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LA THÈSE A ÉTÉ
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HEAT TRANSFER IN TURBULENT RECIRCULATORY FLOWS AFFECTED BY BUOYANCY FORCES IN RECTANGULAR CAVITIES

by

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ABSTRACT

The transport of momentum and heat in turbulent flow is often affected by buoyancy forces. In the present investigation, a turbulent recirculatory flow inside a rectangular cavity is studied. The flow is driven by the combined effects of shear, induced by a moving lower wall, and buoyancy, induced by a temperature difference between the upper and lower walls.

The analysis is based on the development of a two equation turbulence model proposed by Jones and Launder [24]. In this model, the equations for the kinetic-energy of turbulence, dissipation rate of the kinetic-energy of turbulence and mean-square temperature fluctuations are solved numerically, together with the momentum and energy equations, using the control volume approach. A numerical technique based on the Gosman procedure [2] is developed to solve the equations.

The flow of water and liquid sodium is investigated for Reynolds numbers up to 10^6. The effect of buoyancy forces is analyzed. The flow patterns, temperature distributions, average Nusselt numbers and turbulence characteristics are presented.

The obtained results and observed trends are in good agreement with analytical and experimental work reported by other investigators [1, 3 and 15].

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NOMENCLATURE

Symbol English Letters	Definition
A	Control volume cross-sectional area, m^2
Ap	$\sum a_i - Sp$
a	Coefficient of the finite difference equation
C	Convective term, $C = \rho UA$
c_p	Specific heat at constant pressure, $J/kg.^oK$
c	Empirical constant appearing in the K-ε model, see table (3.1)
D	Diffusion term, $D = \Gamma A/\Delta y$
d	Cavity width, m
E	Constant in the logarithmic law of the wall
F	Flux at control volume boundaries
G	Generation term
Gr	Grashof number, $Gr = g\beta\delta T d^3/\nu^2$
Gr/Re^2	Mixed convection parameter
g	Gravitational acceleration, 9.806 m/s^2
h	Cavity height, m
K	Kinetic-energy of turbulence, $K = \frac{1}{2}(\overline{u_i^2})$
ℓ	Characteristic turbulence-length scale
ℓ_m	Prandtl's mixing-length
$\ell_{m,\phi}$	Prandtl's mixing-length of scalar quantity
NI,NJ	Number of grid mesh in x and y direction respectively

Symbol	Definition
$\overline{Nu}$	Average Nusselt number, $\left\|\frac{dT}{dn}\right\|_{n=0}$
Nu_b	Average Nusselt number computed with buoyancy forces
n	Coordinate normal to the wall
P	Pressure, N/m^2
Pr	Prandtl number, $Pr = \frac{c_p \mu}{\lambda}$
Q	Heat flux, W/m^2
q	Mean-square temperature-fluctuation, $^oK^2$
Re	Reynolds number, $Re = \frac{U_o d}{\nu}$
S_ϕ	Source term, see table (3.2)
$S_{\phi V}$	Integral source term, $S_{\phi V} = Sp_1 \phi_p + Su_1$
S_{false}	False source term
Sp	$Sp_1 + \max\left(\sum_i c_i, 0.0\right)$
Sp_1	Component of linearized source term
Su	$Su_1 + \max\left(\sum_i c_i, 0.0\right)$
Su_1	Component of linearized source term
T	Temperature, ^oK
T_1	Upper wall temperature, ^oK
T_o	Moving wall temperature, ^oK
δT	Temperature difference between two successive cells
u	Mean velocity component in the x-direction, m/s
U_o	Moving wall velocity, m/s

Symbol	Definition
URF	Under-relaxation factor
u	Velocity fluctuation component in the x-direction, m/s
V	Mean velocity component in the y-direction, m/s
v	Velocity fluctuation component in the y-direction, m/s
VOL	Cell volume
X,Y	Dimensionless horizontal and vertical cartesian coordinates, x/d, y/d respectively
x,y	Horizontal and vertical cartesian coordinates

Greek Letters

β	Coefficient of volumetric expansion, $1/^{o}K$
ε	Dissipation rate of turbulence kinetic energy $\frac{C_D K^{3/2}}{\ell}$
κ	Constant in Van Driest's formula
λ	Thermal conductivity, $W/m^{o}K$
Γ	Exchange coefficient μ/σ
μ	Dynamic viscosity, kg/m-sec
ν	Kinematic viscosity, m^2/s
ρ	Fluid density, kg/m^3
τ	Shear stress, N/m^2
ϕ	General variable
σ	Prandtl number
$\sum_i$	Summation over control volume sides

Symbol	Definition
Subscripts	
E,W,N,S	East, west, north and south neighbouring nodes of p
e,w,n,s	East, west, north and south side boundaries of control volume
eff	Effective (molecular plus turbulent)
K	For kinetic-energy of turbulence
p	Centre node of control volume
t	Turbulent
x	Local value
W	Wall value
ε	For energy-dissipation
Superscripts	
$\overline{}$	Average value
n	n-th iteration
'	Previous iteration

CHAPTER 1

INTRODUCTION

The transport of momentum, heat and mass are often affected by the gravitational field. Such is the case for recirculatory flows in enclosures, buoyant shear flows or warm jets submerged in cold temperature fluids.

In order to study the effects of buoyancy forces on the transport of momentum and heat, cavity flow was chosen. Its geometric simplicity and practical implications provided a good test for theoretical models and new numerical schemes.

One example of cavity flow may be found in the fast breeder nuclear reactor (Phoenix), where the reactor core is surrounded by a large vessel filled with hot liquid sodium and traversed at the bottom by a cold jet leaving the heat exchanger. The mixing layers of the cold jet create a turbulent recirculatory flow which is affected by buoyancy forces due to density changes induced by the temperature gradient.

The analytical study of this problem was carried out by Grand et al. [1] for laminar flow conditions, using water and liquid sodium, by solving numerically the Navier-Stokes and energy equations in the stream function and vorticity form. They presented the stream lines and the isotherms together with the Nusselt number for Reynolds numbers in the range of 10^2 to 10^3 and for ratios of the mixing convection parameter, Gr/Re^2 between 0 and 1.

The objective of the present work was to study numerically the flow, heat transfer and turbulence characteristics of the cavity flow for high range of Reynolds number up to 10^6. It also represents, to the author's knowledge, the first attempt to predict numerically the turbulent recirculatory flow affected by buoyancy forces in a rectangular cavity.

The numerical calculations were based on the development of the two equation model of turbulence in which the turbulent exchange coefficient (the turbulent viscosity) was determined from the solution of two transport equations for the kinetic energy of turbulence K, and the rate of dissipation of the kinetic energy of turbulence ε.

A finite difference solution for the governing equations of motion in their primitive form (velocities and pressure), the energy equation and the transport equations for the turbulence quantities was obtained by developing a numerical technique suggested by Gosman et al. [2]. A hybrid central and upwind differentiating scheme based on the control volume integration approach was used.

A brief review of related literature in cavity flow problems and mathematical turbulence models is presented in chapter 2. The theoretical developments which led to the momentum equations, the energy equation and the transport equations for the turbulence quantities, together with the boundary conditions, are presented in chapter 3.

The numerical and computational procedures are discussed in chapters 4 and 5 respectively. The numerical results for the flow, heat transfer and turbulence characteristics are given in chapter 6.

Parameters studied include the Reynolds number Re, and the mixed convection parameter Gr/Re^2 for Prandtl numbers of 3 and 0.01.

The numerical results are compared with available data published on the flow and heat transfer in cavity flow for low Reynolds numbers. It should be noted, however, that limited data appear to exist for cavity flow at high Reynolds numbers.

CHAPTER 2

SURVEY OF LITERATURE

2.1 STEADY FLOWS IN RECTANGULAR CAVITIES

One example of a closed streamline problem is the fluid motion generated in a rectangular cavity by the translation of the upper or lower wall. This problem has attracted several investigators because its geometric simplicity is appropriate for testing the theoretical models and new numerical schemes.

Cavity flows driven by shear in the absence of buoyancy forces have been studied for a wide range of Reynolds numbers. Analytical studies for high Reynolds numbers were carried out by Batchelor [3] and Squire [4]. Batchelor's analysis assumed that the flow inside the cavity can be considered as an inviscid core of constant vorticity with a thin layer along the boundaries. He also showed that the separated region has a fixed size at high Reynolds numbers; in other words, the size of the primary eddy is independent of the Reynolds number.

Numerical solutions of the Navier-Stokes equations have been reported by Donovan [5] and Burggraf [6]. They studied the problem of a square cavity and provided a comparison with Batchelor's model. The computations which were carried out at Reynolds number of 400 showed that the flow consists of

three eddies; one primary eddy occupying most of the cavity and two secondary eddies at the upstream and downstream corners of the cavity.

Furthermore, the results obtained analytically and experimentally by Pan and Acrivos [7] and Weiss and Florsheim [8] have provided some information about the flow configuration inside rectangular cavities for Reynolds numbers greater than 400. Their results showed that the size of the primary eddy is dependent on Reynolds number up to a value of at least 2700. The upstream corner eddy increases in size up to a value of 500 and then shrinks continuously.

The nature of the eddies formed in the cavity depends also on the aspect ratio (cavity height to width ratio). Visual studies by Mills [9] and Pan and Acrivos [7], for a cavity in which the upper wall is moving, investigated the behaviour of the eddies for different aspect ratios. The downstream corner eddy becomes larger with increasing aspect ratio. For an aspect ratio of two, the downstream corner eddy grows until it occupies the entire lower portion of the cavity. The relative position of the pair of eddies depends on the Reynolds number.

One of the main problems in computational fluid mechanics is that the grid size must decrease as the Reynolds number increases so that numerical stability is obtained. Greenspan [10] considered the cavity flow problem numerically, using the

generalized Newton's method with over relaxation to speed the convergence. He obtained a convergent solution for Reynolds number up to 10^5 and for a mesh size of 1/20* and 1/50. Greenspan did not observe any secondary eddies when he employed a mesh size of 1/20 for any of the Reynolds numbers he assumed. This mesh size was found to be large. However, at mesh spacing of 1/50, the flow configuration, which he obtained at Reynolds number of 10^4, contains a downstream secondary eddy, whereas at Reynolds number of 10^5 no secondary eddies were observed.

The Imperial College group at London [11, 12 and 13] have conducted a series of investigations relating to the cavity flow driven by shear in the absence of buoyancy forces. These series of studies employed an unequal mesh spacing with decreasing mesh size towards the wall so that the variations near the wall could be predicted accurately. They also used a unidirectional differentiating technique on the vorticity so that the differences were always backwards with reference to the direction of flow and thus accelerated the convergence. These new features were not present in the previous investigations.

Studies on cavity flows driven by the combined effects of shear, induced by the moving top wall, and buoyancy forces, induced by the temperature difference between the top and bottom walls, have been carried out by Torrance et al [14] for Prandtl number of one and Reynolds number of 100 with

* The notation 1/20 means that the ratio of mesh size to cavity dimension is 1:20.

different values of Grashof number. Their results showed that, in the absence of buoyancy forces, for aspect ratios of 1/2 and 1, only a single primary eddy appears with a small corner eddy, whereas for an aspect ratio of 2, a secondary eddy appears within the lower portion of the cavity with the existence of a corner eddy. They also found that the buoyancy forces tended to diminish the strength of the primary eddy.

Grand et al [1] have studied the problem for the two different Prandtl numbers of water and liquid sodium. They numerically solved the Navier-Stokes equations for laminar flow. Velocity and temperature distributions were obtained for Reynolds numbers ranging from 10^2 to 2.5×10^3 and mixed-convection parameter ranging from 0 to 1. They found that both secondary eddies are enhanced due to the buoyancy forces in the case of liquid sodium but, for water, only the downstream corner eddy is enhanced while the upstream one disappears.

Another investigation was carried out by Nallasamy and Prasad [15] for a wide range of Reynolds numbers from 0 to 5×10^4. They solved the full Navier-Stokes and energy equations numerically using a finite-difference technique. They observed the tendency of the secondary eddies to shrink in size with increasing Reynolds number, ending with only the primary eddy at high Reynolds numbers. The tendency of the centre of the primary eddy to move towards the centre of the cavity with increasing Reynolds number was also observed.

Summarizing the previous works on the cavity flow, results have been obtained by solving analytically and numerically the Navier-Stokes and energy equations for laminar flow. To the author's knowledge, no study has been carried out for turbulent flow affected by buoyancy forces in rectangular cavities.

2.2 MATHEMATICAL MODELS OF TURBULENCE

2.2.1 The Basis of Constructing a Turbulence Model

The momentum equation for turbulent flow (the Reynolds equation) can be written as follows:

$$\frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial x_j}(\rho U_i U_j) - \frac{\partial}{\partial x_j}(\mu \frac{\partial U_i}{\partial x_j} - \rho \overline{u_i u_j}) + \frac{\partial P}{\partial x_i} = 0$$

This equation may be solved directly without correlating the fluctuating terms (Reynolds stresses $-\rho \overline{u_i u_j}$). At present, this equation cannot be solved numerically, since the important details of turbulence are small scale in character. For example, eddies responsible for the decay of turbulence are typically about 0.1 mm. Consequently, in order to solve the equations numerically, about 10^5 discrete points may be needed, which would exceed the storage capacity of any existing computer. Therefore, the time-averaged effects of the turbulence are usually considered, rather than their macroscopic details.

Since there is no direct way of knowing the magnitude of these terms, it is necessary to approximate or 'model' their effects in terms of quantities that can be determined. Hence, a 'turbulence model' refers to a set of equations which, when numerically solved with the mean flow equations, provides a solution for turbulent flow problems.

The concept of a turbulence model was first formulated by Boussinesq [16]. As in Stokes's law for laminar flow ($\tau_\ell = \mu \frac{\partial U_i}{\partial x_j}$), he suggested that the turbulent shear stress ($\tau_t = -\rho \overline{u_i u_j}$) was equal to the product of the mean velocity gradient and a turbulent viscosity μ_t. Hence,

$$\tau_t = -\rho \overline{u_i u_j} = \mu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$

where the turbulent viscosity μ_t is not a fluid property as is μ, but its value depends on the structure of turbulence at the point of interest.

The introduction of the concept of turbulent viscosity μ_t was the basis for constructing a turbulence model. Further development involved its expression in terms of calculable quantities. This was accomplished by using different approaches.

2.2.2 Algebraic Turbulence Models

The first attempt to express the turbulent viscosity algebraically as a function of known quantities was attributed to Prandtl in 1925 [17], where an important advance in

modelling a turbulent flow was made by the introduction of a hypothesis which has become known as Prandtl's mixing-length theory. In this hypothesis, the turbulent viscosity is described by the following formula:

$$\mu_t = \rho \ell_m^2 \left| \frac{\partial U}{\partial y} \right|$$

where ℓ_m has been prescribed algebraically.

One of the interesting contributions in describing the mixing-length was the proposal of Von-Kármán in 1930 [16]. He suggested that the mixing-length ℓ_m is proportional to the ratio of the first to the second derivatives of the mean velocity, i.e.

$$\ell_m \alpha \left| \frac{\partial U}{\partial y} \Big/ \frac{\partial^2 U}{\partial y^2} \right|$$

This proposal eliminated the need to prescribe the mixing-length profile, but in many cases where $\frac{\partial^2 U}{\partial y^2} = 0$, at the inflection points of the velocity profile, the formula gives infinite mixing-length, and therefore the turbulent shear stress cannot be computed for the above-mentioned condition.

A large amount of experimental data has been reported by Escudier [18] in the boundary layers near a wall. He concluded that, except in the semi-laminar region close to the wall, the mixing-length distribution could be described

by the following formula,

$$\ell_m = \kappa y$$

where $\kappa = .435$, and y is the perpendicular distance between the wall and the point of interest.

In the semi-laminar region, Van Driest [19] suggested that the effective viscosity is given by the sum of the laminar and turbulent viscosities, thus :

$$\mu_{eff} = \mu + \rho \ell_m^2 \left|\frac{\partial U}{\partial y}\right|$$

where ℓ_m is described by the following algebraic equation

$$\ell_m = \kappa y \left[1 - \exp.- \frac{y \tau_W^{\frac{1}{2}} \rho^{\frac{1}{2}}}{A\mu} \right] \qquad (2.1)$$

with $A = 26.0$.

Modifications to Van Driest's hypothesis were necessary for Van Driest's mixing-length formula to satisfy different cases. Among these modifications, Patankar and Spalding [20] proposed that the local shear stress τ_+ should appear in the formula of ℓ_m instead of the shear stress at the wall τ_W, thus:

$$\ell_m = \kappa y \left[1 - \exp.- y_+ \tau_+^{\frac{1}{2}} / A\right] \qquad (2.2)$$

This formula gave a better correlation with the experimental results which they obtained.

In the same manner as the treatment of the momentum exchange coefficient, the mixing-length hypothesis was extended so that it could deal with the turbulent exchange coefficient of scalar quantities [16]. For a scalar quantity ϕ, the turbulent diffusion flux may be expressed as:

$$J_\phi = -\Gamma_{\phi,t} \text{ grad } \phi \qquad (2.3)$$

where the turbulent exchange coefficient $\Gamma_{\phi,t}$ is given by:

$$\Gamma_{\phi,t} = \rho \ell_{m,\phi} \ell_m \left|\frac{\partial U}{\partial y}\right| \qquad (2.4)$$

and $\ell_{m,\phi}$ is the mixing-length scale of the scalar quantity

2.2.3 Differential Turbulence Models

The expression of the turbulent viscosity through algebraic formulae did not fully describe the turbulent phenomena in most cases, because the algebraic formulae do not take into account the convective and diffusive transport of the turbulence properties. Hence, the turbulent viscosity is described using a differential transport equation, as conjectured by Kolmogorov [21].

2.2.3.1 One equation model of turbulence

Kolmogorov [21] proposed that, instead of relating the turbulence velocity to the mean velocity gradient, a better representation would be to relate it to the kinetic energy of turbulence K [16]. Thus the turbulence velocity may be expressed as:

$$V_t \propto \sqrt{K}$$

where $K = \frac{1}{2}(\overline{u_i^2})$, and the turbulent viscosity becomes

$$\mu_t = \rho V_t \ell_m = \rho \sqrt{K}\, \ell_m$$

The kinetic-energy of turbulence K is determined from the solution of the differential transport equation, but the mixing-length ℓ_m still has to be prescribed algebraically.

Instead of solving the differential equation for the turbulence properties (i.e. K, ℓ, etc.) to obtain the turbulent viscosity and hence the Reynolds stress, the turbulent stress can be obtained directly by solving a differential equation for the transport of the shear stress itself. The first model of this kind was that developed by Bradshaw et al [22]. The model is based on the hypothesis that the turbulent shear stress is proportional to the kinetic energy of turbulence K as follows:

$$\tau_t \alpha \rho K$$

where K and hence τ_t are determined from the differential transport equation. The length scale, which appeared in the equation for K, was described algebraically. Lack of experimental data did not allow them to simulate the τ-equation. Nee and Kovasznay [23] developed a transport equation in which the turbulent viscosity is the dependent variable instead of K. In so doing, the transport effects on the turbulent viscosity were considered, but the assumed length scale profile was not universal.

In order to account for the transport effects on the length scale, the latter must be described through a differential equation. Hence, to obtain a better prediction, it is necessary to consider two equation models.

2.2.3.2 Two equation models of turbulence

The two equation models of turbulence are models in which the kinetic energy of turbulence K and the length scale ℓ are determined from transport equations where the convection and diffusion effects on K and ℓ are considered.

The transport equation for the kinetic energy of turbulence is derived by multiplying the momentum equation for each component of velocity by its fluctuating value, time-averaging and summing the resulting equations. Following Launder and Spalding [16], the form of the equation describing the transport of K is written as:

$$\frac{DK}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial x_k}\left(\frac{\mu_t}{\sigma_K}\frac{\partial K}{\partial x_k}\right) + \frac{\mu_t}{\rho}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right)\frac{\partial U_i}{\partial x_k} - \varepsilon$$

where the various terms are interpreted as follows: The term $\frac{DK}{Dt}$ represents the change in the kinetic energy due to convection. The term $\frac{1}{\rho}\frac{\partial}{\partial x_k}\left(\frac{\mu_t}{\sigma_k}\frac{\partial K}{\partial x_k}\right)$ represents the diffusion of the kinetic energy. The production of the kinetic energy is expressed by the term $\frac{\mu_t}{\rho}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right)$, and the term ε gives the value of the dissipation of the kinetic energy.

The transport equation for the length scale ℓ has been developed in a different form by several authors. Among them, Ng and Spalding [24] have determined the length scale from the solution of two transport equations for $K\ell$ and ℓ. Spalding [25] determined ℓ from the solution of two transport equations for K/ℓ^2 and ℓ.

One of the models, which has successfully predicted turbulent flow, is the K-ε model developed by Jones and Launder [26]. In this model, the turbulent viscosity is determined from the solution of two transport equations, one for the turbulence kinetic energy K and the other for the dissipation rate of the turbulence kinetic energy ε, through the following relation:

$$\mu_t = c'_\mu \rho K^{3/2}/\ell$$

where c'_μ is a constant. The length scale of turbulence ℓ is defined in terms of K and ε as follows:

$$\ell = c_D K^{3/2}/\varepsilon$$

The transport equation for ε was first developed by Hanjalic [27]. It may be written as:

$$\frac{D\varepsilon}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial x_k}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_k}\right) + \frac{c_1 c_\mu}{\rho}\frac{\varepsilon}{K}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right)\frac{\partial U_i}{\partial x_k} - c_2\frac{\varepsilon^2}{K}$$

The interpretation of this equation follows a similar argument to that used for K.

A similar transport equation for the mean-square fluctuation of a scalar property 'F', such as the square of temperature fluctuations may be derived by multiplying the equation for 'F' by its fluctuating quantity 'f'. Spalding [28] has adopted the following form:

$$\frac{Dq}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial x_k}\left(\frac{\mu_t}{\sigma_q}\frac{\partial q}{\partial x_k}\right) + \frac{c_{q_1}}{\rho}\mu_t\left(\frac{\partial F}{\partial x_k}\right)^2 - c_{q_2}\frac{K^2}{\ell}q$$

where q represents the mean square fluctuation of F, i.e. $\overline{f^2}$.

Jones and Launder [26] have applied the "K-ε model" to predict the behaviour of boundary layers in accelerating flows where the boundary layers revert from turbulent to

laminar flow, a phenomenon called "laminarization". Their predictions were in close agreement with the experimental results. Launder and Spalding [29] predicted turbulent boundary layer flows as well as turbulent recirculatory flows using the "K-ε model". The obtained results were in close agreement with experimental data.

The model has also been used by Taminini [30] to predict the turbulent diffusion flame, while Plumb and Kennedy [31] have applied it to natural convection from a vertical isothermal surface. Their results were found to be in reasonable agreement with experiment.

Mujumdar and Li [32] used the model to predict a two-dimensional turbulent flow between parallel plates and their results were found to be in close agreement with experimental data. Salcudean and Guthrie [33] applied the model to describe the recirculatory flow in filling ladles. The model has also been used to predict three-dimensional flow in ducts of rectangular cross-section rotating about an axis normal to the longitudinal direction [34]. Agreement with experimental data was good for constant area ducts at low rotation, but for divergent ducts at high rotation, the results were not in good agreement with experimental data. Modifications to the forms of the K and ε equations were necessary to account for the rotational effect.

The purpose of the present work was to investigate the effects of Reynolds number and buoyancy forces on turbulent recirculatory flows in square cavities. The flow is driven by the combined effects of the moving bottom wall and the temperature difference between the top and bottom walls. In performing the calculations, the K-ε turbulence model was developed to account for the buoyancy forces. The flow patterns, temperature distribution and turbulence characteristics are presented for the two different Prandtl numbers of water and liquid sodium for a wide range of Reynolds numbers up to 10^6, and mixed convection parameters from 0 to 2.

CHAPTER 3

MATHEMATICAL FORMULATION

3.1 INTRODUCTION

The governing equations for steady, incompressible and Newtonian flow with constant physical properties in a rectangular cavity, with the exception of the contribution of the temperature dependent density to the buoyancy forces, are presented in section 3.2. The equations are then represented in a standard form in section 3.3 to simplify the solution procedure. The boundary conditions for the resulting elliptical equations and the treatment of the near wall regions are given in sections 3.4 and 3.5 respectively.

3.2 GOVERNING EQUATIONS

The physical model is shown in Fig. 3.1. A two-dimensional rectangular cavity of width d and height h is considered. The top and bottom walls are maintained at a constant temperature T_1 and T_o respectively, the vertical walls being insulated. The temperature T_1 of the top wall is higher than T_o for all cases. The bottom wall is moving with a constant velocity U_o from left to right.

The conservation of momentum for a turbulent flow can be obtained from the Navier-Stokes equations by expressing the velocities and pressures as mean and fluctuating components; by time-averaging, the resulting equation (the Reynolds equation) excluding the body force will be:

$$\frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial x_j}(\rho U_i U_j) - \frac{\partial}{\partial x_j}(\mu \frac{\partial U_i}{\partial x_j} - \rho \overline{u_i u_j}) + \frac{\partial P}{\partial x_i} = 0 \qquad (3.1)$$

where $-\rho \overline{u_i u_j}$ is the turbulent (Reynolds) stress, which express the superposition of the fluctuation on the mean motion.

It is assumed that the turbulent shear stress is related to the mean rate of strain via a turbulent viscosity μ_t.

i.e. $$-\rho \overline{u_i u_j} = \mu_t (\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}) \qquad (3.2)$$

By substituting the turbulent stress, Eq. 3.2, into Eq. 3.1, the conservation of the momentum for a turbulent flow is obtained:

$$\frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial x_j}(\rho U_i U_j) - \frac{\partial}{\partial x_j}(\mu \frac{\partial U_i}{\partial x_j} + \mu_t \frac{\partial U_i}{\partial x_j} + \mu_t \frac{\partial U_j}{\partial x_i}) + \frac{\partial P}{\partial x_i} = 0 \qquad (3.3)$$

The following assumptions are made:

(i) The flow is two-dimensional and steady,

(ii) the fluid is incompressible and Newtonian,

(iii) the physical properties are independent of the temperature, with the exception of the contribution of the temperature dependent density to the buoyancy forces.

Under the above assumptions and by considering cartesian coordinates due to the geometry of the system, the x and y momentum equations reduce respectively to:

$$\frac{\partial}{\partial x}(\rho U^2) + \frac{\partial}{\partial y}(\rho VU) - \mu_{eff}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) - \mu_{eff}\frac{\partial^2 U}{\partial x^2}$$

$$- \mu_{eff}\frac{\partial^2 V}{\partial x \partial y} + \frac{\partial P}{\partial x} = 0 \qquad (3.4)$$

$$\frac{\partial}{\partial x}(\rho UV) + \frac{\partial}{\partial y}(\rho V^2) - \mu_{eff}\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) - \mu_{eff}\frac{\partial^2 V}{\partial y^2}$$

$$- \mu_{eff}\frac{\partial^2 U}{\partial x \partial y} + \frac{\partial P}{\partial y} = 0 \qquad (3.5)$$

where μ_{eff} is the effective viscosity of the fluid and is taken as the sum of the molecular and turbulent contributions, i.e.,

$$\mu_{eff} = \mu + \mu_t \qquad (3.6)$$

Following Jones and Launder [26], the turbulent viscosity μ_t is determined by the local values of the density ρ, the kinetic energy of turbulence K, and the turbulence length scale ℓ. Thus, for dimensional homogeneity:

$$\mu_t = c'_\mu \rho K^{\frac{1}{2}} \ell \qquad (3.7)$$

where c'_μ is a constant. The turbulence length scale is determined from the solution of the rate of dissipation of

the turbulence kinetic-energy ε, which may be assumed proportional to $K^{3/2}/\ell$.

Thus, the turbulent viscosity may be recast as:

$$\mu_t = c_\mu \rho K^2/\varepsilon \tag{3.8}$$

where c_μ is a constant.

In the present work two equations for K and ε are solved to determine the turbulent viscosity.

An equation for K may be obtained from the Reynolds equation. Launder and Spalding [16] have shown that the transport equation of the kinetic energy of turbulence can be written as:

$$\frac{\partial}{\partial x}(\rho UK) + \frac{\partial}{\partial y}(\rho VK) - \frac{\mu_{eff}}{\sigma_K}\frac{\partial^2 K}{\partial x^2} - \frac{\mu_{eff}}{\sigma_K}\frac{\partial^2 K}{\partial y^2} - \mu_t G + c_D \rho \varepsilon = 0 \tag{3.9}$$

where σ_K is the turbulent Prandtl number

$\mu_t G$ is the turbulence generation

ε is the dissipation of the turbulence kinetic energy.

The transport equation for the dissipation rate of the kinetic energy of turbulence ε, can be written as:

$$\frac{\partial}{\partial x}(\rho U\varepsilon) + \frac{\partial}{\partial y}(\rho V\varepsilon) - \frac{\mu_{eff}}{\sigma_\varepsilon}\frac{\partial^2 \varepsilon}{\partial x^2} - \frac{\mu_{eff}}{\sigma_\varepsilon}\frac{\partial^2 \varepsilon}{\partial y^2} - c_1\frac{\varepsilon}{K}\mu_t G + c_2 \rho \frac{\varepsilon^2}{K} = 0 \tag{3.10}$$

where c_1, c_2 and σ_ε are empirical coefficients [26].

The temperature distribution is computed by solving the following equation for the transport of energy:

$$\frac{\partial}{\partial x}(\rho UT) + \frac{\partial}{\partial y}(\rho VT) - \frac{\mu_{eff}}{\sigma_{eff}}\frac{\partial^2 T}{\partial x^2} - \frac{\mu_{eff}}{\sigma_{eff}}\frac{\partial^2 T}{\partial y^2} = 0 \qquad (3.11)$$

where σ_{eff} is the effective Prandtl number, which is related to the molecular and turbulent values as follows:

$$\frac{\mu_{eff}}{\sigma_{eff}} = \frac{\mu}{\sigma} + \frac{\mu_t}{\sigma_t} \qquad (3.12)$$

The Contribution of Buoyancy Forces

The cavity flow is affected by the temperature difference between the top and bottom walls, which causes changes in density, thereby inducing buoyancy forces. These forces are included in the y-momentum equation by considering them as imposed body forces, and are expressed by the term $\rho g \beta \delta T$ [17].

In a turbulent flow, the fluctuation of the temperature affects the transport of the turbulence energy and its dissipation rate. Due to these effects, a buoyancy term needs to be added to the equations of the kinetic-energy of turbulence and the energy dissipation.

The relation between the temperature fluctuation and the vertical velocity fluctuation, as suggested by Launder and Spalding [16], was used as follows:

$$\overline{vt} = c_4 (qK)^{\frac{1}{2}} \qquad (3.13)$$

where q is the mean-square temperature fluctuation.

The transport equation for the mean-square temperature fluctuation is derived by multiplying the energy equation by the fluctuating quantity of the temperature t, and then time-averaging the resultant equation. Following Launder's suggestion [16], the equation for q can be derived as:

$$\frac{\partial}{\partial x}(\rho Uq) + \frac{\partial}{\partial y}(\rho Vq) - \frac{\mu_{eff}}{\sigma_q}\frac{\partial^2 q}{\partial x^2} - \frac{\mu_{eff}}{\sigma_q}\frac{\partial^2 q}{\partial y^2}$$

$$+ c_{q_1}\mu_t\left[(\frac{\partial T}{\partial x})^2 + (\frac{\partial T}{\partial y})^2\right] - c_{q_2}\rho\frac{\varepsilon}{K}q = 0 \qquad (3.14)$$

Therefore, the complete set of equations describing the turbulent flow with the influence of buoyancy forces may be summarized as:

Conservation of Mass:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \qquad (3.15)$$

This equation is not solved directly, but is used to generate the pressure equation.

Conservation of Momentum:

x-component:

$$\frac{\partial}{\partial x}(\rho U^2) + \frac{\partial}{\partial y}(\rho VU) - \mu_{eff}\frac{\partial^2 U}{\partial x^2} - \mu_{eff}\frac{\partial^2 U}{\partial y^2} - \mu_{eff}\frac{\partial^2 U}{\partial x^2} - \mu_{eff}\frac{\partial^2 V}{\partial x \partial y}$$

$$+ \frac{\partial P}{\partial x} = 0 \qquad (3.16)$$

y-component:

$$\frac{\partial}{\partial x}(\rho UV) + \frac{\partial}{\partial y}(\rho V^2) - \mu_{eff}\frac{\partial^2 V}{\partial x^2} - \mu_{eff}\frac{\partial^2 V}{\partial y^2} - \mu_{eff}\frac{\partial^2 V}{\partial y^2} - \mu_{eff}\frac{\partial^2 U}{\partial x \partial y}$$

$$+ \frac{\partial P}{\partial y} + \rho g \beta \delta T = 0 \qquad (3.17)$$

Conservation of Energy:

$$\frac{\partial}{\partial x}(\rho UT) + \frac{\partial}{\partial y}(\rho VT) - \frac{\mu_{eff}}{\sigma_{eff}}\frac{\partial^2 T}{\partial x^2} - \frac{\mu_{eff}}{\sigma_{eff}}\frac{\partial^2 T}{\partial y^2} = 0 \qquad (3.18)$$

Kinetic Energy of Turbulence:

$$\frac{\partial}{\partial x}(\rho UK) + \frac{\partial}{\partial y}(\rho VK) - \frac{\mu_{eff}}{\sigma_K}\frac{\partial^2 K}{\partial x^2} - \frac{\mu_{eff}}{\sigma_K}\frac{\partial^2 K}{\partial y^2} - \mu_t G$$

$$+ c_D \rho \varepsilon - \rho g \beta c_4 (qK)^{\frac{1}{2}} = 0 \qquad (3.19)$$

Dissipation Rate of the Kinetic Energy of Turbulence:

$$\frac{\partial}{\partial x}(\rho U\varepsilon) + \frac{\partial}{\partial y}(\rho V\varepsilon) - \frac{\mu_{eff}}{\sigma_\varepsilon}\frac{\partial^2 \varepsilon}{\partial x^2} - \frac{\mu_{eff}}{\sigma_\varepsilon}\frac{\partial^2 \varepsilon}{\partial y^2} - c_1 \frac{\varepsilon}{K}\mu_t G$$

$$+ c_2 \rho \frac{\varepsilon^2}{K} - c_3 c_4 \rho g \beta \varepsilon \left(\frac{q}{K}\right)^{\frac{1}{2}} = 0 \qquad (3.20)$$

Mean-Square Temperature-Fluctuation:

$$\frac{\partial}{\partial x}(\rho U q) + \frac{\partial}{\partial q}(\rho V q) - \frac{\mu_{eff}}{\sigma_q}\frac{\partial^2 q}{\partial x^2} - \frac{\mu_{eff}}{\sigma_q}\frac{\partial^2 q}{\partial y^2} - c_{q_1}\mu_t\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right]$$

$$+ c_{q_2}\rho\frac{\varepsilon}{K}\, q = 0 \qquad (3.21)$$

Turbulent Viscosity Equation:

$$\mu_t = c_\mu \rho K^2/\varepsilon \qquad (3.22)$$

where the σ's represent the turbulent Prandtl numbers for the parameters in question and the c's are the constants of the model.

Numerical values for c_D, c_μ, c_1, c_2, σ_K and σ_ε are those recommended by Jones and Launder [26], the values for c_3 and c_4 chosen were based upon experimental data [31] and the constants c_{q_1}, c_{q_2} and σ_q are those used by Taminini [30]. The values of these constants are given in table 3.1.

3.3 GENERAL TRANSPORT EQUATION

The set of equations, Eq.(3.15) - Eq.(3.21), represent the governing equations to be solved numerically. Following Gosman et al. [2], these equations can be represented by a standard form so that a single solution algorithm may be applied. This form is given by:

$$\underbrace{\frac{\partial}{\partial x}(\rho U\phi) + \frac{\partial}{\partial y}(\rho V\phi)}_{\text{convection terms}} - \underbrace{\frac{\partial}{\partial x}(\Gamma\frac{\partial \phi}{\partial x}) - \frac{\partial}{\partial y}(\Gamma\frac{\partial \phi}{\partial y})}_{\text{diffusion terms}} = \underbrace{S_\phi}_{\text{source term}} \qquad (3.23)$$

Eq. (3.23) represents the transport of a general variable ϕ. The first two terms of this equation account for the convection transport due to the fluid motion, where U and V are the horizontal and vertical mean velocities: The third and fourth terms represent the diffusion transport of ϕ, where Γ represents the turbulent exchange coefficient. The term on the right-hand side is the source term, and is specific for all equations.

The substitutions for the ϕ, Γ and S_ϕ terms for each different equation are summarized in table 3.2.

Table 3.1

The Values of the Model Constants

c_μ	c_D	c_1	c_2	c_3	c_4	c_{q_1}	c_{q_2}	σ_K	σ_ε	σ_q
0.09	1.0	1.44	1.92	1.44	0.5	2.8	1.7	1.0	1.3	0.9

Table 3.2

General Form of the Transport Equation

$$\frac{\partial}{\partial x}(\rho U\phi) + \frac{\partial}{\partial y}(\rho V\phi) - \frac{\partial}{\partial x}(\Gamma\frac{\partial \phi}{\partial x}) - \frac{\partial}{\partial y}(\Gamma\frac{\partial \phi}{\partial y}) = S_\phi$$

Equation	ϕ	Γ	S_ϕ
x-Momentum	U	μ_{eff}	$-\frac{\partial P}{\partial x} + \mu_{eff}\frac{\partial^2 U}{\partial x^2} + \mu_{eff}\frac{\partial^2 V}{\partial x \partial y}$
y-Momentum	V	μ_{eff}	$-\frac{\partial P}{\partial y} + \mu_{eff}\frac{\partial^2 V}{\partial y^2} + \mu_{eff}\frac{\partial^2 U}{\partial x \partial y} - \rho g \beta \delta T$
Energy	T	$\frac{\mu}{\sigma} + \frac{\mu_t}{\sigma_t}$	0
Turbulence kinetic energy	K	$\frac{\mu_{eff}}{\sigma_K}$	$\mu_t G - c_D \rho \varepsilon + \rho g \beta c_4 (qK)^{\frac{1}{2}}$
Turbulence energy dissipation rate	ε	$\frac{\mu_{eff}}{\sigma_\varepsilon}$	$c_1 \frac{\varepsilon}{K} \mu_t G - c_2 \rho \frac{\varepsilon^2}{K} + c_3 c_4 \rho g \beta \varepsilon (\frac{q}{K})^{\frac{1}{2}}$
Mean square temperature fluctuation	q	$\frac{\mu_{eff}}{\sigma_q}$	$c_{q_1} \mu_t \left[(\frac{\partial T}{\partial x})^2 + (\frac{\partial T}{\partial y})^2\right] - c_{q_2} \rho \frac{\varepsilon}{K} q$

where $G = 2\left[(\frac{\partial U}{\partial x})^2 + (\frac{\partial V}{\partial y})^2\right] + (\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x})^2$

3.4 BOUNDARY CONDITIONS

The applicable boundary conditions are specified as follows (see Fig. 3.1):

$U = U_o$; $V = 0$	on the moving wall
$U = 0$; $V = 0$	on the stationary walls
$T = T_o$ and $T = T_1$	on the lower and upper walls respectively
$\partial T/\partial n = 0$.	on the vertical sides

3.5 NEAR WALL REGIONS

Far from the solid walls, the velocity and temperature gradients are usually not very steep, a moderately finite difference grid gives accurate solutions. However, close to the solid walls, the gradients of the velocity and temperature are much steeper, and therefore a fine grid is needed to obtain accurate results.

3.5.1 Modification of the Momentum Transport Equation

Close to the walls, the momentum transport processes have been modelled through the wall function method [29].

In this method, the nearest-to-wall point p, as shown in Fig. 3.2, should be sufficiently remote from W, which lies on the wall, so that the term $(\rho K^{\frac{1}{2}} \ell/\mu)_p$ will be much greater than unity. Consequently, the viscous effect will be entirely overwhelmed by the turbulent effect. It is then assumed that

a logarithmic velocity profile prevails in the region between the wall and the node p, the expression being:

$$\frac{U_p}{(\tau/\rho)_W^{\frac{1}{2}}} = \frac{1}{\kappa} \ell n\left(\frac{E\ y_p (\tau\rho)_W^{\frac{1}{2}}}{\mu}\right) \tag{3.24}$$

where the subscript p indicates that the values are those at grid node p, y_p is the distance from the wall, and κ and E are the log-law constants. Furthermore, in the uniform-shear stress layers, the convection and diffusion can be neglected. It may then be shown from Eq.(3.19) that:

$$\tau_p = \tau_W = c_\mu^{\frac{1}{2}} \rho K \tag{3.25}$$

From Eqs.(3.24) and (3.25), the shear stress can be related to the velocity and the kinetic energy of turbulence through the relation:

$$\tau_p = \frac{\rho\kappa\ c_\mu^{\frac{1}{4}}\ K_p^{\frac{1}{2}}\ U_p}{\ell n\left(\frac{E\ y_p\ c_\mu^{\frac{1}{4}}\ \rho K_p^{\frac{1}{2}}}{\mu}\right)} \tag{3.26}$$

3.5.2 Modification of the Energy Transport Equation

Following Patankar and Spalding [20], the heat flux near the wall can be expressed as a function of the wall shear stress as follows:

$$(Q/A)_W = \frac{c_p\sqrt{\tau_W \rho}\ (T_W - T)}{\sigma_t (U_+ + p)} \tag{3.27}$$

where σ_t is the turbulent Prandtl number,

$$U_+ = U/\sqrt{\tau_W/\rho}$$

and p is a constant.

The value recommended by Spalding and Jayatillaka [35] has been used in this work.

i.e.

$$p = 9.24 \left(\frac{\sigma}{\sigma_t}\right)^{0.75} \tag{3.28}$$

The heat flux can be related to the kinetic energy of turbulence as follows:

$$K = (\tau_W/\rho)/c_\mu^{\frac{1}{2}} \tag{3.29}$$

and hence,

$$\sqrt{\tau_W \rho} = c_\mu^{\frac{1}{4}} \rho K^{\frac{1}{2}}$$

Therefore,

$$(Q/A)_W = \frac{c_\mu^{\frac{1}{4}} c_p \rho K^{\frac{1}{2}} (T_W - T)}{\sigma_t (U_+ + p)} \tag{3.30}$$

3.5.3 Modification of the Transport of Dissipation Rate Equation

The rate of dissipation of the kinetic energy of turbulence near the wall ε_p is fixed according to the hypothesis that the length scale varies linearly with the distance from the wall. Following [2], it is assumed that ε_p varies according to the following relation:

$$\varepsilon_p = \sqrt{(\tau_w/\rho)^3}/\kappa y_p \tag{3.31}$$

From Eq. (3.29), $\tau_w/\rho = K\, c_\mu^{\frac{1}{2}}$

Therefore, the expression for ε_p becomes:

$$\varepsilon_p = c_\mu^{3/4}\, K_p^{3/2}/\kappa y_p \tag{3.32}$$

3.5.4 Modification of the Transport of the Mean Square Temperature Fluctuation Equation

In the near-wall regions, the generation and dissipation terms in the transport equation for the mean square temperature fluctuation far outweigh the other terms. Thus, from Eq. (3.21), it can be shown that:

$$q = \frac{c_{q_1}\, K_p \mu_t}{c_{q_2} \rho \varepsilon_p} \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 \right] \tag{3.33}$$

CHAPTER 4

THE FINITE DIFFERENCE CALCULATION PROCEDURE

4.1 INTRODUCTION

In this chapter, the finite difference procedure for the general transport equation, Eq.(3.23), is presented. The finite-difference equations are derived using the control volume integration technique.

The calculation procedure for the general terms is stated in section 4.2, while the source terms are presented in section 4.3. The stability of the final set of algebraic equations in the iterative solution procedure is discussed in section 4.4. Finally, the pressure equation is derived in section 4.5.

4.2 DERIVATION OF THE GENERAL TERMS

4.2.1 The Grid Network

In deriving the finite difference equation, the cavity is represented by a rectangular grid network, as shown in Fig. 4.1. The continuous lines are the grid lines and the intersections of the latter are the nodal points. The dotted lines, which lie mid-way between the grid nodes, form the control volume.

The fluid properties (ρ and μ) and the quantities (P, T, K, ε and q) are computed at each nodal point, while the components of the velocity vector U and V are calculated

at the control volume boundaries. Each variable will therefore be at the centre of its control volume.

The physical boundaries (cavity walls) are chosen to coincide with the control volume boundaries, in order to facilitate application of the boundary conditions.

4.2.2 The Derivation Procedure

Since the variables and the control volume have been specified, the transport equation, Eq.(3.23), can be integrated over the corresponding control volume, within which the fluid properties are assumed constant. Thus,

$$\iint_{VOL} \left[\frac{\partial}{\partial x} (\rho U \phi) + \frac{\partial}{\partial y} (\rho V \phi) - \frac{\partial}{\partial x} (\Gamma \frac{\partial \phi}{\partial x}) - \frac{\partial}{\partial y} (\Gamma \frac{\partial \phi}{\partial y}) \right] d(VOL)$$

$$= \iint_{VOL} S_\phi \, d(VOL)$$

Using Gauss's theorem, the above equation can be written as:

$$\int_s^n \left[(\rho U \phi - \Gamma \frac{\partial \phi}{\partial x})_e - (\rho U \phi - \Gamma \frac{\partial \phi}{\partial x})_w \right] dy$$

$$+ \int_w^e \left[(\rho V \phi - \Gamma \frac{\partial \phi}{\partial y})_n - (\rho V \phi - \Gamma \frac{\partial \phi}{\partial y})_s \right] dx - \iint_{VOL} S_\phi d(VOL) = 0$$

or, in short form,

$$F_e - F_w + F_n - F_s = \iint_{VOL} S_\phi \, d(VOL) \qquad (4.1)$$

where the subscripts e, w, n and s refer to the east, west, north and south boundaries respectively, and

$$F_e = \int_s^n (\rho U\phi - \Gamma \frac{\partial \phi}{\partial x})_e \, dy$$

$$F_w = \int_s^n (\rho U\phi - \Gamma \frac{\partial \phi}{\partial x})_w \, dy$$

$$F_n = \int_w^e (\rho V\phi - \Gamma \frac{\partial \phi}{\partial y})_n \, dx$$

$$F_s = \int_w^e (\rho V\phi - \Gamma \frac{\partial \phi}{\partial y})_s \, dx$$

The last four equations represent the total flux into the control volume from the east, west, north and south boundaries.

