

**FRAMEWORK FOR ESTIMATION OF THE LATERAL
EARTH PRESSURE ON RETAINING STRUCTURES WITH
EXPANSIVE AND NON-EXPANSIVE SOILS AS BACKFILL
MATERIAL CONSIDERING THE INFLUENCE OF
ENVIRONMENTAL FACTORS**

by

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DEDICATION

I dedicate this thesis to my beloved parents

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ABSTRACT

Lateral earth pressures (LEP) that arise due to backfill on retaining structures are typically determined by extending the principles of saturated soil mechanics. However, there is evidence in the literature to highlight the LEP on retaining structures due to the influence of soil backfill in saturated and unsaturated conditions are significantly different. Some studies are reported in the literature to interpret the variation of LEP on the retaining structures assuming that the variation of matric suction in unsaturated backfill material is hydrostatic (i.e. matric suction is assumed to decrease linearly from the surface to a value of zero at the ground water table). Such an assumption however is not reliable when the backfill behind the retaining wall is an expansive soil, which is extremely sensitive to the changes in variation of water content values. Significant volume changes occur in expansive soils due to the influence of environmental factors such as the infiltration and evaporation. In addition to the volume changes, the swelling pressure of the expansive soils also varies with changes in water content and can significantly influence the LEPs behind the retaining wall.

In this thesis, a framework for estimating the LEPs of unsaturated soils is proposed considering the variation of matric suction with respect to various water flow rates (i.e. infiltration and evaporation). The proposed approach is extended for expansive and non-expansive soils in this thesis taking into account of the influence of both the cracks

and the lateral swelling pressure with changes in water content. A program code LEENES (Lateral pressure estimation on retaining walls taking account of Environmental factors for Expansive and Non-Expansive Soils) in MATLAB is written to predict the LEP. The program LEENES is valuable tool for geotechnical engineers to estimate the LEPs on retaining structures for various scenarios that are conventionally encountered in geotechnical engineering practice. The studies presented in this thesis are of interest to the practitioners who routinely design retaining walls with both expansive and non-expansive soils as backfill material.

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NOMENCLATURE

Abbreviations

1-D	One dimensional
CVS	Constant volume swell test
HAE	High air entry
LEFM	Linear elastic fracture mechanics
LEP	Lateral earth pressure
MBV	Methylene blue value
min	Minute
SEM	Scanning electron microscope
SPCC	Soil permeability characteristic curve
SWCC	Soil-water characteristic curve
TSCC	Tensile strength characteristic curve
UDEC	Universal distinct element code

Symbols

$(u_a - u_w)$ (kPa)	Matric suction
$(u_a - u_w)_s$ (kPa)	Matric suction at ground surface
$(\sigma - u_a)$ (kPa)	Net normal stress
a, m, n	Fitting parameters for SWCC
a_b, n_b	Fitting parameters in Brutsaert's (1966) equation
A_c, A	Activity of soils
$As, \lambda s$	Regression analysis parameters for swelling pressure estimation
A_t, B_t, C_t, D_t	Empirical constants in tensile strength estimation
B	Coefficient of unified strength theory
C, W, F', G	Fitting parameters for depth of initial cracking estimation
c' (kPa)	Effective cohesion
CC (%)	Clay content
C_d (kPa)	Total cohesive strength
c_t' (kPa)	Unified effective cohesion
C_u	Coefficient of uniformity

C_w (kPa)	Total adhesive strength
D (m)	Depth of ground water table
d_e (mm)	Dominant particle size diameter
E	Void ratio of soils
e_0	Initial void ratio of soils
E_a (kN/m)	Active earth force
e_f	Final void ratio of soils
EI	Expansion index
E_p (kN/m)	Passive earth force
E_{sat}, E (kPa)	Modulus of elasticity under saturated condition
E_{unsat} (kPa)	Modulus of elasticity under unsaturated condition
F_i	Initial state factor
FSI (%)	Free swell index
H (kPa)	Elastic modulus with respect to a change in matric suction
h_0 (m)	Depth of elastic area
H_e (m)	Depth of expansive layer
IL	Liquidity index
K_a	Coefficient of active earth pressure
K_{at}	Unified coefficient of active earth pressure
K_p	Coefficient of passive earth pressure
K_{pt}	Unified coefficient of passive earth pressure
k_s (m/s)	Saturated coefficient of permeability
k_w (m/s)	Unsaturated coefficient of permeability depends on matric suction.
LL (%)	Liquid limit
LL_w (%)	Weighted liquid limit
LS (%)	Linear shrinkage
M	Coefficient of intermediate principal stress in unified strength theory
m_s	Reduction coefficient of swelling pressure
N_ϕ	Coefficient of earth pressure
P_a (kPa)	Atmospheric pressure
P_f (kPa)	Final lateral swelling pressure
PI (%)	Plastic limit
PI, Ip (%)	Plastic index
P_L (kPa)	Lateral swelling pressure
PS (%)	Probable swell
P_s (kPa)	Vertical swelling pressure
P_s' (kPa)	Corrected swelling pressure

P_{s0} (kPa)	Intercept on the P_s axis at zero suction value
q (m/s)	Flow rate of water in unsaturated soils
q_s (m/s)	Flow rate of water in saturated soils
R_s	Swelling pressure ratio
S, S_r (%)	Degree of saturation
SI (%)	Shrinkage index
SL (%)	Shrinkage limit
SP	Swelling potential
S_R (%)	Residual degree of saturation
u_a (kPa)	Pore-air pressure
u_w (kPa)	Pore-water pressure
W	Gravity of soil mass
w (%)	Gravimetric water content
w_i, w_0 (%)	Natural water content
w_r (%)	Residual gravimetric water content
w_s (%)	Saturated gravimetric water content
z_c (m)	Depth of cracks
z_w (m)	Distance above ground water table
α (°)	Angle of back wall and vertical plane
α_E, β_E	Fitting parameters of unsaturated modulus of elasticity
α_T	Coefficient of tensile strength of soils
β (°)	Angle of the filling plane of back wall and horizontal plane
β_S	Fitting parameter for swelling pressure estimation
γ (kN/m ³)	Unit weight of soils
γ_d (kN/m ³)	Dry unit weight
γ_{unst} (kN/m ³)	Unit weight of unsaturated soils
γ_w (kN/m ³)	Unit weight of water
δ (°)	Friction angle of filling and back wall
ε	Mean-zero Gaussian random error term
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	Total strain in the x-, y- and z-diretion
θ (%)	Volumetric water content
θ (°)	Angle of sliding plane and horizontal plane
K	Fitting parameter for shear strength of unsaturated soils
M	Poisson's ratio
ρ_{dn} (kg/m ³)	Natural dry density
σ (kPa)	Total normal stress
σ' (kPa)	Effective normal stress
σ_0 (kPa)	At-rest earth pressure
σ_c (kPa)	Surcharge stress due to cracks

σ_h (kPa)	Total horizontal stress
σ_h' (kPa)	Effective horizontal stress
σ_{ha} (kPa)	Active earth pressure
σ_{hp} (kPa)	Passive earth pressure
σ_s (kPa)	Surcharge stress
σ_t, t (kPa)	Tensile strength of soils
σ_v (kPa)	Total vertical stress
σ_v' (kPa)	Effective vertical stress
σ_{vs} (kPa)	Vertical self-weight stress
$\sigma_x, \sigma_y, \sigma_z$ (kPa)	Total normal stress in the x-, y- and z-direction
τ_{nat} (kPa)	Natural soil suction
ϕ^b (°)	Angle of shearing resistance with respect to matric suction
ϕ' (°)	Angle of internal friction
ϕ_h' (°)	Unified angle of internal friction
ϕ_t^b (°)	Unified angle of shearing resistance
χ_t	The reduction coefficient of effective cohesion in tensile strength estimation
ψ (°)	Dilation angle
ψ (kPa)	Total suction
ψ_i (kPa)	Initial soil suction

CHAPTER 1

INTRODUCTION

1.1 Statement of the problem

Countless civil infrastructure failures and casualties have been reported due to the problems associated with expansive soils, particularly in arid and semi-arid regions of the world over the past sixty years (Holtz and Gibbs 1954, Krohn and Slosson 1980, Steinberg 1998, Jones and Jefferson 2012). Several countries have reported expansive soils problems, which include: Algeria, Australia, China, Cuba, France, Ghana, India, Indonesia, Iran, Israel, Kenya, Mexico, South Africa, Saudi Arabia, Spain, Turkey, United Kingdom and the U.S.A. The losses associated with expansive soils have been reported as several billions of dollars annually in some of these countries (see Table 1.1, Adem and Vanapalli 2014). It is also reported that the annual losses associated with expansive soils is far greater than the losses associated with natural disasters such as the hurricanes, tornadoes, floods or earthquakes (Jones and Holtz 1973, Nelson and Miller, 1992).

Table 1.1 The annual costs associated with the damages to structures constructed in or with expansive soils for different regions in the world (after Adem and Vanapalli 2014).

Region	Cost of damage/ year	Reference
USA	\$ 13 billion	Puppala and Cerato (2009)
UK	£ 400 million	Driscoll and Crilly (2000)
France	€ 3.3 billion	Johnson (1973)
Saudi Arabia	\$ 300 million	Ruwaih (1987)
China	¥ 100 million	Ng et al. (2003)

The problems to the geotechnical infrastructure such as the slopes, retaining walls, pavements and lightly loaded residential structures and its foundations constructed with or founded within expansive soils may be attributed to the high swelling pressure associated with volume change behavior due to wetting (Chen 1975, Charlie et al. 1984, Cameron and Walsh 1984, Dafalla and Shamrani 2011, Yilmaz 2006, Chen 2012, Jones and Jefferson 2012, Fredlund et al. 2012). In addition, various problems are also reported due to shrinkage associated with drying of expansive soils (Miller et al. 1997, Puppala et al. 2004, Chen 2012).

Retaining walls are widely used soil supporting structures for several civil infrastructure such as the foundations, slopes, tunnels, bridges, pavements and railways. Conventional theoretical and numerical methods are widely used to determine the lateral earth pressures (LEP) on the retaining structures. Rankine or Coulomb's approaches form conventional methods that are widely used in retaining wall design practice by extending the mechanics of saturated soils (Tavakkoli and Vanapalli 2011). These methods provide reasonable estimates of LEPs for soils such as the gravels, sands, silts, glacial tills and clays under dry or saturated conditions. However, these methods are not applicable for expansive soils which are typically in a state of unsaturated condition. Expansive soils are used as backfill material behind the retaining wall in some regions of the world because of non-availability of other favourable soils (Ireland 1964, Pufahl et al. 1983, Lu 2010). These soils are prone to swell upon wetting due to precipitation activities such as the rain or snow or due to water pipe lines leakage within the vicinity of retaining walls. Expansive soils swell upon wetting and exert additional pressure on the retaining walls. In addition, expansive soils crack behind the wall due to drying. In other words, environmental factors have a significant and complex influence on the LEP of a retaining wall with expansive soils as backfill material.

Currently, two approaches are commonly used for estimation of the earth pressure of unsaturated soils for the design retaining walls (Zhu and Liu 2001). The first approach extends Fredlund et al. (1978) strength equation for unsaturated soils into Rankine's earth pressure theory. The Rankine's theory uses Mohr-Coulomb failure criterion assuming plastic equilibrium conditions. The LEPs are calculated using this approach assume that the back of the retaining structure is vertical, its surface is smooth, and the filling surface behind the back wall is horizontal. Because of the simplified assumptions of Rankine theory, this method cannot be used for most cases that are commonly encountered in practice applications (Pufahl et al. 1983, Zhu and Liu 2001, Zhang 2012).

The second approach uses the Coulomb's earth pressure theory by incorporating the influence of matric suction in the unsaturated soil (Zhang 2012). This approach can be used for retaining walls with a frictional surface; however, it only provides resultant pressure instead of lateral earth pressure distribution with respect to depth as in Rankine's method.

In the above two approaches, the additional contribution arising from swelling pressure of expansive soils is typically added to the earth pressure directly, for reliable earth pressure estimation. The laboratory test results (i.e. constant volume test, swell and load-back test and under pressure test) of the vertical swelling pressure determined is used as a tool in the estimation of the lateral swelling pressure. Several researchers suggest a reduction coefficient, typically around 0.2~ 0.6 for estimating lateral pressure from vertical swelling pressure results (Zhang 1995a, Zhu and Liu 2001). Also, Zhu and Liu (2001) suggested another approach for accounting the additional swelling pressure influence on retaining walls with expansive soils as backfill material. In this approach, instead of conventional angle of internal friction of shear strength, an equivalent angle of internal friction is used to take into account the influence of swelling pressure. However,

the equivalent angle of internal friction is related to the normal stress, which varies with depth of the retaining wall. It was reported that two thirds of retaining walls in Lechan and Chenzhou areas in China that used this approach have shown extensive cracks, displacements or even failures due to misjudgement or erroneous estimation of the equivalent internal friction angle (Zhang 1995b).

In a study reported by Ireland (1964), more than half of the retaining walls performance were unsatisfactory which had expansive clays as backfill or are founded upon them. The unsatisfactory performance may be attributed to the propagation of tensile cracks which contribute to water seepage due to which the soil swells and acts as an additional lateral earth pressure. Due to this reason, the influence of swelling pressure towards lateral earth pressure cannot be neglected in the rational design of retaining wall.

The shrink-swell potential of expansive soils is influenced by its initial water content, water content variation, void ratio, internal structure and vertical stresses, as well as the type and amount of clay minerals in the soil (Bell and Culshaw 2001). Of all these parameters, the variation of water content is considered to be the dominant factor that contributes to significant changes of bulk volume and swelling pressure. The water content changes may be due to seasonal variations, or brought about by local site changes such as the leakage from water supply pipes or drains, changes to surface drainage and landscaping or following the planting, removal or severe pruning of trees or hedges (Cheney 1986). As discussed earlier, there are some approaches in the literature to predict the lateral earth pressure with expansive soils as backfill material on the retaining walls, for certain scenarios (Pufahl et al. 1983, Zhu and Liu 2001, Hu 2006, Zhang et al. 2011, Zhang 2012). However, a comprehensive framework taking account of the environmental factors (i.e. drying and wetting conditions) for estimation of the LEP on retaining walls with expansive unsaturated soils as backfill is not available. There is a need for a

comprehensive framework that can be applied to both the fine-grained soils that do not swell and expansive soils under both saturated and unsaturated conditions considering the influence of cracks and other environmental factors extending the mechanics of unsaturated soils. Such a framework will be valuable for practicing geotechnical engineers for the design of retaining walls.

1.2 Research objectives

In this study, a comprehensive framework is proposed for estimating the LEP on retaining walls due to expansive soils by extending the mechanics of unsaturated soils. In this framework, the evaporation or infiltration water flow rates are the key factors to estimate the variation of matric suction profile in the expansive soil, when it is used as a backfill material behind a retaining wall. Under drying conditions, cracks propagate in expansive soils. An approach is presented in this thesis for estimating the crack depth. The depth of cracks is estimated extending the assumption that the tensile strength of soil is equal to the lateral stress. Upon infiltration, the lateral swelling pressure generates as the degree of saturation changes from a state of unsaturated to saturated condition associated with an increase in the water content (i.e. matric suction reduction). The lateral swelling pressure associated with the variation of matric suction profile is estimated from the relation between vertical and lateral swelling pressure. The framework that is developed for expansive soils can also be extended for non-expansive soils (i.e. fine-grained soils such as the clays, glacial tills and silty soils). In other words, fine-grained soils could be treated as a special case for expansive soils that do not swell due to wetting associated with infiltration. The lateral earth pressure of non-expansive soils is estimated in terms of vertical water flow rates, without considering the influence of cracks and swelling pressures.

The key objectives of the present study is summarized below:

- (i) Estimate the matric suction profiles for non-expansive and expansive soils taking account of the local climate records (i.e. monthly evaporation and infiltration water flow rates) (Yeh 1989, Likos and Lu 2004). In this approach, the soil-water characteristic curve (SWCC) is used as tool to estimate variation of the matric suction profile for the soils above the ground water table
- (ii) Propose an equation to estimate the depth of tension cracks taking into account of the influence of various evaporation water flow rates.
- (iii) Present the available approaches for estimating the lateral swelling pressure of expansive unsaturated soils from vertical swelling pressure values determined from laboratory test results.
- (iv) Propose a procedure to estimate the lateral earth pressure (LEP) for unsaturated expansive soils for both drying and wetting conditions.
- (v) Extend the proposed method of lateral earth pressure estimation for non-expansive unsaturated and saturated soils.
- (vi) The earth pressure distributions for both non-expansive and expansive soils according to the proposed approach are determined by using a program code developed using the MATLAB software. The program code is referred to as LEENES in this thesis. LEENES is abbreviated form for Lateral pressure estimation on the retaining walls taking account of Environmental factors for Expansive and Non-Expansive Soils.
- (vii) Discuss and compare the calculation results of LEP results for different retaining walls with both expansive and non-expansive soils as backfill material.

1.3 Novelty of the research

Environmental factors (i.e. wetting-drying and freeze-thaw cycles) have a significant influence on the swell-shrinkage behavior of expansive soils because they are extremely sensitive to the variation of water content. In both natural and compacted expansive soils, cracks propagate in dry seasons and additional swelling pressures generate upon infiltration. However, in most cases, the depth of cracks is typically assumed to be constant and the influence of lateral swelling pressure on the retaining walls is neglected (Morris et al. 1992, Pufahl et al. 1992). Also, at present, there are limited investigations that are undertaken which focus on reliable estimation of the LEP for unsaturated expansive soils extending the mechanics of unsaturated soils (Pufahl et al. 1983, Zhang 1995, Hu 2006, Zhu and Liu 2001, Zhang 2012).

In this thesis, a comprehensive framework is proposed for estimation of LEP on a retaining wall with expansive soil as backfill material. A program LEENES is developed using the MATLAB software incorporating all the features of the proposed framework. LEENES facilitates in calculations and presents the variation of LEP in graphical form. Instead of numerical procedures using complex finite element programs, the proposed approach, LEENES is relatively simple for use in conventional practice by geotechnical engineers. Figure 1.1 provides a summary of the step by procedure is followed using the LEENES for estimating the LEP and plotting the results.

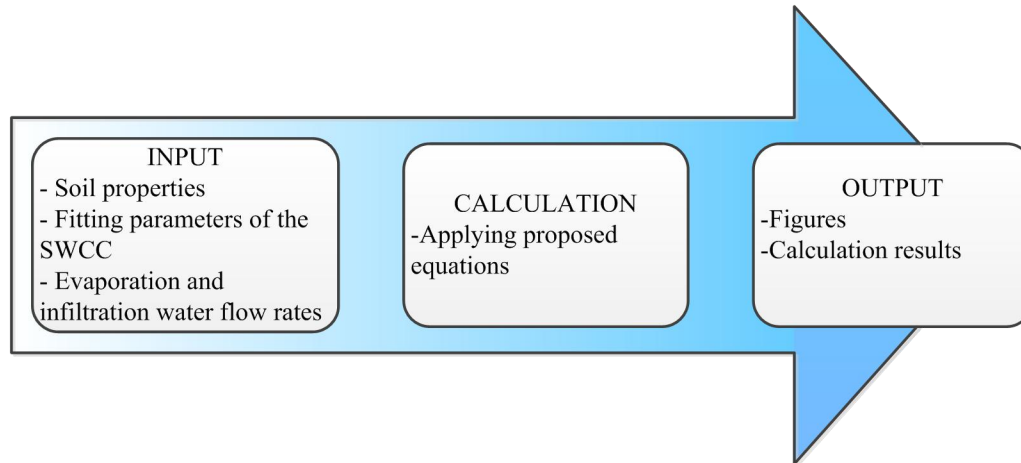


Figure 1.1 The step-by-step procedure followed in LEENES program for lateral earth pressure estimation for retaining walls

The novel features of this study are summarized as follow:

- Simple program code LEENES, which is a MATLAB based program is developed for extending the proposed framework.
- Taking account of the influence environmental factors, the matric suction profiles are estimated for various vertical water flow rates. The suction profiles information is required for reliable estimation of lateral earth pressure.
- The depth of cracks is predicted for the suction profile estimated using different vertical infiltration flow rates.
- The lateral swelling pressure is estimated from a semi-empirical model proposed by Tu and Vanapalli et al. (2016).
- Along with the SWCC, other mechanical properties, which include modulus of elasticity at saturated condition, effective cohesion, effective angle of internal friction, and Poisson's ratio are required along with the local weather data for

implementing the proposed framework for estimation of the lateral earth pressure variation behind the retaining wall with expansive and non-expansive clays.

- Proposed framework can be extended and used for both non-expansive and expansive soils under both unsaturated and saturated conditions using LEENES for estimating the LEP with respect to the depth in retaining walls.

1.4 Thesis layout

This thesis are presented in six chapters as summarized below:

Chapter 1, entitled, “Introduction”, presents a general background information with respect to estimation of the LEP of the presently followed approaches in the literature for both expansive and non-expansive soils taking account of the influence of environmental factors (i.e. infiltration and evaporation conditions). The need for proposing a rational method for LEP estimation extending the mechanics of unsaturated soils is highlighted. The key objectives along with the novelty of this thesis are also summarized in this chapter.

Chapter 2, entitled, “Literature review”, provides up-to-date relevant background information of the mechanics of unsaturated soils required for explaining the proposed framework. Key formulations to calculate the swelling pressure and lateral earth pressure for expansive unsaturated soils are also summarized.

Chapter 3, entitled, “Prediction of the depth of cracks and lateral swelling pressure”, describes the swell-shrinkage behavior of expansive soils under both drying and wetting conditions. The relationship between the environmental factors and matric suction profiles is highlighted in this chapter (Yeh 1989, Likos and Lu 2004). An equation is derived for predicting the depth of cracks associated with evaporation considering the

vertical steady-state water flow rates. The corresponding matric suction profiles are applied to estimate the lateral swelling pressures of expansive soils under wetting conditions.

Chapter 4, entitled “Proposed approach for predicting lateral earth pressure of expansive unsaturated soils”, provides details of the framework for estimating the LEP of unsaturated expansive soils by extending the mechanics of unsaturated soils. In the proposed approach, both the propagation of cracks upon evaporation and lateral swelling pressures development upon infiltration are also presented.

Chapter 5, entitled, “Application of the proposed framework for LEP estimation of expansive and non-expansive soils”, employs the proposed framework to investigate the LEP distributions and calculate lateral earth pressures for hypothetical retaining walls with different backfill soil types (Regina clay and Indian Head till). Furthermore, the results are discussed and compared to highlight the influence of the seasonal water content variation based on the local weather station records.

Chapter 6, “Conclusions and proposed research for future studies”, concisely summarizes the work presented in this thesis and highlights the major conclusions. The future research works that can be undertaken for better understanding of the influence of lateral swelling pressure on the design of the retaining walls are also summarized.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Expansive soils pose significant challenges to geotechnical and structural engineers for providing reliable tools for the design of infrastructure. Significant property losses and human casualties are attributed to improper classification and understanding of the engineering behavior of expansive soils (Dhowian et al. 1988, Chen 1975, Erzin and Erol 2004, Puppala and Cerato 2009, Jones and Jefferson 2012, Qi and Vanapalli 2015). Several failures that were reported in the literature are related to lightly loaded structures such as the pavements, residential and industrial buildings that are constructed in unsaturated expansive soils, which suffer damages caused by vertical volumetric deformations (Vu and Fredlund 2004, Avsar et al. 2009, Kayabali and Demir 2011, Mohamed et al. 2014). The instability of retaining walls, failure of pile foundations and certain slopes may predominantly be attributed to the lateral swelling pressure induced by expansive soils (Chen 1975, Nelson and Miller 1992, Marsh and Walsh 1996). Due to these reasons, geotechnical engineers require proper training and tools to undertake soil investigation studies to identify and classify expansive soils. In addition, they need tools for the proper design, construction and maintenance of the infrastructure to alleviate problems associated with expansive soils (Baker 1981, Bagge 1985, Sapaz 2004, Kayabali and Demir 2011).

This chapter focus is directed towards providing relevant background literature for better understanding expansive soils behavior in general and to understand various properties that have a significant influence in the design of retaining walls with expansive soils as backfill material, in particular. One of the major problems of using expansive soils as backfill material is its high sensitivity with respect to water content changes associated with evaporation and infiltration. In other words, sensitivity associated with water content changes has a significant influence for evaluating the swell-shrink behavior of expansive soils. The key information of interest is how cracks develop in expansive soils in addition to the development of swelling pressure as an additional stress on the retaining wall.

Considerable research has been undertaken during the past six decades which include both laboratory tests and field studies to better understand the influence of the swell-shrink behavior on the performance of structures constructed with expansive soils or structures founded on them (Seed and Chan 1959, Seed et al. 1962, Fredlund 1983, Chen 1988, Nelson and Miller 1992, Morris et al. 1992, Shuai 1996, Jones and Jefferson 2012). Some of the key studies that are of interest in the rational design of retaining walls include: (i) estimation or prediction of the swelling pressure; (ii) estimation of crack depth.

The methods for determining the swelling pressures are commonly divided into two groups: direct (i.e. laboratory and field tests) and indirect methods (i.e. from index properties tests) (Nelson and Miller 1992, Fredlund and Rahardjo 1993, Kayabali and Demir 2011, Vanapalli and Lu 2012, Wang et al. 2013). Both laboratory and field studies are tedious, time consuming and hence expensive. These limitations to certain extent can be overcome by proposing empirical and semi-empirical equations or approaches that are useful for application in engineering practice (Yilmaz 2006). Along similar lines, various laboratory tests, numerical analysis and analytical methods are available to determine the

depth of crack propagation associated with evaporation (Lee et al. 1988, Morris et al. 1992, Konarad and Ayad 1997, Nahlawi and Kodikara 2005, Tavakkoli and Vanapalli 2011, Zhang et al. 2011).

Several investigators have attempted to estimate earth pressure on the retaining walls with expansive soils as backfill by taking account of the influence of both external and internal factors, including soil properties, evaporation and precipitation. Several approaches for lateral earth pressure (LEP) estimation of unsaturated soils that are available in the literature are also summarized (Pufahl et al. 1983, Tavakkoli and Vanapalli 2011, Zhang et al. 2010, Zhang 2012).

In this chapter, the general background of expansive soils and a comprehensive review of its swell-shrink behavior are presented. In order to propose rational approach for LEP estimation of unsaturated soils, background information about the stress state variables for interpretation of the unsaturated soils behavior is presented. In addition, Soil-water Characteristic Curve (SWCC), which can be used as a tool for predicting the unsaturated soils properties, is succinctly summarized in this chapter. Some key properties of unsaturated soils that are of interest in the estimation of earth pressures, which includes the modulus of elasticity, the coefficient of permeability and the shear strength behavior of unsaturated soils, also are discussed. More specifically, the shear strength and tensile strength of unsaturated soils, which form the key properties in the estimation of earth pressure is provided in greater detail.

2.1.1 General

Expansive soils can be categorized as problematic clays which are typically found in nature are in an unsaturated condition with multiple micro and macro cracks or fractures. These soils that exhibit remarkable swell-shrink characteristics due to changes in their

moisture content from their natural environment conditions. In spite of their well-known problems, their use is unavoidable as backfill material for retaining walls when other suitable materials are not available in the close proximity. As a first step to deal with these problematic soils, appropriate identification and classification systems are built to guide engineers for their use (Das 1995).

In typical expansive soils, montmorillonite and illite are the primary minerals that have the capacity to imbibe large amounts of water molecules between their clay sheets (Jia 2010, Zhang 2012). When degree of saturation in expansive soils increases or become saturated, more water molecules are absorbed between the clay sheets, causing the volume of soil mass to increase. This process weakens the inter-clay bonds and causes a reduction in the tensile and shear strength of the soil. When water is removed (i.e. evaporation or gravitational forces), the overall volume of the soil reduces in addition to development of cracks (Jones and Jefferson 2012). For this reason, the water flow and shear strength behavior are the key properties of interest in the design of retaining walls with expansive soils as backfill material. In addition, information of the swelling pressures that will generate when the volumetric change is restrained also are required.

2.1.2 Classification

Several investigators have summarized various criteria that can be used for classifying the swell potential of expansive soils (Nelson and Miller 1992, Yilmaz 2006, Rao et al. 2011 and Çimen et al. 2012). A comprehensive summary of these studies is listed in Table 2.1, Table 2.2 and Figure 2.1)

Table 2.1 Summary of criteria for classifying swell potential of expansive soils (modified after Yilmaz 2006).

Reference	Criteria	Remarks
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Altmeyer (1955)	$LS < 5$, $SL > 12$, $PS < 0.5$ (non-critical) $5 \leq LS \leq 8$, $10 \leq SL \leq 12$, $0.5 \leq PS \leq 1.5$ (marginal) $LS > 8$, $SL < 10$, $PS > 1.5$ (critical)	Based on LS , SL and PS . Remolded sample. Soaked under 6.9 kPa surcharge. LS = linear shrinkage, %. PS = probable swell, %. SL = shrinkage limit, %.
Holtz (1959)	$CC > 28$, $PI > 35$, $SL > 11$ (very high) $22 \leq CC \leq 31$, $25 \leq PI \leq 41$, $7 \leq SL \leq 12$ (high) $13 \leq CC \leq 23$, $15 \leq PI \leq 28$, $10 \leq SL \leq 16$ (medium)	Based on CC , PI and SL . CC = clay content, % (< 0.002 mm). PI = plastic limit, %.
Seed et al. (1962)	See Figure 2.2 (a)	Based on oedometer test using compacted specimen, percentage of clay $< 2 \mu\text{m}$ and activity.
Van Der Merwe (1964)	See Figure 2.2 (c)	Based on PI , percentage of clay $< 2 \mu\text{m}$ and activity.
Raman (1967)	$PI > 32$ and $SI > 40$ (very high) $23 \leq PI \leq 32$, $30 \leq SI \leq 40$ (high) $12 \leq PI \leq 23$, $15 \leq SI \leq 30$ (medium) $PI < 12$ and $SI < 15$ (low)	Based on PI and SI . SI = shrinkage index = $LL - SL$, %. LL = liquid limit, %.
Uniform Building Code (1968)	$EI > 130$ (very high) $91 \leq EI \leq 130$ (high) $51 \leq EI \leq 90$ (medium) $21 \leq EI \leq 50$ (low) $0 \leq EI \leq 20$ (very low)	Based on oedometer test on compacted specimen with degree of saturation close to 50% and surcharge of 6.9 kPa. EI = expansion index = $100 \times$ percent swell $\times$ fraction passing No.4 sieve.
Sowers and Sowers (1970)	$SL < 10$ and $PI > 30$ (high) $10 \leq SL \leq 12$, $15 \leq PI \leq 30$ (moderate) $SL > 12$ and $PI < 15$ (low)	Little swell will occur when w_0 results in LI of 0.25
Dakshanamurthy and Raman (1973)	See Figure 2.2 (b)	Based on plasticity chart.
Snethen (1984)	$LL > 60$, $PI > 35$, $\tau_{nat} > 4$, $SP > 1.5$ (high) $30 \leq LL \leq 60$, $25 \leq PI \leq 35$, $1.5 \leq \tau_{nat} \leq 4$, $0.5 \leq SP \leq 1.5$ (medium)	PS is representative for field condition, can be used without τ_{nat} , but accuracy will be reduced. SP = swelling potential.

	$LL < 30, PI < 25, \tau_{nat} < 1.5,$ $SP < 0.5$ (low)	τ_{nat} = natural soil suction, tsf.
Chen (1988)	$PI \geq 35$ (very high) $20 \leq PI \leq 55$ (high) $10 \leq PI \leq 35$ (medium) $PI \leq 15$ (low)	Based on PI .
McKeen (1992)	See 2.2 (d)	Based on measurements of soft water content, suction and volume change on drying.

Table 2.2 Summary of formulations for swell potential determination (modified after Yilmaz 2006, Nelson and Miller 2007, Rao et al. 2011, Adem 2014, Çimen et al. 2012).

Formulation	Reference and remarks
$SP = 3.6 \times 10^{-5} A_c^{2.44} CC^{3.44}$ $SP = 0.00216 PI^{2.44}$, for undisturbed soils $SP = 0.0036 PI^{2.44}$, for disturbed soils	Seed et al. (1962) A_c = activity
$SP = 0.000413 SI^{2.67}$	Ranganatham and Satyanarayana (1965)
$\log SP = (1/12) (0.44 LL - w_i + 5.5)$ $\log SP = (1/19.5) (6.242 \gamma_d + 0.65 LL - 130.5)$	Vijayvergiya and Ghazzaly (1973) For undisturbed soils. w_i = natural water content, %. γ_d = dry unit weight, lb/ft ³ .
$\log SP = 0.9 (PI / w_i) - 1.10$	Schneider and Poor (1974) For undisturbed soils.
$SP = (0.00229 PI) (1.45 CC) / w_i + 6.38$	Nayak and Christensen (1974) For soils compacted to the maximum standard AASHTO unit weight at optimum water content by free swell test. CC = clay content, % (<0.002 mm).
$SP = 7.5 - 0.8 w + 0.203 CC$	McCormack and Wilding (1975) w = water content, %.
$SP = 2.77 + 0.131 LL - 0.27 w_i$	O'Neil and Ghazzally (1977)
$\log SP = 0.036 LL - 0.0833 w_i + 0.458$	Johnson and Snethen (1978)
$SP = 23.82 + 0.7346 PI - 0.1458 H_e - 1.7 w_0 + (0.0025 PI) w_i - (0.00884 PI) H_e$	Johnson (1978) for undisturbed soils, at $PI \geq 40\%$. H_e = depth of expansive layer, ft.
$SP = -9.18 + 1.5546 PI + 0.08424 H_e + 0.1 w_0 -$	Johnson (1978)

$(0.0432 PI) w_i - (0.01215 PI) H_e$	for undisturbed soils, at $PI \leq 40\%$.
$SP = 0.00411 (LL_w)^{4.17} q^{-3.86} w_0^{-2.33}$	Weston (1980) LL_w = weighted liquid limit, %.
$SP = 0.0000411 A_c^{2.559} CC^{3.44}$	Bandyopadhyay (1981)
$SP = 0.2558 e^{0.0838 PI}$	Chen (1988) Compacted soils with initial condition at $\gamma_d = 15.7 \sim 17.3$ kN/m ³ and $w_i = 15 \sim 20$ % by free swell test.
$SP = 0.00064 PI^{1.37} CC^{1.37}$	Basma (1993) for soils compacted to the maximum standard AASHTO unit weight at optimum water content by free swell test.
$SP = 41.161 A_c + 0.6236$ $SP = 0.0763 \Psi_i - 339.03$	Çokça (2002) Ψ_i = initial soil suction, kPa
$SP = 4.24 \gamma_d - 0.47 w_0 - 0.14 q - 0.06 FSI - 55$	Rao et al. (2004) FSI = free swell index, %.
$SP = 1.0 + 0.06 (CC + PI - w_0)$	Sabtan (2005)
$SP = 0.6 PI^{1.188}$	Azam (2007)
$SP = 2.098 e^{-1.7169 IL}$	Yilmaz (2009) IL = liquidity index
$SP = -57.865 + 37.076 \rho_d + 0.524 MBV + \varepsilon$	Türköz and Tosun (2011) MBV = methylene blue value. ε = mean-zero Gaussian random error term.
$(SP)_1 = (0.3139 \gamma_d^{0.3552} - 0.1177 w_0^{0.4470}) PI^{0.9626}$ $(SP)_2 = (0.4768 \gamma_d^{0.3888} - 0.0033 w_0^{1.6045}) PI^{0.7224}$ $SP = \text{mean}(SP_1, SP_2)$	Çimen et al. (2012)
$SP = 24.5 (q^{-0.26}) (PI \times CC)^{1.26} [F_i - 7.1 (\sigma_s^{0.22}) (PI \times CC)^{1.26}]$	Zumrawi (2013) F_i = initial state factor. σ_s = Surcharge, kPa.

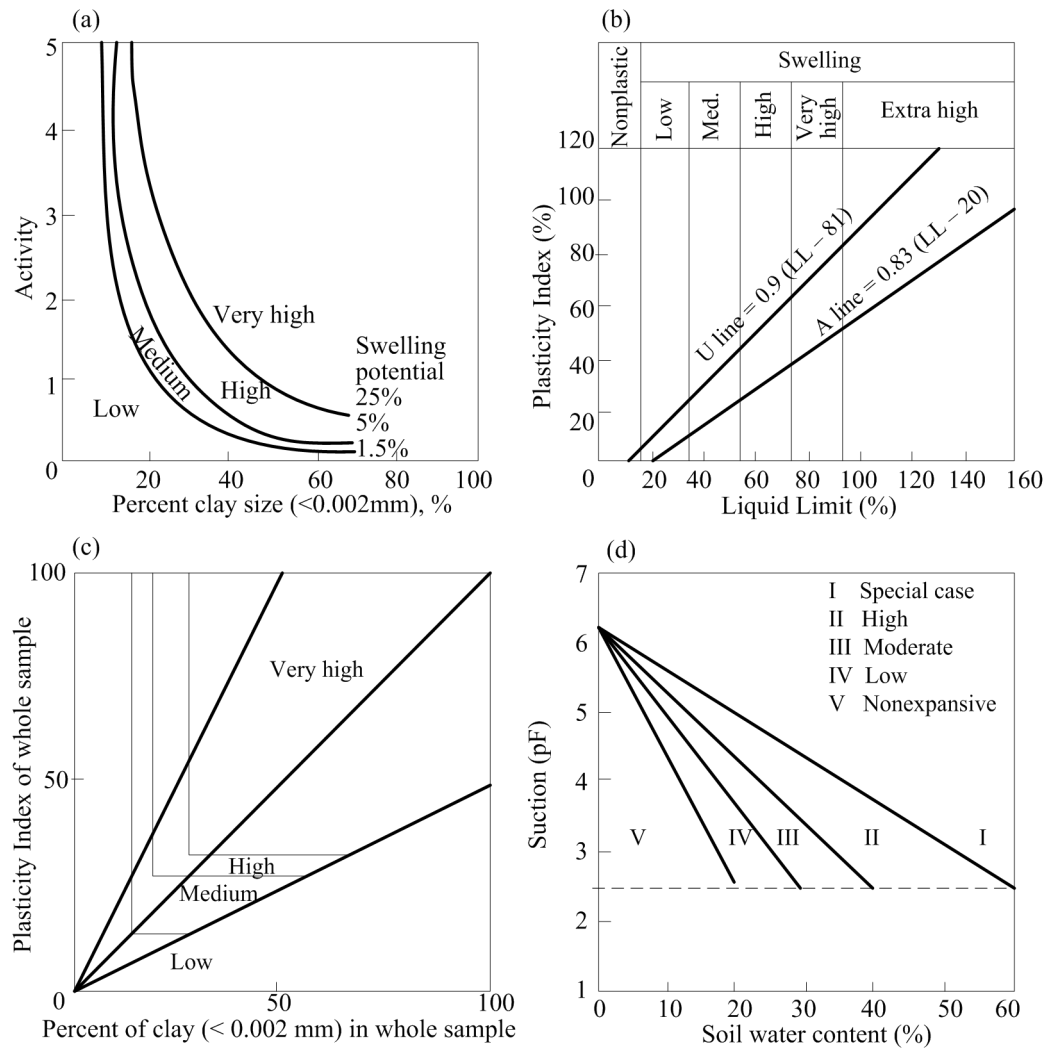


Figure 2.1 Commonly used criteria for determining swell potential (after Yilmaz 2006).

The degree of expansion significantly varies in different expansive soils and depends on various parameters. There are several classification methods that are available in the literature to characterize expansive soils, however, they are typically based on limited experimental data and can't be applied to all the expansive problems in practice (Nelson and Miller 1992).

2.1.3 Mineralogy

Most soil classification systems arbitrarily define clay particles as having an effective diameter of two microns (0.002 mm) or less. However, typical expansive clays which fall in the category of phyllosilicate family, their minerals are commonly made up of combinations of two simple structural units, namely, the silicon tetrahedron and the aluminum or magnesium octahedron (see Figure 2.2 and Figure 2.3). The silica tetrahedron consists of a silicon atom surrounded tetrahedrally by four oxygen ions as shown in Figure 2.2 (a) while the alumina octahedron consists of an aluminum atom surrounded octahedrally by six oxygen ions as shown on Figure 2.3 (a) (Chen 1975).

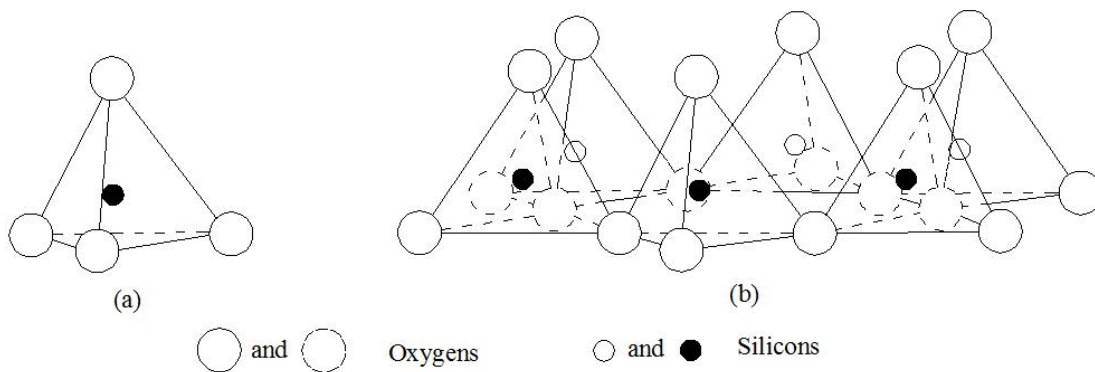


Figure 2.2 Silicon tetrahedron and silicate tetrahedral arranged in a hexagonal network (after Soga and Mitchell 2005).



● Aluminums, magnesiums, etc.

Clay minerals have the property of absorbing certain anions and cations and retaining them in an exchangeable state. The *cation exchange capacity* is defined as the charge or electrical attraction for cation per unit mass as measured in millequivalent per 100 grams of soil.

Montmorillonite, illite and kaolinite are the three major clay minerals. Among them, montmorillonite are commonly considered as the primary mineral, which contributes to unique characteristic behavior of expansive soils (Chen 1975, Jia 2010).

The structure of montmorillonite consists of an octahedral sheet that is sandwiched between two silica sheets (see Figure 2.4a). Bonding between successive layers is

attributed to the van der Waals forces and to cations that balance charge deficiencies in the structure. These bonds are relatively weak and can easily separate by adsorption of water or other polar liquids. Because of large amount of unbalanced substitution in the minerals, montmorillonite has high cation exchange capacity. The hydration energy overcomes the attractive forces between the unit layers. As a result, the montmorillonite mineral is the dominant source that contributes to the swelling behavior in the expansive soils (Soga and Mitchell 2005). Montmorillonite minerals are typically 10 times more active in absorbing cations compared to kaolinite minerals. This is caused by the large net negative charge carried by the montmorillonite particle and its greater specific surface as compared to kaolinite and illite (Chen 1975).

Illite

Illite is the one of the commonly found clay mineral in soils, which usually occurs as very small, flaky particles mixed with other clay and nonclay materials (Soga and Mitchell 2005). It is three-layer silica-gibbsite-silica sandwich, which is similar to that of montmorillonite (see Figure 2.4b). However, some of the silicon atoms are replaced by aluminum, and, in addition, potassium ions are present between the tetrahedral sheet and adjacent crystals (Chen 1975). The cation exchange capacity of illite is less than that of montmorillonite.

Kaolinite

Kaolinite belongs to 1:1 mineral (see Figure 2.4c). The bonding between successive layers can be attributed to the van der Waals forces and the hydrogen bonds. The cation exchange capacity is too weak to resist interlayer bonding. Due to this reason, kaolinite is not an expansive mineral.

