

FREQUENCY DOMAIN ANALYSIS
OF ELECTROCARDIOGRAM

by

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ABSTRACT

The advent of digital computer technology has made possible analysis and computations which would be prohibited if undertaken by hand. The Fourier analysis of a periodic function of time can easily be done by modern computer techniques; thus making this method feasible.

An electrocardiogram consists of several distinct waves with different electrophysiological significance. Such an analog electrocardiogram can be analyzed by means of the Fourier analysis.

This thesis investigates the frequency domain analysis technique in analyzing electrocardiogram. Using this technique the frequency spectra of normal and abnormal cardiac waveforms are obtained. These various spectra are then discussed and an attempt is made to show that the frequency spectrum can be used to characterize the cardiac waveforms and hence to detect the heart diseases.

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CHAPTER 1

INTRODUCTION

The medical importance of most electrocardiologic signals is due to empirical relations established between them and the disease.

The technique of recording of the electrical activities of the human heart and the technique of analysis are developed rapidly since the advent of computer.

Thomas R. Taylor [1] has discussed the technique in computer-assisted ECG analysis. Hubert V. Pipberger [2] presented a paper in the automatic analysis of the P-QRS-T complex of the electrocardiogram by digital computer, and the use of computer in interpretation of electrocardiograms. L. F. Staples, J. E. Gustafson, G. J. Balm and W. A. Tate [3] have joined together to extract the method of computer interpretation of electrocardiogram. Robert J. Dobow [4] and Patrick A. Gorman have investigated the accuracy of the electrocardiographic measurement by computer. H. V. Pipberger [5] also presented a paper in the use of computer in ECG interpretation.

In the interpretation of electrocardiogram [1-10] the computer

is used to identify the wave from formation and calculate the amplitudes and time intervals of the component waves. By a comparison of these calculations with the stored ideal standards a diagnostic report can be given. The ideal standards ^{was} produced by a large well-documented library of electrocardiogram records with suitably established abnormalities confirmed by autopsy. The electrocardiogram records were stored in the computer and ideal average for each condition was produced, eventually providing the diagnostic standards.

As for the frequency domain studies in the electrocardiogram, J. V. der Groeben [11] has studied the spatial frequency distribution of the QRS Loop. Lee D. Cady and Leo J. Tick [12] have investigated that 34 frequencies (up to a maximum of 5 frequencies) are needed to reconstruct an electrocardiogram signal. Paul H. Laugner [13-15] has demonstrated the high-frequency components in the electrocardiogram given by the beading and notching of the QRS complex not recorded by the conventional electrocardiography in common use. A. M. Scher [16] has analysed the electrocardiogram and concluded that none of the curves studied had the amplitude at 100 Hz or at any higher frequency greater than 4 percent of the amplitude of the fundamental, and in each individual studied and in the

entire data, the contribution by frequencies over 70 Hz was less than 2% of the total curve area. Noel P. Thompson [17] has determined the frequencies which must be passed by an electrocardiographic amplifier to keep from disturbing the electrocardiographic signature. He found out that none of his records required a harmonic which had a frequency above 51.6 cycles, and he concluded that 51.6 cycles per second is sufficient as an upper cut-off frequency in all cases, but at worst, it would not tend to exceed this value by much. E. K. Franke [48] has studied the high frequency components in electrocardiogram by power spectrum analysis. A. J. Kerwin [49] has discussed the effect of the frequency response of electrocardiograms and vectrocardiograms.

However, the frequency domain analysis and the discussion of the various spectra of the electrocardiograms contained in this thesis have not been studied in the works already mentioned.

CHAPTER 11

ELECTROCARDIOGRAM

2.1. Electrocardiogram

The electrocardiogram [18-21] is a graphic recording of the electrical potential associated with the heart beat. The heart muscle is unique among the muscles of the body in that it possesses the property of automatic rhythmic contraction. The impulses which precede contraction arise in the conduction system of the heart. The impulses result in excitation of the muscle fibers throughout the myocardium. Impulse formation and conduction produce weak electric currents which spread through the entire body. By applying electrodes to various positions on the body and connecting these electrodes to an electrocardiographic apparatus the electrocardiogram is recorded. The connections of the apparatus are such that in upright deflection indicates positive potential and a downward deflection indicates negative potential.

The electrocardiogram helps in the diagnosis of most cardiac arrhythmias. It yields information about the state of the ventricular myocardium e.g. myocardial infarction, bundle branch block etc., atrial and ventricular hypertrophy, effects of cardiac drugs and disturbances in electrolyte metabolism.

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The electrocardiogram helps in the diagnosis of most cardiac arrhythmias. It yields information about the state of the ventricular myocardium e.g. myocardial infarction, bundle branch block etc., atrial and ventricular hypertrophy, effects of cardiac drugs and disturbances in electrolyte metabolism.

The electrocardiogram, however, tells nothing about the functional capacity of the heart, gives little or no information about the prognosis in the individual case, and does not replace clinical knowledge, experience, and judgment of the doctor. The variations of normal and abnormal electrocardiogram show considerable overlap.

Each portion of the electrocardiogram is associated with a specific anatomical or physiological portion of the heart, and an abnormality of a wave or complex indicates an abnormality in that portion of the heart.

An illustration of the electrical conduction system through the heart is shown in Fig 1.

A normal diagram of electrocardiographic complexes, intervals, and segment is shown in Fig 2.

The amplitude of a complex is measured vertically in millimeters and is a measure of voltage, thus 1 mm may equal 0.1 millivolt.

The duration of a complex is measured horizontally in seconds, from the point where the wave begins to the point where it returns to the base line.

The P-wave represents atrial depolarization, the initiation of the impulse in the S-A (sino atrial) node, its spread through the atrial walls, and contraction of the atria.

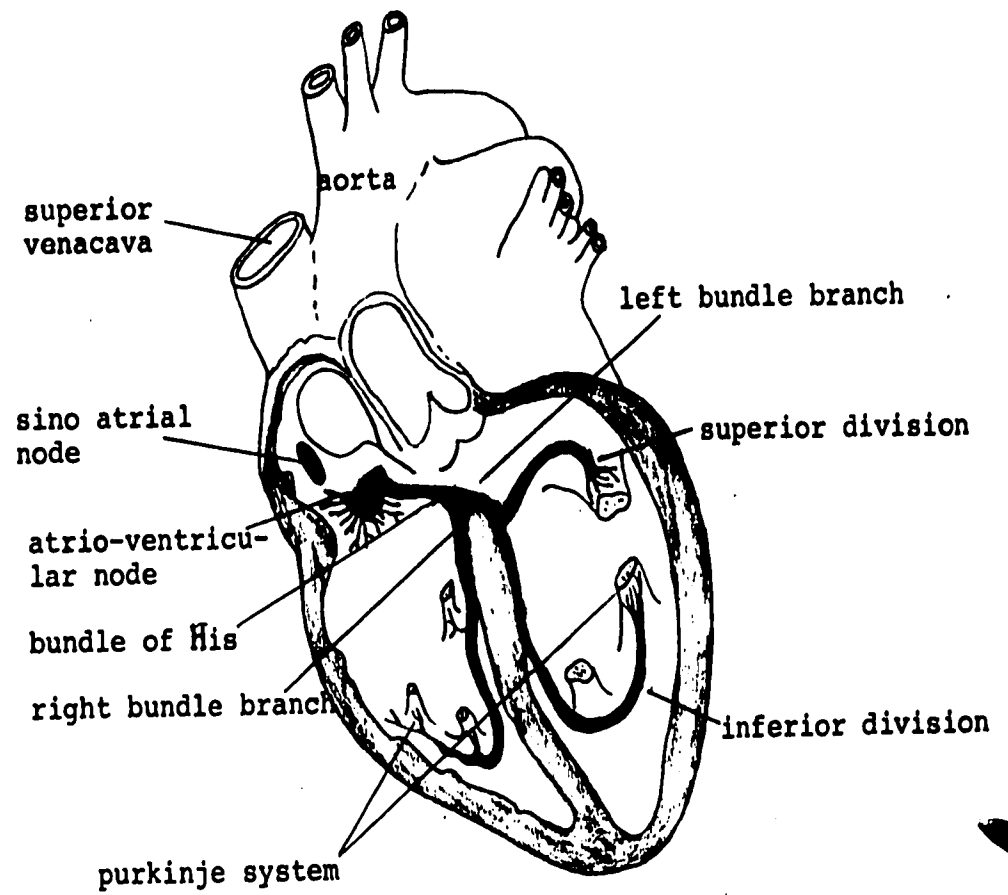


Figure 1. Illustration of Conduction System.

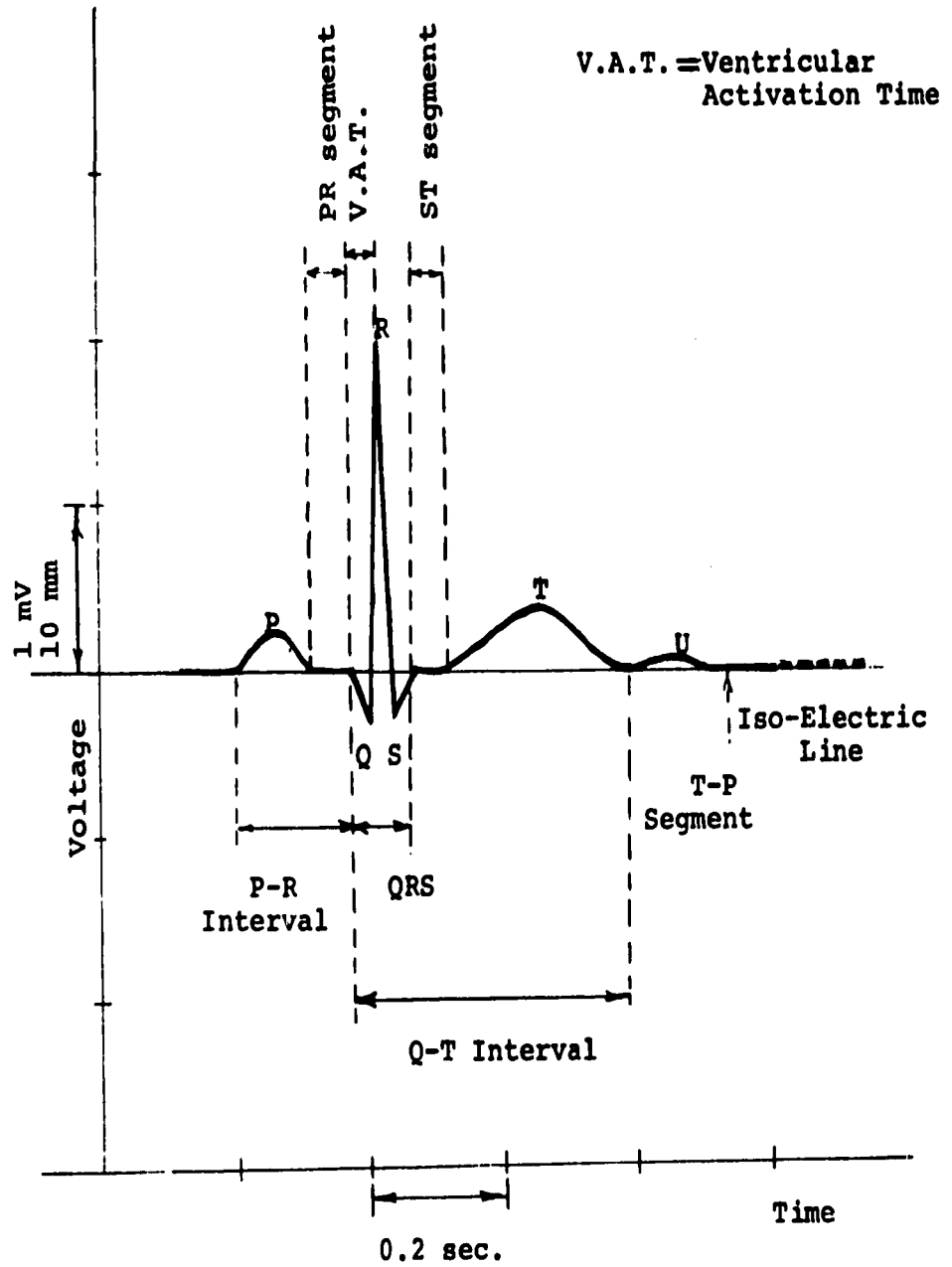


Figure 2. Diagram of Electrocardiographic Complexes, Intervals, and Segments.

The initial spread of the stimulus through the muscle is known as depolarization and the return of the stimulated muscle to the resting state is known as repolarization. An abnormality of P-wave indicates that the impulse has spread in an abnormal fashion, or that the atria are abnormal. P-waves are abnormal if they are more than 2.5 mm tall, or are different from the normal for that individual, or are superimposed on the preceding T-wave (Fig 2). The atrial repolarization is usually not seen in the average 12-lead electrocardiogram.

The P-R interval shows the time taken by the stimulus to travel from the S-A node to the A-V (atrioventricular) node. Prolongation of the P-R interval means delay or blocking of this spread. Shortening of the P-R interval indicates that the impulse has followed a path shorter than usual or that the ventricles has been stimulated before the normal impulse reached the A-V node. Abnormal P-R intervals are less than 0.1 sec or more than 0.2 sec in duration.

The QRS complex represents the spread of the stimulus through the interventricular conduction system and stimulation of the ventricular muscles. This is the ventricular depolarization. Prolongation of QRS indicates delay of conduction through the bundle of His and its arborizations or origination of a new stimulus in an abnormal ventricular

focus with resultant abnormal spread. The amplitude of QRS is due to the mass of active, ventricular myocardium plus the cardiac position. An abnormal QRS complex is more than 0.1 sec in duration (usually 20 mm).

Abnormal Q-waves are indicative of destruction of ventricular tissue. Q-waves are abnormal if they are longer than 0.04 sec or are abnormally deep.

R-wave is the first positive deflection during the ventricular depolarization. S-wave is the first negative deflection of the ventricular depolarization following the first positive deflection R-wave.

T-wave represents the ventricular repolarization. S-T segment and T-wave are associated with contraction of the ventricles. An abnormality of either or both indicates some physiological, organic or chemical change in the ventricular muscles. In the 3 standard leads (see section 2.2), S-T is abnormal if it is elevated more than 1 mm or depressed more than $\frac{1}{2}$ mm. T-wave is usually abnormal if it is less than 1 mm in amplitude in lead I or II.

The U-wave is a rounded wave which follows T-wave and may be fused with it. The U-waves have not been proven of great clinical importance, and their physiological cause is not certain. It may be a result of the failure of restoration of the normal membrane resting potential at com-

pletion of repolarization.

2.2. The Lead Systems

Different electrodes with leads [18,22,26,28] on different position of the human body to pick up the electrical signals generated by the heart, and to show the electrical activity of the heart as viewed from different directions. An electrode picks up the electrical activity of those cardiac structures which it faces. When electrodes are placed on the extremities as in the 3 standard Leads, Lead I, Lead II and Lead III, Figure 3, and the Augmented Unipolar Extremity Leads aVR, aVL and aVF, Figure 4, the activity of the heart is viewed from its lateral aspects. When the electrodes are placed on the anterior chest wall as in the 6 Unipolar Precordial Leads V_1 , V_2 , V_3 , V_4 , V_5 and V_6 , Figures 5a and 5b, the activity of the anterior and posterior surfaces of the heart is explored. Thus the 12 Leads are necessary in order to view the heart in all three dimensions. Each lead shows the difference between the voltages picked up by each of the two electrodes employed.

