INFORMING MATHEMATICS TEACHERS' REFLECTIVE PRACTICE WITH STUDENT SURVEYS ON AFFECTIVE DOMAIN

BRUCE MCLAURIN

A thesis submitted to the University of Ottawa

in partial fulfillment of the requirements for

the degree of Master of Arts in Education

Faculty of Education
University of Ottawa

Thesis Supervisor: Dr. Christine Suurtamm

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Abstract

This thesis examines the potential of a change analysis of student beliefs and attitudes about mathematics to inform teachers' reflective practice and provide the basis for modifying classroom practice. The author and two colleagues were involved in *Math4theNines*, a collaborative inquiry project for Ontario Grade 9 Mathematics. As part of that project, they developed an online survey to track the impact of their classroom practice on how students felt about mathematics and how students felt about themselves as learners of mathematics. The teachers reported that the before-and-after course survey and the accompanying change analysis that indicated any shifts in their students' attitudes and beliefs toward mathematics provided some unique and revealing perspectives on their practice. This study is a retrospective of that experience and an attempt to reproduce the results with three volunteer teachers. Although the results were mixed, there is some evidence to suggest that this approach has the potential to enhance teachers' focus on the new social-emotional strand in Ontario Grades 1 to 9 mathematics which presently is neither evaluated nor reported on. There is also potential to inform teachers in their efforts to develop the positive attitudes that have been shown to improve academic achievement and encourage entry into STEM fields.

Acknowledgements

While completing my Bachelor of Education at the University of Calgary, I was encouraged by Dr. William Hunter to pursue graduate work. My initial intention was to preface a master's degree with some classroom experience. Three decades later I was ready. I retired from the classroom, but my interest in education remained.

I sincerely thank Dr. Suurtamm for agreeing to supervise my thesis, as the opportunity to work with her was my main motivation for completing a master's thesis. She provided direction for my research based on her knowledge of my work as well as an enormous amount of support and patience that allowed me to work at a measured pace. Dr. Suurtamm assembled a supportive thesis committee whose insight added to my research. I am appreciative of the time and guidance that Dr. Ruth Kane and Dr. Raymond Leblanc provided.

I am very grateful to the teachers who volunteered their time and energy for my research. It is a challenge to add one more task to an already demanding occupation. It was a joy to talk 'teaching' with these dedicated educators. They represent the long list of devoted educators who shaped my understanding of education by sharing their ideas and experience.

I wish that I could personally thank all my students who shaped my ideas about teaching and learning. Teachers are not born but made, and I am no exception.

Finally, I would like to thank Cathy, my partner of 38 years and counting, who, after decades of supporting me throughout my teaching career, encouraged me to complete my MA. Better late than never.

Introduction

Statement of the Problem

"*Mathematics is the work of the devil and frankly, I despise it!*" These are the words of a student of mine on his first day of high school. He was responding to an activity in my Grade 9 applied mathematics [MFM1P][1] class. All the students were asked to write down their feelings about mathematics on a sheet of paper without including their name. When everyone finished writing, I asked them to crunch up the paper into a ball and on my signal, they tossed it across the room in a somewhat cathartic act. Then I instructed them to retrieve one of the paper snowballs, return to their desks and read it. Some students offered to read aloud the paper in their possession. We repeated this cycle a few times; crunch up the paper, toss it, then retrieve and read it. After which, I collected the papers so I could share them all the following day. For all the times that I did this activity, none of the authors' identities were ever revealed, except in this example concerning the devils' work. As the students began to write, he was the one near the front who raised his hand and in a strong, defiant voice asked, "How do you spell *despise*?"

My experience as a teacher of mathematics has taught me that student self-preservation can manifest itself in acts of defiance. Negative appraisals are not uncommon, nor surprising. I am very familiar with the sense of helplessness and mistrust that many students carried into Grade 9 applied math. These feelings can be exacerbated with subsequent mathematics courses and can persist into adulthood as revealed by my university students in the education department. I lament that the subject I was employed to deliver was the source of such misery. Yet this student's derision is at the heart of why I entered the teaching profession. My goal was to change the way mathematics was taught and change the way students felt about the subject. Knowing the identity of this student gave me an opportunity to evaluate my teaching.

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[1] In Ontario until 2021, Grade 9 students were *streamed* or divided into two main tracks of mathematics. The *academic* course [MPM1D] was intended to "develop students' knowledge and skills through the study of theory and abstract problems" whereas the *applied* course [MFM1P] was intended to "develop students' knowledge and skills through practical applications and concrete examples" (Ontario Ministry of Education, 2005).
In 2021 the new MTH1W course amalgamated the two to destream Grade 9 mathematics.

Our school used the Ontario provincial exam from EQAO [Education Quality Accountability Office] as our final exam in our Grade 9 courses. As a result, there was a narrow window to evaluate the exams and meet the shipping deadline, leaving minimal time to peruse the accompanying completed student survey. That semester, I made a point of checking one item on one particular student's survey; 'I like mathematics'. The student who had despised mathematics had the option on the 5-point Likert scale to answer 'strongly disagree'. Instead, he penciled in the slightly milder 'disagree'. I accepted that as a small victory.

This marker was my evidence that I had *moved the needle*, and by doing so, I might have improved other students' attitudes toward mathematics as well. Although I was never totally confident about my ability to make a difference, this very small sample size was enough to offer some reassurance that I was doing what I had set out to do; to teach mathematics in such a manner as to improve students' attitudes and possibly increase their appreciation of mathematics. It wasn't until towards the end of my career that I stumbled on a more comprehensive way to assess my impact on students' beliefs and attitudes toward mathematics.

When one student refers to math as the work of the devil, that is a problem. It is also a problem when only 44% of students in applied math achieved the Ontario EQAO provincial standard, versus 84% of those in academic math (People for Education, 2013). I believe that these problems are undoubtedly connected. Destreaming Grade 9 math in 2021 has altered the numbers but the problem persists. Combining the results for academic and applied in 2018-2019 showed that 75% of Grade 9 students met the provincial standard. With destreaming, 52% of Grade 9 students met the provincial standard (EQAO, 2022). Some might blame this dramatic drop on a newly implemented program lacking adequate support, but the negative effects of the pandemic cannot be overlooked. What I found more interesting is that 52% of the students met the standard and 53% indicated they liked math. Coincidence? I think not.

If student beliefs and attitudes are as important in the learning of mathematics as I claim, and when math teachers, novice or experienced, do not have a clear idea about how their classroom practice is affecting their student's beliefs and attitudes about mathematics, that is a problem worth studying.

Beginning in 2014, I was a member of a five-teacher team from my school that joined nine other Ontario schools in *Math4theNines* (2015). This was a two-year government funded collaborative inquiry project to improve achievement in Grade 9 applied mathematics. In an

effort to track the impact of our classroom practice and how the students enrolled in Grade 9 applied felt about themselves as math learners, we developed and administered a before-and-after student survey on beliefs and attitudes toward mathematics. The analysis of the *change* in attitudes proved significantly more insightful than any single course exit survey that I had administered previously.

My previous exit questionnaires had been effective at yielding students' perceptions of the course presentation and content. I expected that the exit surveys provided a general reflection of how students felt about the course, but I also suspected that any judgments were heavily influenced by how they felt about me, as a teacher and maybe as a person as well. The exercise of subjecting oneself to student exit surveys might appear to come with a certain level of risk as it opens up an avenue for criticism. Yet this is a relatively safe activity for a number of reasons. Most high school mathematics courses are compulsory for graduation or a requirement for admission to a postsecondary program. Within that rather rigid framework, students do not have the option to choose their teachers. The conditions are set and students have to resign themselves to the path they have chosen or have been funneled into by previous marks and prerequisites. This could explain why a review of ratemyteacher.com shows that there are very few reviews of math teachers in my school compared to other departments. Another factor which probably softened any criticism in my year-end surveys was a lack of anonymity before grades were finalized. Students' responses were handwritten and therefore never quite anonymous.

I believe that there are certain aspects of courses and teaching which can be assessed honestly and accurately by students in a year-end survey. But a student's evaluation of the general quality of a course is ultimately a guess as to how they feel they might fare in a subsequent course. This comes from the compulsory nature of the mathematics curriculum and speaks to the culture that pervades secondary mathematics. The priority for math teachers is to prepare students for the next grade level. The priority for students is to prepare themselves for the next grade level. Math teachers tend to pass judgment on students' previous teachers based on how prepared students are entering their class. Math teachers also fear that they will be judged by their colleagues in the next grade if they fail to prepare their students adequately. Student performance in subsequent grades can be a measure of teacher effectiveness. Teachers of Grade 8 math were often eager to know how their students were fairing in my Grade 9 class. (This pressure to perform is passed from teachers to students and can cultivate a culture of

memorization, tricks and calculation shortcuts that fail to develop real understanding.) A student may be left to base an evaluation of a course on a high mark which seems to indicate they are adequately prepared for the next level. Experienced teachers understand that is not a guarantee of success.

At the time, I had a sense of the limitations of end-of-course surveys that I have outlined. These surveys provided helpful feedback without any real surprises. It was all that I knew until I was exposed to the change analysis that indicated a shift in student beliefs and attitudes about mathematics. This offered a new form of feedback which I felt had significant potential. It asked students not to judge their teacher or the course but to express their own ideas which could open a window into how my teaching was influencing student attitudes toward math. Since then, I have questioned if the change analysis of a two-part survey of students' beliefs and attitudes might be beneficial for other teachers to monitor and reflect on their own classroom practice. This led me to the following research questions:

1. In what way might a student survey provide teachers insight into their students' beliefs and attitudes about mathematics?
2. How might the change-analysis results of a before-and-after student survey on beliefs and attitudes on mathematics provide teachers with an understanding of how their classroom practice affects students' beliefs and attitudes concerning mathematics?
3. In what manner might the change-analysis of a before-and-after student survey on beliefs and attitudes toward mathematics promote teachers' self-reflective practice?

Background

This research thesis represents a qualitative study of the impact of a student survey on the reflective practice of teachers. It is a monograph which includes academic references and data collected from interviews with teacher volunteers. Yet the foundation of this study relies heavily on my personal and professional experience from nearly three decades in the classroom. I come to this project not as an academic, but as a teacher whose career was characterized by a willingness to experiment with various pedagogies and reflect on their effectiveness. The desire to better understand the complexities of teaching mathematics which dominated my career remains with me. This thesis is evidence of that. The act of incorporating qualitative methods to observe teaching culture defines this as a descriptive ethnography. This study also represents my

chance to reflect on past adventures and reconnect with colleagues who are continuing to refine their teaching methods. Since I am revisiting chapters of my own story, it contains elements of an autoethnography. I insist on the label *story* quite intentionally rather than the more academic *narrative* or *account*. Any attempt on my part to produce a non-biased research thesis will be coloured by my ideas and experiences as a secondary school mathematics teacher. I do not consider this a detriment. It is my belief that teacher stories are an important avenue for change and when teachers share their reflections of their own practice, they have the potential to make significant contributions to the evolution of education. Change can and will happen when teachers become learners of their own teaching and researchers of their own practice (Schön, 1983; Hattie, 2012).

Reflections on a career of teaching mathematics

The reasons that I became a mathematics teacher are central to my story. My story begins with my own experience as a student in high school where my interest in mathematics was eroded by uninspiring lectures and tedious exercises. I liked math and I liked my math teachers, but I found very little joy in the plodding workings of the mathematics classroom. As my interest waned, so did my achievement. With non stellar grades in math, I was accepted into a general science university program. After a few anemic years of university peppered with travel and work, my interest in math was revived by working as an instructional aide of mathematics in an innovative secondary school. My task was to assist students with difficulties as they navigated through a self-paced, preset mathematics program. As I worked alongside students on problems, I began to build mathematical connections that had eluded me in high school. It was then that I began to learn, understand and appreciate mathematics. I understood high school math for the first time in my life and, with that, came the realization of what I wanted to do with my life. The joy of mathematics that I had once experienced was reawakened and, with it, a desire to become a math teacher. I wanted to provide a more varied classroom experience where students constructed a deeper understanding of mathematical concepts and built an appreciation for the beauty of math, and perhaps more importantly, where they found their place within that world. I imagined a classroom experience that I felt that I was denied and that all students deserved; an experience that might foster those feelings of delight that can surface when mathematical connections reveal themselves. I had no illusions that I possessed the answers and was not

confident that I had the talent and prerequisites. What I did have was the desire and the belief that finding solutions can be joyous, but the true joy is in the struggle.

I returned to university and fashioned my unfinished B.A. into a mathematics major, immediately followed by a two-year B.Ed. program. My education major was secondary mathematics, but it was my minor in computer science that opened doors. My computer skills were both a blessing and a curse. They landed me jobs in British Columbia and Ontario, but at the same time, ensured that the majority of my teaching assignments were in computer science rather than mathematics. Although being the lone computer science teacher in the building allowed me to develop a certain level of creativity and independence, it detracted from my focus to develop a better way to teach mathematics. My role as the school's computer contact teacher was an even greater distraction. The unofficial title of 'computer guy' meant that I not only ran the computer labs and the school website but provided information technology support to the main office staff, guidance counsellors and every classroom teacher. Teaching mathematics was relegated to the position of a pleasant diversion which relied heavily on the required textbook.

In the late 1980's when I studied mathematics education there was a focus on problem solving, promoted in textbooks and classroom posters. In my teaching, I emphasized Polya's (1948) problem solving model and numerous problem-solving strategies such as working backward or starting with a simpler problem. My efforts were met with dubious looks from students and a suggestion from a more experienced educator that I should "forget problem solving; give them a few skills and move them along." Yet I remained determined to build a classroom where questioning and thinking were the norm and not the exception. I recognized that my computer science courses involved more authentic problem solving than did the limited number of math courses I was teaching. Still, I took my job seriously, perhaps too seriously. As a teacher of mathematics, my style prompted one student to remark, "McLaurin? Rhymes with borin'." Later in my career I heard a story from a teacher of English Language Learners. She asked her students which teacher epitomized the relationship of a friend. I didn't make the list. But when she asked which teacher best represented the role of parent, my name rose to the top. (This was in direct contrast to my parenting style where I was accused of being a teacher.) When teaching math, I tried to reduce the *chalk and talk*, limit the focus on the textbook and build mathematical connections with problem solving. Change was slow. Despite a year-end greeting card from an academic class of Grade 12 students that proclaimed I was "the best math teacher

ever", my mathematics classes remained stubbornly traditional. Teaching the so-called *non-academic* courses offered new possibilities.

Finding purpose teaching non-academic classes

As a part time member of the math department, I was often assigned singletons and non-academic math courses. To meet the needs of students in the non-academic courses, I found the impetus and the freedom to break from the shackles of convention. My experience was that students who have not had success in the past were more willing to accept a different way of doing things. An acceptance of change also came from the parents who witnessed their children's frustration resulting from a lack of understanding and limited success. It was a case of nothing ventured, nothing gained. The challenges and opportunities bestowed by two particular classes changed the trajectory of my teaching career.

The first was an unruly group of students in a Grade 9 applied class. My established classroom approach proved ineffective. Any attempts to provide detailed instructions were met with raucous outbursts. I had no desire or intention to be a disciplinarian. I harkened back to a Grade 7 class that I volunteered in during my B.Ed.. Student behavior was strictly controlled by the presence of a strong-willed teacher but devolved into total chaos when she was absent. Any notions of being a 'don't-smile-until-Christmas' kind of teacher vapourized with that experience. Hoping to allow students' self-control to develop, I experimented with other approaches to improve engagement. I placed formal lessons and textbooks aside and relied heavily on the Ontario Ministry of Education resource entitled Targeted Implementation and Planning Supports [TIPS4RM]. I also incorporated physical activities such as running and basketball for data collection. The students worked on TIPS4RM materials most days and my direct instruction was limited to individuals and small groups. The elevated noise level, lack of student notebooks and activities that bordered on chaos may have had my colleagues questioning the collective ability of both me and my students. In the end, there was some solace in learning that that particular group of students scored noticeably higher than the school's previous cohorts on the compulsory EQAO provincial exam.

The second class that further shaped my thinking was a locally developed Grade 9 essentials mathematics class [MAT1L]. Math Essentials was offered for students entering high school who lacked the prerequisite skills to enter the academic or applied stream. My small

group of initially timid and wary students gained confidence in their ability to do mathematics over the course of the semester. They insisted that I provide them with a one-and-a-half-hour final exam to show that they could do math just like all the other students in the school. My success with these intrepid students spurred me to seek new opportunities. With that, I began a new chapter by accepting the position as the head of the mathematics department at another school.

Teaching math full-time for the first time and establishing a fortuitous partnership

The move represented several important changes. I shed my computer technology duties and began to teach mathematics full time with a concentration on non-academic courses. It also led to fertile collaborations with inspiring teachers. I requested that my teaching assignment should include all sections of Grade 9 applied. This request was questioned repeatedly by the outgoing department head. This course was rarely requested by any teacher and was regularly assigned to teachers outside of the mathematics department. It had a reputation among teachers, parents and students as a watered-down academic program. Students streamed into the applied program begin high school by being segregated into what often feels like a lesser class while their more successful elementary school cohorts are enrolled in the academic stream. From day one, these students are often distrustful of a system that had seemingly failed them.

These disenfranchised students represented fifty percent of my two classes of Grade 9 applied math during the first semester. The other half of the classes were students who were repeating the course. One unfortunate student was repeating the course for the second time. The fact that such many students in the previous year had failed to attain a Grade 9 math credit was proof positive that a different approach was needed. As I tried to carve out a new pathway, that particular collection of students was able to express their disappointment and resentment in many creative ways. My hands-on activity to develop volume formulas of three-dimensional solids ended abruptly when the dried beans intended to represent volume became projectiles. I thought that the offending materials had been safely collected, but the beans continued to ricochet off the walls into the following week as the students expressed their discontentment. This inauspicious beginning might have been demoralizing for a new teacher or have been evidence to a more experienced teacher that hands-on activities were, at the very least, an effective waste of time. I was not deterred. In time, my students learned to develop concepts with mathematical thinking

tools or manipulatives such as linking cubes and pattern blocks as well as more common items such as string and tape measures. I measured my success in daily student engagement and a vastly improved pass rate. Armed with this evidence I looked for a confident, seasoned teacher to teach the Grade 10 applied [MFM2P or 2P] program.

I found a willing colleague who worked hard to incorporate various approaches such as portfolios in an attempt to engage the students. After three semesters he became frustrated with the results and announced that he was "done" with teaching 2P. He asked me to reassign his second semester's 2P class to someone else to teach. As luck would have it, a second MFM2P section was created in the weeks leading up to the new semester. I suggested that if he held on to his 2P section, I would take the new section and we would work on the course together. His competitive nature drove him to accept this arrangement. What followed was a fortuitous alchemy between two experienced teachers, one who had found some success in an alternative approach and one who was ready to change to meet the needs of students in the applied stream. One teacher had begun to revamp his practice and the other was about to. We began by throwing out all our established teaching norms to create a blank slate. We then developed a fundamentally new approach with two foundational ideas. First, we would attempt to engage students with play during the first month and expose the curriculum through hands-on activities. Second, we wouldn't restrict and define our play within confining units of study. Instead, various topics would intertwine in our activities. We met almost every day at lunchtime to plan our next move in the evolution of the course. Our improvisational process of building the course prompted my colleague to declare that we had no idea what we were doing. In one sense he was absolutely right but I countered by insisting that I knew exactly what we were doing; we were simply sailing in uncharted waters. I was probably more comfortable with route finding. For me, this was part and parcel of action research and to quote Albert Einstein, "If we knew what we were doing, it wouldn't be called research." I equated our undertaking with problem solving and the fact that problem solving is what you do when you don't know what to do. With experience I had learned to embrace the struggle and trust the process.

Our successful collaboration added the important element of spiraling to my practice and fundamentally changed my colleague's approach. His long-established teacher-centred chalk and talk had been supplanted by a student-centered, hands-on approach steeped in inquiry. We began to include spiraling and activities with multiple expectations in our academic courses as well.

