

INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI

A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor MI 48106-1346 USA
313/761-4700 800/521-0600



Université d'Ottawa • University of Ottawa

Thermal Release of Hidden Magnetic Moments in Low and High T_c Type II Superconductors

by

© David LeBlanc

Thesis submitted to the University of Ottawa
in partial fulfillment of the requirements for the
degree of Doctor of Philosophy in Physics

Department of Physics
University of Ottawa , Ottawa Canada
May 1998



National Library
of Canada

Acquisitions and
Bibliographic Services

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale
du Canada

Acquisitions et
services bibliographiques

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-36779-7

Canada

Acknowledgements

I would like to express my gratitude to my research supervisor Dr. Marcel LeBlanc. For his time, patience and excellent guidance I am deeply indebted. As well, his support and indeed encouragement of my numerous alternate avenues of investigation will not be forgotten.

I wish to thank the many past and present members of our research group, in particular Daniel Cameron, for making my tenure here a most enjoyable and productive one.

I would like to also thank Dr. Paul Gierszewski of the Canadian Fusion Fuels Technology Project (CFFTP). His support and encouragement of my investigations has been most important to me.

Finally I would like to dedicate this thesis to my parents. Their love and support throughout my life are truly treasured gifts.

FOREWORD

As a Masters student under the direction of M.A.R. LeBlanc I had the pleasure of fabricating and then conducting a novel experiment which allowed a unique new view of the dynamics of flux lines in type II superconductors. A complex arrangement of toroidal and axial superconducting magnets in combination with a hollow cylinder of a conventional type II superconductor allowed the monitoring of a “reservoir” of magnetic flux within the hollow cylinder whose dynamics provided important new evidence for and details on the phenomena of flux line cutting. This work was reported in the prestigious Physical Review Letters¹ in 1991.

Since the work for my M.Sc thesis I have been involved in numerous research projects. These efforts focused on the mechanisms and properties of superconductivity of both the conventional low T_c materials and the newer class of high T_c ceramic superconductors. In regards to high T_c materials I have directed successful efforts towards both the study of these materials along with improving their utility and broader integration into the scientific and technological arenas.

Much of these efforts have been under the supervision and direction of my supervisor, M.A.R. LeBlanc but I have also been encouraged by Dr. LeBlanc to pursue research efforts and collaborations outside the normal purview of our research group. I have however, decided to focus upon one single but broadly encompassing project for presentation as my Ph.D. thesis. This work includes extensive experimental observation and theoretical modelling and has brought to light new phenomena, which in my view have been successfully interpreted and quantitatively reproduced.

It seems appropriate to enumerate here the other various projects carried out during my candidacy for the Ph.D. Thus early after my Masters work I completed an extensive computational investigation of low frequency A.C. losses where the varying field is parallel to a static bias field². In this work we discovered new aspects of the A.C. loss valley in type II superconductors and corrected certain false assumptions in the literature on some aspects of these phenomena. This work was published in Physical Review B in 1992.

This investigation was continued and broadened into a collaboration with Daniel Cameron and Jinglei Meng of our research group to address a related phenomenon of the reduction of A.C. losses in coaxial cables of type II superconductors by a steady bias current³. This work has both scientific merit and obvious possible technological benefit and was published in the Journal of Applied Physics in 1996.

Again in collaboration with others in our group and under the direction of M.A.R. LeBlanc I have played an important role in numerous other projects⁴⁻⁷. These have generally applied the standard critical state model to account for a wide range of phenomena in both conventional low T_c and the newer high T_c materials.

I have continued experimental investigations on hollow cylinders related to my Masters work and have carried out other investigations under the direction of Dr. LeBlanc. These efforts have yet to be fully completed and reported in the literature.

As previously mentioned my supervisor Dr. LeBlanc has viewed favourably my initiation of research projects not directly linked to our groups main focus. I have by inclination sought practical applications for my scientific efforts. Indeed it was the

possible improvement and application of superconducting materials which led to my initial interest in this field.

After the discovery of high T_c materials an extensive research effort was devoted towards improving the current carrying capacity of these materials. This involves the introduction of more effective pinning sites which in simple terms are normal regions within the material which best pin or trap flux lines. Material scientists accomplish this in a variety of ways, eg. twinned planes, texturing, tangles of dislocations etc. The more "brute force" techniques of fast neutron or high energy ions also achieved excellent results. Ion damage can lead to columnar defects of obvious potential at pinning flux lines parallel to these introduced structures. Fast neutron damage leads to clustering of dislocations and interstitial atoms as recoil atoms give up their energy to the lattice.

After a review of these techniques I proposed a novel method for the introduction of dislocations and interstitials in the 123 class of superconductors typified by $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ (YBCO). The technique seeks to promote lattice displacement by way of the atomic recoil from prompt γ -ray emission following thermal neutron absorption. This technique simply involves the substitution of another rare earth element on the Yttrium lattice site. In particular Gadolinium, with its enormous neutron absorption cross section and energetic prompt γ . The recoil energy of the atom upon prompt γ emission is sufficient for displacement of Gd from the rare earth lattice site, hence introducing dislocations. Thus given the large cross section of Gd and the much higher prevalence of thermal neutrons in a typical reactor spectrum, point defect densities comparable to that achieved with fast neutrons should be attained with far less irradiation time.

With the assistance of Patrick Fournier and his supervisor Marcel Aubin at the Université de Sherbrooke, several superconducting discs incorporating Gadolinium were prepared and then irradiated at the McMaster nuclear reactor. These initial studies were followed up by irradiations at Chalk River Nuclear Laboratories to take advantage of their nearly purely thermal neutron flux. Later, thin films of GdBCO were prepared, irradiated and studied by Darcy Poulin at McMaster University.

This extremely lattice site specific damage was shown not to provide effective pinning sites for the increase of the critical current. This in itself may shed light on the role of the rare earth site in the binding mechanisms of the Cooper pairs. The initial stages of this work⁸ were presented in a poster session at the 1992 International Critical Currents Conference in Vienna, Austria. However, as is often the case, similar independent work was published by a group at Los Alamos National Laboratory at the same time. While low level thermal neutron irradiation was shown not to enhance j_c it had previously been shown that higher thermal neutron irradiation fluxes can destroy the superconducting state in $\text{GdBa}_2\text{Cu}_3\text{O}_{7-x}$. I have proposed that this in itself leads to a novel technique for the selective conversion to the normal state in a three layer thin film system of YBCO-GdBCO-YBCO. This may result in more practical Josephson junctions or Squids which rely on Superconducting-Normal-Superconducting (S-N-S) boundaries with precise interfaces. Such exact interfaces have proven difficult to achieve in other techniques where there is significant lattice parameter mismatch between layers. There is of course a near exact lattice match between $\text{GdBa}_2\text{Cu}_3\text{O}_{7-x}$ and $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$.

I have invested considerable efforts in the area of Quench-Melt-Growth materials. This refers to a fabrication process which results in single crystals, or more precisely,

single domains of the YBCO superconductor of very large dimension (up to 10 cm to date). Since here the weak Josephson junctions between numerous grains of a polycrystalline material are removed, high critical current density over the entire volume of these large crystals are possible, especially at temperatures not far below 77 K. Thus enormous currents can be induced to either shield against entry of flux or more importantly to trap or retain very high levels of flux. Crystals have trapped in excess of 10 Tesla in temperatures of 20 K to 50 K and over 1 Tesla in 77 K. Thus these materials can act as quasi-permanent magnets as long as they are kept at low temperature after the appropriate field is trapped.

My work with these materials has followed several avenues. I have set up collaborations with researchers at Argonne National Laboratory and the University of Cincinnati who are also studying and utilizing these materials. Among other things I suggested a change in the fabrication technique that has greatly simplified their production. Previously, to nucleate crystal growth a special starting seed of 123 superconductors was needed. This consisted of a "pure" crystal of $\text{NdBa}_2\text{Cu}_3\text{O}_{7-x}$ which has the slightly higher melting point compared to YBCO. These seeding crystals are very expensive and can only be fabricated in small dimensions (typically 2-3 mm). My suggestion, which was successfully implemented, was to utilize crystals of near lattice match already employed as substrate crystals in the growth of thin films of YBCO. These crystals are relatively inexpensive and more importantly, can be provided in large sizes to improve and expand the production of these valuable QMG YBCO "quasi" crystals. This work⁹ was published in *Physica C* in 1995.

The possible uses of these crystals as compact and intense field sources has also been the subject of my attentions. In particular I have studied their possible integration as confinement media for plasma research and for magnetically confined fusion. While the possibility of extremely high sustainable fields is attractive of even greater importance are the extremely steep gradients of magnetic field which are possible with these crystals (i.e. >10 Tesla from a crystal only a few cm in diameter). It is the magnetic field gradients which act as the effective plasma mirrors in many past and present fusion reactor designs. My proposals and possibly unique reactor designs incorporating such crystals have been supported and encouraged by the granting of a long term fellowship by the Canadian Fusion Fuel Technology Project in 1993. CFFTP has been a consortium integrating Canadian fusion research and technology, primarily supported by Ontario Hydro and the Federal Government.

Thus the work of during my candidacy for the Ph.D. has been broad and extensive. However for my Ph.D. thesis I have focused upon one of my research endeavours, namely work on the thermal release of hidden magnetic moments.

Reports and Publications

1. M.A.R. LeBlanc, **D. LeBlanc**, A. Golebiowski, G. Fillion, "New Observations on the Dynamics of Flux Lines in Type-II Superconductors." Phys. Rev. Lett. 66:3309-3312 1991 (N.B. Work of Masters thesis at University of Ottawa)
2. **D. LeBlanc**, M.A.R. LeBlanc, "A.C. Loss Valley in Type-II Superconductors" Phys. Rev.B 45:5443-5449, 1992

3. M.A.R. LeBlanc, D.S.M Cameron, **D. LeBlanc**, J. Meng, "Reduction of Low Frequency A.C. Losses in Coaxial Cable of Type-II Superconductors by a Steady Bias Current." J. Appl. Phys. 79: 334-344, 1996
4. M.A.R. LeBlanc, **D. LeBlanc**, D.S.M Cameron, "I_c Hystereis in Granular Type II Superconductors: A Simple Model" CAP Congress, June 1995
5. M.A.R. LeBlanc, S. Wang, **D. LeBlanc**, D. Krzywinski, J. Meng, "Investigation of the dB/dH Effect Using Trapped Flux in Type-II Superconductors. Phys. Rev. B 2:12895-12910, 1996
6. M.A.R. LeBlanc. D.S.M Cameron, **D. LeBlanc**, "Local Magnetic Relaxation and Hysteresis in the Strong and Weak Bose-Glass Regimes of Type-II Superconductors: A Simple Model." Phys.Rev B 55: R6141-6144, 1997
7. M.A.R. LeBlanc, D.S.M Cameron, M. Kryzywinski, **D. LeBlanc** "Magnetic Relaxation on Bridges Between the Envelopes of Major Hysteresis Loops of Type-II Superconductors." Supercond. Sci. Techol. 10: 625-639, 1997
8. **D. LeBlanc**, M.A.R. LeBlanc, P. Fournier, M. Aubin, "Enhancement of j_c by Slow Neutron Irradiation of Gd_xY_{1-x}Ba₂Cu₃O_{7-y}." International Critical Currents Conf. Vienna Austria 1992
9. D. Shi, K. Lahini, J.R. Hull, **D. LeBlanc**, M.A.R. LeBlanc, A. Dabkowski, Y.Chang, Y Jiang, Z. Zhang, H. Fan, "Growth of Large-Domain YBa₂Cu₃O_x with New Seeding Crystals of CaNdAlO₄ and SrLaGaO₄" Physica C 246: 253-261, 1995

ABSTRACT

Measurements of the evolution of the magnetic moment of nonideal (i.e. pinning rich) type II superconductors in stationary magnetic fields H_a as a function of time at constant temperatures $T < T_c$ or as the temperature is varied to or from T_c provide insight into the dynamics of flux lines and their interactions with the pinning sites.

Three concentric magnetic moments coexist in these materials when the isothermal sweep of H_a causes the locus of the magnetization $\langle M \rangle$ to migrate along any path joining the upper with the lower envelopes of the major hysteresis curves. The "outermost" magnetic moment is always diamagnetic and is generated by the field opposing Meissner surface current. The two other magnetic moments fill the bulk of the specimen, currents circulating in one direction in the outer annular volume embrace an inner volume occupied by counter-rotating persistent currents. In semi-reversible type II superconductors the magnitude of the Meissner magnetic moment is important compared to that arising from the currents in the bulk whereas in very hysteretic specimens it is negligible. In both types of material it is a fairly straightforward procedure to establish configurations in various static H_a where the opposing magnetic moments exactly cancel each other so that the macroscopic net magnetization is zero. However the internal pattern of persistent currents and magnetic flux density profiles which prevail in these

special $\langle M \rangle = 0$ states is complicated, fascinating and provides a full crucible for the investigation of flux line behaviour.

In this thesis I present the results of our experimental study of the evolution of the magnetization of these initial $\langle M \rangle = 0$ states during warming to T_c as a function of the static applied field H_a . The phenomena encountered are seen to depend dramatically on the direction of the change of H_a causing $\langle M \rangle$ to migrate from one envelope of the major hysteresis loop to the opposite.

Further we find major differences in the behaviour exhibited by the semi-reversible type II superconductors in comparison with that manifested by the highly hysteretic samples. We also see that the low T_c and high T_c semi-reversible specimens display very similar but highly intricate behaviour.

Finally we show that a simple model which exploits the critical state concept and applies the principle of conservation of flux lines can account for the major features of all our observations.

TABLE OF CONTENTS

1	Introduction	1
2	Framework and General Background	4
2.1	Ideal (Abrikosov) Type II Superconductor	4
2.2	Strongly Hysteretic Type II Superconductor	6
2.2.1	General Considerations	6
2.2.2	Isothermal Magnetic Behaviour	8
2.2.3	Evolution of Magnetization Upon Warming from $\langle M \rangle = 0$	16
2.3	Release of Hidden Magnetic Moment (Semi-Reversible Sample)	19
3	Experimental Set Up and Procedures	33
3.1	Introduction	33
3.2	Samples Employed	34
3.3	Sample Heater Arrangement	37
3.4	External Field Magnets	38
3.5	Pickup Coil Assemblies	40
3.6	Measurement of $\langle M \rangle$ versus H_a	41
3.7	Monitoring $\langle M \rangle$ versus Temperature	43
3.8	Measurement of the Thermal Release of Hidden Moment $\langle M \rangle$	45

3.9	Methods of Measurement of H_{C1} , H_* , and H_{**}	48
3.9.1	H_{C1}	48
3.9.2	H_*	49
3.9.3	H_{**}	52
3.9.4	H_* and H_{**} in the Presence of I_M	53
3.10	Measurement of Meissner Effect	55
4	Observations on a Low T_c Hysteretic Specimen: VTi	67
4.1	Introduction	67
4.2	Isothermal Magnetization Curves	68
4.3	Release of Hidden Magnetic Moments	69
4.4	Conclusions	72
5	Observation of a High T_c Hysteretic Specimen: Large YBCO Single Crystal	79
5.1	Introduction	79
5.2	Isothermal Magnetization Curves	80
5.3	Release of Hidden Magnetic Moments	82
5.4	Summary and Conclusions	84

6	Observations on a High T_c Hysteretic Specimen:	
	Textured YBCO Rod	92
6.1	Introduction	92
6.2	Isothermal Magnetization Curves	93
6.3	Release of Hidden Magnetic Moments	94
6.4	Summary and Conclusions	95
7	Thermal Disclosure of Hidden Magnetic Moments:	
	Model for Very Irreversible Type II Superconductors	101
7.1	Introduction	101
7.2	Framework of the Model	102
7.3	Development of Expressions for the B profiles	103
7.4	Thermal Evolution of the Magnetization	108
8	Observations on a Low T_c Semi-Reversible Type II Superconductor:	
	Niobium Ribbon	115
8.1	Introduction	115
8.2	Isothermal Magnetization and Related Curves	116
8.3	Release of Hidden Magnetic Moment	119
8.4	Summary and Conclusions	120

9	Observations on a High T_c Semi-Reversible Type II Superconductor:	
	BiSSCO Slab	132
9.1	Introduction	132
9.2	Isothermal and Related Curves	133
9.3	Release of Hidden Magnetic Moment	136
9.4	Summary and Conclusions	139
10	Thermal Disclosure of Hidden Magnetic Moment:	
	Model for Semi-Reversible Type II Superconductors	149
10.1	Introduction	149
10.2	Framework of the Analysis	150
10.2.1	H_{C1} and $H^* \ll H_{C2}$	150
10.2.2	H_{C1} and H^* not negligible relative to H_{C2}	152
10.3	Temperature Dependence of j_o and I_M	160
10.4	Dependence of I_M on H_a ($H_{C10} \leq H_a \leq H_{C2}$)	165
11	Summary and Conclusion	170
	REFERENCES	174

CHAPTER 1

INTRODUCTION

Recently Clem and Hao¹ have examined theoretically the evolution of the magnetization of semi-reversible type II superconductors as the temperature T is increased from $T < T_c$ to the critical value T_c after the specimen has been subjected to three different magnetic field – temperature histories which we now outline.

- (i) The specimen, initially in the “virgin state”, hence after cooling through T_c to T_0 in “zero” (i.e. the earth’s) magnetic field, is magnetized by the application of a magnetic field H_a . This magnetization procedure is denoted zero field cooled (ZFC) in the literature.
- (ii) The specimen is cooled from T_c to T_0 in a stationary magnetic field H_a applied with $T > T_c$ hence when the sample was in the normal state. The sample acquires a diamagnetic moment as it expels magnetic flux while it cools from T_c to T_0 . In the literature this procedure is labeled Field Cooled Cooling (FCC)
- (iii) After experiencing the above FCC procedure, now with T maintained fixed at T_0 , the magnetic field H_a is reduced to zero. This procedure leaves the nonideal specimen with some trapped magnetic flux, hence a remanent or residual magnetic moment.

In the theoretical study of Clem and Hao¹ the magnetic field is maintained fixed at H_a in procedures (i) and (ii) and at zero in procedure (iii) as the specimen is now heated

from T_0 to T_c . These workers also investigated in detail the evolution of the magnetic moment as the specimen expels magnetic flux when undergoing the Field Cooled Cooling procedure.

Our work complements the investigation of Clem and Hao¹. In our study we focus on the evolution of the magnetic moment of type II superconductors during warming from T_0 to T_c in static magnetic fields $0 < H_a < H_{c2}$ after magnetic field-temperature histories which have induced two concentric but oppositely directed magnetic moments in the specimens. Such magnetic structures are encountered when the locus of the magnetization of nonideal (i.e. hysteretic) type II superconductors is in transit from one envelope of the isothermal hysteresis curve of the material to the opposite envelope. This situation also occurs periodically when the superconducting specimen is subjected to an alternating (A.C.) magnetic field H_a of amplitude $H_{\max}$ or is made to carry an alternating conduction current I of amplitude $I_{\max}$. Under these circumstances the swings of H_a between $+H_{\max}$ and $-H_{\max}$, or the conduction current I between $+I_{\max}$ and $-I_{\max}$ generate configurations of persistent currents flowing in opposite directions in concentric volumes of the superconducting specimen. The magnetic moment μ associated with these distributions or patterns of circulation of supercurrents varies from $\mu_{\max}$ through zero to $-\mu_{\max}$ concomitantly with the swings of H_a or I . For convenience and to fix ideas with no loss of generality we address the special situation where the net magnetic moment μ as a function of the cycle of H_a is traversing zero. Also, for simplicity, we set aside discussion of the situations where it is the conduction current I which is alternating in static magnetic fields. At the juncture where $\mu = 0$, the

swing of H_a is interrupted and subsequently the applied field is maintained fixed while the specimen is now warmed from T_0 to T_c .

Now starting with this initially zero net magnetic moment, the magnetization of the specimen is observed to exhibit a variety of fascinating and intricate behaviours as the temperature rises from T_0 to T_c . We have investigated these phenomena in both high T_c and conventional type II superconductors. In both types of materials we have examined specimens which,

- (a) show magnetic irreversibility but also exhibit significant flux expulsion during field cooling hence are labeled semi-reversible and,
- (b) very hysteretic samples which show negligible flux expulsion during field cooling.

The thermal evolution of the magnetization from $\mu = 0$ displayed by the semi-reversible samples is seen to be radically different from that manifested by the strong pinning specimens. Further the behaviours are seen to depend on whether the $\mu = 0$ initial configurations have been established by,

- (i) having H_a descend or ascend in magnitude to its final value and also on,
- (ii) the magnitude of the final H_a .

In this thesis we report on these observations and present a simple model which reproduces the major features of the variety of events we have encountered in the thermal evolution of these initially zero magnetization states.

CHAPTER 2

Framework and General Background

2.1 Ideal (Abrikosov) Type II Superconductors

The idealized type II superconductor envisaged in the seminal analysis of Abrikosov² is regarded as pinning free hence thermodynamically reversible. Thus the spatial average of the magnetic flux density $\langle B \rangle$ and therefore the magnetization $\mu_0 \langle M \rangle = \langle B \rangle - \mu_0 H_a$, for a given specimen are dictated by the temperature T and the applied magnetic field H_a and are not dependent on the previous thermomagnetic history. Consequently the locus of the isothermal magnetization when H_a is decreasing in magnitude retraces the curve exhibited when H_a was increased in magnitude as illustrated in Fig 2.1. Isothermal curves of $\langle M \rangle$ or $\langle B \rangle$ for cycles of H_a between arbitrary limits $H_{\max}$ and $H_{\min}$ embrace no area.

Let $\langle M \rangle_{\text{rev}}$ denote the magnetization $\langle M \rangle$ of the ideal reversible type II superconductor and $\langle B \rangle_{\text{rev}}$ the corresponding $\langle B \rangle$. It is useful to note that, for the ideal reversible material measurements of $\langle M \rangle_{\text{rev}}$ versus H_a provide a direct measure of the magnitude of the field opposing, hence diamagnetic, Meissner surface current I_M and its dependence on H_a and T_0 . To illustrate this point it is convenient to consider idealized planar or cylindrical geometries, i.e. either a slab (ribbon) of thickness $D = 2X$ infinite along y and z or a cylinder of radius R , infinite along z , with the applied field directed along $\pm \bar{z}$.

First we write,

$$I'_M = I_M Z = \int \vec{j}_M \cdot d\vec{s} = \langle j_M \rangle \lambda Z \quad (2.1)$$

where $\langle j_M \rangle$ is the spatial average of the Meissner current density j_M in the penetration depth λ . Now we note that the magnetic moment μ' of a section of length z of an infinite cylinder whose cross section $A = \pi R^2$ is embraced by the Meissner current I'_M circulating along its circumference, can be written,

$$\mu' = I'_M A = I_M Z (\pi R^2) = \langle M \rangle V \quad (2.2)$$

For a segment of height Z and width Y of the idealized slab carrying I'_M along its two surfaces, we write,

$$\mu' = I'_M A = I_M (Z 2XY) = \langle M \rangle V \quad (2.3)$$

Thus in both above situations, $I_M = \langle M \rangle = \langle M \rangle_{\text{rev}}$ in standard international units. This simple relationship between I_M and $\langle M \rangle_{\text{rev}}$ occurs because apart from the Meissner surface current, no net macroscopic current exists inside the bulk or body of the ideal, pinning free (Abrikosov) type II superconductor.

When the applied field exceeds the lower critical field H_{C1} , the Abrikosov type II superconductor exists in a vortex state where it is filled with persistent amperian currents

as illustrated in Fig 2.1 (c) and (d). Each amperian vortex contains one quantum of magnetic flux, $\Phi_0 = h/2e = 2 \times 10^{-15}$ Tesla meter² and is called a flux tube or flux line. In the equilibrium situation, there is no transport current flowing in the interior of the ideal (i.e. pinning free) type II superconductor. Flux lines repel each other and a flux line in the interior of the ideal type II superconductor experiences no net force since it is repelled equally by the neighbours which surround it with equal density $N = \langle B \rangle / \Phi_0$ in all directions. From energy considerations Abrikosov predicted that the flux line lattice would adopt a triangular rather than cubic arrangement. This prediction has been amply verified by a variety of observations³⁻⁶. The flux lines adjacent to the surface experience an outward force from the repulsion they feel because of the interior flux lines. This outward force is balanced by the interaction with the Meissner current and the magnetic field permeating the penetration depth. In the vortex state between H_{C1} and the upper critical field H_{C2} for suppression of superconductivity, I_M denotes the effective current resulting from the superposition of the diamagnetic (field opposing) Meissner surface current and the outer edge of the neighbouring flux sustaining (hence paramagnetic) vortex currents. This is illustrated in Fig 2.1 (c) and (d). In our sketches of the flux density profiles in type II superconductors, this effective I_M will be represented by vertical lines just inside or along the boundaries of the specimen, since for our samples $\lambda \ll R$ or X .

2.2 Strongly Hysteretic (Irreversible) Type II Superconductors

2.2.1 General Considerations

In real type II superconductors however, the migration of flux lines into or out of the specimen as the temperature or the applied magnetic field are varied is impeded by pinning sites, i.e. physical imperfections and chemical impurities.

Why the imperfections and impurities tend to immobilize (i.e. pin) flux lines can be understood as follows.

Flux quantization requires that the superconductivity be quenched along the centre of a vortex. Consequently the density of Cooper pairs varies from zero at the axis of the vortex to its full field and temperature dependent value $n_c(B,T)$ at a characteristic distance from the axis, called the coherence length and denoted $\xi(T)$. This situation is sketched in Fig 2.2 (a). Thus the nucleation of a vortex requires that the superconductor sacrifice an amount of condensation energy, $U_c \approx \pi \xi^2 n_c E_c$ per unit length of vortex "core". E_c is the condensation energy associated with each Cooper pair.

Now we consider voids hence normal regions of radius $R \approx \xi$ in the vicinity of the vortex. It is clearly energetically advantageous for the vortex to migrate until its core overlaps the available normal regions provided by the voids (imperfections, impurities, dislocations etc) since in that volume the ransom of condensation energy has already been paid as illustrated in Fig 2.2 (b).

Now in order to move a pinned vortex sufficient force must be applied to overcome the pinning forces. Such a force on vortices can only arise from their mutual repulsion. Thus a vortex will remain pinned until the repulsive forces exerted on it by its neighbours on one side exceeds the forces it experiences from the neighbours on the other side by a threshold amount. Thus the vortex will unpin and migrate only when this force

imbalance, hence the net repulsive force density F_V acting on it, equals the pinning force density F_P . Consequently the required net repulsive force $F_V = F_P$ can only arise if the gradient in the density of flux lines attains a critical value $(dB/dr)_c$ sufficient to generate the threshold driving force F_V . In view of Maxwell's eqn $\nabla \times \vec{B} = \mu_0 \vec{j}$, this means that a critical depinning current density $j_c(B, T)$ needs to be established to initiate the displacement of flux lines in "non ideal" type II superconductors. Careful analysis shows that $\vec{F}_V$ can be written $\vec{j} \times \vec{B}$, hence the driving force on the flux lines is generally called the Lorentz force⁷⁻¹³.

In the ideal specimens we saw that $B(r)$ averaged over the cross section of a few vortices was uniform throughout the specimen and $B(r) = \langle B(r) \rangle$ varied in a simple reversible way as illustrated in Fig 2.1(b) as a function of H_a . Now however, in the body of the superconductor with pinning sites, the sequences of the flux density configurations $B(r)$ are considerably more complicated as H_a is impressed and removed and also as the temperature is varied with H_a fixed. Consequently the locus of $\langle B \rangle$ or $\mu_0 \langle M \rangle = \langle B \rangle - \mu_0 H_a$ versus H_a does not retrace its path but shows hysteresis when the sweep of H_a is reversed in direction. Further the response of the specimen depends on the previous magnetic field-temperature history in the superconducting state.

2.2.2 Isothermal Magnetic Behaviour

To simplify the detailed description of the thermomagnetic behaviour of nonideal type II superconductors we now first focus on the strongly hysteretic materials and then

examine the semi-reversible specimens. In the former the role played by pinning, hence by the flux density gradients overwhelms the reversible contribution made by the Meissner current $I_M = \langle M \rangle_{\text{rev}}$. A convenient measure of the importance of the effects of pinning on the macroscopic properties is the quantity,

$$H_* = \int_0^R j_c dr = 1/\mu_o \int_0^R (dB/dr) dr \quad (2.4)$$

where the induced persistent currents are taken to fill the entire volume of the specimen. In other words H_* is the external field necessary for first penetration of flux lines to the central axis of the specimen as illustrated in Fig 2.3 (b). Here and throughout the rest of our discussions, to fix ideas we consider idealized cylindrical geometry. In specimens, where $H_* \gg H_{C1} = |I_M|_{\text{max}}$ we can ignore the latter. Such specimens are labeled strongly hysteretic or highly irreversible. It is important to note however that H_* depends on both the “size” of the specimen and on j_c . Consequently a weak pinning material, hence one with modest j_c , but large in thickness or radius can possess a value for H_* much greater than H_{C1} and display very irreversible behaviour. Conversely, a strong pinning material, hence one with large j_c but with a radius r comparable to λ will exhibit some tendency towards reversibility. Specimens which exhibit some degree of reversible behaviour are denoted semi-reversible.

In strongly hysteretic specimens where H_{C1} is negligible the magnetic moment $\langle M \rangle$ and its dependence on H_a arise from the induced internal (bulk) currents associated with the pinned or impeded migration of the flux lines in the body of the material. The nature of the pinning dictates the dependence of the critical current density $j_c(r)$ on the

flux density $B(r)$, hence determines the steepness of the gradients of the flux density dB/dr and consequently the structure of the internal profiles of the flux density (B profiles). The resultant locus of $\langle M \rangle$ versus H_a can be depicted qualitatively as in Fig 2.3 (a) and 2.4 (a), and indeed calculated quantitatively for modeling of experimental results. In such an exercise it is convenient to postulate simple formulae for $j_c(B, T)$ and assess their applicability from comparison of the predictions with the observations. This is the route which we will follow in this thesis.

First we consider a “virgin” sample i.e. one cooled from above T_c in zero (earth’s) field. An external field H_a impressed parallel to the length of the cylindrical sample immediately begins to nucleate flux line at the outer surface since here we regard H_{C1} as negligible. These flux lines are driven inwards by the “magnetic pressure” due to the penetration of the external field in the penetration depth. The first battalions of flux lines are then pressed inwards by the newly formed generations of flux lines behind them. Bulk pinning forces impede the inward migration or redistribution of flux lines and result in the advancing flux line density profiles depicted in Fig 2.3 (b), region A.

Note that in Fig 2.3 and 2.4, the numbers (letters) identify specific points (zones) on the $\langle M \rangle$ versus H_a graphs and also the corresponding profiles (sequences of profiles) in the accompanying sketches.

The advance of the impeded flux lines in zone A continues with rising H_a until the flux front reaches the centre of the cylinder as shown by the profile labeled (1) in Fig 2.3 (b). The applied field required for this penetration to the centre (or mid plane) is termed H_* . At this juncture the entire volume of the cylinder is fully occupied by field opposing

induced persistent currents in a critical state $j_c(r)$. Thus the upper envelope of the major hysteresis curve is first reached here.

A continuation of the increase of H_a beyond H_* causes more flux lines to enter the specimen thereby augmenting the density (the packing) of the flux lines, hence producing a rise in the B profiles as displayed at (2) and (6) in Fig 2.3 (b). Here $\langle M \rangle$ in Fig 2.3 (a) is seen to decline since $j_c(B)$ hence $|dB/dr|$ diminish as B is augmented. Now $\langle M \rangle$ traces part of the upper envelope of the major hysteresis curve as H_a progresses from H_* hence point (1) through (2) and (6) and beyond to H_{C2} if attainable. Here, over the range $H_* \leq H_a \leq H_{C2}$, j_c fills the specimen and circulates in a field opposing sense.

We note that during the sweep of H_a from H_* to H_{C2} a special B profile was generated where the flux density at the centre of the specimen became exactly equal to $\mu_0 H_*$ (see (2) of Fig 2.3 (b) and (c)). The applied field existing at the surface when this configuration is encountered is termed H_{**} .

Fig 2.3 (c) shows that if H_a , upon reaching H_{**} , instead of being made to increase further, is now lowered and reduced to zero, the specimen will be left at point (4) with $\langle M \rangle_{\max \text{ rem}}$, a maximum remanent trapped flux arrangement. At point (4), $j_c(r)$ circulates in a flux retaining sense and fills the entire volume of the specimen. Consequently here $B = \mu_0 H_*$ at the centre of the cylinder. Note also that as H_a decreased from H_{**} to 0, the locus of $\langle M \rangle$ traversed the zero axis (point 3). At this juncture the specimen contains two equal but opposing magnetic moments. The outer volume is now occupied by flux retaining currents. The field opposing (diamagnetic) currents remain undisturbed in the inner volume. Also if H_a after reaching zero and $\langle M \rangle_{\max \text{ rem}}$ (point 4), is now made to increase again in the same direction, the sequence of profiles shown in Fig 2.3 (d) will be

generated and $\langle M \rangle$ will travel from point (4) to point (2) crossing the $\langle M \rangle = 0$ axis at point (5). Now the outer volume is filled with field opposing circulating currents while the inner volume is occupied by field trapping circulating currents. Consequently periodic cycles of H_a between $H_{\max} = H_{**}$ and $H_{\min} = 0$ will generate the sequences of B profiles of Fig 2.3 (c) and (d) and the corresponding minor but complete hysteresis loop defined by points 2,3,4,5,2 of Fig 2.3 (a). Hysteresis loops which span the distance between the upper and lower envelopes are termed “complete”.

If the reversal in the sweep of H_a , hence its decrease were to start at fields higher than H_{**} , for instance at point (6), the sequence of B profiles generated during the descent of H_a would evolve as displayed in Fig 2.5 (e) and diminish to that shown by (4) in Fig 2.3 (c) when H_a reaches zero. Again at point (7) where $\langle M \rangle = 0$, the sample contains two equal but opposite and concentric magnetic moments.

When H_a is made to descend from point (8) to (4), $\langle M \rangle$ traces another portion of the lower envelope of the major hysteresis curve. Since here the sample is filled with flux retaining induced currents, $j_c(r)$, hence $\langle B \rangle > \mu_0 H_a$, this part of the envelope is termed paramagnetic. Note that this paramagnetic zone will extend all the way to H_{C2} if the decrease of H_a commences at H_{C2} .

By the same token the upper “zone” of the magnetization, extending from point (1) hence $H_a = H_*$ to $H_a = H_{C2}$, where the sample is filled with field shielding induced currents hence, $\langle B \rangle < \mu_0 H_a$, is denoted, the diamagnetic envelope.

The hysteresis curve enclosed by the sequence of points 6,7,8,9 and 6 is representative of complete hysteresis loops where $H_{\max} > H_{**}$ and $H_{\min} > 0$. The corresponding sequences of B profiles are sketched in Fig 2.5 (c) and (d).

Clearly larger complete hysteresis loops can be traced which possess an upper and a lower segment overlapping parts of the upper and lower hysteresis envelope. The sequence of points 6,7,8,4,5,2 and 6 in Fig 2.3 (a) traces such a representative loop. The corresponding sequences of B profiles are sketched in Fig 2.5 (e) and (f).

Note that the right and left boundaries of all complete hysteresis loops consist of bridges or transits between the envelopes of the major hysteresis loop and traverse the $\langle M \rangle = 0$ axis.

Hysteresis loops which do not extend from one envelope to the opposite are termed incomplete. The sequence of B profiles for these cases are sketched in Fig 2.5 (a) and (b). Depending on the choice of $H_{\max}$ and $H_{\min}$, such loops may or may not traverse the $\langle M \rangle = 0$ axis. The location of the innermost boundary in the sequence of B profiles, i.e. the minimum r_i , is an unknown entity. For this reason we have avoided this “species” of hysteresis curves in our work.

The zone of $\langle M \rangle$ versus H_a labeled (B) in Fig 2.4 (a) and extending from point (10) where $H_a = 0$ to point (1) where $H_a = H_*$ is termed hybrid. Here as illustrated in Fig 2.4 (c), the flux density profiles comprise an outer volume permeated by positively directed flux lines and an inner volume threaded by negatively directed flux lines. The coexistence of the two polarities of flux lines explains the use of the word hybrid. This hybrid zone is traced after H_a having descended in magnitude from a large negative value to zero is now made to increase in the positive direction to H_* .

The “image” of the sequences of hybrid flux density profiles sketched in Fig 2.4 (c) will be encountered and zone C of Fig 2.4 (a) will be traced if H_a had

descended from a large positive value to zero and then augmented in magnitude but in the negative direction.

An important complete minor hysteresis loop is traced when H_a is made to swing to and fro between $H_{\max} = +H_*$ through $H_a = 0$ to $H_{\min} = -H_*$. Here the flux density profiles evolve through the sequences shown in Fig 2.4 (b) and through the “image” of these profiles when H_a returns from $-H_*$ to H_* . The sequence of points 1, 11, 12, 13, 14, 15, 16, 17 and 1 in Fig 2.4 (a) identifies the structure of this “symmetric” complete minor hysteresis loop.

Finally we stress that along the entire upper envelope of the maximum major hysteresis loop extending from $-H_{C2}$ to $+H_{C2}$, the induced persistent currents fill all of the volume of the specimen and circulate all in the same sense (say clockwise) and in the opposite sense (hence counterclockwise) along the entire lower envelope from $+H_{C2}$ to $-H_{C2}$.

Now, for completeness, taking the Meissner current into account at this juncture, we note that I_M will circulate in a clockwise sense along the 1st and 3rd quadrants and counterclockwise in the 2nd and 4th quadrants of Fig 2.4 (a).

In this thesis we focus on magnetic moments encountered when the isothermal change of H_a causes the locus of $\langle M \rangle$ to migrate from one envelope of the major hysteresis loop to the opposite envelope. At all stages during such an excursion the sample is completely filled with concentric regions of counterrotating currents. To fix ideas we focus on a unit length Z of an infinitely long cylinder of radius R filled with azimuthally circulating currents whose density $j_\phi(r)$ is uniform along $\bar{z}$ and $\bar{\phi}$. The applied field is directed along the $\bar{z}$ axis.

Since an elementary magnetic moment $\Delta\vec{\mu}$ is constructed when an element of electric current $\Delta I = \vec{j} \cdot \Delta\vec{s}$ flowing with a density $\vec{j} = \hat{\phi} j(r)$ through a “window” or cross section $\Delta\vec{s} = \hat{\phi} dr dz$ of a ring embracing an area $\vec{A} = \hat{z}\pi r^2$, we write,

$$\Delta\vec{\mu} = \vec{A}\Delta I = \hat{z}\pi r^2 j_{\phi} dr dz \quad (2.5)$$

Since we are ignoring I_M at this time we identify only two contributions to the total magnetic moment, (see Fig 2.6),

$$\vec{\mu}_{total} = \vec{\mu}_i + \vec{\mu}_o = \vec{\mu}_{pinning} \quad (2.6)$$

where the inner magnetic moment,

$$\vec{\mu}_i = \pm \hat{z}\pi \int_0^{r_i} j_i r^2 dr dz \quad (2.7)$$

and the outer bulk magnetic moment

$$\vec{\mu}_o = \pm \hat{z}\pi \int_{r_i}^{r_o} j_o r^2 dr dz \quad (2.8)$$

The induced current densities $j_i(r)$ and $j_o(r)$ are linked via Maxwell's eqn $\nabla X \vec{H} = \vec{j}$ with the flux density gradients hence arise from pinning of the flux lines in the body of the specimen. We let the subscripts i and o denote the inner and outer annular regions and drop the subscript ϕ for brevity and generality.

Among the vast family of magnetic moments which are generated during isothermal swings of H_a which cause the locus of $\vec{\mu}$ to trace a bridge between the envelopes of the major hysteresis loop we select the special but nevertheless representative set where $|\vec{\mu}_{total}| = 0$, hence where $|\vec{\mu}_{total}|$ crosses the horizontal axis during its excursion. In our experimental investigations the sweep of H_a ceases at this juncture. Then with H_a now maintained fixed the temperature of the specimen is gradually raised to T_c . The evolution of the magnetization is continuously monitored and graphed versus T and for various H_a values as these events take place.

2.2.3 Evolution of Magnetization upon Warming from $\langle M \rangle = 0$

Although in the situations which we address, the spatially averaged $\vec{\mu}$ is initially zero, a net magnetic moment now appears when T is raised from T_o and causes a decay of the persistent currents. The increase of T causes $|\vec{\mu}_{total}|$ to grow because the rise in temperature causes the boundary r_i between the two regions to migrate towards the outer radius thereby increasing the cross-section assigned to the inner magnetic moment and correspondingly diminishing the annular volume allocated to the outer moment.

Concomitantly the total current per unit height, z , $I_o(T) = \int_{r_i(T)}^R j_o(r, T) dr$ occupying the

outer annulus is seen to decay by a larger amount than the total current,

$$I_i(T) = \int_0^{r_i(T)} j_i(r, T) dr \text{ occupying a unit length of the inner volume.}$$

Faraday's law of induction dictates the evolution of r_i with temperature. Since in our work the temperature is increased gradually we regard the sequences of flux density profiles and configurations of circulating currents to correspond to quasi-static critical states determined by T and the appropriate parameters characterizing the specimen. The mutual repulsion of flux lines dictates that they can migrate only "down" the slopes of a flux density profile as illustrated in Fig 2.7. By symmetry, the velocity of the flux lines whose location corresponds to the instantaneous position of the top of the peak (or the bottom of the valley) in the flux density profile must be zero. Consequently at r_i , the electric field, $\vec{E} = \vec{B} \times \vec{v} = 0$.

To fix ideas we focus on the configuration of flux density (B profile) displayed in Fig 2.7(a). The rise in temperature causes a decline of j_c hence the field gradients dB/dr must become shallower. Consequently, flux lines are made to migrate down the slopes of the B profiles. The flux contained in the volume $0 < r < r_i$ redistributes while some of the flux permeating the volume $r_i < r < R$ exits from the specimen. Since the expulsion of flux lines causes $\langle B \rangle$, initially equal to $\mu_o H_a$, to fall below $\mu_o H_a$ in the standard definition of the magnetization,

$$\mu_o \langle M \rangle = \langle B \rangle - \mu_o H_a \quad (2.9)$$

a net diamagnetic magnetic moment therefore appears. In Fig 2.7(a) we endeavour to illustrate why the exit and redistribution of flux causes the boundary r_i to migrate outward. Note that the flux lines represented by the stipled area in the sketch and initially threading the annular volume $r_{int} < r < r_i$ has been made to slide down the flux density gradient by the rise in temperature and now occupies the volume $0 < r < r_{int}$ represented by the vertical dashed "area" in the sketch. Conservation of flux lines then requires that these two "areas" must be equal. A careful drawing shows that here r_i must migrate outward.

The magnitude of this thermally released magnetic moment increases until the boundary r_i reaches the surface R . At this juncture the outer moment has been extinguished and the surviving fraction of the inner magnetic moment laid bare. The subsequent rise in T to T_c now causes this remaining magnetization to also disappear as j_c is further diminished to zero. An estimate of $\langle M \rangle_{peak} = \mu_{peak}/V$, the magnetization when this peak is reached, can readily be obtained by assuming that all the excess flux density (i.e. $\langle B(r) \rangle > \mu_0 H_a$) threading the outer façade of the initial B profile has exited from the specimen as T increased to T_{peak} and r_i moved to R . This approach which overestimates $\langle M \rangle_{peak}$ is developed in chapter 4.

The events just described are not affected when the specimen initially contains flux lines of positive and negative polarity as sketched in Fig 2.7(c) although here the redistribution of flux in the volume $0 < r < r_i$ is accomplished by annihilation of flux lines along the boundary where $B(r) = 0$. In these circumstances it is the net amount of magnetic flux permeating the volume embraced by r_i which is conserved as r_i migrates to the surface.

In situations where H_a has been made to increase after reversal of its sense of sweep along the hysteresis envelope, B profiles as sketched in Fig 2.7 (b) and (d) will be encountered. In these circumstances, the B profiles trace a valley instead of a summit. Now as T is raised the accompanying diminution of j_c hence lowering of the slopes of the gradients leads to entry of flux into the specimen. Thus a paramagnetic magnetic moment appears which then vanishes after reaching a peak value.

In our hysteretic specimens (i.e. those exhibiting negligible flux expulsion during field cooling) the locus of $\langle M \rangle$ with heating, hence versus T, traces a V shape if paramagnetism appears or an inverted V, hence Λ shape if diamagnetism is revealed (see Fig 2.8). A graph of the magnitudes of the peak magnetization released by heating starting from $\langle M \rangle = 0$ when plotted versus H_a displays major differences in structure depending on whether μ_{peak} is diamagnetic or paramagnetic. The model we have sketched above for the migration of flux lines and the evolution of the flux density profiles is developed in detail in this thesis and is shown to semi-quantitatively and qualitatively reproduce all the major features of our observations in the strong pinning materials both conventional and high T_c . We will see that the differences between the diamagnetic and paramagnetic cases arise from the sensitivity of j_c on the strength of the magnetic field.

2.3 Release of a Hidden Magnetic Moment (Semi-Reversible Samples)

The evolution of the magnetization with warming from $\langle M \rangle = 0$, but with $\langle M \rangle$ comprising opposing magnetic moments is considerably more intricate in materials where

the Meissner current I_M is comparable in magnitude to the total critical current I_c which the bulk can support.

Our investigations of the thermal release of hidden magnetic moments in semi-reversible type II superconductors, both conventional and high T_c , reveal a novel behaviour not encountered in strongly hysteretic materials. First a paramagnetic moment appears and then vanishes and now a diamagnetic moment emerges which is eventually extinguished as T attains T_c . This S shape behaviour is illustrated in Fig 2.9 (a). This intricate "performance" is encountered when in the final approach to the $\langle M \rangle = 0$ axis H_a has been increasing in magnitude and the final value of $H_a < H_*$. Here to fix ideas we focus on the right hand quadrants of $\langle M \rangle$ versus H_a as displayed in Fig 2.9 (b).

This behaviour can be understood qualitatively by postulating that the Meissner field shielding current $I_M(H_a, T)$ initially diminishes much more gradually with rising temperature than the bulk current density $j_c(B, T)$. The resulting evolution which ensues from this assumption is shown schematically in Fig 2.10. In the sketch, for simplicity, we regard I_M as initially independent of the increase in temperature.

In the H_a ascending to $\langle M \rangle = 0$ case as depicted in Fig 2.10 (a) the decrease of the field shielding j_c in the outer annulus causes the outer diamagnetic moment μ_o to diminish in magnitude and allows flux to enter the specimen. Consequently the specimen acquires a net paramagnetic magnetization. Concurrently the flux permeating the inner magnetic moment redistributes. Once r_i has migrated to the surface, hence the outer diamagnetic moment μ_o has disappeared, the decline of $j_c(B, T)$ still more rapid than that of $I_M(H_a, T)$ leads to a release of trapped flux by the inner paramagnetic moment μ_i , now occupying the entire volume (less the penetration depth λ), as it diminishes. At some

stage in this evolution the rapidly diminishing bulk paramagnetic moment μ_i becomes equal in magnitude to the Meissner moment $|\mu_M| = |I_M(H_a, T)\pi R^2|$ hence $\bar{\mu}_{total} = \bar{\mu}_i + \bar{\mu}_M = 0$. Further decrease of μ_i with rising temperature at a faster rate than the diminution of $|\mu_M|$ leads to the appearance of a net diamagnetic moment which reaches a peak, then declines and disappears as both the surviving j_c and I_M are extinguished. These features are developed in quantitative detail in this thesis.

However we find that only a diamagnetic moment is released when the $\langle M \rangle = 0$ axis has been approached by causing H_a to descend in magnitude. As depicted in Fig 2.10 (b) this is due to the fact that as the flux retaining j_c in the outer annulus declines with temperature, flux will tend to initially migrate out of the specimen as shown and a net diamagnetic moment will develop (assuming that $I_M(T)$ has remained nearly constant as the temperature is first increased). When the boundary r_i reaches the surface R , the maximum diamagnetic moment is revealed. After this point the further decline of $j_c(T)$ and finally of $I_M(T)$ will then promote the reentry of flux until $\langle M \rangle = 0$ is reached at T_c .

It would only be possible for any paramagnetic moment to be revealed if $I_M(T)$ initially declined much more rapidly than $j_c(T)$. Such a rapid initial drop in $I_M(T)$ would effectively mimic a rising surface field initially driving flux into the specimen faster than the diminution of $j_c(T)$ would be promoting flux leakage from the outer paramagnetic moment. As the decrease in $I_M(T)$ with temperature is not seen to be more rapid than for $j_c(T)$, no initial paramagnetic moment is expected or observed and the thermal evolution of $\langle M \rangle$ in these circumstances is thus similar for both semi-reversible and highly hysteretic specimens. Now however we note that the magnitudes of the ratio $\mu_{peak}/\mu_{max \text{ rem}}$ which we observe in semi-reversible specimens are much larger than that

displayed by the irreversible samples. Indeed this ratio is seen to rise as the ratio $I_M(H_a, T)/H_c$ is made larger.

After a brief description of the experimental set up and procedure we will present our observations on very irreversible and semi-reversible conventional and high T_c type II superconductors. Then we will develop in more detail the ingredients of the model we have just conceptually sketched in this overview. This will show that the theoretical curves which our model generates consistently “predict” all of the trends and features encountered in our measurements.