If the transport across the control volume boundary is regarded as one-dimensional, then the flux solution may be expressed as follows [2]:

$$F_s = \rho_s V_s A_s \left[f_s \phi_S + (1 - f_s) \phi_p \right]$$

where

$$f_s = \text{exp. } (Pe)_s / \left[\exp(Pe)_s - 1 \right]$$

$$(Pe)_s = \rho_s V_s \Delta y_{ps} / \Gamma_s$$

$$\rho_s = (\rho_S + \rho_p)/2$$

and

$$\Gamma_s = (\Gamma_S + \Gamma_p)/2$$

In order to save computing time, the exponential terms are approximated in a simple algebraic form as follows:

For any x,

$$\frac{e^x}{e^x-1} \simeq \begin{cases} \frac{1}{2}\left(1+\frac{2}{x}\right) & |x|<2 \\ 1 & x>2 \\ 0 & x<-2 \end{cases}$$

Using this approximation, the expression for f_s becomes,

$$f_s \simeq \begin{cases} \frac{1}{2}\left[1+2(Pe)_s^{-1}\right] & |(Pe)_s|<2 \\ 1 & (Pe)_s>2 \\ 0 & (Pe)_s<-2 \end{cases}$$

By letting $a_s = \rho_s V_s A_s f_s$

$$c_s = \rho_s V_s A_s$$

and $D_s = \Gamma_s A_s/\Delta y_{ps}$

then $(Pe)_s$ becomes c_s/D_s, and the expression for the flux term F_s is given by:

$$F_s = a_s(\phi_s - \phi_p) + c_s\phi_p \qquad (4.2)$$

where

$$a_s = \begin{cases} c_s/2 + D_s & |c_s/2|<D_s \\ c_s & c_s/2>D_s \\ 0 & c_s/2<-D_s \end{cases} \qquad (4.3)$$

From the definitions of the c's and D's, it is clear that they represent the convection and diffusion transfer respectively. Consequently, the parameter Pe, the local Peclet number, is the ratio of the convection to diffusion transfer rate. The F's represent the fluxes across the boundaries into or out of the control volume. They are equal to the weighted sum of the convection and diffusion terms, (see Eq.(4.2)).

The three expressions for a_s in Eq.(4.3) can be represented in one expression as:

$$a_s = \max.(|c_s/2|, D_s) + \frac{1}{2} c_s \tag{4.4}$$

Similarly, by following the above procedure, expressions for F_e, F_w and F_n can be written as:

$$\begin{aligned} F_e &= a_e(\phi_p - \phi_E) + c_e\phi_p \\ F_w &= a_w(\phi_W - \phi_p) + c_w\phi_p \\ F_n &= a_n(\phi_p - \phi_N) + c_n\phi_p \end{aligned} \tag{4.5}$$

with

$$\begin{aligned} a_e &= \max(|c_e/2|, D_e) - \frac{1}{2} c_e \\ a_w &= \max(|c_w/2|, D_w) + \frac{1}{2} c_w \\ a_n &= \max(|c_n/2|, D_n) - \frac{1}{2} c_n \end{aligned}$$

Substitution of the expressions F_s, F_e, F_w and F_n, from Eqs. (4.2) and (4.5), into the conservation equation for the control volume, Eq. (4.1), gives:

$$\phi_p\left[(a_e + a_w + a_n + a_s) + (c_e - c_w + c_n - c_s)\right]$$

$$- (a_e\phi_E + a_w\phi_W + a_n\phi_N + a_s\phi_S) - \iint_{VOL} S_\phi \, d(VOL) = 0$$

The term $(c_e - c_w + c_n - c_s)$ represents the net accumulation of mass and, to fulfil continuity conditions, it must be equal to zero. Therefore:

$$\sum_i a_i\phi_i - \phi_p \sum_i a_i + S_{\phi V} = 0 \qquad (4.6)$$

where $\sum_i$ denotes $\sum_{i=e,w,n,s}$ and $S_{\phi V} = \iint_{VOL} S_\phi \, d(VOL)$ the integral of the source term over the control volume.

4.3 THE CALCULATION OF THE SOURCE TERM

The source term for the transport equation of each of the variables is expressed linearly as follows:

$$S_{\phi V} = \iint_{VOL} S_\phi \, d(VOL) = Sp_1\phi_p + Su_1 \qquad (4.7)$$

where Sp_1 and Su_1 take the following forms:

1. The Transport of U-Momentum Equation

$$Sp_1 = 0$$

$$Su_1 = \iint_{VOL} S_U \, d(VOL)$$

$$\approx \iint_{VOL} \left(- \frac{\partial P}{\partial x} + \mu_{eff} \frac{\partial^2 U}{\partial x^2} + \mu_{eff} \frac{\partial^2 V}{\partial x \partial y}\right) dxdy$$

$$\approx P \Big|_e^w \Delta y + \left(\mu_{eff} \frac{\Delta U}{\Delta x}\right) \Big|_w^e \frac{VOL}{\Delta x} + \left(\mu_{eff} \frac{\Delta V}{\Delta X}\right) \Big|_s^n \frac{VOL}{\Delta y}$$

2. The Transport of V-Momentum Equation

$$Sp_1 = 0$$

$$Su_1 = \iint_{VOL} \left(- \frac{\partial P}{\partial y} + \mu_{eff} \frac{\partial^2 V}{\partial y^2} + \mu_{eff} \frac{\partial^2 U}{\partial x \partial y} + \rho g \beta \delta T\right) dxdy$$

$$\approx P \Big|_n^s \Delta X + \mu_{eff} \frac{\Delta V}{\Delta y} \Big|_s^n \frac{VOL}{\Delta y} + \left(\mu_{eff} \frac{\Delta U}{\Delta y}\right) \Big|_s^n \frac{VOL}{\Delta x}$$

$$+ \rho g \beta \cdot \delta T \cdot VOL$$

3. The Transport of Kinetic-Energy of Turbulence

$$Sp_1 = - \frac{c_\mu \, c_D \, \rho^2 \, K_p}{\mu_t} \cdot VOL - \rho g \beta \, c_4 (q/K)^{\frac{1}{2}} \cdot VOL$$

$$Su_1 = \mu_t G \cdot VOL$$

4. The Transport of Energy Dissipation

$$Sp_1 = - c_2 \frac{\rho \, \epsilon_p}{K_p} \cdot VOL - c_3 c_4 \rho g \beta (q/K)^{\frac{1}{2}} \cdot VOL$$

$$Su_1 = c_1 \frac{\mu_t G \, \epsilon_p}{K_p} \cdot VOL$$

5. The Transport of Mean-Square Temperature-Fluctuation

$$Sp_1 = - c_{q_2} \rho \frac{\varepsilon}{K} \cdot VOL$$

$$Su_1 = c_{q_1} \mu_t \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \cdot VOL$$

Substituting the general form for the source term, Eq. (4.7), into Eq. (4.6), yields:

$$\sum_i a_i \phi_i - \phi_p \sum_i a_i + Sp_1 \phi_p + Su_1 = 0$$

or

$$\phi_p \left(\sum_i a_i - Sp_1\right) = \sum_i a_i \phi_i + Su \qquad (4.8)$$

4.4 STABILITY OF THE SOLUTION PROCEDURE

The finite difference equations derived in the previous section, Eq. (4.8), are a set of non-linear algebraic equations. For each variable, the coefficients a_i, Sp_1 and Su_1 are functions of the other variables. Hence, these equations are solved using an iterative technique, and the stability of the iterative solution is discussed in this section.

Consider a set of algebraic equations of the form:

$$X_i = \sum_{j, j \neq 1} (a_{ij} X_j + b_i) \qquad (4.9)$$

where X_i are unknown vectors. If the coefficients a_{ij} and b_i are constants, then Eq.(4.9) will be a set of linear algebraic equations which may be solved iteratively to yield stable convergent solutions if $\sum_{j,j\neq i} |a_{ij}| \leq 1$ for each i; and for at least one i, $\sum_{j,j\neq i} |a_{ij}| < 1$. In the case where Eq.(4.9) is non-linear, i.e. a_{ij} and b_i are not constants, the above conditions are necessary, but they may not be sufficient for convergence [13].

Comparison of Eqs.(4.8) and (4.9) yields the following,

$$a_{ij} \equiv \sum_i \frac{a_i}{\sum_j a_j - Sp_1}$$

Therefore, in order for Eq.(4.8) to satisfy the conditions for convergence, the following conditions must be met:

$$\sum_i \frac{a_i}{\sum_j a_j - Sp_1} \leq 1$$

or

$$\sum_i |a_i| \leq \sum_i a_i - Sp_1 \qquad (4.10)$$

Since, from Eq.(4.3), the coefficients a_{ij} are always positive, then Eq.(4.10) will be satisfied if $Sp_1 \leq 0$. In the last

section it was shown that Sp_1 is always zero or negative. The conditions for convergence will then be satisfied and the iterative procedure becomes stable. However, during the iteration procedure, the coefficients may equal zero. This will lead to singularities in the finite-difference equations. To avoid this, an additional false term, S_{false}, is added to Eq. (4.8), where

$$S_{false} = \max\left(\sum_i c_i, 0.0\right) \times (\phi_p' - \phi_p) \qquad (4.11)$$

and ϕ_p' is the value from the previous iteration. It should be noted that the false term will not affect the final results for a converging solution, because at convergence $\phi_p' \approx \phi_p$. Adding Eqs. (4.11) and (4.8) yields:

$$\phi_p\left(\sum_i a_i - Sp_1\right) = \sum_i a_i\phi_i + Su_1 + \max\left(\sum_i c_i, 0.0\right)(\phi_p' - \phi_p)$$

and the final form of the finite-difference equations becomes,

$$\phi_p \, Ap = \sum a_i\phi_i + Su \qquad (4.12)$$

where $Su = Su_1 + \max\left(\sum_i c_i, 0.0\right)\phi_p'$

$$Sp = Sp_1 - \max\left(\sum_i c_i, 0.0\right)$$

and $Ap = \sum_i a_i - Sp$

Since Sp is always negative, the iteration procedure for the final form of the finite-difference equations will be stable.

4.5 DERIVATION OF THE PRESSURE EQUATION

In this section, the pressure equation is derived by the use of the continuity equation.

To satisfy the mass conservation in each control volume, the net accumulation of mass inside each control volume is calculated. The local pressure is adjusted to modify the fluid flow velocities in order to conserve the mass in the control volume.

The procedure involves the definition of a new dependent variable called the pressure correction ΔP, so that $P = P' + \Delta P$ where P' is the estimated value of pressure resulting from the previous iteration, and P is the actual local pressure.

An equation containing the pressure correction term ΔP can be obtained by substituting $P' + \Delta P$ for P in the source term of the U-momentum equation, and then subtracting the latter from the U'-equation resulting from the estimated value of P, i.e. P'. This gives

$$Ap(U_p - U'_p) = \sum_i a_i(U_i - U'_i) + \Delta y(\Delta P_w - \Delta P_e)$$

For converging results, $U_i \simeq U'_i$ $(i = E,W,N,S)$ it follows that:

$$U_p = U_p' + (\Delta P_w - \Delta P_e)\, DU_p \tag{4.13}$$

where $DU_p = \Delta y/Ap$

Similarly, $$V_p = V_p' + (\Delta P_s - \Delta P_n)\, DV_p \tag{4.14}$$

where $DV_p = \Delta x/Ap$

In the same manner, the expressions for all the surrounding nodes become:

$$\begin{aligned} U_E &= U_E' + (\Delta P_p - \Delta P_E)\, DU_E \\ U_W &= U_W' + (\Delta P_W - \Delta P_p)\, DU_W \\ V_N &= V_N' + (\Delta P_p - \Delta P_N)\, DV_N \\ V_S &= V_S' + (\Delta P_S - \Delta P_p)\, DV_S \end{aligned} \tag{4.15}$$

From the continuity equation, it follows that:

$$(\rho UA)_W - (\rho UA)_E + (\rho VA)_S - (\rho VA)_N = 0 \tag{4.16}$$

Substituting Eq.(4.15) into Eq.(4.16) yields:

$$\begin{aligned} &\left[(\rho ADU)_E + (\rho ADU)_W + (\rho ADV)_N + (\rho ADV)_S\right]\Delta P_p \\ &\quad = (\rho ADU)_E\, \Delta P_E + (\rho ADU)_W\, \Delta P_W + (\rho ADV)_N\, \Delta P_N \\ &\qquad + (\rho ADV)_S\, \Delta P_S - \sum_i c_i = 0 \end{aligned} \tag{4.17}$$

where $\sum_i c_i$ is the net mass accumulation.

The equation for ΔP, Eq.(4.17), follows the general form of the algebraic equation, Eq.(4.12):

$$\phi_p(\sum_i a_i - Sp) = \sum a_i \phi_i + Su \qquad (4.12)$$

with $\phi = \Delta P$; $a = \rho ADU$; $Sp = 0$ and $Su = 0$.

It can then be solved using the same algorithm.

Having obtained the solution for ΔP, the velocity components can then be updated using Eqs.(4.13) and (4.14).

CHAPTER 5

COMPUTATIONAL PROCEDURE

5.1 INTRODUCTION

In this chapter, a discussion of the grid design is outlined in section 5.2, followed by a presentation of the method used for iterative solutions of the algebraic equations in section 5.3. The selection of the under-relaxation factor is examined in section 5.4. Finally, the construction of the computer program is discussed in section 5.5. A complete listing of the computer program with a sample output print-out is included in Appendix B.

5.2 GRID DESIGN

The accuracy of the solution and the convergence characteristics depend on the grid size and distribution for any numerical method including those which involve discrete domains.

Close to the walls, the gradients of the variables are relatively large. Consequently, a non-uniform grid, with decreasing mesh size towards the walls was used.

In most of the computer runs in the present work an 18×18 grid system was employed.

5.3 SOLUTION OF THE ALGEBRAIC EQUATION

The finite-difference equation previously derived, Eq.(4.12), together with the boundary conditions, constitute,

as already stated, a system of coupled non-linear simultaneous algebraic equations. The solution procedure for these equations is discussed in this section.

If NI and NJ are the number of grid lines in the horizontal and vertical directions respectively, there will be an NI × NJ grid mesh. Subtracting the physical boundaries, then the number of algebraic equations for each of the seven variables U, V, ΔP, T, K, ε and q will be $(NI-2)\cdot(NJ-2)$. If the entire solution for all the variables taken together is considered, then a total of $7(NI-2)(NJ-2)$ simultaneous equations need to be solved.

It is impractical and computationally expensive to solve these equations using matrix inversion methods because of the enormous size of the coefficient matrix. To simplify the problem, the variables should be solved separately, following the cyclic repetition of the following steps:

1. Initial values U', V', P', T', K', ε' and q' are assumed for all the variables.
2. The turbulent viscosity, μ_t is calculated.
3. The coefficients of the U-momentum equation are assembled according to the guessed values and then solved for U.
4. The coefficients of the V-momentum equation are assembled according to the guessed values and then solved for V.
5. ΔP is calculated, followed by the calculation of P and then updated values of U and V according to Eqs. (4.13) and (4.14).

6. The coefficients of the energy equation are assembled and the solution of T is obtained. Similarly, the calculations of K, ε and q follow the same procedure.
7. The modified variables are used as improved guesses and steps 2 to 6 are repeated until convergence is achieved.

Using this iterative procedure (steps 2 - 6), only the equations for each variable need to be solved simultaneously and consequently the number of simultaneous equations to be solved is reduced from $7(NI-2)\cdot(NJ-2)$ to $(NI-2)\cdot(NJ-2)$.

Now, in order to solve $(NI-2)\cdot(NJ-2)$ simultaneous equations, matrix inversion techniques may be employed. However, as previously stated, this procedure is expensive because of the large amount of computing time required.

In the present study, the set of $(NI-2)\cdot(NJ-2)$ simultaneous equations is solved using the tridiagonal matrix algorithm method (TDMA), in which a set of equations, each with three unknowns in a particular order (except the first and last equations, which have only two unknowns due to the specification of the boundary conditions), may be solved sequentially. The values at grid nodes along a vertical grid line i (values at N, p and s for each point p) are considered as unknown, whereas those at each E and W neighbours, using the values from the previous iteration, are taken to be known (see Fig.5.1). The tridiagonal matrix algorithm method is then applied to this grid line i.

Eq. (4.12) may be recalled as follows:

$$\underline{A_p}\phi_p = \underline{a_n}\phi_N + \underline{a_s}\phi_S + (\underline{a_e}\phi_E + \underline{a_w}\phi_W + \underline{Su}) \qquad (5.1)$$

with the underlined quantities assumed to be known. This will result in equations with three unknowns, except for the node adjacent to the two boundaries. By imposing the boundary conditions, there will be two unknowns instead of three per equation. Therefore, for each line there exist (NJ-2) equations with (NJ-2) unknowns which can be solved using the tridiagonal matrix algorithm.

In this way, it is possible to traverse all the vertical grid lines sequentially starting from left to right through the entire mesh representing the cavity. A large number of traverses is required to obtain exact solutions for all the ϕ values. However, since the coefficients of the algebraic equation in a given iteration are only tentative and must be updated before the next iteration, then it is not essential to obtain an accurate solution. Consequently, two to five traverses are sufficient. In the present work, following the recommendation of Gosman et al. [2], three traverses were used for the variables U, V, T, K, ε and q, whereas five traverses were used for ΔP.

A detailed description of the TDMA is presented in Appendix A [36].

5.4 UNDER-RELAXATION FACTOR

Divergence often occurs in the computation if the difference in magnitude of the variables between successive iterations is large. In order to avoid this, the under-relaxation numerical technique is applied.

Assuming that ϕ_p^{n-1} and ϕ_p^n are the values of ϕ_p computed in the (n-1)th and n-th iterations respectively, then, from the under-relaxation method, the value actually used in the following iteration, ϕ_p^n , is given by:

$$\phi_p^n = URF \cdot \phi_p^n + (1 - URF)\ \phi_p^{n-1} \qquad (5.2)$$

where URF is the under-relaxation factor which is a real number between 0 and 1.

Since, for a value of URF equal to unity, one can see from Eq. (5.2) that $\phi_p^n = \phi_p^n$ and for URF = 0, $\phi_p^n = \phi_p^{n-1}$, then, the under-relaxation factor adjusts the degree of advancement of the variable ϕ in the iteration process.

The calculation of ϕ_p^n and its modification using the under-relaxation factor takes a large amount of computer time, particularly for a grid mesh consisting of a large number of nodes. To avoid this, ϕ_p^n is calculated directly as follows.

From chapter 4, the equation which needs to be solved is Eq. (4.12):

$$\phi_p^n = (\sum_i a_i\phi_i + Su)/Ap \qquad (4.12)$$

Substitution of ϕ_p^n from Eq.(4.12) into Eq.(5.2) gives:

$$\phi_p^n = (\sum_i a_i\phi_i + Su)(URF/Ap) + (1 - URF)\ \phi_p^{n-1}$$

and upon arranging, yields:

$$\phi_p = (\sum_i (a_i\phi_i) + Su_2)/Ap_2 \qquad (5.3)$$

where $Ap_2 = Ap/URF$

$$Su_2 = Su + (1 - URF)\ Ap_2\ \phi_p^{n-1} \qquad (5.4)$$

Eqs.(5.3) and (4.12) are of the same form. Therefore, by computing the coefficients a_i, Su and Ap, and modifying them using Eq.(5.4), the ϕ's can be under-relaxed directly by solving the equation for ϕ_p^n following the normal procedure.

5.5 SUMMARY OF THE COMPUTER PROGRAM

A flow chart of the computational procedure is shown in Fig.5.2. The computer program is structured as follows:

1. The grid distribution is set up.
2. Subroutine INIT computes all the geometric quantities for each of the (NI-2)(NJ-2) nodes and stores them in arrays. It then initializes all the variables.

3. Subroutine PROP calculates the turbulent viscosity based on the assumed values of the kinetic energy of turbulence and the dissipation rate.
4. The initial conditions and the prescribed non-iterative boundary conditions are computed.
5. A complete cycle of iteration is performed by calling subsequently the subroutines CALCU, CALCV, CALCP, CALCT; CALCTE, CALCED and CALCQE which calculate the updated distributions of the variables U, V, ΔP, T, K, ε and q for all the nodes of the grid distribution.
6. Based on the updated values of the kinetic energy of turbulence and the dissipation rate, the turbulent viscosity is recalculated by calling the subroutine PROP.
7. The results are printed out after each iteration by calling the subroutine PRINT.

The iteration procedure terminates automatically if the computer time or the number of iterations exceeds the prescribed values. After the total number of iterations required (MAXIT) is performed, the mean Nusselt number is then calculated.

Since the solution procedure for all the variables follows the same steps (see Eq.5.3), only the procedure for the solution of the U-momentum equation, i.e. Subroutine CALCU is indicated in detail.

The subprogram CALCU computes the convective and diffusive terms, the TDMA coefficients, according to Eqs.(4.3)

and (4.5), and then computes the source terms Sp and Su for the entire field. The boundary conditions are prescribed by calling the subprogram MODU. The coefficients are under-relaxed using the method prescribed in section 5.4. The line-by-line solution procedure using the TDMA is finally performed by calling the subprogram LISOLV.

The number of traverses of line-solution is represented as NSWPU in the flow chart shown in Fig.5.3. The complete program is listed in Appendix B, together with the corresponding output listing. Additional information on some aspects of this program can be found in Ref. [2].

CHAPTER 6

DISCUSSION OF RESULTS

6.1 INTRODUCTION

A numerical study of the flow, heat transfer and turbulence characteristics of a turbulent recirculatory flow affected by buoyancy forces in a square cavity was performed using the finite difference equations derived in chapter 4. The solution was obtained using the numerical procedure described in chapter 5. The results obtained were for water (Pr = 3) and liquid sodium (Pr = 0.01). The Reynolds number is varied from 10^2 to 10^6, while the mixed convection parameter is between 0 and 2. The aspect ratio (E = d/h) of the cavity is taken to be unity.

The flow characteristics are presented in section 6.2 in the form of a velocity distribution. The distribution is given by vectors, which indicate the magnitude and direction of the nodal velocity. In section 6.3, the temperature fields are represented by isothermal lines. A discussion of the heat transfer characteristics is outlined in section 6.4. Finally, the turbulence characteristics are discussed in section 6.5.

6.2 FLOW DISTRIBUTION

6.2.1 The Effects of Reynolds Number

For Water of Pr = 3

The effects of Reynolds number on the velocity distribution are illustrated in Figs.6.1a through 6.1c. At low Reynolds

numbers, as shown in Fig.6.1a for Re = 1.3×10^2, the flow is characterized by three eddies, a primary eddy occupying the centre core, and two secondary eddies located at the upper corners. The formation of the secondary eddies is due to the adverse pressure gradient at the corners. The kinetic energy of the flow, which passes through the corners, is insufficient to overcome the adverse pressure. When the Reynolds number is increased, as shown in Fig.6.1b, the upstream corner eddy disappears at a value of Re = 1.3×10^3. With further increase of the Reynolds number, the downstream corner eddy shrinks in size, and it disappears at Reynolds numbers greater than 6×10^4. Fig.6.1c shows that at Re = 10^5, the flow consists of one primary eddy, since the flow passing along the corners has a kinetic energy high enough to overcome the adverse pressure gradient and consequently prevents the appearance of the corner eddies.

The results of Nallasamy and Prasad [15] showed that the downstream secondary eddy vanishes at Re = 3×10^4, while in the present results it disappears at Reynolds numbers greater than 6×10^4. The difference may be explained by the fact that turbulence is considered in the present computations and thus more momentum is diffused from the higher velocity fluid flowing along the cavity sides. As a result of this, the stream possesses insufficient kinetic energy to impede the formation of a secondary eddy.

The behaviour of the primary eddy centre was studied by tracing the location of this centre as a function of Reynolds number. As the Reynolds number increases, the centre of the primary eddy tends to move upstream towards the centre of the cavity as shown in Fig.6.2. The same tendency was observed by Nallasamy and Prasad [15]. Their results, as shown in Fig.6.2 by the dotted line, were obtained for a cavity with a moving top wall.

For Liquid Sodium of Pr = 0.01

For liquid sodium, the effects of Reynolds number on the flow patterns are illustrated in Figs.6.3a, b and c. At Reynolds number of 10^2, the flow field consists of a primary eddy with two corner eddies. This is shown in Fig.6.3a. At the higher Reynolds number of 5×10^3, as shown in Fig.6.3b, the upstream corner eddy disappears with the existence of the downstream one. In Fig.6.3c, where $Re = 10^6$, the flow pattern shows only one primary eddy.

6.2.2 The Effects of Buoyancy Forces

For Water of Pr = 3

The effects of buoyancy forces on the flow patterns are illustrated in Figs.6.4a, b and c for Reynolds number of 8.4×10^4. In the absence of buoyancy forces, where $Gr/Re^2 = 0$ as shown in Fig.6.4a, no secondary eddy appears, since at this Reynolds number, as indicated earlier, the flow consists of one primary eddy. The secondary eddy is present both for

$Gr/Re^2 = 1$ and for $Gr/Re^2 = 2$ in Figs.6.4b and c, respectively.

Fluid from colder regions of the top wall will tend to flow downwards in the direction of the left wall (from A to B) and warm fluid from B will flow along the top wall (to A) enhancing the secondary eddy (Figs.6.4b and c). Computations indicated the presence of a secondary eddy up to $Re = 2 \times 10^5$ for $Gr/Re^2 = 1$ and up to $Re = 10^6$ for $Gr/Re^2 = 2$.

For Liquid Sodium of Pr = 0.01

The computations for Pr = 0.01 showed that the effects of buoyancy forces are similar to that for Pr = 3.

The effects of buoyancy forces on the velocity distribution examined for Pr = 3 and 0.01 were at high Reynolds numbers, at which values the upstream secondary eddy no longer exists. In order to investigate the effects of buoyancy forces at low Reynolds numbers, a hypothetical fluid with high viscosity was examined.

In the absence of buoyancy forces, as shown in Fig.6.5a, the flow consists of a primary eddy and two secondary eddies. With the influence of buoyancy forces, as shown in Fig.6.5b, the upstream corner eddy does not practically change, while the downstream one increases significantly and occupies most of the upper half of the cavity.

The effects of buoyancy forces on the movement of the centre of the primary eddy was observed. This was found to be a downwards movement of the centre.

6.3 TEMPERATURE DISTRIBUTION

6.3.1 The Effects of Reynolds Number For Water of Pr = 3

The isotherms for Pr = 3 are given for different Reynolds numbers in Figs.6.6a and b. At low Reynolds number, Re = 1.3×10^2 as shown in Fig.6.6a, the temperature distribution is affected by both conduction and convection with the effect of convection being relatively higher. As the Reynolds number increases to Re = 1.3×10^3, the transport of heat by convection dominates and the development of a core of constant temperature is obtained. This is shown in Fig.6.6b.

The temperature profilesalong the vertical centre-line, horizontal centre-line, right side wall and left side wall are illustrated in Figs.6.7a, b, c and d respectively. As shown in Fig.6.7a, for low Reynolds numbers, a stratification of the temperature is observed, which reflects the predominance of conduction transfer. With increasing Reynolds number, the convection dominates and the stratification decreases until a core of constant temperature is obtained at high Reynolds numbers. Further increase of Reynolds number leads to a decrease of the core temperature.

The temperature profile along the horizontal centre-line is represented in Fig.6.7b. It is also noted that, as the Reynolds number increases, the temperature decreases with

decreasing stratification. On the right and left side walls, the temperature profiles are illustrated in Figs.6.7c and d. As expected, the temperature of the right side wall is lower than that of the left side wall.

For Liquid Sodium of Pr = 0.01

For liquid sodium of Pr = 0.01, the Peclet number (Pe = Re·Pr) is small for small Reynolds numbers (Re < 10^3) and the heat is transmitted mostly by conduction. This is shown in Fig.6.8a for Re = 10^2, 5×10^2, and 10^3 where the isotherms differ slightly from the case of pure conduction where they are horizontal.

At Re = 8×10^3, as shown in Fig.6.8b, the temperature field is affected by both conduction and convection. As the Reynolds number increases to Re = 5×10^5, the convection effect dominates and a core of constant temperature is obtained, as shown in Fig.6.8c.

6.3.2 The Effects of Buoyancy Forces

For Water of Pr = 3

The temperature profiles along the horizontal centre-line and at the top wall are represented in Fig.6.9a, for flows without the influence of buoyancy forces and for those with buoyancy at $Gr/Re^2 = 1$ and $Gr/Re^2 = 2$. At the horizontal centre-line, the temperature rises with the influence of buoyancy due to decrease of the convection effect. The decrease in convection also causes an enhancement of the stratification

in the right hand side of the cavity with increasing Gr/Re^2 ratio. The left hand side of the cavity exhibits less stratification with increasing Gr/Re^2 ratio. This is due to the relatively lower temperature fluid flowing downwards as a result of the growing downstream secondary eddy.

In Fig.6.9b, the temperature profiles along the vertical centre-line and left and right walls are illustrated, indicating increase of temperature with buoyancy. It is also observed that the stratification increases and is more pronounced in the upper half of the cavity.

For Liquid Sodium of Pr = 0.01

The influence of the buoyancy forces on the temperature distribution corresponding to $Re = 10^3$, with and without the buoyancy forces, is represented in Fig.6.10. The continuous lines represent the case without buoyancy forces, while the dotted lines represent that with buoyancy forces. Since the circulation of the primary eddy is low due to the influence of the buoyancy forces, as stated in section 6.2, the convection transport of energy decreases. Therefore, the isotherms tend to be similar to those for pure conduction.

6.4 THE HEAT TRANSFER CHARACTERISTICS

The variation of the mean Nusselt number, $\overline{Nu}$, along the top wall is plotted against the Reynolds number in Figs.6.11a and b. It is shown in Fig.6.11a, for liquid sodium (Pr = 0.01), that the Nusselt number differs slightly from unity. The high

thermal conductivity of the liquid sodium causes the heat transfer at low Reynolds numbers to be unaffected by convection. However, for water (Pr = 3), the mean Nusselt number is higher than unity and it increases with increasing Reynolds number. The variation of Nusselt number with Reynolds number is fitted by the following correlation:

$$\overline{Nu} = 1.29\ Re^{0.313} \quad \text{for } 10^2 \leq Re \leq 2.5 \times 10^3.$$

This result agrees reasonably with the data obtained by Grand et al. [1] for this range of Reynolds numbers.

For Reynolds numbers in the range $3 \times 10^3 - 8 \times 10^4$, as shown in Fig.6.11b, the variation of the Nusselt number is fitted by the following relation:

$$\overline{Nu} = 1.564\ Re^{0.287} \text{ for } 3 \times 10^3 \leq Re \leq 8 \times 10^4$$

Further increase of Reynolds number does not significantly change the mean Nusselt number.

The influence of buoyancy forces is also illustrated in Fig.6.11b. A decrease of the mean Nusselt number with the increase of the mixed-convection parameter, Gr/Re^2, is observed. There is an approximately 10% decrease in the Nusselt number for the ratio of $Gr/Re^2 = 1$, and about a 20% decrease for $Gr/Re^2 = 2$.

6.5 TURBULENCE CHARACTERISTICS

A discussion of the effect of Reynolds number and buoyancy forces on the turbulence parameters K, μ_{eff} and q is presented in this section.

The maximum turbulent kinetic energy versus Reynolds number is shown in Fig.6.12. Increasing Reynolds number gives rise to an increase in the turbulent kinetic energy because of the increased fluctuating velocities. In Fig.6.13, the maximum effective viscosity is plotted against the Reynolds number. It increases with increasing Reynolds number. The value of the maximum turbulent viscosity is equal to 10 times its molecular value at Re = 10^4, and it reaches 0.5×10^3 times the molecular value at Re = 10^6. This leads to the importance of considering the flow in the cavity for high Reynolds numbers as a turbulent flow.

The effect of buoyancy forces on the turbulent kinetic energy is illustrated in Fig.6.14. The local values of K on two vertical lines with and without buoyancy effect is represented. The maximum value of the turbulent kinetic energy occurs at the moving wall and decreases with increasing distance from the moving wall. The buoyancy effect is represented by curves 1b and 2b at X = 0.8 and X = 0.2 from the left side wall respectively. It reduces the value of the turbulent kinetic energy because the buoyancy forces tend to dampen the fluctuating velocities. It may also be noted that the decrease of K values due to buoyancy downstream, as shown by curve 2b, exceeds that upstream, as shown by curve 1b.

The influence of buoyancy forces on the effective viscosity on two vertical lines is shown in Fig.6.15. Curves 1a, b represent the effective viscosity profiles in the upward moving stream at a distance of $\bar{X} = 0.8$ from the left side wall, without and with buoyancy effect respectively. Curve 1a indicates that the viscosity decreases with increasing distance from the moving wall. It then increases in the middle of the cavity and finally decreases at the top wall. The variation of the effective viscosity downstream at distance $X = 0.21$ from the left side wall is represented by curves 2a, b without and with buoyancy force respectively. Curve 2a follows a similar variation to curve 1a. It is also shown that the buoyancy effect, represented by curves 1b and 2b, decreases the effective viscosity, since it decreases the kinetic energy of turbulence.

The local value of the mean square temperature fluctuation on two vertical lines is represented in Fig.6.16. Since the value of the mean square temperature fluctuation depends on the temperature gradient as well as the convection and diffusion transfer, it follows that near the wall, where high temperature gradients occur, higher values of the mean square temperature fluctuation are observed. The profiles of the two curves show an agreement with the observed temperature gradient in the fluid.

CHAPTER 7

CONCLUSIONS AND SUGGESTIONS

7.1 CONCLUSIONS

A turbulence model was derived based on the K-ε two equation model in order to study the effects of buoyancy forces on the transport of momentum and energy for a turbulent flow in a rectangular cavity. The results obtained lead to the following conclusions:

i) The turbulent viscosity obtained by employing the present model is significant for Reynolds numbers greater than 10^3. Therefore, the use of the turbulence model to compute high Reynolds number cavity flows is fully justified.

ii) For low Reynolds numbers, the flow consists of a primary eddy and two secondary eddies. The upstream corner eddy first disappears at Reynolds numbers greater than 1.3×10^3, followed by the disappearance of the downstream eddy at Reynolds numbers greater than 6×10^4. This study observed the presence of the downstream secondary eddy for higher Reynolds numbers than those based on the Navier-Stokes equations.

iii) The buoyancy forces enhance the downstream secondary eddy. The present results indicated the presence of a secondary eddy up to $Re = 10^6$ for $Gr/Re^2 = 2$.

iv) Stratification of the temperature distribution is obtained for low Reynolds numbers; with further increase of the Reynolds number, a core of constant temperature is observed.

v) The buoyancy forces increase the temperature on both vertical and horizontal centre-lines of the cavity for the range of Re = 10^4 - 10^6. The stratification is increased on the right side of the cavity and decreased on the left.

vi) The mean Nusselt number increases with increasing Reynolds number up to Re = 6 x 10^4. The buoyancy forces act to decrease the mean Nusselt number.

vii) Turbulence characteristics show an increase of the turbulence kinetic energy and effective viscosity with increasing Reynolds numbers. The effective viscosity and turbulence kinetic energy are decreased by buoyancy effects. The square mean fluctuating temperature exhibits high values for high temperature gradients in the fluid.

7.2 SUGGESTIONS FOR FURTHER RESEARCH

i) The results obtained in this work were for a cavity of an aspect ratio of 1; further study is suggested for different aspect ratios, to investigate the effect of the latter on the flow configuration.

ii) Further work is required in order to assess the values of the model constants for application to the flow of liquid sodium.

iii) The buoyancy affected turbulent flow has important applications, and further development of the present model could extend these considerably.