The Leads I, II and III are called the standard Leads because they have been in general used since the beginning of the electrocardiography and have become the standards. Additional leads have been added in order to correct the shortcoming of the standard Leads and add to the information obtained from them.

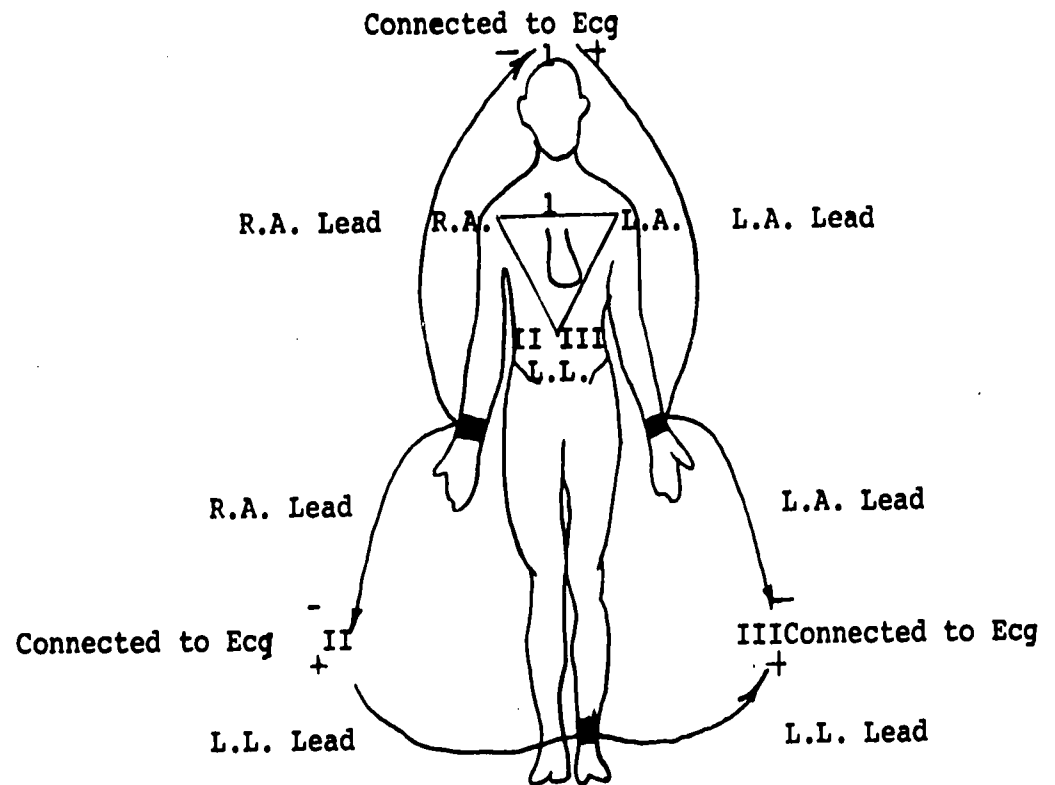


Figure 3. Connections for Bipolar
standard Leads I, II, and III.

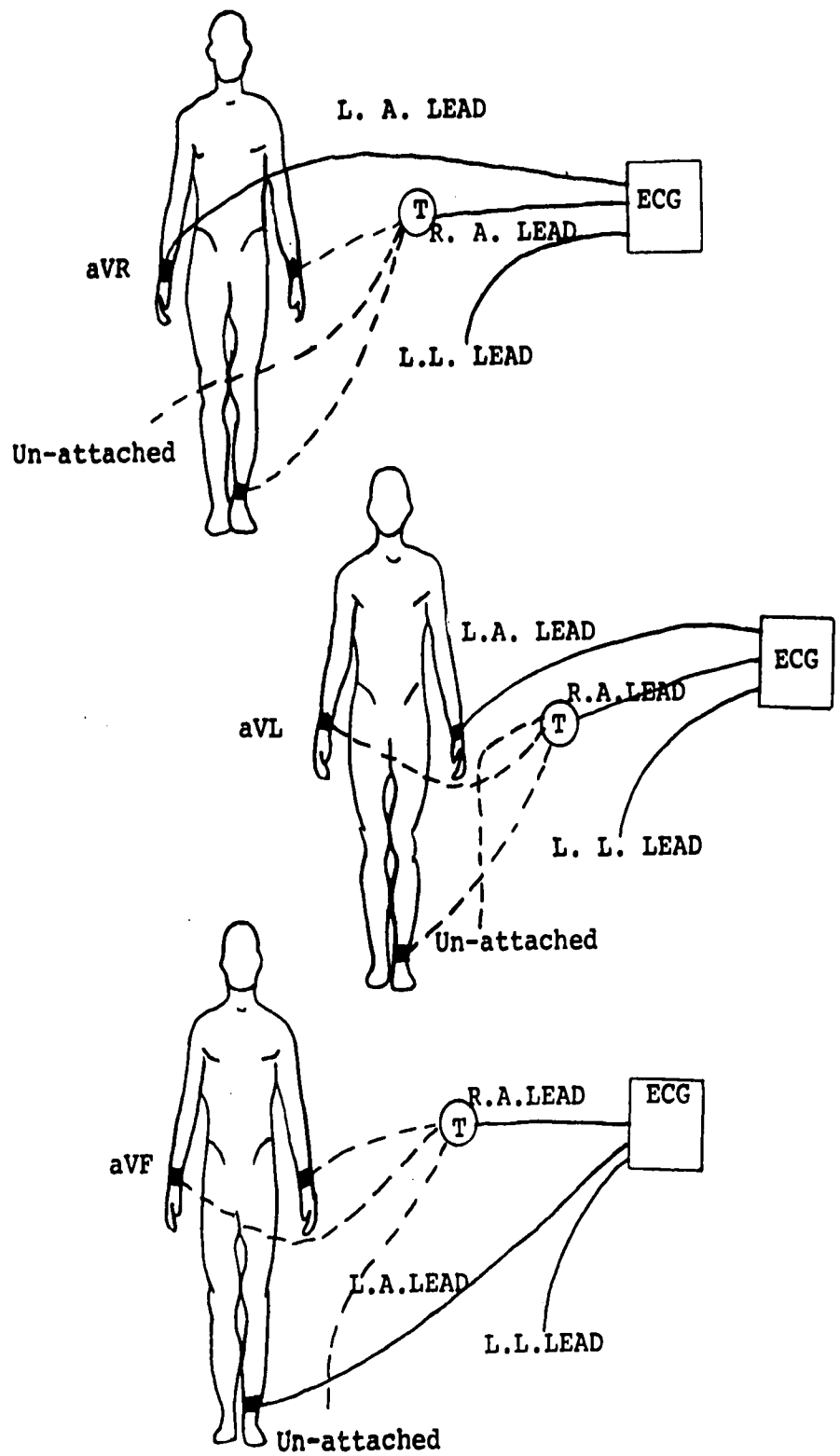


Figure 4. Augmented Unipolar Extremity Leads.

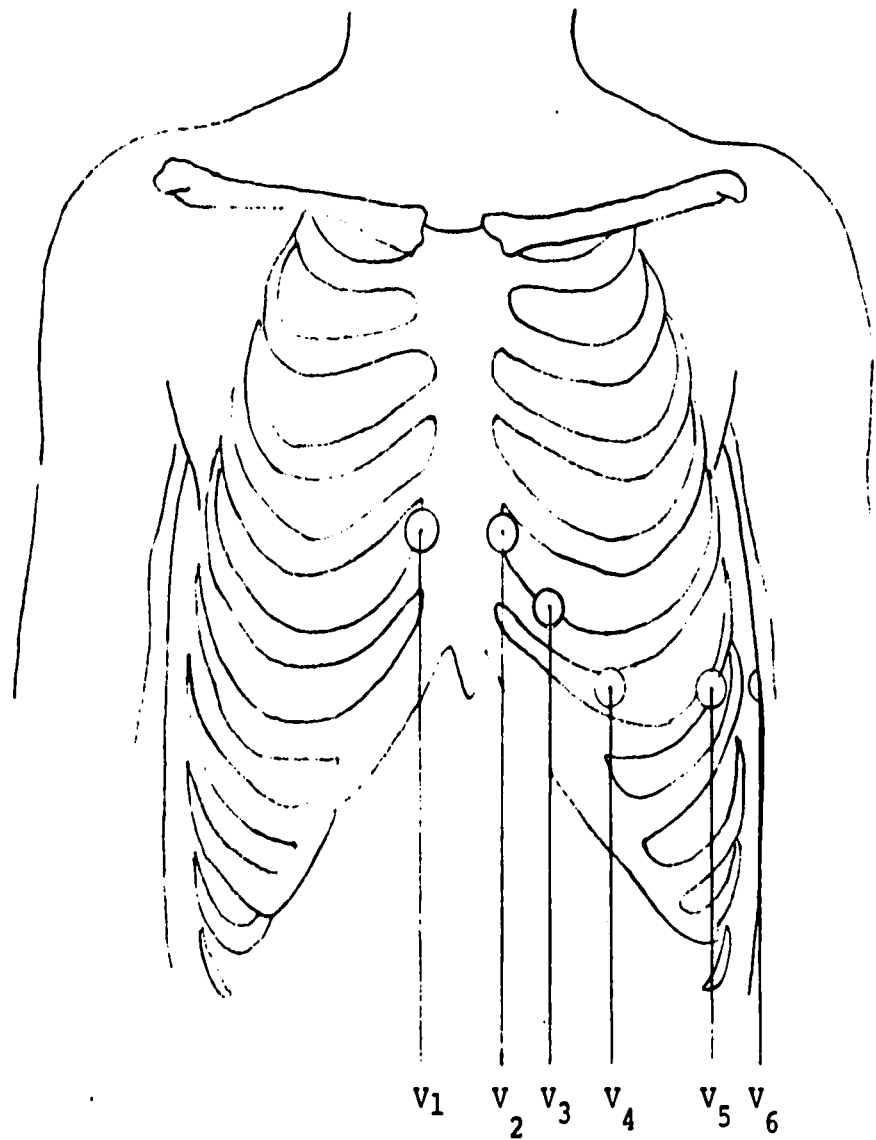


Figure 5a. Location of Unipolar Precordial
Leads.

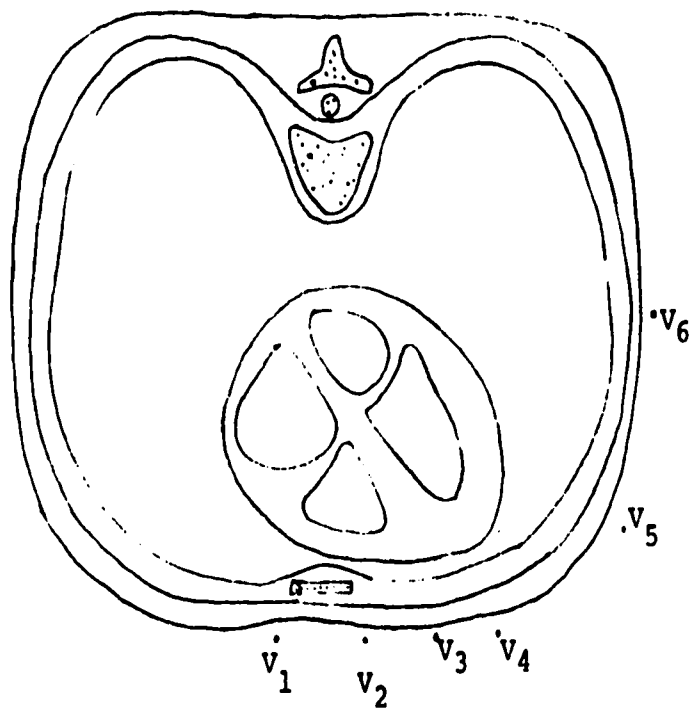


Figure 5b. Transverse section of thorax illustrating position of the Unipolar Precordial Leads.

In the Standard Leads system electrodes are applied to the right forearm left forearm and left leg and connected with the appropriate leads from the electrocardiograph.

The standard bipolar leads represent the difference of potential between two points. The selection is made by the electrocardiograph switch as follows:

$$\text{Lead I} = \text{LA} - \text{RA}$$

$$\text{Lead II} = \text{LL} - \text{RA}$$

$$\text{Lead III} = \text{LL} - \text{LA}$$

where: RA = right forearm
 LA = left forearm
 LL = left leg

Algebraically:

$$\text{Lead II} = \text{Lead I} + \text{Lead III}$$

and:

$$\text{RA} + \text{LA} + \text{LL} = 0$$

The Augmented Extremity Leads are recorded from the right forearm, left forearm and left leg. One electrode is applied to the desired limb and the circuit is completed through the other two electrodes. Thus the Augmented Extremity Leads can be expressed as follows:

$$\begin{aligned} aVR &= \text{RA} - \frac{1}{2} (\text{LL} + \text{LA}) \\ &= \text{RA} + \frac{1}{2} [-(\text{LL} + \text{LA})] \end{aligned}$$

$$= RA + \frac{1}{2} RA$$

$$= \frac{3}{2} RA$$

similarly:

$$aVL = \frac{3}{2} LA$$

$$aVF = \frac{3}{2} LL$$

The Unipolar Precordial Leads are used to obtain the electrical activity of the anterior and posterior surface of the heart by placing the exploring electrode on the anterior surface of the precordium and pairing it with the three-tailed indifferent electrode (RA+LA+LL) as shown in Figure 6. The limb leads primarily reflect only the potentials developed on the lateral, superior and inferior borders of the heart.

Accessory leads are sometimes taken further to the right, or at the back, or in higher interspaces etc.

2.3. Recording of Electrocardiogram

The body surface electrocardiogram is generally recorded with direct writing galvanometers capable of responding linearly to signals at frequencies up to 20 to 40 Hz. The mirror galvanometer recorder can have a frequency response up to 20 Hz. The cathode ray oscillographic recording is capable of responding to much higher frequencies, 1,300 Hz to 20,000 Hz depending on the amplifier in the oscilloscope system.

