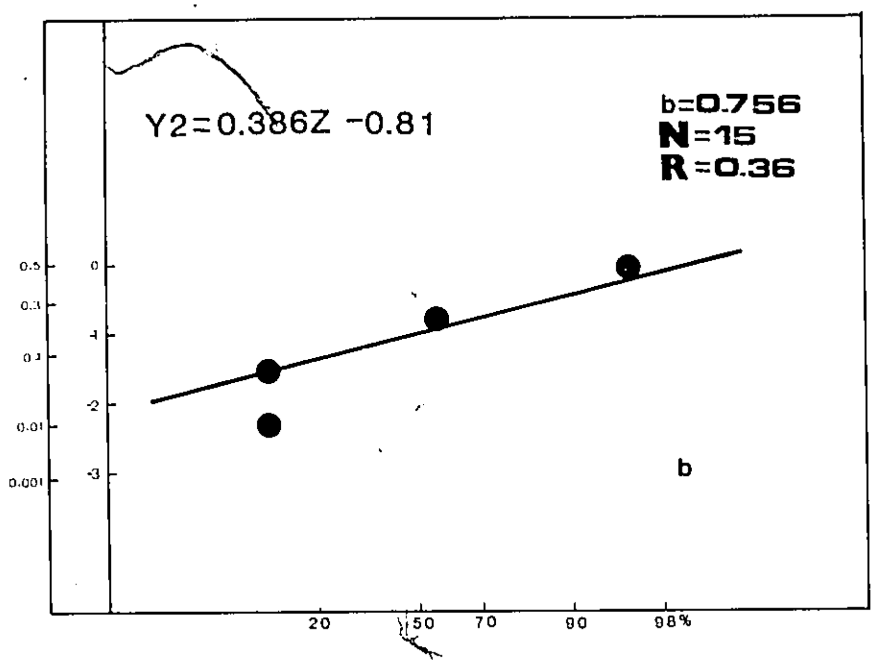
Figure 4.15a, b, c Experimental frequency and theoretical (probnormal) distrbutions for 31(single values) 15(pairs) and 7(sets of four adjacent samples) for A3(E. mosquensis).The two distributions show a good fit.

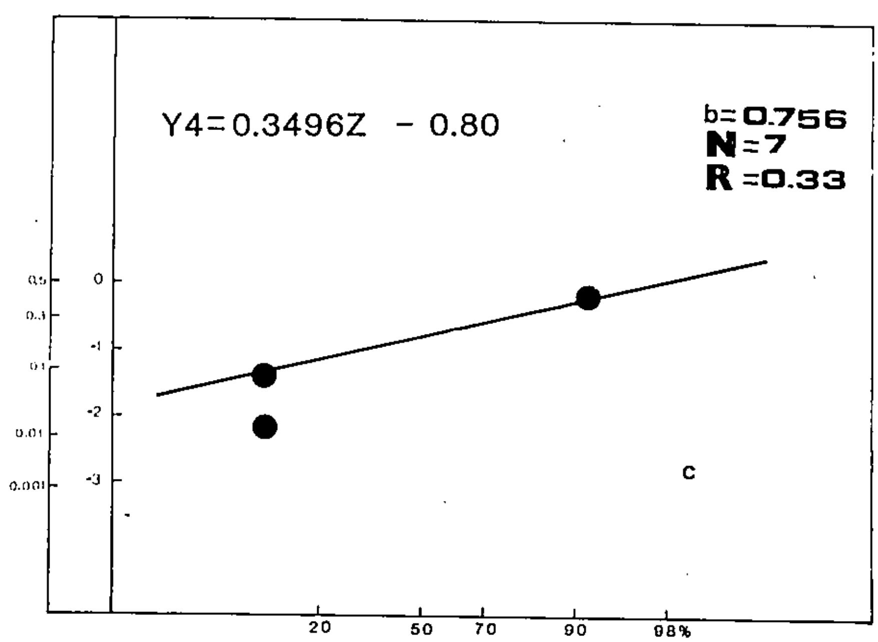


Y = 0.26z - 1.980
b = 1.916
R = 0.25
n = 7
UPPER POINTS OF CLASSES
X = Φ(Y)
Y
0
-1
-2
-3
-4
0.9
0.7
0.5
0.3
0.1
0.01
0.001
c
1 2 5 10 20 40 60 80 95 99 99.9
CUMULATIVE FREQUENCY (%)



A3:E.MOSQUENSIS
Y = 0.4235Z - 0.821
b=0.756
N=31
R=0.39
UPPER POINTS OF CLASSES
X:Φ(Y)
Y
0
-1
-2
-3
0.5
0.3
0.1
0.01
0.001
4.15 a
20 50 70 90 98%
CUMULATIVE FREQUENCY PERCENTAGE


Y2=0.386Z -0.81
b=0.756
N=15
R=0.36
b
20
50
70
90
98%


Y4=0.3496Z - 0.80
b=0.756
N=7
R=0.33
c
20
50
70
90
98%

Z = - 1 y = - 1.244

Z = - 2 y = - 1.668

The distribution of the 15 successive values (31/2 = 15, pairs) of A3 has the same mean as that of the single values of A3. Its variance is

$$S^2(x_2) = \frac{1.49 * 0.01654}{2} = 0.01232$$

For this mean and variance, the R value estimated from Agterberg's (1984) graph is 0.36. The value b is constant and it is dependent on the mean. Eq. 60 becames $Y_2 = 0.386z - 0.81$ for these values. The Y_2 function is also plotted on the probability paper and shown in Fig. 4.15b. The 7 successive relative values of A3 (15/2 = 7 or 31/4 = 7, sets of four of samples) also have the same mean. Its variance is

$$S^2(x_4) = \frac{2.29*0.01654}{4}$$

and by using Agterberg's graph, R = 0.33. The $Y_4 = 0.3496z - 0.80$ function is plotted as the others and shown in Fig. 4.15c. There is good agreement between the observed values and the theoretical prob - normal distribution as shown in Fig. 4.15a, b and c.,.

4.2.3 OPHTHALMIDIUM STRUMOSUM

The ARIMA procedure was applied to A10. The model AR(1) and

Figure 4.16 Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the AR(1) time series model to A10(<u>O. strumosum</u>)

```
SAS                                              15
                      21:31 TUESDAY, JULY 14, 1987
ARIMA PROCEDURE

NAME OF VARIABLE          =      A10
MEAN OF WORKING SERIES=       1.92903
STANDARD DEVIATION        =   2.44503
NUMBER OF OBSERVATIONS=            31
                 AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   5.97819     1.00000  |                    |********************|
  1   2.42279     0.40527  |                    |********            |
  2   1.18282     0.19785  |                    |****                |
  3   2.97305     0.49732  |                    |**********          |
  4    1.9424     0.32491  |                    |******              |
  5  0.788303     0.13186  |                    |***                 |
  6   1.52585     0.25524  |                    |*****               |
  7   1.10028     0.18405  |                    |****                |
  8 -0.0387607   -0.00648  |                    |                    |
  9  0.736675     0.12323  |                    |**                  |
 10  0.282735     0.04729  |                    |*                   |
                   '.' MARKS TWO STANDARD ERRORS

              INVERSE AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1   -0.41223  |            ********|                    |
  2    0.28358  |                    |******              |
  3   -0.34462  |             *******|                    |
  4    0.04717  |                    |*                   |
  5   -0.10078  |                  **|                    |
  6    0.09322  |                    |**                  |
  7   -0.15191  |                 ***|                    |
  8    0.17082  |                    |***                 |
  9   -0.13653  |                 ***|                    |
 10    0.09781  |                    |**                  |

              PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    0.40527  |                    |********            |
  2    0.04022  |                    |*                   |
  3    0.48425  |                    |**********          |
  4   -0.04037  |                   *|                    |
  5   -0.00047  |                    |                    |
  6    0.02176  |                    |                    |
  7   -0.06657  |                   *|                    |
  8   -0.11777  |                  **|                    |
  9    0.08772  |                    |**                  |
 10   -0.14693  |                 ***|                    |

         AUTOCORRELATION CHECK FOR WHITE NOISE

 TO   CHI                            AUTOCORRELATIONS
LAG  SQUARE DF  PROB
  6   23.37  6  0.001  0.405  0.198  0.497  0.325  0.132  0.255
```

```
SAS                                              16
                      21:31 TUESDAY, JULY 14, 1987
ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                            APPROX.
PARAMETER    ESTIMATE    STD ERROR    T RATIO  LAG
MU            1.96197     0.700721       2.80    0
AR1,1          0.4263     0.173122       2.46    1

CONSTANT ESTIMATE   =   1.12558

VARIANCE  ESTIMATE  =   5.24877
STD ERROR ESTIMATE  =   2.29102
AIC                 =   141.505
SBC                 =   144.373
NUMBER OF RESIDUALS=         31

       CORRELATIONS OF THE ESTIMATES

                    MU    AR1,1
MU               1.000    0.077
AR1,1            0.077    1.000

       AUTOCORRELATION CHECK OF RESIDUALS

 TO   CHI                           AUTOCORRELATIONS
LAG  SQUARE DF  PROB
  6   11.20  4  0.024 -0.005 -0.196  0.436  0.166 -0.104  0.185
 12   16.79 10  0.079  0.171 -0.173  0.137  0.116 -0.136 -0.101
 18   21.12 16  0.174  0.080 -0.053 -0.151 -0.081 -0.132 -0.089
 24   24.25 22  0.334 -0.052 -0.057 -0.112 -0.039 -0.050 -0.075

MODEL FOR VARIABLE A10
ESTIMATED MEAN        =   1.96197

AUTOREGRESSIVE FACTORS
            FACTOR 1
1 -0.4263B**(1)
```

ARMA (1,1) were fitted to the data set of Ophthalmidium strumosum AR(1) and ARMA (1, 1) appear to be good models for this series. Their first and third autocorrelations are significantly different from zero as seen in Fig. 4.16. Here we see the first and second asterisks in the a.c.f extend beyond the 95% confidence belt of two standard errors represented by dots. The AR (1) and ARMA (1 , 1) are stationary since the absolute value of $\hat{\phi}_1 < 1$. The key point to remember is that any stationary AR (1) process has a theoretical a.c.f showing exponential decay. ARMA (1, 1) model also shows exponential decay when $\hat{\phi}_1$'s are positive.

The quality of estimated coefficents can be tested with the t- test. For AR (1), $t_1 = 2.46$ (see Fig.4.16) which is greater than 2.0 and $\hat{\phi}_1$ is significantly different from zero at the 5% significance level. For ARMA(1, 1), $t_{\phi_1} = 6.16$, $t_{\theta_1} = 2.58$ and $\hat{\phi}_1$ and $\hat{\theta}_1$ are larger than 2.0. This means that the coefficients of both models are good estimates (see Fig. 4.17).

The t_b - test was used for random shocks, using Eq. 23. For AR (1), $t_{b_1} = -0.279$ at lag 1 and $t_{b_2} = 0.945$ at lag 2. The absolute t_b values of the residual a.c.f are less than 1.25 at lags 1 and 2. The random shocks at those lags are independent and not correlated. For ARMA (1, 1), $t_{b_1} = 0.173$ at lag 1 and $t_{b_2} = 0.533$ at lag 2. The absolute t_b values are less than 1.25, indicating that the random shocks are not correlated for the ARMA (1, 1) model either.

Furthermore, the chi - squared test was applied to the two models. For AR (1), the calculated value is 11.20 for degrees of freedom

df = 4. The critical value from table is 9.49 for df = 4. The calculated value is 16.79 for df = 10. The critical value is 18.3 for df = 10 at the 5% significant level. Since the calculated chi - squared value is less than the critical values it may be concluded that the random shocks are independent (Fig. 4.16).

ARMA (1,1) has a calculated chi - squared value of 8.10 at lag 6 for df = 3 and a calculated value of 12.30 at lag 12 for df = 9. The critical values are 7.81 and 16.9, respectively, at the 5% significance level. The calculated chi - squared value is less than the critical value at lag 6, so the chi - squared statistics suggest that the ARMA (1, 1) model is inadequate (Fig. 4.17).

If AR (1) and ARMA (1, 1) were good models for the working series, AR (1) would be preferred because it has one estimated value fewer than model ARMA (1, 1).

A10 (Ophthalmidium strumosum) has a time series model which is AR (1). AR (1) can be written as $X_t = C + \phi_1 X_{t-1} + a_t$. This model is a signal plus - noise model.

The geostatistical model can be applied as to the previous data set (Epistomina mosquensis). The correlation coefficents were plotted against the lag. Its pattern is an exponential curve. When a logarithmic scale is used for the correlation coefficients, as in Fig. 4.18, this exponential curve plots as a straight line. The a and c values are constant for a stationary series and are estimated from Fig. 4.18. The values are a = 0.08 and c = 0.50. Eq. 47 becomes

$$r(k) = 0.50e^{-0.08k}$$

Figure 4.17 Printout of ARMA(1 1) time series model for A10(_O._ strumosum)

```
                          SAS                                17
                                     21:31 TUESDAY, JULY 14, 1987
                  ARIMA PROCEDURE

            NAME OF VARIABLE        =      A10
            MEAN OF WORKING SERIES=    1.92903
            STANDARD DEVIATION    =    2.44503
            NUMBER OF OBSERVATIONS=         31
                   AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    5.97819    1.00000  |                    |********************|
  1    2.42279    0.40527  |                .   |********            |
  2    1.18282    0.19786  |              .     |****  .             |
  3    2.97305    0.49732  |             .      |**********          |
  4     1.9424    0.32491  |            .       |******    .         |
  5   0.788303    0.13186  |            .       |***       .         |
  6    1.52585    0.25524  |           .        |*****      .        |
  7    1.10028    0.18405  |           .        |****       .        |
  8 -0.0387607   -0.00648  |           .        |           .        |
  9   0.736675    0.12323  |           .        |**         .        |
 10   0.282735    0.04729  |           .        |*          .        |
                    '.' MARKS TWO STANDARD ERRORS

             INVERSE AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1    -0.41223  |             ********|                   |
       2     0.28358  |                    .|******.            |
       3    -0.34462  |              *******|       .           |
       4     0.04717  |                    .|*      .           |
       5    -0.10078  |                  **|        .           |
       6     0.09322  |                    .|**     .           |
       7    -0.15191  |                 ***|        .           |
       8     0.17082  |                    .|***    .           |
       9    -0.13653  |                 ***|        .           |
      10     0.09781  |                    .|**     .           |

             PARTIAL AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1     0.40527  |                   .|********            |
       2     0.04022  |                   .|*      .            |
       3     0.48425  |                   .|**********          |
       4    -0.04037  |                   *|       .            |
       5    -0.00047  |                   .|       .            |
       6     0.02176  |                   .|       .            |
       7    -0.06657  |                   *|       .            |
       8    -0.11777  |                  **|       .            |
       9     0.08772  |                   .|**     .            |
      10    -0.14693  |                 ***|       .            |

          AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                         AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   23.37  6  0.001  0.405  0.198  0.497  0.325  0.132  0.255
```

```
                          SAS                                18
                                     21:31 TUESDAY, JULY 14, 1987
            ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                    APPROX.
       PARAMETER      ESTIMATE   STD ERROR   T RATIO   LAG
       MU              1.88344     1.26202      1.49     0
       MA1,1          0.653912    0.252976      2.58     1
       AR1,1          0.902875     0.14653      6.16     1

       CONSTANT ESTIMATE    =   0.18293

       VARIANCE  ESTIMATE   =   4.99363
       STD ERROR ESTIMATE   =   2.23464
       AIC                  =   141.133
       SBC                  =   145.435
       NUMBER OF RESIDUALS=          31

             CORRELATIONS OF THE ESTIMATES

                       MU      MA1,1     AR1,1
          MU        1.000      0.284     0.375
          MA1,1     0.284      1.000     0.835
          AR1,1     0.375      0.835     1.000

           AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                              AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    8.10  3  0.044  0.031 -0.279  0.304  0.100 -0.160  0.115
 12   12.30  9  0.197  0.096 -0.176  0.113  0.099 -0.131 -0.095
 18   14.60 15  0.480  0.090 -0.025 -0.110 -0.050 -0.097 -0.046
 24   15.22 21  0.812  0.008  0.001 -0.061 -0.004 -0.009 -0.038

            MODEL FOR VARIABLE A10
            ESTIMATED MEAN        =   1.88344

             AUTOREGRESSIVE FACTORS
                          FACTOR 1
            1-.902875B**(1)

             MOVING AVERAGE FACTORS
                          FACTOR 1
            1-.653912B**(1)
```

F_N and F_L were calculated as in section 4.21 and Table 4.7 was constructed.

TABLE 4.7 Theoretical and observed relative variance of Ophthalmidium strumosum

k	$S^2(x)$	F_N	F_L
0	0	0	0.50
1	6.18	1	0.99
2	4.36	1.41	1.45
3	3.54	1.72	1.89
4	3.33	2.16	2.30
5	3.16	2.56	2.70
6	2.96	2.87	3.07

The observed relative variance (F_N) and theoretical variance (F_L) were plotted against lag. The correspondence between values for F_N and F_L is shown in Fig. 4.19. For large values of N or L the curve tends towards a horizontal limit of F_N = 8.38. The standard deviation of 31 uncorrelated data would be $s(\bar{x}_{31}) = 0.004$. The population mean (μ) lies in the interval $\bar{x} \pm 0.004 * t_{0.975}$ with a probability of 95% because $P(0.0102 < \mu < 0.028) = 95\%$ According to Eq. 59, $F_{31} = 8.38$, $s(x_{31})$ must be corrected by the factor $(F_{31})^{1/2} * s(x_{31}) = (8.38)^{1/2} * 0.004 = 0.12$. This results in a new confidence interval of $P(0.008 < \mu < 0.0309) = 95\%$. The estimated mean in Fig. 4.16 is 0.0196 which falls within this confidence belt.

The Ophthalmidium strumosum data, pairs of Ophthalmidium

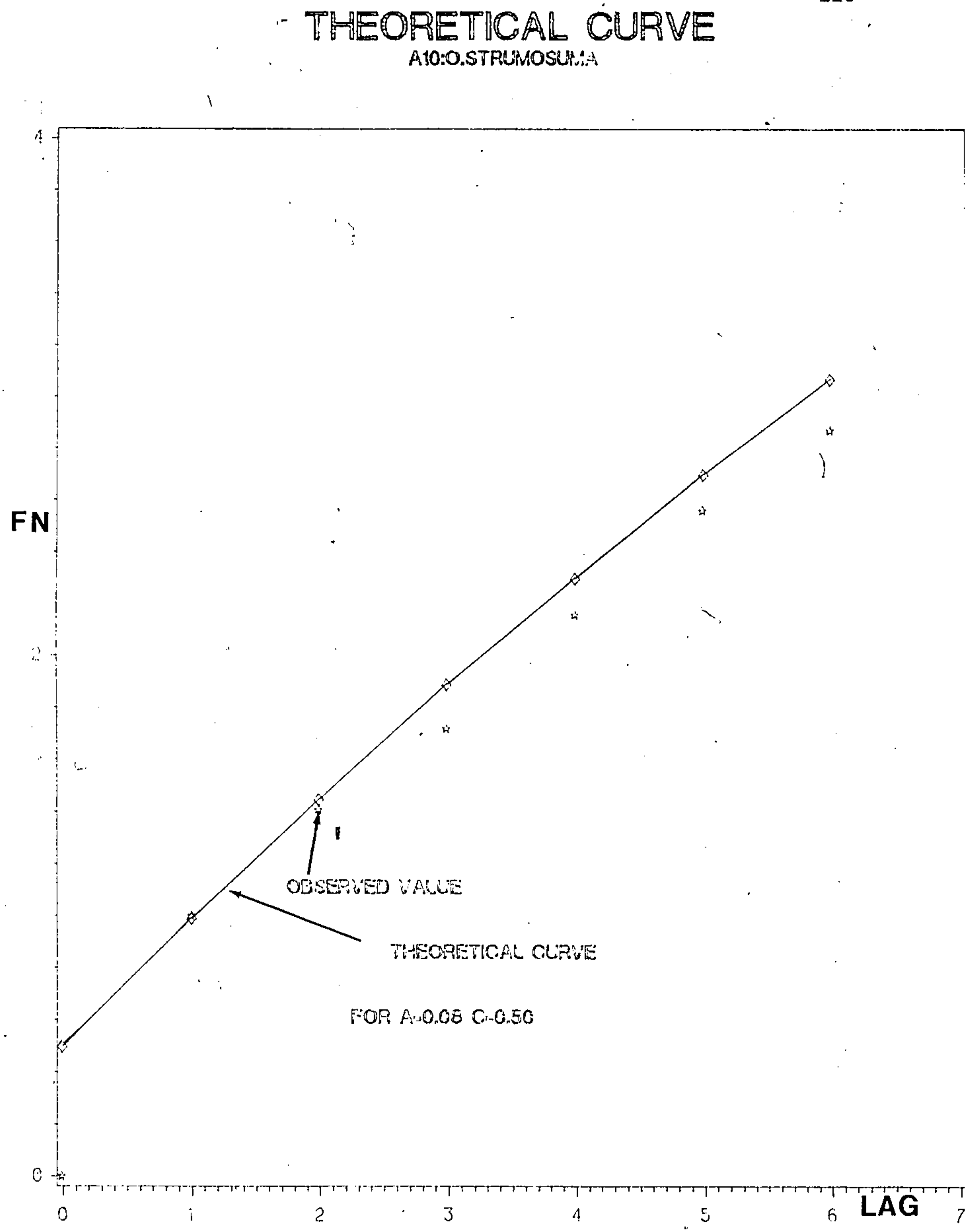


Figure 4.19 Theoretical curve for a = 0.08 and c = 0.50 in comparison with observed values of <u>O. strumosum</u>

Table 4.8 probit transformation of A10 (<u>Ophthalmidium strumosum</u>) from the Tojeira 1 section.

sample single %	probit	sample pairs %	probit	sample set of 4 %	probit
0.0	2.187				
0.0	2.187	0.0	2.187		
0.0	2.187				
0.0	2.187	0.0	2.187	0.0	2.187
0.0	2.187				
0.0	2.187	0.0	2.187		
0.0	2.187				
0.2	2.122	0.1	1.910	0.05	2.187
0.0	2.187				
0.0	2.187	0.0	2.187		
0.3	2.252				
0.2	2.122	0.25	2.187	0.12	1.910
0.0	2.187				
0.0	2.187	0.0	2.187		
2.3	3.005				
0.3	2.252	1.3	2.774	0.65	2.510
2.9	2.634				
8.3	3.615	5.6	3.411		
6.6	3.494				
1.9	2.925	4.2	3.279	4.91	3.345

(continued)

Table 4.8 (concluded)

0.4	2.348				
7.2	3.539	3.8	3.226		
2.4	3.023				
1.9	2.925	2.15	2.976	2.96	3.112
5.8	3.428				
1.9	2.925	3.85	3.232		
2.9	3.104				
5.2	3.374	4.05	3.255	3.95	3.243
2.7	3.073				
1.1	2.710	1.90	2.925		
5.3	3.384				

Table 4.9 Cumulative frequency distributions of A10 (Ophthalmidium strumosum) for 31, 15 and 7 samples.

N = 31

class interval	freq %	cum freq	cum %	plott. value
2.25 - 2.55	16	16	51.61	50.0
2.55 - 2.85	2	18	58.06	56.4
2.85 - 3.15	7	25	80.65	78.7
3.15 - 3.45	3	28	90.32	88.3
3.45 - 3.75	3	31	100.0	97.9

N = 15

1.8 - 2.2	7	7	46.67	43.5
2.2 - 2.6	0	7	46.67	43.5
2.6 - 3.0	3	10	66.67	63.0
3.0 - 3.4	5	15	100.0	95.7

N = 7

1.75 - 2.25	3	3	42.86	36.4
2.25 - 2.75	1	4	57.14	50.0
2.75 - 3.25	3	7	100.0	90.9

strumosum data and set of 4 adjacent Ophthalmidium strumosum data were transformed with the probit transformation (Table 4.8). The frequency distribution table for these data was constructed as Tables 4.9. Their cumulative frequency distributions were plotted on normal probability paper. The mean of the 31 observed values of A10 (Ophthalmidium strumosum) is,

$\bar{x} = 0.01929$

its variance is,

$s^2 (x) = 0.000598$ and

$b = 2.068$ which is constant.

$R = 0.43$ and was derived from the mean and variance, using Appendix II or III. Eq. 60 becomes $Y = 0.4762Z - 2.2290$ which is the prob - normal theoretical distribution model for $N = 31$.

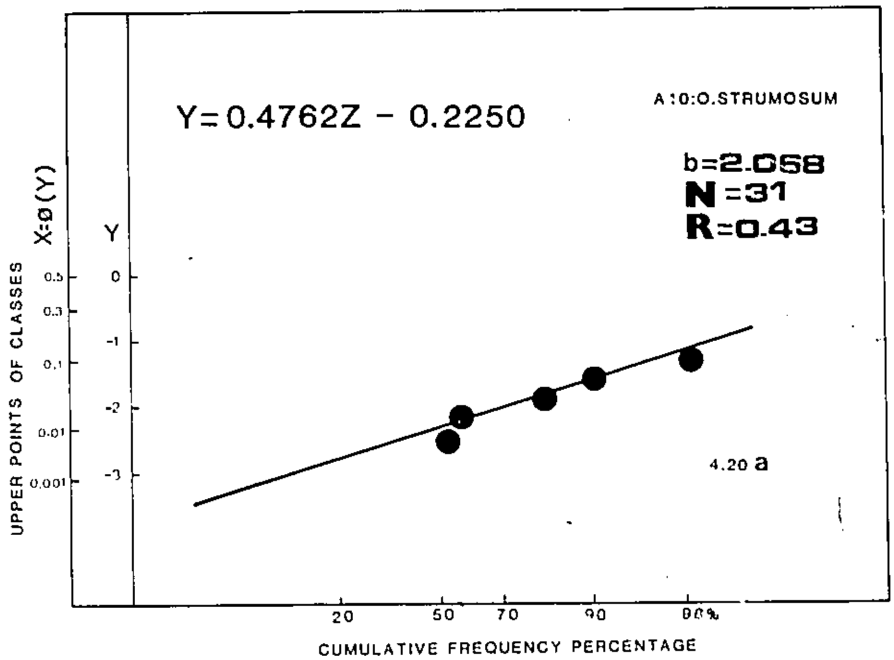
The theoretical distribution of 15 pairs of successive values of A10, shows that the mean is the same, $\bar{x} = 0.01929$ the variance $s^2(x) = 0.0004216$ and $b = 2.068$ $R = 0.37$ and again was derived from the mean and variance using Appendix II or III. The theoretical distribution function is $Y_2 = 0.398Z - 2.2206$

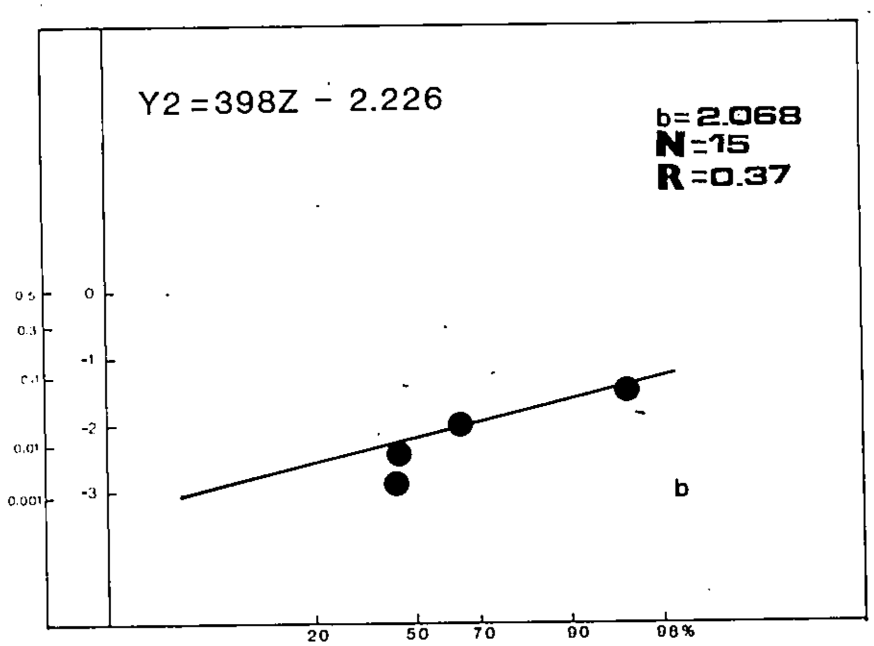
The theoretical distribution of 7 means of four successive relative values of A10 is $Y_4 = 0.349Z - 2.190$ for the mean $\bar{x} = 0.01929$ variance, $s^2(x) = 0.0003229$ and $R = 0.33$ $b = 2.068$

They were plotted on probability paper as can be seen in Fig. 4.20a, b, and c and determine the positions and slopes of the lines for prob - normal distributions satisfying Eq. 60.

4.3 - Tojeira 2 section

Figure 4.20a,b,c Experimental frequency distributions for 31,15 and 7 values of A10(O. strumosum) in comparison with their theoretical distributions (probnormal distributions) satisfying Eq.60


Y= 0.4762Z - 0.2250
A:0:O.STRUMOSUM
b=2.058
N=31
R=0.43
X:ø(Y)
Y
UPPER POINTS OF CLASSES
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
4.20 a
20
50
70
90
98%
CUMULATIVE FREQUENCY PERCENTAGE


Y2 =398Z - 2.226
b=2.068
N=15
R=0.37
0.5
0.3
0.01
0.001
0
-1
-2
-3
b
20
50
70
90
98%

4.3.1 Eoguttulina sp., Epistomina mosquensis, Ophthalmidium strumosum, Spirillina tenüissima and agglutinants

An a.c.f and p.a.c.f were estimated from the data set of Tojeira 2 (see Appendix I). Printout showed that 5 of 14 taxa had theoretical autocorrelations statistically different from zero at lag 1, 2 or 3. The 5 taxa are B1, B2, B11, B13, and B14. Their mean of these realizations is probably stationary. The estimated autocorrelations drop to zero fairly quickly with approximate exponential decay. Stationarity is confirmed since;

for taxon B1 (Eoguttulina sp.) $\hat{\phi}_1 = 0.475$,

for raxon B2 (Epistomina mosquensis) $\hat{\phi}_1 = 0.462$,

for taxon B11 (Spirillina tenuissima) $\hat{\phi}_1 = 0.748$,

for taxon B13 (Ophthalmidium strumosum) $\hat{\phi}_1 = 0.805$ and

for taxon B14 (agglutinants) $\hat{\phi}_1 = 0.51$

These satisfiy the condition $| \hat{\phi}_1 | < 1$. The estimated coefficients are acceptable because the corresponding absolute t - values are equal to or greater than 2.0. All are significantly different from zero at the 5% significance level. However, the t- value of the $\hat{\phi}_2$ for taxon B2 is less than 2.0. In fact, it can be assumed to be equal to 2.0 ($\hat{\phi}_2 = 1.97$)

The purpose of having a good time series model is to compare the estimated a.c.f and p.a.c.f with various theoretical a.c.f.'s and p.a.c.f.'s to find a match. The various theoretical a.c.f. models have been used to find best fitting models for the data sets of B1, B2, B11, B13, and B14 respectively.

Figure 4.21a Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the AR(1) time series model to B14(Agglutinants)

```
                              SAS                               27
                                   21:31 TUESDAY, JULY 14, 1987
                          ARIMA PROCEDURE

                   NAME OF VARIABLE        =       B14
                   MEAN OF WORKING SERIES=    10.4167
                   STANDARD DEVIATION      =   4.89027
                   NUMBER OF OBSERVATIONS=          30
                          AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    23.9147     1.00000 |                    |********************|
  1    12.4696     0.52142 |              .     |**********          |
  2    9.21081     0.38515 |             .      |********.           |
  3    9.81681     0.41049 |            .       |******** .          |
  4    7.29857     0.30519 |          .         |******     .        |
  5    4.29523     0.17961 |          .         |****       .        |
  6    3.49789     0.14627 |          .         |***        .        |
  7   0.223269     0.00934 |      .   .         |           .        |
  8  -0.771685    -0.03227 |          .        *|           .        |
  9    3.53575     0.14785 |          .         |***        .        |
 10  -0.452426    -0.01892 |          .         |           .        |
                          '.' MARKS TWO STANDARD ERRORS

                      INVERSE AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    -0.31910 |              .******|      .            |
  2    -0.01593 |              .      |      .            |
  3    -0.10536 |              .    **|      .            |
  4    -0.13177 |              .   ***|      .            |
  5     0.06135 |              .      |*     .            |
  6    -0.00749 |              .      |      .            |
  7     0.05631 |              .      |*     .            |
  8     0.18166 |              .      |****  .            |
  9    -0.28651 |              .******|      .            |
 10     0.10810 |              .      |**    .            |

                      PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.52142 |              .      |**********         |
  2     0.15557 |              .      |***   .            |
  3     0.22490 |              .      |****  .            |
  4    -0.00035 |              .      |      .            |
  5    -0.07980 |              .    **|      .            |
  6    -0.02123 |              .      |      .            |
  7    -0.16427 |              .   ***|      .            |
  8    -0.02753 |              .     *|      .            |
  9     0.27932 |              .      |******.            |
 10    -0.15514 |              .   ***|      .            |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   25.61  6  0.000  0.521  0.385  0.410  0.305  0.180  0.146
```

```
                              SAS                               28
                                   21:31 TUESDAY, JULY 14, 1987
                   ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                     APPROX.
            PARAMETER    ESTIMATE   STD ERROR   T RATIO  LAG
            MU             10.331     1.54778      6.67    0
            AR1,1        0.510959     0.15981      3.20    1

            CONSTANT ESTIMATE   =   5.05226

            VARIANCE  ESTIMATE =   18.5773
            STD ERROR ESTIMATE =   4.31014
            AIC                =   175.027
            SBC                =    177.83
            NUMBER OF RESIDUALS=        30

                    CORRELATIONS OF THE ESTIMATES

                                MU     AR1,1
                    MU       1.000     0.005
                    AR1,1    0.005     1.000

                 AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                             AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    3.16  4  0.531 -0.073  0.011  0.228  0.113 -0.007  0.129
 12   12.90 10  0.229 -0.067 -0.168  0.293 -0.068 -0.199  0.201
 18   14.70 16  0.547 -0.081 -0.021  0.069 -0.039 -0.115 -0.011
 24   29.29 22  0.137 -0.005 -0.252  0.068 -0.166 -0.134 -0.118

              MODEL FOR VARIABLE B14
              ESTIMATED MEAN     =     10.331

               AUTOREGRESSIVE FACTORS
                           FACTOR 1
              1-.510959B**(1)
```

Figure 4.21b Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the AR(1) time series model to B2(E. mosquensis)

```
                           SAS                                   21
                               21:31 TUESDAY, JULY 14, 1987
                    ARIMA PROCEDURE

           NAME OF VARIABLE         =       B2
           MEAN OF WORKING SERIES=       13.84
           STANDARD DEVIATION    =     11.1162
           NUMBER OF OBSERVATIONS=          30
                    AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    123.57     1.00000  |                    |********************|
  1   82.6732     0.66904  |                    |*************       |
  2   73.2918     0.59312  |                    |************        |
  3   65.7658     0.53221  |                    |***********.        |
  4   44.4355     0.35960  |              .     |*******   .         |
  5   36.0074     0.29139  |             .      |******     .        |
  6   31.5627     0.25542  |             .      |*****      .        |
  7   19.0184     0.15391  |             .      |***        .        |
  8   16.5723     0.13411  |             .      |***        .        |
  9   9.96005     0.08060  |             .      |**         .        |
 10  -3.74567    -0.03031  |             .     *|           .        |
                        '.' MARKS TWO STANDARD ERRORS

                INVERSE AUTOCORRELATIONS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
        1   -0.24444   |               .*****|    .               |
        2   -0.16458   |               .  ***|    .               |
        3   -0.18472   |               . ****|    .               |
        4    0.08161   |               .     |**  .               |
        5    0.07836   |               .     |**  .               |
        6   -0.02635   |               .    *|    .               |
        7    0.02726   |               .     |*   .               |
        8   -0.07548   |               .   **|    .               |
        9   -0.06415   |               .    *|    .               |
       10    0.10547   |               .     |**  .               |

                PARTIAL AUTOCORRELATIONS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
        1    0.66904   |               .     |*************       |
        2    0.26341   |               .     |*****               |
        3    0.12389   |               .     |**  .               |
        4   -0.17430   |               .  ***|    .               |
        5   -0.02600   |               .    *|    .               |
        6    0.06205   |               .     |*   .               |
        7   -0.05618   |               .    *|    .               |
        8    0.01633   |               .     |    .               |
        9   -0.05145   |               .    *|    .               |
       10   -0.14598   |               .  ***|    .               |

           AUTOCORRELATION CHECK FOR WHITE NOISE

 TO   CHI                         AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   47.59  6  0.000   0.669  0.593  0.532  0.360  0.291  0.255
```

```
                          SAS                                        22
                                  21:31 TUESDAY, JULY 14, 1987
              ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                       APPROX.
           PARAMETER      ESTIMATE    STD ERROR    T RATIO   LAG
           MU              13.8358      7.09741       1.95     0
           AR1,1          0.462591     0.174447       2.65     1
           AR1,2          0.367008     0.185876       1.97     2

           CONSTANT ESTIMATE    =  2.35762

           VARIANCE ESTIMATE  =   60.6352
           STD ERROR ESTIMATE =   7.78686
           AIC                =   212.175
           SBC                =   216.378
           NUMBER OF RESIDUALS=        30

                   CORRELATIONS OF THE ESTIMATES

                            MU     AR1,1     AR1,2
              MU         1.000     0.040     0.180
              AR1,1      0.040     1.000    -0.721
              AR1,2      0.180    -0.721     1.000

              AUTOCORRELATION CHECK OF RESIDUALS

 TO     CHI                        AUTOCORRELATIONS
LAG   SQUARE DF   PROB
  6     2.41  3  0.492 -0.092 -0.104  0.171 -0.047 -0.052  0.115
 12     4.86  9  0.846 -0.022 -0.011  0.059  0.006 -0.212  0.002
 18     8.26 15  0.913  0.144  0.060  0.148 -0.048 -0.042 -0.062
 24    12.03 21  0.939  0.056 -0.138 -0.006  0.021 -0.094 -0.051

            MODEL FOR VARIABLE B2
            ESTIMATED MEAN        =   13.8358

             AUTOREGRESSIVE FACTORS
                            FACTOR 1
            1-.462591B**(1)-.367008B**(2)
```

Using the tests that were explained before, it was decided that AR (1) is good model for B14 (Fig. 4.21a), and AR(2) is a good model for taxon B2 (Fig. 4.21b).

According to these tests, AR (1) would also be the best model for taxa B1, B11 and B13 (Figrs. 4.21c, 4.21d and 4.21e). However, the calculated residual of B1 is not statistically zero at lag 2 as seen below.

$$t_{b_1} = \frac{-0.320}{0.1833} = -1.74$$

The absolute t_b value is larger than 1.25 at lag 2. This suggests that the AR(1) model is not an adequate model. Because the residuals are significantly correlated at lag 2, the residuals absolute t_b value of B2 is

$$t_{b_2} = \frac{-0.242}{0.1826} = -1.32$$

at lag 1 (Eq.33). Consequently, the random shock is correlated at lag 1 for taxon B11.

Furthermore, the calculated chi - squared value for taxa B11 is large. This suggests that the residual autocorrelations as a set are significantly different from zero, and the random shocks of the estimated model are probably autocorrelated.

In this case, the residual a.c.f. at lag 2 for taxon B1 and lag 1 for B11 might be ignored, believing that one significant autocorrelation out of 8 could occur just because of sampling error. Alternatively, these models could be assumed statistically inadequate, because some residual a.c.f. t_b - values exceed the suggested critical values.

However, there is no guarantee, of course, that a better model will be discovered.

A statistically adequate model is one whose random shocks are statistically independent, meaning that they are not autocorrelated (Autocorrelation coefficents are all statistically zero). We can not expect all residual autocorrelations to be exactly zero, even for a properly constructed model. The reason is, that the residuals are calculated from a realization (not a process) using only estimates of the ARIMA coefficients (not their true values). Therefore, we expect that sampling error may cause some residual autocorrelations to be nonzero even if a good model has been found.

Unfortunately, there is a potential problem in using Barlett's formula (Eq.33) in testing residual autocorrelation. The estimated standard errors are sometimes seriously overstated when applying Bartlett's formula to residual autocorrelations. This is especially possible at the very short lags. If the estimated standard errors are overstated, then the corresponding t_b - values are underestimated. This is very likely, since finding the exact values for the estimated standard errors and t - values for residual autocorrelations is relatively difficult (Box and Pierce, 1970).

According to the t - test (at lag 2) and chi - squared test (for df = 4 and df = 10), the random shocks of the estimated model of taxon B13 are autocorrelated. The various models tested, which are AR (1), AR (2), ARMA (1, 1), ARMA (2,1), and ARMA (1,2), are not good time series models for taxon B13. However, the AR (1) model

Figure 4.21c Printout of estimated a.c.f.'s and p.a.c.f's after fitting the AR(1) time series model to B1(<u>Eoguttulina</u> sp.)

```
                                  SAS                                   19
                                              21:31 TUESDAY, JULY 14, 1987
                           ARIMA PROCEDURE

               NAME OF VARIABLE        =        B1
               MEAN OF WORKING SERIES=    2.84667
               STANDARD DEVIATION      =    1.43079
               NUMBER OF OBSERVATIONS=         30
                           AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    2.04716     1.00000 |                    |********************|
  1   0.952839     0.46545 |             .      |*********           |
  2   0.638966     0.31212 |           .        |******  .           |
  3   0.440271     0.21506 |           .        |****    .           |
  4   0.728687     0.35595 |          .         |*******  .          |
  5   0.721859     0.35262 |          .         |*******  .          |
  6   0.564276     0.27564 |         .          |******    .         |
  7  -0.148975    -0.07277 |         .         *|          .         |
  8   0.178041     0.08697 |         .          |**        .         |
  9    0.01608     0.00785 |         .          |          .         |
 10  0.0867185     0.04236 |         .          |*         .         |
                            '.' MARKS TWO STANDARD ERRORS

                     INVERSE AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1    -0.35597 |             *******|      .             |
       2    -0.01553 |             .      |      .             |
       3     0.05279 |             .      |*     .             |
       4    -0.05504 |             .     *|      .             |
       5    -0.12524 |             .   ***|      .             |
       6    -0.20078 |             .  ****|      .             |
       7     0.33761 |             .      |*******             |
       8    -0.19931 |             .  ****|      .             |
       9     0.07789 |             .      |**    .             |
      10     0.03349 |             .      |*     .             |

                     PARTIAL AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1     0.46545 |             .      |*********           |
       2     0.12189 |             .      |**    .             |
       3     0.03986 |             .      |*     .             |
       4     0.28239 |             .      |******.             |
       5     0.12452 |             .      |**    .             |
       6     0.00929 |             .      |      .             |
       7    -0.37665 |            ********|      .             |
       8     0.17039 |             .      |***   .             |
       9    -0.16726 |             .   ***|      .             |
      10    -0.05192 |             .    *|      .             |

              AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                          AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
   6   24.65  6  0.000   0.465  0.312  0.215  0.356  0.353  0.276
```

```
                                  SAS                                   20
                                              21:31 TUESDAY, JULY 14, 1987
                 ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                      APPROX.
         PARAMETER      ESTIMATE    STD ERROR    T RATIO  LAG
         MU              2.86394      0.43803       6.54    0
         AR1,1          0.475357     0.165735       2.87    1

         CONSTANT ESTIMATE   =  1.50255

         VARIANCE  ESTIMATE  =  1.69163
         STD ERROR ESTIMATE  =  1.30063
         AIC                 =  103.093
         SBC                 =  105.896
         NUMBER OF RESIDUALS=        30

                  CORRELATIONS OF THE ESTIMATES

                              MU     AR1,1
                 MU        1.000    -0.042
                 AR1,1    -0.042     1.000

                AUTOCORRELATION CHECK OF RESIDUALS

  TO    CHI                         AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
   6    6.29  4  0.178 -0.061  0.079 -0.063  0.223  0.184  0.259
  12   15.86 10  0.104 -0.320  0.179 -0.059  0.171 -0.157 -0.140
  18   25.64 16  0.059 -0.171  0.214 -0.233  0.006 -0.127 -0.096
  24   31.01 22  0.096 -0.006 -0.036 -0.171  0.031 -0.120  0.007

           MODEL FOR VARIABLE B1
           ESTIMATED MEAN          =   2.86394

           AUTOREGRESSIVE FACTORS
                          FACTOR 1
           1-.475357B**(1)
```

Figure 4.21d Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the AR(1) time series model to B11(<u>S. tenuissima</u>)

SAS 21:31 TUESDAY, JULY 14, 1987 23

ARIMA PROCEDURE

```
NAME OF VARIABLE          =  B11
MEAN OF WORKING SERIES=      25.7533
STANDARD DEVIATION        =  17.3525
NUMBER OF OBSERVATIONS=      30
```

AUTOCORRELATIONS

LAG	COVARIANCE	CORRELATION
0	301.11	1.00000
1	203.829	0.67692
2	185.139	0.61485
3	175.348	0.58234
4	127.343	0.42291
5	142.935	0.47469
6	92.4682	0.30709
7	58.3148	0.19367
8	28.4353	0.09443
9	1.65364	0.00549
10	-5.76119	-0.01913

'.' MARKS TWO STANDARD ERRORS

INVERSE AUTOCORRELATIONS

LAG	CORRELATION
1	-0.39549
2	0.13276
3	-0.31940
4	0.31233
5	-0.36177
6	0.09598
7	-0.07369
8	0.16188
9	-0.05475
10	0.02759

PARTIAL AUTOCORRELATIONS

LAG	CORRELATION
1	0.67692
2	0.28910
3	0.18280
4	-0.15411
5	0.20868
6	-0.23593
7	-0.12503
8	-0.22517
9	0.04611
10	-0.04606

AUTOCORRELATION CHECK FOR WHITE NOISE

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	59.22	6	0.000	0.677	0.615	0.582	0.423	0.475	0.307

SAS 21:31 TUESDAY, JULY 14, 1987 24

ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

PARAMETER	ESTIMATE	APPROX. STD ERROR	T RATIO	LAG
MU	27.3794	8.10119	3.38	0
AR1,1	0.747753	0.121978	6.13	1

```
CONSTANT ESTIMATE   =  6.90635

VARIANCE  ESTIMATE  =  152.811
STD ERROR ESTIMATE  =  12.3617
AIC                 =  238.762
SBC                 =  241.564
NUMBER OF RESIDUALS=   30
```

CORRELATIONS OF THE ESTIMATES

	MU	AR1,1
MU	1.000	-0.091
AR1,1	-0.091	1.000

AUTOCORRELATION CHECK OF RESIDUALS

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	11.24	4	0.024	-0.242	-0.011	0.240	-0.239	0.366	0.017
12	13.10	10	0.218	0.024	-0.044	-0.107	0.137	-0.080	-0.002
18	14.05	16	0.595	-0.003	0.010	0.015	-0.111	0.019	-0.028
24	15.97	22	0.817	-0.092	0.009	-0.049	-0.085	-0.015	-0.015

```
MODEL FOR VARIABLE B11
ESTIMATED MEAN      =  27.3794

AUTOREGRESSIVE FACTORS
FACTOR 1
1-.747753B**(1)
```

Figure 4.21e Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the AR(1) time series model to B13(_O. strumosum_)

```
                              SAS                                 25
                    ARIMA PROCEDURE   21:31 TUESDAY, JULY 14, 1987

               NAME OF VARIABLE          =         B13
               MEAN OF WORKING SERIES=           11.25
               STANDARD DEVIATION     =        9.35617
               NUMBER OF OBSERVATIONS=              30
                         AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    87.5378    1.00000  |                    |********************|
  1    69.0171    0.78843  |                    |****************    |
  2    53.5328    0.61154  |                    |************        |
  3    56.5819    0.64637  |                    |*************       |
  4    51.9773    0.59377  |                    |************        |
  5    32.1162    0.36688  |                    |*******             |
  6    26.1013    0.29817  |                    |******              |
  7    24.1731    0.27614  |                    |******              |
  8    11.6782    0.13341  |                    |***                 |
  9   -1.97825   -0.02260  |                    |                    |
 10   -7.87917   -0.09001  |                  **|                    |
                            '.' MARKS TWO STANDARD ERRORS

                      INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.64532  |       *************|                    |
           2    0.21583  |                    |****                |
           3    0.10623  |                    |**                  |
           4   -0.42439  |            ********|                    |
           5    0.43180  |                    |*********           |
           6   -0.22277  |                ****|                    |
           7    0.01671  |                    |                    |
           8    0.05386  |                    |*                   |
           9   -0.09509  |                  **|                    |
          10    0.07555  |                    |**                  |

                      PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.78843  |                    |****************    |
           2   -0.02663  |                   *|                    |
           3    0.45587  |                    |*********           |
           4   -0.18277  |                ****|                    |
           5   -0.32625  |             *******|                    |
           6    0.16977  |                    |***                 |
           7   -0.23351  |               *****|                    |
           8   -0.02593  |                   *|                    |
           9   -0.12438  |                  **|                    |
          10   -0.21573  |                ****|                    |

               AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                              AUTOCORRELATIONS
LAG SQUARE DF   PROB
  6  70.00  6  0.000  0.788  0.612  0.646  0.594  0.367  0.298
```

```
                              SAS                                 26
            ARIMA: MAXIMUM LIKELIHOOD ESTIMATION   21:31 TUESDAY, JULY 14, 1987

             PARAMETER    ESTIMATE     APPROX.
                                     STD ERROR   T RATIO   LAG
             MU            10.5061     4.64023      2.26     0
             AR1,1        0.805088    0.105606      7.62     1

             CONSTANT ESTIMATE    =   2.04778

             VARIANCE  ESTIMATE =    32.2609
             STD ERROR ESTIMATE =    5.67987
             AIC                =    192.327
             SBC                =    195.129
             NUMBER OF RESIDUALS=         30

                     CORRELATIONS OF THE ESTIMATES

                                 MU       AR1,1
                     MU       1.000       0.076
                     AR1,1    0.076       1.000

                  AUTOCORRELATION CHECK OF RESIDUALS

  TO     CHI                              AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
   6   19.82  4  0.001  0.003 -0.414  0.257  0.446 -0.326 -0.066
  12   27.33 10  0.002  0.272  0.072 -0.183  0.131 -0.022 -0.185
  18   28.59 16  0.027 -0.090 -0.039 -0.074  0.024 -0.022 -0.059
  24   29.61 22  0.128 -0.006 -0.039 -0.064 -0.030  0.047 -0.013

               MODEL FOR VARIABLE B13
               ESTIMATED MEAN      =    10.5061

                AUTOREGRESSIVE FACTORS
                         FACTOR 1
               1-.805088B**(1)
```

was selected as a good model (Fig. 4.21e) because it has fewer estimated coefficients than the others. A geostatistical model can be applied to the Tojeira 2 section after a time series model has been recognized for each taxon.

The autocorrelation coefficients were plotted against the lag on semi - logarithmic graph paper. The exponential trends become straight lines as can be seen in Fig 4.21. The best fitting lines were drawn and the constant c and a values can be obtained as shown in Table 4.10.

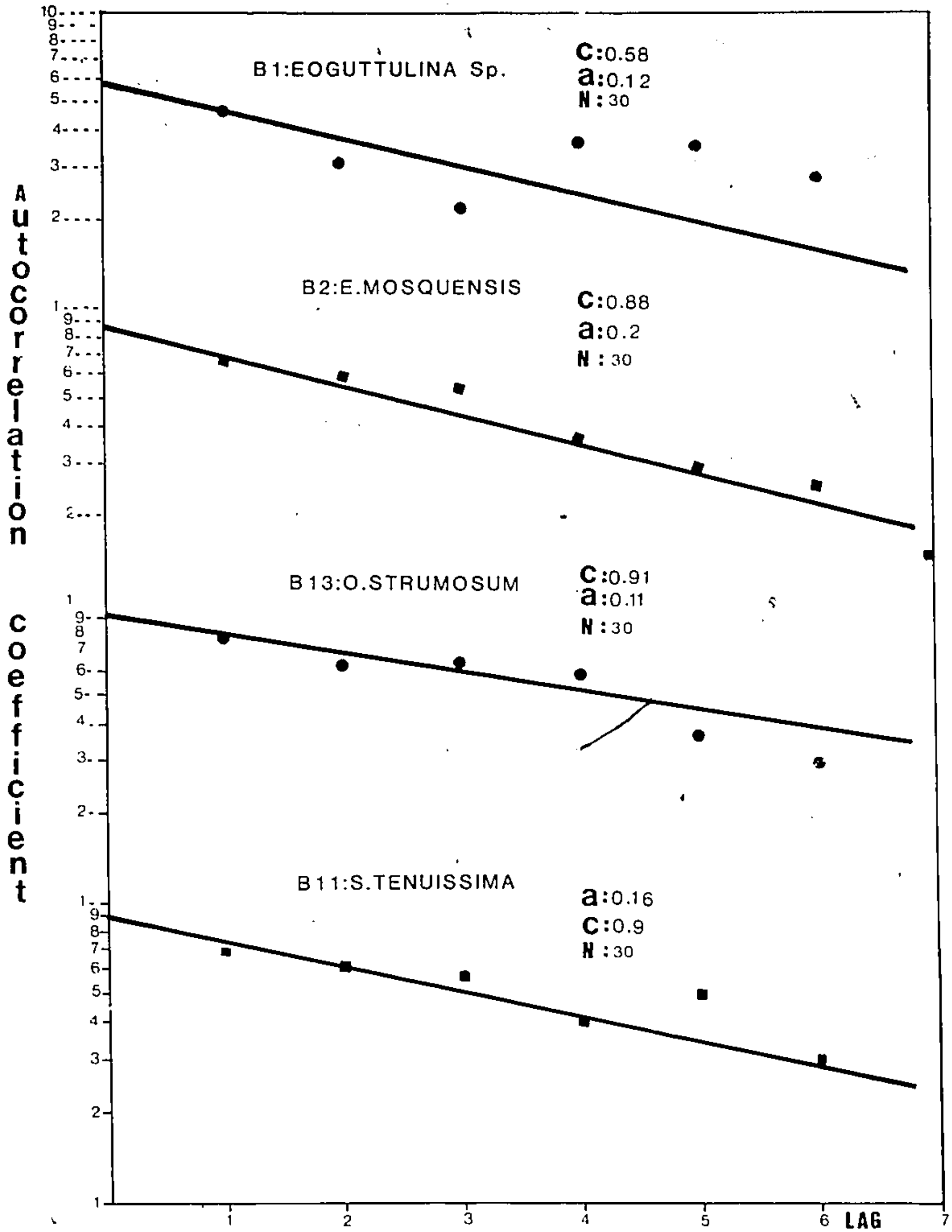
TABLE 4.10 Estimated a's and c's values from best - fitting lines for B1 (Eoguttulina sp.), B2 (Epistomina mosquensis), B11 (Spirillina tenuissima), B13 (Ophthalmidium strumosum) and B14 (agglutinants)

TAXON	B1	B2	B11	B13	B14
c	0.58	0.88	0.90	0.91	0.58
a	0.12	0.20	0.16	0.11	0.12

Table 4.10 shows that the c values for B2, B11 and B13 are quite large or close to 1.0, meaning that these data could fit a time series model which is just a signal (perfect autocorrelation) and not a signal - plus - noise model.

Variances of two or more than two adjacent values were calculated with the F FORTRAN program for each taxon F_N values, which are experimental relative variances, were calculated from variances of adjacent values (Eq. 54). F_L values which are theoretical relative

Figure 4.21 Correlograms of B1(*Eoguttulina* sp.), B2(*E. mosquensis*), B11(*S. tenuissima*), B13(*O. strumosum*) and B14(Agglutinants) with their best - fitting exponential curves. The a's and c's were estimated from the correlograms


Autocorrelation coefficient
B1:EOGUTTULINA Sp.
C:0.58
a:0.12
N:30
B2:E.MOSQUENSIS
C:0.88
a:0.2
N:30
B13:O.STRUMOSUM
C:0.91
a:0.11
N:30
B11:S.TENUISSIMA
a:0.16
C:0.9
N:30
LAG

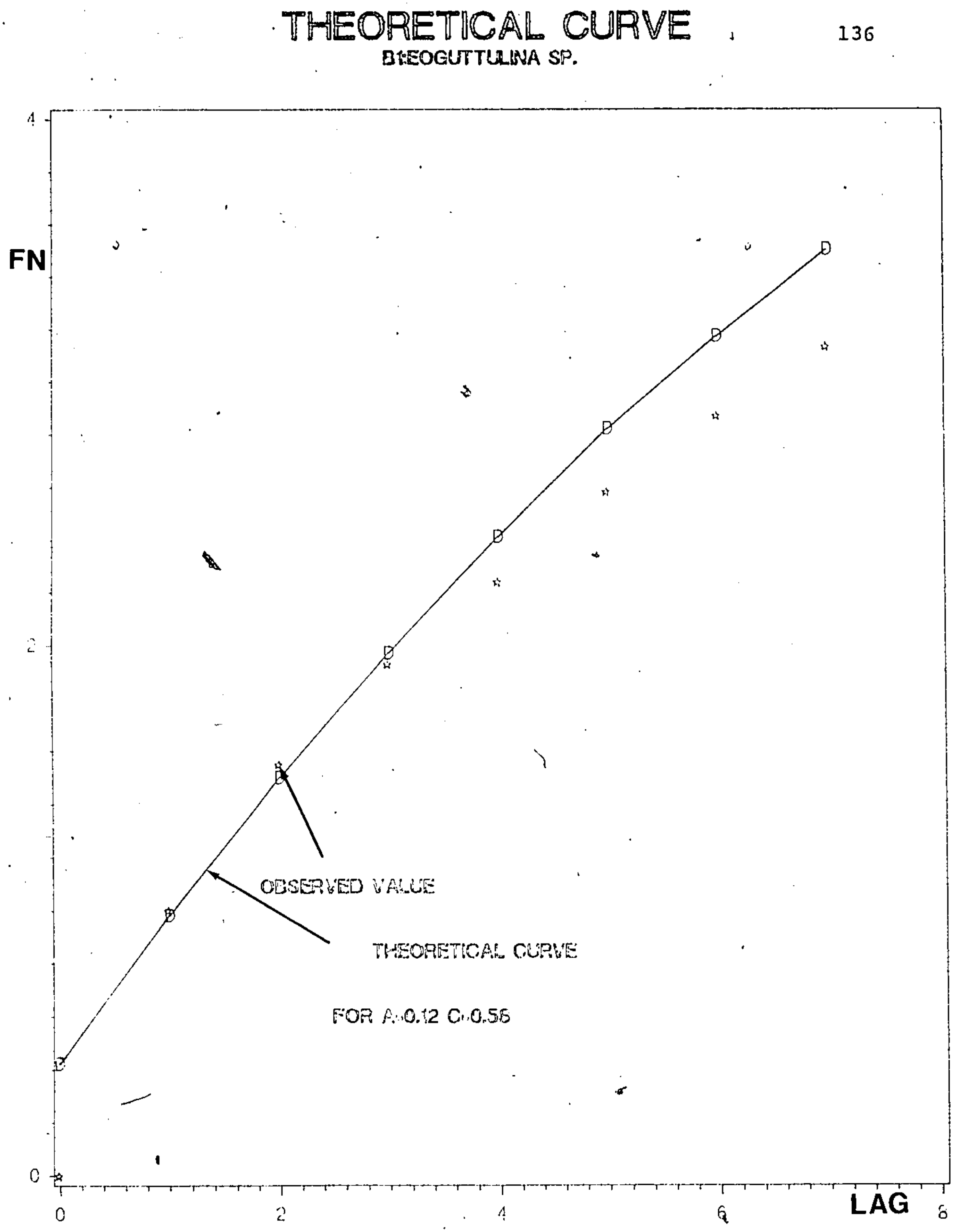


Figure 4.21a Theoretical curve for a = 0.12 and c = 0.58 in comparison with observed values of B1(<u>Eoguttulina</u> sp.)

