THE COMPLEXITY OF MATHEMATICS TEACHERS' COLLABORATIVE PROFESSIONAL LEARNING

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ABSTRACT

This qualitative doctoral dissertation investigates the influence of the characteristics and conditions of complex learning systems on the emergence, sustainability, and diffusion of learning of four professional learning communities (PLCs). These PLCs were associated with a large-scale initiative focused on enhancing mathematics teaching and learning in Grade 9 in Ontario, Canada. This study took place in 2021, five years after completion of the professional learning initiative in 2016 and focused on deepening understanding of how learning emerged during the initiative and how learning was sustained and diffused after the original initiative. The study employed a complexity science informed multiple-case study approach, utilizing a variety of data collection methods including secondary analysis of original project data, surveys, and interviews. Data analysis followed an iterative process, using complexity science as a framework, to analyze data and identify key themes.

Findings revealed that all the characteristics and conditions of complex learning systems manifested across the cases and the whole initiative. Key conditions and characteristics such as decentralized control, self-organization, and nestedness supported neighbour interactions which contributed to teacher learning. The Diffusion of Innovation model was utilized to better describe how and why these themes supported teacher learning. These findings contribute to the understanding of how to support teacher learning in educational contexts and provide insights for educators and researchers seeking to promote effective professional learning initiatives.

In conclusion, this study highlights the need to consider educational settings as interconnected nested systems that interact to influence learning across different systems. Professional learning initiatives within these systems need to incorporate teacher agency over an extended time period to allow for the development of professional relationships and communities

of learners. These communities were observed to self-organize in ways that best supported and sustained their own learning and allowed the learning to diffuse to others in learning systems. These findings underscore the importance of creating supportive environments that fosters collaboration, connections, teacher agency, and shared purpose, thereby enhancing professional learning and educational outcomes.

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4SI Student Success School Support Initiative

BTC Building Thinking Classrooms

CoP Community of Practice

DoI Diffusion of Innovation Model

EPMT Enhancing the Pedagogy of Mathematics Teachers

EQAO Education Quality and Accountability Office

ICMI International Commission on Mathematics Instruction

IEP Individual Educational Plan

IMST^2 Innovations in Math, Science and Technology Teaching

#MTBoS Math Twitter Blogosphere

OAME Ontario Association for Mathematics Educators

OECD Organisation for Economic Cooperation and Development

OME Ontario Ministry of Education

OPC Ontario Principals' Council

OTF Ontario Teachers' Federation

PI Principal Investigator

PLC Professional Learning Community

RA Research Assistant

TIMSS Trends in International Mathematics and Science Study

TLLP Teacher Learning and Leadership Program

VNPS Vertical Non-Permanent Surfaces

VRG Visibly Random Groups

CHAPTER 1: INTRODUCTION

Prologue

In February of 2020, before the world shut down due to COVID-19, I was fortunate to be presenting a paper at the 25th International Commission on Mathematics Instruction (ICMI) Study in Lisbon, Portugal. The paper focused on the collaborative learning activities of a high school mathematics professional learning community (PLC) in Ontario that was part of a project called the OAME Grade 9 Applied Mathematics Inquiry Project. The PLC had been the focus of my MA thesis and I was inspired to pursue a doctoral degree because of this PLC and the research I engaged in. At the conference, when I discussed my plans for future research which would focus on how the learning from the PLC influenced others, a teacher from Oklahoma who was attending the conference responded, “well it influenced me”. This teacher explained to me that she follows participants from the project on Twitter and how the work the PLC members share on social media had influenced how she approached teaching mathematics. This interaction suggested to me that exploring different ways that learning emerges, is sustained, and spreads from initiatives such as the OAME project is more complex than one might originally believe. With this realization I began to reimagine the questions that I wanted to ask for this study and the data that I would need to collect in order to try to answer these questions. I began to reconsider how we define collaborative learning among mathematics teachers and how we determine what counts as effective professional learning for mathematics teachers.

Context

Research on effective professional learning for teachers is well documented (e.g., Anderson et al, 2018; Hord, 2008; jedahljedahl, 2018, & Loucks-Horsley & Matsumoto, 1999).

It is understood that effective professional learning is content focused and aligned with curriculum; collaborative, involves collegial learning and inquiry; and is focused on student learning, while also ongoing, and job-embedded (Anderson et al., 2018; Loucks-Horsley & Matsumoto, 1999). Other conditions such as trust, support, and shared leadership; time and space for professional learning and high levels of teacher agency are also important components of effective professional learning (Day, 2017; Hauge & Wan, 2019). What research has found however, is that even when professional learning programs for teachers are designed using these elements they often produce conflicting results (Goldsmith et al., 2014). Opfer and Pedder (2011) wonder why some initiatives, designed using these effective strategies, are unsuccessful while others, with none of the characteristics of effective professional learning, are more successful. Formal professional learning for teachers generally incorporates at least one of the following models: PLCs, instructional coaching, or professional learning workshops (Slama et al., 2021). The evidence that these different approaches to professional learning are effective is minimal and any evidence that does support their implementation is dependent on abundant financial and professional support (Slama et al., 2021). Lastly, when the support for professional learning is removed the benefits and effectiveness appear to decrease as well (Slama et al., 2021).

Statement of the Problem

Decades of research suggest that collaborative professional learning can provide effective support for mathematics teacher learning (Hargreaves & O'Connor, 2018; Jaworski et al., 2017; Loucks-Horsley & Matsumoto, 1999; ndunda, 2017). While the literature indicates that PLCs are an effective strategy for supporting mathematics teacher learning, it is not known if the learning that emerges is sustained within, or diffused beyond, the PLC after financial and outside support is removed (Slama et al., 2021). What research struggles to define is how and why collaboration, and other identified characteristics and conditions, contribute to teacher learning (Borko & Potari, 2019). Understanding ways that collaborative professional activities can lead to teachers' professional learning may greatly enhance opportunities provided to teachers. Current research on professional learning has begun to consider its complex nature and how the use of complexity science can help the field gain a deeper understanding of ways to support and possibly promote learning (e.g. Cochran-Smith et al., 2014; Davis & Simmt, 2003; Ell et al., 2019; Strom & Viesca, 2020). Complexity provides a framework that allows us to consider not only different factors and conditions of professional learning but how these factors and conditions interact, support teacher learning, and lead to the emergence and sustainability of learning (Boylan, 2021; Cochran-Smith et al., 2014; Ell et al., 2019; Opfer & Pedder, 2011). This study provides an opportunity to consider the learning that emerged from a large-scale, sustained professional learning initiative for mathematics educators in Ontario, Canada. It also explores the sustainability of the learning that emerged after completion of the project and whether or not that learning diffused beyond the original project and the project participants.

The OAME Project

In the fall of 2014, the Ontario Ministry of Education (OME) and the Ontario Association for Mathematics Educators (OAME) launched a project intended to support teachers in enhancing the teaching of Grade 9 Applied Mathematics in Ontario. The OME and the mathematics community had concerns about the achievement levels of some of the students in Grade 9 Applied Mathematics (Macaulay, 2015). They had historically not achieved to the same levels as students in Grade 9 Academic Mathematics, as measured by provincial assessments (EQAO, 2019; Macaulay, 2015). The project consisted of two components, a professional learning component and a research component. The professional learning component of the project was designed and piloted by a steering committee formed by the OAME in partnership with the Principal Investigator (PI), Dr. Christine Suurtamm at the University of Ottawa and included representatives from the OME and the OAME. The steering committee met every 6 weeks during the project and these meetings provided opportunities for discussing how the project was progressing, sharing ways that the different PLCs were engaging in professional learning, and to plan ways to share the learning from the project across the different PLCs.

The research component of the project involved a qualitative multiple-case study led and designed by Dr. Suurtamm to provide insight into the collaborative learning the PLCs engaged in (Suurtamm et al., 2017). A research team, composed of several mathematics education researchers and eight research assistants (RAs), was formed by the PI. The research team provided support to the PLCs during monthly meetings and research and resources when needed. The research team also documented the different ways the PLCs negotiated their goals, how they worked together on their goals, and ways that new learning emerged. The design of the project was influenced by the PI's perspective that, as researchers do, teachers also have an inquiry stance when examining their practice (Cochran-Smith & Lytle, 2009). This inquiry stance involves the active, iterative process of engagement in, and reflection on, practice which results in the emergence of new ideas and actions (Suurtamm & Koch, 2019). The collaborative nature of the project and the use of PLCs was informed by complexity theory (Suurtamm & Koch, 2019). This allowed for the consideration of ways that nested communities, in this case the 10 PLCs in this project, were interconnected and supported by the steering committee and research team. A key condition of complex learning systems is diversity. This condition was purposefully built into the project through the requirement that PLCs consist of individuals with varying roles such as administration, special education, and of course mathematics teachers (Suurtamm et al., 2017). The 10 PLCs represented a diversity of contexts and experiences. Some PLCs were situated in large urban settings and others in more remote settings, or serving smaller populations. Some PLCs were established collaborative learning communities and others were new to the practice. A more in-depth discussion of complexity, complex learning systems, and the ways complexity manifested across different aspects of the project is presented in Chapters 3 and 5. The research team met monthly to discuss what they were observing at different research sites across the project. The research team designed components of the project to align with the recent research on effective professional learning for mathematics teachers outlined earlier in this chapter (Suurtamm et al., 2017). Specifically the research team sough to address the research question:

- How can teachers enhance their understandings of the interactions between Grade 9 Applied Mathematics curriculum, pedagogy, and students' needs?

In the spring of 2014 a call went out to schools across the province of Ontario inviting them to participate in the project. Each school would submit an application on behalf of a PLC formed at their school site. Schools then submitted a proposal outlining their problem of practice and PLC membership, which needed to include teachers, department heads, resource teachers, a mathematics specialist, and an administrator (Suurtamm et al., 2017). The ten selected school PLCs were assigned an RA and provided financial support to meet monthly for a full school day for the two years of the project as well as attend several, multi-day, project-wide meetings. A full timeline of the project meetings, activities, related conferences, and knowledge mobilization initiatives related to the project are outlined in Table 1.

Table 1

Timeline of OAME Math Inquiry Project

Type of Activity	Dates	Participants
Year 1		
Project Launch	Oct 2014	Steering committee, PLCs from 10 schools, research team
Regular monthly individual PLC meetings	Nov 2014-Apr 2015 (Dates determined by PLCs)	PLCs
Adobe Connect sessions with PLC leads	Nov 2014-May 2015 (Every 6 weeks)	PLC leads, steering committee, research team
Steering Committee meetings	Sept 2014-July 2015 (monthly or as needed)	Steering committee
Research Team meetings	Sept 2014-Aug 2015 (monthly)	Research team
Presentations at OAME Conference 2015	May 2015	Various members of PLCs and research team
“Wrap” of Year 1	May 2015	PLCs, steering committee, research team
Year 2		
Summer Institute	Aug 2015	PLCs, steering committee, research team
Regular monthly individual PLC meetings	Sept 2015-May 2016 (Dates determined by PLCs)	PLCs
Adobe Connect sessions with PLC leads	Oct 2015-Apr 2016 (Every 6 weeks)	PLC leads, steering committee, research team
Steering Committee meetings	Sept 2015- Sept 2016 (monthly or as needed)	Steering committee
Research Team meetings	Sept 2015- Sept 2016 (monthly)	Research team
May Institute	May 30-31, 2016	PLC leads, steering committee, research team
August Provincial Conferences	Aug 22-23 2016 (Toronto) Aug 24-25, 2016 (London)	600 Ontario educators (total), representatives from

		school PLCs, research team, steering committee
Workshop development	Aug 2016-Feb 2017	Various members of PLCs and research team
Workshop presentations	May 2017 OAME conference	Various members of PLCs and research team
Knowledge Mobilization	Jan 2017-Apr 2017	Steering committee, Kelly McKie

The project was launched in October of 2014 with a project-wide meeting attended by all PLCs, the steering committee, and several members of the research team. Project-wide meetings, such as the project launch, summer institute, Year 1 “Wrap”, and the final project May Institute brought together all participants in the project including the PLCs, the steering committee, and the research team. These meetings were designed and led by the PI, Dr. Suurtamm, with support from the steering committee and research team and provided time and space for the PLCs to work on developing their own approaches to addressing their problems of practice. At the project-wide meetings the PLCs would engage in professional learning activities designed by the PI that would support their exploration of their own practice. An important component of the project-wide meetings was the time for different PLCs to cross-collaborate and share learning with one another.

Each PLC also scheduled, planned, and led their own monthly PLC meetings on site at their school. Different PLCs chose different professional learning models including engaging in book studies, co-designing, and co-teaching lessons, and integrating technology across the mathematics curriculum (Suurtamm et al., 2017). Each PLC was assigned an RA that would attend, and audio record each meeting. The RA was also tasked with supporting the PLC in terms of research and resources needed. For example, as an RA for two PLCs I was asked to arrange a joint visit between the two PLCs, to draft a survey one PLC would use to track student mindset over the course of a semester, and to collect and document a list of rich tasks that one PLC engaged with in their classes.

Findings from the extensive data collected during the project suggested that each of the PLCs involved in the project interpreted and enacted the objectives of the project in ways that they felt best met the needs of their students. Several themes were consistent across each of the cases including a shift in classroom practice towards the incorporation of rich tasks. Parallel to the use of rich tasks were shifts in teachers' views of assessment and how shifting one's practice often aligns with a shift in assessment (Suurtamm et al., 2017). A focus on making student thinking visible also emerged as a theme across all of the cases. How to effectively use vertical whiteboards as a tool for student collaborative work became a focus of the project. PLCs spent time experimenting with Liljedahl's (2016) research on building thinking classrooms (Suurtamm et al., 2017). Lastly, a focus on supporting students in developing a growth mindset was a common area of focus for the PLCs involved in the OAME project (Suurtamm et al., 2017).

Preliminary analysis of the data revealed that collaboration played a key role in the emergence of learning within the PLCs and also between PLCs and beyond (Lazarus et al., 2017; McKie et al., 2017; Suurtamm et al., 2017). Exit interviews conducted with each of the participants suggested the cross-collaboration between PLCs played an important role in pushing participants to think in different ways. This networked collaborative structure was not a design feature of the original OAME project but was prompted by the participants identifying needs and proposing solutions (Suurtamm & Koch, 2019). The OAME research team supported this networking and provided opportunities for it to occur (McKie, 2016; Suurtamm et al., 2017).

Along with the monthly school PLC meetings, multiple project-wide meetings encouraged cross-collaboration amongst PLCs and with the research team (Suurtamm, 2020).

Upon completion of the project the OAME wanted to mobilize the learning that emerged across the province. Different ways they did this included hiring me to create and lead an online book study for teachers focused on a resource many used during the project, *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2015). I was also tasked with designing and creating a website (www.Math4theNines.ca) that would host some of the resources and learning that emerged from the project. These resources included rich tasks, videos of participants presentations at the August 2016 provincial conferences, and keynote speakers' presentations at these conferences. PLC members were invited to participate in creating a "Workshop-in-a-Bag" to be hosted on the math4theNines website as well. These professional learning modules were intended to be downloaded by administrators, learning consultants, or teachers to support them in leading workshops that would explore some of the learning from the project. The topics of these workshops were selected by the participants, who then designed and created tools they thought others might find useful as well. Topics of these workshops included: assessment, a continuum of learning across different grades including Grade 9, *The Five Practices*, making thinking visible, using rich tasks, rearranging the curriculum, using technology, and effective professional learning.

Knowing that the project resulted in the emergence of specific learning, the purpose of this study was to gain a deeper understanding of how this learning emerged, if and how this learning was sustained by the project participants, if the learning diffused beyond the original project participants, and what conditions supported this emergence, sustainability and diffusion. By doing so I contribute insight into the developing concepts of teacher learning, paying particular attention to how the learning develops and emerges. This insight may help to support teacher learning initiatives in the future (Yin, 2009)

Professional Learning in Ontario

This study took place in Ontario, Canada and it is important to describe the context of education and professional learning in the province at the times the project and the study occurred. In Ontario education is the responsibility of the provincial government and is influenced by the political party in power (Boyd, 2021). During the time of the OAME project the government of Premier Kathleen Wynne, an educator and former Minister of Education under the previous premier, Dalton McGuinty, placed a high priority on “valuing, respecting, and developing teachers’ professional practice” (Campbell et al., 2016, p. 222). One way Wynne’s government did this was through supporting teachers’ professional learning. For example, from 2007-2017 the provincial government, in partnership with the Ontario Teachers’ Federation (OTF) ran the Teacher Learning and Leadership Program (TLLP) with the goals to support “experienced teachers to undertake self-directed professional development; develop teachers’ leadership skills for sharing their professional learning and practices; and facilitate knowledge exchange” (Campbell et al., 2016, p. 219). The OAME project also benefitted from the government’s focus on teachers’ professional learning and was provided a substantial amount of money from the Ministry of Education to support high school mathematics educators (Suurtamm et al., 2017). Since the completion of the OAME project the province has experienced several major changes. The first major change came in 2018 with the election of a new premier, Doug Ford, who cancelled funding for the TLLP, introduced a new Grade 1-8 mathematics curriculum, implemented a standardized math proficiency test for teachers, which was ruled unconstitutional by the provincial courts in 2021 (Campbell, 2023), and implemented a new Grade 9 de-streamed mathematics curriculum during a global pandemic. Premier Ford is known to speak negatively of teachers', questioning their knowledge of mathematics, their ability to teach the subject to students, and supporting his opinions with statements that cannot be factually verified. For example, in the summer of 2018 in the Ontario Legislature he stated:

> We have the greatest teachers around, but one third of the teachers who are teaching our students are failing grade 6 and grade 7 math. How can you teach your students when one third of the teachers are failing math? (Miller, 2019)

In the Spring of 2020 the province, along with the rest of the world, entered into the global COVID-19 pandemic which altered the way students were taught and how teachers collaborated professionally. With multiple province wide school closures and classes moving online over a two-year period teachers were asked to accomplish a difficult task of delivering effective and equitable teaching to their students without the usual support of their colleagues and administration at school (Hargreaves, 2021). Teachers were tasked, overnight, with:

> developing their own and their students' digital competence for learning, enabling students to become self-directed and self-determined learners in remote learning environments, and knowing how to teach material in outdoor environments of physical distancing that had never been offered in that manner before. (Hargreaves, 2021, p. 1839)

All of this was done with minimal professional learning due to the immediacy of the needs to the education system during the pandemic. Ultimately, in Ontario "the combination of the impact of the pandemic and of the current Conservative government's approaches to education [has resulted in] negative consequences for teachers' professional lives" (Campbell, 2023, p. 30).

I mention these different changes as they are important factors in my study. In conducting my study I was reconnecting with educators who had been engaged in a long-term, job-embedded, professional learning initiative at a time when space, time, opportunities, and funding for professional learning was more easily accessible. However, during the time of this doctoral study these same teachers were faced with the extremely difficult challenge of remote and/or bimodal teaching and many were faced with teaching the new de-streamed Grade 9 mathematics curriculum with fewer resources and opportunities for collaborative learning. Add to that the pressures every individual faced of living, working, and often parenting through a global pandemic. As I engaged in a study that sought to gain a deeper understanding of the sustainability and diffusion of learning from a project that occurred during a time that was drastically different from the time during which the study was conducted it was important to situate the study both in the context of the original OAME project and the context during the time this doctoral study was conducted.

Overview of Study

This study was focused on an initiative that was designed using research-based characteristics of effective professional learning. Often, professional learning initiatives are closely examined during their implementation, but their sustainability and their influence is paid little attention (Datnow, 2005; Rogers, 2003; Zehetmeier, 2015). Thus, this research was built on the design and research of the initial OAME project, examined ways learning emerged from the project, and explored aspects of the project that may have contributed to the sustainability and diffusion of teacher learning. This was done through a re-examination of the project's original data, a follow-up survey sent to all original participants, and a set of follow-up case studies with particular PLCs to understand their learning and ways their learning may have influenced other learning systems. By learning systems, I am referring to different systems such as teachers, PLCs, classrooms, schools, or districts. Through these methods I aimed to give voice to the participants to highlight their professional learning as teacher voice has been lacking in research in the area of mathematics teacher professional learning (Jaworski et al., 2017). Through the analysis of different PLCs involved in the project I gained a clearer understanding of the relationship between learning and collaboration. Through the process of interviewing multiple participants from selected PLCs, as well as in the learning systems connected to the PLC, I gained a deeper understanding of the extent of learning and the factors that contribute to its sustainability and influence on learning systems.

This study examined the different ways PLCs in the OAME project experienced, sustained, and diffused learning. Specifically, this study addressed the following research questions:

1. How did the characteristics and conditions of complex learning systems influence the emergence of learning in the different PLCs, and associated learning systems, of the OAME project?
2. In what ways did the characteristics and conditions of complex learning systems influence the sustainability and diffusion of learning in the different PLCs, and associated learning systems, of the OAME project?

Contributions of Study

This study contributes to the existing body of educational research on teachers' professional learning. First, it presents a detailed description of different ways that PLCs in Ontario engaged in the process of collaboratively examining mathematics teaching and learning and how they adapted their teaching based on this collaboration. Second, the study offers the unique perspective of revisiting participants five years after completion of the project to describe ways that the learning that emerged may have been sustained and diffused to others in the province. Third, the study provides insight into why, even though the PLCs were established with the same foundational requirements, the learning that emerged was different in different contexts. The study goes beyond looking for a linear cause and effect of single factors to considering ways different conditions and characteristics of a system interact and lead to learning in different ways. Ultimately this study deepens our understanding of how complexity science can be used to better understand mathematics teacher professional learning.

Thesis Outline

This thesis contains six additional chapters. Chapter 2 – Literature Review, provides an overview of research on current perspectives on mathematics education and the role of the teacher, teacher professional learning, collaborative professional learning, sustainability of professional learning, and the diffusion of professional learning before situating my study in this current research. Chapter 3 – Theoretical Framework, begins with a description of my philosophical and theoretical stances including a discussion on my social constructivist worldview followed by an argument for my use of complexity as a theoretical framework. I begin with an overview of complexity science and complex systems before discussing, in more detail, the characteristics of complex systems and the necessary conditions of complex systems that support the emergence of learning. I then make connections between complexity science, social constructivism, and mathematics education research. A discussion on modelling complex learning systems provides a description of the model I chose to represent this study. I conclude by making connections between complexity and Rogers (2003) Diffusion of Innovation model (DoI) and how I incorporated the DoI model into a discussion of my findings. Chapter 4 – Methodology, presents my two research questions, a description of the research design and defines my role as a researcher and my qualifications to complete this study. Following that I describe the data collection instrument designs, how participants were recruited for the study as well as a brief description of each participant and case. I then provide an overview of my data collection which included survey data and semi-structured interview transcripts with participants from both the original OAME project and those who may have been influenced by the original project and participants. I describe my data analysis process before presenting concluding remarks on the methods used for this study. Chapter 5 – Findings, presents extensive discussion of findings from the secondary analysis of the original project data as well as findings from the analysis of the new data collected for this study. These findings are organized by the two research questions. For each research question I first present findings from the project level before describing findings that emerged at the case level. Chapter 6 – Discussion, makes connections between my findings and my research questions. I also describe how the conditions and characteristics of complex learning systems manifested in different learning systems associated with the OAME project and supported the emergence, sustainability, and diffusion of learning. Lastly Chapter 7 concludes with a discussion of the contributions and limitations of the study and considers what questions still remain upon conclusion of the study, what new questions have emerged, and suggests a direction for future research in the area of mathematics teacher professional learning.

CHAPTER 2: LITERATURE REVIEW

This literature review explores current perspectives on mathematics education, the role of the teacher, and effective professional learning for teachers with a particular focus on sustaining and diffusing collaborative professional learning. First, I situate my study within current literature, and I conclude by arguing for the importance of my study and the ways it can contribute new information to the existing research literature.

Current Perspective on Mathematics Education and the Role of the Teacher

Mathematics education has undergone extensive reform over the last few decades owing, in part, to new insights and research into effective teaching and learning (Goldsmith et al., 2014). These reforms move away from the perspective of mathematics as solely a discipline involving rules, algorithms, and procedures to be memorized towards a discipline that involves inquiry, deep conceptual understanding, and problem solving (Boaler, 2016; NCTM, 2000; Suurtamm et al., 2017). As teaching and learning mathematics has evolved, so has the role of teachers. Teachers were once viewed primarily as technicians, tasked with delivering the curriculum (Cochrane-Smyth & Lytle, 2009). Teachers' roles have evolved from managing students' learning, based on a set mandate, towards being "reflective practitioners or inquirers who make decisions about how students learn, how to assess student learning and appropriate pedagogy for the students s/he teaches based on evidence and experience" (Sachs, 2016, p. 420). This presents the view of teachers as professionals who engage in life-long learning focused on enhancing their practice to best meet the needs of their students (Avalos, 2011). If a curriculum with a focus on inquiry asks that students question, debate, and communicate their ideas then professional learning initiatives designed to support this type of learning would follow a similar model

(Schifter, 1998). This professionalization of teachers prompts a perspective of meaningful and productive professional learning to support teachers and the students they serve (Day, 2017).

Current Perspectives on Teacher Professional Learning

High-quality professional learning for mathematics correlates positively with increased student achievement in mathematics (Desimone & Pak, 2017). Understanding what constitutes high-quality professional learning and ways it influences both teacher and student learning is a difficult task (Guthrie, 1990; Kennedy, 2016; Yoon et al., 2007). In order to discuss current perspectives on professional learning and its effectiveness it is important to first define professional learning and the evolution it has undergone over the last few decades. For the purposes of this literature review I consider professional development and professional learning as two separate phenomena. The Organisation for Economic Cooperation and Development (OECD) defines professional development as “activities that develop an individual’s skills, knowledge, expertise and other characteristics as a teacher” (OECD, 2009). Traditional professional development often consists of workshops held for a school or district wide audience that addresses perceived deficiencies in teacher practice (Bruce et al., 2010). Following workshops, teachers are expected to implement new practices or ideas in their classrooms (Boylan et al., 2018). Workshops as professional development were still the most common form of professional learning in a 2014 survey (n=1300) carried out by the Gates Foundation (2014) in the United States. These workshops align with a deficit model of learning involving the assumption that “teachers are deficient in some way and require ‘topping up’ on the latest pedagogical strategies that the teachers will then translate to their classrooms and implement with success” (Bruce et al., 2010, p. 1599). This view of teacher learning often involves a predetermined product designed and delivered *to* teachers in order to change their thinking and ultimately their practice (Slama et al., 2021). The Gates Foundation (2014) found that only 29% of teachers were highly satisfied with current professional development offerings and only 34% felt that professional development had improved in the years they had been teaching. Alternatively, professional learning suggests a process that is done *by* teachers, and “implies an internal process in which teachers create professional knowledge through interaction with colleagues and other educators in a way that challenges previous assumptions and creates new meanings” (Friesen et al., 2004). Clarke and Hollingsworth (2002) agree and suggest that “the motive for learning more about teaching is not to repair a personal inadequacy as a teacher, but to seek greater fulfillment as a practitioner of the art’’ (p. 948). This professionalization of teaching shifts our perspective towards teachers as professionals seeking out and engaging in life-long learning through experience and practice (Webster-Wright, 2009). Teaching and teacher learning can therefore be thought of as an on-going, iterative, emergent process where a teacher is continually growing. Learning involves the on-going engagement in practice, as well as working in collaborative groups, engaging in professional dialogue, analyzing student work, co-planning, co-teaching, attending conferences, working with knowledgeable others, reflective analysis of teaching, and engaging in action research (Campbell et al., 2016). Professional learning that provides space and time for teachers to inquire into their own practice allows teachers the opportunity to engage in research themselves, rather than be the ones being researched (Sachs, 2016). Professional learning specific to mathematics is ultimately aimed at enhancing student learning and should pay attention to student thinking and closely examine mathematical ideas, including ways to approach them with students (Anderson et al., 2018; Smith et al., 2010).

Research on effective professional learning for teachers suggests it be content focused and aligned with curriculum (Ball et al., 2005; Cohen & Hill, 1998; CPRE, 1998; Loucks-Horsley & Matsumoto, 1999; Shulman, 1987); be collaborative, involve collegial learning and inquiry (Roberts & Pruitt, 2009; Vescio, et al., 2008); and be focused on student learning (Darling-Hammond et al., 2017; Smith et al., 2010; Smith & Stein, 2015), while also ongoing, and job-embedded (Anderson et al., 2018; Loucks-Horsley & Matsumoto, 1999; Simmt et al., 2019). Other conditions such as trust, support (Hord, 2008), and shared leadership (Leithwood & Riehl, 2005); and high levels of teacher agency are also suggested (Day, 2017; Hauge & Wan, 2019) and were encouraged.

Teacher Agency

Calvert (2016) defines teacher agency as "the capacity of teachers to act purposefully and constructively to direct their professional growth and contribute to the growth of their colleagues" (p. 4). Teacher agency is enacted when "professional subjects and/or communities exert influence, make choices, and take stances on their work and/or professional identities" (Eteläpelto et al., 2013, as cited in Day, 2017, p. 37). In any model of professional learning it is ultimately up to the teacher what they choose to incorporate into their own classroom, particularly in a jurisdiction such as Ontario, which has historically stressed the importance of teachers' professional judgement (OME, 2010). When teachers have agency they have the ability to exercise choice and decide what action they will take. This applies to how they teach the mandated curriculum and what they engage with in terms of professional learning. Even if an initiative is mandated how a teacher interacts with, and what they choose to take away from, the experience is their choice. Teacher agency, therefore, plays a crucial role in what is learned through engaging in professional learning of any type. Teachers aren't always necessarily willing participants in professional learning and in fact often attend initiatives with established mindsets (Liljedahl, 2018). A study by Liljedahl (2018) looked at five different professional learning contexts over a two-year period and deduced that there is a taxonomy of mindsets with which teachers approach professional learning. These six mindsets include “resistance”, these teachers do not want to be involved and their contributions are generally negative or disruptive in nature. A second mindset is “do not disturb”, these teachers are generally happy to attend but are not interested in drastic pedagogical changes, rather they want small takeaways such as ready-made lessons or an easily integrated technology or tool. A third mindset is “willing to reorganize”, these teachers are a slightly more open to change than the previous mindset but only on specific, self-identified areas, such as a unit on a specific concept or. Fourth is “willing to rethink”, these teachers are more open to a general reorganization on how they approach larger components of their teaching such as assessment or differentiated learning. Fifth is “inquiry”, in general these teachers are interested in rethinking their ideas of knowledge and pedagogy and may be interested in trying new approaches in multiple areas of their practice. The last mindset is “out with the old”, these teachers generally feel that what they are doing isn’t working and they want to question and dig deeper into how to improve their teaching, they want to replace current practices with new practices (Liljedahl, 2018).

Not only is teacher agency a factor in what is learned from established initiatives, but it also plays an important role in the design of initiatives. The professionalization of teachers and the elements of effective professional learning described in the previous section requires the use of robust models of teacher learning with high levels of teacher agency in the initial design phases. These models position teachers as active participants in the design and facilitation of their own learning (Day, 2017). Teachers bring their own experiences and expertise to the practice of teaching, and this is a valuable resource that should be part of any model of teacher professional learning (Guskey, 1986). Teacher agency involves having the authority to influence ones' learning (Hauge & Wan, 2019) and to actively contribute "to shaping…work and its conditions – for the overall quality of education" (Biesta et al., 2015, p. 624). Campbell et al., (2016) examined the impact that programs such as Ontario's cancelled TLLP program had on developing teacher agency. Findings from their study supported the idea that teachers are and should be leaders in their own professional learning and that a priority in designing and supporting teacher learning is the inclusion of teachers' voice in influencing the methods, goals, and content of that learning (Campbell et al., 2016).

Teacher agency is also emerging in the literature related to the use of technology in the form of social media platforms such as Twitter and Facebook. The Gates Foundation (2014) survey found an overwhelming amount of evidence that suggests teachers are taking their professional learning into their own hands and are turning to others online for support. More recent research has explored these ideas as well. Larsen (2019) suggests that the use of Twitter, and specifically a subsection of the social media platform referred to as the Math Twitter Blogosphere (#MTBoS) avoids the de-professionalizing nature that she suggests occurs during more traditional workshops. Larsen (2019) proposes that the "vast capacities for innovation, craft knowing, and classroom expertise [teachers online] hold when taken together supersedes that which may be offered as a single workshop, or even in a workshop series" (p. 322) and that teacher agency drives this online professional learning space. Anderson (2021) also explored the use of social media as a source for teacher led collaborative professional learning. Anderson (2021) studied the interactions in a Facebook group dedicated to mathematics teacher learning and found that teachers were "able to increase their access to knowledge for mathematics teaching by requesting help through questions posed to the group" (p. 587). Again, teacher agency played an important role in the seeking out of support relevant to teachers' personal contexts in ways that align with their immediate needs, as determined by them and not an outside source. Lastly, in their report on the future of math teacher professional learning Slama and colleagues (2021) suggested that teacher-led online professional learning networks are one of several "points of light" (p. 6) that they referred to as "interesting and important early-stage innovation efforts in math teacher learning" (p. 6). These examples all involve teacher agency and the role it plays in teacher professional learning. They also involve collaborative professional learning and its importance, the focus of the next section of this review of the literature.

Collaborative Professional Learning

Because of the design of the OAME project as a collaborative initiative, this literature review will discuss research related to that method of learning specifically. Inquiring into one's own practice with colleagues through collaborative professional learning has been identified as crucial to supporting and sustaining professional learning (Borko & Potari, 2019; Vescio et al., 2008) as well as contributing to overall satisfaction in the profession (Carroll et al., 2010). Research from the Organisation for Economic Co-operation and Development (OECD) indicates collaboration positively influences teachers' practice and teacher self-efficacy, resulting in increased student achievement (Schleicher, 2015). The publication of the Trends in International Mathematics and Science Study (TIMSS) highlights the effectiveness of collaboration on mathematics teacher learning (Stigler et al., 1999). Since the TIMSS report, research has focused heavily on mathematics teacher collaboration as a tool to enhance teacher learning (Borko & Potari, 2019). There is strong consensus that collaboration is key to supporting professional learning and the formation of PLCs is a proven strategy (Roberts & Pruitt, 2009; Vescio, et al., 2008). When PLCs are interpreted and enacted in ways that position teachers as leaders with high levels of agency, teachers take on the role of designers and facilitators of their own learning, respecting each other's professionalism. A PLC is more than a professional development activity, it can be the framework for how a school operates and transforms (Morrissey, 2000). Stoll and colleagues (2006) define a PLC as a "group of people sharing and critically interrogating their practice in an ongoing, reflective, collaborative, inclusive, learning-oriented, growth-promoting way" (p. 233). Simply 'doing' PLCs will not contribute to teacher learning; there are several characteristics and conditions that are necessary (Hord & Sommers, 2008). Hord (2004) identifies the following characteristics for a PLC to operate effectively: shared beliefs, values, and vision; shared personal practice; shared and supportive leadership; collective learning and its application; and supportive conditions. Kruse and colleagues (1995) also suggest certain conditions for effective PLCs: time and space for teachers to meet and talk about their practice and observe each other teach; teacher empowerment and school autonomy; trust, respect, and support for all members of the school community; and opportunities to seek out innovative practices and acquire knowledge.

A study by Rigelman and Rubin (2012) suggests that modelling collaborative practice in teacher education can encourage teacher candidates to adopt collaborative practices early in their career. Their study focused on a pre-service program that employed a triad model of two teacher candidates and one mentor teacher co-planning, co-teaching, reflecting on observations, and analyzing student work from the lessons. This study suggests that teacher candidates' positive experiences with the collaborative model increased their confidence as teachers. The support of, and discussions with, their partner teacher and classmates led to learning about specific practices to support student learning, notably related to inquiry-based mathematics lessons. Furthermore, teacher candidates sought to sustain these practices in their future classrooms (Rigelman & Rubin, 2012)

Bruce and colleagues (2010) studied the effects of sustained, job embedded, professional learning networks for mathematics teachers in Ontario. This study involved researchers and educators co-planning and co-teaching mathematics lessons in kindergarten to grade 6 classrooms. The study found that participants' prior experience with collaborative professional learning had a positive influence on the levels of learning reported as well as the confidence with which teachers accepted leadership roles within the project (Bruce et al., 2010). The researchers note that the group with prior collaborative experiences were more aware of concepts and practices for which they needed more support. Teachers without prior collaborative experiences "did not recognize that their professional practice was at a fairly low level compared to the expectations embedded in reform mathematics education" (Bruce et al., 2010, p. 1607) and did not identify a need to engage in the initiative to the same extent as the first group. Furthermore, students from the first group had greater student learning improvements than the second group (Bruce et al., 2010). In summary, this study suggests that collaborative professional learning can provide opportunities for teachers to deepen their self-awareness, to engage in reflective practice, and determine for themselves areas of their practice to focus on and enhance. Collaborative professional learning provides opportunities for teachers to learn more about different areas of their practice together.

Providing teachers with time and space to work collaboratively is one of the most challenging conditions to maintain in the realities of today's schools. Teaching continues to be a profession in which one works primarily alone (Rigelman & Rubin, 2012). Carroll and colleagues (2010) report that teachers spend 93% of an average workday isolated from their colleagues. When teachers have the opportunity to engage in professional learning projects and interact in collaborative groups, teachers often go back to their isolation once the project has ended due to lack of supportive conditions to sustain the collaboration (Carroll et al., 2010). Issues of the sustainability and diffusion of professional learning initiatives are therefore important factors to consider when designing and researching mathematics teacher professional learning. Each of these areas will be discussed in the following sections.

Sustainability of Professional Learning Initiatives

From a general perspective sustainability refers to the ability of a phenomenon to endure or continue (Tirosh et al., 2015). Being able to determine exactly what is, and how something is, enduring or continuing is crucial when discussing the sustainability of professional learning. However, determining the sustainability of the learning depends on the aims, content, and processes involved (Tirosh et al., 2015). For example, a traditional professional learning model, which incorporates a process product approach, could be said to be sustained if teachers continue to implement a learned teaching strategy in the future. This model would be in line with the view of sustainability as an end goal achieved and unchanging (Tirosh et al., 2015). Collaborative professional learning models such as PLCs sustain learning in a different way. The sustainability of learning that emerges from a PLC is in line with what Zehetmeier (2015) refers to as a flourishing and enduring phenomenon, one that is growing and progressing, where learning continually emerges. This definition of sustainability is very much in line with complexity science, the theoretical underpinning of my proposed study, which I discuss in Chapter 3.

Specific factors, such as motivation, content, activities, and supportive leadership have been identified as supporting the sustainability of professional learning (Tirosh et al., 2015). Jaworski and Huang (2014) suggest that when an initiative exhibits high levels of teacher agency, is motivated by the needs of the participants, and the content aligns with what teachers identify as a need, they are more likely to participate and sustain the learning. In terms of activities, Kaur's (2015) study found that high levels of collaboration and activities that connected to classroom practice helped to sustain the work of a mathematics professional learning initiative after completion of the project. In a study examining two high schools, Hipp et al. (2008) identified shared leadership as a key factor in sustaining the collaborative work in the schools. Participants in Hipp's study stated that due to school leaders' support, they felt safe to take risks and try new practices without fear of negative consequences (Hipp et al., 2008). This is similar to findings in my MA study of a high school PLC. Teachers stated that they were not afraid to take risks because they felt supported by the administration (McKie, 2016). In both the Hipp et al. (2008) study and my MA study (2016), leadership was instrumental in creating time and space for teachers to collaborate.

Evidence of the role that teacher agency plays in supporting job-embedded, collaborative professional learning emerged from a study in Ontario. A large-scale study by Campbell and colleagues (2016) reported on Ontario's Teacher Learning and Leadership Program (TLLP). As described in a previous chapter, funding for the TLLP was provided yearly from 2007 to 2017 to support teacher-led, school-based professional learning in all discipline areas to foster leadership and share best practices across the province (Campbell et al., 2016). Teacher agency was an important component of the program. Drawing from the work of Hargreaves and Fullan (2012), the TLLP centred teachers as the driving force in their own professional learning. The program design was grounded in international research that suggests "teachers' learning, and leadership requires enabling teachers to be agents at the center of educational changes rather than the subjects or recipients of externally mandated reforms only" (Campbell et al., 2016, p. 219). The

TLLP involved teacher leaders applying for funding from the project. The application included a proposal that outlined the work the teacher would do, who would be involved, and the goals of their project, a budget for the professional learning activities, and a plan for sharing their learning with the wider educational community in Ontario. If teachers were accepted they attended a leadership summit prior to the start of their TLLP projects as well as a sharing summit upon completion of their project. These summits were designed to leverage teacher agency and support collaboration, teacher leadership, and provide mentorship opportunities for those that had completed their TLLP projects.

Although collaborative professional learning was not a requirement of the program, 68.7% of teams reported collaborative learning groups were how they sought to improve their learning (Campbell et al., 2016). Funding was provided in 2013 to conduct a 5-year longitudinal study examining the spread and sustainability of the TLLP program. Teachers self-reported high levels of learning with 77.8% reporting a gain in new knowledge/improved understanding and 74.9% reporting improved instructional practice (Campbell et al., 2016). The authors also shared that 98% of participants self-reported that the learning had been sustained beyond the scope of the funding (Campbell et al., 2016). Lastly, many participants reported sharing their learning from the project within their own schools (77%), with other jurisdictions in Ontario (88%), and even some outside of Ontario and internationally (10%) (Campbell et al., 2016). How the shared learning influenced others beyond the scope of the project is not reported, none the less this study highlights the role that teacher agency plays when establishing professional learning opportunities and sustaining the learning that emerges for teachers in Ontario.

More specific to mathematics was Zehetmeier and Krainer’s (2011) case study of a mathematics teacher in Austria that examined features of professional learning that led to the sustainability of learning that emerged. In 1998, in response to Austria's less than optimal performance in the TIMSS study, the federal ministry of education launched the Innovations in Math, Science and Technology Teaching ($IMST^2$) project (Zehetmeier & Krainer, 2011). This four-year project focused on the "initiation, promotion, dissemination, networking, and analysis of innovations in schools…and support[ing]…the quality development of math, science, and technology teaching" (Zehetmeier & Krainer, 2011, p. 879). Similar to the OAME project reported on in this study, the initiative encouraged collaboration and participants defined their own problems of practice and ways of working. Each year the Austrian government funded 50 projects. The researchers revisited one participant 3 years after the end of the project and again 5 years later to gain insight into the sustainability of the learning that emerged (Zehetmeier & Krainer, 2011). Findings showed that the teacher sustained learning related to pedagogical content knowledge because he observed a positive impact on student learning, and it was aligned with his beliefs (Zehetmeier & Krainer, 2011). He was unable to sustain innovative teaching practices due to a lack of collaboration with colleagues (Zehetmeier & Krainer, 2011). The authors argue that the collaborative nature of the project supported networking during the initiative, but this support ended when the project ended (Zehetmeier & Krainer, 2011). This finding led to the suggestion that follow-up initiatives are vital to the sustainability of professional learning (Zehetmeier & Krainer, 2011).

Teacher Well-Being

A theme that emerged from my review of the literature on the sustainability of teachers' professional learning was the importance of teacher well-being (Clarke & Hollingsworth, 2002; Day, 2017; Lovett, 2020; & Sachs, 2016). Teachers' well-being is negatively influenced by many factors both within and beyond their place of work including loss of motivation, loss of agency, and feelings of isolation (Carroll et al., 2019; Day, 2017; Ingersoll, 2001). These same factors can also lead to teacher burnout or even teachers leaving the profession altogether (Madigan & Kim, 2021). Recent data suggests that teacher attrition is a global phenomenon with different countries impacted at varying rates (Kutsyuruba et al., 2019). In the United States 40% of teachers are leaving the profession in their first five years (Carver-Thomas & Darling-Hammond, 2017), while in the United Kingdom the attrition rate in the first five years of teaching is higher at 50% (Lindqvist & Nordänger, 2016). Here in Canada the attrition rate of teachers in the first five years of their careers is between 30-40% (Kutsyuruba et al., 2019). Prior to 2020 the reasons for leaving the profession included teacher "isolation, lack of support, lack of influence, classroom intrusions, and inadequate time for collaboration" (Carroll et al., 2010). The COVID-19 pandemic added new tensions and stresses to the teaching profession which could impact teacher well-being. Some recent studies, conducted during the pandemic, have suggested that up to "72% of teachers [reported] feeling very or extremely stressed, and 57% feeling very or extremely burned out. Many teachers struggled to have a satisfactory work-family balance (37% never or almost never; 20% only sometimes)" (Kotowski et al., 2022). A recent systematic review of research on teacher burnout and student learning concluded that "students being taught by a teacher suffering from burnout tend to perform worse on exams, tests, and receive lower cumulative grades, than those taught by teachers not experiencing burnout" (Madigan & Kim, 2021, p. 9). The long-term effects of teacher burn-out and a decrease in well-being combined with the already existing high rates of teacher attrition are yet to be fully understood but it stands to reason that teacher well-being will have a direct impact on teacher professional learning and the COVID-19 pandemic extremely limited the types of collaborative professional learning that teachers had previously engaged in.

It is well documented that teachers are one of the most important factors in terms of student success in schools (Burroughs et al., 2019) and teacher well-being has been linked to increases in students' achievement (Bajorek, 2014; Split et al., 2011). Teacher well-being can be supported, in part by effective collaborative professional learning, as "teachers need their colleagues to question, challenge and support them in their professional practice to retain their commitment and job satisfaction" (Lovett 2020, pg. 594). A study carried out by Webb and colleagues (2009) compared the experiences of elementary teachers in both the United Kingdom and Finland as they participated in the development of, and engagement in, PLCs in each country. The study suggested that although traditionally the purpose of engaging in PLCs is to increase student achievement, they found that a focus on teacher well-being and teacher professional learning is just as important as focusing on student achievement, as there is a direct correlation between teachers' well-being and student achievement. Webb and colleagues (2009) explained that "sustaining teachers' motivation, commitment and enjoyment of their work is a crucial goal in itself as well as a means to improving pupil learning" (p. 419).

When teachers are provided opportunities to receive mentorship in their first years of teaching and continued opportunities to network and collaborate with other teachers they are more likely to remain at their respective schools (Borman and Dowling, 2008). Lastly, research has suggested that the social contexts of teaching influenced teacher well-being far more than material resources provided by schools. Johnson and colleague's (2012) study points out that the social context of teachers' work environment matters far more than any material factors. They reported that the school's culture, the principal's leadership, and relationships among colleagues were by far the most important factors influencing teacher job satisfaction and well-being. Teachers with low levels of well-being, or that choose to leave the profession of teaching altogether, are less likely to engage in, or sustain, professional learning (Webb et al., 2009). Issues of diffusion of professional learning are often discussed in parallel to the sustainability of professional learning. I now review literature related to the diffusion of learning that emerges from professional learning initiatives.

Diffusion of Professional Learning Initiatives

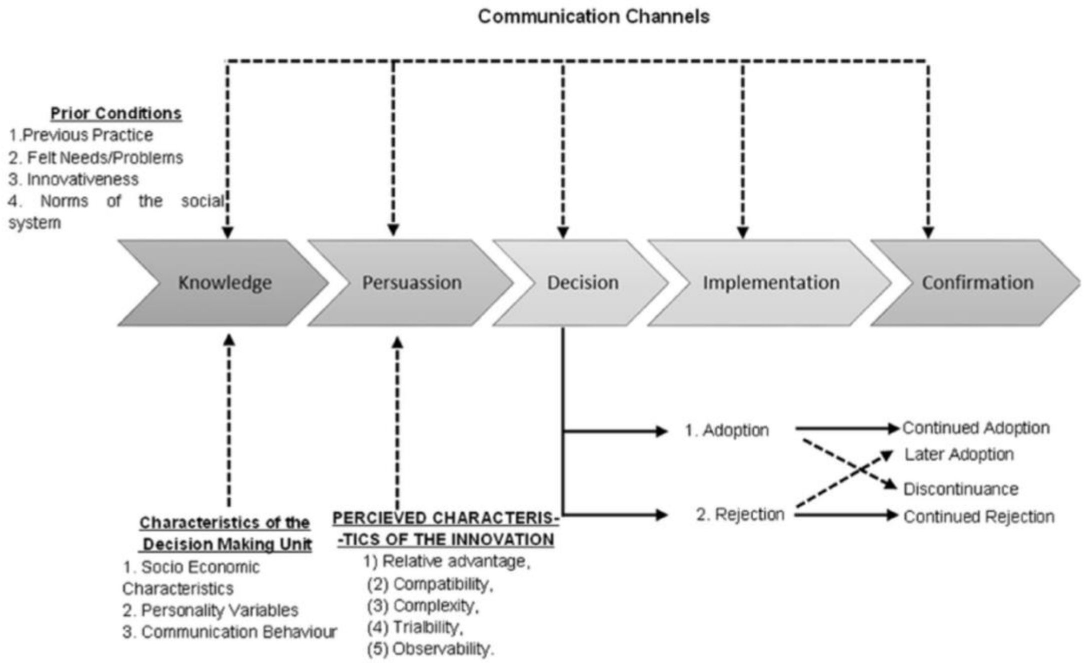
Generally, research literature discusses sustainability in conjunction with the idea of scalability; however, scalability is used and defined in different ways. Often the concept of scalability refers to expanding, the idea that a project carried out with a small group of teachers could be applied to a wider audience (Tirosh et al., 2015). Defining scalability depends on the aims, context, and processes involved in a professional learning initiative. If the end goal is the implementation of a specific classroom practice and it is successful on a smaller scale then scaling up the initiative means implementing it across more schools and districts. In terms of collaborative professional learning that promotes teacher agency, where new ideas and practices are negotiated and learning emerges collectively, the term "scalability" may not apply. Rather, we may need to view ways in which effects of the initiatives spread beyond the boundaries of the initiative. Thus, rather than use the term "scalability", the word "diffusion" seems more appropriate. Diffusion suggests that new ideas and practices are negotiated and adopted in ways that are meaningful to others, beyond the scope of the original project.

The word diffusion used in the research literature on professional learning stems from different fields. Some like Sun et al, (2013) refer to diffusion simply as a *spillover effect*, "the effects of school-based professional development on instructional practices above and beyond the direct effects on teachers who participated in the professional development" (p. 345). Rogers (2003) provides a more comprehensive view of diffusion with his *Diffusion of Innovation* (DoI) theory. Rogers (2004) describes diffusion as “the process through which an innovation, defined as an idea perceived as new, spreads via certain communication channels over time among the members of a social system” (p. 13). The four elements in DoI theory are innovation, communication channels, time, and the social system. In terms of mathematics teacher professional learning the innovation would be an idea or practice that is “perceived as new by an individual or other unit of adoption” (Rogers, 2003, p. 36). A communication channel is the method that messages pertaining to the innovation travel from one individual to others (Rogers, 2003). In education these could include conversations between colleagues, formal professional learning initiatives, conferences, or social media. Time is involved in DoI theory in several ways, including the time it takes to decide to adopt or reject an innovation as well as an innovation’s overall rate of adoption (Rogers, 2003). The social system is a “set of interrelated units that are engaged in joint problem solving to accomplish a common goal” (Rogers, 2003, p. 37). Again, in mathematics education the social system could include a school, a district, or the entire education community including educators, teacher educators, and researchers. In some of Rogers’ final publications before his death he began to argue for the need to consider his DoI model alongside complexity science (Rogers, 2005) and much of this discussion centres on the social system element, which I discuss in more detail in Chapter 3.

Rogers (2003) models the process that individuals experience when deciding whether to adopt or reject an innovation in his model (Figure 1) of the five stages in the innovation-decision process

Figure 1

A Model of the Five Stages in the Innovation-Decision Process (Rogers, 2003)



Note. From "Diffusion of innovations (5th ed.)" by E. Rogers, 2003. Free Press, p. 170

Key stages in this model include knowledge, persuasion, decision, implementation, and confirmation (Rogers, 2003). The knowledge stage involves the introduction of an innovation to an individual or group and they become aware of the innovation. Persuasion is the individual or group forming an opinion of the innovation. Specific characteristics of an innovation that influence persuasion include having an advantage over current ideas, compatibility with one's values and beliefs, ease to comprehend and adapt to one's needs, observable or tangible, and simple to test or trial (Rogers, 2003). The decision phase occurs when a choice is made to either adopt or reject an innovation, and implementation involves putting the new innovation to use (Rogers, 2003). Lastly, confirmation is when a user "seeks reinforcement of an innovation-decision already made" (Rogers, 2003, p. 169). At this stage a user may reverse their original decision on adoption or rejection or maintain their original decision to adopt or reject. In order for an innovation to diffuse it requires effective communication channels and time to spread and be processed by new adopters in an established social system (Rogers, 2003). As I have mentioned, few studies focus specifically on the diffusion of innovations stemming from mathematics professional learning but those that mention diffusion specifically include Kaur (2015), and Zehetmeier (2015) and these provided insights for this study.

Kaur’s (2015) study examined the Enhancing the Pedagogy of Mathematics Teachers (EPMT) project in Singapore (Kaur, 2015). The impetus for the two-year EPMT project was to support teachers in developing tasks that would address reasoning and communication in mathematics classrooms (Kaur, 2011). The ten school PLCs involved in the project were composed of two sets of teaching partners each teaching the same course. These teaching partners engaged in three phases of activities during the project. The professional learning ranged from researcher led training workshops to participant led collaborative professional learning on site (Kaur, 2015). Unlike previous studies I discussed, Kaur (2015) revisited the project participants one year after the project to gain insight into the sustainability and diffusion of the work. Kaur (2015) reported that 80% of the project participants responded to her inquiry and all reported that they were “using the knowledge and skills acquired in the project” (p. 109). The author attributes this sustainability to the relevance of the project to participants’ practice, and the fact that much of the learning was job-embedded (Kaur, 2015). Four schools identified that the work of the project had diffused beyond the original project. New recruits reported they joined the project because of the observed increase in student engagement and learning in the original participants’ classrooms and they wanted to enhance their practice in similar ways (Kaur, 2015). Kaur (2015) references the DoI model explicitly and suggests that mathematics teachers were able to spread their new practices because the ideas (the innovation) exhibited several characteristics identified by Rogers (2003) as influencing diffusion. These characteristics included the fact that the innovation was advantageous for the teachers and easy to test or trial, stating that the innovation “was compatible with the needs of the teachers,…could be experimented in their lessons…[and it] was visible to others when student learning improved” (Kaur, 2015, p. 114).

A study by Zehetmeier (2015) provided another example of the use of the DoI model in mathematics education research. Zehetmeier (2015) used the DoI model as part of a theoretical framework for a qualitative impact analysis of a professional development program in Austria. The framework was used to describe how the different characteristics, identified by Rogers (2003), impact teachers’ knowledge, beliefs, and practice. For example, one teacher shared how the ability to discuss new ideas with colleagues helped to support her learning, Zehetmeier (2015) referred to this as a relative advantage. A second teacher demonstrated the importance of trialability, sharing he felt supported by school leaders to trial new ideas in his classroom which led to his adoption of the new practices. In summary, one of the strengths of the DoI model is its ability to adapt to emergent ideas and innovations that could not have been envisioned in the early years of the models use (Sahin, 2006). This will be discussed further in conjunction with complexity when I discuss my chosen theoretical framework.

Situating Proposed Study

Through a review of existing literature on mathematics teacher professional learning it is evident that there are conflicting findings on the effectiveness of professional learning models and outcomes from similar studies often do not align (Guthrie, 1990; Kennedy, 2016; Yoon et al., 2007). Also there are limited studies that revisit participants after the completion of professional learning projects. Those studies that do revisit participants occur in different contexts with different supporting conditions than my study. This is evident in both the Kaur (2015) and Zehetmeier and Krainer (2011) studies. Kaur’s (2015) study in Singapore revealed high levels of sustainability and some diffusion. The diffusion was due to several factors. The first factor is the observed effects of the project on student engagement and learning in participating teachers' classrooms. The second factor is the willingness of participants to share their learning with individuals beyond the scope of the project. The third factor is the time and space provided to individuals to continue the collaborative professional learning that was initiated through the EPMT project. Kaur's (2015) study differs from the OAME project and Zehetmeier and Krainer's (2011) study due to the different support systems established in the different jurisdictions. The strength of Singapore's professional learning infrastructure has been well documented in the research literature (Bautista et al., 2015; OECD, 2014). Many jurisdictions, like Ontario and Austria, do not provide the same amount of time in a day for teachers to meet and collaborate with colleagues that Singapore does (Darling-Hammond & Richardson, 2009). A focus on determining if and how teachers continue to collaborate was an important consideration when collecting data in order to understand how participants in Ontario find the time and space to collaborate when it is not built into their working day. Zehetmeier and Krainer's (2011) study offered an important perspective on sustainability as they revisited a mathematics teacher several times over an extended period. Their study was limited by their focus on a single teacher at a single site. This reinforced my belief that collecting data from many teachers at several sites, with distinct contexts and conditions, would provide a diverse perspective on the realities of sustaining and diffusing learning that emerges from a professional learning project in Ontario as well as the different levels of systems that may be influenced by such initiatives. These different systems will be explored in more detail in chapter 3.

Summary and Concluding Comments

The research reviewed and discussed above highlights important themes related to mathematics teacher professional learning. What is considered to be effective professional learning for teachers has evolved over the last several decades. Effective professional learning is content focused and aligned with curriculum, collaborative, involves collegial learning and inquiry; and is focused on student learning; and is ongoing, and job-embedded. We now know that teacher agency and well-being are important components related to teacher learning and designing professional learning initiatives that centre the teacher as leaders in their own learning can have a positive impact on both teacher and student learning. One way to centre teachers is to support schools and districts in establishing collaborative learning environments. PLCs can provide time and space for teachers to collaboratively develop their practices, exercise their agency, and pursue problems of practice applicable to their own contexts and students.

The sustainability of professional learning is also a focus of this study and I define it in line with Tirosh et al. (2019) who suggest that sustainability is an initiative's ability to endure or continue. Research in this area suggests that factors, such as motivation, content, activities, and supportive leadership all contribute to the sustainability of professional learning. Teacher agency also supports the sustainability of teacher learning. When teachers play an active role in designing and contributing to the professional learning they engage in with colleagues they are more likely to engage in the initiative and share their learning with others. An outcome of sustaining teacher professional learning can be an increase in teacher well-being. When teachers lose their motivation, feel isolated, and feel a loss of agency in their work they are less likely to continue to engage in, or sustain, professional learning. Teacher well-being has also been directly linked to student achievement, supporting teachers, valuing their agency, and supporting collaborative practices in schools all contribute to the students' achievement.

The diffusion of learning focuses on ways that learning from professional initiatives is shared or spread through a school or other learning systems. Understanding how new ideas and practices are adopted, adapted, and shared can offer insight into how to better support teacher learning. Rogers (2003) DoI model provides a way to analyze ways teacher learning is shared and why certain practices are adopted and not others.

This literature review revealed that often studies designed using similar principles produce conflicting results, in studies deemed successful it is often difficult to understand the key factors that led to the success, and studies on the sustainability and diffusion of learning from initiatives are not as prevalent as studies carried out during initiatives. Following up with a range of participants from the OAME project provided a unique opportunity to compare 10 PLCs created using the same principles, goals, and criteria yet resulted in different ways of working and areas of focus across the PLCs. Some of the PLCs from the original project also agreed to participate in this new study allowing the opportunity to develop a deeper understanding of ways the learning emerged, was sustained, and diffused from the OAME project. I will now move onto discussing the theoretical framework of my study which is directly informed by my review of the research literature of mathematics teacher professional learning.

CHAPTER 3: THEORETICAL FRAMEWORK

In this chapter I present an argument in support of the use of complexity science as a theoretical framework and epistemological approach for this study. I begin this chapter with a discussion of my philosophical and epistemological stances. Following that I provide an overview of complexity science and complex learning systems and describe how my epistemological stance supports the use of complexity as a theoretical framework. I also discuss what complexity asserts about the world and knowledge. I then present a discussion on modelling complex learning systems and how I chose the model of complexity I use to frame this study. Following this I discuss why I chose to incorporate Rogers (2003) DoI model and how it contributes to and compliments the theoretical underpinnings of this study.

Philosophical and Theoretical Stances

My philosophical and theoretical stances are informed by my own experiences as both a learner and a teacher; these experiences contributed to my understanding of the world and fed my curiosity and desire to be an educational researcher. Philosophically I view us, as individuals, as having agency over our learning. Choices, beliefs, and attitudes influence how and what we know. More specifically, teachers are not simply technicians tasked with delivering a mandated curriculum. Rather we are active, reflective, and reactive professionals who make day to day, minute to minute choices that dictate how and what is taught to the students in our classes (Sachs, 2016). Through interactions with students, colleagues, and the world we develop our own individual knowledge of teaching and learning. This philosophical stance aligns with a social constructivist worldview which suggests that the nature of knowledge is based in the exploration of multiple perspectives that participants bring to an experience (Creswell & Poth, 2018).

Educational research is grounded, ultimately, in the study of learning (Jacobson et al., 2019) and when conducted using a social constructivist worldview seeks to understand the multiple perspectives, meanings, and understanding that people have about learning. Rather than seeking one clearly defined, universally agreed upon meaning we look for a “complexity of views rather than narrow[ing] the meanings into a few categories or ideas” (Creswell & Poth, 2018, p. 67). Through my adoption of a social constructivist worldview I acknowledge that I am seeking to understand a phenomenon that is personal to me and one that I also participate in. This being said, I recognize that my own history and current reality as a practicing teacher informs my interpretation of the construction of knowledge that participants share with me (Creswell & Poth, 2018).

As mentioned, educational research is primarily focused on the study of learning. Sharing my stance on the definition of learning is an important component to reading and interpreting this study. According to Jörg and colleagues (2007), within the field of educational research there are over 50 different theories of learning. By adopting a social constructivist worldview I align with theories that view learning as being socially constructed. Researchers such as Dewey, Vygotsky, Bandura, and Wenger have contributed to theories that support this stance.

Dewey (1938) stressed the importance of experiences in education in order to prepare students for an ever-evolving world and future that is unpredictable rather than predetermined. Dewey’s progressive school theory countered the traditional schooling structure which he believed imposed “adult standards, subject-matter, and methods upon those who are only growing slowly toward maturity” (1938, p. 18). Dewey’s description of learning in traditional schools aligns with a deficit model of education. Students were meant to acquire predetermined knowledge rather than participate in the construction of their own meaning of the world. The progressive school philosophy sought to present classrooms as social collectives, where learning was achieved through experiences and interacting with others and their ideas. Ultimately Dewey viewed learning as the creation of new meaning that emerged from an iterative process of experiences. New experiences would influence students' understanding of previous experiences and contribute to the development of understanding of concepts and ultimately the world. The role of the teacher then, was to provide opportunities for experiences that would promote learning in the areas deemed necessary and important.

Vygotsky (1978) was a leading researcher in the development of sociocultural theory which posits that learning is a social process that occurs in social contexts through interaction with others. He believed that knowledge is constructed through these social interactions. In line with Dewey, Vygotsky stressed that learning is not simply acquiring information but rather developing a deeper understanding of the social and cultural contexts in which knowledge is created and used. Similarly Bandura's (1986) social cognitive theory stresses that learning takes place in a social setting and knowledge is created through interacting and observing others in these settings. Bandura differed from Vygotsky in that social cognitive theory suggests that individual agency plays a larger role in his theories of learning while Vygotsky asserts that our social settings play a larger role in what we know. Wenger's (1998) communities of practice (CoP) theory is defined by Wenger as an extension of Vygotsky's sociocultural theory. Wenger (1991) defines CoPs as "groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (p. 1). While Wenger has stressed that CoP theory is not an educational discourse many educational researchers have "interpreted this (CoP theory) description-oriented theory of learning as a prescription-generating theory of teaching" (Davis, 2004, p. 136). One commonality among the different theories discussed is that learning is a social process as is teaching (D'Eon et al., 2000). My definition of learning is derived from the different theories discussed; I view learning as a social process that emerges from experiences. For the purposes of this study I am most interested in the collaborative social processes involved in a community of educators working together to enhance their own learning and the influences on this learning.

Ontologically speaking, a social constructivist worldview suggests that the nature of reality can be different for each individual and that reality for each person is influenced by factors such as race, gender, socio economic status, culture, and age (Creswell & Poth, 2018). The epistemological stance of a social constructivist worldview suggests that reality is co-constructed as the researcher participates in the production of knowledge as they interact with participants, often in the setting being researched, such as a classroom, and construct their own meaning from the data collected, analysed, and described (Yin, 2016). This stance aligns well with my experiences with the OAME project, I was part of two PLCs in the project and was involved in multiple aspects of the project-wide initiative in multiple ways. My own learning was part of the impetus for this study. I next discuss complexity science, complex systems, and how complexity and social constructivism are aligned.

Complexity Science and Complex Systems

Complexity science is a relatively new field which stemmed from systems theory, research on artificial intelligence, and cybernetics in the 1950's (Davis & Simmt, 2003). Complexity science has been used in the social sciences to study phenomena that are not easily understood using positivist, cause-and-effect investigation (Ell et al., 2019). More recently some educational researchers, particularly those in mathematics education, have adopted complexity science as a theoretical framework with a focus on the study of complex learning systems (e.g.