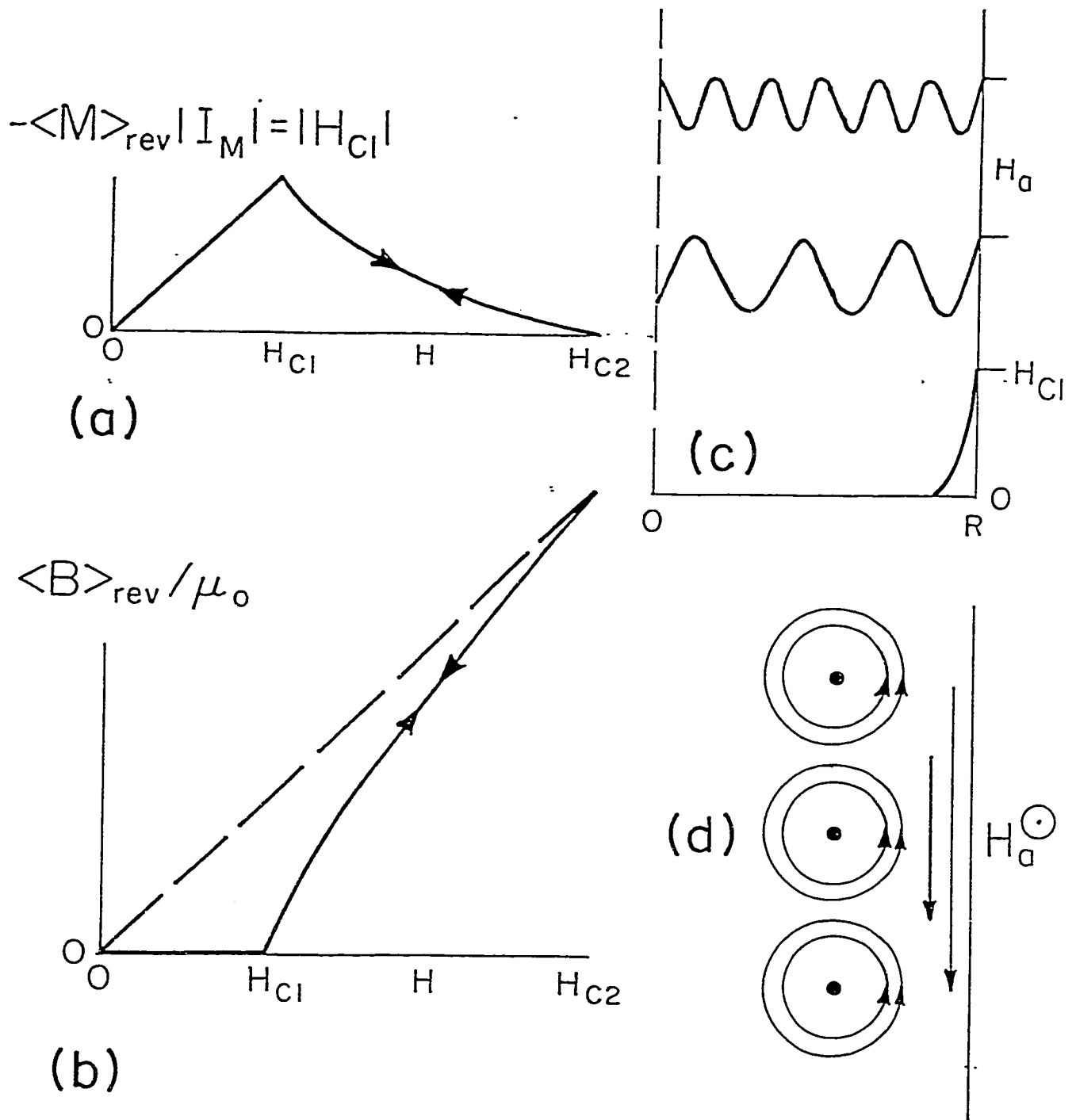


Fig 2.1 (a) Isothermal curve of $\langle M \rangle$ versus H_a for an "ideal" type II superconductor (b) Isothermal Curve of $\langle B \rangle$ versus H_a (c) Magnetic flux density profiles depicting penetration and redistribution of flux lines (d) Current patterns of the paramagnetic vortex currents and the diamagnetic Meissner surface currents

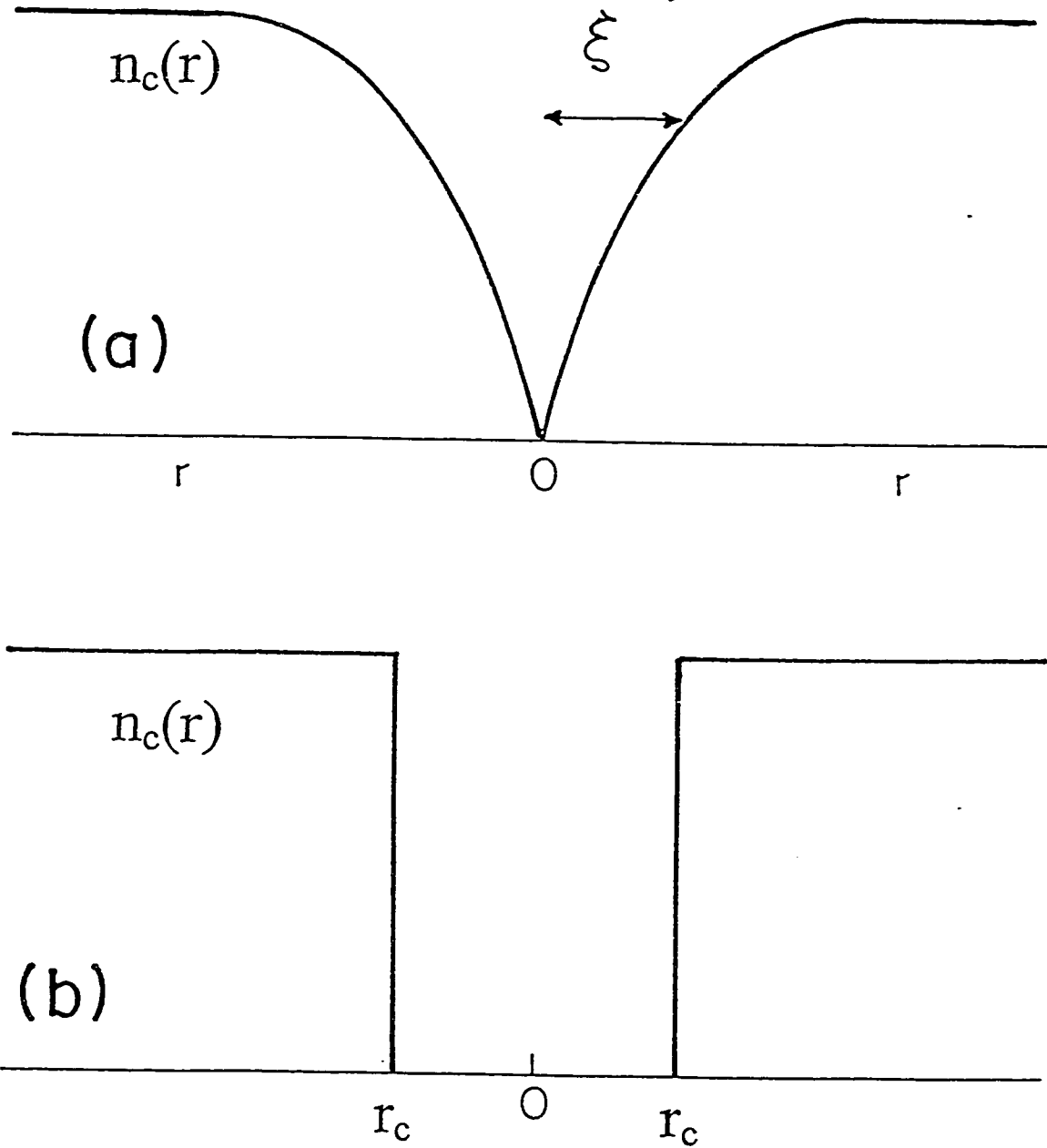


Fig 2.2 (a) Depicts the density of Cooper pairs, $n_c(r)$ surrounding the axis of an individual vortex with the characteristic dimension, ξ . (b) Depicts a void or normal region with associated drop in n_c to zero in the normal zone.

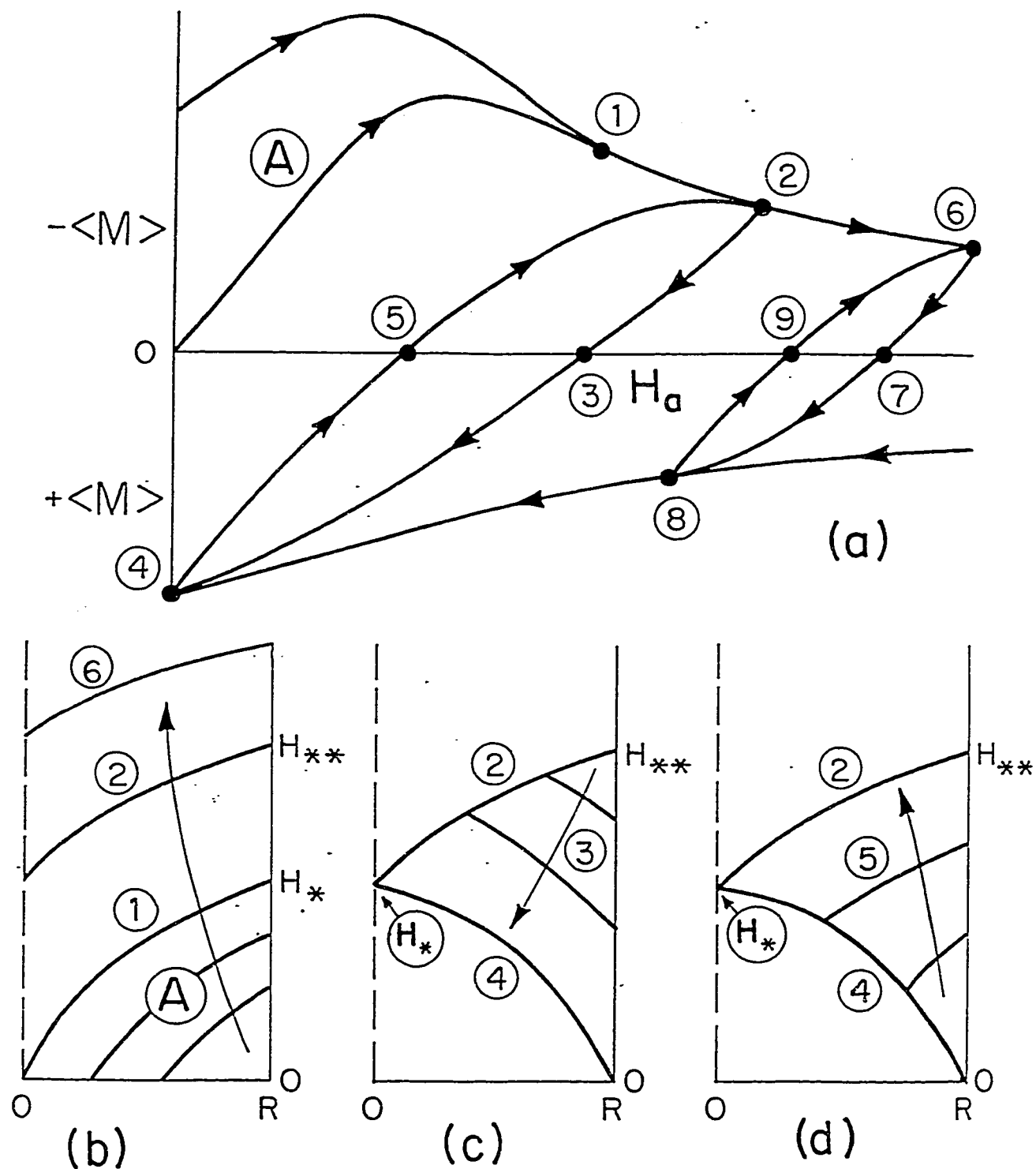


Fig 2.3 (a) Isothermal $\langle M \rangle$ versus H_a for a strongly hysteretic specimen. (b), (c) and (d) associated magnetic flux density profiles (B profiles) for the specimen as discussed in the text. In this and later drawings the numbers (letters) which appear along the flux density profiles correspond to points (zones) along the $\langle M \rangle$ versus H_a curves.

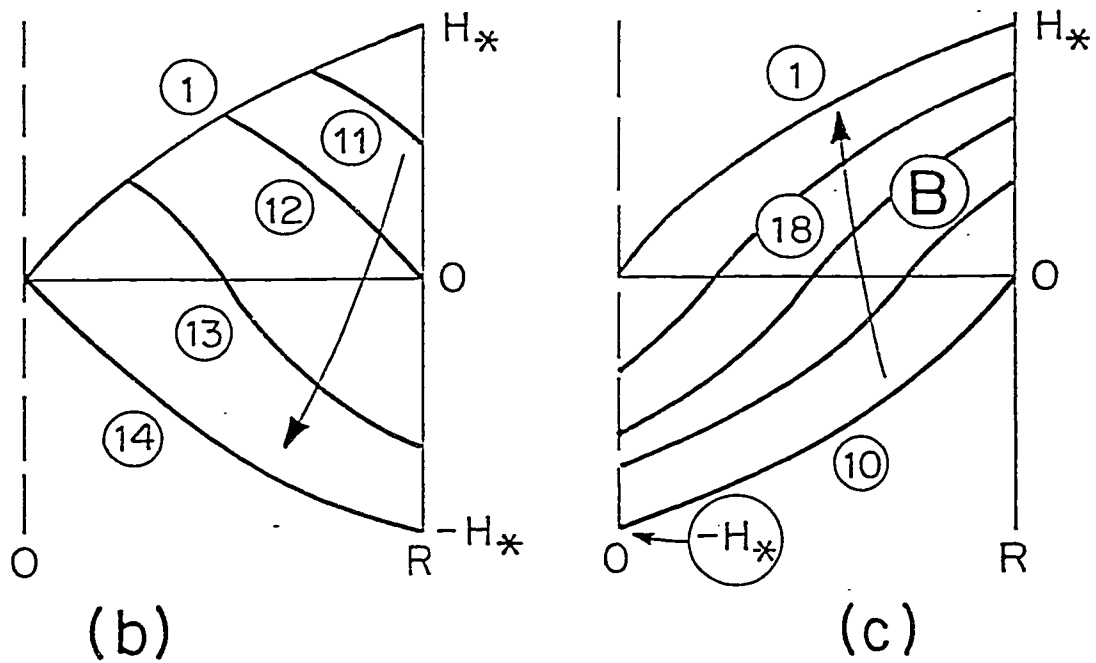
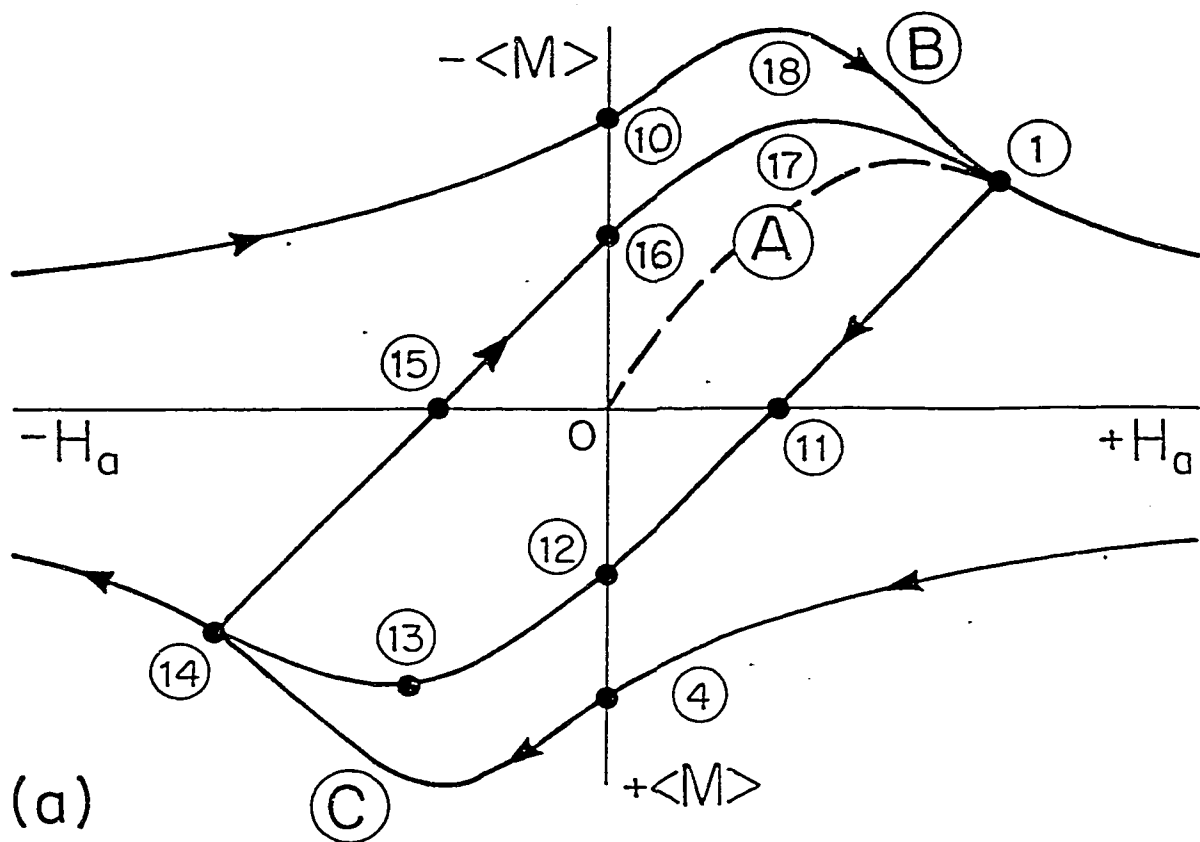


Fig 2.4 (a) Isothermal $\langle M \rangle$ versus H_a for a strongly hysteretic specimen including negative H_a quadrants. (b) and (c) associated magnetic flux density profiles as discussed in the text.

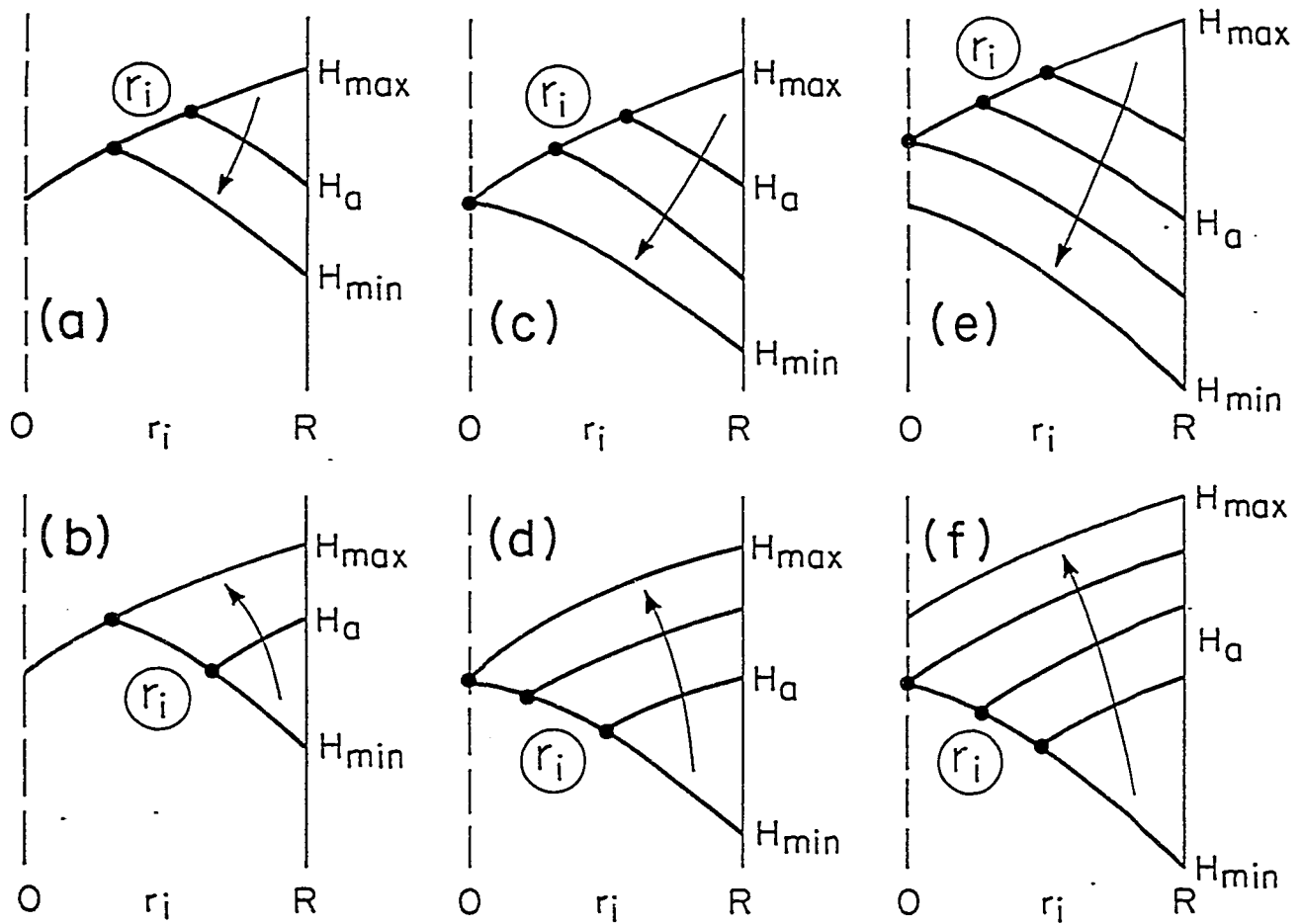


Fig 2.5 Sequences of B profiles as H_a is made to descend from $H_{\max}$ to $H_{\min}$ (a,c,e) and then to ascend from $H_{\min}$ to $H_{\max}$ (b,d,f). The pair (a) and (b) correspond to an incomplete, (c) and (d) exactly complete, and (e) and (f) to an extended complete hysteresis loop

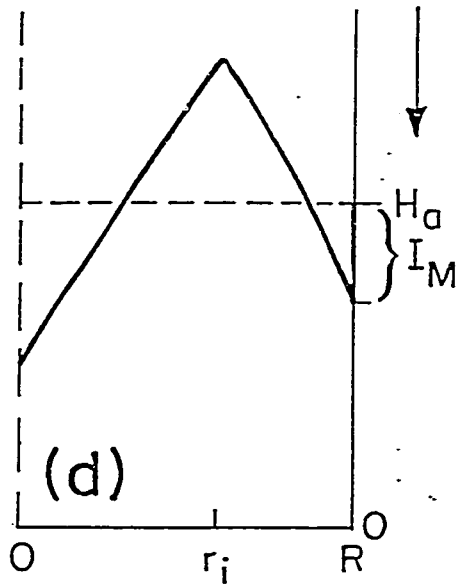
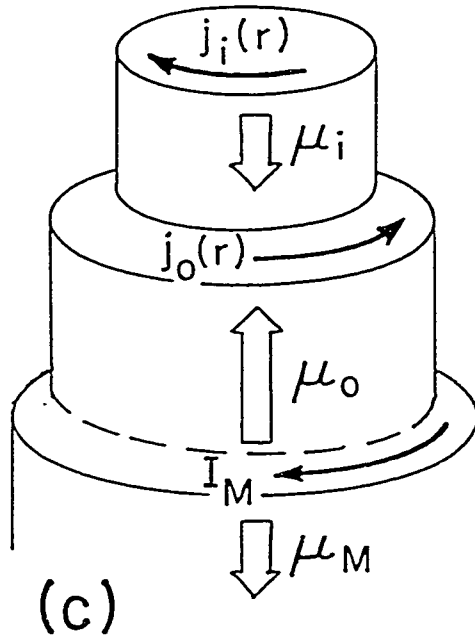
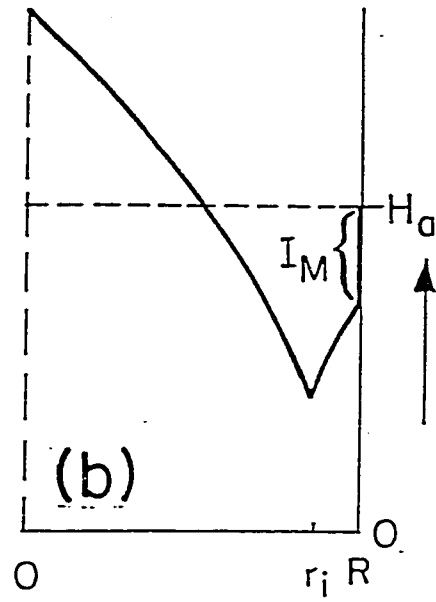
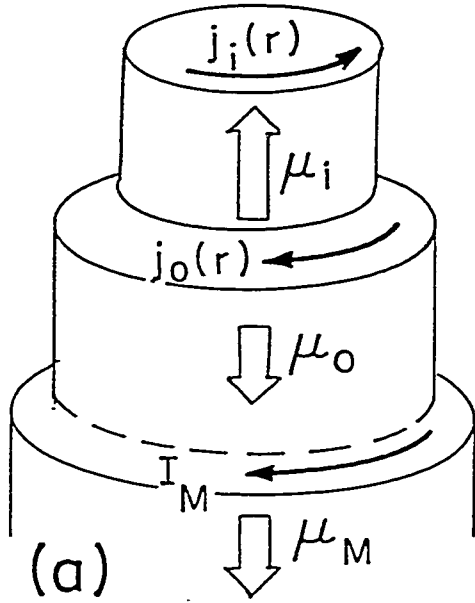


Fig 2.6 (a) Depicts the concentric volumes of the inner and outer bulk critical currents, the Meissner surface current and the three resultant magnetic moments when the $\langle M \rangle = 0$ state is reached by an increasing H_a . (b) The associated B profile (c) Depicts the concentric volumes of the inner and outer bulk critical currents, the Meissner surface current and the three resultant magnetic moments when the $\langle M \rangle = 0$ state is reached by a decreasing H_a (d) The associated B profile

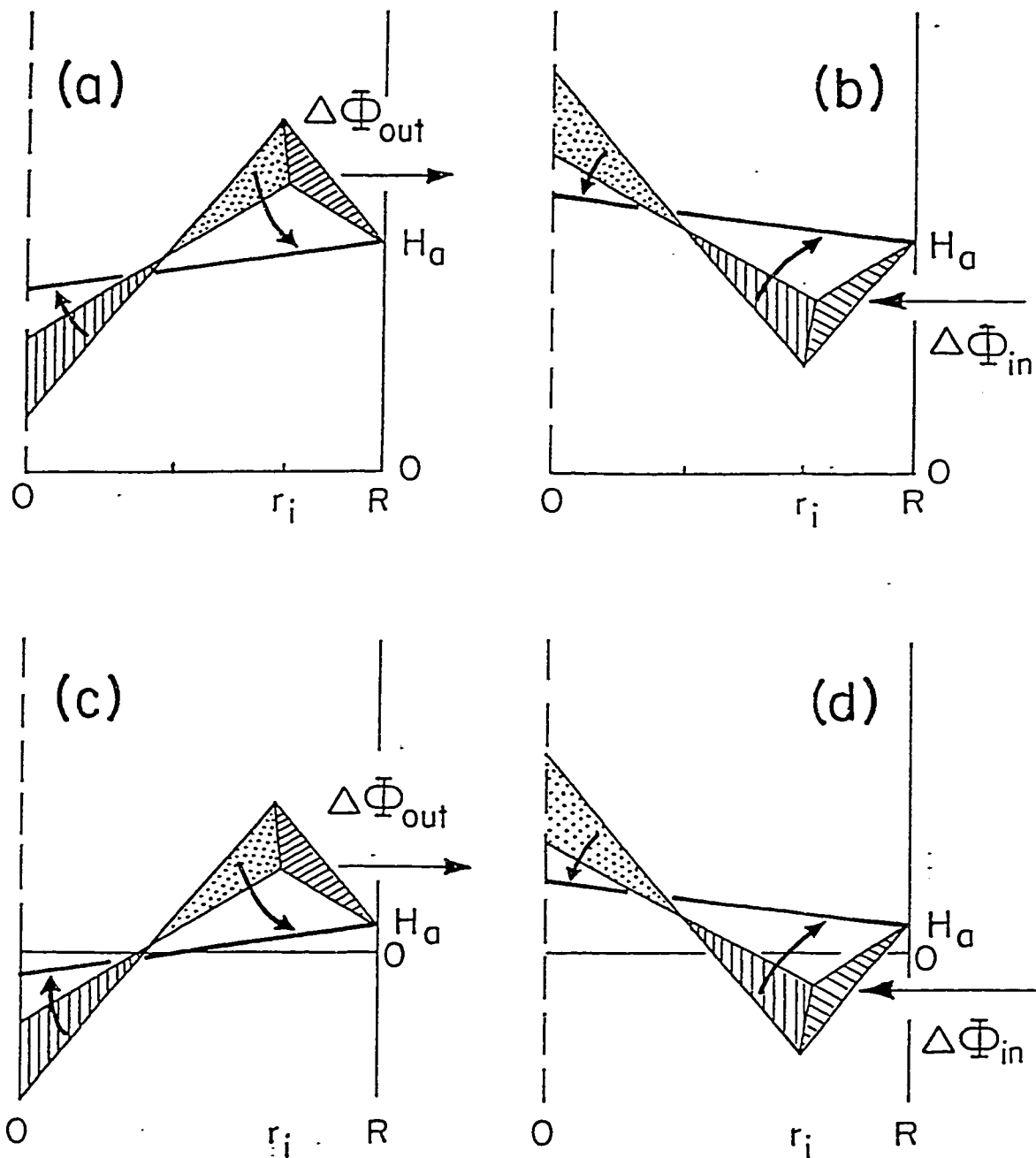


Fig 2.7 (a) and (b) Depicts the evolution of B profiles in relatively high field as a temperature increase causes a lowering or relaxation of the bulk critical current. As discussed in the text flux will move down the “slope” of the dB/dr gradient and flux lines constituting the “stipled” area will migrate into area depicted by “vertical lines”. (c) and (d) A similar action but with redistribution also involving “annihilation” of flux lines of opposing polarity.

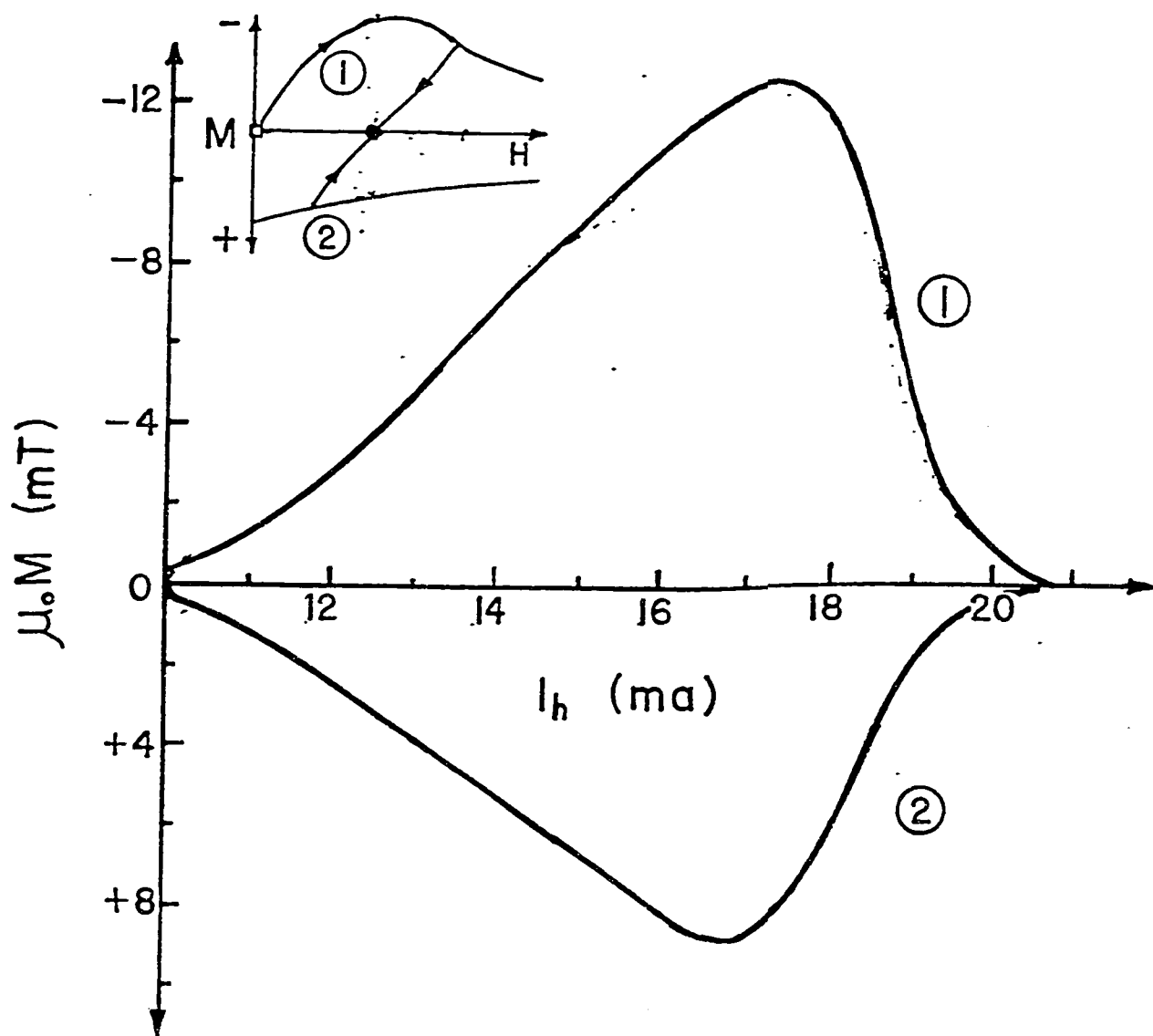


Fig 2.8 Depicts typical evolution of magnetic moment relative the heater current, hence relative to temperature, for the “downswing” case (1) and the “upswing” case (2)

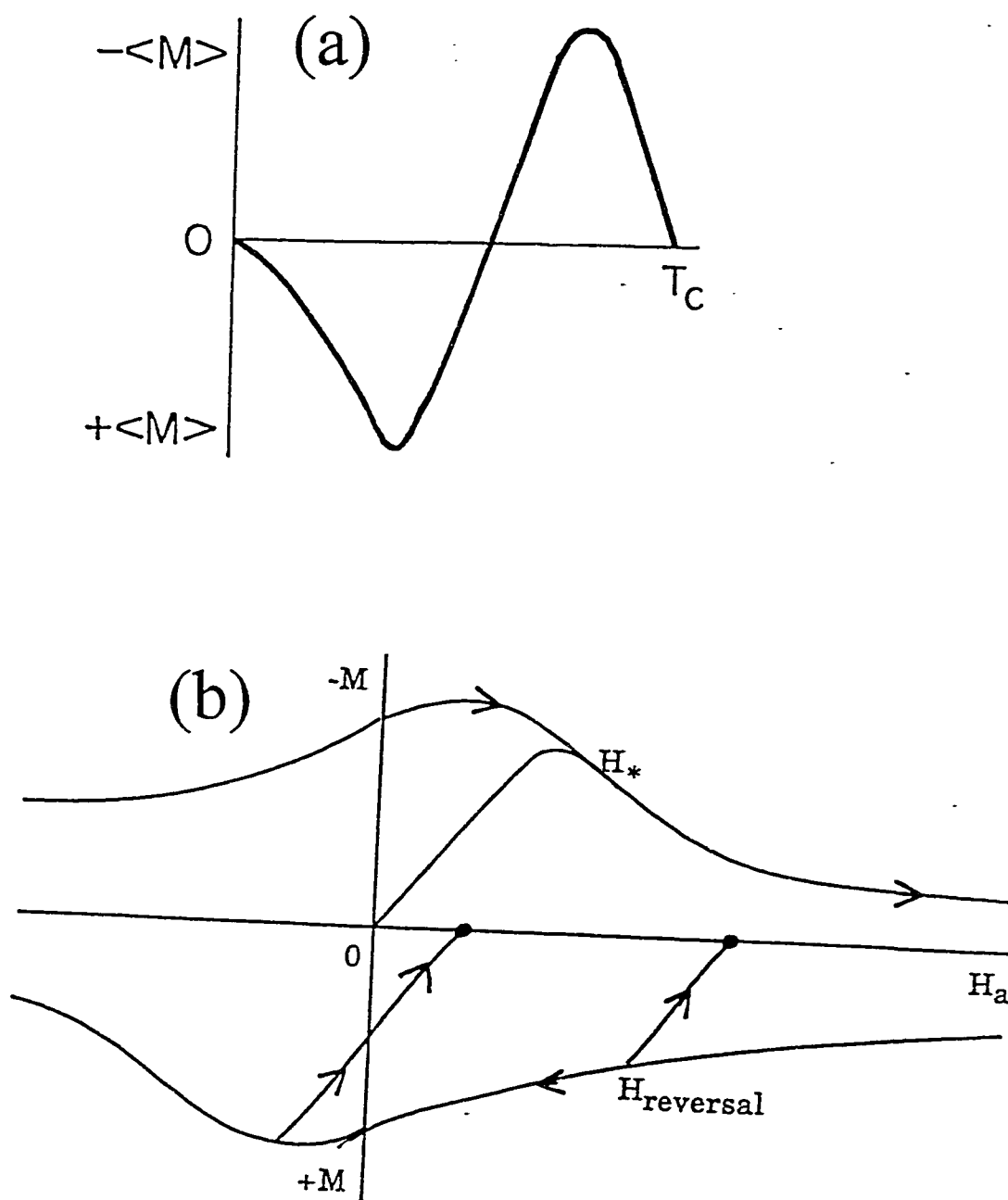


Fig 2.9 (a) Qualitative depiction of S shaped disclosure of first paramagnetic and then diamagnetic moment as temperature is increased for a semi-reversible specimen reaching $\langle M \rangle = 0$ with ascending H_a i.e. upswing, as shown in (b)

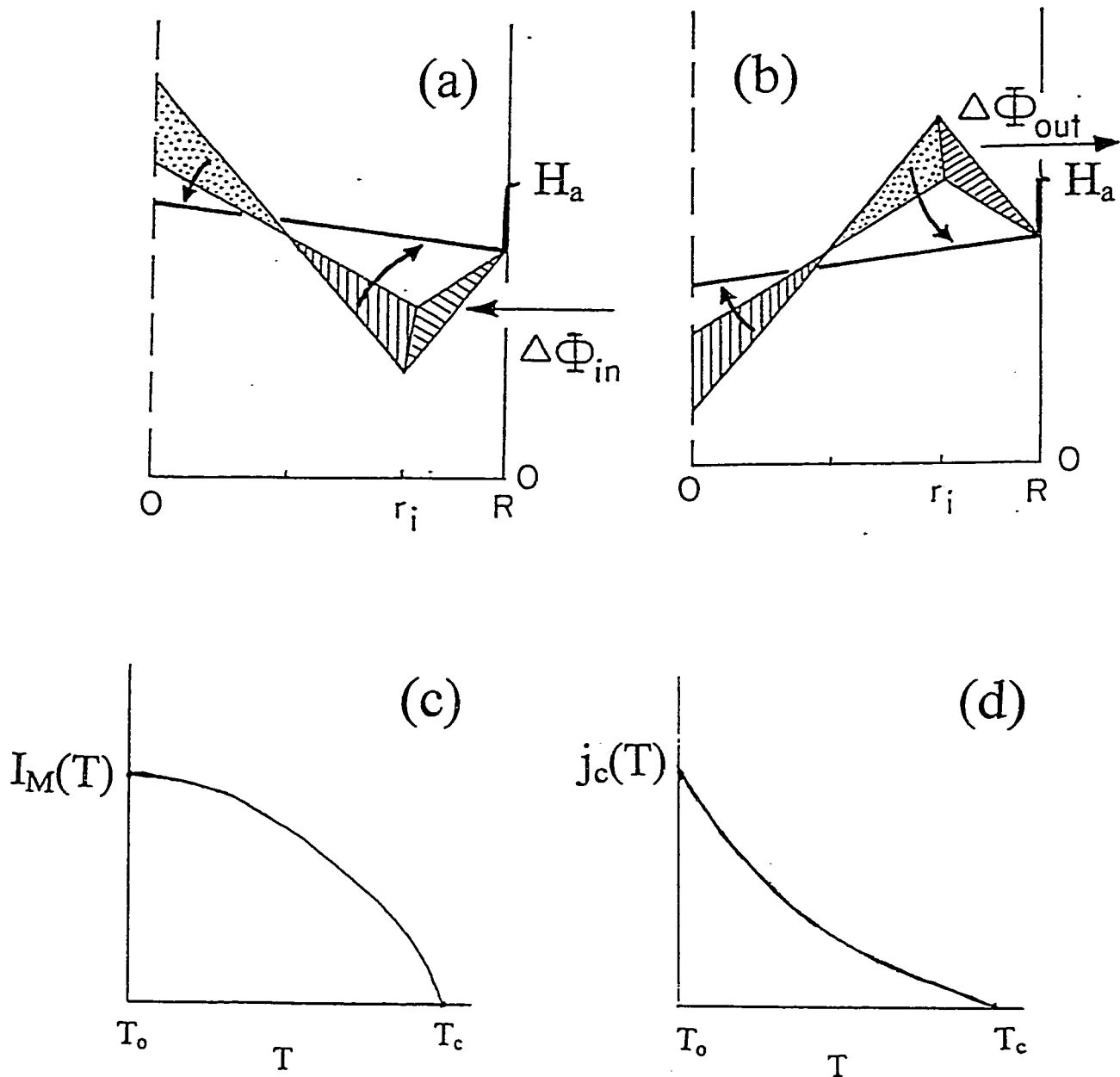


Fig 2.10 Illustrates why as discussed in the text, a decline of I_M versus T which is initially much more gradual than the diminution of j_c versus T (compare (c) and (d)) will lead to an S shaped evolution of $\langle M \rangle$ during warming to T_c starting with $\langle M \rangle = 0$. It is useful to visualize the extreme limiting case where initially I_M is not changing with T . In (a) we see flux entering, hence paramagnetism appearing, as j_c declines and r_i migrates to R . Now further decline of j_c with I_M still unchanged or very slowly diminishing leads to a large release of the flux hence a decrease of paramagnetic moment, then $\langle M \rangle = 0$ and next a growth of a diamagnetic moment. Finally however both j_c and I_M vanish as T_c is approached hence the diamagnetic moment vanishes as flux enters. For the downswing case (b) flux leaves hence diamagnetism appears as j_c declines with I_M nearly constant, and r_i migrates to R . Now the further decrease of j_c and also I_M as T_c is approached allow flux to enter, hence the suppression of the diamagnetic moment.

CHAPTER 3

Experimental Set Up and Procedures

3.1 Introduction

The experimental requirements for our study of thermal release of hidden magnetic moments can be accomplished by various technologies presently available to workers in the field. Primary needs are the ability to accurately monitor the evolution of the magnetization $\langle M \rangle$ of a sample as an external field H_a is varied and more importantly to follow the change in $\langle M \rangle$ as the temperature of the sample is changed at a fixed field.

The methods we shall describe in this chapter involve the use of pick up coils that intimately embrace the length of the sample. This allows the “continuous” monitoring of $\langle M \rangle$ by integrating amplifiers. This ability to “continuously” monitor $\langle M \rangle$ of a sample is for the most part unique to our measurement technique and allows observation of phenomena that might otherwise be missed by the more common techniques of Vibrating Sample Magnetometer, Squid Magnetometer etc¹⁴⁻¹⁶. These alternative techniques build up curves of $\langle M \rangle$ versus T or usually H_a by discrete measurements of $\langle M \rangle$ at a succession of these variables. Our ability of continuous monitoring is especially important where the variation of $\langle M \rangle$ versus T is desired. The evolution of $\langle M \rangle$ versus T can depend upon whether cooling or heating rates are slow, hence quasi-isothermal, or rapid thereby causing a temperature wave or front to pass through the sample. Our ability to obtain a continuous recording of $\langle M \rangle$ regardless of heating rates sets this technique

apart from more common methods which would require stepwise changes in temperature for all discrete (point by point) readings at each specific temperature.

The price paid for these advantages comes primarily in the need for samples of larger size than in the other techniques. As well, due to the fact that “integrating” amplifiers are employed, accumulative noise or signal drift is an ever present factor which means that individual $\langle M \rangle$ vs H_a or T runs should take place within a reasonable time frame. Starting with this choice of $\langle M \rangle$ measurement technique and bearing in mind the limitations it imposes, the overall experimental set up will now be reviewed and the details of procedures necessary for sample characterisation discussed.

3.2 Samples Employed

In order to fully examine the phenomenon of thermal release of magnetic moments for a wide range of superconducting materials, representative specimens were chosen among both high T_c and conventional low T_c type II superconductors. Further in each class, both weakly pinning semi-reversible materials along with stronger pinning irreversible samples have been utilized. As appropriate representatives of highly hysteretic and semi-reversible behaviours respectively we selected conventional low T_c $V_{0.76}Ti_{0.24}$ and Nb and for the high T_c materials $YBa_2Cu_3O_{7-x}$ and $(Bi_{0.9}Pb_{0.1})_2Sr_2Ca_2Cu_3O_{10}$ abbreviated to YBCO and BiSCCO as is customary.

The VTi sample was prepared by the Material Research Corporation by repeated arc melting of a high purity mixture resulting in an ingot of $V_{0.24}Ti_{0.76}$. Slab samples cut from this ingot have been examined previously by Boyer et al¹⁷, LeBlanc et al^{18,19} and

Gandolfini et al²⁰. For our studies a cylinder 2.5 cm long and 0.6 cm in diameter was machined on a lathe from this ingot.

The Nb sample comprised thin ribbons (slabs) cut from a Niobium sheet of high purity niobium metal provided by Kawecki Berylco Industries. Discs and ribbons cut from these sheets have been extensively studied by LeBlanc et al^{18,21,22} and by Cave and LeBlanc²³. Since the sheet thickness was only 0.025 cm and in view of the limitations on measurement sensitivity mentioned earlier, a stack of seven parallel ribbons was prepared. Individual ribbons are 1.0 cm wide and 3.0 cm long. These were electrically insulated from each other by inserting a strip of 0.002 cm thick mylar between each adjacent pair of ribbons. A final rectangular shaped specimen thus had a thickness of $\approx$ 0.185 cm.

We investigated two very different specimens of YBCO. While sharing chemical composition the magnetization curves of these specimens at 77 K are very dissimilar. YBCO samples prepared by the standard co-precipitation method result in a polycrystalline material whose individual grains are only weakly linked in regards to transport critical currents i.e. the intergrain j_c . Thus in these materials the magnetization data comes primarily from the intragrain j_c of the agglomeration of individual small crystals. The two YBCO samples we studied do not fall into this category of weakly linked materials.

The first sample was prepared at Argonne National Laboratories by a special technique known as Quench Melt Growth (QMG)²⁴⁻²⁷ which results in single crystal samples of large dimension. The specific sample obtained is actually an amalgamation of several large crystals formed within a hexagon shape. This hexagon shape is from the die

used to press the raw material before a final heat treatment which utilizes a small seed crystal to nucleate crystal growth. In these QMG samples the high j_c possible within each single crystal is now effectively the bulk macroscopic current. These samples retain a high j_c up to very high magnetic fields. The dimensions of the sample employed are a hexagon of 1 cm thickness and 2 cm between pairs of opposing faces.

The second YBCO sample was a rod prepared via a proprietary melt texture technique by Hoechst of Frankfurt Germany. This technique results in similar medium size grains with their c axis along the length of the rod and with far superior intergrain current carrying capacity than achieved in the co-precipitation method. Consequently the magnetic properties are dominated by these macroscopic currents in the field ranges present in our study. The sample consists of a single rod as extruded by the melt texture technique. Its dimensions are 0.6 cm in diameter and 3.0 cm in length.

The final High T_c specimen was a slab of BiSCCO superconductor $(\text{Bi}_{0.9}\text{Pb}_{0.1})_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$ hence, in the 2223 phase, having dimensions 2.5 cm in height, 2.5 cm in width and 0.21 thickness. It was provided by Dr. Vladimír Plechácěk at the Institute of Physics of the Czech Academy of Science in Prague. Like simple YBCO samples prepared by the coprecipitation method, the BiSCCO specimen is a collection of fine grains only weakly linked electrically (low intergrain j_c). These intergrain currents are easily diminished by modest magnetic fields and make a negligible contribution to the magnetization observed in our studies. Thus it is the strong intragrain j_c within the small grains which is the primary observable quantity. Other similar samples also provided by Dr. Plechácěk have been extensively investigated by LeBlanc et al²⁸ and Celebi and LeBlanc²⁹ for flux trapping properties and flux line cutting phenomena^{3,28,30-41}.

3.3 Sample Heater Arrangement

Measurements on the conventional type II superconductors are performed in a liquid helium bath at atmospheric pressure hence at 4.2 K. The high T_c samples are immersed in a bath of liquid nitrogen at atmospheric pressure hence 77 K. In our experiments it is necessary to heat the specimens to a temperature slightly above their respective critical temperature T_c within a reasonable time frame. After the application of heat, the time required for the specimen to return to the ambient temperature of the cryogenic bath must also occur within a reasonable time frame. Finally the electric heater must not generate magnetic fields that could be sensed by the pick up coils.

In order to accomplish these goals a non-inductive bifilar heater coil is wound directly onto the various specimens. Bifilar refers to the technique in which a doubled length of heater wire is used as a single unit. The heating current enters along one wire of the pair and returns along its twin, thus the magnetic fields generated by the currents are cancelled out in this arrangement. Insulated #38 gauge manganin wire is used and applied uniformly usually as a single layer directly on the sample surface to ensure an intimate thermal contact and uniform heating. Manganin wire is used instead of other metals (e.g. constantan, nichrome etc) because it is not magnetic and only very weakly paramagnetic. Its high resistivity ensures that only a small electric current is needed to provide sufficient I^2R Joule heat. Its negligible change of resistance between 300 and 4.2 K is also of great convenience since the same power supply can be used for liquid nitrogen and helium experiments.

The sample now embraced by the heating wire is then modestly thermally insulated from the surrounding cryogenic bath. The level or thickness of insulation strikes a proper balance between the rates of heating to T_c and of cooling back to 4.2 or 77 K. Heating to above T_c can be performed typically between 5 to 10 seconds in the various specimens while cooling from $T > T_c$ back to ambient range requires from 10 to 20 seconds in the smaller low T_c samples but up to 2 or 3 minutes for the large YBCO crystal specimen.

3.4 External Field Magnets

To provide the necessary uniform applied magnetic field, several solenoids were fabricated to accommodate the various specimens and their associated pick up coil assemblies. For studies at 4.2 K a superconducting solenoid was employed. This magnet was wound onto an aluminium former using 0.5 mm diameter, formvar insulated, multifilamentary twisted NbTi wire embedded in a copper matrix and embraced by a copper sheath and heavy formvar insulation. The large copper to superconductor ratio of 2.5 ensures that the solenoid will not be damaged if driven to critical. However we have never attempted to do so. The maximum residual trapped field at the centre of the solenoid due to persistent currents set up in the NbTi material itself is ≈ 0.2 mT. Its dimensions are 15 cm length, 2.5 cm inner diameter and 5.0 cm outer diameter.

The applied magnetic field experienced by the centrally located specimen is found by the law of Biot Savart,

$$\Delta \vec{H} = \frac{\vec{j} \times \vec{R}}{4\pi R^3} dV \quad (3.1)$$

where dV is an element of volume, which leads to,

$$H_a = \frac{NI}{2(R_o - R_i)} \ln \left\{ \frac{R_o + \sqrt{R_o^2 + (L/2)^2}}{R_i + \sqrt{R_i^2 + (L/2)^2}} \right\} \quad (3.2)$$

where N is the number of turns, L is the length of the solenoid and R_i and R_o are the inner and outer radii of the solenoid. This leads to $B/I = 65.4$ mT/amp for the superconducting solenoid.

For the high T_c samples studied in liquid Nitrogen, conventional solenoids were wound using 14 and 16 gauge copper wire wound onto a Bakelite former of appropriate dimensions. Two separate coils were wound with inner diameters of 5.0 cm and 3.8 cm to better accommodate the variations in the sample and pick up coil arrangements. Both solenoids have an outer diameter of 11.5 cm and a length of 16 cm. From eqn 3.2 one obtains a B/I of 32.84 mT/amp for the solenoid with a larger bore and 38.33 mT/amp for the other.

For all studies a small resistive shunt is placed in series with the power supply and magnet. The voltage drop across this shunt will be proportional to the current flowing through the solenoid, $V_{shunt} = I R_{shunt}$. This voltage is thus used to drive the X axis of an X-Y recorder. Furthermore, the current is recorded at specific points for calibration or conversion to the actual applied field using the respective B/I values listed above.

3.5 Pick Up Coil Assemblies

In order to monitor the evolution of the magnetic moment a main pick up coil surrounds the length of the specimen. Such a coil responds to changes in the magnetic flux density B over the entire volume it encloses. As we require information regarding the magnetic moment or magnetization of the sample, the contribution of the externally applied field can be instrumentally removed giving $\mu_o \langle \vec{M} \rangle = \langle \vec{B} \rangle - \mu_o \vec{H}$ where $\langle B \rangle$ is the spatial average of the magnetic flux permeating the specimen. This negation of the external applied field is accomplished by placing a separate pick up coil of equal dimension (i.e. number of turns and volume embraced) in series opposition with the main coil. However the second coil does not surround the sample although it bathes in the same applied field. These extra coils are termed bucking coils. Where possible a split pair of bucking coils is utilized above and below the main pick up coil. This helps assure that any field non-uniformities or slight movement of the overall assembly within the solenoid will be partially compensated.

Once the main coil is wound we endeavour to wind the bucking coil with an equal number of turns and average embraced area. Generally however the pair of pick up coils do not initially balance each other exactly. With the pick up coil empty or alternatively with the specimen held above T_c the magnitude and polarity of the resulting signal is carefully measured and used to calculate the number of turns that must be either added or removed from the bucking coil. This process is repeated until the desired level of balance is attained. Typically to obtain a satisfactory signal we require a central coil of

several tens of thousands of turns. However a few dozen turns too many or too few in the bucking coil will lead to a significant undesirable background signal.

Once the pick up coil pair are properly balanced the sample-heater arrangement can be mounted within the central pick up coil. In operation any change of the magnetization of the sample with time due to changes in the applied field and/or changes in the temperature of the sample will induce a net emf from the coil,

$$\varepsilon(t) = NA \frac{d\langle M \rangle}{dt} \quad (3.3)$$

N is the number of turns of the pick up coil and A the cross section of the sample. The instantaneous emf is fed to an integrating/amplifier and is thereby continuously integrated or accumulated and produces a net signal S(t),

$$S(t) = c \int_0^t \varepsilon(t) dt = c N A \langle M(t) \rangle \quad (3.4)$$

where c is the amplification factor. The signal S(t) is thus proportional to $\langle M(t) \rangle$ and drives the Y axis of an X-Y recorder. The calibration procedure is discussed below. A typical pick up coil arrangement is depicted in Fig 3.1 which would house the sample and heater within the central coil.

3.6 Measurements of $\langle M \rangle$ vs H_a

As described above, the pick up coil signal proportional to $\langle M \rangle$ from the integrator/amplifier can be used to drive the Y axis of an X-Y recorder and the resistive shunt voltage drop proportional to H_a monitored on the X axis. Thus continuous $\langle M \rangle$ vs H_a curves are obtained in a straightforward manner and relatively rapidly. The experimental set up is shown schematically in Fig 3.2.

Even the most careful of pick up coil balancing will however not entirely remove the contribution of the external field to the net signal. As well, even though great care is taken to ensure that all materials present in and around the experimental set up are non-magnetic, contributions from such stray sources cannot be entirely eliminated. Finally when utilising the superconducting solenoids irreversibility effects from the solenoid itself inevitably arise (i.e. persistent currents arise in the superconducting wire material during field cycling). Fortunately these contributions can be identified, measured and systematically removed from the observations. This is accomplished by repeating the field swings for any $\langle M \rangle$ vs H_a curves with the sample maintained just above T_c by means of the electric heater. This background or normal line contribution which is now obtained must then be subtracted from the superconducting data curve to yield the final data curve.

As shown above the integrator/amplifier signal $S(t)$ is proportional to $\langle M \rangle$. In order to accurately calibrate this signal the standard assumption is made that the "virgin" specimen shields against the entry of magnetic flux in the low field region where $H_a < H_{C1}$. Here the field shielding surface Meissner current assures that B within the specimen is zero and thus $\langle M \rangle = |H_a|$ in this region of applied field. In reality B will be non-zero

within the penetration depth λ of the sample where the shielding currents flow. This dimension however is negligible compared to the cross-section of our samples.

