CALCULATOR USAGE IN LEARNING, RETENTION, AND ATTITUDES IN MATHEMATICS.

by David J. Hunter

Thesis submitted to the School of Graduate Studies of the University of Ottawa as partial fulfillment of the requirements for the degree of Master of Arts in Education.

Ottawa, Ontario, 1977

ACKNOWLEDGMENTS

This thesis was prepared under the supervision of Professor Jean-Paul Dionne, M.A., of the Faculty of Education of the University of Ottawa.

The writer is indebted to Professor Marvin Boss, Ph.D., of the Faculty of Education of the University of Ottawa, for his continued interest and valuable help. Special thanks is also extended to Miss Jean Starrs and Mr. Robert Paterson, both of Canterbury High School in Ottawa, for their cooperation and advice in the conducting of this experiment, and to Mr. Alan Wotherspoon, principal of Canterbury High School, for his cooperation in conducting this study.

CURRICULUM STUDIORUM

David J. Hunter was born October 22, 1936 in Montreal, Canada. He received his Bachelor of Arts in Business Administration from the University of Western Ontario in 1959.

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Calculator Usage in Learning, Retention, and Attitudes in Mathematics

The availability of small electronic calculators raises the question of how they should be used in the classroom in order to improve learning in mathematics. For the present study, a scientific calculator was used for approximately five weeks with senior high school students studying logarithms and their applications.

The learning variable most directly affected when a calculator is used to study mathematics is practice. Ausubel describes practice as a situational variable influenced both by frequency and method. A calculator can affect both the frequency and method of practice.

In each of three regular classes, students were matched according to their recent performance in mathematics. One member from each pair was randomly assigned to an experimental group which used calculators, and the other assigned to a control group. One experimental and one control group were taught by each of three teachers. A fourth class remained undivided, and was taught by one of the above teachers.

When tested on a teacher designed test at the completion of the unit, the matched control groups demonstrated the best performance of all the groups. It was concluded that the calculators may have created more difficulty in learning, may

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have interfered with the material taught in the first lessons, and that the final test may have discriminated against the students using calculators. All groups performed at approximately the same level on an achievement test held five weeks subsequent to the completion of the unit of work.

A semantic differential test indicated no significant differences in attitudes towards logarithms between any of the groups at the completion of the unit.

Further research in this area should utilize calculators over a longer period of time, and the processes and concepts learned with the calculator should be incorporated in the assessment of achievement. Shorter units of work should be used to isolate precisely where the advantages of the calculator can be exploited.

INTRODUCTION

It seems that virtually every high school student will own a small electronic calculator in the near future. Since it is a relatively recent invention, there has been much speculation, on the one hand extolling its virtues and on the other hand condemning it because of the detrimental effects it may have on the student's numerical skills.

The essential problem for the educator is to know what change occurs in the learning situation when a calculator is used on a regular basis. In this research, the use of a scientific calculator within a single unit of work in grade 12 mathematics, namely logarithms, was examined. How the calculator should be used in the classroom is a problem that needs much experimentation. To this date, very few studies have been reported dealing specifically with hand calculator usage. In this experiment, the use of the calculator was emphasized as a learning device, and not merely as a tool for numerical calculation.

It was hypothesized that achievement should improve as a result of a change in the practice structure created when a calculator is introduced on a regular daily basis, and that a positive change in attitude towards the topics covered would be evident in those students using the calculators. A comparison of the resultant learning and retention was made between groups using the calculator and groups not using it.

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Attitudes towards logarithms were also compared at the end of the unit of work.

Theoretical considerations with respect to practice and attitudes towards mathematics are discussed in Chapter 1. This is followed by a review of some studies relating to practice and to the use of calculators in classrooms. Three research hypotheses concerning learning, retention, and attitudes are enunciated as a result of the problems raised.

In Chapter II, the experimental design is presented together with the specific manner in which the calculators were used. Following this is a description of the measuring instruments used to evaluate learning and retention of the concepts dealing with logarithms as well as to evaluate attitudes towards logarithms.

The results of the test on logarithmic concepts and the test on retention of these concepts are examined in a final chapter. Included is a discussion of the results of the attitude tests. This is followed by a general summary along with suggestions for future study.

CHAPTER 1

REVIEW OF THE LITERATURE

This chapter begins with a brief discussion of the consideration given to practice as a learning variable by the behaviorist psychologists. The recent emphasis on the internal processing of information has led to a more complete analysis of practice. Ausubel (1968, pp. 26-27) has classified learning variables into situational and intrapersonal variables and identified practice as a major situational variable. Ausubel sees the effectiveness of practice as being determined by two independent variables, namely its frequency and method. These two independent variables are considered in some detail in conjunction with the learner and the material to be learned. (Ausubel, 1968, chap. 8)

It will be argued that a change in the practice variable is expected to produce a corresponding change in achievement and attitudes towards the topic taught. From the relationship between these variables, the research problems are identified, and the research hypotheses stated.

Historical Perspective

Historically, practice has been considered a relevant factor when examining various types of learning. Whether it be conditioning, chaining, numerical and verbal skill formation, problem solving or concept formation, they all involve

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practice. Indeed, one may speak of a doctor 'practicing medicine', or a lawyer 'practicing law'. One might say that the entire life is spent practicing one skill or another. The end result of practice may be a permanent change in the cogni tive structure so that behaviour becomes permanently modified. Teachers consider practice an integral part of school learning and use it as a device for establishing learning and retention of facts and concepts.

Although it is difficult and somewhat misleading to classify psychological theories, the behaviorists tended to concentrate on the external processes, the observable changes in behavior that could be altered by changing stimuli. They could not ignore the existence of some internal processes, although these were not directly observable.

In reviewing learning theories, Kimble (1975) quotes C.L. Hull as saying that "habit strength" (p. 755) was developed as a function of practice and that the actual response of the learner, rather than the internal perception of the stimulus, participated in the habit formation. Lefrancois (1972, p. 88) reports that Thorndike recognized that the physiological bonds between stimuli and responses would be strengthened through being exercised frequently. Lefrancois (p. 312) notes that B.F. Skinner has shown that practice permits continuous reinforcement which facilitates learning in its initial stages, although intermittent reinforcement increases retention. Lefrancois (p. 314) gives further reference to D.O. Hebb who hypothesized an internal process of neural activity with such activity being facilitated with practice. Thus the importance of practice was recognized in the early theories of learning.

These theories leave some unanswered questions when examining the complex problems of learning school subjects. Essentially most school learning consists of the development of basic linguistic and numerical skills, after which concepts are developed and built upon in a hierarchical manner within each subject field. School learning to a large extent is concerned with the internal processing and verbalization of information and relies on repetition as a standard procedure to establish learning and retention. This suggests that the cognitivist approach to learning is more relevant to the learning of mathematics. According to Lefrancois (1972),

> For example, behaviorism deals largely with investigations of stimuli and responses as well as mediation (neobehaviorism). On the other hand, topics of interest to cognitive psychologists typically do not include stimuli and responses per se, but deal instead with more central processes such as problem solving, decision making, perception, information processing and concept formation. (p. 310)

Although many psychologists have incorporated discussions of practice into their theories, very few have written extensively on practice as a learning variable. An

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exhaustive review of the studies relevant to practice has been made by Ausubel (1968), and he has related practice to the problems of learning in the schools. Ausubel (1968, p. 273) refers to practice as the repetition of some task, and isolates practice as an important variable in learning.

Practice as a Learning Variable

In mathematics, the learning process involves verbal, numerical, and symbolical assimilation. Memory forms an integral part of this process. Ausubel and Robinson (1969) observe that mathematics more than any other school subject, "requires that the learner understand long, sequentially related, and hierarchical organized systems of propositions" (p. 86). The key word is 'understand'. Failure to comprehend one step in the process means that the students would be forced into a rote learning set, usually with undesirable results. Ausubel and Robinson (pp. 86-96) suggest that many mathematical tasks must be over-learned so that they become automatic. Otherwise the sequential connections become too long and complicated and thus interfere with future learning. They emphasize that the material to be learned must be potentially meaningful to the learner, otherwise it would be classified as rote learning. By meaningful learning, Ausubel (1968, pp. 45-56) refers to the ability of the learner to relate the material to his existing experiences or cognitive

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structure. On the other hand, meaningful material refers to material that has some internal logical structure so that it is potentially meaningful.

The first mathematical concepts are learned through generalization from concrete examples and manipulation of objects. However more abstract concepts are conveyed verbally and symbolically. Ausubel and Robinson (1969, p. 92) suggest that learning mathematical symbols represents the same sort of problem as learning a second language. The symbol is initially translated back to simple language, similar to the native tongue, but after much practice this mediational role is no longer necessary. The key idea here is that practice is one of the essential elements in permitting the mediational role to occur. One must conclude that some internal change in the cognitive structure occurs as a result of continued practice with new concepts or ideas that are meaningful to the learner. Once it is recognized that practice is a necessary condition for most mathematical learning, the next step is to examine the variables that contribute to successful practice.

Ausubel (1968, pp. 26-28) classifies learning variables into two broad classes: intrapersonal and situational. The intrapersonal variables include those variables that are learner centered such as age, sex, intelligence, personality, background knowledge, and motivational qualities of the learner. These are essentially internal variables. On the

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other hand, the situational variables refer to the external factors that affect learning in a particular situation such as the teacher, group influences, the subject matter, and practice. Thus Ausubel sees practice as one of the situational variables in the learning process. The above suggests that practice interacts with all the other learning variables. Hence the individual problems of the learner must be taken into account when designing any practice schedule.

In order to analyse practice more completely, Ausubel and Robinson (1969, p. 278) divide practice into two main components:

1. frequency of practice, and
2. method and general conditions of practice.

They refer to these as the two independent variables of practice. These are now discussed in more detail.

Ausubel (1968) suggests that when distributing the available time, the length of the practice session should "conform to empirically established principles of efficient learning and retention" (p. 276). The total time spent in practice and the distribution of that time will determine the efficiency of practice. This must be determined empirically by the teacher. Sufficient practice time must be spent at each stage of the process to establish meaningful learning. A single exposure to many ideas is usually not sufficient to establish a permanent change in the cognitive structure.

The method of practice will be determined by the task to be learned. One does not learn how to play the piano by reading a book. In discussing the type of practice for meaningful learning, Ausubel and Robinson (1969, pp. 289-302) examine the following factors: the type of response made by the learner, the construction or selection of the response, the amount and type of guidance, and the teacher-student characteristics.

The authors state that an overt response makes a more explicit testing of knowledge possible which in turn enhances the motivational effect of feedback to the learner. As to whether the student should select a response or construct one, it is suggested that in the initial learning process, a reformulated response constitutes the more valid measure of understanding and tends to discourage the adoption of a rote learning set by the learner. A third consideration is whether practice occurs with guidance or whether it is undirected. Proper guidance can save much time in meaningful learning by ensuring a balance between an overlearning of particular tasks and a diversity of tasks. The component skills must be mastered before the final skill can be developed. The entire process must be logical and homogeneous in order to be sufficient. Fourthly, the kind of practice must take advantage of the method of presentation, the personality of the teacher, the strengths of the students, and in general should be structured in such a way as to give immediate feedback to the learner. The student should know when and why he is making mistakes as he progresses.

The previous considerations on the frequency and method of practice are dependent upon characteristics of the learner and the specific material to be learned. These are now examined.

Subject matter considerations are largely external in that they can be manipulated by the teacher who will determine the time required to learn and the type of practice required. These considerations include the level of difficulty of the material, the amount of material to be learned, and the degree of continuity with previously learned material.

Learner variables, on the other hand, are internal and usually cannot be altered by the teacher. The students possess certain characteristics that were formed prior to arriving in class, and the teacher must attempt to tailor the practice sessions to the needs of the students. The learner variables that will influence the kind and amount of practice include the age of the students, the intelligence, the stage of emotional and social development, and the sophistication of the subject matter background mastered by the students.

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The major independent variables of practice were considered above. The resultant dependent variables are short term or initial learning, long term retention, and transfer to new learning situations.

Practice, as a situational variable in learning, interacts with the other internal and external variables in the learning process and will have different effects on different people at different times in various situations. Practice plays a vital role in the learning process and is a difficult variable to manipulate because of its many components and possible confounding effects (Ausubel and Robinson, 1969, pp. 274-279).

How does practice influence learning? Ausubel (1968, chap. 8) believes that as a result of practice there are various quantitative and qualitative changes that take place in the cognitive structure. These changes would not occur without practice. He believes that practice can influence the cognitive structure by increasing the ability to transfer learning from a current situation to a new situation. Similarly, practice enhances the learner's responsiveness to subsequent presentations of the same material. In order that practice results in mastery of material, the learning task must be potentially meaningful, the learner must exhibit a meaningful learning set by having mastery of the relevant background concepts, and the practice must be structured in a logical way.

When a calculator is used in the classroom, the frequency and method of practice are altered. This process is examined in the following paragraphs.

Practice With a Calculator

When a scientific calculator is introduced into the mathematics classroom, the student has access to a device that will affect his mode of practice. An examination of the independent variables of practice as outlined by Ausubel (1968) and Ausubel and Robinson (1969), suggests that a calculator, if used properly, should aid in the learning and retention of the concepts taught.

Firstly, the calculator should permit more repetition in certain types of problems within the given class time, and permit more rapid overlearning and reinforcement of background material. Ausubel (1968, p. 289) states that overlearning strengthens the stability and clarity of concepts. This in turn should facilitate the learning of sequentially related material.

Secondly, Ausubel (1968, p. 316) states that the practice that provides immediate feedback in the learning task, should act as a motivating factor providing confirmation, correction, clarification, and evaluation of the adequacy of learning.

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The calculator provides immediate feedback to the learner for computational steps in the problem solving process. The specific calculator used in this study also provides immediate information through some functions built into the memory of the calculator. This is one main advantage that the scientific calculators have over the simpler arithmetic calculators.

Thirdly, Ausubel states that "other factors being equal, the defining attributes of a given concept are learned most readily when the concept if encountered in many diverse contexts" (p. 312). The calculator permits this diverse experience by allowing variations in solutions, and procedures for solving problems that would be impossible or too time consuming without the calculator.

Fourthly, Ausubel states that the "multi-contextual learning... prevents boredom and enhances the exploratory drive" (p. 313). This provides a positive motivating factor.

If in fact all the above factors are at work, then the appropriate learning set should be established. Ausubel (p. 314) suggests than an appropriate learning set can lead to positive transfer by giving the student a general sophistication in approaching a given learning task or attacking a particular type of problem, and by giving the student an appropriate attitude for engaging in a particular kind of activity.

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Although Ausubel has not written extensively on attitudes, he suggests than when attitudes are positive, subjects are highly motivated to learn and will put forth more concentrated effort. Negative attitudes can lead to a close-minded approach that impairs ability to learn new ideas that are contrary to existing beliefs (Ausubel, 1968, pp. 388-93). Thus, motivational effects may influence initial learning either positively or negatively.

The above ideas, advanced by Ausubel, are based on the assumption that practice alters human learning and retention by modifying the cognitive structure (Ausubel, 1968, p. 273). His ideas relate to what he refers to as meaningful learning, and his analysis of practice is made with this in mind. In the present context the mathematics involved in studying logarithms is an example of meaningful material in that it has an internal logical structure and is based on previous knowledge of exponents. This structure is based on concepts that vary in degree of difficulty and complexity and are expressed and interpreted both symbolically and verbally.

The conclusion reached to date with respect to attitudes towards mathematics will be examined in the following section.

Attitudes Towards Mathematics

Many studies on attitudes towards mathematics have been reported in the literature. Since attitudes are influence by many different external factors, it is difficult to isolate any single factor as being of primary importance. Aiken (1970, 1976) has summarized the results of many recent studies on attitudes towards mathematics, and has clarified the problems involved in attempting to measure attitudes. Since this experiment deals with learning mathematics, Aiken is used as the principal reference.

Aiken (1970, p. 551) has defined attitudes as a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept or person. Attitudes are considered to be partly cognitive and partly emotional, and thus can change as the person matures. It is generally agreed that attitudes are learned and that attitudes affect learning. Aiken (pp. 555-6) has found that attitudes towards mathematics could be assessed as early as the third grade, and that attitudes may change over time. Part of the reason for the observed change may be attributable to the precision with which students can express themselves as they mature (Aiken, p. 558).

Aiken (p. 556) suggests that computational errors account for one of the reasons frequently given for disliking arithmetic in Junior High School. If the use of a calculator decreases errors of this sort, then one of the contributing factors to a negative attitude towards mathematics has been reduced.

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It has been suggested that attitude might play a different role at different levels of achievement. Aiken (p. 562) notes that mid-range attitude scores have little relation to achievement. It is only at the extremes that attitude correlates to achievement in any notable way.

Aiken (p. 589) reports that the one consistent conclusion with respect to attitudes is that the teacher rather than the curriculum, or the classroom organization, seems to be the most influential variable on attitude. It has been demonstrated that positive attitudes can be fostered by reducing rote learning, increasing the emphasis on relevance and meaningfulness, and by using activities and games that permit active student participation (Aiken, p. 586).

Using a calculator should contribute to the development of positive attitudes because it minimizes the possibility of incorrect answers in straight computation problems, it demonstrates the relevance of a device that they will be able to use outside the classroom, and it permits an activity different from the normal routine in the mathematics classroom.

There is also the possibility that the calculator may be seen as an intrusion upon the learning process. Some people may have built-in biasses against mechanical devices. There is a dange that some students may treat it as a toy rather than as a serious learning device with the result that students may develop negative attitudes towards mathematics.

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There are no guidelines available at this time to indicate the most efficient ways to utilize the calculator in the classroom, and studies reported later in this chapter indicate no conclusive evidence that attitudes are in fact altered.

In summary, the role of attitudes in learning is not as clear as one might expect. Attitudes do differ, and in most cases, positive attitudes correlate with good performance in mathematics, although the correlations are rather low. Further research is necessary before any firm conclusions can be made with respect to the impact of the calculator on attitudes towards mathematics.

In the remainder of this chapter, the author examines some relevant studies on practice and on the use of the cal culator in the mathematics classroom.

Studies on Practice

There are many studies on practice reported in the literature. The main difficulty is that very few deal with meaningful learning in the school context. In spite of this, one can argue that the practice variables examined in a non-school learning situation will bear the same relationship to learning in a school setting. The following studies are reported to demonstrate the type of research that has been done relating practice to learning.

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Morgan and Alluisi (1971) report that practice at breaking codes improved performance in this problem solving skill. The total number of trials was a relevant factor in performance, and the time required to break the codes reached an asymptotic level after a finite number of trials. Their study also indicated that some internal mediation occurred and that some change in the cognitive structure was achieved through practice.

Morrisett and Hovland (1959) studied the effects of three different kinds of practice in problem solving with twelfth grade students. A problem consisted of 4 different spatial arrangements of a pair of geometrical objects, each arrangement shown one at a time. The particular horizontal or vertical arrangements for any given pair was to be identified through the use of a predetermined code which was maintained throughout all problems, regardless of the stimulus pair used. Group I was given training on 48 presentations of the same problem; Group II was given training on 2 presentations of 24 different problems; and Group III was given training on 16 presentations of 3 different problems. The subjects received 24 test trials on 3 different problems subsequent to the above training.

The performance of Group III on the test was significantly superior to Group I, which in turn was significantly superior to Group II. The authors argued that Group I did not learn to generalize beyond the one problem, and Group II was unable to consolidate learning because too many different problems were presented. The study demonstrates that the frequency of the practice trials is an important factor in developing mastery, which in turn permits the formation of the appropriate learning set for the transfer of skills to related problem solving situations. It also suggests that a sufficient variety of tasks is required to permit the formation of the correct learning set. These ideas are in agreement with Ausubel's ideas that frequency and variety of practice are important factors in establishing an appropriate learning set to effect transfer (Ausubel, 1968, chap. 8).