Davis & Sengupta, 2018; Ell, et al., 2019). Studying complex learning systems involves investigating how “relationships between parts of a system can give rise to collective behaviours” (McGarvey et al., 2021, p. 177). These collective behaviours are considered the learning that emerges from these learning systems (Davis & Sumara, 2006). Complex systems are open, non-linear, not predictable, and not defined by the sum of their parts. They are defined by their different parts and by the ways that these parts interact with, and influence, each other, and the environment. It is due to this combination of factors that complex systems are said to be more than the sum of their parts (Byrne, 1998). Cilliers (1998) suggests that there is no simple definition of complexity; instead he argues that complexity can be best understood through the examination of the characteristics of complex systems. This will be the focus of the next section of this chapter.

Characteristics of Complex Systems

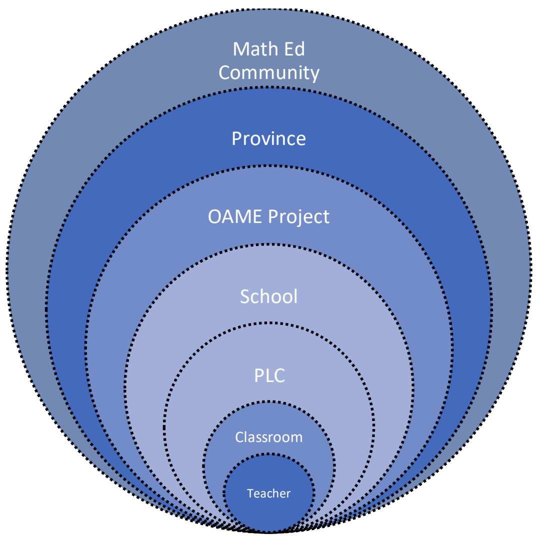
In the literature focused on the study of complex learning systems, different researchers cite characteristics of complex systems in different ways. I align with those who define complex systems by the key characteristics including nestedness, self-organization, and adaptation which interact and result in the emergence of learning (Cilliers, 1998; Davis & Simmt, 2003; Davis & Sumara, 2006). I next discuss each of these characteristics individually.

Nestedness

Complex systems are systems nested within self-similar systems. Self-similarity suggests that smaller components of a system resemble the system as a whole, much like a geometric fractal (Suurtamm, 2020). Nestedness suggests that learning at a smaller scale is similar to the learning of a system as a whole. Understanding the dynamics of a smaller part of a system can lead towards understanding of the larger system (Davis & Sumara, 2006). Within my study this nestedness is represented in the nested nature of education. For example, the teacher, a complex system themself, is nested within different local settings such as their classroom, PLCs, their school, and the OAME project. In this case these systems are all nested within the wider educational system of the province of Ontario, and the greater mathematics education community. A simplistic representation of this nestedness is demonstrated in Figure 2.

Figure 2

Nestedness found in Educational Systems



The boundaries between these nested systems are shown as dotted lines because "boundaries between each entity are permeable, meaning that policies and procedures at any one level can either promote or inhibit actions on one or multiple other levels" (Sands et al., 2022, p. 8).

Self-organization

Self-organization suggests that there is no hierarchical authority imposing instruction on the system; organization is achieved through the non-linear interactions within the system and between the system and its environment (Davis, 2004). This self-organization makes complex systems unpredictable, “they cannot be reduced to, or understood in terms of, straightforward causal inputs and outputs” (McMurtry, 2006, p. 267). Self-organization can occur at multiple levels of a complex system. For example an individual self-organizes at a molecular level as the nervous system and brain respond to different elements (Willshaw, 2006). Teachers collectively self-organize around issues related to their teaching practice and school context. School districts self-organize in response to elements at the district level, the educational system self-organizes at the provincial level based on elements at that level. Most recently this has manifested at the provincial level in Ontario through the cancellation of the Math Proficiency Test for new teachers, the introduction of de-streaming in Grade 9 mathematics, and the provincial response to the COVID-19 pandemic.

Adaptation

Another mechanism by which a complex system responds and evolves is through adaptation. Adaptation within a complex system suggests that a system can change its structure to better fit altered circumstances, but involves more than a predictable, linear response to a system’s environment (Davis & Simmt, 2003). Adaptation occurs within a system as its parts continually interact, respond, and adjust to the dynamic circumstances of the system through iterative feedback loops (Cilliers, 1998). These feedback loops result in adaptation at multiple levels of a system, a simplistic example involves a teacher adapting to a new school setting. As the teacher integrates into the school the teacher is influenced and changed as a result of the school, these changes in turn influence the school, and this continues in a recursive feedback loop (Warren, et al., 1999). Examples of adaptation can be seen in how ideas and experiences influence teachers to adapt their teaching practice, or cultural trends are adopted in society (Davis, 2004). To continue with the example of education, adaptation is represented in the ways that the education system has responded and adjusted to new ideas of equity and inclusivity (OME, 2014), the shift towards the mindset that all students can learn mathematics at high levels (Boaler, 2015), and the role that inquiry-based learning plays in developing deep understanding (Scott et al., 2018). The system has adapted to include a wider perspective of voices and representation. The new de-streamed mathematics curriculum in Ontario is an example of this. The content of the new curriculum highlights how the provincial education system has adapted teaching in order to highlight the importance of culturally responsive and relevant pedagogy (OME, 2021).

Emergence of Learning

All three of these characteristics, nestedness, self-organization, and adaptation interact to support the emergence of learning. Whereas adaptations are the incremental changes that occur in a system due to the interactions and influence of the different components of a system, the emergence of learning is represented in the patterns and collective behaviours that are “learned” as a result of these adaptations (Bar-Yam, 2011; Jacobsen et al., 2019). The emergence of learning is on-going, is unpredictable, and in educational settings such as classrooms or professional learning communities is represented in the collective behaviours that systems adopt. In my study, I am interested in the emergence of learning from the OAME project. I consider this emergence to represent the learning that emerged within different levels of the system, and the system as a whole.

Necessary Conditions for Emergence

Recent research has focused on understanding the conditions necessary to support the self-organization, adaptation, and nestedness of complex learning systems in order to study the emergence of learning within and beyond these systems (Cochran-Smith et al., 2014; Davis et al., 2012; Ell et al., 2019). Key conditions that are necessary to support these characteristics have been identified. I adopt the framework of conditions as developed by Davis and Sumara (2006) these conditions include internal diversity, internal redundancy, neighbour interactions, decentralized control, randomness, and coherence. Davis and Sumara (2006) consider these six conditions as complementary pairs which exist through the tension between each component of the pair:

- specialization—living the tension of internal redundancy and diversity
- trans-level learning—enabling neighbour interactions through decentralized control
- enabling constraints—balancing randomness and coherence (Davis & Sumara, 2006, p. 136).

Considering these conditions as relationships to be negotiated within each pairing highlights the fragility of complex systems and the importance of each condition in supporting the emergence of learning. Without one of these six conditions present in a system, the system may become stagnant or even collapse (Cilliers, 1998). I use the example of a PLC of teachers to discuss how these three sets of complementary pairs manifest in a social grouping and support the emergence of learning.

Specialization

In a PLC internal redundancy is represented by the common traits and common understandings of individuals sharing a profession, such as using the same curriculum, and teaching within the same school system. Redundancy provides commonalities that bring individuals together and provides a reason to interact (Davis & Sumara, 2006). Counter to internal redundancy is internal diversity. Within a PLC this diversity appears in the various experiences, beliefs, and attitudes of different teachers. These differences provide perturbations, allowing for discussion and the possibilities for agreement or disagreement within the community leading to novel responses to the perturbation (Cilliers, 1998). Redundancy and diversity are both essential for a complex system to exist, as together they support the stability of the system while encouraging innovation (Davis & Simmt, 2003).

Trans-level Learning

Trans-level learning in complex systems happens across different levels. For a group to learn, members of that group also individually learn and adapt (Davis & Sumara, 2006). While we often think of neighbours as a physical person or persons, neighbour interactions in a complex system involve more than individuals interacting. More importantly individuals' "ideas, hunches, queries, and other manners of representation" (Davis & Sumara, 2006, p. 142) must bump up against and collide with one another. Continuing with the PLC of teachers example, these interactions could include sharing solutions to problems, co-planning lessons, discussing student work in order to understand student thinking, or engaging in a pedagogical book study. Decentralized control suggests a shared leadership rather than a top-down leadership, where no one individual controls the system. In a PLC trans-level learning might manifest when neighbours, the PLC members, interact with one another and with one another's ideas. These interactions would not be determined by any one individual as "to impose a singular or centralized authority would be to extinguish the potential of the collective as a knowledge producer" (Davis & Sumara, 2008, p. 41).

Enabling Constraints

Enabling constraints involve balancing coherence and randomness. In order for a learning system to endure, coherence to a defined purpose or goal needs to coexist with opportunities for random disruptions which force a system to adjust and adapt in alignment with its goals. (Davis & Sumara, 2006). In terms of a PLC, enabling constraints include teachers alignment in the purpose, goals, and activities of the PLC as well as to the curricular mandates of a jurisdiction. Randomness involves openness and freedom in the ways that the PLC engages in the work of achieving these goals. Strict adherence to one, either coherence or randomness, without balancing the other could lead to little or no emergence from the system and even the collapse of the system (Cilliers, 1998). Having described complexity and complex systems, I next turn to describing how complexity aligns with my social constructivism worldview.

Complexity Science, Social Constructivism, and Mathematics Education Research

I adopted complexity as a theoretical framework as I align with other researchers that consider education as a complex learning system from which learning emerges (e.g., Cochran-Smith et al., 2014, Jacobsen et al., 2019; Opfer & Pedder, 2011). This is in line with my previously discussed definition of learning as a collaborative social process that emerges from experiences. Research in mathematics teacher learning has often focused on specific pieces of professional learning rather than considering its complex nature and the ways that learning is socially constructed (Cochrane-Smith et al., 2014). Reducing teacher learning to individual variables to understand and piece back together does not allow for an understanding of how the different variables interact and influence one another (Opfer & Pedder, 2011). Research on effective professional learning for mathematics teachers is well documented, yet even with this extensive research base different studies have produced conflicting results (Goldsmith et al., 2014). Opfer and Pedder (2011) wonder why some initiatives, designed using these effective strategies, are unsuccessful while others, with none of the characteristics of effective professional learning, are more successful. Perhaps the issue lies in the framework used to analyse these studies as current theoretical models of teachers' professional learning tend towards a linear and reductionist perspective (Cochran-Smith et al., 2014; Garner & Kaplan, 2021; Strom & Viesca, 2020).

The adoption of complexity science as a framework allows for the consideration of not only different characteristics and conditions that influence teacher learning but also how they interact and support learning across multiple levels of the complex system that is education (Jacobsen et al, 2019). Cochran-Smith and colleagues (2014) agree,

> the task of research informed by [complexity science] is describing and explaining how the complexity of factors, levels, interconnections, and systems… function in different local conditions and under differing circumstances to contribute insights about the particular that are also useful beyond the local context and beyond a single moment in time (p. 19).

I have described the characteristics and the conditions necessary to support the emergence of learning from complex learning systems and demonstrated how they might be described in the context of a PLC. I now discuss the compatibility of complexity science and education research focused on professional learning.

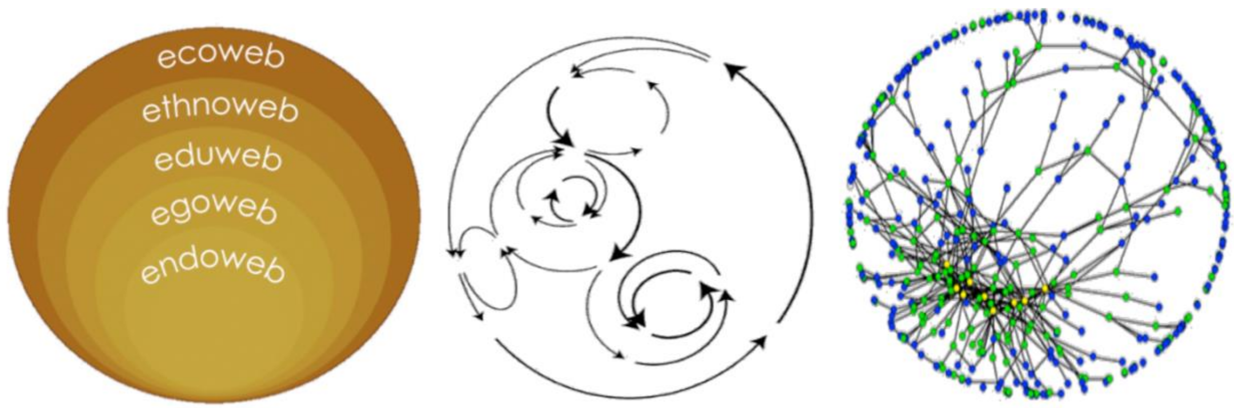
Educational research often reduces phenomenon to a single focus rather than attempting to understand the different factors influencing the phenomenon within each setting (Cochran-Smith et al., 2014). Rather than viewing collaborative learning as occurring within a complicated yet predictable system, made up of many identifiable individual pieces, we need to reimagine it as occurring within a complex system (Clarke & Collins, 2006; Davis & Simmt, 2003). We need to expand our focus beyond the individual and/or the activity (micro) to also consider how the phenomenon is nested within the context of the local setting (meso), the greater educational community (macro), and how all of these levels influence, and are influenced by, the emergent learning (Opfer & Pedder, 2011). For the purposes of this study I define, and focus on, several different levels of a complex system (Figure 2) including the individual teachers (micro), the PLCs, the OAME project (meso), and the greater mathematics education community (macro). Cochrane-Smith and colleagues (2014) argue from a similar stance, "what is needed are new research questions and theoretical frameworks that account for wholes, not just parts, and take complex, rather than reductionist perspectives" (p. 1). The focus on the *whole* of a system of learning led me towards adopting a complexity science framework. The use of a complexity framework will support the identification of "emergent patterns of interaction within and between levels of activity that would constitute an explanatory theory of teacher learning as a complex system" (Opfer & Pedder, 2011, p. 379). This allows me to consider the different characteristics and conditions of the selected PLCs from the OAME that led to the emergence of different outcomes and thus gain some understanding of why.

Modelling Complex Learning System

Davis and Sumara (2012) detail the evolution of modelling complex systems. Early models of complexity focused mainly on the idea that complex systems are composed of self-similar, nested systems. More recently models of complexity have evolved as more fields of research engage with complexity science. The literature has expanded to consider complex systems as entangled dynamics and most recently decentralized networks (Figure 3).

Figure 3

Evolution of Models of Complex Systems



Nested Systems Entangled Dynamics Decentralized Network

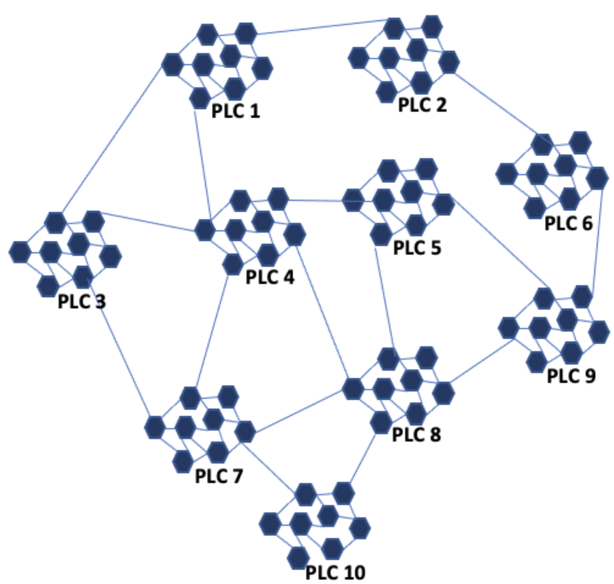
Note. From "Fitting teacher education in/to/for an increasingly complex world," by B. Davis and D. Sumara, 2012, *Complicity: An International Journal of Complexity and Education*, *9*(1), p. 31.

Entangled dynamics introduce the notion that development and learning come about through recursive feedback loops of varying degrees within and between the nested, self-similar systems of earlier complexity models (Davis & Sumara, 2012). Entangled dynamics move beyond the central focus of one individual and consider the ways multiple systems interact within the nested nature of complex systems. This is demonstrated by the nested circular arrows, iterative feedback loops, representing how different pathways of understanding influence other pathways, ultimately moving the system as a whole towards a new state or understanding. Presently models of complexity have evolved to incorporate the idea of decentralized networks and seek to represent the nature of connections between different components of complex systems. Research using complexity frameworks has extended beyond simply describing and analyzing complex systems and has moved towards "occasioning the emergence of complex phenomena… more deliberate efforts to trigger them into being, to support their development, and to sustain their existence" (Davis & Sumara, 2012, p. 31). Studying how nested systems, entangled dynamics, and decentralized networks support the emergence of learning can provide insight into our understanding of mathematics teacher professional learning and ways to create more effective opportunities for teachers.

I now turn to describing how I determined the model of complexity that best represented the individuals and activities involved in the OAME project. I began with a representation of the OAME PLCs (Figure 4) as created by Suurtamm and Koch (2019). Each hexagon represents an individual participant, and each cluster of hexagons represents a PLC. The shorter lines represent the connections between different individuals within a PLC.

Figure 4

Visual representation of OAME PLCs

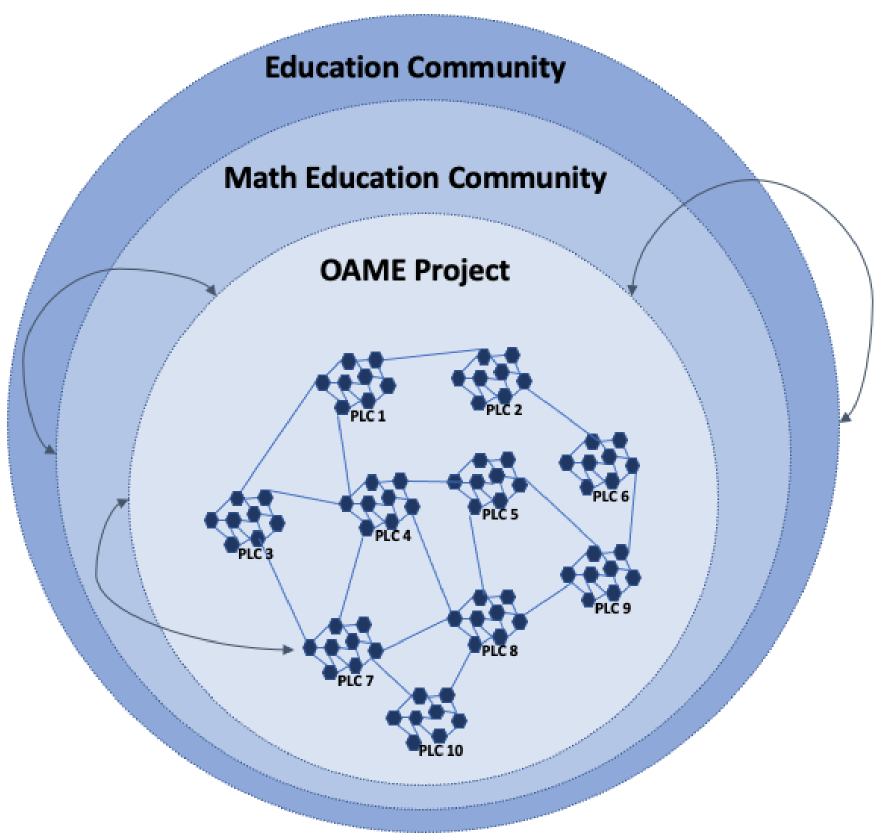


Note. From "*Nested Networks: Creating Space for the Emergence of New Mathematical and Pedagogical Perspectives*," by C. Suurtamm and M. Koch, 2019, Paper presented at the meeting of the American Educational Research Association annual conference, Toronto, Ontario.

As discussed in chapter 1, during the 2 years of the project PLCs began to make connections and collaborate with other PLCs in the project. This is demonstrated through the longer lines connecting the different PLCs. Figure 4 does not represent the entirety of the complex learning system. The PLCs, each made up of individuals, are nested within the complex system of the OAME project. I chose to represent this by overlaying the PLC clusters over the nested representation of the different learning systems associated with the OAME project (Figure 5).

Figure 5

Nested learning systems in OAME project



Interactions across and between different PLCs and their individual members also influenced the learning that emerged from each of these systems, including the project as a whole. The OAME project was nested within the mathematics education community which is nested in the education community. The preliminary analysis of the original OAME data suggested that numerous interactions between project participants, PLCs, and the project influenced the learning that emerged at different levels (Suurtamm et al., 2017). The double headed arrows represent some of the ways that different systems interacted. I suggest that this is still simplistic as it does not account for all of the ways that each different system interacted with other systems even beyond the scope of the systems represented in the diagram. In my opinion, Figure 5 represents a combination of the different models of complex systems shown in Figure 3, nestedness, entangled dynamics, and decentralized networks. This study involved all the components identified in the research of a complexivists study including “co-participations, complex entanglements, decentralized structures, co-adaptive dynamics, self-determination, and non-linear unfoldings” (Davis & Sumara, 2012, p. 30). The study also sought to understand if and how the learning from the project diffused beyond the original project participants. I next describe why I chose to use the DoI model and how it can support our understanding of why learning emerged in the ways it did.

Complexity and Diffusion

This study included developing an understanding of ways that the learning that emerged during the OAME project diffused within the project as well as beyond the project. In order to discuss this diffusion I chose to employ Rogers’ (2003) Diffusion of Innovation (DoI) model in tandem with my use of complexity as a theoretical framework. Through my in-depth review of the literature on DoI theory in Chapter 2 I draw several parallels between the two. Diversity and redundancy in complexity align, respectively, with what Rogers (2003) refers to as heterophily and homophily. Heterophily is the degree to which two individuals are different while homophily is the degree to which two individuals are similar (Rogers, 2003). Rogers suggests that communication is most effective in learning systems with a high degree of homophily, but it is limited to the bounds of that learning system because of the similarities (Rogers, 2003). Diffusion of an innovation requires some degree of heterophilous communication which acts as a bridge allowing an innovation to spread. Similar to the suggestions that a complex system will die if it reaches equilibrium (Cilliers, 1998), an innovation will cease to diffuse beyond a homophilous learning system. Rogers seemingly makes this connection between complexity and DoI, "in cultivating network ties among heterogeneous groups connected by common aims, it is here argued, the innovator may prompt and, to an extent, guide the complex emergence of innovation adoption in social systems" (Rogers, et al., 2004, p. 2). Rogers et al. (2005) expand on this idea when they discuss the compatibility between models that study complex adaptive systems and the DoI model. They suggest that a hybrid approach involving both complexity and DoI is a useful tool for analysing the diffusion of innovations and collaborative networks. Rogers et al. (2005) suggest the rate of adoption in the DoI model is equivalent to the term emergence in complexity. Therefore, I am suggesting that diffusion theory provides language I may use to describe this complex emergence and diffusion of learning that occurred in and from the OAME project.

Summary and Concluding Remarks

An important underpinning of this study was my stance that aligned with others who suggest that education is a complex learning system (e.g., Cochran-Smith et al., 2014; Jacobsen et al., 2019). Another important idea supporting this study was that learning is socially constructed through the iterative process of interacting with others and their ideas and meaning is negotiated through these shared experiences. Learning is different for different people in the same setting; this is part of adopting a social constructivist worldview and as a researcher I am attempting to interpret these different meanings through studying multiple, nested complex learning systems. By adopting a complexity framework I focused on the various characteristics and conditions of complex learning systems that are necessary to support the emergence of learning. The use of the DoI model was used in tandem to help describe how and why learning emerged, was sustained, and diffused through the various learning systems associated with the OAME project. In the next chapter, I describe how I designed this study, including how participants were recruited, as well as how data was collected, and analysed.

CHAPTER 4: METHODOLOGY

This chapter includes a thorough discussion of the study methodology and how the use of complexity science as a theoretical framework informs all aspects of this study design. In Chapter 3 I argued that education is a complex learning system from which learning emerges. Adopting a complexity framework guides me in identifying the different manifestations of complex learning systems in order to better understand the learning that emerged from that system (Davis & Sumara, 2005). Selecting complexity science also impacted the types of questions to ask, how to design the study, what data was collected, how data was analyzed, and how the findings were presented (Ell et al., 2019). Each of these components of the study will be presented and how complexity influenced each component will be described. I first state the research questions which are the foundation of this study and informed all aspects of my methodology. I then describe, in detail, the data collection methods and how these methods connect to the research questions. Next I describe my stance within the study as both a researcher and a participant and how this dual role was supported by my research qualifications and relevant personal experiences. I provide a description of the design of the data collection instruments, recruitment methods, and describe the study participants. I conclude by explaining the process I used to analyse the data that was collected from the study participants.

Research Questions

The purpose of this study was to gain a deeper understanding of the complexity of mathematics teacher collaborative professional learning and the learning that emerged, and continues to emerge, from the OAME project. This study was guided by the following research questions:

1. How did the characteristics and conditions of complex learning systems influence the emergence of learning in the different PLCs, and associated learning systems, of the OAME project?
2. In what ways did the characteristics and conditions of complex learning systems influence the sustainability and diffusion of learning in the different PLCs, and associated learning systems, of the OAME project?

Research Design

I first present a description of the overall structure of this doctoral study before sharing more detail on how I implemented each component of the research design. This qualitative study was a multi-case study involving a total of four cases. Each case included participants who were either original members of a school PLC involved in the OAME project or were somehow connected to an individual who was an original PLC member at that site. Cases were sought that would represent a range of experiences with the OAME project; from those who stated the project did not influence their learning to those who stated the project was very influential to their learning. Data collection and analysis was conducted in four phases beginning with phase 0, a secondary analysis of the original OAME project data set. Phase 1 involved data collection using an online survey tool sent to all original participants involved in the OAME project. This phase was used to identify the four cases that would be involved in this doctoral study. The naming of these four cases, Case 1, 4, 7, and 10 aligned with the original PLC numbers assigned to school PLCs during the OAME project. I chose to maintain this numbering system for logistical purposes as the original data was numbered using the PLC numbers. Phase 2 involved semi-structured interviews with participants from each case. These were individuals who were original OAME project participants. Phase 3 involved data collected through semi-structured interviews with individual participants who were in some way connected to phase 2 participants. Table 2 outlines the different data collection methods, the data collected, and the research questions that the data helped to address. More detail is shared on the collection and analysis of data in each phase in later sections of this chapter. I next situate this study within research on qualitative study methodology.

Table 2

Data Collection Methods, Participants, and Research Question(s) Addressed

Data collection method	Data collection date	Participants	Research question addressed
Phase 0: secondary analysis	Summer 2021	Original OAME project participants	1
Phase 1: survey	August-September, 2021	Original OAME project participants	1 & 2
Phase 2: semi-structured interviews	Fall, 2021	Original OAME project participants	1 & 2
Phase 3: semi-structured interviews	Fall, 2021	Individuals connected to the OAME project through Phase 2 participants	2

When conducting qualitative research the researcher is situated in the world that they are researching and aims to interpret multiple forms of representations of that world (Denzin & Lincoln, 2011). For this study, I sought to describe the learning that emerged from the phenomenon of teachers collaborative professional learning. Through my experience with the OAME Grade 9 Applied Math Inquiry Project I observed, and participated in, different approaches to mathematics teacher collaborative professional learning. My research draws attention to what happened during, and continues to happen because of, a large-scale professional learning initiative focused on supporting mathematics teachers across Ontario.

I selected a multiple-case study approach to guide this research. A multiple-case study provides an approach to qualitative research that explores a phenomenon in context through the collection of a variety of data sources (Baxter & Jack, 2008). Both Stake (1995) and Yin (2014) align a case study approach with a constructivist paradigm which relies on the social construction of reality through a partnership between the researcher and the participants (Baxter & Jack, 2008). The in-context approach to a case study allows the researcher to experience and explore the phenomenon through various lenses. For example, through individual interviews participants are able to share their understanding and experiences with a phenomenon directly with the researcher. Through observations of the phenomenon in context the researcher can describe the phenomenon through a different lens (Baxter & Jack, 2008). A qualitative multiple-case study has been defined by Yin (2014) as "an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (p. 13). My goal for this research was to investigate and describe the experiences of participants, both individually and as collectives, as they worked collaboratively to examine and enhance their own teaching practice and ways they may sustain and diffuse this learning. In this study the phenomenon of teaching and learning is very much dependent on the context within which teachers find themselves and describing the conditions and characteristics that define each case is a central focus of this study. The use of a multiple-case study will then provide opportunities to compare how different contexts, such as PLCs established at different schools across the province, influenced teachers' professional learning.

In this doctoral study a case is defined as being situated or connected to one school PLC that participated in the original OAME project from 2014-2016. Participants in a single case may include any original project participants as well as those who may have been influenced by the work of that PLC. These could include other teachers in the school where the PLC was located, other educators who interacted with that PLC or its members, or researchers who worked in collaboration with the PLC or members of the PLC. By using a multiple-case study I was able to examine across different contexts and describe how these different contexts influenced the emergence of learning at different levels of the OAME project. For this reason I am defining this multiple-case study as exploratory. An exploratory case study "is used to explore those situations in which the intervention being evaluated has no clear, single set of outcomes" (Yin, 2003 as cited in Baxter & Jack, 2008, p. 548).

One of the goals of a multiple-case study in educational research is to provide a description of the phenomena and to offer "something significant to someone (teacher manager, policy-maker, parent, learner etc.) thereby informing her or his work and potentially helping to improve it" (Swann & Pratt, 2004, p. 116). This educational study was ultimately designed to contribute to the educational system in order to enhance how that system operates. Through the use of an exploratory multiple-case study my goal was to understand the different ways that learning can emerge in different contexts. I was influenced by Anderson and colleagues (2005) who suggest combining the case study approach with complexity science to study organizations. They argue that breaking an organization, in this case the education system, into parts to understand will not provide an adequate description of the true nature of the system because it does not consider the relationships, patterns, and influences of different parts on other components and on the system as a whole. By extending a case study approach and applying complexity science, researchers are prompted to focus beyond the boundaries of individual cases to include "the ideas, actions, and interdependencies that occur at and across boundaries" (Cochran-Smith et al., 2014, p. 31). The use of complexity aligns well with multiple-case studies

> because it simultaneously fosters an attitude of attention to emerging patterns, dynamism, and comprehensiveness while focusing attention on defined system properties. The theory suggests that the keys to understanding the system are contained in the patterns of relationships and interactions among the system's agents (Anderson et al., 2005, p. 681).

Complexity provides a framework for studying an education system as "an integrated whole" (Anderson et al., 2005, p. 681). Lastly, extending a case study approach using complexity science requires that we consider multiple cases, Anderson and colleagues explain:

> When more than one case is studied, more than one successful configuration is likely to be found…Complexity theory suggests, however, that there might be more than one way for organizations to be successful. In research, if we seek that one best answer, we will probably find it. Research that is open to more than one way of looking at situations, however, will lead to more useful knowledge. There is likely to be more than one successful process, structure, or configuration of processes and structures (i.e., patterns of organization) within any complex adaptive system. (Anderson et al., p. 679)

Complexity Science offers a tool to move beyond simply describing phenomena towards understanding the dynamic interconnectedness of complex systems such as teaching and learning as well as the possibilities of direction that a phenomenon can take (Jacobsen et al., 2019). Using complexity as a framework requires me to consider not only learning from individuals, but from different nested systems involved in the OAME project, including PLCs, schools, and the greater mathematics education community across the province. Considering findings across the different cases will allow for the exploration of ways that PLCs, and the project as a whole, influenced the learning that emerged both for OAME project participants and for others beyond the scope of the original project.

Researcher's Role and Qualifications

My prior experiences as an educator and researcher allowed me to develop the skills necessary to conduct this doctoral study. As a high school mathematics educator and teacher educator I have extensive knowledge of the Ontario mathematics curriculum and the pedagogical issues related to teaching high school mathematics. Working as an RA on the OAME project provided me the opportunity to develop a deep understanding of conducting research with high school mathematics teachers in Ontario, to develop relationships with many of the project participants, and to combine these experiences to inform my work as a researcher and mathematics teacher educator at the University of Ottawa. More recently these experiences were applied to my work with the OAME to design and create lessons and webinars to support teachers as they implement the new Ontario Grade 9 de-streamed mathematics curriculum introduced in June 2021. My thorough understanding of the Grade 9 de-streamed curriculum, implemented in June of 2021 was an important component of my data collection process as many of the participants were teaching with this curriculum at the time and our conversations often centred on the challenges they were facing with its implementation.

Data Collection and Instrument Design

This study involved the collection of new data during the fall and winter of 2021 which I will describe in detail, and it also involved secondary analysis of the extensive set of data collected during the OAME project between 2014 and 2016 (Table 3). My previous experience as a research assistant with the project as well as a consultant for the OAME after the project, hired to mobilize the knowledge generated, provided me with an extensive foundational understanding of the learning that emerged from different systems in the project. This foundational understanding is a key component of using a complexity framework as establishing the history of a system supports the development of a deeper understanding of the emergence of learning (Ell et al., 2019). Knowing the ways systems existed allowed me to identify the changes in the systems and to also identify the emergence of learning.

I will describe the ways that I incorporated this foundational knowledge in my study when I discuss my data analysis process during each phase. The collection of new data allowed for more specific, intentional collection of data related to the emergence, sustainability, and diffusion of learning from, and beyond, the original project. This new data was collected to deepen my understanding of the learning that emerged during the project, and whether it was sustained and diffused since completion of the project. The data collection for this doctoral study involved purposeful sampling through three phases. It was my goal to offer a range of perspectives on the experiences of the different systems involved in the OAME project and to document ways that the project influenced systems beyond the scope of the original project.

Applying a complexity lens to the multiple-case study approach required that I not only focus on individual cases but also pay close attention to connections between the cases. Data collection during these three phases allowed me to trace the emergence, sustainability, and diffusion of learning from the OAME project. This tracing started with the foundational data collected during the OAME project and continued through each phase of my study. A complete timeline of data collection is provided in Table 6. All data was collected using secure online platforms. As I mentioned, the survey was conducted online using the University of Ottawa's secure *SurveyMonkey* platform. Phase 2 and phase 3 interviews were conducted online using my University of Ottawa secure *Zoom* account. With the consent of the participants, all interviews were video, and audio recorded. Zoom software was used to transcribe the audio from the interviews and reviewed and edited for accuracy by me. I now describe each phase of data collection in more detail.

Phase 0: Data collection

Prior to collecting new data I conducted a secondary analysis of existing data from the OAME project in order to refamiliarize myself with the project (Table 3). To develop a clear understanding of how the project influenced different systems involved in the project I made sure to consider the learning at different levels, including the individual, the PLC, and the project as a whole and therefore reviewed data that targeted all of these different levels. I was fortunate to have been involved in the collection of the original project data in my role as an RA. I was intimately familiar with Case 7 as I had not only been assigned to this specific case but also as Case 7 was the focus of my Master's thesis. For this reason I had a hand in some of the data collection tools designed for the project. Specifically I designed the exit interview questions used at the end of the first year and was involved in designing the exit interviews for the end of the project. I reread individual project exit interviews with participants, the case reports written by each RA for the 10 PLCs involved in the project, and the final project report (Suurtamm et al., 2017). The individual exit interviews provided insight into the perspectives of different participants and highlighted the different experiences that individuals had during the project. The PLC case reports summarized the work of each PLC during the two years of the project and relied on the RA's observational notes as well as audio recordings of PLC monthly meetings. These case reports included a description of the PLC's self-defined problem of practice, how the problem of practice may have evolved during the two years of the project, a description of the PLC membership, the different ways the PLC worked on their problem of practice, and summaries of individual project exit interviews with each PLC member. The final project report, written by the principal investigator with support from the research team, provided a detailed description of the project, an overview of the context and framing of the project, the project research methodology, individual case summaries, analysis of the different themes that emerged across the cases, and detailed recommendations for PLCs beyond the project and ways to support Grade 9 Applied Mathematics education in Ontario. Through this review of the data I was able to add to the extensive foundational knowledge of the project that I mentioned earlier as well as identify specific Cases that could help to fulfill the requirement of recruiting participants with a range of experiences with the project.

Table 3

Secondary Analysis List of Data (collected 2014 - 2016)

Case 1	Case 4	Case 7	Case 10	Project
project application	project application	project application	project application	OAME project final report
Case report year 1	Case report year 1	Case report year 1	Case report year 1	OAME project executive summary
Case report year 2	Case report year 2	Case report year 2	Case report year 2	OAME Year 1 wrap mixed group discussion transcript 1 of 3
	Year 1 School report	Year 2 Summary report	Year 1 meeting 1 notes	OAME Year 1 wrap mixed group discussion transcript 2 of 3
Year 1 meeting 1 transcript	Year 2 summary report	Year 1 meeting 1 transcript	Year 1 meeting 1 transcript	OAME Year 1 wrap mixed

				group discussion transcript 3 of 3
Year 1 meeting 2 transcript	Year 1 meeting 1 transcript	Year 1 meeting 2 transcript	Year 1 meeting 2 transcript	
Year 1 meeting 3 transcript	Year 1 meeting 2 transcript	Year 1 meeting 3 transcript	Year 1 meeting 2 notes	
Year 1 meeting 4 transcript	Year 1 meeting 3 transcript	Year 1 meeting 4 transcript	Year 1 meeting 3 transcript	
Year 2 meeting 1 transcript	Year 1 meeting 4 transcript	Year 1 meeting 5 transcript	Year 2 meeting 1 notes	
Year 2 meeting 2 transcript	Year 1 Summer Institute focus group transcript	Year 1 meeting 6 transcript	Year 2 meeting 1 transcript	
Year 2 meeting 3 transcript	Year 2 meeting 1 transcript	Year 1 meeting 7 transcript	Year 2 meeting 2 notes	
Year 2 meeting 4 transcript	Year 2 meeting 2 transcript	Year 1 meeting 8 transcript	Year 2 meeting 2 transcript	
Year 2 meeting 5 transcript	Year 2 meeting 3 transcript	Year 1 Dolores exit interview transcript		
Year 2 Georgia exit interview transcript	Year 2 meeting 4 transcript	Year 1 Rachel exit interview transcript		
Year 2 Jill exit interview transcript	Year 2 meeting 4 observation notes	Year 1 Tasha exit interview transcript		
Year 2 Drew exit interview transcript	Year 2 meeting 5 transcript	Year 1 Wayne exit interview transcript		
Year 2 Zoe exit interview	Year 2 meeting 5 observation notes	Year 1 William exit interview transcript		
	Year 2 meeting 6 transcript	Year 1 Summer Institute focus group transcript		
	Year 2 meeting 6 observation notes	Year 2 Visit 1 transcript		

	Year 2 meeting 7 transcript	Year 2 Visit 2 transcript		
	Year 2 meeting 7 observation notes	Year 2 Visit 3 transcript		
	Year 2 meeting 8 transcript	Year 2 Visit 4 transcript		
	Year 2 meeting 8 observation notes	Year 2 Visit 5 transcript		
	Year 2 Chrissy exit interview	Year 2 Visit 6 transcript		
	Year 2 Daniel exit interview	Year 2 Visit 7 transcript		
	Year 2 Jennifer exit interview	Year 2 Visit 8 transcript		
	Year 2 Summer Institute Focus Group Interview	Year 2 Visit 9 transcript		
		Year 2 exit interview transcripts Dolores, Lisa, Tasha, Wayne, William		
		Year 2 researcher journal		
		Year 2 Summer Institute Focus Group transcript		

Phase 1: Instrument Design and Data Collection

Instrument design. An online survey was designed for phase 1 data collection in late summer of 2021 (Appendix B). This survey was hosted on *SurveyMonkey;* a secure online resource hosted by the University of Ottawa. There were a total of 12 questions designed to gather information about the participant’s position, role in the PLC formation, how the PLC operated, the influence the project had on the participant's learning, and whether the learning from the project was sustained. Lastly the survey gathered information about the collaborative nature of the project and whether that component was also sustained. The average time to complete the survey was 15 minutes and concluded with an option for individuals to agree to participate in the next phase of the study. Individuals could participate in the survey anonymously or choose to share their name and email address if they agreed to be contacted for phase 2 of the study.

Data collection. Responses to the survey from original participants were reviewed and analysed in order to build my understanding of the experiences of individual participants both during and since the OAME project. The collection of this data served several purposes, the first was it allowed me to add to the foundational knowledge I had established during phase 0 by isolating individual voices. Much of the data reviewed in phase 0 was transcripts of the monthly PLC meetings. These transcripts capture the collective discussions of the group as they focused on their problem of practice as opposed to individual responses to specific questions or prompts from the research team. Another purpose of phase 1 data collection was to gain insight into the sustainability of the learning that emerged from the project and the factors that contributed to this learning. The third purpose was to identify individuals to invite to participate in phase 2. My intention was to identify and invite participants with a range of experiences with the project. Ideally these experiences would include those that stated the project had a minor influence on their professional learning to those that state the project had a major influence on their professional learning.

Phase 2: Instrument Design and Data collection

Instrument design. Semi-structured interviews were used to collect data in phases 2 and 3 of this doctoral study in the fall of 2021. Examples of interview protocols are shared in Appendix D and Appendix E. The interviews were semi-structured in order to allow for inquiry into topics and issues that arose during the course of the interview. I was more familiar with some participants as I had worked closely with their PLCs during the OAME project. Interviews with these participants were less formal as I could ask more detailed questions, such as if the specific work they had begun during the project continued. With other participants, who were unfamiliar with me and my role in the project, the questions were more exploratory and allowed me to develop a deeper understanding of their specific role within the project, the ways that they interacted with the project activities, and the learning that emerged from the project.

Data collection. The second phase of data collection occurred during the fall of 2021 and involved participants from the original project PLCs. These participants were identified through analysis of the phase 1 survey. The participants who were identified agreed to participate in phase 2 of the study and also indicated different experiences with the project. These different experiences are necessary for an exploratory multi-case study in order to highlight the different outcomes experienced through the project. Data collection in phase 2 involved semi-structured individual interviews (Appendix D). These interviews provided a deeper look at the individual, and shared learning experiences within the nested systems of the PLCs and the OAME project as a whole. The interviews provided insight into the emergence and sustainability of learning. A sample letter of information and the consent form for phase 2 are included in Appendix F. Prior to conducting an interview I reviewed the participant’s finished survey to familiarize myself with their responses and to ensure I did not ask the participants questions they had already answered in the survey. I would then use my proposed phase 2 interview protocol to create custom questions that would honour the responses already provided by the participant and to ensure that the questions I was asking were applicable to my study as well as to the circumstances of each individual participant. For instance, with participants from a case that had been the focus of my MA thesis, I did not need to ask for many details on how their PLC worked as I was present at all of their monthly meetings. With other cases that I was not as familiar with it was helpful to ask more detailed questions about how their PLC worked and what they focused on. The purpose of phase 2 data collection was to identify characteristics and conditions of PLCs, and associated learning systems, within the OAME project and to describe how these influenced the emergence, sustainability, and diffusion of learning.

Phase 3: Data collection

Instrument design. Semi-structured interviews (Appendix E) were used in phase 3 of data collection and were similar to the phase 2 interview protocols discussed in the previous section.

Data collection. Phase 3 data collection included semi-structured interviews in late fall of 2021 (Appendix E) with participants recommended by phase 2 participants; this will be described in more detail when I discuss participant recruitment. Phase 3 data collection was designed to gain an understanding of the characteristics and conditions of learning systems associated within, and beyond, the OAME project and how they influenced the emergence, sustainability, and diffusion of learning.

Recruitment of Participants

Phase 1: Recruitment

Participant recruitment for this study also occurred in three phases. For the phase 1 survey I reached out to the executive of the OAME and asked for their support. My continued work with the OAME has allowed me to develop a working relationship with the executive director. The executive director agreed to send out an email (Appendix H) to all of the original OAME project participants on my behalf. This email, sent at the end of August 2021, invited the participants to complete the phase 1 survey. Approximately 80 individuals were invited to complete the survey. An exact number is not known as some participants have retired or moved to new school districts and therefore their emails did not reach the intended recipient. Response to the survey was slow. This could have been due in part to the timing of the survey aligning with a stressful time for mathematics teachers in Ontario. Schools in the province were navigating the global COVID-19 pandemic which resulted in teachers adjusting to online and hybrid teaching. The year leading up to this study saw several school shutdowns and the shift to online learning. Simultaneously, teachers in Grade 9 were adjusting to a newly released de-streamed mathematics curriculum in Ontario. Resources that would support teachers with this new curriculum were slow to be released and professional learning was limited due to the pandemic. In person teacher collaboration that would have normally supported the implementation of the new curriculum was no longer possible in order to reduce the spread of the coronavirus. All of these factors may have impacted the number of teachers willing to participate in a study during this time. A follow-up email was sent in September 2021. In total 14 surveys were completed, five anonymously and nine participants agreeing to be contacted for continued participation in the study (Table 3). Although there were fewer survey responses than I had hoped for, the nine individuals that did respond were from four PLCs and these four PLCs experienced the range of experiences that I had deemed necessary for this study. Each of these four PLCs formed the foundation of the four cases in this study.

Phase 2: Recruitment

Phase 2 recruitment involved contacting participants from phase 1 who indicated on the phase 1 survey they would participate in phase 2 and who met one of the experience requirements during the project. Individual emails were sent to the nine participants that agreed to participate in Phase 2. This email included a letter of information about the study as well as a letter of informed consent (Appendix F). All nine participants agreed to participate in phase 2, dates and times were agreed upon for the semi-structured interviews and these interviews occurred between November and December of 2021. I also posted a message on twitter reminding all participants from the OAME project to check their emails for the link to the survey. This post resulted in a connection being made with a participant who had retired and did not receive the original phase 1 recruitment email. She agreed to participate in phase 2 and an email was then sent to her containing the invitation, letter of information, and letter of informed consent. Another participant was recruited through a colleague who completed the survey and participated in a phase 2 interview. During her interview she mentioned that I should speak to one of her colleagues that participated in the OAME project but had also since retired. She agreed to send an email to this colleague inviting him to get in touch with me. He replied shortly after with an email to me agreeing to participate. Lastly, one of the research assistants from the project agreed to participate in phase 2. He was contacted directly by me through email. In total, there were 12 participants in phase 2 of my study (Table 3).

While the nine participants in phase 2 who did complete the survey did indicate a range of experiences with the project it is important to note that six of these participants indicated that the project had a large influence on their learning. One participant indicated the project had a small influence and two participants indicated the project had a moderate influence. This would suggest that individuals who had a positive experience with the project were more likely to participate in a study and the findings did not necessarily reflect the experiences of the project as a whole. However, listening to those that felt the professional learning was beneficial offers insight into the ways learning emerged, was sustained, and diffused. These findings will be discussed in more detail in Chapter 6.

Phase 3: Recruitment

Phase 3 participant recruitment involved requesting phase 2 participants suggest someone to me, who was not directly involved with the OAME project, who they believe may have been influenced by the project. I then sent a letter of information (Appendix G) to phase 2 participants and asked them to forward the email to those individuals they identified. Phase 2 participants suggested a total of 13 different individuals, and I received emails from six of these individuals agreeing to participate in phase 3 of the study. Two other participants were recruited for phase 3 through my own personal contacts and my knowledge of their connection to a phase 2 participant. One was a teacher candidate who had made a connection with one of the OAME PLCs through my course at the University of Ottawa and the other was the individual I had met at IMCI who I described in Chapter 1. The eight phase 3 semi-structured interviews took place during November and December of 2021, and each lasted between 17-54 minutes. A complete list of the data collected is outlined in Table 4. During the proposal stage of my doctoral study I had suggested that focus group interviews could possibly be used. During the interview process and speaking with the 21 individuals in phase 2 and 3 it became evident that they were under a great deal of stress and overwhelmed with work. I made the decision to not conduct focus group interviews as I determined that the individual interviews would be sufficient in answering my research questions and that it would not be appropriate to ask the participants to give up more of their limited time to attend a focus group interview.

Table 4

Overview of Collected Data

Data	Timeframe	Quantity
Phase 1: survey	August, 2021	14 total surveys completed Case 1: 2 completed Case 4: 1 completed Case 7: 3 completed Case 10: 3 completed Anonymous: 5
Phase 2: semi-structured interviews	Fall, 2021	13 total interviews conducted Case 1: 3 conducted Case 4: 2 conducted Case 7: 5 conducted Case 10: 3 conducted
Phase 3: semi-structured interviews	Fall, 2021	8 total interviews conducted Case 1: 0 conducted Case 4: 1 conducted Case 7: 5 conducted Case 10: 2 conducted

Table 5

Full List of Participants

Pseudonym	Case	Phase	Position	Years Teaching as of 2021
Drew	1	2	School District Lead Teacher	15
Jill	1	2	Mathematics Teacher	20
Tom	1	2	Research Assistant	12
Chrissy	4	2	Mathematics Teacher, Mathematics Department Head	26

Daniel	4	2	Mathematics Teacher	31 (retired)
Martina	4	3	District Math Coach	7
Wayne	7	2	Mathematics Teacher, Mathematics Department Head	27 (retired)
Tasha	7	2	Mathematics Teachers, Special Education Teacher	17
Dolores	7	2	Principal	(retired)
William	7	2	Mathematics Teacher	30
Lisa	7	2	Mathematics Department Head	25
Laurey	7	3	Mathematics Teacher	24
Bridgette	7	3	Teacher Candidate	0
Jeremy	7	3	Principal	30+ (retired)
Dr. Liljedahl	7	3	Researcher	
Mona	7	3	Provincial Principal Coach	25+ (retired)
Charlotte	10	2	Special Education Teacher, Department Head Spec. Ed.	23
Elena	10	2	Mathematics Teacher	21
Maya	10	2	Mathematics Teacher	32
Karen	10	3	Mathematics Teacher	18
Janet	10	3	Mathematics Teacher	10

Table 6

Timeline and Summary of Collected Data

Date	Duration	Data	Participants
Aug -Nov, 2021	N/A	Phase 1 Surveys	Drew, Jill, Wayne, Tasha, Charlotte, Elena, Maya , Chrissy, William, 5 anonymous

Nov 15, 2021	1:00:44	Phase 2 interview	Wayne Case 7
Nov 15, 2021	0:50:52	Phase 2 interview	Tasha Case 7
Nov 16, 2021	0:41:08	Phase 2 interview	Chrissy Case 4
Nov 18, 2021	0:45:32	Phase 2 interview	Dolores Case 7
Nov 19, 2021	0:29:57	Phase 2 interview	Maya Case 10
Nov 22, 2021	0:29:42	Phase 2 interview	William Case 7
Nov 22, 2021	0:31:48	Phase 3 interview	Mona Case 7
Nov 22, 2021	0:25:58	Phase 2 interview	Charlotte Case 10
Nov 23, 2021	0:33:47	Phase 2 interview	Elena Case 10
Nov 24, 2021	0:54:23	Phase 3 interview	Laurey Case 7
Nov 25, 2021	0:36:36	Phase 2 interview	Daniel Case 4
Nov 25, 2021	0:17:33	Phase 3 interview	Karen Case 10
Nov 29, 2021	0:42:19	Phase 2 interview	Drew Case 1
Nov 29, 2021	0:23:04	Phase 3 interview	Jeremy Case 7
Nov 29, 2021	0:25:23	Phase 3 interview	Bridgette Case 7
Nov 30, 2021	0:41:13	Phase 2 interview	Jill Case 1
Dec 3, 2021	0:27:25	Phase 3 interview	Janet Case 10
Dec 6, 2021	0:28:11	Phase 3 interview	Martina Case 4
Dec 7, 2021	0:50:25	Phase 3 interview	Dr. Peter Liljedahl Case 7
Dec 27, 2021	0:45:54	Phase 2 interview	Tom Case 1

Participants

Phase 2 and 3 included 21 semi-structured interviews.Tree diagrams for each of the 4 Cases involved in this study are used to illustrate how these 21 individuals are connected. The light blue rectangles in each tree diagram represents an original OAME project participant and the medium blue rectangles represent a phase 3 participant recruited to participate in the study through their connection with a phase 2 participant.

Figure 6

Case 1 Participant Structure

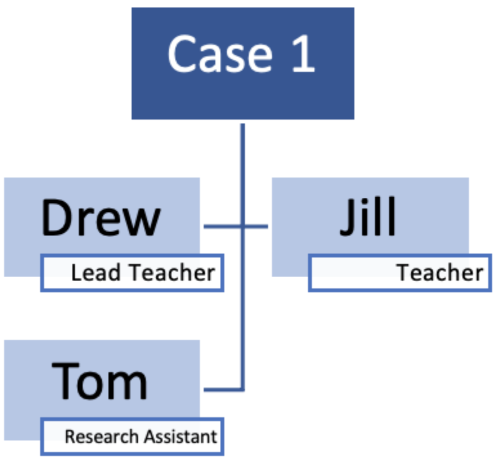


Figure 7

Case 4 Participant Structure

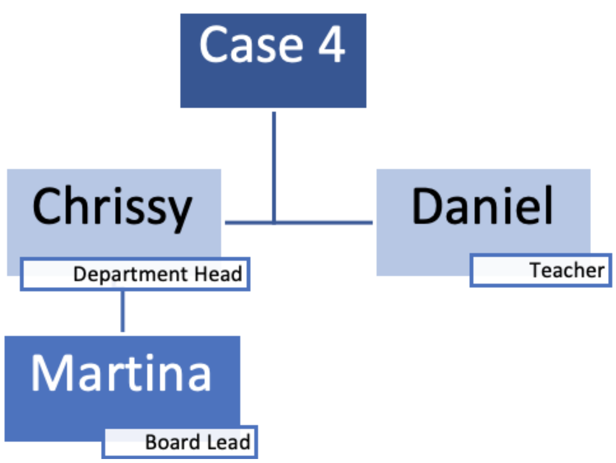


Figure 8

Case 7 Participant Structure

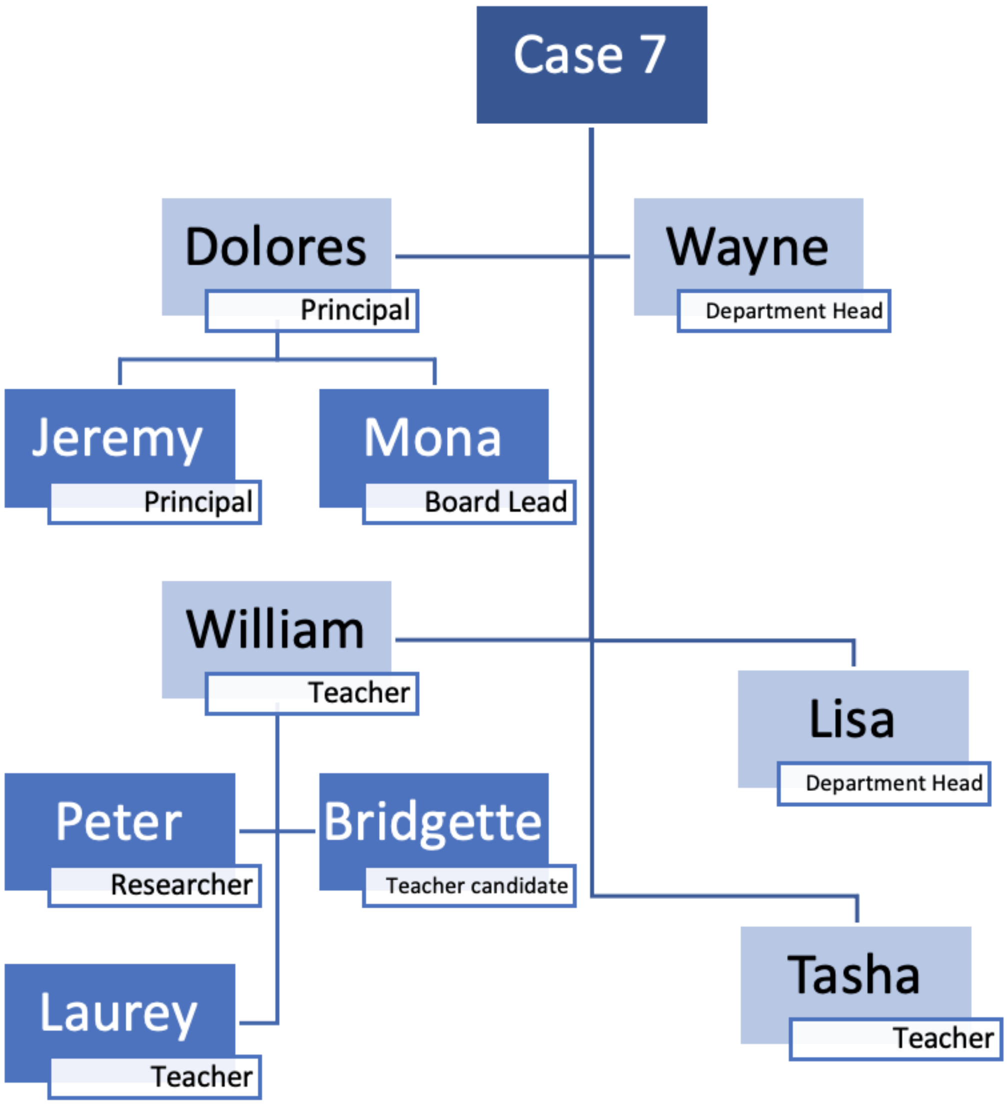
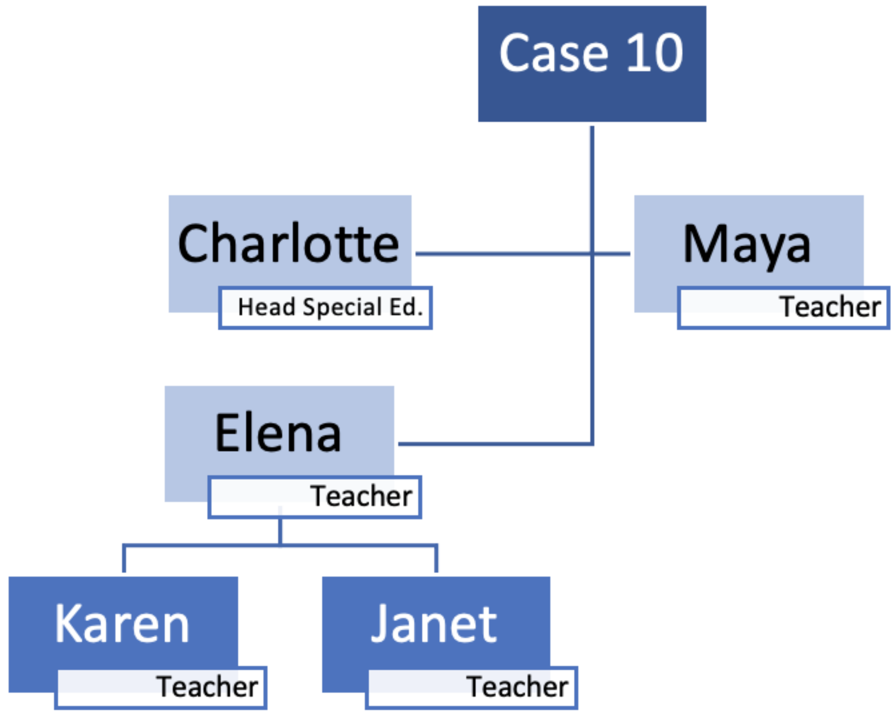


Figure 9
Case 10 Participant Structure



I will next discuss each case briefly (more detail on each case is provided in Chapter 5) and provide a description of the participants associated with each. I include the original OAME project participants as they were part of the secondary analysis of data completed in phase 0. Table 7 provides a summary of the number of participants from each case with background information.

Case 1: Participants

Case 1 began as a PLC located at Weber High School in a rural setting in Ontario and the PLC membership changed significantly between Year 1 and Year 2 of the project. Table 7 outlines the different individuals who were involved during the two years of the project, with participants who also participated in this doctoral study highlighted in bold print.

Table 7

Members of Case 1 OAME Project PLC

Pseudonym	Role Year 1	Role Year 2
Georgia	Principal	Principal
Brenda	Department Head	N/A
Drew	**District Mathematics Lead Teacher**	**District Mathematics Lead Teacher**
Jill	**Mathematics Teacher**	**Mathematics Teacher**
Francis	Community Program (CP) Teacher	CP Teacher
Lucille	Mathematics Teacher	N/A
Barbara	Mathematics Teacher	N/A
Charlotte	Resource Teacher	N/A
Zoe	CP Teacher	CP Teacher
Don	N/A	Temporary Numeracy Coordinator
Jed	N/A	Grade 7/8 Teacher
Josh	N/A	Grade 7/8 Teacher
Lauren	N/A	Grade 7/8 Teacher
Katherine	N/A	Grade 7/8 Teacher
Josh	N/A	Grade 7/8 Teacher
Sandra	N/A	Grade 7/8 Teacher
Gary	N/A	Elementary School Principal
Annika	Research Assistant	N/A
Tom	N/A	Research Assistant

Members from the PLC in Case 1 who participated in my study included Drew, Jill, and Tom. I provided each of these participants with an email to forward to any colleagues who may have been influenced by the OAME project, but I did not receive any replies. Case 1 was the only

Case that did not have participants for phase 3. I will next describe each of these participants and their role in the original OAME project.

Drew was a high school mathematics teacher who held the position of school district lead teacher from 2013-2018 and this was her position during the OAME project. At the time of data collection for this study Drew was in her 15th year of teaching. She joined the PLC in Case 1 for the monthly meetings as well as the project-wide meetings held several times throughout the project. Between Year 1 and Year 2 of the project Drew took a leave from teaching. She returned to participate in the knowledge mobilization component of the project that included planning and presenting at two provincial conferences. These conferences showcased the work and learning from the project. Following the conferences Drew also participated in creating workshops-in-a-bag, which I described in Chapter 1, for the OAME that helped to share the learning from the OAME project across the province. Since completion of the OAME project Drew had returned to the classroom and at the time of data collection was teaching mathematics at a high school in the same school district as PLC 1.

Jill was a mathematics teacher from PLC 1. At the time of data collection Jill was in her 21st year of teaching. It was Jill's idea to apply for the OAME project and after receiving permission from the school principal, she wrote and submitted the application for the Project. Jill shared that she often pushed herself to find professional learning opportunities for herself and her school to participate in. At the time of data collection for this doctoral study Jill continued to teach at the same school and her duties included implementing the Grade 9 de-streamed curriculum released in 2021. Jill was an active participant in online professional learning for mathematics teachers through social media, particularly on twitter through the twitter subgroup

Math Twitter Blogosphere (#MTBoS). It was through the #MTBoS that she became familiar with the work of some of the other project participants, including Tom who I will describe next.

Tom was the research assistant (RA) from the research team at the University of Ottawa and was also a high school mathematics teacher in his 13th year of teaching. Tom was assigned to PLC 1 during Year 2 of the project when the original RA did not continue the second year of the project. Tom was completing his Master of Arts (MA) in education under the supervision of the PI at the time of the OAME project. Upon completion of his MA he returned to the classroom as a mathematics teacher. Tom had a strong connection to many of the teachers in another Case, Case 7, as he had previously engaged in other professional learning opportunities with these teachers.

Case 4: Participants

Case 4 began as a PLC situated in a Grade 7-12 school, Pike Place School, in a rural, northern area of Ontario. PLC 4 had many members come and go over the two years of the project. The full membership list is outlined in Table 8, again those who participated in this study are presented in bold print.

Table 8

Members of Case 4 OAME Project PLC

Pseudonym	Role Year 1	Role Year 2
Chrissy	**Mathematics Department Head**	**Mathematics Department Head**
Daniel	**Mathematics Teacher**	**Mathematics Teacher**
Megan	Special Education Department Head	Special Education Department Head
Joan	Student Success Teacher	Student Success Teacher
Sydney	Grade 7/8 Teacher	Grade 7/8 Teacher

Margaret	Researcher	Researcher
Brad	Mathematics Teacher	N/A
Dean	Vice Principal (attended only Project Launch)	N/A
Alex	N/A	Grade 6 Teacher (Feeder School)
Lynn	N/A	Vice Principal
Alice	N/A	Aboriginal Students Grad Coach
Jennifer	N/A	Grade 6 Teacher (Feeder School)
Quinten	N/A	Grade 7/8 Teacher

Both PLC members who participated in this doctoral study had spent their entire careers teaching at Pike Place School. Chrissy, the head of the mathematics department was in her 27th year of teaching at Pike Place at the time of data collection for this study. She moved to the area after completing her teacher education program and thought she would stay only for a few years to get some teaching experience. Chrissy is responsible for having written and submitted the proposal to participate on behalf of the PLC and worked closely with the school's principal in recruiting the PLC members required to participate. Chrissy was involved in many professional learning initiatives in the province. She played an active role in the knowledge mobilization component of the OAME project, presenting at two provincial conferences that showcased the learning from the project. Chrissy also participated in the creation of the workshops-in-a-bag component of the Math4theNines online resource.

Daniel, like Chrissy, spent his entire teaching career at Pike Place School. He retired shortly after the OAME project with 31 years of teaching experience. Daniel was involved in many extra-curricular activities, he coached sports teams and refereed different sporting events. Pike Place School had a large population of Indigenous students, and it was important to Daniel to educate himself about different traditions and ways of knowing and celebrating in the Indigenous community. Daniel continued to be active in his community since retiring from teaching; at the time of data collection he was teaching at a local community college and his courses focused on healthy living. Daniel was placed in the PLC by Chrissy and the school principal and according to Chrissy he showed up to the first PLC meeting with marking from his courses to complete. When I asked Daniel about this during our interview he laughingly told me that he did not remember that but is not surprised that he did that. He shared that he had been to many Ministry-led professional learning initiatives that would tell him “You must do this” (Daniel, Case 4, phase 2 interview, 2021) and suggested that not all of them were “worth his time” (Daniel, Case 4, phase 2 interview, 2021). He quickly added that the OAME project was totally worth his time. Chrissy saw this change in Daniel, she told me he quickly abandoned bringing marking to the PLC meetings and in fact he “became the anchor for my grade nine programming… by the end of the... math for the nines project” (Chrissy, Case 4, phase 2 interview, 2021).