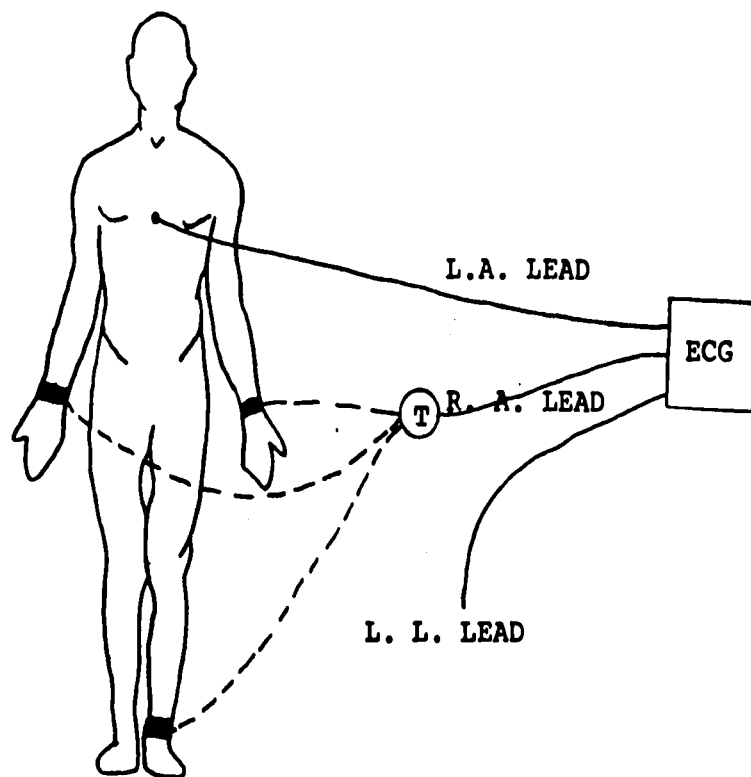


Figure 6. Unipolar Chest Leads.

CHAPTER III

FOURIER ANALYSIS

3.1. Introduction

In order to analyse the electrocardiogram one approach is to fit a mathematical expression to the waveform [29]. Advantage is taken of the fact that any curve may be fitted by a mathematical equation as long as this equation has enough terms in it. One such popular quantitative expression of the electrocardiogram is obtained by the use of Fourier analysis, which takes advantage of the electrocardiogram's cyclic nature. With this technique, it is possible to express any periodic waveform as a sum of sinusoidal waveforms. The number of terms in the resulting expression (Fourier series) will then depend on the complexity of the waveform and the degree of fidelity the user wishes to attain. In the case of electrocardiogram an adequate Fourier series would contain roughly 35 terms.

3.2. The Advantage of the Application of Fourier Analysis

An individual electrocardiogram can be accurately expressed in a mathematical form (e.g. a Fourier series). Electrocardiogram can be classified by comparing the parametric constant c_n of the corresponding Fourier equations and can be grouped statistically (e.g. discriminant analysis). The

comparison of a few key Fourier coefficients is much more efficient than the point by point comparison of the waveform digits. Approximately 35 Fourier coefficients are sufficient to reconstruct a electrocardiographic waveform. Thus instead of the storage of large amount of waveform digits, the storage of these few Fourier coefficients which represent the waveform saves a lot of computer storage space.

3.3. Fourier Series

If $f(t)$ is periodic and satisfies the Dirichlet conditions [30-32], then the Fourier series [30,32,33] is

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \omega_n t + b_n \sin \omega_n t) \quad -(1)$$

where:

$$\omega_n = n\omega_0$$

$$\omega_0 = 2\pi/T$$

T-fundamental period

$$a_n = \frac{2}{T} \int_0^T f(t) \cos \omega_n t dt \quad -(2)$$

$$n=0, 1, 2 \dots$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin \omega_n t dt \quad -(3)$$

$$n=1, 2, \dots$$

Equation (1) can be written compactly as:

$$f(t) = \sum_{n=-\infty}^{\infty} F(n) e^{j\omega n t} \quad - (4)$$

where:

$$\begin{aligned} F(n) &= \frac{1}{2} (a_n - j b_n) \\ &= \frac{1}{2} c_n \\ &= 1/T \int_0^T f(t) e^{-j\omega n t} dt \end{aligned} \quad - (5)$$

$$\theta_n = \tan^{-1} \left(\frac{-b_n}{a_n} \right) \quad - (6)$$

$$c_n = \sqrt{a_n^2 + b_n^2} e^{j\theta_n}$$

$|c_n|$ is the amplitude and θ_n the phase of the nth harmonic.

For a finite equispaced sampled data $x(t)$ with even number of data samples and initial point value to be zero, the Fourier series is:

$$x(k) = A_0 + \sum_{n=1}^{N/2} A_n \cos \left(\frac{2\pi n k}{N} \right) + \sum_{n=1}^{(N/2)-1} B_n \sin \left(\frac{2\pi n k}{N} \right) \quad (7)$$

where:

$$\begin{aligned} A_0 &= \frac{1}{N} \sum_{k=1}^N x_k \\ A_n &= \frac{1}{N} \sum_{k=1}^N x_k \cos \frac{2\pi n k}{N} \end{aligned} \quad (8)$$

$$B_n = \frac{1}{N} \sum_{k=1}^N x_k \sin \frac{2\pi nk}{N} \quad (9)$$

$$n=1, 2, \dots, \frac{(N-1)}{2}$$

$x(k)$ =data value at time kh

$$k = 1, 2, \dots, N$$

~~h = sampling interval in second~~

3.4. Fourier Integral

$$\text{Let } F(n) = \frac{1}{T} \int_0^T f(t) e^{-j\omega n t} dt = \frac{1}{T} F(\omega n)$$

$$\text{As } T \rightarrow \infty, \quad n\omega = \omega \rightarrow \omega$$

We have the Fourier transform pair:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \quad (10)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \quad (11)$$

Subject to the requirement on $f(t)$ that

$$|f(t)| < \infty$$

3.5. The Shifting Theorem

By the Fourier Integral Theorem a time function $f(t)$ starting at $t=0$ and ending at $t=\tau$ has the following Fourier

transform [28,30,33]

$$F(\omega, 0) = \int_0^T f(t) e^{-j\omega t} dt$$

If $f(t)$ is delayed by an instant $t=T_0$ then the Fourier Transform becomes

$$\begin{aligned} F(\omega, T_0) &= \int_{T_0}^{T+T_0} f(t) e^{-j\omega t} dt \\ &= \int_0^T f(t) e^{-j\omega(t+T_0)} dt \\ &= e^{-j\omega T_0} \int_0^T f(t) e^{-j\omega t} dt \\ &= e^{-j\omega T_0} F(\omega, 0) \end{aligned}$$

Thus the shift of a time function along a time axis gives an exponential factor (delay) in the Fourier transform.

3.6. Power Spectrum

$$\text{If } f(t) = \sum_{n=-\infty}^{\infty} F(n) e^{j\omega n t}$$

then the average power of $f(t)$ is

$$\begin{aligned} P_{av} &= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f^2(t) dt \\ &= [F(0)]^2 + 2 \sum_{n=1}^{\infty} |F(n)|^2 \end{aligned}$$

The power spectral density or power spectrum $P(\omega_n)$ of

$f(t)$ is

$$\begin{aligned} P(\omega_n) &= 2T |F(n)|^2 \\ &= \frac{T}{2} |c_n|^2 \end{aligned} \quad (12)$$

3.7. Fast Fourier Transform

The fast Fourier Transform [39-44] is an efficient method for computing the discrete Fourier Transform of a time series with N data points. It saves computer time approximately by a factor of $\frac{N-1}{N}$, where $N=2^r$.

The fast Fourier Transform was discovered independently by Danielson and Lanczos (1942), Good (1958) and Cooley and Tukey (1965).

The Fourier transform of a sampled periodic function of period T is

$$\begin{aligned} F(n) &= \frac{T}{N} \sum_{k=0}^{N-1} x(k) e^{-j2\pi(nk)/N} \\ n &= 0, 1, \dots, (N-1) \end{aligned} \quad (13)$$

$x(k)$ = sampled data value at time t_k

Equation (13) can be represented in matrix form as:

$$\begin{bmatrix} F(n) \end{bmatrix}_{N \times 1} = \begin{bmatrix} U^{nk} \end{bmatrix}_{N \times N} \begin{bmatrix} X_0(k) \end{bmatrix}_{N \times 1}$$

where

$$U = e^{-j2\pi/N}$$

T/N is simply a scaling term and will be eliminated for clarity of presentation.

For example, if we have $N=4$ then

$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ F(3) \end{bmatrix} = \begin{bmatrix} U^0 & U^0 & U^0 & U^0 \\ U^0 & U^1 & U^2 & U^3 \\ U^0 & U^2 & U^4 & U^6 \\ U^0 & U^3 & U^6 & U^9 \end{bmatrix} \begin{bmatrix} X_0(0) \\ X_0(1) \\ X_0(2) \\ X_0(3) \end{bmatrix} \quad (14)$$

Equation (14) requires N^2 complex multiplications and additions.

The following shows the principle of the fast Fourier Transform with no repeated identical computation.

Since $U^{nk} = U^{nk \bmod N}$, $nk \bmod N$ is the remainder upon division of nk by N , Equation (14) can be written as:

$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ F(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & U^1 & U^2 & U^3 \\ 1 & U^2 & U^0 & U^2 \\ 1 & U^3 & U^2 & U^1 \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_0(1) \\ x_0(2) \\ x_0(3) \end{bmatrix}$$

$$\begin{bmatrix} F(0) \\ F(2) \\ F(1) \\ F(3) \end{bmatrix} = \begin{bmatrix} 1 & U^0 & 0 & 0 \\ 1 & U & 0 & 0 \\ 0 & 0 & 1 & U^1 \\ 0 & 0 & 1 & U^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & U^0 & 0 \\ 0 & 1 & 0 & U^0 \\ 1 & 0 & U^2 & 0 \\ 0 & 1 & 0 & U^2 \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_0(1) \\ x_0(2) \\ x_0(3) \end{bmatrix} \quad (15)$$

U^0 and 1 are used to develop a generalized final result.

$$\text{Let } \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & U^0 & 0 \\ 0 & 1 & 0 & U^0 \\ 1 & 0 & U^2 & 0 \\ 0 & 1 & 0 & U^2 \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_0(1) \\ x_0(2) \\ x_0(3) \end{bmatrix} \quad (16)$$

We have

$$x_1(0) = x_0(0) + U^0 x_0(2)$$

In order to develop a generalized result, $U^0 x_0(2)$ is considered as one multiplication (although $U^0 = 1$).

$$x_1(1) = x_0(1) + u^0 x_0(3)$$

That an advantage is taken of the fact

$$u^{nk} = -u^{(nk + N/2)}$$

Therefore

$$u_0^0 = -u^2$$

$$\begin{aligned} x_1(2) &= x_0(0) + u^2 x_0(2) \\ &= x_0(0) - u^0 x_0(2) \end{aligned}$$

$$\begin{aligned} x_1(3) &= x_0(0) + u^2 x_0(3) \\ &= x_0(1) - u^0 x_0(3) \end{aligned}$$

Thus Equation (16) requires only four additions and two multiplications without repeating the identical computation.

From Equations (15) and (16) we have

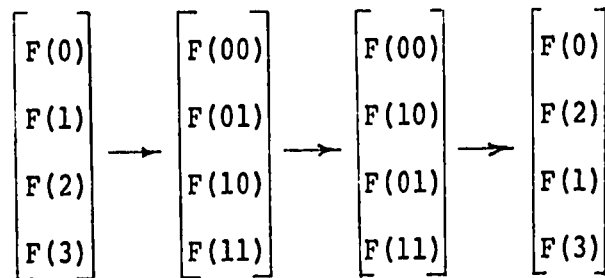
$$\begin{bmatrix} F(0) \\ F(2) \\ F(1) \\ F(3) \end{bmatrix} = \begin{bmatrix} 1 & u^0 & 0 & 0 \\ 1 & u^2 & 0 & 0 \\ 0 & 0 & 1 & u^1 \\ 0 & 0 & 1 & u^3 \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} \quad (17)$$

Similarly Eq. (17) requires two multiplications and four additions.

Therefore, it requires only a total of $N \log_2 N = 8$ complex additions and $\frac{1}{2} N \log_2 N = 4$ complex multiplications instead of N^2 complex multiplications and additions to compute vector $[F(n)]$.

The fast Fourier transform algorithm is a method of factoring a $N \times N$ matrix into r $N \times N$ matrices to minimize the number of calculations.

The vector $[F(n)]$ can be obtained by replacing the argument n in $[F(n)]$ with its binary equivalent and then bit-reversing. For example:



There are general rules to obtain a signal flow graph representation of the factored matrices.

Figure 7. is the signal flow graph representation of factored

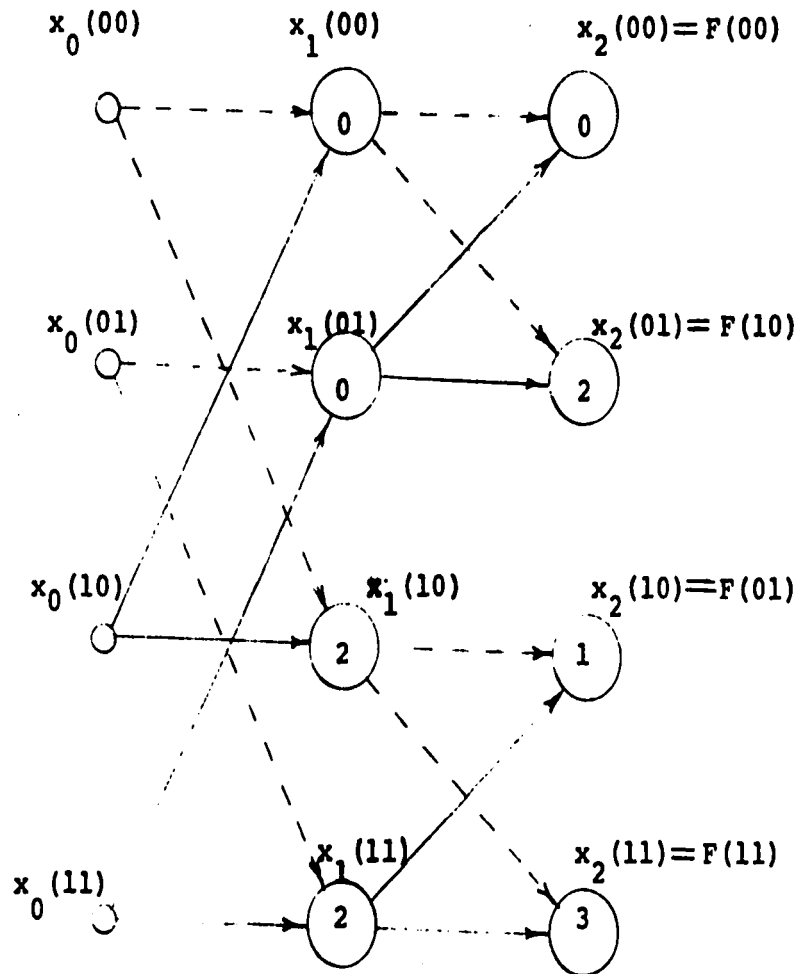


Figure 7. Signal flow graph representation of factored matrices.

matrices for $N=4$:

First column represents the sampled data $X_0(k)$, second column represents the vectors $[X_1(k)]$ and third column represents the vectors $[X_2(k)] = \overline{F(n)}$. Each node is entered by a dashed and a solid line, and within each node is an integer.

The solid line brings a quantity from one of the nodes in a previous column, multiplies the quantity by U^P , where P is the integer in the circle, and the product is added to the quantity brought by the dashed line. Therefore, Figure 7 is a graph of the computational procedure of the factorial matrices, and each vertical column computed corresponds to one of the factored matrices.