Spiraling allowed us to work on the entire curriculum for the entire course. For the first time we could fully honour the directive of the Ontario curriculum document "By the *end* of the course the student will …" (OME, 2005). The corollary being that it allowed students multiple opportunities to demonstrate their learning as advocated in the Ontario Ministry Education document *Growing Success* (OME, 2010). We could also report a somewhat counterintuitive and surprising result. Spiraling gave more time to complete the course material. We attributed this to the fact that students had choice in their problem-solving techniques and progressed at their own speed. Since we were no longer confined to a lockstep approach which required all students to be assessed at the same time, the learning tended to be unfettered and we progressed faster. Buoyed by our success with this new approach, we shared our findings with teachers in other schools and school boards.

Initially, the basis of our presentation and workshops was our experience and our convictions. This was typical of other teacher conference presentations that I attended. In an effort to add legitimacy to our presentations, I searched for research to support spiraling. I discovered Ebbinghaus's (2013) forgetting curve from 1885 and a study by Taylor and Rohrer (2010) on *massed* versus *spaced* practice. I learned how spaced practice requires students to recognize previously studied concepts and retrieve past knowledge and skills. This increased cognitive load is considered as a *desirable difficulty* (Anderson, Bjork & Bjork, 1994). Despite being unfamiliar with Bruner's (1960) work, we stumbled onto his basic principles of spiraling. Our approach was cyclic with increasing depth requiring students to draw from their past knowledge. We also avoided one of the key criticisms of spiraling. We were not *chunking* content into topic blocks. Instead, our lessons required students to make connections between topics in what could be called a *strand curriculum* (Snider, 2004). The interest in educational research demonstrated by our workshop participants was encouraging. Backing our own classroom experience and action research with academic research gave our approach more legitimacy. Our motivation for seeking academic research was not to reassure ourselves that what we were doing was sound practice, it was to give credence to our presentations. Although the research seemed to support our approach, our implementation preceded substantiating research. We remained typical teachers who began with traditional methods and relied on experience and *feel* to guide us.

Combining action research and academic research

I have always considered my teaching model to be *teacher as action-researcher* on a perpetual quest, searching for ways to meet the diverse needs of all my students. I relied on day-to-day student engagement rather than any formal evidence or questionable quasi-formal evidence like test results. Like most secondary teachers who work in the isolation of separate classrooms, my methods were rarely questioned, sometimes by students, rarely by parents, never by administrators. Once I deferred to my students' wishes to have textbooks. That was a week-long experiment that they were happy to end. A parent once challenged our policy of grading with levels rather than marks and percentages. She argued that mathematical calculations require precision and math should be evaluated with equal precision, like when she was in school. I defended assessment with levels, confident with the knowledge that it was a mandated provincial policy. I suggested that despite the appearance of precision in percentages, there had always been very little precision in what was taught, tested, and evaluated. The use of levels was just one area of innovation that we were developing. She then asked, "Are you telling me that you are experimenting with my children's education?". In an uncharacteristically cheeky manner I responded, "Every damn day". And each day I would evaluate my action research by largely relying on what felt right in the classroom. But after our experience with spiraling, I became more aware of educational research and looked for outside evidence of innovative practice. That led me to a new approach for Grade 9 applied mathematics.

I was often disheartened with the sense of helplessness that many students brought to my Grade 9 applied course. Students exhibited a *fixed* mindset rather than a *growth* mindset (Dweck, 2006). My efforts to convince them that they could do math if they tried often fell flat. What I realized is telling students they *can* do math is often no match for all the past evidence of failure that they have experienced. It was my assertion that the way to promote a growth mindset was not by talking about it and putting up posters (as I once did to promote problem solving) but with accessible classroom activities. Engaging activities had improved student mindset somewhat but by streaming students it guaranteed they were starting from a position of 'some can do math, and some can't'. This was a difficult hurdle for students and teachers of Grade 9 applied mathematics. An opportunity to change that came when I read a newspaper article (Rushowy, 2014) on efforts to destream Grade 9 mathematics at Granite Ridge Education Centre. This led me to a report by People for Education (2014) which highlighted the social inequities inherent in

streaming. Their research showed that a higher percentage of students in lower income, non-white neighbourhoods in Toronto were c*hann*eled into Grade 9 applied math rather than into the academic stream. In turn, students in the applied level had lower graduation rates than their cohorts in the academic stream. In Granite Ridge's trial program, streaming was postponed to Grade 10 like other jurisdictions outside of Canada such as Poland. When I met with educators at Granite Ridge, I learned that in their destreamed model, students who struggled in academic math were pulled from French class for extra periods of math instruction.

For our school starting in September of 2015, I proposed a model which would not compromise French credits. Rather than destreaming, we created *combined* classes by distributing students registered in a single section of applied math into three other sections of academic math. These students continued to be registered in the applied level. They worked alongside the students in academic level but were assessed according to the applied level curriculum requirements. To make this model a reality I solicited three teachers who were willing to accept the challenge, namely me and two other experienced teachers. Before we fully implemented our Grade 9 combined classes, we had one more element to add to our practice.

One Saturday in May, I, along with a number of teachers in my department, attended a presentation at the 2014 Canadian Mathematics Educators Forum on vertical non-permanent surfaces and visibly random grouping (Lillejdahl, 2020). The research was so compelling that, with only five weeks remaining in the semester, I along with many of my colleagues implemented these practices the following week. Being handed an evidenced-based approach that had been field tested by others gave us the confidence to execute these changes without hesitation. That proved to be the first time that I implemented changes to my classroom practice that were actually based on extensive academic research, albeit a single source.

The following September I witnessed an immediate change in the attitudes and behaviour of the students of the applied stream in the presence of their cohorts in the academic stream. My energies were freed from being a motivator who was constantly cajoling to being a facilitator developing mathematical thinking. My combined academic and applied class was a mash-up of hands-on activities, spiraling curriculum, vertical non-permanent surfaces and visibly random groups. Within that mix, we added another layer with formalized lesson studies.

A government initiative to improve student achievement of all Grade 9 applied subjects funded the lesson studies of our school's professional learning community. Each lesson study

cycle involved five teachers and one administrator who co-created a lesson for one teacher in a specific class, which was delivered by the teacher of that class while the observing teachers recorded student interactions, followed immediately by a lesson debrief. Student exit cards became an important feedback mechanism of the lesson debriefing sessions. I became more aware of the potential of an underutilized element in my classroom research, student voice.

Throughout my career student feedback in my classes was inferred through my observations, daily reflections on student engagement, when to advance and when to retreat. There were also my informal statistics of improved student attendance, fewer discipline problems and formal statistics such as the reduction in course failures. Direct student feedback was limited to quick checks like thumbs-up / thumbs-down and occasional course exit surveys. Lesson exit cards became an important element in our lesson studies and added another opportunity for student voice. Yet exit cards, exit surveys and quick classroom checks are mere snapshots of student voice that failed to capture any changes in attitudes and beliefs.

As I developed classroom practices that were manageable, based on experience and in accordance with personal beliefs, I was never fully confident that my approaches were fostering positive student attitudes toward mathematics. Was my approach getting at the heart of why I became a math teacher; to instill in students an interest and appreciation for the subject? Throughout my teaching career I have always felt unsure of what effect my class was having on my students. Was I contributing to the overall decline in an interest in mathematics (Singh, Granville & Dika, 2002) that occurs for many students? The importance of students' beliefs has been shown to be an integral part in the choice of science, technology, engineering, mathematics fields [STEM] (Xie, Fang, & Shauman, 2015). If education is failing to inspire students to continue studying mathematics, mathematics teachers are undoubtedly at the centre of the issue. Disenchantment must be occurring in the math classroom staffed by a teacher of mathematics. I am a math teacher. Was my approach helping or hurting? The opportunity to investigate this more closely came when our school partnered with university researchers in a government sponsored project on Grade 9 applied mathematics.

The genesis of the student survey on beliefs and attitudes toward mathematics

Math4theNines was a two-year collaborative inquiry project supported by the Ontario Ministry of Education [OME] with the Ontario Association for Mathematics Education [OAME]

and the Ontario Mathematics Coordinators Association [OMCA] in collaboration with a research team from the University of Ottawa. As one of ten schools from across Ontario, we were assigned a graduate student to document the workings of our professional learning community. At that time, we were spiraling our curriculum with hands-on activities, incorporating visibly random groups working on vertical non-permanent surfaces with combined classes of Grade 9 academic and applied. We wanted an instrument to track the impact of our classroom practice and more specifically on our students enrolled in Grade 9 applied and how they felt about themselves as math learners. Perceptions have their place, but we were seeking more formal evidence. Aided by the graduate student, we developed a student questionnaire on beliefs and attitudes about mathematics (Appendix A). For sample survey questions we referenced a number of sources including Schoenfeld (1989), Boaler (2000), Stipek et al. (2001), Zakaria & Musiran (2010) and Brookstein et al. (2011) as well as the EQAO student survey. We also added some items concerning classroom practices such as group work on whiteboards. Six areas of interest were considered in the construction of the survey. (Table 1)

Table 1: Area of interest and sample survey items

Area of Interest	Sample Item
Math as a set of operations versus a tool for thought	The math that I learn in school is mostly facts and procedures that have to be memorized.
Correct answers versus understanding as primary goal	Making mistakes in math helps me learn.
Enjoyment of math	I like math.
Mindset	Some people are born good at math, some people are not.
Usefulness of math	The math I learn now helps me do work in other subjects.
Classroom practices	I enjoy group work in math class.

The online survey was administered at the beginning of the semester in three Grade 9 classes taught by three different teachers; me and two experienced teachers, both who had been teaching since the year 2000. Using personal cell phones or the classroom computers, students accessed a Google Form where they responded to 23 items on a 4-point Likert scale. The exact same survey was repeated at the end of the semester and our graduate student provided the change analysis which showed the percentage change from beginning to end of students in each

class who agreed or strongly agreed with each item. Each iteration of the survey included the students' names so the graduate student was able to match the data set of students from the beginning of the semester to the set of students who completed the end of semester survey. This allowed her to fix any incomplete or duplicate data that may have skewed the results. We received the results in table form during the first few weeks of the second semester. This allowed us to gauge any shifts in students' attitudes and beliefs and I was enthralled by what I saw.

The outcome of the survey

I remained cautious concerning the validity of the results, yet I was convinced that the survey offered better evidence of change in student attitudes than simply relying on any *feelings* that I might have had about the effectiveness of my approach. I was buoyed by the results. I felt that in terms of fostering a positive attitude toward mathematics and promoting its problem-solving nature, I had 'moved the needle'. This survey did more to relieve my doubts about the effectiveness of my teaching than anything else in my career. Teacher performance appraisals, acknowledgement from colleagues and teaching awards were encouraging but, for me, they do not compare with the level of affirmation that I received from this formal feedback from students about their experience. No single event did more to build my sense of self-efficacy. I was never totally comfortable with feedback about my teaching, however complimentary. The source of my own insecurity was rarely diminished by a focus on what *I* was or wasn't doing. What good teaching looks like will always be open to debate. In my mind, the only real way to measure the effectiveness of classroom practice is to pay attention to the effect on the students. Student test scores and pass rates can be important measures of good teaching but rely in large part on the subjectivity of teachers and can vary widely between classrooms. When the decisions of what to assess, when to assess and how to assess student work is left to the teacher, there are obvious differences in standards. Standardized testing comes with its own set of problems. The focus on test scores and the pursuit of higher marks can have a negative effect on learning (Shepard, 2000) and in the case of mathematics, turn kids off.

We three teachers were not only able to see shifts in student attitudes and beliefs in our own classes, but we also compared classes. Sharing results was explicitly discussed beforehand to ensure that the fear of exposing oneself to judgment was not an issue. These three veteran teachers each expressed total comfort in sharing their own class results. I found the comparison

particularly fascinating. There were similarities that helped to reassure us that there was a certain level of validity in the process. There were also contradictory results that seemed to indicate that students were interpreting the items in a way we had not anticipated. The results also indicated striking differences between the classes.

In most areas of interest, the results were consistent within each class. For example, Table 2 and Table 3 show that Class A showed a percentage increase in students who liked math and also those who considered it one of their favourite subjects. Similarly, Class B showed an increase in students who agreed or strongly agreed to both statements. It is important to note that 8% represents only 2 students in a class of 24. Class C showed a significant drop in students who agreed with the statement 'Math is one of my favourite subjects'.

Table 2: Results from 'I like math.'

Class	Sep	Jan	% Change
A	65	76	+12
B	50	58	+8
C	73	73	0

Table 3: Results from 'Math is one of my favourite subjects.'

Class	Sep	Jan	% Change
A	35	53	+18
B	55	68	+14
C	75	50	-25

Results from the area of interest of mindset were less consistent. These items seemed problematic and were possibly interpreted differently from our original intention. At the start of the semester in one class, most students agreed with the statement 'Some people are good at math and some are not' (Table 4). That seems to indicate a fixed mindset. The results were the same for 'All students would be good at math if they worked at it' (Table 5) which would seem to indicate a growth mindset. There was a significant increase in the number of students who agreed that 'People can't really change how intelligent they are in math' (Table 6). This could be interpreted by a teacher as more evidence of a fixed mindset.

Table 4: Results from 'Some people are good at math and some are not.'

Sep	Jan	% Change
92	83	-8

Table 5: Results from 'All students would be good at math if they worked at it.'

Sep	Jan	% Change
92	83	-8

Table 6: Results from 'People can't really change how intelligent they are in math.'

Sep	Jan	% Change
53	82	+29

In all three classes, the results on the mindset items seemed contradictory and were difficult to interpret. The inclusion of the negative word *can't* in an item is never a good choice, but it does not fully explain the contradictory nature of the results. These items might need to be reworked for clarity for the students and the teachers.

The results of one item seem to be indicative of how any delay in administering the survey might affect results. It seems reasonable to expect students to be attuned to the expectations of their teachers and the culture of the class, especially during the very first week of high school. Our survey was conducted on the fourth day of the semester. In the classes where homework was not emphasized, approximately 60% of the students agreed with the statement 'Doing homework helps me understand math', whereas in one class where the importance of homework was stressed, 92% of the students agreed (Table 7). Interestingly, by the end of the semester the students in all three classes expressed similar sentiments toward the value of homework.

Table 7: Results from 'Doing homework helps me understand math.'

Class	Sep	Jan	% Change
A	64	64	0
B	59	59	0
C	92	67	-25

The area of 'Math as a set of operations versus a tool for thought' revealed a significant difference between the classes (Table 8 and Table 9).

Table 8: Results from 'The math that I learn in school is mostly facts and procedures that have to be memorized.'

Class	Sep	Jan	% Change
A	86	59	-27
B	76	71	-6
C	92	92	0

Table 9: Results from 'You can be creative in math class.'

Class	Sep	Jan	% Change
A	47	76	+29
B	77	95	+18
C	83	67	-17

At the time when we were presented with the change analysis, the three teachers involved did not make the time to discuss the results. This study provided the opportunity to interview the other teachers for the purpose of research.

Analyzing *the change* in attitudes provided significantly more insight than any single course exit survey that I had administered previously. Those previous questionnaires had yielded students' perceptions of the course presentation and content at the end of the semester but did not provide feedback on how their attitudes might have changed for better or worse. Over the years I believe that I made some gains in this regard but despite positive feedback from parents and colleagues as well as some teaching awards, I was never fully confident that I was truly honouring my original commitment to give my students a more positive experience with mathematics. The survey provided important insights into my own teaching on which to reflect. It got to the heart of the question; was I providing the classroom experience that I hoped to create when I entered teaching? I began to think that a before-and-after survey of students' beliefs and attitudes might be an important tool for all teachers to examine and reflect on their own classroom practice.

To study this idea of informing reflective practice with a change analysis on a survey of student beliefs and attitudes about mathematics, it is important to review the research on the role of affective student domain, teacher reflection, feedback and student voice.

Literature Review

This study attempts to provide a mechanism that might encourage teachers to reflect on how their classroom approach is, or is not, fostering positive attitudes toward mathematics. The significance of positive attitudes in education and especially mathematics education is garnering more and more attention in the research community. To study areas of affective domain, we also need to consider how we might gather indications of student attitudes and beliefs. And perhaps most importantly we need to consider the structure and importance of teacher reflection to analyze and interpret student needs.

The role of affective domain in mathematics education

Since Sheila Tobias (1978) made the term *math anxiety* a household term, the area of affective domain in mathematics education involving feelings, emotions, and attitudes has received more attention from researchers and policy makers. Anxiety is one facet of the affective domain as outlined by Hannula et al. (2016) which also includes attitude, beliefs, meaning, self-concept, emotion, interest, motivation, needs, goals, identity, norms, values. For the purpose of this study, the focus was on attitudes and beliefs where beliefs are core convictions that something is true, and attitudes are a way of thinking about something that stems from beliefs and experience.

In the process of attempting to define beliefs in mathematics education, Furinghetti and Pehkonen (2002) found no agreement among 18 specialists. The lack of a clear definition has not deterred researchers, yet Leder (2007) found that most research commonly involved qualitative methods of small sample sizes that often suggest questionable generalizations. Academic research suggests the correlation is not high between attitudes and achievement in mathematics (Eccles & Wigfield, 2002; Zan & Di Martino, 2014), but the general consensus is that motivational factors *do* influence student academic achievement.

Data from the Program for International Student Assessment [PISA] which tests 15-year-olds' academic achievement worldwide, shows that in 63 of 64 education systems that participated in 2012, students reporting higher levels of math anxiety displayed lower levels of math performance than their peers who reported lower levels of math anxiety. Individuals' attitudes, beliefs and emotions play a significant role in their interest and response to

mathematics in general, and their employment of mathematics in their individual lives. Students who feel more confident with mathematics, for example, are more likely than others to use mathematics in the various contexts that they encounter. Students who have positive feelings towards mathematics are more able to grasp mathematical concepts than students who feel anxiety towards the subject. Therefore, one goal of mathematics education is for students to develop attitudes, beliefs and emotions that make them more likely to successfully use the mathematics they know, and to learn more mathematics, for personal and social benefit (OECD, 2013, p. 42).

The Organization for *Economic* Co-operation and Development [OECD] is responsible for administrating PISA and reporting the results. Their mandate is to stimulate economic progress and world trade. When the OECD draws a straight line from math anxiety to economics, it points to the importance of considering the affective domain in mathematics education. A study by Belfield et al. (2015) found that promoting students' social and emotional well-being resulted in significant long-term economic gains. Although this relationship may be bidirectional (math anxiety causes poor performance and poor performance causes math anxiety), math anxiety's negative relation to math performance is striking and apprehension about math should be considered in attempts to improve student achievement (Foley et al., 2017). The affective domain may be more closely linked to students' decision to pursue study in the STEM fields as a lack of achievement in mathematics can be a roadblock (Singh et al., 2002).

Despite the lack of clear definitions and defining theories, the importance of this research remains clear. Students' beliefs and attitudes impact the manner in which they approach the study of mathematics. Attempts to study the affective domain reflect the complex nature of emotions and beliefs. "Research in the 'affective domain' is densely populated with overlapping constructs, partially commensurate methods, and somewhat contradictory findings" (Schoenfeld et al., 2016, p. 395). Possibly with these constraints in mind, Dowker et al. (2016) have argued for more pointed research in linking mathematics anxiety to specific factors as a path forward. Pedagogy and policy can evolve with aspects that have been shown to make a difference. For example, students who consider mathematics as disconnected facts and procedures tend to have a lower academic performance than students who view the subject as an interconnected web of knowledge (Maciejewski & Merchant, 2016).

