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FACULTÉ DES ÉTUDES SUPÉRIEURES
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FACULTY OF GRADUATE AND
POSTDOCTORAL STUDIES

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Ph.D. (Economics)

GRADE / DEGREE

Department of Economics

FACULTÉ, ÉCOLE, DÉPARTEMENT / FACULTY, SCHOOL, DEPARTMENT

Two Country Overlapping-Generations Model on the Global Transmission of Fiscal Policies

TITRE DE LA THÈSE / TITLE OF THESIS

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TWO COUNTRY OVERLAPPING-GENERATIONS MODEL
ON THE GLOBAL TRANSMISSION OF
FISCAL POLICIES

A Thesis

Submitted to the Faculty of Graduate and Postdoctoral Studies of the

University of Ottawa

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in

Economics

By

YOUNG CHEOL JUNG

University of Ottawa

Ottawa, Ontario, Canada

January 2008



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Direction du
Patrimoine de l'édition

395 Wellington Street
Ottawa ON K1A 0N4
Canada

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

ISBN: 978-0-494-50738-4

Our file Notre référence

ISBN: 978-0-494-50738-4

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ACKNOWLEDGEMENT

My long-time work toward a Ph. D was really a challenging experience. Now I realize that I could attain here because of many helpful people. First of all, I feel that I am deeply indebted to my supervisor Professor Nguyen Van Quyen. Without his advices, I might not finish this dissertation. Also I would like to thank my external examiner Professor Nils-Petter Lagerlöf and all members of my thesis committee – Professors Fanny Demers, Nick Rowe, and Marcel Mérette. Their valuable comments helped me to improve the dissertation. My deepest gratitude goes to my family. My father and mother has always been my life-time supporter. Finally, I send my special thank to my wife, Keumjoo. This dissertation is an outcome of her patience and belief.

ABSTRACT

This thesis aims to answer several issues on the global transmission effects of the government fiscal policy. In order to address the issues, this thesis builds a two-country overlapping-generations (OLG) model in which there exist different time preferences, different fiscal policies, and different tax schemes between countries. In particular, the model adopt a utility function with a constant inter-temporal elasticity of substitution (CIES), in which saving positively depends on both labor income and the interest rate.

In chapter 1, we try to answer the question; How does the difference in time preferences of individuals affect economic variables in an open economy? and have shown that; first, when two economies with different time preferences are integrated, the capital labor ratio along the transition path is higher or lower than its steady-state value; second, depending on whether the former or the latter happens, the behavior of the balance of payments and the welfare of two countries also differ; third, the less patient country reveals a surplus of current account in spite of an aging population; lastly, economic integration, even if it worsens the current welfare of the less patient country, ultimately improves the future welfare of the country.

In chapter 2, we try to answer the question; How is the government debt or budget deficit of a country transmitted to other countries? We have first shown that the crowding out effect of the budget deficit is more pronounced in autarky than in open economy. Second, with an aging population, the current account of the debtor country shows a surplus during the transition path and converges to zero in the steady state. With an increasing population, the current account shows an ambiguous sign during the transition path and reveals a deficit in the steady state; lastly, the government debt lowers the welfare of future generations in both countries. In particular, the welfare of the debtor country continuously declines.

In chapter 3, we try to answer the question; Which tax scheme is the best? We have first shown that with respect to capital formation, the consumption tax dominates the capital income tax and the capital income tax dominates the wage tax. Second, the shift to the consumption tax or the capital income tax renders the tax-reformed country a creditor and the other country a debtor. Third, with respect to welfare, the consumption tax is superior to the wage tax but seems not show an advantage over the capital income tax in the tax-reformed country. However, welfare in the other country is somewhat higher under the consumption tax scheme than under other tax schemes of the tax-reformed country; fourth, the tax rate is highest under the capital income tax and is lowest under the consumption tax. Lastly, the transitional analysis shows that the capital income tax can be more rewarding than the consumption tax in the aspect of welfare because the reform to the capital income tax enters its new steady state with a faster speed.

Key words: OLG Model, Time Preference, Capital labor ratio, Budget deficit, Government debt, Tax reform, Wage tax, Capital income tax, Consumption tax, Current Account, Net foreign asset, Welfare.

JEL Classification: C68, C88, F15, F21, F32, F34, F41, H21, H63

INTRODUCTION TO THE THESIS

HOW FISCAL POLICIES AFFECT THE WELFARE of current and future generations has been a long-standing issue in economics. In particular, an intensifying global integration of goods and capital markets has added complexity to the question, and has given a strong motive to look at the issue in the open economy framework. In this context, the aim of this thesis is to answer the following three questions with a two-country model. How does the propensity to save, represented by the rate of time preferences of individual agents, affect the capital stock, the balance of payments, and welfare of countries? How do the government budget deficits and the government debt, affect the macroeconomic variables of countries? Which tax scheme is the best to foster capital formation and to improve the welfare of countries?

The existing literatures on these issues have some limitations in one or more ways. First, the literatures which adopt the infinite horizon model as an analytical tool tend to be confronted with a structural problem; if a country consists of infinitely lived individuals, the interest rate equals the rate of time preference in the steady state because consumption does not vary over time. Therefore, different time preferences cannot be assumed in the open economy in which the interest rates are internationally identical. In addition, the infinite horizon model cannot show the real effects of the government debt or budget deficits since the private sector internalizes the public sector's budget constraint. In these aspects, the overlapping generations (OLG) model can be regarded as a better framework

to cope with the issue on the fiscal policy. However, the literatures adopting the OLG framework also have their own limitations; preferences are frequently assumed to be of a general form or to be logarithmic. For example, in the work of Rankin and Roffia (2003), the assumption of logarithmic preferences simplifies the computations greatly but the simplicity comes with a cost: saving is independent of the interest rate so the role of the interest rate in the capital formation is ignored. The usage of a general preference is not free from difficulties, either. For example, as shown in the work of Persson (1985), the models using the general preference are not feasible for the transition path of the economies, and thus have to rely on comparative analysis that focuses mainly on the steady state. Moreover, most literatures dealing with the issue on the effects of alternative tax schemes are confined to the closed economy analysis; see, for example, Atkinson and Sandmo (1980), Summers (1981) and Auerbach and Kotlikoff (1987). As the global integration of capital markets is progressing, the impact of the tax policy of a country will not be restricted to the domestic economy but will be transmitted to other countries. Then the repercussions from the rest of the world in turn will feedback to the domestic economy. In this sense, the work of Ihori (1996) is noteworthy in that it extends the analysis for the effects of the tax reform to the open economy. However, his work is done with an analytical method and thus, the feedback effect from the repercussions of the tax reform are not properly answered.

The purpose of this thesis is to address the above three questions along with resolving the limitations that previous literatures have been confronted with. As building blocks for the thesis, we formulate a two-country OLG model. Also the model adopts a utility function

with a constant inter-temporal elasticity of substitution (CIES). This type of preferences allows saving to depend on both labor income and the interest rate. More specifically, the model using CIES preferences can accommodate a secondary effect of the interest rate by allowing a positive relationship between the interest rates and saving. Furthermore, by adopting this specific functional form, this study can describe the transition path following the policy shock more precisely.

In Chapter 1, we address the first question: *How does the propensity to save, represented by the rate of time preferences of individual agents, affect the capital stock, the balance of payments, and welfare of countries?* A two-country OLG model is built to examine how the heterogeneity of time preferences affects the economic variables of countries when goods and capital flow freely across national frontiers. Both the short-run and long run implications of an economic integration are analyzed. Initially the world economy composed of two countries – the Home (less patient) and the Foreign (more patient) - is in autarkic long-run equilibrium. In autarky, the less patient country is always poorer than the more patient country in the long run. Now that a border between two countries is open, goods and capital flow freely across the border. Then the results become complicated because the consequences of the impatience of one country can be transmitted to the other country in the open economy. As one of interesting findings, we show that the capital labor ratio along the transition path may be higher or lower than its steady state value. In the normal case, the former holds: the capital labor ratio along the transition path is higher than that of the steady state. However, within some range of the parameters, the latter holds: the capital labor ratio along the transition path is lower than

that of the steady state. Moreover, depending on whether the former or the latter happens, the behavior of the balance of payments and the welfare result of two countries during the transition periods will also differ. For example, contrary to conventional wisdom, we have shown that the less patient country can exhibit current account surplus with an aging population. Also we have proved that economic integration, even if it worsens the welfare of the current young generation, improves the welfare of all future generations of the less patient country.

In Chapter 2, the second question is addressed: *How do the government budget deficits and the government debt affect the macroeconomic variables of countries?* We construct a two-country OLG model in which the government debt induces capital movement between countries. The inter-generational and inter-country effects of the government debt are addressed. Initially the world economy – the Home and the Foreign- is in open-economy long-run equilibrium, and there are no taxes and no government expenditures in the equilibrium. Now the Home government gives a transfer to its old generation by issuing a government debt and thereafter adopts a debt management policy under which the debt per young individual is maintained at a constant level. In contrast, the Foreign government continues to follow a policy of no tax and no government expenditure. A dynamic analysis focusing on the transition paths leads to the interesting findings. First, if the debt is set at a low level, then it crowds out private investment both at home and abroad and induces a fall in the world capital labor ratio through time to a lower level in the long run. The crowding out is more pronounced in autarky than in open economy. Second, the demographic structure affects the current account. With an aging population,

the current account of the debtor country shows a surplus during the transition path and converges to zero in the steady state. With an increasing population, the current account of the debtor country shows an ambiguous sign during the transition path and reveals a deficit in the steady state. Third, the government debt caused by a transfer of a country lowers the welfare of future generations in both countries. Although the welfare of the young generation in the creditor country rises above its old steady-state level initially, after that, the welfare of the successive generations in the creditor country declines steadily through time. In particular, the welfare of the debtor country continuously declines and is lower even under the steady state induced by the debt and the debt management policy than under the old steady state that occurs without debt.

In Chapter 3, the third question is addressed: *Which tax scheme is the best to foster the capital formation and to improve the welfare of countries?* The dynamic effects of tax reform are analyzed in the context of a two-country OLG model. Initially the world economy – the Home and the Foreign- is in open-economy long-run equilibrium. Both countries are in balanced budget under the proportional wage taxes levied on young individuals. Now the Home government unexpectedly shifts its tax scheme to the capital income tax or the consumption tax. In contrast, the Foreign government continues to levy a wage tax. Assuming that both governments maintain a balanced-budget, we examine the effect of each tax reform in the Home country. Simulating the model numerically with plausible parameters, we provide a number of results, which might be useful for policy purposes. First, with respect to capital formation, the consumption tax dominates the capital income tax and the capital income tax dominates the wage tax. The

superiority of the consumption tax scheme comes from its saving boosting effect. Interestingly, due to the transmission effect of the tax policy, the saving rate becomes highest even in the Foreign country in which the consumption tax is not implemented. Second, the tax reform of the Home country affects the net foreign asset position of both countries. The shift to the consumption tax or the capital income tax renders the Home country a creditor and the Foreign country a debtor. In particular, the consumption tax affects the net asset position further. Third, with respect to welfare, the consumption tax is superior to the wage tax but seems not show an advantage over the capital income tax in the tax-reformed country. However, the Foreign country in which the tax reform is not implemented shows a bit higher welfare under the consumption tax scheme than under other tax schemes of the Home country. Fourth, the tax rate is highest under the capital income tax and is lowest under the consumption tax. Fifth, the transitional analysis shows that the capital income tax can be more rewarding than the consumption tax in the aspect of welfare because the reform to the capital income tax enters its new steady state with a faster speed.

CHAPTER 1

TIME PREFERENCES, BALANCE OF PAYMENTS, AND WELFARE

1. INTRODUCTION

THE PURPOSE OF THIS CHAPTER IS TO BUILD a two-country overlapping generations (OLG) model in which difference in time preferences leads to capital movement between countries, and to address how the heterogeneity of time preferences affects macroeconomic variables. In the model, both the short-run and long-run implications of economic integration on consumption, saving, the capital labor ratio, interest rates, the balance of payments, and welfare are analyzed.

The difference in time preferences between countries may come from cultural or institutional factors. Although there exist literatures on endogenous time preferences (see, for example, Uzawa (1968), Lucas and Stokey (1984)), we follow the traditional approach that treats time preferences as given data. Since the pioneering work of Buiter (1981), the open-economy OLG literature has considered the difference in the rates of time preferences as one of the main elements that induce capital flows between countries ;see, for example, Galor (1986), Crettez et al. (1996, 1998), Vidal (2000). This chapter builds on the work of Buiter, but adopts simpler functional forms for preferences and technology. In the macroeconomic literature, preferences are often assumed to be logarithmic while technology to be of the Cobb-Douglas type. When logarithmic

preferences are assumed, saving depends only on labor income, not the interest rate, which, when used with the Cobb-Douglas technology, allow us to obtain a closed-form solution of the model. That saving is independent of the interest rate seems to be a rather restrictive feature of logarithmic preferences, although this functional form for the utility function simplifies the computations greatly. In this respect, the literature that adopts logarithmic preferences suffers from some limitations in that it gains simplicity at the cost of ignoring the role of interest rate in determining saving.

In this thesis, we maintain the assumption of a Cobb-Douglas technology, but adopt a utility function with a constant inter-temporal elasticity of substitution (CIES). A utility function of this type allows for saving to depend on both labor income and the interest rate. Due to the less restrictive assumptions imposed on preferences, the solutions cannot be obtained in a closed form, and thus numerical techniques must be used to compute the equilibrium. Moreover, the functional forms adopted in this study allow us to describe the transition to the long-run equilibrium in contrast with other models that are only amenable to steady-state analysis because of the general functional forms they adopt. As a result of using CIES preferences, we obtain several interesting open-economy outcomes. The capital labor ratio along the transition path might be higher or lower than its steady-state value. Moreover, depending on whether the former or the latter happens, the behavior – during the transition to the steady state – of the balance of payments and the welfare result of two countries might differ as well. In particular, contrary to conventional wisdom, we show that the less patient country can reveal a current account surplus in spite of an aging population. Also we prove that economic integration, even if

it worsens the welfare of the current young generation, improves the welfare of all future generations of the less patient country.

The adoption of the OLG framework is indispensable to introduce different time preferences into the model. In the infinite horizon model, which is another framework used for dynamic macroeconomic analysis, the interest rate equals the rate of time preferences in the steady state since consumption is constant over time. Thus different time preferences cannot be assumed in an open economy model in which the interest rates are identical across countries. The OLG model solves this problem since consumption needs not be constant through time, and, therefore, the interest rate need not equal the rate of time preference. In the OLG model, the equilibrium interest rates in different countries can be the same in spite of different time preferences; see Buiter (1981). By contrast, in the open economy model of infinite horizon, if the world interest rate had differed from the rate of time preference, the country would either accumulate or decumulate her capital forever; see Blanchard and Fischer (1989), p.85.

The rest of this chapter is organized as follows: Section 2 sets up a two-country OLG model. Section 3 describes the autarkic equilibrium. Section 4 presents the main results of this chapter by analyzing the short-run and the long-run equilibrium when the two economies are open for trade in goods and capital. Section 5 summarizes the results.

2. THE MODEL

Time is discrete and denoted by $t, t = 0, 1, \dots$. The world consists of two countries, called H (Home) and F (Foreign), respectively. In each country four classes of economic agents co-exist in each period: a young generation, and old generation, competitive firms, and a government. In the model there is a consumption good that is produced using labor and capital. The consumption good can also be used as investment goods for capital accumulation. An individual works when she is young and retires when she is old. A young individual has one unit of time that she supplies inelastically on the labor market. The individual spends part of the labor income on current consumption; the remaining part is saved to provide for her old-age consumption. The savings of the young generation in any period constitute the economy's capital stock in the following period. In any period, it is the old individuals who are the owners of the economy's stock of capital in that period. In what follows, the consumption good is taken to be the numeraire in each period.

Let $N_{i,t}^0$ and $N_{i,t}^1$ denote, respectively, the number of young individuals and the number of old individuals at the beginning of period t in country i , $i = H, F$. The population in each country is assumed to grow at the same constant rate n . Thus the number of young individuals at the beginning of period t in country i is $N_{i,t}^0 = N_{i,0}^0(1+n)^t, t = 0, 1, \dots$. The preferences of an individual in country i are assumed to be represented by the following utility function:

$$(1) \quad \frac{[c^0]^{1-\sigma}}{1-\sigma} + \beta_i \frac{[c^1]^{1-\sigma}}{1-\sigma}.$$

In (1), c^0 and c^1 denote, respectively, the current consumption and the future consumption. Also, $\sigma, 0 < \sigma < 1$, is the reciprocal of the inter-temporal elasticity of substitution, and $\beta_i, 0 < \beta_i < 1$ is the discount factor for future consumption. We have chosen not to include a bequest motive in the model since such transfers may lead to the neutrality of the government's debt.¹ In what follows, we shall assume that $\beta_H < \beta_F$, i.e., agents in the Home country are less patient than agents in the Foreign country.

In both countries, the technology used to produce the consumption good is of the Cobb-Douglas type, and is assumed to have the following form: $Y = K^\alpha L^{1-\alpha}$, where K , L , and Y denote, respectively, the capital input, the labour input, and the output of the consumption good. Also, $\alpha, 0 < \alpha < 1$, is a parameter representing the share of capital in national income. Letting $k = K/L$ denote the capital labour ratio, we can express the output produced per worker as $f(k) = k^\alpha$. Capital is assumed not to depreciate.

Observe that the two countries differ only in terms of the absolute sizes of their populations and their rates of time preferences. Because our present preoccupation is the influence of time preferences on capital flows, we shall assume that government expenditures and taxes are all zero.

¹ When individuals leave bequests because they are concerned with the utility of their descendents, an OLG model behaves like an infinite horizon model. See Blanchard and Fischer (1989, p.129-130) for the role of bequest in an overlapping-generation model.

3. THE AUTARKIC EQUILIBRIUM

3.1. Utility Maximization and Saving

Consider a young individual of period t in country i who faces the wage rate $\omega_{i,t}$ and the rate of interest $r_{i,t+1}$ on her saving. The individual has to solve the following utility maximization problem

$$(2) \quad \max_{(c_{i,t}^0, c_{i,t+1}^1)} \frac{[c_{i,t}^0]^{1-\sigma}}{1-\sigma} + \beta_i \frac{[c_{i,t+1}^1]^{1-\sigma}}{1-\sigma}$$

subject to

$$(3) \quad c_{i,t}^0 + \frac{c_{i,t+1}^1}{1+r_{i,t+1}} = \omega_{i,t}.$$

It is simple to show that the solution of the maximization problem constituted by (2) and (3) yields the following current consumption and old-age consumption:

$$(4) \quad c_{i,t}^0 = \frac{\omega_{i,t}}{1 + \beta_i^{\frac{1}{\sigma}} (1+r_{i,t+1})^{\frac{1-\sigma}{\sigma}}}$$

and

$$(5) \quad c_{i,t+1}^1 = \frac{\beta_i^{\frac{1}{\sigma}} (1+r_{i,t+1})^{\frac{1}{\sigma}} \omega_{i,t}}{1 + \beta_i^{\frac{1}{\sigma}} (1+r_{i,t+1})^{\frac{1-\sigma}{\sigma}}}.$$

Furthermore, the saving of the individual is given by

$$(6) \quad s_{i,t}(\omega_{i,t}, r_{i,t+1}) = \frac{\omega_{i,t}}{1 + \beta_i^{\frac{1}{\sigma}} (1+r_{i,t+1})^{\frac{1-\sigma}{\sigma}}}.$$

Observe that saving rises with labor income and the rate of return during its investment span.² Also, a more patient individual, i.e., one with a higher value of β_i , saves a higher proportion of labor income. The list $(c_{i,t}^0, c_{i,t+1}^1, s_{i,t}(\omega_{i,t}, r_{i,t+1}))$ is called the optimal lifetime plan of a young individual of period t in country i .

Substituting (4) and (5), respectively, for c^0 and c^1 in (2), we obtain the following expression for the indirect lifetime utility function of a young individual of period t in country i :

$$(7) \quad v_{i,t}(\omega_{i,t}, r_{i,t+1}) = \frac{\omega_{i,t}^{1-\sigma}}{1-\sigma} \left(1 + \beta_i^{\frac{1}{\sigma}} r_{i,t+1}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}.$$

3.2. Profit Maximization

Let $\omega_{i,t}$ and $r_{i,t}$ be the wage rate and the rental rate of capital that prevail in period t in country i . The representative firm that produces the consumption good solves the following profit maximization problem in this period:

$$(8) \quad \max_{(K_{i,t}, L_{i,t})} K^{\alpha} L^{1-\alpha} - \omega_{i,t} L - r_{i,t} K.$$

The following first-order conditions characterize the optimal labor and capital inputs:

$$(9) \quad (1-\alpha) K^{\alpha} L^{-\alpha} - \omega_{i,t} = 0,$$

$$(10) \quad \alpha K^{\alpha-1} L^{1-\alpha} - r_{i,t} = 0.$$

² With the CIES utility function, the interest elasticity of savings is positive in the range of σ , $0 < \sigma < 1$. The positive interest elasticity has been affirmed in many of empirical literatures: see Batina and Ihori (2000, p.47-48) for the savings elasticity controversy.

Let $K_{i,t}$ and $L_{i,t}$ denote, respectively, the capital input and the labor input associated that solves (9) and (10). The output of the consumption good produced from the input combination $(K_{i,t}, L_{i,t})$ is $Y_{i,t} = K_{i,t}^\alpha L_{i,t}^{1-\alpha}$. The list $(Y_{i,t}, K_{i,t}, L_{i,t})$ is called an optimal production plan in period t for the representative firm in country i . The capital labor ratio in period t is then given by $k_{i,t} = K_{i,t} / L_{i,t}$.

3.3. Autarkic Competitive Equilibrium

Let $(K_{i,t}, N_{i,t}^0, N_{i,t}^1)$ denote the state of the economy in country i at the beginning of period t . Here $K_{i,t}$ represents this economy's capital stock in period t , while $N_{i,t}^0$ and $N_{i,t}^1$ – we recall – represent, respectively, the number of young individuals and the number of old individuals in period t in country i . The initial state of the economy of country i , namely, $(K_{i,0}, N_{i,0}^0, N_{i,0}^1)$, is assumed to be given.

DEFINITION 1: By a price system in country i , we mean an infinite sequence $\mathcal{P}_i = (\omega_{i,t}, r_{i,t})_{t=0}^\infty$, where, we recall, $\omega_{i,t}$ and $r_{i,t}$ represent, respectively, the wage rate and the rental rate of capital in period t in country i . An allocation induced by $\mathcal{P}_i$ is an infinite sequence

$$\mathcal{Q}_i = (c_{i,0}^1, (c_{i,t}^0, c_{i,t+1}^1, s_{i,t}(\omega_{i,t}, r_{i,t+1}))_{t=0}^\infty, (Y_{i,t}, K_{i,t}, L_{i,t})_{t=0}^\infty)$$

where

- (i) $c_{i,0}^1$ is the consumption of an old individual of period 0 in country i , when

the price system $\mathcal{P}_i$ prevails;

(ii) $(c_{i,t}^0, c_{i,t+1}^1, s_{i,t})$ is the lifetime plan chosen by a young individual of period t in

country i , when the price system $\mathcal{P}_i$ prevails; and

(iii) $(Y_{i,t}, K_{i,t}, L_{i,t})$ is the production plan chosen by the representative firm in period t in country i when the price system $\mathcal{P}_i$ prevails.

The pair $(\mathcal{P}, \mathcal{Q})$ is said to constitute an autarkic competitive equilibrium if the following market-clearing conditions are satisfied:

(iv) For $t = 0$, we have $(K_{i,t}, L_{i,t}) = (K_{i,0}, N_{i,0}^0)$;

(v) For $t > 0$, we have $(K_{i,t}, L_{i,t}) = (N_{i,t-1}^0 s_{i,t-1}(\omega_{i,t-1}, r_t), N_{i,t}^0)$.

Condition (iv) asserts that the capital and labor markets in period 0 are in equilibrium. As for condition (v), it asserts that the capital and labor markets in period $t, t > 0$, clear. Note that in (v), $N_{i,t-1}^0 s_{i,t-1}(\omega_{i,t-1}, r_t)$ represents the savings of the young generation of period $t - 1$ in country i . Note that due to Walras' law, the goods market also clears when the market for capital and the market for labor clear.

3.4. Existence and Uniqueness of Autarkic Competitive Equilibrium

Let $\hat{r}_{i,0} = D_1 F(K_{i,0}, N_{i,0}^0)$ and $\hat{\omega}_{i,0} = D_2 F(K_{i,0}, N_{i,0}^0)$. If $\hat{r}_{i,0}$ and $\hat{\omega}_{i,0}$ are the rental rate of capital and the wage rate prevailing in period 0 in country i , then the market for capital and the market for labor in period 0 in this country are in equilibrium.

Next, let $r_{i,1}$ denote the rental rate of capital that a young individual of period 0 in country i expects to prevail in period 1. Then using (6), we obtain the following expression for her saving:

$$(11) \quad s_i(\hat{\omega}_{i,0}, r_{i,1}) = \frac{\hat{\omega}_{i,0}}{1 + \beta_i^{\frac{1}{\sigma}} (1 + r_{i,1})^{\frac{1-\sigma}{\sigma}}}.$$

The aggregate saving by the young generation of period 0 in country i is then given by

$$(12) \quad N_{i,0}^0 s_i(\hat{\omega}_{i,0}, r_{i,1}) = N_{i,0}^0 \frac{\hat{\omega}_{i,0}}{1 + \beta_i^{\frac{1}{\sigma}} (1 + r_{i,1})^{\frac{1-\sigma}{\sigma}}},$$

and the capital labor ratio in period 1 that is generated by this maximizing behavior will be

$$(13) \quad \begin{aligned} k_{i,1} &= \frac{N_{i,0}^0 s_i(\hat{\omega}_{i,0}, r_{i,1})}{N_{i,1}^0} \\ &= \frac{1}{1+n} \frac{\hat{\omega}_{i,0}}{1 + \beta_i^{\frac{1}{\sigma}} (1 + r_{i,1})^{\frac{1-\sigma}{\sigma}}}. \end{aligned}$$

Furthermore, the marginal product of capital in period 1 is given by

$$(14) \quad f'(k_{i,1}) = \alpha \left[\frac{1}{1+n} \frac{\hat{\omega}_{i,0}}{1 + \beta_i^{-\frac{1}{\sigma}} (1 + r_{i,1})^{-\frac{1-\sigma}{\sigma}}} \right]^{\alpha-1}.$$

Now let

$$(15) \quad \zeta_i(\hat{\omega}_{i,0}, r_{i,1}) = \alpha \left[\frac{1}{1+n} \frac{\hat{\omega}_{i,0}}{1 + \beta_i^{-\frac{1}{\sigma}} (1 + r_{i,1})^{-\frac{1-\sigma}{\sigma}}} \right]^{\alpha-1}.$$

The function $\zeta_i(\hat{\omega}_{i,0}, \cdot) : r_{i,1} \rightarrow \zeta_i(\hat{\omega}_{i,0}, r_{i,1}), r_{i,1} \geq 0$, is positive, continuous, and strictly decreasing. Furthermore

$$\zeta_i(\hat{\omega}_{i,0}, 0) = \alpha \left[\frac{1}{1+n} \frac{\hat{\omega}_{i,0}}{1 + \beta_i^{-\frac{1}{\sigma}}} \right]^{\alpha-1} > 0$$

and

$$\lim_{r_{i,1} \rightarrow \infty} \zeta_i(\hat{\omega}_{i,0}, r_{i,1}) = \alpha \left[\frac{\hat{\omega}_{i,0}}{1+n} \right]^{\alpha-1}.$$

Hence the curve $\zeta_i(\hat{\omega}_{i,0}, \cdot) : r_{i,1} \rightarrow \zeta_i(\hat{\omega}_{i,0}, r_{i,1}), r_{i,1} \geq 0$, must cross the 45-degree line at a single point, say at $r_{i,1} = \hat{r}_{i,1}$. When $\hat{r}_{i,1}$ is the rate of return to saving for a young individual of period 0 in country i , the capital labor ratio generated by her maximizing behavior is

$$(16) \quad \hat{k}_{i,1} = \frac{1}{1+n} \frac{\hat{\omega}_{i,0}}{1 + \beta_i^{-\frac{1}{\sigma}} (1 + \hat{r}_{i,1})^{-\frac{1-\sigma}{\sigma}}}.$$

Furthermore, because $\hat{r}_{i,1}$ is a fixed point of $\zeta_i(\hat{\omega}_{i,0}, \cdot)$, we must have

$$(17) \quad f'(\hat{k}_{i,1}) = \hat{r}_{i,1}.$$

Next, let

$$(18) \quad \hat{\omega}_{i,1} = f(\hat{k}_{i,1}) - \hat{k}_{i,1}f'(\hat{k}_{i,1}).$$

When $\hat{r}_{i,1}$ and $\hat{\omega}_{i,1}$ are, respectively, the prevailing rental rate of capital and the prevailing wage rate in period 1 in country i , then all the markets in this period in this country clear.