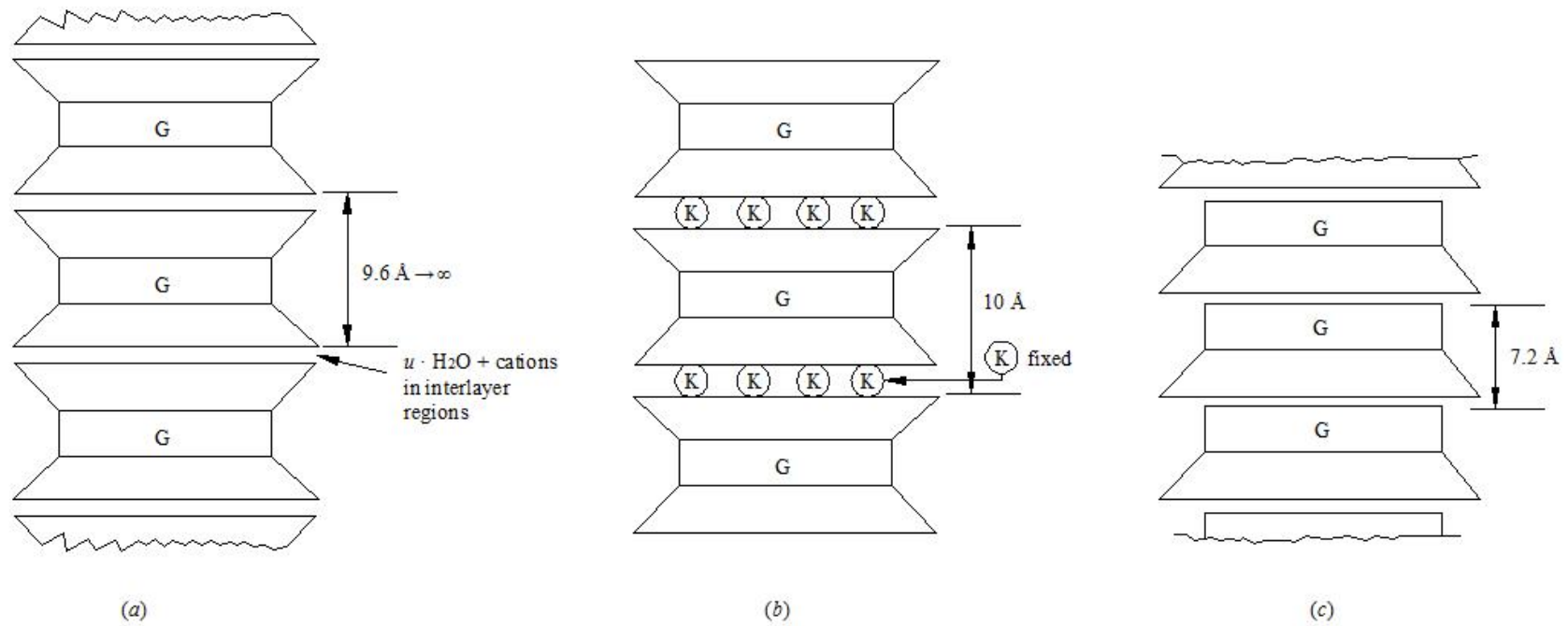


Figure 2.4 Schematic diagram of the structure of clay minerals: (a) montmorillonite, (b) illite, and (c) kaolinite (after Soga and Mitchell 2005).

2.1.4 Swelling Mechanics

Figure 2.5 shows swell process of expansive soils with respect to time. The time-swell curve typically consists of three regions: an initial swell region, primary swell region, and secondary swell region (see Figure 2.5). The minor initial swell is attributed to swelling of the macrostructure, while the primary swell and secondary swell is attributed to microstructural swelling (Rao 2006). From a macro perspective, expansion is caused from absorption of water by clays (Chen 1975).

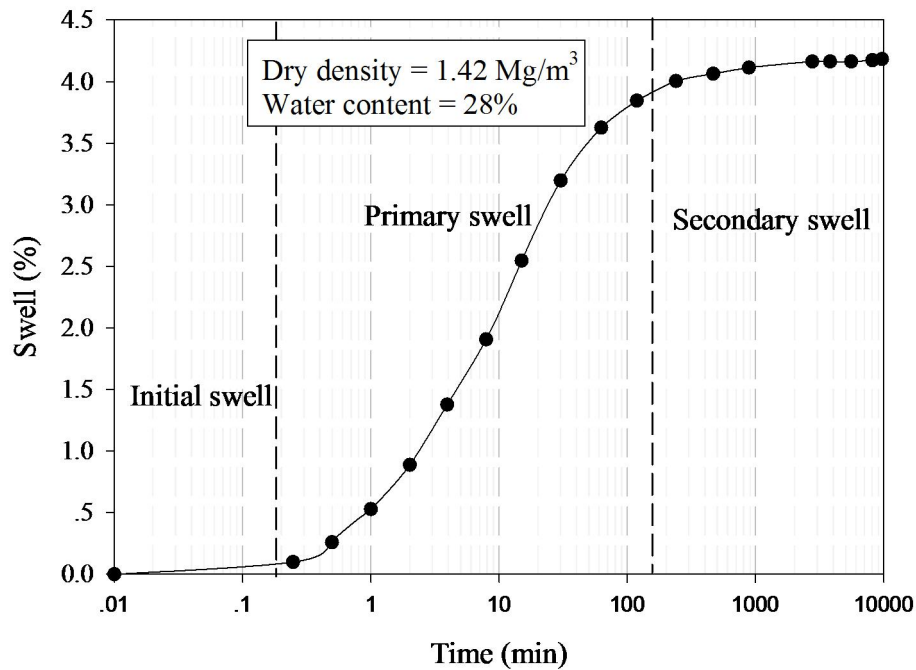


Figure 2.5 Time-swell behavior of compacted cotton soil (after Rao 2006).

Many parameters influence the swelling mechanics of expansive soils. Generally, it is suggested the microstructure swelling process could be divided into two stages: crystalline swelling and osmotic swelling, respectively (Norrish 1954). For better understanding and interpretation of these mechanisms, Low (1992) and Zhang et al. (1995) used the term short-range and long-range swelling for the first and second stage of swelling.

Short-range swelling (Crystalline swelling)

Crystalline swelling is a process that dry expansive clay minerals generally intercalate one, two, three, or four discrete layers of water between the mineral interlayers (Likos 2004). This process depends on the hydration energy of the interlayer cations (Norrish 1954).

With increasing water content, the clay swells but the distance of interlayers remains almost constant. Simultaneously, the increasing water pressure forces the water molecules to form two layers. The spacing between layers increases because of an increased orientation to the counterions, which is the ion that accompanies an ionic species in order to maintain electric neutrality, and a decreased influence of hydrogen bonding to the clay mineral surface (Hensen and Smit 2002).

At this stage, the water molecules, of at least the first layers, are probably arranged in a hexagonal network (Norrish 1954). The hydration is facilitated by the increased interlayer volume and the increased number of intercalated water molecules (Hensen and Smit 2002).

As the interlayer spacing keep increasing, more water molecules enter the crystal layers and the ionic hydration becomes weak, hence the crystalline swelling ends.

Long-range swell

A further expansion of the clay leads to an increasing number of sodium ions in the center of the interlayer and concomitant adsorption of water molecules that hydrate these ions (Hensen and Smit 2002).

The high concentrate absorbed cations try to diffuse away in order to equalize concentrations throughout the pore fluid. However, the cations are restricted by the

negative electrical field originating in the particle surfaces and ion-surface interactions. The escaping tendency due to diffusion and the opposing electrostatic attraction lead to ion distributions adjacent to a single clay particle in suspension. The charged surface and the distributed charge in the adjacent phase are together termed the *diffuse double layer* (Bolt 1956, van Olphen 1963, Mitchell 1993). According to Gouy-Chapman diffuse double layer theory (Gouy 1910, Chapman 1913), the long-range repulsive force between particles depends on the ion concentration between two adjacent parallel layers (Bolt 1956, Tripathy et al. 2004, Soga and Mitchell 2005).

2.2 Steady-state water flow

The key factor, which affects the swell-shrink behavior of expansive soils, is the water content gradient. The water content gradient in unsaturated expansive soils is related to the rate of water flow and soil permeability (Zhang et al. 2011).

The matric suction profiles above ground water table experience considerable changes with environmental factors, as shown in Figure 2.6.

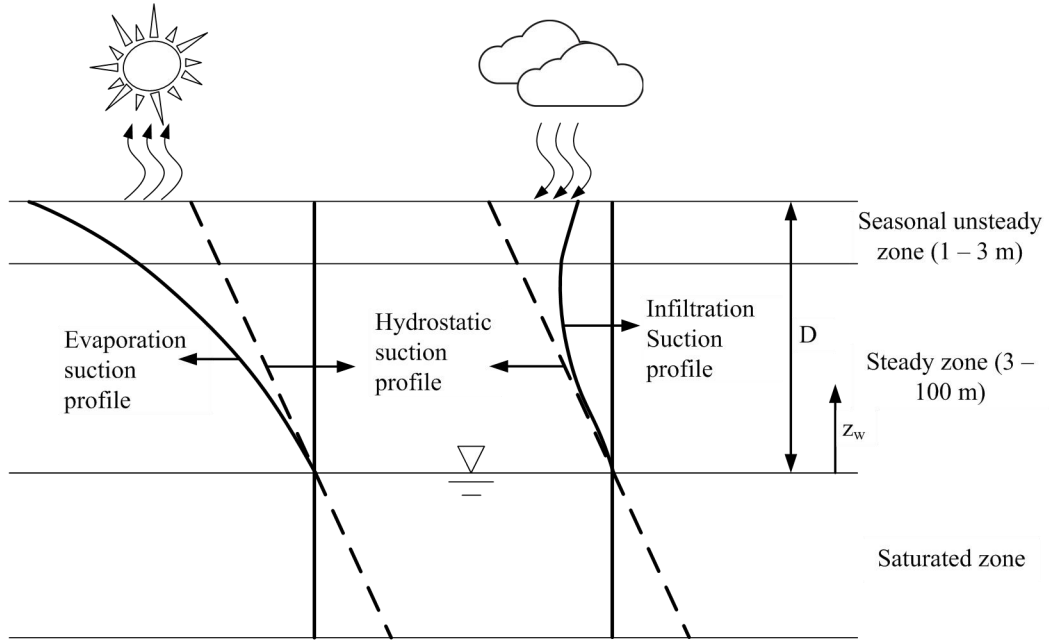


Figure 2.6 Variation of matric suction profiles in unsaturated soil under the influence of various environment conditions.

Lu and Griffiths (2004) proposed the theoretical formulation of matric suction profiles, based on the soil water characteristic curve (SWCC) and soil permeability characteristic curve (SPCC):

Darcy's law is conventionally used to describe the vertical flow of water in saturated soils. In Equation (2.1), the rate of water flow through a soil mass is proportional to the hydraulic head gradient:

$$q_s = -k_s \frac{\partial h_w}{\partial z_w} \quad (2.1)$$

where, q_s is the flow rate of water in saturated soils, k_s is the saturated coefficient of permeability, z_w is the distance above ground water table, as shown in Figure 2.6.

By expanding the Darcy's law, the 1-D vertical steady-state flow rate of unsaturated soils, q can be described as below:

$$q = -k_w \left(\frac{d(u_a - u_w)}{\gamma_w dz_w} + 1 \right) \quad (2.2)$$

where, k_w is the unsaturated coefficient of permeability depends on the matric suction, $(u_a - u_w)$, $(u_a - u_w)$ is the matric suction, γ_w is the unit weight of water.

In Equation (2.2), the unsaturated coefficient of permeability, k_w , is commonly expressed in terms of the void ratio, e , and matric suction, $(u_a - u_w)$, (Fredlund 1983). As shown in Figure 2.7, the unsaturated coefficient of permeability is described using Gardner's model (1958):

$$k_w = k_s e^{-a(u_a - u_w)} \quad (2.3)$$

where, e is the void ratio, a is a fitting parameter of the SWCC.

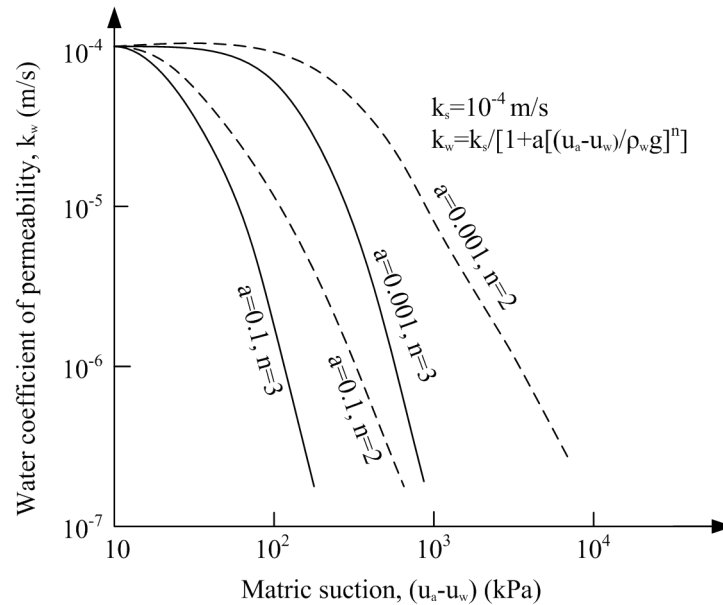


Figure 2.7 Gardner's equation for the water coefficient of permeability as a function of the matric suction (modified from Gardner 1958).

The vertical unsaturated flow rate is described as Equation (2.4) by substituting Equation (2.3) into Equation (2.2). This equation can be used for estimation of the vertical steady-state water flow rate variation with respect to depth.

$$q = -k_s e^{-a(u_a - u_w)} \left(\frac{d(u_a - u_w)}{\gamma_w dz_w} + 1 \right) \quad (2.4)$$

2.3 Swelling pressure

Swelling pressure is defined as the pressure required to hold the soil or restore the soil to its initial void ratio when given access to water (Shuai 1996). The water content and dry density are the two essential factors affecting the magnitude of swelling pressure.

The typical failures observed in engineering practice induced by swelling pressure are due to: (i) uneven heave induced by vertical swelling pressure; (ii) failures of retaining structures and slopes caused by lateral swelling pressures associated with the seasonal precipitation.

Direct measurement methods (i.e. laboratory tests) and indirect determination methods (i.e. semi-empirical and empirical equations) of vertical swelling pressures are employed by geotechnical engineers to address several complex field problems. Laboratory tests measure the swelling pressures directly while semi-empirical or empirical formulas are used when swelling pressures were not measured in the laboratory because of reasons associated with economics.

In recent years, some modified laboratory and field tests have been conducted to determine the differences between the lateral and vertical swelling pressures (Zhang 1993, Sapaz 2004, Xie et al. 2007, Avsar et al. 2009).

2.3.1 Laboratory tests

In a conventional laboratory consolidation test, the swelling pressure is referred as the pressure loaded gradually on the specimen to return to its initial volume after the sample swelled to its maximum volume (ASTM 1996, 2003). Several different types of laboratory methods are available to measure the swelling pressure directly from oedometer tests, which include, free swell test, constant volume and double oedometer tests (Shuai 1996, Attom and Barakat 2000, Kayabali and Demir 2011). Some key advantages and disadvantages of these tests are summarized in Table 2.3.

Table 2.3 Advantages and disadvantages of swelling pressure laboratory tests.

Laboratory test	Advantages and Disadvantages
Free swell test	The swelling process is combined of chemical and physical process while this is a consolidation test. Over-estimate the swelling pressure.
Double oedometer test	Require “identical” soil samples from field Ignores the effect of different dry densities and soil structures.
Constant volume test	The lab instrument should have enough stiffness to prevent any deformation. Easy to conduct tests

The direct laboratory methods are the most convenient way to measure the swelling pressure (Jones and Jefferson 2012). Nevertheless, the values of swelling pressure values measured from laboratory tests suffer uncertainty (Sridharan et al. 1986, Kayabali and Demir 2011). Apart from being quite cumbersome and time consuming, most experimental techniques accompany with several problems, such as the sample preparation and control of parameters such as the water content, dry density, surcharge load, and volume change of the soil specimen (Fredlund 1983, Rao et al. 1988, Tripathy et al. 2002).

Among these problems, the disturbance during sample preparation process in laboratory has a significant influence on the swelling behaviour of expansive soils (Jones and Jefferson 2012). To account for sample disturbance, Fredlund (1983) proposed a procedure to determine the corrected swelling pressure based on the laboratory test results (see Figure 2.8). If this correction is not applied, it is likely that the soil would be misinterpreted as a clay with low swelling pressure.

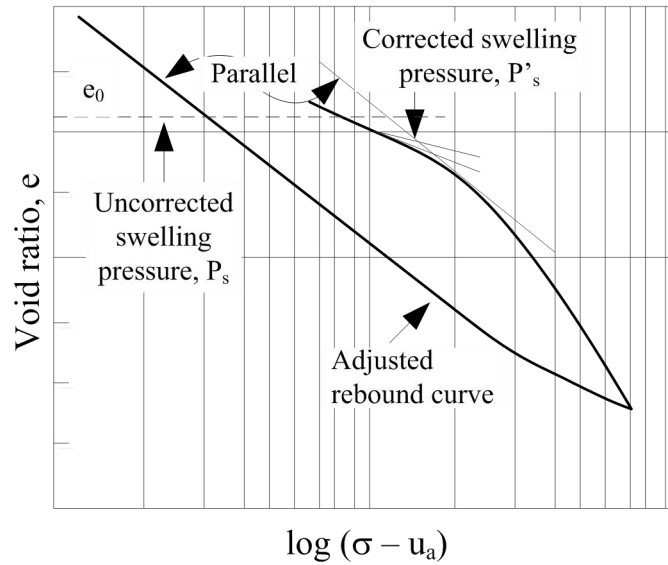


Figure 2.8 Construction procedure to correct for the effect of sampling disturbance (modified from Fredlund and Rahardjo 1993, Adem 2014).

2.3.2 Semi-empirical and empirical equations

Indirect methods to estimate the swelling pressure in expansive soils are widely used in geotechnical engineering practice as they are simple, economical and avoid problems associated with the laboratory tests (Houston et al. 2009, Rao et al. 2011, Vanapalli and Lu 2012).

In Table 2.4, Nelson and Miller (1992), Rao et al. (2011) and Çimen (2012) listed several empirical formulas to determine the swelling pressure.

Table 2.4 Summary of relationships available in the literature (modified after Nelson and Miller 1992, Rao et al. 2011, Çimen 2012).

Relationship	Reference and remarks
$\log(P_s) = 2.132 + 0.0208 LL + 0.000665 \gamma_d - 0.0269 w$	Komornik and David (1969) For undisturbed soils. γ_d = dry unit weight, kg/m ³ . P_s = swelling pressure, kg/cm ² .
$P_s = 3.5817 \times 10^{-2} PI^{1.12} (CC^2 / w_i^2) + 3.7912$	Nayak and Christensen (1971) w_i = initial water content, %. P_s = swelling pressure, psi.
$P_s = 2.29 \times 10^{-2} PI^{1.45} (CC / w) + 6.38$	Nayak and Christensen (1971) PI = plastic index, %.
$\log(P_s) = 0.9(PI / w_i) - 1.19$	Schneider and Poor (1974)
$\log(P_s) = -0.289 - 7w + 6.65 CC$	McCormack and Wilding (1975)
$P_s = 23.82 + 0.7346 PI - 0.1458 H - 1.7w_0 + 0.0025 PI \times w_0 - 0.00884 PI \times H, PI \geq 40$	Johnson (1978)
$P_s = -9.18 + 1.5546 PI + 0.08424 H + 0.1w_0 - 0.0432 PI \times w_0 - 0.01215 PI \times H, PI \leq 40$	Johnson (1978)
$P_s = 0.0446 LL - 1.572$	Nayak (1979)
$P_s = 0.057 PI - 0.566$	Nayak (1979)
$P_s = -227.27 + 2.14 w_i + 1.54 w_l + 72.49 \gamma_d$	Erguler and Ulusay (2003) For remoulded samples. γ_d = dry unit weight, gr/cm ³ . P_s = swelling pressure, N/cm ² .
$\log(P_s) = -4.812 + 0.01405 PI + 2.394 \gamma_d - 0.0163 w_i$	Erzin and Erol (2004) P_s = swelling pressure, kg/cm ² .
$\log(P_s) = -5.197 + 0.01405 PI + 2.408 \gamma_d - 0.819 IL$	Erzin and Erol (2004)
$\log(P_s) = -5.020 + 0.01383 PI + 2.356 \gamma_d$	Erzin and Erol (2004) For constant volume swell test.
$P_s = 63.78 e^{0.15288}$	Sridharan and Gurtug (2004)
$P_s = 12.5(0.001 \psi)^{0.25}$	Thakur and Singh (2005) ψ = total suction, kPa
$P_s = 25(0.001 \psi)^{0.25}$	Thakur and Singh (2005)
$P_s = 135.0 + 2.0(CC + I_p - w_i)$	Sabtani (2005) For undisturbed samples.
$P_s = -8.04 + 0.0177 PI + 4.390 \gamma_d + 0.540 \log \psi$	Erzin and Erol (2007)

$$\begin{aligned}
(\log P_s)_1 &= 0.0276 I_p - 365.2118 \gamma_d^{-2.4616} - 0.0320 w_i + 2.2292 \\
(\log P_s)_2 &= 0.0239 I_p - 1285.3723 \gamma_d^{-3.2768} - 0.0396 w_i + 2.3238 \quad \text{Çimen (2012)} \\
\log P_s &= \text{mean}[(\log P_s)_1, (\log P_s)_2]
\end{aligned}$$

Vanapalli et al. (2012) proposed an empirical equation to estimate the swelling pressure for sand-bentonite mixtures specifically using the SWCC as a tool:

$$P_s = \left(\frac{S_r}{100}\right)^a \cdot (\psi) \quad (2.5)$$

where, S_r is the degree of saturation, a is the fitting parameter, ψ is the total soil suction.

More recently, Tu and Vanapalli (2016), proposed a semi-empirical equation for predicting the variation of swelling pressure with respect to suction for natural expansive soils by modifying Equation (2.5):

$$P_s = P_{s0} + \beta_s \cdot \psi \cdot \left(\frac{S_r}{100}\right)^2 \quad (2.6)$$

where, P_{s0} is the intercept on the P_s axis at zero suction value, β_s is fitting parameter, $\beta_s = 23.05 A^{32.315} (0.237 I_p - 10.278 \rho_{dn} + 0.164)$, A is the activity of soils, $A = I_p / CC$, ρ_{dn} is the natural dry density of soil.

2.3.3 Relationship between the lateral and vertical swelling pressures

Information of lateral swelling pressure is required in the rational design of structures built in expansive soils, such as the shallow foundations, retaining walls, tunnels, canal linings and underground conduits (Sapaz 2004, Avsar et al. 2009). However, limited studies are reported in the literature with respect to the direct measurement of lateral swelling pressure in laboratory or field (Ofer 1981, Joshi and Katti 1984, Fourie 1989, Clayton et al. 1990, Brackley and Sanders 1992, Zhang 1993, Windal and Shahrour 2002, Sapaz 2004, Avsar et al. 2009, Yang et al. 2014, Tang et al. 2015).

Laboratory tests

Zhang (1993) developed a swelling pressure test which is similar to constant volume test to study the influence of anisotropy of expansive soils and to understand the differences in vertical and lateral swelling pressure. Six different expansive soils from China were used in the study. These tests were conducted on undisturbed soil specimens with 55mm diameter and 110mm height dimensions. The average lateral swelling pressure for varies from 7.3kPa to 21.6kPa. The swelling pressure ratio, R_s (which is defined as the ratio between lateral and vertical swelling pressure) lies between 0.343 and 0.646 (see Table 2.5).

Table 2.5 The swelling pressure in three directions of expansive soil (modified from Zhang 1993).

Location	Natural density, γ	Natural water content, w_0	Liquid limit, LL	Plastic limit, PL	Clay content, <0.05 mm	Final water content, w_f	Swelling pressure			Swelling pressure ratio, R_s
							P_z	P_x	P_y	
	kN/m ³						kPa			
Ankan	20.34	20.14	46.8	20.7	45.1~53.5	21.97	28.8	14.3	15.2	0.512
Xixiang	19.96	20.67	39.5	20.1	39.8	21.98	28.0	8.6	10.6	0.343
Lixi	20.47	18.33	40.5	21.5	44.0	20.63	29.7	14.1	14.4	0.480
Lion Mont.	20.29	20.48	39.4	19.2	46.8	20.96	26.1	10.8	8.8	0.376
Mengzi	17.31	48.59	80.7	36.0	83.0	50.69	35.4	24.1	21.6	0.646
Yaquelin	17.71	31.71	60.7	36.9	51.2		14.1	8.6	7.3	0.564

Windal and Shahrour (2002) compared the test results of lateral swelling pressure by using oedometer with a flexible ring and a stiff ring as shown in Figure 2.9. The lateral swelling pressure was found to increase with an increase in the stiffness of the odometer ring.

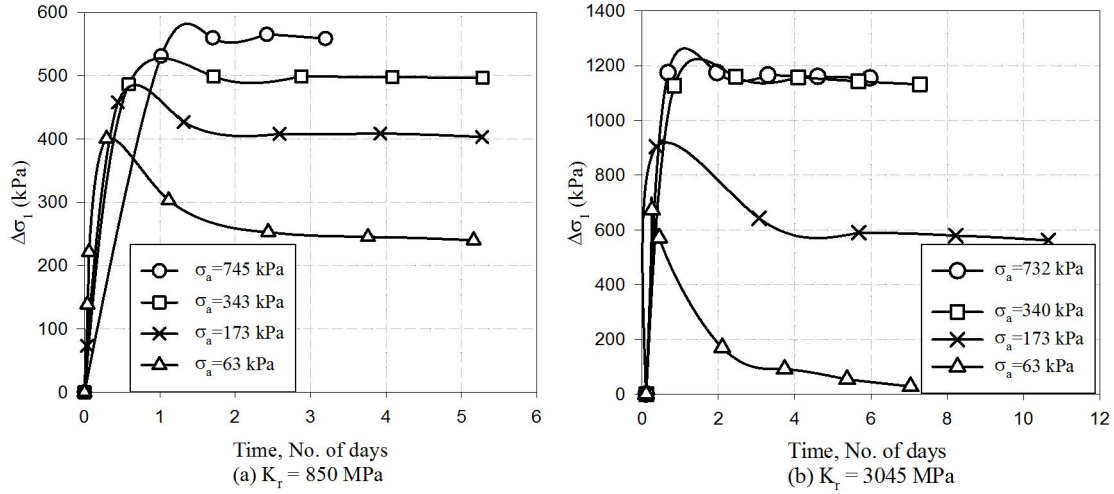


Figure 2.9 Evolution of the lateral swelling pressure: (a) with the flexible ring $K_r = 850$ MPa, (b) with the stiff ring $K_r = 3045$ MPa (modified from Windal and Shahrour 2002).

Sapaz (2004) conducted series of constant volume swell (CVS) tests to determine lateral and vertical swelling pressure associated with the changes of soil properties (i.e. initial dry density and initial water content). It was concluded from this study, that the swelling pressure ratio, R_s , changes from 0.59 to 0.86. Xie et al. (2007) used three-dimensional swell-shrink device to compare the lateral and vertical swelling pressures on remolded expansive soils. The calculated swelling pressure ratio, R_s , from test results lies between 0.367 and 0.679, which shows similarity with the observations reported by Zhang (1993) and Sapaz (2004).

Avsar et al. (2009) used a thin wall oedometer ring to determine the lateral and vertical swelling pressures simultaneously and Scanning Electron Microscope (SEM) to verify the anisotropy swelling behavior. The samples used in this study are the same as Sapaz (2004), known as Ankara clay, which were collected from Ankara Basin, Turkey. The modified thin wall oedometer ring acts as a lateral restraint to measure the maximum value of lateral swelling pressure. The ratio between the swelling pressures in lateral and vertical directions vary between 0.34 and 0.98, these results are in agreement with the conclusions reported by Sapaz (2004) (see Figure 2.10).

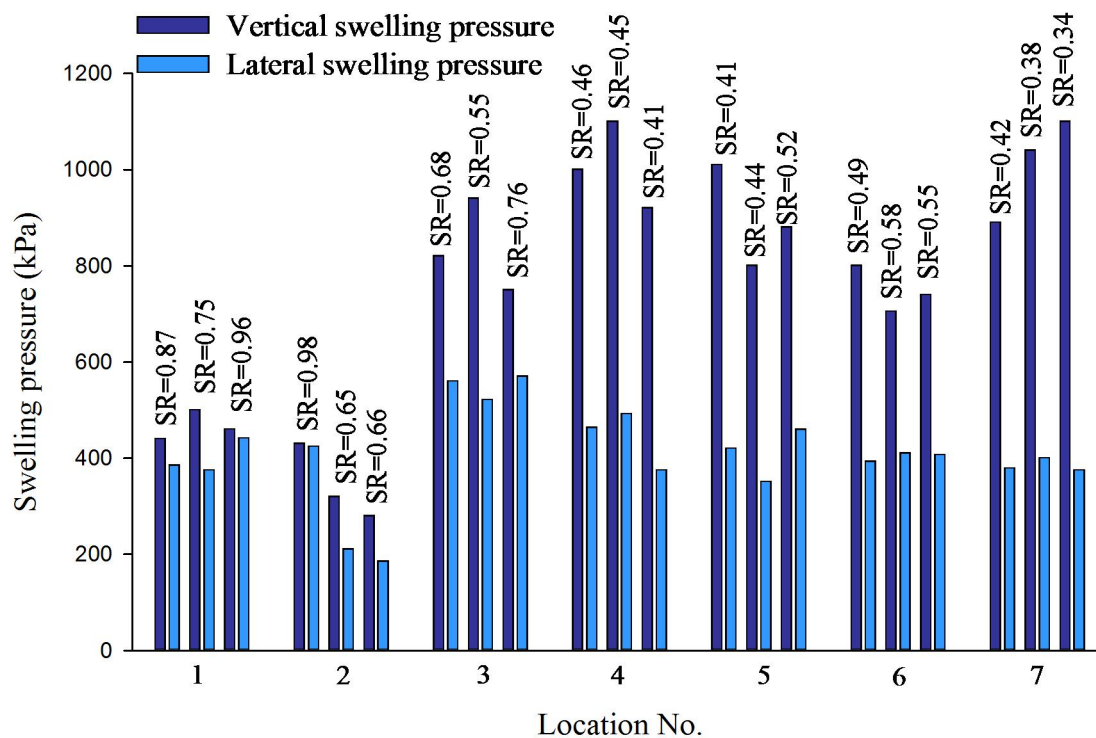


Figure 2.10 Comparison of swelling pressures in vertical and lateral directions (modified from Avsar et al. 2009).

Yang et al. (2014) conducted a constant volume test to measure the lateral swelling force of Guangxi expansive soil and the results show that the lateral expansion force is about 25% of the vertical expansion force.

In situ tests

In addition to the laboratory tests mentioned above, some large scale and in-situ studies are also conducted. Joshi and Katti (1984) built a large scale model with expansive soils in order to simulate field scenario conditions. The development of lateral pressure and vertical movement were presented in the study. The typical curve of the lateral pressure of expansive soils with respect to time is shown in Figure 2.11. The lateral pressure increased rapidly at the beginning of saturation process and then approached a peak value

(i.e. 6.01 kg/cm^2 which is approximately 601 kPa), which shows similar trends of results as Clayton et al. (1991). The lateral swelling pressure decreased slightly and remained at a constant value (i.e. 4.94 kg/cm^2 , which close to 494 kPa). The lateral swelling pressure predominantly contributes for such a development of lateral pressure in expansive soils (Sapaz 2004).

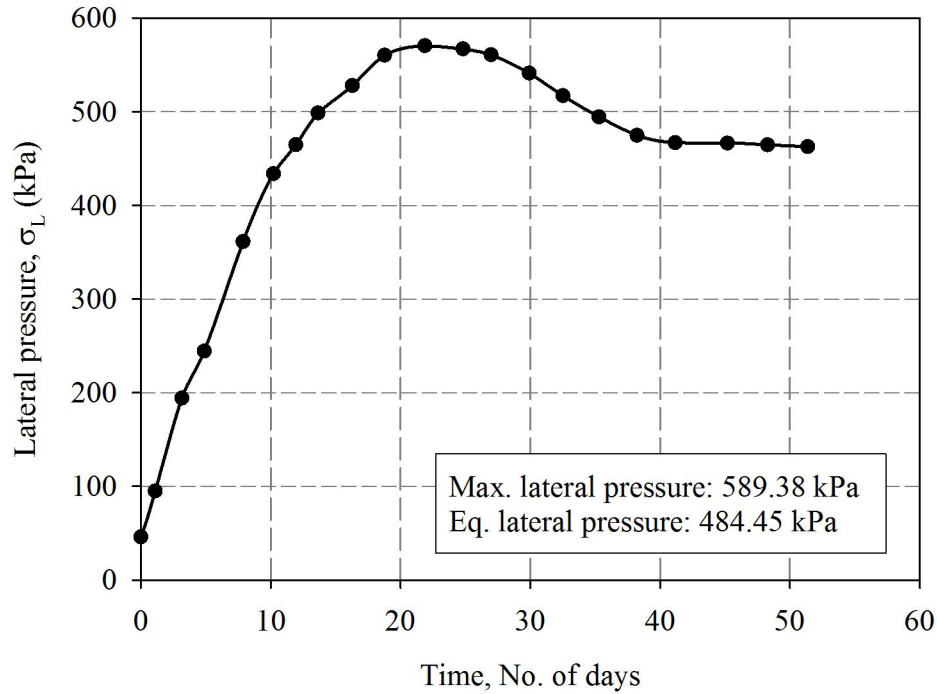


Figure 2.11 Development of lateral pressure with time (modified from Joshi and Katti 1984).

Brackley and Sanders (1992) also monitored the total horizontal pressure of a highly expansive clay in-situ for several years. The large seasonal variations of horizontal pressure were measured in the study with respect to the rainfall or evaporation.

2.4 Fissures and cracks in unsaturated expansive soils

2.4.1 The formation and propagation of fissures and cracks

Griffith (1924) suggested that the macro cracks in soils typically result from the growth of the micro fissures when they are subjected to tensile stresses. The non-uniform shrinkage in expansive soils during the desiccation is also responsible for the occurrence of fissures and cracks (Xu et al. 2011).

The micro cracks occur on the surface of soils first and then extend randomly in different directions. These micro cracks, however, predominantly propagate downwards from ground surface because of the influence of desiccation. As the water evaporates from soil mass, the cracks extend to the underground (Morris et al. 1992). The soil suction profile and the soil properties (i.e. Poisson's ratio and the coefficient of permeability) are the key factors to control the degree of cracks in soils.

Generally, there is a "seasonal unsteady zone" in the surface layer of expansive soil, which is referred as the zone in which water content changes due to climate factors at the ground surface (Nelson et al. 2001). Within this zone (see Figure 2.6), the water content of soil is mainly affected by the external environmental factors. Some studies highlighted the cracks in expansive soils typically occur within the depth of active zone and the maximum depth of cracks could be around 2 to 4 m (Morris et al. 1992, Konrad and Ayad 1997, Kodikara et al. 2000, Erguler 2001, Bao 2004, Jones and Jefferson 2012). However, such estimation is not accurate enough for the rational slope stability analysis and in the calculation of lateral earth pressures in retaining walls.

2.4.2 The effect of fissures and cracks

During the wet seasons, the runoff occurs over the ground surface when the infiltration rate exceeds the soil infiltrability (Oh and Vanapalli 2010). As such, before fissures generate, the infiltration is rather limited since the coefficient of permeability of expansive soils is relatively small.

Upon evaporation, the generation of fissures and cracks in expansive soils provides more channels in soils for water to evaporate and infiltrate (Bao 2004). In other words, the more intensive drying and wetting cycles could lead to further fissures and cracks development (Bao 2004, Ma et al. 2007, Xu et al. 2011).

The matric suction profile experiences significant variation in unsaturated soils associated with wetting and drying conditions. The matric suction variation which is closely connected to the physical and mechanical properties of unsaturated soils, such as the shear strength, soil compressibility and the modulus of elasticity. Apart from the variation of matric suction, fissures and cracks damage the integrity of soils and contribute to the reduction of soils strength significantly (Lu et al. 1997, Bao 2004).

2.4.3 Determination of fissures and cracks

Several research studies focussed on the role of fissures and cracks on the soil behavior during the last two decades from laboratory tests, numerical analysis and mathematical studies (Morris et al. 1992, Nahlawi and Kodikara 2006, Li et al. 2008, Amarasiri and Kodikara 2011, Tavakkoli and Vanapalli 2011). Most researchers focused on the qualitative or statistical analysis of desiccation cracking because the parameters related to the cracks prediction is difficult to capture in quantitative analysis (Amarasiri et al. 2011).

Laboratory tests and numerical analysis

Nahlawi and Kodikara (2005) presented the evolution of the cracking pattern, influence of the desiccation speed and typical crack spacing to depth ratios for soil layers based the experimental results on thin layers of clay soils. Ma et al. (2007) designed an experimental device, which can precisely control the humidity, to reappear the process of crack formation and propagation. In this study, the length and width of the cracks and the average space between cracks are observed and presented with respect to time and humidity. Zhang et al. (2011) used oven drying method and vacuum saturating method to simulate the drying and wetting process of Nanyang expansive soils in laboratory (see Figure 2.12). The vector diagram is employed to study the evolution of the fissures. The propagation of fissures and cracks in expansive soils is mainly governed by the gradient of water content, instead of the water content.

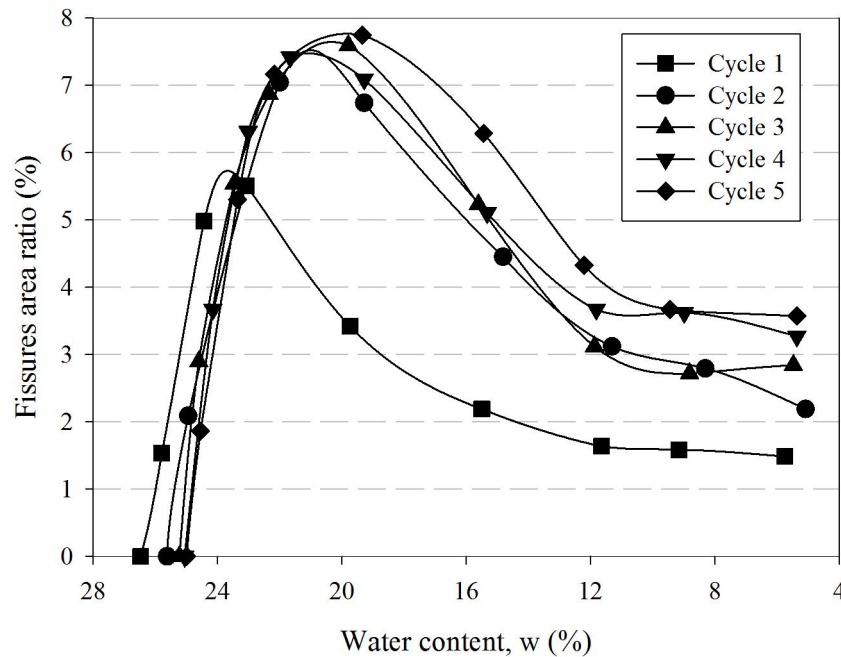


Figure 2.12 Curves of fissures area ratio changes under wetting and drying cycles (modified from Zhang et al. 2011).

Lee et al. (1988) proposed a finite element model for estimating the depth of tension cracking in soils. A stiff embankment on soft soil and an excavated slope are applied to

this finite element model to prove that the fracture mechanics could be used in tension cracking analysis. Amarasiri et al. (2011), in addition, used software Universal Distinct Element Code (UDEC) to analyse desiccation cracking taking account of soil properties which include tensile strength, bulk modulus and suction of the soil. The programming language FISH was embedded in the UDEC code to conduct a sensitive analysis and capture the variation of properties. By replicating six laboratory tests in the numerical model, Amarasiri et al. (2011) captured the evolution of desiccation cracks successfully, for instance, the width and number of cracks and the crack initiation water content.

Mathematical relationships

Morris et al. (1992) proposed mathematical equations to predict the depth of cracks, based on the properties of the soil and the suction profile, based on three theories: (i) elasticity theory, (ii) considering the tensile and shear strength behavior of soils, (iii) linear elastic fracture mechanics (LEFM). Also, Konrad and Ayad (1997) presented a rational idealized framework to predict the spacing between primary shrinkage cracks in cohesive soils and analysed the desiccation crack propagation based on the theory of linear elastic fracture mechanics (LEFM).

However, Prat et al. (2008) and Amarasiri and Kodikara (2011) argued that the applications based on the LEFM are associated with a key disadvantages because it is time-consuming. Also, LEFM may be more valid for brittle soils, because it assumes the infinite tensile stress at the crack tip, which is not consistent with significant plastic behavior around the crack zone.

Zheng et al. (2006) used the linear elastic mechanics to analyze the relations between the depth of initial cracking and soil properties of expansive soils such as tensile strength and Poisson's ratio as follow:

$$z_c = \frac{(u_a - u_w)_s - C\sigma_t}{\frac{(u_a - u_w)_s}{D} + W} \quad (2.7)$$

where, z_c is the depth of initial cracks, $(u_a - u_w)_s$ is the value of matric suction at ground surface, σ_t is the tensile strength of soils, D is the depth of ground water table, C and W are the fitting parameters, $C = (1 - \mu) / (1 - 2\mu)$, $W = \mu\gamma / (1 - 2\mu)$, μ is the Poisson's ratio.

In addition, Zheng et al. (2006) pointed out that relationship between the matric suction value and the depth is complex. Therefore, instead of assuming the matric suction decreases linearly from the surface to a value of zero at the natural ground water table, it would be better to conduct field test to measure the matric suction profile in unsaturated soils.

Li et al. (2008) suggested a relationship for estimation of crack propagation depth based on linear elastic mechanics, which considers the contributions arising from effective cohesion, c' and effective internal friction angle, ϕ' :

$$z_c = \frac{(u_a - u_w)_s - GF'}{\frac{(u_a - u_w)_s}{D} + G} \quad (2.8)$$

where, F' and G are the fitting parameters,

$$G = \frac{\mu\gamma}{1 - 2\mu - (1 - \mu)\alpha_T \tan\phi^b \cot\phi'} \text{ and } F' = \frac{1 - \mu}{\mu\gamma} \alpha_t \chi_t c' \cot\phi', \gamma \text{ is the unit weight of soils, } \alpha_T$$

is the coefficient of tensile strength of soils, χ_t is the reduction coefficient of effective cohesion at the range from 0 to 1, ϕ' is the effective angle of internal friction associated with the net normal stress state variable, $(\sigma - u_a)$, ϕ^b is the angle of shearing resistance with respect to matric suction, $(u_a - u_w)$, c' is the effective cohesion of soil.

Tavakkoli and Vanapalli (2011) suggested that the tension crack extends to a depth where the horizontal active pressure becomes zero by assuming the matric suction decreases

linearly from the surface to zero at the ground water table. Hence, the tension crack for unsaturated soils can be estimated as follow:

$$z_c = \frac{\gamma DS^\kappa \tan \phi'}{\gamma_w S^\kappa \tan \phi' - 0.5 \gamma_{unsat} \sqrt{K_a}} \quad (2.9)$$

where, S is the degree of saturation, κ is the fitting parameter (Garven and Vanapalli, 2006), $\kappa = -0.0016(I_p^2) + 0.0975(I_p^2) + 1$, γ_w is the unit weight of water, γ_{unsat} is the unit weight of unsaturated soils, K_a is the coefficient of active earth pressure, $K_a = \tan^2(45^\circ - \frac{\phi'}{2})$.

2.5 Stress state variables for unsaturated soils

The engineering behavior of both saturated and unsaturated soils can be interpreted in terms of stress state variables (Fredlund and Rahardjo 1993). Effective stress (Equation 2.10) proposed by Terzaghi (1936) is a stress state variable that is conventionally used for rational interpretation of saturated soils:

$$\sigma' = \sigma - u_w \quad (2.10)$$

where, σ' is the effective normal stress, σ is the total normal stress, u_w is the pore-water pressure.