The affective domain is an important, if not critical, element of learning mathematics. This is reflected in an increase in research and shaping of education policy, emerging from the confines of the implied to the explicit. In 2020 the province of Ontario produced a new mathematics curriculum for Grades 1 to 8 followed by a new 2021 curriculum for a destreamed Grade 9. All nine grades specifically include the affective domain in the new social-emotional strand "to help students see themselves as capable and confident math learners" (OME, 2020a) which "reflects current research" in mathematics education. In the previous 2005 guides, mathematical process expectations were placed outside the overall learning expectations and building mathematical confidence was implied. The new guides include tools and strategies embedded in the curriculum to "help students develop confidence, cope with challenges and think critically" (OME, 2020b, p. 80). Strategies are explicitly stated in the new social-emotional curriculum expectations in each grade level to develop a healthy identity as mathematical learners to foster well-being and the ability to learn mathematics.

Feedback and voicing students' attitudes and beliefs

In a Ted Talk, when Bill Gates said everyone needs feedback to do their jobs better but teachers "get almost no systematic feedback to help them do their jobs better" (Gates, 2013, 0:13), he is referring to formal feedback from administrators of *satisfactory* or *not satisfactory.* Teachers do receive almost constant feedback informally from students during every class. In a survey of 307 teachers on subjective teaching success criteria, Jacob et al. (2017) found that less than ten percent of responses focused on teacher behaviours (input perspective) whereas two-thirds of their criteria aligned with student behaviours (output orientation) with student mathematical skills garnering the most responses followed by student engagement. Although student behaviour could be considered feedback, the study placed a third criteria labeled *feedback* outside of input or output perspectives. Feedback, representing twenty-two percent of responses, included test results, feedback from relevant others including parents, teachers and colleagues as well as teachers' own emotions. Curiously there was no mention of student feedback. To Gate's point, the authors of this study speculate that the high frequency of responses in the feedback category might be an indicator that teachers are craving direct external reinforcement. Students should be trusted to have a greater role in providing formal feedback.

Students are the stakeholders with the most to lose and the most to gain and they should be solicited directly.

Traditionally mathematics instruction has emphasized the cognitive domain of Bloom's taxonomy of learning making academic achievement as the focus of assessment in secondary schools. Although aspects of our assessment techniques are not without concern, for the most part, the mechanisms employed are established and considered acceptable if not always valid. Measuring aspects of the affective domain comes with its own challenges. According to Golden (2002) the affective subdomains of emotions, beliefs, attitudes and values cannot be directly or easily measured and must be inferred by behavior or responses to specifically designed instruments (Leder, 2016). As suggested in the study by Jacob et al., classroom teachers make subjective inferences about students' attitudes by noting their level of engagement during a lesson. This measure can take many forms from the type of questions asked by students to the number of student requests for washroom breaks. The data from the Devine et al. (2013) study on what is considered good teaching confirmed the craft-like approach that permeates most teachers' practice which draws on how teachers are 'reaching' students rather than a more systematic reflective approach (p. 105).

In the research community, a diverse range of instruments has been incorporated to measure affect and its role in learning mathematics. The informal approach of the classroom teacher can be improved when observations are carried out by expert observers with a carefully designed observation schedule which can return fairly consistent results (Leder, p. 28). Video recordings of classroom activities can provide rich data in a more convenient and less expensive alternative to direct observation. Larkin & Jorgensen (2016) incorporated student video journals to investigate attitudes of students in year 3 and 6. Interviews can reveal views that are not anticipated in advance unless the interview is so structured that it becomes an oral questionnaire. The self-reporting of actual questionnaires can increase sample sizes while reducing costs. Forgasz et al. (2018) used Facebook to sample beliefs and attitudes from students in nine countries. The majority of studies examining the affective domain incorporate Likert-item surveys. Fennema & Sherman (1973) surveyed thousands of students in their Mathematics Attitudes Scales using Likert items to gain insight into sex-related differences in mathematics achievement. Attitudes toward Mathematics Inventory [ATMI] (Tapia, 1996) or the revised

SHORT ATMI (Lim & Chapman, 2013) also incorporate Likert-type items for students in high school and college. These inventories measure enjoyment, motivation, self-confidence, as well as perceived value and have been shown statistically to be internally consistent and reliable over time (Lim et al.). The Mathematics Attitude and Perceptions Survey [MAPS] (Code et al., 2016) was designed to measure how closely university students' perceptions align with mathematicians. Likert-item surveys, according to Ruthven (2015), offer a convenient technique that serves researchers well in generating results. What is less certain is whether they measure what they purport to measure. Self-reporting surveys may solicit responses that are thought to be socially or culturally acceptable.

Anonymity improves the integrity of responses where students are less likely to answer what is expected of them (Robinson & Taylor, 2007). Just as anonymity is important for student responses, autonomy is important for teachers. Blasé and Blasé (2000), as reported by Donahue, (2016, p. 12) argue that it is important to respect the autonomy of classroom teachers and provide feedback that is non-threatening and encourages them to take risks. A survey on student attitudes places the focus on the students themselves and should be less threatening for teachers than a direct evaluation on the course or teaching methods.

A study involving a teacher evaluation survey by 9765 students by Peterson et al. (2000) found that student responses were reliable and valid. Students responded with "reason, intent and consistent values" (p. 148). In Wilkie's (2016) study, the low incidence of irrelevant responses would suggest that the 3500 students took the open-response survey seriously.

The research shows that Likert-item student surveys can be a simple and effective method for gathering student views but warns that the survey might not always measure what it intends to measure. That is why reflection is such an important element in the act of analysis.

The role of reflection and self-reflection in teaching

There is a difference in *reflection* and *self-reflection*. The mechanism is essentially the same, the only real difference is the object of the gaze. As a classroom teacher, I never made a conscious decision to qualify which reflections were self-reflections and which were more general in nature. In fact, as the adult in the classroom, I felt responsible for all that transpired.

That made all my reflections essentially self-reflections. For any indiscretions, I always blamed my inadequacies rather than blaming students, parents, or administration. My practice of blurring the line between reflection and self-reflection is also often reflected in the literature.

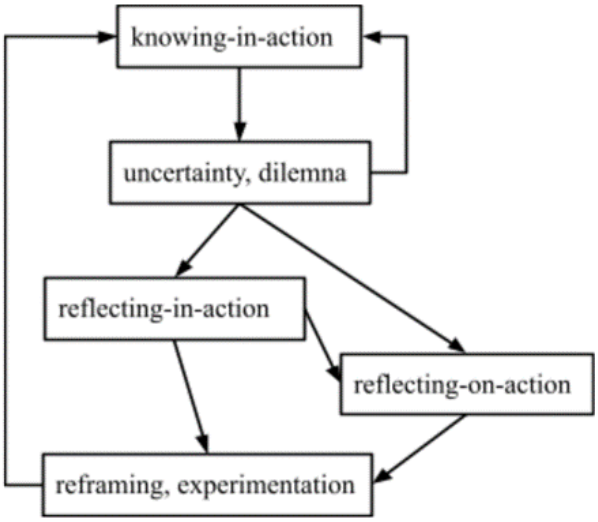
Hattie (2012) encapsulated the results of his meta-meta-analysis of over 50,000 research articles and over 800 meta-analyses with this single statement, "The remarkable feature of the evidence is that the biggest effects on student learning occur when teachers become learners of their own teaching (and when students become their own teachers)" (p. 22). It is my assertion that to be learners of their own teaching, teachers must build their capacity for self-evaluation by developing their powers of self-reflection. Self-reflection is accepted as a critical element in improving teaching methods and is essential in aligning beliefs and teaching practice (Potari & Georgiadou-Kabouridis, 2009 from Fives & Buehl, 2012). Despite that, it appears that when it comes to self-reflection, teachers are left to their own devices. A study by Cavanagh and Prescott (2010), which followed three pre-service teachers though their practicum and into their first year of teaching, noted that there were opportunities to develop reflective practices during the teacher education program, but these were not encouraged or developed by the practicum supervisors. The authors suggested that the pressures of the practicum made the demands of self-reflection too difficult. If preservice teachers are not encouraged to develop self-reflection techniques during their practicum, will they practice self-reflection to effectively self-evaluate their practice?

To reflect on and self-evaluate teaching practice, teachers need to acknowledge what they consider to be good teaching. In a survey of 126 primary and secondary teachers in 12 different Irish schools (Devine et al., 2013) being a reflective practitioner emerged as one of the five top qualities of being a good teacher. The category of reflective practitioner contained the greatest number of individual items, confirming its importance in overall teacher constructs. The list includes; uses a variety of teaching strategies, strives to improve their own teaching, is willing to engage in professional development, seeks advice from colleagues, is self-critical and evaluates own performance, reflects on what is being taught, is able to reflect on their shortcomings, takes risks and experiments in teaching (p. 93). What this extensive list does not explicitly include is any reference to student feedback, although there are opportunities for student feedback within *evaluating own performance.* Soliciting student feedback by its nature might also enter the

category of *risk taking*. The list in this study represents what teachers say are highly important, but contradictions emerged with classroom observations. In this study, the authors noted a conflict between what teachers listed as highly important and what their observed classroom practice emphasized. The mismatch between teachers' stated beliefs and their actual practice has been noted in other studies (Fives & Buehl, 2012; Liljedahl & Oesterle, 2014). The intent of my study is to provide teacher participants with tools that encourage reflective thought to assist in aligning beliefs with practice.

According to Dewey (1910), reflective thought has structure and purpose. Dewey wrote that reflection consists of "active, persistent, and careful consideration of any belief or supposed form of knowledge in light of the grounds that support it and the future conclusions to which it tends" (p. 6). In the seminal work, *The Reflective Practitioner*, Donald Schön (1983) suggests that to engage in the continuous process of self-education, a practitioner must become a researcher into his own practice (p. 299), the same sentiment echoed by Hattie in his meta-analysis. Schön promotes the role of teachers as action researchers, reflecting *on* and *about* the act of teaching. In Schön's model of reflection, teachers, like all professionals, enter a situation with a knowledge base or expertise built on professional norms and personal experience. This is what Schön refers to as *knowing-in-action*. When events unfold as expected, professionals base their actions on this foundation of knowledge, but when the experience does not mesh with what was previously thought or expected, then there is a recognition that things may be amiss and one needs to reframe the situation and connect it to a different knowledge base or past experience. If one cannot find the knowledge to explain or deal with the presented action, then one is left with uncertainty that requires further analysis. If one analyzes thoughts and feelings critically *during* an event, Schön refers to this as *reflection-in-action* whereas *reflection-on-action* occurs *after* an event (Fig. 1).

Figure 1: Schön's (1983) model of reflection



Both allow the individual to gain insight in their behavior and can assist in reframing their thinking. Schön discusses how, when faced with a dilemma, professionals can retreat to the 'high ground of technical rigor' or they can descend into the 'swampy lowlands of uncertainty' and involve themselves in critically important lines of inquiry that require trial and error, intuition, and muddling through (Schön, p. 43), exploits that most teachers can attest to.

Other researchers point to the significance of action in the process of reflection. Finlay (2008) emphasizes that self-reflection is a basic aspect of teaching and learning by making one aware of everyday practice and challenging assumptions. To challenge assumptions, more than a passing thought is required and all models of reflection involve critical analysis and a demand for action. Kolb's (1984) reflective cycle model includes *active experimentation* and Gibb's (1988) model includes an *action plan*. Dricoll's (1994) model includes *actioning the new learning*. To act, a practitioner is required to be aware of their actions and thoughts and be able to analyze and interpret them.

Chatzistamatiou et al. (2014) took Zimmerman's (2002) three phases of self-regulation for students (forethought, performance, and self-reflection) and transcribed it to teacher self-regulation as lesson planning, lesson delivery and post-lesson self-reflection and evaluation. Post-lesson self-reflection would be an example of Schön's *reflection-on-action* (after) and can occur immediately following a lesson, a week or even a year or more later. Although lesson planning occurs prior to the actual lesson, it can also fall into the *reflection-on-action* phase as

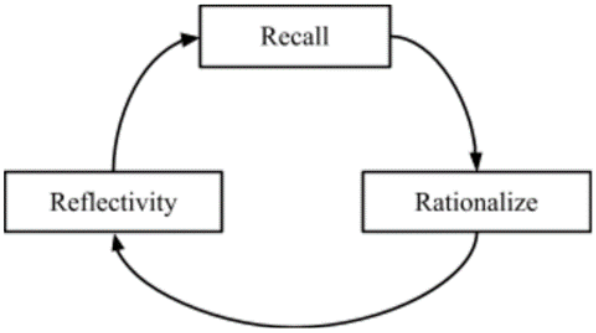
teachers usually base the next lesson on the previous lesson. *Reflection-in-action* happens during a lesson. It seems natural to question the quality of the reflection and the derived interpretation *during* a situation which requires split second decision making. Moon (1999) suggests that this type of reflection is all but impossible, the results of which will be limited. Decades ago, Jackson (1990) made a healthy estimate that a typical teacher can make 1500 decisions per day. Whether you question Jackson's number as too high or guess that there has been an increase due to the infusion of technology into the classroom, it is hard to argue Jackson's point that spontaneity and immediacy are required of teachers during a lesson.

Self-reflection is the final phase of Chatzistamatiou et al.'s self-regulation. They report that studies of strategies of self-regulation in education have often focused on students and they perceive a need to study the growing interest in teachers' self-regulatory process that supports instruction. Their choice to study mathematics instruction was deliberate because of their assertion that mathematics is associated with an increase in anxiety in students *and* teachers. The three cyclic phases of self-regulations; forethought, *performance* and *self-reflection* occur before, during and after the lesson. In teaching with self-regulation, these phases translate roughly to lesson planning, self-observation during the lesson delivery and post-lesson self-reflection and evaluation. The focus on the lesson in this model supports my premise that teachers' self-reflection and self-evaluation tends to revolve around the short-term delivery of lessons rather than more long-term themes of promoting healthy student attitudes and beliefs.

Other models of reflection (Copeland et al., 1993; Kember, 2000; Agustan, et al., 2017) owe much to Polya's (1945) four steps of problem solving: define the problem; design a strategy; carry out the plan; look back. In attempting to define reflective thinking, Copeland et al., for lack of a better term, reluctantly refer to the process of thinking which reflective practitioners engage in as *problem solving* yet they recognize that the term does not capture the more constant nature of learning that reflection involves. Their model places twelve attributes within four categories; *problem identification, generating solutions, testing solutions, learning from reflective practice* which mirror Polya's model. Although not all models have four steps per se, Quinn (2000) suggests that they all involve three fundamental processes; *retrospection, self-evaluation*, and *reorientation*. This model maps directly onto Lee's (2005) model of reflective thinking (Fig. 2)

with its three phases: *recall* your experience to define the problem; *rationalize* by interpreting the situation and looking for connections; *reflectivity* with the intention of change or improvement.

Figure 2: Lee's (2005) reflective thinking model



Lee's uncomplicated model seems ideal for the working classroom teacher, and it dovetails with my research questions. Might the online survey and analysis better allow the teachers to *recall* and define the students' attitudes and beliefs in a manner that might not be possible otherwise? Might the automated change-analysis encourage the teachers to consider and *rationalize* the results? Might the teacher's *reflectivity* be nurtured to the point of encouraging change and improvement? Alternatively, this 3-step cyclic approach can be distilled to three simple questions: What? So what? Now what? (Rolfe et al., 2010). Applying this model to my study could involve the following questions: *What* does the survey indicate about changes in students' attitudes and beliefs and *what* are my feelings about it? *So, what* does this tell me about my approach and the way that the students are responding? *Now what* do I need to change, if anything?

A review of the literature shows numerous models of reflection, none of which were known to me in my teaching career. I, like I suspect all teachers do, recognized the importance of self-reflection and self-evaluation in developing teaching skills. Yet formalized reflective practice is typically not emphasized during teacher education programs nor developed during teaching practicums. Teachers tend to lack formal training in reflective practice and might benefit from any mechanism that helps them recall, rationalize, and reflect. The research shows that teachers tend to rely on surface forms of reflection and evaluate their lessons on student

engagement and how they feel they are reaching the students. In educational settings, models of reflection are often employed for student learning. Researchers point to the need for more emphasis on teacher reflective practice for self-regulation and self-evaluation. Reflective practice for students is often emphasized at the end of a reporting period but when models of reflection are applied to teaching, they tend to focus on individual lessons. Although reflection may happen during an event or long after an event, there is a short-term element to teacher reflection models that emphasize day-to-day post-lesson reflections. This fits with the immediacy required during classroom lessons and pressures to cover the curriculum but it distracts from longer term considerations of student attitudes. All the models stress the active nature of reflective practice with a solid connection to problem solving. Active reflection is a critical element in evaluation with an intention of changing practice. As outlined by Rolfe et al., *now what*?

The '*what'* of this study in the model of reflective practice by Rolfe et al. (2010) focuses on the affective domain. Previous studies have shown the importance of fostering positive attitudes toward mathematics to encourage and prepare students for careers in technology. The OECD argues that this is not only a significant economic consideration but also of social and personal importance. More research is needed but there is enough evidence for jurisdictions such as Ontario to place the affective domain into the mathematics curriculum. I would argue that a generation that understands and appreciates mathematics is critical for a healthy democracy and the survival of the planet.

Researchers have suggested that student attitudes cannot be measured directly but they can be inferred from behaviours or specifically designed instruments. The instruments that researchers often choose are Likert-item student surveys which can be a simple and effective method for gathering student views. Anonymity in surveys has been shown not to degrade the quality of student responses. Although teaching success criteria are centred on student outcomes, direct student feedback is not often sought or considered. Surveys on students' attitudes and beliefs can provide a quick method to attain student feedback and promote active classroom research while maintaining the important element of teacher autonomy.

Research Methodology

Setting

This study incorporated data from over seven years starting with my own experience with the student survey on beliefs and attitudes about mathematics. The interviews with five secondary math teachers occurred during the Covid pandemic, before the World Health Organization declared an end to the global emergency. I incorporated qualitative methods to investigate my research questions on the reflective practices of teachers and the effects of using the before-and-after student survey on beliefs and attitudes.

Participants and Recruitment

In my original proposal, I intended to contact potential participants through email or Twitter. The ethics committee required that I limit any research to individual teachers and avoid any direct connections to schools and school boards. That meant no interviews on school property, no interviews during school and no contact using school board emails. Therefore, contacts were established through social media. I hoped this change would lead to a wider variety of participants and perhaps teachers who I was less familiar with. It did not. Whether that was a factor of my inexperience with social media or that teachers are reluctant to add anything to their workload, I suspect that both factors contributed to a lukewarm response.

While I was teaching, I had a limited presence on Twitter, but I had not been active on any social media since retiring from teaching. It had been six years since my last tweet. When my research proposal was finally approved, the start of the second semester was looming. Over the course of eight days leading up to the semester in February and the start of a new set of courses, I posted three tweets [Appendix B] to my few hundred followers. I received seven likes and ten retweets which reached thousands of followers on Twitter. According to Twitter Analytics, my final tweet was seen by over 2700 people. Over 130 of those Twitter users viewed my blog post pertaining to the study [Appendix C] by the end of January. The response was paltry. The lack of interest may have been partly due to my inexperience with social media but I received one rejection tweet which seemed to point to general fatigue lingering as a symptom of the pandemic measures. The respondent wrote, "I am interested but I am not ready for the amount of accountability it would require. I'm just trying to get through. Another season I would love to. I have heard you speak and would love the opportunity to work with you, just not right now".

My original notion to choose a diverse group of five participants from ten volunteers was overly optimistic. I was keen to have all the participants administer the survey early in the semester so with a limited time for recruiting, I proceeded with only three participants. Rather than a stratified sample of males and females, new and experienced teachers, I continued with convenience sampling as these three were the only teachers accessible and available.

I knew all three volunteers professionally, all working as secondary mathematics teachers in Ontario. My first volunteer was a female teacher with 21 years of experience who I will refer to as Alice. Alice was already familiar with my proposal and was keen to participate. She was teaching a Grade 10 academic [MFM2D] class and a college level Grade 11/12 split class [MBF3C/MAP4C]. My second volunteer was a male teacher who I will refer to as Jeffrey. Jeffrey had been a student in a methods course that I taught at university and had been teaching for eight years. He was teaching one section of the new Grade 9 destreamed course [MTH1W], Grade 10 academic [MPM2D] and Grade 12 data management [MDM4U]. A third colleague volunteered to participate if I needed more teachers in my study. Initially I waited for more willing participants but ultimately, I decided that I needed his involvement. I will refer to him as Paul. Paul had 28 years of experience and was teaching a Grade 11 college level course [MBF3C] and two Grade 12 calculus courses [MCV4U].

