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The Effect of Tool Sharing on Reliability of Flexible Manufacturing Systems

A thesis submitted to the
Faculty of the Graduate Studies and Research
in partial fulfilment of the requirements for the degree of
Master of Applied Science in Mechanical Engineering

By
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ABSTRACT

A major consideration in opting for reliability and flexibility in Flexible Manufacturing Systems (FMSs) is to enhance the availability of resources in order to maintain an uninterrupted production. This means that sufficient redundancies must be foreseen at the preliminary production planning stage to cope with the random breakdowns of components.

In this research effort four mathematical models were developed to determine the spare requirements for tooling system in FMSs, so that a desired system reliability is achieved with minimum cost and/or tool slots occupancy of the system. For the first time, the influences of tool sharing on cost, reliability, spares requirement, and tool magazines capacity of FMSs, in which tools and tool transporter are subject to general failure distributions, were analyzed.

The developed models have been applied to a hypothetical example and the computational results were compared, for the case where tool sharing is not applicable and where tool sharing can be implemented. Several sets of sensitivity analysis were also performed on different system parameters through which the effects of the number of tools shared among machines, the required system reliability, and operating times for tools on the system reliability and cost were assessed.

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TABLE OF CONTENTS

ABSTRACT	i
ACKNOWLEDGEMENT	ii
TABLE OF CONTENTS	iii
LIST OF FIGURES	vi
LIST OF TABLES	vii
CHAPTER 1 INTRODUCTION	1
1.1 General Need and Overview	1
1.2 Objectives of the Research	3
1.3 Organization of the Research	4
CHAPTER 2 FUNDAMENTAL CONCEPTS AND DEFINITIONS	5
2.1 FMS and Tool Management	5
2.2 Reliability and Availability	7
2.3 Types of System Configuration	9

CHAPTER 3	LITERATURE SURVEY	12
3.1	Tool Management in FMS	13
3.1.1	Tool-Life Distributions and Models	13
3.1.2	Tooling Economics and Strategies	17
3.1.3	Spares Management in FMS	23
3.2	Reliability-Based Optimization	25
3.2.1	Optimization of System Reliability with Redundancy Models	25
3.2.2	Spare Provision and Standby Redundancy Allocation	28
CHAPTER 4	MODELS DEVELOPMENT	31
4.1	System Definition	31
4.2	Mathematical Modelling for the Proposed System	33
4.2.1	Model I	37
4.2.2	Model II	43
4.2.3	Model III	46
4.2.4	Model IV	49
CHAPTER 5	COMPUTATIONAL RESULTS AND SENSITIVITY ANALYSIS	56
5.1	Description of the Proposed System	57
5.2	Computational Experiments	63
5.2.1	Minimum Tooling Cost Criterion	66
5.2.2	Minimum Tool Slots Occupation Criterion	69
5.2.3	Joint Consideration of Tooling Cost and Tool Slots Occupancy	71

5.2.4	Maximum Reliability Improvement Rate Criterion	74
5.3	Sensitivity Analysis	78
5.3.1	Sensitivity Analysis of Tool Similarity	78
5.3.2	Sensitivity Analysis of Production Time	82
 CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS		
	FOR FURTHER WORK	86
6.1	Conclusions	86
6.2	Recommendation for Further Studies	88
 REFERENCES		
APPENDIX A Reliability Mathematics		96
APPENDIX B Series Systems With Standby Redundancy		99
APPENDIX C Computer Code for Calculation of the Reliability of Machining Stages Where Tool Sharing is not Possible ...		102
APPENDIX D Computer Code for Calculation of the Reliability of Machining Stages Where Tool Sharing is Possible		110
APPENDIX E Computer Program for Case 1 of Model VI		114
APPENDIX F Computer Program for Case 2 of Model VI		121
APPENDIX G LINDO Inputs for Model I, II, and III		128

LIST OF FIGURES

Figure 2.1	Possible Layout of an FMS Using Tool Sharing	7
Figure 2.2	Basic Types of Parallel Redundancy	10
Figure 2.3	Types of Standby Redundancy at the Component Level	11
Figure 4.1	Schematic Figure of Tooling System with Spares of a Manufacturing Cell	32
Figure 4.2	Block Diagram of Tooling System of Typical Manufacturing System When Tool Sharing is Possible	40
Figure 4.3	Flow Chart of Proposed Heuristic Algorithm for Model IV	53
Figure 5.1	Sensitivity Analysis of the Effect of Tool Similarity on Spares Cost	80
Figure 5.2	Sensitivity Analysis of the Effect of Reliability Requirement on Spares Cost	81
Figure 5.3	Sensitivity Analysis of the Effect of Tool Similarity on Spares Cost with 75% of Production Time	83
Figure 5.4	Sensitivity Analysis of the Effect of Reliability Requirement on Spares Cost with 75% Production Time	83
Figure 5.5	Sensitivity Analysis of the Effect of Tool Similarity on Spares Cost with 50% of Production Time	84
Figure 5.6	Sensitivity Analysis of the Effect of Reliability Requirement on Spares Cost with 50% Production Time	84
Figure 5.7	A Comparison Between the Effect of Production Time and Tool Similarity on Spares Cost	85

LIST OF TABLES

Table 5.1	Part-Machine Assignment	57
Table 5.2	Tool-Machine Allocation	58
Table 5.3	Tool Cost and Tool Slot Requirement for each Tool Type	58
Table 5.4	Operation Time for Tools Assigned to Machine 1 (min.)	59
Table 5.5	Operation Time for Tools Assigned to Machine 2 (min.)	59
Table 5.6	Operation Time for Tools Assigned to Machine 3 (min.)	59
Table 5.7	Operation Time for Tools Assigned to Machine 4 (min.)	59
Table 5.8	Failure Parameters of Tools Mounted on Machine 1	61
Table 5.9	Failure Parameters of Tools Mounted on Machine 2	62
Table 5.10	Failure Parameters of Tools Mounted on Machine 3	62
Table 5.11	Failure Parameters of Tools Mounted on Machine 4	62
Table 5.12	Reliability of Machining Stage When Tool Sharing Is not Applicable	65
Table 5.13	Reliability of each Tool Type When Tool Sharing Is Applicable	66
Table 5.14	Optimum Spares Combination for Case 1. of Model I	67
Table 5.15	Optimum Spares Combination for Case 2. of Model I	68
Table 5.16	Optimum Spares Combination for Case 1. of Model II	69
Table 5.17	Optimum Spares Combination for Case 2. of Model II	70
Table 5.18	Optimum Spares Combination for Case 1. of Model III	72
Table 5.19	Optimum Spares Combination for Case 2. of Model III	73
Table 5.20	Optimum Spares Combination for Case 1. of Model IV	75
Table 5.21	Optimum Spares Combination for Case 2. of Model IV	76
Table 5.22	Optimum System Attributes Obtained with Proposed Models ...	77
Table 5.23	Tool-Machine Allocation, for TS=0	79
Table 5.24	Tool-Machine Allocation, for TS=12	80

**To the Glory of God
who makes everything possible**

and

**To the people of the
Islamic Republic of IRAN**

CHAPTER 1

INTRODUCTION

1.1. General Need and Overview

The consumer oriented production philosophy dictates that the product design be modified often to suit the needs of the customers. The changing market demands, effect of economic recessions, and intense manufacturing competition have all contributed to reveal the need for flexibility and automation in manufacturing systems. Flexible Manufacturing Systems, FMSs, can be an ideal answer for such problems, particularly for a low or medium production environment. But even though these systems promise flexibility, they also pose new problems. Flexible manufacturing systems are capital intensive. Any misconceptions in design or mistakes in implementation can lead to establishment of unreliable systems with low level of availability, inadequate production

efficiency and high operational cost. Hence, a high degree of reliability is crucial for a good return on investment.

A characteristic feature of FMSs is the ability of unmanned operation for some specified production period. This makes it necessary to have structural duplication of equipment which in one hand enables complex production tasks to be resolved and on the other hand enables the FMS continue to operate staidly despite the failure of individual component.

Whenever the FMS is utilized for machining, assembly, or fabrication, it uses essential tools to perform its functions. Such tool wear out, break, or require resetting and maintenance to ensure successful operation of the system. Industry data indicates that tooling accounts for 25-30% of the fixed and variable cost of production in an automated machining environment Ayres(1988b). In any manufacturing system, tools are liable to failure more often than any other components. Shaw (1980) described tool breakage as the single most significant factor that decrease the productivity of manufacturing systems.

Therefore, a certain level of reliability for tooling system must be taken into account in order to make sure having an uninterrupted, reliable production operation. This means that sufficient redundancies must be foreseen at the preliminary production planning stage to cope with the random breakdowns of the tools. The cost of redundancy is a negative factor, whereas the additional reliability gained is a positive one. Whether installation of a redundancy component has to be done is a issue requiring economic justification.

1.2. Objectives of the Research

Literature addressing the part sequencing and tooling issues is extensive. A greater emphasis, however, is needed to be placed on the reliability analysis of the tooling system in FMS. Thus the development of some reliability-based models to evaluate tooling system performance under different tooling strategies is essential.

The major issues deliberated in this dissertation are as follows:

- Determining a set of spares combination so that some specific system attributes are optimized with respect to a particular criterion.
- Evaluation the effects of tool sharing on system reliability and spares requirement.

Specific criteria considered in this research are as follows:

- a) Minimum overall tooling cost criterion;
- b) Minimum occupied tool magazines capacity criterion;
- c) Joint consideration of minimum occupied tool magazine capacity and spares cost criteria;
- d) Maximum reliability improvement rate criterion;

1.3. Organization of the Research

Fundamental concepts and definitions used in this research are elaborated in Chapter 2. The literature review of the research efforts involving the tool management and reliability issues in FMS and the state of the art in reliability modelling as well as optimization of systems with redundancy are given in Chapter 3. Chapter 4 is dedicated to the development of models and required algorithms to implement them. In Chapter 5 some application examples of the proposed models are provided, and the results for the given system are compared for both cases; where tool sharing is not allowed and where it is implemented. Some sensitivity analysis performed on various parameters of the models are also presented in this chapter. Finally the conclusions and the recommendations for further research are discussed in Chapter 6.

CHAPTER 2

FUNDAMENTAL CONCEPTS AND DEFINITIONS

2.1. FMS and Tool Management

Before dealing with the tool management, it must be clear what FMS is all about.

There are several ways to look at FMS, each one yields a different point of view. One of the most popular and quite general is the definition given by The United States National Bureau of Standards, Nagen (1986): "An arrangement of machines (usually numerical control machining centres with tool changers) interconnected by a transport system. The transporter carries work to the machines on pallets or other interface units so that work-machine registration is accurate, rapid and automatic. A central computers controls both machines and transport system. Flexible Manufacturing Systems sometimes process several different workpieces at any one time."

One of the main advantages of a FMS is the capability to process a wide range of different products in random order. Ensuring the availability of the required tools is a critical factor in FMS performance. Even if a specific tool is presented at the machine tool at the start of the day, its unplanned replacement may be necessary upon detecting a problem such as breakage wear, poor quality finish, or excessive cutting temperature.

Broadly defined, tool management is the capability of having the correct tools on the appropriate machines at the right time so that the desired quantities of workpieces are manufactured while maintaining acceptable utilization of assets, Tomek (1986). Tool management issues are particularly visible in the part type selection, machine grouping, and loading problems.

Tool sharing, as a one of the tooling management strategies, has been implemented in practice (Gaalman et al. 1987). In this research the concept of tool sharing is referred to as the situation where a required, but unavailable tool, can be borrowed from other machining centres in the system. In an FMS, tool sharing may or may not be applicable depending on many factors such as the availability of tool transportation facilities. The implementation of tool sharing, however, may affect FMS performance in terms of reliability, flexibility, throughput, tooling cost, etc.

Figure 2.1 shows a possible layout of such a system where a transportation device can substitute tools among the machining centres.

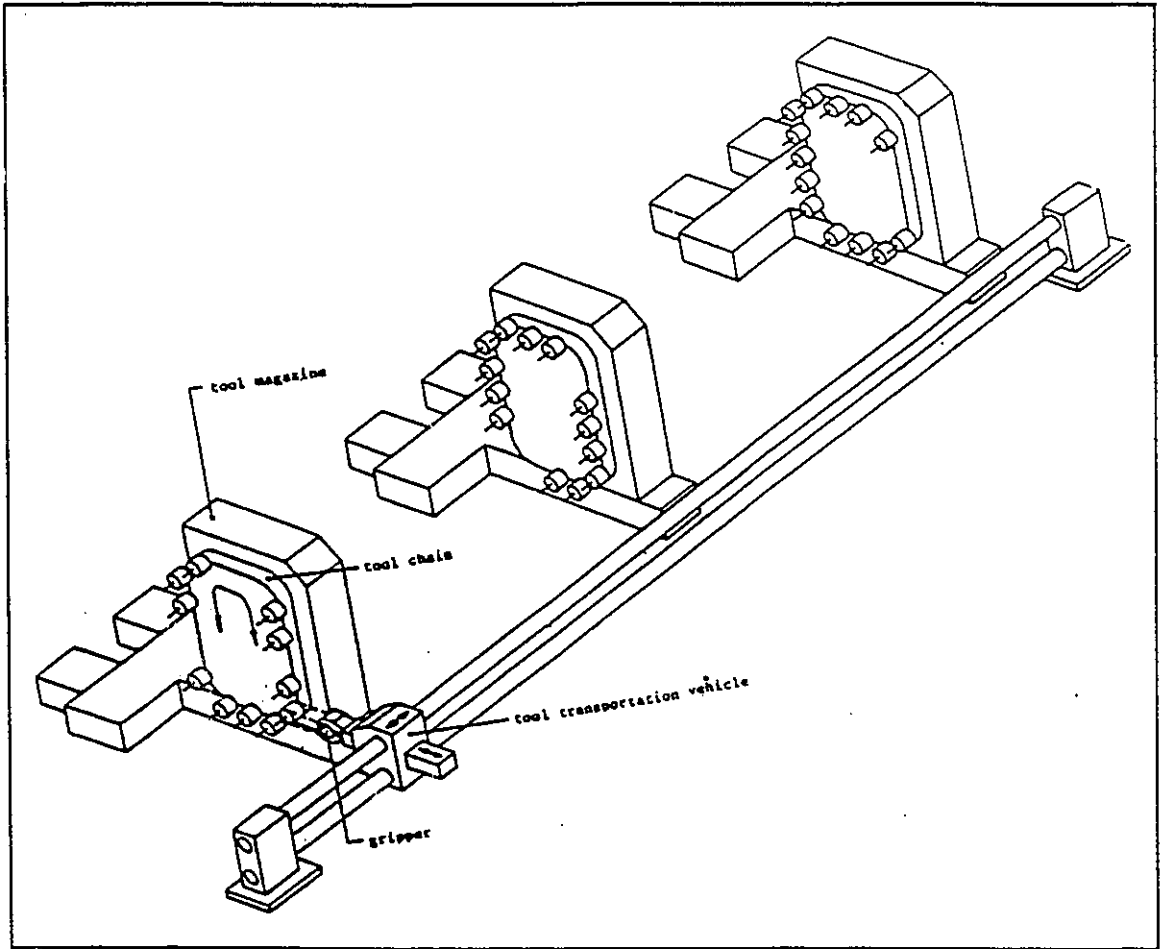


Figure 2.1 Possible layout of an FMS using tool sharing

2.2. Reliability and Availability

Reliability is defined as: "The probability that an item (system) will perform its function adequately for the desired period of time when operated according to the specified conditions", Dhillon(1982). Traditionally, reliability prediction and enhancement have been considered as a matter of experience and common sense. However, the recent rapid

advances in the state-of-art technologies demand a somewhat more exact approach. This has led to the steady development of reliability assessment techniques. There are seven major techniques which are frequently used for evaluation of reliability, these are:

- Block Diagram Analysis (BDA);
- Event Tree Analysis(ETA);
- Fault Tree Analysis(FTA);
- Fault Mode Effect and Criticality Analysis (FMECA);
- Reliability Analysis using Probability Evaluation;
- State-Space (Markov) Analysis;
- Discrete-Change Simulation;

In this research the network modelling or block diagram analysis is utilized to evaluate the reliability at system level; while to determine the reliability for each component, tool, the probability evaluation technique were used.

Availability is the measure of readiness of an item to be put into service when called upon. It can be defined as probability that a system or component is operating satisfactorily at any point in a time interval, where the total time considered includes operating time, repair time, and logistic time.

2.3. Types of System Configuration

The term "system" is taken to mean a collection of component or items, with some relationship among them, which is designed to perform one or more functions (Asher and Feingold, 1983). The system configuration defines the manner in which the system reliability function will behave.

In many complex systems all parts must function successfully in order to get an adequate system performance. That is, the first part failure produces the system failure. Such system is called *series system*. In many cases the problem of achieving high reliability leads to the use of interchangeable modular components intended by some spare parts.

According to ARINC (1964), redundancy can be categorized by three basic criteria: the system level at which redundancy is applied, the state of redundant element while system operating, and the presence or absence of decision and switching devices.

Redundancy may be applied either at the system level or at the component level. The component level redundancy and the system level redundancy have been termed low level and high level redundancy respectively, (Kapur and Lamberson, 1977). If the redundant components are continuously in an operating state and are employed in performing the system function, the redundancy is called *parallel redundancy*. The types of the redundancies for n -component system and m parallel redundant is shown in Figure 2.2.

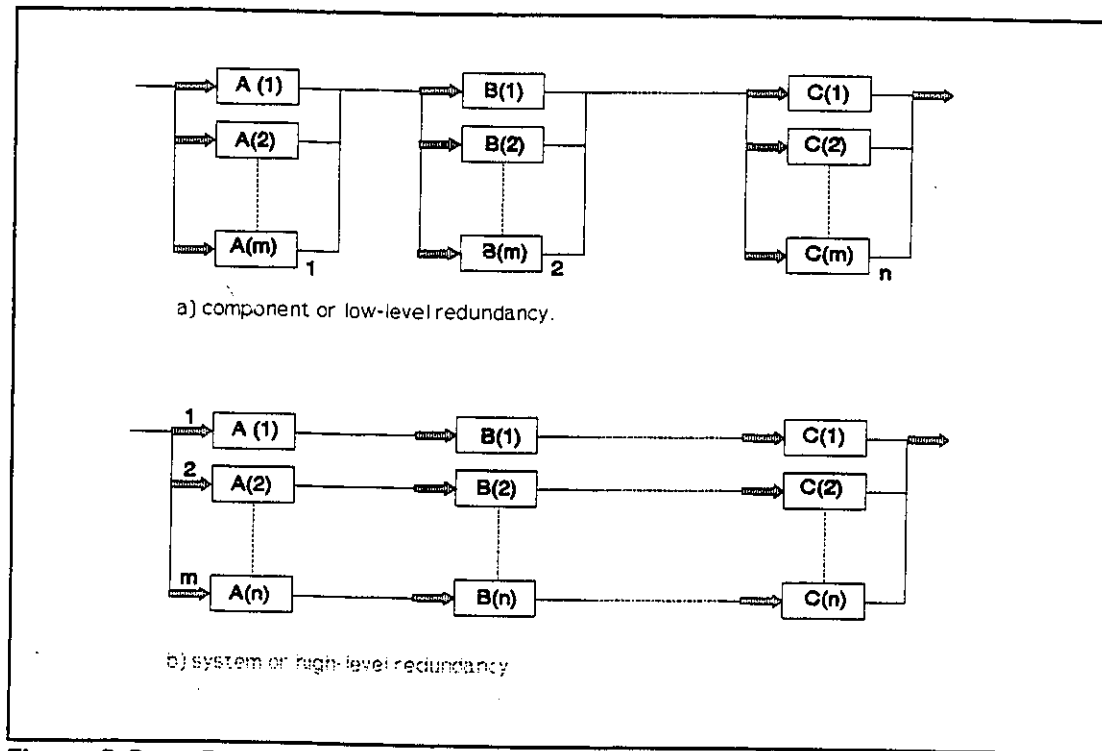


Figure 2.2 Basic Types of Parallel Redundancy.

If the redundant components do not perform any function unless the primary component fails, the redundancy is called *standby redundancy or spare*. If the redundant components are continuously in an active state but do not perform any system function until the primary component fails, the redundancy is called *switching redundancy*. When switching or standby redundancy is employed within a system, it is necessary to have some device that is capable of detecting a failure in the primary element and switching the function to the redundant component. These two types of standby redundancy at component level are shown in Figure 2.3.

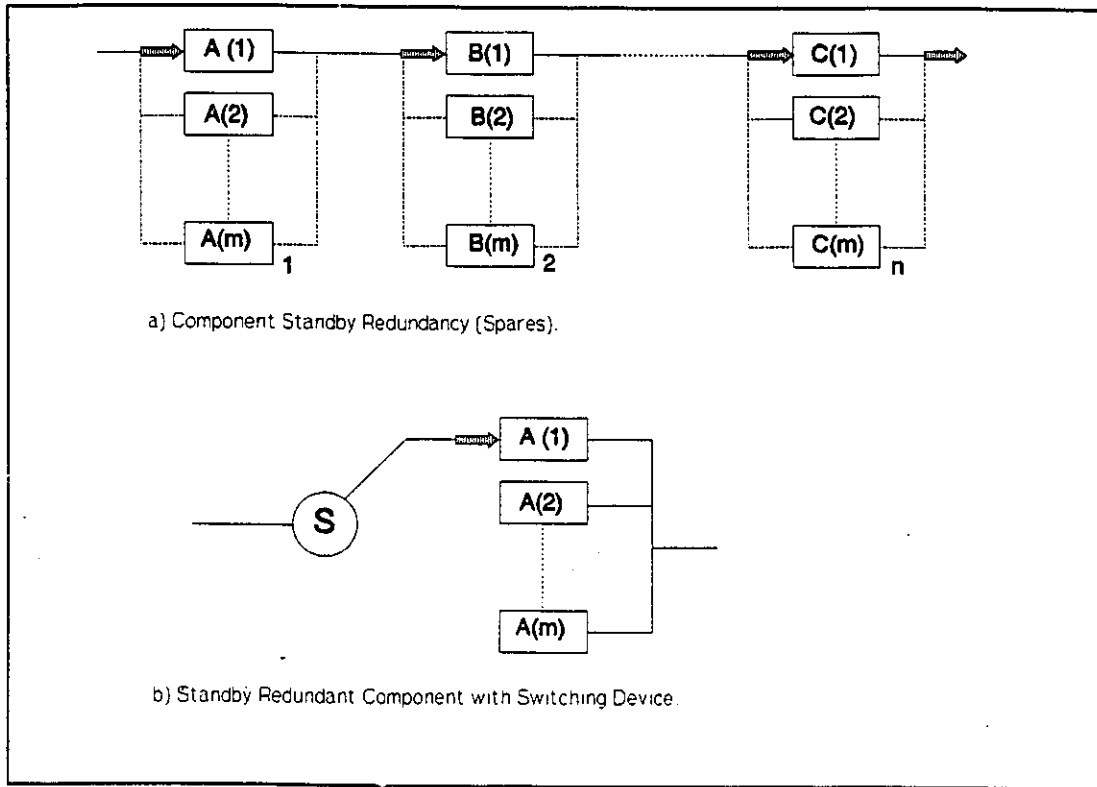


Figure 2.3 Types of Standby Redundancy at the Component Level.

CHAPTER 3

LITERATURE SURVEY

Since the major effort of this dissertation research is directed towards the reliability evaluation of tooling system in FMSs with the emphasis on tool sharing issues, a complete literature survey in the areas of tooling management in FMSs as well as system reliability optimization is necessary. Relevant literature can be classified into two categories:

- 1) Tool management in automated manufacturing systems;
- 2) Optimization of system reliability with redundancy considerations;

In this chapter a review of the major decision problems and research efforts involving the management of tools in automated manufacturing systems is provided first. Then a review of literature specifically related to the optimization of system reliability with redundancy models is presented.