3.7 Monitoring $\langle M \rangle$ vs Temperature

The main goal of this project has been to examine the evolution of $\langle M \rangle$ in a fixed H_a as the temperature is raised to T_c and the persistent currents contributing to $\langle M \rangle$ are affected. Specifically we have focused on the cases where the swings of the field H_a has induced opposing patterns of persistent currents resulting in an overall cancellation of the magnetic moment thus a net $\langle M \rangle$ of zero within the sample. The surprisingly complex behaviour of $\langle M \rangle$, as T is raised, such as the appearance of appreciable net moments and in particular oscillation of the magnetization's polarity provides us with our rich spectrum of information on the individual specimens. The reader may therefore wonder why precise control and monitoring of the temperature of the sample was not pursued in our investigation.

We did indeed monitor the temperature of the specimen during the first phase of our investigation of the thermal release of hidden magnetic moments in the VTi cylinder, the first sample we have studied in the series of five different specimens we have examined. The tip of a ($\text{Au}_{0.93}\text{Fe}_{0.07}$) versus chromel thermocouple was fitted inside a hole of 0.1 cm diameter and 0.5 cm depth drilled at one end of the VTi cylindrical sample. Following standard practice this thermocouple was placed in series with an identical thermocouple immersed in the cryogenic bath. The output voltage was recorded on the Y axis of an XY recorder with the X axis driven by a voltage proportional to the current in the electric

heater. To monitor temperature we also attached a miniature Lakeshore silicon diode to one end surface of the cylinder. A similar approach was also undertaken in the work on the YBCO rod.

In both instances we abandoned the effort to monitor the evolution of the temperature in unison with the evolution of the hidden magnetic moments for the following reasons.

We observed that the peak values of the magnetic moments which appeared upon warming to T_c starting with $\langle M \rangle = 0$ were fairly insensitive to t_H , the duration of the heating interval and $P_H = I_H^2 R$, the joule heating power. We could vary the former quantity over a range of ≈ 4 and the latter over a range of ≈ 2 and obtain data reproducible within an acceptable experimental scatter of a few percent. Note that the minimum values of these two quantities are linked, albeit in a complicated way. With a large P_H , the specimen is warmed to T_c in a shorter time t_H and vice versa. Electronic drift and drift in the desired “stationary” applied magnetic field H_a generated by the solenoid, whether superconducting or copper wound, are proportional to t_H . Consequently we endeavour to shorten t_H without causing the volume of the specimen to heat non-uniformly which would result from using an excessive P_H .

We found however from repeated measurements for any chosen situation that the maximum voltage registered by the thermocouple exhibited proportionately greater scatter than encountered in the corresponding measurements of the peak moments. Further we also found that the position of the peaks in the graphs of the thermocouple voltage V_{th} versus I_H (the heater current) and the corresponding record of $\langle M \rangle$ versus I_H showed poor correspondence. Since our simple model to account for the thermal

disclosure phenomena focused on the magnitudes of the peak moments we decided that our time would be better invested in gaining extensive peak magnetization data on a complete set of specimens representing low and high T_c , hysteretic and semi-reversible type II superconductors.

3.8 Measurement of the Thermal Release of Hidden Magnetic Moments

The experimental work reported in this thesis and the modelling of our observations have focused on the extreme values or peaks that $\langle M \rangle$ attains or passes through during warming to T_c . Experimentally this is a straight forward procedure as we simply record the maximum variation of $\langle M \rangle$ from its initial zero state to a peak value or indeed to both positive and negative peaks as discovered to occur in semi-reversible specimens whether low or high T_c .

The choice of $\langle M \rangle = 0$ as a starting point for heating and thermal disclosure of moment has several positive attributes. From an experimental standpoint one in particular is most beneficial. At the juncture where heating causes T to exceed T_c no persistent currents survive in the specimen hence $B(r) = \mu_0 H_a$ and $\langle M \rangle = 0$. This feature provides a built in reference value hence a test and verification of how close our starting point was to $\langle M \rangle = 0$. Reaching any desired point exactly on the locus of $\langle M \rangle$ versus H_a can be very demanding, whether it is $\langle M \rangle = 0$ we seek or some other end value. The possibility always exists of overshoot or undershoot in our control of H_a and also of

electronic drift during the previous temperature-magnetic field history traversed in order to reach $\langle M \rangle = 0$. If the warming measurement takes place with $\langle M \rangle$ initially not exactly zero any small mismatch between the initial superconducting $\langle M \rangle \neq 0$ and the final normal value $\langle M \rangle_f = 0$ can be taken into account and corrected for in evaluating $\langle M \rangle_{\text{peak}}$. If the mismatch is significant, these data are discarded for the purpose of the project addressed in this thesis.

In our work we want to ensure that the induced currents fill the entire volume of the specimen. Otherwise the inner boundary of the inner region occupied by persistent currents will be unknown. In our approach not only is the sample fully saturated with persistent currents but we also know with certainty that the bulk is occupied by two, and only two, concentric regions of counter-rotating persistent currents, hence by an inner and outer moment of opposite polarity. In the very hysteretic specimens hence with negligible Meissner current $I_M(H_a, T)$, these two moments are equal giving $\langle M \rangle = 0$. In the case where a significant I_M exists, the two moments due to the bulk currents are of the appropriate ratio to yield an overall $\langle M \rangle = 0$ state when combined with the moment due to I_M .

To achieve the objective just described we must ensure that the locus of $\langle M \rangle$ versus H_a is situated on one of the envelopes of the major hysteresis curve when the direction of the sweep of H_a is reversed and the transit to $\langle M \rangle = 0$ takes place. The various scenarios are displayed in Fig 3.3 where for simplicity we have ignored I_M in the sketches. Clearly, appropriate previous temperature-magnetic field histories must be established and followed by the experimentalist in order that $\langle M \rangle$ be made to initially travel on either the upper or lower envelope of the major hysteresis loop. The reader should bear in mind

that the magnetic fields available in our laboratory usually cannot attain H_{C2} of the specimens.

For what we have termed “downswing” trials, the $\langle M \rangle = 0$ state is reached by reversing the sense of sweep of H_a from the upper diamagnetic envelope and lowering the magnitude of H_a to descend to the $\langle M \rangle = 0$ axis as indicated by the paths and points labelled (1) and (2) in Fig 3.3. Experimentally, in order to ensure that the descent has truly begun from the upper saturation envelope, appropriate cycling of the field to high “negative” values or field cooling in large negative H_a must be performed to fully saturate the specimen. Also in order to reach the special (unique) point where both $\langle M \rangle = 0$ and $H_a = 0$, the sweep reversal must begin at some minimum point along the upper envelope. This is depicted as point (1') in Fig 3.3.

Points labelled (3) and (4) display typical paths traced to attain $\langle M \rangle = 0$ starting from the lower envelope. The data obtained starting with $\langle M \rangle = 0$ arrived at through these paths are labelled “upswing”. Note that along some of these paths, (eg. the path ending at (4)), H_a must change sign as it causes $\langle M \rangle$ to migrate from the lower envelope to $\langle M \rangle = 0$.

The profiles sketched in the figures labelled (1), (2), (3) and (4) correspond to the $\langle M \rangle = 0$ states carrying the same number.

The differences in internal B profiles between “upswing” and “downswing” and the role played by I_M give rise to a rich spectrum of observed $\langle M \rangle_{\text{peak}}$ values measured at discrete H_a points. The $\langle M \rangle_{\text{peak}}$ values obtained experimentally are plotted versus the static H_a field present during heating. Our data curves for five different samples are presented and analysed in the subsequent chapters.

3.9 Methods for Measurement of H_{C1} , H^* and H^{**}

3.9.1 H_{C1}

Our work highlights, among other things, the interplay of the reversible Meissner surface current and the internal body currents upon the thermal disclosure of hidden magnetic moments. The contribution of each of these two sources to the overall net magnetization $\langle M \rangle$ may display different temperature dependencies as the specimen is heated. This gives rise to a surprising complexity in the development of the magnetic moments arising during heating from the “apparent” zero moment starting points.

To properly evaluate the spectrum of experimental results and to guide our modelling of the observations, it is important to identify the relative magnitudes of both the Meissner surface currents and the bulk body currents due to internal pinning of flux lines.

If a specimen is found to have a negligible contribution from the Meissner current then it is labeled very hysteretic or irreversible and the modeling will only be concerned with the internal B profiles due to bulk pinning, hence due to the critical current j_c . Alternatively samples displaying significant Meissner current but also with some degree of internal flux pinning are labelled semi-reversible. In modeling the behaviour of these materials we must take into account this Meissner current and its dependence on field and temperature, $I_M(H_a, T)$. We recall that in S.I. units when $H_a = H_{C1}$, the Meissner current I_M flowing along one meter of surface and transverse to H_a is also equal to H_{C1} .

Since $H_{C1}(T)$ is the applied field needed to initiate first entry of flux lines into the specimen this quantity can be estimated by measuring the applied field where the slope of $\langle M \rangle$ for a zero field cooled or virgin specimen versus H_a first deviates from linear. In this segment of the magnetization curve the sample is completely shielded against the external field except in the penetration depth λ .

Alternatively $H_{C1}(T)$ can be established by measuring the minimum sweep of H_a (i.e. application and removal of H_a) after zero field cooling which will cause some flux lines to be trapped in the specimen. Sweeps of H_a up to H_{C1} will be completely reversible, returning to $\langle M \rangle = 0$ at $H_a = 0$. Sweeps of H_a beyond H_{C1} will trap increasing amounts of magnetic flux generating a greater and greater paramagnetic moment when H_a returns to zero. Throughout this discussion we assume that the demagnetization factor is negligible hence the external magnetic field is parallel to the length of a long cylinder or along the flat faces of a ribbon sample. Also we ignore the possible existence of a surface barrier arising from the Bean-Livingston image force⁴², or a de Gennes critical field⁴³, or strong surface pinning⁴⁴⁻⁴⁸. None of our samples show any evidence of any such barriers.

3.9.2 H_*

As before to simplify the discussion we shall first address the case where H_{C1} is negligible. Hence we recall that when a field H_a is impressed upon a “virgin” specimen the field $H_a = H_*$ is reached when the advancing flux front of the flux density profiles has penetrated to the centre of the specimen. As H_a is increased beyond H_* the second point

of interest is reached at $H_a = H_{**}$. At this stage the field at the centre of the specimen has reached H_* .

As mentioned earlier there exist easily identifiable points on the locus of $\langle M \rangle$ versus H_a that can be related to H_* and H_{**} . Experimentally, we trace out the major hysteresis loop hence the upper diamagnetic and lower paramagnetic saturation curves as well as the hybrid section as illustrated in Fig 3.3. With this envelope in place a new sweep of H_a is begun from zero with a virgin sample, i.e. after zero field cooling. Now the locus of $\langle M \rangle$ versus H_a rises and meets or touches the upper diamagnetic envelope and then continues along it. The value of H_a at this meeting point determines H_* . This can be understood by glancing at Fig 3.4 (a) which shows the evolution of B profiles as H_a is increased from zero for the virgin sample. Fig 3.4 (b) shows the sequences of B profiles encountered along the "hybrid" segment of the envelope. The latter is traced when H_a has descended to zero from a large negative value and is now increasing in the positive direction. Note how the B profiles become identical at the field value of H_* .

H_* can also be obtained by measuring $H_{cool\ min}$, the minimum applied field present during field cooling whose subsequent removal will leave the specimen with the maximum amount of trapped flux hence with the flux density configuration as illustrated in sketch (3) of Fig 3.5. The B profiles existing in the specimen after field cooling in $H_{cool\ min}$ and the subsequent lowering to $H_a = 0$ are schematically displayed. The important feature here is that $H_{cool\ min} = H_*$. Any H_{cool} of lower value will not trap the maximum remanent flux and all H_{cool} of higher value will simply trap the same $\langle M \rangle_{max\ rem}$.

Experimentally we cool the specimen in progressively higher fields H_a as illustrated in Fig 3.5 where we also display the corresponding profiles. After returning to $H_a = 0$ the trapped moment is measured by heating to above T_c and recording the released signal. Plotting these moments versus the field of cooling we note a rising trapped moment that reaches a plateau of maximum trapped flux. The field as the plateau is reached indicates H_* .

By symmetry $\mu_0 H_* = B_*$ where B_* is the magnetic flux density at the centre of the specimen when the entire volume has been filled with trapped flux of one polarity hence persistent currents circulating in only one sense. The maximum remanent magnetization $\mu_0 \langle M \rangle_{\max \text{ rem.}}$ provides a measure of the spatial average of the flux density $\langle B \rangle$ for this particular situation since here $\langle B \rangle_{\max \text{ rem.}} = \mu_0 \langle M \rangle_{\max \text{ rem.}}$. Consequently B_* can be estimated from this data. For example in the simplest case of a slab sample with no field dependence on j_c (straight line dB/dx gradients) the profile of maximum remanent flux resembles a triangle with a base the width of the slab and a height of B_* . The spatial average of this will obviously be half of the magnitude of B_* .

It is a straightforward exercise to calculate the amount of flux trapped hence the remanent magnetic moment versus H_{cool} for idealized slab and cylinder geometry exploiting simple analytic expressions for the dependence of j_c on B . In Fig 3.6 we display the result of such calculations where we have used the Bean-London approximation^{49,50}, $j_c = j_o(T)$, hence j_c independent of B , and the simple Kim approximation⁵¹, $j_c = j_o/B$. These theoretical curves can be compared with our pertinent measurements presented later in this thesis.

3.9.3 H_{**}

This quantity can also be determined by two procedures.

First the initial magnetization curve is traced out to a sufficiently high value of H_a to ensure that the field H_{**} has been exceeded, hence that $\langle M \rangle$ has been made to travel far along the upper (diamagnetic) envelope (see Fig 3.7 (a)). The corresponding sequence of B profiles is sketched in Fig 3.7 (b). Then H_a is lowered from this high value. The locus of $\langle M \rangle$ then traces a portion of the lower (paramagnetic) envelope as H_a returns to zero and the maximum remanent flux is trapped in the specimen. Now as depicted in Fig 3.7 (a), the curve labelled (4) (5) (2) (6) is traced. The sequence of B profiles generated during this procedure are depicted in Fig 3.7 (c). Note that the two flux density profiles at $H_a = H_{**}$ are identical when the latter curve joins the upper curve. Therefore this meeting point of the curves signifies H_{**} .

Alternatively H_{**} can be determined by the more tedious procedure depicted in Fig 3.8 (a). A virgin, zero field cooled specimen is subjected to a half cycle (i.e. application and removal) of the applied field whose magnitude is denoted H_{cycle} . After each such half cycle the remanent magnetic moment induced by the swing of H_a is measured and plotted versus H_{cycle} . In Fig 3.8 (b), (c), (d) and (e) we depict the B profiles when $H_a = H_{\text{cycle}}$ and after H_a has been removed. Plots of $\langle M \rangle_{\text{rem}}$ versus H_a display an S shape as shown in Fig 3.9. The onset of the plateau in such graphs determines H_{**} .

Again it is a straightforward exercise to calculate $\langle M \rangle_{\text{rem}}$ versus H_{cycle} with simple analytic expressions for the dependence of j_c on B and for idealized slab and

cylinder geometry. Such calculated curves are displayed in Fig 3.9 where again we have used the Bean-London and the simple Kim approximations for $j_c(B)$.

3.9.4 H_* and H_{**} with $I_M(H_a)$ present

The four procedures we have just described can still be exploited to determine H_* and H_{**} in semi-reversible samples where H_{C1} cannot be neglected. Now however the extraction of these bulk pinning parameters from the observations is considerably more ambiguous and approximate.

However before we address the difficulties in establishing values for H_* and H_{**} we refer the reader to the two procedures described above in section 3.9.1 for the determination of H_{C1} . We then consider that H_{C1} hence $I_M(H_a=H_{C1}) = H_{C1}$ is known.

Now to illustrate the complications encountered in the interpretation of the observations we refer the reader to Figs 3.7 (a) and 3.10 (c). These figures show that at point (1) where the initial magnetization curve (A) touches the upper envelope, we should now write,

$$H_* = H_{a*} - I_M(H_{a*}) \quad (3.5)$$

where as shown in Fig 3.10 (c), $\mu_0 H_*$ denotes as before the magnetic flux density at a distance λ inside the surface of the specimen when the flux front has penetrated to the centre of the specimen. Note that $I_M(H_{a*})$ indicates the Meissner current in equilibrium with the external applied field H_{a*} . Consequently in order to extract H_* from the

measurement of the applied field H_a where the juncture of the magnetization curves occurred we need to estimate the magnitude of I_M in the presence of this applied field. Note that when $H_a > H_{C1}$, the magnitude of the Meissner current is a descending function of H_a (see Fig 2.1), hence $I_M(H_{a*}) < H_{C1}$.

Also in Fig 3.7 (a) and 3.10 (e) we show that for the junction of the bridge labeled (4) (5) (2) between the lower and upper envelope we should now write.,

$$H_{**} = H_{a**} - I_M(H_{a**}) \quad (3.6)$$

where as before $\mu_0 H_{**}$ is the magnetic flux density at a distance λ inside the surface of the specimen, $I_M(H_{a**})$ denotes the Meissner current in equilibrium with the externally applied field $H_a = H_{a**}$. Consequently in order to extract H_{**} from the measurement of the field $H_a = H_{a**}$ where the junction of these two curves takes place we need to separately evaluate I_M in the presence of H_{a**} .

Fig 3.10 (a) displays the kind of S shaped curves that are observed with semi-reversible samples by the H_{cycle} procedure described above. The bottom edge of the S identifies the first entry of flux into the specimen hence H_{C1} . The onset of the plateau determines H_{a**} . Figures 3.10 (b), (c), (d) and (e) depict the evolution of $I_M(H_a)$ and the B profiles when $H_a = H_{cycle}$ and when H_a has been removed.

In order to better determine the onset of penetration and subsequent trapping of flux from the base of the S depicted in Fig 3.10 (a) workers "linearize" the data curve by plotting $\langle M \rangle^n_{rem}$ where n is an arbitrary exponent which achieves this objective. Typically $n = 1/2$ or $1/3$.

To arrive at an estimate of H_* and H_{**} from the measurements obtained by these procedures the researcher usually assumes some simple functional dependence of I_M on $H_a > H_{C1}$.

Finally we note that since there is no Meissner current when $H_a = 0$ therefore $\langle M \rangle_{\max \text{ rem}}$ depends only on $j_c(B)$, hence H_* and whether the specimen is a cylinder or ribbon or has some other simple cross section. Consequently the value of H_* obtained by the procedures discussed above and the “guesses” regarding $I_M(H_{a**})$ and $I_M(H_{a*})$ must be consistent with the value linked to $\langle M \rangle_{\max \text{ rem}}$.

3.10 Measurement of the Meissner Effect

Within the sensitivity of our measuring instruments we detect no expulsion of flux upon cooling from T_c to 77 or 4.2 K for the specimens we have labeled very hysteretic (highly irreversible) i.e. the large YBCO crystal, the YBCO rod and the VTi cylinder. This absence of a Meissner effect arises because here $I_M \ll H_*$, hence the pinning of the flux lines very strongly opposes any expulsion from the sample by the mutual repulsion these exert upon each other.

The Nb ribbons and the BiSCCO slab we have studied exhibit an appreciable Meissner effect, i.e. expel an appreciable fraction of the flux permeating their volume as they are made to cool from T_c to 4.2 or 77 K in various static magnetic fields $H_a < H_{C2}$. For this reason the samples are labeled semi-reversible. For both of these samples we display $\langle M \rangle_{\text{Meissner}}$, the magnitude of the magnetization appearing after field cooling from their respective T_c to the corresponding bath temperature, versus H_a . These data are

recorded in this thesis for completeness and archival purpose. These observations can be modeled by an extension of the thermal disclosure model we present in chapter 10 of this thesis. However we delegate this arduous task to other Ph.D candidates or post-doctoral fellows. We note that Dr. John Clem and his collaborators have made significant progress in solving this basic problem^{1,52}.

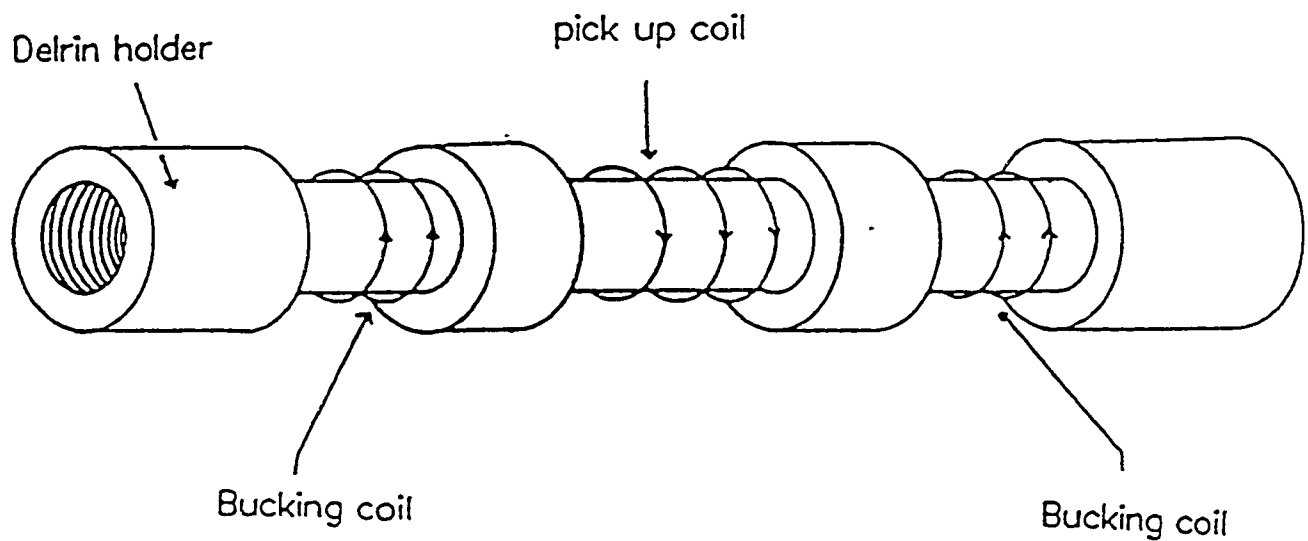


Fig 3.1 Depicts a typical pickup coil arrangement. The sample and associated electric heater are contained within the central main windings of the pickup coil. Note the opposite direction of windings for the bucking coils which do not embrace or "sense" the specimen. In our studies various sizes and shapes of pickup coils have been fabricated to best accommodate the diverse materials studied.

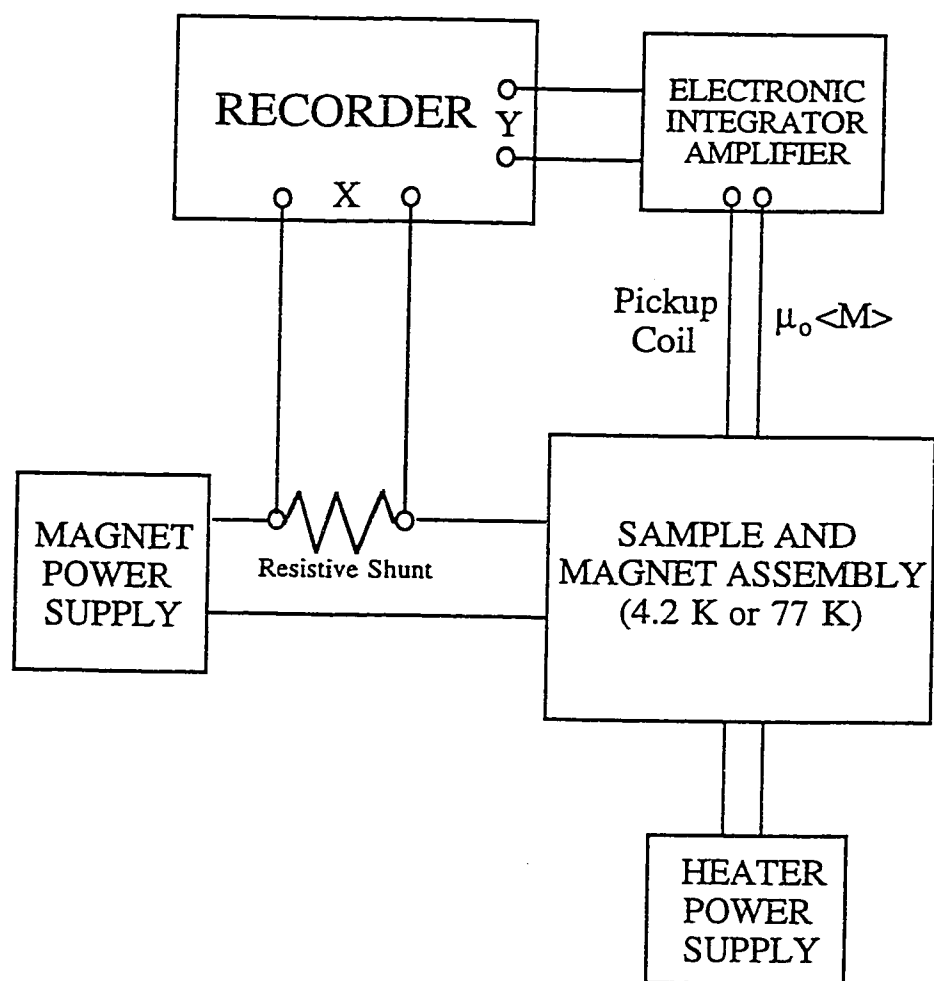


Fig 3.2 Schematic representation the experimental set up.

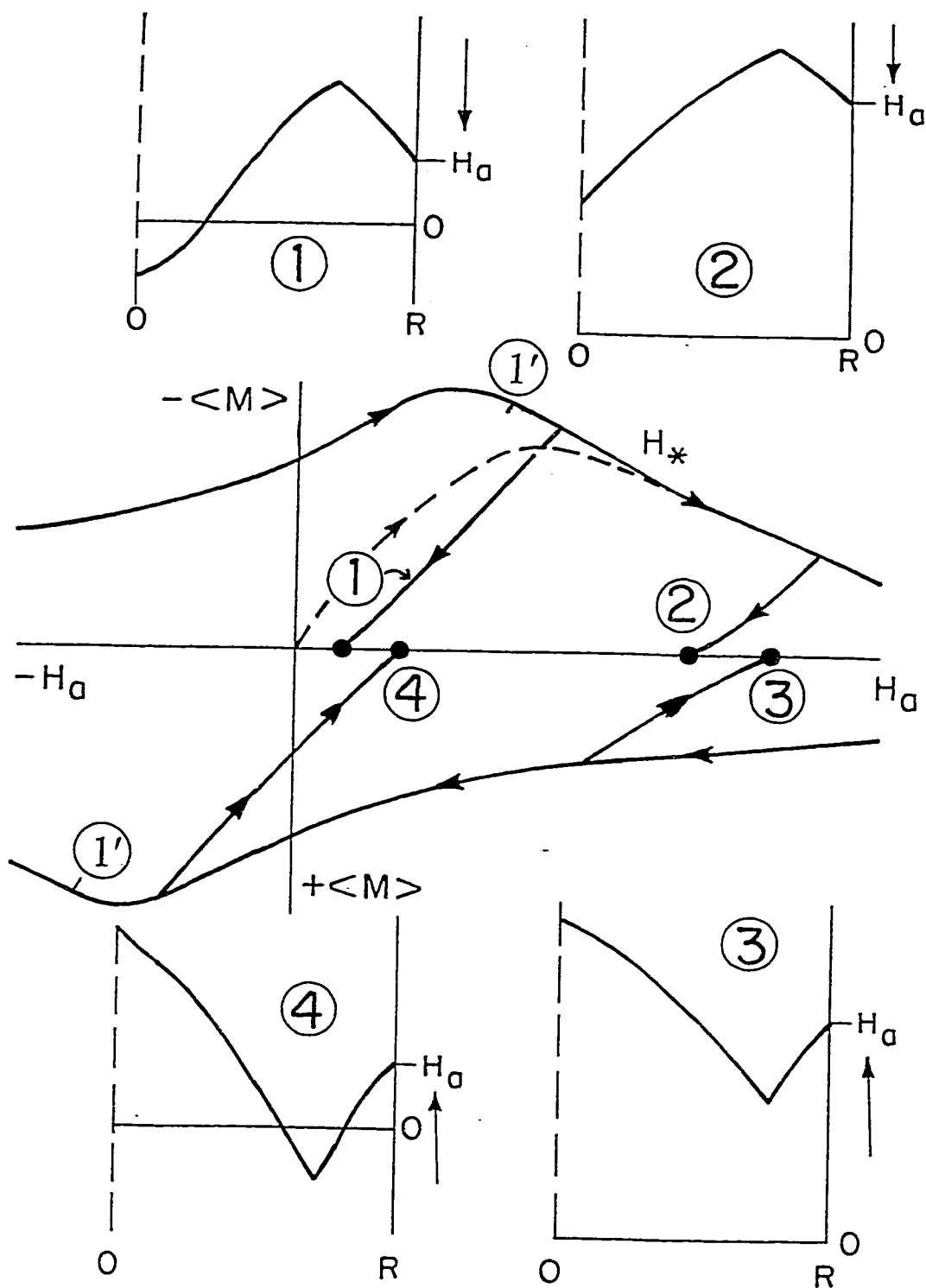


Fig 3.3 Isothermal $\langle M \rangle$ versus H_a for a strongly hysteretic specimen depicting typical “downswing” trials to reach $\langle M \rangle = 0$ denoted by the traces labeled (1) and (2) along with “upswing” trials depicted in (3) and (4). The associated sketches depict the flux density profiles arising when the $\langle M \rangle = 0$ state is reached. Note that in (1) and (4) the specimen will be in a “hybrid” state of both positive and negative polarity of flux lines. Finally the points labeled (1') on the upper and lower envelopes signify the minimum field reversal points that bring either an upswing or downswing trial to $\langle M \rangle = 0$ at $H_a = 0$.

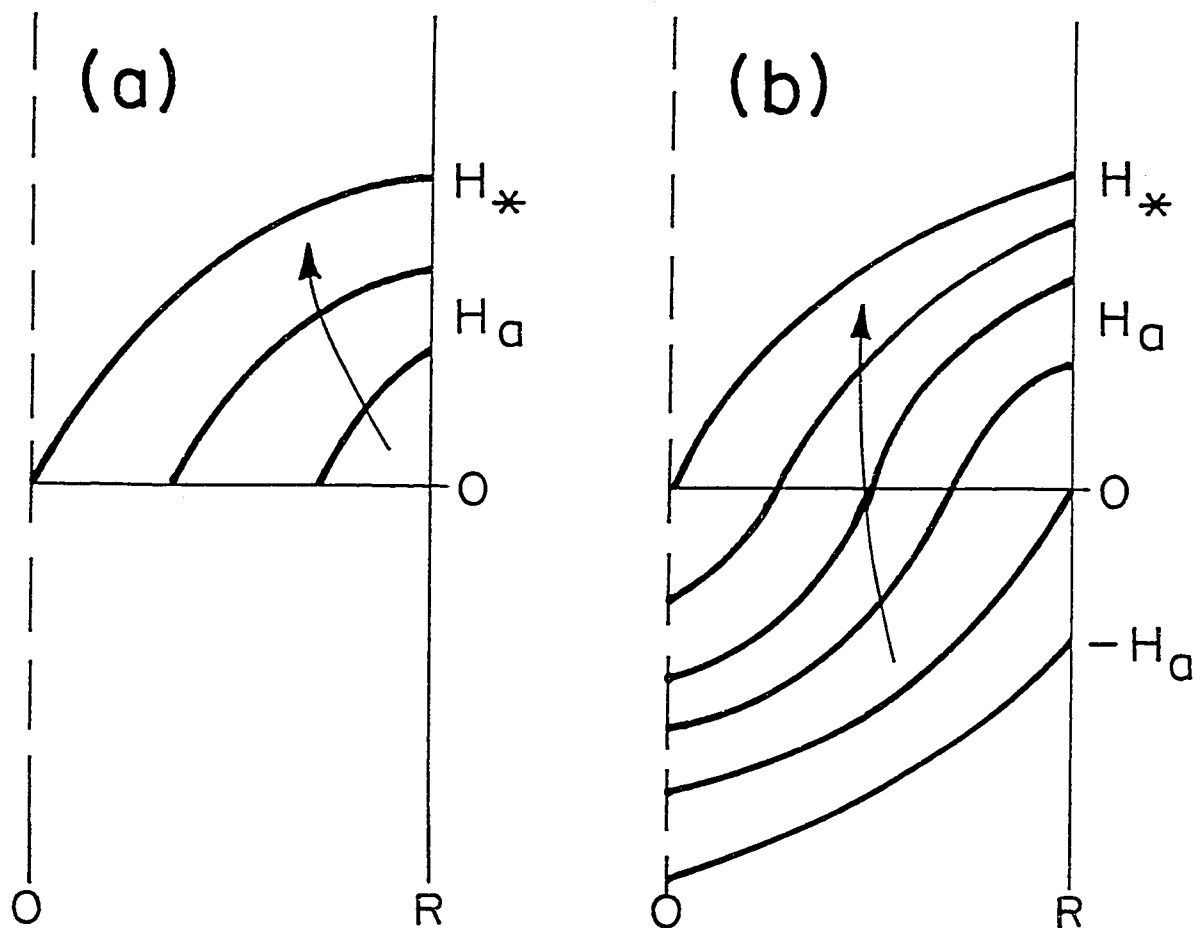


Fig 3.4 (a) Shows the sequence of flux density profiles in a specimen subjected to an increasing field $+H_a$ starting with from the virgin, zero field cooled state. (b) The flux density profiles of a specimen returning from an excursion to a large negative field through zero, and finally increasing in $+H_a$ (thus a specimen traveling along the upper envelope). Note that at $H_a = H_*$ the profiles become identical, this explains the meeting point at H_* of an initial magnetization curve and the upper envelope on plots of $\langle M \rangle$ versus H_a .

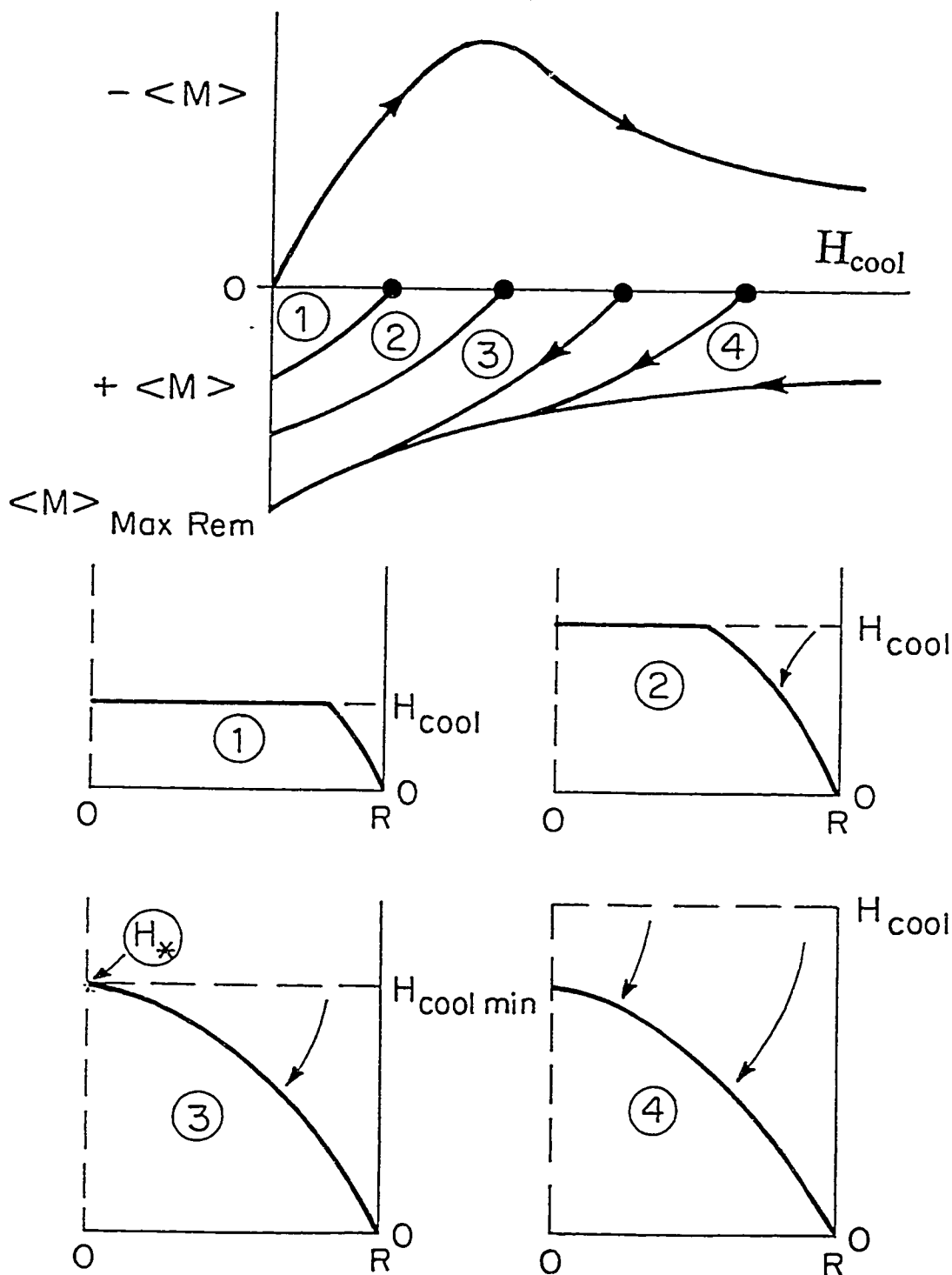


Fig 3.5 With a highly hysteretic sample cooled in progressively higher fields denoted H_{cool} , the lowering of H_a back to zero will follow the locus depicted by the traces (1), (2), (3) and (4). Note that all $H_{cool} \geq H_{cool min}$ will simply result in the same maximum remanent trapped field $\langle M \rangle_{max rem}$ thus $H_{cool min}$ signifies H_* . The associated flux density profiles of (1), (2), (3) and (4) offer a visual guide to this action. If discrete measurements of $\langle M \rangle_{rem}$ versus H_{cool} are plotted, $\langle M \rangle_{rem}$ will reach a plateau at $H_{cool} = H_*$ (see also Fig 3.6)

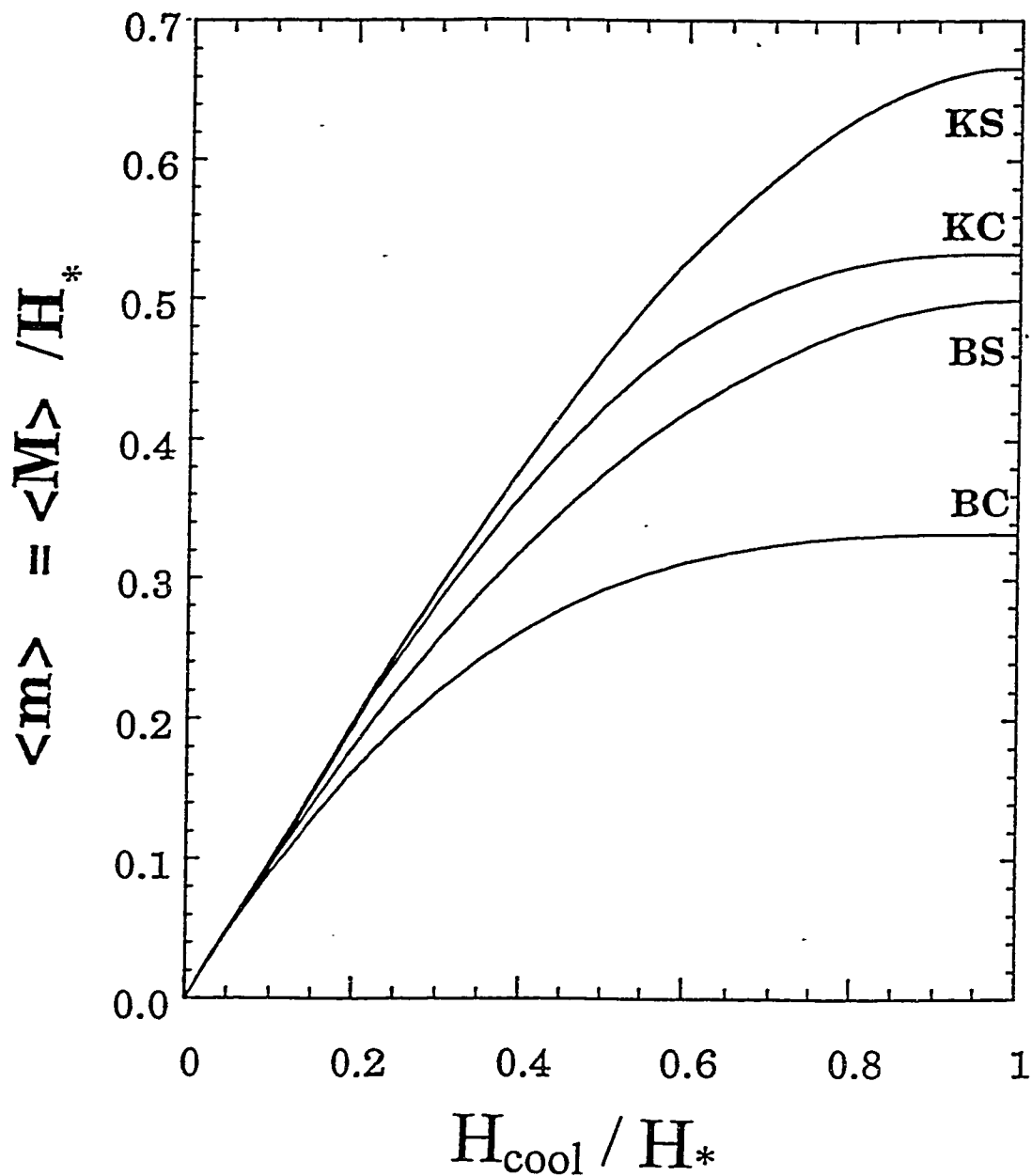


Fig 3.6 Displays $\langle M \rangle_{\text{rem}}$ versus H_{cool} rising to a plateau at $H_{\text{cool}} = H^*$. The calculations used the Kim ($j_c = j_o(T)/B$) and the Bean-London ($j_c = j_o(T)$) approximation for the dependence of j_c on B . The symbols K and B denote these functions, the symbols S and C denote idealized slab and cylinder geometry.

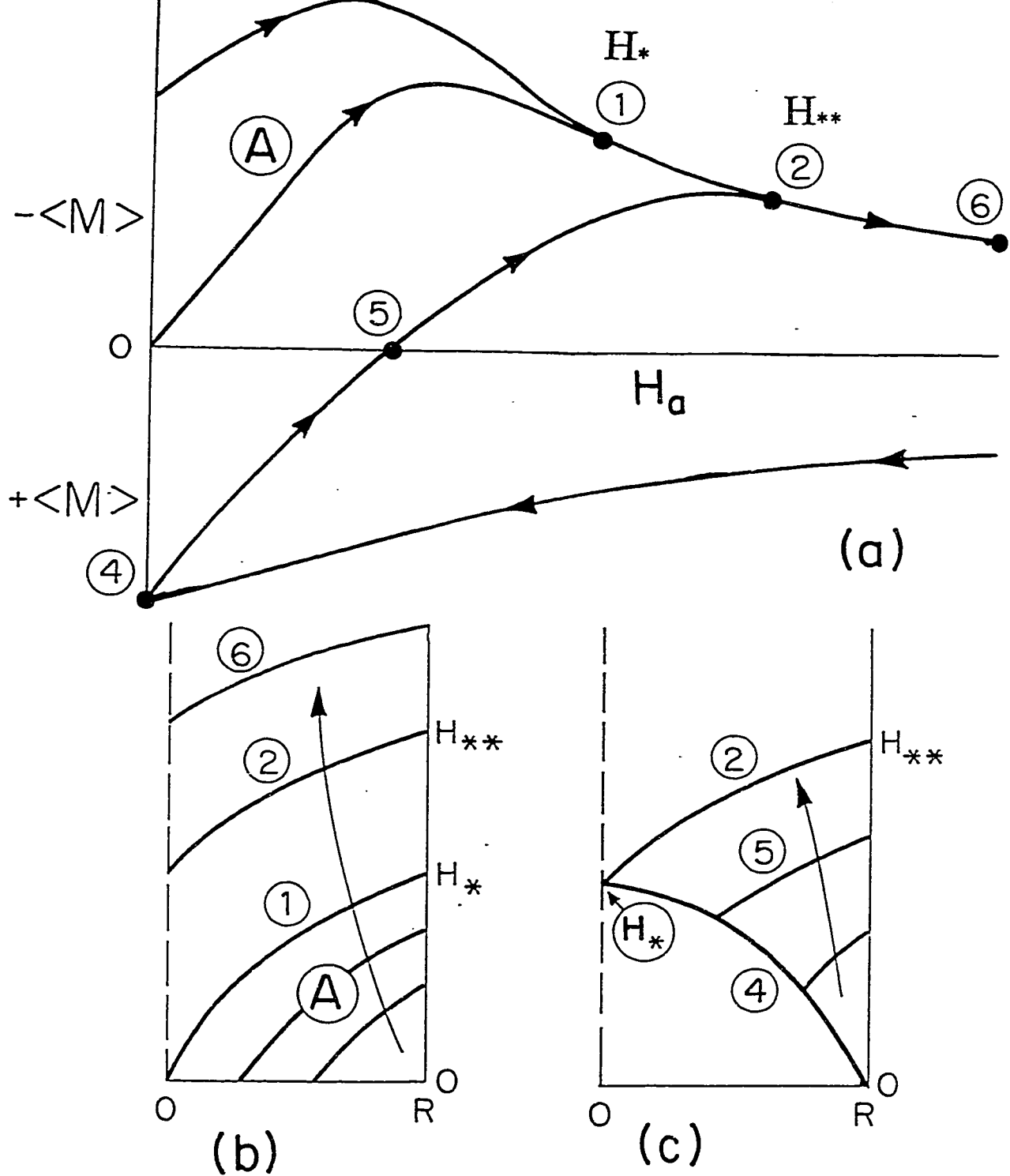


Fig 3.7 (a) depicts a typical $\langle M \rangle$ versus H_a plot for a highly hysteretic specimen. (b) The flux density profiles of a virgin, zero field cooled specimen subjected to an increasing H_a . Note that after the field H_* , the specimen will simply travel along the upper envelope. (c) The flux density profiles of a specimen subjected to an increasing $+H_a$ but starting at the point (4) of figure (a) i.e. one with the maximum trapped paramagnetic moment. Note that the flux density profiles become identical and thus the two $\langle M \rangle$ versus H_a curves "meet" at point (2), thus signifying the field H_{**} .

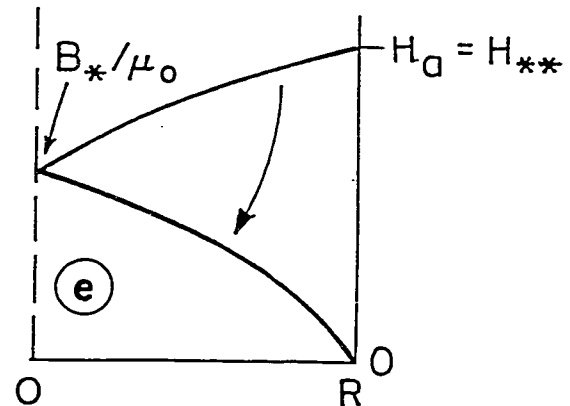
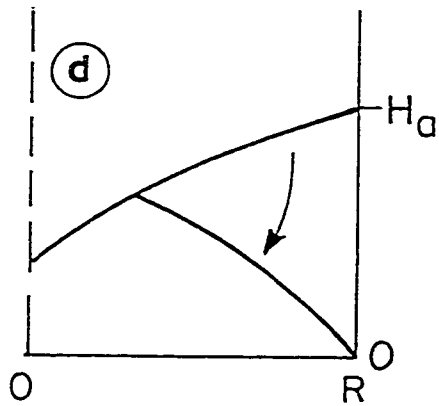
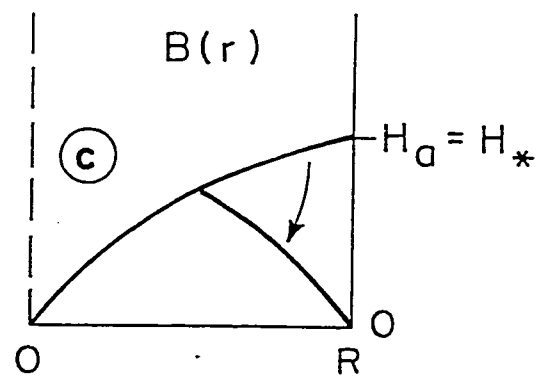
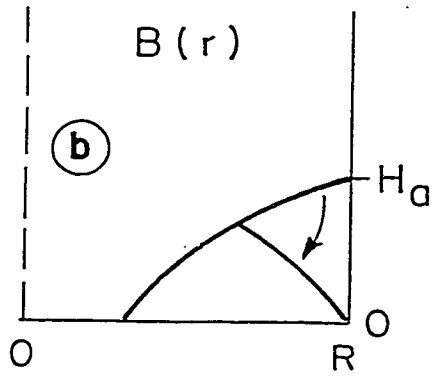
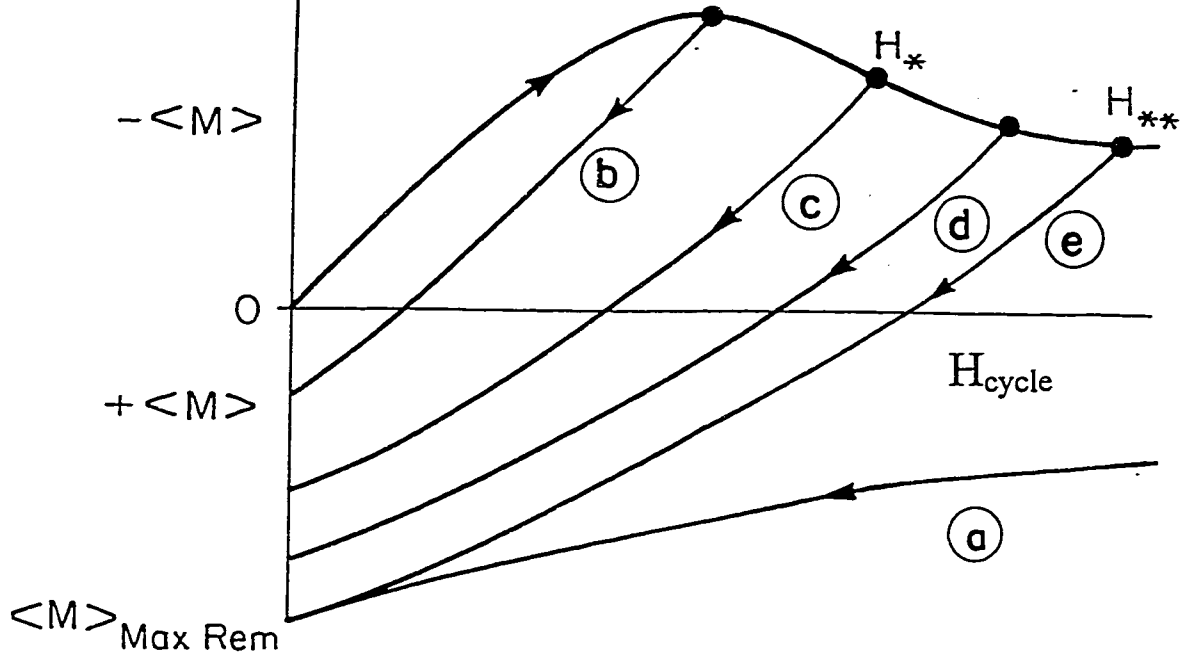


Fig 3.8 Depicts an alternate procedure for determining H_{**} . A virgin, zero field cooled sample is subjected to increasing half cycle swings of the field where H_{cycle} denotes the maximum field attained in each half cycle. Various H_{cycle} swings are depicted by the traces (b), (c), (d) and (e). As the associated flux density profiles display, the maximum remanent flux will only be trapped for cycles out to H_{**} and greater, i.e. $H_{\text{cycle}} \geq H_{**}$. Thus if the spatial average of the remanent field is plotted versus H_{cycle} ($\langle M \rangle_{\text{rem}}$ versus H_{cycle}) then $\langle M \rangle_{\text{rem}}$ will reach a plateau at $H_{\text{cycle}} = H_{**}$ (see also Fig 3.9).