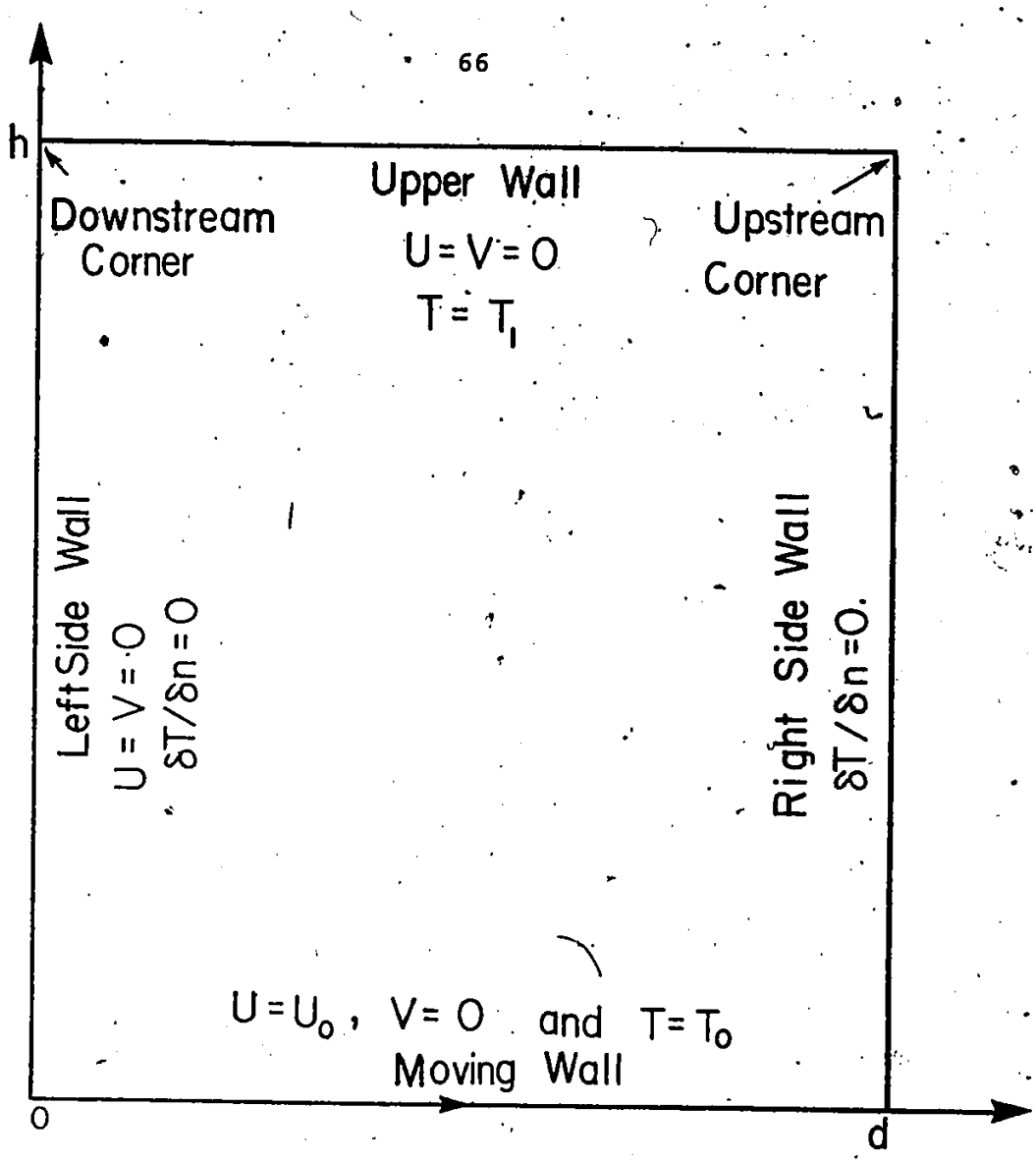


Fig.3.1 Physical Model

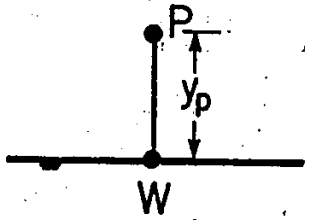


Fig.3.2 Near Wall Region

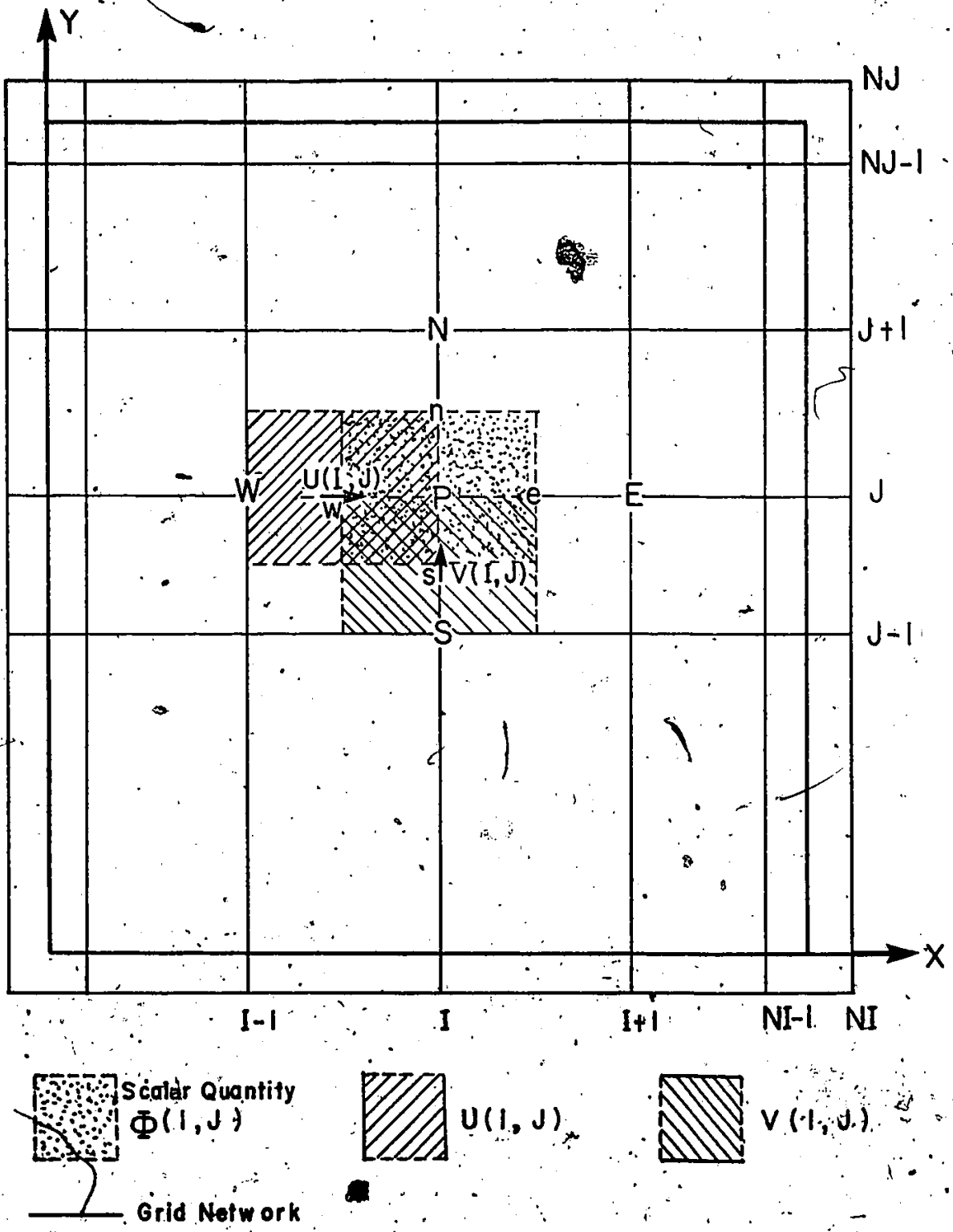


Fig.4.1 Grid Network

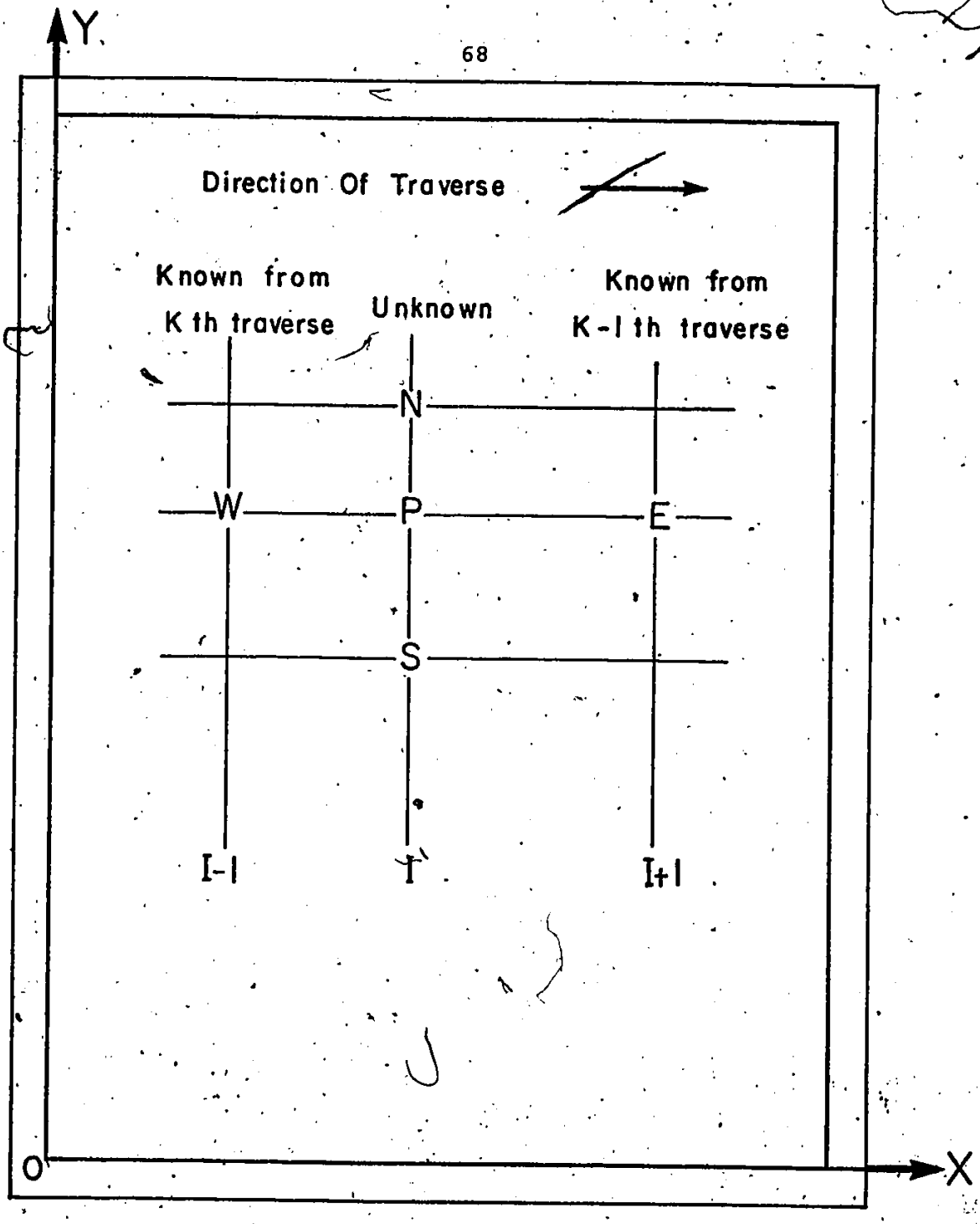


Fig.5.1 Line Iteration Procedure

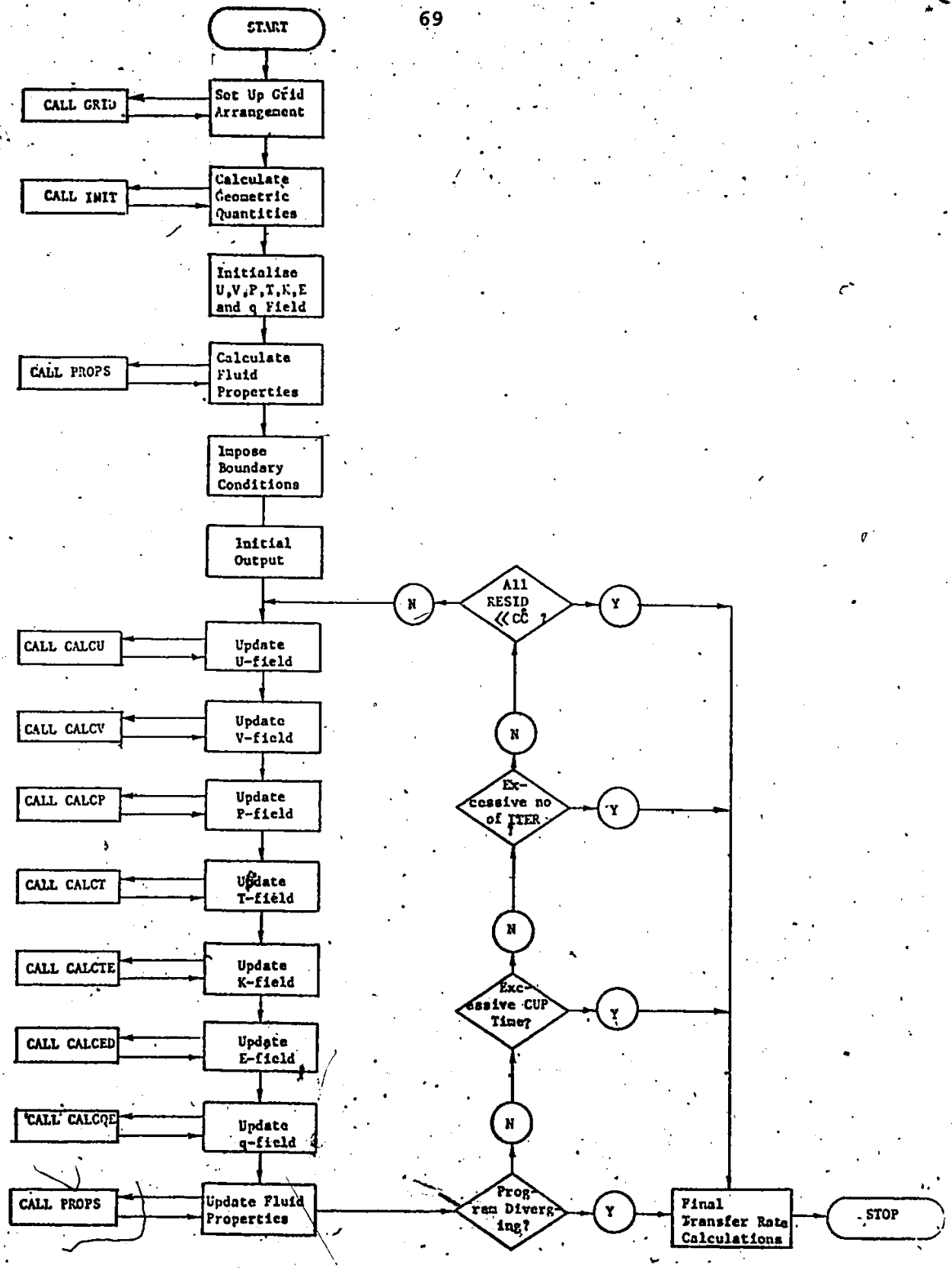


Fig.5.2 Computational Flow Chart: Main Program

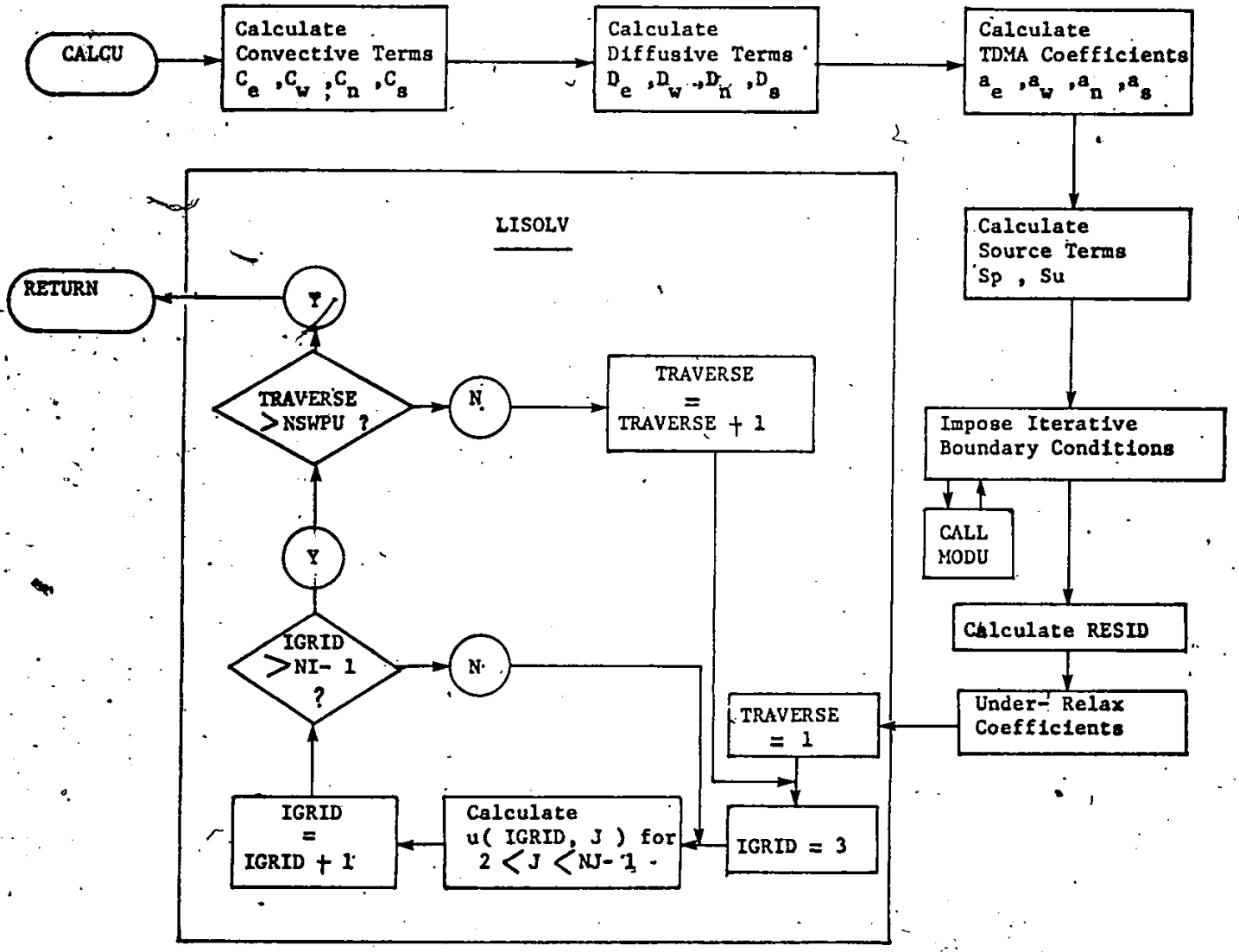


Fig.5.3 Computational Flow Chart: Subroutine CALCU

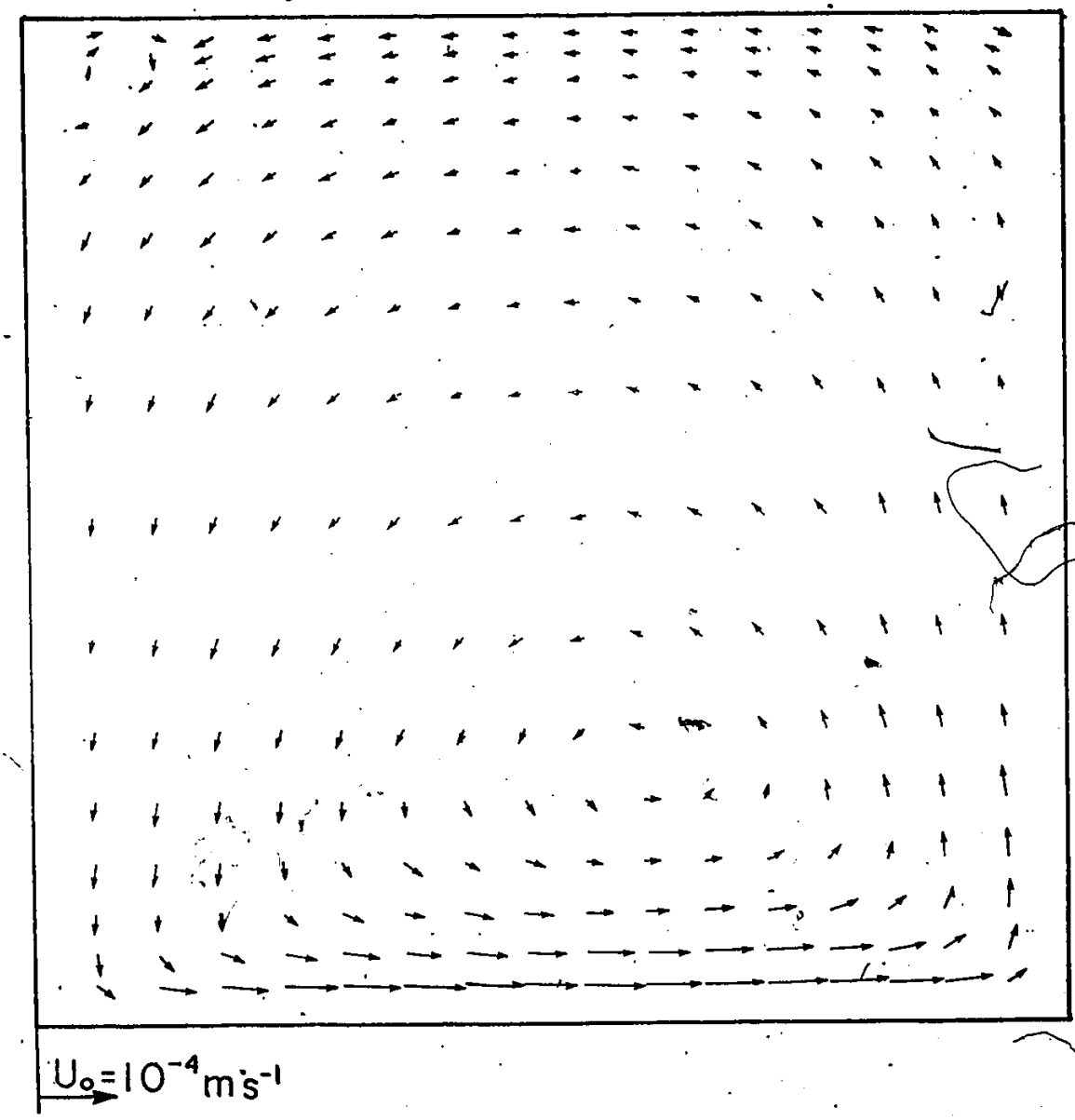


Fig.6.1a Velocity Distribution

Re = 1.3×10^2, Pr = 3, Gr/Re^2 = 0

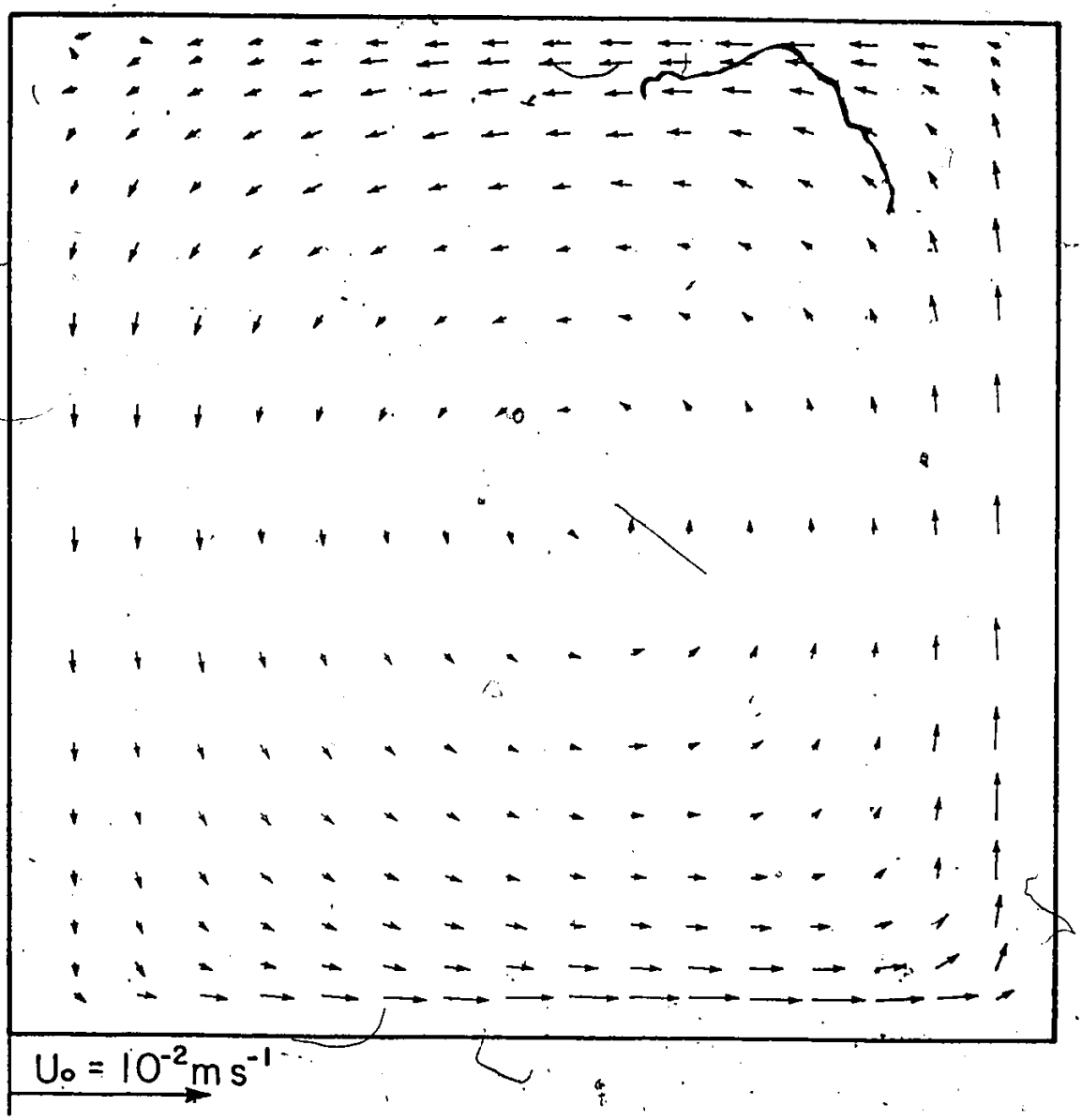


Fig.6.1b Velocity Distribution

$Re = 1.3 \times 10^3$, $Pr = 3$, $Gr/Re^2 = 0$

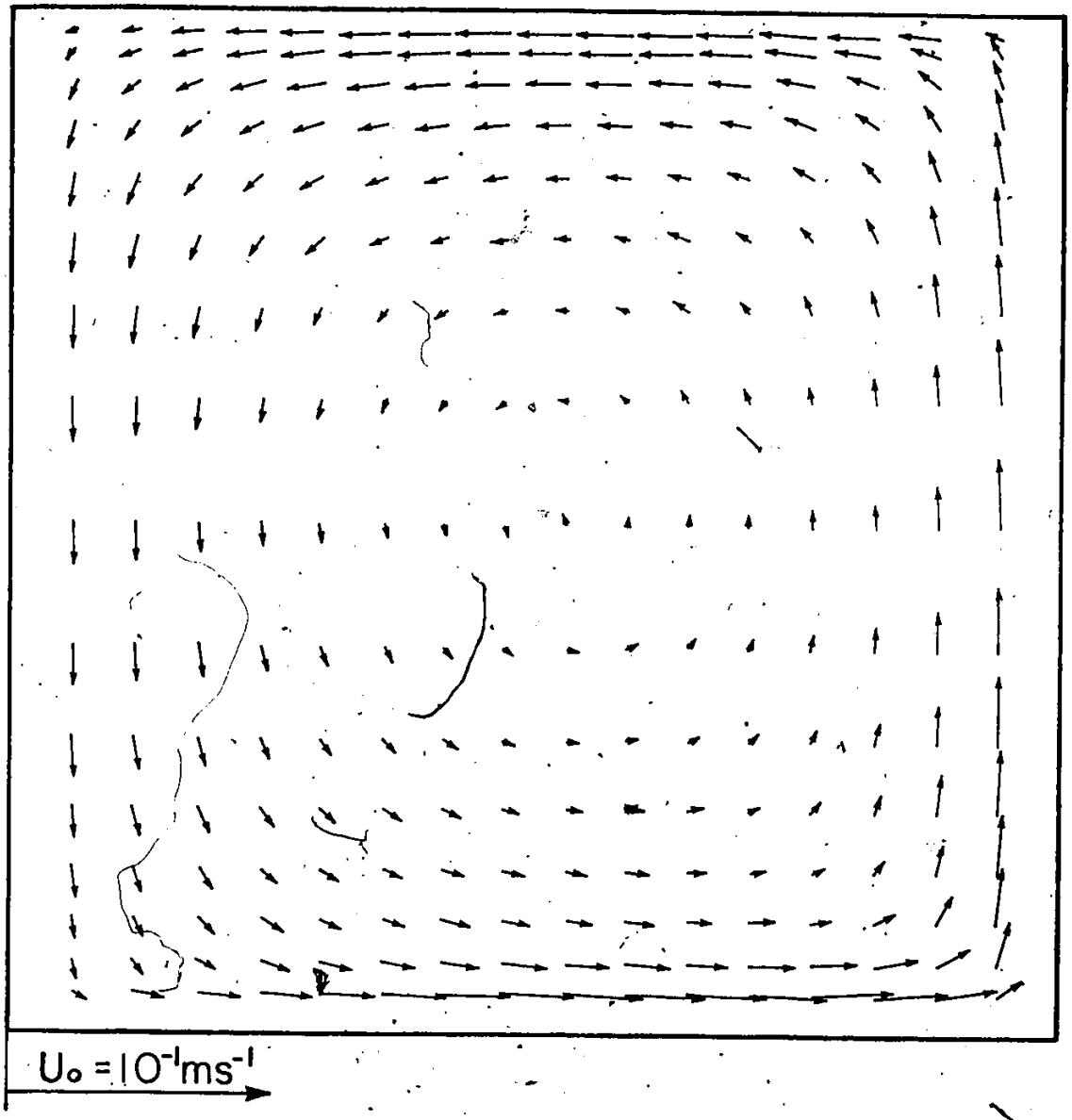


Fig.6.1c Velocity Distribution

$Re = 10^5$, $Pr = 3$, $Gr/Re^2 = 0$

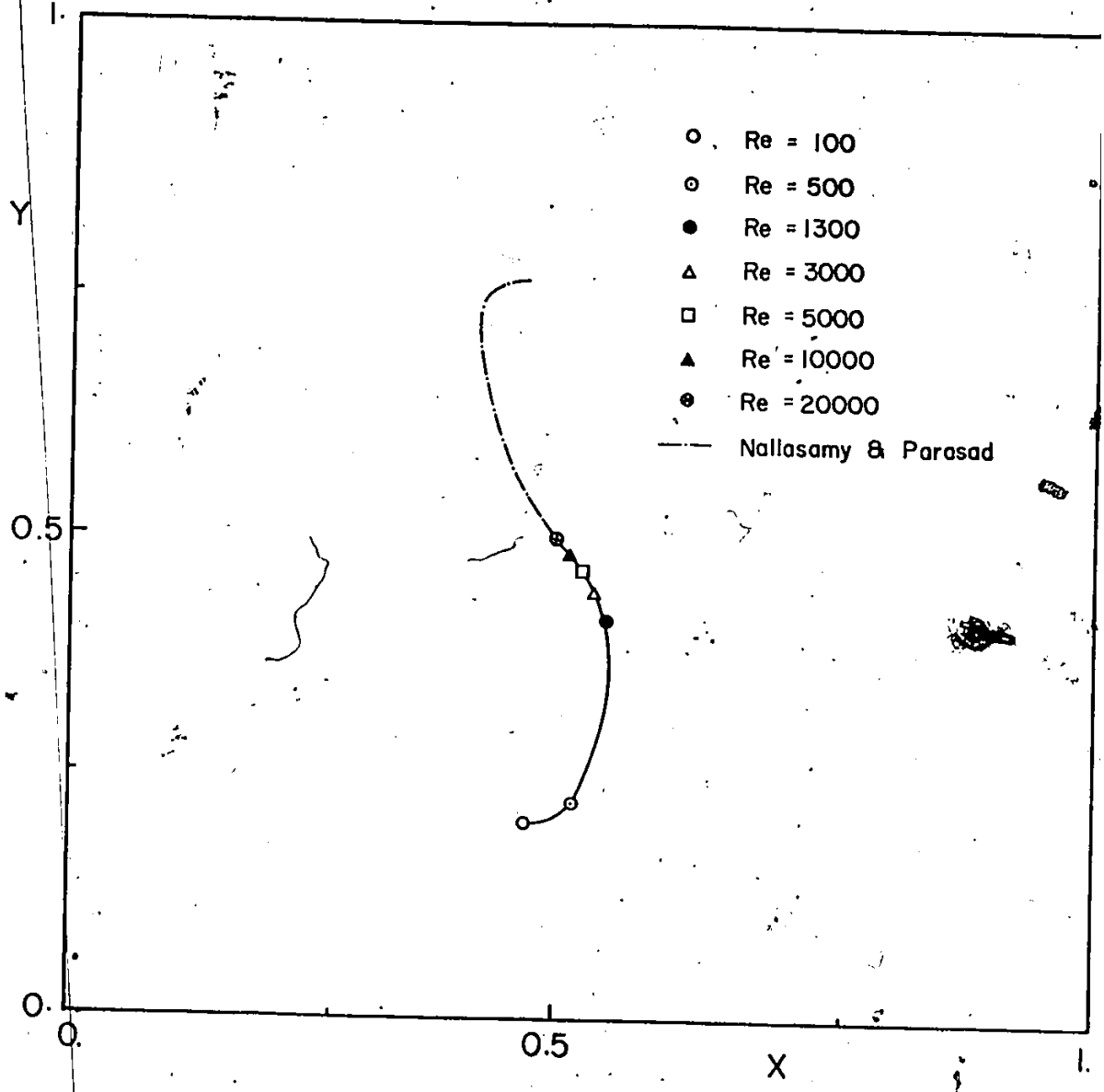


Fig.6.2 Movement of Primary Eddy Centre

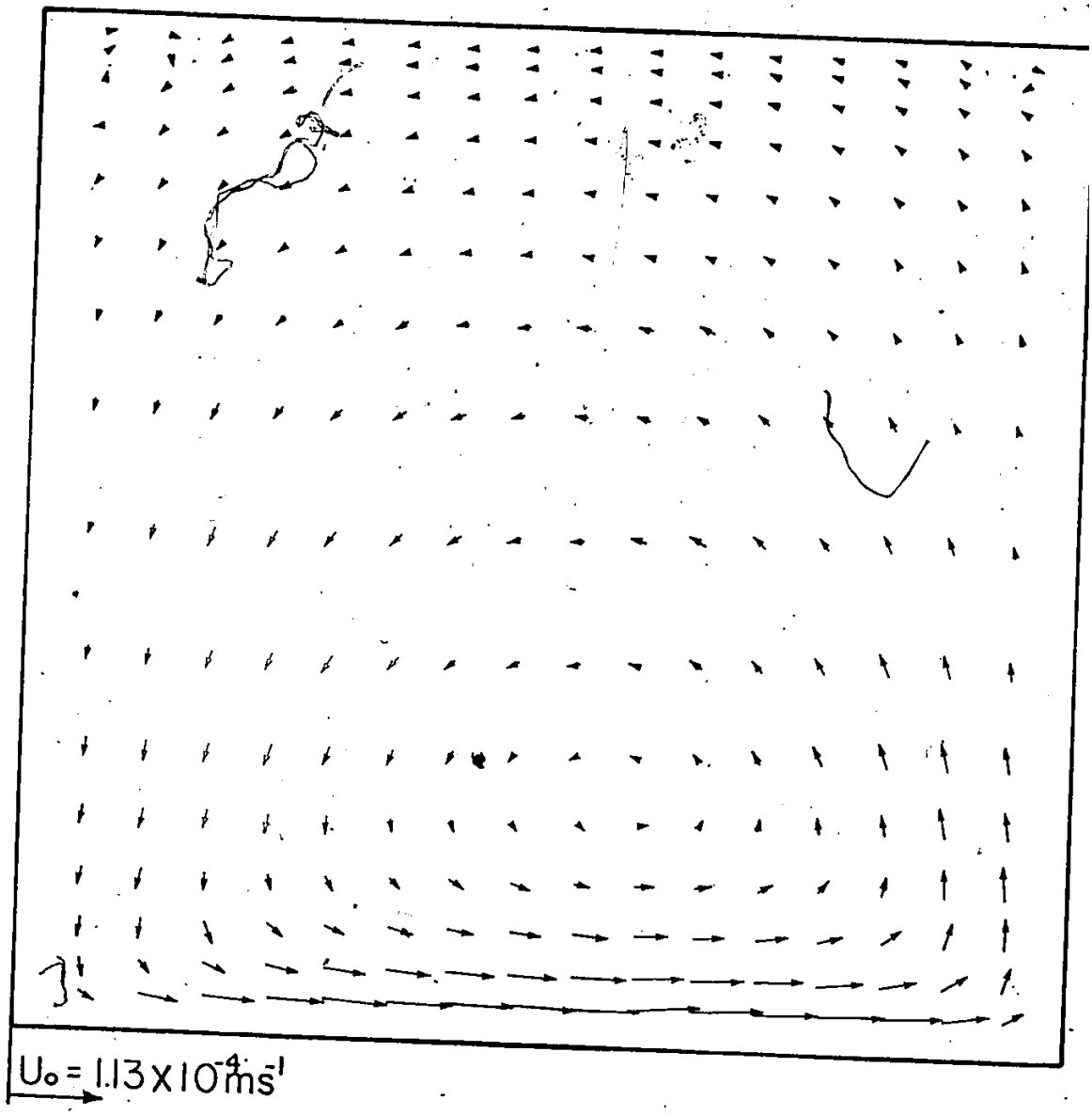


Fig.6.3a Velocity Distribution

$Re = 10^2$, $Pr = 0.01$, $Gr/Re^2 = 0$

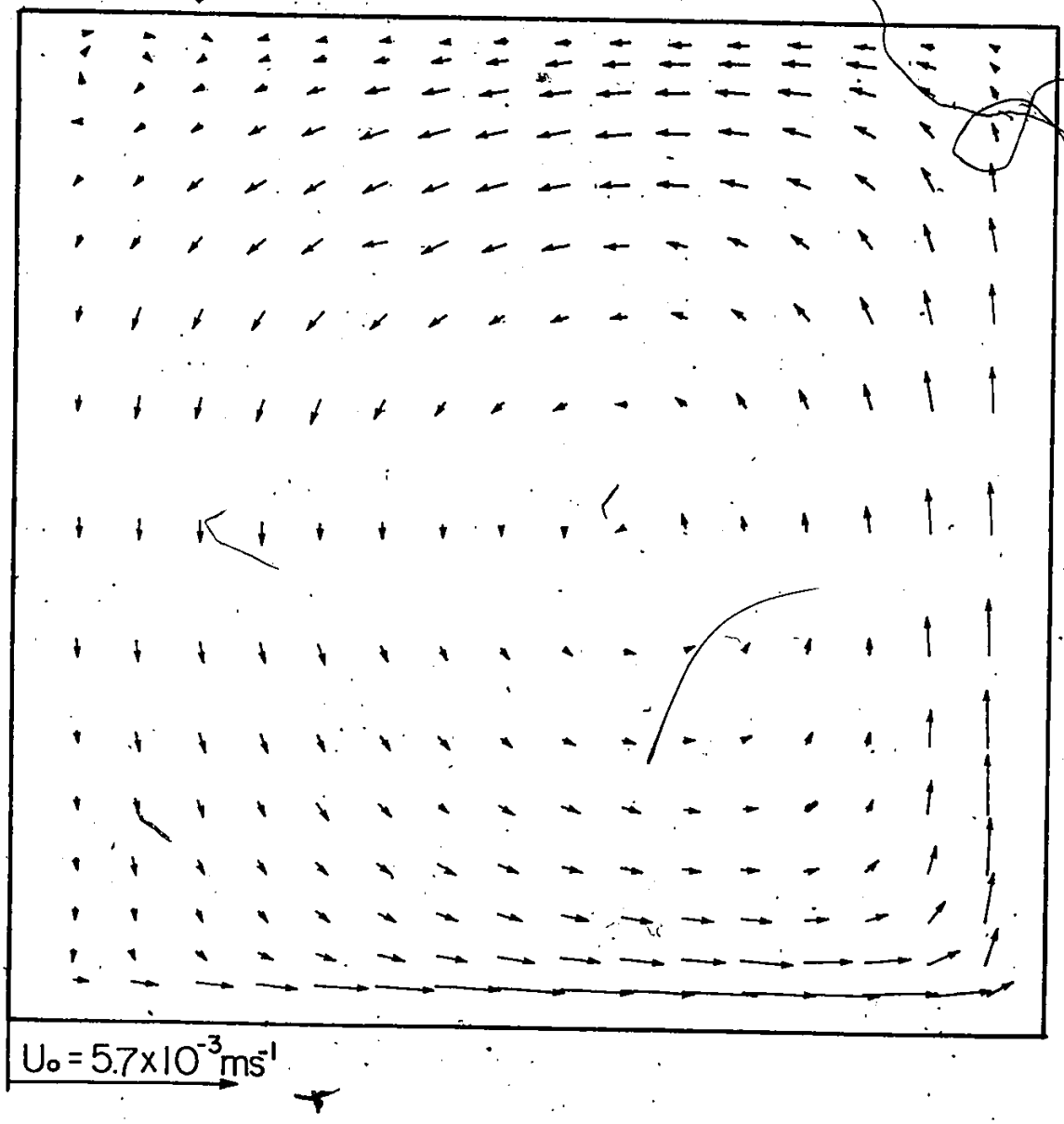


Fig.6.3b Velocity Distribution

$Re = 5 \times 10^3$, $Pr = 0.01$, $Gr/Re^2 = 0$

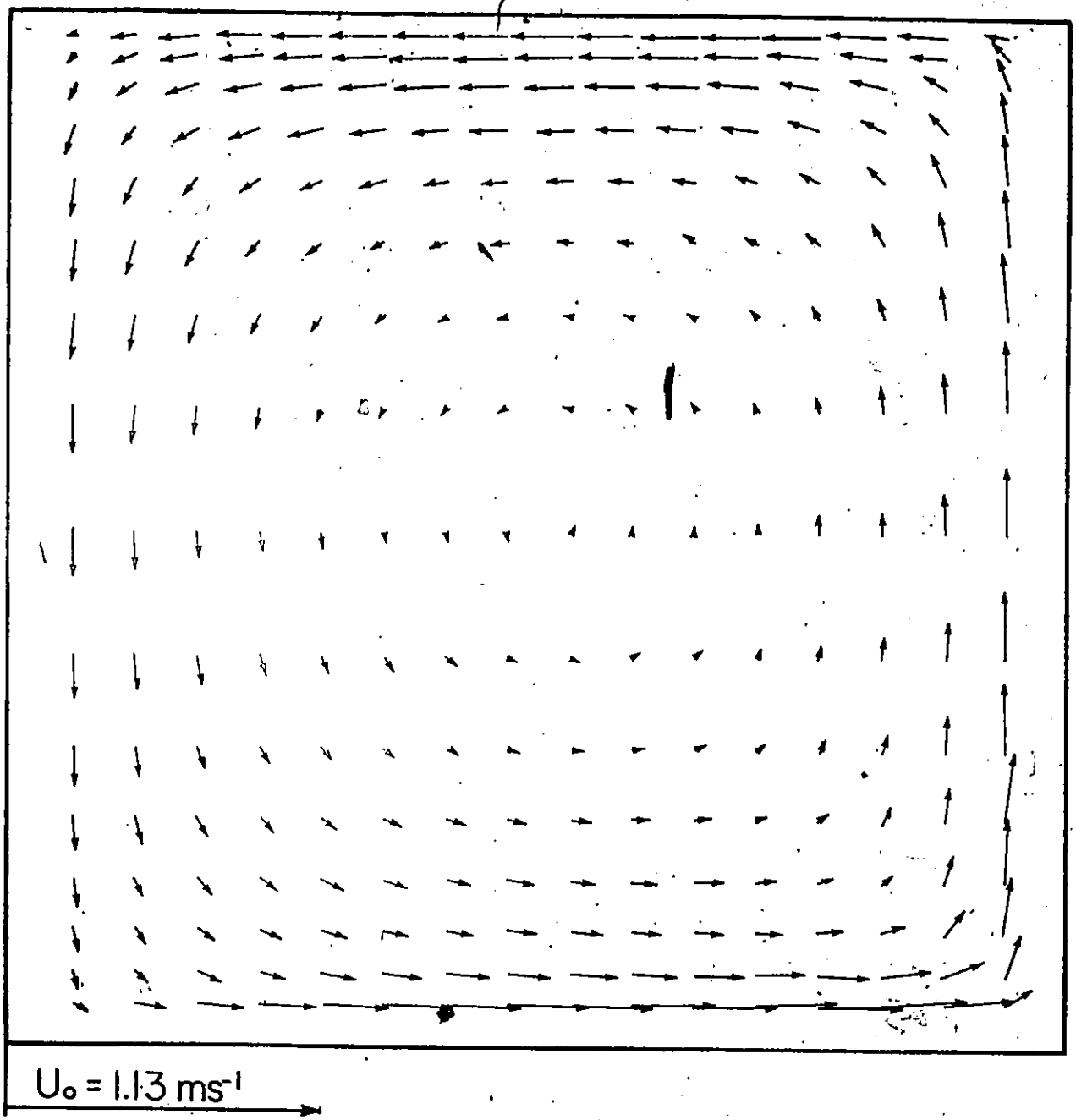


Fig.6.3c Velocity Distribution

$Re = 10^6$, $Pr = 0.01$, $Gr/Re^2 = 0$

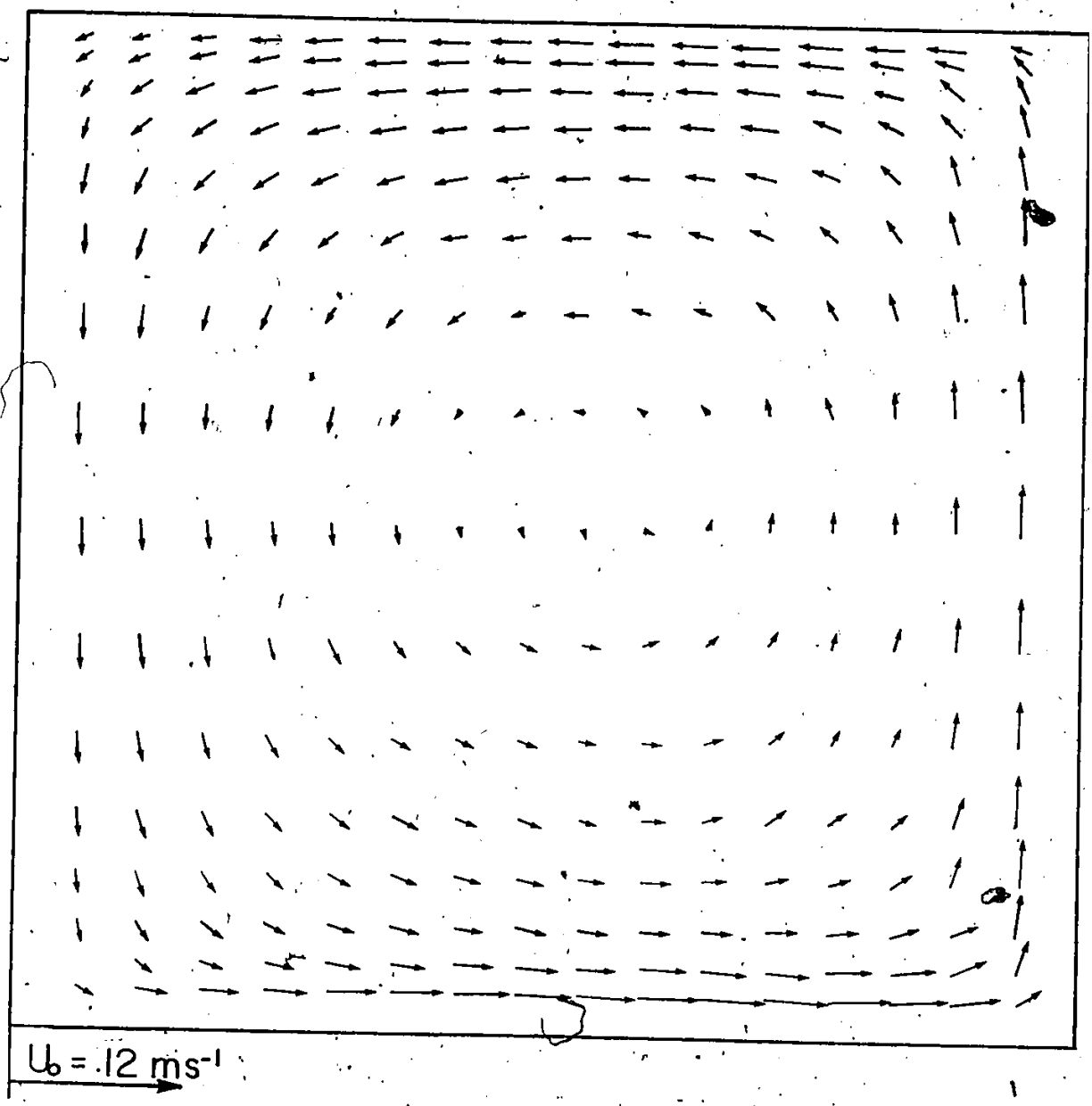


Fig.6.4a Velocity Distribution

$Re = 8.4 \times 10^4$, $Pr = 3$, $Gr/Re^2 = 0$

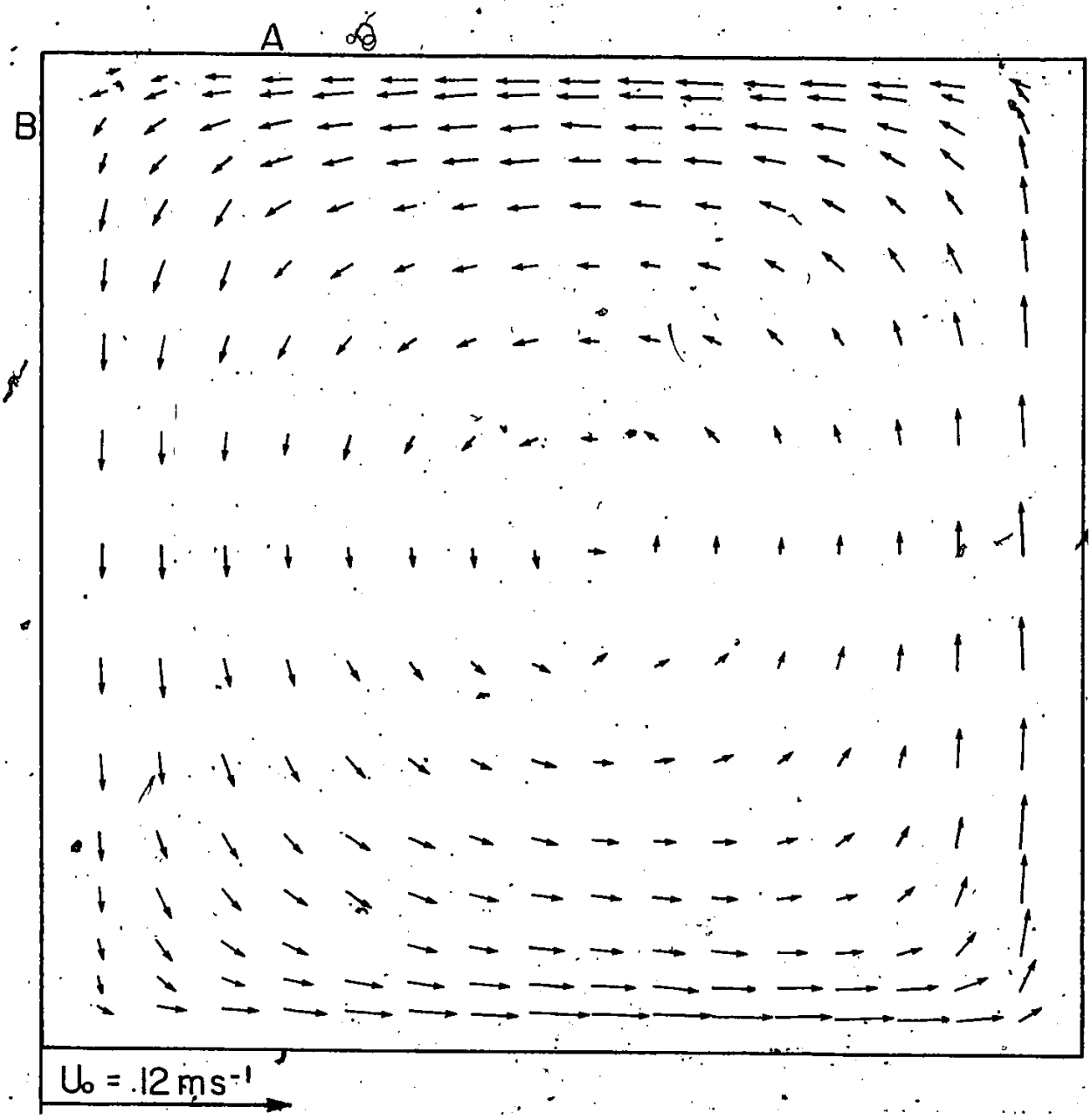


Fig. 6.4b. Velocity Distribution

Re = 8.4×10^4, Pr = 3, Gr/Re^2 = 1