To further expand the scope of my study, I interviewed the two teachers who had been involved with me in the original survey six years previously. The three of us had never made the time to discuss the results. The lack of any exchange of ideas was one of the contributing factors to my lingering questions about the possible role of the survey in teachers' practice. It is important to note that unlike the solo experience of the volunteers in my study, we three teachers had access to each other's results from our own particular classes. Despite that difference and the fact that our experience was six years ago, I felt that insight from these two teachers could apprise my research.

The first teacher of the two involved in the original survey, I will refer to as Hanh. Hanh had taught for twenty-two years and was working as an instructional coach at the time. The second teacher I will refer to as Emma. Emma had been teaching for twenty-two years as well.

Procedure and data collection

Emma and Hanh were interviewed about their experience with the original survey and change analysis. The three volunteer participants, Jeffrey, Alice and Paul, were first interviewed about their reflective practice and invited to use the survey in their classrooms. I interviewed them a second time after they had completed both iterations of the survey and reflected on the change analysis.

My study focused on the reflective practice of the teacher participants and the effect of using the student survey. No student identities were revealed to the researcher and no student survey data on attitudes and beliefs was accessible to anyone other than the students' own teacher. The student survey was to act as a catalyst for teacher self-reflection. This study focused solely on the experience of teachers through the use of individual semi-structured interview questions. (Appendix D). I invited participants to administer the student survey at the beginning and end of at least one course section and to participate in two separate audio-recorded, semi-structured 30 to 45-minute interviews. Initially the first interview was to precede the first administration of the survey but with the semester about to begin, the participants received written directions on how to administer the survey and the first semi-formal interview was scheduled for afterward. The first interview focused on reflective practices. The second interview focused on reflections based on the change analysis of the student survey.

Design of the Student Survey and Change Analysis

Before examining the participants' experiences and reflections with the survey, the design and final form of the student survey and change analysis should be considered.

Our original Beliefs and Attitudes about Mathematics Survey [BAMS] began with our research assistant compiling a sample of items from other surveys (Brookstein et al., 2011; Schoenfeld, 1989; Stipek et al., 2001) and arranging them in categories such as enjoyment, mindset, beliefs, and attributions of success. The three teachers involved then eliminated several of the items before deciding on the final survey 24 item survey. For this study I considered using a more established and vetted instrument such as the ATMI, SHORT ATMI, or the MAPS. Since the focus of my study is on teacher reflection and is not directly measuring students' attitudes and beliefs, I decided that any concerns about the validity of BAMS would not have a major impact on the value of my research. The original BAMS was effective in revealing areas of previously uncovered concern and promoting self-reflection. The fact that BAMS was developed

by teachers for their own use, made it *feel* relevant and valid; important aspects in supporting self-reflective practice. It was unrealistic to require participating teachers in this study to develop their own complete survey, but they were given the option to add as many as five additional survey items not included in BAMS. The 24 core items were to ensure common ground among the participants and the individual items were intended to allow participating teachers to add relevance by investigating their own professional concerns. None of the participants chose the option to add items. So, for the purpose of this study, the core survey included a core set of 23 Likert items, one open response item with the sentence stem "Mathematics is …" (Appendix A).

The response choices in the survey were *strongly agree, agree, disagree, strongly disagree.* According to a study by Lozano, García-Cueto & Muñiz (2008), a 4-point Likert scale is the minimum number of points to ensure reliability and validity. The choice of the 4-point scale was a deliberate strategy to force students to either side of the ledger. A fifth neutral, neither agree or disagree option has been shown to limit skipping of items but this issue was avoided in BAMS as each item was set as *required* within Google Forms. A 4-point scale allows for some detail within the pie charts in the Google Form yet simplifies the final change-analysis. For the change analysis our original research assistant and combined *strongly agree* and *agree* together and *disagree* and *strongly disagree* together. For this study I chose to follow that strategy in my automated change analysis.

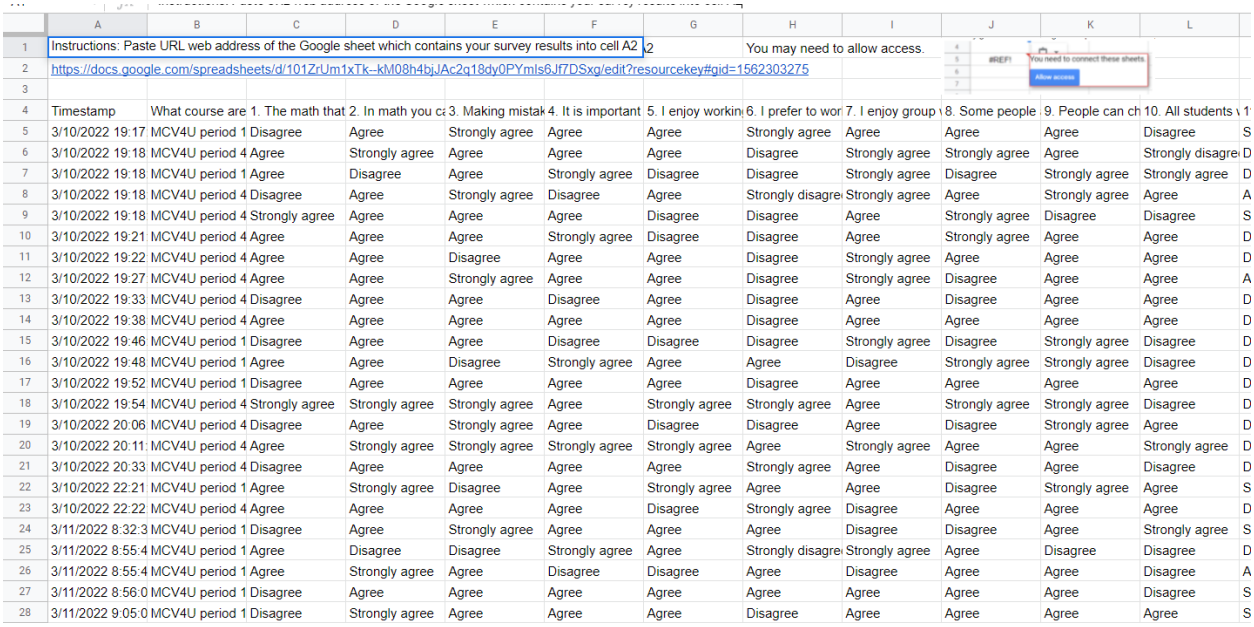
The change analysis was completed on a Google Sheet which the participating teachers copied to create their own version. It consisted of three parts:

1) Import data sheet
2) Summary sheet of the change analysis of all survey items presented in graphical form
3) Twenty-four sheets containing the calculations for each individual survey item

To populate the spreadsheet the teacher was required to copy the address of the Google Sheet which contained the results from the Google Form survey and paste it into their personal copy of the change analysis spreadsheet. This automatically filled the first sheet (Fig. 3) with all the data using the formula `f(x)=importrange(A2,"Form Responses 1!A1:Z300")`. The teacher was prompted to allow access to connect the sheets before the transfer happened. The data imported from the form included a timestamp of when each student answered the survey,

the identity of the particular course, and the individual results for each survey item (Strongly disagree, Disagree, Agree, Strongly agree) (Fig. 3).

Figure 3: Screenshot of a Google Sheet showing the import of data



	A	B	C	D	E	F	G	H	I	J	K	L
1	Instructions: Paste URL web address of the Google sheet which contains your survey results into cell A2						2	You may need to allow access.				
2	https://docs.google.com/spreadsheets/d/101ZrUm1xTk--kM08h4bjJAc2q18dy0PYmIs6Jf7DSxg/edit?resourcekey#gid=1562303275											
3												
4	Timestamp	What course are	1. The math that	2. In math you c	3. Making mistak	4. It is important	5. I enjoy workin	6. I prefer to wor	7. I enjoy group	8. Some people	9. People can ch	10. All students
5	3/10/2022 19:17	MCV4U period 1	Disagree	Agree	Strongly agree	Agree	Agree	Strongly agree	Agree	Agree	Agree	Disagree
6	3/10/2022 19:18	MCV4U period 4	Agree	Strongly agree	Agree	Agree	Agree	Disagree	Strongly agree	Strongly agree	Agree	Strongly disagre
7	3/10/2022 19:18	MCV4U period 1	Agree	Disagree	Agree	Strongly agree	Disagree	Disagree	Strongly agree	Disagree	Strongly agree	Strongly agree
8	3/10/2022 19:18	MCV4U period 4	Disagree	Agree	Strongly agree	Disagree	Agree	Strongly disagre	Strongly agree	Agree	Strongly agree	Agree
9	3/10/2022 19:18	MCV4U period 4	Strongly agree	Agree	Agree	Agree	Disagree	Disagree	Agree	Strongly agree	Disagree	Disagree
10	3/10/2022 19:21	MCV4U period 4	Agree	Agree	Agree	Strongly agree	Disagree	Disagree	Agree	Strongly agree	Agree	Agree
11	3/10/2022 19:22	MCV4U period 4	Agree	Agree	Disagree	Agree	Agree	Disagree	Strongly agree	Agree	Agree	Agree
12	3/10/2022 19:27	MCV4U period 4	Agree	Agree	Strongly agree	Agree	Agree	Disagree	Strongly agree	Disagree	Agree	Agree
13	3/10/2022 19:33	MCV4U period 4	Disagree	Agree	Agree	Disagree	Agree	Disagree	Agree	Disagree	Agree	Agree
14	3/10/2022 19:38	MCV4U period 4	Agree	Agree	Agree	Agree	Agree	Disagree	Agree	Agree	Agree	Agree
15	3/10/2022 19:46	MCV4U period 1	Disagree	Agree	Agree	Disagree	Disagree	Disagree	Strongly agree	Disagree	Strongly agree	Disagree
16	3/10/2022 19:48	MCV4U period 1	Agree	Agree	Disagree	Strongly agree	Agree	Agree	Disagree	Strongly agree	Strongly agree	Disagree
17	3/10/2022 19:52	MCV4U period 1	Disagree	Agree	Agree	Agree	Agree	Disagree	Agree	Agree	Agree	Agree
18	3/10/2022 19:54	MCV4U period 4	Strongly agree	Strongly agree	Strongly agree	Agree	Strongly agree	Strongly agree	Agree	Strongly agree	Strongly agree	Disagree
19	3/10/2022 20:06	MCV4U period 4	Disagree	Agree	Strongly agree	Agree	Disagree	Disagree	Agree	Disagree	Strongly agree	Agree
20	3/10/2022 20:11	MCV4U period 4	Agree	Strongly agree	Strongly agree	Strongly agree	Strongly agree	Agree	Strongly agree	Agree	Agree	Strongly agree
21	3/10/2022 20:33	MCV4U period 4	Disagree	Agree	Agree	Agree	Agree	Strongly agree	Agree	Disagree	Agree	Disagree
22	3/10/2022 22:21	MCV4U period 1	Agree	Strongly agree	Disagree	Agree	Strongly agree	Agree	Agree	Disagree	Strongly agree	Agree
23	3/10/2022 22:22	MCV4U period 4	Agree	Agree	Agree	Agree	Disagree	Strongly agree	Disagree	Agree	Agree	Agree
24	3/11/2022 8:32:3	MCV4U period 1	Disagree	Agree	Strongly agree	Agree	Agree	Agree	Disagree	Disagree	Agree	Strongly agree
25	3/11/2022 8:55:4	MCV4U period 1	Agree	Disagree	Disagree	Strongly agree	Agree	Strongly disagre	Strongly agree	Agree	Disagree	Disagree
26	3/11/2022 8:55:4	MCV4U period 1	Agree	Strongly agree	Agree	Disagree	Disagree	Agree	Disagree	Agree	Agree	Disagree
27	3/11/2022 8:56:0	MCV4U period 1	Disagree	Agree	Agree	Agree	Agree	Agree	Agree	Agree	Agree	Disagree
28	3/11/2022 9:05:0	MCV4U period 1	Disagree	Strongly agree	Agree	Agree	Agree	Disagree	Agree	Agree	Agree	Agree

The change analysis complete with graphs was automatically generated. The data from each item was automatically copied into separate sheets where the analysis of each item was done (Fig 4). The results are tallied in the formula

```
f(x)=countifs($R$4:$R$300,"Strongly disagree",
$T$4:$T$300,"Start",$S$4:$S$300,"MBF3C period 2")
```

which included the criteria of

The response (Strongly Disagree, Disagree, Agree, Strongly Agree)

When the survey was completed (Start or End)

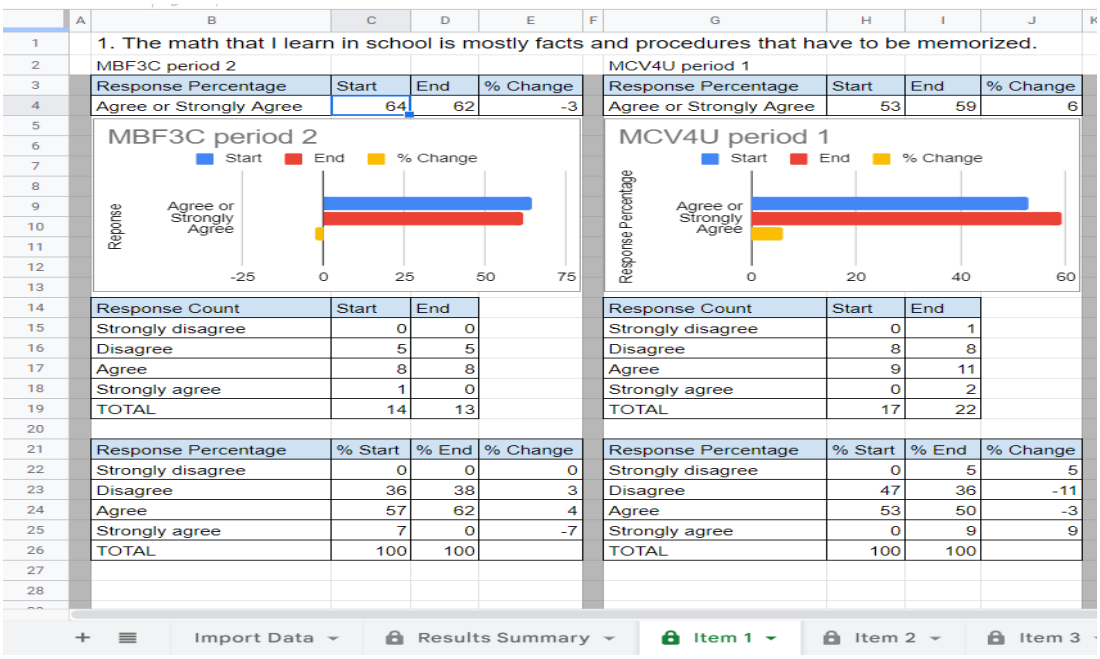
The course name if there was more than one course surveyed in the one Google Form.

In the case where the start/end item was not included in the survey, then the timestamp was converted to 'Start' or 'End' using the formula

```
f(x)=if('Import Data'!A5<>"",if('Import
Data'!A5<date(2022,3,31)+time(0,0,0), "Start","End"),"")
```

The tally of each response was presented in a table and a second table displayed the percentages.

Figure 4: Screenshot detail of a Google Sheet showing the analysis of a single item



1. The math that I learn in school is mostly facts and procedures that have to be memorized.

MBF3C period 2

Response Percentage	Start	End	% Change
Agree or Strongly Agree	64	62	-3

Response Count	Start	End
Strongly disagree	0	0
Disagree	5	5
Agree	8	8
Strongly agree	1	0
TOTAL	14	13

Response Percentage	% Start	% End	% Change
Strongly disagree	0	0	0
Disagree	36	38	3
Agree	57	62	4
Strongly agree	7	0	-7
TOTAL	100	100	

MCV4U period 1

Response Percentage	Start	End	% Change
Agree or Strongly Agree	53	59	6

Response Count	Start	End
Strongly disagree	0	1
Disagree	8	8
Agree	9	11
Strongly agree	0	2
TOTAL	17	22

Response Percentage	% Start	% End	% Change
Strongly disagree	0	5	5
Disagree	47	36	-11
Agree	53	50	-3
Strongly agree	0	9	9
TOTAL	100	100	

The percentages of the responses *Agree* and *Strongly Agree* were combined to compare the *Start* to the *End* in a horizontal bar graph.

The results from each separate item sheet are included in a summary sheet (Fig. 5).

Figure 5: Screenshot of a summary sheet using sample data

Results Summary

1. The math that I learn in school is mostly facts and procedures that have to be memorized.

MBF3C period 2

Response	Start	End	% Change
Agree or Strongly Agree	64	62	-3

MCV4U period 1

Response	Start	End	% Change
Agree or Strongly Agree	53	59	6

MCV4U period 4

Response	Start	End	% Change
Agree or Strongly Agree	76	48	-29

2. In math you can discover things on your own.

MBF3C period 2

Response	Start	End	% Change
Agree or Strongly Agree	79	77	-2

MCV4U period 1

Response	Start	End	% Change
Agree or Strongly Agree	88	86	-2

MCV4U period 4

Response	Start	End	% Change
Agree or Strongly Agree	90	81	-10

Administration of the Survey

Each participant in the study was invited to make their own electronic copy of the Google Form survey which they, and they alone had access to. All participants had Google accounts and did not require an alternative such as Microsoft Forms and Excel. To administer the survey, the teachers shared an access link with their students in Google Classroom or as a printed or projected QR code. Completing the survey took less than 10 minutes and was completed during class time on students' own hand-held devices or on another device provided by the school. The student results of each item on the Google Form are automatically displayed graphically in a pie chart and recorded in a Google Sheet in table form. These are only accessible to the owner of the form, the teacher. Teachers were able to view the initial results of the survey but were not required to do so. These included student individual results (Fig. 6) and collective results as a pie chart (Fig. 7)

Figure 6: Sample individual results available in Google Forms

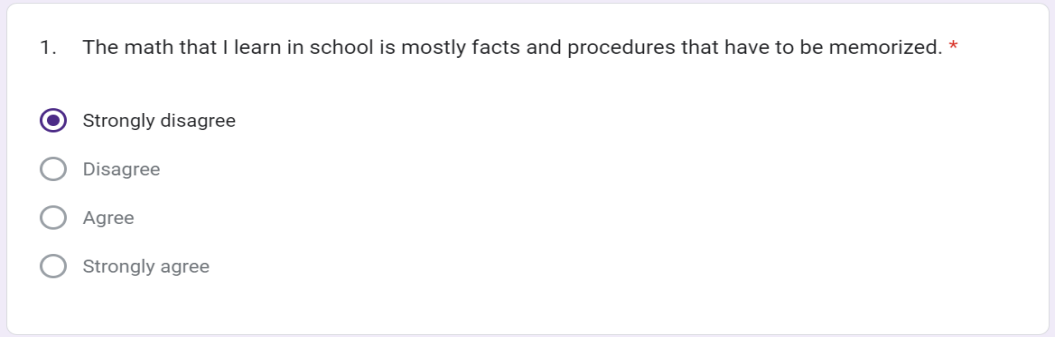
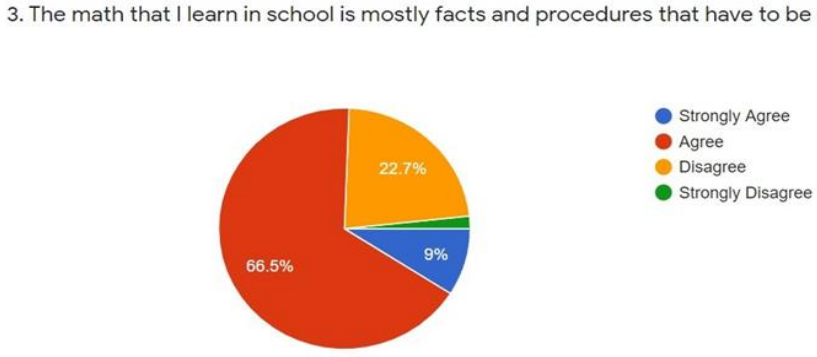


Figure 7: Sample summary results from Google Forms available to the teacher



Alice was the only participant to look at the results as a means to better understand her students. In her copy of the survey, she used the feature in Google Forms that included collecting emails. That allowed her to examine individual results.