Several researchers extended Terzaghi's effective stress concept for unsaturated soils. Of the many equations available in the literature, the equation (Equation 2.11) proposed by Bishop (1959) is more widely used:

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) \quad (2.11)$$

where, u_a is the pore-air pressure, χ is a parameter related to the degree of saturation of the soil.

Fredlund and Morgenstern (1976) proposed an approach for rational interpretation of the engineering behavior of unsaturated soils in terms of two independent stress state variables; namely, net normal stress, $(\sigma - u_a)$, and the matric suction, $(u_a - u_w)$. This approach is consistent with the concepts of continuum mechanics and is more widely used in practice, during the last four decades.

2.6 Soil-water characteristic curve

During the last two decades, the Soil-Water Characteristic Curve (SWCC) has been used as a tool to predict various properties of unsaturated soils including the shear strength of unsaturated soils. The SWCC defines the relationship between the soil suction and the amount of water in the soil (i.e. gravimetric water content, w , or volumetric water content, θ) or degree of saturation, S). Table 2.6 summerizes advantages and disadvantages of each of the designations used to represent the amount of water in a soil (Fredlund et al. 2012).

Table 2.6 Advantages and disadvantages of various designations for amount of water in soil (from Fredlund et al. 2012).

Designation	Advantages	Disadvantages
Gravimetric water content, w	Consistent with usage in classic soil mechanics	Does not allow differentiation between change in volume and change in degree of saturation Does not yield the correct air-entry value when the soil changes volume upon drying
	Most common means of measurement	
	Does not require a volume measurement	
	Reference value is a “mass of soil”	

	which remains constant	
Volumetric water content, θ	Is the basic form that emerges in the derivation of transient seepage and fluid storage in unsaturated soils Commonly used in databases of results obtained in soil science and agronomy	Requires a volume measurement Rigorous definition requires a volume measurement at each soil suction Is the designation least familiar and least used historically in geotechnical engineering
Degree of saturation, S	Most clearly defines the air-entry value Appears to be the variable most closely controlling unsaturated soil property functions	Requires a volume measurement Although volume measurements are required, the degree of saturation variable does not quantify overall volume change

Three stages of a typical unimodal SWCC were identified by Vanapalli et al. (1996): the boundary effect stage, the transition stage, and the residual stage of unsaturation as shown in Figure 2.13. Also, Vanapalli et al. (1999) pointed out that SWCCs show significant differences for different types of soils (i.e. sand, silt, Indian Head Till and Regina Clay) (see Figure 2.14).

Various laboratory methods are available for determining the SWCC. In addition, there are various mathematical equations to represent the SWCC.

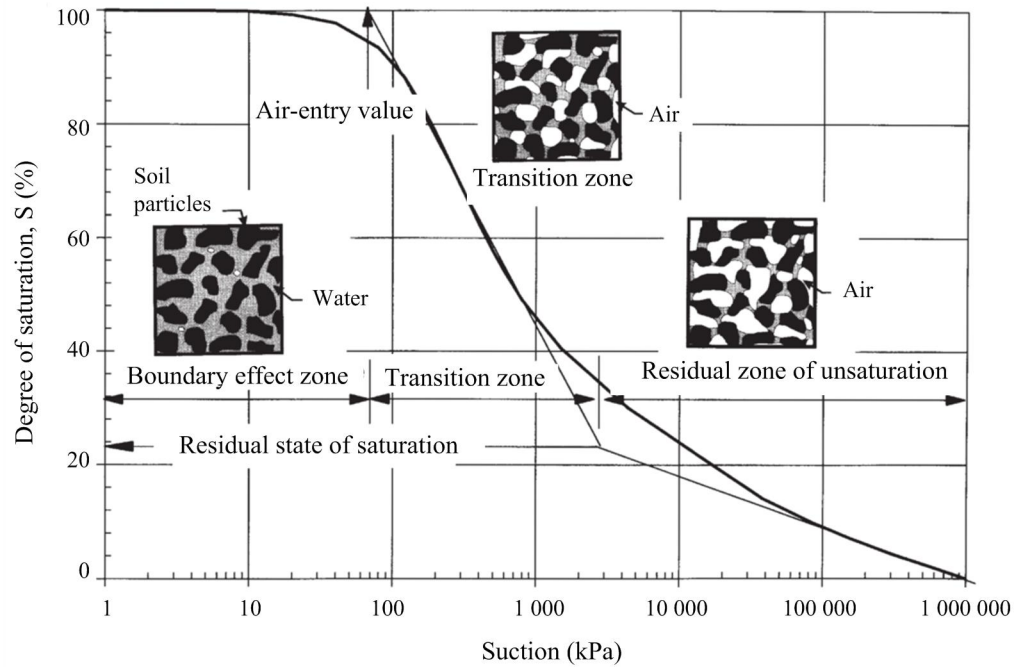


Figure 2.13 Typical soil-water characteristic showing zones of desaturation (from Vanapalli et al. 1999)

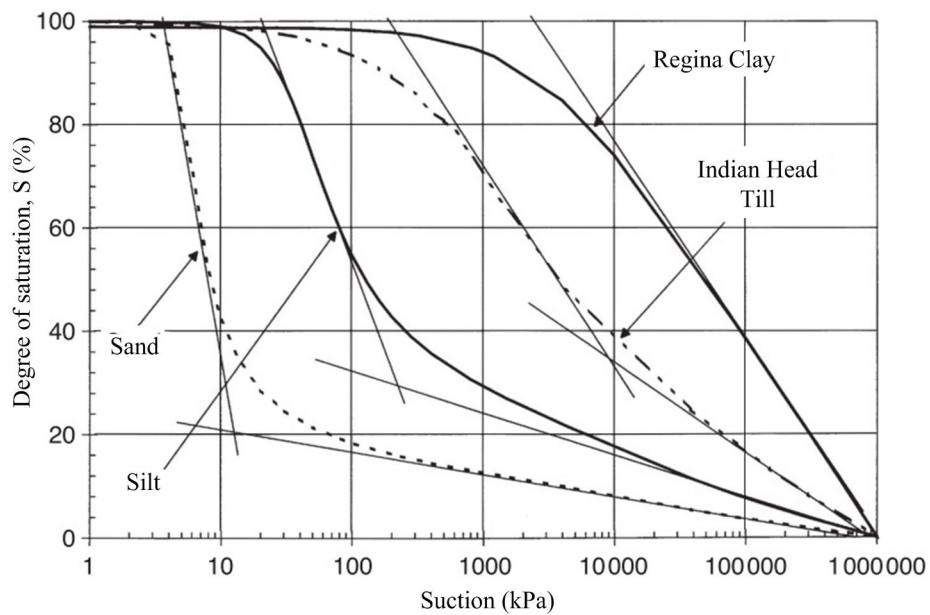


Figure 2.14 Typical soil-water characteristic for four Canadian soils (after Vanapalli et al. 1999).

2.6.1 Laboratory tests

Soil suction can be determined or estimated from laboratory tests using either by direct or indirect methods. Tensiometers and the axis translations technique are commonly used direct methods. However, filter paper technique, thermal conductivity sensor technique, chilled-mirror hygrometer technique are indirect methods that are widely used for estimation of soil suction using different techniques or principles.

Pressure plate and Tempe cell test (Axis translation technique)

The axis translation technique is employed to control matric suction by increasing air pressure while maintaining pore water pressure equal to atmospheric pressure in pressure plate test or Tempe cell test to determine the SWCC behavior. The axis translation technique is widely used to generate matric suction values in the soil specimen while preventing cavitation in the water through the use of high air entry (HAE) ceramic discs (Nam et al. 2010, Fredlund et al. 2012). As a result, the suction values that can be used is limited due to the bubbling pressure of the HAE ceramic disc. In pressure plate test (see Figure 2.16), SWCC of several soil specimens can be determined by placing them on a large saturated HAE ceramic disc in the pressure chamber. The water content changes in the soil specimens are determined after achieving equilibrium condition under each applied suction value. However, in Tempe cell test (see Figure 2.15), only one specimen can be used. The amount of water that expels from the soil specimen for different applied suction values, typically from a lower to higher matric suction value is measured to obtain the SWCC.

The upper limit for suction value for the pressure plate tests is typically 1500 kPa and the Tempe cell tests is 500 kPa.

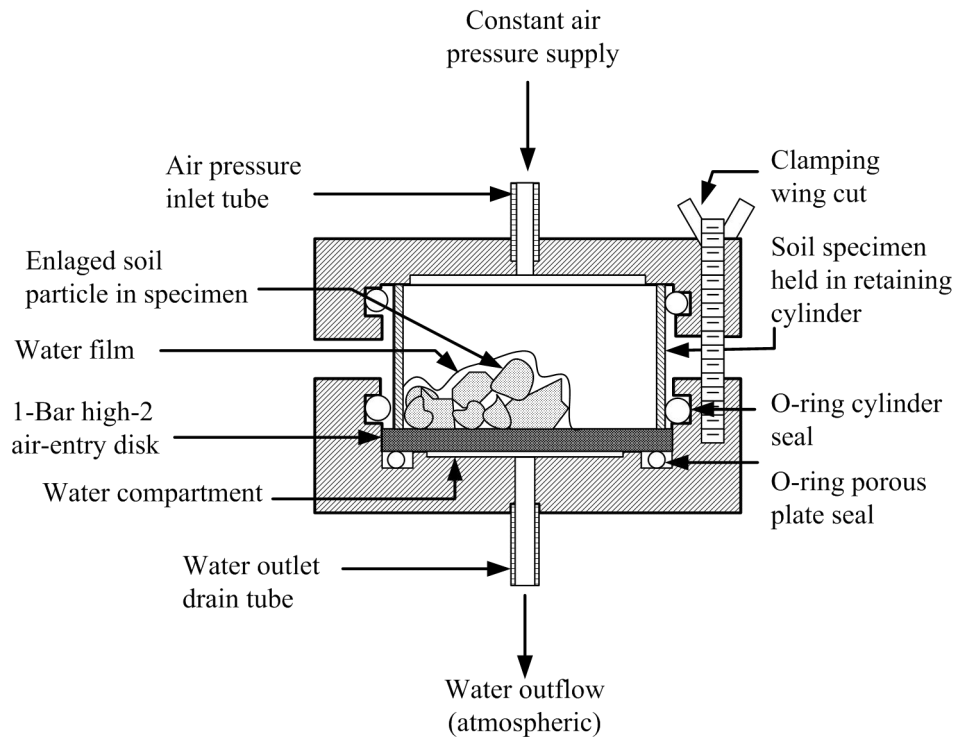


Figure 2.15 Cross-section of a Tempe pressure plate cell manufactured by Soil Moisture Equipment Corporation (modified after Fredlund et al. 2012).

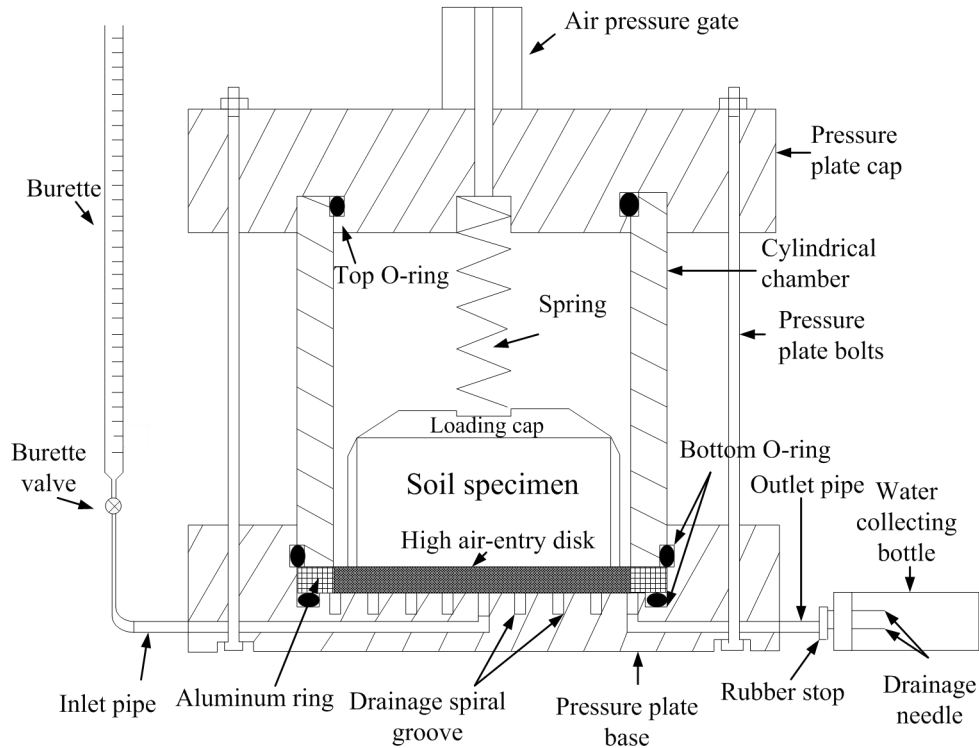


Figure 2.16 Single specimen pressure plate cell developed at University of Saskatchewan, Saskatoon, Canada (after Fredlund et al. 2012).

Filter paper method

Filter paper method (see Figure 2.17), which uses a filter paper to reach equilibrium condition with respect to water content in the soil specimen, using contact or non-contact methods estimate matric or total suction, respectively has been adopted by a number of researchers (Fredlund and Rahardjo 1993, Bulut et al. 2001). It is difficult to ensure adequate contact between the soil specimen and the filter paper for the reliable measurement of matric suction. Based on extensive studies, Power et al. (2008) suggested a contact pressure of 1 kPa to ensure a good contact and obtain reliable measurements of suction. The filter paper method is widely used to measure total suction values using non-contact method. The most commonly used filter papers are the Whatman No. 42, and the Schleicher and Schuell No. 589 papers, both of which have known ASTM calibration curves (ASTM D5298, 2003). The advantages of both the contact and non-contact

methods include its simplicity, low cost, and ability to measure a wide range of suctions. One of the major drawbacks of the filter paper method is that it is time-consuming. One independent test is required to get one data point in the SWCC, which means that a lot of time and effort are required to construct the entire SWCC. Several researchers however have used this method and found to be reliable technique for measuring the SWCC (Bulut et al. 2001, Leong et al. 2002, Dineen et al. 2003, Agus and Schanz 2005, Power et al. 2008, Nam et al. 2010, Fredlund et al. 2012).

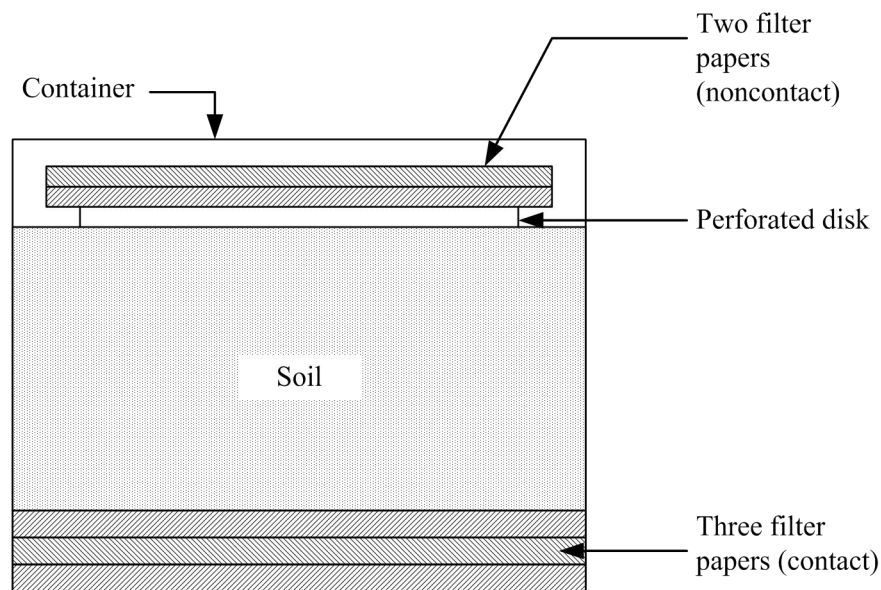


Figure 2.17 Contact and noncontact filter paper methods for measuring matric and total suction (Modified after Fredlund and Rahardjo, 1993)

Potentiometer

The dew point potentiometer, known as a chilled-mirror hygrometer, measures dew point and temperature accurately in a closed space above the soil specimen. The WP4-T dew point potentiometer, manufactured by Decagon Device Inc., measures water potential using the chilled mirror dew point technique. By considering the error of the total suction measurement and the way the isothermal equilibrium between the specimen and the

vapor space is maintained, this technique is considered to be the most reliable for measuring total suction (Decagon Devices Inc. 2003, Petry and Jiang 2003, Agus and Schanz 2005, Thakur et al. 2005, Shah et al. 2006, Sreedeeep and Singh 2006a, b, Thakur et al. 2005 and 2006, Bulut and Leong 2008).

Figure 2.18 shows the approximate range of suctions that can be measured using different techniques for measuring suction. They all have advantages and disadvantages and geotechnical engineers should use appropriate approaches/techniques for measuring suction according to their objectives.

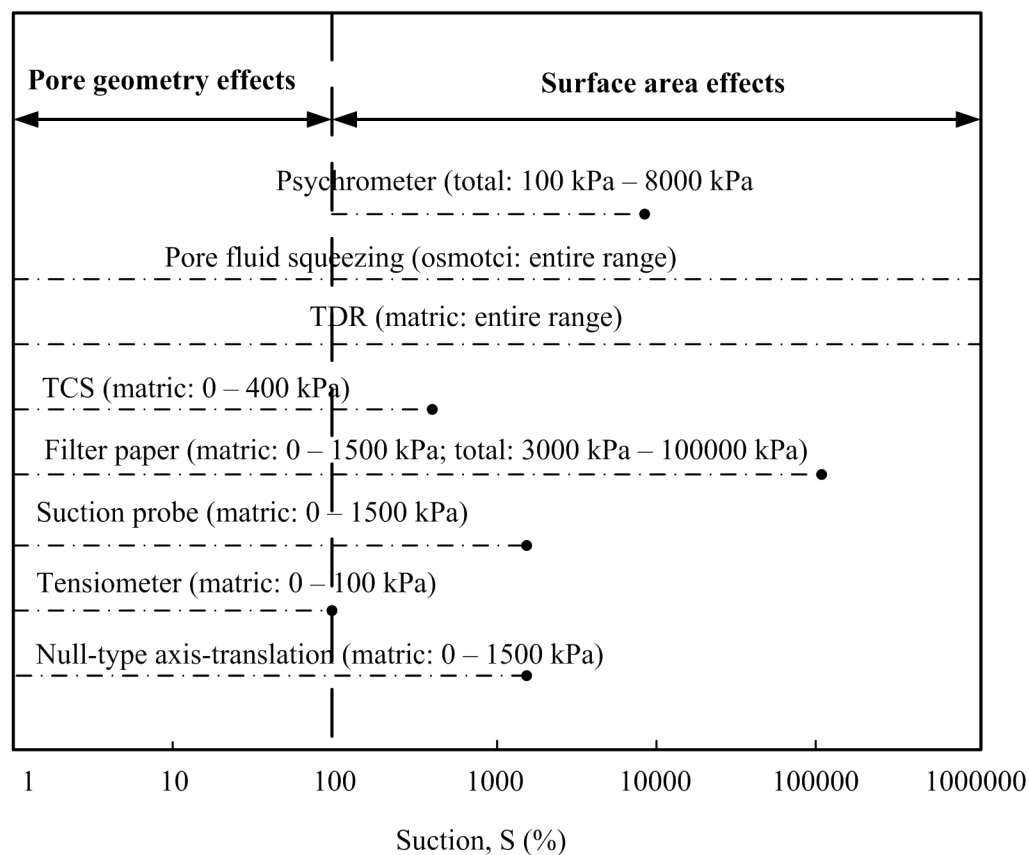


Figure 2.18 Suction measurement range of several available methods (from Agus and Schanz 2005).

More details about other techniques for suction measurement, such as the tensiometer and thermal conductivity sensor, can be found in Dineen et al. (2003), Agus and Schanz (2005), Nam et al. (2010), Fredlund et al. (2012).

2.6.2 Mathematical models for the SWCC

After obtaining the experimental data for the SWCC from laboratory tests, mathematical functions can then be fitted to the SWCC data. Leong and Rahardjo (1997) and Sillers et al. (2001) both presented summaries of the different models available to fit the SWCC data. Among the many formulations available in the literature, Brooks and Corey (1964), van Genuchten (1980) and Fredlund and Xing (1994) equations are commonly used to fit the SWCC data.

Table 2.7 Summary of some SWCC models (modified after Sillers et al. 2001).

Models	Equations	Comments
Burdine (1953) model	$S = \frac{1}{(1 + a\psi^n)^{(1-2/n)}}$	Provides a reasonably accurate representation of data for a variety of soils using two fitting parameters a and n .
Gardner (1956) model	$S = \frac{1}{1 + a\psi^n}$	Is one of the first simple equations used to model the SWCC using two fitting parameters a and n .
Brooks and Corey (1964) model	$S = 1, \psi < a$ $S = \left(\frac{\psi}{a}\right)^{-n}, \psi > a$	Is one of the first models proposed for the SWCC, and still remains popular. Does not provide a continuous mathematical function for the entire SWCC. More suitable for coarse-grained soils
Brutsaert (1966) model	$S = 1 / \left[1 + \left(\frac{\psi}{a} \right)^n \right]$	Was one of the early continuous soil-water characteristic models using only two fitting parameters
van Genuchten (1980) model	$S = \frac{1}{(1 + a\psi^n)^m}$	A versatile model that provides a good fit for a variety of soils over a large suction range. The model parameters have physical

		meaning. The magnitude of the n and m best-fit values may vary somewhat depending on the convergence procedure.
Tani (1982) model	$S = \left(1 + \frac{a - \psi}{a - n}\right) \exp\left(-\frac{a - \psi}{a - n}\right)$	Is inflexible and both of the parameters affect the position and the shape of the curve. Is difficult to mathematically fit to a unique set of parameters. Required several trials to best-fit, with different initial guesses before the model can converge to reasonable values
Boltzman (1984) model	$S = 1, \psi < a$ $S = \exp\left(\frac{a - \psi}{n}\right), \psi > a$	The major disadvantage of this method is that both parameters affect the position and shape of the SWCC. Is not continuous.
Fermi (1987) model	$S = 1 / \left[1 + \exp\left(\frac{\psi - a}{n}\right)\right]$	Is relatively inflexible. The effect each parameter has on the curve is difficult to isolate.
Fredlund and Xing (1994) model	$S = \frac{1}{\left(\ln\left[e + \left(\frac{\psi}{a}\right)^n\right]\right)^m}$	Flexible equation to fit a wide variety of datasets. The soil parameters are meaningful. The effect of one parameter can be distinguished from the effect of the other two parameters.

For the three-parameter models in Table 2.6, the general parameters can be described as a , n and m , which are more flexible; however, they are cumbersome than the two-parameter models (Sillers et al. 2001). Each of the fitting parameters has their own relationships with SWCC as shown in Table 2.8.

Table 2.8 The model parameters (from Sillescu et al. 2001)

Parameters	Meaning
a	Is a suction value corresponding to the inflection point on the SWCC, which has physical meaning in that it bears a relationship to the air-entry value.

n	Is related to the rate of change of the desaturation zone of the SWCC.
m	Is related to the asymmetry of the curve about the inflection point.

Based on Fredlund and Xing (1994) model, Vanapalli and Catana (2005) proposed a simple a method for estimating the SWCC of coarse-grained soils using parameters derived from the grain-size distribution curve and volume mass properties along with one data point of the measured SWCC.

The relationship between the dominant particle size diameter, d_e , and parameter, a , is estimated using the relation below for coarse-grained soils:

$$a = \frac{1.33}{(d_e)^{1.86}} \quad (2.12)$$

The parameter, n , in Fredlund and Xing (1994) model, which corresponds to the slope of the desaturation is predominantly affected by shape, size and distribution of pore space and can be expressed as follow:

$$n = \frac{7.78}{(C_u \times e)^{1.14}} \quad (2.13)$$

where, C_u is the coefficient of uniformity.

The last parameter, m , is related to the asymmetry of SWCC and cannot be estimated by volume mass properties and grain size information (Sillers et al. 2001, Vanapalli and Canata 2005). Hence, based on one measured data point and the equations mentioned above (Equation 2.12 and 2.13), Vanapalli and Canata (2005) presented a mathematical formulation for estimating the SWCC by Fredlund and Xing (1994) model as below:

$$w = w_r + (w_s - w_r) [1/\ln(f)] \quad (2.14)$$

where, w is the gravimetric water content, w_s is the saturated gravimetric water content,

$$w_r \text{ is the residual gravimetric water content, } f = \left(e + \left(\frac{\psi}{1.33/d_e^{0.86}} \right)^{7.78/(C_u \times e)^{1.14}} \right)^{mf}.$$

In addition to the SWCC estimation of coarse-grained soils, Catana et al. (2006) also proposed a simplified method to estimate the SWCC for fine-grained soils by using Brutsaert's (1966) equation:

$$w = w_r + (w_s - w_r) \frac{1}{1 + \left(\frac{\psi}{a_b} \right)^{n_b}} \quad (2.15)$$

where, a_b and n_b are the fitting parameters.

Catana et al. (2006) suggested using only two data points information to estimate the SWCC: one measured point in the suction range of 50 to 500 kPa and another estimated point in the suction range of 1000 to 3000 kPa, as shown in Figure 2.19.

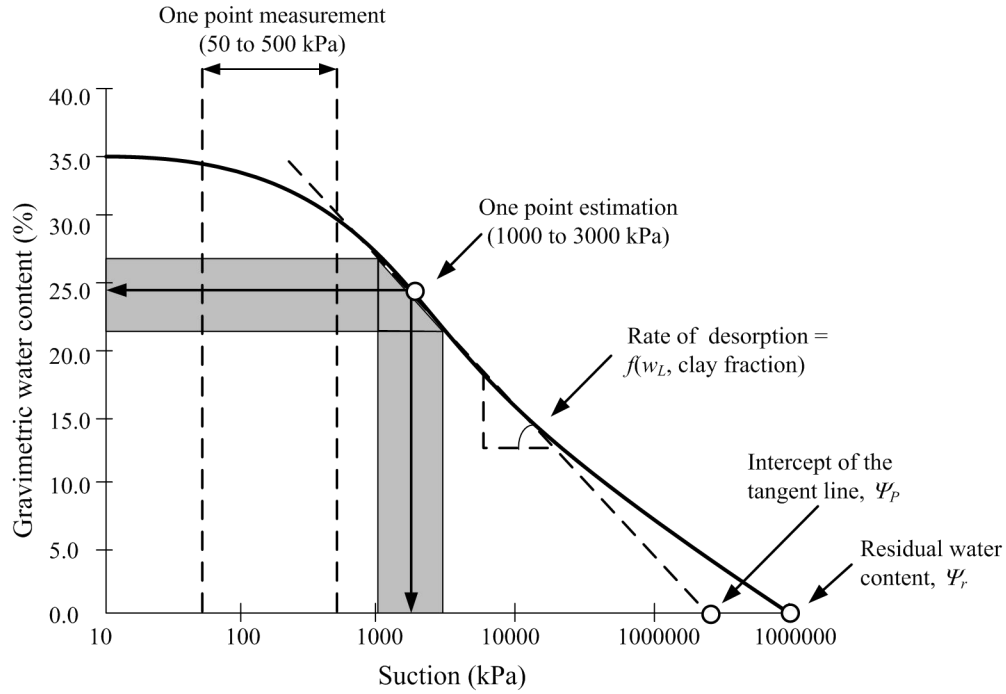


Figure 2.19 The essential features for estimating the SWRC of fine-grained soils (modified from Catana et al. 2006).

2.6.3 Shear strength of unsaturated soils

Bishop (1959) presented the equation of the shear strength of unsaturated soils based on Equation (2.11):

$$\tau = c' + [(\sigma - u_a) + \chi (u_a - u_w)] \tan \phi' \quad (2.16)$$

where, c' is the effective cohesion, ϕ' is the angle of internal friction associated with the net normal stress state variable, $(\sigma - u_a)$.

Later, Fredlund (1975) interpreted the shear strength of unsaturated soils in terms of two independent stress states variables, as below:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad (2.17)$$

where, ϕ^b is the angle of shearing resistance with respect to matric suction, $(u_a - u_w)$.

The experimental results show shear strength behavior with respect suction, ϕ^b is nonlinear with respect to suction when tested over a wide range of suction (see Figure 2.20). The shear strength of unsaturated soils increases linearly up to the air-entry value (AEV) in the boundary effect zone. After the AEV, the shear strength increases non-linearly in the transition zone (i.e. desaturation zone). In the residual zone, the shear strength of an unsaturated soil may increase, decrease, or remain relatively constant beyond the residual suction conditions (Vanapalli et al. 1996).

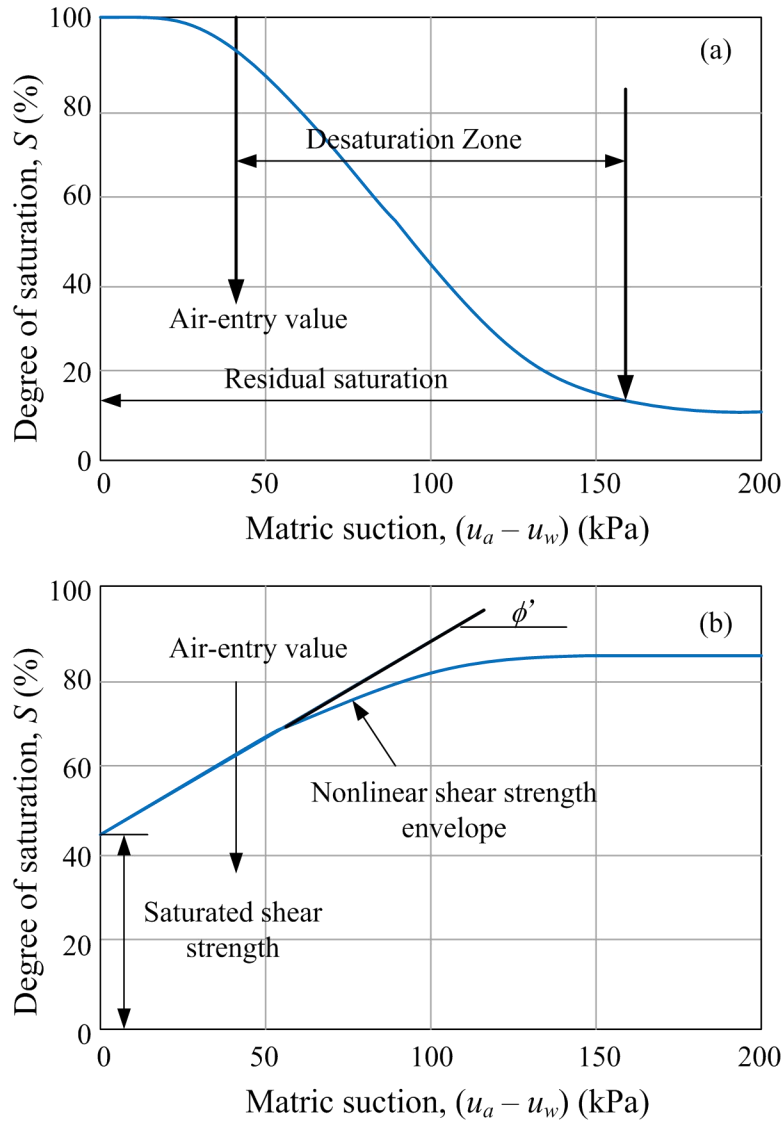


Figure 2.20 (a) A typical soil-water characteristic curve. (b) Shear strength behavior of soil as it relates to the soil-water characteristic curve (from Vanapalli et al. 1996)

To account the nonlinear behavior of shear strength as shown in Figure 2.20. Vanapalli et al. (1996) proposed an equation for predicting the shear strength of unsaturated soils with fitting parameter κ (see Figure 2.21):

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) (S^\kappa) \tan \phi' \quad (2.18)$$

where, κ is the fitting parameter.

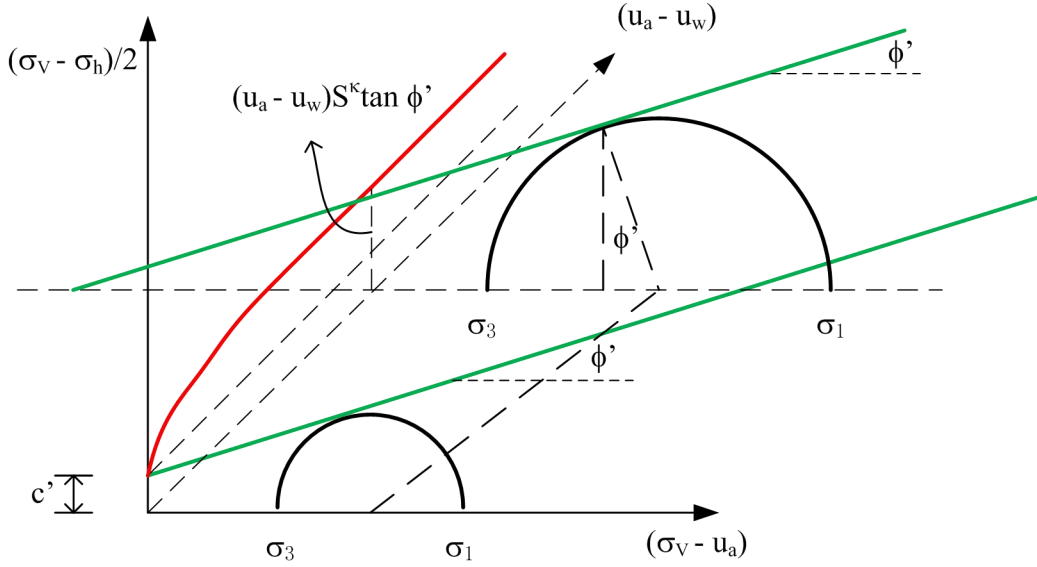


Figure 2.21 The variation of shear strength with respect to the net normal stress and matric suction (from Tavakkoli and Vanapalli 2011).

Garven and Vanapalli (2006) evaluated published experimental data of statically compacted soils and proposed the following equation for the fitting parameter, κ , in Equation (2.17) that can be used for predicting the shear strength of unsaturated soils:

$$\kappa = -0.0016(I_p^2) + 0.0975(I_p) + 1 \quad (2.19)$$

Vanapalli et al. (1996) also presented another equation to predict the shear strength of unsaturated soils, which alleviates the need of using a fitting parameter as below:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \left[\tan \phi' \left(\frac{S - S_R}{100 - S_R} \right) \right] \quad (2.20)$$

where, S_R is the residual degree of saturation.

To interpret the shear strength of expansive unsaturated soils, taking account of the influence of the swelling pressure, Lu et al. (1997) modified Equation (2.16) by replacing matric suction term with swelling pressure as follow:

$$\tau = c' + (\sigma' + u_a) \tan \phi' + m_s P_s \tan \phi' \quad (2.21)$$

where, m_s is the reduction coefficient of swelling pressure, P_s is the swelling pressure, $P_s = A_s \omega^{\lambda_s}$, A_s and λ_s are the regression analysis parameters which can be determined from soil properties.

Zhan and Ng (2006) stated that the contribution of matric suction to shear strength in expansive soils could be interpreted into two parts, namely, the effect of capillary force to the interparticle normal stress, and the effect of suction on soil dilatancy. As such, the dilation angle, Ψ , is introduced in the shear strength equation to account the effect of suction on soil dilatancy, which depends on the soil structure and the value of matric suction as below:

$$\tau = c' + (\sigma' + u_a) \tan(\phi' + \Psi) + (u_z - u_w) \tan \phi^b \quad (2.22)$$

where, Ψ is the dilation angle.

2.6.4 Tensile strength of unsaturated soils

The upper layer of soil is typically subjected to tensile stress due to drying while the lower part is subjected to compressive stress. The influence of water content gradient has a significant influence on the tensile stress. Improved understanding of the tensile strength of unsaturated soils provides better tools for interpretation of the formation and propagation of tension cracks in unsaturated expansive clays. When the tensile stress is greater than the ultimate tensile strength of soils, fissures are generated in unsaturated expansive soils.

Snyder and Miller (1985) suggested that the maximum tensile strength is about half the gauge pressure of the pore water as the desaturation begins. Lu et al. (2007) presented the tensile strength characteristic curve (TSCC) of unsaturated sand and compared it with the SWCC. This study suggests that the maximum value of tensile strength is correlated to the air-entry pressure (see Figure 2.22).

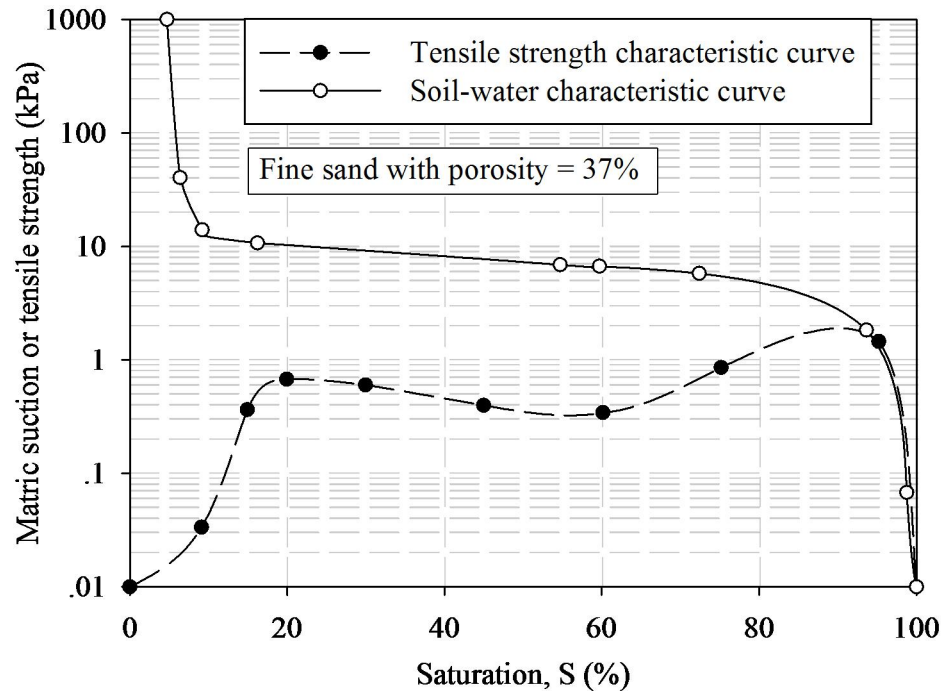


Figure 2.22 Relationship between tensile strength characteristic curve and soil water characteristic curve for the fine sand (Lu et al. 2007)

Tej and Singh (2013) proposed a relatively simple empirical equation (Equation 2.23) to estimate the tensile strength of fine-grained soils. The proposed empirical equation, which is given below, takes into account of the various parameters such as the water content, liquid limit and clay content.

$$\sigma_t = A_t \times w^{-B_t} \times C_t \times w^{D_t} \quad (2.23)$$

where, A_t , B_t , C_t , D_t are empirical constants related to the liquid limit and clay content of the soils: $A_t = 18.6 \times LL^2 \times CC^{-0.02}$, $B_t = 2.6 \times LL^{-0.3} \times CC^{-0.1}$, $C_t = 13.2 \times LL^{-0.3} \times CC^{-0.05}$, $D_t = 1.42 \times LL^{-0.5} \times CC^{-0.3}$.

Fredlund and Rahardjo (1993) suggested that the total cohesion can be estimated by extending the shear stress axis for a specific matric suction at a zero net normal stress:

$$c = c' + (u_a - u_w) \tan \phi^b \quad (2.24)$$

Since the tensile stress is different from the compressive stress, Morris et al. (1992) suggested that the tensile strength, t , can be estimated using the equation below:

$$t = -\alpha_T c \cot \phi' \quad (2.25)$$

where, α_T is the modified coefficient for tensile stress in unsaturated soils, within the range of 0.5-0.7 (Baker 1981, Bagge 1985).

The tensile strength of unsaturated soils can be estimated taking account of the influence of matric suction by substituting Equation (2.23) into Equation (2.24):

$$t = -\alpha_T [c' + (u_a - u_w) \tan \phi^b] \cot \phi' \quad (2.26)$$

2.7 Retaining walls

A retaining wall is a structure that is designed and constructed to resist the lateral pressure of soil that typically arise when soil is deposited on either sides is at different elevations. Various types of retaining walls are used in various geotechnical projects,

such as highways, foundations and dams to prevent fluid or soils damaging the structures (Connor and Faraji 2013, see Figure 2.23).

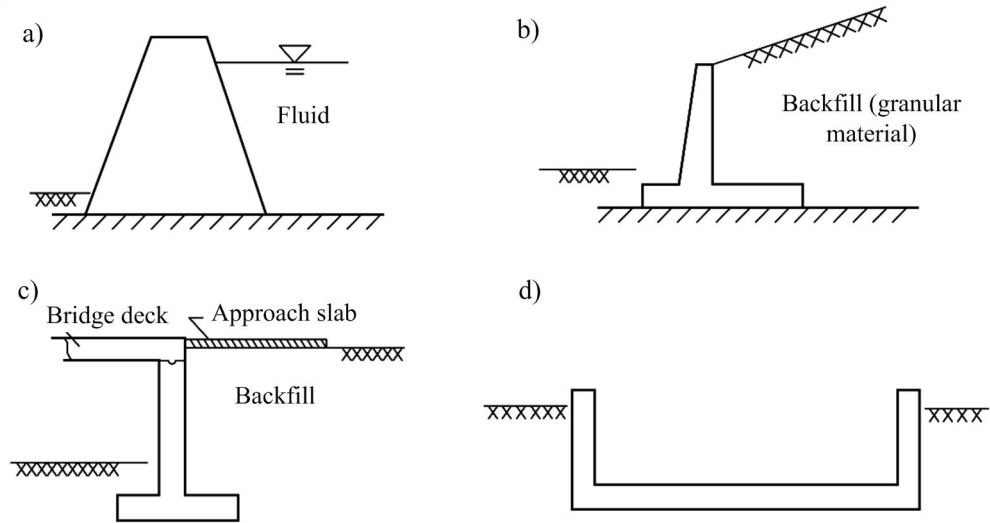


Figure 2.23 Retaining structures: (a) Gravity dam, (b) Cantilever retaining wall, (c) Bridge abutment, (d) Underground basement (from Connor and Faraji 2013).

2.7.1 Categories of retaining walls and their failure modes

Different types of retaining walls can be grouped into two different categories: (i) rigid structures that consist of concrete walls relying on gravity for stability, and (ii) flexible structures which consists of long, slender members of either steel or concrete or wood or plastic and relies on passive soil resistance and anchors for stability (see Figure 2.24 and Figure 2.25, Punmia and Jain 2005). The modes of failure for these two kinds of retaining walls are presented in Figure 2.26 and Figure 2.27, respectively.