Brosgoyle, Hanley, and Angelo (1973) have given some support for the ideas that when considering recall, the frequency of trials, for 'paired associates-verbal learning', is the predominant practice variable. College students were presented with lists of 8 pairs of verbal associates consisting of consonant-vowel-consonant trigrams. One member of the pair was the stimulus item and the other member the response item. They conducted 3 studies, in 2 of which, the inter-stimulus interval was kept constant. In their first study, the subjects who received a series of 3 exposures of 1 second duration for each pair, proved to have superior recall on response items over the subjects who received a single exposure of 1 second, or subjects who received a single exposure of 3 seconds duration. The longer duration of the single exposure did not prove to be beneficial compared to the short single exposure.

In their second study, a series of 3 exposures of 1 second duration, or a series of 3 exposures of 3 seconds duration proved to give superior performance to a single exposure of 3 seconds duration. The longer exposure time in the 3 exposures of 3 seconds each did not prove to be significantly superior when compared to 3 exposures of 1 second each.

In their third study, the inter-stimulus interval of their first study was adjusted so that all three groups had the same length of time for information processing. The group with the 3 exposures of 1 second duration still had significantly greater retention in spite of the equal inter-stimulus interval. This study demonstrates that frequency emerged as being predominant compared to duration in associative learning. The conclusions were based on performance observed on only one presentation of the stimulus item, and thus may not apply to more complex learning situations where practice continues until mastery. However the results support Ausubel's claim that frequency of practice is a component of learning, and that increased frequency may facilitate retention.

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Gagné, Mayor, Garstens, and Paradise (1962) have given further support to Ausubel's contention that an adequate learning set must be established in order that practice results in effective learning and transfer. Two grade 7 classes were taught a unit of work on addition of integers in which a hierarchy of 12 subordinate learning sets was established to facilitate performance on 2 final tasks, one of which was an application of addition of integers, and the other a more theoretical application of the concepts learned.

Four separate programs were developed in which repetition and guidance were varied from low to high amounts. Their results indicated that success in the final tasks were highly correlated with the number of subordinate learning sets acquired, and success at higher learning sets was dependent upon mastery of lower learning sets that were relevant to it. For the theoretically oriented final task, the combination of high guidance and high repetition produced superior performance to those subjects receiving low guidance and low repetition. However, for the other task, which required the mastery of only 3 subordinate learning tasks, no significant difference was observed.

Ausubel (1965) demonstrated that spaced repetition of meaningful material aids retention. Undergraduate college students were randomly assigned to experimental and control groups. The experimental group was given 25 minutes to read and study an unfamiliar learning passage. The control group was given 25 minutes to read and study a different passage. Two days later, both groups were given 25 minutes to read the original familiar passage. After a 48 hour interval, both groups were tested with a multiple choice test. Thus the time of last exposure was kept constant, but the experimental group had been given two exposures, spaced by two days. The experimental group demonstrated superior performance.

Ausubel concludes that spaced repetition directly influenced meaningful learning by permitting the learner to relate potential meanings that he may have missed on the first trial, and secondly, by providing informational feedback to test the correctness of the knowledge acquired on the first trial. Indirectly, the spaced repetition permits actual meanings to emerge on the second trial that otherwise would have remained as potential meanings. Secondly, as a result of prior experiences of trying to remember the material, the learner becomes aware of those elements that promote forgetting. He can guard against this in the second reading.

Gay (1972) has found that each person had an optimum level of practice for learning specific tasks. Three different practice schedules were established for a programmed unit of grade eight mathematics on polynomials. The first group operated under a retention index whereby the individual student was given the optimum number of examples necessary for him to learn and retain concepts. This retention index was determined empirically from a previous unit of work in which the subjects could select from 1, 2, 5, 10, or 15 examples. The optimum number of examples arrived at by all subjects was 5 or 10. The subjects of the second group were allowed to choose as many examples as they felt necessary in order to master each concept, up to a maximum number of 15. The third group was given three examples per concept. Immediate retention was judged at the termination of each of 3 days of instruction. Delayed retention was based on a test held one week subsequent to the termination of the experiment.

It was hypothesized that the retention index group would perform better on measures of both immediate and delayed retention than either of the other groups. This proved to be true for females, but not males. The choice method proved to be better for males on both immediate and delayed retention. The results suggested an interaction between the sex and treatment variables, although cell sizes were too small to draw conclusive results.

It was observed that the group having freedom to choose the number of examples required to learn a task did in fact use different numbers of examples to master those tasks. It is interesting to note that the average number of examples selected by this group was approximately 3, which was equivalent to the number of examples in the group having a fixed number of examples per concept. However, the former group adapted the number of examples to their needs. This suggests that retention may be impaired when an inadequate number of trials or examples are presented to the learner. Gay reports Ausubel (1968) as similarly concluding that if adequate attention was paid to optimal review, students might retain over a lifetime most of the important ideas learned in school.

In summary, it should be stated that the above studies are all limited in that none examines mathematical learning at a senior level, nor do they explicitely examine practice on a scientific calculator. However, they do give support to Ausubel's theory that practice is a variable that cannot be considered in isolation. The efficiency of practice will also be influenced by other variables such as the amount of guidance given the learner, the time required to master the steps in the task, the level of mastery attained, the total number of trials, the nature of the learning task, the availability of appropriate learning sets, the age, intelligence, and sex of the learner.

Since in this research, it is proposed to study practice on the calculator, some recent studies on calculator usage will now be examined.

Studies Related to the Calculator

Cech (1972) worked with 4 grade nine classes consisting of about 20 students per class. The IQ range was 75 to 95. Two teachers each worked with one experimental and one control group over a period of 7 weeks for 45 minutes each day, studying operations on whole numbers. The experimental group, used the calculators to verify paper and pencil computations. The control group was merely told to verify its work manually. All groups were given pretests and post-tests measuring computational skills with whole numbers and attitudes towards mathematics.

Attitudes between experimental and control groups were not significantly different prior to, or subsequent to the treatment. Attitudes were measured on the PY011 Pro-Math Composite Test developed by the School Mathematics Study Group for its longitudinal study. Similarly there was no significant difference in computational skills between the groups when tested at the end without using the calculators. The test used was the Stanford Diagnostic Arithmetic Test 2, parts A, B, and C. When the experimental groups were retested on an alternate form of the test with the calculators, they did score significantly better on computational skills than they had without the calculator.

No attempt was specifically made to use the calculator to build insight into the understanding of mathematical principles. The results suggest that although the calculator used in this fashion may speed up the checking of answers, no apparent improvement in numerical skills is attained when tested without the calculator. Moreover, the above IQ range is hardly representative of a standard class.

Although attitudes were not affected by the use of the calculator, this may be due to the manner in which the calculators were used. The calculator had a very limited role, being used exclusively to check answers. Also, attitudes towards mathematics may be too general a concept to expect it to be altered as a result of using calculators.

Gaslin (1975) worked with 3 grade nine classes at each of 2 schools. The students had been randomly assigned to classes and 3 different treatments were designed for teaching the operations on positive rational numbers. The first group worked with a Conventional Algorithm Set, finding common denominators without the use of calculators. The second group worked with an Alternative Algorithm Set where each fraction was first converted to a decimal on the calculator and then the operations were performed on the calculators. The third group worked with a Calculator Algorithm Set and performed operations in the usual manner by finding common denominators, but with the aid of the calculators.

He was particularly interested in measuring the effect of the calculators on achievement, and change in attitudes

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towards mathematics as a result of being exposed to the calculators. The results of his experiment suggest that the calculator based group using the Alternative Algorithm Set produced success in transfer situations requiring estimating, ordering, and simplifying expressions with more than two operands. However, use of the calculators did not affect development of computational skills with rational numbers.

Retention of skill for performing operations on positive rational numbers was superior for the group using the Calculator Algorithm Set. The fact that the results were not consistent between schools suggests that a teacher effect may have been present. Attitudes were measured on a semantic differential test. There was no significant difference in attitudes between the groups within each school or between schools. Attitudes towards mathematics may be too general a concept to be altered by the use of a calculator over a short period of time.

Advani (1972) worked with a small group of 18 children ages 12 to 15. These students had a history of learning and behavior problems. Four calculators were placed in the classroom and students were encouraged to check answers. The machines were also used as a tool for enrichment and rein forcement on anew unit of work. A comparison of pretest and posttest scores on the Stanford Achievement test showed a significant improvement, and an analysis of questionnaire data indicated an increase in student interest.

However, the change in performance cannot be attributed to the calculator because no control group was used. Advani felt that the attitude change could be attributed to the use of the calculators, as they did help in individualizing instruction and provided variety in the classroom. Some behavior problems were eliminated. The results suggest that calculators may play a very useful role for this type of student.

Schnur and Lang (1976) worked with a group of 60 summer school students needing remedial work. The students were divided into experimental and control groups while keeping a balance between sex and ethnic background. The purpose of the study was to investigate whether elementary school students would achieve greater mathematical computational ability through controlled use of the calculator, and whether such increased ability would transfer to a posttest situation where the calculator was not used. Secondly, they wanted to test whether there would be any significant interaction between sex and calculator usage. Thirdly, they tested for significant interaction between factors of ethnic background with calculator usage.

All students were given uniform instruction on the 4 basic operations with whole numbers over a period of 4 weeks. In the experimental group, calculators were used for 50 minutes each week for verifying correctness of written answers for assigned problems, and on every third set of problems, the students used the calculators exclusively. Use of calculators was not permitted on the tests.

The results indicated that the experimental group was superior to the control group on the posttest. There was no interaction between sex and the use of calculators, nor between ethnic or economic background and use of the calculator. The study indicates that for a remedial individualized mathematics program, the calculator can yield a significant achievement ability growth, that transfers into a non-calculator test situation. The question remains whether the same result will hold in a non-remedial situation and in a non-individualized program.

Schafer, Bell and Crown (1975) have indicated that students using calculators for a short unit of work will perform better on calculation type problems than students not using calculators. This is to be expected since mechanical errors will be minimized with the calculators. Since their experiment was for a very short period of time, it is not possible to draw any conclusive results.

Scandura, Lowerre, Veneski, and Scandura (1976) have done some preliminary work with calculators in the primary classroom. Their observations are subjective but consistent with the observations of other teachers. In their study with 6 year olds in a unit of computational work, the group with calculators showed high enthusiasm. The easily distracted students seemed to benefit by having something to do with their hands. Their concentration span seemed longer than that of students not using the calculators. The students worked independently and were anxious to make up new problems. The absence of paper and pencil helped speed up the activities.

Although the above study concerned younger children, it suggests that it is relevant to examine the change of attitudes with respect to the calculator with older children.

The above studies indicate the direction of current research in the use of the calculator. Most of these studies are exploratory in nature and with one exception, the calculator was used to eliminate tedious computational problems. With respect to the research discussed in the work that follows, a student using a calculator should be able to complete more computational type problems in a given unit of time. Secondly, a student using the calculator should be able to perform more complex computational problems than students without the calculators. The students with the calculators will have the benefit of more practice trials in a given period of time, and a greater variety of problems.

The more complex scientific calculators on the market today provide a wide variety of functions including the trigonometric, exponential, and logarithmic. In this research, the purpose is to exploit the use of the logarithmic and exponential functions of the calculator in addition to its basic arithmetical capabilities, as an aid in teaching the topic of logarithms.

Summary and Basic Hypotheses

Ausubel (1968, chap. 8) has focused attention on variables that determine whether or not practice will lead to effective learning and retention. The writer suggests that when a scientific calculator is used, the method of practice is altered. Although Ausubel does not write extensively on attitudes, he does recognize their importance as a variable related to learning. Aiken (1970) has given support to this idea and although research results on attitudes are inconclusive, Ausubel and Robinson (1969, p. 369) seem to agree that attitudes play an important role in learning. Current studies on the use of the calculator in the classroom are limited in number, and results are not yet conclusive.

In the research presented in this paper, the use of the scientific calculator in grade twelve mathematics classrooms over a period of 5 weeks of regular use is investigated. The topic to be covered is logarithms, a topic requiring theoretical understanding and a considerable amount of computation.

REVIEW OF THE LITERATURE

The various questions raised in the literature suggest at least three problems for research.

1. Will the calculator enhance student understanding of the concepts involved in mathematics?
2. Will the calculator have a positive effect on attitudes towards logarithms?
3. Will the use of the calculator have any effect on retention of the concepts learned?

In order to investigate these problems, the following research hypotheses are formulated.

H_1: When tested without the calculator, students who used the calculator throughout the unit of work on logarithms will have greater understanding of the concepts learned on logarithms than students who have not had practice on the calculator.

H_2: Students using the calculators will exhibit a more positive attitude towards logarithms than students not using the calculators, when tested at the completion of the unit of work.

H_3: Students who have practiced with the calculator will have greater retention of the concepts learned on logarithms than will students who did not practice on the calculator, when tested five weeks after the completion of the unit of work.

CHAPTER II

EXPERIMENTAL DESIGN

In this chapter, the method by which the research subjects were selected, organized into experimental and control groups, and assigned to teachers, is discussed. Three distinct groups of research subjects were isolated. The lesson plans are discussed along with the details of how the calculators were incorporated into these lessons. The chapter includes a description of the four measuring instruments for achievement and attitudes towards logarithms. Since no standardized test on attitudes towards logarithms was available, a test was constructed using the semantic differential technique. The chapter is concluded with a description of how the statistical results were to be analysed.

Research Subjects

The experiment was conducted in a composite high school from a middle class section of a large urban Ontario city. Two phases of mathematics courses are offered at the grade 12 level: General Mathematics, and Advanced Mathematics. The latter course is designed for students planning to take some mathematics in grade 13. The 4 advanced classes in grade 12 were timetabled on the basis of student options so that no intended streaming occurred. All 4 classes were selected for use in this study.

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The mathematics classes were taught by 2 teachers. Periods were of 50 minutes duration, and rotated so that classes met at different times each day, 3 out of 4 days. Three of the classes always met in the afternoon and one class met in the morning. This meant that although the morning class was taught prior to the other classes each day, the rotation ensured that they were not necessarily taught any particular lesson before the afternoon classes.

The students in each of the three afternoon classes were matched on the basis of their December examination scores in mathematics and randomly assigned to one of 2 groups. The December examination scores are included in Appendix 8. The researcher and the 2 regular teachers each taught one experimental group and one control group. The single morning class was not split, and was considered as an external control group. The allocation of students and teachers to classes is shown in Table 1.

From the above division of subjects, three distinct groups of subjects were formed:

(a) those assigned to the experimental groups from the three afternoon classes, to be denoted by EG,

(b) those assigned to the matched control groups from the three afternoon classes, to be denoted by CG, and

(c) those students in the morning class which was not split, and was used as a single class control group, denoted by SCCG.

Table 1

The Numbers of Students Assigned to Teachers in the Experimental and Control groups by Sex, and Class

Class[a]	Experimental Groups				Control Groups			
	Boys	Girls	Total	Teacher	Boys	Girls	Total	Teacher
1	6	9	15	B	10	4	14	A
2	7	7	14	A	4	9	13	C
3	7	6	13	C	4	8	12	B
4					14	10	24	B

[a]Classes 1, 2, and 3 were the split classes.

An analysis of variance comparing the mean scores of the above 4 classes on the December examinations indicated no significant differences between the groups. These results confirmed that the classes were not streamed. The results are summarized in Table 2.

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Table 2

Summary of Analysis of Variance Performed on the December Examination Results for Each of the 4 Mathematics Classes

Source of Variance	SS	df	MS	F
Treatment	634.95	3	211.65	.894
Error	23,676.81	100	236.77	

The Method of the Experiment

Each of the three teachers taught one experimental and one control group, with one of the teachers teaching the single class control group as well, this being his regular class. The experimentor insured daily coordination. Fifteen detailed lesson plans containing specific examples and instructions for both experimental and control groups were developed jointly by the three teachers. To reduce variation in teaching methods, each teacher was involved with an experimental and a control group. Common overhead transparencies of key examples were prepared to ensure uniform presentation of solutions. The lesson plans appear in Appendix 1.

In order to clarify the purpose of the experiment to the students and parents, a letter was sent home with each student prior to the experiment. A copy of the letter appears in Appendix 3.

The students in the EG were not permitted to take the calculators home and were required to write full solutions for problems involving the application of logarithms in their homework. Homework assignments were approximately equivalent for all groups. However, the EG were able to complete more problems during some classes because of the use of the calculators. Each student in the EG worked with an individual calculator in class. The model of the calculator used was the Texas Instruments Scientific Calculator SR-51. Students had received no instruction on the use of the calculator prior to the beginning of the experiment.

How the Calculator Was Used

The actual manner in which the calculator was used is briefly described below. Firstly, students were shown how to generate a table of values for various exponential functions of the form $y=a^x$, $a>0$. The calculator has this particular function built into it. This permitted the EG to generate more values than the CG or SCCG when graphing these functions in class.

Secondly, the EG were told to familiarize themselves with the function $y=10^x$, which was built into the machine.

This calculator generates twelve significant digits, whereas the tables in the text give only five. This provided an opportunity to demonstrate different numbers of significant digits. These students were also encouraged to use the regular arithmetic functions in the calculator whenever possible in class.

The calculator was also employed to generate the logarithm for any positive number. One limitation of the logarithm tables was illustrated, namely that the tables give logarithms only for numbers between 1 and 10, whereas the calculator will give the logarithm for any positive number. Moreover, these students could be exposed to negative logarithms, which was avoided by the CG and SCCG because such logarithms do not occur in the tables.

The fourth use for the calculator was in the application of the rules of logarithms in solving computational problems such as (.06249)X(.4715), or $\frac{36.5}{.00873}$, or $\sqrt[5]{.0756}$ using logarithms.

The fifth use of the calculator was in solving problems on compound interest and present value that normally would require the use of logarithms. The EG were encouraged to use the calculator to evaluate expressions such as $555(1.12)^6$ or $\frac{700}{(1.055)^{12}}$ by using the exponential function on the calculator, whereas the CG and SCCG were required to use logarithms to evaluate the same expressions. Again it must be emphasized that the EG were required to write out the solutions using the logarithm tables for their homework problems. The reader should consult the lesson plans in Appendix 1 for more specific details on how the calculator was used.

In summary, the students using the calculators were able to save time in class by avoiding the necessity to refer to the tables because these values were generated in the calculators. They could avoid computational errors, could generate more values in graphing exponential functions, and in the process could be exposed to more variety in their examples.

Calculators were not permitted on any tests. The final and posttest were designed to measure understanding of concepts and processes, so that the use of the calculator would not have proved helpful for any of the questions.

Measuring Instruments

The measuring instruments are discussed in the following order.

1. Final test on content.
2. Posttest on content.
3. Final test on attitude.
4. Posttest on attitude.

The final test on content consisted of 28 multiple choice questions on exponential functions, logarithmic functions, and logarithms, followed by two parallel questions designed to test the student's ability to set up a computational problem using logarithms. The test was comprised of 6 subtests as illustrated in Appendix 5(c). None of the questions required the use of logarithm tables or any lengthy arithmetic computations. A copy of the final test appears in Appendix 4(a). The questions were designed and selected to cover the content objectives as set out in the lesson plans. A table of specifications was used to aid in this process and appears in Appendix 2. Each concept indicated with an asterisk in the table of specifications was used in the construction of a test item.

The three teachers participated in organizing the objectives and the test items. In this manner it was hoped that content validity would be achieved. The final test was written by all subjects together with ample time to complete the test. The test on content was preceded by a test on attitudes which is discussed later in this chapter. The students indicated their responses on I.B.M. answer sheets and the tests were scored and analysed by computer. Each test item was given equal weighting for scoring purposes.