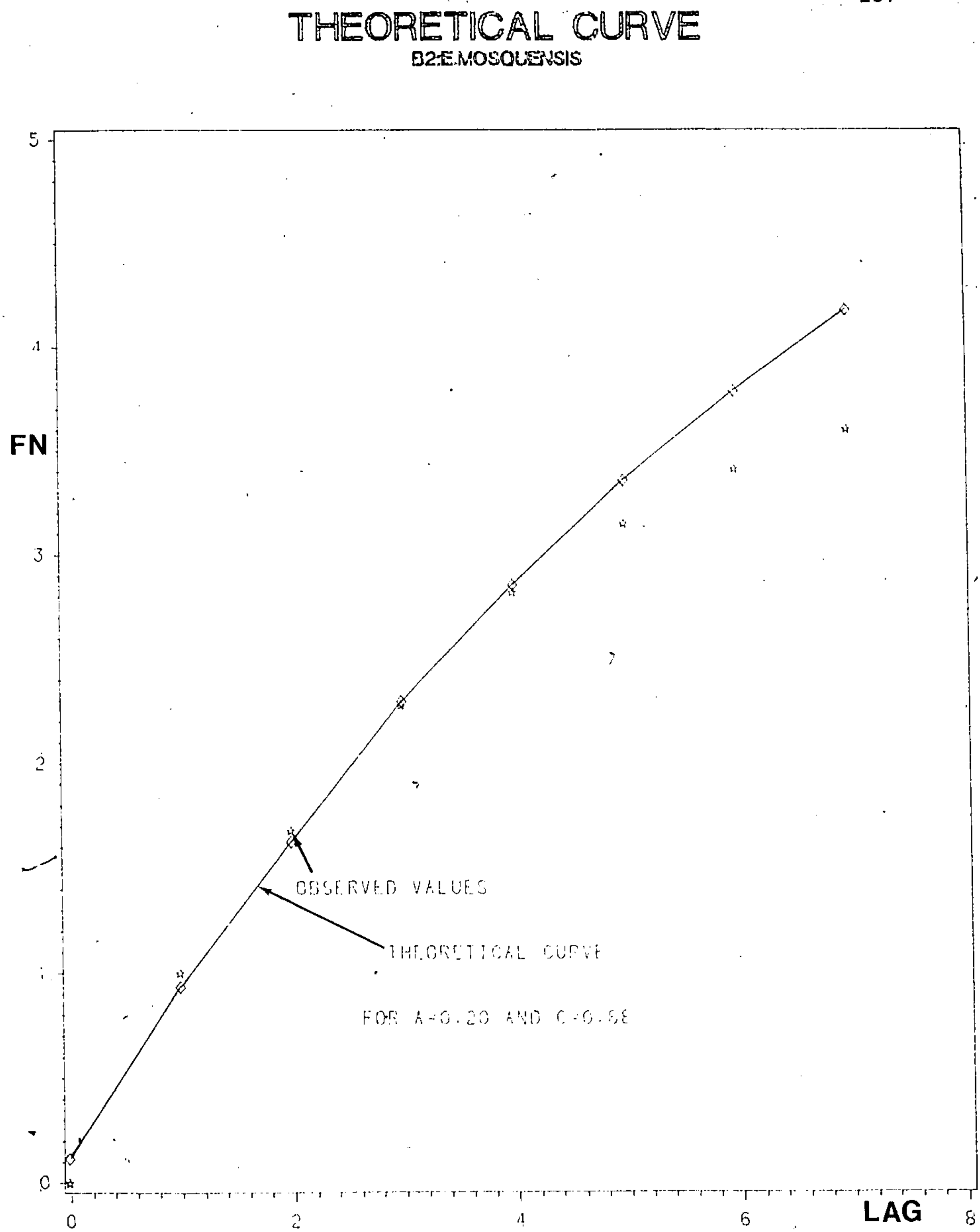


Figure 4.21b$_1$ Theoretical curve for a = 0.20 and c = 0.88 in comparison with observed values of B2(*E. mosquensis*)

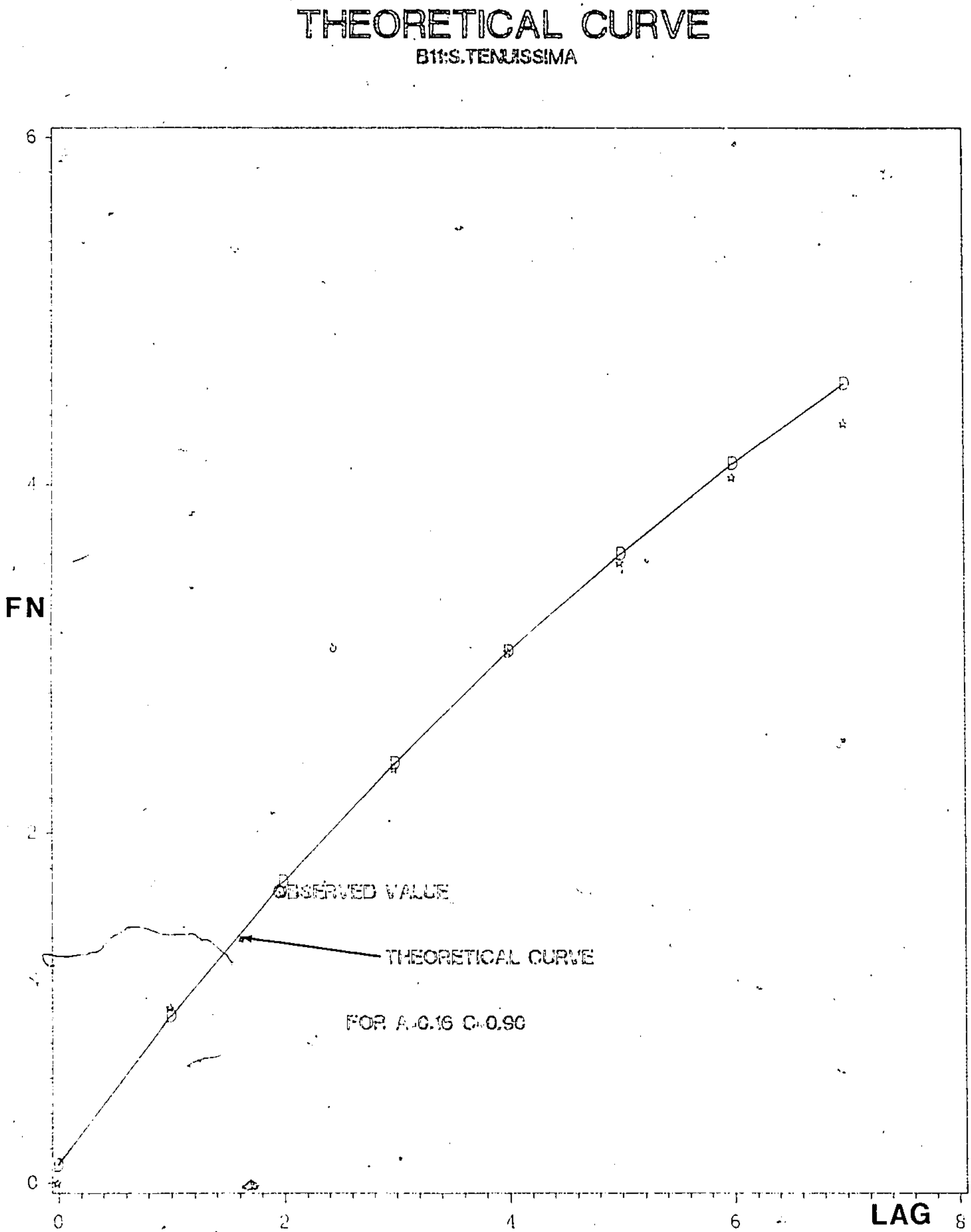


Figure 4.21c1 Theoretical curve for a = 0.16 and c = 0.90 in comparison with observed values of B11 (*S. tenuissima*)

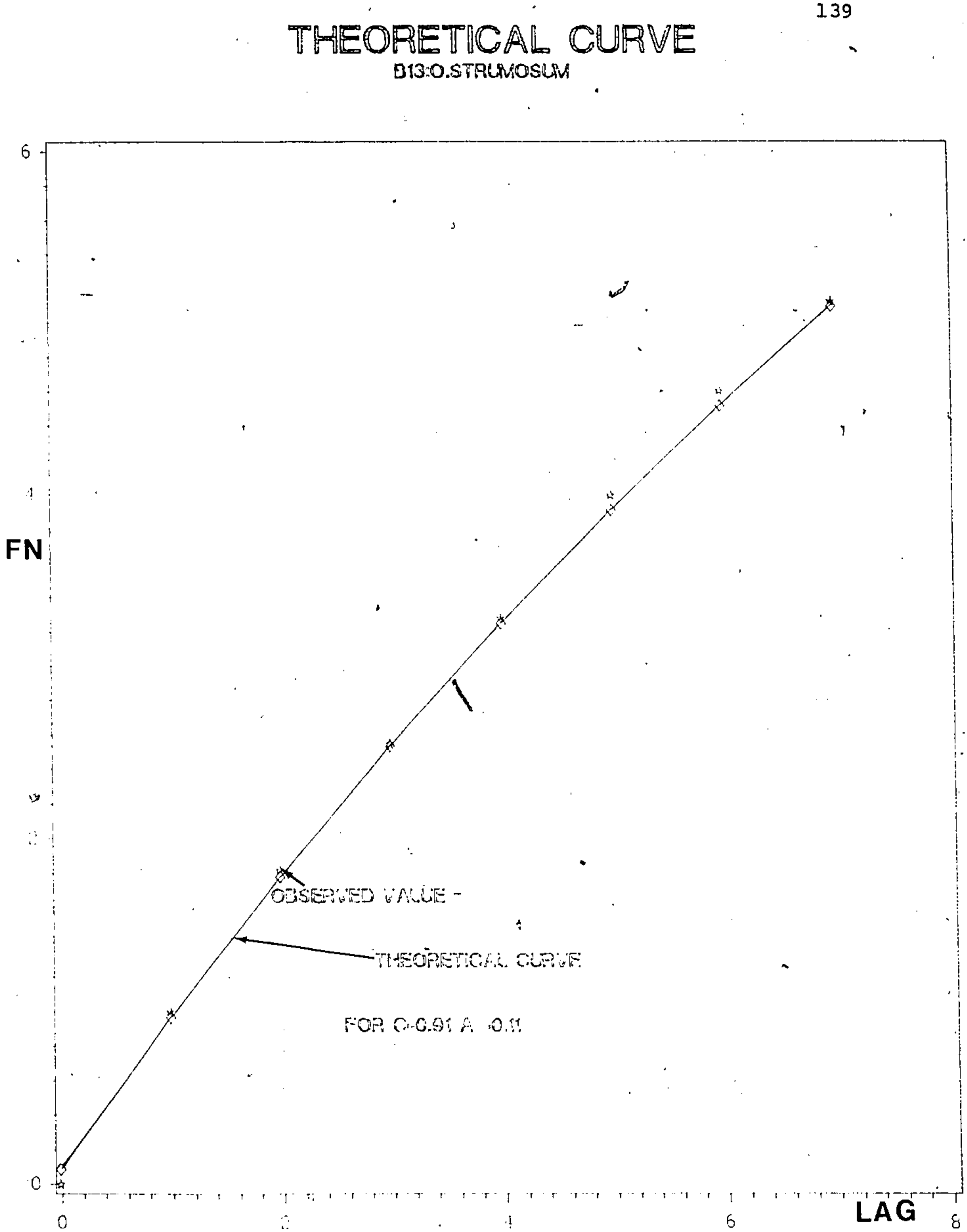


Figure 4.21d_1 Theoretical curve for a = 0.11 and c = 0.91 in comparison with observed values of B13(<u>O. strumosum</u>)

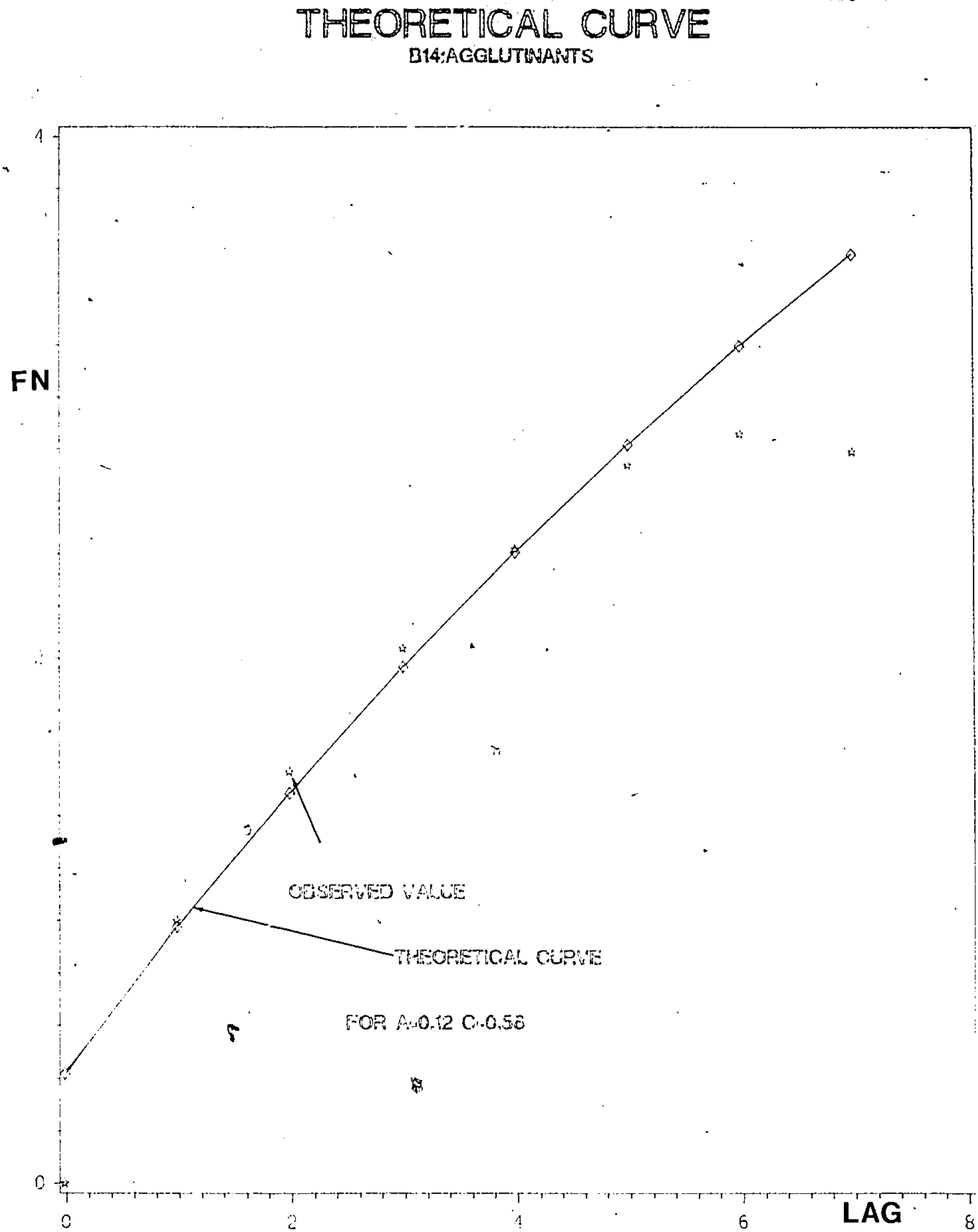


Figure 4.21e$_1$ Theoretical curve for a = 0.12 and c = 0.58 in comparison with observed values of B14 (Agglutinants)

variances, were calculated according to Eq.59 and are shown in Tables 4.11 - 4.14. Theoretical and observed relative variances were plotted against the lag as seen in Fig. $4.21a_1$, $4.21b_1$, $4.21c_1$, $4.21d_1$ and $4.21e_1$.

TABLE 4.11 Theoretical and observed of the relative variance of B1 (Eoguttulina sp.)

k(lag)	$S^2(x)$	F_N	F_L
0	0	0	0.42
1	2.1703	1.00	0.98
2	1.6845	1.55	1.50
3	1.3982	1.93	1.97
4	1.2171	2.24	2.41
5	1.1205	2.58	2.82
6	1.0366	2.87	3.17
7	0.9719	3.13	3.51

TABLE 4.12 Theoretical and observed values of the relative variance of B2 (Epistomina mosquensis)

k(lag)	$S^2(x)$	F_N	F_L
0	0	0	0.12
1	127.83	1	0.94
2	107.66	1.68	1.63

3	97.19	2.28	2.30
4	90.21	2.82	2.86
5	80.51	3.15	3.36
6	72.73	3.41	3.79
7	65.74	3.60	4.18

TABLE 4.13 Theoretical and observed values of the relative variance of B11 (Spirillina tenuissima)

k(lag)	$S^2(x)$	F_N	F_L
0	0	0	0.1
1	311.50	1	0.95
2	259.93	1.66	1.72
3	246.42	2.37	2.40
4	236.71	3.04	3.04
5	221.40	3.55	3.60
6	209.80	4.04	4.12
7	193.60	4.35	4.58

TABLE 4.14 Theoretical and observed values of the relative variance of B13 (Ophthalmidium strumosum)

k(lag)	$S^2(x_1)$	F_N	F_L
0	0	0	0.09
1	90.56	1	0.97

2	82.61	1.82	1.78
3	76.85	2.55	2.54
4	74.17	3.28	3.25
5	72.24	3.99	3.91
6	69.48	4.60	4.52
7	66.23	5.12	5.10

TABLE 4. 15 Theoretical and observed values of the relative variance of B14 (agglutinnants)

k(lag)	S^2 (x)	F_N	F_L
0	0	0	0.42
1	24.74	1	0.98
2	19.41	1.57	1.49
3	16.83	2.04	1.97
4	14.98	2.42	2.41
5	13.54	2.74	2.82
6	11.79	2.86	3.20
7	9.88	2.79	3.55

Taxa B1,B2,B11,B13 and B14 show a good fit between observed and theoretical relative variance curves.

As explained before, the cumulative frequency distribution tables are constructed with single, paired and sets of 4 consecutive values of each taxon respectively (Tables 4.19, 4.21, 4.23, 4.25 and 4.27). Each of these cumulative distributions (Tables 4.20, 4.22, 4.24, 4.26 and 4.28) was plotted on normal probability paper. R values were estimated for single, paired and sets of four values of each taxon, using Appendix II or III, according to their means and variances as shown in Tables 4.16, 4.17 and 4.18.

For single, paired and sets of four values of each taxon Y functions were estimated using Eq. 60. These functions were plotted on normal probability paper which, as before, was used for the probit transformed

TABLE 4.16 Mean, variance, R and b parameters for species B1 (Eoguttulina sp.), B2 (Epistomina mosquensis), B11 (Spirillina tenuissima), B13 (Ophthalmidium strumosum) and B14 (agglutinants) for N=30 samples (single values)

Taxa	B1	B2	B11	B13	B14
$\bar{x}$	0.02847	0.1384	0.25753	0.1125	0.10416
$s^2(x)$	0.00020	0.01236	0.03011	0.00875	0.00235
R	0.23	0.50	0.51	0.51	0.24
b	1.903	1.087	0.644	1.215	1.264
N	30	30	30	30	30

TABLE 4.17 Mean, variance, R, and b parameters for species B1, B2, B11, B13 and B14 for N=15 samples (pairs values)

TAXA	B1	B2	B11	B13	B14
$\bar{x}$	0.02847	0.1384	0.25753	0.1125	0.10416
$s^2(x_2)$	0.00016	0.01038	0.02499	0.00796	0.00188

R	0.18	0.45	0.44	0.45	0.19
b	1.903	1.087	0.644	1.215	1.264
N	15	15	15	15	15

TABLE 4. 18 Mean, variance, R and b parameters for species B1, B2, B11, B13 and B14 for N=7 (sets of four adjacent values)

TAXA	B1	B2	B11	B13	B14
$\bar{x}$	0.02847	0.1384	0.25753	0.1125	0.10416
$s^2(x_4)$	0.000115	0.00871	0.02288	0.00718	0.00145
R	0.16	0.43	0.41	0.44	0.17
b	1.903	1.087	0.644	1.215	1.264
N	7	7	7	7	7

Experimental frequency distributions for five samples of single, paired and sets of 4 microfossil values were plotted on two normal probability scales. The values of b and R were obtained from the means and variances of the microfossil taxon proportion values using Eq. 60 and 62. They determine the positions and slopes of the prob - prob lines which satisfy Eq. 60 as shown in Figs. 4.22 a, b, c - 4.23 a, b , c - 4.24 a, b, c - 4.25 a, b, c and 4.26 a, b, c.

Table 4.19 Probit transformation of B1 (Eoguttulina sp.) from Tojeira 2 section.

sample single %	probit single	sample pairs %	probit pairs	sample set of 4 %	probit set of 4
4.4	3.294				
2.4	3.023	3.4	3.175		
3.8	3.226				
2.3	3.005	3.05	3.120	3.225	3.165
4.2	3.272				
4.8	3.335	4.5	3.305		
3.8	3.226				
3.1	3.134	3.45	3.181	3.975	3.241
4.2	3.272				
3.0	3.119	3.6	3.315		
4.1	3.261				
2.8	3.089	3.45	3.181	3.525	3.191
2.7	3.073				
4.2	3.272	3.45	3.181		
6.9	3.517				
3.4	3.175	5.15	3.369	4.300	3.283
2.5	3.040				
1.1	2.710	1.8	2.903		
3.6	3.201				

(continued)

Table 4.19 (concluded)

3.7	3.213	3.65	3.415	2.725	3.077
2.2	2.986				
1.9	2.925	2.05	2.956		
1.7	2.880				
0.4	2.348	1.05	2.692	1.55	3.077
2.4	3.023				
0.7	2.543	1.55	2.843		
0.4	2.348				
1.4	2.803	0.90	2.634	1.225	2.129
1.4	2.803				
1.9	2.925	1.65	2.868		

Table 4.20 Cumulative frequency distributions of B1 (Eoguttulina sp.) for 30, 15 and 7 samples.

N = 30

class int.	freq.	cum. freq.	perc.	cum. per.	plotting value
2.375 - 2.625	3	3	10.0	10.0	8.80
2.625 - 2.875	3	6	10.0	20.0	18.70
2.875 - 3.125	11	17	36.67	56.67	54.94
3.125 - 3.375	12	29	40.0	96.67	94.50
3.375 - 3.625	1	30	3.33	100.0	97.80

N = 15

2.70 - 2.90	4	4	26.67	26.67	23.91
2.90 - 3.10	2	6	13.33	40.00	36.96
3.10 - 3.30	5	11	33.33	73.33	69.56
3.30 - 3.50	4	15	26.67	100.0	95.65

N = 7

2.20 - 2.60	1	1	14.29	14.29	9.10
2.60 - 3.00	1	2	14.29	28.57	22.73
3.00 - 3.40	5	7	71.43	100.0	90.91

Table 4.21 Probit transformation of B2 (Epistomina mosquensis) from Tojeira 2 section.

sample single %	probit single	sample pairs %	probit pairs	sample set of 4 %	probit set of 4
0.0	2.187				
1.9	2.925	0.95	2.654		
1.0	2.674				
0.0	2.187	0.5	2.424	0.50	2.424
1.1	2.710				
1.6	2.856	1.35	2.788		
2.4	3.023				
5.2	3.374	3.80	3.226	2.575	3.053
0.5	2.424				
0.0	1.187	0.25	2.137		
1.8	2.903				
19.0	4.122	10.4	3.741	5.325	3.386
17.9	4.081				
26.3	4.366	22.1	4.231		
16.6	4.030				
13.8	3.911	15.2	3.972	18.65	4.109
19.0	4.122				
18.6	4.107	18.8	4.115		
16.4	4.100				

(continued)

Table 4.21 (concluded)

6.7	3.501	11.55	3.802	15.175	3.971
31.1	4.507				
16.7	4.034	23.9	4.290		
10.9	3.768				
23.2	4.268	17.05	4.048	20.475	4.175
22.8	4.255				
21.5	4.211	22.15	4.233		
37.6	4.687				
24.9	4.322	31.25	4.511	26.7	4.378
31.7	4.524				
25.0	4.326	28.35	4.427		

Table 4.22 Cumulative frequency distributions of B2 (Epistomina mosquensis) for 30, 15 and 7 samples.

N = 30

class interval	freq	cum. freq.	perc. %	cum. %	plotting value
2.25 - 2.75	6	6	20.00	20.00	18.70
2.75 - 3.25	4	10	13.33	33.33	32.00
3.25 - 3.75	2	12	6.67	40.00	38.50
3.75 - 4.25	10	22	33.33	73.33	71.43
4.25 - 4.75	8	30	26.67	100.0	97.80

N = 15

2.10 - 2.70	3	3	20.00	20.00	17.40
2.70 - 3.30	2	5	13.33	33.33	30.40
3.30 - 3.90	2	7	13.33	46.67	43.50
3.90 - 4.50	8	15	53.33	100.0	95.65

N = 7

2.70 - 3.30	2	2	28.57	28.57	22.80
3.30 - 3.90	1	3	14.29	42.86	36.40
3.90 - 4.50	4	7	57.14	100.0	90.90

Table 4.23 Probit transformation of B11 (Spirillina tenuissima) for N= 30, 15 and 7 samples from the Tojeira 2 section.

sample single %	probit single	sample pairs %	probit pairs	sample set of 4 %	probit set of 4
57.7	5.194				
24.0	4.294	40.85	4.768		
44.8	4.869				
61.0	5.279	52.9	5.073	46.875	4.921
42.0	4.798				
64.1	5.361	53.05	5.076		
38.6	4.710				
30.9	4.501	34.75	4.608	43.90	4.846
46.8	4.920				
52.7	5.068	49.75	4.993		
46.8	4.920				
19.4	4.137	33.1	4.563	41.425	4.784
14.9	3.969				
15.0	3.964	14.95	3.961		
13.8	3.911				
16.2	4.014	15.00	3.964	14.975	3.963
17.0	4.046				
16.7	4.034	16.85	4.040		
12.5	3.850				

(continued)

Table 4:23 (concluded)

11.9	3.820	12.20	3.835	14.525	3.943
17.6	4.069				
11.4	3.794	14.5	3.942		
15.2	3.972				
16.4	4.022	15.8	3.997	15.15	3.970
13.4	3.892				
7.3	3.546	10.35	3.738		
10.4	3.741				
8.8	3.647	9.60	3.695	9.975	3.717
11.8	3.815				
13.5	3.897	12.65	3.856		

Table 4.24 Cumulative frequency distributions of B11 (Spirillina tenuissima) for 30, 15 and 7 samples.

N = 30

class interval	freq.	cum. freq.	perc. %	cum. %	plotting value
3.40 - 3.80	4	4	13.33	13.33	12.10
3.80 - 4.20	15	19	50.00	63.33	61.50
4.20 - 4.60	2	21	6.67	70.00	68.10
4.60 - 5.00	5	26	16.67	86.67	84.60
5.00 - 5.40	4	30	13.33	100.0	97.80

N = 15

class interval	freq.	cum. freq.	perc. %	cum. %	plotting value
3. 75 - 4.05	9	9	60.0	60.0	56.50
4.05 - 4.35	0	9	00.0	60.0	56.50
4.35 - 4.65	2	11	13.33	73.33	69.56
4.65 - 4.95	4	15	26.67	100.0	95.65

N = 7

class interval	freq.	cum. freq.	perc. %	cum. %	plotting value
3.80 - 4.20	4	4	57.14	57.14	50.00
4.20 - 4.60	0	4	0.00	57.14	50.00
4.60 - 5.0	3	7	42.86	100.0	90.91

Table 4.25 Probit transformation of B13 (Ophthalmidium strumosum)
for 30, 15 and 7 samples from Tojeira 2 section.

sample single %	pobit single	sample pairs %	probit pairs	sample set of 4 %	probit set of 4
0.2	2.122				
0.5	2.424	0.35	2.300		
0.0	2.187				
0.0	2.187	0.00	2.187	0.175	2.069
1.1	2.710				
0.0	2.187	0.55	2.456		
1.0	2.674				
0.9	2.634	0.95	2.654	0.75	2.567
2.8	3.089				
0.4	2.348	1.60	2.856		
1.8	2.903				
5.2	3.374	3.50	3.188	2.55	2.456
14.9	3.959				
16.3	4.018	15.6	3.989		
22.4	4.241				
16.7	4.034	19.55	4.156	17.575	4.068
12.0	3.825				
19.4	4.134	15.7	3.993		
27.5	4.402				

(continued)

Table 4.25 (concluded)

14.8	3.955	21.15	4.198	18.425	4.101
16.5	4.026				
27.4	4.399	21.95	4.226		
29.8	4.470				
16.1	4.010	22.95	4.259	22.45	4.242
18.3	4.096				
20.9	4.190	19.60	4.144		
11.5	3.800				
8.3	3.615	9.90	3.713	14.75	3.954
15.4	3.981				
15.4	3.981	15.4	3.981		

Table 4.26 Cumulative frequency distributions of B13 (Ophthalmidium strumosum) for 30, 15 and 7 samples.

N = 30

class interval	freq.	cum. freq.	perc. %	cum. %	plotting value
2.25 - 2.75	9	9	30.00	30,00	28.60
2.75 - 3.25	2	11	6.67	36.67	35.20
3.25 - 3.75	2	13	6.67	43.33	41.80
3.75 - 4.25	14	27	46.67	90.00	88.00
4.25 - 4.75	3	30	10.00	100.0	97.80

N = 15

class interval	freq.	cum. freq.	perc. %	cum. %	plotting value
2.25 - 2.75	4	4	26.67	26.67	23.90
2.75 - 3.25	2	6	13.33	40.0	37.00
3.25 - 3.75	1	7	6.67	46.67	41.80
3.75 - 4.25	8	15	53.33	100.0	95.70

N = 7

class interval	freq.	cum. freq.	perc. %	cum. %	plotting value
2.0 - 2.8	3	3	42.86	42.86	36.40
2.8 - 3.6	0	3	0.00	42.86	36.40
3.6 - 4.4	4	7	57.14	100.0	90.90

Table 4.27 Probit transformation of B14 (agglutinants) for N = 30, 15 and 7 samples from Tojeira 2 section.

sample single	probit single	sample pairs	probit pairs	sample set of 4	probit set of 4
7.5	3.560				
3.9	3.238	5.7	3.420		
3.5	3.188				
3.3	3.162	3.4	3.175	4.55	3.331
4.2	3.283				
2.4	3.023	3.3	3.162		
12.2	3.835				
7.3	3.546	9.75	3.704	6.525	3.488
7.9	3.588				
7.1	3.532	7.5	3.560		
6.6	3.494				
11.2	3.784	8.9	3.653	8.20	3.608
14.9	3.959				
13.4	3.892	14.15	3.926		
5.6	3.411				
14.8	3.955	10.2	3.730	12.175	3.834
13.5	3.897				
10.3	3.735	11.9	3.820		
9.3	3.667				
7.4	3.553	8.35	3.618	10.125	3.725
11.0	3.773				

(continued)

Table 4.27 (concluded)

16.7	4.034	13.85	3.913		
11.8	3.815				
10.5	3.746	11.15	3.781	12.5	3.850
17.8	4.077				
20.8	4.187	19.3	4.133		
12.3	3.840				
20.9	4.190	16.6	4.030	17.95	4.083
13.7	3.906				
10.7	3.757	12.2	3.835		

Table 4.28 Cumulative frequency distributions of agglutinants for 30, 15 and 7 samples.

N=30

class interval	freq.	cum. freq	perc. %	cum. %	plotting value
3.10 - 3.30	5	5	16.67	16.67	15.40
3.30 - 3.50	2	7	6.67	23.33	22.00
3.50 - 3.70	6	13	20.0	43.33	41.80
3.70 - 3.90	10	23	33.33	76.67	74.72
3.90 - 4.10	7	30	23.33	100.0	97.80

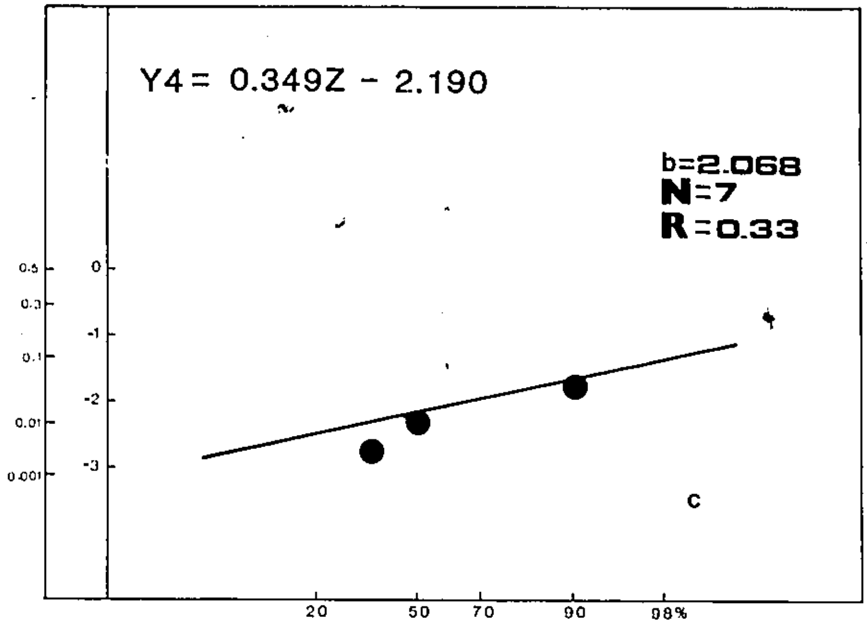
N = 15

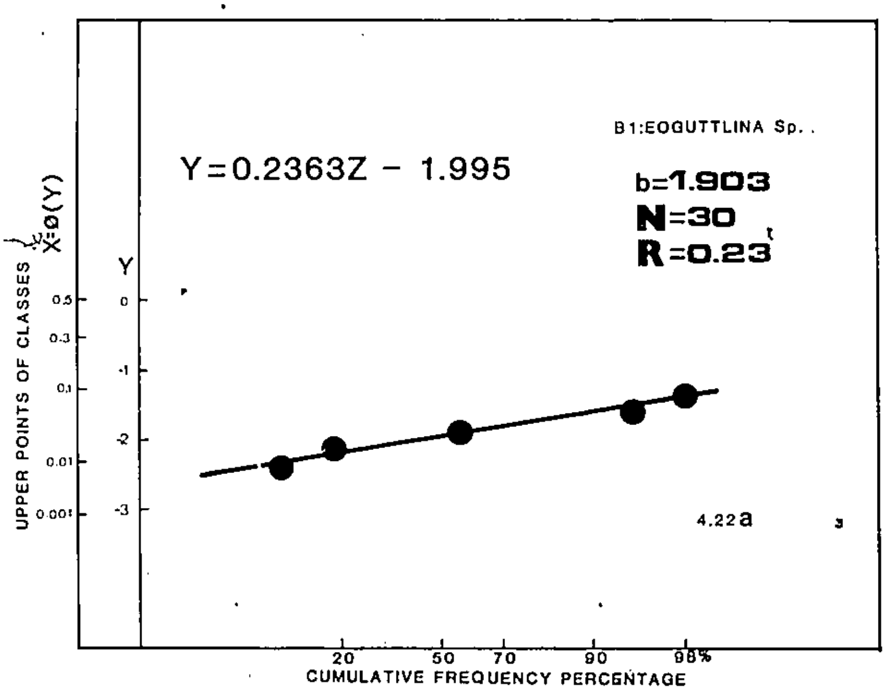
3.15 - 3.45	3	3	20.00	20.00	17.40
3.45 - 3.75	5	8	33.33	53.33	50.00
3.75 - 4.05	5	13	33.33	86.67	82.60
4.05 - 4.35	2	15	13.33	100.0	95.65

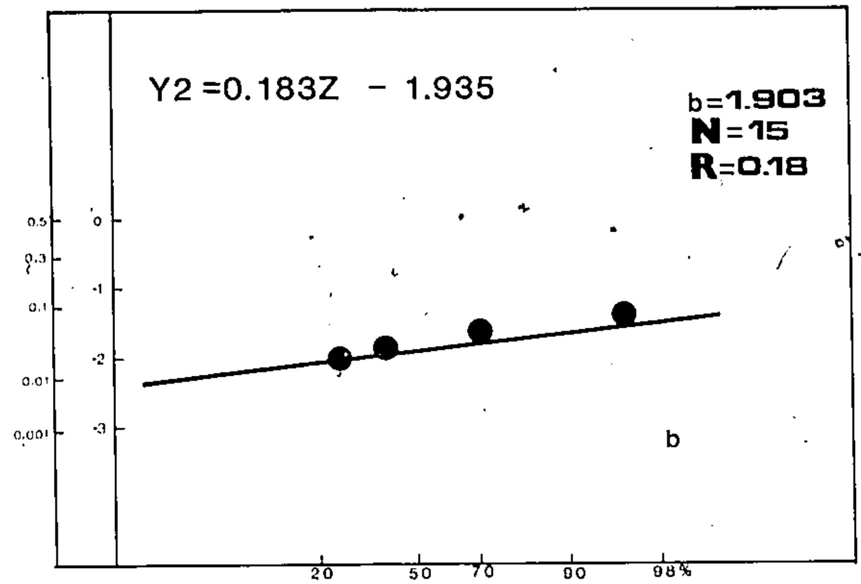
N = 7

3.375 - 3.625	3	3	42.86	42.86	36.40
3.625 - 3.875	3	6	42.86	85.71	77.30
3.875 - 4.125	1	7	14.29	100.0	90.90

Figure 4.22a,b,c Experimental frequency distributions for 30,15 and $^{+}$values of *Eoguttulina* sp.and their theoretical frequency distributions which are probnormal distributions


Y4= 0.349Z - 2.190
b=2.068
N=7
R=0.33
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
c
20
50
70
90
98%


B1:EOGUTTLINA Sp.
Y=0.2363Z - 1.995
b=1.903
N=30
R=0.23
UPPER POINTS OF CLASSES X=ø(Y)
Y
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
4.22a
20
50
70
90
98%
CUMULATIVE FREQUENCY PERCENTAGE


Y2 =0.183Z - 1.935
b=1.903
N=15
R=0.18
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
b
20
50
70
90
98%

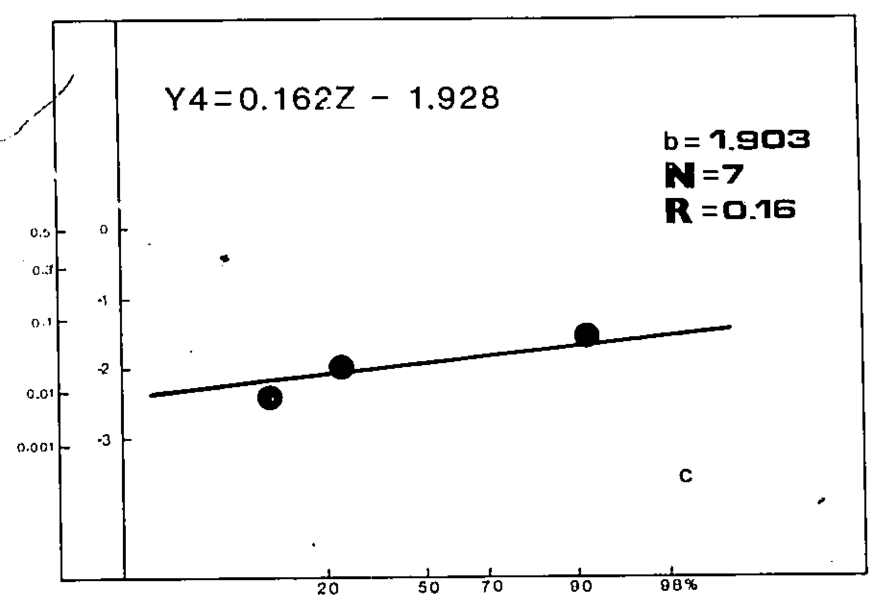
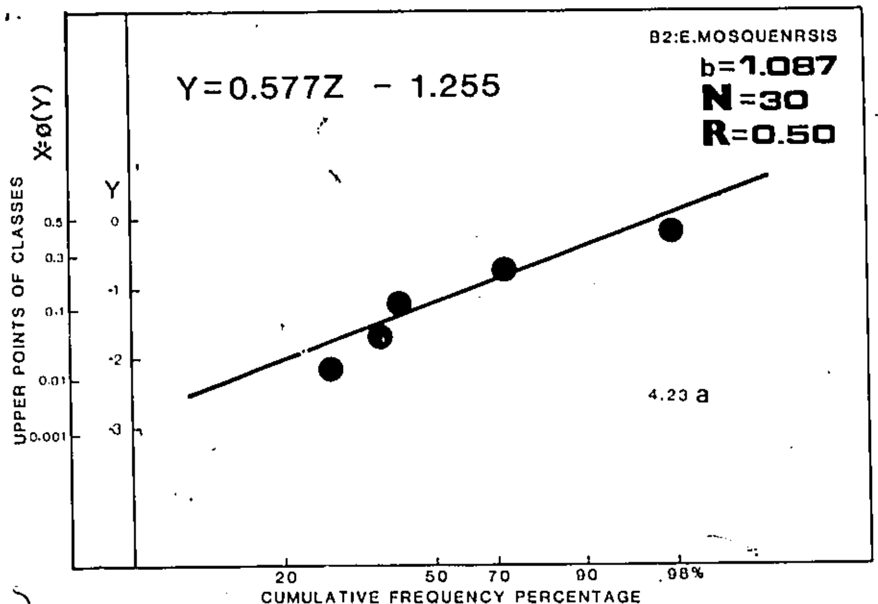

Y4=0.162Z - 1.928
b= 1.903
N=7
R =0.16
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
c
20
50
70
90
98%

Figure 4.23a,b,c Experimental frequency distributions for 30,15 and 7 values of E. mosquensis and their theoretical frequency distributions satisfying Eq.60


B2:E.MOSQUENRSIS
b=1.087
N=30
R=0.50
Y=0.577Z - 1.255
X:ø(Y)
UPPER POINTS OF CLASSES
Y
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
4.23 a
20
50
70
90
98%
CUMULATIVE FREQUENCY PERCENTAGE

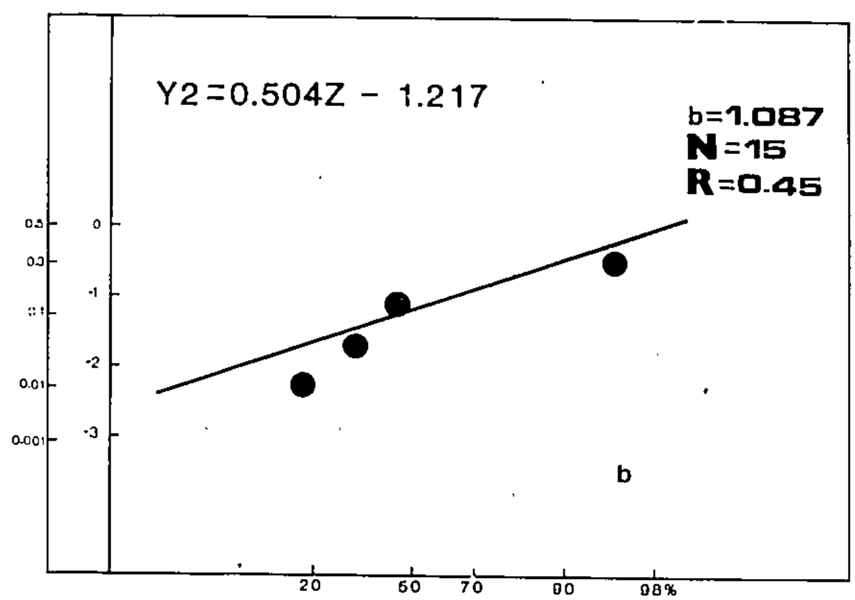
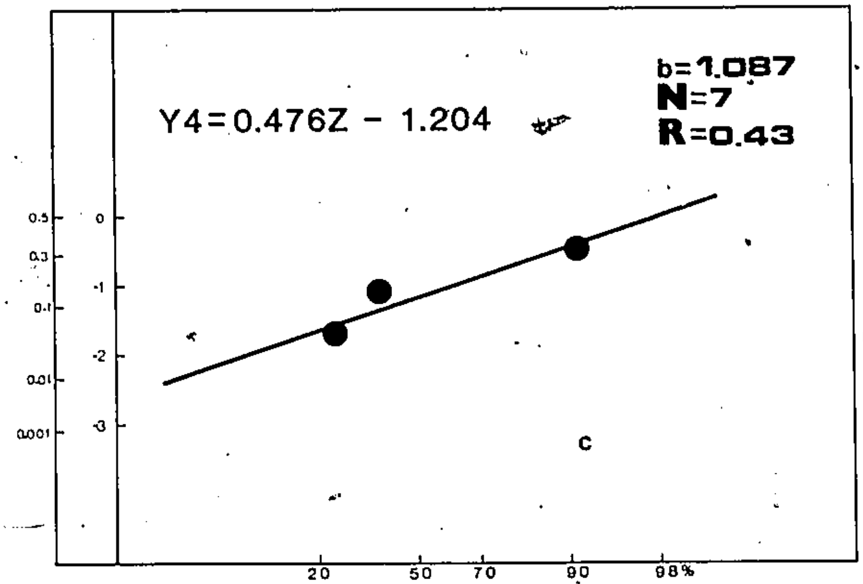
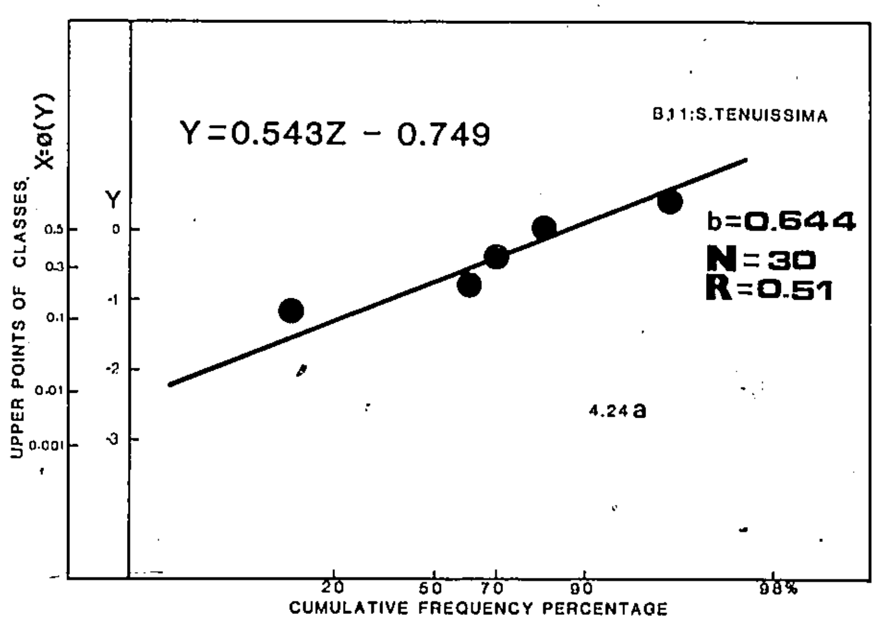
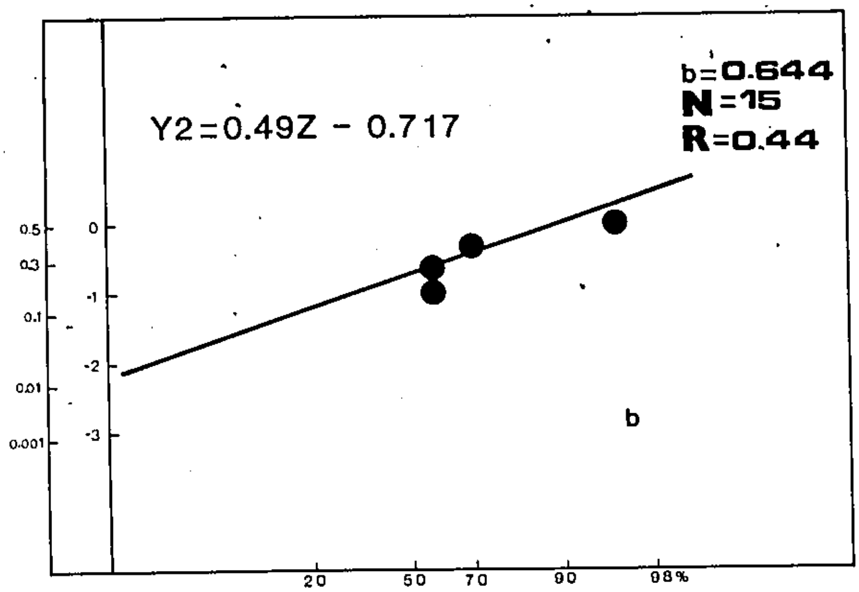

Y2=0.504Z - 1.217
b=1.087
N=15
R=0.45
0.5
0.3
0.01
0.001
0
-1
-2
-3
b
20
50
70
90
98%

Figure 4.24a,b,c Experimental and theoretical distributions for 30,15 and 7 values of S. tenuissima


Y4=0.476Z - 1.204
b=1.087
N=7
R=0.43
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
c
20
50
70
90
98%


Y=0.543Z - 0.749
B11:S.TENUISSIMA
UPPER POINTS OF CLASSES, X:ø(Y)
Y
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
b=0.644
N=30
R=0.51
4.24 a
20
50
70
90
98%
CUMULATIVE FREQUENCY PERCENTAGE


Y2=0.49Z - 0.717
b=0.644
N=15
R=0.44
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
20
50
70
90
98%
b

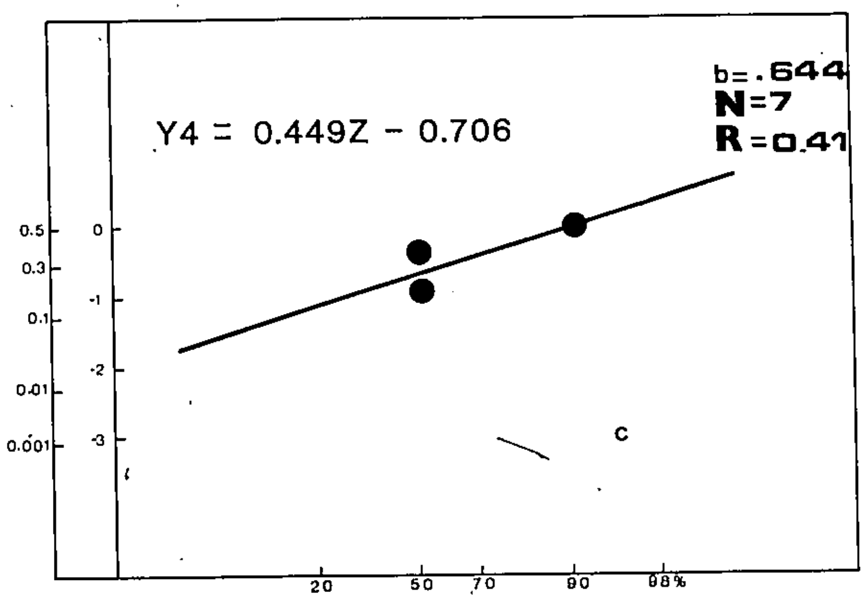
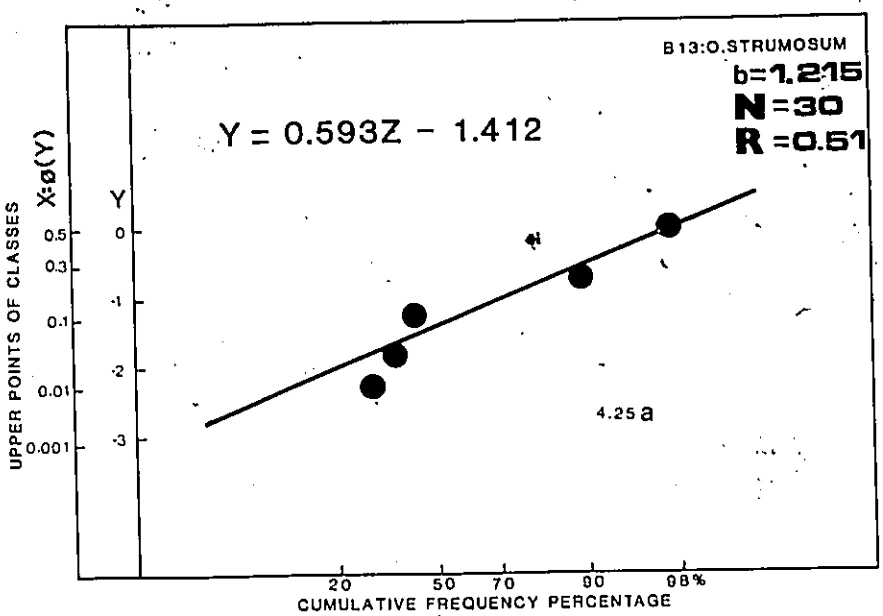

Y4 = 0.449Z - 0.706
b=.644
N=7
R=0.41
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
20
50
70
90
98%
c

Figure 4.25a,b,c Experimental and theoretical distributions for 30,15 and 7 values of O. strumosum


B13:O.STRUMOSUM
b=1.215
N=30
R=0.51
Y = 0.593Z - 1.412
X:ø(Y)
Y
UPPER POINTS OF CLASSES
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
4.25 a
20
50
70
90
98%
CUMULATIVE FREQUENCY PERCENTAGE

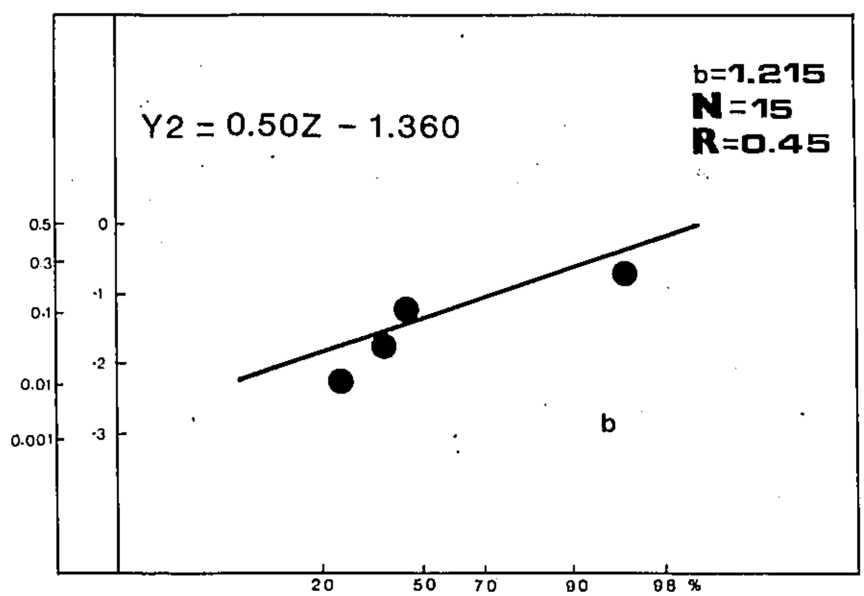
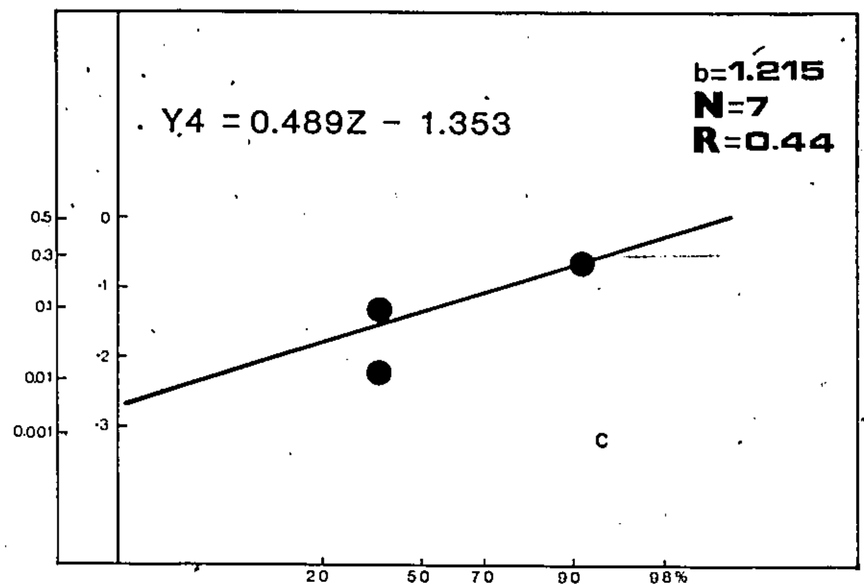
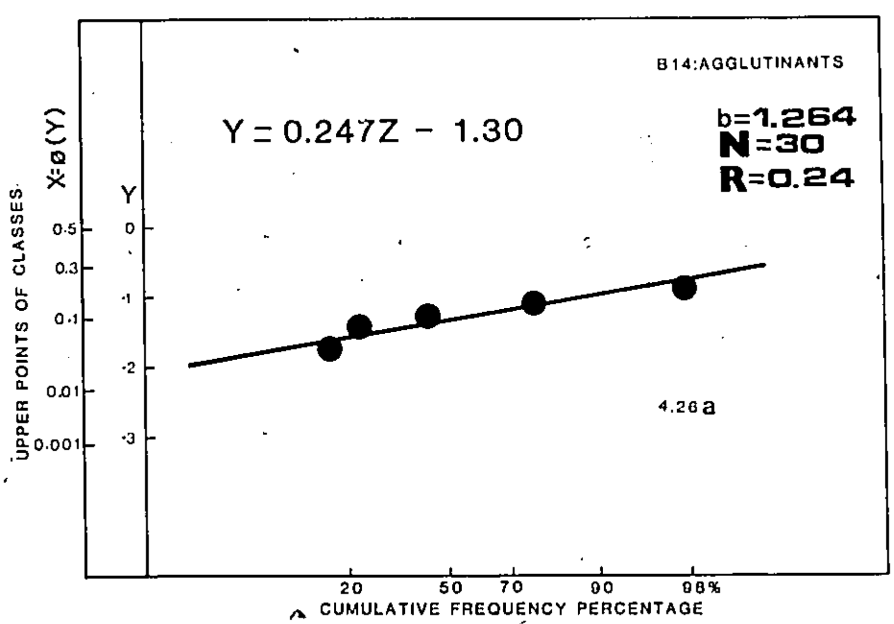
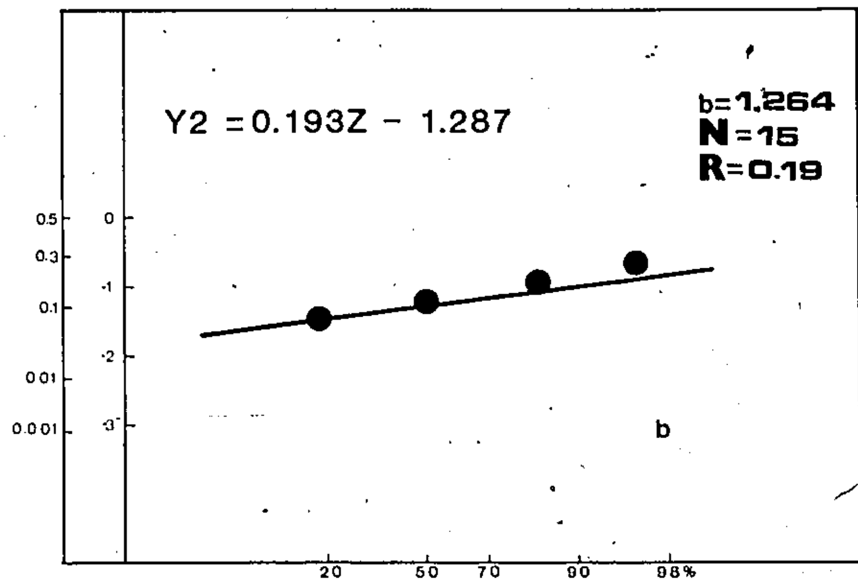
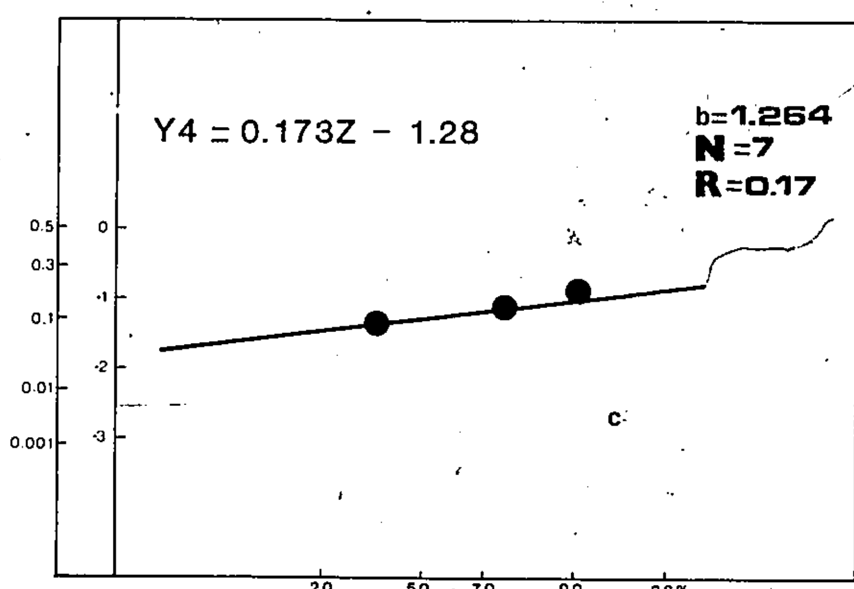

b=1.215
N=15
R=0.45
Y2 = 0.50Z - 1.360
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
b
20
50
70
90
98 %

Figure 4.26a,b,c Experimental and theoretical distributions for 30,15 and 7 values of Agglutinants


Y4 = 0.489Z - 1.353
b=1.215
N=7
R=0.44
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
c
20
50
70
90
98%


B14:AGGLUTINANTS
Y = 0.247Z - 1.30
b=1.264
N=30
R=0.24
X=ø(Y)
UPPER POINTS OF CLASSES
Y
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
4.26a
20
50
70
90
98%
CUMULATIVE FREQUENCY PERCENTAGE


Y2 =0.193Z - 1.287
b=1.264
N=15
R=0.19
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
b
20
50
70
90
98%


Y4 = 0.173Z - 1.28
b=1.264
N=7
R=0.17
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
c
20
50
70
90
98%

CHAPTER 5 New data set and geostatistical modelling

5.1 - Introduction

The Tojeira 1 and Tojeira 2 sections were resampled by this author and F. M. Gradstein during the summer of 1986.

5.2 - Tojeira 1 section

The samples were collected at regular intervals of 5 m. The sampled section starts just below outcrop 17 of Stam (1986). A total of 23 samples were collected, with 5 of these samples coming from the same layer.

As explained before, with only 17 observations, there were less than the minumum (30) recommended for initiating an ARIMA procedure. About 4 autocorrelations would be a safe number to examine, since this is one - fourth of the number of observations. However, this is still not sufficient to draw decisive conclusions about the microfossil abundance.

When ARIMA procedures were applied the autocorrelations of the species turned out to be not significantly different from zero at the 5% significance level. The null hypothesis of absence of autocorrelation was acceptable. All time series of species were white noise. This was expected because 17 samples are fewer than the necessary number for the ARIMA procedure to be considered accurate.

5.3 Tojeira 2 section

A total of 13 samples were recollected from the Tojeira 2 section. They were not collected at a regular interval as explained before. An ARIMA procedure could not be applied to this data set either, because of the small sample size.

5.4 - Correlation of Stam's and Gradstein's data

The new data set was not sufficiently large to construct time series models for either Tojeira 1 or Tojeira 2 separately. To overcome this problem a larger data set was created by combining Stam's and Gradstein's data using samples taken at different locations only. Before creating the enlarged data set, Stam's and Gradstein's data, which were collected at approximately the same location, were correlated to each other. The idea was to see how close the data from the two different data sets related to each other (Tables 5.1 and 5.2).

5.4 1 - Tojeira 1 section:

Table 5.1 was used as input for the SAS CORR procedure to calculate correlation coefficients.

As can be seen in Table 5.3, Ophthalmidium strumosum and the taxa Spirillina elongata and Eoguttulina have a high positive correlation. The taxa Epistomina uhligi, lenticulina, Epistomina mosquensis,

Table 5.1 Comparison of Stam's and Gradstein's data for Tojeira 1 section

Sample No.		A 1		A 2		A 3		A 4		A 5		A 6		A 7	
G	S	G	S	G	S	G	S	G	S	G	S	G	S	G	S
T1/1	6.3	6.1	14.8	0.0	0.0	0.9	0.0	0	0.0	13.8	24.7	3.7	5.7	1.3	4.9
T1/2	6.4	2.0	5.4	0.0	0.2	3.0	1.2	0	0.0	20.2	19.8	6.9	10.6	2.0	3.4
T1/3	6.5	1.9	1.8	28.8	15.1	6.6	15.1	0	13.3	9.0	4.7	3.1	4.9	1.9	4.9
T1/4	6.6	4.4	0.2	11.0	24.2	3.8	11.6	0	0.0	12.3	9.9	1.9	4.8	10.1	2.5
T1/5	6.7	0.4	3.4	15.1	18.1	7.7	3.4	0	0.0	7.4	11.1	0.7	3.1	1.8	3.0
T1/14	6.21	0.0	0.8	18.9	13.5	3.3	1.7	0	1.2	18.4	15.0	2.3	11.3	4.8	1.9
T1/15	6.26A	0.5	1.0	16.6	39.7	4.9	3.7	0	3.4	9.2	16.2	1.8	3.3	2.6	2.5

Sample No.		A 8		A 9		A 10		A 11		A 12		A 13		A 14	
G	S	G	S	G	S	G	S	G	S	G	S	G	S	G	S
T1/1	6.3	0.4	0.0	1.5	0.9	9.2	18.8	30.8	17.9	0.7	0.2	1.5	0.0	25.6	2.5
T1/2	6.4	0.0	0.0	0.5	2.4	5.4	0.4	13.8	26.1	1.5	4.2	0.0	0.0	32.5	5.0
T1/3	6.5	3.8	0.0	0.0	2.9	11.4	7.2	17.7	34.8	3.1	4.9	0.0	0.0	10.6	2.6
T1/4	6.6	7.2	0.0	15.1	10.8	7.9	3.8	15.1	9.1	5.3	14.4	0.6	0.0	12.6	2.3
T1/5	6.7	12.5	0.0	1.5	6.9	0.0	0.6	18.8	29.2	18.1	3.8	4.0	0.2	2.9	5.4
T1/14	6.21	10.1	9.3	3.0	4.2	0.5	0.0	9.8	13.5	0.5	1.9	18.2	7.2	9.6	11.4
T1/15	6.26A	6.9	9.1	3.6	9.1	2.3	0.0	17.2	7.0	26.0	0.3	0.3	5.2	4.3	7.2

Note:

G:F.M.Gradstin

S:B.Stam

Table 5.2 Comparison of Stam's and Gradstein's data for Tojeira 2 section

Sample No.		B1		B2		B3		B4		B5		B6		B7	
G	S	G	S	G	S	G	S	G	S	G	S	G	S	G	S
T2/1	12.2	0.4	2.4	0.0	1.9	69.70	34.6	17.8	6.2	2.4	1.9	0.0	5.4	0.4	3.3
T2/2	12.4	0.9	2.3	0.0	0.0	0.00	0.0	C.0	0.5	1.8	10.3	6.0	3.3	0.9	0.9
T2/3	12.6	0.6	4.8	0.0	1.6	3.60	0.0	1.8	0.4	3.0	1.2	11.2	3.2	1.8	1.6
T2/5	12.7	0.4	3.8	1.0	2.4	28.70	3.8	28.7	0.5	1.4	4.3	15.7	8.7	1.1	4.3
T2/6	12.10	0.5	3.0	0.0	0.0	4.90	0.C	0.0	0.4	11.3	4.6	16.7	6.8	2.0	2.1
T2/8	11.1	0.5	2.8	51.1	19.0	3.69	8.3	8.3	0.8	6.9	5.6	3.7	5.2	1.4	0.4
T2/9	11.4	0.0	6.9	25.2	16.6	10.60	3.7	6.6	0.4	1.3	2.4	3.1	2.8	0.0	2.0
T2/10	11.5	0.0	3.4	0.4	13.8	0.90	3.8	0.0	0.0	2.2	1.9	12.2	4.8	0.9	3.8
T2/11	11.6	0.0	2.5	23.7	19.0	5.20	5.0	7.4	0.0	16.3	3.0	0.0	3.0	1.5	3.0
T2/12	11.12	1.3	1.7	35.3	10.9	3.20	1.1	5.8	0.0	5.1	5.7	7.7	5.2	0.0	2.3
T2/13	11.18	0.0	1.4	54.1	31.7	1.60	1.6	0.0	1.3	9.3	5.2	2.4	5.2	2.0	2.3

Sample No.		B8		B9		B10		B11		B12		B13		B14	
G	S	G	S	G	S	G	S	G	S	G	S	G	S	G	S
T2/1	12.2	0.0	0.0	1.8	2.9	1.8	1.9	1.2	24.0	0.0	1.4	0.0	0.5	1.4	3.9
T2/2	12.4	0.0	0.0	2.7	2.8	13.3	2.3	52.8	61.0	0.3	3.2	0.0	0.0	3.3	3.3
T2/3	12.6	0.0	0.0	3.0	4.8	5.9	3.2	37.3	64.1	0.6	6.5	0.6	0.0	14.2	2.4
T2/5	12.7	0.0	0.5	0.3	1.0	1.0	1.0	3.1	38.6	0.0	5.7	0.0	1.0	7.0	12.2
T2/6	12.10	0.0	0.0	0.0	3.4	2.4	3.8	42.6	52.7	1.0	3.8	0.0	0.4	12.2	7.1
T2/8	11.1	0.0	7.9	0.0	2.0	0.0	1.6	6.0	19.4	2.3	2.0	5.5	5.2	9.2	11.2
T2/9	11.4	5.3	7.3	2.6	4.1	0.0	0.4	12.8	13.8	0.4	3.7	2.6	12.4	25.7	5.6
T2/10	11.5	0.0	2.9	0.9	3.8	6.5	1.9	33.0	15.2	3.4	1.9	3.0	16.7	18.3	14.8
T2/11	11.6	0.0	6.5	1.5	2.5	7.4	1.0	11.9	17.0	1.5	1.5	3.7	12.0	12.6	13.5
T2/12	11.12	0.2	2.6	0.0	1.7	1.3	0.3	8.3	15.2	2.6	4.0	1.9	29.8	9.0	11.8
T2/13	11.18	3.7	2.3	2.4	1.6	0.4	0.0	9.8	11.8	1.6	2.6	1.2	15.4	8.9	13.7

Table 5.3 Correlation coefficients of Stam's and Gradstein's data for Tojeira 1 section

Gradstein's data	Stam's data	Coefficients
L. muensteri	L. muensteri	0.53 0.22
Eoguttulina Sp.	Eoguttulina Sp.	0.68 0.09
S. infima	S. infima	0.68 0.09
E. uhligi	E. uhligi	0.48 0.27
Nodosaria/Dentalina	Nodosaria/Dentalina	-0.54 0.21
S. tenuissima	S. tenuissima	0.09 0.85
S. elongata	S. elongata	0.75 0.05
O. carinatum	O. carinatum	-0.20 0.67
Epistomina Sp.	Epistomina Sp.	0.00 1.00
Lenticulina Sp.	Lenticulina Sp.	0.63 0.13
O. strumosum	O. strumosum	0.73 0.06
Agglutinants	Agglutinants	-0.08 0.87
E. mosquensis	E. mosquensis	0.53 0.22
P. rjasanensis	P. rjasanensis	0.37 0.41

Table 5.4 Correlation coefficients of Stam's and Gradstein's data for Tojeira 2 section

Gradsteins data \ Stam's data	L.muensteri	Eoguttulina Sp.	S.infima	E.uhligi	Nodosaria/Dentalina	S.tenuissima	S.elongata	O.carinatum	Epistomina Sp.	Lenticulina Sp.	O.strumosum	Agglutinants	E.mosquensis	P.rjasanensis
L.muensteri	0.01 0.98													
Eoguttulina Sp.		-0.32 0.33												
S.infima			0.40 0.23											
E.uhligi				0.91 0.00										
Nodosaria/Dentalina					-0.16 0.65									
S.tenuissima						0.73 0.01								
S.elongata							0.49 0.12							
O.carinatum								-0.38 0.24						
Epistomina Sp.									0.38 0.24					
Lenticulina Sp.										0.57 0.07				
O.strumosum											0.43 0.19			
Agglutinants												0.12 0.73		
E.mosquensis													0.85 0.00	
P.rjasanensis														0.39 0.23

Lenticulina muensteri, Pseudolamarckina rjansanensis have a low positive correlation. Spirillina tenussima, Ophthalmidium carinatum and agglutinants have positive or negative correlation coefficients that are close to zero, indicating no correlation between Stam's and Gradstein's data for these taxa. Nodosaria/Dentalina are negatively correlated. The taxa show an inverse relationship in Tojeira 1 section. There is no correlation between Stam's and Gradstein's data for Epistomina sp.. These relationships are shown graphically in Fig. 5.1.

5.4.2 Tojeira 2 section

Epistomina mosquensis, Epistomina uhligi and Spirillina tenuissima have a high positive correlation. Spirillina infima, Spirillina elongata, Epistomina sp., Lenticulina sp., Ophthalmidium strumosum and Pseudolamarckina rjasanensis have a low positive correlation There is no correlation between Stam's and Gradstein's data for Lenticulina muensteri, agglutinants and Nodoseria/ Dentalina as shown in Table 5.4. These relationships are shown graphically in Fig. 5.2.

