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Three Essays on Low-price Guarantees

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Three Essays on Low-Price Guarantees

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Abstract

This thesis consists of three essays and one literature review. The thesis examines theoretically the effects of Low-Price Guarantees (LPGs), which are increasingly common in retail markets and goods industries. Although the essays in this thesis share a common theme, they can be read independently without impeding the readers' understanding of the issues discussed in each.

The first essay studies the effects of LPGs in the search model of Salop and Stiglitz (1977). This model provides a general framework for the comparison of Matching Competition Clauses (MCCs) and Beating Competition Clauses (BCCs): The number of firms in the market is determined by free entry and consumers have incomplete information about prices and differ in their search costs. It is shown that MCCs have stronger collusive effects than BCCs, however, these effects evaporate with the introduction of an arbitrarily small hassle cost.

The second essay extends the Milgrom and Robers model (1982) to study the entry deterrence effects of LPGs. In this model, the potential entrant has no complete information about the true unit cost of the incumbent. It is demonstrated that under certain conditions, the high-cost incumbent can imitate the behavior of the low-cost one and deter entry even in situations where this would not be possible in the absence of a price guarantee. Therefore, the corresponding policy suggestion is to prohibit this pricing behavior when it is exerted by a monopolist.

The third essay studies LPGs in a partial equilibrium model, where consumers can search for lower prices but search takes time and thus delays consumption. A price guarantee to match any lower price offered by the retailer allows a consumer to purchase and consume now while keeping the option of reaping the benefit of a lower

price that he may find later in time. The analysis demonstrates how the following two factors, the variability of market prices and the percentage of bargain hunters, affect the decisions that a firm needs to make in setting its price and the duration of its price guarantee in a competitive environment.

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Introduction

Most consumers have had the personal experience of making purchases from stores that offer low-price guarantees (LPGs). Price-guarantee policies usually take two forms: one is in the form of a matching-price guarantee, which offers to match the lower price charged by any competitor; the other is in the form of a beating-price guarantee, which promises to refund not only the price difference, but more than 100 percent of the difference.

It has also been demonstrated experimentally (Jain and Srivastava, 2000; McConnel et al., 2000) that a price guarantee is used by sellers as a marketing strategy to minimize uncertainty, to reduce regret and ultimately to increase the possibility of a purchase. Accordingly, most market analysts hold the opinion that low-price guarantees promote vigorous competition and benefit consumers.[1] Surprisingly, economic analyses of price-guarantees have generally reached the opposite conclusion. Existing economic theories of price guarantees can be classified into three groups:

A price guarantee is an instrument that can be used to facilitate tacit collusion (Hay, 1982; Salop, 1982; Sargent, 1993 and Chen, 1995). It is argued that low-price guarantees eliminate firms' gain from undercutting rivals' prices and, even worse, LPGs penalize those who undercut price. Consequently, the collusive price can be established.

A price guarantee is a way to discriminate among customers with different search costs (Png and Hirschleifer, 1987; Corts, 1995). It is argued that a price guarantee keeps the benefits of searching within the searchers, i.e., consumers with lower search costs.

[1] The headline of an article in the Financial Times, January 18, 1996, p.8, reads "Petrol Rivals on Price-War Footing" because on the previous day, Esso announced the introduction of its price-matching policy.

As a result, the discipline effect exerted by those searchers on the market price is diminished.

A price guarantee is a way to deter entry (Salop, 1986). It is argued that a low-price guarantee adds credibility to a limit-entry threat. While the study of Arbatskaya (2001) identifies the beat-any-deal guarantee as the only price guarantee, among 14 types commonly observed, that can effectively prevent entry into the market.

In this thesis, we extend the existing theories of price guarantees in a number of directions. The common theme of the three essays in this thesis is imperfect information. In the first essay (Chapter 2), the effects of a matching competition clause (MCC) and a beating competition clause (BCC) are compared in a model where consumers have imperfect information about prices. In the second essay (Chapter 3) the role of price guarantees in entry deterrence is explored in a framework where the entrant has only incomplete information about the incumbent's costs. In the third essay (Chapter 4) consumers do not know prices and it takes time for them to search. Here a price-matching guarantee is used by a firm to induce consumers to purchase right away. The following is a brief review of each essay.

In Chapter 2 we extend the search model of Salop and Stiglitz (1977) to include a low-price guarantee as an option available to firms in a market. To be more specific, each firm in our model offers both a list price and a price guarantee policy. Consumers have imperfect information about prices and differ in their search costs. Each consumer first decides whether to search in order to obtain full information about prices, and then chooses the firm from which he makes his purchase. A consumer may incur a hassle cost when he activates a firm's price guarantee. Firms face U-shaped average cost curves, and entry into the industry occurs until profits are driven to zero.

This framework allows us to compare MCCs and BCCs in terms of their effects on both collusion and price discrimination. In the absence of hassle cost, the collusive effect of MCCs is stronger than BCCs. When firms adopt MCCs, the monopoly price becomes the only possible equilibrium price. When firms adopt BCCs, on the other hand, a range of prices are possible in equilibrium. In contrast to Png and Hirshleifer (1988), the adoption of MCCs eliminates price dispersion and thus leaves no room for price discrimination between consumers with high search cost and those with low search cost. On the other hand, price dispersion is still possible when firms adopt BCCs. In other words, it is possible for firms to practice price discrimination under BCCs but not under MCCs.

These effects of BCCs are robust to the introduction of small hassle costs. However, the same cannot be said about the effects of MCCs. Our analysis confirms Hviid and Shaffer's (1999) finding that the collusive effect of MCCs evaporates with the introduction of an arbitrarily small hassle cost. Perhaps even more interesting is the finding that when MCCs are rendered ineffective by the presence of hassle costs, firms are again able to price discriminate among consumers with different search costs.

In Chapter 3 we study the entry deterrence effects of price guarantees. We extend the Milgrom and Roberts model (1982) of entry deterrence by including price guarantees. In the model, there is one incumbent and one potential entrant. The entrant does not know the true marginal cost of the incumbent. In such a situation, a price guarantee committed by the incumbent would add credibility to incumbent's threat to fight entry. We demonstrate that under certain conditions, a price-matching guarantee can be used in conjunction with a relatively low price to signal low cost and deter entry; while under other conditions, the use of a price guarantee can allow the high-cost incumbent to imitate successfully the behavior of the low-cost one and deter entry even in situations where this would not be possible in the absence of a price guarantee. From a comparison of the equilibrium prices in the situation with a price guarantee and those in the situation without a price guarantee, it can be seen that the availability of a price guarantee as an additional instrument makes entry deterrence possible for a wider range of parameter values.

In Chapter 4 we examine how a firm sets its price and the duration of its price guarantee in a competitive environment. Most of the existing theoretical models on price guarantees are focused on their strategic effects: Low-price guarantees are offered by oligopolistic firms that, by definition, have the ability to influence their rivals' prices. While in reality, most retailers face fierce competition and the price set by a retailer may have little influence on the overall market prices. Furthermore, none of the existing literature has considered the time dimension of low-price guarantees. With the intention to fill the gaps mentioned above, we construct a partial equilibrium model where a retailer sets its own price but it has no influence on the distribution of prices in the market. Consumers can search for lower prices but search takes time and thus delays consumption. A price guarantee to match any lower price offered by the retailer allows a consumer to purchase and consume now while keeping the option of reaping the benefit of a lower price that he may find later in time. Our analysis focuses on how the following two factors, the variability of market prices and the percentage of customers who will continue to search for a lower price irrespective of the price and price guarantee offered by this firm, affect the decisions that a firm needs to make in setting its price and the duration of its price guarantee in a competitive environment. Our analysis shows that the firm will reduce its list price when more of its customers are bargain hunters. Not as obvious is that the effect on the duration of the price guarantee of having more bargain hunters depends on whether the firm faces the prospect of losing customers at the margin. If a marginally higher list price causes a nontrivial fraction of consumers to leave without purchase, the firm will couple the reduction of list price with a shortened price guarantee. Otherwise, the firm will lengthen the duration of price guarantee. We also use a numerical simulation to demonstrate that the adoption of a price guarantee can allow the firm to lower its list price and at the same time earn more profits.

Chapter 1 Literature Review

1.1 Introduction

Low-price guarantees (LPGs) have become increasingly common both in retail markets, such as grocery stores, electronics stores, sports stores, travel agencies and so on, existing in various forms of price-matching or price-beating, and in industries in such forms as most-favored-customer clauses, meet-competition-clauses and competitor-based price clauses. A random Internet search using the keywords "price matching" generated the following examples. Circuit City, an electronics retailer, advertises:

"Nothing's worse than paying too much. That's why we do everything in our power to make sure you don't pay a penny more than you should for anything you buy at Circuit City. If you've seen a lower advertised price from a local store with the same item in stock, we want to know about it. Bring it to our attention, and we'll gladly beat their price by 10% of the difference. Even after your Circuit City purchase, if you see a lower advertised price (including our own sale prices) within 30 days, we'll refund 110% of the difference."

The SFSU bookstore advertises that "All the textbooks you buy at the SFSU Bookstore for your classes are under a Low-Price-Guarantee. Is your textbook being sold cheaper somewhere else? We'll match the price under the following conditions..."

These advertisements sound very attractive to consumers, but invoke alerts to many, especially anti-trust economists. Beginning with Hay (1982), the majority of the literature has seriously questioned this pricing policy by accusing it of facilitating collusion, causing wasteful entry or discriminating against customers who are illinformed. With the evolution of this market practice and the relaxation of some assumptions, more and more studies have explored different aspects of the effects of, and also the incentives for, adopting LPGs. At the same time, researchers from other fields, such as market strategy analysts and psychologists, also give their attention to this market phenomenon.

The purpose of this chapter is to survey both the theoretical and empirical literature on price guarantees. Section 2 reviews the relevant theoretical studies in economics; section 3 reviews the empirical and experimental studies in economics; section 4 reviews the studies on LPGs in other fields; section 5 concludes.

1.2 Theoretical Analysis

Existing theoretical studies on LPGs fall basically into the following categories: (1) collusion theory; (2) entry related theory; and (3) discrimination theory. The majority of the studies argue that LPGs have the potential to turn the mechanism of traditional economics upside down. Only some economists argue for the LPGs but even then only to mitigate the *degree* of its anticompetitive effects, and rarely to reverse the effects.

1.2.1 Collusion Theory

According to this theory, LPGs discourage competitors' incentives to cut prices by committing to a self-binding announcement or a contract, and thus circumvent the use of overt contracts to collude, a practice prohibited by the US Sherman Act (Section 1). They thus facilitate an anti-competitive outcome. Most industrial organization economists hold this view and propose that the antitrust authority ban the practice of offering LPGs.

1.2.1.1 Practice to facilitate collusion

Salop (1986) studies meeting competition clauses together with most-favored-nation clauses to show that these industry practices can increase the likelihood of successful coordination among oligopolists. In his model, he shows with the simplest two by two matrix structure for the Prisoners' Dilemma game that the oligopolists have to use some facilitating practices to get the cooperative outcome when there is no pure Nash equilibrium. It is argued that with the meet-or-release clause as a device to exchange information, the detection lags which cause the time interval spent on off-diagonal price-pair states, (high, low) or (low, high), are shortened or eliminated. Therefore, the firm that offers the matching clauses is immune from the possibility of losing market share to rivals' undetected discounts. Furthermore, by deleting the release option, the seller must meet all rivals' discount clauses, which helps the oligopolists in the market arrive at the cooperative outcome, (high, high).

Holt and Scheffman (1987) study the meet-or-release clause combined with most-favored-customer clause[2] and the advance notification of list-price increases, which is commonly used in producer-goods industries. They present a homogenous-product model in two different scenarios, "when the discounts have to be offered generally (non-selectively) and when the discounts can be offered selectively to a subset of rivals' customers". In their two-stage game, firms select list price in the first stage, and then select discount prices in the second stage. Their analysis shows that the combination of these policies can have anti-competitive effects by having supracompetitive equilibrium lists and transaction prices that are immune to discounting when the discounts can only

[2] Most-Favoured-Customer clauses, which guarantees that the buyer is receiving the lowest price offered to anyone by the seller, have been examined extensively as facilitating practices. Hay (1982) argues that the MFC provision leads a price-cutting firm to lose the profit that it has already received from its previous customers and thus is discouraged from cutting its price. In a one-period differentiated Bertrand model, Cooper (1986) presents that firm which first commits itself to the MFC provision can play as a von Stackelberg leader, rather submitting the right of price setting to the competitors. And this very leadership reduces competition and leads to a cooperative outcome.

be offered generally; but if the discounts can be offered selectively, the anti-competitive effect is dampened by the firms' incentive to make unmatched discount offers and thus steal more sales from competitors.

Sargent (1993) provides a good comparison among the most-favored-nation clauses (MFNs), meeting-the-competition clauses (MCCs), and low-price guarantees (BCCs by our definition). He argues that MFNs can be utilized to maintain the collusive equilibrium, while MCCs can help to increase the price to a supra-competitive level initially. But MCCs are less effective in facilitating collusion in that they send only weak signals to the competitors.

Assuming perfect information and zero transaction costs, i.e., consumers are always able to purchase from sellers offering the lowest net price, Sargent's model shows that if other firms react to the low-price guarantees by cutting price, the only effect is to lose market share to the leader who adopts LPGs first. In his model, LPGs turn economics "upside-down" in three respects:1.The firm posting a higher price can hold all or most market share by offering LPGs at the same time; 2. Competitors cannot gain, or even worse, they may lose market share by a price cut; 3. Firms can earn positive economic profits even in the long run.

Baye and Kovenock (1994) extend the model of Png and Hirshleifer (1987) by including Beat-or-Pay commitments. In the first stage of their model, two firms sequentially announce their advertisements with one of these three options---a price message, a price matching message and a beat-or-pay message, and then in stage two, given knowledge of the first move, firms simultaneously set their prices. Consumers are grouped into uninormed and informed.

After giving payoffs of six pricing subgames, they conclude that the unique Nash equilibrium emerges when the first–mover adopts the beat-or-pay advertisement and the follower sends a price message. In equilibrium, both firms list the monopoly price but the leader profits more by attracting all the informed customers with the advertisement of beat-or-pay. Contrary to the model of Png and Hirshleifer (1987), the price discrimination effect does not exist since all customers, whether informed or uninformed, pay the monopoly price.

1.2.1.2 May facilitate collusion, but not necessarily

Some economists, however, find that the facilitating effect of LPGs is not that strong, taking into consideration other factors, such as different market structures (Simons, 1989), different solution concepts (Chen, 1995) and hassle costs (Hviid & Shaffer, 1999).

Simons (1989) demonstrates that the competitor-based formula pricing clauses, which are usually embedded in long-term contracts, such as most-favored nation clauses and meeting competition clauses, can be either efficient in unconcentrated markets or anti-competitive in concentrated markets.

In unconcentrated industries, competitor-based formula pricing clauses can help the contract to better reflect the market forces by setting the contract price equal to the spot market price; the clauses can improve the efficiency of allocating risk, depending on the degree of risk aversion of the seller and the buyer; the clauses avoid the shortcomings of cost-plus contracts which compromise the incentives of sellers to reduce costs; and also, the clauses may persuade the buyers to purchase sooner if they expect a price decline in the future.

In concentrated industries, competitor-based formula pricing clauses facilitate collusion through producing information exchange and incentive management effects. Simons uses the case of United Stated v. Socony-Vacuum Oil Co. to illustrate that these clauses can compromise competition even when the contract prices are determined by spot market prices. The reason is that the spot market price can be influenced by a small group of agents; and again, MFN clauses increase the sellers' incentive not to lower price or expand products.

Levy (1994) considers mainly two types of price guarantees in an industrial context, i.e., meeting-competition clauses and most-favored-customer clauses, and argues that these price guarantees are advantageous in providing an automatic and straightforward adjustment mechanism especially when prices in the market are significantly volatile.

In his model, when firms are analyzed in an isolated situation, price guarantees can benefit the customers by reducing precontractual costs, such as search cost and thus avoiding delays in purchases, and also by reducing postcontractual costs, such as negotiation efforts along with enforcing and maintaining contracts. Firms can also benefit from this pricing policy by keeping the continuity and adaptation of the contracts.

Putting the firm and the price guarantee back into the environment where rivals anticipate each other's behavior and react accordingly, meeting-competition clauses and most-favored-customer clauses promote higher market prices. Levy asserts that "firms with greater market share often have the most to gain and are likely to have the greater impact on prices; they are most likely to implement MCCs and MFCs for strategic gain." (P.311)

Chen (1995) develops a game based on the Bertrand duopoly model where two firms compete in price guarantees and list prices. He shows that neither is it necessary for both firms to commit themselves to matching or beating competition clauses in equilibrium, nor would a collusive outcome emerge when matching or beating-competition clauses are adopted in equilibrium.

In a market for a homogeneous product, firms compete in a game of two stages: at stage 1, firms choose to adopt matching, beating competition clauses or no price guarantees; and at stage 2, both firms announce their list prices simultaneously. If the solution concept is subgame perfect Nash equilibrium, that no firm adopts price guarantees and both firms set their prices at marginal cost can be part of the equilibrium; however, if forward induction is imposed on the subgame perfect Nash equilibrium, the facilitating effects are more pronounced: both firms offer price guarantees, which can be any combination between matching and beating competition clauses, and monopoly price appears as the effective equilibrium price. Chen also demonstrates that an equilibrium where neither firm commits to price guarantees can arise when either a forward induction or a subgame perfect solution is used, which explains why not every industry offers price guarantees.

Hviid and Shaffer (1999) point out that the existing literature overstates the ability of price-matching to facilitate monopoly price. If consumers must incur some hassle cost, such as an opportunity cost of waiting time and disutility of confronting a sales manager, in order to get the benefit of the firm's price-matching guarantee, consumers would prefer to go directly to the competitors' actual lowest price. Therefore, price matching cannot protect the firm from losing market share and thus the Bertrand equilibrium will prevail in the market.

In their model with two firms selling identical products at different locations, symmetric and asymmetric firms are studied. They indicate that only the firm which has the higher Bertrand-Nash price, or both firms, commits to a price guarantee. In the existence of positive hassle costs, a small price cut will successfully steal customers from high-priced firms. This result implies that, if firms try to collude or sustain higher prices, the best strategy would be hassle free price-matching, i.e., automatically matching the competitor's price. Therefore, Hviid and Shaffer conclude, there may exist reasons other than facilitating collusion for firms to offer price guarantees.

1.2.2 Entry Consideration

Collusive prices often exist in a market where entry is difficult or impossible, while at the same time, a supracompetitive price makes the market attractive to potential entrants. Therefore, some industrial organization economists proceed to study how low-price guarantees can affect entry. One theory argues that the adoption of LPGs can add credibility to a limit-entry threat and thus deter entry (Salop, 1986; Belton, 1987); another theory points out that LPGs drive prices up and help to sustain the higher price, which attracts too many entries and thus results in extra welfare losses (Edlin, 1997).

Salop (1986) also introduces the idea that a no-release matching competition clause adds credibility to a limit-entry threat. Although the incumbent might prefer to accommodate, he has to match the entrant's price required by the no-release MCC. Therefore, the meeting competition clause makes the entry deterrent more credible, and thus the potential entrant can be deterred, even when the incumbent would rather accommodate. In the conclusion, however, Salop suggests some possible benefits stemming from this clause, for example, insurance protection for the buyers and the early purchase instead of searching delay for the sellers.

Belton (1987) extends Salop's analysis to study the meeting and beating competition clauses in a differentiated duopoly model. In his analysis, the incumbent firm chooses whether to adopt price guarantees in the period prior to entry; then the entrant makes its decision on entry and also chooses whether or not to adopt meeting competition clauses. The clause consists of the list price and the guarantee, thus the actual price. He concludes that the commitment to a linear competitor-based price clauses (CPCs) is optimal to the contracting firm. He also provides the conditions under which CPCs can be an entry-deterrent, under which CPCs can facilitate collusive prices, and under which they can result in price discrimination.

Edlin (1997) suggests that the antitrust law authority should ban the usage of price matching policies. He asserts that price matching policies injure competition because two competitive forces, price cutting and entry, do not work well under price matching: a firm that adopts a price-matching policy can raise its price without losing market share and even better, can exploit more profit from the uninformed customers; a firm that does not adopt this pledge cannot attract customers by cutting price and thus it undermines the rivals' incentive to undercut price. In this situation, entry would occur with the supra-competitive price. However, entry cannot drive prices down because entrants are unable to undercut incumbents, again for the existence of price-matching policy. He concludes that the price matching is even worse than ordinary monopoly or cartel because the fact that too many entries lead to extra welfare losses.

In his paper, Edlin considers a new avenue of attack on price matching. With the analogy of price matching to other illegal vertical agreements such as tying sales, refusals to deal with price cutters, exclusive-dealing arrangements, resale price-maintenance agreements and territorial restrictions, he suggests a law to prevent the buyers from invoking a matching policy and to encourage them to buy from firms posting the lowest prices, thus restoring ordinary competition. He also argues that the price-discrimination caused by price matching has anti-competitive effects. Without price matching, informed consumers can play the regulatory effect by forcing list price down but with price matching, the pro-competitive benefits of active searching are limited to informed consumers only.

Focusing on welfare effects of entry deterrence, Edlin and Emch (1999) expand the previous research and analyze the potential welfare losses resulting from price matching. In a homogeneous market where all consumers are fully informed, with the presumption that price matching leads to higher prices, they compare the welfare losses in the cases with price matching and entry, with price matching but no entry, and with monopoly without entry. In industries where the ratio of fixed costs to average costs is small, they conclude that the total welfare losses due to distorted prices and over-entry are much larger under price matching with free entry.

The worst case is given in the Edlin and Emch model and they conclude that attention should be given to a market with a lower fixed cost. This conclusion contradicts traditional theories in this regard.

Prompted by the idea of Edlin and Emch (1999) that a price-matching policy causes over-entry and thus leads to higher welfare losses, Arbatskaya (2001) infers that it would be socially desirable if incumbents could use price-matching to deter entry. With this idea in mind, she provides a general framework for the study of LPGs in terms of their ability to deter entry.

In a perfectly contestable homogenous product market, a single incumbent and entrant are assumed to be competing in prices and LPGs. Arbatskaya covers 14 classes of price guarantees which are commonly observed and studies all the subgame perfect Nash equilibria involving all the strategies involving price and/or these LPGs. She finds that the beat-any-deal guarantee, which intends to beat the competitor's price by some dollar amount or a percentage, is the only type of price guarantee that can effectively prevent entry into the market. Price matching, however, is not able to prevent entry but, rather, it plays the role of facilitating collusion after accommodating the incumbent.

Casado-Izaga (1999) studies the role of price matching policies when an entry decision needs to be made in a circular city. He concludes that price matching leads to more firms selling at a higher price and thus consumer surplus and social welfare are impaired. In addition, his analysis shows that the entry cost has a negative relationship with the social welfare loss, and also the gap between the number of firms in equilibrium with a price matching policy and the optimal number of firms in equilibrium in the absence of a price matching policy. His analysis implies that a zero entry cost market is more vulnerable to the negative effects of price matching guarantees.