The three rules for establishing a signal flow graph for $N=2^r$ are:

1) The sample values $X_0(k)$, where $k=0, 1, \dots, N-1$, of the time function $X_0(t)$ constitute the first vertical array of nodes. Argument k is expressed as a binary number with r bits ($N=2^r$).

Successive arrays $[X_l]$ are drawn to the right. Nodes on the same horizontal level have the same binary address. A circle with a number in it is drawn at each node as explained in rule (2).

2) At k th node in l th array the number in the circle is found by: a) writing the binary number k .

b) sliding this no. $(r-l)$ places to the right and filling the newly opened bit positions on the left by zeros.

c) reversing the order of the bits.

3) In the l th array, node k , in binary from $k_{r-1}, \dots, k_1, k_0$, has a solid line drawn to it from a node in the $(l-1)$ th array. The address of the node in the $(l-1)$ th array is the same as node k , except that bit k_{r-l} must be a one. The dashed line comes from a node in the $(l-1)$ -th array whose address is the same, but bit k_{r-l} must be a zero.

Finally, the finite discrete Fourier Transform places three restrictions upon the data:

1) The data must come from one cycle of a periodic function, and the very favorable situation is when the total number of data N is some power of 2.

2) The number of input data and the number of transform values must be the same.

3) The data must be equispaced in each dimension.

The computational savings depend on the number N and the programming method used.

Since it was not the purpose of this thesis to investigate the fast Fourier Transform and the intent was to discuss the use of the frequency domain analysis in electrocardiograms the classical way of calculating the frequency spectrum was employed.

For further information about the fast Fourier Transform the reader should refer to the references already given at the beginning.

DIGITIZING CONTINUOUS DATA

4.1. Introduction

Digitizing is a sequence of transformations applied to a continuous function of time. The time function is first sampled to a set of discrete points of time, next the sampled function values are quantized into a set of amplitude values. Finally each quantized amplitude is counted into a binary number representation in the computer.

4.2. Sampling

Given a continuous function $f(t)$ over a time interval, the sampling process consists of choosing a set of times $t_0, t_1, \dots, t_n$ within the interval. The function $f(t)$ is then sampled and represented by the values $f(t_0), f(t_1), \dots, f(t_n)$. Usually the sampling is equispaced.

The well known sampling Theorem [33.45] considers the possible error, or loss of information, resulting from preserving only the sample values $\{f(t_i)\}$. The sampling Theorem states that at least $2f_n$ (f_n is the desired maximum frequency) uniformly spaced samples are needed every second in order to reproduce the function $f(t)$. If sampling rate is too slow the sampling values could represent either low or some higher frequencies in the original data. This

phenomenon is called "aliasing".

From Equation (14), the frequencies between 0 and $f_n H_z$ are clearly distinct from one another, but the frequencies in the data which are higher than $f_n H_z$ will be "folded" into the lower frequency range from 0 to $f_n H_z$. This can be seen from the following:

$$\begin{aligned} \text{for } t &= \frac{1}{2f_n}, \quad n=0, 1, \dots \\ \cos 2\pi f t &= \cos 2\pi \left((2nf_n \pm f) \frac{1}{2f_n} \right) \\ &= \cos 2\pi \left(n \pm \frac{f}{2f_n} \right) \\ &= \cos \left(\frac{\pi f}{f_n} \right) \end{aligned}$$

Therefore the cosine function $\cos \left(\frac{\pi f}{f_n} \right)$ could have come from any data at frequencies $(2nf_n \pm f)$. These frequencies (f) , $(2f_n - f)$, $(2f_n + f)$, $(4f_n - f)$, $(4f_n + f)$ and so on are aliases of one another. The frequency f_n is called the Nyquist frequency and the frequency f is called the principle alias.

4.3. Quantization

The purpose of quantization [45,33] is to obtain the discrete amplitude values from a continuous time function at the sampled points. Figure 8 shows that no matter

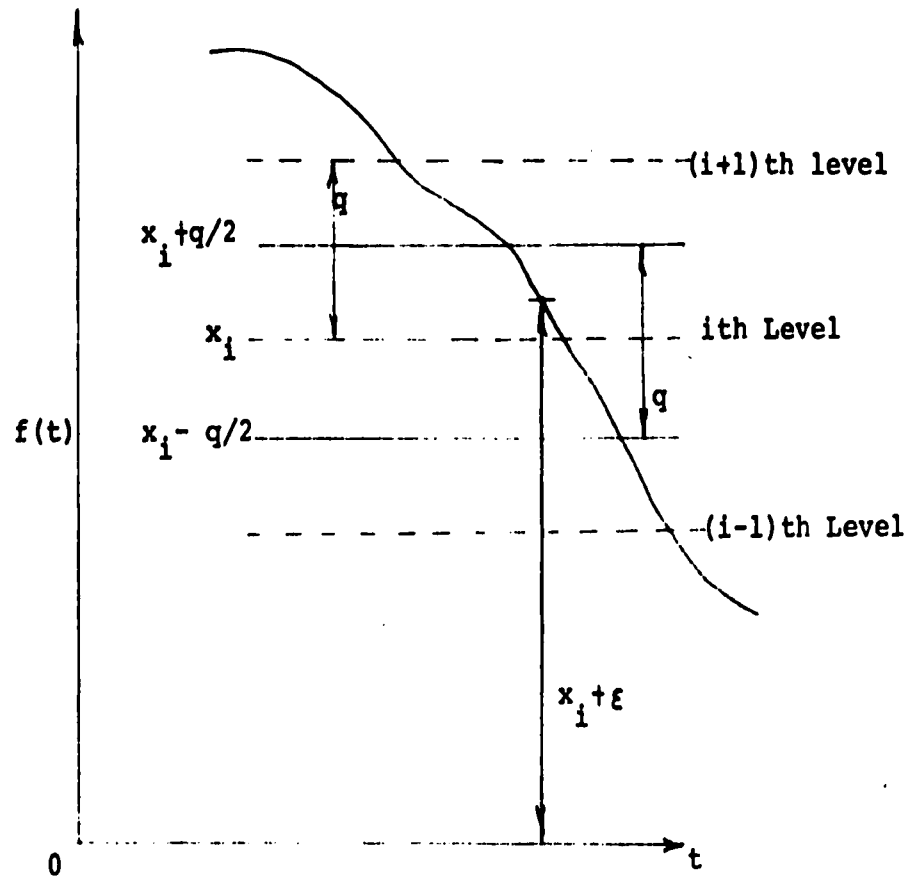


Figure 8. Quantizing error.

how fine the scale, a choice between two consecutive values is required. A mean-squared error value due to quantization can be calculated. The instantaneous value of the function will be $x+\epsilon$, with $-q/2 \leq \epsilon \leq q/2$. represents the error between the instantaneous actual function value and its quantized equivalent.

Let $P(\epsilon)$ be the quantization error probability density function defined by

$$P(\epsilon) = 1 \quad \text{for } -q/2 \leq \epsilon \leq q/2 \\ = 0 \quad \text{otherwise}$$

The mean value $\mu_\epsilon = 0$, since $P(\epsilon)$ is symmetric about $\epsilon = 0$. The mean squared value (variance) of ϵ will then be

$$\overline{\epsilon^2} = \frac{1}{q} \int_{-q/2}^{q/2} \epsilon^2 d\epsilon = \frac{q^2}{12}$$

The rms value (standard deviation) of the quantization error is

$$\sigma_\epsilon = \sqrt{\frac{q^2}{12}} = \frac{q}{2\sqrt{3}} \quad \text{scale unit}$$

This represents the rms "noise". If the full range of $f(t)$ is quantized into N levels then the rms noise to signal ratio will be $\frac{q}{2\sqrt{3}N}$.

Thus the quantizing process introduces some error in the eventual reproduction of the original time function. Once quantized, the instantaneous values of the original function can never be restored exactly.

CHAPTER V

NORMALIZATION OF THE ELECTROCARDIOGRAM

5.1 Amplitude Normalization

In studying the frequency amplitude spectrum, amplitude normalization is required for two reasons. First the amplitude of the cardiac waveform varies with sensitivity of the recording apparatus, even though one attempts to keep the instruments's sensitivity constant. In addition the recording techniques obviously vary from one laboratory to another, further complicating the studies. Second, the amplitude of the records varies with the thickness of the chest wall of a patient. Thin wall normal individuals give rise to large amplitude than those with thick wall, even though the chest wall contour may be similar.

Each digitized amplitude could be divided by the largest amplitude encountered in the record. All reconstructed curves would then have the same maximum height and would lose any variations in amplitude change. Thus another procedure was chosen, which appears to minimize the two variables mentioned, but retains to some extent the variations in amplitudes which may be due to disease. The area under the waveform is obtained by adding

up all digitized values. The mean area is then found by dividing the area by the number of points digitized in the waveform. Each digital amplitude is then divided by the mean area under the curve.

5.2. Time Normalization

Time normalization is not necessary in the frequency domain studies, since we are not concerned about point to point amplitude comparison at a given time in the cardiac cycle to those of another case with a different heart rate or different wave duration. The changes of heart rate and duration have no effect on the shape of the spectrum envelope. The variation on the spectral amplitude due to the change of heart rate or wave duration can be eliminated by the amplitude normalization.

5.2.1 Different Repetition Rate

Let us assume a triangular equally spaced pulse train with period $2L$, base L/c and height A as shown in Fig. 9a.

From Equations (1), (2) and (3) we have

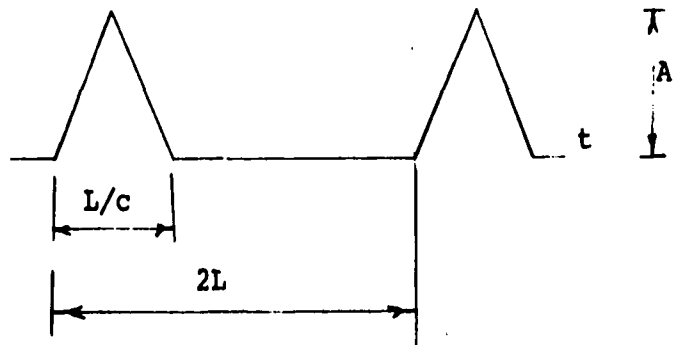


Figure 9a. Wave with period $2L$

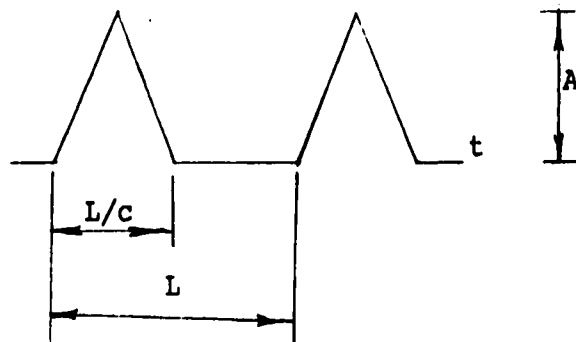


figure 9b. Wave with period L

$$a_n = \frac{2}{2L} \int_0^{2L} f(t) \cos \frac{2n\pi t}{2L} dt$$

$$a_0 = \frac{1}{L} \int_0^{2L} f(t) dt$$

Since the function is zero at all times within any one $2L$ cycle except in a time interval L/c , we may divide this interval into equal parts which correspond with the rising and falling portions of a triangle.

Then we have

$$a_n = \frac{1}{L} \left\{ \int_0^{L/(2c)} S t \cos\left(\frac{n\pi t}{L}\right) dt + \int_{L/2c}^{L/c} 2A \cos\left(\frac{n\pi t}{L}\right) dt - \int_{L/2c}^{L/c} S t \cos\left(\frac{n\pi t}{L}\right) dt \right\}$$

$$\text{where } S = \frac{A}{L/2c} = \frac{2cA}{L}$$

$$\text{Let } x = \frac{n\pi}{L} t$$

$$\begin{aligned} a_n &= \frac{SL}{n^2 \pi^2} \left\{ \int_0^{n\pi/2c} x \cos x \, dx - \int_{n\pi/2c}^{n\pi/c} x \cos x \, dx \right\} \\ &\quad - \frac{2A}{n\pi} \int_{n\pi/2c}^{n\pi/c} \cos x \, dx \\ &= \frac{SL}{n^2 \pi^2} \left\{ \left[x \sin x - \cos x \right]_0^{n\pi/c} - \left[x \sin x - \cos x \right]_{n\pi/2c}^{n\pi/c} \right. \end{aligned}$$

$$+ \frac{2A}{n\pi} \left[\sin x \right]_{n\pi/2c}^{n\pi/c}$$

Now if the same pulses are separated by interval L instead of interval $2L$ (Fig. 9b) we have

$$a'_n = \frac{2}{L} \int_0^L f(t) \cos \frac{2n\pi t}{L} dt$$

$$a'_0 = \frac{2}{L} \int_0^L f(t) dt$$

$$a'_n = \frac{2}{L} \left\{ \int_0^{L/2c} S t \cos \frac{2n\pi t}{L} dt + \int_{L/2c}^{L/c} 2A \cos \frac{2n\pi t}{L} dt \right.$$

$$\left. - \int_{L/2c}^{L/c} S t \cos \frac{2n\pi t}{L} dt \right\}$$

$$= 2 \frac{SL}{(2n)^2 \pi^2} \left\{ \left[x \sin x - \cos x \right]_0^{(2n)\pi/2c} - \left[x \sin x - \cos x \right]_{(2n)\pi/2c}^{(2n)\pi/c} \right\}$$

$$+ 2 \frac{2A}{(2n)\pi} \left[\sin x \right]_{(2n)\pi/2c}^{(2n)\pi/c}$$

We can see that

$$a'_n = 2a_{(2n)}$$

Similarly

$$b'_n = 2b_{(2n)}$$

Thus the new coefficients are equal to twice the $2n$ 'th harmonic corresponding to the original rate. The $2n$ 'th order harmonic corresponding to the original repetition rate of period $2L$ is of the same frequency as the n 'th order harmonic corresponding to the repetition rate of period L . Therefore we may double the amplitude for double the repetition rate, leaving the frequency unchanged. Hence the changes in the heart rate should be without effect on the shape of the spectral envelope. The maxima and minima along the frequency axis are still at the same position. Although the amplitudes of the spectra would alter with the change in the heart rate. This effect on amplitude can be eliminated by the amplitude normalization as mentioned before.

5.2.2. Different Time Duration

Consider two triangular waveforms A and B having the same slope but different time duration as shown in Fig. 10. Since these two waveforms have the same slope and $A_A/A_B = T_A/T_B$ constant, the waveform A is just the magnified graph of waveform B. Thus they will have identical amplitude normalized frequency spectra.

Thus it proves that the time normalization is not necessary in the frequency domain studies and this would simplify any interpretive technique which apply the frequency domain approach.