3.1. Tool Management in FMS

3.1.1. Tool-Life Distributions and Models

In an automated machining environment, it is important to assess the life of a cutting tool during which the quality of the workpiece is acceptable. The ability to effectively predict the end of useful life of a tool would result in increased productivity via diminished inventory costs, optimal replacement policies, improved planning, and reduced waste.

A tool is considered "failed" when it either will not cut, or cuts in a manner grossly different from a sharp tool. It may be removed from service once it produces unsatisfactory parts or prior to this point if its "economic tool life" is first reached. An economic tool life applies to tools which are reground or to the replacement of disposable inserts.

Because of the lack of a universally acceptable physical explanation of tool failure, Taylor (1907), in his well known equation, developed the classical relationship between average tool life and cutting velocity through an empirical study of tool wear. His tool life equation is:

$$VT^n = k \quad \text{or} \quad T = \left(\frac{k}{V} \right)^{\frac{1}{n}} \quad (3.1)$$

Where:

T actual tool life of the cutting time between resharpening (minutes);

V cutting speed (feet/minute);

n,k empirical constant;

The constant value **k** depends on the specific cutting conditions and cutter and raw material variable, and it is numerically equal to the cutting speed that gives a tool life of one minute. In practice, the exponent **n** varies in value from about 0.1 to 0.7 and is a function of the tool material and cutting conditions. It is evident from Equation 2.1 that tool life decreases rapidly with the increase in cutting speed.

To provide a clear picture of the relationship among the feed, speed, and depth of cut for a given tool life value, Cook (1973), proposed the following extended tool life relationship:

$$V_t = \frac{k}{d^x f^y} \quad (3.2)$$

Where

V_t equivalent cutting speed (feet/minute) for a given tool life;

f feed per revolution (inches);

d depth of cut (inches);

x, y, k empirical constants;

Equations 3.1 and 3.2 provide expected values based on random tool life data. However, this is still a deterministic description of what is called a slow death mechanism of tool failure.

Tool life, however, is a random variable with many parts and the uncertainty must be describe probabilistically. These equations account for the effect of cutting variables but fail to account for the aging and wearout characteristics of the tool. To obtain a fuller understanding of tool life, the underlying tool life distribution must be studied. Tool wear is a complicated process involving phenomena such as crater, shank wear, chipping, and tempering.

In general, tool life distributions are mostly dependent on the nature of the failure mechanism. The tool life that is obtained under production machining conditions must in part be the result of a statistical interaction between the variability inherent in the work material and the variabilities in the tool. Therefore, a failure rate function may be developed from tool failure data.

Literature indicates that standard distributions such as normal, log normal, Weibull, exponential, and gamma distributions as well as their combination can be justified and fitted to describe the life of a tool under certain machining conditions. Investigations pertaining to the statistical analysis of reliability and life data have been made in manufacturing processes.

For instance, in high speed cutting or when impact sensitive tools are used in rough machining, tools are more likely to fail due to single-injury than due to gradual wear. Ramalingam and Watson (1977), analyzed the tool life distribution employing both time-dependent and time-independent hazard rates. They have shown that for constant and time-independent hazard machining condition, the probability density function of tool failure is an exponentially distributed function, whereas for single-injury and time-dependent hazard rate, the tool life distribution function can be a Weibull distribution or a combination of Weibull and exponential distributions.

Ramalingam (1977), reported that the expected tool-life distribution in the case of tool failure from multiple injuries due to the constant, time-independent stochastic hazards is a gamma distribution which can be approximated by a normal distribution function. He also suggested that tool-life obeys log-normal distribution when tool failure is due to the crater wear.

In many machining operations the tool quality can be degraded by use. In such an instance, tools fail more probably due to the gradual wear than a single-injury. The manner in which tool wears and the consequences of wear vary with the cutting conditions, the raw material itself, and the quality specifications of the part being machined. Tool wear is a complicated process involving phenomena such as crater information, shank wear, chipping, and tempering. There have been several tool-life studies with the respect to wear mechanisms.

Wager and Barash (1971), reported that for high-speed steel turning tools the tool life values are subjected to a statistical distribution which can be approximated by a normal distribution. Based on experimental investigation, Hitomi et al. (1979), have suggested that the logarithmic normal distribution conform for the tool-wear distribution. Friedman and Zlatin (1974), studied the tool life variation for several metal and cutting tool-workpiece combinations. Ramalingam et al. (1978), carried out a series of test with brittle carbide tool inserts to demonstrate tool life distribution. Their results were in agreement with Weibull distribution.

3.1.2. Tooling Economics and Strategies

It was shown that machining economic calculations based on deterministic tool life concept yields inaccurate results (Fenton and Joseph, 1979). There have been some efforts to accommodate the stochastic nature of tool life in economic modelling. In these studies, tool life is treated as a random variable whose distribution is parameterized by the cutting conditions.

Palei and Zubarev (1990), developed a model to minimize the overall production cost, with optimizing the number of tool changes due to failure to achieve a set of number of good parts, while parts machined with failed tools are considered to be scrapped. In their dynamic programming model, it is assumed that tools fail as a result of fracture or

unacceptable wear caused only by factors relating to the job conditions.

Maccarini et al. (1991), have considered the reliability of tools to evaluate the effect of machining cycle on the final product cost. Billatos and Kendall (1991), have presented a general optimization model to minimize the production cost. In their model premature tool failure cost is taken into account; however they used a linear random tool wear function instead of the actual nonlinear sample function.

In any production system involving the machining of metals, the choices of speed and feed rates, tool selection, and monitoring and control strategies are essential determinations of quality and effective capacity. The dependence of tool life on machining parameters (speed, feed rate, and depth of cut) means that with increasing of these parameters tool must be replaced more often and tooling expenses rise exponentially. With the increase in throughput rates, however, a part requires less machine and labour time and provides a potential for higher revenue. This basic trade-off illustrates that the actual machining parameters should be chosen fully considering the economics of the facility operations. In practice, the production speed, which may represent machining speed, feed rate, or spindle revolution rate, is a decision variable that is used to optimize production.

Hitomi (1976), showed that the cutting speed that minimize unit cost is less than the cutting speed that maximizes profit rate. Consequently, the latter cutting speed is greater

than the cutting speed that maximizes throughput. This model assumes tool replacement can be made within the setup time of a workpiece in a continuous machining operation.

Ermer and Kromodihardjo (1981), have illustrated that due to tool wear and maximum permissible feed and speed a multi-pass turning operation can be cheaper and more economical than single pass operations. Gopalakrishnan and Al-Khayyal (1991), used a geometric programming to select machine speed and feed which minimize total cost of machining, including cost associated with cutting tool wear.

Commare et al. (1983), presented an analytical model to determine optimum cutting conditions to minimize the unit production cost, where in case of tool failure during a operation a penalty cost is applicable both for the rejected workpiece and the time spent to work on the rejected workpiece. The model was implemented on the three proposed strategies and they concluded that the scheduled tool replacement strategy is always more convenient as long as the production cost is to be minimized.

Machining parameters for specific operations such as milling and drilling, that can be optimized are also used in other studies (Crookall and Venkataramani, 1971; Chang et al. 1982; Yellowley, 1983; and Koulamas, 1991).

Part and tool movement policies are among the basic approaches used in loading problems in flexible manufacturing systems. When the required tools to process a part

are not mounted in the tool magazine two alternatives arise; either the part may be sent to another machining centre where those required tools are available or the required tools may be transported from another machining centre.

Han et al. (1989), compare these two strategies. They report the following advantages for tool movement policy:

- Because parts do not move, there is no need to reposition the workpiece or recalibrates the position of the tool head which result in a higher cutting precision;
- Since a part is processed by only one machining centre, a part is delivered into the shop only when a machining centre becomes available, thereby resulting in less work-in process.

However, they also mentioned tool movement policy results in tool-move delay time, which will be increased when tool to be borrowed is in use by other machining centre. They proposed a non linear programming model for the loading of a set of tools to the different machining centres, where each part visits only one of the machining centres for its entire processing. Therefore, a part can remain at the same machining centre while a required but unavailable tools for the processing part are borrowed either from other machining centres or from the tool crib. The quadratic objective function is to minimize the amount of tool traffic among the machining centres and between a machining centre

and tool crib under a reasonable work load balance. Nevertheless, the complexity of the model regarding the various flow-control strategies results in an insufficient dynamically interacting with complex systems.

There are a large number of researches performed on the loading problem in Flexible Manufacturing Systems. Tang (1986), proposed a job scheduling model which aimed to minimize the number of tool changes at a single machine centre, using tool movement policy. Hankis and Rovito (1984), compare two tool allocation and distribution strategies through a case study by using computer simulation. Tang and Denardo (1988), have used a heuristic to minimize the total number of tool switches in a job scheduling problem, where the requisite, but unavailable, tools must be brought to the machine and the time needed to switch tools is significant relative to the processing time.

Carrie and Perera (1986), discussed work scheduling under tool availability constraints. Based on their computational results, they proposed a rough estimation of the tool changes due to tool wear, which is:

$$\textit{number of tool changes} = \textit{total spindle cutting time} / \textit{average tool life}.$$

Another issue is tool search time. This is not the time to find the tool in the magazine (the computer already knows where each tool is), but the time for the magazine to rotate into position for the next tool interchange to take place. Tool search time is relatively

small. However, if aluminum, for instance, are being machined some tools may only be used for 2-3 seconds at a time. Chernvo (1990), based on the solution of the arithmetical equations, describes a method of determining the direction of the rotation which is suitable for the magazines with any number of the sockets.

In many studies tools are considered to occupy equal space on tool magazine. However, it is not the case in many situations. Some tools may have a diameter greater than the spacing of the tool pockets in the magazine, and therefore neighbouring tool pockets can not be used. Carrie and Perera (1986), defined four classes for tool size:

- single tools, which take only one tool position;
- centre tools, which take up to three positions, the pocket the tool placed in and the positions on either side;
- fat tools, which take up only one position, but because of their size can not be positioned in the pocket next to another fat tool;
- handed tools, which are asymmetrical and take two positions, the one the tool is in and the adjacent one on its left or its right hand side depending on the handing of the tool;

In addition, two three-slot tools may take five slots when placed side by side. This means that the number of magazine slots required for each operation depends on the actual placement of the tools in the magazine. These factors are discussed in Stecke

(1983), and Rajagopalan (1986). It is shown that such considerations can be formulated using a number of nonlinear tool magazine capacity constraints.

3.1.3. Spares Management in FMS

During a production period it is possible that several operations have several cutters in common. If these operations are assigned to the same machine, only one copy of each cutter needed to be loaded, which saves some magazine capacity. On the other hand, it is shown that tool changes due to product variety are only a small portion of total tool changes in a production period and tool must be changed more frequently because of wear or breakage, (Carrie and Perera, 1986). Hence, multiple copies may be beneficial or even necessary if certain cutters are used very often or a desirable reliability of the system/machine up to certain time is needed. Then it becomes desirable to load duplicate copies of this tools into the tool magazine. This can reduce the number of times that a machine is stopped to change tools. Adding spares, however, will reduce the effective magazine capacity and the machine flexibility. An important problem is to determine the number of spares of each tool type to load into a magazine.

Optimal allocation of spares is a topic addressed in the several studies of manufacturing systems. Using a hybrid queuing network optimization model, Gross et al. (1983), analyzed the trade off between spares number and the capacities of repair facilities.

Vinod and Sabbagh (1986), studied the relation between stocking spares and the cost of the operating the repair facility in order to determine optimal number of spares and the capacity of the repair facility. To guarantee the availability of tools in the system, their model imposes the restriction that a part can be dispatched to specific machine tool only if the desired cutting tool is already available at that machine.

The performance and reliability of an FMS can be affected by the specific storage locations of spare tools. Kusiak (1983), has characterized some basic tool storage policies, whereby spare tools are stored either on the tool magazine or in a remote tool crib or both. According to his study, the decision about the location of spares must consider the availability and type of tool handling system used as well as the tool magazine type (permanent or interchangeable).

Pan et al.(1986), analyzed the reliability of an automated tool changing system with various carbide inserts and spares subject to Weibull failure distribution. Using a recursive algorithm, they attempted to predict tooling system reliability and determine the desired reliability-cost spares combination. However, no specific model to select the suitable spares combination was proposed. It had to be done through sorting all possible combinations. They also took a numerical approach to integrate the reliability function for each tool with different number of spares. Since they assumed that spares are identical for each tool type, such a function for Weibull distribution (and some other distributions) can be integrated analytically.

3.2. Reliability-Based Optimization

There many researches in the area of optimization of system reliability with redundancy models. The optimization problem is generally formulated to maximize or minimize the objective function subject to certain constraints. In the following some of the accomplishments in the field of reliability optimization will be briefly addressed.

3.2.1. Optimization of System Reliability with Redundancy Models

Complex systems may contain some components which fail more often. It is sometimes impossible to reduce the frequency of the failure for such components by improving quality. Then, the only way to improve the system reliability is to incorporate more redundancies at the locations where failures are expected. Determining the location and the number of redundancies in the system are the issues discussed extensively in literature.

For an N-stage series system, Tillman et al. (1980) treated the problem of allocation redundancy to each of the components so that the system reliability is maximized. The extension of this problem can be stated as finding the optimum number of redundancies which maximizes the system reliability subject to cost constraints (Rao, 1992), where there is a maximum specified cost of the system.

Bellman and Dryefus (1958) were among the first who applied dynamic programming to the solution of the optimal redundancy problem. In their formulation only two types of constraints, cost and weight, were considered. Tillman (1969), has applied integer programming techniques to the problems of maximizing the reliability or minimizing cost subject to several constraints where the components may have different modes of failures. This required a bulky formulation that restricts the size of the system.

Hwang et al. (1971), applied a zero-one integer programming to minimize the weight of subsystems of a life support system subject to several nonlinear constraints while maintaining an acceptable level of reliability of the system. Mizukami (1968), used convex and integer programming techniques. He formulated the allocation problem with integer linear programming by approximating the convex objective function using a piecewise linear function.

Ghare and Taylor (1969), considered the problem of determining the optimum number of redundant components in order to maximize the reliability of series system subject to multiple resource restrictions. After formulating and solving the associated zero-one programming model by a branch-and-bound procedure, they showed that the optimal solution to the associate problem is equivalent to the optimal solution for the optimal redundancy problem.

Federowicz and Mazudar (1968), employed geometric programming to optimize the

redundancy in the series systems. They showed that non-integer solutions obtained through geometric programming techniques, when suitably rounded off, yielded solutions closed to discrete optimal values.

Fan et al. (1967), approached the problem from different viewpoint. Their objective was to maximize expected profit by obtaining optimum redundancy. Where the objective function to be maximized involved terms for total profit gained from the operation of the system minus construction cost for the system.

Many heuristic algorithms have been developed for solving the redundancy allocation problem. For the active-parallel systems, Bala and Aggarwal (1987), proposed a two-phase heuristic algorithm for redundancy optimization in complex networks. The algorithm determines the component to which redundancy should be added. Volkovich and Zaslavskii (1986), proposed an algorithm which solves the optimal redundancy problem with resource constraints for the situations in which the system may operate even when a component fails (active-parallel systems).

Kettelle (1962), suggested a computational approach for maximizing reliability subject to a cost constraint. He solved the problem of redundancy allocation for a series system where a *dominating sequence* described the least costly allocation. This sequence was based on the availability and the cost of each component.

Yasutake and Henley (1985), developed a heuristic-based method for optimizing system availability by redundant allocation. The basis of the proposed heuristic method is to add redundant component to the subsystem which would give the greatest increase in system availability with respect to the cost. To determine such a component a selection factor is calculated as follow:

$$S_i = \Delta \text{ system availability} / \Delta \text{ change in system cost}$$

Where S_i is the selection factor of component i . The component with the largest value of selection factor has a subcomponent redundantly added to it. The process of selecting component for adding redundancy is repeated until the desired system reliability is obtained or resources are consumed.

3.2.2. Spare Provision and Standby Redundancy Allocation

Spare provisioning can be a major issue in designing a new system. A prediction is needed concerning how many backup (spares) each component will need for some defined operating time.

Messinger and Shooman (1970), conducted a tutorial review on techniques for optimum spares allocation. In their paper, they evaluate and compared several techniques presented in the literature for allocating the number of spares of each part type to be assigned in order to maximize the system reliability subject to constraints on resources.

Sasaki and Kaburaki (1977), presented two algorithms for optimum redundancy, namely Lawler-Bell Algorithm and New Lawler-Bell Algorithm. These are nonlinear integer programming techniques. These techniques are computationally precise but complex and often fail when applied to real world situations.

Based on the search procedure, Misra (1991), presented an algorithm which provides an exact solution for general class of integer programming problems in reliability optimization. Lambert et al. (1971), have used dynamic programming for optimally allocating the mean time between failure (MTBF), mean time to repair (MTTR) and the number of redundant units to a multistage system to achieve a given availability at minimum cost.

Taking into consideration the hierarchical organization of a depot and the supply policy, Catunceanu et al. (1987), suggested a model to determine optimum stock quantity in a three level depots system subject to minimum reliability requirement. For each level problem two optimization algorithms were employed. First, a near-optimum solution was reached by "incremental performance per price algorithm" (IPPA), then the "new Lawler-Bell algorithm" (NLB) was used to obtain the optimum solution.

Ghosh and Wells (1990), presented an heuristic algorithm for solving the spares allocation problem to remote machines, in which machines were subsystems of a series system that would be used only for a specified period of time. They assumed that all

spares must be assigned at the beginning of the system's mission. The algorithm determines number of spares for each subsystem (machines) that maximizes the minimum probability of each machine consuming its spares before the useful life of system is completed. Sears (1990), proposed a top-down technique to calculate required number of standby redundancy to increase mean time between failures and to improve system reliability.

There are extensive works in the area of reliability optimization. However, there is a lack of reliability-based modelling in the area of automated manufacturing systems. A greater emphasis should be placed on reliability considerations in the state-of-art of manufacturing systems optimization.

CHAPTER 4

MODEL DEVELOPMENT

This chapter presents and discusses the development of the specific mathematical models necessary to accomplish the research objective stated in section 1.2 of Chapter 1. The major assumptions related to the tooling system configuration with spares are clarified. The mathematical symbols and models parameters are defined. All requisite computer programs are coded in TURBO C language and run on an 386 IBM compatible computer.

4.1. System Definition

It is often the case in FMSs that the parts assigned to a machining centre before completion have to meet a number of tools mounted on the tool magazine. That means, the functional operation of the system depends on the proper operation of all tools. In

this study, the reliability of the system is considered as the probability of successful performance of all required machining operations of the parts assigned to the system.

The Flexible Manufacturing System contemplated in this study consists of several machining centres, where each of them can perform any processing operation of the parts, assigned to it as long as the appropriate tools are supplied. Since the life time of the tools are usually less than the required operation time during a production period, some copies of each tool type may also be provided on tool magazine. Therefore, in term of reliability evaluation, the tooling system of an FMS can be considered as a series system with some spares as standby redundancy. Figure 4.1 shows a schematic representative of a typical system configuration with two machining centre and four tool types. Two spares of each tool type are also placed on the tool magazines.

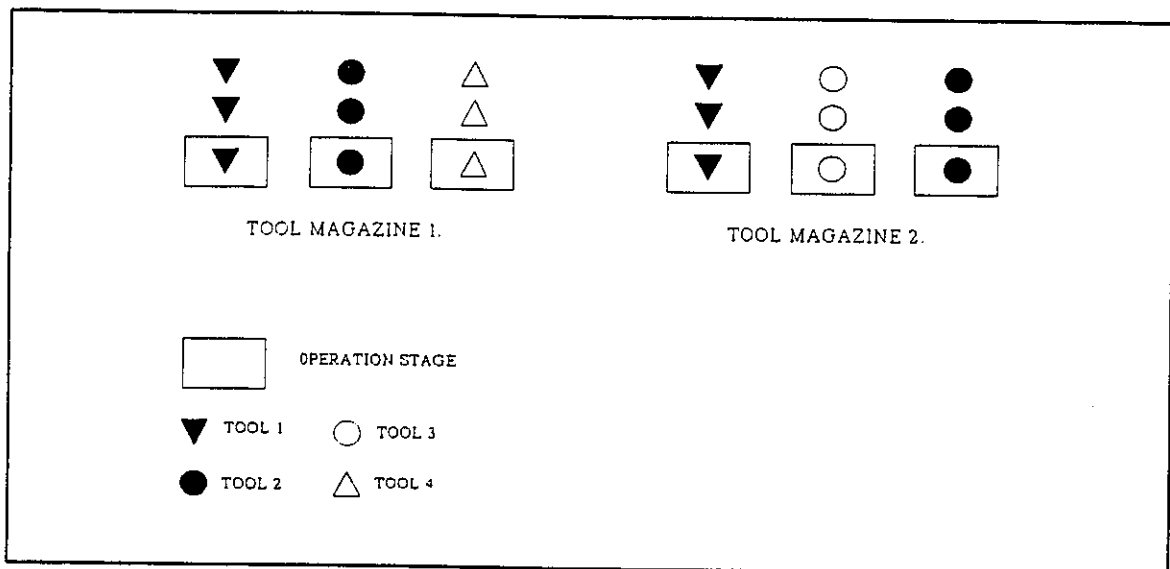


Figure 4.1 Schematic Figure of Tooling System with Spares of a Manufacturing Cell.

As mentioned earlier empirical curve fitting of analytical and laboratory failure data have justified several standard distributions and their combinations to describe the life of a tool under certain machining conditions. In this research three type of distributions (exponential, Wiebull, and Erlangian) are used to analyze the cutting tools reliability. It should be mentioned that the proposed computer programs, with the minor changes, can be modified for other distributions to calculate the system reliability.

4.2. Mathematical Modelling for the Proposed System

When the objective is finding the optimum tools combination, one may simply want to find all possible combinations, sort them and then select the cheapest set of spares for each tool. This approach, however, become extremely difficult when the number of tools in the system as well as the number of possible spares are increased. Both of these parameters will exponentially increase the number of possible combinations. This is also impossible to determine the optimum combination when other priorities are to be optimized, e.g. "maximum reliability improvement rate".

In the following sections, four models are developed to optimize some attributes of the tooling system based on different criteria. These objectives are "Minimum Tooling Cost", "Minimum Tool Slots Occupation", "Minimum Tooling Cost along with Minimum Tooling Slots Occupation", and "Maximum Reliability Improvement Rate". All models

are applied for the system in which tool sharing is allowed and for the system that tool sharing can not be implemented. The proper algorithms and user friendly computer programs are provided whenever necessary. To develop the models the following assumption are made:

- All parts are assigned to the machines in advance, and at least one copy of each required tool type is mounted on the tool magazine.
- A machining centre can perform all required operations of the assigned parts, as long as the required tools are provided on tool magazine.
- Each tool magazine has a limited tool capacity, and only a limited number of copies for each tool type can be mounted onto each machine..
- Operation conditions such as speed and feed rate for each machining stage remain constant during a production period. However, each tool type may have dissimilar operation conditions on different machining centres, i.e. each tool might have different hazard rates on different machines;
- Different tools may be used but all spares for each tool type are identical.
- Each tool may require different number of slots in the tool magazine;
- Switching between the tools on the same machine is perfect, however, the tool transporter device among the machining centres is subject to general failure rate;
- The failure of a tool is not affected by the failure of other tools;
- The detection of a failure is perfect and the replacement of a component is considered a renewal process;

The following notations for indices, parameter and decision variables are introduced for the mathematical formulations.