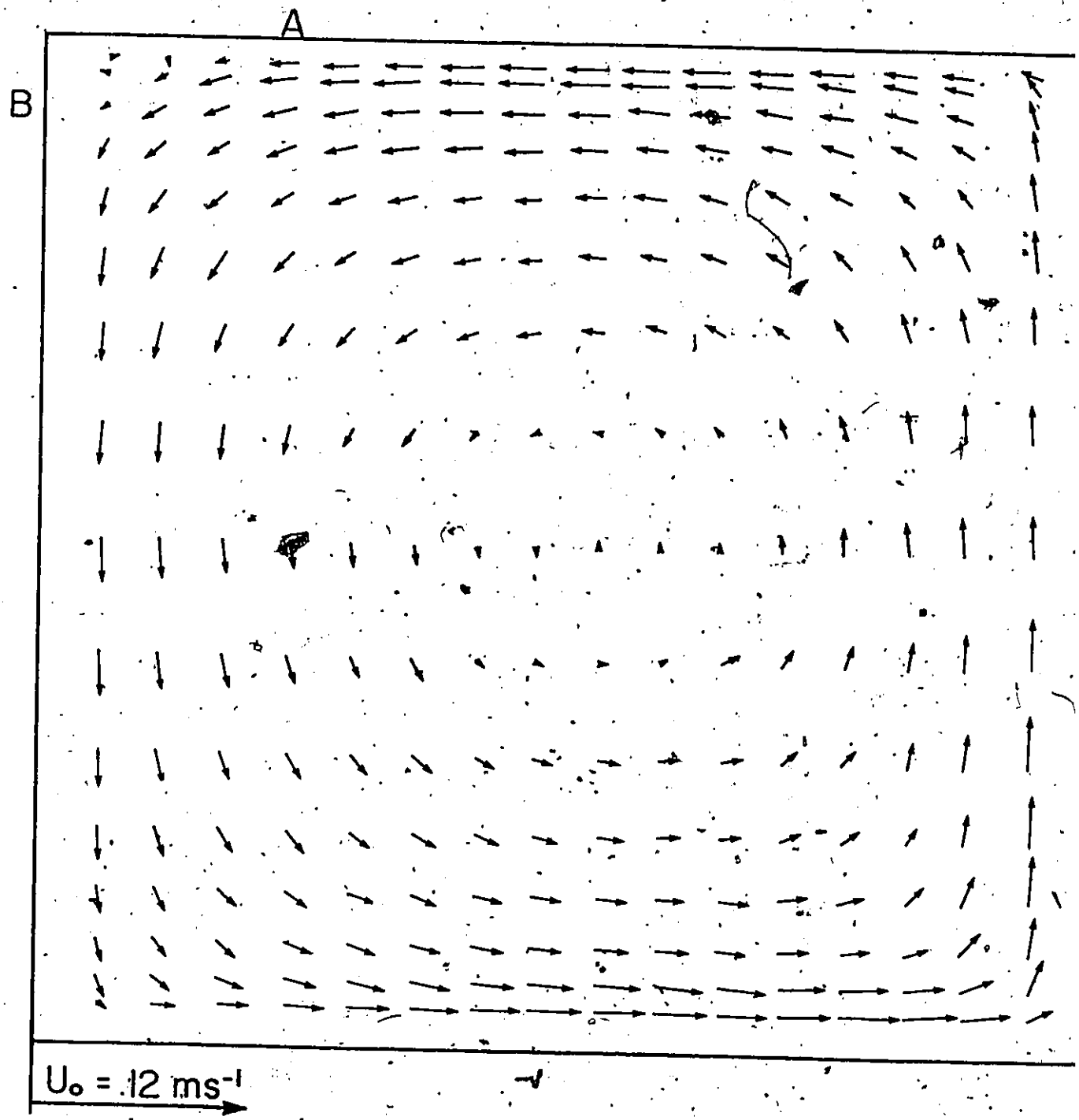


Fig. 6.4c Velocity Distribution

$Re = 8.4 \times 10^4$, $Pr = 3$, $Gr/Re^2 = 2$

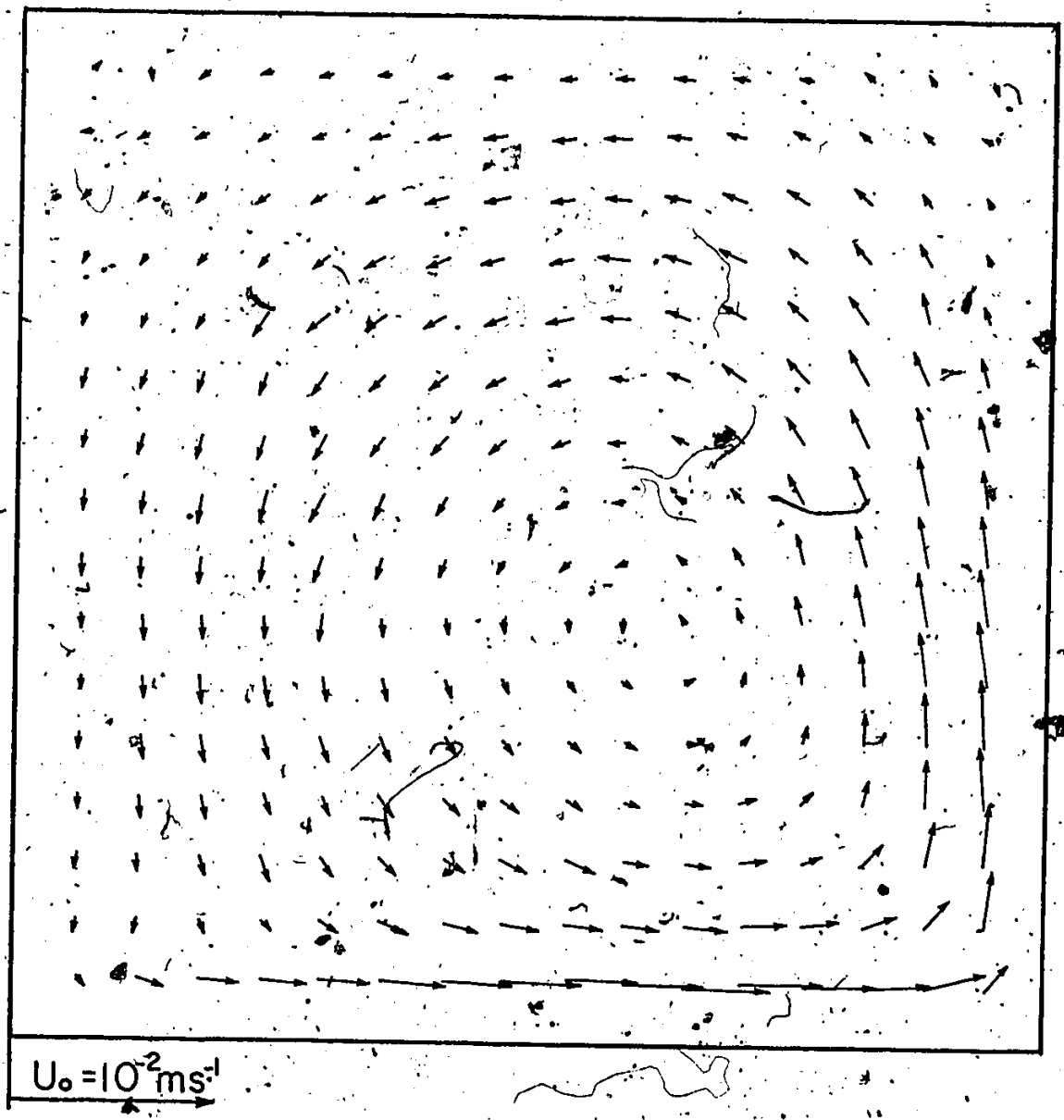


Fig.6.5a Velocity Distribution

$Re = 10^3$, $Pr = 0.01$, $Gr/Re^2 = 0$

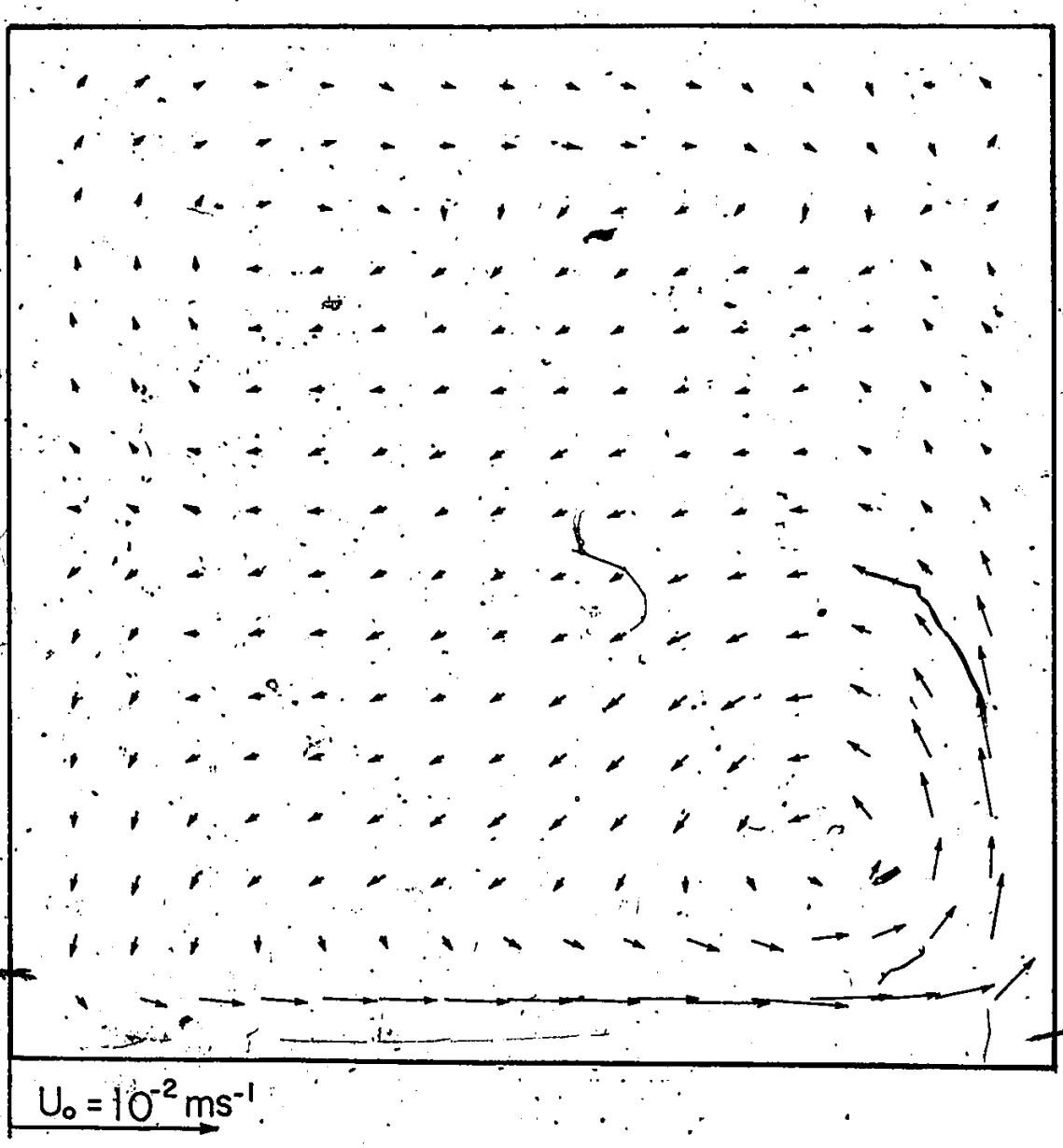


Fig.6.5b Velocity Distribution

$Re = 10^3$, $Pr = 0.01$, $Gr/Re^2 = 1$

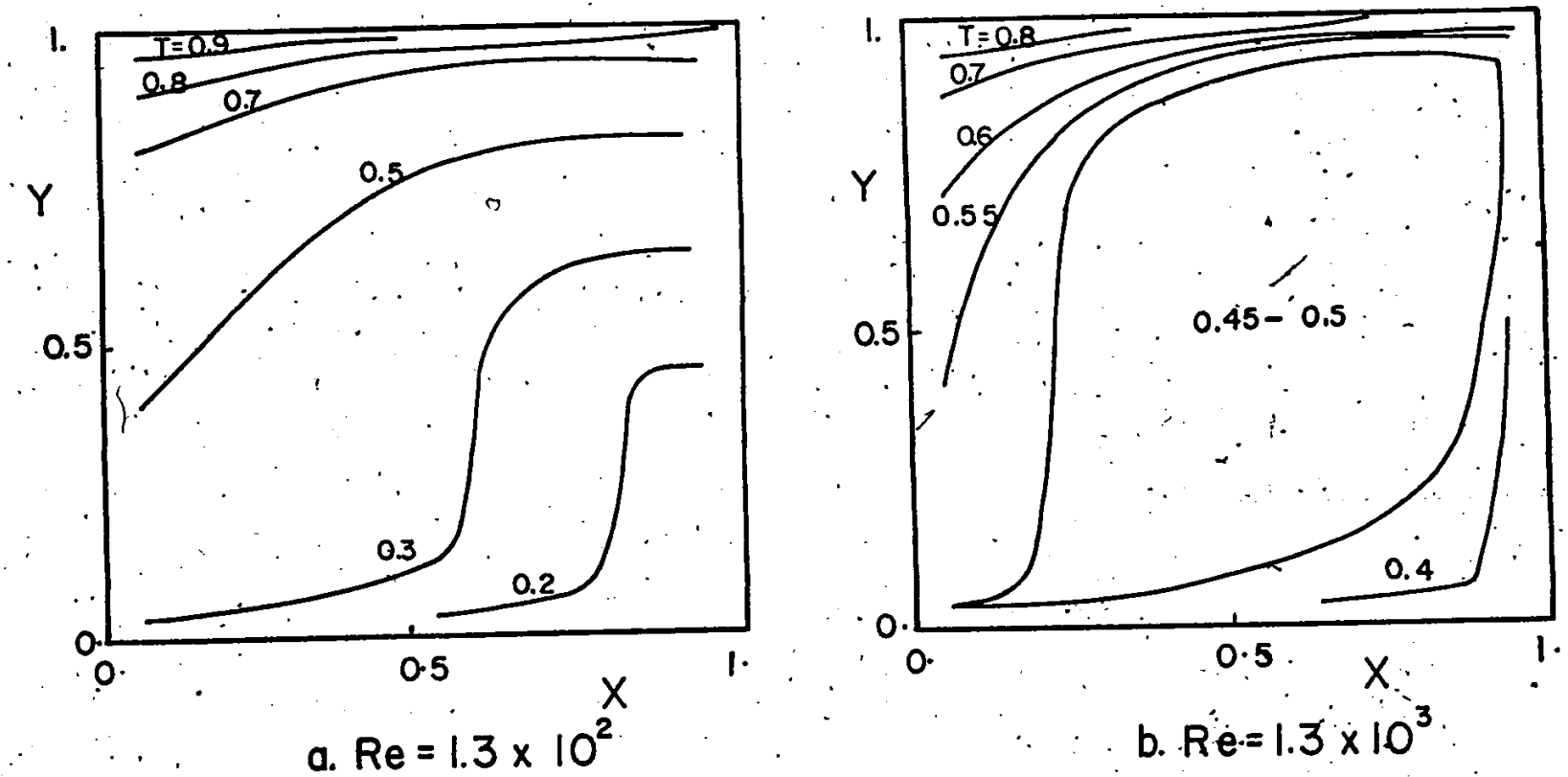


Fig.6.6 Isotherms for Pr = 3, Gr/Re^2 = 0

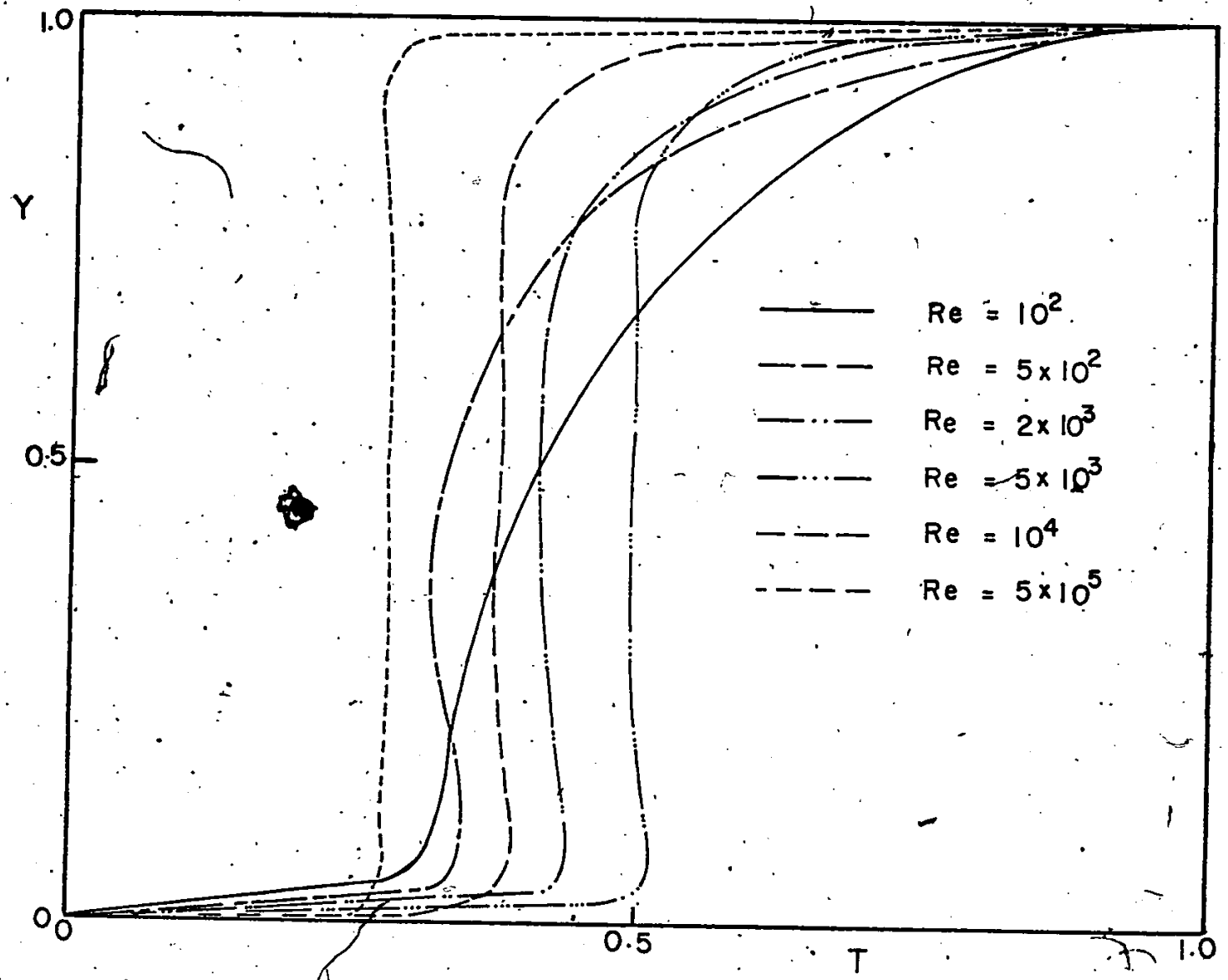


Fig.6.7a Vertical Centre-line Temperature Distribution, Pr = 3, $Gr/Re^2 = 0$

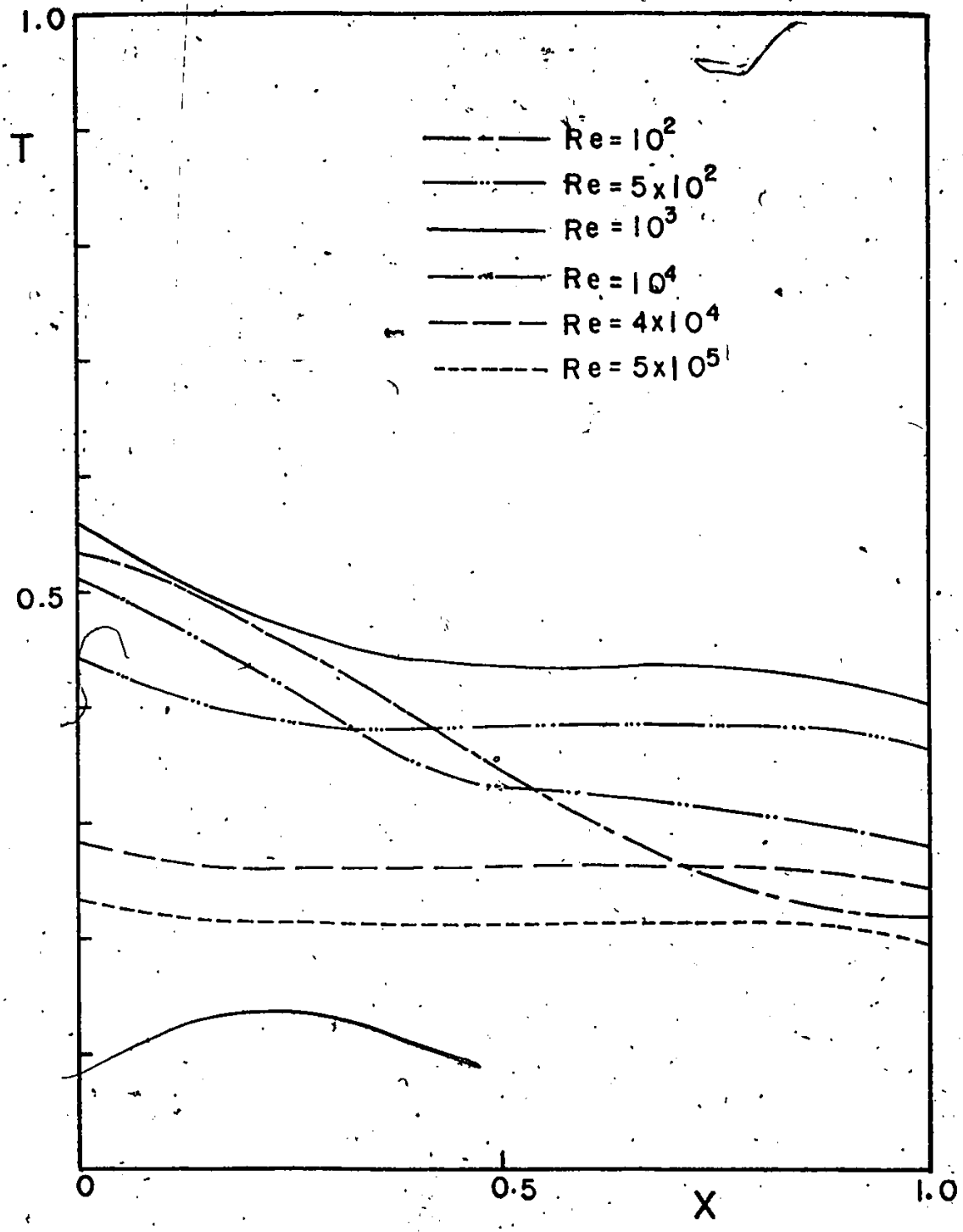


Fig.6.7b Horizontal Centre-line Temperature Distribution
Pr = 3, Gr/Re^2 = .0

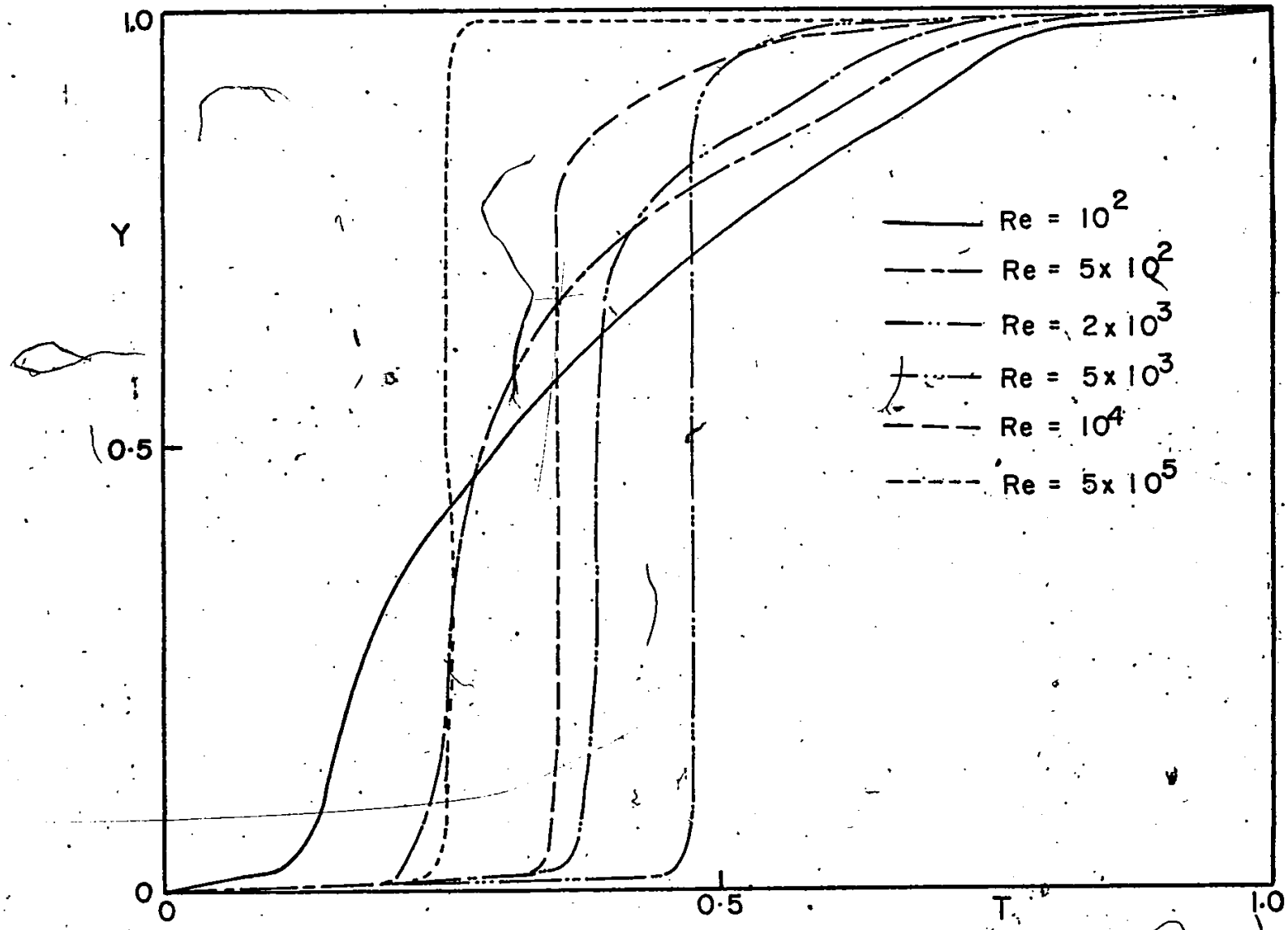


Fig.6.7c Right Side Wall Temperature Distribution, Pr = 3, $Gr/Re^2 = 0$

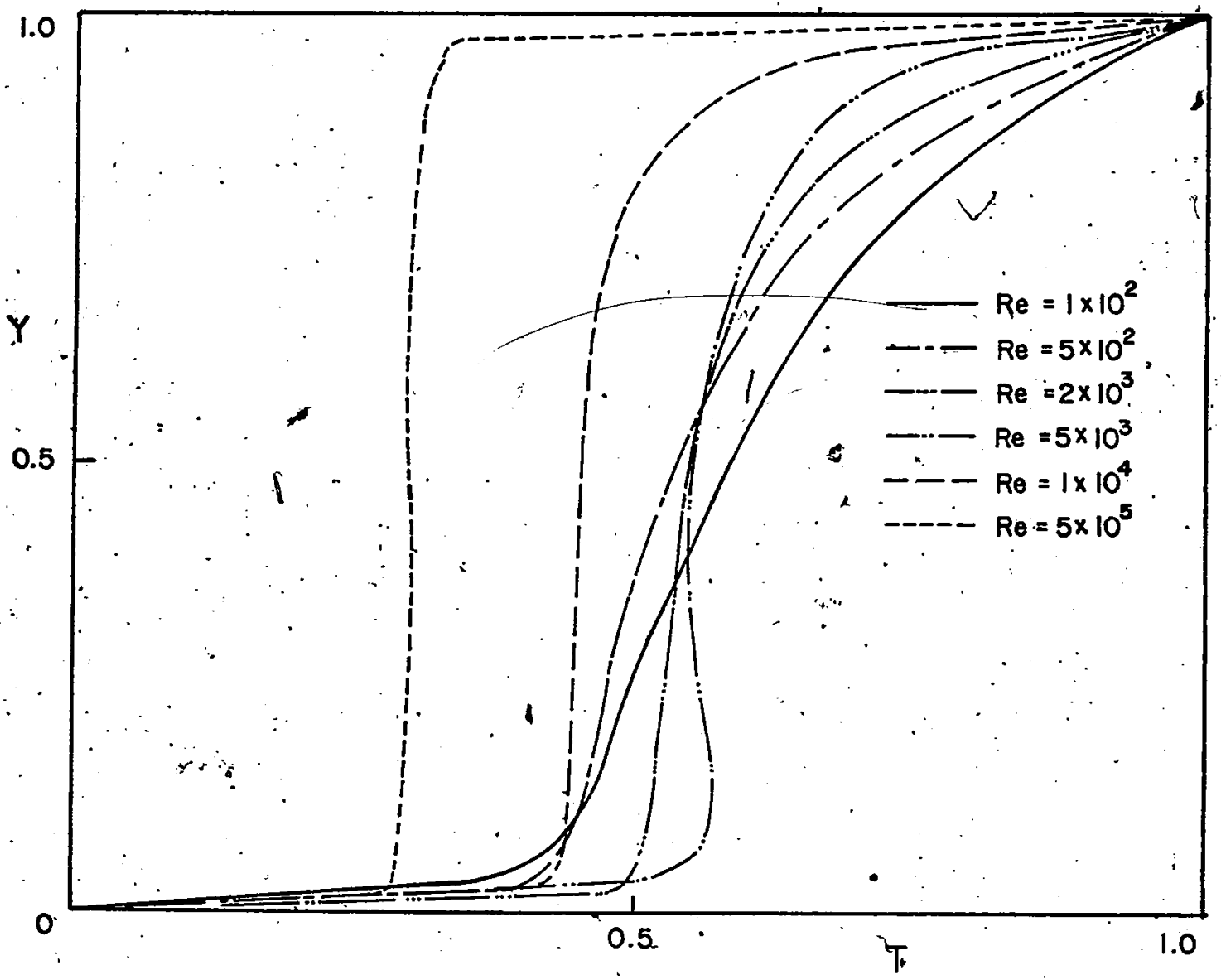


Fig.6.7d Left Side Wall Temperature Distribution, Pr = 3, $Gr/Re^2 = 0$

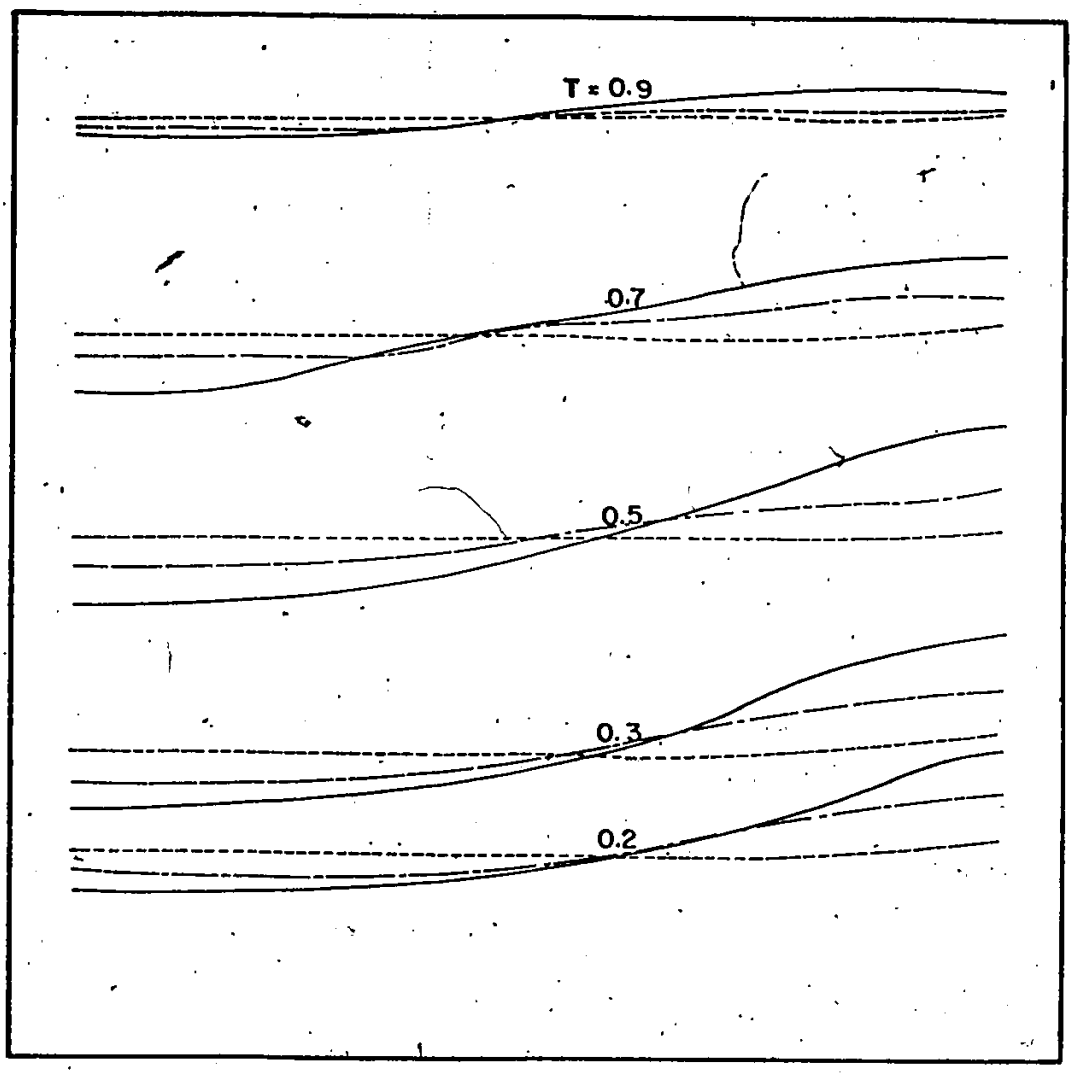


Fig. 6.8a Isotherms, Pr = 0.01, $Gr/Re^2 = 0$

--- Re = 100, ——— - ——— Re = 500, ——— Re = 1000

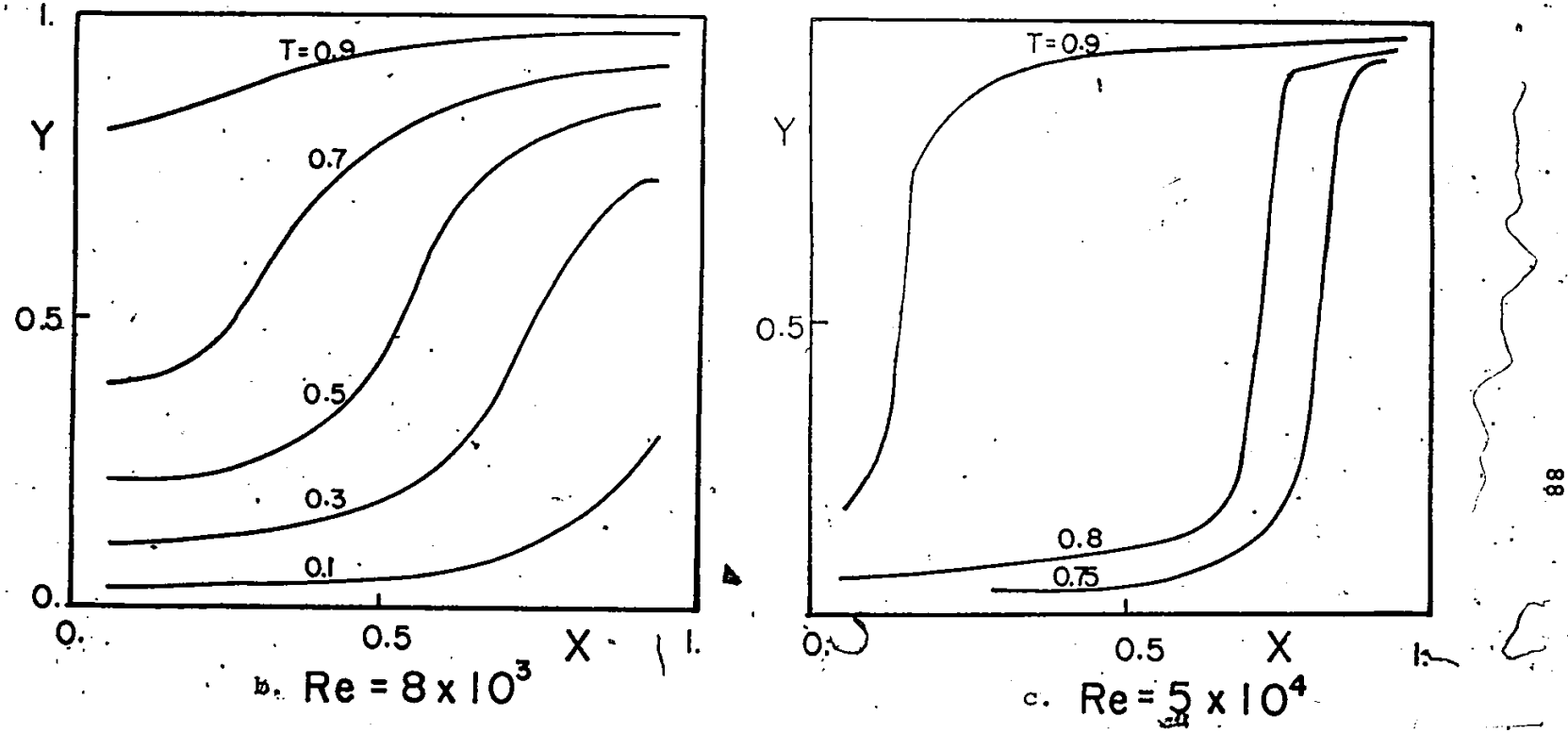


Fig.6.8 Isotherms for Pr = 0.01, $Gr/Re^2 = 0$

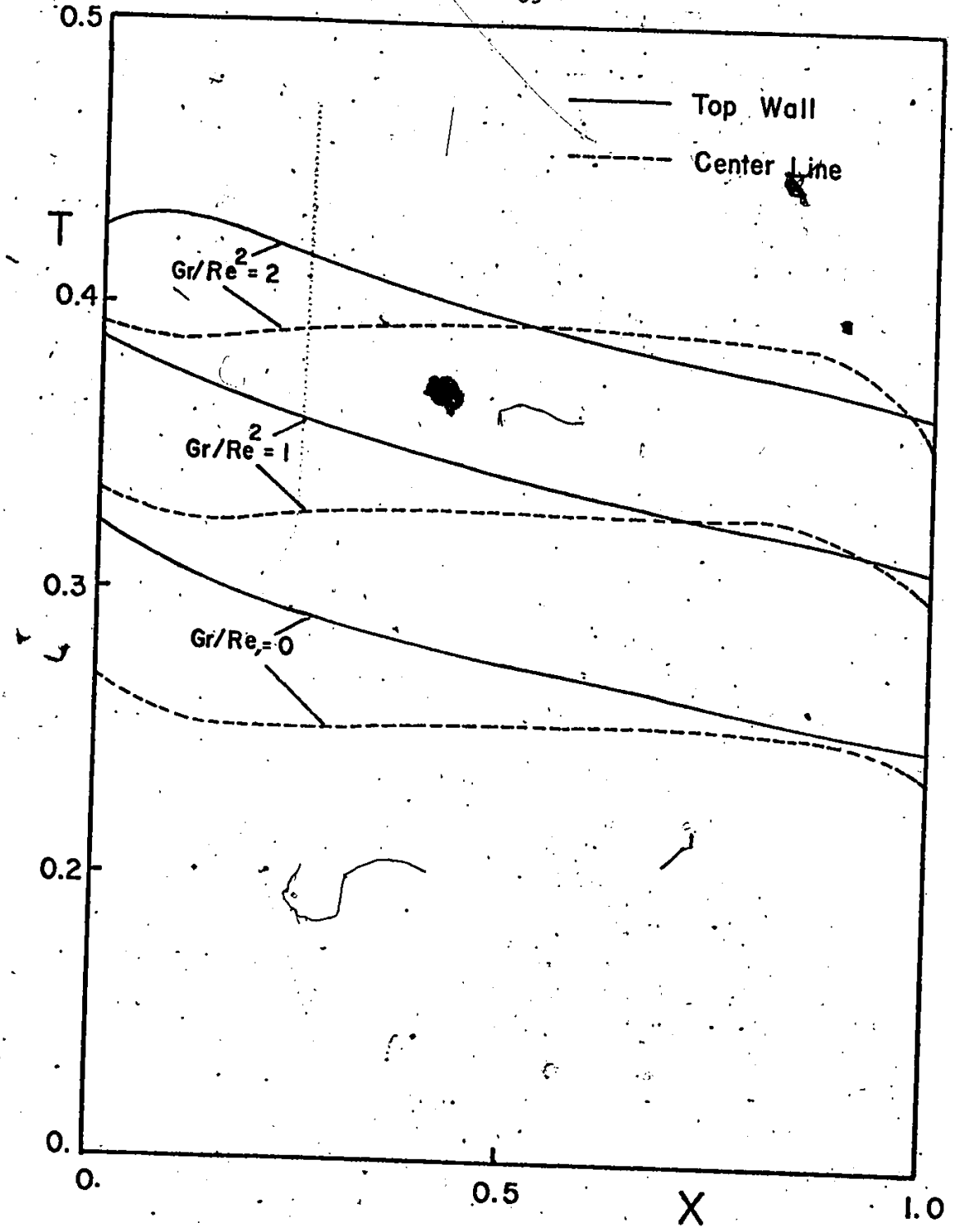


Fig.6.9a Horizontal Temperature Distribution

$Re = 2.7 \times 10^5$, $Pr = 3$

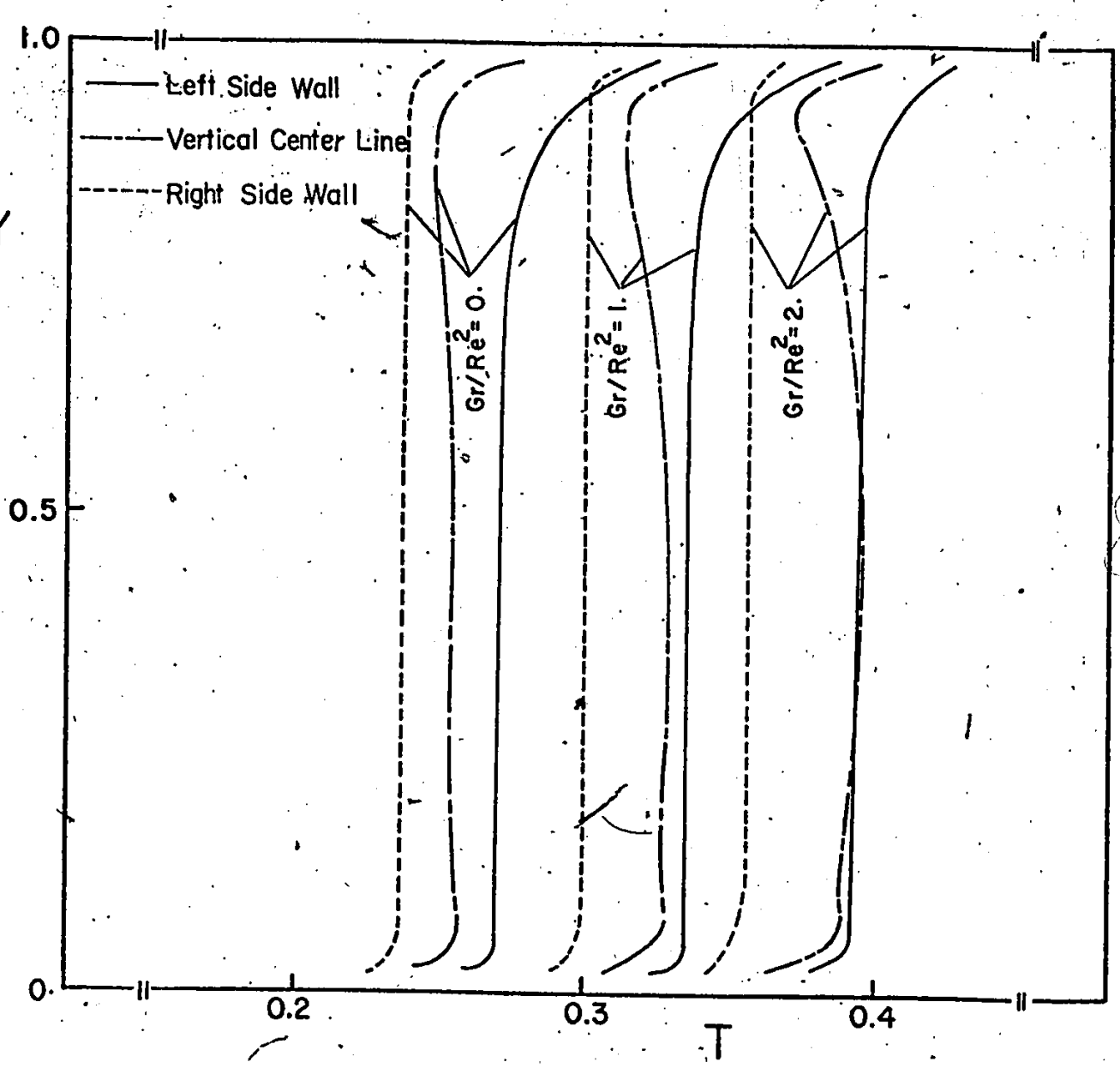


Fig.6.9b Vertical Temperature Distribution

Re = 2.7×10^5, Pr = 3

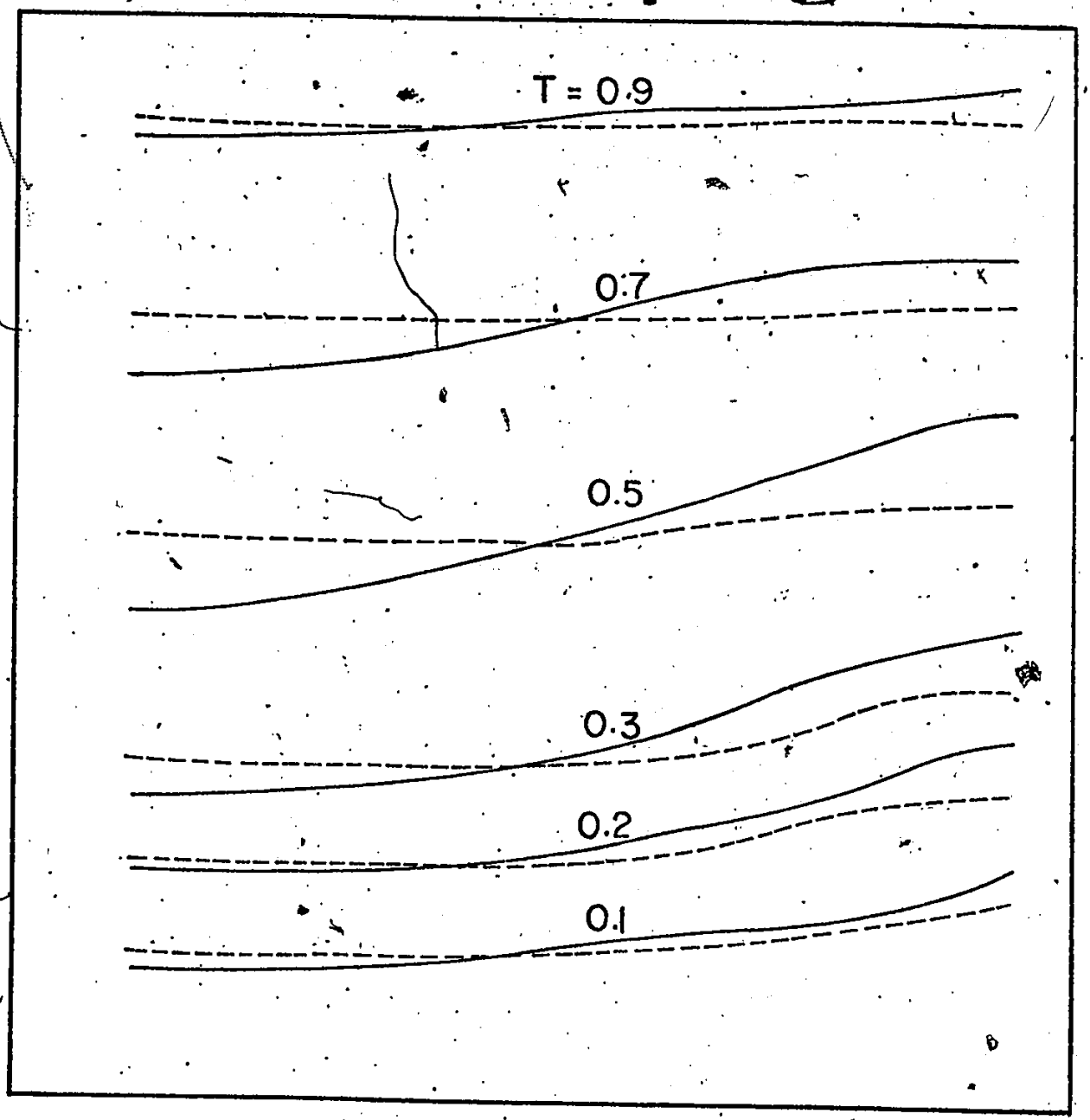
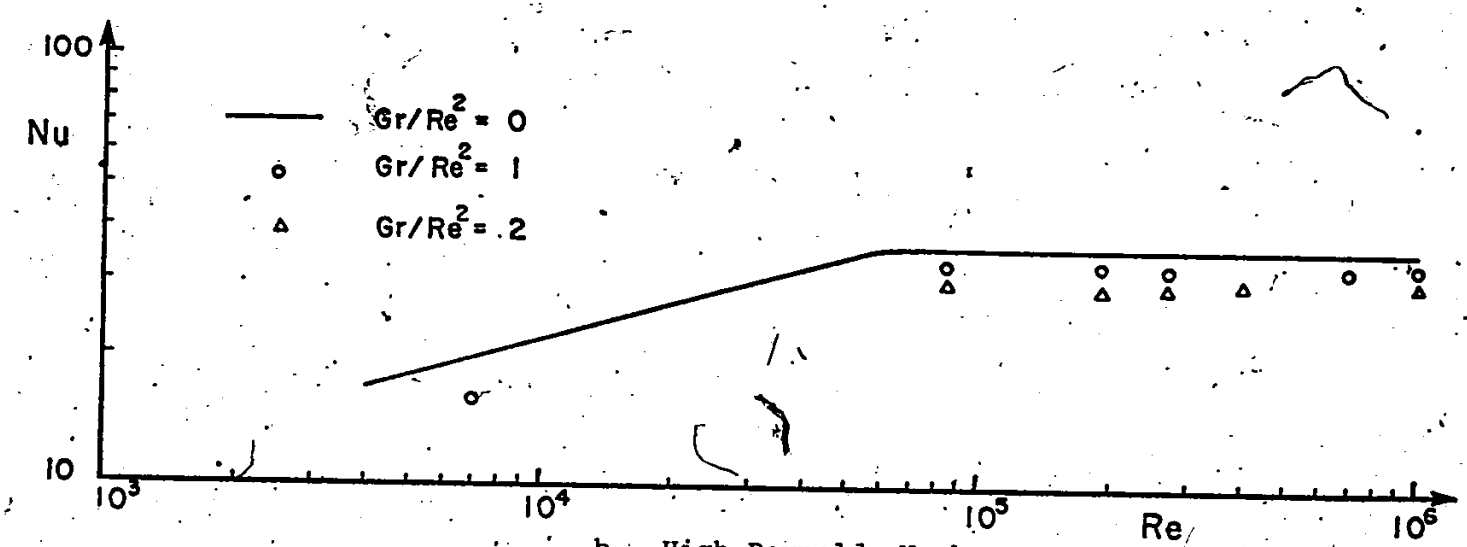
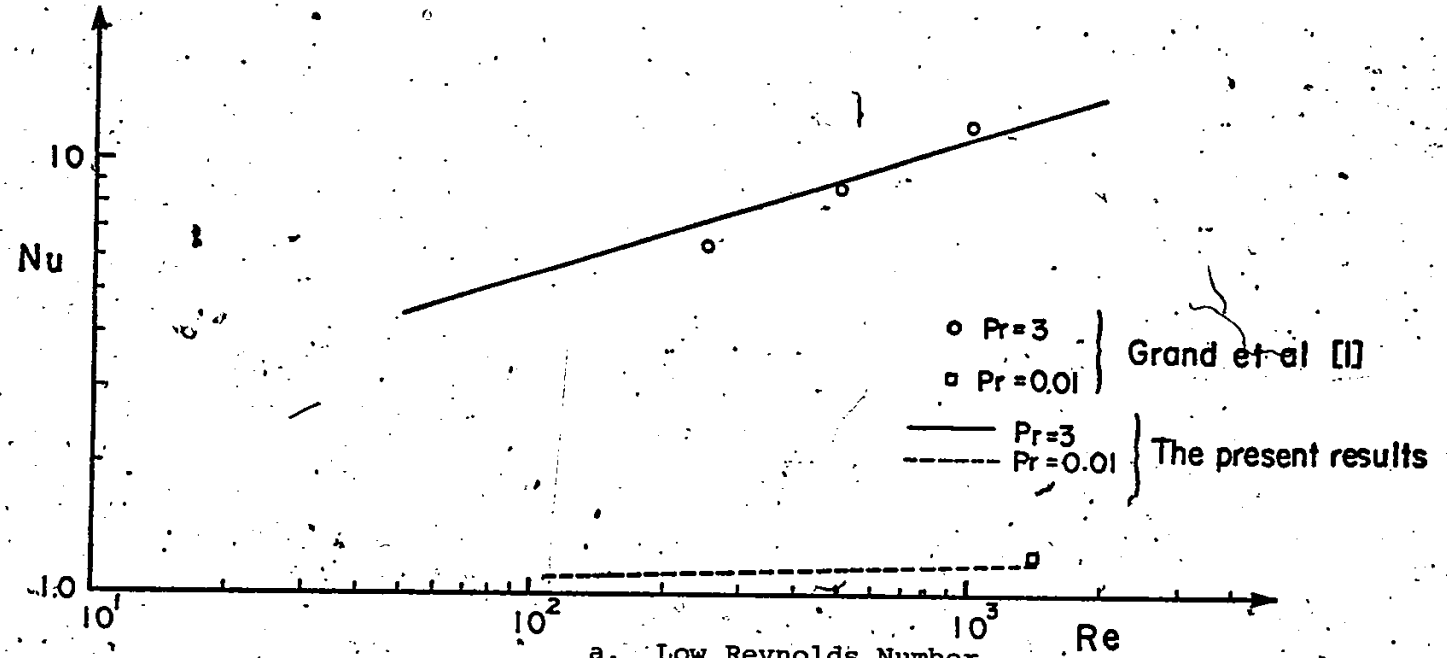