I was able to recruit one individual through PLC 4 to participate in phase 3 of this study. Chrissy mentioned three individuals that I might be interested in speaking to and she invited each of them to reach out to me. Two of these teachers agreed to an interview but only one committed to a specific date to meet. At the time of data collection Martina was a district mathematics coach in the Pike Place School District. Martina met Chrissy when she began her teaching career at Pike Place in 2014. Martina worked closely with Chrissy, who mentored Martina and supported her during her first few years working at Pike Place. After several years Martina was relocated to a different school in the district and eventually landed in the position as a mathematics lead teacher. Martina and Chrissy continued to collaborate, and Martina credited Chrissy with supporting her professional learning.

Case 7: Participants

Case 7 began as a PLC located at Fields High School in a medium size Eastern Ontario urban centre. The PLC membership evolved over the two years of the project. Table 9 presents the individuals involved in the OAME PLC during the two years of the project.

Table 9

Members of Case 7 OAME Project PLC

Pseudonym	Role Year 1	Role Year 2
Dolores	**Principal**	**Principal**
Wayne	**Mathematics Department Head**	**Mathematics Department Co-Head**
Lisa	**N/A**	**Mathematics Department Co-Head**
William	**Mathematics Teacher**	**Mathematics Teacher**
Tasha	**Mathematics Teacher** **Student Success Teacher**	**Mathematics Teacher** **Student Success Teacher**
Rachel	Student Success Teacher	Student Success Teacher
Larry	Science Teacher	N/A
Shirley	Science Teacher	N/A
Anna	Geography Teacher	N/A

Dolores was the principal at Fields High School during the OAME project. She was in her third year as principal at Fields when the project started. Following the completion of the OAME project Dolores was promoted to the position of superintendent within her local school district and in 2017 she retired after over 30 years as an educator. Dolores trained as a French immersion teacher and her teaching experience was in the subject of geography. Dolores explained that she did not have a lot of confidence in the areas of mathematics or science due to a lack of proficient teachers in her childhood. What Dolores saw as a weakness played an important role in the PLC, as she often pushed back against the teachers' ideas when she felt they were too similar to experiences she had as a student. Dolores described her role in the PLC as that of a "mosquito", buzzing ideas into peoples' ears rather than dominating or directing the flow of discussions during PLC meetings. Dolores played another important role in the PLC; she enabled the PLC by providing the time and resources to meet and work collaboratively.

Tasha was a mathematics teacher and special education teacher who, at the time of data collection, was in her 18th year of teaching. Shortly after the completion of the OAME project she left Fields High School for another school in the district. This new role was at a school that specialized in teaching different trades to students and providing classes geared to students who excelled when learning was hands-on. Tasha was interested in teaching at this school due to its speciality programs such as an Autism Program and Behavioural Intervention Program which aligned with her special education training. After one year at this new school Tasha was hired as an instructional coach, working at a district level. Tasha shared that she struggled as a mathematics learner, and this led to her pursuing her certification as a special education teacher.

Wayne was the head of the mathematics department at Fields High School during Year 1 of the project and co-head of the mathematics department during Year 2 of the project. Wayne, like Tasha, struggled through high school for various reasons and shared that he never really understood mathematics. As a university student Wayne had a part-time job as an educational aide helping students in Grades 9 through 12 with mathematics. It was through working with these students that Wayne began to understand mathematics for the first time. This experience inspired Wayne to focus the remainder of his university courses towards mathematics in order to become a mathematics teacher. Wayne told me that he thought being a mathematics teacher would be a good temporary job before moving on to something else, possibly a MA degree after five years of teaching. Interestingly enough, he did pursue a MA degree, only it was after retiring with 30 years of teaching experience. At the time of data collection Wayne was a graduate student in mathematics education. During his years of teaching Wayne became a fairly well-known leader in the mathematics education community. He was invited to lead professional development for other elementary and high school mathematics teachers and has presented his work at national and international education conferences. Wayne was a strong, reassuring presence in the PLC and others often turned to him for words of wisdom during planning meetings when the meetings lost momentum, or the PLC was unsure how to proceed.William was a veteran teacher at Fields High School and, at the time of data collection, was in his 30th year of teaching high school mathematics. William described himself as having been a good enough student, but mathematics was the only subject that engaged him, the only subject for which he would do all his homework. He landed in education after completing a university mathematics degree due to a lack of other career options. William shared that for many years he was very much a “sage on the stage” type of teacher. He described his style of teaching at that time as very traditional. This involved lecturing to his students and following the traditional lesson model of presenting a new mathematical concept to his students, having them listen to him as he explained it, and then watching him do several examples followed by students completing a page of similar examples. After many years of this style of teaching he began to lose his motivation to teach. After William shared this sentiment with Wayne, Wayne suggested that perhaps William needed to change how he was teaching. This was the impetus for what became a turning point in both of their teaching styles. Together they started to develop new approaches to teaching that would engage not only their students but re-engage their own passion for teaching. After hearing Dr. Liljedahl present in 2015 about the Building Thinking Classroom (BTC) (Liljedahl, 2015) model Wayne and William started to incorporate these new ideas with their own ideas. This model involved students working in visibly random groups (VRG) on rich mathematical tasks at vertical non-permanent surfaces (VNPS) such as whiteboards, blackboards, or even windows using non-permanent markers or chalk. The types of tasks used were an important component of the BTC model. The tasks need to be rich with a low floor and high ceiling allowing all students access to the task as well as room for students to dig deeper into the task if possible. The PLC at Fields High School was also experimenting with a method of teaching mathematics referred to in the research as "interleaving" (Brown et al., 2014; Rohrer et al., 2014). The Fields PLC used the term "spiralling" to refer to this approach. Unlike teaching mathematics in blocked units or strands, spiralling involves revisiting different mathematical concepts several times throughout the course. The PLC spiralled using primarily open-ended, activity-based, rich tasks. Different activities often addressed multiple strands of the curriculum simultaneously and required students to retrieve knowledge from prior learning. They soon were sharing their work on social media and personal blogs as well as at local and international conferences. William was a dominant presence during PLC meetings at Fields as well as at the project-wide meetings for the OAME project. Many teachers involved in the project were interested in hearing more about the approaches Fields was taking and many started to incorporate these new approaches at their own schools.Lisa was the fifth member of the original PLC to agree to participate in this study. At the time of data collection for this study Lisa was in her 25th year of teaching and continued in her role as the head of the mathematics department at Fields High School (Wayne had retired leaving Lisa as the sole deparmtnet head). Lisa and I had known each other first as friends and then through working together during the OAME project we developed a professional relationship as well. Lisa is passionate about assessment and evaluation in high school mathematics, and she has worked closely with the Ontario Ministry of Education, as well as her local school district, to develop resources to support teachers as they enhance their approaches in assessment.Tasha, Dolores, and William shared my phase 3 recruitment email with colleagues and acquaintances who they thought were also influenced by the project, even though they were not directly involved. Through Dolores I was able to recruit Jeremy and Mona and through William I was able to recruit Laurey, Bridgette, and Dr. Peter Liljedahl (who agreed to be identified using his name in this study and from now on I will refer to him as Peter). I will next describe each of these connections in more detail.Jeremy was a veteran educator who was connected to Dolores through their work with the Student Success School Support Initiative (4SI). This initiative commenced during the 2012-2013 school year and was a spin-off of the Student Success/Learning to 18 Strategy that the Ontario Ministry of Education began in 2003. The focus of the 4SI project was on creating success for students in Grade 9 and 10 applied courses in mathematics, English, history, geography, and science. Both Dolores and Jeremy were active in the 4SI project as principals at their respective schools. At the time of data collection Jeremy was set to retire in December of 2021. Mona and Dolores met when Mona was a consultant hired through the Ontario Principals' Council (OPC). The OPC supports principals and vice-principals across Ontario. The stated goal of the OPC is "to provide our members with the professional services and supports they need to provide exemplary leadership in public education" (www.principals.ca). Mona, in her role as a consultant with the OPC, would visit school principals in a designated jurisdiction several times a year to offer professional support. When Dolores moved into her role as superintendent Mona and Dolores continued to meet to discuss effective approaches to teaching and learning in schools. Mona has since retired from her OPC role and at the time of data collection was working as the principal at a private high school in Ontario.

Laurey, who at the time of data collection, was in her 25th year of teaching in the United States at the time of data collections for this study. She was also the teacher that I described in the prologue of chapter 1. Laurey and William had been connected through Twitter since 2013 where they were both active in the #MTBoS. The connection I made with Laurey demonstrates the complexity of teachers' professional learning, how connections can be made in multiple ways, and, because of new technologies like social media, connections that were once considered long distance can be thought of as close connections. Laurey's comment to me that the OAME project influenced her own teaching suggested to me that I might want to reconsider how I designed my study and data collection in particular. It was because of my interaction with Laurey that I considered not only studying the sustainability of the learning from the OAME project but the diffusion of learning as well. Hearing that the project influenced teachers across North America suggested that professional learning was not limited to only project participants but included those they interacted with as well.

Bridgette was a teacher candidate in a secondary mathematics course I taught at the University of Ottawa. At the time of data collection Bridgette was working as a temporary teacher and tutoring students in mathematics while pursuing a full-time teaching position. The OAME project influenced my teaching of this course in different ways including using the BTC model, assigning the Five Practices as required reading, and engaging with rich tasks I learned about through the project. One specific way that it influenced my teaching was through building professional relationships with several of the teachers from the PLC in Case 7. When Bridgette was a student in this course my class visited William in his classroom to observe his teaching style, interact with his students, and engage in rich mathematical tasks. The following week William visited our class at the university, and he debriefed with our class about the lesson they observed, answered questions about his approach to teaching, learning, and assessment, and engaged my students in the BTC model. Wayne was also scheduled to visit our classroom but due to job action by Ontario teachers at the time coupled with the arrival of COVID-19, our classes were disrupted. A third member of the PLC, Lisa visited our university classroom. Lisa engaged our university class with activities that explored issues related to assessment and evaluation in mathematics and we discussed how this area had evolved over the last decade. As a student in this course Bridgette shared with me that she was exposed to ideas about teaching and learning mathematics that were new to her. I see this as a direct consequence of the OAME project and Bridgette's perspective on how the OAME project influenced her own teaching and learning since being a teacher candidate would be a valuable perspective to add to the data collection for this study.

Peter is a researcher in mathematics education in British Columbia and is the originator of the BTC model of teaching and learning mathematics. Peter began his career as a high school mathematics teacher before completing his Master's and doctoral studies. His research interests are "creativity, insight, and discovery in mathematics teaching and learning; the role of the affective domain on the teaching and learning of mathematics; the professional growth of mathematics teachers; mathematical problem solving; numeracy; and engaging student thinking" (peterliljedahl.com). He and William developed a close working relationship due in part to a connection made through twitter and the #MTBoS. William was an early adopter of Peter's BTC model and shared his journey of incorporating the model in his classroom on twitter. William, and the rest of the PLC at Fields were strong proponents of this model and often shared their experiences with the other PLCs during project-wide meetings.

Case 10: Participants

Case 10 began as a PLC located at Broadview High School in a medium sized urban centre in Ontario. The PLC maintained a steady core membership over the two years of the project.

Table 10

Members of Case 10 OAME Project PLC

Pseudonym	Role Year 1	Role Year 2
Paul	Principal	Principal
Deidra	Mathematics Department Head	Mathematics Department Head
Marco	Mathematics Teacher	Mathematics Teacher
Maya	**N/A**	**Mathematics Teacher**
Charlotte	**Special Education Department Head**	**Special Education Department Head**
Donald	District Mathematics Leader	District Mathematics Leader
Elena	**Mathematics Teacher**	**Mathematics Teacher**
Claire	N/A	Mathematics Teacher

Charlotte was the special education department head. At the time of data collection Charlotte had been an educator for 23 years. Charlotte was invited to be part of PLC 10 to fulfil the requirement of having a special education teacher involved in the project. Charlotte worked closely with many of the students in Grade 9 Applied Mathematics but had never worked with the mathematics teachers on a project such as the OAME project. Charlotte described her role within the PLC as providing the perspective of students with special needs, individual education plans (IEPs), and accommodations. She suggested that more students in Applied Mathematics fall into these categories than students in Academic mathematics.

At the time of data collection for this doctoral study Elena was in her last year of teaching after 32 years as a mathematics teacher. During the OAME project Elena was finishing her Master of Education degree, and she found that a lot of the discussions within the project aligned with what she was studying in her course. Elena informed me that her school was hesitant to join the OAME project and she was able to convince her colleagues to join. Elena suggested that some of the PLC continued to be hesitant throughout the project and she relied on the energy of other PLCs in the project to inspire some of the work she did towards her own professional learning.

At the time of data collection Maya had been an educator for 21 years and joined the staff of Broadview as a mathematics teacher midway through the OAME project. She was asked to join the PLC and OAME project and she shared that the first time she met her new co-workers was during the summer institute in 2015. Maya's job position has changed several times since the project was completed. In 2019 Maya took on the role of district numeracy support teacher for two years before being promoted to the role of Mathematics and Science Consultant with her school district in the fall of 2021. Maya credited the project with pushing her learning and inspiring her to make these advancements in her career. Through her role as a district numeracy support teacher Maya had supported many mathematics teachers and these connections provided a connection to two participants for phase 3 of this project, Karen, and Janet, both of whom were teachers at Broadview High School.

At the time of data collection Karen had been a teacher for 18 years; the first 12 years of her teaching were spent as an occasional teacher in both elementary and secondary classrooms in her school district. In 2015 Karen was hired as a full-time mathematics teacher having only had experience teaching science. Karen credits Maya's role as a numeracy coach with supporting her teaching and moving her thinking towards her current model of teaching and learning mathematics.

Janet's connection to Maya is similar to Karen's. Janet, a qualified science teacher, had been teaching for ten years at the time of data collection. The first seven years she taught as an occasional teacher in her district before finally securing a full-time teaching position as a mathematics teacher. With no experience teaching mathematics Janet relied on Maya's support to get her to a place where she was comfortable teaching mathematics. She states that if she had to choose between teaching only mathematics or science she would now choose mathematics.

Data Analysis

I now discuss the different phases of this study in terms of the data analysis process. I first discuss my approach to data analysis before discussing the importance of the secondary analysis of the original OAME data. Following that I go through each phase of the analysis of the data collected for this doctoral study.

There were four distinct sets of data analysed during this study. The secondary data analysis of the original OAME project data and the analysis of the phase 1 survey, phase 2 interviews, and phase 3 interviews. Analysing these different sets of data was an iterative process and while I do discuss each phase of analysis in a linear way it is important to stress that analysis of each set of data did not occur in a linear process. While I did begin with a scan of the original OAME project before collecting and analysing different sets of data I returned frequently to each set of data. The foundation of this analysis was the creation of a codebook which I will now describe before discussing each phase of data analysis in more detail.

In order to code all of the data I first created a set of codes, "tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study'' (Miles & Huberman, 1994, p. 56). I created a codebook that was directly informed by my theoretical framework and my research questions; thus my codebook was both structural and theory-driven (DeCuir-Gundy et al., 2011). DeCuir-Gundy and colleagues define structural codes as those that "grow from a specific project's research goals and questions" (p. 137). My research questions were directly informed by my chosen theoretical framework as I was seeking to understand how the conditions and characteristics of complex learning systems influence the emergence, sustainability, and diffusion of learning. Therefore my codebook was also theory-driven as the codes were created using the specific conditions and characteristics of complex systems that influence the emergence, diffusion, and sustainability of learning. These codes are outlined in Table 11.

Table 11

Data Analysis Codebook

Code	Definition	Example
Diversity	Participant makes direct/indirect reference to the differences such as experiences, beliefs, and attitudes. Observation of differences in PLC such as teaching expertise and grade levels taught.	"For us our challenges were partly, at least this year, the mixing of the grades, elementary and high school. We have different timetables; we have different expectations"
Redundancy	Participant makes direct/indirect reference to common traits and understandings. Observation of commonalities that bring individuals together and provides a reason to interact.	"That's right regroup and that you could actually have regular meetings with. People that you've connected with that you really feel you've got some, you know common grounds with"

Neighbour interactions	Participant makes direct/indirect reference to interacting with others, their ideas, or resources. Observation of different ways that participants and ideas interacted.	“It helped to have a close connection with like, Dr. Suurtamm, so that you could actually bounce some ideas off her, which is different than anything else that I’ve had the opportunity to work with”
Decentralized control	Participant makes direct/indirect reference to the ways the PLC was controlled or led. Observation of explicit/implicit examples of leadership style.	“She sits in with the lesson planning and yet I feel will contribute now and then but doesn’t override it and I don’t think anyone feels like they have to impress her whereas I think there are some situations where that might be a different dynamic”
Randomness	Participant makes direct/indirect reference to openness and/or freedom. Observation of explicit/implicit examples of openness and choice amongst participants.	“It shows the importance of letting the teachers pick the topic and direction they want to go”
Coherence	Participant makes direct/indirect reference to the need to align with objectives, mandates, or goals. Observation of explicit/implicit examples of coherence to objectives, mandates, or goals.	“I mean it does, it does apply to the curriculum in the sense that it’s problem solving which is part of the curriculum”
Enabling constraints	Participant makes direct/indirect reference to a balance between randomness and coherence. Examples of a balance of randomness and coherence manifesting were observed by the researcher.	“There is a certain challenge for the principal not to be overbearing, to be able to maintain your relationships of trust but still bring the group along in a way that you see is important”
Self-organization	Participant makes direct/indirect reference to ways that the system organized. Observation of explicit/implicit examples of self-organization.	“We set up the second semester so that we each have an academic class and a split class I think that’s incredibly, that’s been incredibly eye opening”
Adaptation	Participant makes direct/indirect reference to ways that the system adapted to disruptions and/or perturbations. Observation of ways	“So we've changed from being more from being in that position to being more a little bit more progressive thinkers most of us are a little bit

	the system adapted to disruptions and/or perturbations.	better at being open to trying new things”
Nestedness	Participant makes direct/indirect reference to ways they, or their system is connected/nested within a larger system. Observation of explicit/implicit nestedness.	“I love when you can actually hear well on Adobe connect sessions, when people are sharing that, okay well I am trying spiraling, or I am trying to focus on, on mindsets to see if, I don’t know when you just hear what the other people’s journeys are and you are like okay, this is so common”
Emergence	Participant makes direct/indirect reference to the emergence of learning. Observation of explicit/implicit examples of learning at different levels of the system. (e.g., Individual, PLC, school, project).	“For me, it was exposure to PLCs that did not go smoothly. So, how to react to that, in a continuously supportive way, was part of my personal learning”
Sustainability	Participant makes direct/indirect reference to ways the learning from the project has endured or continues for themselves, PLC, or school. Observation of explicit/implicit ways the learning from the project has endured of continued.	“The work, the work from everything so I’m going to expand on that, but like the connections built between Zoe, Jill, Francis, and myself, like that group of teachers has continued”
Diffusion	Participant makes direct/indirect reference to sharing the learning from the project beyond the scope of the original project. Observation of explicit/implicit ways the learning from the project has spread beyond the boundaries of the original project.	“I have used them all the way up until I just left I left the classroom about three year, two years ago, so I was using vertical surfaces, all the way through and that's what I promote as a consultant now”

Once the codebook was established a deep analysis of the transcripts began. This process involved reading through the transcripts several times, making researcher notes, and highlighting ways the different codes manifested and themes that emerged. The first read through of the transcripts was to become more familiar with the transcripts themselves and the overall focus of each. Subsequent readings of the transcripts focused more specifically on coding the transcripts to align with the research questions and my theoretical framework. I used a software program, NVIVO, to organize and code the data. I created codes in the software, based on my codebook and highlighted sections of texts that aligned with these codes. An example of a section of coded transcript is shown in Figure 10 and a summary of the frequency of codes for one case is presented in Figure 11.

Figure 10

Example of Coded Transcript

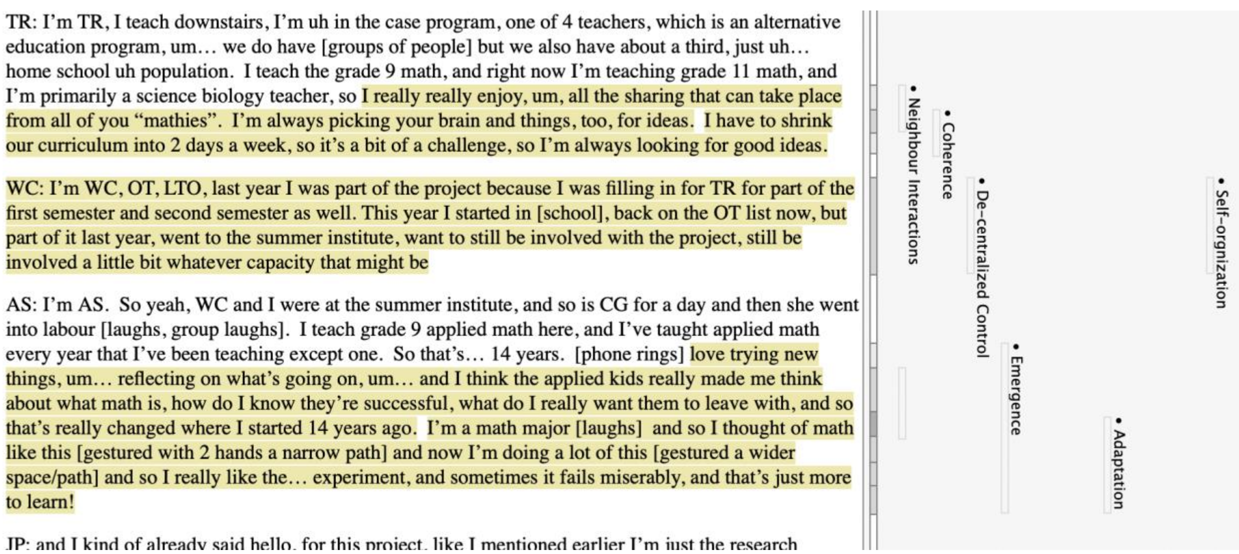


Figure 11

Example of Codes and Frequency

Name	Files	Refer...	Created on	Created...	Modified on	Modified by	Color
Randomness	5	16	2022-02-12, 3:02...	KM	2022-02-15, 11:05...	KM	
Self-organization	6	33	2022-02-12, 3:03...	KM	2022-09-08, 10:09...	KM	
Coherence	7	39	2022-02-12, 2:36...	KM	2022-08-16, 11:07...	KM	
Enabling constraint	7	13	2022-02-15, 11:05...	KM	2022-08-16, 11:07...	KM	
Redundancy	8	38	2022-02-12, 3:00...	KM	2022-08-16, 11:17...	KM	
Decentralized control	9	27	2022-02-12, 3:02...	KM	2022-08-16, 11:10...	KM	
Diversity	11	73	2022-02-12, 2:58...	KM	2022-08-15, 10:07...	KM	
Nestedness	11	24	2022-02-12, 3:03...	KM	2022-09-08, 10:09...	KM	
Adaptation	12	48	2022-02-12, 2:54...	KM	2022-09-07, 9:48...	KM	
Sustainability	12	25	2022-02-12, 3:02...	KM	2022-09-09, 8:51...	KM	
Emergence	15	77	2022-02-12, 2:53...	KM	2022-09-08, 10:09...	KM	
Diffusion	17	65	2022-02-12, 3:03...	KM	2022-09-07, 9:48...	KM	
Neighbour interactions	20	155	2022-02-12, 2:33...	KM	2022-09-08, 10:09...	KM	

Often reading a specific section of a transcript would trigger a memory of something I had read during my review of the original OAME project data, or a recollection I had from my time as an RA. I would add these thoughts, memories, and ideas to my research journal, flagging the transcripts and themes that were connected to the original data. This on-going cross-case, and cross-data, analysis allowed me to build a deeper understanding of not only the learning that emerged during the project but also the learning that was sustained, diffused, and continued to emerge from the project. It also allowed me to build a comprehensive narrative of each PLC. For instance some questions lingered about PLC 10 and the motives that school had for joining the project initially. Through interviews with participants from PLC 10 I was able to clarify, and gain different perspectives on that PLC, including how the PLC worked, how individuals learned, and what learning was sustained and diffused beyond the PLC.

The extended case study approach required that I not only focus on individual cases but also pay close attention to connections between the cases. Data analysis during these three phases allowed me to trace the emergence, sustainability, and diffusion of learning from the OAME project. This tracing started with the foundational data collected during the OAME project and continued through each phase of my study. This tracing was done through the use of NVIVO as the software aggregated coded sections of data together into files. I took extensive notes in my researcher journal that tracked the different ways that the learning emerged, was sustained, and diffused. I now turn to describing each phase of the data analysis process in more detail.

Secondary Analysis of OAME project data

The secondary analysis of data added to my foundational understanding of the learning that emerged from the OAME project, how different PLCs worked, what they worked on, and the different experiences different PLCs had as part of the project. As I mentioned in my discussion of the data collection, I spent a great deal of time engaging in a secondary analysis of the existing data from the OAME project. My first step was a review of the final project report, reviewing the themes that emerged from the project including ways that different PLCs worked and what the PLCs focused on in terms of enhancing their practice (Suurtamm et al., 2017). Once I had recruited participants for this study I was able to identify the PLCs that would be part of this study and pull any relevant data from the original project to conduct a more thorough secondary analysis (Table 3). For example, once participants identified through phase 1 had agreed to participate in phase 2, I read, analysed, and coded their exit interviews from the project, if they existed, and read through the case reports from their PLCs before interviewing them. This process helped to inform the interview questions I asked for each individual. My experience as part of the research team also provided me insight into some of the challenges and successes that each PLC had during the project. This foundational knowledge, together with my review and analysis of the data prior to interviewing the participants ensured that I asked specific questions that could provide insight into the ways the PLC operated, the ways that learning did, or did not emerge, and pointed me towards areas of questioning that would assist me in addressing my research questions.

Throughout the data analysis process I would often return to the original data in order to triangulate my analysis. Triangulation involves consulting various sources of data in order to develop a comprehensive understanding of phenomena; "whether you make use of more than one data collection method, multiple sources of data, multiple investigators, or multiple theories—[triangulation] is a powerful strategy for increasing the credibility of your research" (Merriam & Tisdell, 2016, p. 245). For instance when analyzing new data in terms of the diffusion of learning I knew that several PLCs, during the two years of the OAME project began to collaborate together, visiting one another's schools and learning from each other. One participant referred to this as cross-pollination. My knowledge of this supported the analysis of new data and added a depth to my understanding of comments and ideas shared with participants during this study. This interaction between parts of a system is often missing from data sets yet using complexity means that these interactions are an important component to the analysis process (Hetherington, 2013). While there are no exact analysis methods informed by complexity, Ell et al. (2019) do suggest rich descriptions of the data "are useful for exploring change over time and gaining insight into processes that may have been at work and contributed to system emergence" (p. 10). I therefore employ this method as part of the data analysis process in this study.

Phase 1: Data Analysis

The survey data collected (Appendix B) during phase 1 was analysed using descriptive statistics and provided insight into the ways, and the degree to which, the project facilitated participants' learning. Quantitative questions were graphed (Appendix C) and qualitative questions with an open-ended response were coded using the same codebook that I described earlier (Table 11). The analysis of this data also served to direct me towards possible participants for recruitment for phase 2 of the study. As described previously when I discussed participant recruitment, emails were sent to those individuals that signalled they would be interested in continuing to participate in the study. Once these participants were recruited, I used the analysis of the individual participants' open-ended responses to inform phase 2 interview protocol. I did not want to repeat questions they had already responded to, and I wanted to honour the responses they had provided and asked follow-up questions related to the answers they provided on the survey. For example I would ask participants who indicated the project had a large influence on their learning if they could provide one or two specific examples of ways the project had influenced them. Others responded on the survey they continued to collaborate with others from the project, I made sure to ask questions during phase 2 that would provide more information regarding this theme.

Phase 2 and Phase 3: Data Analysis

Phase 2 and 3 data involved analyzing data collected through semi-structured interviews and was a "multistep 'sense-making' endeavor" (DeCuir-Gundy et al., 2011, p. 137). Data analysis during phase 2 and phase 3 was an iterative process that began during the semi-structured interview when I would respond to participants' comments from the survey with follow-up questions and requests for clarifications on specific topics that aligned with my research questions. The interviews were intended to be semi-structured in order to allow for follow-up questions that would build on responses given by the participant. If a participant provided an answer that could be explored further, for example asking a participant to share specific examples of how the project influenced their learning, I would lead the interview in that way. When an area of inquiry had been exhausted I would return to my interview protocol and continue with the predetermined individualized protocol. Upon completion of the interview, the zoom generated transcript of that interview was reviewed while I listened to the interview again within a 48-hour period. This timing of the review provided me the opportunity to make researcher notes on the interview while it was still fresh in my mind as well as make any corrections to the auto-generated transcript. Listening to the audio while reading the transcript allowed me to add meaning to the transcript by aligning the tone of the voices with the text of the transcript and for deeper meaning to be transmitted. I would add notes to the transcript such as [long pause], [participant sighs], or [said while laughing] while I listened to the interview in order to add to the tone of the written transcript. This tone was then helpful later when deeper analysis was conducted. Once the transcripts were properly formatted, researcher notes generated, and tone of the speaker added I set to the process of coding the data using the codebook I defined.

Concluding Remarks

The research design of this qualitative, multi-case study of four different cases that originated from the OAME Grade 9 Applied project involved the collection and analysis of data in four distinct phases. The methodology used for this research study, from conception to execution was underpinned by my use of complexity science as a theoretical framework. It was paramount to me that I gain the perspective of multiple different components of the complex system that constitutes mathematics education (Jacobsen et al., 2019). Mathematics teacher learning is not a linear process, it exists in a system with many moving parts. Gaining a clearer understanding of these different parts and ways they interact, and influence teacher learning was at the heart of this study and the research methodology reflects this foundation. The next chapter presents the findings from my data analysis.

CHAPTER 5: FINDINGS

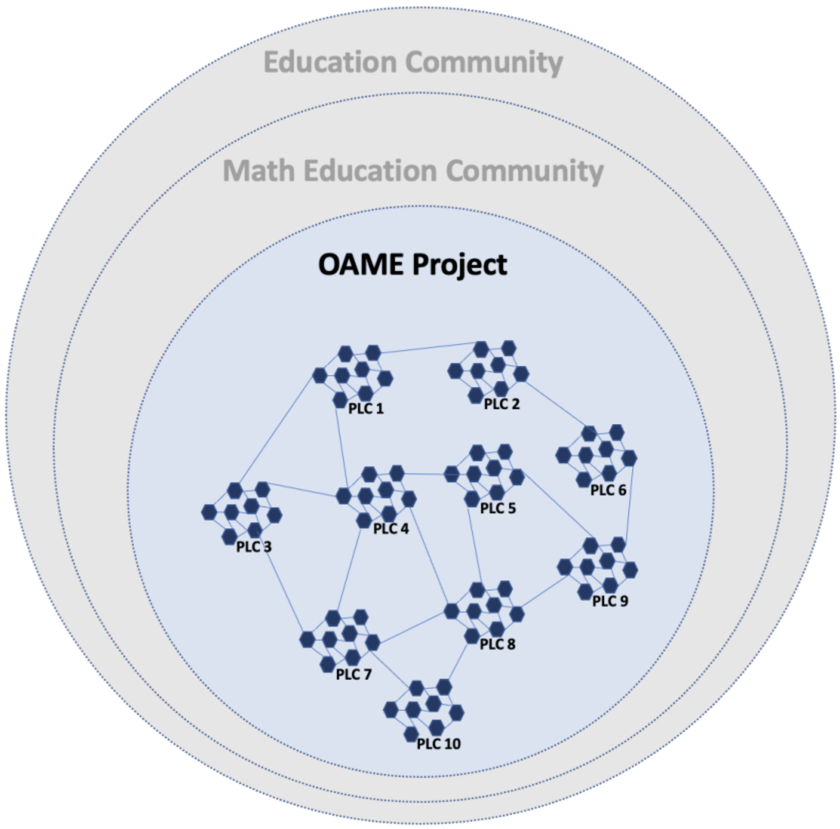
This chapter presents the findings from the data collection and analysis process discussed in Chapter 4. This chapter is framed by the following two research questions:

1. How did the characteristics and conditions of complex learning systems influence the emergence of learning in the different PLCs, and associated learning systems, of the OAME project?
2. In what ways did the characteristics and conditions of complex learning systems influence the sustainability and diffusion of learning in the different PLCs, and associated learning systems, of the OAME project?

As I described in Chapter 4 different data collection instruments were designed to help address different research questions. Addressing research question 1 (RQ1) relies, primarily, on the phase 0 secondary analysis of the original OAME project data, my documented observations and experiences as part of the OAME project research team, the phase 1 survey, and the phase 2 interviews with original OAME project participants. Research question 2 (RQ2) relies primarily on the analysis of the phase 1 survey, phase 2 interviews, and phase 3 interviews. When presenting the findings related to each research question I consider the nested systems associated with the OAME project and the ways that each system may have been influenced by the involvement of that system in the project (Figure 12). In order to do this I discuss findings at the project level and at the PLC level and individual level for each case. However, other systems beyond these play an important role in influencing these three systems including the OAME community and the mathematics education community in the province of Ontario and therefore are important to consider when analysing the data collected to address RQ1.

Figure 12

Question 1: PLCs and Associated Learning Systems of the OAME Project



Addressing Research Question One

RQ1 asks: How did the characteristics and conditions of complex learning systems influence the emergence of learning in the different PLCs, and associated learning systems, of the OAME project? I first present findings from the project level and begin by considering the ways that the conditions and characteristics of complex learning systems manifested in the project and interacted to influence the emergence of learning from the project. The second part of addressing RQ1 involves moving from looking at the project overall to organizing my findings by case. I begin by describing the PLC in each case, the membership of the PLC during the project, the problem of practice identified in the original OAME project application, how the PLC focus may have changed over the two years of the project, and how the PLC worked over the two years to address their problem of practice. I then present findings from the analysis of the data that identify ways the conditions and characteristics of complex systems manifested in each case at different levels and how they interacted to influence the emergence of learning.

The main objectives of the OAME project were "for the school PLCs to focus on enhancing PLC participants' understanding of the Grade 9 Applied Mathematics curriculum, and to focus on implementing the curriculum in ways that will best meet the needs of their students" (Suurtamm et al., 2017, p. 2). I bound my analysis of the data to the following three learning systems: the OAME project, the PLCs, and the individual teachers in order to address RQ1.

Addressing Research Question One: 1: The OAME Project

I begin by describing the findings from the analysis of the data that suggest learning emerged from the project. Understanding the project is an important first step in describing the findings from the analysis of different data sources. My participation in, and documented observations of, monthly research team meetings, project-wide PLC meetings, individual PLC meetings at Case 6 and Case 7, and secondary analysis of the original project data (Table 3) provided evidence that the different conditions and characteristics of complex learning systems manifested at the project level. Analysis of phase 1, 2, and 3 data also provided evidence. The following sections detail my findings from the analysis of the above-mentioned data and are organized by the conditions and characteristics of complexity as outlined in Chapter 3.

Specialization: Redundancy and Diversity. The final OAME project report (Suurtamm et al., 2017) describes how over 52 high schools across Ontario applied to be part of the OAME project in the Spring of 2014. From these 52 applications the project steering committee purposely selected 10 PLCs that represented a range of locations, contexts, and student populations. The various project-wide meeting transcripts and notes, analysis of the individual case reports, and the original project final report provides evidence that these PLCs were both redundant and diverse in their qualities. I first will describe how the findings reveal ways redundancy manifested in the 10 different PLCs before describing how diversity manifested.

Redundancy was manifested in several ways across the PLCs, including the membership of the PLCs, the objectives of the PLCs, and the schedule of activities that the PLCs participated in. A requirement of the project was that each PLC had to have a similar membership structure that included a school administrator, a mathematics lead teacher at the school or district level, two to three teachers of Grade 9 applied mathematics, as well as a special education resource teacher and/or student success teacher from the school (Suurtamm et al., 2017). Individual case reports show that the objectives of each PLC were also aligned with the objective of the project which was to focus on enhancing PLC participants' understanding of the Grade 9 Applied Mathematics curriculum, and to focus on implementing the curriculum in ways that would best meet the needs of their students. Lastly, redundancy manifested in the overall schedule of activities that each PLC participated in. Table 12 outlines the schedule of activities that each PLC participated in over the two years of the project.

Table 12

OAME Project Scheduled of Activities

Type of Activity	Dates	Participants
Year 1		
Project launch	Oct 2014	Steering committee, 10 PLCs, research team

Monthly PLC meetings	Nov 2014 – Apr 2015 (dates determined by PLC)	PLC members, researcher and/or research assistant
Adobe Connect sessions with PLC leads	Nov – May (every 6 weeks)	At least one PLC member, steering committee, research team
Presentations at OAME conference 2015	May 2015	Various members of school PLCs and research team
"Wrap" of Year 1	May 2015	Steering committee, 10 PLCs, research team
Year 2		
Summer institute	Aug 2015	Steering committee, 10 PLCs, research team
Monthly PLC meetings	Sept 2015 – May 2016	PLC members
Adobe connect sessions with school PLC leads	Oct 2015– Apr 2016 (every 6 weeks)	At least one PLC member, steering committee, research team
May institute	May 30 – 31 2016	School PLCs, research team, steering committee
August provincial conference	Aug 22 – 23, 2016 (Toronto) Aug 24-25, 2016 (London)	600 educators (total), representatives from school PLCs, research team, steering committee

Note. Adapted from "*OAME project: Final research report, teaching grade 9 Applied Mathematics: A collaborative inquiry". By* Suurtamm et al., 2017. Report to the Ontario Association for Mathematics Education (OAME) and the Ontario Ministry of Education, January 2017. 163 pages.

While each PLC aligned with the objectives of the project and participated in the same schedule of events, how each PLC operated and what they worked on was different for each PLC (Suurtamm et al., 2017). Thus, diversity manifested in different ways at the project level. Each PLC was required to state their problem of practice when they applied to participate in the

OAME project. Different PLCs selected what they would focus on based on the needs of their school. Many PLCs mentioned a goal of increasing student engagement, but each chose a different way to focus on this goal. Some looked at better defining learning goals and success criteria with their students (Case 9 and Case 10), others focused on helping students to develop a growth mindset through videos and non-mathematical activities (Case 2), and others, like Case 7, looked at introducing new pedagogical models such as the Building Thinking Classroom (BTC) model and spiralling through concepts multiple times during a course. How the PLCs worked when they met for their monthly meetings was also diverse. Some PLCs were already established learning communities that continued using models such as lesson study while others were newly formed PLCs that needed to learn how to work as a PLC before choosing specific models of professional learning. Some PLCs adopted a top-down leadership model while others adopted a decentralized leadership style. Diversity was also represented in the different PLCs' school size, location, and student population. The 10 PLCs were in different locations across Ontario and these locations included large urban city centres, small rural schools, and schools in remote Northern locations. Some schools served students in grades 9 - 12 while others served a wider diversity of ages in grades 7-12.

Trans-level Learning: Neighbour Interactions and Decentralized Control. Findings from the secondary analysis of the OAME project final report suggest that trans-level learning was an important design component of the OAME project (Suurtamm et al., 2017). Ensuring that each PLC had multiple opportunities to interact with other PLCs in a system with decentralized control allowed for neighbour interactions and sharing of knowledge. I present findings from the secondary analysis of the OAME project final report, the individual case reports for each PLC, my observations through my participation as part of the research team, and conversations with the PI. These findings describe ways that the OAME project demonstrated decentralized control as well as the different neighbour interactions that manifested over the two years of the project.

When the steering committee designed the OAME project they leaned on current research on effective professional learning as discussed in Chapters 1 and 2 (Suurtamm et al., 2017). An important component of this research suggests that a shared leadership model is important to support, and encourage, teacher engagement. Although the project schedule of activities and overall goals were determined by the steering committee, each PLC was in charge of their own goals, ways of working, and their focus throughout the two years of the project. In this way the project demonstrated decentralized control. The project-wide meetings brought together all the PLCs and were designed and led by the PI. The structure of these meetings was intentional with scheduled productive working time for the PLCs that allowed for neighbour interactions both within individual PLCs as well as with other PLCS. This time to engage with others is not something that teachers often have during their busy teaching days at school. These meetings were purposely designed to include ample opportunities for the next condition, neighbour interactions.

A deliberate component of the project design was to ensure that neighbour interactions between the different PLCs were enabled and supported during the two years (Suurtamm et al., 2017). For this reason several project-wide meetings were planned. Different neighbour interactions at these project-wide meetings manifested including, within school PLCs, between different PLCs, between PLCs and researchers, and between PLCs and different resources presented by the steering committee, research team, and PLCs. For instance at the project launch in October of 2014 Dr. Suurtamm designed an activity she refers to as speed dating, where different individuals met with others from different PLCs to share their goals, ideas, action plans, and questions with one another. Each person described their PLC's plan and then listened to another PLC's plan through at least 3 rounds with different people each time, ensuring that each PLC was exposed to as many different ideas as possible in a relatively short amount of time. This sharing of ideas could spark interest or questions in others that they could bring back to their own PLC to discuss together. As the project progressed additional activities were designed that would create opportunities for PLCs to interact with others and their ideas. During the project-wide meeting at the end of Year 1 different discussion tables were set up and representatives from each PLC visited these tables. Some topics at these discussion tables included assessment, using rich tasks, and creating positive mindsets with students. After these discussion sessions the PLCs would meet with their own group and share what was discussed across the various tables. This sharing led to many new ideas spreading throughout the project. Findings from the analysis of individual case reports suggest that Case 7 was influential in this regard; their sharing of their spiralling approach to teaching and the BTC model were two ideas that were incorporated in numerous other PLCs. Some PLCs started to work with one another after meeting at province-wide meetings. For example Case 6 travelled to Case 7 to spend the day observing classes and discussing their approaches to teaching mathematics. Case 6 reported that this visit was very influential in helping them better understand the importance of making students' thinking visible. During this visit with the Case 7 PLC one teacher from Case 6 shared:

> there is something about the vertical boards and being able to stand in the middle of the classroom and SEE! Like that observation happens instantaneously. You can have the conversations you need to have instead of going from group to group. It is really cool! (Case 6, Year 2, meeting 5)

Near the end of the project, at the final project-wide meeting PLCs again were mixed and different groups began to prepare presentations on different topics for the province-wide conferences created from the project. These conferences shared the learning that emerged from the project with educators from across Ontario and later, workshops and resources were further shared online at math4thenines.com. These conferences will be discussed in greater detail when I address my second research question.

Opportunities for the PLCs to interact with researchers and resources was another component of the project-wide meetings. Drs. Suurtamm, Macaulay, and Koch all shared research-based presentations on mathematics education in Ontario and mathematics pedagogy. Dr. Suurtamm led the PLCs through various activities incorporating the use of rich tasks, Dr. Macaulay presented her research (2015) that focused on the characteristics of teaching and learning mathematics that led to greater success for students in Grade 9 Applied Mathematics, and Dr. Koch shared a presentation on the *Five Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2015) after the PI noticed several PLCs discussing how best to navigate leading students through rich mathematical tasks. Each participant was provided a copy of the book and analysis of PLC meeting transcripts from Year 2 reveal that this resource was used extensively as a resource throughout the school PLCs. Wayne, from Case 7 shared:

> When we do an activity [in our classrooms] we feel that our consolidation is not as strong as it needs to be and that is something that has been identified in the department as something we need to work on. It’s great to have an activity but how do we know what we’ve got out of it? [taps the book *Five Practices* in front of him to imply that they use the book to work on this]. (Case 7, Year 2, meeting 5)

Some schools engaged with the resource as a book study (e.g., Case 7) and others used the ideas in the book as the foundation of their lesson study (e.g., Case 1).

Through my involvement with the project I observed ways that the project provided opportunities for the PLCs to share different resources. The PI created an online sharing resource where PLCs could upload tasks, and resources to share across the project. Online tools such as Desmos, an online graphing tool, were shared on this online platform and many teachers shared that these resources were a valuable addition to their teaching. I now turn to discussing the findings related to enabling constraints.

Enabling Constraints: Randomness and Coherence. As mentioned, the project was designed to provide PLCs the opportunity to decide how they wanted to work and what they wanted to work on. Randomness therefore was an important component of the OAME project. During the project launch I observed how the PLCs were provided an introduction to the project and the goals and then PLCs were provided time to work on their own action plan. While the steering committee presented options and suggestions for how the PLCs might want to work as a collaborative unit there were no requirements that they follow any specific plan of action. Randomness was also present in other ways such as the choice to apply to be part of the project, whether or not to participate in the research component of the project, and whether or not PLCs wanted to present at the annual OAME conference at the end of Year 1 and Year 2 of the project. Furthermore, PLCs were given a choice to participate in Year 2 and in the province-wide conferences organized at the end of the project.

Some of the ways that coherence manifested at the project level included coherence to the provincial curriculum, the specific PLC membership requirements of the project, a focus on students in Grade 9 Applied Mathematics, attendance at project-wide meetings, and holding monthly PLC meetings at the school site. During project-wide meetings the PLCs were expected to engage in the planned activities, and I observed this coherence as a member of the research team. I was tasked with taking notes and recording the topics the different participants discussed.

Self-organization. As part of the research team I observed and participated in conversations during research team meetings that revolved around how to respond to the needs of the individual participants, and PLCs. I observed the ways the OAME project demonstrated self-organization at the project level in several ways. Overall the project self-organized into a two-year project from the intended one-year project. The steering committee paid close attention to the needs of the PLCs that emerged as the project progressed, as a result different activities and resources were shared with the PLCs. For example, during research team meetings RAs would discuss what their assigned PLCs were working on and if different PLCs were working on similar activities the research team would generate ideas that could support both PLCs. For example the Case 7 PLC had developed a survey that could track student beliefs and attitudes about learning mathematics over the course of the school year. The goal of using this survey was to observe if the activities and work the PLC was doing was supporting students in developing a growth mindset. Through hearing about this survey at a research team meeting the RA from Case 2 adapted the survey and the Case 2 PLC incorporated this tool into their classes as well. This self-organization was not pre-determined at the beginning of the project but rather it was based on a need that arose through the neighbour interactions that were enabled in a network with decentralized control. The RAs from different PLCs were interacting, sharing information and experiences, and adopting different strategies to share with their assigned PLCs. Other ways the project demonstrated self-organization was PLCs seeking out, and making connections with, one another. As an RA at two different PLCs I observed these PLCs discussing different resources such as course plans, lesson ideas, and assessment strategies shared by other PLCs at the project-wide meetings. Different PLCs created shared document folders online that they used to share resources. This self-organization came about from neighbour interactions at the project-wide meetings and the PLCs sharing what they were working on with one another.

During project-wide meetings different PLCs would seek out connections with people and resources that they felt could support their work and their learning. During the final project-wide meeting at the end of Year 2 different PLCs self-organized into working groups to develop presentations for the OAME annual conference as well as for the Grade 9 project conferences that were organized as a way to share the learning from the project with educators across the province. Self-organization aligns with the next characteristic of adaptation. As the project self-organized it also adapted to the changes it experienced.

Adaptation. The project demonstrated adaptation in several ways including introducing resources as the need arose and creating more opportunities for the PLCs to collaborate together. As the project entered Year 2 it was apparent to the steering committee that there was a need for PLCs that were working on similar areas to meet to discuss their challenges and what was working for them (Suurtamm et al., 2017). Project-wide meetings therefore included more cross-case groupings. This way information and ideas became more visible and were shared more easily among the PLCs. Finding from the analysis of participant exit interviews from the cases as well as the phase 1 survey involved in this study revealed that all participants found the sharing between PLCs to be one of the most beneficial components of the project. Chrissy, from Case 4, shared that the connections she made during the project-wide meetings helped her to realize that even though her school was far removed from the rest of the province due to their Northern location they were on the right track and others were having similar issues to the ones her school was experiencing. These connections with others reflect nestedness and the importance of different systems interacting which I will discuss next.

Nestedness. Findings from the analysis of the final project report and the phase 1 survey suggest that the nested nature of the project was an important characteristic influencing the learning of different systems. There were several examples of nestedness at the project level including PLCs being nested within the project, the project being nested within the OAME community, and the OAME community being nested within the greater mathematics education community (Figure 12). The PLCs nested within the project enabled neighbour interactions at both project-wide meetings as well as when PLCs connected and interacted online or visited with each other at their schools. The project was nested within the greater OAME community which involved PLCs presenting their work from the project at annual general conferences. Chrissy, a participant from Case 4, shared that being able to attend the project-wide meetings and the annual OAME conferences was only possible through the funding provided by the project. Neighbour interactions, diversity, and redundancy also worked with the nestedness of the project to support individuals such as Chrissy making connections at conferences. She shared how, when her school presented at the OAME conference at the end of Year 1, there was interest in their work from others in the province. This interest from the mathematics education community buoyed her confidence in the work her school was engaged in. The Case 4 Year 1 report mentioned that the PLC was encouraged that others across the province valued the work they were doing and wanted to learn more. The connections she made at the conferences provided her with learning opportunities. During her phase 2 interview we discussed the ways that the connections she made with others beyond the project due to her involvement with the project has supported her learning:

Chrissy: participation [in the] OAME project has been the one thing that has guided [me] because I was able to go to the annual general [OAME conference] for two years in a row. I was able to make connections with other folks that I've been able to continue on with. I know who to follow. You know?

Researcher: On social media?

Chrissy: Yeah

Researcher: So, is that a big part of it for you?

Chrissy: It is and so now I'm also getting all of the [information from others], like so I've not let my [OAME] membership lapse at all. You know?

The nestedness of the PLCs within the project supported neighbour interactions among the different PLCs as well. Through meeting and discussing issues of teaching and learning at project-wide meetings, individuals and their PLCs made connections that extended beyond their own PLC level. This nestedness provided opportunities for PLCs to interact with ideas and ways of working as a PLC that were new to them. For example, some PLCs had a more rigid, top-down leadership model and the work the PLC focused on was decided by a few individuals rather than the collective (e.g., Case 5 & Case 10). When the PLCs met as a large group the PLCs that were led with a top-down leadership approach were introduced to more collaborative ways of working, different areas of focus, and different examples of rich tasks. One principal shared how during the project-wide meetings he realized his PLC needed to be more collaborative rather than rely on an outside facilitator and therefore adapted their approach in Year 2 to be more collaborative. A member of Case 5 PLC echoed this in the second year.

I was the one that was excited about this project last year at the beginning and then was jaded ever since because it seemed like everybody [in Case 5] was being told to do it a

> certain way and the guy we had last year facilitating was very much like, “This is the way to do it and this is the only way to do it” and the whole thing about learning goals, success criteria kind of stuff, and like it doesn't bring in your individual style. (Rick, Case 5, Year 2, meeting 2)

Without the nestedness of the project the Case 5 PLC may have continued using an approach to professional learning that did not engage the PLC members, possibly leading to less learning. This nestedness along with the other conditions and characteristics all interacted to lead to the emergence of learning at the individual, PLC, and project level. I will next discuss this emergence.

Emergence of Learning: The OAME Project

The above examples are some of the ways the conditions and characteristics of a complex learning system manifested in the OAME project. These conditions and characteristics interacted in ways that influenced the emergence of learning at the project level. I will now describe how these manifestations interacted and supported the emergence of learning.

The condition most coded during my analysis of the data at the project level was neighbour interactions. Analysis of the findings suggest that the nestedness of the project and the way it enabled neighbour interactions was particularly valuable for individuals who did not often have opportunities to participate in provincial initiatives. For instance, Jennifer, a Grade 6 teacher from Case 4 discussed how the opportunity to travel to the project-wide meetings influenced her learning. As mentioned Case 4 was located in a rural area of Ontario and travelling to provincial conferences was not something that Jennifer and her colleagues could often do because of financial barriers. The project provided Case 4 multiple opportunities to travel to meet with others and interact in face-to-face settings. Jennifer found this extremely valuable to her teaching, sharing:

> I think this project, maybe I've just been an ostrich with my head in the sand, but I was completely unaware, I think, of how much math support and math learning was out there, math resources [are] out there. The OAME conference at the beginning of May, I knew nothing about. That blew my mind. It was such great PD for me. And just knowing, having the time to go through some of the Ministry websites and resources that are out there. Just being made aware of what exists already has been super supportive in my math instruction. (Jennifer, Case 4, exit interview)

The majority of the participants interviewed during phase 2 commented on how the opportunity to interact with other individuals and PLCs at the project-wide meetings influenced their learning. This again is an example of how the nestedness of the project supported neighbour interactions. Jill, a mathematics teacher from Case 1 shared that in her rural district most schools are far apart, making attending professional learning sessions after school difficult. The project connected her with a diverse group of individuals all focused on a similar goal, thus diversity and redundancy were important conditions that provided opportunities for rich neighbour interactions across different PLCs. Jill explained how she started to use Twitter as a professional learning tool and connected with many other educators, some of whom were also involved in the OAME Grade 9 Applied project.

> We are kind of far apart in my board right? We are about two and a half hours apart…so like you don't get a lot of collaboration. I remember like actually questing after different types of PD like "Why can't I get any of this?" … but then there were so many people on Twitter in this project… it was really easy [to make connections] and everybody that was

> on Twitter that way was already sharing things because I think we had like a lot of common learning. (Jill, Case 4, phase 2 interview)

Jill's use of Twitter to connect with others from the project is an example of redundancy and diversity. Jill found those on Twitter shared a similar mindset about collaborative learning and they were actively seeking connections with others in order to push their thinking. Diversity was represented in the different experiences and pedagogical knowledge that individual people brought to the collaboration.

Similarly, a district mathematics lead teacher from Case 6 explained how the project provided opportunities for his school district to engage in activities that were new to them and that pushed their thinking. During his exit interview he explained how interacting with other PLCs at the project-wide meetings provided opportunities to learn from and with other PLCs, particularly Case 7 at Fields High School. He explained:

> I think that's it's nice that we were able to cross-pollinate with other schools at the anchor sessions … because this is where that came from. This came from [Grade 9 teacher] seeing the spiralling at Fields at the summer institute and so this is where the idea was born and so I think that cross-pollination this project has provided really got what we needed into [SCHOOL DISTRICT NAME]. (Case 6, District mathematics lead teacher, exit interview)

The cross-pollination he is referring to is an example of neighbour interactions and the nestedness of the project provided opportunities for these interactions. I was the RA assigned to both Case 6 and Case 7 and was therefore able to support a connection between these two PLCs. Case 6 travelled to Case 7 to spend the day learning together and the two schools created a shared drive to share resources and learning online.

Tasha, from Case 7 shared that interacting with others at the project level revealed to her that her PLC was at a different place in their learning than other PLCs and this encouraged her PLC to adopt more a leadership role. Case 7 introduced several new ideas and practices to the other PLCs in the project that influenced the emergence of learning of other PLCs and the project as a whole. Tasha shared in her phase 1 survey that she had hoped she would be able to contribute to building resources for teaching mathematics but instead she and her PLC realized the need to promote their innovative approaches to teaching and learning across the province.

> What attracted me to the project was that I was fortunate to have been at a school who has led the change in teaching practices and our administrator participated and encouraged our innovations. Much of what we were doing in our classrooms were well-received by students and parents. I hoped the project would lead to more collaboration in developing a program of resources to build math confidence in our vulnerable students. Instead, we saw what other schools in the province were doing and realized the importance of promoting change. (Tasha, Case 7, phase 1 survey)

Maya, from Case 10, shared that interacting with other PLCs at the project level inspired her to pursue more professional learning and motivated her to grow professionally. The project provided the PLCs opportunities to work collaboratively with other PLCs and to learn from one another, Maya shared that this was one of the most influential components of the project for her that influenced her own learning, being exposed to new people and their ideas.

> I really loved meeting new people and learning different ways, like, I think that that was the first time I learned what spiralling was. Right? We had never talked about that before. So doing that project was amazing just because I learned so much and met so many people once I really started going to [the] OAME [conferences] and seeing all these

> amazing things. [Now] I am a huge promoter of the thinking classroom and that's something that I learned from that project right? Getting kids up on vertical surfaces I promote that. (Maya, Case 10, phase 2 interview)

Analysis of the project transcripts, research reports, and the collection of phase 1 and phase 2 data overwhelmingly point to the importance of several conditions and characteristics of complexity that supported the emergence of learning at the project level. The diversity and redundancy of the project participants together with opportunities for neighbour interactions in a system with decentralized control led to the emergence of learning. The nested nature of the project also ensured that participants were exposed to different situations that could stimulate their own ideas and goals. I next present findings at the PLC and individual levels by discussing the ways the conditions and characteristics of complex learning systems manifested in each of the four cases included in this study.

Addressing Research Question One: Case 1

I first present an overview of Case 1, drawing from the secondary analysis of phase 0 data. Case 1 began as a PLC located at Weber High School in a rural area of Ontario and had a student population of approximately 650 students at the time of the OAME project. Although not all members of the original PLC were participants in this doctoral study they played a role in the learning that emerged in the PLC and were part of the secondary analysis of the original project data. Thus, it is important to provide details about the membership of the PLC. The PLC membership changed significantly between Year 1 and Year 2 of the OAME project. During Year 1 the PLC members were all from Weber High School. Near the end of Year 1, several members of the PLC decided not to continue with the project into Year 2, and the PLC needed to recruit new members for the second year. Individuals from an elementary feeder school agreed to join the PLC in Year 2 of the project. Therefore, the PLC in Year 2 was made up of educators from both Weber High School and the elementary school. Table 13[1] outlines the individuals who were participants during the two years of the OAME project. Individuals who agreed to participate in this current doctoral study are highlighted in bold print.

Table 13

Members of Case 1 OAME Project PLC

Pseudonym	Role Year 1	Role Year 2
Georgia	Principal	Principal
Brenda	Mathematics Department Head	N/A
Drew	**District Mathematics Lead Teacher**	**District Mathematics Lead Teacher**
Jill	**Mathematics Teacher**	**Mathematics Teacher**
Francis	Community Program (CP) Teacher	CP Teacher
Lucille	Mathematics Teacher	N/A
Barbara	Mathematics Teacher	N/A
Nancy	Special Education Teacher	N/A
Annika	Research Assistant	N/A
Zoe	CP Teacher	CP Teacher
Don	N/A	Temporary Numeracy Coordinator
Jed	N/A	Grade 7/8 Teacher
Josh	N/A	Grade 7/8 Teacher
Lauren	N/A	Grade 7/8 Teacher
Katherine	N/A	Grade 7/8 Teacher
Josh	N/A	Grade 7/8 Teacher
Sandra	N/A	Grade 7/8 Teacher
Gary	N/A	Elementary School Principal

[1] The tables outlining the PLC memberships are reprinted from Chapter 2. I have included them here to remind the reader of the individuals involved in each case.

Tom	**N/A**	**Research Assistant**

During the two years of the project, PLC meetings were generally attended by those listed in Table 13. The exception is Drew, the school district lead teacher, who was on leave for most of Year 2 and Zoe the Community Program teacher who was on leave for most of Year 1 of the project.

The decision to apply to participate in the OAME project was made by the school principal, Georgia, after being asked by a school district superintendent if Weber High School might be interested in participating. Georgia asked Jill, a mathematics teacher, to write and submit the school's application. The original problem of practice identified on the application, written by Jill, was as follows:

> How do we use rich tasks/open problems to diagnose and understand where the students are in their math thinking, increase discussion in class about various solutions, move them forward in their thinking and close the gap while also being able to teach and assess all of the required curriculum for the course? How do we support the students to make connections between topics in math and understand the concepts upon which the topic is built? (Case 1, OAME project Application, June 2014)

The problem of practice evolved as the PLC began to meet and discuss different issues as a group. On the website that the PLC created early in Year 1 to track their learning they identified a different problem of practice that emerged from the group:

> We need to access and develop practices for our 1P [Grade 9 Applied] students that provide for them a context for an authentic understanding of algebraic skills and their application to their lives and learning. This context needs to provide a deeper

> understanding that they can build on in order to develop more complex problem-solving skills (for example, those explored in Grade 10 Applied Math). (Case 1, PLC Website)

Analysis of the transcripts from the first meeting of Year 1 reveal that this newer problem of practice was chosen because the PLC members felt students were struggling with transitions to Grade 10 mathematics and were not carrying forward skills from Grade 9 such as algebraic thinking. The PLC chose an action plan for Year 1 that included holding monthly meetings, cataloguing resources through a website they created, developing a diagnostic tool to assess students' progress, and collecting and recording resources, manipulatives, and apps for use on iPads (Suurtamm et al., 2017).

Over the two years of the project the PLC remained focused on using rich tasks in meaningful ways and also explored issues of assessment (Suurtamm et al., 2017). The PLC incorporated many of the resources suggested by the research team including *Mindsets in the Classroom* by Mary Cay Ricci (2013), *Mindset, the New Psychology of Success: How We Can Learn to Fulfill our Potential* by Carol Dweck (2006) and *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning* by Jo Boaler and Cathy Humphreys (2005). When the book *Five Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2015) was introduced at the summer institute between Years 1 and 2 of the project the PLC decided to incorporate the book into their PLC action plans for Year 2. They also incorporated new teaching techniques they learned about from other PLCs such as using VNPS in the BTC model (Liljedahl, 2015).

Next I present findings that demonstrate how the conditions and characteristics of complex learning systems manifested in Case 1 and how these supported the emergence of learning. I begin with a discussion of the six conditions necessary to support the emergence of learning from a complex system.

Specialization: Redundancy and Diversity. Specialization in a complex system involves balancing two conditions, redundancy and diversity. Because Case 1 experienced a major overhaul to the PLC membership during the two years of the OAME project, I will present evidence of specialization from each year separately, beginning with Year 1. Findings from the analysis of transcripts from Year 1 PLC meetings and exit interviews with several participants at the end of Year 2 suggest that redundancy was evident in the PLC in several ways. During Year 1 PLC members were all from Weber High School, their shared focus was on mathematics teaching and learning, and the majority of the PLC members who were mathematics teachers shared a common pedagogical approach to teaching mathematics. This pedagogical approach to teaching was described by Georgia, the principal, as being “traditional” (Georgia, Case 1, Year 2, exit interview). While I consider this shared approach to teaching mathematics as an example of redundancy I see the fact that one teacher, Jill, was non-traditional as an example of diversity and this represented a difference in the beliefs and attitudes about teaching and learning mathematics that PLC members had. Diversity was also evident in the level of engagement different PLC members had with the goals of the project and the different areas of teaching expertise of the various PLC members. I will describe each of these examples of diversity in more detail.

Findings from the analysis of Year 1 PLC meeting transcripts, Year 2 exit interviews, OAME project final report, and Case 1 report reveal the majority of the teachers shared a traditional approach to teaching mathematics and believed in consistency in mathematics classes across the department, meaning that all teachers should teach the same way using the same course outline. Jill did not share this pedagogical approach. Jill described herself as a progressive teacher who always sought to enhance her practice and experiment with different techniques based on her students' needs (Jill, Case 1, exit interview). This was in direct contrast to the rest of the mathematics teachers who followed a predetermined course plan that was consistent across the department. Georgia confirmed this during her exit interview at the end of Year 2. Georgia shared, "I went through the system as an academic student [and] a traditional math learner. So my vision of math is that of what is consistently being taught by the majority of my teachers" (Georgia, Case 1, exit interview). Zoe, a Community Program (a program that works with students who have not attended traditional schools prior to high school) teacher, shared a similar sentiment during her exit interview. She described what it was like when she started teaching as part of the Weber High School mathematics department:

> **Zoe:** The math department here is very structured, they give you a day-by-day plan
>
> **Researcher:** ah
>
> **Zoe:** And it's like a script.
>
> **Researcher:** Okay
>
> **Zoe:** And you teach from the script.
>
> **Researcher:** hmm
>
> **Zoe:** Complete with problems to be assigned out of the book. Um, so there was no area for deviation at all as a new teacher. (Case 1, Year 2, exit interview)

I see this diversity in pedagogical approaches between Jill and the rest of the PLC as also representing a diversity in teachers' beliefs and attitudes about teaching and learning mathematics. During Year 1 PLC meetings it was evident that the traditional teachers were not as keen to try new approaches to teaching such as using manipulatives or open-ended problems.

Rather, they appeared to be focused on creating consistency across all mathematics classes in the school in order to provide similar ways of speaking about different mathematics concepts in the hopes that this would help with the transition between grade 9 and 10. This focus on consistency was seen in much of the work the PLC engaged in during Year 1 of the project. During the first meeting of Year 1 Brenda, the mathematics Department Head, presented each PLC member with a detailed package that contained the Grade 9 Applied Mathematics curriculum expectations, the mathematical processes, a course outline unit by unit, and an outline of the daily lessons for each unit. Discussions during this first meeting focused on consistency in terms of technology and manipulatives. Many of the PLC members stressed that the classes all had to "match".

