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FACULTÉ DES ÉTUDES SUPÉRIEURES
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FACULTY OF GRADUATE AND
POSTDOCTORAL STUDIES

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GRADE - DEGREE

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TITRE DE LA THÈSE - TITLE OF THE THESIS

Impact of Multipath Angular Distribution on Performance of MIMO Systems

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LE DOYEN DE LA FACULTÉ DES ÉTUDES
SUPÉRIEURES ET POSTDOCTORALES

DEAN OF THE FACULTY OF GRADUATE
AND POSTDOCTORAL STUDIES

IMPACT OF
MULTIPATH ANGULAR DISTRIBUTION
ON PERFORMANCE OF MIMO SYSTEMS

by

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Masters of Applied Science

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April, 2004

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395 Wellington Street
Ottawa ON K1A 0N4
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395, rue Wellington
Ottawa ON K1A 0N4
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Your file Votre référence

ISBN: 0-494-01443-1

Our file Notre référence

ISBN: 0-494-01443-1

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ACKNOWLEDGEMENTS

I would like to express my gratitude to the following people:

- to my supervisor, Professor S. Loyka, for the outstanding guidance and supervision
- to my co-supervisor, Professor O. Yang, for the guidance and support
- to my family, for the love and support throughout my studies.

ABSTRACT

The objective of this thesis is to analyze the impact of multipath angular distribution on the channel correlation and performance of MIMO systems. A full breadth of AOA models is used in this study, and they cover most scattering scenarios of the physical propagation channel. Closed-form expressions for parameters and corresponding correlations of these AOA models are derived. A simulation model was developed and extensive Monte Carlo simulations were performed to obtain the MIMO channel capacity, correlation, and diversity gain for various scenarios. Two major results are (i) the distributions shape has minor impact on the correlation, capacity and diversity gain; the major impact is due to the angular spread; (ii) the continuous and impulsive distributions have dramatically different impacts on the correlation, capacity and diversity gain.

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LIST OF SYMBOLS

Symbol	Meaning	Section
n_{loss}	Path loss exponent	2.1
ρ_{env}	Envelope Correlation	3.2.2
ρ_{real}	Real part of correlation	3.2.2
ρ_{imag}	Imaginary part of correlation	3.2.2
H	Normalized channel matrix	3.2.3
C	Channel capacity of MIMO system	3.2.3
m	Number of receive antenna elements	3.2.3
n	Number of transmit antenna elements	3.2.3
ρ	Signal-to-noise ratio	3.2.3
I_m	Identity matrix with m rows and columns	3.2.3
P_0	Outage Probability	3.2.5
μ	signal-to-noise ratio	3.2.5
μ_p	prescribed signal-to-noise ratio	3.2.5
R	Correlation Matrix	3.3.1
θ_0	Mean AOA of an AOA distribution	4.2.2.1
Δ	One-sided maximum angle of an AOA distribution	4.2.2.1
$p(\varphi)$	Probability density function of an AOA distribution	4.2.2.1
φ	Angle of Arrival	4.2.2.1
S_φ	Angular spread of an AOA distribution	4.2.2.1
Q	Normalization constant	4.2.2.2
σ	Standard deviation of a non-truncated distribution	4.2.2.2
$Erf(.)$	Error function	4.2.2.2
n_c	Exponent in cosine distribution	4.2.2.4
R	Radius of circle in circular scatterer model	4.2.2.5
D	Distance between receiver and transmitter in circular scatterer model	4.2.2.5

θ	Angle of departure from the transmitter in into quadrant 1 and 2 in circular scatterer model	4.2.2.5
α	Angle of departure from transmitter into quadrant 3 and 4 in circular scatterer model	4.2.2.5
L_1	Distance between receiver and the scatterer in quadrant 3 and 4 in circular scatterer model	4.2.2.5
L_2	Distance between receiver and the scatterer in quadrant 1 and 2 in circular scatterer model	4.2.2.5
φ_{max}	Maximum angle of arrival in the circular scatterer model	4.2.2.5
Δ	Maximum angle from broadside for impulsive AOA model	5.2.1
λ	eigenvalue	5.2.3.2
φ	eigenvector	5.2.3.2
Λ	Diagonal matrix of eigenvalues	5.2.3.2
Φ	Eigenvector matrix	5.2.3.2
N	Number of antenna elements	6.2.1
γ	Signal-to-noise ratio	6.2.1
γ_0	Prescribed signal-to-noise ratio	6.2.1
P_{out}	Outage probability	6.2.1
D	Diversity gain	6.5.3
D_0	Diversity gain at broadside	6.5.3
ρ_{env}	Envelope correlation	6.5.5

LIST OF ACRONYMS AND ABBREVIATIONS

Acronyms	Meaning
AOA	Angle of Arrival
MIMO	Multiple-Input-Multiple-Output
LOS	Line-Of-Sight
DIV	Diversity
DOF	Degree of Freedom

1 INTRODUCTION

1.1 MOTIVATION OF THIS RESEARCH

MIMO (multiple-input-multiple-output) systems have emerged as one of the major technological breakthrough in the field of wireless communications in recent history. The transformation of a multipath channel into parallel sub-channels offers tremendous improvement in data rate over what is achievable today. MIMO systems offer a theoretical upper limit on capacity that increases with the number of antenna elements. This has opened up many opportunities and spawned numerous research activities.

Much of the recent research activities on MIMO systems have focused on the development of channel model, derivation of channel capacity, and identifying relationships between system parameters and achievable channel capacity. It is well known that the theoretical upper limit of channel capacity in MIMO systems is only bounded by the number of antenna elements. The challenge is to come up with transmission algorithms to realize this theoretical limit. This cannot be done without first establishing a complete set of channel model that would accurately describe the MIMO wireless channel. A comprehensive study on the impact of AOA models on the performance of MIMO systems would provide a much needed basis for further research and a deeper understanding of MIMO systems.

The study in MIMO systems has only started for a few years and there is a big amount of work to be done to acquire full understanding in this area of wireless communication. The great potential offered by MIMO systems and the need for comprehensive study on channel models are the major motivations behind this research.

1.2 OBJECTIVES

The objective of this research is to produce a comprehensive study on the impact of multipath angular distribution and antenna configurations on the performance of MIMO systems. Different distributions are used to emulate the AOA of multipath signals. They are carefully chosen to represent a full breadth of possible AOA distributions that can occur in real-life environment. To evaluate the performance of MIMO systems, parameters such as correlation, capacity, outage probability, and diversity gain are analyzed. Comparisons are made using AOA distributions of different shapes with the same mean and variance.

Much of the recent works on MIMO system assume the use of a particular AOA distribution and then perform analysis based on the chosen distribution. Different researches also make different choices of angle of arrival distribution, increasing the difficulties to compare results of different papers. The study in this thesis helps to identify the effects of using different AOA distributions, and provide a basis for further work on channel models of MIMO systems.

1.3 METHODOLOGY AND APPROACHES

Both mathematical analysis and Monte Carlo simulations are employed in this study. Simulation results are compared with analytical results to ensure the accuracy of the simulation model and to validate the analytical derivations. To study the impact of multipath angular distribution on performance of MIMO systems, a simulation model is developed in MATLAB to calculate correlation, capacity, outage probability and diversity gain of a MIMO system for different AOA distributions. The simulation model is made up of individual function blocks, each responsible for a particular function. The AOA generator block generates an array of multipath AOA based on the input parameters. The generated

AOA are fed into the MIMO processor block, which simulates the multi-antenna MIMO receiver and produces instantaneous correlation matrix and SNR as outputs. Monte Carlo simulations are performed by repeatedly generating AOA arrays and feeding them into this processor block. The resulting correlation matrix and SNR are used by the capacity calculator and diversity gain calculator blocks to obtain correlation, capacity, outage probability and diversity gain. The high-level flow of the Monte Carlo simulation model is illustrated in Figure 1-1 below.

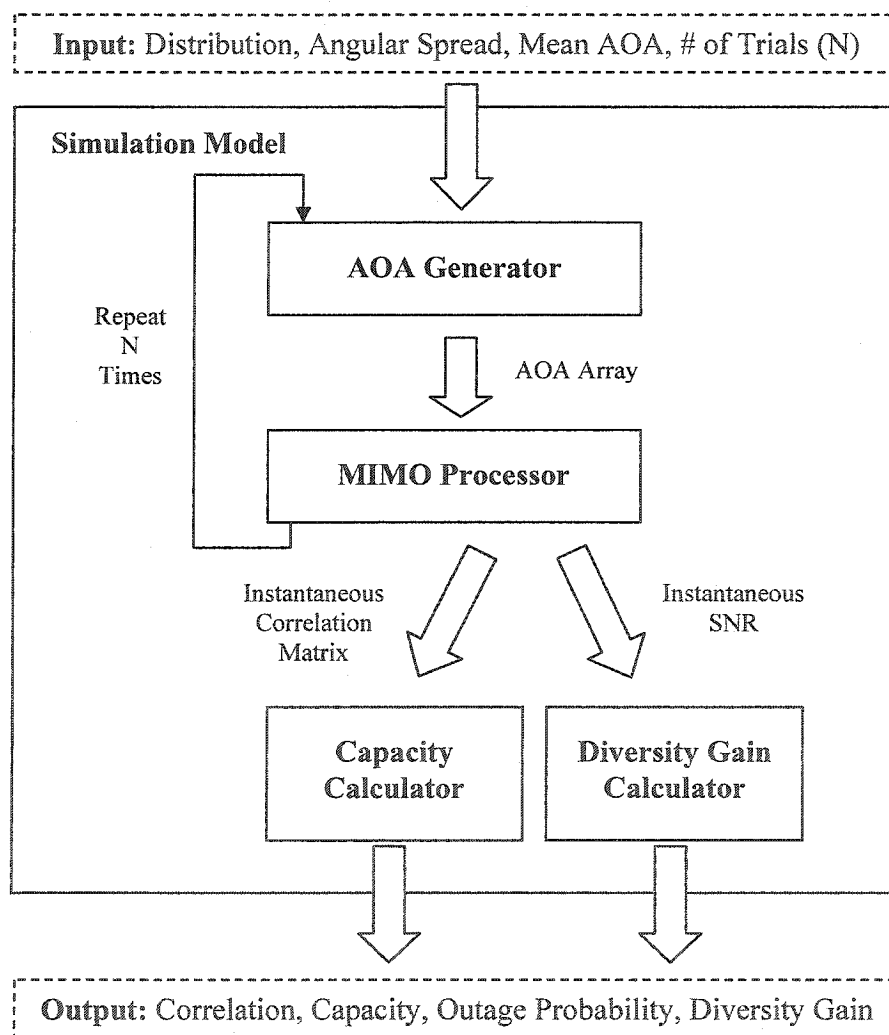


Figure 1-1 High-Level Flow Diagram of Simulation Model

To verify the accuracy of the AOA distributions generated by the simulation model, analytical equations for probability distribution function and angular spread for the studied distributions are derived. The distributions obtained by the analytical equation are then compared to the distributions generated by the simulation model to ensure that there are no discrepancies between them. Analytical equations for parameters such as correlation and outage probability are also used to compare with the simulation results.

Monte Carlo simulations are performed on all the described AOA distributions with different distribution parameters and antenna parameters to provide a full picture of the impact of all these parameters on the performance on MIMO systems. The results are plotted and analyzed. Observations are discussed and conclusions are made.

1.4 OUTLINE OF THESIS

This thesis is divided into seven chapters plus appendices. Chapter 1 is the Introduction. Chapter 2 is an overview of MIMO systems and AOA models. It reviews the results in recent literatures and provides a background for the study carried out in this thesis. Chapter 3 describes the simulation model that is used in this thesis. Chapter 4 describes the impact of continuous AOA distribution on channel capacity of MIMO systems. Chapter 5 describes the impact of impulse AOA distribution on channel capacity of MIMO systems. Chapter 6 describes the impact of AOA distribution on outage probability and diversity gain. Chapter 7 is the Conclusion. Appendix A contains the derivation of AOA distributions used in this study. Appendix B contains all the MATLAB code developed in this study to perform simulations.

1.5 CONTRIBUTION OF THIS WORK

The contribution of this thesis is listed as follows:

- Study of the impact of multipath angular distribution, distribution parameters, and antenna configurations on performance (including channel capacity, outage probability and diversity gain) of MIMO systems (Chapters 4, 5, 6)
- Use of continuous distributions to describe AOA. Performed derivation of analytical equations formulas (normalization constants, angular spread) for AOA distributions (truncated Gaussian, truncated Laplacian, Cosine) under a common reference framework (Chapter 4)
- Use of geometric models to describe AOA. Performed derivation of analytical equation to describe circular scatterer model (Chapter 4.2.2.5)
- Use of delta functions to describe AOA in scenarios when the number of multipath with distinct AOA is limited (Chapter 5.2). Identified of the relationship between number of unique AOA and degrees of freedom in such scenario. (Chapter 5.3.3)
- Development of MATLAB models for MIMO simulation and AOA generation. (Appendix B)

A conference paper titled “Impact of Multipath Angular Distribution on the Performance of MIMO Systems”, based on the research and findings in this thesis, has been accepted by the IEEE Canadian Conference on Electrical and Computer Engineering (CCECE) 2004.

2 REVIEW OF MIMO SYSTEMS AND AOA MODELS

2.1 CHARACTERIZATION OF WIRELESS CHANNELS

The major challenge in wireless communications is to exploit an uncertain wireless channel to maximize the amount of information that can be transmitted across the channel while keeping the number of errors to a minimum at the same time. Previous studies in the area of wireless communications have resulted in comprehensive characterizations of wireless channels. The understanding of wireless channels has also spawned development of techniques and methods to mitigate the obstacles and limitations in wireless channels, and to improve capacity and reduce errors in wireless communications.

Using omni-directional antennas, a transmitted electromagnetic wave is radiated in the three-dimensional space in a spherical fashion around the transmitting antenna. According to the theory of conservation of energy, the sum of all the power on the surface area of the sphere must be equal to the power that is originally transmitted. Thus, at any point in time, the received power at a receiving antenna represents only a small portion of the originally transmitted power. The exact amount of power received is the area of the receive antenna over the surface area of the sphere (where the radius of the sphere is the distance between the transmitter and the receiver), multiplied by the transmitted power. The difference between the transmitted and received power is known as the Free Space Loss, and is represented by the following equation[1]:

$$L_s(d) = \left(\frac{4\pi d}{\lambda} \right)^2 \quad (2.1)$$

where d is the distance between the transmitter and the receiver, and λ is the wavelength of the transmitted electromagnetic wave.

The free space loss describes the loss in transmitted signal power in an open-space wireless environment. However a realistic wireless environment contains many obstacles such as trees and buildings that would introduce complications to the way that signals propagate. The basic mechanisms that impact signal propagation in the wireless communications are reflection, diffraction and scattering [2]. These mechanisms introduce multiple paths for a transmitted signal, and transform a single transmitted electromagnetic wave into many multipath components. These multipath components arrive at the receiver at slightly different time because of the different path lengths. This introduces fluctuations in the power, phase, and angle of arrival of signals received by the receiving antenna, and results in fading.

Fading can be classified into two different types: large-scale fading and small-scale fading. Large-scale fading represents the average loss in signal power over large areas. It is described in terms of mean-path loss and a Gaussian distribution variation about the mean. Large-scale fading is described in the following equation[2]:

$$L_p(d)(dB) = L_s(d_0)(dB) + 10n_{loss} \log_{10} \left(\frac{d}{d_0} \right) + X_\sigma(dB) \quad (2.2)$$

where $L_p(d)$ is the path loss as a function of distance, d is the distance between the transmitter and receiver, $L_s(d_0)$ is the path loss at the reference distance, d_0 is the reference distance, n_{loss} is path loss exponent, X_σ is a zero-mean Gaussian random variable and σ is the standard deviation of the Gaussian random variable. When the path loss exponent is equal to two, the path loss approaches the free space path loss described in equation (2.1). However, in a realistic environment, the path loss exponent is usually larger than two.

Small-scale fading results from the fluctuations in amplitudes and phases of the received signals as a result of small changes in the spatial separation between the transmitter

and the receiver. It is caused by two main phenomena – time spreading of the transmitted signal due to multipath components, and the time variance of the channel due to motions of the transmitter or the receiver. The transmitted signal is spread over time within a delay spread when it is received by the receiving antenna. Depending on the delay spread and symbol period, frequency-selective fading or flat fading may occur. The motion of the transmitter and receiver introduces Doppler shift to the frequency of the received signal, and results in fast or slow fading. Frequency-selective fading and fast fading tend to distort the baseband waveform of the signal, resulting in large error rates regardless of the value of signal-of-noise ratio. Small-scale fading without a line-of-sight signal component can be described using Rayleigh distribution. The probability distribution function of the Rayleigh distribution is listed as follow [2]:

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left[-\frac{r^2}{2\sigma^2}\right], & r \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (2.3)$$

where r is the amplitude of the received signal, and σ is the standard deviation of the amplitude. The path loss of a transmitted signal in the wireless medium can therefore be characterized as the product of large-scale fading and small-scale fading. Large-scale fading provides the average path loss over a large area, and small-scale fading adds fluctuations over the large-scale fading to account for minor spatial changes in the transmitter and the receiver.

2.2 TRADITIONAL APPROACHES TO MITIGATE FADING

Many techniques have been developed to combat fading. A common way to combat frequency-selective fading is to apply adaptive equalization[3], which collects signal

energies that are dispersed over time into the original time interval. Spread spectrum and OFDM are other ways to combat frequency-selective fading. Techniques to combat fast fading include use of robust modulation schemes that does not require phase tracking, increase of signaling rate to make it larger than the channel fading rate, and use of advanced coding and interleaving schemes to reduce the required signal-to-noise ratio.[4]

Improvement of signal-to-noise ratio of the received signal can be done by using diversity and applying advanced coding and interleaving techniques. Diversity provides the receiver with uncorrelated copies of the same signal that would result in higher signal-to-noise ratio if these copies are combined properly. There are many forms of diversity, including time diversity, frequency diversity, spatial diversity, and polarization diversity. Time diversity involves sending of the same signal in different time slots. Frequency diversity involves sending of the same signal using different carrier frequencies. For example, the Rake receiver [5] contains fingers that receiver receive samples of the same signal that arrive at different times and coherently combine them to form a stronger received signal. Polarization diversity [6] provides uncorrelated samples of the same signal by transmitting them in different polarizations from the transmit antenna. Spatial diversity involves the use of multiple antenna systems with antenna elements that are separated by a certain distance. The proper use of signal processing and coding allows exploitation of the spatial diversity in the wireless channel to obtain multiple uncorrelated sub-channels to improve channel capacity.

2.3 CHANNEL CAPACITY OF WIRELESS SYSTEMS

The traditional upper bound of transmission rate known as channel capacity that can be obtained by single-antenna wireless systems under the conditions of frequency-flat channel and additive white Gaussian noise is given by:

$$C = \log_2 (1 + SNR) \quad (2.4)$$

where C is the capacity in bits/sec/Hz and SNR is signal-to-noise ratio. According to this equation, the channel capacity increases with the log of signal-to-noise ratio. Since there is a practical limit to the level of achievable signal-to-noise ratio, and that increase in the signal-to-noise ratio results in diminishing improvements in channel capacity, the upper bound in a practical environment is at a few bits/sec/Hz. In 1998, Foschini and Gans pointed out that in the case when the receiver has complete knowledge of the channel while the transmitter does not, the channel capacity that can be achieved by MIMO systems is [7]:

$$C = \log_2 \det \left[I + \frac{SNR}{n} R \right] \quad (2.5)$$

where I is the identity matrix, n is the number of antenna elements, R is the correlation matrix of the MIMO channel. When there is zero correlation between signals received by the antenna elements, channel capacity can be obtained by:

$$C = n \log_2 \left(1 + \frac{SNR}{n} \right) \quad (2.6)$$

This is a revolutionary discovery in the fundamental limit of capacity in wireless communications. Instead of increasing capacity by the log of SNR as stated in the Shannon's Limit, equation (2.5) indicates that capacity can increase linearly with the number of antenna elements (n). The key to achieving high capacity is to transform the channel into the maximum number of uncorrelated channels. This discovery has opened up big opportunities

for improvement in capacity of wireless communications and spawned a series of research in the area of MIMO system.

2.4 CHANNEL MODELS OF MIMO SYSTEMS

Traditional channel models of wireless communications are adequate in modeling the effects of large-scale and small-scale fading in single-antenna systems. However, the study of MIMO systems necessitates the development of more comprehensive models that account for parameters that are not important in single-antenna systems. According to Clarke's Model[8], multipath signals with complex Gaussian amplitudes and phases arrive at the receiving antenna from all directions. However, the angles of arrival of the multipath signals are not explicitly modeled. In MIMO systems, the angles of arrival of the multipath signals have significant impact on the correlation and channel capacity. Thus the Clarke's Model is not sufficient to describe the wireless environment encountered by MIMO systems.

Ref. [9]-[22] demonstrate the recent development in the channel models applicable to MIMO systems. Ref. [9] and [10] classify the existing channel models into three groups. The first group refers to statistically based models, such as Lee's Model and Uniform Distribution Model. They are based on well-known distribution functions or geometrical models. The second group consists of site-specific models based on measurement data. The third group consists of deterministic models that are entirely site-specific. Geometric theory and techniques such as ray tracing are employed along with site-specific information such as building database and architecture drawings to develop these models.

Ref. [11] and [12] propose a unified channel characterization methodology to include the temporal, frequency and spatial aspects of the wireless channel. Ref. [13] attempts to access the accuracy of statistical channel model by comparing the phase, amplitude, and

capacity distributions of statistical model with those measured using a physical model. It points out that there is a trade-off between model complexity and accuracy. The models that are based on channel physics produces simulation results that match well with those obtained by measurements. The models that use convenient distributions produce simulation results that do not perfectly match the measurements as the channel becomes more complex.

Statistical channel models based on geometric configuration have been proposed in [14] and [15]. Ref. [15] proposes the circular scatterer model and elliptical scatterer model where the scatterers are distributed inside the circle or ellipse surrounding the transmitter. A more sophisticated model is proposed in [14] where transmitted signals bounce off two scatterers in a three-dimensional setting. Statistical channel models based on well-known distribution functions have been proposed in [16] and [17]. Models based on site measurements have been proposed in [18] and [19]. Models obtained using ray-tracing in site-specific environments have been proposed in [20] and [21], and interesting observations were made. Ref. [20] reports that channel capacity would increase when a pedestrian is blocking the light-of-sight signal between the transmitter and receiver. Ref. [21] reports the occurrence of clusters in the angle of arrival. A hardware-based simulator for simulation of MIMO channel is proposed in [22].

2.5 CORRELATION IN MIMO SYSTEMS

Correlation between multipath signals at different antenna elements on the receiver is a very important factor in determining the performance of MIMO systems. MIMO systems achieve high data rate by exploiting multipath components and turning them into independent sub-channels. If the correlations between the multipath components are high, then the ability to achieve high data rate would be severely limited. A lot of researches have

gone into the understanding of correlation in MIMO systems. Ref. [23]-[27] illustrate the recent development in this area.

Ref. [23] provides a fundamental assessment of the impact of correlation on the performance of antenna array. It points out that high correlation affects the ability of antenna array to combat fading but it has negligible impact on its ability to suppress interference. An analytical study of correlation is carried out in [24], which defines correlation as a function of geometric parameters of the multi-antenna array, such as angular spread, angle of arrival, antenna spacing, antenna arrangement. It also points out that large correlation would cause large statistical disparity between the eigenvalues of the correlation matrix, reducing the ability of some sub-channels to convey information. Ref. [25] goes further into identifying the factors that affect correlation by presenting the correlation graphs for different angle of arrival distributions. Ref. [26] presents a general function to describe cross-correlation between links in MIMO systems. Ref. [27] focuses on the error rate of data transmission in MIMO systems and concludes that high correlations would result in higher error rates.

2.6 CHANNEL CAPACITY OF MIMO SYSTEMS

The ultimate goal of MIMO systems is to achieve high data rates. A lot of research activities have been done in determining the upper limit of the channel capacity, identifying the factors that affect channel capacity and developing means to achieve the promised data rate. Ref. [28]- [41] outline the recent development in these areas.

Ref. [28] and [29] present analytical equations to determine the upper bound of mean (ergodic) channel capacity in MIMO systems using channel correlation approach. It is also demonstrated that the impact of channel correlation is negligible when two-antenna beamwidth is smaller than angular spread of incoming multipath signals. Ref. [30] presents

an alternative way to determine channel capacity. It shows the methodology to derive the statistics of eigenvalues and eigenvectors of the correlation matrix and illustrates the use of eigenvalues to calculate average channel capacity and characteristic function. Ref. [31] presents the probability density function of channel capacity achieved in MIMO systems and illustrates that the distribution of the channel capacity can be roughly approximated by the Gaussian distribution.

While many of the researches go into the derivation of the theoretical limit of the channel capacity in MIMO systems, there are others that focus on the derivation of the practical limit of channel capacity in a realistic operating environment. Ref. [32] looks into the practical limit of channel capacity and concludes that the theoretical limit can only be reached in a rich scattering environment that would result in a full rank channel matrix, and providing that both transmitter and receiver have full knowledge of the channel state information. If these conditions exist, then capacity grows linearly with the number of antennas elements. Otherwise, capacity depends on channel state information (from both transmitter and receiver), signal-to-noise ratio and antenna element correlation. It is also noted that correlation sometimes increases and sometimes decreases capacity [33], and that even in low correlation conditions, keyholes effect [34] would reduce the rank of the channel matrix, limiting the achievable capacity. Ref. [35] is another paper that explores the issue on the practical limit of channel capacity. It presents the use of singular value decomposition on channel matrix to decompose the channel into N parallel sub-channels, where N is the number of non-zero singular values. N is also known as the rank of the matrix, and is the minimum value out of the number of transmit antennas, number of receive antennas, and number of multipath components. The paper concludes that in a non-adaptive MIMO system, capacity is limited by the number of scatterers and does not grow indefinitely. In an adaptive

MIMO system, in which the channel is known to the transmitter, the transmitter can transmit only in the significant eigenmodes of the channel, resulting in capacity that increases with the number of antennas.

The study in the limit of the channel capacity leads to the idea of degree of freedom. Ref. [36] and [37] are two papers that attempt to apply the concept of degree of freedom onto MIMO systems. Ref. [36] presents rigorous mathematical definitions for diversity and degree of freedom that are meant to be used in all context of wireless communications. In words, diversity is defined as “redundancy in the system”, and degree of freedom is defined as “number of independent channels available for communications”. Other literatures have defined diversity as the “negative exponent of the signal-to-noise ratio in the probability of error function”[38], and degree of freedom as the “co-efficient of $\log p$ in the capacity function”[39]. High diversity does not necessarily imply high degree of freedom. High diversity results in less error in the data transmission, and high degree of freedom results in high data rate. Ref. [37] presents the space-time MIMO channel in a four-dimensional Fourier series representation, with the four dimensions being frequency, time, and the two spatial dimensions at the transmitter and the receiver. The paper illustrates that essential independent degrees of freedom in the channel can be revealed using this representation.

Ref. [40] and [41] present reviews of the current development in MIMO systems and look into the application of MIMO systems in practice. Both papers acknowledge the tremendous potential increase in capacity that will be brought by MIMO system, but at the same time point out that much research will have to be done, especially in the channel modeling of MIMO systems and the dynamics involved. Existing transmissions algorithms might need to be refined and new algorithm will be developed to fulfill the promise of high data rate brought by MIMO systems. Ref. [42] demonstrates a prototype implementation of

MIMO WCDMA system. This is the first step to the many prototypes and trials that are sure to come in the future.

2.7 DIVERSITY COMBINING

Diversities of various forms have long been used to combat fading and co-channel interference and improve the quality and strength of received signals. Many diversity combining techniques have been developed and used in the time and frequency domains. These techniques are now used in the spatial domain with the emergences of smart antennas and MIMO systems.

Ref. [43] - [47] outline the recent development in the area of diversity combining. The majority of the works revolve around formulation of models to verify the performance of different diversity combining techniques under different environments. Ref. [43] presents an analytical model to obtain the outage probability of maximal ratio combining and optimal combining, providing a basis for comparison between the two diversity combining techniques. Ref. [44] presents an analytical model for maximal ratio combining in presence of multiple co-channel interferers and white Gaussian noise. Ref. [45] studies the performance of optimum combining and maximal ratio combining in wireless environment with Rice fading and co-channel interference. Ref. [46] presents a numerical technique to evaluate outage probability of maximal ratio combining systems over generalized fading channels. Ref. [47] presents the derivation of signal-to-noise ratio of a generalized selection combining system.

2.8 AOA DISTRIBUTIONS AND MODELS

A number of different AOA distributions have been used in previous studies to perform analysis of MIMO systems. Uniform distribution, because of its simplicity, has been used in many papers, including [28] and [48]. However, urban measurements performed by [49] indicated that AOA distributions in a practical environment are non-uniformly distributed. Ref. [18] performed a measurement of the MIMO channel from a mobile receiver in New York, and concludes that the angle of arrival is not uniform distributed, but the angular spread of the received signal is wide enough that there is no significant impact in the channel capacity. It is first shown by [50] in 1998 that the truncated Laplacian distribution can occur in practical situations. Ref. [19] performed measurements in both urban and rural macrocell environment and concludes that Laplacian distribution is a good match with the measured angle of arrival. Using a geometrical model with bi-variate Gaussian distributed scatterers around the mobile station, Ref. [49] showed that AOA can be modeled by truncated Laplacian or truncated Gaussian distribution. Under a wireless LAN setting, Ref. [51] demonstrated that the received signals follow an exponential delay distribution and a Laplacian AOA distribution. Ref. [52] performed an analysis regarding the effect of an one-cluster scatterer on the AOA distribution. It was observed that when the scatterers are Gaussian distributed, then the AOA is Laplacian distributed. When the scatterers are Rayleigh distributed, then the AOA is Gaussian distributed. Laplacian distribution has been used in many other papers, including [48], [53] and [54]. Gaussian distribution has also been used in a large number of publications, including [48], [55], [56], and [57]. Cosine distribution is very similar to Gaussian distribution, and has been used in [58].

Although many distributions have been studied in the past, the equation derived for these distributions for use in the study of MIMO systems are often based on different references and assumptions. This complicates the effort to compare results, and is one of the motivations behind the study carried out in the thesis to perform a comprehensive analysis of these AOA distributions.

2.9 CONCLUDING REMARKS

This chapter provides a review of the development in wireless communications and summarizes the recent research activities being performed in the research community. It starts with the classical characterization of wireless channel, including free space loss, large-scale fading and small-scale fading, and outlines the traditional techniques used to mitigate fading. It then compares the classical Shannon's Limit with the capacity limit of MIMO system, and illustrates the tremendous capacity improvement that can be brought by MIMO systems. Recent literatures in the area of channel models, correlation, channel capacity, diversity combining and AOA distributions and models are also presented and summarized.

Much of the work in the area of MIMO systems have been performed on the MIMO channel model and factors that affect capacity. The existing channel models are far from perfect. Continuous enhancement of these models would bring about new insight into the operation of MIMO systems and lead to new transmission algorithms that would ultimately realize the tremendous promise of MIMO systems. This thesis attempts to produce a comprehensive study on the impact of multipath signal distribution on the channel capacity of MIMO systems. In doing so, it uses the results of some of the recent literature as starting point, and investigates the impact on correlation, capacity, and diversity gain of MIMO

systems as incoming signals and antenna configuration change. Concepts introduced in the literatures will be used in this thesis to explain the observed results.

3 SIMULATION MODEL FOR MIMO SYSTEMS

3.1 INTRODUCTION

The objective of this research is to study the performance of MIMO (Multiple-Input-Multiple-Output) systems as functions of AOA of incoming multipath signals and antenna configuration. A simulation model is developed to facilitate this study. This simulation model takes in as input an array of multipath signal, which is generated according to the specified distribution, angular spread and mean AOA. The adjustable antenna configuration parameters include number of antenna elements and antenna element spacing. The outputs of this model are important performance benchmarks that will serve as the basis of the performance analysis in subsequent chapters. These benchmarks include channel capacity in terms of the data rate of information through the MIMO wireless channel, correlation between antenna array elements, and diversity gain obtained by addition of antenna elements. This chapter provides a mathematical background of the simulation model and the performance benchmarks. The MATLAB code for the simulation models are listed in Appendix B.

3.2 MATHEMATICAL BACKGROUND

3.2.1 Channel Matrix

The MIMO system being considered in this research consists of a transmitter with n antenna elements and a receiver with m antenna elements. Since signals from each of the n transmitting antenna elements are transmitted to all m receiving antenna elements, there is, effectively, a total of $(n \times m)$ different sub-channels from the transmitter to the receiver in the

system. Figure 3-1 illustrates the antenna elements and the sub-channels where $n = m = 4$.

The same idea can be extended to any values of n and m .

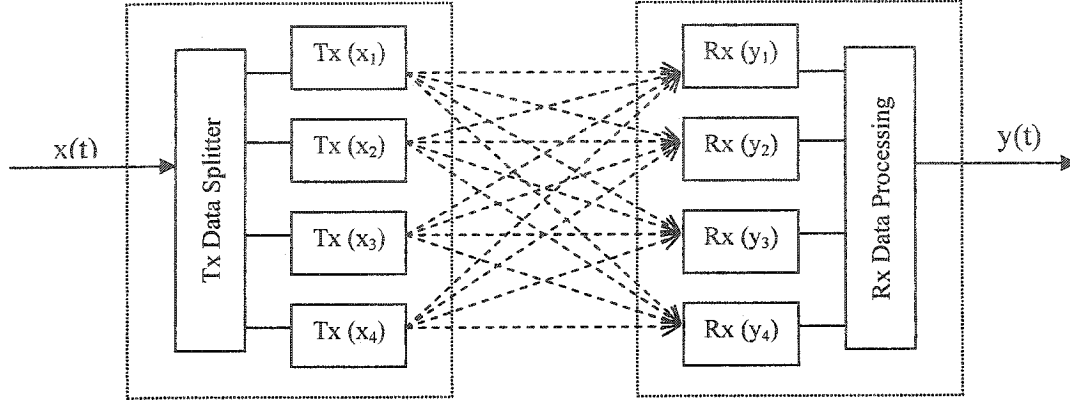


Figure 3-1 MIMO system with four Tx and Rx antenna elements

The received signal is:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (3.1)$$

where $X = [x_1 \ x_2 \ x_3 \ x_4]^T$ is the transmitted signal, $Y = [y_1 \ y_2 \ y_3 \ y_4]^T$ is the received signal, and $H = [h_{ij}]$, $i, j = \{1..4\}$ is the channel matrix. The channel matrix is the transfer function of the channel in matrix form. The entry h_{ij} corresponds to the transfer function from the j -th transmitter antenna element to the i -th receiver antenna element. One row in the channel matrix corresponds to all the transfer functions from all the different transmitter antenna elements into the first receiver antenna element. One column in the channel matrix corresponds to all the transfer functions from one transmitter antenna element into all the different receiver antenna elements. If the channel matrix can be transformed in a diagonal matrix, then we have n independent sub-channels. Otherwise there exist sub-channels that are correlated.