Indices:

i	tool type index,	$i=1, \dots, I$
j	operation index,	$j=1, \dots, J$
k	machine index,	$k=1, \dots, K$
l	part index,	$l=1, \dots, L$
m	spares index,	$m=0, \dots, M_{ik}$

Decision Variables:

$$X_{ikm} \begin{cases} 1 & \text{if } m \text{ spares of tool type } i \text{ are mounted on machine } k; \\ 0 & \text{otherwise;} \end{cases}$$

$$X_{im} \begin{cases} 1 & \text{if there exist } m \text{ spares of tool type } i \text{ in the system;} \\ 0 & \text{otherwise;} \end{cases}$$

Y_{ik} Number of spares of tool type **i** on machine **k**;

R_{ikm} Reliability of tool type **i** on machine **k** with **m** spares;

R_{im} Reliability of tool type **i** in the system with **m** spares;

R_s System reliability;

Parameters:

- A_{ik} $\begin{cases} 1 \text{ if tool type } i \text{ is mounted on machine } k; \\ 0 \text{ otherwise;} \end{cases}$
- C_i Cost of tool type i ;
- E_k Available number of tool slots on machine k after assigning one copy of each required tool to the machine;
- I_k The set of tool types mounted on machine k ;
- M_{ik} Number of allowed spares tool type i for machine k ;
- M_i Number of allowed spares of tool type i in the system;
- R_{Sq} Minimum required system reliability;
- R_r Reliability of tool transportation device;
- ST Tool Similarity in the system;
- t_{ijk} Time that tool i is used to perform operation j of part l on machine k ;
- T_{ik} Cumulative operation time of tool type i on machine k ;
- T_i Required time to transport one spare from one machining centre to the other;
- T_r Cumulative operation time for tool changer device;
- W_i Number of tool magazine slots occupied by tool type i ;
- λ_{ik} Hazard rate of tool i on machine k ;
- λ_i Cumulative hazard rate of tool type i in the system;
- ρ_1 The priority weight assigned to the minimization of spares cost;
- ρ_2 The priority weight associated with the minimization of the tool slots occupancy.

4.2.1. Model I

The purpose of the development of this model is to achieve the required system reliability by allocating a number of spares on tool magazines with minimum overall tooling cost.

CASE 1. TOOL SHARING IS NOT APPLICABLE.

When tool sharing is not allowed, no tool movement among machining centres may be performed. Therefore, the number of spares for each tool is restricted to the number of copies mounted on the same tool magazine. Then, the reliability of system is a function of hazard rate, operation time and number of spares for each tool. Hence,

$$R_s = \prod_{k=1}^K \prod_{i \in I_k} R_{ikm}(\lambda_{ik}, T_{ik}, M_{ik}) \quad (4.1)$$

where

$$T_{ik} = \sum_j \sum_l t_{ijlk} \quad \forall k, i \in I_k \quad (4.2)$$

Utilizing *product rule*¹ for series systems with standby redundancies, reliability of the tooling system, R_s , with associated spares for each machining stage can be calculated as follows:

¹ See Appendix B.

$$R_s(t) = \prod_{k=1}^K \prod_{i \in I_k} \left[\sum_{m \in M_{ik}} \frac{\left(\int_0^t \lambda_{ik}(t) dt \right)^m \exp \left[- \int_0^t \lambda_{ik}(t) dt \right]}{m!} \right] \quad (4.3)$$

Equation 4.3 provides the reliability of tooling system with M spare tools for each machining stage where t , the time interval for each machining stage, is calculated using Equation 4.2. To minimize the overall spares cost of tooling system subject to a minimum reliability requirement, R_{sd} , the model can be written as follows:

OBJECTIVE FUNCTION:

$$\text{MINIMIZE } Z = \sum_{k=1}^K \sum_{i \in I_k} \sum_{m=0}^{M_{ik}} m C_i X_{ikm} \quad (4.4)$$

SUBJECT TO:

$$\prod_{k=1}^K \prod_{i \in I_k} \prod_{m=0}^{M_{ik}} R_{ikm} X_{ikm} \geq R_{sq} \quad (4.5)$$

$$\sum_{i \in I_k} \sum_{m=0}^{M_{ik}} m X_{ikm} W_i \leq E_k \quad \forall k \quad (4.6)$$

$$\sum_{m=0}^{M_{ik}} X_{ikm} = 1 \quad \forall i \in I_k, k \quad (4.7)$$

This model is a mixed-integer program with linear objective function that has 0-1 and integer variables, and a set of nonlinear constraints. This model, with nonlinear constraints, becomes difficult to solve especially for large size problems. The constraint set 4.5 can be linearized by taking the logarithm of both sides. This yields:

$$\sum_{k=1}^K \sum_{i \in I_k} \sum_{m=0}^{M_{ik}} \log R_{ikm} X_{ikm} \geq \log R_{Sg} \quad (4.8)$$

The objective function minimizes the overall spares cost of the tooling system. Constraints 4.8 guaranties minimum reliability requirement. Constraint set 4.6 bonds the tool magazine capacity for each machine. Constraints 4.7 insures that each tool type on each machine can have only 1 or 2 or ,..., M_{ik} spares.

CASE 2. TOOL SHARING IS ALLOWED

In case where tool sharing allowed, the problem can be treated in another way. Let k machines possess tool type i . Also, the transportation device is provided so that standby redundant spares of tool type i can be used in share during production period. Therefore, we can consider these k tool type i as *one* subsystem, instead of k , that are supported by all spares of tool type i in the system. This reduces the number of stages to the number of tool types in the system. In such a system, *in terms of reliability analysis*, the aggregate processing time is independent of the machine on which the tool

is loaded. Figure 4.2 shows the reliability block diagram for the system proposed in section 4.1 when tool sharing is applicable.

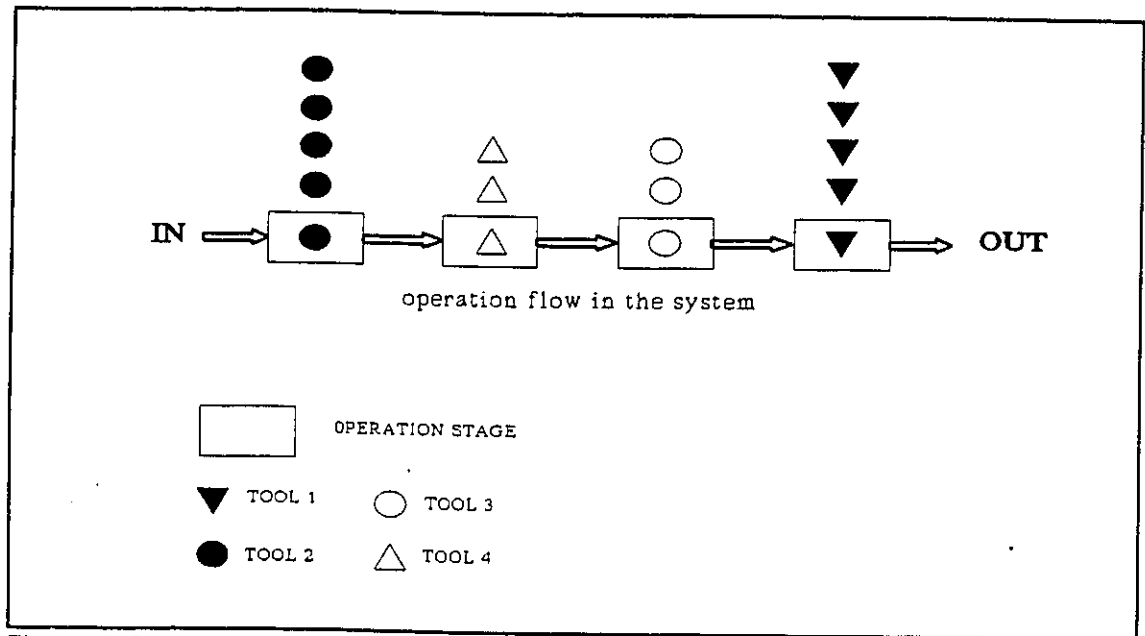


Figure 4.2 Block Diagram of Tooling System of Typical Manufacturing System When Tool Sharing is Possible.

The system reliability in this case is a function of cumulative hazard rates and total number of spares for each tool type and reliability of transportation device. It can be determined as follows:

$$R_s = \prod_{i=1}^I R_{im}(\lambda_i(t), M_i, R_r) \quad (4.9)$$

Where;

$$M_i = \sum_{k=1}^K M_{ik} \quad \forall i \quad (4.10)$$

and

$$\lambda_i(t) = \sum_{k=1}^K \lambda_{ik}(t) \quad \forall i \quad (4.11)$$

The third term, R_r , of tool reliability in Equation 4.9 comes from this fact that, whenever a tool transportation occurs through the system a transportation device must get involved. The reliability of such a device must be taken into account to calculate the reliability of each tool type in the system. The reliability of a tool transporter can be calculated as follows:

$$R_r = P(\lambda_r, T_r) \quad (4.12)$$

Where;

$$T_r = T_t \left(\sum_{k=1}^K \sum_{i=1}^I M_{ik} - \sum_{i=1}^I \text{Min} \{ M_{ik}; k=1, \dots, K \} \right) \quad (4.13)$$

Because of the probabilistic nature of tool failure, it is not clear which tool will fail first and which spare has to be borrowed from other machining centre first. Therefore, to calculate the time that the tool changer will operate for each spare transportation, T_r , it is assumed that the tool changer device has performed all possible transportation preceded to that spare. That is to say, each tool transportation among the machines is considered to be the last one. This is the worst possible situation in terms of failure probability for the tool changer device.

The reliability of tooling system when tool sharing is applicable becomes as follow:

$$R_s(t) = \prod_{i=0}^I \sum_{m \in M_i} \frac{\left(\int_0^t \lambda_i(t) dt \right)^m \exp \left[- \int_0^t \lambda_i(t) dt \right]}{m!} R_r \quad (4.14)$$

Where t is the cumulative working time for the tool type i in the system. It should be noted that in the above expression, as long as no tool transportation between machining centres required, the reliability of tool transporter is being set to 1. To develop the cost optimization model in this case, the objective function and constraint sets should be modified as follows:

OBJECTIVE FUNCTION:

$$\text{MINIMIZE } Z = \sum_{i=1}^I \sum_{m=0}^{M_i} m C_i X_{im} \quad (4.15)$$

SUBJECT TO:

$$\sum_{i=1}^I \sum_{m=0}^{M_i} \log R_{im} X_{im} \geq \log R_{sq} \quad (4.16)$$

$$\sum_{k=1}^K Y_{ik} - \sum_{m=0}^{M_i} m X_{im} = 0 \quad \forall i \quad (4.17)$$

$$\sum_{i \in I_k} Y_{ik} W_i \leq E_k \quad \forall k \quad (4.18)$$

$$Y_{ik} \leq M_{ik} \quad \forall i \in I_k, k \quad (4.19)$$

$$\sum_{m=0}^{M_i} X_{im} = 1 \quad \forall i \quad (4.20)$$

The objective function of this model will minimize the overall spares cost when tool sharing is allowed. Constraint 4.16 provides the minimum reliability requirement. Constraint 4.17 determines the location of the spares in the system. Constraint set 4.18 ensures that required tool slots will not exceed the total available slots on each machine. Due to the presence of tool transporter, spares can be mounted anywhere in the system. However, in order to have a balance spare allocation, constraint set 4.19 bounds the maximum number of spares at each machining stages. Constraint set 4.20 determines the number of spares for each tool type in the system.

4.2.2. Model II

One of the most important flexibility measurement in FMSs is the ability of the system to process different parts in random order. Having the required tools on the tool magazine is the primely condition to provide such ability. Tool magazine capacity is one of the most significant constraints which may prevent loading proper tools. Therefore, minimization of tool slots engaged by spares in the system can be necessary to maintain system flexibility. This is the goal of the subsequent model.

CASE 1. TOOL SHARING IS NOT POSSIBLE

Utilizing the notations described in section 4.2, the following model minimizes the number of tool slots taken by spares in the system.

OBJECTIVE FUNCTION:

$$\text{MINIMIZE } Z = \sum_{k=1}^K \sum_{i \in I_k} \sum_{m=0}^{M_{ik}} m X_{ikm} W_i \quad (4.21)$$

SUBJECT TO:

$$\sum_{k=1}^k \sum_{i \in I_k} \sum_{m=0}^{M_{ik}} X_{ikm} \log R_{ik} \geq \log R_{sq} \quad (4.22)$$

$$\sum_{i \in I_k} \sum_{m=0}^{M_{ik}} m X_{ikm} W_i \leq E_k \quad \forall k \quad (4.23)$$

$$\sum_{m=0}^{M_{ik}} X_{ikm} = 1 \quad \forall i \in I_k, k \quad (4.24)$$

In this model the objective function aims to minimize the number of occupied tool magazine slots subject to system reliability requirement and tool magazines constraints, where some tools may required more than one tool slot on tool magazine. The resultant system configuration yields maximum vacancy on tool magazines.

CASE 2. TOOL SHARING IS APPLICABLE

The above model can be adjusted for the system in which tool sharing can be implemented. This yields:

OBJECTIVE FUNCTION:

$$\text{MINIMIZE } Z = \sum_{i=1}^I \sum_{m=0}^{M_i} m W_i X_{im} \quad (4.25)$$

SUBJECT TO:

$$\sum_{i=1}^I \sum_{m=0}^{M_i} \log R_{im} X_{im} \geq \log R_{sq} \quad (4.26)$$

$$\sum_{k=1}^K Y_{ik} - \sum_{m=0}^{M_i} m X_{im} = 0 \quad \forall i \quad (4.27)$$

$$\sum_{i \in I_k} W_i Y_{ik} \leq E_k \quad \forall k \quad (4.28)$$

$$Y_{ik} \leq M_{ik} \quad \forall i \in I_k, k \quad (4.29)$$

$$\sum_{m=0}^{M_i} X_{im} = 1 \quad \forall i \quad (4.30)$$

The objective function maximizes the available tool slots in the system. Due to the presence of tool transporter, the location of spares has no effect on system reliability.

Here also the same sets of constraints, as those described for the corresponding case of Model I, have to be satisfied.

4.2.3. Model III

One of the shortcomings of the classical linear programming formulation is that they can treat only one objective. The practical situation usually involve multiple conflicting objectives. Such problems involve making trade-off decision to obtain the best compromise solution. Thus in many situations a multiobjectives model may result in more effective solution. To formulate the problem as a multiobjectives model one has to specify a set of objectives along with the priority weight associated with each of them. The magnitude of each priority determines the importance of the corresponded objective. This procedure is shown in following models.

CASE 1. TOOL SHARING IS NOT APPLICABLE

The succeeding two-objective models will provide number of spares in an FMS with respect to the following goals:

- (1) Minimum tooling cost;
- (2) Minimum tool slots engaged in the system;

For the case that tool sharing is not implemented, the model can be formulated as follows:

OBJECTIVE FUNCTION:

$$\text{MINIMIZE } Z = \sum_{k=1}^K \sum_{i \in I_k} \sum_{m=0}^{M_{ik}} m X_{ikm} (\rho_1 C_i + \rho_2 W_i) \quad (4.31)$$

SUBJECT TO:

$$\sum_{k=1}^k \sum_{i \in I_k} \sum_{m=0}^{M_{ik}} X_{ikm} \log R_{ik} \geq \log R_{sq} \quad (4.32)$$

$$\sum_{i \in I_k} \sum_{m=0}^{M_{ik}} m X_{ikm} W_i \leq E_k \quad \forall k \quad (4.33)$$

$$\sum_{m=0}^{M_{ik}} X_{ikm} = 1 \quad \forall i \in I_k, k \quad (4.34)$$

The objective function will minimize both the tooling cost and the number of occupied tool slots simultaneously. Both ρ_1 and ρ_2 are weighted ranking coefficients that show objective priorities. They determine the degree of importance and contribution of each parameter toward the optimum value of the objective function.

CASE 2. TOOL SHARING IS POSSIBLE

In case where tool sharing is applicable, the above model can be modified as follows:

OBJECTIVE FUNCTION:

$$\text{MIN.} \quad Z = \sum_{i=1}^I \sum_{m=0}^{M_i} m X_{im} (\rho_1 C_i + \rho_2 W_i) \quad (4.35)$$

SUBJECT TO:

$$\sum_{i=1}^I \sum_{m=0}^{M_i} \log R_{im} X_{im} \geq \log R_{sq} \quad (4.36)$$

$$\sum_{k=1}^K Y_{ik} - \sum_{m=0}^{M_i} m X_{im} = 0 \quad \forall i \quad (4.37)$$

$$\sum_{i \in I_k} W_i Y_{ik} \leq E_k \quad \forall k \quad (4.38)$$

$$Y_{ik} \leq M_{ik} \quad \forall i \in I_k, k \quad (4.39)$$

$$\sum_{m=0}^{M_i} X_{im} = 1 \quad \forall i \quad (4.40)$$

Again, in this model, the constraint sets which must be satisfied are similar to those described in previous models.

4.2.4. Model IV

In reliability optimization, selecting the cheapest combination of spares may not be always the right choice. Due to the exponential nature of reliability function, there is no one-to-one relationship between the amount of money spent and reliability improvement of the system. In some cases, with the small increase in the cost of tooling a much higher reliability may be achieved. This becomes more significant when a high degree of system reliability, e.g. $R_s > 90\%$, is desired. This is resulted from the exponential nature of reliability function that makes it more difficult to achieve a high system reliability.

In the following model it is attempted to select a combination of spares which minimizes the tooling cost with a new criterion; "*Maximum Reliability Improvement / Unit Cost*". To do so, a dynamic programming model is developed to select the spare with the largest value of system reliability improvement rate. This allocation procedure of selecting a spare for redundant addition is repeated until the desired system reliability is obtained or one of the system resources is exhausted.

CASE 1. TOOL SHARING IS NOT APPLICABLE

To develop the model the following additional notations are required:

- d** Index of spare allocation at each iteration;
- ZO_d** Optimum spares cost of the system at iteration **d**;
- X_{ikd}** $\left\{ \begin{array}{l} 1 \text{ if one spare of tool type } i \text{ is allocated to machine } k \text{ at iteration } d; \\ 0 \text{ otherwise;} \end{array} \right.$
- C_{ikd}** Cost of spare tool type **i** on machine **k** which provides maximum reliability improvement rate at iteration **d**;
- C_{id}** Cost of spare tool type **i** which provides maximum reliability improvement rate at iteration **d**;
- R_{Sikd}** System reliability with adding one spare of tool type **i** mounted on machine **k** at iteration **d**;
- R_{Sid}** System reliability with adding one spare of tool type **i** to the system at iteration **d**;
- R_{Osd}** Optimum system reliability found at iteration **d**;

OBJECTIVE FUNCTION:

$$\text{Min. } ZO_d - ZO_{d-1} + C_{ikd} \Rightarrow \left\{ \text{Max} \left\{ \frac{R_{Sikd} - R_{OS(d-1)}}{C_i} \right\} ; \forall i \in I_k, k \right\} \quad (4.41)$$

SUBJECT TO:

$$R_{OSd} \geq R_{Sg} \quad (4.42)$$

$$\sum_{i \in I_k} \sum_{d=1}^{M_{ik}} W_i X_{ikd} \leq E_k \quad \forall k \quad (4.43)$$

Where

$$d = 1, 2, \dots, M_{ik} \quad \forall i \in I_k, k ; \quad \left\{ \begin{array}{l} i = 1, \dots, I \\ k = 1, \dots, K \end{array} \right\} \quad (4.44)$$

$$R_{OS(d-1)} = R_{Sikd} \rightarrow \{ZO_{d-1}\} \quad (4.45)$$

Based on the reliability improvement rate, the dynamic objective function keeps adding one spare to a machining stage at each iteration until either constraints 4.42 and 4.43 are satisfied or d reaches its limits. In the other words, if one of the machining stages reaches its maximum spares limit, M_{ik} , it has to be excluded from the system in the next spare allocation step until no tool can be allocated to any machining stage. In the above expressions it must be cleared that at $d=1$ system spares cost, ZO_{d-1} , is equal to zero, and the system reliability, R_{OSd-1} , is the product of all stages reliability when no spare is allocated.

The following heuristic algorithm has been developed and coded using Turbo C to solve the above model with the respect to its constraints.

STEP

1. Compute the existence system reliability, (R_{Osd}), with no spares for all stages;
2. Check if the existence system reliability is equal or greater than minimum required reliability, if so go to step 9. otherwise go to step 3.
3. Check for the tool magazines capacity limit and for the stages which reached their spares limitation and exclude them from system for the next iteration;
4. If there left no stage to allocation give an error message regarding the infeasibility of system and stop; otherwise o to step 5.
5. Add one spares to each stage at a time and compute the resultant system reliability (R_{Sikd});
6. Calculate corresponding "*Improved Reliability/Spare Cost*" ratio for each combination.
7. Select the largest ratio and relevant system reliability and spares configuration;
8. Update the existence system reliability and system configuration;
9. Write down cost, reliability of the system, and number of spares for each stage.

The above steps of the heuristic algorithm is summarized in the form of flow chart as shown in Figure 4.3.

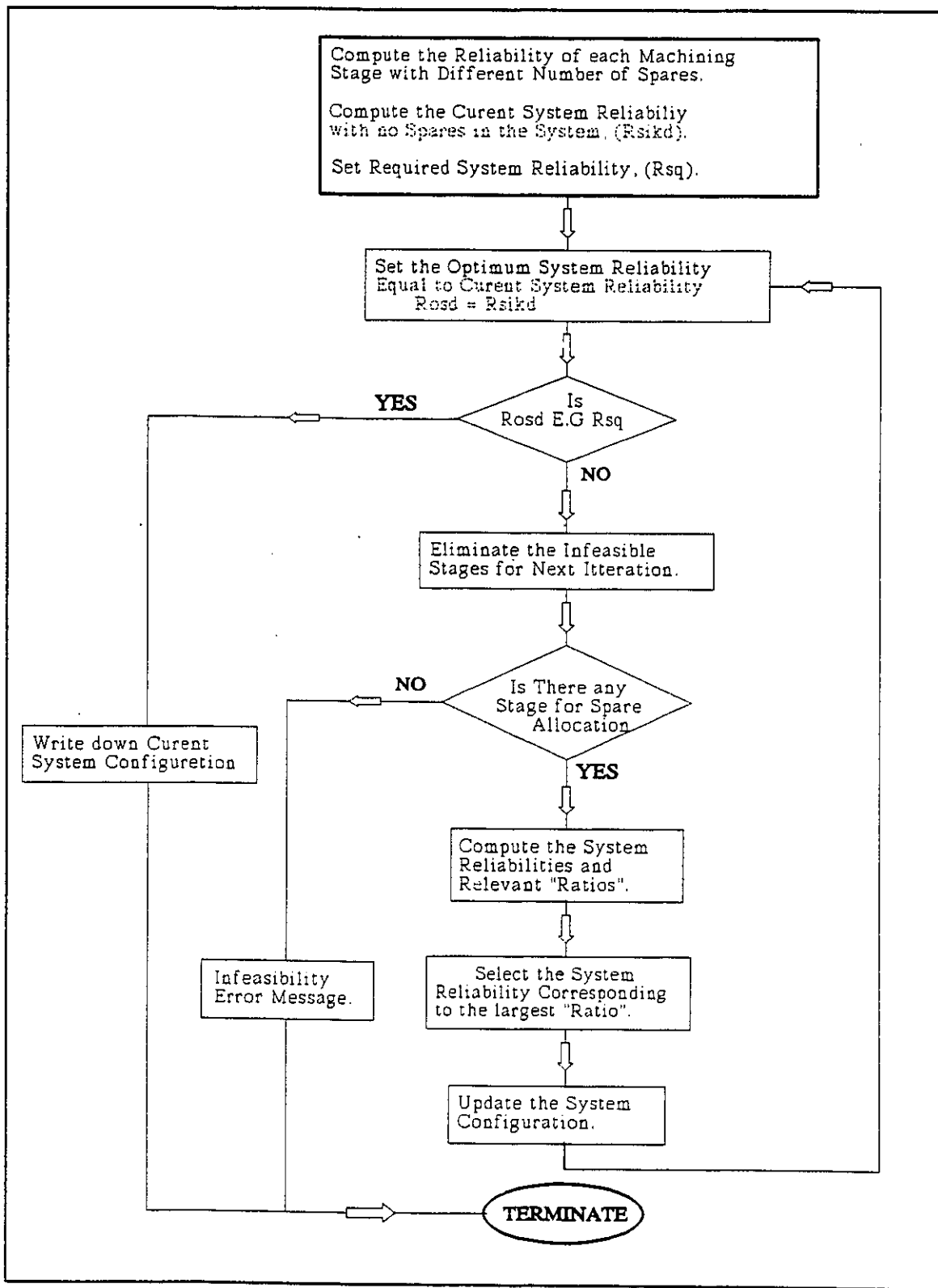


Figure 4.3 Flow Chart of Proposed Heuristic Algorithms for Model IV.