The posttest on content to measure long term retention, consisted of 18 multiple choice items, and was given to all subjects five weeks after the final test. This test was given during their regular mathematics class, was not previously announced, and the EG and CG all wrote within one afternoon. The SCCG wrote the following day. The students were informed that the scores obtained would not count for their term mark. The items were similar in nature to many of the items on the original final test, and covered the same concepts. A copy of the posttest appears in Appendix 5(a). The posttests were scored and analysed by computer in the same manner as the final test. This test was also divided into six subtests as illustrated in Appendix 5(c).

Since no test that measures attitudes towards logarithms was available, the writer designed a test for this purpose. The semantic differential technique was selected. When responding to the semantic differential, the subject is presented with a concept followed by a series of bipolar adjectives (scales), each of which is mainly representative of the evaluative, activity, or potency factors. The subject rates his feelings towards the concept by choosing a position along a seven point scale between two bipolar adjectives. Unfavourable poles receive a score of one, and favourable poles receive a score of seven. The scores on the evaluative scales provide an index for the location of the attitude towards a specific concept. It is usual to include a number of scales representing potency and activity in order to break the response set, obscure the purpose of measurement, and to provide additional information on the meaning of the concept as a whole.

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Lemon (1973, pp. 100-110) reports that while the majority of studies using the semantic differential have been with adults, the scales have been used with children and the results provide confirmation for the stability of the evaluation, potency, and activity framework across different age groups. In all cases, the evaluative factor was clearly differentiated. Test-retest correlations for the semantic differential have been high (greater than .85) based on total test scores. Split halves reliability for evaluative scales ranged from .70 to .76. He suggests that while the instrument appears to give valid measures of the extremity of opinion, its sensitivity to other less well understood components of attitude is more limited. He warns of two sources of bias, namely a response bias based on social desirability, and a bias based on extreme responses caused by polarization of issues. However, he concludes that it is a highly acceptable and much used measure of the evaluative connotations of attitude concepts.

McCallon and Brown (1971) have demonstrated a high correlation for college students (r =.87) between the evaluative scales on the semantic differential and the Mathematics Attitude Scale designed by Aiken and Dreger, which uses the Likert technique.

In conclusion, the consistency of the results in previous research, the age of the respondents in this research, the randomized assignment to experimental and control groups, and the nature of the concepts to be measured, justify the use of the semantic differential as an instrument with which to index attitude on a concept in mathematics.

The scales selected for use in each of the three factors were selected from those listed by Osgood et al. (1971, p. 37) and by Bentler and Lavoie (1972, p. 177). A total of 18 scales were selected, six from each factor, on the basis of their reported high factor loadings in the respective factors, and also for their apparent relevance to the concept of logarithms. The breakdown of the bipolar adjectives into the three factors appears in Appendix 6.

The students were asked to react to two concepts: Quadratic Equations, and Logarithms. Quadratic Equations was a topic taught earlier in the year, and it was included so that the same scales could be examined on more than one concept. A copy of the final test on attitudes appears in Appendix 4(b). The EG and CG were compared on the evaluative scales and a further comparison was made between these two groups and the SCCG.

A factor analysis of the pooled results indicated that some of the scales were not measuring the same factors as was originally thought, (BMD 08M - Factor Analysis, Oblique rotation). The factor loadings are shown in Appendix 7. Since only four of the evaluative scales were consistently measuring the same factor, it was decided to count only these scales in the tests when arriving at a score for the evaluative factor. These four evaluative scales were:

(a) Worthless _____ Valuable

(b) Agreeable _____ Disagreeable

(c) Boring _____ Interesting

(d) Pleasant _____ Unpleasant

In conjunction with the posttest on content, administered five weeks after the final test, the students were given the same 18 pairs of bipolar adjectives and asked to react to the statement 'Working With Logarithms'. A copy of this test appears in Appendix 5(b). Comparisons were made using the same scales as in the final test on attitudes.

Analysis of Data

It was planned to compare the mean scores for the matched experimental and control groups using a two tailed *t* test for correlated groups, (α = .05).

In summary, two matched groups were formed: the experimental group working with calculators, and the control group not working with calculators. As an external control group, a single class, not divided and not using calculators, was used. The groups were tested on attitudes and achievement at the end of the experimental period. This was the procedure for gathering data to test hypotheses 1 and 2. A posttest on the above variables was given five weeks subsequent to the end of the unit of work. This was the procedure used to gather evidence to test hypothesis 3. Three teachers were involved throughout the experiment. Content was examined on a multiple choice test on both final and posttest. Attitudes were measured on a semantic differential test.

The measuring instruments were all designed by the experimentor and were described in some detail in this chapter. Content validity was assessed by the other teachers involved in the experiment.

CHAPTER III

PRESENTATION AND DISCUSSION OF RESULTS

Descriptive data for the achievement and attitude tests are presented, followed by a discussion of the validity and reliability of the measuring instruments. The results of the two achievement tests and attitude tests are then discussed and analysed. The main conclusions and recommendations for future research using calculators are then presented.

Due to the fact that some students dropped the course during the experiment, and that other students were absent for various tests, complete data were available from 27 matched pairs and 18 individuals from the single class control group. The descriptive statistics for all four test results as well as for the December examination in mathematics are presented in Table 3. Raw data for all research subjects appear in Appendix 8.

Validity and Reliability of Measuring Instruments

In an attempt to establish some relationship between the results of the final logarithm test and previous mathematical performance, correlation coefficients between the final logarithm test and the December examination in mathematics were computed for each of the groups. The correlation coefficients are presented in Table 4. There was significant positive correlation for all groups of research subjects, suggesting that both tests were measuring some common mathematical ability.

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Table 3

Descriptive Statistics for December Mathematics Examination, Logarithm Achievement Tests, and Attitude Tests on Logarithms and Quadratic Equations

Test	Group	n	Mean	Mdn.	S.D.
December Examination in Mathematics	Class 1[a]	28	55.96[b]		14.87
	Class 2	27	60.11		17.00
	Class 3	25	59.76		15.20
	Class 4	24	62.88		12.75
Final Logarithm Achievement Test	EG	33	66.70		14.53
	CG	33	74.70		10.56
	SCCG	22	62.50		14.01
Posttest on Logarithms (Achievement)	EG	29	49.66		15.94
	CG	29	49.21		15.17
	SCCG	21	49.38		17.23
Attitudes Towards 'Quadratic Equations'	EG	33	16.06[c]	17.0	6.16
	CG	33	18.64	20.0	4.65
	SCCG	22	17.59	17.0	5.41
Attitudes Towards 'Logarithms'	EG	33	19.12	20.0	5.46
	CG	33	20.33	21.0	4.21
	SCCG	22	17.82	19.5	5.58

(continued on next page)

PRESENTATION AND DISCUSSION OF RESULTS

Table 3 (cont'd)

Test	Group	n	Mean	Mdn.	S.D.
Attitudes Towards 'Working With Logarithms'	EG	29	16.21	16.0	5.83
	CG	29	19.45	20.0	4.48
	SCCG	21	15.43	17.0	4.75

[a] These represent the original 4 mathematic classes before being divided into experimental and control groups.

[b] Achievement scores have all been rescaled to range from 0 to 100.

[c] Attitude scores had a possible range of from 4 to 28.

Table 4

Pearson Correlation Coefficients Between the December Mathematics Examination Scores and the Final Logarithm Test Scores for the Different Groups of Subjects

Group	n	r	Significant (α = .05)
EG	33	.579	Yes
CG	33	.511	Yes
SCCG	22	.753	Yes
Combined Groups	88	.517	Yes

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It will be recalled that all three teachers were involved in the lesson preparations and the test design in an attempt to control content validity.

The descriptive statistics relevant to these tests appear in Appendix 4(c). The standard error of measurement of 2.39, in terms of raw scores, and the KR20 reliability coefficient of .76 as reported in Appendix 4(c), suggest that the final test was sufficiently reliable for its intended purpose.

The items on the posttest on logarithms were prepared by the three teachers and were parallel in design to the items appearing in the final test. The lower KR20 reliability coefficient of .63 could be attributed to the reduced number of items. The test was shorter because the time available was limited to the length of the regular class period. The purpose of the posttest was to measure long term retention. Consequently, no review was held in class prior to the posttest with the result that individual scores were generally lower than on the final test.

With respect to the attitude test, the scales selected for the semantic differential had previously been established as valid and reliable scales for indexing attitudes (Osgood et al., 1971, p. 37, and Bentler and Lavoie, 1972, p. 177). The factor analysis presented in Appendix 7 suggested that for this particular group of subjects, some of the scales were not differentiating along the same factors as those suggested by the above authors. Four scales that measured consistently on the Evaluative factor were used to measure attitudes in this experiment. These scales were:

(a) Worthless --- Valuable

(b) Agreeable --- Disagreeable

(c) Boring --- Interesting

(d) Pleasant --- Unpleasant

This reduced number of scales gave a possible range of raw scores from 4 to 28 for each subject. This range was lower than anticipated and may have affected the reliability of the instrument.

An estimation of the reliability of the attitude test was determined from the scores on the final attitude test and the scores on the posttest measuring attitudes towards logarithms. The concept measured on the final attitude test was 'Logarithms' and the concept measured on the posttest was 'Working With Logarithms'. The same scales were used on both tests. Although the two concepts are not identical, they were considered sufficiently similar to permit a test re-test estimate of reliability. The computation of the Spearman rank correlation gave a value of $r = .62$ which is lower than those of .70 to .76 reported by Lemon (1973, p. 107). However it was concluded that the scales used to index attitude were within an acceptable range to be sufficiently reliable for this experiment.

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Achievement Test Results

Achievement test scores for the final logarithm test and posttest were verified to be normally distributed using a Chi Square test for goodness of fit, (χ^2 = 2.38 for the final test, χ^2 = 4.16 for the posttest, with critical value of ${}_{.95}\chi^2{}_3$ = 7.82). The results with respect to achievement on the final logarithm test are examined first, followed by the posttest results. The results of the comparisons between the mean scores of the various groups in Table 5.

In examining the final test results on logarithms, a two tailed t test for correlated groups indicated a signifi cant difference between the means of the EG and CG, α = .05. It will be recalled that it was predicted in the first research hypothesis that the students in the EG would demonstrate superior performance on the achievement test to those students in the control groups. The results obtained indicated a performance trend opposite in direction to that suggested in the first research hypothesis. Since the EG and CG represented split classes, their results were also compared to the SCCG which was representative of a regular mathematics class. When the means of each of the matched groups were compared separately to the mean of the SCCG using the t test for independent samples, (α = .05), the CG scored significantly higher than the SCCG. It should be emphasized again

PRESENTATION AND DISCUSSION OF RESULTS

Table 5

Comparisons of the Mean Scores on the Final and Posttest on Logarithms for all Combinations of the Three Main Groups Involved in the Experiment

Test	Groups	Observed *t* value	Significant (α = .05)
Final Logarithm Test	EG and CG[a]	-2.87	Yes
	EG AND SCCG[b]	1.07	No
	CG and SCCG[b]	3.67	Yes
Posttest on Logarithms	EG and CG[a]	.11	No
	EG and SCCG[b]	.06	No
	CG and SCCG[b]	-.04	No

[a] Comparison using a two tailed *t* test for correlated groups.

[b] Comparison using a two tailed *t* test for independent groups.

that the SCCG was taught the same material and was considered part of the experiment.

In examining the posttest results on logarithms, no significant differences were found between any of the three groups. The results of the *t* tests were summarized in Table 5. It was hypothesized that students who had used calculators would have greater long term retention of the concepts learned than students who had not used calculators. This hypothesis was not supported.

Since the variations in results on the final test were opposite to the predicted ones, it is necessary to examine the possibility that the use of the calculators had some detrimental effect on achievement. In the initial planning it was anticipated that both the experimental and control groups would receive sufficient practice to master the concepts taught, although the mode of practice would vary. The students using the calculators may have perceived the calculator as an extra task not shared by their fellow students in the control groups. This may have influenced their results.

Students in the EG were able to complete more solutions during approximately one third of the total periods, were able to perform more computations in designing tables of values for graphing functions, and were able to use a greater variety of techniques in working problems applying logarithms. However, none of the above was explicitely measured on the achievement tests as the CG and SCCG were not given this exposure on the calculators. It was the main purpose of this experiment to examine whether such practice with the calculator would clarify the main concepts and processes for working with logarithms. Thus, a second factor that may have influenced the results, was that the EG could not be tested on some of the skills developed through the use of the calculator.

Thirdly, perhaps the calculator when introduced in class, created a difficulty in that the mastery of the sequence of operations on the calculator became the primary objective, rather than understanding the problem solving process. In most applications of the calculator, there was a systematic procedure to follow, and although this sequence was not difficult, it may have been sufficiently distracting to interfere with the understanding of the solution. All groups were given the same exposure to the solutions required without using calculators by following a sequence of examples agreed upon by all teachers. However, the students were not tested for comprehension of the steps involved in the solutions before being allowed to use the calculators. Thus, the students may have been using the calculators to solve problems before they thoroughly understood why they were following the sequence of stpes outlined.

In the first lessons, the concepts of exponential and logarithmic functions were introduced, and it was during these lessons that students were first exposed to the calculators. The concepts were new and perhaps sufficiently complicated without using the calculator. Introducing the calculator was designed to facilitate this learning, but using the calculator at that time may not have done so. Thus, a third factor that may have contributed to the poor performance of the EG was that learning how to use the calculator may have been more demanding than anticipated.

A fourth consideration is that the teachers had not used calculators in teaching logarithms in class prior to this experiment. The teachers were experimenting with a procedure that had not been refined. One of the specific difficulties encountered in the first lessons was that insufficient class time was available for calculator usage after the teacher had presented the relevant material.

Fifthly, the calculator may not have been used for a sufficiently long period of time. The SR51 model has many special functions that were not used in this experiment. The complex appearance of the calculator may have confused some of the students. Of the functions that were used, some students forgot from day to day the correct sequence of steps. Students had no experience with the calculators prior to the experiment. It is possible that if the calculators were used over an extended period of time, any confusion over their operation and any novelty effect would be minimized.

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If testing had occurred immediately after each sub-unit of work, it would be easier to identify the effects of the calculator, whether positive or negative. By examining the aggregate effect on performance over the whole unit of work, any positive effects may have been nullified.

The above arguments are all hypothetical and difficult to measure objectively. A more objective explanation of the results can be found in an analysis of the responses to the test questions.

The achievement tests, both final and posttest, can be subdivided into six main content areas. The specific questions testing these content units, along with a description of the content, appear in Appendix 5(c). In an attempt to isolate any difference in the response pattern within the four largest of these content areas, the mean scores for the EG and CG were compared using a two tailed t test for correlated groups. Raw scores within each content area appear in Appendix 5(d), and a statistical summary is presented in Table 6. Content areas E and F were omitted in this analysis since they contained too few questions, and the specific type of error in the solution would be difficult to isolate.

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Table 6

Comparisons of Final Achievement Test Results on the 4 Main Content Areas for EG and CG

Content Area	Possible Score	EG Mean	EG S.D.	CG Mean	CG S.D.	Observed *t* Value	Significant $\alpha = .05$
A	8	4.97	1.85	6.00	1.52	2.46[a]	Yes
B	8	5.30	2.04	5.70	1.64	.84	No
C	6	4.42	.95	4.97	.72	3.61	Yes
D	11	7.30	1.88	7.88	1.81	1.30	No

[a] These values are determined using a two tailed *t* test for correlated groups.

These results indicate significant differences in performance within content areas A and C with the CG exhibiting superior performance. Content area A examined the student's ability to translate numerical expressions into standard form (scientific notation), in order to obtain the correct characteristic of the logarithm. This was an essential step when writing out a solution using the logarithm tables to evaluate an expression using logarithms. However, it was not an essential step when the students used the calculators because the calculators computed the correct characteristic from the original input.

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Content area C was used to examine the student's comprehension of the logarithmic notation, and his ability to translate from logarithmic notation to exponential notation. This area of content was quite unrelated to the use of the calculator except that the content was taught in the first lessons at the same time that the calculator was introduced to the students.

Content area B examined the student's familiarity with the graphs of the exponential and logarithmic functions. Although the calculator was used briefly in the original graphing of these functions, the concepts to be learned were not directly related to the use of the calculator.

Some of the questions in content area D would have discriminated against the students using the calculators because they involved techniques at which they did not receive as much practice as the control group. Although the CG scored higher in this content area, it was not significantly higher than the EG.

The above analysis supports the argument that the introduction of the calculator in the first lessons had some detrimental effect on the students comprehension of the material taught in those lessons. Secondly, the extra practice received in preparing numerical problems for computation using logarithms, provided the CG with an advantage on the test.

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It is quite a different matter to explain the significant superior performance of the CG over the SCCG in the final achievement test. The reader should recall that neither of these groups used the calculators. An analysis of the responses of the two groups in the four main content areas of the test has been summarized in Table 7.

Table 7

Comparison of Final Achievement Test Results on the 4 Main Content Areas for CG and SCCG

Content Area	Possible Score	CG		SCCG		Observed	Significant
		Mean	S.D.	Mean	S.D.	*t* Value[a]	$\alpha = .05$
A	8	6.00	1.52	4.77	1.73	2.78	Yes
B	8	5.70	1.64	5.10	1.41	1.42	No
C	6	4.97	.72	4.14	1.22	3.20	Yes
C	11	7.88	1.81	6.73	2.16	2.14	Yes

[a] Based on a *t* test for independent groups.

The results indicate that the CG maintained superior performance over the SCCG in the same two areas, (A and C), as they had with the EG. In addition the CG excelled in content area D which examined the ability to simplify logarithmic expressions using the rules of logarithms and some special techniques learned for simplifying these expressions.

One can only speculate as to the causes of these differences. The one obvious difference between the two groups is the fact that the SCCG contained a larger number of students for each meeting. However, this could not be considered a large class since it contained only 24 students.

In brief, some factors would seem to favour superior performance within the SCCG while other factors seem to favour superior performance within the CG. The larger class size in the SCCG, and their possible feeling of exclusion from the experiment, coupled with the greater individual attention available in the CG, would seem to favour performance within the CG. However the morning lessons for the SCCG should have operated in their favour since students tend to become tired and restless as the day progresses.

With respect to the posttest on logarithms, no significant differences were found between any of the three groups. The statistics were reported in Table 5. It would be surprising if the EG had scored higher than the CG on the posttest, since they had scored significantly lower than the CG on the final logarithm test. Thus, in spite of the fact that the CG scored highest on the final test, they did not maintain this advantage on the posttest.

Although the final and posttests on logarithms were different, they were based on the same content. The relative weighting for the content is shown in Appendix 5(c). The performance of the EG and CG on the posttest was quite similar on each test item, so that no analysis of content areas was completed on those results. This similarity of performance is illustrated in Table 8 which shows the responses for both groups to each test item on the posttest.

Table 8

Summary of the Number of Correct Responses to Each Item on the Posttest on Logarithms for EG and CG, n = 29

Question Number	No. of Correct Responses		Question Number	No. of Correct Responses	
	EG	CG		EG	CG
1	27	27	10	18	18
2	24	23	11	15	18
3	15	15	12	9	12
4	8	10	13	14	14
5	16	13	14	2	7
6	5	4	15	19	18
7	20	21	16	4	3
8	11	14	17	22	22
9	24	23	18	8	6

The conclusion with respect to the posttest on achievement is that the EG did not retain more knowledge about logarithms as a result of using the calculators. The superior performance of the CG on the final test was not maintained on the posttest. Consequently, the third research hypothesis was not supported.

The posttest itself may have partially contributed to these results. Although the content coverage on the final test and posttest was similar, the posttest was shorter because less time was available for testing. The posttest also contained different items than those which appeared on the final test. If parallel forms had been used on the final test and posttest, a more reliable comparison of achievement could have been made. Thus it is not clear whether the difference in performance at the two times of testing was due to the test itself, the lack of retention, or both.