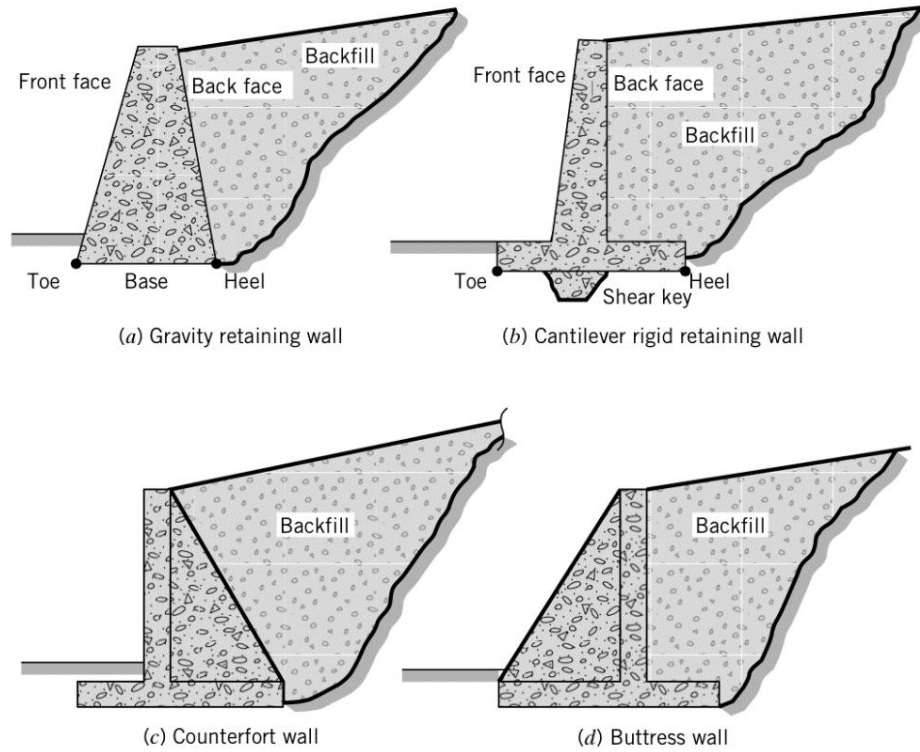


Figure 2.24 Types of rigid retaining walls: (a) Gravity retaining wall, (b) Cantilever rigid retaining wall, (c) Counterfort wall, (d) Buttress wall (from Punmia and Jain 2005).

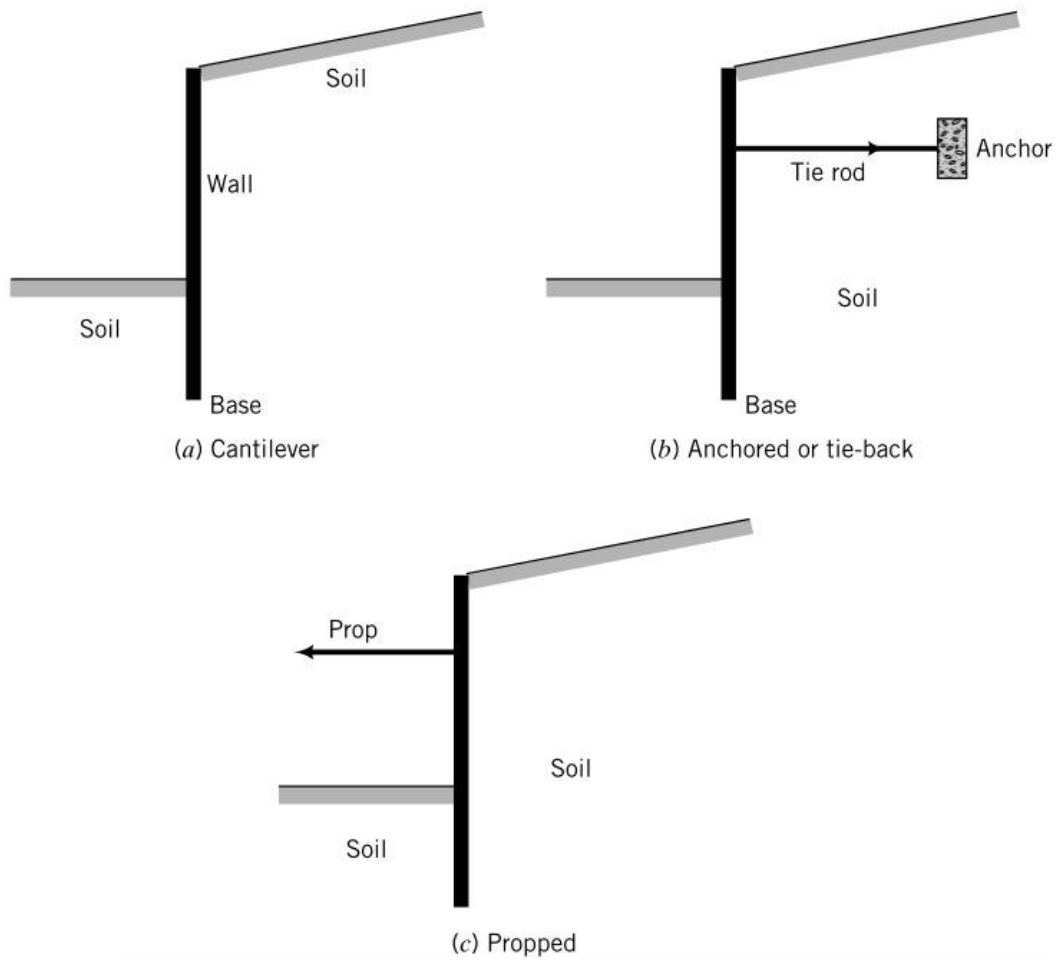


Figure 2.25 Types of flexible retaining walls: (a) Cantilever, (b) Anchored or tie-back, (c) Propped (from Punmia and Jain 2005).

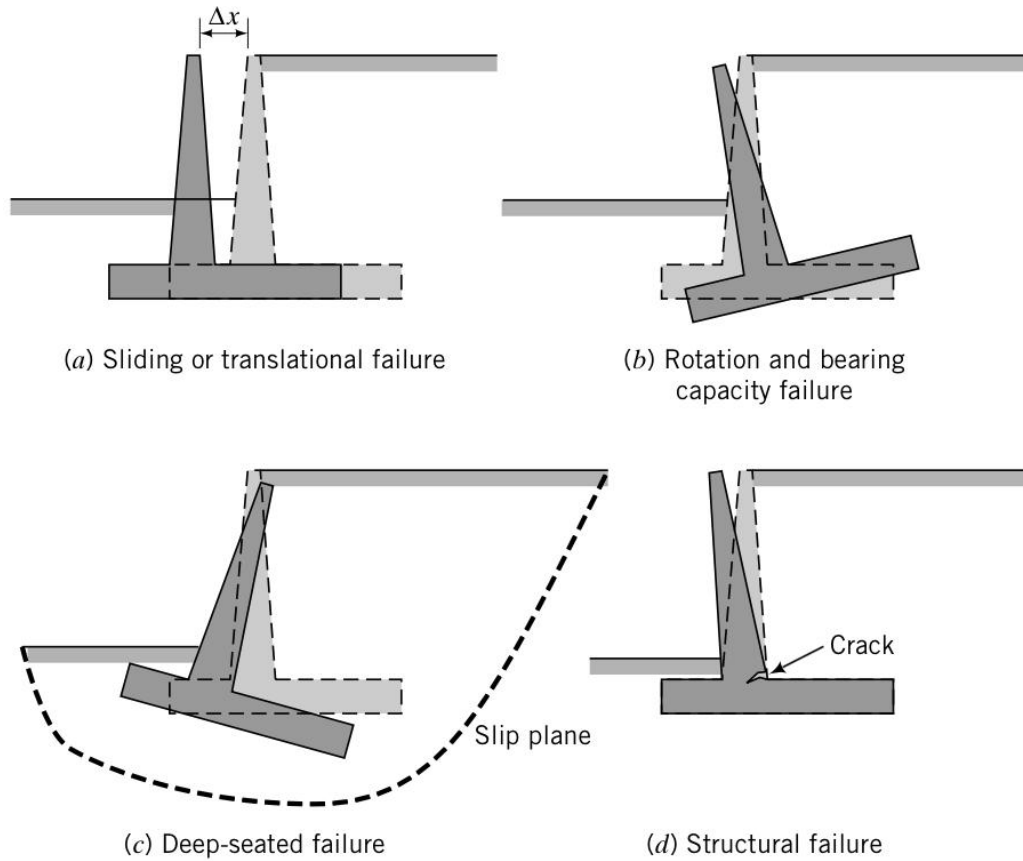


Figure 2.26 Failure modes for rigid retaining walls (the dotted lines show the original position of the wall): (a) Sliding or translational failure, (b) Rotation and bearing capacity failure (c) Deep-seated failure, (d) Structural failure (from Punmia and Jain 2005).

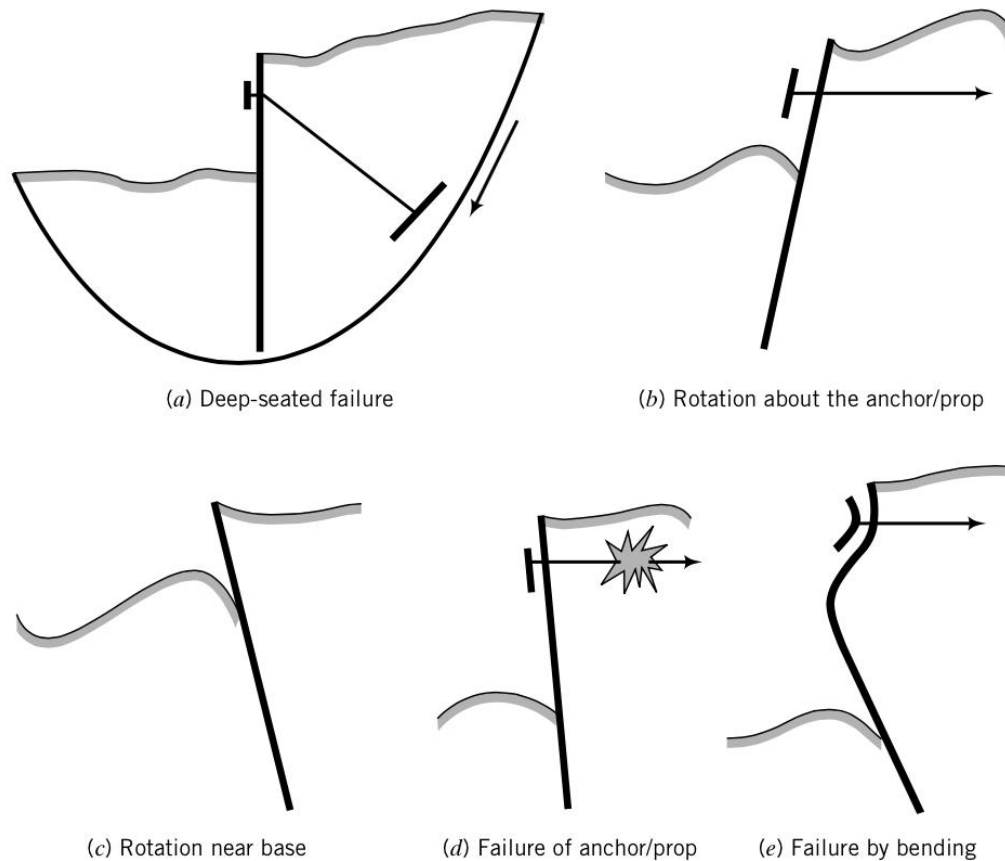


Figure 2.27 Failure modes for flexible retaining walls: (a) Deep-seated failure, (b) Rotation about the anchor/prop, (c) Rotation near base, (d) Failure of anchor/prop, (e) Failure by bending (from Punmia and Jain 2005).

2.7.2 Backfill material

The mechanics of saturated soils are conventionally applied in the design of retaining walls without considering whether the backfill material is in a state of saturated or unsaturated condition. Most commonly used approaches in the design of retaining walls are simple and are based on either Coulomb (1776) or Rankine (1857) theory. The soil density and effective or total shear strength parameters (i.e. friction angle and cohesion) are required for the design of retaining walls. These soil parameters can be determined from conventional laboratory tests.

The presently used approaches for the design of retaining walls are not appropriate because they don't take into account of the influence of soil suction in the backfill material (Tawfik et al. 2007, Tavakkoli and Vanapalli 2011, Vo and Russell 2014). In conventional geotechnical engineering practice, soils are usually assumed to be either saturated or dry in retaining wall analysis. Geotechnical engineers are well aware of swell pressures that can be exerted on the retaining wall with expansive soils as backfill material. However, the influence of swelling is typically not taken into account in conventional earth pressure theories. Pufahl et al. (1983) highlighted the importance of taking account of the influence of swelling pressure in the estimation of the active or passive earth pressure. However, Mohamed et al. (2014) stated that there is no reliable method presently available that allows the designer to reliably predict the pressures on retaining structures with expansive soils as backfill.

2.7.3 Lateral earth pressure

2.7.3.1 Lateral earth pressure of unsaturated soils

Rankine's (1857) and Coulomb's (1776) earth pressure theories are commonly used in the calculation of lateral earth pressure both for saturated and unsaturated soils. The solution from Coulomb's earth pressure theory is analogous to an upper bound solution because it gives a solution that is usually greater than the true solution. On the other hand, the solution for the lateral forces obtained using the Rankine active and passive states is analogous to a lower bound solution - the solution obtained is usually lower than the true solution because a more efficient distribution of stress could exist.

Based on Rankine's earth pressure theory, Pufahl et al. (1983) formulated the expressions for active and passive earth pressures in terms of total stresses for both the saturated and unsaturated states using the Mohr-Coulomb failure criterion and plastic equilibrium (see Figure 2.28):

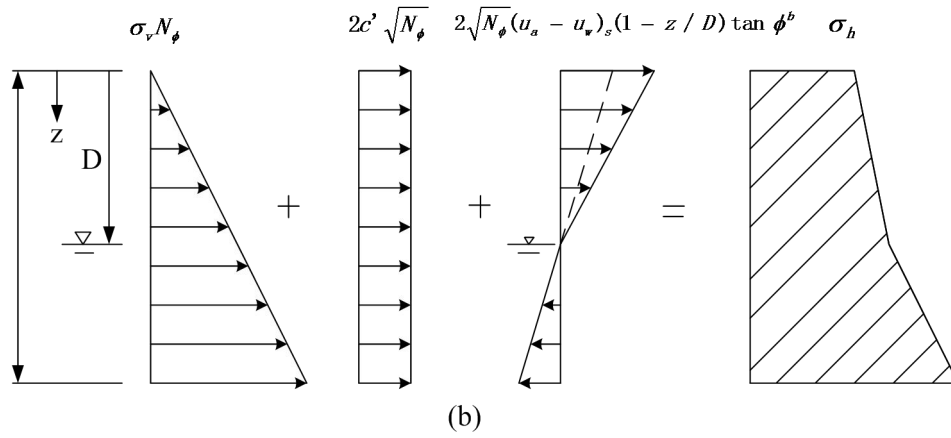
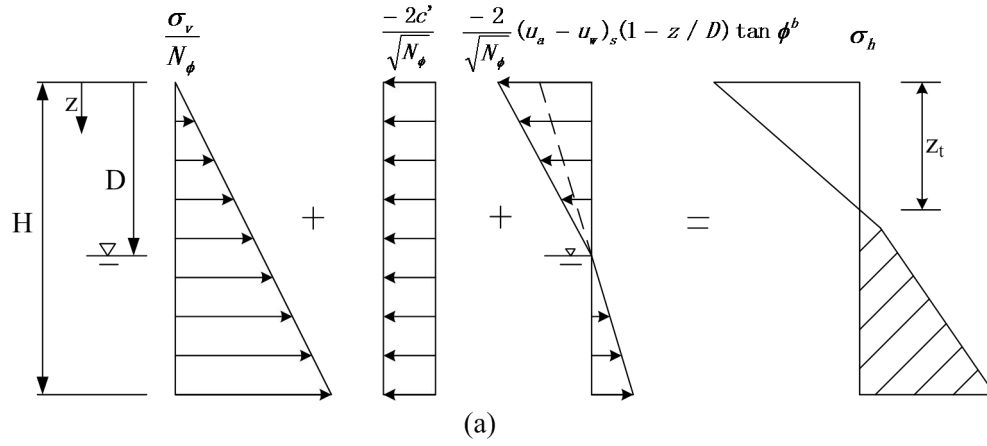


Figure 2.28 Diagrams of earth pressures for unsaturated soils: (a) Active earth pressure, (b) passive earth pressure (after Pufahl et al. 1993)

The total active earth stress at depth, z :

$$\sigma_h = \frac{\sigma_v}{N_\phi} - \frac{2}{\sqrt{N_\phi}} \times [c' + (u_a - u_w)_s (1 - z/D) \tan \phi^b] \quad (2.27)$$

The total passive earth stress at depth, z :

$$\sigma_h = \sigma_v N_\phi - 2\sqrt{N_\phi} \times [c' + (u_a - u_w)_s (1 - z/D) \tan \phi^b] \quad (2.28)$$

where, σ_h is the total horizontal stress, σ_v is the total vertical stress, $(u_a - u_w)_s$ is the matric suction at ground surface, $N_\phi = (1 + \sin \phi') / (1 - \sin \phi')$.

More recently, Tavakkoli and Vanapalli (2011) extended the mechanics of unsaturated soils to estimate the earth pressures on retaining structures by using Equation (2.17) proposed by Vanapalli et al. (1996) and highlighted the conventional approach is conservative in estimating earth pressures for backfills that are in a state of unsaturated condition:

The effective active earth stress at depth, z :

$$\sigma_h' = \frac{\sigma_v'}{N_\phi} - \frac{2}{\sqrt{N_\phi}} \times [c' + (u_a - u_w) S^k \tan \phi'] \quad (2.29)$$

The effective passive earth stress at depth, z :

$$\sigma_h' = \sigma_v' N_\phi - 2\sqrt{N_\phi} \times [c' + (u_a - u_w) S^k \tan \phi'] \quad (2.30)$$

where, σ_h' is the effective horizontal stress; σ_v' is the effective vertical stress.

Considering the linear decrease for matric suction from the value at ground surface to zero at the groundwater table, Zhang et al. (2010) derived a unified solution of active and passive earth pressures for unsaturated soils in terms of two state stress variables:

The active earth force:

$$E_a = \frac{1}{2} \gamma K_{at} (H^2 - h_0^2) - 2c_t' \sqrt{K_{at}} (H - h_0) - 2(u_a - u_w)_0 \sqrt{K_{at}} \tan \phi_t^b \left(H - \frac{H^2}{2D} - h_0 + \frac{h_0^2}{2D} \right) \quad (2.31)$$

where, K_{at} is the unified coefficient of active earth pressure, $K_{at} = \tan^2(45^\circ - \frac{\phi_t'}{2})$, h_0 is the depth of elastic area, $h_0 = 2(c_t' + (u_a - u_w) \tan \phi_t^b) / (\gamma \sqrt{K_{at}})$, c_t' is the unified effective cohesion, $c_t' = \frac{2(1+b)c' \cos \phi}{2+b(1+\sin \phi')}$, b is the coefficient of unified strength theory, ϕ_t' is the unified angle of internal friction associated with the net normal stress state variable, $(\sigma - u_a)$, $\sin \phi_t' = \frac{b(1-m) + (2+b+bm) \sin \phi'}{2+b(1+\sin \phi')}$, m is the coefficient of intermediate principal stress, ϕ_t^b is the unified angle of shearing resistance with respect to the matric suction, $(u_a - u_w)$, $\sin \phi_t^b = \frac{2(1+b) \sin \phi^b}{2+b(1+\sin \phi^b)}$.

The passive earth force:

$$E_p = \frac{1}{2} \gamma K_{pt} H^2 + 2c_t' H \sqrt{K_{pt}} + 2(u_a - u_w)_0 \sqrt{K_{pt}} \tan \phi_t^b \left(H - \frac{H^2}{2D} \right) \quad (2.32)$$

where, K_{pt} is the unified coefficient of passive earth pressure, $K_{pt} = \tan^2(45^\circ + \frac{\phi_t'}{2})$.

However, Zhu et al. (2001) and Hu (2006) stated that the method based on Rankine's earth pressure theory is not practical because of the assumption that the wall of the retaining structures has to be vertical, its surface smooth, and the filling surface of the wall back horizontal. There are deviations between the mathematical results from Rankine's theory and the test results from both lab and field studies. The evidence showed that the values of active earth pressure calculated by Rankine's theory are conservative while the results underestimate the practical situation in passive cases (Xie

et al. 2003, Chen et al. 2005, Zhang et al. 2010). Zhao et al. (2013) considered the unified solution in Equation (2.30) and (2.31) as a special case of Coulomb's earth pressure for unsaturated soils, which has a broader application to the determination of soil pressure.

2.7.3.2 Active earth pressure of expansive unsaturated soils

Vahedifard et al. (2015) addressed that changes in the degree of saturation in the backfill could significantly affect active earth pressures. Although the earth pressure theory of unsaturated soils has been established and developed over 30 years, limited studies focus on the lateral earth pressures of unsaturated expansive soils taking into account of the influence from cracks and swelling pressures.

The active earth pressures associated unsaturated expansive soils should receive more attention as retaining walls typically move away from the soil mass because of the influence of lateral swelling pressure. In addition, fissures and cracks would occur in the surface of expansive soils during the drying process and it is likely that dust or other materials partly fill up the fissures contributing to active earth pressures (Pufahl et al. 1983).

Also, the strains required to achieve the passive state are much larger than those for the active state (see Figure 2.29). For sands, a decrease in lateral earth pressure of 40% of the at-rest lateral earth pressure can be sufficient to reach an active state, but an increase of several hundred percent in lateral earth pressure over the at-rest lateral earth pressure is required to achieve a passive state (Punmia and Jain 2005).

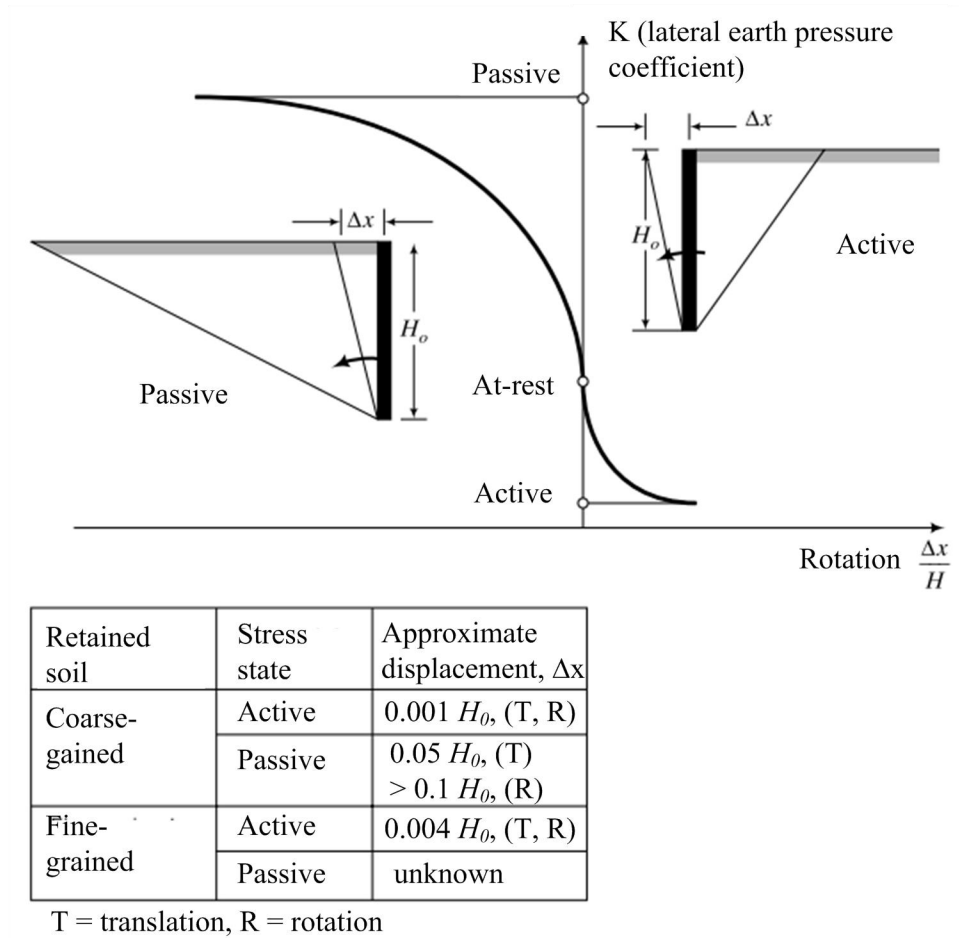


Figure 2.29 Rotation required to mobilize active and passive resistance (after Punmia and Jain 2005).

In order to understand the difference between the lateral earth pressure of expansive soils and conventional soils, Mohamed et al. (2014) used a pressure cell to measure the lateral expansive earth pressure on the retaining structure, and compared the results with the active earth pressure without swelling pressure. This study suggests that the active earth pressure on retaining walls with swelling pressure equals approximately seven times the active earth pressure of soil without swelling pressure (see Figure 2.30).

Also, according to the test results from Sudhindra and Moza (1987), the value of active earth force in expansive soils increases significantly after saturation, however, for sand

and for non-expansive clays, the variation of active earth force is relatively small (Zhu and Liu 2001).

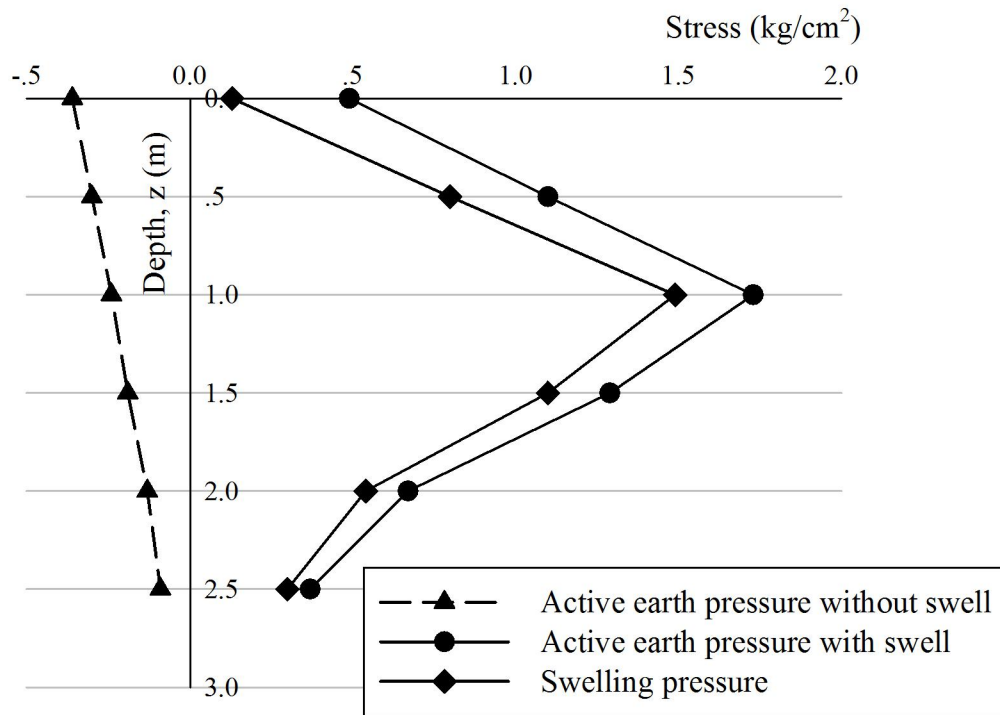


Figure 2.30 The relationship between active earth pressures with/without swelling pressure and depth at water content = 22% and density = 1.43 t/m³ after four days (from Mohamed et al. 2014).

Lu et al. (1997) introduced the reduction coefficient, m_s , to calculate the shear strength of expansive soil (Equation 2.21). Zhang (2012) adopted this equation for active earth pressure calculation for unsaturated expansive soils based on Coulomb's earth pressure theory taking into account of the swelling pressure considering static equilibrium conditions of sliding soil. The reacting force for such a scenario can be expressed as follows:

$$E_a = \frac{W \sin(\theta - \phi') - C_w \sin(\theta - \alpha - \phi') - C_d \cos \phi'}{\cos(\alpha + \delta - \theta + \phi')} \quad (2.33)$$

where, W is the gravity of soil mass, α is the angle of back wall and vertical plane, δ is the friction angle of filling and back wall, θ is the angle of sliding plane and horizontal plane, C_w is the total adhesive strength, $C_w = c' h / \cos \alpha$, C_d is the total cohesive strength, $C_d = (c' + m_s P_s \tan \phi') \left(\frac{h}{\cos \alpha} \frac{\cos(\beta - \alpha)}{\sin(\theta - \beta)} - z_0 \frac{\cos \beta}{\sin(\theta - \beta)} \right)$.

Besides, Zhu and Liu (2001) and Hu (2006) both presented their own approaches to calculate the active earth force of unsaturated expansive soils based on Coulomb's earth pressure theory. As shown in Figure 2.31, by the three methods proposed by Zhu and Liu (2001), Hu (2006) and Zhang (2012), the value of active earth force all share the same increasing tendency with respect to the increased water content.

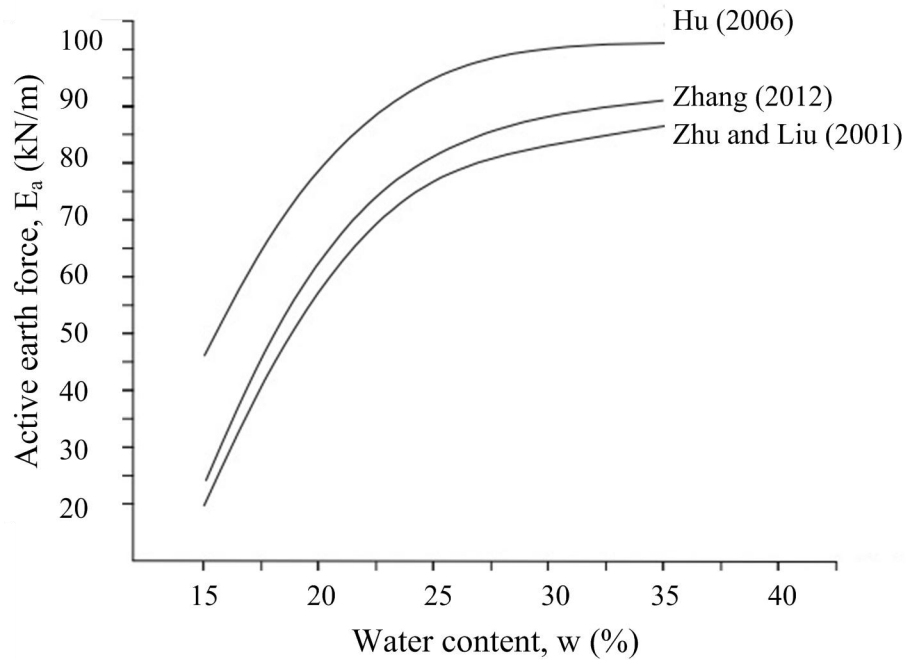


Figure 2.31 Relation between active earth force and water content (modified after Zhang 2012).

2.8 Summary

In an attempt to have a comprehensive understanding of the behavior of retaining walls with expansive soils as backfill, background information of various related topics are presented in this chapter.

Expansive soils preform unique swell-shrink characteristics when they experience water content variations during the drying and wetting circles, which have significant influence to the earth pressure distribution along the retaining walls. Due to this reason, the variation of matric suction profiles with respect to the changes of water content in unsaturated soils is highlighted in this chapter. In addition, the various determination methods of swelling and the propagation of cracks are discussed.

Besides the shrink-swell characteristics of expansive soils, both the tensile and shear strength behavior of unsaturated soils are essential in the earth pressure estimation. Therefore, the details of the SWCC prediction of the strength behavior of unsaturated soils are discussed. In addition, the various formulations available for lateral earth pressure estimation of unsaturated soils from the literature are reviewed as well.

CHAPTER 3

PREDICTION OF THE DEPTH OF CRACKS AND LATERAL SWELLING PRESSURE

3.1 Introduction

Extensive cracks that develop in the shallow layer due to drying have a significant influence on the engineering properties of both expansive and non-expansive soils. For example, cracks affect compressibility, time rate of consolidation, shear strength, and the rate at which water can flow in a fine-grained soil. The performance of most geotechnical constructions are affected directly or indirectly due to the influence of cracks in a soil mass (Morris et al. 1992). Various studies in the literature suggest that the depth of cracks is typically around 2m to 3m for a majority of fine-grained soils (Pufahl et al. 1983, Morris et al. 1992). However, for expansive unsaturated soils, the depth of cracks vary greatly due to complex environmental conditions and influence several other soil properties, such as the coefficient of permeability, the swelling pressure and the modulus of elasticity.

The lateral swelling pressure that arises due to the restriction of soil expansion is a key parameter required in the design of the retaining structures. When the expansion of expansive soil is restricted, it can cause swelling pressure and has a significant effect on the distribution of earth pressure behind the retaining structures (Zhang 2012). Nevertheless, there are limited studies that focused on the relationship between lateral and vertical swelling pressures. From the limited laboratory tests and field studies,

geotechnical engineers realized that there is distinct difference between lateral and vertical swelling pressures (Zhang 1993, Sapaz 2004, Xie et al. 2007, Avsar et al. 2009, Yang et al. 2014).

Prediction of both the crack depth and swelling pressure are required for providing better design and construction techniques for retaining structures with expansive soils as backfill material. In this chapter, an equation for estimating the depth of cracks in expansive soils has been derived. The soil properties, which include the tensile strength, the coefficient permeability, the soil-water characteristic curve (SWCC), and environmental factors which include infiltration and evaporation rate are taken into consideration for estimating the depth cracks. In addition, the relationship between lateral and vertical swelling pressures has been established based on the constitutive relations of unsaturated soils.

3.2 Background

The constitutive relations are useful for providing rational explanation of both the shrinkage and swelling behavior of expansive soils. Extending generalized Hooke's law, the constitutive relationships for a saturated soil can be described in terms of single stress state variable for determining the stress-strain behavior as below.

$$\begin{cases} \varepsilon_x = \frac{(\sigma_x - u_w)}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z - 2u_w) \\ \varepsilon_y = \frac{(\sigma_y - u_w)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z - 2u_w) \\ \varepsilon_z = \frac{(\sigma_z - u_w)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y - 2u_w) \end{cases} \quad (3.1)$$

where, σ_x is the total normal stress in the x-direction, σ_y is the total normal stress in the y-direction, σ_z is the total normal stress in the z-direction, ε_x is the total strain in the

x-direction, ε_y is the total strain in the y-direction, ε_z is the total strain in the z-direction. u_w is the pore-water pressure in soils, E is the saturated elastic modulus. μ is the Poisson's ratio.

Assuming the soil is a linear elastic and isotropic material, the constitutive relations for saturated soils (Equation 3.1) could be extended and applied to unsaturated soils in terms of two independent stress state variables; namely, net normal stress, $(\sigma - u_a)$ and matric suction, $(u_a - u_w)$ as suggested by Fredlund and Morgenstern (1976).

$$\begin{cases} \varepsilon_x = \frac{(\sigma_x - u_a)}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H} \\ \varepsilon_y = \frac{(\sigma_y - u_a)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H} \\ \varepsilon_z = \frac{(\sigma_z - u_a)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y - 2u_a) + \frac{(u_a - u_w)}{H} \end{cases} \quad (3.2)$$

where, u_a is the pore-air pressure and H is the elastic modulus with respect to a change in $(u_a - u_w)$.

The above equations can be used to interpret the shrinkage and swelling behavior of expansive soils. Besides these constitutive relations, the relationship between the matric suction and the steady-state flow profile in unsaturated soils (Equation 2.4) can be used to take into account the effects of infiltration and evaporation. However, for achieving this objective, the 1-D matric suction profile is required. This information can be derived from the steady state flow rate, permeability of unsaturated soils and the fitting parameters of the SWCC as follows (Yeh 1989, Lu and Likos 2004):

$$(u_a - u_w) = -\frac{1}{a} \ln \left[\left(1 + \frac{q_u}{k_s}\right) e^{-\gamma_w a z_w} - \frac{q_u}{k_s} \right] \quad (3.3)$$

where, k_s is the saturated coefficient of permeability, a is one of the parameters for fitting the SWCC relationship (i.e. van Genuchten's model in this thesis), q_u is the unsaturated flow rate of water (negative for infiltration and positive for evaporation), e is the void ratio, z_w is the elevation, which is the distance above ground water table as shown in Figure 2.6, γ_w is the density of water.

The runoff occurs over the ground surface if the rate of water application exceeds the soil infiltrability, which is the saturated coefficient of permeability (Oh and Vanapalli 2010). To avoid such a scenario, the dimensionless flow ratio is considered in the range of $-1 < q_u/k_s$ (Lu and Griffiths 2004).

3.3 Prediction of the depth of cracks

In this section, a mathematical approach is derived to predict the depth of cracks in unsaturated expansive soils as below:

The lateral strain is zero ($\epsilon_x = 0$) in Equation (3.2) when the soil mass is at rest or at K_0 condition before cracking. For this reason, the lateral stress can be expressed in terms of vertical stress and matric suction (Morris et al. 1992) as below:

$$\sigma_x - u_a = \frac{\mu}{1 - \mu} (\sigma_z - u_a) - \frac{E}{H} (1 - \mu) (u_a - u_w) \quad (3.4)$$

The pore-air pressure u_a can be assumed to be equal to zero relative to the atmosphere and so its effect will not be important (Pufahl et al. 1992). Assuming $E/H = (1 - 2\mu)$ and $(\sigma_z - u_a) = \gamma(D - z_w)$, the above equation can be re-written as:

$$\sigma_x = \frac{\mu}{1-\mu} [\gamma(D - z_w)] + (1-2\mu)(1-\mu)(u_a - u_w) \quad (3.5)$$

Recall Equation (2.25) for estimating the tensile strength of soils in Chapter 2:

$$t = -\alpha_T [c' + (u_a - u_w) \tan \phi^b] \cot \phi' \quad (3.6)$$

where, α_T is the modified coefficient for tensile stress in unsaturated soils.

The cracks in expansive soils typically arise at the point where the tensile strength of unsaturated expansive soils is equal to the lateral stress. This assumption leads to the expression below:

$$\sigma_x = t \quad (3.7)$$

Substituting Equation (3.5) and Equation (3.6) into Equation (3.7), an equation can be derived for estimating the depth of cracks in expansive unsaturated soils terms of other soil properties (i.e. Poisson's ratio and effective internal angle) and matric suction.

$$\frac{\mu}{1-\mu} [\gamma(D - z_w)] + (1-2\mu)(1-\mu)(u_a - u_w) = -\alpha_T [c' + (u_a - u_w) \tan \phi^b] \cot \phi' \quad (3.8)$$

In addition, by replacing the $\tan \phi^b$ with $S^\kappa \tan \phi'$ (Vanapalli et al. 1996), the relation between the depth of cracks and water flow rate could be built as below:

$$\frac{\mu}{1-\mu} [\gamma(D - z_w)] + (1-2\mu)(1-\mu)(u_a - u_w) = -\alpha_T [c' + (u_a - u_w) S^\kappa \tan \phi'] \cot \phi' \quad (3.9)$$

By taking account of the environmental factors, such as the evaporation and infiltration flow rates, the matric suction term, $(u_a - u_w)$, could be expressed in terms of the water flow rate q (Equation 3.3).

Instead of measuring the matric suction, this relationship can be used predict the depth of cracks in expansive soil taking into account of the influence of environmental data.

3.4 Prediction of the lateral swelling pressure

3.4.1 Determination of the elastic modulus of unsaturated expansive soil

Modulus of elasticity is a fundamental parameter required for estimating the swelling and shrinkage behaviour of the expansive soils associated with variations of environmental factors such as the drying and wetting (Adem and Vanapalli 2014). Conventionally, the modulus of elasticity is assumed to be a constant value without taking account of the influence of matric suction into consideration. However, modulus of elasticity significantly varies with respect to matric suction in unsaturated soils (see Figure 3.1) (Oh et al. 2009).

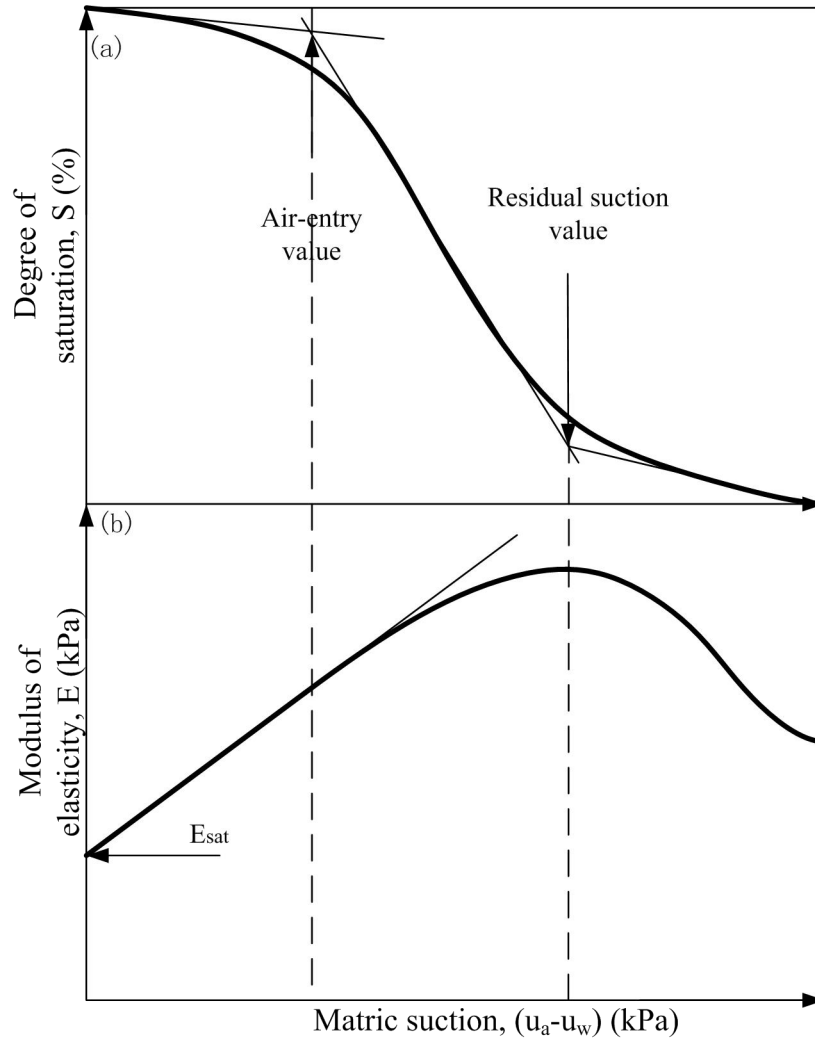


Figure 3.1 The relationship between (a) SWCC and (b) the variation of modulus of elasticity with respect to matric suction (after Oh et al. 2009).

Oh et al. (2009) proposed a semi-empirical model to predict the variation of modulus of elasticity of unsaturated sandy soils with respect to matric suction using the SWCC.

$$E_{unsat} = E_{sat} \left[1 + \alpha_E \frac{(u_s - u_w)}{(P_a / 101.3)} S^{\beta_E} \right] \quad (3.10)$$

where, E_{unsat} is modulus of elasticity under unsaturated conditions, α_E and β_E are fitting parameters, and P_a is atmospheric pressure (i.e. 101.3 kPa).

Vanapalli and Oh (2010) extended Equation (3.10) to a more general model to predict the modulus of elasticity for both coarse- and fine-grained soils. The upper and lower boundary of the fitting parameter, α_E , are developed using the plastic index, I_p , as follow. The upper and lower boundary relationship can be used for low and high matric suction values respectively.

$$1/\alpha_E = 0.5 + 0.312 (I_p) + 0.109 (I_p)^2, 0 \leq I_p(\%) \leq 12 \quad (3.11)$$

$$1/\alpha_E = 0.5 + 0.063 (I_p) + 0.036 (I_p)^2, 0 \leq I_p(\%) \leq 16 \quad (3.12)$$

The fitting parameter, β_E is recommended for coarse- and fine-grained soils to be 1 and 2, respectively.

Adem and Vanapalli (2014) extended Equation (3.10) for unsaturated expansive soils (i.e. $I_p > 16\%$) and achieved a reasonable agreement using the fitting parameters $\alpha_E = 0.05 \sim 0.15$ and $\beta_E = 2$. Therefore, the influence of infiltration or evaporation towards the elastic modulus of unsaturated soils taken into account by substituting Equation (3.3) into Equation (3.10). The modulus of elasticity is a key parameter for predicting the lateral swelling pressure.

3.4.2 Proposed method for the relationship between the vertical and lateral swelling pressure

Beside the cracks, the lateral swelling pressure of expansive soils also has a significant influence on the performance of geotechnical infrastructure. Several empirical equations for the vertical swelling pressure are summarized in Chapter 2. However, it is the lateral swelling pressure that is the key parameter required in the earth pressure estimation for expansive unsaturated soils. The relationship which was derived by Liu and Vanapalli (2016) can be used for estimating magnitude of lateral swelling pressure.