1.2.3 Discrimination Consideration

Without price guarantees, uninformed customers can benefit from the active shopping of informed customers, which drives the price down for everyone, as free-riders. The introduction of price guarantees, however, limits the benefits of searching to the active searching customers themselves, and thus, allows the firm to discriminate among consumers with different search costs.

Png and Hirschleifer (1987) introduce price matching to the strategy space of the traditional price discrimination model. Thus "firms may not only choose price but also they may choose to match the lowest price listed by a competitor" (P.366)

They assume that there exist two classes of consumers with different costs of acquiring information, locals and tourists, and with different demand curves: thus there are two relevant monopoly prices charged to two groups of consumers. In the Nash equilibrium of their model, firms can use price matching to extract more profit from the tourists while still keeping the price for the locals low. The interesting part of their model is that price discrimination cannot be exerted alone by a monopoly, but instead, price guarantees can only enable discrimination with the existence of competitors. They demonstrate that every firm in the market wants to take advantage of setting higher prices for tourists thus reducing total sales if there is no effective coordination.

Corts (1995) extends the model of Salop (1986) to include price-beating which changes the supracompetitive price equilibrium. He reasons that firms with a price-beating guarantee can successfully steal the market and thus make positive profits by undercutting the rival's price.

Corts (1996) further develops a model in which price matching and price beating can be devices to price discriminate. In his model, firms give the right to invoke the price guarantee policy to the consumers and thus consumers are segmented according to their hassle costs which include "the effort expended on producing physical evidence of a lower price and the displeasure experienced in the hassle of making the transaction" (P.285). With only sophisticated consumers utilizing the policy and thus purchasing at the lowest effective price, the equilibrium prices of all firms are lower, which benefit all the consumers. What distinguishes his model from most of the others is that the price guarantee is pro-competitive.

1.2.4 Other Explanations

In addition to the above three theories, other possible explanations have also been provided, for example, LPGs lead to minimized product differentiation as well as price competition (Zhang, 1995); firms have to boost their prices because they need to cover the cost of offering LPGs which can be treated as a contingent liability (Mazmdar & Srivastava, 2001); and LPGs can be utilized to convey price information to uninformed customers (Moorthy & Winter, 2004).

Zhang (1995) adds another possible consequence to the existing literature, the minimization of product differentiation caused by the adoption of price matching policy. In the scenario of a Hotelling location model, two firms differ only in location and play a three-stage game: both firms simultaneously choose locations in the first stage; after locations are chosen, the firms choose whether or not to adopt a price-matching policy simultaneously; in the third stage, the firms choose their prices.

He shows that the only Nash equilibrium occurs when both firms locate at the middle point of the unit line, i.e., firms minimize their differentiation. It is suggested that price matching policies not only suppress the competition in price in the sense that the market price is raised, but also suppress the competition in location in the sense that firms choose independently to locate together at the middle point.

Mazmdar and Srivastava (2001) notice that price-matching refunds can be deemed as a contingent liability for the firms offering such policies. By offering a price-matching policy, a seller is giving customers the option to claim the differential between the price charged by the seller and the lowest price which could be found by the customer.

By implicitly assuming that every customer is identical in having zero search cost, Mazmdar and Srivastava show how the present value of the seller's loss from committing to the price matching guarantee policy, depends on several key factors, such as the lowest price's volatility in the market, the price-matching validity period, and the risk-free rate of return. They conclude that the seller has to charge a higher price with price-matching than otherwise to offset the contingent liability. They claim that their model can be used to compare the cost of employing price-matching guarantees and other pricing strategies and thus help firms improve the design of their strategies.

Chen, Narasimhan and Zhang (2003) acknowledge the competition-dampening effect of price-matching guarantees as argued in the majority of the literature, but more importantly, they claim that price matching guarantees also have a competition-enhancing effect which dominates the former effect when consumers differ in their search cost and store loyalty. They study a model in a market consisting of two identical firms where consumers are segmented as switches, loyals, bargain shoppers and opportunistic loyals. They conclude that average prices and profits are reduced by the adoption of price-matching guarantees when firms are competing; the number of loyals (switchers) positively (negatively) affects the choice of PMG, that is, the increase of switchers enhances the competition-enhancing effect of PMG, and therefore, the gain from increased market share cannot surpass the loss from lower prices.

Moorthy and Winter (2004) extend the study of the price-matching guarantees typical of the traditional duopoly model with homogenous goods and zero transaction costs, to the case with heterogeneous products in location, heterogeneous consumers in information cost, and heterogeneous firms in optimal pricing. Their model demonstrates that, when firms differ in their optimal pricing, price matching serves as a way to convey price information to uninformed consumers. The existence of informed consumers ensures this signal to be credible. They argue that it is costly for the high-priced firm to let the low-priced firm be the price leader, thus the firm that adopts price-matching has to be the one with a lower price.

1.3 Empirical and Experimental Analysis

Compared to the volumes of theoretical work exploring the policies of pricing guarantees, the empirical work is surprisingly scarce. And in addition to that, existing empirical work provides mixed evidence. The increase in market prices when one of the stores adopted a price-matching guarantee in Hess and Gerstner (1991) suggests collusion; the study of Manez (1998) implies that the price-beating policy is used as a loss-leader strategy; while Arbatskaya, Hviid and Shaffer (1999) find a statistically significant lower price for dealers offering low-price guarantee and allowing for post-sale search.

Hess and Gerstner (1991) empirically test the effect of a price matching guarantee, and their analysis supports the theory that the price matching policy promotes a higher degree of price co-ordination. The data used are from prices listed in some supermarkets in North Carolina from 1984 to 1986, where one of them, Winn-Dixie, adopted price matching policies at the end of 1984. After comparison between included product (products that are covered by matching price guarantees) prices and excluded product (products that are excluded from price matching policies) prices at different periods, i.e., before and after the adoption of this policy, their analysis shows that the adoption of price-matching increases the market's average price. And at the same time, the targeted store, the Food Lion, raised the prices for products included, relative to the prices of products excluded, from the price-matching policy.

Manez (1998) examines how a price-beating guarantee and hassle costs (Hviid & Shaffer, 1999) influence the effectiveness of price guarantees as a facilitating device. His data are the prices taken directly from the three supermarkets with the highest market share in 1996 in the UK: Tesco, Sainsbury and Safeway. He considers the supermarket as a multi-product firm, and thus this analysis can illustrate the relationship between low-price guarantees and loss-leaders strategies-defined by Hess & Gerstner, 1987, as a price strategy in which retailers set very low-prices, sometimes below costs, to lure consumers to the shops.

Manez's empirical analysis suggests that Tesco's beating price policy decreases the average price of the basket of products included in the policy but increases the price coordination between supermarkets. Thus, it is concluded that the beating-price policy is used as a loss-leader strategy to attract consumers, while the rivals interpret this action as a threat and thus start a price war over the products included in the price guarantee. Also, he finds that the hassle costs are higher than the expected reward by activating the refund.

Arbatskaya, Hviid and Shaffer (1999) collect data from Sunday newspapers about retail outlets in thirty-seven metropolitan areas in the United States from September 19, 1996 to December 8, 1996, and document the widespread use and variety of LPGs. Their data show that the price-matching guarantee is the most common one and accounts for nearly two-thirds of the LPGs.

In their paper, they use the number of restrictions to proxy the hassle cost and assume that it is in the firm's interest to offer hassle-cost free LPG if it is to facilitate tacit collusion. With this assumption, their sample does not support the tacit collusion story. To test the theory that LPGs are used as a device to price discriminate between informed and uninformed consumers, they consider pairs of price quotes from two tire dealers: the data do not provide evidence that the firm with the LPG has higher prices than the ones without LPG, but on the contrary, the firms with LPGs that allow post-sale search tend to have significantly lower prices than firms with no LPGs.

Fatas and Manez (2001) develop an experimental test to check the ability of the price matching guarantee to raise market price above the Bertrand-Nash equilibrium. Using a Bertrand model as the theoretical framework, they design three treatments: the pure Bertrand price setting; the price matching imposed as an institution; and price matching given as a choice. In each treatment, 18 subjects interact during 50 rounds and in each round the subjects are randomly coupled forming 9 duopolies. Knowing the technical functioning of the game and all available information, the subjects maximize their profits, which are measured by their final monetary payments according to their performances in the experiment.

The results of these treatments indicate that the price matching guarantee has the ability to increase prices to supra-competitive levels and thus provide experimental support to the theoretical analysis that price matching policies are anti-competitive and lead to collusive prices.

1.4 Marketing and Psychological Studies

As we have seen in previous sections, economic research focuses mostly on the perspective of firms; marketing and psychological studies make their contributions by examining consumers' behavior in reacting to the price guarantee.

McConnell et al. (2000) experimentally investigate the role that price guarantees play in the consumers' mental simulation prior to making a purchase. They define "prefactuals" as the mental simulation about future possible events. And upward prefactuals, such as, "if I buy it today and find it for less next week, I'll regret my purchase," and the anticipated regret will reduce the likelihood of purchasing.

Their study reveals that the consumer provided with the price guarantee reduces upward prefactual thinking, dampens anticipated regret, and, even better, the consumer also increases the generation of downward prefactuals (e.g., "if I buy this today and find it cheaper next week, I can get the difference in price refunded to me").

In McConnell et al.'s paper, price guarantees are used by the sellers as a marketing strategy to minimize uncertainty, to reduce anxiety-provoking prefactuals and ultimately to increase the possibility of the purchase. And interestingly, their study also shows that holding a price guarantee, even if not provoked, increases consumer's satisfaction compared with the one without a price guarantee.

Being market researchers, Jain and Srivastava (2000) conduct two experiments to examine the effects of price-matching policies on consumer perceptions of store price, purchase intention and choice of store. In experiment 1, they study consumer perceptions of price expectations, confidence of finding low prices at the store and purchase intention after giving a group of subjects one store description varied in terms of whether a price-matching policy is presented. In experiment 2, they give subjects two store descriptions which consist of three conditions, including neither store offer price-matching policy (PM), store A offers PM and store B offers PM, and then they study subjects' store choice.

Their experimental and theoretical analysis shows that the fact that firms are differentiated and a portion of consumers are uninformed can cause the price-matching policies to be pro-competitive. Furthermore, it explains why all firms will not be attracted to offer price-matching policies. This is consistent with casual observations that not every industry offers price-matching policies.

Srivastava and Lurie (2001) study the impact of price-matching policies from the perspective of customers and focus on consumer perceptions of store prices. By asking consumers to make a purchase or a continue-to-search decision in different set-ups of 2(price-matching present or absent) X 2(base price high or low), they find that the price perceptions were significantly lower in the presence versus absence of a price-matching guarantee and the mean number of additional stores-to-be-visited is significantly lower in the presence of price-matching policies. This study shows that the price-matching policies can be used by firms to signal low price and thus stop consumers' search.

When they replace the base price with the search cost and repeat the experiment, they find that "subjects searched more stores in the presence of a refund in the low search-cost condition and searched fewer stores in the presence of a refund in the high search-cost condition" (P.305). This result contradicts the prediction of signal theory which predicts consumers search less when the search cost is low because the market disciplinary mechanisms are stronger.

Kukar-Kinney and Walters (2003) contribute to the literature by studying how different characteristics of price-matching guarantees, such as the depth of refund and competitive scope, affect consumer behavior. They specify that the PM characteristics influence consumer patronage intentions through affecting consumer perception of the believability and value of the retailer's PM policy. Using similar procedures as those of Jain and Srivastava (2000), their experiments conclude that the depth of refund, i.e., how aggressive the refunding amount, negatively affect the believability of the PM guarantee but positively affect the perceived value of the PM policy, and therefore, retailers are facing a tradeoff when choosing the depth of refund. While on the other hand, the competitive scope, i.e., "the number of competitors eligible for price-matching by the consumer" (P.154), has a highly positive effect on the perceived value of the PM guarantee, but no effect on the believability.

With only two dimensions of PM characteristics examined, their studies suggest that retailers can build consumer patronage by offering price-matching guarantees rather than price-beating guarantees to optimize the effects both on the perceived believability and the value of the pricing policy and the widening of the competitive scopes.

Anderson and Simester (2003) discuss pricing guarantees as one type of pricing cues retailer use, like sale signs, prices that end in nine and signpost items. They find pricing guarantees reassuring to consumers, which is supported by the study of Jain and

Srivastava (2000); but the trust cannot be justified given mixed evidence from existing empirical studies, especially the one of Hess and Gerstner (1991). In conclusion, observing the tendency to make it harder for consumers to enforce pricing guarantees in some industries, they suggest that retailers should be credible in using cues and thus retain consumers' trust.

1.5 Concluding Remarks

Most of the existing economic literature on price guarantees focuses on their strategic effects: price guarantees offered by oligopolistic firms that, by definition, have the ability to influence their rivals' prices. Another common feature is that perfect information is available to the market. In reality, however, not all industries are concentrated enough to accord the firms such market power; neither do markets exist where participants are able to possess perfect information.

With the intention to fill the gaps mentioned above, we develop three models to study low-price guarantees with the common theme of imperfect information; also, the model in chapter four is studied in a framework where the price set by a retailer has little influence on overall market prices.

We provide the comparison of the effects of matching competition clauses and beating competition clauses in a model where consumers have imperfect information about prices and examine the effects to the introduction of consumers' hassle costs in chapter two. Chapter three explores the role of price guarantees to deter entry in a framework where the entrant has only incomplete information about the incumbent's costs. And lastly, chapter four examines how a firm sets its price and the duration of its price guarantee in a competitive environment, where consumers do not possess complete information about price and thus need to spend time to search for a lower price.

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Chapter 2 Low-Price Guarantees in a Search Model

2.1 Introduction

A typical price guarantee policy advertises to match or beat any lower prices offered by its competitors. In the literature, these provisions are called meeting competition clauses (henceforth MCCs) and beating competition clauses (henceforth BCCs).

Since Hay (1982) first pointed out that a price guarantee policy can be used as a facilitating practice, considerable efforts have been devoted to identify the effects of price guarantees. The majority of theoretical analyses have concluded that MCCs and BCCs facilitate collusion among oligopolistic firms (Holt & Scheffman 1987, Belton 1987) and/or allow price discrimination among consumers with different search costs (Png and Hirshleifer 1987, Corts 1996). Most of the analyses, however, are based on a framework of two firms.

In this paper we study price guarantees in a more general framework where the number of firms is determined by free entry. To do so, we extend Salop and Stiglitz's (1977) consumer search model to incorporate price guarantees. To be more specific, each firm in our model offers both a list price and a price guarantee policy. Consumers have imperfect information about prices and differ in their search costs. Each consumer first decides whether to search in order to obtain full information about prices, and then chooses the firm from which he makes his purchase. Firms face U-shaped average cost curves, and entry into the industry occurs until profits are driven to zero.

Our analysis, which focuses primarily on how different parameters affect the price equilibrium, establishes conditions under which BCCs or MCCs result in collusive prices and under which they result in price dispersion. We also consider the role of hassle cost (Hviid and Sheffer 1999) in determining the effects of MCCs and BCCs. We establish conditions under which the existence of hassle cost mitigates the collusive effect of price guarantees and how the difference in price guarantee policies leads to drastically different equilibria.

This chapter is organized as follows. The model is presented in Section 2.2. The equilibrium prices for the model without hassle cost are given in Section 2.3. Section 2.4 examines the effect of hassle cost to the equilibrium prices in the market. Section 2.5 concludes.

2.2 The Model

2.2.1 Firm Behavior

We adopt the search model of Salop and Stiglitz (1977), and analyze the behavior of firms and consumers in a market for the homogeneous product supplied by n firms, ($n \geq 2$). Every firm has an identical technology characterized by a fixed cost T and variable costs $v(q)$. Assume that the marginal cost is an increasing function of q. Then we have a U-shaped average cost curve. We denote the downward-sloping portion of the average cost curve by $AC = A(q)$.

Each firm has perfect information about the prices charged by other firms. Furthermore, all firms have perfect information about consumers' behavior, including their search cost and search decisions.

Central to our analysis we assume that all firms in the market can choose to adopt a price guarantee policy. The price guarantee policy is characterized by the parameter θ, $\theta \in (0,1]$, where $\theta = 1$ corresponds to a price matching guarantee and $0 < \theta < 1$ corresponds to a price beating guarantee, and a consumer who buys from firm i at price P_i will pay a price θP_j if the consumer can find a lower price P_j $(< P_i)$. To simplify the analysis we assume that the value of θ is exogenously fixed.

It is important to distinguish among the price announced by a firm (the list price), the price actually paid by consumers (the effective price), and the final average price received by the firm (the weighted average price). As will be elaborated below, a consumer can choose to stay uninformed about prices or to incur a search cost and become informed. If a consumer chooses to become informed, he finds out the prices offered by all firms in the market. For a firm that offers a list price P_i and adopts a price guarantee policy[3] θ, the effective price paid by uninformed consumers is P_i; and the effective price paid by informed consumers is either $\theta P_{\min}$ (if $P_i > P_{\min}$) or P_i (if $P_i = P_{\min}$), where $P_{\min} = \min\{P_1, \ldots, P_i, \ldots, P_n\}$. The weighted average price is the average price weighted across both informed and uninformed customers, and this is the actual price the firm receives on average. Of course, if no firm in the market offers any price guarantee, the above three prices are equal.

It is assumed that firms follow Nash price setting behavior. Formally, firm i's pricing problem is,

$$\max_{P_i} \pi^i(P_i \mid P^{-i}), P^{-i} = \{P_1, P_2, \ldots, P_{i-1}, P_{i+1}, \ldots, P_n\}$$

[3] In this model, the low price guarantees, MCCs or BCCs, are based on list price. There also exists another type of LPGs which are based on effective prices, claiming "We will match or beat any price". The latter case leads to a vicious circle of undercutting each others' offers if BCC is adopted, and the effective prices would be zero (Arbaskaya, 2001). I focus exclusively on the first type of LPGs.

In addition, we assume that entry is free. Thus, entry occurs until (expected) profits are driven to zero. This assumption implies that every firm makes zero profit in equilibrium. Thus we will be examining a monopolistically competitive equilibrium in pricing strategies.

2.2.2 Consumer Behavior

On the demand side, there are L consumers. They are risk-neutral[4] and view the products of all firms as homogeneous. They do not have perfect price information about the list prices charged by different firms[5]. Suppose there are n stores selling the product at prices $P = (P_1, P_2, ..., P_n)$. Consumers only know the distribution of prices at all stores but not the particular price at each store. We assume that a consumer needs to incur a search cost, C_i, to become informed about prices. Following Salop and Stiglitz (1977), we assume that the consumer becomes fully informed about all n prices once he pays the search costs. What we have here then is a situation where a brochure exists that publishes all the price information. Consumers can choose to purchase the brochure and analyze all the information in the brochure or not. Consumers differ in their information gathering cost due to differences in their analytic ability, the cost of time, and preferences for reading and processing information. In addition, we assume that an informed consumer has to incur a hassle cost, z, if he buys from a higher priced firm and requests the firm to honor its price guarantee policy.

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[4] Risk aversion is effectively captured in searching cost: consumers with higher degree of risk aversion tend to undertake more search activities to reduce the perceived risk level (Dowling and Staelin, 1994), which accordingly is modeled as lower search cost in this paper.

[5] In this model and other two models of this dissertation, there is incomplete information on only one side of the market, i.e., either consumers do not possess complete information about the prices in the market (the first and the third models, or firms do not possess complete information regarding rivals' cost (the second model). An interesting avenue for future research is to study the situation where both sides of the market face incomplete information.

Consumers can be divided into two groups based on their search costs. A portion, α, of them have lower search cost, C_1; and the remaining, $(1-\alpha)$ of them have higher search cost, C_2. To simplify the analysis, we set $C_1 = 0$, so that type one consumers are always informed, and we assume that C_2 is large enough that type 2 consumers are always uninformed[6].

A consumer makes three decisions. First, he must decide whether to enter the market. Second, he must decide whether to purchase the brochure to obtain the price information or to purchase the product at a randomly selected store. Third, in the case of an informed consumer he needs to decide whether to buy from a firm with a higher list price and activate the price guarantee or to go to the lowest-priced firm directly.

We will first analyze the third decision. If consumer i decides to activate the price guarantee at hassle cost z, he will be able to purchase at a price $\theta P_{\min}$. His total expenditure will be E_a^i, where

$$E_a^i = \theta P_{\min} + z$$

Alternatively, he can purchase at the lowest available price and thus, his total expenditure is E_{na}^i, where

$$E_{na}^i = P_{\min}$$

Since he is risk-neutral, he will buy from a higher-priced firm and activate the price guarantee if and only if

$$E_a^i < E_{na}^i \Leftrightarrow \theta P_{\min} + z < P_{\min}$$

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[6] For those goods that are sold and/or advertised through the Internet, the introduction of Internet search engines can dramatically reduce search costs. However, given the fact that search engines only cover a small subset of Internet and provide incomplete or biased information (Kephart and Greenwald, 1999), consumers may still need to search by traditional ways. At the same time, consumers are heterogeneous in terms of their costs of gathering price information on the Internet (Iyer and Pazgal, 2003). Our assumption of two types of consumers still holds.

Knowing his optimal strategy on whether to activate the price guarantee, he decides whether to search or not. If consumer i buys the brochure, he will at least be able to purchase at the lowest available price. His total expenditure will be E_s^i, where

$$E_s^i = P_{\min} + C_i$$

conditional on that he buys from the firm offering the lowest price, or

$$E_s^i = \theta P_{\min} + C_i + z$$

conditional on that he buys from the firm offering a higher price and activates the price guarantee. Combining the above, his total expenditure from the search will be

$$E_s^i = \min\{\theta P_{\min} + C_i + z, P_{\min} + C_i\}$$

Alternatively, he can purchase at a randomly selected store and on average pay a price equal to the mean price, $\bar{P}$. Thus, the total expected expenditure from the no-search strategy is $E_N^i = \bar{P} = (1/n)\sum_{i=1}^{n} P_i$.

The consumer will search if and only if

$$E_s^i < E_N^i \Leftrightarrow \min\left\{\theta P_{\min} + C_i + z, P_{\min} + C_i\right\} < \bar{P}$$

Given the optimal search strategy, the consumer will enter the market if his total cost does not exceed his reservation price. The reservation price for all consumers is the monopoly price, P^m. Then the consumer will enter the market if

$$P^m \geq \min\left\{\theta P_{\min} + z + C_i, P_{\min} + C_i, \bar{P}\right\}$$

2.2.3 Equilibria Obtained in the Case of no Price Guarantee

Without price guarantees, type 1 consumers will go directly to the firm with the lowest price and type 2 consumers will shop randomly when $C_1 = 0$ and C_2 is big enough. This is the case studied in Salop and Stiglitz (1977). For the purpose of comparison we reproduce their results in the following lemmas.