In Tables.5.3 and 5.4, the first number is a correlation coefficient and the second number is probability. The significance probablity of the correlation or PROB > D is the significance probability of Hoeffding's D. This probability is approximate for Spearman, Kendall and Hoeffding statistics.

Stam's and Gradstein's data are only weakly correlated. This is probably due to the relatively large noise component in the data which is caused by counting errors as well as local random variability. However, Eoguttulina sp., Epistomina mosquensis and Ophthalmidium strumosum showed correlations varying from low to high in the Tojeira 1 section The taxa Epistomina uhligi, Epistomina mosquensis and Spirillina tenuissima showed a positve correlation in Tojeira 2. So, these species could be expected to reinforce the autocorrelation structure in the enlarged data set.

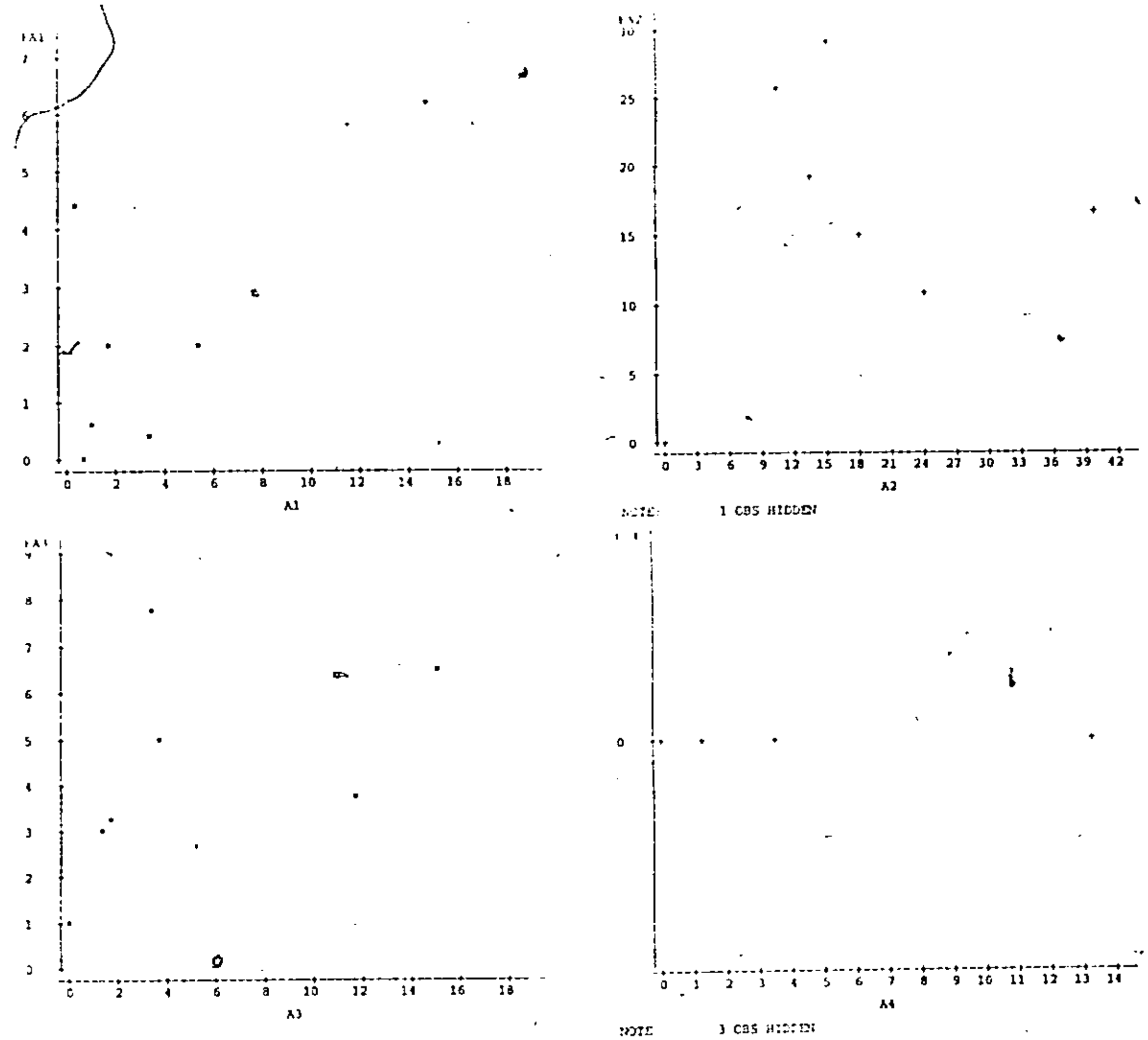


Figure 5.1 Correlation patterns of Stam's and Gradstein's data for Tojeira 1 section.

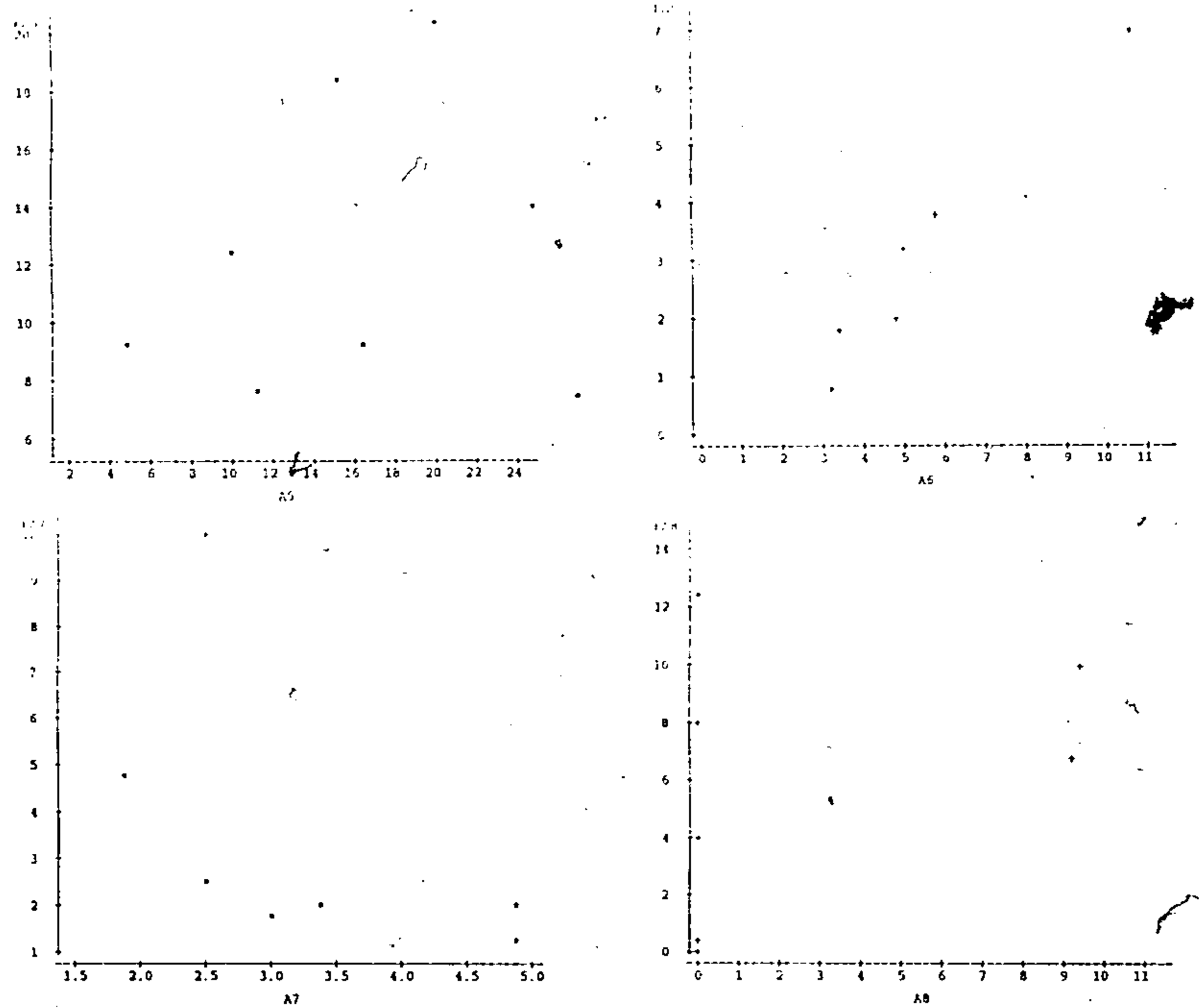


Fig. 5.1 (cont.)

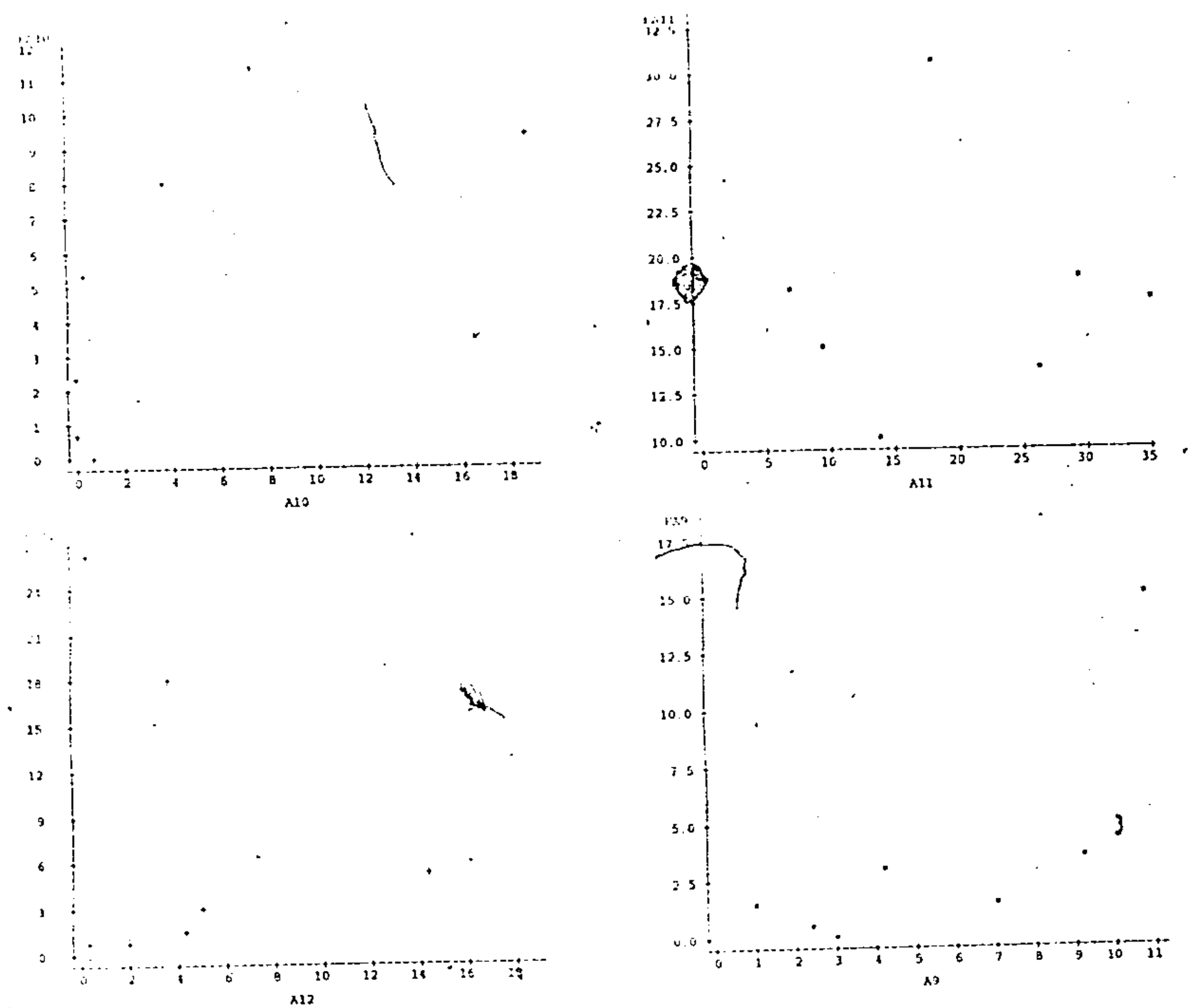


Fig. 5.1 (cont.)

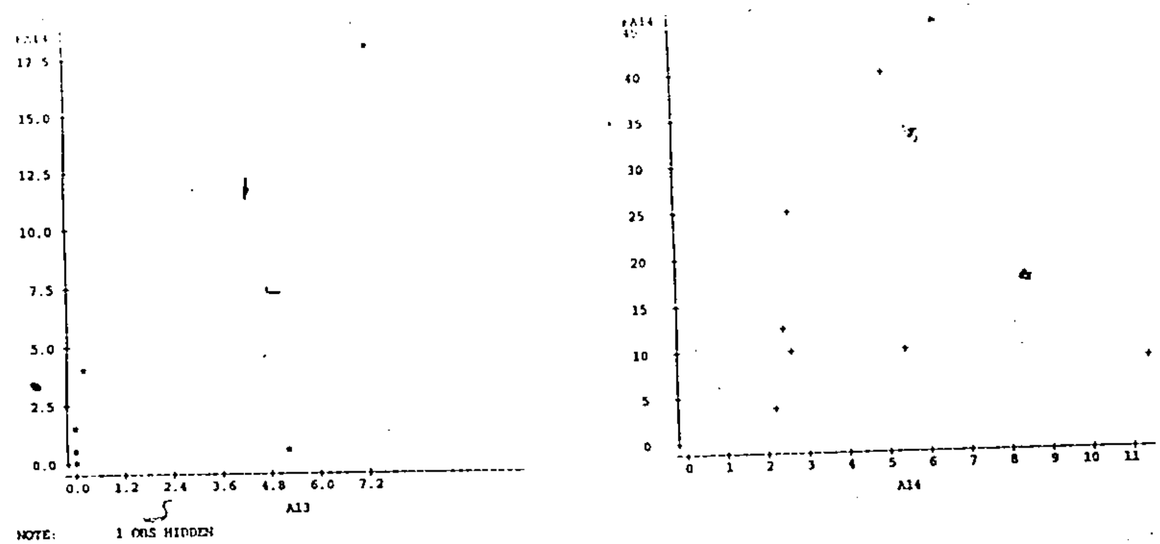


Fig. 5.1 (concluded)

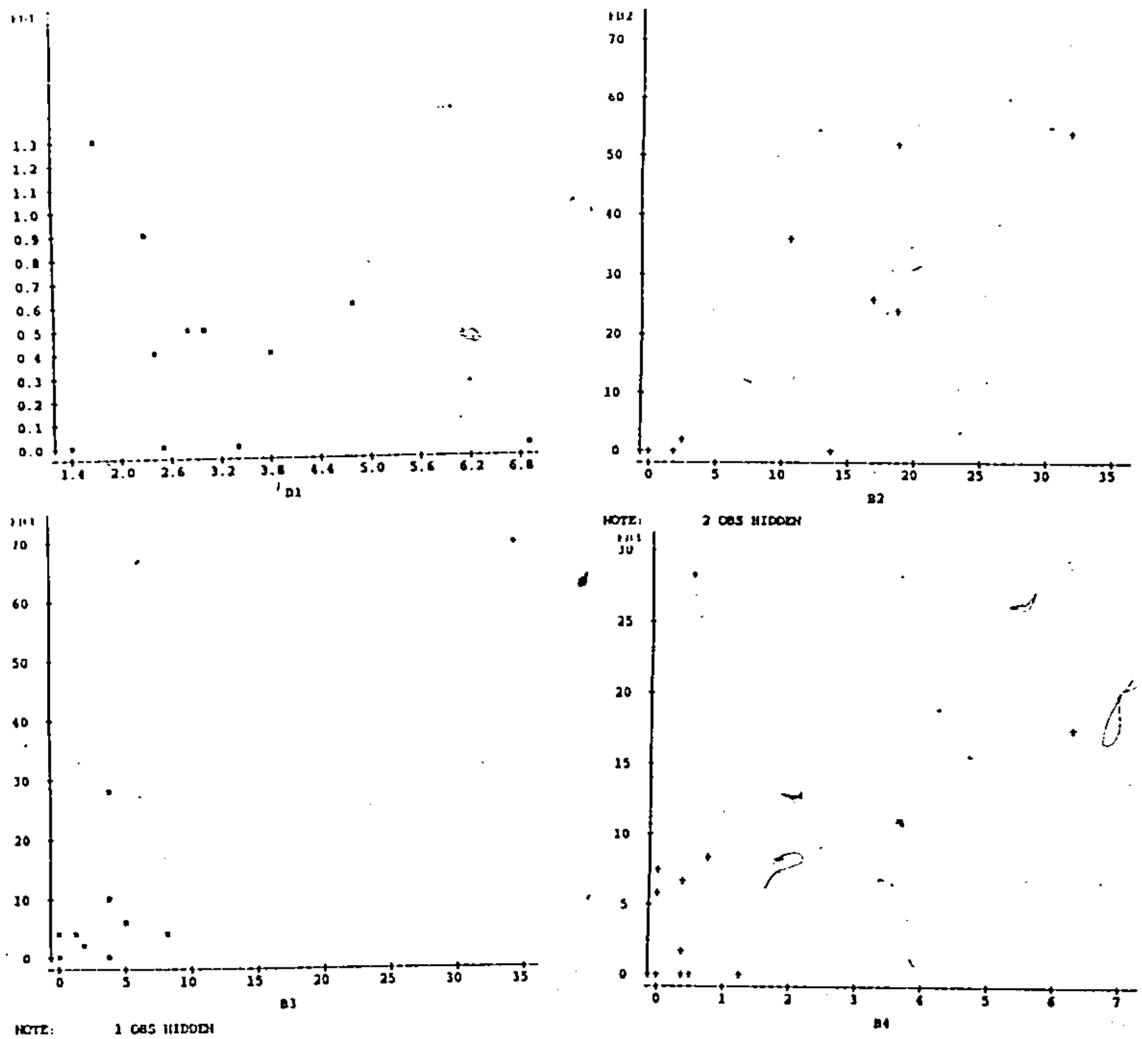


Figure 5.2 Correlation patterns of Stam's and Gradstein's data for Tojeira 2 section

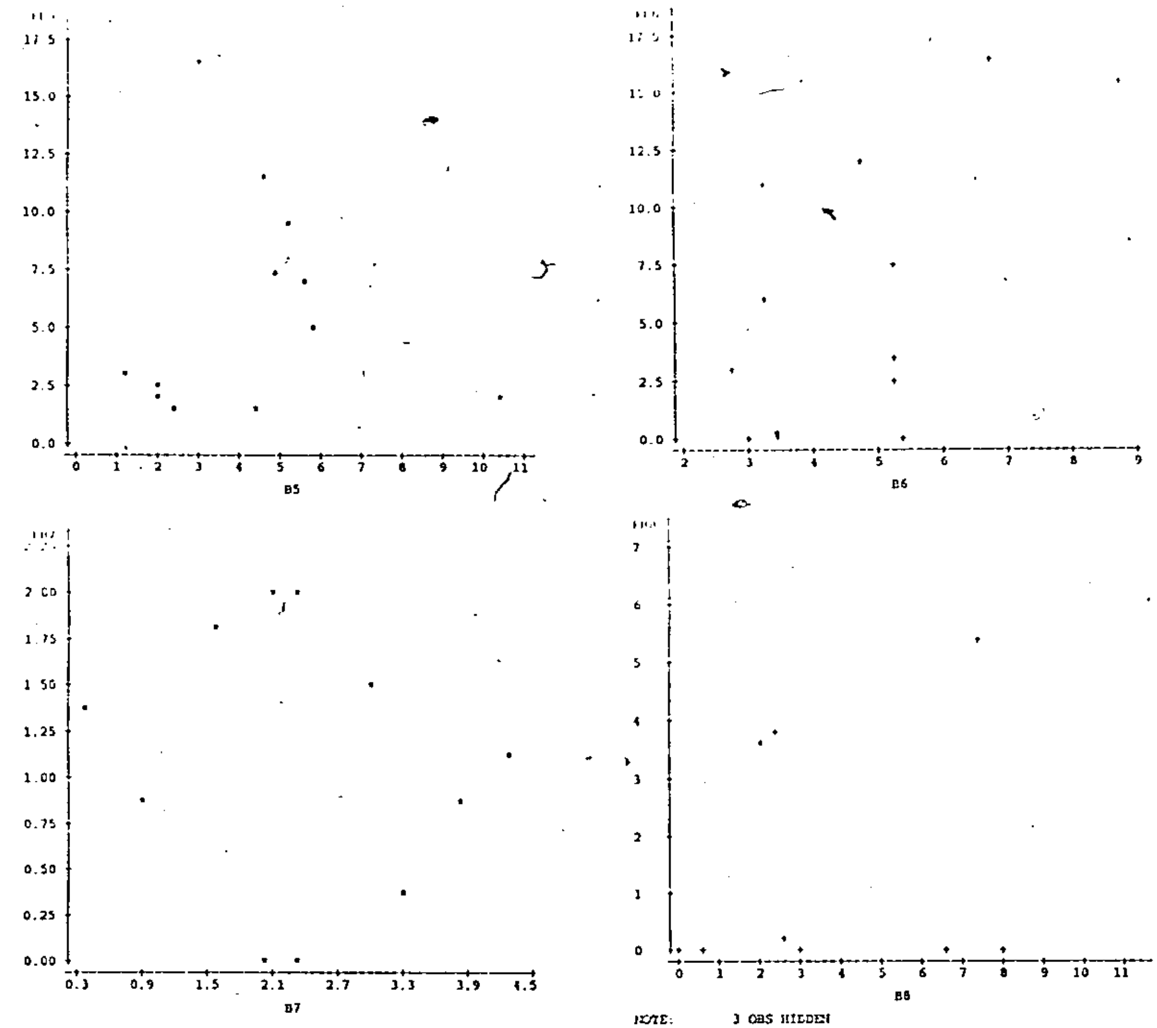


Fig. 5.2 (cont.)

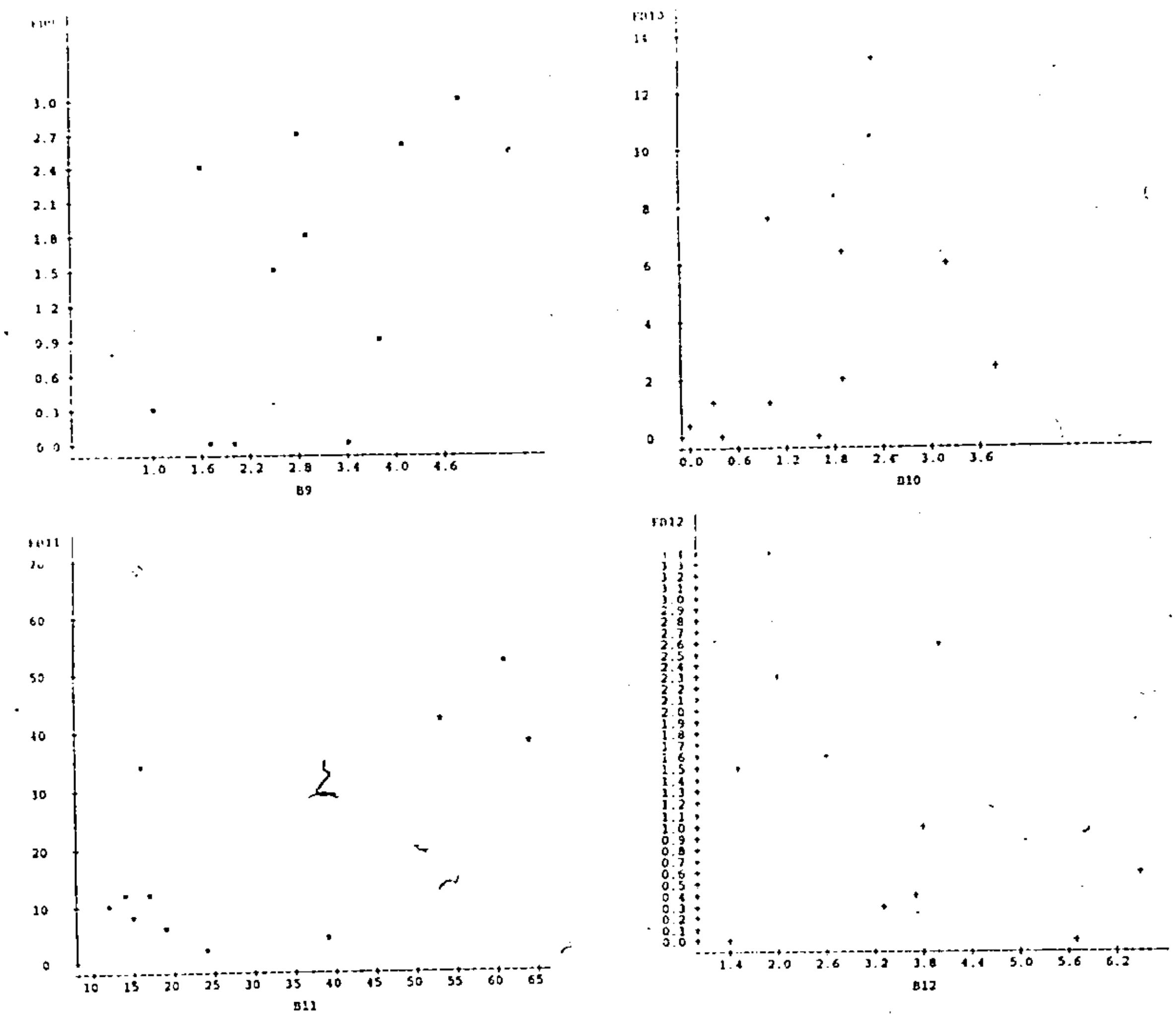


Fig. 5.2 (cont.)

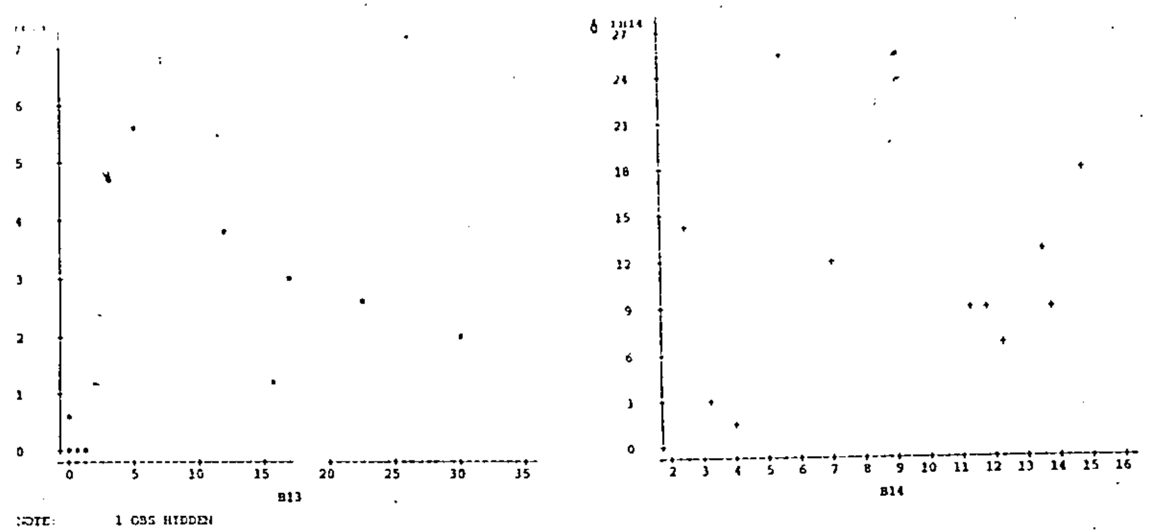


Fig. 5.2 (concluded)

CHAPTER 6: Enlarged data set and geostatistical model

6.1 Introduction

The enlarged data set, (Table 6.1) created by merging Stam's and Gradstein's data, is difficult to work with because of the poor correlation between the two sets. For a linear relationship the relative closeness of the relation is best measured by the coefficient of determination. This measure shows the proportion of the variance in the dependent variable that is associated with differences in the other variable. As seen in Chapter 5, the relative closeness of the relation of Stam's and Gradstein's data is weak.

6.2 - Tojeira 1 section (Eoguttulina sp., Epistomina mosquensis and Ophthalmidium strumosum)

The new data set has N = 41 (Stam's 31 samples plus 17 of Gradstein's sample equals 48 samples, but 7 samples were collected at the same level by both authors.). About 10 autocorrelations is a safe number to examine a taxon using an ARIMA procedure to estimate the a.c.f and p.a.c.f for Tojeira 1.

As seen in Appendix I Eoguttulina sp., Epistomina mosquensis and Ophthalmidium strumosum show an exponential function. The estimated a.c.f and p.a.c.f of Eoguttulina sp., E. mosquensis and O. strumosum suggest two things: 1 - The data of these species have a stationary mean, and 2 - The AR (1) model for Eoguttulina sp.

TABLE 6.1 ENLARGED DATA SET

SAMPLE	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14
S6.2	5.0	4.1	0.0	0.0	1.5	7.9	6.6	4.8	10.4	0.0	0.0	1.8	0.9	45.3
26.3	2.5	14.8	0.0	0.0	0.0	24.7	5.7	4.9	0.2	0.0	0.0	0.9	18.8	17.9
S6.3A	0.5	19.0	0.0	0.5	0.0	22.5	3.8	4.6	1.1	0.0	0.0	0.0	26.8	13.6
S6.4	5.0	5.4	0.2	1.2	0.0	19.8	10.6	3.4	4.2	0.0	0.0	2.4	0.4	26.1
S6.5	2.6	1.8	15.1	0.0	13.3	4.7	4.9	0.0	0.0	0.0	0.0	2.9	7.2	34.8
S6.6A	1.2	13.5	8.4	1.5	16.2	5.4	0.3	0.6	14.4	0.0	0.0	13.8	6.0	11.7
S6.6	2.3	2.2	24.2	11.8	0.0	9.9	4.8	2.5	3.8	0.0	0.0	10.8	3.8	9.1
26.7	5.4	3.4	18.1	3.4	0.0	11.1	3.1	3.8	3.8	0.2	0.0	6.9	0.6	29.2
S6.8	4.5	0.3	12.4	1.0	0.0	5.6	2.1	2.0	35.4	0.0	5.1	5.3	0.0	15.4
S6.9	6.4	2.6	26.1	4.3	0.3	4.6	4.1	2.6	3.7	0.0	10.9	5.7	0.0	23.2
T1/6	7.7	0.5	27.4	5.0	0.0	13.9	1.2	1.0	4.0	1.2	14.4	5.0	3.5	14.2
S6.10	5.1	1.1	27.6	2.5	2.5	1.4	0.6	2.2	1.1	0.3	29.6	1.7	0.3	20.7
T1/7	2.4	0.7	39.5	23.2	0.0	15.2	2.2	0.5	2.7	2.9	6.4	1.0	0.5	2.2
S6.11	3.7	1.1	13.2	17.6	0.0	23.0	6.1	2.0	0.5	0.2	10.9	1.1	0.2	11.1
T1/8	5.0	0.6	25.1	4.7	0.0	24.2	2.4	2.9	4.1	1.2	9.4	3.2	2.6	11.8
S6.12	21.6	1.3	23.6	3.0	3.0	9.3	3.3	0.0	0.0	0.0	12.3	1.3	0.0	15.0
T1/9	32.1	0.0	22.4	8.9	3.0	8.2	2.2	2.2	0.0	0.0	11.2	1.5	1.5	6.7
S6.13	3.9	0.6	23.3	5.7	4.8	8.7	2.7	0.3	1.2	0.0	14.0	1.5	0.0	23.0
S6.14	7.6	1.5	15.5	0.7	0.9	4.8	3.8	1.8	2.4	2.3	4.8	5.0	0.7	31.1
T1/10	7.8	0.4	18.8	15.3	0.4	11.8	1.6	2.7	7.8	0.0	7.8	4.7	5.1	15.7
S6.15	2.4	0.3	35.2	7.8	6.0	8.1	7.2	1.5	0.6	0.3	10.5	1.5	0.0	15.1
T1/11	10.6	1.2	32.4	1.9	0.0	14.8	1.6	3.9	4.7	5.9	5.9	3.5	1.9	9.4
S6.16	7.8	0.5	21.4	3.1	7.7	15.2	5.5	1.4	2.1	2.9	15.0	1.7	0.0	11.2
T1/12	9.4	1.0	25.7	2.0	0.0	7.4	1.7	3.0	6.4	5.7	3.5	2.5	3.2	26.5
S6.17	2.9	1.1	46.3	2.0	1.9	5.1	2.8	1.1	1.4	8.3	6.4	4.1	0.0	13.2
S6.18	14.0	0.3	18.0	1.7	5.3	3.9	7.2	2.8	0.8	6.6	8.0	4.7	0.0	20.8
S6.19	8.5	0.8	39.7	0.6	3.5	3.5	4.4	2.5	1.0	1.9	11.6	4.1	0.0	8.3
T1/13	17.9	0.0	9.2	3.0	0.8	19.8	2.7	3.8	4.9	10.3	3.0	4.2	0.0	19.1
S6.20	6.0	0.8	30.4	6.0	1.4	9.1	7.4	1.5	1.9	0.4	12.6	4.4	1.0	9.2
S6.21	11.4	0.8	13.5	1.7	1.2	15.0	11.3	1.9	1.9	7.2	9.3	4.2	0.0	13.5
S6.22	4.6	3.5	23.0	0.3	0.8	11.8	6.6	1.6	0.5	2.4	11.8	7.6	0.0	22.3
S6.23	3.3	1.5	33.5	0.8	6.2	11.4	9.3	2.1	1.6	1.9	10.9	4.8	0.0	7.7
S6.24	7.1	0.6	37.1	3.2	3.1	6.6	3.6	1.5	1.2	5.8	1.0	3.4	0.0	9.0
S6.25	7.4	0.6	36.6	7.4	8.4	9.1	1.2	3.9	0.6	1.9	0.0	1.9	0.0	13.6
S6.26	17.1	0.9	38.8	4.9	0.6	6.7	2.7	1.7	0.6	2.9	3.2	0.6	0.0	13.3
S6.26A	2.2	1.0	34.8	3.7	3.4	16.2	3.3	2.5	0.3	5.2	9.1	4.6	0.0	7.0
T1/16	3.3	0.0	39.0	1.9	0.0	25.7	2.4	0.9	4.8	2.8	1.9	2.4	2.9	10.5
T1/17	3.3	0.0	38.9	12.9	0.7	16.5	2.6	1.0	0.7	1.3	15.5	0.3	0.0	5.3
S6.27	10.6	0.3	36.5	1.2	1.8	11.9	8.8	0.3	1.2	9.7	6.4	5.5	0.0	8.8
S6.28	11.8	0.0	27.4	3.4	2.3	14.0	2.4	2.0	0.3	1.1	17.1	2.3	0.0	9.4
S6.29	6.2	0.2	15.7	3.2	2.5	6.5	0.6	2.5	0.0	5.3	21.6	4.9	0.4	16.0

(Fig. 6.1), AR(2) model for E. mosquensis (Fig. 6.2), and ARMA (1, 1) model for O. strumosum (Fig. 6.3) are good first choices to try at the estimation stage. The stationarity assumption is confirmed since $\hat{\phi}_1 = 0.510$, $\hat{\phi}_1 = 0.281$ and $\hat{\phi}_1 = 0.935$, respectively for the mentioned species. These satisfy the condition $\hat{\phi}_1 < 1$. The large t - values attached to $\hat{\phi}_1$ indicate that these terms should be kept in the model.

As seen in Figs. 6.1- 6.3 none of the residual autocorrelations have an absolute t_b - value greater than the practical warning levels. The chi-squared values indicate this too.

The constant c and a values were estimated from their best - fitting straight lines as shown in Figs. 6.4 - 6.6. Table 6.2 shows the estimated values of a and c.

TABLE 6.2 Estimated a and c values for best - fitting lines for A2 (Eoguttulina sp.), A3 (Epistomina mosquensis) and A10 (Ophthalmidium stromosum) for N = 41

Taxa	A2(Eoguttulina sp.)	A3(E. mosquensis)	A10(O. strumosum)
c	0.48	0.52	0.60
a	0.08	0.12	0.19

Eq. 56 and Eq. 60 were used to calculate F_N and F_L (Tables 6.3 - 5) respectively. The theoretical and observed relative variances were plotted against the lag as seen in Fig. 6.7 - 6.9 for each species. The correspondence between the experimental and theoretical values are not good for these taxa. As explained above, Stam's and Gradstein's data do

Figure 6.1 Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the AR(1) time series model to A2(<u>Eoguttulina sp.</u>) with N = 41

```
                              SAS                                        1
                                 15:44 TUESDAY, JULY 28, 1987
                       ARIMA PROCEDURE

                 NAME OF VARIABLE         =        A2
                 MEAN OF WORKING SERIES=    2.20244
                 STANDARD DEVIATION     =   4.03343
                 NUMBER OF OBSERVATIONS=         41
                       AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    16.2685    1.00000  |                    |********************|
  1    8.39487    0.51602  |                .   |**********          |
  2     4.3428    0.26694  |              .     |*****               |
  3    5.46661    0.33602  |              .     |*******.            |
  4     5.0157    0.30831  |             .      |******   .          |
  5    1.59018    0.09775  |             .      |**       .          |
  6   0.546632    0.03360  |             .      |*        .          |
  7   0.171189    0.01052  |             .      |         .          |
  8  0.0428721    0.00264  |             .      |         .          |
  9  -0.487336   -0.02996  |             .     *|         .          |
 10  -0.418818   -0.02574  |             .     *|         .          |
                           '.' MARKS TWO STANDARD ERRORS

                   INVERSE AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1   -0.42039  |          ********|***                 |
  2    0.14653  |                . |***   .             |
  3   -0.13343  |              ***|       .             |
  4   -0.16956  |              ***|       .             |
  5    0.12696  |                . |***   .             |
  6   -0.05874  |                .*|      .             |
  7    0.10302  |                . |**    .             |
  8   -0.03658  |                .*|      .             |
  9    0.02154  |                . |      .             |
 10   -0.01973  |                . |      .             |

                   PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    0.51602  |                  |**********          |
  2    0.00091  |                . |      .             |
  3    0.26976  |                . |*****.              |
  4    0.04826  |                . |*     .             |
  5   -0.15205  |              ***|       .             |
  6   -0.02644  |                . *|     .             |
  7   -0.08927  |                .**|     .             |
  8    0.03581  |                . |*     .             |
  9   -0.00216  |                . |      .             |
 10    0.02737  |                . |*     .             |

            AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                            AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   25.25  6  0.000  0.516  0.267  0.336  0.308  0.098  0.034
```

```
                              SAS                                        2
                                 15:44 TUESDAY, JULY 28, 1987
              ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                      APPROX.
           PARAMETER    ESTIMATE    STD ERROR    T RATIO  LAG
           MU            2.19987      1.09416       2.01    0
           AR1,1        0.509558      0.13672       3.73    1

           CONSTANT ESTIMATE   =  1.07891

           VARIANCE  ESTIMATE  =  12.4988
           STD ERROR ESTIMATE  =  3.53536
           AIC                 =  222.154
           SBC                 =  225.581
           NUMBER OF RESIDUALS=        41

                  CORRELATIONS OF THE ESTIMATES

                                MU     AR1,1
                  MU         1.000    -0.024
                  AR1,1     -0.024     1.000

               AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    5.24  4  0.264  0.002 -0.137  0.183  0.232 -0.073 -0.013
 12    5.38 10  0.864 -0.007  0.021 -0.034  0.006 -0.030 -0.009
 18    5.96 16  0.988  0.030 -0.043 -0.048  0.029 -0.016 -0.047
 24    6.60 22  0.999  0.011 -0.046  0.007 -0.040 -0.007 -0.053

             MODEL FOR VARIABLE A2
             ESTIMATED MEAN       =   2.19987

              AUTOREGRESSIVE FACTORS
                           FACTOR 1
             1-.509558B**(1)
```

Figure 6.2 Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the AR(2) time series model to A3(E. mosquensis) with N = 41

```
                         SAS                                    3
                                   15:44 TUESDAY, JULY 28, 1987
                  ARIMA PROCEDURE

           NAME OF VARIABLE        =        A3
           MEAN OF WORKING SERIES=   23.7561
           STANDARD DEVIATION      =   12.1219
           NUMBER OF OBSERVATIONS=        41
                      AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    146.941    1.00000  |                    |********************|
  1    66.4589    0.45228  |                    |*********           |
  2    77.2056    0.52542  |                    |***********         |
  3    41.5462    0.28274  |                .   |******  .           |
  4    31.3431    0.21330  |                .   |****    .           |
  5    22.3485    0.15209  |                .   |***     .           |
  6    7.02954    0.04784  |                .   |*       .           |
  7    -5. 6578  -0.03992  |                .  *|        .           |
  8    5. 7999    0.03593  |                .   |*       .           |
  9    -7.. 1063 -0.05261  |                .  *|        .           |
 10    6.0.891    0.04103  |                .   |*       .           |
                            '.' MARKS TWO STANDARD ERRORS

                 INVERSE AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1   -0.16148  |                 ***|   .                |
       2   -0.39178  |            ********|   .                |
       3    0.10091  |                 .  |** .                |
       4    0.04509  |                 .  |*  .                |
       5   -0.16663  |                 .***|   .                |
       6    0.07035  |                 .  |*  .                |
       7    0.14896  |                 .  |***.                |
       8   -0.09290  |                 . **|   .                |
       9    0.01554  |                 .  |   .                |
      10   -0.01829  |                 .  |   .                |

                 PARTIAL AUTOCORRELATIONS

     LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1    0.45228  |                 .  |*********           |
       2    0.40337  |                 .  |********            |
       3   -0.06228  |                 .  *|   .                |
       4   -0.09450  |                 . **|   .                |
       5    0.03227  |                 .  |*  .                |
       6   -0.06248  |                 .  *|   .                |
       7   -0.13608  |                 .***|   .                |
       8    0.14618  |                 .  |***.                |
       9   -0.00600  |                 .  |   .                |
      10    0.02536  |                 .  |*  .                |

              AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                         AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   28.62  6  0.000  0.452  0.525  0.283  0.213  0.152  0.048
```

```
                         SAS                                    4
                                   15:44 TUESDAY, JULY 28, 1987
            ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                                  APPROX.
     PARAMETER     ESTIMATE     STD ERROR    T RATIO   LAG
     MU             20.8909       5.86549       3.56    0
     AR1,1         0.280763      0.136655       2.05    1
     AR1,2         0.493128      0.139365       3.54    2

     CONSTANT ESTIMATE   =    4.7236

     VARIANCE  ESTIMATE  =   93.6281
     STD ERROR ESTIMATE  =   9.67616
     AIC                 =   306.274
     SBC                 =   311.415
     NUMBER OF RESIDUALS=         41

              CORRELATIONS OF THE ESTIMATES

                       MU      AR1,1     AR1,2
        MU          1.000     -0.012     0.009
        AR1,1      -0.012      1.000    -0.607
        AR1,2       0.009     -0.607     1.000

              AUTOCORRELATION CHECK OF RESIDUALS

  TO    CHI                          AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
   6    1.27  3  0.735 -0.041 -0.006 -0.013 -0.070  0.103 -0.093
  12    5.05  9  0.830 -0.200  0.032 -0.110 -0.031  0.080  0.095
  18    9.31 15  0.861  0.064  0.194 -0.049 -0.060  0.090 -0.082
  24   15.21 21  0.812 -0.099 -0.062 -0.044  0.177 -0.117 -0.060

     MODEL FOR VARIABLE A3
     ESTIMATED MEAN       =   20.8909

      AUTOREGRESSIVE FACTORS
                  FACTOR 1
     1-.280763B**(1)-.493128B**(2)
```

Figure 6.3 Printout of estimated a.c.f.'s and p.a.c.f.'s after fitting the ARMA(1,1) time series model to A10(<u>O. strumosum</u>) with N = 41

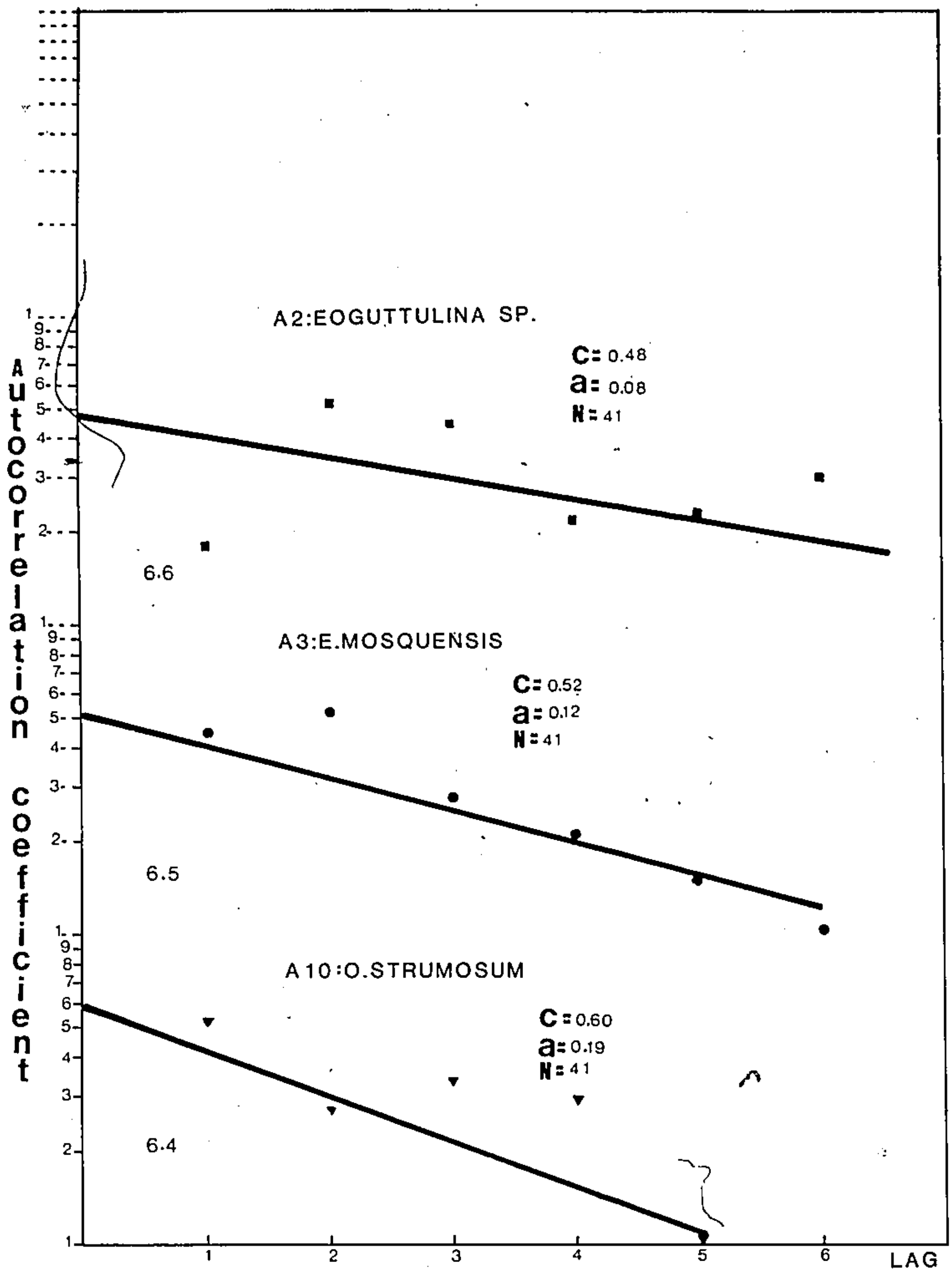
```
                      SAS                                  5
                         15:44 TUESDAY, JULY 28, 1987
                ARIMA PROCEDURE

        NAME OF VARIABLE          =      A10
        MEAN OF WORKING SERIES=      2.39268
        STANDARD DEVIATION        =      2.90906
        NUMBER OF OBSERVATIONS=           41
                   AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0   8.46263    1.00000   |                    |********************|
 1   1.52864    0.18063   |                    |****                |
 2   4.52344    0.53452   |                    |***********         |
 3   3.91379    0.46248   |                    |*********           |
 4   1.8488     0.21847   |                    |****                |
 5   1.94111    0.22937   |                    |*****               |
 6   2.58445    0.30540   |                    |******              |
 7   0.096249   0.01137   |                    |                    |
 8   1.25868    0.14873   |                    |***                 |
 9   1.46956    0.17370   |                    |***                 |
10  -0.903665  -0.10678   |                  **|                    |
                           '.' MARKS TWO STANDARD ERRORS

               INVERSE AUTOCORRELATIONS

    LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
     1    0.19168   |                    |****                |
     2   -0.53654   |         ***********|                    |
     3   -0.41434   |            ********|                    |
     4    0.06252   |                    |*                   |
     5    0.21557   |                    |****                |
     6    0.07009   |                    |*                   |
     7    0.01076   |                    |                    |
     8   -0.07735   |                  **|                    |
     9   -0.06292   |                   *|                    |
    10    0.06077   |                    |*                   |

               PARTIAL AUTOCORRELATIONS

    LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
     1    0.18063   |                    |****                |
     2    0.51882   |                    |**********          |
     3    0.45589   |                    |*********           |
     4   -0.09076   |                  **|                    |
     5   -0.37582   |            ********|                    |
     6    0.06347   |                    |*                   |
     7    0.04744   |                    |*                   |
     8   -0.04929   |                   *|                    |
     9    0.15305   |                    |***                 |
    10   -0.12346   |                  **|                    |

         AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                       AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   33.83  6  0.000  0.181  0.535  0.462  0.218  0.229  0.305
```

```
                     SAS                                  6
                        15:44 TUESDAY, JULY 28, 1987
           ARIMA: MAXIMUM LIKELIHOOD ESTIMATION

                              APPROX.
PARAMETER    ESTIMATE      STD ERROR    T RATIO  LAG
MU            2.29839         1.4931       1.54    0
MA1,1        0.729668        0.18624       3.92    1
AR1,1        0.935098      0.0980471       9.54    1

CONSTANT ESTIMATE  =  0.149172

VARIANCE  ESTIMATE =  6.95289
STD ERROR ESTIMATE =  2.63683
AIC                =  199.284
SBC                =  204.425
NUMBER OF RESIDUALS=       41

          CORRELATIONS OF THE ESTIMATES

                  MU     MA1,1    AR1,1
      MU       1.000     0.309    0.410
      MA1,1    0.309     1.000    0.819
      AR1,1    0.410     0.819    1.000

          AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   13.10  3  0.004 -0.314  0.289  0.258 -0.098 -0.026  0.168
 12   36.25  9  0.000 -0.263 -0.001  0.097 -0.339  0.392 -0.239
 18   43.43 15  0.000  0.125  0.077  0.087 -0.118  0.193 -0.143
 24   46.61 21  0.001 -0.058  0.018 -0.145 -0.046 -0.078 -0.047

     MODEL FOR VARIABLE A10
     ESTIMATED MEAN       =    2.29839

      AUTOREGRESSIVE FACTORS
                     FACTOR 1
    1-.935098B**(1)

      MOVING AVERAGE FACTORS
                     FACTOR 1
     1-.729668B**(1)
```

Figure 6.4 Correlogram for A10 (*O. strumosum*) and best-fitting exponential curve with a = 0.19 and c = 0.60

Figure 6.5 Correlogram for A3 (*E. mosquensis*) and best-fitting exponential curve with a = 0.12 and c = 0.52

Figure 6.6 Correlogram for A2 (*Eoguttulina* sp.) and best-fitting exponential curve with a = 0.08 and c = 0.48


Autocorrelation coefficient
A2:EOGUTTULINA SP.
C= 0.48
a= 0.08
N= 41
6.6
A3:E.MOSQUENSIS
C= 0.52
a= 0.12
N= 41
6.5
A10:O.STRUMOSUM
C= 0.60
a= 0.19
N= 41
6.4
LAG

not show strong linear relationships with one another.

TABLE 6.3 Theoretical and observed values of the relative variance of A2 (Eogiuttulina sp.)

k(lag)	$S^2(x)$	F_N	F_L
0	0	0	0.52
1	16.67	1.0	0.99
2	12.91	1.55	1.43
3	10.22	1.84	1.85
4	7.78	1.87	2.25
5	6.50	1.95	2.63

TABLE 6.4 Theoretical and observed values of the relative variance of A3 (Epistomina mosquensis)

k(lag)	$S^2(x)$	F_N	F_L
0	0	0	0.48
1	150.62	1.0	0.98
2	108.08	1.43	1.44
3	94.30	1.88	1.87
4	80.95	2.15	2.26
5	68.25	2.26	2.63
6	58.61	2.33	2.97

TABLE 6.5 Theoretical end observed values of the relative variance of A10 (Ophthalmidium strumosum)

k(lag)	$S^2(x)$	F_N	F_L
0	0	0	0.40
1	8.67	1.0	0.95
2	5.16	1.2	1.45
3	4.74	1.64	1.90
4	4.49	2.07	2.20
5	4.24	2.44	2.63
6	4.03	2.80	2.97

The probit transformed (Tables 6.6, 6.8 and 6.10) frequency distributions (Tables 6.7, 6.9 and 6.11) for each species were plotted on normal distribution paper for single, paired and sets of 4 consecutive values as done previously (Fig. 6.10a, b, c - 6.11a, b, c and 6.12a, b, c.).

Eq. 60 represents a prob - normal distribution which is a straight line on probability paper. For Eoguttulina sp., Epistomina mosquensis and Ophthalmidium strumosum , the prob - normal distribution parameters, which are x, $s^2(x)$, R and b values, were calculated as shown in Tables 6.12 - 14. Calculated parameters were used with Eq. 60 and Y functions were plotted on normal probability paper as theoretical distributions as shown in Fig. 6.10a, b, c - 6.11a, b, c and 6.12a, b,c.

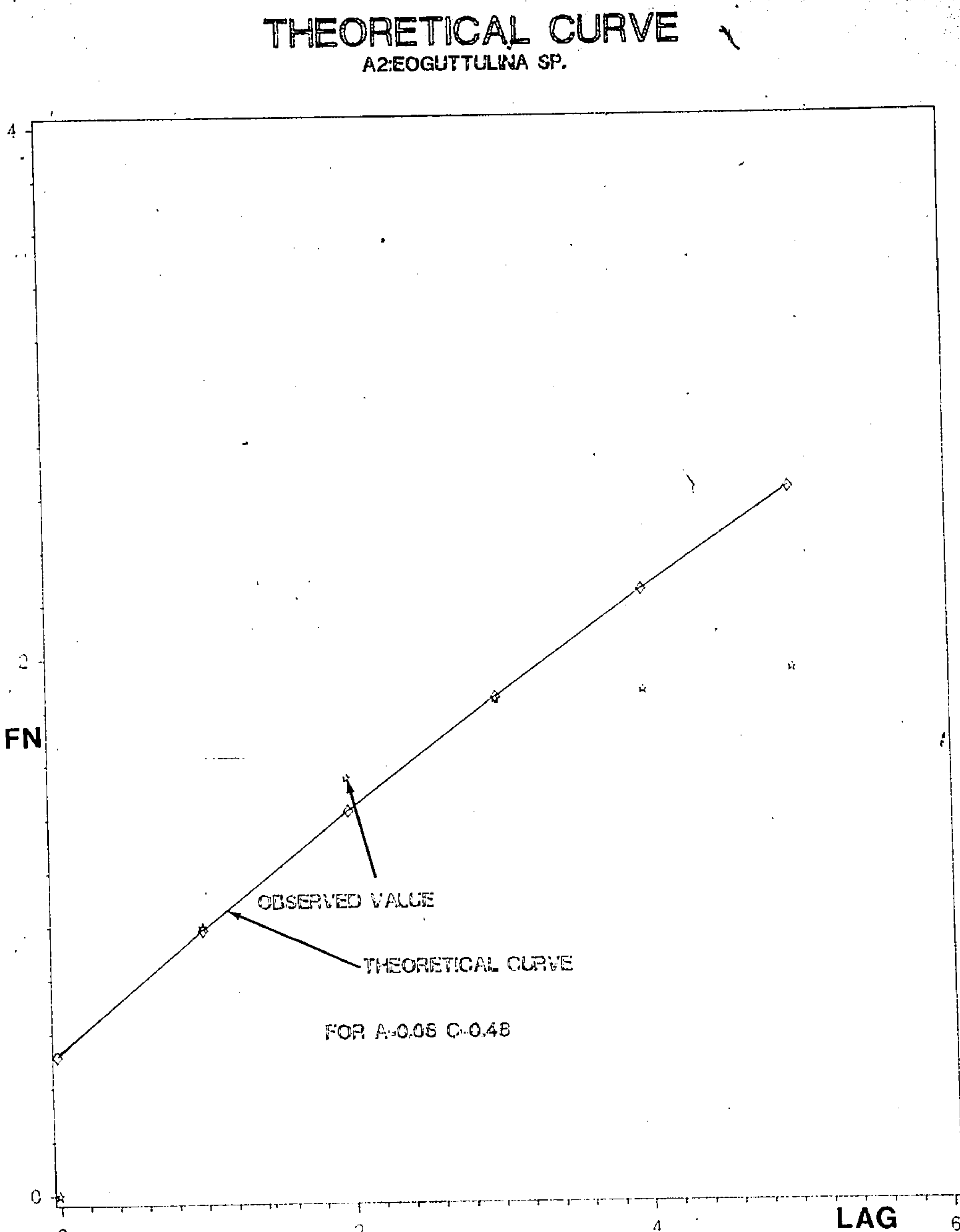


Figure 6.7 Theoretical curve for a = 0.08 and c = 0.48 in comparison with observed values of A2 (*Eoguttulina* sp.)

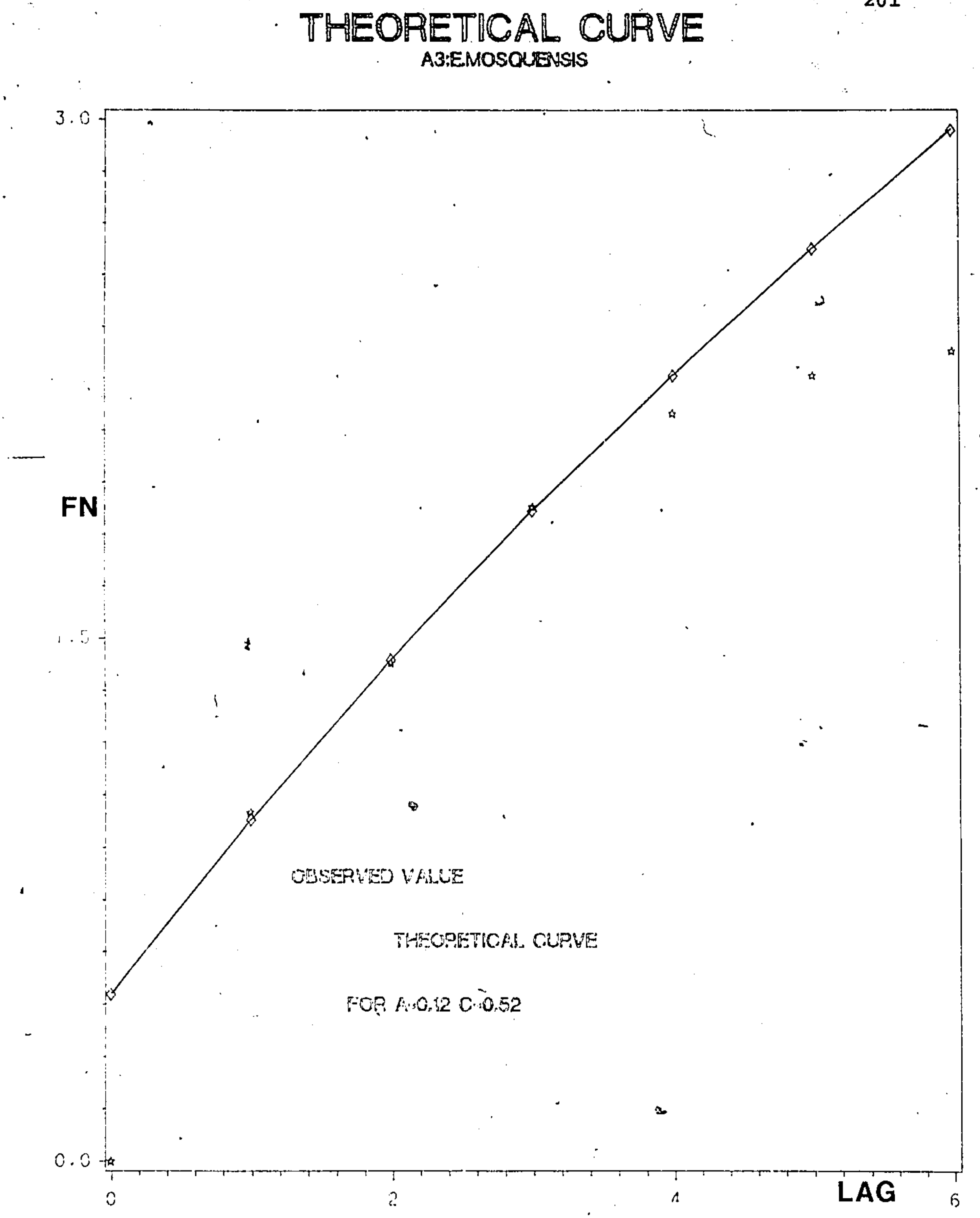


Figure 6.8 Theoretical curve for a 0.12 and c 0.52 in comparison with observed values of E. mosquensis

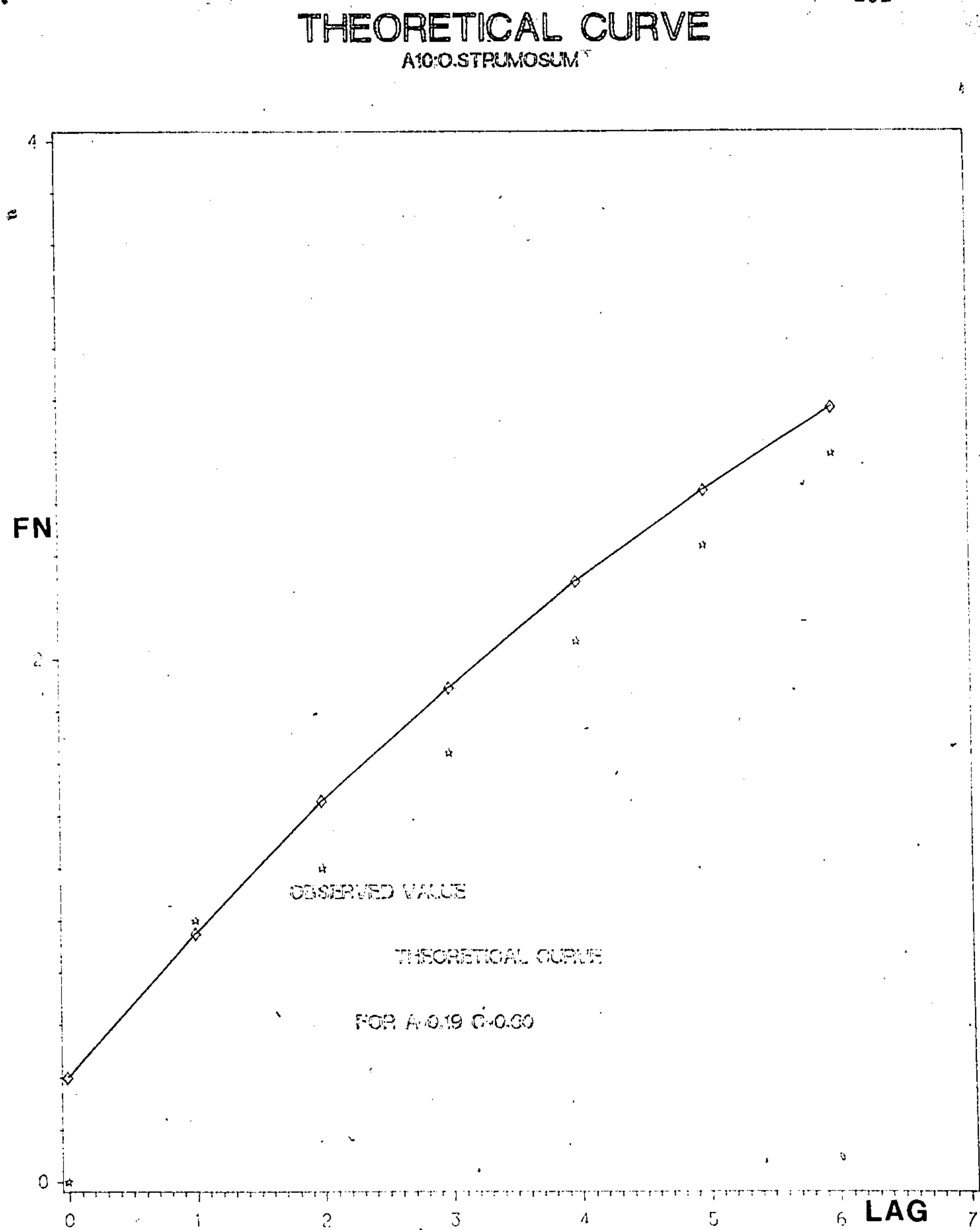


Figure 6.9 Theoretical curve for a 0.19 and c 0.60 in comparison with observed values of O. strumosum

Table 6.6 probit transformation of A2 (Eoguttulina sp.) for N = 41, 20 and 10 samples from the Tojeira 1 section.

sample single	probit single	sample pairs	probit pairs	sample set of 4	probit set of 4
4.1	3.261				
14.8	3.955	9.45	3.686		
19.0	4.122				
5.4	3.393	12.2	3.835	10.825	3.764
1.8	2.903				
13.5	3.897	7.65	3.570		
2.2	2.986				
3.4	3.175	2.80	3.080	5.225	3.376
0.3	2.252				
2.6	3.057	1.45	2.816		
0.5	2.424				
1.1	2.710	0,80	2.591	1.125	2.766
0.7	2.543				
1.1	2.710	0.90	2.634		
0.6	2.488				
1.3	2.774	0.45	2.386	0.675	2.644
0.0	2.187				
0.6	2.488	0.30	2.252		
1.5	2.830				

(continued)

Table 6.6 (concluded)

0.4	2.348	0.85	2.612	0.575	2.475
0.3	2.252				
1.2	2.743	0.75	2.567		
0.5	2.424				
1.0	2.674	0.75	2.567	0.750	2.567
1.1	2.710				
0.3	2.252	0.70	2.543		
0.8	2.591				
0.0	2.187	0.40	2.348	0.550	2.456
0.8	2.591				
0.8	2.591	0.8	2.591		
3.5	3.188				
1.5	2.830	2.50	3.040	1.650	2.868
0.6	2.488				
0.6	2.483	0.60	2.488		
0.9	2.634				
1.0	2.674	0.95	2.654	0.775	2.579
0.0	2.187				
0.0	2.187	0.0	2.187		
0.3	2.252				
0.0	2.187	0.15	2.187	0.00	2.187
0.2	2.187				

Table 6.7 Cumulative frequency distributions of <u>Eoguttulina</u> sp. for N = 41, 20 and 10 samples from the Tojeira 1 section.