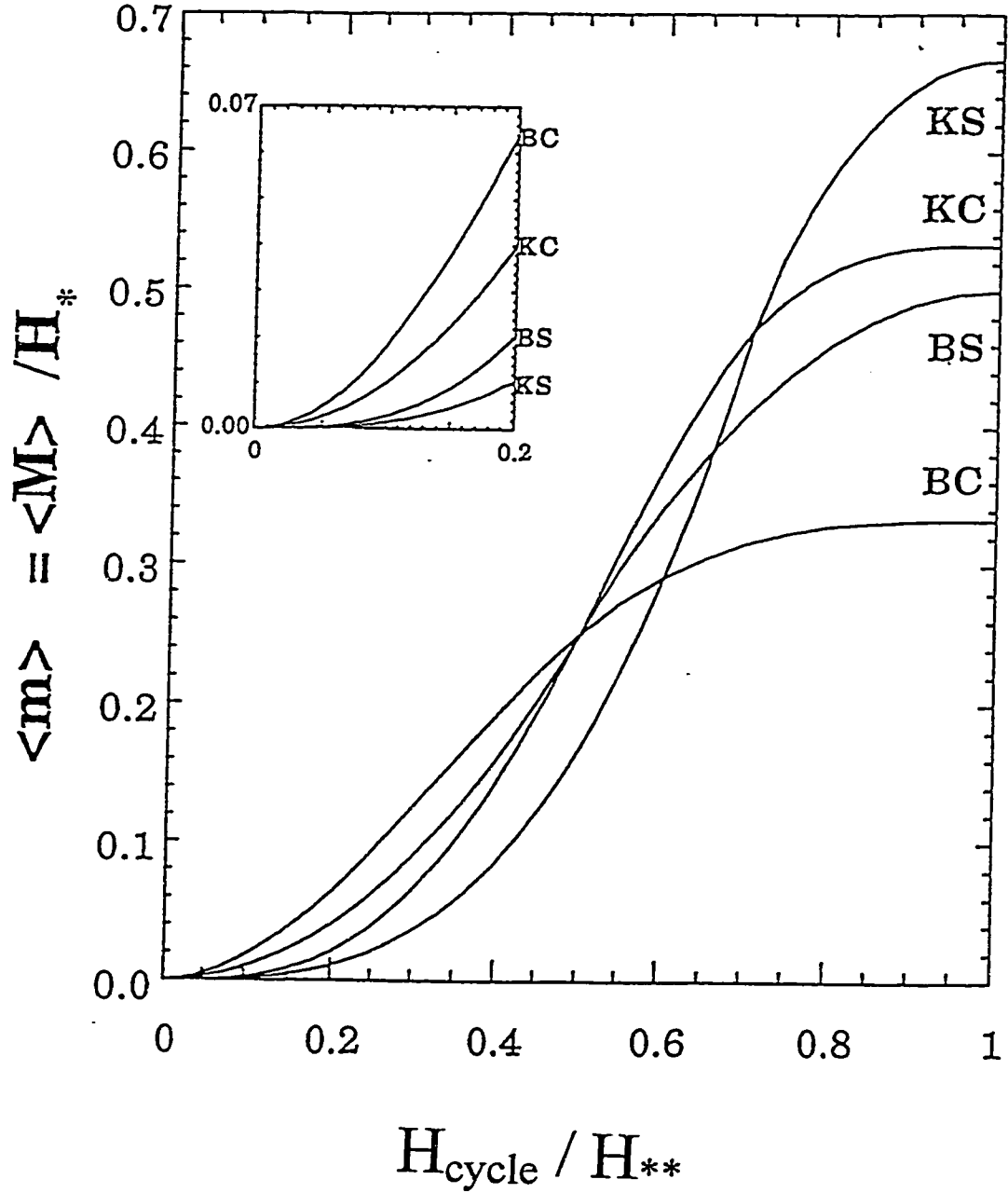


Fig 3.9 Displays $\langle M \rangle_{\text{rem}}$ versus H_{cycle} following an S shaped curve and rising to a plateau at $H_{\text{cycle}} = H_{**}$. The calculations used the Kim ($j_c = j_o(T)/B$) and the Bean-London ($j_c = j_o(T)$) approximation for the dependence of j_c on B . The symbols K and B denote these functions, the symbols S and C denote idealized slab and cylinder geometry.

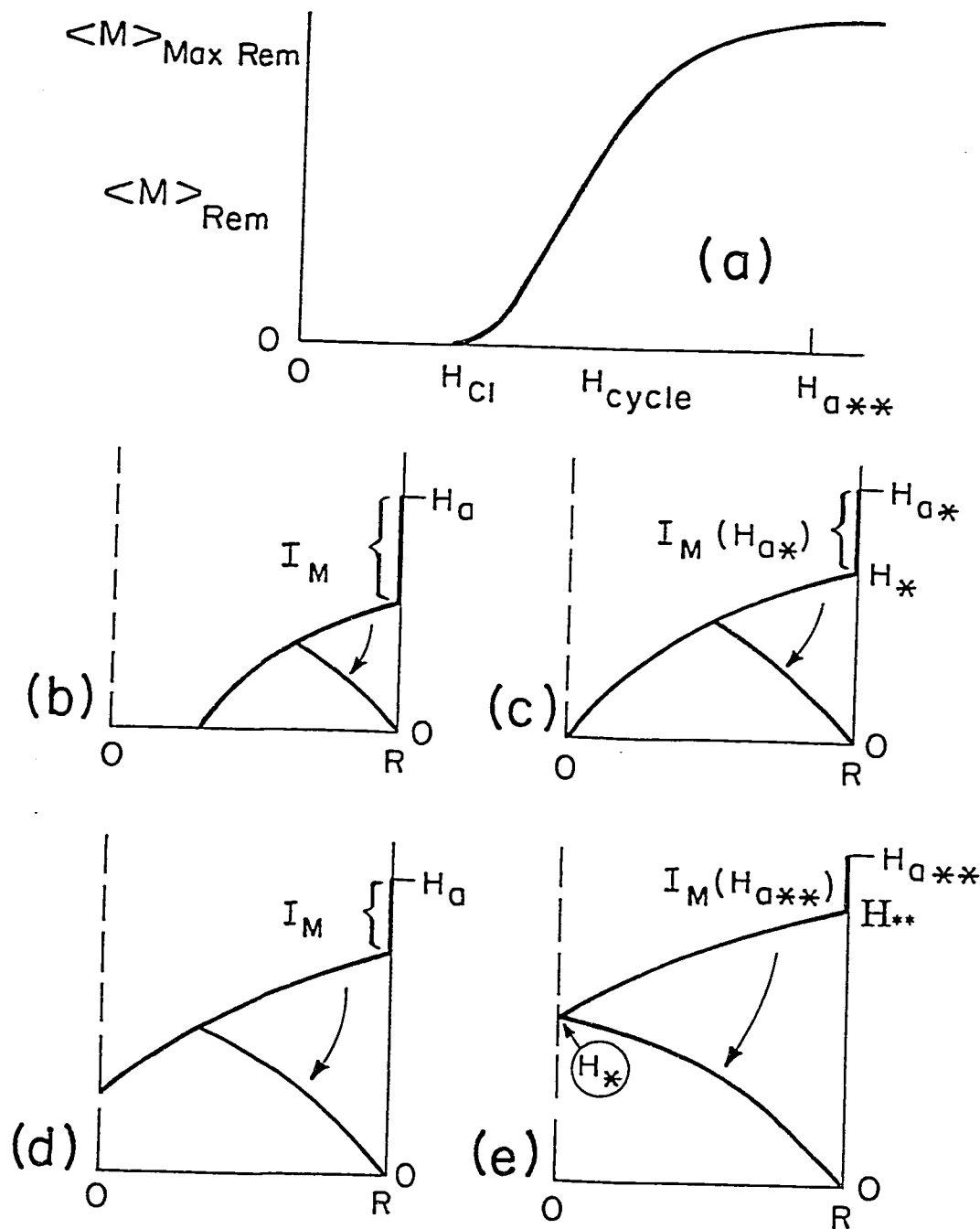


Fig 3.10 The H_{cycle} technique described in the text and in Fig 3.7 leads to similar S shaped curves for plots of $\langle M \rangle_{\text{rem}}$ versus H_{cycle} for hysteretic as well as semi-reversible specimens with appreciable $I_M(H_a, T)$. However for the latter there will be no penetration of flux and thus no trapping of remanent flux up to the point denoted H_{Cl} in (a). Also the plateau of $\langle M \rangle$ versus H_{cycle} now denotes the new quantity H_{a**} . As shown in the associated flux profile of sketch (e), the field H_{a**} will be greater than the value H_{**} by the amount $I_M(H_{a**})$. Similarly in (c), one can see that H_* , the true measure of bulk pinning of a sample of known dimension, will be lower than the experimentally observable quantity H_{a*} . In this case lower by $I_M(H_{a*})$.

CHAPTER 4

Observations on a Low T_c Hysteretic Specimen: VTi

4.1 Introduction

In this chapter we first present the isothermal magnetization measurements carried out on our VTi cylinder sample at 4.2 K. These magnetization curves show that this conventional type II superconducting sample is highly hysteretic (irreversible). From these data we can readily determine values at 4.2 K for the two related parameters H_* and H_{**} , the first and the “double” penetration fields which characterize the bulk critical current of the sample and its low field dependence on the magnetic flux density B .

Then we display our data on the maximum moments μ_{peak} which appear upon warming to T_c from 4.2 K starting with $\langle M \rangle$ initially zero in various final applied fields spanning the range from zero to values appreciably greater than H_{**} . These measurements of the thermal evolution of $\langle M \rangle$ were carried out after the applied field had caused $\langle M \rangle$ to touch the zero axis during its progress from one envelope of the major hysteresis loop towards the opposite envelope. We have obtained extensive data with H_a either descending or ascending in magnitude when it reaches the final values which establish the desired $\langle M \rangle = 0$ configuration. Let the expressions downswing and upswing and the symbols $\mu_{\text{peak}}^{\downarrow} (\langle M \rangle_{\text{peak}}^{\downarrow})$, $\mu_{\text{peak}}^{\uparrow} (\langle M \rangle_{\text{peak}}^{\uparrow})$ denote these two different

sets of data for μ_{peak} ($\langle M \rangle_{\text{peak}}$) versus H_a descending or ascending in magnitude as it approaches the various final values of H_a where $\langle M \rangle$ crosses the zero axis.

The plots of our μ_{peak} data versus H_a reveal remarkable differences between these two sets. In particular, $|\mu_{\text{peak}}^{\downarrow}|$ traces a peak in the low field range whereas $|\mu_{\text{peak}}^{\uparrow}|$ displays a valley in this range. Our model of these observations presented in chapter 6 shows that these features arise from the severe sensitivity of j_c on B in the low field range for this specimen.

4.2 Isothermal Magnetization Curves

The isothermal magnetic behaviour of this specimen is displayed in Fig 4.1. From inspection we can estimate $\mu_0 H_*$ to be ≈ 200 mT. This value emerges from noting that the initial magnetization curve merges with or “touches” the upper envelope of the major hysteresis curve at this applied field. Recall that the initial or virgin magnetization curve is the curve starting at $H_a = 0$ after zero field cooling hence with no magnetic flux in the specimen. We have described in chapters 2 and 3 how the sequences of penetrating magnetic flux density profiles occurring along this initial magnetization curve evolve to a configuration which is identical to a specific B profile encountered along the upper envelope (see Fig 2.3).

From examination of the upper limit of the hysteresis loop starting at $\langle M \rangle_{\text{max rem}}$, hence with $H_a = 0$ we estimate $\mu_0 H_{**}$ to be ≈ 300 mT. The sequence of B profiles encountered along the upper segment of this hysteresis loop are sketched in Fig 2.3 (d).

In this specimen we observed no flux expulsion (i.e. no Meissner effect) upon

cooling from $T_c \approx 7.0$ to 4.2 K greater than the lower limit of our sensitivity of ≈ 0.1 mT. Measurements were performed over the range $0 < H_a < 450$ mT. These measurements by various workers^{53,54} on well annealed samples of VTi in this range of composition indicate that $\mu_0 H_{C1} \approx 5$ mT at 4.2 K. Consequently since $I_c/I_M = H_*/H_{C1} \approx 40$ for this specimen, it is not surprising that it displays negligible Meissner effect and large irreversibility. Theoretical magnetization curves corresponding to the measured curves shown in Fig 4.1 are displayed in Fig 4.2. The framework and procedure for these calculations are presented in chapter 7. In this application of the critical state model^{3,49-51,55,56} we assumed that the j_c dependence on B can be described by,

$$j_c = j_o \left(\frac{B_{ref}}{B} \right)^{1/2} \quad (4.1)$$

The only adjustable parameter which enters into fitting the theoretical curves to the observations is the first penetration field H_* which here reads,

$$H_* = \left(\frac{3}{2} j_o R B_{ref}^{1/2} \right)^{2/3} \quad (4.2)$$

4.3 Release of Hidden Magnetic Moments

Fig 4.3 displays our observations for $\langle M \rangle_{peak} = \mu_{peak}/V$ versus the stationary applied magnetic field H_a present during warming to T_c . It is convenient to normalize

these data with respect to the maximum remanent magnetization $\langle M \rangle_{\text{max rem}}$. This ratio is not dependent on any assumptions entering into the calibration procedure. Also the theoretical calculations of the magnetic moments and their evolution are more easily pursued in this normalized framework. The latter feature will be explained in a later chapter. Further we can readily quantitatively compare the behaviour of different specimens when the thermal release observations are expressed in this normalized form.

Although the magnetic moments which appear during warming to T_c are diamagnetic for the downswing situation and paramagnetic for the upswing case we have superimposed the different sets of data points in Fig 4.3 for purpose of direct comparison and contrast in their behaviour.

Although the description of our model for these data is postponed to chapter 7, at this juncture for completeness we display in Fig 4.4 the corresponding theoretical curves which the model generates. These theoretical curves were calculated using eqn 4.1 and applying the concept of conservation of flux lines as described in detail in chapter 7.

Comparison of Figs 4.3 and 4.4 shows that the theoretical curves reproduce all the major features of the corresponding intricate data curves both qualitatively and quantitatively. Specifically we note the generation by the model of the following items evident in the observations.

- (i) The peak at low field in the $\langle M \rangle_{\downarrow}$ data curve.
- (ii) The valley and the peak at low fields in the $\langle M \rangle_{\uparrow}$ data curve.
- (iii) The close correspondence between the location along the H_a axis of the bottom of the valley and the top of the first peak.

- (iv) The ratio of ≈ 3 for H_a at the position of the second peak relative to the value of H_a when the bottom of the valley is traced.
- (v) The magnitude of the ratio for $\langle M \rangle_{\text{peak}} / \langle M \rangle_{\text{max rem}}$ at $H_a = 0$, and finally,
- (vi) The correct rate of descent of the two sets of data as a function of H_a .

This excellent agreement is in our view quite impressive especially considering the fact that our model for the thermal release of hidden magnetic moments contains only the adjustable parameter H_* and this quantity is determined from the critical state magnetic hysteresis curve (Fig.4.1).

All of the features we have encountered in the thermal evolution of the hidden magnetic moments and just enumerated above are seen to arise from the severe dependence of j_c on B exhibited by the VTi specimen. Since the main purpose of our overall project was to test the validity of the simple model we proposed to describe the release of concentric and oppositely directed magnetic moments in type II superconductors, we have not sought to improve on the theoretical results displayed in Fig 4.4.

We stress that the actual dependence of $j_c(B, T)$ on the temperature plays no role in our successful modelling of our observations of $\langle M \rangle_{\text{peak}}^{\uparrow}$ and $\langle M \rangle_{\text{peak}}^{\downarrow}$ versus H_a on this highly irreversible type II superconductor. For this and many other reasons we did not pursue the time consuming effort required to accurately monitor the temperature of the specimen as the hidden moments were released during warming to T_c .

It is interesting and instructive to examine the flux density profiles corresponding to the features labeled 1, 2, 3 and 4 on Fig 4.4 both for comparison of the initial $\langle M \rangle = 0$

states and also when the thermally disclosed hidden magnetic moments attained a peak magnitude. Fig 4.5 displays the first set and Fig 4.6 the latter set. In Fig 4.6 the vertical position of the four profiles has been adjusted so that they all start from the same baseline at the surface of the specimen (i.e. $\frac{x}{X} = 1.0$) hence can be readily compared with each other. Also for this reason the flux density profile corresponding to the downswing point (2), has been reversed in “polarity”. Consequently in this display the areas under each of the curves can be visually estimated and compared. Note that these areas provide a measure of the spatial average of $B(x)$ relative to a common baseline, hence, a visual picture of $\langle M \rangle_{\text{peak}}$ for each case. From inspection it emerges that profiles 1, 2, and 3 indeed trace a valley and that profile 4 forms a peak above this trio.

4.4 Conclusion

We have reported on extensive observations of the release of the hidden magnetic moments by warming to T_c for a very irreversible low T_c type II superconductor (VTi). A simple model exploiting the concept of conservation of flux lines as the persistent critical currents circulating in the bulk of the specimen are gradually suppressed by the rise in temperature, is shown to reproduce all of the major features of our data. The good agreement between the model and the experimental results arises because j_c is very dependent on the magnetic flux density in this material. Indeed the simple assumption that j_c obeys a simple case of a Kim like formula i.e. $j_c = \text{coefficient}/B^{1/2}$ leads to excellent agreement with our measurements of $\langle M \rangle_{\text{peak}}$ versus H_a “upswing” and “downswing”.

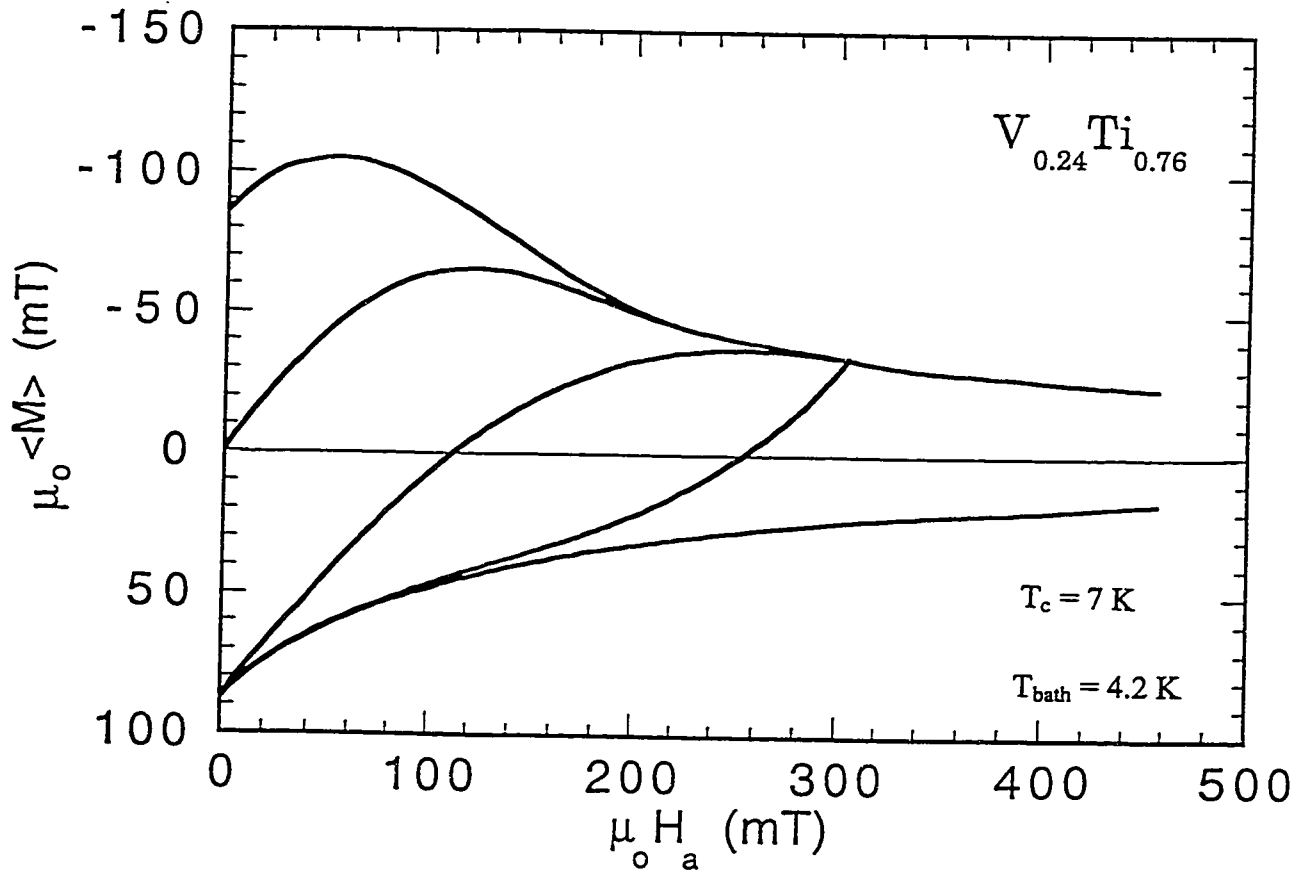


Fig 4.1 Experimentally observed isothermal magnetization curves for the VTi sample. Note the meeting point of the initial magnetization curve and the upper envelope provide an estimate of $\mu_0 H_* \approx 200 \text{ mT}$. Also the curve begun with maximum trapped field near 90 mT meets the upper envelope at a point that denotes $\mu_0 H_{**} \approx 300 \text{ mT}$.

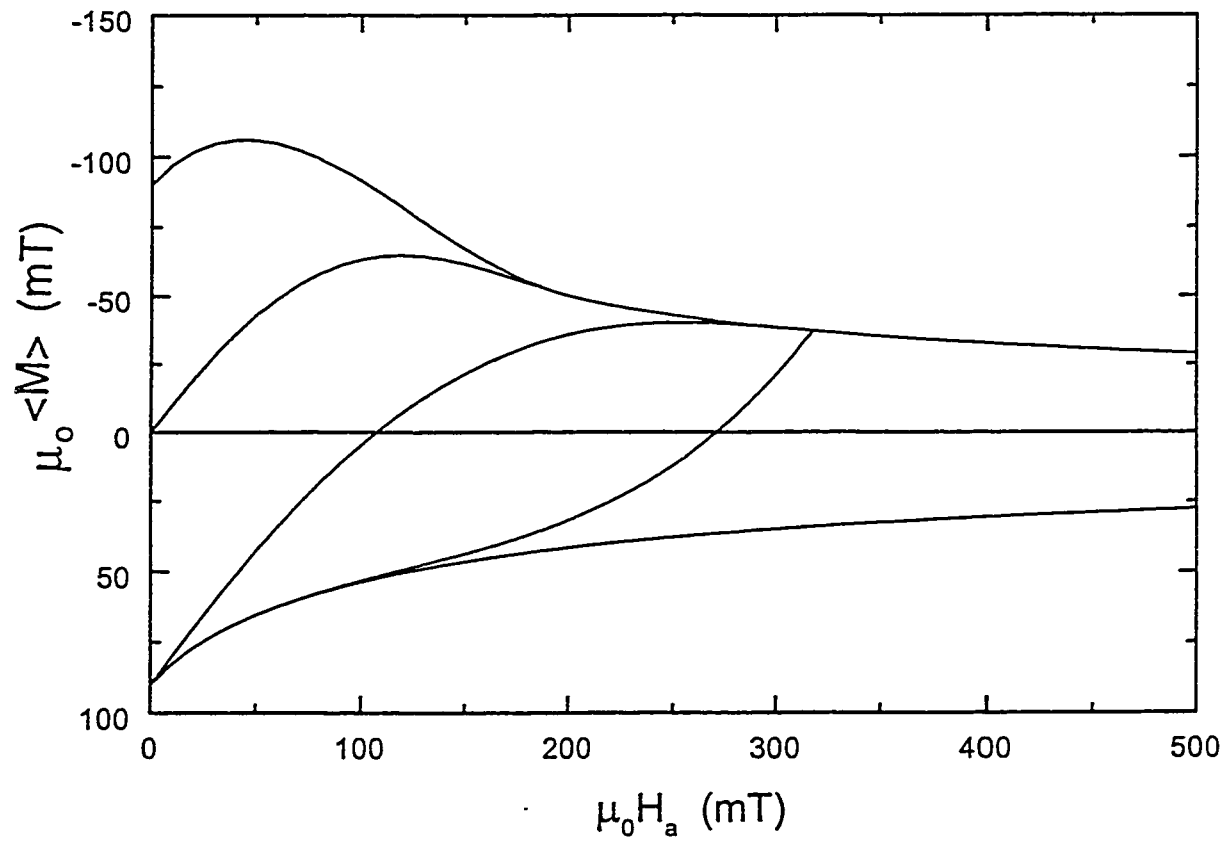


Fig 4.2 Theoretically calculated isothermal magnetization curves for VTi. Calculations were performed using the critical state model and an assumed field dependence of $j_c = j_o(B_{ref}/B)^{1/2}$. The only adjustable parameter is H_* , which is taken to match that determined experimentally in Fig 4.1

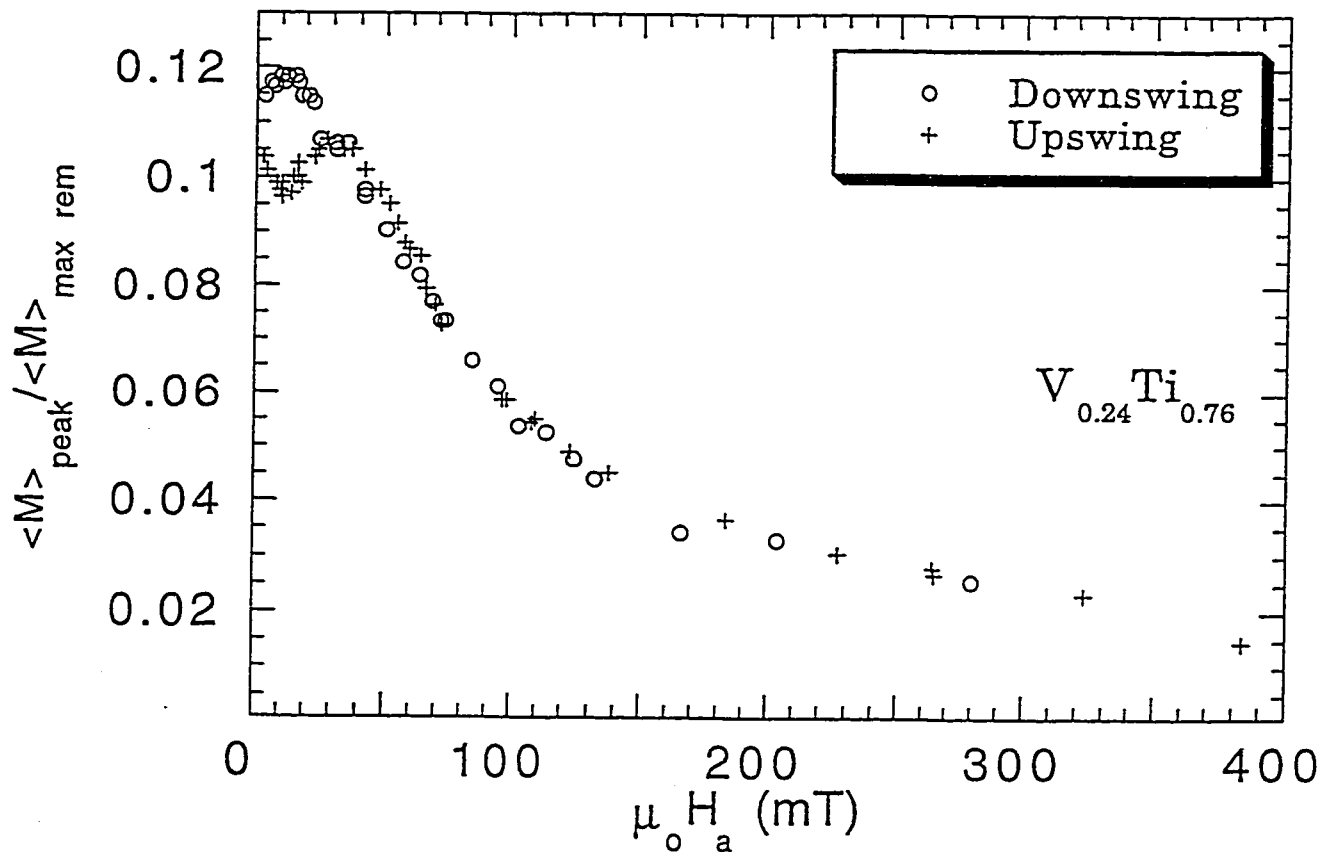


Fig 4.3 Experimental observations of thermally released hidden magnetic moments for VTi, hence $\langle M \rangle_{\text{peak}} = \mu_{\text{peak}}/V$. “Downswing” denotes $\langle M \rangle = 0$ state reached by descending H_a and “upswings” via ascending H_a . The data are normalized to the maximum remanent field $\langle M \rangle_{\text{max rem}}$. Also for convenience of comparison the diamagnetic moment released from “downswing” trials and the paramagnetic moment of “upswing” trials are superimposed.

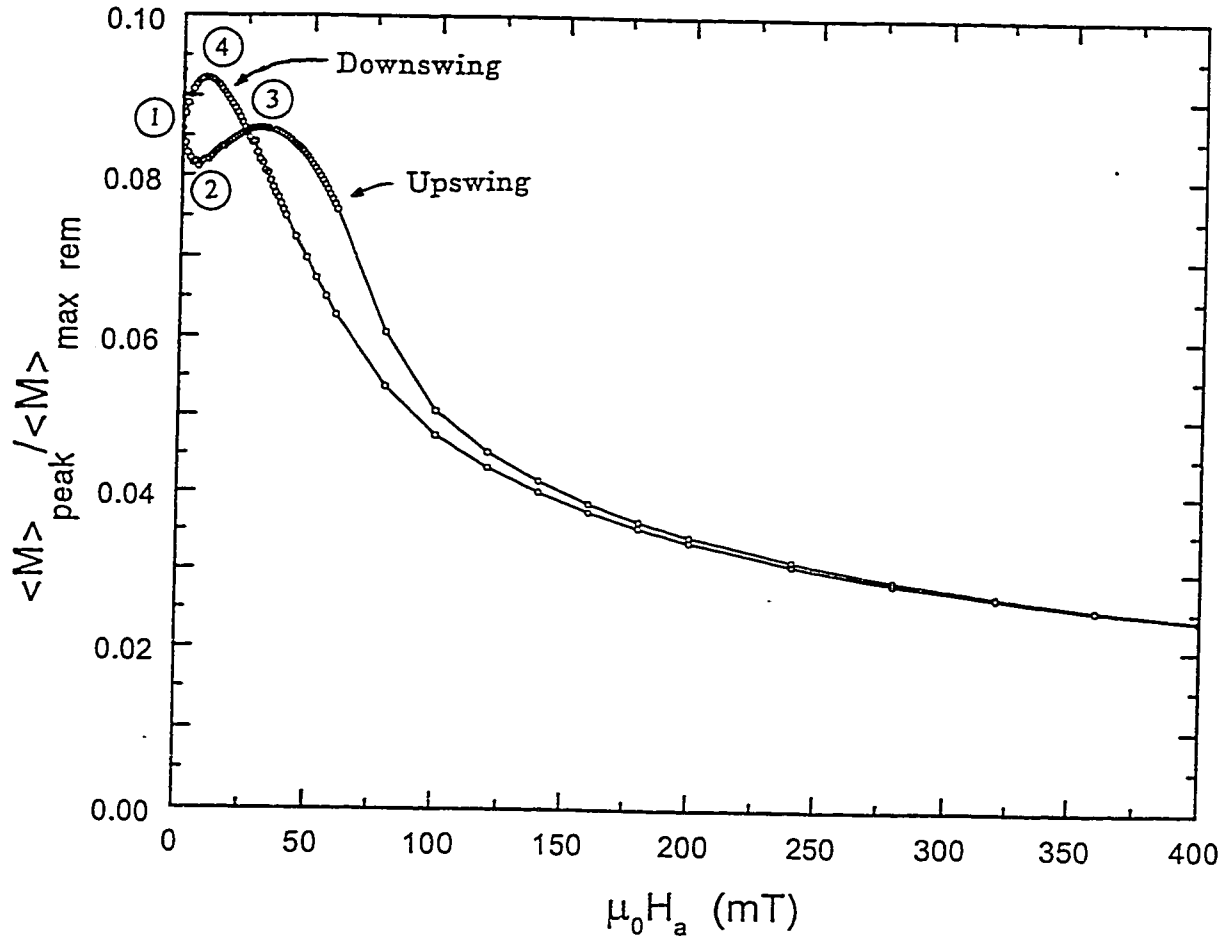


Fig 4.4 Theoretical calculations of the thermally released hidden magnetic moments for VTi. The same j_c dependence $j_c = j_o(B_{ref}/B)^{1/2}$ is employed as in Fig 4.2 for the isothermal magnetization curves. Details of the model are presented in chapter 7. The flux density profiles for the points labeled (1), (2), (3) and (4) are displayed in Figs 4.5 and 4.6

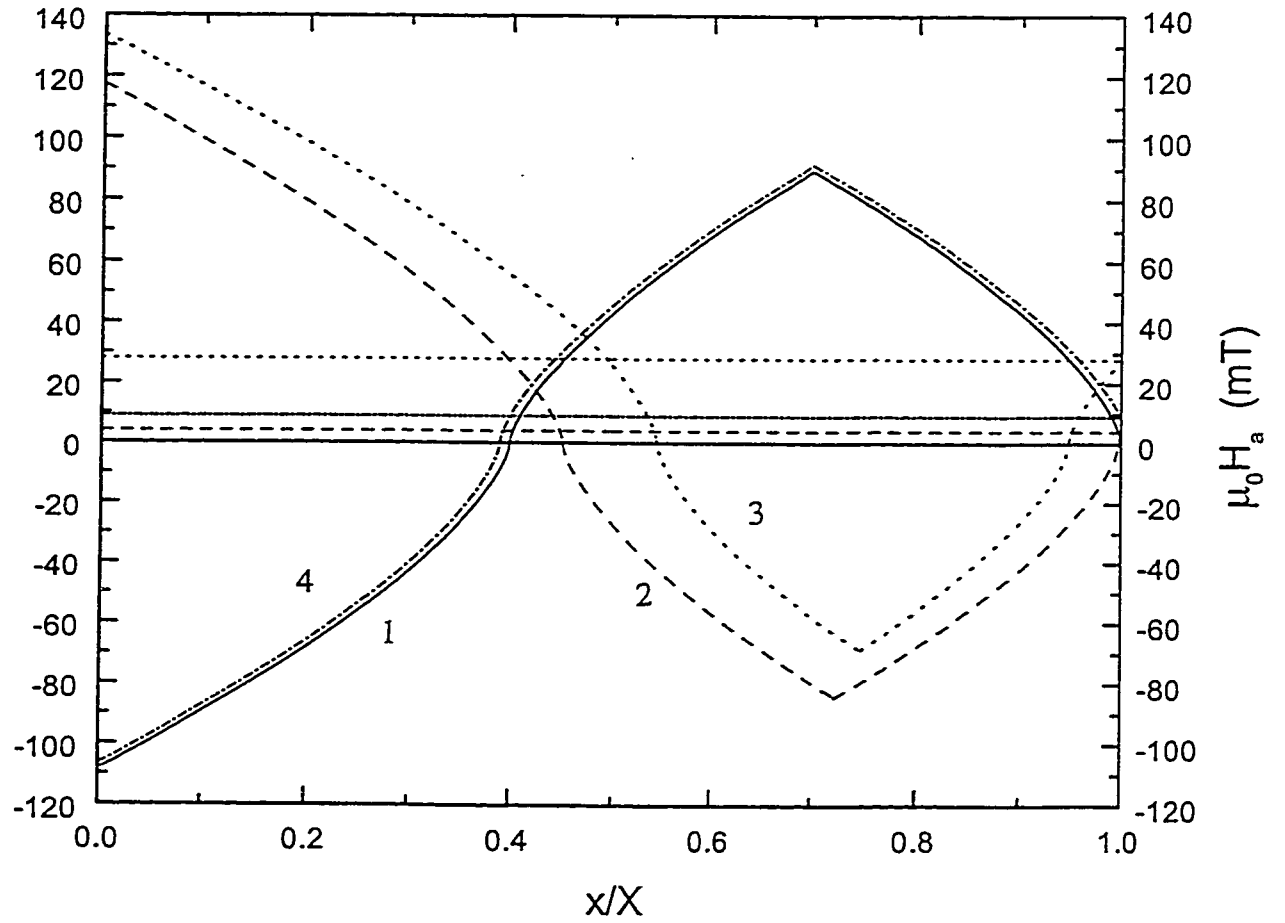


Fig 4.5 Depicts the calculated initial flux density profiles corresponding to the points labeled 1, 2, 3 and 4 of Fig 4.4. For each case $\langle M \rangle = 0$ and the associated baseline $\mu_0 H_a = 0, 4.5, 28$ and 9 respectively are also indicated by the horizontal lines.

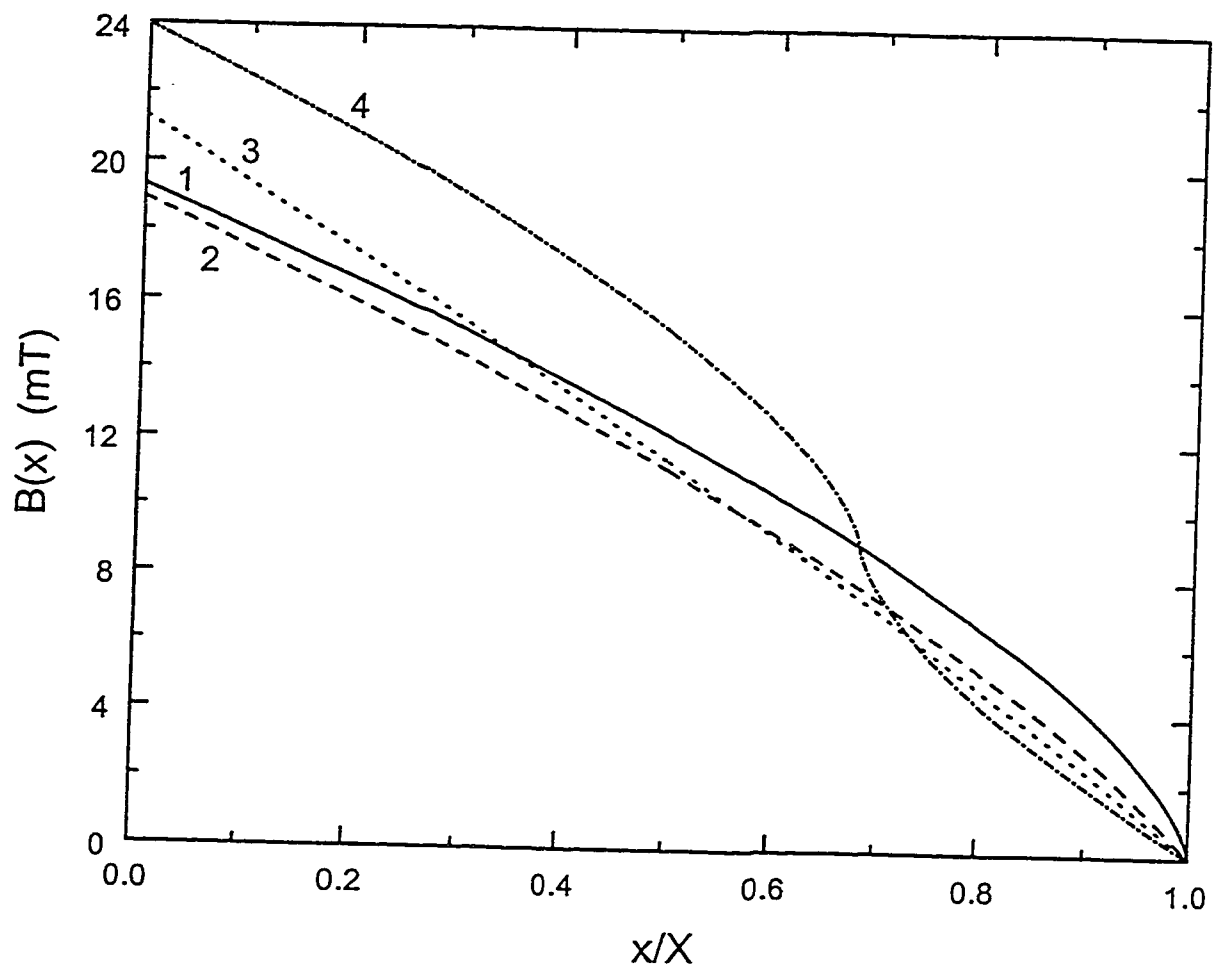


Fig 4.6 Depicts the flux line density profiles for the points 1, 2, 3, and 4 of Fig 4.4. These are the profiles at the instant the sample has attained its largest moment during heating. All profiles are shifted to a common baseline for ease of comparison of the area they embrace, the latter is a measure of $\langle M \rangle$.

CHAPTER 5

Observations on a High T_c Hysteretic Specimen:

Large YBCO Single Crystal

5.1 Introduction

We have studied two very different YBCO samples, both magnetically irreversible. In this chapter we present and discuss the observations on a large hexagonal “single” crystal provided by Argonne National Laboratory. In the next chapter we focus on a “textured” YBCO cylinder consisting of strongly electrically coupled grains.

The 77 K isothermal magnetization curves and flux trapping behaviour which characterize the hysteretic physiognomy of the specimen are presented first. Then we display our data on the maximum moments which appear upon warming from 77 K to $T_c \approx 92$ K with $\langle M \rangle$ initially zero in various final applied fields spanning the range from 0 to H_* . We will see that the thermal disclosure data for this specimen are rather monotonic (i.e. without structure). These observations however are in agreement with the predictions of our model for very hysteretic type II superconductors which have a bulk critical current density j_c nearly independent of B . The isothermal magnetization curves for this sample indeed show that this is the case over the range accessible with our apparatus.

5.2 Isothermal Magnetization Curves

The isothermal magnetization curves of this large crystal at 77 K are displayed in Fig 5.1. The c-axis is $\perp$ to the hexagonal shape of the crystal. The applied magnetic field is directed along the two most distant corners of the hexagon hence parallel to the ab plane and $\perp$ to the c-axis. In YBCO single crystals j_c in the ab plane, denoted j_c^{ab} , is more than an order of magnitude larger than j_c along the c-axis, denoted j_c^c . Consequently the pattern of the quasi-persistent currents induced to circulate all in the same sense and filling the volume of the specimen is complicated^{57,58}. The configuration of j_c in the ab plane along the “waist” of the hexagon is depicted schematically in the inset of Fig 5.2. Although neither idealized planar nor idealized cylindrical geometry are readily applicable to the arrangement encountered here we will nevertheless compare our observations with the predictions for these two tractable schemes.

From inspection of the data of Fig 5.1 and observing where the initial magnetization curve merges with the upper envelope and the upper limit of the hysteresis curve we can estimate H_* and H_{**} as approximately 180 mT and 300 mT respectively. Our apparatus does not allow us to achieve magnetic fields much greater than 300 mT and to stably maintain magnetic fields above 180 mT. The Joule heat injected into the copper wire solenoid becomes excessive when H_a attains ≈ 300 mT. Even though the solenoid is immersed in an open bath of liquid nitrogen its temperature rises appreciably above 77 K hence its resistance increases significantly causing a downward drift in H_a .

Fig 5.2 displays the magnetization curves calculated for idealized cylinder geometry taking j_c independent of B and H to correspond to the measured value of ≈ 180 mT.

In Fig 5.3 we present our measurements of flux trapping by both the H_{cool} and H_{cycle} procedures. The onset of the plateaus in these data provide an alternative estimate of H_* and H_{**} in agreement with the observations from the hysteresis magnetization curves of Fig 5.1. No flux expulsion, i.e. no Meissner effect is detectable within the limits of the sensitivity of our apparatus (i.e. ≈ 0.2 mT) upon cooling from T_c to 77 K in any field $0 < H_a < H_* \approx 180$ mT. This is not surprising since workers report $H_{C1} \leq 5$ mT in small single crystals of YBCO, hence H_{C1} is insignificant compared to H_* in our specimen.

Through the collaboration of Dr. Laurie Wright and his group at the Royal Military College in Kingston, Ontario we have obtained pictures of the pattern of the critical persistent currents of this "crystal". In this study the applied field was directed along the c axis, hence the induced flux retaining currents circulate in the ab plane. As well the study was performed at 20 K such that the critical currents would be orders of magnitude greater than at 77 K. In the sequence of photos shown in Fig 5.4 the sample was first cooled in zero field then subjected to an increasing applied field that would induce diamagnetic shielding currents in the bulk of the crystal. The first photo at 40 mT shows the field to be excluded from nearly the entire volume of the sample. As the field is increased a domain structure of this quasi-single crystal becomes visible. Also the large j_c at 20 K is apparent as the applied field of 1 and then 2 Tesla is completely shielded within the outer millimetre or two of each crystal domain. In this work the Faraday rotation of polarized light reflected from the hexagon surface of the specimen is

measured. These measurements indicate that the large crystal comprises three electrically coupled crystals of comparable size over the ab plane. All our measurements therefore yield averages of the magnetic moments of these three units superimposed on the weaker magnetic moment produced by the "inter-crystal" pattern of induced currents.

5.3 Release of Hidden Magnetic Moments

Our measurements of the peak magnetic moments released during warming from 77 K to T_c in various magnetic fields in the range $0 \leq H_a < H_*$ are displayed in Fig 5.5. Although there is considerable scatter in the data points, it is clear that $\langle M \rangle_{\text{peak}}$ is nearly constant over the range of our measurements with the average $\langle M \rangle_{\text{peak}} / \langle M \rangle_{\text{max rem}} \approx 0.12$. Our simple theoretical model when $j_c = \text{constant}$ predicts a ratio of 0.06 and 0.075, for idealized slab and cylinder geometry respectively. In the framework of the approximation where j_c is independent of B our model for thermal release of magnetic moments predicts that these ratios are independent of H_a and of whether $\langle M \rangle = 0$ was attained by a downswing or upswing. Due to the limitations of our power supplies and copper wire solenoids we could only scan $\langle M \rangle_{\text{peak}}$ for a small range at low fields for the situation where H_a ascends in magnitude. This limitation follows from the constraint that we must ensure that the locus of $\langle M \rangle$ has attained the envelope of the major hysteresis curve when the direction of the sweep of H_a is reversed and is later made to ascend in magnitude when it causes $\langle M \rangle$ to cross the zero axis. Inspection of the two lowest curves of Fig 5.1 shows that this limits our survey for the upswing mode of thermal release starting from $\langle M \rangle = 0$ to the range $0 < H_a \leq 50 \text{ mT}$.

The scatter in these data is appreciable for several experimental reasons.

To raise the temperature of this large crystal from 77 K to above 92 K requires a joule heating rate $P_H \approx 20$ watts over an interval of time t_{heat} of almost 30 seconds. Subsequently the specimen takes almost 3 minutes to cool back to 77 K. By increasing the thermal insulation between the electric heater intimately embracing the crystal and the liquid N_2 bath, we can reduce P_H and t_{heat} , however at the price of appreciably increasing the already lengthy cooling time. The turbulence generated in the liquid N_2 during and immediately after heating of the specimen to T_c causes vibrations of the specimen-pickup coil assembly thereby introducing appreciable noise in the measurements. We note also that the temperature gradient through the volume of the surrounding pickup coil generates a significant Nernst voltage which depends on the strength of the applied magnetic field.

Further, in high T_c materials at 77 K the induced currents in the bulk of the specimen are not persistent but decay logarithmically with time. Consequently the flux density profile when the $\langle M \rangle = 0$ state is reached will exist in the critical state only near the surface of the specimen. The amount of relaxation which has occurred will depend on the time taken to establish the final state, i.e. the duration of the entire procedure whereby $\langle M \rangle$ is made to reach the appropriate major hysteresis envelope and then brought to the $\langle M \rangle = 0$ configuration for the selected final applied field H_a .

In spite of the large scatter it is clear from inspection of Fig 5.5 that our measurements of $\langle M \rangle_{\text{peak}} / \langle M \rangle_{\text{max rem}}$ for this specimen are significantly larger than the predictions of the model. To account for this discrepancy we conjecture that the poor heat conductivity, large heat capacity and high rate of heating encountered in this situation may lead to significant gradients in the profile of the temperature "wave"

advancing into the specimen. Clearly a very non-uniform temperature profile will cause a non-uniform decline in the pattern of the induced currents hence in the thermal evolution of the flux density profiles. In our modelling of these phenomena we can mimic this situation by making j_0 diminish in larger fractions $\Delta j_0/j_0$ in our calculations. This computational procedure does indeed lead to values for $\langle M \rangle_{\text{peak}}$ larger than those calculated exploiting fine-grained steps for $\Delta j_0/j_0$ hence, $\frac{\Delta T}{T_c - T_o}$. Indeed pursuing this procedure we obtain $\langle M \rangle_{\text{peak}}/\langle M \rangle_{\text{max rem}}$ as displayed in Fig 5.6 for idealized slab and cylinder geometry in the Bean-London framework where j_c is independent of B . Here $\Delta n = \Delta j_0/j_0$ denotes the size of the steps used throughout a calculation in the evolution of $\langle M \rangle$ from $\langle M \rangle = 0$ to the peak value $\langle M \rangle_{\text{peak}}$.

5.4 Summary and conclusion

In spite of the considerable scatter in the data points our measurements of the maximum magnetization revealed upon warming for 77 K to $T_c \approx 92$ K for a large hexagonal crystal of YBCO starting with initial states where $\langle M \rangle = 0$ show that $\langle M \rangle_{\text{peak}}/\langle M \rangle_{\text{max rem}} \approx 0.125$ for all values of the applied field whether H_a ascends or descends to the final value over the range, $0 < H_a < H_* \approx 180$ mT. Here H_a was directed along the ab planes.

The hysteresis curves and the flux trapping measurements in this sample over the range $0 < H_a \approx H_{**} \approx 300$ mT indicate that j_c is insensitive to B over this range. Under these circumstances our model for the thermal evolution of the hidden magnetic moments

lead to values of $\langle M \rangle_{\text{peak}}$ which are independent of the magnitude of H_a and whether H_a ascended or descended to the final value during the procedure followed to generate the initial $\langle M \rangle = 0$ configuration. Pursuing the assumptions that (i) $j_c(B,T) = j_o(T)$ and (ii) that the rising temperature profile is uniform over the volume of the specimen during warming, our model leads to $\langle M \rangle_{\text{peak}} / \langle M \rangle_{\text{max rem}} = 0.06$ and 0.075 for idealized slab and cylinder geometry. Mimicking the situations where temperature gradients of various degrees of steepness exist during the penetration of the heat wave and with $j_c(B,T) = j_o(T)$, our model leads to extremal values of $\langle M \rangle_{\text{peak}} / \langle M \rangle_{\text{max rem}} = 0.172$ and 0.203 for these two idealized geometries. These extreme values will be attained if the front of the advancing temperature profile rises abruptly from 77 K to 92 K.

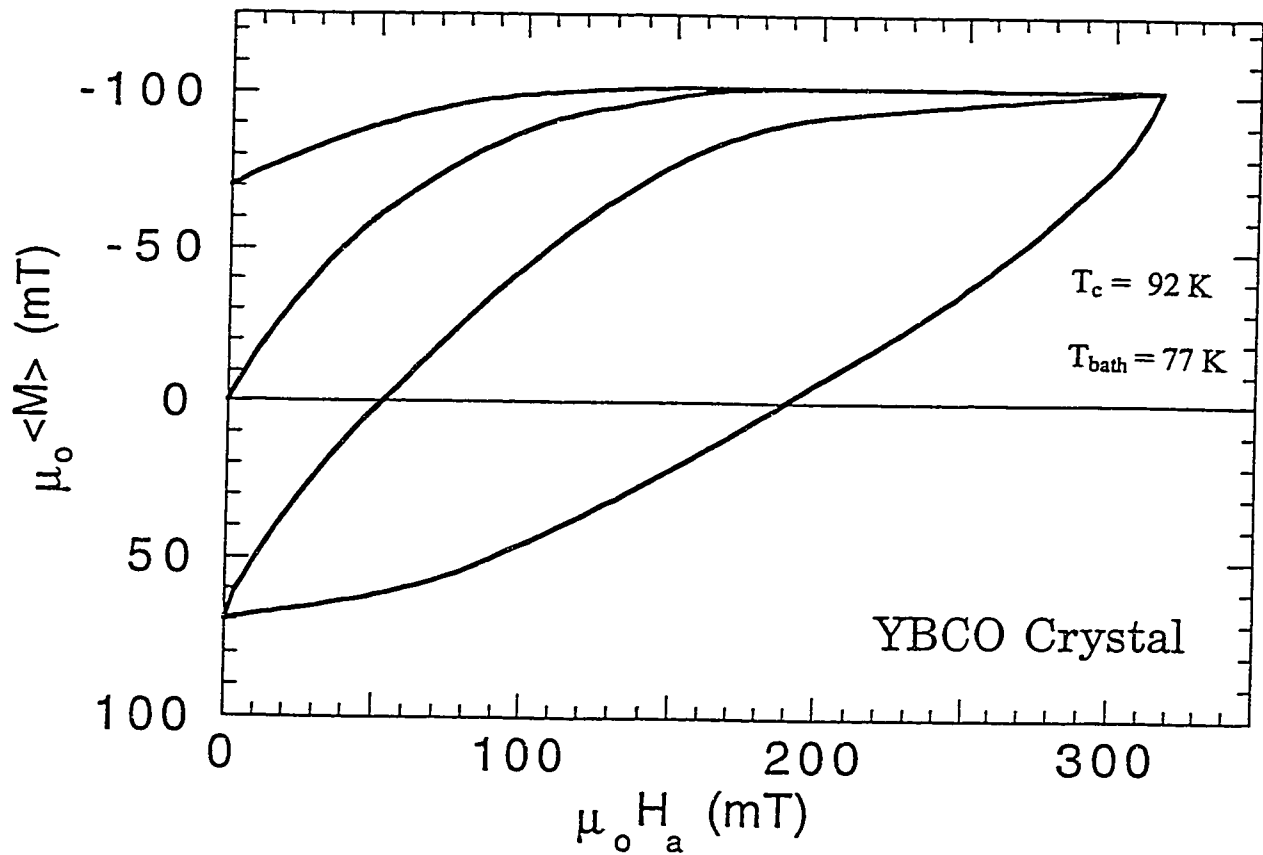


Fig 5.1 Experimentally observed isothermal magnetization curves for the large YBCO crystal. Note the meeting point of the initial magnetization curve and the upper envelope provide an estimate of $\mu_0 H_* \approx 180$ mT. Also the curve begun with maximum trapped field near 70 mT meets the upper envelope at a point that denotes $\mu_0 H_{**} \approx 300$ mT.