For the at-rest or K_0 earth pressure condition, the stress state of soil elements behind the retaining wall are shown in Figure 3.2. As the soil is assumed linear elastic, homogeneous and isotropic, the volume of soil will increase in all directions upon infiltration. As shown in Figure 3.2, the soil expansion in horizontal direction is assumed to be strictly restricted such that a more conservative design can be used for expansive soils. As such, the deformation tendency of soil mass in horizontal direction transforms to be the lateral swelling pressure acting on the retaining structures. In vertical direction, the soil elements at surface layer are allowed to swell freely with zero vertical stress from surcharge load or the gravity stress from upper soil layer. The soil elements at deeper layer are subjected to additional gravity stress, σ_s , which arises mainly from the upper soil layer (Liu and Vanapalli 2016).

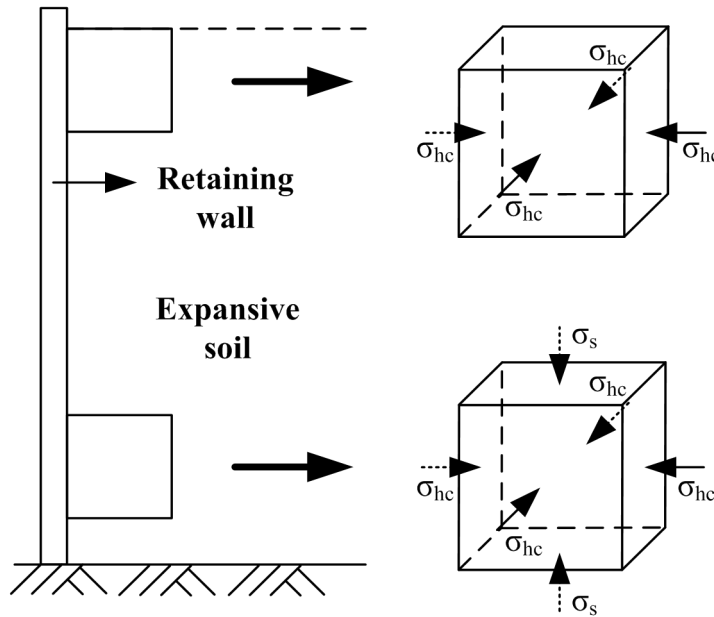


Figure 3.2 Analytical element of expansive soil behind a frictionless retaining wall (from Liu and Vanapalli 2016).

Liu and Vanapalli (2016) simulated the generation of swelling pressure in two stages. Firstly, stage (a) allows the soil elements fully swell vertically due to a reduction in

matric suction while the horizontal deformation is restricted by horizontal confining stress σ_3 . As a result, the length at horizontal direction remains c while in vertical direction it changes from c to b (see Figure 3.3 a). At stage (b), horizontal stress σ_3' and vertical stress σ_1' are applied to the element simultaneously to compress the soil element back to its initial size.

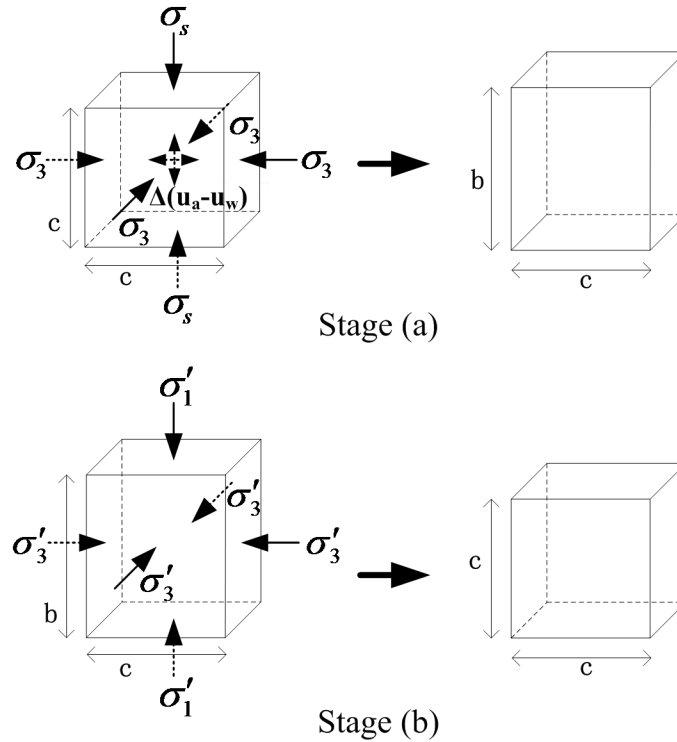


Figure 3.3 Analytical element of expansive soil at deep soil layer (after Liu and Vanapalli 2016).

The constitutive relationships for unsaturated soils (Equation 3.2) proposed by Fredlund and Morgenstern (1976), can be employed to estimate the stress-strain relationship (Equation 3.13 and 3.14) for stage (a) and (b) respectively as suggested by Liu and Vanapalli (2016):

$$\begin{cases} \frac{b-c}{c} = \frac{\sigma_s}{E} - \frac{2\mu}{E} P_L + \frac{u_a - u_w}{H} \\ 0 = \frac{1-\mu}{E} P_L + \frac{u_a - u_w}{H} \end{cases} \quad (3.13)$$

$$\begin{cases} \frac{c-b}{b} = \frac{P_s}{E} - \frac{2\mu}{E} \sigma_3' \\ 0 = \frac{\sigma_3'}{E} - \frac{\mu}{E} (P_s + \sigma_3') \end{cases} \quad (3.14)$$

For lateral earth pressure estimation, the σ_s which represents the influence of the gravity stress of upper soil layer and σ_1' stands for the vertical swelling pressure, P_s , which could be determined by laboratory tests or semi-empirical and empirical equations detailed in Chapter 2. Solving these equations, the relationship between the lateral and vertical swelling pressure, P_s , can be developed as shown in Equation (3.15).

$$P_L = \frac{(1-\mu-2\mu^2)P_s}{\frac{P_s}{E}(1+\mu)(1-\mu-2\mu^2)+1-\mu^2} + \frac{\mu}{1-\mu} \sigma_s \quad (3.15)$$

Tu and Vanapalli (2016) proposed a semi-empirical equation for expansive soils to estimate the vertical swelling pressure:

$$P_s = P_{s0} + \beta_s \cdot \psi \cdot \left(\frac{S_r}{100}\right)^2 \quad (3.16)$$

where, P_{s0} is the intercept on the P_s axis at zero suction value, β_s is fitting parameter, $\beta_s = 23.05A^{32.315} (0.237I_P - 10.278\rho_{dn}) + 0.164$, A is the activity of soils, $A = I_P / CC$.

Therefore, the lateral swelling pressure can be computed when the matric suction changes from initial state to zero:

$$P_{LS} = \frac{(1 - \mu - 2\mu^2)[P_{s0} + \beta_s \cdot \psi \cdot (\frac{S_r}{100})]}{\frac{P_s}{E}(1 + \mu)(1 - \mu - 2\mu^2) + 1 - \mu^2} + \frac{\mu}{1 - \mu} \sigma_s \quad (3.17)$$

For surface soil layers without any surcharge, the σ_s equals to zero. The equation above provides a general relationship between vertical and lateral swelling pressure using only two soil properties, the elastic modulus, E , and the Poisson ratio, μ .

3.5 Example problem

In the earlier sections, the depth of cracks is estimated using the information of tensile strength and the lateral swelling pressure of the expansive soils. In this section, an example problem is illustrated of how this is achieved. All the calculations are accomplished and figures are plotted using the program LEENES that was developed with the MATLAB. The program code of LEENES is summarized in Appendix A.1.

The details of example problem of the retaining wall along with the properties of backfill soils are shown in Figure 3.4.

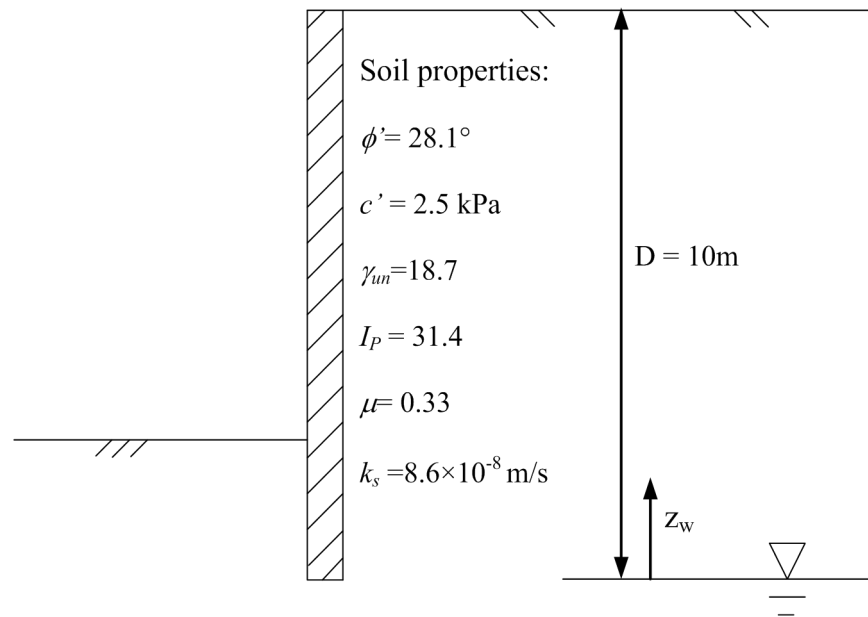


Figure 3.4 The geometry of the example problem.

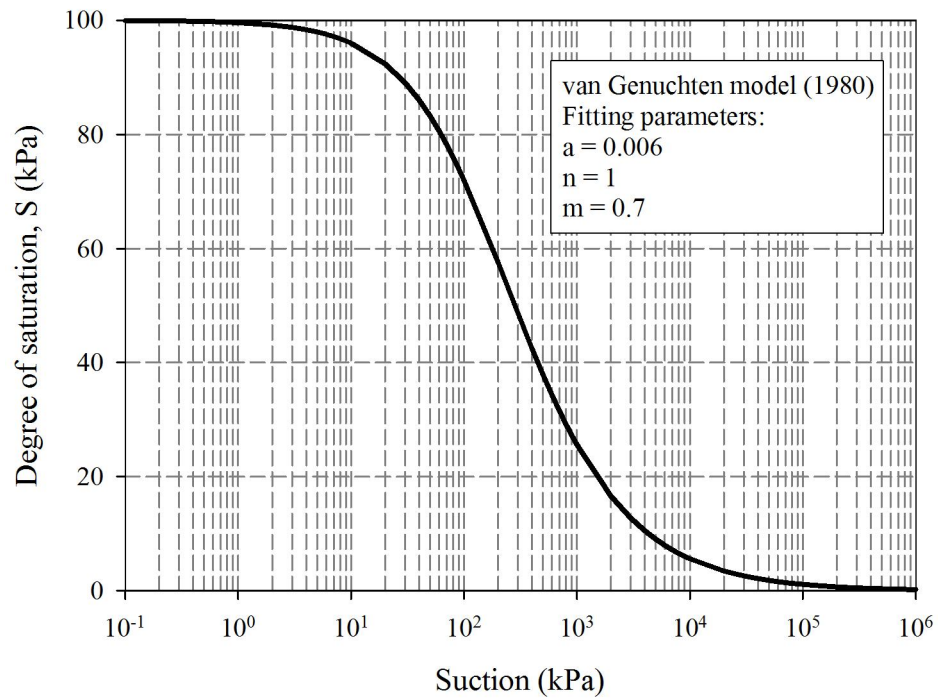


Figure 3.5 The SWCC of the example problem.

Several assumptions used for reducing the complexity of the problem is summarized below:

- The height of backfill is assumed to be 10m and the ground water table is assumed to be at the depth of 10m below backfill surface
- The SWCC of this example problem is shown in Figure 3.5 by applying van Genuchten (1980) equation with the fitting parameters a , n , m of 0.006, 1 and 0.7, respectively.
- The saturated coefficient of permeability, k_s is assumed to be 8.6×10^{-8} m/s.
- The surface of the wall is assumed vertical and smooth (i.e. there no friction between backfill soil and the wall; In other words, Rankine theory is used).

Different water flow rates, q were applied to simulate the effects of environmental factors. The matric suction profiles associated with five different water flow rates (i.e. -3.5×10^{-8} m/s, -1.5×10^{-8} m/s, 0 m/s, 1.5×10^{-8} m/s, 3.5×10^{-8} m/s) were plotted based on Equation (3.3). The positive and negative values of the flow rates represent the evaporation and infiltration conditions, respectively. The matric suction decreases linearly from the ground surface when neglecting the influence of environment (i.e. $q = 0$ m/s). However, Figure 3.6 shows significant changes of matric suction in the active zone. These changes influence both the cracks propagation and the magnitude of lateral swelling pressures of unsaturated expansive soils. For this reason, it is not appropriate to assume a constant value of matric suction or a linear variation of matric suction with respect to depth.

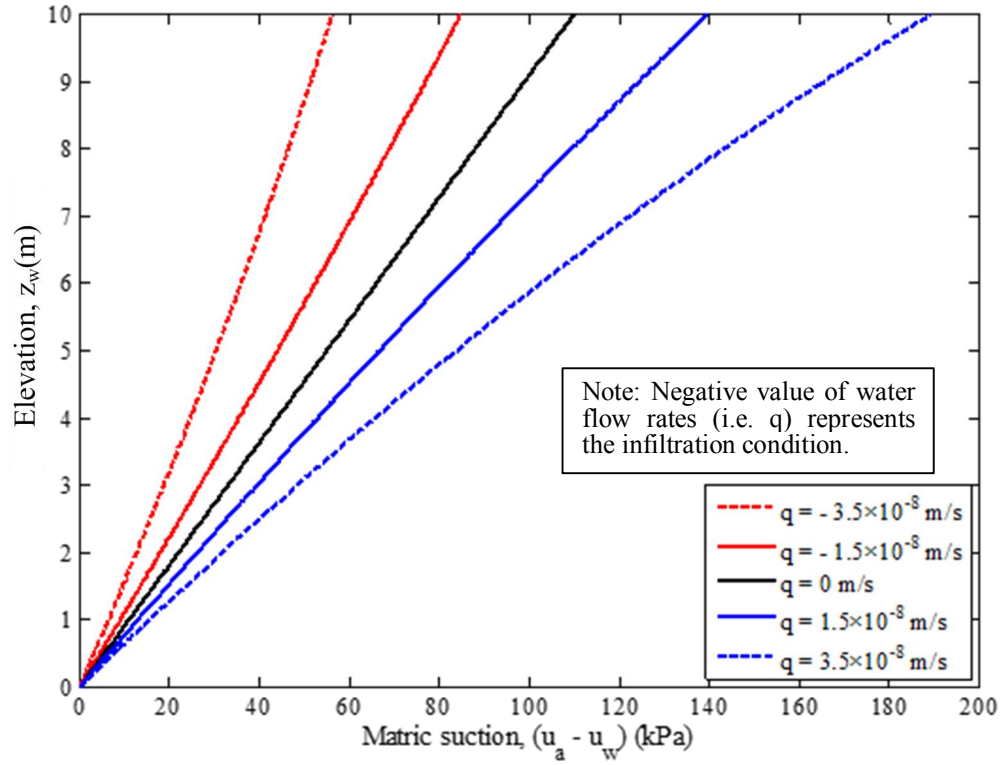


Figure 3.6 Matric suction profile with respect to different flow rates of water.

The ultimate tensile strength of soils at the point where cracks occur for various flow rates is shown in Figure 3.7. The ultimate strength (i.e. absolute value of tensile strength) increases significantly when the values of flow rate of water changes from negative to positive. This curve reveals the significant contribution towards the tensile strength of soils in the drought seasons or periods.

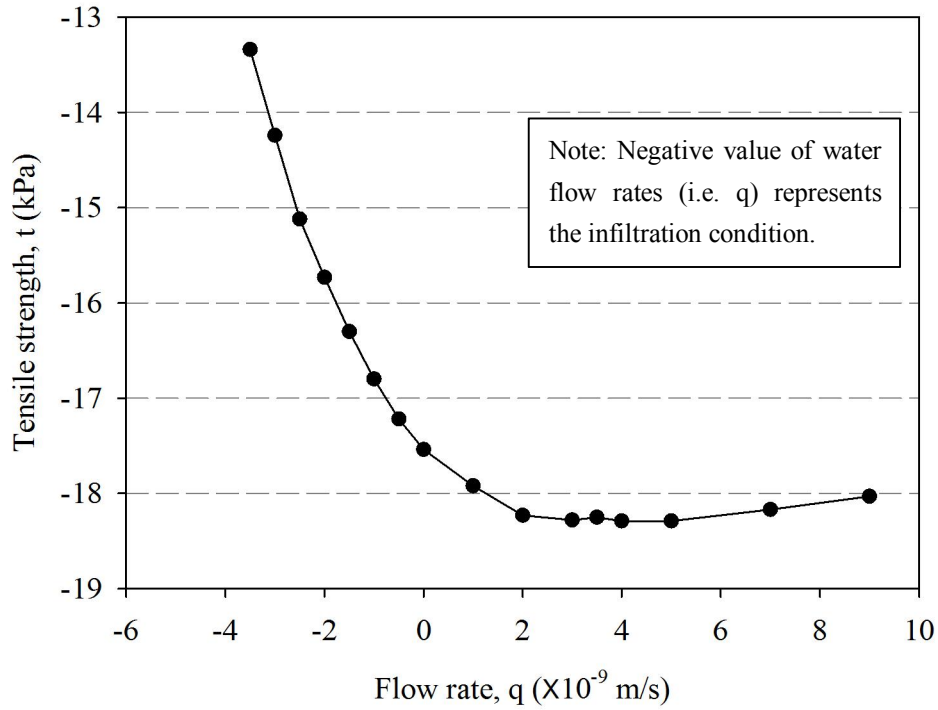


Figure 3.7 The ultimate tensile strength with respect to flow rate of water.

By employing Equation (3.10), the modulus of elasticity for unsaturated conditions corresponding to the five water different flow rates are plotted in Figure 3.8. The largest elastic modulus value for unsaturated condition can reach almost eight times of the saturated one in this example problem. Figure 3.8 highlight the importance of taking account of the influence of suction on the modulus of elasticity in the calculations.

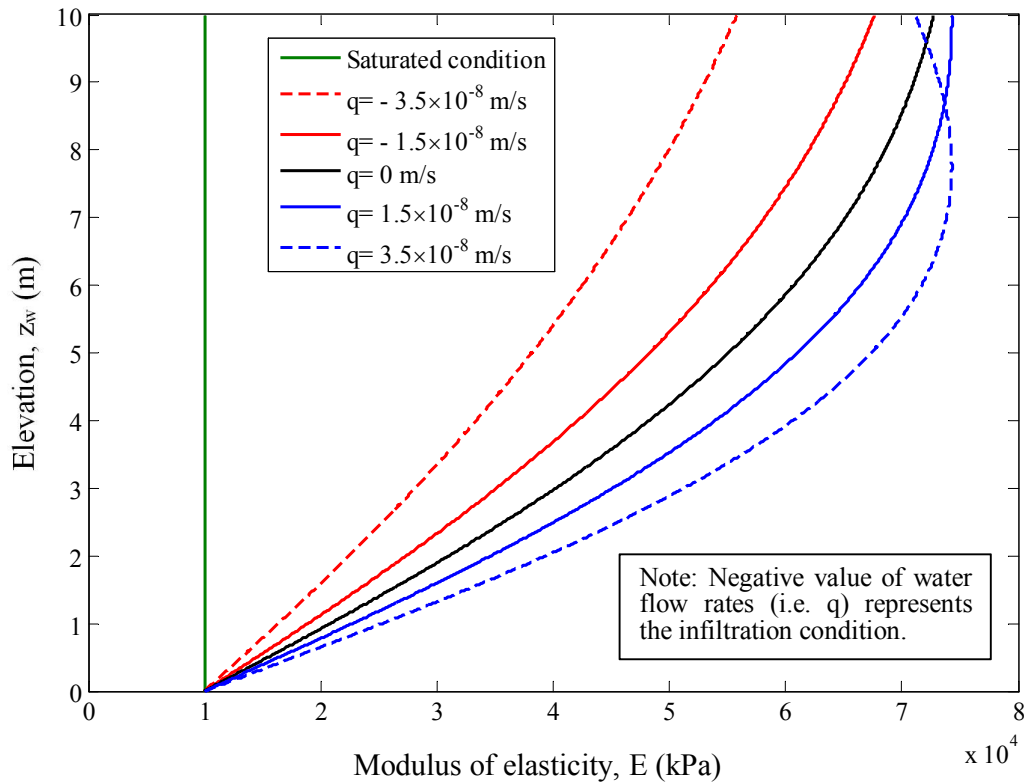


Figure 3.8 The modulus of elasticity for both saturated and unsaturated conditions.

3.6 Summary

Expansive soils undergo significant volume change and during this process cracks propagate. These cracks in turn have significant influence on the lateral swelling pressure on the retaining walls, when expansive soils are used as a backfill material. In this chapter, relationships are developed for predicting the depth of cracks and lateral swelling pressures. These two factors are essential for calculation of the earth pressures behind retaining structures in expansive soils. More details of these calculations are provided in Chapter 4.

The procedural steps followed for predicting the depth of cracks is shown in the flow diagram (Figure 3.9). Infiltration and evaporation are the key factors that affect the water

content. The suction variation with respect to depth is extremely sensitive to water content variation in unsaturated soils. By employing constitutive relations and the SWCC as tools, the depth of cracks is estimated using Equation (3.9).

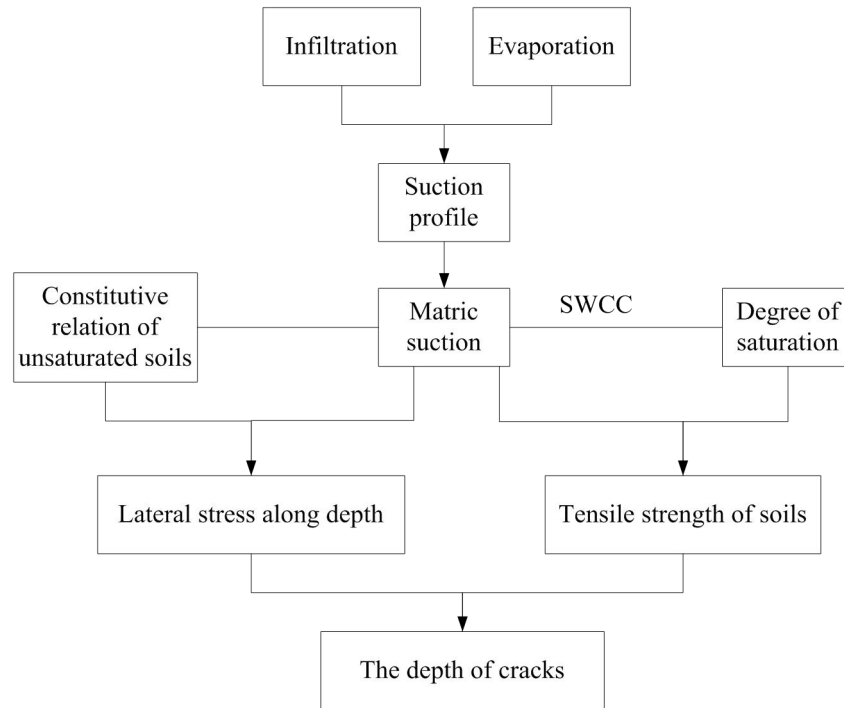


Figure 3.9 The flow diagram for predicting the depth of cracks.

The lateral swelling pressure that develop due to various infiltration rates are also important for the design of retaining wall. Based on the constitutive relationships of unsaturated soils, Equation (3.17) was built to estimate the lateral earth pressure from the information of vertical swelling pressure. The variation of vertical swelling pressure with respect soil suction can be estimated from the SWCC and other soil properties which can be estimated using Equation (3.16).

An example problem is also illustrated to highlight how tensile strength and the modulus of elasticity of unsaturated soils influence the lateral swell pressure (see Figure 3.7 and Figure 3.8).

CHAPTER 4

PROPOSED APPROACH FOR PREDICTING LATERAL EARTH PRESSURE

4.1 Introduction

The volumetric changes associated with variation of water content has a significant influence on the performance of the retaining walls when expansive soils are used as a backfill material. The restricted volumetric changes (i.e. expansive soils movement) typically translate to as lateral earth pressure and act on the retaining wall. In conventional methods, the influence of volumetric changes is not considered in the calculation of lateral earth pressures. For this reason, a comprehensive calculation method is required for reliable calculation earth pressures taking account of changes in matric suctions and lateral swell pressure values associated with changes in water content variations. A reliable design for retaining walls is possible if lateral pressure variation with respect to depth is calculated taking account of the influence of environmental factors which include wetting and drying circles and crack propagation.

In this chapter, extending the relationships that have been developed in Chapter 3, lateral earth pressure distribution is calculated behind the retaining wall taking account of effects of cracks and additional swelling pressure that may arise associated with different infiltration rates. The soil mass of the backfill material when subjected evaporation is typically in a state unsaturated condition. The evaporation rates likely contribute to the development of cracks in expansive soils; however, cracks may not occur in certain non-expansive soils. During wet seasons, the backfill material degree of saturation increases and becomes saturated as water infiltrates into soil A rigorous calculation

framework is proposed for the lateral earth pressures estimation on retaining structures for both expansive and non-expansive soils as backfill material considering both saturated and unsaturated conditions.

4.2 Background

Theoretical treatment of lateral earth pressures in cohesive soils where maximum pressures or resistances are the major unknowns were developed based on the Mohr-Coulomb failure criteria and concepts of plastic equilibrium extending classic Rankine earth pressure theory (Pufahl et al. 1992). In order to extend the saturated soil mechanics for soils that are in a state of unsaturated condition, some assumptions are used in the Rankine's analysis, which include: the surface of the backfill is horizontal; the friction between wall and soils is zero; the failure plane is planar and the soils are elastic in nature.

The parameter, K_0 which is defined as the at-rest earth pressure coefficient, is the ratio of lateral (i.e. horizontal) to vertical pressures when the retaining structures are fixed. In Figure 4.1, circle C_1 represents the fully active condition at a certain matric suction value. As the water content increases, the matric suction decreases and the active earth pressure condition moves from circle C_1 towards circle C_3 , which represents the fully saturated situation. On the other hand, there are scenarios that soils are under passive conditions in practice. For instance, expansive soils under the influence rainfall infiltration develop additional lateral swelling pressures for heavy structures such as the bridges supported with retaining structures, which have a tendency to push the structure into the softened expansive soils. Under these circumstances, circle C_2 and C_4 are in fully passive conditions for unsaturated and saturated soils, respectively (see Figure 4.1).

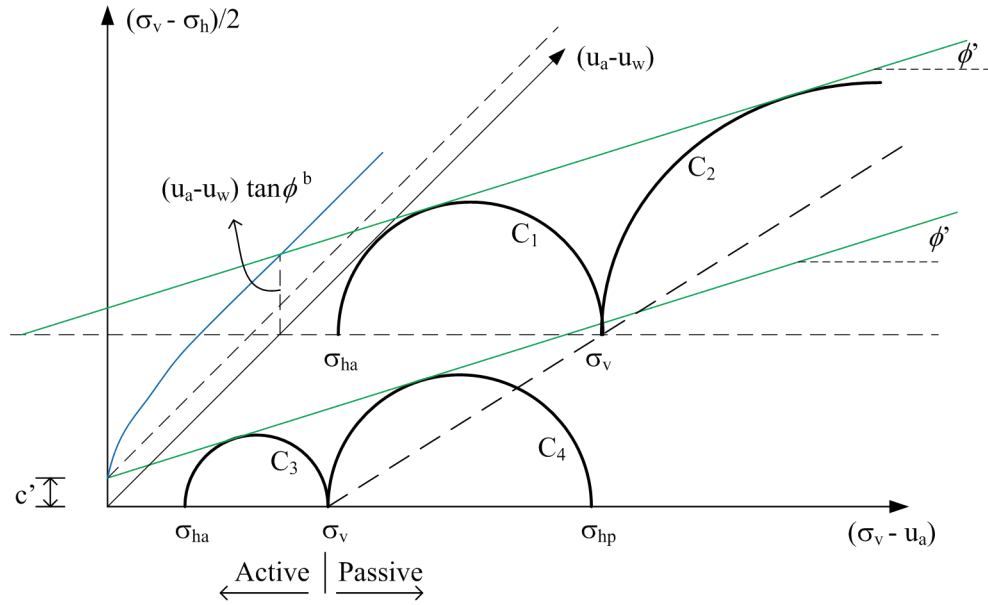


Figure 4.1 Active and passive earth pressures for saturated and unsaturated soils (modified after Pufahl et al. 1992)

Limited research studies were reported in the literature to provide a comprehensive theoretical analysis related to the behavior of retaining structures where unsaturated expansive soils are used as backfill material extending Mohr-Coulomb failure criteria (Pufahl et al. 1992, Fredlund and Rahardjo 1993, Zhu and Liu 2001, Hu 2006, Zhang 2012). Figure 4.2 shows the diagrams of active and passive earth pressure for unsaturated soils based on Rankine theory. It contains three parts: the contribution from vertical stress, cohesion of soils and matric suction.

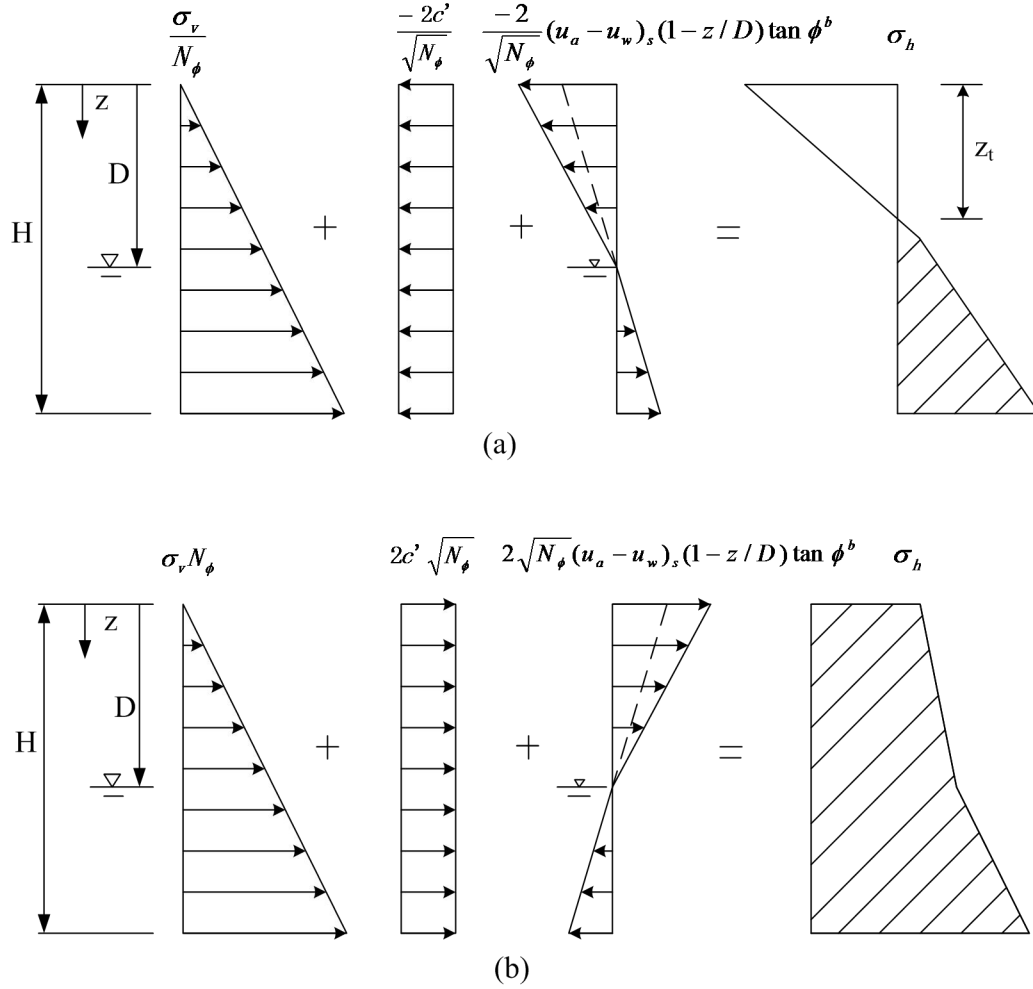


Figure 4.2 Diagrams of earth pressures for unsaturated soils: (a) Active earth pressure, (b) Passive earth pressure (after Pufahl et al. 1992)

4.3 Lateral earth pressure

4.3.1 Earth pressure during drying process

Figure 4.3 shows the horizontal stress behind a retaining wall reduces as the wall moves away from the soil mass or cracks propagate until they reach a limiting value. Such a scenario represents evaporation of water from the soil mass which translates to an increase in the matric suction. The major and minor principal stress can be reasonably well estimated, however, there are no procedures available for estimating intermediate

principal stress. For practice applications, fully active state is typically considered to be the critical scenario (i.e. failure condition) when the maximum lateral earth pressure (i.e. active earth pressure) arises as cracks develop in backfill material of the retaining wall, typically during drought seasons.

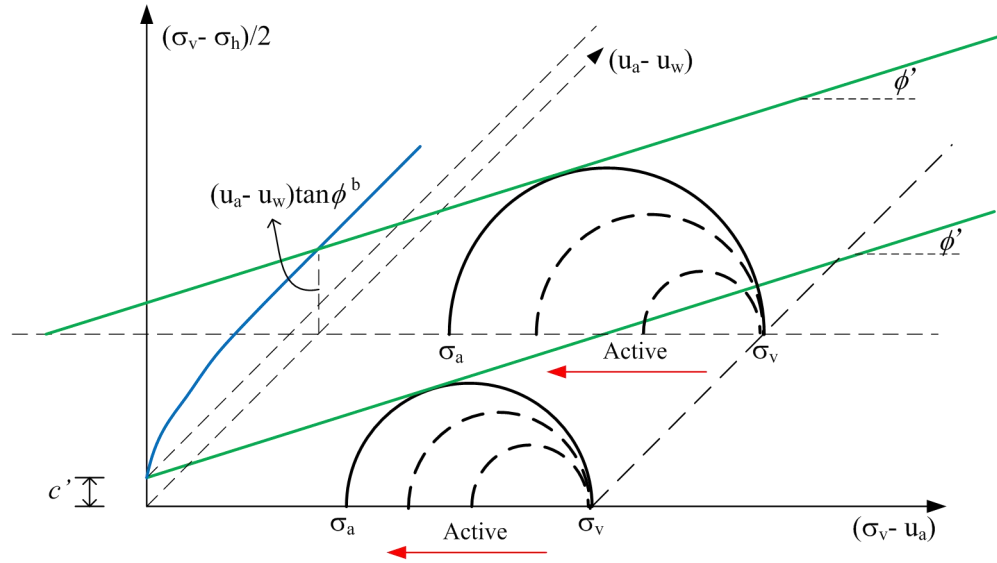


Figure 4.3 Stress states during drying.

Saturated soils

The active earth pressure is conventionally estimated assuming the backfill soil material is in a state of saturated condition. In other words, the contribution from matric suction is disregarded when ground water table is not at the natural ground surface level (Vanapalli and Oh 2012). Terzaghi (1943) proposed Equation (4.1) to estimate the active earth pressure for saturated soils as follow:

$$\sigma_{ha} = (\sigma_{vs} + \sigma_s)K_a - 2\sqrt{K_a}c' \quad (4.1)$$

where, σ_{ha} is the active earth pressure, σ_{vs} is the vertical self-weight stress, $\sigma_{vs} = \gamma z$, σ_s is the surcharge stress, K_a is the coefficient of active earth pressure, $K_a = \tan^2(45^\circ - \frac{\phi'_t}{2})$, c' is the effective cohesion.

Unsaturated soils

The soil behind the retaining wall, however, is typically in a state of unsaturated condition (Tavakkoli and Vanapalli 2011). A typical active stress state for a given matric suction (i.e. soil element at a certain depth) of unsaturated soils is shown in Figure 4.4.

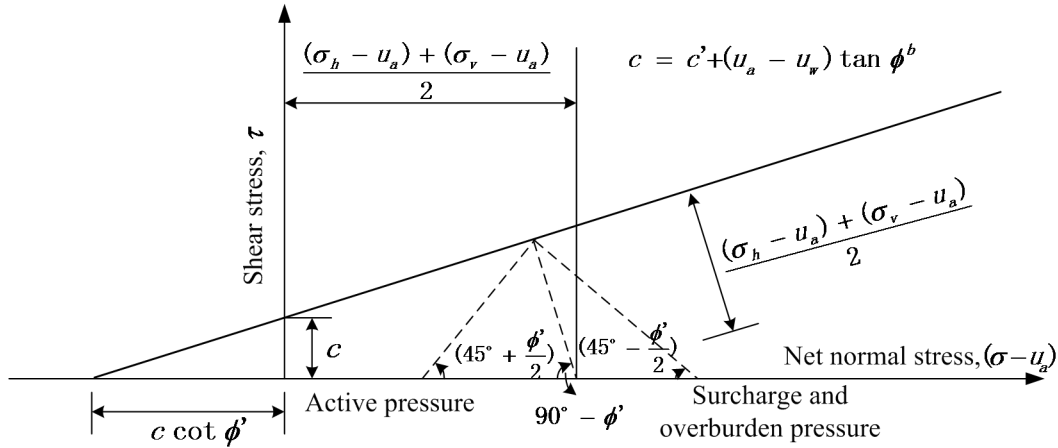


Figure 4.4 Mohr-Coulomb failure envelope at a constant $(u_a - u_w)$ for the active state (after Fredlund and Rahardjo 1993).

The matric suction profile above ground water table can be estimated for different evaporation water flow rates, which is illustrated in the example problem in Chapter 3 (see Figure 3.5). This profile is necessary for the reliable calculation of lateral pressure distribution for unsaturated soils. The active earth pressure can be calculated from vertical stress and the geometric relation (see Figure 4.4) as follow:

$$\sigma_{ha} = (\sigma_{vs} + \sigma_s) K_a - 2\sqrt{K_a} [c' + (u_a - u_w) \tan \phi^b] \quad (4.2)$$

where, ϕ^b is the angle of shearing resistance with respect to matric suction, $(u_a - u_w)$.

The active earth pressure distribution is plotted in Figure 4.5 for both saturated and unsaturated soils. For unsaturated soils, the tensile zone above ground water table, z_t , is mainly caused by the cohesion of soils and the matric suction as shown in Figure 4.5.

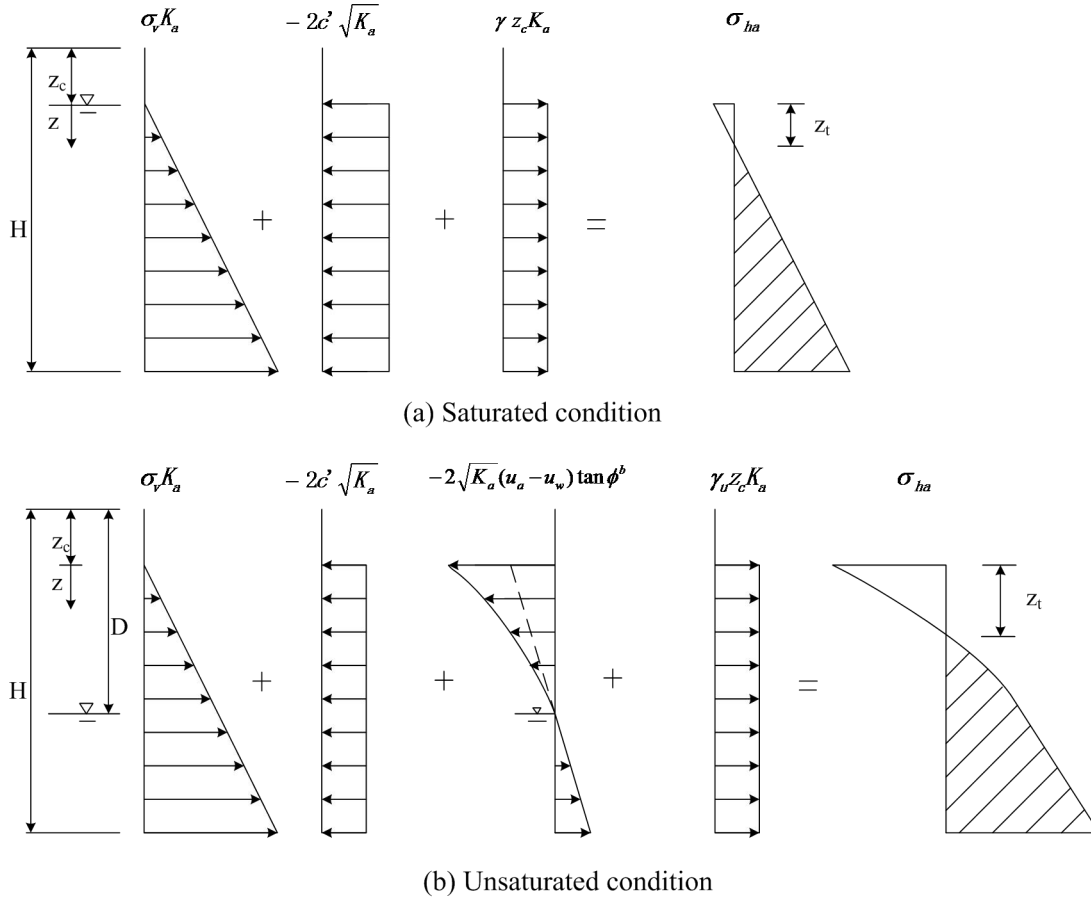


Figure 4.5 Active earth pressure distributions: (a) Saturated condition, (b) Unsaturated condition (modified from Pufahl et al. 1992).

Expansive soils

The influence of extensive cracks in unsaturated soils cannot be ignored when calculating active earth pressure for expansive soils. Pufahl et al. (1983) suggested that the mass of soil up to depth z_c (i.e. the depth of tension crack) can be considered as a surcharge load. Chapter 3, provides details of methodology for estimation of the depth of cracks taking account of the environmental conditions (Equation 3.9). The surcharge stress associated with the soil mass up to the depth of cracks, σ_c , is added to Equation (4.2) to account for vertical stress in expansive soils under drying conditions.

$$\sigma_{ha} = (\sigma_{vs} + \sigma_s + \sigma_c)K_a - 2\sqrt{K_a}[c' + (u_a - u_w)\tan\phi^b] \quad (4.3)$$

where, σ_c is the surcharge stress due to cracks, $\sigma_c = \gamma_u z_c$.

4.3.2 Earth pressure during wetting process

Unsaturated soils

At the beginning of rainy season, the soils above ground water table are typically in a state of unsaturated condition. The matric suction profile can be predicted by Equation (3.3) considering different infiltration water flow rates. A typical at-rest earth pressure distribution behind a fixed wall for unsaturated soils can be simply computed in terms of vertical stress and matric suction as below (Fredlund and Rahardjo 1993):

$$\sigma_0 = \frac{\mu}{1-\mu}\sigma_{vs} - (1-\mu)\frac{E}{H}(u_a - u_w) \quad (4.4)$$

where, σ_0 is the at-rest earth pressure.

When the retaining wall moves towards soil mass, the passive earth pressure for an unsaturated soil can be computed based on Mohr-Coulomb failure criteria (see Figure 4.6) as follow:

$$\sigma_{hp} = (\sigma_{vs} + \sigma_s)K_p + 2\sqrt{K_p}[c' + (u_a - u_w)\tan\phi^b] \quad (4.5)$$

where, σ_{hp} is the passive earth pressure, K_p is the coefficient of passive earth pressure, $K_p = \tan^2(45^\circ + \frac{\phi'_t}{2})$.