CASE 2. TOOL SHARING IS APPLICABLE

The same model can be utilized to optimize the spares cost when tool sharing is possible, except that the number of stages in reliability block diagram will be equal the number of tool type in the system. The maximum possible number of spare allocation steps, d , will be also changed from M_{ik} to M_i . that yields:

$$d = 1, 2, \dots, M_i \quad \forall i ; \{i = 1, \dots, I\} \quad (4.46)$$

The model for the case where tool sharing may be implemented can be modified as follows:

OBJECTIVE FUNCTION:

$$\text{Min. } ZO_d - ZO_{d-1} + C_{id} \Rightarrow \left\{ \text{Max} \left\{ \frac{R_{Sid} - R_{OS(d-1)}}{C_i} ; \forall i \right\} \right\} \quad (4.47)$$

SUBJECT TO:

$$R_{OSd} \geq R_{Sg} \quad (4.48)$$

$$\sum_{i=1}^I \sum_{d=1}^{M_i} W_i X_{ikd} \leq E_k \quad \forall k \quad (4.49)$$

Where

$$R_{OS(d-1)} - R_{sid} \rightarrow \{ZO_{d-1}\} \quad (4.50)$$

Since the tool transportation device is provided, the improvement of system reliability is not affected by the location of the additional spare added on each allocation. Nevertheless, the location of each added spare does affect the tool magazines capacity and the algorithm must take care of this while places spares through the system. Therefore, the proposed heuristic algorithm has to be modified as follow:

- 1-7. Same as step 1-7 in case 1;
8. Assign the selected spare to the stage which have been assigned the least number of spares in previous allocations;
- 9-10. Same as step 8-9 in case 1;

With respect to the changes in reliability calculation formula when tool sharing is allowed, the proposed algorithm can be utilized to determine both the locations and number of spares in the system.

CHAPTER 5

COMPUTATIONAL RESULTS AND SENSITIVITY ANALYSIS

The models proposed in the previous chapter were applied to a hypothetical problem of FMS planning. The results obtained from the example and sensitivity analysis are presented in the following sections.

The proposed system configuration, part types and their process plans, tools parameters, and the specific failure distribution formulas used to calculate the reliability of each tool type are discussed in section 5.1. Section 5.2 includes the computational experiments with the suggested models and discussion of the results. In section 5.3 several sets of sensitivity analysis are performed on different system parameters to demonstrate the influence of tool sharing on reliability of tooling system, in general.

5.1. Description of the Proposed System

In order to better understand the procedure, a small example is shown. For the purposes of illustration, the amount of data is kept minimal. Large amount of data, however, can be handled using the developed models and programs.

The system consist of four general purpose CNC machines. Each machine is equipped with a tool magazine having 16 slots for storing tools. There are four part families which according to characteristics of parts, and operations are assigned to the machines. To perform the required machining operations for each part families a set of tools is mounted on each machine. Listed in Table 5.1 and 5.2 are the information for the relationship of part-machine and tool-machine assignment.

Table 5.1. Part-Machine Assignment.

Machine Type	Part No.									
	1	2	3	4	5	6	7	8	9	10
1	1	0	1	0	1	0	0	0	0	0
2	0	0	0	1	0	1	0	0	0	1
3	0	1	0	0	0	0	1	0	0	0
4	0	0	0	0	0	0	0	1	1	0

Note: "1" indicates an allocation.

Table 5.2. Tool-Machine Allocation.

Machine Type	Tool Type									
	1	2	3	4	5	6	7	8	9	10
1	1	0	0	1	0	0	1	1	0	0
2	1	0	1	0	0	1	0	0	0	1
3	1	1	0	0	1	0	0	0	1	0
4	1	0	1	0	0	0	1	0	1	0

Note: "1" indicates an allocation.

Tool magazine capacity seem to be among the most important parameter for the determination of expected FMS capability. This parameter is affected by the number of tool slots required by each tool type. Another factor to select an optimum combination of tools for a production period is the cost of each tool type. Table 5.3 gives the characteristics of each tool type used in this example.

Table 5.3. Tool Cost and Tool Slot Requirement for each Tool Type.

Tool Type	1	2	3	4	5	6	7	8	9	10
Cost	100	50	200	150	50	100	100	150	100	100
Tool Slot Req.	1	1	2	3	2	1	1	1	2	3

In this example, each batch consists of different items with various machining times for each part. Tables 5.4 through 5.7 show the required operation time for each tool on different part types.

Table 5.4. Operation Time for Tools Assigned to Machine 1 (min.).

Part No.	Tool Type			
	1	4	7	8
1	4	0	6	6
3	3.5	8	3.5	0
5	6.5	6	4.5	5.5

Table 5.5. Operation Time for Tools Assigned to Machine 2 (min.).

Part No.	Tool Type			
	1	3	6	10
4	5	6.5	4	3.5
6	4.5	0	6.5	4.5
10	4	5.5	5	6

Table 5.6. Operation Time for Tools Assigned to Machine 3 (min.).

Part No.	Tool Type			
	1	2	5	9
2	6	5	6	5.5
7	4	6.5	4.5	6.5

Table 5.7. Operation Time for Tools Assigned to Machine 4 (min.).

Part No.	Tool Type			
	1	3	7	9
8	4.5	3.5	6	6
9	6.5	4.5	6	5.5

Although different part types may have various demands, since the total operating time is important to calculate the reliability of each tool, the aggregated process time for each part type is provided in Tables 5.4 to 5.7. From these tables the total operating time, during a production period, for each tool can be easily calculated.

As mentioned earlier the stochastic nature of tool life must be taken into account in order to predict the reliability of each tool type during a production period. This enables us to determine the number of required spares in order to have an uninterrupted production period with a certain probability. In this study three types of failure distribution are considered for the proposed system. They are Weibull, exponential, and Erlangian distributions. The general formula to calculate the reliability of a component with standby redundancy is stated in Chapter 3., that is:

$$R_s(t) = \prod_{k=1}^K \prod_{i \in I_k} \left[\sum_{m \in M_{ik}} \frac{\left(\int_0^t \lambda_{ik}(t) dt \right)^m \exp \left[- \int_0^t \lambda_{ik}(t) dt \right]}{m!} \right] \quad (5.1)$$

For the above mentioned distributions to calculate the integral $\lambda(t) dt$ in the interval $[0, t]$ in the above equation, the results would be as follows:

For Weibull distribution;

$$\int_0^t \lambda(t) dt = \left(\frac{t}{\beta} \right)^b \quad (5.2)$$

Where β is scale and b is shape parameter of Weibull distribution .

For exponential distribution;

$$\int_0^t \lambda(t) dt = \lambda t \quad (5.3)$$

Where λ is constant failure rate for exponential distribution.

For Erlangian distribution;

$$\int_0^t \lambda(t) dt = \frac{t}{\alpha} + \ln \alpha - \ln [\alpha + t] \quad (5.4)$$

Where α is failure parameter for Erlangian distribution.¹

The failure parameters of tools on each machine are shown in Tables 5.8 to 5.11.

Table 5.8. Failure Parameters of Tools Mounted on Machine 1

Tool Type	Failure Parameter			
	Weibull		Exp.	Erlan.
	b	β	λ	α
1	-	-	0.008	-
4	-	-	-	15
7	-	-	0.02	-
8	0.85	86	-	-

¹ See Appendix A.

Table 5.9. Failure Parameters of Tools Mounted on Machine 2

Tool Type	Failure Parameter			
	Weibull		Exp.	Erlan.
	b	β	λ	α
1	-	-	0.0125	-
3	-	-	-	18.5
6	0.632	1245	-	-
10	0.624	480	-	-

Table 5.10. Failure Parameters of Tools Mounted on Machine 3

Tool Type	Failure Parameter			
	Weibull		Exp.	Erlan.
	b	β	λ	α
1	1.09	116	-	-
2	0.531	715	-	-
5	-	-	0.033	-
9	1.6	74	-	-

Table 5.11. Failure Parameters of Tools Mounted on Machine 4.

Tool Type	Failure Parameter			
	Weibull		Exp.	Erlan.
	b	β	λ	α
1	-	-	-	12
3	0.76	120	-	-
7	0.725	423	-	-
9	1.6	74	-	-

The proposed failure parameters are mostly based on published literature describing the experimental tests, Ramalingam et al. (1978). In this study, it is also assumed that one tool may have different failure patterns on different machines. Since one of the most significant factor, which determines failure parameters for a tool, is the machining conditions, that may differ from one machine to another, it is not an unrealistic assumption.

As it is shown in the above tables, some tool types are common among different machines, (e.g. tool 1 on machines 1-4, tool 3 on machine 2 and 4, etc). The number of identical tools has a grate effect on reliability of tooling system when tool sharing is applicable. Therefore, to measure this parameter in the system a new term named *Tool Similarity* has been used which can be defined as:

$$S_i = \sum_{k=1}^K \sum_{i=1}^I A_{ik} - \left[\sum_{i=1}^I A_{ik} ; \{ k-1, \dots, K \} \right] \quad (5.5)$$

Using Equation 5.5, the tool similarity for the proposed configuration is equal to 6.

5.2. Computational Experiments

Several computer programs have been written to prepare the reliability coefficients for the proposed models. These are user friendly programs which accept all the relevant

parameters for tools, parts and different tool failure distributions to determine the reliability of each machining stage/tool type in the system.

The minimum reliability requirement for all models is considered to be at least 90% and for each stage the number of spares, M_{ik} , is supposed to be maximum two copies. For the cases where tool sharing is implemented, the failure distribution of tool transporter is assumed to be exponentially distributed in which $\lambda_r=0.0001$ failure/min. and required time for each spare transportation, T_r , is 0.25 min.

Models I to III were solved using LINDO software on a 386 micro computer. for both cases. To solve Model IV two programs were coded using Turbo C language.

The reliability of each machining stage for the system described in previous section where tool sharing is not applicable is given in Table 5.12.

Table 5.12. Reliability of Machining Stage When Tool Sharing Is not Applicable.

Machining Stage (A_{ik}^2)	Number of Spares on each Machining Stage		
	0	1	2
A_{11}	0.89404	0.99417	0.99978
A_{41}	0.76026	0.96864	0.99720
A_{71}	0.75578	0.96740	0.99703
A_{81}	0.83457	0.98549	0.99914
A_{12}	0.84472	0.98726	0.99929
A_{32}	0.86183	0.98998	0.99951
A_{62}	0.93937	0.99812	0.99996
A_{102}	0.89567	0.99435	0.99979
A_{13}	0.93319	0.99771	0.99994
A_{23}	0.89441	0.99422	0.99978
A_{53}	0.70715	0.95219	0.99464
A_{93}	0.94701	0.99857	0.99997
A_{14}	0.76637	0.97029	0.99742
A_{34}	0.88012	0.99251	0.99968
A_{74}	0.92722	0.99728	0.99993
A_{94}	0.95041	0.99875	0.99997

The results shown in the above table indicate that no tool can have more than two spares even if some identical copies are mounted on some other machines. However, in the case that tool sharing can be implemented, similar spares may be borrowed from other machining centres. Therefore, the reliability computation has to be changed from each machining stage to each tool type. This means that for the given system tool type 1 can have a total of 8 spares in the system, tool type 3 up to 4 spares and so on.

Table 5.13 shows the outputs of the computer program for the proposed system in case where tool sharing can be performed.

² A_{ik} indicates tool type i on machine k .

Table 5.13. Reliability of each Tool Type When Tool Sharing Is Applicable.

Tool Type	Number of Spares for each Tool Type									
	0	1	2	3	4	5	6	7	8	9
1	.54011	.87281	.97527	.99629	.99952	.99992	.99996	.99997	.99998	.99998
2	.89441	.99422	.99978	---	---	---	---	---	---	---
3	.75852	.96816	.99713	.99980	.99998	---	---	---	---	---
4	.76026	.96864	.99720	---	---	---	---	---	---	---
5	.70715	.95219	.99464	---	---	---	---	---	---	---
6	.93937	.99812	.99996	---	---	---	---	---	---	---
7	.70077	.94995	.99424	.99949	.99995	---	---	---	---	---
8	.83457	.98549	.99914	---	---	---	---	---	---	---
9	.90005	.99483	.99982	.99999	.99999	---	---	---	---	---
10	.89567	.99435	.99979	---	---	---	---	---	---	---

The reliabilities given in Tables 5.12 and 5.13 can be used to determine the optimum tooling system configuration with different criteria.

5.2.1. Minimum Tooling Cost Criterion

In this section, the computational results for Model I stated in chapter 3 would be presented. To linearize the minimum reliability requirement constraint, the corresponding logarithm of reliability for each machining stage/tool has been used. Data given in Table 5.14 represents the optimum tooling configuration for tooling system with minimum spare cost when tool sharing is not implemented.

Table 5.14. Optimum Spares Combination for Case 1. of Model I.

Machining Stage (A_{ik})	Number of Spares on each Stage			Stage Reliability	Spares Cost
	0	1	2		
A_{11}	0	1	0	0.99417	100
A_{41}	0	1	0	0.96864	150
A_{71}	0	0	1	0.99703	200
A_{81}	0	1	0	0.98549	150
A_{12}	0	0	1	0.99929	200
A_{32}	0	1	0	0.98998	200
A_{62}	0	1	0	0.99812	100
A_{102}	0	1	0	0.99435	100
A_{13}	0	1	0	0.99771	100
A_{23}	0	1	0	0.99422	50
A_{53}	0	0	1	0.99464	100
A_{93}	0	1	0	0.99857	100
A_{14}	0	0	1	0.99742	200
A_{34}	0	1	0	0.99251	200
A_{74}	0	1	0	0.99728	100
A_{94}	0	1	0	0.99874	100

Note: "1" indicates an allocation.

The system reliability for this configuration is **90.253 %** and total spares cost is equal to **\$ 2150**.

For the same system the optimum spares combination would be changed dramatically if tool sharing is applicable. This is clearly shown in Table 5.15 that provides the results for case 2 of Model I. The information provided in Table 5.15 do not include the reliability of each machining stage, because for this case instead of reliability of each machining stage the reliability of each tool type must be considered. The model,

however, is formulated so that number of spares for each tool type as well as the location of spares can be determined.

Table 5.15. Optimum Spares Combination for Case 2. of Model I.

Machining Stage (A_{ik})	Number of Spares on each Stage			Spare Cost
	0	1	2	
A_{11}	0	0	0	0
A_{41}	0	0	1	300
A_{71}	0	0	0	0
A_{81}	0	1	0	150
A_{12}	0	0	0	0
A_{32}	0	1	0	200
A_{62}	0	1	0	100
A_{102}	0	1	0	100
A_{13}	0	0	0	0
A_{23}	0	1	0	50
A_{53}	0	0	1	100
A_{93}	0	0	0	0
A_{14}	0	0	1	200
A_{34}	0	1	0	200
A_{74}	0	0	1	200
A_{94}	0	1	0	100

Note: "1" indicates an allocation.

The optimum spares cost for this circumstance is **\$1500** and the system reliability is **90.00 %**. Results shown in Table 5.15 indicate that there are significant differences between these two situations in terms of spares cost and number of spares necessary to satisfy the demanded system reliability.

5.2.2. Minimum Tool Slots Occupation Criterion

The second model developed in Chapter 4 aimed to minimize the number of occupied tool slots on tool magazines in order to provide a greater flexibility in the system.

Tabulated in Tables 5.16 and 5.17 are the LINDO computational results of this model for both cases where tool sharing is not allowed and where it is allowed respectively.

Table 5.16. Optimum Spares Combination for Case 1. of Model II.

Machining Stage (A_{ik})	Number of Spares on each Stage			Stage Reliability	No. of Tool Slots
	0	1	2		
A_{11}	0	1	0	0.99417	1
A_{41}	0	1	0	0.96864	3
A_{71}	0	0	1	0.99703	2
A_{81}	0	0	1	0.99913	2
A_{12}	0	1	0	0.98726	1
A_{32}	0	1	0	0.98998	2
A_{62}	0	1	0	0.99812	1
A_{102}	0	1	0	0.99435	3
A_{13}	0	1	0	0.99771	1
A_{23}	0	1	0	0.99422	1
A_{53}	0	0	1	0.99464	4
A_{93}	0	1	0	0.99857	2
A_{14}	0	0	1	0.99742	2
A_{34}	0	1	0	0.99251	2
A_{74}	0	1	0	0.99728	1
A_{94}	0	1	0	0.99874	2

Note: "1" indicates an allocation.

The outcome system reliability for this case is found to be **90.40 %** with the total spares cost of **\$ 2200** and total tool slots involved by spares would be **30** slots.

Table 5.17. Optimum Spares Combination for Case 2. of Model II.

Machining Stage (A_{ik})	Number of Spares on each Stage			No. of Tool Slots
	0	1	2	
A_{11}	0	0	0	0
A_{41}	0	1	0	3
A_{71}	0	0	0	0
A_{81}	0	0	1	2
A_{12}	0	0	0	0
A_{32}	0	1	0	2
A_{62}	0	1	0	1
A_{102}	0	1	0	3
A_{13}	0	1	0	1
A_{23}	0	1	0	1
A_{53}	0	0	1	2
A_{93}	0	0	0	0
A_{14}	0	0	1	2
A_{34}	0	0	0	0
A_{74}	0	0	1	2
A_{94}	0	1	0	2

Note: "1" indicates an allocation.

The optimum tool slots occupancy for this circumstance is **23** slots with a total spares cost of **\$1600** and system reliability equal to **90.62 %**. The outcome tooling combinations provided by this model demonstrate a grate decline of cost and occupied tool slots for both distinct circumstances.

The differences between these configurations and those provided by previous model are that here the model minimizes the number of tool slots occupied by the spares regardless of cost associated with each spare. Therefore, for the first case Model II suggests 2 spares of tool type 8 on machine 1, and 1 copy of tool type 1 on machine 2 instead of having 1 copy of tool type 8, and 2 copies of tool type 1 on machine 2 which are resulted from Model I. For the second case when tool sharing is applicable Model II assigns 1 spare of tool type 4, and 2 spares of tool 8 to machine 1, and 1 spare of tool 1 on machine 3. Whereas, for the same system Model I proposes 2 copies of tool 4, 1 spare of tool 8 and no copy for tool type 1 for the same machines.

5.2.3. Joint Consideration of Tooling Cost and Tool Slots Occupancy

In the previous models only one objective was optimized each time. In order to optimize both tooling cost and tool slots involved by spares, Model III has been developed in Chapter 3. To convert tool cost and tool slot coefficients into one unit, two priority weights, ρ_1 and ρ_2 , are defined. The magnitude of each of them determines the importance of associated criterion in the model and various consequences may be obtain with different weights. In this example, ρ_1 is kept equal to 1 and ρ_2 is set to 100 (no differences were shown for the ρ_2 less than 100). This means that each tool slot has a value equal to \$100 in the system.

In the proposed system if the tool sharing can not be implemented, Model III yields the system configuration shown in Table 5.18.

Table 5.18. Optimum Spares Combination for Case 1. of Model III.

Machining Stage (A_{jk})	Number of Spares on each Stage			Stage Reliability	Spare Cost	No. of Tool Slots
	0	1	2			
A_{11}	0	1	0	0.99417	100	1
A_{41}	0	1	0	0.96864	150	3
A_{71}	0	0	1	0.99703	200	2
A_{81}	0	1	0	0.98549	150	1
A_{12}	0	0	1	0.99929	200	2
A_{32}	0	1	0	0.98998	200	2
A_{62}	0	1	0	0.99812	100	1
A_{102}	0	1	0	0.99435	100	3
A_{13}	0	1	0	0.99771	100	1
A_{23}	0	1	0	0.99422	50	1
A_{53}	0	0	1	0.99464	100	4
A_{93}	0	1	0	0.99857	100	2
A_{14}	0	0	1	0.99742	200	2
A_{34}	0	1	0	0.99251	200	2
A_{74}	0	1	0	0.99728	100	1
A_{94}	0	1	0	0.99874	100	2

Note: "1" indicates an allocation.

This tool combination gives a system reliability equal to **90.25 %** and the optimal spares cost and number of tool slot occupied by spares are **\$ 2150** and **30** tool slots respectively. This indicates an improvement of tooling cost in comparison with Model II where only a single objective, number of occupied tool slots, is minimized.

Table 5.19 shows the computational results of Model III for the case in which tool sharing is possible.

Table 5.19. Optimum Spares Combination for Case 2. of Model III.

Machining Stage (A_{ik})	Number of Spares on each Stage			Spare Cost	No. of Tool Slots
	0	1	2		
A_{11}	0	0	1	200	2
A_{41}	0	1	0	150	3
A_{71}	0	0	0	0	0
A_{81}	0	1	0	150	1
A_{12}	0	0	0	0	0
A_{32}	0	1	1	400	4
A_{62}	0	1	0	100	1
A_{102}	0	1	0	100	3
A_{13}	0	0	0	0	0
A_{23}	0	1	0	50	1
A_{53}	0	0	1	100	4
A_{93}	0	1	0	100	2
A_{14}	0	0	0	0	0
A_{34}	0	0	0	0	0
A_{74}	0	0	1	200	2
A_{94}	0	0	0	0	0

Note: "1" indicates an allocation.

The optimum tooling configuration for this case would cost \$ 1550 for sparring. This provides 90.12 % system reliability. For this configuration the spares occupy 23 slots on the tool magazines. The results for this model also shows a significant reduction in spares cost and subsequently tool slots occupancy for situation where tool sharing is implemented.

When this combination is compared with the corresponding case of Model I, with an increase of \$50 in tooling cost, it yields a saving of one extra tool slot in the system. In comparison with Model II also this model proposed a configuration which costs \$50 less with the same number of tool slots occupancy.

5.2.4. Maximum Reliability Improvement Rate Criterion

There are some instances in which a small increase in cost may result in a much higher system reliability. Model IV in Chapter 4 was developed to handle such situations. The dynamic structure of this model makes it possible to determine optimum tooling cost with respect to the maximum reliability improvement rate. Tables 5.20 and 5.21 contain the computational results of this model.

Table 5.20. Optimum Spares Combination for Case 1. of Model IV.

Machining Stage (A_{ik})	Number of Spares on each Stage			Stage Reliability	Spares Cost
	0	1	2		
A_{11}	0	1	0	0.99417	100
A_{41}	0	0	1	0.99720	300
A_{71}	0	0	1	0.99703	200
A_{81}	0	1	0	0.98549	150
A_{12}	0	1	0	0.98726	100
A_{32}	0	1	0	0.98998	200
A_{62}	0	1	0	0.99812	100
A_{102}	0	1	0	0.99435	100
A_{13}	0	1	0	0.99771	100
A_{23}	0	1	0	0.99422	50
A_{53}	0	0	1	0.99464	100
A_{93}	0	1	0	0.99857	100
A_{14}	0	0	1	0.99742	200
A_{34}	0	1	0	0.99251	200
A_{74}	0	1	0	0.99728	100
A_{94}	0	1	0	0.99874	100

Note: "1" indicates an allocation.

For the first case of Model IV the optimum spares combination will provide **91.79 %** system reliability with **\$ 2200** total spares cost.

Table 5.21. Optimum Spares Combination for Case 2. of Model IV.