Lemma 1 *There are no Three-, Four-,..., -price equilibria. Only Single-Priced Equilibria (SPE) and Two-Price Equilibria (TPE) are possible.*

Proof: Suppose there exists three different prices, P_l, P_m and P_h ($P_l < P_m < P_h$) in equilibrium. At these prices, type one consumers buy from the firm with the lowest price P_l. Type two consumers select firms randomly; thus each firm obtains an equal share of type two consumers' business. Since the firms that charge P_m and P_h sell only to uninformed consumers, their sales are identical. Thus, the firms charging P_h must obtain higher revenue than the firms charging P_m , given $P_m < P_h$, and thus the zero profit condition is broken.

QED

Lemma 2 *There is no single-price equilibrium at any price $\hat{P}$ in the interval $(P^*, P^m]$ where P^* is the competitive price and P^m is the monopoly price.*

Proof: The informed consumers have complete information about the prices and thus they would strictly prefer to buy from the firm with the lowest price. Starting from any price greater than P^*, a firm can profitably deviate by cutting price a little bit and obtaining all the sales to the informed consumers.

QED

Lemma 3 *A single-price equilibrium at competitive price P^* exists if and only if $A[(1-\alpha)\frac{L}{n}] > P^m$.*

Proof: Consider the potential SPE at P^*. First, note that it is not profitable for a firm to reduce its price to below P^* because in a free entry equilibrium $P^* = \min AC$.

Next, we analyze a firm's incentive to raise its price above P^*. A firm that raises its price will lose all of its type 1 consumers, but none of type 2 consumers, as long as $P_d = P^* + \varepsilon \leq P^m$. Such a deviant will be earning higher revenue on fewer sales, which will be $q_d = (1-\alpha)\frac{L}{n}$. In general its profits may rise or fall. This strategy will be profitable if $A(q_d)$, the average cost of producing q_d, is less than the price P_d. Thus, the SPE at P^* prevails if and only if

$$A[(1-\alpha)\frac{L}{n}] > P^m.$$

It is obvious that when $A[(1-\alpha)\frac{L}{n}] < P^m$, the deviant can always earn positive profit by charging monopoly price.

QED

Lemma 4 *There exists a two-price equilibrium with the property that a proportion of firms,* β*, charges a low price* P_l*; and the rest, charges a high price* P_h*, where* $P_h = P^m, P_l = P^*$ *and* β *is defined by* $P^* = A[(1-\alpha+\frac{\alpha}{\beta})\frac{L}{n}]$.

Proof: The lemma describes a two-price equilibrium (TPE) with the property that there exist a number of firms, n, of which a proportion, β, charge a low price P_l; and the rest, a proportion, $1-\beta$, charge a high price P_h. In such a market, the informed consumers will go to the low price firms, while the uninformed consumers distribute evenly between the two types of firms. This proof has two parts.

First, we prove that there is no TPE in which the prices are different from $P_h = P^m$ and $P_l = P^*$. To see this, suppose $P_l > P^*$. Then a firm can profitably deviate by cutting price slightly and obtaining all the sales to the informed consumers. With regard to the high price, suppose $P_h < P^m$. A deviant can raise his price to the monopoly price without losing sales, because high price firms only supply uninformed consumers, and thus increase his profit. Therefore, neither $P_l > P^*$ nor $P_h < P^m$ can be part of a TPE.

Second, we show that the zero profit conditions are satisfied. Given the high price firms sell only to uninformed consumers; the sales per firm are given by $q_h = (1-\alpha)\frac{L}{n}$ and $q_l = (1-\alpha+\frac{\alpha}{\beta})\frac{L}{n}$, respectively. In equilibrium firms offering these prices must earn zero profit. Therefore,

$$P_h = P^m = A(q_h)$$

$$P_l = P^* = A(q_l)$$

The first zero profit condition determines the equilibrium number of firms n. This, along with the second zero profit condition can be used to solve the equilibrium value of β, which is given by $P^* = A[(1-\alpha+\frac{\alpha}{\beta})\frac{L}{n}]$.

2.3 Zero Hassle Cost

Now we turn to the case where firms offer price guarantees. In this section, we consider the case where the hassle cost of activating price guarantees is zero, i.e., $z = 0$. The case of positive hassle costs is analyzed in section 2.4.

In this analysis we consider both types of price guarantees, namely the beating competition clauses (BCCs) and the matching competition clauses (MCCs). As their names suggest a BCC promises its customers that the firm will beat its rivals' lower list price and a MCC promises to match any lower list price. Using our notation, $\theta < 1$ in the case of BCC and $\theta = 1$ in the case of MCC. In what follows we study each of these two types of price guarantees in sequence.

2.3.1 All firms in the Market commit to BCCs

Proposition 1 *In a SPE, all firms set list price at any $\hat{P}$ in the open interval (P^*, P^m) if $\theta \in (0, \theta^0)$, where $\theta^0 = \frac{P^*}{\hat{P}} - \frac{1-\alpha}{\alpha n}\frac{P^m - P^*}{\hat{P}}$. The interval $(0, \theta^0)$ exists if and only if $\alpha > \frac{P^m - P^*}{P^m + (n-1)P^*}$.*

Proof: For a $\hat{P}$ in the open interval (P^*, P^m) to be part of an equilibrium, each firm should have no incentives to either raise or reduce its price starting from $\hat{P}$. A firm has no incentive to reduce its price, because of the price guarantees and zero hassle cost, such a price reduction will not attract any additional customers but lose all informed consumers to other firms, and thus it reduces revenue because of the lower price and the fewer sales.

Now suppose a firm raises its price slightly to $\hat{P} + \varepsilon$. This deviation raises the mean price to

$$\bar{P} = \frac{(n-1)\hat{P} + (\hat{P} + \varepsilon)}{n} = \hat{P} + \frac{\varepsilon}{n}$$

and lowers the lowest effective price to $P_{\min} = \theta\hat{P}$. The benefit of search becomes

$$b = \bar{P} - \theta\hat{P} = \hat{P} + \varepsilon/n - \theta\hat{P} = (1-\theta)\hat{P} + \varepsilon/n$$

If $\varepsilon \le P^m - P^*$, the deviating firm still keep its sales to type 2 consumers, and the price change will attract all type 1 consumers, thus the deviant's sales will jump from $\frac{L}{n}$ to

$$q_d = (1-\alpha)\frac{L}{n} + \alpha L,$$

The actual weighted average price received by the firm becomes

$$P_d = \frac{(1-\alpha)(\hat{P}+\varepsilon)/n + \alpha\theta\hat{P}}{\frac{1-\alpha}{n}+\alpha}$$

Such deviation is profitable if and only if

$$P_d \geq P^* \tag{2.1}$$

Combine (2.1) with the condition that $\varepsilon \leq P^m - P^*$, we have

$$P^m - \hat{P} \geq \varepsilon \geq \frac{\alpha n}{1-\alpha}(P^* - \theta\hat{P}) - (\hat{P} - P^*) \tag{2.2}$$

The deviating firm can never find a satisfactory ε to make the deviation profitable if the given θ is so small that

$$P^m - \hat{P} < \frac{\alpha n}{1-\alpha}(P^* - \theta\hat{P}) - (\hat{P} - P^*) \tag{2.3}$$

From it, we obtain

$$\theta < \frac{P^*}{\hat{P}} - \frac{1-\alpha}{\alpha n}\frac{P^m - P^*}{\hat{P}} = \theta^0 \tag{2.4}$$

Thus, for the given $\theta \in (0, \theta^0)$, $\hat{P}$ is a SPE.

Next, consider the possibility that $\varepsilon > P^m - \hat{P}$. In this case the deviating firm will lose its share of the $(1-\alpha)L$ uninformed consumers. The deviation is a profitable strategy if and only if $\theta\hat{P} \geq P^*$, which implies that $\hat{P}$ is SPE if $\theta < P^*/\hat{P}$. However, from (2.4) we can see that $P^*/\hat{P}$ is greater than θ^0. Therefore, here the condition for $\hat{P}$ to be a SPE is $\theta < \theta^0$.

Finally, we need to ensure that the critical value θ^0 lies between 0 and 1. From (2.4), we observe that $\theta^0 < 1$, since the first term, $\frac{P^*}{\hat{P}}$, is less than 1, and the second term is positive. We have $\theta^0 > 0$ if $\frac{1-\alpha}{\alpha n} < \frac{P^*}{P^m - P^*}$, which implies $\alpha > \frac{P^m - P^*}{P^m + (n-1)P^*}$. If, on the other hand, $\alpha \leq \frac{P^m - P^*}{P^m + (n-1)P^*}$, we have $\theta^0 \leq 0$ and the price $\hat{P}$ cannot be a SPE.

QED

Proposition 2 *Suppose $\theta \in (0, \tilde{\theta})$, where $\tilde{\theta} = 1 - \frac{1-\alpha}{\alpha n P^*}(P^m - P^*)$. There exists a SPE where all firms set list price at the competitive price P^*. The interval $(0, \tilde{\theta})$ exists if and only if $\alpha > \frac{P^m - P^*}{P^m + (n-1)P^*}$.*

Proof: Since $P^* = \min AC$, a price decrease from P^* is never profitable. On the other hand, if a firm raises its price by ε, the deviation will induce type 1 consumers to search and the firm will sell to all the informed consumers at price θP^*. Its sales to type 2 consumers remain unchanged at the higher price $P^* + \varepsilon$ provided that

$$P^* + \varepsilon \leq P^m \tag{2.5}$$

This deviation raises the mean price to

$$\overline{P} = \frac{(n-1)P^* + (P^* + \varepsilon)}{n} = P^* + \frac{\varepsilon}{n}$$

and lowers the lowest effective price to $P_{\min} = \theta P^*$. The benefit of search becomes

$$b = \overline{P} - \theta P^* = \frac{(n-1)P^* + (P^* + \varepsilon)}{n} - \theta P^* = (1-\theta)P^* + \varepsilon/n$$

The number of units sold by the deviating firm becomes

$$q_d = (1-\alpha)\frac{L}{n} + \alpha L$$

The actual weighted average price of this firm becomes

$$P_d = \frac{(1-\alpha)(P^* + \varepsilon)/n + \alpha\theta P^*}{\frac{1-\alpha}{n} + \alpha}$$

This deviation is profitable if and only if $P_d \geq P^*$. Combining this condition with (2.5), we have

$$P^m - P^* \geq \varepsilon > \frac{\alpha n(1-\theta)P^*}{1-\alpha}$$

Thus, to make the deviation profitable we need

$$P^m - P^* > \frac{\alpha n(1-\theta)P^*}{1-\alpha}$$

This implies

$$\theta > 1 - \frac{1-\alpha}{\alpha n P^*}(P^m - P^*) = \tilde{\theta}$$

Therefore, SPE at the competitive price can prevail if θ satisfies $\theta \in (0, \tilde{\theta})$. To guarantee the existence of $\tilde{\theta}$, we need $\alpha > \frac{P^m - P^*}{P^m + (n-1)P^*}$.

QED

It is interesting to note that in both Proposition 1 and Proposition 2 the value of α has to be large enough. Intuitively, the search done by the informed consumers helps keep equilibrium prices low. But their number has to be large enough in order to make it effective and keep price below the monopoly price.

Comparing $\tilde{\theta}$ with θ^0 obtained in propositions 1 and 2, we have

$$\theta^0 = \frac{P^*}{\hat{P}} - \frac{1-\alpha}{\alpha n}\frac{P^m - P^*}{\hat{P}} = \frac{P^*}{\hat{P}}[1 - \frac{1-\alpha}{\alpha n P^*}(P^m - P^*)] = \frac{P^*}{\hat{P}}\tilde{\theta} < \tilde{\theta}$$

The last step comes from that $P^* < \hat{P}$. Therefore, competitive price, P^*, can prevail as a SPE for a wider range of θ than any $\hat{P}$ in the open interval (P^*, P^m) can. Note that when $\alpha \to 1, \hat{\theta} \to 1$ while $\theta^0 \to \frac{P^*}{\hat{P}}$. This suggests that as the proportion of informed consumers approaches 1, SPE at P^* can be obtained for any θ but this is not the case for $\hat{P}$.

Proposition 3 *The monopoly price P^m is a SPE when $\theta \in (0, P^*/P^m)$.*

Proof: Starting from P^m, no firm has an incentive to decrease price, as it will be beaten by other firms and thus only leads to a loss in market share. Suppose instead that a firm raises its price by ε. This raises the average price while lowering the minimum effective price. Since $\bar{P} = \left((n-1)P^m + P^m + \varepsilon\right)/n$, and $P_{\min} = \theta P^m$, the benefit of search rises to

$$\begin{aligned} b = \bar{P} - P_{\min} &= \frac{(n-1)P^m + P^m + \varepsilon}{n} - \theta P^m \\ &= (1-\theta)P^m + \varepsilon / n \end{aligned}.$$

Thus, the firm will attract all type 1 consumers and sell the good at price θP^m. Recall that the number of informed consumer is αL. For large enough L, we have $\alpha L > q^*$. For such a large L, the deviation will be a profitable strategy if and only if

$\theta P^m \geq P^*$, i.e., $\theta \geq P^*/P^m$. In other words, the monopoly price will be a SPE if $\theta < P^*/P^m$.

QED

In addition to the SPEs studied above, there are also Two-Price Equilibriums in this case.

Proposition 4. *Suppose* $\theta > 1 - \frac{(1-\alpha)(P^m - P_l)}{\alpha P_l}$ *for some* $P_l \in [P^*, P^m)$. *There exists a Two-Price equilibrium in which a fraction of firms,* β, *charges a low price* P_l, *and the remaining firms charge a high price* $P_h = P^m$, *where* $\beta = 1 - \frac{(1-\theta)\alpha P_l}{(1-\alpha)(P^m - P_l)}$.

Proof: For the two prices to co-exist in an equilibrium it has to be the case that the firms offering these two prices earn the same expected profits. This implies that the weighted average price received by the high price firms is equal to that received by the low price firms, i.e.,

$$P_d = \frac{P^m(1-\alpha)L/n + \theta P_l \frac{\alpha L}{(1-\beta)n}}{\frac{\alpha L}{(1-\beta)n} + \frac{(1-\alpha)L}{n}} = P_l$$

This implies that the equilibrium β has to satisfy

$$\beta = 1 - \frac{(1-\theta)\alpha P_l}{(1-\alpha)(P^m - P_l)}.$$

Since β is the proportion of firms that charge a low price, β should lie between 0 and 1, i.e.,

$$\theta > 1 - \frac{(1-\alpha)(P^m - P_l)}{\alpha P_l} \tag{2.6}$$

The two-price equilibrium has the property that $P_h = P^m$, as it is to the high price firms' interest to extract the most profit from the uninformed consumers.

The lower price can be any price between the monopoly price and the competitive price which satisfies (2.6), which implies there is no incentive of deviation to increase price. There is no local price reduction, since the price reduction will only lead to loss of profit but no gain of sales.

QED

Proposition 5 *There are no Three-, Four-... -Price Equilibria.*

Proof: Consider a situation where three list prices co-exist in a market, P_l, P_m and P_h. With zero hassle cost, type one consumers will search and obtain full price information. These consumers will buy from higher priced firms and activate price guarantees, thus paying an effective price of θP_l. Every firm gets an equal share of type two consumers. Since the firms that charge P_m and P_h charge the same effective price θP_l to informed consumers but offer P_m and P_h respectively to uninformed consumers, firms with list price P_h obtain higher revenue than firms with P_m, thus violating the zero profit condition.

The same logic can apply to the multi-price equilibriums.

QED

Comparing Propositions 1 to 4 with Lemmas 2 to 4, we make the following two observations. First, the adoption of BCCs tends to push up the prices in an SPE. While the competitive price P^* is still a possible equilibrium, the adoption of BCCs allows higher prices to sustain as an equilibrium as well. Furthermore, the proportion of informed consumers has to be large enough in order for a lower price ($P < P^m$) to prevail as an equilibrium. Second, in the case of TPE the adoption of BCCs pushes up the lower of the two prices (i.e., P_l is higher) but at the same time induces more firms to set the lower price (i.e., β is larger). The relative larger value of β can be seen from that β is determined by the condition $P_l = A[(1-\alpha+\frac{\alpha}{\beta})\frac{L}{n}]$. Since equilibrium occurs at the downward sloping portion of the average cost curve and we have $P_l \geq P^*$, we obtain $(1-\alpha+\frac{\alpha}{\beta^*})\frac{L}{n} \geq (1-\alpha+\frac{\alpha}{\beta_l})\frac{L}{n}$, where β^* indicates the proportion of firms who charge lower price when the equilibrium prices are P^* and P^m; and β_l , when the equilibrium prices are P_l and P^m, therefore, $\beta^* \leq \beta_l$.

2.3.2 All firms commit to MCCs

We consider the case where all firms offer MCC. In this case a firm has no incentive to cut price as other firms will match it and consequently the price reduction will only lead to the same sales at a lower price level, thus lower profit.

Proposition 6 *There are no Two-Price or Multi-Price Equilibria.*

Proof: Consider a situation where some firms offer P_l while other firms offer P_h. With MCC adopted by all firms, type 1 consumers face the same effective price everywhere they buy. So they distribute themselves evenly among the firms. As usual, the uninformed consumers are distributed evenly across firms. Every firm obtains an equal share of customers, L/n. Thus, the P_h firms must obtain higher revenue, breaking the zero profit condition. The same logic applies to multi-price equilibria.

QED

Proposition *7 The only SPE is such that all firms set list price at monopoly price* P^m.

Proof: Consider a potential SPE at $\hat{P} \in [P^*, P^m)$. With MCC in effect, a firm can raise its list price by ε, where $\hat{P} + \varepsilon \le P^m$, without loss of sales. The informed consumers can activate the price guarantee and still purchase at the minimum price $\hat{P}$, while the informed consumers who randomly choose this firm would have to pay a higher price, $\hat{P} + \varepsilon$. Thus, there is an incentive to deviate from $\hat{P} \in [P^*, P^m)$.

At the monopoly price, no firm wants to raise its price. The increase in price makes the firm lose all type 2 consumers and still offer the monopoly price to informed type 1 consumers. Thus it is not a profitable deviation. No firm wants to cut its price, either. The price cut cannot attract more consumers for the deviant and thus only leads to lower profits.

QED

A comparison of Propositions 6 and 7 with Lemmas 3 and 4 shows that in the absence of hassle costs, MCCs can indeed facilitate collusion. Moreover, this facilitating effect is much stronger than what resulted from the adoption of BCCs. While the adoption of BCCs can sometimes lead to higher prices, the adoption of MCCs always leads to the monopoly price.

It is also interesting to note that there is no price dispersion in equilibrium when a MCC is adopted. This is in sharp contrast to Png and Hirshleifer (1987) who show that two different prices prevail as a consequence of price discrimination by firms.

2.3.3 Extension: Endogenizing the Decision to Adopt Price Guarantees

The analysis so far takes as given the assumption that all firms commit to a BCC (or MCC) policy. In this section we endogenize firms' decisions with regard to the adoption of price guarantee policy and show that the results we have obtained still hold with only minor modifications.

Consider the following extension of the model. Suppose that at the beginning of the game firms simultaneously choose a price guarantee policy and a list price. The rest of the game is the same as before. We focus on symmetric equilibria where all firms take the same action with regard to price guarantee policy.

First, consider BCCs. We assume that the parameter θ (<1) is fixed. A firm's choice is between offering no price guarantee or offering a BCC with parameter θ.

Proposition 8 *There exists an equilibrium where all firms commit to BCCs and charge price* $\hat{P}$ *in the open interval* (P^*, P^m) *iff* $A[(1-\alpha)\frac{L}{n}] > P^m$ *and all conditions in Proposition 1 are satisfied.*

Proof: For all firms to choose a price guarantee at $\hat{P}$ to be part of an equilibrium, each firm should have no incentive to deviate from offering BCCs, coupled with, or without, deviation from price $\hat{P}$.

Suppose that a firm deviates by not offering the price guarantee. The firm can choose not to change its list price, in which case its profits would remain the same. Alternatively, the firm can raise its price, but it would lose all informed consumers by doing so. Such a deviation would cause a loss for the firm given that $A[(1-\alpha)\frac{L}{n}] > P^m$. Neither does it have any incentive to cut its price, since the lower price will be beaten by other firms who offer BCCs, which only leads to a decrease of profit because of the loss of informed consumers and the lower profit from uninformed consumers. Therefore, no firm will deviate.

QED

Proposition 9 *There exists an equilibrium where all firms commit to BCCs and charge the competitive price* P^* *iff* $A[(1-\alpha)\frac{L}{n}] > P^m$ *and all conditions in Proposition 2 are satisfied.*

Proof: We can use the same logic as in the proof of Proposition 8 to show that no firm will deviate.

QED

Proposition 10 *Suppose* $\theta \in (0, P^*/P^m)$. *That all firms commit to BCCs and charge the monopoly price* P^m *is a SPE.*

Proof: Starting from the situation where all firms choose to offer price guarantee at monopoly price P^m, no firm has incentive to deviate by not offering BCCs. If this deviation is coupled with a price cut, its lower list price will be beaten by other firms and thus it will lose all informed consumers. On the other hand, it will lose all customers if it raises its price above P^m.

QED

Proposition 11 *There exists a Two-Price equilibrium in which all firms commit to BCCs, and among them, a fraction,* $\beta = 1 - \frac{(1-\theta)\alpha P_l}{(1-\alpha)(P^m - P_l)}$, *charges a low price*

$P_l \in [P^*, P^m)$, and the remaining firms charge a high price $P_h = P^m$ if $A[(1-\alpha)\frac{L}{n}] > P^m$ and all conditions in Proposition 4 are satisfied.

Proof: Starting from the TPE described in Proposition 4, it is to all firms' interest to stay with the choice of offering BCCs. A high price firm has no incentive to deviate because cutting prices in the absence of a price guarantee will cause it to lose all informed customers and raising price above P^m in the absence of a price guarantee drives away all customers. None of the low price firm wants to deviate, either. A firm that raises its price without a BCC cannot increase its profits because $A[(1-\alpha)\frac{L}{n}] > P^m$. With other firms' BCCs in effect, a firm that cuts its price will not attract informed consumers and thus his profit will be lower.

QED

Next, consider MCC. We examine if all firms are willing to offer MCCs given that they can choose not to do so.

Proposition 12 *There exists an equilibrium in which all firms choose to offer MCCs and set list price at monopoly price P^m.*

Proof: At monopoly price P^m, no firm has an incentive to deviate by stop offering matching price guarantee policy. With other firms' MCCs in effect, a deviant cannot attract more consumers by a price cut; and lose all market by price increase.

QED

2.4 Positive Hassle Cost

Up until this point, it has been assumed that firms automatically beat or match any lower price to informed consumers. In this section, we examine the case where informed consumers need to incur a positive hassle cost to activate the price guarantees. A consumer's hassle cost can be quantified as her opportunity cost of time of waiting, the cost of having to bring the rival's advertisement to the store and the disutility in having to confront a salesperson (Hviid and Shaffer, 1999). We will start with an analysis under the assumption that all firms commit to BCCs. Then we will examine firms' decision to adopt BCCs. Finally we will analyze MCCs.