The procedure used to compute $\hat{r}_{i,1}$ and $\hat{\omega}_{i,1}$ can be repeated ad infinitum to obtain a price

system $\hat{\mathcal{P}}_i = (\hat{r}_{i,t}, \hat{\omega}_{i,t})_{t=0}^{\infty}$ and an allocation induced by $\hat{\mathcal{P}}_i$, say

$$\hat{\mathcal{Q}}_i = (\hat{c}_{i,0}^1, (\hat{c}_{i,t}^0, \hat{c}_{i,t+1}^1, s_{i,t}(\hat{\omega}_{i,t}, \hat{r}_{i,t+1}))_{t=0}^{\infty}, (\hat{Y}_{i,t}, \hat{K}_{i,t}, \hat{L}_{i,t})_{t=0}^{\infty})$$

such that the pair $(\hat{\mathcal{P}}_i, \hat{\mathcal{Q}}_i)$ constitute a competitive equilibrium. We summarize the result just established in the following proposition:

PROPOSITION 1: *There exists a unique competitive equilibrium for each economy when it is in autarky.*

3.5. The Dynamics of Capital Accumulation

Using (17) and (18) in (16), we obtain the following expression that defines implicitly the equilibrium capital labor ratio in period 1 in terms of the capital labor ratio in period 0:

$$(19) \quad \hat{k}_{i,1} = \frac{1}{1+n} \frac{(1-\alpha)k_{i,0}^\alpha}{1 + \beta_i^{-\frac{1}{\sigma}} \left(1 + \alpha \hat{k}_{i,1}^{\alpha-1}\right)^{\frac{1-\sigma}{\sigma}}}.$$

In (19), we have let $k_{i,0} = K_{i,0} / N_{i,0}^0$ and substituted $(1-\alpha)k_{i,0}^\alpha$ for the equilibrium wage rate $\hat{w}_{i,0}$.

Now letting $G(k_{i,0})$ denote the value of $\hat{k}_{i,1}$ that solves (22), we can rewrite this expression as

$$(20) \quad G_i(k_{i,0}) = \frac{1}{1+n} \frac{(1-\alpha)k_{i,0}^\alpha}{1 + \beta_i^{-\frac{1}{\sigma}} \left(1 + \alpha G(k_{i,0})^{\alpha-1}\right)^{\frac{1-\sigma}{\sigma}}}.$$

The function $G_i : k_{i,0} \rightarrow G_i(k_{i,0})$ describes the transition of the equilibrium capital labor ratio from period 0 to period 1 in country i . In general, if we substitute $k_{i,t}$ for $k_{i,0}$ in (20) and interpret $k_{i,t}$ as the equilibrium capital labor ratio in period t , then (20) becomes

$$(21) \quad G_i(k_{i,t}) = \frac{1}{1+n} \frac{(1-\alpha)k_{i,t}^\alpha}{1 + \beta_i^{-\frac{1}{\sigma}} \left(1 + \alpha G(k_{i,t})^{\alpha-1}\right)^{\frac{1-\sigma}{\sigma}}},$$

and $G_i(k_{i,t})$ then represents the equilibrium capital labor ratio in period $t+1$. Observe that $G_i(k_{i,t}) \rightarrow 0$ when $k_{i,t} \rightarrow 0$ and $G_i(k_{i,t}) \rightarrow \infty$ when $k_{i,t} \rightarrow \infty$. Furthermore, we also have $\frac{G_i(k_{i,t})}{k_{i,t}} \rightarrow \infty$ when $k_{i,t} \rightarrow 0$ and $\frac{G_i(k_{i,t})}{k_{i,t}} \rightarrow 0$ when $k_{i,t} \rightarrow \infty$. Hence the curve

$G_i : k_{i,t} \rightarrow G_i(k_{i,t}), k_{i,t} > 0$, must cross the 45-degree line from above at one point. We

claim that this curve crosses the 45-degree line only once. Indeed, if we differentiate (21), we obtain

$$(22) \quad G_i'(k_{i,t}) = \frac{\alpha G_i(k_{i,t})}{k_{i,t} \left[1 + \frac{\alpha(1-\alpha)(1-\sigma)}{\sigma} \frac{G(k_{i,t})^{\alpha-1} \beta_i^{\frac{1}{\sigma}} (1 + \alpha G(k_{i,t})^{\alpha-1})^{\frac{1-\sigma}{\sigma}-1}}{1 + \beta_i^{\frac{1}{\sigma}} (1 + \alpha G(k_{i,t})^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \right]}.$$

At a fixed point of G_i , we have $G_i(k_{i,t}) = k_{i,t}$, and (22) is reduced to

$$(23) \quad G_i'(k_{i,t}) = \frac{\alpha}{\left[1 + \frac{\alpha(1-\alpha)(1-\sigma)}{\sigma} \frac{k_{i,t}^{\alpha-1} \beta_i^{\frac{1}{\sigma}} (1 + \alpha k_{i,t}^{\alpha-1})^{\frac{1-\sigma}{\sigma}-1}}{1 + \beta_i^{\frac{1}{\sigma}} (1 + \alpha k_{i,t}^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \right]} < \alpha < 1.$$

Inequality (23) asserts that at a point the curve G_i crosses the 45-degree line, its slope is strictly less than 1, which means that this curve crosses the 45-degree line only once, and the point at which the curve crosses the 45-degree line represents the unique autarkic steady state of the economy of country i . The shapes of the curves $G_i : k_{i,t} \rightarrow G_i(k_{i,t}), k_{i,t} > 0, i = H, F$, are depicted in Figure 1. In the figure, E_H and E_F are the autarkic steady states of the Home country and the Foreign country, respectively. Note that the transition curve for the Foreign country is above that of the Home country because $\beta_F > \beta_H$. Hence the autarkic steady state capital labor ratio is higher in the Foreign country than in the Home country. Also depicted in Figure 1 is the curve $G : k_t \rightarrow G(k_t), k_t > 0$, which describes the transition of the world capital labor ratio from

one period to another when the markets for goods and capital are integrated. The steady state of the world economy is represented by point E in Figure 1.

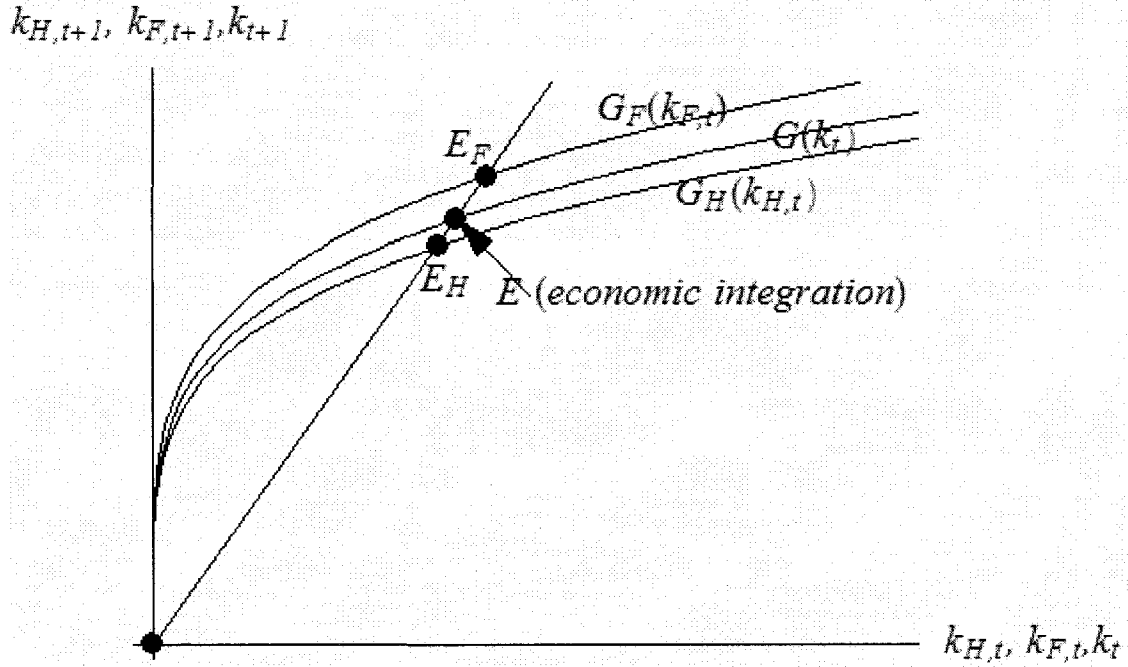


Figure 1.— The capital labor ratios for the Home and Foreign countries: Autarky vs. Open economy

We have just established the following proposition:

PROPOSITION 2: *In autarky, the economy of country $i, i = H, F$, has a unique steady state, say $\bar{k}_i$, which is defined implicitly by*

$$(24) \quad \frac{1}{1+n} \frac{(1-\alpha)\bar{k}_i^{\alpha-1}}{1 + \beta_i^{\frac{1}{\sigma}} (1 + \alpha\bar{k}_i^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} = 1.$$

Furthermore, the autarkic steady state is globally stable and the convergence to this steady state is monotone.

In what follows, we shall let $\bar{r}_i = \alpha \bar{k}_i^{\alpha-1}$ and $\bar{w}_i = (1-\alpha) \bar{k}_i^\alpha$ denote, respectively, the autarkic steady state interest rate and the autarkic steady state wage rate in country $i, i = H, F$. We have the following proposition:

PROPOSITION 3: *In autarky, the Home country, which is less patient than the Foreign country, has a lower capital labor ratio, a lower output, a lower wage rate, and a higher interest rate than the Foreign country in the long run.*

A lower capital labor ratio means a lower saving by a young individual in the Home country and a fortiori a lower capital income for the individual in her old age because capital income accounts for a fraction α of output. Hence the consumption of an old individual is always lower in the Home country than in the Foreign country in autarkic steady state. A lower capital labor ratio also means a lower wage rate.

However, the consumption of a young individual, as given by (4), is not necessarily lower in the Home country because the rise of the current consumption due to a higher rate of time preferences might offset the reduction of the current consumption due to a lower wage rate as well as a higher interest rate. The following proposition summarizes this welfare consequence:

PROPOSITION 4: *In steady state autarky, an old individual in the Home country consumes less than an old individual in the Foreign country. However, the consumption of a young individual in the Home country may be higher or lower than that in the Foreign country.*

In the long run, the more patient country enjoys a higher standard of living than the less patient country. Thriftiness ultimately pays in autarky: even without any altruistic motive, the patience of current generations is rewarded with the prosperity of their future generations. However, when economies are open, the impact will be more complicated since the economic effects of the impatience of one country can be transmitted to other countries.

4. OPEN-ECONOMY EQUILIBRIUM

4.1. *Existence and Uniqueness of Competitive Equilibrium under Integration*

Suppose that initially the two countries are in autarkic long-run equilibrium. Next, suppose that in period 0 the governments of the two countries open their borders and allow goods and capital to flow freely.

In each country, the capital labor ratio in period 0 is equal to its autarkic steady state value, i.e., $k_{i,0} = \bar{k}_i, i = H, F$, with $\bar{k}_H < \bar{k}_F$. The rental rate of capital in period 0 in the Home economy is thus higher than that of the Foreign economy, triggering capital flows

into the Home country. Suppose that capital flows take one period so that at the beginning of period 1 the capital labor ratios at home and abroad are the same, i.e., $k_{H,1} = k_{F,1}$. If we let k_1 denote the common value of $k_{H,1}$ and $k_{F,1}$, which is called the world capital labor ratio in period 1, then the world rate of interest in period 1 is given by $r_1 = \alpha k_1^{\alpha-1}$.

In the Home country, the saving of a young individual of period 0 is given by

$$(25) \quad s_H(\bar{\omega}_H, r_1) = \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}}.$$

Similarly, in the Foreign country, the saving of a young individual of period 0 is given by

$$(26) \quad s_F(\bar{\omega}_F, r_1) = \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}}.$$

The world capital stock in period 1 is then given by

$$(27) \quad \begin{aligned} K_1 &= N_{H,0}^0 s_H(\bar{\omega}_H, r_1) + N_{F,0}^0 s_F(\bar{\omega}_F, r_1) \\ &= N_{H,0}^0 \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} + N_{F,0}^0 \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}}. \end{aligned}$$

The world capital labor ratio in period 1 is

$$(28) \quad \begin{aligned} k_1 &= \frac{K_1}{N_{H,1}^0 + N_{F,1}^0} \\ &= \frac{K_1}{(1+n)(N_{H,0}^0 + N_{F,0}^0)} \\ &= \frac{N_{H,0}^0}{(1+n)(N_{H,0}^0 + N_{F,0}^0)} \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} + \frac{N_{F,0}^0}{(1+n)(N_{H,0}^0 + N_{F,0}^0)} \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} \end{aligned}$$

$$= \frac{1}{n+1} \left[\eta_H \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1+r_1)^{\frac{1-\sigma}{\sigma}}} + \eta_F \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1+r_1)^{\frac{1-\sigma}{\sigma}}} \right].$$

In the expression on the last line of (28), we have let η_H and η_F denote, respectively the population of the Home country and the population of the Foreign country as fractions of the world population. Equation (28) yields the world capital labor ratio in period 1, given that r_1 is the world rate of interest that is expected to prevail in that period. Now let

$$(29) \quad \zeta(\bar{\omega}_H, \bar{\omega}_F, r_1) = \alpha \left(\frac{1}{n+1} \left[\eta_H \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1+r_1)^{\frac{1-\sigma}{\sigma}}} + \eta_F \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1+r_1)^{\frac{1-\sigma}{\sigma}}} \right] \right)^{\alpha-1}.$$

As in the analysis of the competitive equilibrium in autarky, we can show that the map

$$\zeta(\bar{\omega}_H, \bar{\omega}_F, \cdot) : r_1 \rightarrow \zeta(\bar{\omega}_H, \bar{\omega}_F, r_1), r_1 \geq 0,$$

is strictly decreasing and has a unique fixed point, say $r_1 = \hat{r}_1$. Furthermore, because

$$\begin{aligned} \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1+r_1)^{\frac{1-\sigma}{\sigma}}} &< \eta_H \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1+r_1)^{\frac{1-\sigma}{\sigma}}} + \eta_F \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1+r_1)^{\frac{1-\sigma}{\sigma}}} \\ &< \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1+r_1)^{\frac{1-\sigma}{\sigma}}} \end{aligned}$$

for any $r_1 \geq 0$, we must have $\bar{r}_F < \hat{r}_1 < \bar{r}_H$; that is, the equilibrium world interest rate in period 1 is lower than the autarkic steady state interest in the Home country, but higher than the autarkic steady state interest abroad. We shall let $\hat{k}_1 = (\hat{r}_1 / \alpha)^{1/(\alpha-1)}$ denote the equilibrium world capital labor ratio in period 1. The equalization of the capital labor ratios in the two countries in period 1 will lead to the equalization of the wage rates in

period 1 in the Home economy and in the Foreign economy. We shall let $\hat{w}_1 = (1 - \alpha)\hat{k}_1^\alpha$ denote the common wage rate in period 1.

To continue, suppose that r_2 is the interest rate that prevails in the global capital market in period 2. In the Home country, the saving of a young individual of period 1 is given by

$$(30) \quad s_H(\hat{w}_1, r_2) = \frac{\hat{w}_1}{1 + \beta_H^{\frac{1}{\sigma}}(1 + r_2)^{\frac{1-\sigma}{\sigma}}}.$$

Similarly, in the Foreign country, the saving of a young individual of period 1 is given by

$$(31) \quad s_F(\hat{w}_1, r_2) = \frac{\hat{w}_1}{1 + \beta_F^{\frac{1}{\sigma}}(1 + r_2)^{\frac{1-\sigma}{\sigma}}}.$$

The world capital stock in period 2 is then given by

$$(32) \quad \begin{aligned} K_2 &= N_{H,1}^0 s_H(\hat{w}_1, r_2) + N_{F,1}^0 s_F(\hat{w}_1, r_2) \\ &= N_{H,1}^0 \frac{\hat{w}_1}{1 + \beta_H^{\frac{1}{\sigma}}(1 + r_2)^{\frac{1-\sigma}{\sigma}}} + N_{F,1}^0 \frac{\hat{w}_1}{1 + \beta_F^{\frac{1}{\sigma}}(1 + r_2)^{\frac{1-\sigma}{\sigma}}}. \end{aligned}$$

The world capital labor ratio in period 2 is

$$(33) \quad \begin{aligned} k_2 &= \frac{K_2}{N_{H,2}^0 + N_{F,2}^0} \\ &= \frac{\hat{w}_1}{1+n} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}}(1 + r_2)^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}}(1 + r_2)^{\frac{1-\sigma}{\sigma}}} \right]. \end{aligned}$$

Equation (33) yields the world capital labor ratio in period 2, given that r_2 is the world rate of interest that is expected to prevail in that period. Now let ³

$$(34) \quad \zeta(\hat{\omega}_1, r_2) = \alpha \left(\frac{\hat{\omega}_1}{1+n} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1+r_2)^{-\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1+r_2)^{-\frac{1-\sigma}{\sigma}}} \right] \right)^{\alpha-1}.$$

The map $\zeta(\hat{\omega}_1, \cdot) : r_2 \rightarrow \zeta(\hat{\omega}_1, r_2)$, $r_2 \geq 0$, is strictly decreasing and has a unique fixed point, say $r_2 = \hat{r}_2$. We shall let $\hat{k}_2 = (\hat{r}_2 / \alpha)^{1/(\alpha-1)}$ denote the equilibrium world capital labor ratio in period 2 and $\hat{\omega}_2 = (1-\alpha)\hat{k}_2^\alpha$ denote the two countries' common equilibrium wage rate in period 2. At the fixed point $r_2 = \hat{r}_2$, (34) assumes the following form

$$(35) \quad \hat{k}_2 = \frac{(1-\alpha)\hat{k}_1^\alpha}{1+n} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha\hat{k}_2^{\alpha-1})^{-\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha\hat{k}_2^{\alpha-1})^{-\frac{1-\sigma}{\sigma}}} \right].$$

Equation (35) defines implicitly the equilibrium world capital labor ratio in period 2 in terms of its value in period 1. In general, for any $t \geq 1$, if k_t is the equilibrium world capital labor ratio in period t , then the equilibrium world capital labor ratio in period $t+1$ – which we denote by $G(k_t)$ – is defined implicitly by

$$(36) \quad G(k_t) = \frac{(1-\alpha)k_t^\alpha}{1+n} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha G(k_t)^{\alpha-1})^{-\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha G(k_t)^{\alpha-1})^{-\frac{1-\sigma}{\sigma}}} \right].$$

³ Because wages in the two countries are equalized in period 1 after integration, we use only their common value in the definition of the function $\zeta(\hat{\omega}_1, r_2)$.

It is clear that $G: k_t \rightarrow G(k_t), k_t > 0$, is strictly increasing. It is also clear that $G_H(k_t) < G(k_t) < G_F(k_t)$, for all $k_t > 0$, i.e., the transition curve for the world economy is above the Home country's transition curve, but below the Foreign country's transition curve. Furthermore, $\lim_{k_t \rightarrow 0} \frac{G(k_t)}{k_t} = +\infty$ and $\lim_{k_t \rightarrow +\infty} \frac{G(k_t)}{k_t} = 0$. Hence the curve $G: k_t \rightarrow G(k_t), k_t > 0$, must cross the 45-degree line at least once. At such a crossing, brute-force calculations yield the following expression for the slope of the curve:

$$G'(k_t) = \frac{\alpha}{1 + \frac{\frac{\alpha(1-\alpha)(1-\sigma)}{\sigma} k_t^{\alpha-1} \left[\frac{\eta_H \beta_H^{\frac{1}{\sigma}} (1 + \alpha k_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}-1}}{\left(1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha k_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}}\right)^2} + \frac{\eta_F \beta_F^{\frac{1}{\sigma}} (1 + \alpha k_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}-1}}{\left(1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha k_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}}\right)^2} \right]}{\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha k_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha k_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}}}}},$$

which is positive but less than α . Hence the curve $G: k_t \rightarrow G(k_t), k_t > 0$, crosses the 45-degree line only once, and from above. Let $\bar{k}$ be the value of k_t at which this curve crosses the 45-degree line. Then $\bar{k}$ is the steady state world capital labor ratio, and in the long run the world capital labor ratio converges monotonically to $\bar{k}$. More precisely, let $(\hat{k}_t)_{t=1}^{\infty}$ be the sequence of world capital labor ratios defined recursively as follows: For $t=1$ and $t=2$, we have already defined $\hat{k}_1$ and $\hat{k}_2$. For any $t > 2$, let $\hat{k}_t = G(\hat{k}_{t-1})$. Then $\lim_{t \rightarrow \infty} \hat{k}_t = \bar{k}$. Furthermore, we have $\bar{k}_H < \bar{k} < \bar{k}_F$. In what follows, we shall let

$\hat{w}_t = (1 - \alpha)\hat{k}_t^\alpha$ and $\hat{r}_t = \alpha\hat{k}_t^{\alpha-1}$ denote, respectively, the equilibrium wage rate that prevails in both countries and the equilibrium interest rate that prevails on the global capital market – both in period $t, t = 1, 2, \dots$. We summarize the results just obtained in the following proposition:

PROPOSITION 5: *There exists a unique competitive equilibrium under economic integration. The world capital labor ratio in period 1, namely $\hat{k}_1$, satisfies the following condition: $\bar{k}_H < \hat{k}_1 < \bar{k}_F$. That is, one period after capital and goods are allowed to move freely across national frontiers, the world capital labor ratio is higher than the Home country's autarkic steady state capital labor ratio, but lower than the Foreign country's autarkic steady state capital labor ratio. After that the world capital labor ratio converges monotonically to its unique steady state value, say $\bar{k}$, which is defined implicitly by*

$$(39) \quad 1 = \frac{(1 - \alpha)\bar{k}^{\alpha-1}}{1 + n} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha\bar{k}^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha\bar{k}^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \right].$$

Furthermore, $\bar{k}_H < \bar{k} < \bar{k}_F$; that is, the steady state world capital labor ratio is higher than the Home country's autarkic steady state capital labor ratio, but lower than the Foreign country's autarkic steady state capital labor ratio.⁴ Finally, if $\hat{k}_1 > \bar{k}$, i.e., if the world capital immediately after economic integration overshoots its long-run level, then the world capital labor ratio will decline through time to its steady state value in the long

⁴ Here one thing to note is that the world capital labor ratio in the steady state is a weighted average of the capital labor ratios of two countries in autarky steady state. That is, the effects of economic integration on economic growth suggested by Rivera-Batiz and Romer (1991) cannot be considered in the context of this model.

run. On the other hand, if $\hat{k}_1 < \bar{k}$, i.e., if the world capital labor ratio immediately after economic integration falls short of its long-run level, then the world capital labor ratio will rise through time to its steady state value in the long run.

We shall let $\bar{r} = \alpha \bar{k}^{(\alpha-1)}$ denote the steady state world interest rate and $\bar{w} = (1 - \alpha) \bar{k}^\alpha$ denote the common steady state wage rate for the two countries.

Depending on the values of the parameters, the world capital labor ratio in period 1 might be higher or lower than its steady state value.⁵ The following proposition – proved in Appendix A – provides a sufficient condition for the ‘overshoot’ case.

PROPOSITION 6: *If there is a pronounced difference of time preferences between countries: i.e., if the Home country is sufficiently impatient, then the world capital labor ratio in period 1 – one period after economic integration – is higher than its steady state value: $\hat{k}_1 > \bar{k}$.*

The intuition behind Proposition 6 is not hard to grasp. If β_H is small, i.e., if the Home country is quite impatient, then the young individuals in the Home country will shift consumption to the present and save very little for the future. This action will result in a very low capital labor ratio in the long run if the Home country is in autarky. When

⁵ The possibility that the capital labour ratio along the transition path might be higher or lower than its steady-state value is briefly mentioned in Auerbach and Kotlikoff (1998, p.330). However, there is no clarification in their textbook under which conditions the former or the latter happens. Proposition 6 and Proposition 7 in the chapter try to formulate the conditions.

economic integration takes place, the young individuals of the Home country will be able to borrow from abroad to finance their current consumption. Because the autarkic interest rate of the Home country is high, the young individuals of period 0 in the Foreign country will cut back their current consumption and raise their savings to take advantage of the high future rate of interest, and in doing so increase their savings significantly, with the ensuing result that the world capital labor ratio in period 1 overshoots its long-run level, as asserted by Proposition 6.

Depending on the values of parameters, the world capital labor ratio might fall short of its long-run value, as asserted by the following proposition the proof of which is given in Appendix B.

PROPOSITION 7: *If α and η_H are both close to 1, then $\hat{k}_1 < \bar{k}$. That is, if the Home country is large relative to the Foreign country, and if the share in national income of the factor capital is high, then the world capital labor ratio in period 1 – one period after economic integration – is lower than its steady state value.*

The conditions that lead to the result that the world capital labor ratio in period 1 is lower than its long-run level, as stated in Proposition 7, might not hold in reality. When α is close to 1, most of the output goes to the factor capital. The factor income of labor is thus negligible, which in turn implies very little saving and a fortiori a very low capital labor ratio. Also, the condition that η_H is close to 1 implies that the influence of the Foreign country on the Home country is negligible. It is the joint effects of these two conditions

that ensure the ‘undershoot’ of the world capital labor ratio immediately after the economic integration.

4.2. The Current Account and Net Foreign Assets

Let $B_{i,t}$ denote the net foreign assets of country i at the beginning of period t , $i = H, F$, $t = 0, 1, \dots$. Also, let $CA_{i,t}$ denote the current account of country i at the end of period t , $t = 0, 1, \dots$. Note that at the beginning of period 0 – when economic integration begins – we have $B_{H,0} = B_{F,0} = 0$.