> They [apps on the students' iPads] need to match each other, we want that continuity, we don't want like an app that her class uses and is happy with and then next year they go to Barbara's class, and they have a different App. It would be nice to have that continuity and match what we have for hands on toys too. We want to, as much as possible, have things match for continuity. Because that was part of our whole goal with this thing was continuity for 1P and 2P, so it all needs to be continuous right? (Lucille, Case 1, Year 1, meeting 1)

Jill, on the other hand, wanted to consider more student-centred pedagogical approaches. During this same meeting she suggested an idea to co-create diagnostic tools with her students. Jill started to share her idea by saying "I had an idea there, you know me and my ideas, co-creating here, or co-creating with your class for the" but she was cut off by others laughing and an unidentified speaker dismissing her by saying "We don't have time for that". The meeting quickly returned to discussing creating common diagnostic tools for every class in order to assess the areas where students are struggling.

The different PLC members also appeared to have different levels of engagement with the goals of the OAME project. Analysis of the meeting transcripts suggest that the traditional teachers wanted to align their current lessons and assessments in order to ensure consistency. Analysis of the RA's researcher notes from the Year 1 Case report and exit interviews from Year 2 suggest that Jill wanted to spend time trying different professional learning models that provided opportunities to explore different strategies for teaching mathematics. For example Jill wanted other PLC members to observe her teaching as she incorporated rich tasks (see Appendix I). Jill hoped that she and the observing teachers could debrief the lessons together afterwards and share ideas about how it went and perhaps how to adapt the lesson in the future. Jill often invited the other mathematics teachers to observe her new lessons to provide feedback, but most did not accept her offer. Those that did engage with the observation included Drew, Francis, and Zoe. Drew, the district math lead teacher, was supportive of Jill. Francis, a new teacher who was not teaching mathematics, accepted her offer and engaged in the lesson debrief enthusiastically. He, however, did not have a permanent contract with the school and was teaching a modified mathematics class at Weber High School during Year 2 of the project. Zoe also engaged in exploring new learning but due to being on leave for most of Year 1 she was not available to observe and engage with Jill regularly.

Diversity also manifested in the different areas of teaching expertise of the PLC members. Zoe was a science specialist who taught mathematics and both Georgia and Nancy, a special education teacher, often referred to themselves as being "non-Mathies" (Georgia, Case 1, Year 1, meeting 1). They often stressed that others needed to be patient and explain concepts to them. It was evident through analysis of PLC meetings transcripts that this diversity in teaching specialties provided several rich opportunities to consider different ways that students might approach different mathematical concepts as the participants tested out some of the mathematics problems themselves.

Analysis of exit interviews from the end of the project suggest that a dominant issue that impacted how the PLC operated during Year 1 was the lack of balance between redundancy and diversity. This lack of balance was primarily between Jill, the non-traditional mathematics teacher, and the rest of the mathematics department teaching staff. This lack of balance created a great deal of tension and ultimately appeared to lead to the collapse of the PLC near the end of Year 1. Georgia described that although the team acted collegially during the meetings their different approaches were too diverse to sustain the PLC.

> I don't want to use the word "revolted" but that's basically what it was. Unfortunately in my building I have a dynamic where I have traditional math teachers, and I have the non-traditional Jill, the non-traditional math teacher. And the two, don't [pause] operate super well together. They are collegial and all that, and you can see from the operation of the group, you know, they're a great group of people, but um, the belief around a lot of the work that Jill is doing is that it's in theory good, but not necessarily [pause] super [pause] impactful when it's put into practice, and there's challenges with putting it into practice. (Georgia, Case 1, Year 2, exit interview)

For the PLC to continue in Year 2 new members were required and, in her exit interview, Georgia described how new members were found. During Year 1 Jill and Drew had interacted with teachers from a nearby elementary school at a full-day provincial workshop on the EQAO provincial assessment. This workshop was led by the PI, Dr. Suurtamm, but was not connected to the OAME project. This work between the elementary and secondary teachers at the EQAO session helped the two schools build a relationship that led to the PLC reconfiguring with new members from both Weber High School and the elementary school. Year 2, therefore, was a new beginning for the Case 1 PLC and diversity and redundancy manifested in new ways.

Analysis of Year 2 PLC meeting transcripts, Year 2 exit interviews, OAME project final report, and Case 1 case reports suggest that during Year 2 redundancy manifested in all PLC members teaching in the same school district with a similar population of students. Oftentimes the students in the Grade 9 Applied Mathematics classes were students who had attended the elementary school and had been taught by an elementary school PLC member. The PLC in Year 2 shared goals of focusing on student thinking and exploring the continuum of learning across the different grades; the teachers were also all engaged in the same professional learning model. During Year 2 this learning model involved one teacher teaching an agreed upon rich task during the meeting time and others observing. The PLC meeting would begin with the group engaging in the *Five Practices* framework and anticipating students' responses to the problem prior to observing the lesson. Following the lesson observation the PLC would debrief what they observed, discussing student thinking, teacher moves, and planning possible next steps for the teacher to take.

Diversity manifested in the different grade levels the PLC members taught, the different schools the teachers taught at, and the different beliefs and attitudes about teaching and learning mathematics. During Year 2 Jill was again the only high school mathematics teacher teaching Grade 9 Applied Mathematics. Francis and Zoe were teaching a modified version of Grade 9 Applied Mathematics in the CP classrooms. The other teachers in the group taught at the elementary school and were not mathematics specialists. They each taught a split Grade 7/8 class of students and a range of subjects. Diversity in the teachers' beliefs and attitudes about teaching mathematics was evident in the analysis of the Year 2 PLC monthly meeting transcripts. The transcripts show tension arising during discussions when group members disagreed on issues such as how to group students, whether or not to have students working in groups at all, and the types of learning activities the teachers used in their classrooms (Case 1, Year 2, meeting 1). The teachers appeared collegial during these discussions however Georgia and Jill both commented on some tensions among the group in their individual interviews at the end of the project. Jill hinted that the lack of familiarity between the high school teachers and the elementary teachers may have created a hesitancy for the new members to ask questions. Georgia suggested that some of the elementary teachers were more resistant than Jill to trying new approaches and were hesitant to make changes to their teaching. This diversity was described by Georgia:

> I feel that the women were absolutely more engaged than the men in that group, I felt that there was an undertone of resistance with those men. And with the ladies on there, they were authentically trying to look to improve their teaching practices and question it and were thinking deeply about it. (Georgia, Case 1, Year 2, exit interview)

During Year 2 there did appear to be more balance between redundancy and diversity and this helped to sustain the PLC. At the end of Year 2 the PLC was making plans for a third year to continue their collaboration. Issues of diversity and redundancy were hinted at during Jill's exit interview at the end of the project. Jill was asked about some of the challenges the PLC faced, and she discussed how the different membership during the two years influenced how the team interacted. She mentioned that during Year 1 the familiarity among PLC members and a shared expertise as high school mathematics teachers made it easier to ask questions of one another during Year 1 yet the learning during Year 2 seem to be richer in her opinion. This suggests to me that there was greater balance between diversity and redundancy during Year 2.

> I think that last year when I had a group of people that I already knew that I was working with, there definitely was, like if there's a question people asked, nobody was going to get offended. Nobody was going to worry that "oh you're dumb" whatever. But I think there was a safer space last year [Year 1], but there was also maybe not as much progress? I don't know. (Jill, Case 1, Year 2, exit interview)

Trans-level Learning: Neighbour Interactions and Decentralized Control. In order for learning to emerge from a complex system opportunities for neighbour interactions in a system with decentralized control are necessary (Davis & Sumara, 2006). I first describe the ways trans-level learning manifested in Year 1 before describing the ways it manifested during Year 2. Findings from the analysis of the final project report, the PLC meeting transcripts, individual exit interviews, and the case report are presented. During Year 1 examples of neighbour interactions included interactions between different PLC members and their ideas, interactions with others beyond the PLC such as researchers and other PLCs and their respective ideas, interactions with resources such as books and research articles, technology, rich tasks, manipulatives, as well as interactions with students and student thinking.

Analysis of PLC meeting transcripts reveal that neighbour interactions among the whole PLC during Year 1 tended to be limited to discussing creating consistency through course outlines, diagnostic tests, summative assessments, and choosing ways to incorporate the new school iPads into the mathematics classrooms. Analysis of the RA's research notes, the case report, and exit interviews suggest that a subgroup of the PLC appeared to form that included Jill, Drew, and Francis. As mentioned, this subgroup explored rich tasks and how to implement them in the classroom. They also sought neighbour interactions with researchers and other PLCs. This type of neighbour interaction carried over into Year 2 when the PLC reformed. The *Five*

Practices framework became the dominant professional learning model used by the PLC throughout Year 2. The framework prompted many neighbour interactions including interacting as a PLC with a rich task to anticipate students' responses to the task prior to observing a teacher using that task in their classroom, interacting with students and their thinking during the lesson, and interacting as a PLC after the lesson when the lesson was debriefed.

Drew shared that an influential component of the project for her was neighbour interactions with other participants "the other teachers that we were able to network with and learn alongside with was amazing!! Really challenged our thinking on spiralling and rich tasks…The in person learning weekend in London. Working so closely with Chris Suurtamm was an amazing opportunity. The group PD was always amazing!" (Drew, Case 1, phase 1 survey). These different examples from Drew highlight the different neighbour interactions at the project level that influenced her. Being able to share ideas with others beyond their school perturbed Drew's thinking and learning from the PI enhanced Drew's experience with the project.

Another example of neighbour interactions was with PLC members and students and student thinking. These interactions with student thinking occurred somewhat during Year 1 when Jill, Francis, and Drew observed Jill teaching and became a major focus of the PLC in Year 2. During lesson debriefs at the monthly PLC meetings members shared that being a fly on the wall in someone else's classroom during a lesson provided them new perspectives on student thinking as well as teaching and learning. Without the responsibility of teaching, the PLC members who were observing were free to circulate and engage in conversations with students. These observers could share observations with the teacher that the teacher may have missed due to the responsibilities of teaching.

In the Case 1 PLC there were several examples of decentralized control as well as examples of a lack of decentralized control. I begin by discussing the ways a lack of decentralized control may have contributed to the collapse of the PLC during Year 1 before describing ways this influenced the learning that emerged from this PLC. Following that I describe ways that decentralized control did manifest in this case. Analysis of the exit interviews suggest that the formation of the Case 1 PLC lacked decentralized control as it was formed through top-down control. When Georgia decided to apply to the project she "went to the department and placed members on it (the PLC)" (Georgia, Case 1, Year 2, exit interview). This represents two examples where decentralized control was lacking. The decision to apply and who would be involved in the project was a top-down decision. Jill defining the problem of practice independent of the PLC also represented a lack of decentralized control. Analysis of the transcript meetings from Year 1 suggest that the problem of practice selected by Jill may not have reflected the needs of the PLC as a whole.

Analysis of the monthly meeting transcripts suggest that decentralized control was more evident once the Year 1 monthly meetings began. For instance, PLC members were free to contribute their thoughts and opinions about different areas of focus to pursue during the PLC meetings. Determining an area of focus proved to be a difficult task. The meetings were generally led by Drew during Year 1 and her leadership style reflected a very open and collaborative approach. As she attempted to focus the group on setting a clear goal during the first meeting of Year 1 she mentioned "there was a lot, it's been a lot of really rich discussion, but hard to actually nail down what it is that the problem is to move forward" (Drew, Case 1, Year 1, meeting 1). Drew demonstrated decentralized control through her solicitation of ideas and asked questions of the other PLC members regarding what they should focus on during their

PLC meetings and how they wanted to work. As the year progressed it was evident that most of the PLC was not interested in trying new pedagogical approaches, as mentioned in previous sections. It did allow the smaller group to align with a similar goal of integrating some of the new ideas they learned about through the project-wide meetings. In general, Drew's leadership approach seemed to encourage some neighbour interactions and led to slightly more focused discussions, yet, in an interview, Jill shared that a lack of engagement from the members limited the neighbour interactions and may have led to many leaving the project after the first year. "Nobody owned the project at the beginning, so that was a bit weird. Yeah, so that may have actually contributed to what happened later down the road with how many people stayed on the project [after] the first year" (Jill, Case 1, phase 2 interview). I interpret this comment from Jill as an example of a lack of commitment to the goals of the project from individuals, relating back to a redundancy in the minimal engagement of participants with the project during Year 1. When we consider that learning emerges through neighbour interactions that disrupt in a system with decentralized control this appears to be limited in Case 1 during the first year of the project. This was evident in the analysis of meeting transcripts as well as the exit interviews with Jill, Georgia, and Zoe.

Findings from the analysis of the Year 2 meeting transcripts indicate this decentralized leadership approach was continued during Year 2 of the project with Don, the temporary numeracy coordinator, and Jill taking on more of a leadership role yet still using a collaborative approach. Analysis of Year 2 transcripts revealed that the engagement from the PLC members was higher in Year 2 leading to more neighbour interactions and rich discussions about teaching and learning. Engagement of PLC members seemed to play an important role in trans-level learning. The transcripts reveal that there was more push back and productive disagreement among the group that required them to explain their ideas and negotiate together as a group. This engagement was evident in the ways that the group explored the use of different tasks throughout Year 2.

Both years had a similar leadership approach yet the engagement among PLC members was higher in Year 2 leading to more neighbour interactions. It is apparent from the analysis of meeting transcripts that during Year 2 meetings this balance between diversity and redundancy better supported neighbour interactions and participants' willingness to consider different approaches to teaching and learning mathematics. Analysis of Year 2 meeting transcripts show that the diversity in grade levels taught provided a natural focal point for discussions. Different participants could share their opinions and ideas on how mathematical ideas could be approached at various grade levels, providing opportunities for rich discussions. The elementary teachers and secondary teachers had different curriculum objectives to teach and different ways of looking at the rich tasks they were exploring together. This led to more opportunities for neighbour interactions in a system with decentralized control.

Enabling Constraints: Randomness and Coherence. Enabling constraints support the emergence of learning through balancing randomness and coherence (Davis & Sumara, 2006). Analysis of the final project report, the RA's notes in the case 1 report, the meeting transcripts, and the exit interview transcripts suggest a lack of balance between these two conditions. During Year 1 of the project coherence manifested in many more ways than randomness. Coherence manifested in the presence of provincial mandates such as the curriculum and EQAO tests and also in locally imposed coherence to the strict course plan determined by the mathematics departments and the quest for consistency, for all the teachers to "match" by using the same terminology, technology, and manipulatives in every class. As described previously, during Year

1, the PLC in Case 1 lacked a shared belief about teaching and learning mathematics and this resulted in different goals for different members of the PLC, demonstrating a lack of coherence.

Randomness during Year 1 was limited to Jill operating differently than the rest of the department and experimenting with new pedagogical approaches in her classroom, with little support or interest from the other mathematics teachers. During her exit interview Georgia described this dynamic, "it was driven very, very much by the idea that Jill would [say] 'we could do this, we could do that' and she would kind of [lead] and there was a lack of engagement from the other people" (Georgia, Case 1, Year 2, exit interview). Although Jill was supported by Drew and Francis, these two individuals did not play a big role within the school or the mathematics department on a daily basis, leaving Jill as the sole PLC member who was trying new practices.

During Year 2 there appeared to be more balance between randomness and coherence. Although there were still some teachers reluctant to change their practice, analysis of the meeting transcripts shows a coherence around the idea of experimenting with rich tasks and examining their practice. During Year 2 coherence was again present in the form of the curriculum and EQAO expectations as well as the coherence to the PLC's professional learning model. At the start of Year 2 Jill took more of a leadership role in the PLC as she wanted to avoid the issues the PLC had during Year 1 when they lacked coherence to a clearly defined, agreed upon, goal. Jill and Georgia discussed this issue of a lack of a defined goal and set agenda prior to the first meeting of Year 2 as they were the only Year 1 PLC members attending during most of Year 2.

> When we were talking about what we are going to do for the meeting, first of all, we decided we wanted to be a little bit more formal. Last year when we were doing investigations we had lots of big ideas that we want to sort of touch on, but there was no

> consolidation. We decided we wanted to have an agenda [this year] so that we can make sure that we feel like we've accomplished stuff on a day-to-[day], meeting-to-meeting basis as well as making plans forward. (Jill, Case 1, Year 2, meeting 1)

The PLC maintained a more formalized professional learning model of using the *Five Practices* framework as described previously. This created coherence to an agenda or plan of engaging with rich tasks followed by observing each other teaching with these rich tasks.

Analysis of the meeting transcripts from Year 2 suggest that randomness also manifested in the ways that teachers chose to align the rich tasks with their own students' needs and the expectations of the different curricula. Often randomness and coherence co-existed, achieving the necessary balance that defines enabling constraints. For instance the PLC agreed that all teachers would use the Border Problem (Appendix I) in their different classrooms including several grade 7/8 classrooms and in Jill's Grade 11 mathematics classroom. Randomness was present as not all teachers were observed teaching since it was their choice to invite others into their classrooms. Lastly, the PLC collaboratively chose which tasks would be used throughout the year. This represents randomness in the sense that the PLC was free to choose the tasks they thought would best meet the needs of their students yet still align with the goals of the PLC and the project.

These six different conditions, diversity, redundancy, neighbour interactions, decentralized control, coherence, and randomness manifested differently in the PLC of Case 1 which supported the characteristics of complex systems in specific ways. I next describe the ways that the conditions described supported self-organization, adaptation, and nestedness before describing how these conditions and characteristics, at times, led to the emergence of learning in Case 1.

Self-organization. The PLC in Case 1 demonstrated different degrees of self-organization over the two years of the project. In Year 1, the PLC appeared to have competing ways they wanted to organize, one way being to "match" and the other way to try new ideas. This tension ultimately led to a lack of self-organization that may have contributed to the collapse of the PLC. Jill made attempts to organize professional learning opportunities such as classroom observations, but most of the PLC members declined her invitations. There was also the need for the PLC to regroup through self-organization after the disintegration of the PLC in Year 1. Georgia described how the traditional teachers were "not seeing the gains that they should have, or they thought they should have. [I] basically had all my people going 'yeah, I'm out' all of them were out, with the exception of Jill" (Georgia, Case 1, exit interview). This lack of engagement led to the need to reorganize the PLC in order to continue with the project. Georgia attempted, during Year 1, to replace the members who quit with staff from a nearby elementary school but was unsuccessful. However, the interactions with elementary teachers at the EQAO session was serendipity and contributed to the formation of the Year 2 PLC, an example of self-organization.

During Year 2 the PLC demonstrated higher levels of self-organization. This is apparent in analysis of the transcripts of the PLC meetings. Year 2 started out with more self-organization and a focus on organizing the goals of the PLC. Diversity and redundancy among the PLC members supported more neighbour interactions that resulted in novel responses from the PLC members. These novel responses led to a discussion on defining clear goals and ultimately the PLC decided to have a more structured agenda in order to make sure they were focusing on the goals of the project. Self-organization was also observed in how the PLC worked together to determine the direction of the PLC activities, whose classroom the PLC would observe, and the different tasks that they would use in the classes. Different members discussed different strategies before deciding together the best course of action for their professional learning activities.

Adaptation. There are different examples of adaptation in the learning systems associated with Case 1. The PLC and individuals, specifically Jill, exhibited adaptations in different ways. The PLC had to adapt between Year 1 and Year 2 of the project as the majority of the PLC members decided not to continue with the project near the end of Year 1. Georgia and Jill attempted to recruit new members several times and when the first recruits were not interested in participating the PLC adapted again and sought out others to join the PLC. This adaptation occurred simultaneously with the self-organization described previously in order to maintain the PLC's involvement with the project and ensuring that the system was sustained. During her exit interview at the end of Year 2 Drew was asked how the PLC had evolved over time and she shared how every part of the PLC had shifted:

> **Drew:** How has the PLC evolved over time? Um, well its content area shifted and got to more of a [mathematical process focus]. The people within the group shifted [laughs]; the role of the administrator shifted. You know like, I'm thinking like
>
> **Tom:** [laugh] Like everything?
>
> **Drew:** [laughs] Yeah, like everything changed over two years. Right? The people, the administrators, the content area. Me, Don, right? Yeah. Everything. Researchers. Yeah everyone was new, and the idea is we were changing. The only people who were consistent through the entire two years, would have been Francis, Jill, and Georgia. (Drew, Case 1, exit interview)

On an individual level, Jill shared during her exit interview how she adapted her teaching as well as beliefs about teaching and learning mathematics. Through an iterative process of planning, teaching, and reflecting on her teaching Jill continually adapted her teaching on a professional level each school year. Neighbour interactions within the project and her own PLC exposed her to new ideas and teaching practices and this influenced her to adapt her teaching as well. “I definitely think that my students have a richer understanding of their math, but I think I feel like that every year…I’m always tweaking things, seeing things from a new perspective, and changing things” (Jill, Case 1, exit interview). She continued by sharing that, because of the project, she had incorporated spiralling, the process of revisiting the same mathematical concepts several times throughout the year, as well as the use of whiteboards and more student collaboration. Analysis of Jill and Drew’s phase 2 interview transcripts suggest that they have adapted their teaching because of the project. This influence from other PLCs in the project leads to the next characteristic I will describe, nestedness.

Nestedness. Case 1 demonstrated nestedness in several ways during the two years of the project. Individuals were nested within the PLC in Case 1 which was nested within Weber High School and the school district. During Year 2 the PLC was nested within the district as members were from several different schools. The PLC was also nested within the OAME project. Lastly, the different members of the PLC were nested within the mathematics education community of the province, which influenced the membership of the PLC during Year 2.

The individuals who are nested within the Case 1 PLC were themselves complex systems which played an important role in how the PLC operated. Each participant was influenced by their own set of conditions and characteristics which in turn influenced how they interacted with others. Most of the participants had different goals for their own learning. For example the lack of balance between diversity and redundancy during Year 1 led to a lack of engagement from most of the PLC members during Year 1. This led to the self-organization and adaptation of the PLC during Year 2 which influenced how the PLC operated and what they focused on. The PLC was nested within different schools during the two years of the project and the diversity of the teachers' grade levels influenced how the teachers worked and what they chose to work on. During Year 2 specifically the PLC focused on the continuum of learning from grade 7/8 through to grade 11 and they examined the different ways that rich tasks can be used across this continuum. The PLC was nested within the OAME project, and this influenced the types of resources the PLC used. The PLC was influenced a great deal by two of the books recommended by Dr. Suurtamm, *Connecting Mathematical Ideas* (Boaler & Humphreys, 2005) and *Five Practices* (Smith & Stein, 2011). Jill was also greatly influenced by interactions with other PLCs in the project, primarily Case 7. She adopted many of the teaching practices that Case 7 shared at the project-wide meetings such as the BTC model, spiralling through the curriculum, teaching with rich tasks, and also encouraged her PLC to adopt the lesson study model of professional learning that Case 7 used.

Lastly, the individuals and PLC from Case 1 were nested within the mathematics education community of the province of Ontario which provided multiple opportunities for the PLC members to interact with others. For example, several of the PLC members from Year 1 interacted with the PI and other educators from their district at the provincial EQAO workshop. This workshop allowed Jill and Francis to make connections with their colleagues from a local elementary school leading to them to join the PLC in Year 2 and as a result the PLC was able to continue participating in the OAME project. The PLC interacted with educators beyond the project when they presented at the OAME provincial conferences as part of the OAME project, as well as at the Grade 9 Applied conferences that stemmed from the project. This will be discussed in more detail when I describe the findings related to RQ2.

Emergence of Learning: Case One

The above conditions and characteristics of a complex learning system manifested in Case 1 in the different ways described. These conditions and characteristics interacted in ways to influence the emergence of learning at the individual and PLC level in various ways, which I will now describe. Analysis of the findings suggests that there is more evidence of the emergence of learning at the individual level than at the PLC level. I will first describe the emergence of learning evident in the findings at the PLC level before focusing on individual learning, especially that of Drew, Georgia, Jill, and Zoe.

All six conditions that support the emergence of learning as well as the characteristics of adaptation, self-organization, and nestedness were coded in the data from Case 1. The condition most coded was neighbour interactions, followed closely by diversity and redundancy which were coded with the same frequency. As I shared when discussing the six conditions that support the emergence of learning from a complex system, the PLC did not function well during Year 1 of the project. A lack of balance between diversity and redundancy limited the opportunities for neighbour interactions at the PLC level yet the nestedness of the project provided other ways for neighbour interactions to manifest and to influence the learning of the PLC. Jill interacting with others, including the researchers, resources, and other PLCs and their ideas disrupted Jill's thinking enough for her to bring back new information into her own PLC. This resulted in Jill sharing rich tasks such as the Border Problem (Appendix I) with her small group including Drew and Francis.

During Year 2, after the PLC had reorganized and achieved a better balance between diversity and redundancy the PLC established an iterative process of planning, teaching, observing, and debriefing lessons. This process led to learning about how better to prepare to teach using rich tasks and ways that they might assess students who were working with rich tasks. This was evident in the ongoing discussions and engagement among the PLC. The use of resources such as the *Five Practices* influenced the PLC's learning. The PLC also learned that without a clear agenda and plan that focused on agreed upon goals and their model of learning that the PLC wasn't as successful. This was evident during meeting 5 of Year 2 when Jill reminded the PLC they had not adequately anticipated how students would interact with a specific rich task and for the next lesson they should spend more time thinking about how students would respond

> Remember, we already said that some things didn't go super well with the Super Bear [activity]. So, I'm just wanting to make sure that we are framing it ok this time. So, everybody here except for Francis has done the Border Problem before, but I'm hoping that you'll take a moment to rethink [the problem] because we need to have like actually done the question [ourselves]. We learned from the Super Bear [problem] and we want to anticipate the student responses. (Jill, Case 1, Year 2, meeting 5)

The emergence of learning was evident in the findings of analysis of Year 2 monthly PLC meetings when the PLC interacted with rich tasks and with students' thinking related to these tasks. Teachers would share their observations of the lesson and students' interaction with the concepts when the PLC debriefed after the lesson was taught. The PLC's learning related to assessment was enhanced when teaching with these activities. Discussing what they saw students doing led the teachers to make connections to the mathematical processes in the curriculum.

During a debrief at the second meeting of Year 2 the PLC discussed how observing the students working and engaging in problem solving reminded them of the front matter of the curriculum:

> **Don**: in front of every course is
>
> **Jill**: the math processes
>
> **Don**: the same processes, you could go to, just one of them, like problem solving…you could just fall back on that
>
> **Lauren**: which is covered in every strand, there's always a problem-solving piece, and apply it to real life situation. I guess that was sort of my other inkling in my mind of how this is relating to real life, why do we need to do this, or is it relevant? (Case 1, Year 2, meeting 2)

The teachers in the PLC realized that teaching using these rich tasks not only connected to the overall and specific curriculum expectations but also to the processes that support mathematics learning including problem solving; reasoning and proving; reflecting; connecting; communicating; representing; and selecting tools and strategies. One teacher pointed out that the key learning using these rich tasks involved problem solving strategies that were transferrable to other types of problems. This is an example of the emergence of learning through neighbour interactions with rich tasks, among PLCs members, and with student thinking. Coherence to the mandated curricular expectations also influenced the emergence of learning in this example.

Findings from the analysis of exit interviews, phase 1 survey, and phase 2 interviews provide explicit evidence of the emergence of learning at the individual level. Several of the PLC members shared examples of their learning from participating in the OAME project. Drew shared that she learned how to be a better facilitator and to push teachers' thinking through observing and interacting with the PI, Dr. Suurtamm.

> So my personal growth as a leader, I was able to watch how the conversations were facilitated and how [Dr. Suurtamm] could challenge the conversation. How she could model using these tasks and ramping up the degree of difficulty with the questions as well. (Drew, Case 1, phase 2 interview)

The conditions and characteristics that supported this learning include trans-level learning, enabling constraints, specialization, and nestedness. Trans-level learning occurred at the different meetings that Drew attended through the project including at monthly PLC meetings at Weber High School as well as at project-wide meetings with the research team and steering committee, representing neighbour interactions for Drew in a system with decentralized control. Enabling constraints manifested through the coherence to the goals of the project balance with randomness in choice with regards to how the individuals chose to align with these goals. Lastly, for Drew, the nestedness of the project supported neighbour interactions with others beyond Drew's own PLC, creating opportunities for her to interact with a diverse group of people that could push her thinking and present new ideas and opportunities. This diversity is an example of specialization, balancing the tension between redundancy and diversity.

Georgia shared that her experience with the PLC was challenging, and she experienced growth as an administrator as well as an educator. She learned the importance of establishing a clearly defined goal and having authenticity among the participants in terms of commitment to the process of professional learning, both of which were lacking in Year 1. Analysis of her exit interview suggests that Georgia learned that her role as a principal in a PLC is more than simply administrative but also requires her to model the type of engagement she wanted to see from participants. Georgia modelled adaptation when she adapted her leadership style in Year 2 in response to her experiences in Year 1.

> For me, it was exposure to PLCs that did not go smoothly. So, how to react to that, in a continuously supportive way, was part of my personal learning. That was the collaboration when it comes to setting the agenda, making sure that I'm there modelling that, I'm prioritizing it. (Georgia, Case 1, exit interview)

Georgia also shared how her understanding of teaching and learning mathematics was influenced by her involvement with the OAME project. Georgia shared that she had a shift in her own beliefs and attitudes about pedagogical approaches. She thought of herself as a traditional mathematics learner and had not considered other ways to approach teaching mathematics. After engaging with the project, and learning with other project participants, she felt differently about the best way to teach mathematics, specifically using group work to encourage collaborative learning among the students.

> [The OAME project] has impacted me deeply [exhale]…the exposure to the group work has been eye opening for me. So the shift in pedagogy, and I'm very much more aware of a problem that I didn't know existed, but it's one of those things that you don't know what you don't know until you're involved. (Georgia, Case 1, exit interview)

Specialization and trans-level learning is evident in Georgia's comments about learning different approaches to teaching. The diversity of pedagogical approaches that she was exposed to through neighbour interactions in a system with decentralized control supported the emergence of learning that she described.

Jill shared how the purposeful collaborative professional learning activities the PLC engaged in influenced her teaching and learning. By the PLC engaging collaboratively with the *Five Practices* and doing mathematics problems before bringing them to the students helped to clarify learning in ways that simply reading about it in a book did not. When asked how the project influenced her learning and teaching, Jill shared:

> So like purposeful [professional learning] because you can always read [about professional learning] but you can't get meaning out of it, right? The more that I do stuff, the more I really sort of say "this is my mindset and I'm going to do this now, I'm going to try it like a learner". And if you're just reading [about it] you're not really doing that. And to be doing that with somebody else, at the same place, it's just so much more powerful. (Jill, Case 1, exit interview)

Neighbour interactions with other PLCs, project participants, and the PI proved to be an important component for Jill's learning. Analysis of the phase 1 survey suggests that interacting with others across the province was one of the most influential parts of Jill's learning. In response to the question "To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?" from the phase 1 survey, Jill responded that the project had a large influence, and shared several examples of neighbour interactions within the project that influenced her:

> We learned about using rich tasks to drive instruction from William (Case 7) and Chris Suurtamm suggested a Jo Boaler book for our group to read and try. Following that our group continued to learn about Thinking Classrooms and spiralling curriculum mostly being influenced by the group from Fields (Case 7) and William and Wayne's (Case 7) energy and excitement about their work.

It is evident that the different neighbour interactions influenced her own learning and also influenced the ways that the Case 1 PLC operated. These neighbour interactions with others beyond her own PLC were brought into the PLC and were discussed in relation to their own specific school setting and student needs.

Jill was aware of areas of her teaching that she wanted to improve, and she intended to keep working with others beyond her school who were trying similar pedagogical approaches such as teaching through rich tasks using a BTC model. The OAME project provided her the opportunity to engage with others beyond her own school and to consider different ways of teaching and learning mathematics. In her survey response for this current study Jill shared that interacting with Wayne, William, and Tasha from Case 7 pushed her thinking in new directions and she continued to maintain these connections. Jill's learning was influenced by the nestedness of the project that provided opportunities for rich neighbour interactions with a diverse group of mathematics educators and resources from across the province.

Zoe was absent for much of Year 1 of the project and returned to the PLC during Year 2. Zoe shared that her participation in the project had a positive influence on her teaching and her professional growth. Zoe felt that the project gave her the opportunity to engage in mathematics herself as a learner and to observe different grades of students engaging in the same problem. For her this work with mathematical tasks was the most influential activity the PLC engaged with. She felt she developed a deeper appreciation for the need to differentiate teaching mathematics to address the different learning needs of her class. Resources such as the *Five Practices* were particularly influential. Zoe said she appreciated the time to work through these new ideas with her own PLC members as well as those from different PLCs across the province.

> I really became more conscious of how different students will view the same problem and the different analysis methods, sort of going back to that *Five Practices* book, really hammered home how important that is. So I was just really fascinated with how, a group

> of students can look at the same problem and attack it from completely different angles. I can't emphasize enough how much it's helped me grow professionally and how much I think that can mean in my classroom to my kids. [It] just increased my scope and helped me build relationships across the province with teachers that I wouldn't have known prior to this, so it really opened up a bigger picture of the learning community for me as well. (Zoe, Case 1, exit interview)

For Zoe then, her learning was influenced by engaging in neighbour interactions with a diverse group of educators, with resources such as the *Five Practices*, and with student thinking as they engaged in rich tasks that she learned about through the OAME project.

The nested nature of the OAME project provided the needed balance between diversity and redundancy during both years of the project for PLC 1. Even though not all members of the PLC engaged during Year 1, the nestedness of the project offered a balance between redundancy and diversity needed through the other PLCs in the project. This allowed the PLC in Case 1 to persevere, adapt, and self-organize in order to continue their participation in the project which ultimately led to the emergence of learning in this Case.

Addressing Research Question One: Case 4

I first present an overview of Case 4 drawing from the phase 0 secondary analysis of the original project data which includes the OAME project final report, monthly meeting transcripts, RA notes in the PLC case report, and exit interviews with project participants. Following that I present findings from the analysis of data that describe how the conditions and characteristics of complexity manifested in this case during the OAME project. I conclude with a description of the ways that these conditions and characteristics interacted to support the emergence of learning.

Case 4 emerged from a PLC located at Pike Place School, a school situated in a small Northern Ontario city. The high school served students in grades 7 through 12 and at the time of the OAME project had a student population of approximately 700 students. The PLC membership evolved over the two years of the OAME project. This evolution was due in part to the shifting goals of the PLC that came to include considering the continuum of mathematics learning across multiple grades at the school. Thus, in Year 2 additional PLC members from the elementary (Grades 7 and 8) teaching staff at Pike Place High School joined the PLC. Table 14 presents the individuals involved in the PLC during the two years of the OAME project, the names in bolded text are those that also agreed to participate in this current follow up study.

Table 14

Members of Case 4 OAME Project PLC

Pseudonym	Role Year 1	Role Year 2
Chrissy	**Mathematics Department Head**	**Mathematics Department Head**
Daniel	**Mathematics Teacher**	**Mathematics Teacher**
Valerie	Special Education Department Head	Special Education Department Head
Joan	Student Success Teacher	Student Success Teacher
Sydney	Grade 7/8 Teacher	Grade 7/8 Teacher
Brad	English Teacher (teaching Math)	N/A
Dean	Vice Principal	N/A
Margaret	Researcher	Researcher
Alex	N/A	Grade 6 Teacher (Feeder School)
Lynn	N/A	Vice Principal
Alice	N/A	Aboriginal Students Grad Coach
Jennifer	N/A	Grade 6 Teacher (Feeder School)
Quinten	N/A	Grade 7/8 Teacher

A core group of five participants were constant throughout the project. These included Chrissy, the Head of the Mathematics Department, Daniel, a veteran mathematics teacher of 31 years, Sydney, a grade 7/8 teacher, Valerie, the Special Education Department Head, and Joan, the Student Success Teacher.

Case 4 faced several challenges during the years they were involved in the OAME project. The remote location of the town Pike Place School served was experiencing declining enrolment due to competition for students with the only other secondary school in the area. PLC members claimed this other school had a better reputation for academic achievement and competitive sports performance. The declining enrollment at Pike Place meant that the school was also experiencing a decline in staff and the mathematics department had only three mathematics teachers during their involvement with the OAME project. Other staff were tasked with teaching mathematics, like Brad who was technically an English teacher but was known for his expertise in communicating with students. The third mathematics teacher at Pike Place declined to participate and was described by others in the PLC as a traditional mathematics teacher who was not interested in examining or changing his practice. The school also served a high population of Indigenous students who started at Pike Place in Grade 9 after attending federally funded First Nations Schools. The PLC members shared that these schools did not necessarily follow Ontario curricula and therefore the students had different levels and experiences with learning mathematics prior to attending Pike Place School. These combined challenges contributed to the formulation of the PLC’s problem of practice.

Case 4 PLC’s initial problem of practice was to "[e]stablish a growth mindset through the creation of a co-learning environment where all are responsible for the learning.” (Chrissy, Case

4, Year 1, meeting 1). They expanded on this in their end of Year 1 professional learning project report:

> Our overall focus was about increasing student engagement. Specifically we planned to establish a growth mindset through the creation of a co-learning environment where all are responsible for the learning. This plan lived within our school focus about creating a community of learners where we could focus on critical thinking. In mathematics, we interpret critical thinking to mean explaining your work, specifically, in problem solving (Case 4, Year 1 school report).

During the first PLC meeting of the project the group discussed how important it was to support students, especially in Grade 9 Applied Mathematics, in developing a positive mindset. They also chose to focus on activities that were “more problems-based… more group tasks, and small groups” (Chrissy, Case 4, Year 1, meeting 1).

Findings from the analysis of the RA notes from the case report as well as monthly meeting transcripts suggest that the PLC meetings generally ran the same way during the two years of the project. Chrissy, the Mathematics Department Head, was the leader of the PLC. She scheduled and led the PLC meetings, created a set, but flexible agenda, organized tasks, and resources for PLC meetings, and was the PLC’s link to the OAME project team and PI (Suurtamm et al., 2017). Although the PLC meetings were led by Chrissy the meetings were collaborative and all members contributed to deciding next steps, goals, and activities. PLC meetings involved different activities during the two years of the project. Some meetings involved sharing tasks, planning lessons, units, or supporting each other with learning new technologies. Other meetings involved co-planning and co-teaching lessons in a Grade 9 Applied Mathematics classroom. As the PLC worked and focused on their problem of practice their focus shifted to coherence to mathematical concepts across a continuum of learning from Grades 6 through 9. During Year 2 of the project the PLC membership adapted to reflect this focus. Because Pike Place School served Grades 7 through 12, it was not difficult for the Grade 7 and 8 teachers to join the PLC in Year 2 as they were all in the same building following similar timetables.

Next I present findings from the analysis of the original project data as well as the phase 1 survey and phase 2 interviews that demonstrate how the conditions and characteristics of complex learning systems manifested in Case 4. I begin with a discussion of the conditions necessary to support the emergence of learning from a complex system

Specialization: Redundancy and Diversity. Redundancy and Diversity manifested in several ways in Case 4. In this section I begin by describing ways redundancy was present followed by ways diversity was present. Redundancy manifested in the shared beliefs and attitudes about teaching mathematics, a shared focus on a common goal of increasing student engagement through the use of rich tasks, and on preparing students to be successful when completing the Grade 9 EQAO provincial assessment. Redundancy was also represented in the fact that the teachers all taught at the same school and therefore at times had taught the same students given that the school was a Grade 7 through 12 school. This redundancy in teaching experiences with the same students over a six-year period provided a common area of discussion and focus during the PLC meetings.

A consistent theme that emerged from analysis of the transcript data and the Case 4 reports completed for the OAME project was that the members of the PLC were aligned in their beliefs and attitudes about teaching mathematics. The PLC members spent many meetings discussing the shared challenges of engaging students who had a negative mindset about mathematics in their classrooms. The PLC was focused on finding and implementing rich tasks that would allow students hands-on, collaborative experiences. Another shared focus was preparing the students at Pike Place to write the EQAO assessment. Analysis of PLC meeting transcripts revealed that members often spent meetings discussing how their students struggled with staying focused for the duration of the EQAO assessment; the teachers felt that the students lacked the motivation and positive mindset to complete the test in the required time, and that students struggled with the mathematics the test was assessing. Because PLC members had often taught the same students across different grades, the familiarity with students and even their siblings made discussions of students' needs easier at times.

In terms of diversity the PLC members had different experiences teaching mathematics, different opinions on the role of professional learning, different grades they taught, and different roles within the school. Chrissy and Daniel were the only members of the PLC who were specifically mathematics teachers. Others, like Brad who was primarily an English teacher, were recruited to teach mathematics due to a shortage of mathematics teachers at the school. Brad, therefore had less experience teaching mathematics while Chrissy and Daniel were both veteran mathematics teachers. Daniel shared that due to his years of involvement with different professional learning initiatives he was more hesitant to jump into new ideas and was often skeptical of new ideas. He felt that his presence provided a diversity in attitudes that was a positive for the group. He would question the direction of the PLC and push back when ideas did not align with his beliefs. When referring to his colleagues he explained:

> They are willing to jump on and they go full bore in one direction and then a year later they're going full bore in another direction; that, that's just not who I am, and I think

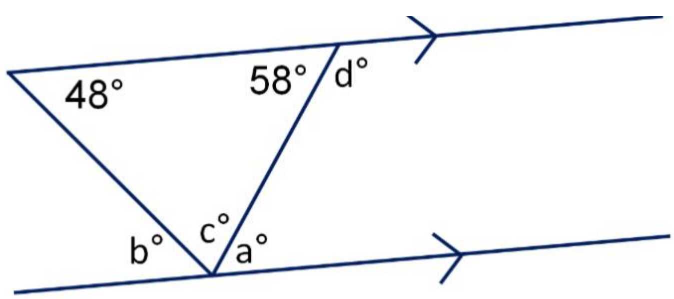
> that's why sometimes I'm not a bad person to have in a PLC because I think I'm a bit of a practical person and you know I call a [spade] a [spade]. (Daniel, Case 4, exit interview)

The other members of the PLC included elementary teachers who were not mathematics specialists and Margaret, the researcher assigned to support the PLC. Margaret was a mathematics education researcher, and her teaching was primarily in a teacher education program, preparing teacher candidates.

Analysis of meeting transcripts reveal that the PLC demonstrated a balance between redundancy and diversity. An example of this balance manifested during the second PLC meeting of Year 1 of the project. The PLC was discussing different lesson ideas for teaching that the sum of the interior angles of a triangle equals 180° (Figure 13).

Figure 13

Finding angles in triangles and parallel lines



The redundancy in the PLC involved the basic understanding of the mathematical concept and the diversity manifested in the different teaching experiences. This diversity provided different perspectives on how to teach this concept. This balance between redundancy and diversity provided the opportunity for rich conversations that saw different teachers building on the ideas of others. After much collaborative sharing Daniel shared how the lesson idea the PLC had generated resulted in a lesson idea that he hadn't thought of before and that he wanted to use it as a Minds On activity, a task that teachers use to activate students' thinking at the start of a mathematics lesson.

> That's a great 'Minds On', I like it. Here it is. I'm going to hand it to you. Here's your stickies or you can just take a blue pen and a red pen, any two colors and cut it…. I think that's great. I want to try that now (Daniel, Case 4, Year 1, meeting 2).

The PLC stayed focused on developing this lesson idea for the remainder of the meeting with different teachers sharing various ways they had taught this concept and adding to a shared new way of teaching the concept. This balance between diversity and redundancy also supported the next set of conditions of trans-level learning.

Trans-level Learning: Neighbour Interactions and Decentralized Control. Trans-level learning involves neighbour interactions between different components in a system with decentralized control (Davis & Sumara, 2006). I begin by describing the ways decentralized control manifested in the findings from analysis of different data sources in Case 4 before describing the different neighbour interactions that were identified in the PLC.

Although this PLC was led by Chrissy , her leadership approach was very much a shared leadership style. The final project report detailed how Chrissy applied for the project, wrote the project application, and invited individuals to join. During the two years of the project Chrissy "facilitated meetings, reminded people of meeting dates, set a flexible agenda, brought materials for members to work on, booked meeting space, arranged lunch, corresponded with the OAME team and Researcher, and attended the project's Adobe Connect meetings" (Suurtamm et al., 2017).

During the meetings Chrissy would remind the PLC of what they had discussed at previous meetings and would look to the group for feedback on what they should work on next.

In my reading of the transcripts, Chrissy never appeared to be prescriptive or controlling. She used an open approach that allowed topics and focus to emerge from the PLC. There was an instance during a Year 2 PLC meeting when the new vice-principal, Lynn, who had recently joined the PLC, attempted to exercise more control over the group. Chrissy intervened because she understood that the direction the PLC took needed to come from the PLC as a whole, not from one individual. After speaking with the school administration about this issue Chrissy shared that "[the administration] see the research too that says if teachers get to choose what they want to be involved in, that they will engage in it more fully, and so they have tried to give us that freedom" (Chrissy, Case 4, Year 2, meeting 3). Analysis of Margaret's notes from this meeting reveal that after this discussion between the administration and Chrissy the vice-principal agreed to be more of an observer in the PLC.

The condition of diversity was important in supporting the model of decentralized control. Analysis of Daniel's exit interview from Year 2 of the project supports this claim. Daniel shared that the PLC operated well because of the diversity of different teachers' beliefs about teaching and learning as well as the collaborative leadership approach that Chrissy adopted. The collaborative nature of the way the staff at Pike Place worked together had been developed through their involvement in other projects and this allowed a certain level of trust and respect to be cultivated. Daniel explained that the decentralized approach to their collaboration made him more willing to be involved:

> Our department works a lot together on different thing. I mean, we all have different philosophies and that's fine but a different philosophy and not being as rigid and well you can only have success this way. Chrissy is very good that way about you know this is what, this is a good thing you did. You had success at this. But if it was, if it was very

> dictated and regimented, I doubt I would have been around the second year. (Daniel, Case 4, exit interview)

This commitment to decentralized control made room for many different types of neighbour interactions to evolve over the duration of the project, which was an important component of the way the PLC operated. These different types of neighbour interactions included interactions with other PLC members during meetings, through observing each other teach, with researchers and resources, with other OAME PLCs, and with student thinking.

Analysis of meeting transcripts shows that the first few PLC meetings during Year 1 involved hours of rich discussions about different issues the PLC members faced in their teaching of mathematics, the context specific issues their student population faced, ways to engage their students, and co-designing different approaches to teaching mathematics that might address these issues. These meetings allowed the PLC members to get to know each other professionally. Daniel explained that even though he and Chrissy had known each other outside of school for many years, this project allowed him to get to know more about how Chrissy thought about teaching and provided opportunities for the group to learn from each other in terms of mathematical pedagogy.

> It was an interesting thing because working with Chrissy, Chrissy and I have been friends for a million years right? And like we fished together, and we hang out together as much as we can right? But I think [the OAME project] made me realize how she thought too, like it not only helped me identify the strengths of myself but of the people that were working with me in the project (Daniel, Case 4, phase 2 interview)

As the project continued the PLC also incorporated neighbour interactions that involved observing each other teach. Several teachers shared during PLC meetings that being able to observe their colleagues teach was extremely beneficial to their own learning and often observing their peers was more beneficial than some other formal professional learning initiatives that teachers had been involved in. During Year 1 meeting 2, Brad shared a specific example about watching a colleague interact with students:

> And I just sat, and I just watched, and it was so much more beneficial than a lot of the things we do, and it was so neat to just see, because he's so good at what he does, and just to be able to, [I] didn't bother the kids, didn't bother anybody, they're comfortable having me around and I just, you know, you could participate up to a point, but just to be able to sit and observe, just the dynamic (Brad, Case 4, Year 1, meeting 2).

Margaret also played an important role in the PLC learning. Neighbour interactions with Margaret pushed the group to consider new ideas they may not have been exposed to. In the PLC's report of their activities at the end of Year 1, they commented: :

> Our research assistant was critical in capturing our conversations and observations. She was able to ask questions that helped us build on our observation, give more depth to our thinking, and help us record things that we didn't think were significant (Case 4, Year 1, School Report).

Furthermore, during the summer institute at the end of Year 2 of the project the PLC shared with Margaret how important her presence had been, that her connections with other mathematics educators in the province allowed the PLC to feel more connected to the greater mathematics community, and to feel that they were on the right track in terms of the work they were doing. Chrissy shared that hearing from Margaret that other schools were interested in what the Pike Place PLC were doing gave them a feeling of pride, "even to hear you say to us yesterday I think you said, 'people are talking about what your group is doing' and Valerie has told us 'The folks in CITY NAME really want to play with you!'" (Chrissy, Case 4, Year 2, summer institute). Margaret's feedback helped the PLC feel like a contributing part of the project and connected to the work going on in the province at that time.

Findings from the analysis of the school's case report, RA notes, and meeting transcripts describe how interacting with resources was another type of neighbour interactions the PLC experienced. Resources they interacted with included online resources such as NRich.org and rich task ideas from researchers such as Jo Boaler and Dan Meyer. The OAME project also shared many resources, and similar to Case 1, Case 4 engaged with the *Five Practices* book during their PLC meetings. Interacting with other PLCs and researchers at the OAME project-wide meetings were also important neighbour interactions in Case 4. As mentioned, the PLC's remote location gave them limited access to provincial professional learning events such as the OAME annual conference or connections to other schools. Chrissy described in her exit interview that the funding the project provided for the PLC to travel to the larger meetings helped the PLC feel that their work mattered and made them feel validated. Interactions with others across the province provided new opportunities to interact with different perspectives on teaching and to share their own experiences. At the summer institute between Year 1 and Year 2 of the project some members shared that hearing about the struggles other PLCs were having made them feel less isolated and buoyed their confidence. They felt less alone in their struggles and saw that they were headed in the right direction in terms of their professional learning.

The analysis of PLC meeting transcripts suggests that interacting with students and their thinking was another example of neighbour interactions that manifested in this case. The diversity of the grade levels the different participants taught meant that the PLC members could interact with students and their thinking across a continuum of learning. Different ways the PLC interacted with student thinking were observing teachers' lessons in their classrooms, examining and discussing examples of student work during PLC meetings, and observing video from lessons. During a Year 1 PLC meeting Brad shared a clip of students working on a problem collaboratively at a whiteboard. He described how he had never observed a student solve a particular problem in the way this student had. He brought the clip to the PLC meeting to share with the group as he wanted to solicit feedback on what his next steps should be.

> **Brad:** [name of student] was looking at things like completely out of the box. Like, I have three years of doing this, I've never seen someone do [that] and it works. But it's a third way of doing it
>
> **Chrissy:** Yeah.
>
> **Brad:** You know, any of us would see an option there, but he's
>
> **Chrissy:** Even look at the body language.
>
> **Brad:** Mm hm. (Case 4, Year 1, meeting 3)

The PLC discussed the clip for a portion of the meeting and offered feedback and ideas to Brad that he could use in supporting that students' thinking. The conversation shifted to focus on the fact that they all agreed having students engage in collaborative work was a positive change, but they all struggled with the fact that the assessments they were giving did not align with how the students worked during class time. Interacting with the video clip therefore pushed the PLC to consider different assessment strategies that aligned with the new teaching practices they are experimenting with. The tests they were giving were paper and pencil and done individually. The teachers found that the students were much more willing to work with non-permanent markers on the whiteboards than they were with making permanent marks on paper without collaborating with classmates. The teachers shared their perspectives from the different grade levels they taught. This prompted PLC participants to reflect on ways they could create a more cohesive program of study for Pike Place School. This leads to the next set of conditions, randomness and coherence.

Enabling Constraints: Randomness and Coherence. Enabling constraints involves balancing randomness and coherence. Findings from the analysis of the project final report, the PLC's case report, and meeting transcripts suggest that both of these conditions manifested in this case. Randomness manifested in the PLC's choice to apply to be a part of the OAME project, in their self-identified problem of practice, and the freedom to choose how they worked as a group. Coherence manifested in the PLC's focus on the goals of the OAME project and their own self-identified problem of practice, in the PLCs commitment to the provincial curriculum, and on preparing students to write the EQAO provincial assessment. The PLC began the project with a focus of increasing student engagement and developing a positive mindset. These goals were maintained during the two years of the project. The PLC tracked their goals, actions, and progress over the two years and created a table for their final project summary that outlined their journey. Table 15 is adapted from the final report (Suurtamm et al., 2017) and it presents ways the needs of the PLC represented coherence to the goals of the project and how randomness was represented in how they focused on these needs. I adapted the table simply by adding the terms "coherence" and "randomness" to the column headings as in my opinion these terms correspond to the ideas shared by the PLC.

Table 15

Case 4 OAME Project Goals - Identifying Needs and Actions

Needs (Coherence)	**Action (Randomness)**

Identify Learner Needs Strengths, needs, barriers to learning, learning style, interests	Grade 8 assessment Diagnostic assessment Learner profile cards Individual Education Plan review Student intake meetings
Student/Teacher Needs Common math language. Problem solving and instructional strategies. Number sense skill building. Math tools and technology	Cross panel meetings. Collaborative planning. Boot Camp Class Rich tasks and problems Modelling with manipulatives.
Emotional Needs Flexible groupings and table partnerships. Building community and relationships as a priority. Open communication.	Use of erasable whiteboards and vertical writing spaces. Note taking and keeping. Purposeful staffing. Other school initiatives
Independence Needs Resiliency, Hope, Ownership Empowerment Confidence Learning Skills	Purposeful planning. Descriptive and timely feedback. Opportunity for risk, thinking and inquiry in real-life situations.

Note. Adapted from "Report to the Ontario Association for Mathematics Education (OAME) and the Ontario Ministry of Education," by C. Suurtamm, J. Lazarus, M. Koch, K. McKie, J. Pai, B. Quigley, E. Morrison, I. Lazarescu, A. M. Goss, T. Sibbald, & K. Knowles, January 2017. p. 49.

Randomness and coherence were also balanced in how the PLC addressed the provincial curriculum. Analysis of meeting transcripts show that the PLC worked collaboratively to discuss different ways to engage students using rich tasks and problems that would also align with the curriculum requirements. When discussing how different tasks could be used the PLC consulted the overall and specific expectations in the curriculum and also considered the mathematical processes outlined in the curriculum. Balancing randomness and coherence was more difficult when it came to EQAO testing. The PLC spent a great deal of time during the beginning of Year 1 trying to reconcile that how they were attempting to shift their teaching towards more rich tasks and open problems that had students working collaboratively at vertical surfaces, did not fit with how the students were expected to complete the individual provincial assessments. These assessments required students to sit for long periods of time and focus on completing many multiple choice and longer form response questions than they were used to. Brad and Chrissy discussed this challenge during the first meeting of Year 1.

> **Brad:** So for me that is something that I've been thinking a lot about is the idea that our instructional strategies, even the things we are talking about doing that we have said so far as a group.
>
> **Chrissy:** Yeah.
>
> **Brad:** Smaller pieces. But then we're getting the actual provincial testing and it's the whole seven-course meal sitting in front of you.
>
> **Chrissy:** Because it is very artificial compared to what we do in our classrooms. Because I've been doing that [working on rich tasks] knowing that we've got to be ready to work for an hour non-stop (Case 4, Year 1, meeting 1).

Analysis of meeting transcripts and the final project report revealed that this challenge of balancing the coherence to the EQAO tests with their new approaches to teaching in a more engaging way became a major focus during the duration of the project. The group worked on different ways to support students in writing the EQAO assessments.

I next describe the ways that the different characteristics that define a complex learning system manifested in Case 4. These characteristics are self-organization, adaptation, and nestedness.

Self-organization. As the project progressed, the PLC members self-organized through conversations and reflections on what it was they were doing. Table 15 reflects the ways that the members chose their own focus and actions related to this focus. The different conditions described above supported the characteristic of self-organization. For example neighbour interactions in a system with decentralized control played an important part of the self-organization that was evident in analysis of the transcripts. These neighbour interactions were supported by a balance between diversity and redundancy. The PLC began their work with the project by sharing some of the practices and challenges they were having and similar experiences within their specific context seemed to unify the PLC. The members were able to share different approaches they had taken in the past to deal with some of their challenges. The PLC appeared to be a safe place to discuss their beliefs and the group shared a common goal of increasing engagement for the students in Grade 9 Applied Mathematics. This demonstrates self-organization in the ways the PLC organized their PLC meetings to address the areas of their practice that needed attention and how they focused on the one individual that was teaching Grade 9 Applied Mathematics at the time. In Year 1 the PLC self-organized around the needs of Brad and in Year 2 they organized their discussions on the needs of Daniel. The group chose to design lessons for Brad and Daniel, observe them teach that lesson, and discuss student thinking they observed. The PLC also self-organized after Brad's departure at the end of Year 1. Brad returned to teaching English in Year 2 and the PLC invited new members from the elementary grades to join the PLC.

Adaptation. During the two years of the project both the PLC and individuals adapted in several ways. As mentioned, the PLC adapted their focus during the two years of the project to focus on the teachers that were actually teaching a Grade 9 Applied classroom. With the addition of elementary colleagues to the PLC adapted their focus to include a deeper examination of the continuum of learning in Grades 6 through 9. Chrissy shared during the summer institute between Year 1 and Year 2 that they felt it would be helpful in deepening their understanding of the journey students took mathematically through these years of schooling leading up to Grade 9 and this would better prepare teachers to teach these students when they reached high school.

Analysis of the exit interviews show that individual teachers also adapted during the two years of the project. Jennifer, a Grade 6 teacher who joined the project in Year 2 explained in her exit interview how she adapted her teaching based on her experiences with the project. Before the project she primarily taught using a textbook, following the order of the chapters, and assigning problems from the book. After working with the PLC, interacting with the other members, and with different resources shared through the project she began teaching with a more student-centred focus and felt more responsive to the needs of the students she was teaching.

> I think over the last year or so I have changed my practice in the sense that I am doing more diagnostic assessments with my kids, I'm assigning less questions in math, and for two reasons, giving the kids better questions, not just repetition but also I find if I'm giving them smaller tasks where there is exit cards or just quick check ins, I can be way more responsive in my teaching practices (Jennifer, Case 4, exit interview).

Chrissy also felt that she was able to adapt her teaching because of the project. She shared that the project provided her time to try out different ideas collaboratively with others and practices that she had only ever read about but never had the time to put in place. The project provided the support to work with the PLC and implement different ideas, such as *The Five Practices* into her classroom.

> Knowing them [new ideas] and being able to put them in place in your classroom are two different things…Same as the *Five Practices*, I've read the book, I've got the stuff, but I haven't been able to use it well. So it [working as a collaborative group in the project] gives you another chance to say okay no wait, try it again and some of it is because you need a partner (Chrissy, Case 4, exit interview).

Lastly, Daniel also adapted his teaching after nearly three decades of teaching. The project provided a space for Daniel to experiment in his classroom, with the support of his colleagues, and try practices he was not normally comfortable with. During his phase 2 interview Daniel suggested that the project encouraged him to completely change his teaching style.

> So I had lots of things that were ingrained in what I did, and so it was an interesting project for me in the sense that I think it encouraged me to do things that maybe I didn't normally do, that I wasn't normally comfortable with, and it would have been easier, not to commit. I could do my last three years [of teaching] and just not do [the project], but you know, rather than go back and more or less reinvent the wheel. Right? (Daniel, Case 4, phase 2 interview)

When asked why he thought he did participate in the project rather than take the easier route of not engaging he suggested that it was because he likes to try new things and to get involved.

> I don't know [pause] like I said, I like different things, I like to, you know, to try different things [pause] and I was glad that I jumped on board, I mean I said it could have been easy to say "whatever I'm just going to just coast through here" but it's kind of not how I roll. (Daniel, Case 4, phase 2 interview)

Next I discuss the last characteristic of complex systems, nestedness.

Nestedness. Different levels of nestedness manifested in Case 4. Individuals were nested within the PLC, which was nested within the OAME project, consisting of many PLCs, as well as the research team and steering committee. The PLC was also nested within the school and school district. This nestedness influenced the ways that the PLC worked and what the PLC focused on during the project. It is evident in analysing the case report and meeting transcripts that the OAME project influenced the PLC. For instance, the PLC's use of resources shared during the project, such as *The Five Practices*, was a result of the PLC being provided the resources as well as time to interact with them. Several times throughout the project the PLC mentioned that they were integrating different ideas and practices they learned from other PLCs including spiralling and rearranging the order of the curriculum when teaching Grade 9 Applied.

During the final project-wide meeting, in a focus group interview with Case 4, self-similarity of components of a system and nestedness were mentioned by Chrissy when she described how important Margaret was to the PLC. She compared Margaret's role as being similar to teachers' roles in the classroom and how Margaret helped to validate the work of the PLC..

> **Chrissy:** I still come back again to the cheerleader that we try to be in our classrooms is how I feel you [Margaret] are to us. You know that we need someone as well like this, to just say "yeah you're on the right track, keep going, yeah keep going" You know, that it validates our [work]
>
> **Valerie:** Oh, you're off track, or prompts us.
>
> [multiple speaking at same time]
>
> **Chrissy:** Right and can redirect us. You know? Even in some of those other discussions about "Well, have you thought about making this a more rich task by going down this

> direction?" and I'm like "Oh shoot, no". But you have the questions too (Case 4, Year 2, summer institute focus group).

The project nested within the OAME community meant that the PLC from Pike Place was able to attend, and present at, the OAME annual conference for two years. Jennifer and Chrissy both shared how important this experience was to their learning and to validate their experiences When Chrissy heard others, from more central locations in Ontario, struggled in similar ways she realized that the work they were doing was important and not solely because of their rural location; she was able to make connections with the Ontario mathematics education community that supported her learning. Chrissy shared:

> That was a huge revelation for me the first time I ever went to OAME, I came from my school in TOWN NAME thinking "Nobody's got a set of applied kids like I do." Right? You know I've got kids who had this happen, they do this, just the way the community is. And the very first workshop I go into in 9 applied, people start talking about their class and I'm like "Holy crap, that's the same as mine." Like you start realizing yeah, this is just the learner, it's not the situation, it's not the environment, it's just the learner. (Chrissy, Case 4, Year 2, exit interview)

The nestedness in Case 4 was an important characteristic for the teachers in a rural Northern town. This characteristic helped them realize that they were not alone and that others, beyond their own unique setting, were experiencing similar challenges.

Emergence of Learning: Case 4

These conditions and characteristics described above interacted in ways to influence the emergence of learning at the PLC level and the individual teacher level. I now describe how these manifestations interacted and supported the emergence of learning. When looking at the frequency of terms coded in the Case 4 data, emergence was the most frequently coded followed closely by redundancy, and neighbour interactions suggesting that these were the key themes that influenced the emergence of learning from this case.

As described, redundancy manifested in the PLC's shared commitment to the goals of the OAME project as well as the PLC's goal of increasing student engagement at Pike Place. This commitment to the goals was evidenced in the neighbour interactions in a system with decentralized control. These neighbour interactions were supported by the balance the PLC had established between diversity and redundancy. Jennifer shared how the diversity of grade levels that different teachers taught brought something unique to the project and provided opportunities for learning for each group.

> I think that we are, as elementary teachers, affecting them [secondary teachers in the PLC] as far as a whole child approach and I think [the] secondary [teachers] are influencing elementary [teachers] as far as the curriculum rigour and math demand part of things if that makes sense. So it's, I think we are coming up with a more balanced and effective math instruction (Jennifer, Case 4, exit interview).

The PLC focused on developing a deeper understanding of what their students needed and ways to better support student learning.. Through experimentation, observation, and collaborative discussions, learning about effective teaching in mathematics emerged. Engaging with one another, trusting one another, and working collaboratively resulted in a more trusting atmosphere among the members of the PLC that better supported learning. Chrissy shared the following during her exit interview

> We have learned that our forward motion happens in the strength of our team. It is through conversations and openness to growth that we have felt the growth in our team

> this year. As far as evidence goes, I'm not sure how to capture the collegial atmosphere that has returned to our staff. We are open to becoming vulnerable – which is what happens when another adult observes your teaching practice. It is through that trusting and trusted team that we can ask difficult questions and accept more difficult answers (Chrissy, Case 4, exit interview).

At the individual level, through analysis of the transcripts, it was evident that many members of the PLC felt more confident in their teaching, were more willing to take on new challenges, and to push themselves to change their practice due to the influence of the collaboration of the PLC. Daniel, who was close to retirement demonstrated high levels of engagement and shared that the project was one of the most influential professional learning experiences he had been a part of. Chrissy shared a similar sentiment and reflected on how the project had influenced how the mathematics department at her school felt validated by their involvement in the project, that they were on the right track now, and she was willing to take more risks in her teaching. Jennifer shared that she felt inspired to try new things and even to apply for the mathematics lead teacher position at the elementary level in her school district.

> It has definitely inspired me to want to dig deeper, learn more, figure things out more to the point where, as I was mentioning earlier, I kind of put my name out there as a math lead for my school for next year. Completely not of the mindset that I know so much about math but that I want to know so much more about math, and I think that that's a way for me to do it, given how comfortable I felt with this particular group of people and this project. It's created a nice safety net for me." (Jennifer, Case 4, exit interview).

The PLC in Case 4 demonstrated the ways that the conditions and characteristics of complexity supported each other and led to the emergence of learning at both the PLC level as well as the individual level. An important observation is how a shared commitment to the goals of the project led to the development of trust among the PLC members which supported the emergence of learning.