Participating teachers were asked to use the exact same Google Form for the second administration of the survey at the end of the course. Again, this was intended to be a 10 minute in-class activity. The change analysis portion of the study happened when the participants had time after completing their regular end-of-year duties. Each teacher made their own copy of a change analysis Google Sheet. To activate the automated display of the changes, each teacher was required to copy and paste the URL of the Google Sheet which contained the responses from both iterations of the survey. This generated a spreadsheet of the results of each item as well as a summary sheet. The response of *strongly agree* was combined with *agree* in a single percentage. A table for each item displayed the starting percentage of students who responded with *agree* and *strongly agree*, the final percentage and the subsequent percentage change (Fig. 8).

Figure 8: Sample change analysis results spreadsheet

4					
5	Item	2. The math that I learn in school is mostly facts and procedures that have to be memorized			
6			Start	End	% Change
7		Agree or Strongly Agree	82	65	-17
8					
9	Item	3. You can be creative in math class			
10			Start	End	% Change
11		Agree or Strongly Agree	64	96	32
12					

This is similar to the analysis of the original survey provided by the graduate student and was sufficient to show overall changes in student attitudes and beliefs rather than a more detailed analysis which involves scoring items.

Every attempt was made to design the survey form and change analysis for ease of administration and interpretation. The resulting simplicity of the product had certain drawbacks. Changes in class enrollment can affect the accuracy of the survey results. Secondary school class rosters often change in the first week of classes and continue to evolve over the semester. The original survey required the students to include their names to allow the research assistant to

scrub the data to eliminate duplicate or erroneous data. To address the issue of changes in enrollment in this study, I considered including self-generated identification codes [SGIC] (Direnga et al., 2016) for a more accurate analysis while preserving students' anonymity. But for the purpose of simplicity as well as the advantages of anonymity as stated previously, I simply chose not to identify the student respondents. Without connecting the final individual survey results directly to the initial survey, the results were assumed to be less accurate. The participating teachers were made aware of this potential drawback when considering the analysis in their reflections.

The decision to have the participants administer the first iteration of the survey with very little lead time was not without issue. I chose to have limited contact with the participants while they prepared first semester final marks and began their second semester course load. Instead I relied on written instructions to guide them through the process and had no contact with them during the semester.

Knowing Paul's teaching assignment ahead of time allowed me to add a survey item to the form to delineate his three classes. Unfortunately, Paul did not administer the survey until almost four weeks into the semester.

Jeffrey successfully administered the initial survey to his Grade 9 destreamed class [MAT1W] at the beginning of the semester. He had a leave of absence during the semester and although he did give the students the second iteration of the survey at the end of the course, he did not complete the change analysis.

Alice chose to survey both her classes and created separate copies of the Google Form for each. This led to issues later with the change analysis. I designed the change analysis spreadsheet to draw all the data from a single Google Form. Both surveys, beginning and end, were to be contained within the same Form. Alice had a form for each of two classes and at the beginning of the semester and the results of two classes combined into one form at the end of the semester. To get all the data into one place for the change analysis, Alice had to copy all the data from three spreadsheets into a single sheet, which she did successfully. There was a second problem that I had not anticipated. Alice included the feature in Google Forms that requested student emails. The result was that the emails became item one in the analysis and the last item, number twenty-

four, 'You can be creative in math class', was bumped out of the analysis. Despite this, Alice was able to interpret the results and do her own analysis of item 24.

The process that the three participants went through was different from the original three teachers but the survey and the information they received from the change analysis was virtually the same. The participating volunteers were not involved in the compilation of the original survey items. They were required to copy survey files and change analysis files to get results, unlike their counterparts who were provided the results in a link. Although the procedure of scrubbing the data was not involved in this study, the survey and change analysis output for this study were essentially the same as in the original experience. The three volunteer participants received similar data about the beliefs and attitudes of their students from the change analysis as the three teachers in the original survey.

Data Analysis and Interpretation

Interviews were audio recorded online or in-person and transcribed with an assist from Google Recorder. What began as semi-structured interviews often devolved into two teachers chatting about education. Attempts to create a written record of some aspects of the interviews were lost to conversation. I did not code the transcripts. Instead, I looked for any mention of specific ideas and more general themes which I tried to allow to emerge naturally. What transpired were five somewhat varied interviews with five experienced educators.

Within the transcripts, I gathered participants' accounts of their understanding of how they engaged in reflective practice. I also noted when the conversation shifted to reflective thoughts and participants might be thinking critically about an event or some aspect of their teaching experience. I looked for evidence of established practices of student feedback that might garner insight into their students' beliefs and attitudes. Accounts of student interviews or student surveys were considered evidence of more formalized feedback techniques. I scanned the transcripts for words such as *feel* and *sense* when referring to reflective practice. These passages were analyzed for evidence of less formal student feedback that leaned toward the craft-like nature of teaching.

I also searched for evidence within the transcripts that the change-analysis results of the student survey might provide teachers with a more informed awareness of how their classroom practice affects students' beliefs and attitudes towards mathematics. Finding evidence of the manner in which the change-analysis of a before-and-after student survey on beliefs and attitudes on mathematics might contribute to teachers' self-reflective practice was challenging. This was not always readily evident in the transcripts and was occasionally inferred from the voice intonations within reactions.

Results

First Interview: Reflective practices of Jeffery

When asked to talk about his reflective practice, Jeffrey focused on his classroom exchanges and how he was guided by *feel*. He spoke about "reflecting" on specific students, "I spend the first 15, 20 minutes of my time really observing those kids and seeing how well I'm catching them. And if I'm capturing them in the beginning, I've usually got them for the whole period." Although this might be considered *reflecting-in-action* I questioned whether this was more about focus and observation than reflection. He explained that "I am reflecting on their ability and then shifting my focus based on reflection. I am thinking I'm using it as a choice tool, to inform where I'm spending my time. That reflection is necessary." When asked about reflective practice outside of the classroom he responded, "I am spending a lot of my efforts ensuring that I'm actually going to hit all the big points of the curriculum, but in some way that these kids can actually access them is meaningful." Inquiring about how reflection fit into his planning, I asked if he did more of his thinking before or after class. He replied, "Definitely before. I am a planner, naturally a planner so I like to spend the time there." When asked if the previous lesson affected his planning, his response indicated that any reflection on the previous lesson tended to be brief and somewhat intuitive, "I can *feel* it. It's almost like at the end of the lesson that went well, or that did not go well and I don't think it takes a lot of reflection to know that when I'm in the flow." "I don't know if it affects exactly the next day, but I know that if I *felt* like a lesson was dissonant or I didn't *feel* like it went as well as it should have." When Jeffery feels that the lesson did not go well, he has a number of informal techniques to check for understanding the following day.

When asked to consider self-reflection versus reflection Jeffery appeared uncomfortable. He admitted to having "a lot of social anxiety" and referred to his own self-reflection as "harsh". He explained that he can't handle compliments or positive feedback, "I've always thrived on the negative feedback. It's a competitive thing. I react better to critical feedback than I do positive feedback. So I think that I grow more with that than I do with the positive." When asked about what gave him joy and offered reassurances that he was doing a good job, Jeffery was unsure how to answer the question, and said that he would have to think about it.

Jeffery stated that at the end of the week or at the conclusion of a topic, he solicits student feedback by drawing three faces on the whiteboard; sad, neutral, and happy, and asks students to raise their hands as he points to each face. Whether this is more of a measure of understanding than an indication of happiness may be irrelevant. One could argue that in math class, understanding and happiness tend to be inseparable as are misunderstanding and misery. For more formal feedback on tasks that are submitted for grading, Jeffery places a similar question that asks, "how are you feeling about your understanding of a certain topic". Students place a dot on a linear scale as an indication of their confidence.

Jeffery said that his after-school reflections tend to be more closely connected to the final conversations that he has had with students rather than anything which may have occurred during the course of the day earlier. At the end of the course, his students self-assess their learning skills as listed on the report cards. He does not do end-of-course student surveys and tends not to think about the job after the last day of school until three weeks before the start of the new year.

Regarding students' attitudes and beliefs, Jeffery understands that students can enter Grade 9 with a sense that they are not good at math. He conveyed that these students tend to shut down when faced with equations and theorems. Instead, he incorporates games and "engaging" activities that aren't "intimidating, math wise". He explains, "I take it from a place that doesn't start off with math". He expressed that because of his approach, "I think that they have a better experience in my course". Jeffery thought that having a student that was unhappy in his course was a rarity. He bases that on his perception of student engagement; students who "are unwilling to try".

First interview: Reflective practices of Alice

Due to technical difficulties not all of Alice's initial interview was recorded. She repeated sentiments that Jeffery expressed about the lack of time for quality reflections, but she includes more formal student feedback in her reflective practice. Alice expressed how the sense of immediacy that characterizes the classroom experience leaves little time for in-depth reflections during class time. The focus during class and between classes each day tends to be a matter of assessing "where the students are and what to do next". A significant concern that occupies her reflections is finding the balance in her approach that will meet the varied needs of her students. How to support those who struggle with anxiety and mental health while challenging confident students with more demanding problems?

Alice referenced *thin slicing*, figuring out patterns based on narrow windows of experience. The level of student engagement is her go to measure of progress and success. When she does self-reflect, it can take the form of an unproductive negativity loop. To break out of the negative self-talk, she finds that conversing with colleagues allows her to escape the negative feedback loop and move to more positive and constructive thoughts. In the area of formalized student feedback, Alice no longer does exit cards in lessons. Instead, she relies on questions on the bottom of her weekly quizzes or *check-ins* as she calls them. She asks students to self-assess their progress and gives them an opportunity to demonstrate learning that was not tested on the quiz. This takes the form of a direct question, 'What have you learned that I haven't asked you about'?

At the end of the course, Alice's students do an extensive exit survey on an online Google Form. Students provide feedback on a number of classroom practices including the online Desmos activities and graphing calculator, working in random groups on vertical whiteboards, and the blended learning platform Google Classroom. They also rate the usefulness of six different online resources to check their understanding. These include Quizizz, The Knowledge Network, Khan Academy, The Centre for Education in Mathematics and Computing [CEMC], Explore Learning, and practice pages that Alice posts in Google Classroom. The survey also asks about the choice of strategies when a poor score on these exercises indicated that more practice was required. The last two items in the survey are open-ended; 'Ideas you haven't shared yet' and 'This semester I learned . . .'. Once her final evaluations are completed, Alice spends a couple of hours examining the survey results.

To encourage all students to answer the survey, Alice tracked who answered the survey and who did not. Alice identified students using the collect email feature in Google Forms. This eliminated the element of anonymity, which may have affected student responses.

First interview: Reflective practices of Paul

When I first asked about the role of reflection in Paul's teaching, he talked about not doing the course the same from semester to semester or even from morning to afternoon. I thought nothing of this seemingly innocuous crossover or connection between the term *reflect*ion and the idea of *repetition* until after his account of a particularly successful Grade 11 lesson. He said, "It was pretty magical. So therefore, when you say, 'Do you reflect on that and would I ever do that activity again?', I'll do that activity with Grade 12 kids."

Paul's reflective practice became more evident when he discussed thinking, "I think about stuff all the time. It doesn't really turn off. I'm always thinking." He explained that he does his preplanning on Sundays. He thinks about what he is going to do the following week "based on what I have done, what we have done". He further adjusts his weekly plan during his quiet moments in his classroom after school each day and while walking home. He said that reflection happens 'in the moment'. Paul explained this in the context of his classroom dynamics. He follows Liljedahl's (2020) *Building Thinking Classrooms* model where he is "trying hard to have them construct the mathematics so that there's a deeper understanding versus me telling them it." To be effective, this approach demands a continuous interchange between teacher and students. While students work as a group of three or four on solving problems on vertical whiteboards, the teacher should constantly monitor the students' thinking by listening to conversations among students and observing their written work. Liljedahl's approach offers a number of teaching strategies to encourage mathematical thinking and guide students on the path to solving problems. Unanticipated situations tend to emerge when students are encouraged to play with a variety of problem-solving strategies. Responding to observations and conversations is what Paul considers reflecting "in the moment".

Paul also considers as reflections, his accounts of his morning lessons shared with colleagues at lunch time; "I think that's a good thing when you can get people to chit chat about what you are doing."

When asked about self-reflection Paul reacted with, "Oh, like how I am doing?" After a pause, he said "I'm pretty confident where I'm at." One source of this confidence is Paul's end-of-course student survey. Paul's students fill out an anonymous hand-written survey on the topics of whiteboarding, random grouping, spiraling, assessment and anything else they wish to expound on. Discussing the survey results, Paul says, "the feedback I get from kids at the end of course is now so positive. Sure, there's some negative stuff there but I'm pretty confident what I'm doing is working. So, I don't think I spend a whole lot of time worried about what parents think or what kids think anymore." Paul clarified this statement by saying that he is not worried as much as he was when he first shifted away from his traditional didactic instructional approach and began spiraling the curriculum and building *the thinking classroom*. Initially he was concerned about how to fairly assess student achievement in this new approach, but he claimed, "I don't worry about that at all anymore." He explained that what his students might lose in the ability to speed through algorithms to answer typical secondary textbook and test problems, they gain in critical thinking skills. Paul explained, "I'm super confident that I'm preparing kids for the future and for thinking."

Not only do Paul's students provide feedback with the end-of-course survey, they also have three student-teacher interviews each semester. While the class is engaged in group work on the whiteboards, Paul calls each student out into the hallway for one-on-one check-ins where each student has an opportunity to express how they believe that they are progressing and convey any concerns that they might have. Paul said he can usually complete the entire roster of interviews in one 75-minute period. He claims that there is a social emotional component to these interviews when he prompts his students with "Tell me how you *feel* the course is going". Paul reiterated, "It's a check-in to see how they're feeling about mathematics and their ability in it." He feels that he is making a much stronger personal connection than he ever did when his classroom practice was teacher centred. He explained, "It's me caring more about them, than I would have before. I have way better relationships with kids now than back, then. Way better relationships, for sure."

When asked more about his teaching methods and how they affect students' attitudes, Paul explained, "We are trying to develop ideas, develop thoughts, collectively versus me being a gatekeeper of the knowledge. I do think it changes their attitude towards mathematics. I think

they realize that it's not just about getting the right answers. It's not just about following a set of procedures." As far as student appreciation is concerned, Paul stated, "I do *feel* like lots of kids start liking it more. It starts becoming their favourite class." When asked how he assesses how well a particular class has gone, he answered, "You can tell by the excitement in the room. I can *feel* when light bulbs are going on." After reflecting for a moment, he defined the source of those feelings more specifically by stating, "I can tell if it went really well based on what they're saying to me and based on where some of their ideas took us."

As an illustration, Paul provided an account of when student ideas took the class on an enriching journey. He explained how a Grade 11 college level [MBF3C] review of the Pythagorean Theorem using fraction sums took an unexpected turn when one group of students organized their fractions into a table and a pattern emerged. Paul explained, "Well, unexpected mathematics that rolled out, and in the moment, seeing that kid organizing in a table and looking at those patterns. Let's see where this goes!" With Paul's encouragement, all the groups were soon implementing this strategy. This rather innocuous decision led to a series of mathematical connections that included linear and quadratic equations, the area model, trigonometry, and an exploration of angle patterns that could be modeled by an exponential function. Paul confirmed that the observation that led to this "in the moment" decision to chart a divergent course is a hallmark of his experience and confidence.

Despite expressing confidence in his abilities, Paul reflected on two areas of his practice that he felt needed work. His analysis of the *thinking classroom* reveals a weakness in the model that he continues to struggle to compensate for. Also, by his own self-assessment, there is an advanced teacher move required of the model that he feels he is "not very good at". He related that he spends more time thinking about his craft than he did when he simply lectured, and his reflections are more in-depth and complex than during his time as a more traditional chalk-and-talk teacher. "Back then the kids came to watch me do it [math]. My reflection would be; should I add another example?"

Recollections and reflections of the users of the 0riginal survey and change analysis

Despite the time elapsed, Hanh and Emma had clear memories and provided valuable insight into the experience. After an extended time to reflect on the original experience and subsequent iterations, they expressed strong convictions about the value of the venture.

Hanh had taught for twenty-two years and was working as an instructional coach at the time of the interview. She had an extensive history of informing her practice with student surveys and student-teacher interviews. Her experience with the original survey was the first time she had done a before-and-after survey with a change analysis component. I interviewed Hanh about the impact of the survey at the time and about its potential in informing teachers' reflective practice. Hanh said that she had no qualms about sharing her results of the survey at the time with her colleagues. She surmised that each of us had enough experience and confidence in our own abilities as not to possess any misgivings about sharing the results. When asked if she found anything interesting or surprising in the results of the survey Hanh replied, "nothing". Despite her claim the survey seemed to provide some important feedback. Hanh explained, "I know there were some things that I took from it to make changes in my practice." I was curious about one particular result from her class that differed significantly from the other two classes. That result showed that the majority of her students agreed with the statement that 'The math that I learn in school is mostly facts and procedures that have to be memorized.' She explained it this way, "There is one thing that I learned. Students learn differently, right? So, there were some students that just like to do the repeated practice over and over again until [they] get it. There's some students that like to take notes and then study their notes whereas, we were, I was doing a lot of the inquiry kind of collaborative stuff and that just wasn't their learning style. So, trying to find something that kind of meshed the two or allowed for more diversity and that is what I delivered."

When asked how we get more teachers to reflect on and question the overall effect on their students' beliefs, Hanh responded, "That's the million-dollar question." She said that she thought that a tool such as this survey had a "hundred percent" place in exposing and changing students' attitudes toward mathematics. When asked if it was better suited for newer teachers rather than more experienced teachers she disagreed and felt that it might even be more illuminating and effective for established teachers. In her words, "Because I think the biggest

problem with our education system and teaching in general, is people try to find the easy way out, so they plan their course, and then use that over and over and over again, without considering who is in front of them necessarily. And what's good for one student isn't necessarily good for another student. And if you're only delivering the same thing that is only accessible to one or a minority of students, what happens to the rest? I don't feel like teaching should ever be stagnant and the only way to know that you're not, and that you're constantly evolving is through these feedback forms from students. If I'm not making a change from the beginning to the end of the semester, then I've failed those kids. I want kids to want to progress and want to better themselves, and why wouldn't I model that?"

Hanh reiterated that teachers are not lazy, they just get "too comfortable" and they need to be more aware of the importance of meeting the students where they are. She offered an account of some of the challenges that emerge from her instructional workshops on destreaming, "There is push back from teachers that some kids just aren't ready, or they don't have the prerequisites or skills, or they can't do it and they're blaming the child and not necessarily looking at the bigger picture." Hanh suggested that the bigger picture included the teachers' role in assessing the students' needs and adjusting for that. That not only includes students' mathematical skills but also their attitudes and beliefs. As far as the role of a before-and-after student survey is concerned, she declared, "I think it should be part of the TPA [Teacher Performance Appraisal]". Hanh continues to use a number of the original items to survey her students.

Emma had been teaching for 22 years at the time of the interview. Emma's account offered a different perspective but also emphasized the value of such an enterprise. Emma's recollection was that during the first experience with the survey she didn't consciously think about the nature of the items on the survey and did not adjust her practice because of it. She explained, "We didn't tailor the way we ran our classes to align them with the questions on the survey. When you include an item like 'I prefer to work on math problems by myself' and we were running math class in collaborative groups, we knew that we would be inviting students to challenge that assumption." Emma's claim that the presence of the survey did not influence her teaching approach mirrors my own experience. I did not give a thought about the survey items during the semester and only considered the results after the change analysis was revealed.