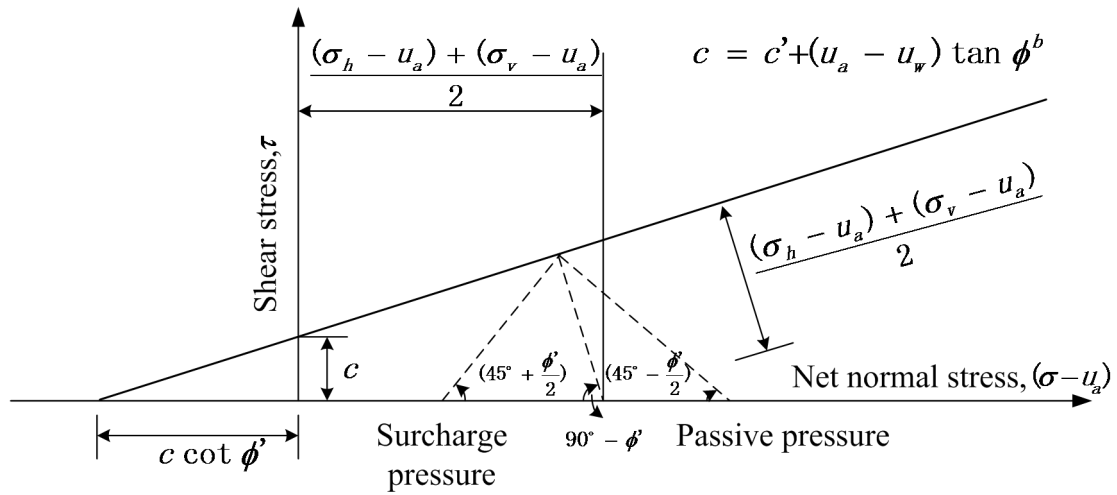


Figure 4.6 Mohr-Coulomb failure envelope at a constant $(u_a - u_w)$ for the passive state (after Fredlund and Rahardjo 1993).

Saturated soils

As a result of infiltration, matric suction reduces to zero when soil becomes fully saturated (i.e. $(u_a - u_w) = 0$). The formulas for at-rest and passive earth pressure of saturated soils can be derived from Equation (4.4) and (4.5) as below, respectively:

$$\sigma_0 = \frac{\mu}{1 - \mu} \sigma_{vs} \quad (4.6)$$

$$\sigma_{hp} = (\sigma_{vs} + \sigma_s) K_p + 2\sqrt{K_p} c' \quad (4.7)$$

Expansive soils

In wetting season, the matric suction varies significantly and the greatest variation usually arises in the upper soil layer (see Figure 4.7a). Figure 3.5 indicates the variation of matric suction profiles corresponding to different environment conditions for a hypothetical fixed wall. For extending conservative design in practice, the matric suction is assumed to change from its initial value to zero. Under such a scenario, the maximum

lateral swelling pressure is computed by Equation (3.17) and shown in Figure 4.7 (b). Figure 4.7 (c) shows the typical at-rest earth pressure distribution for saturated soils behind a retaining wall (Equation 4.4). The environmental factors have more significant influence on the water content variation in the upper layer of soils due to the high permeability of expansive soils, associated with the influence of cracks. As a result, additional lateral swelling pressure exhibits more remarkable effects on the upper soil layer. At greater depth, the influence of lateral swelling pressure can be neglected and the conventional method can be used to estimate the at-rest earth pressure (Sahin 2011). It should be noted that the soils are in a passive state due to additional lateral swelling pressure. Therefore, the passive earth pressure is still governed by the failure state of stress, especially for the upper zone (see Figure 4.7d).

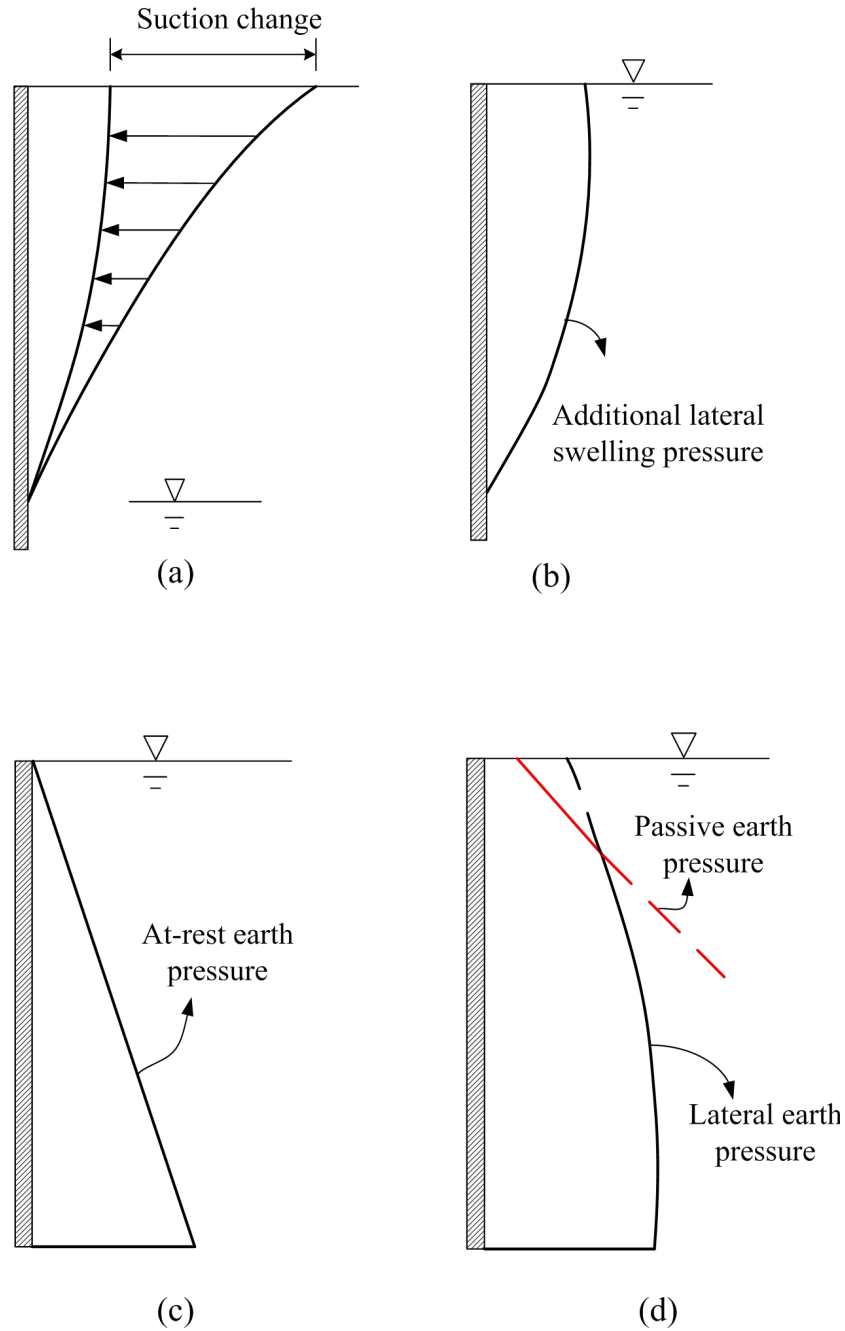


Figure 4.7 Schematic diagram of earth pressure during wetting process: (a) Variation of matric suction, (b) Lateral swelling pressure, (c) At-rest earth pressure, (d) Typical distribution of lateral earth pressure during infiltration.

In wet seasons, the soil elements of expansive soils exhibit swell tendency and are in a passive state due to the increased lateral stress. Higher swelling pressure generates with

greater variation of suction values. Therefore, the most critical scenario occurs when the matric suction changes from initial condition to zero (i.e. saturated condition). As such, the lateral earth pressure can be estimated as follow:

The maximum lateral swelling pressure in Figure 4.7 (b) could be calculated by Equation (3.17) with zero surcharge load:

$$P_{LS} = \frac{(1 - \mu - 2\mu^2)[P_{s0} + \beta_s \cdot \psi \cdot (\frac{S_r}{100})]}{\frac{P_s}{E}(1 + \mu)(1 - \mu - 2\mu^2) + 1 - \mu^2} \quad (4.8)$$

For this scenario, when the soil is in a state of saturated condition (i.e. $(u_a - u_w) = 0$), the at-rest earth pressure in Figure 4.7 (c) can be computed using Equation (4.6).

In Figure 4.7 (d), the final lateral earth pressure is the sum of additional lateral swelling pressure (Equation 4.8) and at-rest earth pressure (Equation 4.6) as follow:

$$\sigma_{hp} = \frac{(1 - \mu - 2\mu^2)P_{s0} + \beta_s \cdot \psi \cdot (\frac{S_r}{100})^2}{\frac{P_s}{E}(1 + \mu)(1 - \mu - 2\mu^2) + 1 - \mu^2} + \frac{\mu}{1 - \mu} \sigma_{vs} \quad (4.9)$$

In addition, in order to avoid shear failure of retaining walls after long period of infiltration, the lateral pressure is limited by the passive earth pressure of saturated soil as shown in Figure 4.7(d) (Equation 4.7).

4.4 Summary

The lateral earth pressure distribution for retaining wall are illustrated in this chapter taking account of the influence of cracks and lateral swelling pressure as per the

discussions presented in Chapter 3. Table 4.1 summarizes the various equations that can be used for active and pressure conditions. The proposed approach can be applied for both expansive and non-expansive soils under unsaturated and saturated conditions. The suction profile corresponding to different water flow rates is the essential variable in the estimation of the lateral pressure distribution behind the retaining wall.

Table 4.1 Equations for lateral earth pressure estimation.

Stress state \ Soil types	Non-expansive soils		Expansive soils
	Saturated condition	Unsaturated condition	
Active earth pressure	Equation (4.1)	Equation (4.2)	Equation (4.3) (Evaporation)
At-rest earth pressure	Equation (4.6)	Equation (4.4)	Equation (4.9) (Infiltration)
Passive earth pressure	Equation (4.7)	Equation (4.5)	

Under drying scenario, as cracks propagate in expansive soils, the soil mass is in an active state. It is difficult to predict the matric suction in the soil layer since the water keeps evaporating from the soil. Therefore, the active failure stress state is considered to be the most critical situation for practice applications. The corresponding lateral pressure distribution for saturated and unsaturated soils can be estimated by Equation (4.1) and (4.2). The proposed theory can be extended for unsaturated expansive soils by considering the soil mass up to crack depth as a surcharge load (Equation 4.3).

In wet seasons, soil mass gradually becomes saturated. For unsaturated soils, the at-rest and passive earth pressure are computed by Equation (4.4) and (4.5), respectively. The matric suction gradually decreases to zero due to moisture infiltration as the backfill soil gets saturated. The at-rest and passive lateral pressure distribution for saturated soils are computed by Equation (4.6) and (4.7). For expansive soils, additional lateral swelling pressure generates because the horizontal volumetric change is restrained by the retaining

walls. In order to provide a conservative design approach, the distribution of lateral earth pressure for expansive soils is predicted by Equation (4.9) for the scenario of matric suction which changes from a known initial value to zero (i.e. soil is fully saturated). Moreover, for frictionless surface, the maximum lateral earth pressure is limited by the Mohr-Coulomb passive earth pressure to avoid shear failure Equation (4.7).

CHAPTER 5

APPLICATION OF THE PROPOSED FRAMEWORK FOR LATERAL EARTH PRESSURE ESTIMATION OF EXPANSIVE AND NON-EXPANSIVE SOILS

5.1 Introduction

Details of expansive soils crack depth prediction and lateral swelling pressure estimation were presented in Chapter 3. A framework for lateral earth pressure estimation behind a retaining wall for both unsaturated expansive and unsaturated non-expansive soils has been proposed in Chapter 4. In this chapter, the proposed framework is employed on a hypothetical retaining wall with two different backfill soil types (i.e. Regina clay, which is an expansive soil and Indian Head till, which is a glacial till, and is a non-expansive soil) taking account of the influence of environmental factors.

Regina clay, is a typical expansive soil that undergoes significant volumetric changes due to the influence of environmental factors (i.e. evaporation and infiltration conditions). Active and passive states are the most critical scenarios for a fixed retaining wall during drought and wet seasons, respectively because of the generation of cracks and additional swelling pressure. Details of these investigations are summarized as Example A in this chapter.

In addition, the proposed approach is extended for lateral earth pressure estimation for a typical non-expansive soil, Indian Head till (i.e. Example B) by taking into account of the influence of environmental factors considering the variation of matric suction profile. The stress state of soil element is related to the displacement of retaining structures.

The at-rest, active and passive earth pressures for unsaturated soils are calculated for both drying and wetting conditions and compared with saturated condition for both Examples A and B.

The climate records from local weather stations (i.e. Regina Int'l A station and Indian Head CDA Station) is also summarized and used in the analyses to illustrate the changes of lateral earth pressure for different scenarios that are typically encountered in practice. The calculation results for both expansive and non-expansive soils are also analyzed and discussed in the following sections.

The following calculations are conducted using the proposed program LEENES in MATLAB for each of the examples:

- The fitting parameters of the Soil-Water Characteristic Curve (SWCC) are predicted in SigmaPlot to best-fit the experimental data of soils by employing van Genuchten model (1980).
- Matric suction profiles are plotted corresponding to different water flow rates under drying and wetting conditions as well as the hydrostatic condition for both examples.
- For Example A: (i) The depth of cracks in expansive soils is calculated under drying conditions. The active earth pressure distributions are plotted and the active earth forces are integrated with respect to different evaporation flow rates taking account of the influence of crack depth; (ii) The additional lateral swelling pressures that arise due to wetting conditions are computed for different infiltration flow rates. The lateral earth pressure distributions under wetting conditions are obtained by summing up the additional lateral swelling pressure and at-rest earth pressure condition.
- For Example B: Based on the matric suction profile that were estimated; the at-rest, active and passive lateral earth pressure distribution of a non-expansive soil is plotted. The lateral earth forces were determined by integration of earth pressures for both drying and wetting conditions.

5.2 Proposed program LEENES used in software MATLAB

There are several commercial software available for researchers to simulate complex geotechnical problems; however, tremendous time investment are required for engineers to write codes and build models (William et al. 2010). There is a need for simple and efficient methods to achieve the numerical analysis rapidly for researchers and practitioners in the field of geotechnical engineering. In the present study, MATLAB has been chosen as the software to write the program LEENES because of following reasons:

- Ubiquitous use in engineering studies as well as practice;
- Users with engineering background are familiar with MATLAB or can easily get familiar;
- High efficiency in numerical calculation and economical;
- Comprehensive graphic processing functions that facilitates quick analyses and interpretation of numerical results.

LEENES here is used as a tool to handle the complex iteration and integral computations using an efficient programming code. The influence of each of the parameters towards the results is considered independently during the calculation process. The methodology presented in Chapter 4 is implemented by LEENES and two comprehensive examples are illustrated using different kind of soils as backfill. This approach is of interest for practitioners in the rational of design of retaining walls using the mechanics of saturated and unsaturated soils.

5.3 Example A: Regina clay, Saskatchewan, Canada

5.3.1 Meteorological data and soil properties

The climate of the Regina area is classified as a cool, semi-arid to sub-humid type. The average monthly precipitation for the Regina area, which is recorded from 1981 to 2010 are shown in Figure 5.1 (Government of Canada 2015). Maximum and minimum average monthly precipitation rate occur typically in June and February are equal to 2.74×10^{-8} m/s and 3.63×10^{-9} m/s, respectively. Average monthly lake evaporation rates are listed in Table 5.1 and the maximum value of evaporation rate in this area is 2.28×10^{-9} m/s.

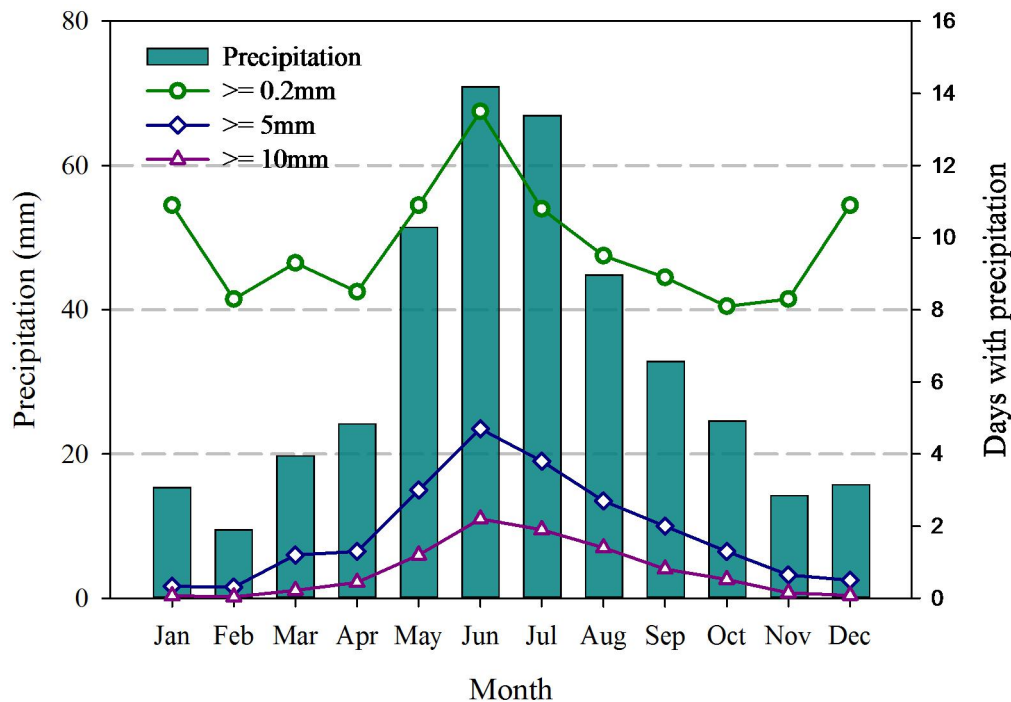


Figure 5.1 Average precipitation data for 1981 to 2010 Canadian Climates Normals from Regina Int'l A Station (modified from Government of Canada 2015).

Table 5.1 Lake Evaporation data for 1981 to 2010 Canadian Climates Normals from Regina Int'l A Station (modified from Government of Canada 2015).

Month	Jan	Feb	Mar	Apr	May	Jun
Lake Evaporation (mm)	0	0	0	0	5.2	5.8
Month	Jul	Aug	Sep	Oct	Nov	Dec
Lake Evaporation (mm)	5.9	5.1	3.6	1.9	0	0

In addition to external factors (i.e. the climate data for Regina area presented above), the basic soil properties is also essential in lateral earth pressure estimation due to their significant influence towards the swell-shrink behavior. The predominant mineral in Regina clay is montmorillonite, with a high swelling potential (Fredlund 1975). Widger and Fredlund (1979) provided the strength parameters of the soil sample from this area corresponding to large strain conditions. Shuai (1996) used the falling head permeability test to determine the saturated coefficient of permeability for Regina clay with respect to void ratio. Other investigators, Fredlund (1967), Vu and Fredlund (2004) conducted several lab tests on the Regina clay samples. The key soil properties Regina clay are summarized in Table 5.2.

Table 5.2 Soil properties of Regina clay

Soil properties		References
<i>Atterberg Limits</i>		
Liquid limit, <i>LL</i> (%)	77	Vu and Fredlund (2004)
Plastic limit, <i>PL</i> (%)	33	
Plasticity index, <i>I_P</i> (%)	44	
<i>Grain-size Distribution</i>		
Sand sizes, (%)	8	Fredlund (1975)
Silt sizes, (%)	41	
Clay sizes, (%)	51	
<i>Basic soil properties</i>		
Effective cohesion, <i>c'</i> (kPa)	17	Widger and Fredlund

Effective angle of internal friction, ϕ' (°)	15	(1979)
Max. dry density, γ_d (kN/m ³)	14.01	Shuai (1996)
Optimum moisture content, OMC (%)	28.5	
Specific gravity, G_s	2.82	Fredlund (1967)
Modulus of elasticity (saturated), E_s (kPa)	500	Vu and Fredlund (2004)
Saturated coefficient of permeability, k_s (m/s)	7.76×10^{-10}	Shuai (1996)
Void ratio, e	0.62	Barbour and Yang (1993)

5.3.2 Matric suction profile

Fredlund (1967) collected samples from the glacial Lake Regina sediment to determine the relationship between the soil suction and water content by using a pressure plate extractor and a pressure membrane extractor. Based on the experimental results from Fredlund (1967), the SWCC of Regina clay can be plotted to fit the experimental data by employing the fitting parameters a , n and m as 0.0001 kPa^{-1} , 0.89974 and 0.4388 in van Genuchten model (1980), respectively (see Figure 5.2).

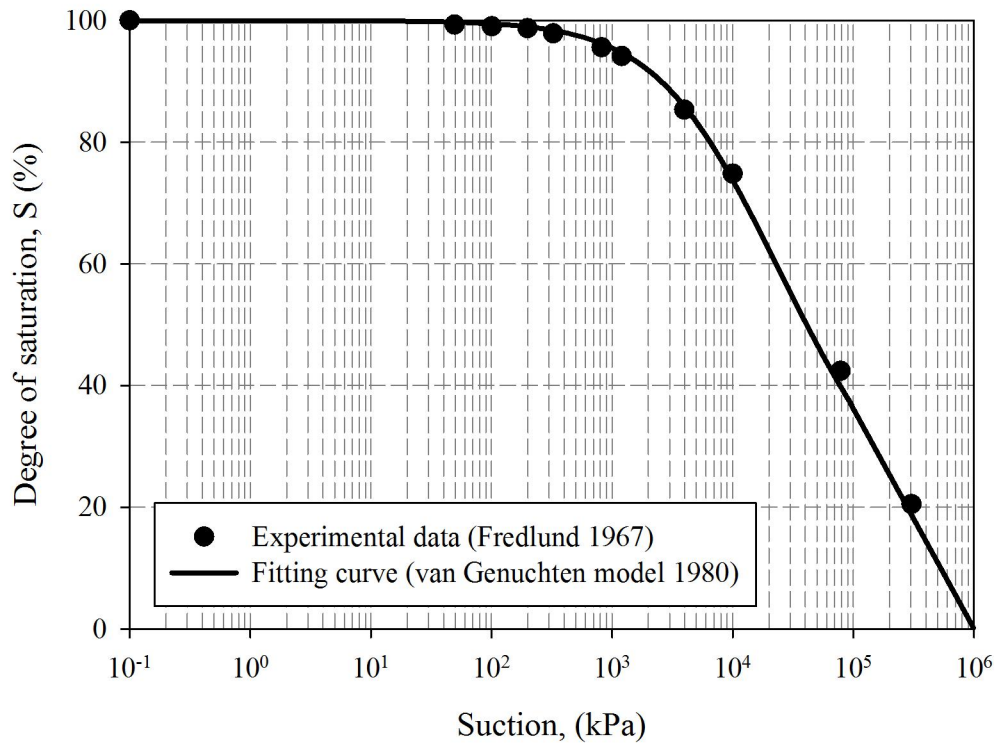


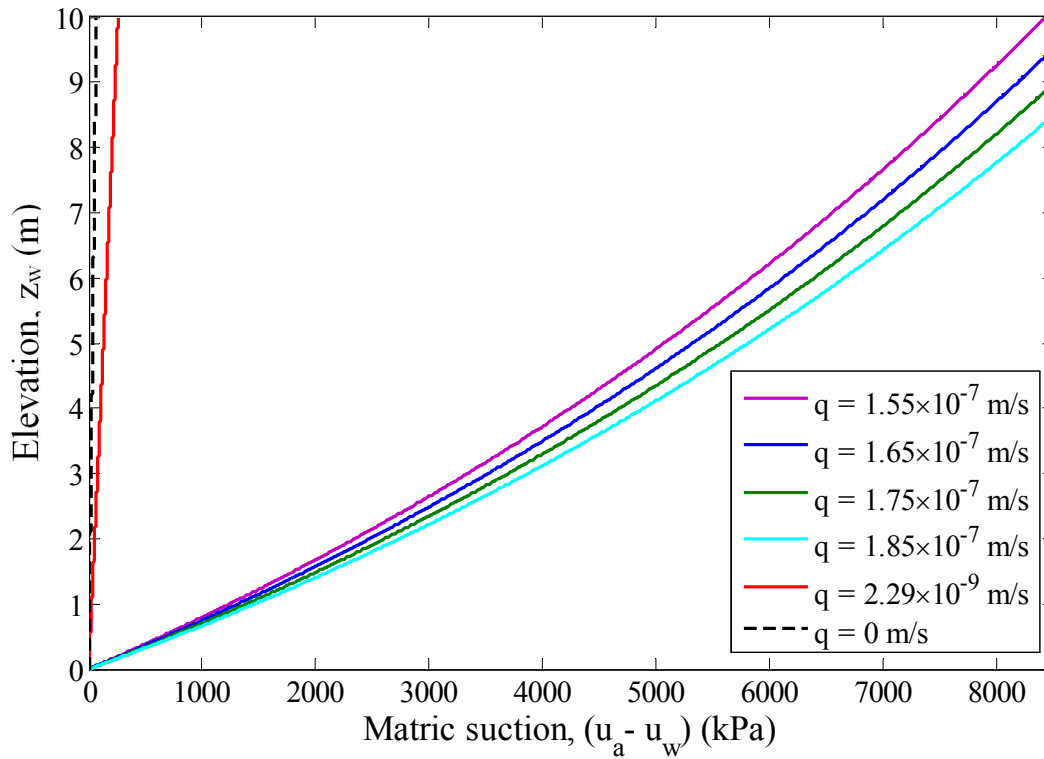
Figure 5.2 Soil-water characteristic curves of Regina clay.

The developed approach is employed on a 10m hypothetical retaining wall, assuming the ground water table to be at a depth of 10m. To evaluate the effects of environmental factors, the matric suction profiles are plotted under both drying and wetting conditions with the aid of Equation (3.3) using program LEENES (see Appendix A.2 and Figure 5.3). In Figure 5.3, $q = 0$ m/s represents the hydrostatic condition (i.e. the dotted black line).

For drying conditions, several different evaporation water flow rates are used as input to plot the matric suction profile. Among the input evaporation rates, $q = 2.28 \times 10^{-9}$ m/s is also used which represents average monthly evaporation rate (i.e. the red line in Figure 5.3a) (see also Table 5.1). Several other evaporation rates also used in in LEENES, including, $q = 1.55 \times 10^{-7}$ m/s which represents a value at which cracks generate (i.e. the purple line in Figure 5.3a). Besides, the matric suction profiles corresponding to $q =$

1.65×10^{-7} m/s, 1.75×10^{-7} m/s and 1.85×10^{-7} m/s are also plotted in order to discuss the influence of cracks propagation on the performance of the retaining wall in the following section (i.e. the dark blue, green and light blue lines in Figure 5.3a, respectively).

Five different water flow rates are employed to simulate the wetting scenarios from no infiltration (i.e. hydrostatic condition, $q = 0$ m/s) to fully saturation condition (i.e. the infiltration rate equals to the saturated coefficient of permeability, $q = 7.76 \times 10^{-10}$ m/s).



(a) Drying conditions

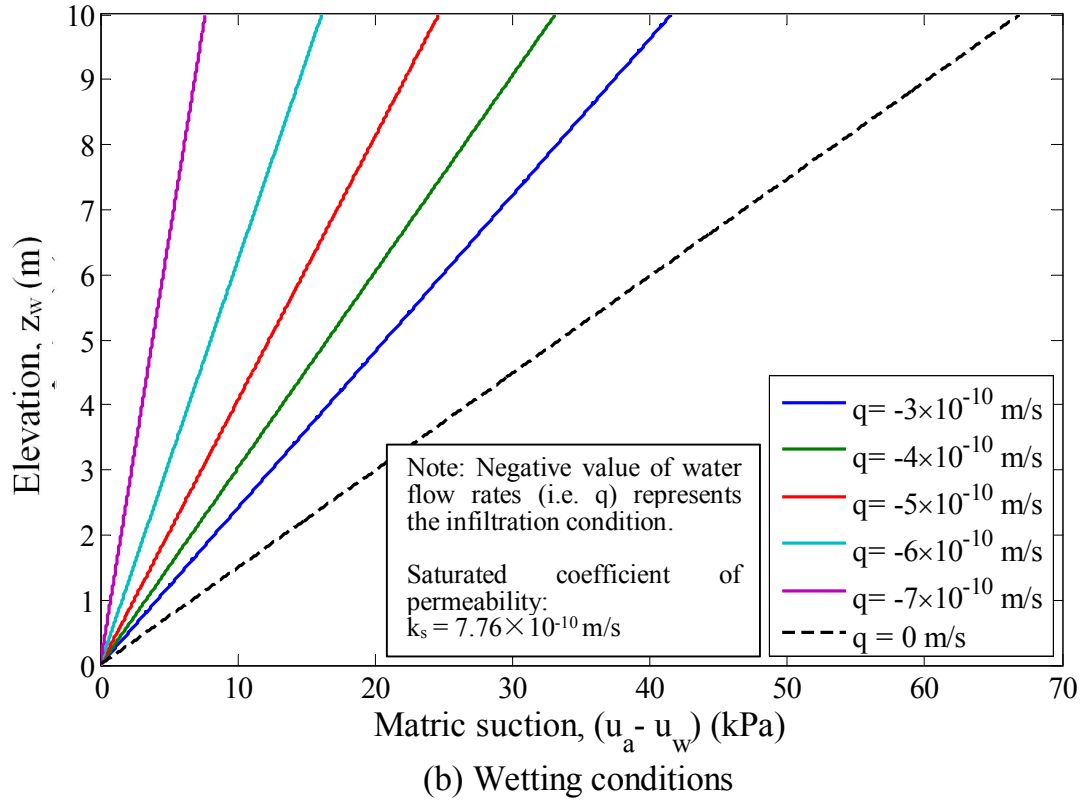


Figure 5.3 The matric suction profiles for Example A: (a) Drying conditions, (b) Wetting conditions.

5.3.3 Drying conditions

Upon evaporation, extensive cracks generate in expansive soils when the lateral stress exceeds the tensile strength of soils (Equation 3.7). Therefore, based on the matric suction profile under drying conditions (see Figure 5.3a), the tensile strength and lateral stress distributions are plotted along the retaining wall in Figure 5.5 with respect to various evaporation rates. As shown in Figure 5.4, several trials are required to find the intersection point. In Example A, crack initiates at the point of intersection of the two lines when the evaporation flow rate equals to $1.55 \times 10^{-7} \text{ m/s}$ and propagates after that. The detailed calculation procedures by LEENES for estimating the depth of cracks are presented in Appendix A.1. The depth of cracks and corresponding ultimate tensile strength of soils are summarized in Appendix Table A.1.

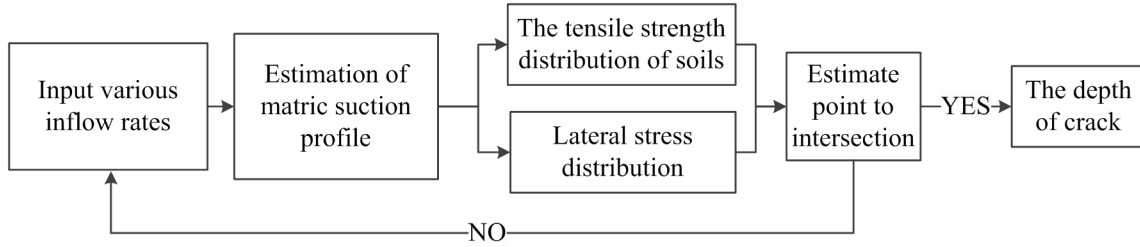


Figure 5.4 The flow diagram for trial procedures conducted in LEENES to estimate the depth of cracks and the corresponding evaporation flow rate in expansive soils.

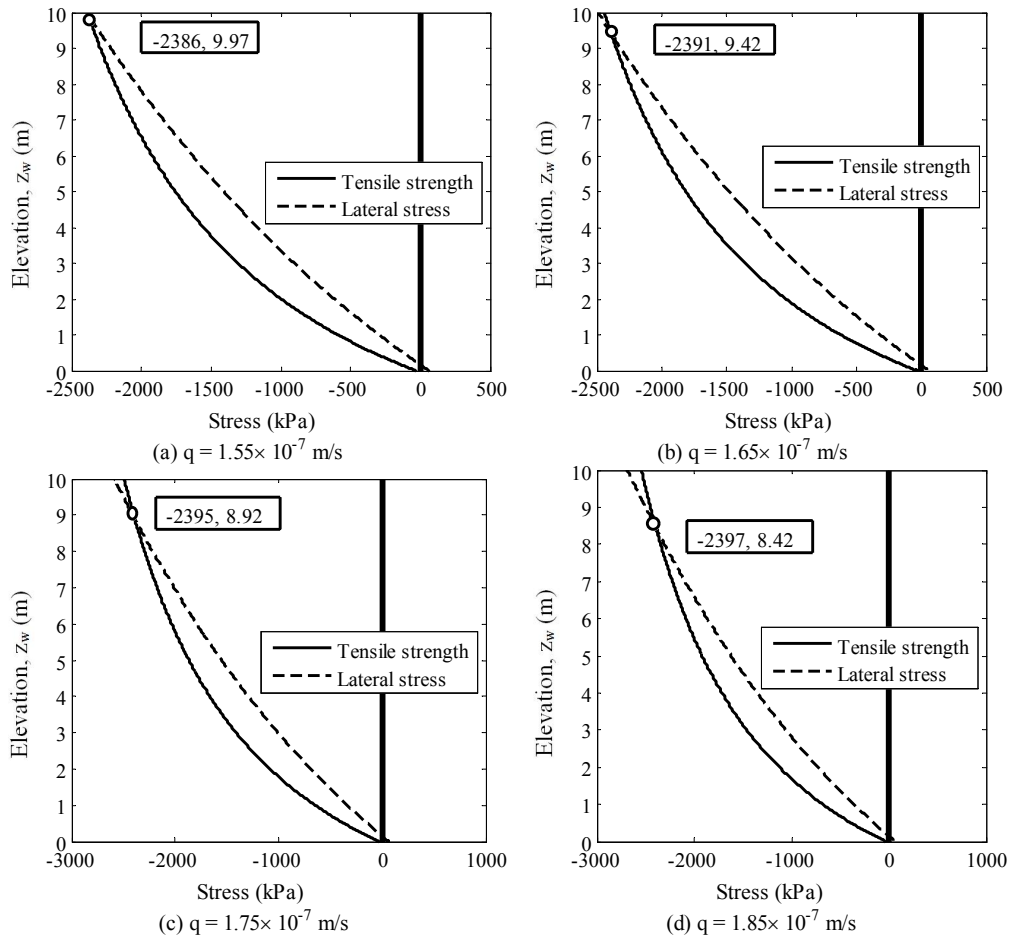


Figure 5.5 The tensile strength and lateral stress distribution with respect to different evaporation rates for Example A (i.e. $q = 1.55 \times 10^{-7}$ m/s, 1.65×10^{-7} m/s, 1.75×10^{-7} m/s, 1.85×10^{-7} m/s).

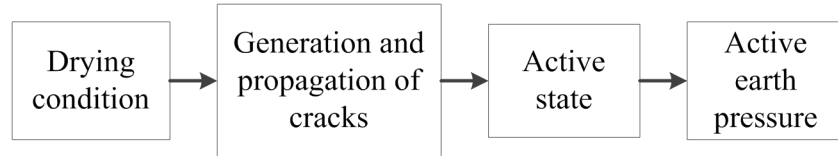


Figure 5.6 The flow diagram for lateral earth pressure estimation under drying condition in expansive soils.

As shown in Figure 5.6, the retaining wall is in a state of active condition after generation of cracks in expansive soil. Under such condition, the soil mass within the crack depth zone is considered as surcharge load to compute the lateral earth pressure distributions (Pufahl et al. 1983). By using the Equation (4.1), (4.2) and (4.3), the lateral earth pressure distributions with respect to different water flow rates including hydrostatic condition (i.e. $q = 0$ m/s) and saturated condition are calculated in LEENES (see Appendix A.2) and plotted in Figure 5.7. The detailed analysis and discussion are presented in the following section.

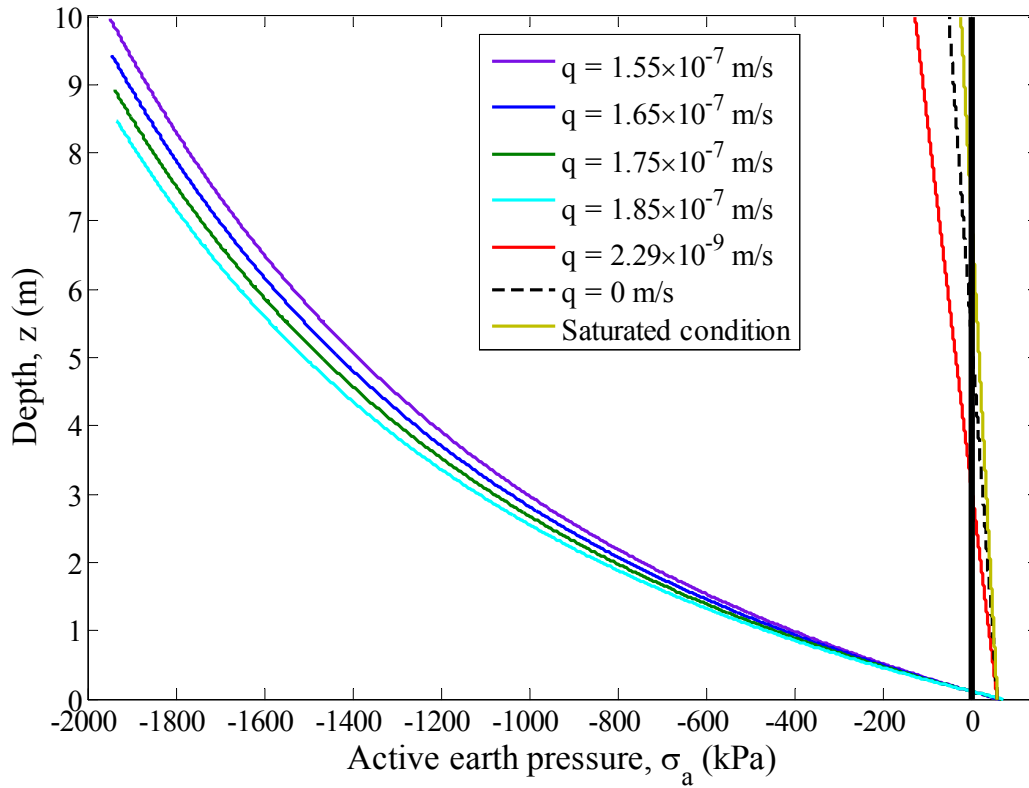


Figure 5.7 The active earth pressure distributions under drying conditions for Example A.

5.3.4 Wetting conditions

As the infiltration continues, the unsaturated soil mass behind the retaining wall gradually reaches saturated condition. The soil elements are typically at at-rest stress state behind a fixed wall. However, as discussed in Chapter 4, the soil elements in expansive soils are in passive state behind a fixed wall due to the additional swelling pressure corresponding to the reduction of matric suction. As illustrated in Chapter 4, for providing a conservative approach in practice, the matric suction profiles are assumed to change from initial unsaturated state (see Figure 5.3b) to fully saturated state (i.e. $(u_a - u_w) = 0$) for expansive soils which results in the development of the maximum lateral swelling pressure. The calculation procedures of the proposed approach is presented in Figure 5.8 to determine the final lateral earth pressure of expansive soils.

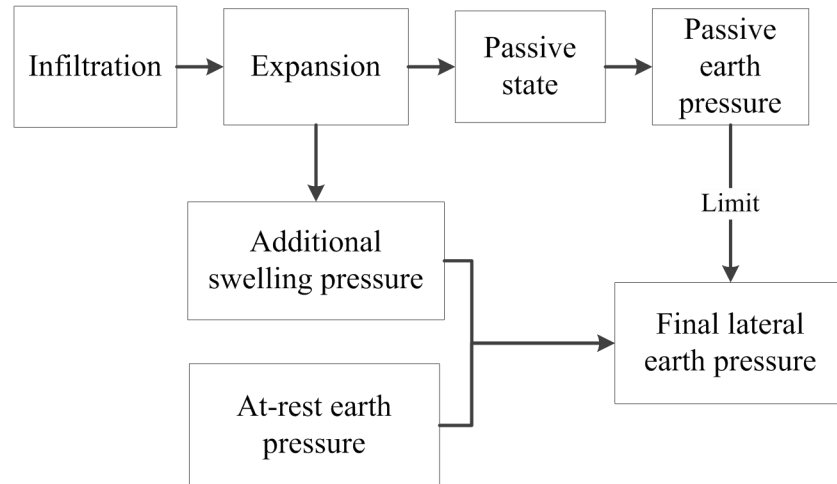


Figure 5.8 The flow diagram for lateral earth pressure estimation under wetting condition in expansive soil.

The final lateral earth pressure in expansive soils under wetting condition (see Figure 5.9 c) can be calculated as the sum of at-rest earth pressure at saturated condition (see Figure 5.9a) and the additional lateral swelling pressure (see Figure 5.9b). The soil elements in the upper layer are governed by the passive earth pressure at saturated condition (Equation 4.7) as shown in Figure 5.9 (c) (Hong 2008, Sahin 2011).

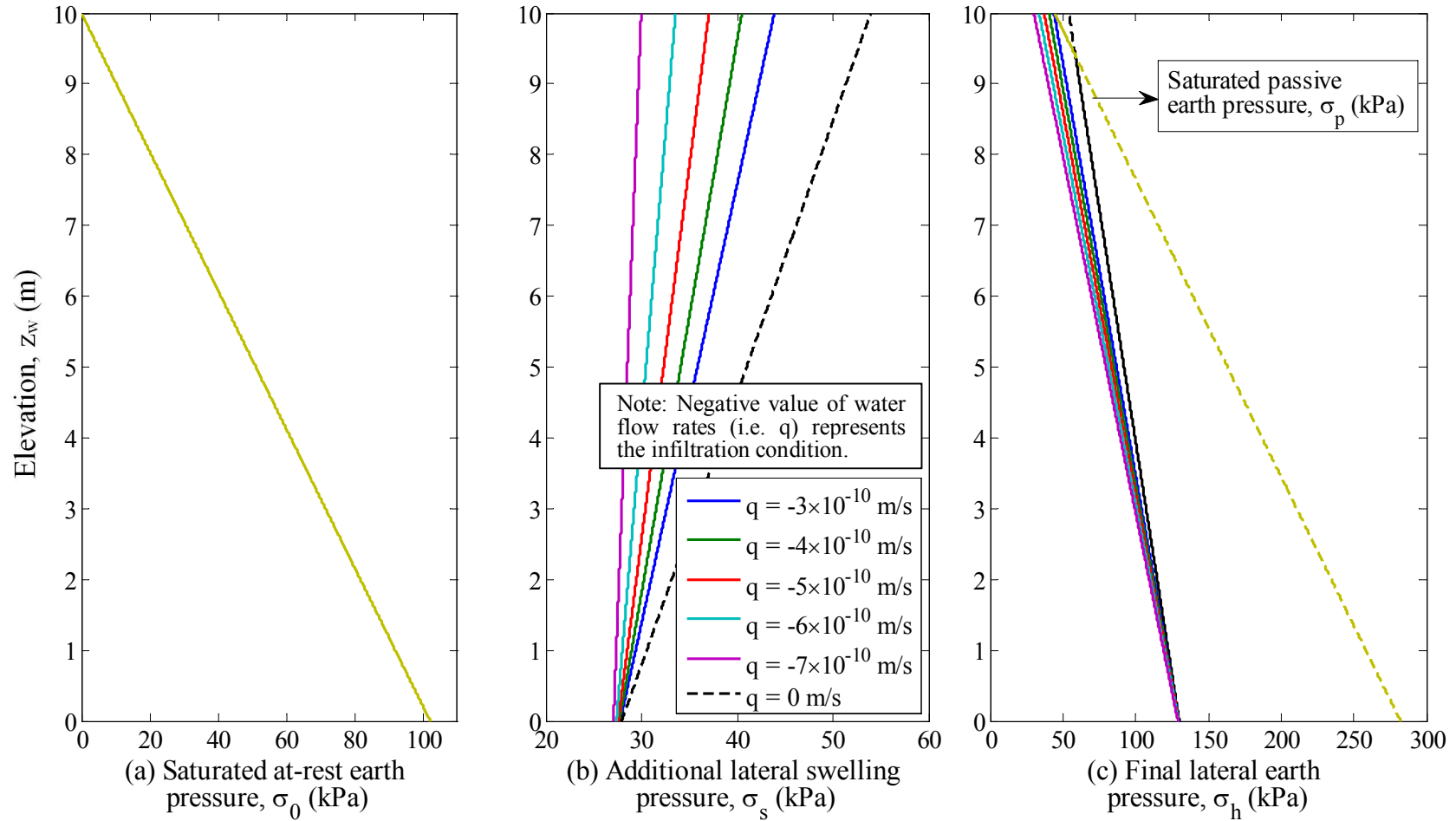


Figure 5.9 Lateral earth pressure distributions under wetting conditions for Example A: (a) Saturated at-rest earth pressure; (b) Additional swelling pressure due to variation of matric suction; (c) Final lateral earth pressure under wetting condition.