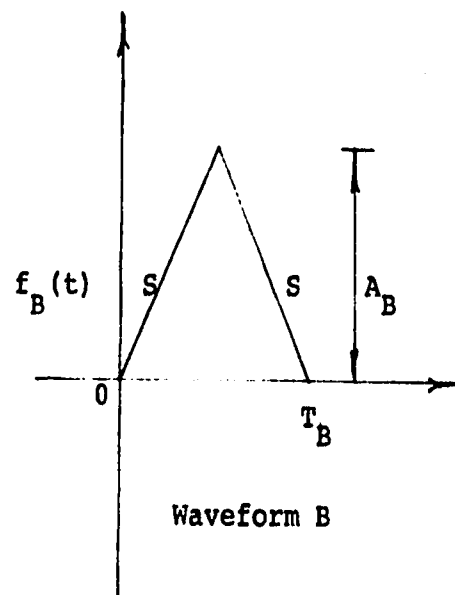
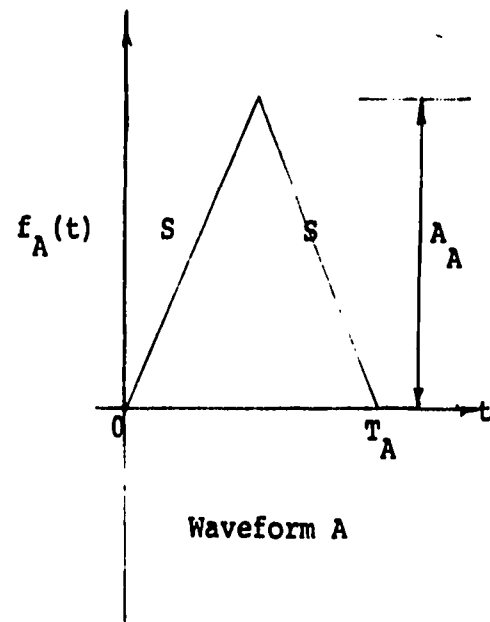


Figure 10. Waveforms A and B with same slope S but different duration.

5.3. Determination of the Electrocardiogram Baseline

The ^V[24,23] is constructed as the straight line connecting two ^{baseline} points on the T-P interval of two consecutive cardiac cycles. These points are taken just prior to the onset of the P-wave. All amplitudes of the electrocardiogram are referred to that baseline. If no P-wave is found, then the baseline is defined as the straight line connecting the Q wave onsets of two consecutive cardiac cycles. If no Q wave is present the baseline is established between two consecutive R wave onsets.

CHAPTER VI

THE WAVEFORM MODELS

6.1. Introduction

The electrocardiogram consists of several distinct waveforms P, QRS and T waves.

For studying the spectrum contributed by each individual waveform, i.e., the QRS complex, the P-wave and the T-wave, we analyze some waveform models. The QRS complex can be approximated by linear or triangular functions and the P-wave and T-wave by a sinusoidal function.

6.2. Inverted V Pulse

The Fourier transform of an inverted V pulse [30,45,46] (Fig. 11) is

$$F(w) = \int_0^{\tau/2} \frac{A}{\tau/2} t e^{-jwt} dt + \int_{\tau/2}^{\tau} (2A - \frac{At}{\tau/2}) e^{-jwt} dt$$

$$\text{Let } x = jwt, \quad t = \frac{x}{jw}$$

$$F(w) = \frac{2A}{jw\tau} \int_0^{jw\tau/2} \frac{x}{jw} e^{-x} dx - \frac{2A}{jw\tau} \int_{jw\tau/2}^{jw\tau} \frac{x}{jw} e^{-x} dx + \frac{2A}{jw} \int_{jw\tau/2}^{jw\tau} e^{-x} dx$$

$$\begin{aligned}
 &= \frac{2A}{(j\omega)^2 \tau} \left[\begin{array}{cc} -x & -x \\ (xe & +e) \end{array} \right] \begin{array}{cc} j\omega\tau & \\ & j\omega\tau/2 \end{array} \begin{array}{cc} -x & -x \\ -(xe & +e) \end{array} \begin{array}{cc} j\omega\tau/2 & \\ & 0 \end{array} \\
 &\quad + j\omega\tau(-e) \begin{array}{cc} -x & j\omega\tau \\ & j\omega\tau/2 \end{array} \left. \right] \\
 &= \frac{4A}{(j\omega)^2 \tau} \left[e^{-j\omega\tau/2} (1 - \cos(\omega\tau/2)) \right] \\
 &= A\tau e^{-j\omega\tau/2} \left(\frac{1 - \cos(\omega\tau/2)}{(\omega\tau/2)^2} \right)
 \end{aligned}$$

Therefore

$$2 |F(\omega)| = 2A\tau \frac{1 - \cos(\omega\tau/2)}{(\omega\tau/2)^2}$$

The sketch of the spectrum is shown in Fig. 11 for $\tau = 0.04$ sec.

6.3. Diphasic QRS Complex

Some QRS complexes may be approximated by two V pulse with the first one inverted as shown in Fig. 12.

The Fourier transform [46] is

$$\begin{aligned}
 F(\omega, 0) - F(\omega, \tau) &= (1 - e^{-j\omega\tau}) F(\omega, 0) \\
 &= (1 - e^{-j\omega\tau}) A\tau e^{-j\omega\tau/2} \left(\frac{1 - \cos(\omega\tau/2)}{(\omega\tau/2)^2} \right)
 \end{aligned}$$

FIGURE 11 SEMI LOGARITHMIC 46 6213
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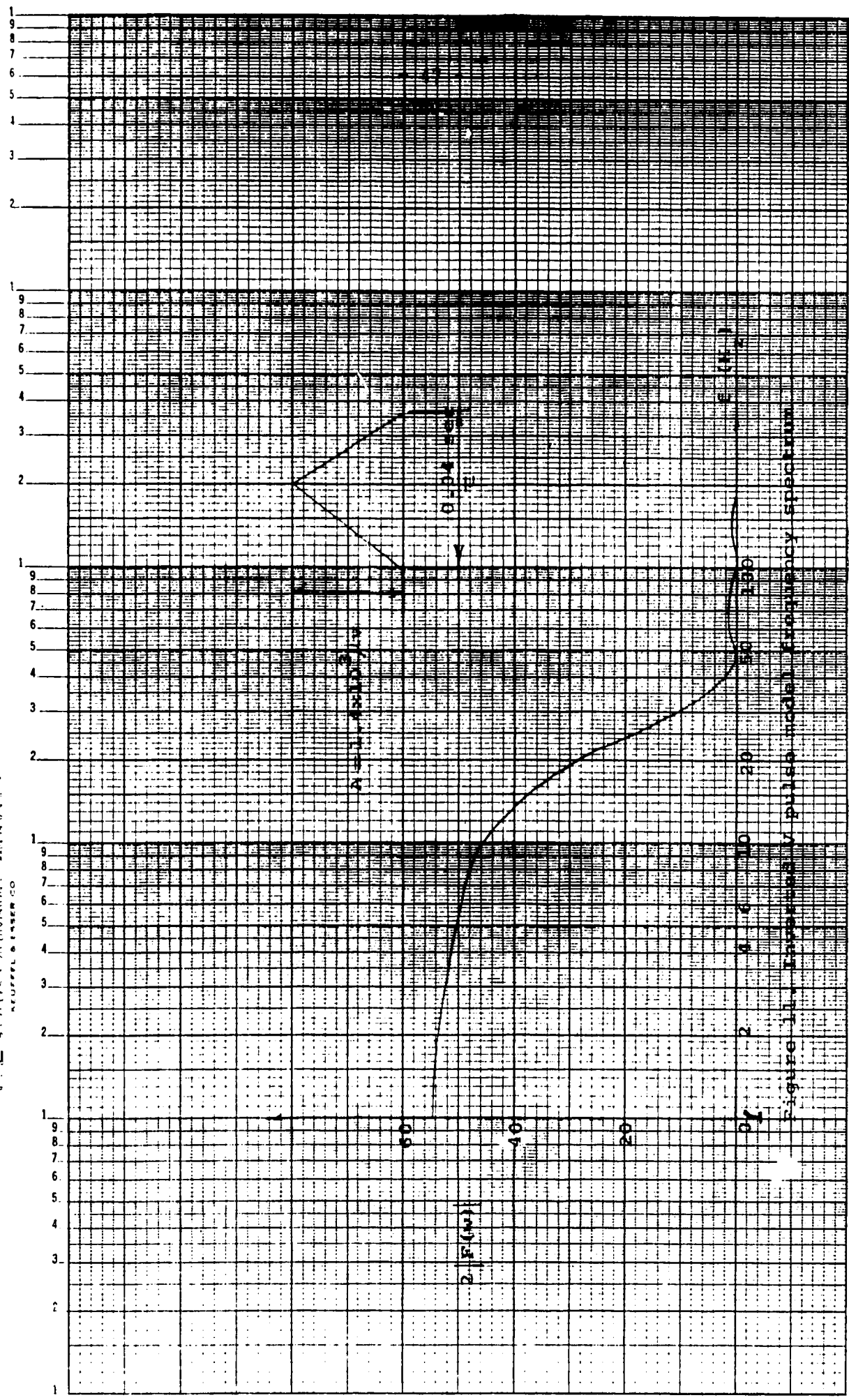


Figure 11. Inverse of pulse model frequency spectrum

1
2
3
4
5
6
7
8
9
10

100
50
20
10
5
2
1

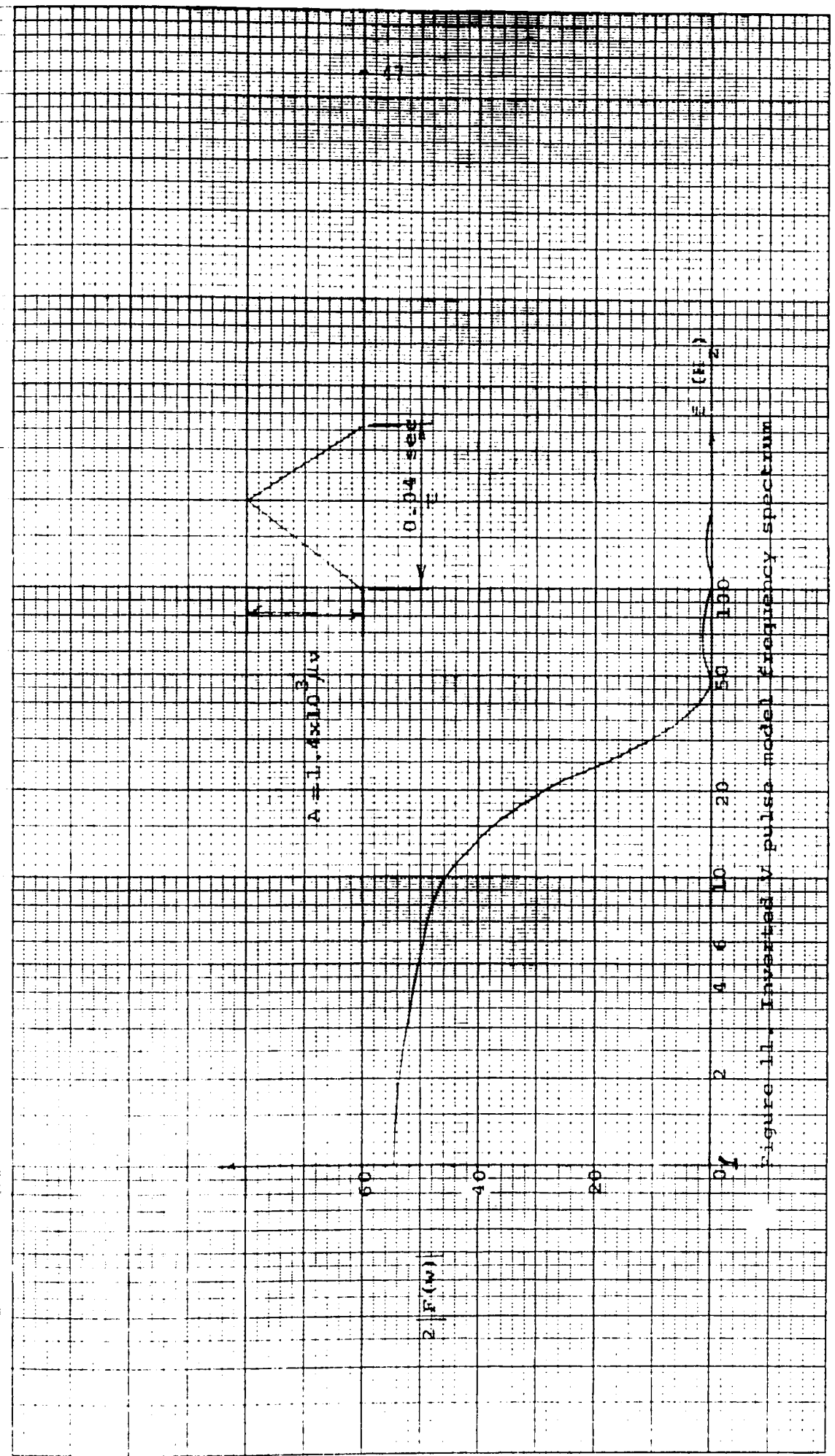


Figure 11. Inverted V pulse model frequency spectrum

$$= A\tau \left(\frac{\sin w\tau/4}{w\tau/4} \right)^2 (j \sin(w\tau/2)) e^{-jw\tau}$$

$$2 |F(w,0) - F(w,\tau)| = 2A\tau \frac{2(1 - \cos(w\tau/2))}{(w\tau/2)^2} |\sin(w\tau/2)|$$

In Fig. 12 a sketch of this spectrum is given again for $\tau = 0.04 \text{ sec}$.

Some QRS complexes are such that the slope of the RS section is double of that of the QRS or ST section. This feature is approximated by two V pulses with the first one inverted and the second one advanced a time $\tau/2$, and the summation of these two pulses gives the approximate model (Fig. 13).