When asked about her own reaction to the results of the survey, Emma said, "You look for the results you want to see, right? I remember I found it challenging to be objective about my own class because I really wanted what I was doing to have made a difference. And so when I would see a question like 'Making mistakes in math helps me learn', I want to feel like I had fostered that environment and I would have really wanted to see that go up by the end of this semester. I found myself really feeling nervous about looking at those results because what if this reveals that I did something that was detrimental to the students." Emma expressed that feeling of uncertainty that seems to characterize the complex act of teaching and learning when she added, "Certainly we work hard as educators. We were all trying a bunch of different things and genuinely caring about what the outcome would be."

Emma's disclosure of anxiety contrasts with Hanh's more self-assured report. More importantly it emphasizes the challenge of being objective that all teachers face. Teachers, being human, tend to make interpretations that fit their own belief system. Whether it is students' test results or survey results, teachers will always maintain the option to ignore or misinterpret or shape the results to suit their needs.

Besides confirming that she was "on the right track", Emma agreed that what made this particular survey more impactful was that we had the opportunity to compare three different classes. She felt that the trust that allowed this to occur was developed through lesson study, and we already had a sense of how our classes differed. The survey exposed some differences in our beliefs and what we emphasized. The example that Emma pointed to concerned how we approached homework. Hanh delivered a program which very intentionally included twenty minutes of homework every night and she reminded the students that it was an important part of the experience. Emma, on the other hand, actually tried not to give homework, instead offering some optional practice or a single question that some students might want to mull over. Emma recalled that the survey showed that initially Hanh's students mirrored their teacher's sentiments by showing a higher regard for the importance of homework. The differences in our philosophies were never explicitly stated but the survey revealed differences in the way we structured our classes. Emma noted, "what I find is interesting, when you talk to teachers, they don't always say the same things that are reflected in their classroom practice. If you had more anonymous results and then you surveyed the teacher to say how much homework you give or what you value in

your classroom, the teacher might respond in a way that is either idealized or reflects what they think they do." This phenomenon of the disconnect between what teachers say they do and what they actually do has been studied by Fives and Buehl (2012) and Liljedahl and Oesterle (2014).

Emma employed a similar version of the survey the following year to evaluate the effectiveness of a new, innovative program. She collaborated with a colleague to create a section of techno-math where Grade 9 applied math was combined with computer programming. This new course emphasized *application* in the *applied* math course. To measure the impact, the teachers reused the before-and-after student survey with some additional items about technology. Emma, who has a background in statistics, did the final change analysis herself and reported some "really very important results" about this particular group of students who were chosen for this techno-math program. She reported "quite startling comparisons" between the beginning of the year and the end of the year on how students felt about themselves as learners and how they perceived the usefulness of math in real life. For the survey item 'I can see how useful math is in real life', Emma said, "there was a big jump by the end and it felt really good to feel like that. It felt reassuring to me as an educator, that I was, we were, on the right track." Emma explained that this second instance of the survey felt more purposeful. She suggested that the initial survey felt that we were grasping at straws but in this follow-up was much more directed. Emma explained, "The things that we wanted students to feel, they were feeling in this program. I was able to feel that that was a bit more intentional than when we first started this."

Emma acknowledged that although teachers will have the option to interpret survey results in their own fashion, it still "has the potential to challenge what you might think is going on in your classroom". Emma went on to consider the character of a teacher who would even consider such a survey. In her view, "I think it's complicated because what I think, and I think you observe this a little bit too, is and the teachers who are receptive to the idea of having a survey in their class and actually seeing results at the end of the of a class that might already be indicative of a teacher who's already fairly reflective in their practice." Emma asked, "How open is the teacher to begin with to examine their practice?" To her point, all six of the teachers [I am including myself] who used the survey, in my own estimation, are open to examining their practice in an effort to improve.

This idea led Emma to articulate what she considers a significant moment in a teacher's career. She mused, "I think about this a lot and because I just wonder about that moment where you become an educator and where does that turning point [happen]? I call them turning points in your practice where you are now open to this idea of what is happening in your classroom and the idea that you might need to change some things and you are now all of a sudden thinking about students differently and what their needs are differently." When asked about the conditions that might lead to such a turning point, Emma considered a teacher she knows that is new to the profession and responded with a question, "Is it new teacher versus experienced teacher or is it the fixed mindset versus growth mindset teacher that would be interested in this kind of survey?"

Emma was presented with Hanh's opinion that many experienced teachers are comfortable with what they're doing, and they don't feel a need to change. She provided the perspective that, "Education is changing. More and more students are coming into classes with IEPs [Individualized Education Plans]. More and more students are coming in with very diverse backgrounds, where now [we] are recognizing what a disservice we've done with our racialized and marginalized students, which is part of this conversation because your Grade 9 applied classes had all of our brown and black students in them" as reported by People for Education (2014) on Toronto schools.

Emma argued that the traditional one-size-fits-all model of education that many of us experienced as students no longer applies. "Teachers are really feeling like they want to just stay in their 'I'm going to teach' lane and they're finding that they can't just stay in there. Because with what I was doing before, I really never did think about some of these other students and what their needs and their backgrounds [are]. All of these students who have IEPs and have diverse gender expression and all of these kinds of things." By Emma's estimation, "It's a very uncomfortable time for a lot of teachers and education right now."

Emma suggested that this survey "is shining a spotlight on teacher learning. When we talk about the teachers' pedagogical practice, it's about our thinking about the nature of teaching as a reflective practice and really it is shining light on. What is it that we're doing this for? We're not a factory where students come in, we put a stamp on them, and students go out. And we have a career that spans twenty to thirty years. So, we should be learning during that time as well.

We're part of society, and society is changing, and we should be changing. And how are we capturing what our learning is doing during that time? And how we capture that typically is that our learning gets reflected in what our students are doing. It's like it's becoming personalized somehow and you're really internalizing. What is it that you do in your profession? How are you making meaning out of it? And as soon as you start making meaning out of it, then you do start caring about these things." Emma continued, "I'm not saying we didn't care at the beginning of our careers. I hope it's not that I didn't care, but it felt very superficial compared to the way I think about my students and my classes."

Reflections of the Volunteer Participants on the Student Survey and Change Analysis

During interviews, participating teachers were reminded not to disclose any specific student information and of the option to withhold any results from the researcher. The results of the survey were not data for this research study. The data came from interviews of the teachers' reflections on their interpretation of the results.

Second interview: Jeffrey's reflections on the survey and change analysis

During the second interview Jeffrey outlined changes that he was making in his practice. He had incorporated more student feedback into his assessments with one-on-one interviews with his students at midterm and at the end of the course. He interviewed the students in the hallway as the class solved problems in small groups. The students were given their evidence records ahead of time and they were asked how they felt the course was going for them. Jeffrey stressed that this process is especially important for the Grade 9 students who generally lack the skills to judge their own performance. It is also an opportunity to express any concerns they might have. Jeffrey has found that the student feedback that these interviews provide has added a significant element to his practice.

I took the opportunity to clarify some of his statements concerning his reflective practice from the first interview. Jeffrey's responses in the first interview on issues of feedback and self-reflection left me with more questions. During a more relaxed second interview I learned that my comments on *positive feedback* were interpreted by Jeffery as *positive comments* or *compliments*. Compliments make Jeffrey uncomfortable for two reasons. First, an element of critical self-

evaluation creeps in, knowing that he could do better. Second, he also feels that compliments hint at flattery which may be insincere. He said, "I think the default in giving feedback is to be nice and to water it down if it's bad. What is their intention? What do they gain by being negative other than having to explain themselves further?" He applied this thinking to the survey and questioned whether students would voice their real opinion on a survey.

As mentioned, Jeffrey did not do the change analysis with his survey. To facilitate the discussion concerning the change analysis, I presented Jeffrey with an example of a change analysis output from my university math course for non-math teachers. The survey items were the same. Because the results were not from students, he was familiar with, he was left to speculate on the validity of the survey. When considering the item 'I enjoy group work in math class', he said, "Is there significant change because the kids know I value that and do it every day or is that because they are enjoying it." On the item 'It's important to get the right answer in math' Jeffery had similar thoughts, "This is what they understand as what I value, but once again, do they value it?" He considered the item 'I like math' at length. "I can see how that would be beneficial to me if I reflect on that question. Let's say it's neutral or went down. I'd be pretty concerned about the moves I can make there as a teacher. So, they're not hands on enough? Was it too theoretical? Did they not see themselves in what they were doing? Are there things that I would try next time? Would I have to get them outside and throw a baseball around to do it? Like what will it take to get a kid to say that?" Jeffery continued his reflections by questioning the importance of liking math. "I don't know if that is super important to me. I think about whether or not they would jump into a task and try it and try to figure it out and be intrigued by it. I want them to see that there's a value in a puzzle to be solved and if they don't like it …, that's kind of how I frame a lot of my work. It's a puzzle and if they don't like puzzles …". He agreed that he would be more interested in any change with the item 'I like to work on challenging math problems'. He reflected, "Like it or dislike it. Okay, is that math? Is that something else?"

When first presented the example change analysis, Jeffery expressed reservations on the validity of the survey. As he spent more time considering each item, his focus shifted, and he began to speculate on the overall value of the results. He concluded, "I can see how digging into

those questions would help me decide where I need to put my effort in the future. I see how it has value now."

Second interview: Alice's reflections on the survey and change analysis

Alice commented that the items in the survey reflected her values on growth mindset, collaborative learning, and the creative nature of mathematics, "I think I'm seeing mostly the things that I wanted them to value are being reflected". Although Alice's second interview occurred two weeks after the end of the school year, she had not yet examined the results of the change analysis in detail. During the interview she proceeded to go through the results from each item one-by-one and reflect on each item individually. She reported that the results of certain items made her "happy". All her students agreed that 'You can discover things in mathematics on your own' and the majority agreed that 'You can be creative in math class'.

She reported that there was a significant increase of students who agreed that 'I enjoy sharing ideas with my peers in math class' as well as 'I like to do math on the whiteboard'. Alice speculated that this student endorsement of collaborative learning may be the result of returning to a more normal classroom without the restriction of Covid protocols during the pandemic. The exchange between students was sparse during virtual online classes and whiteboard work was limited by social distancing practices during the initial return to in-class learning.

Items that tried to distinguish between growth and fixed mindset proved to be inconsistent and problematic. An increased number of students agreed with the statement 'Some students are good at math, and some are not'. Alice was surprised by this since she thought that this indicated that her students had shifted toward an attitude of fixed mindset. She then noticed that this was contradicted with the response to 'People can change how intelligent they are in mathematics'. Nearly all of her students agreed or strongly agreed with this statement, and she surmised that this indicated a growth mindset. The contradictory nature of her results on mindset mirrors the difficulties that occurred during the first application of the original survey. Removing the negative from 'People can't change how intelligent they are in math' may have made a difference yet there remain issues with the difference between what the teachers intended with the items on mindset and how the students interpreted them.

Alice spent time reflecting on the results of the item 'The math I learn now helps me with other subjects'. The decrease in student agreement concerned her and had her wondering about the possible causes. Analysis on her part was hampered by the fact that within a single set of survey results she had two separate classes and three separate grades. She speculated that the change might be driven by the responses of the Grade 12 students who are beginning to focus on a particular area of study. To investigate further, Alice vowed to parse out the responses using the email addresses to identify the source of the decrease.

There were results from another item that Alice found puzzling. As stated earlier, Alice's own exit survey includes several questions on the perceived usefulness of the substantial array of online homework help that she provides. The results of the survey item 'Doing homework helps me understand math' showed that less than half of students agreed to this statement at both the beginning and the end of the course. She was visibly taken aback by this and mused, "They really don't think doing homework helps them understand math at all". These results brought to her mind the research of Alfie Kohn (2007). Her understanding of Kohn's message was that requiring homework created inequity so either, in her words, "there really shouldn't be homework, or homework should be meaningful". By offering varied options of extra practice but not evaluating homework, Alice hoped that more students would find homework meaningful. Considering how few students indicated that they agreed that homework helps them understand math, Alice seemed perplexed. Deliberating on this, Alice said, "I'm kind of conflicted because I still do have my own evidence. I've seen this growth in them, when they have decided to commit x amount of time to focused practice. So, I don't know. I don't know what that lower amount means".

Second interview: Paul's reflections on the survey and change analysis

In Paul's case, he had no difficulties in administering both iterations of the form and transferring the results to the change analysis spreadsheet. Unfortunately, it was approximately four weeks into the semester when the students answered the first survey. He understood that the analysis might not show as much change as if he had administered it earlier. When he first reviewed the change analysis there were results that seemed suspect. I realized that there was a problem with a formula in my version of the change analysis spreadsheet that he had copied.

Before any more analysis could be considered, I had to fix the problem and send him an undated version. By that time summer holidays had started, and he ignored the updated change analysis until we met again during the school year. He examined the results of each item during the second interview and afterward we discussed the overall process.

Paul had the advantage of seeing the results of all three of his classes. Unfortunately, too much time had passed to allow him to effectively connect the results to the students in the classes. Despite that, he was able to speculate on the results. In the process, he demonstrated how a teacher is able to interpret the results to suit their own ideas. For example, why did the students in his Grade 11 college level [MBF3C] show an increase in agreeing that getting the right answer was important whereas his Grade 12 calculus class [MCV4U] did not? The results from the 4U students fit with Paul's emphasis on the importance of process. Why was that different for the 3C's? Paul speculated that the 3C students were doing better than they had in their previous courses and were enjoying the fact that they were getting correct answers.

As a teacher whose classes revolved around group work, Paul was pleased with results from the item 'I enjoy group work in math class'. There was an increase in all his classes with one hundred percent in 3C. He had an interesting interpretation of the positive change for 'All students would be good at math, that they work hard at it'. "I think they feel like they probably did more math in my class than they have at any other time in their lives because they're doing it for 75 minutes a day and they feel like if they worked hard on it, they got better." Paul was surprised that there was an increase in students who reported that when they see a math problem, they get nervous. He made an analogy to sports and decided a certain level of stress or nervousness was not a bad thing and it can enhance performance. Having to work collaboratively with peers and perform might also add an element of stress. There was strong agreement among all three classes that they like to work on the whiteboards with two classes showing one hundred percent by the end.

In the 3C class there was an increase in students who agreed with the statement 'I forgot how to do problems that I have solved before'. All the students agreed or strongly agreed to the statement by the end of the semester. Paul reasoned, "I think that's reflective of not teaching algorithms. They don't feel like they know how to solve it because everything feels unique. Even

though they have some of the tools they're never really sure where to start." He added, "We lose some automaticity doing it the way we do it for sure but the stuff that we gain is worth it. Being able to solve a problem quickly, being ready for a test and being automatic is different than what we're doing every day."

There was minimal change of 4U students who generally reported that they liked math. Although the final count for the 3C students was below fifty percent by the end, there was a significant twenty-five percent increase in students who reported that they liked math. This difference in the classes and change in the 3C class was mirrored in the results for 'Math is one of my favourite subjects'. Paul was not surprised that the results showed high agreement among the 4C students for 'The math I learn helps me with other subjects' and 'I need to do well in math to study what I want later'. What gave him pause to reflect was the difference with the 3C class. Although their results for 'The math I learn helps me with other subjects' increased by twenty-five percent, the results showed a twenty percent drop in students that agreed that they needed to do well in math to study what they want later. Paul surmised, "I feel like that's reflective of the attitude towards it. Even though they don't think they feel like they have to do well in it to use it. I think they don't feel like it's the mark that matters to do well. I just think after having spent the semester with me they're just like, it's okay, right? I'm doing math every day. I'm getting better at it. Do I have to really do well in the course?"

Paul felt that the survey did capture his students' attitudes and beliefs about mathematics, but he re-emphasized that he might have seen more change had he given the initial survey at the start of the semester. He also suggested that the survey items were better suited for the intermediate grades and that it could be modified for the senior courses. Overall, he agreed that the before-and-after change analysis offered insight that was not available by other means. He said, "It would be a good thing to do at the start of every course and at the end of every course". "I think it's useful data, regardless, whether it moves in the right direction or not. If it doesn't move in the right direction, what can we do differently? And if it is moving in the right direction, let's find out what it is that made them say that and do more of that. Would it be useful? For sure it would be useful." He added that he would be curious to see what the data looked like for the new Grade 9 destreamed course. "It would be super useful for those teachers to get a feel for what they are doing in that course".

Paul repeated that it would be useful for all teachers no matter what level they were teaching. He suggested, "Why wouldn't we? We don't know their background and where they come from, so why would I not want to know their attitudes about it? Walking into my course and then what their attitude was walking out of my course. That is what we're doing, changing their beliefs about math, right?"

When asked if he might change his practice based on what the survey revealed, Paul took a defensive stance that pointed to the challenges he has faced over the years to transform his classroom from teacher-centred to student-centred. "I feel pretty strongly about where this is at and where this is going and that it's good for education, good for their learning. And I'm not going to back down just because they've been learning by memorizing verses understanding all the way through." He illustrated his position with an account of a Grade 12 student who recently announced to the entire class that, "This is different, I'm understanding stuff. It's starting to make sense to me."

Although Paul espoused the benefits of such a survey, the fact that he did not follow up with the change analysis without my intervention suggests that there is a fundamental weakness in the process. A student survey which involves two ten-minute sessions on cell phones is easy to forget and easy to ignore. In my own experience, I did not think about the survey until I was sent the results in an email. The fact that student responses reside electronically can be a benefit when there are five months between iterations. This allows the teacher to retrieve the data more easily perhaps than a paper record. *Recall* is the first phase in Lee's model of reflective practice and is critical for change analysis. But because the data exists in a virtual state, it can more easily be ignored than other forms. In contrast, Paul's end of course surveys are handwritten and the pile of paper they create is very evident on his desk. He feels compelled to read them within a day or two. Paul also feels that written feedback allows the student a richer form of feedback which makes for more interesting reading for him. "It's scales that kids just tick versus when you ask them to articulate you get something different from them. [You] get what it was that they didn't like what it was that they liked. It's a little bit more personal, a little bit more meaningful than raw data like that." He did acknowledge that feedback in written form doesn't allow for the same level of change analysis. "The beginning and end are both there if you want it, right in one sort of simple package."

Paul considered how peer support could add a level of accountability with the potential to add another level of analysis with shared results. He suggested, "If you're doing this with another person or the department, then there could be a little bit of check and balance. A way to say, what's going on?"

There is little doubt that Paul will continue to use his student interviews and end-of-course exit surveys, but it seemed that after his brief reflection on the change analysis, Paul realized the potential and said, "I really wish I had done it with my 2D's [MFM2D] this semester."

Discussion

Reexamining the Original Research Questions

My study began with three questions. The first question was 'In what way might a student survey provide teachers insight into their students' beliefs and attitudes about mathematics?' My conclusion is that the survey itself provided limited insight for both groups of the teachers in this study for the simple reason that only two of the six teachers actually looked at the initial survey results. Alice, one of those two teachers, used the survey to identify students who might require more attention at the start of the course. Because she included emails in her version of the survey, Alice could identify the students who did not like math and she made a point of observing their early progress. Hanh gained insight into the attitudes of her class which led her to adjust her approach. Ninety-two percent of her students agreed with the statement 'The math that I learn in school is mostly facts and procedures that have to be memorized'. When the class struggled with "the inquiry kind of collaborative stuff", Hanh decided "that just wasn't their learning style" and turned to an approach based on facts and procedures. Others, me included, who have faced pushback on inquiry-based learning might not have been swayed as Hanh was but it is interesting to note that Hanh relied on this approach during her own schooling.