2.4.1 All firms commit to BCCs

Proposition 13 *Any list price* $\hat{P}$ *in the open interval* (P^*, P^m) *can be part of a SPE if* $\theta \in (0, \theta^0)$ *and* $z < \hat{z}$ *, where* $\theta^0 = \frac{P^*}{\hat{P}} - \frac{1-\alpha}{\alpha n}\frac{P^m - P^*}{\hat{P}}$ *and* $\hat{z} = (\hat{P} - P^*) + \frac{1-\alpha}{\alpha n}(P^m - P^*)$. *The interval of* θ *exists if and only if* $\alpha > \frac{P^m - P^*}{P^m + (n-1)P^*}$.

Proof: First, we consider whether a firm wants to deviate by raising its list price above $\hat{P}$. A price increase to $\hat{P} + \varepsilon$ is a profitable deviation if and only if both of the following two conditions are satisfied.

a. The price increase will induce the type one consumers to activate the BCC price guarantee. i.e.

$$g = (1-\theta)\hat{P} \geq z \tag{2.7}$$

b. The firm's weighted average price associated with this deviation is at least as high as the competitive price. In succeeding to attract type 1 consumers to activate the BCC and also keep the uninformed customers, the deviant's sales will jump up to $Q_d = \alpha L + (1-\alpha)L/n$ and the weighted average price will be

$$P_d = \frac{\alpha\theta \hat{P} + {(1-\alpha)}/{n}\,(\hat{P}+\varepsilon)}{\alpha + \frac{1-\alpha}{n}}$$

Thus, the deviation is profitable if

$$P_d \geq P^* \tag{2.8}$$

And the condition to keep the uninformed consumers is

$$\varepsilon \leq P^m - \hat{P}$$

Combining it with (2.8), we have

$$P^m - \hat{P} \geq \varepsilon \geq \frac{\alpha n}{1-\alpha}(P^* - \theta\hat{P}) - (\hat{P} - P^*)$$

For ε to exist, we need

$$P^m - \hat{P} \geq \frac{\alpha n}{1-\alpha}(P^* - \theta\hat{P}) - (\hat{P} - P^*)$$

Thus, for the price increase to be profitable, θ must satisfy

$$\theta > \frac{P^*}{\hat{P}} - \frac{1-\alpha}{\alpha n}\frac{P^m - P^*}{\hat{P}} = \theta^0 \tag{2.9}$$

Inserting (2.9) into (2.7), we have

$$z < (1-\theta^0)\hat{P} = (\hat{P} - P^*) + \frac{1-\alpha}{\alpha n}(P^m - P^*) = \hat{z} \tag{2.10}$$

Therefore, firms will not deviate by raising prices if either (2.7) or (2.9) is not satisfied, i.e. if $z > g = (1-\theta)\hat{P}$, or when the given θ lies in the range of $\theta \in (0, \theta^0)$.

Second, consider whether a firm wants to deviate by reducing its list price. In the case that $z > g = (1-\theta)\hat{P}$ for a given θ, the price reduction is profitable. When the hassle cost is high, type one consumers have no incentive to incur the hassle cost to activate the higher-price-firms' beating price clause, therefore the deviating firm can steal all the informed consumers by a small price reduction. Conversely, the firm will not want to reduce its list price if $z < g$.

Therefore, there is a SPE at $\hat{P}$ if both $z < g = (1-\theta)\hat{P}$ and $\theta < \theta^0$ are satisfied. Combining them, we have the sufficient condition of hassle cost to obtain the SPE, i.e., $z < (1-\theta^0)\hat{P} = \hat{z}$ and $\theta < \theta^0$.

QED

Proposition 14 *The competitive price P^* can be part of a SPE if either of the following two conditions is satisfied:*

(1) $A[(1-\alpha)\frac{L}{n}] > P^m$ and $z > (1-\theta)P^*$, for the given θ ; or

(2) $z \le (1-\theta)P^*$ with the condition that $\theta < \tilde{\theta}$ where $\tilde{\theta} = 1 - \frac{1-\alpha}{\alpha n P^*}(P^m - P^*)$ and the interval $(0, \tilde{\theta})$ exists if and only if $\alpha > \frac{P^m - P^*}{P^m + (n-1)P^*}$.

Proof: First, consider the case $z > (1-\theta)P^*$. In this case no consumer will activate the price guarantee. As a result, a firm that raises its price will not attract any additional type one customers. Thus, there is no incentive for a firm to raise its price above P^* given that $A[(1-\alpha)\frac{L}{n}] > P^m$ is satisfied. On the other hand, price reduction is not profitable because $P^* = \min AC$. Therefore, P^* can be sustained as a SPE.

Second, consider the case $z \le (1-\theta)P^*$, in which case an informed consumer will activate the BCC if he buys from a firm offering a price higher than P^*. Then a firm may find it profitable to raise its list price by ε in order to attract all the informed consumers. This will indeed be the case if

$$P_d = \frac{(1-\alpha)(P^*+\varepsilon)/n + \alpha\theta P^*}{\frac{1-\alpha}{n}+\alpha} \ge P^*$$

This implies

$$\theta > 1 - \frac{1-\alpha}{\alpha n P^*}(P^m - P^*) = \tilde{\theta}$$

Therefore, in this case the competitive price can be sustained as a SPE if $\theta < \tilde{\theta}$.

QED

Proposition 15 *The monopoly price P^m is part of a SPE if both $\theta \in (0, \frac{P^*}{P^m})$ and $z < (1-\theta)P^m$ are satisfied.*

Proof: Given that $z < (1-\theta)P^m$, informed consumers will buy from a firm offering a price higher than P^m and activate the BCC. Using the same arguments as those in the proof of Proposition 3, we can show that a firm has no incentive to raise its list price above P^m. On the other hand, if the firm reduces its list price to P', it will lose all informed customers to other firms because of the latter's BCC policy. The informed consumers will activate the price guarantee at these other firms because the hassle cost z is less than the gain from lower effective price, which is equal to $P^m - \theta P'$.

QED

Proposition 16 *If* $z > (1-\theta)P^m$, *a TPE exists with the property that* $P_h = P^m$ *and* $P_l = P^*$. *If* $z \le (1-\theta)P^m$, *a TPE exits with the property that* $P_h = P^m$, $P_l \in [P^*, P^m)$ *and* $\beta = 1 - \frac{(1-\theta)\alpha P_l}{(1-\alpha)(P^m - P_l)}$.

Proof: In the case $z \le (1-\theta)P^m$, the hassle cost is low enough that consumers will activate the price guarantee when they are from a firm offering a list price P^m. Then the analysis in the proof of Proposition 4 applies. The TPE exists only if the firms in the market are indifferent between P_h and P_l. We restate the conditions as following,

$$P_h = P^m,\ P_l \in [P^*, P^m),\ \beta = 1 - \frac{(1-\theta)\alpha P_l}{(1-\alpha)(P^m - P_l)} \text{ where } \frac{\alpha}{(1-\alpha)}\frac{P_l}{P^m - P_l} < \frac{1}{1-\theta}.$$

The condition $z > (1-\theta)P^m$ implies that $z > P_l(1-\theta)$. In this case the hassle cost is high enough that the beating price guarantee is not going to be activated. Then the price guarantee has no effect on equilibrium; we are effectively in the same situation as the one studied in Salop and Stiglitz (1977). As the proof of Lemma 4 shows, the TPE has the property that: $P_h = P^m$ and $P_l = P^*$.

QED

The above results show that if the hassle cost is too high, BCCs become ineffective as no consumers will want to activate them. Consequently the equilibria are the same as in the case where no BCC is offered.

The more interesting case is the one where the hassle cost is small but not trivial. Here the presence of a hassle cost does not dampen the facilitating effects of BCCs. This is in contrast to Hviid and Shaffer (1999) where an arbitrarily small hassle cost can render BCCs ineffective in facilitating collusion. In our model, the additional gains from the activation of a BCC keep it still effective when the hassle cost is not too high.

2.4.2 Endogenization of firms' decision to adopt BCCs

Here we consider the same extension as the one in section 2.3.3. Suppose that firms choose both prices and whether to offer BCCs. As will be shown below, it can indeed be an equilibrium that all firms adopt BCCs.

Note from the discussion in section 2.4.1 that no consumers will activate BCCs if the hassle cost is too high. In such a case, the endogenization of the choice to offer BCCs is trivial. Since the BCC has no real effect and the adoption of BCC does not cost the firms anything, that all firms adopt BCCs can be an equilibrium.[7]

The more interesting case is when the hassle cost is relatively small so that consumers will activate BCC when they find a lower price. This is the case our analysis will focus on.

Proposition 17 *There exists an equilibrium where all firms commit to BCCs and list price* $\hat{P}$ *in the open interval* (P^*, P^m) *if* $z < \hat{z}$ *, where* $\hat{z} = (\hat{P} - P^*) + \frac{1-\alpha}{\alpha n}(P^m - P^*)$, $A[(1-\alpha)\frac{L}{n}] > P^m$ *and all conditions in Proposition 13 are satisfied.*

Proof: For all firms to choose a beating price guarantee at $\hat{P}$ to be part of an equilibrium, each firm should have no incentive to deviate from offering BCCs, coupled with, or without, deviation from price $\hat{P}$.

[7] In this case, that no firm adopts BCC can also be an equilibrium.

When the hassle cost is less than the value of $\hat{z}$, given by $\hat{z} = (\hat{P} - P^*) + \frac{1-\alpha}{\alpha n}(P^m - P^*)$, our proof of proposition 13 shows that informed consumers activate price guarantees. Suppose that a firm deviates by not offering the price guarantee. The firm can choose not to change its list price, in which case its profits would remain the same. Alternatively, the firm can raise its price, but it would lose all informed consumers by doing so. Such a deviation would cause a loss for the firm given that $A[(1-\alpha)\frac{L}{n}] > P^m$ is satisfied. Neither does it have any incentive to cut its price, since the lower price will be beaten by other firms who offer BCCs, which only leads to a decrease of profit because of the loss of informed consumers and the lower profit from uninformed consumers. Therefore, no firm will deviate.

QED

Proposition 18 *There exists an equilibrium where all firms commit to BCCs and charge the competitive price* P^* *if* $z \leq (1-\theta)P^*$, $A[(1-\alpha)\frac{L}{n}] > P^m$ *and all conditions in Proposition 14(2) are satisfied..*

Proof: The same logic in the proof of proposition 17 applies to show that no firm will deviate.

QED

Proposition 19 *Suppose* $\theta \in (0, \frac{P^*}{P^m})$. *There exists an equilibrium where all firms offer BCCs at the monopoly price* P^m *if* $z < (1-\theta)P^m$ *is satisfied.*

Proof: As shown in the proof of proposition 15, when $z < (1-\theta)P^m$, consumers are going to activate the price guarantee under this circumstance. At monopoly price

P^m, no firm has an incentive to deviate by not offering BCCs: it will lose all informed consumers if this deviation is coupled with price cut; or lose all market if the price is raised above P^m.

QED

Proposition 20 *There exists a Two-Price equilibrium in which all firms commit to BCCs, and among them, a fraction,* $\beta = 1 - \frac{(1-\theta)\alpha P_l}{(1-\alpha)(P^m - P_l)}$ *, charges a low price* $P_l \in [P^*, P^m)$ *, and the remaining firms charge a high price* $P_h = P^m$ *if* $z \le (1-\theta)P^m, \theta > 1 - \frac{(1-\alpha)(P^m - P_l)}{\alpha P_l}$ *and* $A[(1-\alpha)\frac{L}{n}] > P^m$ *are satisfied.*

Proof: With $z \le (1-\theta)P^m$, a high price firm sells to uninformed consumers at price P^m, and also sells to informed consumers at price θP^m; a low price firm serves only uninformed consumers. Assume one high price firm deviates by not offering price guarantees, it will lose all informed consumers and further raising price above P^m in the absence of a price guarantee simply drives away all consumers. Assume one low price firm stops offering price guarantee, a price cut at the same time cannot attract more customers and a price increase cannot improve its profits because $A[(1-\alpha)\frac{L}{n}] > P^m$.

QED

2.4.3 MCCs and hassle cost

When firms commit to meet competitors' lower prices, the gain for a consumer from activating the price guarantee is only the difference between the two list prices. The consumer can avoid the hassle cost by purchasing from the firm offering the lower list price directly. As a result, no consumer will activate the price guarantee and thus the situation is the same as the one without price guarantee. Therefore, the equilibrium obtained is the same as stated in Lemma 1 to 4.

Proposition 21 *A SPE at competitive price* P^* *exists if and only if* $A[(1-\alpha)\frac{L}{n}] > P^m$. *There is no SPE at any other list prices.*

Proof: Follows from the Lemmas 2 and 3 and the observation that no consumer will activate the MCC.

QED

Proposition 22 *A TPE exists with the property* $P_h = P^m$ *and* $P_l = P^*$.

Proof: Follows from Lemma 4 and the observation no consumer will activate the MCC.

QED

In the case of matching price guarantee, the mere existence of a hassle cost renders price matching guarantees ineffective. This confirms Hviid and Saffer's (1999) finding.

The endogenization of firms' choice to adopt MCCs in this case is also trivial, since MCCs have no real effect. A firm can either adopt or not adopt MCCs, and that decision does not change the outcome of the game in any way.

2.5 Conclusions

In this paper, we have examined the equilibria in a search model where consumers differ in search costs and firms offer MCCs or BCCs. This framework has allowed us to compare MCCs and BCCs in terms of their effects on both collusion and price discrimination. In the absence of hassle cost, the collusive effect of MCCs is stronger than BCCs. When firms adopt MCCs, the monopoly price becomes the only possible equilibrium price. When firms adopt BCCs, on the other hand, a range of prices are possible in equilibrium. In contrast to Png and Hirshleifer (1988), the adoption of MCCs eliminates price dispersion and thus leaves no room for price discrimination between consumers with high search cost and those with low search cost. On the other hand, price dispersion is still possible when firms adopt BCCs. In other words, it is possible for firms to practice price discrimination under BCCs but not under MCCs.

These effects of BCCs are robust to the introduction of small hassle costs. However, the same cannot be said about the effects of MCCs. Our analysis confirms Hviid and Shaffer's (1999) finding that the collusive effect of MCCs evaporates with the introduction of an arbitrarily small hassle cost. Perhaps even more interesting is the finding that when MCCs are rendered ineffective by the presence of hassle cost, price dispersion is possible in equilibrium, but still there is no room for discrimination, since no informed consumers will activate the MCCs offered by higher-price firms.

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Chapter 3 Low-Price Guarantee and Entry Deterrence under Incomplete Information

3.1 Introduction

In the literature it has been widely recognized that the use of price guarantees can facilitate tacit collusion among oligopolistic firms. This proposition has been studied and has found support in most of the theoretical analyses on this subject (see, for example, Holt and Scheffman 1987, Belton 1987 and Chen 1995). Another role of price guarantee that has received quite a bit of attention in the literature is price discrimination. Firms can use price guarantees along with high list prices to price-discriminate between uninformed and informed consumers (Png and Hirschleifer, 1987 and Corts, 1996).

Much less studied is the role of price guarantees in entry deterrence. The only paper that has analyzed this role is Arbatskaya (2001). Starting from Bain's (1949) idea that an incumbent firm can use low prices to deter entry, Arbatskaya (2001) shows that low price guarantees can be used as a credible threat to fight entry, and she identifies situations where such a commitment can indeed deter entry even in a perfectly contestable market. One notable feature of her model is that firms and consumers have perfect information.[8]

The objective of this paper is to analyze the role of price-matching guarantees in entry deterrence using a model of asymmetric information. Specifically, we use a variation of Milgrom and Roberts' (1982) model of limit pricing. In the model there is

[8] In fact, this is a common feature shared by almost all existing models of low price guarantees. The only exception is Moorthy and Winter (2004) in which consumers have imperfect information about prices and price guarantees are used by firms to signal low prices.

one incumbent and one potential entrant. The asymmetric information comes from the fact that the entrant does not know the true marginal cost of the incumbent. We demonstrate that in such a situation the incumbent can use a price matching guarantee to deter entry under certain conditions.

This chapter is organized as follows. We present the model in section 2, and we study the benchmark case where the incumbent is not allowed to use price guarantee in section 3. Then in section 4 we study the equilibrium in the case where the incumbent firm can adopt a matching price guarantee. We use a specific model to illustrate the results in section 5 and conclude in section 6.

3.2 The Model

We consider a variation of the model by Milgrom and Roberts (1982) as simplified by Fudenberg and Tirole (1986). In this model there are two firms, an incumbent and a potential entrant, interacting in a market for differentiated products. They play a two-period game. In period one firm 1, the incumbent, is a monopolist in the market. In period two firm 2, the entrant, decides to enter or stay out of the market. If entry occurs, there is duopolistic competition in period 2. Otherwise, the incumbent remains a monopolist.

There is asymmetric information about firm 1's unit cost of production, which takes on two possible values: c_1^L and c_1^H. Firm 1 has perfect information about its cost from the beginning. Firm 2 does not know firm 1's unit cost and thus makes its decision without having complete information; however, it might be able to infer cost information by observing the incumbent's behavior in period one. *A priori* firm 2 believes that the probability of firm 1's cost being c_1^L is ρ and c_1^H, $1-\rho$. Firm 2's unit cost of production is c_2. We assume that $0 < c_1^L < c_2 < c_1^H$.

Equivalently we can restate this model as a four-stage game. At stage 1, "Nature" chooses the incumbent's unit cost, with ρ being the probability that c_1^L is chosen. At stage 2, the incumbent observes c_1^i with certainty, chooses P_1 and price guarantee policy, and serves the whole market. At stage 3, the entrant observes P_1 and the incumbent's price guarantee policy, but not c_1^i, and chooses either to enter or stay out. Finally, at stage 4 the firms earn duopoly profits, if entry has occurred, and otherwise the incumbent continues to receive monopoly profits. Note that stages 1 and 2 are in period one, and stages 3 and 4 are in period two, with production taking place in each period. Period-two profits are discounted by a factor δ. We shall consider pure-strategy sequential equilibria of this game.

We let $M_1^i(P_1, NPG)$ denote the incumbent's period-one monopoly profit when it charges P_1 and offers no price guarantee policy, where $i = L$ or H (indicating low or high cost). Then

$$M_1^i(P_1, NPG) = (P_1 - c_1^i)X(P, NPG) \tag{3.1}$$

Similarly, we let $M_1^i(P_1, PG)$ ($i = L$ or H) denote the incumbent's monopoly profit when it charges P_1 and offers a price guarantee policy. Then,

$$M_1^i(P_1, PG) = (P_1 - c_1^i)X(P, PG) - c_1^{PG} \tag{3.2}$$

We assume that both types of firm 1 have to incur a cost of c_1^{PG} to commit a price-matching guarantee[9].

We use $D_1^i, i = L, H$ to denote the incumbent's duopoly profits in the second period if entry occurs, and we assume that $M_1^i(P_1, PG) > D_1^i$ and that $D_1^L > D_1^H > 0$. Let $F > 0$ represent the start-up cost for the entrant, and denote by D_2^i the entrant's duopoly profits when the incumbent's unit costs are c_1^i. It is obvious that $D_2^H > D_2^L > 0$.

We note that if the start up cost is so large that $F > D_2^H$, the expected profit for the potential entrant would be $\rho D_2^L + (1-\rho)D_2^H < F$ for all $1 \geq \rho \geq 0$, therefore, no entry would occur. When the start up cost is so small that $F < D_2^L$, the potential entrant's expected profit would be positive no matter which type of incumbent it faces, i.e., $\rho D_2^L + (1-\rho)D_2^H > F$ for all $1 \geq \rho \geq 0$, thus entry cannot be deterred. Therefore,

Lemma 1 *If $F > D_2^H$, no entry occurs; the start-up cost alone will deter entry. If $F < D_2^L$, entry always occurs; no strategy can deter entry.*

In the following analysis we assume that the start-up cost of the entrant satisfies $D_2^H > F > D_2^L > 0$.

3.3 Equilibrium without Price Guarantees

As a benchmark, we first consider the equilibrium in the case where the incumbent is not allowed to offer a price-matching guarantee. In this case the model is reduced to that of Fudenberg and Tirole (1986). As they and others (Milgrom and

[9] Chen (1995) also assumes that firms need to make extra efforts to establish a price guarantee policy; for example, a retail store may have to put up a sign in the store that states ' we offer our customers a price guarantee.'

Roberts 1982) have shown, in such a situation the incumbent can use limit pricing to signal to the entrant that it has low cost, thus deterring entry. For completeness we produce here the results from Fudenberg & Tirole (1986).

Let P_1^L and P_1^H denote the period-one prices charged by the incumbent when its cost is low and high respectively. Let M_1^L and M_1^H denote the incumbent's profit when it maximizes its short run profit, depending on its type ($M_1^i \equiv M_1^i(p_m^i)$) and P_m^i denote the price charged when type i incumbent maximizes its monopoly profit.

There are two types of equilibria in this model: separating equilibrium and pooling equilibrium. In a separating equilibrium, the high-cost incumbent and the low-cost incumbent charge different prices in period one. As a result, the incumbent's type is revealed to the potential entrant at the end of period one, and thus entry occurs exactly when it would have occurred under symmetric information.

Proposition 1 If and only if P_1^L satisfies

$$M_1^H - M_1^H(P_1^L) \geq \delta(M_1^H - D_1^H) \tag{3.3}$$

$$M_1^L - M_1^L(P_1^L) \leq \delta(M_1^L - D_1^L) \tag{3.4}$$

, a separating equilibrium exists where the high-cost incumbent chooses P_m^H in period one and the entrant enters in period two, while the low cost incumbent chooses P_1^L in period one and the entrant stays out in period two.

Proof: First, we prove that conditions *(3.3)* and *(3.4)* are necessary conditions for separation equilibria. A high cost incumbent faces two choices when there exists a potential entrant: Induce entry by charging its monopoly price P_m^H , and then obtain monopoly profit M_1^H at period one and D_1^H at period two after entry occurs; Or limit price to p_1^L, which the low-cost incumbent would have charged to signal its low cost[10], and thus deter entry obtaining $M_1^H(P_1^L)+\delta M_1^H$. Therefore, a necessary condition for a separating equilibrium to exist is that $M_1^H+\delta D_1^H \geq M_1^H(P_1^L)+\delta M_1^H$, rearranging, we have condition (3.3).

Similarly, for the low-cost incumbent, it can offer price p_1^L, deter entry and obtain $M_1^L(P_1^L)+\delta M_1^L$; or offer monopoly price, induce entry and obtain $M_1^L+\delta D_1^L$. And thus a necessary condition for the low-cost incumbent to choose P_1^L in period one and the entrant stays out in period two is condition (3.4).

Secondly, we prove that the necessary conditions are also sufficient. The high-cost type of incumbent obtains its monopoly profit in the first period and thus is not willing to deviate to another price that induces entry. Nor does he deviate to p_1^L, given condition *(3.3)* is satisfied. And similarly, the low-cost type is not willing to deviate.

QED

In a pooling equilibrium, the entrant can infer nothing from incumbent's pricing behavior in period one. So it enters the market if its expected profit is positive; otherwise, it stays out of the market.

Proposition 2 *No pooling equilibrium exists when* $\rho D_2^L+(1-\rho)D_2^H>F$.