5.3.5 Analysis and discussion

The formation and propagation of cracks is significantly influenced by the tensile strength of unsaturated soils (Amarasiri and Kodikara 2011). The tensile strength of unsaturated soils can be considered as the sum of bridge water stress (i.e. capillary suction) and negative pore pressure (Schubert 1975, Lu et al. 2007). The development of the crack depth and ultimate tensile strength in Example A with respect to various evaporation rates are presented in Figure 5.10. Higher evaporation flow rate results in a large magnitude of matric suction which contributes to an increase in the ultimate tensile strength of soils (see Figure 5.10). In addition, cracks propagate to a greater depth at higher evaporation water flow rates as shown in Figure 5.10. The results of the present study are consistent with the observations of Wu et al. (2014).

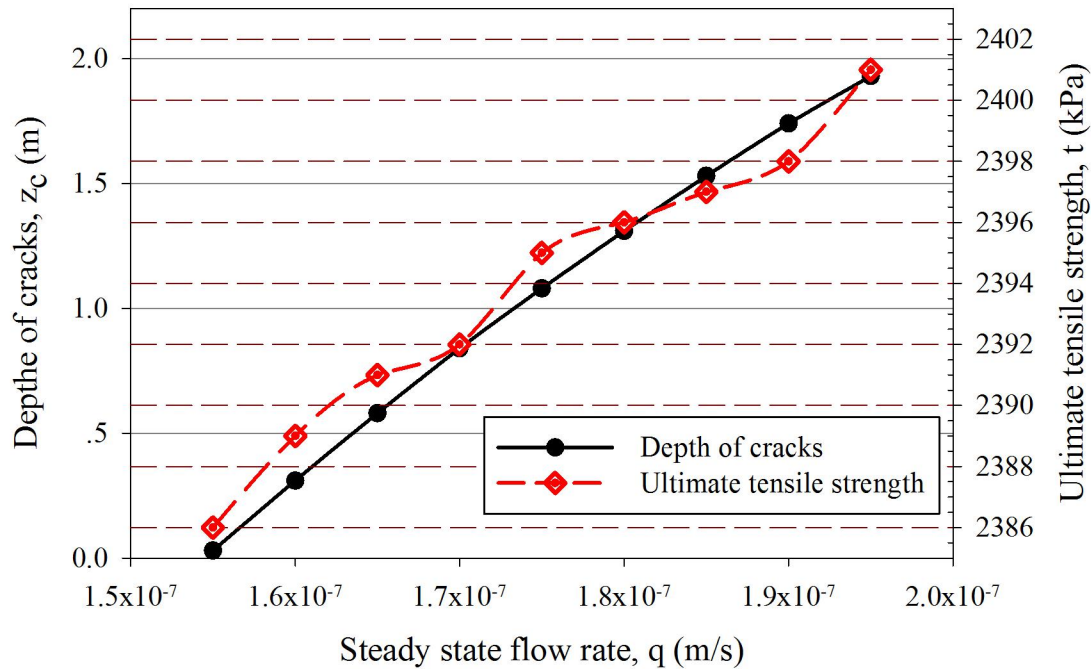


Figure 5.10 The depth of cracks and ultimate tensile strength with different steady state flow rate for Example A.

The depth of crack, the corresponding active earth forces, E_a , against a smooth wall with respect to various evaporation flow rates that are integrated over the depth are shown in Figure 5.7. Table 5.3 also summarizes the results of active earth force for saturated soil and also for soil under unsaturated condition for different flow rates.

Table 5.3 The active earth forces under drying conditions for Example A.

Flow rate, q (m/s)		4.1×10^{-8}	4.5×10^{-8}	4.9×10^{-8}	5.3×10^{-8}	2.29×10^{-9}	0
Active earth force, E_a (kN/m)	Unsaturated soils	2.94	3.22	3.47	3.67	83.99	144.43
	Saturated soils	192.55					

The maximum active force (i.e. 192.55 kN/m) occurs when the backfill soil behind the retaining wall is in a state of saturated condition (Table 5.3). For unsaturated expansive soils, the resultant active earth force reduced to 83.99 kN/m when the maximum recorded evaporation flow rate in Regina area (i.e. $q = 2.29 \times 10^{-9}$ m/s). Cracks generate only under extremely dry condition in Regina clay due to the high values of matric suction. For such a scenario, the active earth force decreased significantly with extensive cracks. In other words, in comparison to the conventional approach for active lateral earth pressure estimation, which assumed a hydrostatic condition, the proposed approach can better reflect the actual field scenario when cracks occur in expansive soils.

The additional lateral swelling forces and final lateral earth forces that act on the retaining wall under wetting conditions can be determined by integrating the lateral pressure distributions along the retaining wall as shown in Figure 5.9. The integration results are plotted with respect to various infiltration rates in Figure 5.11. The detailed integration results are summarized in Appendix Table A.2.

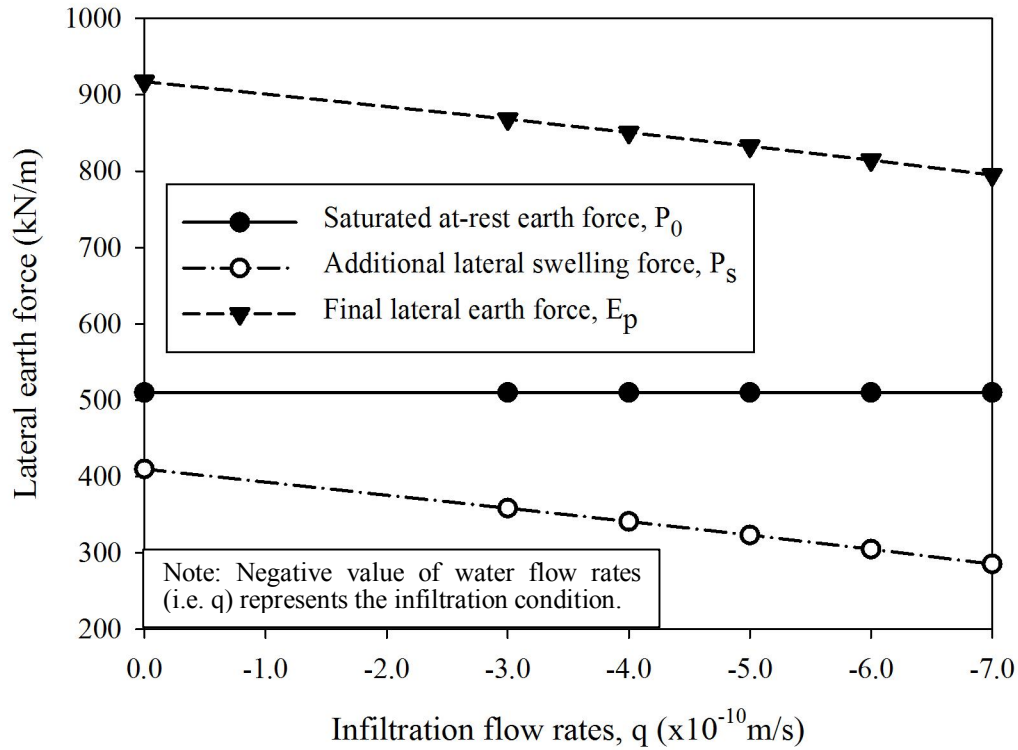


Figure 5.11 The lateral forces with respect to various infiltration flow rates for Example A.

A reduction in additional swelling force can be observed from Figure 5.11 with the increased infiltration flow rates while the saturated at-rest earth force remains constant (i.e. 510 kN/m). Such a behavior could be explained as follows: relative drying condition (i.e. lower infiltration flow rate) results in a higher matric suction in the retaining wall backfill (see Figure 5.3b). This in turn will lead to an increase in the affinity of soil to imbibe water. Due to this reason, an expansive soil backfill material, with a high initial value of matric suction contributes to an additional lateral swelling pressure as it moves from a state of unsaturated to saturated condition (Erol and Ergun 1994, Sapaz 2004). Overall, in comparison to the saturated at-rest earth force, the final lateral earth force of expansive soils under wetting conditions is significantly increased because of the additional lateral swelling pressure.

Under such scenario, as illustrated in Chapter 3, the soil expansion at horizontal direction is assumed to be strictly restricted. The deformation tendency of expansive soil mass in horizontal direction transforms to be the lateral swelling pressure acting on the retaining structures. As such, the soil elements are typically at passive state in expansive soils. Due to this reason, the upper layer of expansive soils often experienced a passive failure (Hong 2008, Sahin 2011). Due to this reason, the saturated passive earth pressure for saturated soils (Equation 4.7) should be taken under consideration in practical retaining wall design to prevent shear failure on the upper layer of expansive soil.

The traditional method for calculating the lateral earth pressures for unsaturated expansive soils is conservative for evaporation conditions. The lateral earth pressure in expansive soils significantly reduces due to the influence of cracks. On the other hand, under wetting conditions, the lateral earth pressure of expansive soils calculated by conventional method is not accurate enough without considering the swelling mechanism of expansive soils. In addition to the earth pressure, additional lateral swelling pressure generates in expansive soils and acts on the fixed retaining wall due to the variation of matric suction.

5.4 Example B: Indian Head till, Saskatchewan, Canada

5.4.1 Meteorological data and soil properties

In this Example B, the soil properties from various sources for glacial clay till from Indian Head, Saskatchewan, Canada are used for analysis (Vanapalli 1994, Vanapalli et al. 1999 and 2007, Oh and Vanapalli 2010, Fredlund et al. 2012). This soil is classified as a CL according to Unified Soil Classification System (USCS). The index properties, grain size distribution and other basic soil properties are summarized in Table 5.4.

The meteorological data of monthly precipitation is plotted in Figure 5.12, from which the maximum precipitation value was found to be 2.99×10^{-8} m/s. The Indian Head CDA Station also records the data for monthly lake evaporation rates; the maximum value monthly evaporation was equal to 1.20×10^{-9} m/s (see Table 5.5).

Table 5.4 Soil properties of Indian Head till

Soil properties		References
<i>Atterberg Limits</i>		
Liquid limit, <i>LL</i> (%)	35.5	Oh and Vanapalli (2010)
Plastic limit, <i>PL</i> (%)	16.8	
Plasticity index, <i>I_P</i> (%)	18.7	
<i>Grain-size Distribution</i>		
Sand sizes, (%)	28	Fredlund et al. (2012)
Silt sizes, (%)	43	
Clay sizes, (%)	30	
<i>Basic soil properties</i>		
Effective cohesion, <i>c'</i> (kPa)	10	Fredlund et al. (2012)
Effective angle of internal friction, <i>ϕ'</i> (°)	22.5	
Modulus of elasticity (saturated), <i>E_s</i> (kPa)	2000	Vanapalli et al. (2007)
Specific gravity, <i>G_s</i>	2.72	Oh and Vanapalli (2010)
Max. dry density, <i>γ_d</i> (kN/m³)	18.0	
Optimum moisture content, <i>OMC</i> (%)	16.3	
Initial void ratio, <i>e₀</i>	0.474	Vanapalli et al. (1999)

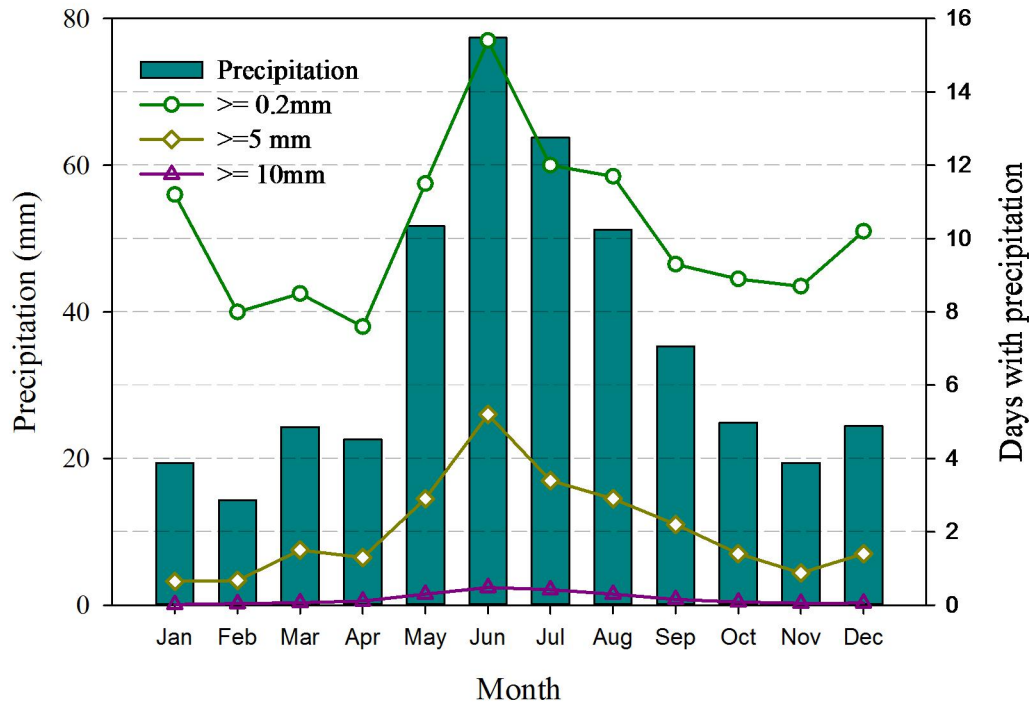


Figure 5.12 Precipitation data from 1981 to 2010 from Canadian Climate Normals in Indian Head CDA Station (modified from Government of Canada 2015).

Table 5.5 Lake evaporation data for 1981 to 2010 Canadian Climate Normals from Regina Gilmour Station (modified from Government of Canada, 2015).

Month	Jan	Feb	Mar	Apr	May	Jun
Lake Evaporation (mm)	0	0	0	0	2.6	3.1
Month	Jul	Aug	Sep	Oct	Nov	Dec
Lake Evaporation (mm)	3.1	2.8	0	0	0	0

5.4.2 Matric suction profile

The experimental data of the SWCC for Indian Head till measured using a pressure plate apparatus on soil specimens compacted at the optimum moisture content condition (Vanapalli et al. 1996) for establishing the matric suction profile. Same as the previous example, the van Genuchten (1980) model is used to fit the tested points and is shown in

Figure 5.13. The fitting parameters in the SWCC, namely a , n and m are 0.001 kPa^{-1} , 0.6584 and 0.5852 , respectively.

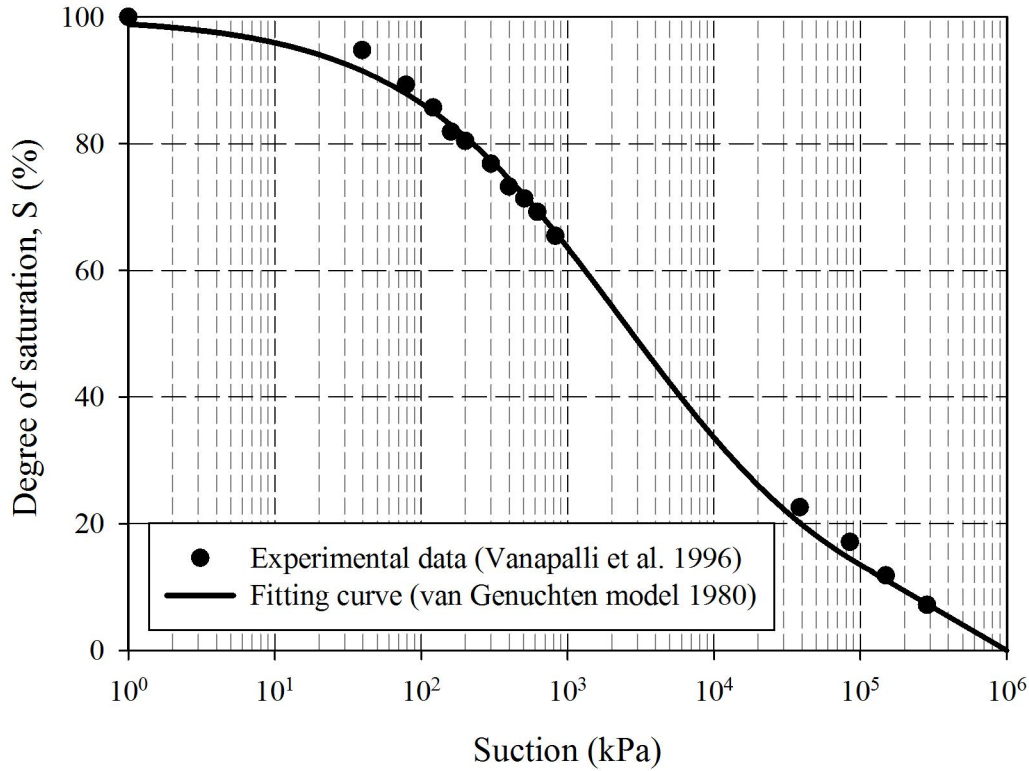
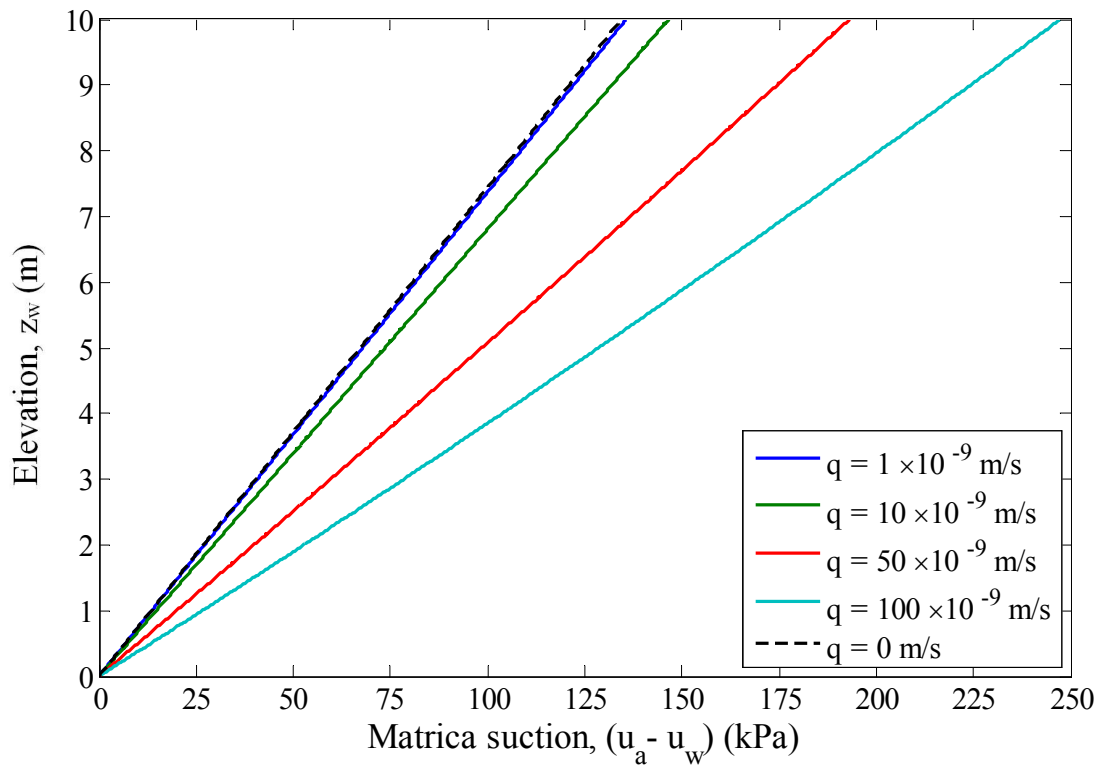


Figure 5.13 Soil-water characteristic curves of Indian Head till (modified from Vanapalli et al. 1996).

Using LEENES, the hypothetical retaining wall with the same geometrical dimensions as detailed for Example A is modeled. The fitting parameters for the SWCC and the soil properties of Indian Head till are summarized in Table 5.4. The saturated coefficient of permeability, k_s , is assumed of $1 \times 10^{-7} \text{ m/s}$ (Oh and Vanapalli 2010). The matric suction profiles are established under both drying and wetting conditions using Equation (3.3) (see Appendix A.3 and Figure 5.14). The continuous black line represents suction profile for hydrostatic condition (i.e. $q = 0 \text{ m/s}$).

Several evaporation rates which include, $q = 1 \times 10^{-9}$ m/s (i.e. the blue line in Figure 5.14 a) $q = 10 \times 10^{-9}$, 50×10^{-9} and 100×10^{-9} m/s) are used to plot corresponding matric suction profiles understand the influence on environmental factors on the performance of retaining wall. Four different water flow rates were employed as shown in Figure 5.14 (b) between no infiltration (i.e. hydrostatic condition, $q = 0$ m/s) to fully saturation condition (i.e. the infiltration rate equals to the saturated coefficient of permeability, $q_s = 1 \times 10^{-7}$ m/s) to understand the influence of wetting conditions.



(a) Drying conditions

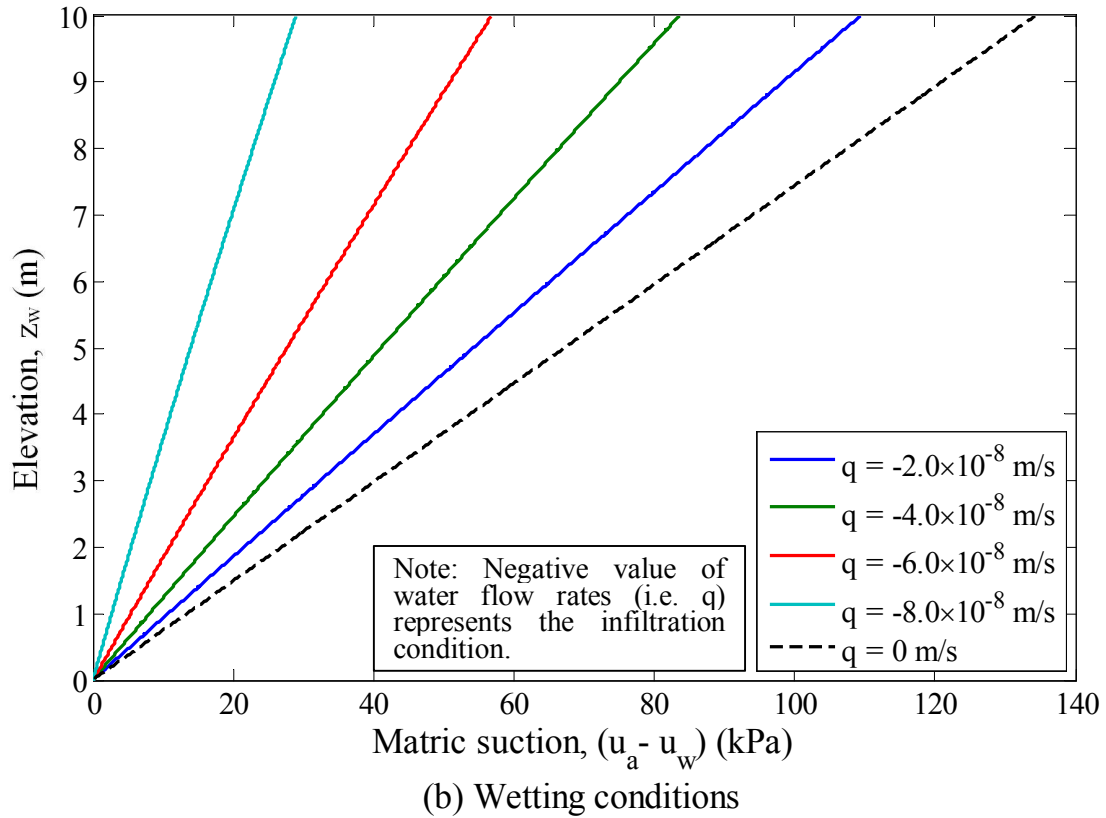


Figure 5.14 The matric suction profiles for Example B: (a) Drying conditions, (b) Wetting conditions.

5.4.3 Drying and wetting conditions

When there is no significant swell-shrink behavior, the stress state of non-expansive soil elements is only dependent on the movement of the retaining wall. The at-rest, active and passive earth pressure distributions for Example B are discussed performing the analyses following the various steps shown in flow chart (see Figure 5.15) for both drying and wetting conditions.

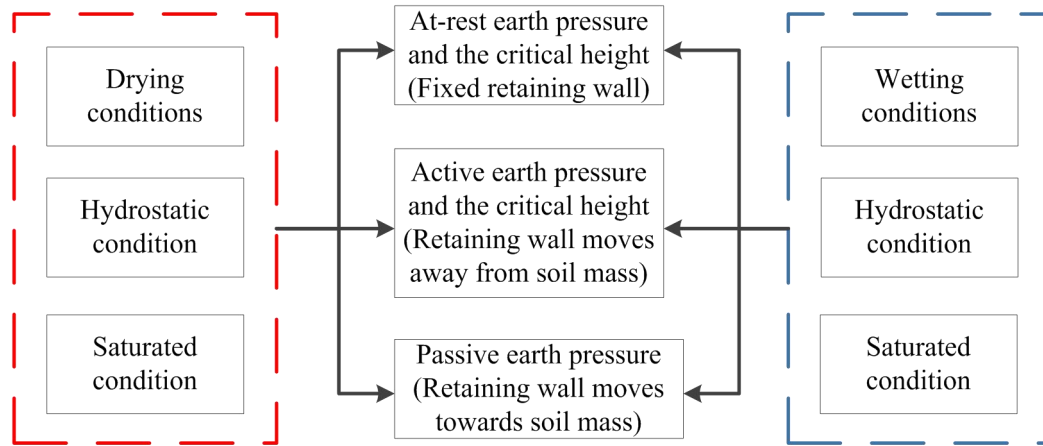


Figure 5.15 The flow diagram for lateral earth pressure estimation under drying and wetting conditions of Example B.

The matric suction profiles for Example B for different drying conditions are plotted with respect to various evaporation and infiltration flow rates as well as hydrostatic condition in Figure 5.14. The at-rest, active and passive earth pressure distributions are plotted in Figure 5.16 and Figure 5.17, respectively for different environmental factors (i.e. evaporation and infiltration flow rates) along with the hydrostatic and saturated conditions.

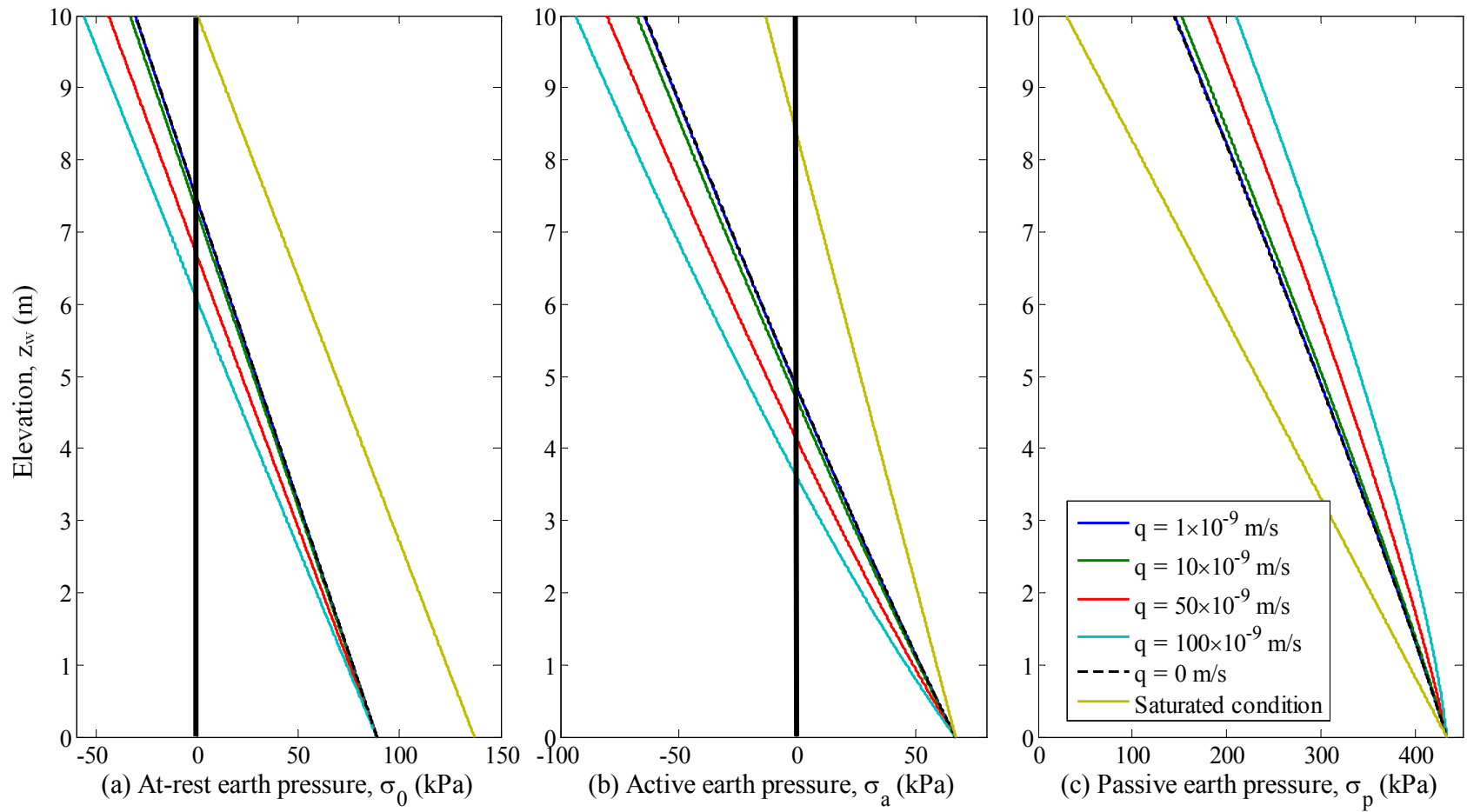


Figure 5.16 Lateral earth pressure distributions under drying and saturated conditions: (a) At-rest earth pressure; (b) Active earth pressure; (c) Passive earth pressure.

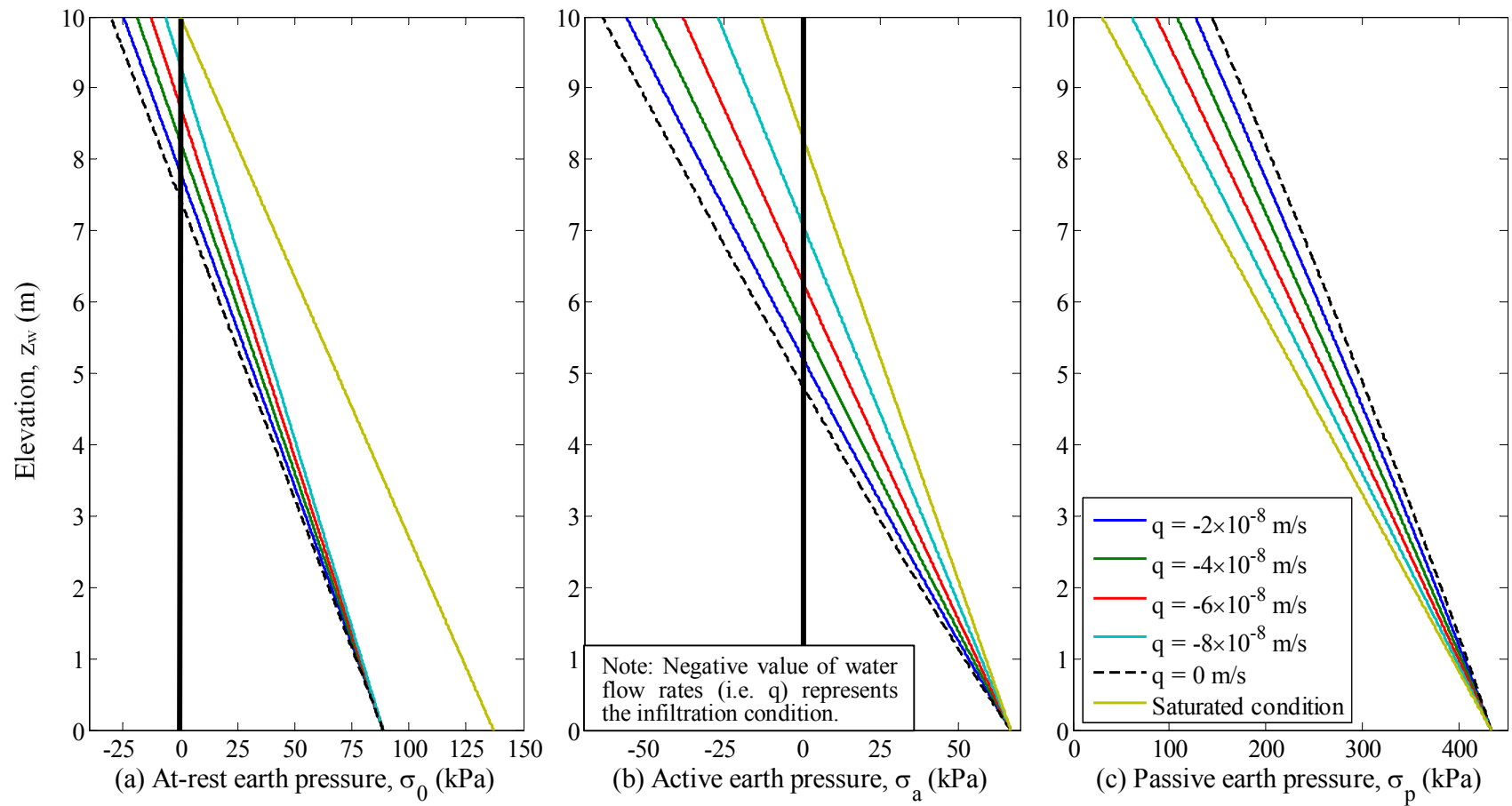


Figure 5.17 Lateral earth pressure distributions under wetting and saturated conditions: (a) At-rest earth pressure; (b) Active earth pressure; (c) Passive earth pressure.

5.4.4 Analysis and discussion

The critical height (i.e. depth of tensile zone, z_t) is an important factor that should be considered in temporary cuts or excavation projects. The changes of critical height for various water flow rates is plotted in Figure 5.18 (b) for Example B under at-rest and active conditions. The detailed calculation results are presented in Appendix Table A.3. It can be observed from Figure 5.18 (b), the critical height in unsaturated soils increases (i.e. changes from 0.68m to 3.95m) at-rest condition (i.e. the retaining wall is fixed) when the water flow rate changes from infiltration to evaporation condition. Similar increasing trends in critical height (i.e. changes from 2.89m to 6.40m see Figure 5.18b) when the soil elements are at active stress state (i.e. retaining wall moves away from soil mass).

It is widely acknowledged that matric suction is the main reason that enable the cohesive soils to stand unsupported (Vanapalli and Oh 2012). However, during the design life period of retaining wall, the soils usually experience several drying and wetting cycles, which changes the matric suction profile considerably as shown in Figure 5.14. The value of matric suction rises as water keeps evaporating and contributes to the self-supporting ability of soil. Vanapalli and Oh (2012) highlighted the importance of reliable estimation of the critical height such that geotechnical structures can be designed and constructed based on rational procedures. The proposed approach in this thesis provides geotechnical engineers with a relative simple and practical method for estimation of the critical height by taking into account of the evaporation and infiltration flow rates.

In addition to the critical height, the changes of lateral earth forces for Example B are plotted in Figure 5.18 (a) and (c) with respect to different water flow rates while the detailed information could be found in Appendix Table A.4.

When soil elements are at a state of at-rest or active conditions, the resultant earth force reduces due to the decrease in matric suction value for unsaturated soils when the

influence of environment changes from evaporation to infiltration (see Figure 5.18 a). The resultant lateral earth force for at-rest hydrostatic conditions (i.e. 329.82 kN/m) is approximately 50% in comparison to saturated condition (i.e. 684.65 kN/m), which is consistent with the results found by Tavakkoli and Vanapalli (2011). Under active condition, the maximum lateral earth force corresponding to infiltration condition (i.e. $q = -8 \times 10^{-8}$ m/s) is 238.19 kN/m, which is almost double the value under assumed drying condition (i.e. $E_a = 120.6$ kN/m when $q = 10 \times 10^{-8}$ m/s), it is however still less than the saturated active earth force, which is equal to 279.39 kN/m.

On the other hand, the limiting passive stresses of retaining wall can be increased greatly due to the influence of suction (Sahin 2011, Fredlund et al. 2012, Vo and Russell 2014). The passive forces acting on the retaining wall in Example B are integrated from the passive earth pressure over the depth of retaining wall. The passive forces are increased from 2469.50 kN/m to 3214.95 kN/m when the matric suction increased due to the evaporation process as shown in Figure 5.18 (c).

The variation in matric suction profile of unsaturated soils due to changes in the environment has significant influence on the lateral earth pressure behind a retaining structures (Lu and Griffiths 2004). It can be concluded from earlier discussion that geotechnical engineers are encouraged to maintain the soils at unsaturated condition when a temporary retaining structure is under construction to achieve benefits both with respect to safety aspects and relative economics. This is because, under such scenario, the magnitude of active or at-rest earth pressure against the retaining structures is relatively small due to the contribution from matric suction. So the soil mass shows better self-support ability when soil backfill is in a state of unsaturated condition. On the other hand, the passive resistance is generally considered when permanent retaining structures is designed to avoid severe failures of soils or structures (Fredlund et al. 2012). In practice, the retaining structures are typically experience several drying and wetting

cycles. The simple procedure proposed in the present study is useful to take account of the influence of matric suction profile in the backfill which can be estimated considering different evaporation and infiltration flow rates based on the environmental data for rational design of retaining walls.

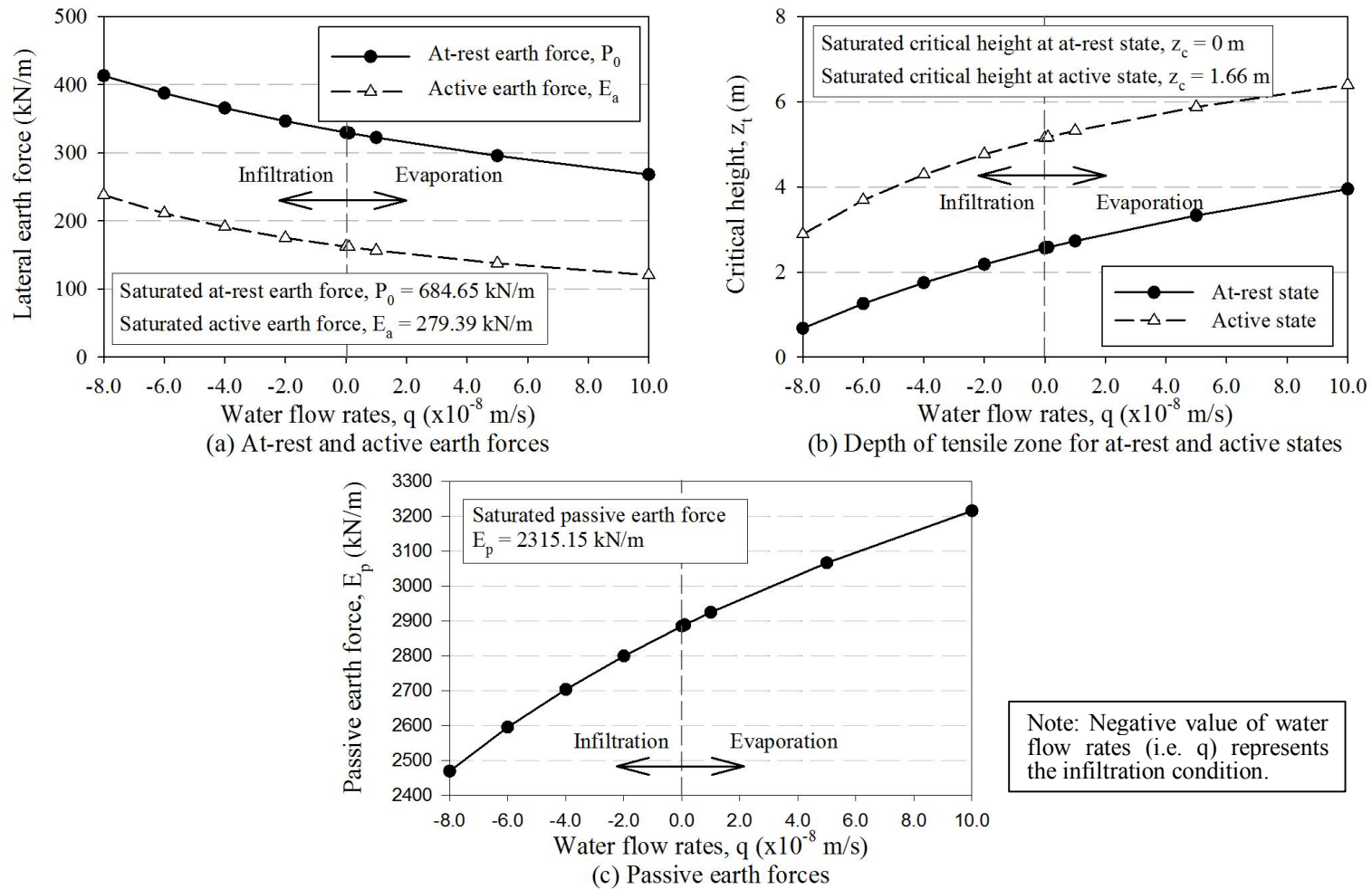


Figure 5.18 Lateral earth forces and depth of tensile zone under drying and wetting conditions for Example B: (a) At-rest and active earth forces; (b) Depth of tensile zone for at-rest and active states; (c) Passive earth forces.

5.5 Summary

The framework of lateral earth pressure estimation proposed in the thesis extending the mechanics of unsaturated soils is demonstrated on a fixed retaining wall with expansive soils in Example A. The propagations of cracks and additional swelling pressures change the stress state of expansive soil elements behind the retaining wall. The limiting lateral earth pressures therefore is discussed in this chapter taking account of variation of matric suction profile within the backfill which is estimated for different environmental conditions scenarios (i.e. wetting and drying) using the program code LEENES developed using MATLAB.

In addition, an extension of the proposed approach is applied to a non-expansive soil as backfill material in Example B. Lateral earth pressure for three typical stress states (i.e. at-rest, active and passive state) are presented associated with evaporation and infiltration water flow rates.

The proposed framework is simple and can be used by practicing geotechnical engineers in the retaining wall design taking account of the influence of various environmental factors (i.e. associated with different drying and wetting conditions) of expansive and non-expansive soils.

CHAPTER 6

CONCLUSIONS AND PROPOSED RESEARCH FOR FUTURE STUDIES

6.1 General

Rational procedures for lateral earth pressure estimation of unsaturated expansive soils that take account of swelling and shrinkage characteristics due to environmental factors are valuable for practicing engineers to provide reliable methods for design of retaining walls. Conventional methods that are used for estimation of lateral earth pressures of unsaturated soils, the matric suction variation is typically assumed to decrease linearly from surface to a value of zero as it approaches groundwater table condition, assuming hydrostatic condition (Fredlund and Rahardjo 1993, Hadži-Niković et al. 2015). However, the environmental factors (i.e. infiltration and evaporation) have a significant influence on the suction profiles of unsaturated soils, which is typically non-linear.