Fig.6.10 Isotherms, Pr = 0.01, Re = 10^3

——— $Gr/Re^2 = 0$, ------ $Gr/Re^2 = 1$



b. High Reynolds Number



a. Low Reynolds Number

Fig.6.11 Mean Nusselt Number

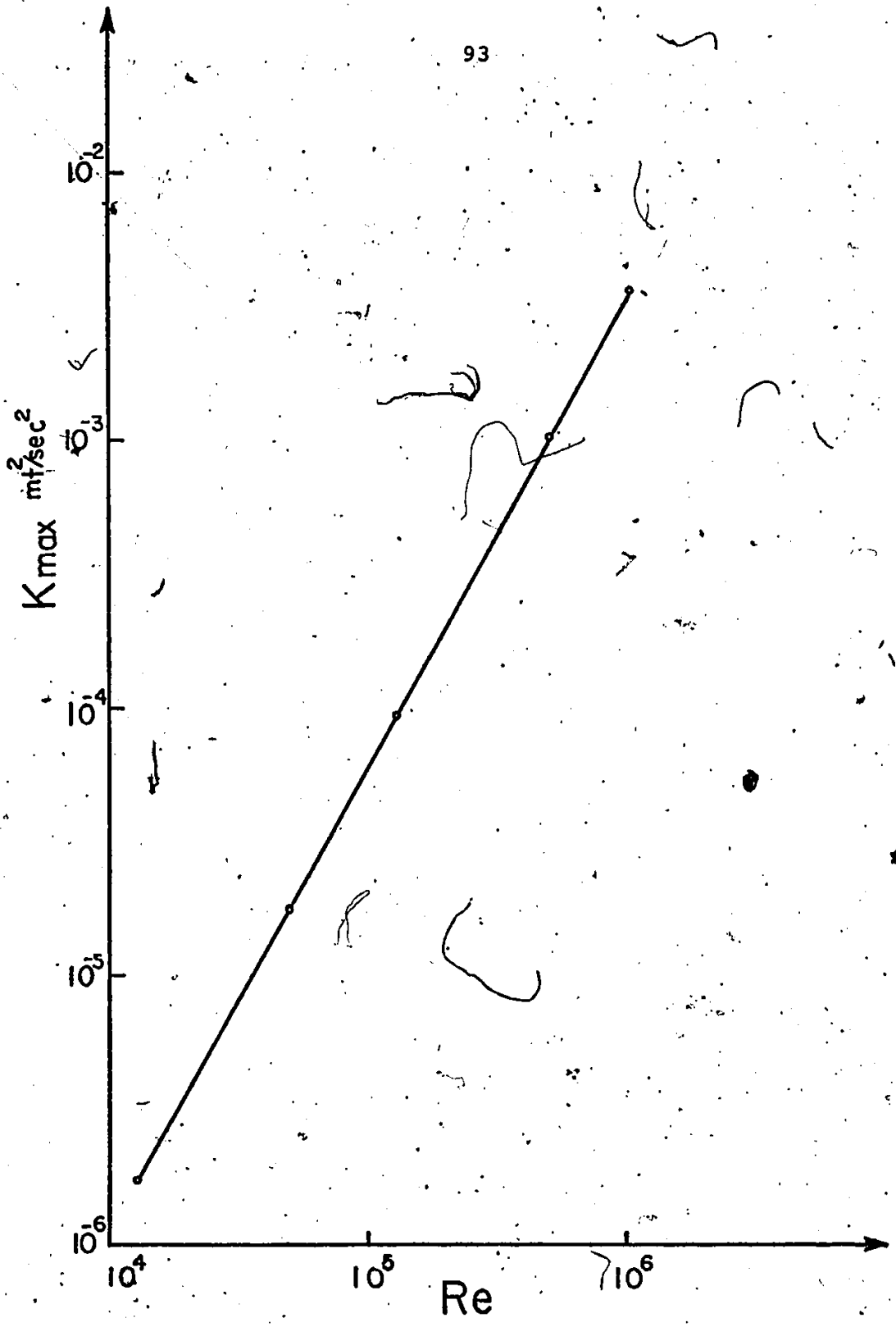


Fig.6.12 Variation of the Turbulence Kinetic Energy with Reynolds Number

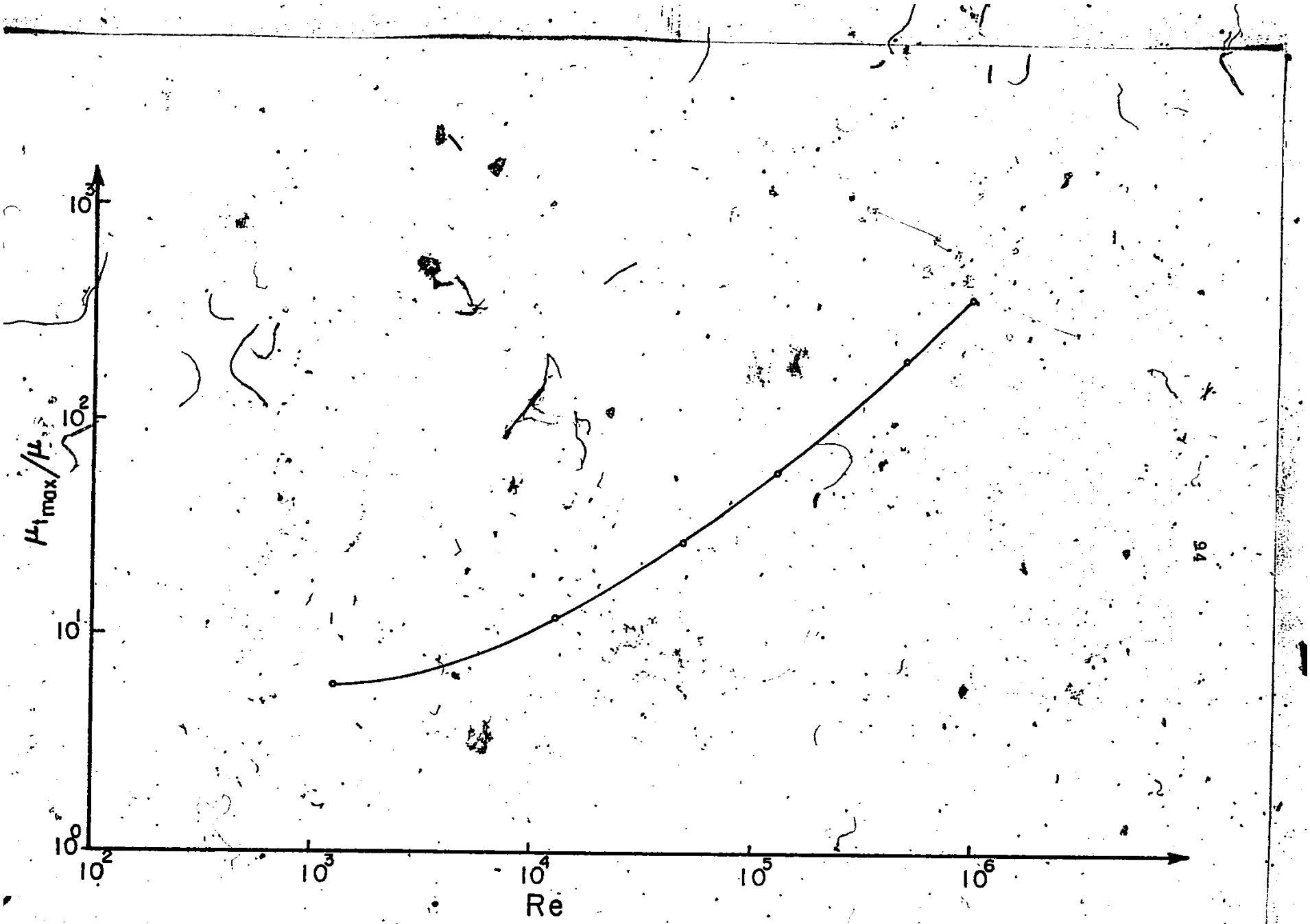


Fig.6.13 Variation of the Effective Viscosity with Reynolds Number

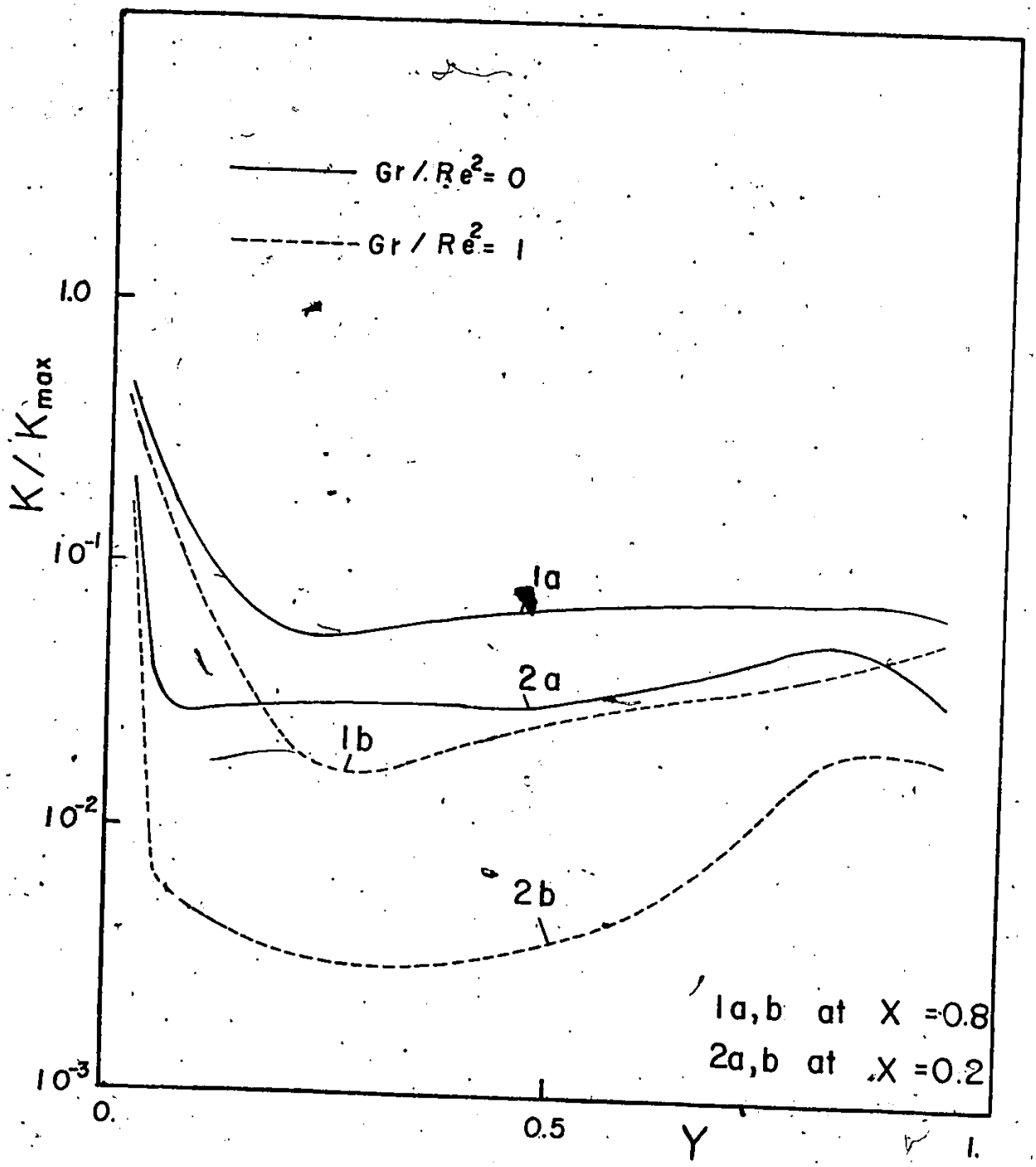


Fig.6.14 Vertical Turbulence Kinetic Energy Distribution

Re = 8.4×10^4

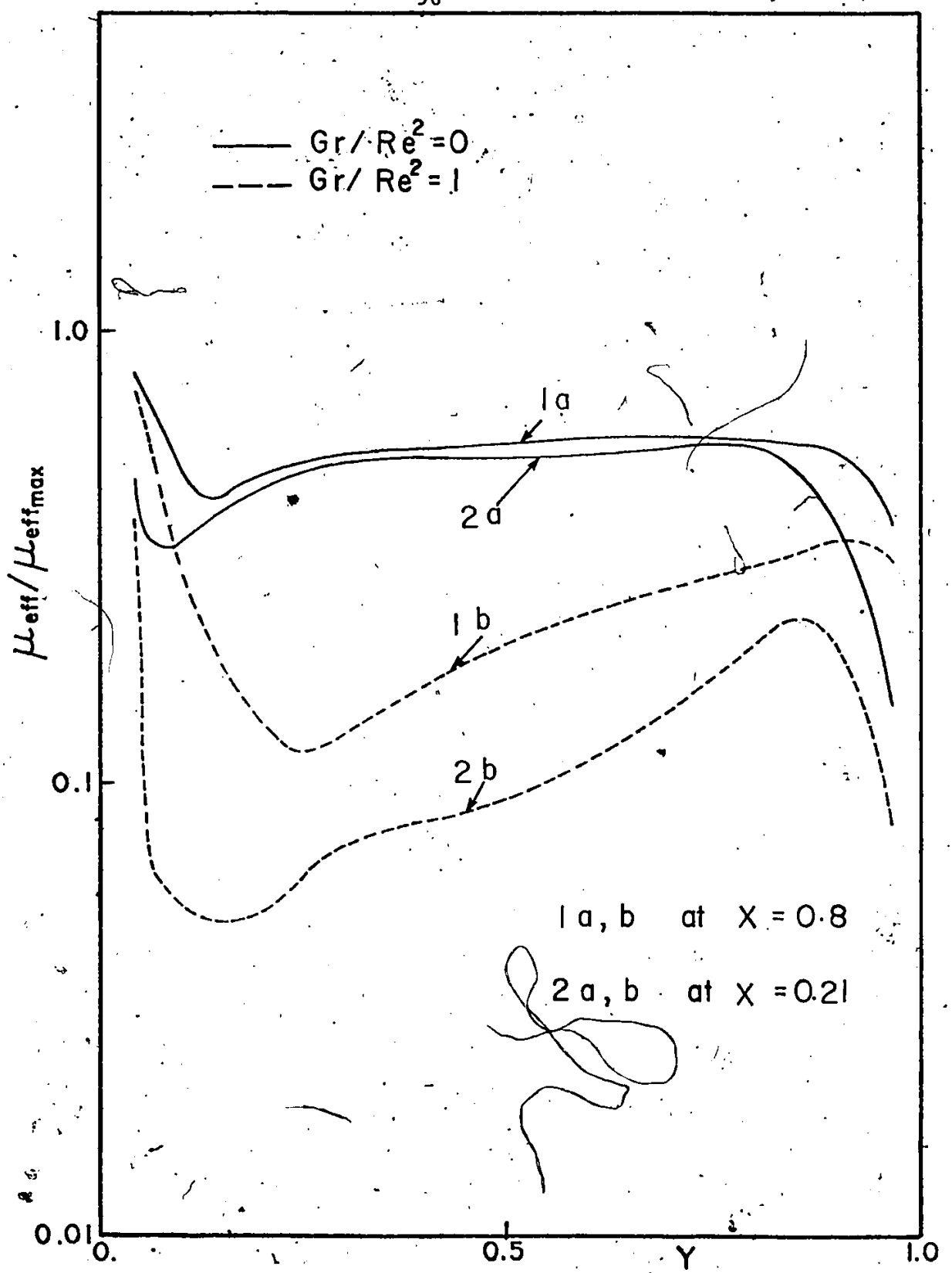


Fig.6.15 Vertical Effective Viscosity Distribution

Re = 8.4×10^4

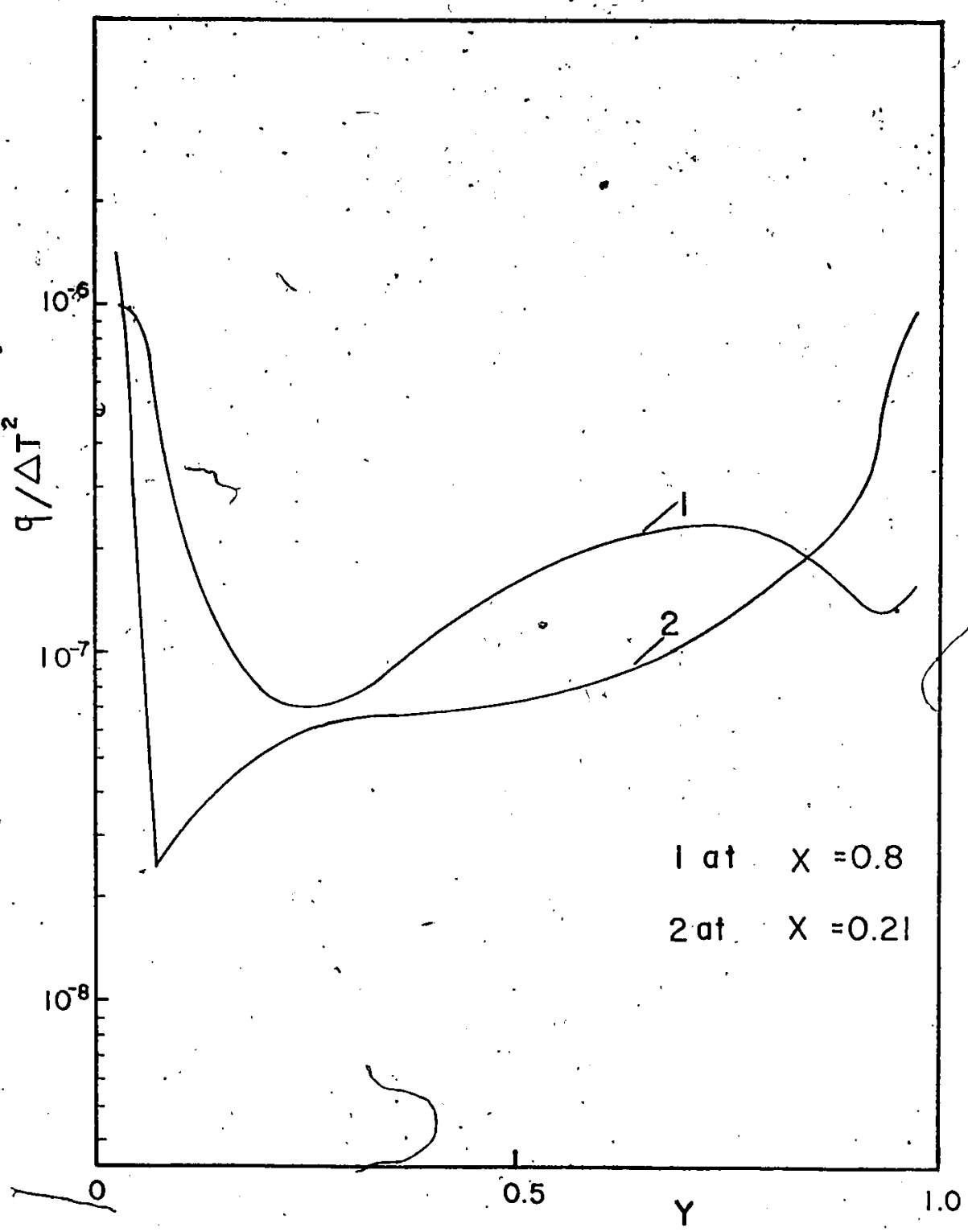


Fig.6.16 Vertical Distribution of the Mean Square Temperature Fluctuation, Re = 8.4×10^4

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APRENDIX A

Tridiagonal Matrix Algorithm (TDMA)

Starting with Eq.(5.1):

$$\underline{A_p}\phi_p = \underline{a_n}\phi_N + \underline{a_s}\phi_S + (\underline{a_e}\phi_E + \underline{a_w}\phi_W + Su)$$

with the underlined quantities assumed known. This equation can be represented by a system of Nj-2 equations of the general form:

$$d_i\phi_i = a_i\phi_{i+1} + b_i\phi_{i-1} + c_i$$

or

$$-a_i\phi_{i+1} + d_i\phi_i - b_i\phi_{i-1} = c_i \qquad (A.1)$$

where $d_i = A_p$; $a_i = a_n$; $b_i = a_s$;

$$c_i = a_e\phi_E + a_w\phi_W + Su$$

and the values of ϕ_1 and ϕ_{Nj} are specified.

The set of equations (A.1) can be written in the following matrix form:

$$P\phi = Q$$

i.e.

$$\begin{bmatrix} d_2 & -a_2 & & & & \\ -b_3 & d_3 & -a_3 & & & \\ & -b_4 & d_4 & -a_4 & & \\ & \cdots & \cdots & \cdots & & \\ & & -b_j & d_j & -a_j & \\ & & \cdots & \cdots & \cdots & \\ & & & -b_{Nj-2} & d_{Nj-2} & -a_{Nj-2} \\ & & & & -b_{Nj-1} & d_{Nj-1} \end{bmatrix} \begin{bmatrix} \phi_2 \\ \phi_3 \\ \phi_4 \\ \cdots \\ \phi_j \\ \\ \phi_{Nj-2} \\ \phi_{Nj-1} \end{bmatrix} = \begin{bmatrix} c_2 + b_2\phi_1 \\ c_3 \\ c_4 \\ \cdots \\ c_j \\ \cdots \\ c_{Nj-2} \\ c_{Nj-1} + a_{Nj-1}\phi_{Nj} \end{bmatrix}$$

where P is a tridiagonal matrix.

To solve this system of equations, factorize P into the products:

$$P = LU \tag{A.2}$$

where L and U are two $Nj-1 \times Nj-1$ matrices with the following form:

$$L = \begin{bmatrix} T_2 & & & & & \\ -B_3 & T_3 & & & & \\ & -B_4 & T_4 & & & \\ & \cdots & \cdots & \cdots & & \\ & & -B_j & T_j & & \\ & & \cdots & \cdots & \cdots & \\ & & & -B_{Nj-1} & T_{Nj-1} \end{bmatrix} ; \quad U = \begin{bmatrix} 1 & -A_2 & & & & \\ & 1 & -A_3 & & & \\ & \cdots & \cdots & \cdots & & \\ & & 1 & -A_j & & \\ & & \cdots & \cdots & \cdots & \\ & & & 1 & -A_{j-2} & \\ & & & & 1 & \end{bmatrix}$$

provided that all the principal minors of P are non-singular.

With these forms for L and U, the product LU is itself tridiagonal. By direct multiplication:

$$LU = \begin{bmatrix} T_2 & -T_2A_2 & & & & \\ -B_3 & B_3A_2+T_3-T_3A_3 & & & & \\ & -B_4 & B_4A_3+T_4-T_4A_4 & & & \\ & \cdots\cdots & \cdots\cdots & \cdots\cdots & & \\ & & -B_j & B_jA_{j-1}+T_j & -T_jA_j & \\ & & \cdots\cdots & \cdots\cdots & \cdots\cdots & \\ & & & -B_{Nj-2} & B_{Nj-2}A_{Nj-3}+T_{Nj-2} & -T_{Nj-2}A_{Nj-2} \\ & & & & & B_{Nj-1}A_{Nj-2}+T_{Nj-1} \end{bmatrix}$$

By equating the non-zero elements of LU with those of P, the following recursive formulae for B_j, T_j and A_i are obtained:

	$T_2 = d_2$	$A_2 = a_2/T_2$
$B_3 = b_3$	$T_3 = d_3 - B_3A_2$	$A_3 = a_3/T_3$
.....	.....	.....
$B_j = b_j$	$T_j = d_j - B_jA_{j-1}$	$A_j = a_j/T_j$
.....	.....	.....
$B_{Nj-1} = b_{Nj-1}$	$T_{Nj-1} = d_{Nj-1} - B_{Nj-1}A_{Nj-2}$	

provided that $T_j \neq 0$ for $j = 2, \ldots, Nj-1$.

Substituting b_j for B_j in T_j and defining $A_1 = 0$, gives:

$$B_j = b_j \quad ; \quad T_j = d_j - B_jA_{j-1} \quad ; \quad A_j = a_j/T_j \qquad (A.3)$$

Now, a solution to the tridiagonal system may be obtained. From the definition of L and U as expressed in Eq.(A.1), it follows that:

$$P = LU$$

$$P\phi = LU\phi = Q \tag{A.4}$$

Let

$$U\phi = C \tag{A.5}$$

where C is a column matrix with elements $C_2, C_3, \ldots, C_{Nj-1}$.

Substituting Eq.(A.5) into Eq.(A.4) gives:

$$LC = Q$$

i.e.

$$\begin{bmatrix} T_2 & & & & & \\ -B_3 & T_3 & & & & \\ & -B_4 & T_4 & & & \\ & \cdots\cdots & & & & \\ & & -B_j & T_j & & \\ & & \cdots\cdots & & & \\ & & & -B_{Nj-1} & T_{Nj-1} \end{bmatrix} \begin{bmatrix} C_2 \\ C_3 \\ C_4 \\ \cdots \\ C_j \\ \cdots \\ C_{Nj-1} \end{bmatrix} = \begin{bmatrix} c_2 + b_2\phi_1 \\ c_3 \\ c_4 \\ \cdots \\ c_j \\ \cdots \\ c_{Nj-1} + a_{Nj-1}\phi_{Nj} \end{bmatrix}$$

Hence,

$$T_2C_2 = c_2 + b_2\phi_1$$

$$-B_3C_2 + T_3C_3 = c_3$$

$$-B_4C_3 + T_4C_4 = c_4$$

$$\cdots\cdots\cdots\cdots$$

$$-B_jC_{j-1} + T_jC_j = c_j$$

$$\cdots\cdots\cdots\cdots$$

$$-B_{Nj-1}C_{Nj-2} + T_{Nj-1}C_{Nj-1} = c_{Nj-1} + a_{Nj-1}\phi_{Nj}$$

with $B_j = b_j$ as expressed in Eq.(A.3) and defining $C_1 = \phi_1$, the solution of C can be obtained directly. This yields the following, provided that $T_j \neq 0$:

$$C_j = (c_j + b_jc_{j-1})/T_j \qquad 2 \leq J \leq Nj-2$$

$$C_{Nj-1} = (c_{Nj-1} + b_{Nj-1}c_{Nj-2})/T_{Nj-1} + a_{Nj-1}\phi_{Nj}/T_{Nj-1} \qquad (A.6)$$

By defining $A_{Nj-1} = a_{Nj-1}/T_{Nj-1}$, the second term on the right-hand side of C_{Nj-1} becomes $\phi_{Nj}A_{Nj-1}$.

With all the C's defined as in Eq.(A.6), the ϕ's can be solved using Eq.(A.5), which gives:

$$\begin{bmatrix} 1 & -A_2 & & & & \\ & 1 & -A_3 & & & \\ & & \cdots\cdots & & & \\ & & & 1 & -A_j & \\ & & & & \cdots\cdots & \\ & & & & 1 & -A_{Nj-2} \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} \phi_2 \\ \phi_3 \\ \cdots \\ \phi_j \\ \cdots \\ \phi_{Nj-2} \\ \phi_{Nj-1} \end{bmatrix} = \begin{bmatrix} c_2 \\ c_3 \\ \cdots \\ c_j \\ \cdots \\ c_{Nj-2} \\ c_{Nj-1} \end{bmatrix}$$

Finally ϕ is solved by back substitution, beginning with ϕ_{Nj-1} giving,

$$\left.\begin{array}{l} \phi_{Nj-1} = c_{Nj-1} \\ \phi_{Nj-2} - A_{Nj-2}\phi_{Nj-1} = c_{Nj-2} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \phi_j - A_j\,\phi_{j+1} = c_j \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \phi_2 - A_2\phi_3 = c_2 \end{array}\right\} \equiv \left\{\begin{array}{l} \phi_{Nj-1} = c_{Nj-1} \\ \phi_{Nj-2} = c_{Nj-2} + A_{Nj-2}\phi_{Nj-1} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \phi_j = c_j + A_j\,\phi_{j+1} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \phi_2 = c_2 + A_3\phi_3 \end{array}\right.$$

with the C's defining as in Eq.(A.6), the ϕ's can be written as:

$$\phi_j = (c_j + b_j c_{j-1})/T_j + A_j\phi_{j-1} \qquad 2 \le j \le Nj-1$$

with

$$T_j = d_j - b_j\,A_{j-1} \qquad 2 \le j \le Nj-1$$

$$A_j = a_j/T_j \qquad 2 \le j \le Nj-1$$

$$C_j = (c_j + b_j\,c_{j-1})/T_j \qquad 2 \le j \le Nj-2$$

$$A_1 = 0 \;;\; C_1 = \phi_1$$

Note that the calculation of the coefficients c_j, A_j and T_j has to be calculated from j = 2 to Nj-1, while the calculation of the ϕ's is from j = Nj-1 to 2.