Machining Stage (A_{ik})	Number of Spare on each Stage			Spares Cost
	0	1	2	
A_{11}	0	1	0	100
A_{41}	0	0	1	150
A_{71}	0	1	0	200
A_{81}	0	1	0	150
A_{12}	0	1	0	200
A_{32}	0	1	0	200
A_{62}	0	1	0	100
A_{102}	0	1	0	100
A_{13}	0	1	0	100
A_{23}	0	1	0	50
A_{53}	0	0	1	100
A_{93}	0	1	0	100
A_{14}	0	0	0	200
A_{34}	0	0	0	200
A_{74}	0	1	0	100
A_{94}	0	0	0	100

Note: "1" indicates an allocation.

The resultant system reliability where tool sharing is allowed yields 92.02 % with the total spares cost of \$1600.

With comparison to the same cases of Model I, the results of this model clearly indicates the possibility of having large reliability improvement with small increase in sparing cost. For the case where tool sharing is not possible the optimum spares cost found by Model I is \$2150. In contrast, the outcomes of Model IV yield a combination of spares which costs only \$50 more but results in more than 1.5% increase in the system reliability.

The models also give dissimilar tooling configuration for the case where tool sharing is applicable. The optimum spares combination resulted from Model costs \$1500 and gives 90.00% system reliability. Whereas, for the same circumstances, the outcomes of Model IV assigns the spares so that more than 2% growth in reliability can be obtained with an additional cost of \$100.

Another advantage of this model is that, for the system in which tool sharing cannot be implemented, it assigns the spares to stages which have got the minimum number of spares in previous allocations. In other words, there is always a balance for spare allocation for each tool type. For instance, Model I selected 2 copies of tool type 7 which both of them were allocated to machine 4. Model IV also took 2 spares of tool type 7, however one of them was assigned to machine 1 and the other to machine 4. This may result in less tool transportation during a production period. Table 5.22 summarizes the optimum sparing cost, tool slots occupancy, and system reliability resulting from the proposed models.

Table 5.22. Optimum System Attributes Obtained with Proposed Models.

Model	Tool sharing is not Possible			Tool sharing is Possible		
	System Reliability	Spares Cost	Slots Req.	System Reliability	Spares Cost	Slots Req.
I	90.25%	\$2150	30	90.08%	\$1500	24
II	90.40%	\$2200	30	90.62%	\$1600	23
III	90.25%	\$2150	30	90.12%	\$1550	23
IV	91.79%	\$2200	32	92.02%	\$1600	25

5.3. Sensitivity Analysis

The influences of tool sharing on reliability, flexibility, and tooling cost for a given FMS have been illustrated in the previous section. In the following sections, some sensitivity analysis will be performed to illustrate the influence of the number of tools shared among machining centres, minimum system reliability requirement, and the length of production periods on the optimum spares cost.

In order to show, more clearly, the effects of tool sharing for the system in which tool sharing is applicable, it is assumed that tool cost and required number of tool slot for each tool is identical for all tool types, \$100 and 1 slot respectively. The rest of tools and parts characteristics, except the number of similar tools in the system, are kept the same as those described in section 5.1.

5.3.1. Sensitivity Analysis of Tool Similarity

For minimum system reliability requirement ranging from 75% to 98%, a demonstration has been conducted with Model IV. Since the tool slot requirement and tool cost are considered to be identical for all tool types, Model I and Model IV would result in the same spares combination and cost.

In the following sensitivity analysis, 13 distinct Tool Similarities have been considered. This range covers from the situation that there is no common tools in the system, $TS=0$, to the case where all machines have an identical set of tools, $TS=12$. It should be noted that $TS=0$ can refer to the case that tool sharing is not applicable regardless of the number of similar tools in the system. Table 5.23 shows the tooling configuration of the system for $TS=0$.

Table 5.23. Tool-Machine Allocation, for $TS=0$.

Machine Type	Tool Type															
	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0	1 1	1 2	1 3	1 4	1 5	1 6
1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1

Note: "1" indicates an allocation.

As it was noted earlier, the failure parameters for each stage would remain the same as those described in section 5.1. To perform the sensitivity analysis it is supposed that at each step one machining stage can use the same tool type as another stage. For instance, if tool type 5 on machine 2 can be replaced by tool type 1, the Tool Similarity of the system would be equal to 1 and Tool Similarity would be equal to 2 if tool type 1 can also perform the job assigned to tool type 9 on machine 3, and so on. If it is possible to use similar tool type among all machining centres, the highest Tool Similarity that can

be achieved would be 12 which corresponds to the tooling system showed in Table 5.24.

Table 5.24. Tool-Machine Allocation, for TS=12.

Machine Type	Tool Type			
	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1

Note: "1" indicates an allocation.

Figures 5.1 and 5.2 show the optimum spares cost for different Tool Similarities and reliability requirements.

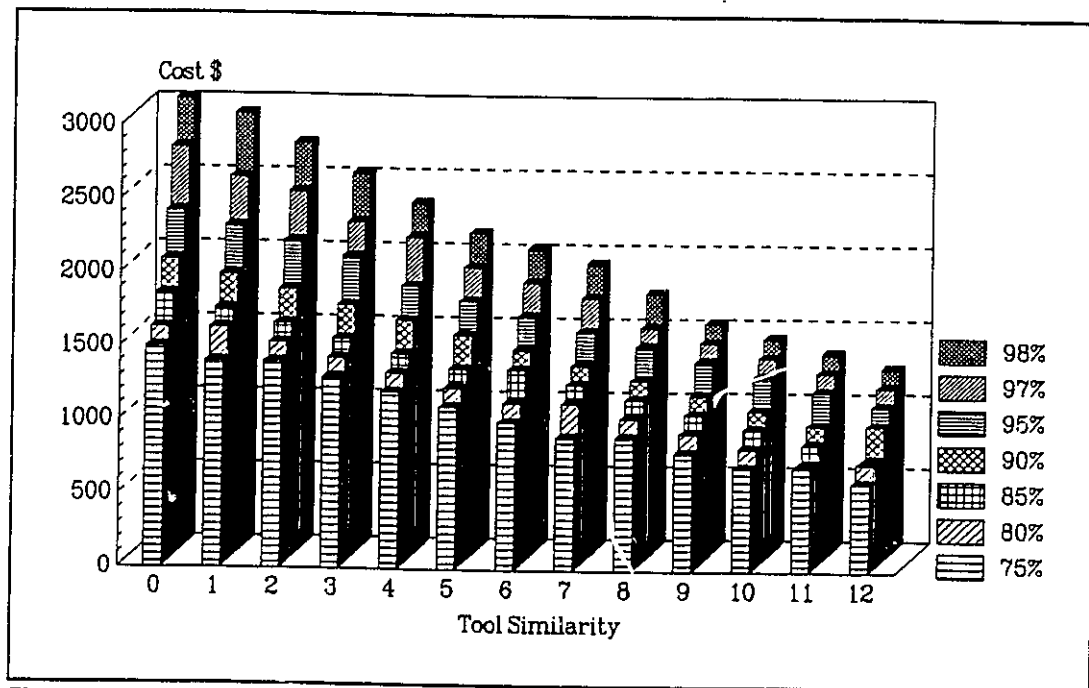


Figure 5.1 Sensitivity Analysis of the Effect of Tool Similarity on Spares Cost.

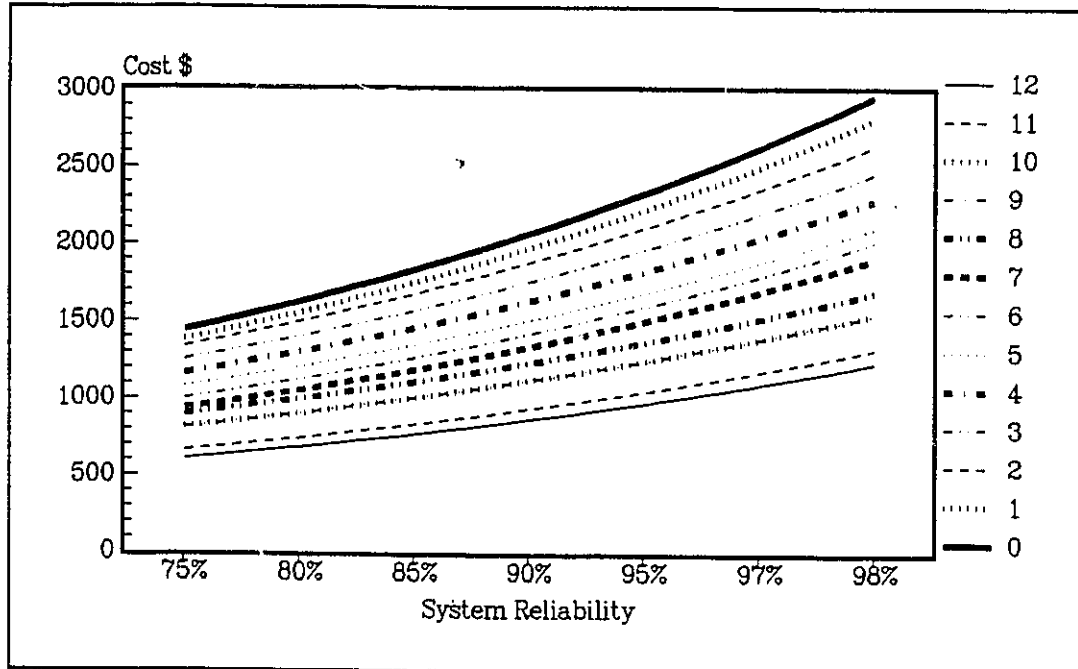


Figure 5.2 Sensitivity Analysis of the Effect of Reliability Requirement on Spares Cost.

Figures 5.1 and 5.2 explicitly illustrate the significant reduction of spares cost as Tool Similarity is increased. It is also shown that as the minimum system reliability requirement becomes closer to 100%, the spares cost is increased exponentially. This agrees with the exponential nature of reliability function. However, this growth of cost is slower for the systems with higher Tool Similarity than those with smaller Tool Similarity.

5.3.2. Sensitivity Analysis of Production Time

One of the major factors which determines the tool reliability, is the operation time assigned to it. Accordingly, there is a trade off between the production time and the spare requirement to obtain a certain system reliability.

This section is dedicated to examine the consequences of various production time on spares cost of the system with different reliability requirements and Tool Similarities.

Figures 5.3 and 5.4 show the spares cost of the system, versus different reliability requirement and Tool Similarities, when the production time is reduced to 75%. In other words, they show the cost of spares for a production period when only 75% of demand for each part type is produced.

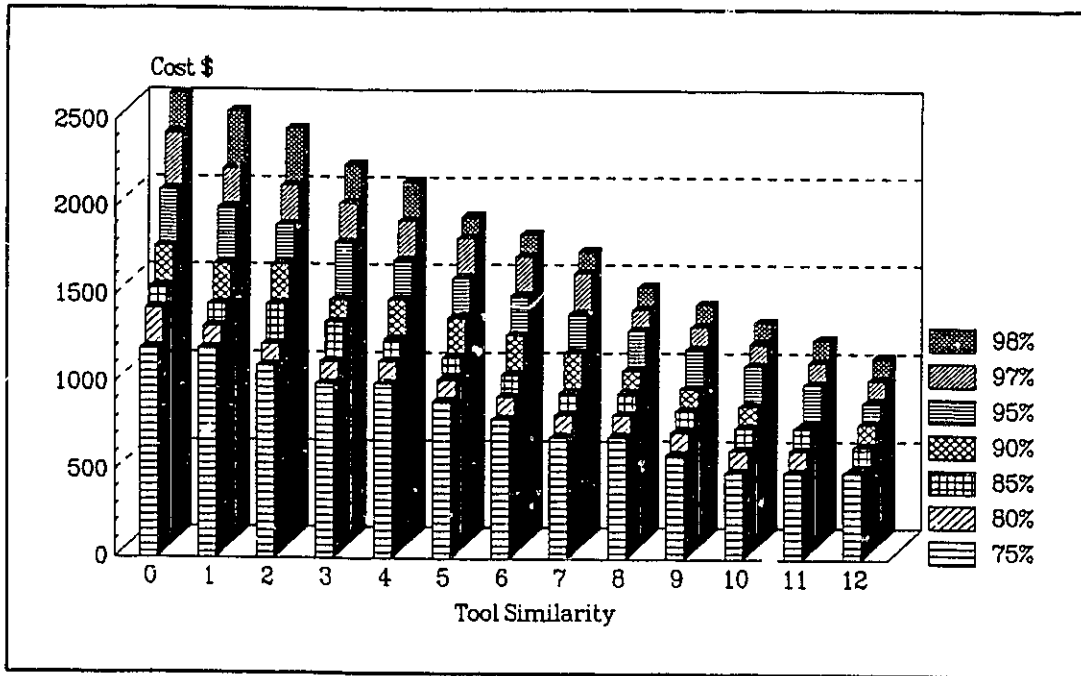


Figure 5.3 Sensitivity Analysis of the Effect of Tool Similarity on Spares Cost with 75% of Production Time.

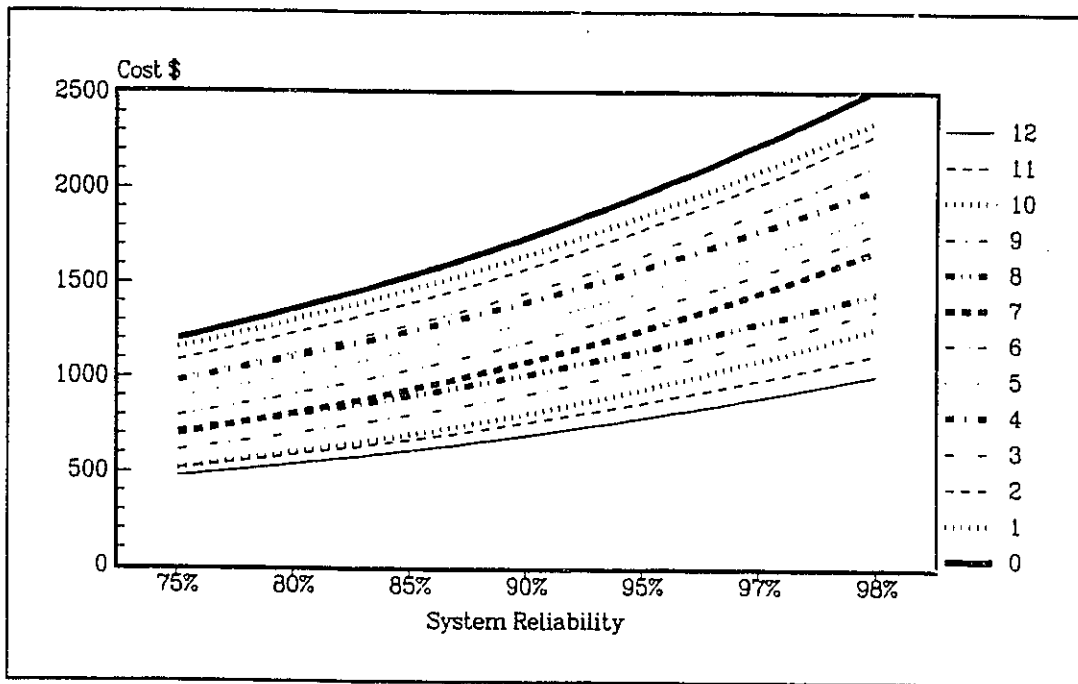


Figure 5.4 Sensitivity Analysis of the Effect of Reliability Requirement on Spares Cost with 75% Production Time.

Figures 5.5 and 5.6 contains the same information when production period is reduced to 50% from the original production time.

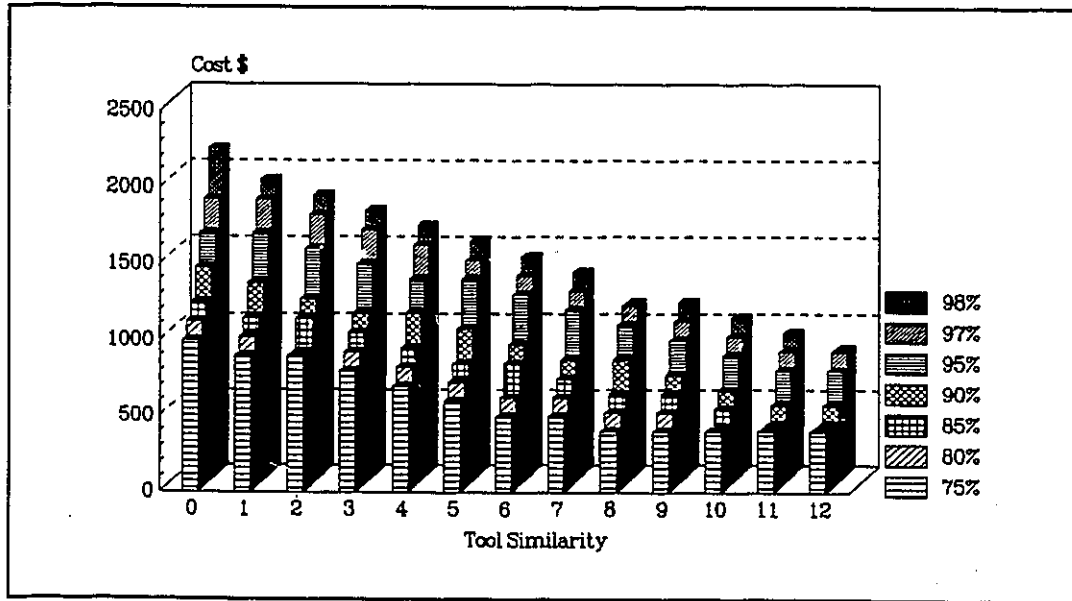


Figure 5.5 Sensitivity Analysis on the Effect of Tool Similarity on Spares Cost with 50% of Production Time.

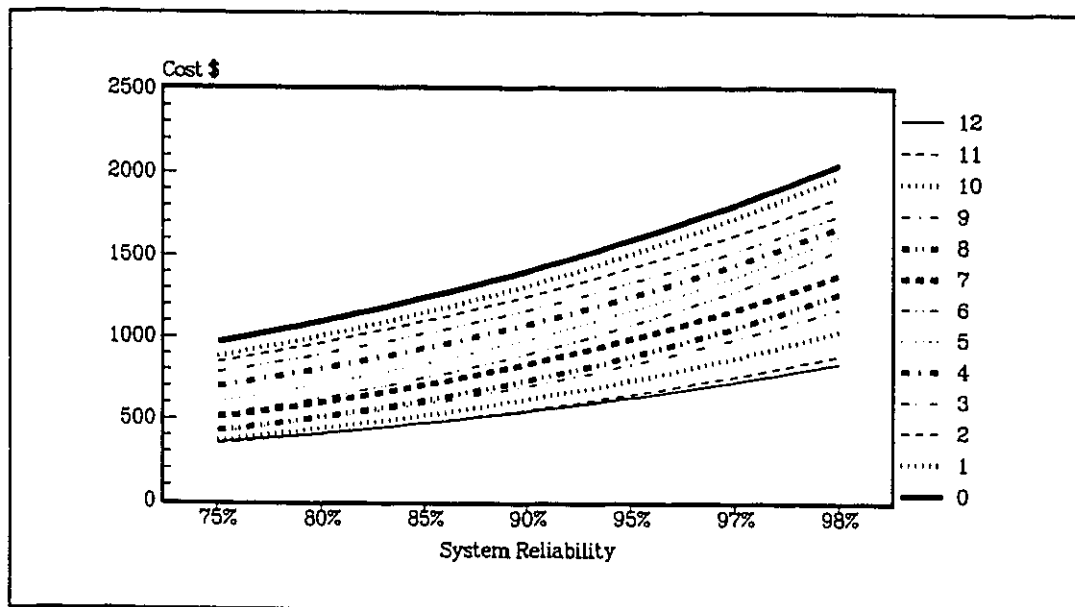


Figure 5.6 Sensitivity Analysis of the Effect of Reliability Requirement on Spares cost with 50% of Production Time.

A cost comparison is shown in Figure 5.7 for different production times, system reliabilities and Tool Similarities.

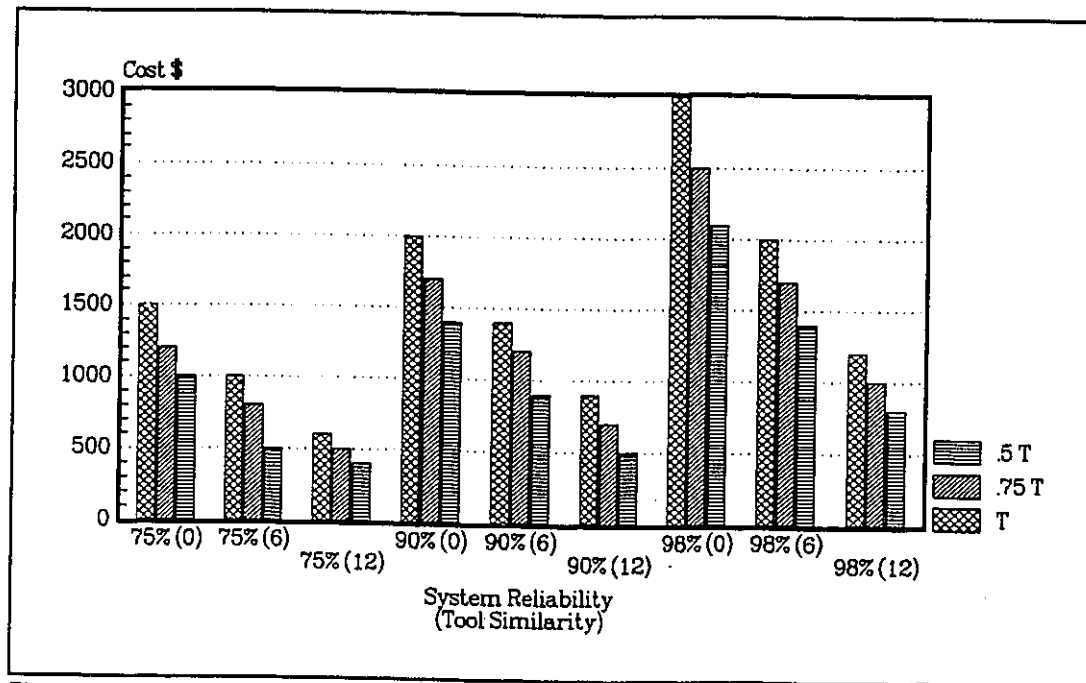


Figure 5.7 A Comparison Between the Effect of Production Time and Tool Similarity on Spares Cost.

Figure 5.7 clearly indicates that, for the given range of production time (from 100% to 75%, and 50%), cost reduction resulted from utilizing tool sharing in the system is more significant than those due to the shortened production time. For instance, for 90% required reliability, if tool sharing is not applicable, reducing the production period by 50% would result in \$600 cost reduction. For TS=6, the same amount could be saved if tool sharing can be implemented, and for TS=12 this cost reduction would be \$1100 which is more than 50% lower than spares cost for the case that tool sharing is not applicable.

CHAPTER 6

Conclusions and Recommendations for Further Works

6.1. Conclusions

With the advent of complex and expensive manufacturing systems, reliability, one of their essential performance measures, has become highly important. Tool reliability is one of the most essential aspects of an effective Flexible Manufacturing System, and has a significant influence on overall system reliability. This research aimed to evaluate the impacts of tool sharing on performance of FMSs with respect to reliability requirement.

Several conclusions may be drawn on the basis of the work and results presented in this dissertation :

1. It has been shown that, in term of reliability assessment, the tooling system of FMS can be treated as a series system with standby redundancy. It is also shown that reliability of such systems can be predicted with various failure distributions.
2. Based on distinct criteria, four models were developed to optimize the tooling cost of FMS. These models can provide the number and location of required spares when a desired level of tooling system reliability is specified.
3. The effects of tool sharing on cost, reliability and capacity of tooling system have been analyzed. The proposed models were justified to handle systems in which tool sharing is implemented.
4. This research has demonstrated the application of proposed model with a realistic example. It is shown that, with an additional cost of tool transportation device, a great reduction in spares cost and required tool magazines capacity can be achieved.
5. Several sets of sensitivity analysis were conducted to assess the importance of various parameters on system reliability and tooling cost. These parameters include number of tools shared among machining centres, minimum system reliability requirement, and length of production period.