Immediately after economic integration, a young individual of period 0 in the Home country sees her labor income remaining at the autarkic level $\bar{\omega}_H$, but a drop in the rate of return to her saving from the autarkic level $\bar{r}_H$ to the world level $\hat{r}_1$. The drop in the rate of interest induces her to raise current consumption and save less. Her saving after economic integration is

$$(40) \quad s_H(\bar{\omega}_H, \hat{r}_1) = \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} < s_H(\bar{\omega}_H, \bar{r}_H).$$

A young individual of period 0 in the Foreign country also sees her labor income remaining the same at the autarkic level $\bar{\omega}_F$, but a rise in the rate of return she obtains for her saving from the autarkic level $\bar{r}_F$ to the world level $\hat{r}_1$. Thus a young individual of period 0 abroad will cut back her current consumption and save more. Her saving after economic integration is

$$(41) \quad s_F(\bar{\omega}_F, \hat{r}_1) = \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} > s_F(\bar{\omega}_F, \bar{r}_F)$$

Because in autarkic steady state the capital labor ratio is higher in the Foreign country than in the Home country, we must have

$$(42) \quad s_H(\bar{\omega}_H, \hat{r}_1) < s_H(\bar{\omega}_H, \bar{r}_H) < s_F(\bar{\omega}_F, \bar{r}_F) < s_F(\bar{\omega}_F, \hat{r}_1).$$

In period 1, the equilibrium condition on the global capital market is

$$(43) \quad N_{H,0}^0 \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} + N_{F,0}^0 \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} = (N_{H,1}^0 + N_{F,1}^0) \hat{k}_1.$$

Observe that the first expression on the left side of (43) represents the national saving in period 0 of the Home economy and the second expression the national saving in period 0 of the Foreign economy. The expression on the right side of (43) represents the world's capital stock in period 1.

Equation (43) can be rearranged as

$$(44) \quad N_{H,0}^0 \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} - (1+n)N_{H,0}^0 \hat{k}_1 = - \left(N_{F,0}^0 \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} - (1+n)N_{F,0}^0 \hat{k}_1 \right).$$

Observe that on the left side of (44) the expression $(1+n)N_{H,0}^0 \hat{k}_1$ represents the capital stock located inside the Home country in period 1. The left side of (44) thus represents the net foreign assets of the Home country in period 1. Similarly, the expression inside the grand brackets on the right side of (44) represents the net foreign assets of the Foreign country. Since the saving of the individual in period 0 is lower in the Home country than in the Foreign country as represented in (42), we can assert that the left side of (44) must be negative. That is, immediately after economic integration capital will start flowing into

the Home country, and the Home country will run a current account deficit at the end of period 0, which is given by

$$\begin{aligned}
 (45) \quad CA_{H,0} &= B_{H,1} - B_{H,0} \\
 &= B_{H,1} \\
 &= N_{H,1}^0 \left(\frac{1}{1+n} \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} - \hat{k}_1 \right) \\
 &= \frac{N_{H,1}^0}{1+n} \left(\frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} - \frac{\eta_H \bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} - \frac{\eta_F \bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} \right) \\
 &= \eta_F N_{H,0}^0 \left(\frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} - \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} \right).
 \end{aligned}$$

The Home country's net foreign assets at the beginning of period $t, t \geq 2$, is given by

$$\begin{aligned}
 (46) \quad B_{H,t} &= N_{H,t}^0 \left(\frac{1}{1+n} \frac{(1-\alpha)\hat{k}_{t-1}^\alpha}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha\hat{k}_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} - \hat{k}_t \right) \\
 &= N_{H,t}^0 \left[\frac{1}{1+n} \frac{(1-\alpha)\hat{k}_{t-1}^\alpha}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha\hat{k}_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} - \frac{(1-\alpha)\hat{k}_{t-1}^\alpha}{1+n} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha\hat{k}_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha\hat{k}_t^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \right] \right] \\
 &= \eta_F (1+n)^{t-1} N_{H,0}^0 (1-\alpha) \phi(\hat{k}_{t-1}), \\
 &(t = 2, \dots).
 \end{aligned}$$

In (46), we have let⁶

$$(47) \quad \phi(k) = k^\alpha \left(\frac{1}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} - \frac{1}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \right).$$

The Home country's net foreign assets per old individual at the beginning of period $t, t \geq 2$, is then given by

$$(48) \quad b_{H,t} = \frac{B_{H,t}}{(1+n)^{t-1} N_{H,0}^0} = \eta_F (1-\alpha) \phi(\hat{k}_{t-1}).$$

Now the Home country's current account at the end of period 1 is given by⁷

$$(49) \quad \begin{aligned} CA_{H,1} &= B_{H,2} - B_{H,1} \\ &= \eta_F (1-\alpha) (1+n) N_{H,0}^0 \left(\phi(\hat{k}_1) - \frac{1}{1+n} \left(\frac{\bar{k}_H^\alpha}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} - \frac{\bar{k}_F^\alpha}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} \right) \right) \\ &= \eta_F (1-\alpha) N_{H,0}^0 \left(\frac{(1+n) \hat{k}_1^\alpha - \bar{k}_H^\alpha}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} + \frac{\bar{k}_F^\alpha - (1+n) \hat{k}_1^\alpha}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}} \right) \end{aligned}$$

For $t = 2, 3, \dots$, the Home country's current account is given by

$$(50) \quad \begin{aligned} CA_{H,t} &= B_{H,t+1} - B_{H,t} \\ &= \eta_F (1-\alpha) (1+n)^t N_{H,0}^0 \left[\phi(\hat{k}_t) - \frac{1}{1+n} \phi(\hat{k}_{t-1}) \right] \end{aligned}$$

⁶ In (47), $\phi(k)$ is always negative because $\beta_H < \beta_F$. Thus, in (46), the Home country will continue to reveal its net foreign debts: i.e., $B_{H,t} < 0$.

⁷ Note from (49) that if there is no population growth ($n = 0$), the current account of the Home country at the end of period 1 is always in surplus because $\bar{k}_H < \hat{k}_1 < \bar{k}_F$. However, if $n > 0$, the result depends on the magnitude of n .

The behavior of the Home country's capital account and net foreign assets during the transition depend on the behavior of $\phi(\hat{k}_t)$ through time. We have already shown that the Home country's current account at the end of period 0 is negative. To know whether the Home country's net foreign assets rise or decline through time, let us differentiate $\phi(k)$ with respect to the world capital labor ratio. We have

$$\begin{aligned}
 (51) \quad \frac{\phi'(k)}{\alpha k^\alpha} &= \frac{1}{k} \left(\frac{1}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} - \frac{1}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \right) \\
 &\quad + (\alpha - 1) \frac{1-\sigma}{\sigma} G'(k) G(k)^{\alpha-2} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}-1} \left(\frac{\beta_H^{\frac{1}{\sigma}}}{\left(1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}\right)^2} \right. \\
 &\quad \left. - \frac{\beta_F^{\frac{1}{\sigma}}}{\left(1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}\right)^2} \right) \\
 &= \frac{1}{k} A_1 + (\alpha - 1) \frac{1-\sigma}{\sigma} G'(k) G(k)^{\alpha-2} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}-1} A_2,
 \end{aligned}$$

where we have let

$$(52) \quad A_1 = \frac{1}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} - \frac{1}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}}$$

and

$$(53) \quad A_2 = \frac{\beta_H^{\frac{1}{\sigma}}}{\left(1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}\right)^2} - \frac{\beta_F^{\frac{1}{\sigma}}}{\left(1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha G(k)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}\right)^2}.$$

Observe that $A_1 < 0$ because $\beta_H < \beta_F$. Hence the first expression on the right side of (51) is negative. As for A_2 , its sign is ambiguous. Hence the sign of the second expression on the right side of (51) is ambiguous. However, if α is close to 1, then the second expression will be dominated by the first expression, and the derivative $\phi'(k)$ will be negative.

In the long run, the net foreign assets per old individual in the Home country is given by

$$\begin{aligned}
 \bar{b}_H &= \lim_{t \rightarrow +\infty} b_{H,t} \\
 (54) \quad &= \eta_F (1 - \alpha) \phi(\bar{k}) \\
 &= \eta_F (1 - \alpha) \bar{k}^\alpha \left(\frac{1}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \alpha \bar{k}^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} - \frac{1}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \alpha \bar{k}^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \right) < 0.
 \end{aligned}$$

If there is no population growth, i.e., if $n = 0$, then in the long run the Home country's current account is equal to zero,⁸ and its foreign debt per old (or young) individual is given by (54). Also, in the long run, the Home country runs a trade surplus, which represents the capital income that the Foreign country repatriates for its investments in the Home country. We have the following proposition:

PROPOSITION 8: *Suppose that there is no population growth. Also, suppose that the share of capital remuneration in national income is sufficiently high.*

⁸ From (50), we find out that the current account of a country is zero in the steady state if $n=0$. The result is in contrast to the infinite horizon model in which the annuity value of net foreign assets is consumed each period, leading to a constant stream of current account imbalances. The deviating result of the OLG model stems from the ownership on net foreign assets. In the OLG model, net foreign assets are owned by the current old generations. Therefore, the consumption of current young generations cannot be affected by the net foreign assets.

- (a) *If the world capital labor ratio in period 1 is above its steady state value, then for all periods $t \geq 1$ during the transition to the steady state, the Home country's foreign debts decline through time, with the ensuing consequence that its current account is in surplus in each of these periods. In the long run, the Home country's net foreign assets will attain a constant negative value, and its current account will be zero.*
- (b) *If the world capital labor ratio in period 1 is below its steady state value, then for all periods $t \geq 0$ during the transition to the steady state, the Home country's foreign debts rise through time, with the ensuing consequence that its current account is in deficit in each of these periods. In the long run, the Home country's net foreign assets will attain a constant negative value, and its current account will be zero.*

PROOF: First, note that if α is close to 1, then $\phi'(k) < 0$. To prove (a), note that if the world capital labor in period 1 is above its long-run level, then it must be declining along the transition path, and this means $\phi(\hat{k}_t), t \geq 1$, and a fortiori $B_{H,t}, t \geq 1$, will rise through time. The Home country's foreign debts will decline through time, which means that its current account must be in surplus in each period to reduce its reliance on foreign capital, although in the long run its net foreign assets are still negative. To prove (b), note that if the world capital labor ratio in period 1 is below its long-run level, then it must rise through time along the transition path, and this implies that $\phi(\hat{k}_t), t \geq 2$, and a fortiori $B_{H,t}, t \geq 2$, will decline through time. The Home country's rising foreign debts imply that

its current account must be in deficit in each period, starting with period 0, except possibly for period 1, to import foreign capital. ■

The following proposition presents the evolution of the Home country's current account and net foreign assets for the case of positive population growth.

PROPOSITION 9: *Suppose that in each country the population grows at a constant rate $n > 0$. Also, suppose that the share of capital remuneration in national income is sufficiently high.*

- (a) *If the world capital labor ratio in period 1 is above its steady state value, then for all periods $t \geq 1$ during the transition to the steady state, the Home country's foreign debts might rise or decline through time. This ambiguous result is due to the fact that although the capital labor ratio is declining through time, the Home country's rising population, ceteris paribus, requires more capital to be imported to provide the rising labor force with adequate capital input. The temporal behavior of its current account is also ambiguous during the initial periods of the transition phase. However, in the long run, the Home country's net foreign assets and current account will both be negative and their absolute values both rise geometrically at the rate of population growth.*
- (b) *If the world capital labor ratio in period 1 is below its steady state value, then for all periods $t \geq 0$ during the transition to the steady state, the Home country's foreign debts rise through time, with the ensuing consequence that its current*

account is in deficit in each of these periods. In the long run, the Home country's net foreign assets and its current account will both be negative and rise geometrically at the rate of population growth.

PROOF: To prove (a) of Proposition 9, note that the population growth exerts an opposite impact on the decline in the world capital labor ratio during the transition. However, because the world capital labor ratio converges to its steady state value in the long run, at the aggregate level, the Home country's foreign debts and its current account behave qualitatively in the same manner in the long run. To prove (b) of Proposition 9, note that population growth reinforces the conclusion of (b) of Proposition 8. ■

According to conventional wisdom, an aging population deteriorates the current account of a country since it dampens national saving. In our model, the demographic effect on the current account is due to the existence of different time preference. In the normal case where the world capital labor ratio in the transition path is above its steady state, the demographic structure may have an opposite effect on the current account in the transition path, depending on time preference. When the demographic structure is aging, say $n = 0$, the less patient country reveals a surplus of current account while the more patient country shows a deficit. When the demographic structure is still young, say $n > 0$, both countries show an ambiguous sign in their current account. The long-run outcome also differs. A higher population growth may deteriorate the current account of the less patient country rather than improve it in the long run because the rising labor force

requires more capital to be imported for adequate capital input.⁹ In sum, the conventional wisdom holds true for the more patient country but it may not hold for the less patient country.¹⁰

4.3. *Welfare Analysis*

In our argument leading to the existence and uniqueness of the open-economy competitive equilibrium, we have shown that the world interest rate in period 1 is lower than the autarkic long-run interest rate in the Home country, but higher than that in the Foreign country. Given that it takes one period for capital flows to equalize the interest rates in both countries, the wage rate in each country immediately after integration will remain at its autarkic steady-state level. Then a young individual of period 0 in the Home country will be worse-off: her wages are still the same, but the interest she obtains for her saving has declined from $\bar{r}_H$ to $\hat{r}_1$. Her optimal choice now is to consume more and save less. For a young individual of period 0 in the Foreign country, the opposite results apply. Her labor income is still at the autarkic steady-state level $\bar{w}_F$, but the interest she obtains for her saving has risen from $\bar{r}_F$ to $\hat{r}_1$. She is better-off and will shift her consumption from the present to the future by cutting back her current consumption.

⁹ That can explain in part why the United States have shown a large current account deficit despite her relatively higher population growth than other developed countries. A rising labour force in the United States might have required more capital to be imported. As a result, the current account deficits have been revealed.

¹⁰ Despite the results derived from our model, we cannot fully convince the above intuitions. Our model has a limitation by assuming an identical population growth between countries. Thus in order to obtain a more intuitive result, different rate of population growth between countries should be allowed in the model.

During the transition, the lifetime utility of a young individual of period $t, t = 1, \dots$, in country $i, i = H, F$, according to (7), is given by ¹¹

$$(55) \quad v_{i,t}(\hat{\omega}_{i,t}, \hat{r}_{i,t+1}) = \frac{\hat{\omega}_{i,t}^{1-\sigma}}{1-\sigma} \left(1 + \beta_i^{\frac{1}{\sigma}} \hat{r}_{i,t+1}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}.$$

Because $\bar{k}_H < \hat{k}_t < \bar{k}_F$, for $t = 1, 2, \dots$, we have

$$(56) \quad \bar{\omega}_H < \hat{\omega}_t < \bar{\omega}_F,$$

and

$$(57) \quad \bar{r}_F < \hat{r}_t < \bar{r}_H,$$

($t = 1, \dots$).

Thus along the transition to the long-run equilibrium and relative to the autarkic equilibrium, a young individual of period $t, t = 1, 2, \dots$, in the Home country earns a higher wage, but a lower interest rate for her saving. The opposite results hold for a young individual in the Foreign country. To determine the net impact of a higher labor income and a lower rate of return to saving on the welfare of a young individual of period $t, t = 1, 2, \dots$, in the Home country requires some brute-force computations to relate the current wage rate with the equilibrium world interest rate in the next period. To this end, suppose that ω_t is the current prevailing wage rate, and $k_t = [\omega_t / (1 - \alpha)]^{1/\alpha}$ be the current world capital labor ratio. The equilibrium capital labor ratio in the next period is

¹¹ As shown in Blanchard and Fischer (1989, p. 97-98), it may be arbitrary to choose a social time preference factor at which the utility of future generations is discounted. Therefore, for simplification, we just assume that the life time welfare function of a young individual born in period t is a social welfare function of period t .

then given by $k_{t+1} = G(k_t)$, and the equilibrium world rate of interest in the next period by

$$(58) \quad r_{t+1} = \alpha k_{t+1}^{\alpha-1} = \alpha \left(G([\omega_t / (1-\alpha)]^{1/\alpha}) \right)^{\alpha-1}.$$

Because G is strictly increasing, (58) defines a unique relationship between ω_t and r_{t+1} , say $\omega_t = \varphi(r_{t+1})$, with $\varphi'(r_{t+1}) < 0$. Using the definition of φ , we can then express the welfare of a young individual of period $t > 0$ in country i as a function of the equilibrium world rate of interest in the following period as follows

$$(59) \quad \begin{aligned} v_{i,t}(\omega_{i,t}, r_{i,t+1}) &= \frac{\omega_{i,t}^{1-\sigma}}{1-\sigma} \left(1 + \beta_i^{\frac{1}{\sigma}} r_{i,t+1}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma} \\ &= \frac{\varphi(r_{t+1})^{1-\sigma}}{1-\sigma} \left(1 + \beta_i^{\frac{1}{\sigma}} r_{i,t+1}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}. \end{aligned}$$

The following lemma is proved in Appendix C:

LEMMA: *The map*

$$(60) \quad r_{i,t+1} \rightarrow \frac{\varphi(r_{t+1})^{1-\sigma}}{1-\sigma} \left(1 + \beta_i^{\frac{1}{\sigma}} r_{i,t+1}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}$$

is strictly decreasing in r_{t+1} . That is, for a young individual of period $t > 0$ in country i , a rise in the equilibrium world rate of interest in the next period lowers her lifetime utility.

Using the lemma and (57), we can assert that relative to the autarkic equilibrium all the young generations of period $t > 0$ in the Home (Foreign) country win (lose) with economic integration. In particular, in the long run the residents of the Home (Foreign) country are better-off (worse-off) with economic integration. Furthermore, if the world

capital labor ratio in period 1 overshoots its long-run level, then in its long descent to the long-run equilibrium the standard of living of the successive young generations – both at home and abroad – will be declining through time. On the other hand, if the world capital labor ratio in period 1 is below its long-run value, then in its monotone convergence to its steady state level, the world capital labor ratio will rise through time, and the standard of living of the successive young generations – both at home and abroad – will be rising through time. We summarize the results just obtained in the following proposition:

PROPOSITION 10: *Starting from the autarkic long-run equilibria in both countries, an economic integration of the two economies will lower (raise) the life time welfare of the current young generation of the Home (Foreign) country. However, the economic integration will raise (lower) the life time welfare of all future generations of the Home (Foreign) country. Furthermore, if the world capital labor ratio in period 1 overshoots its long-run value, then as the world capital labor ratio descends to its long-run level, the welfare of the successive young generations – both at Home country and abroad – declines steadily. On the other hand, if the world capital labor ratio in period 1 is below its long-run value, then as the world capital labor ratio climbs to its long-run level, the welfare of the successive young generations – both at Home and abroad – steadily improves through time.*

PROPOSITION 10 coincides with the welfare result of Buiter in the case of dynamic efficiency. Buiter's analysis, however, is limited to long-run welfare comparison. This chapter extends his proposition by showing how transitional welfare of two countries

differs depending on the path of their capital labour ratio. PROPOSITION 10 also provides a useful intuition on the issue of the capital market opening in developing countries. Granted that the real interest rates are higher in developing countries than in developed countries, the capital market opening of the former will lower the real interest rates in the developing countries. Then the current young generation of the developing countries will be worse off since a lower interest rate may dampen their capital income in the future. In this sense, a young generation of the developing countries may have enough reasons for opposing contemporaneous capital market opening or globalization. However, according to our model, their future generations will be definitely better off with capital market opening, as the effect of higher wage rate dominates the effect of lower interest rate. In sum, the unfavourable effects of impatience in a country can be transmitted to the other country in the open economy so that the less patient country is ultimately better off with the economic integration.¹²

5. CONCLUDING REMARKS

This chapter has examined the manner in which an economic integration of two large economies affects the macroeconomic variables. In particular, we have focused on the role of time preferences as a factor that leads to capital movement between countries. The model we formulate maintains the traditional assumption of a Cobb-Douglas technology, but adopts a utility function with constant inter-temporal elasticity of substitution. The

¹² The model also can be applied to the problems among the developed countries. Suppose that the United States have perpetually shown a higher time preference, i.e., less patient than Japan. Then based on our model, it is possible to say that the current prosperity of American would have been indebted in large part to the patience of Japanese.

assumption of preferences with a constant elasticity of substitution made saving depend positively on both labor income and the rate of interest, and allowed us to obtain several interesting results.

In autarkic equilibrium, the more patient country enjoys a higher standard of living than the less patient country. Considering that the rate of time preference is inversely related to the propensity to save, we can assert that the thriftiness of the current generations ultimately determines the welfare of their future generations.

In the open-economy setting, the outcomes become slightly more complicated because the effects of the impatience of one country can be transmitted to other country. The world capital labor ratio on the transition path, which always lies between the autarkic capital labor ratios of the two countries, might be higher or lower than its steady-state value. If there is a pronounced difference in time preferences between the two countries, the world capital labor ratio in the period that follows economic integration overshoots its long-run level. Then the less patient country is overheated on the initial transition periods. Depending on the values of the parameters, the world capital labor ratio in the period that follows economic integration might fall short of its long-run level. Under this scenario, which is less likely expected to occur, the more patient country experiences a depression during the initial transition periods.

In particular, this chapter has shown how economic integration affects the balance of payments and the welfare of a country differently conditional on the population growth.

In the transition path, the less patient country can reveal a current account surplus while the more patient country shows a deficit as long as both countries are aging with no population growth. However, when both countries have a high population growth, the direction of the current account is ambiguous. In the long run, the outcome is more apparent; the current account of the less patient country will be zero with no population growth and will be negative with a high population growth.

Also economic integration has different welfare results conditional on the time preference. With economic integration, the less patient country is better off while the more patient country is ultimately worse off. Even if the current young generation of the less patient country is worse off with integration, her future generations are better off with their precedent generation's decision to open their border.

APPENDIX A

THE PROOF OF PROPOSITION 6

Recall that the world interest rate in period 1, namely $\hat{r}_1$, is the fixed point of the following map:

$$(A.1) \quad \begin{aligned} & \zeta(\bar{\omega}_H, \bar{\omega}_F, \cdot) : r \rightarrow \zeta(\bar{\omega}_H, \bar{\omega}_F, r) \\ & = \alpha \left(\frac{1}{n+1} \left[\eta_H \frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}} (1+r)^{-\frac{1-\sigma}{\sigma}}} + (1-\eta_H) \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}} (1+r)^{-\frac{1-\sigma}{\sigma}}} \right] \right)^{\alpha-1}. \end{aligned}$$

Also, the world steady state interest rate is the fixed point of the following map:

(A.2)

$$\zeta(\bar{w},.) : r \rightarrow \zeta(\bar{w}, r) = \alpha \left(\frac{1}{n+1} \left[\eta_H \frac{\bar{w}}{1 + \beta_H^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} + (1-\eta_H) \frac{\bar{w}}{1 + \beta_F^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} \right] \right)^{\alpha-1}.$$

Now let

$$\begin{aligned} (A.3) \quad I &= \left(\eta_H \frac{\bar{w}_H}{1 + \beta_H^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} + \eta_F \frac{\bar{w}_F}{1 + \beta_F^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} \right) \\ &\quad - \left(\eta_H \frac{\bar{w}}{1 + \beta_H^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} + \eta_F \frac{\bar{w}}{1 + \beta_F^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} \right) \\ &= \eta_H \frac{\bar{w}_H - \bar{w}}{1 + \beta_H^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} + \eta_F \frac{\bar{w}_F - \bar{w}}{1 + \beta_F^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}}. \end{aligned}$$

Now note that for any given value of r , we have $\bar{w}_H \rightarrow 0$ and $\beta_H^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}} \rightarrow \infty$ when $\beta_H \rightarrow 0$. As for $\bar{w}$, the common value of the steady state wage rates in the two countries, it is strictly less than $\bar{w}_F$ and will remain outside a left neighborhood of $\bar{w}_F$ as $\beta_H \rightarrow 0$. Thus I will be positive when β_H is sufficiently small, which implies that $\hat{r}_1 < \bar{r}$, or equivalently $\hat{k}_1 > \bar{k}$ ■

APPENDIX B

THE PROOF OF PROPOSITION 7

To prove Proposition 7, we shall show that $I < 0$ when η_H is close to 1. Differentiating

I – the function defined in by A.3 Appendix A – with respect to η_H , we obtain

$$(B.1) \quad \frac{\partial I}{\partial \eta_H} = \left(\frac{\bar{\omega}_H}{1 + \beta_H^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} - \frac{\bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} \right) - \left(\frac{\bar{\omega}(\eta_H)}{1 + \beta_H^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} + \eta_H \frac{\bar{\omega}'(\eta_H)}{1 + \beta_H^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} - \frac{\bar{\omega}(\eta_H)}{1 + \beta_F^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} + (1 - \eta_H) \frac{\bar{\omega}'(\eta_H)}{1 + \beta_F^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} \right).$$

Note that in (B.1) we have made explicit the dependence of the world steady state wage on η_H , the relative size of the Home country's population.