APPENDIX B

A Sample Computer Program

```
C*********************************************************************C
C              TWO-DIMENSIONAL(PLANE OR AXISYMMETRICAL)               C
C              BASIC TEST CASE :                                      C
C                                                                     C
C              PRIDICTION OF A TURBULENT RECIRCULATORY FLOW           C
C              IN RECTANGULAR CAVITY AFFECTED BY BUOYANCY FORCES.     C
C*********************************************************************C
C
1       DIMENSION HEDU(9),HEDV(9),HEDP(9),HEDT(9),HEDK(9),HEDD(9),HEDM(9)     0340
       1         ,HEDA(9),HEDB(9),HEDQ(9),HEDR(9),HEDG(9),HEDE(9)             0350
2       COMMON                                                                0360
       1/UVEL/RESORU,NSWPU,URFU,DXEPU(32),DXPWU(32),SEWU(32)                  0370
       1/VVEL/RESORV,NSWPV,URFV,DYNPV(32),DYPSV(32),SNSV(32),RCV(32)          0380
       1/PCOR/RESORM,NSWPP,URFP,DU(30,30),DV(30,30),IPREF,JPREF               0390
       1/TEN/RESORK,NSWPK,URFK                                                0400
       1/TDIS/RESORE,NSWPD,URFE                                               0410
       1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)         0420
       1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT                                      0430
3       COMMON
       1/GEOM/INDCOS,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),         0440
       1      SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)                      0450
       1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
       2        VISD(30,30)
       1/PROB1/UIN,TEIN,EDIN,FLOWIN,ALAMDA,
       2      RSMALL,RLARGE,AL1,AL2,JSTEP,ISTEP,JSTP1,JSTM1,ISTP1,ISTM1,      0480
       3    NITER,US
       1/TURB/GEN(30,30),CD,CMU,C1,C2,CAPPA,ELOG,PRED,PRTE                     0490
       1/WALLF/YPLUSN(30,30),XPLUSW(30,30),TAUN(30,30),TAUW(30,30)
       1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),     0510
       1      SP(30,30)
4       COMMON
       1/SUS/SU1(30,30),SU2(30,30),SU3(30,30),SU4(30,30)
       1/SUSP/SUKD(30,30),SPKD(30,30)
       1/TEMP/T(30,30),GAMH(30,30),RESORT,NSWPT,URFT,PRANDL,PFUN,
       2    TWALLN,TIN,GAMQ(30,30),TBOTOM
       1/MSTF/QE(30,30),PRQE,CQ1,CQ2,RESQE,URFQ,BETA,GRAVTY,C3,C4,NSWPQ,
       1       SUQE(30,30),GENQ(30,30),SPQE(30,30),RESORQ
       1/AREA1/INCT
       1/FINAL/ALNUUL(32),ALNUWL(32),ALNUUT(32),ALNUWT(32),
       2      VAVR(30,30),RA(30,30),ANGLE(30,30),UAVR(30,30),FENRG(30,30)
```

```
      LOGICAL INCALU,INCALV,INCALP,INPRO,INCALK,INCALD,INCALM,INCALA,      0530
     1    INCALB,INCALT,INCT,INCALQ,INCALR,INCALG,INCAFE
      REAL NU
      READ(5,010)HEDU,HEDV,HEDP,HEDT,HEDK,HEDD,HEDM,HEDA,HEDB,HEDQ        0650
     1,HEDR,HEDG,HEDE
  010 FORMAT(9A4)
C       ************          TEACH-T          **************        C    0070
CHAPTER  1  1  1  1  1  PARAMETERS AND CONTROL INDICES  1  1  1  1  1  1    0680
C                                                                          0690
C-----GRID                                                                 0700
      GREAT=1.E30                                                          0550
      NITER=0                                                              0570
      IT=30
      JT=30
      NSWPU=3                                                              0600
      NSWPV=3                                                              0610
      NSWPP=5                                                              0620
      NSWPT=3
      NSWPK=3                                                              0630
      NSWPD=3                                                              0640
      NSWPQ=3
      NI=18
      NJ=18
      NIM1=NI-1
      NJM1=NJ-1                                                            0740
      IMAX=NI
      JMAX=NJ
      INDCOS=1
      DY=.03
      DX=0.03
      READ(5,020) (Y(J),J=1,NJ)
  020 FORMAT(10F6.4)
      DO 100 I=1,NI
      X(I)=Y(I)
  100 CONTINUE
C
C-----DEPENDENT VARIABLE SELECTION                                         1010
C
      INCALU=.TRUE.                                                        1020
      INCALV=.TRUE.                                                        1030
      INCALP=.TRUE.                                                        1040
      INCALT=.TRUE.
```

```
      INCALD=.TRUE.                                                    1060
      INCALK=.TRUE.                                                    1050
      INPRO=.TRUE.                                                     1070
      INCT=.TRUE.
      INCALQ=.TRUE.
      INCALR=.TRUE.
      INCALG=.TRUE.
      INCAFE=.TRUE.
C
C-----FLUID PROPERTIES                                                 1080
C
      DENSIT=974.
      VISCOS=3.72E-4
      PRANDL=2.33
      TK=0.16
      CPP=1002.3
      BETA=6.72E-4
      NU=VISCOS/DENSIT
C
C-----TURBULENCE CONSTANTS                                             1110
C
      CMU=0.09                                                         1120
      CD=1.00                                                          1130
      C1=1.44                                                          1140
      C2=1.92                                                          1150
      C3=1.44
      C4=0.5
      CAPPA=.4187                                                      1160
      ELOG=9.793                                                       1170
      PRED=1.3
      PRTE=1.0                                                         1190
      PRANDT=0.9
      PFUN=PRANDL/PRANDT
      PFUN=9.24*PFUN**0.75
C
C-----BOUNDARY VALUES                                                  1200
C
      UIN=0.1E-9
      TURBIN=.03                                                       1230
      TEIN=1.E-6
      QEIN=0.001*TURBIN
      ALAMDA=0.005                                                     1250
      EDIN=1.E-4
```

```
      C
      C-----PRESSURE CALCULATION                                          1270
      C
   72       JPREF=9
   73       IPREF=9
   74       TIN=350.
      C
      C-----PROGRAM CONTROL AND MONITOR                                   1300
      C
   75       MAXIT=75
   76       IMON=4
   77       JMON=4
   78       URFU=0.5                                                      1340
   79       URFV=0.5                                                      1350
   80       URFP=1.0                                                      1360
   81       URFT=1.0
   82       URFE=0.7                                                      1370
   83       URFK=0.7                                                      1380
   84       URFQ=0.7
   85       URFVIS=0.7                                                    1390
   86       TWALEN=400.
   87       TBOTOM=300.
   88       INDPRI=100                                                    1400
   89       SORMAX=0.01
   90       GRAVTY=-9.806
      C                                                                   1420
      CHAPTER 2 2 2 2 2 2 INITIAL OPERATIONS 2 2 2 2 2 2 2 2 2 2 2 2      1430
      C                                                                   1440
      C-----CALCULATE GEOMETRICAL QUANTITIES AND SET VARIABLES TO ZERO    1450
      C
   91       CALL INIT                                                     1460
      C------SPECIFY INLET CONDITIONS                                     1470
      C                                                                   1480
   92       US=500.*NU/X(NI)
   93       U(2,2)=UIN
   94       DO 202 I=2,NI
   95       DO 202 J=2,NJM1
   96       T(I,J)=TIN
   97       T(2,1)=TBOTOM
   98       TE(I,J)=1.E-10
   99       QE(I,J)=QEIN
  100       QE(I,J)=1.E-6
```

```
      ED(I,J)=1.E-6
      U(I,1)=US
  202 U(I,J)=UIN
      TE(1,2)=TE(2,2)
      ED(1,2)=ED(2,2)
      DO 203 I=1,NI
      T(I,NJ)=TWALLN
      DO 203 J=1,NJ
  203 YPLUSN(I,J)=12.
      DO 204 J=1,NJ
      DO 204 I=1,NI
      XPLUSW(I,J)=12.
  204 CONTINUE
      CALL PROPS                                                        1680
C                                                                       1690
C-----INITIAL OUTPUT
C
      WRITE(6,220) UIN                                                  1720
      WRITE(6,221) TIN
      RENOLD=US*X(NI)/NU
      GR=-GRAVTY*BETA*(TWALLN-T(2,1))*Y(NJ)**3/NU**2
      WRITE(6,222) TWALLN
      WRITE(6,223) PRANDL
      WRITE(6,231) RENOLD
      WRITE(6,232) GR
      WRITE(6,250) VISCOS
      WRITE(6,260) DENSIT                                               1770
      IF(INCALU) CALL PRINT(2,2,NI,NJ,IT,JT,XU,Y,U,HEDU)                1780
      IF(INCALV) CALL PRINT(2,2,NI,NJ,IT,JT,X,YV,V,HEDV)                1790
      IF(INCALP) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,P,HEDP)                 1800
      INCT=.TRUE.                                                       1810
      IF(INCALT) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,T,HEDT)
      INCT=.FALSE.
      IF(INCALK) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,TE,HEDK)                1820
      IF(INCALD) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,ED,HEDD)                1830
      IF(INCALQ) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,QE,HEDQ)
      RESORT=0.
C                                                                       1920
C                                                                       1930
CHAPTER  3  3  3  3  3  3  3  ITERATION LOOP  3  3  3  3  3  3  3  3  3  1940
C                                                                       1950
      WRITE(6,310) IMON,JMON                                            1960
  300 NITER=NITER+1
```

```
      C
      C-----UPDATE MAIN DEPENDENT VARIABLES                                  1980
      C
  137       IF(INCALU) CALL CALCU                                           1990
  138       IF(INCALV) CALL CALCV                                           2000
  139       IF(INCALP) CALL CALCP                                           2010
  140       IF(INCALT) CALL CALCT
  141       IF(INCALK) CALL CALCTE                                          2020
  142       IF(INCALD) CALL CALCED                                          2030
  143       IF(NITER.LE.150) GO TO 201
  144       IF(INCALQ) CALL CALCQE
      C
      C-----UPDATE FLUID PROPERITIES                                         2040
      C
  145   201 IF(INPRO) CALL PROPS
      C
      C-----INTERMEDIATE OUTPUT                                              2060
      C
  146       WRITE(6,399) NITER
  147   399 FORMAT(I4)
  148       WRITE(6,3333) RESORM,RESORU,RESORV,RESORT,RESORQ
  149  3333 FORMAT(2X,F20.5)
  150       DUMMY=0.0
  151       DUMMY=T(IMON,JMON)                                              2100
  152       IF(NITER.LE.150) GO TO 301
  153       IF(MOD(NITER,100).NE.0) GO TO 301
  154       WRITE(6,311) NITER,RESORU,RESORV,RESORM,RESORT,RESORK,RESORE    2110
     1                ,U(IMON,JMON),V(IMON,JMON),P(IMON,JMON),DUMMY,       2120
     1                     TE(IMON,NJM1),ED(IMON,NJM1)                     2130
  155       IF(INCALU) CALL PRINT(2,2,NI,NJ,IT,JT,XU,Y,U,HEDU)              2150
  156       IF(INCALV) CALL PRINT(2,2,NI,NJ,IT,JT,X,YV,V,HEDV)              2160
  157       IF(INCALP) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,P,HEDP)               2170
  158       INCT=.TRUE.
  159       IF(INCALT) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,T,HEDT)
  160       INCT=.FALSE.
  161       IF(INCALK) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,TE,HEDK)              2180
  162       IF(INCALD) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,ED,HEDD)              2190
  163       IF(NITER.LE.25) GO TO 301
  164       IF(INCALQ) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,QE,HEDQ)
  165       WRITE(6,310) IMON,JMON
  166   301 CONTINUE                                                        2210
```

```
      C
      C-----TERMINATION TESTS                                                2220
      C
  167       SORCE=AMAX1(RESORM,RESORU,RESORV,RESORT)
  168       IF(NITER.EQ.20.AND.SORCE.GT.1.0E4*SORMAX) GO TO  302             2240
  169       IF(NITER.EQ.MAXIT) GO TO 302                                     2250
      C                                                                      2290
  170       GO TO 300
  171     302 CONTINUE                                                       2310
      C                                                                      2320
      CHAPTER  4  4  4  4  4  4  FINAL OPERATIONS AND OUTPUT  4  4  4  4  4  4   2330
      C                                                                      2340
  172       IF(INCALU) CALL PRINT(2,2,NI,NJ,IT,JT,XU,Y,U,HEDU)               2350
  173       IF(INCALV) CALL PRINT(2,2,NI,NJ,IT,JT,X,YV,V,HEDV)               2360
  174       IF(INCALP) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,P,HEDP)                2370
  175       INCT=.TRUE.
  176       IF(INCALT) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,T,HEDT)
  177       INCT=.FALSE.
  178       IF(INCALK) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,TE,HEDK)               2380
  179       IF(INCALD) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,ED,HEDD)               2390
  180       IF(INPRO ) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,VIS,HEDM)              2400
  181       IF(INCALQ) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,QE,HEDQ)
  182       J=NJM1
  183       CDTERM=CMU**0.25
  184       DYN=YV(NJ)-Y(NJM1)
      C
      C     CALCULATION OF THE LOCAL AND AVERAGE NUSSELT NUMBER ALONG THE TOP WALL
      C
  185       XREF=X(NIM1)-X(2)
  186       TREF=TWALLN-TBOTOM
  187       AVNUUL=0.
  188       AVNUWT=0.
  189       AVNUUT=0.
  190       AVNUWL=0.
  191       WRITE(6,402)                                                     2490
  192       DO  401  I=2,NIM1                                                2500
  193       UPLUSN=ALOG(ELOG*YPLUSN(I,NJM1))/CAPPA
  194       HFLUXT=DEN(I,J)*CPP*SQRT(TE(I,J))*CDTERM*(TWALLN-T(I,NJM1))/
           1(PRANDT*(UPLUSN+PFUN))
  195       HFLUXL=TK*(TWALLN-T(I,NJM1))/DYNP(NJM1)
  196       HFLUXL=ABS(HFLUXL)
  197       HFLUXT=ABS(HFLUXT)
```

```
198         ALNUUL(I)=HFLUXL*1./(TK*TREF)
199         ALNUWL(I)=HFLUXL*XREF/(TK*TREF)
200         ALNUUT(I)=HFLUXT*1./(TK*TREF)
201         ALNUWT(I)=HFLUXT*XREF/(TK*TREF)
202         AVNUUL=AVNUUL+ALNUUL(I)*DXEP(I)
203         AVNUWL=AVNUWL+ALNUWL(I)*DXEP(I)
204         AVNUWT=AVNUWT+ALNUWT(I)*DXEP(I)
205         AVNUUT=AVNUUT+ALNUUT(I)*DXEP(I)
206         WRITE(6,403) I,ALNUUL(I),ALNUWL(I),ALNUUT(I),ALNUWT(I)
207    401  CONTINUE
208         AVNUUL=AVNUUL/XREF
209         AVNUWL=AVNUWL/XREF
210         AVNUUT=AVNUUT/XREF
211         AVNUWT=AVNUWT/XREF
212         WRITE(6,410) AVNUUL,AVNUWL,AVNUUT,AVNUWT
      C
      C     CALCULATION OF NON-DIMENSIONAL TEMPERATURE AND RESULTANT VELOCITY
      C
213         TREF=TWALLN-TBOTOM
214         DO 510 I=1,NI
215         DO 510 J=1,NJ
216         RA(I,J)=0.
217         ANGLE(I,J)=0.
218    510  CONTINUE
219         DO 500 I=2,NIM1
220         DO 500 J=2,NJM1
221         T(I,J)=(T(I,J)-TBOTOM)/TREF
222         UAVR(I,J)=0.5*(U(I,J)+U(I+1,J))
223         VAVR(I,J)=0.5*(V(I,J)+V(I,J+1))
224         RA(I,J)=SQRT(UAVR(I,J)**2+VAVR(I,J)**2)
225         ANGLE(I,J)=ATAN(VAVR(I,J)/UAVR(I,J))
226         ANGLE(I,J)=ANGLE(I,J)*180./3.14159
227         IF(UAVR(I,J).LT.0.) ANGLE(I,J)=ANGLE(I,J)+180.
228    500  CONTINUE
229         IF(INCALT) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,T,HEDT)
230         IF(INCALR) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,RA,HEDR)
231         IF(INCALG) CALL PRINT(2,2,NI,NJ,IT,JT,X,Y,ANGLE,HEDG)
232         STOP
```

```
      C
      C-----FORMAT STATEMENTS                                                    2560
      C
  233   221 FORMAT(1H0,25X,'INITIAL FLUID TEMPERATURE ',T60,1H=,3X,1PE11.3)
  234   220 FORMAT(//1H0,15X,'INITIAL FLUID VELOCITY',T60,3X,1PE11.3)
  235   222 FORMAT(///1H0,15X,'THERMAL BOUNDARY CONDITIONS ARE _ _ _'//
           11H0,25X,'PRESCRIBED TOP  WALL TEMPERATURE= ',1PE11.3//
           11H0,25X,'ADIBATIC SIDE WALL '///)
  236   223 FORMAT(1H0,15X,'RPANDTL NUMBER',T60,1H=,3X,1PE11.3)
  237   231 FORMAT(1H0,15X,16HREYNOLDS NUMBER ,T60,1H=,3X,1PE11.3)                2
  238   232 FORMAT(1H0,15X,16HGRASHOF  NUMBER ,T60,1H=,3X,1PE11.3)                26
  239   250 FORMAT(1H0,15X,18HLAMINAR VISCOSITY ,T60,1H=,3X,1PE11.3)              2630
  240   260 FORMAT(1H0,15X,14HFLUID DENSITY ,T60,1H=,3X,1PE11.3)                  2640
  241   310 FORMAT(1H0,4HITER,4X,106HI------------ABSOLUTE RESIDUAL SOURCE SUM    2650
           1S-------------I       I-------FIELD VALUES AT MONITORING LOCATION,  2660
           2     1H(,I2,1H,,I2,1H),9H--------I/2X,2HNO,6X,4HUMOM,6X,4HVMOM,6X,  2670
           34HMASS,6X,4HENER,6X,4HTKIN,6X,4HDISP,10X,1HU,9X,1HV,9X,1HP,9X,1HT,  2680
           49X,1HK,9X,1HD/)                                                     2690
  242   311 FORMAT(1H ,I3,4X,1P6E10.3,3X,1P6E10.3)                             2700
  243   410  FORMAT(//5X,'AV.LAM.NU/UL =',F10.5,2X,'AV.LAM.NU/L =',F10.5,
           1'AV.TUR.NU/UL =',F10.5,2X,'AV.TUR.NU/L =',F10.5)
  244   402  FORMAT(///5X,1HI,5X,
           1'L.LAM.NU/UL',2X,'L.LAM.NU/L',2X,'L.TUR.NU/UL',2X,
           2'L.TUR.NU/L')
  245   403  FORMAT(/5X,I2,6X,E10.3,5X,E10.3,3X,E10.3,2X,E10.3)
  246        END                                                                 2730
```

```
      SUBROUTINE INIT                                                    INIT
C                                                                        INIT
CHAPTER  0  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0  0   INIT
C                                                                        INIT
      COMMON                                                             INIT
     1/UVEL/RESORU,NSWPU,URFU,DXEPU(32),DXPWU(32),SEWU(32)               INIT
     1/VVEL/RESORV,NSWPV,URFV,DYNPV(32),DYPSV(32),SNSV(32),RCV(32)       INIT
     1/PCOR/RESORM,NSWPP,URFP,DU(30,30),DV(30,30),IPREF,JPREF              0390
     1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)        0420
     1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
      COMMON
     1/GEOM/INDCOS,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),      INIT
     1      SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)                    INIT
     1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
     2        VISD(30,30)
     1/PROB1/UIN,TEIN,EDIN,FLOWIN,ALAMDA,
     2       RSMALL,RLARGE,AL1,AL2,JSTEP,ISTEP,JSTP1,JSTM1,ISTP1,ISTM1,  MODA
     3    NITER,US
     1/TURB/GEN(30,30),CD,CMU,C1,C2,CAPPA,ELOG,PRED,PRTE                   0490
     1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),   0510
     1      SP(30,30)
     1/TEMP/T(30,30),GAMH(30,30),RESORT,NSWPT,URFT,PRANDL,PFUN,
     2     TWALLN,TIN,GAMQ(30,30),TBOTOM
     1/MSTF/QE(30,30),PRQE,CQ1,CQ2,RESQE,URFQ,BETA,GRAVTY,C3,C4,NSWPQ,
     1       SUQE(30,30),GENQ(30,30),SPQE(30,30),RESORQ
C                                                                        INIT
      DO 100 J=1,NJ                                                      INIT
      R(J)=Y(J)                                                          INIT
  100 IF(INDCOS.EQ.1)R(J)=1.0                                            INIT
      DXPW(1)=0.0                                                        INIT
      DXEP(NI)=0.0                                                       INIT
      DO 101 I=1,NIM1                                                    INIT
      DXEP(I)=X(I+1)-X(I)                                                INIT
  101 DXPW(I+1)=DXEP(I)                                                  INIT
      DYPS(1)=0.0                                                        INIT
      DYNP(NJ)=0.0                                                       INIT
      DO 102 J=1,NJM1                                                    INIT
      DYNP(J)=Y(J+1)-Y(J)                                                INIT
  102 DYPS(J+1)=DYNP(J)                                                  INIT
      SEW(1)=0.0                                                         INIT
      SEW(NI)=0.0                                                        INIT
```

```
      DO 103 I=2,NIM1                                                   INIT
  103 SEW(I)=0.5*(DXEP(I)+DXPW(I))                                      INIT
      SNS(1)=0.0                                                        INIT
      SNS(NJ)=0.0                                                       INIT
      DO 104 J=2,NJM1                                                   INIT
  104 SNS(J)=0.5*(DYNP(J)+DYPS(J))                                      INIT
      XU(1)=0.0
      DO 105 I=2,NI                                                     INIT
  105 XU(I)=0.5*(X(I)+X(I-1))                                           INIT
      DXPWU(1)=0.0                                                      INIT
      DXPWU(2)=0.0                                                      INIT
      DXEPU(1)=0.0                                                      INIT
      DXEPU(NI)=0.0                                                     INIT
      DO 106 I=2,NIM1                                                   INIT
      DXEPU(I)=XU(I+1)-XU(I)                                            INIT
  106 DXPWU(I+1)=DXEPU(I)                                               INIT
      SEWU(1)=0.0                                                       INIT
      SEWU(2)=0.0                                                       INIT
      DO 107 I=3,NIM1                                                   INIT
  107 SEWU(I)=0.5*(DXEPU(I)+DXPWU(I))                                   INIT
      YV(1)=0.0                                                         INIT
      RV(1)=0.0                                                         INIT
      DO 108 J=2,NJ                                                     INIT
      RV(J)=0.5*(R(J)+R(J-1))                                           INIT
      RCV(J)=0.5*(RV(J)+RV(J-1))                                        INIT
  108 YV(J)=0.5*(Y(J)+Y(J-1))                                           INIT
      DYPSV(1)=0.0                                                      INIT
      DYPSV(2)=0.0                                                      INIT
      DYNPV(NJ)=0.0                                                     MODA
      DO 109 J=2,NJM1                                                   9ER&T
      DYNPV(J)=YV(J+1)-YV(J)                                            INIT
  109 DYPSV(J+1)=DYNPV(J)                                               INIT
      SNSV(1)=0.0                                                       INIT
      SNSV(2)=0.0                                                       INIT
      SNSV(NJ)=0.0                                                      INIT
      DO 110 J=3,NJM1                                                   INIT
  110 SNSV(J)=0.5*(DYNPV(J)+DYPSV(J))                                   INIT
```

```
      C                                                                  ----------
      CHAPTER  2  2  2  2  2  2  SET VARIABLES TO ZERO  2  2  2  2  2  2      INIT
      C                                                                      INIT
  302       DO 200 I=1,NI                                                    INIT
  303       DO 200 J=1,NJ                                                    INIT
  304       U(I,J)=0.0                                                       INIT
  305       V(I,J)=0.0                                                       INIT
  306       P(I,J)=0.0                                                       INIT
  307       PP(I,J)=0.0                                                      INIT
  308       TE(I,J)=0.0
  309       ED(I,J)=0.0
  310       QE(I,J)=0.0
  311       DEN(I,J)=DENSIT                                                  MODA
  312       VIS(I,J)=VISCOS                                                  MODA
  313       GAMH(I,J)=VISCOS/PRANDL
  314       GAMQ(I,J)=VISCOS/PRANDL
  315       DU(I,J)=0.0                                                      INIT
  316       DV(I,J)=0.0                                                      INIT
  317       SU(I,J)=0.0                                                      INIT
  318       SP(I,J)=0.0                                                      INIT
  319   200 CONTINUE
  320       RETURN                                                           INIT
  321       END                                                              INIT
```

```
      SUBROUTINE PROPS                                                  PROPS
C                                                                       PROPS
CHAPTER  0  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0  0  PROPS
C                                                                       PROPS
      COMMON                                                            PROPS
     1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
     2        VISD(30,30)
     1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)        0420
     1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
     1/TURB/GEN(30,30),CD,CMU,C1,C2,CAPPA,ELOG,PRED,PRTE                   0490
     1/TEMP/T(30,30),GAMH(30,30),RESORT,NSWPT,URFT,PRANDL,PFUN,
     2     TWALLN,TIN,GAMQ(30,30),TBOTOM
C                                                                       PROPS
CHAPTER   1   1   1   VISCOSITY   1   1   1
C                                                                       PROPS
      PRQE=0.9
      DO 100 I=2,NIM1
      VIS(I,1)=VIS(I,2)
      DO 100 J=2,NJM1
      VISOLD=VIS(I,J)                                                   PROPS
      IF(ED(I,J).EQ.0.) GO TO 102                                       PROPS
      VIS(I,J)=DEN(I,J)*TE(I,J)**2*CMU/ED(I,J)+VISCOS
      GO  TO  101                                                       PROPS
  102 VIS(I,J)=VISCOS                                                   PROPS
  101 VIS(I,J)=URFVIS*VIS(I,J)+(1.-URFVIS)*VISOLD                       PROPS
      GAMH(I,J)=VISCOS/PRANDL+(VIS(I,J)-VISCOS)/PRANDT
      GAMQ(I,J)=VISCOS/PRANDL+(VIS(I,J)-VISCOS)/PRQE
C-----UNDER-RELAX VISCOSITY                                             PROPS
  100 CONTINUE
      RETURN                                                            PROPS
      END                                                               PROPS
```

```
      SUBROUTINE CALCU                                                  CALCU
C                                                                       CALCU
CHAPTER  0  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0  0  CALCU
C                                                                       CALCU
      COMMON                                                            CALCU
     1/UVEL/RESORU,NSWPU,URFU,DXEPU(32),DXPWU(32),SEWU(32)              CALCU
     1/PCOR/RESORM,NSWPP,URFP,DU(30,30),DV(30,30),IPREF,JPREF             0390
     1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)       0420
     1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
      COMMON
     1/GEOM/INDCOS,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),     CALCU
     1      SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)                  CALCU
     1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
     2       VISD(30,30)
     1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),   0510
     1      SP(30,30)
     1/PROB1/UIN,TEIN,EDIN,FLOWIN,ALAMDA,
     2       RSMALL,RLARGE,AL1,AL2,JSTEP,ISTEP,JSTP1,JSTM1,ISTP1,ISTM1,  MODA
     3    NITER,US
C                                                                       CALCU
CHAPTER  1  1  1  1  1  1  ASSEMBLY OF COEFFICIENTS  1  1  1  1  1  1  1 CALCU
C                                                                       CALCU
      DO 100 I=3,NIM1                                                   CALCU
      DO 101 J=2,NJM1                                                   CALCU
C-----COMPUTE AREAS AND VOLUME                                          CALCU
      U(I,1)=US
      AREAN=RV(J+1)*SEWU(I)                                             CALCU
      AREAS=RV(J)*SEWU(I)                                               CALCU
      AREAEW=R(J)*SNS(J)                                                CALCU
      VOL=R(J)*SEWU(I)*SNS(J)                                           CALCU
C-----CALCULATE CONVECTION COEFFICIENTS                                 CALCU
      GN=0.5*(DEN(I,J+1)+DEN(I,J))*V(I,J+1)                             CALCU
      GS=0.5*(DEN(I,J-1)+DEN(I,J))*V(I,J)                               CALCU
      GNW=0.5*(DEN(I-1,J)+DEN(I-1,J+1))*V(I-1,J+1)                      CALCU
      GSW=0.5*(DEN(I-1,J)+DEN(I-1,J-1))*V(I-1,J)                        CALCU
      GE=0.5*(DEN(I+1,J)+DEN(I,J))*U(I+1,J)                             CALCU
      GP=0.5*(DEN(I,J)+DEN(I-1,J))*U(I,J)                               CALCU
      GW=0.5*(DEN(I-1,J)+DEN(I-2,J))*U(I-1,J)                           CALCU
      CN=0.5*(GN+GNW)*AREAN                                             CALCU
      CS=0.5*(GS+GSW)*AREAS                                             CALCU
      CE=0.5*(GE+GP)*AREAEW                                             CALCU
      CW=0.5*(GP+GW)*AREAEW                                             CALCU
```

```
      C-----CALCULATE DIFFUSION COEFFICIENTS                              CALCU
  360       VISN=0.25*(VIS(I,J)+VIS(I,J+1)+VIS(I-1,J)+VIS(I-1,J+1))       CALCU
  361       VISS=0.25*(VIS(I,J)+VIS(I,J-1)+VIS(I-1,J)+VIS(I-1,J-1))       CALCU
  362       DN=VISN*AREAN/DYNP(J)                                         CALCU
  363       DS=VISS*AREAS/DYPS(J)                                         CALCU
  364       DE=VIS(I,J)*AREAEW/DXEPU(I)                                   CALCU
  365       DW=VIS(I-1,J)*AREAEW/DXPWU(I)                                 CALCU
      C-----CALCULATE COEFFICIENTS OF SOURCE TERMS                        CALCU
  366       SMP=CN-CS+CE-CW                                               CALCU
  367       CP=AMAX1(0.0,SMP)                                             CALCU
  368       CPO=CP                                                        CALCU
      C-----ASSEMBLE MAIN COEFFICIENTS                                    CALCU
  369       AN(I,J)=AMAX1(ABS(0.5*CN),DN)-0.5*CN                          CALCU
  370       AS(I,J)=AMAX1(ABS(0.5*CS),DS)+0.5*CS                          CALCU
  371       AE(I,J)=AMAX1(ABS(0.5*CE),DE)-0.5*CE                          CALCU
  372       AW(I,J)=AMAX1(ABS(0.5*CW),DW)+0.5*CW                          CALCU
  373       DU(I,J)=AREAEW                                                CALCU
  374       SU(I,J)=CPO*U(I,J)+DU(I,J)*(P(I-1,J)-P(I,J))                  CALCU
  375       SP(I,J)=-CP                                                   CALCU
  376       DUDXP  =(U(I+1,J)-U(I,J))/SEW(I)
  377       DUDXM  =(U(I,J)-U(I-1,J))/SEW(I-1)
  378       SU(I,J) =(VIS(I,J)*DUDXP-VIS(I-1,J)*DUDXM)/SEWU(I)*VOL+SU(I,J)
  379       GAMP   =0.25*(VIS(I,J)+VIS(I-1,J)+VIS(I,J+1)+VIS(I-1,J+1))
  380       DVDXP  =RV(J+1)*(V(I,J+1)-V(I-1,J+1))/DXEP(I)
  381       GAMM   =0.25*(VIS(I,J)+VIS(I-1,J)+VIS(I,J-1)+VIS(I-1,J-1))
  382       DVDXM  =RV(J)*(V(I,J)-V(I-1,J))/DXEP(I)
  383       SU(I,J) =SU(I,J)+(GAMP*DVDXP-GAMM*DVDXM)/SNS(J)/R(J)*VOL
  384     101 CONTINUE                                                    CALCU
  385     100 CONTINUE                                                    CALCU
      C                                                                   CALCU
      CHAPTER  2  2  2  2  2  2  2  PROBLEM MODIFICATIONS  2  2  2  2  2  2  2 CALCU
      C                                                                   CALCU
  386       CALL MODU                                                     CALCU
      C                                                                   CALCU
```

```
      CHAPTER  3  FINAL COEFF. ASSEMBLY AND RESIDUAL SOURCE CALCULATION  3  3  CALCU
      C             3                                                          CALCU
      C                                                                        CALCU
387         RESORU=0.0                                                       CALCU
388         DO 300 I=3,NIM1                                                  CALCU
389         DO 301 J=2,NJM1                                                  CALCU
390         AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)                  CALCU
391         DU(I,J)=DU(I,J)/AP(I,J)                                          CALCU
392         RESOR=AN(I,J)*U(I,J+1)+AS(I,J)*U(I,J-1)+AE(I,J)*U(I+1,J)         CALCU
           1     +AW(I,J)*U(I-1,J)-AP(I,J)*U(I,J)+SU(I,J)
393         VOL=R(J)*SEW(I)*SNS(J)
394         SORVOL=GREAT*VOL
395         IF(-SP(I,J).GT.0.5*SORVOL)  RESOR=RESOR/SORVOL                   CALCU
396         RESORU=RESORU+ABS(RESOR)                                         CALCU
      C-----UNDER-RELAXATION                                                 CALCU
397         AP(I,J)=AP(I,J)/URFU                                             CALCU
398         SU(I,J)=SU(I,J)+(1.-URFU)*AP(I,J)*U(I,J)                         CALCU
399         DU(I,J)=DU(I,J)*URFU                                             CALCU
400       301 CONTINUE                                                       CALCU
401       300 CONTINUE                                                       CALCU
      C                                                                      CALCU
      CHAPTER  4  4  4  SOLUTION OF DIFFERENCE EQUATION  4  4  4  4  4  4  4  CALCU
      C                                                                      CALCU
      C                                                                      CALCU
402         DO 400 N=1,NSWPU                                                 CALCU
403       400 CALL LISOLV(3,2,NI,NJ,IT,JT,U)                                 CALCU
404         RETURN                                                           CALCU
405         END
```

```
406         SUBROUTINE CALCV                                                    CALCV
      C                                                                         CALCV
      CHAPTER  0  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0  0   CALCV
      C                                                                         CALCV
407         COMMON                                                            CALCV
           1/VVEL/RESORV,NSWPV,URFV,DYNPV(32),DYPSV(32),SNSV(32),RCV(32)      CALCV
           1/PCOR/RESORM,NSWPP,URFP,DU(30,30),DV(30,30),IPREF,JPREF              0390
           1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)        0420
           1/GEOM/INDCOM,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),    CALCV
           1      SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)                 CALCV
           1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
           1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
           2       VISD(30,30)
408         COMMON
           1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),    0510
           1      SP(30,30)
           1/PROB1/UIN,TEIN,EDIN,FLOWIN,ALAMDA,
           2       RSMALL,RLARGE,AL1,AL2,JSTEP,ISTEP,JSTP1,JSTM1,ISTP1,ISTM1,  MODA
           3    NITER,US
           1/TEMP/T(30,30),GAMH(30,30),RESORT,NSWPT,URFT,PRANDL,PFUN,
           2     TWALLN,TIN,GAMQ(30,30),TBOTOM
           1/MSTF/QE(30,30),PRQE,CQ1,CQ2,RESQE,URFQ,BETA,GRAVTY,C3,C4,NSWPQ,
           1      SUQE(30,30),GENQ(30,30),SPQE(30,30),RESORQ
      C                                                                         CALCV
      CHAPTER  1  1  1  1  1  1  ASSEMBLY OF COEFFICIENTS  1  1  1  1  1  1  1 CALCV
      C                                                                         CALCV
409         DO 100 I=2,NIM1                                                   CALCV
410         DO 101 J=3,NJM1                                                   CALCV
      C-----COMPUTE AREAS AND VOLUME                                          CALCV
411         AREAN=RCV(J+1)*SEW(I)                                             CALCV
412         AREAS=RCV(J)*SEW(I)                                               CALCV
413         AREAEW=RV(J)*SNSV(J)                                              CALCV
414         VOL=RV(J)*SEW(I)*SNSV(J)                                          CALCV
      C-----CALCULATE CONVECTION COEFFICIENTS                                 CALCV
415         GN=0.5*(DEN(I,J+1)+DEN(I,J))*V(I,J+1)                             CALCV
416         GP=0.5*(DEN(I,J)+DEN(I,J-1))*V(I,J)                               CALCV
417         GS=0.5*(DEN(I,J-1)+DEN(I,J-2))*V(I,J-1)                           CALCV
418         GE=0.5*(DEN(I+1,J)+DEN(I,J))*U(I+1,J)                             CALCV
419         GSE=0.5*(DEN(I,J-1)+DEN(I+1,J-1))*U(I+1,J-1)                      CALCV
420         GW=0.5*(DEN(I,J)+DEN(I-1,J))*U(I,J)                               CALCV
421         GSW=0.5*(DEN(I,J-1)+DEN(I-1,J-1))*U(I,J-1)
```

```
      CN=0.5*(GN+GP)*AREAN                                                  CALCV
      CS=0.5*(GP+GS)*AREAS                                                  CALCV
      CE=0.5*(GE+GSE)*AREAEW                                                CALCV
      CW=0.5*(GW+GSW)*AREAEW                                                CALCV
C-----CALCULATE DIFFUSION COEFFICIENTS                                      CALCV
      VISE=0.25*(VIS(I,J)+VIS(I+1,J)+VIS(I,J-1)+VIS(I+1,J-1))               CALCV
      VISW=0.25*(VIS(I,J)+VIS(I-1,J)+VIS(I,J-1)+VIS(I-1,J-1))               CALCV
      DN=VIS(I,J)*AREAN/DYNPV(J)                                            CALCV
      DS=VIS(I,J-1)*AREAS/DYPSV(J)                                          CALCV
      DE=VISE*AREAEW/DXEP(I)                                                CALCV
      DW=VISW*AREAEW/DXPW(I)                                                CALCV
C-----CALCULATE COEFFICIENTS OF SOURCE TERMS                                CALCV
      SMP=CN-CS+CE-CW                                                       CALCV
      CP=AMAX1(0.0,SMP)                                                     CALCV
      CPO=CP                                                                CALCV
C-----ASSEMBLE MAIN COEFFICIENTS                                            CALCV
      AN(I,J)=AMAX1(ABS(0.5*CN),DN)-0.5*CN                                  CALCV
      AE(I,J)=AMAX1(ABS(0.5*CE),DE)-0.5*CE                                  CALCV
      AS(I,J)=AMAX1(ABS(0.5*CS),DS)+0.5*CS                                  CALCV
      AW(I,J)=AMAX1(ABS(0.5*CW),DW)+0.5*CW                                  CALCV
      DV(I,J)=0.5*(AREAN+AREAS)                                             CALCV
      SU(I,J)=CPO*V(I,J)+DV(I,J)*(P(I,J-1)-P(I,J))                          CALCV
      SP(I,J)=-CP                                                           CALCV
      IF(INDCOS.EQ.2) SP(I,J)=SP(I,J)-2.*VIS(I,J)*VOL/RV(J)**2
      DUDYP  =(U(I+1,J)-U(I+1,J-1))/DYPS(J)
      GAMP   =0.25*(VIS(I,J)+VIS(I+1,J)+VIS(I,J-1)+VIS(I+1,J-1))
      GAMN   =0.25*(VIS(I,J)+VIS(I-1,J)+VIS(I,J-1)+VIS(I-1,J-1))
      DUDYM  =(U(I,J)-U(I,J-1))/DYPS(J)
      SU(I,J) =SU(I,J)+(GAMP*DUDYP-GAMM*DUDYM)/SEW(I)*VOL
      DVDYP  =(V(I,J+1)-V(I,J))/SNS(J)
      RGAMP  =VIS(I,J)*R(J)
      DVDYM  =(V(I,J)-V(I,J-1))/SNS(J-1)
      RGAMM  =VIS(I,J-1)*R(J-1)
      SU(I,J) =SU(I,J)+(RGAMP*DVDYP-RGAMM*DVDYM)/(R(J)*SNS(J))*VOL
      IF(NITER.LE.150) GO TO 101
      DENU=(DEN(I,J)+DEN(I,J-1))/2.
      TMEAN=0.5*(TWALLN+TBOTOM)
      SU(I,J)=SU(I,J)+DENU*GRAVTY*BETA*(T(I,J+1)-T(I,J))*VOL
  101 CONTINUE                                                              CALCV
  100 CONTINUE                                                              CALCV
```

```
      C                                                                 CALCV
      CHARTER  2  2  2  2  2  2  2  PROBLEM MODIFICATIONS  2  2  2  2  2  2  2 CALCV
      C                                                                 CALCV
  459 *     CALL MODV                                                   CALCV
      C                                                                 CALCV
      CHAPTER  3  FINAL COEFF. ASSEMBLY AND RESIDUAL SOURCE CALCULATION  3  3  CALCV
      C                                                                 CALCV
  460       RESORV=0.0                                                  CALCV
  461       DO 300 I=2,NIM1                                             CALCV
  462       DO 301 J=3,NJM1                                             CALCV
  463       AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)             CALCV
  464       RESOR=AN(I,J)*V(I,J+1)+AS(I,J)*V(I,J-1)+AE(I,J)*V(I+1,J)    CALCV
           1     +AW(I,J)*V(I-1,J)-AP(I,J)*V(I,J)+SU(I,J)               CALCV
  465       DV(I,J)=DV(I,J)/AP(I,J)                                     CALCV
  466       VOL=R(J)*SEW(I)*SNS(J)
  467       SORVOL=GREAT*VOL
  468       IF(-SP(I,J).GT.0.5*SORVOL)  RESOR=RESOR/SORVOL
  469       RESORV=RESORV+ABS(RESOR)                                    CALCV
      C-----UNDER-RELAXATION                                            CALCV
  470       AP(I,J)=AP(I,J)/URFV                                        CALCV
  471       SU(I,J)=SU(I,J)+(1.-URFV)*AP(I,J)*V(I,J)                    CALCV
  472       DV(I,J)=DV(I,J)*URFV                                        CALCV
  473     301 CONTINUE                                                  CALCV
  474     300 CONTINUE                                                  CALCV
      C                                                                 CALCV
      CHAPTER  4  4  4  SOLUTION OF DIFFERENCE EQUATION  4  4  4  4  4  4  4  CALCV
      C                                                                 CALCV
  475       DO 400 N=1,NSWPV                                            CALCV
  476     400 CALL LISOLV(2,3,NI,NJ,IT,JT,V)                            CALCV
  477         RETURN                                                    CALCV
  478         END                                                       CALCV
```

```
      SUBROUTINE CALCP                                                  CALCP
C                                                                       CALCP
CHAPTER  0  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0  0  CALCP
C                                                                       CALCP
      COMMON                                                            CALCP
     1/PCOR/RESORM,NSWPP,URFP,DU(30,30),DV(30,30),IPREF,JPREF            0390
     1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)      0420
     1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
      COMMON
     1/GEOM/INDCOS,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),     CALCP
     1      SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)                  CALCP
     1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
     2       VISD(30,30)
     1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),  0510
     1      SP(30,30)
      RESORM=0.0                                                        CALCP
C                                                                       CALCP
CHAPTER  1  1  1  1  1  1  ASSEMBLY OF COEFFICIENTS  1  1  1  1  1  1  1 CALCP
C                                                                       CALCP
      DO 100 I=2,NIM1                                                   CALCP
      DO 101 J=2,NJM1                                                   CALCP
C-----COMPUTE AREAS AND VOLUME                                          CALCP
      AREAN=RV(J+1)*SEW(I)                                              CALCP
      AREAS=RV(J)*SEW(I)                                                CALCP
      AREAEW=R(J)*SNS(J)                                                CALCP
      VOL=R(J)*SNS(J)*SEW(I)                                            CALCP
C-----CALCULATE COEFFICIENTS                                            CALCP
      DENN=0.5*(DEN(I,J)+DEN(I,J+1))                                    CALCP
      DENS=0.5*(DEN(I,J)+DEN(I,J-1))                                    CALCP
      DENE=0.5*(DEN(I,J)+DEN(I+1,J))                                    CALCP
      DENW=0.5*(DEN(I,J)+DEN(I-1,J))                                    CALCP
      AN(I,J)=DENN*AREAN*DV(I,J+1)                                      CALCP
      AS(I,J)=DENS*AREAS*DV(I,J)                                        CALCP
      AE(I,J)=DENE*AREAEW*DU(I+1,J)                                     CALCP
      AW(I,J)=DENW*AREAEW*DU(I,J)                                       CALCP
C-----CALCULATE SOURCE TERMS                                            CALCP
      CN=DENN*V(I,J+1)*AREAN                                            CALCP
      CS=DENS*V(I,J)*AREAS                                              CALCP
      CE=DENE*U(I+1,J)*AREAEW                                           CALCP
      CW=DENW*U(I,J)*AREAEW                                             CALCP
      SMP=CN-CS+CE-CW                                                   CALCP
      SP(I,J)=0.0                                                       CALCP
      SU(I,J)=-SMP                                                      CALCP
```

```
      C-----COMPUTE SUM OF ABSOLUTE MASS SOURCES                          CALCP
  504       RESORM=RESORM+ABS(SMP)                                        CALCP
  505   101 CONTINUE                                                      CALCP
  506   100 CONTINUE                                                      CALCP
      C                                                                   CALCP
      CHAPTER  2  2  2  2  2  2  2  PROBLEM MODIFICATIONS  2  2  2  2  2  2  2 CALCP
      C                                                                   CALCP
  507       CALL MODP                                                     CALCP
      C                                                                   CALCP
      CHAPTER  3  3  3  3  3  FINAL COEFFICIENT ASSEMBLY  3  3  3  3  3  3  3  CALCP
      C                                                                   CALCP
  508       DO 300 I=2,NIM1                                               CALCP
  509       DO 301 J=2,NJM1                                               CALCP
  510   301 AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)                CALCP
  511   300 CONTINUE                                                      CALCP
      C                                                                   CALCP
      CHAPTER  4  4  4  4  4  SOLUTION OF DIFFERENCE EQUATIONS  4  4  4  4  4  CALCP
      C                                                                   CALCP
      C                                                                   CALCP
      CHAPTER  5  5  5  5  CORRECT VELOCITIES AND PRESSURE  5  5  5  5  5  5  3+CLG-
  512       DO 400 N=1,NSWPP                                              CALCP
  513   400 CALL LISOLV(2,2,NI,NJ,IT,JT,PP)                               CALCP
      C                                                                   CALCP
      C-----VELOCITIES                                                    CALCP
  514       DO 500 I=2,NIM1                                               CALCP
  515       DO 501 J=2,NJM1                                               CALCP
  516       IF(I.NE.2) U(I,J)=U(I,J)+DU(I,J)*(PP(I-1,J)-PP(I,J))          CALCP
  517       IF(J.NE.2) V(I,J)=V(I,J)+DV(I,J)*(PP(I,J-1)-PP(I,J))          CALCP
  518   501 CONTINUE                                                      CALCP
  519   500 CONTINUE                                                      CALCP
      C-----PRESSURES (WITH PROVISION FOR UNDER-RELAXATION)               CALCP
  520       PPREF=PP(IPREF,JPREF)                                         CALCP
  521       DO 502 I=2,NIM1                                               CALCP
  522       DO 503 J=2,NJM1                                               CALCP
  523       P(I,J)=P(I,J)+URFP*(PP(I,J)-PPREF)                            CALCP
  524       PP(I,J)=0.0                                                   CALCP
  525   503 CONTINUE                                                      CALCP
  526   502 CONTINUE                                                      CALCP
  527       RETURN                                                        CALCP
  528       END                                                           CALCP
```

```
      SUBROUTINE CALCT
      COMMON
     1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)          0420
     1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
      COMMON
     1/GEOM/INDCOS,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),
     1      SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)                       0450
     1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
     2        VISD(30,30)
     1/PROB1/UIN,TEIN,EDIN,FLOWIN,ALAMDA,
     2       RSMALL,RLARGE,AL1,AL2,JSTEP,ISTEP,JSTP1,JSTM1,ISTP1,ISTM1,
     3    NITER,US
     1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),     0510
     1      SP(30,30)
     1/TEMP/T(30,30),GAMH(30,30),RESORT,NSWPT,URFT,PRANDL,PFUN,
     2     TWALLN,TIN,GAMQ(30,30),TBOTOM
      LOGICAL INCT
C
CHAPTER  1  1  1  1  1  1  ASSEMBLY OF COEFFICIENTS  1  1  1  1  1  1
C
      DO 100 I=2,NIM1
      DO 101 J=2, NJM1
C_____COMPUTE AREAS VOLUME
      AREAN=RV(J+1)*SEW(I)
      AREAS=RV(J)*SEW(I)
      AREAEW=R(J)*SNS(J)
      VOL=R(J)*SNS(J)*SEW(I)
C     CALCULATE CONVECTION COEFFICIENTS
      GN=0.5*(DEN(I,J)+DEN(I,J+1))*V(I,J+1)
      GS=0.5*(DEN(I,J)+DEN(I,J-1))*V(I,J)
      GE=0.5*(DEN(I,J)+DEN(I+1,J))*U(I+1,J)
      GW=0.5*(DEN(I,J)+DEN(I-1,J))*U(I,J)
      CN=GN*AREAN
      CS=GS*AREAS
      CE=GE*AREAEW
      CW=GW*AREAEW
C     CALCULATE DIFFUSION COEFFICIENTS
      GAMN=0.5*(GAMH(I,J)+GAMH(I,J+1))
      GAMS=0.5*(GAMH(I,J)+GAMH(I,J-1))
      GAME=0.5*(GAMH(I,J)+GAMH(I+1,J))
      GAMW=0.5*(GAMH(I,J)+GAMH(I-1,J))
      DN=GAMN*AREAN/DYNP(J)
      DS=GAMS*AREAS/DYPS(J)
```

```
      DE=GAME*AREAEW/DXEP(I)
      DW=GAMW*AREAEW/DXPW(I)
C     SOURCE TERMS
      SMP=CN-CS+CE-CW
      CP=AMAX1(0.0,SMP)
      CPO=CP
C     ASSEMBLE MAIN COEFFICIENTS
      AN(I,J)=AMAX1(ABS(0.5*CN),DN)-0.5*CN
      AS(I,J)=AMAX1(ABS(0.5*CS),DS)+0.5*CS
      AE(I,J)=AMAX1(ABS(0.5*CE),DE)-0.5*CE
      AW(I,J)=AMAX1(ABS(0.5*CW),DW)+0.5*CW
      SU(I,J)=CPO*T(I,J)
      SP(I,J)=-CP
  101 CONTINUE
  100 CONTINUE
      INCT=.TRUE.
C
C CHAPTER  2  2  2  2  2  2  PROBLEM MODIFICATIONS  2  2  2  2  2  2
C
      CALL MODT
C CHAPTER 3 FINAL COEFFICIENT ASSEMBLY AND RESIDUAL SOURCE CALCULATION 3
C
      RESORT=0.0
      DO 300 I=2,NIM1
      DO 301 J=2,NJM1
      AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)
      RESOR=AN(I,J)*T(I,J+1)+AS(I,J)*T(I,J-1)+AE(I,J)*T(I+1,J)
     1      +AW(I,J)*T(I-1,J)-AP(I,J)*T(I,J)+SU(I,J)
      VOL=R(J)*SEW(I)*SNS(J)
      SORVOL=GREAT*VOL
      IF(-SP(I,J).GT.0.5*SORVOL) RESOR=RESOR/SORVOL
      RESORT=RESORT+ABS(RESOR)
C     UNDER-RELAXATION
      AP(I,J)=AP(I,J)/URFT
      SU(I,J)=SU(I,J)+(1.0-URFT)*AP(I,J)*T(I,J)
  301 CONTINUE
      INCT=.FALSE.
  300 CONTINUE
C
C CHAPTER  4  4  4  4  4  4  SOLUTION OF DIFFERENCE EQUATIONS  4  4  4  4  4
C
      DO 400 N=1,NSWPT
  400 CALL LISOLV(2,2,NI,NJ,IT,JT,T)
      RETURN
      END
```

```
586         SUBROUTINE CALCTE                                                  KINE
            C                                                                  KINE
            CHAPTER  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0   KINE
            C                                                                  KINE
587          COMMON                                                            KINE
            1/TEN/RESORK,NSWPK,URFK                                            KINE
            1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)       0420
            1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
588          COMMON
            1/GEOM/INDCOS,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),     KINE
            1      SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)                  KINE
            1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
            2        VISD(30,30)
            1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),    0510
            1      SP(30,30)
            1/TURB/GEN(30,30),CD,CMU,C1,C2,CAPPA,ELOG,PRED,PRTE                    0490
            1/WALLF/YPLUSN(30,30),XPLUSW(30,30),TAUN(30,30),TAUW(30,30)
            1/PROB1/UIN,TEIN,EDIN,FLOWIN,ALAMDA,
            2       RSMALL,RLARGE,AL1,AL2,JSTEP,ISTEP,JSTP1,JSTM1,ISTP1,ISTM1,  MODA
            3    NITER,US
            1/SUSP/SUKD(30,30),SPKD(30,30)
            1/MSTF/QE(30,30),PRQE,CQ1,CQ2,RESQE,URFQ,BETA,GRAVTY,C3,C4,NSWPQ,
            1      SUQE(30,30),GENQ(30,30),SPQE(30,30),RESORQ
            C
            CHAPTER  1  1  1  1  1  1  ASSEMBLY OF COEFFICIENTS  1  1  1  1  1  1   KINE
            C                                                                  KINE
589         PRTE=1.0                                                           KINE
590         DO 100 I=2,NIM1                                                    KINE
591         DO 101 J=2,NJM1                                                    KINE
            C-----COMPUTE AREAS AND VOLUME                                     KINE
592         U(I,1)=US                                                          KINE
593         AREAN=RV(J+1)*SEW(I)                                               KINE
594         AREAS=RV(J)*SEW(I)                                                 KINE
595         AREAEW=R(J)*SNS(J)                                                 KINE
596         VOL=R(J)*SNS(J)*SEW(I)                                             KINE
            C-----CALCULATE CONVECTION COEFFICIENTS                            KINE
597         GN=0.5*(DEN(I,J)+DEN(I,J+1))*V(I,J+1)                              KINE
598         GS=0.5*(DEN(I,J)+DEN(I,J-1))*V(I,J)                                KINE
599         GE=0.5*(DEN(I,J)+DEN(I+1,J))*U(I+1,J)                              KINE
600         GW=0.5*(DEN(I,J)+DEN(I-1,J))*U(I,J)                                KINE
601         CN=GN*AREAN                                                        KINE
602         CS=GS*AREAS                                                        KINE
```

```
603         CE=GE*AREAEW                                                    KINE
604         CW=GW*AREAEW                                                    KINE
      C-----CALCULATE DIFFUSION COEFFICIENTS                                KINE
605         GAMN=0.5*(VIS(I,J)+VIS(I,J+1))/PRTE                             KINE
606         GAMS=0.5*(VIS(I,J)+VIS(I,J-1))/PRTE                             KINE
607         GAME=0.5*(VIS(I,J)+VIS(I+1,J))/PRTE                             KINE
608         GAMW=0.5*(VIS(I,J)+VIS(I-1,J))/PRTE                             KINE
609         DN=GAMN*AREAN/DYNP(J)                                           KINE
610         DS=GAMS*AREAS/DYPS(J)                                           KINE
611         DE=GAME*AREAEW/DXEP(I)                                          KINE
612          DW=GAMW*AREAEW/DXPW(I)                                         KINE
      C-----SOURCE TERMS                                                    KINE
613         SMP=CN-CS+CE-CW                                                 KINE
614         CP=AMAX1(0.0,SMP)                                               KINE
615         CPO=CP                                                          KINE
616         DUDX=(U(I+1,J)-U(I,J))/SEW(I)                                   KINE
617         DVDY=(V(I,J+1)-V(I,J))/SNS(J)                                   KINE
618         DUDY=((U(I,J)+U(I+1,J)+U(I,J+1)+U(I+1,J+1))/4.-(U(I,J)+U(I+1,J)+  MODA
           1U(I,J-1)+U(I+1,J-1))/4.)/SNS(J)                                 KINE
619         DVDX=((V(I,J)+V(I,J+1)+V(I+1,J)+V(I+1,J+1))/4.-(V(I,J)+V(I,J+1)+V(  MODA
           1I-1,J)+V(I-1,J+1))/4.)/SEW(I)                                   KINE
620         GEN(I,J)=(2.*(DUDX**2+DVDY**2)+(DUDY+DVDX)**2)*VIS(I,J)
621         VDR=V(I,J)/(RV(J)+1.E-20)
622         IF(J.EQ.2)  VDR=0.0                                             MODA
623         IF(INDCOS.EQ.2) GEN(I,J)=GEN(I,J)+VIS(I,J)*0.5*(VDR+V(I,J+1)/
           1                RV(J+1))**2
      C-----ASSEMBLE MAIN COEFFICIENTS                                      KINE
624         AN(I,J)=AMAX1(ABS(0.5*CN),DN)-0.5*CN                            KINE
625         AS(I,J)=AMAX1(ABS(0.5*CS),DS)+0.5*CS                            KINE
626         AE(I,J)=AMAX1(ABS(0.5*CE),DE)-0.5*CE                            KINE
627         AW(I,J)=AMAX1(ABS(0.5*CW),DW)+0.5*CW                            KINE
628         SU(I,J)=CPO*TE(I,J)                                             KINE
629         SUKD(I,J)=SU(I,J)                                               MODA
630         SU(I,J)=SU(I,J)+GEN(I,J)*VOL                                    KINE
631         SP(I,J)=-CP                                                     KINE
632         SPKD(I,J)=SP(I,J)                                               MODA
633         SP(I,J)=SP(I,J)-CD*CMU*DEN(I,J)**2*TE(I,J)*VOL/VIS(I,J)         KINE
634         IF(NITER.LE.150) GO TO 101
635         QEPTE=QE(I,J)/TE(I,J)
636         SP(I,J)=SP(I,J)+DEN(I,J)*GRAVTY*BETA*C4*VOL*SQRT(QEPTE)
637       101 CONTINUE                                                      KINE
638       100 CONTINUE                                                      KINE
```

```
C                                                                       KINE
CHAPTER  2  2  2  2  2  2  PROBLEM MODIFICATIONS  2  2  2  2  2  2      KINE
C                                                                       KINE
      CALL MODTE                                                        KINE
C                                                                       KINE
CHAPTER 3 FINAL COEFFICIENT ASSEMBLY AND RESIDUAL-SOURCE CALCULATION 3--KINE
C                                                                       KINE
      RESORK=0.0                                                        KINE
      DO 300 I=2,NIM1                                                   KINE
      DO 301 J=2,NJM1                                                   KINE
      AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)                   KINE
      RESOR=AN(I,J)*TE(I,J+1)+AS(I,J)*TE(I,J-1)+AE(I,J)*TE(I+1,J)       KINE
     1      +AW(I,J)*TE(I-1,J)-AP(I,J)*TE(I,J)+SU(I,J)                  KINE
      VOL=R(J)*SEW(I)*SNS(J)
      SORVOL=GREAT*VOL
      IF(-SP(I,J).GT.0.5*SORVOL)  RESOR=RESOR/SORVOL
      RESORK=RESORK+ABS(RESOR)                                          MODA
C-----UNDER-RELAXATION                                                  KINE
      AP(I,J)=AP(I,J)/URFK                                              KINE
      SU(I,J)=SU(I,J)+(1.-URFK)*AP(I,J)*TE(I,J)                         MODA
  301 CONTINUE                                                          KINE
  300 CONTINUE                                                          KINE
C                                                                       KINE
CHAPTER  4  4  4  4  4  SOLUTION OF DIFFERENCE EQUATIONS  4  4  4  4  4 KINE
C                                                                       KINE
      DO 400 N=1,NSWPK                                                  MODA
  400 CALL LISOLV(2,2,NI,NJ,IT,JT,TE)                                   KINE
      RETURN                                                            KINE
      END                                                               KINE
```

```
657         SUBROUTINE CALCED                                                   DISP
      C                                                                         DISP
      CHAPTER  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0        DISP
      C                                                                         DISP
658         COMMON
           1/TDIS/RESORE,NSWPD,URFE
           1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
659         COMMON
           1/GEOM/INDCOS,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),       DIS
           1      SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)
           1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
           2       VISD(30,30)
           1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),     0510
           1      SP(30,30)
           1/TURB/GEN(30,30),CD,CMU,C1,C2,CAPPA,ELOG,PRED,PRTE                     0490
           1/WALLF/YPLUSN(30,30),XPLUSW(30,30),TAUN(30,30),TAUW(30,30)
           1/SUSP/SUKD(30,30),SPKD(30,30)
           1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)          0420
           1/PROB1/UIN,TEIN,EDIN,FLOWIN,ALAMDA,
           2       RSMALL,RLARGE,AL1,AL2,JSTEP,ISTEP,JSTP1,JSTM1,ISTP1,ISTM1,   MODA
           3    NITER,US
           1/MSTF/QE(30,30),PRQE,CQ1,CQ2,RESQE,URFQ,BETA,GRAVTY,C3,C4,NSWPQ,
           1       SUQE(30,30),GENQ(30,30),SPQE(30,30),RESORQ
      C                                                                         DISP
      CHAPTER  1  1  1  1  1  1  ASSEMBLY OF COEFFICIENTS  1  1  1  1  1  1    DISP
      C                                                                         DISP
660         DO 100 I=2,NIM1                                                     DISP
661         DO 101 J=2,NJM1                                                     DISP
      C-----COMPUTE AREAS AND VOLUME                                            DISP
662         AREAN=RV(J+1)*SEW(I)                                                DISP
663         AREAS=RV(J)*SEW(I)                                                  DISP
664         AREAEW=R(J)*SNS(J)                                                  DISP
665         VOL=R(J)*SNS(J)*SEW(I)                                              DISP
      C-----CALCULATE CONVECTION COEFFICIENTS                                   DISP
666         GN=0.5*(DEN(I,J)+DEN(I,J+1))*V(I,J+1)                               DISP
667         GS=0.5*(DEN(I,J)+DEN(I,J-1))*V(I,J)                                 DISP
668         GE=0.5*(DEN(I,J)+DEN(I+1,J))*U(I+1,J)                               DISP
669         GW=0.5*(DEN(I,J)+DEN(I-1,J))*U(I,J)                                 DISP
670         CN=GN*AREAN                                                         DISP
671         CS=GS*AREAS                                                         DISP
672         CE=GE*AREAEW                                                        DISP
673         CW=GW*AREAEW                                                        DISP
```

```
      C-----CALCULATE DIFFUSION COEFFICIENTS                              DISP
  674       GAMN=0.5*(VIS(I,J)+VIS(I,J+1))/PRED                           DISP
  675       GAMS=0.5*(VIS(I,J)+VIS(I,J-1))/PRED                           DISP
  676       GAME=0.5*(VIS(I,J)+VIS(I+1,J))/PRED                           DISP
  677       GAMW=0.5*(VIS(I,J)+VIS(I-1,J))/PRED                           DISP
  678       DN=GAMN*AREAN/DYNP(J)                                         DISP
  679       DS=GAMS*AREAS/DYPS(J)                                         DISP
  680       DE=GAME*AREAEW/DXEP(I)                                        DISP
  681        DW=GAMW*AREAEW/DXPW(I)                                       DISP
      C-----SOURCE TERMS                                                  DISP
  682       SMP=CN-CS+CE-CW                                               DISP
  683       CP=AMAX1(0.0,SMP)                                             DISP
  684       CPO=CP                                                        DISP
      C-----ASSEMBLE MAIN COEFFICIENTS                                    DISP
  685       AN(I,J)=AMAX1(ABS(0.5*CN),DN)-0.5*CN                          DISP
  686       AS(I,J)=AMAX1(ABS(0.5*CS),DS)+0.5*CS                          DISP
  687       AE(I,J)=AMAX1(ABS(0.5*CE),DE)-0.5*CE                          DISP
  688       AW(I,J)=AMAX1(ABS(0.5*CW),DW)+0.5*CW                          DISP
  689       SU(I,J)=CPO*ED(I,J)                                           DISP
  690       SUKD(I,J)=SU(I,J)                                             MODA
  691       SU(I,J)=SU(I,J)+C1*CMU*GEN(I,J)*VOL*DEN(I,J)*TE(I,J)/VIS(I,J)
  692       SP(I,J)=-CP
  693       SPKD(I,J)=SP(I,J)                                             MODA
  694       SP(I,J)=SP(I,J)-C2*DEN(I,J)*ED(I,J)*VOL/TE(I,J)               DISP
  695       IF(NITER.LE.150) GO TO 101
  696       QEPTE=QE(I,J)/TE(I,J)
  697       SP(I,J)=SP(I,J)+C3*C4*VOL*DEN(I,J)*GRAVTY*BETA*SQRT(QEPTE)
  698     101 CONTINUE                                                    DISP
  699     100 CONTINUE                                                    DISP
      C                                                                   DISP
      CHAPTER  2  2  2  2  2  2  PROBLEM MODIFICATIONS  2  2  2  2  2  2  DISP
      C                                                                   DISP
  700       CALL MODED                                                    DISP
```

```
C                                                                       DISP
CHAPTER 3 FINAL COEFFICIENT ASSEMBLY AND RESIDUAL SOURCE CALCULATION 3  DISP
                                                                        DISP
701       RESORE=0.0                                                    DISP
C                                                                       DISP
702       DO 300 I=2,NIM1                                               DISP
703       DO 301 J=2,NJM1                                               DISP
704       AP(I,J)=AN(I,J)+AS(I,J)+AE(I,J)+AW(I,J)-SP(I,J)               DISP
705       RESOR=AN(I,J)*ED(I,J+1)+AS(I,J)*ED(I,J-1)+AE(I,J)*ED(I+1,J)   DISP
         1      +AW(I,J)*ED(I-1,J)-AP(I,J)*ED(I,J)+SU(I,J)             DISP
706       VOL=R(J)*SNS(J)*SEW(I)                                        MODA
707       SORVOL=GREAT*VOL
708       IF(-SP(I,J).GT.0.5*SORVOL)  RESOR=RESOR/SORVOL
709       RESORE=RESORE+ABS(RESOR)                                      MODA
C-----UNDER-RELAXATION                                                  DISP
710       AP(I,J)=AP(I,J)/URFE                                          DISP
711       SU(I,J)=SU(I,J)+(1.-URFE)*AP(I,J)*ED(I,J)                     MODA
712     301 CONTINUE                                                    DISP
713     300 CONTINUE                                                    DISP
                                                                        DISP
C                                                                       DISP
CHAPTER  4  4  4  4  4  SOLUTION OF DIFFERENCE EQUATIONS  4  4  4  4  4 DISP
C                                                                       DISP
714       DO 400 N=1,NSWPD
715     400 CALL LISOLV(2,2,NI,NJ,IT,JT,ED)                            DISP
716       RETURN                                                        DISP
717       END                                                           DISP
```

```
      SUBROUTINE CALCQE
C
CHAPTER  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0
C
      COMMON
     1/MSTF/QE(30,30),PRQE,CQ1,CQ2,RESQE,URFQ,BETA,GRAVTY,C3,C4,NSWPQ,
     1    SUQE(30,30),GENQ(30,30),SPQE(30,30),RESORQ
     1/TEMP/T(30,30),GAMH(30,30),RESORT,NSWPT,URFT,PRANDL,PFUN,
     2    TWALLN,TIN,GAMQ(30,30),TBOTOM
     1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)          0420
     1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
      COMMON
     1/GEOM/INDCOS,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),
     1     SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)
     1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
     2       VISD(30,30)
     1/PROB1/UIN,TEIN,EDIN,FLOWIN,ALAMDA,
     2       RSMALL,RLARGE,AL1,AL2,JSTEP,ISTEP,JSTP1,JSTM1,ISTP1,ISTM1,
     3    NITER,US
     1/TURB/GEN(30,30),CD,CMU,C1,C2,CAPPA,ELOG,PRED,PRTE                     0490
     1/WALLF/YPLUSN(30,30),XPLUSW(30,30),TAUN(30,30),TAUW(30,30)
     1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),     0510
     1      SP(30,30)
C
CHAPTER  1  1  1  1  1  1  ASSEMBLY OF COEFFICIENTS  1  1  1  1  1  1
C
      PRQE=0.9
      DO 100 I=2,NIM1
      DO 101 J=2,NJM1
C-----COMPUTE AREAS AND VOLUME
      AREAN=RV(J+1)*SEW(I)
      AREAS=RV(J)*SEW(I)
      AREAEW=R(J)*SNS(J)
      VOL=R(J)*SNS(J)*SEW(I)
C-----CALCULATE CONVECTION COEFFICIENTS
      GN=0.5*(DEN(I,J)+DEN(I,J+1))*V(I,J+1)
      GS=0.5*(DEN(I,J)+DEN(I,J-1))*V(I,J)
      GE=0.5*(DEN(I,J)+DEN(I+1,J))*U(I+1,J)
      GW=0.5*(DEN(I,J)+DEN(I-1,J))*U(I,J)
      CN=GN*AREAN
      CS=GS*AREAS
      CE=GE*AREAEW
      CW=GW*AREAEW
```

```
744         SMP=CN+CS+CE+CW
745         CP=AMAX1(0.0,SMP)
746         CPO=CP
747         CQ1=2.8
748         CQ2=1.7
749         DTDY=((T(I,J+1)+T(I,J))/2.-(T(I,J-1)+T(I,J))/2.)/SNS(J)
750         DTDX=((T(I+1,J)+T(I,J))/2.-(T(I-1,J)+T(I,J))/2.)/SEW(I)
751         IF(J.EQ.NJM1) DTDY=0.
752         GENQ(I,J)=(DTDY**2+DTDX**2)*VIS(I,J)
      C_____ASSEMBLE MAIN COEFFICIENTS
753         AN(I,J)=AMAX1(ABS(0.5*CN),DN)-0.5*CN
754         AS(I,J)=AMAX1(ABS(0.5*CS),DS)+0.5*CS
755         AE(I,J)=AMAX1(ABS(0.5*CE),DE)-0.5*CE
756         AW(I,J)=AMAX1(ABS(0.5*CW),DW)+0.5*CW
757         SU(I,J)=CPO*QE(I,J)
758         SUQE(I,J)=SU(I,J)
759         SU(I,J)=SU(I,J)+CQ1*GENQ(I,J)*VOL
760         SP(I,J)=-CP
761       2 SPQE(I,J)=SP(I,J)
762         SP(I,J)=SP(I,J)-CQ2*ED(I,J)*DEN(I,J)*VOL/TE(I,J)
763     101 CONTINUE
764     100 CONTINUE
      C
      CHAPTER  2  2  2  2  2  2  PROBLEM MODIFICATIONS  2  2  2  2  2  2
      C
765         CALL MODQE
```

```
      C
      CHAPTER 3 FINAL COEFFICIENT ASSEMBLY AND RESIDUAL SOURCE CALCULATION 3
  766       RESORQ=0.0
  767       DO 300 I=2,NIM1
  768       DO 301 J=2,NJM1
  769     5 AP(I,J)=AN(I,J)+AS(I,J)+AW(I,J)+AE(I,J)-SP(I,J)
  770       RESOR=AN(I,J)*QE(I,J+1)+AS(I,J)*QE(I,J-1)+AE(I,J)*QE(I+1,J)
            *AW(I,J)*QE(I-1,J)-AP(I,J)*QE(I,J)+SU(I,J)
  771       VOL=R(J)*SEW(I)*SNS(J)
  772       SORVOL=GREAT*VOL
  773       IF(-SP(I,J).GT.0.5*SORVOL) RESOR=RESOR/SORVOL
  774       RESORQ=RESORQ+ABS(RESOR)
      C_____UNDER_RELAXATION
  775       AP(I,J)=AP(I,J)/URFQ
  776       SU(I,J)=SU(I,J)+(1.-URFQ)*AP(I,J)*QE(I,J)
  777   301 CONTINUE
  778   300 CONTINUE
      C
      CHAPTER  4  4  4  4  4  SOLUTION OF DIFFERENCE EQUATIONS  4  4  4  4  4
      C
  779       DO 400 N=1,NSWPQ
  780   400 CALL LISOLV(2,2,NI,NJ,IT,JT,QE)
  781       RETURN
  782       END
```