In conclusion, with respect to achievement, neither of the two research hypotheses was supported. There was a significant difference between the EG and CG in the final achievement results, but opposite in direction to the one anticipated. There was no significant difference between the two groups on retention over a five week period.

The achievement results have now been summarized and discussed. The results of the attitude tests are now examined.

PRESENTATION AND DISCUSSION OF RESULTS

Results of Attitude Tests

The second dependent variable in this experiment was attitudes towards logarithms as measured on the semantic differential. It will be recalled that the experimental and control groups were matched on the basis of performance on their December examination in mathematics. It was expected that this match would create two groups that were also reasonably matched on attitudes. However, no measure of attitudes was taken at the time of the December examination.

When the December examination scores between the EG and CG were correlated, the value of r = .98 indicated that in fact a good match had been obtained on the basis of achievement. When the final attitude test scores between the EG and CG were correlated on the same matching as above, the value of r = .09 indicated that the groups should not be considered as matched with respect to attitudes towards logarithms (n = 66).

A Chi Square test indicated significant departure from normality for the posttest attitude results, (χ^2 = 9.65 with the critical value of ${}_{.95}\chi^2_3$ = 7.82). A similar of the final attitude scores suggested a normal distribution (χ^2 = 7.41, with the critical value of ${}_{.95}\chi^2_4$ = 9.49).

The skewed distribution of the posttest scores led to the use of a non-parametric technique when examining the attitude results. A non-parametric technique was used on both sets of scores in order to be consistent.

Since the groups were considered as independent groups with respect to attitudes, the final attitude scores, which are summarized in Table 3, were analysed using the Kruskal-Wallis test. There were no significant differences in attitude towards 'Logarithms' at the time of the final test, ($H = 2.7$, $\alpha = .05$). Thus the second research hypothesis, that the EG would demonstrate a more positive attitude towards logarithms than the CG at the completion of the experiment, was not supported.

Although a posttest on attitudes was not an integral part of the proposed research, a comparison of attitudes was made at the time of the posttest so that any change in attitudes could be examined after a five week interval. The scales used on the semantic differential were identical to the final test, however the concept examined was "Working With Logarithms". The posttest on attitudes indicated a significant difference in attitudes between the three groups, ($H = 7.29$, $\alpha = .05$). Simultaneous confidence intervals using simple contrasts of the rank means for each group indicated that the attitude of the CG was more positive than that of the SCCG. (The contrast estimate was 16.98, and the 95% simultaneous confidence intervals are 16.98 ± 15.28).

PRESENTATION AND DISCUSSION OF RESULTS

It had been anticipated that using calculators would have some positive impact on attitudes towards logarithms. Since students using the calculators did not demonstrate more positive attitude towards logarithms on the final test, a questionnaire was given to these students mid-way between the final test and the posttest. The questionnaire was given at this time because the experience of working with the calculators was still relatively fresh in their minds. Students were asked to respond to five statements. They indicated a degree of agreement or disagreement with each statement on a scale from 1 to 7. A sixth question solicited student comments. A copy of the questionnaire and a summary of the results appear in Appendix 9.

Over 60% of the students indicated that they would like to continue using the calculators, suggesting a positive, but not overwhelming reaction in favour of their use. Similarly only 25% of the respondents seemed to feel that the calculators had a negative influence on their test scores. The majority of subjects did not find the calculator confusing, thought it was a good time saver, and felt it was helpful in the study of logarithms. The student comments to question 6 appear in Appendix 10, and indicate some agreement with the responses to the other five questions.

There can be no conclusive reasons given to explain why the students using calculators did not exhibit a more positive attitude towards logarithms than the students in the control groups. The questionnaire results indicated that although the students enjoyed using the calculators, this was not sufficient to create a more positive attitude towards logarithms. The only conclusion that can be reached at this time is that the use of the calculators in this experiment had no positive effect on attitudes towards the material studied.

With respect to the posttest on attitudes, it was reported earlier that the CG scored significantly higher than the SCCG but not higher than the EG. There were three factors that may have influenced these results. Firstly, the fact that the questionnaire discussed above was given to the EG prior to the posttest on attitudes, while it was not given to the other groups, may have influenced the performance of the EG on the attitude test. Secondly, the posttest on attitudes was written subsequent to the achievement test, and thus attitudes may have been influenced by posttest performance. Thirdly, the concept 'Working With Logarithms' was a different concept from 'Logarithms', which had been used on the final test. These three sources of contamination may partially explain the differences between the groups at the time of the posttest, and also differences in results between the final

PRESENTATION AND DISCUSSION OF RESULTS

test and the posttest on attitudes.

Practice was isolated as a variable affecting learning and retention of meaningful material. The purpose of this research was to explore whether or not the use of a small scientific calculator would improve achievement and retention of the concepts learned as well as attitudes towards logarithms, within a unit of work in grade twelve advanced mathematics classes. The calculator was used to diversify practice in the classroom and allow more problems to be solved in a given period of time. The arithmetic capabilities of the calculator also allowed extra computations and verification of arithmetical operations where these were necessary. With the EG, the exponential and logarithmic functions built into the calculator allowed exposure to concepts and methods that were avoided by students not using calculators.

There were three dependent variables examined in the research:

(a) achievement on the concepts and application of logarithms,

(b) retention of those concepts over a five week period, and

(c) attitudes towards logarithms.

The independent variable that was manipulated was the use of the calculator in studying logarithms.

It was hypothesized that students using calculators throughout the unit of work on logarithms would demonstrate greater understanding of the concepts learned than students not having such exposure. This hypothesis was not supported and in fact, the matched control groups achieved significantly higher results than either the experimental or single class control groups. It should be noted that the calculators were not used on any of the tests.

Several possible reasons for the poor performance of the experimental groups were presented. Firstly, students using the calculators may have perceived their use as an extra task over and above the requirements of the students in the other groups, and they may in fact have had a more difficult learning experience. Secondly, they were not tested on the skills developed in using the calculators. Thirdly, an analysis of the responses to the questions indicated that learning how to use the calculator may have interfered with the topics taught in the early lessons. Fourthly, the teachers were inexperienced at using calculators in class. Lastly, the short period of exposure to the calculator along with the novelty effect may have distracted the students in class. A further analysis of the final test results indicated that lack of specific practice in writing out complete solutions as a result of using the calculators may have contributed to the generally lower scores within the calculator groups. This analysis also indicated that the calculator groups had a lower achievement particularly on the material taught in the lessons used to introduce the calculator.

According to Ausubel (1968, chap. 8), practice must be directly related to the learning task or at least be related in a meaningful way by the learner. The principal deficiency in this experiment was that some of the specific learning that did take place with the calculator was not directly tested. The time available in class was insufficient to master the processes on the calculator, and also master the techniques that were needed without the calculator. This gives further support to Ausubel's contention that the distribution of the available time is a critical factor in efficient practice, and that efficient practice schedules must be empirically established.

The use of the calculator as a time saving device and a computational aid was judged to be successful by most students, but a different measure of achievement would be necessary in order to evaluate that success. In general, the use of the calculator in the initial presentation of ideas was not successful. Learning how to use the calculator takes time which has the effect of reducing the time available for presentation of the regular lesson. There was probably some novelty effect present as a result of having smaller groups and working with new teachers for the duration of the experiment.

There is one question that remains to be answered. Why did both control groups not have a performance superior to the experimental group's on the final achievement test? If the use of the calculator had some negative influence on performance, then it should follow that the SCCG would have scored higher than the EG on the final logarithm test. This was not the case. The fact that both the EG and CG consisted of smaller groups, and the possible feeling of belonging to a special group, may have contributed to their superior achievement, although the difference was only significant in the case of the CG.

It was secondly hypothesized that students who had used calculators would exhibit a more positive attitude towards logarithms than students who had not used calculators. This hypothesis was not supported. The lack of any significant differences in attitudes towards 'Logarithms' between the EG and CG led to the development of a questionnaire given to students who had used the calculators. The students indicated a general positive response towards using the calculators, although there were some negative and indifferent attitudes expressed.

Aiken (1970, p. 589) has suggested that changes in the curriculum or classroom organization have little apparent impact on attitudes. This experiment seems to verify this statement. The confounding effects of different teacher personalities, and different student reactions to the use of the calculator may have contributed to the lack of a detectable difference in attitudes between the EG and CG.

The posttest on attitudes towards 'Working With Logarithms' indicated more positive attitudes within the CG over the SCCG. However, there were sufficient contamination effects present to suggest that these results are open to question.

The last hypothesis stated that students using the calculators would have greater retention of the concepts learned, and thus achieve higher on a posttest on logarithms than students not using a calculator. This hypothesis was not supported with the results indicating no significant differences between any of the three groups.

In brief, this research has demonstrated that using a scientific calculator essentially as an enrichment device, or as a time saving device to study logarithms, has no positive effect on achievement or retention over a five week period. Also, no positive effect on attitudes towards logarithms was observed.

It is recommended that in future studies where the scientific calculator is to be introduced into the learning of mathematical concepts, that sufficient time be allowed in the initial stages to allow students to become familiar with the operation of the calculator so that this learning does not interfere with the mathematical concepts themselves. Secondly, if the calculator contributes to the understanding of some mathematical concepts, then these concepts should be clearly identified. Thirdly, if the calculator has been used in the regular classes, then the calculator has been part of the learning experience, and should be incorporated in the evaluation of achievement. The final achievement test in this experiment probably discriminated against the groups using the calculators simply because some of the processes tested were techniques at which the calculator groups did not receive sufficient practice in class. Fourthly, any future experiment should attempt to measure treatment in smaller components so that the work can be assessed on only a few concepts at a time, rather than on a total unit of work. This would permit a more refined analysis of exactly where the calculator can be used to advantage. Lastly, the time required to master a concept should be studied in relation to the use of a calculator. The speed of the calculator may permit a more rapid learning of some concepts, especially those that require repetition.

Computational problems of a complex nature are not common in the advanced mathematics program, and thus the calculator will likely have limited application to these students. This experiment supports the idea that the calculator has great computational and time saving possibilities. The experiment failed to demonstrate that using a calculator as a computational aid and a time saving device will improve comprehension of the concepts of logarithms compared to students not using the calculator.

Finally, the semantic differential technique needs further development for measuring attitudes towards mathematical concepts. It has potential because of its simplicity of construction, administration, and scoring. However, more research must be done in isolating good bipolar adjectives that are relevant to specific mathematical concepts.

REFERENCES

Advani, Ken. The effect of desk calculators on achievement and attitude of children with learning and behavior problems. A research report. Dec. 1972. (ERIC Document Reproduction Service No. ED 077 160).

Aiken, Lewis R. Jr. Attitudes towards mathematics. Review of Educational Research, Oct. 1970, 40(4), 551-96.

Aiken, Lewis R. Jr. Update on attitudes and other affective variables in learning mathematics. Review of Educational Research, Spring 1976, 46(2), 293-311.

Ausubel, David P. Educational psychology. A cognitive view. New York: Holt, Rinehard & Winston, 1968.

Ausubel, David P., and Robinson, Floyd G. School learning; an introduction to educational psychology. New York: Holt, Rinehard & Winston, 1969.

Ausubel, David P., Youssef, Mohammed. The effect of spaced repetition on meaningful retention. The Journal of General Psychology, 1965, 73, 147-50.

Bentler, P.M., & LaVoie, Allan L. An extension of semantic space. Journal of Verbal Learning and Verbal Behavior, 1972, 11, 174-182.

Brosgoyle, Leonard; Hanley, Michael J. & Angelo, F. Contino. An analysis of repetition. Psychological Reports, 1973, 32, 835-39.

Callantine, Mary F. & Warren, J.M. Learning set in human concept formation. Psychological Reports, 1955, 1, 363-67.

Cech, Joseph P. The effect of the use of desk calculators on attitude and achievement with low achieving ninth graders. The Mathematics Teacher, Feb. 1972, LXV (2), 183-6.

Gagné, Robert M., Mayer, John R., Garstens, Helen L. & Paradise, Noel E. Factors in acquiring knowledge of a mathematical task. Psychological Monographs, General & Applied, 1962, 76(7), Whole No. 526.

REFERENCES

Gaslin, William L. A comparison of achievement and attitudes of students using conventional or calculator based algorithms for operations on positive rational numbers in ninth grade mathematics. Journal For Research in Mathematics Education, March 1975, 6, 95-108.

Gay, Lorraine R. Use of a retention index for mathematical instruction. Journal of Educational Psychology. 1972, 63(5), 466-72.

Kimble, Gregory A. Learning Theories. Encyclopedia Britannica, 1975, 10, 15th ed., 754-59.

Lefrancois, Guy R. Psychological theories and human learning: Kongor's report. California: Brooks/Cole, 1972.

Lemon, Nigel. Attitudes and their measurement. New York: John Wiley & sons, 1973, 100-110.

McCallon, Earl L., & Brown, John D. A semantic differential instrument for measuring attitude toward mathematics. The Journal of Experimental Education, Summer 1971, 39(4), 69-72.

Morgan, Ben B. Jr., & Alluisi, Earl A. Acquisition and performance of a problem solving skill. Perceptual And Motor Skills, 1971, 33, 515-23.

Morrisett, Lloyd, Jr., & Hovland, Carl I. A comparison of the variates of training in human problem solving. Journal of Experimental Psychology, 1959, 58(1), 52-55.

Osgood, Charles E., Suci, George J., & Tannenbaum, Percy H. The Measurement of Meaning. Urbana: 1971, University of Illinois Press.

Scandura, Alice M., Lowerre, George F., Veneski, Jacqueline, & Scandura, Jos. F. Using electronic calculators with elementary school children. Educational Technology, Aug. 1976, XVI(8), 14-18.

Schafer, Pauline, Bell, Max S. & Crown, Warren D. Calculators in some fifth grade classrooms. Elementary School Journal, Oct. 1975, 76, 27-31.

Schnur, James O. & Lang, Jerry W. Just pushing buttons or learning? A case for mini-calculators. The Arithmetic Teacher, Nov. 1976, 23(7), 559-62.

APPENDIX 1

Lesson Plans

Lesson 1.

Objectives.

1. To define an exponential function.
2. To have students graph various exponential functions by obtaining a table of values.
3. To familiarize students with the domain and range of exponential functions.
4. To define an increasing or decreasing function.
5. To define an asymptote and recognize the asympototic nature of the X axis for exponential functions.
6. To have the calculator introduced and be able to use the y^x function to generate a table of values.

ORAL REVIEW.

Definition of a function.

Types of functions met so far. i.e. linear, quadratic, and trigonometric.

Domain and range of functions, Vertical line test.

How to find the inverse of a function.

Relation of the inverse to the line y=x.

PRESENTATION.

(a) Define an exponential function, $y=a^x$, where 'a' is a positive constant. Tell students that the best way to get a feel for a function is to graph a few of them. We shall vary the constant (parameter) and see what happens to the shape of the curve as 'a' changes. The following chart will be given to the students on a ditto, and each student should fill in the values for one whole number and one fraction.

X	-3	-2	-1	0	1	2	3
$y = 2^x$							
$y = 5^x$							
$y = .5^x$							
$y = .1^x$							

THE CALCULATOR GROUP CAN BE SHOWN HOW TO USE THE y^x FUNCTION. THESE STUDENTS CAN OBTAIN MORE VALUES, AND SHOULD COMPLETE THE FOLLOWING TABLE OF VALUES INSTEAD OF THE ONE ABOVE.

X	-3	-2	-1	0	.5	1	1.5	2	2.5	3
$y = 2^x$										
$y = 5^x$										
$y = .5^x$										
$y = .1^x$										
$y = 10^x$										

SHOW THESE STUDENTS THE SPECIAL FUNCTION 10^x BUILT INTO THE CALCULATOR, AND HOW TO USE IT.

Graph paper is to be supplied to all students.

(b) Draw the graph of $y = 5^x$ with them in class. Have it on an overhead and discuss: the domain, the range, intercepts, and the increasing nature of $y = a^x$ for this example.

(c) Define an increasing function.

ASSIGNMENT.

(a) Complete the above graphs, all on the same set of axes.

(b) Read section 5:6, p. 192-4 in the text. Refer to your graphs as you read.

(c) CALCULATOR GROUPS COULD FAMILIARIZE THEMSELVES WITH THE SIMPLE ARITHMETIC FUNCTIONS ON THE CALCULATORS IF ANY TIME REMAINS IN CLASS.

Lesson 2.

Objectives.

1. To review the concept of an exponential function.
2. To establish that $a>0$ for all exponential functions as an integral part of the definition.
3. To review the notation and concept of the inverse of a function.
4. To define the logarithmic function and explain the notation $y = \log_a x$.

ORAL REVIEW.

Definition of a function.

Definition of an exponential function.

Domain, range, intercepts, and asymptotes.

Have homework graphs out, and compare to the graphs on the overhead.

PRESENTATION.

(a) In order to establish the fact that $y = a^x$ is only continuous when $a>0$, have the students attempt to complete a table of values for $y = (-2)^x$.

x	-2	-1	0	1	2
$(-2)^x$					

Then have them try to find values for:

x	$-\frac{1}{2}$	$\frac{1}{2}$
$(-2)^x$		

Conclusion? $y = a^x$ is only continuous for all x if $a>0$.

(b) Now the idea here is to graph the inverse of an exponential function in order to see what it looks like.

(1) Firstly have them build up a table of values. Do this on the board with them.

x	-4	-2	0	2	4	6	8	16
$y=2^x$								

THE CALCULATOR GROUP CAN WORK WITH MORE VALUES:

x	-4	-2	0	2	3	4	5	6	7	8	10	12	16
$y=2^x$													

(2) Hand them out graph paper with the axes marked as required, and have them graph $y = 2^x$.

(3) Now ask them to graph the inverse, $x = 2^y$ by simply interchanging the x and y values in the ordered pairs.

(4) Draw the line $y = x$ and review the relationship between this line, and the graphs of a function and its inverse.

(c) How can we find the equation of this inverse? How can we express 'y' in terms of 'x'? Here we must review how

APPENDIX 1

this problem was handled in the past. Complete the following chart with them as a review.

f(x)	$f^{-1}(x)$
$y = 3x + 4$	$x = 3y + 4$ OR $y = \frac{x-4}{3}$ Thus $f^{-1}(x) = \frac{x-4}{3}$
$y = 2^x$	$x = 2^y$ OR y = ? ....
$y = 5x - 3$	$x = 5y - 3$ OR $y = \frac{x+3}{5}$ Thus $f^{-1}(x) = \frac{x+3}{5}$
$y = 5^x$	$x = 5^y$ OR y = ? ....

This should be sufficient to show the need for some new notation. We need a statement explaining 'y' is. The expression for $f^{-1}(x)$ in the second line of the above table can be read as: " y is the exponent that 2 must be raised to, in order to give x."

Explain that mathematicians have agreed to shorten this statement to the following: $y = \log_2 x$.

Show the degree of similarity between this notation and the notation y = f(x) by recalling that y= cos(x) was shortened to y = cosx. Similarly, $y = \log_a(x)$ is shortened to $y = \log_a x$.

ASSIGNMENT.

(a) Study your notes from class.

(b) Page 194, question 4 is to be completed on a set of axes provided by the teacher.

(c) Tell the students that we will be dealing with the new notation above in greater detail at a later date, and they should simply understand its meaning at this time.

Lesson 3.

Objectives.

1. To review exponential and logarithmic functions.
2. To review the notation for these functions.
3. To emphasize the graphical relationship between these two functions.
4. To use the graph of $y = 10^x$ to evaluate arithmetical problems by approximation.

ORAL REVIEW.

The types of functions studied so far: exponential,and logarithmic along with their domain, range, and intercepts. The notation $y = \log_a x$. The relationship between the graphs of the two functions.

PRESENTATION.

(a) Have the students examine the graph of $y = 10^x$ from their text book, and show how to read off the exponential form of various numbers. Note that all the numbers are between 1 and 10.