3.2.2 Correlation Matrix and Correlation

Correlation matrix is a measure of correlation of the channel matrix. It is the product of the channel matrix and its Hermitian conjugate. In the simulation model, instantaneous correlation matrix is used, and it is defined as:

$$R = HH^+ \quad (3.2)$$

In the analysis of MIMO system, it is necessary to study correlation as a function of antenna spacing. The correlation used in this case is the correlation of signals received by two antenna elements in the receiver antenna array system, and is defined as:

$$\rho = \frac{E\{x_1 x_2\}}{E\{x_1^2\}} \quad (3.3)$$

where x_1 is the real value of the signal at the first antenna element, x_2 is the real value of the signal at the second antenna element, and $E\{\cdot\}$ is the expectation function. Correlation can vary between $[-1, 1]$. Sometimes it is more convenient to use envelope correlation. Envelope correlation is defined as:

$$\rho_{env} = \frac{E\{x_1 x_2\}^2 + E\{x_1 y_2\}^2}{E\{x_1^2\}^2} \quad (3.4)$$

where y_2 is the imaginary value of the signal at the second antenna element.

3.2.3 Channel Capacity

The advantage of using MIMO system is to capability to achieve high capacity. The channel capacity of MIMO system is given by the Foschini-Telatar formula[7], and is defined as:

$$C = \log_2 \left[\det \left(I_m + \frac{\rho}{n} HH^+ \right) \right] \text{bit} / \text{s} / \text{Hz} \quad (3.5)$$

where I_m is the identity matrix with m rows and columns, m is the number of receive antenna elements, ρ is the SNR, n is the number of transmit antenna elements, H is the normalized channel matrix, and H^+ is the Hermitian conjugate of normalized channel matrix.

Channel capacity of MIMO system can also be calculated using eigenvalues as follow:

$$C = \sum_{i=1}^m \log \left(1 + \frac{\rho}{n} \lambda_i \right) \quad (3.6)$$

where m is the number of eigenvalues of the channel matrix, n is the number of transmit antenna elements, and λ_i is the i -th eigenvalue of the channel matrix.

3.2.4 Maximal Ratio Combining

It is an important part of a complete study of system performance to analyze the impact of diversity combining in MIMO systems. Maximal ratio combining is the most effective linear combining technique for coherent reception with independent fading at each antenna element in the presence of white Gaussian noise. In this research, maximal ratio combining is used to perform diversity combining in a system configuration with one transmit antenna and n receive antennas. The signal received by each antenna element is:

$$y_i = h_i x + \zeta_i \quad (3.7)$$

where h_i is the frequency response of the channel for the i -th receive antenna, x is the transmitted signal, and ζ_i is the noise at the i -th receive antenna. The output of the maximal ratio combiner is the weighted sum of the signal at each receive antenna:

$$y_{out} = \sum_{i=1}^n y_i h_i \quad (3.8)$$

Using this combining technique, the output SNR is the sum of the SNR of all branches:

$$\gamma_{out} = \sum_{i=1}^n \gamma_i \quad (3.9)$$

where γ_i is the SNR of the received signal at the i -th receive antenna.

3.2.5 Outage Probability

The performance of a diversity combining system can be measured by outage probability and diversity gain. Outage probability is defined as the probability of a receiver system to achieve a signal-to-noise ratio that is less than the prescribed value. Given the probability distribution of the receiving signal-to-noise ratio, the outage probability can be determined using the following relationship:

$$P_o = \Pr(\gamma < \gamma_p) \quad (3.10)$$

where $\Pr$ is the probability function, and γ_p is the prescribed SNR. Outage probability is expressed as a function of SNR. The outage probability curves describe the outage probability of the diversity combining system at each value of signal-to-noise ratio. As parameters of the system varies the curve changes correspondingly.

3.2.6 Diversity Gain

Diversity gain is the gain in SNR of a diversity combining system when the number of receiving antennas is increased from one value to a bigger value at a particular outage probability level. The diversity gain can be obtained by:

$$g = \gamma_{pd}(P_o) - \gamma_{pi}(P_o) \quad (3.11)$$

where $\gamma_{pd}(P_o)$ and $\gamma_{pi}(P_o)$ are the SNR values for a prescribed outage probability for the diversity combining system and the non-diversity combining system respectively. Diversity gain shows the increase in performance as the number of receiving antenna elements is

increased. It is generally known that the biggest improvement can be observed when the number of antennas is increased from one to two. Then the improvement starts to slow down. After a while, the improvement is so small that it is not worth implementing any more additional receiving antennas.

3.3 SIMULATION MODEL

3.3.1 Channel Capacity

The simulation model for channel capacity provides a realistic simulation environment that takes into account of real life wireless channel effects. The wireless channel is assumed to undergo Rayleigh fading. Without loss of generality, the number of antenna elements in the transmitter and receiver are set to ten. Figure 3-2 is the graphical illustration of the simulation model.

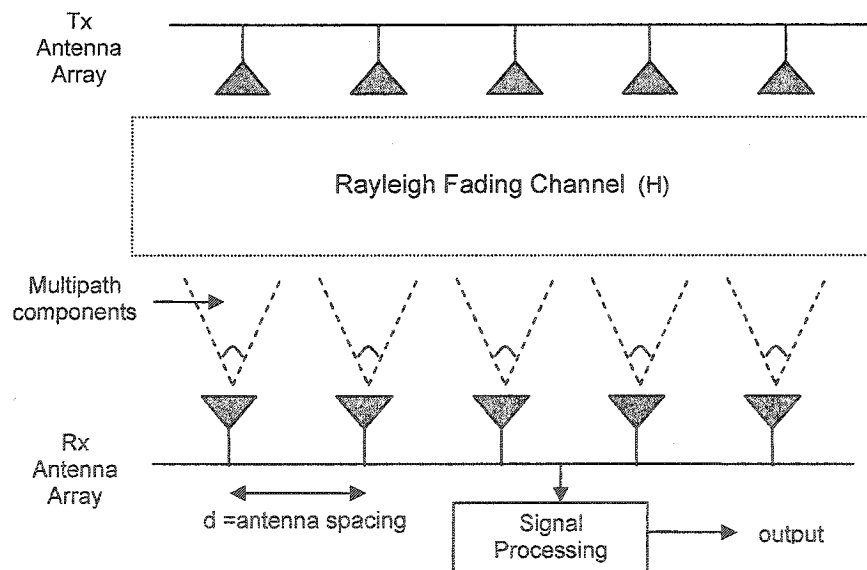


Figure 3-2 Simulated MIMO Channel

Since the equation to obtain the channel capacity given the correlation matrix is already known, we perform Monte Carlo simulations to obtain many samples of correlation matrices, and use the correlation matrices to calculate capacity. Both the mean capacity and the upper bound of the mean capacity [29] are calculated in this study. The mean capacity can be obtained by averaging the values of channel capacity obtained over the number of runs of simulation. The upper bound on the mean capacity can be obtained by first averaging the channel matrix over the number of runs of simulation, and then calculating the channel capacity using the averaged channel matrix.

We need to obtain the channel matrix in order to determine the correlation matrix. The channel matrix in our model is a 10-by-10 matrix similar to the one illustrated in (3.1). Each (i,j) entry in the matrix represents the received signal at the i^{th} receive antenna element from the j^{th} transmit antenna element is:

$$h_{ij} = \sum_k a_k e^{j(\varphi_k - \beta_{ik})} \quad (3.12)$$

where k is 10, the number of multipath components for each transmitted signal, a_k and φ_k are the amplitude and phase of the k^{th} multipath component, and β_{ik} is the phase shift of the k^{th} multipath component in the i^{th} antenna element with respect to the first one.

The amplitude of each multipath component, a_k , is obtained using complex Gaussian distribution with zero mean and unit variance: $CN(0,1)$. The direction of the multipath component is generated according to a specific distribution, and will be discussed in more details in Section 4. Assuming a quasi-static and frequency-flat channel, the phase shift, β_{ik} , is defined as:

$$\beta_i = (2\pi / \lambda) d(i-1) \cos(\theta) \quad (3.13)$$

where λ is the wavelength of the incoming signal, and d is the distance between adjacent antenna elements, i is the antenna element index and θ is the angle where the multipath signal hits the antenna element. The graphical illustration of the phase shift in the receive antenna array is shown in Figure 3-3.

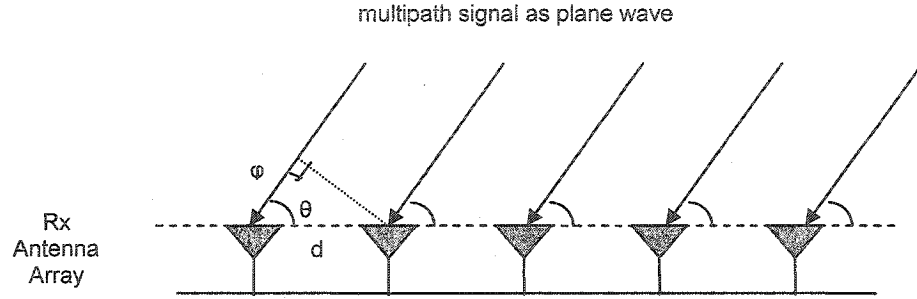


Figure 3-3 Receive Antenna Array

Having obtained the channel matrix, the correlation matrix can be determined using (3.2). The correlation matrix is then normalized according to the number of multipath components and antenna elements such that $\text{Tr}(\mathbf{R}) = n$, where $\text{Tr}(\cdot)$ is the trace function, $\mathbf{R}$ is the correlation matrix and n is the number of antenna elements.. The channel capacity can be determined using (3.5). The mean value is obtained by averaging over many samples of correlation matrix, and then calculating the capacity using the averaged correlation matrix, as follows:

$$\langle C \rangle = \log_2 \left[\det \left(\mathbf{I}_m + \frac{\rho}{n} \langle \mathbf{R} \rangle \right) \right] \quad (3.14)$$

where $\mathbf{I}_m$ is the identity matrix with m rows and columns, m is the number of receive antenna elements, ρ is the SNR, n is the number of transmit antenna elements, $\mathbf{R}$ is the correlation matrix and $\langle \cdot \rangle$ denotes the expectation of the function inside the bracket. The upper bound value can be obtained by first calculating the capacity for each instantaneous correlation matrix, and then determine the average over all the samples of channel capacities, as follows:

$$\langle C \rangle_{upperbound} = \left\langle \log_2 \left[\det \left(I_m + \frac{\rho}{n} R \right) \right] \right\rangle \quad (3.15)$$

3.3.2 Diversity Gain

The diversity gain model assumes the use of maximal ratio combining at the receiver. The transmitter and receiver contain one and m antenna elements respectively. The equation for the received signal is the same as (3.1) with j set to 1. The channel capacity simulation model developed in Section 3.3.1 is re-used, except that the output SNR, rather than the channel matrix or the correlation matrix is processed. After generating enough samples of the output signal, the outage probability at a prescribed SNR can be determined using equation (3.10). Diversity gain can be determined using equation (3.11). The prescribed outage probability is set to 10^{-2} in our simulations.

3.4 CONCLUDING REMARKS

This chapter gives the objective of the research, provides an introduction to the mathematical concepts and equations that are employed in the study, and explain in details the simulation models that are used to facilitate the study. This chapter is intended to provide a foundation for the rest of the thesis, as the results and observations illustrated in the next few chapters are all based on the simulation models described in this chapter.

The objective of this research is to understand the performance of MIMO systems by varying multipath angular distributions and antenna system parameters. The two major benchmarks that are studied in details are channel capacity and diversity gain. Other benchmarks that are studied include the correlation matrix, its eigenvalues, and the outage probability. The system parameters that are analyzed include antenna element spacing, mean AOA, and angular spread.

4 EFFECT OF CONTINUOUS AOA DISTRIBUTION ON MIMO CAPACITY

4.1 INTRODUCTION

The wireless medium contains natural landscape and man-made buildings that can introduce scattering, shadowing and reflections to electromagnetic waves. Because of these effects, a single transmitted signal can bounce off different objects and become multiple multipath components, which results in fading. These multipath components arrive at the receiver from different angles. Most simulation models emulate this phenomenon by using distribution functions to describe the angles of arrival of the multipath signals.

In this study, we analyze the effects of multipath angular distribution on channel capacity of MIMO systems. A number of AOA models have been presented in the literatures. However, the models presented by different papers were often based on different assumptions, complicating the effort to compare results. This chapter extends the existing work by performing comprehensive mathematical analysis on all the major AOA distributions and models using the same references and assumptions. All the AOA distributions are fed into the same simulation model to obtain channel capacity and diversity gain results. These results are then analyzed and conclusions are made regarding the effects of multipath angular distribution on MIMO systems.

Two categories of AOA models are employed in this study. The first category involves continuous distribution functions. This includes modification of well-known distributions such as Gaussian and Laplacian distributions. Distributions derived from practical geometrical models are also used. The second category involves impulsive functions. Impulsive functions are distributions where the multipath signals arrive

exclusively from a few specific angles, resulting in distribution functions containing impulses at a few angles, but zeros at all other angles.

In this chapter, background information on the geometrical definition of angle of arrival is given. Then the AOA models with continuous distributions are discussed. The study of continuous AOA distributions begins with a detailed discussion of the mathematical and simulation models. The simulation results are provided and analyzed. Conclusions are drawn. The AOA models with impulse functions are discussed in the next chapter.

4.2 ANGLE OF ARRIVAL MODELS

4.2.1 AOA Reference Framework

A common reference framework is essential to facilitate a comprehensive study on AOA. Figure 4-1 shows the geometrical illustration of the arrival of multipath signals at the receive antenna array. The line that is perpendicular to the linear antenna array is referred to as the broadside direction, and the angle at that line is defined as 0 degrees. Angles on the right hand side of the broadside direction are denoted as positive, and angle of the left hand side of the broadside direction are denoted as negative. All multipath signals are assumed to come from within $\pm 90^\circ$ of the mean AOA, which is the statistical mean of an AOA distribution, and is defined as:

$$\theta_{mean} = \int_{-\pi/2}^{\pi/2} \theta f(\theta) d\theta \quad (4.1)$$

An AOA distribution contains multiple multipath signals that come from angles within a range of 2Δ degrees. The boundaries of the AOA distribution are defined by $(\theta_0 - \Delta)$ degrees and $(\theta_0 + \Delta)$ degrees, where θ_0 is the center angle of the distribution. The mean AOA does not necessarily have to be 0 degree, and can be at any angles within plus and minus 90

degrees. Although distributions can be characterized by their maximum ranges, the maximum range is not a good measure for comparison between different distributions because two distributions can be very different statistically while still having the same maximum range. To allow different distributions to be compared, angular spread is used. Angular spread is defined as the square root of the variance of a given AOA distribution. It is a good measure to allow fair comparison of different distributions.

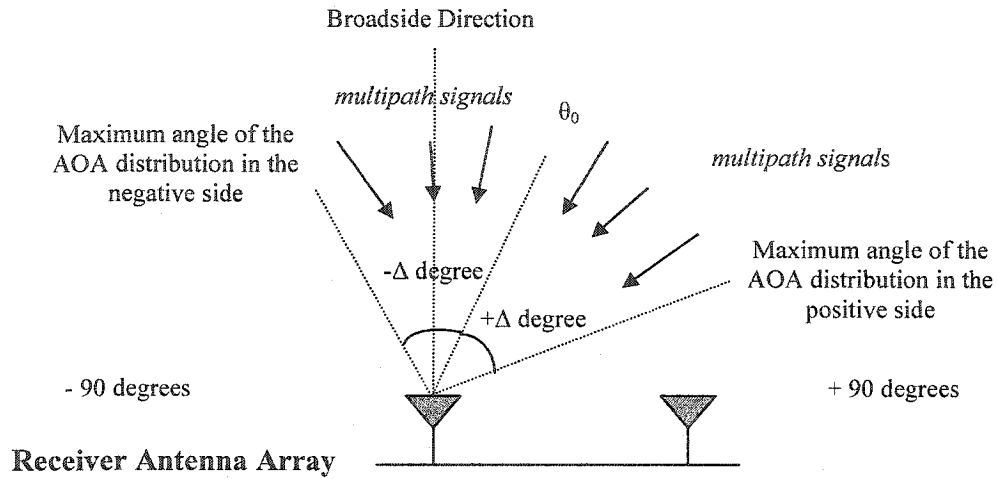


Figure 4-1 Geometrical Model of AOA

4.2.2 Mathematical Background

4.2.2.1 Uniform Distribution

The simplest model to describe angles of arrivals is to assume that all multipath signals are equally likely to arrive from any angles within a range. This means that the multipath signals are uniformly distributed across a range of angles. The following equation defines the probability density function for uniform distribution:

$$p(\varphi) = \frac{1}{2\Delta} \text{ for } -\frac{\pi}{2} + \varphi_0 \leq \varphi \leq \frac{\pi}{2} + \varphi_0 \quad (4.2)$$

where Δ is one half of the maximum range, and can also be interpreted as the angle from the center of the distribution to the boundary of the distribution, and φ_0 is the mean AOA. As it is well-known, the angular spread is defined as:

$$S_\varphi = \frac{\Delta}{\sqrt{3}} \quad (4.3)$$

The derivation of the angular spread is given in Appendix A.

4.2.2.2 Gaussian Distribution

The Gaussian distribution employed in this study is a truncated Gaussian distribution. The distribution is truncated in the sense that all values outside the range $[-90^\circ, 90^\circ]$ around the mean AOA have zero probability density. This distribution is first proposed by Adachi, Feeney, Williamson and Parsons[60]. The probability density function is defined as:

$$p(\varphi) = \left(Q / \sqrt{2\pi\sigma^2} \right) e^{-(\varphi - \varphi_0)^2 / 2\sigma^2} \quad \text{for } -\frac{\pi}{2} + \varphi_0 \leq \varphi \leq \frac{\pi}{2} + \varphi_0 \quad (4.4)$$

where Q is the normalization constant, $Q = 1 / \text{erf}\left(\frac{\pi}{\sqrt{8}\sigma}\right)$, φ_0 is the mean AOA, and σ is the standard deviation of the corresponding non-truncated Gaussian distribution. The normalization constant (Q) ensures that the probability distribution function integrates to unity. Previous literatures have presented many different equations for the normalization constant, creating confusions and doubts about the validity of those equations. In light of this, the equations for normalization constant of all AOA models presented in this study are results of independent derivations. The derivation is given in Appendix A. The standard deviation (σ) of the non-truncated Gaussian distribution is related to the angular spread (S_φ) of the truncated Gaussian distribution by the following relationship:

$$S_{\varphi} = \left\{ Q\sigma^2 \left[-\frac{\sqrt{\pi}}{\sqrt{2}\sigma} e^{-\frac{\pi^2}{8\sigma^2}} + \operatorname{erf}\left(\frac{\pi}{\sqrt{8}\sigma}\right) \right] \right\}^{\frac{1}{2}} \quad (4.5)$$

The derivation is given in Appendix A.

4.2.2.3 Laplacian Distribution

The Laplacian distribution employed in this study is a truncated Laplacian distribution with a range of $[-90^\circ, 90^\circ]$. It was shown to be distribution that can occur in practical situations, and had been used in [50], [52] and [53]. The following equation defines the probability density function for truncated Laplacian AOA distribution:

$$p(\varphi) = \left(Q / \sqrt{2\pi\sigma^2} \right) e^{-\frac{\sqrt{2}|\varphi-\varphi_0|}{\sigma}} \quad \text{for } -\frac{\pi}{2} + \varphi_0 \leq \varphi \leq \frac{\pi}{2} + \varphi_0 \quad (4.6)$$

where $Q = 1 / \left(1 - e^{-\frac{\pi}{\sqrt{2}\sigma}} \right)$ is the normalization constant, and φ_0 is the center angle. The relationship between the angular spread of the truncated Laplacian distribution and the standard deviation of the non-truncated Laplacian distribution is defined as:

$$S_{\varphi} = \left\{ Q \left[-e^{-\frac{\pi}{\sqrt{2}\sigma}} \left(\frac{\pi^2}{4} + \frac{\pi\sigma}{\sqrt{2}} + \sigma^2 \right) + \sigma^2 \right] \right\}^{\frac{1}{2}} \quad (4.7)$$

The derivation is given Appendix A.

4.2.2.4 Cosine Distribution

The cosine distribution contains a set of multipath signals that are distributed according to the cosine distribution within the range of $[-90^\circ, 90^\circ]$ around the mean AOA. The use of cosine distribution for angle of arrival is first proposed by Lee[59]. The probability density function as defined as:

$$p(\varphi) = Q \cos^{n_c}(\varphi_0) \text{ for } -\frac{\pi}{2} + \varphi_0 \leq \varphi \leq \frac{\pi}{2} + \varphi_0 \quad (4.8)$$

where the normalization constants Q for even and odd value of n_c are defined as:

$$Q_{\text{even}-n_c} = \frac{\pi}{2^{2n_c}} \binom{2n_c}{n_c} \quad Q_{\text{odd}-n_c} = \sum_{k=0}^n \left\{ \binom{2n_c+1}{k} \frac{2(-1)^{n_c-k}}{2n_c-2k+1} \right\} \quad (4.9)$$

The derivations are given in Appendix A. The angular spread of the cosine distribution is determined by the value of exponent (n_c). As the value of n_c increases, the angular spread decreases. The angular spread for even and odd values of n are:

$$S_{\varphi, \text{even}-n_c} = \left[2 \left(\frac{\pi}{2} \right)^3 (3\pi)^{-1} + 8 \sum_{k=0}^{n_c-1} \left\{ \binom{2n_c}{k} \left[\frac{(n_c-k)(-1)^{n_c-k}}{(2n_c-2k)^3} \right] \right\} \binom{2n_c}{n_c}^{-1} \right]^{\frac{1}{2}} \quad (4.10)$$

$$S_{\varphi, \text{odd}-n_c} = \left[\sum_{k=0}^{n_c} \left\{ \binom{2n_c+1}{k} \left(\frac{1}{2n_c-2n_c+1} \right)^3 \left[2 \left[\left((2n_c-2k+1) \frac{\pi}{2} \right)^2 - 2 \right] (-1)^{n_c-k} \right] \right\} \left[\sum_{k=0}^{n_c} \left\{ \binom{2n_c+1}{k} \frac{2(-1)^{n_c-k}}{2n_c-2k+1} \right\} \right] \right]^{\frac{1}{2}} \quad (4.11)$$

The derivations are given in Appendix A.

4.2.2.5 Circular Scatterer Model

The circular scatterer model is a geometrical model that emulates a realistic wireless environment [2] [8] [26]. In an urban setting, there are many buildings and objects around the transmitter. These buildings and objects represent the primary source of scatterers encountered by the transmitted signal. Since the transmitter is surrounded by the scatterers, these scatterers can be interpreted by equivalent scatterers located on a circle around the transmitter. The transmitted signals originate from the transmitter in the middle of the circle, bounce off the scatterers on the circle, and heads towards the receiver.

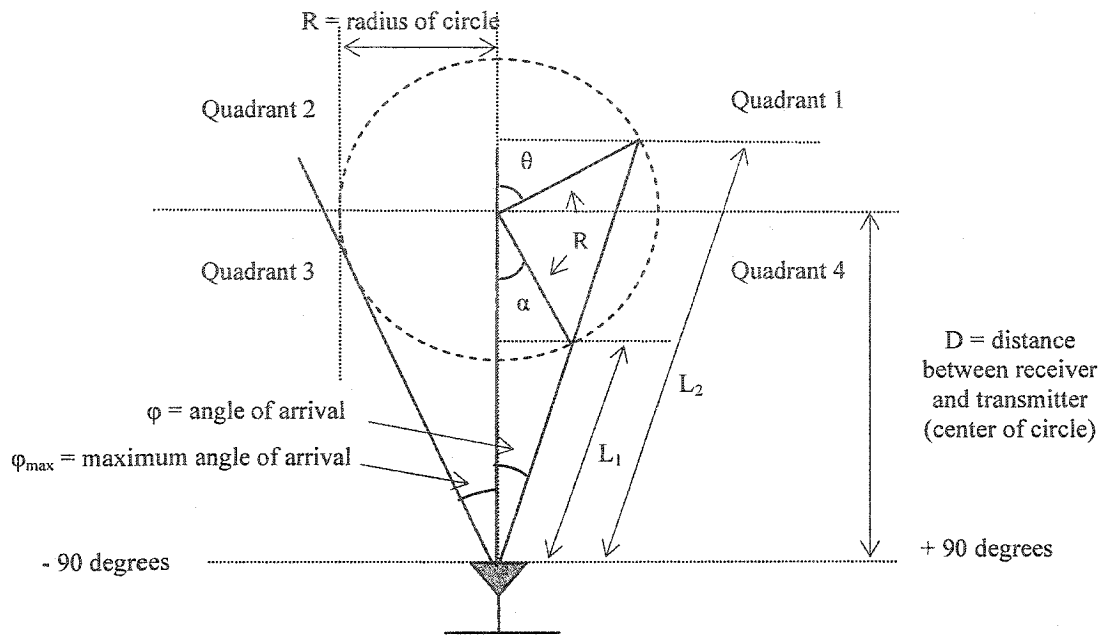


Figure 4-2 Geometrical Illustration of Circular Scatterer Model

Figure 4-2 shows the geometrical illustration of the circular scatterer model. The figure assumes a mean AOA of 0° , but the same set of geometry applies to any value of mean AOA. In the figure, ϕ is the AOA from the perspective of the receiver. The signal coming straight from the transmitter would have an AOA of 0° . The symbol $\phi_{\max}$ is the maximum AOA, which is determined by the angle between the broadside direction and the tangent line from the receiver to the circle.

As illustrated in the figure, the signal of each AOA can come from one of two different scatterers, one located in the semi-circle that is further away from the receiver (i.e. quadrant 1 and 2), and another located in the semi-circle that is closer to the receiver (i.e. quadrant 3 and 4). The only exception is when the signal is at maximum AOA, where the signal comes from the scatterer at the tangent point of the circle. The symbol α represents the angle between the broadside direction and the direction of original signal from the

transmitter to the scatterer in quadrant 3 and 4. The symbol θ represents the angle between the broadside direction and the direction of original signal from the transmitter to the scatterer in quadrant 1 and 2. The symbol L_1 represents the distance between the receiver and the scatterer in quadrant 3 and 4. The symbol L_2 represents the distance between the receiver and the scatterer in quadrant 1 and 2. The distance between the transmitter and any scatterer is always R , which is the radius of the circle. The AOA is defined as:

$$\varphi(L) = \sin^{-1}(R \sin \alpha / L) \quad (4.12)$$

where L = distance between scatterer and receiver. The probability density function is defined as:

$$p(\varphi) = \frac{1}{\pi} [A(L_1) - A(L_2)], \quad \varphi \in [-\varphi_{\max}, \varphi_{\max}] \quad (4.13)$$

where $A(L) = \left[1 - \left(\frac{L \sin \varphi}{R} \right)^2 \right]^{\frac{1}{2}} \left[\frac{L \cos \varphi}{R} + \frac{\sin \varphi}{R} \left(-D \sin \varphi - \frac{1}{2} (D^2 \cos^2 \varphi - D^2 + R^2)^{\frac{1}{2}} (-2D^2 \cos \varphi \sin \varphi) \right) \right]$.

For most practical situations when the radius of circle is smaller than the distance between transmitter and receiver, the angular spread can be determined by $\sigma_\varphi = \Delta / \sqrt{2}$ where Δ is the maximum AOA. The derivations are given in Appendix A.

4.2.3 Simulation Models

4.2.3.1 Uniform Distribution

The simulation model for uniform distribution uses the build-in MATLAB function *rand()* to generate an array of ten AOAs within the range $(-\Delta, \Delta)$ according to uniform distribution. The parameter Δ is the maximum one-sided range of the distribution obtained from the angular spread using the relationship given in (4.3). The AOA array is then shifted

according to the desired value of mean AOA. Figure 4-3 shows the empirical probability density function of uniform distribution with different values of angular spread. The PDFs integrate to one when the angles are considered in radians. The angles are shown in degrees for illustration purpose.

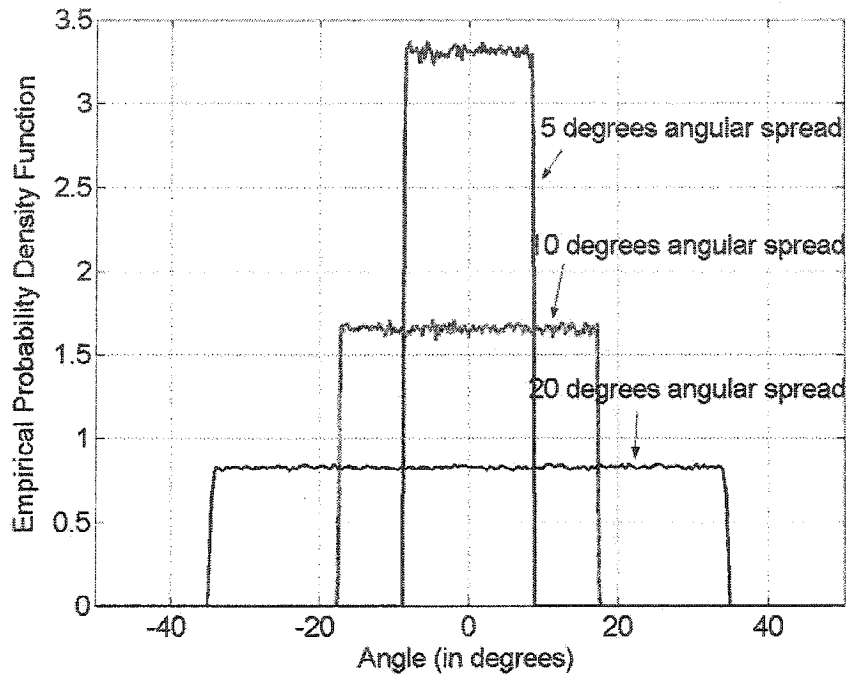


Figure 4-3 Empirical PDF of Uniform Distribution

4.2.3.2 Gaussian Distribution

The AOA samples for Gaussian distribution is generated using the build-in MATLAB function *normrnd()*. This function generates an array of ten AOA samples by specifying the mean and standard deviation of the distribution. The standard deviation is obtained from the angular spread using the relationship given by (4.5). Figure 4-4 shows the empirical probability density function of Gaussian distribution for a few different values of angular spread. The PDFs integrate to one when the angles are considered in radians. The angles are shown in degrees for illustration purpose. Figure 4-5 shows the empirical angular

spread vs. standard deviation relationship for Gaussian distribution. The angular spread and the standard deviation are exactly the same for small values of standard deviation (less than 30 degrees). The reason is that for small standard deviations, all of the angles of arrivals are within the range of $[-90^\circ, 90^\circ]$. Thus, the angular spread is equal to standard deviation. As the standard deviation increases, the number of out-of-range angles of arrivals that are truncated increases. This results in values of angular spread that are smaller than standard deviations. When the standard deviation approaches 90 degrees, the angular spread converges to 50 degrees. This is the maximum angular spread that can be achieved under the constraint that angles of arrivals are limited to $[-90^\circ, 90^\circ]$. In this study, only angular spreads of less than 50 degrees will be used.

4.2.3.3 Laplacian Distribution

There are no build-in MATLAB functions to generate Laplacian distribution. Each sample of a Laplacian distribution is obtained by first generating a value (x) with the range $[0,1]$ using the uniform distribution generation function *rand()*. Then the Laplacian sample (y) is determined using the following relationship:

$$\begin{aligned} y &= \left(\sigma/\sqrt{2}\right)\log(2x) \quad \text{for } x < 0.5 \\ y &= -\left(\sigma/\sqrt{2}\right)\log(2x) \quad \text{otherwise} \end{aligned} \tag{4.14}$$

Figure 4-6 shows the empirical PDF of Laplacian distribution for a few different values of angular spread. The PDFs integrate to one when the angles are considered in radians. The angles are shown in degrees for illustration purpose. Figure 4-7 shows the relationship between angular spread of truncated Laplacian distribution and standard deviation of non-truncated Laplacian distribution.