N = 41

Class interval	Freq.	cum. freq	perc. %	cum. %	plotting value
2.25 - 2.55	18	18	43.90	43.90	42.74
2.55 - 2.85	13	31	31.71	75.61	74.19
2.85 - 3.15	3	34	7.32	82.93	81.45
3.15 - 3.45	4	38	9.76	92.68	91.13
3.45 - 3.75	0	38	0.00	92.68	91.13
3.75 - 4.05	3	41	7.32	100.00	98.40

N = 20

2.2 - 2.6	11	11	55.00	55.00	52.50
2.6 - 3.0	4	15	20.00	75.00	72.13
3.0 - 3.4	2	17	10.00	85.00	82.00
3.4 - 3.8	2	19	10.00	95.00	91.80
3.8 - 4.2	1	20	5.00	100.00	96.72

N = 10

2.2 - 2.6	5	5	50.00	50.00	45.16
2.6 - 3.0	3	8	30.00	80.00	74.19
3.0 - 3.4	1	9	10.00	90.00	83.87
3.4 - 3.8	1	10	10.00	100.00	93.55

Table 6.8 Probit transformation of A3 (Epistomina mosquensis) for N = 41,20 and 10 samples from the Tojeira 1 section.

sample single	probit single	sample pairs	probit pairs	sample set of 4	probit set of 4
0.0	2.187				
0.0	2.131	0.00	2.187		
0.0	2.131				
0.2	2.187	0.00	2.187	0.00	2.187
15.1	3.968				
8.4	3.621	11.75	3.812		
24.2	4.300				
18.1	4.081	21.15	4.198	16.45	4.024
12.4	3.845				
26.1	4.360	19.25	4.131		
27.4	4.399				
27.6	4.405	27.5	4.402	23.375	4.273
38.5	4.708				
13.2	3.883	25.85	4.357		
25.1	4.329				
23.6	4.281	24.35	4.304	25.10	4.329
22.4	4.241				
23.3	4.271	22.85	4.258		
15.5	3.985				

(continued)

Table 6.8 (concluded)

18.8	4.115	17.15	4.047	20.0	4.158
35.2	4.620				
32.4	4.543	33.8	4.582		
21.4	4.207				
25.7	4.347	23.55	4.280	28.675	4.437
46.3	4.907				
18.0	4.085	32.15	4.535		
39.7	4.739				
9.2	3.671	24.45	4.308	28.30	4.426
30.4	4.487				
13.5	3.897	21.95	4.226		
23.0	4.261				
33.5	4.574	28.25	4.424	25.10	4.329
37.1	4.671				
36.6	4.658	36.85	4.638		
38.8	4.715				
34.8	4.609	36.8	4.636	36.825	4.463
39.0	4.721				
38.9	4.718	38.95	4.718		
36.5	4.655				
27.4	4.399	31.95	4.532	35.45	4.626
16.7	4.34				

Table 6.9 Cumulative frequency distribution of A3 (Epistomina mosquensis) for N = 41, 20 and 10 samples for Tojeira 1 section

N= 41

class interval	freq.	cum. freq.	perc. %	cum. %	plotting value
2.25 - 2.75	4	4	9.76	9.76	8.87
2.75 - 3.25	0	4	0.00	9.76	8.87
3.25 - 3.75	2	6	4.88	14.63	13.70
3.75 - 4.25	11	17	26.83	41.46	40.32
4.25 - 4.75	23	40	56.10	97.56	95.96
4.75 - 5.25	1	41	2.44	100.00	98.40

N = 20

2.25 - 2.75	3	3	15.00	15.00	13.11
2.75 - 3.25	0	3	0.00	15.00	13.11
3.25 - 3.75	0	3	0.00	15.00	13.11
3.75 - 4.25	4	7	20.00	35.00	32.80
4.25 - 4.75	13	20	65.00	100.00	96.72

N = 10

2.1 - 2.7	1	1	10.00	10.00	6.45
2.7 - 3.3	0	1	0.00	10.00	6.45
3.3 - 3.9	0	1	0.00	10.00	6.45
3.9 - 4.5	9	10	90.00	100.00	93.55

Table 6.10 Probit transformation of A10 (Ophthalmidium strumosum)
for N = 41, 20 and 10 samples from Tojeira 1 section.

sample single	probit single	sample pairs	probit pairs	sample set of 4	probit set of 4
0.0	2.187				
0.0	2.187	0.00	2.187		
0.0	2.187				
0.0	2.187	0.00	2.187	0.00	2.187
0.0	2.187				
0.0	2.187	0.00	2.187		
0.0	2.187				
0.2	2.187	0.00	2.187	0.00	2.187
0.0	2.187				
0.0	2.187	0.00	2.187		
1.2	2.187				
0.3	2.252	0.75	2.567	0.375	2.324
2.9	3.104				
0.2	2.187	1.55	2.843		
1.2	2.187				
0.0	2.187	0.60	2.488	1.075	2.701
0.0	2.187				
0.0	2.187	0.00	2.781		
2.3	3.005				

(continued)

Table 6.10 (concluded)

0.0	2.187	1.15	2.715	0.575	2.472
0.3	2.252				
5.9	3.437	3.10	3.134		
2.9	3.104				
5.7	3.420	4.30	3.283	3.70	3.213
8.3	3.615				
6.6	3.494	7.45	3.556		
1.9	2.925				
0.3	2.252	1.10	2.710	4.275	3.280
0.4	2.348				
7.2	3.539	3.80	3.226		
2.4	3.023				
1.9	2.925	2.15	2.969	2.975	3.115
5.8	3.428				
1.9	2.925	3.85	3.236		
2.9	3.104				
5.2	3.374	4.05	3.255	3.95	3.243
2.8	3.089				
1.3	2.774	2.05	2.956		
9.7	3.701				
1.1	2.710	5.40	3.393	3.725	3.216
5.3	3.384				

Table 6.11 Cumulative frequency distributions of Ophthalmidium strumosum for N = 41, 20 and 10 samples

N = 41

class interval	freq.	cum. freq.	perc. %	cum. %	plotting value
2.125 - 2.375	19	19	46.34	46.34	45.16
2.375 - 2.625	0	19	0.00	46.34	45.16
2.625 - 2.875	4	23	9.76	56.10	54.84
2.875 - 3.125	9	32	21.95	78.05	76.60
3.125 - 3.375	1	33	2.44	80.49	79.00
3.375 - 3.625	8	41	19.51	100.00	98.40

N = 20

2.25 - 2.55	7	7	35.00	35.00	32.80
2.55 - 2.85	4	11	20.00	55.00	52.46
2.85 - 3.15	3	14	15.00	70.00	67.21
3.15 - 3.45	5	19	25.00	95.00	91.80
3.45 - 3.75	1	20	5.00	100.00	96.72

N = 10

2.25 - 2.55	4	4	40.00	40.00	35.50
2.55 - 2.85	1	5	10.00	50.00	45.20
2.85 - 3.15	1	6	10.00	60.00	54.80
3.15 - 3.45	4	10	40.00	100.00	93.55

TABLE 6.12 Means, variances,R and b parameters for species A2,A3 and A10 for N=41

Taxon	Eoguttulina sp.	E.mosquensis	O. Strumosum
$\bar{x}$	0.022025	0.23756	0.023927
$s^2(x)$	0.001627	0.014694	0.000846
b	2.014	0.714	1.977
R	0.60	0.39	0.47
N	41	41	41

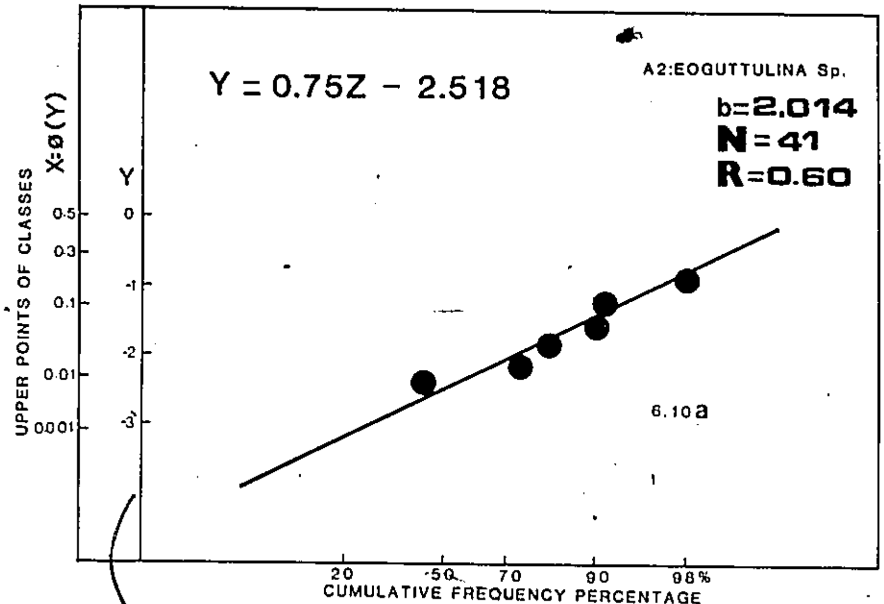
TABLE 6.13 Means, variances, R and b parameters for species A2, A3 and A10 for N=20

Taxa	Eoguttulina sp.	E.mosquensis	O.strumosum
$\bar{x}$	0.022025	0.23756	0.023927
$s^2(x)$	0.001277	0.011020	0.0006347
b	2.014	0.714	1.977
R	0.55	0.34	0.42
N	20	20	20

TABLE 6.14 Means, variances, R and b parameters for species A2, A3 and A10 for N=10

Taxa	Eoguttulina sp.	E.mosquensis	O.strumosum
$\bar{x}$	0.022025	0.23756	0.023927
$s^2(x)$	0.001037	0.008816	0.000487
b	2.014	0.714	1.977
R	0.51	0.29	0.38
N	10	10	10

Figure 6.10a,b,c Experimental frequency distributions for 41,20 and 10 values of Eoguttulina sp. plotted on two normal probability scales. The values of b and R were obtained from the means and variances. Theoretical distributions are the probnormal distributions which satisfy Eq.60


Y = 0.75Z - 2.518
A2:EOGUTTULINA Sp.
b=2.014
N=41
R=0.60
UPPER POINTS OF CLASSES X:ø(Y)
Y
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
6.10a
20
50
70
90
98%
CUMULATIVE FREQUENCY PERCENTAGE

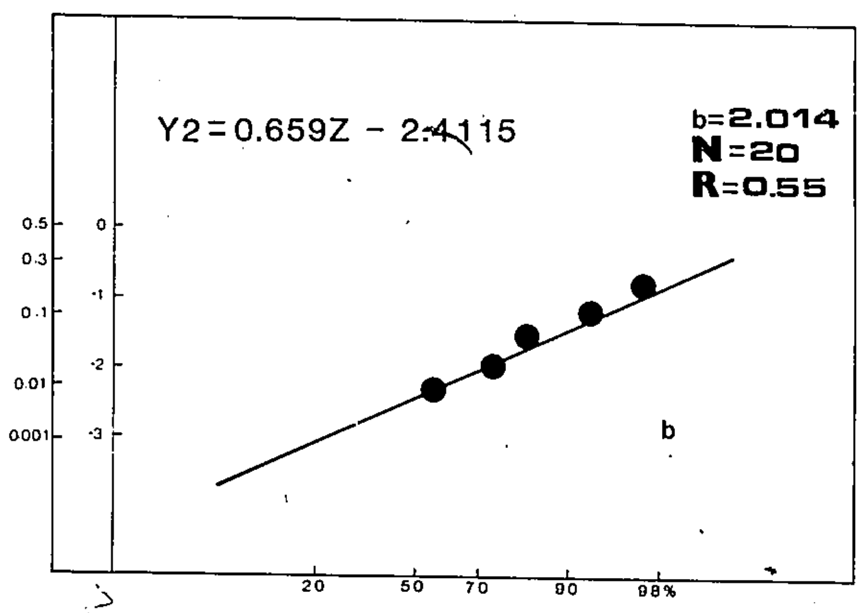
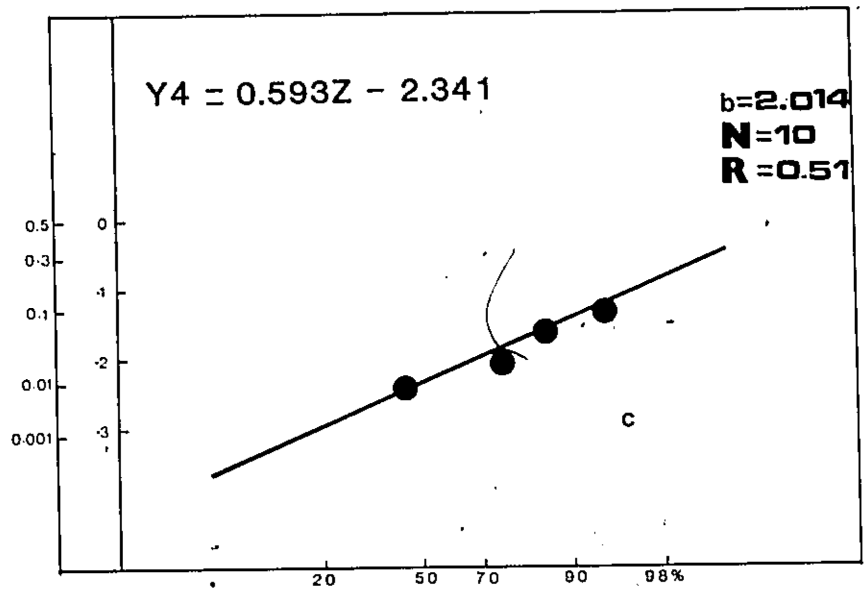
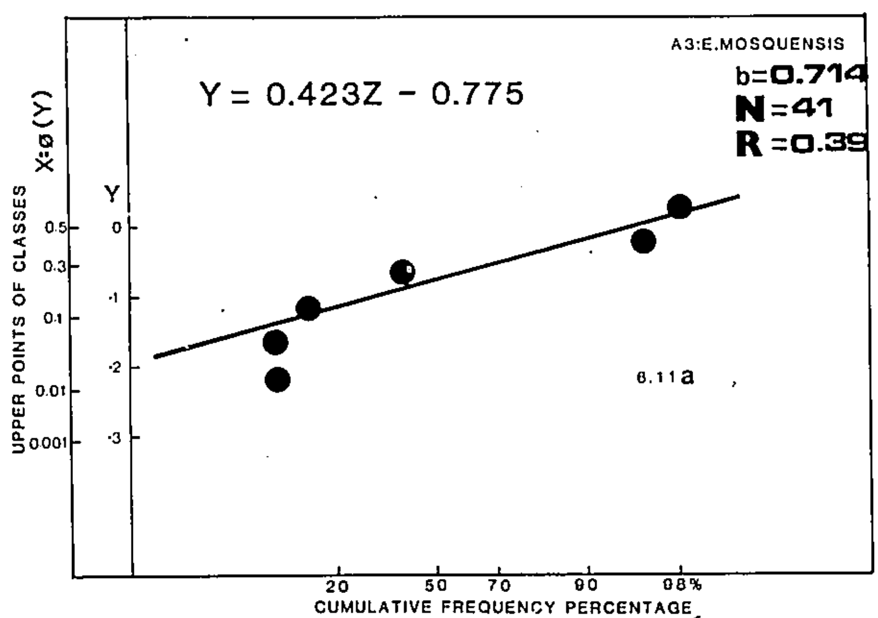
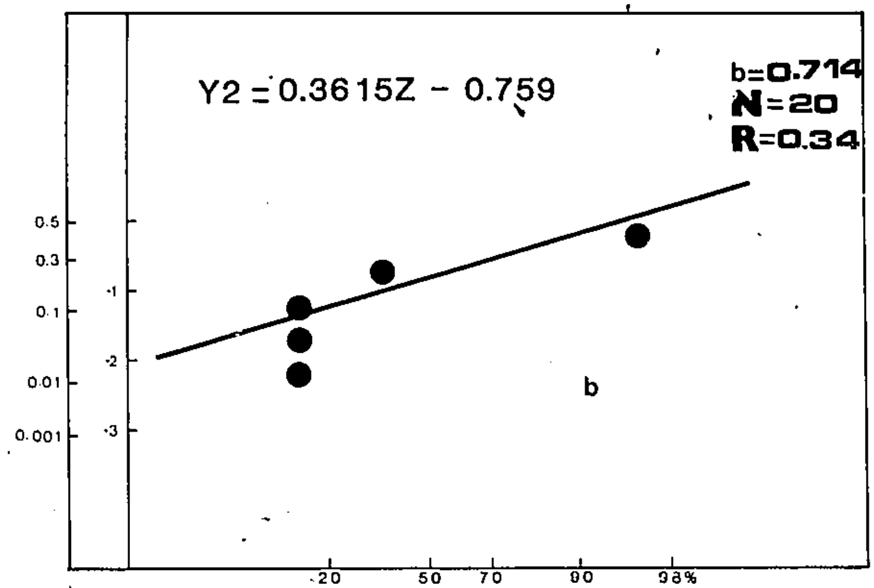

Y2=0.659Z - 2.4115
b=2.014
N=20
R=0.55
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
b
20
50
70
90
98%

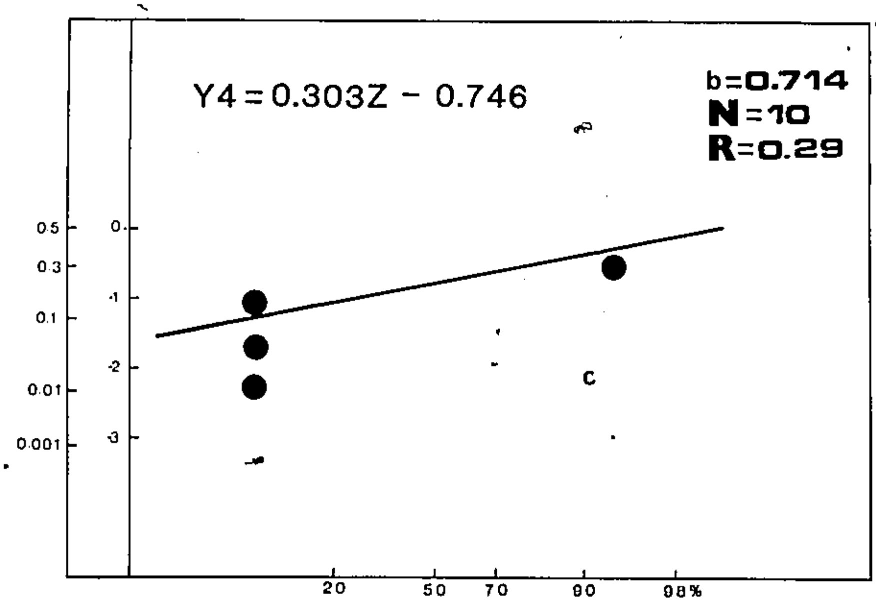
Figure 6.11a,b,c Experimental and theoretical frequency distributions for 41(single values),20 (pairs) and 10(sets of four adjacent samples) for E. mosquensis


Y4 = 0.593Z - 2.341
b=2.014
N=10
R=0.51
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
c
20
50
70
90
98%


A3:E.MOSQUENSIS
Y = 0.423Z - 0.775
b=0.714
N=41
R=0.39
X:ø(Y)
UPPER POINTS OF CLASSES
Y
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
6.11a
20
50
70
90
98%
CUMULATIVE FREQUENCY PERCENTAGE

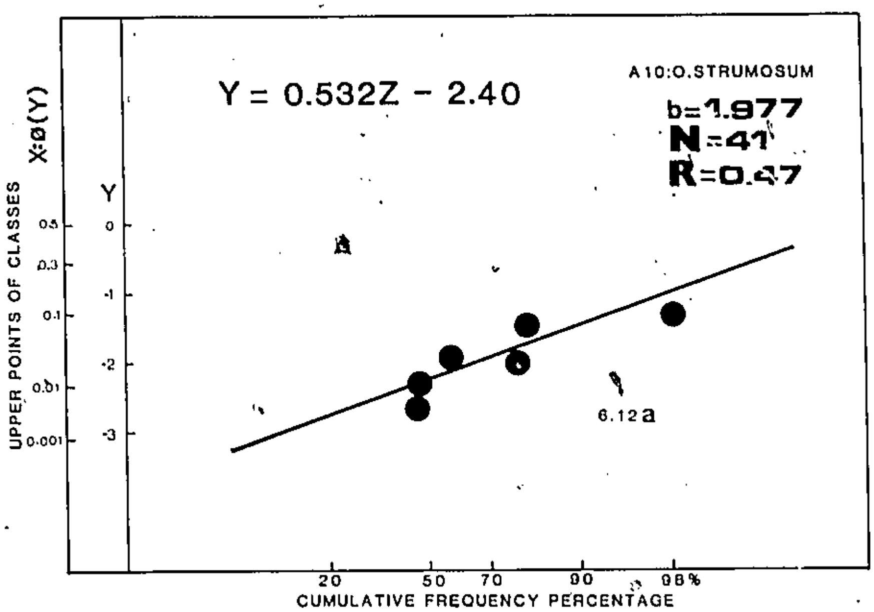


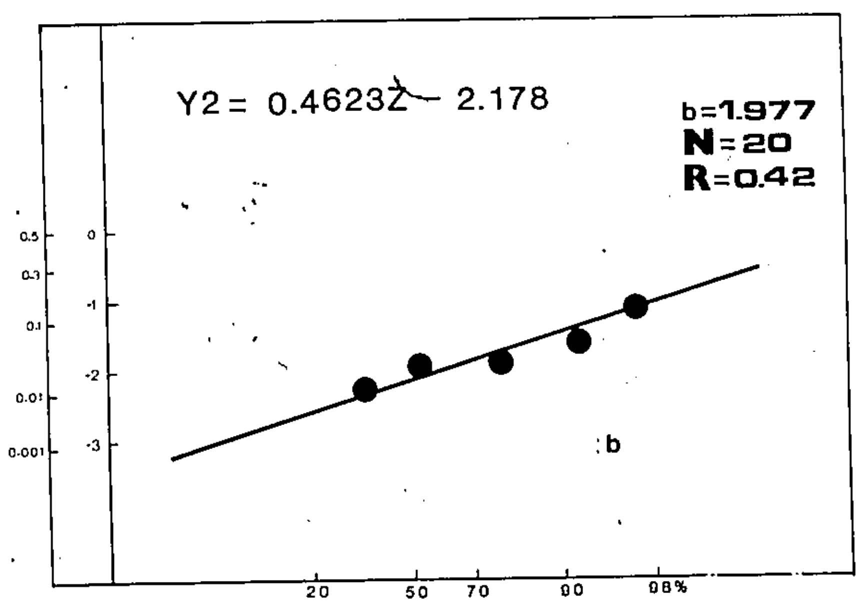
Y2 = 0.3615Z - 0.759
b=0.714
N=20
R=0.34
0.5
0.3
0.1
0.01
0.001
-1
-2
-3
b
20
50
70
90
98%

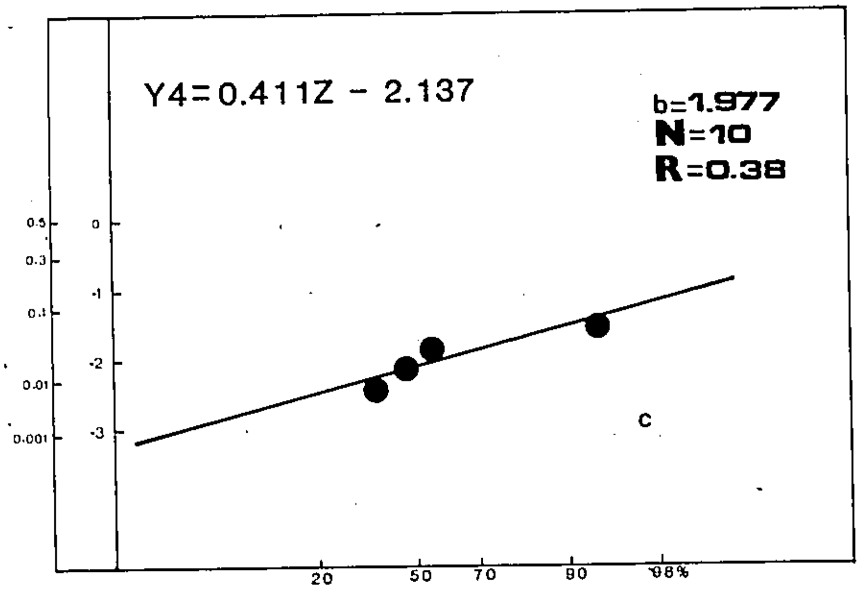


Y4 = 0.303Z - 0.746
b=0.714
N=10
R=0.29
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
c
20
50
70
90
98%

Figure 6.12a,b,c Experimental and theoretical distributions for 41, 20 and 10 values of O. strumosum They were plotted on probability paper as explained in the text


Y = 0.532Z - 2.40
A10:O.STRUMOSUM
b=1.977
N=41
R=0.47
X:ø(Y)
UPPER POINTS OF CLASSES
Y
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
6.12 a
20
50
70
90
98%
CUMULATIVE FREQUENCY PERCENTAGE


Y2 = 0.4623Z - 2.178
b=1.977
N=20
R=0.42
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
:b
20
50
70
90
98%



Y4=0.411Z - 2.137
b=1.977
N=10
R=0.38
0.5
0.3
0.1
0.01
0.001
0
-1
-2
-3
C
20
50
70
90
98%

Observed frequency distributions of Eoguttulina sp. and Epistomina mosquensis have a good fit with the theoretical prob - normal distributions. The observed frequency of Ophthalmidium strumosum a good fit with its theoretical distributions, but not as good as those of Epistomina mosquensis and Eoguttulina sp.

6.3 Tojeira 2 section

Gradstein's samples matched Stam's samples at many levels. Just 2 of Gradstein's samples were collected from different levels than Stam's. This results in a sample number of 32 for the enlarged data set of Tojeira 2. This newly created data set is not much different from the old data set of Tojeira 2. For this reason the enlarged data set was not studied by author.

Five samples were collected in the same layer at the Tojeira 1 section. Each sample contains the same percent of different species. If the percent of species were plotted against sample numbers, the lines should be horizontal. Actually such a plot shows that they are not horizontal or that they are not even close to straight lines as shown in Fig. 6.13. The reason for this may be that each group of fossils presents counting problems or some species were grouped together as one species. In addition, during the sampling process,some species could be lost.

6.4 Binomial distribution and standard deviation of noise

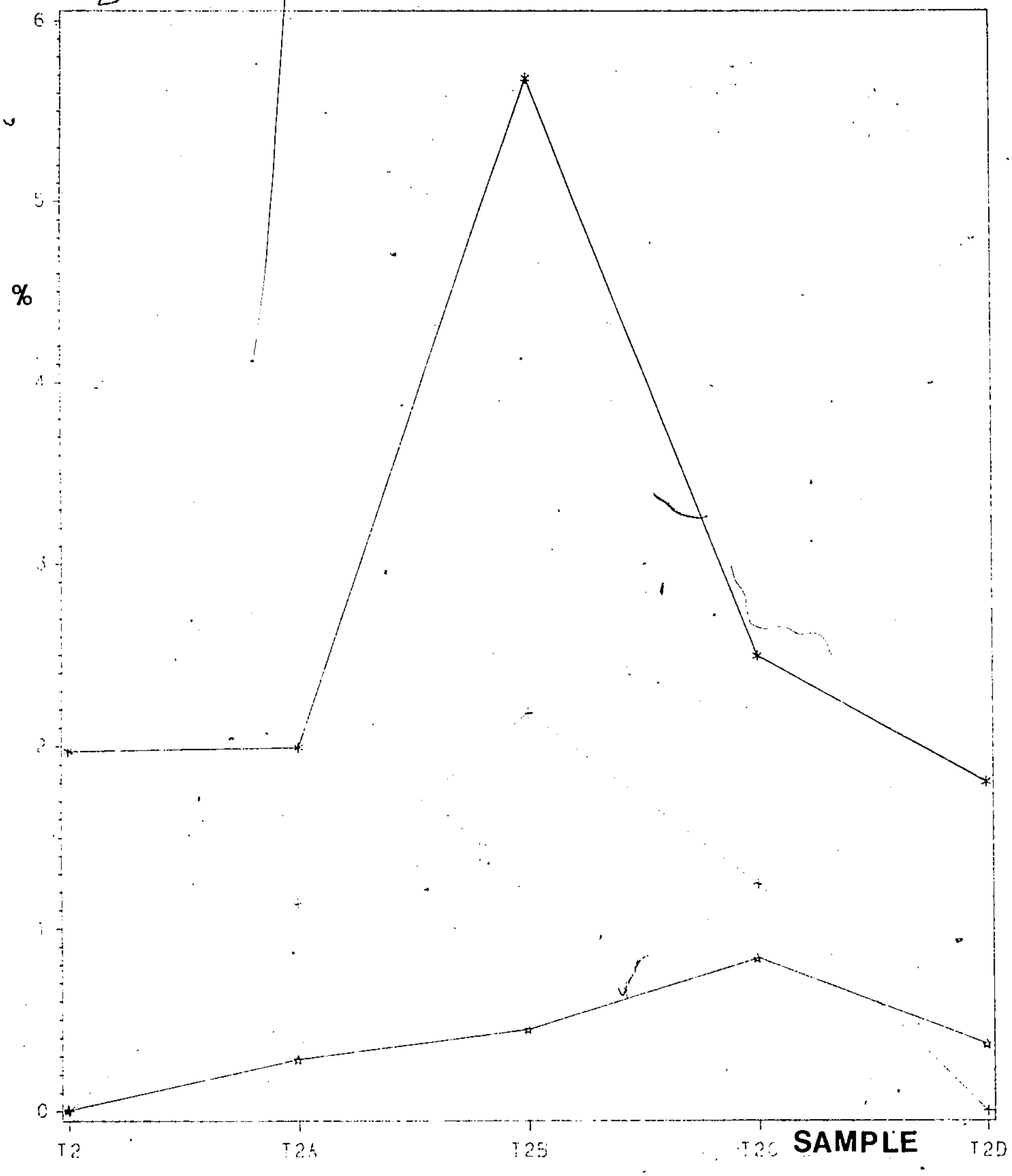


Figure 6.13 Five samples from Tojeira 1 section; which were collected in the same layer, were plotted with respect to their percent values

If frequency distributions of species were binomial distributions, the binomial mean and standard deviation are,

$\mu = n\,p$

and

$\sigma = (n\,p\,q)^{1/2}$

where, μ is a binomial mean, σ is a binomial standard deviation, n is a grade average of samples and p is a percent of a microfossil content of a given species. From p it follows that $q = 1 - p$.

$p = 0.0277$, $q = 0.9733$ and $n = 443$ for Eoguttulina sp.

The mean is

$\mu = 0.0277 * 443 = 12.27$ and

the standard deviation is

$\sigma = (443*0.0277*0.9733)^{1/2} = 3.45\%$

Binomial noise, (σ_B) $= \dfrac{100*\sigma}{\bar{n}}$

for Eoguttulina sp. is $\sigma_B = 0.78\%$

Noise of variance (σ_N^2) $= (1 - c)\,s^2$

where c is constant value which was mentioned before, s^2 is variance of species.

$\sigma_N = 4.824$

Noise of standard deviation (σ_N) $= (\sigma_N^2)^{1/2}$

$\sigma_N = 2.19\%$ for Eoguttulina sp.

The ratio of noise of standard deviation and binomial noise is

$\sigma_B / \sigma_N = 0.36$

The same procedures have been followed for all studied series and the results are shown in Table 6.15. Here, the ratio of the Eoguttulina sp. is larger than 1.00 but, this must be less than 1.00. This discrepancy suggests that the c value of this species was underestimated.

TABLE 6.15 BINOMIAL DISTRIBUTION AND STANDARD DEVIATION OF NOISE

Species	P	Q	c	S^2	$\bar{n}$	$\mu=\bar{n}P$	$\sigma=\sqrt{\bar{n}PQ}$	$\sigma_B=\frac{100\sigma}{\bar{n}}$	$\sigma^2_N=(1-c)S^2$	$\sigma_N=\sqrt{\sigma^2_N}$	N/B	Section	Sample No:
A2: Eoguttulina spp.	0,0277	0,9723	0,76	0,0020	443	12,27	3,45	0.78	4,8	2,2	0,36	Tojeira 1	30
A3: E. mosquensis	0,2247	0,7753	0,76	0,0160		99,54	8,78	1,98	38,4	6,2	0,32		
A10: O. strumosum	0,01929	0,8071	0,50	0,0006		8,54	2,62	0,59	3,0	1,7	0,34		
B1: Eoguttulina spp.	0,0285	0,9715	0,58	0,0002	250	7,12	2,63	1,05	0,9	0,9	1,12	Tojeira 2	31
B2: E. mosquensis	0,1384	0,8616	0,88	0,0134		34,65	5,46	2,19	14,8	3,8	0,57		
B11: S. tenuissima	0;2575	0,7425	0,90	0,0301		64,38	6,91	2,76	30,1	5,5	0,50		
B13: O. strumosum	0,1125	0,8875	0,91	0,0087		28,12	4,99	2,00	7,9	2,8	0,71		
B14: Agglutinants	0,1042	0,8958	0,58	0,0024		26,04	4,83	1,93	10,0	3,2	0,61		
A2: Eoguttilina spp.	0,0220	0,9780	0,48	0,0016	408	8,99	2,96	0,71	8,3	2,9	0,25	Tojeira 1	41
A3: E. mosquensis	0,2376	0,7624	0,52	0,0147		96,92	8,60	2,11	70,6	8,4	0,25		
A10: O. strumosum	0,0239	0,9761	0,60	0,0008		9,76	3,09	0,76	3,4	1,8	0,41		

CHAPTER 7: Correlation between taxa of Tojeira 1 and 2 sections end Stam's data

7.1 Introduction

Before correlation, Stam's (1986) data set (Table 1.1 and 1.2) was transformed with the logit transformation which is close to the probit transformation.

Instead of using Table XI, for logists in Fisher and Yates(1963), a small FORTRAN program was prepared and used as shown below.

The logit transformation, $Z = (1/2) LOG_e (P/Q)$., is equivalent to the R, Z transformation with $R = 2P - 1$. For values of $P < 0.5$ logits are negative and numerically equal to the tabulated values for 1 - P.

```
      DIMENSION P(40,40),Q(40.40),R(40,40),A(40,40),Z(40,40)
      READ (5,1000) N,M
      DO 200 I = 1,N
      READ ( 5,1001) ( P ( I,J),J = 1,M)
 200  CONTINUE
1000  FORMAT ( 2I2)
1001  FORMAT ( 14F4.1)
      DO 100 I=1,N
      DO 101 J=1,M
      Q(I,J)= 100.0-P(I,J)
```

TABLE 7.1 CORRELATION COEFFICIENTS OF 14 SPECIES IN TOJEIRA 1 SECTION (STAM'S DATA)

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14
A1														
A2	-0.586 0.001													
A3	0.394 0.028	-0.679 0.000												
A4	0.182 0.328	-0.450 0.011	0.528 0.002											
A5	0.110 0.555	-0.359 0.047	0.433 0.015	-0.010 0.957										
A6	-0.223 0.228	0.280 0.127	-0.438 0.014	0.029 0.876	-0.380 0.035									
A7	0.147 0.429	0.004 0.982	-0.175 0.348	-0.182 0.326	-0.269 0.143	0.461 0.009								
A8	-0.165 0.376	0.225 0.224	-0.395 0.028	-0.033 0.861	-0.482 0.001	0.192 0.300	0.065 0.727							
A9	-0.170 0.359	0.197 0.289	-0.193 0.299	-0.067 0.718	-0.239 0.196	-0.181 0.329	-0.060 0.751	0.181 0.331						
A10	0.393 0.029	-0.494 0.005	0.462 0.009	0.024 0.898	0.341 0.061	-0.099 0.595	0.067 0.720	0.055 0.767	-0.280 0.127					
A11	0.436 0.014	-0.659 0.000	0.591 0.001	0.303 0.097	0.221 0.232	-0.246 0.182	0.019 0.921	-0.222 0.231	-0.258 0.161	0.402 0.025				
A12	0.081 0.666	-0.149 0.423	0.407 0.023	0.024 0.899	0.158 0.396	-0.323 0.076	-0.103 0.582	-0.118 0.529	0.379 0.036	0.212 0.252	0.689 0.714			
A13	-0.660 0.000	0.731 0.000	-0.625 0.000	-0.423 0.018	-0.206 0.267	0.216 0.243	-0.101 0.590	0.101 0.587	0.027 0.886	-0.484 0.006	-0.688 0.000	-0.199 0.283		
A14	0.013 0.946	0.300 0.101	-0.451 0.011	-0.429 0.016	-0.148 0.427	-0.227 0.219	0.029 0.879	0.043 0.819	0.166 0.372	-0.368 0.042	-0.318 0.082	-0.074 0.692	0.188 0.312	

```
      R(I,J)= P(I,J)/Q(I,J)
      A(I,J)= LOG(R(I,J))
      Z(I,J)= A(I,J)/2.0
  101 CONTINUE
  100 CONTINUE
      DO 90 I =1,N
      WRITE (6,2000) (Z(I,J), J =1,M )
   90 CONTINUE
 2000 FORMAT ( 14 ( 1x,F7.4))
      STOP
      END
```

NOTE : Table 1.1 and 1.2 was thought of as a matrix (N = 31, M = 14)

Logit transformed data were used as input to the CORR SAS package program to calculate correlation coefficients.

7.2 Correlation of taxa in Tojeira 1 section

Significant positive and negative correlation coefficents between the various taxa are given in Table. 7.1.

Pseudolmarckina rjasanensis, Epistomina mosquensis and Epistomina uhligi are known to characterize deeper shelf deposits (Pazdro, 1969; Gradstein, 1983; Lloyd, 1970). All species of this group are positively correlated to one the other.

Spirillina tenuissima fits in well and actually confirms the idea of this species preferring shallow water (Stam, 1986). Spirillina infima is positively correlated with Spirillina tenuissima but Spirillina elongata is negatively correlated to it. This was not really expected. However, this correlation coefficient is very close to 0 and it is not significantly different from zero.

Lenticulina muensteri muensteri is positively correlated with the Lenticulina sp. and negatively correlated with Epistomina sp.. Epistomina sp. has a moderate degree of positive correlation with Epistomina mosquensis and a negative correlation with Lenticulina muensteri and Nodosaria/Dentalina sp. which indicates that they are closer to the deep water assocation rather than the shallow water one.

There is no correlation between Lenticulina muensteri and Epistomina uhligi, or between Epistomina uhligi and Spirillina infima.

The group formed by Epistomina sp. and Spirillina elongata is, considering their negative correlations with the shallow water group, closer to the deeper group.

The correlation coefficents of the species are usually low, but, as seen in Table 7.1, Spirillina infima has a high positive correlation with Eoguttulina sp.. Epistomina mosquensis with agglutinants and Epistomina mosquensis with Eoguttulina have a high negative correlation.

7.3 Correlation of taxa in Tojeira 2 section

Psedolamearckina rjasanensis, Epistomina mosquensis, Ophtalmidium strumosum and agglutinants have a high positive correlation (Table 7.2)

TABLE 7.2 CORRELATION COEFFICIENTS OF 14 SPECIES IN TOJEIRA 2 SECTION (STAM'S DATA)

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14
B1														
B2	-0.483 0.007													
B3	0.085 0.653	0.316 0.089												
B4	0.366 0.047	-0.424 0.020	0.444 0.014											
B5	-0.161 0.396	-0.010 0.960	-0.167 0.377	0.120 0.527										
B6	0.040 0.834	-0.236 0.210	-0.149 0.432	0.127 0.503	0.045 0.813									
B7	-0.066 0.728	-0.015 0.936	0.265 0.157	0.110 0.562	-0.178 0.348	0.117 0.539								
B8	-0.445 0.014	0.788 0.000	0.268 0.153	-0.367 0.046	0.030 0.875	-0.152 0.423	0.055 0.771							
B9	-0.034 0.859	0.096 0.613	-0.325 0.080	-0.241 0.200	-0.023 0.902	-0.064 0.736	-0.141 0.457	0.001 0.996						
B10	0.455 0.011	-0.662 0.000	-0.010 0.960	0.325 0.080	-0.188 0.319	0.019 0.921	-0.145 0.445	-0.673 0.000	0.061 0.750					
B11	0.502 0.005	-0.909 0.000	-0.369 0.045	0.343 0.064	-0.057 0.765	0.188 0.320	-0.131 0.491	-0.827 0.000	-0.021 0.911	0.749 0.000				
B12	0.454 0.012	-0.257 0.170	-0.213 0.259	0.036 0.851	0.070 0.714	-0.263 0.160	-0.309 0.097	-0.172 0.363	-0.325 0.080	0.046 0.807	0.264 0.158			
B13	-0.385 0.036	0.870 0.000	0.163 0.390	-0.515 0.004	0.138 0.468	-0.236 0.210	0.031 0.869	0.774 0.000	0.041 0.829	-0.726 0.000	-0.899 0.000	-0.018 0.923		
B14	-0.460 0.011	0.699 0.000	0.138 0.467	-0.533 0.002	0.153 0.418	-0.106 0.578	0.048 0.801	0.616 0.000	-0.100 0.601	-0.632 0.000	-0.754 0.000	-0.120 0.528	0.745 0.000	

Consistent with this are the negative correlations between Epistomina mosquensis, Eoguttulina sp. and Spirillina infima.

The group formed by Epistomina mosquensis, Pseudolamerckina rjasanesis, Ophthalmidium strumosum and agglutinants has a preference for relatively deeper water. A second group formed by Eoguttulina sp., Spirillina infima and Spirillina tenuissima has a preference for shallow water. Epistomina mosquensis is negatively correlated with Lenticulina and Epistomina sp.

Epistomina uhligi, Lenticulina muensteri, Nodosaria/Dentalina sp., Spirillina elongata have a preference for deeper water. Lenticulina, Epistomina sp. and Ophthalmidium carinatum have a preference for shallow water as suggested by the correlations in Table 7.2.

The result of correlating taxa within the Tojeira 1 and Tojeira 2 sections shows that the bentnonic foraminifers microfauna is the same for both sections, but species of Tojeira 2 are more strongly correlated than the species of Tojeira 1 as seen in Tables 7.1 and 7.2.

In general, the correlations of this section, based on logits of proportion values for the species, confirm the correlations previously obtained by Stam (1986) who used Drooger's (1982) BALANC and DISTUR computer programs.

CHAPTER 8: Conclusions

In this new geostatistical approach to biostratigraphic sampling and spatial analysis, two stratigraphic sections were studied in detail. They are the so - called Tojeira 1 and Tojeira 2 sections located in central Portugal. The geology and macropaleontology of area have been studied since 1880. The lithology and micropaleontology have been clearly established by Stam (1986).

For statistical analysis, the count is considered to be the numerical proportion of a taxon from an assemblage of foraminifera. It is characterized as a series. These series have a logical order (from bottom to top or from top to bottom of sections) and are related to time. Time series analysis shows how a time series variable (X_t) can be related to its own past values or that there may be an interdependence in the sense that some taxon property (its proportion) in a certain sample is strongly related to the one(s) in the sample(s) immediately below, even if the proportions of species are zero in some of the samples.

The duration of time from Middle to Late Jurassic age is approximately 11 m.y. Microfossil sample data for this time interval were statistically analyzed as time series. The samples plotted were proportion values of species. At Tojeira 1 section, the proportions of taxon Epistomina mosquensis and Ophthalmisium strumosum seem to show an increasing trend, and Eoguttulin sp. a decreasing trend towards the late Jurassic age (Figs. 8.1 - 8.3) with Stam's data and Figs.8.4

Figure 8.1 Time series of 31 Eoguttulina sp. percent values from Tojeira 1 section. Data are autocorrelated according to Table 1.1

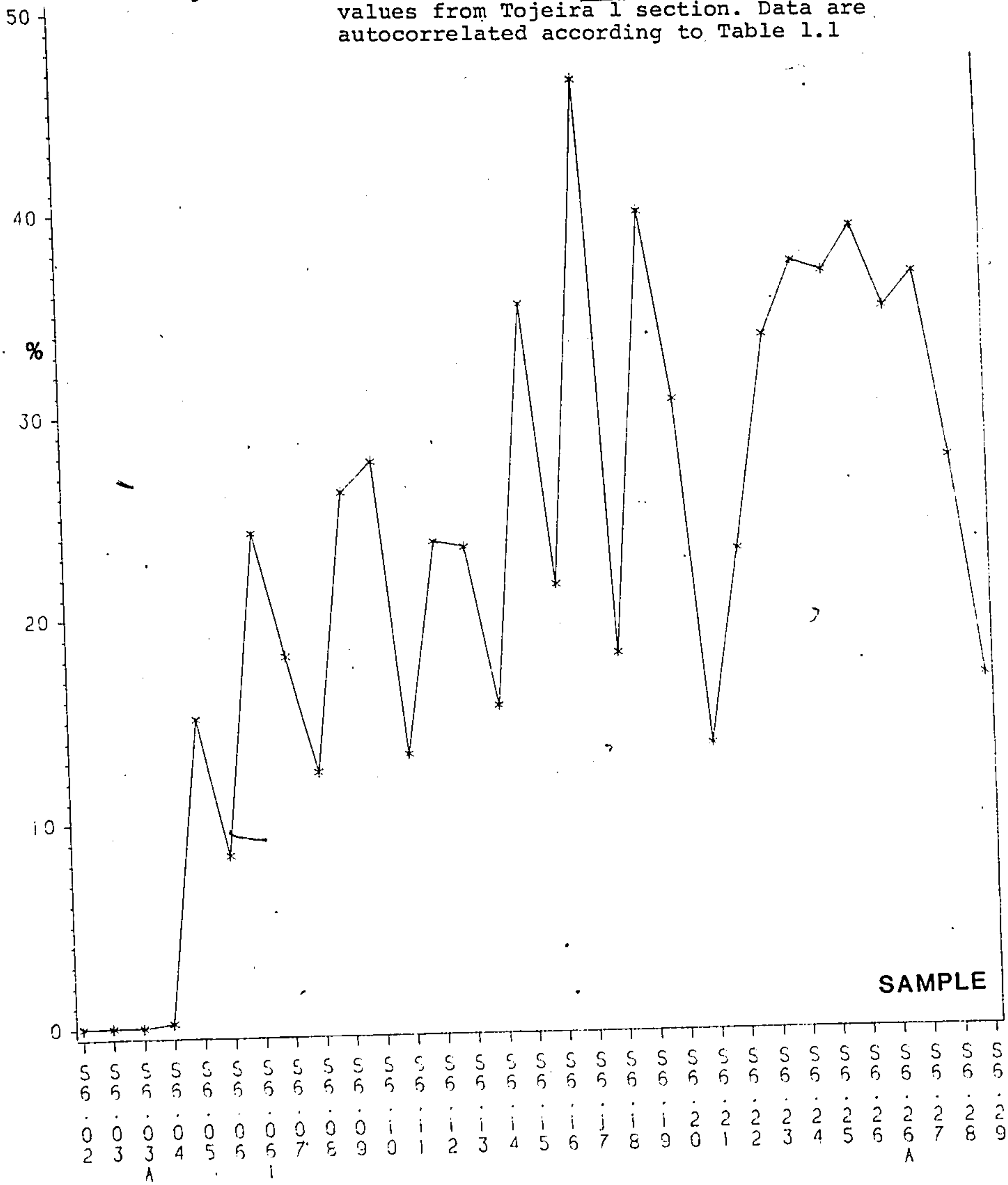


Figure 8.2 Time series of 31 E. mosquensis percent values from Tojeira 1 section. Data are autocorrelated according to Table 1.1

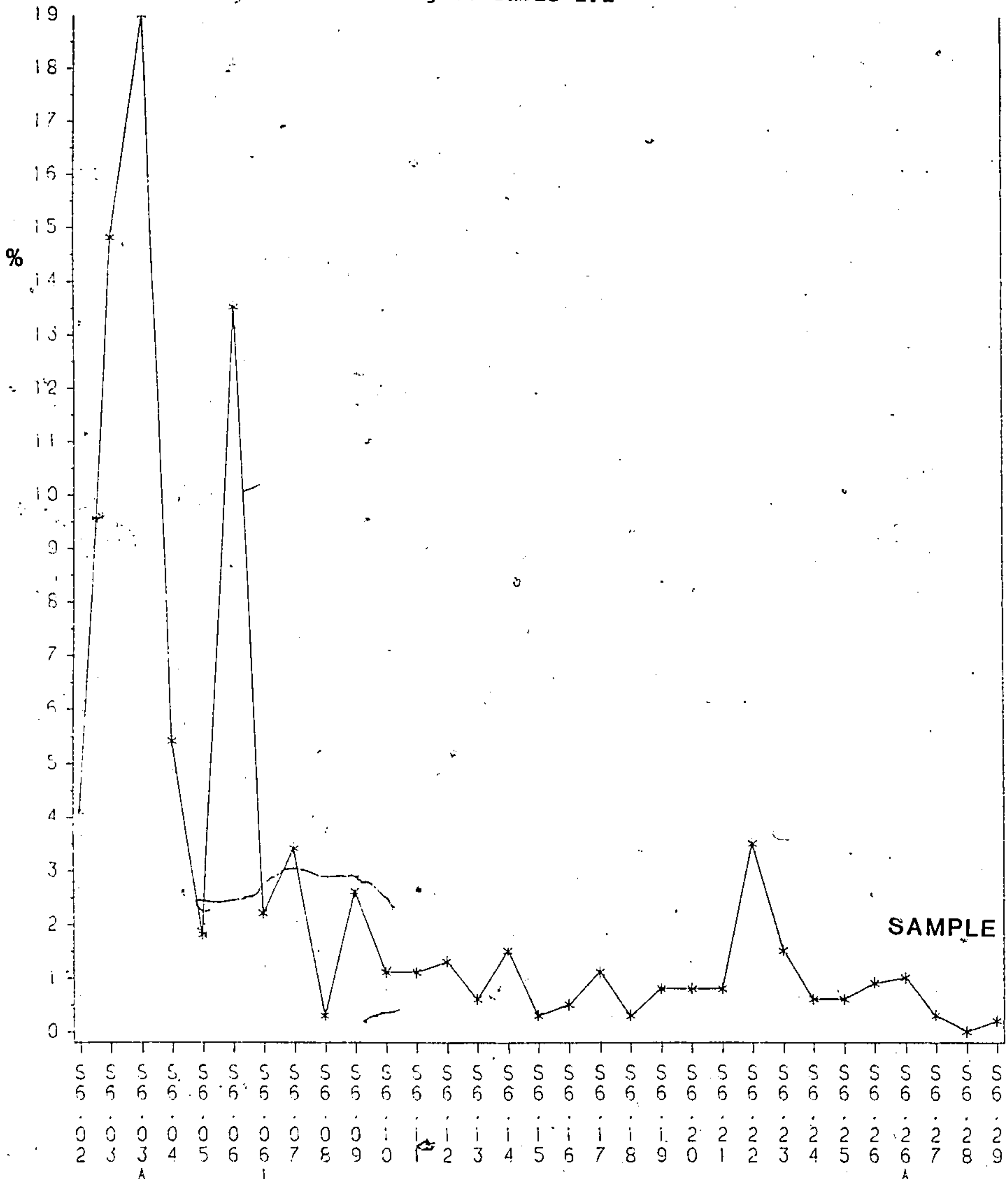
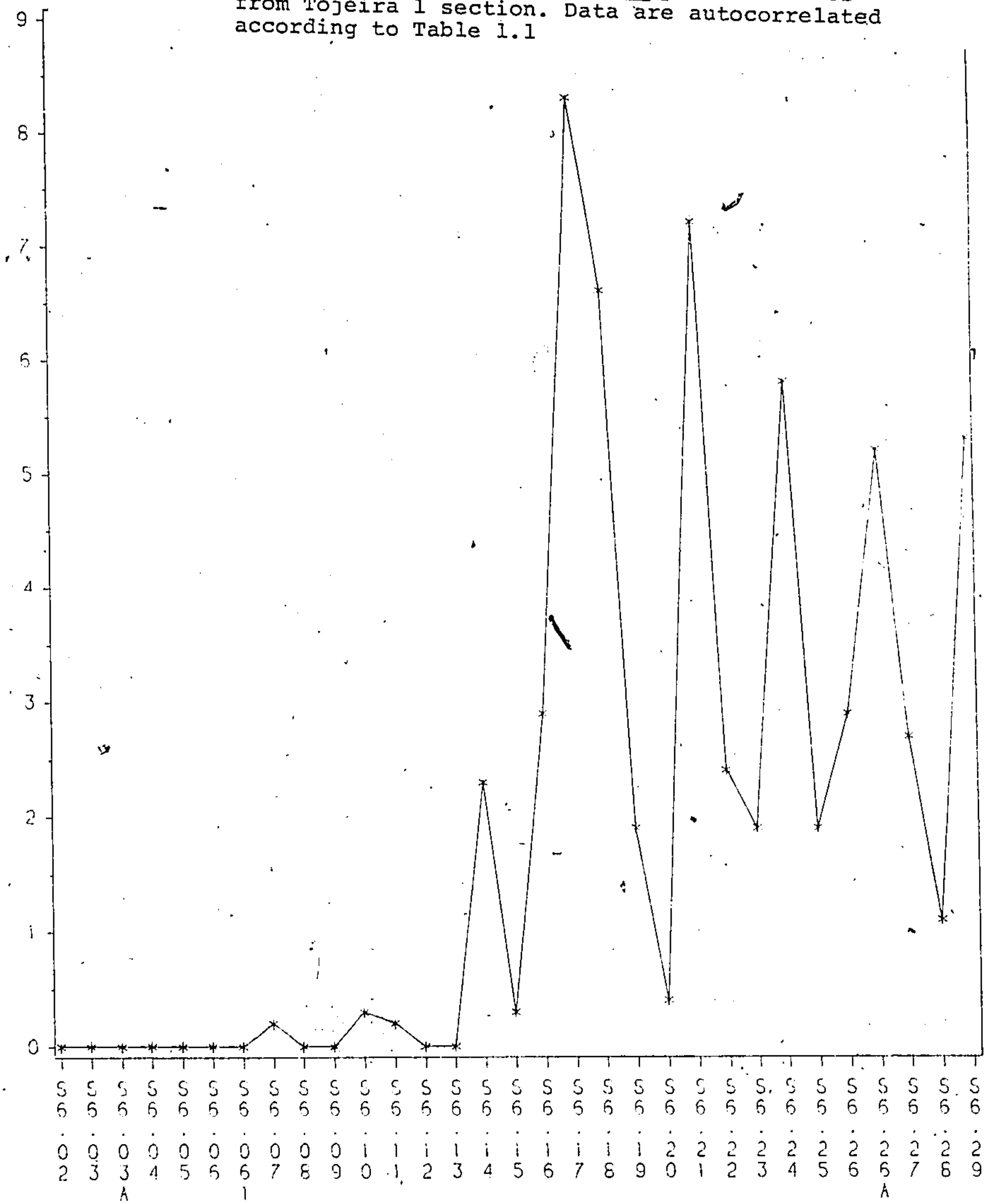


Figure 8.3 Time series of 31 O. strumosum percent values from Tojeira 1 section. Data are autocorrelated according to Table 1.1



- 8.6 with enlarged data). At Tojeira 2 section, the proportions of taxa Epistomina mosquensis, Ophthalmidium strumosum and agglutinants may show an increasing trend and Eoguttulina sp. and Spirillina tenuissima show a decreasing trend from Middle Jurassic to late Jurassic age (Figs. 8.4 - 8.8)

The ARIMA process, formed from a combination of autoregressive and moving average processes, was used to calculate autocorrelation coefficients for 14 species in both the Tojeira 1 and Tojeira 2 sections (Appendix I).

Before starting the new geostatistical model, the data was tested in terms of a time series model. The three time series models that were considered were the AR (p, q) , p = 1 and q = 0, autoregressive model, the MA (p, q), p = 0 and q = 1, moving average model and the ARMA (p, q) p = 1, 2 and q = 1, 2 autoregressive and moving average model. It was determined that only the AR (p, q) and ARMA (p, q) models explained the data suitably. This was because these two models are signal - plus - noise models.

The preparation and counting procedures play a primary role in the magnitude of the white noise (error). The white noise was tested according to t and chi - squared tests, as was the periodogram which was calculated by the SAS SPECTRA procedure.

If the observations during the count really are random, the statistical error may be considered to be equal to the theoretical standard deviation of the binomial distribution model. The ratio of the theoretical standard deviation from the binomial distribution to the

Figure 8.4 Time series of 30 Eoguttulina sp. percent values from Tojeira 2 section. Data are autocorrelated according to Table 1.2

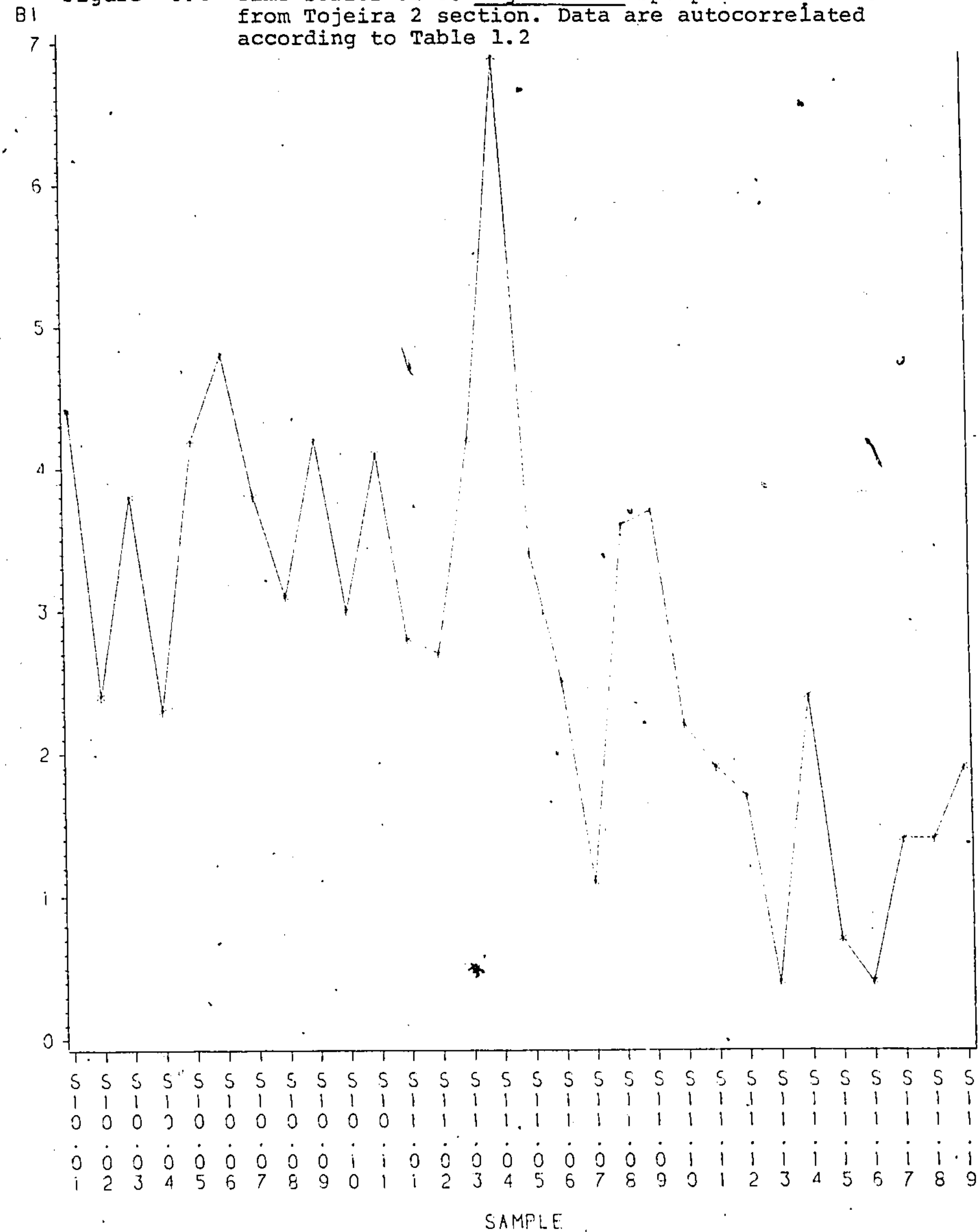


Figure 8.5 Time series of 30 E. mosquensis percent values from T ojeira 2 section. Data are autocorrelated according to Table 1.2

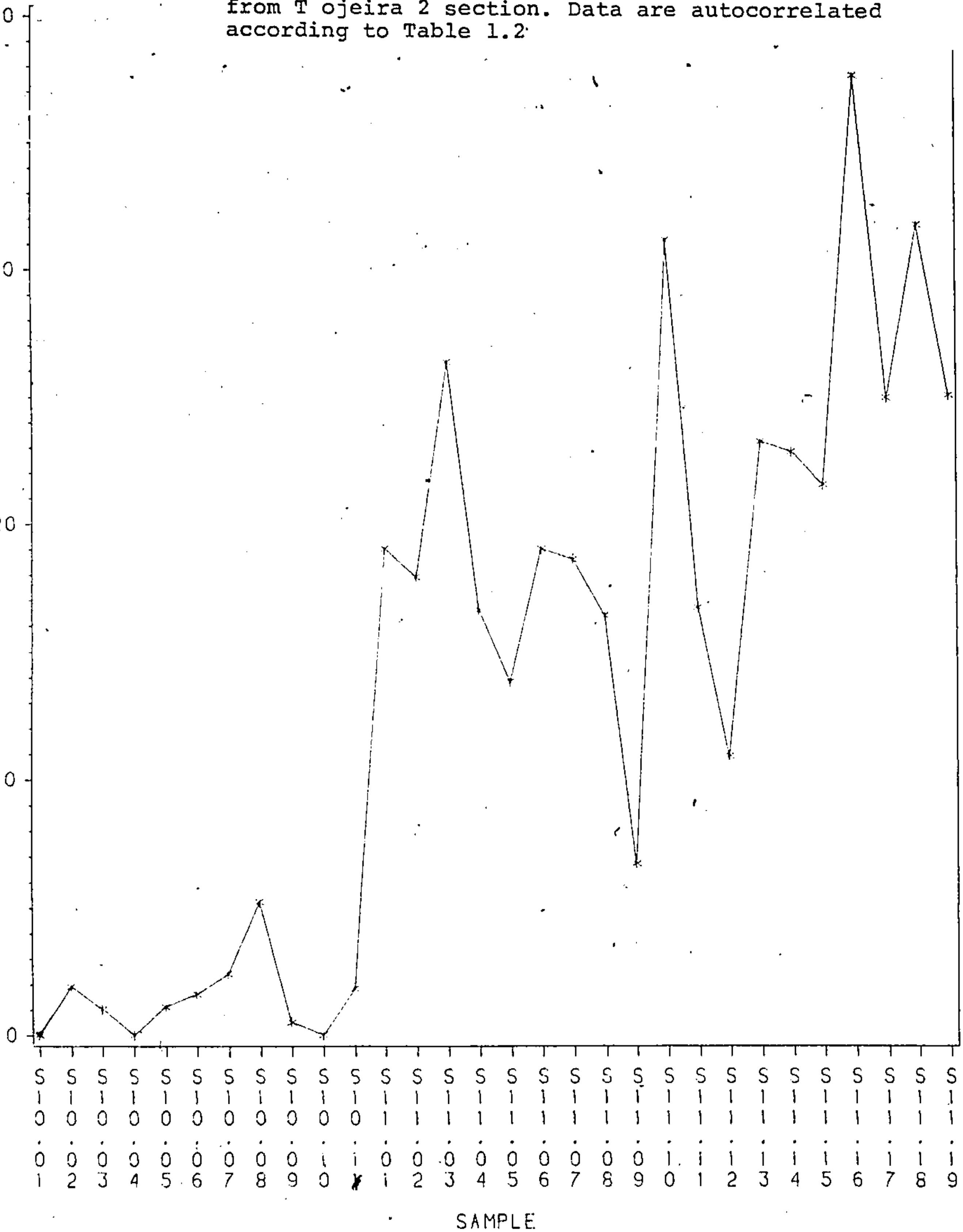
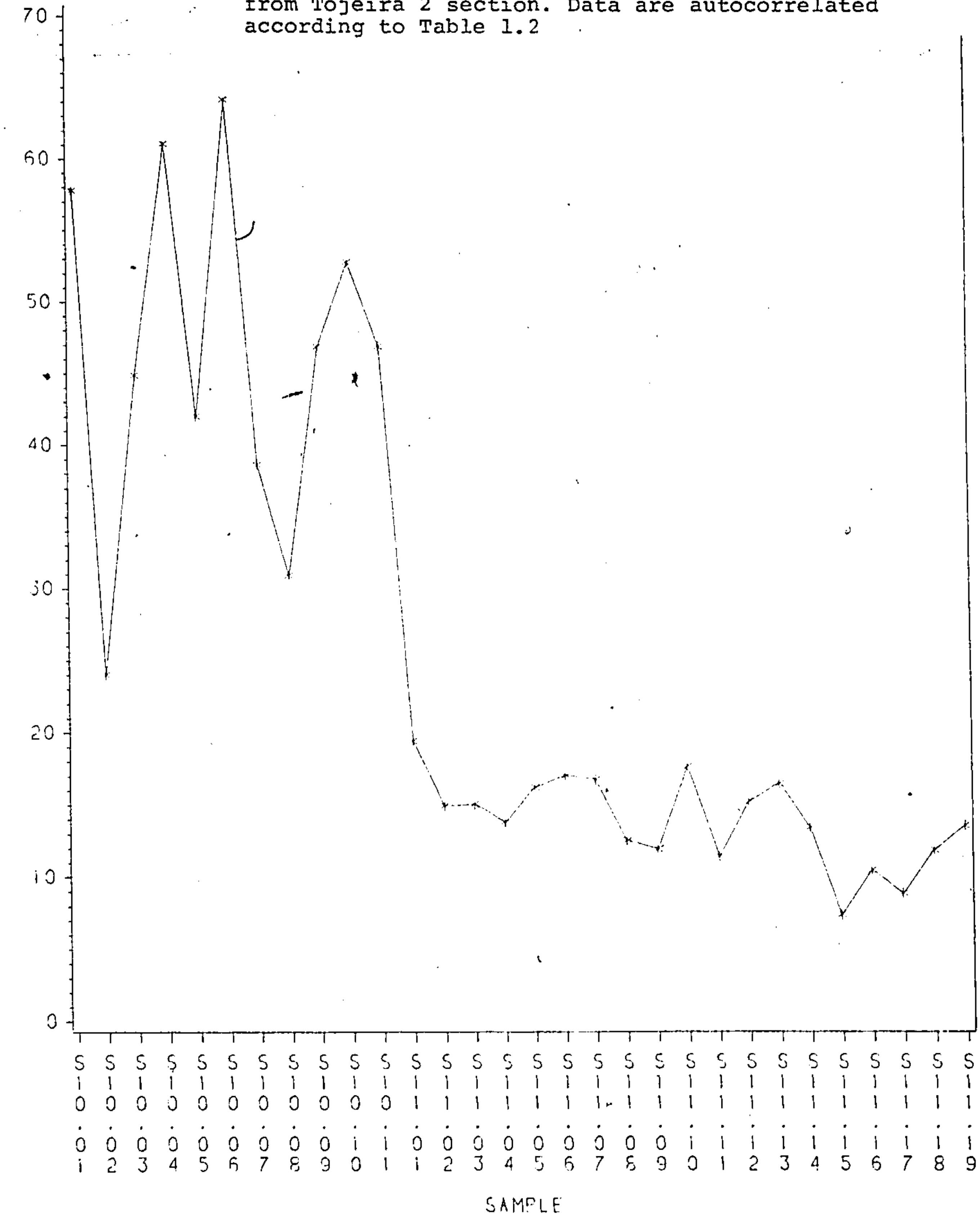


Figure 8.6 Time series of 30 S. tenuissima percent values from Tojeira 2 section. Data are autocorrelated according to Table 1.2



B13 Figure 8.7 Time seris of 30 O. strumosum percent values from Tojeira 2 section. Data are autocorrelated according to Table 1.2

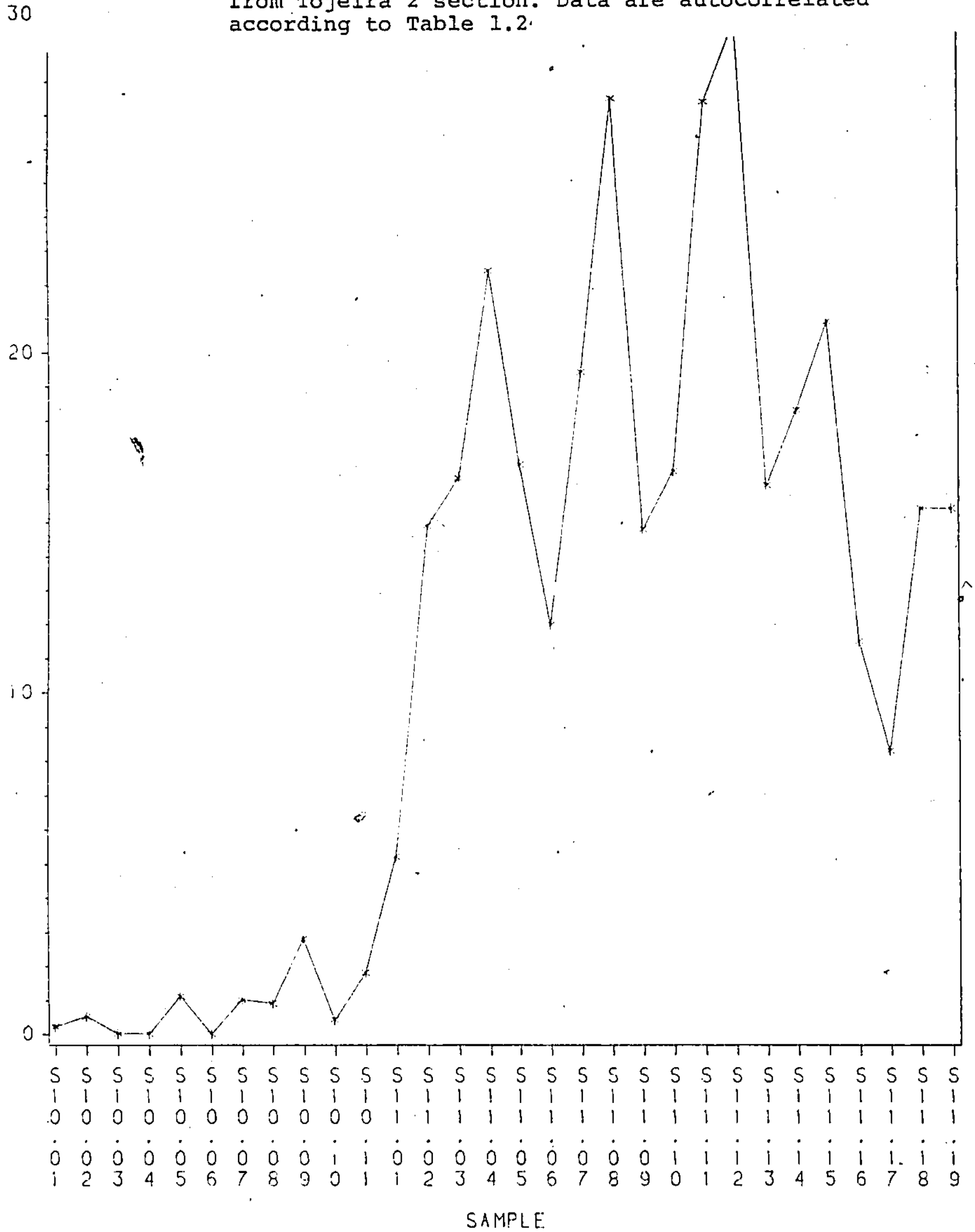
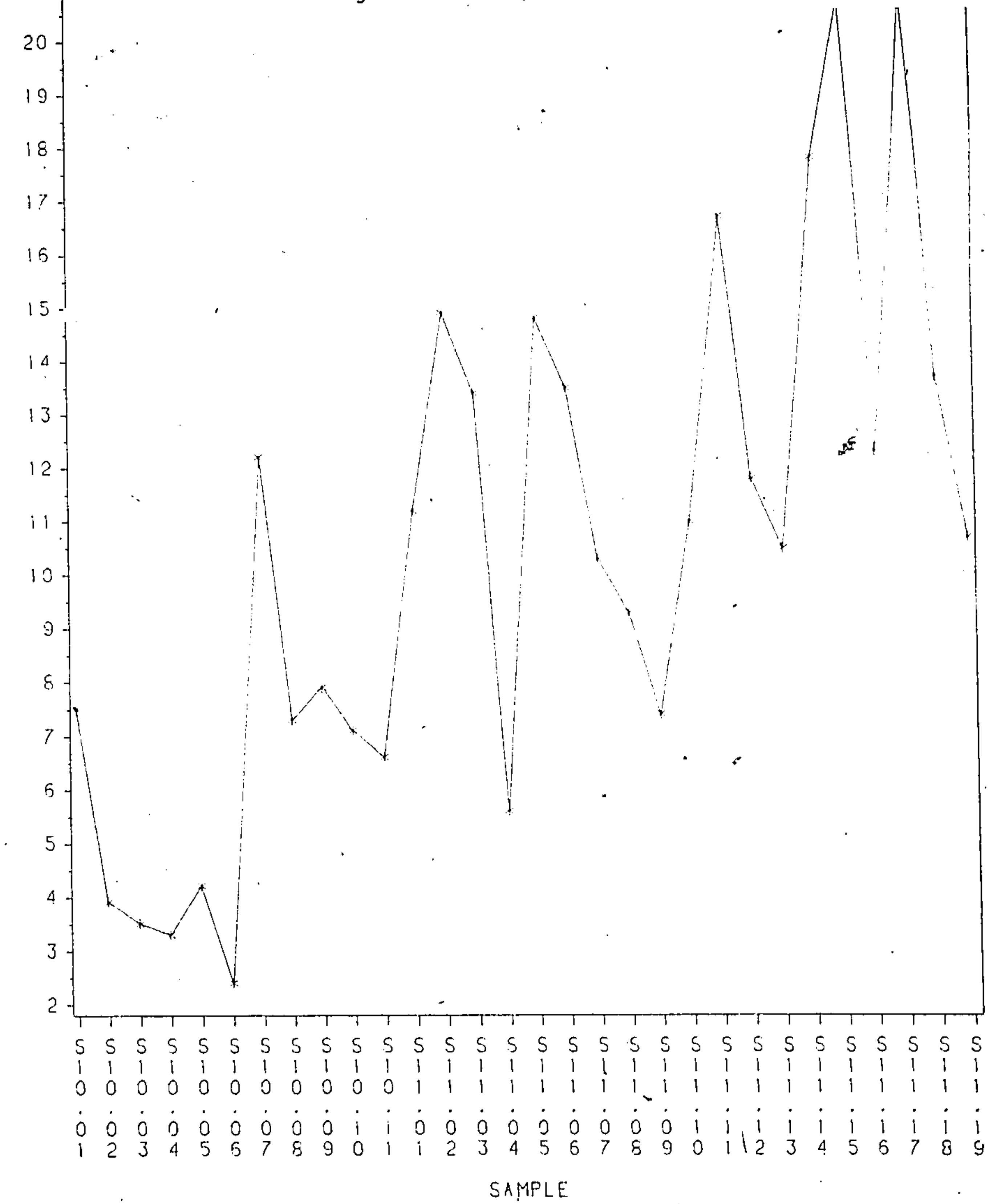


Figure 8.8 Time series of 30 Agglutinants percent values from Tojeira 2 section. Data are autocorrelated according to Table 1.2



standard deviation of the probnormal distribution (geostatistical model) was calculated and shown in Table 6.15.