The transform is now

$$F(w,0) - F(w,\tau/2) = (1 - e^{-jw\tau/2}) F(w,0)$$

$$= A\tau e^{-jw\tau/2} \left(\frac{\sin(w\tau/4)}{w\tau/4} \right)^2 (1 - e^{-jw\tau/2})$$

$$= A\tau e^{-j(w\tau/2)(3/2)} \left(\frac{\sin(1/2(w\tau/2))}{(1/2)(w\tau/2)} \right)^2 \sin(1/2)(w\tau/2)$$

$$2 |F(w,0) - F(w,\tau/2)|$$

$$= 2A\tau \frac{2}{(w\tau/2)^2} (1 - \cos(w\tau/2)) |\sin(1/2)(w\tau/2)|$$

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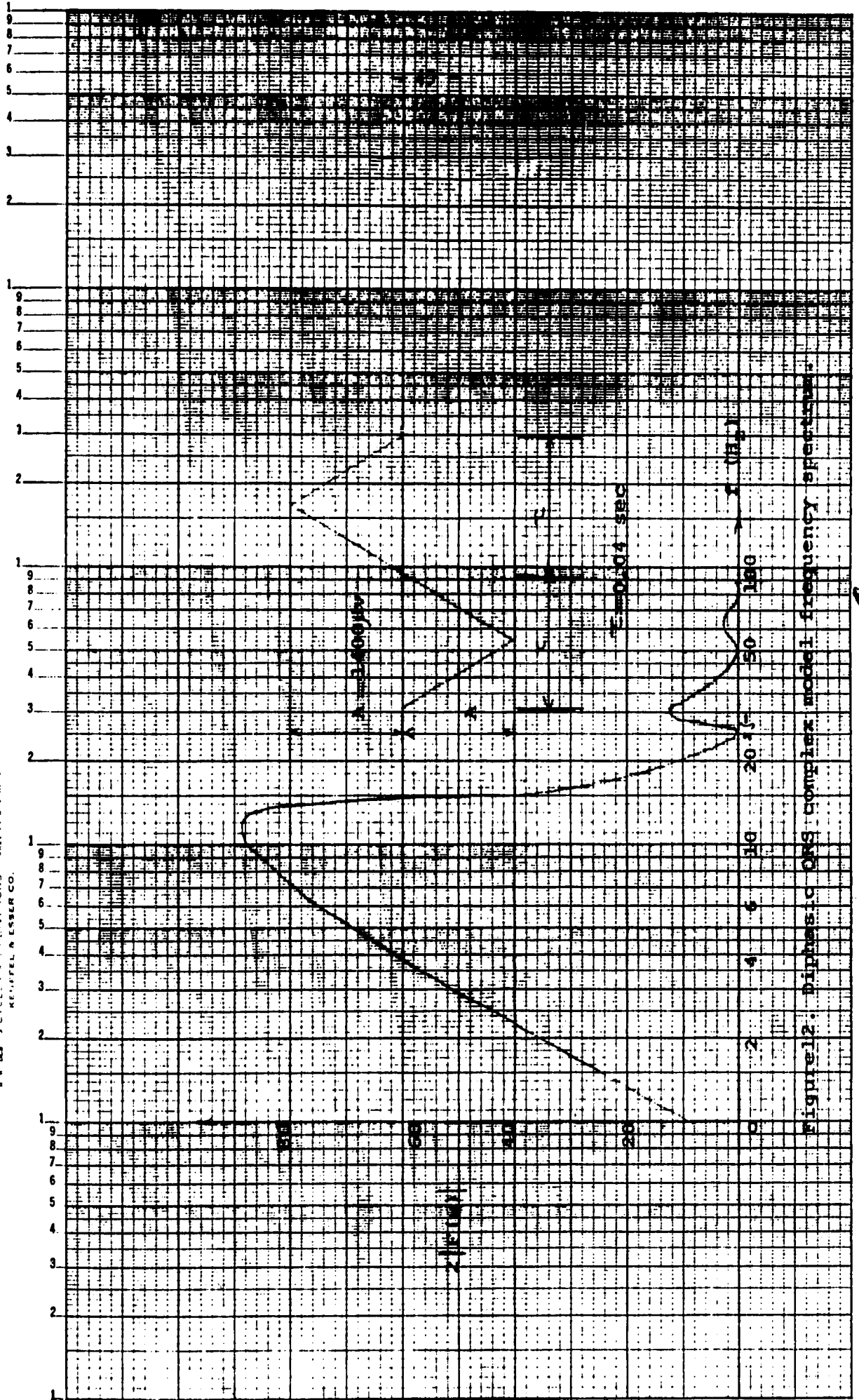


Figure 12. Diphasic QRS complex model frequency spectrum

Fig. 13 shows the sketch of the spectrum of this waveform model.

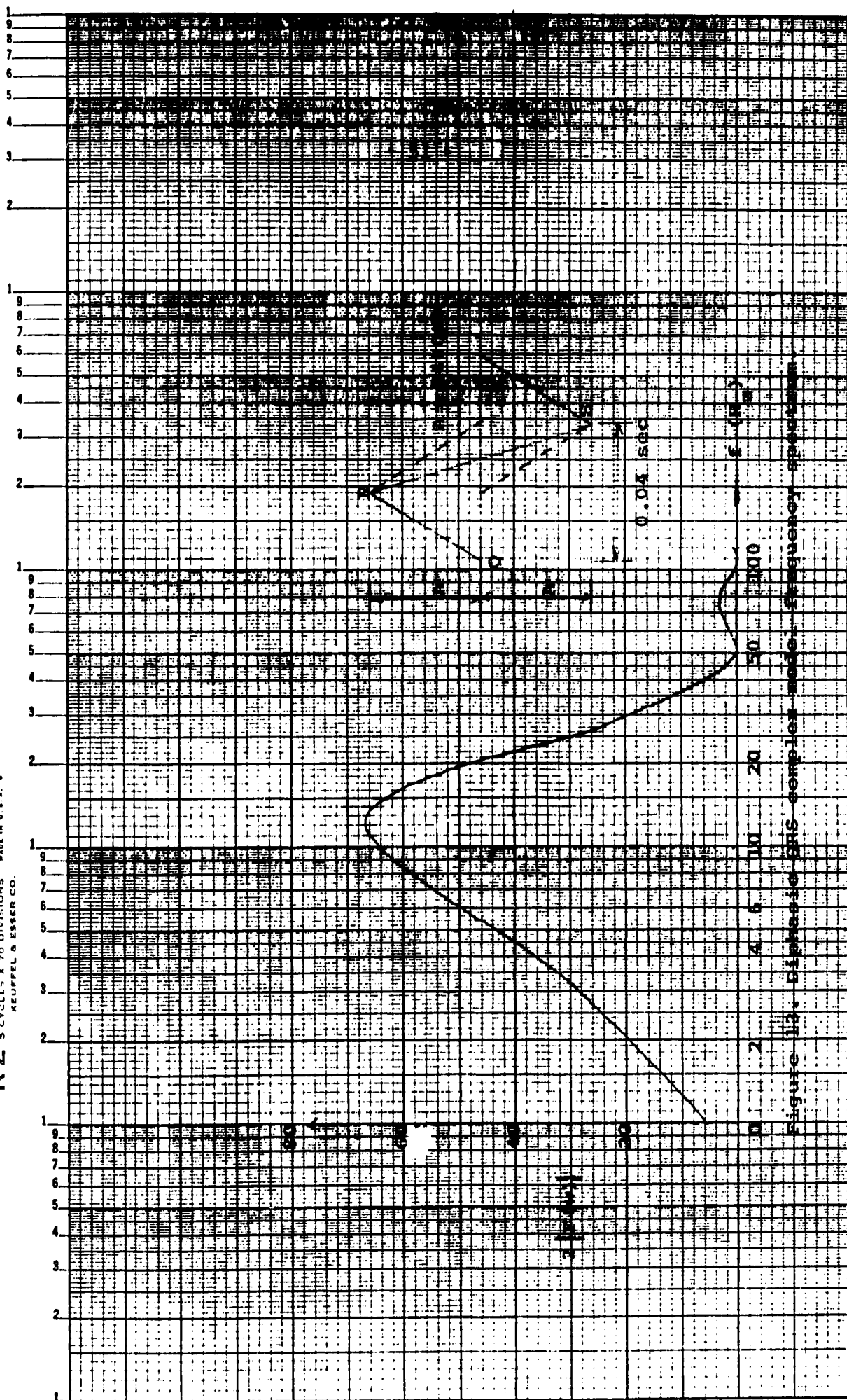
6.4. Triphasic QRS Complex

More generally, the QRS complex is triphasic with the slope of the sides varying from top to bottom. This QRS complex feature is approximated with the model shown in Fig. 14.

The Fourier transform [46] of this triphasic QRS model is

$$\begin{aligned}
 F(w, 0) &= - \int_0^{\tau/2} \frac{2A}{\tau} t e^{-j\omega t} dt - \int_{\tau/2}^{\tau} (2A - \frac{2A}{\tau} t) e^{-j\omega t} dt \\
 &= \frac{2A}{(j\omega\tau)^2} e^{-j\omega\tau/2} 2(1 - \cos(\omega\tau/2)) \\
 F(w, -\tau) &= - \int_{-\tau}^{-\tau/2} (\frac{2A}{\tau}) t e^{-j\omega t} dt + \int_{-\tau/2}^0 (2A - \frac{2A}{\tau} t) e^{-j\omega t} dt \\
 &= \frac{2A\tau}{(j\omega)^2} \begin{bmatrix} -x & -x & 0 & -x & -x & -j\omega\tau/2 & -x & 0 \\ (xe^{-x} & -e^{-x}) & & -(xe^{-x} & -e^{-x}) & +j\omega(-e^{-x}) & & \\ & & -j\omega\tau/2 & & -j\omega\tau & & -j\omega\tau/2 \end{bmatrix} \\
 &= \frac{-2A\tau}{(j\omega\tau)^2} e^{j\omega\tau/2} (1 - j\omega\tau)^2 (1 - \cos(\omega\tau/2)) \\
 F(w, 0) + F(w, -\tau) &= 2A e^{j\omega\tau/2} (1 - \cos(\omega\tau/2)) \frac{2}{(j\omega)^2} (j\omega\tau - 1 + e^{-j\omega\tau})
 \end{aligned}$$

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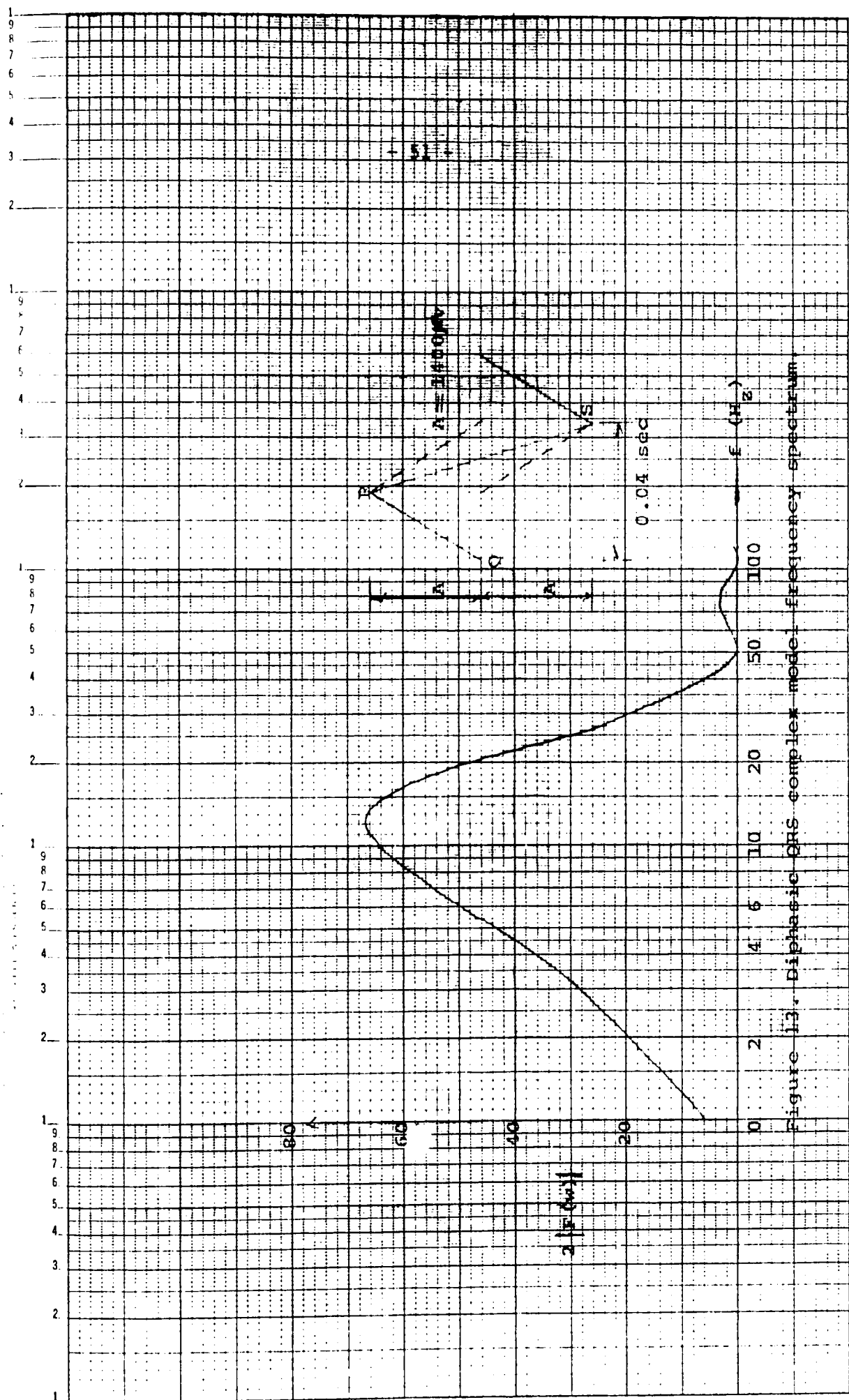


Figure 13. Diphase QRS complex model frequency spectrum.

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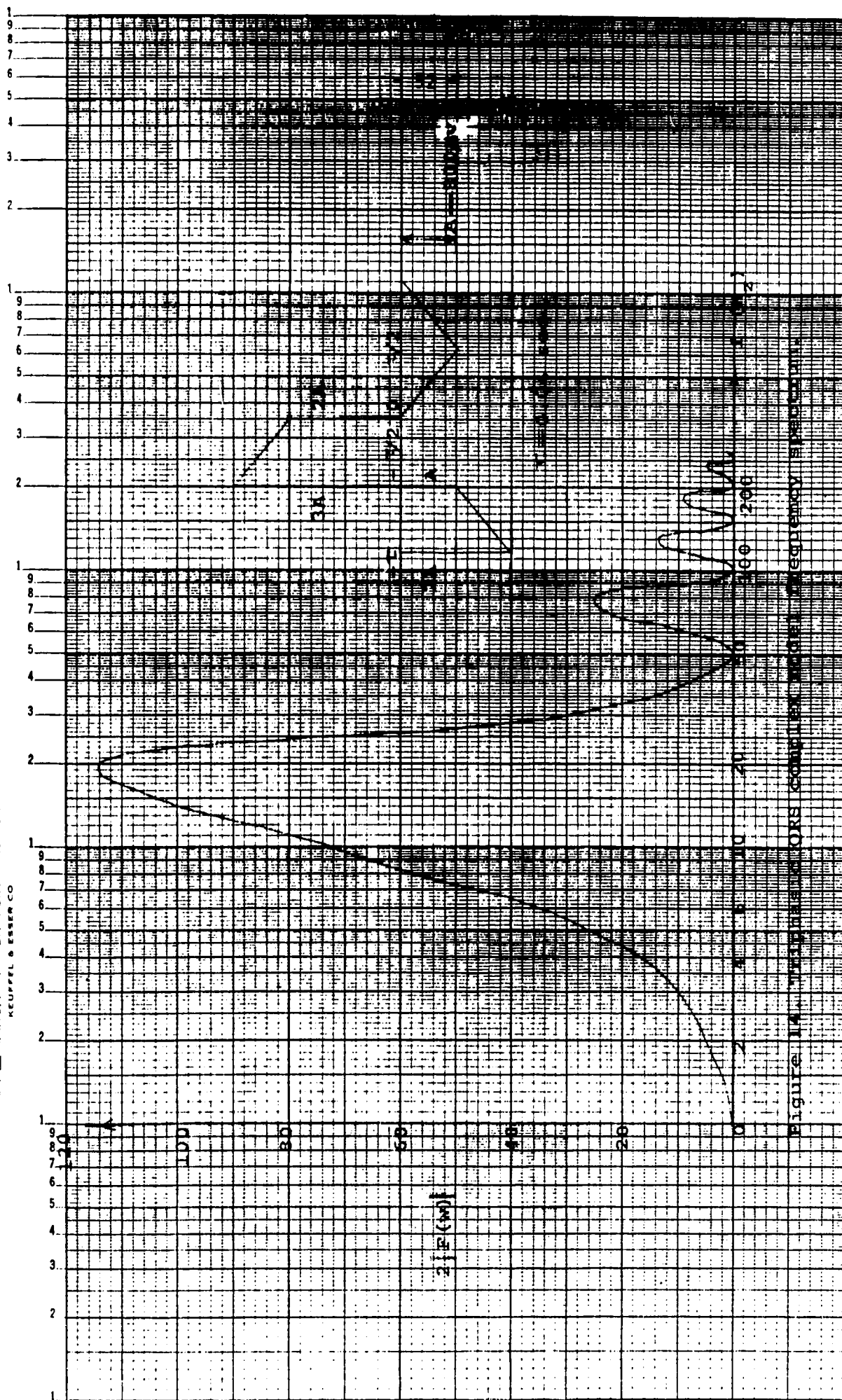


Figure 14. This is a QRS complex model frequency spectrum.