Examples: $3.4 \doteq 10^{.53}$, thus $\log_{10} 3.4 \doteq .53$

$2.7 \doteq 10^{.43}$, thus $\log_{10} 2.7 \doteq .43$

$8.9 \doteq 10^{.95}$, thus $\log 8.9 \doteq .95$

Emphasize that a logarithm is just an exponent.

(b) Now show the class how to use the graph of $y = 10^x$ to approximate the value of some arithmetic problems. Use the following examples.

APPENDIX 1

(i) $3.4 \times 2.7 \doteq 10^{.53} \times 10^{.43}$

$= 10^{.96}$

$\doteq 9.1$ (from the graph)

(ii) $870 \times 2600 = 8.7 \times 2.6 \times 10^5$

$\doteq 10^{.94} \times 10^{.46} \times 10^5$ (from graph)

$\doteq 10^{.36} \times 10^6$

$\doteq 2.3 \times 10^6$

At this point it can be indicated why we need only a graph for numbers between 1 and 10.

THE CALCULATOR GROUPS CAN BE GIVEN THE CHANCE TO CHECK THEIR APPROXIMATIONS BY USING THE CALCULATORS. THIS WILL HELP THEM BECOME FAMILIAR WITH THE ARITHMETIC FUNCTIONS ON THE CALCULATORS.

(iii) $24.0 \div 8.6 = \dfrac{2.4 \times 10}{8.6}$

$\doteq \dfrac{10^{.38} \times 10^1}{10^{.93}}$

$= 10^{.46}$

$\doteq 2.9$

(iv) $\sqrt{14} = \sqrt{1.4 \times 10}$

$= 1.4^{\frac{1}{2}} \times 10^{\frac{1}{2}}$

$\doteq 10^{.07} \times 10^{.5}$

$= 10^{.57}$

$\doteq 3.7$

ASSIGNMENT.

p. 200, #2,3,5,6.

<u>Lesson 4.</u>

Objectives.

1. To review again that $y = 10^x$ and $y = \log x$ are inverse functions.
2. To review the use of the graph of $y = 10^x$ in approximating

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arithmetic computations.

3. To examine the graph of $y = \log x$ and show how the computations are related to the graph.

4. To give more practice with the notation $y = \log x$, and its relation to the exponential notation $y = a^x$.

REVIEW.

(a) Firstly take up the following questions from the homework. p. 200, # 2(e),(f), (i); 3(e),(h); 5(b),(e); 6(c),(h).

HAVE THE CALCULATOR GROUPS CHECK THEIR ANSWERS WITH THE CALCULATORS TO COMPARE THE TWO METHODS. IN QUESTION 6, THEY WILL NEED TO USE THE $\sqrt[x]{y}$ FUNCTION.

(b) Have the students graph the function $y = \log x$, $1 \leq x \leq 10$ by using the following table of values. They can be shown the logarithm tables on page 564 and obtain the 'y' values there. Emphasize that they are finding the exponents to base 10 that will give these numbers.

THE CALCULATORS GROUPS CAN ALSO OBTAIN THE LOG VALUES FROM THE CALCULATORS.

x	0	1	2	3	4	5	6	7	8	9	10	
y=logx												(tables)
y=logx												(calcul.)

Emphasize that these are approximate numbers.

(c) Again emphasize that we only need logarithms for numbers between 1 and 10, because every number in our number system can be written in that form; i.e. $.00872=8.72X10^{-3}$.

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PRESENTATION.

(a) Use the preceding graph to discuss the manner in which logarithms work. This graph should be on an overhead transparency so that the following demonstration can be made.

Evaluate 2 X 3

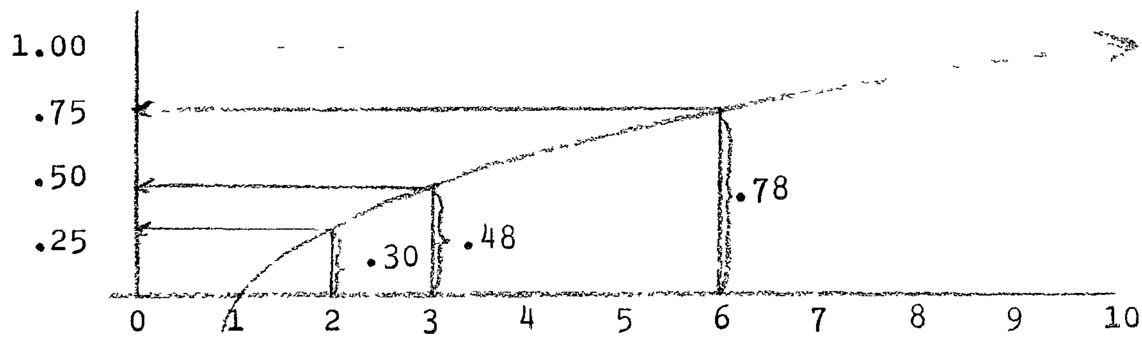
Let x = 2 X 3

$\doteq 10^{.30} \times 10^{.48}$

$= 10^{.78}$

$= 6.$

The purpose of the graph is to illustrate the addition of logarithms when you multiply.



(b) Now evaluate 2000 X 300.

Let x = 2000 X 300

$= 2.0 \times 3.0 \times 10^5$

$\doteq 10^{.30} \times 10^{.48} \times 10^5$

$= 10^{.78} \times 10^5$

$= 6 \times 10^5$

This example emphasizes again the fact that we need logs

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only for numbers between 1 and 10.

(c) Since the exponential and logarithmic functions are inverses, the students must be able to transform from the exponential form into the logarithmic form very quickly. The student should realize that both forms are equivalent, but that one might be more useful than the other, depending upon the nature of the problem. The following examples will help illustrate this.

Exponential Form	Logarithmic form
$32 = 2^5$	$5 = \log_2 32$
$64 = 2^6$	$6 = \log_2 64$
$3^{-2} = \frac{1}{9}$	$-2 = \log_3(\frac{1}{9})$

(d) Have the students work the following:

1. $\log_2 8$ 2. $\log_5 1$ 3. $\log_{10} 100$ 4. $\log_7 7$

SHOW THE CALCULATOR GROUPS THE LIMITATIONS OF THE CALCULATORS IN THAT THEY CANNOT REASON BUT CAN ONLY DO WHAT YOU TELL THEM TO DO.

(e) Do the oral exercise on page 216 in class.

ASSIGNMENT.

Write out solutions for the oral exercise, and on p. 216 complete #4,5,6,7, 10.

Lesson 5.

Objectives.

1. To review and consolidate understanding of the notation

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$y = \log_a x$ and its relation to the exponential form.

2. To ensure that students know what is meant by the word logarithm.
3. To demonstrate the rules for using logarithms for multiplication and division.
4. To solve equations using logarithms.

N.B. CALCULATORS ARE NOT NEEDED FOR THIS LESSON.

REVIEW.

Orally review the notation: if $2^3=8$, then $\log_2 8 = 3$.

if $10^{.5} \doteq 3.16$, then ______?

Again emphasize that the notation for common logarithms omits the base 10.

Conversely, if $\log 8.42 = .9253$, then $10^{.9253} = 8.42$

and if $\log_7 49 = 2$, then ______?

PRESENTATION.

(a) Tell the class that we have seen in the last few days that multiplication and division can be reduced to simple addition and subtraction of exponents provided we convert the numbers to the same base.

Recall that $2 \times 3 \doteq 10^{.3} \times 10^{.48} = 10^{.78} = 6$ and that $3 \div 2 \doteq 10^{.48} \div 10^{.30} = 10^{.16} \doteq 1.5$

Tell class that we now wish to formalize these rules.

Although we will work with base 10, the student should understand that the rules that we develop will hold for any base, provided all the numbers are written in the same base.

Thus if $3 \times 2 = 10^{.48} \times 10^{.30}$

$= 10^{.78}$

$= 6$

then, $\log(3 \times 2) = .48 + .30$

$= \log 3 + \log 2$

and in general, if $3 \times 2 = a^m \times a^n$

$= a^{m+n}$

then $\log_a(3 \times 2) = m + n$

$= \log_a 3 + \log_a 2.$

To generalize even further, if $p.q = a^m \times a^n$

$= a^{m+n}$

then $\log_a(p.q) = m + n$

$= \log_a p + \log_a q$

This leads to a general rule for the logarithm of a product: 'The logarithms of a product is the sum of the logarithms of its factors.'

Emphasize that this is really nothing new. It is merely the rule for adding exponents where powers are multiplied. The only difference is that it is dealing with the exponents as separate entities.

(b) Now we shall examine an example with division.

Recall again that $3 \div 2 \doteq 10^{.48} \div 10^{.30}$

$= 10^{.48-.30}$

Thus, $\log(3 \div 2) = .48 - .30 = \log 3 - \log 2.$

APPENDIX 1

This is exactly as one would expect since when dividing powers to the same base, we subtract the exponents,(logs).

Thus if $3 \div 2 = a^m \div a^n$

$= a^{m-n}$

then $\log_a(3 \div 2) = m - n$

$= \log_a 3 - \log_a 2$

In general, if $p \div q = a^m \div a^n$

$= a^{m-n}$

then $\log_a(p \div q) = m - n$

$= \log_a p - \log_a q.$

This leads to the rule for the logarithm of a quotient; 'The logarithm of a quotient is the difference of the logarithms of the two numbers.'

(c) Now work the example of page 219 and 221 with the class. These should be on transparancies.

ASSIGNMENT.

p. 220, #3,4,5,6. and p. 222, # 3,4,5,6.

Lesson 6.

Objectives.

1. To review the graphs of $y = 10^x$ and $y = \log x$ and their relation to each other.
2. To review the laws for logarithms for multiplication and division.
3. To establish the laws for operating with powers.
4. To solve some more difficult problems.

APPENDIX 1

ORAL REVIEW.

(a) Have a brief oral review of the l.l.m. and l.l.d. Ask students for the reasons for these rules.

(b) Have the homework put on the board. Ask specifically for at least one part of #6, 5(e), (f), on page 222.

PRESENTATION.

(a) Hand out graph paper and have students quickly sketch the graph of $y = 10^x$, $0 \leq x \leq 1$ along with the graph of its inverse, $y = \log x$, $1 \leq x \leq 10$.

The students should be shown the tables at the back of the text from which the values may be found.

THE CALCULATOR GROUPS CAN GENERATE THE VALUES FROM THE CALCULATORS. THEY SHOULD ALSO BE SHOWN THE TABLES.

(b) Tell the class that we want them to be familiar with the above graphs so that they should be able to sketch them without the help of tables.

Now we wish to formalize the rule for evaluating powers by means of logarithms.

Example. Evaluate $(4.7)^2$.

Firstly, using the graph of $y = 10^x$ we can see that

$4.7 \doteq 10^{.67}$

$$\begin{aligned} \text{Thus, } 4.7^2 &\doteq (10^{.67})^2 \\ &= 10^{1.34} \\ &= 10^{.34} \times 10^1 \\ &\doteq 2.2 \times 10 \end{aligned}$$

APPENDIX 1

Have them check the above estimation on the graphs that they just completed. Then go through the following steps:

Since $(4.7)^2 \doteq 10^{1.34}$

$\log(4.7)^2 = 1.34$

$= 2 \times .67$

$= 2 \times \log 4.7$

In general, if $4.7 = a^m$

then $(4.7)^2 = (a^m)^2$

$= a^{2m}$

thus, $\log_a(4.7)^2 = 2m$

$= 2\log_a 4.7$

OR

if $p = a^x$

then $p^n = (a^x)^n$

$= a^{nx}$

thus, $\log_a p^n = nx$

$= n(\log_a p)$

This gives rise to a general rule, namely:

'The logarithm of a number raised to a power, is the power times the logarithm of the number'.

Again emphasize that this is nothing new. The students have already learned this in their power rule for exponents.

The only difference is that when working with logarithms, we are dealing only with the exponents.

(c) Work the example on p. 223. This is on an overhead.

(d) Do the oral exercise on p. 223.

ASSIGNMENT.

p. 224, #3,4,5. p. 222, #8, p. 220, #8.

Lesson 7.

Objectives.

1. To summarize the rules for working with logarithms.

APPENDIX 1

2. To show the students how to use the logarithm tables of common logarithms, and explain the mean difference columns.
3. TO SHOW THE CALCULATOR GROUPS HOW THE CALCULATOR GENERATES ALL THE NUMBERS IN THE LOGARITHM TABLES AND ALSO THE FACT THE CALCULATOR IS NOT RESTRICTED IN GIVING LOGARITHMS FOR NUMBERS BETWEEN 1 AND 10.

ORAL REVIEW.

(a) Quickly review the three rules developed for operating with logarithms. i.e.

$\log_a(MXN) = \log_a M + \log_a N$ because $a^x \times a^y = a^{x+y}$

$\log_a \frac{M}{N} = \log_a M - \log_a N$ " $a^x \div a^y = a^{x-y}$

$\log_a M^n = n\log_a M$ " $(a^x)^y = a^{xy}$

(b) Have the homework difficulties put on the board.

PRESENTATION.

(a) Emphasize again that every number in our base ten system can be written in standard form and thus expressed as a number between 1 and 10.

Examples: $347.2 = 3.472 \times 10^2$

$.003472 = 3.472 \times 10^{-3}$ etc.

Explain that this is the reason why the tables only give logarithms for the numbers between 1 and 10. Every number between 1 and 10 can be written to base ten using the exponents found in the logarithm tables. At this point, the graph of y = logx should be on the overhead so that the values in the tables can be related to the values on the graph.

(b) The following examples will illustrate how to find the logarithm corresponding to any number.

$$\log 347.2 = \log(3.472 \times 10^2)$$
$$= \log 3.472 + \log 10^2$$
$$= .5406 + 2$$

Mention that .5406 is called the mantissa and the 2 is called the characteristic. The mantissa is found from the tables, but the characteristic is found from observation.

(c) THE STUDENTS WITH THE CALCULATORS ARE TO BE SHOWN HOW TO OBTAIN log347.2. THEY SIMPLY INPUT 347.2 AND CALL FOR THE LOGARITHM. THUS log347.2 = 2.540579717

(d) This is a good time to have a brief discussion about approximate numbers. From this point on, all our calculations will be approximate, and the student should realize that all the numbers in the tables are approximations.

(e) Example. Find log(.003472)

$$\log(.003472) = \log(3.472 \times 10^{-3})$$
$$= \log 3.472 + \log 10^{-3}$$
$$= .5406 - 3 \quad \text{or} \quad \bar{3}.5406$$

THE CALCULATOR GROUP SHOULD TRY THIS QUESTION, AND OF COURSE THEY WILL OBTAIN A DIFFERENT ANSWER, i.e. -2.45942083. RELATE THIS VALUE TO THE GRAPH TO SHOW THAT IT IS A

APPENDIX 1

LEGITIMATE VALUE BUT DOES NOT APPEAR IN THE LOG TABLES.

(f) Summarize with the following drill.

What is the mantissa?

What does it tell us?

What is the characteristic?

What does it tell us?

Why do log347.2 and log(.003472) have the same mantissa?

(g) The students should be given the following chart on a ditto and asked to fill it in.

NUMBER	S. FORM	LOGARITHM	MANTISSA	CHARACTERISTIC
987.3				
2.756				
.793				
.0534				
.00104				
279.38				
6,374.2				
26.4×10^{2}				
.000052				
386×10^{-4}				

ASSIGNMENT.

Finish the above table, and the problems on p. 230, #6,7,8.

<u>Lesson.</u>

A class test was given to all groups during this period.

<u>Lesson 8.</u>

Objectives.

1. To summarize the process for finding the logarithm for any number.

APPENDIX 1

2. To review what is meant by the logarithm of a number.
3. To review the mantissa, characteristic, and the use of the log tables, and the exponential tables.
4. To multiply using logarithms.
5. TO HAVE THE CALCULATOR GROUPS LEARN HOW TO USE THE CALCULATORS TO MULTIPLY WITH LOGARITHMS, AND TO VERIFY THE COMPUTATIONS.

ORAL REVIEW.

(a) Ask the students to find the log(.004826) from the tables, AND FROM THE CALCULATORS.

Ask them what is meant by a logarithm?

What is the mantissa and the characteristic?

What do they mean?

WHY DOES THE ANSWER ON THE CALCULATOR DIFFER FROM THE ANSWER IN THE TABLES? SHOW THE RELATIONSHIP ON THE GRAPH AGAIN.

(b) Have selected questions from the homework put on the board. THE STUDENTS WITH THE CALCULATORS SHOULD DO THE QUESTIONS BOTH WAYS.

PRESENTATION.

(a) Tell the class that the principal reason for developing logarithms was to simplify long computations. The purpose was to exploit the rules for exponents that involve simple addition and subtraction. Today we shall look at how logarithms can be used to simplify multiplication .

Work the following examples in class.

APPENDIX 1

Example 1. Evaluate 463.5 X 17.39 using logarithms.

Solution. Let $P = 463.5 \times 17.39$

$$= 4.635 \times 1.739 \times 10^3$$

$$\log P = \log 4.635 + \log 1.739 + 3$$

$$= .6661 + .2402 + 3$$

$$= .9063 + 3$$

Emphasize that this is an exponent.

$P = 8.060 \times 10^3$ from the exponential tables.

Example 2. Evaluate .06349 X .4715

Solution. Let $P = 6.349 \times 4.715 \times 10^{-3}$

(Save a step by putting in std. form)

$$\log P = \log 6.349 + \log 4.715 - 3$$

$$= .8027 + .6735 - 3$$

$$= 1.4762 - 3$$

$$= .4762 - 2$$

$$P = 2.993 \times 10^{-2}$$

(b) Let the class try one on their own now.

Example 3. Evaluate 67.4 X .000973

(c) THE CALCULATOR GROUPS WILL NOW BE SHOWN HOW TO USE THE CALCULATOR TO OBTAIN THE VALUES INSTEAD OF THE TABLES. EMPHASIZE THAT THEY MUST STILL SET THE PROBLEM UP IN A REASONABLE FASHION. TELL THEM THEY MUST STILL BE ABLE TO USE THE TABLES FOR ANY TESTS AND FOR THE HOMEWORK, SO DO NOT FORGET THE ABOVE METHOD.

APPENDIX 1

Example. Evaluate 463.5 X 17.39

Solution. Let P = 463.5 X 17.39

logP = log463.5 + log17.39

P = 8060

Note the following:

(i) it is not necessary to put the numbers in std. form,

(ii) all the calculations are done in the calculator so that three lines are all that are required in the solution.

NOW HAVE THEM CHECK THEIR ANSWER BY PERFORMING THE ORIGINAL COMPUTATION WITHOUT LOGARITHMS.

Example 2. Evaluate .06349 X .4715

Solution. Let P = .06349 X .4715

logP = log.06349 + log.4715

P = .02994

(d) THE CALCULATOR GROUPS SHOULD BE ABLE TO FINISH #2,3,4 IN CLASS, FROM p. 235.

ASSIGNMENT.

p. 235 #2,3,4,5,6,7.

Note that the idea here is to give the calculator groups more practice in class. If they have not completed 2,3, and 4 in class they must finish them for homework the long way. If they manage to finish all the questions, give them some extra multiplication for homework where they must use the tables. ... p.235 #15.

APPENDIX 1

Lesson 9.

Objectives.

1. To review multiplication with logarithms.

2. To learn how to divide with logarithms.

3. To give the CALCULATOR GROUPS more practice with the calculators.

ORAL REVIEW.

Put a few selected questions from the homework on the board and have the CALCULATOR GROUPS DO THEM BOTH WAYS. The remaining students can be given a review question such as .387 X .007924 X 644.3.

PRESENTATION.

(a) Explain to the class that logarithms can also be used to simplify division. Division is reduced to subtraction when dealing with logarithms. Why?