A good starting point for this type of modeling is to construct the covariance of a pattern. The covariance can be converted into an autocorrelation coefficient by using standard statistical formulae or the SAS package program ARIMA process.

The autocorrelation function (r_l) is exponential for all directions. The autocorrelation coefficients (r_l) plotted along the vertical axis with a logarithmic scale. The geostatistical model or the signal - plus noise model with $r = c \exp.(- al)$ where a and c are two constants characterizing the stationary series. The a and c are estimated from a correlogram (see Figs.4.8, 4.13, 4.18, 4.21, 6.4, 6.5 and 6.6). If noise is absent, c = 1 and the line passes through the point r(0) = 1 for zero lag. If the series consists of noise only, c = 0.

Estimated a and c values were used for the calculation of the F_L function (Eq.54). The F function was also determined experimentally by averaging an increasingly greater number of adjacent values of species. This gave the factors F_N calculated by the F FORTRAN program (see Appendix I) for relative variance of the mean multiplied by the number of observations (N). For uncorrelated data, F_N is equal to one. The F_N and F_L functions were plotted against the lag as an observation and as a theoretical curve. They all show a good fit for the species studied. (see Figs. 4.9, 4.14, 4.19, 4.27, 4.28,4.29, 4.30, 4.31, 6.7, 6.8, and 6.9).

In the model used for this study, the frequency distribution of a

Figure 8.9 Time series of 41 Eoguttulina sp. percent values from Tojeira 1 section. Data are autocorrelated according to Table 6.1

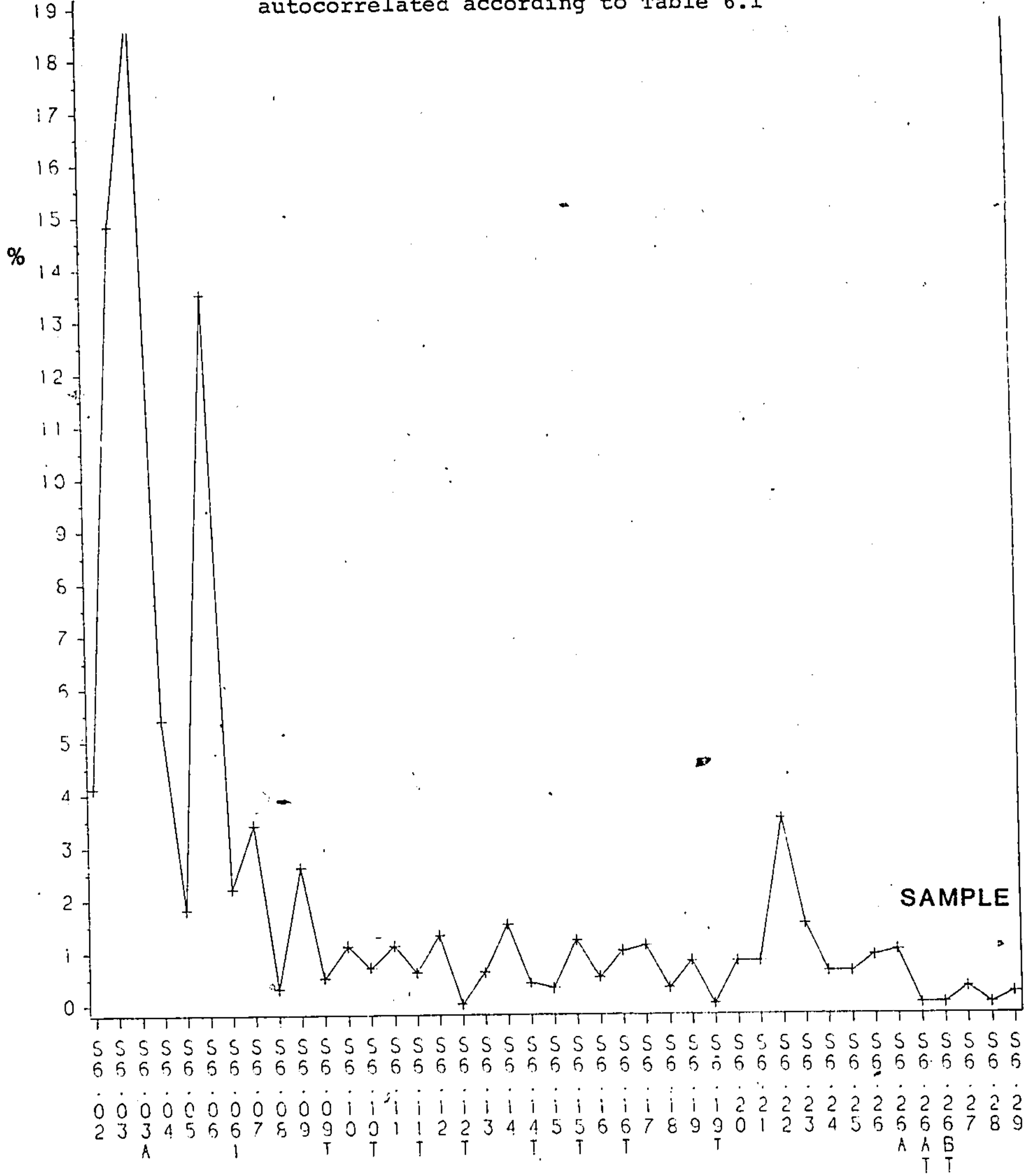


Figure 8.10 Time series of 41 E. mosquensis percent values from Tojeira 1 section. Data are autocorrelated according to Table 6.1

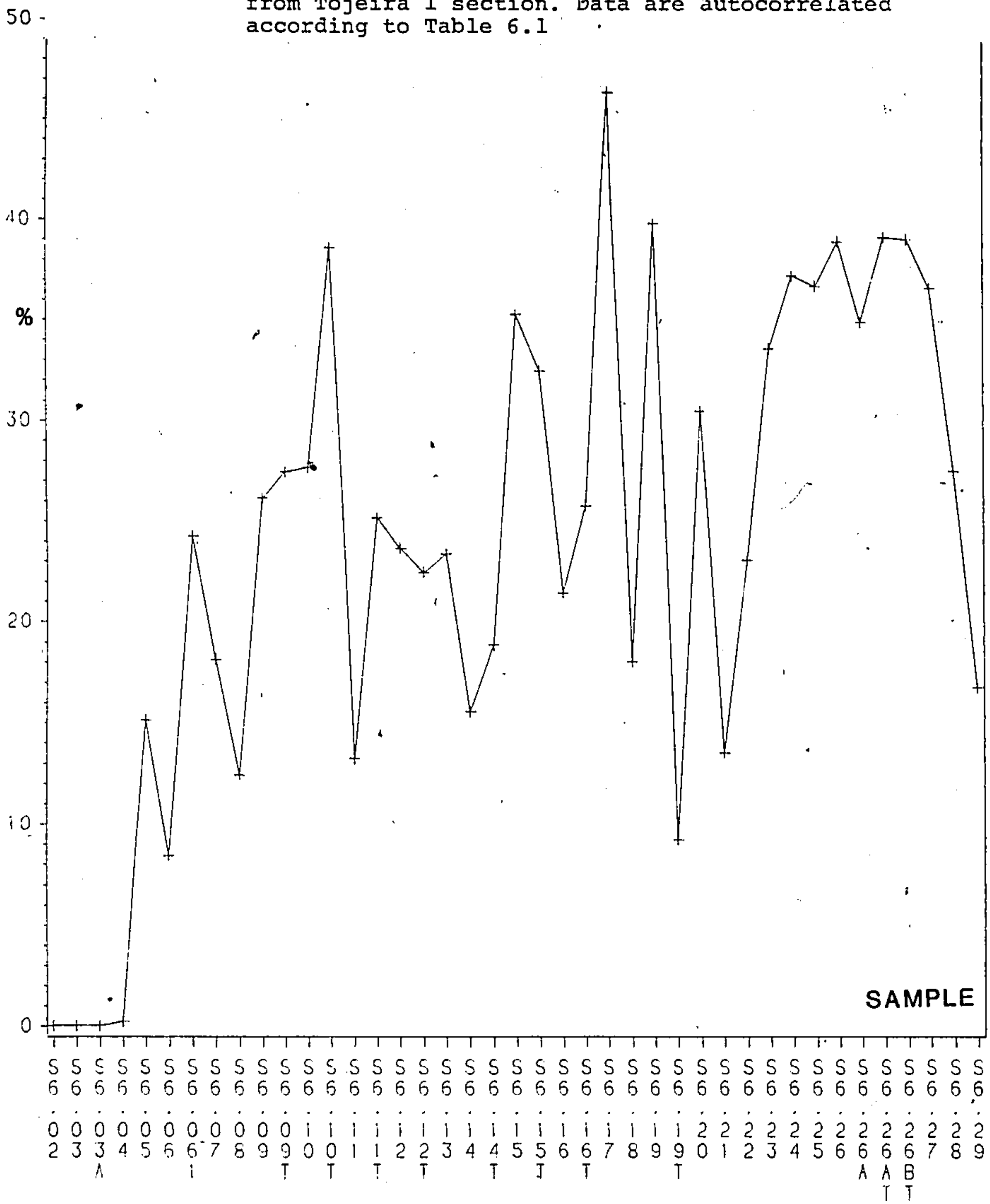
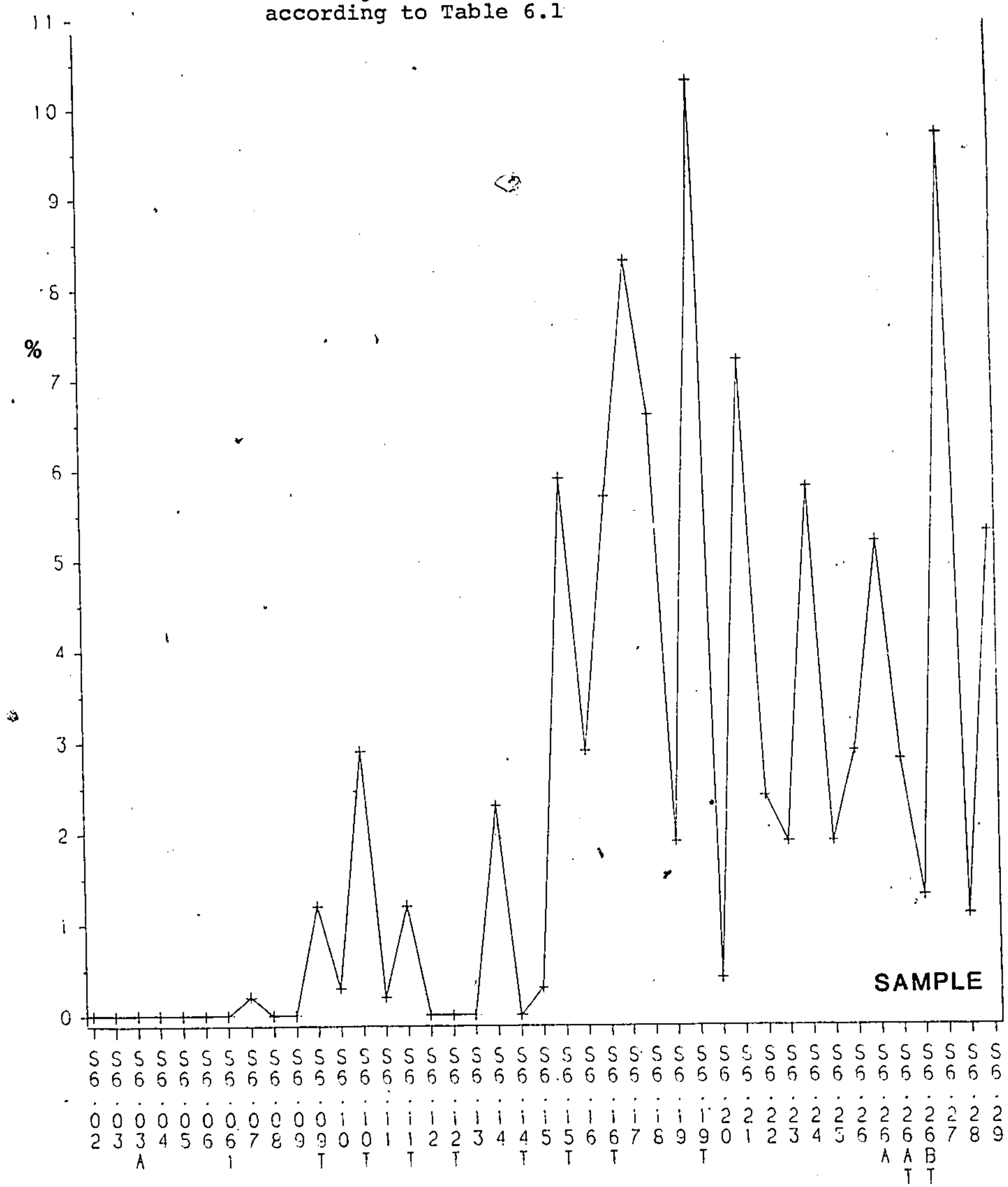


Figure 8.11 Time sereis of 41 O. strumosum percent values from Tojeira 1 section. Data are autocorrelated according to Table 6.1

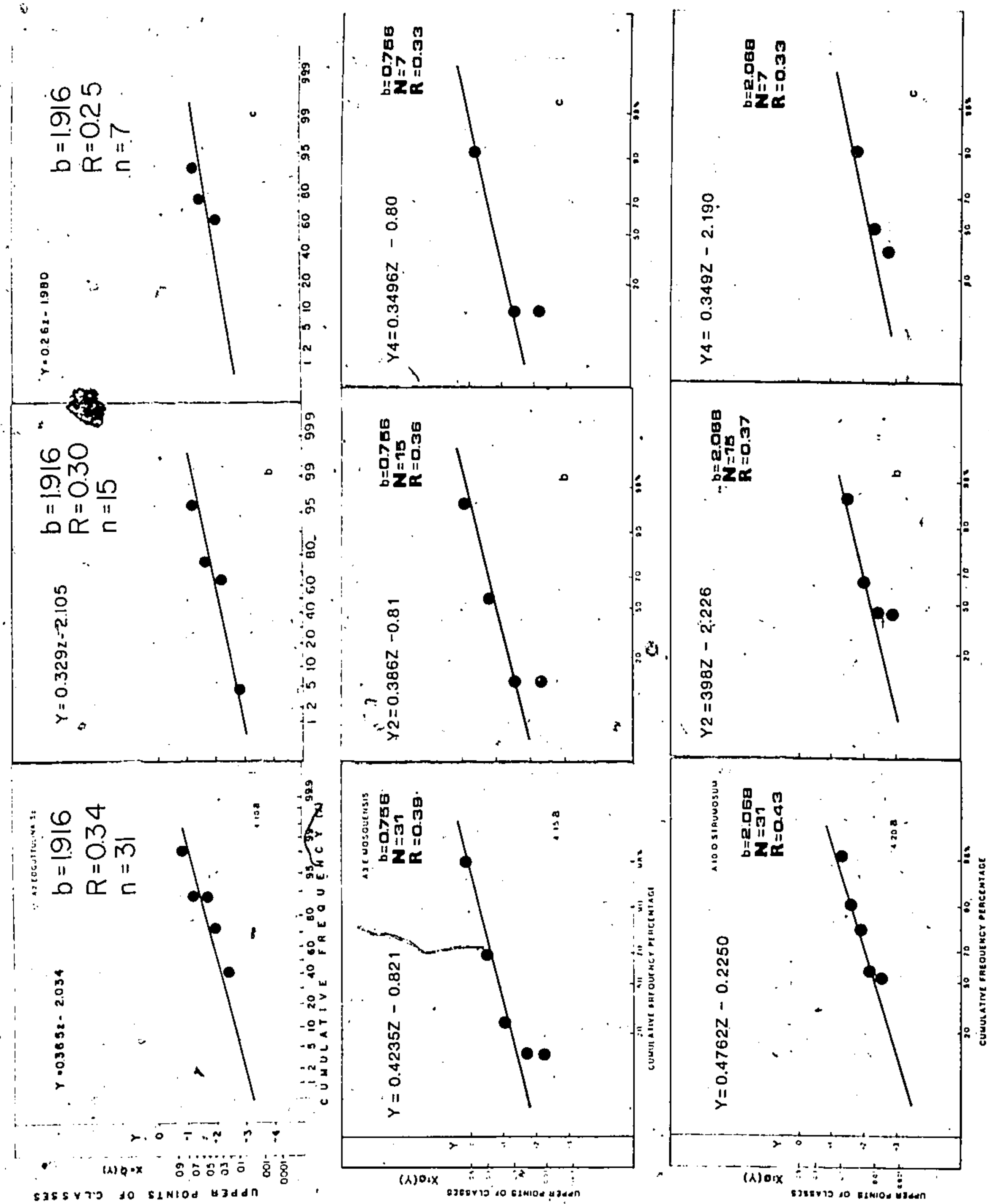


random variable (X_t) in Eq. 60 becomes a straight line if the function is plotted on paper that has a normal probability scale both for the ordinate and for the abscissa representing the cumulative frequency percentage. If the model is good, a straight line is also obtained by plotting the probit of the proportion value of the taxon (see probit transformation tables for each taxon) on normal probability paper. This procedure facilitates the comparison between the model given by Eq.60 (the theoretical distribution) and any observed frequency distribution for a sample obtained by superimposing a grid on the pattern and computing the average proportion values of groups of adjoining original samples.

In our statistical model study three different data sets were applied successively to both sections.

First, Stam's data (Tables 1.1 and 1.2) were used for application of the geostatistical model. Nine examples of the graphical test of the model are shown in Fig. 8.12. These nine plots are for the previously discussed foraminifer species of Upper Jurassic age in the Tojeira 1 section. The first set of 3 plots is for Eoguttulina sp.. The mean $\bar{x}$ = 0.0277 has a corresponding variance of S^2 = 0.00201 for 31 samples, S^2_2 = 0.00154 for 15 samples which are combinations of pairs of adjacent single samples and S^2_4 = 0.00052 for 7 samples which are combinations of groups of four adjacent single samples. These data yield the values of b and R shown in Fig. 8.12. The second set of 3 plots is for Epistomina mosquensis with the mean $\bar{x}$ = 0.2247 and variances S^2 = 0.01654, S^2_2= 0.01232 and S^2_4=0.00947 for 31,

Figure 8.12 Experimental frequency distributions and their theoretical distributions for 31,15 and 7 values of *Eoguttulina* sp., *E. mosquensis* and *O. strumosum*. The values of b's and R's obtained from their mean and variance for Tojeira 1 section with Stam's data


Y = 0.365z - 2.034
b = 1.916
R = 0.34
n = 31
Y = 0.329z - 2.105
b = 1.916
R = 0.30
n = 15
Y = 0.26z - 1.980
b = 1.916
R = 0.25
n = 7
Y = 0.4235Z - 0.821
b=0.756
N=31
R=0.39
Y2=0.386Z -0.81
b=0.756
N=15
R=0.36
Y4=0.3496Z - 0.80
b=0.756
N=7
R=0.33
Y = 0.4762Z - 0.2250
b=2.068
N=31
R=0.43
Y2=398Z - 2.226
b=2.068
N=15
R=0.37
Y4= 0.349Z - 2.190
b=2.068
N=7
R=0.33
CUMULATIVE FREQUENCY (%)
CUMULATIVE FREQUENCY PERCENTAGE
UPPER POINTS OF CLASSES
X=Q(Y)
X(Q)(Y)
a
b
c

15 and 7 samples. respectively. The third set of 3 plots are for Ophthalmidium stromosum with mean $\bar{x} = 0.01929$, and variances $S^2 = 0.02445$, $S^2{}_2 = 0.00042$ and $S^2{}_4 = 0.00032$.

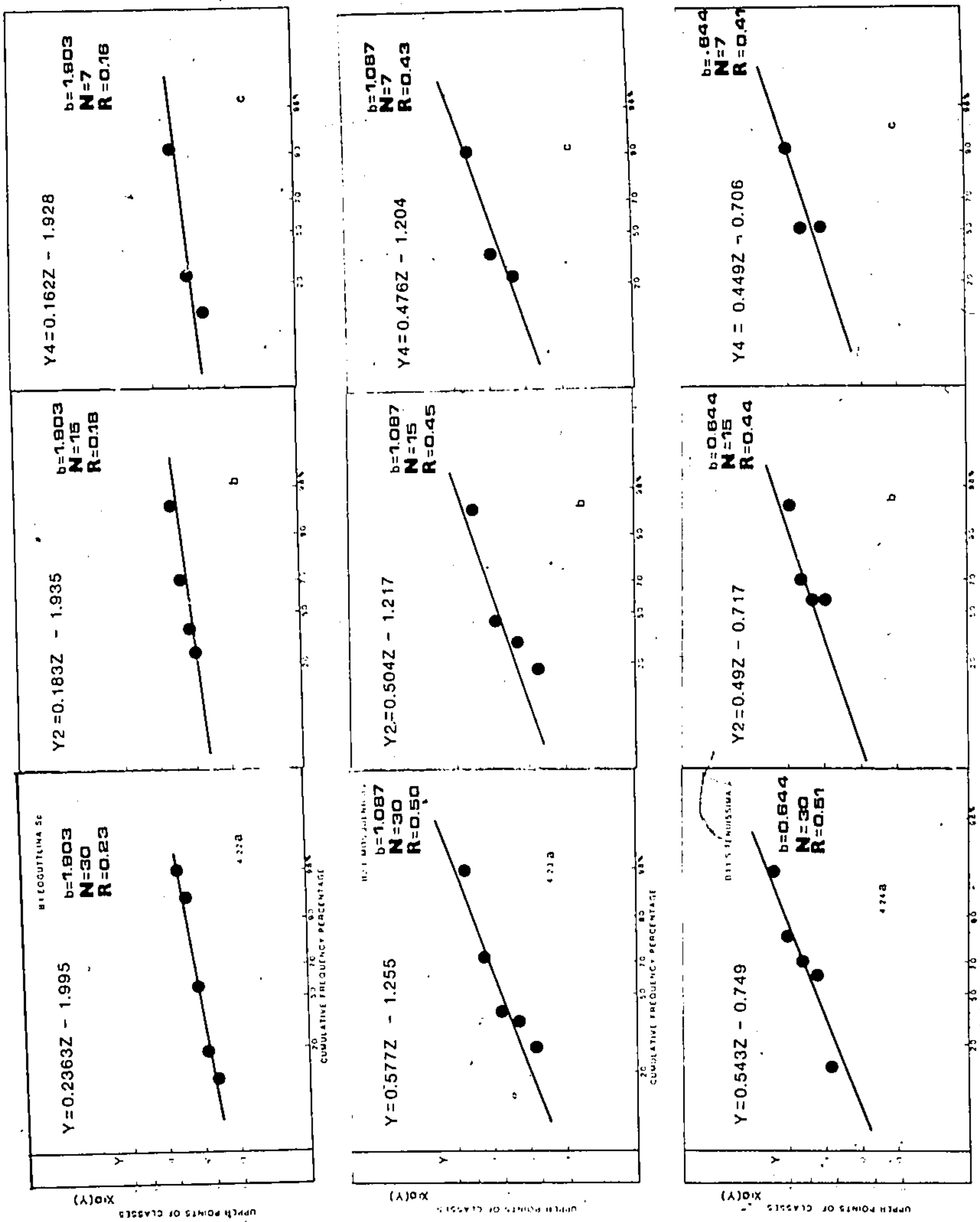
The next fifteen examples of the model are shown in Fig. 8.13. These fifteen plots of species of Middle - Upper Jurassic age are for the Tojeira 2 section.

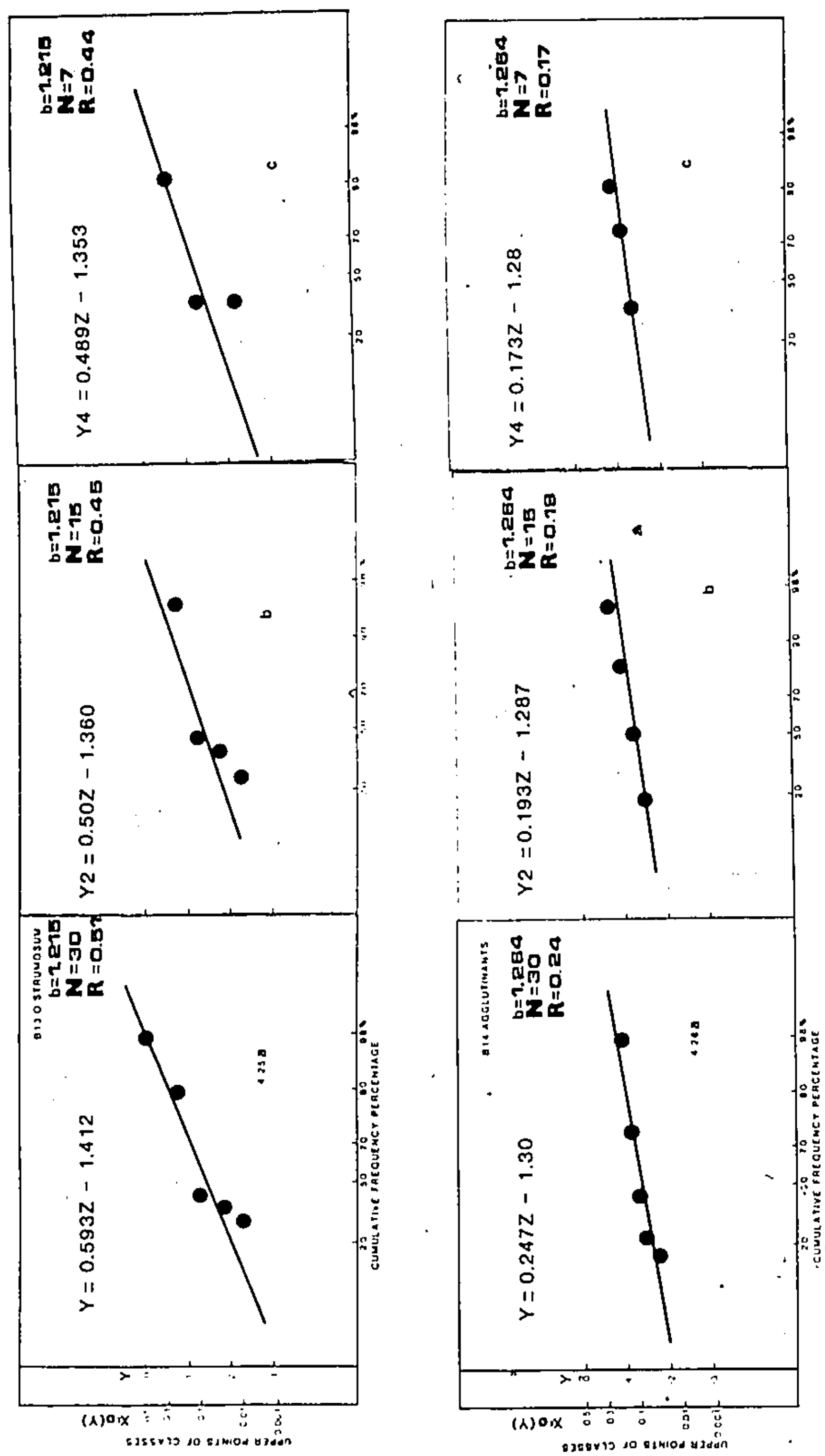
The first set of 3 plots is for B1 (Eoguttulina sp.). Eoguttulina sp. has mean a equal to 0.02847 and sample variances equal to 0.00020 0.00016 and 0.00011 for 7, 15 and 30 samples respectively. The second set of 3 plots is for B2 (Epistomina mosquensis). Epistomina mosquensis has a mean equal to 0.13840 and sample variances equal to 0.01236, 0.01038 and 0.00871. As shown in Fig 8.13, the third set of 3 plots is for B11 (Spirillina tenuissima), the fourth set of 3 plots is for B13 (Ophthalmidium strumosum) and the fifth set of 3 plots is for B14 (agglutinants). Their means, variances, b's and R's values were also estimated for 7, 15, and 30 samples, respectively.

The second set of data consisted of Gradstein's data. This data set included seventeen samples from Tojeira 1 and thirteen samples from Tojeira 2 . In general, the ARIMA process requires a minimum of 30 samples for reliable results. Since each section had less than 30 samples the results of the ARIMA process on Gradstein's data must be considered with caution.

Third, an enlarged data set, which consisted of both Stam's and Gradstein's data sets, was applied to 14 species for both sections.

Figure 8.13 Experimental frequency distributions and their theoretical distributions for 30,15 and 7 values of Eoguttulina sp., E. mosquensis, S. tenuissima, O. strumosum and Agglutinants for Tojeira 2 section with Stam's data


Y=0.2363Z - 1.995
b=1.803
N=30
R=0.23
Y2=0.183Z - 1.935
b=1.803
N=15
R=0.18
Y4=0.162Z - 1.928
b=1.803
N=7
R=0.16
Y=0.577Z - 1.255
b=1.087
N=30
R=0.50
Y2=0.504Z - 1.217
b=1.087
N=15
R=0.45
Y4=0.476Z - 1.204
b=1.087
N=7
R=0.43
Y=0.543Z - 0.749
b=0.644
N=30
R=0.51
Y2=0.49Z - 0.717
b=0.644
N=15
R=0.44
Y4 = 0.449Z - 0.706
b=.644
N=7
R=0.41
CUMULATIVE FREQUENCY PERCENTAGE
UPPER POINTS OF CLASSES
X10(Y)


Y = 0.593Z − 1.412
b=1.215
N=30
Y2 = 0.50Z − 1.360
b=1.215
N=15
R=0.45
Y4 = 0.489Z − 1.353
b=1.215
N=7
R=0.44
CUMULATIVE FREQUENCY PERCENTAGE
UPPER POINTS OF CLASSES
B14 AGGLUTINANTS
Y = 0.247Z − 1.30
b=1.264
N=30
R=0.24
Y2 = 0.193Z − 1.287
b=1.264
N=15
R=0.18
Y4 = 0.173Z − 1.28
b=1.264
N=7
R=0.17
CUMULATIVE FREQUENCY PERCENTAGE
UPPER POINTS OF CLASSES

Nine examples as previously are explained for the Tojeira 1 section using Stam's data only are now used for a graphical test of the model (Fig. 8.14) using Stam's and Gradstein's data together As before, the first set of 3 plots is for A2(Eoguttulina sp.). They now have an average value equal to 0.02202 and variances equal to 0.00163, 0.00130 and 0.00104 for 10, 20 and 41 samples, respectively. The new results can be seen in Fig. 8.14. The second set of 3 plots is for A3 (Epistomina mosquensis) and the third set of 3 plots are for A10 (Ophthalmidium strumosum)

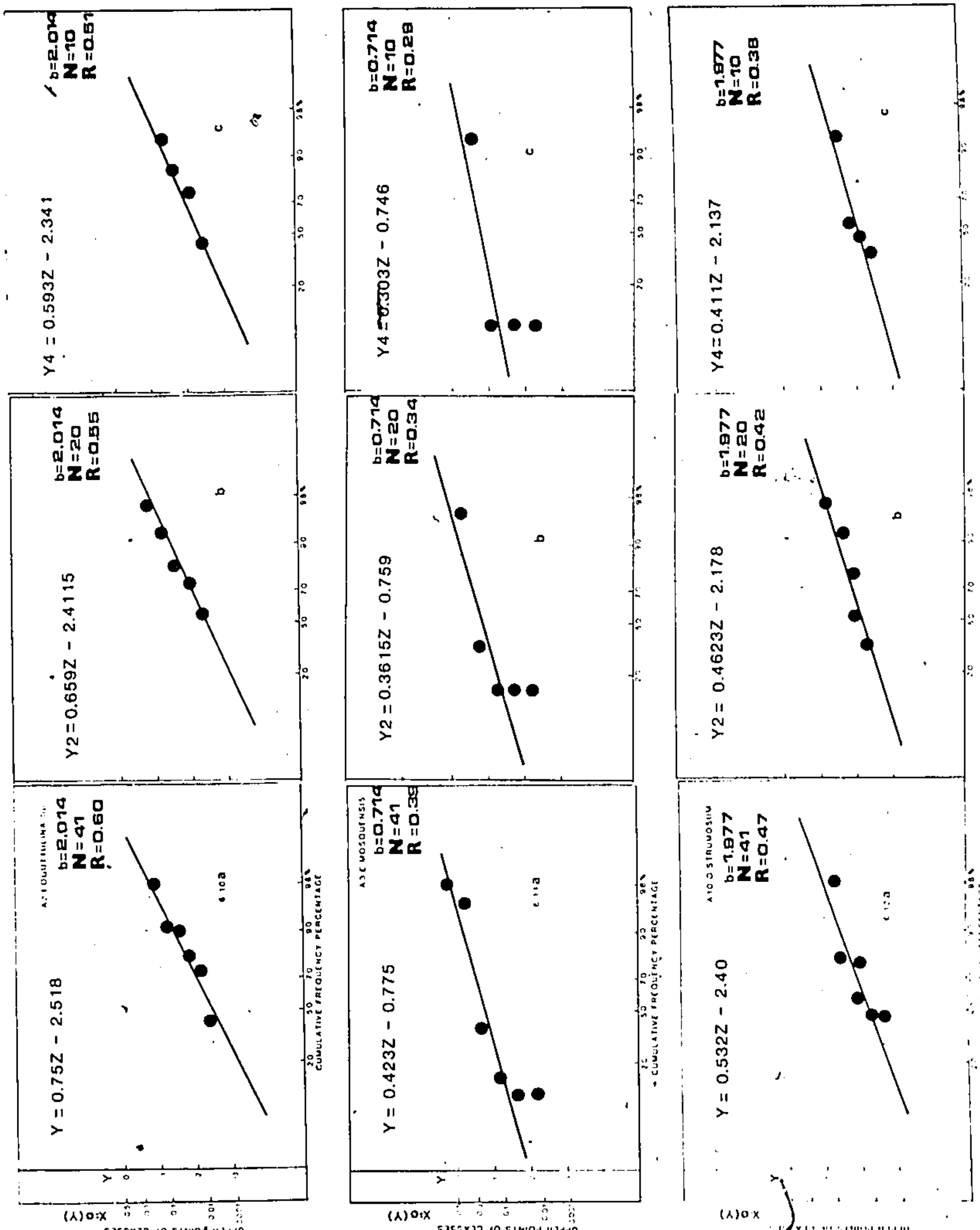
It may be concluded that the model given by equation (60) provides a good fit to the data used in Fig. 8.12, 8.13 and 8.14. The observed data tend to lie about the line approximated by the theoretical line described by Eq. 60 with b and R estimated from the means and variances of the proportion values.

The R values were tabulated with respect to their means and variances, their logarithmic means and variances or their means and quantities equal to (variance/(mean (1.0 - mean)) (see appendix II). R values can also be estimated from plots that were constructed according to mean and variance relationships (Appendix III).

The results shown in Figs. 8.12, 8.13 and 8.14 indicate that the new model provides a good fit for all sample sizes. Furthermore. the model given by Eq. 60 remains applicable with decreasing sample size.

The Tojeira 1 and Tojeira 2 sections were correlated with each other using Stam's data. Before correlation, the raw data were transformed with a logistical transformation. Positive correlation

Figure 8.14 Experimental frequency distributions and their theoretical distributions (probnormal) for 41, 20 and 10 values of Eoguttulina sp., E. mosquensis and O. strumosum with enlarged data


Y4 = 0.593Z - 2.341
b=2.014
N=10
R=0.51
c
Y2 = 0.659Z - 2.4115
b=2.014
N=20
R=0.55
b
Y = 0.75Z - 2.518
b=2.014
N=41
R=0.60
Y4 = 0.303Z - 0.746
b=0.714
N=10
R=0.28
c
Y2 = 0.3615Z - 0.759
b=0.714
N=20
R=0.34
b
Y = 0.423Z - 0.775
b=0.714
N=41
Y4 = 0.411Z - 2.137
b=1.977
N=10
R=0.38
c
Y2 = 0.4623Z - 2.178
b=1.977
N=20
R=0.42
b
Y = 0.532Z - 2.40
b=1.977
N=41
R=0.47
CUMULATIVE FREQUENCY PERCENTAGE
UPPER POINTS OF CLASSES
X:α(Y)

does not mean that the taxa and biological environments are dependent on each other. It means that they have similar relations with the same set of environmental conditions.

Epistomina mosquensis, Ophthalmidium strumosum and agglutinants show a positive correlation (Table 7.1 and 7.2). Epistomina mosquensis has been reported from deeper shelf deposits (Pazdro, 1969). Epistomina mosquensis shows negative correlation with Eoguttulina sp. and with Spirillina tenuissima which prefer shallow water (Stam, 1986).

Eoguttulina sp., Epistomina mosquensis, Ophthalmidium strumosum Spirillina tenuissima and agglutinants are unique species. The distribution of these taxa could be of bistratigraphic values. they could be used for correlation between sections using the abundance data for this species as well as for correlation with other species within the same samples.

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APPENDIX I : SAS and FORTRAN programs are used in this project. Listing of FORTRAN programs

1 - FF FORTRAN

2 - F FORTRAN

3 - ARIMA for Tojeira 1 and 2 sections with Stam's data

4 - ARIMA for Tojeira 1 and 2 sections with Gradstein's data

5 - ARIMA for Tojeira 1 section with enlarged data

1 - FF FORTRAN

```
C     PROGRAM HERMIT
      DO 100 K=1,11
      READ 1,A,G,X
    1 FORMAT (F6.4,F10.8,F6.4)
      IF (G.EQ.0.0) GO TO 999
      DO 10 I= 1,70
      RI=FLOAT (I)-1.0
      RI=RI/100.0
      R=0.3+RI
      SMR=R**2+R**4*A**2/2.0
      SMR=SMR+R**6*(A**2-1.0)**2/6.0
      SMR= SMR+R**8*(A**3-3.0*A)**2/24.0
      SMR=SMR+R**10*(A**4-6.0*A**2+3.0)**2/120.0
      SMR=SMR+R**12*(A**5-10.0*A**3+15.0*A)**2/720.0
      SMR=SMR+R**14*(A**6-15.0*A**4+45.0*A**2-15.0)**2/5040.0
      SMR=SMR+R**16*(A**7-21.0*A**5+105.0*A**3-105.0*A)**2/40320.0
      SMR9=R**18*(A**8-28.0*A**6+210.0*A**4-420.0*A**2+105.0)**2
      SMR9=SMR9/362880.0+SMR
      SMR10=A**9-36.0*A**7+378.0*A**5-1260.0*A**3+945.0*A
      SMR10=SMR10**2
      SMR10=R**20*SMR10
      SMR10=SMR10/3628800.0+SMR9
      SMR11=A**10-45.0*A**8+630.0*A**6-3150.0*A**4+4725.0*A**2-945.0
      SMR11=SMR11**2
      SMR11=R**22*SMR11
      SMR11=SMR11/39916800.0
      SMR11=SMR11+SMR10
      SMR12=A**11-55.0*A**9+990.0*A**7-6930.0*A**5+17325.0*A**3-
     +10395.0*A
      SMR12=R**24*SMR12
      SMR12=SMR12/479000000.0+SMR11
      SMR13=A**12-66.0*A**10+1485.0*A**8-13860.0*A**6+51975.0*A**4-
     +62370.0*A**2+10395.0
      SMR13=SMR13**2
      SMR13=R**26*SMR13
      SMR13=SMR13/6227000000.0+SMR12
      SMR11=SMR11*G**2
      SMR12=SMR12*G**2
      SMR13=SMR13*G**2
      Y=SMR12/(X*(1.0-X))
   10 PRINT 110,A,R,SMR11,SMR12,SMR13,Y
  110 FORMAT (2(F6.4,5X),4(F10.8,5X))
  100 CONTINUE
  999 STOP
      END
```

2 - F FORTRAN

```
      DIMENSION X(50),A1(50),B2(50),C3(50),D4(50),E5(50),F6(50),H7(50)
      DIMENSION P8(50),R9(50),A11(50),B22(50),C33(50),D44(50),E55(50)
      DIMENSION F66(50),H77(50)
      DO 110 MM=1,3
      READ(5,105) N
  105 FORMAT(I2)
      READ(5,103) (X(I),I=1,N)
  103 FORMAT(13F4.1)
      SUMX=0.0
      SUMXX=0.0
      SUMAM=0.0
      SUMAA=0.0
      SUMBN=0.0
      SUMBB=0.0
      SUMCT=0.0
      SUMCC=0.0
      SUMDF=0.0
      SUMDD=0.0
      SUMEX=0.0
      SUMEE=0.0
      SUMFZ=0.0
      SUMFF=0.0
      SUMHW=0.0
      SUMHH=0.0
      DO 102 I=1,N
      SUMX=SUMX+X(I)
      SUMXX=SUMXX+X(I)**2
  102 CONTINUE
      DO 101 K=1,N-1
      A1(K)=X(K)+X(K+1)
      A11(K)=(A1(K))/2.0
      SUMAM=SUMAM+A11(K)
      SUMAA=SUMAA+A11(K)**2
  101 CONTINUE
      DO 100 L=1,N-2
      B2(L)=A1(L)+X(L+2)
      B22(L)=(B2(L))/3.0
      SUMBN=SUMBN+B22(L)
      SUMBB=SUMBB+B22(L)**2
  100 CONTINUE
      DO 99 M=1,N-3
      C3(M)=B2(M)+X(M+3)
      C33(M)=(B2(M)+X(M+3))/4.0
      SUMCT=SUMCT+C33(M)
      SUMCC=SUMCC+C33(M)**2
   99 CONTINUE
      DO 98 J=1,N-4
      D4(J)=C3(J)+X(J+4)
      D44(J)=D4(J)/5.0
      SUMDF=SUMDF+D44(J)
      SUMDD=SUMDD+D44(J)**2
   98 CONTINUE
      DO 97 NN=1,N-5
      E5(NN)=D4(NN)+X(NN+5)
      E55(NN)=E5(NN)/6.0
      SUMEX=SUMEX+E55(NN)
      SUMEE=SUMEE+E55(NN)**2
   97 CONTINUE
      DO 96 JJ=1,N-6
      F6(JJ)=E5(JJ)+X(JJ+6)
      F66(JJ)=F6(JJ)/7.0
      SUMFZ=SUMFZ+F66(JJ)
      SUMFF=SUMFF+F66(JJ)**2
   96 CONTINUE
      SO=(SUMXX-(SUMX**2 /FLOAT(N)))/FLOAT(N-1)
      S1=(SUMAA-(SUMAM**2/FLOAT(N-1)))/FLOAT(N-2)
      S2=(SUMBB-(SUMBN**2 /FLOAT(N-2)))/FLOAT(N-3)
      S3=(SUMCC-(SUMCT**2 /FLOAT(N-3)))/FLOAT(N-4)
      S4=(SUMDD-(SUMDF**2 /FLOAT(N-4)))/FLOAT(N-5)
      S5=(SUMEE-(SUMEX**2 /FLOAT(N-5)))/FLOAT(N-6)
      S6=(SUMFF-(SUMFZ**2 /FLOAT(N-6)))/FLOAT(N-7)
      WRITE(6,*) SO,S1,S2,S3,S4,S5,S6
  110 CONTINUE
      STOP
      END
```