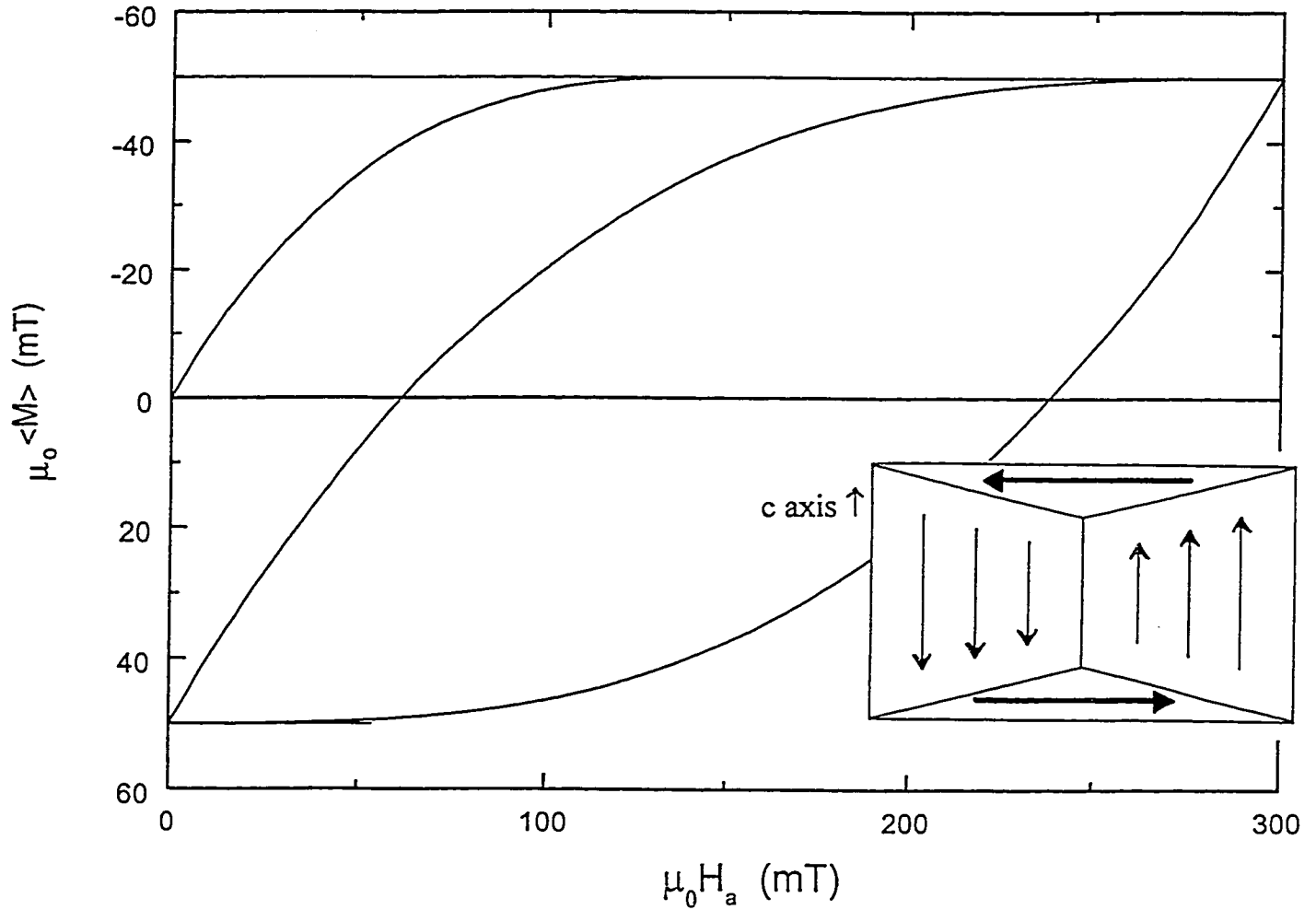


Fig 5.2 Theoretically calculated isothermal magnetization curves for YBCO crystal. Calculations were performed using the critical state model and the assumption of a constant j_c hence the Bean approximation. The adjustable parameter H_* is taken to match that determined experimentally in Fig 5.1. The inset depicts saturated circulating current patterns in a finite sized specimen with different critical current along the ab and c axis. With a larger critical current density for flow along the ab plane, a much smaller cross section is needed for the current flowing in that direction to “complete the circuit” and feed the larger cross section of weaker current density along the c axis.

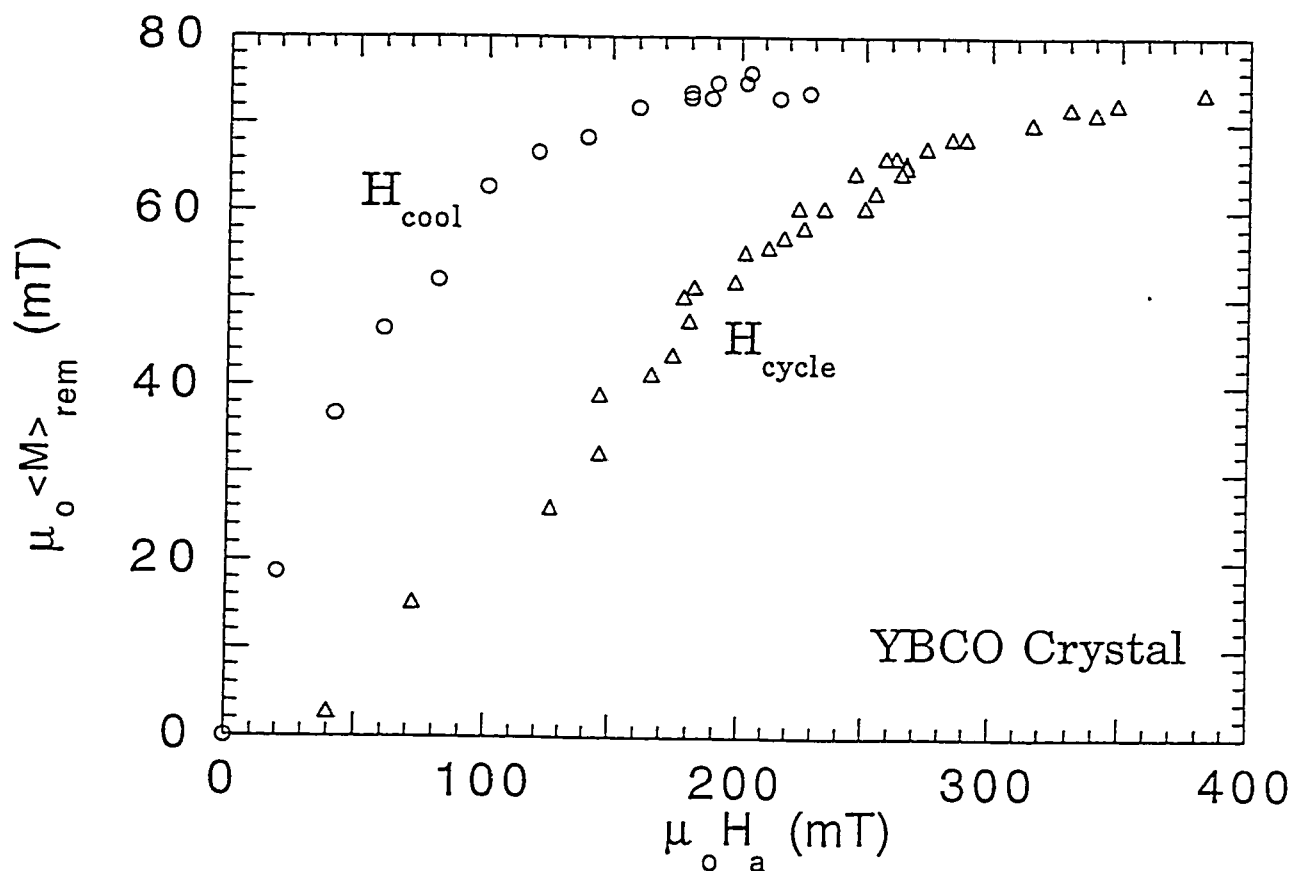


Fig 5.3 H_{cool} and H_{cycle} measurements for the YBCO crystal. Recall H_{cool} denotes the field in which the specimen is cooled from above T_c to 77 K before the field is returned to zero and the trapped flux measured by heating the sample to above T_c . H_{cycle} values denote the maximum field impressed upon a virgin sample before returning to $H_a = 0$. Again the trapped field is recorded and plotted. As described in chapter 3 the onset of the plateau for H_{cool} trapping the maximum remanent magnetization, $\langle M \rangle_{max rem}$, determines H_* . In this case $\mu_0 H_* \approx 180$ mT. Also the onset of the plateau for the H_{cycle} measurements indicates H_{**} , in this case $\mu_0 H_{**} \approx 300$ mT. Thus matching the values obtained from Fig 5.1.

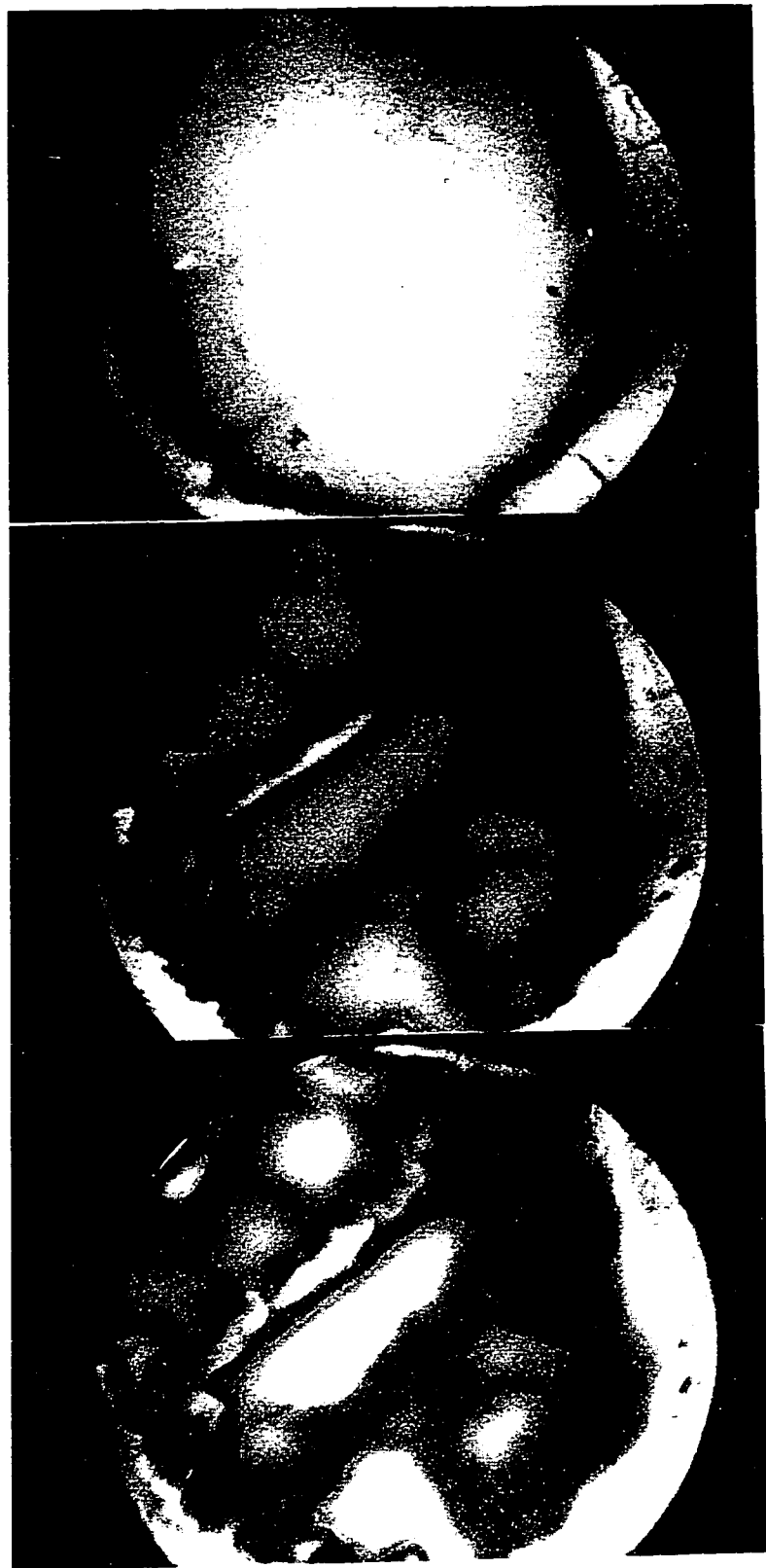


Fig 5.4 The photos above depict the Quench-Melt-Growth crystal shielding an external field of increasing magnitude as described in the text. First with near complete shielding of a 40 mT field in the top photo and then modest field penetration between crystal domains at 1 and then 2 Tesla. The crystal is at approximately 20 K in all photos.

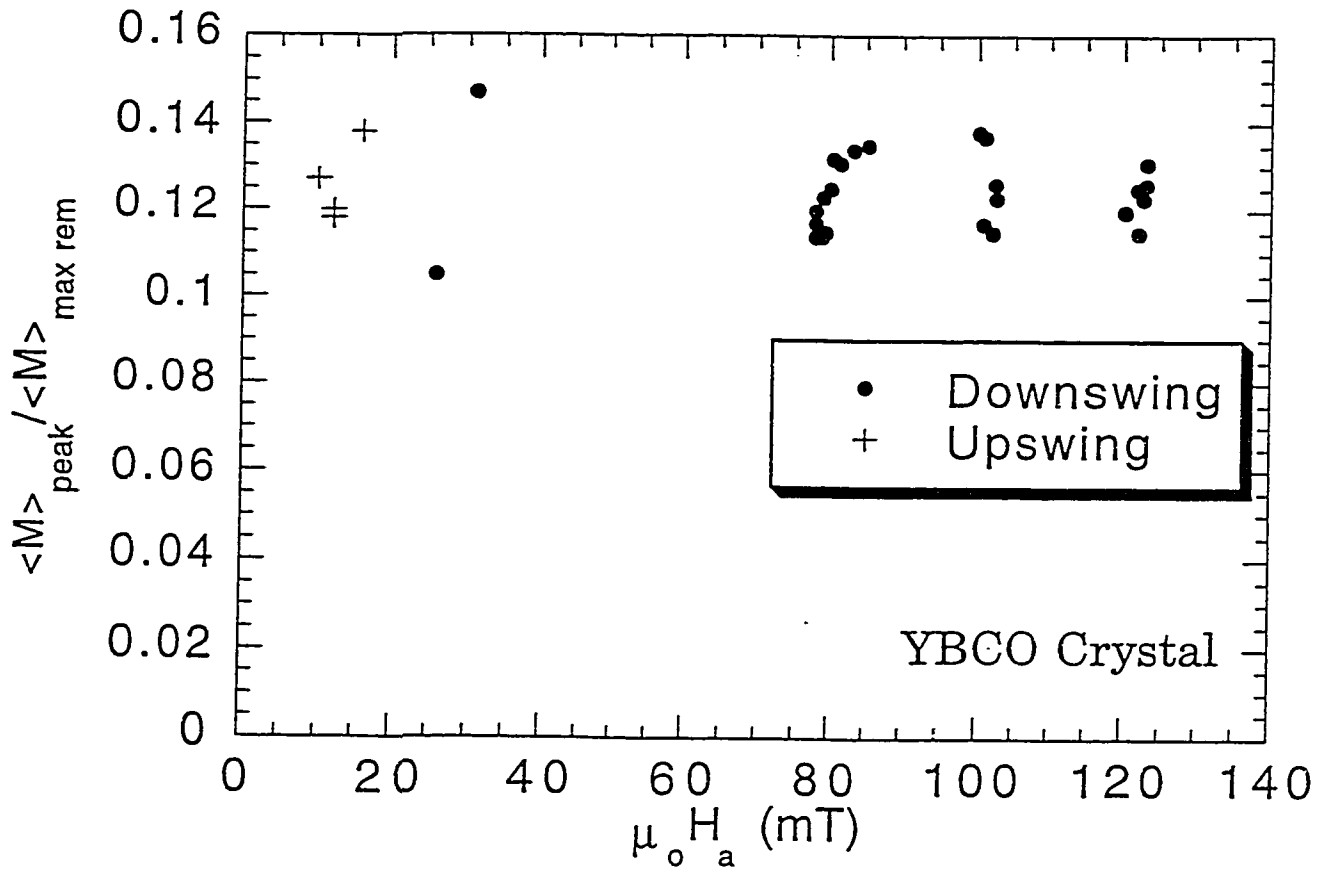


Fig 5.5 Experimental observations of the thermally disclosed hidden magnetic moments for the large YBCO crystal. "Downswing" denotes $\langle M \rangle = 0$ state reached by descending H_a and "upswings" via ascending H_a . The data are normalized to the maximum remanent field $\langle M \rangle_{\max \text{ rem}}$. For convenience of comparison the diamagnetic moment released from "downswing" trials and the paramagnetic moment of "upswing" trials are superimposed. While experimental difficulties lead to a large scatter in the observations, it is quite obvious that as expected the thermally disclosed moments appear equal regardless of the applied field and whether trials are for "downswings" or "upswings"

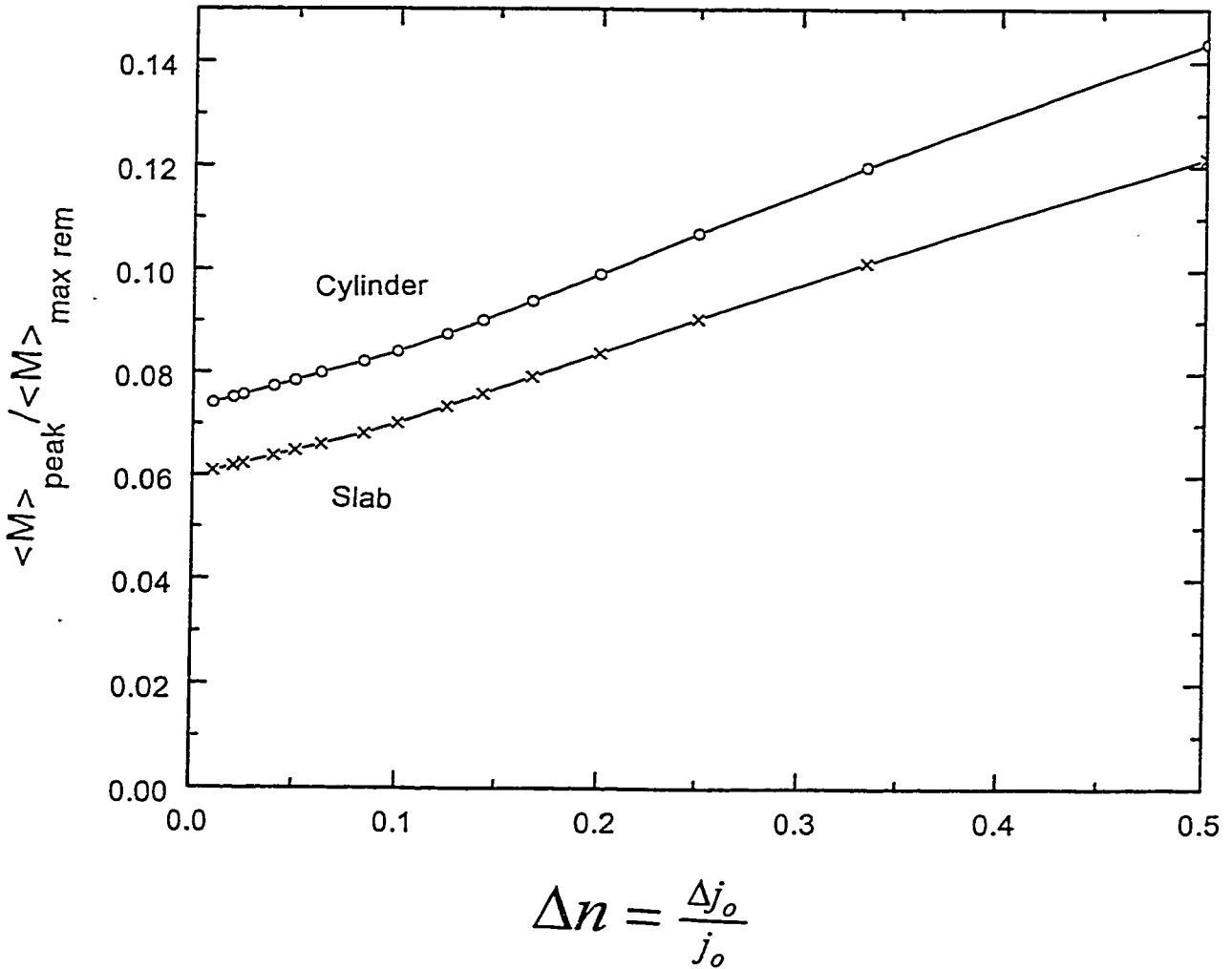


Fig 5.6 Our theoretical model results in universal values of $\langle M \rangle_{\text{peak}} / \langle M \rangle_{\text{max rem}}$ for either idealized slab or cylinder geometry that are lower than the value found experimentally. However if the step size for the decrements of the critical current are increased (larger $\Delta j_o / j_o$), then larger thermally released moments are calculated. These larger step sizes mimic a nonuniform temperature rise and a temperature “wave” moving into the specimen. Such possible nonuniformity is not surprising for the large YBCO crystal with its large heat capacity, poor thermal conductivity and hence requirement of high joule heating to gradually raise its temperature towards T_c .

CHAPTER 6

Observation on a High T_c Hysteretic Specimen

Textured YBCO Rod

6.1 Introduction

In this chapter we present and discuss our observations on a textured YBCO rod provided by Hoechst. Exploiting a proprietary procedure Hoechst fabricates rods and hollow cylinders (tubes) where the grains (platelets of a few dozen microns along the ab plane) are preferentially aligned with the c-axis along the axis of the rod or tube. By careful control of heat/pressure treatment, oxygenation and composition Hoechst achieves good electrical coupling between the platelets. Consequently the magnetic behaviour of these rods and tubes in magnetic fields directed along their length is dominated by the intergrain, hence bulk, supercurrents and therefore exhibits strong hysteresis.

Comparison of our observations on this high T_c specimen with the corresponding measurements on the low T_c cylinder of VTi presented in Chapter 4 will show that these two samples are almost identical twins in regards to their magnetic hysteresis curves. The dependence of the thermally released magnetic moments on H_a also show striking similarities. However the small valley in $\langle M \rangle_{\text{peak}}^{\uparrow}$ encountered in the VTi data and reproduced by our model is not seen in the measurements on the YBCO rod. In contrast

the YBCO rod traces a higher summit for $\langle M \rangle_{\text{peak}}^{\downarrow}$ versus H_a than displayed by the VTi cylinder.

6.2 Isothermal Magnetization Curves

Fig 6.1 displays the major hysteresis envelope, initial magnetization curve and a hysteresis loop extending between $H_{\min} = 0$ and $H_{\max} > H_{**}$. Again inspection of these curves shows that the merger of the initial magnetization curve with the upper envelope occurs when $H_a \approx 50$ mT thereby providing a value for the first penetration field H_* . Further inspection shows that the hysteresis loop starting at $H_a = 0$ and $\langle M \rangle_{\max \text{ rem}}$ touches the upper envelope at $H_a \approx 80$ mT thereby yielding a value for the double penetration field H_{**} . The specimen, within the sensitivity of ≤ 0.2 mT of our measurements, exhibits no flux expulsion (i.e. no Meissner effect) upon cooling from T_c to 77 K in any $H_a < H_{**}$. Consequently $H_{C1}(T)$ can clearly be ignored in our analysis of the behaviour of this sample.

Fig 6.2 displays the magnetization curves corresponding to Fig 6.1 calculated for idealized cylinder geometry and the simple assumption that,

$$j_c = \frac{j_o}{(B/B_{ref})^{1/2}} \quad (6.1)$$

The single adjustable parameter $H_* = (\frac{3}{2} R j_o B_{ref}^{1/2})^{2/3}$ appearing in the calculation is chosen to correspond to the observed value for H_* .

Fig 6.3 displays the evolution of the trapped flux as a function of the magnitude of the magnetic field applied to the virgin (i.e. zero field cooled) sample and then removed. H_{cycle} denotes the magnitude of the magnetic field impressed on the sample during each such swing of H_a . The onset of the plateau for $\langle M \rangle_{\text{rem}}$ indicates the double penetration field H_{**} as explained earlier in the thesis. From inspection of Fig 6.3 we estimate $\mu_0 H_{**} \approx 80$ mT in agreement with the estimate obtained from inspection of the corresponding junction of the magnetization curves in Fig 6.1.

6.3 Release of Hidden Magnetic Moments

Our measurements of the peak magnetic moments released during warming from 77 K to T_c starting with $\langle M \rangle = 0$ in various magnetic fields over the range $0 \leq H_a < H_*$ are displayed in Fig 6.4. Again for purpose of direct comparison of the two sets of data we plot the absolute values of $\langle M \rangle_{\text{peak}}$ versus H_a ascending (upswing) and descending (downswing) in magnitude to the final value where $\langle M \rangle = 0$. Note however that $\langle M \rangle_{\text{peak}}^{\uparrow}$ is paramagnetic whereas $\langle M \rangle_{\text{peak}}^{\downarrow}$ is diamagnetic. The $\uparrow$ and $\downarrow$ arrows indicate that H_a ascended or descended in magnitude. Fig 6.4 should be compared with the corresponding data for VTi, its low T_c “twin”, displayed in Fig 4.3.

The valley evident in the data for $\langle M \rangle_{\text{peak}}^{\downarrow}$ for the VTi sample is not found in the YBCO rod. Both materials however display a summit in the data for $\langle M \rangle_{\text{peak}}^{\downarrow}$ versus H_a . For the YBCO rod, H_{summit} , the position of this summit, normalized to H_* yields a value ≈ 0.22 whereas this ratio $H_{\text{summit}}/H_* \approx 0.06$ for the VTi sample. The latter corresponded nicely with the value generated by our simple model. Fig 6.5 displays the predictions of

our simple model calculated using $j_c = j_o/(B/B_{ref})^{1/2}$ (hence eqn 6.1). Note that this is the same expression exploited for the description of the VTi data.

6.4 Summary and Conclusion

Clearly the agreement between the theoretical curves and the data for $\langle M \rangle_{peak}$ versus H_a is less impressive in the case of the YBCO rod than for the low T_c type II superconductor VTi. We note that in the YBCO rod, the magnetic moment arises from azimuthally circulating intergrain currents coexisting with intragrain induced currents of high critical density. Our model assumes a monolithic and isotropic cylinder hence ignores any intragrain contribution and does not address the extremely complicated situation encountered here. Consequently it is nevertheless gratifying and indeed perhaps surprising that our crude model manages to describe the observations as well as it does.

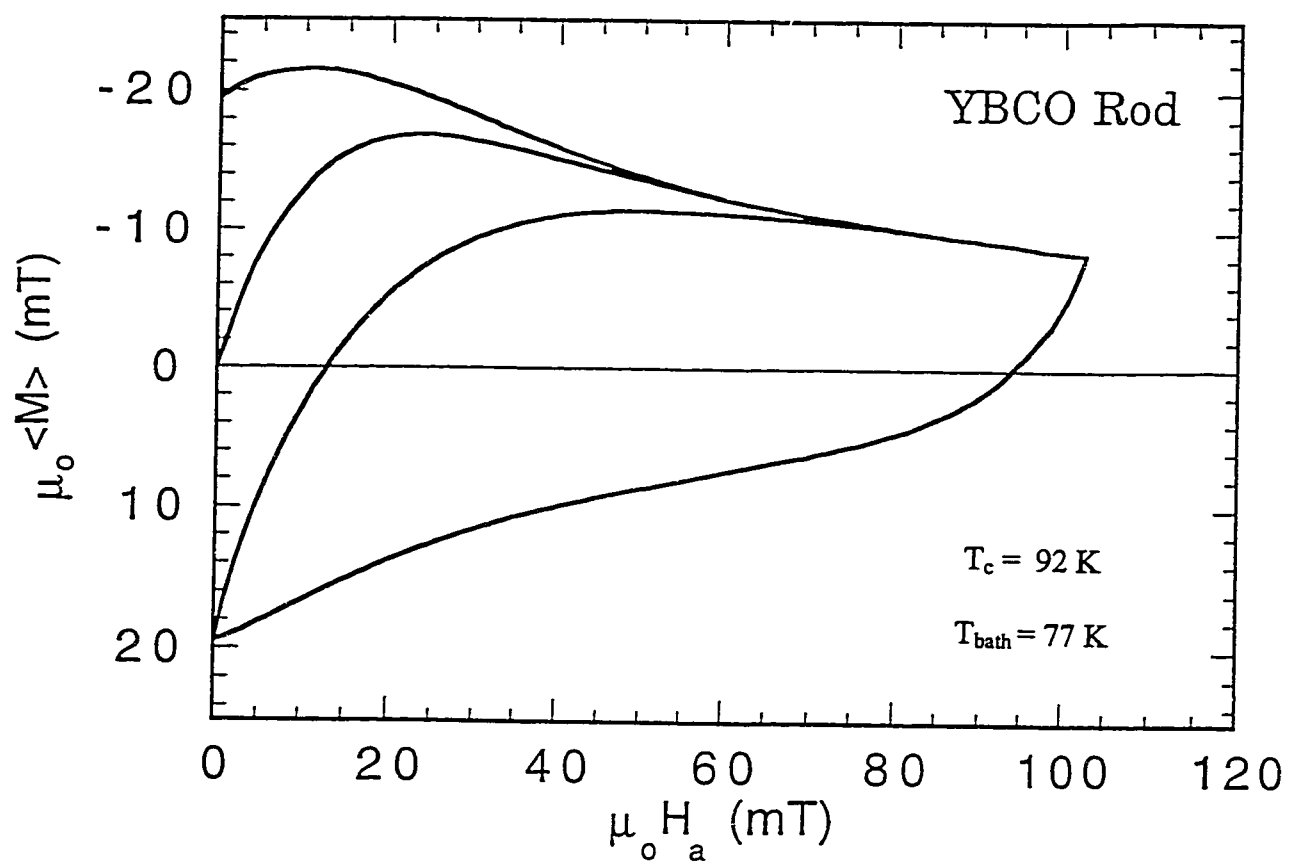


Fig 6.1 Experimentally observed isothermal magnetization curves for the YBCO rod sample. Note the meeting point of the initial magnetization curve and the upper envelope provide an estimate of $\mu_0 H_* \approx 50$ mT. Also the curve begun with maximum trapped field near 20 mT meets the upper envelope at a point that denotes $\mu_0 H_{**} \approx 80$ mT.

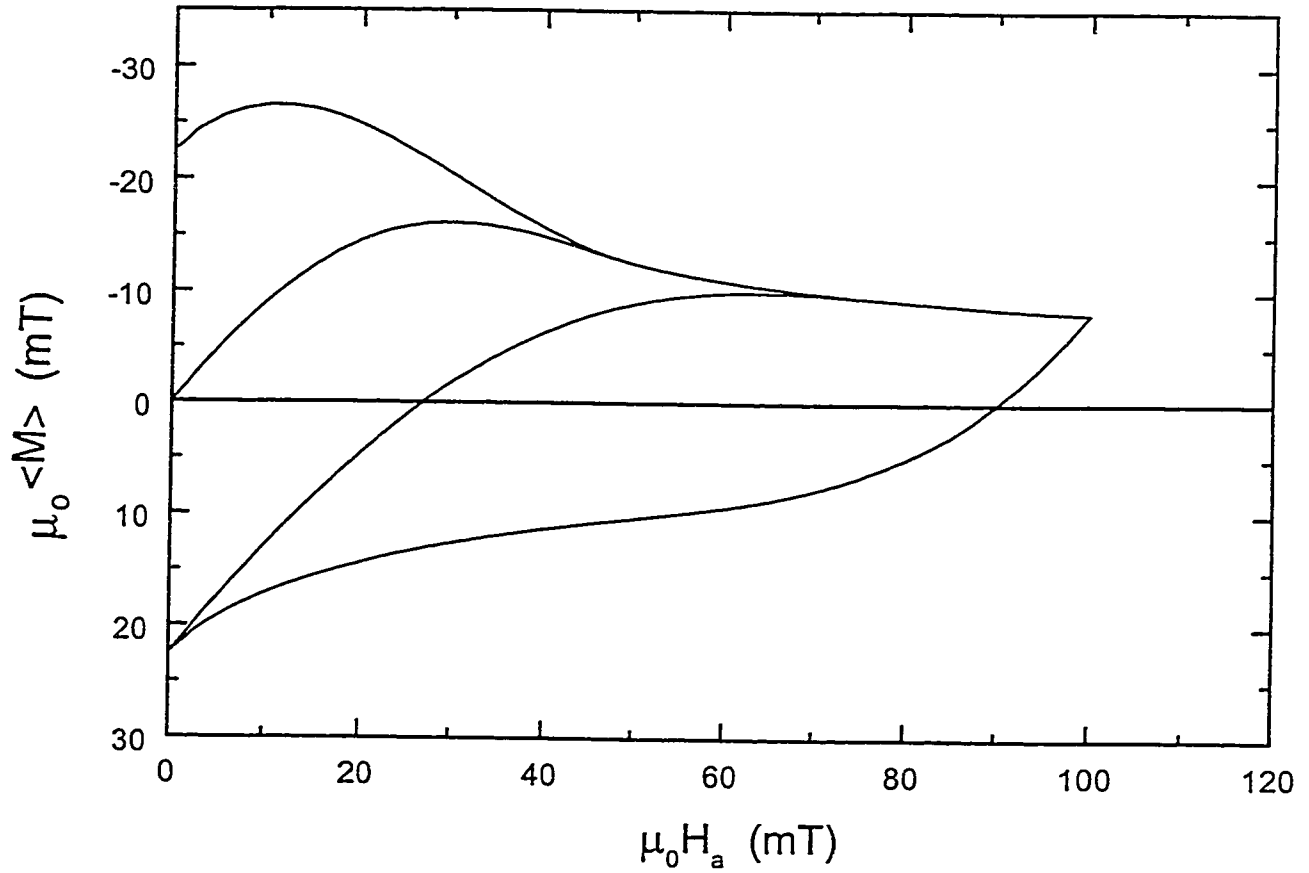


Fig 6.2 Theoretically calculated isothermal magnetization curves for the YBCO rod sample. Calculations were performed using the critical state model and an assumed field dependence of $j_c = j_o(B_{ref}/B)^{1/2}$. The only adjustable parameter is H_* , which is taken to match that determined experimentally in Fig 6.1

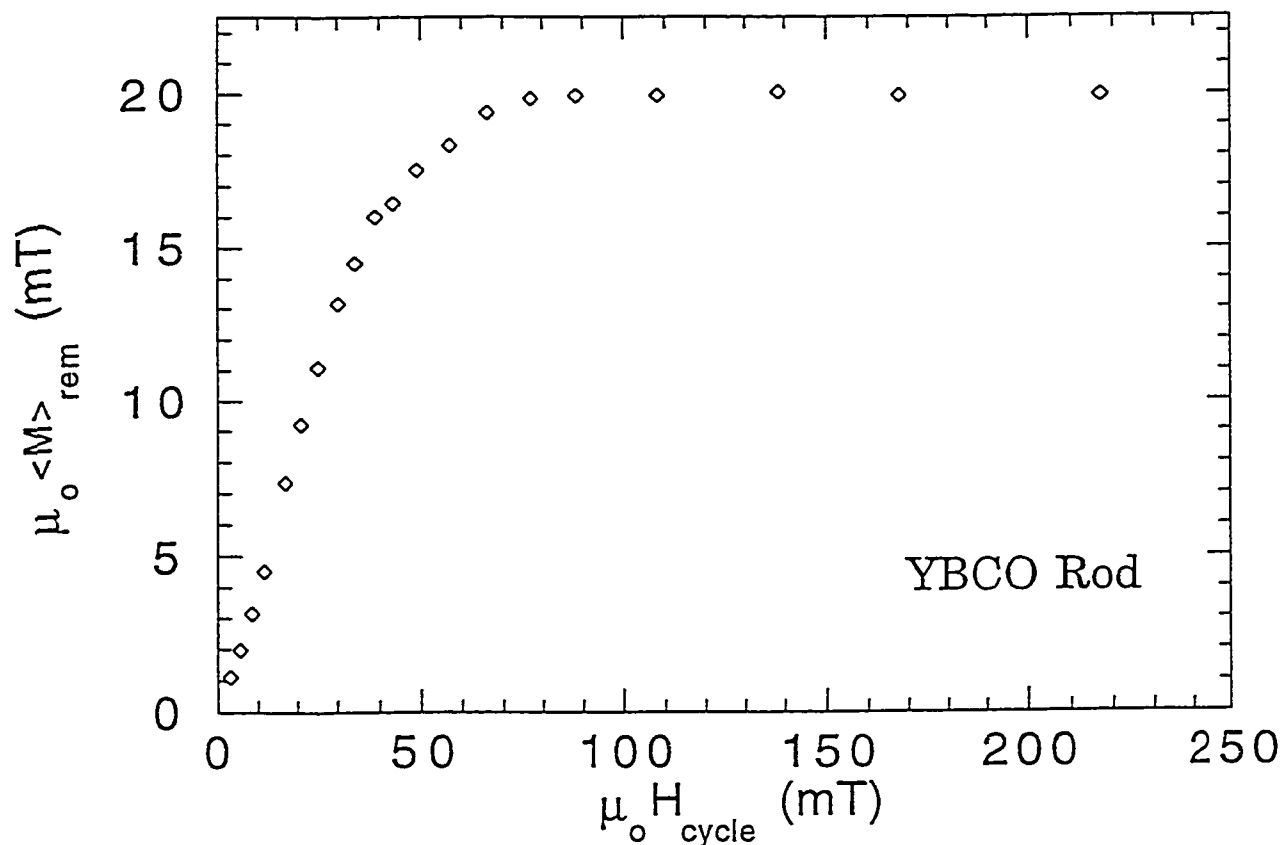


Fig 6.3 H_{cycle} measurements for the YBCO rod. Recall H_{cycle} denotes the maximum field impressed upon a virgin sample before returning to $H_a = 0$. At zero field the trapped field is released by heating to above T_c , recorded and plotted. As described in chapter 3 the onset of the plateau for H_{cycle} measurements indicates H_{**} , in this case $\mu_0 H_{**} \approx 80$ mT. Thus matching the values obtained from Fig 6.1.

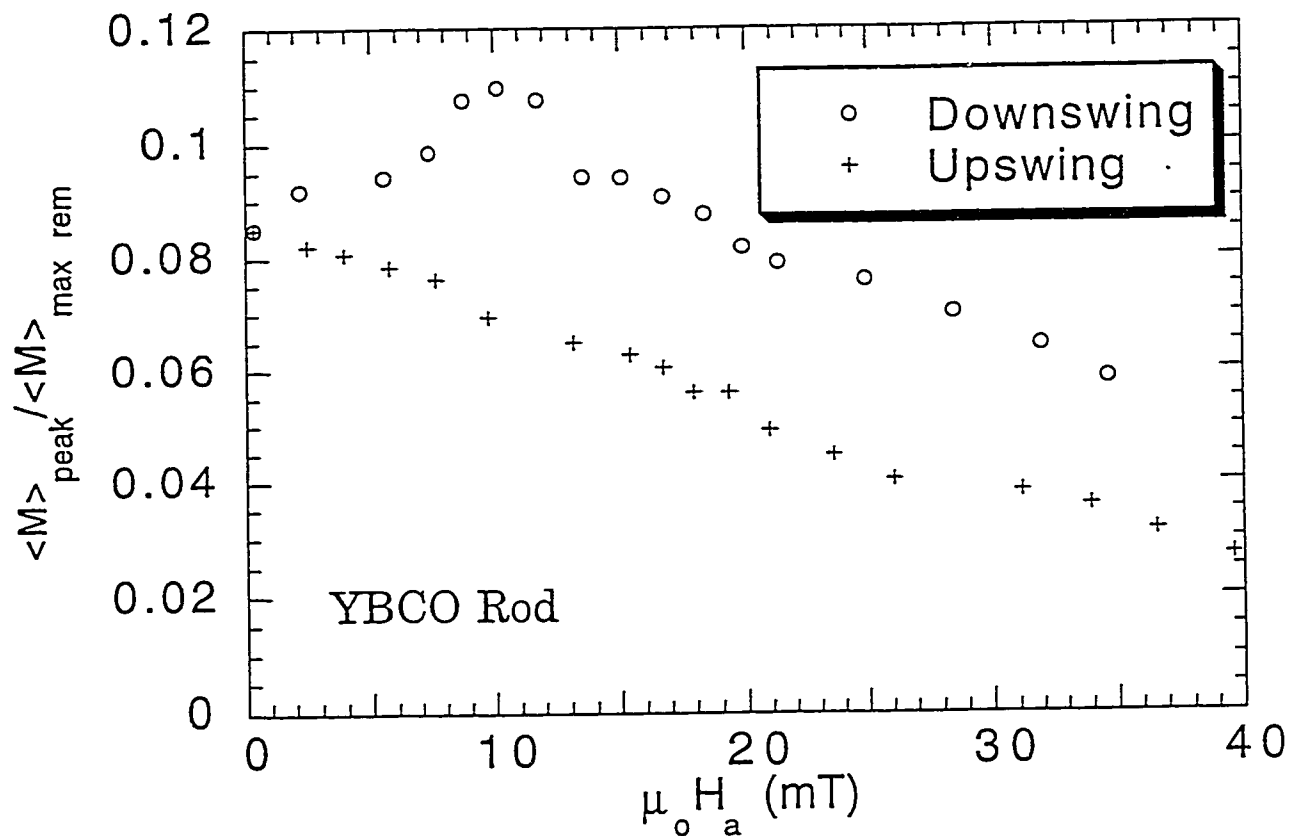


Fig 6.4 Experimental observations of thermally released hidden magnetic moments for the YBCO rod, hence $\langle M \rangle_{\text{peak}} = \mu_{\text{peak}}/V$. “Downswing” denotes $\langle M \rangle = 0$ state reached by descending H_a and “upswings” via ascending H_a . The data are normalized to the maximum remanent field $\langle M \rangle_{\text{max rem}}$. Also for convenience of comparison the diamagnetic moment released from “downswing” trials and the paramagnetic moment of “upswing” trials are superimposed.

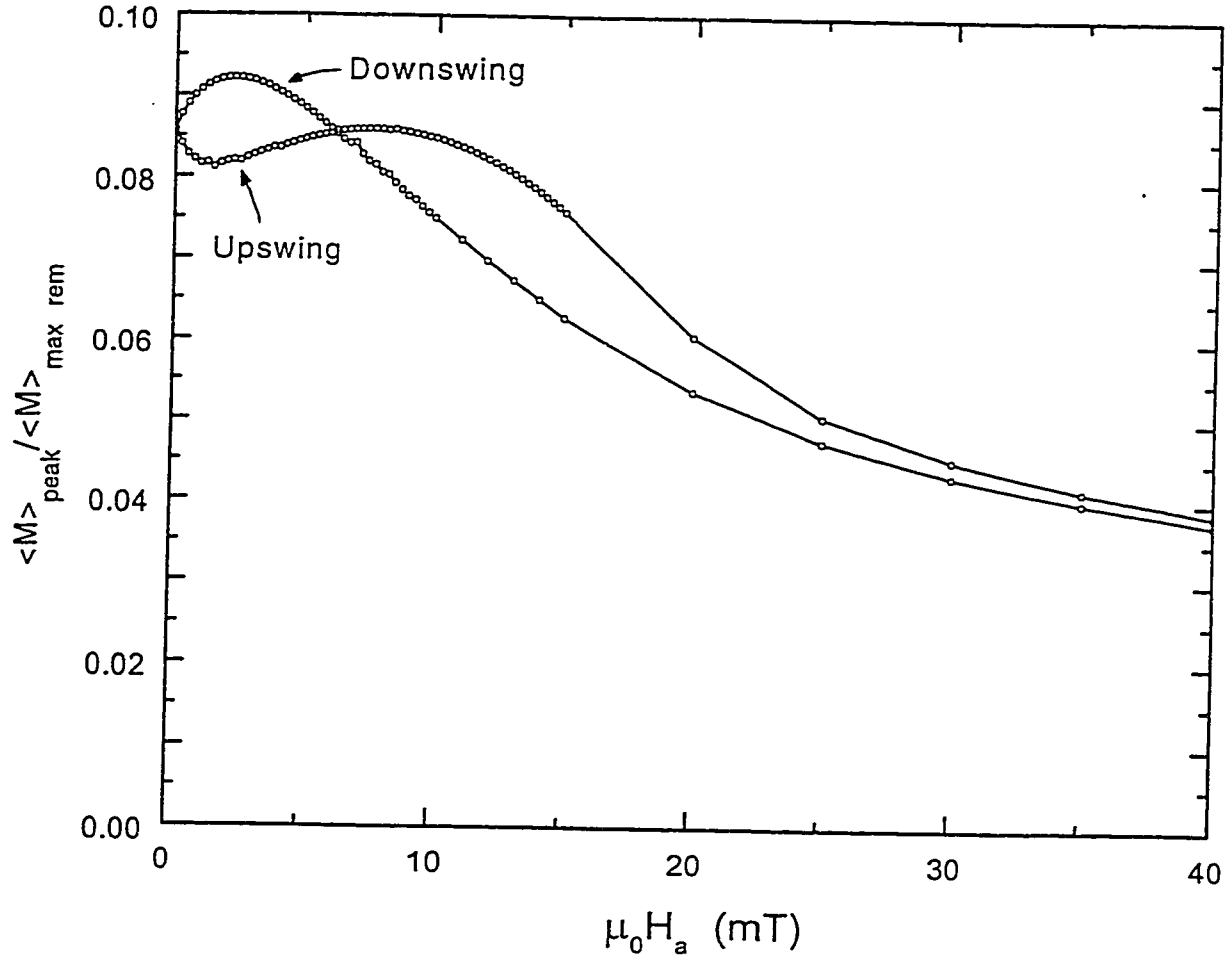


Fig 6.5 Theoretical calculations of the thermally released hidden magnetic moments for the YBCO rod. The same j_c dependence $j_c = j_o(B_{\text{ref}}/B)^{1/2}$ is employed as in Fig 6.2 for the isothermal magnetization curves. Details of the model are presented in chapter 7.

CHAPTER 7

Thermal Disclosure of Hidden Magnetic Moments: Model for Very Irreversible Type II Superconductors

7.1 Introduction

In this chapter we present our model to describe the evolution of the magnetization of highly hysteretic type II superconductors, whether low or high T_c , during warming to T_c in stationary applied magnetic fields H_a . The specimen has previously undergone an appropriate magnetic field history in the superconducting state such that it is filled with induced counterrotating persistent currents in a critical state. To fix ideas we focus on an infinitely long isotropic cylinder of radius R with H_a directed along the axis. The magnetization procedure has ensured that an inner volume extending from the centre to a radius r_i is occupied by currents circulating azimuthally in one sense while in the surrounding annular volume, $r_i < r \leq R$, the induced currents flow in the opposite direction. In this first basic version of our model we will completely ignore the existence of the equilibrium field shielding (diamagnetic) Meissner current $I_M(H_a, T)$, hence take $H_{c1}(T) = 0$. In chapter 10 we will extend our model to incorporate the presence of this thermodynamically reversible surface current.

7.2 Framework of the Model

Soon after the discovery of type II superconductors capable of supporting large lossless currents in intense magnetic fields, hence material exhibiting strong hysteresis, workers have found that their magnetization curves and critical current properties far below H_{C2} could be conveniently and satisfactorily described by exploiting simple analytic expressions for the critical current density of the general form,

$$j_c(B, T) = \frac{j_o(T) B_{ref}^P(T)}{(B(T) + B_o)^P} \quad (7.1)$$

where usually $0 \leq P \leq 1$. The “parameter” B_{ref} is introduced so that the important quantity $j_o(T)$ has the units of current density. The physical origin or basis for the parameter B_o is obscure. Its presence in the formula avoids the appearance of an infinite current density along a tube or inner surface where the magnetic flux density $B(r) = 0$. However this is arithmetically immaterial since such “sheets” of infinite current density will have zero volume.

In work where the object is to optimize the description of the isothermal magnetization curves and thereby obtain the best estimate of $j_c(B)$ at the chosen temperature in the context of eqn 7.1, researchers have dedicated much computational effort to identifying the appropriate values for the parameters n , B_o and $j_o B_{ref}^P$. The cases where $P = 0$, $\frac{1}{2}$ and 1 have been frequently encountered and exploited. In these circumstances eqn 7.1 is referred to as the Bean⁴⁹-London⁵⁰, Yasukochi⁶¹ and Kim⁵¹

formula or approximation. Since our objective is mainly to examine the merits of a simple model which describes the evolution with warming of an important category of flux density profiles, we let $B_o = 0$ in eqn 7.1 and focus on the three basic cases just mentioned.

For idealized planar and cylindrical geometry, Maxwell's eqn, $\nabla \times \vec{B} = \mu_o \vec{j}$, combined with the assumption that $\vec{j}$ exists in a critical state, $\vec{j} = \vec{j}_c(B, T)$, reads

$$\frac{dB}{dr} = \frac{dB}{dx} = \pm \mu_o j_c(B, T) = \pm \frac{\mu_o j_o}{(B / B_{ref})^P} \quad (7.2)$$

where we have introduced a simplified form of eqn 7.1.

We now simply apply the requirements that the magnetic flux density profile must be continuous and the boundary condition that just inside the surface of the specimen, $B_R(R) = \mu_o H_a$. The latter follows because we let $H_{C1}(T) = 0$ hence ignore $I_M(H_a, T)$.

7.3 Development of Expressions for the B profiles

We need to address the four basic flux density configurations (B profiles) sketched in Fig 7.1 and develop expressions for the different segments of these B profiles. Consequently we write eqn 7.2 in integral form,

$$\int_{B(r)}^{\mu_o H_a} B^P dB = \pm \mu_o j_o B_{ref}^P \int_r^R dr \quad (7.3)$$

after integration we obtain,

$$B_a(r) = \left\{ (\mu_o H_a)^{P+1} + (P+1) \mu_o j_o B_{ref}^P R \left(1 - \frac{r}{R}\right) \right\}^{1/(P+1)} \quad (7.4)$$

where the subscript a refers to the right hand side segments of the B profiles in Fig 7.1 (a).

It is convenient and useful to normalize all the quantities with respect to the first penetration field (at the bath temperature, T_o).

$$\mu_o H_* = B_* = \left\{ (P+1) \mu_o j_o B_{ref}^P R \right\}^{1/(P+1)} \quad (7.5)$$

Consequently eqn 7.4 now reads,

$$b_a(r) / \mu_o = h_a(r) = \left\{ h_a^{P+1} + \frac{j_n}{j_o} (1-r) \right\}^{1/(P+1)} \quad (7.6)$$

where $b(r) = B(r)/B_*$, $h_a = H_a/H_*$, $h(r) = H(r)/H_*$ and we let r denote the normalized position r/R . In eqn 7.6 we have also introduced the ratio j_n/j_o . Consequently when the temperature of the specimen is increased we cause the flux density gradients to relax to a new critical state by letting j_n diminish from the initial j_o value to 0, hence j_n/j_o to descend from 1.0 to 0 by convenient discrete amounts.