Addressing Research Question One: Case 7

Case 7 started as a PLC located at Fields High School, a school situated in a medium size Eastern Ontario urban centre. The high school served students in grades 9 through 12 and at the time of the OAME project had a population of approximately 1500 students. The PLC membership evolved over the two years of the OAME project. This evolution was due to the ways that the principal, Dolores, used funding from various projects to support professional learning for school staff. During Year 1 of the OAME project the PLC addressed two different professional learning initiative agendas, the OAME project and the Student Success School Support Initiative (4SI) Project. The 4SI project, a cross-curricular initiative, focused on creating success for students in Grade 9 and 10 Applied Mathematics, English, History, Geography, and Science. During Year 2 of the OAME project the PLC split into two different PLCs in order to involve more individuals. The 4SI PLC focused on the Grade 10 Applied courses while the OAME PLC focused on Grade 9 mathematics classes Table 16 presents the individuals involved in the Case 7 PLC during the two years of the OAME project. The names in bolded text are those that also agreed to participate in this current follow up study.

Table 16

Members of Case 7 OAME Project PLC

Pseudonym	Role Year 1	Role Year 2
Dolores	**Principal**	**Principal**
Wayne	**Mathematics department head**	**Mathematics department co-head**

Lisa	**N/A**	**Mathematics department co-head**
William	**Mathematics teacher**	**Mathematics teacher**
Tasha	**Mathematics teacher** **Student success teacher**	**Mathematics teacher** **Student success teacher**
Rachel	Student success teacher	Student success teacher
Larry	Science teacher	N/A
Shirley	Science teacher	N/A
Anna	Geography teacher	N/A
Kelly	Research Assistant	Research assistant

During Year 1 the cross-curricular focus of the PLC meant that some PLC meetings focused on subjects other than mathematics. As the RA assigned to this school for the OAME project I only attended PLC meetings that focused on mathematics. PLC meetings were attended by all of the teachers identified in Table 1, however the OAME project-wide meetings were attended by only Dolores, Wayne, William, Rachel, Tasha, and Lisa (starting at the end of Year 1). During Year 2 the PLC meetings held at Fields High School were attended by Wayne, Lisa, Tasha, and Rachel. Dolores would occasionally attend briefly but she aligned more with the 4SI PLC, as did William. For the project-wide meetings during Year 2, Wayne, Lisa, Tasha, Rachel, Dolores, and William attended.

The application for the OAME project was written by the special education teacher, Tasha, with the full support of the mathematics department head, Wayne, and the principal, Dolores. On the application the Case 7 PLC submitted, they identified their problem of practice as follows:

> According to EQAO questionnaire results, two-thirds of our Applied students come to Grade 9 with a defeatist attitude, little to no confidence in their ability to do and hating

> math. They rely on memorized algorithms fed to them and have difficulty making math connections. All these factors result in behaviour and attendance concerns and ultimately sabotaging any success in high school math. Leading MATH Success has motivated change in our curriculum delivery and using the TIPS4RM resources (teaching the strands by units through the activities and worksheets) have increased engagement; however, they guide students instead of allowing them to develop their innate critical thinking and problem-solving skills to take ownership and determine the boundaries of their learning. Students with attendance issues would miss entire units and not be able to figure out the material in the few classes left for preparing for the EQAO. Also, teaching through units didn’t necessarily address students’ gaps, nor allow them to see connections between the strands. Can we find the right recipe to build our Grade 9 Applied students’ confidence, keep them engaged, help them take responsibility of their learning and realize success, while not neglecting students with attendance issues? (OAME project Application)

The PLC’s approach to addressing this problem of practice was to continue with an established professional learning model they referred to as lesson study. The PLC had been engaged in this model for a year before joining the OAME project. The PLC’s approach to lesson study was inspired by a Japanese model of professional learning (Stigler & Hiebert, 1999; Yoshida, 2008) and involved the PLC spending half a day designing a lesson for one teacher’s class on a topic or concept of that teacher’s choosing. The group would then observe the lesson being taught the following week. The PLC would debrief immediately after the lesson to discuss what they observed, listen to the teacher’s experience in teaching the lesson, and discuss how to adapt the lesson to better meet students’ needs. Because of the cross-curricular nature of the PLC during Year 1, the PLC focused on metacognition and designing engaging, hands-on activities rather than subject specific tasks. In this way the big ideas from each lesson designed could be applied across different subjects.

When the PLC from Case 7 joined the OAME project they were already incorporating their own practice of spiralling as described in Chapter 4 and the Building Thinking Classrooms (BTC) model (Liljedahl, 2015). During Year 2, when the PLC split into two, the OAME PLC focused mainly on the school's aspiration to de-stream Grade 9 Applied Mathematics. It is important to remember that during the OAME project Grade 9 students in Ontario were still streamed into Academic and Applied Mathematics courses[2]. Fields High School approached this streaming differently than most high schools in the province. The mathematics department was concerned with how labelling students as Applied or Academic impacted students' engagement and mental well-being. The school had begun what they termed "combined" classes. All students who registered in Grade 9 Applied Mathematics were placed in a combined class with students who were registered in Grade 9 Academic Mathematics but who had been identified by the guidance department as needing extra support. The intention of the combined classes was to remove some of the stigma that students, and often teachers, associated with being registered in the Applied stream. Students in the combined classes did not know who was registered in Applied or Academic and were not told that the class was a combined class. Students' designation as either being registered in Applied or Academic was left unconfirmed for as long as possible during the term in order to allow students to switch into a different stream if possible. At the summer institute, between Years 1 and 2 of the project, the PLC revisited their original

[2] In the Fall of 2021 Ontario implemented a de-streamed Grade 9 mathematics curriculum for all students across the province.

OAME project problem of practice. Through a group discussion they identified areas of focus for Year 2 as follows:

- Consolidate our approach to provide an online framework for other educators who are interested in developing activity-based lessons within a spiralling curriculum.
- Develop instructional and assessment practices that meet the requirements of both our applied and academic program, both within the split class model and in the academic-specific program (e.g. algebraic thinking and analytical geometry for students in the Academic-level in our split classes).
- Track the previous year's grade 9 students to study their progress in grade 10 (Year 1 summer institute focus group).

Some of the ways the PLC worked towards their focus during Year 2 included co-creating a framework of lessons, activities, and assessments for teaching all Grade 9 mathematics classes at Fields High School, including the combined Grade 9 mathematics classes. The first meetings of Year 2 involved the PLC finalizing their Grade 9 course plan. During subsequent meetings the PLC co-designed common assessments for all Grade 9 classes and at later meetings debriefed how their students performed on these assessments. Each member would bring examples of student work to share and discuss. They reflected on ways to improve the common assessments to address any issues that arose in terms of student misunderstandings, the time it took students to complete the assessments, and how they assessed student work. These common assessments, unlike the consistency that Case 1 was striving for, were created collaboratively and in response to student work and thinking. Another activity the PLC members engaged in during Year 2 was the creation of a student beliefs and attitudes survey (Appendix J) . This survey was intended to track the well-being of their students and to determine if their teaching approach had positively influenced the ways students viewed themselves as mathematics learners.

Next I present findings that demonstrate how the conditions and characteristics of complex learning systems manifested in Case 7 and how these supported the emergence of learning in the different learning systems associated with the OAME project. I begin with a discussion of the conditions necessary to support the emergence of learning from a complex system.

Specialization: Redundancy and Diversity. During the two years of the project, specialization manifested in different ways. I begin by describing how redundancy and diversity were represented in Case 7 during Year 1 of the OAME project before describing these two conditions during Year 2. During Year 1 the members of the PLC at Fields High School were aligned in their focus on better serving students in Grade 9 Applied courses. Through analysis of the meeting transcripts and my observational notes as the RA it was evident that redundancy was present with this PLC. Redundancy manifested in all the participants being from the same school, teaching the same students, sharing a similar perspective with regards to teaching and learning, engaging in an agreed upon model of professional learning (lesson study), and focusing on the same goal of co-designing lessons to best meet students' needs. Evidence from exit interviews and phase 2 interviews with participants suggest that the PLC members were purposefully invited to participate in the PLC by Dolores because they shared a similar views regarding teaching or were open to considering new ways of teaching. William described how Dolores looked for people who would work well together based on their similar approaches to teaching and learning:

> I think that our leader at the time did a really good job of picking the people that she thought would move [their practice], you know she's like if someone was on the wrong side of the fence she didn't even bother with them if she thought they were on the fence and had a chance, then she would try to involve them (William, Case 7, phase 2 interview)

Dolores confirmed the importance of a PLC sharing similar attitudes towards professional learning. “The biggest challenge for a PLC is the nay sayer…other schools had nay sayers and they just submarined the whole thing [collaborative learning]…I don’t do that, I don’t spend a lot of energy on people who are pushing back” (Dolores, Case 7, Year 1, exit interview). Furthermore, analysis of Year 1 PLC meeting transcripts revealed another commonality; members willingly engaged in the work of lesson study focused these co-designed lessons to enhance students’ learning.

Diversity manifested in the PLC during Year 1 in several ways, including diversity in the subject of teaching expertise and training, diversity in the beliefs and attitudes about teaching and learning mathematics, diversity in individuals’ experiences as mathematics learners, and lastly, diversity in the personalities of the PLC members. The first example of diversity is that different PLC members taught different subjects including mathematics, science, and geography. This cross-curricular nature of the PLC provided a diversity of subject area expertise that Wayne noted was beneficial to his learning and the learning of the PLC “that was great, to have somebody from the outside of math to really to help us, you know? You get a bit myopic when you're all talking the same stuff, that was good” (Wayne, Case 7, phase 2 interview).

Each PLC member had different beliefs and attitudes about teaching and learning mathematics. Rachel shared how hearing about different ways that members thought about a topic made her realize that people think differently about concepts.

> It’s neat to get to know my colleagues better in terms of, not just personal, well I know them as people, but how they think and I think sometimes as teachers you think that everyone thinks the same way as you do (Rachel, Case 7, Year 1, exit interview)

It is possible that this diversity in beliefs and attitudes about teaching and learning mathematics was influenced by PLC members’ experiences with learning mathematics themselves. This diversity in experiences was evident throughout Year 1 meetings. During an exit interview at the end of Year 1 Dolores addressed how the diversity of the teachers’ experiences as both teachers and learners can challenge the group to reconsider their approaches to teaching. For instance, during PLC meetings when the group was designing lessons similar to ones she experienced as a learner she would point out how those lessons had not helped her develop an understanding of the concept being taught. Dolores shared:

> It's handy because I’ll be in a [meeting] and I’ll be able to say “You know what? I remember learning electricity and not getting it and you’re doing it; you’re doing exactly what they did when I was in school. That doesn’t work” (Dolores, Case 7, Year 1, exit interview).

This diversity in experiences as learners disrupted the assumptions teachers had about successful approaches to teaching. These perturbations led to exchanges within the group to clarify the goals of the activity and create lessons that better addressed their teaching goal.

The previous examples show how diverse experiences of PLC members as learners led to the group engaging in rich dialogue about teaching and learning. There were other instances when the diversity did not lead to opportunities for learning for everyone in the group. Tasha considered herself a unique mathematics teacher as she struggled through high school mathematics. “I memorized my way through everything, like I memorized the algorithm and that’s how I got through math …I always tell my students… [that] in grade 9 I actually had to repeat grade 9 math a few times” (Tasha, Case 7, Year 1, exit interview). This experience as a learner influenced how she approached teaching mathematics. Tasha aimed to design lessons that would focus on understanding mathematical concepts rather than memorizing them. For instance she mentioned a lesson study she wanted to do that allowed students to explore a misunderstanding she had observed. “I wanted to come up with an idea like that [plotting points on a grid] and I kind of got shot down by everyone in the group, ‘well we can’t do that’, well ok” (Tasha, Case 7, Year 1, exit interview). This example suggests that diversity did not always lead to rich dialogue and also highlights the diverse personalities of PLC members. Tasha shared that she is quite introverted so did not push her ideas. She suggested the strong personalities of some of the members meant not all voices were heard during the PLC meetings; she shared “especially in this group here, it’s hard to get any word in edgewise and I’m an introverted thinker so I have to think through and understand the process before I give my input so it’s difficult” (Tasha, Case 7, Year 1, exit interview).

When the focus of the PLC shifted to focus on the combined Grade 9 mathematics classes in Year 2 redundancy manifested in new ways. These new ways included the shared focus of the PLC on creating a common framework for teaching all students in Grade 9, as well as all of the PLC members being from the same school and teaching the same course. Redundancy was further increased during Year 2, as only those teaching Grade 9 mathematics were involved in the monthly PLC meetings. Diversity was still represented in their different beliefs about teaching and learning mathematics as well as diversity in their experience teaching a combined Grade 9 mathematics class. This diversity in beliefs and attitudes about teaching mathematics was clearly evident when the transcripts from each PLC meeting were analysed. Wayne, Tasha, Lisa, and Rachel had different opinions on most topics they discussed. Generally the four PLC members' beliefs and attitudes on how to teach mathematics fell along a spectrum with Wayne at one end with the belief that lessons should have no scaffolding and limited written instructions, homework should be non-existent, and assessment should be based primarily on professional judgement of observation of students' work and conversations with students. Tasha was on the other end of the spectrum, insisting that lessons should have extensive scaffolding and detailed written instructions, homework was an essential component of her teaching, and assessment was very thorough and came in many forms including observations and conversations alongside written tests. Lisa, a new member of the mathematics department at the end of Year 1, landed somewhere in the middle. She often acted as a moderator when the PLC was discussing different topics. Rachel, the fourth member of the PLC during Year 2, was co-teaching mathematics with Tasha. She had not taught mathematics in many years and admitted, during an early meeting in Year 2, that she was just trying to keep up with the lessons on a day-by-day basis. An example of the PLC's diversity in beliefs is demonstrated in this exchange comparing what each teacher left for their supply teacher for the day. They are reviewing what Tasha left and Wayne is critiquing the amount of detail, with others adding comments.

Wayne: It's a lot of words Tasha

Tasha: It's a lot of words, yes, but I discuss it at the beginning, like I talk it over with my kids at the beginning. So we talk about each of the primary questions and then I give them the sheet afterwards. They can choose to fill it out, whatever, right?

Wayne: Sure

Tasha: The data collection is what I find most important

Lisa: Is it too many words?

Wayne: Well, for me ya.

Rachel: He doesn't do sheets.

Wayne: I don't do sheets, I say "Figure it out kids. Come back to me when you, let me know" [starting to laugh]

[…]

Lisa: Like this might be too many words for me. Like I might be kind of in between no words and all of this. (Case 7, Year 2, meeting 3)

This conversation led to a lengthy exchange between the group members which again highlights the diversity of the group and their approaches to teaching and learning and how much direct instruction one should use. Wayne felt teachers provided students with too many step-by-step instructions which limits students' thinking "do this do this do this, follow exactly what I am thinking here. I'll lead you by the nose [and] when you are done you will know where you are. No? You are lost? Because you really don't [know where you are] I led you through the whole thing, the process" (Wayne, Case 7, Year 2, meeting 3). After a back and forth between Tasha and Wayne debating the amount of direction students need the group laughs about how different their opinions are. The laughter highlights the comfort, trust, and respect that existed in the group which I interpret as a sign of achieving a balance between diversity and redundancy. The group moved beyond being simply collegial and was focused on the hard work of examining their practice which involves disagreeing and pushing against each other's ideas.

The four PLC members also had diverse experiences teaching grade 9. Rachel for instance had not taught any mathematics in several years and was relying on Tasha to help "keep her afloat" (Rachel, Case 7, Year 2, meeting 1) during Year 2 of the project. Lisa, an experienced teacher was new to teaching at Fields High School in the combined mathematics classes. She often struggled to reconcile that the Academic and Applied Mathematics curricula were different, and students registered in each had to meet different expectations, yet the combined approach meant all students were being taught the same lessons. Wayne and Tasha had experience teaching the combined class the previous year and Lisa would turn to them to ask for their opinion on how best to deal with different challenges, such as the analytical geometry unit that was in the Academic curriculum and not the Applied. This topic was an ongoing point of discussion among the PLC members, Lisa talked about this struggle during the seventh meeting of Year 2.

> Like, this is a real conflict for me because I see huge differences between the academic and the applied class. Like I actually, it's really hard for me to reconcile this in my mind. How we are making, how we can make these parallel? When I see like the analytic geometry strand…I'm just thinking about the curriculum, and that particular strand, and the number sense and algebra in Grade 9 Academic requires a significant amount of practice, like it's a considerable, conceptual abstract leap that the applied students, the applied curriculum does not compel the students to go to, so I don't know what to do about that (Case 7, Year 2, meeting 7).

This diversity in experience teaching a combined Grade 9 class pushed the PLC to discuss and analyze their approaches and Wayne admitted the analytical geometry strand was an area he struggled to teach. The PLC spent a great deal of time discussing this issue and working towards a clearer approach to teaching analytical geometry in their combined classes. This diversity in approaches was a source of stress for Lisa and she admitted trying to pretend it wasn't hard was difficult for her. She struggled as she attempted to change her approach to teaching mathematics to fit in with the other teachers at Fields. "It took a lot of energy to try and pretend I knew what I was doing [laughing]. It took a lot of energy, oh my goodness. But I feel like I can fit on sturdier ground this semester" (Lisa, Case 7, Year 2, meeting 7).

Through analysis of the transcripts from the two years of the OAME project and relying on my researcher notes from observing this PLC in meetings it was evident their redundancy and diversity interacted in ways that pushed their thinking, perhaps more in Year 1 than in Year 2. This balance between diversity and redundancy supported rich conversations. These conversations lead to the next set of complementary conditions, neighbour interactions in a system with decentralized control.

Trans-level Learning: Neighbour Interactions and Decentralized Control. Through analysis of the original OAME project data as well as observing each PLC meeting over the two years of the OAME project it is evident that two of the key conditions of complexity, decentralized control and neighbour interactions, manifested in several ways in Case 7. I first describe the ways decentralized control manifested in Case 7 before I describe the neighbour interactions exhibited.

Decentralized control was evident in the ways informal collaboration began among the PLC members and in how PLC members worked together to decide the focus of their professional learning. Prior to Dolores joining Fields High School and the OAME project Wayne and William had started to collaborate informally and were "pretty much meeting every day at lunchtime" to discuss issues they felt were important (Wayne, Case 7, phase 2 interview). Choosing to meet during their lunch hours demonstrates decentralized control. When Dolores arrived at Fields she encouraged the teachers to collaborate during school hours and supported the collaboration without controlling it, "before, we were just, we kind of were doing things on the sly" (Wayne, Case 7, phase 2 interview). When Dolores became principal she fully supported the collaboration and helped to establish the PLC at Fields. Wayne explained that "it was just so freeing to have your administrator being part of the team…there was no [leader], it came from all of us; it was a very, very communal and committee kind of thing" (Wayne, Case 7, phase 2 interview). This exhibits the way the PLC began as a decentralized network. Decentralized control was observed during Year 1 when a specific teacher would choose the focus of their lesson for a lesson study and the lesson was designed collaboratively with feedback from each PLC member. This decentralized control continued through Year 2 as the PLC shifted to designing a framework for teaching Grade 9 mathematics, suitable for both a combined class as well as an Academic class. Decentralized control allowed for many types of neighbour interactions, which I now describe in more detail.

Neighbour interactions manifested in several ways in Case 7 including when individuals interacted with each other and other's ideas during PLC meetings, when individuals interacted with others and their ideas beyond the PLC, when individuals interacted with different types of resources, and when individuals interacted with students and student thinking. The first type of neighbour interaction, interactions among the PLC members, was the driving force of the PLC in Case 7. In fact the formation of the PLC emerged from neighbour interactions between several

PLC members. William described how, prior to the start of the OAME project he turned to Wayne for support as William was struggling to find motivation to continue teaching. William's teaching at the time was "very teacher directed" (William, Case 7, Year 1, exit interview). Together, Wayne and William began rethinking their approach to teaching mathematics and they reported they were greatly influenced by interactions with people and ideas beyond their own school, "lots of things happened at the same time, there was a viral video of Dan Meyer; Wayne and I started talking about teaching with activities and spiralling at that time. Like a lot of things all happened at the same time" (William, Case 7, Year 1, exit interview). I classify these "lots of different things" as neighbour interactions, ideas bumping up against his ideas and causing a novel response. As Wayne and William began to shift their practice towards a more student-centered model, they also began sharing their learning at local and international educational conferences where they were exposed to even more new ideas and perturbations. These new ideas were shared with the PLC when the school joined the OAME project.

During Year 2 the PLC did not engage in lesson study but still worked collaboratively, with an abundance of neighbour interactions to push their thinking. For example, during one meeting Lisa and Wayne debated the value of always having students working at the VNPS. Lisa wondered whether there might be some benefit to providing opportunities for students to work individually. Wayne and Lisa shared their different ideas on this topic:

> **Wayne:** I find that I get much more production [with students at VNPS working in VRG], as in the questions get done faster, than if they sit at their desk with a piece of paper and pencil
>
> **Lisa:** Who is doing the learning for some of those? That's what I don't know. That's a question that I have

Rachel: That's a good question (Case 7, Year 2, meeting 3)

The group discussed this issue for quite some time. Analysis of the transcript shows this exchange to be respectful yet still pushes the thinking of each member of the group.

There were also neighbour interactions between the Case 7 PLC members and specific individuals such as researchers active in the field of mathematics education, other educators, and the researchers involved in the OAME project. An influential interaction for the PLC occurred immediately prior to the OAME project. William and Wayne attended a mathematics education conference and saw a presentation by Dr. Peter Liljedahl about his BTC model. This interaction had a profound influence on them and consequently influenced the PLC. Tasha explained how Wayne and William's interaction with Peter influenced the PLC.

Tasha: So the vertical classroom idea and having the kids problem solving on the boards, has been such a brilliant idea

Researcher: And where did that come from?

Tasha: Wayne and William and some other math teachers went to a conference that's held every four years or something. There was a professor from Simon Fraser University… it's vertical, non-permanent surfaces. So he showed his research and data on it. So William and Wayne were so excited about the idea that they started on Monday; they came in and got these whiteboards and so it [started right away] (Tasha, Case 7, Year 1, exit interview).

Wayne and William immediately installed whiteboards around their classrooms and soon nearly all the mathematics teachers in the school had incorporated at least some parts of the BTC model.

Wayne shared that attending conferences at both local and international levels greatly influenced his learning and the learning of the PLC as a whole. These types of neighbour interactions occurred in a network with decentralized control, meaning the teachers were free to choose the types of conference presentations to attend. Wayne expanded on how they networked with others beyond their school and how these neighbour interactions directly influenced the PLC.

> When our provincial conference [OAME annual conference] was in Kingston I had almost everyone in the department down there and we have been presenting. This year we have five different presentations we are giving at our provincial conference with different teachers, so we understand the value of that; but we pick up, we learn from, we get exposed to these people at these conferences and then most of us took the Jo Boaler course at Stanford, the online course, and Dolores, our principal, has provided us with a lot of books to read on visible learning, essential questions, visible teaching and she feeds, as a department head I get these things and the 4SI group we get these books from her and so we read about it and it's great. I mean it used to be we'd get a little thing in our mailbox with a few photocopied pages of some article but now we are getting books [laughing] (Wayne, Case 7, Year 1, exit interview)

The OAME project also provided opportunities for participants to interact with resources provided by research the team. For instance, Dr. Koch's presentation on the *Five Practices* influenced the PLC members to incorporate the resource into their PLC work. During Year 2 the PLC used the *Five Practices* as the focus of a book study. The practices influenced their lesson planning and guided how the teachers consolidated lessons. During meeting five of Year 2 Lisa described how the book influenced her thinking in terms of spending more time thinking about the goals of a lesson prior to teaching it and what she would look for in student responses that connected to this goal.

> The authors of the [*Five Practices*] talk a lot about how the work in groups often ends up being like a show and tell instead of getting at the real mathematical ideas and I see that in my own classes. You want to honour student voice, you want to honour everyone's thinking, so you go around the room and you point out different things that students have done, which is all great but the ideas presented [in the book] talk about how you can be much more purposeful in how you choose which group's work to discuss and it all comes from a common thread of what mathematical idea are you trying to convey by the end of that lesson or that activity, so a lot of emphasis on pre-planning (Lisa, Case 7, Year 2, meeting 5).

The PLC had neighbour interactions with many resources during the OAME project. Dolores was constantly reading current research on pedagogy and sharing books, research articles, and websites with Fields teachers. Throughout the project, participants interacted with many resources including books such as *Make it Stick* (Brown et al., 2014), *Visible Learning For Teaching: Maximizing Impact on Learning* (Hattie, 2013), *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching* (Boaler, 2016) and websites such as (wodb.ca), *Estimation 180* (estimation180.com), and a database of rich mathematical lessons created and compiled by educator Dan Myers. Lisa also mentioned that she followed Tasha's website and used resources in her classroom that she found on her colleague's website.

Interacting with students and student thinking were essential to the Case 7 PLC, and a main impetus for the PLC applying to the OAME project. Analysis shows that interacting with students and their thinking was a key component to the PLC work during both years of the OAME project. PLC members interacted with students during their day-to-day teaching activities. They also interacted with student thinking through assessment and evaluation practices such as observations and conversations with students, tests, quizzes, exit cards, and student journal entries. During Year 1 each lesson planning meeting began with the classroom teacher, for whom the lesson was being designed, describing each of their students in detail. This allowed the PLC to have a full picture of the students and the type of lesson that might best fit their learning needs. During Year 2 PLC members shared student work with one another during PLC meetings and discussed what the student thinking revealed about the lessons and activities the PLC had designed. Tasha shared that she always interviewed students after a written test to help clarify what students understood; this idea was adopted by the others in the PLC. As a group, the PLC agreed that interviewing students would be an important component of the ways the teachers assessed students, as interviews provided opportunities for the teachers to interact with students and their thinking in powerful ways. Wayne shared:

> I was able to get a real sense of their strategy and how inefficient or efficient it was which was great because the interview brought that out… The after piece is so valuable and it cleared up a few ideas for me and it also exposed some things (Case 7, Year 2, meeting 6)

Later during this same PLC meeting Wayne reflected on the discussion and the idea of interviewing students:

> I really liked that part of it [the discussion about interviews with students] which got me thinking about how I don't really have interviews with students until halfway through the semester and realizing that I'm missing out, so I think that next semester I'd like to have

> that interview in the first week or two, talk to everyone, have an interview with every one of my students very early (Case 7, Year 2, meeting 6)

It is clear that these neighbour interactions with colleagues and students influenced his thinking.

Another form of interacting with students and student thinking was through the student survey mentioned in a previous section. The survey was to track any changes in attitudes over a semester to see whether their lessons were helping students increase their confidence in doing mathematics. The survey was used primarily in Year 2 near the end of the project. It was evident that student survey influenced the PLC members in planning their approach teaching Grade 9 mathematics the following year. Wayne shared "I think this is good feedback for me and areas that I can work on and areas that I thought I was doing much better on… I've been blown away by a lot of the things I'm reading here" (Case 7, Year 2, meeting 9). In Wayne's interview for phase 2 of this study, he shared that the survey was so influential to his learning that he decided to pursue a master's degree in education to follow up on that work. The focus of his MA is on ways teachers can use information from a similar survey to learn how their teaching influences student thinking. When discussing his work Wayne shared "I think that's an incredibly important thing that teachers don't spend enough time thinking about" (Wayne, Case 7, phase 2 interview).

When analyzing the data it was clear that there were many neighbour interactions occurring within a network with decentralized control. Analysis also showed that these neighbour interactions were made possible, and supported through the enabling constraints that existed in this PLC. I next discuss how randomness and coherence manifested in the PLC at Fields High School.

Enabling Constraints: Randomness and Coherence. The PLC in Case 7 demonstrated both coherence to a defined purpose and goal as well as opportunities for randomness to disrupt the PLC, forcing the system to adjust and adapt (Davis & Sumara, 2006). I first discuss coherence before discussing the randomness demonstrated in Case 7. Coherence was demonstrated in the ways the PLC focused on the goals of the OAME project, set meeting agendas, maintained alignment with the Ontario curriculum and provincial testing requirements, and followed perceived norms established at Fields High School.

The PLC's original problem of practice sought to move students beyond memorization towards development of "their innate critical thinking and problem-solving skills to take ownership and determine the boundaries of their learning…. [and to] build our grade 9 applied students' confidence, keep them engaged, help them take responsibility of their learning and realize success" (Case 7, OAME project application). Throughout Year 1 the PLC revisited this goal and discussed different ways to support students. Analysis of the transcript from the second meeting of Year 1 highlights the importance the PLC put on enhancing teaching and learning. Dolores began the meeting by discussing strategies the PLC had been using such as focusing on accountable talk and asking good questions and she prompted the PLC to also consider student metacognition to help students understand how they learn best. Dolores pointed out that some teachers were focusing on metacognition, and she stressed that if the whole PLC was to align with this focus it could have a positive impact on the students.

> A couple of people do it explicitly in their classes and my question as we go into this cycle was what about if we were all doing that? Basically these kids are seeing [a PLC teacher] in three or four of their classes. I think that's very powerful if everybody were addressing [metacognition]. I think that over the long term we would see growth for them (Case 7, Year 1, meeting 2)

Transcripts from subsequent meetings clearly show that the PLC remained focused on the goal of enhancing students' metacognition. The lessons that the PLC co-designed all had a specific focus on metacognition.

Another example of coherence was the PLC's focus on mandates from the government including the curriculum and standardized provincial tests. As the PLC was attempting to combine the two Grade 9 curricula into one course they paid particular attention to how their common framework addressed the expectations from both the Applied and Academic courses. Since the provincial assessment (EQAO) addressed the curriculum, some were concerned about preparing all students adequately for the assessment. For instance, during meeting 3 of Year 2 Tasha shared that she worried about parents looking at the school's overall performance on EQAO and felt it was her responsibility to prepare students adequately. The PLC also employed the curriculum from previous grades to help understand what expectations the students should have been exposed to in previous years. They built their framework from these expectations and constantly referred to different curricula to support the work they were doing.

> I actually went back and looked at the Grade 8 curriculum. I've been studying the Grade 8 curriculum because I do think we have to be a little bit careful in our course. I do think we spend a bit too much time reviewing Grade 8 even though we always want to say that we don't do review…so I was kind of feeling like we need to be careful (Lisa, Case 7, Year 2, meeting 4).

Lastly, Lisa shared that she felt pressure to align her teaching to others at Fields High School. Prior to coming to Fields Lisa knew that the school was known for teaching in particular ways and that some teaching practices, such as having students practice specific types of problems was seen as unacceptable. At one point, when the PLC was discussing the importance of consolidating specific mathematical concepts before moving on to a new concept, Lisa brought up the idea of student practice:

> Well for me that's where a lot of the opportunities for students to practice comes in, and I don't want that to feel like [that's] a bad word. I feel bad saying [that] because we don't seem to do that here, but like you know, sometimes I think they need, I do actually think they need some problems to work on (Lisa, Case 7, Year 2, meeting 7)

This diversity that Lisa brought to the PLC and the mathematics department represented randomness that provided disruption to the PLC which allowed for the learning system, in this case the PLC, to react in novel ways including Lisa challenging the norms of the system. This led the PLC to discuss the role that having students practice mathematical concepts played in their spiralled approach to teaching the curriculum and they explored ways to enhance their model. They revisited their approach to spiralling through the curriculum and looked for ways to introduce more practice for students.

Randomness manifested in several ways in the PLC in Case 7 including pedagogical choices of individual teachers and professional learning models chosen by the PLC during the project. During Year 1 randomness was seen in the different lessons teachers chose to design for their classes and ultimately had final say in how the lesson would be taught in their class. During Year 2 the PLC developed the common framework with common assessments yet each teacher chose how to teach the lessons in the framework and how to incorporate the assessments designed by the PLC. For example during one meeting in Year 2 the PLC discussed the different ways the teachers employed an assessment designed by the PLC. The assessment had six questions in total. Tasha and Rachel gave the students unlimited days to complete the assessment individually. Lisa allowed students to work independently for the first day and on subsequent days students could pair up to work collaboratively while Lisa interviewed individual students about the work they had completed. Wayne chose only two of the six questions and allowed students only one day to complete the two questions. He felt it was “cruel” to prolong the assessment any longer than one day. In terms of the common assessments the PLC designed during Year 2 Wayne described how they provided opportunities demonstrating both coherence and randomness

> I just want to emphasize if we have a common assessment, like a common piece of paper, we can evaluate it anyway we want to really, right? Depending on what our kids have done and who they are and what’s in front of us…You can differentiate in classes, differentiate students, whatever, but if we have some kind of mechanism that’s common with all of us that will help (Wayne, Case 7, Year 2, meeting 1)

Randomness also manifested in the professional learning models the PLC chose during the two years of the OAME project. Year 1 they continued with their established lesson study model while during Year 2 they chose to work on developing the common framework. During each meeting of Year 2 the PLC discussed next steps and these next steps were determined by the PLC as a group based on identified needs of the PLC members.

These 6 conditions discussed support the characteristics of a complex system in different ways. I next discuss the characteristics of self-organization, adaptation, and nestedness before discussing how all of these conditions and characteristics influenced the emergence of learning in Case 7.

Self-organization. The PLC in Case 7 demonstrated self-organization prior to the OAME project and throughout the two years of the project. Examples of this include the ways that Wayne and William organized themselves into a collaborative learning system before the project and the ways the PLC membership changed over time. All six conditions described above supported self-organization in Case 7 and I will describe different examples of self-organization and the ways the conditions necessary to support learning in a complex system supported that self-organization. Wayne and William organizing themselves into a collaborative system was supported by trans-level learning involving neighbour interactions in a network with decentralized control. Enabling constraints also supported their self-organization. The enabling constraints were represented in their requirement for coherence to the provincial curriculum, yet randomness in the new teaching practices and learning tasks they experimented with. These new practices disrupted their ideas of effective teaching and learning. This self-organization between Wayne and William led to further self-organization within the mathematics department and the school. PLCs were established to support the new practices and to engage in more collaborative learning as a larger learning system within the school.

As described, the PLC at Fields reorganized itself into two PLCs between Years 1 and 2 of the OAME project and I consider this another example of self-organization. Trans-level learning and enabling constraints supported this self-organization. Through neighbour interactions in a network with decentralized control the PLC members collectively made the decision to split the PLC into two. Each PLC member had the choice to continue the work each PLC was engaged in. Furthermore, the self-organization into two PLCs allowed the school to better cohere to the objectives of each professional learning initiative while still maintaining randomness in how they went about working towards these objectives.

Adaptation. Adaptation was evident at the individual and PLC levels. Individually, different teachers adapted their teaching practice to align with their evolving beliefs and attitudes. As individuals adapted so too did the PLC as the individuals influenced the ways the

PLC worked collaboratively and what they focused on collectively. Tasha is an example of adaptation at the individual level; she spoke about how her involvement in the lesson study in Year 1 led her to adapt how she incorporated different tasks in her classroom. She gave the example of a lesson that asked students to justify their answers, mathematically to the questions "Which One Would You Rather - $20 in Timbits or $20 in donuts?". Tasha shared how she adapted her teaching approach based on the lesson developed through interactions with the PLC

> **Tasha:** In the past, I would not have spent the entire period on that, and I wouldn't have thought about how they were thinking throughout the process. Because, in that lesson we had them think about their answer immediately and then decide. Then do a little bit of work and then look at each other's work and then reanalyze whether or not their decision was the best and then continue on with their analysis. I don't think I would have; I think I just would have had them do the math and then…
>
> **Researcher:** move on into the "real lesson"?
>
> **Tasha:** Yes, exactly (Case 7, Year 1, exit interview)

This adaptation Tasha described was supported through different conditions described above, including redundancy in the goal of designing a rich lesson and diversity in the different perspectives which encouraged neighbour interactions in a decentralized network. This combination of redundancy and diversity supported neighbour interactions that pushed the lesson to be more than Tasha intended to design. Lastly the PLC aligned the lesson's focus to what Tasha had chosen yet randomness was demonstrated in the variety of ideas and suggestions the PLC generated.

Dolores also provided examples of adaptation at the individual level. She adapted her leadership style to align with the lesson study model while at Fields and she changed how she carried out teacher performance appraisals. Rather than simply sitting in a classroom observing a teacher, she engaged in what she termed “a lesson study process” (Dolores, Case 7, Year 2, exit interview). Dolores and a teacher would co-design a lesson that dealt with an area of teaching that the teacher wanted to focus on. Dolores would then observe the teacher teaching that lesson and then the two would debrief afterwards. She described how she turned the teacher performance appraisals into an opportunity for learning rather than purely an evaluation.

Lastly, William shared how he has adapted his teaching practice based on his experiences with the PLC and the lesson study process in particular. When asked during his Year 2 exit interview what he thought the biggest successes of the PLC were he suggested it was the way it influenced his own teaching. He described that designing a lesson as a group changed the way he taught when he was planning his next semester courses independent of the PLC.

> We worked on this and now there is carry over to the rest of my lessons that I do on my own, right? I’m now implementing the strategies that I’m learning about in the PLC in my everyday teaching so that would be the biggest positive thing for me is I now teach this way, right? It’s the way I teach (William, Case 7, Year 2, exit interview).

The PLC also adapted different approaches to collaborative professional learning during the two years of OAME project. During Year 1 the PLC adapted the co-designed lessons to focus on teaching practices, such as a focus on metacognition, that could be applied to all subject areas rather than focus on subject specific content. The diversity in teaching expertise in the PLC as well as the redundancy in the shared goals of enhancing the teaching and learning across several subject areas help to support these adaptations. During Year 2 the focus on the de-streamed classrooms also represented an adaptation. The school was adapting to respond to an identified need in their student population. This adaptation was supported by a balance between the diversity of students' learning needs, the redundancy that all students are required to register and achieve success in Grade 9 mathematics, and the overlapping redundancy in the two curricula.

Overall, the PLC in Case 7 adapted their teaching in numerous ways. During his Year 1 exit interview William succinctly listed many of these adaptations:

> We've really changed the way our classrooms operate, it's way more student centred, way more rich tasks, way more productive struggle, better attitudes towards mathematics in our classes. Because of lesson study, because of spiralling, because of good tasks, because of work with Peter Liljedahl and learning how to conduct a thinking classroom. All those different things have affected our change. And then this project, Year 1 looking at lesson study and changing instructional strategies was very beneficial and then Year 2 where they looked at blurring the lines between 9 academic and 9 applied (William, Case 7, Year 2, exit interview).

In reviewing this list of adaptations it is clear that the conditions of complexity supported these changes. The diversity and redundancy of the PLCs, which demonstrated decentralized control, created opportunities for rich neighbour interactions with others such as Dr. Liljedahl greatly influenced the PLC to adapt their teaching practices to align with the BTC model. These adaptations occurred in systems supported by the enabling constraints of coherence to the mandated curricula of the province and randomness to experiment with different approaches to teaching the curricula.

Nestedness. The PLC at Fields High School was nested within the OAME project. In terms of the PLC and Fields High School the PLC was influenced by the way the PLC was nested within the school which was, in turn, nested within the greater educational community. In discussing trans-level learning, this nestedness supported neighbour interactions between the

PLC in Case 7 and others in the education community. The members of the PLC spoke about the different interactions that influenced their learning including other teachers in different schools, attending educational conferences, and even interacting with other educators online through social media platforms such as Twitter. These different influences represented the nested nature of education as demonstrated in Figure 12.

There are some notable differences in the ways the PLC in Case 7 operated during Year 1 and Year 2 of the OAME project that relate to nestedness. The process of lesson study involved a nestedness that was not demonstrated in Year 2 of the project. During Year 1 the PLC met as a group of professionals during school hours outside of the classroom. They also engaged in professional learning as a group within each other’s classrooms, observing teaching and interacting with students’ thinking. In this example the PLC was nested within the classroom, which is nested within the school. Analysis of the data identified this nestedness as being influential in teacher learning. Lisa spoke about the isolation of teaching and how not being able to learn from each other by observing each other teach was a drawback to the approach the PLC took in Year 2. Year 1 allowed individuals to interact with students and student thinking in different ways than simply teacher-student interactions. Observing others teach and observing ways students responded to the co-created lesson provided a diversity in perspectives of student thinking. Being able to observe students’ thinking without having to be concerned with pedagogical next steps provided a different perspective to the PLC teachers. The enabling constraint of coherence was more prominent in interactions with students during Year 2 as teachers needed to cohere to their lesson plan and respond to student thinking. These types of neighbour interactions with students and student thinking were not available to teachers in Year

2, during Year 2 individuals were limited to the interactions with students that occurred during their own teaching.

Emergence of Learning: Case 7

The above conditions and characteristics interacted in different ways to support each other and combine in various ways to lead to the emergence of learning. While all the conditions and characteristics of complex learning systems were coded in the Case 7 data, neighbour interactions were the most frequently coded theme followed by diffusion then emergence. I next discuss a few examples of the emergence of learning in Case 7 at both the individual and PLC level and how the different conditions and characteristics of complexity science influenced this emergence.

The PLC's lesson study process was noted by several participants as having a large influence on their professional learning. This process involved a balance between redundancy and diversity which encouraged the rich conversations that were described in previous sections of this case. For Tasha, as described previously, her learning was influenced when the PLC worked together to adapt a lesson for her class. Tasha's example of developing her lesson from a 10-minute activity to a lesson that took the entire class period stresses the value of interacting with colleagues. It shows how the balance between redundancy and diversity allows ideas and representations to interact with one another, leading to the creation of new understanding. I suggest that this example aligns with the idea that a complex system is more than the sum of its parts, when parts of a system interact the learning that emerges from individual parts and the system as a whole can be described as greater than the sum of its parts.

William also shared how the lesson study process influenced his thinking and explained how during his exit interview at the end of Year 1:

William: The very first time we had a lesson study we did cup stacking (similar task available at https://blog.mrmeyer.com/2008/linear-fun-2-stacking-cups/). They [the PLC] completely changed the way I do that lesson

Researcher: They did?

William: That group did, I usually do cup stacking where it's one particular way and here's two English teachers and Wayne and Dolores saying "Well, why are you telling them how to stack them? Let them stack them however they want". That just opened it up. "Oh that's a great idea!" and so this activity that I have done a few semesters all of a sudden got all these teacher moves and all this richness to it and it was the very first one that I did and it was a very powerful moment for me where like here are all these people in my class and I remember being nervous too, all these people in my classroom watching this and like [it was] completely different, it was still activity based. I'd been playing around with activities but there was something different about that particular day because it was so planned.

Researcher: So you were teaching the lesson that you had designed together with them?

William: Ya, now it was a lesson I had done before …but it was very teacher scaffolded [before] and all of a sudden we took this activity and opened it right up and then where it went was great. It literally ended up taking a week to get through it all. We thought we would get through it in a period, [but] it was a week-long lesson, and I learned a lot about students there. They brought a lot more to the plate than we thought [they would]. They had more prior knowledge than we thought, that multiple ways of solving things is good, it doesn't always have to be algebra, every kid has a voice you know, there was all this stuff, differentiation, there was all this stuff because we videotaped the entire thing and I

> got to go back and look at that video with the math coach, and so it was very eye opening to watch that lesson unfold on video and I would never have been able to do that if it weren't for [MATH COACH], she edited it all and cut it all down and made it visible for me so it was a very changing moment for …but the group has really been the most intense lesson study for sure if there has ever been a group of people that have gotten together and really thought hard about how we teach and what we teach. (William, Case 7, Year 1, exit interview).

Evident in the above quote are each condition of complexity and the ways they support the characteristics of complex systems. The importance of balancing diversity and redundancy in order to support neighbour interactions in a self-organizing system with decentralized control is evident. Also evident is the importance of enabling constraints, balancing randomness with coherence, in order to allow for adaptations to existing teacher practices to meet the needs of the nested systems. These nested systems include the teachers, the PLC, and the classroom of students that include both teachers and students. The importance of interacting with students and their thinking is also evident.

The emergence of learning at the PLC level was described during an exit interview with Dolores when she succinctly described the PLC in Case 7 as a complex system. Through a question about her own background in teaching she shared that even though she was not a math and science person by training, her knowledge of teaching was a valuable contribution to the group. Each member of the PLC brought something important to the work they did. When designing a mathematics lesson, for example, Dolores talked about how her contributions provided a different perspective:

> I'll bring them back to "what are those, what's that high yield teaching strategy we're looking at?" Oh yeah, it's got to be self-verbalisation, it's got to be higher order thinking"…. It's a very respectful group and what's powerful about it is it's all about ideas, there's nothing personal about any of this and that's the force of the lesson study, is that we are not looking at the teacher, we are looking at the kids. "Was our lesson impacting those kids?" It's not about you. Now there have been some A-ha moments where a teacher did something and we all went "gasp" and the teacher went "gasp" and we all realized the impact of the teacher in that particular situation but it could have been any of us and we all just revelled in this discovery that we had and it has been very much that kind of lab that we are running and seeing the effect of this element and that little piece and the recipe and how does it change the final formula and the impact, it's very powerful (Dolores, Case 7, Year 1, exit interview)

To me this quote represents every condition and characteristic of a complex system. Balancing the tension between diversity and redundancy was one of the key factors in driving the learning in the PLC. Trans-level learning was represented through the neighbour interactions in a system that exhibited decentralized control. No one teacher was in charge of the lessons they designed, they each contributed to what they referred to as "our lesson".. Enabling constraints, of which Dolores plays a crucial role, helped to support the learning of the PLC. The teachers were nested within the PLC which was nested within the school. Each of these systems is learning as a result of the other systems learning. When the PLC had an "Aha!" moment the PLC learned by the individuals learning. The PLC self-organized around a particular focus and adapted their lesson to align with their chosen goals. They learned from each lesson they designed and adapted the next lesson based on the learning that resulted from previous lessons. All of these conditions and characteristics lead to the emergence of learning in the PLC in Case 7, as Dolores calls it, the "final formula". During her exit interview Dolores also specifically credited funding and support through projects such as the OAME and 4SI projects as supporting the emergence of learning. She shared that the ability to interact with others across the province in the ways that these projects provided was directly related to the learning that emerged:

> There is no doubt about it, the funding and the push from 4SI and OAME and anything that happens to be out [there] keeps those teachers on track and seeking new and better ways of doing it. The opportunities to hear the big minds changes their behaviour, changes their practice. (Dolores, Case 7, Year 2, exit interview)

Overall, the PLC in Case 7 demonstrated how the different conditions and characteristics of complexity supported the emergence of learning in both the teachers and the collaborative groups they were a part of.

Addressing Research Question One: Case 10

Case 10 started as a PLC located at Broadview High School in a medium sized urban centre in Ontario. At the time, the school had a population of approximately 1200 students and the PLC maintained a steady core membership over the two years of the project. Two members joined between Year 1 and Year 2 of the project. As I will discuss, the Case 10 PLC worked collaboratively with another school in the project, Case 9. For that reason I also include a member of PLC 9 in the table below that describes the active participants in the Case 10 PLC.

Table 17

Members of Case 10 OAME project PLC

Pseudonym	Role Year 1	Role Year 2

Paul	Principal	Principal
Deirdra	Mathematics Department Head	Mathematics Department Head
Marco	Mathematics Teacher	Mathematics Teacher
Elena	**Mathematics Teacher**	**Mathematics Teacher**
Charlotte	**Special Education Department Head**	**Special Education Department Head**
Donald	District Mathematics Leader Case 10	District Mathematics Lead Teacher Case 10
Gabby	District Mathematics Leader Case 9	District Mathematics Lead Teacher Case 9
Maya	**N/A**	**Mathematics Teacher**
Claire	N/A	Mathematics Teacher

The PLC's application, which was identical to Case 9's application, was written by the two district mathematics lead teachers Donald and Gabby. The problem of practice identified on both applications was as follows:

> Grade 9 Applied students, in general, tend to be less engaged with their learning than the average student. Through the use of learning goals and constructing success criteria, students will develop deeper understanding of the Grade 9 Applied Math curriculum and become more focused learners. (PLCs 9 and 10 OAME project Applications, June 2014)

The goals of the PLC stated in both applications were:

- Increased collaboration between teacher partners
- Increased student engagement through the use of manipulatives/technology
- Increased confidence in teaching the math curriculum
- Increased evidence of growth mindset in both teachers and students
- Increased connection of math concepts to big ideas for both teachers and students

(PLC 9 and 10 OAME project Applications, June 2014)

During the project the PLC at Broadview met monthly, almost always with PLC 9. The PLC meetings were led, often in a workshop style, by Donald and Gabby. Analysis of the meeting transcripts and the RA notes suggest that the meetings involved primarily Gabby sharing lesson and activity ideas that she had developed. Often, after presenting ideas of different ways to teach mathematical concepts, or looking at student work, PLC members were given time to plan how they might incorporate these ideas into their own classrooms. These PLCs explored several topics that included developing engaging activities, implementing technology in the classroom through the use of iPads, and improving assessment practices. In reviewing meeting transcripts, it appears that Donald and Gabby were seen as experts in teaching and learning mathematics and often the other PLC members asked their thoughts and opinions on different tools and practices. I next describe the ways that the conditions of complex systems manifested in PLC 10.

Specialization: Redundancy and Diversity. The PLC at Broadview High School demonstrated the conditions of redundancy and diversity to varying degrees. Redundancy in this PLC manifested in all of the PLC members being from the same school, focused on the same subgroup of students, and focused on the teaching and learning of Grade 9 Applied Mathematics. There were also some common areas of agreement in terms of pedagogy, primarily all the PLC members agreed that students needed time to practice their mathematical skills in order to gain mastery of these skills. This was evident during the first meeting of Year 2 when the group was discussing the advantages of different tools such as notebooks and whiteboards for students to practice with. Although there was disagreement about which tool or approach was better they all agreed that practice is essential for student learning.

Diversity manifested in several ways in this PLC including individuals' different teaching specializations, different members' level of engagement with the project, and different beliefs and attitudes about teaching and learning mathematics. Diversity in teaching specialization was most notable in the presence of Charlotte, the head of the special education department. Her role meant that she was not in a classroom with the same students everyday but rather spent her time supporting students in the special education office on an as needed basis. Prior to the project she did not spend time as part of the planning in the mathematics department. As with the other PLCs in the OAME project this PLC met the requirement of having a diverse membership in terms of individuals' roles in the school (as shown in Table 17)

There was also diversity in the level of engagement among participants. Elena, a mathematics teacher in Case 10, suggested that not all the teachers were as keen to be involved with the project as she was. "We had different members that had different levels of engagement" and her high level of engagement with the project was not "uniform across all the group members" (Elena, Case 10, phase 2 interview). One teacher, Marco, was in his last year of teaching and was just "enjoying the ride" (Elena, Case 10, phase 2 interview). Elena suggested this lack of engagement on Marco's part posed some problems in terms of collaboration. Marco was not interested in any classroom observation or co-teaching strategies that were suggested by the PLC. Analysis of the Year 1 meeting transcripts show that other than the two district mathematics lead teachers, Elena contributed to the discussions more frequently than other PLC members. Maya, also a mathematics teacher, joined the PLC at the summer institute between Year 1 and 2. She was very interested in exploring new practices and engaging in collaborative discussions about different teaching strategies. It appears that during Year 2, with the addition of Maya, the PLC experienced more of a balance between redundancy and diversity. Maya shared that she saw her presence as causing a positive disruption in the PLC that pushed the PLC in their thinking, “me going there was a huge part of that [change]. Just changing the [dynamics of the group]. Right?” (Maya, Case 10, phase 2 interview). Analysis of the Year 2 transcripts support Maya’s claim; she often would push back against what others were saying in a respectful, professional way. For example, during the first meeting of Year 2 Maya challenged a point that Elena and Gabby made about not needing to have students make notes. She acknowledged her views might change as she engages in the work the PLC was doing but she wasn’t opposed to challenging other’s views.

> You made a comment about not necessarily having to have it written down on paper anymore. And I know when Gabby was here last week she had that same feeling. And I don’t agree with that. I think, and I’m new to this whole thing, so maybe I’ll learn something, I don’t know, differently. (Maya, Case 10, Year 2, meeting 1)

Analysis of transcripts also revealed diversity was present in the PLC members’ beliefs and attitudes about teaching and learning mathematics. Elena described herself as a “progressive thinker” (Elena, Case 10, phase 2 interview) and was completing her Master’s degree in education as the project began. She pushed her school to apply to the OAME project and had to convince others the project was a good idea. Maya shared she “loved meeting new people and learning different ways [of teaching]” (Maya, Case 10, phase 2 interview). She was always looking for new ways to learn and engage in professional learning. Both Elena and Maya described the department head, Deirdra, as being a traditional teacher and not as open to teaching using practices such as games and rich tasks. During Year 2, Deirdra bluntly shared her opinion on the ways that Maya was teaching, and this was hard for Maya. “There were days that I, it was tough sometimes because she would say ‘Oh, you and your games you play in class’” (Maya, Case 10, phase 2 interview).

Trans-level Learning: Neighbour Interactions and Decentralized Control. Neighbour interactions in Case 10 manifested in different ways including interactions between different PLC members and their ideas, others beyond the PLC such as researchers and other PLCs and their respective ideas, resources such as videos, websites, technology applications, and rich tasks, and interactions with students and student thinking. The majority of neighbour interactions between PLC members and their ideas were instigated by Donald and Gabby. As mentioned, the first meetings of the PLC involved Donald and Gabby presenting resources and activities they had already created and used in various classrooms to the group. They would show videos of students working on tasks and then they presented reasons why the other PLC members should use the same, or similar tasks. As the project evolved other individuals began to share more during the PLC meeting, particularly when they had Gabby or Donald visit their class and lead an activity for their students. Another example of neighbour interactions related to sharing knowledge with one another regarding students and their needs. Charlotte, the head of the special education department, provided time and space for teachers to discuss specific students who Charlotte worked with in her department. Maya shared having time to sit and discuss students and their needs with Charlotte was helpful for preparing to teach and provided her a deeper understanding of some of the students’ needs. One example involved Charlotte sharing information about a student who was determined to be successful in Maya’s class:

> **Maya:** I’m definitely approaching him differently because of the background you gave me, after that conversation we had about that he wants this really badly and he wants to go to college

Charlotte: And you wouldn't have gotten that from an IEP (individual education plan).

Maya: I wouldn't have gotten that from an IEP. All I would have gotten is that the kid is deaf, and he has an interpreter. Right. I wouldn't have gotten any of that from an IEP. (Case 10, Year 1, meeting 2)

The PLC also interacted with others, and their ideas, at project-wide meetings. Ideas such as the BTC model, using vertical surfaces, and spiralling, the concept of revisiting mathematical ideas multiple times throughout a course, appealed to the PLC and they introduced some of these ideas in their own classrooms. There did appear to be a difference in the ways the PLC in Case 10 discussed ideas. Whereas other PLCs would talk about different areas they were working through collaboratively, such as ways to improve engagement, in this PLC, Gabby would share ways she had seemingly already solved these issues. It was more a directed discussion, led by one rather than an exploratory collaborative discussion about possible strategies.

The PLC interacted with many different resources. Donald and Gabby showed numerous videos at each PLC meeting focused on mathematical pedagogy, making mistakes, and asking good questions. These different videos included researchers and professional learning facilitators such as Jo Boaler, Dan Meyer, Damian Cooper, and Marian Small. Donald and Gabby also shared rich tasks and mathematical activities from websites such as Desmos, learnteachlead.ca, ontariomath.blogspot.com, and EduGains.ca. The Donald and Gabby spent a lot of time showing the PLC different applications they could use on the iPads such as Explain Everything that allows a teacher or a student to record audio over a video image as they work, SeeLevel, an app that measures the vertical distance of a tall object such as a tree or a bell tower, and Nearpod, an application that allows a teacher to gamify lessons and assessment through the use of quizzes and games. Other apps included CubeThink, NotePad, Google Apps, including YouTube and

YouTube Capture, GeoBoards, Base Ten Blocks, Algebra Tiles, Pattern Blocks, Number Lines, Fractions, Number Rods, DragonBox, BaiBoards, and Khan Academy. Lastly, the PLC members interacted with students and their thinking. During PLC meetings the PLC was shown videos of student thinking and student work and the members were led through a discussion by Donald and Gabby. As the project continued the videos shown at the PLC meetings were from lessons some PLC members participated in, although these lessons were always co-taught with, or led by, Donald or Gabby.

In terms of control of the PLC, analysis of transcript meetings suggests the two district mathematics lead teachers led most meetings. The PLC operated primarily through centralized control rather than decentralized. Analysis of the project application reveals perhaps Donald and Gabby viewed the teachers involved in both schools as needing change in some way, the applications stated, “by working in small groups, teachers will feel more willing to talk about refining their practice,…[develop] deeper understanding of the curriculum,…[and that] a shift in teacher practice [would allow] for a greater emphasis on student-centered learning” (PLC 9 and 10 OAME project Application, June 2014). It is evident through analysis of the meeting transcripts and project interviews with Gabby that the expectation from the two leaders was that teachers would align their practice to what was modelled by Gabby and at times Donald. Gabby explained she found if a teacher is shown what to do it is better than the teacher trying to determine what are best practices by themselves.

> I think if you just have them create something, sometimes they just don't know how to do that or what you could do…I think just giving them stuff and coming in and doing it with them is kind of the bridge into them noticing, you know, “wow I can change what I was doing”. (Gabby, Case 10, Year 2, exit interview)

It seemed many PLC members were on board with this approach to working, but it did not appeal to every member of the PLC. Meeting notes documented by the research assistant show Elena did not like the top-down approach the PLC was taking:

> [Elena] commented that they would have liked to have more opportunities to work on their own ideas but felt that being required to meet with the other school prevented them from having adequate time to work on their own ideas. She felt that the PLC meetings were directed from above and felt it should be more self-directed. She stated that the meetings began to feel more like a workshop where the other school would spend much of the time showing what they were working on. Though she enjoyed working with the other school and sharing ideas, she felt it went a little overboard. She described this as a sore point. (PLC 10, Case Study Report)

It is evident Elena was comfortable sharing her opinions during PLC meetings. Elena, and later Maya, were vocal during PLC meetings where other PLC members did not contribute as much.

Although the PLC meetings were led by the district mathematics lead teachers, what the different teachers chose to work on during these meetings, when time was allotted for working in school PLCs, was left up to the PLC members. In this way the PLCs demonstrated some decentralized control. During this work time members of the PLC would check with Donald to make sure what they were doing was in line with what he viewed as the goal of the project and the PLC. For instance, during the first meeting of Year 2 Donald was talking about the different roles technology played in the classroom and describing that at times technology can provide ways to teach mathematics that are not possible without technology such as graphing and exploring functions using Desmos. Elena asks:

> **Elena:** Is that what we are supposed to be striving for when we use technology?

> **Donald:** I don't know, that's sort of like the gold standard, but not everything can be that. And I think if you are at least striving towards, like moving away from just replacing, like if you are just reading a book on an iPad. That's not really a great use of technology there. (Case 10, Year 2, meeting 1)

I will discuss the influence the leadership in the PLC had on the learning when I discuss the emergence of learning in a later section.

Enabling Constraints: Randomness and Coherence. The PLC in Case 10 appeared to lack a balance between randomness and coherence, perhaps due to the lack of decentralized control it experienced. Randomness was limited to the ways Donald and Gabby chose to lead the PLC, the ways other PLC members chose to adopt the proposed teaching practices the district mathematics lead teachers suggested, and in the ways individuals engaged with others in the project beyond their own PLC. It is unclear if PLC members had a choice on whether Donald and Gabby would visit their class to lead a lesson and it appears this co-teaching happened less in Case 10 than it did in Case 9. The PLC members who agreed to be part of the work did not have a lot of choice in how the PLC worked or what it worked on. In fact it was suggested by Maya, during her exit interview, that the PLC was chosen to apply for the OAME project by the school district because of the more traditional approaches of many teachers in the mathematics department and the hope was the project would influence the teachers to shift their practices.

> I think one of the reasons that our school was chosen for [the OAME project], it was mostly the consultant [Donald] at the time kind of said this school needs help because the teachers were very traditional, and they had a bad reputation. Like they were just tough on their students, they felt like they were trying to have integrity in their classroom, but it was not working. Like kids were failing and they were dropping out, and so there needed

> to be some sort of change there and I think that that's why that school was chosen for the OAME project to get, you know, the numeracy support teacher in there and start giving them different strategies. (Maya, Case 10, phase 2 interview)

Overall, it would appear that Donald and Gabby may have adopted a deficit model approach to professional learning, and this suggests a lack of balance between coherence and randomness. During a Year 1 interview with Donald and Gabby it was evident they viewed the teachers as needing to be shown the possibilities of teaching and learning mathematics in different ways and Donald and Gabby saw their role as demonstrating these ways. During this interview Gabby shared that rather than create something collaboratively with the teachers she shares with them the lessons she has already created, explaining “that's where we've come from is taking everything that I… already had… I'm going to share everything, and I think that's the start, you know. They have to see that before there's going to be a change” (Gabby, Case 10, Year 1, interview). Later in the interview Gabby explained that teachers need to be given activities in small steps because “it's a lot for the teacher to make that change, right? So it's like okay one step at a time, right? So here, try that activity but while you're doing this activity, what kind of questions could you ask the student?” (Gabby, Case 10, Year 1, interview).

Coherence manifested in the ways the PLC focused on the goals of the OAME project, aligned with the provincial curriculum and provincial standardized testing, and adhered to a particular way of working as a PLC. Both Donald and Gabby were committed to the goals of the OAME project which included enhancing the teaching and learning of Grade 9 Applied Mathematics in their school district. The PLC spent time discussing the ways the specific tasks and activities that Donald and Gabby shared aligned with the provincial curriculum and how these tasks and activities could support students as they prepared to take the EQAO test. The

PLC overall worked in the same way over the course of the two years of the OAME project. Donald and Gabby would present examples of lessons and student work and the PLC would discuss what they were observing. As the project progressed the PLC began to design new ideas based on the examples they had been shown with Donald and Gabby playing a guiding role in these new lessons.

Self-organization. There were several examples of a lack of self-organization in Case 10. Analysis of meeting transcripts suggests that the top-down leadership model and the choice to combine Case 9 and 10 represents imposed organization. These decisions were made by the district mathematics lead teachers with little to no input from the other PLC members. Donald hints at a lack of self-organization from the PLC when he complains the teachers did not work on new lesson ideas in between PLC meetings, "so, you know, when I would go in last semester it would be like 'Yep, let's do this', planning and planning and planning and then it wouldn't seem like anything happened in between me coming in the next time" (Donald, Case 10, Year 1, interview). Perhaps the lack of decentralized control in the PLC influenced the members and this limited their engagement with the work of the project. Some evidence of self-organization was mentioned by Elena. During her phase 2 interview Elena suggested that the project did provide time for the participants to navigate issues of their own choosing and she appreciated that time to work together as a department. She shared that this time together encouraged participants to be more open to new ideas and to discuss practices that might influence their practice.

Adaptation. There was evidence of adaptation by some of the members of the PLC and the PLC as a whole. During her phase 2 interview, Charlotte, the Head of Special Education at Broadview High School shared that the time she spent with the PLC helped her to understand some of the strategies the mathematics teachers were using in their classrooms, and the different manipulatives and technologies teachers were using with the students. For example Charlotte was introduced to a manipulative, algebra tiles, through working with the PLC and she now has these tools in her room for students to use when they visit her for extra support. Charlotte therefore adapted how she interacts with students through her involvement with the project. Prior to the PLC Charlotte wasn't clear on how the teachers were teaching and often found it difficult to support students using tools and technologies she was not familiar with.

Both Elena and Maya adapted their teaching styles to incorporate the use of vertical non-permanent surfaces, manipulatives, and more open problems in their classroom. During her phase 2 interview it was clear Maya had adapted her teaching because of the project. She shared how she used to be the type of teacher who would lecture her students for 45 minutes while they took notes but now her class works collaboratively using the BTC model and she has the class create meaningful notes at the end of class; "the teachers shouldn't be writing notes for 45 minutes and then having students practice for 15. Like it's just not working" (Maya, Case 10, phase 2 interview). Elena also shared she was still incorporating several of the strategies she learned about from other PLCs in the project. She continues to use the BTC model and encourages new teachers at her school to adapt in similar ways.

Nestedness. Like other PLCs in the project the PLC in Case 10 demonstrated different levels of nestedness. The individuals from Case 10 were nested within the PLC, which was nested within Broadview High School, the 2 combined PLCs, and the school district. The PLC was also nested within the OAME project which provided opportunities for neighbour interactions with a more diverse population of individuals and PLCs. These opportunities for neighbour interactions with others beyond their own school and the other district school involved in the project was one of the components of the project both Maya and Elena shared was most important in their experience with the project. Elena explained:

> I got a lot of energy from people from other schools that had the same kind of excitement about things, as I did, but within my own board I don't know whether I felt that same level of excitement by others…I really enjoyed being able to get that perspective, like I really enjoyed meeting people from other schools and thought about "wow everybody should, we all should know what's going on out in all of Ontario" so we do have a better perspective [about] where we're at compared to them and where we should be at or what we should be, you know, aiming to be at. (Elena, Case 10, phase 2 interview)

This nestedness, combined with the other conditions and characteristics combined to lead to the emergence of learning among different individuals and the PLC as a whole.

Emergence of Learning: Case 10

Analysis of the project meeting transcripts and the individual phase 2 interviews revealed the emergence of learning at different levels including the individual level, the PLC level, and the school level. All three PLC members interviewed for phase 2 of this study shared how their involvement in the project led to new learning for them individually. Similar to other cases, the most frequently used code was neighbour interactions. For Charlotte, neighbour interactions with others, with mathematical ideas and tools, and student thinking influenced her learning. She shared how having the ongoing, sustained, and embedded time to engage in conversations with the teachers in the mathematics department influenced how she engaged with students when they visited her in the resource room at Broadview High School. She shared how she had a better understanding of working with mathematical tools such as algebra tiles and the whiteboards with students. She also had a better understanding of the types of problems the students were being exposed to and the ways the teachers were working with students in the classroom. This allowed Charlotte to address the students' needs in ways that aligned with what was happening in the classroom. She was able to work with the same tools as the student in ways she hadn't been able to before. In addition to neighbour interactions, Charlotte's learning was also supported by the nested nature of the PLC within the school.