Expansive soils typically undergo extensive volumetric changes due to changes in matric suction associated with the variation of water content due to wetting or drying conditions. Extensive cracks develop due to evaporation conditions; however, additional lateral swelling pressure arises on the retaining structures due to the influence of infiltration. In simple terms, the soil stress state behind the retaining wall is significantly influenced due to environmental factors.

The key objective of this thesis is to provide a comprehensive framework for estimating the lateral pressures of unsaturated expansive soils taking into account of influence of

both the cracks and the swelling pressure due to evaporation and infiltration, respectively. In addition, the framework is also extended to apply for non-expansive soils.

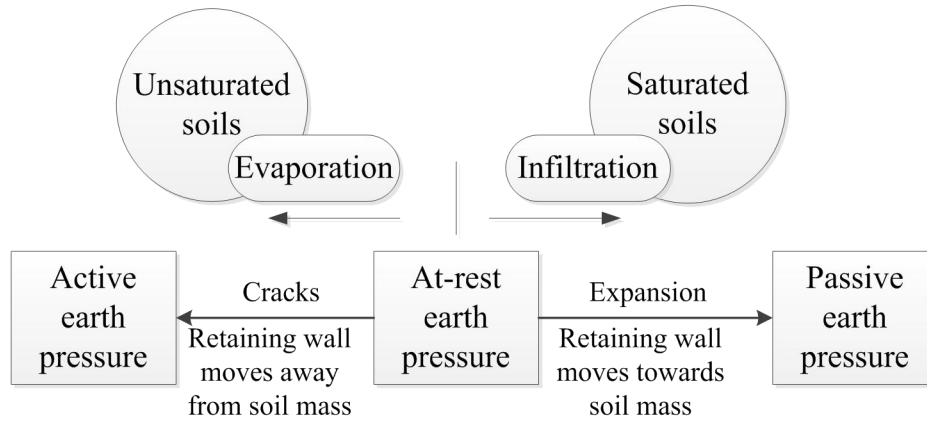


Figure 6.1 Schematic diagram of the proposed framework.

As shown in Figure 6.1, soils mass changes from unsaturated condition to saturated condition during drying-wetting process.

For non-expansive soils, the stress state of soil depends on the relative movement of retaining wall with respect to the soil mass (i.e. backfill). The soil behind the retaining wall is typically in an active state when retaining wall moves away from soil mass. The active earth pressure is computed by Equation 4.1 and 4.2, for saturated and unsaturated soils, respectively. On the other hand, when the retaining wall has a tendency to move towards the backfill, the soil behind the wall is in a state of passive condition. The passive earth pressures for saturated and unsaturated soils are computed by Equation 4.7 and 4.5, respectively. Besides, the at-rest earth pressures for saturated and unsaturated soils are calculated by Equation 4.6 and 4.4.

For expansive soils, in addition to the movements of retaining walls, the soil stress state also is significantly influenced to environmental factors as summarized earlier due to the changes in water content. The vertical steady-state flow rate is employed to account the

influence of environmental factors and estimate the variation of suction in the backfill along the depth of retaining wall (Equation 3.3). The depth of cracks in drought or dry season and additional lateral swelling pressures in rainy or wet season are estimated using Equation 3.9 and 3.17, respectively. The lateral earth pressure distributions for expansive soils under both drying and wetting conditions are computed by Equation 4.3 and 4.9, respectively.

In this thesis, a program LEENES is developed using the MATLAB software to facilitate calculations of all elements of the proposed framework, which is discussed in the earlier paragraph, and plot figures of lateral pressure estimation. Once the information of the input parameters of soil properties and the SWCC (in the form fitting parameters for the SWCC equations, a , n and m using the models of van Genuchten (1980) or Fredlund and Xing (1994), the proposed framework can be realized following the step-by-step approach shown in a flow chart form in Figure 6.2.

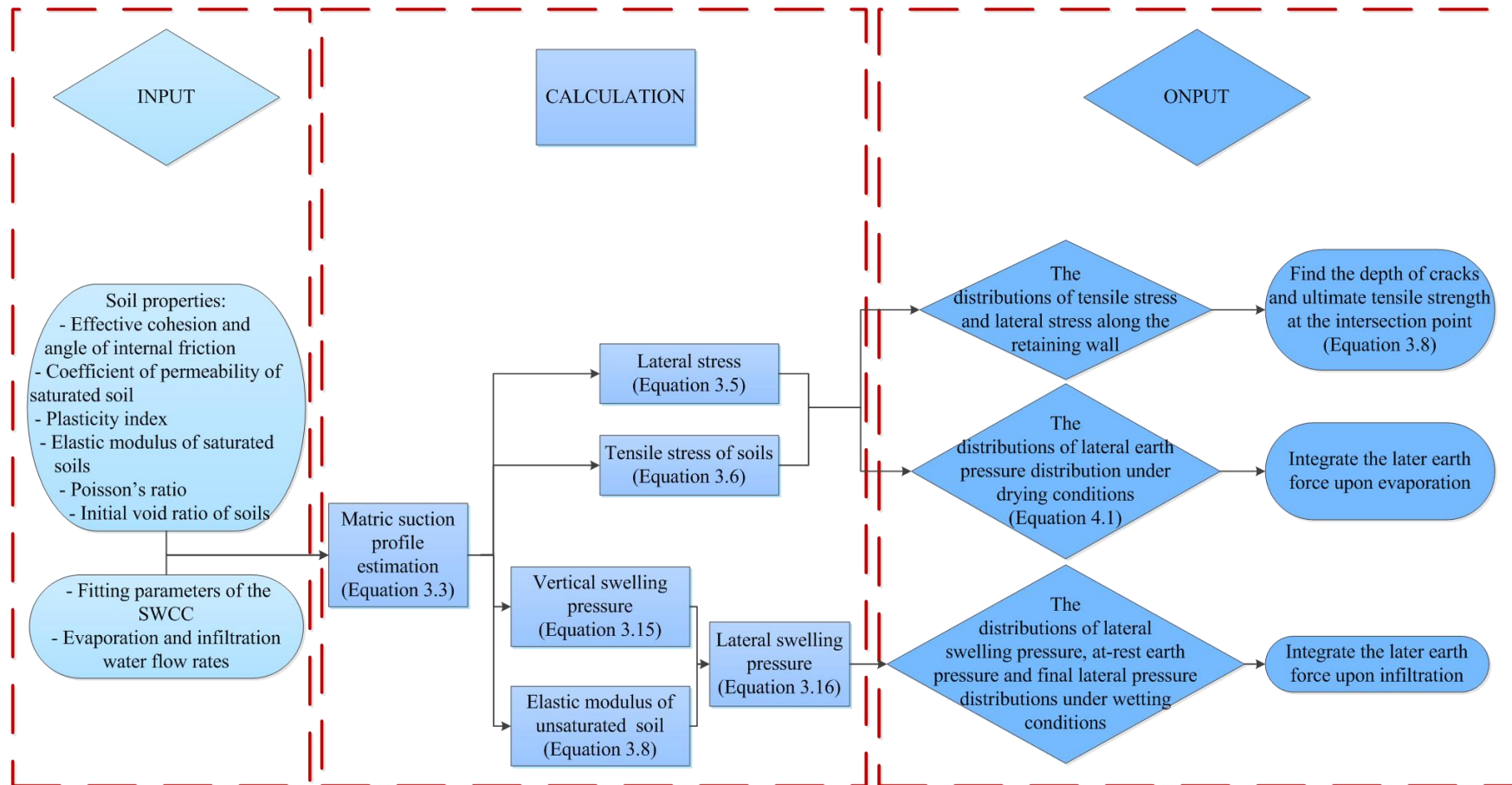


Figure 6.2 Flow chart for the proposed program LEENES.

Examples with two different types of soils (i.e. Example A: Regina clay, a typical expansive soil and Example B: Indian Head till, a glacial till which is non-expansive fine-grained soil) are illustrated to present the calculation procedures and results using the proposed program LEENES (see Appendix). Combined with the local climate data, analysis and discussion are performed to compare the computation results for both examples under drying and wetting conditions.

In the following sections, detailed conclusions of the thesis are summarized. In addition, future studies that can be undertaken with respect to estimation of the lateral earth pressures of expansive soils is also suggested.

6.2 Conclusions

- The suction profile variation associated with water content variation is required for predicting the shrinkage and swelling behavior of expansive soils. The variation of matric suction profile along the retaining wall depth is estimated taking account of environmental factors (i.e. evaporation or infiltration using Equation 3.3). The fitting parameters of the soil-water characteristic curve (SWCC) (i.e. a , n , and m) and the coefficient of permeability of saturated soils are required for estimation of the matric suction profile in the backfill.
- Cracks generate in expansive soils under drying conditions to a depth where the lateral stress equals to tensile strength of soils. The depth of cracks is estimated using Equation 3.9 with the aid of LEENES.
- Fully active state, which is the critical condition in the design of retaining wall with expansive soil as a backfill material arises due to evaporation condition. The most critical scenario arises when cracks propagate. At this condition, the lateral stress (i.e. minor principal stress) and the vertical stress (i.e. major principal stress) decrease as shown in Figure 4.3.

- The soil mass within the zone of crack depth, z_c , in unsaturated expansive soils is considered as a surcharge load (Pufahl et al. 1983) for the estimation of lateral earth pressure distribution extending the Mohr-Coulomb failure criterion (Equation 4.3).
- Under wetting conditions, the maximum swelling pressure occurs when there is greatest variation of matric suction in expansive soils from its initial unsaturated condition. However, for extending a conservative approach, soils are assumed to be in a fully saturated condition after prolonged period of rainfall or gradual snow melting. As a result, the matric suction is assumed to change from its initial condition value to zero (i.e. soil is fully saturated).

A semi-empirical model proposed by Tu and Vanapalli (2016) to predict the variation of vertical swelling pressure with respect to suction using SWCC as a tool is used in the present study. The lateral swelling pressure, which is a key parameter expressed in terms of vertical swelling pressure for unsaturated soils (Equation 3.17) (Liu and Vanapalli 2016). The lateral earth pressure distribution for expansive saturated soils under wetting condition is proposed in Equation (4.9).

- Combined with local climate data for monthly infiltration and evaporation rates from Regina Int'l A Station and Indian Head CDA Weather Station, two examples (namely, Example A and Example B) are summarized using LEENES, a MATLAB program code developed for implementing the framework proposed in this thesis. The calculation results are analyzed and discussed in this thesis to evaluate the influence of the environmental factors (i.e. infiltration and evaporation flow rates) towards expansive and non-expansive soils.
- The evaporation rates of recorded climate rates are not able to trigger cracks generated in a non-expansive soil analyzed in the present study (Example B: Indian Head till). However, the situation is different for expansive soils investigated in the present study (Example A: Regina Clay). Relatively, low evaporation rates are sufficient for crack generation and propagation in expansive unsaturated soils. In

other words, extensive cracks propagate with relative ease in expansive soils. In addition, large values of matric suction arise in expansive unsaturated soils upon evaporation, which provide the high shear strength of soils. Due to this reason, the corresponding lateral earth pressure is reduced significantly under drying condition in expansive soils.

- Although the precipitation data for both examples are basically the same, the difference of calculation results between Example A: Regina Clay and Example B: Indian Head till are considerable. Additional lateral swelling pressures act on the retaining walls when expansive soils are used as backfill. Consequently, soil elements are in a passive state and retaining wall are subjected to larger lateral pressure.

6.3 Proposed future studies for estimating lateral earth pressure of unsaturated expansive soils

- The fitting parameters of the SWCC have significant influence on matric suction profile in unsaturated soils. However, SWCCs are expected to undergo hysteresis during drying and wetting circles. Hence, effects of hysteresis of the SWCCs should be taken into consideration.
- The elastic modulus of unsaturated soil is a key parameter in the proposed approach and has great influence to the calculation results. However, limited research is available to estimate the elastic modulus of unsaturated soils with respect to the change of matric suction (Oh et al. 2009, Adem and Vanapalli 2014). More laboratory tests and field studies are required to provide a reliable estimation of the elastic modulus of unsaturated expansive soils.
- After drying seasons, extensive cracks contribute to the development of preferential pathways for water to infiltrate into the soil. Due to this reason, expansive soils will

be significantly influenced both during the drying and wetting cycles. A more comprehensive model is required to take into account of influence of both drying and wetting cycles for expansive soils.

- Upon evaporation, the suction profiles are assumed to change from initial state to zero (i.e. fully saturated condition) for conservative estimation. Nevertheless, expansive soils rarely reach fully saturated conditions in practice because of low coefficient of permeability. There is a need for a method for estimating swelling pressures when the suction changes from initial state to another intermediate stage and associated changes taking account of wetting and drying cycles.

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APPENDIX

A.1 Program code for example problem

```
%----- Input soil properties and fitting parameters
%-----
>> syms a n m z ks gamaw pr gamau D pr ip e vr
% Fitting parameters of SWCC
>> a=0.006;
>> n=1.7;
>> m=0.7;
% Depth along the retaining wall
>> z=[0:0.01:10];
% Coefficient of permeability of saturated soils
>> ks=8.6*10(-8);
% Density of water
>> gamaw=9.8;
% Poisson's ratio
>> pr=0.33;
% Dry density of soils
>> gamau=18.7;
% Depth of ground water table
>> D=10;
% Plastic index
>> ip=31.4;
% Saturated elasticity of modulus
>> e=1*104;
% Void ratio
vr=1.8;

% ----- Input various water flow rates -----
>> syms q1 q2 q3 q4 q5
>> q1=-3.5*10(-8);
>> q2=-1.5*10(-8);
>> q3=0;
>> q4=1.5*10(-8);
>> q5=3.5*10(-8);
```

```

% ----- Calculating matric suction profiles corresponding to different water flow rates ----
>> syms ms1 ms2 ms3 ms4 ms5
>> ms1=(-log((1+q1/ks)*vr.^(-gammau*a*z)-q1/ks))/a;
>> ms2=(-log((1+q2/ks)* vr.^(-gammau*a*z)-q2/ks))/a;
>> ms3=(-log((1+q3/ks)* vr.^(-gammau*a*z)-q3/ks))/a;
>> ms4=(-log((1+q4/ks)* vr.^(-gammau*a*z)-q4/ks))/a;
>> ms5=(-log((1+q5/ks)* vr.^(-gammau*a*z)-q5/ks))/a;

% ----- Plot the matric suction profiles -----
>> plot(ms1,z,ms2,z,ms3,z,ms4,z,ms5,z)

% ----- Calculate degree of saturation -----
>> syms ds1 ds2 ds3 ds4 ds5
>> ds1=1./(1+(a*ms1).^n).^m;
>> ds2=1./(1+(a*ms2).^n).^m;
>> ds3=1./(1+(a*ms3).^n).^m;
>> ds4=1./(1+(a*ms4).^n).^m;
>> ds5=1./(1+(a*ms5).^n).^m;

% ----- Calculate the unsaturated elasticity of modulus along
the retaining wall with respect to different water flow rates
-----
>> syms eun1 eun2 eun3 eun4 eun5
>> eun1=e*(1+0.1*ms1.*ds1.^2);
>> eun2=e*(1+0.1*ms2.*ds2.^2);
>> eun3=e*(1+0.1*ms3.*ds3.^2);
>> eun4=e*(1+0.1*ms4.*ds4.^2);
>> eun5=e*(1+0.1*ms5.*ds5.^2);

% -----Elastic modulus under saturated condition-----
>> syms x y
>> y=z;
>> x=0*y+e;

% ----- Plot the elastic modulus of unsaturated and saturated soils
-----
>> plot(x,y,eun1,z,eun2,z,eun3,z,eun4,z,eun5,z),axis([0 8*10^4 0 10])

```

A.2 Program code for Example A

```
%----- Input soil properties and fitting parameters
-----
>> syms c fi D ks gamau a n m ip k Ka Kp e pr vr
% Effective cohesion of soils
>> c=17;
% Effective angle of internal friction
>> fi=15;
% Depth of ground water table
>> D=10;
% Coefficient of permeability of saturated soil
>> ks=7.9*10^(-10);
% Dry density of soils
>> gamau=14;
% Fitting parameters of SWCC
>> a=0.0001;
>> n=0.9974;
>> m=0.4388;
% Plasticity index
>> ip=44;
% Fitting parameters
>> k=-0.0016*ip^2+0.0975*ip+1;
% Elastic modulus of saturated soils
>> e=500;
% Poisson's ratio
>> pr=0.3;
% Void ratio
>> vr=0.62;

% Coefficient of active earth pressure
>> Ka=(tand(45-fi/2))^2;
% Coefficient of passive earth pressure
>> Kp=(tand(45+fi/2))^2;

%----- Input the water flow rates under drying condition -----
>> syms q1 q2 q3 q4 q5 qh
>> q1=1.55*10^(-7);
```

```

>> q2=1.65*10^(-7);
>> q3=1.75*10^(-7);
>> q4=1.85*10^(-7);
%----- Input the water flow rates in practice
-----
>> q5=2.29*10^(-9);
%----- Input the water flow rates under hydrostatic condition -----
>> qh=0;

%----- Input the depth of cracks corresponding to various evaporation flow
rates-----
% Depth of cracks
>> syms zc1 zc2 zc3 zc4
>> zc1=0.03;
>> zc2=0.58;
>> zc3=1.08;
>> zc4=1.53;
>> zc5=2.22;

>> syms z z1 z2 z3 z4
>> z=[0:0.01:10];
>> z1=[0:0.01:9.97];
>> z2=[0:0.01:9.42];
>> z3=[0:0.01:8.92];
>> z4=[0:0.01:8.47];

% ----- Calculate the matric suction profiles
-----
>> syms ms1 ms2 ms3 ms4 ms5 msh
>> ms1=log((1+q1/ks)*vr.^(-gamau*a.*z1)-q1/ks)/a;
>> ms2=log((1+q2/ks)* vr.^(-gamau*a.*z2)-q2/ks)/a;
>> ms3=log((1+q3/ks)* vr.^(-gamau*a.*z3)-q3/ks)/a;
>> ms4=log((1+q4/ks)* vr.^(-gamau*a.*z4)-q4/ks)/a;
>> ms5=log((1+q5/ks)* vr.^(-gamau*a.*z)-q5/ks)/a;
>> msh=log((1+qh/ks)* vr.^(-gamau*a.*z)-qh/ks)/a;

% ----- Calculate degree of saturation -----
>> syms ds1 ds2 ds3 ds4 ds5 dsh
>> ds1=1./(1+(a*ms1).^n).^m;
>> ds2=1./(1+(a*ms2).^n).^m;
>> ds3=1./(1+(a*ms3).^n).^m;

```

```

>> ds4=1./(1+(a*ms4).^n).^m;
>> ds5=1./(1+(a*ms5).^n).^m;
>> dsh=1./(1+(a*msh).^n).^m;

% ----- Calculate lateral earth pressure distributions under drying conditions -----
>> syms pa1 pa2 pa3 pa4 pa5 pah pa
>> pa1=gamau.*(D-z1+zc1)*Ka-2*(Ka)^0.5.*(c+ms1.*ds1.^k*tand(fi));
>> pa2=gamau.*(D-z2+zc2)*Ka-2*(Ka)^0.5.*(c+ms2.*ds2.^k*tand(fi));
>> pa3=gamau.*(D-z3+zc3)*Ka-2*(Ka)^0.5.*(c+ms3.*ds3.^k*tand(fi));
>> pa4=gamau.*(D-z4+zc4)*Ka-2*(Ka)^0.5.*(c+ms4.*ds4.^k*tand(fi));
>> pa5=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+ms5.*ds5.^k*tand(fi));
>> pah=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+msh.*dsh.^k*tand(fi));
>> pa=gamau.*(D-z)*Ka-2*(Ka)^0.5.*c;

% Draw the line to show the retaining wall
>> syms ul
>> ul=0*z;

% ----- Plot matric suction profiles under drying conditions -----
>> plot(ms1,z1)
>> hold on
>> plot(ms2,z2)
>> plot(ms3,z3)
>> plot(ms4,z4)
>> plot(ms5,z)
>> plot(msh,z)

% ----- Plot lateral earth pressure distributions under drying conditions -----
>> plot(pa1,z1)
>> hold on
>> plot(pa2,z2)
>> plot(pa3,z3)
>> plot(pa4,z4)
>> plot(pa5,z)
>> plot(pah,z)
>> plot(pa,z)
>> plot(ul,z)

% ----- Calculate the lateral forces under drying conditions -----
>> syms num1 pa11

```

```

>> num1=size(find(pa1>0));
>> num1=num1(1,2);
>> pa11=trapz(z(:,1:num1), pa1(:,1:num1))

>> syms num2 pa21
>> num2=size(find(pa2>0));
>> num2=num2(1,2);
>> pa21=trapz(z(:,1:num2), pa2(:,1:num2))

>> syms num3 pa31
>> num3=size(find(pa3>0));
>> num3=num3(1,2);
>> pa31=trapz(z(:,1:num3), pa3(:,1:num3))

>> syms num4 pa41
>> num4=size(find(pa4>0));
>> num4=num4(1,2);
>> pa41=trapz(z(:,1:num4), pa4(:,1:num4))

>> syms num5 pa51
>> num5=size(find(pa5>0));
>> num5=num5(1,2);
>> pa51=trapz(z(:,1:num5), pa5(:,1:num5))

% Saturated condition
>> syms nums pas
>> nums=size(find(pa>0));
>> nums=nums(1,2);
>> pas=trapz(z(:,1:nums), pa(:,1:nums))

% Hydrostatic condition
>> syms numh pah1
>> numh=size(find(pah>0));
>> numh=numh(1,2);
>> pah1=trapz(z(:,1:numh), pah(:,1:numh))

%----- Input the water flow rates under wetting condition
-----
>> syms q7 q8 q9 q10 q11
>> q7=-3*10^(-10);
>> q8=-4*10^(-10);

```

```

>> q9=-5*10^(-10);
>> q10=-6*10^(-10);
>> q11=-7*10^(-10);

% ----- Calculate the matric suction profiles
-----
>> syms ms7 ms8 ms9 ms10 ms11
>> ms7=log((1+q7/ks)*vr.^(-gamau*a.*z)-q7/ks)/a;
>> ms8=log((1+q8/ks)*vr.^(-gamau*a.*z)-q8/ks)/a;
>> ms9=log((1+q9/ks)*vr.^(-gamau*a.*z)-q9/ks)/a;
>> ms10=log((1+q10/ks)*vr.^(-gamau*a.*z)-q10/ks)/a;
>> ms11=log((1+q11/ks)*vr.^(-gamau*a.*z)-q11/ks)/a;

% ----- Calculate degree of saturation -----
>> syms ds7 ds8 ds9 ds10 ds11
>> ds7=1./(1+(a*ms7).^n).^m;
>> ds8=1./(1+(a*ms8).^n).^m;
>> ds9=1./(1+(a*ms9).^n).^m;
>> ds10=1./(1+(a*ms10).^n).^m;
>> ds11=1./(1+(a*ms11).^n).^m;

% ----- Calculate vertical swelling pressures -----
>> syms ps7 ps8 ps9 ps10 ps11
>> ps7=50+0.7.*ms7.*ds7.^2;
>> ps8=50+0.7.*ms8.*ds8.^2;
>> ps9=50+0.7.*ms9.*ds9.^2;
>> ps10=50+0.7.*ms10.*ds10.^2;
>> ps11=50+0.7.*ms11.*ds11.^2;

% ----- Calculate unsaturated modulus of elasticity -----
>> syms e7 e8 e9 e10 e11 eh
>> e7=e*(1+0.1*ms7.*ds7.^2);
>> e8=e*(1+0.1*ms8.*ds8.^2);
>> e9=e*(1+0.1*ms9.*ds9.^2);
>> e10=e*(1+0.1*ms10.*ds10.^2);
>> e11=e*(1+0.1*ms11.*ds11.^2);
>> eh=e*(1+0.1*msh.*dsh.^2);

% ----- Calculate additional lateral swelling stress due to suction changes -----
>> syms plsh pls7 pls8 pls9 pls10 pls11
>> plsh=(1-pr-2*pr^2).*psh/(psh/eh.*(1+pr).*(1-pr-2*pr^2)+1-pr^2);

```

```

>> pls7=(1-pr-2*pr^2).*ps7/(ps7/e7.*(1+pr).*(1-pr-2*pr^2)+1-pr^2);
>> pls8=(1-pr-2*pr^2).*ps8/(ps8/e8.*(1+pr).*(1-pr-2*pr^2)+1-pr^2);
>> pls9=(1-pr-2*pr^2).*ps9/(ps9/e9.*(1+pr).*(1-pr-2*pr^2)+1-pr^2);
>> pls10=(1-pr-2*pr^2).*ps10/(ps10/e10.*(1+pr).*(1-pr-2*pr^2)+1-pr^2);
>> pls11=(1-pr-2*pr^2).*ps11/(ps11/e11.*(1+pr).*(1-pr-2*pr^2)+1-pr^2);

% ----- Calculate at-rest earth pressure for saturated soils -----
>> syms par
>> par= pr/(1-pr)*(gamau+9.8)*(D-z);

% ----- Calculate final lateral earth pressures under wetting conditions -----
>> syms pl7 pl8 pl9 pl10 pl11 plh
>> pl7=pls7+par;
>> pl8=pls8+par;
>> pl9=pls9+par;
>> pl10=pls10+par;
>> pl11=pls11+par;
>> plh=plsh+par;

% ----- Calculate passive earth pressures -----
% Passive earth pressure for saturated soils
>> syms pp
>> pp=gamau.*(D-z)*Kp+2*(Kp)^0.5.*c;

% Final lateral earth pressures are limited by the passive earth pressure
>> zh1=[0:0.01:9.40];
>> plhdown=plh(:,1:941);
>> zh2=[9.39:0.01:10];
>> plhup=plh(:,940:1001);
>> ppdown=pp(:,1:941);
>> ppup=pp(:,940:1001);

% ----- Plot the matric suction profiles with respect to different water flow rates-----
>> plot(ms7,z,ms8,z,ms9,z,ms10,z,ms11,z,ms12,z)

% -----Plot the at-rest earth pressure, lateral swelling pressure and
final lateral earth pressure distributions under wetting conditions-----
>> subplot(1,3,1)
>> plot(par,z)

```

```

>> subplot(1,3,2)
>> plot(pls7,z,pls8,z,pls9,z,pls10,z,pls11,z,plsh,z)
>> subplot(1,3,3)
>> plot(plhdown,zh1,plhup,zh2)
>> hold on
>> plot(pl7,z,pl8,z,pl9,z,pl10,z,pl11,z)
>> plot(ppdown,zh1,ppup,zh2)

% ----- Calculate additional lateral swelling forces -----
>> syms pls71 pls81 pls91 pls101 pls111 plsh1
>> pls71=trapz(z,pls7)
>> pls81=trapz(z,pls8)
>> pls91=trapz(z,pls9)
>> pls101=trapz(z,pls10)
>> pls111=trapz(z,pls11)
>> plsh1=trapz(z,plsh)

% ----- Calculate final lateral forces under wetting conditions -----
>> syms pl71 pl81 pl91 pl101 pl111 pl121
>> pl71=trapz(z71,pl7)
>> pl81=trapz(z81,pl8)
>> pl91= trapz(z91,pl9)
>> pl101= trapz(z101,pl10)
>> pl111=trapz(z111,pl11)
>> plh1=trapz(zh2,ppup)+trapz(zh1,plhdown)

```

A.3 Program code for Example B

```
%----- Input soil properties and fitting parameters
-----
>> syms c fi D ks gamau a n m ip k Ka Kp e pr vr
% Effective cohesion of soils
>> c=10;
% Effective angel of internal friction
>> fi=22.5;
% Depth of ground water table
>> D=10;
% Coefficient of permeability of saturated soils
>> ks=1*10^(-7);
% Dry density of soils
>> gamau=18;
% Fitting parameters of SWCC
>> a=0.0014;
>> n=0.6584;
>> m=0.5852;
% Plasticity index
>> ip=18.7;
% Fitting parameter
>> k=-0.0016*ip^2+0.0975*ip+1;
% Elastic modulus of saturated soils
>> e=2000;
% Poisson's ratio
>> pr=0.33;
% Void ratio
>> vr=0.474;

% Coefficient of active earth pressure
>> Ka=(tand(45-fi/2))^2;
% Coefficient of passive earth pressure
>> Kp=(tand(45+fi/2))^2;

>> syms z
>> z=[0:0.01:10];

%----- Input the water flow rates under drying condition -----
```

```

>> syms q1 q2 q3 q4 qh
>> q1=1*10^(-9);
>> q2=2*10^(-9);
>> q3=3*10^(-9);
>> q4=4*10^(-9);
>> qh=0;

% ----- Calculate the matric suction profiles -----
>> syms ms1 ms2 ms3 ms4 msh
>> ms1=log((1+q1/ks)*vr.^(-gamau*a.*z)-q1/ks)/a;
>> ms2=log((1+q2/ks)*vr.^(-gamau*a.*z)-q2/ks)/a;
>> ms3=log((1+q3/ks)*vr.^(-gamau*a.*z)-q3/ks)/a;
>> ms4=log((1+q4/ks)*vr.^(-gamau*a.*z)-q4/ks)/a;
>> msh=log((1+qh/ks)*vr.^(-gamau*a.*z)-qh/ks)/a;

% ----- Calculate degree of saturation -----
>> syms ds1 ds2 ds3 ds4 dsh
>> ds1=1./(1+(a*ms1).^n).^m;
>> ds2=1./(1+(a*ms2).^n).^m;
>> ds3=1./(1+(a*ms3).^n).^m;
>> ds4=1./(1+(a*ms4).^n).^m;
>> dsh=1./(1+(a*msh).^n).^m;

% ----- Calculate active earth pressure distributions under drying conditions -----
>> syms pa1 pa2 pa3 pa4 pah pa
>> pa1=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+ms1.*ds1.^k*tand(fi));
>> pa2=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+ms2.*ds2.^k*tand(fi));
>> pa3=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+ms3.*ds3.^k*tand(fi));
>> pa4=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+ms4.*ds4.^k*tand(fi));
>> pah=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+msh.*dsh.^k*tand(fi));
>> pa=gamau.*(D-z)*Ka-2*(Ka)^0.5.*c;

% ----- Calculate at-rest earth pressure distributions under drying conditions -----
>> syms par1 par2 par3 par4 parh par
>> par1=pr/(1-pr)*gamau*(D-z)-(1-pr)*(1-2*pr)*ms1;
>> par2=pr/(1-pr)*gamau*(D-z)-(1-pr)*(1-2*pr)*ms2;
>> par3=pr/(1-pr)*gamau*(D-z)-(1-pr)*(1-2*pr)*ms3;
>> par3=pr/(1-pr)*gamau*(D-z)-(1-pr)*(1-2*pr)*ms4;
>> parh=pr/(1-pr)*gamau*(D-z)-(1-pr)*(1-2*pr)*msh;
>> par=pr/(1-pr)*(gamau+9.8)*(D-z);

```

```

% ----- Calculate passive earth pressure distributions under drying conditions
% -----
>> syms pp1 pp2 pp3 pp4 pph pp
>> pp1=gamau.*(D-z)*Kp+2*(Kp)^0.5.*(c+ms1.*ds1.^k*tand(fi));
>> pp2=gamau.*(D-z)*Kp+2*(Kp)^0.5.*(c+ms2.*ds2.^k*tand(fi));
>> pp3=gamau.*(D-z)*Kp+2*(Kp)^0.5.*(c+ms3.*ds3.^k*tand(fi));
>> pp4=gamau.*(D-z)*Kp+2*(Kp)^0.5.*(c+ms4.*ds4.^k*tand(fi));
>> pph=gamau.*(D-z)*Kp+2*(Kp)^0.5.*(c+msh.*dsh.^k*tand(fi));
>> pp=gamau.*(D-z)*Kp+2*(Kp)^0.5.*c;

% ----- Plot matric suction profiles under drying conditions -----
>> plot(ms1,z, ms2,z,ms3,z,ms4,z,msh,z)

% ----- Plot lateral earth pressure distributions under drying conditions -----
>> subplot(1,3,1)
plot(par1,z,par2,z,par3,z,par4,z,parh,z,par,z)
>> subplot(1,3,2)
>> plot(pa1,z1,pa2,z,pa3,z,pa4,z,pah,z,pa,z)
>> subplot(1,3,3)
>> plot(pp1,z,pp2,z,pp3,z,pp4,z,pp,z,pph,z)

% ----- Calculate the at-rest earth forces under drying conditions -----
>> syms num1 par11
>> num1=size(find(par1>0));
>> num1=num1(1,2);
>> par11=trapz(z(:,1:num1), par1(:,1:num1))

>> syms num2 par21
>> num2=size(find(par2>0));
>> num2=num2(1,2);
>> par21=trapz(z(:,1:num2), par2(:,1:num2))

>> syms num3 par31
>> num3=size(find(par3>0));
>> num3=num3(1,2);
>> par31=trapz(z(:,1:num3), par3(:,1:num3))

>> syms num4 par41
>> num4=size(find(par4>0));
>> num4=num4(1,2);

```

```

>> par41=trapz(z(:,1:num4), par4(:,1:num4))

% Saturated condition
>> syms pars
>> nums=size(find(par>0));
>> nums=nums(1,2);
>> pars=trapz(z(:,1:nums), par(:,1:nums))

% Hydrostatic condition
>> syms numh parh1
>> numh=size(find(parh>0));
>> numh=numh(1,2);
>> parh1=trapz(z(:,1:numh), parh(:,1:numh))

% ----- Calculate the active earth forces under drying conditions -----
>> syms num1 pa11
>> num1=size(find(pa1>0));
>> num1=num1(1,2);
>> pa11=trapz(z(:,1:num1), pa1(:,1:num1))

>> syms num2 pa21
>> num2=size(find(pa2>0));
>> num2=num2(1,2);
>> pa21=trapz(z(:,1:num2), pa2(:,1:num2))

>> syms num3 pa31
>> num3=size(find(pa3>0));
>> num3=num3(1,2);
>> pa31=trapz(z(:,1:num3), pa3(:,1:num3))

>> syms num4 pa41
>> num4=size(find(pa4>0));
>> num4=num4(1,2);
>> pa41=trapz(z(:,1:num4), pa4(:,1:num4))

% Saturated condition
>> syms pas
>> nums=size(find(pa>0));
>> nums=nums(1,2);
>> pas=trapz(z(:,1:nums), pa(:,1:nums))

```

```

% Hydrostatic condition
>> syms numh pah1
>> numh=size(find(pah>0));
>> numh=numh(1,2);
>> pah1=trapz(z(:,1:numh), pah(:,1:numh))

% ----- Calculate the passive earth forces under drying conditions
-----
>> syms pp11 pp21 pp31 pp41 pps pph1
>> pp11=trapz(z, pp1)
>> pp21=trapz(z, pp2)
>> pp31=trapz(z, pp3)
>> pp41=trapz(z, pp4)
>> pps=trapz(z, pp)
>> pph1=trapz(z, pph)

%----- Input the water flow rates under wetting condition
-----
>> syms q7 q8 q9 q10
>> q7=-2*10^(-8);
>> q8=-4*10^(-8);
>> q9=-6*10^(-8);
>> q10=-8*10^(-8);

% ----- Calculate the matric suction profiles
-----
>> syms ms7 ms8 ms9 ms10
>> ms7=-log((1+q7/ks)*exp(-gamau*a.*z)-q7/ks)/a;
>> ms8=-log((1+q8/ks)*exp(-gamau*a.*z)-q8/ks)/a;
>> ms9=-log((1+q9/ks)*exp(-gamau*a.*z)-q9/ks)/a;
>> ms10=-log((1+q10/ks)*exp(-gamau*a.*z)-q10/ks)/a;

% ----- Calculate degree of saturation -----
>> syms ds7 ds8 ds9 ds10
>> ds7=1./(1+(a*ms7).^n).^m;
>> ds8=1./(1+(a*ms8).^n).^m;
>> ds9=1./(1+(a*ms9).^n).^m;
>> ds10=1./(1+(a*ms10).^n).^m;

% -----Active earth pressure distributions under wetting
conditions-----

```

```

>> syms pa7 pa8 pa9 pa10
>> pa7=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+ms7.*ds7.^k*tand(fi));
>> pa8=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+ms8.*ds8.^k*tand(fi));
>> pa9=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+ms9.*ds9.^k*tand(fi));
>> pa10=gamau.*(D-z)*Ka-2*(Ka)^0.5.*(c+ms10.*ds10.^k*tand(fi));

% -----At-rest lateral earth pressures under wetting conditions-----
>> syms par7 par8 par9 par10
>> par7= pr/(1-pr)*gamau*(D-z)-(1-pr)*(1-2*pr)*ms7;
>> par8= pr/(1-pr)*gamau*(D-z)-(1-pr)*(1-2*pr)*ms8;
>> par9= pr/(1-pr)*gamau*(D-z)-(1-pr)*(1-2*pr)*ms9;
>> par10= pr/(1-pr)*gamau*(D-z)-(1-pr)*(1-2*pr)*ms10;

% -----Passive lateral earth pressures under wetting conditions-----
>> syms pp7 pp8 pp9 pp10
>> pp7=gamau.*(D-z)*Kp+2*(Kp)^0.5.*(c+ms7.*ds7.^k*tand(fi));
>> pp8=gamau.*(D-z)*Kp+2*(Kp)^0.5.*(c+ms8.*ds8.^k*tand(fi));
>> pp9=gamau.*(D-z)*Kp+2*(Kp)^0.5.*(c+ms9.*ds9.^k*tand(fi));
>> pp10=gamau.*(D-z)*Kp+2*(Kp)^0.5.*(c+ms10.*ds10.^k*tand(fi));

% ----- Plot the matric suction profiles with respect to different water flow rates
-----
>> plot(ms7,z,ms8,z,ms9,z,ms10,z,ms11,z,ms12,z)

% ----- Plot the lateral earth pressure distributions under wetting conditions
-----
>> subplot(1,3,1)
>> plot(pa7,z,pa8,z,pa9,z,pa10,z,pah,z,pa,z)
>> subplot(1,3,2)
>> plot(par7,z,par8,z,par9,z,par10,z,parh,z,par,z)
>> subplot(1,3,3)
>> plot(pp7,z,pp8,z,pp9,z,pp10,z,pph,z,pp,z)

% ----- Calculate the at-rest earth forces under wetting conditions
-----
>> syms num7 par71
>> num7=size(find(par7>0));
>> num7=num7(1,2);
>> par71=trapz(z(:,1:num7), par7(:,1:num7))

```

```

>> syms num8 par81
>> num8=size(find(par8>0));
>> num8=num8(1,2);
>> par81=trapz(z(:,1:num8), par8(:,1:num8))

>> syms num9 par91
>> num9=size(find(par9>0));
>> num9=num9(1,2);
>> par91=trapz(z(:,1:num9), par9(:,1:num9))

>> syms num10 par101
>> num10=size(find(par10>0));
>> num10=num10(1,2);
>> par101=trapz(z(:,1:num10), par10(:,1:num10))

% ----- Calculate the active earth forces under wetting conditions -----
>> syms num7 pa71
>> num7=size(find(pa7>0));
>> num7=num7(1,2);
>> pa71=trapz(z(:,1:num7), pa7(:,1:num7))

>> syms num8 pa81
>> num8=size(find(pa8>0));
>> num8=num8(1,2);
>> pa81=trapz(z(:,1:num8), pa8(:,1:num8))

>> syms num9 pa91
>> num9=size(find(pa9>0));
>> num9=num9(1,2);
>> pa91=trapz(z(:,1:num9), pa9(:,1:num9))

>> syms num10 pa101
>> num10=size(find(pa10>0));
>> num10=num10(1,2);
>> pa101=trapz(z(:,1:num10), pa10(:,1:num10))

% ----- Calculate the passive earth forces under wetting conditions -----
>> syms pp71 pp81 pp91 pp101
>> pp71=trapz(z, pp7)

```

```
>> pp81=trapz(z, pp8)
>> pp91=trapz(z, pp9)
>> pp101=trapz(z, pp10)
```

A.4 Program code for estimating the depth of crack

```
%----- Input soil properties and fitting parameters
-----
syms a n m z ks gamaw q pr gamau D ms k ip alfat fi c gamaw ds t ls;
% Fitting parameters of SWCC
a=0.001;
n=0.5991;
m=0.5;
% Depth along the retaining wall
z=[0:0.01:10];
% Coefficient of permeability of saturated soils
ks=5*10^(-8);
% Density of water
gamaw=9.8;
% Water flow rate
q=1.6*10^(-7);
% Poisson's ratio
pr=0.33;
% Dry density of soils
gamau=18;
% Depth of ground water table
D=10;
% Fitting parameters
>> k=-0.0016*ip^2+0.0975*ip+1;
% Modified coefficient for tensile stress in unsaturated soil
>> alfat=0.5;
% Effective angle of internal friction
>> fi=22.5;
% Effective cohesion of soils
>> c=10;
% Density of water
>> gamaw=9.8;

% ----- Calculate the corresponding matric suction profile -----
ms=(-log((1+q/ks)*exp(-gamau*a*z)-q/ks))/a;

% ----- Calculate the degree of saturation -----
>> ds=1./(1+(a*ms).^n).^m;
```

```

% ----- Calculate the tensile stress distribution -----
>> t=-alfat.*cotd(fi).*(c+ms.*tand(fi).*ds.^k);

% ----- Calculate the lateral stress along the retaining wall -----
ls=pr/(1-pr)*gamau*(D-z)-(1-2*pr)*(1-pr)*ms;
% Draw the line to show the retaining wall
syms ul;
ul=0*z;

% ----- Plot the tensile stress of soil and lateral stress distributions -----
plot(t,z,ls,z,ul,z)

```

A.5 Detailed calculation results for both examples

Table A.1 The depth of cracks and ultimate tensile strength under drying conditions (Example A).

Flow rate, q ($\times 10^{-7}$ m/s)	1.55	1.60	1.65	1.70	1.75	1.80	1.85
Crack depth, z_c (m)	0.03	0.31	0.58	0.84	1.08	1.31	1.53
Ultimate tensile strength, t (kPa)	2386	2389	2391	2392	2395	2396	2397

Table A.2 The resultant lateral forces under wetting conditions (Example A).

Flow rate, q ($\times 10^{-10}$ m/s)	0	-3.0	-4.0	-5.0	-6.0	-7.0
Saturated at-rest earth force, P_o (kN/m)	510					
Additional lateral swelling force, P_{LS} (kN/m)	409.72	358.40	340.92	323.08	304.65	285.16
Final lateral earth force, E_p (kN/m)	917.43	868.40	850.92	833.08	814.65	795.16

Note: Negative value of water flow rates represents the infiltration condition.

Table A.3 The critical height under at-rest and active stress states (Example B).

Water flow rates, q ($\times 10^{-8}$ m/s)		Saturated condition	-8	-6	-4	-2	0	0.1	1	5	10
Critical height, z_c (m)	At-rest state	0	0.68	1.26	1.75	2.18	2.56	2.58	2.73	3.33	3.95
	Active state	1.66	2.89	3.69	4.29	4.77	5.15	5.17	5.32	5.88	6.4

Note: Negative value of water flow rates represents the infiltration condition.

Table A.4 The resultant lateral earth forces under drying and wetting conditions (Example B).

Water flow rates, q ($\times 10^{-8}$ m/s)	Saturated condition	-8	-6	-4	-2	0	0.1	1	5	10
At-rest earth force, P_0 (kN/m)	684.65	413.16	387.44	365.72	346.66	329.82	328.93	322.28	295.68	268.20
Active earth force, E_a (kN/m)	279.39	238.19	211.39	191.29	175.21	162.48	161.81	156.78	138.02	120.60
Passive earth force, E_p (kN/m)	2315.15	2469.5	2595.3	2703.5	2799.0	2884.5	2888.5	2924.1	3065.7	3215.0

Note: Negative value of water flow rates represents the infiltration condition.