Proof: When $\rho D_2^L+(1-\rho)D_2^H>F$ is satisfied, firm 2 makes a strictly positive expected profit if it enters (as posterior beliefs are the same as prior beliefs) at the

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[10] We assume that this price is lower than P_m^L, the monopoly price of low-cost type incumbent, to illustrate that the low-cost incumbent has to sacrifice some profit to signal its type. Detailed proof is given in Condition 1.

pooling price. This means that entry can not be deterred, so that the incumbent of either type would maximize its profit by charging its monopoly price in period one. As the monopoly prices of two type incumbents differ, no pooling equilibrium exists.

QED

***Proposition** 3 Suppose $\rho D_2^L + (1-\rho) D_2^H < F$. In a pooling equilibrium, the entrant does not enter iff the price chosen by both types of incumbent, P_1^L, satisfies $M_1^L - M_1^L(P_1^L) \le \delta(M_1^L - D_1^L)$, and*

$$M_1^H - M_1^H(P_1^L) \le \delta(M_1^H - D_1^H) \tag{3.5}$$

Proof: With $\rho D_2^L + (1-\rho) D_2^H < F$ being satisfied, the possibility of absolutely profitable entry is eliminated. Continuing the proof of Proposition 1, we can have that, a high-cost incumbent would want to pool if the condition $M_1^H + \delta D_1^H \le M_1^H(P_1^L) + \delta M_1^H$ is met. And thus a pooling equilibrium exists if both types of incumbents want to play price P_1^L and deter entry, which conditional on $M_1^L - M_1^L(P_1^L) \le \delta(M_1^L - D_1^L)$ and $M_1^H - M_1^H(P_1^L) \le \delta(M_1^H - D_1^H)$ are satisfied.

And following the same logic in the proof of proposition 2, none of the types would want to deviate from P_1^L under conditions *(3.4)* and *(3.5)*.

QED

To characterize in more detail the set of P_1^L that supports a separating equilibrium, we note the following conditions on the demand and cost functions that would ensure a separating equilibrium.

Condition 1 *The high-cost type incumbent would wish to pool if* $P_1^L = P_m^L$, *that is,* $M_1^H - M_1^H(P_m^L) \le \delta(M_1^H - D_1^H)$.

This assumption makes intuitive sense. Under incomplete information, the low cost firm has to sacrifice its short-run profits to signal its type and make it very costly for the high-cost type to imitate. Therefore, the separating equilibrium under incomplete information is not the same as the one in the complete information context.

Condition 2 *It is less costly for the low-cost type to charge a low price. That is,* $\dfrac{\partial\left[M_1^{H(}(P_1) - M_1^L(P_1)\right]}{\partial P_1} > 0$.

This condition is called single crossing condition, or sorting condition, which ensures that the curves $y = M_1^L - M_1^L(P_1^L)$ and $y = M_1^H - M_1^H(P_1^L)$ cross at most once in the $\{P_1^L, y\}$ space. It is satisfied in our model since $\dfrac{\partial^2\left[(P_1 - c_1)X_1^m(P_1)\right]}{\partial P_1 \partial c_1} = -\dfrac{dX_1^m}{dP_1} > 0$.

Condition 3 *The lower the incumbent's cost, the more it benefits from remaining a monopolist, i.e.,* $M_1^L - D_1^L > M_1^H - D_1^H$

This condition can be satisfied under fairly mild restrictions on the demand functions. Letting $X_1(P_1, P_2)$ denote the duopoly demand for firms 1 (notice that $X_1(P_1, +\infty) = X_1^m(P_1)$), $M_1(c_1)$ and $D_1(c_1)$ the monopoly and duopoly profits associated with cost c_1, P_1^d and P_2^d the equilibrium duopoly prices (functions of c_1), and using the envelope theorem, we have

$$\frac{d[M_1(c_1)-D_1(c_1)]}{dc_1}=\frac{d}{dc_1}(\max_{P_1}\left[(P_1-c_1)X_1^m(P_1)\right]-\max_{P_1}\left[(P_1-c_1)X_1(P_1,P_2^d)\right])$$
$$=-X_1^m(P_1^m)+X_1(P_1^d,P_2^d)-(P_1^d-c_1)\frac{\partial X_1}{\partial P_2}\frac{\partial P_2^d}{\partial c_1}$$

The third term in this equality is presumably negative (because $P_1^d-c_1>0$, $\partial X_1/\partial P_2>0$ and – under mild conditions - $\partial P_2^d/\partial c_1>0$). Thus, if monopoly demand for firm 1 exceeds its duopoly demand,[11] the difference between M_1 and D_1 decreases with c_1 and therefore we have $M_1^L-D_1^L>M_1^H-D_1^H$.

Given the above conditions, we can construct Figure 1 to illustrate the set of separating equilibria. The price $\tilde{P}_1$ satisfies *(3.3)* with equality while $\tilde{\tilde{P}}_1$ satisfies *(3.4)* with equality. Therefore, the interval $\left[\tilde{\tilde{P}}_1,\tilde{P}_1\right]$ represents the set of all possible separating equilibria. The price $\tilde{P}_1$ is called the least-cost separating price because, of all potential separating prices, the low cost type would prefer the one at $\tilde{P}_1$ (the closest to P_m^L).

The set of prices P_1 that supports a pooling equilibrium also depends on the cost and the demand functions. Let us simply note that, from Condition 1, there exists an interval of prices around P_m^L that satisfy these two inequalities.

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[11] This condition holds in a more specific model presented in section 5.

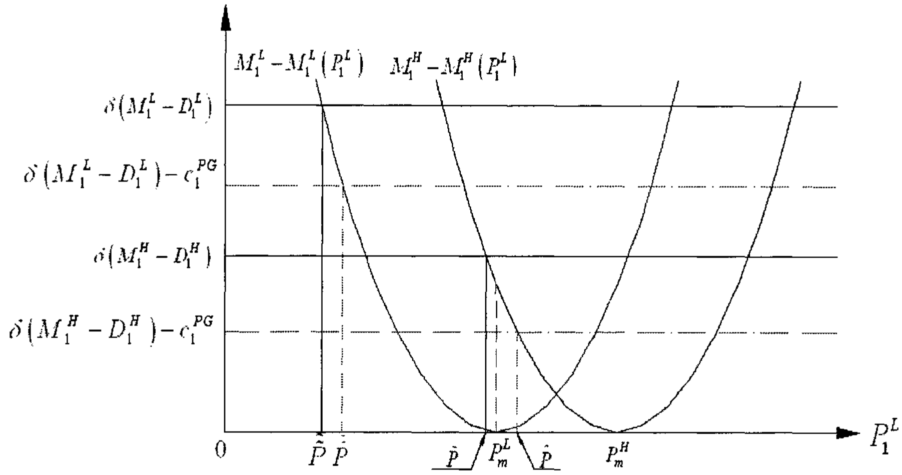

Figure 1 Equilibrium with/without matching price guarantee

Source: Tirole, J., The Theory of Industrial Organization, 1988, P370

Now it is easy to see that if P_1 satisfies inequalities *(3.4)* and (3.5), P_1 can be made part of a pooling equilibrium. Suppose that whenever firm 1 chooses a price differing from P_1 (an off-the-equilibrium path price), firm 2 believes that firm 1 has a high cost. Firm 2 then enters, and firm 1 might as well play its monopoly price. Thus, from inequalities *(3.4)* and (3.5), neither type would want to deviate from P_1.

3.4 Equilibrium with Price Guarantees

Now that the option of a price guarantee is available, the incumbent is no longer confined to the use of limit pricing as a way to send a signal about his cost to the potential entrant. In addition to a low price, the incumbent can also use the price-matching guarantee to signal his low cost. We will consider separating equilibrium and pooling equilibrium in sequence.

3.4.1 Separating Equilibrium

In a separating equilibrium, the high cost type incumbent does not deter entry in period two. Thus, in period one it chooses the monopoly price without price guarantee, i.e. it chooses the strategy (P^m, NPG). Its payoff is $M_1^H + \delta D_1^H$. The low cost type incumbent, on the other hand, commits itself to (P_1^{LPG}, PG) in period one and thus deters entry. Its payoff is $M_1^L(P_1^{LPG}) + \delta M_1^L - c_1^{PG}$.

For a separating equilibrium to occur, the incumbent's strategies must satisfy the incentive compatibility conditions. That is, the high cost type does not want to pick the low cost type's equilibrium strategy and vice-versa. Therefore, for the high cost type we have $M_1^H(P_1^{LPG}) + \delta M_1^H - c_1^{PG} \le M_1^H + \delta D_1^H$, i.e.,

$$M_1^H - M_1^H(P_1^{LPG}) \ge \delta(M_1^H - D_1^H) - c_1^{PG} \tag{3.6}$$

Similarly, the low cost type's incentive compatibility condition implies $M_1^L(P_1^{LPG}) + \delta M_1^L - c_1^{PG} \ge M_1^L + \delta D_1^L$, i.e.,

$$M_1^L - M_1^L(P_1^{LPG}) \le \delta(M_1^L - D_1^L) - c_1^{PG} \tag{3.7}$$

With the aid of Figure 1 we can compare the separating equilibrium in the case of price guarantee with that in the case of no price guarantee. As shown in Figure 1, with price guarantee the two horizontal curves move downward by the amount c_1^{PG}. We obtain the interval $\left[\hat{\hat{P}}_1, \hat{P}_1\right]$ from figure 1. $\hat{P}_1$ is such that the equation (3.6) is satisfied with equality; of all potential separating prices, the low cost type would prefer the one at $\hat{P}_1$ (the closest to P_m^L).

From Figure 1 we see that $\hat{P}_1 > \tilde{P}_1$. This implies that the availability of a price guarantee tends to push up the equilibrium price when the incumbent is a low cost type.

3.4.2 Pooling Equilibrium

In a pooling equilibrium, the two types of incumbent choose the same strategy and consequently the entrant learns nothing from observing the first period strategy of the incumbent. Therefore, it is possible to have a pooling equilibrium where both the high-cost and the low-cost type incumbents offer the same price and commit themselves to price guarantees in the first period.

With a price guarantee, the condition for the existence of pooling equilibrium becomes,

$$\rho D_2^{LPG} + (1-\rho) D_2^{HPG} < F \tag{3.8}$$

where $D_2^{LPG} = D_2^L$ and $D_2^{HPG} < D_2^H$.[12] When entry occurs in the second period, the two firms compete as duopolists. From our assumption $0 < c_1^L < c_2 < c_1^H$, we have $P_1^{LPG} < P_2 < P_1^{HPG}$.

We note that the use of price guarantee has no effect on firm 2's profit when firm 1's cost is low, since $P_1^{LPG} < P_2$. But when firm 1's unit cost is high, which implies $P_1^{HPG} > P_2$, the commitment of matching competitor's lower price will reduce the incumbent's effective price and thus lower D_2^H to D_2^{HPG}. This, in turn, lowers firm 2's expected profit and allows a larger range of ρ to satisfy condition (3.8). Therefore, the

[12] We will illustrate these claims in the context of a simple model with unit demand on a circular city in next section. Here we present only the main idea about how this works.

availability of price guarantee makes entry deterrence in a pooling equilibrium possible for a wider range of parameter values.

Assume that condition (3.8) is satisfied, so that a pooling strategy (P_1^{LPG}, PG) deters entry. A necessary condition for this choice to be a pooling equilibrium strategy is that neither type wants to deviate to choose the monopoly price in period 1. If one of them were to do so, it would at worst allow entry. Therefore, P_1^{LPG} must satisfy condition (3.7) and the analogous condition for the high-cost type:

$$M_1^H - M_1^H(P_1) \leq \delta(M_1^H - D_1^H) - c_1^{PG} \tag{3.9}$$

Now it is easy to see that if P_1^{LPG} satisfies conditions (3.7) and (3.9), P_1^{LPG} can be made part of a pooling equilibrium. Suppose that whenever firm 1 plays a strategy differing from (P_1^{LPG}, PG), the entrant believes that the incumbent has a high cost. The entrant then enters, in which case the incumbent would have been better off choosing its monopoly price without committing to a price guarantee in period 1. Thus, from conditions (3.7) and (3.9), neither type of incumbents would want to deviate from (P_1^{LPG}, PG).

3.5 A Model with a Specific Demand Function

Consider a model in which consumers are distributed uniformly on a "circle city" with circumference equal to 1. Density is unitary around this circle. We assume that all travels occur along the circle. Because of a transportation cost, the products are differentiated even though they are physically identical. The higher the transportation cost, the more differentiated the products.

Each consumer has a unit demand and a unit transportation cost of t. The total cost of a unit to a consumer is then equal to the sum of the purchase price and the transportation cost. The consumer's utility maximization problem is equivalent to the minimization of this total cost so long as the latter does not exceed his reservation price, $\bar{R}$. We assume the transportation cost is linear. Thus, a consumer living at the distance $x \in (0, 1/2]$ incurs a cost of tx to go to the store.

The two firms are located at half a circle away from each other.[13] They play the same game as specified in the general model in section 2. Because of the unit demand function of individual consumers, we have to consider the question whether all consumers are served by the incumbent monopolist. Depending on the answer to this question, we study three different cases.

3.5.1 Neither Type of Incumbent Serves the Whole Market in Period 1

Suppose that the monopolist does not serve the whole market and that the consumer located at m from firm 1 is indifferent between purchasing and not purchasing. Then m satisfies the condition $P_m^i + tm = \bar{R}$. From this we derive the demand function facing the monopolist $X_1^{i,m} = 2m = \dfrac{2(\bar{R} - P_m^i)}{t}$.

Therefore, firm 1's optimization problem can be written as $\max\limits_{P_m^i}[(P_m^i - c_1^i)X_1^{i,m}]$.

Solving the first order condition, we have $P_m^i = \dfrac{\bar{R} + c_1^i}{2}$, $X_1^{i,m} = \dfrac{\bar{R} - c_1^i}{t}$ and

[13] It is clear that if the location of the entrant were endogenous, its profit would be maximized if it were to locate at half a circle away from the incumbent. Nevertheless, we do not endogenize the location choice here because we want to be consistent with the general model where the demand function is exogenously given.

$$M_1^i = \frac{(\bar{R} - c_1^i)^2}{2t} \tag{3.10}$$

If firm 1 limits his price and charge P_1^L to deter entry, his profit would be

$$M_1^i(P_1^L) = \frac{2}{t}(P_1^L - c_1^i)(\bar{R} - P_1^L) \tag{3.11}$$

If entry occurs in period 2, the marginal consumer who is indifferent between buying from these two firms is located at x, where x satisfies the condition $P_1^i + tx = P_2 + t(1-x)$. From this we can derive the demand functions facing the two firms:

$X_1^i(P_1^i, P_2) = 2x = \frac{1}{2} + \frac{P_2 - P_1^i}{t}$ and $X_2(P_1^i, P_2) = 1 - 2x = \frac{1}{2} + \frac{P_1^i - P_2}{t}$.

Therefore, two firms respectively seek to maximize

$$\max_{P_1^i}[(P_1^i - c_1^i)(\frac{1}{2} + \frac{P_2 - P_1^i}{t})]$$

and

$$\max_{P_2}[(P_2 - c_2)(\frac{1}{2} + \frac{P_1^i - P_2}{t})]$$

Solving the first-order conditions of these two maximization problems, we obtain the prices, quantities, and the profits for this case, namely

$P_1^i = \frac{t}{2} + \frac{2c_1^i + c_2}{3}; P_2 = \frac{t}{2} + \frac{2c_2 + c_1^i}{3}$;

$X_1^i(P_1^i, P_2) = \frac{1}{2} + \frac{c_2 - c_1^i}{3t}; X_2(P_1^i, P_2) = \frac{1}{2} + \frac{c_1^i - c_2}{3t}$;

and

$$D_1^i = \frac{1}{t}(\frac{t}{2} + \frac{c_2 - c_1^i}{3})^2; D_2^i = \frac{1}{t}(\frac{t}{2} + \frac{c_1^i - c_2}{3})^2 \tag{3.12}$$

To proceed we make the following four assumptions:

Assumption 1 $c_1^L + t > \overline{R} > c_1^H + t/2$

Assumption 2 $c_1^L + \frac{3}{2}t > c_2 > c_1^H - \frac{3}{2}t$

Assumption 3 $c_1^L < 3\sqrt{tF} + c_2 - \frac{3t}{2} < c_1^H$.

Assumption 4 $c_1^H - c_1^L < 2k$,

where $k = \sqrt{(\overline{R} + c_1^H)^2 - 4c_1^H\overline{R} - (1-\delta)(\overline{R} - c_1^H)^2 - 2\delta(\frac{t}{2} + \frac{c_2 - c_1^H}{3})^2}$.

Assumption 1 ensures that the monopolist indeed serves only part of the market and that the monopoly demand exceeds the incumbent's demand as a duopolist. From $c_1^L + t > \overline{R}$ we obtain $P_m^i + t/2 > \overline{R}$, which implies that the monopolist does not serve the whole market. From $X_1^m(P_1^m) > X_1(P_1^d, P_2^d)$, which states that the monopoly demand exceeds its duopoly demand, we have $\frac{\overline{R} - c_1^i}{t} > \frac{1}{2} + \frac{c_2 - c_1^i}{3t}$, which is satisfied given the assumption that $\overline{R} > c_1^H + t/2$ and $c_2 < c_1^H$. We note that the assumption $\overline{R} > c_1^H + t/2$ also implies that the whole market would be served if the incumbent were to set its price equal to its marginal cost.

Assumption 2 ensures that two firms share the market and compete oligopolistically if entry occurs. In our model, the following two conditions should be satisfied, i.e., $P_2 + \frac{t}{2} > P_1^H$ and $P_1^L + \frac{t}{2} > P_2$.

Assumption 3 ensures that the condition $D_2^L < F < D_2^H$ is satisfied. From $D_2^H > F$, we have $D_2^H = \frac{1}{t}(\frac{t}{2} + \frac{c_1^H - c_2}{3})^2 > F$, rearrange, we obtain $c_1^H > 3\sqrt{tF} + c_2 - \frac{3t}{2}$; From $D_2^L < F$, and thus $D_2^L = \frac{1}{t}(\frac{t}{2} + \frac{c_1^L - c_2}{3})^2 < F$, we obtain $c_1^L < 3\sqrt{tF} + c_2 - \frac{3t}{2}$.

As we will see from the proof of Proposition 4, assumption 4 ensures that Condition 1 in the general model, $M_1^H - M_1^H(P_m^L) \le \delta(M_1^H - D_1^H)$, is satisfied.

Proposition 4 *Suppose the option of offering a price guarantee is not available to the incumbent. Let*

$$\tilde{P}_1 = \frac{\bar{R} + c_1^H}{2} - \sqrt{(\bar{R} + c_1^H)^2 - 4c_1^H \bar{R} - (1-\delta)(\bar{R} - c_1^H)^2 - 2\delta(\frac{t}{2} + \frac{c_2 - c_1^H}{3})^2}$$

There exists a separating equilibrium where the low cost incumbent sets a limiting price at $P_1^L = \tilde{P}_1$ and deters entry while the high cost incumbent chooses its monopoly price $P_m^H = \frac{\bar{R} + c_1^H}{2}$ and entry occurs.

Proof: Recall that a separating equilibrium must satisfy

$M_1^H - M_1^H(P_1^L) \ge \delta(M_1^H - D_1^H)$ and $M_1^L - M_1^L(P_1^L) \le \delta(M_1^L - D_1^L)$

Inserting (3.10), (3.11) and (3.12), and solving, we have

$$\tilde{\tilde{P}}_1 < P_1^L < \tilde{P}_1,$$

where

$$\tilde{P}_1 = \frac{\bar{R}+c_1^H}{2} - \sqrt{(\bar{R}+c_1^H)^2 - 4c_1^H\bar{R} - (1-\delta)(\bar{R}-c_1^H)^2 - 2\delta(\frac{t}{2}+\frac{c_2-c_1^H}{3})^2}$$

and

$$\tilde{\tilde{P}}_1 = \frac{\bar{R}+c_1^L}{2} - \sqrt{(\bar{R}+c_1^L)^2 - 4c_1^L\bar{R} - (1-\delta)(\bar{R}-c_1^L)^2 - 2\delta(\frac{t}{2}+\frac{c_2-c_1^L}{3})^2} \qquad (3.13)$$

Assumption 4 ensures that $\tilde{P}_1 < P_m^L$, and thus the condition $M_1^H - M_1^H(P_m^L) \le \delta(M_1^H - D_1^H)$ is satisfied.

Eliminating dominated prices in the interval $[\tilde{\tilde{P}}_1, \tilde{P}_1)$ for firm 1, we narrow down the equilibrium price to $\tilde{P}_1$.

QED

Proposition 5 *Suppose the option of offering a price guarantee is not available to the incumbent. There exists a pooling equilibrium where both types of incumbent choose the price* $P_1^L = P_m^L$ *and deter entry if* $\rho > \dfrac{(\frac{3t}{2}+c_1^H-c_2)^2 - 9tF}{(3t+c_1^H+c_1^L-2c_2)(c_1^H-c_1^L)}$.

Proof: For a pooling equilibrium to exist, we must have the following two conditions:

(1) $\rho D_2^L + (1-\rho)D_2^H < F$. This implies that $\rho > \dfrac{D_2^H - F}{D_2^H - D_2^L}$. This, along with our assumption, $D_2^H > F > D_2^L$, ensures that ρ lies in the interval $(0,1)$. Inserting equation(3.12), we obtain $\rho > \dfrac{(\frac{3t}{2}+c_1^H-c_2)^2 - 9tF}{(3t+c_1^H+c_1^L-2c_2)(c_1^H-c_1^L)}$.

(2) The pooling equilibrium price P_1^L satisfies inequalities (3.4) and (3.5). We combine equations (3.10), (3.11), (3.12) and Condition 1, we have $\tilde{P}_1 < P_1^L < P_m^L$, where $P_m^L = \dfrac{\overline{R} + c_1^L}{2}$ and $\tilde{P}_1 = \dfrac{\overline{R} + c_1^H}{2} - k$.

Again, we eliminate other less reasonable equilibria and have the pooling equilibrium price $P_1^L = P_m^L$.