Elena also credits her learning to neighbour interactions. She shared that the ways she adapted her teaching led to her having a more chaotic classroom at times because students were talking and working in groups at the vertical surfaces. She felt able to manage this chaos because of some of the resources and practices she interacted with through the project, specifically the *Five Practices* framework.

> It's chaotic especially because many of us have grown up in a traditional [classroom] and so it's hard to imagine a chaotic classroom.[But] I realized [it was a] good thing, when you have the confidence of being able to pull things together or take a look at five different solutions and look at different aspects of it, it helps students kind of, you know, gather whatever it is that they're supposed to gather. It's very helpful. (Elena, Case 10, phase 2 interview)

Elena discussed how her understanding of assessment of student learning was also influenced by the project and her integration of the BTC model into her practice. She felt the project increased her confidence in assessing students using more than paper and pencil tests. The BTC model also allowed her to see and hear more of the students' thinking in ways that demonstrated their understanding without the need to test them.

> Well, for me, even to assess them, I can see a lot more how kids are understanding. One of the things that I really liked about vertical surfaces is the assessment aspect of it. Now

> I know that there were so many different discussions on how to deal with assessment and collect documentation and stuff, but I was much less formal but still it allowed me to get to know kids on a level where I felt a lot more comfortable about my other measurements and it was the first time that I felt comfortable being able to do that and not having to rely on just you know your paper and pencil tools to figure out. (Elena, Case 10, phase 2 interview)

When Maya joined the project during the summer institute between Year 1 and 2, she found meeting new people and hearing their different approaches to teaching helped to push her teaching in different ways. She learned about spiralling through the curriculum from Case 7 and began to integrate the concept into her teaching. She also adopted the BTC model other schools were talking about at the summer institute. Through collaboration with her own PLC she began to see the benefits of having students collaborate and work at VNPS.

Marco, a mathematics teacher at Broadview shared that he felt the students were more engaged with the way he was teaching mathematics and his incorporation of new practices such as open tasks and using vertical surfaces. They were more excited to be in class than before, even at the end of the day when he found students were typically less engaged. This excitement from his students was encouraging Marco to try new things and perhaps keep teaching a little bit longer than he had planned, as he was eligible for retirement the year prior to the project

> My students are excited now, and that's what I find. Even being a Period 4 class or Grade 9 Applied, Period 4 class, I've noticed in past years that students were kind of sluggish, they were ready to quit or whatever, but now, students are coming to my class excited. They come in, they look at my Smart Board, and say, "What are we doing today, Sir?" You know, "Are we going to do something exciting," or whatever. And they are all

> excited. And that's what I'm finding really amazing about this little program that we are doing. The students are getting excited, at math. I think that's the whole purpose behind this, to get them involved, to get them going, to try new things, to get them to think. And for me personally, it's been a change in my career as well. I've been looking at retiring, and I'm eligible to retire, last June, but this project has given me some more inspiration to keep on going a little bit longer and try out new things, and so far that's what's happening. I'm getting excited, the students are getting excited. And I think that's all we need to do. Get everyone excited. (Marco, Case 10, Year 1, meeting 3)

This response from Marco seems to contradict Elena's assessment of his coasting through the project. Perhaps, even though he didn't engage in the co-planning and co-teaching with the others, he still learned through listening and negotiating meaning for himself. He was persuaded to try out some new ideas and his interactions with students influenced his learning.

At the PLC and school level several individuals shared how they felt the project influenced learning at Broadview High School. At the end of Year 1 of the project, during a discussion with the research assistant assigned to Case 10, the department head shared how she felt the project was influencing the PLC and the mathematics department at Broadview. She didn't define what was making the teachers better but clearly she felt something was happening, because of the project, that was spreading through the school:

> I'm finding that our teachers are becoming better teachers, and all of the information that we're gathering as part of the project, we're disseminating into the department and other department members are gaining from our knowledge as well. So, overall, the whole department is becoming better. (Deirdra, Case 10, Year 1, meeting 3)

Elena echoed Marco’s comments and suggested decentralized control can be credited for some of the learning that emerged from case 10. She felt the project had encouraged the PLC members to be more open minded about professional learning. Prior to the project she felt the mathematics department at Broadview was “anti-establishment” (Elena, Case 10, phase 2 interview) and they were skeptical of professional learning initiatives that told teachers what to do and how to change. Elena suggested that this project was different in that it involved teachers in the decision-making process and provided space for the PLCs to discuss and navigate their own problems of practice. Even though she felt Donald and Gabby were, at times, too dominant in their workshop style approach to the PLC meetings, she still felt there was opportunity for her and the PLC to take some ownership in their learning. Elena shared:

> I definitely think that our department for a number of reasons, not just the project but I’m sure the project contributed to it, has evolved from being more traditionalist and probably, I don't want to say anti-progressive but against the establishment like “Hey you're going to make us do that now? What?” Change things, you know? So we've changed from being in that position to being a little bit more progressive thinkers. Most of us are a little bit better at being open to trying new things. (Elena, Case 10, phase 2 interview)

Even though the PLC was led, at times, through a top-down leadership, the nested nature of the OAME project provided opportunities for the PLC at Broadview to interact with other PLCs and be exposed to alternative models of teaching and learning mathematics. Analysis of the Case report mentioned how the Case 10 PLC had been influenced by a different PLC in the project to integrate a focus on growth mindset into their classroom practice. The PLC also began Year 2 by meeting separately from Case 10, suggesting that perhaps they were starting to forge a different direction for themselves. The randomness to choose what models of learning they felt would best meet their needs and the needs of their students was apparent in the different strategies they adopted in their classrooms. Some strategies, such as engaging in hand-on rich tasks seemed to stem from the leadership of Donald and Gabby while other practices, such as spiralling, multiple forms of assessment, and adopting the BTC model came from others beyond the PLC but from within the nested structure of the project. The on-going, job-embedded nature of the project provided the time for the PLC to gain confidence and experience with new teaching practices and support to try new ideas. This, in turn, provided positive experiences for the PLC members that encouraged them to continue to pursue professional learning models that aligned with their own needs.

Addressing Research Question Two

The second research question asks: In what ways did the characteristics and conditions of complex learning systems influence the sustainability and diffusion of learning in different PLCs, and associated learning systems, of the OAME project? For RQ2 I expand the list of associated learning systems to include those systems, beyond the original project, that may have been influenced by the project or individuals involved with the project. I begin with a discussion of the findings at the OAME project level before describing findings from each case at different levels of systems including individuals, PLCs, schools, school districts, mathematics education system, and the educational system as a whole (Figure 5). Findings addressing RQ2 are drawn primarily from the analysis of the phase 1 survey and phase 2 and 3 interview transcripts. The exit interviews carried out by the RAs at the end of the project also provided insight into the sustainability and diffusion of learning from the project. I describe ways the learning that emerged from the OAME project was sustained and ways that the conditions and characteristics of complexity may have supported this sustainability. Following that I discuss ways that the learning from the project may have been diffused beyond the original project participants and the ways that the conditions and characteristics of complexity may have supported this diffusion.

Addressing Research Question Two: OAME Project

I begin with describing the ways the learning that emerged from the OAME project was sustained and diffused at the project level both during the project and since completion of the project. In terms of sustainability I would like to recall from Chapter 2 how I define sustainability. I align with both Tirosh et al. (2015) and Zehetmeier (2015) and define sustainability as the ability of a phenomenon to endure or continue, a flourishing and enduring phenomenon, one that is growing and progressing, where learning continually emerges. Additionally, I define diffusion as the sharing of new ideas and practices, and these ideas being negotiated and adopted in ways that are meaningful to others.

Sustainability. During the project the learning that emerged was sustained in different ways. From my own experiences and observations I know that the project extended from a one-year project to a two-year project, and then expanded to include knowledge mobilization through workshops, conference presentations, and web-based resources. These are examples of how the learning continued to emerge. Through the secondary analysis of phase 0 project data and analysis of phase 1 survey and phase 2 interviews it is evident that learning was sustained during the project. As described in previous sections several participants shared that their learning was influenced by the nested nature of the project, the opportunities to engage in neighbour interactions with other project participants, and the willingness of the participants to continue to engage in the project over an extended period of time. In this way the ongoing emergence of learning from the project described when addressing RQ1 are also examples of the way the collaborative learning was sustained during the project.

There is limited evidence to suggest that the learning that emerged continued to be sustained at the project level after completion of the project. This was due to the PLCs no longer meeting as a whole group to share and continuing to engage in the professional learning in the ways that they did when funding was available. There is clear evidence that the learning is being sustained at the PLC and individual level and this will be discussed in more detail when I address RQ2 in terms of the four cases involved in this study.

Diffusion. The learning that emerged from, and across the OAME project, diffused in two distinct ways. The first way the learning diffused was during the project, primarily through project-wide meetings and the interactions between different PLCs and individual project participants. This type of diffusion was discussed in detail when addressing RQ1 and ways learning emerged at the project level.

The second way the learning could have possibly been diffused was beyond the boundaries of the original project to others that interacted with the learning from the project in different ways. These different ways included when PLCs presented learning from the project at the OAME annual conferences during the project, the two province wide conferences organized to share the learning from the OAME project, the project participants creating workshops-in-a-bag after the completion of the project, and when the OAME engaged in knowledge mobilization from the project. This knowledge mobilization included initiatives such as creating the Math4theNines online resources, professional learning initiatives such as a *Five Practices* book study, and the OAME hiring some project participants to facilitate professional learning sessions after the project in schools across the province. It is not possible for me to describe specific findings related to the diffusion of learning at this level as I did not collect data during the events and activities I listed. Therefore I will only present findings at the Case level.

Addressing Research Question Two: Case 1

As described previously, the PLC in Case 1 struggled to form a cohesive collaborative group during Year 1 of the project. During Year 2 the PLC worked more collaboratively after inviting new members to join. Analysis of transcripts from the original project data exit interviews, phase 1 survey response, and phase 2 transcripts revealed that there was both sustainability and diffusion of learning occurring at different levels and to different degrees in Case 1. First I will discuss the sustainability that is evident at different levels including the individual, the PLC, school, school district, and the mathematics education community. After that I will discuss how the learning that emerged from the PLC in Case 1 has diffused beyond the original project.

Case 1: Sustainability. At the individual level Drew and Jill both responded to the invitation to complete the phase 1 survey for this study that provided some insight into whether the learning that emerged from the OAME project is being sustained at the individual level. Their participation in the survey also led them to being invited to participate in phase 2 of the study, and both accepted this invitation. At the time of data collection Drew was no longer a district mathematics lead teacher but had returned to working as a classroom teacher in the same school district as Weber High School. Drew responded to the phase 1 survey question that asked if the project was still influencing her practice with “I still use many of the resources created [through the project] in my own classroom now that I returned to my classroom” (Drew, Case 1, phase 1 survey). She also reported that the practices learned from the project are having a positive influence on student thinking as Drew commented during her phase 2 interview that students were more engaged and would gravitate towards using the whiteboards as a tool, suggesting they were a beneficial tool that increased student engagement. Drew wondered how to possibly measure the impact of these practices as it is difficult to measure. When asked specifically during her phase 2 interview if she felt the project influenced student learning Drew replied:

> Math is challenging and challenging to teach, so I would say that the students have loved, like just through their comments out loud or what you'd see in class, to challenge each other's thinking through those tasks. I think they found those fun. But like does it last for deeper learning? I'm not sure. I can't help but say "yes", because you're trying to encourage problem solving, critical thinking, reasoning, and proving, like these are all the front matter of our document. And again today that's what we all believe we wanted our kids to be better at. (Drew, Case 1, phase 2 interview)

Drew shared that the connections she made with others across the province through her involvement in the project helped to sustain her learning after the project was completed. She was able to continue to push her thinking with these individuals when they would reconnect at the OAME annual conferences. She would seek out these connections and attend their sessions if they were presenting, she explained:

> [The OAME project] was the best PD that we had because it felt genuine, it was in the moment of what we were trying to dig into, lots of learning came from that afterwards, so when you were away at the OAME [annual] conference you could dig into some of those sessions that you met some of the people earlier. We are able to like listen to their perspective, a different way, or how other people challenge their thinking. (Drew, Case 1, phase 2 interview)

Jill similarly shared in her survey response that she continued to incorporate many resources and practices that she interacted with during the project, sharing “I am still utilizing and learning about the Thinking Classroom, and any course that I have pedagogical control over I use spirals and rich tasks” (Jill, Case 1, phase 1 survey). Jill also shared that she continued to use the BTC model as it helps to build community in her classroom which, due to the COVID-19 pandemic and school shutdowns, has been extremely difficult to build. Jill suggested that the learning from the project influenced her students’ learning. She shared, “I think the kids built their community, so they feel more connected, and I think my kids currently feel more connected” (Jill, Case 1, phase 2 interview). Jill also suggested that the project had influenced her approach to her own professional learning describing how her confidence had increased, and she felt better prepared to defend her pedagogical choices with parents and administrators who might challenge her.

> Finally, I have the confidence to stick to my new convictions - utilizing 5 Practices, spiralling, Thinking Classroom etc. I feel informed/empowered about the pedagogical choices I make, so I can defend them to parents/admin[istration] or try and bring them on board. My professional learning has mostly continued along the pathway introduced to me by the Math 4 The Nines project (the ideas shared by Chris Suurtamm, Tom, you, and Alison Macauley have been very impactful). (Jill, Case 1, phase 1 survey)

This increased confidence and empowerment could also support Jill in sustaining the learning that emerged for her from the OAME project. When prompted to complete the following sentence on the phase 1 survey “Some of the highlights of the OAME Grade 9 Applied Project for me included…”, Jill responded by listing different ways the project made her feel:

- Learning together.

- Feeling supported for our learning by the principal and the project researchers. (Empowering!!)
- Collegial discussions - with my PLC and from around the province. Having discussions with my principal about how to best support student learning in mathematics. (I felt heard!!)
- Summer institute where I felt valued for the work I do and where my learning is supported. (Jill, Case 1, phase 1 survey)

Interacting with others beyond their own PLC influenced both Drew and Jill in terms of being exposed to new ideas and practices. Therefore the conditions and characteristics that supported this sustainability of learning included the nested nature of the OAME project that brought participants from different PLCs together. This supported neighbour interactions between a diverse population of educators (diversity) sharing a common goal of enhancing the teaching and learning of mathematics (redundancy). The opportunity and freedom to meet and discuss issues with others during the project represented the enabling constraints of randomness and coherence. Both Drew and Jill adapted their teaching to align with new ideas and practices. Ongoing experimentation in their classrooms and salient outcomes of increased student engagement supported their beliefs that the rich tasks, spiralling techniques, and BTC model are beneficial to their student learning.

Analysis of Drew and Jill's phase 2 interviews reveal sustainability of learning from the project at the PLC level, both within their own PLC and with other PLCs from the project. Jill and Drew shared they were still collaborating in different ways with some of their own school PLC members and other participants from the project. In their survey responses both Drew and Jill shared that the core group of the PLC that was consistent through both years of the OAME project, Drew, Jill, Francis, and Zoe, continued to collaborate when they could. Drew and Francis were at different schools while Jill and Zoe remained at Weber High School. Drew shared that they kept in touch primarily through emails and texts and Jill offered that this core group continue to share ideas and collaborate.

> Yes, [we still collaborate] but not all [PLC] members. Francis and Zoe and I applied for a TLLP grant to continue our learning about consolidation of thinking tasks. We continue to collaborate even when Francis has been at another school. Last year Zoe and I received a grant to incorporate Indigenous Knowledge in our math classes with help from a knowledge keeper. We continue to seek counsel from each other. The others in the group found the work to be too much as life threw other challenges at them. And the Student Success teacher and the Special Education teacher moved to difference roles. I do wish we had been able to maintain this connection. (Jill, Case 1, phase 1 survey)

The learning that emerged from Case 1 was also sustained through their involvement in the greater mathematics education community in the province. There was some indication at the end of the project that Georgia planned to support the sustainability of the learning that emerged in Case 1. She shared in her exit interview that the PLC was operating well, and she planned to apply for more funding in order to continue the work the PLC was doing into a third year. Analysis of her exit interview reveals that Georgia was focused on sustaining the collaboration among the PLC members at her school and also to continue the collaboration with other PLCs in the project, primarily the PLC in Case 7. Georgia made a connection with the Case 7 PLC principal, Dolores, at the summer institute at the end of Year 2 and they discussed the possibility of having the Case 1 members visit Fields High School in order to learn more about their approach to spiralling the curriculum.

> One of the things that I've asked my board to do, because this is a 2-year project and now the funding has dried up, I think my team would benefit from sustaining it for one more year… I've asked for us to be able to have some funding to support a trip to CITY NAME where I believe the principal that we met and had conversations around it, in that consolidation piece, in London. They're doing some, um, spiralling is what they're calling it, but they're doing the rounds, and um… I believe that they're more successful. (Georgia, Case 1, year 2 exit interview)

During her phase 2 interview Jill confirmed that Georgia secured funding to continue the collaboration and that a connection with Case 7 continued after year two of the project. A few members from the Case 1 PLC visited with William at Fields High School in his classroom and both Tasha, William, (Case 7 teachers) and Tom (Case 1 RA) visited Weber High School individually on separate occasions. Tom worked in the same school district as the Case 7 school and worked closely with many of the participants from Case 7. Jill described during her phase 2 interview that the funding Georgia secured was used by these specific teachers to self-organize professional learning days after the project ended. These sessions included engaging in the lesson study process, designing a lesson, and observing each other teach as well as debriefing afterwards. Jill also continued to collaborate with Tasha and the two presented some of their learning at different mathematics education events in the province. Jill continued to pursue funding opportunities and used these funds to continue collaborating with colleagues at Weber High School who she formed a professional relationship with through the OAME project. Lastly, in her survey response Jill mentioned that she continued to connect with and follow several of the participants from the OAME project on Twitter including Gabby and Donald from Case 10 and the others from Case 7.

Jill suggests that the rigour, scale, and duration of the project allowed the participants to make deep and meaningful connections. The OAME project was not a typical one-day professional learning event with little support or funding. The project, with extensive funding from the Ontario Ministry of Education, provided long term, well-supported professional learning. Jill shared how the quality of the project allowed participants to make not just connections with others across the province but to develop meaningful relationships that extended beyond professional learning. Because of these connections, the learning is sustained through the participants self-organizing and continuing to collaborate both online and face-to-face. She shared:

> I think we had a lot of common learning but also because that sharing was happening, I think that really did cement like these relationships like Tasha (Case 7) has been to my house right? And she picked me up from the airport for OAME when it was in CITY NAME. Like I mean we just, we got to meet Tom's (Case 1 RA) baby, remember? There was this really supportive [group]… there was a lot of connection and sharing that was happening and also all of the organization showed that what was going on, was important, like a lot of the times as a teacher your PD is very low funding right? You're…you don't count for much, you don't get a snack and they're like "bring your own coffee", right? Like there's no appreciation of the work that's going into actually just being there….So the fact that people share what they're doing, everyone knows how much time that takes, and people are gentle with the share right? There's all of that underlying community that comes from connecting but also sharing your own time, the time that was yours, right? We got to come together [summer institute] in a really nice

place and have good food to talk about [teaching and learning]. (Jill, Case 1, phase 2 interview)

The conditions and characteristics that supported this connection include diversity and redundancy, different schools exploring different practices that are all focused on a shared goal of enhancing the teaching and learning of mathematics. This redundancy supported neighbour interactions in a system with decentralized control. The project was designed intentionally to nest the PLCs within the project which provided opportunities for different PLCs to meet and discuss with one another any issues that they deemed relevant and important to their specific school context. These interactions also involved randomness and coherence. The different PLCs could discuss any topic of interest to them but generally discussions centred around issues that related to the teaching and learning of mathematics. Lastly, self-organization was evident in the ways that the two PLCs (PLC 1 and PLC 7) made arrangements to meet and continue to interrogate their practices.

There are several key conditions and characteristics of complex systems that supported the sustainability of learning in Case 4 including trans-level learning - neighbour interactions in a system with decentralized control, self-organization, and nestedness. Table 18 summarizes these key conditions and characteristics and provides examples of each.

Table 18

Conditions and Characteristics that Influenced Sustainability - Case 1

Condition/Characteristics	**Example of sustainability**
Trans-level learning – neighbour interactions in a system with decentralized control	*Jill* and *Drew's* learning was sustained through ongoing neighbour interactions with others from their own PLC as well as others from other PLCs. These interactions occurred in a system with decentralized control.

	Drew reported that neighbour interactions with her students revealed to her that students were more engaged and motivated in math class when she used new approaches she learned about through the OAME project. This outcome encouraged her to sustain these practices.
Self-organization	*Jill* and other project members self-organized their own collaborative learning opportunities, lesson study, conference presentations etc. Self-organization was evident when Georgia was able to secure additional funding to support continued collaborative professional learning at the *PLC* level.
Nestedness	The nestedness of the Case 1 PLC within the OAME project allowed the Case 1 participants to establish professional and personal relationships with other individuals and PLCs from the project.

Case 1: Diffusion. In terms of diffusion, Drew and Jill were hesitant to suggest that the learning from the project has directly influenced others in their schools or districts. Drew did say that she shared her learning with colleagues at her new school, that she encouraged student teachers in her class to use the BTC model of incorporating VNPS, VRG, and rich tasks, and that she routinely presented the work that emerged from the OAME project at district-wide professional learning initiatives. Jill mentioned in her survey response that she felt she had spread the learning to others in her school district primarily when planning courses together. In response to a question regarding how the project has influenced her teaching Jill mentioned that "when I am working with colleagues to develop a course plan I have influenced many to spiral their curriculum. I also have been able to add rich tasks when co-planning" (Jill, Case 1, phase 1 survey response). Jill shared that perhaps due to the COVID-19 pandemic there have been more closed doors than in the past and that teachers are not as keen to collaborate with one another. She worried that the collaborative nature of teaching has "fizzled" (Jill, Case 1, phase 2

interview) a bit and teachers are simply trying to get through the new realities of teaching through a global pandemic.

These examples of sharing the learning from the OAME project are supported by neighbour interactions in a network with decentralized control. Both Drew and Jill are free to share their learning with others in their school district. This represents a nestedness as different schools are nested in the district and they come together to share ideas and learn from one another. Drew and Jill both suggested others who they felt may have been influenced by the work they were doing as part of the OAME project and agreed to reach out to others to invite them to participate in phase 3 of this study. I did not receive any responses from others who they sent this invitation to, therefore there is no phase 3 data to analyze for this case.

Addressing Research Question Two: Case 4

Two members of the original Case 4 PLC agreed to participate in phase 2 of this study, Chrissy and Daniel. At the time of data collection for this doctoral study, Chrissy continued as the Head of the Mathematics department at Pike Place School and Daniel had retired from teaching in 2016. Chrissy responded to the request for participants to complete the phase 1 survey and then agreed to participate in the phase 2 interview. During this interview Chrissy suggested that I speak to Daniel to provide a different perspective from her own. She offered to reach out to Daniel, and he agreed to participate in a phase 2 interview as well; he did not complete the phase 1 survey. Through analysis of these participants' exit interviews from the original project data, phase 1 survey responses, and their phase 2 interview transcripts it is evident that some of the learning was sustained and diffused at different levels.

Case 4: Sustainability. Analysis of data suggests that there has been sustainability of the learning that emerged from Case 4. This sustainability is evident at the individual level, the PLC level, and at the school level. I will discuss each of these levels of sustainability in more detail, beginning with the individual level.

During his phase 2 interview Daniel shared that the learning from the project influenced his teaching after the project and in some ways he was able to sustain what he learned and in some ways he was not. Overall the project changed how he viewed his role in the classroom and how he approached teaching his students. Rather than seeing the curriculum as something to be delivered to students regardless of their own experiences and background he saw the students as the driving force of what he teaches. He realized that it was his job to provide meaningful opportunities for all students to learn. He described that after the project he had a better awareness of the whole student.

> **Daniel:** I was more aware of what was going on in my class and the struggles of some of my students and how maybe I can reach some of them, as opposed to just well here's what it is [the mathematical lesson and concepts]. I'll help you if you want, but that's [it]. You know? I think I was always a reasonably empathetic teacher and would go out of my way to do extra but at the same time there were still students that I wasn't getting [to]. And why?
>
> **Researcher:** Okay, interesting, so the work you were doing kind of revealed more of the diversity of the student thinking?
>
> **Daniel:** Exactly. Not only that, but the issues they had, you know, at large too. (Daniel, Case 4, phase 2 interview)

This description of his learning connects to a discussion shared in RQ1 when Jennifer, an elementary teacher in the PLC, mentioned the ways the different grade levels of teachers were influencing one another. She suggested that the elementary teachers were influencing the high school teacher to consider "the whole child" (Jennifer, Case 4, Year 2, exit interview). Daniel suggested in his phase 2 interview that this idea stuck with him and continued to influence his teaching.

Daniel also suggested that some of the learning from the project was difficult to sustain. He described during his phase 2 interview how the project provided ample opportunities for the PLC to support teachers in their own classrooms with co-designing and co-teaching lessons. After the project was finished the financial support for release time for teachers to meet, plan, and observe in each other's classrooms was gone. Daniel reflected that he found it difficult to maintain the type of teaching they engaged in during the project without other educators in his classroom. He explained in his phase 2 interview, during the project "we were working in groups, there was three of us or two of us or depending on the class I was dealing with, and we had a lot more support, so it was a lot more team oriented… there was way more support than I normally had in a 1P (Grade 9 Applied) class" (Daniel, Case 4, phase 2 interview). When I asked him to explain what he meant by more support he described how, because of the project there would be more than one teacher in his class working with students when he would be teaching some of the activities the PLC had created during their planning meetings. These other teachers were other project participants who had release time provided from the project. The PLC meetings provided time for Daniel and his colleagues to discuss ideas and lessons that would benefit the students they were working with; he shared he felt the PLC meetings provided "time to plan lessons that would be more appropriate for these kids" (Daniel, Case 4, phase 2 interview). He described how he was able to spend more time with individual students during a lesson because of the other adults in the room. Through that experience he learned more about the students and their needs. The hands-on projects the PLC had developed collaboratively that both he and the students enjoyed became too much for him to orchestrate on his own without the support of others. He shared "when I went back to just the regular classroom without the support it became difficult to maintain all of that, you still try to take parts of it, but you can't maintain the same level" (Daniel, Case 4, phase 2 interview).

When asked if the learning from the project was sustained Chrissy shared that the project influenced her teaching and leadership on an individual level and had also influenced how the mathematics department operated. A response on her phase 1 survey suggested that being part of the project increased Chrissy's confidence and pushed her to continue to collaborate more with others outside of her school and district. She shared that a highlight of being a part of the project included "identifying my place as a member of the provincial math community and valuing my contributions (not being intimidated that we're not ready to try)" (Chrissy, Case 4, phase 1 survey). In terms of her teaching, Chrissy shared that the project provided her with numerous connections with educators, researchers, and organizations around the province. In her phase 2 interview Chrissy mentioned that the OAME project provided her an opportunity to be part of the OAME annual conferences. She also shared she had maintained a close connection with the work the OAME continued to do. This connection helped her to sustain and continue her learning. When discussing the role the OAME played in her learning she shared:

> So I'm always getting the [OAME] *Gazette*. I'm always getting the emails that say there's a speaker session coming or there's a new project coming… so I know what's going on, which makes us feel like we're kind of on the leading edge of how things are happening. (Chrissy, Case 4, phase 2 interview)

When asked to clarify how she hears about different learning opportunities she mentions:

> Twitter, Facebook through the *Gazette* and through who's speaking at the OAME sessions that I've joined, the TEAMS (a chapter of the OAME) sessions that were coming up this last block. Like anything that's coming out online we're joining. (Chrissy, Case 4, phase 2 interview)

If there were opportunities to participate in collaborative learning initiatives, Chrissy generally signed up and encouraged her colleagues to do the same. In this way she continued to learn and participate in the mathematics education community in Ontario.

At the end of the project Chrissy shared that the project provided more opportunities for the PLC, and as an extension, the mathematics department, to meet and discuss issues of importance to their school context. She predicted this would be one of the lasting impacts of their involvement in the project.

> Well, I think communication between our department is the biggest success that's going to last the longest. I mean you get to know the people you're working with, right? You also get to a better appreciation for what they do, and you get to see what they do, and you know it's easy to dismiss somebody if you don't have that close connection. And the conversations we've had like so many like we meet far more than I've met with any other group in any other thing ever [with emphasis], right, which is good, right? (Chrissy, Case 4, Year 2, exit interview)

Analysis of Chrissy's phase 1 survey responses and phase 2 interview transcript suggests that she was correct in her prediction that the project increased communication within the mathematics department at Pike Place High School. In reporting the highlights of the OAME project in her survey response, Chrissy listed "building a collaborative team in my school" (Chrissy, Case 4, phase 1 survey). She shared that since the project finished, the mathematics department started what they called learning labs which were a continuation of the collaborative learning they began with the project. The teachers continued to co-design lessons and then observe each other teaching. When she was asked about these, Chrissy shared:

> We've always wanted to because we knew what the power of bringing people into one another's classrooms has been. So, because of the OAME project that's given us license to be able to do this and now we knew that it was beneficial. (Chrissy, Case 4, phase 2 interview)

Chrissy reported that the department has seen less staff turnover because of the project and the collaborative environment it nurtured. She suggested that the department became more of a community, leading to more student success which in turn has had a positive impact on teachers' engagement and their excitement about teaching at Pike Place School.

> Since the project happened we haven't had a revolving door of staff coming through the math department and I have trained and trained and trained people to be in the department. One of them was bringing in someone else who was as excited about it as me. Another part of it is that we are having success in our class the way others are not, and that makes a big difference. I see that success in the room [and] then at the end of the day, our failure rates have declined, you know, we had promotion meetings on Monday. I had three students to talk about out of seven…eight classes. (Chrissy, Case 4, phase 2 interview)

Chrissy suggested that the success of the project at Pike Place had a long-term impact on how the administration staffs the school. Since the project, she has been involved in the hiring process for the mathematics department, and she suggested that the administrative team realized the importance of building a strong community to support teaching and learning and that Chrissy plays an important role in helping to build that community. She shared:

> [The project] has changed so many different things because it changes the way we even hire in the math department, because if you are not willing to be collaborative you're not allowed to be in the department. So this siloed behavior that we used to have where nobody would even enter someone else's classroom, let alone actually ask about “I don't know how to do this, can you help me?” is completely gone… and our administration doesn't hire without asking us what we recommend as well, they come to me and say ‘which one of these [people] would you like in your math department. (Chrissy, Case 4, phase 2 interview)

There are several key conditions and characteristics of complex systems that supported the sustainability of learning in Case 4 including trans-level learning - neighbour interactions in a system with decentralized control, nestedness, and self-organization. Table 19 summarizes these key conditions and characteristics and provides examples of each.

Table 19

Conditions and Characteristics that Influenced Sustainability - Case 4

Condition/Characteristics	**Example of sustainability.**
Trans-level learning – neighbour interactions in a system with decentralized control	*Daniel's* interactions with others in the PLC influenced the level to which he was able to sustain his learning and adaptation of his practice. When he had others supporting him is his classroom he was better able to sustain the new approaches to teaching. Without the support of other teachers in his classroom Daniel found it difficult to incorporate all of the activities the PLC had co-created. He therefore adapted to a style of teaching that included the new practices that he could sustain while also including some of his previous practices.

	Chrissy shared how neighbour interactions with other project participants increased her confidence when she realized that others were in the same place in their learning as she was, and she felt that she was on the right track. This increased confidence led to adaptations in how she approached professional learning. She was more confident in seeking out and participating in professional learning opportunities. At the *school* level decentralized control played an important role in sustaining the learning. The administration did not dictate who the school would hire rather it was a collaborative decision between the mathematics department and the administration. Having conversations about who would best align with the staff and their ways of working allowed the mathematics department to maintain a more collaborative culture.
Nestedness	The nestedness of the OAME project was an important component in terms of sustainability in *Chrissy's* situation. The PLCs being nested within the project, the project being nested within the OAME community which in turn is nested within the greater mathematics education community provided opportunities for Chrissy to make, and sustain, connections with educators engaged in similar professional learning journeys as well as to stay engaged in discussions and collaborative learning. At the *school* level sustaining the learning is supported by the nestedness and this is evident in how the administration includes Chrissy in these important decisions. The mathematics department is nested within the school and Chrissy's involvement spans across these different systems; she is involved at the administration level as well as at the mathematics department level.
Self-organization	At the *school* level the administration was making a choice to organize their school in ways that best meet their needs and the needs of their students. They had observed positive outcomes from the collaborative learning model and therefore self-organized to hire new teachers that would adopt a collaborative approach to teaching.

Case 4: Diffusion. It is evident through analysing Chrissy's interview transcripts from phase 2 that Chrissy felt the project had a positive impact on the community at Pike Place. When

asked if there are others that Chrissy felt the OAME project may have influenced she suggested at least three individuals. Chrissy sent a letter of invitation to these individuals and one individual, Martina, agreed to participate in phase 3 of this study. Martina was a first year, occasional teacher in the Pike Place school district during the time of the OAME project. She connected with Chrissy through district initiatives, and they formed a mentor/mentee relationship. In Martina's interview, Martina described how Chrissy was instrumental in supporting her teaching during the first few years of her career. These first years of Martina's teaching aligned with Chrissy's involvement with the project and Chrissy shared different practices with her such as the BTC model and different resources she learned about and developed through her involvement with the project. Martina claimed that they connected and worked well together because they shared similar beliefs about teaching and learning mathematics. Therefore redundancy played an important role in the connection that was formed between these two teachers and this redundancy provided a common area of focus for neighbour interactions.

> I'm no longer in the same building with [Chrissy] but I still work closely with her and a few other teachers in the board. Like we will chat daily and so what I found is like Chrissy is a really good collaborator and her and I think the same way on a lot of things. (Martina, Case 4, phase 3 interview)

Martina took on the role of a mathematics coach for the school district, and she shared how she has used ideas and practices in her coaching that she learned from and with Chrissy. Specifically Martina used the learning lab model that Chrissy also used with the mathematics department at Pike Place and the BTC model that Chrissy shared with her. In this sense the learning from the OAME project had diffused and been adopted by Martina and through Martina into other classrooms in the school district. Nestedness helped to enable this diffusion from Chrissy to Martina and then outwards to others in the school district. Self-organization was also evident when other teachers from the Pike Place school district, like Martina, were able to interact with those teachers who participated in the OAME project allowing for the learning to diffuse. Redundancy also appeared to have played an important role as Martina and Chrissy beliefs and attitudes about teaching and learning mathematics were in alignment.

Addressing Research Question Two: Case 7

Analysis of the original OAME project data, phase 1 survey responses, and phase 2 and 3 interview transcripts suggest the learning that emerged from Case 7 during the OAME project had been sustained and diffused in different ways. First I will discuss ways that the learning from the PLC's involvement in the project was being sustained at various levels including the individual, the PLC, school, and school district level. Following that I will discuss ways that the learning from the project diffused beyond the original PLC to other individuals who form part of Case 7.

Case 7: Sustainability. I begin with a discussion of the sustainability of learning at the individual level in Case 7 before describing ways the learning was sustained at the PLC, school, and district levels. During her OAME project exit interview Dolores suggested that staying up to date on current research on effective pedagogy was essential to her learning and was one way she sustained the learning that emerged from the project. Dolores transitioned into a new role of superintendent at the school district the year after the OAME project was finished. During her exit interview I asked her if she might apply the learning from the OAME project in her new role. She explained part of the sustainability of the learning involved the "habit of, the constant learning, the reading. You can't do this job right without being on point with the research. So if you are not reading about it you're not knowing about it, then you can't really speak to it" (Dolores, Case 7, Year 2, exit interview).

For Tasha, during her phase 2 interview, she explained how she sustained some of the collaborative learning that emerged since completion of the project. Tasha's connection with Jill in Case 1 helped to sustain her learning from the project. They both focused on students' well-being in mixed ability classes and how best to meet the needs of all students. Outside of sustaining collaborative professional relationships with other project participants Tasha explained some of the challenges she experienced since completion of the project. Following the project Tasha transferred to a different high school for one year and then another high school the following year. After that, in 2021, Tasha began a role as instructional mathematics coach at the school district level. Analysis of Tasha's survey responses showed she felt the learning that emerged from the project, specifically her PLC's focus on de-streaming Grade 9 mathematics, continued to support her work. Analysis of Tasha's phase 2 interview transcript revealed that her year at a new high school in 2017 was not a positive experience and she found it difficult to sustain the types of teaching practices and ideas that had emerged during her time with the project. She shared it "was horrific, everything that we had done at Fields, everything that we've implemented, all the strategies and things like that, it seemed like the staff there (at her new school) had no idea about any of these" (Tasha, Case 7, phase 2 interview). Not only had the staff not heard of any of the ideas and practices but they appeared very resistant to collaborating with Tasha to develop strategies that could work at the new school. It was not a positive supportive environment, and she went so far as to say, "it was a very toxic environment" (Tasha, Case 7, phase 2 interview) and the leadership at the school was "antagonistic" (Tasha, Case 7, phase 2 interview). Tasha relied on the learning from the project to get her through the year and to help her engage her students in meaningful ways. She found two other staff members who shared her interests in collaborating, and they developed a working relationship. Tasha explained:

> It was that collaboration piece of the OAME project that I valued so much. Like the discussions, the ideas that generated from our discussions, the support, everything is what got me through the year at NEW SCHOOL. Like I found my niche of people that I could collaborate with… I got to collaborate and bounce ideas off of these other teachers. We had discussions, and I could play off ideas from them, and that kind of got me through the first semester of the year. (Tasha, Case 7, phase 2 interview)

This situation lacked some of the conditions and characteristics that support collaborative professional learning. For instance there appeared to be a lack of balance between diversity and redundancy at the school level. There were only a few teachers interested in working with Tasha collaboratively and she mentioned that the majority of staff made it difficult for this collaborative work to happen. Finding two others who shared her interests allowed Tasha to develop a small collaborative group who could sustain her well-being for the school year. However, the toxic environment Tasha mentioned led her to find a new position at another high school in the district the following year.

Wayne retired a year after the conclusion of the OAME project. Analysis of Wayne's original project exit interview, phase 1 survey responses, and phase 2 interview transcript reveal that Wayne sustained the learning that emerged from the OAME project. Even though he retired from teaching Wayne continued to participate in collaborative professional learning and began a graduate degree in mathematics education. During his phase 2 interview Wayne described how, after retirement, he intended to take time off from thinking about teaching and learning but he found his mind kept wandering back to the work that he and the PLC had engaged in. He decided to continue to explore the learning and enrolled in a graduate program to research teachers' reflective practice and how teachers can use student feedback to inform their teaching. He was inspired by the work the PLC had started during Year 2 of the project that used a student survey to gain a deeper understanding of how students' beliefs and attitudes about mathematics and learning mathematics evolved during the school year. The survey was intended to capture the influence the combined Grade 9 mathematics class had on students' beliefs and attitudes about their own ability to do math. At the time of the phase 2 interview Wayne was collecting data from classroom teachers who used a version of the beliefs and attitudes survey in their classes. He was interviewing teachers on how the survey data might influence their practice. During his interview Wayne explained his research and why it is important to him to use student voice to influence mathematics teaching. He credited the OAME project with influencing his ongoing learning, "so here I am how many years later? I'm still [learning] and that's the influence of that project" (Wayne, Case 7, phase 2 interview). During this interview he mentioned cautiously that, at times, he considers continuing his studies with a doctoral degree.

William continued to teach at Fields High School and remained active in the mathematics education community. Analysis of his phase 1 survey responses suggests he sustained learning that emerged from the project. He continued to use and extend effective teaching practices that were developed through the collaborative lesson study the PLC used at Fields High School. He stated that "I still use some of the pedagogy that we tested in the lesson studies. Snowballing (students sharing learning among groups), VNPS, VRG, spiralling to name a few" (William, Case 7, phase 1 survey). When asked to expand on this during his phase 2 interview William said the collaborative learning he took part in during the project was beneficial to his practice and he realized this type of learning needed to continue at his school. While the PLC as it existed during the project had evolved, William tried to continue using the model with the current mathematics department teachers. He found the lack of funding for release time and the challenges that COVID-19 placed on teachers impacted their ability to collaborate. William explained:

> I try really hard to include people around the building now in things, but of course there's no release time and everybody is spending all their time… prepping, marking tests, and doing all that sort of stuff. So to get them together now on their own time, is a very difficult thing to have happen…and I think a lot of them are drowning. So, you know I definitely realize that when good things are happening in a building that was because we had PLCs that were rocking it. (William, Case 7, phase 2 interview)

Analysis of project transcripts revealed that an important part of the learning from Case 7, and the project as whole, was connected to the BTC model and William played a leading role in this. The relationship William formed with Peter provided William opportunities to continue to develop the model alongside the researcher, with Peter, at times, visiting William's classroom. Through this collaboration William was able to sustain and develop the professional learning that occurred during the OAME project. This will be discussed more in upcoming sections related to both sustainability and diffusion.

At the time of data collection Lisa remained in her position as head of the mathematics department at Fields and was still actively involved in collaborative learning within the school. She also had started a Master's degree in education and credited the project with encouraging this step in her learning. During her phase 2 interview for this study Lisa spoke almost exclusively about ways the mathematics department and school sustained the learning from the OAME project, rather than for her, at the individual level. I therefore will discuss analysis of

Lisa’s data in terms of sustainability at those different levels. Two key areas of sustainability of learning from the OAME project were evident in the analysis of data from Case 7 at the PLC and school level. These two areas include collaborative professional learning and a focus on student well-being. I will next describe ways these areas of learning were sustained at these various levels in Case 7.

Collaboration among colleagues appeared to be sustained after the initial project and it was evident that Dolores made this a priority at Fields. Analysis of her project exit interview revealed how Dolores intentionally focused on sustaining the learning that had emerged from the PLC. She wanted to ensure that any changes to the administration would not disrupt their established professional learning model. Dolores described how empowering teachers as leaders could ensure that the professional learning model was deeply entrenched at Fields High School at different levels including PLCs, departments, and the school as a whole. She explained:

> Well that is my approach to leading…that you have to, if you want sustainability, you have to deepen the leadership and build the leadership cadres…It is not unique to the math initiatives in the school or the 4SI initiatives, it’s how I run the school. (Dolores, Case 7, Year 2, exit interview)

Analysis of Wayne and Lisa’s phase 2 interview transcripts provide evidence that Dolores was successful in establishing sustainability of the collaborative learning model. Wayne discussed how the project legitimized the work they were doing at Fields High School; it brought the learning out into the open and changed the culture of the mathematics department. The project allowed collaborative learning to become the way the PLC and, as an extension, the mathematics department worked. When discussing how they sustained the collaborative learning after the release time and funding were gone Wayne said the culture that had been established remained and, while they did not have large chunks of time together as a group, the closeness and trust they had developed allowed the mathematics teachers, including Wayne, William, and Lisa, to continue to push their thinking and to turn to one another for support. He shared:

> It builds your sense of the fact [of] how important it is, the work you do outside the classroom and not just on your own, but with others. Right? And certainly Lisa and I were fortunate to share [preparation time], and so we just used to talk about this stuff all the time…We just continued that journey so even without the release time, things would happen after school, before school, whatever, and I think it just felt like, to just realize, that those discussions, how important those discussions are. (Wayne, Case 7, phase 2 interview)

Analysis of Lisa's phase 2 interview also provided evidence that the collaborative learning environment was being sustained. She shared this was possible with continued support from the administration. The processes Dolores established at the school were still in place and the administrative team looked for specific qualities when hiring new staff. Lisa described how the administration did this:

> The administration understands that the Fields math department prioritises collaborative learning, collaborative problem solving, thinking, active learning. So some of these tenets that we've been developing. Admin[istration], because they were on board from the beginning, are now able to perpetuate that. They put questions about that into the questions that they pose to the LTOs (long term occasional teachers) and to potential contract teachers during interviews. The vice principal is like "we've got your back" she said "we, the questions we asked…we were really impressed with this teacher with their willingness to try new things, and you know, maybe they didn't know a lot about, maybe

> they haven't done spiralling before, but we made sure that we were picking somebody who was really open to learning and open to sharing and connecting with colleagues". (Lisa, Case 7, phase 2 interview)

Lisa explained how this involvement from the administration made her feel supported and ensured that the department they were building was open to collaboration. This also demonstrates how the mathematics department and school were sustaining the collaborative learning that emerged from the OAME project.

The learning from the project was not only sustained for the members of the PLC and the mathematics department but also was sustained at the school level. A focus on addressing Grade 9 students' needs continued to be a school-wide focus. According to Lisa's phase 2 interview, during the year after the OAME project the PLC continued to evolve; this time it returned to the original cross-curricular focus of Year 1. The PLC once again included teachers who were teaching other Grade 9 Applied subjects such as French, Geography, English, and Science. Lisa explained the goal of the PLC expanded and focused on better understanding students' experiences across different Applied courses. The teachers collaboratively discussed the strengths of individual students in different courses and adopted practices that could better support students. Lisa shared that through several "collaborative meetings, where we actually focused on key students, we followed key students with their timetable… throughout the course of the day…that really brought our collaboration to a new level in my mind, that was an amazing next step" (Lisa, Case 7, phase 2 interview). Through observing students in different courses using different teaching practices the teachers were able to build a clearer picture of the students' day, how to engage them more, and help students move away from being passive learners. Lisa explained, "we recognize[d] their strengths and… we were able to come up with some really specific strategies to support [students] and that we agreed that we would try in all of our classes" (Lisa, Case 7, phase 2 interview).

Analysis of the data suggests the learning that emerged from Case 7 was also sustained at the school district level. As mentioned previously, the year directly following the project Dolores moved into the role of superintendent at the school district level. She stayed in this role for two years before retiring from education in 2017. Analysis of her exit interview transcript and phase 2 interview transcript revealed ways the learning that emerged at Fields was being sustained at the district level through Dolores' work as a superintendent and in the years that followed her retirement. During her time as superintendent Dolores applied the learning from the OAME project to her new role. As discussed when addressing RQ1, Dolores, when the principal at Fields, adapted her teacher appraisal process to align with the lesson study model the PLC had developed. Dolores explained during her phase 2 interview that this same lesson study process was used at the district level when she met with groups of principals and they co-designed approaches to some administrative activities used at their individual schools. Dolores mentioned that, after she retired, she still connected with individuals from her superintendent role, and acted as a mentor to principals who seek her out for advice, "I feel very honoured, I have a lot of contact still, with particularly principals [who] come to me for mentoring and stuff. So I love that" (Dolores, Case 7, phase 2 interview). Dolores' work with principals is an example of how the learning from the OAME project was sustained at the school district level.

It is evident that the learning that emerged from Case 7 during the OAME project was sustained, and that conditions and characteristics of complexity played an important role in this sustainability. Although all conditions and characteristics of complex learning systems manifested in ways that influenced the sustainability of the learning, several key conditions and characteristics stood out as essential, including the importance of balancing diversity and redundancy (specialization), the importance of neighbour interactions in a system with decentralized control (trans-level learning), and the way that the nested nature of the learning systems supported specialization and trans-level learning. Table 20 summarizes the ways these key conditions and characteristics manifested in Case 7.

Table 20

Conditions and Characteristics that Influenced Sustainability - Case 7

Condition/Characteristics	**Example of sustainability.**
Specialization – balancing diversity and redundancy	*Tasha* lacked the balance between diversity and redundancy at the school level when she moved to a new school but was able to find a balance between diversity and redundancy with a small group of individuals at her new school which allowed her to persevere through a difficult situation. This was enough to support her well-being and she left this situation after only one year. *Lisa* shared how the PLC at Fields evolved to include a more diverse group of educators that were teaching different subjects yet were all teaching a similar cohort of students in Grade 9 Applied classes. This balance between diversity and redundancy provided opportunities for the PLC to continue to navigate teaching practices that would best meet the needs of their students across a range of classes. At the *school* level, the Fields' administration team recognized the importance of redundancy when staffing the mathematics department and would seek out individuals who were open to teaching and learning using new ideas and would collaborate with the rest of the department.
Trans-level learning – neighbour interactions in a system with decentralized control	*Dolores* sustained the learning that emerged from the PLC through continually interacting with research materials. She was able to interact with research that was important to her and was not mandated to focus on any particular area, thus she interacted with research in a system with decentralized control. *Wayne's* interaction with the research team and with student thinking was sustained as evident in his pursuit of a graduate

	degree. He continued to interact with the research team following the project and focused on the student survey the PLC had designed. The survey, and survey responses from students, prompted him to further explore how this tool might support teacher learning. Again, his neighbour interactions with the research team occurred in a system with decentralized control. There was no top-down authority imposing a focus on the individuals or the PLC. *William's* interactions with the BTC model and with Dr. Liljedahl occurred in a system with decentralized control. These interactions helped to sustain the learning that emerged from the project. His interactions with his students also supported the sustainability of learning as he observed the increase engagement his students. The collaboration within the *PLC* and the sharing of information about specific students represents neighbour interactions in a system with decentralized control.
Nestedness	*Wayne's* learning was sustained in part through the nested nature of the project and the way the PLC was nested within the greater mathematics education community within the province. This community includes the research team, in particular the PI, who supported Wayne's continued pursuit of learning in a graduate program. The *school's* focus on building a collaborative community represents the sustainability of learning that emerged from the project. This sustainability at the school level can be attributed to nestedness and the way the PLC was nested within the school.
Self-organization	*Dolores* made it a goal to create a collaborative environment among the staff and spent energy building the leadership capacity of the teachers. The *school* self-organized into a collaborative community, focused on hiring new staff that would align with this organization of the school, and the *PLC* self-organized into new forms each year to continually address the needs of students as they arose.

These examples provide insight into how the key conditions and characteristics supported the sustainability of learning that emerged from the OAME project and also provide examples of instances where the learning was not sustained due to the lack of these conditions and characteristics. I now will describe how the learning was diffused beyond the original members of the PLC in Case 7.

Case 7: Diffusion. Analysis of phase 2 and 3 interview transcripts reveal learning from the OAME project diffused at the individual, school, district, and mathematics community levels. Phase 2 data reveals ways that the learning diffused beyond the project and phase 3 data offers the perspective of those that the learning diffused to. Participants for phase 3 of data collection were recruited by the original members of the Fields PLC who were also participants in this study. Phase 3 participants include Mona, Jeremy, Bridgette, Laurey, and Peter. I next discuss different ways that the learning diffused from the project from both phase 2 and phase 3 participants' perspectives.

At the end of the OAME project Dolores shared that the diffusion of learning from Fields PLC was the biggest success they experienced through their involvement with the project. Dolores suggested that the learning from the PLC influenced other individuals and departments in the school as well as educators beyond their school. Dolores felt that the professional learning that emerged should be documented and shared with others. She explained:

> The success in the school is evident…The biggest success is the way it has permeated outside of math and across the curriculum and across the school but also what I see outside of our school, outside of our board and across Ontario and even elsewhere. It's crazy how a great idea has taken hold. Personally, I think somebody ought to find money to give these two men [Wayne and William] a semester off to get their work done formally and published. We will still continue to try and have that happen in my [district] but I think this should be validated and formalized. It's too rich, the work is too rich to

> simply be housed in one school and these guys having to rush away from their classrooms to share it. (Dolores, Case 7, Year 2, exit interview)

Dolores' claim is supported by analysis of the data as well. Dolores invited Mona and Jeremy to speak with me and both agreed to participate in this study. As described in Chapter 4, Mona was working with the Ontario Principals Council (OPC) as a coach and mentor. Her work included supporting high school principals in the school district that Fields High School was part of. Dolores and Mona would connect several times a year when Mona would meet with a group of principals working in the Fields' School District. During those meetings principals would share what their schools were working on, what challenges they were facing, and the group would work together to provide support to one another. It was during these meetings that Mona would learn more about the PLC model that Dolores was implementing at Fields and the way the PLC used lesson study as a model of professional learning. During her phase 3 interview, Mona claimed she was influenced by the work that Dolores was engaging in with the PLC. Mona then incorporated this learning into her work as a principal mentor. Specifically, Mona stated that she learned about lesson study and building collaborative working groups from Dolores, and she shared these ideas when working with other superintendents and principals in the province. She gained an understanding of how this model provided participants a focus and goal for collaborating and that the professional learning resulted in a product rather than simply bringing people together to talk. Mona described the process of lesson study as providing an "opportunity to share, discuss, and collaborate, even though you talk about collaboration you don't really collaborate until you find the thing that can bring everyone together. They all take ownership, rather than just having a conversation they had a product" (Mona, Case 7, phase 3 interview). Mona described how other school districts she was working with were hearing about the work that Fields was doing. These districts asked Mona to help them connect with Fields High School so that they could learn from them. She told me how she had requests from superintendents and principals from schools in other parts of the province asking her about Fields and their work with the BTC model and spiralling through the curriculum. Mona explained she was able to make arrangements for Fields to visit these other schools to share the work they were doing. In this way the learning from the project diffused across the province.

Jeremy, the second participant recruited by Dolores for phase 3 of this study, shared a similar sentiment about working with Dolores. When I interviewed Jeremy he was in his last year as a principal in the same school district that Dolores had worked in. Both Jeremy and Dolores were principals at schools that participated in the 4SI project discussed previously and through this work they had collaborated professionally with a focus on supporting the teachers who had students in Applied level courses at their school. Jeremy referred to Dolores multiple times throughout his phase 3 interview as "a great mentor" and it was evident that they worked together closely in their roles as principals. He explained how Dolores was a strong leader in the education community and that many benefitted from working with her:

> She's a great mentor, right? If you were anybody who wants to learn, if you don't attach your wagon to her man like then you're not wanting to learn and so that's part of the instructional leadership, I think, is to show your vulnerabilities. Also we really worked hard at helping our staff come together as staff. (Jeremy, Case 7, phase 3 interview)

Jeremy does not directly link his experiences with Dolores to the learning that emerged from the OAME project. Rather he comments on how working with Dolores resulted in an emergence of learning for a principal PLC that they were both a part of. It stands to reason that as Dolores engaged in multiple projects she was influenced and in turn influenced others through sharing her knowledge from these experiences.

Through William, three other participants were recruited, Bridgette, Laurey, and Peter. The first participant I will discuss is Bridgette. As discussed in Chapter 4 Bridgette was a teacher candidate (TC) in the same city as Fields High School. She was also a student in a course I taught on teaching senior secondary mathematics. Part of this course included Bridgette and her classmates observing William teaching in his classroom at Fields. Following this observation, William visited our class on campus to debrief what they observed. Also, Lisa, on another occasion, visited Bridgette's teacher education class to share her expertise in assessment. These visits occurred because of my connections with William and Lisa and the professional and personal relationships that were established during the OAME project. Analysis of Bridgette's phase 3 interview transcript reveals that these experiences influenced Bridgette's learning, suggesting that the learning from the OAME project diffused beyond the project. Bridgette shared that the interactions with both William and Lisa influenced her course work in a graduate program following completion of her Bachelor of Education degree as well as her teaching upon completion of her Master's degree. Bridgette explained that during course work for her Master's degree she referenced the work of both William and Lisa in several of her courses. She used William's presentation he shared with the TCs and the notes she took during Lisa's presentation in several of her graduate course assignments.

> I referenced William's PowerPoint I think like three times throughout my master's I went back through my emails to look at the PowerPoints that you had shared with us and I looked at the notes that I took on Lisa's visit about the assessments and how she [uses] levels and, like the thinking shift from percentages to levels and from like punitive

> feedback or punitive marking into feedback and I have used those throughout the last year in my master's assignments, so when you said [your study] is all about this learning spreading, 100% [it has]. (Bridgette, Case 7, phase 3 interview)

The interactions with William and Lisa also influenced her teaching. The work she engaged in during her senior mathematics course was influential because it connected her with practicing teachers who were engaged in practices she was not familiar with. She found the connection to real classrooms provided her opportunities to gain a deeper understanding of how teaching high school mathematics had evolved in some schools and this influenced how she worked with students that she tutored. She shared that she incorporated ideas that she learned from both William and Lisa into her teaching. She used Lisa's feedback approach, explaining "[I use] Lisa's idea of written feedback instead of marks all the time with [students] … it really has influenced me, and I know I'm going to be trying to do things their way when I have my own classroom" (Bridgette, Case 7, phase 3 interview).

Diffusion beyond Bridgette is also evident in analysis of her phase 3 transcript. Bridgette shared two examples of how she shared her learning with others. The first example was during one of her graduate courses. The professor was interested to learn more about William's work that Bridgette referenced and asked if Bridgette would share William's PowerPoint presentation with her as well. Bridgette explained:

> [For] one of my final papers for [INSTRUCTOR NAME] class actually I referenced William's presentation and she said "Oh!" and I like took the screenshot from one of his slides where he was talking about, like the flow where if something is too easy [students] get bored… and I included it in my paper and she said, "I've never seen this before I'm going to start using this now" (Bridgette, Case 7, phase 3 interview).

A second example of diffusion beyond Bridgette involved a colleague of hers who Bridgette collaborated with and shared some of the learning from interactions with William and Lisa. Bridgette told me how she shared many of the BTC, spiralling, and assessment practices with a peer who is also a high school mathematics teacher. Together the two teachers developed a spiralled course outline for the new Grade 9 de-streamed mathematics curriculum. The colleague then shared this course outline with the staff at her school which is in a different school district than Fields High School.

> I also have a friend, talking about the spread, who's a math teacher in CITY NAME and this year they gave her a grade nine de-streamed [class], two sections of it. She is a new teacher as well, but this summer I sat with her and we planned the course together using tons of William's ideas about, like questions on the board…not taking notes…using questions with the low floor high ceiling, so that you can make it harder…we planned her whole course around these…ideas. Then we used some of Lisa's …teaching as well [on assessment]. So it totally spread to CITY NAME. And then it grew within her math department, the [other] teachers have it now…and are using this stuff that she prepared. (Bridgette, Case 7, phase 3 interview)

Bridgette is an example of the learning from Case 7 diffusing through multiple pathways. Because of the project's influence on myself and my own learning I incorporated many of the new ideas and practices into my teaching and through my connections with William and Lisa I was able to sustain our collaborative learning after the completion of the project. By inviting William and Lisa into our university classroom, as well as visiting Fields High School, the learning diffused to TCs. Bridgette's experience continued to sustain and diffuse the learning that emerged from the project as she shared the learning with her colleague and to her colleague's math department.

Laurey was a second participant recruited through connections with William and is the teacher I describe in the opening vignette of this dissertation. There is evidence from analysis of Laurey's phase 3 interview that the work William engaged in during the OAME project diffused to his online community, including Laurey and the #MTBoS. Laurey explained that she was limited in the number of different perspectives on teaching and learning at her school and she relied on Twitter and the #MTBoS to provide the diversity needed to push her thinking. She claimed with collaborative online learning "everything is dynamic, and everything is about growth because you have this idea and you talk to someone else about it, and from that this new idea emerges or your original idea is improved and strengthened and validated" (Laurey, Case 7, phase 3 interview).

Laurey became interested in the BTC model after William presented the idea at Twitter Math Camp (TMC) in 2014. TMC was a face-to-face collaborative events organized by teachers who met through the #MTBoS. This was the same time that the OAME project started, and William began sharing lots of his activities and interactions with the BTC model on Twitter. Laurey was interested in the BTC model and was inspired by the work that William was posting on Twitter. "[BTCs] became this huge thing…reading BTC and everyone was asking for videos and of course there's a time lapse video of William's classroom where [he] takes a 50-minute class and puts it into seven minutes" (Laurey, Case 7, phase 3 interview). This sharing of learning helped others to develop a better understanding of what the BTC looked like and how they might be able to incorporate it as well. Seeing William engage in the BTC inspired Laurey to try it and to read more of the research behind the model. This led her to interact with Peter's research as well and engage in her own action research in her classroom to see if her students responded similarly to what the research showed. She found that yes, her students were quicker to engage in mathematical tasks if they had VNPS and if they were standing at these surfaces with groups of their peers. When asked to explain how William, and others online influenced her thinking Laurey concluded by sharing “being able to reach into this depth of knowledge from people with all sorts of backgrounds and experiences and variety of environments that just brings a richness to it” (Laurey, Case 7, phase 3 interview).

The final participant recruited through William is Dr. Peter Liljedahl. As mentioned throughout the findings chapter Peter’s BTC model was a consistent part of the learning that emerged from the OAME project. William and Wayne’s interactions with Peter at a mathematics conference in April of 2014 influenced their own teaching of mathematics and this influenced others in their PLC to adapt their practices. The PLC shared these practices with others in the OAME project. Analysis of the phase 3 interview with Peter suggests that interactions with William and William’s learning that emerged from the OAME project, diffused to Peter, and influenced his research in some ways.

During his interview Peter described how he became acquainted with William. William began to share the work he was doing in his classroom that involved the BTC model on Twitter using the #MTBoS and also tagging Peter in his tweets. Peter described how William became a leader in the BTC movement and was vocal about it on Twitter. Peter describes how others would reach out to him after reading and learning about the BTC through William on Twitter and at TMC.

> I started hearing through people who are on Twitter that William had gone to Twitter Math Camp, he had talked about thinking classrooms at Twitter Math Camp. So this is

> how William, in my mind, how he started to emerge from the crowd, so to speak, as someone who had embraced it [BTC model]. (Peter, Case 7, phase 3 interview)

Peter explained how William, and others, sharing their BTC experiences online, helped to mobilize his research and soon others from across North America were reaching out to him and inviting him to speak in their school districts.

Peter and William developed a close working relationship that involved not only collaborating but also co-teaching in William's class. Peter described that being in William's class provided him with experiences of the BTC model that he might not have had without the connection with William. This collaboration provided Peter the opportunity to engage with the BTC model with students and teachers and to develop a deeper understanding of ways the BTC model could be implemented. The experience also provided him with authentic experiences with the BTC model he could share with others in the education community. When asked if he felt working with teachers, and William in particular, influenced his research Peter responded:

> What are some of the ways that work with William has influenced me? I guess, we can answer that in different ways, one is "Did I learn anything about thinking classrooms from William? Are there any micro tools that exist that William does that I've learned about?" So one is the way thinking classrooms can be implemented in the context of interleaving (spiralling). So William was really the first person I met, who was hardcore into interleaving and seeing how thinking classrooms operates within that. The other thing is William was the first teacher, I saw who would have a little whiteboard to himself, that he would put up tasks and that students would take from and so on, and so forth, so I learned about that there. (Peter, Case 7, phase 3 interview)

Peter credits William with being an important influence in terms of the diffusion of the BTC model in the mathematics education community.

> I think one of the things that's most impactful is William is an amazing advocate for thinking classrooms and his name has power, so what he lends to thinking classrooms is that authenticity of someone who has a voice and then he aims that voice at thinking classrooms which gives a greater credibility for a wider audience. And that audience comes to me because they want to learn, they go to William because they want to learn. (Peter, Case 7, phase 3 interview)

Not only is William's voice as an advocate for the BTC model an important component of the diffusion of the model but it is the stories that Dr. Liljedahl has through working with William that help to support his work with teachers. It is the collaborative work that the two have done together that provide many real examples that Peter can share with others in the mathematics education community.

> Then there's the stories, you know. I think one of the most powerful things that we can give teachers when we're working with them in professional settings is stories. And I have stories from being in William's classroom that I can give to teachers, that I tell. Stories about things we do, and I tell stories about things that William does and so on and so forth and, and this lends credence to the work. (Peter, Case 7, phase 3 interview)

The interview concluded with a discussion about the fact that the BTC model is still emerging, and that William and Peter continue to collaborate and discuss ways the model can be adapted to include new ideas such as thin slicing which involves breaking bigger tasks into smaller tasks that still maintain the richness of the larger task. In terms of sustainability and diffusion of learning from the project it is evident that through the collaborative relationship between

William and Peter the learning with the BTC model was sustained and diffused across the OAME project participants and beyond to others, including phase 3 participants Bridgette and Laurey, through tools such as Twitter, face-to-face professional learning opportunities such as the OAME conferences, and Peter's book *Building Thinking Classrooms in Mathematics, Grades K-12: 14 Practices for Enhancing Learning* (Liljedahl, 2015).

As with my description of the sustainability of the learning that emerged from Case 7, the diffusion of learning from this case was supported by all the conditions and characteristics of complex learning systems in some way. The key conditions and characteristics that influenced the diffusion of learning were specialization - balancing redundancy and diversity, trans-level learning - neighbour interactions in a system with decentralized control, nestedness, and self-organization. These key elements are summarized in Table 21.