Introducing the appropriate limits of integration in eqn 7.3 we now proceed to develop expressions for the other segments of the B profiles displayed in Fig 7.1. Thus,

$$h_b(r) = \left\{ h_a^{P+1}(r_i) - \frac{j_n}{j_o}(r_i - r) \right\}^{1/(P+1)} \quad (7.7 \text{ a})$$

Alternatively this can be written,

$$h_b(r) = \left\{ h_a^{P+1} - \frac{j_n}{j_o}(2r_i - r - 1) \right\}^{1/(P+1)} \quad (7.7 \text{ b})$$

where we have introduced $h_a(r_i)$ from eqn 7.6 in 7.7 (a).

$$|h_c(r)| = \left\{ \frac{j_n}{j_o}(r_o - r) \right\}^{1/(P+1)} \quad (7.8)$$

where, $r_o = 2r_i - 1 - \frac{h_a^{P+1}}{j_n/j_o}$ and since $h_b(r) = 0$ at $r = r_o$ (see eqn 7.7 (b)).

$$h_d(r) = \left\{ h_a^{P+1} - \frac{j_n}{j_o}(1 - r) \right\}^{1/(P+1)} \quad (7.9)$$

$$h_e(r) = \left\{ h_d^{P+1}(r_i) + \frac{j_n}{j_o}(r_i - r) \right\}^{1/(P+1)} \quad (7.10 \text{ a})$$

Alternatively, introducing $h_d(r_i)$ from eqn 7.9 the above eqn reads,

$$h_e(r) = \left\{ h_a^{P+1} + \frac{j_n}{j_o} (2r_i - r - 1) \right\}^{1/(P+1)} \quad (7.10 \text{ b})$$

$$|h_f(r)| = \left\{ \frac{j_n}{j_o} (r'_o - r) \right\}^{1/(P+1)} \quad (7.11)$$

where,

$$r'_o = 1 - \frac{h_a^{P+1}}{j_n / j_o} \quad (7.12)$$

since $h_d(r) = 0$ at $r = r'_o$ (see eqn 7.9)

$$|h_g(r)| = \left\{ h_f^{P+1}(r_i) - \frac{j_n}{j_o} (r_i - r) \right\}^{1/(P+1)} \quad (7.13 \text{ a})$$

Alternatively, introducing $h_f(r_i)$ from eqn 7.11 and r'_o from eqn 7.12 we obtain,

$$|h_g(r)| = \left\{ \frac{j_n}{j_o} \left(1 - \frac{h_a^{P+1}}{j_n / j_o} - 2r_i + r \right) \right\}^{1/(P+1)} \quad (7.13 \text{ b})$$

Finally,

$$h_h(r) = \left\{ \frac{j_n}{j_o} (r'_{oo} - r) \right\}^{1/(P+1)} \quad (7.14 \text{ a})$$

where,

$$r'_{oo} = \frac{h_a^{P+1}}{j_n/j_o} + 2r_i - 1 \quad (7.15)$$

since $h_g(r) = 0$ at r'_o (see eqn 7.13 (b)). Note also by symmetry, $r'_o - r_i = r_i - r'_{oo}$, hence $r'_{oo} = 2r_i - r'_o$.

In our work we have focused on initial flux density configurations where $\langle M \rangle = 0$ hence where Φ_{total} , the total (net) magnetic flux permeating the specimen be made equal to the applied magnetic flux $\mu_o H_a \pi R^2$. Consequently in a specified magnetic field H_a/H_* we determine r_i by solving the integral equation,

$$\Phi_{total} = 2\pi \int_0^R B(r) r dr = \mu_o H_a \pi R^2 \quad (7.16)$$

which in our normalized form reads,

$$2 \int_0^1 h(r) r dr = h_a = \int_0^1 h(x) dx \quad (7.17)$$

where the right hand expression applies to idealized planar geometry. $B(r)$, $h(r)$ and $h(x)$ in eqn 7.16 and 7.17 represent the sum of the appropriate segments of B profiles.

7.4 Thermal Evolution of the Magnetization

Once the interface r_i which makes the initial magnetization $\langle M \rangle = 0$ has been found by exploiting Mathematica® to solve the appropriate integral equation for a specified exponent P , geometry and applied field h_a , after a downswing or upswing approach to $\langle M \rangle = 0$ we proceed to let the B profiles relax by incrementally diminishing the normalized critical current density parameter j_n/j_o and let r_i migrate as this takes place.

We have pursued two different scenarios in our calculations of the relaxation of the B profiles. The sequences of events envisaged in each of these two scenarios are illustrated in Figs 7.2 and 7.3.

In the first scenario we visualize that the advancing temperature profile first causes the outward facing façade of the B profile to relax to a shallower gradient as j_n/j_o decreases to j_{n+1}/j_o by a discrete amount $\Delta j/j_o$. In the figures, to fix ideas, we write $n = 1$ and $n + 1 = 2$. The “new” diminished façade joins the pre-existing profile at r_{int} . A straightforward computation exploiting the appropriate expressions enumerated above determines r_{int} .

Now we let the flux which is permeating the volume $0 \leq r \leq r_{s2}$ at this juncture to redistribute by “sliding” down the declining flux density gradient as j changes from j_1 to j_2 in this volume. In this manner we are applying the principle of conservation of flux lines and determine r_{s2} the location of the new summit of the B profile (i.e. the new r_i boundary) by solving the integral,

$$\int_0^{r_{\text{int}}} h_1(r) r dr + \int_{r_{\text{int}}}^{r_{s2}} h_2(r) r dr = \int_0^{r_{s2}} h_2(r) r dr \quad (7.18)$$

where the subscripts 1 and 2 denote the profiles corresponding to j_1 and j_2 .

Since the new boundary $r_i \equiv r_{s2} = r_{s,n+1}$ between the two regions of counter circulating currents is now known it is a straightforward exercise to calculate $\langle m \rangle_{n+1} = \langle M \rangle_{n+1}/H_*$ corresponding to j_{n+1}/j_0 as it evolves from $\langle M \rangle = 0$ to a peak value. It is clear from inspection of Fig 7.2 that this peak diamagnetic value will be attained when r_i has migrated to R . Subsequently as j_n further diminishes flux now enters the specimen causing the peak diamagnetic moment to vanish gradually.

The scenario just outlined can readily be applied, mutans mutandis, to the different initial B profiles displayed in Fig 7.1 and encountered when $\langle M \rangle = 0$ has been reached by an up-sweep of H_a to its final value.

In the second scenario we have exploited to investigate the thermal evolution of the B profiles we visualize that the temperature has increased uniformly throughout the specimen thereby causing j_n to diminish to j_{n+1} “simultaneously” throughout the volume of the specimen. This is the framework envisaged by Clem and Hao in their paper¹.

Fig 7.3 illustrates the computational method whereby we implement this scenario. In (a) we guess that the decline of the flux density gradients to the new value has caused the boundary between the outer and inner magnetic moments, $r_i = r_{s1}$ to migrate inward to r_{s2} whereas in (b) we stipulate an outward displacement. We will see in chapter 10 some circumstances where inward migration of the interface is encountered.

For the situations depicted in (a) of Fig 7.3 where r_i moves inward, r_{s2} is obtained by solving, via Mathematica®, the integral equation,

$$\int_0^{r_{s2}} h_1(r) r dr = \int_0^{r_{s2}} h_2(r) r dr \quad (7.19)$$

which expresses the conservation of the flux lines inside the volume bounded by r_{s2} ,

For the situations displayed in (b) where r_i moves outward, r_{s2} is obtained by solving the integral equation,

$$\int_0^{r_{s1}} h_1(r) r dr + \int_{r_{s1}}^{r_{s2}} h_1(r) r dr = \int_0^{r_{s2}} h_2(r) r dr \quad (7.20)$$

For small decrements i.e. $\Delta j/j_0 \leq 0.005$ we find that the two scenarios we have outlined yield the same values for $\langle M \rangle_{\text{peak}}$ within $\approx 1\%$. However for large decrements they predict significantly different peak magnetizations. As expected, the results generated by scenario 2 do not change markedly as $\Delta j/j_0$ is made bigger. For scenario 1 however and as anticipated from inspection of Fig 7.3, larger decrements for $\Delta j/j_0$ lead to appreciably larger magnitudes for the $\langle M \rangle_{\text{peak}}$. In chapter 5 we have exploited this feature to account for our observations on the large YBCO crystal where the penetrating temperature profile is believed to have very steep gradients because of the high wattage required to warm the sample, its poor thermal conductivity and large heat capacity.

Because we find the first scenario more plausible, more versatile and easier to implement, this is the scheme we have exploited for the results presented in this thesis.

Finally we note that the values for $\langle M \rangle_{\text{peak}}$ predicted by the model do not depend on the actual relationship between the parameter j_n/j_o and T/T_c . The model however does provide accurate values for j_n/j_o when $\langle M \rangle_{\text{peak}}$ is attained as a function of H_a ascending or descending. Thus had we invested the time and care needed to monitor T as the peaks in the magnetization were traversed during warming, these data curves could be compared with the predictions of our model and provide information on $j_o(T,B)$ in the vicinity of T_c .

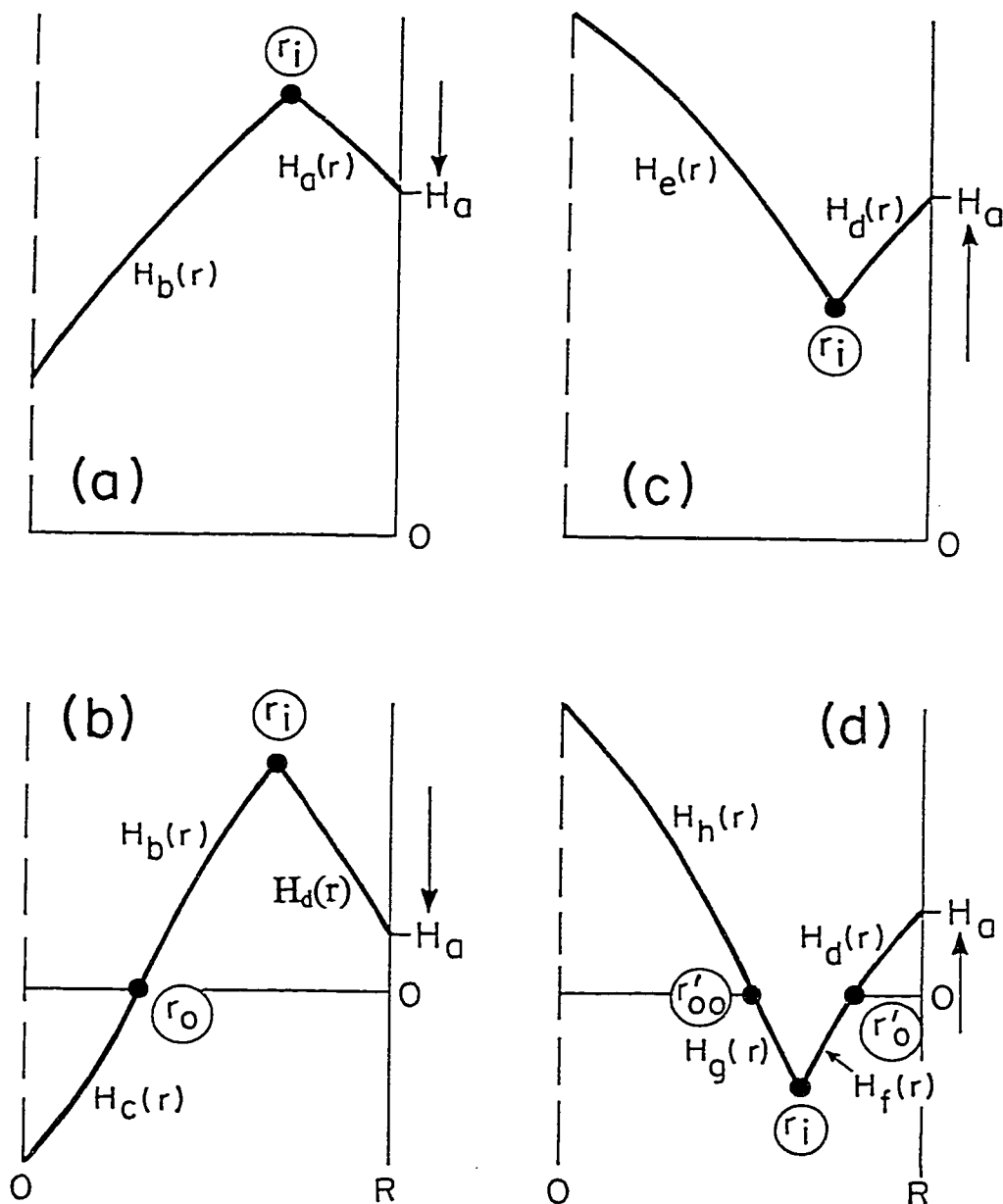


Fig 7.1 Flux density profiles with the associated functions and boundaries needed to describe and thus model the spectrum of profiles giving the $\langle M \rangle = 0$ state. (a) and (b) display profiles for H_a descending, thus downswings. In (b) with $\langle M \rangle = 0$ reached in low fields the specimen is in the hybrid state with both positive and negative flux. This introduces a new boundary (r_o) required at the $B = 0$ point. (c) and (d) are for H_a ascending or upswings. Again in (d) the specimen is in a hybrid state and requires the determination of several boundaries to properly express individual profiles used in the integral relations eqn 7.16 to obtain the $\langle M \rangle = 0$ state.

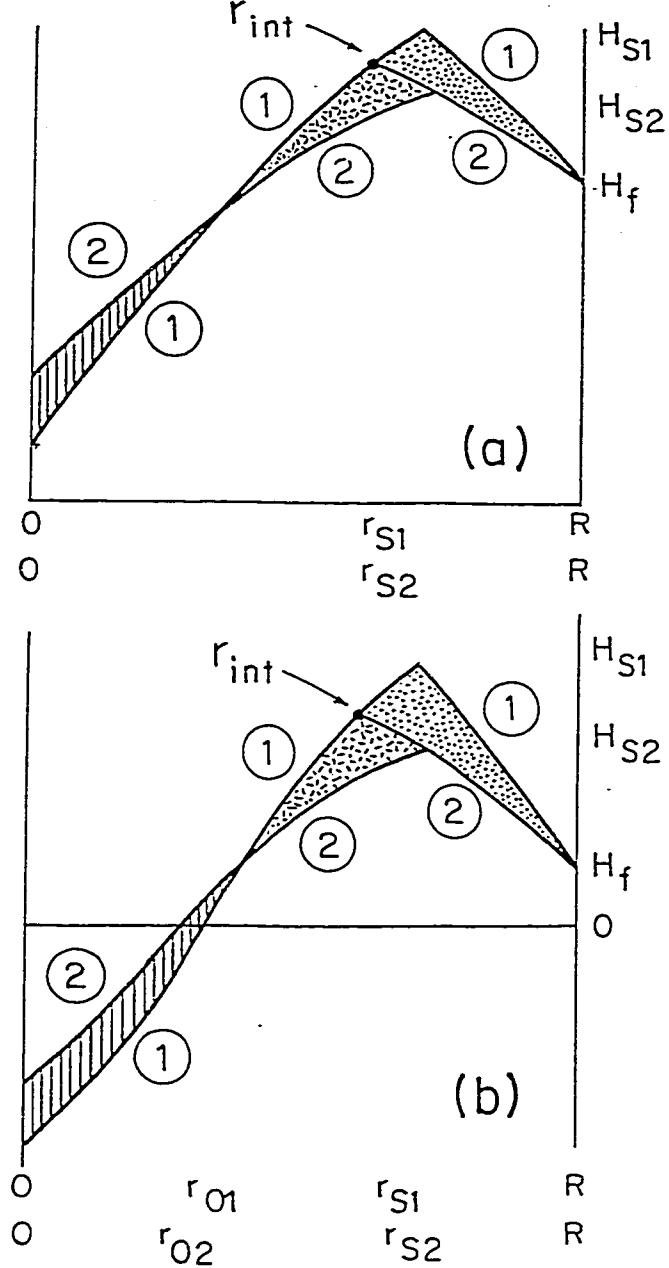


Fig 7.2 (a) Outlines the procedure used in scenario 1 described in the text, in this case for a "downswing" trial. In the first step a temperature change and thus relaxation of flux density gradients is assumed only in the outer façade. Flux can only flow "downhill" and thus an amount of flux relative to the "dotted" area leaves the sample. The point r_{int} is then easily determined. Next, relaxation is allowed for the remaining volume and conservation of flux dictates that the flux represented by the "stippled" area will flow "downhill" and thus fill the area represented by vertical lines. Since these areas are equal the integral eqn 7.18 is used to solve for the new peak position r_{S2} . These steps are repeated until r_{sn} becomes R and represents the peak paramagnetic moment possible. The spatial average is calculated giving $\langle M \rangle_{peak}$ for a specific value of H_a . In the hybrid case (b) the procedure is identical but leads to more cumbersome integrals to solve.

CHAPTER 8

Observation on a Low T_c Semi-Reversible Type II

Superconductor: Niobium Ribbon

8.1 Introduction

In many respects, niobium is perhaps the “ideal” type II superconductor. The magnetization of high purity, well annealed polycrystalline samples exhibits nearly perfect reversible behaviour $\mu_0 H_{C1} \approx 120$ mT at $T = 0$ and is the largest found in nature. With $\mu_0 H_{C1} \approx 100$ mT and $\mu_0 H_{C2} \approx 500$ mT at 4.2 K, even a modest superconducting solenoid or a copper wire solenoid immersed in liquid N_2 can provide sufficient magnetic field to enable even a third world laboratory (i.e. ours) the opportunity to probe its properties over the entire superconducting state.

A modest degree of cold rolling of wires or rods into ribbons or sheets creates a sufficient high density of imperfections (i.e. tangles of dislocations)) in this metal that it can support bulk current densities of $\approx 10^9$ A/m² in $\mu_0 H \leq 500$ mT. Indeed the first superconducting solenoids were made using unannealed Nb wires in 1958-60 independently by G. Yntema, S.H. Autler and M.A.R. LeBlanc⁶²⁻⁶⁴ generating ≈ 0.5 T at 4.3 K and 0.7 T at 1.8 K. These Nb specimens display very large hysteresis and the presence of the large Meissner current becomes difficult to discern.

Thin sheets (0.025 cm thick) of high purity Nb obtained from Kawecki Berylco were annealed in their laboratory at our request in order to weaken the strong pinning generated by the cold rolling hence reduce the bulk critical current density j_c sufficiently so as to endow the final sample with a first penetration field $H_* = \langle j_c \rangle X$ comparable to H_{C1} . Consequently the specimens we studied exhibit the desired semi-reversible magnetic behaviour. We will show in this chapter that this state of affairs enables us to clearly identify the contributions of both the Meissner current and the bulk induced currents to the peak magnetic moments appearing upon warming to T_c starting with $\langle M \rangle = 0$. Now three concentric regions of circulating currents initially coexist, two in the bulk and one along the surface, each giving rise to a magnetic moment. In the initial $\langle M \rangle = 0$ states, whether attained via H_a descending or ascending in magnitude to the final value, two of these magnetic moments combine to exactly cancel the magnitude of the third as illustrated in Fig 2.6.

8.2 Isothermal and related Magnetization Curves.

The basic four isothermal magnetization curves measured at 4.2 K are displayed in Fig 8.1. Here we also present, for comparison purposes, the Meissner data i.e. the spatial average of the magnetic flux expelled upon cooling from $T_c \approx 9.0$ to 4.2 K in various static magnetic fields $0 < H_a < H_{C2}$. For archival purposes we again display the Meissner data in an enlarged format in Fig 8.2.

It is far beyond the scope of this thesis to endeavour to reproduce this Meissner flux expulsion data. We note that to our knowledge such tasks have not yet been reported in

the literature. Clem and his collaborators¹⁻⁵² have described the elements of the procedure to be followed in accomplishing such a task. Clem and Hao have implemented this scheme analytically for a specific simple situation¹. In their calculation they have calculated $\langle M \rangle$ versus T diminishing from T_c in stationary $H_a \ll H_{C2}$ assuming,

- (i) $j_c(T)$ independent of B , hence for the Bean-London approximation
- (ii) $I_M = f H_{C1}$ when $H_a > H_{C1}$, where f is an adjustable fraction, and,
- (iii) the same dependence on temperature for $j_c(T)$ and $H_{C1}(T)$ namely $1 - (T/T_c)^2$.

These crude assumptions are clearly incapable of accounting for our magnetization curves. We note however that the model we develop in chapter 10 to account for the thermal release of the hidden magnetic moments bears close kinship to the framework proposed by Clem and collaborators^{1,52} to account for the Meissner flux expulsion in semi-reversible type II superconductors. It is more difficult to model the latter phenomenon, both computationally and analytically, because regions with subcritical gradients develop in the flux density profiles during field cooling whereas during slow warming the entire volume exists in temperature dependent critical states. Nevertheless our group has initiated efforts and made some progress in computationally solving this problem for more realistic analytic dependences of j_c and I_M on B .

In Fig 8.3 we display our measurements of $\langle M \rangle_{\text{rem}}$ versus H_{cool} and H_{cycle} . From inspection of this figure and Fig 8.1 we estimate $H_* \approx 170$ mT, $H_{**} \approx 240$ mT and $H_{C1} \approx 100$ mT. Clearly then bulk pinning critical currents and the equilibrium Meissner current play comparable roles in the magnetic behaviour of this specimen as desired.

To apply our model for thermal release of hidden magnetic moments to this specimen we want analytic expressions for the dependences of $j_c(B, T)$ on B and of

$I_M(H_a, T)$ on H_a which are simple and take into account the fact that our magnetization measurements extend close to H_{C2} . Consequently we select the simple linear relations,

$$j_c(B, T) = j_o(T) \left\{ 1 - \left(\frac{B}{B_{C2}(T)} \right) \right\} \quad (8.1)$$

and

$$I_M(H_a, T) = H_{C1}(T) \left\{ \frac{H_{C20} - H_a}{H_{C20} - H_{C10}} \right\} \quad (8.2)$$

when, $H_{C10} \leq H_a \leq H_{C20}$ where H_{C10} and H_{C20} denote the values at the bath temperature. Recall that $I_M = H_a$ in the ranges, $0 < H_a < H_{C1}$.

Fig 8.4 displays the isothermal magnetization curves corresponding to those presented in Fig 8.1 which are calculated by exploiting these simple ingredients for idealized planar geometry. In these calculations we have taken $\mu_o H_{C1} = 80$ mT, $\mu_o H_{C2} = 460$ mT and $\mu_o H_* = 180$ mT. The analytic expressions for the various flux density profiles which follow from eqns 8.1 and 8.2 when H_a is made to trace the various magnetization curves shown in Fig 8.4 are given in chapter 10 for semi-reversible type II superconductors.

We consider the correspondence between the theoretical curves displayed in Fig 8.4 and the measured curves presented in Fig 8.1, although rather modest, as nevertheless adequate for our main purpose. We emphasize again that our objective is to test the ability of our model for the thermal release of opposing concentric magnetic moments to

account for observations when both I_M and bulk critical currents are equally important.

We therefore now proceed to this crucial new aspect.

8.3 Release of Hidden Magnetic Moments

The uppermost data points (open circles) in Fig 8.5 display our measurements for $\langle M \rangle_{\text{peak}}^{\downarrow} / \langle M \rangle_{\text{max rem}}$ versus H_a descending. One obvious feature of interest here is the broad hump traced by these data points versus H_a . For ease of comparison we display again the corresponding data for the hysteretic low T_c (VTi) and high T_c (YBCO rod) specimens in Fig 8.6 (see the $\circ$ data points).

For H_a ascending to final values in the range $0 < H_a < H_{C1}$, we report two sets of data points from the evolution of $\langle M \rangle$ during warming to T_c starting with $\langle M \rangle = 0$. Here as sketched in the insert, $\langle M \rangle$ traces an S shape as T is made to rise to T_c . Initially the specimen acquires a paramagnetic moment, hence flux is entering until point 2 is reached. Then flux is seen to exit from the sample in sufficient amount that this paramagnetic moment disappears (point 3). The exit of flux however continues upon further rise in temperature, consequently the specimen next develops a diamagnetic moment. The magnitude of this moment attains a maximum (point 4) and is then made to vanish (point 5) as the superconducting state is extinguished by the final rise of T to T_c . The triangles, Δ , and the diamonds, $\diamond$, in Fig 8.5 display the observed $\langle M \rangle$ at points 2 and 4 on the S shaped curve of the insert. In contrast the hysteretic low and high T_c specimens during their excursion to T_c exhibit only a V shaped curve, hence only the

appearance of a paramagnetic magnetization whose peak values are displayed versus H_a in the lower quadrants of Fig 8.6.

Further we note that when the final values of H_a ascending exceed $\approx H_{C1}/2$, the semi-reversible Nb only exhibits a diamagnetic moment during the entire warming history to T_c . This stands in dramatic contrast with the behaviour of the hysteretic samples which disclose only paramagnetism during their excursion to T_c in the higher range of H_a ascending (see the lower data curves in Fig 8.6).

Fig 8.7 displays the corresponding theoretical values for $\langle M \rangle_{\text{peak}}^{\uparrow}$ and $\langle M \rangle_{\text{peak}}^{\downarrow}$ generated by the model described in chapter 10. In these calculations we exploited eqns 8.1 and 8.2 to describe the dependence of $j_c(B,T)$ on B and $I_M(H_a,T)$ on H_a . For the variation of I_M with temperature we assumed a simple linear dependence, hence,

$$I_M(H_a, T) = I_M(H_a) \left\{ 1 - \frac{T}{T_c} \right\} \quad (8.3)$$

when $H_a > H_{C10}$. For the bulk current density we assumed that the current density parameter $j_o(T)$ obeyed the following prescription,

$$j_o(T) = j_{oo} \left\{ 1 - \left(\frac{T}{T_c} \right)^{1/2} \right\} \quad (8.4)$$

We have extensively explored the effect of various combinations of different temperature dependences for I_M and j_o . These calculations show that the important feature here is the relative rate of decrease of H_{C1} and j_o with temperature. The behaviour

displayed in Fig 8.7 is encountered provided that I_M initially declines much more slowly with increasing T than j_o .

We have also investigated the effect of introducing various more realistic dependences for j_c on B and for I_M on H_a than the crude linear approximations (eqns 8.1 and 8.2). The more "sophisticated" analytic functions we have exploited in this enterprise have however not led to any significant improvement in the overall description of the thermal disclosure observations.

Fig 8.8 and 8.9 display a typical sequence of B profiles calculated with our model as $\langle M \rangle$ progresses through points 1, 2, 3, 4 and 5 of the S curve traversed by the magnetization during warming to T_c starting with $\langle M \rangle = 0$ and sketched in the inset of Fig 8.5. From careful inspection of the sequence of curves shown in Fig 8.9 we can see why $\langle M \rangle$ has traced an S curve when I_M is much less sensitive to temperature than j_o over the range of temperature lower than that existing when point 4 is reached on the curve.

8.4 Summary and Conclusion

Exploiting simple analytical expressions to describe the field and temperature dependence of the two ingredients needed to account for the basic magnetization curve of a semi-reversible low T_c type II superconductor we have successfully reproduced the salient features of the phenomena encountered during the thermal release of hidden magnetic moments in a Nb ribbon during warming from 4.2 K to T_c in a broad range of

magnetic fields ascending and descending to the final value where $\langle M \rangle$ crosses the zero axis.

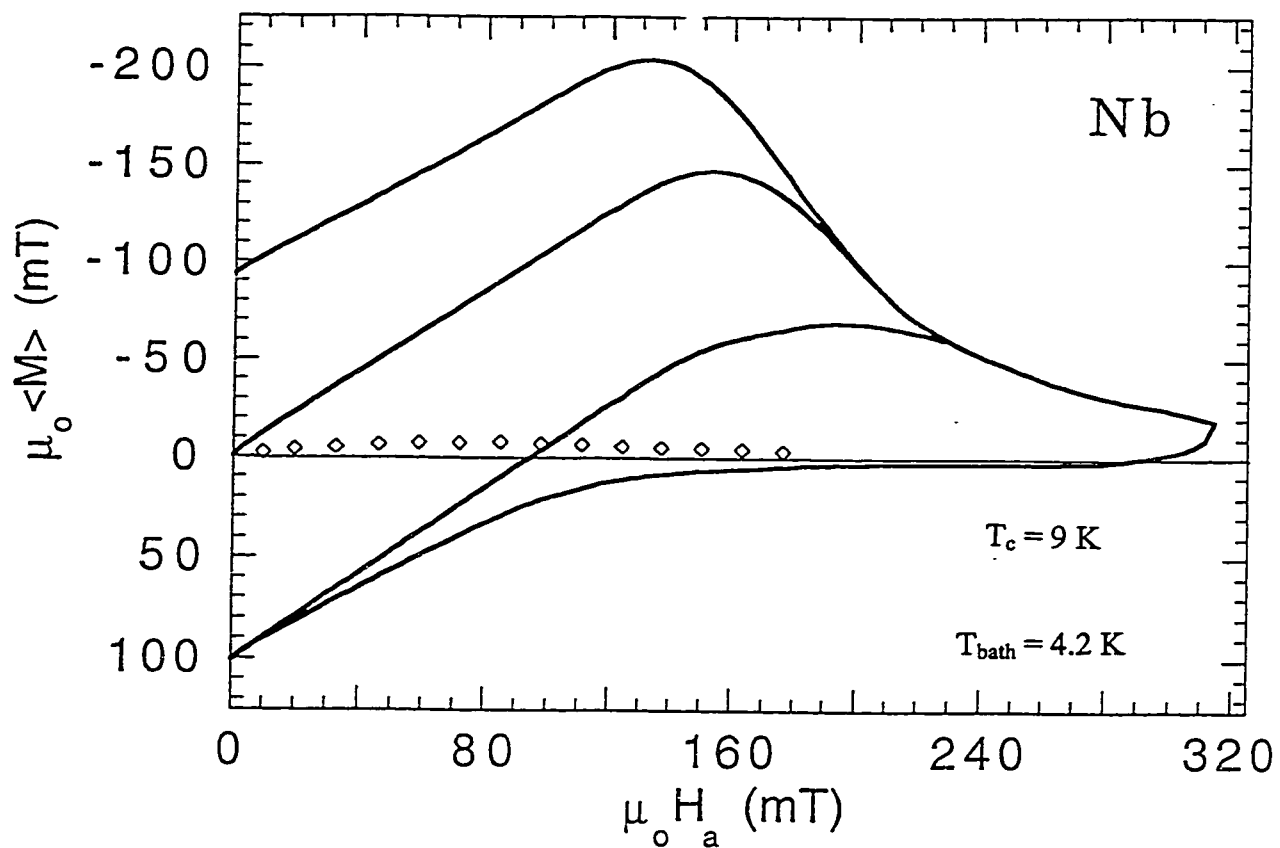


Fig 8.1 Experimentally observed isothermal magnetization curves for the semi-reversible Nb sample. For comparison the discrete measurements of the Meissner flux expulsion are also displayed. Note that at the higher fields the lower envelope lies only slightly below the $\langle M \rangle = 0$ axis.

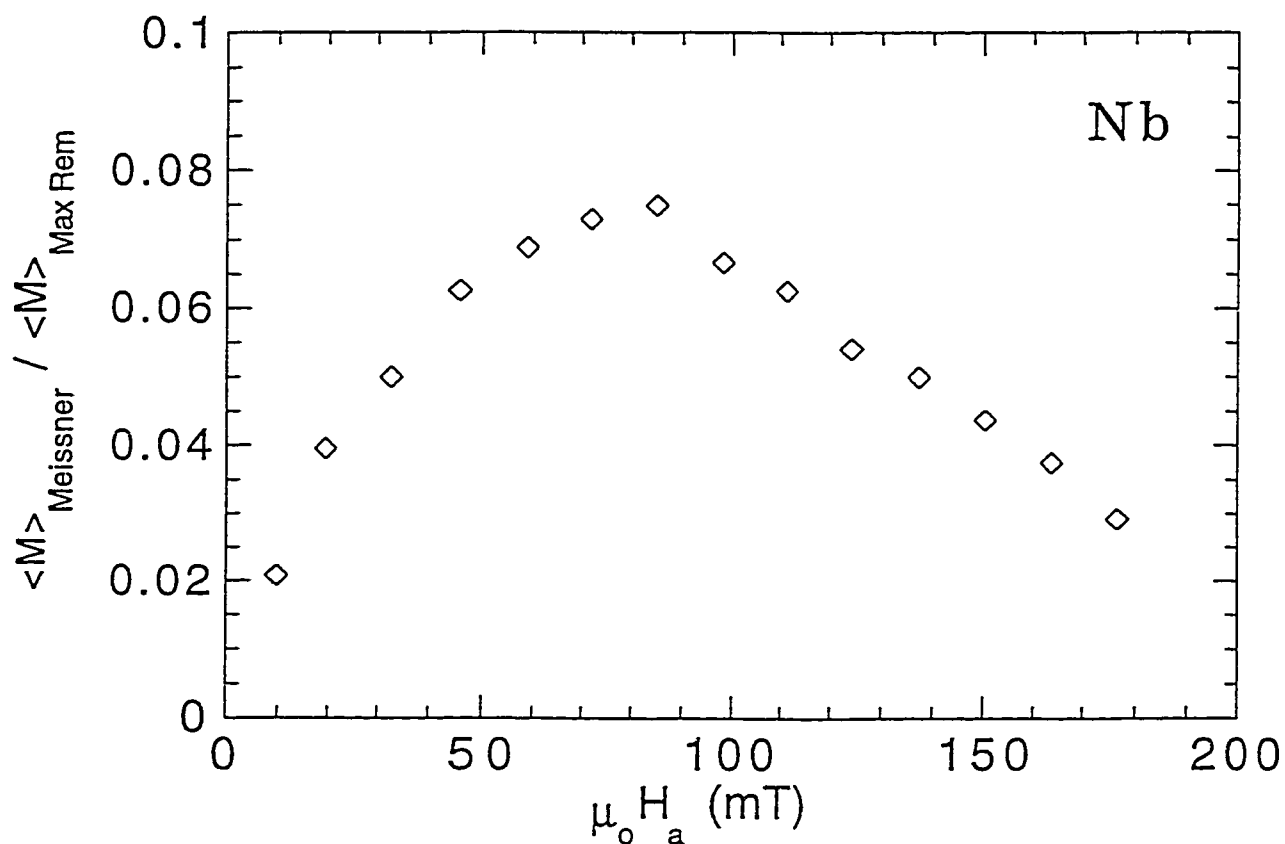


Fig 8.2 An expanded view of the spatial average of the Meissner flux expulsion normalized to $\langle M \rangle_{\text{max rem}}$. For each discrete data point the sample is cooled from above T_c to 4.2 K in a static field H_a . Recall that this quantity does not provide a direct measure of the actual Meissner current $I_M(H_a)$ unless the specimen is ideal hence pinning free.

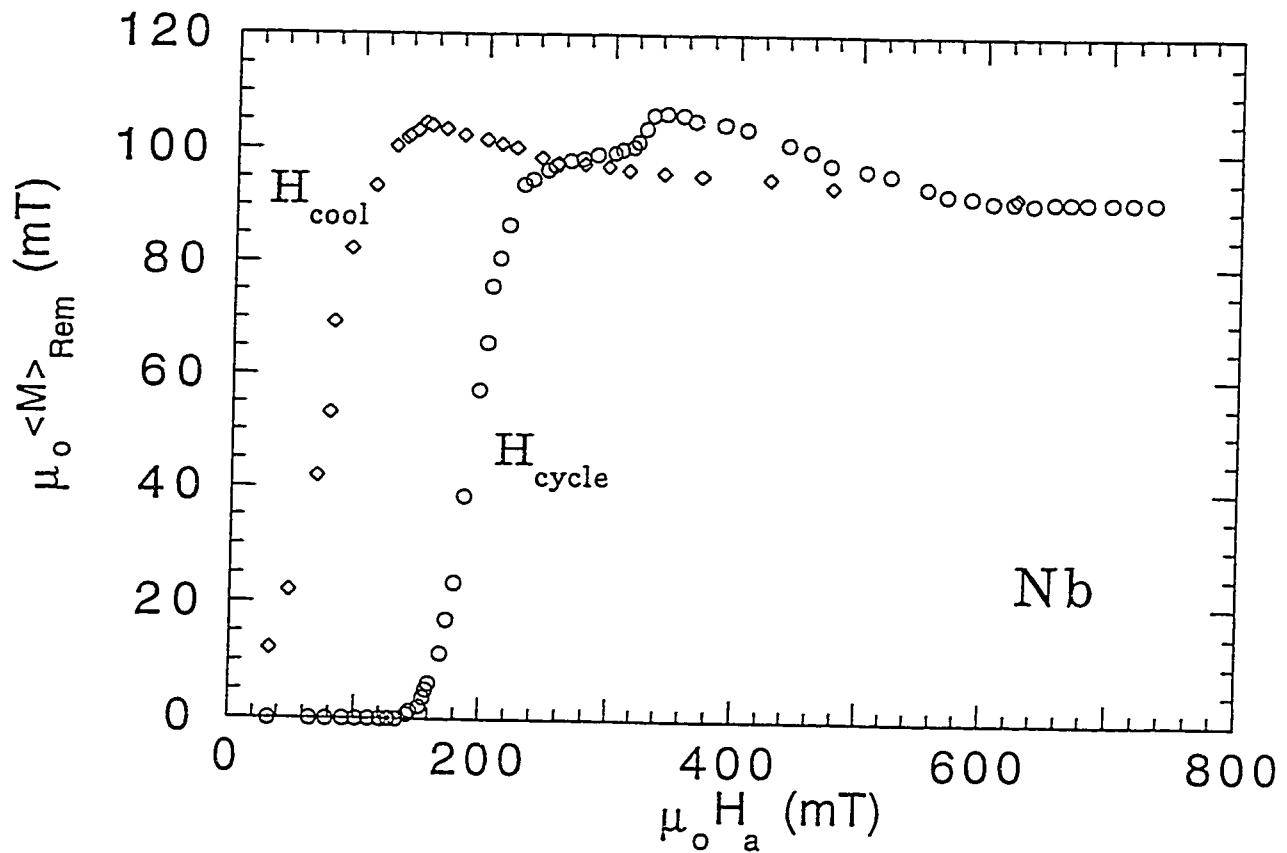


Fig 8.3 H_{cool} and H_{cycle} measurements for the Nb. H_{cool} denotes the field in which the specimen is cooled from above T_c to 4.2 K the field is removed and the trapped field measured by heating the sample to above T_c . H_{cycle} denote the maximum field impressed upon a virgin sample before returning to $H_a = 0$. The onset of the plateau for the H_{cool} trapping data, $\langle M \rangle_{\text{max rem}}$, determines H^* . In this case $\mu_0 H^* \approx 170$ mT. Also the onset of the plateau for H_{cycle} measurements indicates H^* , in this case $\mu_0 H^* \approx 240$ mT. These results roughly match those obtained from Fig 8.1. The initial region of zero trapped field for H_{cycle} can provide an estimate of when flux first begins to penetrate the sample hence a measurement of H_{Cl} . However as even a highly hysteretic specimen will have a quite flat initial segment, obtaining H_{Cl} in this manner must be done with some care.

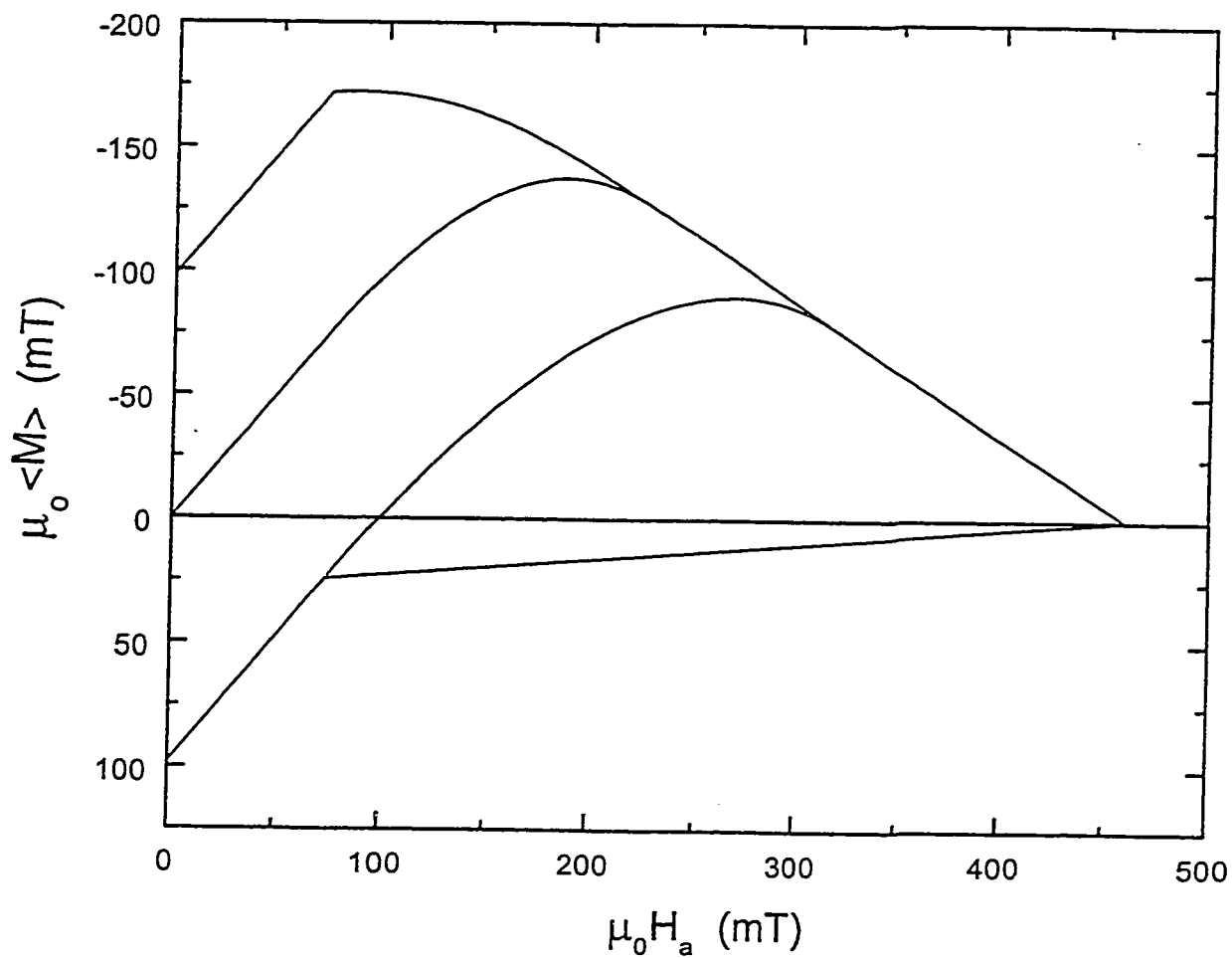


Fig 8.4 Theoretically calculated isothermal magnetization curves for the Nb specimen as described in the text.

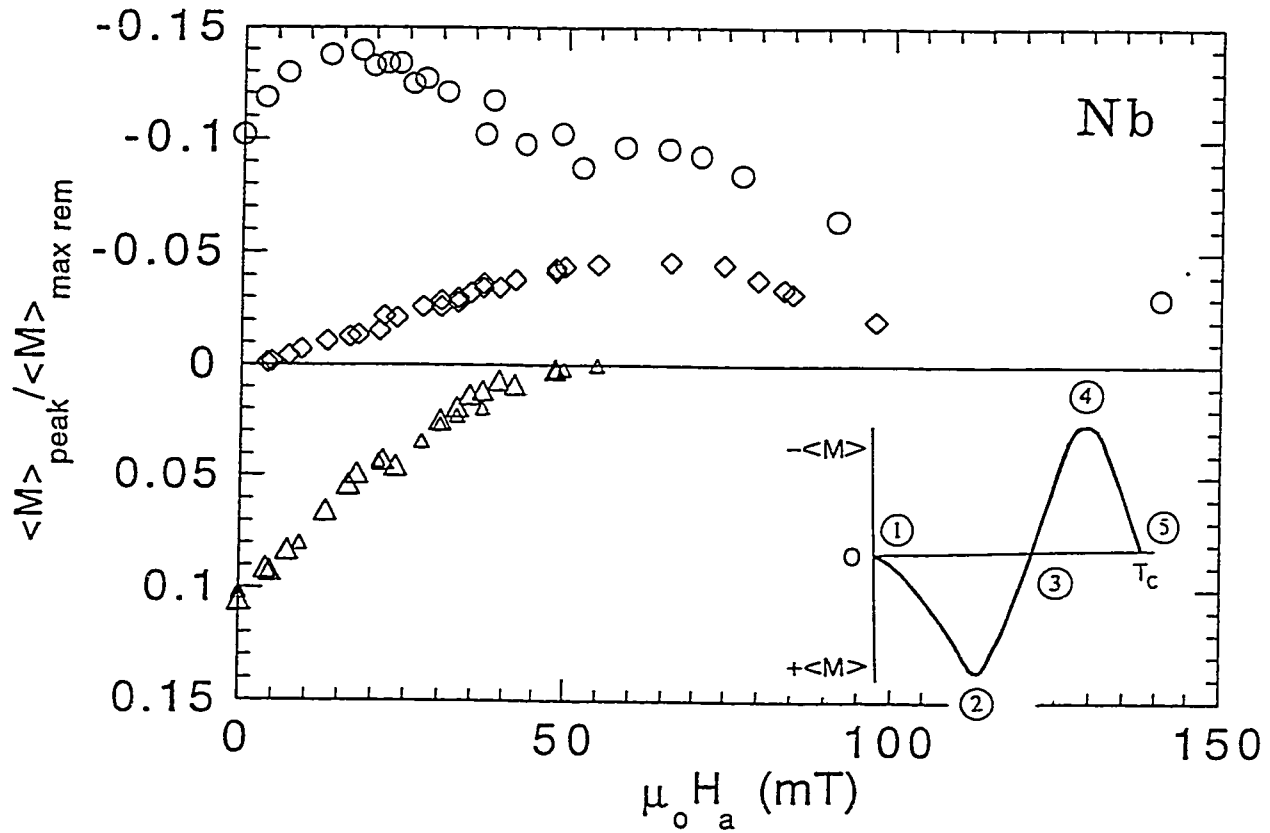


Fig 8.5 Experimental observations of the thermally disclosed hidden magnetic moments for the Nb specimen. The open circles are for “downswing” trials and show a similar broad peak as seen in the highly hysteretic specimens (see Fig 8.6 for comparison). The “upswing” trials bring forth the novel occurrence of an S shaped thermal release as depicted in the sketch. Warming of the specimen after an upswing first leads to a paramagnetic moment which attains a maximum denoted by 2 and plotted as the triangle, Δ , data points. As the temperature continues to rise, the sample sheds its paramagnetic moment, passes through $\langle M \rangle = 0$ at point 3 and then develops a diamagnetic moment reaching a maximum at point 4 and recorded by a diamond, $\diamond$, in the plot. Finally T will attain T_c and the specimen is in the normal state at point 5.

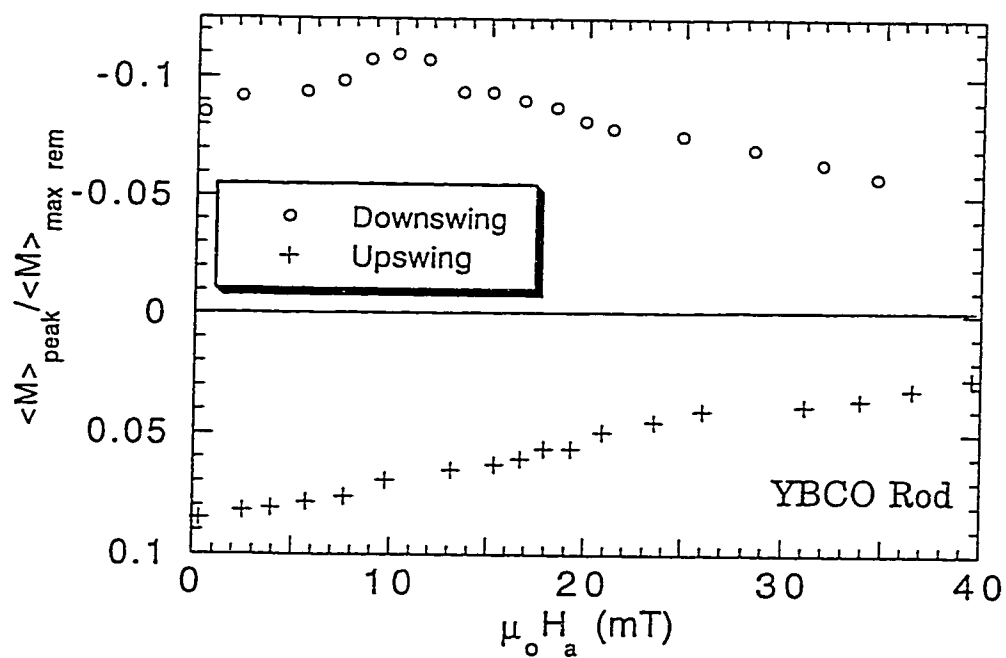
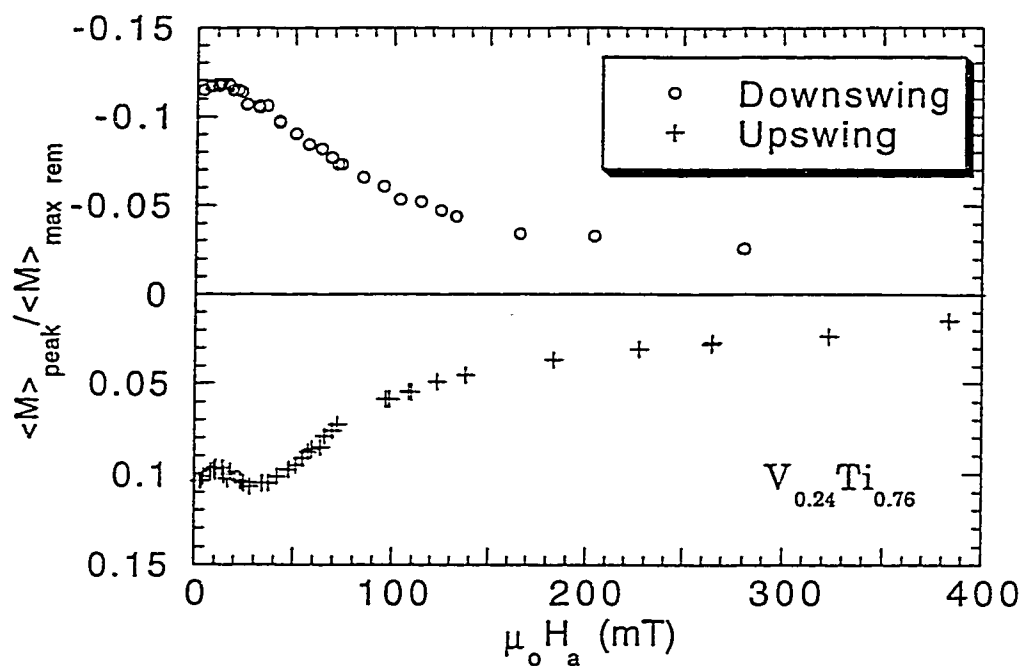


Fig 8.6 We reproduce the thermal release of magnetic moments for the VTi and YBCO rod specimens. Here we plot the diamagnetic and paramagnetic results in the appropriate quadrants for comparison with the Nb data.

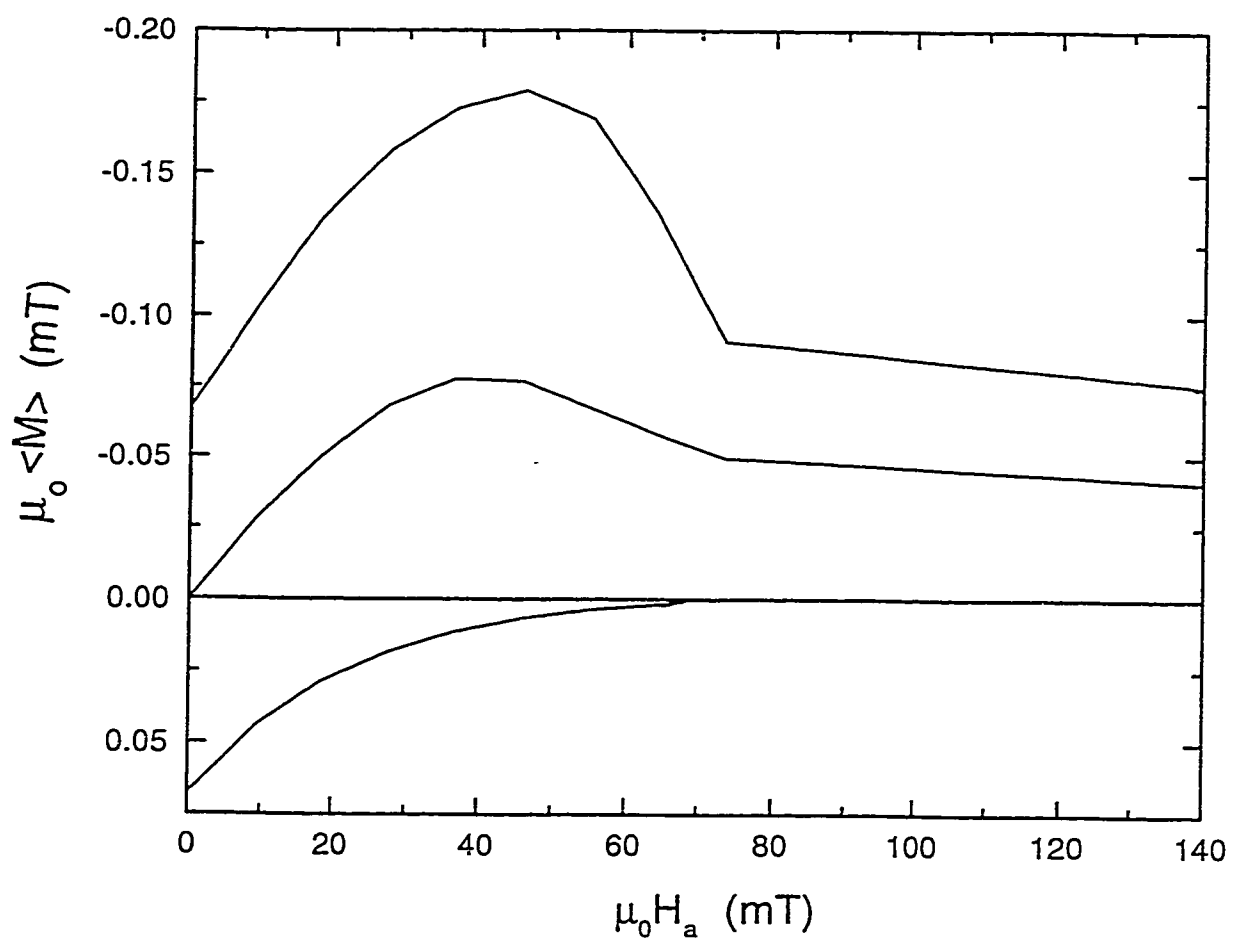


Fig 8.7 Theoretical calculations of the thermally released hidden magnetic moments for Nb as described in the text.