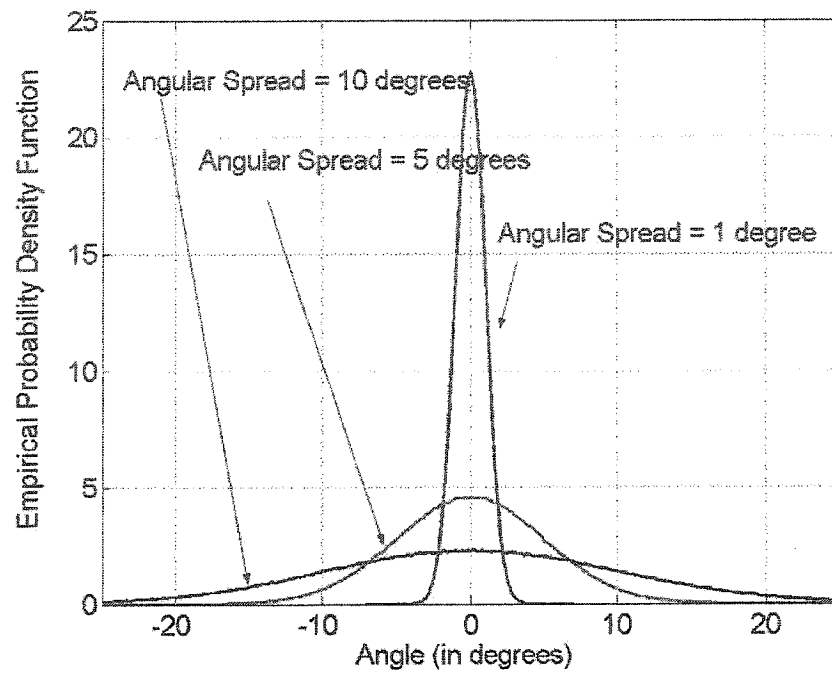


Figure 4-4 Empirical PDF of Gaussian Distribution

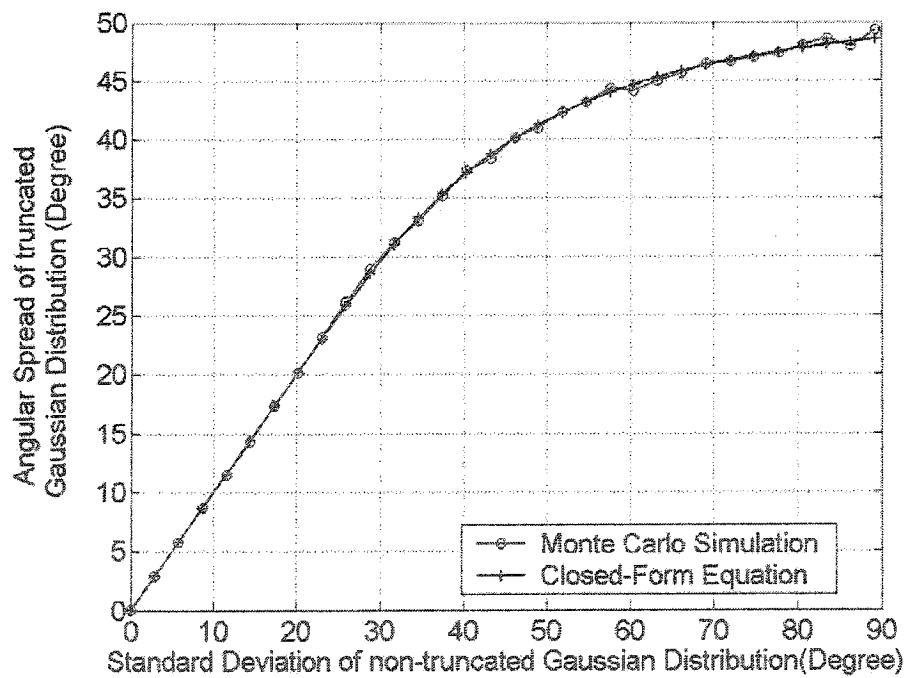


Figure 4-5 Angular Spread vs. Standard Deviation for Gaussian Distribution

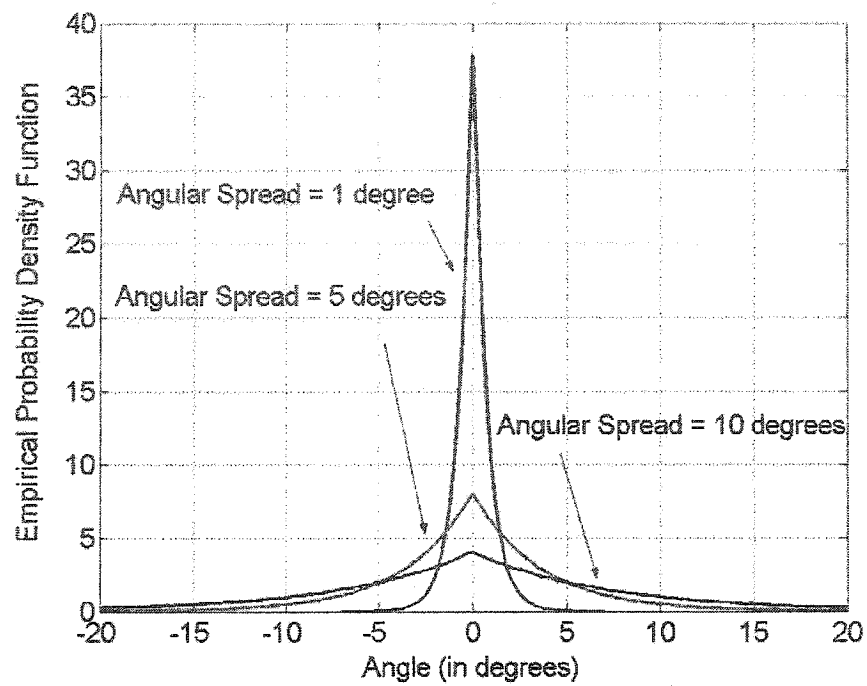


Figure 4-6 Empirical PDF of Laplacian Distribution

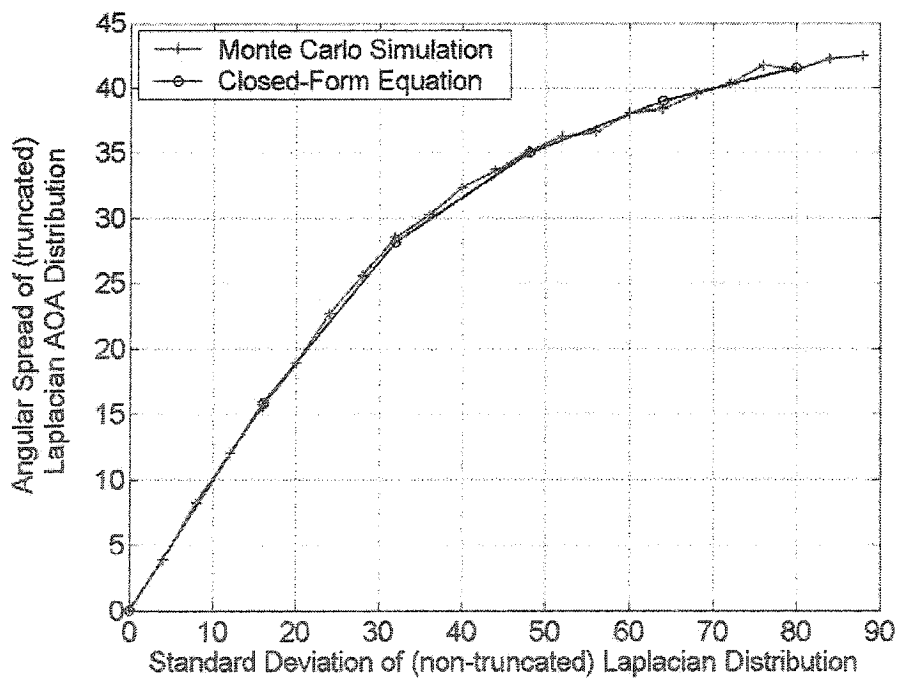


Figure 4-7 Angular Spread vs. Standard Deviation for Laplacian Distribution

4.2.3.4 Cosine Distribution

The procedure to generate cosine distribution is relatively more complicated than other distributions. There are no build-in MATLAB functions to generate cosine distribution. Also, the generation function cannot be described by a standard closed-form equation. To generate cosine distribution, we divide the range of AOA into m equal regions. Using equation (4.8), we obtain the probability of each region using the mean value of each region as ϕ_0 . Then we generate samples according to these probabilities. When we make the number of regions infinitely large (making the size of each region indefinitely small), the generated distribution would approach the cosine distribution. Figure 4-8 shows the probability density function of the cosine distribution obtained by simulations. The PDFs integrate to one when the angles are considered in radians. The angles are shown in degrees for illustration purpose. Figure 4-9 shows the angular spread versus value of exponent of Cosine distribution. The angular spread reaches its maximum value when the value of n_c is equal to 0, and it slowly converges to zero as the value of n_c approaches infinity. The maximum angular spread can be derived mathematically. When the value of n is 0, all the cosine values become 1 and the cosine distribution becomes a uniform distribution in the range of $[-90^\circ, 90^\circ]$. The angular spread of a uniform distribution can be calculated by $S_\phi = \frac{\Delta}{\sqrt{3}} = \frac{90^\circ}{\sqrt{3}} = 51.96^\circ$. Thus 51.96° is the maximum angular spread that can be achieved.

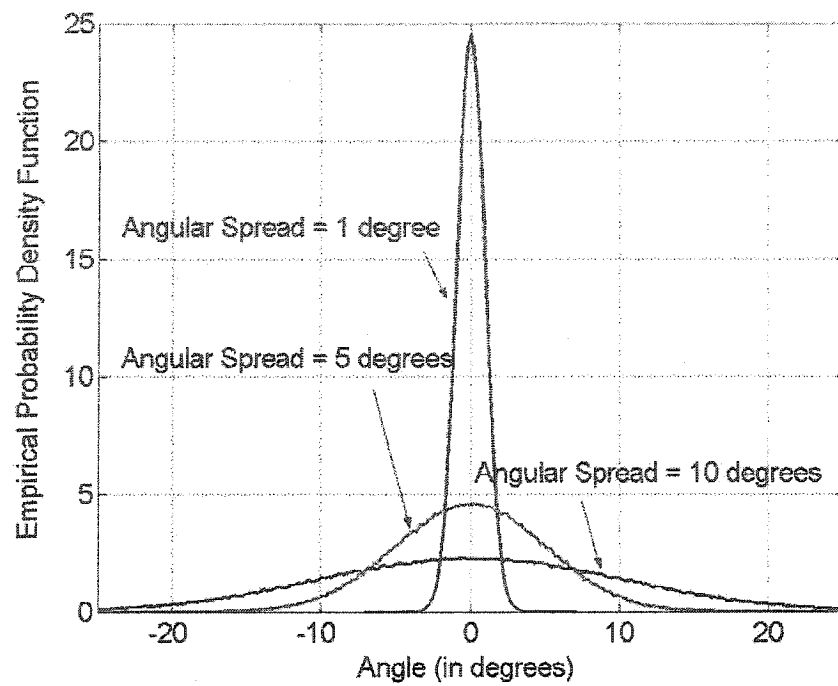


Figure 4-8 Empirical PDF of Cosine AOA Distribution

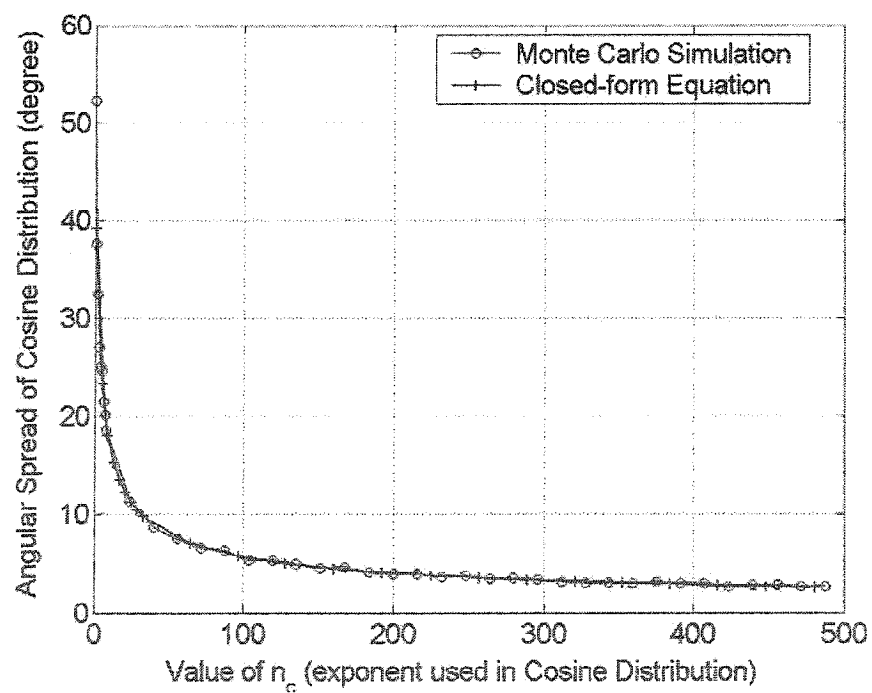


Figure 4-9 Angular Spread vs. Exponent Value (n_c) for Cosine Distribution

4.2.3.5 Circular Scatterer Model

The procedure to generate angle of arrivals of the circular scatter model starts with first generating the scatterers along the circle using uniform distribution. Then the angles of arrivals are calculated based on the position of the scatterers, using equation (4.12). Figure 4-10 shows the empirical probability density function of the circular scatterer model for different values of angular spread. The PDFs integrate to one when the angles are considered in radians. The angles are shown in degrees for illustration purpose. It can be seen that most of the samples are concentrated near the maximum angles. The reason behind this is that when the scatterers are uniformly distributed along the circle, there are more scatterers on the two sides of the circle that would produce large angle of arrivals.

Figure 4-11 illustrates the angular spread / maximum angle ratio versus radius / distance ratio. It can be shown that for radius / distance ratio of less than 0.5 (which corresponds to most practical situations), the angular spread is roughly equal to 0.7 of the maximum angle. This relationship provides a simple yet accurate method to obtain the angular spread once the maximum angle is known.

4.2.4 Summary of Continuous AOA Models

The previous sections discuss the continuous angle of arrival distributions and models that are used in the simulation to evaluate their effects on channel capacity of MIMO systems. These distributions include Uniform, Gaussian, Laplacian, Cosine, and the circular scatterer model. These distributions are truncated in the sense that the range is limited to $\pm 90^\circ$ of the center angle.

Analytical equations for the probability density function and angular spread are derived for each of the distributions. The effects of these AOA distributions can be compared

if the distributions are generated with the same angular spread. Simulation procedures are also discussed. The empirical probability density functions of the studied distributions are compared with the analytical-determined probability density functions. The two functions are perfect match for all the continuous distributions investigated in this study. This verifies the correctness of the Monte Carlo simulations to generate the distributions. Figure 4-12, and Figure 4-13 show the comparison of all five AOA distributions with angular spread of 1 and 10 degrees.

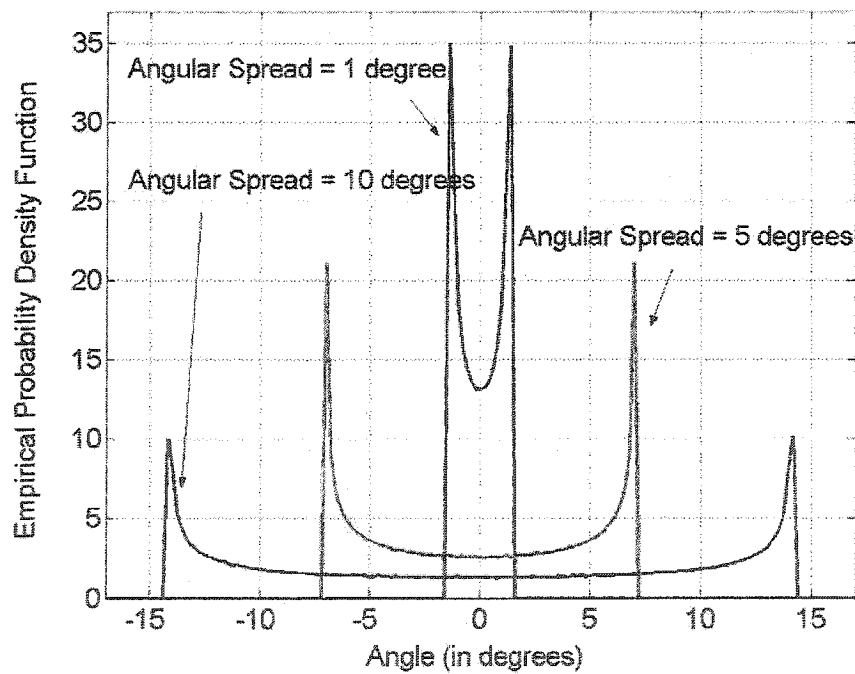


Figure 4-10 Empirical Probability density function of Circular Scatterer Model

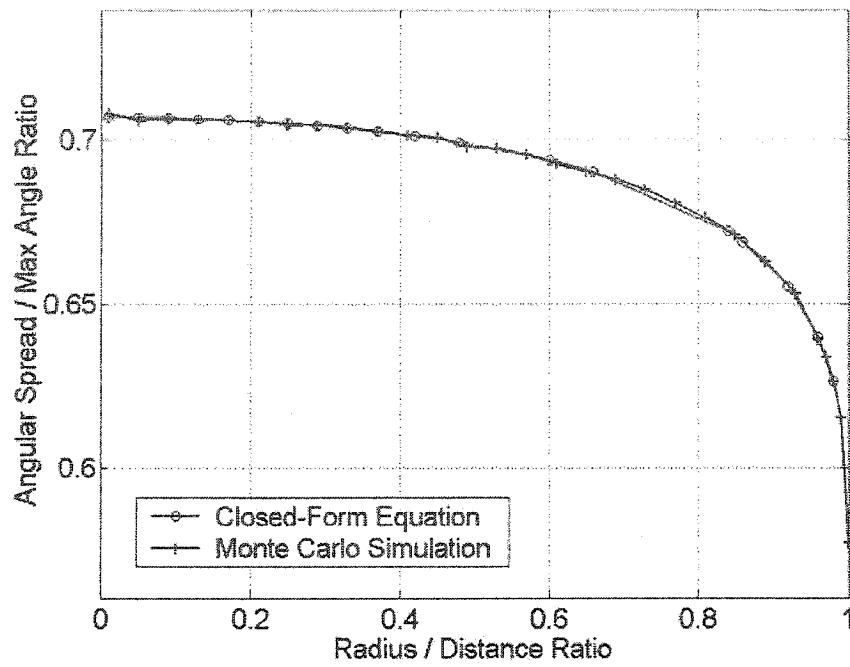


Figure 4-11 Angular Spread / Maximum Angle Ratio vs. Radius / Distance Ratio

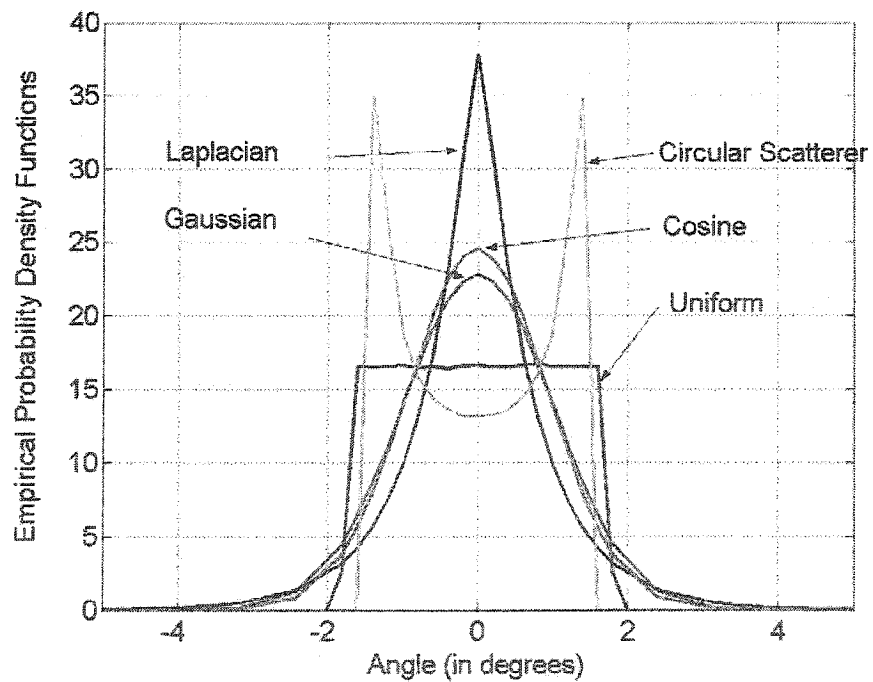


Figure 4-12 Comparison of all AOA distributions with angular spread of 1 degree

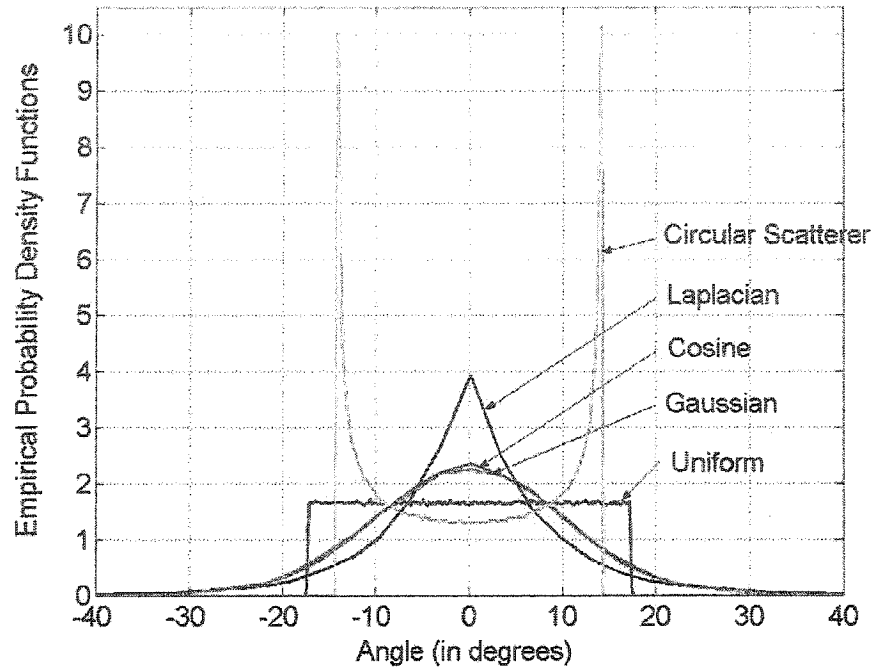


Figure 4-13 Comparison of all AOA distributions with angular spread of 10 degrees

4.3 SIMULATION RESULTS AND OBSERVATIONS

4.3.1 Channel Capacity

4.3.1.1 Effects of AOA Distribution

Figure 4-14, and Figure 4-15 show the channel capacity versus antenna spacing graphs for all five continuous angle of arrival distributions considered in this study, with angular spread of 1° and 10° respectively. The setting of the simulation is as follow: Number of Trials = 10000, Mean AOA = 0° , Number of Transmit Antennas = 10, Number of Receive Antennas = 10, Number of Multipath Components per Transmitted Signal = 10, Signal-to-noise Ratio = 30 dB.

It is clear that the channel capacities as functions of antenna spacing obtained by using different AOA distributions exhibit very similar trends, with slight differences. In general, the channel capacity increases as antenna spacing increases until the maximum value is reached. After reaching the maximum value, the channel capacity for most AOA distributions stays at that value, while some others exhibit minor fluctuations around the maximum value. The maximum values of mean channel capacity achieved by all five distributions are the same, at 55 bits/second/Hz. The upper bounds of mean channel capacity achieved by all five distributions are also the same, at 66 bits/second/Hz. These results match well with the results obtained by the equation $C_{\max} = n \log_2(1 + \rho/n)$.

For all values of angular spread considered in this study, the channel capacity curves of Gaussian, Laplacian and Cosine AOA distributions are almost identical as the channel capacity rises from 10 to 50 bits/sec/Hz. After that, they approach the maximum value at slightly different pace. All three AOA distributions achieve higher capacity at smaller values of antennas spacing than uniform distribution and circular scatterer model. Also, these three AOA distributions do not exhibit fluctuations about the maximum channel capacity than are present in the uniform distribution and circular scatterer model. Gaussian, Cosine and Laplacian AOA distributions can be categorized as distributions where the majority of angle of arrivals are near the mean value. Uniform AOA distribution contains a more even distribution of angle of arrivals across all angles. Circular scatterer model is a distribution where the majority of the angles of arrivals are near the two ends.

It can be concluded that AOA distributions with signals concentrated in the center of the distribution generally achieves better channel capacity than distributions with more spread-out signals at the same value of antenna spacing. Also AOA distributions with signals

concentrated in the center do not exhibit fluctuations around the maximum channel capacity that are present in distributions with more spread-out signals. Gaussian, Laplacian and Cosine AOA distributions provide better performance and stability than Uniform AOA distribution and circular scatterer model. This observation is consistent with all values of angular spreads that are considered in this study.

Another interesting observation of the AOA distributions is that the trend of the channel capacity curve of each AOA distribution relative to other AOA distributions remains the same regardless of the value of angular spread. This indicates that the effect of the AOA distribution of channel capacity is the same regardless of the values of angular spreads.

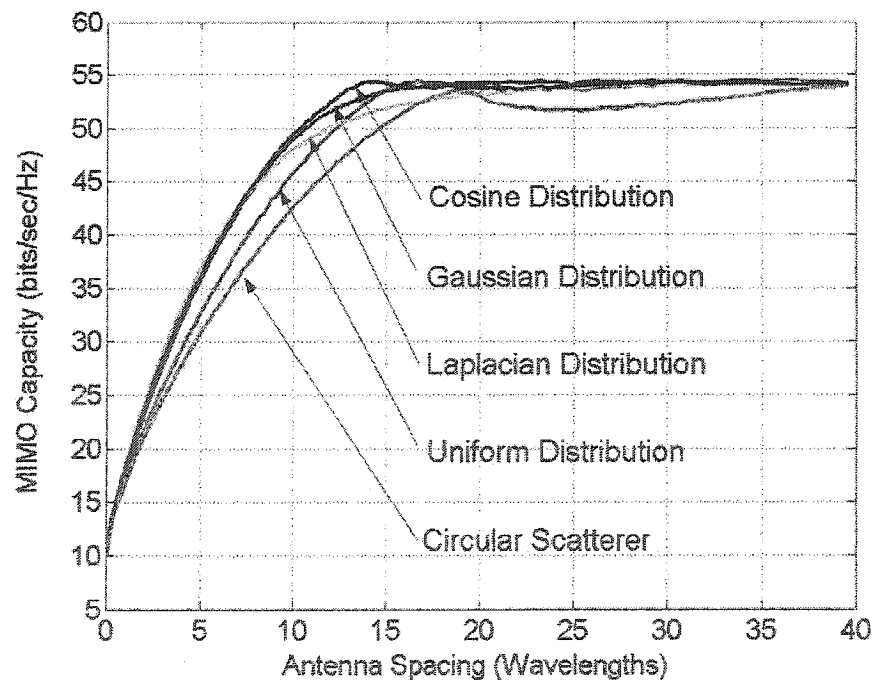


Figure 4-14 MIMO Capacity vs. Antenna Spacing for Angular Spread of 1 degree

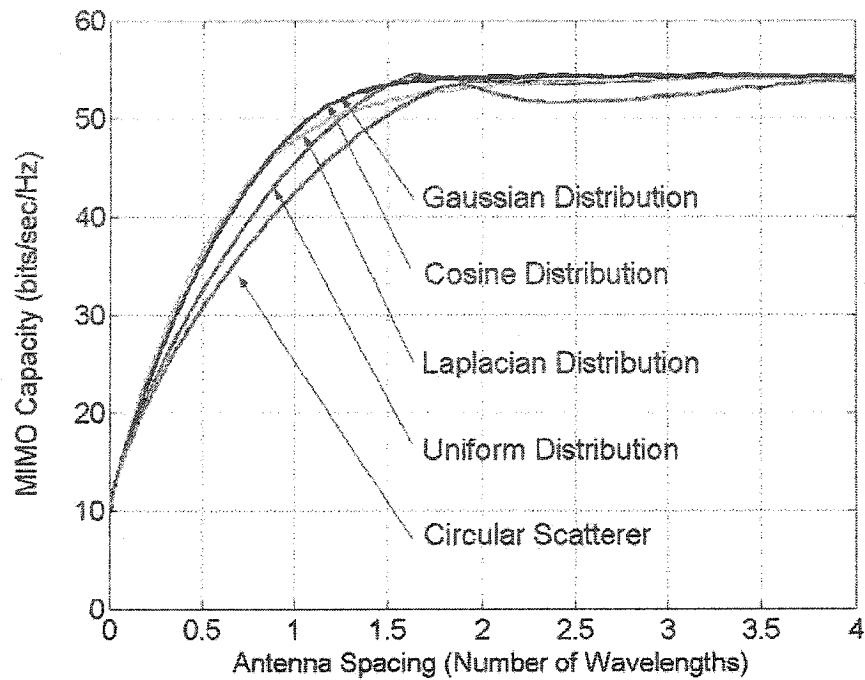


Figure 4-15 MIMO Capacity vs. Antenna Spacing for Angular Spread of 10 degrees

4.3.1.2 Effects of Angular Spread

Figure 4-16 and Figure 4-17 show the MIMO capacity versus antenna spacing graphs for different values of angular spreads, and for Uniform and Gaussian distributions respectively. For both AOA distributions, it is obvious that distributions with larger angular spreads achieve higher channel capacities at a given antenna spacing than distributions with smaller angular spreads. As shown in Figure 4-16, AOA distribution with angular spread of 40 degrees achieves the maximum capacity at antenna spacing of 0.5λ . AOA distribution with angular spread of 20 degrees achieves the maximum capacity at antenna spacing of 0.8λ . For AOA distributions with angular spreads of 10 and 3, the values of antenna spacing at which maximum capacity are reached are 1.5 and 5.3 respectively.

Comparing Figure 4-16 with Figure 4-17, it is clear that there are minor fluctuations around the maximum channel capacity when uniform AOA distribution is used, but these

fluctuations are not present when Gaussian AOA distribution is used. It is another indication that the fluctuation around the maximum channel capacity caused by the AOA distribution and is independent of the angular spread. It can be concluded that as the angular spread increases, the performance of the MIMO system increases, meaning that the maximum channel capacity can be reached at a smaller value of antenna spacing. Also, the fluctuation of channel capacity around the maximum value is an effect of the AOA distribution, and not an effect of the value of angular spread.

4.3.1.3 Effects of mean AOA

Figure 4-18 and Figure 4-19 show the MIMO capacity versus antenna spacing graphs for different values of mean AOA, using uniform distribution and angular spreads of 1 degree and 20 degrees respectively. It is clear that the best performance is obtained when the mean AOA is at 0 degree, representing the broadside direction. As the mean AOA moves away from the broadside direction, the performance becomes worse and it requires a larger value of antenna spacing for the system to achieve the maximum channel capacity. Any interesting observation can be seen in Figure 4-18, when the angular spread is 1 degree and the mean AOA is 90 degrees. With this configuration, the range of angles that signals can arrive from is from 88.3 degrees to 91.7 degrees. At these values of angle of arrivals, the signals are essentially coming at the same direction as the linear antenna array. The signals received at different antenna elements would be highly correlated and as a result, the improvement in capacity with the increase of antenna spacing is very slow. On the other hand, when the angular spread is set to 20 degrees in Figure 4-19, the improvement in capacity with the increase of antenna spacing is much faster, because of the wider range of incoming signals resulting in lower correlations between antenna elements.

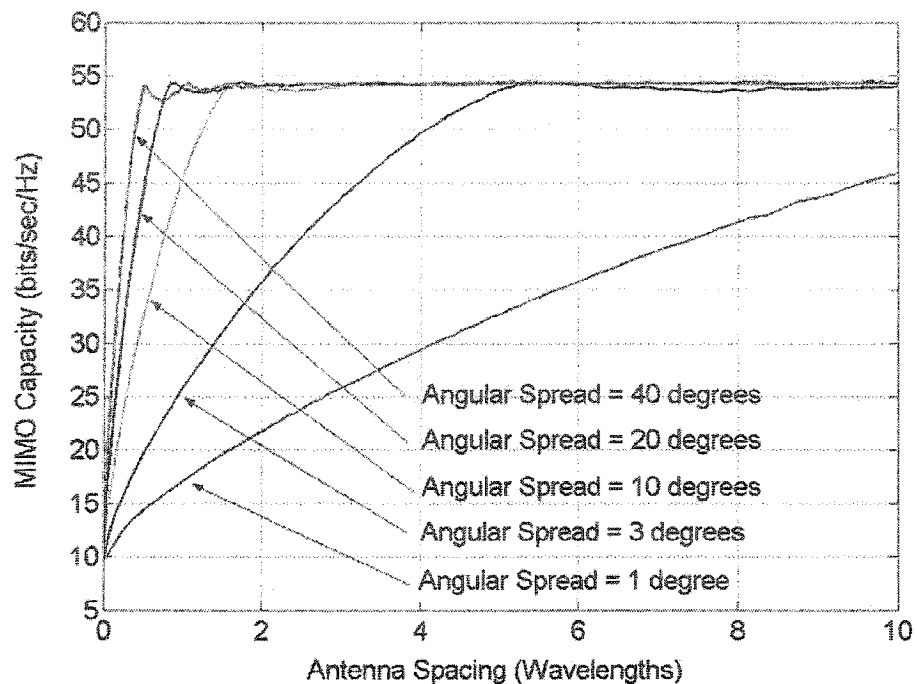


Figure 4-16 MIMO Capacity vs. Antenna Spacing for different angular spreads using uniform distribution

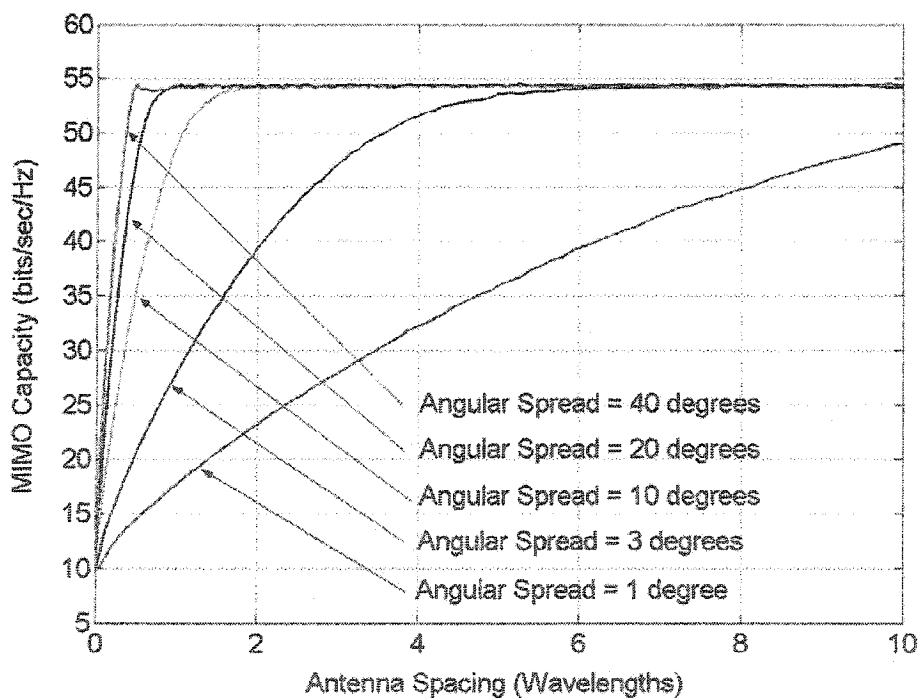


Figure 4-17 MIMO Capacity vs. Antenna Spacing for different angular spreads using Gaussian distribution

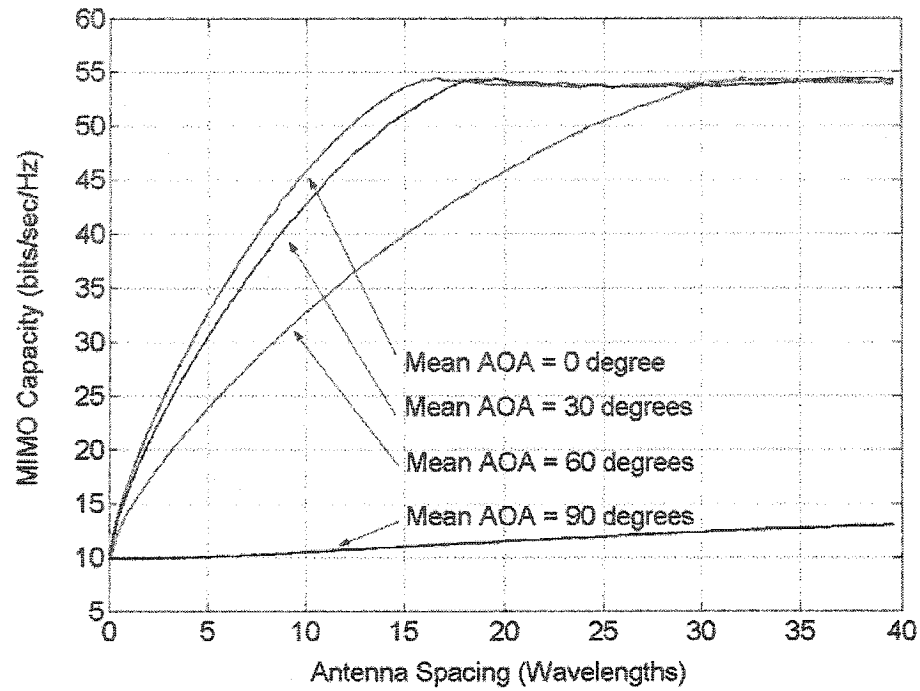


Figure 4-18 MIMO Capacity vs. Antenna Spacing for uniform distribution with angular spread of 1 degree and different values of mean AOA

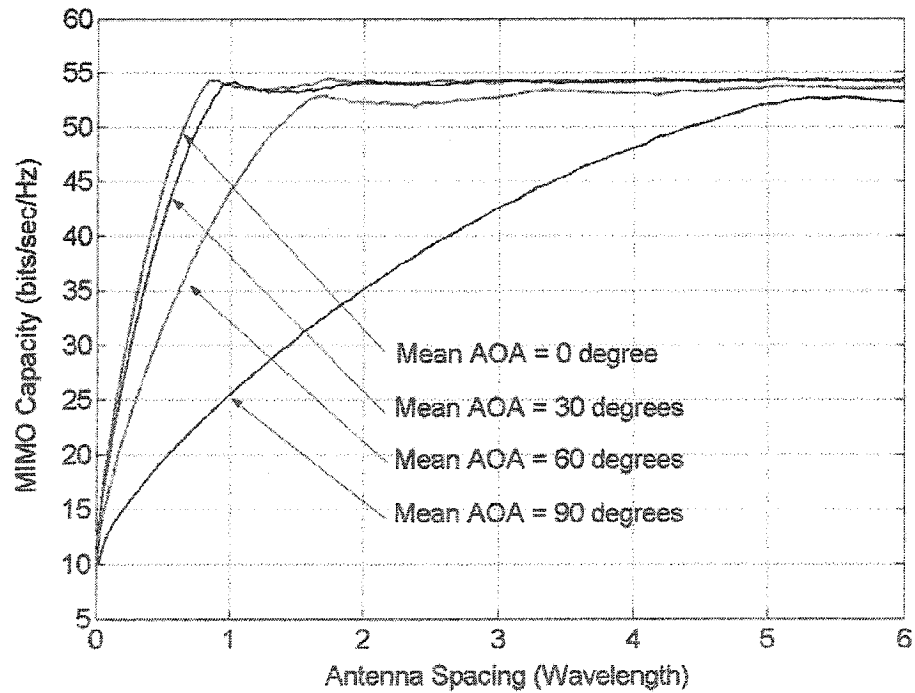


Figure 4-19 MIMO Capacity vs. Antenna Spacing for uniform distribution with angular spread of 20 degrees and different values of mean AOA

4.3.1.4 Effects of Number of Rx Antennas

Figure 4-20 shows the MIMO capacity versus antenna spacing graphs for different number of Rx antenna elements. It is clear that the maximum channel capacity is a function of the number of antenna elements. As the number of antenna element increases, the maximum channel capacity increases. The following table summarizes the maximum channel capacity for all numbers of antenna elements. Note that these values are the maximum channel capacity as determined by the Foschini-Telatar equation (3.5) when the SNR is assumed to be 30 dB.