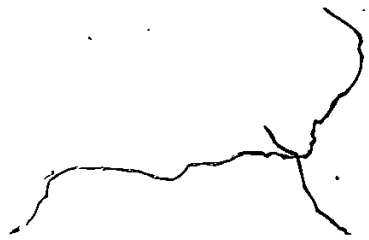
```
      SUBROUTINE LISOLV(ISTART,JSTART,NI,NJ,IT,JT,PHI)                  LISOLV
C                                                                       LISOLV
CHAPTER  0  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0  0  LISOLV
C                                                                       LISOLV
      DIMENSION PHI(IT,JT),A(32),B(32),C(32),D(32)                      LISOLV
      COMMON                                                            LISOLV
     1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30),    0510
     1      SP(30,30)
      LOGICAL INCT
      INCT=.TRUE.
      NIM1=NI-1                                                         LISOLV
      NJM1=NJ-1                                                         LISOLV
      JSTM1=JSTART-1                                                    LISOLV
      A(JSTM1)=0.0                                                      LISOLV
C-----COMMENCE W-E SWEEP                                                LISOLV
      DO 100 I=ISTART,NIM1                                              LISOLV
      C(JSTM1)=PHI(I,JSTM1)                                             LISOLV
C-----COMMENCE S-N TRAVERSE                                             LISOLV
      DO 101 J=JSTART,NJM1                                              LISOLV
C-----ASSEMBLE TDMA COEFFICIENTS                                        LISOLV
      A(J)=AN(I,J)                                                      LISOLV
      B(J)=AS(I,J)                                                      LISOLV
      C(J)=AE(I,J)*PHI(I+1,J)+AW(I,J)*PHI(I-1,J)+SU(I,J)                LISOLV
      D(J)=AP(I,J)                                                      LISOLV
C-----CALCULATE COEFFICIENTS OF RECURRENCE FORMULA                      LISOLV
      TERM=1./(D(J)-B(J)*A(J-1))                                        LISOLV
      A(J)=A(J)*TERM                                                    LISOLV
  101 C(J)=(C(J)+B(J)*C(J-1))*TERM                                      LISOLV
C-----OBTAIN NEW PHI'S                                                  LISOLV
      DO 102 JJ=JSTART,NJM1                                             LISOLV
      J=NJ+JSTM1-JJ                                                     MODA
  102 PHI(I,J)=A(J)*PHI(I,J+1)+C(J)                                     LISOLV
  100 CONTINUE                                                          LISOLV
      INCT=.FALSE.
      RETURN                                                            LISOLV
      END                                                               LISOLV
```

```
      SUBROUTINE PRINT(ISTART,JSTART,NI,NJ,IT,JT,X,Y,PHI,HEAD)          0010
      COMMON
     1/AREA1/INCT
      DIMENSION PHI(IT,JT),X(IT),Y(JT),HEAD(9),STORE(50)                0020
      DIMENSION F(7),F4(11)                                             0030
      LOGICAL INCT
      DATA F/4H(1H ,4H,A4,,4HI3, ,4H11I ,4H10, ,4H7X, ,
     14HA6) /                                                           0040
      DATA F4/4H 1I ,4H 2I ,4H 3I ,4H 4I ,4H 5I ,4H 6I ,                 0050
     1        4H 7I ,4H 8I ,4H 9I ,4H10I ,4H11I /                       0060
      DATA HI,HY/4H  I=,4H Y =/                                         0070
      ISKIP=1                                                           0080
      JSKIP=1                                                           0090
      WRITE(6,110)HEAD                                                  0100
      ISTA=ISTART-12                                                    0110
  100 CONTINUE                                                          0120
      ISTA=ISTA+12                                                      0130
      IEND=ISTA+11                                                      0140
      IEND=MINO(NI,IEND)                                                0150
      F(4)=F4(IEND-ISTA)                                                0160
      WRITE(6,F) HI, (I,I=ISTA,IEND,ISKIP), HY                          0170
      WRITE(6,112)                                                      0180
      DO 101 JJ=JSTART,NJ,JSKIP                                         0190
      J=JSTART+NJ-JJ                                                    0200
      DO 120 I=ISTA,IEND                                                0210
      A=PHI(I,J)                                                        0220
      IF(ABS(A).LT.1.E-20) A=0.0                                        0230
  120 STORE(I)=A                                                        0240
      IF(INCT) GO TO 200                                                0250
      WRITE(6,113)  J,(STORE(I),I=ISTA,IEND,ISKIP),Y(J)
      GO TO 300                                                         0260
  200 WRITE(6,150)  J,(STORE(I),I=ISTA,IEND,ISKIP),Y(J)
  300 CONTINUE
  101 CONTINUE
      WRITE(6,114) (X(I),I=ISTA,IEND,ISKIP)
C-----------------------------------------------------                  0270
      IF(IEND.LT.NI)GO TO 100                                           0280
      RETURN                                                            0290
  110 FORMAT(1H0,20(2H*-),7X,9A4,7X,20(2H-*))                           0300
  112 FORMAT(3H  J)                                                     0310
  113 FORMAT(1H ,I3,1P12E10.2,0PF7.3)                                   0320
  114 FORMAT(6H0X=   ,F7.3,11F10.3)                                     0330
  150 FORMAT(1H ,I3,1P12E10.3,0PF7.3)                                   0340
      END                                                               0330
                                                                       PROMOD
```

```
      SUBROUTINE PROMOD                                                 PROMOD
C                                                                       PROMOD
CHAPTER  0  0  0  0  0  0  0  PRELIMINARIES  0  0  0  0  0  0  0  0  0  PROMOD
C                                                                       PROMOD
      COMMON                                                            PROMOD
     1/VVEL/RESORV,NSWPV,URFV,DYNPV(32),DYPSV(32),SNSV(32),RCV(32)      PROMOD
     1/UVEL/RESORU,NSWPU,URFU,DXEPU(32),DXPWU(32),SEWU(32)              PROMOD
     1/PCOR/RESORM,NSWPP,URFP,DU(30,30),DV(30,30),IPREF,JPREF             0390
     1/VAR/U(30,30),V(30,30),P(30,30),PP(30,30),TE(30,30),ED(30,30)       0420
     1/ALL/IT,JT,NI,NJ,NIM1,NJM1,GREAT
      COMMON
     1/GEOM/INDCOS,X(32),Y(32),DXEP(32),DXPW(32),DYNP(32),DYPS(32),     PROMOD
     1      SNS(32),SEW(32),XU(32),YV(32),R(32),RV(32)                  PROMOD
     1/FLUPR/URFVIS,VISCOS,DENSIT,PRANDT,DEN(30,30),VIS(30,30),
     2        VISD(30,30)
     1/PROB1/UIN,TEIN,EDIN,FLOWIN,ALAMDA,
     2       RSMALL,RLARGE,AL1,AL2,JSTEP,ISTEP,JSTP1,JSTM1,ISTP1,ISTM1,  MODA
     3    NITER,US
     1/SUSP/SUKD(30,30),SPKD(30,30)
     1/COEF/AP(30,30),AN(30,30),AS(30,30),AE(30,30),AW(30,30),SU(30,30);   0510
     1      SP(30,30)
      COMMON
     1/TURB/GEN(30,30),CD,CMU,C1,C2,CAPPA,ELOG,PRED,PRTE                   0490
     1/WALLF/YPLUSN(30,30),XPLUSW(30,30),TAUN(30,30),TAUW(30,30)
     1/TEMP/T(30,30),GAMH(30,30),RESORT,NSWPT,URFT,PRANDL,PFUN,
     2     TWALLN,TIN,GAMQ(30,30),TBOTOM
     1/MSTF/QE(30,30),PRQE,CQ1,CQ2,RESQE,URFQ,BETA,GRAVTY,C3,C4,NSWPQ,
     1       SUQE(30,30),GENQ(30,30),SPQE(30,30),RESORQ
C                                                                       PROMOD
CHAPTER  1  1  1  1  1  1  1  1  PROPERTIES  1  1  1  1  1  1  1  1  1  PROMOD
C                                                                       PROMOD
      ENTRY MODPRO                                                      PROMOD
C-----NO MODIFICATIONS FOR THIS PROBLEM                                 PROMOD
      RETURN
C                                                                       PROMOD
CHAPTER  2  2  2  2  2  2  2  2  U MOMENTUM  2  2  2  2  2  2  2  2  2  PROMOD
C                                                                       PROMOD
      ENTRY MODU                                                        PROMOD
C-----TOP WALL                                                          MODA
      NNIM1=NI-2
      NNJM1=NJ-2
      CDTERM=CMU**0.25                                                  MODA
      YP=YV(NJ)-Y(NJM1)                                                 MODA
      J=NJM1                                                            MODA
```

```
      DO  210  I=3,NIM1                                                  MODA
      SQRTK=SQRT(0.5*(TE(I,J)+TE(I-1,J)))
      DENU=0.5*(DEN(I,J)+DEN(I-1,J))                                     MODA
      YPLUSA=0.5*(YPLUSN(I,NJM1)+YPLUSN(I-1,NJM1))
      IF(YPLUSA.LE.11.63)     GO  TO  211                                MODA
      TMULT=DENU*CDTERM*SQRTK*CAPPA/ALOG(ELOG*YPLUSA)                    MODA
      GO  TO  212                                                        MODA
  211 TMULT=VISCOS/YP                                                    MODA
  212 TAUN(I,NJM1)=-TMULT*U(I,J)
      SP(I,J)=SP(I,J)-TMULT*SEWU(I)*RV(NJ)                               MODA
  210 AN(I,J)=0.0                                                        MODA
      TAUN(2,NJM1)=TAUN(3,NJM1)
      TAUN(NI,NJM1)=TAUN(NIM1,NJM1)
C-----LEFT SIDE WALL                                                     MODA
      DO 213 J=2,NJM1
      U(1,J)=0.
      AW(2,J)=0.
  213 CONTINUE
C  BOTTOM WALL
      YP=YV(2)-Y(1)
      CDTERM=CMU**0.25                                                   MODA
      J=2
      DO 4210 I=3,NIM1
      SQRTK=SQRT(0.5*(TE(I,J)+TE(I-1,J)))                                MODA
      DENU=0.5*(DEN(I,J)+DEN(I-1,J))                                     MODA
      YPLUSA=0.5*(YPLUSN(I,2)+YPLUSN(I-1,2))
      IF(YPLUSA.LE.11.63) GO TO 4211
      TMULT=DENU*CDTERM*SQRTK*CAPPA/ALOG(ELOG*YPLUSA)                    MODA
      GO TO 4212
 4211 TMULT=VISCOS/YP
 4212 TAUN(I,2)=-TMULT*U(I,J)+TMULT*US
      SP(I,J)=SP(I,J)-TMULT*SEWU(I)*RV(NJ)                               MODA
      SU(I,J)=SU(I,J)+TMULT*US*SEWU(I)*RV(NJ)
      U(I,1)=US
      U(2,1)=US
      AS(I,J)=0.
 4210 CONTINUE
      TAUN(2,2)=TAUN(3,2)
      TAUN(NI,2)=TAUN(NIM1,2)
C-----RIGHT SIDE WALL                                                    MODA
      DO 3203 J=1,NJ
      U(NI,J)=0.0
      AE(NIM1,J)=0.0
 3203 CONTINUE
      RETURN                                                             PROMOB
```

```
      C                                                                 PROMOD
      CHAPTER  3  3  3  3  3  3  3  3  V MOMENTUM  3  3  3  3  3  3  3  3  3   PROMOD
      C                                                                 PROMOD
  903       ENTRY MODV                                                  PROMOD
      C-----LEFT SIDE WALL                                              MODA
  904       CDTERM=CMU**0.25                                            MODA
  905       XP=X(2)-XU(2)                                               MODA
  906       I=2                                                         MODA
  907       DO 310 J=3,NJM1
  908       SQRTK=SQRT(0.5*(TE(I,J)+TE(I,J-1)))                         MODA
  909       XPLUSA=0.5*(XPLUSW(2,J)+XPLUSW(2,J-1))
  910       DENV=0.5*(DEN(I,J)+DEN(I,J-1))                              MODA
  911       IF(XPLUSA.LE.11.63)     GO  TO  311                         MODA
  912       TMULT=DENV*CDTERM*SQRTK*CAPPA/ALOG(ELOG*XPLUSA)             MODA
  913       GO  TO  312                                                 MODA
  914   311 TMULT=VISCOS/XP                                             MODA
  915   312 TAUW(2,J)=-TMULT*V(I,J)
  916       SP(I,J)=SP(I,J)-TMULT*SNSV(J)*RV(J)                         MODA
  917   310 AW(I,J)=0.
  918  1111 CONTINUE
  919       TAUW(2,2)=TAUW(2,3)
  920       TAUW(2,NJ)=TAUW(2,NJM1)
      C-----RIGHT SIDE WALL
  921       CDTERM=CMU**0.25
  922       XP=X(NI)-XU(NI)
  923       I=NIM1
  924       DO 2310  J=2,NJM1
  925       SQRTK=SQRT(0.5*(TE(I,J)+TE(I,J-1)))                         M
  926       XPLUSA=0.5*(XPLUSW(NIM1,J)+XPLUSW(NIM1,J-1))
  927       DENV=0.5*(DEN(I,J)+DEN(I,J-1))                              MODA
  928       IF(XPLUSA.LE.11.63) GO TO 2311
  929       TMULT=DENV*CDTERM*SQRTK*CAPPA/ALOG(ELOG*XPLUSA)             MODA
  930       GO TO 2312
  931  2311 TMULT=VISCOS/XP
  932  2312  TAUW(NIM1,J)=-TMULT*V(I,J)
  933       SP(I,J)=SP(I,J)-TMULT*SNSV(J)*RV(J)                         MODA
  934  2310 AE(I,J)=0.0
  935       TAUW(NIM1,2)=TAUW(NIM1,3)
  936       TAUW(NIM1,NJ)=TAUW(NIM1,NJM1)
      C-----TOP WALL                                                    MODA
  937       DO  313  I=2,NIM1                                           MODA
  938   313 AN(I,NJM1)=0.0                                              MODA
```



```
      C-----BOTTOM WALL                                                  MODA
  939       DO  302  I=2,NIM1                                           MODA
  940     302 AS(I,3)=0.0                                               MODA
  941       RETURN                                                      PROMOD
      C                                                                 PROMOD
      CHAPTER  4  4  4  4  4  4  PRESSURE CORRECTION  4  4  4  4  4  4  4  4  PROMOD
      C                                                                 PROMOD
  942       ENTRY MODP                                                  PROMOD
  943       RETURN                                                      PROMOD
      C                                                                 PROMOD
      CHAPTER  5  5  5  5  5  5  5  THERMAL ENERGY  5  5  5  5  5  5  5  5  PROMOD
      C                                                                 PROMOD
      C-----NO MODIFICATIONS FOR THIS PROBLEM
      C                                                                 PROMOD
      CHAPTER 6 6 6 6 6 6 6 TURBULENT KINETIC ENERGY                    PROMOD
      C                                                                 PROMOD
  944       ENTRY MODTE                                                 PROMOD
      C-----TOP WALL                                                    MODA
  945       CDTERM=CMU**0.25                                            MODA
  946       YP=YV(NJ)-Y(NJM1)                                           MODA
  947       J=NJM1                                                      MODA
  948       DO  610  I=2,NIM1                                           MODA
  949       U(I,1)=US
  950       DENU=DEN(I,J)                                               MODA
  951       SQRTK=SQRT(TE(I,J))                                         MODA
  952       VOL=R(J)*SNS(J)*SEW(I)                                      MODA
  953       GENCOU=0.5*(ABS(TAUN(I+1,NJM1)*U(I+1,J))+ABS(TAUN(I,NJM1)*U(I,J)))
           1/YP
  954       YPLUSN(I,NJM1)=DENU*SQRTK*CDTERM*YP/VISCOS
  955       DUDY=((U(I,J)+U(I+1,J)+U(I,J+1)+U(I+1,J+1))/4.-(U(I,J)+U(I+1,J)+   MODA
           1U(I,J-1)+U(I+1,J-1))/4.)/SNS(J)                             MODA
  956       GENRES=GEN(I,J)-VIS(I,J)*DUDY**2
  957       GEN(I,J)=GENRES+GENCOU                                      MODA
  958       IF(YPLUSN(I,NJM1).LE.11.63) GO TO 611
  959       DITERM=DEN(I,J)*(CMU**.75)*SQRTK*ALOG(ELOG*YPLUSN(I,NJM1
           1))/(CAPPA*YP)
  960       GO TO 612                                                   MODA
  961   611 CONTINUE                                                    MODA
  962       DITERM=DEN(I,J)*(CMU**.75)*SQRTK*YPLUSN(I,NJM1)/YP
  963   612 CONTINUE                                                    MODA
  964       SU(I,J)=GEN(I,J)*VOL+SUKD(I,J)                              MODA
  965       SP(I,J)=-DITERM*VOL+SPKD(I,J)                               MODA
  966   610 AN(I,J)=0.0                                                 MODA
```

```
      C-----LEFT SIDE WALL                                                    MODA
  967       XP=X(2)-XU(2)                                                     MODA
  968       I=2
  969       DO 620 J=2,NJM1
  970       DENV=DEN(I,J)                                                     MODA
  971       SQRTK=SQRT(TE(I,J))                                               MODA
  972       VOL=R(J)*SNS(J)*SEW(I)                                            MODA
  973       XPLUSW(2    ,J)=DENV*SQRTK*CDTERM*XP/VISCOS
  974       GENCOU=0.5*(ABS(TAUW(2    ,J+1)*V(I,J+1))+ABS(TAUW(2    ,J)
           1*V(I,J)))/XP
  975       DVDX=((V(I,J)+V(I,J+1)+V(I+1,J)+V(I+1,J+1))/4.-(V(I,J)+V(I,J+1)+V( MODA
           1I-1,J)+V(I-1,J+1))/4.)/SEW(I)                                    MODA
  976       GENRES=GEN(I,J)-VIS(I,J)*DVDX**2
  977       GEN(I,J)=GENRES+GENCOU                                            MODA
  978       IF(XPLUSW(2    ,J).LE.11.63) GO TO 621
  979       DITERM=DEN(I,J)*(CMU**.75)*SQRTK*ALOG(ELOG*XPLUSW(2    ,J))/(
           1CAPPA*XP)
  980       GO TO 622                                                         MODA
  981   621 CONTINUE                                                          MODA
  982       DITERM=DEN(I,J)*(CMU**.75)*SQRTK*XPLUSW(2    ,J)/XP
  983   622 CONTINUE                                                          MODA
  984       SU(I,J)=SUKD(I,J)+GEN(I,J)*VOL
  985       SP(I,J)=SPKD(I,J)-DITERM*VOL
  986   620 AW(I,J)=0.0                                                       MODA
      C-----RIGHT SIDE WALL
  987       I=NIM1
  988       DO 1620 J=2,NJM1
  989       DENV=DEN(I,J)                                                     MODA
  990       SQRTK=SQRT(TE(I,J))                                               MODA
  991       XPLUSW(NIM1,J)=DENV*SQRTK*CDTERM*XP/VISCOS
  992       VOL=R(J)*SNS(J)*SEW(I)                                            MODA
  993       GENCOU=0.5*(ABS(TAUW(NIM1,J+1)*V(I,J+1))+ABS(TAUW(NIM1,J)*V(I,J
           1)))/XP
  994       DVDX=((V(I,J)+V(I,J+1)+V(I+1,J)+V(I+1,J+1))/4.-(V(I,J)+V(I,J+1)+V( MODA
           1I-1,J)+V(I-1,J+1))/4.)/SEW(I)                                    MODA
  995       GENRES=GEN(I,J)-VIS(I,J)*DVDX**2
  996       GEN(I,J)=GENRES+GENCOU                                            MODA
  997       IF(XPLUSW(NIM1,J).LE.11.63) GO TO 1621
  998       DITERM=DEN(I,J)*(CMU**.75)*SQRTK*ALOG(ELOG*XPLUSW(NIM1,J))
           1/(CAPPA*XP)
  999       GO TO 1622
 1000   1621 CONTINUE
```

```
1001          DITERM=DEN(I,J)*(CMU**.75)*SQRTK*XPLUSW(NIM1,J)/XP
1002    1622  CONTINUE
1003          SU(I,J)=SUKD(I,J)+GEN(I,J)*VOL
1004          SP(I,J)=SPKD(I,J)-DITERM*VOL
1005    1620  AW(I,J)=0.0
      C-----BOTTOM WALL                                                  MODA
1006          J=2                                                        MODA
1007          YP=YV(2)-Y(1)
1008          DO 3630 I=2,NIM1
1009          DENU=DEN(I,J)
1010          SQRTK=SQRT(TE(I,J))
1011          VOL=R(J)*SNS(J)*SEW(I)
1012          GENCOU=0.5*(ABS(TAUN(I+1,2)*(U(I+1,J)-US))
             2+ABS(TAUN(I,2)*(U(I,J)-US)))/YP
1013          YPLUSN(I,2)=DENU*SQRTK*CDTERM*YP/VISCOS
1014          DUDY=((U(I,J)+U(I+1,J)+U(I,J+1)+U(I+1,J+1))/4.
             1-(U(I,J)+U(I+1,J)+U(I,J-1)+U(I+1,J-1))/4.)/SNS(J)
1015          GENRES=GEN(I,J)-VIS(I,J)*DUDY**2
1016          GEN(I,J)=GENRES+GENCOU
1017          IF(YPLUSN(I,2).LE.11.63) GO TO 3631
1018          DITERM=DEN(I,J)*(CMU**.75)*SQRTK*ALOG(ELOG*YPLUSN(I,2))
             1/(CAPPA*YP)
1019          GO TO 3632
1020    3631  CONTINUE
1021          DITERM=DEN(I,J)*(CMU**.75)*SQRTK*YPLUSN(I,2)/(CAPPA*YP)
1022    3632  CONTINUE
1023          SU(I,J)=GEN(I,J)*VOL+SUKD(I,J)
1024          SP(I,J)=-DITERM*VOL+SPKD(I,J)
1025    3630  AS(I,J)=0.0
1026          RETURN
      C                                                                  PROM
      CHAPTER  7 7 7 7 7 7 7 7 DISSIPATION                               PROMOD
      C                                                                  PROMOD
1027          ENTRY MODED                                                PROMOD
      C-----TOP WALL
1028          YP=YV(NJ)-Y(NJM1)                                          MODA
1029          J=NJM1                                                     MODA
1030          TERM=(CMU**.75)/(CAPPA*YP)                                 MODA
1031          DO  710  I=2,NIM1                                          MODA
1032          SU(I,J)=GREAT*TERM*TE(I,J)**1.5                            MODA
1033     710  SP(I,J)=-GREAT
```

```
      C-----LEFT SIDE WALL                                              MODA
 1034       XP=X(2)-XU(2)                                               MODA
 1035       I=2                                                         MODA
 1036       TERM=(CMU**.75)/(CAPPA*YP)
 1037       NJM2=NJ-2
 1038       DO  720  J=2,NJM2
 1039       AW(2,J)=0.0
 1040       SU(I,J)=GREAT*TERM*TE(I,J)**1.5
 1041   720 SP(I,J)=-GREAT
      C-----RIGHT SIDE WALL
 1042       NJM2=NJ-2
 1043       I=NIM1
 1044       DO 1720 J=2,NJM2
 1045       XP=X(2)-XU(2)
 1046       TERM=(CMU**.75)/(CAPPA*XP)
 1047       SU(I,J)=GREAT*TERM*TE(I,J)**1.5
 1048  1720 SP(I,J)=-GREAT
      C-----BOTTOM WALL
 1049       J=2
 1050       DO 1730 I=2,NIM1
 1051       YP=YV(2)-Y(1)
 1052       J=2
 1053       TERM=(CMU**.75)/(CAPPA*XP)
 1054       SU(I,J)=GREAT*TERM*TE(I,J)**1.5
 1055  1730 SP(I,J)=-GREAT
 1056       RETURN
      C
 1057       ENTRY MODT
      C-----TOP WALL (CONSTANT TEMPERATURE)
 1058       CDTERM=CMU**0.25
 1059       J=NJM1
 1060       DYN=YV(NJ)-YV(NJM1)
 1061       DO 500 I=2,NIM1
 1062       T(I,NJ)=TWALLN
 1063       AN(I,J)=0.0
 1064       AREA=RV(J+1)*SEW(I)
 1065       IF(YPLUSN(I,NJM1).LE.11.63) GO TO 510
 1066       UPLUS=ALOG(ELOG*YPLUSN(I,NJM1))/CAPPA
 1067       GT=DEN(I,J)*CDTERM*SQRT(TE(I,J))/(PRANDT*(UPLUS+PFUN))
 1068       GO TO 511
 1069   510 GT=VISCOS/(PRANDL*DYN)
 1070   511 TERM=GT*AREA
```

```
1083
1084          AREA=A(I,J+1)*S(J+1)
1085          IF(YPLUSM(I,2).LE.11.63) GO TO 5510
1086          UPLUS=ALOG(ELOG*YPLUSM(I,2))/CAPPA
1087          GT=DEN(I,J)*CDTERM*SQRT(TE(I,J))/(PRANDT*(UPLUS+PFUN))
1088          GO TO 5511
1089    5510  GT=VISCOS/(PRANDL*DYN)
1090    5511  TERM=GT*AREA
1091          SU(I,J)=SU(I,J)+TERM*TBOTOM
1092          SP(I,J)=SP(I,J)-TERM
1093          AS(I,J)=0.0
1094    1502  CONTINUE
      C-----RIGHT SIDE WALL
1095          I=NIM1
1096          DYN=XU(NI)-XU(NIM1)
1097          DO 7500 J=2,NJM1
1098          AE(I,J)=0.
1099          T(NI,J)=T(NIM1,J)
1100    7500  CONTINUE
1101          RETURN
      C
      CHAPTER  8 8 8 8 8  MEAN SQURE TEMPERATURE FLACTUATION
      C
1102          ENTRY MODQE
1103          CQ1=2.8
1104          CQ2=1.7
```

```
C-----LEFT SIDE WALL
      XP=X(3)-X(2)
      I=2
      DO 6610 J=2,NJM1
      VOL=R(J)*SNS(J)*SEW(I)
      DTDX=((T(I+1,J)+T(I,J))/2.-(T(I-1,J)+T(I,J))/2.)/SEW(I)
      GENQ(I,J)=(DTDX**2)*VIS(I,J)
      QETERM =CQ1*TE(I,J)*GENQ(I,J)/(CQ2*DEN(I,J)*ED(I,J))
      SU(I,J)=GREAT*QETERM
      SP(I,J)=-GREAT
      AW(I,J)=0.
 6610 CONTINUE
C-----RIGHT SIDE WALL
      I=NIM1
      XP=X(NI)-XU(NI)
      DO 6310 J=2,NJM1
      VOL=R(J)*SNS(J)*SEW(I)
      DTDX=((T(I+1,J)+T(I,J))/2.-(T(I-1,J)+T(I,J))/2.)/SEW(I)
      GENQ(I,J)=(DTDX**2)*VIS(I,J)
      QETERM =CQ1*TE(I,J)*GENQ(I,J)/(CQ2*DEN(I,J)*ED(I,J))
      SU(I,J)=GREAT*QETERM
      SP(I,J)=-GREAT
      AE(I,J)=0.
 6310 CONTINUE
C-----BOTTOM WALL
      J=2
      DO 9630 I=2,NIM1
      YP=Y(3)-Y(2)
      VOL=R(J)*SNS(J)*SEW(I)
      DTDY=(T(I,J+1)-T(I,J))/DYNP(J)
      GENQ(I,J)=(DTDY**2)*VIS(I,J)
      QETERM =CQ1*TE(I,J)*GENQ(I,J)/(CQ2*DEN(I,J)*ED(I,J))
      SU(I,J)=GREAT*QETERM
      SP(I,J)=-GREAT
      AS(I,J)=0.
 9630 CONTINUE
C-----TOP WALL
      J=NJM1
      DO 8610 I=2,NIM1
      DTDY=(T(I,J)-T(I,J-1))/DYPS(J)
      GENQ(I,J)=(DTDY**2)*VIS(I,J)
```

```
      QETERM =CQ1*TE(I,J)*GENQ(I,J)/(CQ2*DEN(I,J)*ED(I,J))
      SU(I,J)=GREAT*QETERM
      SP(I,J)=-GREAT
      AN(I,J)=0.
 8610 CONTINUE
      RETURN
      END
```

PROMOD

U VELOCITY

	I= 2	3	4	5	6	7	8	9	10	11	Y =
J											
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	1.00E-10	2.79E-07	5.46E-07	3.99E-07	-5.27E-07	-2.15E-06	-4.31E-06	-6.87E-06	-8.27E-06	-6.75E-06	4.80E-01
16	1.00E-10	4.33E-07	6.56E-07	-2.40E-08	-2.35E-06	-6.21E-06	-1.13E-05	-1.70E-05	-1.97E-05	-1.61E-05	4.70E-01
15	1.00E-10	1.99E-07	-1.08E-07	-1.83E-06	-5.77E-06	-1.18E-05	-1.94E-05	-2.76E-05	-3.08E-05	-2.51E-05	4.55E-01
14	1.00E-10	-4.80E-07	-1.84E-06	-4.95E-06	-1.03E-05	-1.77E-05	-2.67E-05	-3.58E-05	-3.87E-05	-3.21E-05	4.35E-01
13	1.00E-10	-1.37E-06	-3.98E-06	-8.47E-06	-1.48E-05	-2.26E-05	-3.19E-05	-4.10E-05	-4.40E-05	-3.79E-05	4.10E-01
12	1.00E-10	-2.14E-06	-5.76E-06	-1.13E-05	-1.81E-05	-2.60E-05	-3.50E-05	-4.41E-05	-4.81E-05	-4.37E-05	3.80E-01
11	1.00E-10	-2.51E-06	-6.67E-06	-1.27E-05	-1.97E-05	-2.74E-05	-3.60E-05	-4.54E-05	-5.12E-05	-4.93E-05	3.45E-01
10	1.00E-10	-2.35E-06	-6.26E-06	-1.21E-05	-1.89E-05	-2.62E-05	-3.44E-05	-4.40E-05	-5.21E-05	-5.36E-05	3.00E-01
9	1.00E-10	-1.40E-06	-3.90E-06	-8.02E-06	-1.33E-05	-1.93E-05	-2.59E-05	-3.44E-05	-4.40E-05	-5.01E-05	2.40E-01
8	1.00E-10	-4.21E-07	-1.40E-06	-3.31E-06	-5.80E-06	-8.32E-06	-1.08E-05	-1.42E-05	-2.06E-05	-3.06E-05	1.80E-01
7	1.00E-10	-2.35E-07	-6.58E-07	-1.12E-06	-9.98E-07	2.23E-07	2.55E-06	5.77E-06	6.64E-06	-1.02E-07	1.35E-01
6	1.00E-10	-1.19E-06	-2.21E-06	-1.81E-06	1.14E-06	6.30E-06	1.38E-05	2.44E-05	3.36E-05	3.25E-05	1.00E-01
5	1.00E-10	-4.15E-06	-6.91E-06	-5.07E-06	1.83E-06	1.31E-05	2.83E-05	4.85E-05	6.57E-05	7.04E-05	7.00E-02
4	1.00E-10	-1.02E-05	-1.25E-05	-4.12E-06	1.27E-05	3.43E-05	6.00E-05	8.75E-05	1.09E-04	1.17E-04	4.50E-02
3	1.00E-10	-5.77E-06	6.96E-06	3.20E-05	6.16E-05	9.09E-05	1.20E-04	1.49E-04	1.70E-04	1.79E-04	2.50E-02
2	1.00E-10	5.31E-05	9.58E-05	1.32E-04	1.62E-04	1.88E-04	2.13E-04	2.36E-04	2.53E-04	2.59E-04	1.00E-02
X=	0.0	0.017	0.035	0.057	0.085	0.117	0.157	0.210	0.270	0.322	

	I= 14	15	16	17	18	Y =
J						
18	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	2.45E-07	6.74E-07	4.29E-07	1.19E-07	0.0	4.80E-01
16	-7.15E-07	6.85E-07	6.55E-07	2.66E-07	0.0	4.70E-01
15	-3.36E-06	-5.43E-07	2.81E-07	2.05E-07	0.0	4.55E-01
14	-7.22E-06	-2.93E-06	-8.18E-07	-1.17E-07	0.0	4.35E-01
13	-1.12E-05	-5.47E-06	-2.09E-06	-5.27E-07	0.0	4.10E-01
12	-1.49E-05	-7.48E-06	-2.93E-06	-7.65E-07	0.0	3.80E-01
11	-1.89E-05	-9.39E-06	-3.54E-06	-8.88E-07	0.0	3.45E-01
10	-2.47E-05	-1.29E-05	-5.01E-06	-1.28E-06	0.0	3.00E-01
9	-3.13E-05	-1.84E-05	-7.99E-06	-2.34E-06	0.0	2.40E-01
8	-3.56E-05	-2.41E-05	-1.17E-05	-3.78E-06	0.0	1.80E-01
7	-3.15E-05	-2.72E-05	-1.59E-05	-5.77E-06	0.0	1.35E-01
6	-1.41E-05	-2.23E-05	-1.77E-05	-7.67E-06	0.0	1.00E-01
5	1.91E-05	-5.50E-06	-1.43E-05	-8.52E-06	0.0	7.00E-02
4	7.11E-05	3.17E-05	7.92E-07	-5.66E-06	0.0	4.50E-02
3	1.41E-04	1.01E-04	4.43E-05	1.06E-05	0.0	2.50E-02
2	2.26E-04	1.92E-04	1.41E-04	6.47E-05	0.0	1.00E-02
X=	0.422	0.445	0.462	0.475	0.485	