Example 1. Simplify $\frac{62.75}{27.56}$ using logarithms.

solution. Let $x = \frac{62.75}{27.56}$

$$= \frac{6.275}{2.756} \times 10^0$$

$$\log x = \log 6.275 - \log 2.756 + 0 \text{ (why?)}$$

$$= .7976 - .4402$$

$$= .3574$$

$$x = 2.277$$

Example 2. Simplify $\frac{16.75 \times 49.65}{75.36}$

Solution. Let $x = \frac{1.1675 \times 4.965}{7.536} \times 10^1$

$$\log x = \log 1.675 + \log 4.965 - \log 7.536 + 1$$
$$= .2240 + .6959 - .8721 + 1$$
$$= .0428 + 1$$
$$x = 1.104 \times 10^{1}$$

Let them try one on their own.

Example 3. Evaluate $\frac{34.5}{175.2 \times 453}$

Solution. Let $x = \frac{3.45}{1.752 \times 4.53} \times 10^{-3}$

$$\log x = \log 3.45 - \log 1.752 - \log 4.53 - 3$$
$$= .5378 - .2435 - .6561 - 3$$
$$= .5378 - 3.8996$$

Since the tables do not provide logarithms for negative numbers (refer to the graph) we avoid negative numbers by adding 4 and then subtracting it again as follows:

$$\log x = 4.5378 - 3.8996 - 4$$

$= .6382 - 4$ giving a positive mantissa,

$$x = 4.347 \times 10^{-4}$$

(b) WITH THE CALCULATOR THE COMPUTATION BECOMES MUCH SIMPLER. THE ABOVE EXAMPLE CAN BE DONE IN THREE STEPS.

$$\text{Let } x = \frac{34.5}{175.2 \times 453}$$

$$\log x = \log 34.5 - \log 175.2 - \log 453$$

IT IS NOT NECESSARY TO WORRY ABOUR NEGATIVE LOGARITHMS.

$$x = .0004347$$

APPENDIX 1

ASSIGNMENT.

p. 237, #2,3. WORK AS MANY AS POSSIBLE USING THE CALCULATOR. THE CALCULATOR GROUPS SHOULD BE ABLE TO COMPLETE MOST OF THESE IN CLASS. THEY SHOULD BE GIVEN SOME EXTRAS TO DO FOR HOMEWORK, i.e. p. 253, #16 d,e,f.

<u>Lesson 10.</u>

Objectives.

1. To consolidate computation on Multiplication and division with logarithms.
2. To use logarithms to solve problems with powers and roots.

ORAL REVIEW.

Put selected homework questions on the board.

HAVE THE CALCULATOR GROUPS DO THEM THE LONG WAY ON THE BOARD AND THEN CHECK THEM IN CLASS WITH THE CALCULATORS.

PRESENTATION.

(a) Explain that logarithms really save a great deal of time in calculations requiring powers and roots.

Example 1. Evaluate $(2.683)^6$

Solution. Let $x = 2.683^6$

$$\log x = 6 \log 2.683$$
$$= 6 \times .4286$$
$$= 2.5716$$
$$= .5716 + 2$$
$$x = 3.729 \times 10^2$$

APPENDIX 1

Example 2. Evaluate the following: $\sqrt[3]{.4679}$

Solution. Let $x = (4.679 \times 10^{-1})^{1/3}$

$$\log x = \frac{1}{3}(\log 4.679 - 1)$$
$$= \frac{1}{3}(.6701 - 1)$$

Since 3 does not divide evenly into (-1), we simply convert to a number that 3 will divide into, as follows:

$$\log x = \frac{1}{3}(2.6701 - 3)$$
$$= .8900 - 1$$
$$x = 7.762 \times 10^{-1}$$

(b) USING THE CALCULATOR.

Example 1. Let $x = 2.683^6$

$$\log x = 6 \log 2.683$$
$$x = 373.0$$

Example 2. Let $x = (.4679)^{1/3}$

$$\log x = \frac{1}{3}(\log .4679)$$
$$x = .7763$$

HAVE THEM CHECK THIS ANSWER BY USING THE $\sqrt[x]{y}$ FUNCTION ON THE CALCULATOR.

ASSIGNMENT.

p. 237, #2(e),(f),(g),(h). p. 240, #2(g),(h),(i).

p. 240, #3(f),(g),(i), and #4.

THE CALCULATOR GROUP WOULD BE ABLE TO COMPLETE MOST OF THESE IN CLASS. THEY CAN BE ASSIGNED SOME EXTRAS FOR H.W. TO BE DONE WITH THE TABLES. i.e. p. 240, #7(a),(b),(c) IS TO BE HANDED IN TOMORROW.

Lesson 11.

Objectives.

1. To consolidate skills in computing with logarithms,
2. To evaluate more complicated expressions.
3. TO GIVE CALCULATOR GROUPS MORE PRACTICE WITH THE CALCULATORS.

ORAL REVIEW.

Have selected homework problems on the board.

THE CALCULATOR GROUPS SHOULD DO THEM THE LONG WAY ON THE BOARD, AND THEN CHECK THEM WITH THE CALCULATORS.

PRESENTATION.

(a) Tell the class to imagine how long it would take them to simplify the following expression if they had to do it the long way without the use of logarithms.

Evaluate. $\sqrt{\dfrac{84.3 \times .5973}{(4.235)^3}}$

Note that this will be worked out on an overhead transparency.

Solution. Let $s = \left(\dfrac{8.43 \times 5.973}{4.235)^3}\right)^{\frac{1}{2}}$

$$\begin{aligned} \log s &= \tfrac{1}{2}(\log 8.43 + \log 5.973 - 3\log 4.235) \\ &= \tfrac{1}{2}(\ .9258 + .7762 - 3(.6268)) \\ &= \tfrac{1}{2}(\ 1.7020 - 1.8804) \\ &= \tfrac{1}{2}(3.7020 - 1.8804 - 2) \text{ avoids negatives,} \\ &= .9108 - 1 \end{aligned}$$

$$s = 8.143 \times 10^{-1}$$

APPENDIX 1

WITH THE CALCULATOR:

$$\log s = \tfrac{1}{2}(\log 84.3 + \log .5973 - 3\log 4.235)$$

$$s = .8142$$

All the calculations are done without clearing the calculator. It is basically a two step solution.

(b) Now have the students try this one.

The solution will be worked out on an overhead.

Evaluate: $\dfrac{(1.732)^2 \times \sqrt[3]{.7125}}{\sqrt{.03743}}$

Solution. Let $x = \dfrac{1.732^2 \times (7.125 \times 10^{-1})^{1/3}}{(3.743 \times 10^{-2})^{\frac{1}{2}}}$

$$\begin{aligned}
\log x &= 2\log 1.732 + \tfrac{1}{3}(\log 7.125-1) - \tfrac{1}{2}(\log 3.743-2)\\
&= 2(.2385) + \tfrac{1}{3}(2.8528-1) - \tfrac{1}{2}(.5732-2)\\
&= .4770 + \tfrac{1}{3}(2.8528-3) - .2866 + 1\\
&= .4770 + .9509 - .2866\\
&= 1.1413\\
x &= 1.385 \times 10^1
\end{aligned}$$

(c) WITH THE CALCULATORS.

$$\log x = 2\log 1.732 + \tfrac{1}{3}\log .7125 - \tfrac{1}{2}\log .03743$$

$$x = 13.85$$

AGAIN, ALL THE CALCULATIONS ARE DONE WITHOUT CLEARING THE CALCULATOR SO THAT A GREAT DEAL OF TIME IS SAVED.

ASSIGNMENT. CALCULATOR GROUPS SHOULD BE ABLE TO COMPLETE p. 240 #7 IN CLASS. The remainder should be done for H. W. p. 255 #28(k) is to be done with the tables and handed in by all students tomorrow. Solution will be on a ditto.

APPENDIX 1

Lesson 12.

Objectives.

1. To review simple interest and develop compound interest.
2. To apply logarithms to the solution of such problems.
3. To show the connection between compound interest and exponential functions.

ORAL REVIEW.

(a) Discuss situations where interest is charged, i.e. for loans, credit buying, and savings accounts.

(b) Show that the formula for simple interest is really stating that the interest paid is a function of 3 factors: the principal, rate, and time.

(c) Work an example.

How much interest will a person receive on a savings account paying 7½% interest if he has $75.00 in the account of 8 months?

Solution.

$$I = PRT$$
$$= 75 \times .075 \times \frac{8}{12}$$
$$= \$3.75$$

LET THE CALCULATOR GROUPS WORK THIS OUT ON THE CALCULATOR.

PRESENTATION.

(a) Tell the class that compound interest occurs when interest is left with the principal so that new interest is paid on the old interest as well as the principal.

Example. Start with $100.00 at 8% p.a.

APPENDIX 1

PRINCIPAL	INTEREST	
\$100.	\$8.00	
108.00	8.64	
116.64	9.33	etc...

(b) Tell the class that we would like to develop a formula for arriving at the amount after a certain number of compound interest periods. The following example demonstrates how such a formula can be found.

Using the data from the previous example,

At the end of 1 year, $A = 100 + 100(.08)$

$$= 100(1.08)$$

At the end of the 2 nd. year, $A = 100(1.08) + 100(1.08)(1.08)$

$$= 100(1.08)(1 + .08)$$

$$= 100(1.08)^2$$

At the end of the 3 rd. year, $A = 100(1.08)^2 + 100(1.08)^2(.08)$

$$= 100(1.08)^2(1 + .08)$$

$$= 100(1.08)^3$$

and so on.... At the end of n years, the value of the deposit is $A = 100(1.08)^n$

(c) In general, $A = P(1 + i)^n$ where:

A = the accumulated amount at the end of the period of time,

P = the original principal,

i = the rate of interest per period, expressed as a decimal,

n = the number of interest periods.

APPENDIX 1

(d) Work the following example. To save time, this will be done on an overhead.

Example. Find the amount of $500. in 6 years at 12% per annum (p.a.) compounded annually.

Solution. $A = P(1 + i)^n$

$P = 500$, $n=6$, and $i= .12$

$$A = 500(1.12)^6$$

$$\log A = \log 500 + 6\log 1.12$$

$$= .6990 + 2 + 6(.0492)$$

$$= .9942 + 2$$

$$A = 9.968 \times 10^2$$

$$\doteq \$986.80$$

SHOW THE CALCULATOR GROUP THAT THEY CAN FIND THE ANSWER VERY QUICKLY WITHOUT LOGARITHMS.

$$A = 500(1.12)^6$$

$$= \$986.91$$

NOTE. THEY MUST FIRSTLY EVALUATE 1.12^6 USING THE y^x BUTTON, THEN MULTIPLY BY 500.

Example 2. Find the amount of $500. in 6 years at 12% compounded semi-annually.

Solution. This will also be done on an overhead. You will need to discuss what is meant by semi-annual computation.

$P = 500$, $n = 12$, $i = .06$

$$A = P(1 + i)^n$$

APPENDIX 1

$$= 500(1.06)^{12}$$

$$\log A = \log 500 + 12\log 1.06$$

$$= .6990 + 12(.0253) + 2$$

$$= .0026 + 3$$

$$A = 1.006 \times 10^3$$

$$= \$1,006.00$$

WITH THE CALCULATOR, $A = 500(1.06)^{12}$

$$= \$1006.10$$

(e) Have the students refer to the graphs on page 245 to compare the growth of $1.00 at different rates of compound interest. THE CALCULATOR GROUPS CAN ACTUALLY GRAPH THE GROWTH OF $1.00 AT 4%, 8% and 12% OVER A 20 YEAR PERIOD BY COMPLETING THE FOLLOWING TABLE. (DIVIDE UP THE WORK SO THAT IT IS DONE QUICKLY).

n	$A = (1.04)^n$	$A = (1.08)^n$	$A = (1.12)^n$
0			
5			
10			
15			
20			

HAVE THE ABOVE VALUES ON OVERHEADS. PLOT THESE GRAPHS ON THE SAME SET OF AXES.

(f) Do the oral exercise on p. 246.

ASSIGNMENT.

p. 246 #4. THE CALCULATOR GROUP WILL EASILY FINISH THESE QUESTIONS IN CLASS. THEY SHOULD DO AT LEAST ONE QUESTION

APPENDIX 1

AT HOME USING LOGARITHMS. i.e. p. 254, #18(a).

<u>Lesson 13.</u>

Objectives.

1. To review compound interest.
2. To provide more practice with compound interest and logarithm problems.

ORAL REVIEW.

(a) Discuss the concept of compound interest to make sure that the students understand the idea.

(b) Have the homework put on the board. STUDENTS WITH THE CALCULATORS SHOULD DO IT BOTH WAYS TO BE SURE THAT THEY CAN DO THE PROBLEMS WITH LOGARITHMS.

PRESENTATION.

This period is essentially a work period. The assignment is to finish all the questions on pages 246-247.

THE CALCULATOR GROUPS SHOULD SET UP EACH QUESTION AND DO THE CALCULATIONS ON THE CALCULATOR. IF THEY FINISH ALL THE QUESTIONS IN CLASS, THEN THEY SHOULD DO p.255, #28.

ASSIGNMENT.

p. 254, #18 c,d,e, and 19 to be done with logarithms.

The non-calculator groups may not finish the assignment on page 246-7 in which case they will simply be assigned that for their homework.

APPENDIX 1

Lesson 14.

Objectives.

1. To acquaint students with the concept of Present Value.

2. To gain practice in solving problems using logarithms.

ORAL REVIEW.

Selected problems from the homework may be put on the board. Stress the proper form in setting up the solution.

PRESENTATION.

(a) Discuss the concept of Present Value. Tell the students that it is really looking at the same problem as the amount problems, only from a different point of view.

Thus if $A = P(1 + i)^n$ then $P = \frac{A}{(1+i)^n}$

(b) Explain that we are interested in knowing how much must be invested today in order to have a certain amount in the future.

Example. (This will be on an overhead.)

How much must you invest today at 12% p.a. compounded annually so that you will have $1,000. in five years?

Solution. $P.V. = \frac{A}{(1+i)^n}$

$A = \$1,000.$, $i = .12$, and $n = 5$.

$$P.V. = \frac{1,000}{(1.12)^5}$$

$$\log P.V. = \log 1,000 - 5\log 1.12$$

$$= 3 - 5(.0492)$$

$$= 2.7540$$

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$$P.V. = 5.675 \times 10^2$$

$$\doteq \$567.50$$

(c) THE CALCULATOR GROUPS CAN DO IT BOTH WAYS FOR PRACTICE.

WITH THE CALCULATOR, $P.V. = \frac{1000}{(1.12)^5}$

$$\log P.V. = \log 1{,}000 - 5\log 1.12$$

$$P.V. = \$567.43$$

AND WITHOUT LOGARITHMS, $P.V. = \frac{1000}{(1.12)^5}$

$$= \$567.43$$

(d) Example 2. Have on an overhead.

Find the present value of $3,500 due in 9 years if money is worth 11% p.a. compounded semi-annually.

Solution. $P.V. = \frac{A}{(1+i)^n}$

$$A = 3{,}500,\ i = .055,\ \text{and } n = 18.$$

$$P.V. = \frac{3500}{(1.055)^{18}}$$

$$\log P.V. = \log 3500 - 18\log 1.055$$

$$= .5441 + 3 - 18(.0233)$$

$$= .1247 + 3$$

$$P.V. = 1.332 \times 10^3$$

$$= \$1{,}332.00$$

THE CALCULATOR GROUP CAN CHECK THIS ANSWER WITH THE CALCULATOR.

$$P.V. = \frac{3500}{(1.055)^{18}}$$

$$= \$1{,}335.13$$

APPENDIX 1

ASSIGNMENT.

p. 249, Ex. B. p. 253, #11, 12, 13.

THE CALCULATOR GROUPS WILL BE ABLE TO FINSIH ALL OF QUESTIONS ON p. 249 IN CLASS. THEY CAN DO p. 254, #23, 24 USING LOGS FOR HOMEWORK.

Lesson 15.

Objectives.

1. To summarize the main points of the chapter.
2. To prepare the students for the test on the chapter.

ORAL REVIEW.

The following points should be quickly reviewed;

1. $y = a^x$ and $y = \log_a x$ are inverse functions.
2. The common logarithm function is the inverse of $y = 10^x$ and is denoted by $y = \log x$.
3. There are an infinite number of exponential and logarithmic functions, but the one above is the most useful.
4. The three basic rules for working with logs are:
   $\log MN = \log M + \log N$, $\log \frac{M}{N} = \log M - \log N$, $\log M^n = n\log M$.
5. These rules are used to evaluate complicated expressions.

PRESENTATION.

(a) Do the oral exercise on p. 251-2, #1-7, and 10.

Take up the homework. Work # 28(1) in class.

THE CALCULATOR GROUP CAN DO IT BOTH WAYS.

ASSIGNMENT.

Study for the test.

APPENDIX 2

TABLE OF SPECIFICATIONS FOR THE UNIT EXPONENTIAL AND LOGARITHMIC FUNCTIONS GRADE 12.

Cognitive Level	Main Concepts To Be Studied
	Exponential Functions
Recall of Information <u>Knowledge</u> of specifics, terminology, conventions, facts, sequences, methods, and processes, categories, principles and theories.	DEfinition of a function. Defn. of an exponential function. *Defn. of inverse of a function. General form of $y = a^x$, $a>0$. *Intercepts of $y = a^x$. *Domain and range of a function. Fractional exponents. Asymptote of $y = a^x$.
<u>Comprehension.</u> Translation. Interpretation. Extrapolation.	*Shape of the exponential curve as 'a' changes. Why 'a' must be greater than 0.
<u>Application</u>. Ability to apply the above knowledge to novel situations.	*Ability to graph any exponential function by building up a table of values. Ability to generate the graph of $y = 10^x$ quickly, $0 \leq x \leq 1$. Use of the graph of $y = 10^x$ to approximate products and quotients. Why standard form is necessary.

APPENDIX 2

Logarithmic Functions	Theory Underlying the Rules For Working With Logarithms
*Definition *Notation The fact that a logarithm is an exponent. *Use of standard form. *Domain and range of this function. *Intercepts.	Review of the rules for exponents: $x^a \times x^b = x^{a+b}$ $x^a \div x^b = x^{a-b}$ $(x^a)^b = x^{ab}$
*What does $\log_2 x$ mean? *Evaluation of logs of the form $\log_2 8$. What do the numbers in the tables represent? *Ability to transform from $x = a^m$ into $m = \log_a x$.	Explaining the rules: $\text{Log } MN = \log M + \log N$ $\text{Log}\frac{M}{N} = \log M - \log N$ $\log M^n = n \log M$
Ability to translate from exponential notation to logarithmic notation in new situations.	*Simplification of expressions using the above rules. *Solving equations using logarithms.

APPENDIX 2

Working With Common Logarithms	Computations Using Common Logarithms
Definition of Common Logs. Using the tables. *Definition of Mantissa and Characteristic. Discussion of Approximate Numbers. Notation: $\bar{3}.425 = .425-3$.	Definition of Compound Interest. Use of the formula $A = P(1+i)^n$. Present Value. Use of the formula $P.V. = \frac{A}{(1+i)^n}$ Development of computational skills in evaluating logs. * i.e. $\log x = \frac{1}{3}(.625 - 1)$ $= \frac{1}{3}(2.625 - 3)$
Reason why tables only have logarithms for numbers from 1 to 10. Why do 3.14 ahd .000314 have the same mantissa? *Understanding of the range of values in the tables. Can you find log(-4)? Why not? *What does the characteristic tell you?	Use of the anti-log tables to evaluate logarithmic answers. Graphical representation of compound interest related to exponential functions. * Process involved in solving exponential equations.*
*Ability to write any number in standard form. *Ability to estimate the log of any number.	*Multiplication using common logarithms, along with applications to division, roots, and powers. *Ability to complete complex computational problems. *Ability to use logarithms to evaluate compound interest and present value problems.