When $\eta_H = 1$, (B.1) becomes

$$(B.2) \quad \left. \frac{\partial I}{\partial \eta_H} \right|_{\eta_H=1} = \frac{\bar{\omega}_H - \bar{\omega}_F}{1 + \beta_F^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}} - \frac{\bar{\omega}'(1)}{1 + \beta_H^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}}.$$

Next, recall that the steady state world interest rate $\bar{r}$ and the steady state world capital labor ratio is linked by the following relation:

$$\begin{aligned}
(B.3) \quad 1 &= \frac{(1-\alpha)\bar{k}^{\alpha-1}}{1+n} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}}(1+\bar{r})^{\frac{1-\sigma}{\sigma}}} + \frac{(1-\eta_H)}{1 + \beta_F^{\frac{1}{\sigma}}(1+\bar{r})^{\frac{1-\sigma}{\sigma}}} \right] \\
&= \frac{(1-\alpha)\bar{r}(\eta_H)}{\alpha(1+n)} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}}} + \frac{(1-\eta_H)}{1 + \beta_F^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}}} \right].
\end{aligned}$$

Note that on the second line of (B.3) we have made explicit the dependence of the steady state world interest rate on η_H . Differentiating (B.3) with respect to η_H , we obtain

$$\begin{aligned}
(B.4) \quad 0 &= \frac{\partial \bar{r}}{\partial \eta_H} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}}} + \frac{(1-\eta_H)}{1 + \beta_F^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}}} \right] + \\
&\quad \left[\frac{1 + \beta_H^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}} + \eta_H \frac{1-\sigma}{\sigma} \beta_H^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}-1} \frac{\partial \bar{r}}{\partial \eta_H}}{\left(1 + \beta_H^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}}\right)^2} \right. \\
&\quad \left. \bar{r}(\eta_H) \left[- \left(1 + \beta_F^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}}\right) + (1-\eta_H) \frac{1-\sigma}{\sigma} \beta_F^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}-1} \frac{\partial \bar{r}}{\partial \eta_H} \right] \right. \\
&\quad \left. + \frac{\left(1 + \beta_F^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}}\right)^2}{\left(1 + \beta_F^{\frac{1}{\sigma}}(1+\bar{r}(\eta_H))^{\frac{1-\sigma}{\sigma}}\right)^2} \right].
\end{aligned}$$

For $\eta_H = 1$, (B.4) is reduced to

$$(B.5) \quad 0 = \left(\frac{\partial \bar{r}}{\partial \eta_H} \Big|_{\eta_H=1} \right) \left[\frac{1}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}}} + \frac{\frac{1-\sigma}{\sigma} \beta_H^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}-1} \bar{r}(1)}{\left(1 + \beta_H^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}} \right)^2} \right] \\ + \bar{r}(1) \left[\frac{1}{\left(1 + \beta_H^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}} \right)} - \frac{1}{\left(1 + \beta_F^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}} \right)} \right].$$

Now the equilibrium wage rate, the interest rate, and the equilibrium capital labor ratio are linked by the following relations $\omega = (1 - \alpha)k^\alpha$ and $r = \alpha k^{\alpha-1}$, from which we obtain

$\frac{\partial \omega}{\partial r} = -\alpha^{\frac{1}{1-\alpha}} r^{\frac{1}{\alpha-1}}$. Using this partial derivative in (B.5), we obtain

$$(B.6) \quad \bar{\omega}'(1) = -\alpha \frac{\bar{\omega}(1)}{1-\alpha} \left[\frac{\left[\frac{1}{\left(1 + \beta_F^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}} \right)} - \frac{1}{\left(1 + \beta_H^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}} \right)} \right]}{\left[\frac{1}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}}} + \frac{\frac{1-\sigma}{\sigma} \beta_H^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}-1} \bar{r}(1)}{\left(1 + \beta_H^{\frac{1}{\sigma}} (1 + \bar{r}(1))^{\frac{1-\sigma}{\sigma}} \right)^2} \right]} \right].$$

Observe that when $\alpha \uparrow 1$, the right side of (B.6) will tend to $-\infty$. Thus when the share in national income of the factor capital is high enough, the expression on the right side of (B.2) will be positive. Thus I is increasing when η_H is close to 1. Finally, because $I = 0$ when $\eta_H = 1$, we can now conclude that $I < 0$ when η_H is close to 1. ■

APPENDIX C

PROOF OF THE LEMMA

According to (6), the saving of a young individual of period t in country i is given by

$$(C.1) \quad s_{i,t}(\omega_t, r_{t+1}) = \frac{\omega_t}{1 + \beta_i^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}}.$$

The world capital stock in period $t+1$ is given by

$$(C.2) \quad \begin{aligned} K_{t+1} &= N_{H,t}^0 s_{H,t}(\omega_t, r_{t+1}) + N_{F,t}^0 s_{F,t}(\omega_t, r_{t+1}) \\ &= N_{H,t}^0 \frac{\omega_t}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} + N_{F,t}^0 \frac{\omega_t}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}}. \end{aligned}$$

The world capital labor ratio in period $t+1$ is

$$(C.3) \quad \begin{aligned} k_{t+1} &= \frac{K_{t+1}}{(1+n)(N_{H,t}^0 + N_{F,t}^0)} \\ &= \frac{\omega_t}{1+n} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} \right]. \end{aligned}$$

Substituting $[r_{t+1}/\alpha]^{\frac{1}{\alpha-1}}$ for k_{t+1} in (C.3), we obtain

$$(C.4) \quad \left[\frac{r_{t+1}}{\alpha} \right]^{\frac{1}{\alpha-1}} = \frac{\omega_t}{1+n} \left[\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} \right],$$

which can be rewritten as

$$(C.5) \quad \omega_t = \frac{(1+n) \left[\frac{r_{t+1}}{\alpha} \right]^{\frac{1}{\alpha-1}}}{\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}}}.$$

Now recall from (7) that the indirect lifetime utility function of a young individual of period t in the Home country is given by

$$(C.6) \quad v_{H,t}(\omega_t, r_{t+1}) = \frac{\omega_t^{1-\sigma}}{1-\sigma} \left(1 + \beta_H^{\frac{1}{\sigma}} r_{t+1}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}.$$

Using (C.5) in (C.6), we obtain

$$(C.7) \quad \begin{aligned} & v_{H,t}(\varphi(r_{t+1}), r_{t+1}) \\ &= \left[\frac{(1+n) \left[\frac{r_{t+1}}{\alpha} \right]^{\frac{1}{\alpha-1}}}{\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}}}} \right]^{1-\sigma} \left(1 + \beta_H^{\frac{1}{\sigma}} r_{t+1}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}. \end{aligned}$$

Note that in (C.7), we have used the definition of the function $\omega_t = \varphi(r_{t+1})$.

Taking the logarithm of (C.7), we obtain

$$(C.8) \quad \begin{aligned} & \text{Log}[v_{H,t}(\varphi(r_{t+1}), r_{t+1})] = -\text{Log}[1-\sigma] + \sigma \text{Log} \left(1 + \beta_H^{\frac{1}{\sigma}} r_{t+1}^{\frac{1-\sigma}{\sigma}} \right) \\ & + (1-\sigma) \text{Log} \left[\frac{(1+n) \left[\frac{r_{t+1}}{\alpha} \right]^{\frac{1}{\alpha-1}}}{\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}}}} \right]. \end{aligned}$$

Differentiating (C.8) with respect to r_{t+1} , then simplifying the result¹³ we obtain

¹³ All calculations, derivations and simulations in this thesis were carried out by *Mathematica*.

(C.9)

$$\begin{aligned}
& \frac{d}{dr_{t+1}} \text{Log}[v_{H,t}(\varphi(r_{t+1}), r_{t+1})] \\
&= - \frac{(1-\sigma) \left(r_{t+1} + \alpha \beta_H^{\frac{1}{\sigma}} r_{t+1}^{\frac{1}{\sigma}} \right)}{(1-\alpha) r_{t+1} \left(r_{t+1} + \beta_H^{\frac{1}{\sigma}} r_{t+1}^{\frac{1}{\sigma}} \right)} \\
& \quad - \frac{(1-\sigma) \left(\frac{\eta_H (1-\sigma) \beta_H^{\frac{1}{\sigma}} (1+r_{t+1})^{-1-\frac{1-\sigma}{\sigma}}}{\sigma \left(1 + \beta_H^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}} \right)} + \frac{\eta_F (1-\sigma) \beta_F^{\frac{1}{\sigma}} (1+r_{t+1})^{-1-\frac{1-\sigma}{\sigma}}}{\sigma \left(1 + \beta_F^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}} \right)} \right)}{\frac{\eta_H}{1 + \beta_H^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1 + \beta_F^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}}}}
\end{aligned}$$

The right side of (C.9) is negative. The lemma is proved for a young individual of period t in the Home country. The proof can be repeated for a young individual of period t in the Foreign country. ■

CHAPTER 2

THE GLOBAL TRANSMISSIONS OF GOVERNMENT DEBT

I. INTRODUCTION

THE OBJECTIVE OF THIS CHAPTER IS TO STUDY how the government debt of a country is transmitted to other countries in open economy. Governments, unlike individuals, do not have a finite life, and can borrow permanently. This perpetuity of government adds a number of interesting features to the overlapping-generations (OLG) model. If generations are not linked with intergenerational altruism, the issuance of government debt influences the welfare of current and future generations by redistributing resources across generations. Suppose that a government gives a transfer to its old generation by issuing government debt and adopts a debt management policy under which the debt per young individual is maintained at a constant level. If the young individuals as prime-age workers bear all the taxes, current and future young individuals of the country have to bear the burden of the debt service payment. Given that the government must satisfy its inter-temporal budget constraint, clearly the issuance of government debt, other things equal, benefits the current old generation at the expense of the current young generation and all future generations. Since young individuals are an exclusive source of saving in two-period OLG model, the reduction of their disposable income will retard the formation of capital stock in a country. This is the mechanism by which the fiscal policy is non-neutral in the basic OLG models, such as Diamond (1965) and Blanchard (1985).

A two-country OLG model adds another interesting feature: an inter-country redistribution of resources. The government debt of a country can be transmitted to the other country if capital can flow freely across national frontiers. The current and future generations of the other country without taxes and government expenditures might have to share the burden of the debt of their neighboring government. In analyzing the effects of fiscal policy, an important merit of the OLG model is that the model can demonstrate a non-neutrality of government debt; that is, the OLG model is able to show the real effects of the fiscal policy. On the contrary, in the infinite horizon model, which is an alternative framework for modeling dynamic macroeconomic behavior, the private sector fully internalizes the public sector's budget constraint. Thus government expenditures completely crowd out private consumption, and have no effect on the capital stock in the steady state; that is, Ricardian equivalence holds in the infinite horizon model.¹⁴ Due to this neutrality, fiscal policy has received relatively less attention than monetary policy in recent theoretical works of the infinite horizon model based on micro-foundations.¹⁵ In addition, on the aspects of theoretical feasibility, the OLG model allows for a precise discussion of intergenerational welfare distributions in a relatively simple way by studying a sequence of overlapping generations. Thus the OLG model makes it possible to pose a central question about the burden of government debt in terms of lifetime utilities of welfare-maximizing consumers.

¹⁴ However, Ricardian equivalence may not hold in the infinite horizon model under some conditions: i.e., rigidities, capital market imperfection and distortionary taxation. See Bernheim (1987) for a more detailed discussion. Similarly, as mentioned in Chapter 1, Ricardian equivalence may hold in the OLG model if intergenerational bequests are allowed.

¹⁵ See, for example, the new open-economy macroeconomics literatures originated by Obstfeld and Rogoff (1996).

Since the seminal work of Diamond, the global effects of the government budget deficit or government debt has been examined in several open-economy OLG models. The existing literature, however, reveals some limitations; For example, Persson (1985) and Dornbusch (1984) focus on steady state and comparative analysis. Therefore, the dynamic transition path toward a new steady state is less obviously addressed; Fried and Howitt (1988) exclude the growth factor in the model by supposing that the capital stock is fixed and thus the effects of falling capital labor ratio due to the budget deficit are not properly answered; Frenkel and Razin (1987, 1992), and Ganelli (2005) do not include the capital stock as a factor of production. Then the crowding-out effects of the government debt on capital, and subsequent distributional effects cannot be fully analyzed.

Probably, the methodology of this chapter is closest to that of Obstfeld and Rogoff (1996). However, this chapter enriches their results particularly by tracing the transitional dynamics of capital labor ratio, balance of payments and welfare in two countries and as well, by assuming a less restrictive form of preference. In our model, we assume a Cobb-Douglas technology, but adopt a utility function with a constant inter-temporal elasticity of substitution (CIES). A utility function of this type allows for saving to depend on both labor income and the interest rate. By accommodating a positive effect of interest rate on saving, this chapter can trace the transition dynamics with more precision.¹⁶ Due to the

¹⁶ Suppose that the government debt crowds out investment. The rise in the interest rate induced by the issuance of the government debt will encourage the saving of the current young individuals. Thus ultimately, the capital stock in the economy will fall by less than predicted by the model with logarithmic preferences in which the interest rate has no feedback on the saving decision.

less restrictive preferences, the solutions are given only in an implicit form and thus, a numerical iteration method must be used to derive the equilibrium values. In addition, this chapter briefly deals with the sustainability of the government debt following Rankin and Roffia (2003). As well, unlike their analysis, the model of this chapter is not restricted to steady state outcomes or a logarithmic preference in the sustainability issue.

In this chapter, we assume that the world economy – the Home and the Foreign- is initially in long-run equilibrium, and that there are no taxes and no government expenditures in that long-run equilibrium. Next, starting from that long-run equilibrium, the Home government makes a transfer to the old generation, and the transfer is funded through the issuance of government debt. Thereafter, the Home government maintains a constant debt per young individual by levying a lump sum tax on the wage income of its current young and all future young generations. In contrast, the Foreign government continues to follow the policy of no tax and no government expenditure. Under this circumstance, the current old generation of the Home country will apparently achieve a free-ride welfare gain. The burden of the free-ride gain will be transferred to and shared with the current young generation of the Home country and all subsequent generations in both countries.

The rest of this chapter is organized as follows. Section 2 sets up a two-country model without taxes and government expenditures. Section 3 analyzes debts, deficits, and crowding out of private investment in the presence of debts and taxes. Section 4 shows how the current account and the net foreign assets are changed with the debts. Section 5

presents the welfare results. Section 6 provides numerical examples for the analysis. Lastly, Section 7 is given to the concluding remarks.

2. A TWO-COUNTRY GLOBAL MODEL WITHOUT TAXES AND GOVERNMENT EXPENDITURES

2.1. *Preferences and Technology*

Time is discrete and denoted by $t, t = 0, 1, \dots$. The world consists of two countries, called H (Home) and F (Foreign), respectively. In each country four classes of economic agents co-exist in each period: a young generation, and old generation, competitive firms, and a government. In the model there is a consumption good that is produced using labor and capital. The consumption good can also be used as investment goods for capital accumulation. An individual works when she is young and retires when she is old. A young individual has one unit of time that she supplies in-elastically on the labor market. The individual spends part of the labor income on current consumption; the remaining part is saved to provide for her old-age consumption. The savings of the young generation in any period constitute the economy's capital stock in the following period. In any period, it is the old individuals who are the owners of the economy's stock of capital in that period. In what follows, the consumption good is taken to be the numeraire in each period.

Let $N_{i,t}^0$ and $N_{i,t}^1$ denote, respectively, the number of young individuals and the number of old individuals at the beginning of period t in country i , $i = H, F$. The population in each country is assumed to grow at the same constant rate n . Thus the number of young individuals at the beginning of period t in country i is $N_{i,t}^0 = N_{i,0}^0 (1+n)^t$, $t = 0, 1, \dots$. The preferences of an individual in country i are assumed to be represented by the following utility function:

$$(1) \quad \frac{[c^0]^{1-\sigma}}{1-\sigma} + \beta_i \frac{[c^1]^{1-\sigma}}{1-\sigma}.$$

In (1), c^0 and c^1 denote, respectively, the current consumption and the future consumption. Also, σ , $0 < \sigma < 1$, is the reciprocal of the inter-temporal elasticity of substitution, and β_i , $0 < \beta_i < 1$ is the discount factor for future consumption. We have chosen not to include bequest motive in the model since such transfers may lead to the neutrality of the government's debt. In what follows, we shall assume that $\beta_H < \beta_F$, i.e., agents in the Home country are less patient than agents in the Foreign country.

In both countries, the technology used to produce the consumption good is of the Cobb-Douglas type, and is assumed to have the following form: $Y = K^\alpha L^{1-\alpha}$, where K , L , and Y denote, respectively, the capital input, the labour input, and the output of the consumption good. Also, α , $0 < \alpha < 1$, is a parameter representing the share of capital in national income. Letting $k = K/L$ denote the capital labour ratio, we can express the output produced per worker as $f(k) = k^\alpha$. Capital is assumed not to depreciate.

Observe that the two countries differ only in terms of the absolute sizes of their populations and their rates of time preferences.¹⁷

2.2. Utility Maximization and Saving

Consider a young individual of period t in country i who faces the wage rate $\omega_{i,t}$ and the rate of interest $r_{i,t+1}$ on her saving. A young individual of period t in country i solves the following utility maximization problem:

$$(2) \quad \max_{(c_{i,t}^0, c_{i,t+1}^1)} \frac{[c_{i,t}^0]^{1-\sigma}}{1-\sigma} + \beta_i \frac{[c_{i,t+1}^1]^{1-\sigma}}{1-\sigma}$$

subject to

$$(3) \quad c_{i,t}^0 + \frac{c_{i,t+1}^1}{1+r_{i,t+1}} = \omega_{i,t}.$$

As we have shown in section 3.1 of Chapter 1, it is simple to show that the solution of the maximization problem constituted by (2) and (3) yields the following current consumption and old-age consumption:

$$(4) \quad c_{i,t}^0 = \frac{\omega_{i,t}}{1 + \beta_i^{\frac{1}{\sigma}} (1+r_{i,t+1})^{\frac{1-\sigma}{\sigma}}}$$

and

$$(5) \quad c_{i,t+1}^1 = \frac{\beta_i^{\frac{1}{\sigma}} (1+r_{i,t+1})^{\frac{1}{\sigma}} \omega_{i,t}}{1 + \beta_i^{\frac{1}{\sigma}} (1+r_{i,t+1})^{\frac{1-\sigma}{\sigma}}}.$$

Furthermore, the saving of the individual is given by

¹⁷ For ease of exposition, we repeat in the next two sections the analysis in sections 3.1 and 3.2 of Chapter 1.

$$(6) \quad s_{i,t}(\omega_{i,t}, r_{i,t+1}) = \frac{\omega_{i,t}}{1 + \beta_i^{\frac{1}{\sigma}} (1 + r_{i,t+1})^{\frac{1-\sigma}{\sigma}}}.$$

Observe that saving rises with labor income and the rate of return during its investment span. Also, a more patient individual, i.e., one with a higher value of β_i , saves a higher proportion of labor income. The list $(c_{i,t}^0, c_{i,t+1}^1, s_{i,t}(\omega_{i,t}, r_{i,t+1}))$ is called the optimal lifetime plan of a young individual of period t in country i .

Substituting (4) and (5), respectively, for c^0 and c^1 in (2), we obtain the following expression for the indirect lifetime utility function of a young individual of period t in country i :

$$(7) \quad v_{i,t}(\omega_{i,t}, r_{i,t+1}) = \frac{\omega_{i,t}^{1-\sigma^{1-\sigma}}}{1-\sigma} \left(1 + \beta_i^{\frac{1}{\sigma}} r_{i,t+1}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}.$$

2.3. Profit Maximization

Let $\omega_{i,t}$ and $r_{i,t}$ be the wage rate and the rental rate of capital that prevail in period t in country i . The representative firm that produces the consumption good solves the following profit maximization problem in this period:

$$(8) \quad \max_{(K_{i,t}, L_{i,t})} K^{\alpha} L^{1-\alpha} - \omega_{i,t} L - r_{i,t} K.$$

The following first-order conditions characterize the optimal labor and capital inputs:

$$(9) \quad (1-\alpha) K^{\alpha} L^{-\alpha} - \omega_{i,t} = 0,$$

$$(10) \quad \alpha K^{\alpha-1} L^{1-\alpha} - r_{i,t} = 0.$$

Let $K_{i,t}$ and $L_{i,t}$ denote, respectively, the capital input and the labor input associated that solves (9) and (10). The output of the consumption good produced from the input combination $(K_{i,t}, L_{i,t})$ is $Y_{i,t} = K_{i,t}^\alpha L_{i,t}^{1-\alpha}$. The list $(Y_{i,t}, K_{i,t}, L_{i,t})$ is called an optimal production plan in period t for the representative firm in country i . The capital labor ratio in period t is then given by $k_{i,t} = K_{i,t} / L_{i,t}$.

2.4. Autarkic Equilibrium

Let $(K_{i,t}, N_{i,t}^0, N_{i,t}^1)$ denote the state of the economy in country i at the beginning of period t . Here $K_{i,t}$ represents this economy's capital stock in period t , while $N_{i,t}^0$ and $N_{i,t}^1$ – we recall – represent, respectively, the number of young individuals and the number of old individuals in period t in country i . The initial state of the economy of country i , namely, $(K_{i,0}, N_{i,0}^0, N_{i,0}^1)$, is assumed to be given. In Chapter I, we have shown (Proposition 1) that there exists a unique competitive equilibrium for each economy when it is in autarky. Furthermore, in autarky, the Home country, which is less patient than the Foreign country, has a lower capital labor ratio (Proposition 3) than the Foreign country in the long run.

2.5. Open-Economy Equilibrium

Suppose that initially the two countries are in autarkic long-run equilibrium. Next, suppose that in period 0 the governments of the two countries open their borders and

allow goods and capital to flow freely. In Chapter 1, we have shown (Proposition 5) that there exists a unique competitive equilibrium under economic integration. The world capital labor ratio in the long run, say $\bar{k}$, is higher than $\bar{k}_H$, the autarkic steady state capital labor of the Home country, but lower than $\bar{k}_F$, the autarkic steady state capital labor of the Foreign country: $\bar{k}_H < \bar{k} < \bar{k}_F$, and this means that in steady state the Home country is a net debtor.

3. DEBTS, DEFICITS, AND CROWDING OUT OF PRIVATE INVESTMENT

The model formulated in Section 2 can be used to analyze the global impact of government deficits and debts. The natural way to address this question is to introduce into that model a particular type of debt instrument and a policy to manage the debt over time. We could then study the dynamic adjustments of the world economy in its convergence to the steady state. To this end, we shall assume that the world economy is initially in long-run equilibrium, and that there are no taxes and government expenditures in that long-run equilibrium. Next, starting from that long-run equilibrium, the Home government issues – as a gift¹⁸ to the old generation of period 0 – a quantity of claims, say $D_{H,0}$, on itself. The national debt per young individual of period 0 in the Home country is then given by $\bar{d} = D_{H,0} / N_{H,0}^0$. To finance this gift, the Home government chooses to tax the current young and all future young generations. Since the young

¹⁸ The gift can be interpreted as a social security payment.

individuals in our model can be interpreted as prime-age workers, it is reasonable to assume that they bear all the taxes. In what follows, we shall denote by $\tau_{H,t}^0$ the tax (lump sum) imposed on a young individual of period t in the Home country. Letting $D_{H,t}$ denote the national debt of the Home country in period $t, t = 0, 1, \dots$, we can represent the financial constraint of the Home government by the following difference equation:

$$(11) \quad D_{H,t+1} = (1 + r_t)D_{H,t} - N_{H,t}^0 \tau_{H,t}^0,$$

$$(t = 0, 1, \dots).$$

Following Diamond, op cit., we shall assume that the taxes in each period $t = 0, 1, \dots$, say $\tau_{H,t}^0$, are chosen so that the debt per worker, $D_{H,t} / N_{H,t}^0, t = 0, 1, \dots$, is held at the constant level $\bar{d}$. It then follows from (11) that

$$(12) \quad \begin{aligned} \bar{d} &= \frac{D_{H,t+1}}{N_{H,t+1}^0} \\ &= (1 + r_t) \frac{D_{H,t}}{N_{H,t+1}^0} - \frac{N_{H,t}^0}{N_{H,t+1}^0} \tau_{H,t}^0 \\ &= \frac{1 + r_t}{1 + n} \bar{d} - \frac{\tau_{H,t}^0}{1 + n}. \end{aligned}$$

Note that in (12) r_t represents the world interest rate in period $t, t = 0, 1, \dots$. Solving (12) for $\tau_{H,t}^0$, we obtain

$$(13) \quad \tau_{H,t}^0 = (r_t - n)\bar{d}.$$

Equation (13) gives the tax imposed on a young individual of period t in the Home country so that the debt per young individual in the Home country remains at the level $\bar{d}$ through time.

Under the above policy of debt management, the solution of the utility maximization of the successive young generations in the Home country must be modified to account for the tax $\tau_{H,t}^0$. The labor income, net of taxes, of a young individual of period t in the Home country is now given by

$$(14) \quad \omega_t - \tau_{H,t}^0 = \omega_t - (r_t - n)\bar{d}.$$

In (14), ω_t represents the wage rate that prevails in each country in period t when capital markets are integrated. The solution of the utility maximization problem of a young individual of period t in the Home country now becomes

$$(15) \quad c_{H,t}^0 = \frac{\omega_t - \tau_{H,t}^0}{1 + \beta_H^{\frac{1}{\sigma}}(1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}},$$

$$(16) \quad c_{H,t+1}^1 = \frac{\beta_H^{\frac{1}{\sigma}}(1 + r_{t+1})^{\frac{1}{\sigma}}(\omega_t - \tau_{H,t}^0)}{1 + \beta_H^{\frac{1}{\sigma}}(1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}},$$

and

$$(17) \quad s_{H,t}(\omega_t - \tau_{H,t}^0, r_{t+1}) = \frac{\omega_t - \tau_{H,t}^0}{1 + \beta_H^{\frac{1}{\sigma}}(1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}}.$$

As for the problems faced by the representative firms in the Home and Foreign countries, they are still the same and given by (8), (9), and (10). For the successive generations abroad, the solution of their lifetime utility maximization problem is still given (4), (5), and (6). However, now their savings, besides capital, might also contain part of the national debts of the Home country.

Now because we assume that initially the world economy is in steady state, the world capital labor ratio in period 0 is equal to $\bar{k}$. The initial wage rate in both countries is then given by $\bar{w} = (1 - \alpha)\bar{k}^\alpha$, and the initial rental rate of capital on the global capital market by $\bar{r} = \alpha\bar{k}^{\alpha-1}$. To avoid dynamic inefficiency, we shall assume that $\bar{r} > n$. This assumption also ensures that the tax paid by the successive young generations in the Home country is positive.

If k_1 is the world capital labor ratio in period 1, then the world rate of interest in period 1 is given by $r_1 = \alpha k_1^{\alpha-1}$. In the Home country, the saving of a young individual of period 0 is given by

$$(18) \quad s_H(\bar{w} - \tau_{H,0}^0, r_1) = \frac{\bar{w} - \tau_{H,0}^0}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}},$$

and in the Foreign country, the saving of a young individual of period 0 is given by

$$(19) \quad s_F(\bar{w}, r_1) = \frac{\bar{w}}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}}.$$

The introduction of national debts into the model changes the saving decisions of the young individuals in both countries in a fundamental manner. Now the saving of a young individual – in the Home country or in the Foreign country – might contain both capital and debts. In period 1, the equilibrium condition for the world capital market is given by¹⁹

¹⁹ Since the national debt of the Home country, D_H is included in the capital market clearing condition of the world, it is not meaningful to make a distinction of national debt between the internal debt and the external debt as shown in Diamond (1965).

$$\begin{aligned}
(20) \quad & N_{H,0}^0 s_H(\bar{\omega} - \tau_{H,0}^0, r_1) + N_{F,0}^0 s_F(\bar{\omega}, r_1) \\
&= N_{H,0}^0 \frac{\bar{\omega} - \tau_{H,0}^0}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} + N_{F,0}^0 \frac{\bar{\omega}}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} \\
&= (N_{H,0}^0 + N_{F,0}^0)(1 + n)k_1 + D_{H,0}.
\end{aligned}$$

The equilibrium condition for the world capital market, as represented by the second equality in (20), can be rewritten as follows

$$\begin{aligned}
(21) \quad k_1 &= \frac{N_{H,0}^0}{(N_{H,0}^0 + N_{F,0}^0)(1 + n)} \frac{\bar{\omega} - \tau_{H,0}^0}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} \\
&+ \frac{N_{F,0}^0}{(N_{H,0}^0 + N_{F,0}^0)(1 + n)} \frac{\bar{\omega}}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} \\
&- \frac{D_{H,0}}{(N_{H,0}^0 + N_{F,0}^0)(1 + n)} \\
&= \frac{1}{1 + n} \left[\eta_H \left(\frac{\bar{\omega} - (r_1 - n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right) + \eta_F \frac{\bar{\omega}}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} \right].
\end{aligned}$$

In the expression on the last line of (21), we have let η_H and η_F denote, respectively the population of the Home country and the population of the Foreign country as fractions of the world population. Equation (21) yields the world capital labor ratio in period 1, given that r_1 is the world rate of interest that is expected to prevail in that period. Now let

$$(22) \quad \zeta(\bar{\omega}, r_1) = \alpha \left(\frac{1}{n+1} \left[\eta_H \left(\frac{\bar{\omega} - (r_1 - n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right) + \eta_F \frac{\bar{\omega}}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} \right] \right)^{\alpha-1}.$$

As defined, $\zeta(\bar{w}, r_1)$ represents the interest rate in period 1 that is generated by the maximizing behavior of the young generations of period 0 in both countries, given that (i) the initial wage rate is $\bar{w}$, and (ii) the world interest rate that is expected to prevail in period 1 is r_1 . To ensure that the tax paid a young individual of period 0 in the Home is non-negative, we shall only consider the values of r_1 that satisfies $r_1 > n$. Furthermore, in order for the tax policy to be sustainable in period 0, we must have $\bar{w} - \tau_{H,0}^0 = (r_1 - n)\bar{d} > 0$; that is, $r_1 < n + (\bar{w}/\bar{d})$.

Note that a rise in $\bar{d}$ shifts the curve

$$(23) \quad \zeta(\bar{w}, \cdot) : r_1 \rightarrow \zeta(\bar{w}, r_1), \quad n < r_1 < n + (\bar{w}/\bar{d}),$$

upward. Furthermore, when $\bar{d} = 0$ the upper constraint on r_1 ceases to be binding, and the map has a fixed point, which is $\bar{r}$, the initial world interest rate. Thus when $\bar{d}$ is not too large, the map will have a fixed point, say $r_1 = \hat{r}_1 > \bar{r}$. However, if $\bar{d}$ is not small, the expression on the last line of (21) will not be positive, which means that the debt management policy cannot be sustained even temporarily: the global capital market in period 1 cannot be in equilibrium.