100 90 80 70 60 50 40 30 20 10 0

100 90 80 70 60 50 40 30 20 10 0

1 2 3 4 5 6 7 8 9

1 2 3 4 5 6 7 8 9

1 2 3 4 5 6 7 8 9

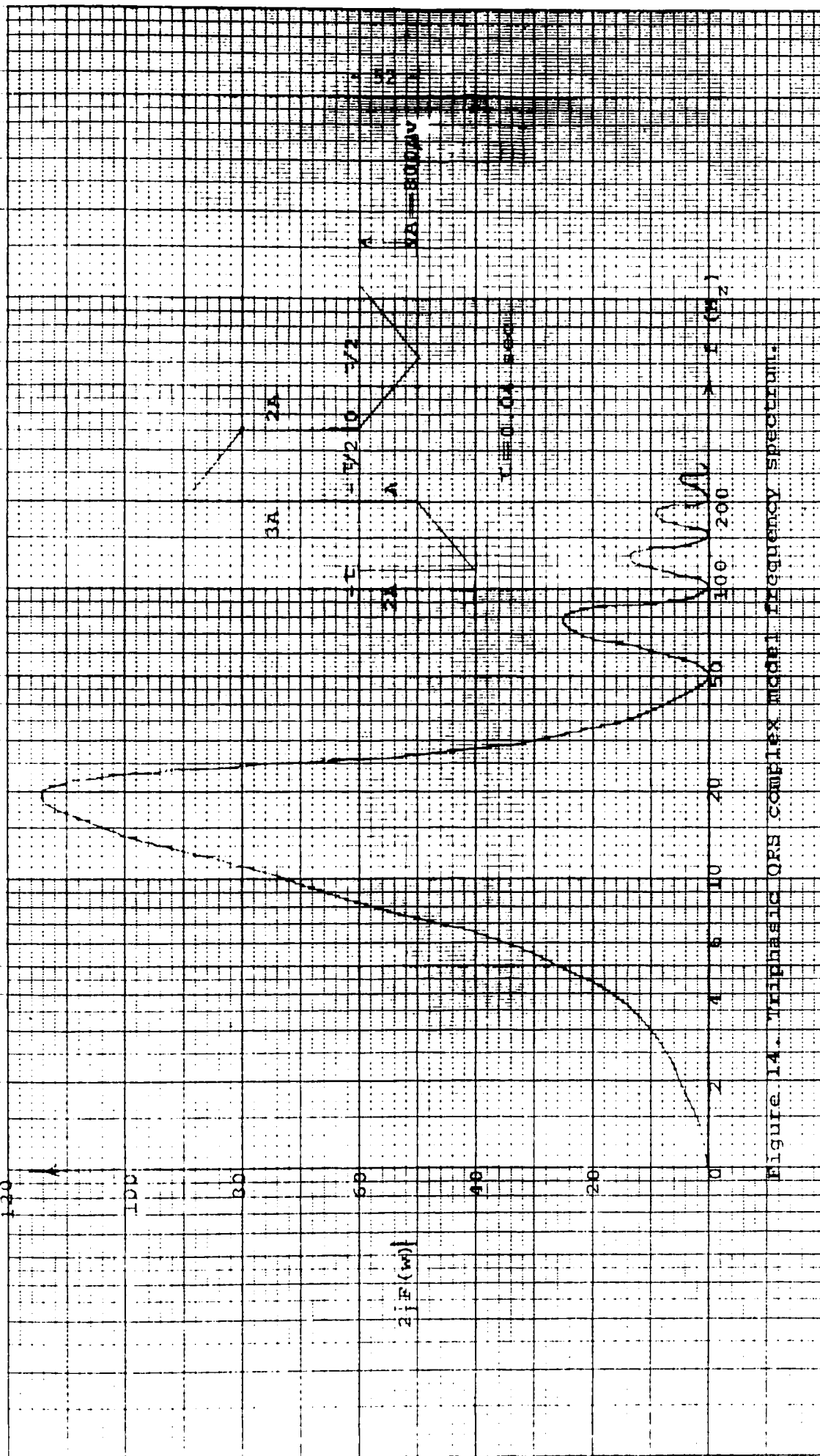


Figure 14. Triphasic QRS complex model frequency spectrum.

$$\begin{aligned}
 & 2 |F(w, 0) - F(w, -\tau)| \\
 &= 4A\tau(1 - \cos(w\tau/2)) \left| \frac{-1}{(w\tau)^2} (jw\tau - 1 + e^{-jw\tau}) \right| \\
 &= \frac{4A}{(w\tau)^2} (1 - \cos(w\tau/2)) \left[2(1 - \cos(w\tau/2)) + (w\tau)^2 \right]^{1/2}
 \end{aligned}$$

In Fig. 14 a sketch of the spectrum is given.

6.5. T-wave and P-wave

The T-wave and P-wave can usually be approximated to a half sin wave or the square thereof.

The P-wave contributes very little to the spectrum of the electrocardiogram in comparison with the QRS complex.

For $f_1(t) = A \sin \frac{\pi}{\tau} t$

$$\begin{aligned}
 F_1(w) &= A \int_0^\tau (\sin \frac{\pi}{\tau} t) e^{-jw t} dt \\
 &= \frac{A}{2j} \int_0^\tau \left[e^{j(\pi/\tau - w)t} - e^{-j(\pi/\tau + w)t} \right] dt \\
 &= \frac{A}{2j} \left[\frac{1}{j(\pi/\tau - w)} e^{j(\pi/\tau - w)t} + \frac{1}{j(\pi/\tau + w)} e^{-j(\pi/\tau + w)t} \right]_0^\tau
 \end{aligned}$$

$$= \frac{-A}{2} \frac{1}{(\pi/\tau)^2 - w^2} \left[(\pi/\tau - w) (e^{-j(\pi/\tau - w)\tau} - 1) + (\pi/\tau + w) (e^{-j(\pi/\tau + w)\tau} - 1) \right]$$

$$= \frac{A/2}{(\pi/\tau)^2 - w^2} e^{-jw\tau} \left[e^{jw\tau} + 1 \right] (2 \frac{\pi}{\tau})$$

putting $w = 2\pi f$

$$= \frac{A\pi/\tau}{(\pi/\tau)^2 - 4\pi^2 f^2} e^{-j2\pi f\tau} \left[e^{j\pi f\tau} + e^{-j\pi f\tau} \right]$$

$$= \frac{2A}{(1 - 4f^2\tau^2)} e^{-j\pi f\tau} \cos \pi f\tau$$

$$2|F_1(w)| = \frac{4A\tau}{\pi(1 - 4f^2\tau^2)} \cos \pi f\tau$$

For $f\tau = 1/2$, $2|F_1(w)| \rightarrow A\tau$

Similary for $f(t) = A \sin^2 \frac{\pi}{\tau} t$ we have the Fourier transform

$$F_2(w) = \left(\frac{Ae^{-j\pi f\tau}}{2\pi f} \right) \left(\frac{\sin \pi f\tau}{1 - f^2\tau^2} \right)$$

$$2|F_2(w)| = \frac{A}{\pi f} \left| \frac{\sin \pi f\tau}{1 - f^2\tau^2} \right|$$

when $f\tau = 1$, $2|F_2(w)| \rightarrow \frac{A}{2f} = \frac{A\tau}{2}$

Fig. 15 and 16 show the spectrum of $f_1(t)$ and $f_2(t)$ respectively with $\tau = 0.2$ sec, the normal T-wave duration.

The frequency spectra of the waveforms previously discussed were plotted on the same scale for comparison.

The Fourier transform of the electrocardiogram $F(\omega)$ is therefore

$$F(\omega) = F_P(\omega) + e^{-j\omega T_Q} F_{QRS}(\omega) + e^{-j\omega T_T} F_T(\omega)$$

Taking the P-wave onset as the origin on the time axis, T_Q and T_T are the time delay of the QRS and T waves respectively.

With the phase shift the spectrum of individual waveforms may be added to or subtracted from one another depending on the phase angles.

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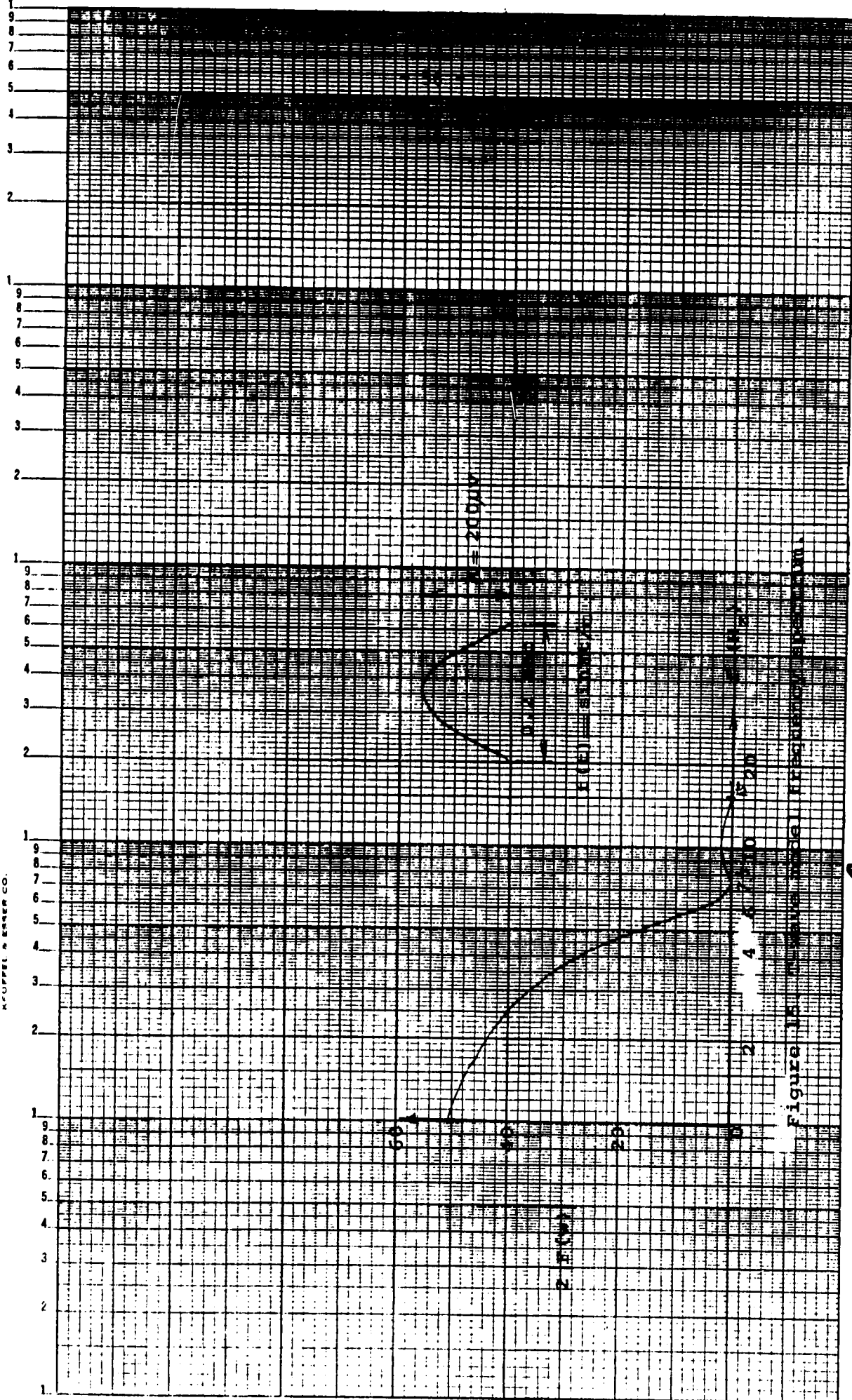


Figure 15. Resonance Model Frequency Response.

Figure 15. T-wave model frequency spectrum.

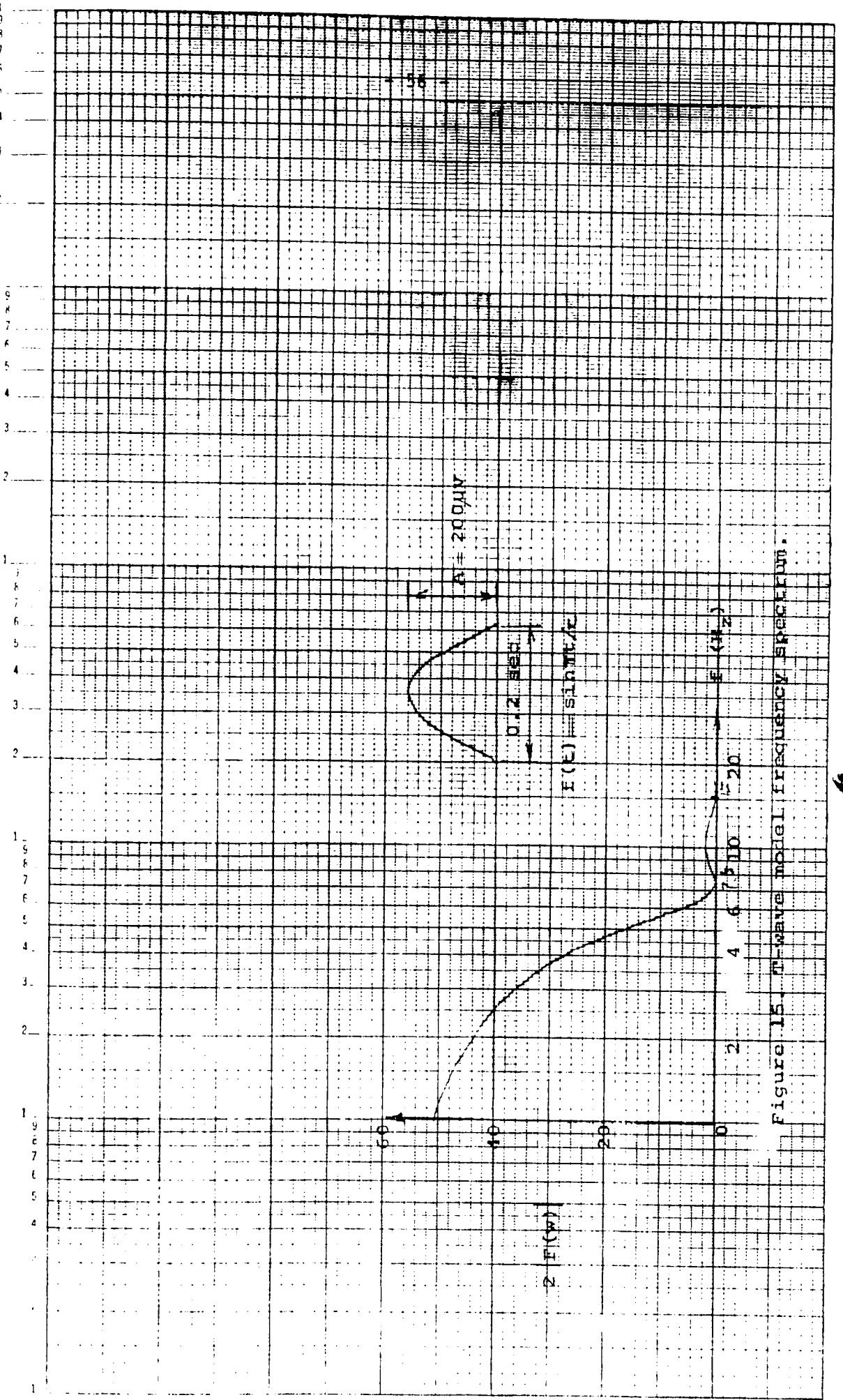


Figure 15. T-wave model frequency spectrum.