Unlike Hanh's experience, the survey offered no insight for most of the teachers involved since they did not examine the results of the first survey. My interpretation of this indifference is based on my own experience as someone who also failed to review the results of the initial survey. For teachers as well as students, the launch of a new course brings the promise of a fresh

start with new beginnings. Although there is value in acknowledging and even honouring attitudes and beliefs that students bring with them into a new course, these feelings represent beliefs and attitudes based on earlier experiences. With a new course there is an inclination to look forward rather than backward. A ten-minute online survey tends to fade in importance when faced with the enormity of the task of establishing classroom routines to complete a course within the allotted time. There were no reports of any students in any of the classes inquiring about the results and most of the teachers did not take time to look at the initial results. The initial survey proved to be something of little consequence and as such was easily ignored. This sentiment was expressed by Emma when she declared that in no manner did, she consciously adjust her teaching to align with any of the survey items.

During the interviews a related question emerged: would this survey be appropriate for all secondary math teachers and their math courses? The teachers in this study shared significant common ground pedagogically and the survey items seem to align with their values on mindset, collaborative learning, the value of making mistakes and the creative nature of mathematics. After examining the results of the change analysis, Paul felt that the survey did capture his students' beliefs and attitudes. Alice commented, "I think I'm seeing mostly the things that I wanted them to value are being reflected". So, the question is, would a different cross section of teachers share her sentiment and feel that the survey items reflect what they would like to know about their students' beliefs and attitudes? Although certain items such as, 'I like to do math on the black/white board' might not be applicable to all classrooms, there is nothing to limit students responding to this statement even if it was not an element of a particular class situation. When the survey was designed, there was an effort to include alternative viewpoints which had the added benefit of providing checks and balances. The item 'I enjoy group work in math class' was balanced by 'I prefer to work on math problems by myself'. Although the survey may have a certain ideological slant, the 4-point Likert scale hopefully provides an opportunity for students to reliably express their attitudes. Perhaps adding alternative items that relate to a more traditional approach, such as working from textbooks, might make the survey more inclusive with a wider application. Alice believed that this survey was open enough to meet the requirements of all teachers and students, but I remain uncertain how it would be received by teachers and students in a traditional teacher-centred situation.

My second research question was 'How might the change analysis results of a before-and-after student survey on beliefs and attitudes on mathematics provide teachers with an understanding of how their classroom practice affects students' beliefs and attitudes concerning mathematics?' Based on the experience of the three teachers in the original experience, the change analysis provided an informed understanding of the change in students' attitudes. Although one of those teachers reported no real surprises, she revealed that she made adjustments to her classroom approach based on those results. The other two teachers were startled by what the change analysis produced. It gave them a new perspective of their teaching. Those two teachers found reassurance that their approach had led to a significant improvement in student attitudes about mathematics. When one of those teachers was involved in piloting a new course the following year, the change analysis provided "really very important results". It showed a "startling" "big jump" in how students felt about themselves as learners and how they perceived the usefulness of math in real life. That version of the survey was more intentional and provided an even greater reassurance for both teachers piloting the new course that the things that they "wanted students to feel, they were feeling in this program".

The information provided from the change analysis was less significant for two of the three teachers who volunteered for this study. In my estimation, these teachers may not have been as invested in the process. One teacher examined the results while being interviewed months after the semester had ended. As the interview progressed, he was considering the nature of the items and results of the change analysis for the first time. His experience was less about providing him with an understanding of how his classroom practice affects his students' beliefs and attitudes and more about the realization of how this tool could effectively do that. This was also the experience of the participating teacher who did not complete the change analysis but was presented with sample results from my university class. At first, he had difficulty relating to the results and questioned the validity but during the course of the interview he began to trust the results and realized the value in the process.

Alice was the one volunteer who was able to complete the change analysis within two weeks of the end of the semester and benefitted more significantly. As she examined the results during the course of the interview, she saw a variety of different outcomes that informed her practice. Because she was able to complete the change analysis on the before-and-after survey as

intended, it provided a much richer understanding of how her classroom practices affected her students' beliefs and attitudes about mathematics. She made quick reflections as she inspected each item. Certain items were reassuring, some were perplexing, and others were problematic. She was reassured when all of her students agreed that 'You can discover things in mathematics on your own' and the majority agreed that 'You can be creative in math class'. Yet these two items revealed little or no change. Alice speculated that this may be attributed to the fact that she had taught most of the students in her classes previously and they came into her classes instilled with these values. Collaborative learning items such as 'I enjoy sharing ideas with my peers in math class' and 'I like to do math on the black/white board' showed a significant positive increase which she attributed to the easing of covid restrictions where students had either been isolated in virtual classes and not been able to freely intermix. She was able to justify the decrease in agreement with 'The math I learn now helps me with other subjects' based on how the content of the senior courses diverged from the science program. Alice felt that unfavourable results demanded more scrutiny. One item that required further attention showed that less than half her students agreed that homework helps them understand math. This she found perplexing given the time and effort she spends providing a variety of homework opportunities. She vowed to look more closely at this quandary. The items on mindset proved contradictory, as was the experience of the original three teachers. Although Paul was able to justify the contradictions apparent in the items on mindset, Alice found the results to be problematic, confusing and of little value.

As previously stated, the survey itself had limited value. On its own the survey is simply a snapshot and snapshots fail to reveal direction or rate. The real power lies in the before-and-after nature of the survey and the subsequent change analysis. As Alice surmised, "It's okay if they don't start off with the view of mathematics and learning that we might have, but it'd be nice to know before the end if they are gaining that confidence." Although all three volunteers in this study appreciated the possibilities of the process, this study failed to reproduce the conditions that the original three teachers experienced that were so impactful.

My third research question was 'In what manner might the change-analysis of a before-and-after student survey on beliefs and attitudes toward mathematics promote teachers' self-reflective practice?' All three volunteers in the study reflected on the nature of the information

provided by the change analysis. Items that confirmed what they believed to be true provided reassurance but items that conflicted with what they had expected were greeted with more scrutiny. Some conflicts could be rationalized but others tended to encourage an examination of the volunteer teachers' classroom practice. Bringing attention to an unexpected result is a start, but simply thinking about situations or events without action amounts to reminiscing. According to all the academic models, true reflection must include action. My intention for examining a mechanism for self-reflection is to offer a pathway leading to action; leading to change. Paul's account of sharing the details of a successful lesson with his colleagues at lunchtime may not qualify as reflective practice unless it leads to change. So rather than asking 'in what manner might the change analysis promote self-reflection', it seems pertinent to ask if there has been any action that might indicate the presence of productive self-reflection.

For all three teachers in the original iteration of the survey there was action that could be directly attributed to the results of the change analysis. Although the specifics were not revealed in the course of this study, Hanh reported that her reaction was to make changes to her classroom practice based on the change analysis. Emma found inspiration in her results. She took the reassurances garnered from the change analysis of the Grade 9 academic/applied course to develop and pilot a new course that went further in meeting the needs of those learners in applied mathematics. As the third teacher in the group, the change analysis gave me significant insight that no other experience had come close to providing. The feeling that this was an idea worth pursuing persisted which manifested itself in this research.

As with the second question involving the change analysis, the experience of the three volunteer participants was not quite as significant as it was of the original three teachers. Jeffrey did not complete the change analysis. Without results from his own class, he was left to consider sample results from my university class. After accepting that the results might be valid, he speculated that examining the change analysis could offer a framework for future efforts. As the least experienced of all six teachers, Jeffrey's interviews hinted at a lower level of confidence than his counterparts and he might have had the most to gain from the change analysis of student beliefs and attitudes. During the course of this study, he did make some changes that cannot be attributed to the change analysis but may have been influenced by this study's focus on student feedback and reflective practice. Jeffrey added one-on-one student interviews at midterm and at

semester's end. Unlike Jeffrey, Paul readily expressed confidence in his classroom approach. As he looked at each item, he was able to explain the results in the context of his classes, yet because of the length of time that had elapsed his analysis was neither extensive nor deep. There was nothing in the exercise that compelled him to alter his classroom approach. After sifting through the change analysis, he recognized its potential to inform teachers of potential changes needed. His endorsement included a regret that he did not apply the process to a new course with which he was less familiar.

Alice's change analysis did inform her self-reflective practice which resulted in tangible changes in her classroom approach, and which were revealed in a second iteration of the change analysis of the before-and-after survey. In an email she described how, after better understanding the negative effects of pandemic restrictions, she made a more focused effort to support group work with her new cohort of students. These measures included developing group thinking, communication skills and social skills. The change analysis confirmed the effectiveness of her efforts as there was a twenty-five percent increase in students who agreed that they like to work on the whiteboard and a thirty-nine percent increase in students who agreed with the statement 'I enjoy sharing ideas with my peers in math class'. Another focus was offering challenging and engaging problems daily and emphasizing the value of learning from mistakes in these challenges. The result was a twenty-nine percent increase in students who agreed that they like to work on challenging math problems and a thirty-four percent increase in students who agreed that making mistakes help them learn math. Concerned that the minority of her students in the previous semester agreed that homework helps them learn math, Alice made significant changes to her approach. The survey in the subsequent semester showed that the semester started with similar numbers with only forty-four percent agreeing that homework helps learn math. Unlike the previous semester which saw little change, there was a twenty percent increase in students who agreed that homework helps them learn math. This is an example of a teacher recognizing an issue as a result of the change analysis, reflecting on it and making active changes to produce benefits for her students. This is a clear demonstration of the problem-solving potential of this process.

Additional Considerations

Teachers in this study and in other research (e.g., Potari & Georgiadou-Kabouridis, 2009 as cited in Fives & Buehl, 2012; Devine et al., 2013) consider reflection an important element of effective teaching but what became apparent over the course of this study were the differences in understanding of what reflective practice is and what it entails. When asked directly about *reflection*, Paul's initial interpretation seemed more closely connected to the term *repetition*. He explained how his lessons did not necessarily repeat from day to day and from semester to semester but when a lesson worked well, he was happy to repeat it. He interpreted my question on reflection by asking "So therefore when you say, do you reflect on that and would I ever do that activity again?" and answering his own version of the question with, "I'll do that activity with Grade 12 kids." Another example that Paul offered as reflection is when he describes a successful lesson to his colleagues at lunchtime. Although repeating the lesson may be an appropriate action, recounting a lesson might not include the action that is required in models of self-reflection (Kolb, 1984; Gibb, 1988; Dricoll, 1994).

When asked about reflection, Jeffery had a different take on the term. He spoke about reflecting on specific students, spending the first fifteen or twenty minutes of the lesson "really observing those kids and seeing how well I'm catching them". Jeffrey further explained his position, "I am reflecting on their ability and then shifting my focus based on that reflection. I am thinking I'm using it as a choice tool, to inform where I'm spending my time. That reflection is necessary." Although this might be considered *reflecting-in-action*, complete with serious thought and consideration associated with reflection, this could also be described as focused observation. As the interviews progressed and reflection was discussed, both participants shifted their notion of reflection to match my own definition more closely. Why such a critical element of teaching does not have a more shared definition is worth exploring.

Common teacher duties that are easily recognized include lesson planning, recording attendance, delivering lessons, managing the classroom, assessing student achievement. And although reflective practice is an essential element of being a teacher and a component of everything a teacher does, it is not included in the list of teacher's duties. It is not observed nor assessed. There is no bell to signal the start of the reflection period and there are no deadlines for completing reflections. Although all teachers do it, they have no formal framework or model for

reflective practice. Until a teacher shares their reflections with colleagues, reflection practice is not visible. Certain reflective practice seems more readily identifiable such as when Jeffery reports thinking about an issue while alone in his workshop or Hanh remains awake at night with a problem. Yet reflective thinking can intertwine with the myriad of decisions that occupy a teacher's day and may not be recognized as reflective thought. Paul expressed it well when he said, "I am thinking about stuff all the time. It doesn't really turn off. I'm always thinking." Teachers are no different than most experts who are challenged to explain how they do what they do.

Another aspect of reflective practice hinted at in this study is the somewhat anxiety-inducing nature of looking inwardly at self-reflection. The lines between reflection and self-reflection may be blurred but most of the teachers in this study who talked freely about reflection had to be prompted to talk about self-reflection. As the least experienced teacher in the study, Jeffery seemed the least comfortable turning the conversation inward. When asked about self-reflection Jeffery immediately mentioned having "a lot of social anxiety". He expressed a self-critical nature when referring to his own self-reflection as "harsh". He justified that by suggesting that he found negative feedback more motivating than positive feedback. With over two decades of classroom experience, Alice's narrative seemed more introspective and more comfortable in considering self-reflection and her role in the workings of the classroom. This was apparent when she described how she dissipated self-doubt by talking to her peers. Paul, the most experienced educator of the group, did not comment on self-reflection until asked directly. He seemed unsure of the question and for clarification, he asked, "Like, how am I doing?" A look at Paul's history may help to explain his response. During his initial years of teaching, Paul honed his craft and was confident in his ability to teach through lectures. When he shelved his well-established teacher-centred approach, he did a great deal of work to quell self-doubts and justify his new student-centred approach to students, parents and colleagues. Since redefining his practice and reestablishing his confidence, his focus has been on his students and improving technical aspects of his *thinking classroom* with little thought to what he might consider as self-reflection or self-evaluation. He declared that he is "pretty confident" in his own abilities and "super confident" in what he is providing to his students.

Despite Paul's assertion of his confidence, for many teachers, self-reflection is not necessarily an exercise in self-confidence. When teaching coach Martin Odima (2023) posed the

Twitter question, *How many years in the teaching profession did it take for you to feel like a competent teacher? Like, you felt like you knew what you were doing?* Typical responses included, *I'm in year 20 and I still have days where I don't feel like I don't know what I am doing*, as well as the rather sardonic *The year after I left*. Educators on Twitter may not represent a cross section of teachers, but if this is an indication of the level of uncertainty that emerges from teachers' self-reflections, then the change analysis on a before-and-after survey might provide an increase in self-awareness and self-confidence, as was the case with twenty-year veteran participant Emma who found reassurances that she was "on the right track".

Connections to the Literature

Participants in this study aligned with other studies (e.g. Potari & Georgiadou-Kabouridis, 2009 from Fives & Buehl, 2012; Devine et al, 2013) that demonstrated that participants understood that self-reflection was a critical element in improving teaching methods. In general, the practice of these participants was similar to teachers in other studies with one notable difference. The participants in this study reflect an increasing focus on the affective domain and the role of student feedback in that shift. They support the notion that promoting student's social and emotional well-being can improve student achievement, which has been found in other studies (e.g., Foley et al., 2017). To that end, a number of the teachers in this study incorporate student surveys and one-on-one interviews. They also understood that students who struggle with mathematical concepts often approach mathematics as a series of disconnected facts as shown by Maciejewski & Merchant (2016) and they understood the importance of promoting mathematics as an interconnected web of knowledge. This study echoed the work of Jacob et al. (2017) in that teachers judge their performance mostly on student behaviors. Unlike the work of Jacob et al., it did not show teachers were more concerned with skill attainment than student engagement. This is perhaps because this study was focused on the affective domain rather than mathematical skill development. Without specifically designed instruments recommended by Leder (2016), these teachers infer affective sub-domains of beliefs and attitudes indirectly from behaviour or responses as shown in the study by Golden (2002). This process of 'reading the room' confirms the craft-like nature of teaching as described by Devine et al. (2013). Participants reported they could "feel it" when students were fully engaged in the

lessons. Where these feelings come from is less clear. Judging engagement by observing students' eyes (Brookfield, 1998) might be a source of feedback in a lecture style classroom where students are facing the teacher. This does not apply to the classrooms of these teachers who tend to involve students working in small groups at the whiteboard in what is termed *defronting* the classroom (Liljedahl, 2020). One of the participants was able to articulate the feeling that indicates student engagement by pointing to more specific indicators such as the quality of student questions and their ability to articulate their ideas.

Studies by Wilkie (2016) and Peterson et al. (2000) showed that students respond to surveys with reason and intent. The teachers in this study did not question the validity of the results from the survey with one exception. The participant who did not complete the change analysis was initially reluctant to accept that students would respond to the survey truthfully and might be inclined to provide responses that are socially acceptable or that align with his own ideals, a similar result as found by Ruthven (2015). This particular teacher expressed a similar suspicious attitude when discussing his view on feedback in general. He offered, "I think the default in giving feedback is to be nice and to water it down if it's bad." The discussion that followed included the findings by Wilkie that showed that students tend to take questionnaires seriously and the study by Peterson et al. that students responded to surveys with reason and intent. He seemed more accepting of the results after I pointed to the fact that the survey protected the anonymity of the students which was demonstrated by Robison and Taylor (2007) to improve the integrity of survey responses.

The participants in this study agreed with the teachers in the Devine et al. study (2013), that being a reflective practitioner is an important quality of good teaching. Their experience mirrored the findings of Cavanagh and Prescott (2010) which noted that there were limited opportunities to develop reflective practices during teacher education programs, but these were not encouraged by their practicum supervisors. All of the teachers in this study engage in many if not all of the strategies of good teaching listed in the Devine et al. study. In the realm of reflection, they seek advice from colleagues, are self-critical, evaluate their own performance and reflect on their shortcomings. They also continue to take risks and experiment with teaching methods, despite years of experience. One aspect of good teaching which the Devine study failed to mention was something that all of these teachers emphasize: soliciting student feedback. In the

decade since the Devine et al. study, it is possible that more and more teachers are learning the value of student feedback while experimenting with new techniques. If a teacher is comfortable in their practice and has no intention of changing, there is less to be gained from soliciting student feedback. When a professional operates in this steady state, it is considered *knowing-in-action* (Schön, 1983) where there are no surprises and events unfold as expected. A teaching model which involves a one-way transmission of information adapts more easily to detailed planning whereas an open model which is built on student involvement is less predictable. The teachers in this study rely on their experience for much of their practice yet their openness to experiment with new methods demands that they embrace moments of uncertainty that require reflection. How to define these moments of reflection is less clear.

The model for teacher self-regulation (Chatzistamatiou et al., 2014) involves three clearly defined steps; lesson planning, lesson delivery and post-lesson self-reflection and evaluation, all of which have loosely defined boundaries. The experience of the participants would point to the fact that reflection can and does occur throughout the process and is not confined to the post-lesson. When a teacher makes decisions during lesson delivery, can that be considered as *reflection-in-action* (Schön, 1983)? Intuition, trial and error, general muddling through are certainly involved but these teachers generally did not define these moments as *reflection*. Attempts by two of the participants to identify reflection within classroom activities led to somewhat imprecise definitions. As reflections that occur after an event, post-lesson reflections are more readily recognized as *reflection-on-action* (Schön) yet even then, they were not always defined by the participants as *reflection*. Reflection is part and parcel of the process of information gathering, interpretation, and planning. It is blended into the thinking that these teachers do when lesson planning and is not an easily defined separate process. Lesson planning is a recognized duty of all teachers and reflective practice is what transpires in the background during this process. Participants' accounts of thinking about issues while alone in a workshop or awake at night may be easily categorized as reflections-on-action yet there may be another way to think about Schön's distinction between reflection-*in*-action and reflection-*on*-action. Rather than a model of reflection that revolves around a single lesson (Chatzistamatiou et al) could we consider a model of reflection that involves an entire semester?

While an educator is immersed in the process of teaching a course, could thinking about how the course is delivered over the entire duration of a semester be considered reflection-in-

action? When the bell signals the end of a class session or the end of the school day, teachers can still be immersed in the task. My own teaching career was characterized by the participant who said "I think about stuff all the time. It doesn't really turn off. I'm always thinking". If teachers are constantly engrossed in their craft throughout the semester, could all their reflections be considered reflections-in-action? Should we reserve the term *reflections-in-action* for those on-the-fly decisions made during class time? Or might the entire semester-long process in which a teacher is engaged be considered *in-action* until the final class is complete?

Perhaps when the grades have been submitted, there is an opportunity to engage in the post-mortem reflection-on-action more fully. Participants who received feedback from the change analysis between semesters reported gaining reassurance that they were on the right track but did not spend an inordinate amount of time analyzing the results. Detailed analysis at the end of the school year has its own particular challenges. As one participant expressed, when the summer holiday starts, the thinking stops, and July signals a time to recuperate rather than cogitate. As planning begins for upcoming courses, August may be an opportunity for more in-depth reflection-on-action. *Recall* is the first step in Lee's (2005) reflective thinking model. A change analysis of student attitudes and beliefs can assist in recall. With accurate recall, the second step to *rationalize* effectively is feasible. With recall and rationalization, reflectivity with the intention of change becomes achievable. The change analysis on student beliefs and attitudes could play an important role in teachers' self-reflection and self-assessment.