To continue, suppose that the debt incurred by the Home government in period 0 is not too heavy and that $\hat{r}_1$ is the unique equilibrium world interest rate in period 1. The equilibrium world capital labor ratio in period 1 is then given by $\hat{k}_1 = (\hat{r}_1 / \alpha)^{\frac{1}{\alpha-1}}$, and wage rate that prevails in both countries in that period by $\hat{w}_1 = (1 - \alpha)\hat{k}_1^\alpha$. Because

$\hat{r}_1 > \bar{r}$, we have $\hat{\omega}_1 < \bar{\omega}$. If r_2 is the interest rate that is expected to prevail in the global capital market in period 2, then the saving of a young individual of period 1 in the Home country is given by

$$(24) \quad s_H(\hat{\omega}_1 - \tau_{H,1}^0, r_2) = \frac{\hat{\omega}_1 - \tau_{H,1}^0}{1 + \beta_H^{\frac{1}{\sigma}}(1+r_2)^{\frac{1-\sigma}{\sigma}}},$$

Similarly, in the Foreign country, the saving of a young individual of period 1 is given by

$$(25) \quad s_F(\hat{\omega}_1, r_2) = \frac{\hat{\omega}_1}{1 + \beta_F^{\frac{1}{\sigma}}(1+r_2)^{\frac{1-\sigma}{\sigma}}}.$$

The equilibrium world capital labor ratio in period 2, if an equilibrium exists, is then given by

$$(26) \quad k_2 = \frac{1}{1+n} \left[\eta_H \left(\frac{\hat{\omega}_1 - (r_2 - n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}}(1+r_2)^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right) + \eta_F \frac{\hat{\omega}_1}{1 + \beta_F^{\frac{1}{\sigma}}(1+r_2)^{\frac{1-\sigma}{\sigma}}} \right].$$

Equation (26) yields the world capital labor ratio in period 2, given that r_2 is the world rate of interest that is expected to prevail in that period. Next, let

$$(27) \quad \zeta(\hat{\omega}_1, \cdot) : r_2 \rightarrow \zeta(\hat{\omega}_1, r_2), n < r_2 < n + (\hat{\omega}_1 / \bar{d}),$$

be the map defined by

$$(28) \quad \zeta(\hat{\omega}_1, r_2) = \alpha \left(\frac{1}{1+n} \left[\eta_H \left(\frac{\hat{\omega}_1 - (r_2 - n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}}(1+r_2)^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right) + \eta_F \frac{\hat{\omega}_1}{1 + \beta_F^{\frac{1}{\sigma}}(1+r_2)^{\frac{1-\sigma}{\sigma}}} \right] \right)^{\alpha-1}.$$

As in the case of period 1, if $\bar{d}$ is not small, then $\zeta(\hat{\omega}_1, r_2)$ will not have a fixed point: the debt management policy is sustainable in period 1, but not sustainable in period 2. On

the other hand, if $\bar{d}$ is small enough, then it has a unique fixed point, say $r_2 = \hat{r}_2$.²⁰ We shall let $\hat{k}_2 = (\hat{r}_2 / \alpha)^{1/(\alpha-1)}$ denote the equilibrium world capital labor ratio in period 2 and $\hat{\omega}_2 = (1 - \alpha)\hat{k}_2^\alpha$ denote the two countries' common equilibrium wage rate in period 2. Note that $\hat{r}_2 > \hat{r}_1$ because $\hat{\omega}_1 < \bar{\omega}$.

The procedure just described for periods 1 and 2 can be repeated ad infinitum to obtain an increasing sequence $(\hat{r}_t)_{t=1}^\infty$, with $\bar{r} < \hat{r}_1$, of interest rates that clear the world capital market in each period $t = 1, 2, \dots$, if the debt management policy of the Home government can be sustained indefinitely. In the long run, the equilibrium world interest rate, say $\bar{r} = \lim_{t \rightarrow \infty} \hat{r}_t$, is the value of $\bar{r}$ that is defined implicitly by the following relation:

$$(29) \quad \bar{r} = \alpha \left(\frac{1}{1+n} \left[\eta_H \left(\frac{(1-\alpha) \left[\frac{\bar{r}}{\alpha} \right]^{\frac{\alpha}{\alpha-1}} - (\bar{r}-n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}} (1+\bar{r})^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right) + \eta_F \frac{(1-\alpha) \left[\frac{\bar{r}}{\alpha} \right]^{\frac{\alpha}{\alpha-1}}}{1 + \beta_F^{\frac{1}{\sigma}} (1+\bar{r})^{\frac{1-\sigma}{\sigma}}} \right] \right)^{\alpha-1}.$$

In what follows, we shall let

$$(30) \quad \bar{k} = \left[\frac{\bar{r}}{\alpha} \right]^{\frac{1}{\alpha-1}}$$

and

$$(31) \quad \bar{\omega} = (1 - \alpha)\bar{k}^\alpha$$

²⁰ We ignore equilibrium with a lower value of capital labour ratio in the presence of the government debts as shown in Obstfeld and Rogoff (1996).

denote, respectively, the equilibrium world capital labor ratio and the equilibrium wage rate in the long run that are induced by the debt management policy adopted by the Home government. We summarize the preceding results in the following proposition:

PROPOSITION 1: *Suppose that the Home government makes a gift to its old generation of period 0 by issuing a quantity of claims on itself, and then adopts a debt management policy under which the debt per young individual in each period $t = 0, 1, \dots$ is maintained at a constant level $\bar{d}$. Also, suppose that $\bar{d}$ is not too high and that this policy is sustainable in the long run. Then in the period that follows the period that the gift is made, the equilibrium world interest rate will rise above its initial level of $\bar{r}$, and will continue to rise through time to the long-run level $\bar{r}^*$, as defined implicitly by (29). In the long run, the equilibrium world capital labor ratio, namely $\bar{k}^*$, will be lower than $\bar{k}$, its initial level before the debt is incurred. Furthermore, the higher is the initial debt incurred by the Home government, the higher will be the equilibrium world interest rate in the long run, and a fortiori the lower will be the world capital labor ratio in the long run.*

Proposition 1 describes how the impact of the debt incurred by the Home government is transmitted abroad: it crowds out private investment both at home and abroad and induces a fall in the world capital labor ratio through time to a lower level in the long run. Intuitively, we expect that if the debt is incurred in autarky, then there will be a more pronounced crowding out of private investment in the Home country. Indeed, the result for debt and debt management for autarky can be obtained by setting $\eta_H = 1$ and $\eta_F = 0$

in the open-economy analysis. If we let $\bar{r}_H$ denote the long-run equilibrium interest rate in the Home country if the debt management policy is carried out in autarky, then the following version of (29) holds

$$(32) \quad \bar{r}_H = \alpha \left(\frac{1}{1+n} \frac{(1-\alpha) \left[\frac{\bar{r}_H}{\alpha} \right]^{\frac{\alpha}{\alpha-1}} - (\bar{r}_H - n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \bar{r}_H)^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right)^{\alpha-1}.$$

We claim that $\bar{r} < \bar{r}_H$. To prove this, suppose that the Home country is in autarky and that the current prevailing wage rate is $\bar{\omega}$. Under this scenario, the equilibrium interest rate in the next period in the Home country is the fixed point of the map

$$(33) \quad r \rightarrow \alpha \left(\frac{1}{1+n} \frac{\bar{\omega} - (r-n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}} (1+r)^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right)^{\alpha-1}.$$

Next, note that

$$\begin{aligned} & \alpha \left(\frac{1}{1+n} \frac{\bar{\omega} - (r-n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}} (1+r)^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right)^{\alpha-1} > \\ & \alpha \left(\frac{1}{1+n} \left[\eta_H \left(\frac{\bar{\omega} - (r-n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}} (1+r)^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right) + \eta_F \frac{\bar{\omega}}{1 + \beta_F^{\frac{1}{\sigma}} (1+r)^{\frac{1-\sigma}{\sigma}}} \right] \right)^{\alpha-1}. \end{aligned}$$

Furthermore because $\bar{r}$ is the fixed point of the map

$$(34) \quad r \rightarrow \alpha \left(\frac{1}{1+n} \left[\eta_H \left(\frac{\bar{\omega} - (r-n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}} (1+r)^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right) + \eta_F \frac{\bar{\omega}}{1 + \beta_F^{\frac{1}{\sigma}} (1+r)^{\frac{1-\sigma}{\sigma}}} \right] \right)^{\alpha-1},$$

the fixed point of the map defined by (33) must be greater than $\bar{r}$. Hence if the Home country is in autarky, and if $\bar{\omega}$ is the current prevailing wage rate, then the equilibrium interest rate in the next period will be higher than $\bar{r}$. The argument just presented can be repeated ad infinitum to show that in autarky the interest rate in the Home country will rise monotonically through time to a unique value $\bar{r}_H > \bar{r}$. We state the result just obtained more formally in the following proposition:

PROPOSITION 2: *Suppose that the Home country is in autarky and that government makes a gift to its old generation of period 0 by issuing a quantity of claims on itself, and then adopts a debt management policy under which the debt per young individual in each period $t = 0, 1, \dots$ is maintained at a constant level $\bar{d}$. Also, suppose that $\bar{d}$ is not too high and that this policy is sustainable in the long run. Let $\bar{r}_H$ denote the autarkic interest rate in the long run under this policy of debt management. Then $\bar{r}_H > \bar{r}$; that is, in the long-run the crowding out of private investment is more pronounced in autarky than in open economy.*

4. THE CURRENT ACCOUNT AND NET FOREIGN ASSETS

Let $B_{i,t}$ denote the net foreign assets of country i at the beginning of period t , $i = H, F$, $t = 0, 1, \dots$. Also, let $CA_{i,t}$ denote the current account of country i at the end of period t , $t = 0, 1, \dots$. Note that at the beginning of period 0 – when the Home government makes the gift $D_{H,0}$ to its old generation of period 0 – the Home country's net foreign assets are given by

$$(35) \quad \begin{aligned} B_{H,0} &= -B_{F,0} \\ &= -N_{F,0}^1 [s_F(\bar{\omega}, \bar{r}) - (1+n)\bar{k}] \end{aligned}$$

For a young individual of period 0 in the Foreign country her labor income is still $\bar{\omega}$, but the rate of return to her saving has risen from $\bar{r}$ to $\hat{r}_1$. Her saving is now

$$(36) \quad s_F(\bar{\omega}, \hat{r}_1) = \frac{\bar{\omega}}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}}.$$

The net foreign assets of the Foreign country at the beginning of period 1 is thus given by

$$(37) \quad \begin{aligned} B_{F,1} &= N_{F,0}^0 [s_F(\bar{\omega}, \hat{r}_1) - (1+n)\hat{k}_1] \\ &= (1+n)N_{F,0}^1 [s_F(\bar{\omega}, \hat{r}_1) - (1+n)\hat{k}_1] \\ &> (1+n)N_{F,0}^1 [s_F(\bar{\omega}, \bar{r}) - (1+n)\bar{k}] = (1+n)B_{F,0}. \end{aligned}$$

Hence the net foreign assets of the Foreign country rises one period after the debt is incurred by the Home government, which implies that the Home country runs a current account deficit at the end of period 0: $CA_{H,0} < 0$.

To analyze the current account of the Home country and its net foreign assets for $t = 1, 2, \dots$ requires some more work. To this end consider a young individual of period $t > 0$ in the Foreign country, and suppose that the equilibrium wage rate in period t is ω_t and the equilibrium world rate of interest in period $t + 1$ is r_{t+1} . Her saving is then given by

$$(38) \quad \begin{aligned} s_{F,t} &= \frac{\omega_t}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} \\ &= \frac{\varphi(r_{t+1})}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}}. \end{aligned}$$

Out of this saving, she has to provide capital for the next young generation in her own country and export the rest to the Home country. The capital export of a young individual is then given by

$$(39) \quad \begin{aligned} e_{F,t} &= s_{F,t} - (1 + n) \left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}} \\ &= \frac{\varphi(r_{t+1})}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} - (1 + n) \left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}}. \end{aligned}$$

The following lemma is proved in Appendix A.

LEMMA 1: *If $\bar{d}$ is small, then the map*

$$(40) \quad r_{t+1} \rightarrow \frac{\varphi(r_{t+1})}{1 + \beta_F^{\frac{1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} - (1 + n) \left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}}$$

is strictly decreasing.

Now recall from Proposition 1 that the equilibrium world rate of interest is increasing with time. Then Lemma 1 implies that the resource for the saving is consistently depleted as the interest rate goes up. Applying Lemma 1 to a rising interest rate, we obtain the following proposition:

PROPOSITION 3: *Suppose that under the debt and the debt management policy adopted by the Home government the debt per young individual in the Home country is small and constant through time. Then the capital export by a young individual of period $t, t = 0, 1, \dots$, in the Foreign country declines with t . Hence if there is no population growth, the net foreign assets of the Foreign country, which has shown a large increase in period 0, will decline through time, but remains positive in the long run, or equivalently the current account of the Home country will be positive for all $t > 0$ and tends to 0 in the long run. However, if the population growth rate is positive, the capital required by the successive young generations in the Home country might demand an increasing inflow of capital by foreign residents, with the ensuing result that the Home country's current account exhibits an ambiguous sign during the transition path even if it is negative in the long run.*

When a once-and-for-all transfer increases the government debt of a country by an equal amount, the demographic structure of the world affects the current account of two countries. With an aging population, say $n = 0$, the net foreign debt of the debtor country converges to a lower value as the economy approaches its steady state. Then the debtor

country must show a surplus in her current account on the transition path toward the steady state. With an increasing population, say $n > 0$, the net foreign debt of the debtor country may increase since the rising labor force requires more capital to be imported for adequate capital input. Then the current account of the debtor country during the transition path depends on the parameter values.

5. WELFARE ANALYSIS

Relative to the steady state that prevailed before the Home government makes the gift to its old generation of period 0, the old generation of period 0 in the Home country is clearly better-off: the consumption of this generation now consists of the old-age consumption of the old steady state plus the gift. As for the old generation of period 0 in the Foreign country, which does not receive any gift from its own government, its situation is still the same as in the old steady state, and is thus neither better-off nor worse-off.

For a young individual of period 0 in the Foreign country, she is clearly better-off. Her labor income is still $\bar{w}$, but the rate of return to her saving has risen from $\bar{r}$ to $\hat{r}_1$. Her current consumption is given by

$$(41) \quad c_{F,0}^0 = \frac{\bar{w}}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}},$$

and her saving by

$$(42) \quad s_{F,0}(\bar{\omega}, \hat{r}_1) = \frac{\bar{\omega}}{1 + \beta_F^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}}.$$

Relative to the steady state level before the Home government incurs the debt, she now consumes less, but saves more. Her lifetime utility is now

$$(43) \quad \begin{aligned} v_{F,0}(\bar{\omega}, \hat{r}_1) &= \frac{\bar{\omega}^{1-\sigma}}{1-\sigma} \left(1 + \beta_F^{\frac{1}{\sigma}} \hat{r}_1^{\frac{1-\sigma}{\sigma}} \right)^{\sigma} \\ &> \frac{\bar{\omega}^{1-\sigma}}{1-\sigma} \left(1 + \beta_F^{\frac{1}{\sigma}} \bar{r}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}. \end{aligned}$$

As for a young individual of period 0 in the Home country, we claim that her situation deteriorates with the debt. Immediately after the gift has been made, a young individual of period 0 in the Home country sees her disposable income reduced from the level $\bar{\omega}$ to $\bar{\omega} - \tau_{H,0} = \bar{\omega} - (\hat{r}_1 - n)\bar{d}$. Her current consumption is given by

$$(44) \quad c_{H,0}^0 = \frac{\bar{\omega} - (\hat{r}_1 - n)\bar{d}}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}},$$

and her old-age consumption by

$$(45) \quad c_{H,1}^1 = \frac{\beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}} [\bar{\omega} - (\hat{r}_1 - n)\bar{d}]}{1 + \beta_H^{\frac{1}{\sigma}} (1 + \hat{r}_1)^{\frac{1-\sigma}{\sigma}}}.$$

Because $\hat{r}_1 > \bar{r} > n$, her current consumption is lower than the corresponding steady state level she enjoys if the Home government had not incurred the debt. As for her old-age consumption, the rise in the future world rate of interest above the steady state level that prevailed before period 0 has two opposite effects. First of all, the rise in the world

interest rate reduces her disposable income, which, everything else equal, reduces her saving. However, the rise in the future world rate of interest, again everything else equal, induces her to save more. To obtain a more definite answer to the combined impact of these two opposing effects on her old-age consumption, note that in period 1 the combined consumption of the two old generations – both at home and abroad – is given by $(N_{H,1}^0 + N_{F,1}^0)[\hat{k}_1 + \alpha \hat{k}_1^\alpha]$, and the total consumption of the old generation in the Foreign country by $N_{F,1}^1(1 + \hat{r}_1)s_{F,0}(\bar{\omega}, \hat{r}_1)$. The total consumption of the old generation of period 1 in the Home country is then given by

$$(46) \quad \begin{aligned} & (N_{H,1}^0 + N_{F,1}^0)[\hat{k}_1 + \alpha \hat{k}_1^\alpha] - N_{F,1}^1(1 + \hat{r}_1)s_{F,0}(\bar{\omega}, \hat{r}_1) \\ & < (N_{H,1}^0 + N_{F,1}^0)[\bar{k} + \alpha \bar{k}^\alpha] - N_{F,1}^1(1 + \bar{r})s_{F,0}(\bar{\omega}, \bar{r}). \end{aligned}$$

The right side of inequality (46) represents the total consumption of the old generation of period 1 in the Home country if there is no public debt in the Home country. Hence with the debt incurred by the Home government, the consumption – current as well as future – of a young individual of period 0 in the Home country is lower than its corresponding level in the old steady state. The public debt thus makes the young generation of period 0 in the Home country worse-off.

For the young generations that come after period 0, the welfare analysis is more complicated. Consider a young generation of period $t, t = 0, 1, \dots$, in country $i, i = H, F$. Suppose that ω_t is the current prevailing wage rate in both countries and r_{t+1} is the world rate of interest prevailing in the next period. Then the lifetime utility of a young individual of period t in the Home country is given by

$$(47) \quad v_{H,t}(\omega_t - (r_{t+1} - n)\bar{d}, r_{t+1}) = \frac{[\omega_t - (r_{t+1} - n)\bar{d}]^{1-\sigma}}{1-\sigma} \left(1 + \beta_H^\sigma r_{t+1}^{\frac{1-\sigma}{\sigma}} \right)^\sigma.$$

Now given $\bar{d}$, we can use the equilibrium condition on the global capital market in period t to assert that there exists a functional relationship that links r_{t+1} to ω_t , and vice versa. For our own purpose, we shall express this functional relationship as $\omega_t = \varphi(r_{t+1})$, then rewrite (47) as follows:

$$(48) \quad v_{H,t}(\varphi(r_{t+1}) - (r_{t+1} - n)\bar{d}, r_{t+1}) = \frac{[\varphi(r_{t+1}) - (r_{t+1} - n)\bar{d}]^{1-\sigma}}{1-\sigma} \left(1 + \beta_H^\sigma r_{t+1}^{\frac{1-\sigma}{\sigma}} \right)^\sigma.$$

For a young individual of period t in the Foreign country, the following version of (48) expresses her lifetime utility:

$$(49) \quad \begin{aligned} v_{F,t}(\omega_t, r_{t+1}) &= v_{F,t}(\varphi(r_{t+1}), r_{t+1}) \\ &= \frac{[\varphi(r_{t+1})]^{1-\sigma}}{1-\sigma} \left(1 + \beta_F^\sigma r_{t+1}^{\frac{1-\sigma}{\sigma}} \right)^\sigma. \end{aligned}$$

The following lemma is proved in Appendix B.

LEMMA 2: *If the debt per young individual $\bar{d}$ is small, then both (48) and (49) are strictly decreasing in r_{t+1} .*

The following proposition describes the impact of the debt on the current and future young generations of the Home country.

PROPOSITION 4: *Suppose that the debt per young individual in each period $t = 0, 1, \dots$, in the Home country is small. Then the following chain of inequalities describes the welfare of the successive young generations of the Home country:*

$$(50) \quad v_{H,-1}(\bar{\omega}, \bar{r}) > v_{H,0}(\bar{\omega}, \hat{r}_1) > v_{H,1}(\hat{\omega}_1, \hat{r}_2) > \dots > v_{H,t}(\hat{\omega}_t, \hat{r}_{t+1}) > \dots$$

In (50), we have let $v_{H,-1}(\bar{\omega}, \bar{r})$ denote the lifetime utility of a young individual in the steady state that prevailed before the Home government incurred the debt. That is, the debt and the debt management policy adopted by the Home government reduce the welfare of its young generation of period 0, and the welfare of all the generations that follows. Furthermore, the standard of living in the Home country declines through time. In particular, the welfare of the Home country is lower under the steady state induced by the debt and the debt management policy than under the old steady state that occurs without debt.

PROOF: Recall from Proposition 1 that the sequence $(\hat{r}_t)_{t=1}^{\infty}$ of equilibrium world interest rates begins with $\hat{r}_1 > \bar{r}$, then rises steadily through time to the new equilibrium level $\bar{r}$ in the long run. The chain of inequalities now follows from Lemma 1 and from the fact that the welfare of the young generation of period 0 in the Home country is lower than its corresponding level in the old steady state. ■

For the Foreign country, the following version of (50) holds:

$$(51) \quad v_{F,0}(\bar{\omega}, \hat{r}_1) > v_{F,1}(\hat{\omega}_1, \hat{r}_2) > \dots > v_{F,t}(\hat{\omega}_t, \hat{r}_{t+1}) > \dots$$

That is, the standard living in the Foreign country also declines through time. However, because the welfare of the young generation of period 0 is higher than the level in the old steady state, it is not clear from (51) that the welfare of the Foreign country is higher or lower in the new steady state than in the old steady state. The following lemma, proved in Appendix B, gives a condition that guarantees that the debt and debt management policy adopted by the Home country also lowers the welfare of the Foreign country in the new steady state.

LEMMA 3: *Consider a young individual of the Foreign country who lives in a steady state under which the world interest is r . Then his lifetime utility, as a function of r , is given by*

$$(52) \quad v_F(r) = \frac{\left((1-\alpha)[r/\alpha]^{\frac{\alpha}{\alpha-1}} \right)^{1-\sigma}}{1-\sigma} \left(1 + \beta_F^{\frac{1}{\sigma}} r^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}.$$

If $\alpha \geq 1/2$, then (52) is strictly decreasing in r .

Using (51) and Lemma 2, we obtain the following Proposition:

PROPOSITION 5: *Suppose that the debt per young individual in each period $t = 0, 1, \dots$, in the Home country is small. Then the welfare of the young generation of period 0 in the Foreign country rises above its old steady-state level. After that the standard of living in the Foreign country declines steadily through time. Furthermore if the income share of the factor capital is at least $1/2$, then the welfare of the Foreign country is lower in the steady state with debt than in the steady state without debt.*

PROPOSTION 4 and PROPOSITION 5 describe about how the economic effects of a transfer can be transmitted across generations and countries. The welfare of all current and future generations of a country can be invariably reduced by a once-and-for-all transfer in the country which creates the national debts. Moreover, the welfare of all future generations of the other country that is immune to its own national debt, can be lowered by the fiscal policy of her neighbor country. The only groups who could be better off from the transfer are the direct beneficiary of the transfer and the current young generation of the neighbor country. All others are worse off.

6. NUMERICAL EXAMPLES

6.1. Sustainability calculation

In the example the values assumed for the parameters are as follows:

$$\alpha = \frac{1}{2}; \sigma = \frac{1}{2}; \beta_H = \frac{1}{2}; \beta_F = \frac{4}{5}; \eta_H = \frac{5}{9}; \eta_F = \frac{4}{9}; n = \frac{35}{100};$$

The debt per worker in the Home country is held at the constant level $\bar{d}$, which is assumed to be 7.5 percent of the wage rate of period 0; $\bar{d} = 0.075$ $\omega_0 = 0.007783$.

Based on the parameter values and the ratio of the debt per worker, we can trace the transition paths of the wage rate, the interest rate and the capital labor ratio toward the steady state in the open economy.

wages = Table[{t, $\omega[t]$ }, {t, -1, 15}]

```
{{-1, 0.103775}, {0, 0.103775}, {1, 0.0966878}, {2, 0.0930676}, {3, 0.0911352},
{4, 0.0900781}, {5, 0.0894919}, {6, 0.0891643}, {7, 0.0889804}, {8, 0.0888769}, {9, 0.0888186},
{10, 0.0887857}, {11, 0.0887672}, {12, 0.0887567}, {13, 0.0887508}, {14, 0.0887475}, {15, 0.0887456}}
```

Interestrates = Table[{t, $r[t]$ }, {t, -1, 15}]

```
{-1, 2.40906}, {0, 2.40906}, {1, 2.58564}, {2, 2.68622}, {3, 2.74318}, {4, 2.77537}, {5, 2.79355}, {6, 2.80381}, {7, 2.80961},
{8, 2.81288}, {9, 2.81473}, {10, 2.81577}, {11, 2.81636}, {12, 2.81669}, {13, 2.81688}, {14, 2.81698}, {15, 2.81704}
```

capitallaborratios = Table[{t, kx[ω [t]]}, {t, -1, 15}]

```
{{-1, 0.0430768}, {0, 0.0430768}, {1, 0.0373941}, {2, 0.0346463}, {3, 0.0332225},
{4, 0.0324563}, {5, 0.0320352}, {6, 0.0318011}, {7, 0.03167}, {8, 0.0315964}, {9, 0.031555},
{10, 0.0315316}, {11, 0.0315185}, {12, 0.031511}, {13, 0.0315068}, {14, 0.0315045}, {15, 0.0315031}}
```

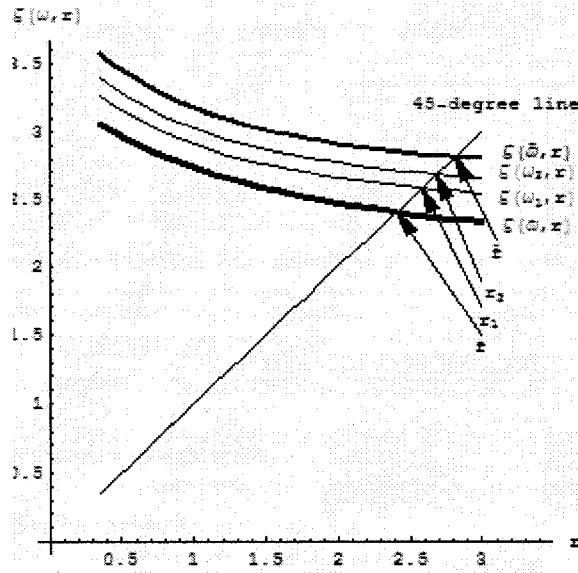


Figure 1.- The dynamics of the equilibrium world interest rate with sustainable debt

On the contrary, when $\bar{d} = 0.15$ $\omega_0 = 0.015662$, the debt and debt management policy cannot be sustained beyond period 2. In period 1, the equilibrium world interest rate is $r_1 = 3.10649$, which is above the initial long-run equilibrium value of $\bar{r} = 2.40906$. The equilibrium wage rate in period 1 is $\omega_1 = 0.08048$. In period 2, the equilibrium world interest rate is $r_2 = 4.33587$ and the equilibrium wage rate is $\omega_2 = 0.05766$.