Table 21

Conditions and Characteristics that Influenced Diffusion - Case 7

Condition/Characteristic	Example of Diffusion
Specialization – balancing diversity and redundancy	The common goal of examining practices that would enhance the teaching and learning of mathematics was a condition that manifested in all the instances of diffusion in Case 7 and represents redundancy among all of the participants. These instances also included diversity in how these practices were examined. With *Dolores, Mona,* and *Jeremy*, school administrators and those that supported them were coming together to collaborate. *William, Bridgette, Laurey,* and *Peter* also balanced the redundancy to the focus of improving teaching and learning of mathematics with the diversity of the different roles that each participant had and the level of experience as educators.
Trans-level learning – neighbour interactions in a system with decentralized control	Each instantiation of diffusion in Case 7 involved neighbour interactions in a system with decentralized control. *Dolores* interacted with *Mona* and *Jeremy* in a professional learning and leadership capacity. These interactions occurred in a system that did organize support for educators but did not

	dictate what it was that they worked on. *William* interacted with *Bridgette* through her university course work, with *Laurey* online through the #MTBoS, and with *Peter* through both Twitter and face-to-face professional learning initiatives. All of these systems can be described as decentralized.
Nestedness	Nestedness, combined with neighbour interactions in a system with decentralized control, was a key characteristic that supported the diffusion of learning. Different individuals, and learning systems are nested in the greater education community and this nestedness provides opportunities for people and their ideas to bump up against one another. These different levels of nestedness for Case 7 are presented in Figure 14.
Self-organization	*Mona, Bridgette, Laurey,* and *Peter* described ways the individuals and systems self-organized when presented with new ideas and practices. For example both Mona and Peter describe how others reached out to them when they heard about work that each was doing connected with Fields. Mona in terms of the collaborative learning the PLC was engaged in and Peter in terms of the BTC work that William was doing. Bridgette and Laurey both described how they were involved in a self-organization process as well. Bridgette through working with colleagues to design a course that reflected the learning that emerged from the OAME project and Laurey when she was involved in the #MTBoS and engaging with the work that William, and others from the project were sharing online.

Addressing Research Question Two: Case 10

Three members of the PLC in Case 10 agreed to participate in phase 2 of this study, Elena, Maya, and Charlotte. Through Maya two other participants were recruited for phase 3, Janet and Karen. At the time of data collection Charlotte and Elena were still in the same roles that they were in during the project. Charlotte as the head of the special education department at Broadview School and Elena as a mathematics teacher. Maya was in her new role as a district mathematics and science consultant after having taken over Gabby’s role as district mathematics lead teacher for a few years after the project. Through analysis of exit interview transcripts from the original project data, phase 1 survey responses, and phase 2 and 3 interview transcripts it is evident that there has been sustainability of learning at different levels including the individual and the school as well as diffusion of learning at different levels including individual, school, and district.

Sustainability. In terms of the sustainability of individual learning, in her phase 1 survey Elena listed several ways that the project was still influencing her practice including how she organizes her classroom and how she assesses students:

> I do use the learning strategies [learned through the project] in all my teaching, my classroom is set up to allow for group work on non-permanent vertical surfaces. While I have not eliminated traditional furniture in my classroom as some have, I try to incorporate group work at least once for each unit I cover in the math course, depending on the content. Also, I changed my assessment strategies to de-emphasize marks (although I was already on this path prior to participation, my learning during this project empowered me to stay on track with this). I became more flexible with how students showed me they had met learning goals, creating assessments that were less test-like for some of the units I taught and allowing students to re-take tests after they showed evidence that they had spent more time practicing skills. (Elena, Case 10, phase 1 survey)

Elena also shared that she continued to be influenced by others in the project and educators who were referred to during the project through Twitter and at the OAME annual conferences.

During the project Maya was in the role of mathematics teacher and credits the OAME project with giving her the confidence to pursue mentoring positions within her school district. In her phase 1 survey, when asked if the project influenced her approach to her own professional learning she responded “absolutely, it ignited my love of learning and changed my teaching practice and ultimately the project was a kickstart to my career advancement” (Maya, Case 10, phase 1 survey). When asked to expand on this survey response during her phase 2 interview Maya reiterated “I learned so much from this project, because this probably was the first time that I did something like this and then it just kick started my thirst for learning new things” (Maya, Case 10, phase 2 interview). Through her survey responses and follow up questions during her phase 2 interview Maya discussed how, when the project finished, she felt the need to continue to work collaboratively with her department and she sought out opportunities that would provide the time and space for her and her colleagues to continue collaborating. She received close to $50,000 from the Ministry of Education in the form of TLLP[3] grants and Maya used these funds to sustain the collaborative learning that the Broadview mathematics department had established through the project. Three years after the project she took over the role of district mathematics lead teacher that Gabby had previously held and the next year she became a mathematics and science consultant for her school district. Through the project she gained confidence in her own teaching, and this pushed her to continue to collaborate, attend conferences, and learn from others in the province.

> I never would have thought, five years ago, that this would be where I was. I guess I just never thought that I could do it, right? And then, once I really started like, you know going away and going to [the] OAME [annual conference] and seeing all these amazing things. Well, I mean I am a huge promoter of the thinking classroom and that's something that I learned from that project, right? Getting kids up on vertical surfaces, I promote that…. When I came back [from the summer institute after Year 2], I just wanted more so

[3] The TLLP grant program ended in 2017 when the Conservative Party was elected in Ontario.

> I applied for lots of grants…I got release time for four of my teachers and we did planning on creating rich tasks. (Maya, Case 10, phase 2 interview)

Through analysis of both Maya's phase 1 survey responses and phase 2 interview transcripts it is evident that Maya is sustaining the learning that emerged from the OAME project and the continued collaboration she engaged in with her department. For example, she shared that during the project she was just beginning to learn about the BTC model of teaching mathematics and the different components, some of which she wasn't sure how to implement in her classroom. Over time, through experimentation, she started to gain a deeper understanding of why different components were part of the model, such as the process of generating meaningful notes with students rather than lecturing students for long periods of time. She has also started to experiment with the BTC model in other mathematics classes including her grade 11 and 12 courses that she teaches. She shared during her phase 2 interview that she feels the BTC model needs to be adapted in different ways for different classes and grade levels. For example Maya found that the meaningful notes are more important in higher grade levels and the rich tasks are important in Grade 9 specifically in order to introduce students to working collaboratively and learning how to learn to "do math" (Maya, Case 10, phase 2 interview).

Another tool that Maya used to sustain her learning and to continue to engage in collaborative learning is the use of Twitter. Like the majority of teachers who completed the survey Maya shared that she is able to stay connected with other project participants through the #MTBoS on Twitter and relies on Twitter for her professional learning, especially during the pandemic when all face-to-face initiatives had to be canceled. When asked to expand on this more during her phase 2 interview Maya explained that:

> Just making the connections right? Like one thing, they [the teachers in the project] all started following them [other teachers] on Twitter and like Twitter is amazing… and even now like, with this new de-streamed [Grade 9 mathematics] course because we have no resources and not even the curriculum in full, math teachers are amazing, like the amount of stuff that people put out there, I'm thankful. (Maya, Case 10, phase 2 interview)

Maya found that she could learn through other educators online who were taking some of the ideas that emerged during the time of the OAME project and working on teaching plans and lesson ideas that align with the learning that emerged. For example some teachers and schools were publishing course plans that spiral through the curriculum using a set of rich tasks. Maya used these plans within her own school district and in this way continued to sustain and adapt the learning that was emerging.

Although Charlotte did not discuss any specific individual learning that she herself was sustaining she did share in both her phase 1 survey and phase 2 interview that it is evident that learning from the project is being sustained at the mathematics department level. Charlotte shared in her phase 1 survey response that while she is no longer collaborating with the PLC at Broadview she does think that the learning from the PLC is being sustained. She interacts with students in the special education room when they visit her and her team for extra support with their schoolwork. She sees evidence of the open tasks, use of manipulatives, and the continued role that the whiteboards play in students' learning. Even though she no longer collaborated with the rest of the PLC due to time restraints, she felt that the work they did as a PLC was being sustained as the culture of the mathematics department overall had evolved. When asked about this during her phase 2 interview Charlotte explained:

> It kind of changed the whole way that we approach teaching math right? So now, when students are coming to the resource room to get support from me I'm seeing them come with algebra tiles, some of them bring whiteboards with them to try their project or their questions before they write them on the paper. (Charlotte, Case 10, phase 2 interview)

All of the conditions and characteristics of complex learning systems supported the sustainability of learning in Case 10. This is evident particularly in the use of Twitter as a tool to stay connected with others from the project. Twitter represents a balance between diversity and redundancy as it involved a diverse population of mathematics educators congregating around a shared goal of exploring issues related to the teaching and learning of mathematics. Through neighbour interactions in the decentralized network of social media individuals randomly choose what conversations to follow, pay attention to, or contribute to. The hashtag #MTBoS provided a set of rules for the community to follow, the focus of the hashtag is mathematics education, yet topics of discussion are as varied as the participants beliefs, attitudes, and specific contexts. The online group self-organized, adapted, and learning emerged as the conditions and characteristics interacted. Nestedness also played an important role in sustaining the learning that emerged from Case 10. Nestedness manifested in the ways that Elena and Maya connected with others from the project on Twitter representing the ways that the project was nested within the greater mathematics education community. Nestedness also manifested in the example that Charlotte shared. The work of the PLC at Broadview was being sustained in the mathematics department and evidence of that sustainability was being seen in the special education department, this represents the influence different nested systems, in this case departments in a school, have on each other. Lastly, self-organization also supported the sustainability of the learning that emerged. Through seeking out funding and continuing to organize opportunities for collaborative professional learning the PLC and mathematics department at Broadview was able to sustain their collaborative learning processes.

The key conditions and characteristics that influenced the sustainability of learning were trans-level learning - neighbour interactions in a system with decentralized control, nestedness, and self-organization. These key elements are summarized in Table 22.

Table 22

Conditions and Characteristics that Influenced Sustainability - Case 10

Condition/Characteristic	Example
Trans-level learning – neighbour interactions in a system with decentralized control	*Elena* and *Maya* sharing how they used Twitter to support their learning is an example of trans-level learning. The two teachers engaged in neighbour interactions in a system with decentralized control.
Nestedness	The use of Twitter to engage in professional learning is an example of the nestedness of participants in the greater mathematics education community. This nestedness supported the sustainability of the collaborative learning that was established during the OAME project. *Charlotte's* participation in the OAME project provided opportunities for different departments (special education and mathematics) in the school to be nested within the PLC. This nestedness supported the learning of students across these different departments. Charlotte was better positioned to support students who came to the special education department for math help because of the learning that she sustained from the project.
Self-organization	*Maya* and her colleagues engaged in self-organization when she applied for funding to sustain the collaborative professional learning the OAME project had supported at Case 10.

Diffusion. Diffusion of learning in Case 10 was present at several levels including the individual, the school, and the district. I next discuss diffusion at these three levels in more detail. Two individuals, Karen and Janet, were recruited through Maya for participation in phase 3 from Case 10, Both of these mathematics teachers joined the full-time staff at Broadview High School shortly after the end of the OAME project. Maya shared during her phase 2 interview that she encouraged both of these teachers to collaborate with her and the PLC members when they joined the mathematics department. Through their collaboration with the mathematics department, the learning from the project diffused. Both Karen and Janet enthusiastically credit the OAME project, and the members of the original PLC at Case 10, with influencing their learning. I will first describe Karen's experiences before describing Janet's.

During her phase 3 interview Karen explained that she had not taught mathematics before she was hired as a full-time staff member at Broadview. Gabby, who was the district mathematics lead teacher and worked with the Case 10 PLC during the OAME project, worked closely with Karen to mentor her in teaching. Analysis of Karen's phase 3 transcript revealed that Gabby used a similar approach to mentoring Karen that she used during the OAME project. Gabby would teach in Karen's class while Karen observed and learned from her. Karen described how she had never seen lessons taught in the way Gabby was teaching her students and it was eye opening to learn from Gabby in this way.

> I obviously wasn't at that project, but I learned tons from teachers from that project. She [Gabby] basically worked with me daily to teach my nine academic [class] and apply those concepts that they learned for the nine applied [classrooms] to the nine academics… I was blessed to work with her, and she like totally opened my eyes. [I] would have never thought to teach math like that. It was interesting because I came from, I mean when I was in high school, it was take up examples, here's a note, do your homework. (Karen, Case 10, phase 3 interview)

When asked if Maya also worked with her in this way she shared that at the time she started Maya was a teacher at Broadview and she guided Karen in her teaching as well. The three of them worked well together and started to collaborate on designing courses together using the BTC model, “me, Maya, and Gabby actually got together and designed the Grade 10 course following those kind of guidelines” (Karen, Case 10, phase 2 interview). Karen shared that Maya pushed her to think differently about teaching mathematics and she appreciated the time that Maya spent with her. During her phase 3 interview I sensed that Karen did not ask for this type of support from Maya and Gabby yet did appreciate the support she received:

Researcher: So, did you seek out that support or was it kind of?

Karen: [laughing] no they gifted it to me as I walked in the door.

Researcher: Okay

Karen: I took whatever I could at that point. Yeah I mean I had two other classes, and I was just trying to survive. (Karen, Case 10, phase 3 interview)

When asked specifically what she learned from her school being involved in the OAME project Karen mistakenly thought that Peter was a guest speaker and shared his BTC model approach as part of the project. It is obvious the BTC model was an important part of the learning that emerged from the project and that this learning had diffused to others at Broadview, Karen in particular. She discussed how she was using components of the BTC model in her classroom, and she had changed how she taught to align with the model. Her attitude about her role as a teacher evolved through working with Gabby and Maya. Karen described how she now saw her role in the classroom as more of a facilitator and as a result the students were more engaged. She explained that she was “not even teaching the kids [I’m] more being a facilitator, let them do the thinking on their own and learning on their own. It's nice to have them active and they don't need these big, long notes” (Karen, Case 10, phase 3 interview). She also shared that she had learned more about different ways to assess students, the benefits of spiralling through mathematics courses, the importance of using tasks with a low floor and a high ceiling which provided multiple entry points for students to engage, and the role that Twitter and the #MTBoS played in mathematics teachers’ professional learning. “The first thing I’ll tell a new teacher is join Twitter and put in this hashtag [#MTBoS]… there’s tons of Ontario teachers that are very powerful on Twitter… Maybe they got sent to the OAME and they got the spark” (Karen, phase 3 interview). Lastly Karen shared that she has learned that collaborative learning with others in her building is the best way to enhance her own learning. She and the other phase 3 participant, Janet developed a close working relationship with both Gabby and Maya, and they continued to collaborate and develop course ideas together.

Janet became a full-time mathematics teacher in the same school district at the same time as Karen but at a different High School. Like Karen, Janet did not have much experience teaching high school mathematics, as she had majored in biochemistry in university. Janet first collaborated with Maya when Maya was a district mathematics lead teacher. During her phase 3 interview Janet described how Maya asked her if she would like support and Janet gladly accepted. Working with Maya exposed Janet to different ways of teaching mathematics, particularly in Grade 9. Janet described how she came to work with Maya:

> She asked if I would be willing to welcome her into my room to help me and I needed it because I’d never taught the grade nine math curriculum before, and I was scared about EQAO… she was just really helpful. I learned a lot about even just the way to be up, I mean I’ve taught other math courses and been up at the front of the room all the time, but just the way in which you should approach teaching. (Janet, Case 10, phase 3 interview)

Janet continued to share that she was now trying a new approach that Maya had introduced her too, spiralling. At first she did not think it would be an effective way to teach mathematics but after working with Maya, planning a course, and seeing the results she couldn't imagine teaching the new de-streamed curriculum any other way. The following is an exchange between Janet and I during her phase 2 interview which describes how she learned about spiralling through Maya who had learned about spiralling through the OAME project.

> **Janet**: Now we're into this and I've taken on a new kind of approach it's the first time I've ever using a technique called spiralling.
>
> **Researcher**: OK.
>
> **Janet**: I was terrified. I was going to try spiralling … it was new, and I was like "I'm not doing this!" Because I didn't think I had time and now I can't imagine not ever doing it. Like I can't imagine a time when I wasn't doing this
>
> **Researcher**: So where did you learn about spiralling if you don't mind me asking?
>
> **Janet**: Maya, Maya, yeah a hundred percent. (Janet, Case 10, phase 3 interview)

Janet proceeded to explain how spiralling benefits students in school after two years of online learning. The ability for students to revisit concepts several times provided her the opportunity to address the many gaps that have been created due to the COVID-19 pandemic. She saw many benefits in her classroom to using this approach and others in her school were also interested in learning more about it. In this sense the learning had diffused beyond Janet to others in her school. For example the head of the mathematics department at Janet's school wants to use a similar approach to Janet when he teaches the new de-streamed course during the next semester. "My department head is like very, very supportive but he's also not teaching grade nines right now so he's like ‘you're going to be the person I’m coming to when I go to teach this’” (Janet, Case 10, phase 3 interview).

There is evidence in the phase 2 and 3 transcripts that the learning is being diffused at the school level. Both Elena and Maya felt that the learning had spread beyond the original project to others at Broadview. Elena described how she would encourage student teachers that she worked with to incorporate the BTC model. Elena shared during her phase 2 interview that other teachers at their school had started to use the model as well, “we did extend our ideas to new teachers that are using it now” (Elena, Case 10, phase 2 interview). As mentioned when discussing sustainability, Maya, inspired by the project, continued to find ways to sustain the collaboration that had started at the school. This also led to diffusion of the learning beyond the original PLC members. She invited Karen and other teachers in the mathematics department to join the collaborative learning that had started, this is an example of the learning that emerged from the project diffusing.

During her phase 3 interview Karen shared that the learning is being sustained and diffused through not only her interactions with project participants but through Karen to others in the school. Karen is no longer a new teacher at her school and has become one of the more senior teachers at Broadview and she shares what she learned with others in her school. In this way the learning from the project is being diffused across the school

> It's even nice now because, like over this short span of six years I’m now, like the one of the most senior teachers here in the math department. Yeah, so there [are] new people that are constantly coming to me asking me for advice. So they're learning from [the OAME project] too, through a secondary source. (Karen, Case 10, phase 3 interview)

At the school district level there is evidence that the learning from the project diffused beyond the PLC in Case 10 and into the school district. Analysis of both the phase 1 survey responses and the phase 2 and 3 interview transcripts provides findings that support this claim. During her phase 2 interview Elena shared that after completion of the project Broadview shared the learning that emerged from their involvement with the OAME project to others in the school district. She felt that the work they presented on using the BTC model, particularly the VNPS was well received and even led to Broadview leading professional learning sessions for their school district.

> The [district] had some initiative, where we, each school, had to had to pick one thing that they thought was working in their department and share it with the rest of the math departments at the [district level]. So we presented the vertical surfaces and got really good reviews from the superintendent at the time…Many of the members that were watching, they were really impressed with what we were doing, because it was, at that time, it was kind of a new idea for them too…A lot of the stuff that we did at the OAME project ended up filtering down. We were the people that were doing the PD at our [district]. (Elena, Case 10, phase 2 interview)

While I cannot speak to what Gabby has been doing since the completion of the project it is clear that she continued supporting teachers after the project completed. Karen described the ways that Gabby supported her and shared her expertise. It is most likely that Karen was not the only teacher that Gabby was supporting in her role as district mathematics lead teacher. As Maya took over Gabby’s role three years ago she shared how she encouraged others across her district to incorporate the learning that emerged from the project into their classrooms and mathematics departments. Specifically she endorses the BTC model and the use of open rich tasks in their classroom:

> So I was using vertical surfaces, all the way through and that's what I promote as a consultant now. I would attribute a huge part of me being a part of the OAME project to my growth as an educator. Because I loved the learning, I did a lot to further my career and currently I am the Math Science consultant in my [district] so I have the pleasure of sharing my experience with the entire [district]. (Maya, Case 10, phase 2 interview)

There are several key conditions and characteristics of complex learning systems that supported the diffusion of learning in Case 10, including neighbour interactions in a system with decentralized control, nestedness, and self-organization. Neighbour interactions were an important condition that supported the diffusion of learning at the individual, school, and district level. The learning that emerged from the project was diffused to Karen and Janet through neighbour interactions between the original project participants, Elena, Maya, and Gabby. At the school level Karen shared how she is diffusing the learning from the project across the mathematics department to the school department head and new teachers that join the staff. Elena presenting the learning from the project at the district level is another example of the role that neighbour interactions played in diffusing the learning. All of these examples of neighbour interactions occurred in a system with decentralized control meaning that no higher authority was implementing structure over the system. The nestedness of the education system also appears to have supported the diffusion of learning in Case 10. The OAME project was designed using a nested model. PLCs were nested within schools that were nested within different school districts across the province. This nestedness supported the sharing of information across different systems.

Table 23

Conditions and Characteristics that Influenced Diffusion - Case 10

Condition/Characteristic	**Example**
Trans-level learning – neighbour interactions in a system with decentralized control	*Elena*, *Maya*, and *Gabby* shared their learning with other members of the Broadview mathematics department and other high schools in the district through neighbour interactions in a system with decentralized control. *Karen* then also shared what she learned from *Maya* and *Gabby* with others in her school including the mathematics department head and other teachers.
Nestedness	The *PLC* being nested within Broadview which is nested within the school district supported the diffusion of learning beyond the PLC and Broadview into other schools in the *school district*. Maya's new role as a learning consultant meant that she could share the learning beyond her school PLC and across the *school district*.
Self-organization	The participants in this study all spoke of ways they shared the learning with others across different learning systems including their *schools* and *school district*. This is an example of individuals self-organizing into collectives and sharing learning such as using the BTC model and spiralling.

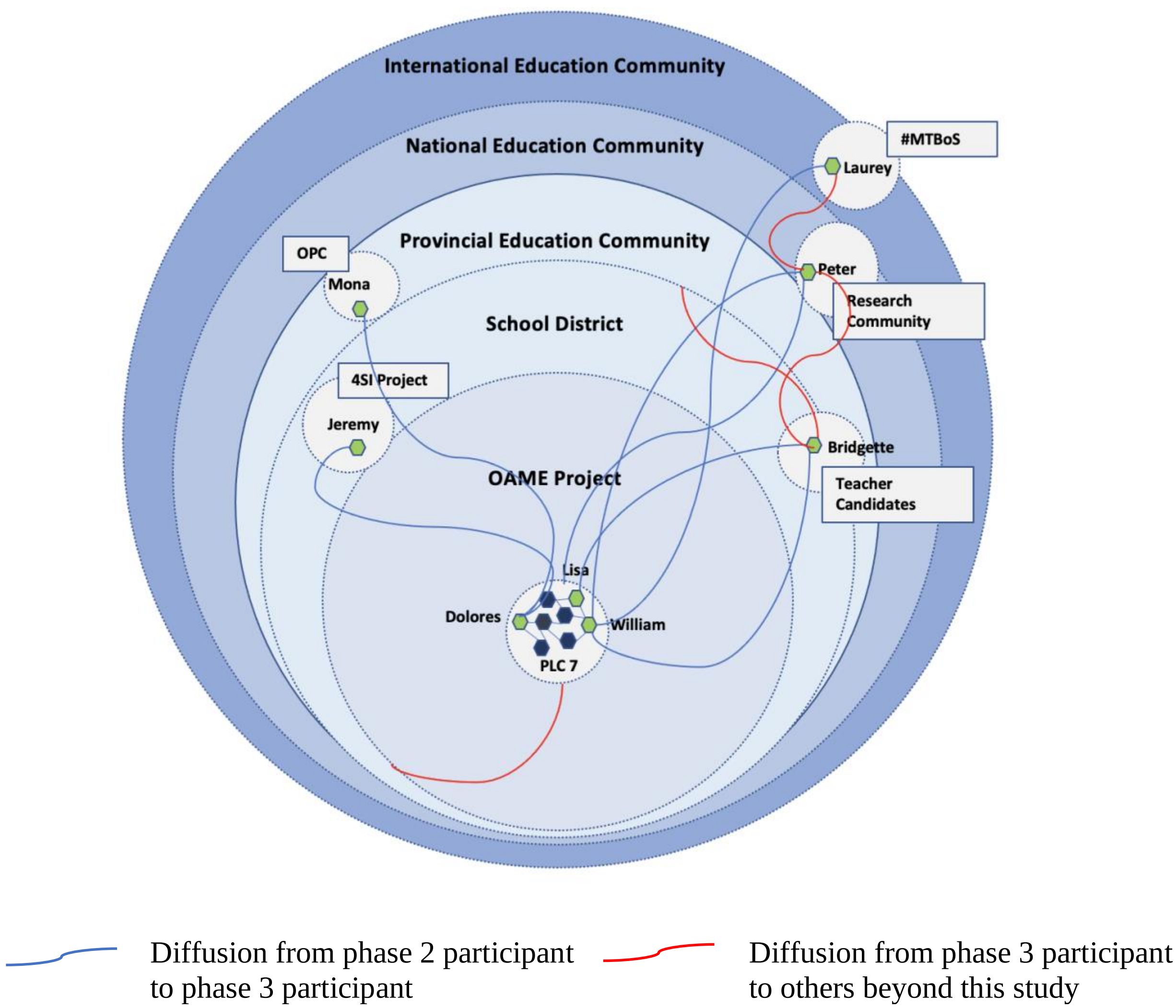
Concluding Comments

This chapter presented the extensive findings from each of the four phases of data collection and analysis in this doctoral study. Even with the detail that I included in this chapter there are many more examples of the ways the conditions and characteristics of complex learning systems manifested across and between the four cases of this study. In summary I would like to share a diagram that I find helpful to attempt to visualize the ways learning emerged, was sustained, and diffused. Figure 14 showcases some of the connections between different individuals and the nested systems in which they are located, as represented by this study's data.

I use Case 7 as an example as that case had the most participants at each level and across the most types of learning systems. These are, by no means, all the connections that exists as the learning emerged, was sustained, and diffused from the OAME project, but it does offer an overview of some of the ways. The diagram also considers the possibility that the learning diffused beyond phase 3 participants, as mentioned by some during their phase 3 interviews.

Figure 14

Nestedness and Neighbour Interactions Between Different Levels



CHAPTER 6: DISCUSSION

When we try to pick out anything by itself –
we find it hitched to everything else in the universe (John Muir)

This quote, by John Muir, was used by Davis and Sumara (2006) in their book *Complexity and Education* when introducing a chapter describing the network of complexity. For me, it adequately sets the stage for a discussion on the findings presented in Chapter 5. In Chapter 5 I presented a somewhat linear discussion of the ways the conditions and characteristics necessary for the emergence of learning manifested in and across the OAME project and the four cases in this study. In this chapter, I attempt to present a “big picture” discussion of how the network of complexity manifested across this study and influenced the emergence, sustainability, and diffusion of learning in different systems across the original OAME project, as well as the four cases. I use the word attempt as it would be impossible to describe all the ways that different components of systems interacted and influenced one another. I organize the discussion in two sections. The first section answers the two research questions that asked:

1. How did the characteristics and conditions of complex learning systems influence the emergence of learning in the different PLCs, and associated learning systems, of the OAME project?
2. In what ways did the characteristics and conditions of complex learning systems influence the sustainability and diffusion of learning in the different PLCs, and associated learning systems, of the OAME project?

When answering each question I discuss the key conditions and characteristics that supported the emergence, sustainability, and diffusion of learning. I also employ the DoI model (Rogers, 2003) to describe how and why the learning emerged as well as how it was sustained and diffused five years after the project. As discussed throughout this dissertation the conditions and characteristics of complex learning systems do not exist independent of one another. They interact, influence, and support one another. Therefore, as I present a discussion I will also describe other conditions and characteristics that supported the major themes that are the focus of the discussion. For example the theme of neighbour interactions in a decentralized network was also supported by diversity and redundancy, and vice-versa.

In the second section of this chapter I connect the findings of this doctoral study to the literature reviewed in Chapter 2. I focus on ways the findings from this doctoral study align with research on effective professional learning. I also discuss how this doctoral study supports the claim that a hybrid approach involving both complexity and the DoI model is a useful tool for analysing the diffusion of innovations through collaborative networks (Rogers et al., 2005).

Answering Research Question One

When considering the extensive findings detailed in Chapter 5 I focus on four key conditions and characteristics that interacted and supported the emergence of learning from the OAME project and associated learning systems. These include neighbour interactions, decentralized control, self-organization, and nestedness.

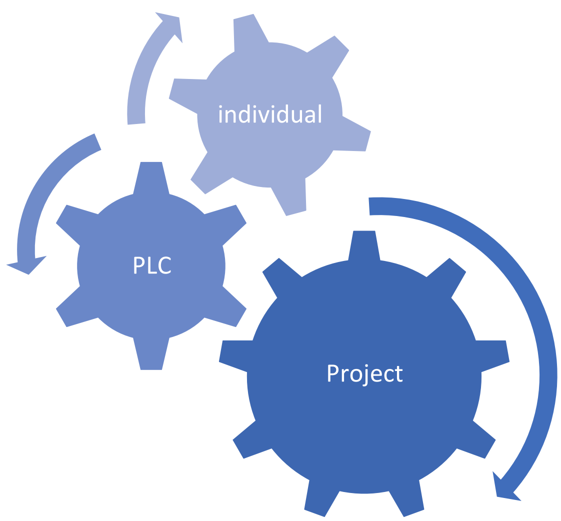
Neighbour Interactions in a System with Decentralized Control

An important theme evident in the findings was the positive role that neighbour interactions in a network with decentralized control played in supporting the emergence of learning from the OAME project. Neighbour interactions included time to interact with other individuals and their ideas, hunches, and representations in ways that were determined by the individuals involved. These individuals included other educators, researchers, and students.

Numerous neighbour interactions occurred, and were supported, during the OAME project at multiple levels. Neighbour interactions at the personal level involved an individual negotiating ideas internally while neighbour interactions at higher levels such as the PLC or project level included conversations, observations, and negotiations between project members and their ideas. As the level increased outwards (Figure 5) to include more individuals, and therefore more ideas, so did opportunities for interactions. All of the different levels of interactions contained the possibilities to influence other levels, similar to cogs turning a system (Figure 15).

Figure 15

Relationship Between Different Neighbour Interactions of the OAME Project



Some of the key neighbour interactions that stand out include PLC members interacting with student thinking either during classroom observations or examination of student work during PLC meetings, PLCs discussing and negotiating new practices and ideas within their own context, and neighbour interactions across PLCs at the project-wide meetings and sessions.

Neighbour interactions with student thinking were prominent in all four cases. For instance, cases 1, 4, and 7 co-designed tasks and activities as a group and then observed a member teaching that task to a class of students, followed by a de-brief as a PLC. Interacting with student thinking presented the teachers with different perspectives on how students learned and created opportunities to disrupt their thinking regarding how students think. Findings from Cases 1, 4, and 7 discussed explicit examples of this. These disruptions represented neighbour interactions in a system with decentralized control. Decentralized control was evident in all three of these cases as they collaboratively decided to determine a goal and co-designed tasks to address that goal. Case 10 interacted with student thinking by observing videos of lessons taught by Gabby prior to the start of the OAME project. Observing students through video provided an area of focus for discussion and negotiation of understanding what it meant to teach using rich tasks. Case 10 demonstrated less decentralized control than the other cases, yet they still engaged in examining student thinking as a focus for group discussions. Observing students and having time to discuss, reflect, and fine tune the tasks students engaged with were important components of the work of all of the cases in this study. This leads to the next type of neighbour interactions, those that occurred at the PLC level.

Decentralized control and the balance between redundancy and diversity seemed to influence the extent and nature of neighbour interactions. Neighbour interactions between individuals were most frequent during PLC meetings. While each PLC was supported to meet once a month throughout the two years of the project; some met more frequently and others less. The influence that these meetings had on the emergence of learning varied across the four cases. For instance at the PLC level Case 1 lacked redundancy in terms of a shared goal during Year 1; this created an imbalance between the diversity and redundancy of the group which limited opportunities for neighbour interactions and ultimately led to the reconfiguration of the PLC. When the PLC reformed with new members in Year 2 the group aligned in terms of goals which resulted in an increase of neighbour interactions, particularly around issues of student thinking and how best to design a rich task. Case 4 and Case 7 both established PLCs with a balance between redundancy and diversity which was evident in how the PLCs were aligned with their goals. Neighbour interactions in these cases also occurred within a system with decentralized control. While both PLCs had someone that could be identified as a leader, Chrissy in Case 4 and Dolores in Case 7, their leadership style encouraged distributed leadership. The direction and focus of these two PLCs were always determined by the group. Lastly, Case 10 was led with a top-down approach throughout much of the project and often the opportunities for neighbour interactions that pushed the thinking of the group were guided by Gabby and Donald. However, the project supported neighbour interactions with others outside of their own PLC and these neighbour interactions were in a system with a more decentralized control. This leads to a discussion of neighbour interactions that occurred at the project level.

Opportunities to engage in neighbour interactions through sharing experiences at the project level were intentionally designed into the project and it would appear that this level of interaction significantly influenced the learning that emerged from the project. All the participants from the original project who were also involved in this doctoral study reflected on how connecting with others across the province influenced their learning. For Case 1, in particular, the project level interactions provided the balance between diversity and redundancy that was missing for the high school mathematics teachers. When Jill learned of different approaches to teaching Grade 9 mathematics from others in the project she was motivated to incorporate these into her teaching when she returned to Weber High School. She credits the OAME project with encouraging her to continue to push her practice and try new ideas. For others, such as members of the Case 7 PLC, interreacting with others at the project level reinforced their beliefs that they were on the right track with their new approaches to teaching mathematics, including using the BTC model and spiralling through the curriculum using rich tasks. Tasha remarked that her goals for being involved in the OAME project evolved when she met the other participants and realized that the mathematics department at Fields was at the forefront of shifting practice in the province. This encouraged her to continue to push the work the PLC was doing and to share their work with others. Similarly, Chrissy from Case 4 shared how interacting with other teachers from different parts of the province and school contexts boosted her confidence and she realized that Pike Place High School wasn't the only mathematics department struggling with particular issues in teaching mathematics; she was buoyed by the fact that others were interested in hearing more about their ideas. This increased Chrissy's confidence and encouraged her to continue to push for more collaborative practices within her school and school district.

Diversity and redundancy were important conditions that supported these neighbour interactions at various levels of the system. At the PLC level the diversity in teachers' own experiences and perspectives about teaching and learning provided the necessary perturbations to the group as they interacted and designed lessons, discussed student learning, and shared ideas allowing for discussion and negotiations within the community, leading to novel responses to the perturbation. Simultaneously the redundancy that manifested in the shared focus and goal of enhancing the teaching and learning of mathematics provided the necessary common ground that allowed the individuals to come together as a collective learning system. This was also true at the project level. The diversity of different contexts and models of professional learning adopted by the different PLCs provided opportunities for the sharing of new ideas and ways of working to be introduced across the project. The redundancy in the shared focus of all the participants engaged in enhancing the teaching and learning of mathematics provided the common ground and common language necessary to support neighbour interactions.

Evident in all of the neighbour interactions described is the importance of decentralized control. The findings demonstrate that decentralized control helped to support teachers and PLCs in making their own choices – about their goals, their focus, the tasks they chose to use, how they used the tasks in their classes, and even the ways in which the PLCs worked. Thus the teachers had ownership of their own learning. This decentralized control also supported the self-organization that was observed across the project and the systems nested within the project.

Self-organization and Nestedness

Providing space for decentralized control and setting up the project as nested communities helped to support characteristics for learning such as self-organization. As discussed previously self-organization occurred at multiple levels including the project and PLC level. Self-organization was supported by adaptation as the different systems self-organized they adapted to their new structures through an iterative process of seeking out and testing new ideas. The project itself adapted to the needs of the participants by self-organizing into a two-year project rather than a one-year project to meet the evolving needs of the participants. PLCs connected and formed new collectives such as Case 6 and 7 connecting and cross-pollinating during Year 2 and members from different PLCs forming groups to present learning from the project at provincial conferences. This self-organization would not have been possible without the nestedness of the project that supported neighbour interactions in a system with decentralized control. The PLCs being nested within the project provided opportunities for neighbour interactions at the project level described above. These interactions supported individuals and PLCs to meet and to push each other's thinking.

Drawing from the discussion above it is evident that neighbour interactions in a system with decentralized control, self-organization, and nestedness were the key conditions and characteristics that interacted to influence the emergence of learning at the individual, PLC, and project level of the OAME project. When considering the findings and viewing the OAME project as a complex learning system it is evident that relationships between different conditions and characteristics gave rise to new learning and collective behaviours at the project level.

There were several practices that emerged from the project that were incorporated by all of the cases involved in this study. These collective practices, or behaviours are examples of learning that emerged. These collective practices included incorporating the *5 Practices* as a framework for planning and leading classroom discussion around the use of rich tasks, the BTC model as a way to engage students in rich mathematical tasks, and the spiralling approach to teaching the mathematics curriculum in order to revisit concepts multiple times throughout a course. As I discussed in Chapter 3, self-similarity suggests that smaller components of a system resemble the system as a whole. The collective behaviours that emerged from the project are examples of the self-similar nature of nested complex learning systems and suggest that learning at the individual level is similar to the learning of a system as a whole. Each participant in this doctoral study discussed their implementation of at least one of these collective behaviours at the individual level and these collective behaviours emerged across all the cases involved in this study.

In answering RQ1 I analyzed the ways that the different conditions and characteristics of complexity manifested across the OAME project and the four cases in this study. I also examined ways that these conditions and characteristics influenced the learning that emerged. In doing so I considered the similarities between what my findings were pointing to and how these findings were similar to the different stages and components of the DoI model. The DoI model seemed useful in organizing why and how learning emerged from the OAME project. I began to consider whether diffusion and emergence might be similar, if not in fact the same phenomenon. For instance, learning was diffused (or emerged) both during the project, within the PLCs and across PLCs, and as you will see when I discuss RQ2, beyond the project. When the DoI model is used in parallel with the complexity framework the reasons for how and perhaps why this learning emerged also become more evident. As explained in previous chapters the adoption of innovations, in this case the collective behaviours described, involves diffusion which is “the process through which an innovation, defined as an idea perceived as new, spreads via certain communication channels over time among the members of a social system” (Rogers, 2003, p. 13). Through my analysis of the data and identifying key conditions and characteristics of complex systems I make a connection between the adoption of new ideas (innovations) and the emergence of collective behaviours (learning) across the cases in this study.

Learning occurred during the two years of the project and was supported by the design of the project. As mentioned, the project, which was informed by complexity theory, was intentionally designed to provide opportunities for the iterative, collaborative process of relfection and action. As described, each case in this study had different contexts, structures, leadership approaches, and participation levels yet similar collective behaviours emerged across them all. Combining the DoI model with complexity science allows me to consider how and why this happened. The four elements in DoI theory are the innovation, communication channels, time, and the social system. These four elements are represented in the complex system of the OAME project as follows: the innovations are the collective behaviours that emerged, the communication channels are the different neighbour interactions described, time to interact with these innovations was provided through the two years of the project, and the social system was the complex learning system of the OAME project. This social system cannot be underestimated in terms of its influence on the learning that emerged. The intentional design of the project established the nested nature of the different learning systems that was crucial in making the connections among and between different individuals and collectives.

Key stages in the DoI model include knowledge, persuasion, decision, implementation, and confirmation (Rogers, 2003). Table 24 presents these key stages by organizing the ways they manifested in the OAME project, connecting them to the conditions and characteristics of complex learning systems, and to the relevant ideas reviewed in the literature for this study.

Table 24

Manifestation of DoI Model Key Stages in OAME Project

Key Stages (Rogers, 2003)	Manifestation in the OAME Project	Connections to Complexity Science	Connections to Literature
Knowledge (introduction of innovation)	Collective behaviours that emerged from the OAME: • 5 Practices • Building Thinking Classrooms • Use of Rich Tasks • Spiralling	Perturbations Neighbour interactions	Learning emerges from a complex system
Persuasion (forming an opinion on the innovation)	Individuals and groups were persuaded to experiment with the innovations listed above because they appeared to: • have an advantage over current practices • be compatible with individuals and groups values and beliefs	Diversity Redundancy Neighbour Interactions Decentralized Control Self-organization Adaptation	Shared goals Agency Collaboration Job-embedded Time

	• be easy to comprehend and adapt to individual and groups needs • be observable or tangible (others shared their success with them) • be easy to test and try out in difference contexts and settings		
Decision (a choice to adopt or reject the innovation)	The four PLCs in this study all made the choice to adopt some or all of the innovations.	Decentralized Control Adaptation (if adopted) Self-organization	Agency
Implementation (putting innovation to use)	The time, space, and agency to implement these innovations were important components of this stage of the model. Each PLC that implemented these innovations did so freely. It was not a requirement of the project but rather the innovations were persuasive enough for advantages to seem as possible and worth the effort to implement them	Neighbour Interactions (interacting with students' thinking) Decentralized Control Adaptation Self-organization	Agency Long-term Job-embedded
Confirmation (seeks reinforcement of an innovation-decision already made)	Time to grapple with the outcomes of implementing these innovations and to engage in observations, discussions, and negotiations about whether they were advantageous was possible because of the conditions and characteristics in place at each PLC. This confirmation component appears to be a crucial stage in the adoption of an innovation.	Diversity Redundancy Neighbour Interactions (discussing and negotiating as a PLC) Decentralized Control Self-organization Adaptation	Agency Job-embedded Collaborative

In terms of this discussion, knowledge of the innovation was presented through the sharing of ideas at the PLC and project levels. Persuasion, the stage where an opinion of the innovation is formed, was observed to occur both at the individual, PLC, and project level, as presented in Chapter 5. Neighbour interactions such as those discussed earlier, sharing ideas at PLC meetings, and interacting with student thinking provided opportunities for participants to consider the specific characteristics of an innovation, such as having an advantage over current ideas, compatibility with one's values and beliefs, ease to comprehend and adapt to one's needs, observable or tangible, and simple to test, or trial. The nested nature of the project provided multiple opportunities for individuals to form an opinion of an innovation, they could interact with these ideas during project-wide meetings as well as engage in discussions and negotiations about the innovation with their PLC. The decision phase occurred when a choice was made to either adopt or reject an innovation. With some PLCs this was made at an individual level such as Case 1 (Jill) and 10 (Elena and Maya), and with others the PLC made the decision, like Case 4 and 7, to incorporate the practice into PLC planning of lessons. The implementation stage involved the time to put the new innovation to use at the different schools involved. Lastly, confirmation is when a user "seeks reinforcement of an innovation-decision already made" (Rogers, 2003, p. 169), different PLCs chose what innovations they would continue to use and how they would use it based on their experiences in the testing stage. In summary, the DoI model provides a way to relate how and why the PLCs adopted the new innovations described. The PLCs saw an advantage to implementing the innovations and through testing, observing, discussing, and negotiating their understanding of the innovations the individuals and PLCs confirmed that the innovations would enhance the teaching and learning of mathematics and therefore adopted them.

Answering Research Question Two

The discussion on the sustainability and diffusion of learning that emerged from the OAME project focuses on a period of time five years (2021) after the completion of the OAME project (2016). During the project, time, space and funding were available for PLCs to meet both in their own schools and with other PLCs involved in the project. As I discussed previously, sustaining connections, sharing of ideas, and collaborative learning during the project were perhaps easier because of this funding. After the project, funding that provided time to meet was no longer available, yet all of the PLCs involved in this doctoral study managed to sustain some of the learning and all shared that the learning was diffused in some way. Similar to RQ1, the key conditions and characteristics of neighbour interactions, decentralized control, self-organization, and nestedness supported this sustainability and diffusion. In the following sections I discuss ways these key conditions and characteristics influenced the sustainability and diffusion of learning.

Neighbour Interactions and Decentralized Control

A theme from the findings was the key role neighbour interactions across systems of learning with decentralized control played in terms of supporting the sustainability of learning from the OAME project. These neighbour interactions occurred between different individuals, PLCs, and other learning systems associated with the OAME project and project participants. Neighbour interactions online through Twitter, face-to-face collaborative learning, and with students and student thinking are some examples of neighbour interactions that supported the sustainability of learning, and all occurred in systems with decentralized control.

At the individual level all OAME project participants in this doctoral study mentioned following others from the project as well as other mathematics educators on Twitter either in their phase 1 survey or in their phase 2 interview. These interactions via social media channels allowed individuals to continue to share, and learn about, experiences of teaching and learning mathematics with others is a system with no hierarchical authority structure. Neighbour interactions through Twitter were important as they often provided the necessary balance between diversity and redundancy that some individuals were lacking in their personal teaching contexts. Individuals like Jill for example needed to look beyond their own PLC and school setting to interact with others who could continue to push her thinking in ways that she deemed important. She then could work with her core group of interested colleagues, Drew, Zoe, and Francis, to discuss and interact with these new ideas in ways that made sense to their school setting and student needs.

Face-to-face neighbour interactions also supported the sustainability of learning from the OAME project. As mentioned in Chapter 5 several participants, including Jill, Tom, Tasha, and William continued to collaborate professionally, some of them met at one another’s schools to work together and others presented at local conferences to share their learning together. These connections across different systems supported the sustainability of the learning that emerged. Several participants, such as Chrissy and Elena, discussed how they made it a point to attend mathematics education conferences and seminars. These face-to-face neighbour interactions were a continuation of the collaborative learning that had emerged from the project. Decentralized control in the learning systems was exhibited in the teachers being able to choose their own professional learning opportunities, and in fact was very much teacher driven. These individuals were choosing to continue to seek out professional learning opportunities, both informal and formal, on their own.

Lastly, positive outcomes from neighbour interactions with students and student thinking encouraged participants to continue to use the collective practices that emerged from the OAME project. For example Chrissy, Daniel, and Drew all shared how their students were more engaged when taught using new practices such as the BTC model and using rich tasks. Chrissy reported that the student failure rate at Pike Place decreased, and she credited the OAME project and the collaborative professional learning her PLC had engaged in with supporting this success.

Neighbour interactions in the systems within and beyond the OAME project also supported the diffusion of learning. Again, the use of Twitter was mentioned by several of the phase 3 participants, perhaps most notably by Laurey, a mathematics teacher from the United States. Laurey followed several educators from Ontario, including William on Twitter under the #MTBoS hashtag. These connections through social media allow us to redefine community to include others we perhaps have never met or worked with yet share similar goals and interests (Larsen, 2019). As I described in Chapter 5, these online communities can be viewed as a complex learning system when we consider the diversity and redundancy they demonstrate, the opportunities for neighbour interactions in a system with decentralized control, as well as the enabling constraints of coherence to the focus on mathematics teaching and learning coupled with the randomness to engage in any topic of discussion within that focus.

Peter’s connection with William and other educators across the province provided another perspective on how neighbour interactions in systems with decentralized control can influence the diffusion of learning. There were several interactions that supported the diffusion of learning related to the BTC model. William sharing his experiences with the BTC on Twitter in the #MTBoS community meant that many others beyond the project also learned about the BTC model perhaps initially through William and not Peter, the creator of the model. This diffusion of learning connected William and Peter and their work continued to diffuse across the province after the project ended through channels that originated through the OAME project. Phase 3 participants Karen and Janet shared how Maya suggested they follow Peter and William on Twitter in order to enhance their own teaching and learning. These types of neighbour interactions in systems with decentralized control, such as providing time and space for teachers to attend conferences, engaging on Twitter, and opportunities for teachers to share their learning within their schools through talk in the math office or staff room were key in the sustainability and diffusion of learning within and beyond the OAME project.

Self-organization

Self-organization is a characteristic of complex learning systems and is an outcome of a system responding to new ideas and connections formed between different parts of a learning system (Cochran-Smith et al., 2014). In the case of this doctoral study the individual teachers and PLCs were self-organizing in response to the learning that emerged from the project and the new relationships that were formed. Evident throughout the findings was the role that self-organization played in sustaining the learning that emerged from the OAME project. Self-organization occurred in several ways, including teachers shifting their practice to include the collective behaviours that emerged from the project, teachers and administrators applying for, and receiving, funding to support continued collaborative professional learning at their sites, as well as creating new hiring practices to support the sustainability of a collaborative school culture. I will discuss each of these examples in the following sections.

Participants in phase 2 all shared that they have shifted their practice to include some or all or the collective behaviours that emerged from the OAME project. Some, like Daniel, found it difficult to engage in all of these practices without the support that the project provided yet he maintained that he views his role as a teacher differently now. He focused more on teaching the whole child, rather than simply delivering the curriculum. Others, like Jill and Chrissy were buoyed by an increase in confidence through the project to continue to teach in these new ways because of the outcomes they observed through interacting with their students. Additionally, Case 7 continually self-organized the structure of their PLC. They sustained the collaborative approach they took to identify ways to support students across a range of applied grade 9 classes for the years following the completion of the OAME project.

PLCs also self-organized in terms of how they financially sustained collaborative professional learning. Without the financial and structural support that the OAME project provided individuals, PLCs, and schools needed to self-organize and find ways to sustain the learning that was meaningful to them. Jill, Georgia, and Maya all spoke explicitly about the efforts they made in order to secure funding to support their PLC and professional learning. This funding helped them continue to collaborate as a PLC and to connect with other schools and districts in meaningful ways, such as individuals from Case 1 visiting the Case 7 school site to engage in professional learning.

In two different cases, Chrissy and Lisa shared how their schools self-organized in terms of who they would hire to join the mathematics department. They learned that to sustain a collaborative community they would need to hire individuals who were open to working as a team and trying new ideas. Thus, this self-organization is an outcome of the learning that emerged from the PLCs through the neighbour interactions in a system with decentralized control. The two cases learned that developing a collaborative environment supported their learning and therefore sought to continue to build a collaborative community for their schools.

Self-organization similarly influenced the diffusion of learning from the OAME project. Several phase 3 participants self-organized their practice in response to interacting with phase 2 participants. Mona and Jeremy both shared how their involvement with Dolores influenced how they approached their leadership roles supporting teachers and principals. Teachers such as Laurey, Bridgette, Karen, and Janet shifted their practice based on their interactions with phase 2 participants online and in face-to-face interactions. Having the opportunity to learn about the collective behaviours that emerged from the project and the freedom to choose to adapt their teaching to align with these practices represents self-organization. Interestingly, the learning continued to be diffused beyond phase 3 participants. Laurey, Bridgette, Janet, and Karen for example all described how they had shared these collective practices with others beyond those included in phase 3 of this study. This suggests that the learning from the OAME project continued to emerge and diffuse.

It is important to note that self-organization happens within, and across different learning systems, thus nestedness also plays an important role in supporting the sustainability of learning. I now turn to discussing how nestedness supported the sustainability and diffusion of learning.

Nestedness

A theme that was noted in the findings of this study was that the sustainability and diffusion of learning from the OAME project was influenced by the nestedness of the education system. Opportunities to engage across different systems such as schools, districts, and the greater educational community (Figure 2) provided opportunities for neighbour interactions in systems with decentralized control which could lead to self-organization of different systems.

While our education system ultimately has a defined structure through mandated curricula and jurisdictional rules and regulations, teachers historically have been supported in exercising their professional judgement in terms of how they engage in professional learning and how they implement the intended curricula. In terms of sustainability, nestedness allowed individuals to find a better balance between diversity and redundancy. For instance both Jill and Chrissy looked beyond her own school setting to sustain their learning, and engaged in the greater mathematics community through workshops, conferences, and the connections they made.

The nestedness of the education community also supported the diffusion of learning from the project. Connections between phase 2 and 3 participants were generally across different learning systems. For example, William and Lisa’s connection with Bridgette was an example of two different learning systems, a university, and a secondary school, nested within the Ontario education community. Peter and Laurey were also nested in #MTBoS on Twitter with others from the OAME project and this nestedness supported the diffusion of learning from the project. Lastly, the different roles of participants within the different nested systems provided opportunities to establish connections across learning systems. For instance, Maya’s role as a math and science consultant in her district meant that she could share the learning from the OAME project with others across the district, such as Karen and Janet.

In considering the findings it is clear that the complex learning system that is the OAME project is nested in and influences the larger educational system and all the systems within it. These two systems cannot be separated, the relationship of influence is reciprocal, and learning is not a linear one-way process. An example of this was evident in the interactions of William and Peter. While William was clearly influenced by the model Peter had created the interactions that occurred between the two of them influenced both of their understanding of teaching, learning, and in Peter's case, researching with the model. The ongoing interactions between them produced new ideas that they both shared beyond their own interactions. The ideas were shared with others within the OAME project by the Case 7 PLC and Peter also shared his experiences working with William and other teachers with educators across North America. The BTC model was shared and tested by others who decided to adapt their teaching to include the BTC model. This is true, to different degrees, for the other collective behaviours discussed in the previous sections as evident in the phase 2 and 3 interviews with all of the cases. This suggests that learning that emerged from the OAME project has been sustained at the project level and has spread beyond.

I again consider the DoI model and how it can lead to a better understanding of why the learning that emerged was sustained and diffused. I begin by considering the key stages of the DoI model (knowledge, persuasion, decision, implementation, and confirmation). I present these key stages by organizing the ways they manifested in the OAME project, connecting them to the conditions and characteristics of complex learning systems, and to the relevant literature reviewed for this study. This table differs slightly from Table 24 in the sense that I consider both the sustainability of the learning by phase 2 participants and the diffusion of learning to the phase 3 participants.

Table 25

Manifestation of DoI Model Key Stages Supporting Sustainability and Diffusion

Key Stages (Rogers, 2003)	Manifestation in the OAME Project	Connections to Complexity Science	Connections to Literature
Knowledge (introduction of innovation)	Sustainability: Phase 2 participants were aware of the collective behaviours	Perturbations Neighbour interactions	Learning emerges from a complex system

	that emerged from the OAME project Diffusion: Collective behaviours that emerged from the OAME project were introduced to phase 3 participants by phase 2 participants.		
Persuasion (forming an opinion on the innovation)	Sustainability: Phase 2 participants who continued to engage with the practices had been persuaded to do so because of the points listed below. Diffusion: Phase 3 participants engaged in forming an opinion through experimentation. They were persuaded to try these innovations because they appeared to: • have an advantage over current practices • be compatible with individuals and groups values and beliefs • be easy to comprehend and adapt to individual and groups needs • be observable or tangible (others shared their success with them) • be easy to test and try out in difference contexts and settings	Diversity Redundancy Neighbour Interactions Decentralized Control Self-organization	Shared goals Agency Collaboration Job-embedded Time
Decision (a choice to adopt or reject the innovation)	Sustainability: Phase 2 participants continued to adopt the innovations because of the points listed above. Diffusion: Phase 3 participants in this study	Decentralized Control Adaptation (if adopted) Self-organization	Agency

	made the choice to adopt some or all of the innovations.		
Implementation (putting innovation to use)	Sustainability: Phase 2 participants continued to implement the innovations. Diffusion: Phase 3 participants implemented some or all of the innovations through trialling and testing and decided the innovations were persuasive enough for advantages to seem as possible and worth the effort to implement them	Neighbour Interactions (interacting with students' thinking) Decentralized Control Adaptation Self-organization	Agency Long-term Job-embedded
Confirmation (seeks reinforcement of an innovation-decision already made)	Sustainability: Through continued implementation of the innovations phase 2 participants were continually seeking confirmation that the innovations were advantageous. Diffusion: Unlike phase 2 participants, phase 3 participants did not have the same levels of time and space to grapple with the innovations. Yet, through interactions with phase 2 participants and the innovation they still confirmed that the innovations were advantageous.	Diversity Redundancy Neighbour Interactions (discussing and negotiating as a PLC) Decentralized Control	Agency Job-embedded Collaborative

In terms of sustainability, individuals from the OAME project had knowledge of the innovation through the learning that emerged from the project, through experience they continued to be persuaded to engage with the innovation and continually decided to implement the innovations. Through this continued implementation they received confirmation that the innovation was advantageous. This suggests that sustainability of the learning exists in the confirmation stage of the DoI model. In terms of the diffusion of the collective behaviours beyond the OAME project, Table 25 reflects the process involved in the diffusion of learning to the phase 3 participants. Knowledge of the innovations were shared across different learning channels (connection with phase 2 participants) and the individuals were persuaded to implement the innovation because of perceived advantages over their current practices. After deciding to implement the innovation the users received confirmation that the innovations proved to be advantageous. The individuals that I interviewed for phase 3 confirmed that they had been introduced to these innovations, had trialled, and tested them and found the innovations to be advantageous to their practice.

Throughout this study I was struck with some similarities between my findings and the research literature I reviewed in Chapter 2. I now connect the findings of this study to the literature. I focus on the ways the findings align with research on effective professional learning and discuss how both complexity and the DoI model are useful tools for deepening our understanding of the complexity of mathematics teachers' collaborative professional learning.

Connections to the Literature

In answering RQ1 and RQ2 it appears that similar conditions and characteristics of complex learning systems supported the emergence, sustainability, and diffusion of learning from the OAME project and associated learning systems. These include neighbour interactions in a system with decentralized control, self-organization, and nestedness. I will now discuss how these conditions and characteristics align with research on professional learning.

Neighbour Interactions

As discussed throughout this doctoral study, neighbour interactions involve more than two individuals interacting. They can also involve an individual interacting with a resource, an idea, or a new practice. When I consider the ways that neighbour interactions manifested in the OAME project and supported the emergence, sustainability, and diffusion of learning I see several correlations to the research literature. My findings align with research that demonstrates the importance of collaborative professional learning, the act of inquiring into one's practice with colleagues. Phase 2 participants in this study overwhelmingly stated that collaborating with others in their PLC, and with others across the project had the biggest influence on their learning and maintaining these interactions after the project supported the sustainability of the learning. Similarly phase 3 participants shared that interacting with others and the collective behaviours that emerged from the project influenced their own learning, this represents ways that diffusion of learning was supported. This is in line with numerous researchers who have reported on the crucial role that teacher collaboration plays in supporting, sustaining, and diffusing teacher professional learning (e.g., Borko & Potari, 2019; Schleicher, 2015; Stigler et al., 1999; Vescio et al., 2008). Several studies I reviewed shared similar findings to my study. For example Rigelman & Rubin (2012) suggested that modelling collaborative professional learning with teacher candidates during their practicum led to teacher candidates seeking to sustain practices they were interrogating collaboratively during practicum in their future classrooms. Campbell and colleagues' (2016) study, investigating the impact of Ontario's TLLP program, found that collaborative learning was the method adopted most often by TLLP participants and an overwhelming 98% of participants reported they were able to sustain the learning that was generated by their involvement in the TLLP. Diffusion of the learning was also reported to have occurred at high frequency (77% within their own schools, 88% with others in Ontario, and 10% outside of Ontario/internationally). Another study, by Zehetmeier and Krainer (2011), was similar to the OAME project in design, with collaboration encouraged and participants defining their own problems of practice and ways of working. The one participant that the researchers followed-up with several times (5 and 8 years after completion of the project) experienced similar outcomes to Daniel from Case 4. Daniel sustained some of the practices that he developed such as focusing on the whole child but was unable to sustain some of the innovative teaching practices due to a lack of collaborative learning time with the PLC, similar outcomes as Zehetmeier and Krainer's participant.

A second type of neighbour interactions observed in this doctoral study was participants interacting with mathematical content such as rich tasks and the curriculum. This aligns with researchers who suggest that effective professional learning should focus on content and be aligned with the curriculum (e.g., Ball et al., 2005; Cohen & Hill, 1998; CPRE, 1998; Darling-Hammond et al., 2017; Loucks-Horsley & Matsumoto; 1999). Ball and colleagues (2005) suggest implementing a curriculum effectively requires skilled teachers who understand the subject matter, stating "how well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing" (p. 14). When teachers engage with content and curricula, specifically mathematical content, they are provided opportunities to develop a deeper understanding of concepts and even to change their own identities as learners of mathematics (Anderson et al., 2018). Developing a deep understanding of mathematical content allows teachers to better understand how students are interacting with this content and to navigate and support students as they learn mathematics (Ball et al., 2005; Shulman, 1987). This leads to the next type of neighbour interactions, those between the participants and students and students' thinking.

All of the PLCs involved in this study engaged in lesson observation with the goal of better understanding how students were interacting with lessons the PLCs had designed. After lessons were observed the PLCs would debrief about how the students interacted with the lesson and how to refine the lesson in order to better meet the needs of their students. The collective behaviour of using the *Five Practices* was one way that teachers supported these neighbour interactions. Several researchers have identified a focus on student thinking as an effective strategy for teacher professional learning (Darling-Hammond et al., 2017; Roth et al., 2011; Smith & Stein, 2015). Focusing on student thinking can provide teachers opportunities to better understand student ideas in order to better anticipate and plan responses and prompts to students questions or misunderstanding (Roth et al., 2011; Smith & Stein, 2015). A study by Roth et al. (2011) compared two groups of teachers involved in a professional learning initiative. One group focused only on subject content during their learning session and the other group included collaborative video analysis of lessons and lesson refinement in addition to a focus on content. The group involved in video analysis reported higher levels of student learning gains.

Decentralized Control

Simply interacting with others, and with new ideas, practices, and resources does not guarantee that learning will emerge (Hord & Sommers, 2008). For learning to emerge from a complex system these interactions must be enabled in a system with decentralized control (Davis & Sumara, 2006). Decentralized control implies a shared leadership with no one individual making decisions and controlling the system. Davis and Sumara (2006) go so far as to say that a single leader choosing the direction in a system could eliminate the potential for learning to emerge from a system. Similarly, research suggests that for collaborative professional learning to be effective a PLC requires a shared leadership approach (Hipp et al., 2008; Leithwood & Riehl, 2005). Shared leadership implies that teachers participate in determining the focus and direction of professional learning activities (Calvert, 2016). This shared leadership which has surfaced in my findings to be a stimulus for learning to emerge, aligns with what some researchers refers to as teacher agency (e.g., Biesta et al., 2015; Campbell et al., 2016; Day, 2017; Hauge & Wan, 2019). Teacher agency refers to teachers having the autonomy to influence ones' learning and make their own choices regarding the focus and method of their professional learning (Hauge & Wan, 2019). Teacher agency is deemed to be an important characteristic of effective professional learning (e.g., Biesta et al., 2015; Day, 2017; Hauge & Wan, 2019 Sachs, 2016). Shared leadership and space for teacher agency were part of the initial OAME project design. In its final recommendations for professional learning, the OAME final project report recommended that that PLCs use a distributed leadership approach to make space for teacher directed learning (Suurtamm et al., 2017). Other research, such as the study of Ontario's TLLP by Campbell et al. (2016) shared similar findings. The TLLP was designed such that teachers were the designers and leaders of their own professional learning and prioritized the inclusion of teachers' voice in influencing the methods, goals, and content of that learning (Campbell et al., 2016). Campbell et al. (2016) shared that "enabling teachers as developers of actionable knowledge is powerful for sharing improvements in practices" (p. 219). The findings of my doctoral study overwhelmingly pointed to teachers sustaining and diffusing the learning that emerged from their self-designed projects.

Self-Organization

In this doctoral study self-organization manifested across different levels of the OAME project, including ways the project self-organized in response to PLC's needs, the ways different PLCs restructured to achieve a better balance between diversity and redundancy, and the ways that different individuals adapted their practices in response to neighbour interactions across the nested systems of the project. These examples of self-organization are similar to a claim by Davis and Simmt (2003) that systems in education are adaptive self-organizing systems. Self-organization is an ongoing phenomenon that arises from the interactions of parts of a system with decentralized control and the "process of self-organization changes the relationships among the elements of the system as new relationships emerge" (Cochran-Smith et al., 2014, p. 8). These new relationships were apparent in the ways the different PLCs made, and sustained, connections with others within and beyond the project. Thus, connections to teacher agency are present in this characteristic as well. Teachers choosing the tasks to engage with, the content to focus on, the models of professional learning to employ, and the learning experiences they chose to share with others in the project all influenced the self-organization of the learning systems involved in the OAME project. All of these actions were determined and negotiated by the participants in the project rather than through top-down instruction.

Self-organization aligns with the DoI model (Rogers, 2003) and includes the choice to engage with new ideas, the freedom to decide if new ideas are beneficial, and the choice to share new ideas with others. Time to trial and test the innovations was also key in allowing individuals to determine for themselves if the innovation was advantageous. Self-organization is apparent in the DoI model as the decision to adopt a new practice is determined not through a top-down process but through a choice made by each individual or group who chooses to test and trial a new idea before its adoption. It is through the adoption of new ideas that systems self-organize to include these new ideas.

Some researchers suggest that teachers' freedom to choose and set a direction for their learning, both individually and collectively can be connected to teacher well-being as it mitigates teachers' frequent feelings of isolation and loss of agency (e.g., Day, 2017; Carroll et al., 2019; Ingersoll, 2001). A decrease in teacher well-being has been shown to negatively impact student performance (Bajorek et al., 2014; Madigan & Lee, 2021). In my study neighbour interactions in a system with decentralized control supported self-organization in ways that may have had a positive influence on teachers' well-being. Case 4 and 7 specifically mentioned how the project offered them the opportunity to get to know their colleagues at a more personal level, they better understood who their colleagues were as people and not just teachers, and they reported a strong sense of community. The interactions between PLCs provided opportunities for connections for those who felt isolated in their own PLC. They realized that they were not alone, that they were on the right track, and what they were doing mattered. This can be seen as contributing to teachers' well-being, participants reported feeling heard and valued and this could be a reason they engaged in, and sustained learning that emerged from the project. Similar findings were evident in the research. Webb et al. (2009) found that when a PLC focused on teacher well-being and professional learning the impact on student achievement was equivalent to PLCs that focused primarily on content. Similarly, Johnson et al., (2012) suggested that developing healthy relationships with colleagues can positively influence teacher well-being and as an extension student achievement.