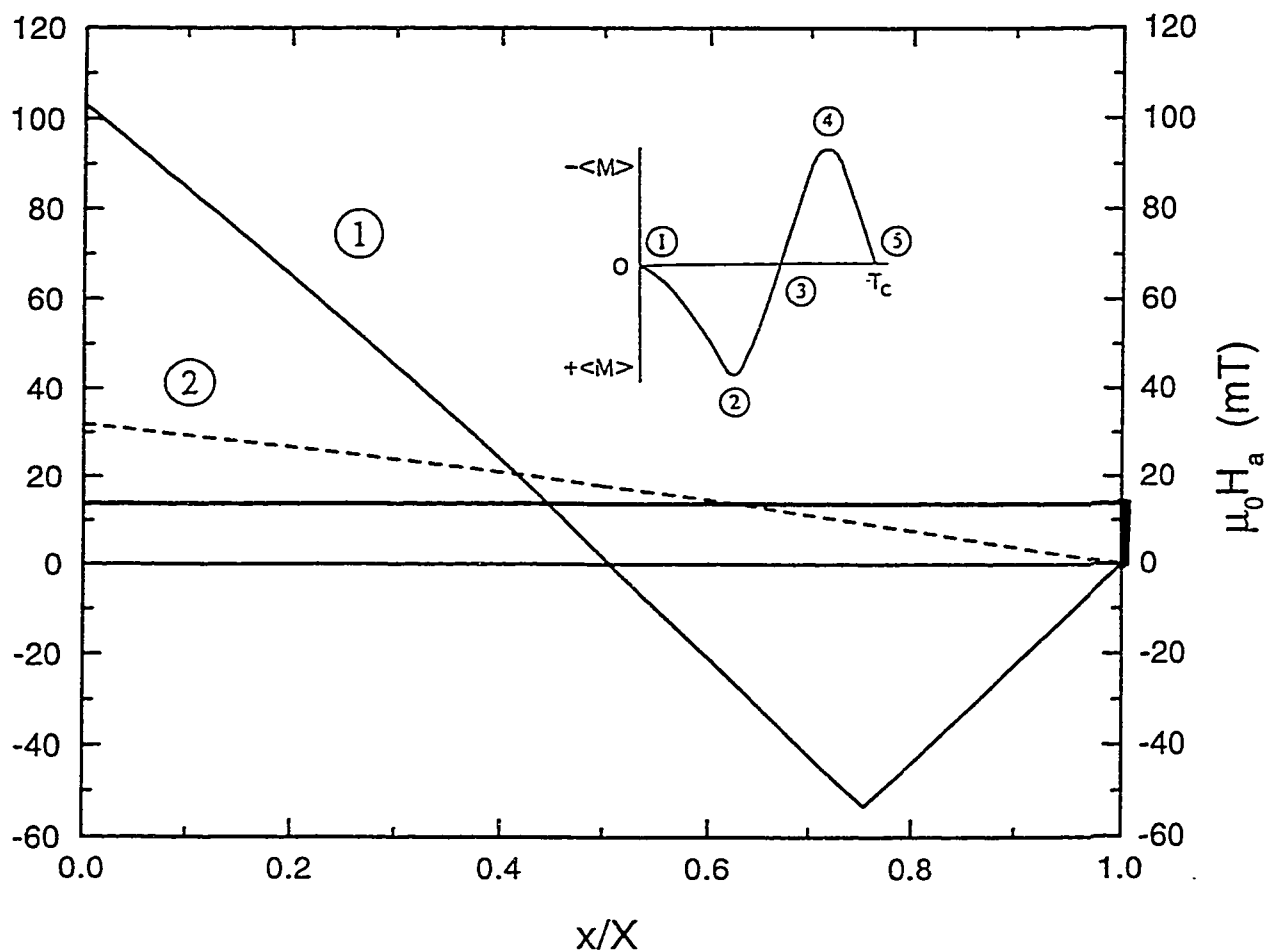


Fig 8.8 As an informative exercise we display the first flux density profile for the series of points 1, 2, 3, 4 and 5 associated with our successful theoretical reproduction of the S shaped thermal release for Nb. The profile represented by point 1 is when $\langle M \rangle = 0$ has been reached by an upswing of the external field. Point 2 is after the temperature has risen enough to allow the maximum paramagnetic moment to develop.

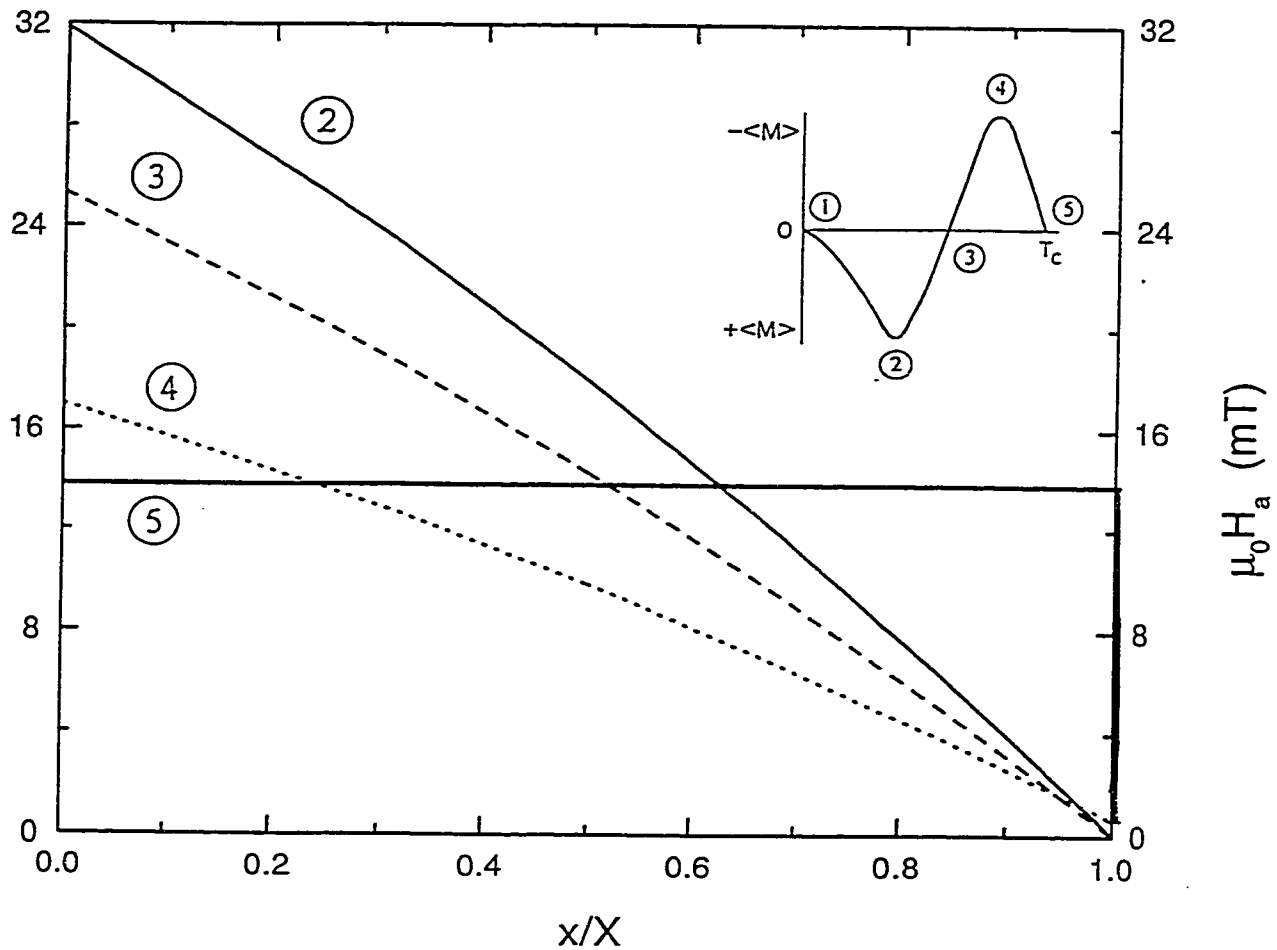


Fig 8.9 An extension of Fig 8.8 showing the flux density profiles for the extremal and $\langle M \rangle = 0$ points as H_a is held static while the specimen is heated and traces an S shaped curve of $\langle M \rangle$ versus T , again shown in the inset. As can be elicited from the profiles, at 2 (shown again) the sample attains is largest paramagnetic moment. At 3, the overall spatial average is $\langle M \rangle = 0$. At point 4 the sample attains the largest diamagnetic moment before reaching T_c and the baseline $\langle M \rangle = 0$ shown by 5.

CHAPTER 9

Observations on a High T_c Semi-Reversible Type II

Superconductor: BiSCCO Slab

9.1 Introduction

The disks of YBCO which are commonly sold to schools and Universities with the kits to demonstrate the Meissner effect in high T_c superconductors consist of weakly connected grains. A strong rare earth permanent magnet included in the kit and usually in the shape of a disk or cube is placed on top of and in contact with the YBCO disk. A substantial fraction of the return flux of the permanent magnet which threads the body of the YBCO disk in the normal state is expelled by the grains as the disk is cooled by liquid nitrogen from above $T_c = 92$ K to 77 K. The critical current of the Josephson junctions which provide the electrical link between the superconducting grains is effectively suppressed, initially by the field of the permanent magnet then by the superposition of this field and the return field of the grains as they become diamagnetically magnetized by the Meissner effect. Let us suppose that the face of the permanent magnet cube or disk which is adjacent to the YBCO disk is a North pole. Because of the Meissner effect the YBCO grains become an agglomeration of individual magnetic dipoles whose North

poles face the permanent magnet. Consequently the mutual repulsion between identical poles causes the permanent magnet to rise and float above the cold YBCO disk.

These commercial YBCO disks exhibit semi-reversible magnetic behaviour, partly because at low fields intergrain currents can circulate through the specimen and also large intragrain current densities can exist because of strong pinning inside the grains. These irreversible induced currents compete with the thermodynamically reversible Meissner currents circulating at the periphery of the individual grains^{60,65}.

However since we had already investigated highly hysteretic YBCO specimens in our survey of thermal disclosure of hidden magnetic moments we elected to study a different magnetically semi-reversible high T_c material. Consequently we obtained BiSCCO slabs from Dr. Vladimír Plecák at the Czech Institute of Physics in Prague which exhibit the desired semi-reversible properties.

9.2 Isothermal and related Magnetization Curves

The four basic isothermal magnetization curves measured at 77 K are displayed in Fig 9.1. From inspection of these magnetization curves and the values of the applied field where they join and also from examination of the data for $\langle M \rangle_{\text{rem}}$ versus H_{cycle} displayed in Fig 9.2 we can estimate $\mu_0 H_* \approx 20$ mT and $\mu_0 H_{**} \approx 30$ mT.

In Fig 9.1 for comparison purpose we also present the Meissner data, i.e. the spatial average of the magnetic flux expelled upon cooling from $T_c \approx 110$ K to 77 K in several static magnetic fields. We note that the maximum $\mu_0 \langle M \rangle_{\text{Meissner}}$ is almost

$(\frac{1}{2}) \langle M \rangle_{\max \text{ rem.}}$. Thus although here $\mu_0 H_{C1} \approx 6 \text{ mT}$ at $77 \text{ K}^{60,61}$ is much smaller than $\mu_0 H_{C1} \approx 100 \text{ mT}$ at 4.2 K for our Nb specimen, the small size of the BiSCCO grains, typically $\approx 20 \text{ }\mu\text{m}$ diameter, the large penetration depth $\lambda \approx 0.5 \text{ }\mu\text{m}$ at $77 \text{ K}^{60,61}$ leads upon cooling from T_c to the expulsion of an appreciable fraction of the flux initially permeating the grains in the normal state. We also note that in this specimen the lower envelope of the major hysteresis curve lies in the diamagnetic quadrant when H_a descending in magnitude is larger than H_* . This behaviour occurs because in magnetic fields $H_a \geq H_*$, the diamagnetic moment generated by I_M circulating along the surface of the individual grains is larger in magnitude than the paramagnetic moment produced by the flux retaining induced currents circulating inside the volume of each grain. It is useful at this juncture to recall that the Meissner effect in the ideal type I and type II superconductors manifests itself in two ways. (a) Expulsion of flux upon cooling into the superconducting state in static magnetic fields $H_a \leq H_{C2}(T)$, and, (b) expulsion of magnetic flux when H_a is decreased below $H_{C2}(T)$ at a fixed temperature $T < T_c(H_a)$ where $T_c(H_a)$ is the critical temperature for the normal-superconducting transition in the magnetic field H_a .

The steep descent from diamagnetism to paramagnetism seen in Fig 9.1 as H_a is lowered below H_* can be attributed to three factors.

- (i) $I_M(H_a, T)$ hence the diamagnetic moment of the grains, is expected to decrease linearly with H_a in the range $0 < H_a < H_{C1}$ thereby allowing the paramagnetic moment generated by the flux retaining induced currents to become manifest,
- (ii) the intragrain $j_c(B)$ increases steeply with decreasing B in the low field range and,

- (iii) the intergrain critical current (the current carrying capacity of the Josephson junctions) is partly restored in the low field range.

We stress however that the intergrain current density is not completely restored even when H_a is returned to zero after excursions of the magnetization to the envelopes of the major hysteresis curve. The reason for this is that now the links between the grains (the Josephson junctions) bathe in the return fields of the fully magnetized grains²⁹.

In view of the preceding, in our analysis of the thermal disclosure of hidden magnetic moments in this specimen we will view the sample as a collection of electrically decoupled grains which for simplicity we take to be all of the same size. Further to make the problem tractable and since this better corresponds to the external shape of the slab sample we will apply idealized planar geometry.

Fig 9.3 shows the theoretical magnetization curves we calculated in this idealized framework. In these calculations, for simplicity and convenience, we,

- (i) let $I_M(H_a) = H_{C1}$ hence be independent of H_a when $H_a \geq H_{C1}$ and ,
- (ii) introduce again the simple analytic function exploited earlier in our analysis of the observations on the VTi and YBCO cylinders, namely,

$$j_c(B, T) = \frac{j_o(T)}{(B / B_{ref})^{1/2}} \quad (9.1)$$

We find good agreement with observation staking $H_{C1} = H_*/3$. In these calculations the adjustable parameters are H_* and H_{C1} . The former is chosen to yield $\langle M \rangle_{\max rem}$ in good agreement with the measured value and the ratio $H_{C1}/H_* \approx 1/3$, is selected to generate semi-reversible behaviour corresponding semi-quantitatively with the

observations, i.e. a lower envelope for the major hysteresis curve which lies in the diamagnetic quadrant when H_a descending is $\geq H_*$.

9.3 Release of Hidden Magnetic Moments

Fig 9.4 displays our observations of peak or extremal values in the excursions of the magnetization during warming from 77 to $T_c \approx 110$ K starting with $\langle M \rangle = 0$ in various H_a descending and ascending to the final values. We note the similarities in the behaviour at low fields of this granular highly anisotropic high T_c semi-reversible sample and that of the homogeneous and isotropic low T_c Nb in the same field range (see Fig 8.5). In the downswing data for both materials the diamagnetic moment (filled circles) generated upon warming grows in magnitude as a function of H_a in the low field range.

However, in final fields $H_a \geq H_*$ we encounter a new situation in the BiSCCO specimen which did not occur in the Nb sample. The lower envelope of the major hysteresis curve for the Nb (see Fig 8.1) although lying close to the $\langle M \rangle = 0$ axis remained in the paramagnetic quadrant over the entire range $0 \leq H_a \leq H_{C2}$. In contrast, the corresponding magnetization curve for the BiSCCO sample lies in the diamagnetic quadrant when $H_a \geq H_*$. Consequently it is not possible to generate $\langle M \rangle = 0$ states at 77 K in this specimen when $H_a \geq H_*$. Therefore the range of initial $\langle M \rangle = 0$ configurations available for thermal disclosure investigations is restricted in this specimen to the range $0 \leq H_a \leq H_*$ where H_* denotes the applied field where the lower hysteresis envelope crosses the $\langle M \rangle = 0$ axis and indicated by the • in Figs 9.1 and 9.3.

In view of these circumstances we chose to extend the thermal disclosure study to the adjacent magnetic moment configurations encountered along the lower envelope, hence where $H_a > H_*$ and is descending in magnitude. The type of B profiles existing here is sketched in Fig 9.5 (e). Note that the diamagnetic moment due to the Meissner current dominates the paramagnetic moment generated by the flux retaining currents induced in the body of the grain hence the sample exhibits a net diamagnetic moment. In these situations we find that, upon warming, the magnitude of the existing diamagnetic magnetization initially increases, rises to a maximum and then descends to zero at T_c as sketched in Fig 9.5 (b). For comparison in Fig 9.5 (a) we sketch the locus of $\langle M \rangle$ upon warming for the range $0 \leq H_a \leq H_*$. In Fig 9.4 the solid circles $\bullet$ (empty diamonds $\diamond$) data points indicate the peak value attained by the diamagnetic magnetization during warming after a downswing of H_a for H_a final less (greater) than H_* . The empty circle (o) data points display the increment, $\Delta \langle M \rangle = \langle M \rangle_{\text{peak}}^\downarrow - \langle M \rangle_{\text{initial}}$ (see Fig 9.5 (b)). We can readily understand the behaviour displayed in Fig 9.5 (b) from consideration of Fig 9.5 (e) if we assume that I_M is initially insensitive to the rise of temperature whereas j_c is concurrently decreasing more rapidly. Consequently magnetic flux trapped by j_c is released from the specimen as j_c diminishes thereby augmenting the net diamagnetic moment due to the Meissner current which remains essentially unchanged. Eventually however I_M must also diminish and therefore now at a higher rate than j_c as T_c is approached and hence the large diamagnetic moment is made to vanish.

The behaviour observed for H_a ascending for the BiSCCO sample is seen to follow the same sequence as a function H_a that we encountered in the Nb specimen. This is illustrated in the sketches of Fig 9.5 in the order (c), (d) then (a). The empty triangles Δ

(full diamonds ♦) in Fig 9.4 display the maximum excursion of the S sketched in Fig 9.5 (d), i.e. points 2 and 4. Note that the downward excursion vanishes when $\mu_0 H_a > 8 \text{ mT}$. Now however, the data for H_a ascending can be regarded as terminating at $H_a = H_*$ since the initial $\langle M \rangle = 0$ state cannot be established when $H_a > H_*$. Alternatively, when $H_a > H_*$ we can regard the measurements of thermal disclosure of hidden magnetic moments to belong to an “extension” of the upswing data points. Thus in this perspective the sequence of (c), (d) and (a) of Fig 9.5 is followed by the behaviour sketched in (b).

Fig 9.6 displays the theoretical curves for the thermal release, $\langle M \rangle_{\text{peak}} / \langle M \rangle_{\text{max rem}}$ versus H_a ascending and descending. These curves were calculated using the same simple expressions and the parameters applied to the calculation of the hysteresis curves of Fig 9.3, namely,

- (i) a bulk current density $j_c(B, T) = \frac{j_o(T)}{(B/B_{cT})^{1/2}}$ (eqn 9.1) and ,
- (ii) a Meissner current $I_M = H_a$ when $H_a \leq H_{C1}$ and

$$I_M(H_a) = H_{C1} = \frac{H_*}{3} \quad (9.2)$$

hence field independent when $H_a > H_{C1}$.

Fig 9.6 should be compared with Fig 9.4.

The important ingredients required in reproducing the major features of the thermal disclosure data is that j_c initially decrease more rapidly with increasing temperature than I_M . In this regard we explored several scenarios for the relative temperature variation of these two quantities, namely,

$$j_c = j_o \{1 - (T/T_c)\}, \quad I_M = I_{Mo} \{1 - (T/T_c)^2\} \quad (9.3 \text{ a})$$

$$j_c = j_o \{1 - (T/T_c)\}, \quad I_M = I_{Mo} \{1 - (T/T_c)^4\} \quad (9.3 \text{ b})$$

$$j_c = j_o \{1 - (T/T_c)^{1/2}\}, \quad I_M = I_{Mo} \{1 - (T/T_c)^2\} \quad (9.3 \text{ c})$$

$$j_c = j_o \{1 - (T/T_c)^{1/2}\}, \quad I_M = I_{Mo} \{1 - (T/T_c)^4\} \quad (9.3 \text{ d})$$

We obtained the best agreement with observations exploiting eqn 9.3 (d) and these are the results shown in Fig 9.6. We note however that in all four cases, the theoretical curves generate all the salient features of the data curves.

For completeness in Fig 9.7 and 9.8 we again display a sequence of calculated flux density configurations which trace out the important points 1, 2, 3, 4 and 5 of an S shaped curve encountered upon warming to T_c typical for H_a upswing. Here $\mu_o H_a = 4\text{mT}$.

9.4 Summary and Conclusion

Considering that the theoretical calculations we have presented address a specimen which is isotropic and has idealized geometry whereas the actual sample is a slab containing a tightly packed collection of highly anisotropic grains consisting of irregularly shaped platelets with nearly random orientation of their c-axis and spanning a range of sizes around $\approx 20 \mu\text{m}$, it is remarkable, in our view, that we obtain such a degree of correspondence with observations both for the standard hysteresis curves and for the thermal release of the hidden magnetic moments. This encouraging agreement indicates

that our simple model for the phenomena encountered during thermal disclosures is fundamentally quite robust. In the next chapter to account for the behaviour of semi-reversible type II superconductors where the Meissner current $I_M(H_a, T)$ plays an important role we present our extension of the basic model described in chapter 7.

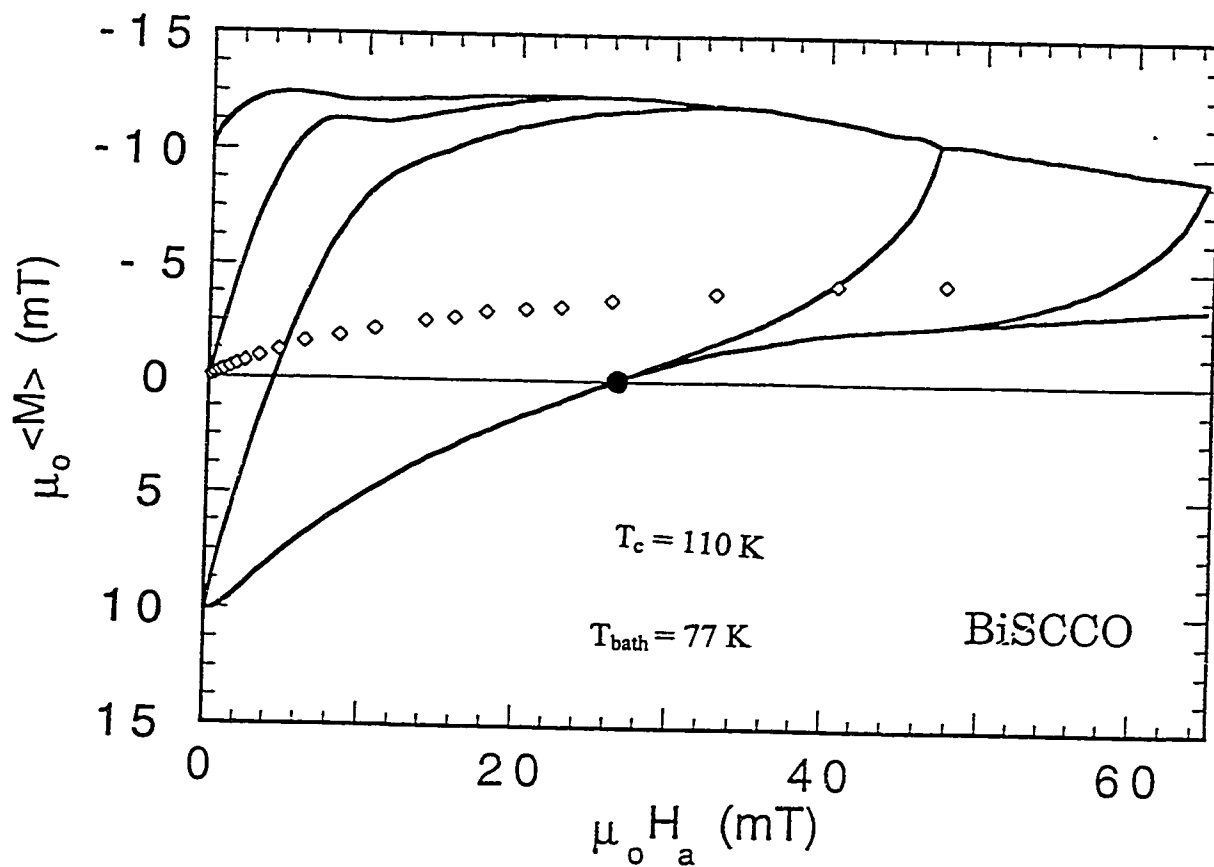


Fig 9.1 Experimentally observed isothermal magnetization curves for the semi-reversible BiSCCO sample. For comparison the discrete measurements of the Meissner flux expulsion are also displayed. Note that at fields greater than ≈ 20 mT the lower envelope lies above the $\langle M \rangle = 0$ axis.

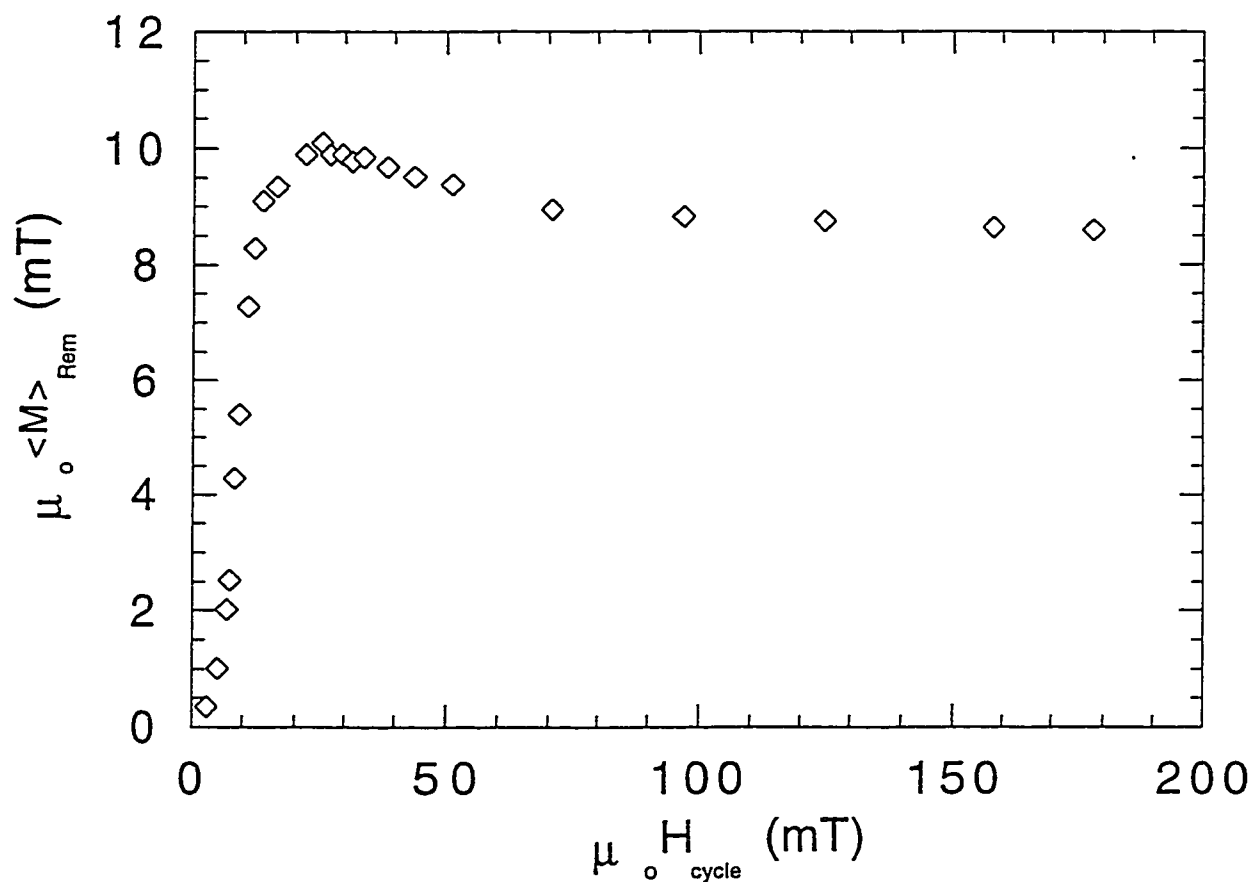


Fig 9.2 H_{cycle} measurements for the BiSCCO specimen. H_{cycle} denotes the maximum field impressed upon a virgin sample before returning to $H_a = 0$. The onset of the plateau for H_{cycle} measurements indicates H_{**} , in this case $\mu_o H_{**} \approx 25$ to 30 mT. This result roughly matches the H_{**} apparent in Fig 9.1.

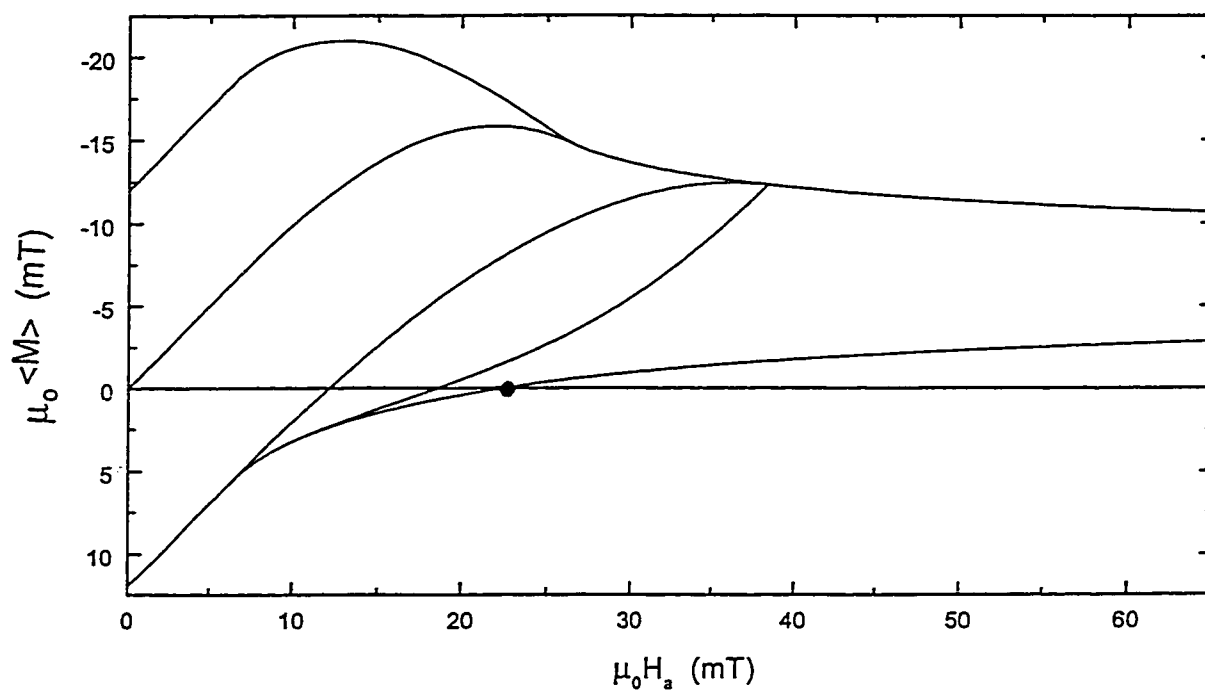


Fig 9.3 Theoretically calculated isothermal magnetization curves for the BiSCCO specimen as described in the text.

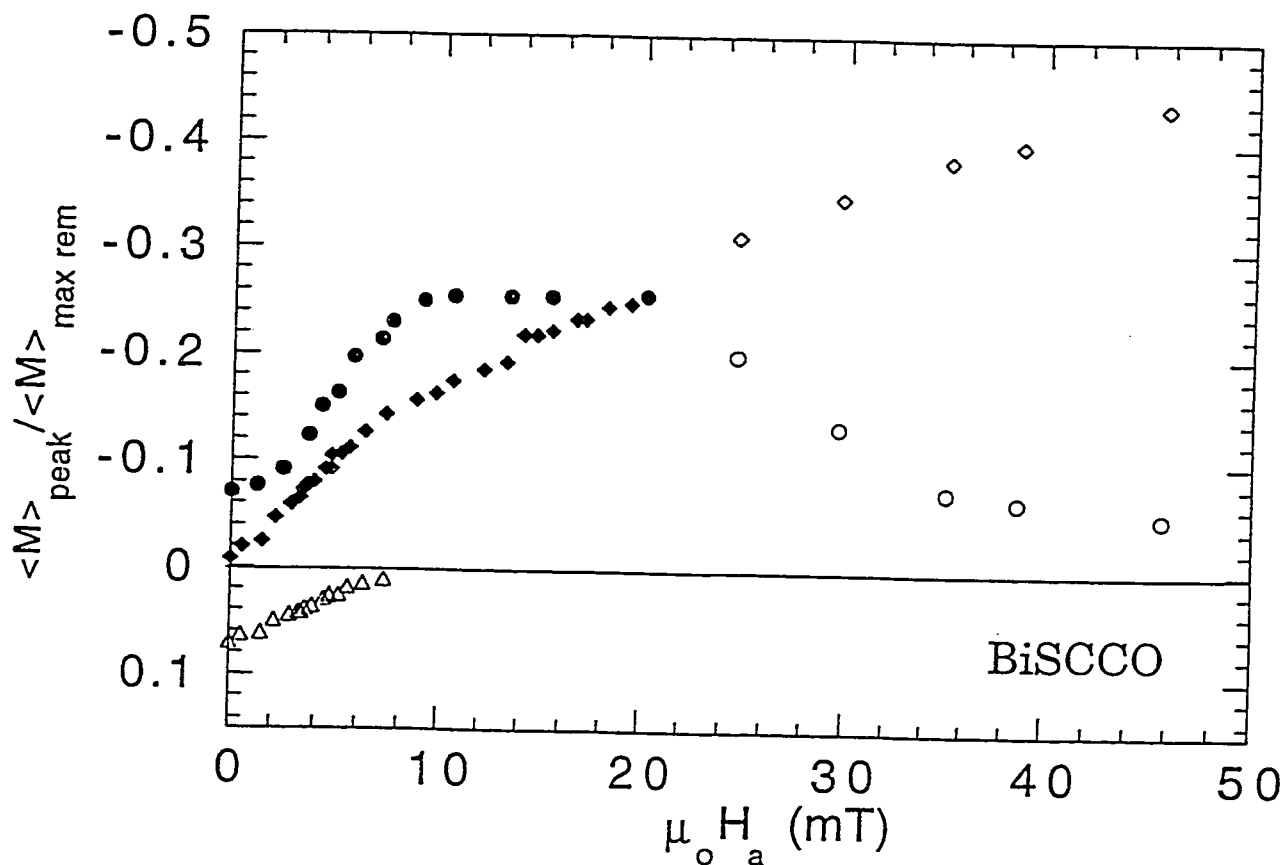


Fig 9.4 Experimental observations of the thermally disclosed hidden magnetic moments for the BiSCCO specimen. As outlined in the text and apparent by Fig 9.1, $\langle M \rangle = 0$ is only attainable below ≈ 20 mT. Up to this value the results are quite similar to those found for Nb. Downswing data (solid circles) giving a broad peak and upswing data giving the dual release S shape disclosure in low fields (solid diamonds and empty triangles). Refer to the text for details pertaining to all the various regions.

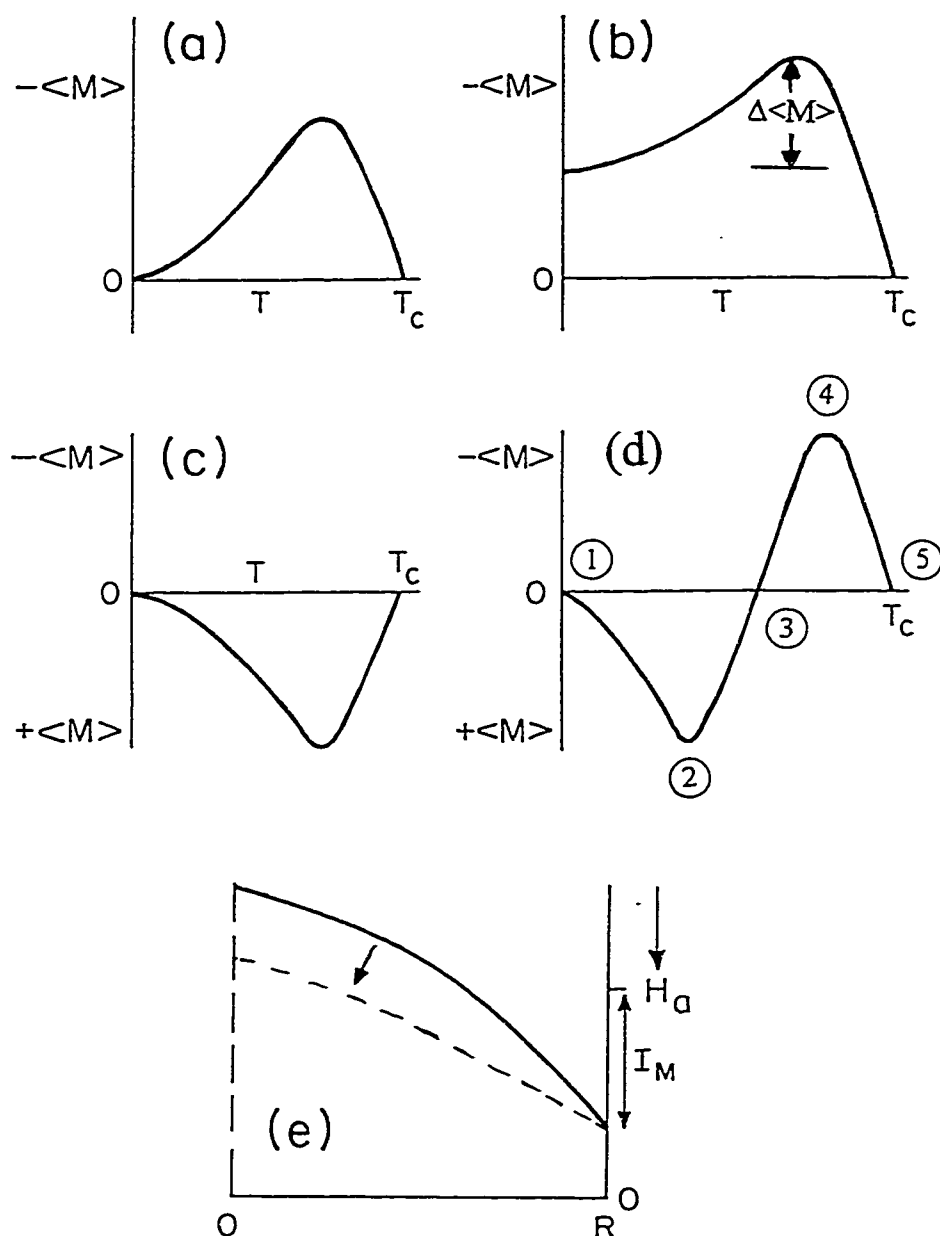


Fig 9.5 (a) to (d) The family of $\langle M \rangle$ versus T curves that show the variety of experimentally observed thermal disclosures for the BiSCCO specimen. These are related to the various regions of the experimental data shown in Fig 9.4. Above 20 mT the $\langle M \rangle = 0$ state cannot be reached but an explanation for the observed increase of the already diamagnetic moment for downswings (see (b)) can be obtained from the flux density profile shown in (e). Here if the bulk pinning decreases more rapidly with temperature than I_M , then flux will exit the specimen and increase its diamagnetic moment.

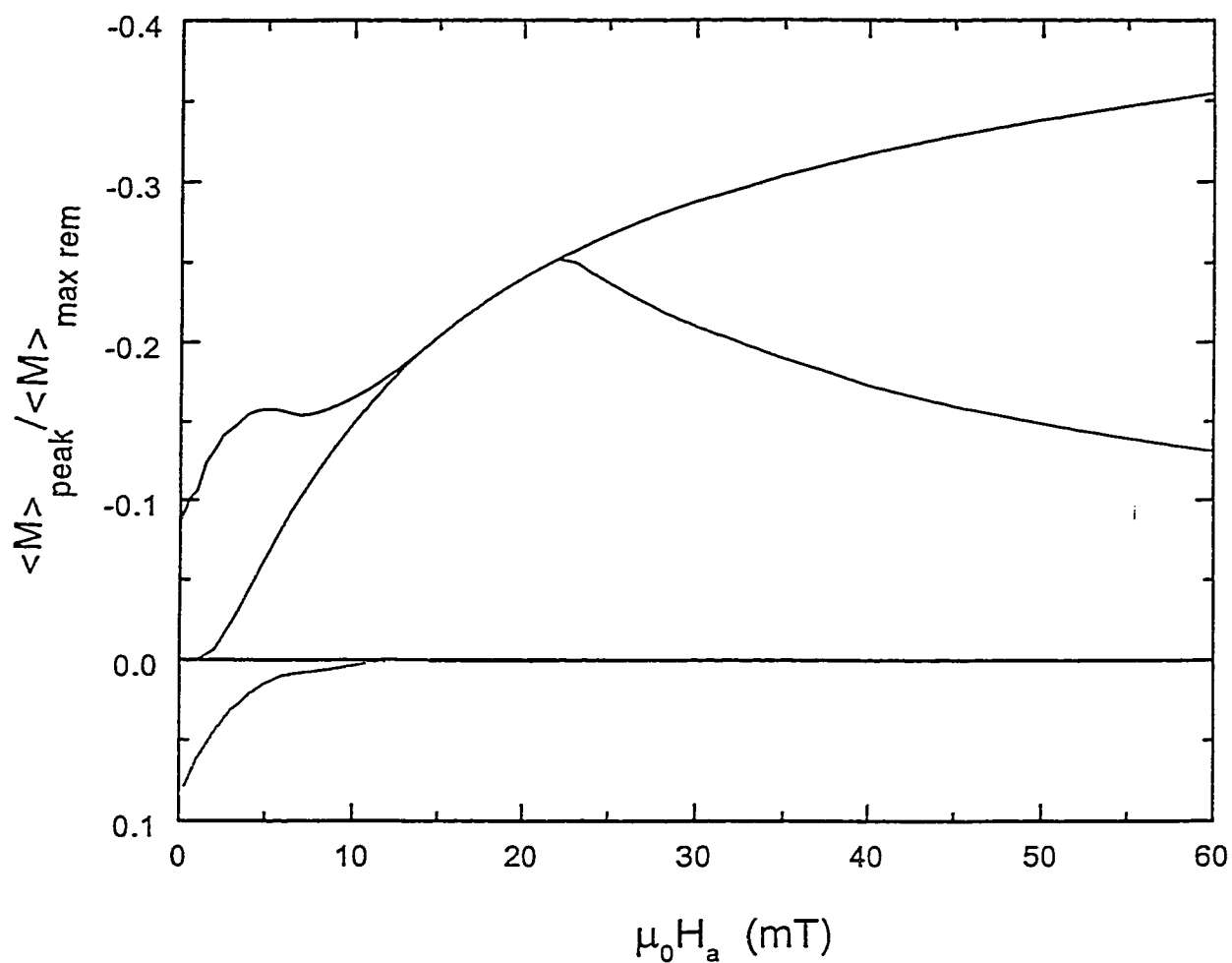


Fig 9.6 Theoretical calculations of the thermally released hidden magnetic moments for BiSCCO as described in the text. Note the region by region agreement with the experimental data of Fig 9.4.

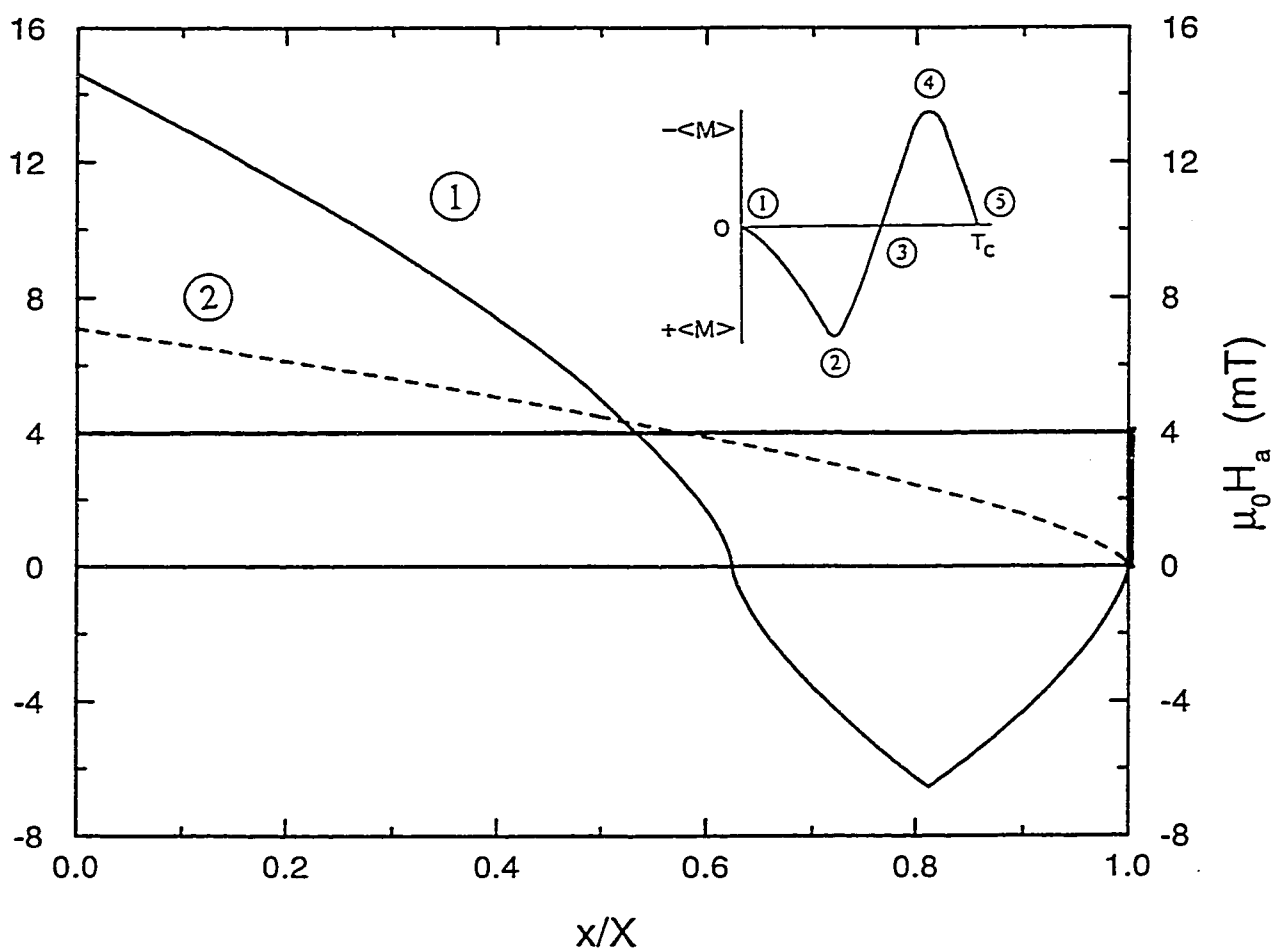


Fig 9.7 Again as an informative exercise we display the first flux density profile for the series of points 1, 2, 3, 4 and 5 associated with our successful theoretical reproduction of the S shaped thermal release for BiSSCO. The profile represented by point 1 is when $\langle M \rangle = 0$ has been reached by an upswing of the external field. Point 2 is after the temperature has risen enough to allow the maximum paramagnetic moment to develop.

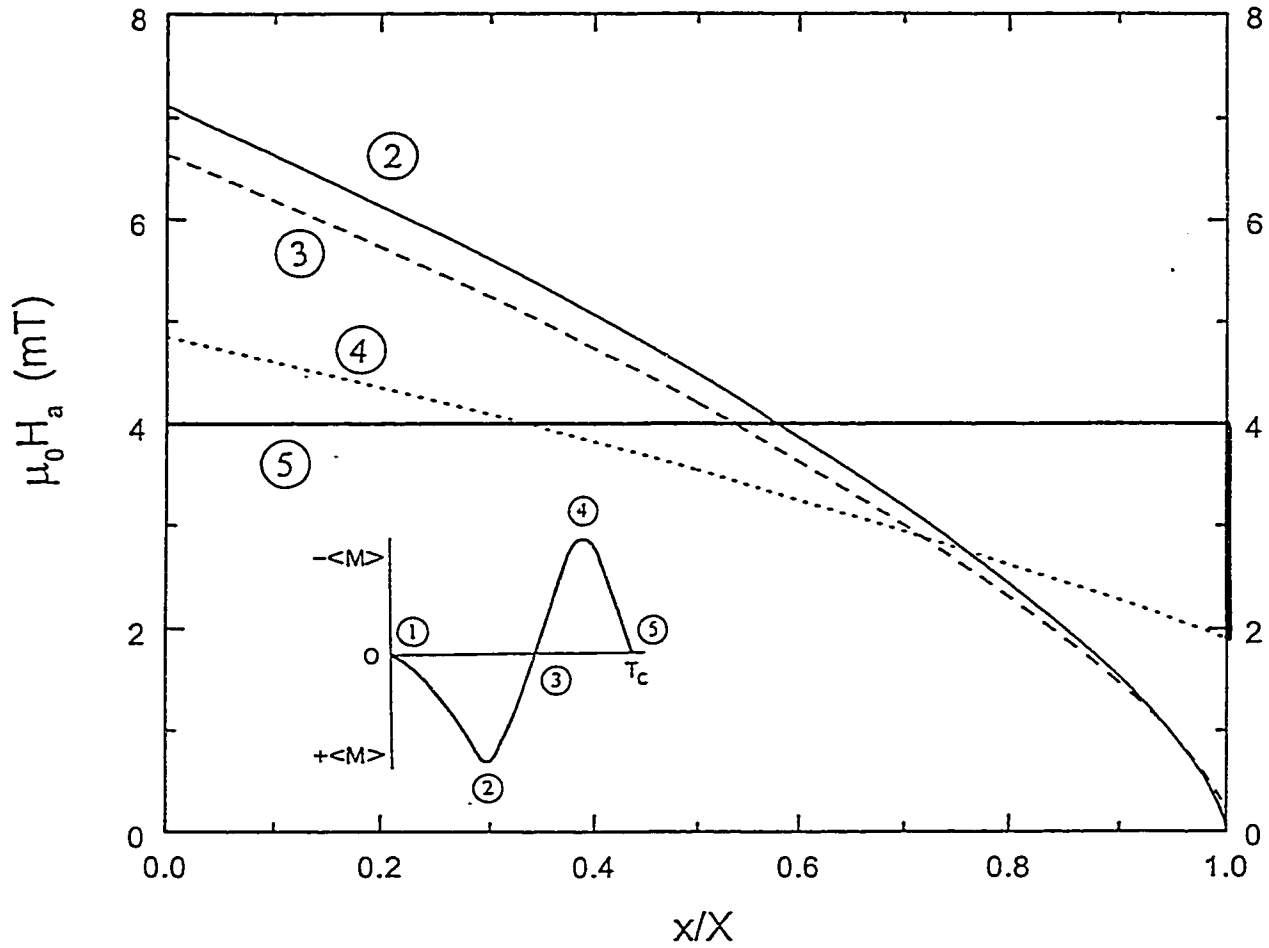


Fig 9.8 An extension of Fig 9.7 showing the flux density profiles for the extremal and $\langle M \rangle = 0$ points as H_a is held static while the specimen is heated and traces an S shaped curve of $\langle M \rangle$ versus T , again shown in the inset. As can be elicited from the profiles, at 2 (shown again) the sample attains its largest paramagnetic moment. At 3, the overall spatial average is $\langle M \rangle = 0$. At point 4 the sample attains the largest diamagnetic moment before reaching T_c and the baseline $\langle M \rangle = 0$ shown by 5.