3 - ARIMA for Tojeira 1 and 2 sections with Stam's data

```
                                   SAS                          12:42 MONDAY, OCTOBER 5, 1987

                             ARIMA PROCEDURE

                   NAME OF VARIABLE         =        A1
                   MEAN OF WORKING SERIES=   6.47097
                   STANDARD DEVIATION      =   4.66713
                   NUMBER OF OBSERVATIONS=        31
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0    21.7821     1.00000  |                    |********************|          0
  1   -1.72983    -0.07942  |              .   **|        .           |   0.179605
  2    3.91989     0.17996  |              .     |****    .           |   0.180734
  3    1.36552     0.06269  |              .     |*       .           |   0.186425
  4    1.39608     0.06409  |              .     |*       .           |   0.187104
  5   0.476232     0.02186  |              .     |        .           |   0.187811
  6    1.31162     0.06022  |              .     |*       .           |   0.187893
  7   0.798186     0.03664  |              .     |*       .           |   0.188514
  8     1.1767     0.05402  |              .     |*       .           |   0.188744
  9    1.07484     0.04935  |              .     |*       .           |   0.189242
 10   -1.22516    -0.05625  |              .    *|        .           |   0.189657
                            '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.08837  |              .     |**      .           |
            2    -0.16292  |              .  ***|        .           |
            3    -0.08739  |              .   **|        .           |
            4    -0.02719  |              .    *|        .           |
            5     0.01360  |              .     |        .           |
            6    -0.02652  |              .    *|        .           |
            7    -0.04160  |              .    *|        .           |
            8    -0.06838  |              .    *|        .           |
            9    -0.02290  |              .     |        .           |
           10     0.07444  |              .     |*       .           |

                          PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.07942  |              .   **|        .           |
            2     0.17476  |              .     |***     .           |
            3     0.09172  |              .     |**      .           |
            4     0.04602  |              .     |*       .           |
            5     0.00317  |              .     |        .           |
            6     0.03885  |              .     |*       .           |
            7     0.03329  |              .     |*       .           |
            8     0.03971  |              .     |*       .           |
            9     0.03825  |              .     |*       .           |
           10    -0.07885  |              .   **|        .           |

                     AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                               AUTOCORRELATIONS
  LAG  SQUARE  DF   PROB
   6     1.82   6  0.935  -0.079  0.180  0.063  0.064  0.022  0.060
                                   SAS                          12:42 MONDAY, OCTOBER 5, 1987
```

```
                              ARIMA PROCEDURE

                    NAME OF VARIABLE          =        A2
                    MEAN OF WORKING SERIES=    2.77097
                    STANDARD DEVIATION      =  4.48713
                    NUMBER OF OBSERVATIONS=        31
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    20.1343     1.00000  |                    |********************|          0
  1    9.83615     0.48853  |                    |**********          |   0.179605
  2    4.60299     0.22861  |              .     |*****     .         |   0.218301
  3    6.48433     0.32205  |              .     |******    .         |   0.225892
  4    5.80549     0.28834  |             .      |******     .        |   0.240247
  5    1.74721     0.08678  |             .      |**         .        |   0.251162
  6 -0.0175513    -0.00087  |             .      |           .        |   0.252128
  7  -0.106569    -0.00529  |             .      |           .        |   0.252128
  8  -0.377554    -0.01875  |             .      |           .        |   0.252131
  9  -0.502992    -0.02498  |             .      |           .        |   0.252176
 10  -0.975797    -0.04846  |             .     *|           .        |   0.252256
                           '.' MARKS TWO STANDARD ERRORS

                           INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.39887  |            ********|           .        |
            2     0.15046  |              .     |***        .        |
            3    -0.17302  |              .  ***|           .        |
            4    -0.12338  |              .   **|           .        |
            5     0.05685  |              .     |*          .        |
            6     0.03231  |              .     |*          .        |
            7     0.02655  |              .     |*          .        |
            8     0.03444  |              .     |*          .        |
            9    -0.04236  |              .    *|           .        |
           10     0.01083  |              .     |           .        |

                           PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.48853  |              .     |**********          |
            2    -0.01319  |              .     |           .        |
            3     0.28289  |              .     |******     .        |
            4     0.03506  |              .     |*          .        |
            5    -0.12060  |              .   **|           .        |
            6    -0.07626  |              .   **|           .        |
            7    -0.06344  |              .    *|           .        |
            8     0.00149  |              .     |           .        |
            9     0.04306  |              .     |*          .        |
           10    -0.01458  |              .     |           .        |

                     AUTOCORRELATION CHECK FOR WHITE NOISE

        TO     CHI                          AUTOCORRELATIONS
       LAG   SQUARE  DF   PROB
         6    17.22   6  0.009   0.489  0.229  0.322  0.288  0.087 -0.001
                                        SAS                12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =        A3
                    MEAN OF WORKING SERIES=      22.471
                    STANDARD DEVIATION    =     12.6522
                    NUMBER OF OBSERVATIONS=          31
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    160.079     1.00000 |                    |********************|          0
  1    79.9485     0.49943 |                .   |**********          |   0.179605
  2    85.2347     0.53245 |              .     |***********         |   0.219887
  3    58.3794     0.36469 |             .      |*******    .        |    0.25815
  4    32.2471     0.20145 |            .       |****        .       |   0.274266
  5    27.9955     0.17489 |            .       |***         .       |   0.278998
  6    14.9058     0.09312 |            .       |**          .       |   0.282512
  7    25.9934     0.16238 |            .       |***         .       |   0.283501
  8    23.4033     0.14620 |            .       |***         .       |   0.286485
  9    19.8307     0.12388 |           .        |**           .      |   0.288882
 10    12.4919     0.07804 |           .        |**           .      |    0.29059
                            '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    -0.05880 |              .    *|          .         |
           2    -0.39872 |           ********|          .         |
           3    -0.21208 |              . ****|          .         |
           4     0.10481 |              .     |**        .         |
           5     0.14240 |              .     |***       .         |
           6     0.13767 |              .     |***       .         |
           7    -0.12063 |              .   **|          .         |
           8    -0.13749 |              .  ***|          .         |
           9    -0.00195 |              .     |          .         |
          10     0.08213 |              .     |**        .         |

                        PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1     0.49943 |              .     |**********          |
           2     0.37708 |              .     |********            |
           3     0.01664 |              .     |          .         |
           4    -0.18030 |              . ****|          .         |
           5     0.00757 |              .     |          .         |
           6     0.03097 |              .     |*         .         |
           7     0.16038 |              .     |***       .         |
           8     0.06029 |              .     |*         .         |
           9    -0.06668 |              .    *|          .         |
          10    -0.11801 |              .   **|          .         |

                   AUTOCORRELATION CHECK FOR WHITE NOISE

  TO      CHI                           AUTOCORRELATIONS
 LAG   SQUARE DF    PROB
   6    26.46  6   0.000   0.499  0.532  0.365  0.201  0.175  0.093
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                                   ARIMA PROCEDURE

                           NAME OF VARIABLE        =        A4
                           MEAN OF WORKING SERIES=   3.36129
                           STANDARD DEVIATION    =   3.70368
                           NUMBER OF OBSERVATIONS=        31
                                   AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0    13.7172    1.00000   |                    |********************|         0
  1 -0.0383418   -0.00280   |             .      |        .           |  0.179605
  2   0.841339    0.06133   |             .      |*       .           |  0.179607
  3   -2.75041   -0.20051   |             .  ****|        .           |  0.180281
  4     1.6674    0.12156   |             .      |**      .           |  0.187337
  5    3.96199    0.28883   |             .      |******  .           |  0.189864
  6   -2.44681   -0.17838   |            .   ****|         .          |  0.203545
  7   -1.97001   -0.14362   |            .    ***|         .          |  0.208527
  8   -3.54132   -0.25817   |            .  *****|         .          |  0.211693
  9    1.22872    0.08958   |           .        |**        .         |  0.221617
 10   -1.91904   -0.13990   |           .     ***|          .         |  0.222782
                            '.' MARKS TWO STANDARD ERRORS

                             INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1   -0.04815   |                   *|        .           |
             2   -0.23355   |             . *****|        .           |
             3    0.30592   |             .      |******  .           |
             4    0.01545   |             .      |        .           |
             5   -0.38451   |            ********|        .           |
             6    0.14956   |             .      |***     .           |
             7    0.19767   |             .      |****    .           |
             8    0.00011   |             .      |        .           |
             9    0.02264   |             .      |        .           |
            10    0.18367   |             .      |****    .           |

                             PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1   -0.00280   |             .      |        .           |
             2    0.06133   |             .      |*       .           |
             3   -0.20093   |             .  ****|        .           |
             4    0.12469   |             .      |**      .           |
             5    0.32729   |             .      |*******             |
             6   -0.28702   |             .******|        .           |
             7   -0.16683   |             .   ***|        .           |
             8   -0.08904   |             .    **|        .           |
             9   -0.04150   |             .     *|        .           |
            10   -0.26761   |             . *****|        .           |

                      AUTOCORRELATION CHECK FOR WHITE NOISE

    TO     CHI                                AUTOCORRELATIONS
   LAG   SQUARE DF    PROB
     6     6.75  6   0.345 -0.003   0.061 -0.201  0.122  0.289 -0.178
                                    SAS                                   12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                        NAME OF VARIABLE        =       A5
                        MEAN OF WORKING SERIES=    3.18065
                        STANDARD DEVIATION      =  3.83426
                        NUMBER OF OBSERVATIONS=         31
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    14.7016     1.00000  |                    |********************|          0
  1    3.21369     0.21860  |               .    |****    .           |   0.179605
  2   -2.91795    -0.19848  |              . ****|         .          |   0.187992
  3   -3.86058    -0.26260  |              .*****|         .          |   0.194634
  4   -4.18162    -0.28443  |             .******|         .          |   0.205746
  5   -1.53857    -0.10465  |             .    **|          .         |   0.218061
  6  -0.892851    -0.06073  |             .     *|          .         |   0.219676
  7  -0.395641    -0.02691  |             .     *|          .         |   0.220217
  8    1.14899     0.07815  |             .      |**        .         |   0.220323
  9   0.118924     0.00809  |             .      |          .         |   0.221215
 10    0.52068     0.03542  |             .      |*         .         |   0.221225
                          '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.46397  |              .     |*********           |
             2     0.62334  |              .     |************        |
             3     0.51071  |              .     |**********          |
             4     0.50650  |              .     |**********          |
             5     0.35377  |              .     |*******             |
             6     0.32722  |              .     |*******             |
             7     0.25312  |              .     |*****   .           |
             8     0.13863  |              .     |***     .           |
             9     0.12349  |              .     |**      .           |
            10     0.09072  |              .     |**      .           |

                          PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.21860  |              .     |****    .           |
             2    -0.25862  |              .*****|        .           |
             3    -0.17046  |              .  ***|        .           |
             4    -0.26489  |              .*****|        .           |
             5    -0.10707  |              .   **|        .           |
             6    -0.23353  |              .*****|        .           |
             7    -0.19589  |              . ****|        .           |
             8    -0.13432  |              .  ***|        .           |
             9    -0.26045  |              .*****|        .           |
            10    -0.19116  |              . ****|        .           |

                   AUTOCORRELATION CHECK FOR WHITE NOISE

 TO   CHI                               AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    9.19  6  0.163   0.219 -0.198 -0.263 -0.284 -0.105 -0.061   12:42 MONDAY, OCTOBER 5, 1987
                               SAS
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                              ARIMA PROCEDURE

                       NAME OF VARIABLE        =        A6
                       MEAN OF WORKING SERIES=    10.2419
                       STANDARD DEVIATION      =    5.9591
                       NUMBER OF OBSERVATIONS=        31
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      STD
  0    35.5108     1.00000  |                    |********************|         0
  1    7.74813     0.21819  |                .   |****    .           |  0.179605
  2   -2.21072    -0.06225  |               .   *|        .           |  0.187961
  3   -12.2193    -0.34410  |               .*******|     .           |  0.188625
  4     1.7834     0.05022  |               .    |*       .           |  0.207891
  5    7.55795     0.21284  |               .    |****    .           |  0.208282
  6   -2.76041    -0.07773  |               .  **|         .          |  0.215183
  7   -9.98317    -0.28113  |               . ******|      .          |  0.216087
  8    -3.3978    -0.09568  |               .  **|         .          |   0.22758
  9    3.65937     0.10305  |               .    |**       .          |  0.228874
 10    9.38114     0.26418  |               .    |*****    .          |  0.230366
                            '.' MARKS TWO STANDARD ERRORS

                           INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.29908  |              .******|      .            |
            2    -0.38702  |             ********|      .            |
            3     0.54935  |              .      |***********        |
            4    -0.02219  |              .      |      .            |
            5    -0.36353  |              .******|      .            |
            6     0.26237  |              .      |*****  .           |
            7     0.10517  |              .      |**    .            |
            8    -0.16990  |              .   ***|      .            |
            9     0.07907  |              .      |**    .            |
           10    -0.01973  |              .      |      .            |

                           PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.21819  |              .      |****  .            |
            2    -0.11535  |              .    **|      .            |
            3    -0.32327  |              .******|      .            |
            4     0.22505  |              .      |*****  .           |
            5     0.14637  |              .      |***   .            |
            6    -0.35507  |              *******|      .            |
            7    -0.13750  |              .   ***|      .            |
            8     0.25738  |              .      |*****  .           |
            9    -0.12726  |              .   ***|      .            |
           10     0.03779  |              .      |*     .            |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

      TO     CHI                         AUTOCORRELATIONS
     LAG  SQUARE DF    PROB
       6    8.21  6   0.223   0.218 -0.062 -0.344   0.050   0.213 -0.078
                                   SAS                        12:42 MONDAY, OCTOBER 5, 1987
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                                ARIMA PROCEDURE

                       NAME OF VARIABLE        =        A7
                       MEAN OF WORKING-SERIES=   4.73548

                       STANDARD DEVIATION      =   2.81477
                       NUMBER OF OBSERVATIONS=        31
                                AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    7.92293     1.00000  |                    |********************|          0
  1    1.10392     0.13933  |             .      |***   .             |   0.179605
  2 -0.0276046    -0.00348  |             .      |      .             |   0.183059
  3    1.22344     0.15442  |             .      |***   .             |   0.183061
  4   -1.09178    -0.13780  |             .   ***|      .             |   0.187216
  5   0.754833     0.09527  |             .      |**    .             |   0.190459
  6  -0.727153    -0.09178  |             .    **|      .             |   0.191991
  7  -0.841242    -0.10618  |             .    **|      .             |   0.193401
  8  -0.607848    -0.07672  |             .    **|      .             |   0.195272
  9   -1.96482    -0.24799  |             . *****|      .             |   0.196242
 10   -1.67154    -0.21097  |             .  ****|      .             |   0.206103
                            '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

             LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
               1    -0.38413  |            ********|      .             |
               2     0.40548  |             .      |********            |
               3    -0.37965  |            ********|      .             |
               4     0.37481  |             .      |*******             |
               5    -0.30921  |             .******|      .             |
               6     0.25903  |             .      |***** .             |
               7    -0.14434  |             .   ***|      .             |
               8     0.15599  |             .      |***   .             |
               9     0.00262  |             .      |      .             |
              10     0.16411  |             .      |***   .             |

                          PARTIAL AUTOCORRELATIONS

             LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
               1     0.13933  |             .      |***   .             |
               2    -0.02335  |             .      |      .             |
               3     0.16139  |             .      |***   .             |
               4    -0.19177  |             .  ****|      .             |
               5     0.16925  |             .      |***   .             |
               6    -0.20026  |             .  ****|      .             |
               7     0.03173  |             .      |*     .             |
               8    -0.18538  |             .  ****|      .             |
               9    -0.11470  |             .    **|      .             |
              10    -0.25130  |             . *****|      .             |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

            TO    CHI                          AUTOCORRELATIONS
           LAG  SQUARE DF   PROB
             6    2.95  6  0.814  0.139 -0.003  0.154 -0.138  0.095 -0.092
                                         SAS                     12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                        NAME OF VARIABLE       =       A8
                        MEAN OF WORKING SERIES=   2.13871
                        STANDARD DEVIATION     =  1.28557
                        NUMBER OF OBSERVATIONS=        31
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0     1.6527    1.00000  |                    |********************|         0
  1   0.679098    0.41090  |                  . |********            |  0.179605
  2   0.107905    0.06529  |                 .  |*  .                |  0.207728
  3   -0.36514   -0.22094  |                 .****  .                |  0.208389
  4  -0.119194   -0.07212  |                 .   *|  .               |  0.215813
  5 -0.0759129   -0.04593  |                .    *|   .              |  0.216589
  6  0.0579618    0.03507  |                .     |*  .              |  0.216903
  7   0.322502    0.19514  |                .     |****.             |  0.217086
  8   0.265222    0.16048  |                .     |***  .            |  0.222673
  9  0.0828116    0.05011  |                .     |*    .            |  0.226373
 10  -0.298475   -0.18060  |                . ****|    .             |   0.22673
                           '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    -0.42496  |            ********|      .            |
           2    -0.10749  |               .  **|      .            |
           3     0.37282  |               .    |*******            |
           4    -0.27712  |               .****** |    .           |
           5     0.04671  |               .    |*     .            |
           6     0.13220  |               .    |***   .            |
           7    -0.16498  |               .  ***|     .            |
           8     0.04037  |               .    |*     .            |
           9    -0.04214  |               .   *|      .            |
          10     0.06562  |               .    |*     .            |

                          PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1     0.41090  |               .    |********           |
           2    -0.12459  |               .  **|      .            |
           3    -0.24432  |               .*****|     .            |
           4     0.16168  |               .    |***   .            |
           5    -0.08013  |               .  **|      .            |
           6     0.00688  |               .    |      .            |
           7     0.26953  |               .    |***** .            |
           8    -0.07033  |               .   *|      .            |
           9    -0.03590  |               .   *|      .            |
          10    -0.10181  |               .  **|      .            |

                   AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI
LAG  SQUARE DF   PROB                 AUTOCORRELATIONS
  6    8.02  6  0.237   0.411  0.065 -0.221 -0.072 -0.046  0.035
                               SAS                  12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =        A9
                    MEAN OF WORKING SERIES=    3.16774
                    STANDARD DEVIATION      =    6.59889
                    NUMBER OF OBSERVATIONS=         31
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
 0    43.5454    1.00000   |                    |********************|        0
 1    2.56896    0.05899   |                .   |*    .              |  0.179605
 2    1.32893    0.03052   |                .   |*    .              |  0.180229
 3    11.1393    0.25581   |                .   |*****.              |  0.180396
 4   -6.02295   -0.13831   |                . ***|     .             |  0.191741
 5    2.81595    0.06467   |               .    |*     .             |  0.194933
 6   -1.75948   -0.04041   |               .   *|      .             |  0.195623
 7   -4.63903   -0.10653   |               .  **|      .             |  0.195893
 8    7.75166    0.17801   |               .    |****  .             |  0.197753
 9 -0.0426226   -0.00098   |               .    |      .             |  0.202856
10   -1.93053   -0.04433   |               .   *|      .             |  0.202856
                            '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1   -0.15858   |                . ***|     .              |
             2    0.04762   |                .    |*    .              |
             3   -0.28828   |                ******|    .              |
             4    0.13699   |                .    |***  .              |
             5   -0.04629   |                .   *|     .              |
             6    0.13468   |                .    |***  .              |
             7    0.01087   |                .    |     .              |
             8   -0.11214   |                .  **|     .              |
             9   -0.03875   |                .   *|     .              |
            10    0.04062   |                .    |*    .              |

                          PARTIAL AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    0.05899   |                .    |*    .              |
             2    0.02713   |                .    |*    .              |
             3    0.25352   |                .    |*****.              |
             4   -0.17877   |                .****|     .              |
             5    0.08499   |                .    |**   .              |
             6   -0.12817   |                . ***|     .              |
             7   -0.00705   |                .    |     .              |
             8    0.14075   |                .    |***  .              |
             9    0.03408   |                .    |*    .              |
            10   -0.04747   |                .   *|     .              |

                     AUTOCORRELATION CHECK FOR WHITE NOISE

     TO     CHI                            AUTOCORRELATIONS
    LAG   SQUARE DF   PROB
      6     3.50  6  0.744   0.059  0.031  0.256 -0.138  0.065 -0.040
                                     SAS               12:42 MONDAY, OCTOBER 5, 1987
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                                ARIMA PROCEDURE

                     NAME OF VARIABLE        =        A10
                     MEAN OF WORKING SERIES=    1.92903
                     STANDARD DEVIATION    =    2.44503
                     NUMBER OF OBSERVATIONS=         31
                                 AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    5.97819     1.00000 |                    |********************|          0
  1    2.42279     0.40527 |             .      |********            |   0.179605
  2    1.18282     0.19786 |            .       |****   .            |   0.207013
  3    2.97305     0.49732 |           .        |**********          |   0.213026
  4     1.9424     0.32491 |          .         |******   .          |   0.247662
  5   0.788303     0.13186 |          .         |***      .          |   0.261051
  6    1.52585     0.25524 |         .          |*****     .         |    0.26319
  7    1.10028     0.18405 |         .          |****      .         |   0.271057
  8 -0.0387607    -0.00648 |         .          |          .         |   0.275059
  9   0.736675     0.12323 |         .          |**        .         |   0.275064
 10   0.282735     0.04729 |         .          |*         .         |   0.276839
                             '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1    -0.41223 |            ********|      .             |
              2     0.28358 |             .      |******.             |
              3    -0.34462 |             *******|      .             |
              4     0.04717 |             .      |*     .             |
              5    -0.10078 |             .    **|      .             |
              6     0.09322 |             .      |**    .             |
              7    -0.15191 |             .   ***|      .             |
              8     0.17082 |             .      |***   .             |
              9    -0.13653 |             .   ***|      .             |
             10     0.09781 |             .      |**    .             |

                          PARTIAL AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1     0.40527 |             .      |********            |
              2     0.04022 |             .      |*     .             |
              3     0.48425 |             .      |**********          |
              4    -0.04037 |             .     *|      .             |
              5    -0.00047 |             .      |      .             |
              6     0.02176 |             .      |      .             |
              7    -0.06657 |             .     *|      .             |
              8    -0.11777 |             .    **|      .             |
              9     0.08772 |             .      |**    .             |
             10    -0.14693 |             .   ***|      .             |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

   TO     CHI                               AUTOCORRELATIONS
  LAG   SQUARE DF    PROB
    6    23.37  6   0.001   0.405   0.198   0.497   0.325   0.132   0.255
                                     SAS                        12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                         NAME OF VARIABLE        =       A11
                         MEAN OF WORKING SERIES=    7.80968
                         STANDARD DEVIATION      =  7.16498
                         NUMBER OF OBSERVATIONS=         31
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
 0    51.337     1.00000   |                    |********************|          0
 1   26.3863     0.51398   |                    |**********          |   0.179605
 2   13.0168     0.25356   |              .     |*****     .         |    0.22204
 3   9.56743     0.18637   |              .     |****      .         |   0.231192
 4   -3.5746    -0.06963   |              .    *|          .         |   0.235988
 5   -7.5117    -0.14632   |              .  ***|          .         |    0.23665
 6   -2.3727    -0.04622   |              .    *|          .         |    0.23955
 7  -1.99501    -0.03886   |              .    *|          .         |   0.239838
 8  -7.45035    -0.14513   |              .  ***|          .         |   0.240041
 9  -7.73027    -0.15058   |              .  ***|          .         |   0.242855
10  -3.14883    -0.06134   |              .    *|          .         |   0.245848
                            '.' MARKS TWO STANDARD ERRORS

                            INVERSE AUTOCORRELATIONS

      LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1   -0.38910   |              ********|        .           |
       2    0.00328   |              .       |        .           |
       3   -0.24165   |              .  *****|        .           |
       4    0.24735   |              .       |*****   .           |
       5    0.08416   |              .       |**      .           |
       6   -0.08535   |              .     **|        .           |
       7   -0.10398   |              .     **|        .           |
       8    0.04695   |              .       |*       .           |
       9    0.13012   |              .       |***     .           |
      10   -0.06895   |              .      *|        .           |

                            PARTIAL AUTOCORRELATIONS

      LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
       1    0.51398   |              .       |**********          |
       2   -0.01444   |              .       |        .           |
       3    0.08371   |              .       |**      .           |
       4   -0.26738   |              .  *****|        .           |
       5   -0.02047   |              .       |        .           |
       6    0.09151   |              .       |**      .           |
       7    0.01603   |              .       |        .           |
       8   -0.19356   |              .   ****|        .           |
       9   -0.08524   |              .     **|        .           |
      10    0.10330   |              .       |**      .           |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

 TO      CHI                                AUTOCORRELATIONS
LAG    SQUARE DF    PROB

 6     13.66   6   0.034   0.514   0.254   0.186 -0.070 -0.146 -0.046
                                     SAS                         12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =       A12
                    MEAN OF WORKING SERIES=   3.91613
                    STANDARD DEVIATION     =   2.92928
                    NUMBER OF OBSERVATIONS=        31
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0    8.58071    1.00000  |                    |********************|          0
  1    4.29819    0.50091  |             .      |**********          |   0.179605
  2    1.17676    0.13714  |           .        |***       .         |   0.220105
  3  -0.962835   -0.11221  |           .      **|          .         |   0.222844
  4   -2.43778   -0.28410  |           .  ******|          .         |   0.224659
  5   -3.16228   -0.36853  |           . *******|          .         |   0.235964
  6    -2.8985   -0.33779  |          .  *******|           .        |   0.253853
  7   -2.55518   -0.29778  |         .    ******|            .       |   0.267961
  8  -0.618964   -0.07213  |         .         *|            .       |   0.278431
  9   0.143681    0.01674  |         .          |            .       |   0.279033
 10  -0.374912   -0.04369  |         .         *|            .       |   0.279066
                           '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.26717 |             . *****|       .            |
            2     0.09241 |             .      |**     .            |
            3     0.15864 |             .      |***    .            |
            4     0.08799 |             .      |**     .            |
            5     0.16314 |             .      |***    .            |
            6    -0.00021 |             .      |       .            |
            7     0.24329 |             .      |*****  .            |
            8    -0.04734 |             .     *|       .            |
            9    -0.06288 |             .     *|       .            |
           10     0.19272 |             .      |****   .            |

                         PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.50091 |             .      |**********          |
            2    -0.15188 |             .   ***|       .            |
            3    -0.15750 |             .   ***|       .            |
            4    -0.18886 |             .  ****|       .            |
            5    -0.18326 |             .  ****|       .            |
            6    -0.12623 |             .   ***|       .            |
            7    -0.19568 |             .  ****|       .            |
            8     0.06038 |             .      |*      .            |
            9    -0.15329 |             .   ***|       .            |
           10    -0.27604 |             .******|       .            |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                                 AUTOCORRELATIONS
LAG   SQUARE DF    PROB
  6    22.75  6   0.001   0.501   0.137 -0.112 -0.284 -0.369 -0.338
                                  SAS                 12:42 MONDAY, OCTOBER 5, 1987
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                                   ARIMA PROCEDURE

                         NAME OF VARIABLE        =       A13
                         MEAN OF WORKING SERIES=     2.16452
                         STANDARD DEVIATION      =    5.7693
                         NUMBER OF OBSERVATIONS=          31
                                   AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1         STD
  0    33.2849     1.00000  |                    |********************|          0
  1    14.6964     0.44154  |                    |*********           |   0.179605
  2    4.73734     0.14233  |                    |***   .             |   0.211744
  3    7.90004     0.23735  |             .      |*****   .           |   0.214808
  4    5.20103     0.15626  |             .      |***     .           |   0.223108
  5    1.44456     0.04340  |             .      |*       .           |    0.22661
  6  -0.762471    -0.02291  |             .      |        .           |   0.226878
  7   -1.17215    -0.03522  |             .     *|        .           |   0.226953
  8   -1.01493    -0.03049  |             .     *|        .           |   0.227129
  9   -1.01704    -0.03056  |             .     *|        .           |   0.227261
 10   -1.34978    -0.04055  |             .     *|        .           |   0.227394
                           '.' MARKS TWO STANDARD ERRORS

                              INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.49257  |          **********|      .             |
            2     0.27666  |                    |******.             |
            3    -0.27158  |             . *****|      .             |
            4     0.08508  |             .      |**    .             |
            5    -0.07421  |             .     *|      .             |
            6     0.07744  |             .      |**    .             |
            7    -0.01609  |             .      |      .             |
            8     0.02684  |             .      |*     .             |
            9    -0.02013  |             .      |      .             |
           10     0.01104  |             .      |      .             |

                              PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.44154  |                    |*********           |
            2    -0.06537  |             .     *|      .             |
            3     0.24858  |             .      |*****  .            |
            4    -0.05725  |             .     *|      .             |
            5    -0.00425  |             .      |      .             |
            6    -0.09391  |             .    **|      .             |
            7    -0.01292  |             .      |      .             |
            8    -0.01486  |             .      |      .             |
            9     0.01273  |             .      |      .             |
           10    -0.01542  |             .      |      .             |

                     AUTOCORRELATION CHECK FOR WHITE NOISE

     TO    CHI                                AUTOCORRELATIONS
    LAG   SQUARE DF   PROB
      6    10.44  6  0.107    0.442  0.142  0.237  0.156  0.043 -0.023
                                      SAS                              12:42 MONDAY, OCTOBER 5, 1987
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                           ARIMA PROCEDURE

                  NAME OF VARIABLE        =        A14
                  MEAN OF WORKING SERIES=   16.9581
                  STANDARD DEVIATION     =   8.85819
                  NUMBER OF OBSERVATIONS=        31
                           AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0    78.4676    1.00000  |                    |********************|         0
  1    11.3129    0.14417  |                .   |***    .            |  0.179605
  2   -11.1688   -0.14234  |                . ***|       .            |  0.183301
  3    18.3317    0.23362  |                .   |*****  .            |  0.186832
  4    33.6093    0.42832  |                .   |*********           |  0.196029
  5    2.83115    0.03608  |               .    |*       .           |  0.224195
  6   -1.99423   -0.02541  |               .   *|        .           |  0.224382
  7    13.2224    0.16851  |               .    |***     .           |  0.224475
  8    1.55163    0.01977  |               .    |         .          |  0.228519
  9    8.10947    0.10335  |               .    |**       .          |  0.228574
 10   -9.15769    0.11671  |               .    |**       .          | -0.230077
                            '.' MARKS TWO STANDARD ERRORS

                       INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1   -0.11838  |                .  **|       .            |
             2    0.06853  |                .    |*      .            |
             3   -0.10076  |                .  **|       .            |
             4   -0.38181  |             ********|       .            |
             5    0.03583  |                .    |*      .            |
             6    0.09942  |                .    |**     .            |
             7   -0.06439  |                .   *|       .            |
             8    0.17800  |                .    |****   .            |
             9   -0.12390  |                .  **|       .            |
            10   -0.02677  |                .   *|       .            |

                       PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    0.14417  |                .    |***    .            |
             2   -0.16658  |                .  ***|      .            |
             3    0.29576  |                .    |******.             |
             4    0.35002  |                .    |*******             |
             5    0.00122  |                .    |       .            |
             6    0.03817  |                .    |*      .            |
             7   -0.00334  |                .    |       .            |
             8   -0.20574  |                . ****|      .            |
             9    0.17752  |                .    |****   .            |
            10    0.03608  |                .    |*      .            |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

        TO    CHI                        AUTOCORRELATIONS
       LAG  SQUARE DF   PROB
         6   10.45  6  0.107  0.144 -0.142  0.234  0.428  0.036 -0.025
                                     SAS                      12:42 MONDAY, OCTOBER 5, 1987
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                                ARIMA PROCEDURE

                      NAME OF VARIABLE        =        B1
                      MEAN OF WORKING SERIES=    2.84667
                      STANDARD DEVIATION      =    1.43079
                      NUMBER OF OBSERVATIONS=         30
                                AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      STD
  0    2.04716     1.00000  |                    |********************|         0
  1   0.952839     0.46545  |              .     |*********           |  0.182574
  2   0.638966     0.31212  |            .       |******  .           |  0.218577
  3   0.440271     0.21506  |            .       |****    .           |  0.232961
  4   0.728687     0.35595  |           .        |******* .           |  0.239487
  5   0.721859     0.35262  |           .        |*******  .          |  0.256517
  6   0.564276     0.27564  |          .         |******    .         |  0.272195
  7  -0.148975    -0.07277  |          .        *|          .         |  0.281345
  8   0.178041     0.08697  |          .         |**        .         |  0.281972
  9    0.01608     0.00785  |          .         |          .         |  0.282865
 10  0.0867185     0.04236  |          .         |*         .         |  0.282872
                            '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.35597  |             *******|         .          |
            2    -0.01653  |             .      |         .          |
            3     0.05279  |             .      |*        .          |
            4    -0.05504  |             .     *|         .          |
            5    -0.12524  |             .   ***|         .          |
            6    -0.20078  |             .  ****|         .          |
            7     0.33761  |             .      |*******  .          |
            8    -0.19931  |             .  ****|         .          |
            9     0.07789  |             .      |**       .          |
           10     0.03349  |             .      |*        .          |

                          PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.46545  |             .      |*********           |
            2     0.12189  |             .      |**       .          |
            3     0.03986  |             .      |*        .          |
            4     0.28239  |             .      |******   .          |
            5     0.12452  |             .      |**       .          |
            6     0.00929  |             .      |         .          |
            7    -0.37665  |            ********|         .          |
            8     0.17039  |             .      |***      .          |
            9    -0.16726  |             .   ***|         .          |
           10    -0.05192  |             .     *|         .          |

                     AUTOCORRELATION CHECK FOR WHITE NOISE

    TO     CHI                                AUTOCORRELATIONS
   LAG   SQUARE DF    PROB
     6    24.65  6   0.000   0.465   0.312   0.215   0.356   0.353   0.276
                                      SAS                    12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                         NAME OF VARIABLE        =        B2
                         MEAN OF WORKING SERIES=       13.84
                         STANDARD DEVIATION      =   11.1162
                         NUMBER OF OBSERVATIONS=          30
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0     123.57    1.00000  |                    |********************|        0
  1    82.6732    0.66904  |               .    |**************      |  0.182574
  2    73.2918    0.59312  |             .      |*************       |  0.251345
  3    65.7658    0.53221  |            .       |***********.        |  0.294324
  4    44.4355    0.35960  |           .        |*******    .        |  0.324823
  5    36.0074    0.29139  |          .         |******      .       |  0.337832
  6    31.5627    0.25542  |          .         |*****       .       |  0.346109
  7    19.0184    0.15391  |          .         |***         .       |  0.352336
  8    16.5723    0.13411  |          .         |***         .       |   0.35457
  9    9.96005    0.08060  |          .         |**          .       |  0.356257
 10   -3.74567   -0.03031  |          .        *|            .       |  0.356864
                           '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    -0.24444  |               .*****|    .               |
          2    -0.16458  |               .  ***|    .               |
          3    -0.18472  |               . ****|    .               |
          4     0.08161  |               .     |**  .               |
          5     0.07836  |               .     |**  .               |
          6    -0.02635  |               .    *|    .               |
          7     0.02726  |               .     |*   .               |
          8    -0.07548  |               .   **|    .               |
          9    -0.06415  |               .    *|    .               |
         10     0.10547  |               .     |**  .               |

                         PARTIAL AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1     0.66904  |               .     |*************       |
          2     0.26341  |               .     |*****               |
          3     0.12389  |               .     |**  .               |
          4    -0.17430  |               .  ***|    .               |
          5    -0.02600  |               .    *|    .               |
          6     0.06205  |               .     |*   .               |
          7    -0.05618  |               .    *|    .               |
          8     0.01633  |               .     |    .               |
          9    -0.05145  |               .    *|    .               |
         10    -0.14598  |               .  ***|    .               |

                   AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                             AUTOCORRELATIONS
 LAG  SQUARE  DF   PROB
   6   47.59   6  0.000   0.669   0.593   0.532   0.360   0.291   0.255
                                  SAS              12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =        B3
                    MEAN OF WORKING SERIES=    3.92667
                    STANDARD DEVIATION     =    6.01049
                    NUMBER OF OBSERVATIONS=         30
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0     36.126     1.00000  |                    |********************|         0
  1    1.51753     0.04201  |               .    |*        .          |  0.182574
  2   -3.53858    -0.09795  |               .  **|         .          |  0.182896
  3   0.127884     0.00354  |               .    |         .          |  0.184636
  4   -2.46216    -0.06815  |               .   *|         .          |  0.184639
  5    1.63624     0.04529  |               .    |*        .          |  0.185475
  6     1.5387     0.04259  |               .    |*        .          |  0.185844
  7    -4.5629    -0.12631  |               . ***|         .          |  0.186169
  8   -4.00759    -0.11093  |              .   **|          .         |  0.189004
  9    1.83885     0.05090  |              .     |*         .         |  0.191162
 10    4.40425     0.12191  |              .     |**        .         |  0.191613
                            '.' MARKS TWO STANDARD ERRORS

                           INVERSE AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    -0.04220  |               .    *|         .          |
          2     0.06401  |               .     |*        .          |
          3    -0.03641  |               .    *|         .          |
          4     0.09306  |               .     |**       .          |
          5    -0.03051  |               .    *|         .          |
          6    -0.03120  |               .    *|         .          |
          7     0.10229  |               .     |**       .          |
          8     0.08175  |               .     |**       .          |
          9    -0.03291  |               .    *|         .          |
         10    -0.10205  |               .   **|         .          |

                           PARTIAL AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1     0.04201  |               .     |*        .          |
          2    -0.09989  |               .   **|         .          |
          3     0.01241  |               .     |         .          |
          4    -0.07963  |               .   **|         .          |
          5     0.05490  |               .     |*        .          |
          6     0.02278  |               .     |         .          |
          7    -0.12017  |               .   **|         .          |
          8    -0.10147  |               .   **|         .          |
          9     0.04419  |               .     |*        .          |
         10     0.10754  |               .     |**       .          |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

     TO    CHI                            AUTOCORRELATIONS
    LAG  SQUARE DF   PROB
      6    0.71  6  0.994  0.042 -0.098  0.004 -0.068  0.045  0.043
                                SAS                               12:42 MONDAY, OCTOBER 5, 1987
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                                 ARIMA PROCEDURE

                         NAME OF VARIABLE        =        B4

                         MEAN OF WORKING SERIES=       0.78
                         STANDARD DEVIATION    =     1.1791
                         NUMBER OF OBSERVATIONS=         30
                                 AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    1.39027     1.00000  |                   |********************|           0
  1   0.219053     0.15756  |            .      |***   .             |    0.182574
  2  0.0451067     0.03244  |            .      |*     .             |    0.187052
  3    0.47116     0.33890  |            .      |*******             |    0.187239
  4 -.00145333    -0.00105  |           .       |       .            |    0.206677
  5    -0.0312    -0.02244  |           .       |       .            |    0.206677
  6   0.175253     0.12606  |           .       |***    .            |    0.206758
  7 -.00469333    -0.00338  |           .       |       .            |    0.209305
  8 -0.0341733    -0.02458  |           .       |       .            |    0.209306
  9   0.192547     0.13850  |           .       |***    .            |    0.209403
 10  0.0543333     0.03908  |           .       |*      .            |    0.212434
                           '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
         1    -0.23235  |            . *****|      .             |
         2     0.11859  |            .      |**    .             |
         3    -0.33148  |            *******|      .             |
         4     0.12301  |            .      |**    .             |
         5    -0.04440  |            .     *|      .             |
         6     0.05921  |            .      |*     .             |
         7    -0.02410  |            .      |      .             |
         8     0.04745  |            .      |*     .             |
         9    -0.08985  |            .    **|      .             |
        10    -0.00517  |            .      |      .             |

                          PARTIAL AUTOCORRELATIONS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
         1     0.15756  |            .      |***   .             |
         2     0.00781  |            .      |      .             |
         3     0.34108  |            .      |*******             |
         4    -0.12231  |            .    **|      .             |
         5     0.00314  |            .      |      .             |
         6     0.01399  |            .      |      .             |
         7     0.01224  |            .      |      .             |
         8    -0.01454  |            .      |      .             |
         9     0.11015  |            .      |**    .             |
        10     0.00620  |            .      |      .             |

                     AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                              AUTOCORRELATIONS
 LAG   SQUARE DF    PROB
   6     5.60  6   0.470   0.158   0.032  0.339 -0.001 -0.022  0.126
                                   SAS                          12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =        B5
                    MEAN OF WORKING SERIES=   4.49667
                    STANDARD DEVIATION      =   2.12783
                    NUMBER OF OBSERVATIONS=        30
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      STD
  0    4.52766     1.00000 |                    |********************|        0
  1   0.709677     0.15674 |                .   |***    .            | 0.182574
  2   -1.67178    -0.36924 |            ******* |       .            | 0.187006
  3  -0.614223    -0.13566 |              . *** |        .           | 0.209905
  4   0.292387     0.06458 |             .      |*        .          | 0.212808
  5   0.139454     0.03080 |             .      |*        .          |  0.21346
  6   0.213731     0.04721 |             .      |*        .          | 0.213608
  7 -0.0601248    -0.01328 |             .      |         .          | 0.213956
  8  -0.643481    -0.14212 |             .  *** |         .          | 0.213983
  9  -0.522414    -0.11538 |             .   ** |         .          | 0.217107
 10  0.0617407     0.01364 |             .      |         .          | 0.219141
                          '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.21865  |               .****|       .            |
            2     0.36645  |               .    |*******             |
            3     0.02426  |               .    |       .            |
            4     0.02148  |               .    |       .            |
            5     0.08662  |               .    |**     .            |
            6     0.01198  |               .    |       .            |
            7     0.05845  |               .    |*      .            |
            8     0.10023  |               .    |**     .            |
            9     0.05235  |               .    |*      .            |
           10     0.04799  |               .    |*      .            |

                          PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.15674  |               .    |***    .            |
            2    -0.40372  |             ********       .            |
            3     0.01085  |               .    |       .            |
            4    -0.06937  |               .   *|       .            |
            5    -0.02711  |               .   *|       .            |
            6     0.06307  |               .    |*      .            |
            7    -0.05135  |               .   *|       .            |
            8    -0.11133  |               .  **|       .            |
            9    -0.09157  |               .  **|       .            |
           10    -0.06065  |               .   *|       .            |

                   AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                                AUTOCORRELATIONS
 LAG  SQUARE  DF   PROB
   6    6.42   6  0.378   0.157 -0.369 -0.136  0.065  0.031  0.047   12:42 MONDAY, OCTOBER 5, 1987
                                  SAS
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                           ARIMA PROCEDURE

                 NAME OF VARIABLE          =        B6
                 MEAN OF WORKING SERIES=   3.90333
                 STANDARD DEVIATION    =   2.06292
                 NUMBER OF OBSERVATIONS=        30
                           AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      STD
  0   4.25566    1.00000   |                    |********************|       0
  1  -0.552789  -0.12990   |              .  ***|      .             |  0.182574
  2  -0.406812  -0.09559   |              .   **|      .             |  0.185629
  3   0.437443   0.10279   |              .     |**    .             |  0.187263
  4   0.085343   0.02005   |             .      |       .            |  0.189134
  5   0.817843   0.19218   |             .      |****   .            |  0.189205
  6  -1.27698   -0.30007   |             .******|       .            |  0.195604
  7   0.236197   0.05550   |             .      |*      .            |  0.210389
  8   0.0830526  0.01952   |             .      |       .            |  0.210876
  9  -0.385826  -0.09066   |             .    **|       .            |  0.210936
 10  -0.292304  -0.06869   |             .     *|       .            |  0.212231
                           . MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    0.01110  |              .      |       .            |
          2    0.17511  |              .      |****   .            |
          3   -0.18919  |              .  ****|       .            |
          4    0.03787  |              .      |*      .            |
          5   -0.21713  |              .  ****|       .            |
          6    0.22299  |              .      |****   .            |
          7   -0.03498  |              .     *|       .            |
          8    0.10951  |              .      |**     .            |
          9    0.04786  |              .      |*      .            |
         10    0.11592  |              .      |**     .            |

                        PARTIAL AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1   -0.12990  |              .   ***|       .            |
          2   -0.11440  |              .    **|       .            |
          3    0.07636  |              .      |**     .            |
          4    0.03533  |              .      |*      .            |
          5    0.22582  |              .      |*****  .            |
          6   -0.26738  |              . *****|       .            |
          7    0.03030  |              .      |*      .            |
          8   -0.08778  |              .    **|       .            |
          9   -0.03499  |              .     *|       .            |
         10   -0.13952  |              .   ***|       .            |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

 TO   CHI                             AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    6.28  6  0.392 -0.130 -0.096  0.103  0.020  0.192 -0.300
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SAS 12:42 MONDAY, OCTOBER 5, 1987

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                                ARIMA PROCEDURE

                       NAME OF VARIABLE        =        B7
                       MEAN OF WORKING SERIES=   2.75667
                       STANDARD DEVIATION      =   1.57452
                       NUMBER OF OBSERVATIONS=        30
                                AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      STD
  0   2.47912     1.00000   |                    |********************|      0
  1  -0.511729   -0.20642   |                ****|      .             |  0.182574
  2   1.02654     0.41407   |                    |********            |  0.190194
  3  -0.437066   -0.17630   |            .   ****|        .           |  0.218184
  4   0.537716    0.21690   |            .       |****    .           |  0.222882
  5  -0.248813   -0.10036   |            .     **|        .           |  0.22981
  6   0.31268     0.12613   |            .       |***     .           |  0.231267
  7  -0.870349   -0.35107   |            .*******|        .           |  0.233548
  8  -0.33489    -0.13508   |           .     ***|         .          |  0.250522
  9  -0.209986   -0.08470   |           .      **|         .          |  0.252939
 10  -0.288159   -0.11623   |           .      **|         .          |  0.253882
                            '.' MARKS TWO STANDARD ERRORS

                           INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    0.10318   |                    |**     .            |
            2   -0.37825   |            ********|       .            |
            3    0.02025   |             .      |       .            |
            4    0.03499   |             .      |*      .            |
            5   -0.21618   |             .  ****|       .            |
            6   -0.11661   |             .    **|       .            |
            7    0.36096   |             .      |*******.            |
            8    0.19961   |             .      |****   .            |
            9   -0.09582   |             .    **|       .            |
           10   -0.00428   |             .      |       .            |

                           PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.20642   |             .  ****|       .            |
            2    0.38800   |             .      |********            |
            3   -0.05398   |             .     *|       .            |
            4    0.03618   |             .      |*      .            |
            5    0.01862   |             .      |       .            |
            6    0.01705   |             .      |       .            |
            7   -0.35606   |             *******|       .            |
            8   -0.37136   |             *******|       .            |
            9    0.14940   |             .      |***    .            |
           10    0.00678   |             .      |       .            |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                            AUTOCORRELATIONS
 LAG   SQUARE DF   PROB
   6    11.15  6   0.084 -0.206   0.414 -0.176   0.217 -0.100   0.126
                                  SAS                                  12:42 MONDAY, OCTOBER 5, 1987
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                         ARIMA PROCEDURE

                   NAME OF VARIABLE        =        B8
                   MEAN OF WORKING SERIES=    2.73333
                   STANDARD DEVIATION      =    3.06446
                   NUMBER OF OBSERVATIONS=         30
                          AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1          STD
  0    9.39089     1.00000 |                    |********************|          0
  1     3.2033     0.34111 |              .     |*******             |   0.182574
  2    1.99281     0.21221 |             .      |****    .           |   0.202707
  3    2.60356     0.27724 |             .      |******  .           |   0.209982
  4   0.445519     0.04744 |            .       |*        .          |   0.221848
  5   0.984148     0.10480 |            .       |**       .          |   0.222186
  6  -0.473333    -0.05040 |            .      *|         .          |   0.223828
  7   0.308296     0.03283 |            .       |*        .          |   0.224206
  8    1.90059     0.20239 |            .       |****     .          |   0.224366
  9  -0.924556    -0.09845 |            .     **|         .          |   0.230371
 10  -0.733148    -0.07807 |            .     **|         .          |   0.231769
                          '.' MARKS TWO STANDARD ERRORS

                    INVERSE AUTOCORRELATIONS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
         1    -0.29149 |              .******|     .              |
         2    -0.01466 |              .      |     .              |
         3    -0.22760 |              . *****|     .              |
         4     0.11758 |              .      |**   .              |
         5    -0.03351 |              .     *|     .              |
         6     0.15140 |              .      |***  .              |
         7    -0.03746 |              .     *|     .              |
         8    -0.22547 |              . *****|     .              |
         9     0.10198 |              .      |**   .              |
        10     0.06744 |              .      |*    .              |

                    PARTIAL AUTOCORRELATIONS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
         1     0.34111 |              .      |*******             |
         2     0.10847 |              .      |**   .              |
         3     0.20121 |              .      |**** .              |
         4    -0.13067 |              .   ***|     .              |
         5     0.08424 |              .      |**   .              |
         6    -0.17821 |              .  ****|     .              |
         7     0.13725 |              .      |***  .              |
         8     0.16688 |              .      |***  .              |
         9    -0.20288 |              .  ****|     .              |
        10    -0.09083 |              .    **|     .              |

              AUTOCORRELATION CHECK FOR WHITE NOISE

    TO    CHI                        AUTOCORRELATIONS

   LAG  SQUARE DF   PROB
     6    8.74  6  0.189  0.341  0.212  0.277  0.047  0.105 -0.050
                                  SAS                 12:42 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =         B9
                    MEAN OF WORKING SERIES=         3.17
                    STANDARD DEVIATION      =    1.47313
                    NUMBER OF OBSERVATIONS=           30
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0     2.1701     1.00000 |                    |********************|        0
  1   -0.29153    -0.13434 |                .***|   .                |  0.182574
  2  -0.525627    -0.24221 |               *****|   .                |   0.18584
  3  0.0997767     0.04598 |              .     |*   .               |  0.196081
  4  -0.481353    -0.22204 |              . ****|    .               |   0.19644
  5    0.46615     0.21481 |              .     |****.               |  0.204635
  6    0.34162     0.15742 |              .     |***.                |  0.212018
  7  -0.267277    -0.12316 |             .    **|    .               |  0.215879
  8   -0.32724    -0.15079 |             .   ***|    .               |  0.218208
  9 -0.0766367    -0.03531 |             .     *|    .               |  0.221655
 10   0.246933     0.11379 |             .      |**  .               |  0.221842
                            '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.12628 |              .     |***  .              |
            2     0.28686 |              .     |******.             |
            3    -0.02893 |              .    *|     .              |
            4     0.25037 |              .     |*****.              |
            5    -0.11637 |              .   **|     .              |
            6     0.00631 |              .     |     .              |
            7     0.03441 |              .     |*    .              |
            8     0.13900 |              .     |***  .              |
            9     0.01242 |              .     |     .              |
           10    -0.03098 |              .    *|     .              |

                          PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.13434 |              .  ***|     .              |
            2    -0.26504 |              .*****|     .              |
            3    -0.03373 |              .    *|     .              |
            4    -0.31135 |             ******|     .              |
            5     0.15251 |              .     |***  .              |
            6     0.08178 |              .     |**   .              |
            7     0.03100 |              .     |*    .              |
            8    -0.19181 |              . ****|     .              |
            9    -0.02789 |              .    *|     .              |
           10     0.03916 |              .     |*    .              |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                           AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    7.27  6  0.297 -0.134 -0.242  0.046 -0.222  0.215  0.157
                              SAS                                   12:42 MONDAY, OCTOBER 5, 1987
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                                  ARIMA PROCEDURE

                          NAME OF VARIABLE        =        B10
                          MEAN OF WORKING SERIES=         1.75
                          STANDARD DEVIATION      =    2.43143
                          NUMBER OF OBSERVATIONS=           30
                                  AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1          STD
  0    5.91183    1.00000  |                    |********************|          0
  1    1.15208    0.19488  |                  . |****  .             |   0.182574
  2    1.47583    0.24964  |                 .  |*****  .            |   0.189381
  3    1.41358    0.23911  |                 .  |*****  .            |    0.20005
  4   0.314667    0.05323  |                 .  |*      .            |    0.20936
  5    1.52342    0.25769  |                 .  |*****  .            |    0.20981
  6   0.593667    0.10042  |                .   |**      .           |   0.220107
  7    1.06442    0.18005  |                .   |****    .           |   0.221629
  8   0.916667    0.15506  |                .   |***     .           |   0.226452
  9 0.00891667    0.00151  |                .   |        .           |   0.229964
 10  0.0388333    0.00657  |                .   |        .           |   0.229964
                           '.' MARKS TWO STANDARD ERRORS

                             INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    -0.08506  |                .  **|      .             |
             2    -0.10941  |                .  **|      .             |
             3    -0.13372  |                . ***|      .             |
             4     0.09586  |                .    |**    .             |
             5    -0.14349  |                . ***|      .             |
             6     0.02797  |                .    |*     .             |
             7    -0.13122  |                . ***|      .             |
             8    -0.07424  |                .   *|      .             |
             9     0.04813  |                .    |*     .             |
            10     0.10112  |                .    |**    .             |

                             PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.19488  |                .    |****  .             |
             2     0.22002  |                .    |****  .             |
             3     0.17291  |                .    |***   .             |
             4    -0.06315  |                .   *|      .             |
             5     0.19122  |                .    |****  .             |
             6     0.00800  |                .    |      .             |
             7     0.10181  |                .    |**    .             |
             8     0.02725  |                .    |*     .             |
             9    -0.08498  |                .  **|      .             |
            10    -0.11571  |                .  **|      .             |

                      AUTOCORRELATION CHECK FOR WHITE NOISE

         TO     CHI                           AUTOCORRELATIONS
        LAG   SQUARE DF   PROB
          6     8.48  6  0.205   0.195   0.250   0.239   0.053   0.258   0.100     12:42 MONDAY, OCTOBER 5, 1987
                                         SAS
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =        B11
                    MEAN OF WORKING SERIES=      25.7533
                    STANDARD DEVIATION      =    17.3525
                    NUMBER OF OBSERVATIONS=           30
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    301.11     1.00000  |                    |********************|         0
  1   203.829     0.67692  |                .   |**************      |  0.182574
  2   185.139     0.61485  |              .     |************        |  0.252748
  3   175.348     0.58234  |             .      |************        |   0.29847
  4   127.343     0.42291  |            .       |********   .        |  0.334204
  5   142.935     0.47469  |           .        |*********   .       |  0.351591
  6   92.4682     0.30709  |           .        |******       .      |  0.372342
  7   58.3148     0.19367  |           .        |****         .      |  0.380691
  8   28.4353     0.09443  |           .        |**           .      |  0.383961
  9   1.65364     0.00549  |           .        |             .      |  0.384734
 10  -5.76119    -0.01913  |           .        |             .      |  0.384737
                           '.' MARKS TWO STANDARD ERRORS

                           INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.39549  |           ********|           .        |
            2     0.13276  |           .       |***        .        |
            3    -0.31940  |           .*******|           .        |
            4     0.31233  |           .       |******     .        |
            5    -0.36177  |           ********|           .        |
            6     0.09598  |           .       |**         .        |
            7    -0.07369  |           .      *|           .        |
            8     0.16188  |           .       |***        .        |
            9    -0.05475  |           .      *|           .        |
           10     0.02759  |           .       |*          .        |

                           PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.67692  |           .       |**************      |
            2     0.28910  |           .       |******.             |
            3     0.18280  |           .       |****   .            |
            4    -0.15411  |           .    ***|       .            |
            5     0.20868  |           .       |****   .            |
            6    -0.23593  |           .  *****|       .            |
            7    -0.12503  |           .    ***|       .            |
            8    -0.22517  |           .  *****|       .            |
            9     0.04611  |           .       |*      .            |
           10    -0.04606  |           .      *|       .            |

                     AUTOCORRELATION CHECK FOR WHITE NOISE

      TO   CHI                            AUTOCORRELATIONS
     LAG  SQUARE DF   PROB
       6   59.22  6  0.000  0.677   0.615  0.582  0.423  0.475  0.307
                                    SAS                                 12:42 MONDAY, OCTOBER 5, 1987
```

```
                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =      B12
                    MEAN OF WORKING SERIES=    3.06667
                    STANDARD DEVIATION      =  2.19884
                    NUMBER OF OBSERVATIONS=        30
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    4.83489    1.00000  |                    |********************|          0
  1   0.954185    0.19735  |                .   |****      .         |   0.182574
  2    2.13293    0.44115  |                .   |*********           |   0.189552
  3   0.564889    0.11684  |              .     |**        .         |   0.221143
  4  0.0462963    0.00958  |              .     |          .         |   0.223191
  5  -0.402852   -0.08332  |              .   **|          .         |   0.223205
  6   -1.87056   -0.38689  |              .*****|***       .         |   0.224239
  7  -0.359481   -0.07435  |             .     *|           .        |   0.245483
  8   -1.37507   -0.28441  |             .******|           .        |   0.246233
  9   0.456444    0.09441  |             .      |**         .        |    0.25695
 10  -0.600593   -0.12422  |             .    **|           .        |   0.258103
                           '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    0.08776  |              .     |**      .           |
             2   -0.34865  |              *******|       .           |
             3   -0.37417  |              *******|       .           |
             4   -0.10116  |              .   **|       .           |
             5    0.13671  |              .     |***     .           |
             6    0.36373  |              .     |*******             |
             7    0.04028  |              .     |*       .           |
             8   -0.11128  |              .   **|        .           |
             9   -0.17617  |              . ****|        .           |
            10    0.08286  |              .     |**      .           |

                         PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    0.19735  |              .     |****    .           |
             2    0.41850  |              .     |********            |
             3   -0.02067  |              .     |        .           |
             4   -0.23528  |              . *****|       .           |
             5   -0.13688  |              .   ***|       .           |
             6   -0.36804  |              *******|       .           |
             7    0.15339  |              .     |***     .           |
             8    0.10261  |              .     |**      .           |
             9    0.27290  |              .     |*****   .           |
            10   -0.13705  |              .   ***|       .           |

                     AUTOCORRELATION CHECK FOR WHITE NOISE

        TO    CHI                        AUTOCORRELATIONS
       LAG  SQUARE DF   PROB
         6   14.70  6  0.023  0.197  0.441  0.117  0.010 -0.083 -0.387
                                     SAS                         12:42 MONDAY, OCTOBER 5, 1987
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```
                              ARIMA PROCEDURE

                     NAME OF VARIABLE       =       B13
                     MEAN OF WORKING SERIES=       11.25
                     STANDARD DEVIATION     =    9.35617
                     NUMBER OF OBSERVATIONS=         30
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0    87.5378     1.00000 |                    |********************|          0
  1    69.0171     0.78843 |                .   |****************    |   0.182574
  2    53.5328     0.61154 |             .      |************        |   0.273449
  3    56.5819     0.64637 |           .        |*************       |   0.315763
  4    51.9773     0.59377 |          .         |************ .      |   0.357155
  5    32.1162     0.36688 |     .              |*******       .     |   0.388669
  6    26.1013     0.29817 |   . .              |******        .     |   0.400046
  7    24.1731     0.27614 |     .              |******        .     |   0.407387
  8    11.6782     0.13341 |    .               |***        .   .    |    0.41358
  9   -1.97825    -0.02260 |    ..              |                .   |   0.415011
 10   -7.87917    -0.09001 |                  **|                .   |   0.415052
                            '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.64532 |       *************|       .            |
            2     0.21583 |             .      |****   .            |
            3     0.10623 |             .      |**     .            |
            4    -0.42439 |            ********|       .            |
            5     0.43180 |             .      |*********           |
            6    -0.22277 |             .  ****|       .            |
            7     0.01671 |             .      |       .            |
            8     0.05386 |             .      |*      .            |
            9    -0.09509 |             .    **|       .            |
           10     0.07555 |             .      |**     .            |

                          PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.78843 |             .      |****************    |
            2    -0.02663 |             .     *|       .            |
            3     0.45587 |             .      |*********           |
            4    -0.18277 |             .  ****|       .            |
            5    -0.32625 |             *******|       .            |
            6     0.16977 |             .      |***    .            |
            7    -0.23351 |             . *****|       .            |
            8    -0.02593 |             .     *|       .            |
            9    -0.12438 |             .    **|       .            |
           10    -0.21573 |             .  ****|       .            |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

   TO     CHI                           AUTOCORRELATIONS
  LAG  SQUARE DF    PROB
    6   70.00  6   0.000   0.788   0.612   0.646   0.594   0.367   0.298
                                   SAS                          12:42 MONDAY, OCTOBER 5, 1987
```

```
                              ARIMA PROCEDURE

                       NAME OF VARIABLE        =       B14
                       MEAN OF WORKING SERIES=    10.4167
                       STANDARD DEVIATION     =   4.89027
                       NUMBER OF OBSERVATIONS=         30
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    23.9147    1.00000  |                    |********************|          0
  1    12.4696    0.52142  |                .   |**********          |   0.182574
  2    9.21081    0.38515  |              .     |********.           |   0.226845
  3    9.81681    0.41049  |             .      |********.           |   0.247685
  4    7.29857    0.30519  |            .       |****** .            |    0.26941
  5    4.29523    0.17961  |            .       |****    .           |   0.280698
  6    3.49789    0.14627  |            .       |***     .           |   0.284502
  7   0.223269    0.00934  |            .       |        .           |   0.286998
  8  -0.771685   -0.03227  |            .      *|        .           |   0.287008
  9    3.53575    0.14785  |            .       |***     .           |   0.287129
 10  -0.452426   -0.01892  |            .       |        .           |   0.289656
                           '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    -0.31910  |              .******|   .                |
           2    -0.01593  |              .      |   .                |
           3    -0.10536  |              .    **|   .                |
           4    -0.13177  |              .   ***|   .                |
           5     0.06135  |              .      |*  .                |
           6    -0.00749  |              .      |   .                |
           7     0.05631  |              .      |*  .                |
           8     0.18166  |              .      |****                |
           9    -0.28651  |              .******|   .                |
          10     0.10810  |              .      |** .                |

                        PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1     0.52142  |              .      |**********          |
           2     0.15557  |              .      |***.                |
           3     0.22490  |              .      |****                |
           4    -0.00035  |              .      |   .                |
           5    -0.07980  |              .    **|   .                |
           6    -0.02123  |              .      |   .                |
           7    -0.16427  |              .   ***|   .                |
           8    -0.02753  |              .     *|   .                |
           9     0.27932  |              .      |******.             |
          10    -0.15514  |              .   ***|   .                |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                               AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
   6   25.61  6  0.000   0.521  0.385  0.410  0.305  0.180  0.146
```