V VELOCITY

I=	2	3	4	5	6	7	8	9	10	11	Y =
J											
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.85E-01
17	1.60E-07	1.53E-07	-6.48E-08	-3.36E-07	-4.98E-07	-5.42E-07	-4.86E-07	-2.33E-07	2.88E-07	7.06E-07	4.75E-01
16	4.71E-07	3.14E-07	-4.42E-07	-1.39E-06	-1.98E-06	-2.12E-06	-1.84E-06	-8.02E-07	1.15E-06	2.62E-06	4.62E-01
15	6.72E-07	9.86E-09	-1.78E-06	-3.90E-06	-5.23E-06	-5.47E-06	-4.57E-06	-1.73E-06	3.03E-06	6.36E-06	4.45E-01
14	6.04E-08	-1.73E-06	-4.89E-06	-8.26E-06	-1.03E-05	-1.05E-05	-8.50E-06	-2.81E-06	5.89E-06	1.16E-05	4.22E-01
13	-2.08E-06	-5.82E-06	-1.04E-05	-1.45E-05	-1.70E-05	-1.69E-05	-1.33E-05	-4.18E-06	9.09E-06	1.80E-05	3.95E-01
12	-6.05E-06	-1.25E-05	-1.83E-05	-2.26E-05	-2.49E-05	-2.41E-05	-1.90E-05	-6.34E-06	1.18E-05	2.51E-05	3.62E-01
11	-1.18E-05	-2.20E-05	-2.89E-05	-3.29E-05	-3.44E-05	-3.28E-05	-2.61E-05	-1.03E-05	1.33E-05	3.30E-05	3.22E-01
10	-1.88E-05	-3.37E-05	-4.26E-05	-4.59E-05	-4.61E-05	-4.35E-05	-3.57E-05	-1.74E-05	1.19E-05	4.04E-05	2.70E-01
9	-2.36E-05	-4.23E-05	-5.36E-05	-5.75E-05	-5.71E-05	-5.35E-05	-4.53E-05	-2.70E-05	4.84E-06	4.14E-05	2.10E-01
8	-2.49E-05	-4.52E-05	-5.80E-05	-6.23E-05	-6.12E-05	-5.68E-05	-4.87E-05	-3.27E-05	-5.15E-06	3.05E-05	1.57E-01
7	-2.55E-05	-4.62E-05	-5.89E-05	-6.21E-05	-5.96E-05	-5.45E-05	-4.62E-05	-3.21E-05	-1.03E-05	1.80E-05	1.17E-01
6	-2.77E-05	-4.81E-05	-5.83E-05	-5.87E-05	-5.45E-05	-4.84E-05	-3.97E-05	-2.71E-05	-1.10E-05	9.21E-06	8.50E-02
5	-3.42E-05	-5.24E-05	-5.60E-05	-5.18E-05	-4.50E-05	-3.80E-05	-2.91E-05	-1.92E-05	-8.51E-06	4.11E-06	5.75E-02
4	-4.74E-05	-5.54E-05	-4.76E-05	-3.80E-05	-3.00E-05	-2.35E-05	-1.73E-05	-1.11E-05	-5.06E-06	2.00E-06	3.50E-02
3	-5.31E-05	-4.26E-05	-2.82E-05	-1.91E-05	-1.43E-05	-1.08E-05	-7.67E-06	-4.82E-06	-2.15E-06	9.89E-07	1.75E-02
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

I=	14	15	16	17	18	Y =
J						
18	0.0	0.0	0.0	0.0	0.0	4.85E-01
17	1.91E-07	-1.39E-07	-2.46E-07	-1.19E-07	0.0	4.75E-01
16	9.70E-07	-1.59E-07	-6.32E-07	-4.51E-07	0.0	4.62E-01
15	3.17E-06	6.67E-07	-7.32E-07	-8.09E-07	0.0	4.45E-01
14	7.46E-06	3.38E-06	5.39E-07	-5.46E-07	0.0	4.22E-01
13	1.45E-05	8.70E-06	3.98E-06	9.02E-07	0.0	3.95E-01
12	2.52E-05	1.72E-05	9.60E-06	3.39E-06	0.0	3.62E-01
11	4.21E-05	3.05E-05	1.81E-05	6.94E-06	0.0	3.22E-01
10	6.95E-05	5.43E-05	3.38E-05	1.37E-05	0.0	2.70E-01
9	1.04E-04	8.99E-05	6.09E-05	2.77E-05	0.0	2.10E-01
8	1.31E-04	1.27E-04	9.42E-05	4.75E-05	0.0	1.57E-01
7	1.39E-04	1.53E-04	1.26E-04	7.06E-05	0.0	1.17E-01
6	1.27E-04	1.61E-04	1.53E-04	9.55E-05	0.0	8.50E-02
5	9.66E-05	1.47E-04	1.65E-04	1.19E-04	0.0	5.75E-02
4	5.72E-05	1.08E-04	1.54E-04	1.32E-04	0.0	3.50E-02
3	2.60E-05	5.13E-05	1.07E-04	1.13E-04	0.0	1.75E-02
2	0.0	0.0	0.0	0.0	0.0	0.0
X=	0.435	0.455	0.470	0.480	0.490	

PRESSURE

J \ I=	2	3	4	5	6	7	8	9	10	11	Y =
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	3.12E-06	3.09E-06	3.01E-06	2.88E-06	2.75E-06	2.69E-06	2.75E-06	3.08E-06	3.64E-06	4.01E-06	4.80E-01
16	3.11E-06	3.09E-06	3.02E-06	2.92E-06	2.81E-06	2.75E-06	2.81E-06	3.11E-06	3.60E-06	3.93E-06	4.70E-01
15	3.14E-06	3.12E-06	3.06E-06	2.97E-06	2.88E-06	2.82E-06	2.86E-06	3.11E-06	3.52E-06	3.79E-06	4.55E-01
14	3.21E-06	3.17E-06	3.11E-06	3.03E-06	2.95E-06	2.90E-06	2.90E-06	3.07E-06	3.36E-06	3.56E-06	4.35E-01
13	3.31E-06	3.25E-06	3.16E-06	3.06E-06	2.98E-06	2.92E-06	2.88E-06	2.95E-06	3.13E-06	3.33E-06	4.10E-01
12	3.38E-06	3.30E-06	3.16E-06	3.01E-06	2.90E-06	2.80E-06	2.70E-06	2.70E-06	2.86E-06	3.12E-06	3.80E-01
11	3.31E-06	3.22E-06	3.04E-06	2.81E-06	2.63E-06	2.47E-06	2.30E-06	2.23E-06	2.42E-06	2.79E-06	3.45E-01
10	2.94E-06	2.85E-06	2.65E-06	2.35E-06	2.08E-06	1.83E-06	1.55E-06	1.36E-06	1.56E-06	2.07E-06	3.00E-01
9	2.12E-06	2.05E-06	1.89E-06	1.60E-06	1.26E-06	8.88E-07	4.36E-07	0.0	4.68E-08	6.32E-07	2.40E-01
8	1.09E-06	1.06E-06	9.55E-07	7.47E-07	4.38E-07	3.43E-08	-5.76E-07	-1.33E-06	-1.59E-06	-9.53E-07	1.80E-01
7	3.00E-07	2.88E-07	2.60E-07	1.93E-07	4.48E-08	-3.01E-07	-9.85E-07	-1.98E-06	-2.53E-06	-1.96E-06	1.35E-01
6	-3.20E-07	-3.08E-07	-2.12E-07	-5.55E-08	-1.07E-08	-2.52E-07	-9.23E-07	-2.02E-06	-2.75E-06	-2.17E-06	1.00E-01
5	-1.00E-06	-9.24E-07	-5.82E-07	-1.17E-07	2.18E-07	1.48E-07	-4.58E-07	-1.64E-06	-2.51E-06	-1.87E-06	7.00E-02
4	-2.11E-06	-1.75E-06	-8.37E-07	1.67E-07	7.98E-07	7.64E-07	1.32E-07	-1.14E-06	-2.05E-06	-1.28E-06	4.50E-02
3	-4.22E-06	-2.55E-06	-5.96E-07	7.76E-07	1.47E-06	1.42E-06	7.16E-07	-6.58E-07	-1.65E-06	-8.20E-07	2.50E-02
2	-6.68E-06	-9.74E-07	1.51E-06	2.52E-06	2.82E-06	2.48E-06	1.52E-06	-8.99E-08	-1.28E-06	-5.95E-07	1.00E-02
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

J \ I=	14	15	16	17	18	Y =
18	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	3.82E-06	3.71E-06	3.67E-06	3.68E-06	0.0	4.80E-01
16	3.79E-06	3.71E-06	3.68E-06	3.68E-06	0.0	4.70E-01
15	3.72E-06	3.66E-06	3.64E-06	3.64E-06	0.0	4.55E-01
14	3.60E-06	3.56E-06	3.53E-06	3.50E-06	0.0	4.35E-01
13	3.47E-06	3.40E-06	3.34E-06	3.25E-06	0.0	4.10E-01
12	3.34E-06	3.22E-06	3.11E-06	2.97E-06	0.0	3.80E-01
11	3.21E-06	3.02E-06	2.85E-06	2.67E-06	0.0	3.45E-01
10	2.88E-06	2.66E-06	2.43E-06	2.18E-06	0.0	3.00E-01
9	2.08E-06	1.92E-06	1.64E-06	1.24E-06	0.0	2.40E-01
8	1.89E-06	1.94E-06	1.64E-06	1.07E-06	0.0	1.80E-01
7	2.97E-06	3.65E-06	3.53E-06	2.81E-06	0.0	1.35E-01
6	5.33E-06	7.20E-06	7.59E-06	6.88E-06	0.0	1.00E-01
5	8.95E-06	1.30E-05	1.47E-05	1.44E-05	0.0	7.00E-02
4	1.29E-05	2.04E-05	2.54E-05	2.69E-05	0.0	4.50E-02
3	1.59E-05	2.72E-05	3.86E-05	4.51E-05	0.0	2.50E-02
2	1.65E-05	2.87E-05	4.45E-05	6.03E-05	0.0	1.00E-02
X=	0.435	0.455	0.470	0.480	0.490	

TEMPERATURE

J \ I=	2	3	4	5	6	7	8	9	10	11	Y =
18	4.000E 02	4.000E 02	4.000E 02	4.000E 02	4.000E 02	4.000E 02	4.000E 02	4.000E 02	4.000E 02	4.000E 02	4.900E-01
17	3.932E 02	3.926E 02	3.915E 02	3.901E 02	3.882E 02	3.859E 02	3.831E 02	3.802E 02	3.778E 02	3.770E 02	4.800E-01
16	3.878E 02	3.869E 02	3.852E 02	3.829E 02	3.800E 02	3.765E 02	3.720E 02	3.678E 02	3.641E 02	3.627E 02	4.700E-01
15	3.806E 02	3.796E 02	3.777E 02	3.748E 02	3.710E 02	3.664E 02	3.607E 02	3.555E 02	3.514E 02	3.500E 02	4.550E-01
14	3.729E 02	3.721E 02	3.702E 02	3.671E 02	3.630E 02	3.578E 02	3.517E 02	3.463E 02	3.425E 02	3.414E 02	4.350E-01
13	3.660E 02	3.652E 02	3.634E 02	3.604E 02	3.563E 02	3.512E 02	3.453E 02	3.400E 02	3.366E 02	3.355E 02	4.100E-01
12	3.603E 02	3.594E 02	3.575E 02	3.547E 02	3.508E 02	3.460E 02	3.406E 02	3.355E 02	3.324E 02	3.313E 02	3.800E-01
11	3.554E 02	3.544E 02	3.526E 02	3.499E 02	3.464E 02	3.421E 02	3.372E 02	3.325E 02	3.297E 02	3.286E 02	3.450E-01
10	3.506E 02	3.497E 02	3.478E 02	3.455E 02	3.425E 02	3.388E 02	3.345E 02	3.305E 02	3.283E 02	3.274E 02	3.000E-01
9	3.474E 02	3.465E 02	3.447E 02	3.423E 02	3.397E 02	3.367E 02	3.332E 02	3.299E 02	3.284E 02	3.278E 02	2.400E-01
8	3.456E 02	3.448E 02	3.431E 02	3.410E 02	3.386E 02	3.361E 02	3.334E 02	3.308E 02	3.296E 02	3.289E 02	1.800E-01
7	3.444E 02	3.438E 02	3.423E 02	3.404E 02	3.383E 02	3.363E 02	3.340E 02	3.318E 02	3.307E 02	3.301E 02	1.350E-01
6	3.436E 02	3.430E 02	3.417E 02	3.400E 02	3.383E 02	3.366E 02	3.346E 02	3.327E 02	3.316E 02	3.309E 02	1.000E-01
5	3.429E 02	3.424E 02	3.412E 02	3.398E 02	3.384E 02	3.369E 02	3.351E 02	3.333E 02	3.322E 02	3.314E 02	7.000E-02
4	3.423E 02	3.418E 02	3.407E 02	3.394E 02	3.381E 02	3.366E 02	3.349E 02	3.333E 02	3.320E 02	3.311E 02	4.500E-02
3	3.414E 02	3.403E 02	3.390E 02	3.376E 02	3.362E 02	3.349E 02	3.333E 02	3.317E 02	3.305E 02	3.295E 02	2.500E-02
2	3.353E 02	3.349E 02	3.341E 02	3.330E 02	3.319E 02	3.307E 02	3.293E 02	3.279E 02	3.267E 02	3.258E 02	1.000E-02
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

J \ I=	14	15	16	17	18	Y =
18	4.000E 02	4.000E 02	4.000E 02	4.000E 02	4.000E 02	4.900E-01
17	3.817E 02	3.842E 02	3.861E 02	3.870E 02	3.869E 02	4.800E-01
16	3.687E 02	3.721E 02	3.748E 02	3.761E 02	3.761E 02	4.700E-01
15	3.554E 02	3.586E 02	3.612E 02	3.626E 02	3.625E 02	4.550E-01
14	3.449E 02	3.469E 02	3.485E 02	3.494E 02	3.494E 02	4.350E-01
13	3.369E 02	3.379E 02	3.386E 02	3.390E 02	3.389E 02	4.100E-01
12	3.311E 02	3.314E 02	3.316E 02	3.317E 02	3.317E 02	3.800E-01
11	3.275E 02	3.274E 02	3.275E 02	3.275E 02	3.274E 02	3.450E-01
10	3.259E 02	3.257E 02	3.256E 02	3.256E 02	3.256E 02	3.000E-01
9	3.259E 02	3.254E 02	3.252E 02	3.252E 02	3.251E 02	2.400E-01
8	3.260E 02	3.252E 02	3.248E 02	3.247E 02	3.246E 02	1.800E-01
7	3.264E 02	3.251E 02	3.243E 02	3.241E 02	3.240E 02	1.350E-01
6	3.268E 02	3.250E 02	3.239E 02	3.235E 02	3.234E 02	1.000E-01
5	3.272E 02	3.250E 02	3.235E 02	3.228E 02	3.228E 02	7.000E-02
4	3.272E 02	3.250E 02	3.230E 02	3.221E 02	3.221E 02	4.500E-02
3	3.263E 02	3.245E 02	3.224E 02	3.213E 02	3.212E 02	2.500E-02
2	3.235E 02	3.226E 02	3.213E 02	3.201E 02	3.201E 02	1.000E-02
X=	0.435	0.455	0.470	0.480	0.490	

TURBULENCE ENERGY

I=	2	3	4	5	6	7	8	9	10	11	Y =
J											
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	6.72E-10	7.08E-10	7.90E-10	9.27E-10	1.12E-09	1.36E-09	1.66E-09	1.94E-09	2.00E-09	1.82E-09	4.80E-01
16	6.86E-10	7.27E-10	8.19E-10	9.70E-10	1.17E-09	1.41E-09	1.69E-09	1.94E-09	2.03E-09	1.90E-09	4.70E-01
15	7.28E-10	7.83E-10	9.01E-10	1.08E-09	1.31E-09	1.54E-09	1.80E-09	2.03E-09	2.19E-09	2.13E-09	4.55E-01
14	8.15E-10	8.93E-10	1.05E-09	1.27E-09	1.51E-09	1.75E-09	1.98E-09	2.24E-09	2.49E-09	2.50E-09	4.35E-01
13	9.52E-10	1.06E-09	1.26E-09	1.50E-09	1.75E-09	1.98E-09	2.22E-09	2.52E-09	2.85E-09	2.93E-09	4.10E-01
12	1.15E-09	1.27E-09	1.49E-09	1.74E-09	1.99E-09	2.22E-09	2.48E-09	2.84E-09	3.25E-09	3.41E-09	3.80E-01
11	1.41E-09	1.50E-09	1.72E-09	1.97E-09	2.21E-09	2.45E-09	2.73E-09	3.16E-09	3.66E-09	3.95E-09	3.45E-01
10	1.83E-09	1.80E-09	1.95E-09	2.18E-09	2.41E-09	2.65E-09	2.96E-09	3.45E-09	4.06E-09	4.52E-09	3.00E-01
9	2.25E-09	2.03E-09	2.05E-09	2.24E-09	2.47E-09	2.71E-09	3.05E-09	3.59E-09	4.33E-09	5.02E-09	2.40E-01
8	2.50E-09	2.15E-09	2.03E-09	2.14E-09	2.33E-09	2.56E-09	2.89E-09	3.45E-09	4.32E-09	5.28E-09	1.80E-01
7	2.60E-09	2.19E-09	1.99E-09	2.05E-09	2.24E-09	2.50E-09	2.86E-09	3.44E-09	4.27E-09	5.37E-09	1.35E-01
6	2.66E-09	2.20E-09	1.97E-09	2.06E-09	2.34E-09	2.69E-09	3.19E-09	3.87E-09	4.65E-09	5.62E-09	1.00E-01
5	2.98E-09	2.30E-09	2.14E-09	2.48E-09	2.98E-09	3.56E-09	4.31E-09	5.13E-09	5.81E-09	6.52E-09	7.00E-02
4	3.98E-09	3.19E-09	3.69E-09	4.54E-09	5.43E-09	6.24E-09	6.98E-09	7.54E-09	7.92E-09	8.27E-09	4.50E-02
3	8.47E-09	8.83E-09	9.86E-09	1.07E-08	1.11E-08	1.11E-08	1.09E-08	1.06E-08	1.03E-08	1.02E-08	2.50E-02
2	2.77E-08	2.47E-08	2.17E-08	1.93E-08	1.72E-08	1.53E-08	1.36E-08	1.21E-08	1.12E-08	1.08E-08	1.00E-02
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

I=	14	15	16	17	18	Y =
J						
18	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	9.41E-10	6.17E-10	3.19E-10	9.97E-11	1.00E-10	4.80E-01
16	1.00E-09	6.51E-10	3.37E-10	9.99E-11	1.00E-10	4.70E-01
15	1.16E-09	7.59E-10	3.91E-10	1.00E-10	1.00E-10	4.55E-01
14	1.44E-09	9.54E-10	4.97E-10	1.01E-10	1.00E-10	4.35E-01
13	1.82E-09	1.24E-09	6.58E-10	1.03E-10	1.00E-10	4.10E-01
12	2.35E-09	1.64E-09	8.89E-10	1.17E-10	1.00E-10	3.80E-01
11	3.09E-09	2.24E-09	1.25E-09	1.70E-10	1.00E-10	3.45E-01
10	4.18E-09	3.24E-09	1.94E-09	3.63E-10	1.00E-10	3.00E-01
9	5.48E-09	4.75E-09	3.30E-09	1.11E-09	1.00E-10	2.40E-01
8	6.88E-09	6.61E-09	5.44E-09	2.90E-09	1.00E-10	1.80E-01
7	8.26E-09	8.48E-09	8.01E-09	5.77E-09	1.00E-10	1.35E-01
6	9.65E-09	1.03E-08	1.05E-08	9.29E-09	1.00E-10	1.00E-01
5	1.11E-08	1.22E-08	1.28E-08	1.29E-08	1.00E-10	7.00E-02
4	1.23E-08	1.46E-08	1.55E-08	1.59E-08	1.00E-10	4.50E-02
3	1.26E-08	1.68E-08	1.94E-08	1.93E-08	1.00E-10	2.50E-02
2	1.38E-08	1.71E-08	2.47E-08	2.91E-08	1.00E-10	1.00E-02
X=	0.435	0.455	0.470	0.480	0.490	

ENERGY DISSIPATION

J	I= 2	3	4	5	6	7	8	9	10	11	Y =
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	1.37E-12	1.47E-12	1.74E-12	2.21E-12	2.93E-12	3.92E-12	5.29E-12	6.67E-12	7.00E-12	6.06E-12	4.80E-01
16	1.41E-12	8.62E-13	8.66E-13	1.06E-12	1.37E-12	1.79E-12	2.33E-12	2.82E-12	2.91E-12	2.54E-12	4.70E-01
15	1.54E-12	6.90E-13	5.22E-13	5.81E-13	7.35E-13	9.36E-13	1.17E-12	1.34E-12	1.37E-12	1.22E-12	4.55E-01
14	1.82E-12	7.23E-13	4.36E-13	4.18E-13	5.00E-13	6.12E-13	7.27E-13	8.05E-13	8.38E-13	7.97E-13	4.35E-01
13	2.30E-12	8.56E-13	4.55E-13	3.80E-13	4.16E-13	4.84E-13	5.55E-13	6.22E-13	7.01E-13	7.35E-13	4.10E-01
12	3.04E-12	1.07E-12	5.22E-13	3.91E-13	3.91E-13	4.36E-13	5.00E-13	5.95E-13	7.38E-13	8.55E-13	3.80E-01
11	4.17E-12	1.37E-12	6.20E-13	4.24E-13	3.95E-13	4.27E-13	5.01E-13	6.44E-13	8.68E-13	1.10E-12	3.45E-01
10	6.13E-12	1.89E-12	7.85E-13	4.83E-13	4.16E-13	4.40E-13	5.31E-13	7.28E-13	1.05E-12	1.44E-12	3.00E-01
9	8.38E-12	2.46E-12	9.54E-13	5.37E-13	4.32E-13	4.43E-13	5.39E-13	7.72E-13	1.20E-12	1.76E-12	2.40E-01
8	9.83E-12	2.84E-12	1.07E-12	5.63E-13	4.19E-13	4.07E-13	4.83E-13	7.05E-13	1.18E-12	1.94E-12	1.80E-01
7	1.04E-11	3.00E-12	1.13E-12	5.77E-13	4.16E-13	4.00E-13	4.80E-13	7.06E-13	1.17E-12	2.00E-12	1.35E-01
6	1.09E-11	3.07E-12	1.13E-12	5.97E-13	4.73E-13	5.17E-13	6.95E-13	1.05E-12	1.58E-12	2.37E-12	1.00E-01
5	1.27E-11	3.27E-12	1.20E-12	8.20E-13	9.74E-13	1.31E-12	1.85E-12	2.49E-12	3.12E-12	3.83E-12	7.00E-02
4	1.97E-11	4.57E-12	2.89E-12	3.76E-12	4.82E-12	5.75E-12	6.51E-12	7.04E-12	7.45E-12	7.96E-12	4.50E-02
3	6.12E-11	2.33E-11	2.27E-11	2.38E-11	2.36E-11	2.25E-11	2.09E-11	1.92E-11	1.81E-11	1.79E-11	2.50E-02
2	1.81E-10	1.52E-10	1.25E-10	1.05E-10	8.88E-11	7.46E-11	6.20E-11	5.21E-11	4.63E-11	4.43E-11	1.00E-02
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

J	I= 14	15	16	17	18	Y =
18	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	2.26E-12	1.20E-12	4.47E-13	7.82E-14	1.00E-06	4.80E-01
16	1.05E-12	6.03E-13	2.58E-13	3.92E-14	1.00E-06	4.70E-01
15	5.52E-13	3.37E-13	1.62E-13	3.95E-14	1.00E-06	4.55E-01
14	4.04E-13	2.58E-13	1.35E-13	3.97E-14	1.00E-06	4.35E-01
13	4.44E-13	2.93E-13	1.58E-13	4.10E-14	1.00E-06	4.10E-01
12	6.42E-13	4.39E-13	2.38E-13	4.98E-14	1.00E-06	3.80E-01
11	1.07E-12	7.69E-13	4.29E-13	8.71E-14	1.00E-06	3.45E-01
10	1.95E-12	1.58E-12	9.79E-13	2.71E-13	1.00E-06	3.00E-01
9	3.34E-12	3.29E-12	2.62E-12	1.45E-12	1.00E-06	2.40E-01
8	5.33E-12	6.39E-12	6.71E-12	6.11E-12	1.00E-06	1.80E-01
7	7.59E-12	1.06E-11	1.39E-11	1.72E-11	1.00E-06	1.35E-01
6	9.91E-12	1.51E-11	2.30E-11	3.51E-11	1.00E-06	1.00E-01
5	1.27E-11	2.05E-11	3.32E-11	5.72E-11	1.00E-06	7.00E-02
4	1.69E-11	2.85E-11	4.68E-11	7.89E-11	1.00E-06	4.50E-02
3	2.65E-11	4.34E-11	7.33E-11	1.05E-10	1.00E-06	2.50E-02
2	6.34E-11	8.81E-11	1.52E-10	1.95E-10	1.00E-06	1.00E-02
X=	0.435	0.455	0.470	0.480	0.490	

VISCOSITY

	I= 2	3	4	5	6	7	8	9	10	11	Y =
J											
18	3.72E-04	3.72E-04	3.72E-04	3.72E-04	3.72E-04	3.72E-04	3.72E-04	3.72E-04	3.72E-04	3.72E-04	4.90E-01
17	4.01E-04	4.02E-04	4.03E-04	4.06E-04	4.09E-04	4.13E-04	4.18E-04	4.21E-04	4.22E-04	4.20E-04	4.80E-01
16	4.01E-04	4.26E-04	4.40E-04	4.50E-04	4.60E-04	4.70E-04	4.80E-04	4.89E-04	4.96E-04	4.96E-04	4.70E-01
15	4.02E-04	4.50E-04	5.08E-04	5.49E-04	5.76E-04	5.95E-04	6.14E-04	6.40E-04	6.78E-04	6.99E-04	4.55E-01
14	4.04E-04	4.69E-04	5.93E-04	7.08E-04	7.71E-04	8.08E-04	8.46E-04	9.17E-04	1.02E-03	1.06E-03	4.35E-01
13	4.07E-04	4.87E-04	6.75E-04	8.89E-04	1.01E-03	1.08E-03	1.15E-03	1.27E-03	1.39E-03	1.39E-03	4.10E-01
12	4.10E-04	5.04E-04	7.45E-04	1.05E-03	1.26E-03	1.36E-03	1.45E-03	1.56E-03	1.63E-03	1.57E-03	3.80E-01
11	4.14E-04	5.17E-04	7.90E-04	1.18E-03	1.46E-03	1.60E-03	1.68E-03	1.73E-03	1.72E-03	1.62E-03	3.45E-01
10	4.20E-04	5.22E-04	7.97E-04	1.24E-03	1.59E-03	1.77E-03	1.82E-03	1.81E-03	1.75E-03	1.62E-03	3.00E-01
9	4.25E-04	5.19E-04	7.57E-04	1.19E-03	1.61E-03	1.83E-03	1.88E-03	1.84E-03	1.74E-03	1.62E-03	2.40E-01
8	4.28E-04	5.14E-04	7.07E-04	1.08E-03	1.51E-03	1.79E-03	1.89E-03	1.85E-03	1.75E-03	1.63E-03	1.80E-01
7	4.29E-04	5.12E-04	6.80E-04	1.01E-03	1.43E-03	1.74E-03	1.87E-03	1.84E-03	1.73E-03	1.63E-03	1.35E-01
6	4.30E-04	5.10E-04	6.72E-04	9.98E-04	1.39E-03	1.60E-03	1.65E-03	1.62E-03	1.58E-03	1.54E-03	1.00E-01
5	4.33E-04	5.14E-04	7.09E-04	1.03E-03	1.17E-03	1.22E-03	1.25E-03	1.30E-03	1.32E-03	1.35E-03	7.00E-02
4	4.43E-04	5.67E-04	7.84E-04	8.54E-04	9.08E-04	9.65E-04	1.03E-03	1.08E-03	1.11E-03	1.12E-03	4.50E-02
3	4.75E-04	6.65E-04	7.48E-04	7.93E-04	8.27E-04	8.52E-04	8.70E-04	8.82E-04	8.87E-04	8.87E-04	2.50E-02
2	7.44E-04	7.23E-04	7.01E-04	6.82E-04	6.65E-04	6.49E-04	6.32E-04	6.17E-04	6.08E-04	6.05E-04	1.00E-02
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

	I= 14	15	16	17	18	Y =
J						
18	3.72E-04	3.72E-04	3.72E-04	3.72E-04	3.72E-04	4.90E-01
17	4.06E-04	4.00E-04	3.92E-04	3.83E-04	3.72E-04	4.80E-01
16	4.55E-04	4.34E-04	4.11E-04	3.94E-04	3.72E-04	4.70E-01
15	5.86E-04	5.22E-04	4.55E-04	3.94E-04	3.72E-04	4.55E-01
14	8.19E-04	6.80E-04	5.32E-04	3.94E-04	3.72E-04	4.35E-01
13	1.03E-03	8.31E-04	6.12E-04	3.95E-04	3.72E-04	4.10E-01
12	1.13E-03	9.11E-04	6.63E-04	3.96E-04	3.72E-04	3.80E-01
11	1.15E-03	9.42E-04	6.91E-04	4.01E-04	3.72E-04	3.45E-01
10	1.16E-03	9.55E-04	7.10E-04	4.15E-04	3.72E-04	3.00E-01
9	1.16E-03	9.72E-04	7.37E-04	4.46E-04	3.72E-04	2.40E-01
8	1.15E-03	9.71E-04	7.58E-04	4.92E-04	3.72E-04	1.80E-01
7	1.16E-03	9.70E-04	7.77E-04	5.42E-04	3.72E-04	1.35E-01
6	1.20E-03	9.82E-04	7.91E-04	5.87E-04	3.72E-04	1.00E-01
5	1.22E-03	1.01E-03	8.06E-04	6.25E-04	3.72E-04	7.00E-02
4	1.15E-03	1.02E-03	8.24E-04	6.54E-04	3.72E-04	4.50E-02
3	8.94E-04	9.42E-04	8.22E-04	6.82E-04	3.72E-04	2.50E-02
2	6.34E-04	6.64E-04	7.23E-04	7.53E-04	3.72E-04	1.00E-02
X=	0.435	0.455	0.470	0.480	0.490	

MEAN SQUARE TEMPERATURE FLUCTUATION

I=	2	3	4	5	6	7	8	9	10	11	Y =
J											
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.80E-01
16	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.70E-01
15	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.55E-01
14	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.35E-01
13	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.10E-01
12	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	3.80E-01
11	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	3.45E-01
10	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	3.00E-01
9	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	2.40E-01
8	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.80E-01
7	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.35E-01
6	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-01
5	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	7.00E-02
4	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.50E-02
3	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	2.50E-02
2	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-02
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

I=	14	15	16	17	18	Y =
J						
18	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.80E-01
16	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.70E-01
15	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.55E-01
14	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.35E-01
13	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.10E-01
12	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	3.80E-01
11	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	3.45E-01
10	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	3.00E-01
9	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	2.40E-01
8	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.80E-01
7	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.35E-01
6	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-01
5	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	7.00E-02
4	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	4.50E-02
3	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	2.50E-02
2	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-06	1.00E-02
X=	0.435	0.455	0.470	0.480	0.490	

I	L.LAM.NU/UL	L.LAM.NU/L	L.TUR.NU/UL	L.TUR.NU/L
2	0.681E 01	0.320E 01	0.323E 00	0.152E 00
3	0.744E 01	0.350E 01	0.361E 00	0.170E 00
4	0.850E 01	0.400E 01	0.433E 00	0.204E 00
5	0.992E 01	0.466E 01	0.542E 00	0.255E 00
6	0.118E 02	0.553E 01	0.698E 00	0.328E 00
7	0.141E 02	0.661E 01	0.910E 00	0.428E 00
8	0.169E 02	0.795E 01	0.120E 01	0.563E 00
9	0.198E 02	0.929E 01	0.150E 01	0.704E 00
10	0.222E 02	0.104E 02	0.171E 01	0.802E 00
11	0.230E 02	0.108E 02	0.170E 01	0.798E 00
12	0.223E 02	0.105E 02	0.153E 01	0.721E 00
13	0.206E 02	0.970E 01	0.129E 01	0.607E 00
14	0.183E 02	0.859E 01	0.101E 01	0.473E 00
15	0.158E 02	0.743E 01	0.722E 00	0.340E 00
16	0.139E 02	0.654E 01	0.476E 00	0.224E 00
17	0.130E 02	0.613E 01	0.269E 00	0.126E 00

AV.LAM.NU/UL = 16.74229 AV.LAM.NU/L = 7.86888 AV.TUR.NU/UL = 1.10472 AV.TUR.NU/L = 0.51922

TEMPERATURE

I=	2	3	4	5	6	7	8	9	10	11	Y =
J											
18	4.00E 02	4.00E 02	4.00E 02	4.00E 02	4.00E 02	4.00E 02	4.00E 02	4.00E 02	4.00E 02	4.00E 02	4.90E-01
17	9.32E-01	9.26E-01	9.15E-01	9.01E-01	8.82E-01	8.59E-01	8.31E-01	8.02E-01	7.78E-01	7.70E-01	4.80E-01
16	8.78E-01	8.69E-01	8.52E-01	8.29E-01	8.00E-01	7.65E-01	7.20E-01	6.78E-01	6.41E-01	6.27E-01	4.70E-01
15	8.06E-01	7.96E-01	7.77E-01	7.48E-01	7.10E-01	6.64E-01	6.07E-01	5.55E-01	5.14E-01	5.00E-01	4.55E-01
14	7.29E-01	7.21E-01	7.02E-01	6.71E-01	6.30E-01	5.78E-01	5.17E-01	4.63E-01	4.25E-01	4.14E-01	4.35E-01
13	6.60E-01	6.52E-01	6.34E-01	6.04E-01	5.63E-01	5.12E-01	4.53E-01	4.00E-01	3.66E-01	3.55E-01	4.10E-01
12	6.03E-01	5.94E-01	5.75E-01	5.47E-01	5.08E-01	4.60E-01	4.06E-01	3.55E-01	3.24E-01	3.13E-01	3.80E-01
11	5.54E-01	5.44E-01	5.26E-01	4.99E-01	4.64E-01	4.21E-01	3.72E-01	3.25E-01	2.97E-01	2.86E-01	3.45E-01
10	5.06E-01	4.97E-01	4.78E-01	4.55E-01	4.25E-01	3.88E-01	3.45E-01	3.05E-01	2.83E-01	2.74E-01	3.00E-01
9	4.74E-01	4.65E-01	4.47E-01	4.23E-01	3.97E-01	3.67E-01	3.32E-01	2.99E-01	2.84E-01	2.78E-01	2.40E-01
8	4.56E-01	4.48E-01	4.31E-01	4.10E-01	3.86E-01	3.61E-01	3.34E-01	3.08E-01	2.96E-01	2.89E-01	1.80E-01
7	4.44E-01	4.38E-01	4.23E-01	4.04E-01	3.83E-01	3.63E-01	3.40E-01	3.18E-01	3.07E-01	3.01E-01	1.35E-01
6	4.36E-01	4.30E-01	4.17E-01	4.00E-01	3.83E-01	3.66E-01	3.46E-01	3.27E-01	3.16E-01	3.09E-01	1.00E-01
5	4.29E-01	4.24E-01	4.12E-01	3.98E-01	3.84E-01	3.69E-01	3.51E-01	3.33E-01	3.22E-01	3.14E-01	7.00E-02
4	4.23E-01	4.18E-01	4.07E-01	3.94E-01	3.81E-01	3.66E-01	3.49E-01	3.33E-01	3.20E-01	3.11E-01	4.50E-02
3	4.14E-01	4.03E-01	3.90E-01	3.76E-01	3.62E-01	3.49E-01	3.33E-01	3.17E-01	3.05E-01	2.95E-01	2.50E-02
2	3.53E-01	3.49E-01	3.41E-01	3.30E-01	3.19E-01	3.07E-01	2.93E-01	2.79E-01	2.67E-01	2.58E-01	1.00E-02
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

I=	14	15	16	17	18	Y =
J						
18	4.00E 02	4.00E 02	4.00E 02	4.00E 02	4.00E 02	4.90E-01
17	8.17E-01	8.42E-01	8.61E-01	8.70E-01	3.87E 02	4.80E-01
16	6.87E-01	7.21E-01	7.48E-01	7.61E-01	3.76E 02	4.70E-01
15	5.54E-01	5.86E-01	6.12E-01	6.26E-01	3.63E 02	4.55E-01
14	4.49E-01	4.69E-01	4.85E-01	4.94E-01	3.49E 02	4.35E-01
13	3.69E-01	3.79E-01	3.86E-01	3.90E-01	3.39E 02	4.10E-01
12	3.11E-01	3.14E-01	3.16E-01	3.17E-01	3.32E 02	3.80E-01
11	2.75E-01	2.74E-01	2.75E-01	2.75E-01	3.27E 02	3.45E-01
10	2.59E-01	2.57E-01	2.56E-01	2.56E-01	3.26E 02	3.00E-01
9	2.59E-01	2.54E-01	2.52E-01	2.52E-01	3.25E 02	2.40E-01
8	2.60E-01	2.52E-01	2.48E-01	2.47E-01	3.25E 02	1.80E-01
7	2.64E-01	2.51E-01	2.43E-01	2.41E-01	3.24E 02	1.35E-01
6	2.68E-01	2.50E-01	2.39E-01	2.35E-01	3.23E 02	1.00E-01
5	2.72E-01	2.50E-01	2.35E-01	2.28E-01	3.23E 02	7.00E-02
4	2.72E-01	2.50E-01	2.30E-01	2.21E-01	3.22E 02	4.50E-02
3	2.63E-01	2.45E-01	2.24E-01	2.13E-01	3.21E 02	2.50E-02
2	2.35E-01	2.26E-01	2.13E-01	2.01E-01	3.20E 02	1.00E-02
X=	0.435	0.455	0.470	0.480	0.490	

RESULTANT VELOCITY

I=	2	3	4	5	6	7	8	9	10	11	Y =
J											
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	1.61E-07	4.19E-07	4.73E-07	1.80E-07	1.36E-06	3.24E-06	5.60E-06	7.57E-06	7.51E-06	5.36E-06	4.80E-01
16	3.83E-07	5.93E-07	4.05E-07	1.47E-06	4.46E-06	8.84E-06	1.42E-05	1.83E-05	1.79E-05	1.31E-05	4.70E-01
15	5.80E-07	1.68E-07	1.47E-06	4.63E-06	9.49E-06	1.61E-05	2.37E-05	2.92E-05	2.80E-05	2.13E-05	4.55E-01
14	4.38E-07	1.44E-06	4.76E-06	9.75E-06	1.60E-05	2.36E-05	3.19E-05	3.74E-05	3.57E-05	2.88E-05	4.35E-01
13	1.22E-06	4.63E-06	9.85E-06	1.63E-05	2.32E-05	3.05E-05	3.81E-05	4.27E-05	4.16E-05	3.64E-05	4.10E-01
12	4.21E-06	1.00E-05	1.67E-05	2.37E-05	3.04E-05	3.67E-05	4.27E-05	4.64E-05	4.70E-05	4.48E-05	3.80E-01
11	9.01E-06	1.79E-05	2.55E-05	3.21E-05	3.79E-05	4.26E-05	4.65E-05	4.90E-05	5.18E-05	5.89E-05	3.45E-01
10	1.54E-05	2.82E-05	3.69E-05	4.24E-05	4.62E-05	4.87E-05	4.99E-05	5.00E-05	5.44E-05	6.27E-05	3.00E-01
9	2.13E-05	3.81E-05	4.85E-05	5.28E-05	5.41E-05	5.35E-05	5.05E-05	4.50E-05	4.78E-05	6.45E-05	2.40E-01
8	2.43E-05	4.38E-05	5.59E-05	6.01E-05	5.95E-05	5.60E-05	4.87E-05	3.45E-05	2.56E-05	5.00E-05	1.80E-01
7	2.52E-05	4.57E-05	5.85E-05	6.22E-05	6.04E-05	5.56E-05	4.77E-05	3.30E-05	8.38E-06	2.51E-05	1.35E-01
6	2.66E-05	4.72E-05	5.86E-05	6.04E-05	5.72E-05	5.24E-05	4.70E-05	4.14E-05	3.47E-05	3.03E-05	1.00E-01
5	3.10E-05	5.06E-05	5.75E-05	5.52E-05	5.03E-05	4.79E-05	5.15E-05	6.16E-05	6.87E-05	6.70E-05	7.00E-02
4	4.11E-05	5.51E-05	5.25E-05	4.51E-05	4.43E-05	5.63E-05	7.73E-05	9.95E-05	1.13E-04	1.15E-04	4.50E-02
3	5.03E-05	4.90E-05	4.26E-05	5.48E-05	7.94E-05	1.07E-04	1.35E-04	1.60E-04	1.75E-04	1.78E-04	2.50E-02
2	3.76E-05	7.74E-05	1.15E-04	1.47E-04	1.75E-04	2.01E-04	2.25E-04	2.44E-04	2.56E-04	2.58E-04	1.00E-02
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

I=	14	15	16	17	18	Y =
J						
18	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	4.70E-07	5.56E-07	3.01E-07	8.44E-08	0.0	4.80E-01
16	5.81E-07	6.86E-07	6.36E-07	3.15E-07	0.0	4.70E-01
15	2.84E-06	2.86E-07	7.24E-07	6.39E-07	0.0	4.55E-01
14	7.35E-06	2.76E-06	4.77E-07	6.80E-07	0.0	4.35E-01
13	1.38E-05	7.13E-06	2.61E-06	3.18E-07	0.0	4.10E-01
12	2.28E-05	1.39E-05	7.04E-06	2.18E-06	0.0	3.80E-01
11	3.65E-05	2.47E-05	1.40E-05	5.18E-06	0.0	3.45E-01
10	5.89E-05	4.33E-05	2.61E-05	1.03E-05	0.0	3.00E-01
9	9.03E-05	7.33E-05	4.76E-05	2.07E-05	0.0	2.40E-01
8	1.21E-04	1.10E-04	7.79E-05	3.77E-05	0.0	1.80E-01
7	1.38E-04	1.42E-04	1.11E-04	5.92E-05	0.0	1.35E-01
6	1.34E-04	1.58E-04	1.40E-04	8.32E-05	0.0	1.00E-01
5	1.12E-04	1.55E-04	1.59E-04	1.07E-04	0.0	7.00E-02
4	9.25E-05	1.29E-04	1.60E-04	1.25E-04	0.0	4.50E-02
3	1.28E-04	1.08E-04	1.33E-04	1.23E-04	0.0	2.50E-02
2	2.09E-04	1.68E-04	1.16E-04	6.52E-05	0.0	1.00E-02
X=	0.435	0.455	0.470	0.480	0.490	

ANGLE OF INCLINATION

J \ I=	2	3	4	5	6	7	8	9	10	11	Y =
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	2.98E 01	1.05E 01	-3.92E 00	2.49E 02	1.91E 02	1.85E 02	1.82E 02	1.81E 02	1.79E 02	1.76E 02	4.80E-01
16	5.55E 01	2.32E 01	-3.87E 01	2.16E 02	1.96E 02	1.89E 02	1.85E 02	1.82E 02	1.78E 02	1.73E 02	4.70E-01
15	8.01E 01	7.43E 01	2.29E 02	2.15E 02	2.02E 02	1.94E 02	1.88E 02	1.82E 02	1.76E 02	1.68E 02	4.55E-01
14	1.23E 02	2.17E 02	2.25E 02	2.19E 02	2.09E 02	2.00E 02	1.92E 02	1.83E 02	1.73E 02	1.62E 02	4.35E-01
13	2.36E 02	2.35E 02	2.31E 02	2.24E 02	2.16E 02	2.07E 02	1.97E 02	1.85E 02	1.70E 02	1.56E 02	4.10E-01
12	2.55E 02	2.47E 02	2.39E 02	2.32E 02	2.24E 02	2.14E 02	2.02E 02	1.87E 02	1.67E 02	1.51E 02	3.80E-01
11	2.62E 02	2.55E 02	2.48E 02	2.40E 02	2.32E 02	2.22E 02	2.09E 02	1.90E 02	1.66E 02	1.47E 02	3.45E-01
10	2.66E 02	2.61E 02	2.56E 02	2.48E 02	2.41E 02	2.32E 02	2.18E 02	1.96E 02	1.67E 02	1.44E 02	3.00E-01
9	2.68E 02	2.66E 02	2.63E 02	2.58E 02	2.52E 02	2.45E 02	2.33E 02	2.10E 02	1.70E 02	1.41E 02	2.40E-01
8	2.70E 02	2.69E 02	2.68E 02	2.66E 02	2.63E 02	2.60E 02	2.55E 02	2.40E 02	1.80E 02	1.34E 02	1.80E-01
7	2.70E 02	2.69E 02	2.69E 02	2.69E 02	2.70E 02	-8.86E 01	-8.50E 01	-7.91E 01	-6.70E 01	1.05E 02	1.35E-01
6	2.69E 02	2.68E 02	2.68E 02	2.70E 02	-8.63E 01	-7.90E 01	-6.61E 01	-4.56E 01	-1.78E 01	2.67E 01	1.00E-01
5	2.66E 02	2.64E 02	2.64E 02	2.68E 02	-8.15E 01	-6.44E 01	-4.18E 01	-2.21E 01	-8.14E 00	5.70E 00	7.00E-02
4	2.63E 02	2.58E 02	2.61E 02	-8.45E 01	-5.79E 01	-3.31E 01	-1.74E 01	-8.77E 00	-3.43E 00	1.52E 00	4.50E-02
3	2.67E 02	-8.93E 01	-6.28E 01	-3.14E 01	-1.62E 01	-9.26E 00	-5.30E 00	-2.86E 00	-1.18E 00	4.82E-01	2.50E-02
2	-4.50E 01	-1.60E 01	-7.05E 00	-3.72E 00	-2.33E 00	-1.54E 00	-9.78E-01	-5.65E-01	-2.40E-01	1.10E-01	1.00E-02
X=	0.010	0.025	0.045	0.070	0.100	0.135	0.180	0.240	0.300	0.345	

J \ I=	14	15	16	17	18	Y =
18	0.0	0.0	0.0	0.0	0.0	4.90E-01
17	1.17E 01	-7.19E 00	-2.42E 01	-4.50E 01	0.0	4.80E-01
16	9.15E 01	-1.25E 01	-4.36E 01	-6.50E 01	0.0	4.70E-01
15	1.33E 02	1.17E 02	-7.04E 01	-8.08E 01	0.0	4.55E-01
14	1.34E 02	1.33E 02	1.92E 02	2.65E 02	0.0	4.35E-01
13	1.27E 02	1.22E 02	1.20E 02	1.46E 02	0.0	4.10E-01
12	1.19E 02	1.12E 02	1.05E 02	1.00E 02	0.0	3.80E-01
11	1.13E 02	1.05E 02	9.91E 01	9.49E 01	0.0	3.45E-01
10	1.09E 02	1.02E 02	9.69E 01	9.36E 01	0.0	3.00E-01
9	1.06E 02	1.00E 02	9.62E 01	9.32E 01	0.0	2.40E-01
8	1.04E 02	9.94E 01	9.57E 01	9.29E 01	0.0	1.80E-01
7	1.02E 02	9.87E 01	9.56E 01	9.28E 01	0.0	1.35E-01
6	9.78E 01	9.73E 01	9.52E 01	9.26E 01	0.0	1.00E-01
5	8.65E 01	9.37E 01	9.41E 01	9.23E 01	0.0	7.00E-02
4	5.63E 01	8.28E 01	9.09E 01	9.13E 01	0.0	4.50E-02
3	1.90E 01	4.77E 01	7.81E 01	8.75E 01	0.0	2.50E-02
2	3.56E 00	8.77E 00	2.74E 01	6.03E 01	0.0	1.00E-02
X=	0.435	0.455	0.470	0.480	0.490	