QED

Proposition 6 *Suppose the option of offering a price guarantee is available to the incumbent. Let*

$$\hat{P}_1 = \frac{\overline{R} + c_1^H}{2} - \sqrt{(\overline{R} + c_1^H)^2 - 4(c_1^H \overline{R} + c_1^{PG}) - (1-\delta)(\overline{R} - c_1^H)^2 - 2\delta(\frac{t}{2} + \frac{c_2 - c_1^H}{3})^2},$$

thus $\hat{P}_1 > \tilde{P}_1$. *Furthermore, there exists a separating equilibrium where the low cost incumbent chooses a limiting price* $P_1^L = \hat{P}_1$ *and deters entry while the high cost incumbent offers his monopoly price and does not deter entry.*

Proof: With a price guarantee, the conditions for the existence of a separating equilibrium become,

$$M_1^H - M_1^H(P_1^{LPG}) \geq \delta(M_1^H - D_1^H) - c_1^{PG}$$

$$M_1^L - M_1^L(P_1^{LPG}) \leq \delta(M_1^L - D_1^L) - c_1^{PG}$$

We insert (3.10), (3.11) and (3.12) into above conditions, and thus we obtain the interval that the limit price should satisfy $\hat{\hat{P}}_1 < P_1^L < \hat{P}_1$,

where

$$\hat{P}_1 = \frac{\overline{R} + c_1^H}{2} - \sqrt{(\overline{R} + c_1^H)^2 - 4(c_1^H \overline{R} + c_1^{PG}) - (1-\delta)(\overline{R} - c_1^H)^2 - 2\delta(\frac{t}{2} + \frac{c_2 - c_1^H}{3})^2}$$

and

$$\widehat{\widehat{P}}_1 = \frac{\overline{R}+c_1^L}{2} - \sqrt{(\overline{R}+c_1^L)^2 - 4(c_1^L\overline{R}+c_1^{PG}) - (1-\delta)(\overline{R}-c_1^L)^2 - 2\delta(\frac{t}{2}+\frac{c_2-c_1^L}{3})^2}$$

A comparison of $\widehat{P}_1$ and $\widehat{\widehat{P}}_1$ with $\tilde{P}_1$ and $\tilde{\tilde{P}}_1$, respectively, where

$$\tilde{P}_1 = \frac{\overline{R}+c_1^H}{2} - \sqrt{(\overline{R}+c_1^H)^2 - 4c_1^H\overline{R} - (1-\delta)(\overline{R}-c_1^H)^2 - 2\delta(\frac{t}{2}+\frac{c_2-c_1^H}{3})^2}$$

and

$$\tilde{\tilde{P}}_1 = \frac{\overline{R}+c_1^L}{2} - \sqrt{(\overline{R}+c_1^L)^2 - 4c_1^L\overline{R} - (1-\delta)(\overline{R}-c_1^L)^2 - 2\delta(\frac{t}{2}+\frac{c_2-c_1^L}{3})^2}$$, shows that

$\widehat{P}_1 > \tilde{P}$ and $\widehat{\widehat{P}}_1 > \tilde{\tilde{P}}$.

QED

Under a separating equilibrium, the low-cost incumbent can charge a higher price, compared to the situation without price guarantee, to limit entry; since the costs of offering a price guarantee and limiting price together prevent high-cost incumbent from imitating the behavior of the low-cost one.

***Proposition** 7 Suppose the option of offering a price guarantee is available to the incumbent. A pooling equilibrium does not exist if the cost of offering a price guarantee is so large that* $\widehat{P}_1 > P_m^L$.

Proof: In this case, the low cost firm can still charge his monopoly price but distinguish himself from the high cost firm by committing to a price-matching guarantee while the high cost firm cannot afford to imitate this strategy.

QED

Proposition 8 *Suppose the option of offering a price guarantee is available to the incumbent. There exists a pooling equilibrium where both firms choose* $P_1^L = P_m^L$ *and deter entry if the cost of offering the price guarantee is so low that* $\widehat{P}_1 < P_m^L$.

Proof: With a price guarantee, firm 2 knows that the high cost firm 1 will match his price, and thus from his reaction function, we have

$$P_2^{PG} = \frac{t}{2} + c_2$$

$$D_2(P_1^H, P_2^{PG}, PG) = \frac{1}{2}$$

In this scenario, the profit of firm 2 becomes $D_2^{H,PG} = \frac{t}{4} - \frac{c_2}{2}$

Recall from equation (3.12) that the profit of firm 2 in the case where there is no price guarantee can be rewritten as $D_2^i = \frac{t}{4} - \frac{c_2}{3} + \frac{c_1^i}{3} + \frac{(c_1^i - c_2)^2}{9t}$. Obviously, we have $D_2^{H,PG} < D_2^L < D_2^H$.

Therefore, from our assumption, $D_2^{H,PG} < D_2^L < F$, we have $\rho D_2^L + (1-\rho) D_2^{H,PG} < F$ for any $1 > \rho > 0$.

QED

With the introduction of a price guarantee, the incumbent can manipulate its price in a way that does not reveal his cost information yet deters entry more effectively. In this model, entry is deterred even in the case that the entrant believes that there is only very small possibility that the incumbent has low cost.

3.5.2 Low-Cost Incumbent Serves the Whole Market but the High-Cost One Not

The condition for this scenario to occur is $c_1^H + t > \overline{R} > c_1^L + t$. Here the high cost incumbent's one-period monopoly price is the same as before at $P_m^H = \frac{\overline{R} + c_1^H}{2}$. Because $\overline{R} > c_1^L + t$, the low cost firm will charge $P_m^L = \overline{R} - \frac{t}{2}$ and thus maximize his profit by extracting the last unit of consumer surplus from the last consumer.

We examine if the sorting condition is satisfied in this context. Since the low cost incumbent serves the whole market and the total demand is 1, we have $M_1^L = \overline{R} - \frac{t}{2} - c_1^L$. From last subsection, we have $M_1^H = \frac{(\overline{R} - c_1^H)^2}{2t}$. Suppose that firm 1 would limit its price at P_1^L and the profit of the incumbent would be $P_1^L - c_1^i$. It is straightforward to verify that

$$M_1^L - M_1^L(P_1^L) = \overline{R} - \frac{t}{2} - P_1^L$$

$$M_1^H - M_1^H(P_1^L) = \frac{(\overline{R} - c_1^H)^2}{2t} - P_1^L + c_1^H$$

And thus the curves of $M_1^L - M_1^L(P_1^L)$ and $M_1^H - M_1^H(P_1^L)$ become straight lines as shown by Figure 2.

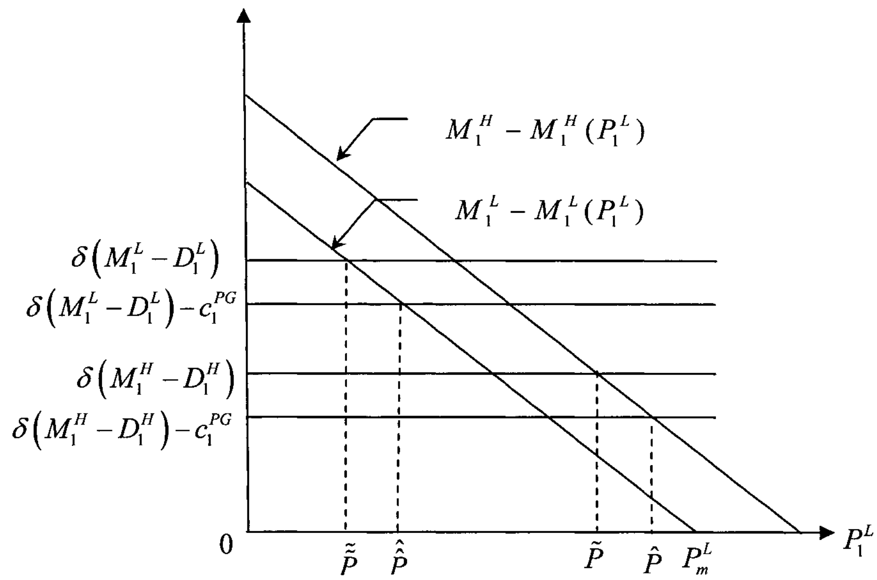

Figure 2 Only the low-cost incumbent serves the whole market

In the situation where the option of offering matching price guarantee is not available, the interval $\left[\tilde{\tilde{P}}_1, \tilde{P}_1\right]$ represents the set of all possible separating equilibria. The price $\tilde{P}_1$ is the least-cost separating price because, of all potential separating prices, the low cost type would prefer the one at $\tilde{P}_1$ (the closest to P_m^L). And the interval $\left[\tilde{P}_1, P_m^L\right]$ represents the set of all possible pooling equilibria. Following the same logic of proof of Proposition 4 and 5, we obtain the following two propositions.

Proposition 9 *Suppose the option of offering a price guarantee is not available to the incumbent. Let*

$$\tilde{P}_1 = (1-\delta)\frac{(\bar{R}-c_1^H)^2}{2t} + c_1^H + \frac{\delta}{t}\left(\frac{t}{2} + \frac{c_2 - c_1^H}{3}\right)^2$$

There exists a separating equilibrium where the low cost incumbent sets a limiting price at $P_1^L = \tilde{P}_1$ *and deters entry while the high cost incumbent chooses its monopoly price* $P_m^H = \dfrac{\bar{R} + c_1^H}{2}$ *and entry occurs.*

Proposition 10 *Suppose the option of offering a price guarantee is not available to the incumbent. There exists a pooling equilibrium where both types of incumbent choose the price* $P_m^L = \bar{R} - \dfrac{t}{2}$ *and deter entry if* $\rho > \dfrac{(\frac{3t}{2} + c_1^H - c_2)^2 - 9tF}{(3t + c_1^H + c_1^L - 2c_2)(c_1^H - c_1^L)}$.

Now suppose the incumbent can use a matching price guarantee together with limiting price to deter entry, the two horizontal curves representing $\delta\left(M_1^L - D_1^L\right)$ and $\delta\left(M_1^H - D_1^H\right)$ move downward by the amount of c_1^{PG}, and therefore, we have the interval $\left[\hat{\hat{P}}_1, \hat{P}_1\right]$ representing the set of all possible separating equilibria, and $\left[\hat{P}_1, P_m^L\right]$ the set of all possible pooling equilibria. Using the same arguments of Proposition 6,7 and 8, we obtain the following two propositions:

Proposition 11 *Suppose the option of offering a price guarantee is available to the incumbent. Let*

$$\hat{P}_1 = (1-\delta)\frac{(\bar{R} - c_1^H)^2}{2t} + c_1^H + \frac{\delta}{t}\left(\frac{t}{2} + \frac{c_2 - c_1^H}{3}\right)^2 + c_1^{PG}$$

There exists a separating equilibrium where the low cost incumbent sets a limiting price at $P_1^L = \hat{P}_1$ *and deters entry while the high cost incumbent chooses its monopoly price* $P_m^H = \dfrac{\bar{R} + c_1^H}{2}$ *and entry occurs.*

Proposition 12 *When the option of offering a price guarantee is available to the incumbent, there exists a pooling equilibrium where both types of incumbents set the price at* $P_m^L = \bar{R} - \frac{t}{2}$ *and deter entry if* $\rho > \dfrac{(\frac{3t}{2} + c_1^H - c_2)^2 - 9tF}{(3t + c_1^H + c_1^L - 2c_2)(c_1^H - c_1^L)}$.

We observe that the matching price guarantee helps the low cost firm in the separating equilibrium by a higher price, since $\hat{P}_1 > \tilde{P}_1$, but make no difference in the pooling equilibrium.

3.5.3 Both Types of Incumbents Serve the Whole Market

It is easy to see that if the reservation price is big enough that $\bar{R} > c_1^H + t$, both types of incumbent serve the whole market. In this case, we have $P_m^i = \bar{R} - \frac{t}{2}$, and thus $M_1^i = \bar{R} - \frac{t}{2} - c_1^i$. At the entry-limiting price P_1^L, the profit of the incumbent would be $P_1^L - c_1^i$. It costs both types of incumbent the same amount $M_1^i - M_1^i(P_1^L) = \bar{R} - \frac{t}{2} - P_1^L$ if the low cost one wants to use the entry-limiting price to signal its type. We have Figure 3 to illustrate this situation: compared with figure 1 and 2, the curves of $M_1^L - M_1^L(P_1^L)$ and $M_1^H - M_1^H(P_1^L)$ combine to one straight line.

In the situation where the option of offering matching price guarantee is not available, the interval $\left[\tilde{\tilde{P}}_1, \tilde{P}_1\right]$ represents the set of all possible separating equilibria. And the interval $\left[\tilde{P}_1, P_m^i\right]$ represents the set of all possible pooling equilibria. Using the same arguments of Proposition 4 and 5, we obtain the following two Propositions.

Proposition 13 *Suppose the option of offering a price guarantee is not available to the incumbent. Let*

$$\tilde{P}_1 = (1-\delta)(\bar{R}-\frac{t}{2}) + c_1^H + \frac{\delta}{t}(\frac{t}{2}+\frac{c_2-c_1^H}{3})^2$$

There exists a separating equilibrium where the low cost incumbent sets a limiting price at $P_1^L = \tilde{P}_1$ *and deters entry while the high cost incumbent chooses its monopoly price* $P_m^H = \bar{R}-\frac{t}{2}$ *and entry occurs.*

Proposition 14 *Suppose the option of offering a price guarantee is not available to the incumbent. There exists a pooling equilibrium where both types of incumbent choose the price* $P_1^L = P_m^i = \bar{R}-\frac{t}{2}$ *and deter entry if* $\rho > \dfrac{(\frac{3t}{2}+c_1^H-c_2)^2 - 9tF}{(3t+c_1^H+c_1^L-2c_2)(c_1^H-c_1^L)}$.

Now suppose the incumbent can use a matching price guarantee together with limiting price to deter entry, the two horizontal curves representing $\delta\left(M_1^L - D_1^L\right)$ and $\delta\left(M_1^H - D_1^H\right)$ move downward by the amount of c_1^{PG}, and therefore, we have the interval $\left[\hat{\hat{P}}_1, \hat{P}_1\right]$ representing the set of all possible separating equilibria, and $\left[\hat{P}_1, P_m^i\right]$ the set of all possible pooling equilibria. Following the same logic as Proposition 6,7 and 8, we have the following two propositions:

Proposition 15 *Suppose the option of offering a price guarantee is available to the incumbent. Let*

$$\hat{P}_1 = (1-\delta)(\bar{R}-\frac{t}{2}) + c_1^H + \frac{\delta}{t}(\frac{t}{2}+\frac{c_2-c_1^H}{3})^2 + c_1^{PG}$$

There exists a separating equilibrium where the low cost incumbent sets a limiting price at $P_1^L = \hat{P}_1$ and deters entry while the high cost incumbent chooses its monopoly price $P_m^H = \overline{R} - \frac{t}{2}$ and entry occurs.

Proposition 16 *When the option of offering a price guarantee is available to the incumbent, there exists a pooling equilibrium where both types of incumbents set the price at $P_m^i = \overline{R} - \frac{t}{2}$ and deter entry if $\rho > \frac{(\frac{3t}{2} + c_1^H - c_2)^2 - 9tF}{(3t + c_1^H + c_1^L - 2c_2)(c_1^H - c_1^L)}$.*

Again, the matching price guarantee helps the low cost firm in the separating equilibrium by allowing it to set a higher limiting price. But the guarantee makes no difference in the pooling equilibrium.

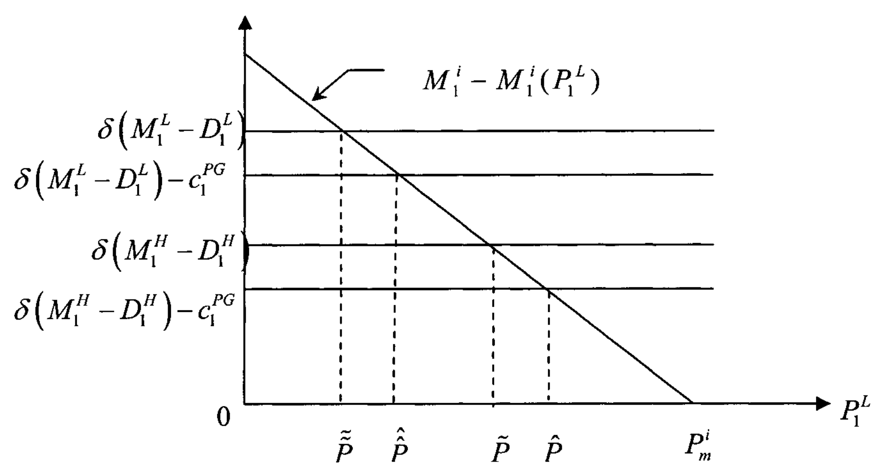

Figure 3 Both types of incumbents serve the whole market

3.6. Conclusions

In this paper we introduce a price-matching guarantee into a simplified version of Milgrom and Roberts' (1982) limit pricing model and demonstrate the entry deterrence effects of price-matching guarantee. Our analysis shows that under certain conditions a price-matching guarantee can be used in conjunction with/without a relatively low price to signal low cost and deter entry. However, under some other conditions, the use of price guarantees can allow the high-cost incumbent to imitate successfully the behavior of the low-cost one and deter entry even in situations where this would not have been possible in the absence of price guarantee. Overall, the availability of price guarantee as an additional instrument makes entry deterrence possible for a wider range of parameter values.

In this paper we have not explicitly studied the welfare effect of price guarantees. However, it is interesting to examine a few welfare implications from our analysis. First of all, social welfare, compared to the case without price guarantees, is lowered: pre-entry price is increased for the separating equilibrium. Secondly, to the extent that the benefits of entry, namely increased product variety and lower post-entry prices, are higher than the costs of entry, entry deterrence is welfare-reducing. Consequently, the use of price guarantees reduces welfare by making entry deterrence more likely.

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Chapter 4 Low-Price Guarantee as a Way to Induce Purchase

4.1 Introduction

As the literature survey in Chapter 1 has shown, low-price guarantees (LPGs), such as matching competition clauses (MCCs) and beating competition clauses (BCCs), have been extensively studied in the economics literature. However, none of the existing theoretical models has considered the non-strategic effects of LPGs or the time dimension.

First, with respect to the non-strategic effects of LPGs, a common feature of existing models is that LPGs are offered by oligopolistic firms that, by definition, have the ability to influence their rivals' prices. In reality, not all industries are concentrated enough to accord firms such ability. In retail markets where competition is fierce (e.g. the retailing of consumer electronics in large North American cities), the price set by a retailer may have little influence on overall market prices. In such a situation, the strategic effects of LPGs (i.e. facilitation of tacit collusion) are absent, and LPGs are likely to be used as a device to attract customers. Indeed, the latter is typically how LPGs are portrayed in the popular press. Yet such a simple explanation of LPGs has very much been ignored in the economics literature.

Second, regarding the time dimension of LPGs, price guarantees are usually valid for a stated length of time, e.g. 30 days after purchase. Indeed, in one of the few empirical works on low-price guarantees, Arbatskaya, Hviid and Shaffer (1999) find the length of price guarantees tends to be longer when the prices are more volatile. This time dimension of LPGs, however, has not been studied in the existing literature.

The objective of this chapter is to analyze the matching competition clauses (MCCs) taking into consideration non-strategic effects and time dimention of price guarantees. To do so we construct a partial equilibrium model where a retailer sets its price but it has no influence on the distribution of prices in the market. Consumers can search for lower prices but search takes time which delays consumption. A guarantee to match any lower price offered by the retailer allows a consumer to purchase and consume now while keeping the option of reaping the benefit of a lower price that he may find later in time.[14] In this framework we analyze the retailer's decision to set its list price and the duration of its price-matching guarantee.

The central question that we are interested in answering is, how does a firm set its price and the duration of its price matching guarantee in a competitive environment? Our analysis focuses on two factors that measure the degree of competition: the variability in market prices and the percentage of customers who will continue to search for a lower price irrespective of the price and price guarantee offered by this firm ("bargain hunters"). We demonstrate that increased variability in market prices causes the firm to raise its list price and at the same time prolong the length of its matching price guarantee. As expected, the firm will reduce its list price when more of its customers are bargain hunters. Not as obvious is that the effect of having more bargain-hunting customers on the duration of the price guarantee depends on whether the firm faces the prospect of losing customers at the margin. If a marginally higher list price causes a nontrivial fraction of consumers to leave without purchase, the firm will couple the reduction of list price with a shortened price guarantee. Otherwise, the firm will lengthen the duration of the price guarantee. Using a numerical simulation we also

[14] As a recent psychological study (McConnell et al, 2000) shows, when making purchase decision, consumers mentally simulate about future events and anticipate regrets, such as what if "I find lower price elsewhere", with the result of reduced likelihood of buying. Price-matching, however, reduces the anticipated regret and increases the likelihood of purchase.

demonstrate that the adoption of a price matching guarantee can allow the firm to lower its list price and at the same time earn higher profits.

This chapter is organized as follows. The model is presented in Section 4.2. The firm's decision with regard to list price and price guarantee is analyzed in Section 4.3 by means of comparative statics. Results from numerical simulation are presented in Section 4.4 to further illustrate factors that influence the firm's list price and price guarantee policies. Conclusions are in section 4.5.

4.2 The Model

Consider a market with a large number of consumers. Each consumer buys one unit, and his reservation price is $\bar{w}$. Consumers have imperfect information about the prices offered by different firms. The only way to find out the prices offered by firms is through search. Search, of course, takes time. However, a consumer does not have to make his purchase right away. If he does not want to purchase at a price offered by the firm that he has just searched, he can delay his purchase until he finds the "right" price. What we have in mind here, therefore, are markets for goods such as consumer electronics, fashions, and books, in which consumers can usually delay their purchase for a period of time[15].

A consumer's choice over the timing of purchase is analogous to a financial call option. A call option gives the holder the right, for some specified amount of time, to pay an exercise price and in return receive an asset (say, a share of stock) that has some value (Hull, 1997). A consumer with a purchase plan likewise has the option to spend money now or in the future, in return for the product needed. By offering a matching

[15] Empirical studies (Arbatskaya, Hviid and Shaffer, 1999) show that low price guarantees are more common for durable goods, other examples including cars, computers, books, office products, pool tables, satellite TVs, tires and cellular phones, etc. With this in mind, the option of delay of purchase makes sense for such goods.

price guarantee, a firm allows the consumer to make the purchase now and still reap the benefit of any lower price that he may find later. This price guarantee policy allows the consumer to enjoy the consumer surplus at present time and also keep the options alive at the consumer's discretion. After the purchase, the consumer can continue to search for a lower price. If he does not find a lower price, the option expires without value. If he does find a lower price, on the other hand, he goes back to the firm to claim the price difference and the value of the option is positive.[16]

There are a large number of firms in this market. The probability distribution of prices offered by all these firms is uniform with support [0, m], where $m \le \bar{w}$.

We focus our analysis on the decision-making of a single firm in this market. Suppose that in each period this firm is visited by N consumers. These N consumers can be partitioned into two groups. In the absence of a price guarantee, aN of these consumers leave the firm and continue to search, while $(1-a)N$ of these consumers stop searching and make the purchase at this firm. We assume that a is a function of p, the firm's price. To be more specific, we assume that a takes the form $a = \alpha_0 + \beta(p)$, where α_0 is a positive constant, which is the fraction of those consumers who will continue to search regardless of the firm's price, and we shall call these consumers "bargain hunters". The function $\beta(p)$ is an increasing function in p. Then $\frac{da}{dp} = \beta' > 0$; in other words, if the firm raises its price, more consumers will leave and continue to search.[17]

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[16] Mazumdar and Srivastava (2001) also use the analogy between price guarantees and financial options. Their focus is on how to calculate the cost implied by a price guarantee policy. In doing so they view a LPG as a put option granted by the firm to all of its customers, and they argue that a firm offering a LPG needs to raise its list price to cover the cost of the put option.

[17] This specification is consistent with the finding from the search literature. For example, Stigler (1961) shows the number of buyers from a dealer decreases as his price increases in the form of $N_i = KNn(1-p)^{n-1}$.