Summary

In *The Reflective Practitioner: How professionals think in action*, Schön (1983) defined the need for all professionals to research their own practice through reflexivity. Almost 30 years later Hattie (2012), in *Visible learning for teachers: maximizing impact on learning*, encapsulated the results of his meta-meta-analysis by claiming that "the biggest effects on student learning occur when teachers become learners of their own teaching and when students become their own teachers" (p. 22). To that end, teachers should reflect, think about and evaluate their attitudes, motivations and behaviours. Teachers, in this and other studies, understand the importance of reflection in developing their practice. Yet teachers have very little training in reflective practice and little or no knowledge of models of reflective thought. Although every teacher may report that reflective practice is an important element of what they do, what is considered reflection and what form it takes varies among teachers. Self-reflection can be unproductive and have the unwanted potential to be self-deflating. For teachers in this study, reflective practice remains an essential element in the process of thinking extensively about the most effective methods to convey the curriculum and support their students. When teachers act on these thoughts and adjust their actions, they are engaging in the reflective thought process. The act of converting these thoughts to actions is the foundation of reflexivity. To date, academic research and the fostering of reflective thought has often centred on student learning. More work on how to develop teachers' reflective practice is needed at the academic level, but this work may be more effective at the school level. The development of more systematic reflexivity should be applied to the affective domain of learners.

In recent years there has been an increasing awareness of the importance of fostering positive attitudes toward the learning of mathematics. Positive attitudes about mathematics have far reaching consequences for individual students and society. This plays out in short-term day-to-day student engagement and longer-term interest in careers in the STEM fields. The province of Ontario has acknowledged this by including a social-emotional strand in the mathematics curriculum (OME, 2020b). Awareness of the issue may help to create the environment where students' social-emotional well-being is not trampled in the march to develop mathematical skills by memorizing algorithms and focusing on computational efficiency. Although the social-emotional strand is neither assessed nor reported on, increased awareness of its importance by

students, parents and teachers could lead to more inclusive pedagogy which fosters mathematical thinking and cultivates an appreciation of the subject in all students.

Based on my own experience and my interviews with five other teachers, I believe that this study shows that tracking changes in student beliefs and attitudes has the potential to be a more systematic mechanism for addressing students' social-emotional well-being. When properly implemented, this mechanism proved helpful in two ways. In some cases, it provided reassurances that the teacher's approach was having a positive effect on students' beliefs and attitudes. In other cases, it exposed areas that demanded a closer examination and possible changes.

Referring to Lee's (2005) model of reflection, the initial survey results combined with the end results residing safely in the cloud give teachers the power to *recall* students' experience to define the situation and identify any problems. The change analysis can provide the basis for *rationalizing* and interpreting the situation and making connections. The last step in the constantly revolving cycle is *reflectivity* with the intention of change or improvement. Improvement should be at the core of the life-long process of becoming a mathematics teacher.

Teaching mathematics to students who appreciate the subject is a joy. There can also be joy in the challenging but important work of teaching students who were indifferent or loathsome. My student who wrote that mathematics was the work of the devil was far from alone in his condemnation. Other anonymous comments were even more difficult to read like one student's rather disturbing proclamation that if he was in a relationship with mathematics, he would beat it every day. How is it that such a subject of such potential beauty can be the source of such derision? I do not dismiss these as misguided rants from a few rash adolescents. I do not accept these attitudes as collateral damage in the pursuit of producing the next generation of engineers. It is hard to imagine that many in my fraternity of math teachers seemingly accept this situation. Action starts with awareness; awareness of every math teacher's role in promoting healthy attitudes about mathematics.

Would you believe that my student who reported that he despised mathematics went on to study mathematics in post-secondary education? Don't believe it. In reality I have no idea about his fate, but I doubt it centered on academics and the study of mathematics. The last time I saw him he looked very much like a rock star with his body piercings and electric guitar. At that time, I felt nothing but fondness for him, as I do now. Given the opportunity, he was brave enough to

let me know where he stood on the study of mathematics. I accepted his position and welcomed him as my student, and he reciprocated by accepting me as his teacher. I was not totally successful in reversing past harms, but I had evidence that his experience in my class reduced his math class angst. In turn, he helped me on my journey to become a better math teacher. Distinguished educator John Mason said that a brilliant math teacher is not born but made. Becoming a math teacher is a "life-long process of learning to balance care for learner(s) and care for mathematics" (Majewska, 2020). A teaching approach that promotes positive student attitudes about mathematics is one method to balance care for learners and care for mathematics. A method to track changes in students' beliefs and attitudes could play an important role in such an effort. If we value the social-emotional needs of our math learners, then we need to develop methods to track beliefs and attitudes. Conversely, if we have a method of tracking student attitudes, we will more readily value social-emotional student needs.

Limitations

The limitations of this study begin with the small sample size. The three volunteers provided valuable insight into their understanding of reflective thought but for two of the three teachers, their experience with the survey and change analysis was limited. For one volunteer, a delayed administration of the survey and a delayed change analysis reduced the overall usefulness of the process. Any perceived lack of commitment to the project by the volunteers can be attributed to my reluctance to initiate any contact that might increase their workload. These teachers had given a great deal of themselves to persevere during the difficult conditions of the pandemic. I can only speculate if the stresses of the pandemic had some bearing on one volunteer's leave-of-absence and failure to complete the change analysis. More timely communication and guidance on my part could have helped participants in completing the survey and change analysis, resulting in more informed data. This was somewhat counterbalanced by including the experience of two teachers who were able to apply the survey in more than one course, in more than one semester.

I missed an opportunity while interviewing my colleagues who developed and administered the original iteration of the survey. Their recollections on the change analysis were surprisingly informative but I failed to inquire about their reflective processes, thereby limiting a potential source of data.

Another concern is sampling bias. I was familiar with two of the volunteers and had worked extensively with the two teachers who I collaborated with to create the survey and one of the volunteer participants. Without a real choice of volunteers, this is essentially convenience sampling. All the teachers interviewed share similar classroom approaches. They are all concerned with students' affective domain and their classrooms are decidedly student centred. Although not every teacher knows every other teacher, every teacher has had professional dealings with at least three others. This adds an element of self-selection bias where the participants share similar characteristics with each other and with the researcher.

Recommendations and Next Steps

Research on reflective practice in education often focuses on developing reflexivity in students. More research at the academic level needs to focus on teacher reflective practice for self-regulation and self-evaluation. The type of research that I would argue for is the development and testing of effective techniques to support teachers in more systematic approaches to self-reflection and self-evaluation. These techniques should harness student feedback while supporting teachers' autonomy. This type of research need not rely on pending academic research to develop more effective working models of self-reflection. Teachers have a role to play. Education is what happens between a teacher and their students and innovation happens when there are positive changes in that interchange.

In my limited years as a full-time mathematics teacher, I witnessed and participated in important changes in mathematics education. These reforms tended to be more practical than philosophical in nature. Web-based graphing apps on student-owned cellular phones replaced the use of school-based graphing calculators. A district-developed online assessment tool resulted in many more teachers adopting mandated assessment reforms. In both these cases, teachers and students were given better tools to do their work. Despite the important contributions that can come from outside the classroom, the most significant changes in my classroom were driven by teachers collaborating with teachers. This included the development of activity-based learning with manipulatives and hands-on activities; spiraling and the interlacing of strands; combining academic and applied classes; and the adoption of the thinking classroom with its visibly random groups and vertical non-permanent surfaces. Although these strategies were ultimately based on sound academic research, they demanded a healthy dose of classroom action-research to be fully

realized. In the same manner, teachers collaborating with teachers can pilot more systematic approaches to support informed reflexivity.

Developing and encouraging effective teacher reflection will support Hattie's (2012) ascertain that teachers must be learners of their teaching. The first step in advancing more structured approaches to reflection is to define reflective thought, not with definitions but with discussion amongst teachers as to what it is and what form it can take in teachers' practice. Every teacher practices reflective thought and understands its value but reflection as well as self-reflection mean different things to different teachers. Before developing more systematic approaches to reflection, teachers must construct a working understanding of what it is. Understanding is the critical first step in solving any problem and as a way of illustration, I can offer an account of a different professional development initiative.

One year my school made critical thinking the focus of staff development. As an initial first step, I surveyed the teaching staff, the majority of whom reported that they promoted critical thinking in their classroom. Months later after discussions about critical thinking and what it entailed in classroom activities, a follow-up survey revealed that there was a significant drop in teachers who reported they involved their students in critical thought. What changed? A shared understanding of critical thinking had emerged.

Once reflective thought is better understood, educators can examine and develop methods to solicit student feedback to inform their reflections. Student feedback places students at the center of the solution and teaching can evolve from its craft-like nature to be more evidence based. Teachers can begin to self-reflect not on 'how things feel to me' but on 'how do my students feel about what we are doing'. Tracking changes in attitudes with a student survey can serve to support innovative practice and provide reassurances that established practices and/or innovations pedagogy are having a positive effect on learners' social-emotional well-being.

The results of this study are not decisively conclusive but encouraging. I no longer have the same professional connections that I once had but I intend to present my findings in an article in the *OAME Gazette*, at conferences and on the web and promote the use of the survey. I will make available the student survey on beliefs and attitudes about mathematics and the accompanying automated change analysis for other educators to use and modify. If the need arises, I plan to offer a version based on Microsoft Forms and Excel. If more teachers choose to use the survey, they might provide important feedback for its use and improvements. A pipe

dream would be to have a web-based version which would be fully customizable for any and all subject areas, allowing a user to choose the number of items, add/delete or modify items, preset data collection times, send reminders to students, and automatically email the change analysis results directly to the teacher.

In the effort to innovate and build improved teaching and learning strategies, the day-to-day feedback that teachers imply from student engagement remains important, yet reflective practice must not be confined to single lessons. A mechanism to track longer trends is important. To avoid the deterioration that characterizes some students' appreciation of mathematics, teachers must become more aware of the longer-term effect of their established routines and new approaches on students' attitudes and beliefs.

Conclusion

Reflective teaching is examining one's underlying beliefs about teaching and learning and aligning them with actual classroom practice before, during and after a course is taught. Classroom practice should include an awareness of how students' social-emotional needs are critical in developing positive attitudes in learning mathematics. Although the province of Ontario has included a social-emotional strand in the mathematics curriculum, its importance is diminished since it is neither assessed nor reported on. Yet any decline of student interest in mathematics presents a problem for teachers to address. Teachers who ignore this issue contribute to the problem. Teachers who address this issue can contribute to the solution. The key to improving student interest lies in teachers' reflexivity and the ability for teachers to be learners of their teaching. The reflective thought that teachers practice tends not to be systematic or supported by effective models. Teachers' self-reflection and self-evaluation are inclined to be limited to the informal 'feel' for student engagement in day-to-day lessons. This study examines the effect on teacher reflections with the change in student attitudes and beliefs over the duration of an entire course. The results showed that the online survey and accompanying automated change analysis on students' attitudes and beliefs about mathematics increased teachers' awareness of potential issues and provided reassurances that innovative practice was having positive effects on how students felt about mathematics and how they felt about themselves as learners. The process protected the anonymity of the students and the autonomy of the teachers.

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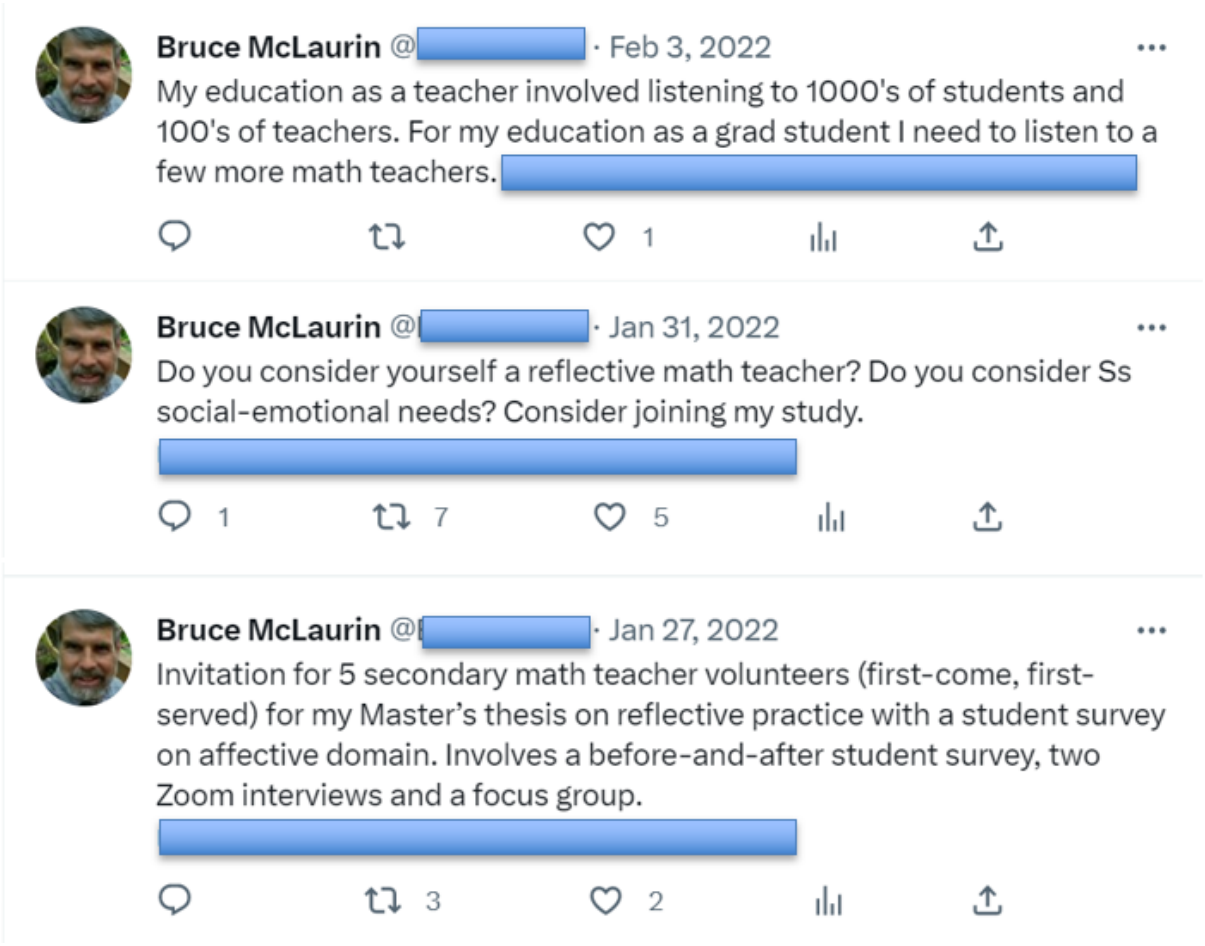
Appendix A: Items of the Beliefs and Attitudes about Mathematics Survey [BAMS]

1. The math that I learn in school is mostly facts and procedures that have to be memorized.
2. In math you can discover things on your own.
3. Making mistakes in math helps me learn.
4. It is important to get the right answer in math.
5. I enjoy working on challenging math problems.
6. I prefer to work on math problems by myself.
7. I enjoy group work in math class.
8. Some people are good at math and some are not.
9. People can change how intelligent they are in math.
10. All students would be good at math if they worked hard at it.
11. When I see a math problem, I get nervous.
12. I like to do math on the black/white board.
13. I enjoy sharing ideas with my peers in math class.
14. I forget how to do problems that I have solved before.
15. If I get stuck on a math problem I ask for help.
16. If I get stuck on a math problem I usually try to figure out a different way that works.
17. I like math.
18. Math is one of my favourite subjects.
19. The math I learn now helps me do work in other subjects.
20. I need to do well in math to study what I want later.
21. Doing homework helps me understand math.
22. I talk about math when I am not at school.
23. You can be creative in math class.
24. Please complete the statement. *Mathematics is …*

Appendix B: Invitation for volunteers in a series of tweets



Appendix C: Blog post published on 1/27/22 11:51 AM to solicit potential volunteers

Still reflecting on students' beliefs and attitudes about mathematics

During my teaching career I spent a great deal of time reflecting on the attitudes that students developed toward mathematics. That hasn't changed in retirement.

So now I am studying students' beliefs and attitudes and teachers' reflective practice in my Master's thesis with Dr. Christine Suurtamm as my academic supervisor.

During the Ontario-based Math 4 the Nines project, I, and two other teachers of grade 9 mathematics worked with a graduate student to develop and administer a before-and-after student survey on beliefs and attitudes about mathematics. A change-analysis of the survey revealed some interesting results that gave us pause to think and reflect.

In my research study, entitled *Informing mathematics teachers' reflective practice with student surveys on affective domain,* I am inviting secondary math teachers to talk about their reflective practices, with a focus on the information gathered through a before-and-after student survey on beliefs and attitudes about mathematics (see below).

Participation in this study will involve a 30 to 40-minute Zoom interview at the beginning of the semester on reflective practices. Participants will then be invited to offer their students an online survey on beliefs and attitudes about mathematics. I will provide a Google Form for you to copy*. The survey will take about 10 minutes at the beginning of the semester and again at the end of the semester. A before-and-after change-analysis will be automatically generated. At the end of the semester, participating teachers will be invited to a second 30 to 40-minute Zoom interview and a 45-minute online Zoom focus group.

***NOTE: By making a copy of the Google Form, you and you alone will have access to the student data. Neither my supervisor nor I will have any access to student data or student information.**

I appreciate that there are added time pressures on classroom teachers these days but if you are interested in participating or would like more information, please send me a DM @[redacted] or an email to [redacted] or [redacted].

Thank you for your interest.

~Bruce

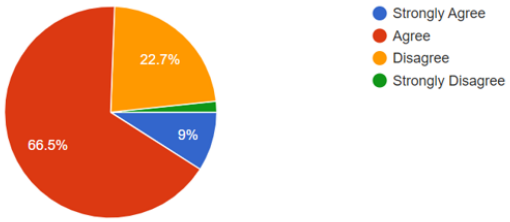
Appendix C: (cont.)

Sample items from the student survey on Beliefs and Attitudes about Mathematics

1. The math that I learn in school is mostly facts and procedures that have to be memorized.
2. In math you can discover things on your own.
3. Making mistakes in math helps me learn.

Sample results from the Google Form survey

2. The math I learn in school is mostly facts and procedures that have to be memorized. *



Sample results from the automated change-analysis

2. The math I learn in school is mostly facts and procedures that have to be memorized. *

	Start	End	% Change
Agree or Strongly Agree	82	65	-17

Appendix D: Questions for semi-structured interviews

The first semi-structured interview will be conducted before the start of the course before the first iteration of the student survey. Sample questions might include but will not be limited to:

- Please tell me about your experiences with self-reflection.
    - Where do you place the importance of self-reflection in the evolution of your teaching practice?
    - What aspects of your teaching experience tend to promote self-reflection?
    - When do you tend to self-reflect?
- Please tell me about your experiences with student feedback.
    - Describe the role of student feedback in your teaching practice.
    - Describe any student feedback that you have used such as exit surveys, journals, questionnaires, etc.
- How do you view the role of students' attitudes and beliefs in mathematics?
    - Describe the role of students' attitudes and beliefs in academic achievement.

The second semi-structured interview will be conducted after the participants have viewed the change analysis of the survey and had time to reflect on the results. Sample questions might include but will not be limited to:

- What can you tell me about the results of the student survey?
    - Which class(es) did you survey and why?
    - Do you think that results accurately reflect the change in students' attitudes and beliefs?
- What can you tell me about any reflections that you might have had after viewing the results?
    - Describe any aspects of the results that stand out in your mind or gave you pause to reflect.
- How might this experience affect your classroom practice?