6.2. Recommendation for Further Studies

Several problem areas directly associated with spares modelling were found during this study which require further investigation :

1. The reliability prediction models developed in this research have been applied for Weibull, exponential, and Erlangian failure distributions. With a minor extension of the computer codes the reliability of the system whose components are subject to other distributions can be computed.
2. It is possible to consider preventive maintenance and corrective maintenance policies for the modelling of tooling system in FMSs.
3. It is also possible to extend the reliability-based optimization models into availability-based models, to consider restoration aspects of the systems.
4. One may incorporate the inventory cost of spares, machine down time cost, and part defect cost into cost optimization models.
5. The problem of machine loading in FMS can be modeled where tooling system reliability and spares provision are taken into account.
6. Implementing tool sharing in an FMS environment, requires some sort of tool transportation facilities which impose additional costs and complexity into the system. These aspects of tool sharing and its impacts on FMS performance can be studied in future research.

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APPENDIX A

Reliability Mathematics

Some basic concepts used in reliability engineering are as follows:

○ **Failure Distribution Function $F(t)$:** Is the probability of failure of a component within the time interval $[0,t]$. The probability of failure as a function of time can be defined by:

$$P(0 \leq t \leq t) = F(t), \quad t \geq 0.$$

where t is a random variable denoting the failure time.

○ **Reliability Function $R(t)$:** Reliability can be defined as the probability of success, or the probability that the system will perform its intended function within the time interval $[0,t]$. It is expressed as: $R(t) = 1 - F(t) = P(t \geq t \geq 0)$.

If the time to failure random variable t has a density function $f(t)$, then:

$$R(t) = 1 - F(t) = 1 - \int_0^t f(t) dt = \int_t^{\infty} f(t) dt \quad (1)$$

○ **Hazard Rate $h(t)$:** The hazard rate is the conditional failure rate of the component which is usually expressed in failure per unit time. That is, if t represents the time to failure of a component, $h(t) dt$ is the probability that a component that has survived up to time t will fail in the next time interval dt . The function $h(t)$ can be defined by

$$h(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)} \quad (2)$$

Considering the above relationship, a general formula of reliability function in term of hazard rate can be expressed by (Dhillon, 1982).

$$R(t) = \exp \left(- \int_0^t h(t) dt \right) \quad (3)$$

USEFUL STATISTICAL DISTRIBUTIONS

A number of statistical distributions have been used to model failure characteristics. Table 1. summarises most of the more widely used distributions including those which are applied more often in mechanical reliability assessment. The associated reliability and failure functions, hazard rate, and the range of parameters variation are also listed.

Table 1. Some Statistical Distributions Useful in Reliability Engineering.

Type of distribution	Probability density Function $f(t)$	Cumulative Distribution Function $F(t)$	Reliability Function $R(t)$	Hazard Rate $h(t)$
Normal	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]$ $-\infty < t < \infty$	$\int_0^t f(t) dt$	$\int_t^{\infty} f(t) dt$	$\frac{f(t)}{R(t)}$
Lognormal	$\frac{1}{t\sigma\sqrt{2\pi}} \exp\left[-\frac{(\log t - \mu)^2}{2\sigma^2}\right]$ $0 < t < \infty$	$\int_0^t f(t) dt$	$\int_t^{\infty} f(t) dt$	$\frac{f(t)}{R(t)}$
Weibull two-parameter	$\frac{\beta t^{\beta-1}}{\lambda^\beta} \exp\left[-\left(\frac{t}{\lambda}\right)^\beta\right]$ $t \geq 0, \lambda > 0, \beta > 0$	$1 - \exp\left[-\left(\frac{t}{\lambda}\right)^\beta\right]$	$\exp\left[-\left(\frac{t}{\lambda}\right)^\beta\right]$	$\frac{\beta t^{\beta-1}}{\lambda^\beta}$
Exponential	$\lambda \exp(-t\lambda)$ $t \geq 0$	$1 - \exp(-t\lambda)$	$\exp(-\lambda t)$	λ
Gamma	$\frac{\lambda^\eta}{\Gamma(\eta)} t^{\eta-1} \exp(-t\lambda)$ $t \geq 0, \eta > 0$	$\sum_{k=1}^{\infty} \frac{(\lambda t)^k \exp[-\lambda t]}{k!}$	$\sum_{k=0}^{\eta-1} \frac{(\lambda t)^k \exp[-\lambda t]}{k!}$	$\frac{f(t)}{R(t)}$
Special Erlangian	$\frac{t}{\alpha^2} \exp\left(-\frac{t}{\alpha}\right)$ $t \geq 0$	$1 - \left(1 + \frac{t}{\alpha}\right) \exp\left(-\frac{t}{\alpha}\right)$	$\left(1 + \frac{t}{\alpha}\right) \exp\left(-\frac{t}{\alpha}\right)$	$\frac{t}{\alpha(t+\alpha)}$

APPENDIX B

Series Systems With Standby Redundancy

Figure 1. illustrates a series configuration reliability diagram which consists of I units.

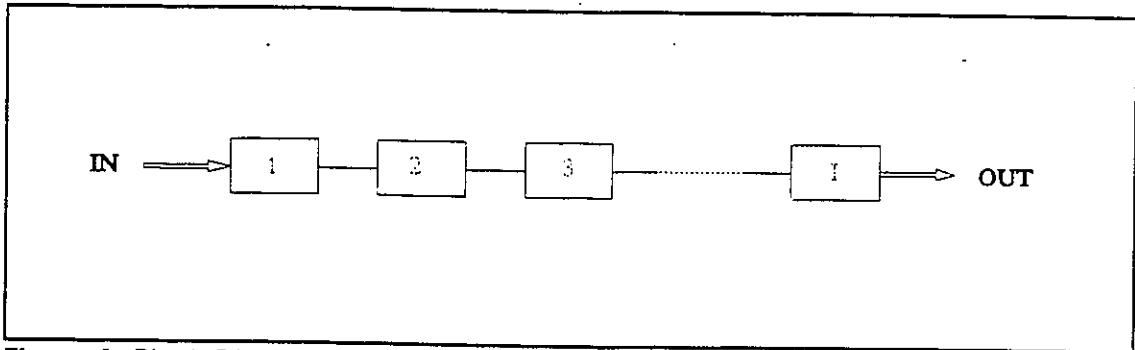


Figure 1. Block Diagram for Series Configuration of I Components.

To calculate the reliability of the such a system, let assume E_i denotes the successful operation of each unit. The probability $P(t)$, of successful operation of the system for a time t is given as follow:

$$P_s(t) = P(E_1 \cap E_2 \cap E_3 \dots \cap E_I) \quad (4)$$

$P_s(t)$ gives the reliability of the system $R_s(t)$; thus, $R_s(t) = P_s(t)$. If the units do not interact and their failures are independent, then Equation 4. can be rewritten to:

$$R_s(t) = P(E_1) P(E_2) \dots P(E_l) \quad (5)$$

Thus;

$$R_s(t) = R_1(t) R_2(t) \dots R_l(t). \quad (6)$$

Then,

$$R_s(t) = \prod_{i=1}^l R_i(t) \quad (7)$$

Equation 7. constitutes what is commonly called the *product rule* in reliability. Since the reliability of each component can not be more than 1, the system reliability decreases rapidly as the number of series components increases.

The reliability function for a component that has M standby redundant unit can be denoted by $R^M(t)$, if one assumes perfect failure detection and replacement. This is equivalent to the assuming that switch is 100% reliable. A two-unit standby component, leads to the following mathematical formulation, Pan et al. (1986).

$$\begin{aligned} R^2(t) &= P[(t_1 > t) \text{ or } (t_1 \leq t \text{ and } t_2 > t - t_1)] \\ &= P(t_1 > t) + P(t_1 \leq t \text{ and } t_2 > t - t_1) \end{aligned} \quad (8)$$

If the success modes are mutually exclusive, Equation 8. can be written as follows:

$$R^2(t) = R_1(t) + \int_0^t f_1(t_1) R_2(t-t_1) dt_1 \quad (9)$$

If one considers the general case where each standby redundant unit is subject to general failure distribution, and not necessary a same distribution, the reliability of the subsystem with M standby spares with the perfect switching becomes: Dhillon and Singh (1981).

$$R^{M+1}(t) = R^M(t) + \int_0^t f_1(t_1) \int_0^{t-t_1} f_2(t_2) \int_0^{t-t_1-t_2} f_3(t_3) \dots \int_0^{t-t_1-\dots-t_{M-1}} f_M(t_M) R_{M+1}(t-t_1-\dots-t_M) dt_M \dots dt_1 \quad (10)$$

Equation 10. is the general formula for calculation the reliability of standby redundant systems whose standby components are not identical. If the standby redundancies are identical for each components, the following system reliability expression, with general unit hazard rate $\lambda(t)$, can be developed, Dhillon (1982).

$$R^M(t) = \sum_{m=0}^M \frac{\left(\int_0^t \lambda(t) dt \right)^m \exp \left[- \int_0^t \lambda(t) dt \right]}{m!} \quad (11)$$

Equation 11. gives the reliability of a component with any number of standby redundancies, M , as long as the spares are identical.

APPENDIX C

Computer Code for Calculation of the Reliability of Machining Stages Where Tool Sharing is not Possible

The following computer program developed in TURBO C will compute the reliability of machining stages with different number of spares for the case that tool sharing is not possible.

A user friendly interface accepts all required information through an interaction with the user. The outputs of this program can be used as the reliability coefficients for the first cases in models I to III proposed in Chapter 3. Parameters defined in this Appendix includes those used in other computers codes developed in this research.

/***** PARAMETERS DEFINITION *****/

Aik[] : 0 if tool i assigned to machine k, -1 otherwise;
 Bik[] : Scale parameter of Weibull distribution;
 bik[] : Shape parameter of Weibull distribution;
 capk[] : Tool magazine capacity of machine k;
 COMBIN22 : Output file name for reliability of machining stages;
 COMBIN11 : Output file name for reliability of tool types;
 COST[] : Cost of tool type i;
 DISTik[] : Type of failure distribution of tool type i on machine k;
 Eik[] : Failure parameter of Erlangian distribution;
 factrl[] : Factorial number of spares;
 Fik[] : Failure rate of tool type i;
 Fr : Failure rate of transportation device;
 hold[][] : Existing number of spares for each stage;
 JOB : Number of parts in the system;
 MAC : Number of machines in the system;
 Mi[] : Maximum number of spares of tool type i in the system;
 Mmax : Maximum number of spares for each machining stages;
 NS : Number of machining stages in the system;
 OPTIM-22 : Output file name for optimum solution when tool sharing is not allowed;
 penalt : Penalty for assigning more than possible number of spares to each stage;
 Plk[] : 0-1 part machine assignment;
 pos[][] : Number of spares added to each each stage at each iteration (0-1);
 ratio[] : Spare Cost/Improve Reliability for each iteration;
 Rd : Minimum reliability requirement;
 Rex : Optimum existence reliability;
 Rikm[][][] : Reliability of tool type i on machine k with m spares;
 Rik[] : Reliability of tool type i on machine k at each iteration;
 Rop[] : Reliability of the system with one additional spares for each stage;
 Rr : Reliability of tool transportation device;
 Rsm[][] : Reliability of each machining stage with m spares;
 SCOST : Total spares cost for the system for the optimum reliability;
 Smax : Maximum number of spares for machining stage;
 Tik[] : Total operating time of tool i on machine k;
 TOOL : Number of tools type in the system;
 tr : Tool transportation time;
 Tr : Maximum possible operation time for tool transportation device;
 Wi[] : Number of tool slots required by tool type i;
 i : Tool index;
 k : Machine index;
 M : Spare index:

```

#include <stdio.h>
#include <math.h>
#define TOOL 10
#define MAC 4
#define JOB 10
#define Smax 2
#define NS 16

main ()

{

int a, b, i, j, l, k, d, p = 1, q = 1, M, COST[NS], DISTik[TOOL][MAC], factrl[TOOL];

float t, h, c, b, p, B, E;

double Rik[NS][MAC], Rikm[NS][MAC][Smax + 1], ri, haz;

FILE *REL_NTS;

static float Fik[TOOL][MAC], Bik[TOOL][MAC], bik[TOOL][MAC], Eik[TOOL][MAC],
Tik[TOOL][MAC];

        /***** INPUT DATA *****/

        for (k = 1; k <= MAC; ++k)
        for (i = 1; i <= TOOL; ++i)
        {
                clrscr ();
                printf("IS TOOL TYPE %d MOUNTED ON MACHINE %d ? ( 0 or -1 ) [ ]\n", i,k);
                gotoxy (50,1);
                scanf("%d", & a);
                Aik[i-1][k-1] = a;
        }

        for (i = 1; i <= TOOL; ++i)
        {
                clrscr ();
                printf("WHAT IS THE COST OF TOOL TYPE %d ? $/tool [ ]\n",i);
                gotoxy (50,1);
                scanf ("%d" ,& c);
                COST[i-1] = c;
        }

```

```

for (k=1; k<=MAC; ++k)
for (i=1; i<=TOOL; ++i)
if (Aik[i-1][k-1]==0)
{
clrscr();
printf("WHAT IS THE FAILURE DISTRIBUTION OF TOOL TYPE %d ON MACHINE
%d ? (1 EXP., 2 WEIB., 3 ERLA.) [ ] \n",i,k);
gotoxy (78,1);
scanf ("%d",& d);
DISTik[i-1][k-1]=d;
}

```

```

for (k=1; k<=MAC; ++k)
for (i=1; i<=TOOL; ++i)
if (Aik[i-1][k-1]==0)
{
if (DISTik[i-1][k-1]==1)
{
clrscr();
printf("WHAT IS THE FAILURE RATE OF TOOL TYPE %d ON MACHINE %d ? [ ]
\n",i,k);
gotoxy (57,1);
scanf ("%f" ,& h);
Fik[i-1][k-1]=h;
}

```

```

if (DISTik[i-1][k-1]==2)
{
clrscr();
printf("WHAT IS THE SCALE PARAMETER OF TOOL TYPE %d ON MACHINE %d ?
[ ] \n",i,k);
gotoxy (47,1);
scanf ("%f" ,& B);
Bik[i-1][k-1]=B;

```

```

printf("WHAT IS THE SHAPE PARAMETER OF TOOL TYPE %d ON MACHINE %d ?
[ ] \n",i,k);
gotoxy (47,2);
scanf ("%f" ,& b);
bik[i-1][k-1]=b;
}

```

```

if (DISTik[i-1][k-1]==3)
{
clrscr();

```

```

printf("WHAT IS THE ERLANGIAN PARAMETER OF TOOL TYPE %d ON MACHINE %d?
[ ] \n",i,k);
    gotoxy (51,1);
    scanf ("%f" ,& E);
    Eik[i-1][k-1]=E;
}
}

```

```

    factrl[0] = 1;
    factrl[1] = 1;
    for(M=2;M<=10; ++M)
    factrl[M] = M*factrl[M-1];

```

```

    for(l = 1; l <= JOB; ++l)
    for(k = 1; k <= MAC; ++k)
    Plk[l-1][k-1]=0;

```

```

    for(l = 1; l <= JOB; ++l)
    z:for(k = 1; k <= MAC; ++k)
    {
    clrscr();
    printf ("IS PART NUMBER %d ASSIGNED TO MACHINE %d ? (0 or 1) [ ] \n",l,k);
    gotoxy (52,1);
    scanf("%d",&p);
    Plk[l-1][k-1]=p;
    if (p!=0 && l < JOB)
    {
        l=l+1;
        goto z;
    }
}

```

```

    for(k = 1; k <= MAC; ++k)
    for(i = 1; i <= TOOL; ++i)
    {
    if (Aik [i-1][k-1] == 0)
    Tik[i-1][k-1]=0;
    else Tik[i-1][k-1]=-1;
    }

```

```

for(k = 1; k <= MAC; ++k)
for(l = 1; l <= JOB; ++l)
for(i = 1; i <= TOOL; ++i)
{
if (Plk[l-1][k-1] == 1 && Aik [i-1][k-1] == 0)
{
clrscr();

```

```

printf ("WHAT IS THE OPERATING TIME OF PART %d WITH TOOL %d ON MACHINE
%d ? (min.) [  ]\n",l,i,k);
gotoxy (73,1);
scanf ("%f", & t);
Tik[i-1][k-1] = Tik[i-1][k-1] + t;
}
}

```

/***** RELIABILITY CALCULATION *****/

```

for(k = 1;k <= MAC; ++k)
for(i = 1;i <= TOOL; ++i)
for(M = 0; M <= Smax; ++M)
{
Rikm[i-1][k-1][M] = 0;
Rik[i-1][k-1] = 0;
}

clrscr();

for(k = 1; k <= MAC; ++k)
for(i = 1; i <= TOOL; ++i)
if (Aik[i-1][k-1] != -1)
{
if (DISTik[i-1][k-1] == 1)
for(M = 0;M <= Smax; ++M)
{
haz = Fik[i-1][k-1] * Tik[i-1][k-1];
ri = (pow ( haz , M ) * exp( -haz )) /factrl[M];
Rik[i-1][k-1] = ri + Rik[i-1][k-1];
Rikm[i-1][k-1][M] = Rik[i-1][k-1];
}

if (DISTik[i-1][k-1] == 2)
for(M = 0;M <= Smax; ++M)
{
haz = pow( (Tik[i-1][k-1]/Bik[i-1][k-1]) , bik[i-1][k-1]);
ri = ( pow ( haz , M ) * exp( -haz)) /factrl[M];
Rik[i-1][k-1] = ri + Rik[i-1][k-1];
Rikm[i-1][k-1][M] = Rik[i-1][k-1];
}
}
}

```

```

if (DISTik[i-1][k-1] == 3)
  for(M=0;M<=Smax; ++M)
  {
    haz = (Tik[i-1][k-1]/Eik[i-1][k-1]) + (log(Eik[i-1][k-1]) -
(log(Eik[i-1][k-1] + Tik[i-1][k-1]));
    ri = ( pow ( haz , M ) * exp (-haz)) /factrl[M];
    Rik[i-1][k-1] = ri + Rik[i-1][k-1];
    Rikm[i-1][k-1][M] = Rik[i-1][k-1];
  }
}

```

***** FORMAT OF OUTPUT *****

```
REL_ANTS = fopen ("COMBIN22", "w");
```

```

fprintf (REL_ANTS, " STAGE NUMBER OF SPARES ON EACH
MACHINING STAGE \n\n");
fprintf (REL_ANTS, " 0 1 2 \n");
fprintf (REL_ANTS, " ----- \n\n");
fprintf (REL_ANTS, " %5d",p);

```

```
p += 1;
```

```

for(k = 1; k <= MAC; ++k)
for(i = 1; i <= TOOL; ++i)
for(M=0; M <= Smax; ++M)
if(Rikm[i-1][k-1][M] != 0)
{
  fprintf (REL_ANTS, " %20.8f",Rikm[i-1][k-1][M]);
  if (M == Smax)
  if (p <= NS)
  {
    fprintf (REL_ANTS, "\n %5d",p);
    p += 1;
  }
}

```

```

fprintf (REL_ANTS, "\n\n CORRESPONDING NATURAL LOGARITHMS
X -1000000\n ");
fprintf (REL_ANTS, " STAGE NUMBER OF SPARES ON EACH
MACHINING STAGE \n\n");
fprintf (REL_ANTS, " 0 1 2 \n");
fprintf (REL_ANTS, " ----- \n\n");
fprintf (REL_ANTS, " %5d",q);

```

```

q += 1;
for(k = 1; k <= MAC; ++k)
for(i = 1; i <= TOOL; ++i)
for(M = 0; M <= Smax; ++M)
if(Rikm[i-1][k-1][M] != 0)
{
fprintf (REL_NTS, " %20.0f", (log10 (Rikm[i-1][k-1][M]) * -1000000));
    if (M == Smax)
    if (q <= NS)
    {
fprintf (REL_NTS, "\n %5d", q);
q += 1;
    }
}
}
}

```

APPENDIX D

Computer Code for Calculation of the Reliability of Machining Stages Where Tool Sharing is Possible

This subroutine computes the reliability of each tool type in the system when tool sharing can be performed. The definitions of variables are stated in Appendix C.

/****** MAIN PROGRAM *****/

```
#include <stdio.h>
#include <math.h>
#define TOOL 10
#define MAC 4
#define Smax 2
```

```
main ()
```

```
{
```

```
int i, j, k, l, b, d, n, q = 1, p = 1, M, Mmax = 0;
```

```
double Ri[TOOL], Rim[TOOL][9], ri, temp, tr = .25, Tr, Fr = .0001, Rr, HAZi[TOOL],
HAZik[TOOL][MAC];
```

```
static long int Mi[TOOL] = {8,2,4,2,2,2,4,2,4,2},
factrl[9] = {1,1,2,6,24,120,720,5040,40320},
```

```
int DISTik[10][4] = {{1,1,2,3},{0,0,2,0},{0,3,0,2},{3,0,0,0},{0,0,1,0},{0,2,0,0},
{1,0,0,2},{2,0,0,0},{0,0,2,2},{0,2,0,0}};
```

```
static float Fik[10][4] = {{.008,.0125,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0},
{0,0,.033,0},{0,0,0,0},{.02,0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0}};
```

```
Bik[10][4] = {{0,0,116,0},{0,0,715,0},{0,0,0,120},{0,0,0,0},{0,0,0,0},
{0,1245,0,0},{0,0,0,423},{86,0,0,0},{0,0,74,74},{0,480,0,0}};
```

```
bik[10][4] = {{0,0,1.09,0},{0,0,.531,0},{0,0,0,.76},{0,0,0,0},{0,0,0,0},
{0,.632,0,0},{0,0,0,.725},{.85,0,0,0},{0,0,1.6,1.6},{0,.624,0,0}};
```

```
Eik[10][4] = {{0,0,0,12},{0,0,0,0},{0,18.5,0,0},{15,0,0,0},{0,0,0,0},{0,0,0,0},
{0,0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0}};
```

```
Tik[10][4] = {{14,13.5,10,11},{-1,-1,11.5,-1},{-1,12,-1,8},{14,-1,-1,-1},{-1,-1,10.5,-1},
{-1,15.5,-1,-1},{14,-1,-1,12},{11.5,-1,-1,-1},{-1,-1,12,11.5},{-1,14,-1,-1}};
```

```
FILE *REL_PTS;
```

```
clrscr();
```

/****** RELIABILITY CALCULATION *****/

```
for(i = 1; i <= TOOL; ++i)
for (k = 1; k <= MAC; ++k)
{
Ri[i-1] = 0;
HAZi[i-1] = 0;
HAZik[i-1][k-1] = 0;
}
```

```
clrscr();
```

```
for (k = 1; k <= MAC; ++k)
for (i = 1; i <= TOOL; ++i)
{
if (DISTik[i-1][k-1] == 1)
HAZik[i-1][k-1] = Fik[i-1][k-1] * Tik[i-1][k-1];

if (DISTik[i-1][k-1] == 2)
HAZik[i-1][k-1] = pow( (Tik[i-1][k-1]/Bik[i-1][k-1]) , bik[i-1][k-1]);

if (DISTik[i-1][k-1] == 3)
HAZik[i-1][k-1] = (Tik[i-1][k-1]/Eik[i-1][k-1]) + (log(Eik[i-1][k-1]) -
(log(Eik[i-1][k-1] + Tik[i-1][k-1])));
}
```

```
for(k = 1; k <= MAC; ++k)
for(i = 1; i <= TOOL; ++i)
HAZi[i-1] = HAZi[i-1] + HAZik[i-1][k-1];
```

```
for(i = 1; i <= TOOL; ++i)
Mmax = Mmax + (Mi[i-1] - Smax);
Tr = tr * Mmax;
```

```
for(i = 1; i <= TOOL; ++i)
{
for(M = 0; M <= Mi[i-1]; ++M)
{
if (M > Smax)
Rr = exp (-Fr * Tr);
else Rr = 1;
ri = ((pow ( HAZi[i-1] , M ) * exp (-HAZi[i-1] )) /factrl[M] )) * Rr;
Ri[i-1] = ri + Ri[i-1];
Rim[i-1][M] = Ri[i-1];
}
}
```

/****** FORMAT OF OUTPUT *****/

REL_PTS = fopen ("COMBIN11", "w");

```
fprintf (REL_PTS," STAGE NUMBER OF SPARES ON EACH MACHINING STAGE \n\n");
fprintf (REL_PTS," 0 1 2 3 4 5 6 7 8 \n");
fprintf (REL_PTS," -----\n\n");
fprintf (REL_PTS,"%4d",p);
p += 1;
```

```
for(i = 1; i <= TOOL; ++i)
for(M = 0; M <= Mi[i-1]; ++M)
if(Rim[i-1][M] != 0)
{
fprintf (REL_PTS," %12.8f",Rim[i-1][M]);
if (M == Mi[i-1])
if (p <= TOOL)
{
fprintf (REL_PTS,"\n%4d",p);
p += 1;
}
}
}
```

```
fprintf (REL_PTS,"\n\n\n CORESPONDING NATURAL LOGARITHMS X -1000000\n" );
fprintf (REL_PTS," STAGE NUMBER OF SPARES ONEACH MACHINING STAGE \n\n");
fprintf (REL_PTS," 0 1 2 3 4 5 6 7 8 \n");
fprintf (REL_PTS," ----- \n\n");
fprintf (REL_PTS,"%4d",q);
q += 1;
```

```
for(i = 1; i <= TOOL; ++i)
for(M = 0; M <= Mi[i-1]; ++M)
if(Rim[i-1][M] != 0)
{
fprintf (REL_PTS,"%12.0lf", (log10 (Rim[i-1][M]) * -1000000));
if (M == Mi[i-1])
if (q <= TOOL)
{
fprintf (REL_PTS,"\n%4d",q);
q += 1;
}
}
}
```

APPENDIX E

Computer Program for Case 1 of Model VI

The purpose of this computer program is to provide the optimum spares combination with "maximum reliability improvement rate" criterion for the case that tool sharing is not implemented. The user interface section is removed for simplicity. However the same interface as that stated in Appendix C can be used.