# of Antennas	1	2	3	4	5	6	7	8	9	10
Highest Capacity (bits/s/Hz)	9.29	15.88	21.75	27.10	32.16	37.04	41.59	46.06	50.37	54.54
Increment	-	6.59	5.87	5.35	5.06	4.88	4.55	4.47	4.31	4.17

This observation validates the theory that MIMO capacity is limited by the minimum of the three factors - number of transmit antennas, number of receive antenna, and number of multipath signals. Figure 4-20 clearly shows that maximum MIMO capacity increases as the number of antenna increases. Also, it is observed that regardless of the number of antennas, the maximum capacity corresponding to the number of antennas is reached at the same value of antenna spacing.

4.3.2 Correlation

4.3.2.1 Effect of AOA Distribution

Figure 4-21 shows the correlation vs. antenna spacing graphs for different AOA distributions with angular spreads of 5 degrees. It is observed that the correlations for Cosine

and Gaussian AOA distributions converge to zero at the lowest value of angular spread when compared to other AOA distributions. The correlation of Laplacian AOA distribution approaches zero at bigger values of antenna spacing than Cosine and Gaussian AOA distributions. The correlations of Uniform AOA distribution and circular scatterer model exhibit substantial amount of fluctuations around zero and eventually converge to zero. The degree of fluctuation of circular scatterer model is larger than that of the Uniform AOA distribution.

The MIMO capacities of the different AOA distributions observed in Section 4.3.1.1 are direct consequences of the correlations studied in this chapter. It is clear that a large value of channel capacity can be reached when correlation is low, and vice-versa. Since the correlations of Cosine and Gaussian AOA distributions converge to zero at relatively small values of antenna spacing, the channel capacities of these two distributions also approach the maximum value at relatively small values of antenna spacing. Since the correlations of Uniform AOA distribution and circular scatterer model converge to zero at relatively large values of antenna spacing, the channel capacities of these two distributions also approach the maximum value at relatively large values of antenna spacing. The fluctuations around zero in the correlation of Uniform AOA distribution and circular scatterer model result in the fluctuations around the maximum value in the channel capacities of these two distributions.

4.3.2.2 Effect of Angular Spread

Figure 4-22 and Figure 4-23 show the correlation versus antenna spacing for different value of angular spreads for Uniform and Gaussian AOA distribution respectively. It is observed that the initial rate of decrease in correlation as function of antenna spacing is bigger when the angular spread is larger. When the angular spread is small, the correlation

comes down at a slower rate. For Uniform AOA distribution, it is clear that the correlations for all values of angular spread fluctuate around zero before converging to it. For Gaussian AOA distribution, the correlations for most values of angular spread do not exhibit any fluctuations and converge to zero directly. The only noticeable fluctuation occurs when the angular spread is at 40 degrees, and this fluctuation is of smaller scale than that of the Uniform AOA distribution at the same angular spread. The effect of this fluctuation can be seen in terms of MIMO capacity in Figure 4-17, where the MIMO capacity for angular spread of 40 degrees exhibit a small but noticeable amount of fluctuation that is not present in all other values of angular spreads. However, the fluctuation is very minor, when compared to fluctuations seen in Uniform AOA distribution and circular scatterer model.

4.3.2.3 Effect of Mean AOA

Figure 4-24 and Figure 4-25 show the correlation vs. antenna spacing for different values of mean AOA, for Uniform and Gaussian AOA distributions respectively. The correlations for small values of mean AOA approaches zero at smaller values of antenna spacing than those for larger values of mean AOA. For large values of mean AOA such as 90 degrees, the correlation essentially fluctuates between the maximum and minimum values (1 and -1), with local maximum and minimum values slowly decreasing. Figure 4-26 and Figure 4-27 offer a better picture by illustrating the envelope correlation vs. antenna spacing for the same sets of environment. It is very clear that the envelope correlations of AOA distributions with smaller angles of incidence converge to zero at much lower value of antenna spacing than those with larger angles of incidence.

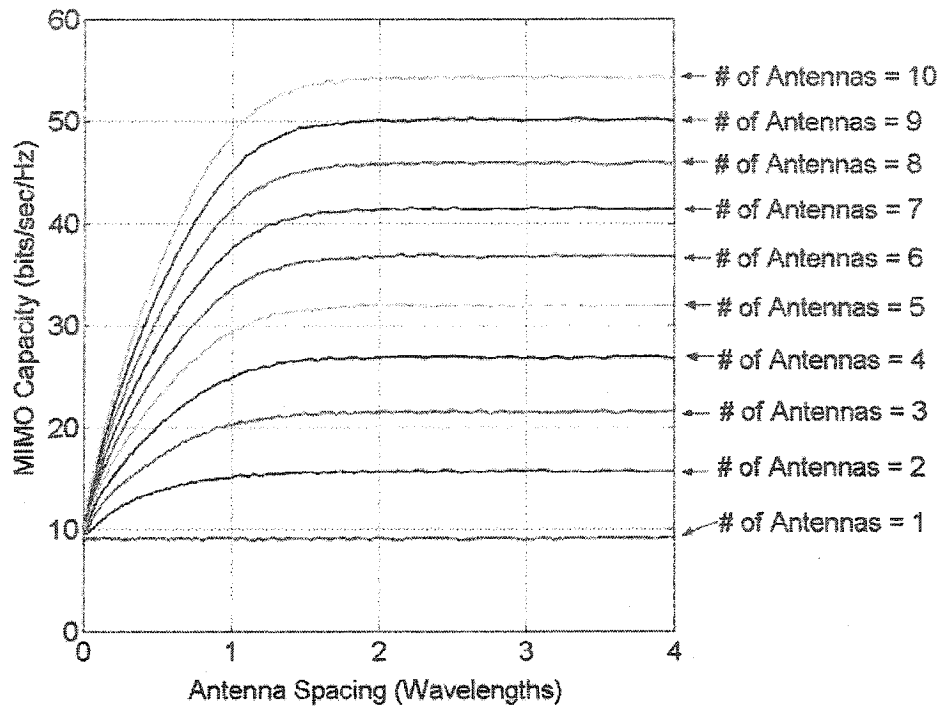


Figure 4-20 MIMO Capacity vs. Antenna Spacing for Gaussian AOA distribution with angular spread of 10 degrees and different number of antenna elements in the receiver antenna array

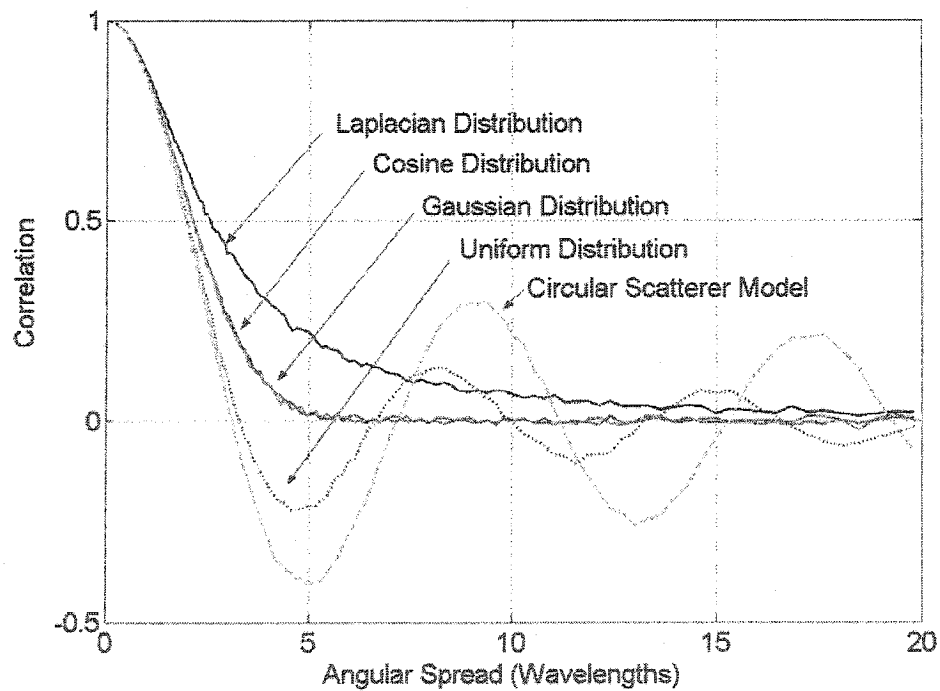


Figure 4-21 Correlation vs. Antenna Spacing for different AOA distributions with angular spread of 5 degrees and mean AOA of 0 degree

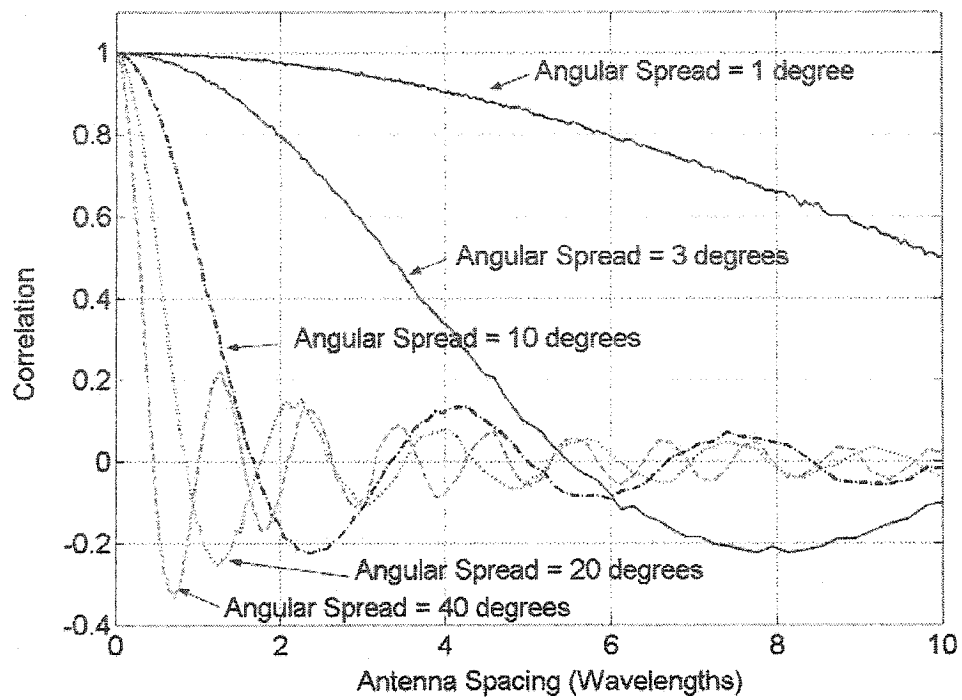


Figure 4-22 Correlation vs. Antenna Spacing for uniform distribution with different values of angular spread and mean AOA of 0 degree

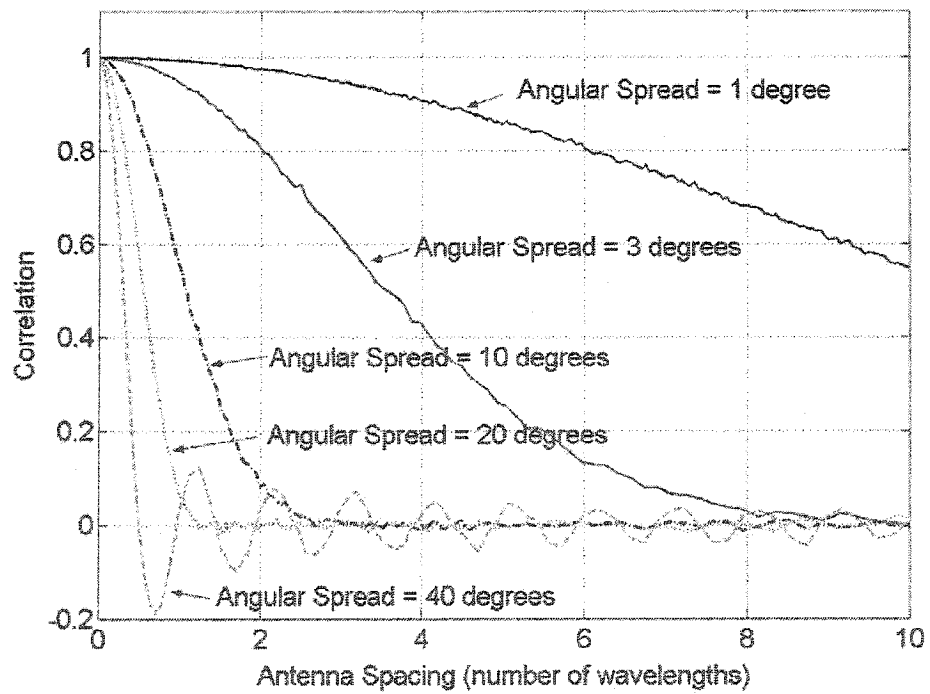


Figure 4-23 Correlation vs. Antenna Spacing for Gaussian distribution with different values of angular spread and mean AOA of 0 degree

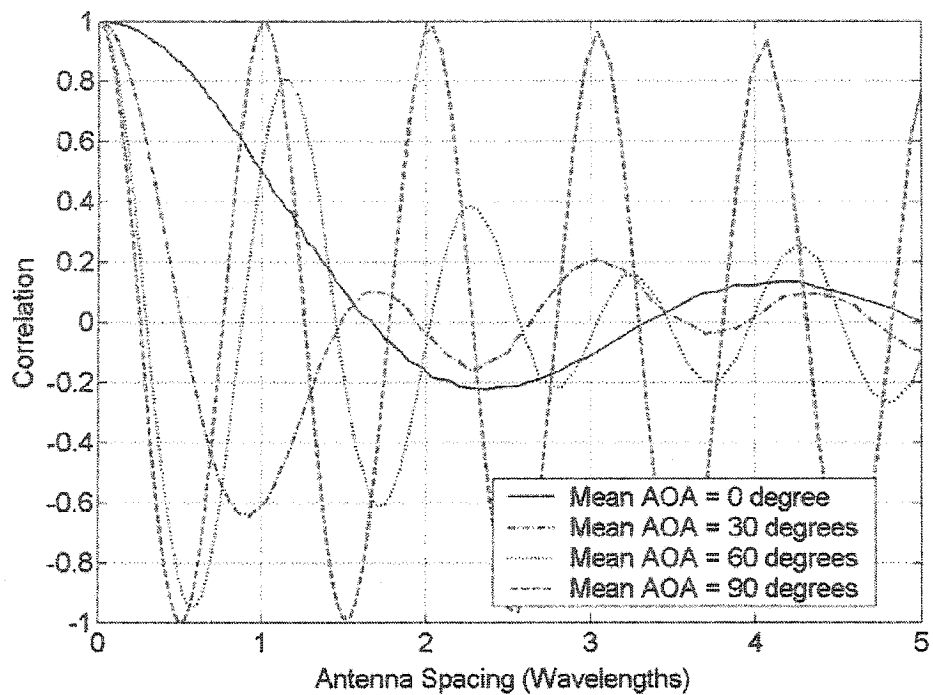


Figure 4-24 Correlation vs. Antenna Spacing for uniform AOA distribution with angular spread of 10 degrees and different values of mean AOA

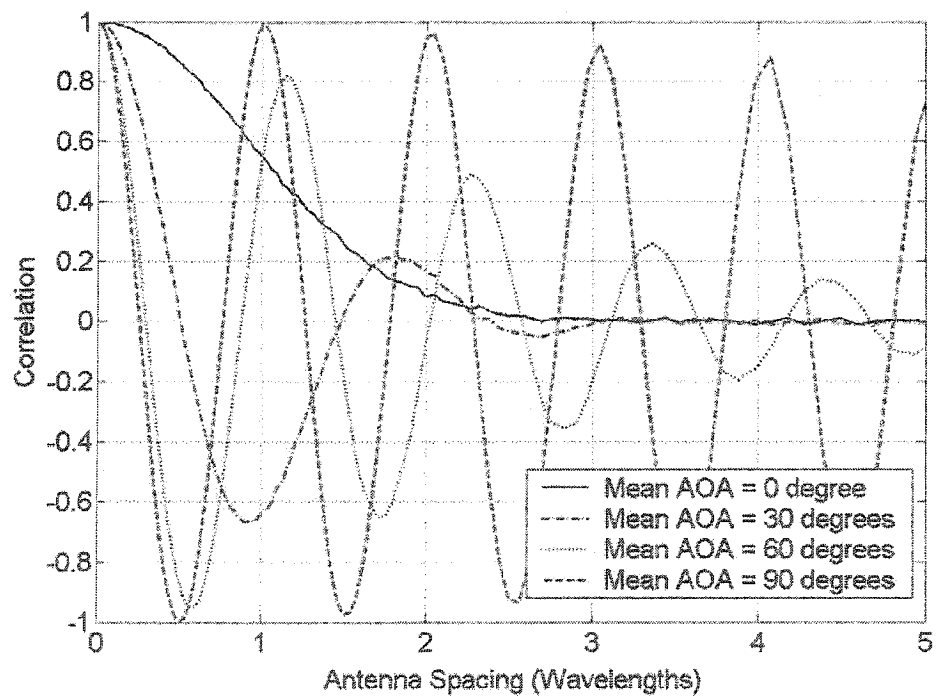


Figure 4-25 Correlation vs. Antenna Spacing for Gaussian AOA distribution with angular spread of 10 degrees and different values of mean AOA

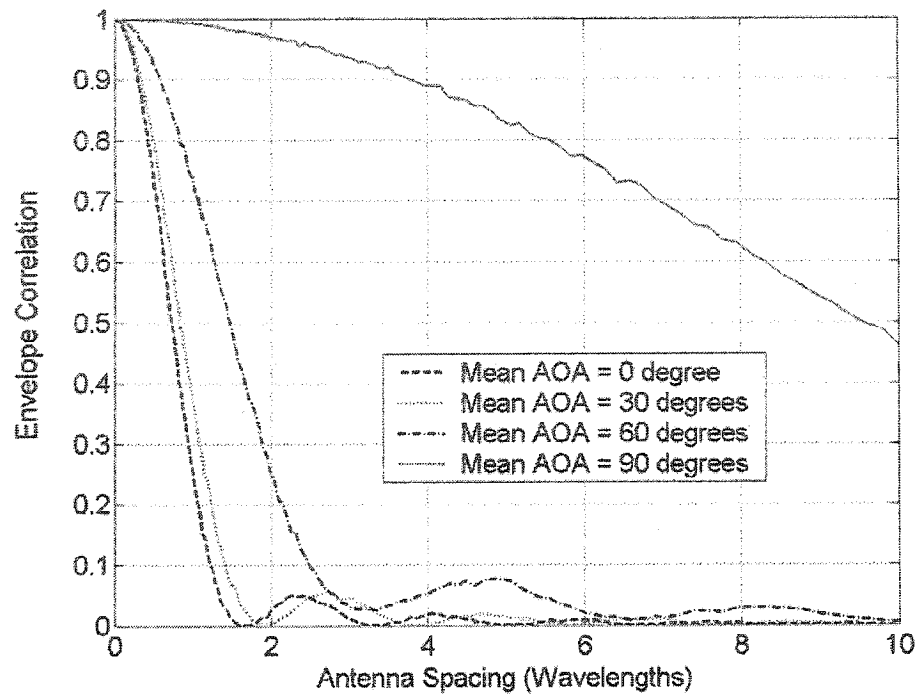


Figure 4-26 Envelope Correlation vs. Antenna Spacing for Uniform distribution with angular spread of 10 degrees and different values of mean AOA

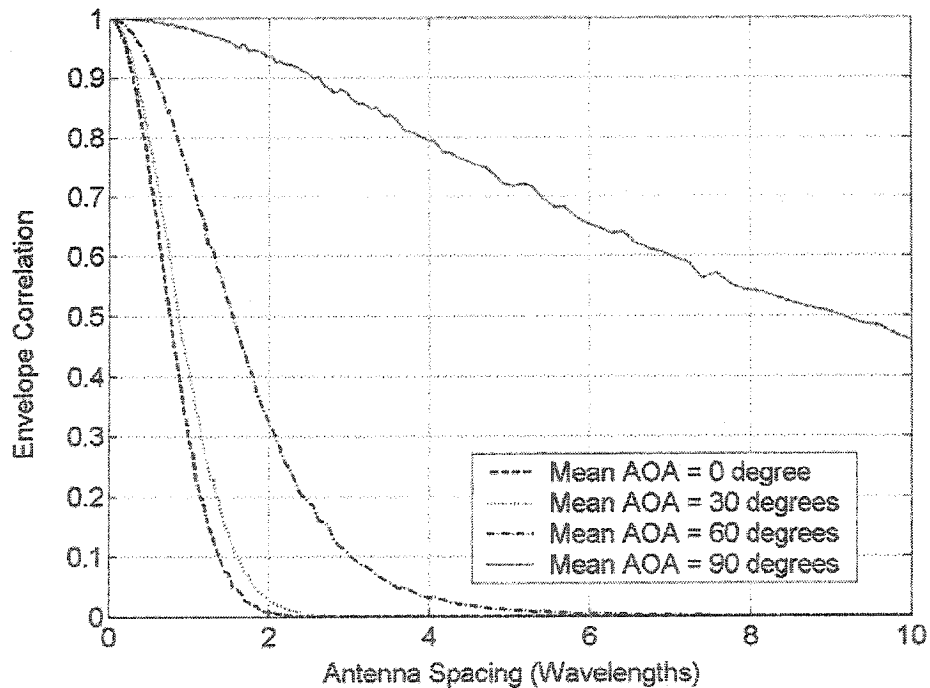


Figure 4-27 Envelope Correlation vs. Antenna Spacing for Gaussian distribution with angular spread of 10 degrees and different values of mean AOA

4.4 CONCLUDING REMARKS

4.4.1 Summary of this Chapter

This chapter explores the impact of continuous AOA distributions on the performance of MIMO systems in terms of channel capacity and correlation. Five distributions, including Uniform, Gaussian, Laplacian and Cosine AOA distributions, as well as the circular scatterer model, are introduced and employed in the studies. These distributions are truncated in the sense that the allowable angles are limited to $\pm 90^\circ$ of the mean AOA. For comparison purposes, different AOA distributions with the same angular spread are being compared, and closed-form equations of angular spreads are developed for all the AOA distributions.

The results of the simulations performed in this chapter can be categorized into two areas: (1) the impact of AOA distribution shape on MIMO system performance, and (2) the impact of AOA distribution parameters and antenna settings on MIMO system performance. For the first category, it is observed that all five distributions result in very similar MIMO capacity and correlation curves. AOA distributions with signals concentrated around the mean AOA (uniform, Gaussian and Cosine) tend to attain the maximum capacity at smaller values of antenna spacing, and there are no fluctuations of channel capacity after the maximum value is obtained. AOA distributions with signals that are more spread out or concentrated at areas far away from the mean AOAs (Uniform and circular scatterer model) tend to attain the maximum capacity at relatively larger values of antenna spacing, and there are noticeable amount of fluctuations around the maximum value after the maximum value is first achieved.

The results of the second category of observation involving distribution parameters and antenna settings are summarized in the following table:

	Correlation	Channel Capacity
Angular Spread $\uparrow$	$\downarrow$	$\uparrow$
Mean AOA $\uparrow$ (moving away from the broadside direction)	$\uparrow$	$\downarrow$
Number of Rx Antennas $\uparrow$	Not Applicable	$\uparrow$

4.4.2 Contribution of this Chapter

The contribution of this chapter is in the comprehensive study of the impact of AOA distributions on MIMO system performance. Most literatures assume a particular AOA distribution and use it to derive other results. It would be of interest to analyze the effects of using different AOA distributions and changing distribution parameters. This is the main motivation for the study that is done in this chapter.

Another contribution of this chapter is the development of a comprehensive set of equations of AOA distribution equations that allows comparisons between different distributions. Equations (4.5), (4.7), (4.9), (4.10), (4.11), and (4.13) are new equations developed in this study. Their derivations are given in Appendix A. These equations provide a foundation for further analysis in the study of AOA distributions.

5 EFFECT OF IMPULSIVE AOA MODELS ON MIMO CAPACITY

5.1 INTRODUCTION

As mentioned in Chapter 4, two categories of angle of arrival models are investigated in this study. Chapter 4 discusses the first category - continuous AOA distributions. This chapter focuses on the second category - impulsive AOA models. The term “impulsive” in this context implies impulses of AOA in the angular domain. The mathematical background and simulation methodologies of the impulsive AOA models are provided. They are followed by an illustration of simulation results and observations.

At the end, conclusions are drawn. The objective of the work with impulse models in this chapter is to analyze the performance and behavior of MIMO systems when the number of multipath components is small. Traditional studies on wireless systems assume a rich scattering environment with large numbers of multipath components arriving at the receiver. It would be of interest to find out the effects on MIMO systems if this condition does not hold.

5.2 AOA MODELS

5.2.1 Definition of Impulsive AOA Models

The impulsive AOA models emulate situations where a limited number of multipath signals arrive at the receiver antenna system. Each multipath signal arriving from a specific angle represents one impulse in the impulsive AOA model. The numbers of impulses being investigated are set to values that are smaller than the number of antenna elements in the receiver antenna array. Since ten antenna elements are used in our study, the number of

impulses that will be investigated ranges from one to ten. The probability density functions of impulsive models are:

$$p_{N=odd}(\varphi) = \frac{1}{N} \left[\sum_{n=1}^{(N-1)/2} \left[\delta\left(\varphi - \frac{2n\Delta}{N-1}\right) + \delta\left(\varphi + \frac{2n\Delta}{N-1}\right) \right] + \delta(\varphi) \right] \quad (5.1)$$

$$p_{N=even}(\varphi) = \frac{1}{N} \left[\sum_{n=1}^{N/2} \left[\delta\left(\varphi - \frac{2n\Delta}{N}\right) + \delta\left(\varphi + \frac{2n\Delta}{N}\right) \right] \right] \quad (5.2)$$

where N is the number of unique AOA in the AOA distribution, $\delta(\varphi)$ is the delta function, and Δ is the one-sided range of the distribution. For illustration purpose, Figure 5-1 shows the geometrical representation of a 3-Impulse Model.

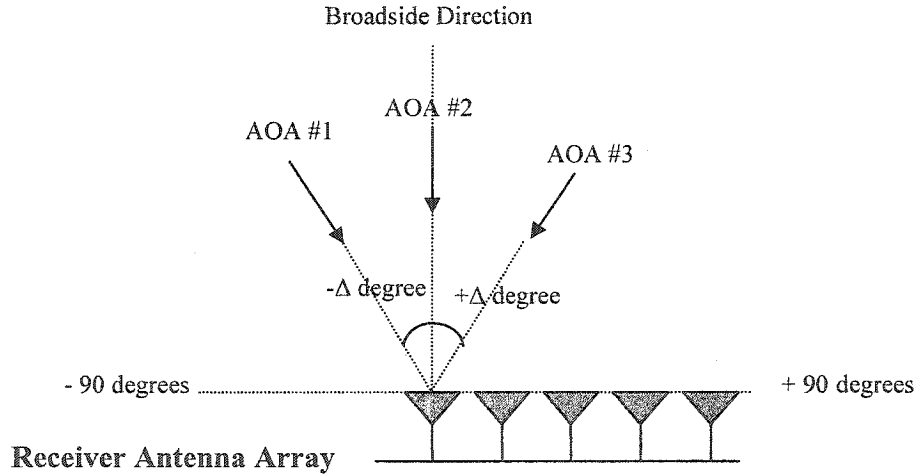


Figure 5-1 Geometric Illustration of 3-Impulse Model

Since the focus of this study is to investigate the effect of impulsive AOA distributions on MIMO capacity, the angular spreads and distributions of the various impulse models are insignificant as long as the number of impulses is set to the desired value. For convenience, the impulsive models are defined as even function, where the locations of impulses of the left side of the broadside direction are reflections of those on the right side of

the broadside direction. The maximum angle is set to the same value for all the impulse models to keep all the impulses within the same range.

5.2.2 Physical Meaning of Impulsive AOA Models

It is important to illustrate the physical meaning of impulsive AOA models in a practical wireless environment. Impulsive AOA models represent wireless channels with very few multipath components. This implies situations where there is a direct line-of-sight path between the transmitter and the receiver, and very few (or even zero) scatterers in the neighborhoods of both the transmitter and the receiver. The One-Impulse model is the special case when there is only one path between the transmitter and the receiver. This scenario is not very likely in an urban area, but is possible in a rural area, when there is a huge open space between the transmitter and the receiver, with no mountains or big obstacles nearby. One-Impulse model is also possible when the transmitter, surrounded by many scatterers nearby, is very far away from the receiver, and that the area between the transmitter and the receiver is free of scatterers in the azimuth. In this case, all the multipath components resulting from the scatterers close to the transmitter would arrive at essentially the same angle at the receiver.

Multiple-Impulse models correspond to situations with one line-of-sight signal and a few multipath signals. Again, this is not very likely in an urban area, but can happen in a rural area, where there is a line-of-sight path between transmitter and receiver, and mountains or big obstacles far away from the transmitter. The mountains would reflect the signal, introducing multipath components that arrive at the receiver at a different angle than the original signal. Figure 5-2 illustrates the physical representation of Two-Impulse model, where there is a line-of-sight signal and a multipath signal resulted from a distant scatterer.

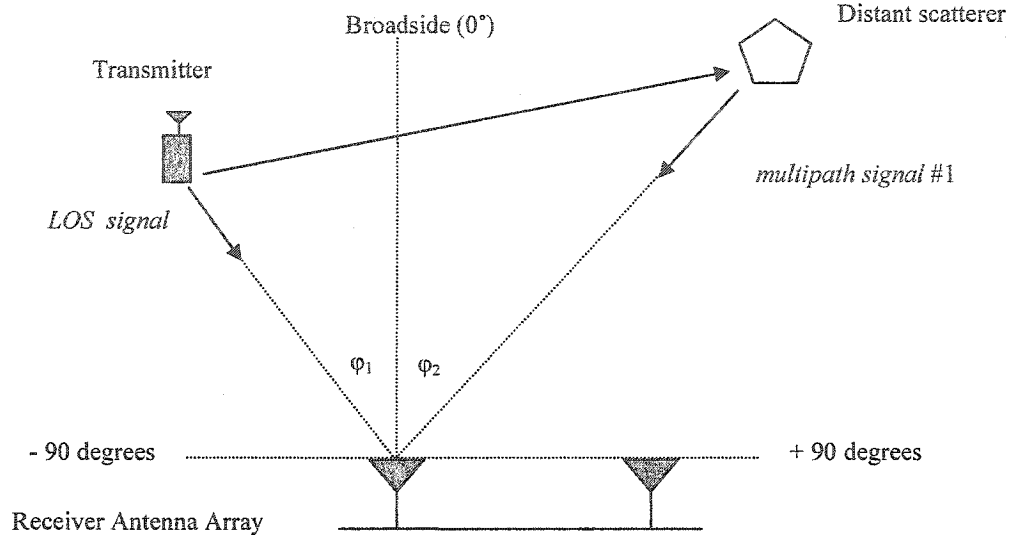


Figure 5-2 Physical Meaning of Two-Impulse AOA Model

5.2.3 Mathematical Analysis

5.2.3.1 Received Signals and their Correlations

The signal received by the j -th transmitter antenna element to the i -th receiver antenna element is defined as by equation (3.12) where the amplitude of the signal is a complex Gaussian distribution with zero mean and unit variance, as described in Section 3.3.1. The AOAs are fixed to a few directions as specified by the chosen impulsive model. The correlation between the signals received by adjacent receive antenna elements is defined as:

$$\rho = \int \exp(j2\pi d \sin \varphi) p(\varphi) d\varphi \quad (5.3)$$

where $p(\varphi)$ is the probability density function of the chosen impulsive model given in (5.1) and (5.2). The correlation functions for N -impulse models for both even and odd values of N are given below. The derivations are given in Appendix A.

$$\rho_{N=odd} = \frac{2}{N} \sum_{n=1}^{(N-1)/2} \cos\left(2\pi d \sin\left(\frac{2n\Delta}{N-1}\right)\right) + \frac{1}{N} \quad (5.4)$$

$$\rho_{N=even} = \frac{2}{N} \sum_{n=1}^{N/2} \cos\left(2\pi d \sin\left(\frac{2n\Delta}{N}\right)\right) \quad (5.5)$$

5.2.3.2 Eigen Decomposition of Correlation Matrix and Degrees of Freedom

Eigen decomposition can be performed on the correlation matrix R to obtain a set of eigenvalues and eigenvectors. Each eigenvector represents the subspace of an incoming signal. The eigenvalue decomposition of the correlation matrix is given as:

$$R = \sum_{i=1}^N \lambda_i \varphi_i \varphi_i^+ = \Phi \Lambda \Phi^+ \quad (5.6)$$

where λ_i is the i -th eigenvalue, φ_i is the i -th eigenvector, $\Lambda = \text{diag}[\lambda_1, \lambda_2 \dots \lambda_N] =$ diagonal matrix of eigenvalues, and $\Phi = [\varphi_1, \varphi_2 \dots \varphi_N] =$ eigenvector matrix. Eigen decomposition provides a mean to analyze impulsive AOA models. The eigenvalues and eigenvectors of the correlation matrix of the impulsive AOA models provide information about the number of parallel sub-channels and their gains.