By offering a price-matching guarantee the firm can induce all N consumers to purchase at this firm. Some of these consumers will continue to search after the purchase, and, if they find a lower price within time t, they can come back to this firm to invoke the price guarantee. As indicated earlier, the fraction of these active searchers is $\alpha_0 + \beta(p)$. We must also take into consideration that of the remaining consumers, some may run into a lower price during that period even though they are not actively searching. More consumers will find a lower price this way the longer the period t. Let $\gamma(t)$ be the percentage of such "passive searchers." The γ is an increasing function of t. Here we use αN to denote the number of consumers who potentially will come back to the firm to invoke the price guarantee. Then $\alpha = \alpha_0 + \beta(p) + \gamma(t)$ where $\alpha_1 = \dfrac{d\alpha}{dp} = \beta' > 0$ and $\alpha_2 = \dfrac{d\alpha}{dt} = \gamma' > 0$. Subscripts 1 and 2 are used to denote the derivative with respect to p and t, respectively. Note that the above specification implies $\alpha_{12} = \alpha_{21} = \dfrac{d^2\alpha}{dpdt} = 0$.

Following Hviid and Shaffer (1999), we assume that consumers have to incur a hassle cost c_s if they want to invoke the price guarantee. The value of c_s is exogenously fixed.

Lemma 1 *The firm can use a MCC to induce all N consumers to purchase at this firm by setting p and t in such way that* $(e^{rt} - 1)(\overline{w} - p) \geq c_s$.

Proof: By definition those $(1-\alpha)$N consumers who will not continue to search will buy from this firm. What we need to prove is that the price guarantee will induce purchase by those αN consumers who would have left the firm in the absence of the price guarantee. Consider one such consumer. If the firm does not offer a MCC, she leaves the firm and continues to search. Suppose that she eventually makes the purchase at time t at a price x, the lowest price from her search. The present value of the surplus she gets would be $(\bar{w}-x)e^{-rt}$. Now consider the case where the firm offers a MCC. The consumer can choose to ignore the MCC and continue the search, in which case her payoff would remain the same as before. If, on the other hand, she purchases at this firm and finds a lower price later through her continued search, her surplus would be $(\bar{w}-p)+(p-x-c_s)e^{-rt}$, which differs from the surplus in the previous case by the amount of $(1-e^{-rt})(\bar{w}-p)-c_s e^{-rt}$. Therefore, she will purchase from this firm as long as this difference is nonnegative, which implies $(e^{rt}-1)(\bar{w}-p)\geq c_s$.

QED

To keep the analysis simple, we assume that all of the $\alpha(p,t)N$ consumers come to activate the price guarantee at time t, at the end of the guarantee period.[18] Given the uniform distribution of prices, the expected value of the refunds that the firm is going to pay at time t is,

$$\int_0^p (p-x)\frac{x-0}{m-0}dx = \frac{p^3}{6m} \tag{4.1}$$

In equation(4.1), x stands for the lowest price that a consumer finds in the market. Obviously, there is a difference between the lowest price in the market and the lowest price that a particular consumer finds. The former is 0 while the latter follows a uniform distribution.

[18] This is a reasonable assumption in this context because the actual duration of a price guarantee is usually less than 30 days. The interest income lost due to the delay in claiming the refund is usually small relative to the expected gain from finding a lower price, especially for large ticket items such as consumer electronics. In such a situation, consumers prefer waiting till the end of the guarantee period to invoke the price guarantee.

The firm's optimization problem is to maximize its expected profit, which can be written as

$$\underset{p,t}{Max}\,\pi = Np - \alpha(p,t)N\frac{p^3 e^{-rt}}{6m} \tag{4.2}$$

Subject to

$$p \le \overline{w} \tag{4.3}$$

$$p \le \overline{w} + (1-e^{-rt})(\overline{w}-p) - c_s e^{-rt} \tag{4.4}$$

$$(e^{rt}-1)(\overline{w}-p) \ge c_s \tag{4.5}$$

This optimization problem can be simplified by noting the following redundancy in constraints. Rewriting (4.4), we have

$$p \le \overline{w} - \frac{c_s}{2e^{rt}-1} \tag{4.6}$$

Rewriting of (4.5) yields,

$$p \le \overline{w} - \frac{c_s}{e^{rt}-1} \tag{4.7}$$

From(4.6) and (4.7), we have $p \le \overline{w} - \frac{c_s}{e^{rt}-1} < \overline{w} - \frac{c_s}{2e^{rt}-1} < \overline{w}$, the second and third inequalities coming from our assumption that the hassle cost is positive, $c_s > 0$. Thus, constraints (4.3) and (4.4) are not binding if constraint (4.5) is satisfied. In other words, (4.3) and (4.4) are redundant. The firm's optimization problem is then simplified to the maximization of (4.2) subject to (4.5). Rearrange, and the Lagrangean associated with this problem is

$$L = f(p,t) - \lambda g(p,t) \tag{4.8}$$

where $f(p,t) = 6mp - \alpha(p,t)p^3 e^{-rt}$ and $g(p,t) = (e^{rt} - 1)(p - \overline{w}) + c_s \leq 0$.

By Kuhn-Tucker conditions, we have

$$\frac{\partial L}{\partial p} = 6m - e^{-rt}(\alpha_1 p^3 + 3\alpha p^2) - \lambda(e^{rt} - 1) = 0 \tag{4.9}$$

$$\frac{\partial L}{\partial t} = (\alpha r - \alpha_2)p^3 e^{-rt} + \lambda(\overline{w} - p)re^{rt} = 0 \tag{4.10}$$

$$(e^{rt} - 1)(p - \overline{w}) + c_s \leq 0 \tag{4.11}$$

$$\lambda > 0 \text{ if } g(p,t) = (e^{rt} - 1)(p - \overline{w}) + c_s = 0 \tag{4.12}$$

$$\lambda = 0 \text{ if } g(p,t) = (e^{rt} - 1)(p - \overline{w}) + c_s < 0 \tag{4.13}$$

In the following section, we study how the firm's profit, price and duration of price guarantee are affected by various exogenous variables.

4.3 Comparative Statics

The two exogenous variables that are of particular interest are m and α_0, which measure, respectively, the variability in market prices and the percentage of active searchers who continue search regardless of p and t (bargain hunters). By applying the envelop theorem to the firm's objective function (4.2), that is $\underset{p,t}{Max}\,\pi = Np - \alpha(p,t)N\frac{p^3e^{-rt}}{6m}$, it is straightforward to find out how the firm's profits are affected by these two parameters.

$$\frac{\partial \pi}{\partial m} = \alpha(p,t)N\frac{p^3e^{-rt}}{6m^2} > 0 \tag{4.14}$$

$$\frac{\partial \pi}{\partial \alpha_0} = -N\frac{p^3e^{-rt}}{6m} < 0 \tag{4.15}$$

Accordingly, we have our first proposition:

Proposition 1 *The firm's profit is an increasing function of m but a decreasing function of* α_0.

More importantly, we are interested in how the price and the duration of the matching price guarantee, p and t, are affected by m and α_0. To do so, we must consider separately the case where constraint (4.5) is binding and the case where it is not.

4.3.1 Binding Constraint

When constraint (4.5) is binding, we have,

$$g(p,t) = (e^{rt} - 1)(p - \overline{w}) + c_s = 0 \tag{4.16}$$

By taking the total differential of equation(4.16),(4.9) and(4.10), we arrive at,

$$
\begin{aligned}
-g_1 dp - g_2 dt &= 0 \\
-g_1 d\lambda + (f_{11} - \lambda g_{11})dp + (f_{12} - \lambda g_{12})dt &= -6dm + 3p^2 e^{-rt} d\alpha_0 \\
-g_2 d\lambda + (f_{21} - \lambda g_{21})dp + (f_{22} - \lambda g_{22})dt &= 0 - p^3 r e^{-rt} d\alpha_0
\end{aligned} \tag{4.17}
$$

where $g_1 = e^{rt} - 1$, $g_2 = -re^{rt}(\overline{w} - p)$, $g_{11} = 0$, $g_{12} = g_{21} = re^{rt}$, $g_{22} = -r^2 e^{rt}(\overline{w} - p)$, $f_{11} = -6pe^{-rt}(\alpha + \alpha_1 p)$, $f_{12} = f_{21} = p^2 e^{-rt}(3\alpha r + \alpha_1 rp - 3\alpha_2)$ and $f_{22} = p^3 re^{-rt}(2\alpha_2 - \alpha r)$.

To ensure a unique local interior maximum, subject to $g(p,t) = 0$, the second order condition must satisfy,

$$
|H| = \begin{vmatrix} 0 & g_1 & g_2 \\ g_1 & (f_{11} - \lambda g_{11}) & (f_{12} - \lambda g_{12}) \\ g_2 & (f_{21} - \lambda g_{21}) & (f_{22} - \lambda g_{22}) \end{vmatrix} > 0 \tag{4.18}
$$

Combining with (4.10) and rearranging, we have

$$
\begin{aligned}
|H| = {} & 2p^2 r[(\overline{w} - p)\alpha_1 re^{rt} + (e^{rt} - 1)(\alpha_2 - \alpha r)][3(\overline{w} - p) - p(1 - e^{-rt})] \\
& + p^3 r(e^{rt} - 1)(\alpha_2 - 2\alpha r + \alpha_2 e^{-rt}) + 6\alpha pr^2 e^{rt}(\overline{w} - p)^2 > 0
\end{aligned} \tag{4.19}
$$

We have $\alpha_2 - \alpha r > 0$ from(4.10), and thus for (4.19) to be positive, a set of sufficient conditions are

$$
3(\overline{w} - p) - (1 - e^{-rt})p > 0 \tag{4.20}
$$

$$
\alpha_2 > 2\alpha r \tag{4.21}
$$

Inequality (4.20) implies that $\dfrac{\overline{w}}{p} > \dfrac{4 - e^{-rt}}{3} = k$. Note that $\dfrac{4}{3} > k > 1$ because $0 < e^{-rt} < 1$. Thus, a sufficient condition for (4.20) to hold is $p < \dfrac{3}{4}\overline{w}$.

Proposition 2 *In the case where constraint (4.5) is binding, p and t are increasing functions of m.*

Proof: To study the comparative statics with respect to m, first, we let $d\alpha_0 = 0$, but $dm \neq 0$, rearrange and we have the matrix equation:

$$\begin{pmatrix} 0 & -g_1 & -g_2 \\ -g_1 & (f_{11} - \lambda g_{11}) & (f_{12} - \lambda g_{12}) \\ -g_2 & (f_{21} - \lambda g_{21}) & (f_{22} - \lambda g_{22}) \end{pmatrix} \begin{pmatrix} \partial\lambda/\partial m \\ \partial p/\partial m \\ \partial t/\partial m \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 0 \end{pmatrix} \tag{4.22}$$

By Cramer's rule, we can have all three comparative statics, but we have interest in only the following two,

$$\frac{\partial p}{\partial m} = \frac{1}{|J|} \begin{vmatrix} 0 & 0 & -g_2 \\ -g_1 & -6 & (f_{12} - \lambda g_{12}) \\ -g_2 & 0 & (f_{22} - \lambda g_{22}) \end{vmatrix} = \frac{6r^2 e^{2rt} (\overline{w} - p)^2}{|J|} > 0 \tag{4.23}$$

$$\frac{\partial t}{\partial m} = \frac{1}{|J|} \begin{vmatrix} 0 & -g_1 & 0 \\ -g_1 & (f_{11} - \lambda g_{11}) & -6 \\ -g_2 & (f_{21} - \lambda g_{21}) & 0 \end{vmatrix} = \frac{6re^{rt} (\overline{w} - p)(e^{rt} - 1)}{|J|} > 0 \tag{4.24}$$

The Jacobian $|J|$ is positive since

$$|J| = \begin{vmatrix} 0 & -g_1 & -g_2 \\ -g_1 & (f_{11} - \lambda g_{11}) & (f_{12} - \lambda g_{12}) \\ -g_2 & (f_{21} - \lambda g_{21}) & (f_{22} - \lambda g_{22}) \end{vmatrix} = |H| > 0.$$

QED

Proposition 2 states that when the variability in market prices increases (i.e. m rises), the firm will raise its list price and at the same time lengthen the duration of the price guarantee. Intuitively the firm can increase its profits by taking advantage of the increased variability and raising its list price. A higher list price, however, reduces consumers' surplus and the searchers will leave the firm without purchasing unless the reduction is offset by the improvement in price guarantee. Hence the longer duration of the matching price guarantee.

Proposition 3 *In the case where constraint (4.5) is binding, p and t are decreasing functions of α_0 given that the second-order sufficient condition(4.20) is satisfied, i.e.,* $3(\overline{w}-p)-(1-e^{-rt})p>0$.

Proof: we let $d\alpha_0 \neq 0$, but $dm=0$, rearrange and we have the matrix equation

$$\begin{pmatrix} 0 & -g_1 & -g_2 \\ -g_1 & (f_{11}-\lambda g_{11}) & (f_{12}-\lambda g_{12}) \\ -g_2 & (f_{21}-\lambda g_{21}) & (f_{22}-\lambda g_{22}) \end{pmatrix} \begin{pmatrix} \partial\lambda/\partial\alpha_0 \\ \partial p/\partial\alpha_0 \\ \partial t/\partial\alpha_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3p^2e^{-rt} \\ -p^3re^{-rt} \end{pmatrix} \tag{4.25}$$

By Cramer's rule, we can have all three comparative statics, and again, we are interested in only the following two,

$$\begin{aligned} \frac{\partial p}{\partial\alpha_0} &= \frac{1}{|J|}\begin{vmatrix} 0 & 0 & -g_2 \\ -g_1 & 3p^2e^{-rt} & (f_{12}-\lambda g_{12}) \\ -g_2 & -p^3re^{-rt} & (f_{22}-\lambda g_{22}) \end{vmatrix} \\ &= \frac{-p^2r^2e^{rt}}{|J|}(\overline{w}-p)[3(\overline{w}-p)-p(1-e^{-rt})] \end{aligned} \tag{4.26}$$

$$\begin{aligned} \frac{\partial t}{\partial\alpha_0} &= \frac{1}{|J|}\begin{vmatrix} 0 & -g_1 & 0 \\ -g_1 & (f_{11}-\lambda g_{11}) & 3p^2e^{-rt} \\ -g_2 & (f_{21}-\lambda g_{21}) & -p^3re^{-rt} \end{vmatrix} \\ &= \frac{-p^2r}{|J|}(e^{rt}-1)[3(\overline{w}-p)-p(1-e^{-rt})] \end{aligned} \tag{4.27}$$

The signs of $\frac{\partial p}{\partial \alpha_0}$ and $\frac{\partial t}{\partial \alpha_0}$ are ambiguous. But given that condition (4.20) is satisfied, we have $\frac{\partial p}{\partial \alpha_0} < 0$, and $\frac{\partial t}{\partial \alpha_0} < 0$.

QED

When there are more bargain hunters, the firm faces more intense competition with the rest of the market. It then has to improve its offer to its potential customer by either reducing its list price or lengthening the price guarantee or both. It is interesting to note that under the condition specified in Proposition 3, the firm reduces the list price but shortens the duration of price guarantee. It is not surprising that the firm cuts the list price, due to the discipline effects exerted by the active search of more consumers; what is surprising is that the firm chooses to shorten rather than lengthen the duration of price guarantee. To understand the intuition behind this, note from the objective function (4.2) and (4.10) that the present value of the refunds given to the consumers under the price guarantee is an increasing function of t. As α_0 becomes larger and hence more customers invoke the price guarantee, the policy (for a given t) becomes more costly to the firm. This leads the firm to cut back on price guarantee and retain customers through a lower list price.

4.3.2 Slack Constraint

When constraint (4.5) is not binding, $g(p,t) = (e^{rt} - 1)(p - \overline{w}) + c_s < 0$. Thus, $\lambda = 0$. The firm's optimization problem becomes the following one:

$$\underset{p,t}{Max}\, \pi = 6mp - \alpha(p,t)p^3 e^{-rt} \tag{4.28}$$

The first-order conditions are

$$\frac{\partial \pi}{\partial p} = 6m - e^{-rt}(\alpha_1 p^3 + 3\alpha p^2) = 0 \tag{4.29}$$

$$\frac{\partial \pi}{\partial t} = (\alpha r - \alpha_2) p^3 e^{-rt} = 0 \tag{4.30}$$

To ensure that the second order conditions are satisfied, we need,

(A1) $\pi_{11} = \dfrac{\partial^2 \pi}{\partial p^2} = -e^{-rt} p(6\alpha + 6\alpha_1 p + \alpha_{11} p^2) < 0$;

(A2) $\pi_{22} = \dfrac{\partial^2 \pi}{\partial t^2} = p^3 e^{-rt}(2\alpha_2 r - \alpha_{22} - \alpha r^2) = p^3 e^{-rt}(\alpha_2 r - \alpha_{22}) < 0$; and

(A3) $\pi_{11}\pi_{22} - (\pi_{12})^2 > 0$, where $\pi_{12} = \dfrac{\partial^2 \pi}{\partial p \partial t} = e^{-rt}(3\alpha r p^2 + \alpha_1 r p^3 - 3\alpha_2 p^2) = \alpha_1 r p^3 e^{-rt}$ (the last equality comes from (4.30)).

A sufficient condition for (A1) to hold is that the elasticity of α with respect p is greater than -6, i.e.,

$$\frac{\alpha_{11}}{\alpha_1} p > -6 \tag{4.31}.$$

A sufficient condition to ensure (A2) is $\dfrac{\alpha_{22}}{\alpha_2} > r$. Assumption (A3) can be written as

$$p^4 e^{-2rt}[(6\alpha + 6\alpha_1 p + \alpha_{11} p^2)(\alpha_{22} - \alpha_2 r) - \alpha_1^2 r^2 p^2] > 0 \tag{4.32}$$

Rearranging and simplifying, we have

$$(6\alpha + 3\alpha_1 p + \alpha_{11} p^2)(\alpha_{22} - \alpha_2 r) - \alpha_1 p(3\alpha_2 r - 3\alpha_{22} + \alpha_1 r^2 p) > 0 \tag{4.33}$$

Condition (4.33) is satisfied if

$$6\alpha + 3\alpha_1 p + \alpha_{11} p^2 > 0 \tag{4.34}$$

$$3\alpha_2 r - 3\alpha_{22} + \alpha_1 r^2 p < 0 \tag{4.35}$$

Combined with our assumption that $\alpha_1 > 0$, (4.34) can be written in the elasticity form as

$$\frac{6\alpha}{\alpha_1 p} + \frac{\alpha_{11} p}{\alpha_1} > -3 \tag{4.36}$$

To conduct comparative statics, totally differentiate the first-order conditions (4.29) and (4.30),

$$\pi_{11} dp + \pi_{12} dt = -6dm + 3p^2 e^{-rt} d\alpha_0 \tag{4.37}$$

$$\pi_{21} dp + \pi_{22} dt = 0dm - rp^3 e^{-rt} d\alpha_0 \tag{4.38}$$

***Proposition** 4 Suppose that constraint (4.5) is not binding and that the second-order conditions (A1)-(A3) are satisfied. The firm's p and t are increasing functions in m.*

Proof: Let $dm \neq 0$, but $d\alpha_0 = 0$, rearrange and we have the matrix equation

$$\begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} \begin{pmatrix} dp/dm \\ dt/dm \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} \tag{4.39}$$

The solution, by Cramer's rule, is found to be

$$\frac{\partial p}{\partial m} = \frac{1}{|J|}\begin{vmatrix} -6 & \pi_{12} \\ 0 & \pi_{22} \end{vmatrix} = \frac{-6\pi_{22}}{|J|}$$
$$\frac{\partial t}{\partial m} = \frac{1}{|J|}\begin{vmatrix} \pi_{11} & -6 \\ \pi_{21} & 0 \end{vmatrix} = \frac{6\pi_{21}}{|J|} = \frac{6e^{-rt}\alpha_1 rp^3}{|J|} \tag{4.40}$$

Given that the second-order conditions are satisfied, π_{11} and π_{22} are negative and the Jacobian, $|J| = \pi_{11}\pi_{22} - (\pi_{12})^2 > 0$, in the denominator is positive. Thus, $\frac{dp}{dm} > 0$ and $\frac{dt}{dm} > 0$.

QED

On the surface, Proposition 4 appears to show the same comparative static results as Proposition 2; the price and duration of price guarantee rise with the variability of market prices. In fact, the results in these two propositions operate through different channels. Note that constraint (4.5) is binding in Proposition 2 but not binding in Proposition 4. When the constraint is not binding, the firm does not have to worry about losing customers when it raises its list price marginally. Consequently, the lengthening of price guarantee accompanied the price increase is not there to retain customers. Rather, it is there to mitigate, in present value term, the expected increase in customer refunds as more customers will come back to activate the price guarantee. As can be seen from the firm's profit function in (4.2), an increase in t reduces the present value of any given amount of refund to be given out at time t.[19]

***Proposition** 5 Suppose that constraint (4.5) is not binding. The firm's p is a decreasing function but t is an increasing function of α_0.*

[19] An increase in t also raises the value of $\alpha(p,t)$ At the optimal solution, this effect is dominated by the change in the present value term e^{-rt}.

Proof: Now we let the exogenous variable α_0 vary alone, rearrange and we have the matrix equation

$$\begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} \begin{pmatrix} dp/d\alpha_0 \\ dt/d\alpha_0 \end{pmatrix} = \begin{pmatrix} 3p^2e^{-rt} \\ -rp^3e^{-rt} \end{pmatrix} \tag{4.41}$$

The solution, by Cramer's rule, is found to be

$$\frac{\partial p}{\partial \alpha_0} = \frac{1}{|J|} \begin{vmatrix} 3p^2e^{-rt} & \pi_{12} \\ -rp^3e^{-rt} & \pi_{22} \end{vmatrix} = \frac{p^5e^{-2rt}(3\alpha_2 r - 3\alpha_{22} + \alpha_1 r^2 p)}{|J|} \tag{4.42}$$

$$\begin{aligned} \frac{\partial t}{\partial \alpha_0} = \frac{1}{|J|} \begin{vmatrix} \pi_{11} & 3p^2e^{-rt} \\ \pi_{21} & -rp^3e^{-rt} \end{vmatrix} &= \frac{p^4e^{-2rt}(\alpha_{11}p^2r + 3\alpha_1 pr - 3\alpha r + 9\alpha_2)}{|J|} \\ &= \frac{p^4re^{-2rt}(\alpha_{11}p^2 + 3\alpha_1 p + 6\alpha)}{|J|} \end{aligned} \tag{4.43}$$

The last equality comes from(4.30), i.e., $\alpha r = \alpha_2$.

The signs of $\frac{\partial p}{\partial \alpha_0}$ and $\frac{\partial t}{\partial \alpha_0}$ are in general ambiguous. However, given that conditions (4.34) and (4.35) are satisfied, we have $\frac{\partial p}{\partial \alpha_0} < 0$ and $\frac{\partial t}{\partial \alpha_0} > 0$.