APPENDIX 3

LETTER TO PARENTS

Dear Parent:

During the next few weeks, some of the students in Grade 12 mathematics will be given an opportunity to work in class with small calculators. The purpose of the study is to examine the feasibility of classroom use of the calculators in a unit of work, and to gain experience with some of the problems encountered by the students and teachers when the calculators are used.

Groups of students will be assigned at random to work with the calculators, and the calculators will <u>not</u> be permitted on any test or examination, so that no student will have any advantage over any other on a test.

If you have any questions, please do not hesitate to call Miss Jean Starrs or Mr. Bob Paterson.

Sincerely,

Principal.

APPENDIX 4(a)

FINAL LOGARITHM TEST

INSTRUCTIONS: You are to indicate the correct answer on the I.B.M. marking sheets. Fill in the bubble corresponding to the correct answer. Be sure to clearly erase any errors that you make.

1. When 347.2×10^{-4} is expressed in standard form, it becomes
   A. .3472
   B. 3.472×10^{-6}
   C. 3.472×10^{-2}
   D. 3.472×10^{2}
   E. 3.472×10^{1}
2. $\frac{.00872 \times 36.4}{57.83 \times 20.5} = \frac{8.72 \times 3.64}{5.783 \times 2.05} \times 10^{n}$
   The correct value of 'n' is:
   A. -2
   B. -4
   C. -6
   D. -5
   E. -3
3. When you evaluate log(13.72 X .00273), the characteristic is:
   A. 1
   B. -4
   C. -2
   D. 2
   E. -3
4. Five different students look up the value of log8.54 in the tables. Which of the five answers below is <u>likely</u> correct?
   A. .9314 B. -.3914 C. -.9314 D. 1.9314 E. .09314

APPENDIX 4(a)

5. The domain of the function defined by the equation y=logx is:
   A. $\{x|x\varepsilon R\}$
   B. $\{x|x\varepsilon R,\ x \neq 0\}$
   C. $\{x|x\varepsilon R,\ x<0\}$
   D. $\{x|x\varepsilon R,\ 0\leq x\leq 10\}$
   E. $\{x|x\varepsilon R,\ x>0\}$

6. The range of the function defined by the equation $y = 10^x$ is:
   A. $\{y|y\varepsilon R,\ y>0\}$
   B. $\{y|y\varepsilon R\}$
   C. $\{y|y\varepsilon R,\ y \neq 0\}$
   D. $\{y|y\varepsilon R,\ 1\leq y\leq 10\}$
   E. $\{y|y\varepsilon R,\ y\geq 0\}$

7. If $6 \doteq 2^{2.6}$, which of the following statements is true:
   A. $2.6 = \log 6$
   B. $6 = \log_2 2.6$
   C. $2 = \log_{2.6} 6$
   D. $2.6 = \log_6 2$
   E. $2.6 = \log_2 6$

8. If $x = \log_2 128$, then the correct value of x is:
   A. 2^7
   B. 2^6
   C. 6
   D. 7
   E. 8

9. The value of $\log_b b^2$ is:
   A. 2
   B. b
   C. 2b
   D. b^2
   E. 2^b

APPENDIX 4(a)

10. The value of $10^{\log 6}$ is:

A. 60

B. 6

C. 10^6

D. cannot be found.

E. cannot be found without tables.

11. The value of $\log_3 9^6$ is:

A. 3^{12}

B. 6

C. 3

D. 12

E. 8

12. Given the function y=logx, indicate which statement is correct:

A. The y intercept is 1.

B. The negative Y axis is an asymptote.

C. The X intercept is undefined.

D. The negative X axis is an asymptote.

E. None of the above is true.

13. The point common to $y=\log_3 x$ and $y=\log_5 x$ is:

A. 1

B. (0,1)

C. (0,0)

D. (1,0)

E. (1,1)

14. The value of $\log_2 \sqrt[3]{128}$ is:

A. 1/3

B. 2/3

C. $2^7/3$

D. 7/3

E. 2^7

APPENDIX 4(a)

15. When evaluating a logarithm, a student reaches the following point: $\log x = \frac{1}{3}(.6702 - 2)$.
The next line in his calculation when simplified should be:
A. $\log x = .2234 - 2$
B. $\log x = .5567 - 1$
C. $\log x = .5568 - 1$
D. $\log x = .5567 - 3$
E. none of these.

16. If $2.7 \doteq 10^{.43}$ and $\log 8.9 \doteq .95$, then the value of $\log(8.9 \div 2.7)$ is:
A. .52
B. .58 - 1
C. 1.38
D. $10^{.52}$
E. $10^{1.38}$

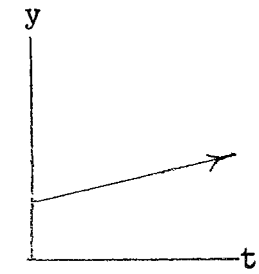
17. If $\log_2(\frac{x^2}{9}) = 2$, then x equals:
A. $\pm 3\sqrt{2}$
B. $3\sqrt{2}$
C. ± 6
D. 6
E. 36

18. if $\log(x+2) + \log(x-1) = 1$, then the value of x is:
A. -3
B. 4
C. -4
D. 3 or -4
E. 3

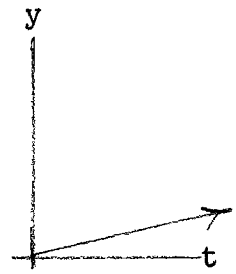
APPENDIX 4(a)

19. Which of the following graphs could represent the changing value of \$1,000 invested today at 8% p.a. if interest is compounded annually.

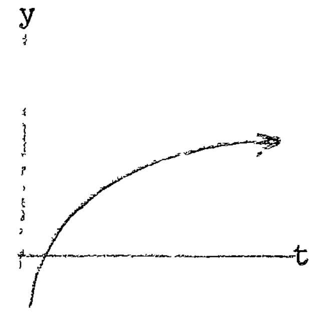
A.



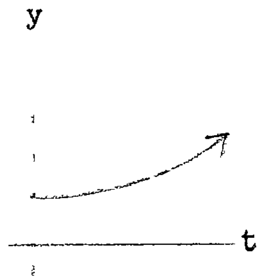
B.



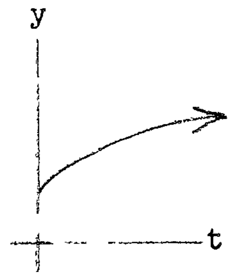
C.



D.



E.



20. Which of the following statements is <u>incorrect</u>.

A. $\log_a \frac{PQ}{R} = \log_a P + \log_a Q - \log_a R$

B. $\log_b \frac{M}{NP} = \log_b M - \log_b N - \log_b P$

C. $\log_p M^n = n\log_p M$

D. $\log_a (MN)^x = x\log_a M + x\log_a N$

E. $\log_a (\frac{P}{QR})^m = m\log_a P - m\log_a Q + m\log_a R$

21. A possible equation for the curve at the right is:

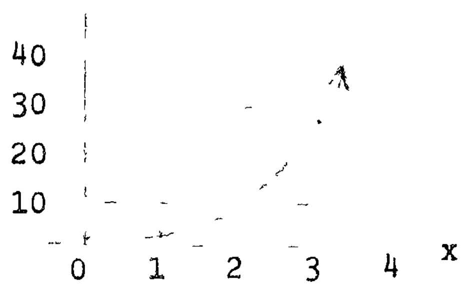
A. $y = 3^3$

B. $y = 2^x$

C. $y = \log_3 x$

D. $y = x^3$

E. $y = 3^x$



APPENDIX 4(a)

22. The equation of the inverse of the function defined by the given curve is:

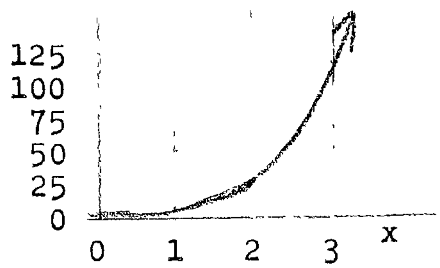
A. $y = \log_5 x$

B. $y = x^5$

C. $y = 5^x$

D. $y = \log_x 5$

E. $y = \log 5x$



23. If logA = 3.5421 and logB = .5421-3, then

A. $A = B \times 10^5$

B. $A = B \times 10^3$

C. $A = B \times 10^{-3}$

D. $A = B \times 10^6$

E. $A = B \times 10^{-6}$

24. If $\log_a m = x$ and $\log_a p = y$, then the value of $\log_a mp$ is:

A. $x + y$

B. xy

C. a^{xy}

D. $\log_a(x+y)$

E. $\log_a xy$

25. If $x = \frac{274.2}{.00971}$, then which of the following statements is correct:

A. log2.742 - log9.71 + 5 = logx

B. log2.742 - log9.71 - 1 = logx

C. log2.742 - log9.71 + 1 = logx

D. log2.742 - log9.71 + 4 = logx

E. none of the above is correct.

APPENDIX 4(a)

26. When evaluating $x = \sqrt[3]{\frac{.0845^4}{27.8}}$, which logarithmic statement is correct:

A. $\log x = 3(4\log .0845 - \log 27.8)$

B. $\log x = \frac{1}{3}(\log 8.45 - \log 2.78)$

C. $\log x = \frac{4}{3}\log 8.45 - \frac{1}{3}\log 2.78 - 3$

D. $\log x = \frac{1}{3}(4\log 8.45 - \log 2.78 - 7)$

E. None of the above is correct.

27. If $250 was invested today at 10% p.a. compounded semi-annually for 11 years, it would amount to:

A. $250(1.10)^{11}$

B. $\frac{250}{(1.10)^{22}}$

C. $250(1.055)^{20}$

D. $250(1.05)^{22}$

E. $\frac{250}{(1.05)^{22}}$

28. $x invested today at 11% p.a. in order to grow to $1,000 in four years would be found by evaluating which of the following logarithmic statements.

A. $\log x = 3 - 4\log 1.11$

B. $\log x = \log 1000 \div \log(1.11)^4$

C. $\log x = \log 1000 \div 4\log 1.11$

D. $\log x = 3 \div 4\log 1,11$

E. $\log x = 3 + 4\log 1.11$

The following questions are designed to test your ability to set up a correct solution. You are <u>not</u> required to work out the final answer. You will use logarithms to establish a method <u>up to the point where you would normally look up the logarithms in the tables</u>. At that point you <u>stop.</u> The following example will illustrate what is required.

APPENDIX 4(a)

EXAMPLE.	PROBLEM.	Evaluate 3.15×4.28
	SOLUTION.	Let $x = 3.15 \times 4.28$
		$\log x = \log 3.15 + \log 4.28$
		STOP!

29. Evaluate using logarithms. $\sqrt[3]{\dfrac{83.72}{249.4}}$

30. Evaluate using logarithms. $\dfrac{.00973 \times 48.2}{(0.0845)^3}$

APPENDIX 4(b)

SEMANTIC DIFFERENTIAL ATTITUDE TEST

GENERAL INSTRUCTIONS.

The purpose of this exercise is to measure the meanings of certain concepts to various people by having them judge them against a series of descriptive scales. In taking this test please make your judgments on the basis of what these things mean to you. On the top of each page of this booklet you will find a different concept to be judged and beneath it a set of scales. You are to rate the concept on each of these scals in order.

Here is how you are to use the scales:

If you feel that the concept at the top of the page is very closely related to one end of the scale, you should place your check-mark as follows:

Fair _X_:___:___:___:___:___:___ Unfair

OR

Fair ___:___:___:___:___:___:_X_ Unfair

If you feel that the concept is quite closely related to one or the other end of the scale (but not extremely), you should put your check-mark as follows:

Active ___:_X_:___:___:___:___:___ Passive

Active ___:___:___:___:___:_X_:___ Passive

If you feel that the concept seems only slightly related to one side as opposed to the other side (but is not really

APPENDIX 4(b)

neutral), then you should check as follows:

Strong ___:___:_X_:___:___:___:___ Weak

OR

Strong ___:___:___:___:_X_:___:___ Weak

The direction toward which you check, of course, depends upon which of the two ends of the scale seem most characteristic of the thing you are judging.

If you consider the concept to be neutral, or both sides of the scale equally associated with the concept, or if the scale is completely irrelevant to the concept, then you should place your check mark in the middle space.

Safe ___:___:___:_X_:___:___:___ Dangerous

IMPORTANT: (1) Place your check-marks in the middle of spaces and not on the boundaries:

___:_X_:___:___:___:X__:___

THIS NOT THIS

(2) Be sure to check every scale for each of the concepts. Do not omit any.

(3) Never put more than one check mark on a single scale.

Sometimes you may feel as though you have had the same item on the test. This will not be the case, so do not look back and forth through the items. Do not try to remember how you checked similar items earlier in the test. Make each item a

APPENDIX 4(b)

a separate and independent judgment. Work at fairly high speed through the two items. Do not worry or puzzle over individual items. It is your first impressions, the immediate feelings about the items that we want. On the other hand, please do not be careless, because we want your true impressions.

There are only two concepts to react to on this test. Each concept is written at the top of the page. You may begin whenever you are ready.

APPENDIX 4(b)

1. LOGARITHMS.

WORTHLESS	___:___:___:___:___:___:___	VALUABLE
FAST	___:___:___:___:___:___:___	SLOW
PASSIVE	___:___:___:___:___:___:___	ACTIVE
AGREEABLE	___:___:___:___:___:___:___	DISAGREEABLE
SMOOTH	___:___:___:___:___:___:___	ROUGH
SHORT	___:___:___:___:___:___:___	LONG
FAIR	___:___:___:___:___:___:___	UNFAIR
DELICATE	___:___:___:___:___:___:___	RUGGED
BORING	___:___:___:___:___:___:___	INTERESTING
VIGOROUS	___:___:___:___:___:___:___	LIFELESS
DULL	___:___:___:___:___:___:___	SHARP
FEROCIOUS	___:___:___:___:___:___:___	PEACEFUL
DYNAMIC	___:___:___:___:___:___:___	STAGNANT
RELAXED	___:___:___:___:___:___:___	TENSE
DEEP	___:___:___:___:___:___:___	SHALLOW
PLEASANT	___:___:___:___:___:___:___	UNPLEASANT
STRONG	___:___:___:___:___:___:___	WEAK
MASCULINE	___:___:___:___:___:___:___	FEMININE

continue on next page...

APPENDIX 4(b)

2. QUADRATIC EQUATIONS.

INTERESTING	___:___:___:___:___:___:___	BORING
ROUGH	___:___:___:___:___:___:___	SMOOTH
STAGNANT	___:___:___:___:___:___:___	DYNAMIC
UNFAIR	___:___:___:___:___:___:___	FAIR
SLOW	___:___:___:___:___:___:___	FAST
MASCULINE	___:___:___:___:___:___:___	FEMININE
DULL	___:___:___:___:___:___:___	SHARP
VALUABLE	___:___:___:___:___:___:___	WORTHLESS
WEAK	___:___:___:___:___:___:___	STRONG
ACTIVE	___:___:___:___:___:___:___	PASSIVE
UNPLEASANT	___:___:___:___:___:___:___	PLEASANT
LONG	___:___:___:___:___:___:___	SHORT
LIFELESS	___:___:___:___:___:___:___	VIGOROUS
AGREEABLE	___:___:___:___:___:___:___	DISAGREEABLE
FEROCIOUS	___:___:___:___:___:___:___	PEACEFUL
SHALLOW	___:___:___:___:___:___:___	DEEP
TENSE	___:___:___:___:___:___:___	RELAXED
RUGGED	___:___:___:___:___:___:___	DELICATE

3. How would you rate your overall feelings about working with logarithms:

Strongly enjoyed working with logarithms.	___:___:___:___:___:___:___	Very much disliked working with logarithms.

APPENDIX 4(c)

INFORMATION RELEVANT TO THE FINAL TEST AND THE POST-TEST ON LOGARITHMS FOR ALL RESEARCH SUBJECTS WRITING TESTS

	Final Test	Post-Test
Number of Subjects	98	93
Number of Test Items	36	18
Mean Score[a]	25.54	9.10
Standard Deviation	4.88	2.95
Highest Score	34	16
Lowest Score	7	3
Perfect Score	36	18
Reliability Coeff. (KR20)	.76	.63
Standard Error of Measurement	2.39	1.79

[a] These are raw scores before being converted into percentages.

APPENDIX 5(a)

POST-TEST ON LOGARITHMS

The following questions are designed to see how much you remember about logarithms. You have probably forgotten quite a few things, so do not be upset if you miss some of the questions. The results of this test will not be held against you, however you are encouraged to do as well as you can as the information will be helpful to your teachers. You might consider this test merely as a good review at this time.

INSTRUCTIONS.

Shade the letter corresponding to the correct answer to each question. Use the I.B.M. answer sheets.

1. If $\log_a N = M$, then which of the following is correct:

   A. $N = a^M$
   B. $N = M^a$
   C. $M = a^N$
   D. $M = N^a$
   E. $M = N$

2. The equation $x = a^y$ when written in the form $y = f(x)$ becomes:

   A. $y = a^x$
   B. $y = x^a$
   C. $y = \log_a x$
   D. $y = \log_a x$
   E. $y = \log x$

APPENDIX 5(a)

3. The domain and range respectively of $y = \log_{10}x$ are:

A. R, R

B. $\{x|x>0\}$, R

C. $\{x|x\geq 0\}$, R

D. $\{x|x>0\}$, $\{y|0\leq y\leq 10\}$

E. $\{x|x\geq 1\}$, $\{y|y\geq 0\}$

4. If $y = \log_4 9$, then 4^{2y} equals:

A. 18

B. 162

C. 16

D. 36

E. 81

5. The value of $\log_2 \sqrt[5]{\frac{1}{16}}$ is:

A. $-\frac{1}{5}$

B. 4

C. -20

D. $-\frac{4}{5}$

E. 20

6. The value of $\log_2 8 + \log_2 20 - \log_2 5$ is:

A. 32

B. 5

C. 2.5

D. $\log_2 23$

E. $\frac{\log_2 28}{\log_2 5}$

APPENDIX 5(a)

7. When .00873 X 10^{-2} is expressed in standard form it becomes:

A. .873
B. .0000873
C. 8.73 X 10^{-3}
D. 8.73 X 10^{-1}
E. none of these.

8. When you evaluate $\log\left[\frac{246.5 \times .0872}{.00732}\right]$, the characteristic is:

A. -2
B. 3
C. 4
D. -4
E. none of these.

9. The value of $\log_b(b^a)^x$ is:

A. ax
B. x
C. a
D. b^x
E. b^a

10. Given the function $y = a^x$, $a>0$, indicate which statement is correct:

A. The x intercept is 1.
B. The positive x axis is an asymptote.
C. The y intercept is 1.
D. The negative y axis is an asymptote.
E. The positive y axis is an asymptote.

APPENDIX 5(a)

11. The point in common to $y = \log_4 x$ and $y = 4^x$ is:

A. (0,1)
B. (1,0)
C. (1,1)
D. There are no points in common.
E. There are an infinite number of points in common.

12. When evaluating a logarithm, a student reaches the following point: $\log x = \frac{1}{4}(1.3251 - 3)$

The next line in his calculations <u>when simplified</u> should be:

A. $\log x = .3313 - 3$
B. $\log x = .3312 - 3$
C. $\log x = .5813 - 4$
D. $\log x = .5813 - 1$
E. $\log x = .3313 - .75$

13. If $4.9 \doteq 10^{.69}$ and $\log 7.6 \doteq .88$, then the value of $\log(4.9 \times 7.6)$ will be:

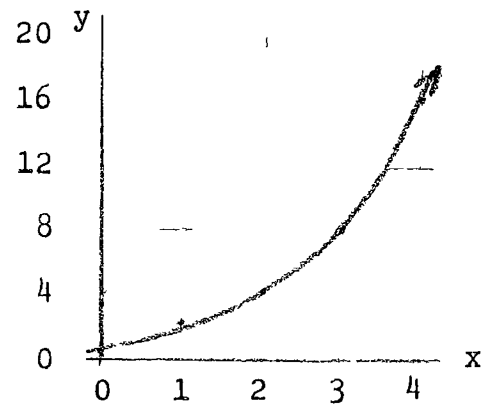
A. 1.57
B. .19
C. $10^{1.57}$
D. .81 - 1
E. cannot be found.