However, in period 3 there is no level of world interest rate that clears the world capital market: the map $r_3 \rightarrow \zeta(\omega_2, r_3)$ has no fixed point, as can be seen from the following figure:

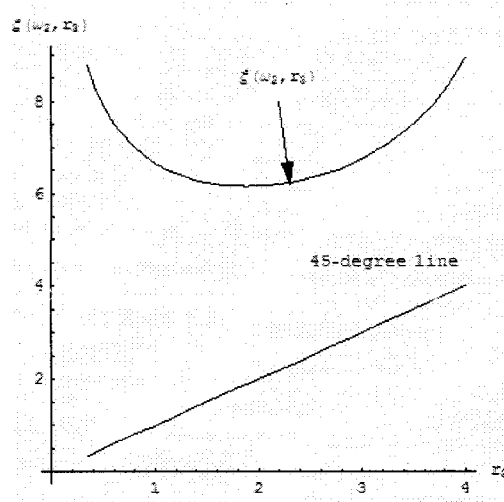


Figure 2.- Unsustainable debt and debt management policy

6.2 Net foreign assets and current account calculation

If the rate of population growth is positive, i.e., $n = 35\%$, the net foreign assets of the Home country are

$$B_{H,t+1} = \text{Table} \left[(1+n)^t \left(\frac{\omega[t]}{1 + (\beta_H)^{-\frac{1}{\sigma}} (1+r[t+1])^{-\frac{1-\sigma}{\sigma}}} - (1+n) \left(\frac{r[t+1]}{\alpha} \right)^{\frac{1}{\alpha-1}} \right), \{t, -1, 10\} \right]$$

{{-1, 0}, {0, -0.0218}, {1, -0.02853}, {2, -0.03792}, {3, -0.05079}, {4, -0.06827},
{5, -0.09194}, {6, -0.12395}, {7, -0.16721}, {8, -0.22564}, {9, -0.30454}, {10, -0.41107}}

and the current account of the Home country is

$$CA_{H,t} = \text{Table} \left[\left((1+n)^t \left(\frac{\omega[t]}{1 + (\beta_H)^{-\frac{1}{\sigma}} (1+r[t+1])^{-\frac{1-\sigma}{\sigma}}} - (1+n) \left(\frac{r[t+1]}{\alpha} \right)^{\frac{1}{\alpha-1}} \right) - (1+n)^{t-1} \left(\frac{\omega[t-1]}{1 + (\beta_H)^{-\frac{1}{\sigma}} (1+r[t])^{-\frac{1-\sigma}{\sigma}}} - (1+n) \left(\frac{r[t]}{\alpha} \right)^{\frac{1}{\alpha-1}} \right) \right), \{t, -1, 10\} \right]$$

{{-1, 0}, {0, -0.0218}, {1, -0.00673}, {2, -0.0094},
{3, -0.01286}, {4, -0.01748}, {5, -0.02367}, {6, -0.03201},
{7, -0.04326}, {8, -0.05843}, {9, -0.0789}, {10, -0.10653}}

The current account of the Home country with a positive population growth, beginning with the generation of period $t = -1$ (the old steady state) can be seen from the following figure:

```
ListPlot[CA[H], PlotStyle→PointSize[0.03], PlotRange→{{-1, 10}, {-0.15, 0.1}}, AxesLabel→{"t", "CAH,t"}]
```

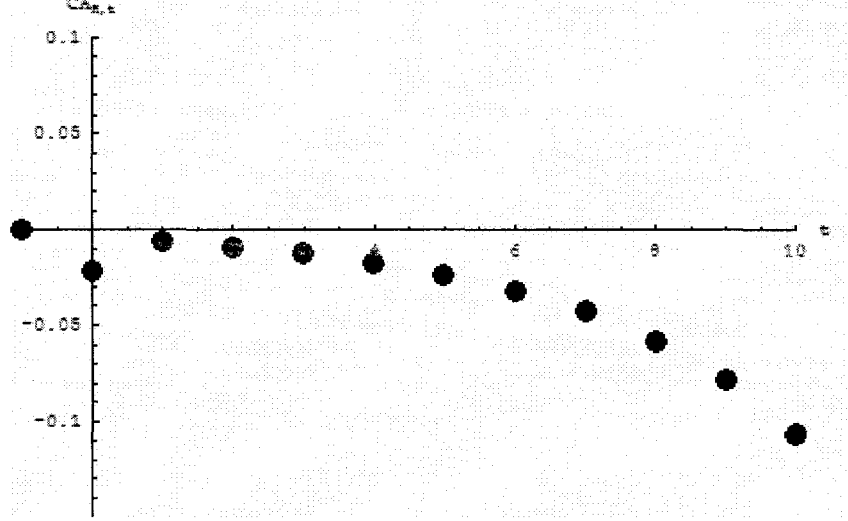


Figure 3.- The current account of the Home country with a positive population growth

If the rate of population growth is zero, the net foreign assets of the Home country are

$$B_{H,t+1} = \text{Table} \left[\left(\frac{\omega[t]}{1 + (\beta_H)^{-\frac{1}{\sigma}} (1+r[t+1])^{-\frac{1-\sigma}{\sigma}}} - \left(\frac{r[t+1]}{\alpha} \right)^{\frac{1}{\alpha-1}} \right), \{t, -1, 10\} \right]$$

```
{{-1, 0}, {0, -0.03488}, {1, -0.03326}, {2, -0.03244},
 {3, -0.032}, {4, -0.03176}, {5, -0.03163}, {6, -0.03156},
 {7, -0.03152}, {8, -0.0315}, {9, -0.03148}, {10, -0.03148}}
```

and the current account of the Home country is

$$CA_{H,t} = \text{Table} \left[\left(\frac{\omega[t]}{1 + (\beta_H)^{-\frac{1}{\sigma}} (1+r[t+1])^{-\frac{1-\sigma}{\sigma}}} - \left(\frac{r[t+1]}{\alpha} \right)^{\frac{1}{\alpha-1}} \right) - \left(\frac{\omega[t-1]}{1 + (\beta_H)^{-\frac{1}{\sigma}} (1+r[t])^{-\frac{1-\sigma}{\sigma}}} - \left(\frac{r[t]}{\alpha} \right)^{\frac{1}{\alpha-1}} \right), \{t, -1, 10\} \right]$$

```
{{-1, 0}, {0, -0.03488}, {1, 0.00163}, {2, 0.00082},
 {3, 0.00044}, {4, 0.00024}, {5, 0.00013}, {6, 0.00007},
 {7, 0.00004}, {8, 0.00002}, {9, 0.00001}, {10, 0, 0}}
```

The current account of the Home country with no population growth, beginning with the generation of period $t = -1$ (the old steady state) can be seen from the following figure:

```
ListPlot[CA[H0], PlotStyle→PointSize[0.03], PlotRange→{{-1, 10}, {-0.04, 0.002}}, AxesLabel→{"t", "CAH,t"}]
```

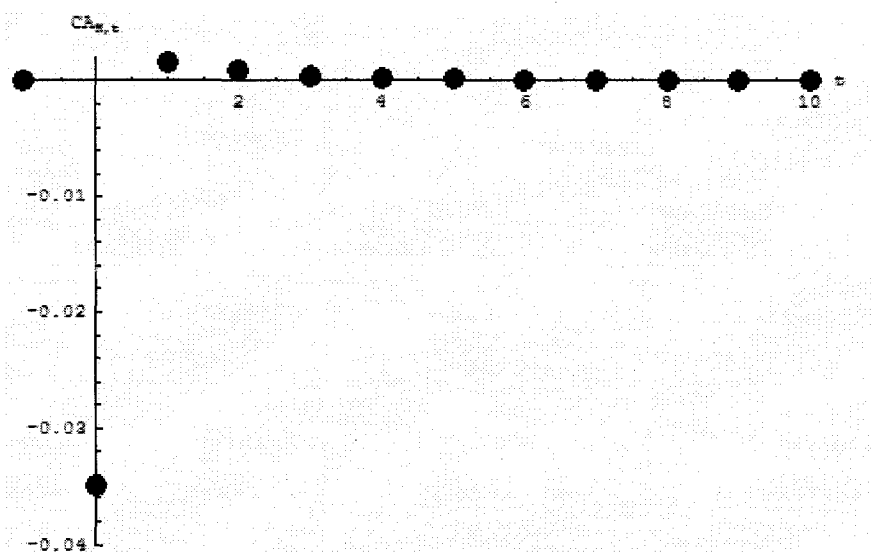


Figure 4.- The current account of the Home country with no population growth

6.3 Welfare calculation

The indirect lifetime utility functions of the successive generations are

$$v[H][t_-] := \frac{(\omega[t] - (r[t+1] - n) d)^{1-\sigma} \left(1 + r[t+1]^{\frac{1-\sigma}{\sigma}} \beta_H^{\frac{1}{\sigma}}\right)^{\sigma}}{1 - \sigma},$$

$$v[F][t_-] := \frac{\omega[t]^{1-\sigma} \left(1 + r[t+1]^{\frac{1-\sigma}{\sigma}} \beta_F^{\frac{1}{\sigma}}\right)^{\sigma}}{1 - \sigma}$$

The lifetime utility of a young individual in the steady state that prevails before the gift is made is

$$\begin{aligned} v[H][-1] &= \frac{(\omega[-1])^{1-\sigma} \left(1 + r[0]^{\frac{1-\sigma}{\sigma}} \beta_H^{\frac{1}{\sigma}}\right)^{\sigma}}{1 - \sigma} \\ &= 0.815546 \end{aligned}$$

The lifetime utility of the successive generations in the Home country is

```
v[H] = Table[{t, v[H][t]}, {t, -1, 9}]
```

```
{{-1, 0.815536}, {0, 0.754209}, {1, 0.7245}, {2, 0.708499}, {3, 0.699698},  
{4, 0.694801}, {5, 0.692059}, {6, 0.690518}, {7, 0.68965}, {8, 0.689161}, {9, 0.688886}}
```

The lifetime utility of the successive young generations in the Home country, beginning with the generation of period $t = -1$ (the old steady state) can be seen from the following figure:

```
ListPlot[tv[H], PlotStyle -> PointSize[0.03], AxesLabel -> {"t", "vH,t( $\omega_t, x_{t+1}$ )"}]
```

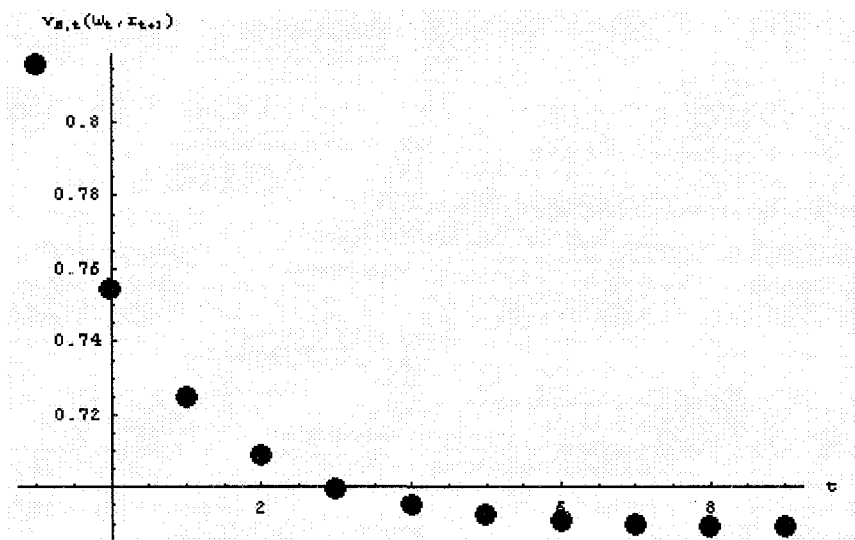


Figure 5.- The lifetime utility of the successive young generations in the Home country

The lifetime utility of the successive generations in the Foreign country is

```
v[F] = Table[{t, v[F][t]}, {t, -1, 9}]
```

```
{{-1, 1.02718}, {0, 1.04977}, {1, 1.0255},  
{2, 1.01284}, {3, 1.00601}, {4, 1.00225}, {5, 1.00016},  
{6, 0.99899}, {7, 0.99833}, {8, 0.99796}, {9, 0.99775}}
```

The lifetime utility of the successive young generations in the Foreign country, beginning with the generation of period $t = -1$ (the old steady state) can be seen from the following figure:

```
ListPlot[tv[F], PlotStyle -> PointSize[0.03], AxesLabel -> {"t", "vF,t( $\omega_t, x_{t+1}$ )"}]
```

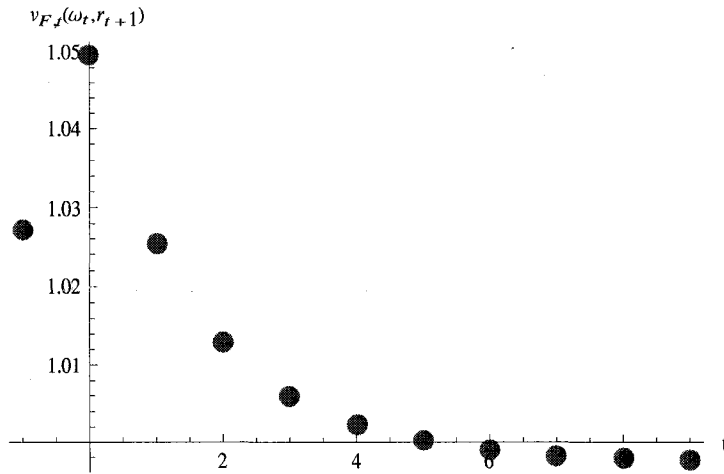


Figure 6.- The lifetime utility of the successive young generations in the Foreign country

7. CONCLUDING REMARKS

In this chapter, we have explored how the government debt is globally transmitted in the two-country environment. By adopting a CIES preference into the model, we could accommodate a positive effect of interest rate on saving and also could trace the transition paths of the economic variables more precisely. A dynamic analysis focusing on the transition paths has led to the major finding as follows.

First, the government debt of a country is transmitted over countries as well as over generations. If the debt is set at a high level, the debt management policy cannot be sustained even temporarily. Therefore, the analysis is no longer feasible for the case. If the government debt of a country is set at a low level, then it crowds out private investment both in the debtor country and the creditor country and induces a fall in the world capital labor ratio through time to a lower level in the long run. The crowding out

is more pronounced in autarky than in open economy. Since the influence of the government debt on domestic economy is less severe, countries may rely on the government debt too much for their deficit financing in the open economy.

Second, the demographic structure affects the current account. With an aging population, the current account of the debtor country shows a surplus during the transition path and converges to zero in the steady state. With an increasing population, the current account of the debtor country shows an ambiguous sign during the transition path and reveals a deficit in the steady state.

Third, the government debt caused by a transfer of a country lowers the welfare of future generations in both countries. Although the welfare of the young generation in the creditor country rises above its old steady-state level initially, after that, the welfare of the successive generations in the creditor country declines steadily through time. In particular, the welfare of the debtor country continuously declines and is lower in the steady state induced by the debt and the debt management policy than under the old steady state that occurs without debt.

APPENDIX A

THE PROOF OF LEMMA 1

The capital export of a young individual of period t in the Foreign country is

(A.1)

$$\begin{aligned}
 e_{F,t} &= s_{F,t} - (1+n) \left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}} \\
 &= -(1+n) \left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}} + \left(\left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}} + \frac{(1-\eta_F)\bar{d}}{1+n} + \frac{(1-\eta_F)(r_{t+1}-n)\bar{d}}{(1+n) \left(1 + \beta_H^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}} \right)} \right) \div \\
 &\quad \left(\left(1 + \beta_F^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}} \right) \left(\frac{1-\eta_F}{(1+n) \left(1 + \beta_H^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}} \right)} + \frac{\eta_F}{(1+n) \left(1 + \beta_F^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1-\sigma}{\sigma}} \right)} \right) \right)
 \end{aligned}$$

Differentiating the right side of the second equality in (A.1), then setting $\bar{d} = 0$ in the resulting derivative, we obtain

$$\begin{aligned}
 \frac{d}{dr_{t+1}} [e_{F,t} |_{\bar{d}=0}] &= \\
 &= - \left((1+n) \left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}} \left(\beta_F^{\frac{1}{\sigma}} - \beta_H^{\frac{1}{\sigma}} \right) (-1 + \eta_F) \left(\beta_H^{\frac{1}{\sigma}} \left((1+r_{t+1})^{\frac{1}{\sigma}} (\sigma + r_{t+1}(1-\alpha) + \alpha \sigma r_{t+1}) \beta_F^{\frac{1}{\sigma}} + \sigma (1+r_{t+1})^2 (1-\eta_F) \right) \right. \right. \\
 &\quad \left. \left. + \sigma (1+r_{t+1})^2 \beta_F^{\frac{1}{\sigma}} \eta_F \right) \right) / \left((-1 + \alpha) \sigma r_{t+1} \left(\beta_H^{\frac{1}{\sigma}} \left((1+r_{t+1})^{\frac{1}{\sigma}} \beta_F^{\frac{1}{\sigma}} + (1+r_{t+1}) (1-\eta_F) \right) + (1+r_{t+1}) \beta_F^{\frac{1}{\sigma}} \eta_F \right)^2 \right) < 0.
 \end{aligned}$$

(A.2)

Hence for small $\bar{d}$ we still have $\frac{d}{dr_{t+1}}e_{F,t} < 0$, which implies that a rise in the equilibrium world interest rate in the next period reduces the capital export of a young individual of period t in the Foreign country. ■

APPENDIX B

THE PROOF OF LEMMA 2

In period t , the saving of a young individual in the Home country is given by

$$(B.1) \quad s_{H,t} = \frac{\omega_t - (r_{t+1} - n)\bar{d}}{1 + \beta_H^{\frac{-1}{\sigma}}(1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}},$$

while the saving of a young individual in the Foreign country is given by

$$(B.2) \quad s_{F,t} = \frac{\omega_t}{1 + \beta_F^{\frac{-1}{\sigma}}(1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}}.$$

The equilibrium condition on the global capital market in period $t+1$ is given by

$$(B.3) \quad k_{t+1} = \frac{N_{H,t}^0 \left(\frac{\omega_t - (r_{t+1} - n)\bar{d}}{1 + \beta_H^{\frac{-1}{\sigma}}(1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right) + \frac{N_{F,t}^0 \omega_t}{1 + \beta_F^{\frac{-1}{\sigma}}(1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}}}{(1+n)(N_{H,t}^0 + N_{F,t}^0)}.$$

Substituting $\left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}}$ for k_{t+1} in (B.3), we obtain

$$(B.4) \quad \left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}} = \frac{N_{H,t}^0 \left(\frac{\omega_t - (r_{t+1} - n)\bar{d}}{1 + \beta_H^{\frac{-1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}} - \bar{d} \right) + \frac{N_{F,t}^0 \omega_t}{1 + \beta_F^{\frac{-1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}}}}{(1+n)(N_{H,t}^0 + N_{F,t}^0)}.$$

Solving (B.4) for ω_t , we obtain

$$(B.5) \quad \omega_t = \varphi(r_{t+1}) = \frac{\frac{\eta_H (r_{t+1} - n)\bar{d}}{(1+n) \left(1 + \beta_H^{\frac{-1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}} \right)} + \left(\frac{r_{t+1}}{\alpha} \right)^{\frac{1}{\alpha-1}} + \frac{\eta_H \bar{d}}{1+n}}{\frac{\eta_H}{(1+n) \left(1 + \beta_H^{\frac{-1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}} \right)} + \frac{\eta_F}{(1+n) \left(1 + \beta_F^{\frac{-1}{\sigma}} (1 + r_{t+1})^{\frac{1-\sigma}{\sigma}} \right)}}.$$

The lifetime utility of a young individual of period t in the Home country is given by

$$(B.6) \quad v_{H,t}(\varphi(r_{t+1}) - (r_{t+1} - n)\bar{d}, r_{t+1}) = \frac{[\varphi(r_{t+1}) - (r_{t+1} - n)\bar{d}]^{1-\sigma}}{1-\sigma} \left(1 + \beta_H^{\frac{1}{\sigma}} r_{t+1}^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}.$$

Taking the logarithm of (B.6), then differentiating the result with respect to r_{t+1} , we obtain

(B.7)

$$\begin{aligned}
& \frac{d}{dr_{1+t}} \text{Log} \left[\frac{\left(\frac{1-\sigma}{1+r_{1+t}} \frac{1}{\beta_H^\sigma} \right)^\sigma (-\bar{d}(-n+r_{1+t})+\varphi(r_{1+t}))^{1-\sigma}}{1-\sigma} \right] = \\
& \frac{(1-\sigma)r_{1+t}^{-1+\frac{1-\sigma}{\sigma}} \frac{1}{\beta_H^\sigma}}{1+r_{1+t} \frac{1-\sigma}{\sigma} \frac{1}{\beta_H^\sigma}} + \left((1-\sigma) \left(-\bar{d} - \left(\alpha^{-\frac{1}{-1+\alpha}} r_{1+t}^{-\frac{1}{-1+\alpha}} - \frac{\bar{d}\eta_H}{1+n} + \frac{\bar{d}(-n+r_{1+t})\eta_H}{(1+n) \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_H^{-1/\sigma} \right)} \right) \right. \right. \\
& \left. \left. \left(\frac{(1-\sigma)(1+r_{1+t})^{-1-\frac{1-\sigma}{\sigma}} \beta_H^{-1/\sigma} \eta_H}{(1+n) \sigma \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_H^{-1/\sigma} \right)^2} + \frac{(1-\sigma)(1+r_{1+t})^{-1-\frac{1-\sigma}{\sigma}} \beta_F^{-1/\sigma} \eta_F}{(1+n) \sigma \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_F^{-1/\sigma} \right)^2} \right) \right) / \\
& \left(\frac{\eta_H}{(1+n) \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_H^{-1/\sigma} \right)} + \frac{\eta_F}{(1+n) \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_F^{-1/\sigma} \right)} \right)^2 + \\
& \left. \frac{\alpha^{-\frac{1}{-1+\alpha}} r_{1+t}^{-1+\frac{1}{-1+\alpha}}}{-1+\alpha} + \frac{(1-\sigma) \bar{d} (1+r_{1+t})^{-1-\frac{1-\sigma}{\sigma}} (-n+r_{1+t}) \beta_H^{-1/\sigma} \eta_H}{(1+n) \sigma \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_H^{-1/\sigma} \right)^2} + \frac{\bar{d} \eta_H}{(1+n) \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_H^{-1/\sigma} \right)} \right) / \\
& \frac{\eta_H}{(1+n) \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_H^{-1/\sigma} \right)} + \frac{\eta_F}{(1+n) \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_F^{-1/\sigma} \right)} \left. \right) / \\
& \left(-\bar{d}(-n+r_{1+t}) + \frac{\alpha^{-\frac{1}{-1+\alpha}} r_{1+t}^{-\frac{1}{-1+\alpha}} - \frac{\bar{d}\eta_H}{1+n} + \frac{\bar{d}(-n+r_{1+t})\eta_H}{(1+n) \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_H^{-1/\sigma} \right)}}{\frac{\eta_H}{(1+n) \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_H^{-1/\sigma} \right)} + \frac{\eta_F}{(1+n) \left(1+(1+r_{1+t})^{-\frac{1-\sigma}{\sigma}} \beta_F^{-1/\sigma} \right)}} \right)
\end{aligned}$$

For $\bar{d} = 0$, the right side of (B.7) is reduced to

(B.8)

$$\frac{(1-\sigma)\left(r_{t+1} + \alpha\beta_H^{\frac{1}{\sigma}}r_{t+1}^{\frac{1}{\sigma}}\right)}{(1-\alpha)r_{t+1}\left(r_{t+1} + \beta_H^{\frac{1}{\sigma}}r_{t+1}^{\frac{1}{\sigma}}\right)} \frac{(1-\sigma)\left(\frac{\eta_H(1-\sigma)\beta_H^{\frac{1}{\sigma}}(1+r_{t+1})^{-1\frac{1-\sigma}{\sigma}}}{\sigma\left(1+\beta_H^{\frac{1}{\sigma}}(1+r_{t+1})^{\frac{1-\sigma}{\sigma}}\right)^2} + \frac{\eta_F(1-\sigma)\beta_F^{\frac{1}{\sigma}}(1+r_{t+1})^{-1\frac{1-\sigma}{\sigma}}}{\sigma\left(1+\beta_F^{\frac{1}{\sigma}}(1+r_{t+1})^{\frac{1-\sigma}{\sigma}}\right)^2}\right)}{\frac{\eta_H}{1+\beta_H^{\frac{1}{\sigma}}(1+r_{t+1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_F}{1+\beta_F^{\frac{1}{\sigma}}(1+r_{t+1})^{\frac{1-\sigma}{\sigma}}}}$$

which is negative. Hence for small $\bar{d}$ the derivative (B.7) is negative, which means that the lifetime utility of a young individual of period t in the Home country is a decreasing function of the equilibrium world rate of interest in the next period. This result can also be established for a young individual of period t in the Foreign country by the same proof technique. ■

APPENDIX C

THE PROOF OF LEMMA 3

Taking the logarithm of (52), then differentiating the result with respect to r , we obtain

$$(C.1) \quad \frac{d}{dr} \text{Log} \left(\frac{\left((1-\alpha) \left(\frac{r}{\alpha} \right)^{\frac{1}{\alpha-1}} \right)^{\alpha} \left(1 + \beta_F^{\frac{1}{\sigma}} r^{\frac{1-\sigma}{\sigma}} \right)^{\sigma}}{1-\sigma} \right) \\ = - \frac{(1-\sigma) \left(\alpha r + (2\alpha-1) \beta_F^{\frac{1}{\sigma}} r^{\frac{1}{\sigma}} \right)}{r(1-\alpha) \left(r + \beta_F^{\frac{1}{\sigma}} r^{\frac{1}{\sigma}} \right)}.$$

Observe that the right side of (C.1) shifts downward when α rises. Furthermore, for $\alpha = 1/2$, (C.1) is reduced to

$$(C.2) \quad - \frac{1-\sigma}{r + \beta_F^{\frac{1}{\sigma}} r^{\frac{1}{\sigma}}} < 0.$$

Hence in steady state the lifetime utility of a young individual in the Foreign country is a decreasing function of the equilibrium world rate of interest. ■

CHAPTER 3

THE DYNAMIC EFFECTS OF TAX REFORM IN THE OPEN ECONOMY

1. INTRODUCTION

TAXATION HAS AN IMMEDIATE IMPACT on the capital stock of an economy by affecting the resource allocation between generations. Given the importance of tax policy, there has been a long-standing argument on which tax instruments are better for capital formation in a country. To address this issue, two streams of literature have been developed during the past two decades. The first stream focuses on the comparison between the capital income tax and the wage tax in an infinite horizon model; see, for example, Judd (1985) and Chamley (1986). This stream of the literature leads to the prediction that the optimal tax rate on capital income is zero; only labor income should be taxed in the long run. The second stream is built on an overlapping-generations (OLG) model; see, for example, Atkinson and Sandmo (1980) and Summers (1981). Since the second stream models explicitly the behavior of agents in successive generations, consumption is naturally included as an additional tax base for comparison. Before the incidence of “tax reform” in favor of the consumption tax in the 1990s²¹, the potential benefits of switching from “income taxes” to “consumption taxes” have generated a considerable amount of debate on the efficiency of each tax instrument. For example, Summers (1981) reported that switching to a consumption tax would raise consumer

²¹ For example, the GST is introduced in Canada in 1991, and the VAT is introduced in Japan in 1989.

lifetime income by 6 to 16 percent, and boost the capital-output ratio 40 to 60 percent. In addition, for reasonable parameter values, the annual welfare gain is estimated at 10 percent of GNP. Auerbach and Kotlikoff (1987) incorporated endogenous labor supply and multi-period lifetime, and reported smaller, but still substantial gains for the consumption tax.²²

One of the main reasons we use the OLG model is that the observed lifetime consumption profile is not flat but has a life-cycle pattern. Since agents face a period of retirement at the end of their life, they have a strong motive for saving for their old age. The OLG framework is suitable for constructing a dynamic general equilibrium model that involves this kind of behavior in a tractable way. This is a critical feature underlying the OLG model when we analyze the effects of changes in tax policy. Despite the fact that the optimal tax is derived in an inter-temporal setting, the main results of the second stream do not deviate much from those of the first stream: the consumption tax is superior to a wage tax, and the capital income tax is inferior to both the wage tax and the consumption tax, as far as capital formation is concerned. However, due to the lack of a method to solve for the dynamic path of the economy, the two streams, until the early 1980s, focused on analytical methods. Since then, keeping pace with the progress of computer technology, a number of researchers have turned to numerical methods for analyzing and evaluating complex policy issues. Auerbach and Kotlikoff (1987) pioneered the use of numerical simulation model in addressing this issue in the context of

²² See Aaron and Gale (1996) for a comprehensive discussion of the economic effects of tax reform; Batina and Ihori (2000), Erosa and Gervais (2001) for a good theoretical survey on the issue; and Erosa and Gervais (2001) for a comparison of the optimal taxation between the infinite horizon model and the OLG model.

a dynamic general equilibrium model. These researchers extended the OLG model of Diamond (1965) to analyze the effects of dynamic fiscal policies by endogenizing the labor supply of households who live many periods. Simulating the impact of changes in the tax bases, Auerbach and Kotlikoff found that the consumption tax generates significantly higher capital formation than either the wage tax or the capital income tax.²³ The literatures mentioned above, however, deal only with these matters in the context of a closed economy. As the global integration of capital markets is progressing, the impact of the tax policy of a country will not be restricted to the domestic economy, but will be transmitted to other countries. The repercussions from the rest of the world in turn will feedback to the domestic economy. In this sense, the work of Ithori (1996) is noteworthy in that it extends the analysis for the effects of tax reform to the open economy. However, his work is not free from the limitations, either. Since the research is done with analytical methods mainly focusing on the steady state, the feedback effect from the repercussions of the tax reform cannot properly be addressed.