QED

No matter whether the constraint is binding or not, the increase in the percentage of bargain hunters dampens the price charged by this firm. It is also interesting to note the difference between Proposition 5 and Proposition 3. When constraint (4.5) is not binding, the firm lengthens (rather than shortening) the effective period of the matching price guarantee when there are more bargain hunters. Since (4.5) is not binding, this is done not because the firm is concerned about losing customers. Rather it is done to reduce the current value of increased customer refund resulted from an increase in the number of customers claiming the refund.

4.4 Numerical Simulation

In this section, we present the results from numerical simulations of the model. The purposes of the numerical simulations are, first, to confirm the comparative static results obtained in Section 4.3, and second, to illustrate the effects of adopting a price guarantee policy on the firm's list price and profit.

For numerical simulations we chose a functional form for $\alpha(p,t)$ that satisfies the assumptions of the model. Specifically,

$$\alpha(p,t) = \alpha_0 + bp^{1/2} + ct^2 \tag{4.44}$$

Function (4.44) satisfies $\alpha_1 > 0$, $\alpha_2 > 0$, and the second-order conditions which guarantee that the firm can maximize its object function.

With the aid of a computer program it is straightforward to calculate the profit-maximizing p and t using the function (4.44). The numerical results show that constraint (4.5) is binding if the reservation price of consumers is low and/or the hassle cost to activate the price guarantee is high; otherwise, the constraint is not binding.

Recall from Propositions 2 and 3 that p and t are increasing functions of m, and that when condition (4.20) is satisfied, p and t are decreasing functions of α_0. These results are illustrated in Figures 4 and 5. The parameter values used in this set of simulations are $b = 0.06$, $c = 0.01$, $c_s = 3$, $r = 0.1$ and $w = 16$.

Figures 6 and 7 illustrate the comparative static results in the case where constraint (4.5) is not binding. Figure 6 corresponds to the parameter values

$m = 15$, $\beta = 0.08$, $\gamma = 0.01$ and $r = 0.1$; it shows how p and t responds to changes in α_0. We see that an increase in α_0 results in a decrease in p but an increase in t. Figure 7 plots the changes of t and p with the change of m associated with parameter values $\alpha_0 = 0.3$, $b = 0.08$, $c = 0.01$ and $r = 0.1$. As m is increased, both p and t increase.

In the remainder of this section, we use the numerical example to illustrate how the adoption of a price guarantee policy affects the firm's price and profit. To do so, we compare the solution to firm's optimization problems when the firm adopts the price guarantee policy with that when the firm does not adopt the policy.

If the firm does not adopt the price guarantee policy, its optimization problem is

$$\underset{p}{Max}\, Np - \alpha(p,0)Np \tag{4.45}$$

subject to

$$p \leq \bar{w} \tag{4.46}$$

Combining $\alpha(p,0) = \alpha_0 + bp^{\frac{1}{2}}$ and first order condition $N(1 - \alpha_1 p - \alpha) = 0$, we obtain

$$p^* = \frac{4(1-\alpha_0)^2}{9b^2} \tag{4.47}$$

Inserting this solution into (4.45), the maximum profit is

$$\pi^* = \frac{4N}{27b^2}(1-\alpha_0)^3 \tag{4.48}$$

Figures 8, 9 and 10 illustrate the case where constraint (4.5) is binding. In figure 6, the effects of adopting price matching guarantee on price are given by comparing the list price with LPGs and without LPGs. And the effects on the profits are shown in Figure 9 and Figure 10. Similar comparisons are done for the case where the constraint is not binding, as illustrated in Figures 11, 12 and 13.

These figures show that the adoption of price guarantee policy can improve the firm's profit at the same time as it reduces its list price. It is perhaps not surprising that a firm can increase its profit by offering a price guarantee since the policy increases its sales. What is interesting is that the firm's list price falls, in contrast to the existing models in the literature where the adoption of a price guarantee policy raises firms' prices. The key factor here, of course, is that the firm in our model operates in a competitive environment. The firm's choice of list price and price guarantee policy has no strategic effect on other firms in the market. Consequently, the price guarantee is used only as a way to induce customers to make the purchase; it is not used as a strategic instrument to facilitate collusion.

4.5 Conclusions

In this chapter we study a firm's decision with regard to list price and price-matching policy in a competitive environment. In contrast to existing models of price guarantees where firms have market power, the firm in our model is unable to act strategically and its actions has no impact on other firms in the market. In such an environment the firm can use the price-matching policy only as a way to induce more customers to purchase. Another contribution of this chapter is that we study a dimension of the matching price guarantee that has so far been ignored in the literature, namely the duration of the matching price guarantee.

This new framework allows us to study price guarantees from a perspective very different from the existing literature. Unlike the price discrimination theory, our analysis shows that price guarantee policy does not weaken the discipline on prices brought about by bargain-hunting consumers. In fact, the firm's list price is lower when there are more bargain hunters among its potential customers. Furthermore, in contrast to collusion theory, our analysis demonstrates that the adoption of a price guarantee can in fact lead the firm to reduce its list price. An obvious conclusion one can draw from our analysis, then, is that the effects of a price guarantee policy in a competitive market are very different from those in an oligopolistic market. Consequently, the policy prescriptions for dealing with price guarantees should be cognizant of market structure.

On the time dimension of price guarantee, our analysis shows that increased variability in market prices causes the firm to prolong the length of its matching price guarantee, and this accords with the results of Arbatskaya, Hviid and Shaffer (1999). More interestingly, the effect of having more bargain hunters among its customers on the duration of price guarantee depends on whether the firm faces the prospect of losing customers at the margin. If a marginally higher list price causes a nontrivial fraction of consumers to leave without purchase, the duration of price guarantee is shortened. Otherwise, the firm will lengthen the duration of price guarantee. These results suggest that the suppression of time dimension of price guarantees, as done in the existing models, is not necessarily without loss of generality.

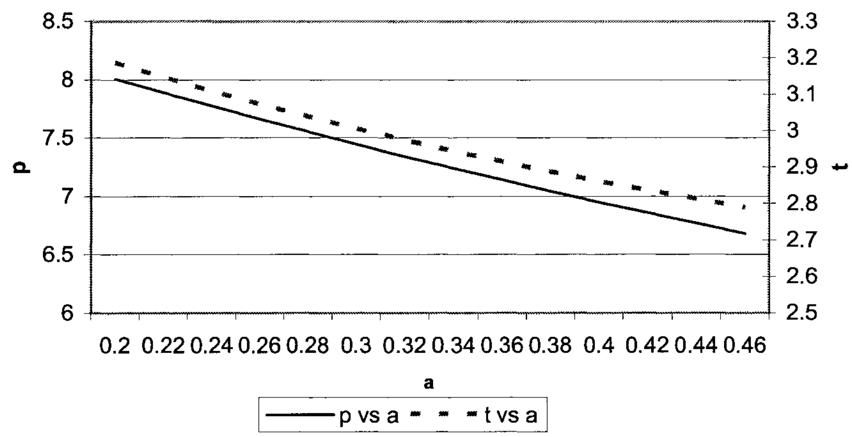

Figure 4 p^* and t^* vs α_0 (binding constraint)

Note: $m = 12$, $b = 0.06$, $c = 0.01$, $c_s = 3$, $r = 0.1$ and $w = 16$

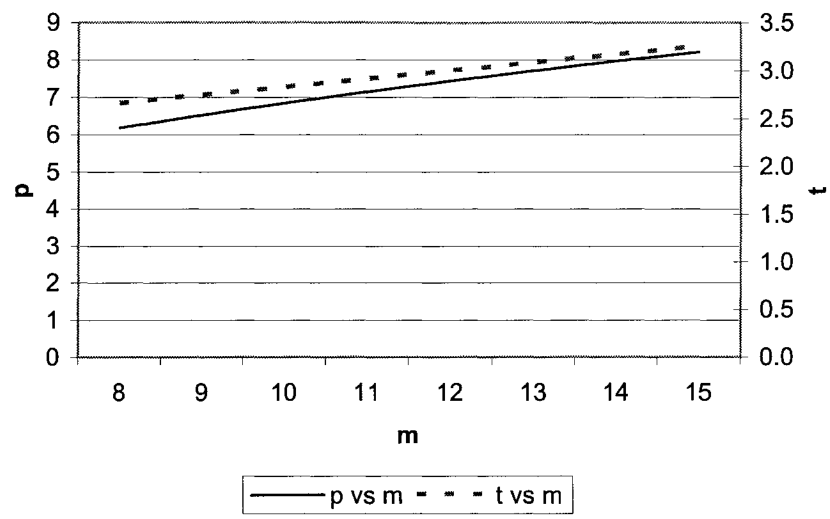

Figure 5 p^* and t^* vs. m (binding constraint)

Note: $\alpha_0 = 0.3$, $b = 0.06$, $c = 0.01$, $c_s = 3$, $r = 0.1$ and $w = 16$

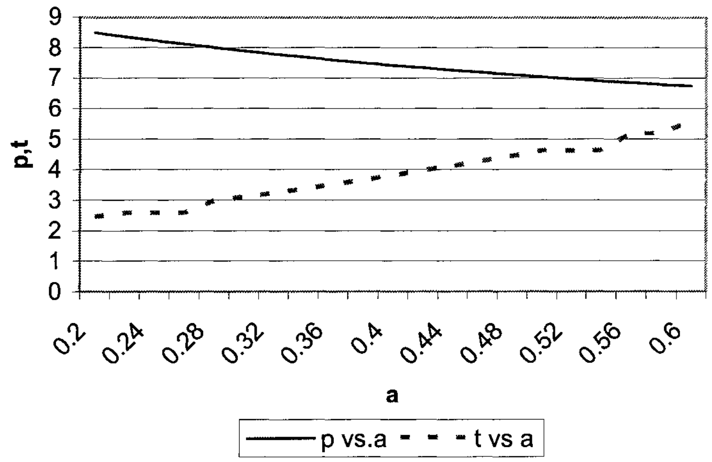

Figure 6 p^* and t^* vs. α_0 (slack constraint)

Note: $m = 15$, $b = 0.08$, $c = 0.01$ and $r = 0.1$

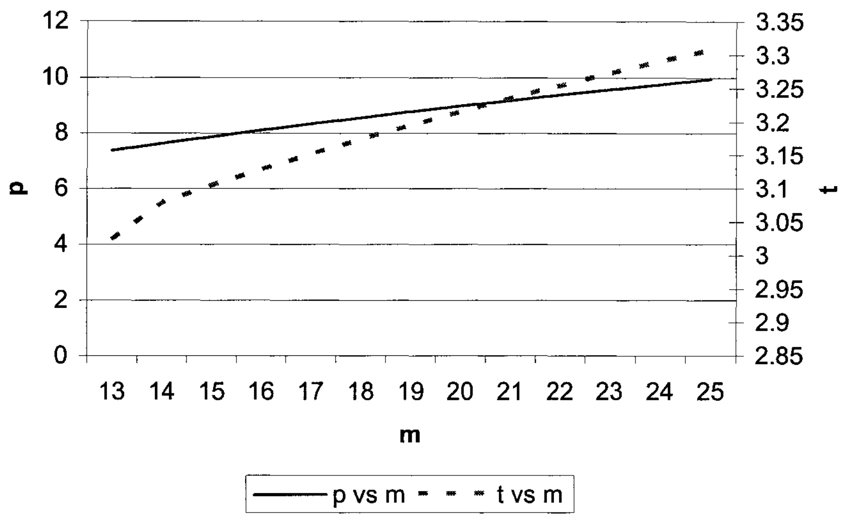

Figure 7 p^* and t^* vs. m (slack constraint)

Note: $\alpha_0 = 0.3, b = 0.08, c = 0.01$ and $r = 0.1$

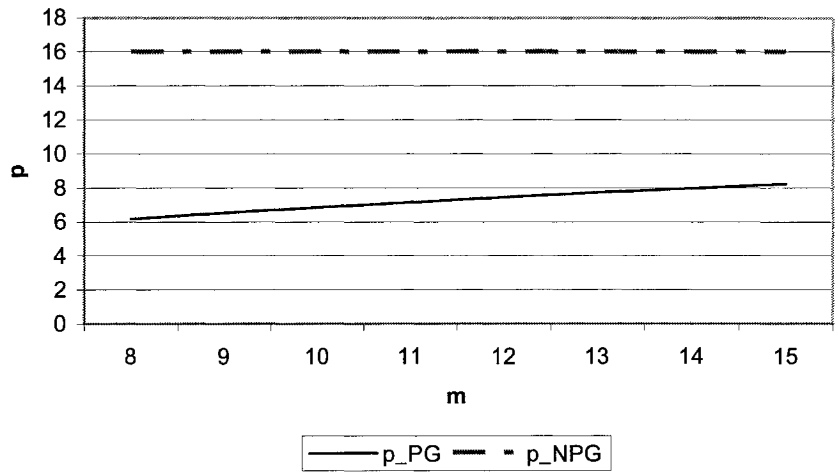

Figure 8 $p^* \_PG$ and $p^* \_NPG$ vs. m (binding constraint)

Note: $\alpha_0 = 0.3$, $b = 0.08$, $c = 0.02$, $c_s = 3$, $r = 0.1$ and $w = 16$

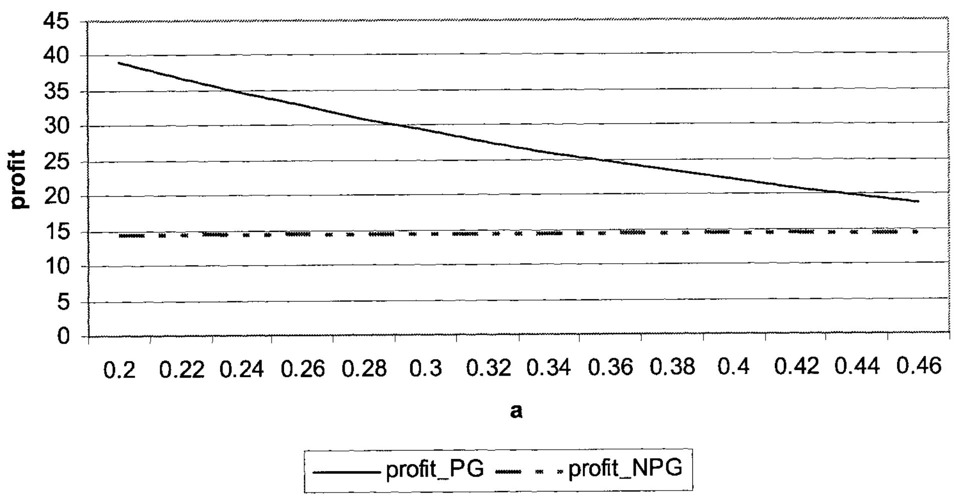

Figure 9 π^*_PG and π^*_NPG vs. α_0 (binding constraint)

Note: $m = 12$, $b = 0.06$, $c = 0.01$, $c_s = 3$, $r = 0.1$ and $w = 16$

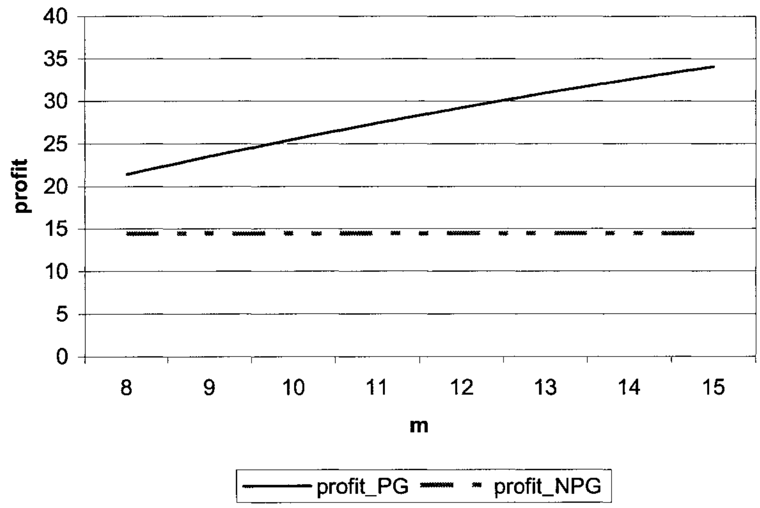

Figure 10 π^*_PG and π^*_NPG vs. m (binding constraint)

Note: $\alpha_0 = 0.3$, $b = 0.06$, $c = 0.01$, $c_s = 3$, $r = 0.1$ and $w = 16$

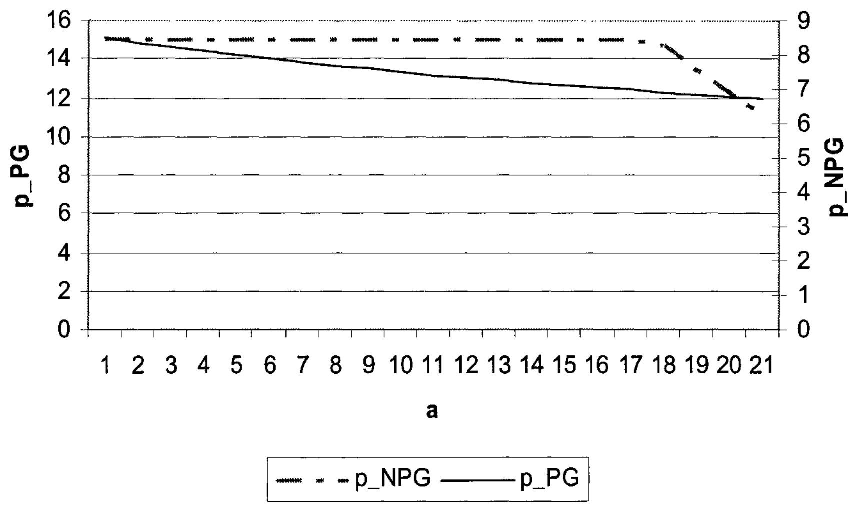

Figure 11 p^{*}_PG and p^{*}_NPG vs. α_0 (slack constraint)

Note: $m = 15, b = 0.08, c = 0.01$ and $r = 0.1$

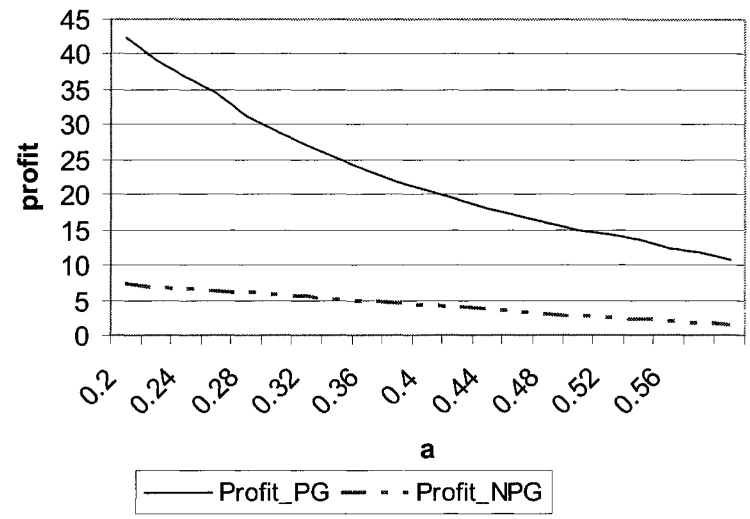

Figure 12 π^*_PG and π^*_NPG vs. α_0 (slack constraint)

Note: $m = 15, b = 0.08, c = 0.01$ and $r = 0.1$

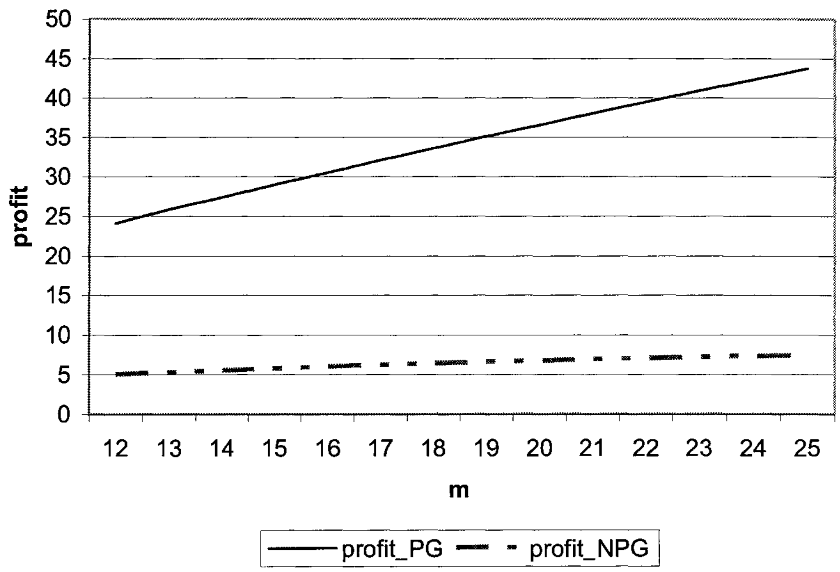

Figure 13 π^*_PG and π^*_NPG vs. m (slack constraint)

Note: $\alpha_0 = 0.3, b = 0.08, c = 0.01$ and $r = 0.1$

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General Conclusions

The widespread use of low-price guarantee in many retail markets intrigued volumes of economic study. The general conclusion of the existing theoretical literature is that the low-price guarantees result in prices that are not low at all. However, the empirical studies and our casual observations do not provide strong grounds. The three models in this thesis extend previous literature by studying the effects of low-price guarantee in more general frameworks where incomplete information being the common theme.

In chapter two, I study the effects of low-price guarantee in the search model of Salop and Stiglitz (1977). This model provides a general framework for the comparison of matching price guarantee and beating price guarantee: The number of firms in the market is determined by free entry and consumers have incomplete information about prices and differ in their search costs. It is shown that MCCs have stronger collusive effects than BCCs, however, these effects evaporate with the introduction of an arbitrarily small hassle cost. A possible extension would be to endogenize the choice of offering low-price guarantee in a richer dynamic search model. And instead of assuming fixed hassle costs, another avenue for future research is to model the hassle costs imposed by firms as a way to mitigate risk which firms occurred because of the adoption of low-price guarantees.

In chapter three, I extend the Milgrom and Robers model (1982) to study the entry deterrence effects of low-price guarantee. In this model, the potential entrant has no complete information about the true unit cost of the incumbent. It is demonstrated that under certain conditions, the high-cost incumbent can imitate the behavior of the low-cost one and deter entry even in situations where this would not be possible in the absence of a price guarantee. Therefore, the corresponding policy suggestion is to prohibit this pricing behavior when it is exerted by a monopolist.

In chapter four, the non-strategic effects and the time dimension of low-price guarantee are explored in a competitive environment. The analysis shows that the discipline effects of bargain hunters on the market prices do not diminish as predicted by the price discriminating theory. The adoption of price guarantee is a way to induce purchase which can allow the firm to lower its list price and at the same time earn higher profits. A possible line of further extension is to assume that $\ln P$ follows a log normal distribution, i.e., $\ln P \sim N(m, \sigma^2)$, and allow firms to choose the types of low-price guarantees, BCCs or MCCs, together with the time dimension of post-sale search.