The number of degrees of freedom is defined as “the number of independent channels available for communication in a particular communication systems” in [36]. According to this definition, as the number independent channels increases, the total channel capacity increases. As a result, the higher the number of degree of freedom, the better is the channel capacity of a given MIMO system.

5.2.4 Simulation Models

The simulation model for channel capacity employed in the studies of impulsive AOA models is the same as the one used in continuous AOA distribution. The details of the

simulation model are described in Section 3.2.3. This channel capacity simulation model takes an array of multipath signals with angle of arrivals and signal magnitudes as input, and returns the channel capacity as function of antenna spacing as output. To investigate the effect of impulsive AOA models on channel capacity, we generate an array of multipath signals according to the impulsive AOA models, and feed the array input the simulation model.

The angles of arrivals of the multipath signals of impulsive AOA models are defined in Section 5.2.1. To generate an array of samples for one-impulse model, the angles of arrivals of all the samples are set to 0° . To generate an array of samples for two-impulse model, the angles of arrivals of half the samples are set to $-\Delta^\circ$, and the angles of arrivals of the other half are set to Δ° . The same methodology applies to models of other number of impulses. Since the purpose of the study is to study the effect of multipath angular distribution, and not the multipath amplitude distribution, the model is designed to have equal number of multipath signals representing each angle in a given impulsive AOA model.

5.3 SIMULATION RESULTS AND OBSERVATIONS

5.3.1 Channel Capacity

Figure 5-3 and Figure 5-4 below show the channel capacities of the MIMO system versus antenna spacing in the receive antenna array for all ten impulsive AOA models. Models with one to five impulses are shown in Figure 5-3, and models with six to ten impulses are shown in Figure 5-4. The following configuration is used: # of Trials = 10000, Mean AOA = 0 degree, # of Transmit Antennas = 10, # of Receive Antennas = 10, Signal-to-noise Ratio = 30 dB, Δ used in PDF of impulsive models = 10° .

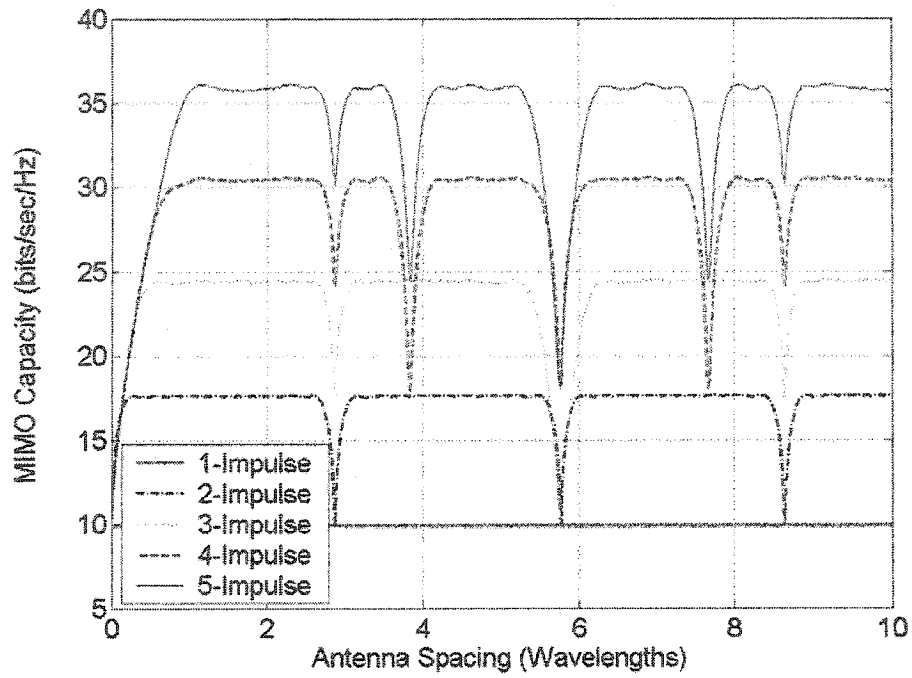


Figure 5-3 MIMO Capacity vs. Antenna Spacing for Impulse Models with 1 to 5 Impulses

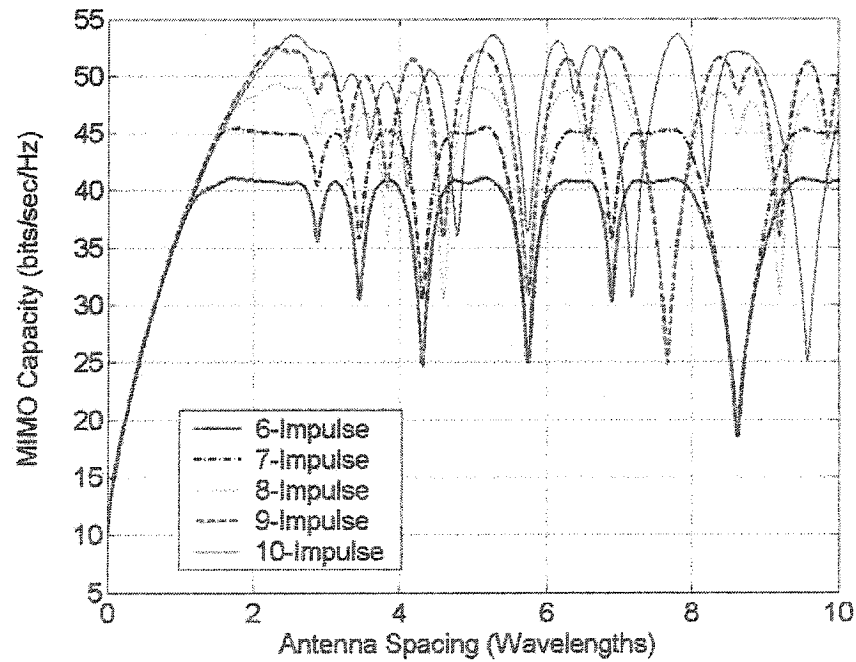


Figure 5-4 MIMO Capacity vs. Antenna Spacing for Impulse Models with 6 to 10 Impulses

It is clear that the MIMO capacity curves obtained using impulsive AOA models are quite different from those obtained using continuous AOA distributions. Using continuous AOA distributions, the MIMO capacity rises from the lowest achievable data rate to the highest achievable data rate within a certain range of antenna spacing which is largely dependent upon the angular spread of the incoming signal distribution. After achieving the highest achievable data rate, the MIMO capacity stays there regardless of any further increase in antenna spacing. Using impulsive AOA models, the MIMO capacity rises from the lowest achievable data rate to the highest achievable data rate within a certain range of antenna spacing. However, the MIMO capacity does not stay at that highest achievable data rate after that data rate is reached. There are periodical drops in capacity as the antenna spacing increases.

As described in the previous chapter, there is an explicit relationship between correlation and capacity in MIMO systems. Capacity is low when correlation is high and capacity is high when correlation is low. For continuous AOA distributions, correlation converges to zero as antenna spacing increases. Therefore, the channel capacity converges to the highest achievable value and stays there. The correlation of impulsive AOA models has been investigated. It is found that the trends of correlation match the trend of channel capacity in any given impulsive model. The results and detailed discussions are presented in Section 5.3.2. Another interesting observation of impulsive AOA models is that the highest achievable capacities are different for different number of impulses. As the number of impulse increases, the maximum achievable capacity increases. Since there are ten transmit and receive antennas, substantial improvement in channel capacity is not expected when the number of impulses increases beyond ten. The following table and Figure 5-5 show the highest achievable MIMO capacity for different values of impulses.

# of impulses	1	2	3	4	5	6	7	8	9	10
Max Capacity	9.94	17.69	24.49	30.63	36.16	41.17	45.54	49.34	52.57	53.7
Increment	-	7.75	6.80	6.14	5.53	5.01	4.37	3.80	3.23	1.13

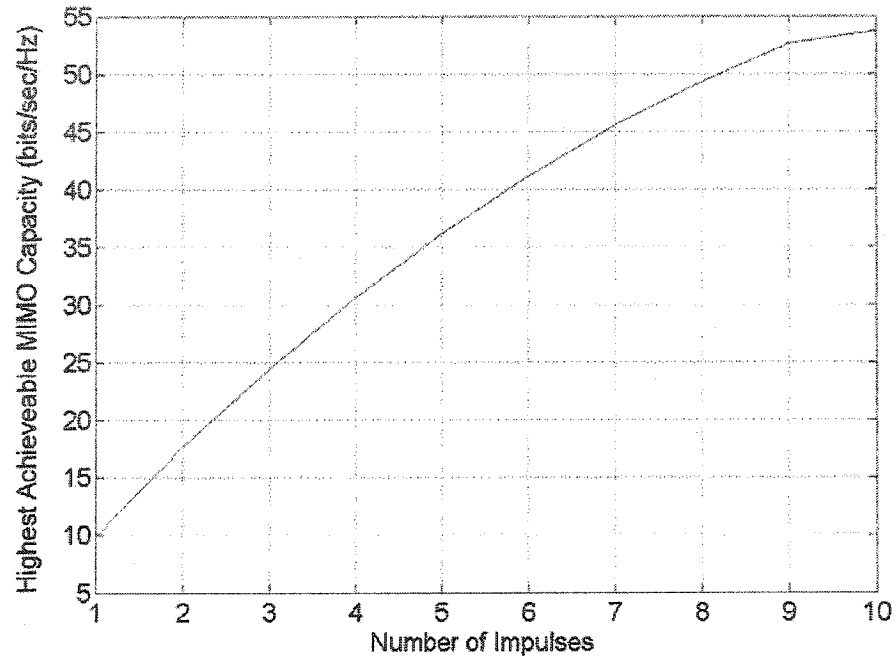


Figure 5-5 Highest Achievable MIMO Capacity in Impulsive AOA Models

MIMO system achieves high data rate because of its ability to exploit the space diversity made available by multipath signals. Without enough multipath signals, the ability of MIMO system to achieve high data rate would be limited. With impulsive AOA models, the number of multipath signals is limited, and this puts an upper bound on to the highest achievable data rate of the MIMO system. As the number of multipath signals increases, the highest achievable data rate increases. This explains why the MIMO capacity is higher as the number of impulses increase. Another interesting observation of the impulsive AOA model is that the lowest value of the drop in capacity of a model with a given number of impulses is always equal to the highest achievable capacity of another model with the smaller number of impulses. As an example, the highest achievable capacity of one-impulse model is 10

bits/sec/Hz. While the highest achievable capacity of two-impulse model is about 17.7 bits/sec/Hz, there are periodic drops in capacity where the lowest value is always 10 bits/sec/Hz – the highest achievable capacity of one-impulse model. Similarly, the three-impulse model has drops in capacity where the lowest values are equal to the highest data rates of two-impulse model and one-impulse model. As a result, at certain values of antenna spacing, the capacity of an impulsive AOA model is the same as that of another impulsive AOA model with fewer impulses.

5.3.2 Correlation

Figure 5-6 shows the correlation versus antenna spacing for impulsive AOA models with two to five impulses. It can be seen that the correlation for all the four impulse models fluctuates between 1 and -1, with peaks and valleys of various scales. As the number of impulse increases, the fluctuations can be seen to be closer to zero. Figure 5-7 shows the same graph for impulsive AOA models with six to nine impulses. Figure 5-8 shows the correlation and MIMO capacity as functions of antenna spacing for the two-impulse model. It is evident that there is a clear relationship between the drops in the MIMO capacity and the correlation. The drops in MIMO capacity coincides with the maxima and minima of the correlation, which are at 1 and -1 respectively. As the correlation moves away from the maxima and minima, the MIMO capacity quickly returns to the highest achievable data rate. Figure 5-9 shows the correlation and MIMO capacity as functions of antenna spacing for the three-impulse model. The correlation oscillates between the maxima of 1 and the minima of -0.3. There are two levels of drops in the MIMO capacity curve. The bigger drop coincides with the maxima of the correlation curve, while the smaller drop coincides with the minima of the correlation curve.

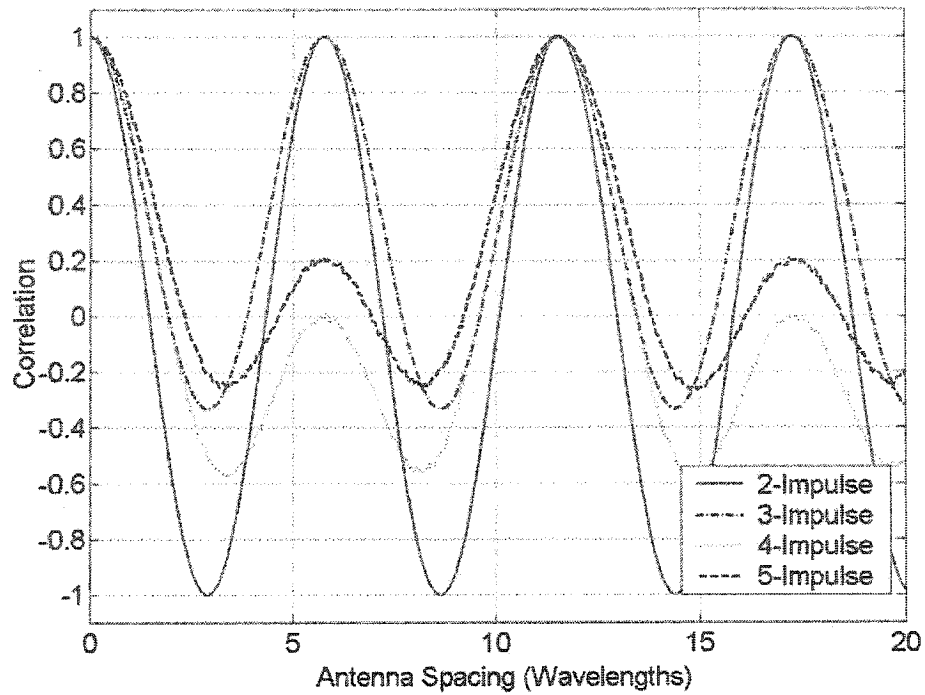


Figure 5-6 Correlation vs. Antenna Spacing for Impulsive Models with 2 to 5 Impulses

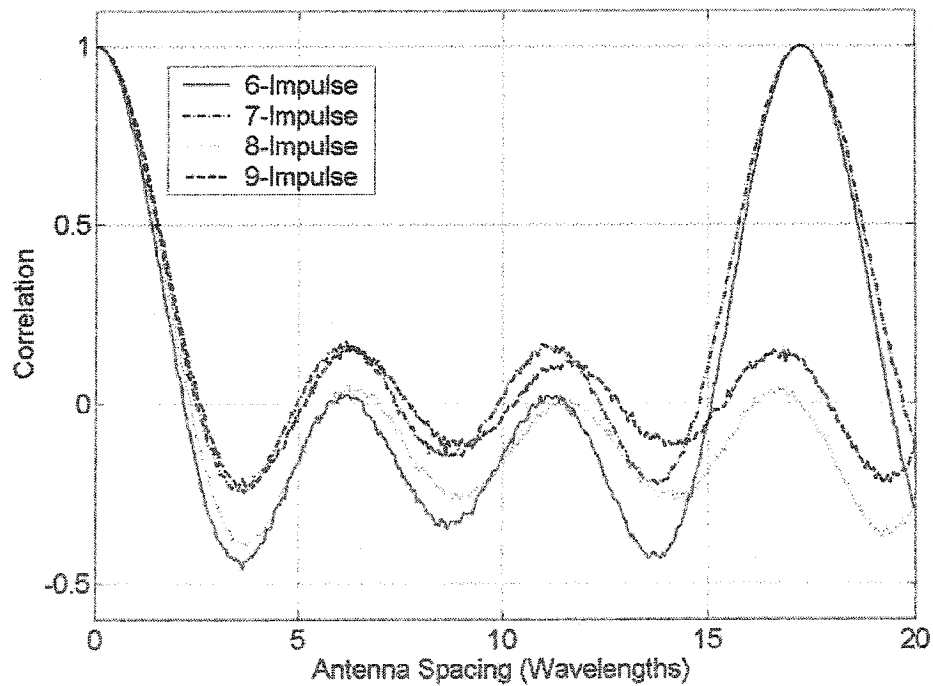


Figure 5-7 Correlation vs. Antenna Spacing for Impulsive Models with 6 to 9 Impulses

Figure 5-10, Figure 5-11 and Figure 5-12 show the correlation and MIMO capacity vs. antenna spacing for four-impulse model, five-impulse model, and six-impulse model respectively. It can be shown that the maxima and minima of the correlation coincide with drops in all the MIMO capacity. However there are some drops that do not have a corresponding maxima or minima in the correlation curve. The reason behind this is that the correlation curve represents the correlation between signals received by adjacent antenna elements. Correlations exist between antenna elements that are two elements apart, three elements apart, or more. Those correlations also affect the MIMO capacity and can result in drops. A full explanation behind all the drops in MIMO capacity is presented in Section 5.3.3, when eigenvalues of the correlation matrix are analyzed.

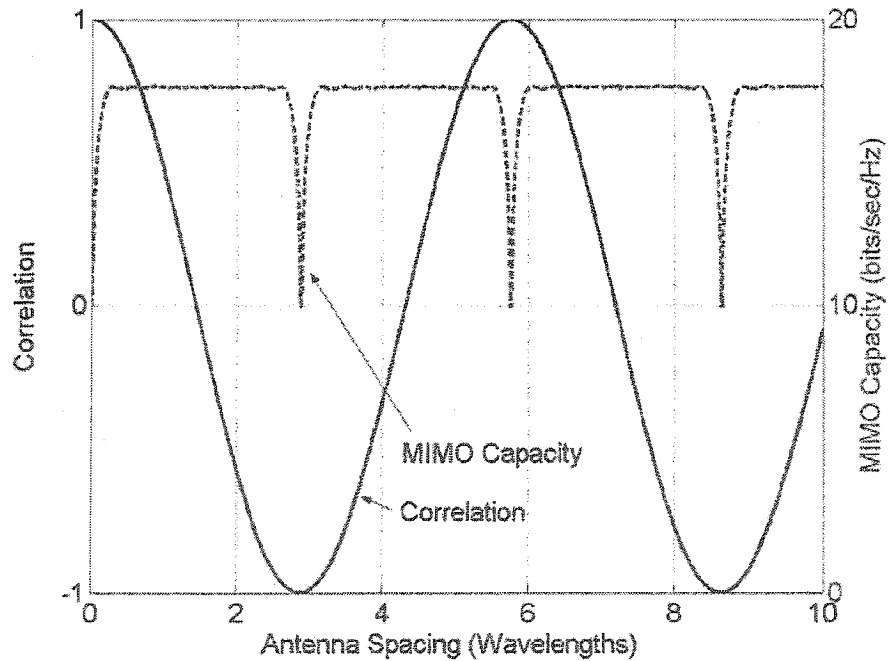


Figure 5-8 Correlation and MIMO Capacity vs Antenna Spacing for 2-Impulse Model

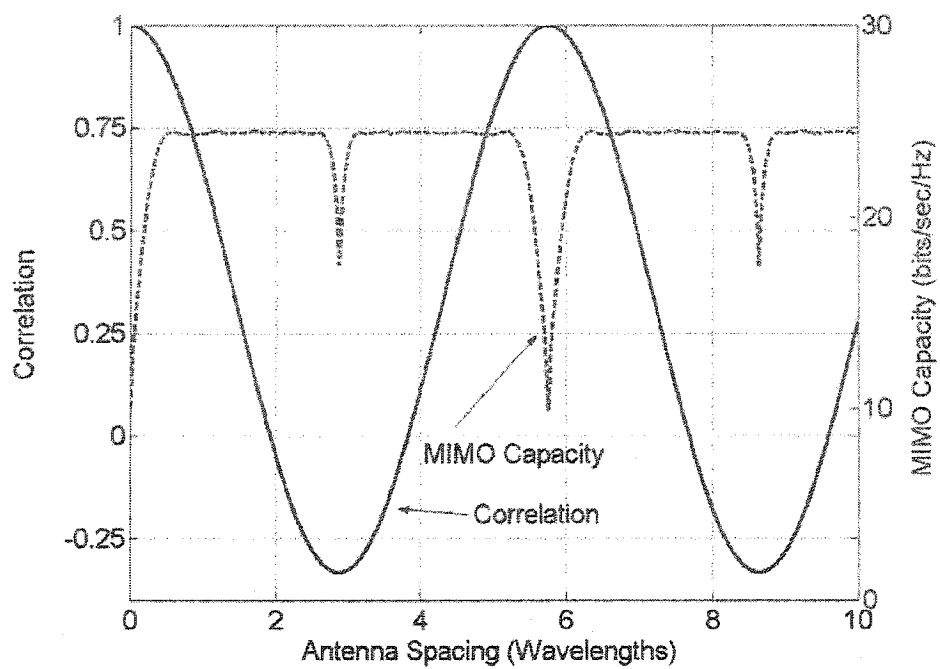


Figure 5-9 Correlation and MIMO Capacity vs Antenna Spacing for 3-Impulse Model

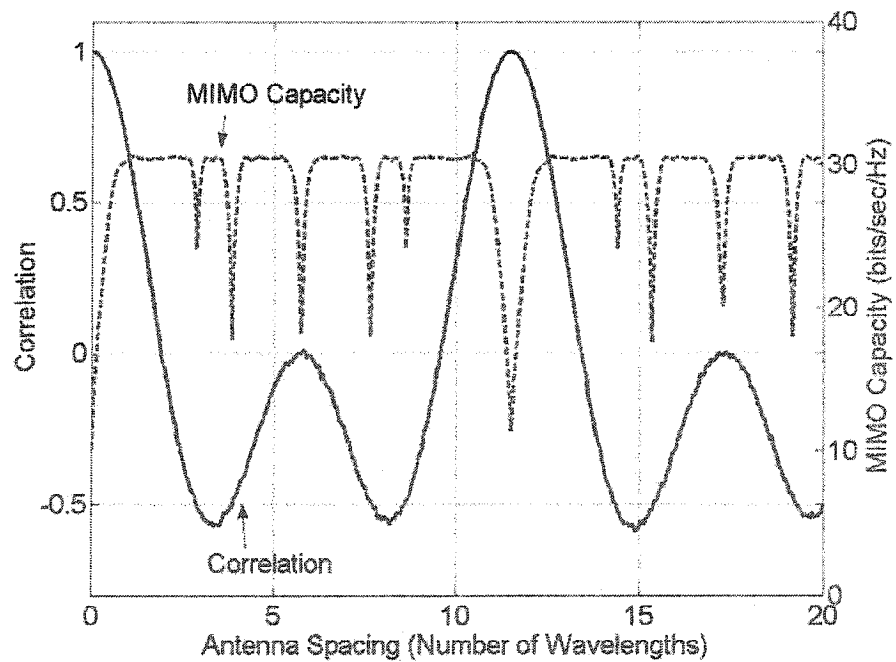


Figure 5-10 Correlation and MIMO Capacity vs Antenna Spacing for 4-Impulse Model

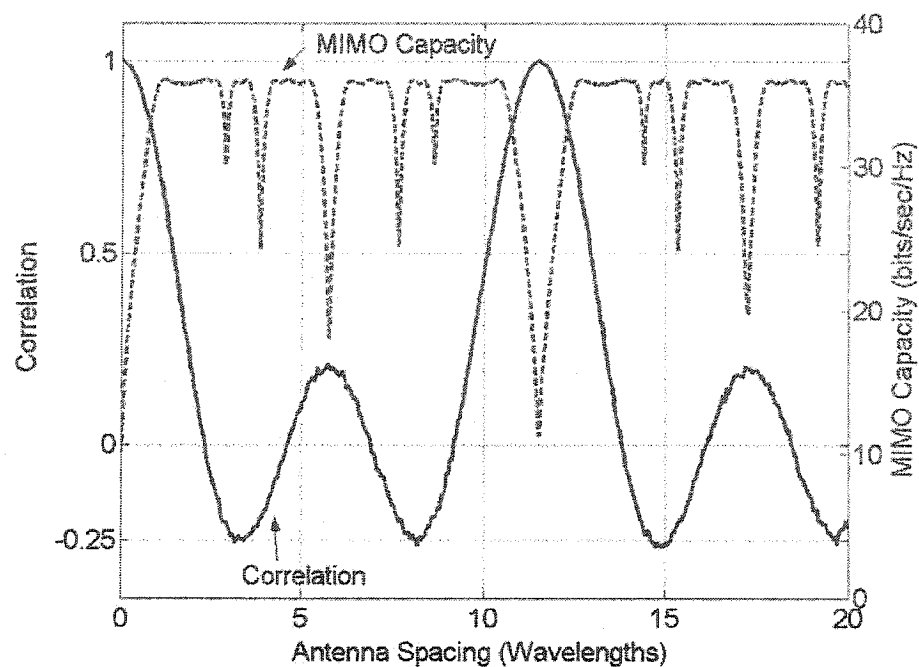


Figure 5-11 Correlation and MIMO Capacity vs. Antenna Spacing for 5-Impulse Model

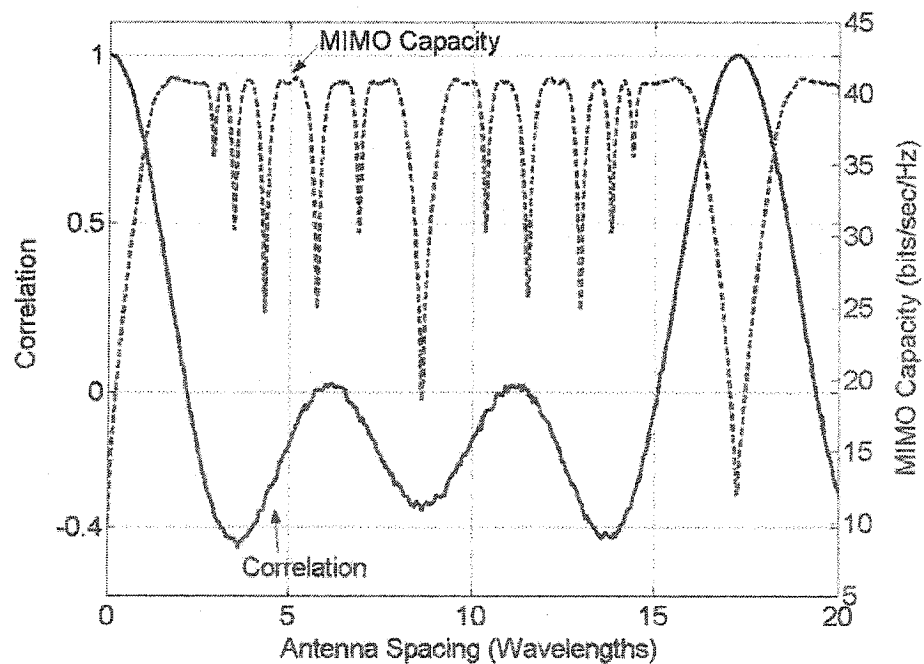


Figure 5-12 Correlation and MIMO Capacity vs. Antenna Spacing for 6-Impulse Model

5.3.3 Eigenvalues of Correlation Matrix

Eigen decompositions are performed on the correlation matrix of the MIMO system with different values of antenna spacing in the receiver antenna array, while all other parameters are fixed. The resulting eigenvalues as functions of antenna spacing reveal important information about the fluctuations of MIMO capacity in impulsive AOA models.

Figure 5-13, Figure 5-14, Figure 5-15, Figure 5-16, Figure 5-17 and Figure 5-18 show the eigenvalues and MIMO capacity as functions of antenna spacing for impulsive AOA models with two, three, four, five, six, and seven impulses respectively. The eigenvalues and the sum of eigenvalues are located at the bottom part of the graphs; while the MIMO capacity is located at the top part of the graphs. An important observation that can be seen from the graphs is that the number of non-zero eigenvalues of a particular impulsive AOA model is the same as the number of impulses of the impulse model. As explained in Section 5.2.3.2, each eigenvector of the correlation matrix represents the subspace of an incoming signal that can be detected by the receiver. This means that the number of non-zero eigenvalues of the correlation matrix is equivalent to the number of independent signals that can be received by the receiver array. And the maximum number of independent signals that can be received is the number of incoming multipath signal, which is represented by the number of impulses in the impulsive AOA system. Even though the maximum number of non-zero eigenvalues is fixed for a given impulse model regardless of changes in antenna spacing, the values of these eigenvalues vary as function of antenna spacing. Since the maximum number of possible orthogonal sub-channels in a MIMO system is bound by the number of antenna elements, the sum of the eigenvalues is always equal to the total number of antenna elements in the receiver antenna array.

It can also be shown that there is a clear relationship between the fluctuations in the MIMO capacity and the fluctuations in the eigenvalues. As antenna spacing varies, the values of eigenvalues for a given impulse model vary as well. At certain antenna spacing, the value of one or more eigenvalues can drop to zero, thus reducing the total number of non-zero eigenvalues. The reduction in the number of non-zero eigenvalues coincides with the drop in MIMO capacity. In the two-impulse model, it is evident that the MIMO capacity stays at the maximum value at values of antenna spacing where there are two non-zero eigenvalues. The MIMO capacity drops to the minimum value when there is only one non-zero eigenvalue. In the three-impulse model, the MIMO capacity stays at the maximum value at values of antenna spacing where there are three non-zero eigenvalues. The MIMO capacity encounters a small drop at values of antenna spacing where there are two non-zero eigenvalues. The MIMO capacity drops to the minimum value at values of antenna spacing where there is only one non-zero eigenvalue. The same phenomenon happens in all other impulsive AOA models, as shown in the graphs below.

Another interesting observation that can be seen from the graphs is the value of MIMO capacity at each drop. In each impulsive AOA model with more than two impulses, there are multiple levels of drops in MIMO capacity, and the value of MIMO capacity of each drop is always equal to the maximum achievable MIMO capacity of another impulsive AOA model with less impulses. As an example, the graph for seven-impulse mode, as shown in Figure 5-18, contains six different levels of drops. These six different levels of drops correspond to the maximum capacity of six-impulse mode, five-impulse model, four-impulse model, three-impulse model, two-impulse model, and one-impulse model respectively. The MIMO capacity drops to the one-impulse model level at the value of antenna spacing where there is only one non-zero eigenvalue. Having one non-zero eigenvalue means that the

receiver antenna array can only correctly detect one incoming signal. Since MIMO systems achieve high data rate by exploiting space diversity made available by multipath signals, the maximum capacity attained it can only detect one signal is the same as the maximum capacity of the one-impulse model, when there is only one incoming signal. The MIMO capacity drops to the two-impulse model level at the value of antenna spacing where there are only two non-zero eigenvalues. The MIMO capacity drops to the three-impulse model level at the value of antenna spacing where there are only three non-zero eigenvalues. The same phenomenon applies to other levels of drops.

The number of non-zero eigenvalues in the correlation matrix of a given impulsive AOA model represents the degrees of freedom, which is “the number of independent channels available for communication in a particular communication systems” as defined in [36]. Even though the receiver antenna array of the MIMO system is capable of achieving a maximum data rate as determined by the number of antenna elements, this maximum data rate cannot be achieved if the number of multipath signals with different angles of arrival is less than the number of antenna elements. In situations where the number of multipath signals with different angles of arrival is limited, increasing the number of antenna elements in the receiver does not result in increase in channel capacity. Also, the maximum capacity does not stay at a saturation level in such situations. There are periodic drops in the capacity as the antenna spacing varies. Thus the decision of the optimal antenna spacing is very important in such situations. A large antenna spacing does not always guarantee maximum capacity when the number of multipath signals is limited.