4 - ARIMA for Tojeira 1 and 2 sections with Gradstein's data

```
                          NAME OF VARIABLE       =        A1
                          MEAN OF WORKING SERIES=    1.15882
                          STANDARD DEVIATION     =   1.63529
                          NUMBER OF OBSERVATIONS         17
                                   AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    2.67419    1.00000   |                    |********************|
 1    0.686302   0.25664   |          .         |*****               |
 2    0.496583   0.18569   |          .         |****     .          |
 3    1.10119    0.41178   |         .          |********  .         |
 4   -0.07552   -0.02824   |        .          *|           .        |
 5   -0.268423  -0.10038   |        .         **|           .        |
 6   -0.188695  -0.07056   |        .          *|           .        |
                           '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.12821   |          .      ***|          .         |
           2   -0.02767   |          .        *|          .         |
           3   -0.46485   |          .*********|          .         |
           4    0.10400   |          .         |**        .         |
           5    0.06690   |          .         |*         .         |
           6    0.13544   |          .         |***       .         |

                        PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.25664   |          .         |*****     .         |
           2    0.12828   |          .         |***       .         |
           3    0.36714   |          .         |*******   .         |
           4   -0.25633   |          .    *****|          .         |
           5   -0.17154   |          .      ***|          .         |
           6   -0.18741   |          .     ****|          .         |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                            AUTOCORRELATIONS
LAG   SQUARE DF   PROB
 6      6.42  6  0.378   0.257   0.186   0.412 -0.028 -0.100 -0.071
```

```
                    NAME OF VARIABLE          =        A2
                    MEAN OF WORKING SERIES=     21.6412
                    STANDARD DEVIATION        =  11.8928
                    NUMBER OF OBSERVATIONS=          17
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    141.439     1.00000   |                    |********************|
 1    38.1513     0.26974   |          .         |*****     .         |
 2   -20.0548    -0.14179   |          .      ***|          .         |
 3   -4.20078    -0.02970   |          .        *|          .         |
 4    4.50278     0.03184   |          .         |*         .         |
 5   -5.85604    -0.04140   |          .        *|          .         |
 6    -31.831    -0.22505   |          .    *****|          .         |
                            '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1   -0.38177    |          . ********|          .         |
          2    0.26639    |          .         |*****     .         |
          3   -0.12548    |          .      ***|          .         |
          4    0.10370    |          .         |**        .         |
          5   -0.08331    |          .       **|          .         |
          6    0.17286    |          .         |***       .         |

                          PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    0.26974    |          .         |*****     .         |
          2   -0.23138    |          .    *****|          .         |
          3    0.09094    |          .         |**        .         |
          4   -0.02217    |          .         |          .         |
          5   -0.04675    |          .        *|          .         |
          6   -0.21617    |          .     ****|          .         |

                   AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                                AUTOCORRELATIONS
LAG   SQUARE  DF   PROB
 6     3.48    6  0.747   0.270 -0.142 -0.030  0.032 -0.041 -0.225
```

```
                    NAME OF VARIABLE       =        A3
                    MEAN OF WORKING SERIES=   6.40588
                    STANDARD DEVIATION     =  5.68118
                    NUMBER OF OBSERVATIONS=       17
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    32.2758     1.00000   |                    |********************|
 1   -1.97734    -0.06126   |           .       *|         .          |
 2    2.95346     0.09151   |           .        |**       .          |
 3    5.42415     0.16806   |           .        |***      .          |
 4   -6.07136    -0.18811   |           .    ****|         .          |
 5   -8.05679    -0.24962   |           .   *****|         .          |
 6   -13.7782    -0.42689   |         .*********|          .          |
                             '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.30357   |           .        |******   .          |
           2   -0.13706   |           .     ***|         .          |
           3   -0.29889   |           .  ******|         .          |
           4    0.10453   |           .        |**       .          |
           5    0.32608   |           .        |*******  .          |
           6    0.33585   |           .        |*******  .          |

                         PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.06126   |           .       *|         .          |
           2    0.08808   |           .        |**       .          |
           3    0.18064   |           .        |****     .          |
           4   -0.18192   |           .    ****|         .          |
           5   -0.32762   |           . *******|         .          |
           6   -0.55264   |         ***********|         .          |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                              AUTOCORRELATIONS
LAG   SQUARE DF    PROB
 6      8.82  6   0.184 -0.061  0.092  0.168 -0.188 -0.250 -0.427
```

```
                         NAME OF VARIABLE         =        A4
                         MEAN OF WORKING SERIES=  0.288235
                         STANDARD DEVIATION       =  0.722644
                         NUMBER OF OBSERVATIONS=        17
                                  AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    0.522215     1.00000  |                    |********************|
 1 -.00550987     -0.01055  |           .        |        .           |
 2 -0.0809851     -0.15508  |           .     ***|        .           |
 3 -0.0670486     -0.12839  |           .     ***|        .           |
 4  0.0833584      0.15962  |           .        |***     .           |
 5 -0.0820822     -0.15718  |           .     ***|        .           |
 6 -0.0869693     -0.16654  |           .     ***|        .           |
                            '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.03359   |           .        |*       .           |
            2     0.13275   |           .        |***     .           |
            3     0.21052   |           .        |****    .           |
            4    -0.07173   |           .       *|        .           |
            5     0.18067   |           .        |****    .           |
            6     0.13800   |           .        |***     .           |

                        PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.01055   |           .        |        .           |
            2    -0.15521   |           .     ***|        .           |
            3    -0.13519   |           .     ***|        .           |
            4     0.13530   |           .        |***     .           |
            5    -0.20316   |           .    ****|        .           |
            6    -0.15596   |           .     ***|        .           |

                   AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                          AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
  6     3.01  6  0.807 -0.011 -0.155 -0.128  0.160 -0.157 -0.167
```

```
                    NAME OF VARIABLE        =       A5
                    MEAN OF WORKING SERIES=   14.5647
                    STANDARD DEVIATION      =  5.49309
                    NUMBER OF OBSERVATIONS=        17
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0     30.174     1.00000 |                    |********************|
  1   -7.01419    -0.23246 |          .    *****|         .          |
  2   -0.62423    -0.02069 |          .         |         .          |
  3    2.02853     0.06723 |          .         |*        .          |
  4   -9.33289    -0.30930 |          .   ******|         .          |
  5   0.590222     0.01956 |          .         |         .          |
  6     8.3642     0.27720 |          .         |******   .          |
                            '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.20588 |          .         |****     .          |
            2    -0.05378 |          .        *|         .          |
            3     0.07858 |          .         |**       .          |
            4     0.26487 |          .         |*****    .          |
            5     0.00316 |          .         |         .          |
            6    -0.21255 |          .     ****|         .          |

                        PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.23246 |          .    *****|         .          |
            2    -0.07899 |          .       **|         .          |
            3     0.04646 |          .         |*        .          |
            4    -0.30199 |          .   ******|         .          |
            5    -0.13775 |          .      ***|         .          |
            6     0.26070 |          .         |*****    .          |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

    TO     CHI                              AUTOCORRELATIONS
   LAG  SQUARE  DF   PROB
     6    5.85   6  0.440  -0.232  -0.021   0.067  -0.309   0.020   0.277
```

```
                         NAME OF VARIABLE        =        A6
                         MEAN OF WORKING SERIES=   2.41176
                         STANDARD DEVIATION      =   1.31412
                         NUMBER OF OBSERVATIONS=        17
                                   AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0     1.72692     1.00000  |                    |********************|
 1    0.763244     0.44197  |             .      |*********.          |
 2   -0.072473    -0.04197  |            .      *|          .         |
 3   -0.481824    -0.27901  |            .  ******|         .         |
 4   -0.333424    -0.19307  |           .     ****|          .        |
 5   0.0297171     0.01721  |           .        |           .        |
 6    0.124865     0.07230  |           .        |*          .        |
                            '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.43875  |          .*********|         .          |
            2     0.05191  |          .        |*         .          |
            3     0.12306  |          .        |**        .          |
            4     0.06143  |          .        |*         .          |
            5    -0.09927  |          .      **|          .          |
            6     0.04964  |          .        |*         .          |

                        PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.44197  |          .        |*********.           |
            2    -0.29491  |          .  ******|          .          |
            3    -0.16966  |          .     ***|          .          |
            4     0.02709  |          .        |*         .          |
            5     0.06389  |          .        |*         .          |
            6    -0.06758  |          .       *|          .          |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

   TO    CHI                          AUTOCORRELATIONS
  LAG   SQUARE DF   PROB
   6      6.86  6  0.334  0.442 -0.042 -0.279 -0.193   0.017   0.072
```

```
                        NAME OF VARIABLE       =       A7
                        MEAN OF WORKING SERIES=   2.72941
                        STANDARD DEVIATION     =   2.1687
                        NUMBER OF OBSERVATIONS=       17
                                  AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0     4.70325   1.00000   |                    |********************|
 1 -0.0227498   -0.00484   |          .         |          .         |
 2  -0.925361   -0.19675   |          .     ****|          .         |
 3   -1.59756   -0.33967   |          .  *******|          .         |
 4   0.032357    0.00688   |          .         |          .         |
 5   -0.35676   -0.07585   |          .       **|          .         |
 6  -0.129821   -0.02760   |          .        *|          .         |
                           '.' MARKS TWO STANDARD ERRORS

                             INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    0.39905   |          .         |********  .         |
            2    0.42832   |          .         |*********.          |
            3    0.49050   |          .         |**********          |
            4    0.19623   |          .         |****      .         |
            5    0.21231   |          .         |****      .         |
            6    0.15815   |          .         |***       .         |

                             PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.00484   |          .         |          .         |
            2   -0.19678   |          .     ****|          .         |
            3   -0.35554   |          .  *******|          .         |
            4   -0.07108   |          .        *|          .         |
            5   -0.26194   |          .    *****|          .         |
            6   -0.25259   |          .    *****|          .         |

                      AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                         AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
  6     3.67  6  0.721 -0.005 -0.197 -0.340  0.007 -0.076 -0.028
```

```
                    NAME OF VARIABLE       =          A8
                    MEAN OF WORKING SERIES=    7.09412
                    STANDARD DEVIATION     =   4.53878
                    NUMBER OF OBSERVATIONS=         17
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    20.6006    1.00000  |                    |********************|
  1    4.72761    0.22949  |          .         |*****     .         |
  2  -0.795194   -0.03860  |          .        *|          .         |
  3   -0.18575   -0.00902  |          .         |          .         |
  4   -5.54001   -0.26893  |          .    *****|          .         |
  5   -4.23105   -0.20539  |          .     ****|          .         |
  6   -3.65351   -0.17735  |          .     ****|          .         |
                           '.' MARKS TWO STANDARD ERRORS

                      INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.24570  |          .    *****|          .         |
            2    0.24780  |          .         |*****     .         |
            3   -0.15264  |          .      ***|          .         |
            4    0.26483  |          .         |*****     .         |
            5   -0.00675  |          .         |          .         |
            6    0.14920  |          .         |***       .         |

                      PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    0.22949  |          .         |*****     .         |
            2   -0.09634  |          .       **|          .         |
            3    0.02430  |          .         |          .         |
            4   -0.29442  |          .   ******|          .         |
            5   -0.07585  |          .       **|          .         |
            6   -0.18121  |          .     ****|          .         |

                AUTOCORRELATION CHECK FOR WHITE NOISE

   TO     CHI                        AUTOCORRELATIONS
  LAG   SQUARE DF    PROB
    6     4.95  6   0.550   0.229 -0.039 -0.009 -0.269 -0.205 -0.177
```

```
                    NAME OF VARIABLE          =        A9
                    MEAN OF WORKING SERIES=   3.14706
                    STANDARD DEVIATION    =   3.33363
                    NUMBER OF OBSERVATIONS=        17
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    11.1131    1.00000  |                    |********************|
 1   -3.12923   -0.28158  |          .   ******|          .         |
 2   0.370916    0.03338  |          .         |*         .         |
 3     -3.017   -0.27148  |          .    *****|          .         |
 4   0.334046    0.03006  |          .         |*          .        |
 5  -0.891586   -0.08023  |          .       **|           .        |
 6    1.18718    0.10683  |          .         |**         .        |
                           '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    0.51174   |          .         |**********          |
          2    0.38165   |          .         |********  .         |
          3    0.39483   |          .         |********  .         |
          4    0.23210   |          .         |*****     .         |
          5    0.16276   |          .         |***       .         |
          6    0.05390   |          .         |*         .         |

                          PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1   -0.28158   |          .   ******|          .         |
          2   -0.04986   |          .        *|          .         |
          3   -0.30014   |          .   ******|          .         |
          4   -0.16052   |          .      ***|          .         |
          5   -0.18010   |          .     ****|          .         |
          6   -0.08077   |          .       **|          .         |

                   AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                             AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  5    3.86  6  0.696 -0.282  0.033 -0.271  0.030 -0.080  0.107
```

```
                    NAME OF VARIABLE        =        A10
                    MEAN OF WORKING SERIES=          3.4
                    STANDARD DEVIATION      =    3.29349
                    NUMBER OF OBSERVATIONS=           17
                             AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    10.8471    1.00000  |                    |********************|
  1    3.67412    0.33872  |          .         |*******  .          |
  2    3.52882    0.32533  |          .         |*******  .          |
  3   0.995294    0.09176  |          .         |**       .          |
  4   -1.19529   -0.11020  |          .       **|         .          |
  5   -1.03412   -0.09534  |          .       **|         .          |
  6  -0.149412   -0.01377  |          .         |         .          |
                            '.' MARKS TWO STANDARD ERRORS

                      INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1   -0.12361  |          .       **|         .          |
             2   -0.38734  |          .********|         .          |
             3   -0.06015  |          .        *|         .          |
             4    0.24724  |          .         |*****    .          |
             5    0.07393  |          .         |*        .          |
             6   -0.12352  |          .       **|         .          |

                      PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    0.33872  |          .         |*******  .          |
             2    0.23789  |          .         |*****    .          |
             3   -0.08717  |          .       **|         .          |
             4   -0.23041  |          .    *****|         .          |
             5   -0.01366  |          .         |         .          |
             6    0.16058  |          .         |***      .          |

                AUTOCORRELATION CHECK FOR WHITE NOISE

   TO    CHI                        AUTOCORRELATIONS
  LAG  SQUARE DF   PROB
    6    5.34  6  0.501   0.339  0.325  0.092 -0.110 -0.095 -0.014
```

```
                   NAME OF VARIABLE        =       A11
                   MEAN OF WORKING SERIES=   14.4235
                   STANDARD DEVIATION      =   7.03241
                   NUMBER OF OBSERVATIONS=        17
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    49.4547    1.00000   |                   |********************|
  1   0.644812    0.01304   |         .         |          .         |
  2    4.94326    0.09996   |         .         |**        .         |
  3  -0.633246   -0.01280   |         .         |          .         |
  4   -7.33985   -0.14842   |         .      ***|          .         |
  5    -12.558   -0.25393   |         .    *****|          .         |
  6   -16.3663   -0.33093   |         .  *******|          .         |
                            '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    0.18872    |         .         |****      .         |
            2   -0.02907    |         .        *|          .         |
            3   -0.02585    |         .        *|          .         |
            4    0.10697    |         .         |**        .         |
            5    0.25151    |         .         |*****     .         |
            6    0.27632    |         .         |******    .         |

                        PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

            1    0.01304    |         .         |          .         |
            2    0.09980    |         .         |**        .         |
            3   -0.01544    |         .         |          .         |
            4   -0.15969    |         .      ***|          .         |
            5   -0.25656    |         .    *****|          .         |
            6   -0.33594    |         .  *******|          .         |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                              AUTOCORRELATIONS
  LAG  SQUARE DF   PROB
   6     5.72  6  0.455   0.013   0.100 -0.013 -0.148 -0.254 -0.331
```

```
                  NAME OF VARIABLE        =        A12
                  MEAN OF WORKING SERIES=    5.60588
                  STANDARD DEVIATION     =   6.50325
                  NUMBER OF OBSERVATIONS=        17
                           AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    42.2923    1.00000  |                    |********************|
  1   -6.11872   -0.14468  |           .     ***|         .          |
  2   -8.38865   -0.19835  |           .    ****|         .          |
  3   -1.09246   -0.02583  |           .       *|         .          |
  4   -8.35198   -0.19748  |           .    ****|         .          |
  5    7.22092    0.17074  |           .        |***       .         |
  6   -4.75818   -0.11251  |           .      **|          .         |
                            '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1    0.28760  |           .        |******   .          |
              2    0.43729  |           .        |*********.          |
              3    0.20307  |           .        |****     .          |
              4    0.30856  |           .        |******   .          |
              5    0.02857  |           .        |*        .          |
              6    0.16185  |           .        |***      .          |

                        PARTIAL AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1   -0.14468  |           .     ***|         .          |
              2   -0.22397  |           .    ****|         .          |
              3   -0.10039  |           .      **|         .          |
              4   -0.28798  |           .  ******|         .          |
              5    0.05283  |           .        |*        .          |
              6   -0.22173  |           .    ****|         .          |

                AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                               AUTOCORRELATIONS
LAG   SQUARE DF    PROB
  6     3.41  6   0.756 -0.145 -0.198 -0.026 -0.197  0.171 -0.113
```

```
                    NAME OF VARIABLE       =       A13
                    MEAN OF WORKING SERIES=   3.28824
                    STANDARD DEVIATION     =  4.60395
                    NUMBER OF OBSERVATIONS=        17
                             AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    21.1963    1.00000  |                    |********************|
  1    6.96954    0.32881  |          .         |*******    .        |
  2    2.68746    0.12679  |          .         |***        .        |
  3   -1.21996   -0.05756  |          .        *|           .        |
  4   -4.94429   -0.23326  |          .    *****|           .        |
  5   -2.81963   -0.13302  |          .      ***|           .        |
  6  -0.211018   -0.00996  |          .         |           .        |
                            '.' MARKS TWO STANDARD ERRORS

                       INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.23231  |          .    *****|         .          |
           2   -0.12077  |          .       **|         .          |
           3   -0.01969  |          .         |         .          |
           4    0.19486  |          .         |****     .          |
           5    0.02428  |          .         |         .          |
           6   -0.06713  |          .        *|         .          |

                       PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.32881  |          .         |*******  .          |
           2    0.02094  |          .         |         .          |
           3   -0.11807  |          .       **|         .          |
           4   -0.20845  |          .     ****|         .          |
           5    0.01896  |          .         |         .          |
           6    0.07759  |          .         |**       .          |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                          AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
   6    4.44  6  0.618   0.329  0.127 -0.058 -0.233 -0.133 -0.010
```

```
                      NAME OF VARIABLE       =        A14
                      MEAN OF WORKING SERIES=     12.3882
                      STANDARD DEVIATION     =    10.2683
                      NUMBER OF OBSERVATIONS=          17
                               AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    105.438    1.00000    |                    |********************|
 1    21.6447    0.20528    |         .          |****      .         |
 2   -4.39074   -0.04164    |         .         *|          .         |
 3   -8.37037   -0.07939    |         .        **|          .         |
 4  -0.807299   -0.00766    |         .          |          .         |
 5   -17.6811   -0.16769    |         .       ***|          .         |
 6   -28.1517   -0.26700    |         .     *****|          .         |
                             '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.15208    |         .       ***|          .         |
           2    0.05062    |         .          |*         .         |
           3    0.11229    |         .          |**        .         |
           4   -0.04696    |         .         *|          .         |
           5    0.09117    |         .          |**        .         |
           6    0.19157    |         .          |****      .         |

                          PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.20528    |         .          |****      .         |
           2   -0.08747    |         .        **|          .         |
           3   -0.05485    |         .         *|          .         |
           4    0.01880    |         .          |          .         |
           5   -0.18809    |         .      ****|          .         |
           6   -0.21330    |         .      ****|          .         |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                              AUTOCORRELATIONS
 LAG  SQUARE  DF    PROB
   6    3.89   6   0.692   0.205 -0.042 -0.079 -0.008 -0.168 -0.267
```

```
                    NAME OF VARIABLE        =          B1
                    MEAN OF WORKING SERIES=     13.5615
                    STANDARD DEVIATION      =   12.6542
                    NUMBER OF OBSERVATIONS=          13
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0     160.129    1.00000  |                    |********************|
 1   -0.878516   -0.00549  |                    |                    |
 2    -42.7192   -0.26678  |          .    *****|          .         |
 3    -69.7566   -0.43563  |          . *********|         .         |
 4    -14.6981   -0.09179  |       .          **|           .        |
 5     47.7994    0.29851  |       .            |******      .       |
 6      32.498    0.20295  |       .           .|****          .     |
                            '.' MARKS TWO STANDARD ERRORS

                       INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    0.50643   |          .         |**********.         |
            2    0.45188   |          .         |*********  .        |
            3    0.43911   |          .         |*********  .        |
            4    0.23761   |          .         |*****      .        |
            5    0.04899   |          .         |*          .        |
            6    0.05282   |          .         |*          .        |

                       PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.00549   |          .         |           .        |
            2   -0.26682   |          .    *****|           .        |
            3   -0.47261   |          .*********|           .        |
            4   -0.31109   |          .   ******|           .        |
            5   -0.03116   |          .        *|           .        |
            6   -0.09025   |          .       **|           .        |

                AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                          AUTOCORRELATIONS
 LAG   SQUARE  DF   PROB
   6     8.46   6  0.206 -0.005 -0.267 -0.436 -0.092  0.299  0.203
```

```
                    NAME OF VARIABLE        =        B2
                    MEAN OF WORKING SERIES=  0.346154
                    STANDARD DEVIATION     =  0.397328
                    NUMBER OF OBSERVATIONS=        13
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    0.15787    1.00000  |                    |********************|
  1 -0.0376195   -0.23829  |          .    *****|               .    |
  2 -0.0174875   -0.11077  |          .       **|               .    |
  3 -.00688?11   -0.04359  |          .        *|               .    |
  4  0.0223623    0.14165  |          .         |***            .    |
  5 -0.0342513   -0.21696  |          .     ****|               .    |
  6  0.0293127    0.18568  |          .         |****           .    |
                           '.' MARKS TWO STANDARD ERRORS

                      INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    0.25588  |          .         |*****          .    |
            2    0.22695  |          .         |*****          .    |
            3    0.11221  |          .         |**             .    |
            4   -0.03220  |          .        *|               .    |
            5    0.11236  |          .         |**             .    |
            6   -0.10546  |          .       **|               .    |

                      PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.23829  |          .    *****|               .    |
            2   -0.17764  |          .     ****|               .    |
            3   -0.12810  |          .      ***|               .    |
            4    0.08263  |          .         |**             .    |
            5   -0.19695  |          .     ****|               .    |
            6    0.12355  |          .         |**             .    |

               AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                          AUTOCORRELATIONS
 LAG  SQUARE  DF   PROB
   6    3.72   6  0.715  -0.238 -0.111 -0.044  0.142 -0.217  0.186
```

```
                    NAME OF VARIABLE        =        B3
                    MEAN OF WORKING SERIES=   18.3385
                    STANDARD DEVIATION      =   21.2601
                    NUMBER OF OBSERVATIONS=        13
                             AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0     451.99    1.00000   |                    |********************|
 1    213.069    0.47140   |          .         |*********  .        |
 2   -18.7645   -0.04152   |          .        *|           .        |
 3   -85.7151   -0.18964   |          .     ****|           .        |
 4    52.6769    0.11654   |          .         |**          .       |
 5    73.3606    0.16231   |          .         |***         .       |
 6   -22.4497   -0.04967   |          .        *|            .       |
                            '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1   -0.50849   |          .**********|          .         |
          2   -0.04805   |          .         *|          .         |
          3    0.34868   |          .          |*******   .         |
          4   -0.25925   |          .     *****|          .         |
          5    0.01449   |          .          |          .         |
          6    0.06786   |          .          |*         .         |

                        PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    0.47140   |          .          |*********  .        |
          2   -0.33909   |          .   *******|           .        |
          3   -0.00521   |          .          |           .        |
          4    0.32606   |          .          |*******    .        |
          5   -0.18957   |          .      ****|           .        |
          6   -0.12023   |          .        **|           .        |


                  AUTOCORRELATION CHECK FOR WHITE NOISE

    TO     CHI                         AUTOCORRELATIONS
   LAG   SQUARE  DF    PROB
    6     5.35    6   0.500   0.471 -0.042 -0.190  0.117  0.162 -0.050
```

```
                         NAME OF VARIABLE        =         B4
                         MEAN OF WORKING SERIES=    11.2769
                         STANDARD DEVIATION      =    18.3383
                         NUMBER OF OBSERVATIONS=         13
                                  AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    336.294     1.00000 |                   |********************|
  1   -43.2555    -0.12862 |         .      ***|         .         |
  2   -12.4749    -0.03710 |         .        *|         .         |
  3   -46.4074    -0.13800 |         .      ***|         .         |
  4    97.8625     0.29100 |         .         |******   .         |
  5    -29.228    -0.08691 |         .       **|         .         |
  6    9.77839     0.02908 |         .         |*        .         |
                            '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    0.06120  |         .         |*        .         |
             2    0.06660  |         .         |*        .         |
             3    0.08174  |         .         |**       .         |
             4   -0.23384  |         .    *****|         .         |
             5    0.03087  |         .         |*        .         |
             6   -0.02726  |         .        *|         .         |

                        PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1   -0.12862  |         .      ***|         .         |
             2   -0.05454  |         .        *|         .         |
             3   -0.15302  |         .      ***|         .         |
             4    0.25968  |         .         |*****    .         |
             5   -0.03937  |         .        *|         .         |
             6    0.02984  |         .         |*        .         |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                        AUTOCORRELATIONS
 LAG  SQUARE DF    PROB
   6    2.71  6   0.845 -0.129 -0.037 -0.138  0.291 -0.087  0.029
```

```
                    NAME OF VARIABLE          =          B5
                    MEAN OF WORKING SERIES=         6.93846
                    STANDARD DEVIATION    =         8.36297
                    NUMBER OF OBSERVATIONS=              13
                                 AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0     69.9393     1.00000  |                    |********************|
 1     -26.019    -0.37202  |          .  *******|        .           |
 2     4.71492     0.06741  |         .          |*        .          |
 3    -15.7526    -0.22523  |         .     *****|         .          |
 4     21.8879     0.31296  |         .          |******   .          |
 5    -23.0055    -0.32893  |        .    *******|          .         |
 6     6.56121     0.09381  |        .           |**         .        |

                          '.' MARKS TWO STANDARD ERRORS

                      INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.37556   |          .         |********   .        |
            2     0.25657   |          .         |*****      .        |
            3     0.24582   |          .         |*****      .        |
            4     0.05299   |          .         |*          .        |
            5     0.23583   |          .         |*****      .        |
            6     0.09791   |          .         |**         .        |

                      PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.37202   |          .  *******|           .        |
            2    -0.08239   |          .       **|           .        |
            3    -0.26729   |          .    *****|           .        |
            4     0.16445   |          .         |***        .        |
            5    -0.22125   |          .     ****|           .        |
            6    -0.13083   |          .      ***|           .        |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                                AUTOCORRELATIONS
 LAG  SQUARE DF    PROB
   6    8.32  6   0.215 -0.372   0.067 -0.225   0.313 -0.329   0.094
```

```
                         NAME OF VARIABLE       =         B6
                         MEAN OF WORKING SERIES=    5.08462
                         STANDARD DEVIATION     =   4.49013
                         NUMBER OF OBSERVATIONS=         13
                                   AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0     20.1613    1.00000  |                    |********************|
 1    -1.50665   -0.07473  |          .        *|          .         |
 2     1.42801    0.07083  |          .         |*         .         |
 3   -0.471238   -0.02337  |          .         |          .         |
 4    -3.58055   -0.17759  |          .     ****|          .         |
 5      6.6337    0.32903  |          .         |*******   .         |
 6    -2.18964   -0.10861  |          .       **|          .         |
                            '.' MARKS TWO STANDARD ERRORS

                              INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.06574   |          .        *|          .         |
            2   -0.08259   |          .       **|          .         |
            3    0.07048   |          .         |*         .         |
            4    0.13715   |          .         |***       .         |
            5   -0.28085   |          .   ******|          .         |
            6    0.05656   |          .         |*         .         |

                              PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.07473   |          .        *|          .         |
            2    0.06561   |          .         |*         .         |
            3   -0.01366   |          .         |          .         |
            4   -0.18668   |          .     ****|          .         |
            5    0.32033   |          .         |******    .         |
            6   -0.06448   |          .        *|          .         |

                       AUTOCORRELATION CHECK FOR WHITE NOISE
```

```
 TO    CHI                                AUTOCORRELATIONS
LAG  SQUARE DF    PROB
  6    3.84  6   0.698 -0.075  0.071 -0.023 -0.178  0.329 -0.109
```

```
                     NAME OF VARIABLE       =        B7
                     MEAN OF WORKING SERIES=    7.29231
                     STANDARD DEVIATION     =   5.64971
                     NUMBER OF OBSERVATIONS=         13
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0    31.9192     1.00000 |                    |********************|
  1      7.301     0.22873 |          .         |*****     .         |
  2    3.80751     0.11929 |          .         |**        .         |
  3   -10.4451    -0.32724 |        .    *******|           .        |
  4   -4.82239    -0.15108 |       .         ***|            .       |
  5   -8.55718    -0.26809 |       .       *****|            .       |
  6   0.624766     0.01957 |       .            |            .       |
                            '.' MARKS TWO STANDARD ERRORS

                    INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1    -0.22379 |         .      ****|          .         |
             2    -0.08884 |         .        **|          .         |
             3     0.28361 |         .          |******    .         |
             4    -0.06324 |         .         *|          .         |
             5     0.14317 |         .          |***       .         |
             6    -0.01220 |         .          |          .         |

                    PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1     0.22873 |         .          |*****     .         |
             2     0.07066 |         .          |*         .         |
             3    -0.39107 |         .  ********|          .         |
             4    -0.00172 |         .          |          .         |
             5    -0.16838 |         .       ***|          .         |
             6     0.01496 |         .          |          .         |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    5.45  6  0.488  0.229  0.119 -0.327 -0.151 -0.268  0.020
```

```
                    NAME OF VARIABLE        =        B8
                    MEAN OF WORKING SERIES=   1.18462
                    STANDARD DEVIATION     =   0.80369
                    NUMBER OF OBSERVATIONS=        13
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   0.645917    1.00000  |                    |********************|
  1  -0.138302   -0.21412  |          .     ****|        .           |
  2   0.106532    0.16493  |          .         |***     .           |
  3  -0.082244   -0.12733  |          .      ***|        .           |
  4   -0.14741   -0.22822  |          .    *****|        .           |
  5 -0.0596177   -0.09230  |          .       **|        .           |
  6  -0.185435   -0.28709  |          .   ******|        .           |
                            '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.38862  |          .         |********.           |
           2    0.13082  |          .         |***     .           |
           3    0.19806  |          .         |****    .           |
           4    0.27880  |          .         |******  .           |
           5    0.27946  |          .         |******  .           |
           6    0.24361  |          .         |*****   .           |

                         PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.21412  |          .     ****|        .           |
           2    0.12481  |          .         |**      .           |
           3   -0.07420  |          .        *|        .           |
           4   -0.30518  |          .   ******|        .           |
           5   -0.19340  |          .     ****|        .           |
           6   -0.34483  |          .  *******|        .           |

                AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                              AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    5.18  6  0.522 -0.214  0.165 -0.127 -0.228 -0.092 -0.287
```

```
                    NAME OF VARIABLE        =        B9
                    MEAN OF WORKING SERIES=    1.14615
                    STANDARD DEVIATION      =  1.05219
                    NUMBER OF OBSERVATIONS=          13
                             AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0     1.1071     1.00000 |                    |********************|
  1   0.232499     0.21001 |          .         |****                |
  2   0.607128     0.54839 |          .         |***********.        |
  3   0.119686     0.10811 |         .          |**         .        |
  4   0.151475     0.13682 |       .            |***         .       |
  5  -0.129695    -0.11715 |       .          **|             .      |
  6  -0.292877    -0.26454 |    .  .       *****|             .      |
                              '.' MARKS TWO STANDARD ERRORS

                    INVERSE AUTOCORRELATIONS

               LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
                1     0.07525  |          .         |**         .        |
                2    -0.40286  |          .   ********|         .        |
                3    -0.16361  |          .       ***|          .        |
                4    -0.13153  |          .       ***|          .        |
                5     0.11450  |          .         |**         .        |
                6     0.21603  |          .         |****       .        |

                    PARTIAL AUTOCORRELATIONS

               LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
                1     0.21001  |          .         |****       .        |
                2     0.52756  |          .         |***********         |
                3    -0.08276  |          .       **|           .        |
                4    -0.22632  |          .    *****|           .        |
                5    -0.19996  |          .     ****|           .        |
                6    -0.32061  |          .   ******|           .        |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                                 AUTOCORRELATIONS
LAG   SQUARE DF    PROB

 6     8.97   6   0.176   0.210   0.548   0.108   0.137  -0.117  -0.265
```

```
                    NAME OF VARIABLE        =        B10
                    MEAN OF WORKING SERIES=    1.57692
                    STANDARD DEVIATION      =     1.6821
                    NUMBER OF OBSERVATIONS=         13
                              AUTOCORRELATIONS

LAG COVARIANCE  CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   2.82947     1.00000   |                    |********************|
  1    0.8299     0.29331   |         .          |******     .        |
  2  0.296013     0.10462   |         .          |**         .        |
  3 0.0594037     0.02099   |         .          |           .        |
  4  0.142321     0.05030   |         .          |*          .        |
  5 -0.462039    -0.16330   |         .       ***|.          .        |
  6 -0.566163    -0.20010   |         .      ****|           .        |
                             '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.23801   |         .     *****|           .        |
           2   -0.03799   |         .         *|           .        |
           3    0.05151   |         .          |*          .        |
           4   -0.14489   |         .       ***|           .        |
           5    0.12467   |         .          |**         .        |
           6    0.10326   |         .          |**         .        |

                         PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.29331   |         .          |******     .        |
           2    0.02034   |         .          |           .        |
           3   -0.01645   |         .          |           .        |
           4    0.05087   |         .          |*          .        |
           5   -0.20854   |         .      ****|           .        |
           6   -0.11708   |         .        **|           .        |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                           AUTOCORRELATIONS
 LAG  SQUARE DF   PROB
  6     3.42  6  0.754  0.293  0.105  0.021  0.050 -0.163 -0.200
```

```
                    NAME OF VARIABLE          =        B11
                    MEAN OF WORKING SERIES=    0.846154
                    STANDARD DEVIATION        =    1.65837
                    NUMBER OF OBSERVATIONS=           13
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    2.75018    1.00000   |                    |********************|
 1    -0.4733   -0.17210   |          .      ***|         .          |
 2   0.161566    0.05875   |          .         |*        .          |
 3  -0.545817   -0.19847   |          .     ****|         .          |
 4   0.826031    0.30036   |         .          |******   .          |
 5  -0.364843   -0.13266   |         .       ***|          .         |
 6  0.0646973    0.02352   |         .          |          .         |
                          '.' MARKS TWO STANDARD ERRORS

                       INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.04736   |          .         |*        .          |
           2    0.01085   |          .         |         .          |
           3    0.12475   |          .         |**       .          |
           4   -0.21809   |          .     ****|         .          |
           5    0.05690   |          .         |*        .          |
           6    0.04180   |          .         |*        .          |

                       PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.17210   |          .      ***|         .          |
           2    0.03002   |          .         |*        .          |
           3   -0.18926   |          .     ****|         .          |
           4    0.25233   |          .         |*****    .          |
           5   -0.05330   |          .        *|         .          |
           6   -0.04601   |          .        *|         .          |

                AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                              AUTOCORRELATIONS
 LAG   SQUARE  DF   PROB.
  6     3.71    6   0.716  -0.172  0.059  -0.198  0.300  -0.133  0.024
```

```
                         NAME OF VARIABLE        =         B12
                         MEAN OF WORKING SERIES=    1.36923
                         STANDARD DEVIATION      =  1.08087
                         NUMBER OF OBSERVATIONS=         13
                                  AUTOCORRELATIONS

LAG COVARIANCE  CORRELATION  -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0   1.16828     1.00000    |                    |********************|
  1   0.211821    0.18131    |           .        |****          .     |
  2   0.174292    0.14919    |           .        |***           .     |
  3  -0.467497   -0.40016    |           . ********|               .    |
  4  -0.156268   -0.13376    |           .     ***|               .    |
  5  -0.238885   -0.20447    |           .    ****|                .   |
  6   0.0384388   0.03290    |           .        |*               .   |
                               '.' MARKS TWO STANDARD ERRORS

                            INVERSE AUTOCORRELATIONS

          LAG CORRELATION  -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.21255   |          .     ****|              .     |
            2   -0.20411   |          .     ****|              .     |
            3    0.42260   |          .         |********      .     |
            4   -0.04496   |          .        *|              .     |
            5   -0.00511   |          .         |              .     |
            6    0.07887   |          .         |**            .     |

                            PARTIAL AUTOCORRELATIONS

          LAG CORRELATION  -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    0.18131   |          .         |****          .     |
            2    0.12027   |          .         |**            .     |
            3   -0.46767   |          . *********|              .     |
            4    0.01114   |          .         |              .     |
            5   -0.03989   |          .        *|              .     |
            6   -0.10672   |          .       **|              .     |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

   TO     CHI                                AUTOCORRELATIONS
  LAG  SQUARE DF    PROB
    6    5.49  6   0.483    0.181   0.149 -0.400 -0.134 -0.204   0.033
```

```
                        NAME OF VARIABLE    =         B13
                        MEAN OF WORKING SERIES=    3.16923
                        STANDARD DEVIATION  =     3.83734
                        NUMBER OF OBSERVATIONS=        13
                             AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    14.7252    1.00000   |                    |********************|
 1    2.48354    0.16866   |          .         |***      .          |
 2   -5.25287   -0.35673   |          .  *******|         .          |
 3   -2.11158   -0.14340   |         .       ***|           .        |
 4    0.80965    0.05498   |         .          |*          .        |
 5   -1.19457   -0.08112   |         .        **|           .        |
 6   -3.91594   -0.26593   |         .    *****|           .        |
                            '.' MARKS TWO STANDARD ERRORS

                     INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.14213   |          .       ***|          .         |
           2    0.39361   |          .          |********  .         |
           3    0.06957   |          .          |*         .         |
           4    0.17761   |          .          |****      .         |
           5    0.04167   |          .          |*         .         |
           6    0.20250   |          .          |****      .         |

                     PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.16866   |          .          |***       .         |
           2   -0.39645   |          . ********|           .         |
           3    0.00914   |          .          |          .         |
           4   -0.06651   |          .         *|          .         |
           5   -0.16575   |          .       ***|          .         |
           6   -0.26785   |          .     *****|          .         |

                AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                          AUTOCORRELATIONS
LAG   SQUARE DF   PROB
 6      5.32  6  0.504   0.169 -0.357 -0.143  0.055 -0.081 -0.266
```

```
                      NAME OF VARIABLE          =        B14
                      MEAN OF WORKING SERIES=   19.7154
                      STANDARD DEVIATION        =   18.2136
                      NUMBER OF OBSERVATIONS=        13
                               AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
 0    331.734    1.00000   |                    |********************|
 1   -36.3239   -0.10950   |          .       **|          .         |
 2   -31.2764   -0.09428   |          .       **|          .         |
 3   -45.2304   -0.13635   |          .      ***|          .         |
 4    131.488    0.39637   |         .          |********    .       |
 5   -114.864   -0.34625   |        .    *******|             .      |
 6    -23.085   -0.06959   |        .          *|              .     |
                            '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.13984   |          .      ***|          .         |
            2    0.08227   |          .         |**        .         |
            3    0.16055   |          .         |***       .         |
            4   -0.26739   |          .    *****|          .         |
            5    0.27948   |          .         |******    .         |
            6    0.03312   |          .         |*         .         |

                         PARTIAL AUTOCORRELATIONS

           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.10950   |          .       **|          .         |
            2   -0.10756   |          .       **|          .         |
            3   -0.16338   |          .      ***|          .         |
            4    0.36563   |          .         |*******   .         |
            5   -0.36007   |          . *******|           .         |
            6   -0.04222   |          .        *|          .         |

                  AUTOCORRELATION CHECK FOR WHITE NOISE

  TO     CHI                            AUTOCORRELATIONS
 LAG   SQUARE DF    PROB
  6     7.18   6   0.305 -0.109 -0.094 -0.136  0.396 -0.346 -0.070
```

5 - ARIMA for Tojeira 1 section with enlarged data

```
                                        SAS                         12:55 MONDAY, OCTOBER 5, 1987

                                 ARIMA PROCEDURE

                        NAME OF VARIABLE        =        A1
                        MEAN OF WORKING SERIES=    7.31951
                        STANDARD DEVIATION      =    6.05049
                        NUMBER OF OBSERVATIONS=         41
                                 AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    36.6084     1.00000  |                    |********************|          0
  1    8.54141     0.23332  |                  . |*****.              |   0.156174
  2    1.10711     0.03024  |                .   |*    .              |   0.164456
  3   -4.74188    -0.12953  |                . ***|     .              |   0.164591
  4   -2.14367    -0.05856  |                .   *|     .              |   0.167059
  5    2.84364     0.07768  |                .    |**   .              |   0.167559
  6    3.96367     0.10827  |                .    |**   .              |   0.168435
  7    3.71424     0.10146  |                .    |**   .              |   0.170124
  8   -4.37248    -0.11944  |                .  **|     .              |   0.171594
  9   0.815104     0.02227  |                .    |     .              |   0.173609
 10   -4.14431    -0.11321  |                .  **|     .              |   0.173679
                             .' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.27292  |                .*****|    .               |
            2    -0.00133  |                .     |    .               |
            3     0.07235  |                .     |*   .               |
            4     0.03253  |                .     |*   .               |
            5    -0.05115  |                .    *|    .               |
            6    -0.04249  |                .    *|    .               |
            7    -0.08777  |                .   **|    .               |
            8     0.18613  |                .     |****.               |
            9    -0.15838  |                .  ***|    .               |
           10     0.11190  |                .     |**  .               |

                          PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.23332  |                .     |*****.              |
            2    -0.02559  |                .    *|     .              |
            3    -0.13842  |                .  ***|     .              |
            4     0.00437  |                .     |     .              |
            5     0.10324  |                .     |**   .              |
            6     0.05408  |                .     |*    .              |
            7     0.05345  |                .     |*    .              |
            8    -0.15057  |                .  ***|     .              |
            9     0.11702  |                .     |**   .              |
           10    -0.13093  |                .  ***|     .              |

                      AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                          AUTOCORRELATIONS
LAG   SQUARE  DF   PROB
  6     4.27   6  0.640   0.233  0.030 -0.130 -0.059  0.078  0.108
                                 SAS                         12:55 MONDAY, OCTOBER 5, 1987
```

```
                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =         A2
                    MEAN OF WORKING SERIES=    2.20244
                    STANDARD DEVIATION      =  4.03343
                    NUMBER OF OBSERVATIONS=         41
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0    16.2685     1.00000 |                    |********************|         0
  1    8.39487     0.51602 |                    |**********          |  0.156174
  2     4.3428     0.26694 |              .     |*****    .          |  0.193337
  3    5.46661     0.33602 |              .     |*******  .          |  0.202127
  4     5.0157     0.30831 |             .      |******    .         |  0.215321
  5    1.59018     0.09775 |             .      |**        .         |  0.225832
  6   0.546632     0.03360 |             .      |*         .         |  0.226861
  7   0.171189     0.01052 |             .      |          .         |  0.226983
  8  0.0428721     0.00264 |             .      |          .         |  0.226995
  9  -0.487336    -0.02996 |             .     *|          .         |  0.226995
 10  -0.418818    -0.02574 |             .     *|          .         |  0.227092
                            '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1    -0.42039  |           ********|          .        |
              2     0.14653  |              .    |***       .        |
              3    -0.13343  |              .  ***|         .        |
              4    -0.16956  |              .  ***|         .        |
              5     0.12696  |              .    |***       .        |
              6    -0.05874  |              .    *|         .        |
              7     0.10302  |              .    |**        .        |
              8    -0.03658  |              .    *|         .        |
              9     0.02154  |              .    |          .        |
             10    -0.01973  |              .    |          .        |

                         PARTIAL AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1     0.51602  |              .    |**********         |
              2     0.00091  |              .    |          .        |
              3     0.26976  |              .    |*****     .        |
              4     0.04826  |              .    |*         .        |
              5    -0.15205  |              .  ***|         .        |
              6    -0.02644  |              .    *|         .        |
              7    -0.08927  |              .   **|         .        |
              8     0.03581  |              .    |*         .        |
              9    -0.00216  |              .    |          .        |
             10     0.02737  |              .    |*         .        |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

      TO     CHI                         AUTOCORRELATIONS
     LAG  SQUARE DF   PROB
       6   25.25  6  0.000  0.516  0.267  0.336  0.308  0.098  0.034
                                   SAS                12:55 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =        A3
                    MEAN OF WORKING SERIES=   23.7561
                    STANDARD DEVIATION      =   12.1219
                    NUMBER OF OBSERVATIONS=        41
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1          STD
  0    146.941    1.00000  |                    |********************|            0
  1    66.4589    0.45228  |                    |*********           |     0.156174
  2    77.2056    0.52542  |                    |***********         |     0.185388
  3    41.5462    0.28274  |           .        |******  .           |     0.218713
  4    31.3431    0.21330  |           .        |****    .           |     0.227453
  5    22.3485    0.15209  |           .        |***     .           |     0.232281
  6    7.02954    0.04784  |           .        |*       .           |     0.234697
  7   -5.86578   -0.03992  |           .       *|        .           |     0.234935
  8    5.27999    0.03593  |           .        |*       .           |       0.2351
  9   -7.73063   -0.05261  |           .       *|        .           |     0.235234
 10    6.02891    0.04103  |           .        |*       .           |     0.235521
                            '.' MARKS TWO STANDARD ERRORS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
         1   -0.16148  |                 ***|        .           |
         2   -0.39178  |            ********|        .           |
         3    0.10091  |           .        |**      .           |
         4    0.04509  |           .        |*       .           |
         5   -0.16663  |           .     ***|        .           |
         6    0.07035  |           .        |*       .           |
         7    0.14896  |           .        |***     .           |
         8   -0.09290  |           .      **|        .           |
         9    0.01554  |           .        |        .           |
        10   -0.01829  |           .        |        .           |

                         PARTIAL AUTOCORRELATIONS

       LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
         1    0.45228  |           .        |*********           |
         2    0.40337  |           .        |********            |
         3   -0.06228  |           .       *|       .            |
         4   -0.09450  |           .      **|       .            |
         5    0.03227  |           .        |*      .            |
         6   -0.06248  |           .       *|       .            |
         7   -0.13608  |           .     ***|       .            |
         8    0.14618  |           .        |***    .            |
         9   -0.00600  |           .        |       .            |
        10    0.02536  |           .        |*      .            |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                           AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6   28.62  6  0.000   0.452  0.525  0.283  0.213  0.152  0.048   12:55 MONDAY, OCTOBER 5, 1987
                               SAS
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE         =        A4
                    MEAN OF WORKING SERIES=     4.46341
                    STANDARD DEVIATION       =   5.04597
                    NUMBER OF OBSERVATIONS=          41
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    25.4618    1.00000  |                    |********************|         0
  1    6.36527    0.24999  |                .   |*****.              |  0.156174
  2   -1.38245   -0.05430  |               .   *|    .               |  0.165647
  3     3.2394    0.12723  |               .    |***  .              |   0.16608
  4    2.26237    0.08885  |               .    |**   .              |  0.168441
  5   -3.02327   -0.11874  |               .  **|     .              |   0.16958
  6    3.87057    0.15201  |               .    |***  .              |  0.171596
  7    6.58025    0.25844  |               .    |*****.              |  0.174849
  8   -2.63504   -0.10349  |               .  **|     .              |   0.18393
  9   -3.73567   -0.14672  |               . ***|     .              |  0.185345
 10     -4.281   -0.16813  |               . ***|     .              |  0.188156
                            '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    -0.30566  |              ******|     .              |
          2     0.10194  |              .     |**   .              |
          3    -0.03355  |              .    *|     .              |
          4    -0.17787  |              .  ****|    .              |
          5     0.19071  |              .     |**** .              |
          6    -0.10213  |              .    **|    .              |
          7    -0.21402  |              .  ****|    .              |
          8     0.17069  |              .     |***  .              |
          9    -0.08118  |              .    **|    .              |
         10     0.20016  |              .     |**** .              |

                         PARTIAL AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1     0.24999  |              .     |*****.              |
          2    -0.12458  |              .   **|     .              |
          3     0.18813  |              .     |**** .              |
          4    -0.00737  |              .     |     .              |
          5    -0.12468  |              .   **|     .              |
          6     0.24357  |              .     |*****.              |
          7     0.11698  |              .     |**   .              |
          8    -0.18142  |              . ****|     .              |
          9    -0.05843  |              .    *|     .              |
         10    -0.27370  |              .*****|     .              |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                                AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    5.87  6  0.438   0.250 -0.054  0.127  0.089 -0.119  0.152
                                  SAS
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12:55 MONDAY, OCTOBER 5, 1987

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                                     ARIMA PROCEDURE

                         NAME OF VARIABLE         =       A5
                         MEAN OF WORKING SERIES=   2.52439
                         STANDARD DEVIATION    =   3.55574
                         NUMBER OF OBSERVATIONS=        41
                                     AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    12.6433    1.00000  |                    |********************|          0
  1    2.24441    0.17752  |                .   |****    .           |   0.156174
  2   -1.30087   -0.10289  |                . **|        .           |    0.16102
  3   -3.11962   -0.24674  |               *****|         .          |   0.162616
  4   -2.44439   -0.19333  |               .****|         .          |   0.171504
  5  -0.848978   -0.06715  |               .   *|         .          |    0.17674
  6   0.133595    0.01057  |               .    |         .          |   0.177361
  7  -0.184646   -0.01460  |               .    |         .          |   0.177376
  8   -1.29605   -0.10251  |               .  **|         .          |   0.177406
  9    -0.5111   -0.04042  |               .   *|         .          |   0.178845
 10  -0.888027   -0.07024  |               .   *|         .          |   0.179067
                            '.' MARKS TWO STANDARD ERRORS

                               INVERSE AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1     0.08220 |                .   |**    .             |
           2     0.26103 |                .   |*****.              |
           3     0.29717 |                .   |******              |
           4     0.24542 |                .   |*****.              |
           5     0.19106 |                .   |****  .             |
           6     0.14600 |                .   |***   .             |
           7     0.13330 |                .   |***   .             |
           8     0.16722 |                .   |***   .             |
           9     0.03215 |                .   |*     .             |
          10     0.12786 |                .   |***   .             |

                               PARTIAL AUTOCORRELATIONS

         LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1     0.17752 |                .   |****  .             |
           2    -0.13878 |                . ***|      .             |
           3    -0.21194 |                .****|      .             |
           4    -0.13554 |                . ***|      .             |
           5    -0.06908 |                .   *|      .             |
           6    -0.05981 |                .   *|      .             |
           7    -0.10565 |                .  **|      .             |
           8    -0.17006 |                . ***|      .             |
           9    -0.06913 |                .   *|      .             |
          10    -0.16106 |                . ***|      .             |

                      AUTOCORRELATION CHECK FOR WHITE NOISE

          TO    CHI                          AUTOCORRELATIONS
         LAG  SQUARE DF   PROB
           6    6.70  6  0.350  0.178 -0.103 -0.247 -0.193 -0.067  0.011
                                     SAS                                12:55 MONDAY, OCTOBER 5, 1987
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                                ARIMA PROCEDURE

                      NAME OF VARIABLE          =        A6
                      MEAN OF WORKING SERIES=    11.5854
                      STANDARD DEVIATION    =    6.37172
                      NUMBER OF OBSERVATIONS=         41
                                 AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0    40.5988    1.00000  |                    |********************|          0
  1    12.1566    0.29943  |                    |******              |   0.156174
  2    -1.2492   -0.03077  |             .     *|      .             |   0.169599
  3   -10.8471   -0.26718  |             .*****|      .             |   0.169735
  4   -10.1845   -0.25086  |             .*****|      .             |     0.1797
  5   -7.27753   -0.17925  |            .  ****|       .            |   0.188048
  6   -1.41672   -0.03490  |            .     *|       .            |    0.19217
  7     2.3766    0.05854  |            .      |*      .            |   0.192325
  8    2.82059    0.06947  |            .      |*      .            |   0.192759
  9    1.55476    0.03830  |            .      |*      .            |   0.193368
 10   -5.70755   -0.14058  |            .   ***|       .            |   0.193553
                            '.' MARKS TWO STANDARD ERRORS

                            INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1   -0.16618  |             .   ***|      .             |
            2    0.08851  |             .      |**    .             |
            3    0.21336  |             .      |****  .             |
            4    0.08943  |             .      |**    .             |
            5    0.14137  |             .      |***   .             |
            6    0.05964  |             .      |*     .             |
            7    0.07467  |             .      |*     .             |
            8    0.06213  |             .      |*     .             |
            9   -0.05682  |             .     *|      .             |
           10    0.16444  |             .      |***   .             |

                            PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    0.29943  |             .      |******              |
            2   -0.13229  |             .   ***|      .             |
            3   -0.24277  |             .*****|      .             |
            4   -0.11607  |             .    **|      .             |
            5   -0.11543  |             .    **|      .             |
            6   -0.03955  |             .     *|      .             |
            7   -0.02149  |             .      |      .             |
            8   -0.04282  |             .     *|      .             |
            9   -0.03244  |             .     *|      .             |
           10   -0.19869  |             .  ****|      .             |

                      AUTOCORRELATION CHECK FOR WHITE NOISE

  TO    CHI                                AUTOCORRELATIONS
 LAG  SQUARE  DF   PROB
   6   11.94   6  0.063   0.299 -0.031 -0.267 -0.251 -0.179 -0.035
                                   SAS                12:55 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                      NAME OF VARIABLE       =        A7
                      MEAN OF WORKING SERIES=   4.08293

                      STANDARD DEVIATION     =      2.714
                      NUMBER OF OBSERVATIONS=          41
                                  AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      STD
  0    7.36581    1.00000   |                    |********************|        0
  1    1.09548    0.14873   |                .   |***   .             |  0.156174
  2    0.21186    0.02876   |                .   |*     .             |  0.159591
  3    1.43856    0.19530   |                .   |****  .             |  0.159717
  4  -0.336459   -0.04568   |               .   *|       .            |  0.165439
  5  -0.823165   -0.11175   |               .  **|       .            |  0.165747
  6  0.0338657    0.00460   |               .    |       .            |  0.167575
  7   0.231467    0.03142   |               .    |*      .            |  0.167578
  8   -1.63057   -0.22816   |               .*****|      .            |  0.167721
  9   0.580703    0.07884   |               .    |**     .            |  0.175128
 10  -0.635859   -0.08633   |               .  **|       .            |  0.175991
                            '.' MARKS TWO STANDARD ERRORS

                           INVERSE AUTOCORRELATIONS

               LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
                 1  -0.31871   |               ******|              |
                 2   0.15854   |               .    |***  .         |
                 3  -0.23596   |               .*****|      .       |
                 4   0.15216   |               .    |***   .        |
                 5  -0.04968   |               .   *|      .        |
                 6   0.13650   |               .    |***   .        |
                 7  -0.19177   |               . ****|      .       |
                 8   0.24107   |               .    |*****.         |
                 9  -0.18119   |               . ****|      .       |
                10   0.14007   |               .    |***   .        |

                           PARTIAL AUTOCORRELATIONS

               LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
                 1   0.14873   |               .    |***   .        |
                 2   0.00679   |               .    |      .        |
                 3   0.19435   |               .    |****  .        |
                 4  -0.10899   |               .  **|      .        |
                 5  -0.09594   |               .  **|      .        |
                 6  -0.00186   |               .    |      .        |
                 7   0.06613   |               .    |*     .        |
                 8  -0.22337   |               . ****|      .       |
                 9   0.15267   |               .    |***   .        |
                10  -0.17870   |               . ****|      .       |

                      AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    3.49  6  0.745   0.149  0.029  0.195 -0.046 -0.112  0.005
                               SAS                                  12:55 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                  NAME OF VARIABLE        =        AB
                  MEAN OF WORKING SERIES=   2.15122
                  STANDARD DEVIATION    =   1.26436
                  NUMBER OF OBSERVATIONS=        41
                             AUTOCORRELATIONS
                                                                                            STD
LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1                         0
  0     1.5986     1.00000 |                    |********************|               0.156174
  1   0.290697     0.18185 |                  . |****  .             |               0.161255
  2   0.301811     0.18880 |                  . |****  .             |               0.166559
  3  -0.431037    -0.26963 |                 *****|      .             |               0.176886
  4   -0.23768    -0.14868 |                 .  ***|      .             |               0.179908
  5  0.0227675     0.01424 |                 .    |      .             |               0.179935
  6   0.225232     0.14089 |                 .    |***   .             |               0.182606
  7    0.30029     0.18785 |                 .    |****  .             |                0.18726
  8   0.015086     0.00944 |                 .    |      .             |               0.187272
  9  -0.280487    -0.17546 |                 . ****|      .             |               0.191239
 10  -0.236024    -0.14764 |                   ***|      .             |
                            '.' MARKS TWO STANDARD ERRORS

                        INVERSE AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    -0.14426 |                  ***|                    |
  2    -0.36151 |              *******|                    |
  3     0.23642 |                .    |*****.              |
  4     0.08860 |                .    |**   .              |
  5    -0.05723 |                .   *|     .              |
  6     0.00630 |                .    |     .              |
  7    -0.11918 |                .  **|     .              |
  8     0.01418 |                .    |     .              |
  9     0.13225 |                .    |***  .              |
 10    -0.01834 |                .    |     .              |

                        PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.18185 |                .    |****  .             |
  2     0.16106 |                .    |***   .             |
  3    -0.34796 |              *******|      .             |
  4    -0.08406 |                .  **|      .             |
  5     0.22041 |                .    |****  .             |
  6     0.08156 |                .    |**    .             |
  7     0.01606 |                .    |      .             |
  8    -0.06229 |                .   *|      .             |
  9    -0.16564 |                . ***|      .             |
 10     0.02412 |                .    |      .             |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                              AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    8.50  6  0.203   0.182  0.189 -0.270 -0.149  0.014  0.141   12:55 MONDAY, OCTOBER 5, 1987
                               SAS
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                          ARIMA PROCEDURE

                NAME OF VARIABLE        =          A9
                MEAN OF WORKING SERIES=     3.37317
                STANDARD DEVIATION      =       5.856
                NUMBER OF OBSERVATIONS=           41
                          AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0    34.2927    1.00000   |                    |********************|         0
  1    0.39959    0.01165   |              .     |      .             |  0.156174
  2    2.69638    0.07863   |              .     |**    .             |  0.156195
  3    7.08272    0.20654   |              .     |****  .             |  0.157157
  4   -2.64996   -0.07727   |             .    **|       .            |  0.163644
  5    1.88719    0.05503   |             .      |*      .            |  0.164531
  6   -1.36239   -0.03973   |             .     *|       .            |   0.16498
  7   -4.46498   -0.13020   |             .   ***|       .            |  0.165213
  8    2.77023    0.08078   |             .      |**     .            |  0.167697
  9   0.398515    0.01162   |             .      |       .            |  0.168643
 10   -1.05833   -0.03086   |             .     *|       .            |  0.168663
                            '.' MARKS TWO STANDARD ERRORS

                          INVERSE AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    -0.00220  |              .     |      .             |
          2    -0.08767  |              .   **|      .             |
          3    -0.20863  |              . ****|      .             |
          4     0.02158  |              .     |      .             |
          5    -0.01919  |              .     |      .             |
          6     0.10036  |              .     |**    .             |
          7     0.10788  |              .     |**    .             |
          8    -0.06717  |              .    *|      .             |
          9    -0.05697  |              .    *|      .             |
         10     0.00400  |              .     |      .             |

                          PARTIAL AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1     0.01165  |              .     |      .             |
          2     0.07850  |              .     |**    .             |
          3     0.20608  |              .     |****  .             |
          4    -0.08886  |              .   **|      .             |
          5     0.02515  |              .     |*     .             |
          6    -0.07404  |              .    *|      .             |
          7    -0.10716  |              .   **|      .             |
          8     0.07403  |              .     |*     .             |
          9     0.06176  |              .     |*     .             |
         10    -0.00434  |              .     |      .             |

                AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                                 AUTOCORRELATIONS
LAG   SQUARE DF    PROB
  6     2.78  6   0.836   0.012   0.079   0.207 -0.077   0.055 -0.040
                                  SAS                          12:55 MONDAY, OCTOBER 5, 1987
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                              ARIMA PROCEDURE

                    NAME OF VARIABLE        =        A10
                    MEAN OF WORKING SERIES=    2.39268
                    STANDARD DEVIATION      =    2.90906
                    NUMBER OF OBSERVATIONS=         41
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0    8.46263     1.00000 |                    |********************|           0
  1    1.52864     0.18063 |                .   |****  .             |    0.156174
  2    4.52344     0.53452 |                .   |***********         |    0.161189
  3    3.91379     0.46248 |              .     |*********           |    0.199797
  4     1.8488     0.21847 |              .     |****    .           |    0.224394
  5    1.94111     0.22937 |             .      |*****    .          |    0.229523
  6    2.58445     0.30540 |             .      |******   .          |    0.235047
  7   0.096249     0.01137 |             .      |          .         |    0.244534
  8    1.25868     0.14873 |             .      |***       .         |    0.244547
  9    1.46996     0.17370 |             .      |***       .         |    0.246743
 10  -0.903666    -0.10678 |             .    **|          .         |    0.249708
                           '.' MARKS TWO STANDARD ERRORS

                         INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.19168 |                .   |****  .             |
            2    -0.53654 |        ***********|       .             |
            3    -0.41434 |            ********|      .             |
            4     0.06252 |                .   |*     .             |
            5     0.21557 |                .   |****  .             |
            6     0.07009 |                .   |*     .             |
            7     0.01076 |                .   |      .             |
            8    -0.07735 |                .  **|     .             |
            9    -0.06292 |                .   *|     .             |
           10     0.06077 |                .   |*     .             |

                         PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.18063 |                .   |****  .             |
            2     0.51882 |                .   |**********          |
            3     0.45589 |                .   |*********           |
            4    -0.09076 |                .  **|     .             |
            5    -0.37582 |            ********|      .             |
            6     0.06347 |                .   |*     .             |
            7     0.04744 |                .   |*     .             |
            8    -0.04929 |                .   *|     .             |
            9     0.15305 |                .   |***   .             |
           10    -0.12346 |                .  **|     .             |

                     AUTOCORRELATION CHECK FOR WHITE NOISE

      TO     CHI                        AUTOCORRELATIONS
     LAG  SQUARE DF    PROB
       6   33.83  6   0.000   0.181   0.535   0.462   0.218   0.229   0.305
```

```
                              ARIMA PROCEDURE

                        NAME OF VARIABLE        =       A11
                        MEAN OF WORKING SERIES=    7.83171
                        STANDARD DEVIATION      =  6.60799
                        NUMBER OF OBSERVATIONS=         41
                                AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
 0     43.6656    1.00000  |                    |********************|        0
 1     15.9143    0.36446  |                .   |*******             |  0.156174
 2     14.8708    0.34056  |              .     |*******             |  0.175698
 3     5.99078    0.13720  |            .       |***     .           |  0.191122
 4    0.544578    0.01247  |            .       |        .           |  0.193509
 5   -0.696286   -0.01595  |            .       |        .           |  0.193528
 6   -0.975483   -0.02234  |            .       |        .           |   0.19356
 7    -9.55388   -0.21880  |            .   ****|        .           |  0.193623
 8    -9.92328   -0.22726  |            .  *****|        .           |  0.199563
 9    -1.11285   -0.02549  |            .      *|        .           |  0.205778
10    -10.5178   -0.24087  |            .  *****|        .           |  0.205855
                           '.' MARKS TWO STANDARD ERRORS

                           INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1   -0.25272   |              .*****|       .            |
           2   -0.34140   |             *******|       .            |
           3    0.15694   |              .     |***    .            |
           4    0.16400   |              .     |***    .            |
           5   -0.12574   |              .  ***|       .            |
           6   -0.17890   |              . ****|       .            |
           7    0.24895   |              .     |*****  .            |
           8    0.11475   |              .     |**     .            |
           9   -0.26906   |              .*****|       .            |
          10    0.15120   |              .     |***    .            |

                           PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
           1    0.36446   |              .     |*******             |
           2    0.23955   |              .     |*****  .            |
           3   -0.05444   |              .    *|       .            |
           4   -0.11561   |              .   **|       .            |
           5   -0.01146   |              .     |       .            |
           6    0.02921   |              .     |*      .            |
           7   -0.24283   |              .*****|       .            |
           8   -0.14338   |              .  ***|       .            |
           9    0.26400   |              .     |*****  .            |
          10   -0.23326   |              .*****|       .            |

                    AUTOCORRELATION CHECK FOR WHITE NOISE

 TO     CHI                               AUTOCORRELATIONS
LAG   SQUARE DF    PROB

 6     12.02  6   0.062   0.364   0.341   0.137   0.012 -0.016 -0.022
```

```
                              ARIMA PROCEDURE

                      NAME OF VARIABLE         =        A12
                      MEAN OF WORKING SERIES=    3.65122
                      STANDARD DEVIATION      =    2.69327
                      NUMBER OF OBSERVATIONS=         41
                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      STD
  0    7.25372    1.00000  |                    |********************|         0
  1    3.51527    0.48434  |              .     |**********          |  0.156174
  2    1.45065    0.19999  |            .       |****   .            |  0.189297
  3  -0.117878   -0.01625  |            .       |       .            |  0.194382
  4    -1.0721   -0.14780  |            .    ***|       .            |  0.194415
  5   -1.31381   -0.18112  |            .   ****|       .            |  0.197137
  6   -2.27622   -0.31380  |            . ******|       .            |  0.201155
  7   -2.19445   -0.30253  |           .  ******|        .           |   0.21276
  8   -1.39861   -0.19281  |           .    ****|        .           |  0.223005
  9  -0.705341   -0.09724  |           .      **|        .           |  0.227035
 10  -0.655185   -0.09032  |           .      **|        .           |  0.228048
                              '.' MARKS TWO STANDARD ERRORS

                              INVERSE AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1    -0.32680  |             *******|     .              |
            2    -0.03779  |              .    *|     .              |
            3     0.08899  |              .     |**   .              |
            4     0.14713  |              .     |***  .              |
            5    -0.13618  |              .  ***|     .              |
            6     0.14404  |              .     |***  .              |
            7     0.10209  |              .     |**   .              |
            8    -0.00514  |              .     |     .              |
            9    -0.07150  |              .    *|     .              |
           10     0.12032  |              .     |**   .              |

                              PARTIAL AUTOCORRELATIONS

          LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
            1     0.48434  |              .     |**********          |
            2    -0.04520  |              .    *|     .              |
            3    -0.12515  |              .  ***|     .              |
            4    -0.10996  |              .   **|     .              |
            5    -0.05595  |              .    *|     .              |
            6    -0.24555  |              .*****|     .              |
            7    -0.09228  |              .   **|     .              |
            8    -0.00203  |              .     |     .              |
            9    -0.04510  |              .    *|     .              |
           10    -0.15738  |              .  ***|     .              |

                      AUTOCORRELATION CHECK FOR WHITE NOISE

 TO    CHI                                AUTOCORRELATIONS
LAG   SQUARE DF    PROB
  6    19.77  6   0.003    0.484  0.200 -0.016 -0.148 -0.181 -0.314   12:55 MONDAY, OCTOBER 5, 1987
                                  SAS
```

```
                              ARIMA PROCEDURE

                     NAME OF VARIABLE        =        A13
                     MEAN OF WORKING SERIES=    2.15366
                     STANDARD DEVIATION      =  5.07668
                     NUMBER OF OBSERVATIONS=         41
                             AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0    25.7727     1.00000 |                    |********************|         0
  1    10.1469     0.39371 |                    |********            |  0.156174
  2    3.06424     0.11889 |                .   |**   .              |   0.17875
  3    5.17424     0.20076 |                .   |****  .             |  0.180668
  4    3.83261     0.14871 |                .   |***   .             |   0.18603
  5     1.1649     0.04520 |               .    |*      .            |  0.188907
  6  -0.758916    -0.02945 |               .   *|       .            |  0.189171
  7   -1.31927    -0.05119 |               .   *|       .            |  0.189283
  8   0.844001     0.03275 |               .    |*      .            |   0.18962
  9   0.934288     0.03625 |               .    |*      .            |  0.189758
 10   -1.02274    -0.03968 |               .   *|       .            |  0.189927
                           '.' MARKS TWO STANDARD ERRORS

                       INVERSE AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1    -0.38930 |           ********|                    |
              2     0.15081 |               .   |***  .              |
              3    -0.17483 |               .***|     .              |
              4    -0.01312 |               .   |     .              |
              5     0.00752 |               .   |     .              |
              6     0.01971 |               .   |     .              |
              7     0.06654 |               .   |*    .              |
              8    -0.03965 |               .  *|     .              |
              9    -0.03865 |               .  *|     .              |
             10     0.03133 |               .   |*    .              |

                       PARTIAL AUTOCORRELATIONS

            LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
              1     0.39371 |                   |********            |
              2    -0.04273 |               .  *|     .              |
              3     0.20011 |               .   |**** .              |
              4     0.00079 |               .   |     .              |
              5    -0.01989 |               .   |     .              |
              6    -0.07818 |               . **|     .              |
              7    -0.04449 |               .  *|     .              |
              8     0.07790 |               .   |**   .              |
              9     0.01562 |               .   |     .              |
             10    -0.03899 |               .  *|     .              |

                 AUTOCORRELATION CHECK FOR WHITE NOISE

     TO    CHI                          AUTOCORRELATIONS
    LAG  SQUARE DF   PROB
      6   10.54  6  0.104   0.394  0.119  0.201  0.149  0.045 -0.029
```

SAS 12:55 MONDAY, OCTOBER 5, 1987

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                                 ARIMA PROCEDURE

                        NAME OF VARIABLE       =       A14
                        MEAN OF WORKING SERIES=    15.7829
                        STANDARD DEVIATION     =   8.64627
                        NUMBER OF OBSERVATIONS=         41
                                 AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1        STD
  0     74.758     1.00000 |                    |********************|          0
  1    6.26782     0.08384 |                  . |**   .              |   0.156174
  2   0.896922     0.01200 |                  . |     .              |   0.157268
  3    10.5644     0.14131 |                  . |***  .              |    0.15729
  4    20.4752     0.27389 |                  . |*****.              |   0.160357
  5   -3.53611    -0.04730 |                 . *|      .             |   0.171387
  6   -1.66831    -0.02232 |                 .  |      .             |   0.171705
  7    17.0853     0.22854 |                 .  |***** .             |   0.171776
  8   -4.33257    -0.05795 |                 . *|      .             |   0.179039
  9    0.30896     0.00413 |                 .  |      .             |   0.179496
 10    2.06242     0.02759 |                 .  |*     .             |   0.179498
                           '.' MARKS TWO STANDARD ERRORS

                            INVERSE AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1    -0.14297  |               .***|   .                |
          2    -0.01420  |               .   |   .                |
          3     0.02498  |               .   |   .                |
          4    -0.28409  |          ******   |   .                |
          5     0.08802  |               .   |** .                |
          6     0.05981  |               .   |*  .                |
          7    -0.17890  |             ****  |   .                |
          8     0.14685  |               .   |***.                |
          9    -0.05857  |               .  *|   .                |
         10     0.00309  |               .   |   .                |

                            PARTIAL AUTOCORRELATIONS

        LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
          1     0.08384  |               .   |** .                |
          2     0.00500  |               .   |   .                |
          3     0.14089  |               .   |***.                |
          4     0.25735  |               .   |*****.              |
          5    -0.09059  |               . **|   .                |
          6    -0.03660  |               .  *|   .                |
          7     0.18226  |               .   |****.               |
          8    -0.15832  |               .***|   .                |
          9     0.06816  |               .   |*  .                |
         10    -0.00364  |               .   |   .                |