14. If $\log x + \log 2x = \log 36$, then the correct value of x is:

A. 12
B. 18
C. $\pm 3\sqrt{2}$
D. $3\sqrt{2}$
E. none of these.

APPENDIX 5(a)

15. A possible equation for the inverse of the function defined by the accompanying graph is:



A. $y = x^2$
B. $y = 2^x$
C. $y = \log x$
D. $y = \log_2 x$
E. $y = \log 2x$

16. If $x = \dfrac{549.2}{(.00791)^3}$ then:

A. $\log x = \frac{1}{2}\log 5.492 - 3\log 7.91 - 8$

B. $\log x = \frac{1}{2}\log 5.492 - 3\log 7.91 + 10$

C. $\log x = \frac{1}{2}\log 5.492 + 3\log 7.91 + 10$

D. $\log x = \frac{1}{2}\log 5.492 + 3\log 7.91 + 11$

E. none of the above is correct.

17. If $700 was invested today at 12% p.a. compounded semi-annually for 7 years, it would amount to:

A. $\dfrac{700}{(1.06)^{14}}$
B. $700(1.12)^7$
C. $\dfrac{700}{(1.12)^7}$
D. $700(1.06)^7$
E. $700(1.06)^{14}$

18. A person wishes to find how long it will take $1,000 to grow to $5,000, if money earns 8% p.a. compounded annually. Which of the following logarithmic statements would be needed in order to solve the problem. (We let 'n'

APPENDIX 5(a)

represent the number of years.)

A. $n = \log\frac{5}{1.08}$

B. $n = \frac{\log 5}{\log 1.08}$

C. $n = \log 5 - \log 1.08$

D. $n = \frac{\log 1.08}{\log 5}$

E. none of these.

Since you were unaware of this test, it is requested that you say absolutely nothing about it to any students who may take mathematics in another class. They are also going to write this test, and you will not be helping them in any way by telling them about it. On the other hand, you will be doing us a big favour if you keep it to yourself. Thank you for your cooperation.

APPENDIX 5(b)

SEMANTIC DIFFERENTIAL ATTITUDE POST-TEST

INSTRUCTIONS. (These were essentially the same as for the final attitude test and have not been repeated here.)

'WORKING WITH LOGARITHMS"

Pleasant	___:___:___:___:___:___:___	Unpleasant
Long	___:___:___:___:___:___:___	Short
Smooth	___:___:___:___:___:___:___	Rough
Worthless	___:___:___:___:___:___:___	Valuable
Rugged	___:___:___:___:___:___:___	Delicate
Peaceful	___:___:___:___:___:___:___	Ferocious
Shallow	___:___:___:___:___:___:___	Deep
Agreeable	___:___:___:___:___:___:___	Disagreeable
Fast	___:___:___:___:___:___:___	Slow
Masculine	___:___:___:___:___:___:___	Feminine
Unfair	___:___:___:___:___:___:___	Fair
Vigorous	___:___:___:___:___:___:___	Lifeless
Weak	___:___:___:___:___:___:___	Strong
Dynamic	___:___:___:___:___:___:___	Stagnant
Interesting	___:___:___:___:___:___:___	Boring
Sharp	___:___:___:___:___:___:___	Dull
Relaxed	___:___:___:___:___:___:___	Tense
Active	___:___:___:___:___:___:___	Passive

APPENDIX 5(c)

THE CONTENT OF THE FINAL AND POSTTEST ON LOGARITHMS WHEN SUBDIVIDED INTO 6 MAIN CONTENT AREAS INCLUDING THE RELATIVE WEIGHTING OF THESE AREAS

Concept	Questions Covering This Concept		Percentage Weighting	
	Final Test	Post-Test	Final Test	Post-Test
A. Translating expressions into standard form, and knowledge of the meaning of the characteristic, and the ability to obtain the correct characteristic.	#1,2,3, 23, 29, 32, 33, 36	#7,8	22%	12%
B. Knowledge of the range of values of logarithms in the tables along with the properties of the function $y = \log x$ and the function $y = a^x$,	#4,5,6, 12, 13, 19, 21, 22	#3,10, 11, 15	22%	23%
C. Translating from logarithmic notation to exponential notation and comprehension of the logarithmic notation.	#7,8,9, 10, 11, 14	#1,2,4 5, 9	17%	28%
D. Ability to simplify expressions using the rules of logarithms and the special techniques learned.	# 15,16,20 24, 25, 26, 28, 30, 31, 34, 35	# 6, 12 13, 16	27%	22%
E. Solving equations involving logarithms	#17, 18	#14	6%	6%
F. Applications to compound interest	#27, 28	#17, 18	6%	11%

APPENDIX 5(d)

RAW SCORES ON EACH OF THE 4 MAIN SUBTESTS FOR THE FINAL LOGARITHM TEST FOR EACH OF THE THREE GROUPS OF SUBJECTS

Experimental Group Sub-Tests				Control Group Sub-Tests				Single Class Control Group Sub-Tests			
A	B	C	D	A	B	C	D	A	B	C	D
5	5	4	8	7	8	5	9	6	4	2	7
1	4	5	6	7	4	4	7	5	5	5	8
2	5	3	5	6	5	4	4	7	8	5	8
5	4	5	8	4	3	6	7	3	5	3	6
5	6	4	6	6	2	4	8	3	3	1	7
5	3	4	7	4	7	6	8	5	4	5	11
3	2	3	6	6	3	4	7	3	4	4	6
4	7	6	7	6	4	6	6	8	7	5	9
4	7	4	6	5	6	5	6	3	5	4	9
6	8	6	8	8	8	5	7	3	6	2	5
5	8	5	8	7	7	6	8	4	6	5	8
5	7	5	9	7	7	6	8	3	5	4	5
5	6	4	8	7	4	5	9	5	2	5	7
2	6	5	7	5	5	5	9	3	5	3	5
1	0	3	1	7	8	4	10	3	4	5	2
4	8	2	8	6	3	4	9	8	8	5	9
4	7	4	9	5	4	5	9	4	4	5	3
4	5	4	4	7	6	5	10	7	6	5	9
4	5	4	9	8	5	5	6	6	5	5	4
3	4	5	6	6	7	5	8	4	5	3	5
7	7	5	6	7	7	5	9	5	6	5	7
6	7	5	8	6	7	6	11	7	5	5	8

continued...

APPENDIX 5(d)

Experimental Group Sub-Tests				Control Group Sub-Tests			
A	B	C	D	A	B	C	D
6	1	5	8	0	6	5	4
6	4	5	6	6	4	4	3
5	3	3	7	4	6	5	8
7	5	3	9	5	6	5	10
6	6	5	10	5	7	4	8
5	6	4	6	6	6	4	9
7	2	5	10	7	7	6	10
8	7	5	7	6	5	5	9
8	6	5	8	7	7	5	9
8	7	5	9	7	6	5	7
7	7	6	11	8	8	6	9

APPENDIX 6

SELECTED BIPOLAR ADJECTIVES FOR THE SEMANTIC DIFFERENTIAL ACCORDING TO EVALUATIVE, POTENCY, AND ACTIVITY FACTORS.

Evaluative Factor.	Valuable	Worthless
	Agreeable	Disagreeable
	Fair	Unfair
	Interesting	Boring
	Relaxed	Tense
	Pleasant	Unpleasant
Potency Factor.	Rough	Smooth
	Long	Short
	Rugged	Delicate
	Masculine	Feminine
	Deep	Shallow
	Strong	Weak
Activity Factor.	Fast	Slow
	Active	Passive
	Vigorous	Lifeless
	Sharp	Dull
	Ferocious	Peaceful
	Dynamic	Stagnant

APPENDIX 7

FINAL TEST FACTOR LOADINGS

SEMANTIC DIFFERENTIAL ATTITUDE TEST

Scales	Code	CONCEPTS: Logarithms Factor Loadings 1 st.	2 nd.	3 rd.	Quadratic Eqns. Factor Loadings 1 st.	2 nd.	3rd.
Worthless-Valuable	E[a]	.43	-.22	.06	.54	.01	-.03
Fast-Slow	A	.01	-.61	.02	.09	-.12	.82
Passive-Active	A	.34	-.11	.09	.41	.65	.11
Agreeable-Disag.	E	.63	-.21	-.19	.64	-.14	.20
Smooth-Rough	P	-.24	.62	.13	-.18	50	-.27
Short-Long	P	.12	.75	.26	-.52	.13	-.06
Fair-Unfair	E	.34	-.32	-.12	.30	-.04	.44
Delicate-Rugged	P	-.17	.31	-.10	-.11	.23	-.28
Boring-Interesting	E	.79	.12	.19	.71	-.24	-.06
Vigorous-Lifeless	A	.76	.17	.19	.58	.25	.00
Dull-Sharp	A	.91	.22	-.20	.97	.03	-.29
Ferocious-Peaceful	A	.01	.37	-.19	-.14	.74	-.09
Feminine-Masculine	P	-.04	.09	.68	-.16	.03	.49
Dynamic-Stagnant	A	.67	-.14	.13	.43	-.21	.45
Relaxed-Tense	E	.23	-.50	.38	.19	-.61	.15
Deep-Shallow	P	.35	.00	.03	.00	.17	.13
Pleasant-Unpleasant	E	.62	-.20	.04	.68	-.17	.21
Strong-Weak	P	.40	-.18	.11	.39	.24	.34

a The code refers to the expected factor, E(Evaluative), A(Activity), and P(Potency).

APPENDIX 8

TEST SCORES FOR ALL SUBJECTS SEPARATED BY TEACHER AND GROUP.

Teacher	Subject	Dec. Math Exam	Attit. Quad.	Final Log Test	Final Attit. Test	Post-Test Logs	Post-Test Attit.
B	E1[a]	38	20	67	19	28	11
	E2	39	12	50	22	44	22
	E3	41	11	50	24	28	12
	E4	53	7	67	18	44	19
	E5	57	11	64	21	28	25
	E6	58	ab.	ab.	ab.	28	10
	E7	59	7	61	28	56	15
	E8	60	8	42	10	39	7
	E9	67	19	72	16	50	16
	E10	68	10	69	12	ab.	ab.
	E11	70	20	86	22	67	21
	E12	72	23	81	23	39	22
	E13	81	26	75	24	67	25
A	C1[b]	35	8	83	20	72	20
	C2	38	21	67	19	44	14
	C3	40	13	61	17	50	18
	C4	50	20	64	18	17	20
	C5	56	24	64	16	61	24
	C6	58	21	67	20	28	12
	C7	59	20	75	23	39	25
	C8	63	19	64	19	44	10
	C9	64	16	69	12	33	14
	C10	70	20	67	19	72	20
	C11	71	22	86	25	50	26
	C12	71	13	78	12	ab.	ab.
	C13	76	27	86	24	50	21

a 'E' denotes that the subject belongs to the experimental group, and the numbers indicate the matching.

b 'C' denotes control group.

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Teacher	Subject	Dec. Math Exam	Attit. Quad.	Final Log Test	Final Attit. Test	Post-Test Logs	Post-Test Attit.
A	E14	35	21	72	22	ab.[c]	ab.
	E15	44	14	61	13	39	20
	E16	46	11	19	11	33	13
	E17	55	17	67	14	50	12
	E18	58	13	69	15	ab.	ab.
	E19	58	15	53	16	56	10
	E20	63	9	72	13	ab.	ab.
	E21	66	8	64	8	78	7
	E22	72	14	56	23	50	21
	E23	74	22	78	23	72	19
	E24	78	9	81	25	67	12
	E25	89	ab.	ab.	ab.	83	15
C	C14	37	23	75	24	56	23
	C15	38	23	75	23	39	23
	C16	50	19	86	22	44	17
	C17	57	19	64	25	61	23
	C18	57	7	67	14	44	12
	C19	58	20	86	23	50	25
	C20	64	ab.	ab.	ab.	33	23
	C21	67	23	75	22	44	24
	C22	68	16	81	24	44	20
	C23	73	11	83	20	ab.	ab.
	C24	78	21	89	20	56	20
	C25	85	21	69	16	28	12

c 'ab.' denotes absent for that test.

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Teacher	Subject[e]	Dec. Math Exam	Attit. Quad.	Final Log Test	Final Attit. Test	Post-Test Logs	Post-Test Attit.
C	E26	35	4	61	8	39	4
	E27	45	19	56	17	33	11
	E28	50	13	58	17	44	20
	E29	52	21	72	27	39	24
	E30	55	25	72	sp.[d]	39	sp.
	E31	65	25	86	17	72	20
	E32	65	20	58	19	44	14
	E33	65	19	69	20	56	17
	E34	70	20	78	22	67	24
	E35	70	23	78	23	39	22
	E36	75	28	89	28	78	28
	E37	83	20	92	24	72	21
B	C26	35	23	56	18	50	21
	C27	40	18	69	19	50	20
	C28	47	21	47	24	39	22
	C29	52	18	78	21	44	19
	C30	55	19	44	10	56	9
	C31	60	21	72	22	67	20
	C32	65	25	89	24	50	23
	C33	70	18	89	25	78	22
	C34	70	14	75	9	39	9
	C35	73	21	81	22	67	21
	C36	80	12	72	28	ab.	ab.
	C37	87	19	92	18	89	19

d 'sp.' denotes spoiled answer sheet.

e There were two subjects who dropped the course during the experiment, and three subjects for which no match was available. Their results are not included in the above data.

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Teacher	Subject	Dec. Math Exam	Attit. Quad.	Final Log Test	Final Attit. Test	Post-Tes Logs	Post-Test Attit.
B	SCC1[f]	42	11	50	20	ab.	ab.
	2	45	10	47	6	44	8
	3	47	ab.	ab.	ab.	50	21
	4	50	17	56	19	50	18
	5	50	14	67	19	50	15
	6	50	17	44	18	ab.	ab.
	7	52	12	67	18	17	14
	8	52	17	61	17	44	19
	9	55	16	50	21	39	20
	10	57	7	44	24	ab.	ab.
	11	60	12	53	11	33	12
	12	63	17	67	14	33	18
	12	65	21	47	19	28	16
	14	67	13	56	8	50	13
	15	67	21	50	25	50	17
	16	67	24	58	26	44	17
	17	70	26	75	27	44	20
	18	70	18	67	11	50	8
	19	75	24	83	20	78	19
	20	77	ab.	ab.	ab.	33	8
	21	78	26	75	22	83	19
	22	80	26	83	12	67	8
	23	80	17	86	14	67	10
	24	90	21	89	21	83	24

f These are the results for the single class control group.

APPENDIX 9

A COPY OF THE QUESTIONNAIRE GIVEN TO SUBJECTS OF THE EXPERIMENTAL GROUP BETWEEN THE FINAL TEST AND POST-TEST, FOLLOWED BY A SUMMARY OF THE RESULTS

1. Questionnaire.

The following questions are designed to determine your feelings toward using the calculator while studying logarithms. Please indicate your feelings toward each statement by marking the appropriate space for each statement. Use the following guide.

AA	Agree Absolutely	DA	Disagree Absolutely
SA	Strongly Agree	SD	Strongly Disagree
A	Agree	D	Disagree
U	Undecided		

1. I would like to continue to use the calculator in other topics in mathematics.

   _____ _____ _____ _____ _____ _____ _____

   AA SA A U D SD DA

2. I probably could have scored higher on the final test on logarithms if I had <u>not</u> used the calculator while studying logarithms.

   _____ _____ _____ _____ _____ _____ _____

   AA SA A U D SD DA

3. The calculator tended to confuse me when used in studying logarithms.

   _____ _____ _____ _____ _____ _____ _____

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4. I felt that the calculator saved time in class in working solutions to the problems involving logarithms because I did not need to look up the numbers in the tables.

____	____	____	____	____	____	____
AA	SA	A	U	D	SD	DA

5. The calculator was generally helpful in studying the unit on logarithms.

____	____	____	____	____	____	____
AA	SA	A	U	D	SD	DA

6. Do you have any opinions on the use of the calculator in class. Please feel free to express them below.

B. Summary of Results.(N = 32)

The results of each question are summarized by grouping the three scales at each end into one, thus forming three general areas of opinion: Agree, Undecided, and Disagree.

	Agree	Undecided	Disagree
Question 1	62.5%	21.8%	15.6%
2	25	25	50
3	25	3.9	62.5
4	87.5	6.3	6.3
5	65.6	21.8	12.5

APPENDIX 10

A SUMMARY OF STUDENT COMMENTS TO QUESTIONNAIRE

"The calculator definitely saved time by finding the proper logarithms without using the tables. Looking up in tables is for kindergarten students. However, I believed it would be more useful if the student had to add, multiply numbers instead of just pushing buttons. To teach logarithms efficiently I believe one must use a blend of both techniques."

"As long as the class has a teacher who cares enough to teach the log method both ways as it was taught to us, I feel the calculator is an excellent tool which should be used not only in mathematics but in other areas such as chemistry and physics. The calculator cut down in wasted time and the class benefitted from this enormously."

"I felt I was getting confused with the calculator because there were so many buttons to press. It helped alot in the basic math like multiplying, dividing, adding, subtracting etc."

"I think we did a lot more in class than we could have otherwise, which gave us more time to ask questions. I cannot understand why the calculator is not used more in schools as it is a readily available piece of equipment and is fairly cheap. It can be argued that it is a crutch in studying

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logarithms, but so are log tables."

"I think in the calculator group we learned just as much and more than the non-calculator group. We did the work using the charts and learned how to use a calculator at the same time."

"The only place I feel calculators would help would be in Chemistry."

"I enjoyed this calculator class very much. It saved alot of time consuming search through tables. This enabled me to do more problems and to have more practice. It also saved me alot of errors in looking up in the tables."

"I feel using the calculator saved time, and did not interfere with my learning of the subject. As long as the student is taught how to do the problems in longhand, there should be no problems in using the calculators."

"I think the calculators should be used only when the student understands how to do the same work by hand."

"The calculator did not help in understanding logarithms as such, but perhaps it was a doorway to better understanding because it allowed more concentration on the main question without interference from tedious arithmetic."

"They were helpful in the way we could do more homework. I still feel that If I had worked with the book and no

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calculator, I would have done better."

"By the time I figured our how to use the calculator, I would have been further ahead to use the tables."

"I did not feel that the calculator helped at all. If anything it did harm."

"After using the calculator to figure out answers, I couldn't get used to not using the calculator during tests."

"They were faster but no help because you had to know how to do it out the long way which is needed more."

"I did not think that the experiment did justice to the calculator because we did not have use of them for homework and there was not any testing when we were allowed to use them."

"I found that trying to use and remember the buttons on the calculator a big enough feat, as there were so many equations to do and thus so many buttons to push."

"They aren't good in a classroom situation because they don't give the students the practice they need."

"I feel that calculators should only be used by students who have good background in basic mathematics, otherwise I feel that the basic math will be deteriorated and students will become dependent on the calcualtor to do their math for them."

"I feel that the calculators were helpful only up to a certain

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point. By this I mean that it was good to learn logs, or any new material by working hard at it by yourself without relying on a calculator, which is what we did. Then when we had learned how to use logs confidently and were forced with tedious calculations was the time when we should have used calculators."

"I feel that we used the calculators too much in doing problems in the class and not enough of working them out."

"I found using calculators a good experience and it didn't hinder me from fully understanding logarithms."

"Since we were not allowed to use calculators for the test, some people were more interested in using the tables."