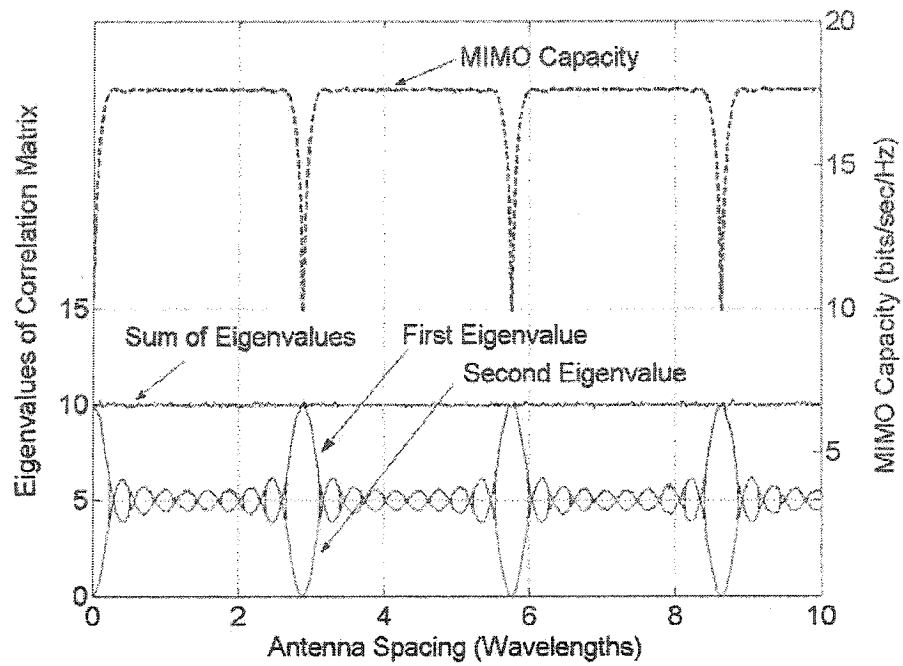


Figure 5-13 Eigenvalues and MIMO Capacity vs. Antenna Spacing for 2-Impulse Model

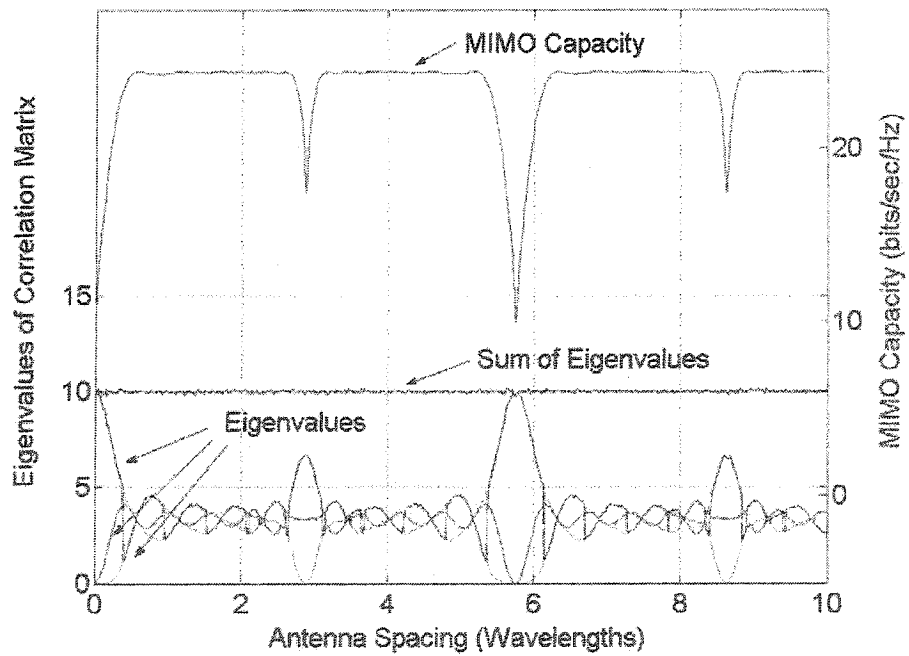


Figure 5-14 Eigenvalues and MIMO Capacity vs. Antenna Spacing for 3-Impulse Model

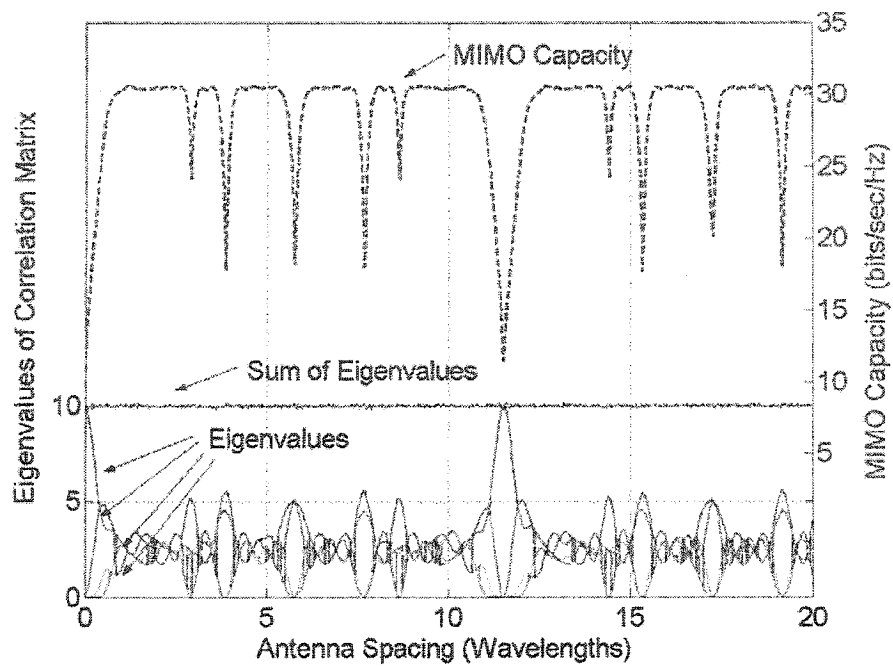


Figure 5-15 Eigenvalues and MIMO Capacity vs. Antenna Spacing for 4-Impulse Model

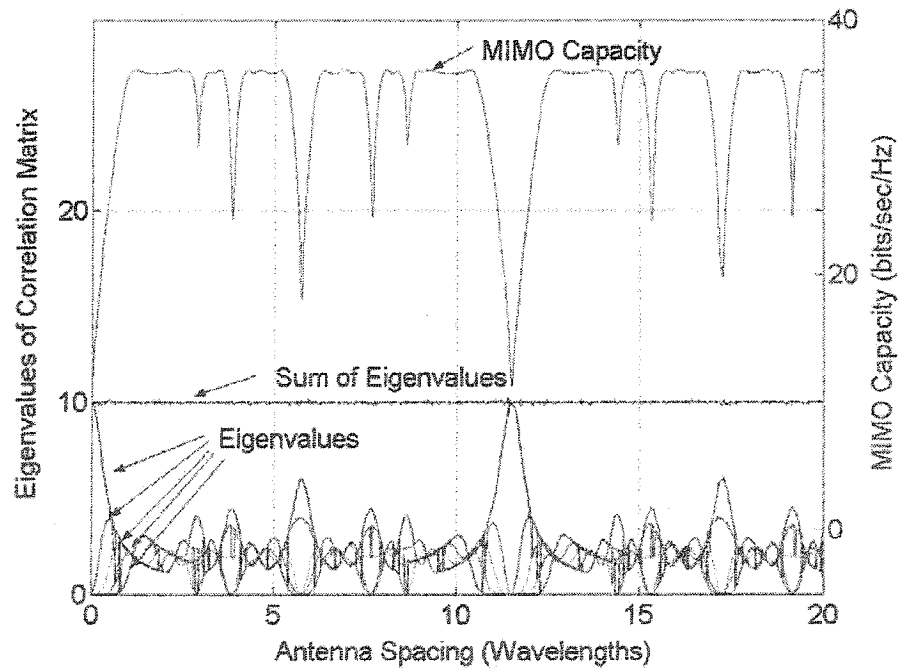


Figure 5-16 Eigenvalues and MIMO Capacity vs. Antenna Spacing for 5-Impulse Model

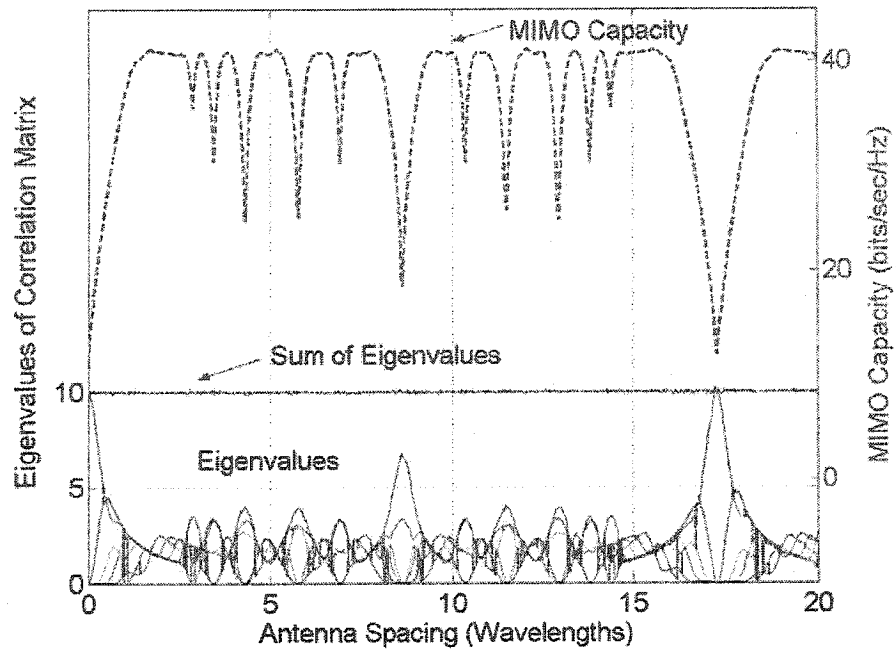


Figure 5-17 Eigenvalues and MIMO Capacity vs. Antenna Spacing for 6-Impulse Model

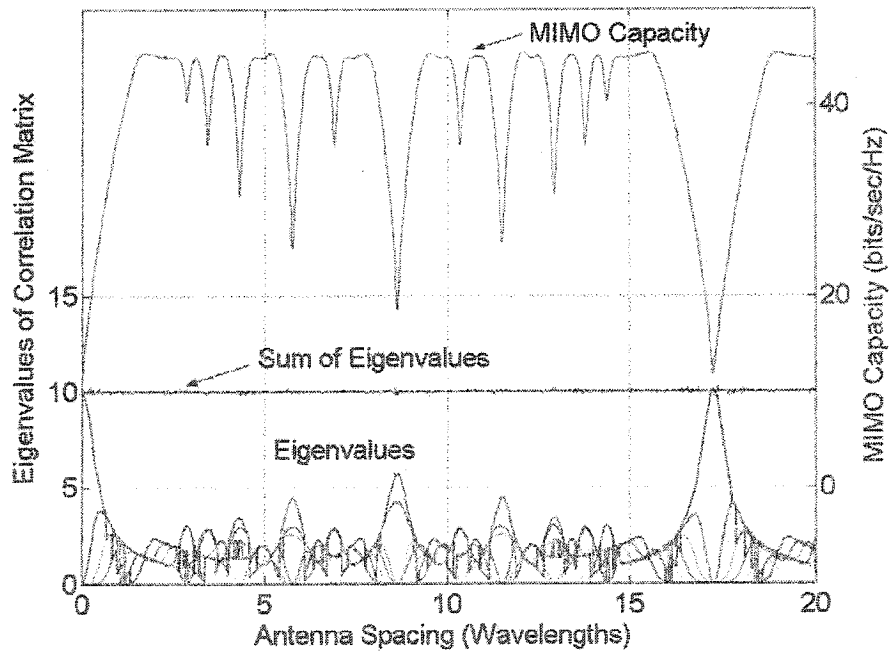


Figure 5-18 Eigenvalues and MIMO Capacity vs. Antenna Spacing for 7-Impulse Model

5.4 CONCLUDING REMARKS

5.4.1 Summary of this Chapter

Buildings and natural landscape provide scatterers that turn one transmitted signal into many multipath signals with different angles of arrival at the receiver end. MIMO systems achieve high data rate by exploiting the space diversity made available by these multipath signals in the wireless environment. Traditional studies of MIMO systems assume large numbers of multipath signals coming from different angles of arrivals at the receiver. This chapter extends these studies by considering the performance and behavior of MIMO systems in situations where the number of multipath signals coming from different angles of arrivals is limited. Such situation is possible in large rural area with open space, or when all the multipath components originate from an area far away from the receiver, and are seen by the receiver as coming from essentially the same angle.

This chapter introduces impulsive AOA models and provides the results of feeding multipath signals generated by these models into the MIMO capacity simulation model. Impulsive AOA models are developed for values of impulses from one to ten, representing one to ten multipath signals. The MIMO capacity simulation model used here is the same as the one used for continuous AOA distributions. The resulting MIMO capacities are different from those resulting from continuous AOA distributions in the following areas:

1. The MIMO capacity does not stay at the maximum value with the increase of antenna spacing. Rather, there are periodic drops in the MIMO capacity.
2. The MIMO capacities at the drops of a given impulsive AOA model correspond to the maximum MIMO capacities of another impulsive AOA model with fewer impulses.

Eigen decompositions are performed on the correlation matrix of the MIMO systems to explain the pattern of drops in MIMO capacities. It is observed that the maximum number of non-zero eigenvalues of a given impulsive AOA model is the same as the number of impulses in that model. The non-zero eigenvalues represent the degrees of freedom, which is defined as “the number of independent channels available for communication in a particular communication systems” [36]. The drops in MIMO capacity occur at values of antenna spacing where the number of non-zero eigenvalues is less than the maximum number. At any given antenna spacing, the MIMO capacity is determined by the non-zero eigenvalues.

It is concluded that even though the receiver antenna array of the MIMO system is capable to achieve a maximum data rate as determined by the number of antenna elements, this maximum data rate cannot be achieved if the number of multipath signals with different angles of arrival is less than the number of antenna elements. Also, large antenna spacing does not guarantee maximum capacities because of the periodic drops in MIMO capacities.

5.4.2 Contributions of this Chapter

The principal contribution of this chapter is in the introduction of impulsive AOA models to simulate a multipath-limited wireless channel and the application of this kind of channel in the calculation of channel capacity and correlation of MIMO systems. In doing so, this chapter identifies the limitation of MIMO systems when the number of multipath signals is limited and points out the instability in maximum achievable capacity as function of antenna spacing. The results in this chapter also confirm the relationship between degrees of freedom and the number of non-zero eigenvalues of the correlation matrix, and illustrate the importance of degree of freedom in determining the capacity of MIMO systems. In addition,

Equations for correlation of impulsive AOA models (equations (5.4) and (5.5)) are new equations developed in this study.

6 EFFECTS OF AOA DISTRIBUTION ON OUTAGE PROBABILITY AND DIVERSITY GAIN

6.1 INTRODUCTION

The study of channel capacity represents one aspect of the study on MIMO systems. Another aspect of the study that would lead to a more comprehensive understanding of MIMO systems is in the area of diversity combining. In wireless communications, diversities of various forms are used to combat fading and co-channel interference and improve the quality and strength of received signals. Diversity combining can be implemented in different domains such as frequency, time or space. The interest of this study is primarily in the use of diversity combining techniques to exploit spatial diversity introduced by the multipath components of the transmitted signals. Many diversity combining techniques are known for many years, but the challenge in recent years is the formulation of models and simulations to apply these diversity combining techniques in the spatial domain.

The work presented in this chapter is an extension of the analysis of effects of angle of arrivals on MIMO systems that is done in previous chapters. It provides a systematic simulation methodology to quantitatively measure the effects of changes in angle of arrivals in incoming signals on MIMO systems in terms of outage probability and diversity gain. The method of diversity combining being used in this study is maximal ratio combining, which is described in Section 3.2.4. The objective is to find out the effects of changes in AOA models and antenna configurations on outage probability and diversity gain. The parameters that are evaluated include AOA distribution, angular spread, mean AOA and antenna spacing of the receive antenna array. The simulation results are presented, observations are made and conclusions are drawn.

6.2 MATHEMATICAL ANALYSIS

6.2.1 Outage Probability

The outage probability is the probability of an instantaneous SNR to be below a prescribed level. At a given average signal-to-noise ratio, the lower the outage probability, the better is the performance of the system. For example, when the outage probability is 10%, it means that 90% of the time, the incoming signal is below the prescribed threshold. The details of outage probability can be found in Section 3.2.5. The following equation is the outage probability formula for maximal ratio combining under Rayleigh fading channel, assuming no correlation between receiver antenna elements.

$$P_{out} = 1 - e^{-\frac{\gamma}{\gamma_0}} \sum_{k=1}^N \frac{(\gamma / \gamma_0)^{k-1}}{(k-1)!} \quad (6.1)$$

where N = number of receive antenna elements. Figure 6-1 shows the outage probability vs. signal-to-noise ratio graph obtained by equation (6.1). Note that the SNR in the x-axis is normalized to the average SNR. For instance, a value of -10 dB refers to 10 dB below average SNR. It is clear that as the number of receive antenna elements increases, the outage probability at a given signal-to-noise ratio decreases. It means that as the number of receive antenna elements increases, the performance of the maximal ratio combining system improves.

6.2.2 Diversity Gain

Diversity gain is the improvement in signal-to-noise ratio of a diversity combining system when the number of receiving antennas is increased from a certain value to a larger value. The details of diversity gain can be found in Section 3.2.6. Figure 6-1 illustrate how to

obtain the diversity gain once the outage probabilities for one- and two-antenna systems are available. A specific value of outage probability is set as the reference, and in this study, the reference is set to be at 1% outage probability. The diversity gain is the differences in the signal-to-noise ratios of the one- and two- antenna systems that correspond to 1% outage probability. It can be shown in Figure 6-1 that at 1% outage probability, the SNR of the two-antenna system is -8 dB, and the SNR of the one-antenna system is -20 dB. Therefore the diversity gain is 12 dB.

6.3 ANGLE OF ARRIVALS MODEL

The angle of arrival models that are used in this study are uniform, Gaussian and Laplacian AOA distributions. These distributions are continuous AOA distributions described in Section 4.2. The focus on this study is not only on the angle of arrival distribution, but also on attributes of the AOA distribution, such as angular spread and mean AOA, and the antenna spacing of the receive antenna array.

6.4 SIMULATION MODEL

The details of the simulation model are described in Section 3.3.2. The MATLAB code for the simulation is listed in Appendix B. The simulation model takes in as input a sample of incoming signals according to the chosen angle of arrival distributions, and returns as output the outage probability and diversity gain. In the process, outage probabilities for different number of receive antenna elements are obtained, and the outage probabilities are used to obtain the diversity gains. For all the simulations in this chapter, the following configuration is used: Number of Trials = 50000, Number of Transmit Antennas = 1,

Number of Receive Antennas = depends on simulation, Number of Multipath Components per Transmitted Signal = 10, Outage probability to calculate diversity gain = 10^{-2} .

6.5 SIMULATION RESULTS AND OBSERVATIONS

6.5.1 Effects of AOA Distribution

Figure 6-2 shows the outage probability vs. antenna spacing graph for different AOA distributions when the SNR is fixed to 10 dB below average SNR. The four distributions result in very similar outage probability curves. The maximum difference in the outage probability between the four distributions at all values of antenna spacing is 0.004, and is considered negligible. Since distribution shape has negligible impact on outage probability, the same conclusion applies to diversity gain.

6.5.2 Effects of Angular Spread

Figure 6-3 shows the effect of angular spread on outage probability for Gaussian distribution. Simulations have been performed on other continuous distributions (such as uniform and Laplacian), and they all exhibit the same behavior. It is shown that at a given SNR, the outage probability for a larger angular spread is lower than that for a smaller angular spread. The differences in outage probabilities for different angular spreads are small when the number of receive antenna element is one. However, as the number of antenna elements increases, the difference is more obvious.

Figure 6-4 shows the diversity gain vs. antenna spacing graph for different values of angular spreads. Note that the unit of diversity gain is dB. For all values of angular spread, diversity gain increases as antenna spacing increases. Also, it is evident that when the diversity gain saturates at a maximum value. When the angular spread is 10 degrees, the

saturation value is reached at relatively small antenna spacing of 1.5λ . When the angular spread is 3 degrees, the saturation value is reached at an antenna spacing of 5λ . Based on this trend, as the angular spread increases, the value of antenna spacing at which the saturation value of diversity gain is reached decreases. It is also shown that there exist a direct relationship between diversity gain and antenna spacing. As antenna spacing increases, diversity gain increases as well.

6.5.3 Effects of Mean AOA

Figure 6-5 show the outage probability versus SNR with varying mean AOA (0° , 60° and 90°) for Gaussian distribution. The same simulation was performed on other continuous distributions (such as uniform and Laplacian), and all the studied distributions exhibit the same pattern of outage probability. At a given SNR, as the mean AOA moves away from the broadside direction, the outage probability curve shifts to the left, meaning higher outage probabilities, and therefore worse performance. When the AOA comes from the broadside direction, the direction of the signal is perpendicular to the linear antenna array (i.e. the line formed by joining all the antenna elements of the antenna array). Since the angles to the right of the broadside are referred to as positive, and the angles to the left are referred to as negative, an increase in the mean AOA means that the mean angle of the spread of signals is shifting to the right. When the mean AOA reaches 90° , the signals are parallel to the linear antenna array. The physical meaning of this result is that at a given signal-to-noise ratio, the outage probability is the lowest when the signals are coming directly into the receiver from the broadside, and that the outage probability is the highest when the signals are coming sideways into the receiver. For optimal performance, the incoming signal and the linear antenna array should be perpendicular to each other.

Figure 6-6 shows the diversity gain vs. mean AOA for uniform distribution with angular spread of 3° and antenna spacing of one wavelength. It can be seen that as mean AOA moves from 0° (broadside) to 50° , diversity gain decreases. The reason behind this is that the diversity gain being calculated here is the difference of signal-to-noise ratios between the one-antenna system and the two-antenna system at an outage probability of 10^{-2} . The outage probability curve of the one-antenna system is fixed regardless of changes in other parameters. The outage probability curve of the two-antenna system is highly sensitive to changes in other parameters. As the mean AOA increases, the outage probability curve of the two-antenna system shifts to the left, resulting in the drop in diversity gain. It is also shown that diversity gain as a function of mean AOA can be approximated by:

$$D = D_0 \cos(\varphi) \quad (6.2)$$

where D = diversity gain, D_0 = diversity gain at broadside, and φ = mean AOA.

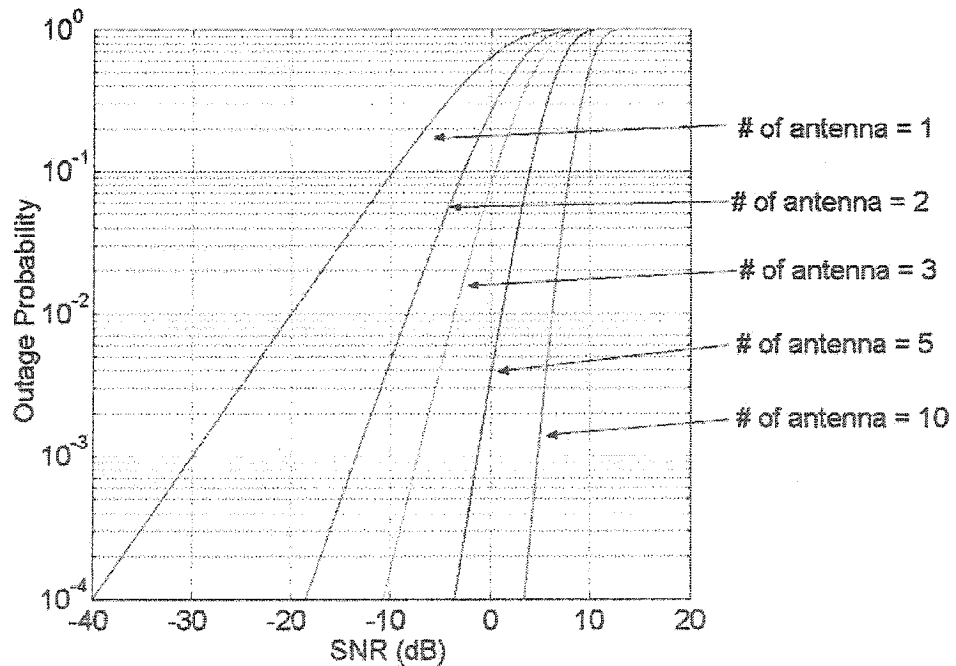


Figure 6-1 Outage Probability vs. SNR obtained by analytical equation

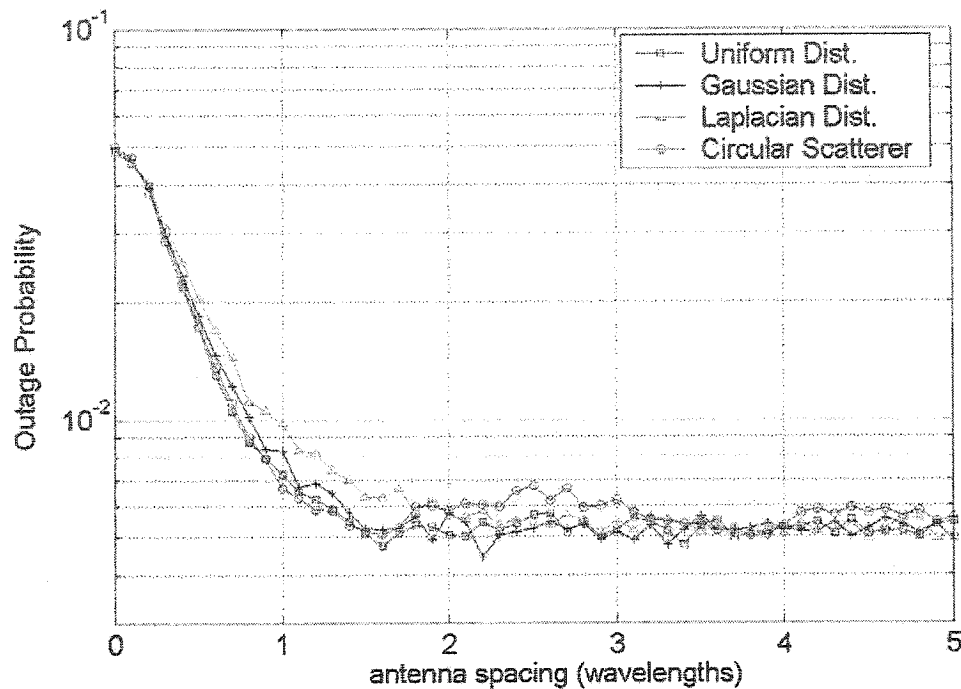


Figure 6-2 Outage Probability vs. Antenna Spacing for different AOA distributions

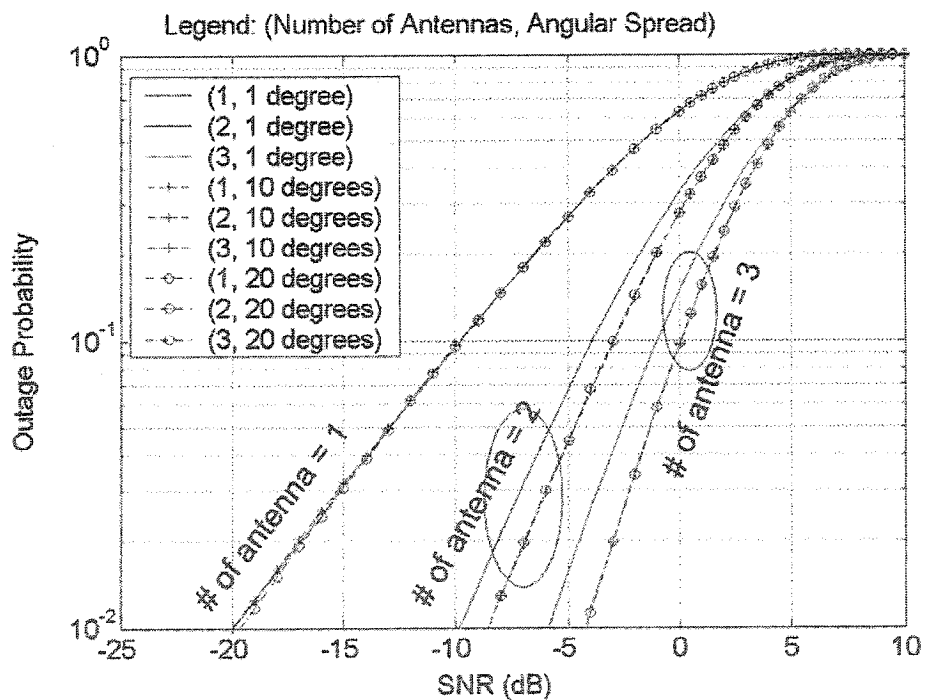


Figure 6-3 Outage Probability vs. SNR for different Angular Spread for Gaussian distribution

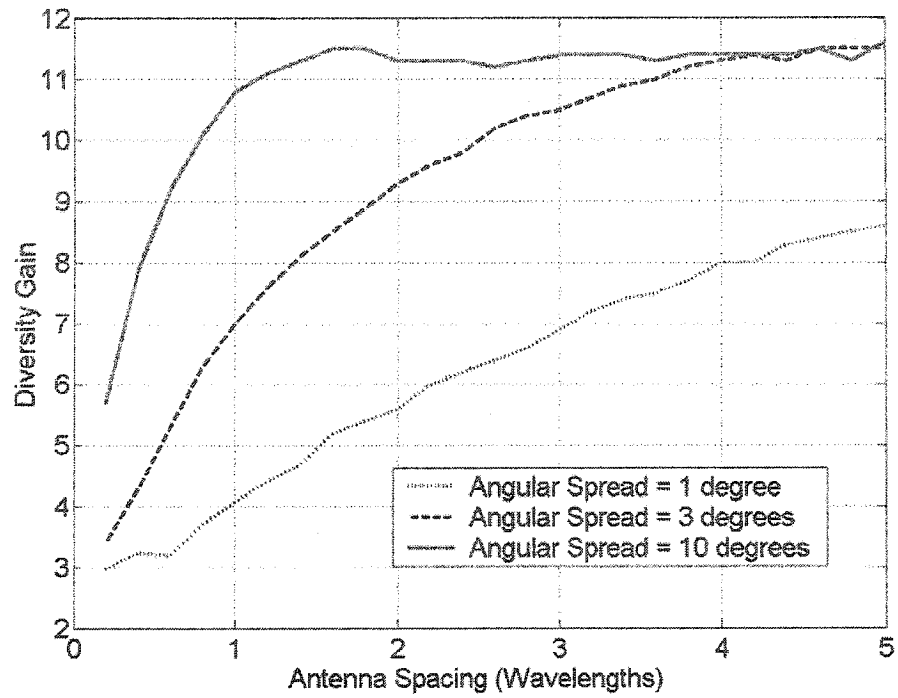


Figure 6-4 Diversity Gain vs. Antenna Spacing for different Angular Spreads

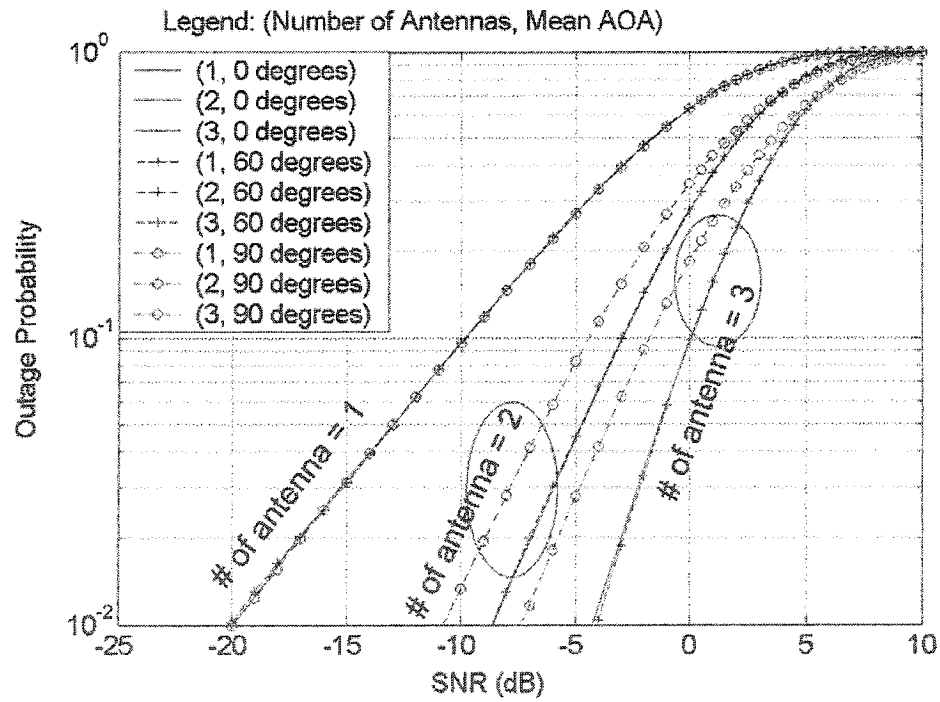


Figure 6-5 Outage Probability vs. SNR for Gaussian AOA Distribution with different Mean AOA

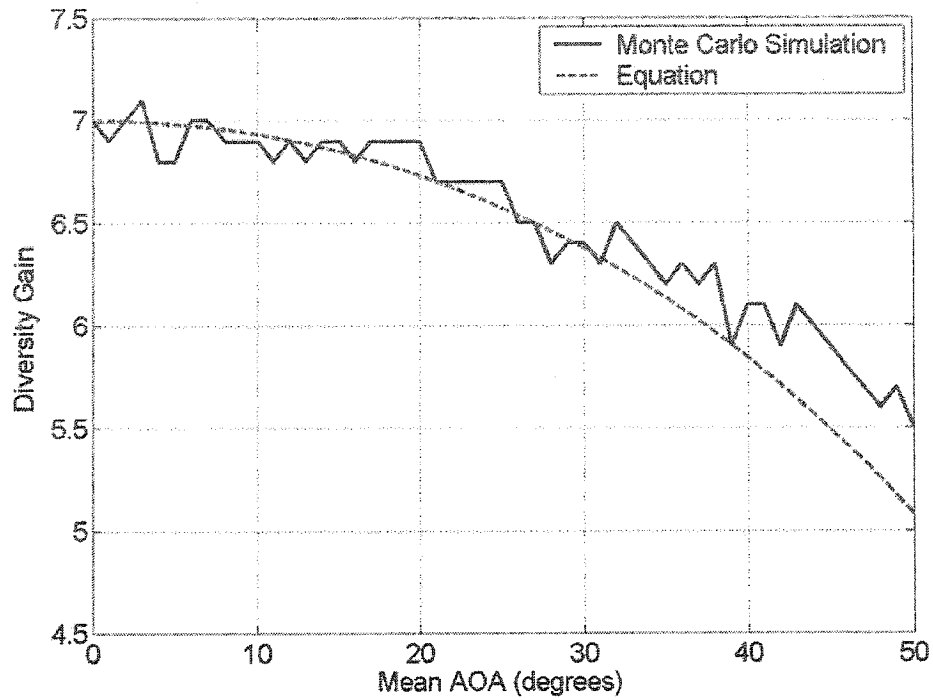


Figure 6-6 Diversity Gain vs. Mean AOA for Uniform Distribution – Comparison of simulation results and analytical equation

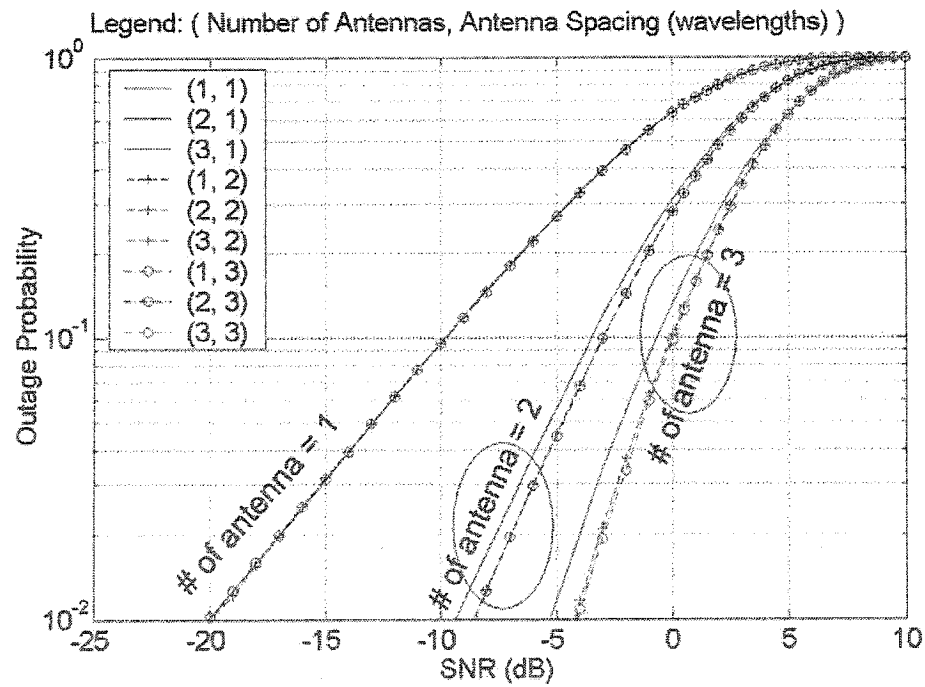


Figure 6-7 Outage Probability vs. SNR for different Antenna Spacing for Uniform Distribution

6.5.4 Effects of Antenna Spacing

Figure 6-7 shows the outage probability vs. SNR for different values of antenna spacing and different number of receive antenna elements. For systems with two or more antenna elements, it is clear that at a given SNR, the outage probability for bigger antenna spacing is lower than that for smaller antenna spacing. Therefore bigger antenna spacing results in better performance and lower outage probability.

6.5.5 Envelope Correlation

All the AOA distribution parameters and antenna configuration parameters discussed in this chapter have effects on the envelope correlation. These parameters include angular spread, mean AOA and antenna spacing. It would be of interest to find out that relationship between diversity gain and envelope correlation. This explains the relationships between all the discussed parameters and diversity gain. The envelope correlation coefficient is defined as:

$$\rho_{env} = \frac{E[x_1 x_2]^2 + E[x_1 y_2]^2}{E[x_1^2]^2} = \rho_{real}^2 + \rho_{imag}^2 \quad (6.3)$$

where x_i is the real value of the complex-valued signal at antenna i , and y_i is the imaginary part of the complex-valued signal at antenna i . Since it is not possible to directly specify a value of envelope correlation to a system, a new simulation methodology is introduced. The new simulation calculates both of envelope correlation and the diversity gain of the MIMO system. To obtain different values of envelope correlation, many trials are run as different combinations of values of angular spread, mean AOA and antenna spacing are used. Figure 6-8 is the result of the simulation and illustrates a clear relationship between diversity gain and envelope correlation.