The purpose of this chapter is to analyze the impact – on the country itself and the rest of the world – of a tax reform that a country carries out. To study the problem of tax reform, we formulate a two-period and two-country OLG model. In each economy, two generations – one young and one old – coexist in each period. An individual lives two periods, working when she is young, and retiring when she is old. Part of her labor income is spent on current consumption, and the remaining part is saved – under the form of capital investment – to provide for her old-age consumption. Besides their private

²³ See Kotlikoff (2000) for a detailed explanation of the origin, structure, and a future improvement of their dynamic simulation OLG model.

consumption, individual agents – young and old – pay taxes to the government. In order to finance the provision of public goods, the government levies proportional taxes on all or one of the tax bases: wages, capital income, and consumption. Simulating the model with plausible values for the parameters, this chapter tries to answer the following questions: Which tax scheme is better for capital formation and economic growth? Which tax scheme is better for improving the welfare of a country? And how is the tax reform of one country transmitted to the rest of the world?

The rest of the paper is organized as follows. In Section 2, the basic two-period and two-country OLG model that include all tax schemes is presented. In Section 3, the basic model is specialized to trace the effects of the tax reform toward the capital income tax and the consumption tax. In the Section 4, the simulation of the model is explained, and the results of the simulation are presented. Section 5 contains a summary of the results and some concluding remarks.

2. THE MODEL

2.1. *Economic Agents, Preferences, and Technology*

In the model, time is discrete and denoted by $t, t = 0, 1, \dots$. There are two countries denoted by $i, i = H(Home), F(Foreign)$. In each period, four classes of economic agents co-exist in the each country: a young generation, an old generation, a representative firm that produces a consumption good from labour and capital, and a government. Each government provides a public good that is financed by taxing its citizens. The consumption good can also be used as an investment good to augment the stock of capital as well as to produce the public good. For simplicity, we assume that the public good enters the utility function of a consumer additively, and thus will be suppressed in the utility maximization of an individual because it does not influence her private decision.

In each period t , the population structure is represented by a list, say $(N_{i,t}^0, N_{i,t}^1)$, where $N_{i,t}^0$ and $N_{i,t}^1$ denote, respectively, the number of young and the number of old individuals in country i . The initial population, i.e., $(N_{i,0}^0, N_{i,0}^1)$ is assumed to be given. The population of both countries is assumed to grow at a constant rate n ; that is, the population structure in period $t \geq 1$ is represented by $N_{i,t} = (N_{i,t}^0, N_{i,t}^1) = (N_{i,t-1}^0(1+n), N_{i,t-1}^1)$. An individual lives two periods, working when she is young, and retiring when she is old. A young individual owns nothing except for one unit of labour that she supplies inelastically on the labour market. Part of the wages

she earns is spent on current consumption; the remaining part is saved – under the form of capital investment – to provide for her old-age consumption. Since the bequest motive is not assumed in the model, an old individual consumes all her savings. In each period, the savings of the young generation constitute the capital stock in the next period. The preference of a young individual of period t in country i is assumed to be representable by the following CIES utility function:

$$u_i(c_{i,t}^0, c_{i,t+1}^1) = \frac{1}{1-\sigma} \left([c_{i,t}^0]^{1-\sigma} + \beta_i [c_{i,t+1}^1]^{1-\sigma} \right),$$

where $c_{i,t}^0$ and $c_{i,t+1}^1$ denote, respectively, the consumption when she is young (“current consumption”) and when she is old (“old-age consumption”). As for the parameters of the utility function, σ , $0 < \sigma < 1$, is the reciprocal of the inter-temporal elasticity of substitution. Also, β_i , $0 < \beta_i < 1$ is the discount factor used to discount future consumption.

The technology used to produce the consumption good is of the Cobb-Douglas type, and is assumed to have the following form: $Y = AF(K, L) = AK^\alpha L^{1-\alpha}$, where K , L , and Y denote, respectively, the capital input (non-depreciable), the labour input, and the output of the consumption good. Also, α , $0 < \alpha < 1$, is a parameter representing the factor share of capital in national income. The parameter A , which is assumed to be constant, is a scale factor for the production technology. The costs of installing and disposing of capital are ignored. There is no tax on the firm’s profits.

2.2. Lifetime Utility Maximization

We shall denote by $\omega_{i,t}$ and $r_{i,t}$, respectively, the wage rate and the rate of interest that prevail in period t in country i . Three types of proportional tax – wage tax, capital income tax, and consumption tax – are levied on individual agents by the government, and the tax rates are denoted, respectively, by $\tau_{i,t}^\omega$, $\tau_{i,t}^r$, and $\tau_{i,t}^c$. In period t , a young individual in country i solves the following lifetime utility maximization problem:

$$(1) \quad \max_{(c_{i,t}^0, c_{i,t+1}^1)} \frac{1}{1-\sigma} ([c_{i,t}^0]^{1-\sigma} + \beta [c_{i,t+1}^1]^{1-\sigma})$$

subject to

$$(2) \quad (1 + \tau_{i,t}^c) c_{i,t}^0 + \frac{(1 + \tau_{i,t+1}^c) c_{i,t+1}^1}{1 + (1 - \tau_{i,t+1}^r) r_{i,t+1}} - (1 - \tau_{i,t}^\omega) \omega_{i,t} = 0.$$

Letting λ denote the multiplier associated with the budget constraint (2), we have the following first-order conditions that characterize the preceding utility maximization problem:

$$(3) \quad (c_{i,t}^0)^{-\sigma} - \lambda_{i,t} (1 + \tau_{i,t}^c) = 0,$$

$$(4) \quad \beta (c_{i,t+1}^1)^{-\sigma} - \lambda_{i,t} \frac{(1 + \tau_{i,t+1}^c)}{1 + (1 - \tau_{i,t+1}^r) r_{i,t+1}} = 0,$$

$$(5) \quad (1 + \tau_{i,t}^c) c_{i,t}^0 + \frac{(1 + \tau_{i,t+1}^c) c_{i,t+1}^1}{1 + (1 - \tau_{i,t+1}^r) r_{i,t+1}} - (1 - \tau_{i,t}^\omega) \omega_{i,t} = 0.$$

From (3) and (4), we obtain the following relation between $c_{i,t}^0$ and $c_{i,t+1}^1$:

$$(6) \quad c_{i,t+1}^1 = \left(\beta_i \left[1 + (1 - \tau_{i,t+1}^r) r_{i,t+1} \right] \left[\frac{1 + \tau_{i,t}^c}{1 + \tau_{i,t+1}^c} \right] \right)^{\frac{1}{\sigma}} c_{i,t}^0.$$

Using (6) in the lifetime budget constraint (5), we obtain

$$(7) \quad (1 + \tau_{i,t}^c) c_{i,t}^0 + \frac{(1 + \tau_{i,t+1}^c)}{1 + (1 - \tau_{i,t+1}^r) r_{i,t+1}} \left(\beta_i \left[1 + (1 - \tau_{i,t+1}^r) r_{i,t+1} \right] \left[\frac{1 + \tau_{i,t}^c}{1 + \tau_{i,t+1}^c} \right] \right)^{\frac{1}{\sigma}} c_{i,t}^0 - (1 - \tau_{i,t}^\omega) \omega_{i,t} = 0.$$

Solving (7) for $c_{i,t}^0$, we obtain

$$(8) \quad c_t^0 = \frac{(1 - \tau_{i,t}^\omega) \omega_{i,t}}{(1 + \tau_{i,t}^c) + (\beta_i)^{\frac{1}{\sigma}} (1 + \tau_{i,t}^c)^{\frac{1}{\sigma}} \left[\frac{1 + (1 - \tau_{i,t+1}^r) r_{i,t+1}}{1 + \tau_{i,t+1}^c} \right]^{\frac{1-\sigma}{\sigma}}}.$$

Using (8), we obtain the following expression for $c_{i,t+1}^1$:

$$(9) \quad c_{i,t+1}^1 = \left(\beta_i \left[1 + (1 - \tau_{i,t+1}^r) r_{i,t+1} \right] \left[\frac{1 + \tau_{i,t}^c}{1 + \tau_{i,t+1}^c} \right] \right)^{\frac{1}{\sigma}} \times \left(\frac{1}{(1 + \tau_{i,t}^c) + (\beta_i)^{\frac{1}{\sigma}} (1 + \tau_{i,t}^c)^{\frac{1}{\sigma}} \left[\frac{1 + (1 - \tau_{i,t+1}^r) r_{i,t+1}}{1 + \tau_{i,t+1}^c} \right]^{\frac{1-\sigma}{\sigma}}} \right) (1 - \tau_{i,t}^\omega) \omega_{i,t}.$$

Since old-age consumption is funded by principal and net interest on savings, the savings of a young individual of period t are related to old-age consumption by

$$\begin{aligned}
s_{i,t} &= \frac{(1 + \tau_{i,t+1}^c)}{1 + (1 - \tau_{i,t+1}^r)r_{i,t+1}} c_{i,t+1}^1 \\
(10) \quad &= \frac{(1 + \tau_{i,t+1}^c)}{1 + (1 - \tau_{i,t+1}^r)r_{i,t+1}} \left(\beta_i \left[1 + (1 - \tau_{i,t+1}^r)r_{i,t+1} \right] \left[\frac{1 + \tau_{i,t}^c}{1 + \tau_{i,t+1}^c} \right] \right)^{\frac{1}{\sigma}} \\
&\quad \times \left(\frac{1}{(1 + \tau_t^c) + (\beta_i)^{\frac{1}{\sigma}} (1 + \tau_t^c)^{\frac{1}{\sigma}} \left[\frac{1 + (1 - \tau_{t+1}^r)r_{t+1}}{1 + \tau_{t+1}^c} \right]^{\frac{1-\sigma}{\sigma}}} \right) (1 - \tau_t^\omega) \omega_t.
\end{aligned}$$

The list $(c_{i,t}^0, c_{i,t+1}^1, s_{i,t})$ is called the optimal lifetime plan of a young individual of period t in country i .

2.3. Profit Maximization

In period t , the representative firm of country i that produces the consumption good solves the following profit maximization problem:

$$(11) \quad \max_{(K_{i,t}, L_{i,t})} AK_{i,t}^\alpha L_{i,t}^{1-\alpha} - \omega_{i,t} L_{i,t} - r_{i,t} K_{i,t}.$$

The following first-order conditions characterize the optimal labor and capital inputs:

$$(12) \quad (1 - \alpha)AK_{i,t}^\alpha L_{i,t}^{-\alpha} - \omega_{i,t} = 0,$$

$$(13) \quad \alpha AK_{i,t}^{\alpha-1} L_{i,t}^{1-\alpha} - r_{i,t} = 0.$$

The list $(Y_{i,t}, L_{i,t}, K_{i,t})$, with $Y_{i,t} = AK_{i,t}^\alpha L_{i,t}^{1-\alpha}$, is called the optimal production plan in period t of the representative firm of country i that produces the consumption good.

Letting $k_{i,t} = K_{i,t} / L_{i,t}$ denote the capital labor ratio chosen by the firm, we obtain the following expression for the output of the consumption good per unit of labor input in period t :

$$(14) \quad y_{i,t} = Ak_{i,t}^\alpha.$$

Also, the first-order conditions (13) and (14) can be rewritten in terms of the capital labor ratio k_t as follows:

$$(15) \quad \omega_{i,t} = A(1 - \alpha)k_{i,t}^\alpha,$$

$$(16) \quad r_{i,t} = A\alpha k_{i,t}^{\alpha-1}.$$

2.4. Government Policy

To simplify matters, we assume that each government maintains a balanced budget in each period $t = 0, 1, \dots$. Also, each government levies taxes to keep its expenditure per capita constant, say $g_{i,t} = g_i, t = 0, 1, \dots, i = H, F$. Letting $T_{i,t}$ and $S_{i,t}$ denote, respectively, the tax revenues collected by the government of country i in period t and the savings of the young generation of country i in period t , we obtain the following constraint of a balanced budget:

$$(17) \quad \begin{aligned} T_{i,t} &= \tau_{i,t}^\omega \omega_{i,t} N_{i,t}^0 + \tau_{i,t}^r r_{i,t} S_{i,t-1} N_{i,t}^1 + \tau_{i,t}^c (c_{i,t}^0 N_{i,t}^0 + c_{i,t}^1 N_{i,t}^1) \\ &= g_i (N_{i,t}^0 + N_{i,t}^1). \end{aligned}$$

By a tax policy of the government, we mean an infinite sequence $\mathfrak{T}_i = (\tau_{i,t}^\omega, \tau_{i,t}^r, \tau_{i,t}^c)_{t=0}^\infty$.

3. TAX REFORMS

In this section, we investigate the effects of a tax reform in an open economy. Initially, two countries are in a steady state in which a proportional tax is levied only on the wage. Next from the steady state under the proportional wage tax, the tax scheme of the Home country is unexpectedly shifted to a capital income tax or a consumption tax – as a result of a tax reform – while the Foreign country maintains her wage tax scheme.

3.1. *The Competitive Equilibrium under a Proportional Wage Tax*

First, note that under a proportional wage tax, (17) is reduced to

$$(18) \quad T_{i,t} = \tau_{i,t}^{\omega} \omega_{i,t} N_{i,t}^0 = g_i(N_{i,t}^0 + N_{i,t}^1), \quad (t = 0, 1, \dots, i = H, F).$$

The proportional wage tax levied by the government of country i is then given by

$$(19) \quad \tau_{i,t}^{\omega} = \frac{g_i(N_{i,t}^0 + N_{i,t}^1)}{\omega_{i,t} N_{i,t}^0} = \frac{g_i(2 + n)}{\omega_{i,t}(n + 1)}$$

Because capital is assumed to be perfectly mobile, capital flows in each period – assumed to occur instantaneously – will take place until the rental rates of capital in both countries are equalized. Furthermore, because both countries have the same production technology, the equalization of the rental rates of capital in the two countries imply the equality of the capital labor ratios both at home and abroad, and their common value is called the world capital labor ratio. The equalization of the capital labor ratios in the two countries in turn implies the equalization of wages both at home and abroad.

The world capital labor ratio in period 0 is given by

$$(20) \quad k_0 = \frac{N_{H,0}^1 s_{H,-1} + N_{F,0}^1 s_{F,-1}}{N_{H,0}^0 + N_{F,0}^0},$$

and the equilibrium rental rate of capital in period 0 in the global capital market is given by

$$(21) \quad r_0 = \alpha A(k_0)^{\alpha-1}.$$

The equilibrium wage rate in each country in period 0 is given by

$$(22) \quad \omega_{H,0} = \omega_{F,0} = \omega_0 = (1 - \alpha) A(k_0)^\alpha.$$

Also, the proportional wage tax in period 0 in country i is given by

$$(23) \quad \tau_{i,0}^\omega = \frac{g_i(2+n)}{\omega_0(1+n)}.$$

Let k_1 be the equilibrium world capital labor ratio in period 1. The equilibrium rental rate of capital in the global capital market in period 1 is then given by

$$(24) \quad r_1 = \alpha A(k_1)^{\alpha-1}.$$

Using (11), we obtain the following expression for the saving of a young individual of period 0 in country i :

$$(25) \quad s_{i,0} = \frac{(1 - \tau_{i,0}^\omega) \omega_0}{1 + (\beta_i)^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}}.$$

The world capital stock in period 1 that is generated by the maximizing behavior of the young generations of period 0 in both countries – when they face the wage rate ω_0 , the proportional wage tax $\tau_{i,0}$, $i = H, F$, and the interest rate r_1 is then given by

$$\begin{aligned}
& N_{H,0}^0 s_{H,0} + N_{F,0}^0 s_{F,0} \\
(26) \quad & = N_{H,0}^0 \frac{(1 - \tau_{H,0}^\omega) \omega_0}{1 + (\beta_H)^{\frac{1}{\sigma}} (1 + \alpha A(k_1)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} + N_{F,0}^0 \frac{(1 - \tau_{F,0}^\omega) \omega_0}{1 + (\beta_F)^{\frac{1}{\sigma}} (1 + \alpha A(k_1)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}}.
\end{aligned}$$

Hence the world capital labor ratio in period 1 that is generated by the maximizing behavior of the young generations of period 0 in both countries – when they face the wage ω_0 , the proportional wage tax $\tau_{i,0}, i = H, F$, and the interest rate r_1 is given by

$$\begin{aligned}
(27) \quad k_1 &= \frac{N_{H,0}^0 s_{H,0} + N_{F,0}^0 s_{F,0}}{N_{H,1}^0 + N_{F,1}^0} \\
&= \frac{N_{H,0}^0}{N_{H,1}^0 + N_{F,1}^0} \frac{(1 - \tau_{H,0}^\omega) \omega_0}{1 + (\beta_H)^{\frac{1}{\sigma}} (1 + \alpha A(k_1)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \\
&\quad + \frac{N_{F,0}^0}{N_{H,1}^0 + N_{F,1}^0} \frac{(1 - \tau_{F,0}^\omega) \omega_0}{1 + (\beta_F)^{\frac{1}{\sigma}} (1 + \alpha A(k_1)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \\
&= \frac{\omega_0}{1+n} \left(\frac{\eta_{H,0} (1 - \tau_{H,0}^\omega)}{1 + (\beta_H)^{\frac{1}{\sigma}} (1 + \alpha A(k_1)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_{F,0} (1 - \tau_{F,0}^\omega)}{1 + (\beta_F)^{\frac{1}{\sigma}} (1 + \alpha A(k_1)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}} \right)
\end{aligned}$$

Note that in (27) we have let $\eta_{H,0} = N_{H,0}^0 / (N_{H,0}^0 + N_{F,0}^0)$ denote the population of the Home country relative to the world population, and $\eta_{F,0} = 1 - \eta_{H,0}$.

Now note that given k_0 , the initial world capital labor ratio, the equilibrium wage rate in period 0 and the proportional wage tax in period 0 can be computed from (22) and (23), respectively. Hence (27) implicitly defines k_1 , and can be interpreted as the transition equation for the equilibrium world capital labor ratio from one period to the next. Thus starting from any initial value for the world capital labor ratio, the temporary equilibria of the system can be computed recursively.

3.2. Tax Reform: The Home Government Shifts to the Capital Income Tax

If the Foreign country maintains the proportional wage tax while the Home country shifts to the capital tax, then the Foreign country's proportional wage tax is still given by (19). More specifically, in each period, the Foreign country's proportional wage tax is given by

$$(28) \quad \tau_{F,t}^{\omega} = \frac{g_F(N_{F,t}^0 + N_{F,t}^1)}{\omega_t N_{F,t}^0} = \frac{g_F(2+n)}{\omega_t(1+n)}, \quad (t = 0, 1, \dots).$$

As for the Home country, the shift to the capital income tax reduces (17) to

$$(29) \quad T_{H,t} = \tau_{H,t}^r r_t s_{H,t-1} N_{H,t}^1 = g_H(N_{H,t}^0 + N_{H,t}^1),$$

from which we obtain the following expression for the income tax rate in period t :

$$(30) \quad \tau_{H,t}^r = \frac{g_H(N_{H,t}^0 + N_{H,t}^1)}{r_t s_{H,t-1} N_{H,t}^1} = \frac{g_H(2+n)}{r_t s_{H,t-1}}, \quad (t = 0, 1, \dots).$$

The world capital labor ratio in period 0 is given by

$$(31) \quad k_0 = \frac{N_{H,0}^1 s_{H,-1} + N_{F,0}^1 s_{F,-1}}{N_{H,0}^0 + N_{F,0}^0},$$

and the equilibrium rental rate of capital in period 0 in the global capital market is given by

$$(32) \quad r_0 = \alpha A(k_0)^{\alpha-1}.$$

The equilibrium wage rate in each country in period 0 is given by

$$(33) \quad \omega_{H,0} = \omega_{F,0} = \omega_0 = (1 - \alpha) A(k_0)^{\alpha}.$$

Also, the proportional wage tax in period 0 in the Foreign country is given by

$$(34) \quad \tau_{F,0}^{\omega} = \frac{g_F(2+n)}{\omega_0(1+n)}.$$

Let k_1 be the equilibrium world capital labor ratio in period 1. The equilibrium rental rate of capital in the global capital market in period 1 is then given by

$$(35) \quad r_1 = \alpha A(k_1)^{\alpha-1}.$$

Using (10), we obtain the following expression for the saving of a young individual of period 0 in the Foreign country and a young individual of period 0 in the Home country, respectively,

$$(36) \quad s_{F,0} = \frac{(1 - \tau_{F,0}^\omega) \omega_0}{1 + (\beta_F)^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}}.$$

$$(37) \quad s_{H,0} = \frac{\omega_0}{1 + (\beta_H)^{\frac{1}{\sigma}} [1 + (1 - \tau_{H,1}^r) r_1]^{\frac{1-\sigma}{\sigma}}}.$$

The world capital stock in period 1 that is generated by the maximizing behavior of the young generations of period 0 in the two countries is then given by

$$(38) \quad \begin{aligned} & N_{H,0}^0 s_{H,0} + N_{F,0}^0 s_{F,0} \\ &= N_{H,0}^0 \frac{\omega_0}{1 + (\beta_H)^{\frac{1}{\sigma}} [1 + (1 - \tau_{H,1}^r) r_1]^{\frac{1-\sigma}{\sigma}}} + N_{F,0}^0 \frac{(1 - \tau_{F,0}^\omega) \omega_0}{1 + (\beta_F)^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}}. \end{aligned}$$

Hence the world capital labor ratio in period 1 that is generated by the maximizing behavior of the young generations of period 0 in both countries is given by

$$(39) \quad \begin{aligned} k_1 &= \frac{N_{H,0}^0 s_{H,0} + N_{F,0}^0 s_{F,0}}{N_{H,1}^0 + N_{F,1}^0} = \\ &= \frac{N_{H,0}^0}{N_{H,1}^0 + N_{F,1}^0} \frac{\omega_0}{1 + (\beta_H)^{\frac{1}{\sigma}} [1 + (1 - \tau_{H,1}^r) r_1]^{\frac{1-\sigma}{\sigma}}} + \frac{N_{F,0}^0}{N_{H,1}^0 + N_{F,1}^0} \frac{(1 - \tau_{F,0}^\omega) \omega_0}{1 + (\beta_F)^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} \end{aligned}$$

$$= \frac{\omega_0}{1+n} \left[\frac{\eta_{H,0}}{1 + (\beta_H)^{\frac{1}{\sigma}} [1 + (1 - \tau_{H,1}^r) r_1]^{\frac{1-\sigma}{\sigma}}} + \frac{\eta_{F,0} (1 - \tau_{F,0}^\omega)}{1 + (\beta_F)^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} \right].$$

Also, for $t = 1$ (30) becomes

$$(40) \quad \tau_{H,1}^r = \frac{g_H(2+n)}{r_1 s_{H,0}}.$$

For any given value of the world capital labor ratio k_0 , the system consisting of (35), (37), and (40) is a system of three equations in the three unknowns r_1 , $s_{H,0}$, and $\tau_{H,1}^r$, and in principle can be solved for these variables in terms of k_0 . In this manner, the temporary equilibria of the world economy can be obtained, recursively, starting from any value for the initial world capital labor ratio.