CHAPTER 10

Thermal Disclosure of Hidden Magnetic Moments: Model for Semi-Reversible Type II Superconductors

10.1 Introduction

In chapter 7 we described in some detail the standard critical state framework applied by researchers to calculate magnetization curves for highly hysteretic type II superconductors. In this approach H_{C1} is ignored hence the magnetic moments are due entirely to induced currents $j_c(B,T)$ circulating in the bulk of the material.. Then focusing on the $\langle M \rangle = 0$ states where the specimen is filled with concentric regions of counter-rotating currents we examined the evolution as $j_c(B,T)$ was made to diminish as the temperature was increased in stationary applied fields. As this occurred the concept of conservation of flux line in the body of the type II superconductor was applied in order to follow the redistribution of flux inside the sample. In this simple but basic situation the peak values which appeared in the thermal evolution of the net magnetic moments did not depend on the variation of j_c with temperature. Now however in the description of the thermal disclosure in semi-reversible type II superconductors where the Meissner current $I_M(H_a, T)$ plays an important role we need to take the temperature dependence of both $j_c(B, T)$ and $I_M(H_a, T)$ into account.

10.2 Framework of the Analysis

10.2.1 H_{C1} and $H_* \ll H_{C2}$

It is a straightforward exercise to introduce the Meissner current $I_M(H_a, T)$ into the framework of the critical state for highly hysteretic type II superconductors which we have presented in chapter 7. From examination of Fig 10.1 and its earlier version where H_{C1} was ignored (see Fig 7.1) we see that the role of the Meissner current and its dependence on H_a can be taken into account simply by writing,

$$H_s = H_a - I_M(H_a) \quad (10.1)$$

instead of H_a in the formulae developed in chapter 7. Note that $\mu_0 H_s$ now indicates the magnetic flux density just inside the specimen at a distance of the order of the penetration depth λ from the surface. Note also that $H_s = 0$ when $H_a \leq H_{C1}$ since then $I_M(H_a) = H_a$.

For instance eqn 7.4 now reads,

$$B_a(r) = \left\{ (\mu_0 H_a - \mu_0 I_M(H_a))^{P+1} + (P+1) \mu_0 j_o B_{ref}^P R \left(1 - \frac{r}{R}\right) \right\}^{1/(P+1)} \quad (10.2)$$

and in normalized form,

$$h_a(r) = \left\{ (h_a - i_{Mn}(h_a))^{P+1} + \frac{j_n}{j_o} (1-r) \right\}^{1/(P+1)} \quad (10.3)$$

where,

$$i_{Mn} \equiv I_{Mn}(H_a) / H_* \equiv \frac{I_{M0}(H_a) f_M(n)}{H_*} \quad (10.4)$$

and,

$$j_n = j_o f_j(n) \quad (10.5)$$

where $f_M(n)$ and $f_j(n)$ are arbitrary functions varying from 1.0 when the pure number $n = 0$ to 0.0 when $n = 1.0$. These functions respectively describe the dependence on temperature of the Meissner current and of the bulk critical current density parameter j_o .

We continue to write,

$$\mu_o H_* = B_* = \{(P+1) \mu_o j_o B_{ref}^P R\}^{1/(P+1)} \quad (10.6)$$

We can ignore the effect of H_{C2} in the analysis of the thermal disclosure phenomena in semi-reversible high T_c superconductors such as the BiSCCO slab where $\mu_o H_{C2} > 10$ Tesla at 77 K since the peak moments released upon warming to T_c are seen to diminish appreciably when H_a exceeds H_{C1} and H_* and both of these quantities are minute compared to H_{C2} . Further in the calculations for the BiSCCO sample, for simplicity and in harmony with observations we let $I_M(H_a) = H_{C1}$ when $H_a > H_{C1}$, hence independent of H_a .

10.2.2 H_{C1} and H_* not negligible relative to H_{C2}

It is clear from inspection of the basic magnetization curves for the Nb sample (see Fig 8.1) that here H_{C1} and H_* are appreciable fractions of H_{C2} . Consequently the upper critical field H_{C2} should be taken into account in the modelling of the thermal disclosure data even though the peak moments released by warming have already diminished significantly when $H_a > (H_* + H_{C1})$ in this specimen.

At this juncture we can return to the Kim like expression for $j_c(B, T)$ (eqn 7.1) and introduce factor functions $F(B/B_{C2})$ which are physically plausible, hence write,

$$j_c(B, T) = \frac{j_o(T) B_{oT}^p}{(B + B_o(T))^p} F(B / B_{C2}(T)) \quad (10.7)$$

However to pursue this approach we also demand for computational convenience and economy that tractable closed form expressions exist for the integral,

$$\int \frac{(B + B_o)^p}{F(B / B_{C2})} dB \quad (10.8)$$

which yields formulae for the $B(r)$ profiles, and, for the subsequent integrals,

$$\int B(r) r dr \quad \text{or} \quad \int B(x) dx \quad (10.9)$$

which are needed in calculating the redistribution of flux line and the magnetic moments.

After a modest but unsuccessful effort to find functions of the modified Kim type (i.e. eqn 10.7) which met these constraints we turned our attention to other types of simpler expressions which satisfy these criteria, namely expressions of the form,

$$j_c(H, T) = j_o \{1 - H / H_{C2}(T)\}^P \quad (10.10)$$

In this category we focused on the linear case ($P = 1$) and a simple convex downward situation ($P = 2$).

For archival purpose and for completeness we record here the expressions for the various flux density profiles for both cases. We refer the reader to Fig 10.1 where the segments $H_a(r)$ through $H_h(r)$ of the various flux density profiles which can be encountered are identified by the subscripts a, b, c, d, e, f, g and h.

$$\text{Linear Case: } j_c = j_o(T) \{1 - (H/H_{C2}(T))\}$$

Following the procedure described in chapter 7 we introduce j_c into Maxwell's eqn, integrate, rearrange and obtain,

$$H_a(r) = H_{C2} - \{H_{C2} - (H_a - I_M)\} e^{-\frac{j_o R}{H_{C2}}(1-r/R)} \quad (10.11)$$

Here we find it convenient to normalize all fields to H_{C20} , the upper critical field at the initial temperature, hence write, $h_{C2}(T) = H_{C2}(T)/H_{C20}$, $h_a = H_a/H_{C20}$ and $i_M(H_a) = I_M(H_a)/H_{C20}$.

Further we wish that $j_c(H,T)$ retains its slope, $dj_c/dH = j_o(T)/H_{C2}(T) = j_{oo}/H_{C20}$, hence “shrink” symmetrically along the ordinate and abscissa as the temperature increases. Here j_{oo} denotes $j_o(T)$ at the initial temperature.

We write

$$j_o(T) = j_{oo} f_j(T) \quad \text{and} \quad H_{C2}(T) = H_{C20} f_H(T) \quad (10.12)$$

where $f_j(T)$ and $f_H(T)$ are arbitrary monotonic functions varying from 1.0 at the initial temperature to 0 at T_c .

The above requirement dictates that,

$$f_j(T) \equiv f_H(T) \quad (10.13)$$

hence, the exponent in eqn 10.11 reads,

$$\frac{j_o(T)R}{H_{C2}(T)} = \frac{j_{oo} R f_j(T)}{H_{C20} f_H(T)} = \frac{j_{oo} R}{H_{C20}} = a \quad (10.14)$$

and is seen to be independent of temperature.

In normalized form, eqn 10.11 now reads,

$$h_a(r) = f_j(T) - \left\{ f_j(T) - [h_a - i_M(h_a) f_M(T)] \right\} e^{-a(1-r)} \quad (10.15)$$

where $f_M(T)$ is also an arbitrary monotonic function of temperature descending from 1.0 at the initial temperature to 0 at T_c . For brevity henceforth we will write f_j instead of $f_j(T)$, f_M instead of $f_M(T)$ and i_M instead of $i_M(h_a)$. Next,

$$h_b(r) = f_j - \left\{ f_j - h_a(r_i) \right\} e^{+a(r_i-r)} \quad (10.16)$$

which can be rewritten,

$$h_b(r) = \left\{ f_j - [h_a - i_M f_M] \right\} e^{+a(2r_i-r-1)} \quad (10.17)$$

where we have introduced $h_a(r_i)$ from eqn 10.15.

$$|h_c(r)| = f_j \left\{ 1 - e^{-a(r_{oo}-r)} \right\} \quad (10.18)$$

where r_{oo} is obtained by letting $h_b(r) = 0$ at $r = r_{oo}$ in eqn 10.17.

$$h_d(r) = f_j - \{f_j - (h_a - i_M f_M)\} e^{+a(1-r)} \quad (10.19)$$

$$h_e(r) = f_j - \{f_j - h_d(r_i)\} e^{-a(r_i-r)} \quad (10.20)$$

which can be rewritten,

$$h_e(r) = \{f_j - (h_a - i_M f_M)\} e^{-a(2r_i-r-1)} \quad (10.21)$$

where we have introduced $h_d(r_i)$ from eqn 10.19

$$|h_f(r)| = f_j \{1 - e^{-a(r'_o-r)}\} \quad (10.22)$$

where r'_o is obtained by letting $h_d(r_i) = 0$ at $r = r'_o$ in eqn 10.19.

$$|h_g(r)| = f_j - \{f_j - |h_f(r_i)|\} e^{+a(r_i-r)} \quad (10.23)$$

$$h_h(r) = f_j \{1 - e^{-a(r'_{oo}-r)}\} \quad (10.24)$$

where r'_{∞} is determined by letting $h_g(r) = 0$ at $r = r'_{\infty}$ in eqn 10.23.

The initial value for the only unknown quantity, i.e. the interface r_i between the concentric regions of counter-rotating currents in the bulk follows from our stipulation that initially $\langle M \rangle = 0$. Consequently via Mathematica, for cylindrical geometry we solve the integral $2 \int H(r) r dr = H_a R^2$ where $H(r)$ represents the spatial sequence of all the segments of profiles established by the swing of H_a from one envelope of the hysteresis curve towards the opposite, i.e. the appropriate flux density configuration among the four displayed in Fig 10.1

Convex Downward Case: $j_c = j_o(T) \{1 - [H/H_{C2}(T)]\}^2$

Introducing j_c into the Maxwell/critical state eqn, $dH/dr = \pm j_c$, integrating and rearranging leads to

$$H_a(r) = H_{C2} - \frac{1}{\left\{ \frac{1}{H_{C2} - (H_a - I_M)} \right\} + \frac{j_o R}{(H_{C2})^2} (1 - r/R)} \quad (10.25)$$

As before the requirement that $j_c(H, T)$ “shrink symmetrically” to zero as the temperature increases to T_c dictates that,

$$\frac{j_o(T)R}{H_{C2}(T)} = \frac{j_o R f_j(T)}{H_{C20} f_H(T)} = \frac{j_{o0} R}{H_{C20}} = a \quad (10.26)$$

hence,

$$f_j(T) \equiv f_H(T) \quad (10.27)$$

Where again j_{∞} and H_{C20} denote these quantities at the initial temperature and $f_j = f_H$ varies monotonically from 1.0 at T_{initial} to 0 at T_c .

Again normalizing field quantities to H_{C20} eqn 10.25 reads,

$$h_a(r) = f_j - \frac{1}{\left\{ \frac{1}{f_j - (h_a - i_M f_M)} \right\} + \frac{a}{f_j} (1-r)} \quad (10.28)$$

where $a = j_{\infty} R / H_{C20}$, $i_m = I_M(h_a) / H_{C20}$ and $f_M(T)$ is an arbitrary monotonic function varying from 1.0 at T_{initial} to 0 at T_c which describes the temperature dependence of the Meissner current.

$$h_b(r) = f_j - \frac{1}{\left\{ \frac{1}{f_j - h_a(r_i)} \right\} - \frac{a}{f_j} (r_i - r)} \quad (10.29)$$

which can be rewritten,

$$h_b(r) = f_j - \frac{1}{\left\{ \frac{1}{f_j - (h_a - i_M f_M)} \right\} - \frac{a}{f_j} (2r_i - r - 1)} \quad (10.30)$$

where we have introduced $h_a(r_i)$ from eqn 10.28 into eqn 10.29.

$$|h_c(r)| = f_j \left\{ 1 - \frac{1}{1+a(r_{oo}-r)} \right\} \quad (10.31)$$

where r_{oo} is obtained by letting $h_b(r_{oo}) = 0$ at $r = r_{oo}$ in eqn 10.30.

$$h_d(r) = f_j - \frac{1}{\left\{ \frac{1}{f_j - (h_a - i_M f_M)} \right\} - \frac{a}{f_j} (1-r)} \quad (10.32)$$

$$h_e(r) = f_j - \frac{1}{\left\{ \frac{1}{f_j - h_d(r_i)} \right\} + \frac{a}{f_j} (r_i - r)} \quad (10.33)$$

which can be rewritten,

$$h_e(r) = f_j - \frac{1}{\left\{ \frac{1}{f_j - (h_a - i_M f_M)} \right\} + \frac{a}{f_j} (2r_i - r - 1)} \quad (10.34)$$

where we have introduced $h_d(r_i)$ from eqn 10.32.

$$\left| h_f(r) \right| = f_j \left\{ 1 - \frac{1}{1+a(r'_o-r)} \right\} \quad (10.35)$$

where r'_o is obtained by letting $h_d(r) = 0$ at $r = r'_o$ in eqn 10.32

$$\left| h_g(r) \right| = f_j \left\{ 1 - \frac{1}{\left\{ \frac{1}{f_j - h_d(r_i)} \right\} - \frac{a}{f_j} (r_i - r)} \right\} \quad (10.36)$$

$$h_h(r) = f_j \left\{ 1 - \frac{1}{1+a(r'_{oo}-r)} \right\} \quad (10.37)$$

where r'_{oo} is obtained by letting $h_g(r) = 0$ at $r = r'_{oo}$.

As indicated previously, the only unknown in these expressions is r_i , the boundary between the counter circulating currents in the bulk of the specimen. The initial value for r_i is determined by the prescription that initially $\langle M \rangle = 0$.

10.3 Temperature Dependence of j_o and I_M

The fascinating excursions of $\langle M \rangle$ which we observe during warming to T_c starting with $\langle M \rangle = 0$ arise because the bulk critical current density parameter $j_o(T)$ and a significant Meissner current $I_M(T)$ decrease at different rates as the temperature is increased. We have consequently explored the effect of a variety of different temperature dependencies of these two quantities on the phenomena. In this exercise we used simple

arbitrary expressions chosen to emphasize the differences in the variations with temperature for $j_o(T)$ and $I_M(T)$.

For purpose of discussion we will write these functions of the temperature as if the initial temperature were $T = 0$. For instance for a parabolic dependence we write,

$$f(T) = 1 - \left(\frac{T}{T_{co}}\right)^2 \quad (10.38)$$

Since the initial temperature T_i in our experiments is not zero, this approach would lead to,

$$f(T_i) = 1 - \left(\frac{T_i}{T_{co}}\right)^2 = \text{fraction} \quad (10.39)$$

hence inconvenient for purpose of discussion and calculation.

Obviously, the parabolic dependence could read,

$$f(T) = 1 - \left(\frac{T - T_i}{T_{co} - T_i}\right)^2 \quad (10.40)$$

which is cumbersome to write. Since in the calculations the quantity in brackets is a decimal fraction which is incremented by an integral multiplier from 0 to 1.0, it is immaterial which form we use.

For these reasons, in the discussion which now follows we use the simple form illustrated by eqn 10.38.

For clarity and to fix ideas we now address a specific but illustrative case, namely where,

$$j_o(T) = j_{oo} \left\{ 1 - \left(\frac{T}{T_{co}} \right)^{1/2} \right\} = j_{oo} f_j(T) \quad (10.41)$$

and hence,

$$H_{c2}(T) = H_{c20} \left\{ 1 - \left(\frac{T}{T_{co}} \right)^{1/2} \right\} = H_{c20} f_H(T) \quad (10.42)$$

Next for purpose of illustration we choose and write,

$$I_M(T) = I_{Mo} \left\{ 1 - \left(\frac{T}{T_{c2}} \right)^2 \right\} = I_{Mo} f_M(T) \quad (10.43)$$

where I_{Mo} denotes the initial magnitude of the Meissner current in the selected final applied field H_a .

The reader will note that in eqn 10.43 the temperature T is normalized with respect to a critical temperature T_{c2} and not with respect to T_{co} , the critical temperature of the bulk of the superconductor in zero field. We now explain the introduction of T_{c2} here instead. For the remainder of this discussion we refer the reader to Figs 10.2 and 10.3.

From inspection of the expression for the segments of the critical state flux density profile, for instance, eqn 10.11 and its normalized form eqn 10.15, the reader will

note that when the increase of temperature will have caused $H_{C2}(T)$ to diminish until it is equal to H_a and concurrently I_M has been made to drop to zero, the superconducting state is now quenched and all the B profiles have consequently become horizontal and equal to $\mu_0 H_a$. Formally this defines a critical temperature, denoted T_{c2} which is dictated by H_a as follows,

$$H_{C2}(T_{c2}) = H_a = H_{C20} \left\{ 1 - \left(\frac{T_{c2}}{T_{co}} \right)^{1/2} \right\} \quad (10.44 \text{ a})$$

hence,

$$\frac{T_{c2}}{T_{co}} = \left\{ 1 - \frac{H_a}{H_{C20}} \right\}^2 = \left\{ 1 - h_a \right\}^2 \quad (10.44 \text{ b})$$

This formalism ensures that $I_M(T)$ vanishes in unison with $j_0(T)$. These features are illustrated in Fig 10.2 when $H_a > H_{C10}$ and in Fig 10.3 when $0 < H_a < H_{C10}$.

It is also useful to identify a critical temperature, denoted T_{c10} corresponding to H_{C10} , the lower critical field at the initial temperature (i.e. at $T = 0$ in our discussion). Therefore,

$$H_{C2}(T_{c10}) = H_{C10} = H_{C20} \left\{ 1 - \left(\frac{T_{c10}}{T_{co}} \right)^{1/2} \right\} \quad (10.45 \text{ a})$$

hence,

$$\frac{T_{c1o}}{T_{co}} = \left\{ 1 - \frac{H_{C10}}{H_{C20}} \right\}^2 = \left\{ 1 - h_{C10} \right\}^2 \quad (10.45 \text{ b})$$

For purpose of computation however it is useful and convenient to express $f_M(T)$ in harmony with $f_j(T)$ and $f_H(T)$. Hence we write,

$$\frac{T}{T_{c2}} = \frac{T/T_{co}}{T_{c2}/T_{co}} = \frac{T/T_{co}}{(1-h_a)^2} \quad (10.46)$$

where we introduce eqn 10.44 (b). Eqn 10.43 now reads

$$I_M(T) = I_{Mo} \left\{ 1 - \left(\frac{T/T_{co}}{(1-h_a)^2} \right)^2 \right\} = I_{Mo} f_M(T) \quad (10.47)$$

When $H_a < H_{C10}$, $I_M = H_a$ until a new critical temperature $T_{c1} < T_{c2} \equiv T_{c1o}$ is reached as illustrated in Fig 10.3 (b). Now in order that,

- (i) $I_M(T)$ and $j_o(T)$, vanish at the same temperature $T_{c2} \equiv T'_{c1o}$, and,
- (ii) the specified temperature dependence $f_M(T)$ remain in force in the range $T_{c1} \leq T \leq T_{c2}$,

it is necessary here to adjust the coefficient I_{Mo} in front of $f_M(T)$, in eqn 10.43 to a new value denoted H'_{C10} . To do this we note, as illustrated in Fig 10.3 (b) that the following ratios must hold,

$$\frac{H'_{C10}}{H_{C10}} = \frac{T'_{c10}}{T_{c10}} = \frac{T_{c2}}{T_{c10}} = \frac{T_{c2}/T_{co}}{T_{c10}/T_{co}} = \frac{(1-h_a)^2}{(1-h_{C10})^2} \quad (10.48)$$

where we have introduced, $T'_{c10} \equiv T_{c2} = T_{co}(1 - h_a)^2$ (eqn 10.44 b) and $T_{c10} = T_{co}(1 - h_{C10})^2$ (eqn 10.45 b).

Consequently, $I_M(T)$ now reads,

$$I_M(T) = H'_{C10} f_M(T) = H_{C10} \frac{(1-h_a)^2}{(1-h_{C10})^2} \left\{ 1 - \frac{(T/T_{co})^2}{(1-h_a)^2} \right\} \quad (10.49)$$

which applies in the range $T_{c1} \leq T \leq T_{c2}$ where T_{c1} is determined by letting $I_M(T_{c1}) = H_a$ in the above eqn.

10.4 Dependence of I_M on H_a ($H_{C10} \leq H_a \leq H_{C20}$)

Several empirical expressions have been proposed and exploited in the literature to describe the variation of I_M over the range $H_{C10} \leq H_a \leq H_{C20}$. In our analysis we have applied three such functions. A simple relation,

$$I_M = H_{C10} \left(\frac{H_{C20} - H_a}{H_{C20} - H_{C10}} \right) \quad (10.50)$$

a convex downward variation,

$$I_M = H_{C10} \left[1 - \sqrt{\frac{H_a - H_{C10}}{H_{C20} - H_{C10}}} \right] \quad (10.51)$$

and a formula proposed by Clem⁶⁶,

$$I_M = H_a - H_{C20} \sqrt{\frac{H_a^2 - H_{C10}^2}{H_{C20}^2 - H_{C10}^2}} \quad (10.52)$$

which exhibits a more severe convex downward structure than eqn 10.51. Although the latter two correspond more closely to observations, we did not achieve any significant improvement in the agreement with our observations on the Nb sample when these more sophisticated and realistic approximations were introduced into the model we have just presented. Indeed the Clem formula led to a deterioration of the agreement with the thermal disclosure data in comparison with the results obtained using the linear approximation (eqn 10.50).

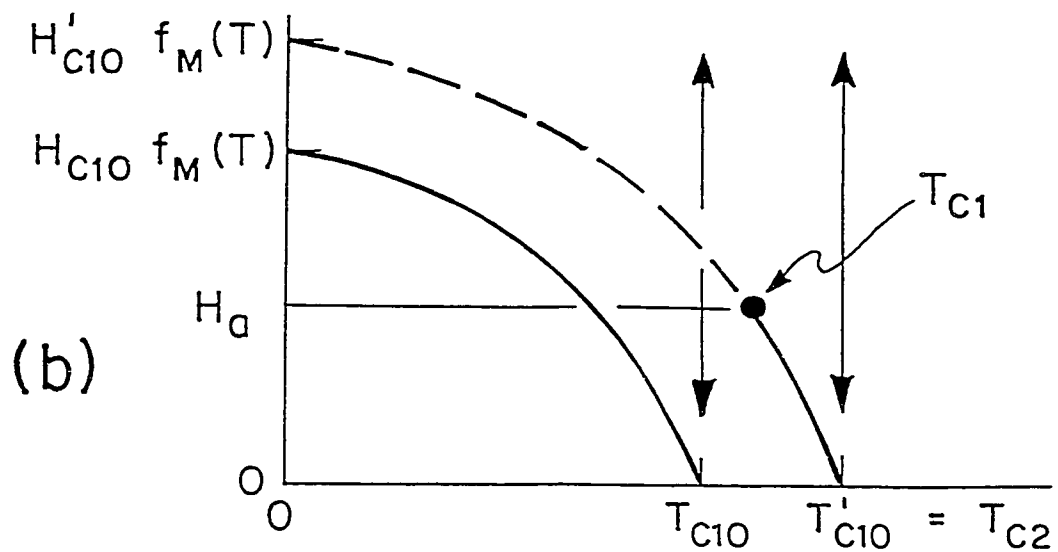
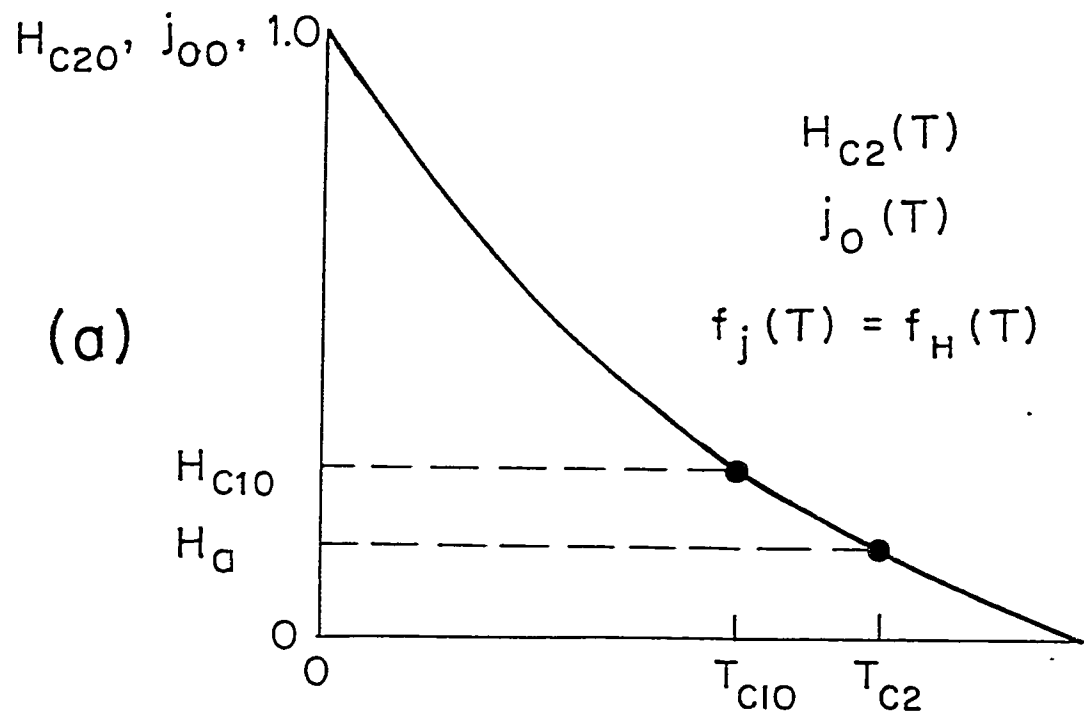
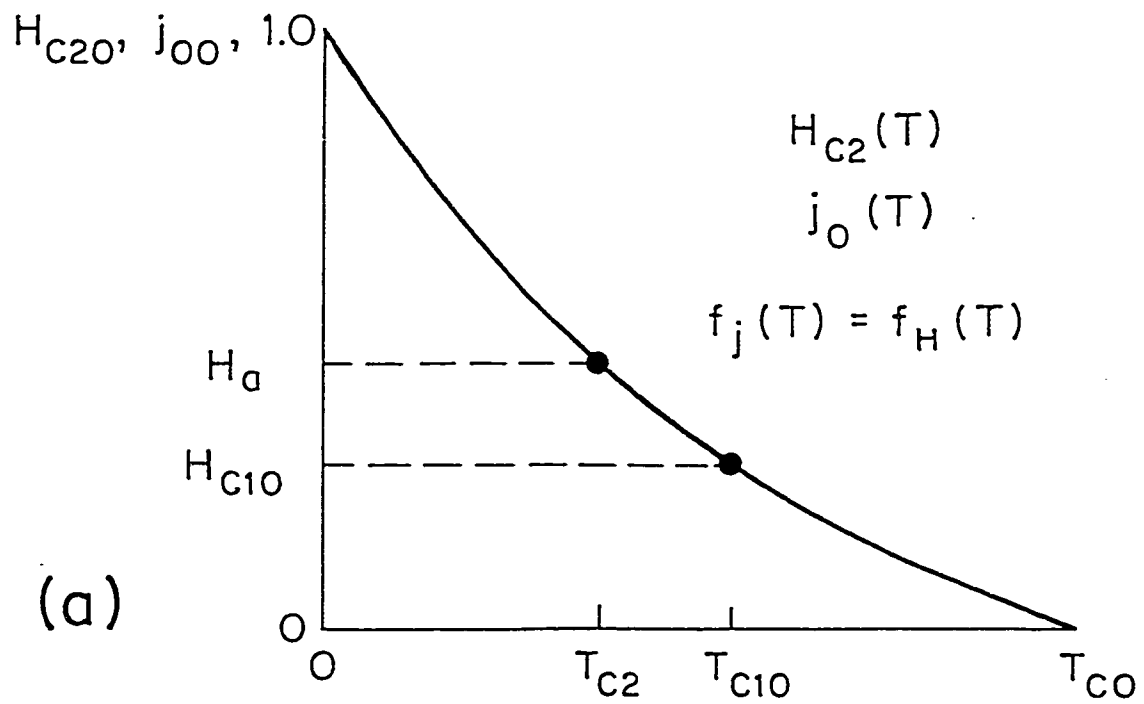
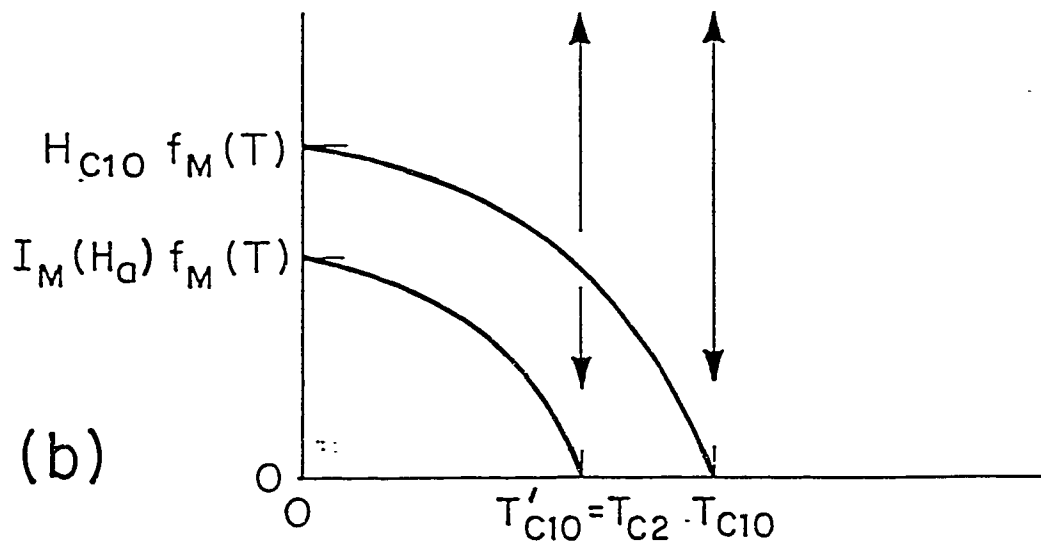


Fig 10.3 For the case $0 < H_a < H_{C10}$, a visual guide to the development in the text of the expressions which assure that $I_M(T)$ vanishes in unison with $j_0(T)$.



(a)



(b)

Fig 10.2 For the case $H_a > H_{C10}$, a visual guide to the development in the text of the expressions which assure that $I_M(T)$ vanishes in unison with $j_o(T)$.

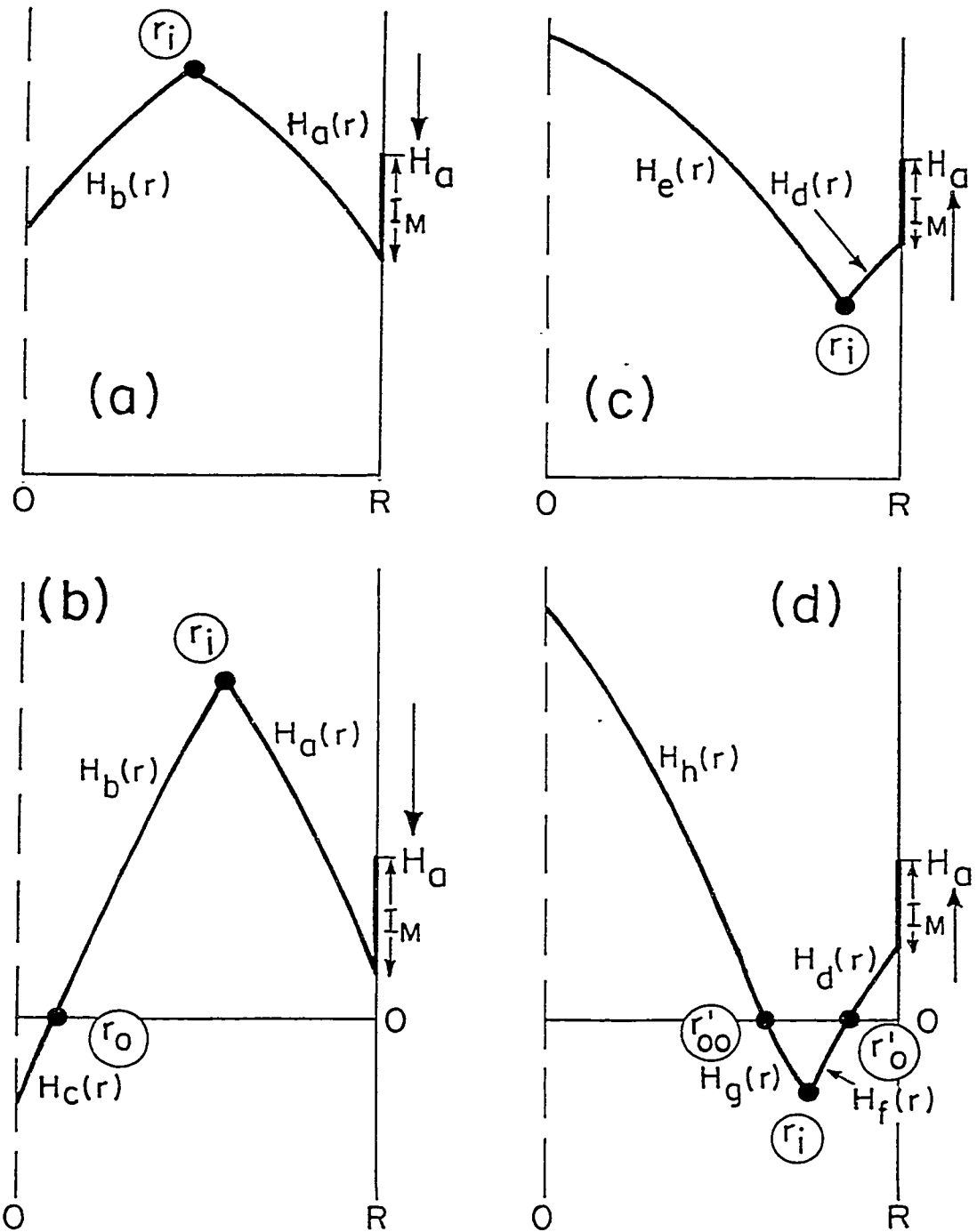


Fig 10.1 Displays various flux density profiles with the presence of significant $I_M(H_a, T)$ resulting in $\langle M \rangle = 0$ for both downswing, (a) and (b) as well as upswing, (c) and (d) application of H_a . Note in (b) and (d) the occurrence of negative flux regions (hybrid zones).

Chapter 11

Summary and Conclusion

Many workers, especially since the discovery of high T_c superconductors, have studied the evolution of the magnetic moment of nonideal type II superconductors (i.e. material “impregnated” with numerous pinning sites) as a function of time or of temperature in stationary applied magnetic fields $0 \leq H_a \leq H_{C2}$. In these investigations, in order to compare apples with apples, judicious experimenters ensure that the bulk of the specimen is completely occupied with persistent currents circulating in the same direction, hence that the locus of the magnetization $\langle M \rangle$ is situated either on the upper or the lower envelopes of the major hysteresis loop. A few researchers have also addressed experimentally⁶⁷⁻⁶⁹ and theoretically^{1,52,70,71} the situations where the locus of $\langle M \rangle$ is situated at various positions along the bridges traced by $\langle M \rangle$ as it is made to migrate from one envelope to the opposite envelope. In these circumstances the net bulk magnetic moment consists of two concentric volumes each filled with persistent currents but circulating in opposite directions. In samples termed semi-reversible where the Meissner surface current I_M is important, a third magnetic moment, hence that generated by this equilibrium current makes a significant, always diamagnetic, contribution to the total moment.

Among the vast array of $\langle M \rangle$ states encountered along the innumerable possible traversals between the two major hysteresis envelopes we have focused on the set where the three contributions to the total magnetic moment exactly cancel each other and produce a complex initial flux density configuration where the resultant $\langle M \rangle = 0$. We note that this spectrum of $\langle M \rangle = 0$ states can be obtained by H_a descending (downswing) or ascending (upswing) to its final stationary value. These two approaches to $\langle M \rangle = 0$ states lead to distinctly different sets of configurations of flux density profiles and consequently to dramatic differences in the phenomena encountered upon warming of the specimens to the normal state in various stationary applied fields. We examined these fascinating phenomena over the full range of fields where these are manifest, namely $0 \leq H_a \approx H_*$ where H_* is the first full penetration field into the specimen.

In a broad qualitative sense, nonideal type II superconductors can be classified as semi-reversible and hysteretic depending on whether the equilibrium Meissner contribution to their magnetic behaviour is significant or negligible. Consequently we have addressed representative specimens of these two categories in our work. Further since low T_c and high T_c type II superconductors may possess fundamentally different properties besides the large difference in their transition temperatures we have studied representative samples of both "species". Thus we envisaged examining the thermal release of hidden magnetic moments in four representative materials.

It is standard practice in the analysis of magnetic behaviour and critical current properties of type II superconductors to first exploit the simple Bean-London approximation^{49,50} which assumes j_c to be independent of B . Back of an envelope applications of this crude framework often leads to predictions in remarkable qualitative

and quantitative agreement with a large variety of observations^{55,56}. Pursuing our model for the release of hidden magnetic moments in a very hysteretic specimen using the Bean-London approximation we immediately saw that the peak magnetic moments revealed upon warming would not depend on the magnitude of H_a and whether H_a ascended or descended to the final $\langle M \rangle = 0$ state but would depend to some extent on the geometry of the specimen. For this reason we studied these phenomena in a large YBCO crystal where j_c was observed to be insensitive to the applied fields in the range extending to H_{*} and beyond.

For all of our samples we measured the four basic magnetization curves and the Meissner flux expulsion when this phenomenon was observable within the sensitivity of our system. The main purpose here was to illustrate the “character” of our samples in the panorama of semi-reversible and large hysteretic behaviour. These data enable us to decide whether the Meissner current plays a negligible role and can be ignored in the modeling. These curves also provide us with estimates of the relative magnitude of H_{*} , H_{**} and I_M for the semi-reversible samples. In some cases to complement the estimates of H_{*} and H_{**} we exploited the tedious procedures termed H_{cycle} and H_{cool} .

In our analysis of the thermal disclosure of hidden magnetic moments, for convenience, simplicity and computational economy we only used simple analytic expressions to describe the dependence of j_c on B and of I_M on H_a .

Pursuing well established procedures we introduce these functions in calculations of the four basic magnetization curves for all our specimens. Our main and only purpose in this exercise was to assess how well the simple and crude analytic expressions we chose described hence characterized the magnetic properties of the samples. Then

satisfied that these simple ingredients were acceptable approximations we proceeded to exploit them and the estimates for I_M and H_* in the calculations of the thermal disclosure phenomena within the framework of the simple model we proposed to account for these events.

The model we put forward to explain our observations is conceptually very simple. For hysteretic samples the model relies entirely on an application of the principle of flux line conservation as the critical current density is made to diminish and the associated flux density profiles are made to relax during gradual warming to T_c . For semi-reversible samples, the physical situation is more complicated. Nevertheless the only basically new element which enters into the model is that the variation of I_M and j_c with temperature needs to be quite different in order to account for the rich spectrum of behaviour we have encountered in our measurements. By comparing the five pertinent figures displaying our observations of thermal release phenomena with the corresponding theoretical curves, the reader can readily accept the validity and strength of our simple model.

REFERENCES

1. John R. Clem and Zhidong Hao, Phys Rev B 48, 13774 (1993)
2. A.A. Abrikosov, Sov. Phys. JETP 5 1174 (1957), J. Phys. Chem Solids 2, 199 (1957)
3. A.M. Campbell and J.E. Evetts, "Critical Currents in Superconductors" Taylor and Francis 1972, Advances in Physics 21, 199 (1972)
4. R.P. Huebener "Magnetic Flux Structures in Superconductors" Springer Series in Solid State Sciences. Springer-Verlag (1979)
5. E.H. Brandt "The Flux-Line Lattice in Superconductors" Rep. Prog. Phys. 58, 1465-1594, (1995)
6. M. Tinkham "Introduction to Superconductivity" 2nd Edition, International Series in Pure and Applied Physics, McGraw-Hill Inc. (1996)
7. P.G. De Gennes "Superconductivity of Metals and Alloys" W.A. Benjamin Inc (1966)
8. D. Saint-James, E.J. Thomas and G. Sarma "Type II Superconductivity" Pergamon Press (1969)
9. "Superconductivity" Vol 2, Edited by R.D. Parks, Marcel Dekker Inc. (1969)
10. D.R. Tilley and J. Tilley, "Superfluidity and Superconductivity" 3rd Edition Adam Hilger Pub. (1990)
11. T.P. Sheahen, "Introduction to High-Temperature Superconductivity" Selected Topics in Superconductivity. Plenum Press, (1994)

12. G. Blatter, M.V. Feigel'man, V.B. Geshkenbein, A.I. Larkin and V.M. Vinokur
"Vortices in High-Temperature Superconductors" Rev Mod Phys **66**, 1125-1388
(1994)
13. D.X. Chen, J.J. Moreno, A. Hernando, A. Sanchez and B.Z. Li "Nature of the driving
force on an Abrikosov vortex" Phys Rev B **57**, 5059-5063, (1998)
14. J.E.C. Williams "Superconductivity and its Applications" Pion Limited 1970
15. T. Orlando and K. Delin, "Foundations of Applied Superconductivity" Addison-
Wesley Publishing Company, (1991)
16. "Magnetic Susceptibility of Superconductors and Other Spin Systems" Edited by
R.A. Hein, T.L. Francavilla and D.H. Liebenberg. Plenum Press (1991)
17. R. Boyer, G. Fillion and M.A.R. LeBlanc J. Appl. Phys **51**, 1692 (1980)
18. M.A.R. LeBlanc and J.P. Lorrain, J. Appl. Phys **55**, 4035 (1984)
19. M.A.R. LeBlanc, G. Fillion and J.P. Lorrain J. Appl. Phys **59**, 3208 (1986)
20. G. Gandolfini, M.A.R. LeBlanc and J. Sekerka, Cryogenics **29**, 373 (1989)
21. M.A.R. LeBlanc, G. Fillion, W.E. Timms, A. Zahradnitsky and J.R. Cave
Cryogenics **21**, 491 (1981)
22. M.A.R. LeBlanc, S.X. Wang, D. LeBlanc, M. Krzywinski and J. Meng, Phys Rev B
52, 12895 (1995)
23. J.R. Cave and M.A.R. LeBlanc J. Appl. Phys. **53**, 1631 (1982)
24. D. Shi, K. Lahini, J.R. Hull, D. LeBlanc, M.A.R. LeBlanc, A. Dabkowski, Y.Chang,
Y Jiang, Z. Zhang, H. Fan, "Growth of Large-Domain YBa₂CU₃O_x with New
Seeding Crystals of CaNdAlO₄ and SrLaGaO₄" Physica C **246**: 253-261, (1995)

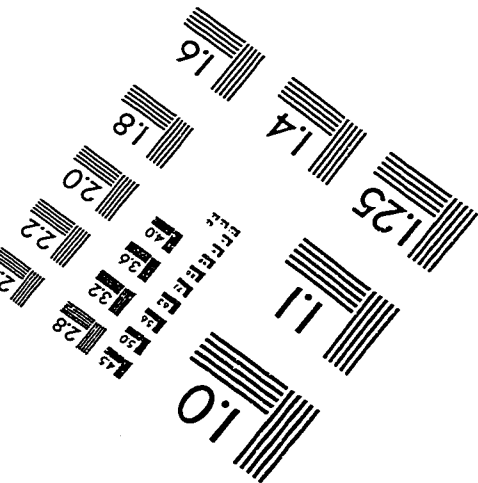
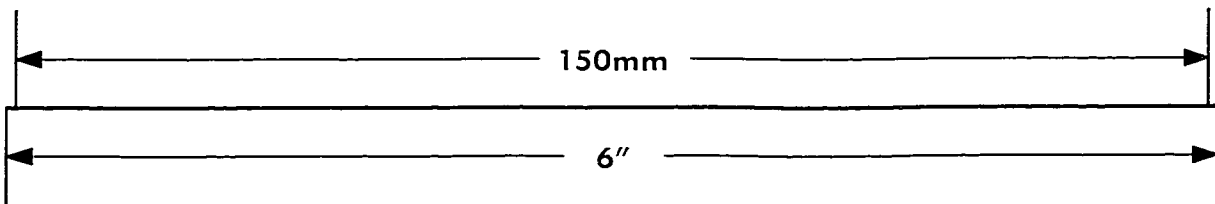
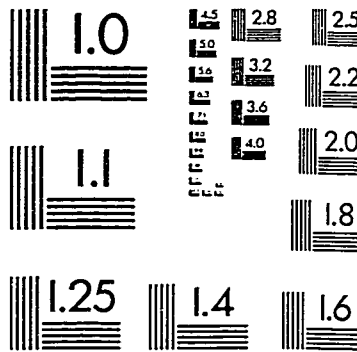
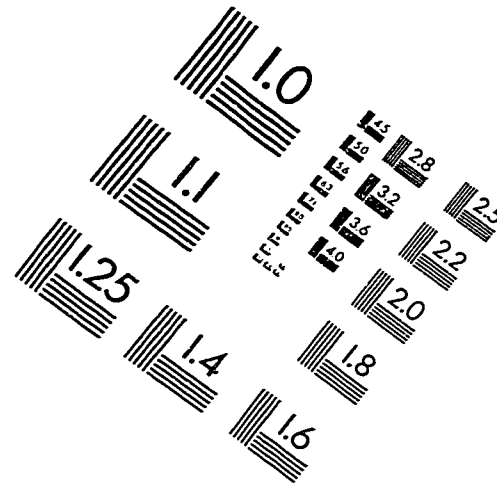
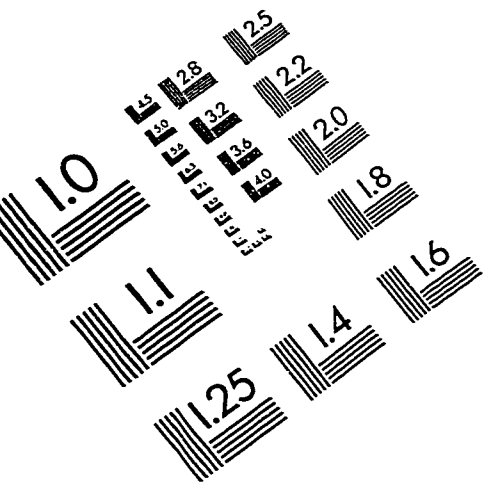
25. M. Murakami, T. Oyama, H. Fujimoto, T. Takahiro, S. Gotoh, Y. Shiohara, N. Koshizuka and S. Tanaka, Jpn. J. Appl. Phys. **29** L1991 (1990)
26. M. Morita, K. Miyamoto, K. Doi, M. Murakami, K. Sawano and S. Matsuda, Physica C **172**, 383-387 (1990)
27. S. Gotoh, M. Murakami, H. Fujimoto and N. Koshizuka, J. Appl. Phys. **72** (6), 2404 (1992)
28. M.A.R. LeBlanc, S. Celebi, S.X. Wang, and Vladimir Plecháč, Phys. Rev. Lett. **71**, 367 (1993)
29. S. Celebi and M.A.R. LeBlanc, Phys. Rev. B **49**, 16009 (1994)
30. L.M. Fisher, A.V. Kalinov, S.E. Savel'ev, I.F. Voloshin, V.A. Yampol'skii, M.A.R. LeBlanc and S. Hirscher, Physica C **278**, 169 (1997)
31. F. Pérez-Rodríguez and A. Pérez-González, J.R. Clem, G. Gandolfini and M.A.R. LeBlanc, Phys. Rev. B **56**, 3473 (1997)
32. D.G. Walmsley, J. Phys. F **2**, 510 (1972)
33. J.R. Clem, Phys. Rev. B **26**, 2463 (1982)
34. J.R. Clem and A. Pérez-González, Phys. Rev. B **30**, 5041 (1984)
35. A. Pérez-González and J.R. Clem, Phys. Rev. B **31**, 7048 (1985)
36. A. Pérez-González and J.R. Clem, J. Appl. Phys. **58**, 4326 (1985)
37. J.R. Clem and A. Pérez-González, Phys. Rev. B **33**, 1601 (1986)
38. A. Pérez-González and J.R. Clem, Phys. Rev. B **43**, 7792 (1991)
39. J.R. Cave and M.A.R. LeBlanc, J. Appl. Phys. **53**, 1631 (1982)
40. R. Boyer and M.A.R. LeBlanc, Solid State Commun. **24**, 261 (1977)
41. R. Boyer, G. Fillion and M.A.R. LeBlanc, J. Appl. Phys. **51**, 1692 (1980)

42. C.P. Bean and J.D. Livingston, Phys. Rev. Lett. **12**, 14 (1964)
43. P.G. de Gennes, Solid State Comm. **3**, 127 (1965)
44. Y. Ren Sun, J.R. Thompspon, H.R. Kerchner, D.K. Christen, M. Paranthaman and J. Brynestad, Phys. Rev. B **50**, 3330 (1994)
45. F. Zuo, D. Vacaru, H.M. Duan and A.M. Hermann, Phys. Rev **B 47**, 5535 (1993)
46. S. Kim, R.E. Howard and M.R. Beasley, J. Appl. Phys. **49**, 730 (1978)
47. J.F. Bussière and V.T. Kovachev, J. Appl. Phys **49**, 2526 (1979)
J.F. Bussière and M. Suenaga, *ibid*, **47**, 707 (1976)
48. D.G. Schweitzer and M. Garber, Phys. Rev. **160**, 348 (1967)
D.G. Schweitzer and B. Bertman, *ibid*. **152**, 293 (1966)
49. C.P. Bean, Rev. Mod. Phys. **36**, 31, (1964)
50. H. London, Phys. Lett. **6**, 162 (1963)
51. Y.B. Kim, C.F. Hempstead and A.R. Strnad, Phys. Rev. Lett. **9**, 306 (1962),
Phys.Rev. **131**, 2486 (1963)
52. L. Krusin-Ellbaum, A.P. Mazolemoff, D.C. Cronemeyer, F. Holtzberg, John R. Clem and Zhidong Hao, J. Appl. Phys **67**, 4670 (1990)
53. R.R. Hake, Phys. Rev. **158**, 356 (1967)
54. "The Effect of Metallurgical Variables on Superconducting Properties", J.D. Livingston and H.W. Shadler, Progress in Materials Science, Vol 12 Pergamon Press, (1965)
55. H. Ullmaier, "Irreversible Properties of Type II Superconductors", Springer Tracts in Modern Physics, Springer-Verlag (1975)

56. W.J. Carr. "A.C. Loss and Macroscopic Theory of Superconductors" Gordon and Breach Science Publishers, (1983)
57. E.M. Gyorgy, R.B. van Dover, K.A. Jackson, L.F. Schneemeyer and J.V. Waszczak, Appl. Phys. Letts **55**, 283 (1989)
58. R.L. Peterson, J. Appl. Phys **67**, 6930 (1990)
59. Studies in High Temperature Superconductors, Ed. Anant Narlikar, Vol 14 "Field Penetration and Magnetization of High Temperature Superconductors" Nova Science Publishers, Inc. (1995)
60. "Processing and Properties of High- T_c Superconductors" Vol 1, Bulk Materials Ed. Sungho Jin, World Scientific (1993)
61. K. Yasukochi, T. Ogasawara, N. Usui, H. Kolayashi and S. Ushio, J. Phys. Soc. Japan **19**, 1649 (1964), *ibid*, **21**, 80 (1966)
62. "From High-Temperature Superconductivity to Microminiature Refrigeration" Eds, B. Cabrera, H. Gutfreund and K. Vladimir, Plenum Press, (1996)
63. M.A.R. LeBlanc and W.A. Little, Proc. 7th Int Conf. Low Temp. Physics, Toronto, 1960, p.362, University of Toronto Press (1961)
64. S.H. Autler, Rev. Sci. Instr. **31**, 369 (1960)
65. E.H. Brandt, Am. J. Phys. **58**, 43 (1990)
66. John. R. Clem J. Appl. Phys. **50**, 3518 (1979)
67. M.R. Beasley, R. Labusch and W.W. Webb, Phys Rev. **181**, 682 (1969)
68. K. Kwasnitza and C.H. Widmer, Physica C **184**, 341 (1991)
69. K. Kwasnitza and C.H. Widmer, Cryogenics **33**, 378 (1993)

70. M.A.R. LeBlanc, Daniel S.M. Cameron, Martin Krzywinski and David LeBlanc,
Supercond. Sci. Technol. **10**, 625 (1997)
71. M.A.R. LeBlanc, D.S.M Cameron, S. Celebi and J.P. Pascal, Supercond. Sci.
Technol. **11**, 359 (1998)

IMAGE EVALUATION TEST TARGET (QA-3)



APPLIED IMAGE, Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989

© 1993, Applied Image, Inc., All Rights Reserved