It is clear that as envelope correlation increases, the diversity gain decreases. When there is no correlation between signals received by adjacent antenna elements, the diversity gain is at the maximum value, as determined by (6.1). It is 11.5 dB. This is the maximum diversity gain that can be obtained when increasing the number of antenna elements from one to two. When the envelope correlation is at the maximum value, the diversity gain is at 3 dB. This is the minimum diversity gain that can be obtained when increasing the number of antenna elements from one to two. The minimum gain is gain associated with the addition of antenna element without taking into account the full benefit of diversity combining. Figure 6-8 also shows the analytical results given by $D = D_0 \cos(\phi)$ where D is the diversity gain, D_0 is the diversity gain at zero correlation and ϕ is the envelope correlation. Although the simulation and analytical results are not exactly the same, they illustrate the same trend.

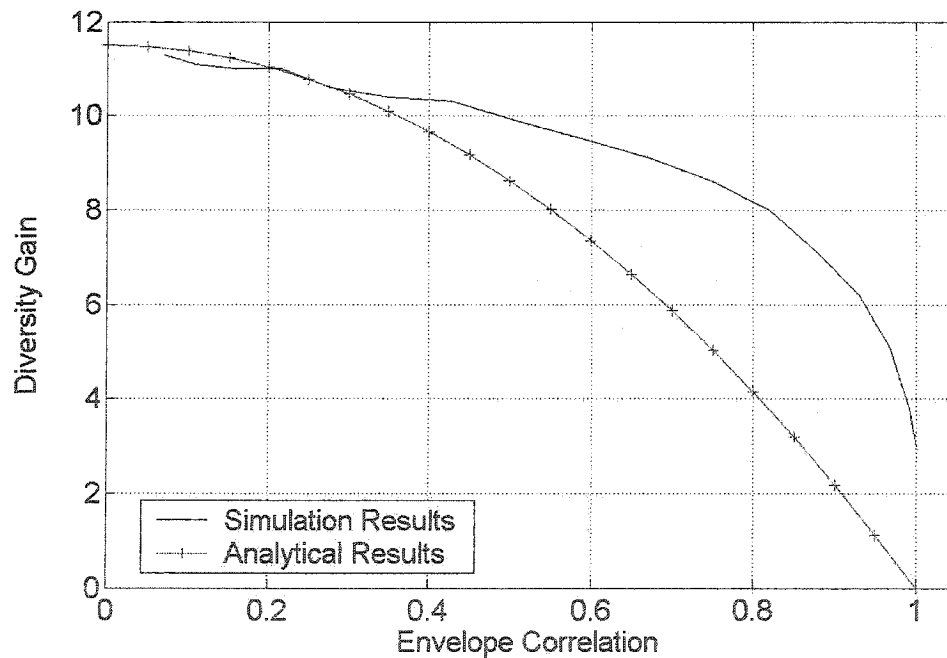


Figure 6-8 Diversity Gain vs. Envelope Correlation (Outage Probability Threshold = 10^{-2})

6.5.6 Summary of Observations

The results in the previous section can be summarized into following table.

Variables	Outage	Diversity Gain
Angle of Arrival Distribution	Negligible Effect	Negligible Effect
Angular Spread $\uparrow$	$\downarrow$	$\uparrow$
Mean AOA $\uparrow$	$\uparrow$	$\downarrow$
Antenna Spacing $\uparrow$	$\downarrow$	$\uparrow$

As a summary, angle of arrival distribution has negligible effects on the performance of the maximal ratio combiner. However, angular spread, mean AOA and antenna spacing affect the performance of the maximal ratio combiner. As angular spread or antenna spacing increases, the performance of the maximal ratio combiner improves. As the mean AOA moves away from the broadside direction, the performance of the maximal ratio combiner deteriorates. The relationship between envelope correlation and other parameters can be summarized into the following table.

Variables	Envelope Correlation
Angular Spread $\uparrow$	$\downarrow$
Mean AOA $\uparrow$	$\uparrow$
Antenna Spacing $\uparrow$	$\downarrow$

The following conclusion can be made regarding the relationship of envelope correlation to performance of maximal ratio combining.

Variables	Outage	Diversity Gain
Envelope Correlation $\uparrow$	$\uparrow$	$\downarrow$

6.6 CONCLUDING REMARKS

6.6.1 Summary of this Chapter

An important area of research in the recent development of wireless communication is in the area of diversity combining. Diversity combining can be implemented in different domains such as frequency, time or space. The study in this thesis is focused on the formation of models and simulations to understand the role of diversity combining in exploiting spatial diversity in SIMO (Single-Input-Multiple-Output) system. The analysis of diversity combining in SIMO system is an extension to previous chapters, which concentrate on channel capacity. The objective of this chapter is to evaluate the effects of AOA distribution parameters and antenna configuration parameters on maximal ratio combining. The parameters considered include AOA distributions, angular spreads, mean AOA and antenna spacing. The means to analyze maximal ratio combining include outage probability and diversity gain. The AOA models established in earlier chapters are reused and the simulation model is modified to implement functional blocks for maximal ratio combining. It is concluded that there exist relationships between envelope correlation and diversity gain, and between envelope correlation and outage probability. As envelope correlation increases, outage probability increases and diversity gain decreases. The effects of angular spread, mean AOA, and antenna spacing on outage probability and diversity gain are all consequences of the effect of envelope correlation.

6.6.2 Contribution of this Chapter

The principal contribution of this chapter is the extension of study of AOA distributions into the area of diversity combining, and to evaluate the impact on outage

probability and diversity gain as the AOA distribution changes, and also as configuration parameters change.

7 CONCLUSION

The ability of MIMO systems to turn multipath signals into parallel channels for increased capacity has fueled many new researches in this area. The work of this thesis provides a basis for further work in MIMO systems by presenting an analysis on the impact of multipath signal distribution and antenna configuration on the performance of MIMO systems. The understanding of the relationship between multipath signal distribution and performance of MIMO systems is a important step in the complete understanding of the MIMO channel. This study also yields important information about the behavior of MIMO systems when a specific angle of arrival distribution is anticipated, or when a specific antenna configuration is used. This knowledge is critical for design of MIMO system that can adapt to different distributions of multipath signals.

The thesis introduces a flexible simulation model that can take in any kind of angle of arrival samples, and produces the required results, such as channel capacity or diversity gain. The AOA distributions used in this study include modifications of well-known continuous distribution functions, distributions based on geometrical model, and impulsive models. Distributions with samples concentrated at the two edges, at the center, and evenly distributed across the range are considered. These distributions result in different correlation functions, and slightly different capacity curves. Some distributions might result in oscillation of capacity about the maximum value, and all the distributions achieve maximum capacity at values of antenna spacing that are fairly close. Impulsive models have larger effects on correlation and capacity. These models result in periodic variations of correlation function and capacities, which are much less than the maximum values. The main reason is that impulsive models limit the angle of arrival to a few angles, thus reducing the degree of

freedom of the channel. This places a limit on the capacity that can be achieved regardless of the number of antennas in the MIMO system.

Other than the choice of AOA distributions, impact of parameters such as angular spread, mean AOA and antenna spacing are observed. Variations of these parameters change the correlation curves, which ultimately affect the achievable channel capacity. As a rule of thumb, MIMO system performs the best when the mean AOA is from the broadside direction, with large angular spread and larger antenna spacing.

The impacts on outage probability and diversity gain have also been evaluated. It is observed that angle of arrival distributions do not have a significant effect on outages and diversity gains. The outage is the lowest, and the diversity gain is at its high when the angular spread is large, antenna spacing is large, and when the mean AOA is from the broadside direction.

The analysis carried out in this thesis focuses on linear antenna array with equal spacing between all adjacent antenna elements. One possible extension of this research is to explore the impact of multipath angular distribution on other antenna array configurations, such as circular or rectangular antenna arrays. The research in this thesis is only a beginning step towards the full understanding of MIMO channel and MIMO systems. The continuous works towards full understanding of MIMO systems and development of transmission algorithms to fulfill the MIMO data rates will certainly yield fruitful results, and revolutionize the wireless communications that we know today.

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Appendix A – Mathematical Derivations

A.1 Derivation of Angular Spread for Uniform Distribution

Angular spread of the Uniform distribution is the same as the standard deviation of the distribution. It is defined as follow:

$$s_\varphi = \left[\int_{-\Delta}^{\Delta} \varphi^2 f_\varphi(\varphi) d\varphi \right]^{\frac{1}{2}} = \left[\int_{-\Delta}^{\Delta} \frac{\varphi^2}{2\Delta} d\varphi \right]^{\frac{1}{2}} = \left(\frac{1}{2\Delta} \left[\frac{\varphi^3}{3} \right]_{-\Delta}^{\Delta} \right)^{\frac{1}{2}} \quad (\text{A.1})$$

$$s_\varphi = \left(\frac{1}{2\Delta} \cdot \frac{2\Delta^3}{3} \right)^{\frac{1}{2}} = \frac{\Delta}{\sqrt{3}} \quad (\text{A.2})$$

A.2 Derivation of Normalization Constant for Truncated Gaussian Distribution

The probability distribution function of the Gaussian AOA Distribution is defined as:

$$f_\varphi(\varphi) = \frac{Q}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\varphi_l - \varphi_0)^2}{2\sigma^2}}, \quad -\frac{\pi}{2} + \varphi_0 \leq \varphi_l \leq \frac{\pi}{2} + \varphi_0 \quad (\text{A.3})$$

Without loss of generality, we set the center angle φ_0 to 0. To obtain the normalization constant of the Gaussian AOA Distribution, the integration of the probability distribution function from $-\pi/2$ to $\pi/2$ is set to unity.

$$\int_{-\pi/2}^{\pi/2} \frac{Q}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\varphi_l)^2}{2\sigma^2}} d\varphi_l = 1 \quad (\text{A.4})$$

$$Q = \sqrt{2\pi\sigma^2} \left[\int_{-\pi/2}^{\pi/2} e^{-\frac{(\varphi_l)^2}{2\sigma^2}} d\varphi_l \right]^{-1} \quad (\text{A.5})$$

Setting $x = \varphi_l / \sqrt{2}\sigma$, then $\varphi_l = \sqrt{2}\sigma x$, $d\varphi_l = \sqrt{2}\sigma dx$

$$Q = \frac{\sqrt{2\pi\sigma^2}}{2\sqrt{2}\sigma \int_0^{\frac{\pi}{2\sqrt{2}\sigma}} e^{-x^2} dx} = \frac{\sqrt{2\pi\sigma^2}}{\sqrt{2\pi}\sigma \operatorname{erf}\left(\frac{\pi}{\sqrt{8}\sigma}\right)} = \frac{1}{\operatorname{erf}\left(\frac{\pi}{\sqrt{8}\sigma}\right)} \quad (\text{A.6})$$

where $\operatorname{erf}(\cdot)$ is the error function and is defined as: $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

A.3 Derivation of Angular Spread for Truncated Gaussian

Distribution

Angular spread of the Gaussian AOA Distribution is defined as the standard deviation of the truncated Gaussian distribution within the range of $[-\pi/2, \pi/2]$, and is defined as follow:

$$s_\varphi = \left[\int_{-\pi/2}^{\pi/2} \varphi^2 f_\varphi(\varphi) d\varphi \right]^{\frac{1}{2}} = \left[\int_{-\pi/2}^{\pi/2} \varphi^2 \frac{Q}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\varphi-\varphi_0)^2}{2\sigma^2}} d\varphi \right]^{\frac{1}{2}} \quad (\text{A.7})$$

where Q is the normalization constant defined in the previous section and φ_0 is the center angle. Without loss of generality, we set the center angle φ_0 to 0. Then the angular spread is derived as follow:

$$s_\varphi = \left[\int_{-\pi/2}^{\pi/2} \varphi^2 \frac{Q}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\varphi)^2}{2\sigma^2}} d\varphi \right]^{\frac{1}{2}} \quad (\text{A.8})$$

$$s_\varphi = \left[\frac{Q}{\sqrt{2\pi\sigma^2}} \int_{-\pi/2}^{\pi/2} \varphi^2 e^{-\frac{(\varphi)^2}{2\sigma^2}} d\varphi \right]^{\frac{1}{2}} \quad (\text{A.9})$$

Setting $x = \varphi / \sqrt{2\sigma}$, then $\varphi = \sqrt{2\sigma}x$, $d\varphi = \sqrt{2\sigma}dx$

$$s_\varphi = \left[\frac{Q}{\sqrt{2\pi\sigma^2}} \int_{-\frac{\pi}{2\sqrt{2\sigma}}}^{\frac{\pi}{2\sqrt{2\sigma}}} 2\sqrt{2\sigma} x^2 e^{-x^2} dx \right]^{\frac{1}{2}} \quad (\text{A.10})$$

Since the probability density function is an even function,

$$s_\varphi = \left[\frac{Q4\sigma^2}{\sqrt{\pi}} \int_0^{\frac{\pi}{2\sqrt{2\sigma}}} x^2 e^{-x^2} dx \right]^{\frac{1}{2}} \quad (\text{A.11})$$

Using the formula $\int x^2 e^{-x^2} dx = -\frac{1}{2}xe^{-x^2} + \frac{1}{2} \int e^{-x^2} dx$, we get the following:

$$s_\varphi = \left[\frac{Q4\sigma^2}{\sqrt{\pi}} \left\{ -\frac{\pi}{2\sqrt{8\sigma}} e^{-\frac{\pi^2}{8\sigma^2}} + \frac{\sqrt{\pi}}{4} \operatorname{erf}\left(\frac{\pi}{\sqrt{8\sigma}}\right) \right\} \right]^{\frac{1}{2}} \quad (\text{A.12})$$

$$s_\varphi = \left[Q\sigma^2 \left\{ -\frac{\sqrt{\pi}}{\sqrt{2\sigma}} e^{-\frac{\pi^2}{8\sigma^2}} + \operatorname{erf}\left(\frac{\pi}{\sqrt{8\sigma}}\right) \right\} \right]^{\frac{1}{2}} \quad (\text{A.13})$$

A.4 Derivation of Normalization Constant for Truncated Laplacian Distribution

The probability distribution function of the Laplacian AOA Distribution is defined as:

$$p(\varphi) = \frac{Q}{\sqrt{2\sigma^2}} e^{-\frac{\sqrt{2}|\varphi-\varphi_0|}{\sigma}}, \quad -\frac{\pi}{2} + \varphi_0 \leq \varphi \leq \frac{\pi}{2} + \varphi_0 \quad (\text{A.14})$$

Without loss of generality, we set the center angle φ_0 to 0. To obtain the normalization constant of the Laplacian AOA Distribution, the integration of the probability distribution function from $-\pi/2$ to $\pi/2$ is set to unity.

$$\int_{-\pi/2}^{\pi/2} \frac{Q}{\sqrt{2\sigma^2}} e^{-\frac{\sqrt{2}|\varphi|}{\sigma}} d\varphi = 1 \quad (\text{A.15})$$

Setting $x = \frac{\sqrt{2}\varphi}{\sigma}$, therefore $\varphi = \frac{x\sigma}{\sqrt{2}}$ and $d\varphi = \frac{\sigma}{\sqrt{2}} dx$. Since the probability density function

is an even function, we get the following:

$$Q = \frac{\sqrt{2\sigma^2}}{2 \int_0^{\frac{\pi}{\sqrt{2}\sigma}} \frac{\sigma}{\sqrt{2}} e^{-x} dx} = \frac{1}{\int_0^{\frac{\pi}{\sqrt{2}\sigma}} e^{-x} dx} \quad (\text{A.16})$$

$$Q = \frac{1}{\left[-e^{-x}\right]_0^{\frac{\pi}{\sqrt{2}\sigma}}} = \frac{1}{1 - e^{-\frac{\pi}{\sqrt{2}\sigma}}} \quad (\text{A.17})$$

A.5 Derivation of Angular Spread for Truncated Laplacian Distribution

Angular spread of the Laplacian AOA Distribution is defined as the standard deviation of the truncated Laplacian distribution within the range of $[-\pi/2, \pi/2]$, and is defined as follow:

$$s_\varphi = \left[\int_{-\pi/2}^{\pi/2} \varphi^2 f_\varphi(\varphi) d\varphi \right]^{\frac{1}{2}} = \left[\int_{-\pi/2}^{\pi/2} \varphi^2 \frac{Q}{\sqrt{2\sigma^2}} e^{-\frac{\sqrt{2}|\varphi-\varphi_0|}{\sigma}} d\varphi \right]^{\frac{1}{2}} \quad (\text{A.18})$$

where Q is the normalization constant defined in the previous section and φ_0 is the center angle. Without loss of generality, we set the center angle φ_0 to 0. Then the angular spread is derived as follow:

$$s_{\varphi} = \left[\int_{-\pi/2}^{\pi/2} \varphi^2 \frac{Q}{\sqrt{2\sigma^2}} e^{-\frac{\sqrt{2}|\varphi|}{\sigma}} d\varphi \right]^{\frac{1}{2}} \quad (\text{A.19})$$

Since the probability density function is an even function, and the value of φ above zero is always positive, we get the following:

$$s_{\varphi} = \left[2 \int_0^{\pi/2} \varphi^2 \frac{Q}{\sqrt{2\sigma^2}} e^{-\frac{\sqrt{2}\varphi}{\sigma}} d\varphi \right]^{\frac{1}{2}} \quad (\text{A.20})$$

Setting $x = \frac{\sqrt{2}\varphi}{\sigma}$, therefore $\varphi = \frac{x\sigma}{\sqrt{2}}$ and $d\varphi = \frac{\sigma}{\sqrt{2}} dx$,

$$s_{\varphi} = \left[\frac{Q\sigma^2}{2} \int_0^{\pi/2} x^2 e^{-x} dx \right]^{\frac{1}{2}} \quad (\text{A.21})$$

Using the equation $\int x^2 e^{-x} dx = -e^{-x}(x^2 + 2x + 2)$

$$s_{\varphi} = \left[\frac{Q\sigma^2}{2} \left[-e^{-x}(x^2 + 2x + 2) \right]_0^{\frac{\pi}{\sqrt{2}\sigma}} \right]^{\frac{1}{2}} \quad (\text{A.22})$$

$$s_{\varphi} = \left[\frac{Q\sigma^2}{2} \left[-e^{-\frac{\pi}{\sqrt{2}\sigma}} \left(\frac{\pi^2}{2\sigma^2} + \frac{\sqrt{2}\pi}{\sigma} + 2 \right) + 2 \right] \right]^{\frac{1}{2}} \quad (\text{A.23})$$

$$s_{\varphi} = \left\{ Q \left[-e^{-\frac{\pi}{\sqrt{2}\sigma}} \left(\frac{\pi^2}{4} + \frac{\pi\sigma}{\sqrt{2}} + \sigma^2 \right) + \sigma^2 \right] \right\}^{\frac{1}{2}} \quad (\text{A.24})$$

A.6 Derivation of Normalization Constant for Truncated Cosine

Distribution

The normalization constant is defined as $Q = 1 / \int_{\frac{\pi}{2} + \varphi}^{\pi + \varphi} \cos^{n_c}(\varphi) d\varphi$. Using the following

two formulas, we can derive closed-form equations for even and odd values of n .

$$\int \cos^{2n} x dx = \frac{1}{2^{2n}} \binom{2n}{n} x + \frac{1}{2^{2n-1}} \sum_{k=0}^{n-1} \binom{2n}{k} \frac{\sin(2n-2k)x}{2n-2k} \quad (\text{A.25})$$

$$\int \cos^{2n+1} x dx = \frac{1}{2^{2n}} \sum_{k=0}^n \binom{2n+1}{k} \frac{\sin(2n-2k+1)x}{2n-2k+1} \quad (\text{A.26})$$

For even value of n_c :

$$Q_{\text{even}-n_c} = \frac{1}{2^{2n}} \binom{2n_c}{n_c} \pi + \frac{1}{2^{2n_c-1}} \sum_{k=0}^{n_c-1} \left\{ \binom{2n_c}{k} \frac{1}{n_c-k} \sin \left[(2n_c-2k) \frac{\pi}{2} \right] \right\}$$

$$\text{Since } \sin \left[(2n_c-2k) \frac{\pi}{2} \right] = 0 \text{ for even value of } n, Q_{\text{even}-n_c} = \frac{1}{2^{2n_c}} \binom{2n_c}{n_c} \pi. \quad (\text{A.27})$$

For odd value of n_c :

$$Q_{\text{odd}-n_c} = \sum_{k=0}^{n_c} \left\{ \binom{2n_c+1}{k} \frac{1}{2n_c-2k+1} 2 \sin \left[(2n_c-2k+1) \frac{\pi}{2} \right] \right\} \quad (\text{A.28})$$

A.7 Derivation of Angular Spread for Truncated Cosine

Distribution

$$\text{The angular spread is defined as } S_\varphi = \frac{1}{Q} \int_{\frac{\pi}{2} + \varphi}^{\pi + \varphi} \varphi^2 \cos^{n_c}(\varphi) d\varphi \quad (\text{A.29})$$

Using the following two formulas, we can derive closed-form equations for angular spread:

$$\int x^2 \cos^{2n} x dx = \left(\frac{2n}{k} \right) \frac{x^3}{2^{2n} \cdot 3} + \frac{1}{2^{2n-1}} \sum_{k=0}^{n-1} \left(\frac{2n}{k} \right) \int x^2 \cos[(2n-2k)x] dx \quad (\text{A.30})$$

$$\int x^2 \cos^{2n+1} x dx = \frac{1}{2^{2n}} \sum_{k=0}^n \left(\frac{2n+1}{k} \right) \int x^2 \cos[(2n-2k+1)x] dx \quad (\text{A.31})$$

For even value of n_c :

$$S_\varphi = \left\{ \frac{1}{Q} \left[\left(\frac{2n_c}{n_c} \right) \frac{2 \left(\frac{\pi}{2} \right)^3}{3 \times 2^{2n_c}} + \frac{1}{2^{2n_c-1}} \sum_{k=0}^{n_c-1} \left\{ \left(\frac{2n_c}{k} \right) \left[2(2n_c-2k) \pi \cos \left[(2n_c-2k) \frac{\pi}{2} \right] + 2 \left\{ \left[(2n_c-2k) \frac{\pi}{2} \right]^2 - 2 \right\} \sin \left[(2n_c-2k) \frac{\pi}{2} \right] \right\} \right] \right\}^{\frac{1}{2}}$$

$$S_{\varphi, \text{even}-n_c} = \left[2 \left(\frac{\pi}{2} \right)^3 (3\pi)^{-1} + 8 \sum_{k=0}^{n_c-1} \left\{ \left(\frac{2n_c}{k} \right) \left[\frac{(n_c-k)(-1)^{n_c-k}}{(2n_c-2k)^3} \right] \right\} \left(\frac{2n_c}{n_c} \right)^{-1} \right]^{\frac{1}{2}} \quad (\text{A.32})$$

For odd value of n_c :

$$S_\varphi = \left[\frac{1}{Q} \sum_{k=0}^{n_c} \left\{ \left(\frac{2n_c+1}{k} \right) \left(\frac{1}{2n_c-2k+1} \right)^3 \left[2(2n_c-2k+1) \pi \cos \left[(2n_c-2k+1) \frac{\pi}{2} \right] + 2 \left\{ \left[(2n_c-2k+1) \frac{\pi}{2} \right]^2 - 2 \right\} \sin \left[(2n_c-2k+1) \frac{\pi}{2} \right] \right] \right\} \right]^{\frac{1}{2}}$$

$$S_{\varphi, \text{odd}-n_c} = \left[\sum_{k=0}^{n_c} \left\{ \left(\frac{2n_c+1}{k} \right) \left(\frac{1}{2n_c-2k+1} \right)^3 \left[2 \left\{ \left[(2n_c-2k+1) \frac{\pi}{2} \right]^2 - 2 \right\} (-1)^{n_c-k} \right] \right\} \left[\sum_{k=0}^{n_c} \left\{ \left(\frac{2n_c+1}{k} \right) \frac{2(-1)^{n_c-k}}{2n_c-2k+1} \right\} \right]^{-1} \right]^{\frac{1}{2}} \quad (\text{A.33})$$

A.8 Derivation of Angle of Arrival for Circular Scatterer Model

Figure 8-1 is the geometrical illustration for signals coming from quadrant 3 and 4 of the circle. The objective here is to derive an equation for angle of arrival (φ) as a function of α . There are two ways to derive the relationship. The derivation of the first equation is as follows:

$$D_1 = R \cos(\alpha), \quad Y = R \sin(\alpha), \quad D_2 = D - D_1 = D - R \cos(\alpha)$$

$$\varphi = \tan^{-1} \left(\frac{Y}{D_2} \right) = \tan^{-1} \left(\frac{R \sin(\alpha)}{D - R \cos(\alpha)} \right) \quad (\text{A.34})$$

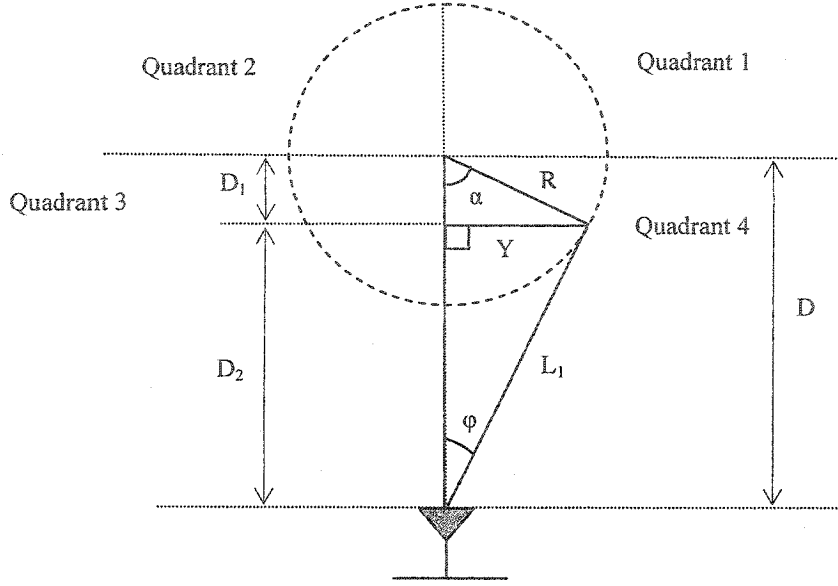


Figure 8-1 Geometrical Illustration of Circular Scatterer Model for Rays coming from Quadrant 3 and 4

The derivation of the second equation is as follows:

Let L = distance from the scatterer (regardless of which quadrant) to the receiver, therefore

$$L=L_1$$

$$R^2 = L^2 + D^2 - 2LD \cos(\varphi)$$

$$0 = L^2 - 2LD \cos(\varphi) + (D^2 - R^2)$$

$$L = D \cos(\varphi) \pm \sqrt{R^2 - D^2 \sin^2(\varphi)} \quad (\text{A.35})$$

Since each angle of arrival (φ) can be attributed to two different scatter, L_1 is the distance between the receiver and the closer scatterer (in quadrant 3 and 4), and L_2 is the distance between the receiver and the further away scatterer (in quadrant 1 and 2).

$$L_1 = D \cos(\varphi) - \sqrt{R^2 - D^2 \sin^2(\varphi)} \quad (\text{A.36})$$

$$L_2 = D \cos(\varphi) + \sqrt{R^2 - D^2 \sin^2(\varphi)} \quad (\text{A.37})$$

In this section, we are only interested in L_1 .

$$\varphi = \sin^{-1}\left(\frac{Y}{L_1}\right) = \tan^{-1}\left(\frac{R \sin(\alpha)}{D \cos(\varphi) - \sqrt{R^2 - D^2 \sin^2(\varphi)}}\right) \quad (\text{A.38})$$

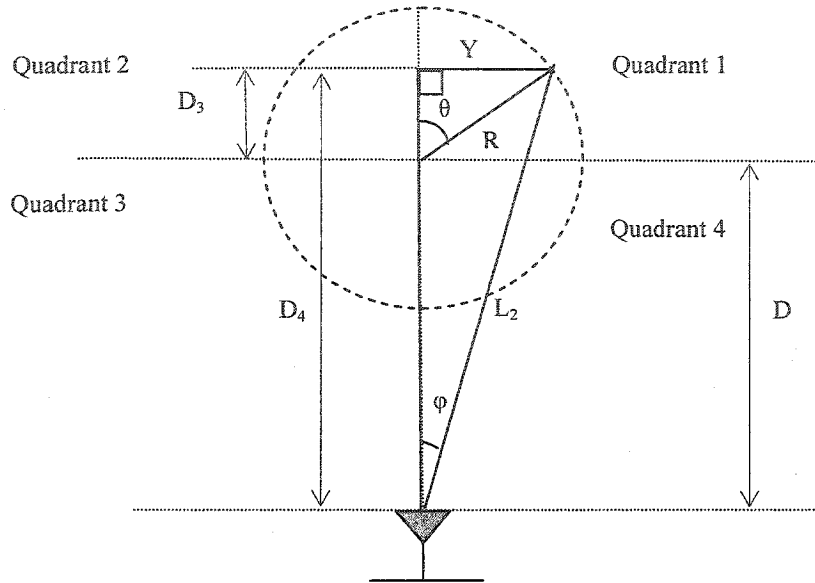


Figure 8-2 Geometrical Illustration of Circular Scatterer Model for Rays coming from Quadrant 1 and 2

Figure 8-2 is the geometrical illustration for signals coming from quadrant 1 and 2 of the circle. There are two ways to derive the relationship between θ and ϕ . The derivation of the first equation is as follows:

$$D_3 = R \cos(\theta), Y = R \sin(\theta), D_4 = D + D_3 = D + R \cos(\theta)$$

$$\varphi = \tan^{-1}\left(\frac{Y}{D_4}\right) = \tan^{-1}\left(\frac{R \sin(\theta)}{D + R \cos(\theta)}\right) \quad (\text{A.39})$$

The derivation of the second equation is as follows:

$$L_2 = D \cos(\varphi) + \sqrt{R^2 - D^2 \sin^2(\varphi)} \quad (\text{A.40})$$

$$\varphi = \sin^{-1}\left(\frac{Y}{L_1}\right) = \sin^{-1}\left(\frac{R \sin(\theta)}{D \cos(\theta) + \sqrt{R^2 - D^2 \sin^2(\varphi)}}\right) \quad (\text{A.41})$$

A.9 Derivation of Angular Spread for Circular Scatterer Model

Equation for Angular Spread/Maximum Angle Ratio in Integration Form

$$L_1 = D \cos(\varphi) - \sqrt{R^2 - D^2 \sin^2(\varphi)} \quad (\text{A.42})$$

$$L_2 = D \cos(\varphi) + \sqrt{R^2 - D^2 \sin^2(\varphi)} \quad (\text{A.43})$$

Using Sine Law:

$$\alpha = \sin^{-1}\left(\frac{L_1 \sin(\varphi)}{R}\right) \quad (\text{A.44})$$

$$\theta = \sin^{-1}\left(\frac{L_2 \sin(\varphi)}{R}\right) \quad \text{when } L_2 > \sqrt{D^2 + R^2} \quad (\text{A.45})$$

$$\theta = \pi - \sin^{-1}\left(\frac{L_2 \sin(\varphi)}{R}\right) \quad \text{when } L_2 \leq \sqrt{D^2 + R^2} \quad (\text{A.46})$$

Cumulative Probability Function:

$$F_\varphi(\varphi) = \int_{\alpha_{\max}}^{\alpha_{\max}} f_\theta(\theta) d\theta + \int_{\alpha_{\max}}^{\alpha_{\max}} f_\alpha(\alpha) d\alpha$$

$$F_\varphi(\varphi) = \frac{1}{2\pi} \left[\int_{\alpha_{\max}}^{\alpha_{\max}} d\theta + \int_{\alpha_{\max}}^{\alpha_{\max}} d\alpha \right]$$

$$F_\varphi(\varphi) = \frac{1}{\pi} (\alpha + \theta)$$

(A.47)

Take derivative to obtain Probability Density Function:

$$f(\varphi) = \frac{1}{\pi} (A - B) \quad (\text{A.48})$$

$$A = \left[1 - \left(\frac{\sin \varphi (L_1)}{R} \right)^2 \right]^{\frac{1}{2}} \left[\frac{\cos \varphi}{R} (L_1) + \frac{\sin \varphi}{R} \left(-D \sin \varphi - \frac{1}{2} (R^2 - D^2 \sin^2(\varphi))^{\frac{1}{2}} (-2D^2 \cos \varphi \sin \varphi) \right) \right]$$

$$B = \left[1 - \left(\frac{\sin \varphi (L_2)}{R} \right)^2 \right]^{\frac{1}{2}} \left[\frac{\cos \varphi}{R} (L_2) + \frac{\sin \varphi}{R} \left(-D \sin \varphi + \frac{1}{2} (R^2 - D^2 \sin^2(\varphi))^{\frac{1}{2}} (-2D^2 \cos \varphi \sin \varphi) \right) \right]$$

Since the mean of the distribution is 0, the variance is:

$$\sigma_\varphi^2 = \int_{-\varphi_{\max}}^{\varphi_{\max}} \varphi^2 f(\varphi) d\varphi$$

The angular spread is equal to σ and the angular spread/maximum angle is equal to:

$$\frac{\text{angular spread}}{\text{maximum angle}} = \frac{\sigma_\varphi}{\sin^{-1}(R/D)} \quad (\text{A.49})$$

In order to obtain a deterministic equation, some simplifications are performed.

Assumptions:

It is assumed that the radius is small compared to the distance ($R/D < 0.1$)

When R/D is small, the angles theta and alpha are roughly be the same.

When R/D is small, $D = l_1 = l_2$

Derivation:

$$\alpha = \theta = \sin^{-1} \left(\frac{\sin(\varphi) D}{R} \right) \approx \sin^{-1}(\varphi D / R) \quad \text{Note: } \sin(\varphi) = \varphi \text{ for small } \varphi$$

Cumulative Distribution Function:

$$F_\varphi(\varphi) = \frac{1}{\pi} (\alpha + \theta)$$

$$F_\varphi(\varphi) = \frac{2}{\pi} [\sin^{-1}(\varphi D / R)] \quad (\text{A.50})$$

Taking derivative to obtain probability density function:

$$f_\varphi(\varphi) = \frac{D}{\pi R} \left(1 - \frac{D^2 \varphi^2}{R^2} \right)^{\frac{1}{2}} \quad (\text{A.51})$$

Let $B = D/R$

$$\text{Variance} = \sigma_{\varphi}^2 = \frac{B}{\pi} \int_{-\varphi_{\max}}^{\varphi_{\max}} \frac{\varphi^2}{\sqrt{1-(B\varphi)^2}} d\varphi = \frac{2 \sin^{-1}(B\varphi_{\max}) - \sin[2 \sin^{-1}(B\varphi_{\max})]}{2\pi B^2} \quad (\text{A.52})$$

The angular spread is equal to σ and the angular spread/maximum angle is equal to:

$$\frac{\text{angular spread}}{\text{maximum angle}} = \frac{\sigma_{\varphi}}{\sin^{-1}(R/D)} = \frac{\sqrt{\{2 \sin^{-1}(B\varphi_{\max}) - \sin[2 \sin^{-1}(B\varphi_{\max})]\} / 2\pi B^2}}{\sin^{-1}(R/D)}$$

$$\varphi_{\max} = \sin^{-1}(R/D) = \sin^{-1}(1/B) \approx 1/B, \text{ therefore } B\varphi_{\max} = 1 \quad (\text{A.53})$$

$$\frac{\text{angular spread}}{\text{maximum angle}} = \frac{\sqrt{\{2 \sin^{-1}(1) - \sin[2 \sin^{-1}(1)]\} / 2\pi}}{B(1/B)} = \sqrt{\frac{(\pi - 0)}{2\pi}} = \frac{1}{\sqrt{2}} \quad (\text{A.54})$$

Therefore, the (angular spread / maximum angle) ratio for small values of (radius / distance) is equal to $1/\sqrt{2}$.