3.3. Tax Reform: The Home Government Shifts to the Consumption Tax

If the Foreign country maintains the proportional wage tax while the Home country shifts to the consumption tax, then the Foreign country's proportional wage tax in each period is given by

$$(41) \quad \tau_{F,t}^\omega = \frac{g_F(2+n)}{\omega_t(1+n)}, \quad (t = 0, 1, \dots).$$

As for the Home country, the shift to the consumption tax reduces (17) to

$$(42) \quad T_{H,t} = \tau_{H,t}^c (c_{H,t}^0 N_{H,t}^0 + c_{H,t}^1 N_{H,t}^1) = g_H(N_{H,t}^0 + N_{H,t}^1),$$

from which we obtain the following expression for the consumption tax rate in period t :

$$(43) \quad \tau_{H,t}^c = \frac{g_H(N_{H,t}^0 + N_{H,t}^1)}{c_{H,t}^0 N_{H,t}^0 + c_{H,t}^1 N_{H,t}^1} = \frac{g_H(2+n)}{(1+n)c_{H,t}^0 + c_{H,t}^1}, \quad (t = 0, 1, \dots).$$

The world capital labor ratio in period 0 is given by

$$(44) \quad k_0 = \frac{N_{H,0}^1 s_{H,-1} + N_{F,0}^1 s_{F,-1}}{N_{H,0}^0 + N_{F,0}^0},$$

and the equilibrium rental rate of capital in period 0 in the global capital market is given by

$$(45) \quad r_0 = \alpha A(k_0)^{\alpha-1}.$$

The equilibrium wage rate in each country in period 0 is given by

$$(46) \quad \omega_{H,0} = \omega_{F,0} = \omega_0 = (1 - \alpha)A(k_0)^\alpha.$$

Also, the proportional wage tax in period 0 in the Foreign country is given by

$$(47) \quad \tau_{F,0}^\omega = \frac{g_F(2+n)}{\omega_0(1+n)}.$$

Let k_1 be the equilibrium world capital labor ratio in period 1. The equilibrium rental rate of capital in the global capital market in period 1 is then given by

$$(48) \quad r_1 = \alpha A(k_1)^{\alpha-1}.$$

Using (10), we obtain the following expression for the saving of a young individual of period 0 in the Foreign country and a young individual of period 0 in the Home country, respectively,

$$(49) \quad s_{F,0} = \frac{(1 - \tau_{F,0}^\omega)\omega_0}{1 + (\beta_F)^{\frac{1}{\sigma}}(1 + r_1)^{\frac{1-\sigma}{\sigma}}}.$$

$$\begin{aligned}
s_{H,0} &= \frac{(1+\tau_{H,1}^c)}{1+r_1} \left(\beta_H [1+r_1] \left[\frac{1+\tau_{H,0}^c}{1+\tau_{H,1}^c} \right] \right)^{\frac{1}{\sigma}} \times \\
(50) \quad & \left(\frac{1}{(1+\tau_{H,0}^c) + (\beta_H)^{\frac{1}{\sigma}} (1+\tau_{H,0}^c)^{\frac{1}{\sigma}} \left[\frac{1+r_1}{1+\tau_{H,1}^c} \right]^{\frac{1-\sigma}{\sigma}}} \right) \omega_0.
\end{aligned}$$

The world capital stock in period 1 that is generated by the maximizing behavior of the young generations of period 0 in the two countries is then given by

$$\begin{aligned}
& N_{H,0}^0 s_{H,0} + N_{F,0}^0 s_{F,0} \\
&= N_{H,0}^0 \frac{(1+\tau_{H,1}^c)}{1+r_1} \left(\beta_H [1+r_1] \left[\frac{1+\tau_{H,0}^c}{1+\tau_{H,1}^c} \right] \right)^{\frac{1}{\sigma}} \times \\
(51) \quad & \left(\frac{1}{(1+\tau_{H,0}^c) + (\beta_H)^{\frac{1}{\sigma}} (1+\tau_{H,0}^c)^{\frac{1}{\sigma}} \left[\frac{1+r_1}{1+\tau_{H,1}^c} \right]^{\frac{1-\sigma}{\sigma}}} \right) \omega_0 \\
&+ N_{F,0}^0 \frac{(1-\tau_{F,0}^\omega) \omega_0}{1 + (\beta_F)^{\frac{1}{\sigma}} (1+r_1)^{\frac{1-\sigma}{\sigma}}}.
\end{aligned}$$

Hence the world capital labor ratio in period 1 that is generated by the maximizing behavior of the young generations of period 0 in both countries is given by

$$\begin{aligned}
k_1 &= \frac{N_{H,0}^0 s_{H,0} + N_{F,0}^0 s_{F,0}}{N_{H,1}^0 + N_{F,1}^0} \\
&= \frac{N_{H,0}^0}{N_{H,1}^0 + N_{F,1}^0} \frac{(1 + \tau_{H,1}^c)}{1 + r_1} \left(\beta_H [1 + r_1] \left[\frac{1 + \tau_{H,0}^c}{1 + \tau_{H,1}^c} \right] \right)^{\frac{1}{\sigma}} \times \\
&\quad \left(\frac{1}{(1 + \tau_{H,0}^c) + (\beta_i)^{\frac{1}{\sigma}} (1 + \tau_{H,0}^c)^{\frac{1}{\sigma}} \left[\frac{1 + r_1}{1 + \tau_{H,1}^c} \right]^{\frac{1-\sigma}{\sigma}}} \right)^{\frac{1-\sigma}{\sigma}} \omega_0 \\
&\quad + \frac{N_{F,0}^0}{N_{H,1}^0 + N_{F,1}^0} \frac{(1 - \tau_{F,0}^\omega) \omega_0}{1 + (\beta_F)^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}}. \\
&= \frac{\omega_0}{1 + n} \left[\eta_{H,0} \frac{(1 + \tau_{H,1}^c)}{1 + r_1} \left(\beta_H [1 + r_1] \left[\frac{1 + \tau_{H,0}^c}{1 + \tau_{H,1}^c} \right] \right)^{\frac{1}{\sigma}} \times \right. \\
&\quad \left(\frac{1}{(1 + \tau_{H,0}^c) + (\beta_i)^{\frac{1}{\sigma}} (1 + \tau_{H,0}^c)^{\frac{1}{\sigma}} \left[\frac{1 + r_1}{1 + \tau_{H,1}^c} \right]^{\frac{1-\sigma}{\sigma}}} \right)^{\frac{1-\sigma}{\sigma}} \\
&\quad \left. + \eta_{F,0} \frac{(1 - \tau_{F,0}^\omega) \omega_0}{1 + (\beta_F)^{\frac{1}{\sigma}} (1 + r_1)^{\frac{1-\sigma}{\sigma}}} \right]
\end{aligned}
\tag{52}$$

In the case of proportional wage tax, an individual only has to pay taxes when she is working. When the home government switches from the proportional wage tax to the capital income tax, an individual only pays taxes on her old-age income. We have seen in these two cases that the series of temporary equilibria can be computed recursively: knowledge of the world capital labor in one period can be used to compute the world capital labor ratio in the next period. In contrast with the proportional wage tax and the

capital income tax, the tax on consumption affects an individual in both periods of her life cycle, and this makes the recursive computation of the series of temporary equilibria impossible. To see why, suppose that the current state of the system is known. For a young individual of the current period in the Home country to plan her life cycle, she must know the consumption tax in the next period. Due to the balanced budget constraint in each period, the consumption tax rate in the next period depends on the consumption of the young generation of that period, and the consumption of a young individual in the next period depends on the consumption tax rate two periods into the future. Thus in order to study the transition of the world economy to its long-run equilibrium, we have to trace out the convergence to the long-run equilibrium backwardly.

4. SIMULATION

In this section, we simulate the model to study the economic effects of the tax reform. To simplify the matter, we assume that preference, technology and the size of population are identical between two countries.

4.1. *Parameter Values*

Table I summarizes the parameter values used to generate the numerical simulation results. Although replicating a real economy is not the objective of this chapter, the explanatory power of the model will be enhanced if parameters and outcomes are within

reasonable ranges. The scale parameter on technology, A , is set to 4 and the reciprocal of the inter-temporal elasticity of substitution σ is set to 0.5.²⁴ The share of capital in output, α , is chosen to be 0.25, following Auerbach and Kotlikoff (1987) and Summers (1981). The discount factor for future consumption, β , is selected to be the reciprocal of the population growth factor, i.e., $1/1.01$ in one year, then for a period of one generation of 30 years, it is given to $\frac{1}{(1.01)^{30}} \approx 0.75$.²⁵ The population growth rate, n , is assumed to be equal to 1% par annum, which becomes 0.35 for a period of one generation. Lastly, the government expenditure per capita, g , is chosen to be 0.2, which can be translated into the government expenditure over GDP of about 10 percent.

TABLE I
PARAMETER VALUES

Parameters	A	α	β	σ	n	g
Value	4	0.25	0.75	0.5	0.35	0.2

4.2. Initial Steady State before Tax Reform

The second columns of Table II and Table III show the steady state values when the proportional wage taxes are initially levied in both countries. Since the two countries are assumed to be identical, their initial steady-state values will be identical. According to

²⁴ Note that the scale parameter A is arbitrarily chosen for computing convenience since technological progress is assumed away in this model. Also note $\sigma = 0.5$ means an agent's willingness to shift consumption between young age and old age is moderate.

²⁵ In the literature on simulating the two-period OLG model, a period of 30 years is normally chosen as the time span of one generation. See, for example, De la Croix and Michel (2002).

Table II, the capital labor ratio under the proportional wage tax is 1.0448. The output per worker is 4.0441. The tax on wages is 11.5%, and the interest rate is 0.9676.²⁶ For a young individual, her current consumption is 1.2744, and her old-age consumption is 2.7754, yielding a lifetime utility of 4.7567. For the entire economy, some figures might be excessive, compared to those observed for real economies. In the simulation, the capital/GDP ratio (K/Y) is 7.75; the consumption/GDP ratio (C/Y) is 82.4%; the government expenditure/GDP ratio (G/Y) is 8.6%; and the gross saving/GDP ratio (S/Y) of 34.9%.²⁷ Lastly, the net foreign assets/GDP ratio (B/Y) is zero since there is no capital flow between countries that are identical.

4.3. *Steady States under Tax Reform*

The third and the fourth column of Table II and Table III display the impact that the shifts of tax base have on two economies' capital labor ratio and other related variables in the steady state. First note that the consumption tax scheme generates better outcomes than other tax schemes as far as capital formation and economic growth are concerned. Under the consumption tax, the capital labor ratio is 1.1196, and output per worker is

²⁶ The interest rate of 0.9676 is equivalent to 2.28% annually, which seems to be within a reasonable range of the real interest rate. This interest rate fulfils the condition of the dynamic efficiency, $\bar{r} > n$, since we are assuming that the population growth rate is 1% annually.

²⁷ Again GDP in two-period OLG model is assumed to be equivalent to GDP over 30 years. Therefore, to obtain the capital/GDP ratio of the economy, the GDP (Y) is adjusted on an equivalent annual basis by dividing the output per period by 30. The capital/GDP ratio of 7.75 is higher than the ratios (between 3 and 6) typically observed in developed countries. Compared to the ratios computed from real data, the simulated ratios appear to overestimate the consumption/GDP ratio and the gross saving/GDP ratio, but underestimate the government expenditure/GDP ratio. For instance, according to the figures of year 2005 in *Economic Report of the President* (2006, Table B-1 and B-35), U.S. federal government spending absorbs about 7 % of GDP, and the spending of all levels of the U.S. government spending is 18.9 % of GDP. Private consumption accounts for 70% of GDP, and the national saving accounts for 11 % of GDP. However, it should be noticed that the main purpose of our simulation is not to fully reflect the real world, but to estimate the directional change of economic variables after a shift of the tax schemes.

4.1146, which are higher than those under the other tax schemes. That is, the long-run capital labour ratio under the consumption tax is 7.2 % higher than under the wage tax and 5.3 % higher than under the capital income tax.²⁸ The output per worker under the consumption tax is 1.7 % higher than that under the wage tax and 1.3 % higher than that under the capital income tax.

TABLE II
STEADY STATE VALUES: HOME COUNTRY

Economic variables of the Home country	Tax system of the Home country		
	Wage tax	Capital income tax	Consumption tax
Saving (s)	1.4105	1.4584	1.6018
Capital/labor ratio (k)	1.0448	1.0636	1.1196
Output per worker (y)	4.0441	4.0621	4.1146
Welfare (u)	4.7567	4.835	4.826
Current consumption (c^0)	1.2744	1.5882	1.3467
Old-age consumption (c^1)	2.7754	2.381	2.789
Tax rate (τ)	0.1148	0.3375	0.102
Interest rate (r)	0.9676	0.9548	0.9188
Net foreign asset per an old individual (b)	0.0000	0.0167	0.0669
Capital/GDP (K/Y)	7.7508	7.8548	8.1632
Saving/GDP (S/Y)	0.3488	0.359	0.3893
Consumption/GDP (C/Y)	0.8235	0.8251	0.8294
Gov't Expenditure/GDP (G/Y)	0.0861	0.0857	0.0846
Net Foreign Asset/GDP (B/Y)	0.0000	0.0041	0.0163

²⁸ If leisure is included in the utility function, the capital labour ratio will be much lower under the wage tax than under other tax schemes since the workers tend to allocate more of their time endowment to leisure in order to evade the wage tax. In addition, if the tax reform happens in a closed economy, the superiority of the consumption tax scheme will be much higher since the favourable effects on the capital stock is not transmitted to other countries.

TABLE III
STEADY STATE VALUES: FOREIGN COUNTRY

Economic variables of the Foreign country	Tax system of the Home country		
	Wage tax	Capital income tax	Consump- tion tax
Saving (s)	1.4105	1.4132	1.4211
Capital/labor ratio (k)	1.0448	1.0636	1.1196
Output per worker (y)	4.0441	4.0621	4.1146
Welfare (u)	4.7567	4.7605	4.7719
Current consumption (c^0)	1.2744	1.2852	1.3167
Old-age consumption (c^1)	2.7754	2.7626	2.7268
Tax rate (τ)	0.1148	0.1143	0.1128
Interest rate (r)	0.9676	0.9548	0.9188
Net foreign asset per an old individual (b)	0.0000	-0.0167	-0.0669
Capital/GDP (K/Y)	7.7508	7.8548	8.1632
Saving/GDP (S/Y)	0.3488	0.3479	0.3454
Consumption/GDP (C/Y)	0.8235	0.8202	0.8109
Gov't Expenditure/GDP (G/Y)	0.0861	0.0857	0.0846
Net Foreign Asset/GDP (B/Y)	0.0000	-0.0041	-0.0163

The superiority of the consumption tax comes from its effect on saving. In the Home country, the saving rate under the consumption tax is 13.6 % higher than under the wage tax, and 9.8 % higher than under the capital income tax. A priori, there is no reason why a tax on consumption will have a larger impact on saving because the tax affects both current and future consumption, and does not offer any incentive to shift consumption within the life cycle. Also, under the consumption tax, the taxpayers have to save more when working in order to pay the consumption tax imposed on their old-age consumption, and this means that saving and the capital labor ratio should be higher. In addition, as noted by Summers (1981), Ihori (1996), and Batina and Ihori (2000), the consumption tax

can reduce the present value of taxation by postponing the tax payments, and thus contributes to capital formation. This tax deferral effect is one of the contributors of the higher capital labor ratio under the consumption tax scheme. One interesting result is that the saving rate of the Foreign country is highest (although the difference is not outstanding) when the Home country adopts the consumption tax, even though the tax reform is not implemented in the Foreign country. The higher world capital labor ratio fostered by the consumption tax increases the wage rate common in both countries and the higher wage rate, in turn, boosts the saving of the Foreign country, as well.

Second, note that the tax reform of the Home country affects its net foreign asset position. Since the rise in saving that is induced by the tax reform is higher in the Home country than in the Foreign country, the Home country becomes a creditor and the Foreign country a debtor. As can be seen from Table III, the net foreign asset per old individual of the Foreign country is -0.0669 under the consumption tax and -0.0167 under the capital income tax. Similarly, the net foreign asset per GDP of the Foreign country is -1.6% under the consumption tax and -0.4% under the capital income tax. That is, without any change in preferences or technology, the Foreign country becomes a debtor country simply due to the tax reform of the Home country. In particular, the consumption tax deteriorates the net asset position of the Foreign country even further.

Third, note that with respect to the welfare of the Home country, the consumption tax is superior to the wage tax, but seems not to show an advantage over the capital income tax. The welfare is 4.826 under the consumption tax, 4.835 under the capital income tax and

4.7567 under the wage tax; in other words, the welfare is 1.5% higher under the consumption tax than under the wage tax, but 0.2% lower than under the capital income tax. However, the welfare result that shows the inferiority of the consumption tax relative to the capital income tax might have originated from the structure of our lifetime utility function in which a higher weight is given to current consumption than to old-age consumption.²⁹ The capital income tax by taxing an individual's income in her old age induces the individual to shift consumption to the present. The consumption taxes, however, are levied on both generations, and therefore, there is no incentive to shift consumption between current and future. In this sense, the welfare improvement under the consumption tax can be considered as a result of the rise in the lifetime consumption.³⁰ In particular, it is noteworthy to see that the old-age consumption of the Home country is highest under the consumption tax. The interests earned on their net foreign assets are a main contributor to the highest old-age consumption.

The welfare of the Foreign country is also affected by the tax reform of the Home country. Note from the Table III that the welfare of the Foreign country is slightly higher under the consumption tax than under other tax schemes adopted by the Home country. The welfare of the Foreign country is 4.7719 under the consumption tax, 4.7605 under the capital income tax, and 4.7567 under the wage tax. It is interesting to see that the welfare of the Foreign country can be improved by the tax reform in the Home country. The tax

²⁹ As shown in equation (1), welfare is defined as the lifetime utility of a young individual in period t . In (1), the old-age consumption is always discounted in the welfare calculations, since β is assumed to be less than 1.

³⁰ The increase of the lifetime consumption in the Home country can be verified by the ratio of Consumption/GDP in Table II. In this table, the ratio is highest under the consumption tax even after considering that GDP is highest under the consumption tax scheme.

reform induces capital flows into the Foreign country, and now the young individuals of the Foreign country can increase their current consumption simply by borrowing from the young individuals of the Home country. However, the young individuals of the Foreign country will have to reduce their old-age consumption so as to pay the interests of their foreign debts. Thus the lifetime consumption of a young individual of the Foreign country might be deteriorated by the consumption tax of the Home country.³¹

Fourth, the tax rate is highest under the capital income tax, and is lowest under the consumption tax. In Table II, the rate of the Home country is 11.5% under the wage tax, 33.8% under the capital income tax, and 10.2% under the consumption tax. The capital income tax is more heavily imposed than the wage tax or the consumption tax because capital income has a much smaller tax base.

4.4. *The Transition to the New Steady States*

In the overlapping generations model in which one period has about 30 years span, the current generations may be less interested in the steady state outcomes of the tax reform because the long-term results are not linked to their utility unless the welfare of their offspring are included in their utility function. Therefore, the transitional analysis on the current or near future generations is more demanding. Table IV shows the transition values after the tax reform has been implemented. Comparing the steady state values of

³¹ From Table III, we can see that the total consumption over GDP of the Foreign country deteriorates with the adoption of the consumption tax in the Home country: the ratio of 81.09% is lowest among the tax schemes.

Table II and Table III to the sixth and the seventh column of Table IV, we find that the capital labor ratio and the utility enter their steady state in two periods under the consumption tax and in one period under the capital income tax.³²

TABLE IV
THE TRANSITION VALUES

			$t = -1$	$t = 0$	$t = 1$	$t = 2$
The Home Country	Consumption Tax	Capital labor ratio (k)	1.0448	1.0448	1.1027	1.1196
		Utility (u)	4.6295	4.776	4.8105	4.826
	Capital Income Tax	Capital labor ratio (k)	1.0448	1.0448	1.0636	-
		Utility (u)	4.5397	4.8243	4.835	-
The Foreign Country	Consumption Tax	Capital labor ratio (k)	1.0448	1.0448	1.1027	1.1196
		Utility (u)	4.7567	4.7323	4.7617	4.7719
	Capital Income Tax	Capital labor ratio (k)	1.0448	1.0448	1.0636	-
		Utility (u)	4.7567	4.7486	4.7605	-

Regarding the welfare of the Home country, it is easy to say that the current old generation will be worse off with the tax reform- whether it is shifting to the consumption tax or to the capital income tax. For an old individual of the Home country, she has already paid the tax on wages. Thus when the tax reform is implemented by shifting from the proportional wage tax to the consumption tax or the capital income tax, she shall pay taxes again in the second period of her life cycle. In the fourth column of Table IV, the welfare of the generation born in period $t = -1$, is 4.6295 under the consumption tax and 4.5397 under the capital income tax, which is lower than the welfare of 4.7567 under the wage income tax in Table II.³³ However, the welfare of the generation born in period

³² The tax on consumption affects the individuals in both periods of the life cycle. Thus the consumption tax may need a longer time to converge to the steady state.

³³ In other words, the welfare of the current old generation falls by 2.7% under the consumption tax and by 4.6% under the capital income tax.

$t = 0$, when the tax reforms are implemented, improves with the tax reform. In the fifth column of Table IV, it is 4.776 under the consumption tax and 4.8243 under the capital income tax, both of which are higher than the welfare of 4.7567 under the wage tax found in Table II. Also the welfare of the generation born in period $t = 1$ is higher than that of the preceding generation. In the sixth column of Table IV, it is 4.8105 under the consumption tax and 4.835 under the capital income tax, both of which are higher than that of the generation born in period $t = 0$. Here note that the welfare of the Home country is always higher under the capital income tax than under the consumption tax in the short-run as well as in the long-run.

As for the welfare of the Foreign country, it is interesting to see that the welfare of the generation born in period $t = 0$ in which the tax reforms are implemented, worsens with the tax reform of the Home country. In the fifth column of Table IV, it is 4.7323 under the consumption tax and 4.7486 under the capital income tax, both of which are slightly lower than the initial welfare of 4.7567. In particular, the consumption tax by the Home country will worsen the welfare of the current young generation of the Foreign country further. The major culprit will be the fall in the old-age consumption due to lower interest revenue caused by the tax reform of the Home country. The welfare of the generation born in period $t = 1$, however, recovers and even surpasses their initial level as the higher capital labor ratio raises the wage rate.

4.5. Sensitivity Analysis

The magnitude of the estimated effects induced by a shift in the tax base clearly depends on the values of the parameter of the model. Since the accuracy of the estimated values of the parameters is normally uncertain, sensitivity analysis is inevitable to ensure the “correctness” of conclusions in simulation analysis. Table V- VIII, which are presented in the Appendix, show how the steady state values change with the key parameters of the model. To simplify the analysis, we use two values- one is low and the other is high- within the reasonable range of each parameter while holding all the other parameters at their original values. The sensitivity analysis indicates that the main results summarized in Table II and Table III are robust: The consumption tax generates the highest capital labour ratio, regardless of the parameter values. The sensitivity analysis, however, shows that the ranking of the capital labour ratios between the wage tax scheme and the capital income tax scheme can be reversed, depending on the parameter values. For example, in Table V, when g is high (say $g = 0.5$), we find that the capital labour ratio under the wage tax surpasses that under the capital income tax. The reason is in a high tax rate in the capital tax scheme. Facing with the extremely high rate of capital tax, individuals may have a lower propensity to save in order to avoid a severe taxation. Also, in Table VI, when β is low (say $\beta = 0.5$), a wage tax might be better than a capital income tax, at least by not discouraging the propensity to save. In addition, in Table VII, when α is low (say $\alpha = 0.1$), the share of wage in output is high and workers have a higher propensity to save, which encourages the accumulation of capital under the wage tax. Lastly, In Table VIII, when σ is low (say $\sigma = 0.1$), the wage tax outperforms the capital income

tax. That is, if an agent's willingness to shift consumption between current and old-age consumption is high, the tax on wage discourages capital formation less.

The sensitivity analysis also verifies the welfare results of the Table II and Table III: As seen in Table V-VIII, the welfare of both countries always is better under the consumption tax than under the wage tax regardless of the parameter values. In the Home country, the consumption tax is dominated by the capital income tax. In particular, the welfare of the Foreign country is highest under the consumption tax. However, the ranking of the welfare between the consumption tax and the capital income tax scheme can be reversed in the Home country, depending on the parameter values. For example, in Table V, when g is high (say $g = 0.5$), we find that the welfare under the consumption tax surpasses that under the capital income tax. The lower welfare under the capital income tax seems to be an outcome of a lower capital labor ratio induced by a high capital tax rate. Also in Table VII, when α is low (say $\alpha = 0.1$), the consumption tax dominates the capital income tax. As the share of wage in output is lower, the effect of higher capital labor ratio dominates the consumption shifting effect discussed in section 4.3. In addition, in Table VIII, when σ is low (say $\sigma = 0.1$), an agent's willingness to shift consumption between current and old-age consumption is high. Then, the tax on capital discourages capital formation, which results in a lower welfare than under the consumption tax. Finally, in Table V-VIII, it is confirmed that the tax rate is always lowest under the consumption tax and highest under the capital income tax, except for the case of a low α .

5. CONCLUDING REMARKS AND FURTHER EXTENSION

In this chapter, we have examined the dynamic effects of tax reform. From the open-economy setting of the model, we have obtained several interesting results that are summarized as below. First, with respect to capital formation, the consumption tax dominates the capital income tax, and the capital income tax dominates the wage tax. The superiority of the consumption tax scheme comes from its saving-boosting effect. Interestingly, due to the transmission effects of the tax policy, the saving rate becomes highest even in the other country in which the consumption tax is not implemented. Second, the tax reform in one country affects the net foreign asset position of the two countries. The shift to the consumption tax or the capital income tax renders the tax-reformed country a creditor and the other country a debtor. In particular, the consumption tax affects the net asset position further. Third, with respect to welfare, the consumption tax is superior to the wage tax but seems not show an advantage over the capital income tax in the tax-reformed country. However, in the other country, the welfare is slightly higher under the consumption tax than under other tax schemes adopted by the tax-reformed country. Fourth, the tax rate is highest under the capital income tax, and is lowest under the consumption tax. Lastly, the steady state outcomes are also applied to the transition path. In particular, the transitional analysis shows that the capital income tax can be more rewarding than the consumption tax especially in the aspect of the short-run welfare because the capital income tax enters its new steady state with a faster speed.

The economies in this model, however, differ from real economies in many ways. Thus, the policy implications that are appropriate to the present model must be accepted with a caveat. Obviously, the use of a model in which one period represents approximately thirty years will limit the applicability of model as far as timely policy is concerned. Also the assumption that agents receive wage income only in the first period of their lives might not be realistic. When we add some reality to the model by assuming that agents receive wage income in both periods of their lives, we may make up for the shortcoming of two-period model. In addition, the introduction of endogenous labor will enrich the results in this model. If endogenous labor is allowed, the capital labor ratio is much lower under the wage tax than under other tax schemes since the workers tend to allocate more of their time endowment to leisure in order to evade the wage tax. These further extensions will improve the applicability of the model for policy purpose.

APPENDIX

TABLE V: RESULTS OF THE SENSITIVITY ANALYSIS FOR g

		$g = 0.5$			$g = 0.05$		
		Wage tax	Capital income tax	Consumption tax	Wage tax	Capital income tax	Consumption tax
Home country	Capital labor ratio	0.8035	0.8019	1.0066	1.1562	1.1617	1.1744
	Utility	4.1875	4.2822	4.4284	5.0000	5.0205	5.0152
	Tax rate	0.3064	0.9212	0.3008	0.028	0.0835	0.0237
Foreign country	Capital labor ratio	0.8035	0.8019	1.0066	1.1562	1.1617	1.1744
	Utility	4.1875	4.1869	4.2568	5.0001	5.001	5.003
	Tax rate	0.3064	0.3066	0.2896	0.028	0.028	0.0279

TABLE VI: RESULTS OF THE SENSITIVITY ANALYSIS FOR β

		$\beta = 0.9$			$\beta = 0.5$		
		Wage tax	Capital income tax	Consumption tax	Wage tax	Capital income tax	Consumption tax
Home country	Capital labor ratio	1.2541	1.2913	1.343	0.6484	0.6377	0.6963
	Utility	5.3096	5.3834	5.377	3.8682	3.9589	3.944
	Tax rate	0.1097	0.3187	0.0988	0.1294	0.3962	0.112
Foreign country	Capital labor ratio	1.2541	1.2913	1.343	0.6484	0.6377	0.6963
	Utility	5.3096	5.3156	5.324	3.8682	3.8642	3.8859
	Tax rate	0.1097	0.1089	0.1078	0.1293	0.1299	0.1270

TABLE VII: RESULTS OF THE SENSITIVITY ANALYSIS FOR α

		$\alpha = 0.5$			$\alpha = 0.1$		
		Wage tax	Capital income tax	Consumption tax	Wage tax	Capital income tax	Consumption tax
Home country	Capital labor ratio	0.5886	0.7145	0.7513	1.0593	1.0231	1.1138
	Utility	3.791	4.2266	4.2172	4.8219	4.7818	4.8235
	Tax rate	0.2269	0.1891	0.1153	0.0962	0.9013	0.1053
Foreign country	Capital labor ratio	0.5886	0.7145	0.7513	1.0593	1.0231	1.1138
	Utility	3.791	3.9416	3.9814	4.8219	4.8218	4.8224
	Tax rate	0.2269	0.2059	0.2008	0.0962	0.0965	0.0957

TABLE VIII: RESULTS OF THE SENSITIVITY ANALYSIS FOR σ

		$\sigma = 0.9$			$\sigma = 0.1$		
		Wage tax	Capital income tax	Consumption tax	Wage tax	Capital income tax	Consumption tax
Home country	Capital labor ratio	0.8203	0.8886	0.8962	1.9047	1.7211	1.9881
	Utility	18.525	18.619	18.591	3.711	3.69	3.7642
	Tax rate	0.1219	0.3391	0.1064	0.0988	0.3479	0.0933
Foreign country	Capital labor ratio	0.8203	0.8886	0.8962	1.9047	1.7211	1.9881
	Utility	18.525	18.542	18.543	3.711	3.6992	3.7183
	Tax rate	0.1219	0.1195	0.1193	0.0988	0.1013	0.0977

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