Nestedness

The nestedness of the OAME project supported important neighbour interactions between individuals and PLCs. The opportunities for individuals in settings that lacked the necessary balance between conditions such as diversity and redundancy to find this balance with others across the nested system supported the emergence, sustainability, and diffusion of learning. How the nestedness influenced the learning from the project was a significant finding in this doctoral study. Studies that focused on collaboration and supporting communities of practice (Wenger, 1998) generally focused on what an individual learned through their involvement in a project rather than how the project influenced their learning, and the learning of a larger system (e.g., Zehetmeier, 2015). While it would appear that nestedness played a role in teacher learning in the Zehetmeier study, nestedness was not explicitly discussed or dissected. Rather other teachers at a school starting to adopt some of the changes one teacher had made were only briefly discussed. Interestingly, in a previous paper studying the impact of a different learning initiative Zehetmeier and Krainer (2011) suggested that "the taxonomy of levels of impact needs to be extended" (p. 876) to better understand the full scope of teacher learning from the initiative. Understanding the full scope of a project at a variety of levels was a major focus of my study. I have found that considering nestedness as a significant characteristic of complex learning systems was critical when studying teacher learning and considering different levels of learning in order to have a full picture of the influence of an initiative.

Complexity and Diffusion of Innovation Model – A Hybrid Approach

When I began this doctoral study I planned to use two separate frameworks, complexity to discuss the emergence and sustainability of learning and the DoI model to discuss the diffusion of learning. These two frameworks have some parallels, as I discussed in Chapter 3.

Complexity provides language to discuss ways that learning emerges from complex systems while the DoI model provides language to discuss how and why a new idea spreads through a social system. Rogers et al. (2005) suggests that a hybrid approach that considers both complexity and DoI could be helpful to better understand how and why ideas emerge and spread through collaborative learning systems. This doctoral study supports this suggestion. The use of the DoI model was helpful in discussing the diffusion of learning and it also supported my understanding of the emergence of learning from complex learning systems. When considering the ways learning emerged from the OAME project and associated learning systems I found that the DoI model provided language to discuss why this learning emerged. The key stages of knowledge, persuasion, decision, implementation, and confirmation in the DoI model offered more specific descriptions of what was happening in a complex learning system when the conditions and characteristics interact. This specific language was expressly helpful when describing the persuasion stage that involves testing, trialling, and observing that an innovation is more advantageous than a previous innovation. I see these different verbs as different types of neighbour interactions to be negotiated within a system such as a PLC. At the same time complexity supports the DoI model by offering a clearer description of the processes that support the key stages. For example testing and trialling an innovation in a system with a lack of diversity would limit the ways the system interacted with the innovation due to lack of perturbations to the system. Thus, a hybrid approach that describes the characteristics and conditions of complex learning systems combined with the DoI model offers us a more detailed perspective on how and why learning emerges, is sustained, and diffuses from complex learning systems.

CHAPTER 7: CONCLUSION

This chapter begins with a summary of this doctoral study. I then present a discussion on the study's contributions to research on mathematics teacher professional learning as well as implications for professional learning. I then discuss limitations to this study that should be taken into consideration. This chapter concludes with reflections on my learning as a researcher and teacher educator, the next steps in my research career, and some final concluding remarks.

Summary of Study

This qualitative multi-case study involved four cases that originated from different school PLCs involved in the OAME Grade 9 Applied Project. The study focused on understanding how the characteristics and conditions of complex learning systems influenced the emergence, sustainability, and diffusion of learning across different learning systems involved in the project. The study was guided by the following research questions:

1. How did the characteristics and conditions of complex learning systems influence the emergence of learning in the different PLCs, and associated learning systems, of the OAME project?
2. In what ways did the characteristics and conditions of complex learning systems influence the sustainability and diffusion of learning in the different PLCs, and associated learning systems, of the OAME project?

For this study I adopted a social-constructivist worldview that aligned with my beliefs that learning is socially constructed (Bandura, 1986; Dewey, 1938; Vygotsky, 1978; Wenger, 1998). Participants in this study included educators from across Ontario, Canada. Some of the participants had participated in the OAME Grade 9 Applied Inquiry Project between the years of 2014-2016, and others were individuals who interacted with the project participants in various ways since completion of the project between the years of 2016-2021. Data for this multi-case study was collected and analysed through a four-phase process. Phase 0 involved secondary analysis of the original OAME project data. Phase 1 data was collected through the use of an online survey. Phase 2 data was collected through semi-structured interviews with 11 original OAME project participants, and phase 3 data was collected through semi-structured interviews with participants who were connected to phase 2 participants in some way. Phase 2 and 3 interviews occurred in the winter of 2021.

In addressing my first research question I suggest that while all the characteristics and conditions necessary to support the emergence of learning from complex systems were present across the different learning systems associated with the OAME project, specific characteristics and conditions were the most influential. These included neighbour interactions in a system with decentralized control, self-organization, and nestedness. The OAME project provided ample opportunity for teachers to interact with one another across multiple levels of the systems involved; each of these systems had decentralized control. These interactions with other educators and their ideas, resources, and students and student thinking. Self-organization included responding to different interactions and new ideas to form new connections and ways of working. This self-organization was enhanced by the nestedness the OAME project provided opportunities for participants to interact with others beyond their own PLC as well as with new ideas that could push the thinking of the participants. The numerous opportunities for neighbour interactions contributed to lessening feelings of isolation and buoyed the confidence of participants who felt valued and heard through participation in the project.

In addressing the second research question I again noted that all of the characteristics and conditions of complex learning systems manifested across the cases in this study. However, key themes were identified that contributed to the sustainability and diffusion of learning from the OAME project. As with RQ1, the key themes included neighbour interactions, decentralized control, self-organization, and nestedness. Neighbour interactions in systems with decentralized control were mentioned by all participants in either phase 1 or phase 2 and also by several phase 3 participants when they described their use of Twitter to connect with educators across the greater mathematics education learning system. Phase 2 participants shared how they continued to collaborate online with OAME project participants through Twitter. Phase 3 participants reported how they were connected to the OAME twitter accounts and also followed some of the OAME project participants on Twitter. Peter suggested that the widespread diffusion of the BTC model was due, in part, to people like William and Tom who posted their BTC experiences online and shared their positive experiences with others. Self-organization and nestedness also supported sustainability and diffusion of learning after completion of the OAME project. Teachers self-organized in ways that supported their new learning and provided opportunities to diffuse the learning across the nested systems of the OAME project. Understanding why teachers chose specific practices can be explained, in part, by incorporating the DoI model.

The four elements of the DoI model were used as a framework to describe how and why certain collective behaviours were adopted by the participants. These four elements include the innovation(s), communication channels, time, and social system. When analysing the data from this study it became evident that the innovations (the collective behaviours outlined in Chapter 6) were introduced through communications channels (neighbour interactions), over time, in the social system (nested complex learning systems) of the greater mathematics education learning system associated with the OAME project. The key stage of persuasion in the DoI model played a significant role in the adaptation of teachers' practice to include the collective behaviours that emerged from the OAME project. Persuasion appeared to be a key stage in the DoI model that supported the sustainability and diffusion of learning from the OAME project. This research confirms that individuals were persuaded to incorporate an innovation when they are convinced of several factors, the most important being that the innovation is more advantageous than current practices. When considering the findings from the analysis of data it is evident that persuasion was a key stage that supported the sustainability and diffusion of learning from the learning systems associated with the OAME project.

Contributions to Research

This doctoral study contributes to research in several significant ways including contributing to the field of research that employs complexity science as a theoretical framework. Through the adoption of a complexity lens, this study advances our understanding of teacher learning, moving beyond individual cases to explore learning across the nested levels of complex learning systems. This study sheds light on the interconnectedness of teacher learning and emphasizes the importance of considering the nested nature of teacher professional learning. Through acknowledging and interrogating the nested nature of mathematics teacher professional learning this study provided a more comprehensive understanding of the factors influencing that learning.

In order to study complex learning systems and account for the nested nature of teacher learning I adopted a case study approach that was informed by complexity science. Several researchers (e.g., Anderson et al., 2005; Cochran-Smith et al., 2014; Ell et al., 2019; Heatherington, 2013) argue that traditional case studies reduce complex systems to simple linear reductionist process and suggest that many individuals who adopt complexity do not address this issue. Ell and colleagues (2019) suggest that there is a growing body of research in education using a complexity lens. This study therefore contributes to this growing body of research by describing how I adapted a case study approach to not only consider individual cases but also honour the nested nature of teacher learning by considering the connections between and across these cases.

Another contribution to educational research was providing an example of a study that incorporated a hybrid approach of complexity science and the DoI model. Rogers et al. (2005) suggested that combining these two approaches might provide insight into how to better plan and implement effective collaborative processes for social groups such as mathematics teachers. The use of this hybrid approach has not been widely explored in educational research. Findings from this study suggest that this approach offers valuable insight towards understanding both how an innovation is adopted, sustained, and diffused and the conditions and characteristics that can support a learning system in navigating the adoption of these innovations.

Perhaps one of the most significant contributions of this study was the perspectives provided on mathematics teacher learning over an extended period of time. The secondary analysis of the extensive data from the original OAME project collected between 2014 and 2016 offered insight into how learning emerged from a professional learning initiative. The collection of new data in 2021 allowed a more in depth understanding of the ways that teachers sustained and diffused this learning once supports from the project were no longer available. In order to appreciate the impact of professional learning, research needs to take place over extended periods of time. Revisiting participants allows us the opportunity to better understand the effectiveness of initiatives and what teachers deemed as meaningful to their practice.

As I discussed in my review of the literature the role of mathematics teachers has evolved over the last several decades. The professionalization of teachers has emerged as an important area of focus in research and centering the voice of the teacher was an important component of this study. This study highlights the importance of conducting research that centres teachers' voices as teachers' perspectives, expertise, and experience are invaluable assets that enrich the research process and contribute to more meaningful understanding of the complexity of mathematics teachers' collaborative professional learning.

Implications for Mathematics Teacher Professional Learning

The opportunity to understand how learning emerged during the original OAME project and how this learning was sustained and diffused has important implications for mathematics teacher professional learning. When we consider how the different PLCs and cases interacted and learned both during and after the OAME project it is clear that those who achieved a balance between conditions like diversity and redundancy and operated in a system with decentralized control were better positioned within their own contexts to engage in neighbour interactions which contributed to effective professional learning.

Supporting and establishing the key conditions and characteristics of complex learning systems aligns with much of the research on the characteristics of effective professional learning (e.g., Anderson et al, 2018; Hord, 2008; Liljedahl, 2018, & Loucks-Horsley & Matsumoto, 1999). Providing the time, space, and agency to inquire into ones' practice with colleagues is paramount to supporting the emergence of learning but is often not sufficient to support teachers in their learning and in sustaining their learning (Day, 2017; Hauge & Wan, 2019). My study demonstrates that considering the ways to establish a balance of conditions and characteristics necessary to support learning is crucial. Thus, whether designing professional learning explicitly or considering how to establish environments conducive to teacher learning, we should consider, and strive to put in place, the conditions and characteristics of complex learning systems.

Particularly, those in positions to influence learning systems should be conscious of the ways that decentralized control, self-organization, and nestedness are manifested in their context and strive to support these factors in order to support neighbour interactions.

When a learning system such as a teacher, PLC, or school is unable to achieve this balance of conditions within itself, access and agency to connect with others within the larger educational setting may be able to establish this balance. Thus, organizing teacher learning within a nested complex learning system can best support teacher learning. Opportunities to meet and interact with others beyond a teacher's own context can provide the necessary perturbations to encourage teachers to continually push their thinking and engage in lifelong learning.

This study focused on mathematics teacher learning in two different contexts. One context being during a long-term professional learning initiative and the other being five years following the initiative. What was evident in this study was that the professional learning initiative provided teachers the time and opportunity to self-organize and create collaborative professional relationships and communities within their own schools and across the greater mathematics education community. These relationships and communities contributed to not only the emergence of learning during the project but teacher agency which helped to foster the sustainability and diffusion of learning after the initiative. Thus, an implication from this study is that designing and supporting professional learning initiatives should provide time and space for the establishment of relationships, community, and teacher agency. These may have a long-term impact on teacher learning, providing sustainability and diffusion beyond the scope of the original project investment.

Limitations

Limitations of this study stem from several areas including participant recruitment, the differences in data available for analysis from each case, and participant bias. As I mentioned in Chapter 4, participant recruitment was a limitation of this study. Over 80 individuals participated in the OAME project between the years of 2014 and 2016 yet only 14 individuals completed the online survey and nine agreed to be contacted for phase 2. I was able to recruit three additional participants for phase 2 bringing the total phase 2 participants to 12. Among the 14 individuals who completed the survey seven of the participants reported that the project had a large influence on their professional learning, five reported a moderate influence, and 3 reported a small influence, (one participant chose both large and moderate). Therefore, 64% of participants surveyed agreed that the project had a large to moderate influence on their professional learning. Of those who stated the project had a small influence on their learning only one agreed to participate in phase 2 of the study. This limited data collection to those who felt the project had a large to moderate influence and possibly limiting the diversity. Creswell and Poth (2021) refer to this as selection bias, that it is possible that those "who desired to participate were somehow different than…those that decided not to participate" (p. 598). While I see this as a limitation to the study, I argue that those that did participate offer us insight into components that supported teachers who found benefit from professional learning.

A second limitation is the differences in the detail and amount of data collected from each case during the OAME project. For example, Case 7 had more detail as I was the RA at that site for two years and was also the focus of my MA research. Thus, I collected additional data from that case and developed a deeper understanding of that case. Other cases were more remote, and it was difficult for RAs to attend every monthly meeting. In cases like this the RA would discuss a summary of the monthly meeting with one or two participants. This altered the level of detail in the data for these cases. Also different RAs had different levels of experience and interest in the research being conducted. Margaret for instance, at Case 4 was an experienced researcher and mathematics teacher educator with a vested interest in the work that was being done. She planned to publish research articles based on her involvement at that site so the detail in her notes and data collection is richer than at other sites. Margaret also engaged in the learning alongside the PLC, as reported in Chapter 5. This level of engagement was not consistent across all research sites during the OAME project. To be clear, it was not a requirement that RAs engage in the learning to the degree that Margaret did, none the less, this could have altered the data collection process.

Finally, the process of talking about one's learning with someone outside of their own personal practice is not always an easy thing to do. Participant bias may have influenced the responses that participants provided. Participant bias refers to "participants being aware of what the researcher is trying to investigate, or anticipates finding, and what this implies for how participants are expected to behave" (McCambridge et al., 2012, p.1). It is impossible for me to know if participant bias played a role in influencing my data collection process, but it is important to consider this as a limitation to this study.

Reflections and Future Research

As I engaged in this research and in writing up the findings and discussion I realized that my understanding of what it means to engage in research using a complexity lens evolved and expanded. As I finish this study I step away with an understanding that complexity and complex systems are several things all at once. Complexity is a theory, it is a way of viewing the world that forces us to break out of linear cause and effect patterns, and it is also a goal, a state to strive for in order to support the emergence of learning. By understanding the importance of achieving a balance between key conditions such as diversity and redundancy, neighbour interactions in a system with decentralized control, and coherence and randomness we can perhaps use this understanding to better support teachers as they engage in collaborative professional learning. This is true across many phenomena including classroom learning, teacher professional learning, and in educational research as a whole.

Through discussions with my thesis supervisor I now appreciate that the use of complexity as a lens is not only limited to gaining a deeper understanding of teacher learning it can also be applied to how I view the process of engaging in educational research. When I think back to how this doctoral journey started I realize the pivotal role the ICMI study played in forming my own understanding of mathematics education research. The focus of the ICMI study was *Teachers of Mathematics Working and Learning in Collaborative Groups.* Of course, this aligned perfectly with what I felt was the important area to focus on in terms of supporting teacher learning, but it was also the focus of how we, as researchers, engaged in the ICMI study. We were researchers from all over the world with diverse backgrounds, engaged in deep and rich discussion through a shared focus of mathematics teachers' collaborative learning. By the end of the study I had formed professional relationships and a community with several of the other participants that continue to inspire and drive my own research. One researcher in particular, Dr. Robin Anderson, has become an important co-collaborator and together we have connected with others in the field of complexity studies in education.

Robin and I have been engaged with colleagues to begin to model complex learning systems. We have presented at the American Educational Research Association (AERA) conference in both 2022 and 2023 as part of the Complexity Theories in Education Special Interest Group. This work has connected us with others who are also engaging in research using complexity as a theoretical framework. Together at AERA we recently orchestrated a workshop titled *Utilizing Visualizations of Complex Educational Phenomena as a Way to Justify Data Sources and Sampling* (Bullock et al., 2023). This work has contributed to my understanding of the methodological considerations when conducting research using complexity, an understanding that I did not have when I began this study. I have aligned with others who are examining this phenomenon and we continue to develop methodological underpinnings to support research using a complexity science framework from which I can draw from.

As I continue to engage in researching and designing professional learning for pre-service and in-service teachers I am drawing on the conclusions from this study and my experiences collaborating with others. Most notably, that for learning to emerge individuals and groups require opportunities to interact with others' ideas, hunches, and representations in a system with high degrees of agency. Also notable were the key stages of the diffusion of an innovation and the time, space, and agency for individuals and groups to determine for themselves if an innovation is advantageous and ultimately might benefit teaching, learning, and even research in their own contexts. I am part of a newly formed working group exploring how we can understand and build pre-service and in-service teacher capacity related to teaching and learning mathematics through coding and computational thinking in the province of Ontario. This future research will incorporate both the DoI model and complexity science as theoretical frameworks. Again this work came about through interacting with other researchers from different contexts who were all focusing on a shared goal.

Concluding Comments

Interacting with and listening to teachers has always played a key role in my own learning and has been a driving force in my research. As I described in the introduction of this dissertation, meeting and learning with Laurey at ICMI had a profound impact on how I focused this dissertation. I began to reconsider how I define teacher learning and the role that teachers play in seeking out and designing their own learning. My understanding of the type of study that could capture the essence of teacher learning expanded when I realized the complex nature of this learning. Finding a framework that could account for the different components of teacher learning was of paramount importance. Through engaging in course work that exposed me to different theories and philosophies of education I was exposed to different ideas that aligned with my worldviews of teaching and learning. Through the iterative process of reading, thinking, and discussing these ideas with my supervisor and committee and others in the field I was able to develop a deeper understanding of the role that both complexity and the DoI model play in presenting this study in a way that both honoured teacher voice and was supported through current educational research. What started as a conversation with one teacher led to an understanding of the many ways that teachers and researchers are interconnected and how learning does not stop once professional learning initiatives conclude. Respecting and honouring teacher agency, centering teacher voice, and putting in place conditions that support the emergence of teacher learning can greatly enhance opportunities for teachers' professional learning. As William stated many years ago when I first began my research journey, we really are "Better Together".

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Appendix A: Original OAME Project Data Collected

Case level data (not every case had every item listed):

- PLC application form
- Ethics application for each school district
- Monthly case reports from 10 PLCs for two years of the project
- Monthly PLC meeting data
  - audio files
  - transcripts
  - photos
  - artefacts such as lesson plans, rubrics, resources, research articles
- researcher observational notes
- Summer institute team meeting data
  - audio files
  - transcripts
  - photos
  - artefacts such as lesson plans, rubrics, resources, research articles
- researcher observational notes
- Exit interviews with individual PLC members
- Email correspondence with research teams
- EQAO school report results (2011-2012, 2012-2013, 2013-2014)
- EQAO school district results (2011-2012, 2012-2013, 2013-2014)

Project level data:

- Adobe Connect meeting audio files
- Project-wide meetings
  - presentation slides
  - schedule of events
  - discussion panel audio files
  - RA field notes
- Project-wide collaborative group session
  - audio files
  - partial transcriptions
  - photos
- Facilitator focus group sessions
  - audio files
  - partial transcriptions
- Summer institute feedback response form submissions
- Summer institute collaborative activity descriptions
- EQAO provincial reports (2011-2012, 2012-2013, 2013-2014)
- Final project report (Suurtamm et al., 2017)

Appendix B: Phase 1 Survey Responses

Respondent 1: Drew - Case 1	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	School Board Lead Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A large influence. The other teachers that we were able to network with and learn alongside with was amazing!! Really challenged our thinking on spiralling and rich tasks
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	Still use many of the resources created in my own classroom now that I returned to my classroom.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	Yes, just though email and texts.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	Just through Twitter and email
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	The in person learning weekend in London. Working so closely with Chris Suurtamm was an amazing opportunity. The group PD was always amazing!
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	Respondent skipped this question
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	The teacher led approach to this was great!
Respondent 2: Anonymous	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher

Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching,
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A small influence.
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	not teaching grade 9 currently
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	teaching more Phys ed than math right now so no
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	teaching more Phys ed than math right now so no
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	Respondent skipped this question
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	Respondent skipped this question
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	more focus on process and not product less answers and more questioning

Respondent 3: Jill - Case 1	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students Because I love learning about the ideas others are researching. And I love working with teachers from around the province. (And I applied for the project because I am always on the lookout for funding)

Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I wrote and submitted the proposal on behalf of the PLC
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A large influence. We learned about using rich tasks to drive instruction from William and Chris Suurtamm suggested a Jo Boaler book for our group to read and try. Following that our group continued to learn about Thinking Classrooms and spiraling curriculum mostly being influenced by the group from Fields and William and Wayne's energy and excitement about their work.
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	Absolutely. As stated above, I am still utilizing and learning about the Thinking Classroom, and any course that I have pedagogical control over I use spirals and rich tasks. When I am working with colleagues to develop a course plan, I have influenced many to spiral their curriculum. I also have been able to add rich tasks when co-planning
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	Yes, but not all members. [Teacher Name] and [Teacher Name] and I applied for a TLLP grant to continue our learning about consolidation of thinking tasks. We continue to collaborate even when [Teacher Name] has been at another school. Last year [Teacher Name] and I received a grant to incorporate Indigenous Knowledge in our math classes with help from a knowledge keeper. We continue to seek counsel from each other. The others in the group found the work to be too much as life threw other challenges at them. And the Student Success teacher and the Special Education teacher moved to difference roles. I do wish we had been able to maintain this connection.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	Yes. Tasha has presented with me at the CEMC summer mathematics teacher conference. [Teacher Name] and I communicate over Twitter. Some of the funds for the TLLP project paid for our group to see William at Field's and then for Tasha and [Teacher Name] to come to see us at [school district]. I also continue to follow the work of [Teacher Name], [Teacher Name], [Teacher Name] and [Teacher Name], [Teacher Name], [Teacher Name] via twitter. All of whom I met or reconnected with in this project.
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	Learning together. Feeling supported for our learning by the principal and the project researchers. (empowering!!). Collegial discussions - with my PLC and from around the

	province. Having discussions with my principal about how to best support student learning in mathematics. (I felt heard!!). Summer Institute where I feel valued for the work I do and where my learning is supported.
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	Change the end date. I wish it had continued. Change I guess the original framing of the project. Year 1 was not as life changing as the following years. Perhaps because our first year was a distributed leadership and our learnings were not immediately apparent. After year 1 the PLC no longer included the Special education teacher, the student success teacher, and several secondary math teachers. Also, the math coordinator went on maternity leave. Year 2 we gained teachers from the elementary panel and their administrators. Also, the leadership for the PLC changed to a coleader from distributed leadership.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	Yes, I look wider than my school board for community. (I did a little before, but I feel much more connected to this community now). Yes, I try to build collaborative groups in a more formal way now, including applying for grants and utilizing release time. (for those in my school or school group). Finally, I have the confidence to stick to my new convictions - utilizing 5 Practices, spiralling, Thinking Classroom etc. I feel informed/empowered about the pedagogical choices I make, so I can defend them to parents/admin or try and bring them on board. My professional learning has mostly continued along the pathway introduced to me by the Math 4 The Nines project (the ideas shared by Chris Suurtamm, [Teacher Name], you, and Alison Macauley have been very impactful).

Respondent 4: Elena - Case 10	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students, To increase my opportunity for career advancement, I looked forward to getting together with teachers outside out district and learning about what was happening in other parts of the province to get a broader perspective of education in Ontario
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator

Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A moderate influence, The components of the project that were most influential were the opportunities to share innovative ideas that teachers felt were making a difference in student learning. The workshops/sessions that most stuck with me were those that involved orchestrating group work in a way that lowered the floor and raised the ceiling for all learners - including learning about the work of Margaret Smith and Mary Kay Stein and Peter Liljedahl. It is most meaningful for me to get new information from the original source where nothing is lost in translation.
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	Yes, it is. I have not taught the Grade 9 Applied course for a couple of years, but I do use the strategies learning in all my teaching – my classroom is set up to allow for group work on non-permanent vertical surfaces. While I have not eliminated traditional furniture in my classroom as some have, I try to incorporate group work at least once for each unit I cover in the math course, depending on the content. Also, I changed my assessment strategies to de-emphasize marks (although I was already on this path prior to participation, my learning during this project empowered me to stay on track with this). I became more flexible with how students showed me they had met learning goals, creating assessments that were less test-like for some of the units I taught and allowing students to re-take tests after they showed evidence that they had spent more time practicing skills.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	While we do not have a formal PLC, I do think the project allowed our department to evolve. Our department head was a traditionalist but the project opened her mind to new ideas and she was a little more open to progressive ideas and supportive of teachers who wanted to try them by providing the required tools - white boards, technology such as ipads, etc.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	Currently, I only follow other participants on Twitter to see what they are doing. However, I don't typically share what I am doing beyond the math teachers in my school board.

Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	-watching teachers putting their ideas into practice in presentations/workshops and experiencing the student perspective to get a better understanding of why some strategies work better than others -being able to present our own ideas to colleagues from around the province and get feedback on our work -meeting and sharing with teachers in informal settings, ex. over lunch
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	While I really enjoyed in-person meetings, the cost of the project was always at the back of my mind. I think virtual sessions might be as effective for some aspects of the project.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	In some ways...I follow many educators that I met or heard about during the project on Twitter to get ideas to incorporate into my own classroom.

Respondent 5: Chrissy - Case 4	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher, Department Head
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I wrote and submitted the proposal on behalf of the PLC
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A large influence. I wished that I had staff that were as excited about the project as I was, but the excitement grew as we went and saw positive results in our students. Most influential component was the opportunity for TIME. Although it was work for us to prepare to be away, we felt valued that our time was honoured and valuable enough to use school time and not additional time.
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	We are still using the whiteboards (both individual and on the wall) and have continued to value non-permanent and low-risk methods of communication for students. We have also had a greater buy-in for collaborative projects since the project finished.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now	As the team changes due to retirements and new teaching assignments, we have extended our discussions to include partner schools in our board.

and how it may have evolved? If no, please explain why not.	
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	I have continued to share emails and follow the work of other participants through their work with other OAME projects. Our team as a whole is more confident to reach out to OAME documents and best practices of other teachers and I have had many great discussions with colleagues about what they are reading and what they are trying.
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	Building a collaborative team in my school. Identifying my place as a member of the provincial math community and valuing my contributions (not being intimidated that we're not ready to try)
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	Having more connections online. Maybe it's because we are that much more comfortable now, but we could have made more frequent contact. That's not to say that our coach was unavailable, because she was always responsive to emails in a timely manner.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	Because we got so much from the project and our participation in the annual general conference, we have continued to participate in online conferences, webinars, podcasts, and professional reading both as individuals and in small groups.

Respondent 6: Tasha - Case 7	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher, Special Education Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students, Other (please specify): To collaborate with colleagues and see what others across Ontario are doing
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I wrote and submitted the proposal on behalf of the PLC
Q4 How would you describe the leadership of your PLC?	Other: I wrote and submitted my proposal, but I was just a classroom teacher, so the leadership role was assumed by the math department heads. I could describe it as shared, in that I was able to collaborate in different aspects. The direction I intended for the project was not how it played out.
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A small influence. Professional learning covers a broad spectrum, so my response is direct at my teaching practices. What attracted me to the project was that I was fortunate to have been at a school who has led the change in teaching practices and our administrator participated and

	encouraged our innovations. Much of what we were doing in our classrooms were well-received by students and parents. I hoped the project would lead to more collaboration in developing a program of resources to build math confidence in our vulnerable students. Instead, we saw what other schools in the province were doing and realized the importance of promoting change.
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	A lot of what came out of the project is being encouraged today, especially with de-streaming of math. I take every opportunity I can to share the resources produced from it.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	I have switched school, so collaboration is limited. Also, within our district, math PD tends to first be shared with administrators and department heads. From there, the opportunities are often only shared with chosen staff, so anyone else might not be aware of them.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	I still keep in touch with a few of the participants. We have collaborated on other projects and PD workshops, as well as connect at OAME's annual conference, through email and social media.
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	...meeting and sharing experiences, perspectives, and resources with colleagues all over the province.
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	...more people of diverse backgrounds, varying perspectives, and experiences. This would have been a great opportunity for student teachers to be a part of.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	The project has emphasized the importance and need for professional learning if we want our future generations (everyone) to have a voice and live in a just and equitable society.

Respondent 7: Maya - Case 10	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students,
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC

Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A large influence. The project was extremely influential for myself and my department. There was a huge culture shift in my department on teaching practices and how students learn best.
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	The OAME project was a huge influence in my career, it ignited me to move forward in my learning and I continued work with a few grants. I was granted over $50000 in a TLLP that enabled my department to further research and grow in our teaching practices.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	I would attribute a huge part of me being a part of the OAME project to my growth as an educator. Because I loved the learning, I did a lot to further my career and currently I am the Math/Science consultant in my board, so I have the pleasure of sharing my experience with the entire board.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	I have kept in contact with a few people involved but mostly through Twitter.
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	I enjoyed all of the learning but more importantly the connections that I made with other educators throughout Ontario.
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	I think that we did a good job of getting the word out to teachers, but I suppose sharing more frequently and getting the information to more teachers.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	Absolutely, like I stated before it ignited my love of learning and changed my teaching practice and ultimately the project was a kickstart to my career advancement

Respondent 8: William - Case 7	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students,
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A large influence. Lesson study helped me grow in my knowledge of pedagogy. The project also allowed me to share with people all across

	Ontario, this benefitted me as a leader in the math community.
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	Yes. I still use some of the pedagogy that we tested in the lesson studies. Snowballing, VNPS, VRG, spiraling to name a few.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	No most of them have moved on from our school-still collaborating but with new teachers to our school.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	Yes-many of us are connected on Twitter. Also, prior to the pandemic I was able to go to one of their school and work with them for a day.
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	Collaboration with colleagues. Changing to effective pedagogy. Presenting findings.
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	Being able to visit other schools involved in the project.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	More collaborative-and more learning of pedagogy close to the classroom.

Respondent 9: Wayne – Case 7	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Math Department Head
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students, Other (please specify): To work closely with colleagues
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	Involved in the submission process
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A large influence. Time to discuss ideas with colleagues is invaluable and this project allowed a great amount of time for in-depth discussions and exchange of ideas.

Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	Yes. Although I am no longer teaching, this project has been a major influence on my graduate work.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	Although I no longer work with them directly, I continue to have email and in-person discussions with members of my school PLC.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	Not anymore.
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	having our principal directly involved in our work was incredibly important to support and legitimize our efforts.
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	building in an element of sharing of learning with colleagues within our school and district who we know and who know us.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	Yes. I am more inclined to freely discuss ideas with colleagues and better equipped to deal with disagreements.

Respondent 10: Anonymous	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher, Student Success Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students,
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A moderate influence. A large influence. (recipient chose both). Most influential were the shared ideas resulting in many different approaches in the classroom.
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	I have moved on to different role, special education but can still draw on knowledge for our students

Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	Not really, more involved in other areas. Some of our PLC are still involved in math education and there is now an emphasis on destreamed math.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	Respondent skipped this question
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	seeing the enthusiasm throughout the province and realizing that we were already an active part of the change in approach to the applied students
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	Respondent skipped this question
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	Respondent skipped this question

Respondent 11: Anonymous	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students,
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A moderate influence. I was a grade 7 math teacher in a K-8 school, working with math teachers from the secondary panel so this project allowed me to better understand what math looked like on the grade 9 applied stream. Having time to collaborate on what to anticipate from students when provided a variety of problem-based questions was very helpful. It allowed me as a teacher to better prepare myself for asking students questions and guiding them toward a better understanding of math.

Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	Yes, I'm still using more open based problems to guide student learning.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	I have since switched schools and PLCs were cancelled due to the pandemic.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	I follow some of the other PLC members through social media.
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	Gaining a better understanding of what my students could expect in secondary school, creating a folder of open problems to use with our students and attending/presenting at the OAME conference.
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	Seemed repetitive at times. Sometimes I felt the conversations circled back to the same thing, however I really enjoyed seeing the math in action during classroom visits.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	I was a brand-new contract teacher when I was involved in this project so it's tough to say.

Respondent 12: Anonymous	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices, To improve learning for my students, To increase my opportunity for career advancement
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC.
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A moderate influence. It influenced me to take risks and introduce new strategies in my teaching. I had a better understanding of what to look for to determine whether or not my students were gathering a conceptual understanding of

	mathematics. It gave me ideas on how to make lessons more student centered.
Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	Yes. I have incorporated many of the new learnings into my teaching. I often use VNPS in my units of study. I use the 5 practices of orchestrating math discussion in conjunction with VNPS. I follow on Twitter those teachers in the project that continue to share their ideas to gain fresh ideas.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	We have very informal collaboration - I have shared with new teachers the ideas/strategies that I've gathered on assessment and curriculum implementation both verbally in informal discussions over lunch or after school and electronically by sharing my files.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	Only via social media and it is very one-way. I rarely post on Twitter, but I often go to Twitter to keep abreast the kinds of conversations that are happening in the math world.
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	Meeting teachers across the province to have broader perspective of what math learning looks like across the province.
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	Respondent skipped this question
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	I am at the end of my career, so I am no longer looking into professional learning for a high school setting. I may pursue a position in a Faculty of Education and I think that my experience would help me teach teachers better...in this case, I would look for opportunities to learn how to teach adults.

Respondent 13: Anonymous	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Mathematics Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve my teaching, To learn new ideas and practices
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator
Q4 How would you describe the leadership of your PLC?	Single/one individual leader that determined the direction of the activities of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A small influence.

Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	As a person who sat in on a couple of meetings - the valuable part was the time to consider different teaching strategies for different students. However, it was like much of teaching resource opportunities - a one and done event. We consider possibilities and return to the same practices in the classrooms. I may have adopted some changes - but really cannot think of anything specific.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	No - transfers and staff reassignments have made this difficult.
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	no
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	Time to reflect on learning practices with other teachers and trying to brainstorm solutions for specific students or classes.
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	Monthly meetings were hard to make as I missed half due to coaching and other school events. With meetings scheduled for the majority - I felt little continuity with group.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	The best ideas come from collaborating with experienced teachers - new fangled approaches usually don't last and quickly are dispensed with if they are not practical on a daily basis.

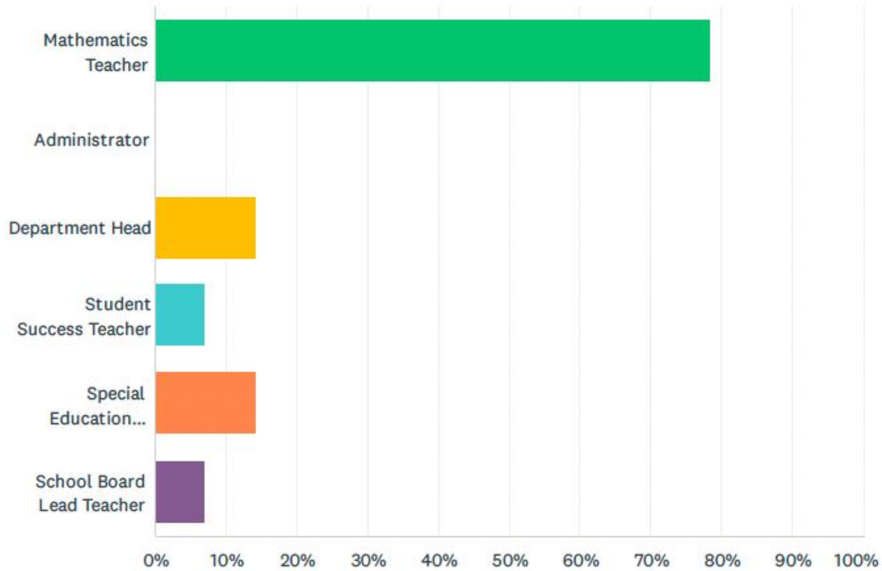
Respondent 14: Charlotte – Case 10	
Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?	Special Education Teacher
Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)	To improve learning for my students
Q3 How did you come to be involved in the OAME Grade 9 Applied Project?	I was invited to participate by my department/school/district/administrator
Q4 How would you describe the leadership of your PLC?	Horizontal/Distributed/Shared leadership across all members of the PLC
Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?	A moderate influence.

Q6 Is the OAME Grade 9 Applied Project still influencing your practice? If yes, please describe how.	I see it in the Math work my students are doing in the Resource Room.
Q7 Are you continuing to collaborate with members from your school PLC? If yes, please describe what this collaboration looks like now and how it may have evolved? If no, please explain why not.	Not really
Q8 Are you still collaborating with participants from other PLCs involved with the project? If yes, please describe what this collaboration looks like now. (ex. Face to face, email, shared drives, social media)	No
Q9 Please complete this statement "Some of the highlights of the OAME Grade 9 Applied Project for me included…"	learning new and different ways to reach learners who had a fixed mindset about mathematics. The use of non-permanent vertical surfaces really engaged the students and gave them the ability to try new things.
Q10 Please complete this statement "Some of the features of the project that I would change would include…"	More ability for the special education teacher to be in the classrooms and engaging students that way.
Q11 Has the OAME Grade 9 Applied Project influenced how you approach your own professional learning? If yes, in what ways?	Yes, it allowed us to collaborate with people from other boards and glean best practices which helped in our professional learning.

Appendix C: Quantitative Survey Data

Q1 What was your role during your involvement with the OAME Grade 9 Applied Project?

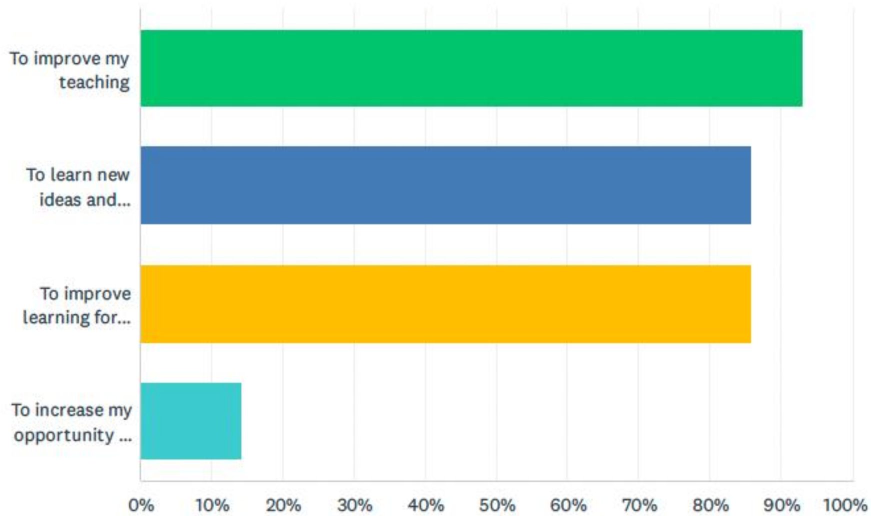
Answered: 14 Skipped: 0



ANSWER CHOICES	RESPONSES	
Mathematics Teacher	78.57%	11
Administrator	0.00%	0
Department Head	14.29%	2
Student Success Teacher	7.14%	1
Special Education Teacher	14.29%	2
School Board Lead Teacher	7.14%	1
Total Respondents: 14		

Q2 Why did you participate in the OAME Grade 9 Applied Project? (choose all that apply)

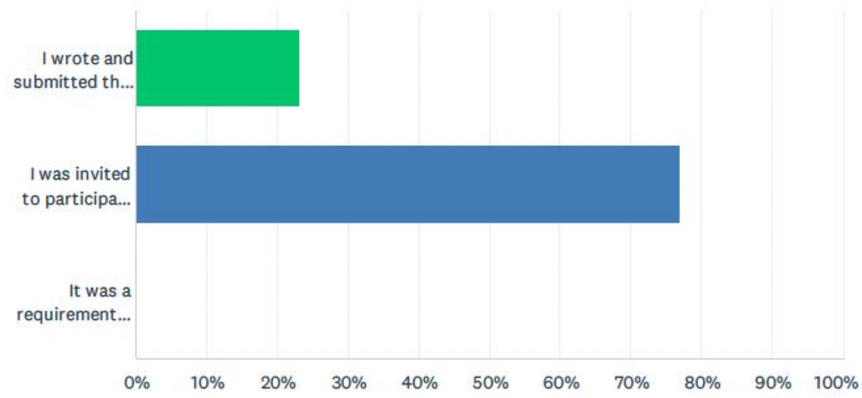
Answered: 14   Skipped: 0



ANSWER CHOICES	RESPONSES	
To improve my teaching	92.86%	13
To learn new ideas and practices	85.71%	12
To improve learning for my students	85.71%	12
To increase my opportunity for career advancement	14.29%	2
Total Respondents: 14		

Q3 How did you come to be involved in the OAME Grade 9 Applied Project?

Answered: 13 Skipped: 1

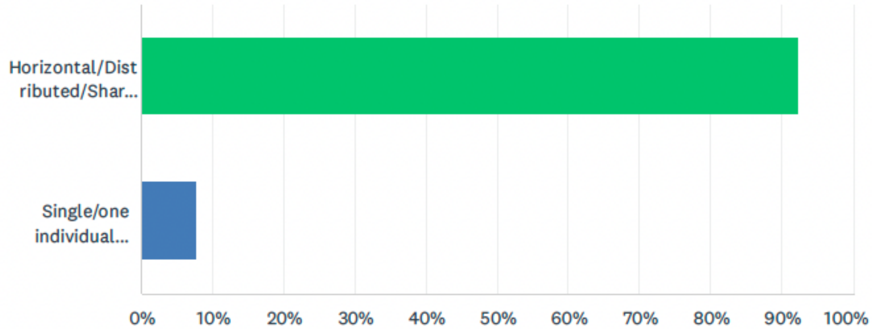


ANSWER CHOICES	RESPONSES	
I wrote and submitted the proposal on behalf of the PLC	23.08%	3
I was invited to participate by my department/school/district/administrator	76.92%	10
It was a requirement from my department/school/district/administrator	0.00%	0
Total Respondents: 13		

#	OTHER (PLEASE SPECIFY)	DATE
1	Involved in the submission process	9/23/2021 10:31 PM

Q4 How would you describe the leadership of your PLC?

Answered: 13   Skipped: 1

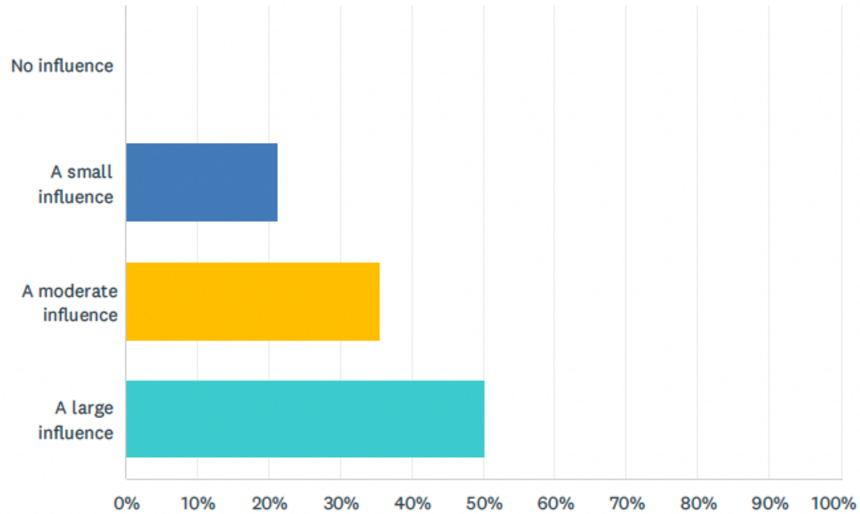


ANSWER CHOICES	RESPONSES	
Horizontal/Distributed/Shared leadership across all members of the PLC	92.31%	12
Single/one individual leader that determined the direction of the activities of the PLC	7.69%	1
TOTAL		13

#	OTHER (PLEASE SPECIFY)	DATE
1	I wrote and submitted my proposal, but I was just a classroom teacher, so the leadership role was assumed by the math department heads. I could describe it as shared, in that I was able to collaborate in different aspects. The direction I intended for the project was not how it played out.	8/31/2021 8:23 PM

Q5 To what degree did the OAME Grade 9 Applied Mathematics Project influence your professional learning?

Answered: 14 Skipped: 0



ANSWER CHOICES	RESPONSES	
No influence	0.00%	0
A small influence	21.43%	3
A moderate influence	35.71%	5
A large influence	50.00%	7
Total Respondents: 14		

Appendix D: Example of Phase 2 Interview Protocol

Thank you for agreeing to be part of my research project. I greatly appreciate you taking the time to meet with me to discuss the work you engaged in with the OAME project. The focus of my study is to reconnect with participants from the project and follow up on ways that the project may have influenced them. In particular I'm curious about the learning that emerged from the project, whether or not you are able to sustain the collaborative professional learning that the project supported, and whether any specific parts of the project have diffused beyond the original participants.

1. I realize that your own professional learning did not begin with the OAME project, particularly because I was part of your PLC as an RA with the project. So maybe we can begin with talking about your response to the survey where you responded that the project had a large influence on your PL.
    a. In what ways did your involvement with the project influence your learning and practice? Can you provide a specific example?
    b. In what ways could your experience have been different? Can you provide a specific example?
    c. Lastly you talked about how having your principal involved "legitimized" your efforts. Can you tell me a bit more about that?

2. Can you tell me a bit about how decisions were made regarding the activities you engaged in during your monthly PLC meetings? Can you provide a specific example?

3. Do you think that your participation in the OAME project had an influence on your students? If so, in what ways? Can you provide a specific example?

4. Can you tell me about ways that you think the project influenced you since we last met together as a project? Why did it influence you?

5. Do you still collaborate with any of the original members of your PLC? Can you tell me more about this?

6. Do you still collaborate with any of the project participants beyond your original PLC? Can you tell me more about this?

7. Do you feel that the work you and your PLC engaged in influenced others that were not directly involved with the OAME project?

8. What does your PL look like now? (now that funding is gone, time to meet reduced etc.)

9. Has your knowledge, or your beliefs an attitudes about teaching and learning mathematics evolved since the OAME project, or during your career as a teacher?

10. Lastly, can you think of anyone that may have been directly influenced by the work that emerged from the OAME project? Do you think that they might be interested in speaking with me for this study?

Appendix E: Example of Phase 3 Interview Protocol

1. Can you tell me about yourself and your involvement in mathematics education?

2. Can you tell me about your connection to Dolores?

3. In what ways did your connection with Dolores, and the OAME Grade 9 Applied project influence your own learning and practice?

4. How did you incorporate the influence of the OAME project into your own practice?

5. Are you still collaborating with Dolores? If so, can you tell me more about that?

6. What other ways do you collaborate with others and/or engage in professional learning and development?

Appendix F: Phase 2 Letter of Information and Informed Consent

Dear colleagues,

Thank you for taking the time to complete the OAME Grade 9 Applied project survey sent to you earlier this year. Hearing more about your experiences, and those of your professional learning community, will contribute to a robust description of the OAME project, how participants engaged in the project, and the influence the project may, or may not, have had on individual participant's professional learning and practice. I would like to invite you to participate in the second phase of this research project. As I mentioned previously, the purpose of this research study is to gain an understanding of the different ways that collaborative settings may contribute to, and support, mathematics teacher learning and practice and the influence that this learning has on different components of a learning system. These different components can include, but are not limited to teachers, schools, professional learning communities, and school districts.

This second phase consists of an individual interview which will take approximately 30-45 minutes, and an optional focus group interview which will take approximately 30-45 minutes. Both of these interviews will occur virtually using software such as Zoom, or Microsoft Teams. If you prefer, the individual interview can be conducted over the telephone. With your permission, the interviews will be audio recorded in order for me to fully engage in our conversation, without having to take detailed notes. Following the interviews, I will transcribe the interview myself, and will offer you a copy for your records. Your participation in this study will include you responding to questions about your teaching practice and professional learning. You have my assurance that every effort will be made to respect your personal reflections in the writing and reporting of the research. If, at any time, you wish to end the interview, and/or withdraw from the study your request will be respected. Should you choose to withdraw from the study and prefer that I not use your interview data collected up to that point, I will remove it from the research.

This study will be carried out under the supervision of Dr. Chris Suurtamm at the University of Ottawa, Faculty of Education with funding provided by the Social Sciences and Humanities Research Council of Canada. The data is being collected for the purposes of my PhD thesis in mathematics education and perhaps for subsequent research articles.

If you are interested in participating in this research study please review and sign the attached consent form and email a signed copy to me at _________ Upon receipt of your signed consent form I will email you to schedule an interview to take place at a time convenient for you. If you have any questions please do not hesitate to get in touch with either myself or my supervisor, Dr. Chris Suurtamm.

Sincerely,

Kelly McKie

Informed Consent:

University of Ottawa, Faculty of Education
Telephone:____________
145, Jean-Jacques Lussier
Ottawa ON K1N 6N5 Canada

Name of Professor:
Dr. Christine A. Suurtamm

Name of Student:
Kelly McKie

Invitation to Participate: I am invited to participate in the research study entitled *Deepening Our Understanding of the Complexity of Mathematics Teachers' Collaborative Professional Learning* conducted by Kelly McKie in partial fulfilment of the requirements for the degree of Doctor of Philosophy of Education. This research project is funded by the Social Sciences and Humanities Research Council of Canada.

Purpose of the Study: I understand that the purpose of this research study is to gain a fundamental understanding of the different ways that collaborative settings contribute to, and support, mathematics teacher learning and practice and the influence that this learning has on different components of a learning system

Participation: My participation will consist essentially of one audio-recorded session of approximately 30-45 minutes during which I will be interviewed and possibly one audio-recorded focus group interview lasting approximately 30-45 minutes. The interviews will be scheduled for a date and time that are convenient for me. I agree that the researcher may contact me, via email, to ask any follow up questions or to verify content from the interview.

Risks: I have received assurance from the researcher that every effort will be made to respect these personal reflections in the writing and reporting of the research. If at any time I wish to end the interview, and/or withdraw from the study I may do so.

Benefits: My participation in this study will provide information to the research community regarding collaborative professional learning for mathematics teachers and may contribute to ongoing professional development initiatives in the province of Ontario. The opportunity to discuss my own professional learning may also provide me with time to reflect on my own experiences as well as personal and professional learning.

Confidentiality and Anonymity: I have received assurance from the researcher that the information I share will remain strictly confidential. No data, such as school name or school board name, will be used that might identify me. I can decide whether or not my name can be used in the reporting of the research. The contents of the interview will be used for the proposed research study as well as future research articles in the area of mathematics teachers' professional learning. In the case of the optional focus group interview there are limits to confidentiality due to the online format and that fact that other participants are involved. We ask that all participants contribute to maintaining the confidentiality of the other participants and not share the names of others involved with anyone not participating in the study.

Conservation of data: The audio recordings and transcripts will be kept in a secure manner on a password protected computer. Hard copies of the transcripts will be kept in a locked filing cabinet. The data will be kept indefinitely.

Voluntary Participation: I am under no obligation to participate and if I choose to participate, I may withdraw from the study at any time and/or refuse to answer any questions. If I choose to withdraw, all of my data gathered until the time of withdrawal will be destroyed.

Acceptance: I, __________________________, agree to participate in selected components of the above research study conducted by Kelly McKie of the Faculty of Education, University of Ottawa, which research is under the supervision of Dr. Christine A. Suurtamm. I understand that by accepting to participate I am in no way waiving my right to withdraw from the study.

☐ I AGREE to participate in one individual interview.

If I have any questions about the study, I may contact Kelly McKie and/or Dr. Suurtamm at the numbers mentioned above.

If I have any ethical concerns regarding my participation in this study, I may contact the Protocol Officer for Ethics in Research, University of Ottawa, 550 Cumberland Street, Room 154, (613) 562-5387 or ethics@uottawa.ca.

I am invited to print two copies of the consent form, one of which is mine to keep, the other is to be signed, dated, and emailed to Kelly McKie at ________. Upon receipt of the signed consent form, I will be contacted by the researcher to schedule the individual interview at a time that is convenient for me.

Participant's signature: _______________________ Date: ___________

Researcher's signature: _______________________ Date: ___________

Appendix G: Phase 3 Letter of Information and Informed Consent

Dear

This is an invitation to participate in a research project that seeks to gain an understanding of the ways that collaboration may contribute to, and support, mathematics teacher learning and practice. The project also seeks to understand the influence that this learning has on different components of a learning system. These different components can include, but are not limited to teachers, schools, professional learning communities, and school districts. Your name was suggested due to your involvement with ________, at ________ High School

This study will be carried out by Kelly McKie, a PhD candidate at the University of Ottawa, Faculty of Education under the supervision of Dr. Chris Suurtamm. This research project is funded by the Social Sciences and Humanities Research Council of Canada. The data is being collected for the purposes of a PhD thesis in mathematics education and perhaps for subsequent research articles.

As part of the research, ________ was interviewed because of his involvement in the OAME Grade 9 Applied Project. He was asked to identify other individuals that may have been influenced by the collaborative learning of the project. I would like to invite you to participate in this research project in order to add your experiences to the data collected. Your participation would involve an approximately 30–45-minute individual interview. This interview will be conducted by the researcher, Kelly McKie, and will occur with the use of software such as Zoom, or Microsoft Teams. If you prefer, the individual interview can be conducted over the telephone. With your permission, the interview will be audio recorded so the researcher may fully engage in the conversation. Following the interview, the researcher will transcribe the interview herself, and will offer you a copy for your records. Your participation in this study will include you responding to questions about your teaching practice and professional learning. You have the researcher's assurance that every effort will be made to respect your personal reflections in the writing and reporting of the research. If, at any time, you wish to end the interview, and/or withdraw from the study your request will be respected. Any data collected will not be used in the research study if you so choose.

If you are interested in participating in this research study please review and sign the attached consent form. If you have any questions please do not hesitate to get in touch with either the researcher or her supervisor, Dr. Chris Suurtamm.

Sincerely,

Kelly McKie

Informed Consent
University of Ottawa, Faculty of Education
Telephone:
145, Jean-Jacques Lussier
Ottawa ON K1N 6N5 Canada

Name of Professor:
Dr. Christine A. Suurtamm
Faculty of Education, University of Ottawa

Name of Student:
Kelly McKie
Faculty of Education, University of Ottawa

Invitation to Participate: I am invited to participate in the research study entitled *Deepening Our Understanding of the Complexity of Mathematics Teachers' Collaborative Professional Learning* conducted by Kelly McKie in partial fulfilment of the requirements for the degree of Doctor of Philosophy of Education. This research project is funded by the Social Sciences and Humanities Research Council of Canada.

Purpose of the Study: I understand that the purpose of this research study is to gain a fundamental understanding of the different ways that collaborative settings contribute to, and support, mathematics teacher learning and practice and the impact that this learning has on different components of a learning system.

Participation: My participation will consist essentially of one audio-recorded session of approximately 30-45 minutes during which I will be interviewed and possibly one audio-recorded focus group interview (30-45 minutes). The interviews will be scheduled for a date and time that are convenient for me. I agree that the researcher may contact me, via email, to ask any follow up questions or to verify content from the interview.

Risks: I have received assurance from the researcher that every effort will be made to respect these personal reflections in the writing and reporting of the research. If at any time I wish to end the interview, and/or withdraw from the study I may do so.

Benefits: My participation in this study will provide information to the research community regarding collaborative professional learning or mathematics teachers and may contribute to ongoing professional development initiatives in the province of Ontario. The opportunity to discuss my own professional learning may also provide me with time to reflect on my own experiences as well as personal and professional learning.

Confidentiality and Anonymity: I have received assurance from the researcher that the information I will share will remain strictly confidential. I can decide whether or not my name can be used in the reporting of the research. The contents will be used for the proposed research study as well as future research articles in the area of mathematics teachers' professional learning. In the case of the optional focus group interview there may be limits to confidentiality due to the online format and the fact that other participants are involved. We ask that all participants contribute to maintaining the confidentiality of the other participants and not share the names of others involved with anyone not participating in the study.

Conservation of data: The audio recordings and transcripts will be kept in a secure manner on a password protected computer. Hard copies of the transcripts will be kept in a locked filing cabinet indefinitely.

Voluntary Participation: I am under no obligation to participate and if I choose to participate, I may withdraw from the study at any time and/or refuse to answer any questions. If I choose to withdraw, all data gathered until the time of withdrawal will be destroyed.

Acceptance: I, __________________________, agree to participate in the above research study conducted by Kelly McKie of the Faculty of Education, University of Ottawa, whose research is under the supervision of Dr. Christine A. Suurtamm. I understand that by accepting to participate I am in no way waiving my right to withdraw from the study.

☐ I AGREE to participate in one individual interview.

☐ I AGREE to participate in one focus group interview.

If I have any ethical concerns regarding my participation in this study, I may contact the Protocol Officer for Ethics in Research, University of Ottawa, 550 Cumberland Street, Room 154, (613) 562-5387 or ethics@uottawa.ca.

There are two copies of the consent form, one of which is mine to keep, the other is to be signed and dated and emailed to Kelly McKie at___________. Upon receipt of the signed consent form, I will be contacted by the researcher to schedule the individual interview at a time that is convenient for me.

Participant's signature: ______________________ Date: ___________

Researcher's signature: ______________________ Date: ___________

Appendix H: Phase 1 Recruitment Email

May 26, 2021

Hi______,

I have recently defended my PhD proposal (attached) and have received approval to proceed with my research on the sustainability and diffusion of learning from the OAME Grade 9 Applied project. I am currently completing my ethics application and was hoping that the OAME might be able to assist me in recruiting participants.

My request would be that OAME send a short survey (see proposal for a draft of the questions), via email, to all the participants involved in the OAME Grade 9 Applied project. Completion of the survey is completely voluntary. On this survey individuals could supply their name and email if they agree to being contacted for follow up questions.

I appreciate you taking the time to consider my request and would be happy to discuss any questions or concerns you may have.

Have a great day.

Kelly McKie BSc Eng, BEd, MA
PhD candidate
University of Ottawa
Faculty of Education, Mathematics

Recruitment Email sent Aug 26, 2021

Dear OAME Grade 9 Applied Project participant,

I am writing to invite you to participate in a research project that involves reconnecting with participants of the OAME Grade 9 Applied Mathematics Inquiry Project which ran from the fall of 2015 to the summer of 2017. The project was led by Dr. Chris Suurtamm from the University of Ottawa, _____________from the OAME, in partnership with the Ontario Ministry of Education.

The purpose of this research project is to gain an understanding of the different ways that collaborative settings might contribute to, and support, mathematics teacher learning and practice and the influence this learning has on teachers, schools, and school districts. I am carrying out this project as a graduate student, currently working toward completing my PhD in Education at the University of Ottawa.

This component of the project involves a short survey (see link below) sent out to all participants involved in the OAME Grade 9 Applied project. The goal of this survey is to gain a sense of the influence of the OAME Grade 9 Applied project and to understand if and how collaboration, and related professional learning, continues. Your participation would involve completing this online survey (time to complete is approximately 10 minutes) at a time and location that is most convenient for you. A second, optional, component of the project will involve short interviews with some of the participants, and a possible focus group interview. If you would consider taking part in the second component of this project, please indicate your interest when asked within the survey. All data gathered will be kept on a secure, password protected computer. Any research findings that may be used in research related to my doctoral studies, or research related writings, will maintain participants' anonymity.

This research has been cleared by the University of Ottawa Research Ethics Board. By completing and submitting this online survey you are agreeing to participate in phase 1 (the survey) of this research study. Please complete and submit this survey at your earliest convenience.

If you have any questions about this research, please contact me, Kelly McKie, at _______ or my supervisor, Dr. Christine Suurtamm at _________. If you have questions, concerns, or complaints about the research ethics of this study, contact the Protocol Officer for Ethics in Research at 613-562-5387 (ethics@uOttawa.ca).

Acceptance: By completing and submitting the survey, I am consenting to participate in this research study.

Begin Survey

If the link above does not take you to the survey, please copy and paste this URL into your internet browser. https://www.surveymonkey.ca/r/3RZPJ8R

Sincerely,

Kelly McKie BSc Eng, BEd, MA

PhD candidate

University of Ottawa

Faculty of Education, Mathematics

Reminder Email:

Subject line: INVITATION - Research Project

Dear OAME Grade 9 Applied project participant.

Thank you to those of who have completed the follow up survey for the OAME Grade 9 Applied Math Inquiry Project (Survey link). As part of my doctoral research I am studying whether, and what, learning emerged from the OAME Grade 9 Applied project that started in 2015 and was led by Dr. Christine Suurtamm, the OAME, and the Ministry of Education. I am interested in **all** experiences and perspectives so please consider adding **your** voice to the project. Your responses are gathered anonymously. I will not know your identity unless you choose to provide it.

Complete Survey

If you have any questions, comments, or concerns please don't hesitate to contact me, Kelly McKie, at ____________
Thank you for considering participating in my study.

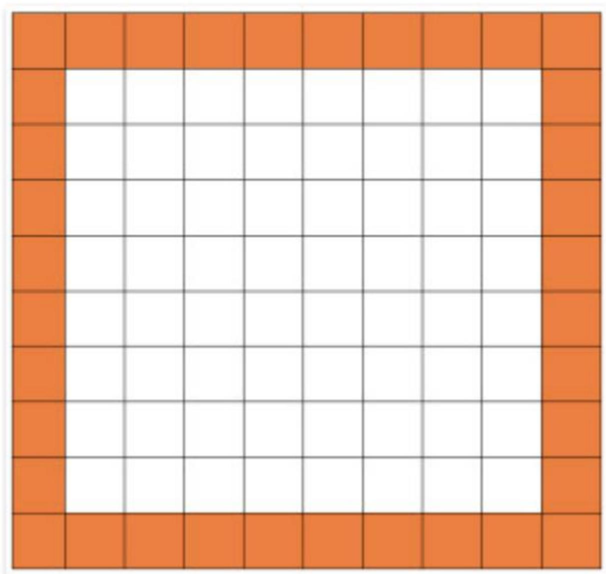
Sincerely,

Kelly McKie

Appendix I: Examples of Rich Tasks

The Border Problem (Boaler & Humphries, 2005)

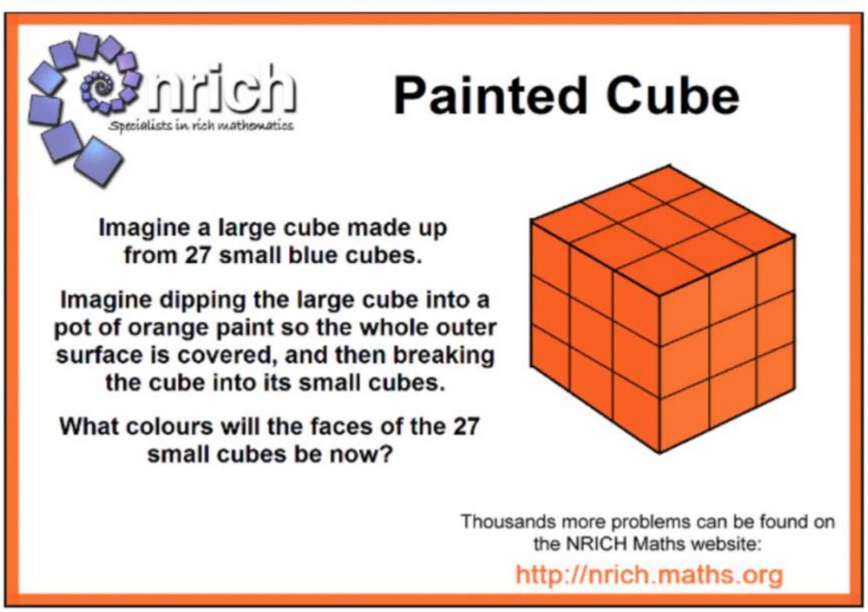
Without counting one by one, determine how many shaded squares are in this 10×10 grid.



How many squares would be shaded around the border of a 6 x 6 grid?

What about a 100 x 100 grid?

The Painted Cube (nrich.maths.org)



Appendix J: Student Beliefs and Attitudes Survey

1. (the following questions had the options Strongly Agree, Agree, Disagree, or Strongly Disagree)
2. The math that I learn in school is mostly facts and procedures that have to be memorized.
3. You can be creative in math class.
4. In math you can discover things on your own.
5. Making mistakes in math helps me learn.
6. It is important to get the right answer in math.
7. I enjoy working on challenging math problems.
8. I prefer to work on math problems by myself.
9. I enjoy group work in math class.
10. Some people are good at math, and some are not.
11. People can't really change how intelligent they are in math.
12. All students would be good at math if they worked hard at it.
13. When I see a math problem, I get nervous.
14. I like to go to the board or share my answers with peers in math class.
15. I enjoy hearing the thoughts and ideas of my peers in math class.
16. I forget how to do problems that I have solved before.
17. If I get stuck on a math problem I ask for help.
18. If I get stuck on a math problem I usually try to figure out a different way that works.
19. I like math.
20. Math is one of my favourite subjects.
21. The math I learn now helps me do work in other subjects.

22. I need to do well in math to study what I want later.

23. Doing homework helps me understand math.

24. I talk about math when I am not at school.

25. Write 2 words to describe math *