/****** MAIN PROGRAM *****/

```
#include <stdio.h>
#include <math.h>
#define TOOL 10
#define MAC 4
#define Smax 2
#define penalt 100000
#define NS 16
main ()

{
FILE *PERF;

int i, j, k, b, c, d, g, p, q, M, swich, sw, su, sv, sx, m, u, n=0, SCOST=0, pos[NS][NS],
hold[NS][NS], res[NS], cont[NS];

float Rik[TOOL][MAC], Rikm[TOOL][MAC][Smax + 1], Rsm[NS][Smax + 1], Rop[NS], Rt,
Rex = 1, Rd, haz, ri, ratio[NS], temp;

static int factrl[5] = {1,1,2,6,24}, Wi[TOOL] = {1,1,2, 3,2,1,1,1,2,3},
COST[NS] = {100,150,100,150,100,200,100,100,100,50,50,100, 100,200,100,100},

DISTik[TOOL][MAC] = {{1,1,2,3},{0,0,2,0},{0,3,0,2},{3,0,0,0},{0,0,1,0},{0,2,0,0},
{1,0,0,2},{2,0,0,0},{0,0,2,2},{0,2,0,0}}, capk[MAC] = {10,9,10,10},

Aik[10][4] = {{0,0,0,0},{-1,-1,0,-1},{-1,0,-1,0},{0,-1,-1,-1},{-1,-1,0,-1},{-1,0,-1,-1},
{0,-1,-1,0},{0,-1,-1,-1},{-1,-1,0,0},{-1,0,-1,-1}};

static float Fik[TOOL][MAC] = {{.008,.0125,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0},
{0,0,.033,0},{0,0,0,0},{.02,0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0}},

Bik[TOOL][MAC] = {{0,0,116,0},{0,0,715,0},{0,0,0,120},{0,0,0,0},{0,0,0,0},
{0,1245,0,0},{0,0,0,423},{86,0,0,0},{0,0,74,74},{0,480,0,0}},

bik[TOOL][MAC] = {{0,0,1.09,0},{0,0,.531,0},{0,0,0,.76},{0,0,0,0},{0,0,0,0},
{0,.632,0,0},{0,0,0,.725},{.85,0,0,0},{0,0,1.6,1.6},{0,.624,0,0}},

Eik[TOOL][MAC] = {{0,0,0,12},{0,0,0,0},{0,18.5,0,0},{15,0,0,0},{0,0,0,0},{0,0,0,0},
{0,0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0}},

Tik[TOOL][MAC] = {{14,13.5,10,11},{-1,-1,11.5,-1},{-1,12,-1,8},{14,-1,-1,-1},
{-1,-1,10.5,-1},{-1,15.5,-1,-1},{14,-1,-1,12},{11.5,-1,-1,-1},{-1,-1,12,11.5},
{-1,14,-1,-1}};

clrscr();
```

/****** RELIABILITY CALCULATION *****/

```

for(k = 1;k <= MAC; ++k)
for(i = 1;i <= TOOL; ++i)
for(M = 0; M <= Smax; ++M)
{
Rikm[i-1][k-1][M] = 0;
Rik[i-1][k-1] = 0;
}

```

```
clrscr();
```

```

for(k = 1; k <= MAC; ++k)
for(i = 1; i <= TOOL; ++i)
if (Tik[i-1][k-1] != -1)
{

```

```

if (DISTik[i-1][k-1] == 1)
for(M = 0; M <= Smax; ++M)
{
haz = Fik[i-1][k-1] * Tik[i-1][k-1];
ri = (pow ( haz , M ) * exp( -haz )) /factrl[M];
Rik[i-1][k-1] = ri + Rik[i-1][k-1];
Rikm[i-1][k-1][M] = Rik[i-1][k-1];
}

```

```

if (DISTik[i-1][k-1] == 2)
for(M = 0; M <= Smax; ++M)
{
haz = pow( (Tik[i-1][k-1]/Bik[i-1][k-1]) , bik[i-1][k-1]);
ri = ( pow ( haz , M ) * exp (-haz)) /factrl[M];
Rik[i-1][k-1] = ri + Rik[i-1][k-1];
Rikm[i-1][k-1][M] = Rik[i-1][k-1];
}

```

```

if (DISTik[i-1][k-1] == 3)
for(M = 0; M <= Smax; ++M)
{
haz = (Tik[i-1][k-1]/Eik[i-1][k-1]) + (log(Eik[i-1][k-1]) - (log(Eik[i-1][k-1] +
Tik[i-1][k-1])))
ri = ( pow ( haz , M ) * exp (-haz)) /factrl[M];
Rik[i-1][k-1] = ri + Rik[i-1][k-1];
Rikm[i-1][k-1][M] = Rik[i-1][k-1];
}

```

```
}
```

```

for (k = 1; k <= MAC; ++k)
for (i = 1; i <= TOOL; ++i)
if (Aik[i-1][k-1] != -1)
{
su = 1;
while (su == 1)
{
for (M = 0; M <= Smax; ++M)
Rsm[n][M] = Rikm[i-1][k-1][M];
su = 0; n += 1;
}
}

```

/****** FINDING OUT OPTIMUM SOLUTION *****/

```

PERF = fopen ("OPTIM-22", "w");

```

```

NS = 1;
for(i = 0; i <= NS; ++i)
for(j = 0; j <= NS; ++j)
hold[i][j] = 0;

```

```

for(i = 0; i <= NS; ++i)
Rex = Rex * Rsm[i][0];

```

```

printf("PLEASE ENTER REQUIRE SYSTEM RELIABILITY. ( )\n\n");
gotoxy (43,1);
scanf ("%f", & Rd);

```

```

if(Rex >= Rd)
goto s;

```

```

M = 1; swich = 1;
while(swich == 1)
{
for(i = 0; i <= NS; ++i)
for(j = 0; j <= NS; ++j)
pos[i][j] = 0; c = 0;

```

```

for (k = 0; k <= (MAC-1); ++k)
for (i = 0; i <= (TOOL-1); ++i)
if (Aik[i][k] != -1)
{
if(Wi[i] > capk[k])
cont[c] = -1;
if(Wi[i] <= capk[k])
cont[c] = 0; c++;
}
}

```

```

for(i=0; i <= NS; ++i)
{
Rt = 1;
    for(j=0; j <= NS; ++j)
    {
        if(i!=j)
        {
            if (hold[i][i] >= Smax || cont[i] == -1)
            {
                ratio[i] = penalt;
                goto w;
            }
            Rt = Rt * Rsm[i][M + hold[i][i]] * Rsm[j][(M-1) + hold[j][j]];
        }
    }

    Rop[i] = Rt/(pow (Rsm[i][M + hold[i][i]],(NS-1)));
    ratio[i] = COST[i]/(Rop[i]-Rex);
    pos[i][i] += 1;
w: ;
}

for(i=0; i <= NS; ++i)
    hold[i][i] += 1;

for(d=0; d <= (NS-1); ++d)
for(b=d+1; b <= NS; ++b)
{
if (ratio[b] < ratio[d])
    {
temp = ratio[d];
ratio[d] = ratio[b];
ratio[b] = temp;

temp = Rop[d];
Rop[d] = Rop[b];
Rop[b] = temp;

        for(k=0; k <= NS; ++k)
        {
            q = pos[d][k];
            pos[d][k] = pos[b][k];
            pos[b][k] = q;
        }
    }
}

sw = 1;    j=0;

```

/****** REARRANGING SYSTEM CONFIGURATION *****/

```

while(sw == 1)
{
    for(i=0; i <= NS; ++i)
    if(pos[j][i] != 0)
    if(hold[i][i] <= Smax)
        {
            for(k=0; k <= NS; ++k)
                {
                    if(k == i)
                    m = k;
                    if(k != i)
                    hold[k][k] -= 1;
                }
            Rex = Rop[j];
            sw = 0;
        }
    j += 1;
    if(j > NS)
    {
        sw = 0;
        swich = 0;
        printf("THERE IS NO FEASIBLE SOLUTION FOR THIS SYSTEM !!\n");
    }
}

```

```

sx = 1; n = 0;
while (sx == 1)
{
    for (k=0; k <= (MAC-1); ++k)
    for (i=0; i <= (TOOL-1); ++i)
    if (Aik[i][k] != -1)
        {
            if (n == m)
                {
                    Aik[i][k] += 1;
                    capk[k] -= (1 * Wi[i]);
                    sx = 0;
                }
            ++n;
        }
}

```

```

if(Rex >= Rd)
{
s:    swich = 0;

```

```
/****** OUTPUT FORMAT *****/
```

```
fprintf(PERF, "\n\nTHE MINIMUM REQUIRED RELIABILITY IS %8.5f\n", Rd);  
fprintf(PERF, "\nOPTIMUM SYSTEM RELIABILITY IS [%8.6f]\n\n", Rex);
```

```
for(j=0; j<=NS; ++j)  
fprintf(PERF, "X%d = [%2d ] " , (j+1), hold[j][j]);
```

```
for(j=0; j<=NS; ++j)  
{  
    u=hold[j][j] * COST[j];  
    SCOST=u+SCOST;  
}
```

```
fprintf(PERF, "\nTOTAL COST OF SPARES FOR THE SYSTEM IS [$%3d ]\n\n", SCOST);  
}  
}
```

APPENDIX F

Computer Program for Case 2 of Model VI

This computer program uses the iterative procedures to optimize the sparing cost of the system, based on Model VI proposed in Chapter 3. This program provides the optimum tooling configuration for the case in which tool sharing is possible.

/****** MAIN PROGRAM *****/

```
#include <stdio.h>
#include <math.h>
#define penalt 100000
#define Smax 2
#define TOOL 10
#define MAC 4

main ()

{

int pos[TOOL][TOOL], hold[TOOL][TOOL], swich, sw, u, SCOST=0, i, j, k, l, b, d, n, m,
q, M, cont[TOOL], sel = 10, Mmax = 0;

double Ri[TOOL], Rim[TOOL][9], ri, temp, tr = .25, Tr, Fr = .0001, Rr, HAZi[TOOL],
HAZk[TOOL][MAC], Rt, Rex = 1, Rd, ratio[TOOL], Rop[TOOL];

static int COST[TOOL] = {100,50,200,150,50,100,100,150,100,100},

factrl[9] = {1,1,2,6,24,120,720,5040,40320}, Mi[TOOL] = {8,2,4,2,2,2,4,2,4,2},

DISTik[TOOL][MAC] = {{1,1,2,3},{0,0,2,0},{0,3,0,2},{3,0,0,0},{0,0,1,0},{0,2,0,0},
{1,0,0,2},{2,0,0,0},{0,0,2,2},{0,2,0,0}},

Aik[TOOL][MAC] = {{0,0,0,0},{-1,-1,0,-1},{-1,0,-1,0},{0,-1,-1,-1},{-1,-1,0,-1},
{-1,0,-1,-1},{0,-1,-1,0},{0,-1,-1,-1},{-1,-1,0,0},{-1,0,-1,-1}},
Wi[10] = {1,1,2,3,2,1,1,1,2,3}, capk[MAC] = {10,9,10,10};

static float Fik[TOOL][MAC] = {{.008,.0125,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0},
{0,0,.033,0},{0,0,0,0},{.02,0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0}},

Bik[TOOL][MAC] = {{0,0,116,0},{0,0,715,0},{0,0,0,120},{0,0,0,0},{0,0,0,0},
{0,1245,0,0},{0,0,0,423},{86,0,0,0},{0,0,74,74},{0,480,0,0}},

bik[TOOL][MAC] = {{0,0,1.09,0},{0,0,.531,0},{0,0,0,.76},{0,0,0,0},{0,0,0,0},
{0,.632,0,0},{0,0,0,.725},{.85,0,0,0},{0,0,1.6,1.6},{0,.624,0,0}},

Eik[TOOL][MAC] = {{0,0,0,12},{0,0,0,0},{0,18.5,0,0},{15,0,0,0},{0,0,0,0},{0,0,0,0},
{0,0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0}},

Tik[TOOL][MAC] = {{14,13.5,10,11},{-1,-1,11.5,-1},{-1,12,-1,8},{14,-1,-1,-1},
{-1,-1,10.5,-1},{-1,15.5,-1,-1},{14,-1,-1,12},{11.5,-1,-1,-1},{-1,-1,12,11.5},
{-1,14,-1,-1}};

FILE *REL_PTS;
```

/****** RELIABILITY CALCULATION *****/

```

for(i = 1; i <= TOOL; ++i)
for (k = 1; k <= MAC; ++k)
{
Ri[i-1] = 0;
HAZi[i-1] = 0;
HAZik[i-1][k-1] = 0;
}

```

clrscr();

```

for (k = 1; k <= MAC; ++k)
for (i = 1; i <= TOOL; ++i)
{
if (DISTik[i-1][k-1] == 1)
HAZik[i-1][k-1] = Fik[i-1][k-1] * Tik[i-1][k-1];

if (DISTik[i-1][k-1] == 2)
HAZik[i-1][k-1] = pow( (Tik[i-1][k-1]/Bik[i-1][k-1]) , bik[i-1][k-1]);

if (DISTik[i-1][k-1] == 3)
HAZik[i-1][k-1] = (Tik[i-1][k-1]/Eik[i-1][k-1]) + (log (Eik[i-1][k-1])) -
(log(Eik[i-1][k-1] + Tik[i-1][k-1]));
}

```

```

for(k = 1; k <= MAC; ++k)
for(i = 1; i <= TOOL; ++i)
HAZi[i-1] = HAZi[i-1] + HAZik[i-1][k-1];

```

```

for(i = 1; i <= TOOL; ++i)
Mmax = Mmax + (Mi[i-1] - Smax);
Tr = tr * Mmax;

```

```

for(i = 1; i <= TOOL; ++i)
{
for(M = 0; M <= Mi[i-1]; ++M)
{
if (M > Smax)
Rr = exp (-Fr * Tr);
else Rr = 1;

ri = ((pow ( HAZi[i-1] , M ) * exp ( -HAZi[i-1] )) /factrl[M]) * Rr;
Ri[i-1] = ri + Ri[i-1];
Rim[i-1][M] = Ri[i-1];
}
}

```

```
/****** FINDING OUT OPTIMUM SOLUTION *****/
```

```
REL_PTS = fopen ("OPTIM-11","w");

TOOL -= 1;  MAC -=1;

    for(i=0; i <= TOOL; ++i)
    for(j=0; j <= TOOL; ++j)
    hold[i][j]=0;

    for(i=0; i <= TOOL; ++i)
    Rex = Rex*Rim[i][0];

printf("PLEASE ENTER REQUIRE SYSTEM RELIABILITY. [  ]\n\n");
gotoxy (43,1);
scanf ("%lf", & Rd);

    if(Rex >= Rd)
    goto s;

M=1; swich=1;
while(swich == 1)
{
for(i=0; i <= TOOL; ++i)
for(j=0; j <= TOOL; ++j)
pos[i][j]=0;

    for (i=0; i <= TOOL; ++i)
    for (k=0; k <= MAC; ++k)
    if (Aik[i][k] != -1)
    {
        if (Wi[i] > capk[k])
        cont[i] = -1;
        if (Wi[i] <= capk[k])
        {
            cont [i] = 0; k = 10;
        }
    }
}
```

```

for(i=0; i <= TOOL; ++i)
{
    Rt = 1;
    for(j=0; j <= TOOL; ++j)
    {
        if(i=j)
        {
            if (hold[i][i] >= Mi[i] || cont[i] == -1)
            {
                ratio[i] = penalt;
                goto w;
            }
            Rt = Rt * Rim[i][M + hold[i][i]] * Rim[j][(M-1) + hold[j][j]];
        }
    }

    Rop[i] = Rt / (pow (Rim[i][M + hold[i][i]], (TOOL-1)));
    ratio[i] = COST[i] / (Rop[i] - Rex);
    pos[i][i] += 1;
w: ;
}

for(i=0; i <= TOOL; ++i)
    hold[i][i] += 1;

```

```

for(d=0; d <= (TOOL-1); ++d)
for(b=d+1; b <= TOOL; ++b)
{
    if (ratio[b] < ratio[d])
    {
        temp = ratio[d];
        ratio[d] = ratio[b];
        ratio[b] = temp;

        temp = Rop[d];
        Rop[d] = Rop[b];
        Rop[b] = temp;

        for(k=0; k <= TOOL; ++k)
        {
            q = pos[d][k];
            pos[d][k] = pos[b][k];
            pos[b][k] = q;
        }
    }
}

```

```

sw = 1;    j = 0;

```

/****** REARRANGING SYSTEM CONFIGURATION *****/

```

while(sw == 1)
{
    for(i=0; i <= TOOL; ++i)
    if(pos[j][i] != 0)
    if(hold[i][i] <= Mi[i])
        {
            for(k=0; k <= TOOL; ++k)
                {
                    if(k == i)
                    m = k;
                    if(k != i)
                    hold[k][k] -= 1;
                }
            Rex = Rop[j];
            sw = 0;
        }

    i += 1;
    if(j > TOOL)
    {
        sw = 0;        swich = 0;
        printf("THERE IS NO FEASIBLE SOLUTION FOR THIS SYSTEM !!\n");
    }
}

```

```

for (k=0; k <= MAC; ++k)
if (Aik[m][k] != -1)
if (Wi[m] <= capk[k])
{
    if (Aik[m][k] < sel)
        {
            temp = Aik[m][k];
            n = k;
        }
    sel = temp;
}
Aik[m][n] += 1;
capk[n] -= (1 * Wi[m]);
sel = 10;

```

```

if(Rex >= Rd)
{
s:    swich = 0;

```

```
/****** OUTPUT FORMAT *****/
```

```
fprintf(REL_PTS, "\n\nTHE MINIMUM REQUIRED RELIABILITY IS %8.5f\n", Rd);  
fprintf(REL_PTS, "\nOPTIMUM SYSTEM RELIABILITY IS [%8.6f]\n\n", Rex);
```

```
    j = 1;  
    for (i=0; i <= TOOL; ++i)  
    for (k=0; k <= MAC; ++k)  
    if (Aik[i][k] != -1)  
    fprintf(REL_PTS, "X%d%d = [%2d ] ", i+1, k+1, Aik[i][k]);
```

```
        for (i=0; i <= TOOL; ++i)  
        for (k=0; k <= MAC; ++k)  
        if (Aik[i][k] != -1)
```

```
        {  
        u = Aik[i][k] * COST[i];  
        SCOST = u + SCOST;  
        }
```

```
    fprintf(REL_PTS, "\nTOTAL COST OF SPARES FOR THE SYSTEM IS  
    [ $%3d ]\n\n", SCOST);
```

```
    }  
}  
}
```

APPENDIX G

LINDO Inputs for Model I, II, and III.

The following computer codes are used as inputs for LINDO to solve the example problem with the proposed linear models stated in Chapter 4.