                      AUTOCORRELATION CHECK FOR WHITE NOISE

 TO      CHI                                AUTOCORRELATIONS
LAG   SQUARE DF   PROB
  6     4.95  6  0.550   0.084  0.012  0.141  0.274 -0.047 -0.022   12:55 MONDAY, OCTOBER 5, 1987
                                SAS
```

APPENDIX II: Relationships between mean and variance for various R values (GRAPHS)

1 - Relationships between mean and variance

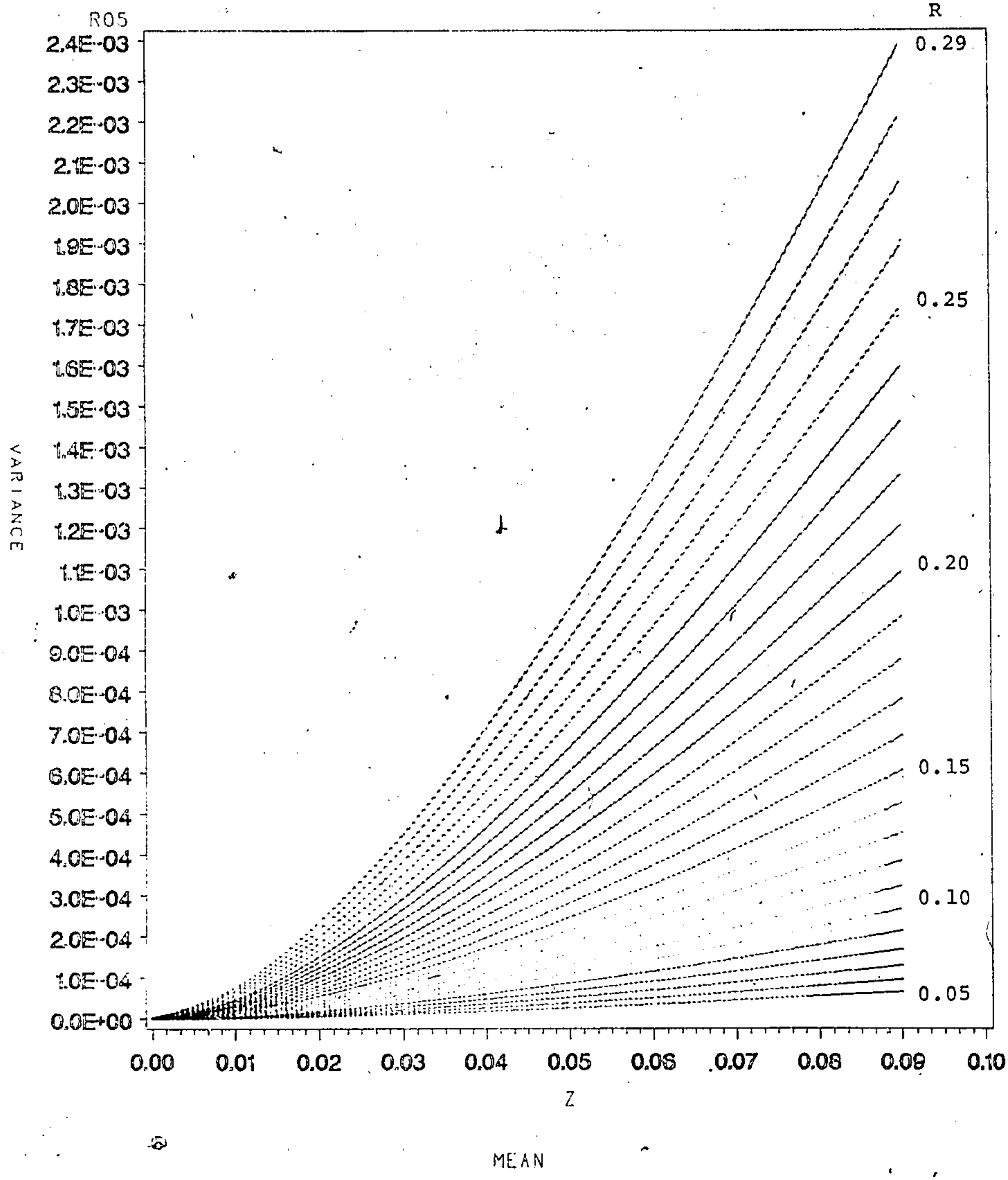
2 - Relationships between mean and variance/(mean(1.0 - mean))

3 - Relationships between logarithmic mean and variance/(mean(1.0 - mean))

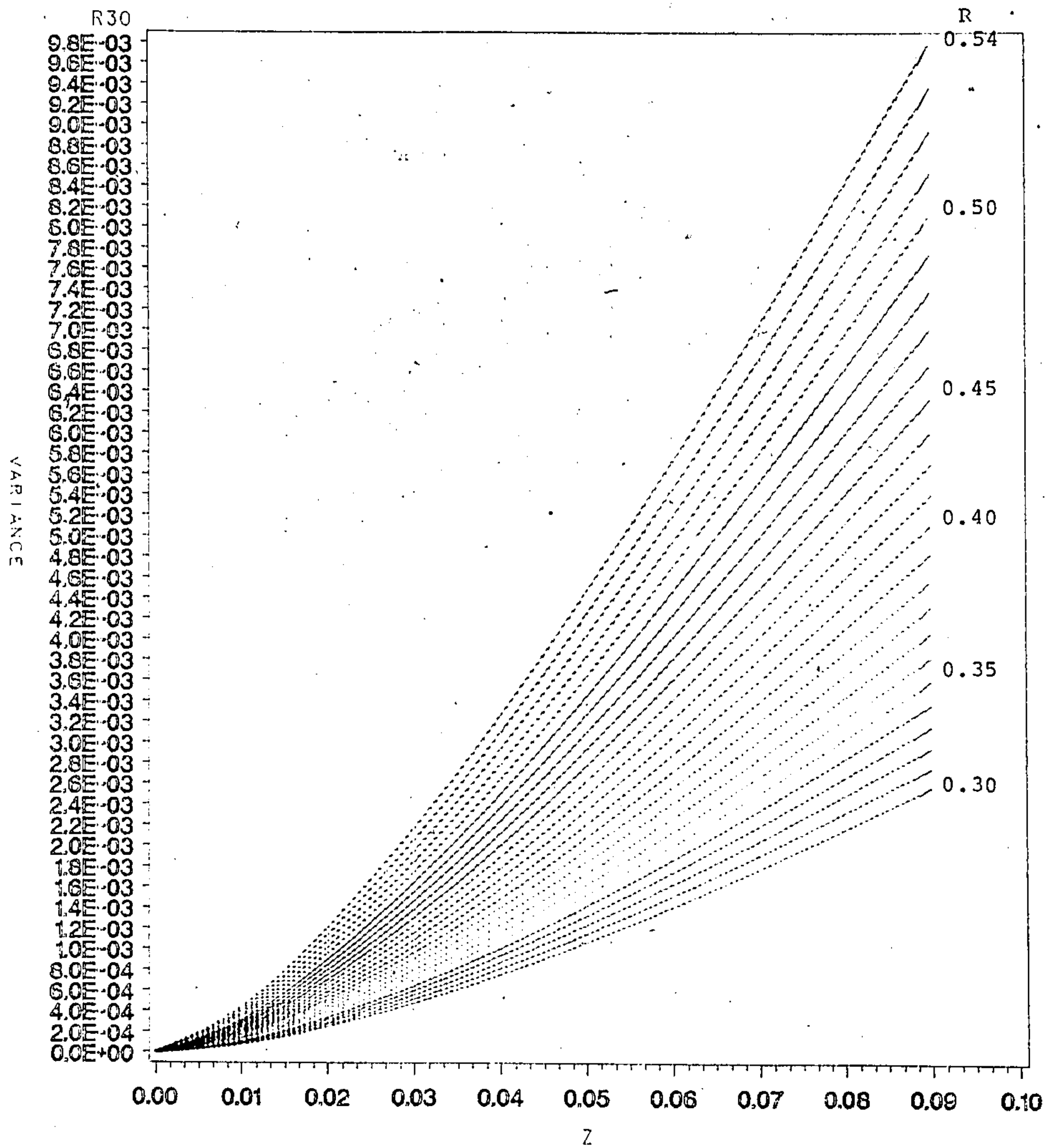
1 - Relationships between mean and variance

[redacted]

WITH SPLINE FITTING



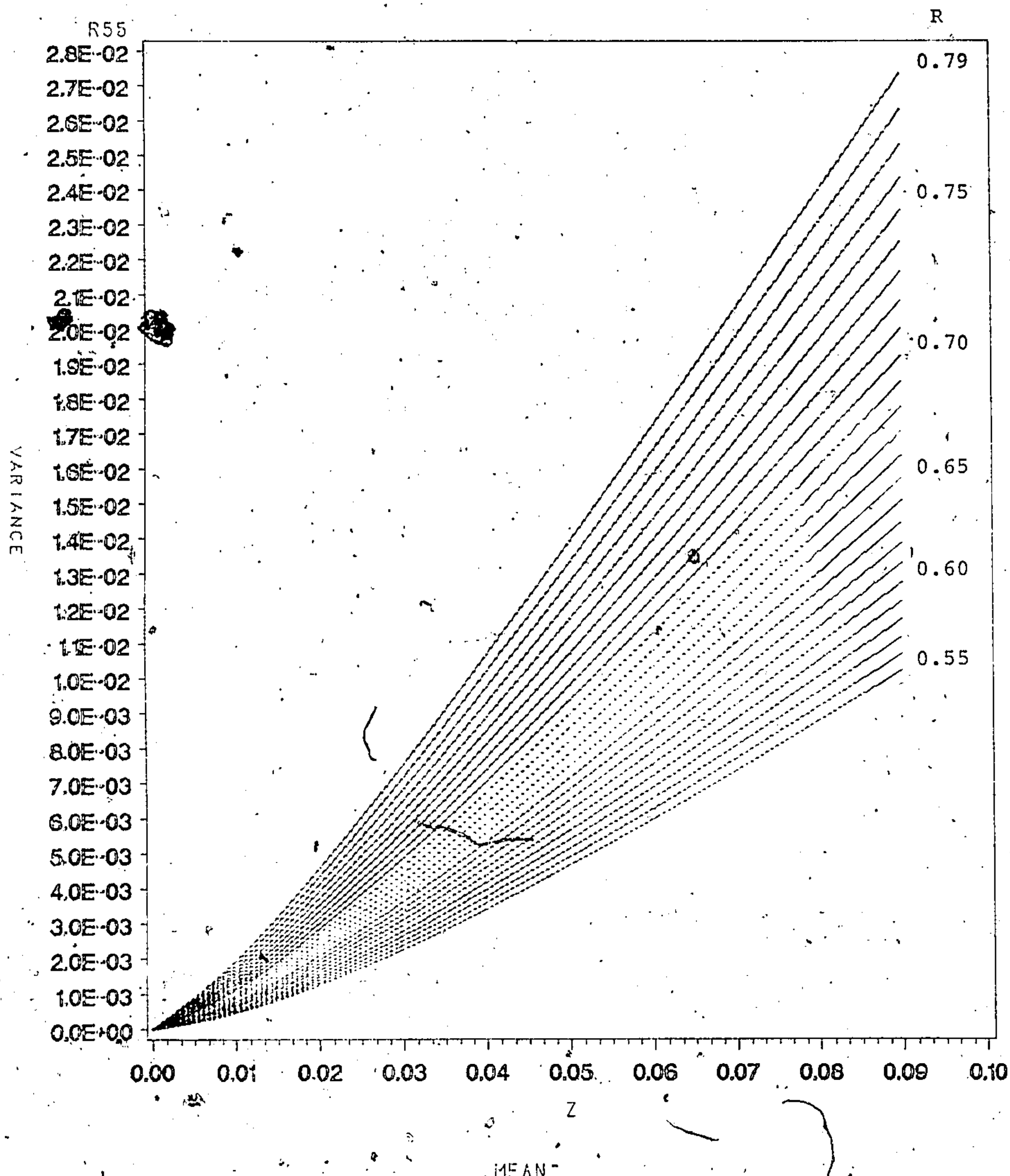
WITH SPLINE FITTING



MEAN

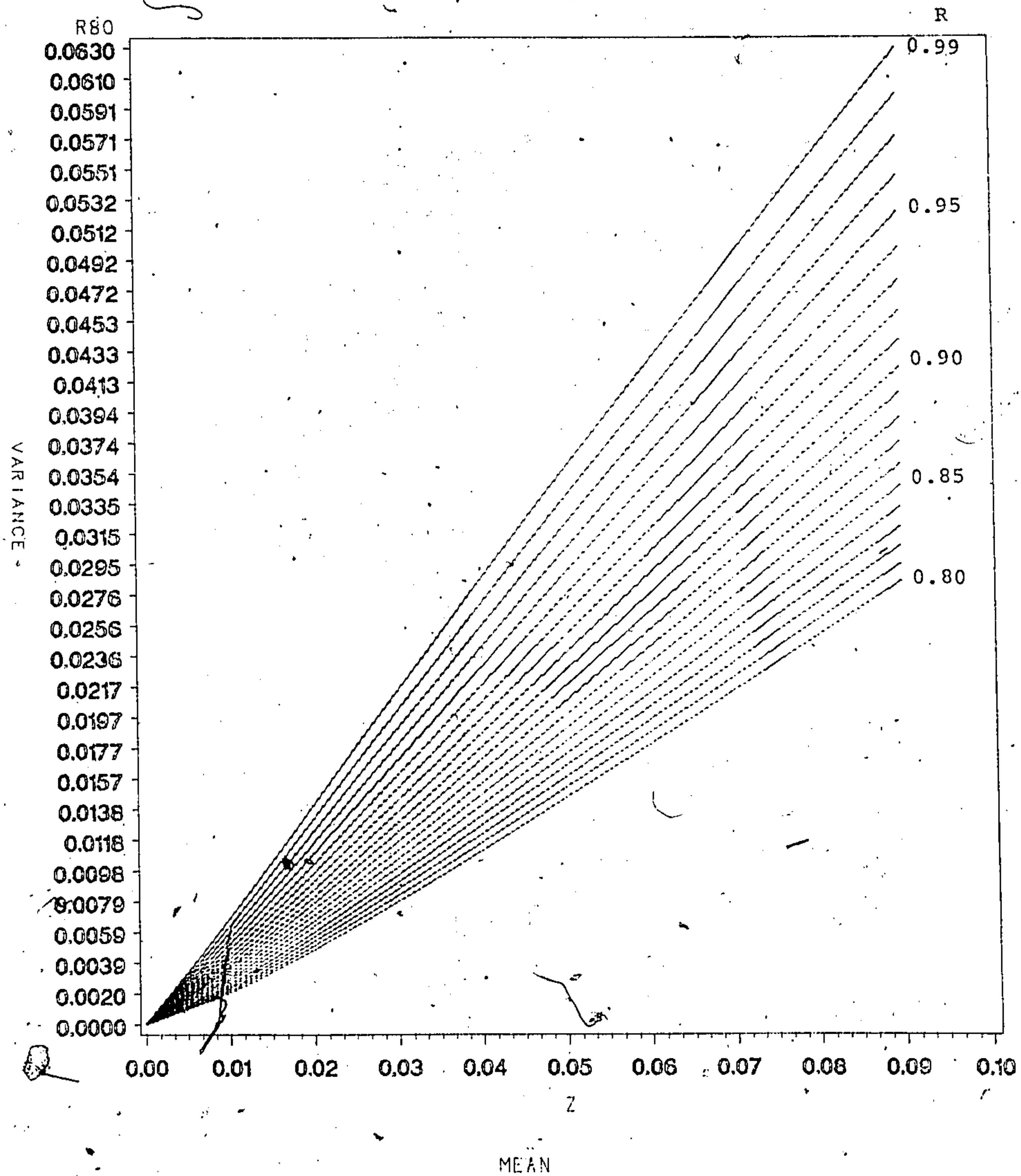
[redacted]

WITH SPLINE FITTING



[redacted]

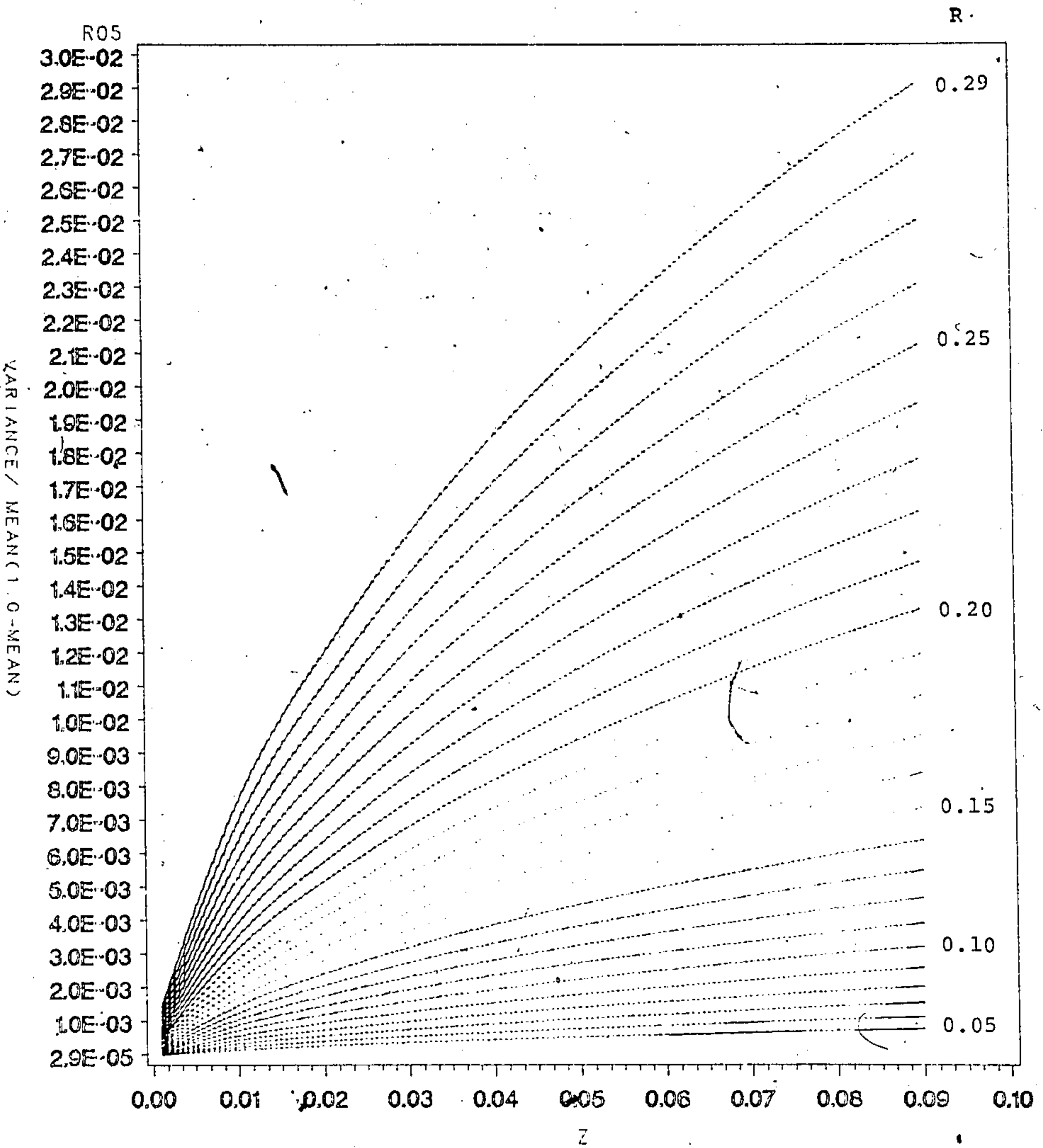
WITH SPLINE FITTING



2 - Relationships between mean and variance/(mean(1.0 - mean))

[redacted]

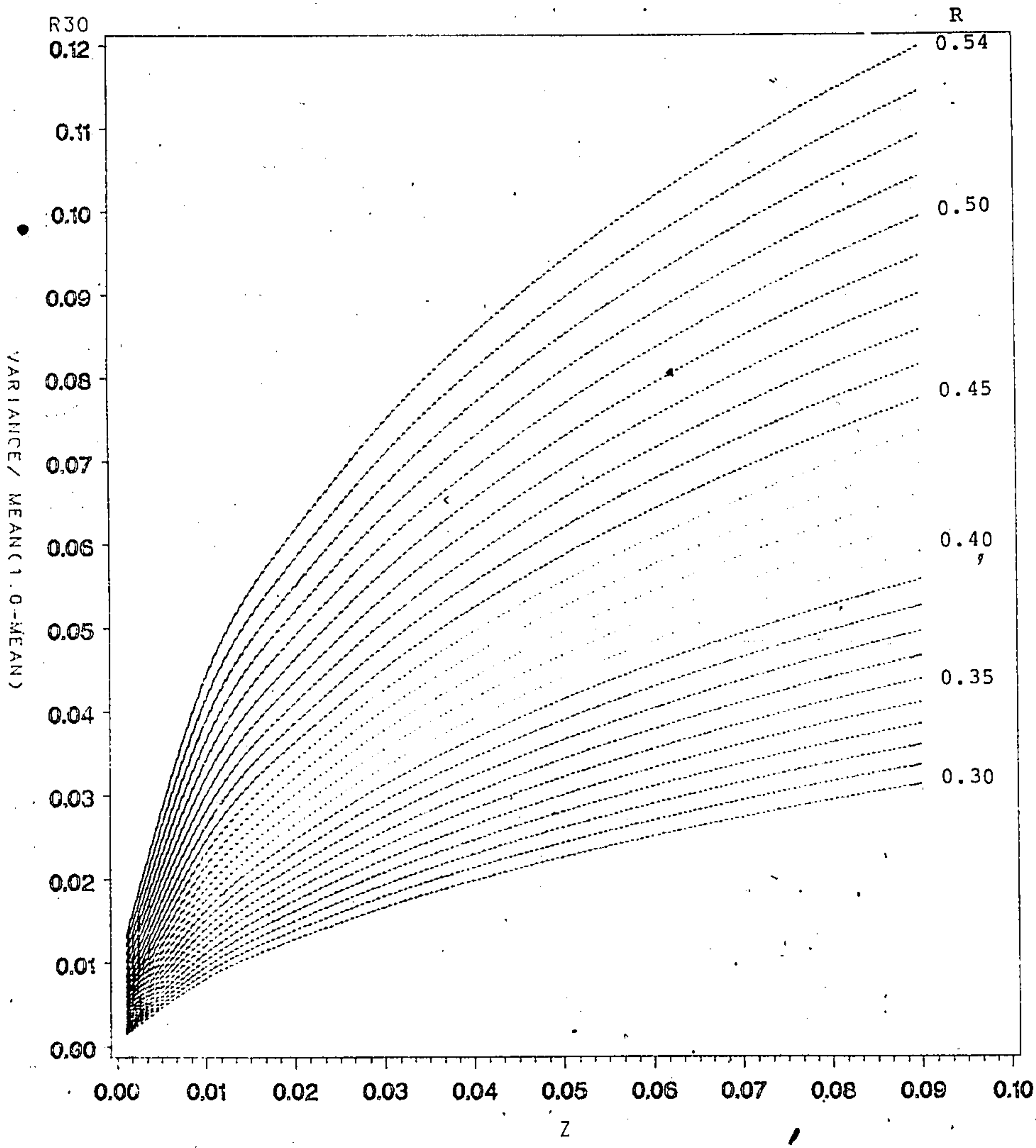
WITH SPLINE FITTING



MEAN

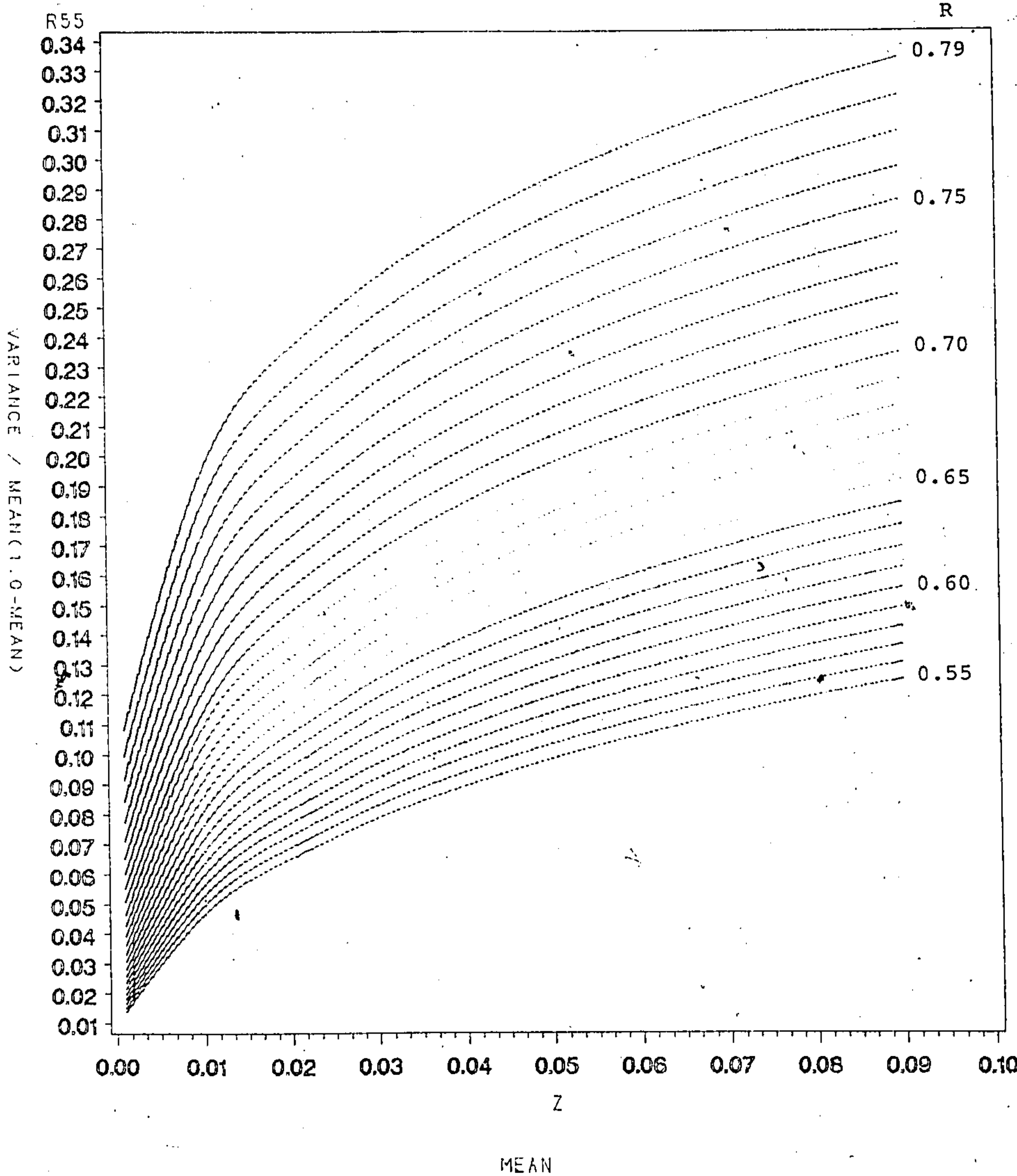
[redacted]

WITH SPLINE FITTING



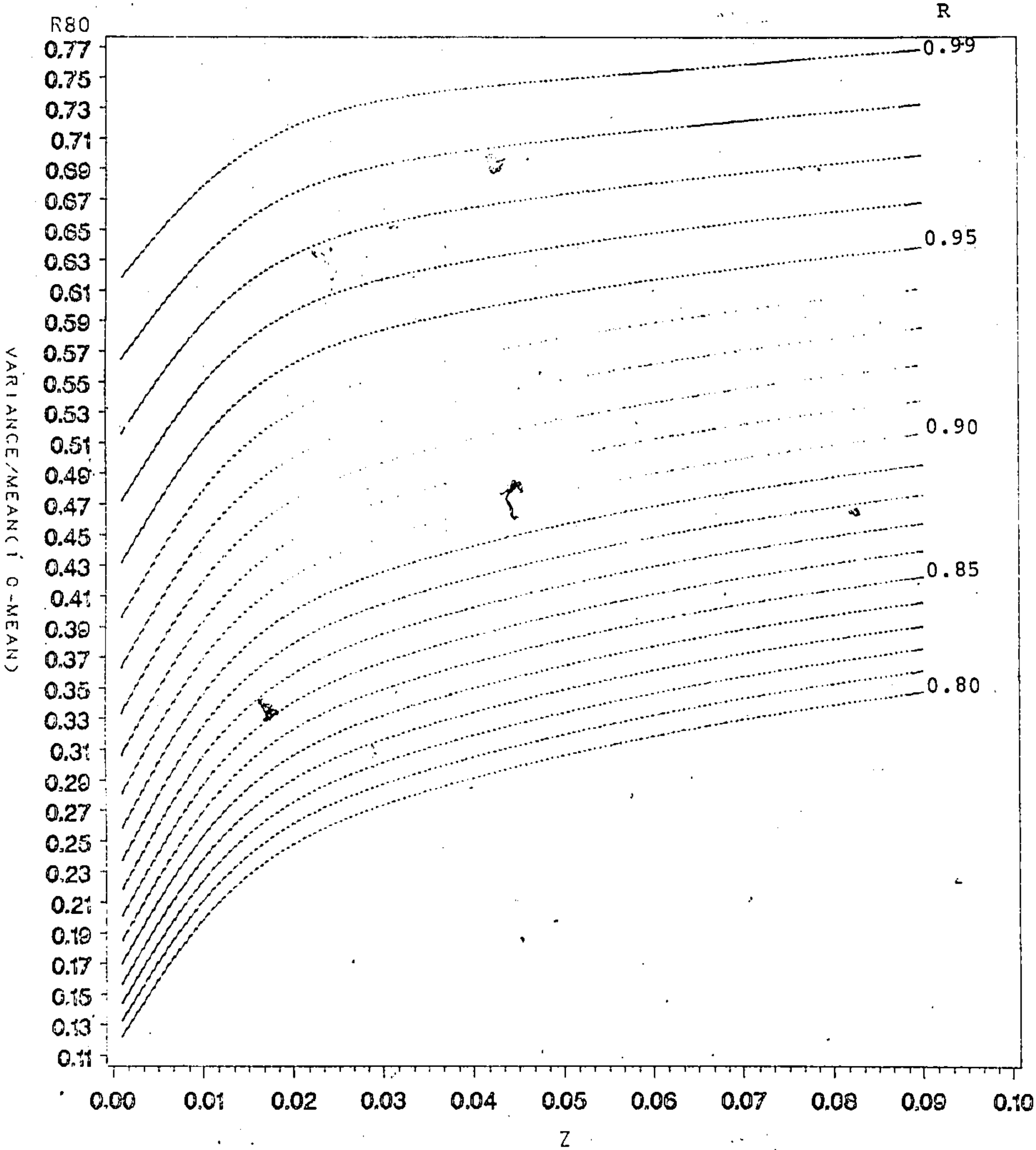
MEAN

[redacted]


R55
R
0.79
0.75
0.70
0.65
0.60
0.55
VARIANCE / MEAN(1.0-MEAN)
0.34
0.33
0.32
0.31
0.30
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
0.21
0.20
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
Z
MEAN

[redacted]

WITH SPLINE FITTING

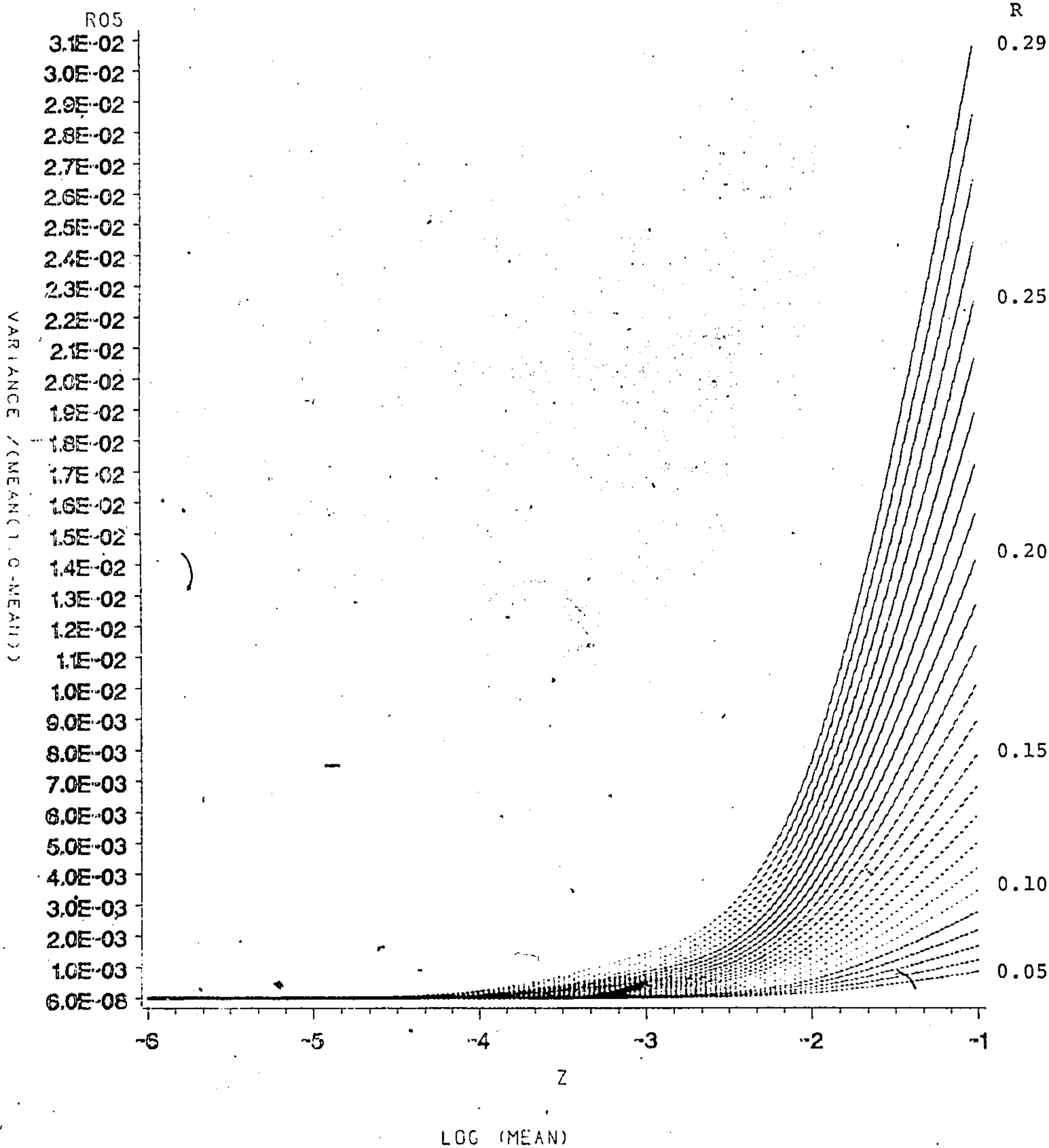


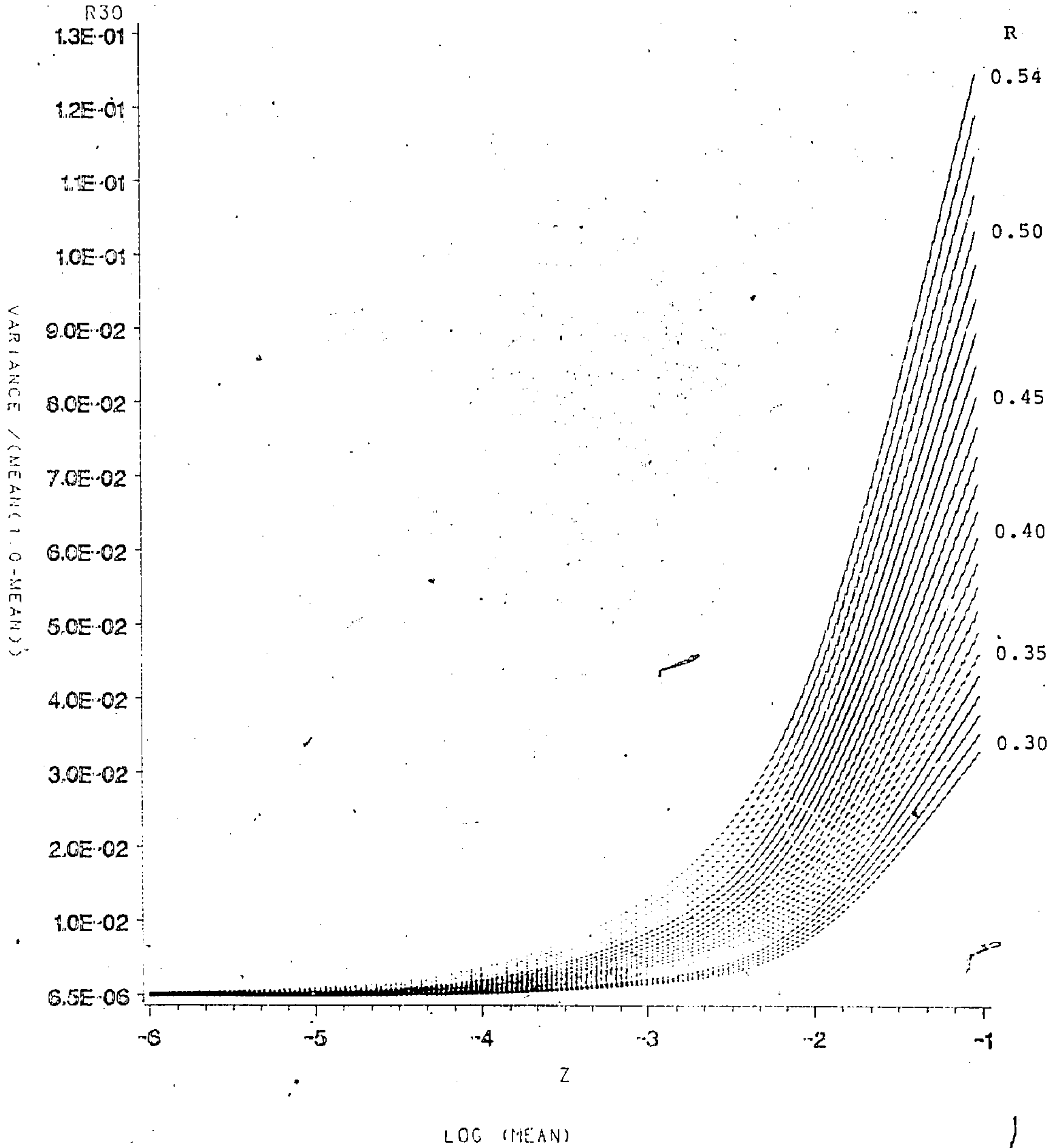
MEAN

3 - Relationships between logarithmic mean and variance/ (mean(1.0 - mean))

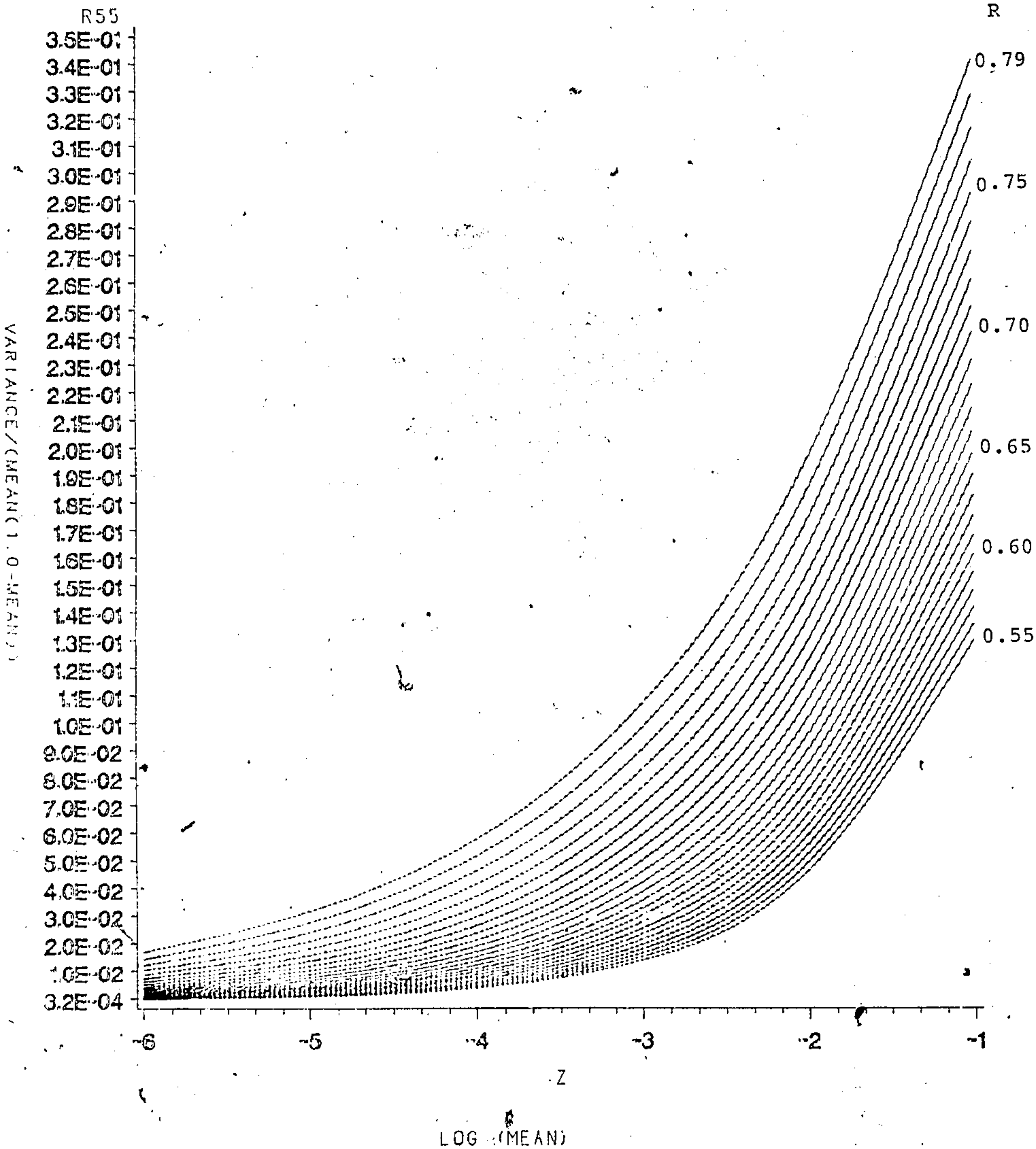
[redacted]

WITH SPLINE FITTING

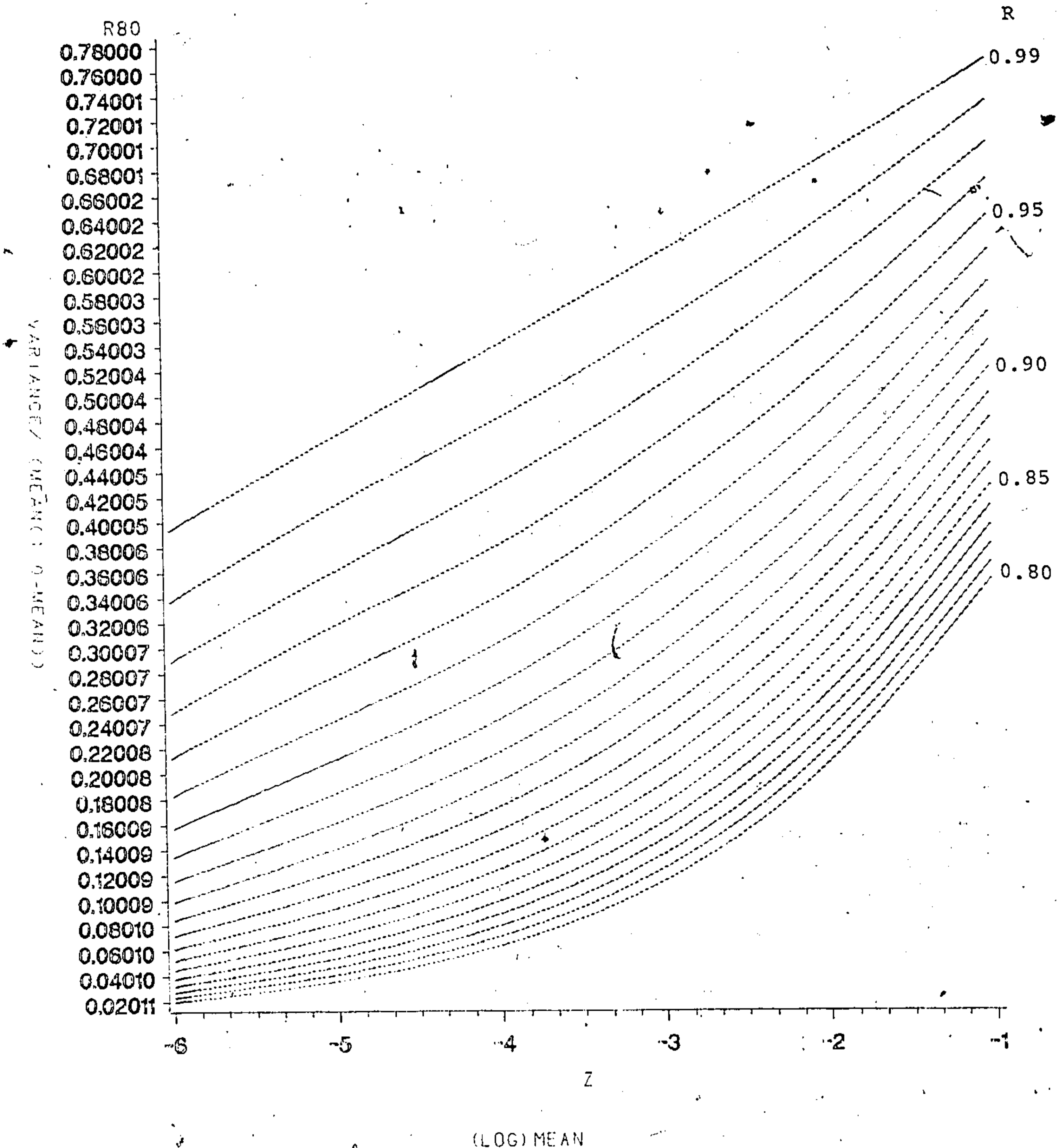



R30
1.3E-01
1.2E-01
1.1E-01
1.0E-01
9.0E-02
8.0E-02
7.0E-02
6.0E-02
5.0E-02
4.0E-02
3.0E-02
2.0E-02
1.0E-02
6.5E-06
VARIANCE /(MEAN(1.0-MEAN))
R
0.54
0.50
0.45
0.40
0.35
0.30
-6
-5
-4
-3
-2
-1
Z
LOG (MEAN)

WITH SPLINE FITTING



WITH SPLINE FITTING



APPENDIX III: Relationships between mean and variance for various R values (TABLES)

1 - Relationships between mean and variance

2 - Relationships between mean and variance/(mean (1.0 - mean))

3 - Relationships between logarithmic mean and variance/ (mean(1.0 - mean))

1 - Relationships between mean and variance

Relationships between mean and variance

R \ $\bar{X}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.05	0.000000	0.000002	0.000006	0.000012	0.000019	0.000027	0.000036	0.000045	0.000055	0.000066
0.06	0.000000	0.000003	0.000008	0.000017	0.000027	0.000038	0.000051	0.000065	0.000080	0.000095
0.07	0.000000	0.000004	0.000012	0.000023	0.000037	0.000052	0.000070	0.000089	0.000109	0.000130
0.08	0.000000	0.000005	0.000015	0.000030	0.000048	0.000069	0.000091	0.000116	0.000142	0.000170
0.09	0.000000	0.000006	0.000019	0.000038	0.000061	0.000087	0.000116	0.000147	0.000180	0.000215
0.10	0.000000	0.000007	0.000024	0.000047	0.000075	0.000108	0.000144	0.000182	0.000223	0.000266
0.11	0.000000	0.000009	0.000029	0.000057	0.000091	0.000131	0.000174	0.000221	0.000271	0.000322
0.12	0.000000	0.000011	0.000035	0.000068	0.000109	0.000156	0.000208	0.000264	0.000323	0.000384
0.13	0.000000	0.000013	0.000041	0.000085	0.000129	0.000184	0.000245	0.000310	0.000380	0.000452
0.14	0.000000	0.000015	0.000048	0.000094	0.000150	0.000214	0.000285	0.000361	0.000442	0.000526
0.15	0.000000	0.000017	0.000055	0.000108	0.000173	0.000247	0.000328	0.000361	0.000442	0.000526
0.16	0.000000	0.000020	0.000063	0.000124	0.000197	0.000282	0.000374	0.000474	0.000580	0.000690
0.17	0.000000	0.000022	0.000072	0.000141	0.000224	0.000319	0.000424	0.000537	0.000657	0.000781
0.18	0.000000	0.000025	0.000081	0.000159	0.000252	0.000360	0.000478	0.000604	0.000739	0.000879
0.19	0.000000	0.000028	0.000091	0.000178	0.000283	0.000403	0.000534	0.000676	0.000827	0.000982
0.20	0.000000	0.000032	0.000102	0.000198	0.000315	0.000449	0.000595	0.000752	0.000919	0.001092
0.21	0.000000	0.000035	0.000113	0.000220	0.000350	0.000497	0.000659	0.000833	0.001018	0.001209
0.22	0.000000	0.000039	0.000125	0.000244	0.000386	0.000549	0.000727	0.000918	0.001121	0.001331
0.23	0.000000	0.000043	0.000138	0.000268	0.000425	0.000603	0.000799	0.001009	0.001231	0.001461
0.24	0.000000	0.000048	0.000152	0.000295	0.000466	0.000661	0.000875	0.001104	0.001346	0.001597
0.25	0.000000	0.000053	0.000167	0.000322	0.000509	0.000722	0.000954	0.001204	0.001468	0.001740
0.26	0.000000	0.000058	0.000182	0.000352	0.000555	0.000786	0.001038	0.001309	0.001595	0.001891
0.27	0.000000	0.000063	0.000199	0.000383	0.000603	0.000854	0.001127	0.001419	0.001729	0.002048
0.28	0.000000	0.000069	0.000216	0.000415	0.000654	0.000924	0.001219	0.001535	0.001869	0.002213
0.29	0.000000	0.000075	0.000234	0.000450	0.000707	0.000999	0.001317	0.001657	0.002016	0.002386
0.30	0.000000	0.000081	0.000254	0.000486	0.000764	0.001077	0.001419	0.001784	0.002169	0.002566
0.31	0.000000	0.000088	0.000274	0.000525	0.000823	0.001159	0.001525	0.001916	0.002329	0.002754
0.32	0.000000	0.000096	0.000296	0.000565	0.000884	0.001245	0.001637	0.002055	0.002496	0.002950
0.33	0.000000	0.000103	0.000319	0.000607	0.000949	0.001335	0.001754	0.002200	0.002671	0.003155
0.34	0.000000	0.000112	0.000343	0.000652	0.001018	0.001429	0.001876	0.002352	0.002853	0.003368
0.35	0.000000	0.000120	0.000369	0.000699	0.001089	0.001528	0.002003	0.002510	0.003042	0.003590
0.36	0.000000	0.000130	0.000395	0.000748	0.001164	0.001631	0.002136	0.002674	0.003240	0.003820
0.37	0.000000	0.000140	0.000424	0.000800	0.001242	0.001739	0.002275	0.002846	0.003445	0.004060
0.38	0.000000	0.000150	0.000454	0.000854	0.001324	0.001851	0.002420	0.003024	0.003659	0.004309
0.39	0.000000	0.000161	0.000485	0.000911	0.001410	0.001968	0.002571	0.003210	0.003881	0.004568
0.40	0.000000	0.000173	0.000518	0.000970	0.001500	0.002091	0.002728	0.003403	0.004112	0.004837
0.41	0.000000	0.000186	0.000553	0.001033	0.001594	0.002216	0.002892	0.003604	0.004351	0.005116
0.42	0.000000	0.000199	0.000590	0.001098	0.001692	0.002352	0.003062	0.003814	0.004600	0.005405

MEAN

R \ $\bar{X}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.43	0.000000	0.000213	0.000628	0.001167	0.001794	0.002491	0.003240	0.004031	0.004859	0.005705
0.44	0.000000	0.000228	0.000669	0.001239	0.001901	0.002636	0.003424	0.004257	0.005127	0.006016
0.45	0.000000	0.000244	0.000712	0.001315	0.002013	0.002787	0.003616	0.004491	0.005405	0.006338
0.46	0.000000	0.000261	0.000757	0.001393	0.002130	0.002944	0.003816	0.004735	0.005693	0.006672
0.47	0.000000	0.000278	0.000804	0.001476	0.002252	0.003108	0.004023	0.004987	0.005993	0.007017
0.48	0.000000	0.000297	0.000853	0.001563	0.002379	0.003279	0.004239	0.005250	0.006303	0.007375
0.49	0.000000	0.000317	0.000906	0.001653	0.002512	0.003456	0.004464	0.005522	0.006624	0.007745
0.50	0.000000	0.000338	0.000961	0.001748	0.002650	0.003641	0.004697	0.004805	0.006957	0.008129
0.51	0.000000	0.000360	0.001018	0.001847	0.002795	0.003834	0.004939	0.006098	0.007302	0.008526
0.52	0.000000	0.000384	0.001079	0.001951	0.002945	0.004034	0.005190	0.006402	0.007659	0.008936
0.53	0.000000	0.000409	0.001142	0.002060	0.003102	0.004242	0.005451	0.006717	0.008029	0.009361
0.54	0.000000	0.000435	0.001209	0.002173	0.003266	0.004459	0.005723	0.007043	0.008412	0.009800
0.55	0.000000	0.000463	0.001280	0.002292	0.003437	0.004685	0.006004	0.007382	0.008809	0.010254
0.56	0.000000	0.000493	0.001353	0.002416	0.003615	0.004919	0.006297	0.007733	0.009219	0.010723
0.57	0.000000	0.000525	0.001431	0.002546	0.003801	0.005163	0.006600	0.008097	0.009644	0.011209
0.58	0.000000	0.000558	0.001512	0.002682	0.003995	0.005417	0.006915	0.008474	0.010084	0.011711
0.59	0.000000	0.000593	0.001598	0.002824	0.004197	0.005681	0.007242	0.008865	0.010539	0.012230
0.60	0.000000	0.000630	0.001688	0.002972	0.004407	0.005956	0.007582	0.009271	0.011011	0.012767
0.61	0.000000	0.000670	0.001782	0.003127	0.004626	0.006241	0.007934	0.009691	0.011499	0.013321
0.62	0.000000	0.000711	0.001881	0.003290	0.004855	0.006538	0.008300	0.010126	0.012003	0.013895
0.63	0.000000	0.000756	0.001985	0.003459	0.005093	0.006847	0.008680	0.010577	0.012526	0.014488
0.64	0.000000	0.000802	0.002094	0.003637	0.005342	0.007168	0.009074	0.011045	0.013067	0.015101
0.65	0.000000	0.000852	0.002209	0.003822	0.005601	0.007502	0.009484	0.011530	0.013627	0.015735
0.66	0.000000	0.000904	0.002330	0.004017	0.005871	0.007849	0.009909	0.012032	0.014207	0.016391
0.67	0.000000	0.000960	0.002456	0.004220	0.006153	0.008211	0.010350	0.012553	0.014808	0.017069
0.68	0.000000	0.001018	0.002589	0.004432	0.006446	0.008587	0.010809	0.013094	0.015429	0.017771
0.69	0.000000	0.001080	0.002729	0.004654	0.006753	0.008978	0.011285	0.013654	0.016074	0.018497
0.70	0.000000	0.001146	0.002876	0.004887	0.007072	0.009386	0.011780	0.014235	0.016741	0.019249
0.71	0.000000	0.001216	0.003030	0.005130	0.007406	0.009810	0.012294	0.014839	0.017433	0.020027
0.72	0.000000	0.001290	0.003193	0.005385	0.007754	0.010252	0.012828	0.015465	0.018150	0.020832
0.73	0.000000	0.001368	0.003363	0.005651	0.008118	0.010712	0.013384	0.016116	0.018894	0.021667
0.74	0.000000	0.001451	0.003543	0.005931	0.008497	0.011192	0.013962	0.016791	0.019665	0.022531
0.75	0.000000	0.001539	0.003732	0.006224	0.008894	0.011692	0.014564	0.017493	0.020466	0.023428
0.76	0.000000	0.001632	0.003931	0.006531	0.009309	0.012213	0.015191	0.018223	0.021297	0.024358
0.77	0.000000	0.001730	0.004141	0.006853	0.009743	0.012758	0.015843	0.018982	0.022161	0.025322
0.78	0.000000	0.001835	0.004362	0.007191	0.010197	0.013326	0.016524	0.019772	0.023058	0.026324
0.79	0.000000	0.001947	0.004594	0.007546	0.010672	0.013920	0.017233	0.020595	0.023992	0.027365
0.80	0.000000	0.002065	0.004840	0.007919	0.011170	0.014541	0.017974	0.021452	0.024964	0.028448

MEAN

$\bar{X}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.81	0.000000	0.002191	0.005099	0.008311	0.011692	0.015190	0.018747	0.022346	0.025976	0.029575
0.82	0.000000	0.002324	0.005374	0.008724	0.012240	0.015871	0.019556	0.023280	0.027032	0.030748
0.83	0.000000	0.002466	0.005663	0.009159	0.012816	0.016583	0.020401	0.024255	0.028134	0.031971
0.84	0.000000	0.002618	0.005970	0.009618	0.013422	0.017331	0.021287	0.025275	0.029284	0.033248
0.85	0.000000	0.002780	0.006295	0.010102	0.014059	0.018117	0.022216	0.026343	0.030488	0.034581
0.86	0.000000	0.002952	0.006640	0.010613	0.014731	0.018943	0.023191	0.027462	0.031747	0.035977
0.87	0.000000	0.003136	0.007006	0.011155	0.015439	0.019813	0.024216	0.028637	0.033068	0.037438
0.88	0.000000	0.003333	0.007396	0.011729	0.016189	0.020730	0.025294	0.029871	0.034455	0.038970
0.89	0.000000	0.003544	0.007810	0.012338	0.016981	0.021699	0.026431	0.031171	0.035913	0.040581
0.90	0.000000	0.003769	0.008253	0.012986	0.017822	0.022723	0.027631	0.032541	0.037448	0.042275
0.91	0.000000	0.004012	0.008726	0.013676	0.018715	0.023809	0.028901	0.033988	0.039067	0.044061
0.92	0.000000	0.004273	0.009232	0.014412	0.019666	0.024961	0.030245	0.035518	0.040779	0.045947
0.93	0.000000	0.004553	0.009775	0.015199	0.020679	0.026187	0.031673	0.037141	0.042592	0.047943
0.94	0.000000	0.004855	0.010358	0.016044	0.021762	0.027494	0.033192	0.038864	0.044515	0.050060
0.95	0.000000	0.005182	0.010987	0.016951	0.022923	0.028890	0.034811	0.040699	0.046560	0.052310
0.96	0.000000	0.005536	0.011665	0.017928	0.024169	0.030385	0.036541	0.042656	0.048741	0.054708
0.97	0.000000	0.005919	0.012400	0.018983	0.025511	0.031990	0.038394	0.044749	0.051070	0.057268
0.98	0.000000	0.006335	0.013197	0.020126	0.026960	0.033718	0.040384	0.046993	0.053565	0.060009
0.99	0.000000	0.006787	0.014064	0.021367	0.028529	0.035582	0.042526	0.049405	0.056245	0.062953

2 - Relationships between mean and variance/(mean(1.0 - mean))

Relationships between mean and variance/(mean(1.0-mean))

R \ $\bar{X}$	0.001	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.05	0.000029	0.000181	0.000300	0.000399	0.000485	0.000561	0.000630	0.000694	0.000753	0.000806
0.06	0.000042	0.000261	0.000433	0.000576	0.000699	0.000810	0.000909	0.001000	0.001085	0.001162
0.07	0.000057	0.000357	0.000592	0.000786	0.000954	0.001104	0.001239	0.001364	0.001479	0.001584
0.08	0.000075	0.000468	0.000775	0.001029	0.001248	0.001445	0.001622	0.001784	0.001934	0.002071
0.09	0.000096	0.000595	0.000984	0.001306	0.001584	0.001833	0.002057	0.002262	0.002452	0.002625
0.10	0.000119	0.000738	0.001220	0.001618	0.001961	0.002269	0.002545	0.002798	0.003033	0.003247
0.11	0.000146	0.000898	0.001483	0.001965	0.002390	0.002753	0.003087	0.003393	0.003678	0.003936
0.12	0.000175	0.001076	0.001773	0.002348	0.002843	0.003286	0.003684	0.004048	0.004387	0.004694
0.13	0.000208	0.001271	0.002092	0.002768	0.003350	0.003870	0.004336	0.004764	0.005161	0.005521
0.14	0.000245	0.001485	0.002440	0.003225	0.003901	0.004504	0.005045	0.005541	0.006001	0.006418
0.15	0.000285	0.001718	0.002818	0.003721	0.004498	0.005191	0.005812	0.006380	0.006909	0.007387
0.16	0.000329	0.001971	0.003227	0.004257	0.005142	0.005930	0.006637	0.007284	0.007884	0.008428
0.17	0.000377	0.002245	0.003669	0.004833	0.005833	0.006724	0.007522	0.008252	0.008929	0.009542
0.18	0.000430	0.002541	0.004143	0.005452	0.006574	0.007574	0.008468	0.009286	0.010044	0.010730
0.19	0.000488	0.002859	0.004552	0.006113	0.007366	0.008480	0.009477	0.010387	0.011231	0.011995
0.20	0.000551	0.003201	0.005196	0.006820	0.008209	0.009445	0.010549	0.011557	0.012492	0.013336
0.21	0.000619	0.003568	0.005777	0.007572	0.009107	0.010469	0.011686	0.012797	0.013826	0.014756
0.22	0.000694	0.003961	0.006397	0.008372	0.010059	0.011555	0.012891	0.014109	0.015237	0.016256
0.23	0.000775	0.004381	0.007057	0.009222	0.011068	0.012704	0.014164	0.015494	0.016726	0.017837
0.24	0.000863	0.004830	0.007758	0.010123	0.012135	0.013918	0.015507	0.016954	0.018293	0.019501
0.25	0.000959	0.005309	0.008503	0.011076	0.013264	0.015199	0.016922	0.018491	0.019942	0.021250
0.26	0.001063	0.005821	0.009293	0.012085	0.014454	0.016548	0.018412	0.020106	0.021674	0.023085
0.27	0.001176	0.006365	0.010131	0.013150	0.015709	0.017969	0.019977	0.021803	0.023491	0.025009
0.28	0.001299	0.006945	0.011017	0.014275	0.017031	0.019462	0.021621	0.023582	0.025394	0.027023
0.29	0.001432	0.007562	0.011956	0.015461	0.018422	0.021031	0.023345	0.025447	0.027386	0.029130
0.30	0.001577	0.008219	0.012948	0.016711	0.019885	0.022677	0.025153	0.027398	0.029470	0.031331
0.31	0.001734	0.008917	0.013996	0.018027	0.021421	0.024404	0.027045	0.029439	0.031647	0.033628
0.32	0.001904	0.009658	0.015103	0.019412	0.023034	0.026213	0.029025	0.031572	0.033919	0.036025
0.33	0.002089	0.010446	0.016272	0.020869	0.024727	0.028108	0.031096	0.033800	0.036290	0.038522
0.34	0.002290	0.011283	0.017505	0.022401	0.026501	0.030091	0.033260	0.036125	0.038762	0.041124
0.35	0.002508	0.012171	0.018806	0.024011	0.028362	0.032165	0.035520	0.038550	0.041337	0.043831
0.36	0.002745	0.013113	0.020178	0.025702	0.030310	0.034334	0.037878	0.041078	0.044018	0.046648
0.37	0.003003	0.014113	0.021623	0.027477	0.032351	0.036600	0.040339	0.043711	0.046809	0.049576
0.38	0.003282	0.015174	0.023146	0.029340	0.034486	0.038968	0.042906	0.046454	0.049711	0.052619
0.39	0.003586	0.016300	0.024750	0.031294	0.036721	0.041439	0.045580	0.049309	0.052728	0.055779
0.40	0.003916	0.017493	0.026439	0.033344	0.039058	0.044019	0.048367	0.052279	0.055864	0.059059
0.41	0.004274	0.018758	0.028217	0.035493	0.041501	0.046710	0.051270	0.055368	0.059121	0.062464
0.42	0.004664	0.020099	0.030088	0.037746	0.044055	0.049517	0.054293	0.058580	0.062504	0.065995

R \ $\bar{X}$	0.001	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.43	0.005087	0.021520	0.032057	0.040107	0.046724	0.052443	0.057438	0.061918	0.066015	0.069657
0.44	0.005547	0.023026	0.034129	0.042581	0.049512	0.055494	0.060711	0.065386	0.069658	0.073452
0.45	0.006048	0.024622	0.036308	0.045171	0.052424	0.058673	0.064116	0.068988	0.073438	0.077385
0.46	0.006592	0.026313	0.038600	0.047884	0.055464	0.061984	0.067657	0.072729	0.077357	0.081460
0.47	0.007183	0.028105	0.041010	0.050725	0.058638	0.065434	0.071338	0.076612	0.081421	0.085680
0.48	0.007826	0.030002	0.043543	0.053699	0.061950	0.069026	0.075165	0.080643	0.085634	0.090049
0.49	0.008525	0.032012	0.046207	0.056811	0.065407	0.072766	0.079142	0.084825	0.089999	0.094572
0.50	0.009286	0.034141	0.049006	0.060069	0.069014	0.076659	0.083274	0.089164	0.094523	0.099253
0.51	0.010112	0.036395	0.051947	0.063477	0.072776	0.080711	0.087567	0.093665	0.099209	0.104097
0.52	0.011011	0.038782	0.055039	0.067042	0.076701	0.084928	0.092026	0.098333	0.104062	0.109108
0.53	0.011989	0.041309	0.058286	0.070772	0.080793	0.089315	0.096657	0.103174	0.109088	0.114292
0.54	0.013052	0.043984	0.061699	0.074674	0.085063	0.093880	0.101466	0.108193	0.114293	0.119654
0.55	0.014208	0.046817	0.065283	0.078755	0.089512	0.098629	0.106459	0.113396	0.119681	0.125199
0.56	0.015464	0.049815	0.069048	0.083022	0.094152	0.103568	0.111644	0.118790	0.125260	0.130933
0.57	0.016831	0.052989	0.073003	0.087485	0.098989	0.108705	0.117026	0.124381	0.131034	0.136862
0.58	0.018317	0.056348	0.077157	0.092152	0.104031	0.114048	0.122613	0.130176	0.137011	0.142992
0.59	0.019933	0.59904	0.081520	0.097031	0.109288	0.119605	0.128413	0.136182	0.143198	0.149330
0.60	0.021689	0.063667	0.086102	0.102133	0.114768	0.125384	0.134433	0.142406	0.149602	0.155882
0.61	0.023599	0.067650	0.090914	0.107468	0.120479	0.131393	0.140682	0.148858	0.156230	0.162656
0.62	0.025675	0.071865	0.095967	0.113046	0.126434	0.137644	0.147169	0.155544	0.163091	0.169660
0.63	0.027931	0.076326	0.101275	0.118878	0.132640	0.144144	0.153903	0.162474	0.170192	0.176901
0.64	0.030385	0.081048	0.106849	0.124977	0.139111	0.150904	0.160894	0.169658	0.177543	0.184388
0.65	0.033051	0.086045	0.112704	0.131355	0.145857	0.157936	0.168152	0.177104	0.185153	0.192131
0.66	0.035950	0.091335	0.118854	0.138025	0.152890	0.165251	0.175688	0.184824	0.193032	0.220138
0.67	0.039100	0.096934	0.125314	0.145001	0.160224	0.172861	0.183514	0.192828	0.201190	0.208420
0.68	0.042525	0.102861	0.132102	0.152298	0.167873	0.180779	0.191641	0.201129	0.209640	0.216987
0.69	0.046247	0.109137	0.139234	0.159933	0.175852	0.189020	0.200084	0.209738	0.218393	0.225852
0.70	0.050293	0.115782	0.146729	0.167922	0.184176	0.197597	0.208857	0.218670	0.227461	0.235027
0.71	0.054691	0.122819	0.154607	0.176284	0.192862	0.206528	0.217974	0.227938	0.236859	0.244525
0.72	0.059472	0.130273	0.162891	0.185039	0.201929	0.215829	0.227451	0.237559	0.246603	0.254361
0.73	0.064670	0.138170	0.171602	0.194206	0.211396	0.225519	0.237307	0.247549	0.256706	0.264549
0.74	0.070322	0.146539	0.180767	0.203811	0.221284	0.235617	0.247560	0.257925	0.267188	0.275108
0.75	0.076467	0.155410	0.190411	0.213875	0.231617	0.246144	0.258229	0.268708	0.278066	0.286054
0.76	0.083152	0.164817	0.200564	0.224427	0.242418	0.257125	0.269338	0.279917	0.289362	0.297407
0.77	0.090423	0.174795	0.211259	0.235496	0.253716	0.268585	0.280911	0.291577	0.301095	0.309189
0.78	0.098336	0.185385	0.222528	0.247111	0.265537	0.280550	0.292973	0.303713	0.313293	0.321424
0.79	0.106948	0.196628	0.234409	0.259310	0.277916	0.293051	0.305553	0.316351	0.325980	0.334135
0.80	0.116326	0.208571	0.246945	0.272128	0.290887	0.306121	0.318682	0.329522	0.339186	0.347353

R \ $\bar{X}$	0.001	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.81	0.126540	0.221265	0.260179	0.285608	0.304488	0.319796	0.332395	0.343259	0.352943	0.361109
0.82	0.137671	0.234766	0.274162	0.299794	0.318761	0.334116	0.346729	0.357598	0.367285	0.375436
0.83	0.149806	0.249135	0.288949	0.314739	0.333756	0.349125	0.361727	0.372579	0.382251	0.390371
0.84	0.163045	0.264441	0.304601	0.330498	0.349522	0.364872	0.377434	0.388245	0.397885	0.405958
0.85	0.177496	0.280757	0.321185	0.347133	0.366119	0.381411	0.393902	0.404648	0.414234	0.422242
0.86	0.193281	0.298168	0.338776	0.364715	0.383611	0.398803	0.411188	0.421841	0.431351	0.439276
0.87	0.210537	0.316764	0.357460	0.383322	0.402071	0.417116	0.429357	0.439886	0.449296	0.457118
0.88	0.229417	0.336649	0.377329	0.403043	0.421580	0.436425	0.448480	0.458851	0.468135	0.475832
0.89	0.250093	0.357935	0.398492	0.423976	0.442230	0.456816	0.468636	0.478813	0.487943	0.495492
0.90	0.272758	0.380751	0.421066	0.446233	0.464125	0.478384	0.489915	0.499857	0.508802	0.516178
0.91	0.297630	0.405240	0.445188	0.469943	0.487380	0.501238	0.512421	0.522081	0.530808	0.537986
0.92	0.324956	0.431561	0.471011	0.495246	0.512130	0.525501	0.536266	0.545594	0.554064	0.561018
0.93	0.355014	0.459895	0.498709	0.522311	0.538525	0.551308	0.561579	0.570517	0.578691	0.585390
0.94	0.388123	0.490445	0.528479	0.551322	0.566735	0.578819	0.588506	0.596990	0.604825	0.611238
0.95	0.424640	0.523438	0.560548	0.582493	0.596957	0.608211	0.617213	0.625171	0.632617	0.638712
0.96	0.464978	0.559134	0.595172	0.616070	0.629414	0.639687	0.647888	0.655236	0.662240	0.667981
0.97	0.509602	0.597824	0.632648	0.652334	0.664360	0.673478	0.680743	0.687389	0.693891	0.699244
0.98	0.559048	0.639840	0.673312	0.691607	0.702089	0.709850	0.716021	0.721857	0.727791	0.732718
0.99	0.613927	0.685557	0.717555	0.734261	0.742937	0.749102	0.753999	0.758903	0.764198	0.768657

3 - Relationships between logarithmic mean and variance/
(mean(1.0 - mean))

Relationships between logarithmic mean and variance/(mean(1.0-mean))

R \ $\bar{X}$	-1	-2	-3	-4	-5	-6
0.05	0.000000	0.000001	0.000004	0.000029	0.000181	0.000856
0.06	0.000000	0.000001	0.000006	0.000042	0.000261	0.001234
0.07	0.000000	0.000001	0.000008	0.000057	0.000357	0.001682
0.08	0.000000	0.000001	0.000010	0.000075	0.000468	0.002199
0.09	0.000000	0.000002	0.000013	0.000096	0.000595	0.002787
0.10	0.000000	0.000002	0.000017	0.000119	0.000738	0.003447
0.11	0.000000	0.000003	0.000021	0.000146	0.000898	0.004177
0.12	0.000000	0.000003	0.000025	0.000175	0.001076	0.004981
0.13	0.000000	0.000004	0.000030	0.000208	0.001271	0.005857
0.14	0.000001	0.000005	0.000035	0.000245	0.001485	0.006808
0.15	0.000001	0.000006	0.000041	0.000285	0.001718	0.007834
0.16	0.000001	0.000007	0.000048	0.000329	0.001971	0.008936
0.17	0.000001	0.000008	0.000055	0.000377	0.002245	0.010114
0.18	0.000001	0.000009	0.000064	0.000430	0.002541	0.011371
0.19	0.000001	0.000010	0.000073	0.000488	0.002859	0.012708
0.20	0.000002	0.000012	0.000083	0.000551	0.003201	0.014125
0.21	0.000002	0.000013	0.000094	0.000619	0.003568	0.015624
0.22	0.000002	0.000015	0.000107	0.000694	0.003961	0.017206
0.23	0.000002	0.000017	0.000121	0.000775	0.004381	0.018873
0.24	0.000003	0.000020	0.000136	0.000863	0.004830	0.020627
0.25	0.000003	0.000023	0.000153	0.000959	0.005309	0.022469
0.26	0.000004	0.000026	0.000171	0.001063	0.005821	0.024400
0.27	0.000004	0.000029	0.000192	0.001176	0.006365	0.026424
0.28	0.000005	0.000033	0.000215	0.001299	0.006945	0.028540
0.29	0.000006	0.000038	0.000240	0.001432	0.007562	0.030752
0.30	0.00001	0.00004	0.00027	0.00158	0.00822	0.03306
0.31	0.00001	0.00005	0.00030	0.00173	0.00892	0.03547
0.32	0.00001	0.00006	0.00033	0.00190	0.00966	0.03798
0.33	0.00001	0.00006	0.00037	0.00209	0.01045	0.04060
0.34	0.00001	0.00007	0.00042	0.00229	0.01128	0.04332
0.35	0.00001	0.00008	0.00046	0.00251	0.01217	0.04615
0.36	0.00002	0.00009	0.00052	0.00275	0.01311	0.04909
0.37	0.00002	0.00010	0.00057	0.00300	0.01411	0.05214
0.38	0.00002	0.00012	0.00064	0.00328	0.01517	0.05531
0.39	0.00002	0.00014	0.00071	0.00359	0.01630	0.05860
0.40	0.00003	0.00015	0.00079	0.00392	0.01749	0.06202
0.41	0.00003	0.00018	0.00088	0.00427	0.01876	0.06555
0.42	0.00004	0.00020	0.00098	0.00466	0.02010	0.06922
0.43	0.00005	0.00023	0.00110	0.00509	0.02152	0.07302
0.44	0.00005	0.00026	0.00122	0.00555	0.02303	0.07695
0.45	0.00006	0.00030	0.00136	0.00605	0.02462	0.08103
0.46	0.00007	0.00034	0.00152	0.00659	0.02631	0.08524
0.47	0.00009	0.00039	0.00169	0.00718	0.02810	0.08960
0.48	0.00010	0.00044	0.00189	0.00783	0.03000	0.09412
0.49	0.00012	0.00051	0.00211	0.00853	0.03201	0.09878
0.50	0.00014	0.00058	0.00235	0.00929	0.03414	0.10361
0.51	0.00016	0.00067	0.00262	0.01011	0.03659	0.10859
0.52	0.00019	0.00076	0.00293	0.01101	0.03878	0.11375
0.53	0.00023	0.00087	0.00327	0.01199	0.04131	0.11907
0.54	0.00027	0.00100	0.00365	0.01305	0.04398	0.12458
0.55	0.00032	0.00115	0.00407	0.01421	0.04682	0.13026
0.56	0.00038	0.00132	0.00455	0.01546	0.04982	0.13613
0.57	0.00044	0.00152	0.00508	0.01683	0.05299	0.14220
0.58	0.00052	0.00174	0.00567	0.01832	0.05637	0.14847

MEAN

R \ $\overline{X}$	-1	-2	-3	-4	-5	-6
0.59	0.00062	0.00200	0.00634	0.01993	0.05990	0.15494
0.60	0.00073	0.00230	0.00708	0.02169	0.06367	0.16163
0.61	0.00086	0.00264	0.00791	0.02360	0.06765	0.16853
0.62	0.00102	0.00303	0.00884	0.02567	0.07187	0.17566
0.63	0.00121	0.00348	0.00987	0.02793	0.07633	0.18303
0.64	0.00143	0.00400	0.01103	0.03038	0.08105	0.19063
0.65	0.00169	0.00460	0.01232	0.03305	0.08605	0.19849
0.66	0.00199	0.00528	0.01377	0.03595	0.09133	0.20661
0.67	0.00236	0.00607	0.01539	0.03910	0.09693	0.21500
0.68	0.00278	0.00697	0.01719	0.04252	0.10286	0.22368
0.69	0.00329	0.00801	0.01920	0.04625	0.10914	0.23264
0.70	0.00389	0.00920	0.02145	0.05029	0.11578	0.24191
0.71	0.00459	0.01057	0.02397	0.05469	0.12282	0.25149
0.72	0.00542	0.01214	0.02677	0.05947	0.13027	0.26141
0.73	0.00639	0.01394	0.02990	0.06467	0.13817	0.27167
0.74	0.00754	0.01600	0.03340	0.07032	0.14654	0.28230
0.75	0.00889	0.01836	0.03730	0.07647	0.15541	0.29330
0.76	0.01047	0.02107	0.04166	0.08315	0.16482	0.30470
0.77	0.01234	0.02418	0.04652	0.09042	0.17480	0.31653
0.78	0.01453	0.02773	0.05195	0.09834	0.18538	0.32879
0.79	0.01709	0.03181	0.05800	0.10695	0.19663	0.34152
0.80	0.02011	0.03647	0.06476	0.11633	0.20857	0.35475
0.81	0.02363	0.04181	0.07231	0.12654	0.22126	0.36850
0.82	0.02777	0.04792	0.08074	0.13767	0.23477	0.38282
0.83	0.03260	0.05491	0.09015	0.14981	0.24914	0.39772
0.84	0.03826	0.06291	0.10066	0.16304	0.26444	0.41327
0.85	0.04487	0.07207	0.11240	0.17750	0.28076	0.42950
0.86	0.05259	0.08254	0.12552	0.19328	0.29817	0.44646
0.87	0.06160	0.09452	0.14018	0.21054	0.31676	0.46421
0.88	0.07211	0.10823	0.15657	0.22942	0.33665	0.48282
0.89	0.08435	0.12391	0.17490	0.25009	0.35794	0.50236
0.90	0.09861	0.14185	0.19540	0.27276	0.38075	0.52290
0.91	0.11519	0.16237	0.21836	0.29763	0.40524	0.54455
0.92	0.13448	0.18585	0.24406	0.32496	0.43156	0.56739
0.93	0.15689	0.21270	0.27285	0.35501	0.45990	0.59156
0.94	0.18291	0.24344	0.30513	0.38812	0.49044	0.61719
0.95	0.21309	0.27862	0.34132	0.42464	0.52344	0.64442
0.96	0.24809	0.31889	0.38195	0.46498	0.55913	0.67342
0.97	0.28862	0.36499	0.42756	0.50960	0.59782	0.70440
0.98	0.33555	0.41778	0.47881	0.55905	0.63984	0.73757
0.99	0.38982	0.47826	0.53643	0.61393	0.68556	0.77319