Derivation of Correlation for One-Impulse Model

$$\text{Correlation: } \rho = \int \exp(j2\pi d \sin \varphi) f(\varphi) d\varphi \quad (\text{A.55})$$

$$\rho = \int \exp(j2\pi d \sin \varphi) \delta(0) d\varphi$$

$$\rho = \int \exp(j2\pi d \sin(0)) d\varphi = \exp(0) = 1$$

$$\rho_{\text{one-impulse}} = 1 \quad (\text{A.56})$$

Derivation of Correlation for N-Impulse Model

For even values of N:

$$p_{N=\text{even}}(\varphi) = \frac{1}{N} \left[\sum_{n=1}^{N/2} \left[\delta\left(\varphi - \frac{2n\Delta}{N}\right) + \delta\left(\varphi + \frac{2n\Delta}{N}\right) \right] \right]$$

$$\rho = \int \exp(j2\pi d \sin \varphi) p(\varphi) d\varphi$$

$$\begin{aligned}
\rho_{N=even} &= \int \frac{\exp(j2\pi d \sin \varphi)}{N} \left[\sum_{n=1}^{N/2} \left[\delta\left(\varphi - \frac{2n\Delta}{N}\right) + \delta\left(\varphi + \frac{2n\Delta}{N}\right) \right] \right] d\varphi \\
\rho_{N=even} &= \frac{1}{N} \sum_{n=1}^{N/2} \left[\exp\left(j2\pi d \sin\left(-\frac{2n\Delta}{N}\right)\right) + \exp\left(j2\pi d \sin\left(\frac{2n\Delta}{N}\right)\right) \right] \\
\rho_{N=even} &= \frac{2}{N} \sum_{n=1}^{N/2} \cos\left(2\pi d \sin\left(\frac{2n\Delta}{N}\right)\right) \tag{A.57}
\end{aligned}$$

For odd values of N:

$$\begin{aligned}
\rho_{N=odd}(\varphi) &= \frac{1}{N} \left[\sum_{n=1}^{(N-1)/2} \left[\delta\left(\varphi - \frac{2n\Delta}{N-1}\right) + \delta\left(\varphi + \frac{2n\Delta}{N-1}\right) \right] + \delta(\varphi) \right] \\
\rho &= \int \exp(j2\pi d \sin \varphi) p(\varphi) d\varphi \\
\rho_{N=odd} &= \int \frac{\exp(j2\pi d \sin \varphi)}{N} \left[\sum_{n=1}^{(N-1)/2} \left[\delta\left(\varphi - \frac{2n\Delta}{N-1}\right) + \delta\left(\varphi + \frac{2n\Delta}{N-1}\right) \right] + \delta(\varphi) \right] d\varphi \\
\rho_{N=odd} &= \frac{1}{N} \left\{ \sum_{n=1}^{(N-1)/2} \left[\exp\left(j2\pi d \sin\left(-\frac{2n\Delta}{N-1}\right)\right) + \exp\left(j2\pi d \sin\left(\frac{2n\Delta}{N-1}\right)\right) \right] + \exp(j2\pi d \sin(0)) \right\} \\
\rho_{N=odd} &= \frac{2}{N} \sum_{n=1}^{(N-1)/2} \cos\left(2\pi d \sin\left(\frac{2n\Delta}{N-1}\right)\right) + \frac{1}{N} \tag{A.58}
\end{aligned}$$

Appendix B - MATLAB Code

B.1 MATLAB Code to Generate Samples of Uniform AOA

Distribution

```
% fi0 = center angle
% dfi = angular spread in radian
% N = number of trials
% fi = generated samples according to Gaussian AOA distribution

uniform_max_angle = dfi * sqrt(3); % calculate max angle
fi = rand(N,1)*2*uniform_max_angle-uniform_max_angle+fi0; % generate samples
```

B.2 MATLAB Code to Generate Samples of truncated Gaussian

AOA Distribution

```
% fi0 = center angle
% sigma = standard deviation of corresponding non-truncated Gaussian distribution
% N = number of trials
% fi = generated samples according to Gaussian AOA distribution

fi = normrnd(fi0,sigma,N,1); % generate a sample of Gaussian AOA
```

B.3 MATLAB Code to Generate Samples of truncated Laplacian

AOA Distribution

```
% fi0 = center angle
% sigma = standard deviation of corresponding non-truncated Laplacian distribution
% N = number of trials
% fi = generated samples according to truncated Laplacian AOA distribution

% logic to generate laplacian distribution
x = rand(N,1); % generate N uniformly-distributed samples
j = find(x<1/2); % find entries that are smaller than 1/2
k = find(x>=1/2); % find entries that are larger than 1/2
fi(j) = (sigma/sqrt(2)).*log(2.*x(j)); % generate samples for x < 1/2
fi(k) = -(sigma/sqrt(2)).*log(2.*(1-x(k))); % generate samples for x >= 1/2
fi = reshape(fi,N,1); % reshape the samples
```

B.4 MATLAB Code to Generate Samples of Cosine AOA

Distribution

```
% fi0 = center angle
% dfi = angular spread in radians
```



```

% N          = number of trials
% num_section = number of sections to divide x-axis into
% fi         = generated samples according to truncated Laplacian AOA distribution

lowerbound = -pi/2;
upperbound = pi/2;
step = pi/num_section;

n = cosn_getn(df1);          % obtain value of n (exponent) from angular spread

x = lowerbound+step/2:step:upperbound-step/2;          % divide x-axis into regions
y = (cos(x).^n);          % obtain probability of all regions

for loop1 = 1:1:size(y,2)
    for loop2 = 1:1:loop1
        z(loop1) = z(loop1) + y(loop2);
    end
end
z = z/sum(y);          % z is the cumulative density function

result(N) = 0;          % resulting cosine-distributed samples
a=rand(N,1);          % generate N uniformly-distributed samples
% (used to determine number of samples for each region)
b=rand(N,1);          % generate N uniformly-distributed samples
% (used to determine distribution of samples in each region)
j=find(a>=0 & a<z(1));          % find samples the fall into the probability range of the
% first region

% generate samples that are uniformly-distributed within the region, the number
% of samples in the region correspond to the probability of that region according
% to cosine distribution
result(j) = (0+b(j))/((upperbound-lowerbound)/step);

% repeat the same process for each region
for loop4 = 1:1:((upperbound-lowerbound)/step)-1
    j=find(a>=z(loop4) & a<z(loop4+1));
    result(j) = (loop4+b(j))/((upperbound-lowerbound)/step);
end

result = reshape(result,N,1);          % reshape the sample
result = (result - 0.5)*pi;          % shift to center at 0 from -pi/2 to pi/2
fi = result;

```

B.5 MATLAB Code to Generate Samples of Circular Scatterer

Model

```

% distance = distance between transmitter and receiver
% radius   = radius of circle
% number_of_scatterers = number of scatterers
% angles   = generated angle of arrivals

array_a = rand(number_of_scatterers,1) * 2 * pi - pi;
angles = array_a * 0; % set angles to have the same dimension

q1 = find(array_a<=-pi);
q2 = find(array_a>=-pi & array_a <0);
q3 = find(array_a>=0 & array_a <=pi);
q4 = find(array_a>=pi);

angles(q1) = - atan((radius*sin(array_a(q1)+pi))./(distance+radius*cos(array_a(q1)+pi)));
angles(q2) = - atan((radius*sin(-array_a(q2)))./(distance-radius*cos(-array_a(q2))));
angles(q3) = atan((radius*sin(array_a(q3)))./(distance-radius*cos(array_a(q3))));
angles(q4) = atan((radius*sin(-array_a(q4)+pi))./(distance+radius*cos(-array_a(q4)+pi)));
angles = reshape(angles,number_of_scatterers,1);

```

B.6 MATLAB Code of Simulation Model to Obtain Channel Capacity and Correlation

The following is a list of files that are part of the simulation model:

Filename	Description
gen_mimo_cap.m	Top level simulation model to obtain channel capacity
gen_env_cor_mag.m	Top level simulation model to obtain correlation
Aver_rayT.m	Function that returns channel capacity and/or correlation given the configuration as input
RayT.m	Function to generate instantaneous capacity and/or instantaneous correlation

GEN_MIMO_CAP.M

```
function output = gen_mimo_cap(input)

% -----
% Function: generate mimo system channel capacity
% -----

% input parameters
% -----
% dist : distribution (0=uniform, 1=gaussian, 2=laplacian
%           3=cosine, 4=circular scatterer)
% df   : angular spread in degrees
% fi0  : the center angle
% ro   : average SNR (ro/n - average SNR per Rx branch)
% M    : number of trials
% N    : number of multipath components per transmitted signal
% n    : the number of antennas
% lbound_spacing : lowerbound of antenna spacing (in wavelength)
% ubound_spacing : upperbound of antenna spacing (in wavelength)
% step_spacing  : step of antenna spacing (in wavelength)

dist      = input{1};
df        = input{2};
fi0       = input{3}*pi/180; % convert to radian
ro        = input{4};
M         = input{5};
N         = input{6};
```

```

n          = input{7};
lbound_spacing = input{8};
ubound_spacing = input{9};
step_spacing  = input{10};
directory     = input{11};

% processing of input parameters
% -----
dfi = df/180*pi;          % conversion from degrees to radian

% calculate the standard deviation (sigma) for gaussian and laplacian
% and the distance-radius ratio for circular scatterer model
% -----
% dist : 0 = uniform      1 = gaussian      2 = laplacian
%        3 = cosine       4 = circular scatterer

if (dist == 1) | (dist == 2)
    sigma = cal_sigma(dist,dfi);
    fprintf('dist %f, angular spread = %f, sigma = %f\n', dist, dfi, sigma)
    cosn_array = 0;
elseif (dist == 3)
    cosn_array = cosn_dist_prep(dfi,90);
    sigma = 0;
elseif (dist == 4)
    % in this case, sigma is the distance when radius is one
    sigma = cal_dist_radius_ratio(dfi);
    cosn_array = 0;
else
    % for all other distributions, these two variables
    % are not used
    sigma = 0;
    cosn_array = 0;
end

% output to screen the current configuration
% -----
fprintf('Generating samples for\n\tDistribution #\td\n\tCenter Angle = %d\n\tAngular Spread = %d\n\tNumber of Trials = %d\n\tNumber of Multipath Signals = %d\n\tNumber of Antenna = %d\n\tSNR = %d\n', dist, fi0*180/pi, dfi*180/pi,M,N,n,ro);

% put all input parameters into an array
% -----
InDat = [M N n dfi*180/pi fi0*180/pi ro];

% opening of file
% -----
s = [ directory 'rayT_gauss_' int2str(M) '_' int2str(N) '_' int2str(n) '_' int2str(dfi*180/pi) '_' int2str(fi0*180/pi) '_dist' int2str(dist) '_snr' int2str(ro) '.dat'];
fid = fopen(s,'w');

% write configuration information into file
% -----
fprintf(fid, 'M=%g\t N=%g\t n=%g\t dfi=%g\t fi0=%g\t ro=%g\n\n', InDat);
fprintf('\n\n M=%g\t N=%g\t n=%g\t dfi=%g\t fi0=%g\t ro=%g\n\n', InDat);
fprintf(fid, '\t d\t\t ctr\t\t cavtr\t\t time\n\n');
fprintf(' \t\t\t ctr\t cavtr \t\t\t time\n\n');

tic;          % start timer

% generating channel capacities
% -----
for i = lbound_spacing:step_spacing:100
    % calculate antenna spacing
    d = (ubound_spacing*10^(-1/50))-(ubound_spacing/100)+10^(-15);
    % call function to obtain channel capacity
    Out = Aver_rayT(M,N,n,d,dfi,fi0,ro,dist,0,sigma,cosn_array);
    Rtr = Out{1};          % mean Rx correlations (over M trials)
    ctr = Out{2};          % mean capacity (by averaging M trials)
    cavtr = Out{3};        % upper bound of capacity
end

```

```

cav = Out{4};          % not used
RtrT = Out{5};         % mean Tx correlation (over M trials)
Out_data = [d ctr cavtr toc];
% print [antenna spacing, mean capacity, upper bound capacity, time] to file
fprintf(fid, '%8.3f\t %8.2f\t %8.2f\t %8.2f\n', Out_data);
% print [antenna spacing, mean capacity, upper bound capacity, time] to screen
fprintf('%8.3f %8.2f %8.2f %8.2f\n', Out_data);
end

status = fclose(fid);   % close file
output = {s};          % output results

```

GEN_ENV_COR_MAG.M

```

function output = gen_env_cor_mag(input)

% -----
% Function: generate mimo system correlation coefficient
% -----

% input parameters
% -----
% dist : distribution (0=uniform, 1=gaussian, 2=laplacian
%           3=cosine, 4=circular scatterer)
% df   : angular spread in degrees
% fi0  : the center angle
% ro   : average SNR (ro/n - average SNR per Rx branch)
% M    : number of trials
% N    : number of multipath components per transmitted signal
% n    : the number of antennas
% lbound_spacing : lowerbound of antenna spacing (in wavelength)
% ubound_spacing : upperbound of antenna spacing (in wavelength)
% step_spacing   : step of antenna spacing (in wavelength)

dist = input{1};
df   = input{2};
fi0  = input{3}*pi/180; % convert to radian
ro   = input{4};
M    = input{5};
N    = input{6};
n    = input{7};
lbound_spacing = input{8};
ubound_spacing = input{9};
step_spacing   = input{10};
directory      = input{11};

% processing of input parameters
% -----
n    = 2; % hardcode number of antenna to 2
dfi = df/180*pi; % conversion from degrees to radian
fi0_d = fi0*180/pi;

% calculate the standard deviation (sigma) for gaussian and laplacian
% and the distance-radius ratio for circular scatterer model
% -----
if (dist == 1) | (dist == 2)
    sigma = cal_sigma(dist, dfi);
    fprintf('dist %f, angular spread = %f, sigma = %f\n', dist, dfi, sigma)
    cosn_array = 0;
elseif (dist == 3)
    cosn_array = cosn_dist_prep(dfi, 90);
    sigma = 0;
elseif (dist == 4)
    % in this case, sigma is the distance when radius is one
    sigma = cal_dist_radius_ratio(dfi);
    cosn_array = 0;
else

```

```

sigma = 0;
cosn_array = 0;
end

% output to screen the current configuration
% -----
fprintf('Generating envelope correlation magnitudes for\n\tDistribution #d\n\tCenter
Angle = %d\n\tAngular Spread = %d\n\tNumber of Trials = %d\n\tNumber of Multipath Signals
= %d\n\tNumber of Antenna = %d\n\tSNR = %d\n', dist, fi0*180/pi, dfi*180/pi,M,N,n,ro);

% put all input parameters into an array
% -----
InDat = [M N n dfi*180/pi fi0*180/pi ro];

% opening of file
% -----
s = [ directory 'env_cor_mag_dist_' int2str(dist) '_angularspread_' int2str(df)
'_angleincidence_' int2str(fi0_d) '_trials_' int2str(M) '.dat'];
fid = fopen(s,'w');

% write configuration information into file
% -----
fprintf(fid,'M=%g\t N=%g\t n=%g\t dfi=%g\t fi0=%g\t ro=%g\n\n',InDat);
fprintf(fid,'antenna_spacing, correlation_magnitude, time\n');

tic;          % start timer

% generating correlation coefficients
% -----
for i = lbound_spacing:0.5:100
    % calculate antenna spacing
    d = (ubound_spacing*10^(-i/50)-(ubound_spacing/100)+10^(-15));
    % call function to obtain correlation
    Out = Aver_rayT(M,N,n,d,dfi,fi0,ro,dist,1,sigma,cosn_array);
    cor_mag = Out{6};          % correlation coefficient
    fprintf (fid,'%f, %f, %f\n', d, cor_mag, toc);
end

status = fclose(fid);          % close file
output = {s};                  % output results

```

AVER_RAYT.M

```

function A = Aver_rayT(M,N,n,d,dfi,fi0,ro,dist,cal_cor,sigma,cosn_array)

% -----
% Function: this is a file for computing the average correlation matrix
% and the mean (ergodic) capacity of a multipath matrix channel.
% Instantaneous correlation matrix R is computed by RrayT.
%
% It also computes envelope correlation coefficient.only work for n=2
% -----

% input parameters
% -----
% N          - the number of rays
% M          - the number of trials
% d          - element spacing of Rx array
% dfi        - angular spread
% fi0        - average AOA
% ro        - average SNR (ro/n - average SNR per Rx branch)
% cal_cor    - calculate correlation, 0=no, 1=yes
% sigma      - sigma for distribution
% cosn_array - cosine n distribution array

% internal and output parameters
% -----
% R and Rt   - instantaneous Rx and Tx brach correlations
% ctr        - mean capacity (by averaging M trials)

```

```

% Rtr & RtrT - mean Rx and Tx correlations (over M trials)

% initialize the parameters
% -----
ctr=0;           % mean capacity, start with 0
Rtr(n,n)=0;      % mean Rx correlation, start with 0
RtrT(n,n)=0;     % mean Tx correlation, start with 0
Hout(n,n)=0;     % channel matrix, start with 0
cor_x=0;         % x component of envelope correlation
cor_y=0;         % y component of envelope correlation
cor_z=0;         % z component of envelope correlation
%A = zeros(n,n,2);

% obtain instantaneous results for M trials
% -----
for m=1:M        % repeat the procedure for M trials

    Rout = RrayT(N,n,d,dfi,fi0,dist,sigma,cosn_array);
    R = Rout{1}; % instantaneous Rx correlation matrix

    % ----- start of code for calculation of correlation -----
    if (cal_cor == 1)
        x1=abs(R(1,1)); % received signal at antenna 1
        x2=real(R(2,1)); % real part of complex correlation
        y2=imag(R(2,1)); % imag part of complex correlation
        x=x1*x2;         % instantaneous x component of envelope correlation
        y=x1*y2;         % instantaneous y component of envelope correlation
        z=x1*x1;         % instantaneous z component of envelope correlation
    end
    % ----- end of code for calculation of correlation -----

    Rt = Rout(2); % instantaneous Tx correlation matrix
    H = Rout(3); % channel matrix
    ctr = log2(det(eye(n)+ro/n*R))+ctr; % sum of channel capacity
    Hout = H + Hout; % sum of channel matrix
    Rtr = R+Rtr; % sum of Rx correlation matrix
    RtrT = Rt+RtrT; % sum of Tx correlation matrix

    % ----- start of code for calculation of correlation -----
    if (cal_cor == 1)
        cor_x=cor_x+x; % sum of x component
        cor_y=cor_y+y; % sum of y component
        cor_z=cor_z+z; % sum of z component
        cor_enve=(cor_x.^2+cor_y.^2)/(cor_z.^2); % envelope correlation
        cor_mag = (cor_x)/(cor_z); % correlation coefficient
    end
    % ----- end of code for calculation of correlation -----

end

% average the results over M trials
% -----
Rtr=Rtr/M; % average Rx correlation matrix (over M trials)
RtrT=RtrT/M; % average Tx correlation matrix (over M trials)
Hout=Hout/M; % average channel matrix (over M trials)
cavtr = log2(det(eye(n)+ro/n*Rtr)); % average channel capacity
cav = 0;

% output results
% -----
% Rtr - average Rx correlation matrix
% ctr/M - upper bound of channel capacity
% cavtr - average channel capacity
% RtrT - average Tx correlation matrix
% cor_mag - correlation coefficient
% cor_enve - envelope correlation

if (cal_cor == 1)
    A = {Rtr ctr/M cavtr cav RtrT cor_mag cor_enve};
else

```

```

A = {Rtr ctr/M cavtr cav RtrT Hout};
End

```

RRAYT.M

```

function R = RrayT(N,n,d,dfi,fi0,dist,sigma,cosn_array)

% -----
% Function: this is a file for computing correlation matrix of a
%           multipath matrix channel. N rays exist for every Tx, with
%           independent AOA set for every Tx, and with (i) independent
%           phases and the same magnitude or (ii) independent
%           complex Gaussian gains, uncorrelated from Tx to Tx.
% -----

% input parameters
% -----
% d           - element spacing of rx antenna
% fi          - angle-of-arrival (AOA)
% dfi         - angular spread
% fi0         - mean angle of arrival
% ksi        - phase shift between two adjacent elements
% n           - the number of antennas
% N           - the number of rays
% psi        - their phases
% a           - Tx complex gains
% sigma      - sigma (std deviation)
% cosn_array - cosine n distribution array

format compact

uniform_max_angle = dfi * sqrt(3); % calculate max angle for
                                   % uniform distribution

for j = 1:n % for each transmitting antenna

    % -----
    % uniform AOA distribution
    % -----
    if (dist == 0)
        fi = rand(N,1)*2*uniform_max_angle-uniform_max_angle+fi0;

    % -----
    % gaussian AOA distribution
    % -----
    elseif (dist == 1)
        % generate samples of gaussian distribution
        fi = normrnd(fi0,sigma,N,1);

    % -----
    % laplacian AOA distribution
    % -----
    elseif (dist == 2)
        % generate samples of laplacian distribution
        fi = laplace_noise(N,1,sigma)+fi0;

    % -----
    % cosine AOA distribution
    % -----
    elseif (dist == 3)

        % generate sample of cosine distribution
        fi = cosn_dist_gen(cosn_array,N)+fi0;

    % -----
    % circular scatterer AOA distribution
    % -----
    elseif (dist == 4)

```

```

% generate samples of circular scatterer model
fi = circular_scatterers(sigma,1,N)+fi0;

% -----
% two impulses AOA distribution
% -----
elseif (dist == 5)

    fi = two_impulses(dfi, N)+fi0;

% -----
% three impulses AOA distribution
% -----
elseif (dist == 6)

    fi = multiple_impulses(3, dfi, N)+fi0;

% -----
% four impulses AOA distribution
% -----
elseif (dist == 7)

    fi = multiple_impulses(4, dfi, N)+fi0;

% -----
% one impulse AOA distribution
% -----
elseif (dist == 8)

    fi = multiple_impulses(1, dfi, N)+fi0;

% -----
% five impulse AOA distribution
% -----
elseif (dist == 9)

    fi = multiple_impulses(5, dfi, N)+fi0;

% -----
% six impulse AOA distribution
% -----
elseif (dist == 10)

    fi = multiple_impulses(6, dfi, N)+fi0;

% -----
% seven impulse AOA distribution
% -----
elseif (dist == 11)

    fi = multiple_impulses(7, dfi, N)+fi0;

% -----
% eight impulse AOA distribution
% -----
elseif (dist == 12)

    fi = multiple_impulses(8, dfi, N)+fi0;

% -----
% nine impulse AOA distribution
% -----
elseif (dist == 13)

    fi = multiple_impulses(9, dfi, N)+fi0;

% -----
% ten impulse AOA distribution
% -----
elseif (dist == 14)

```



```

        fi = multiple_impulses(10, dfi, N)+fi0;

    end

    % calculate the phase arriving at each antenna element
    ksi = 2*pi*d*sin(fi);
    % generate an amplitude for each multipath signal
    a = 1/sqrt(2)*(randn(N,1) + 1i*randn(N,1));

    for i = 1:n
        % calculate the entry in the channel matrix
        % corresponding to each receiving antenna element
        h(i,j) = trace(diag(a.*exp(-1i*i*ksi)));
    end
end

Rr = 1/(n*N)*h*h'; % Rx correlation matrix
H = 1/(n*N).*h; % channel matrix
h = h.'; % complex conjugate of channel matrix
Rt=1/(n*N)*h*h'; % Tx correlation matrix
R = [Rr Rt H]; % output

```

B.7 MATLAB Code of Simulation Model to Obtain Outage

Probability and Diversity Gain

The following is a list of files that are part of the simulation model:

Filename	Description
mrc_outage.m	Top level simulation model to obtain outage probability
diversity_gain.m	Top level simulation model to obtain diversity gain
RrayT_mrc.m	Function to generate instantaneous SNR

MRC_OUTAGE.M

```

% -----
% This is the working file to generate the outage probability curve
% -----
%
% filename: mrc_outage.m
%
% variables:
% -----
% N - the number of rays
% n_rx - the number of receive antennas
% n_tx - the number of transmit antennas

```

```

% d - element spacing of rx antenna
% dfi - angular spread
% fi0 - mean angle of arrival
% dist = distribution
% sigma - sigma (std deviation)
% cosn_array - cosine n distribution array
% -----

clear all;
M = 5*10^4;           % number of samples
N = 10;               % number of rays
n_rx = 5;             % number of receive antenna elements
n_tx = 1;             % number of transmit antenna elements
fi0 = 0*pi/180;       % center angle
min_snr = 1;          % minimum signal to noise ratio
dist = 1;             % gaussian distribution
df = 10;              % angular spread in degrees
dfi = df/180*pi;      % angular spread in radians

tic;                  % start timer
format short;
format compact;

dist = 1;             % distribution
fi0 = 0;              % mean AOA
dfi = 10/180*pi;      % angular spread
nb_antenna = 1;       % number of antenna
antenna_spacing = 5;  % antenna spacing
snr_lowerbound = -40; % lowerbound of SNR
snr_upperbound = 20;  % upperbound of SNR
snr_step = 0.5;       % step of SNR

% open data file
% -----
filename = ['mrc_outage_dist_' int2str(dist) '_nb_antenna_' int2str(nb_antenna)
'_angular_spread_' int2str(dfi*180/pi) '_center_' int2str(fi0*180/pi) '_spacing'
int2str(antenna_spacing) '.dat'];
fid = fopen(filename,'w');

% putting settings onto file
% -----
fprintf('...Processing %s\n', filename);
fprintf(fid,'// Maximal Ratio Combining\n');
fprintf(fid,'// Distribution      : %d\n',dist);
fprintf(fid,'// Number of Antenna : %d\n',nb_antenna);
fprintf(fid,'// Angular Spread    : %d degrees\n',dfi*180/pi);
fprintf(fid,'// Center Angular    : %d degrees\n',fi0*180/pi);
fprintf(fid,'// Element Spacing   : %d\n',antenna_spacing);
fprintf(fid,'// Lowerbound SNR    : %d\n',snr_lowerbound);
fprintf(fid,'// Upperbound SNR     : %d\n',snr_upperbound);
fprintf(fid,'// SNR Step          : %d\n',snr_step);
fprintf(fid,'%f, %f\n',0,0);

% calculate the standard deviation (sigma) for gaussian and laplacian
% -----
if (dist == 1) | (dist == 2)
    sigma = cal_sigma(dist,dfi);
    cosn_array = 0;
elseif (dist == 3)
    cosn_array = cosn_dist_prep(dfi,90);
    sigma = 0;
elseif (dist == 4) | (dist == 5)
    % in this case, sigma is the distance when radius is one
    sigma = cal_dist_radius_ratio(dfi);
    cosn_array = 0;
else
    sigma = 0;
    cosn_array = 0;
end

```

```

% get SNR samples
% -----
for sample_index = 1:M
    snr(sample_index) =
    RrayT_mrc(N,n_tx,nb_antenna,antenna_spacing,dfi,fi0,dist,sigma,cosn_array);
end %for sample_index = 1:M

% process the SNR samples to get outage probability
% -----
snr_index = snr_lowerbound; % set to lowerbound

% repeat the process until upperbound is reached
while (snr_index < snr_upperbound)
    snr_index_scaler = 10.^(snr_index/10); % convert to scaler
    j=find(snr<snr_index_scaler); % find entires below SNR level
    prob_outage = size(j,2)/size(snr,2); % convert to percentage
    fprintf(fid,'%f, %f\n', snr_index_scaler, prob_outage); % print to file
    snr_index = snr_index + snr_step; % repeat for next SNR level
end

% close file
% -----
status = fclose(fid);

```

DIVERSITY_GAIN.M

```

% -----
% This is the working file to calculate the diversity gain
% -----
%
% filename: diversity_gain_spacing.m
%
% variables:
% -----
% N - the number of rays
% n_tx - the number of transmit antennas
% d - element spacing of rx antenna
% dfi - angular spread
% fi0 - mean angle of arrival
% dist = distribution
% sigma - sigma (std deviation)
% cosn_array - cosine n distribution array
%
% -----

clear all;
M = 10^6; % number of samples
N = 10; % number of rays
n_tx = 1; % number of transmit antenna elements
fi0 = 0*pi/180; % center angle
min_snr = 1; % minimum signal to noise ratio
dist = 0; % gaussian distribution
df = 10; % angular spread in degrees
dfi = df/180*pi; % angular spread in radians

tic;
format short;
format compact;

dist = 1; % distribution
fi0 = 0; % mean AOA
dfi = 10/180*pi; % angular spread
nb_antenna = 1; % number of antenna
antenna_spacing = 5; % antenna spacing
snr_lowerbound = -45; % lowerbound for SNR
snr_upperbound = 20; % upperbound for SNR
outage_threshold = 10.^(-2); % threshold for outage probability
snr_step = 0.1; % step for SNR

```

```

found = 0; % flag: 1=found, 0=not found

% open data file
% -----
filename = ['diversity_gain_vs_antenna_spacing_dist_' int2str(dist) '_angular_spread_'
int2str(dfi*180/pi) '_center_' int2str(fi0*180/pi) '.dat'];
fid = fopen(filename,'w');

% print setting to file
% -----
fprintf(fid,'// -----\n');
fprintf(fid,'// Distribution : %d\n',dist);
fprintf(fid,'// Angular Spread : %d degrees\n',dfi*180/pi);
fprintf(fid,'// Center Angular : %d degrees\n',fi0*180/pi);
fprintf(fid,'// Outage Threshold : %d\n',outage_threshold);
fprintf(fid,'// -----\n');
fprintf(fid,'spacing, snr_for_1_antenna, snr_for_2_antenna,diversity_gain\n');

% calculate the standard deviation (sigma) for gaussian and laplacian
% -----
if (dist == 1) | (dist == 2)
    sigma = cal_sigma(dist,dfi);
    cosn_array = 0;
elseif (dist == 3)
    cosn_array = cosn_dist_prep(dfi,90);
    sigma = 0;
elseif (dist == 4) | (dist == 5)
    % in this case, sigma is the distance when radius is one
    sigma = cal_dist_radius_ratio(dfi);
    cosn_array = 0;
else
    sigma = 0;
    cosn_array = 0;
end

% -----
% processing 1 antenna element
% -----
nb_antenna = 1; % set number of antenna to 1

% get SNR samples
% -----
for sample_index = 1:M
    snr(sample_index) =
RrayT_mrc(N,n_tx,nb_antenna,antenna_spacing_index,dfi,fi0,dist,sigma,cosn_array);
end

snr_index = snr_lowerbound; % set to lowerbound
found = 0; % set flag to not found

% repeat loop until found or upperbound reached
while ( (snr_index < snr_upperbound) & (found == 0) )
    snr_index_scaler = 10.^(snr_index/10); % convert to scaler
    j=find(snr<snr_index_scaler); % find entries below SNR level
    prob_outage = size(j,2)/size(snr,2); % convert to percentage

    if (prob_outage >= outage_threshold) % if threshold reached
        snr_index_1_antenna = snr_index; % save SNR level
        found = 1; % set found flag
    end
    snr_index = snr_index + snr_step; % increment SNR level
end

% -----
% processing 2 antenna element
% -----
nb_antenna = 2; % set number antenna to 2

% get SNR samples
% -----

```

```

for sample_index = 1:M
    snr(sample_index) =
RrayT_mrc(N,n_tx,nb_antenna,antenna_spacing_index,dfi,fi0,dist,sigma,cosn_array);
end

snr_index = snr_lowerbound;          % set to lowerbound
found = 0;                            % set flag to not found
while ( (snr_index < snr_upperbound) & (found == 0) )
    snr_index_scaler = 10.^(snr_index/10); % convert to scaler
    j=find(snr<snr_index_scaler);        % find entries below SNR level
    prob_outage = size(j,2)/size(snr,2); % convert to percentage

    if (prob_outage >= outage_threshold) % if threshold reached
        snr_index_2_antenna = snr_index; % save SNR level
        found = 1;                     % set found flag
    end
    snr_index = snr_index + snr_step;    % increment SNR level
end

% print SNR and diversity gain to file
% -----
fprintf(fid,'%f, %f, %f, %f\n',antenna_spacing_index, snr_index_1_antenna,
snr_index_2_antenna, (snr_index_2_antenna - snr_index_1_antenna));

% close file
% -----
status = fclose(fid);

```

RRAYT_MRC.M

```

function R = RrayT_mrc(N,n_tx,n_rx,d,dfi,fi0,dist,sigma,cosn_array)

% -----
% This function returns the resulting SNR (in DB) of an n_rx-branch receive
% antenna array using maximal ratio combining from a n_tx-branch
% transmit antenna array
% -----
% function name: RrayT_mrc
%
% output:
% -----
% snr = resulting signal-to-noise ratio of maximal ratio combining
%
% input:
% -----
% N          - the number of rays
% n_rx       - the number of receive antennas
% n_tx       - the number of transmit antennas
% d          - element spacing of rx antenna
% dfi        - angular spread
% fi0        - mean angle of arrival
% dist       - distribution
% sigma      - sigma (std deviation)
% cosn_array - cosine n distribution array
%
% variables:
% -----
% fi         - angle-of-arrival (AOA)
% ksi        - phase shift between two adjacent elements
% a          - Tx complex gains
% noise_power - noise power
%
% -----

format compact

uniform_max_angle = dfi * sqrt(3); % max angle of uniform dist

```

```

noise_power = 1; % noise power
transmit_power = 1; % transmit power

for j = 1:n_tx % for each tx antenna

    % -----
    % uniform AOA distribution
    % -----
    if (dist == 0)
        fi = rand(N,1)*2*uniform_max_angle-uniform_max_angle+fi0;

    % -----
    % gaussian AOA distribution
    % -----
    elseif (dist == 1)
        fi = normrnd(fi0,sigma,N,1);

    % -----
    % laplacian AOA distribution
    % -----
    elseif (dist == 2)
        fi = laplace_noise(N,1,sigma)+fi0;

    % -----
    % cosine AOA distribution
    % -----
    elseif (dist == 3)
        fi = cosn_dist_gen(cosn_array,N)+fi0;

    % -----
    % circular scatterer AOA distribution
    % -----
    elseif (dist == 4)
        fi = circular_scatterers(sigma,1,N)+fi0;

    % -----
    % two impulses AOA distribution
    % -----
    elseif (dist == 5)
        fi = two_impulses(sigma,1,N)+fi0;

    end

    ksi = 2*pi*d*sin(fi); % phase shift
    a = 1/sqrt(2)*(randn(N,1) + 1i*randn(N,1)); % amplitude for signal
    for i = 1:n_rx % for each rx antenna
        h(i,j) = trace(diag(a.*exp(-1i*i*ksi))); % generate channel matrix
    end
end

y = (h'*h)/N; % normalize power
snr = transmit_power * abs(y)/noise_power; % get SNR
R = snr; % return SNR

```