LINDO Input for Case 1 of Model I

MINIMIZE Z

SUBJECT TO

Z- 0X110- 100X111- 200X112- 0X410- 150X411- 300X412- 0X710- 100X711-
200X712- 0X810- 150X811- 300X812- 0X120- 100X121- 200X122- 0X320- 200X321-
400X322- 0X620- 100X621- 200X622- 0X1020- 100X1021- 200X1022- 0X130-
100X131- 200X132- 0X230- 50X231- 100X232- 0X530- 50X531- 100X532- 0X930-
100X931- 200X932- 0X140- 100X141- 200X142- 0X340- 200X341- 400X342- 0X740-
100X741- 200X742- 0X940- 100X941- 200X942 = 0

48641X110 + 2536X111 + 94X112 + 119035X410 + 13835X411 + 1217X412 +
121602X710 + 14392X711 + 1292X712 + 78533X810 + 6346X811 + 374X812 +
73287X120 + 5566X121 + 307X122 + 64576X320 + 4373X321 + 213X322 +
27160X620 + 815X621 + 17X622 + 47849X1020 + 2457X1021 + 89X1022 +
30028X130 + 993X131 + 23X132 + 48459X230 + 2518X231 + 93X232 +
150483X530 + 21277X531 + 2334X532 + 23643X930 + 621X931 + 11X932 +
115557X140 + 13096X141 + 1120X142 + 55457X340 + 3266X341 + 137X342 +
32817X740 + 1181X741 + 30X742 + 22087X940 + 543X941 + 9X942 < = 45757.49

48641X110 + 2536X111 + 94X112 + 119035X410 + 13835X411 + 1217X412 +
121602X710 + 14392X711 + 1292X712 + 78533X810 + 6346X811 + 374X812 +
73287X120 + 5566X121 + 307X122 + 64576X320 + 4373X321 + 213X322 +
27160X620 + 815X621 + 17X622 + 47849X1020 + 2457X1021 + 89X1022 +
30028X130 + 993X131 + 23X132 + 48459X230 + 2518X231 + 93X232 +
150483X530 + 21277X531 + 2334X532 + 23643X930 + 621X931 + 11X932 +
115557X140 + 13096X141 + 1120X142 + 55457X340 + 3266X341 + 137X342 +
32817X740 + 1181X741 + 30X742 + 22087X940 + 543X941 + 9X942 -R = 0

X110 + X111 + X112 = 1
X410 + X411 + X412 = 1
X710 + X711 + X712 = 1
X810 + X811 + X812 = 1
X120 + X121 + X122 = 1
X320 + X321 + X322 = 1
X620 + X621 + X622 = 1
X1020 + X1021 + X1022 = 1
X130 + X131 + X132 = 1
X230 + X231 + X232 = 1
X530 + X531 + X532 = 1
X930 + X931 + X932 = 1
X140 + X141 + X142 = 1
X340 + X341 + X342 = 1
X740 + X741 + X742 = 1
X940 + X941 + X942 = 1

0X110+ 1X111+ 2X112+ 0X410+ 3X411+ 6X412+ 0X710+ 1X711+ 2X712+
0X810+ 1X811+ 2X812 <= 10
0X120+ 1X121+ 2X122+ 0X320+ 2X321+ 4X322+ 0X620+ 1X621+ 2X622+
0X1020+ 3X1021+ 6X1022 <= 9
0X130+ 1X131+ 2X132+ 0X230+ 1X231+ 2X232+ 0X530+ 2X531+ 4X532+
0X930+ 2X931+ 4X932 <= 10
0X140+ 1X141+ 2X142+ 0X340+ 2X341+ 4X342+ 0X740+ 1X741+ 2X742+
0X940+ 2X941+ 4X942 <= 10

End

Int X_{ikm}

$\forall i \in I_k, k, M_{ik}$

Leave

LINDO Input for Case 2 of Model I

MINIMIZE Z

SUBJECT TO

Z-0X10-100X11-200X12-300X13-400X14-500X15-600X16-700X17-800X18-0X20
-50X21-100X22-0X30-200X31-400X32-600X33-800X34-0X40-150X41-300X42
-0X50-50X51-100X52-0X60-100X61-200X62-0X70-100X71-200X72-300X73
-400X74-0X80-150X81-300X82-0X90-100X91-200X92-300X93-400X94-0X100
-100X101-200X102 = 0

267513X10+59079X11+10872X12+1606X13+196X14+23X15+5X16+3X17+
2.5X18+48459X20+2518X21+93X22+120033X30+14051X31+1246X32+
85X33+5X34+119035X40+13835X41+1217X42+150483X50+21277X51+
2334X52+27160X60+815X61+17X62+154419X70+22299X71+2505X72+
219X73+16X74+78533X80+6346X81+374X82+45730X90+2251X91+
78X92+2X93+0.5X94+47849X100+2457X101+89X102 <= 45757.49

267513X10+59079X11+10872X12+1606X13+196X14+23X15+5X16+3X17+
2.5X18+48459X20+2518X21+93X22+120033X30+14051X31+1246X32+
85X33+5X34+119035X40+13835X41+1217X42+150483X50+21277X51+
2334X52+27160X60+815X61+17X62+154419X70+22299X71+2505X72+
219X73+16X74+78533X80+6346X81+374X82+45730X90+2251X91+
78X92+2X93+0.5X94+47849X100+2457X101+89X102-R=0

Y11+Y12+Y13+Y14-0X10-1X11-2X12-3X13-4X14-5X15-6X16-7X17-8X18=0
Y23-0X20-1X21-2X22=0
Y32+Y34-0X30-1X31-2X32-3X33-4X34=0
Y41-0X40-1X41-2X42=0
Y53-0X50-1X51-2X52=0
Y62-0X60-1X61-2X62=0
Y71+Y74-0X70-1X71-2X72-3X73-4X74=0
Y81-0X80-1X81-2X82=0
Y93+Y94-0X90-1X91-2X92-3X93-4X94=0
Y102-0X100-1X101-2X102=0
1Y11+3Y41+1Y71+1Y81 <= 10
1Y12+2Y32+1Y62+3Y102 <= 9
1Y13+1Y23+2Y53+2Y93 <= 10
1Y14+2Y34+1Y74+2Y94 <= 10

X10+X11+X12+X13+X14+X15+X16+X17+X18=1
X20+X21+X22=1
X30+X31+X32+X33+X34=1
X40+X41+X42=1
X50+X51+X52=1
X60+X61+X62=1
X70+X71+X72+X73+X74=1
X80+X81+X82=1

$$X90 + X91 + X92 + X93 + X94 = 1$$
$$X100 + X101 + X102 = 1$$

$$Y11 \leq 2$$
$$Y12 \leq 2$$
$$Y13 \leq 2$$
$$Y14 \leq 2$$
$$Y23 \leq 2$$
$$Y32 \leq 2$$
$$Y34 \leq 2$$
$$Y41 \leq 2$$
$$Y53 \leq 2$$
$$Y62 \leq 2$$
$$Y71 \leq 2$$
$$Y74 \leq 2$$
$$Y81 \leq 2$$
$$Y93 \leq 2$$
$$Y94 \leq 2$$
$$Y102 \leq 2$$

End

$$Gin Y_{ik} \quad \forall i \in I_k, k$$

$$Int X_{ik} \quad \forall i, M_i$$

Leave

LINDO Input for Case 1 of Model II

MINIMIZE W

SUBJECT TO

W-0X110 -1X111 -2X112 -0X410 -3X411 -6X412 -0X710 -1X711 -2X712 -0X810
-1X811 -2X812 -0X120 -1X121 -2X122 -0X320 -2X321 -4X322 -0X620 -1X621 -2X622
-0X1020 -3X1021 -6X1022 -0X130 -1X131 -2X132 -0X230 -1X231 -2X232 -0X530
-2X531 -4X532 -0X930 -2X931 -4X932 -0X140 -1X141 -2X142 -0X340 -2X341 -4X342
-0X740 -1X741 -2X742 -0X940 -2X941 -4X942 = 0

0X110 +100X111 +200X112 +0X410 +150X411 +300X412 +0X710 +100X711
+200X712 +0X810 +150X811 +300X812 +0X120 +100X121 +200X122 +0X320
+200X321 +400X322 +0X620 +100X621 +200X622 +0X1020 +100X1021
+200X1022 +0X130 +100X131 +200X132 +0X230 +50X231 +100X232 +0X530
+50X531 +100X532 +0X930 +100X931 +200X932 +0X140 +100X141
+200X142 +0X340 +200X341 +400X342 +0X740 +100X741 +200X742 +0X940
+100X941 +200X942 - Z = 0

48641X110+ 2536X111+ 94X112+ 119035X410+ 13835X411+ 1217X412+
121602X710+ 14392X711+ 1292X712+ 78533X810+ 6346X811+ 374X812+
73287X120+ 5566X121+ 307X122+ 64576X320+ 4373X321+ 213X322+
27160X620+ 815X621+ 17X622+ 47849X1020+ 2457X1021+ 89X1022+
30028X130+ 993X131+ 23X132+ 48459X230+ 2518X231+ 93X232+
150483X530+ 21277X531+ 2334X532+ 23643X930+ 621X931+ 11X932+
115557X140+ 13096X141+ 1120X142+ 55457X340+ 3266X341+ 137X342+
32817X740+1181X741+ 30X742+ 22087X940+ 543X941 + 9X942 <= 45757.49

48641X110+ 2536X111+ 94X112+ 119035X410+ 13835X411+ 1217X412+
121602X710+ 14392X711+ 1292X712+ 78533X810+ 6346X811+ 374X812+
73287X120+ 5566X121+ 307X122+ 64576X320+ 4373X321+ 213X322+
27160X620+ 815X621+ 17X622+ 47849X1020+ 2457X1021+ 89X1022+
30028X130+ 993X131+ 23X132+ 48459X230+ 2518X231+ 93X232+
150483X530+ 21277X531+ 2334X532+ 23643X930+ 621X931+ 11X932+
115557X140+ 13096X141+ 1120X142+ 55457X340+ 3266X341+ 137X342+
32817X740+ 1181X741+ 30X742+ 22087X940+ 543X941 + 9X942 -R =0

+X110 +X111 +X112 = 1
+X410 +X411 +X412 = 1
+X710 +X711 +X712 = 1
+X810 +X811 +X812 = 1
+X120 +X121 +X122 = 1
+X320 +X321 +X322 = 1
+X620 +X621 +X622 = 1
+X1020 +X1021 +X1022 = 1
+X130 +X131 +X132 = 1
+X230 +X231 +X232 = 1

+X530 +X531 +X532 = 1
+X930 +X931 +X932 = 1
+X140 +X141 +X142 = 1
+X340 +X341 +X342 = 1
+X740 +X741 +X742 = 1
+X940 +X941 +X942 = 1

+0X110 +1X111 +2X112 +0X410 +3X411 +6X412 +0X710 +1X711 +2X712
+0X810 +1X811 +2X812 <= 10
+0X120 +1X121 +2X122 +0X320 +2X321 +4X322+ 0X620 +1X621 +2X622
+0X1020 +3X1021 +6X1022 <= 9
+0X130 +1X131 +2X132 +0X230 +1X231 +2X232 +0X530+ 2X531 +4X532
+0X930 +2X931 +4X932 <= 10
+0X140 +1X141 +2X142 +0X340 +2X341 +4X342 +0X740+ 1X741 +2X742
+0X940 +2X941 +4X942 <= 10

End

Int X_{ikm}

$\forall i \in I_k, k, M_{ik}$

Leave

LINDO Input for Case 2 of Model II

MINIMIZE W

SUBJECT TO

$$\begin{aligned} W - 0X10 - 1X11 - 2X12 - 3X13 - 4X14 - 5X15 - 6X16 - 7X17 - 8X18 - 0X20 - 1X21 - 2X22 \\ - 0X30 - 2X31 - 4X32 - 6X33 - 8X34 - 0X40 - 3X41 - 6X42 - 0X50 - 2X51 - 4X52 \\ - 0X60 - 1X61 - 2X62 - 0X70 - 1X71 - 2X72 - 3X73 - 4X74 - 0X80 - 1X81 - 2X82 \\ - 0X90 - 2X91 - 4X92 - 6X93 - 8X94 - 0X100 - 3X101 - 6X102 = 0 \end{aligned}$$

$$\begin{aligned} 0X10 + 100X11 + 200X12 + 300X13 + 400X14 + 500X15 + 600X16 + 700X17 \\ + 800X18 + 0X20 + 50X21 + 100X22 + 0X30 + 200X31 + 400X32 + 600X33 + 800X34 \\ + 0X40 + 150X41 + 300X42 + 0X50 + 50X51 + 100X52 + 0X60 + 100X61 + 200X62 \\ + 0X70 + 100X71 + 200X72 + 300X73 + 400X74 + 0X80 + 150X81 + 300X82 + 0X90 \\ + 100X91 + 200X92 + 300X93 + 400X94 + 0X100 + 100X101 + 200X102 - Z = 0 \end{aligned}$$

$$\begin{aligned} 267513X10 + 59079X11 + 10872X12 + 1606X13 + 196X14 + 23X15 + 5X16 + 3X17 + \\ 2X18 + 48459X20 + 2518X21 + 93X22 + 120033X30 + 14051X31 + 1246X32 + \\ 85X33 + 5X34 + 119035X40 + 13835X41 + 1217X42 + 150483X50 + 21277X51 + \\ 2334X52 + 27160X60 + 815X61 + 17X62 + 154419X70 + 22299X71 + 2505X72 + \\ 219X73 + 16X74 + 78533X80 + 6346X81 + 374X82 + 45730X90 + 2251X91 + \\ 78X92 + 2X93 + .5X94 + 47849X100 + 2457X101 + 89X102 \leq 45757.49 \end{aligned}$$

$$\begin{aligned} 267513X10 + 59079X11 + 10872X12 + 1606X13 + 196X14 + 23X15 + 5X16 + 3X17 + \\ 2X18 + 48459X20 + 2518X21 + 93X22 + 120033X30 + 14051X31 + 1246X32 + \\ 85X33 + 5X34 + 119035X40 + 13835X41 + 1217X42 + 150483X50 + 21277X51 + \\ 2334X52 + 27160X60 + 815X61 + 17X62 + 154419X70 + 22299X71 + 2505X72 + \\ 219X73 + 16X74 + 78533X80 + 6346X81 + 374X82 + 45730X90 + 2251X91 + \\ 78X92 + 2X93 + .5X94 + 47849X100 + 2457X101 + 89X102 - R = 0 \end{aligned}$$

$$\begin{aligned} Y11 + Y12 + Y13 + Y14 - 0X10 - 1X11 - 2X12 - 3X13 - 4X14 - 5X15 - 6X16 - 7X17 - 8X18 = 0 \\ Y23 - 0X20 - 1X21 - 2X22 = 0 \\ Y32 + Y34 - 0X30 - 1X31 - 2X32 - 3X33 - 4X34 = 0 \\ Y41 - 0X40 - 1X41 - 2X42 = 0 \\ Y53 - 0X50 - 1X51 - 2X52 = 0 \\ Y62 - 0X60 - 1X61 - 2X62 = 0 \\ Y71 + Y74 - 0X70 - 1X71 - 2X72 - 3X73 - 4X74 = 0 \\ Y81 - 0X80 - 1X81 - 2X82 = 0 \\ Y93 + Y94 - 0X90 - 1X91 - 2X92 - 3X93 - 4X94 = 0 \\ Y102 - 0X100 - 1X101 - 2X102 = 0 \end{aligned}$$

$$\begin{aligned} 1Y11 + 3Y41 + 1Y71 + 1Y81 \leq 10 \\ 1Y12 + 2Y32 + 1Y62 + 3Y102 \leq 9 \\ 1Y13 + 1Y23 + 2Y53 + 2Y93 \leq 10 \\ 1Y14 + 2Y34 + 1Y74 + 2Y94 \leq 10 \end{aligned}$$

$$\begin{aligned} X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 = 1 \\ X20 + X21 + X22 = 1 \end{aligned}$$

X30 + X31 + X32 + X33 + X34 = 1
X40 + X41 + X42 = 1
X50 + X51 + X52 = 1
X60 + X61 + X62 = 1
X70 + X71 + X72 + X73 + X74 = 1
X80 + X81 + X82 = 1
X90 + X91 + X92 + X93 + X94 = 1
X100 + X101 + X102 = 1

Y11 <= 2
Y12 <= 2
Y13 <= 2
Y14 <= 2
Y23 <= 2
Y32 <= 2
Y34 <= 2
Y41 <= 2
Y53 <= 2
Y62 <= 2
Y71 <= 2
Y74 <= 2
Y81 <= 2
Y93 <= 2
Y94 <= 2
Y102 <= 2

End

Gin Y_{ik} $\forall i \in I_k, k$

Int X_{ik} $\forall i, M_i$

Leave

LINDO Input for Case 1 of Model III

MINIMIZE Q

SUBJECT TO

Q-0X110-200X111-400X112-0X410-450X411-900X412-0X710-200X711-400X712
-0X810-250X811-500X812-0X120-200X121-400X122-0X320-400X321-800X322
-0X620-200X621-400X622-0X1020-400X1021-800X1022-0X130-200X131
-400X132-0X230-150X231-300X232-0X530-250X531-500X532-0X930-300X931
-600X932-0X140-200X141-400X142-0X340-400X341-800X342-0X740-200X741
-400X742-0X940-300X941-600X942 = 0

0X110 + 100X111 + 200X112 + 0X410 + 150X411 + 300X412 + 0X710 + 100X711
+ 200X712 + 0X810 + 150X811 + 300X812 + 0X120 + 100X121 + 200X122 + 0X320
+ 200X321 + 400X322 + 0X620 + 100X621 + 200X622 + 0X1020 + 100X1021
+ 200X1022 + 0X130 + 100X131 + 200X132 + 0X230 + 50X231 + 100X232 + 0X530
+ 50X531 + 100X532 + 0X930 + 100X931 + 200X932 + 0X140 + 100X141 + 200X142
+ 0X340 + 200X341 + 400X342 + 0X740 + 100X741 + 200X742 + 0X940 + 100X941
+ 200X942 - Z = 0

48641X110 + 2536X111 + 94X112 + 119035X410 + 13835X411 + 1217X412 +
121602X710 + 14392X711 + 1292X712 + 78533X810 + 6346X811 + 374X812 +
73287X120 + 5566X121 + 307X122 + 64576X320 + 4373X321 + 213X322 +
27160X620 + 815X621 + 17X622 + 47849X1020 + 2457X1021 + 89X1022 +
30028X130 + 993X131 + 23X132 + 48459X230 + 2518X231 + 93X232 +
150483X530 + 21277X531 + 2334X532 + 23643X930 + 621X931 + 11X932 +
115557X140 + 13096X141 + 1120X142 + 55457X340 + 3266X341 + 137X342 +
32817X740 + 1181X741 + 30X742 + 22087X940 + 543X941 + 9X942 < = 45757.49

48641X110 + 2536X111 + 94X112 + 119035X410 + 13835X411 + 1217X412 +
121602X710 + 14392X711 + 1292X712 + 78533X810 + 6346X811 + 374X812 +
73287X120 + 5566X121 + 307X122 + 64576X320 + 4373X321 + 213X322 +
27160X620 + 815X621 + 17X622 + 47849X1020 + 2457X1021 + 89X1022 +
30028X130 + 993X131 + 23X132 + 48459X230 + 2518X231 + 93X232 +
150483X530 + 21277X531 + 2334X532 + 23643X930 + 621X931 + 11X932 +
115557X140 + 13096X141 + 1120X142 + 55457X340 + 3266X341 + 137X342 +
32817X740 + 1181X741 + 30X742 + 22087X940 + 543X941 + 9X942 -R = 0

+X110 + X111 + X112 = 1
+X410 + X411 + X412 = 1
+X710 + X711 + X712 = 1
+X810 + X811 + X812 = 1
+X120 + X121 + X122 = 1
+X320 + X321 + X322 = 1
+X620 + X621 + X622 = 1
+X1020 + X1021 + X1022 = 1
+X130 + X131 + X132 = 1
+X230 + X231 + X232 = 1

$$\begin{aligned}
&+X530 + X531 + X532 = 1 \\
&+X930 + X931 + X932 = 1 \\
&+X140 + X141 + X142 = 1 \\
&+X340 + X341 + X342 = 1 \\
&+X740 + X741 + X742 = 1 \\
&+X940 + X941 + X942 = 1
\end{aligned}$$

$$\begin{aligned}
&+0X110+ 1X111+ 2X112+ 0X410+ 3X411+ 6X412+ 0X710+ 1X711+ 2X712+ \\
&0X810+ 1X811+ 2X812 \leq 10 \\
&+0X120+ 1X121+ 2X122+ 0X320+ 2X321+ 4X322+ 0X620+ 1X621+ 2X622+ \\
&0X1020+ 3X1021+ 6X1022 \leq 9 \\
&+0X130+ 1X131+ 2X132+ 0X230+ 1X231+ 2X232+ 0X530+ 2X531+ 4X532+ \\
&0X930+ 2X931+ 4X932 \leq 10 \\
&+0X140+ 1X141+ 2X142+ 0X340+ 2X341+ 4X342+ 0X740+ 1X741+ 2X742+ \\
&0X940+ 2X941+ 4X942 \leq 10
\end{aligned}$$

End

$$Int X_{ikm} \quad \forall i \in I_k, k, M_{ik}$$

Leave

LINDO Input for Case 2 of Model III

MINIMIZE Q

SUBJECT TO

Q- 0Y10 -200Y11 -400Y12 -600Y13 -800Y14 -1000Y15 -1200Y16 -1400Y17 -1600Y18
-0Y20 -150Y21 -300Y22 -0Y30 -400Y31 -800Y32 -1200Y33 -1600Y34 -0Y40 -450Y41
-900Y42 -0Y50 -250Y51 -500Y52 -0Y60 -200Y61 -400Y62 -0Y70 -200Y71 -400Y72
-600Y73 -800Y74 -0Y80 -250Y81 -500Y82 -0Y90 -300Y91 -600Y92 -900Y93 -1200Y94
-0Y100 -400Y101 -800Y102 = 0

0Y10 +100Y11 +200Y12 +300Y13 +400Y14 +500Y15 +600Y16 +700Y17
+800Y18 +0Y20 +50Y21 +100Y22 +0Y30 +200Y31 +400Y32 +600Y33 +800Y34
+0Y40 +150Y41 +300Y42 +0Y50 +50Y51 +100Y52 +0Y60 +100Y61 +200Y62
+0Y70 +100Y71 +200Y72 +300Y73 +400Y74 +0Y80 +150Y81 +300Y82 +0Y90
+100Y91 +200Y92 +300Y93 +400Y94 +0Y100 +100Y101 +200Y102 -Z = 0

267513Y10+ 59079Y11+ 10872Y12+ 1606Y13+ 196Y14+ 23Y15+ 5Y16+ 3Y17
+3Y18+ 48459Y20+ 2518Y21+ 93Y22+ 120033Y30+ 14051Y31+ 1246Y32+
85Y33+ 5Y34+ 119035Y40+ 13835Y41+ 1217Y42+ 150483Y50+ 21277Y51+
2334Y52+ 27160Y60+ 815Y61+ 17Y62+ 154419Y70+ 22299Y71+ 2505Y72+
219Y73+ 16Y74+ 78533Y80+ 6346Y81+ 374Y82+ 45730Y90+ 2251Y91+ 78Y92+
2Y93+ .3Y94+ 47849Y100+ 2457Y101+ 89Y102 <= 45757.49

267513Y10+ 59079Y11+ 10872Y12+ 1606Y13+ 196Y14+ 23Y15+ 5Y16+ 3Y17
+3Y18+ 48459Y20+ 2518Y21+ 93Y22+ 120033Y30+ 14051Y31+ 1246Y32+
85Y33+ 5Y34+ 119035Y40+ 13835Y41+ 1217Y42+ 150483Y50+ 21277Y51+
2334Y52+ 27160Y60+ 815Y61+ 17Y62+ 154419Y70+ 22299Y71+ 2505Y72+
219Y73+ 16Y74+ 78533Y80+ 6346Y81+ 374Y82+ 45730Y90+ 2251Y91+ 78Y92+
2Y93+ .3Y94+ 47849Y100+ 2457Y101+ 89Y102 -R = 0

X11 + X12 + X13 + X14 -0Y10 -1Y11 -2Y12 -3Y13 -4Y14 -5Y15 -6Y16 -7Y17 -8Y18 = 0
X23 -0Y20 -1Y21 -2Y22 = 0
X32 + X34 -0Y30 -1Y31 -2Y32 -3Y33 -4Y34 = 0
X41 -0Y40 -1Y41 -2Y42 = 0
X53 -0Y50 -1Y51 -2Y52 = 0
X62 -0Y60 -1Y61 -2Y62 = 0
X71 + X74 -0Y70 -1Y71 -2Y72 -3Y73 -4Y74 = 0
X81 -0Y80 -1Y81 -2Y82 = 0
X93 + X94 -0Y90 -1Y91 -2Y92 -3Y93 -4Y94 = 0
X102 -0Y100 -1Y101 -2Y102 = 0

1X11 + 3X41 + 1X71 + 1X81 <= 10
1X12 + 2X32 + 1X62 + 3X102 <= 9
1X13 + 1X23 + 2X53 + 2X93 <= 10
1X14 + 2X34 + 1X74 + 2X94 <= 10
Y10 + Y11 + Y12 + Y13 + Y14 + Y15 + Y16 + Y17 + Y18 = 1
Y20 + Y21 + Y22 = 1

$Y_{30} + Y_{31} + Y_{32} + Y_{33} + Y_{34} = 1$
 $Y_{40} + Y_{41} + Y_{42} = 1$
 $Y_{50} + Y_{51} + Y_{52} = 1$
 $Y_{60} + Y_{61} + Y_{62} = 1$
 $Y_{70} + Y_{71} + Y_{72} + Y_{73} + Y_{74} = 1$
 $Y_{80} + Y_{81} + Y_{82} = 1$
 $Y_{90} + Y_{91} + Y_{92} + Y_{93} + Y_{94} = 1$
 $Y_{100} + Y_{101} + Y_{102} = 1$

$X_{11} \leq 2$
 $X_{12} \leq 2$
 $X_{13} \leq 2$
 $X_{14} \leq 2$
 $X_{23} \leq 2$
 $X_{32} \leq 2$
 $X_{34} \leq 2$
 $X_{41} \leq 2$
 $X_{53} \leq 2$
 $X_{62} \leq 2$
 $X_{71} \leq 2$
 $X_{74} \leq 2$
 $X_{81} \leq 2$
 $X_{93} \leq 2$
 $X_{94} \leq 2$
 $X_{102} \leq 2$

End

$Gin Y_{ik} \quad \forall i \in I_k, k$

$Int X_{ik} \quad \forall i, M_i$

Leave