$\frac{P_{\text{avg}}}{P_{\text{avg}}}$ SEMI LOGARITHMIC 46 6213
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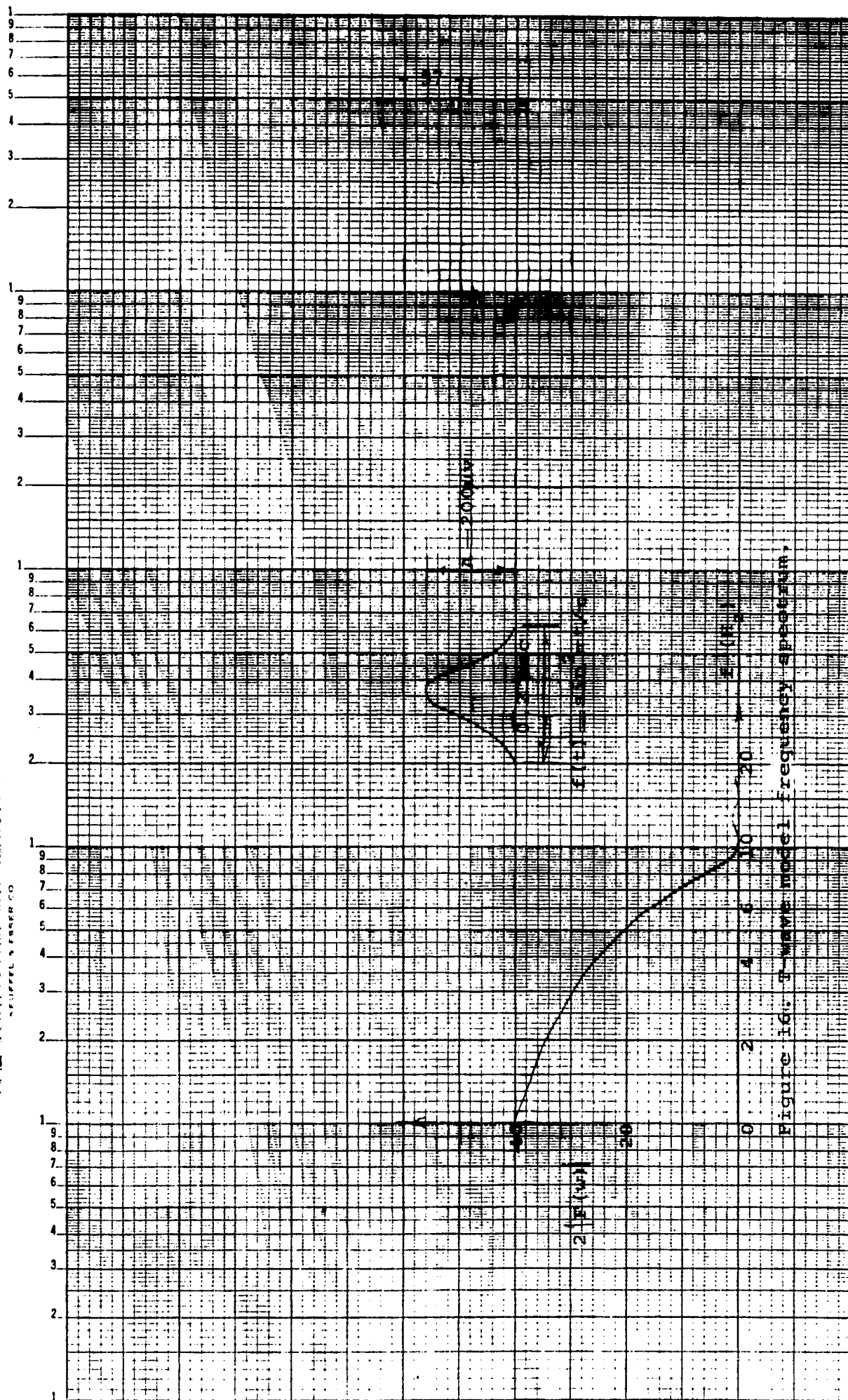


Figure 16: T-wave model frequency spectrum.

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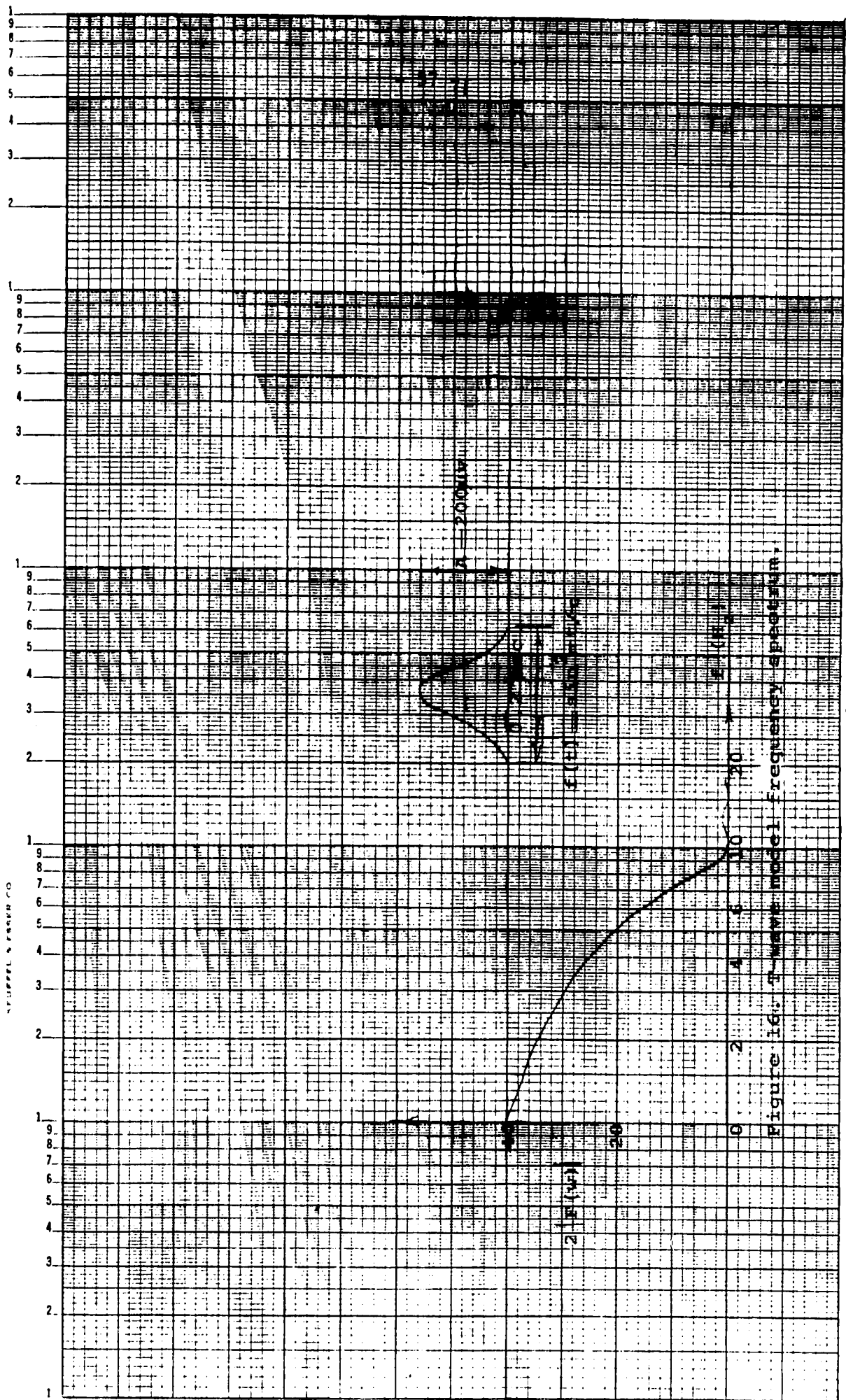
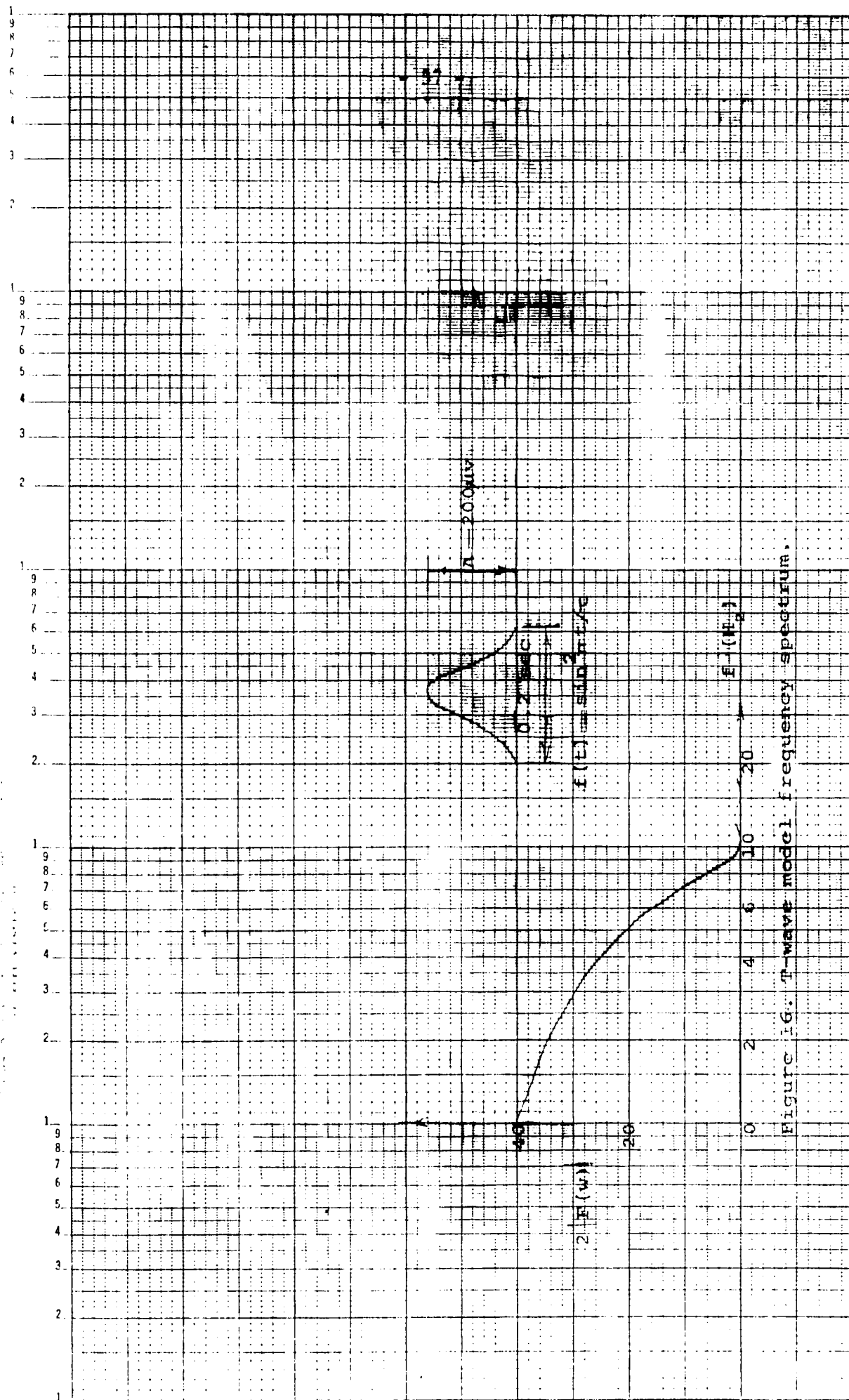


Figure 16. T-wave model frequency spectrum.



CHAPTER VII

INVESTIGATION OF THE ELECTROCARDIOGRAPHY

7.1. Introduction

The electrocardiograms investigated were taken from the synoptic electrocardiogram presented at the symposium on Advanced Electrocardiography at the Emory School of medicine, Atlanta [6] .

The electrocardiograms recorded from standard lead I were chosen for studies, since standard lead I is the most significant diagnostic single lead among the lead system, and it is easy to apply in practice.

The electrocardiograms studied were digitized and then normalized to find the frequency spectrum and phase spectrum.

Appendix I listed the cases under study. The cases were grouped into seven categories:

- (1) Normal
- (2) Left Ventricular Hypertrophy
- (3) Right Ventricular Hypertrophy
- (4) Right Bundle Branch Block
- (5) Left Bundle Branch Block
- (6) Myocardial Infraction
- (7) Chronic Coronary Insufficiency

The magnified cardiac waveforms of the cases studied and the corresponding frequency and phase spectra are shown

in Figures 17a to 36a, Figures 17b to 36b and Figures 17c to 36c respectively.

The cardiac waveforms investigated were taken from the onset of the P-wave to the end of the T-wave.

7.2. Normal (Cases 1 and 2)

Case 1 (Figure 17a) is taken as a typical normal case for comparison with the other abnormal cases.

In general the normal electrocardiogram has the QRS complex with sharp peak and relatively fast rise time in R-wave portion while the P-wave and T-wave are slow. The P-wave and T-wave, and the positioning of the P-wave, T-wave and the QRS complex would favor the first five harmonics. The higher harmonics are mostly contributed by the QRS complex.

In Case 1 the spectral pattern (Figure 17b) of a small first harmonic followed by harmonic peaks at the second, fourth and seventh harmonics could be taken as a standard normal case of spectral pattern for comparison with other abnormal cases of spectral patterns. However, there exists a wide waveform variations within the normal and abnormal waveforms.

In Figure 18a, case 2, the wider and smaller R-wave and the smaller Q and S-waves contribute to the third and

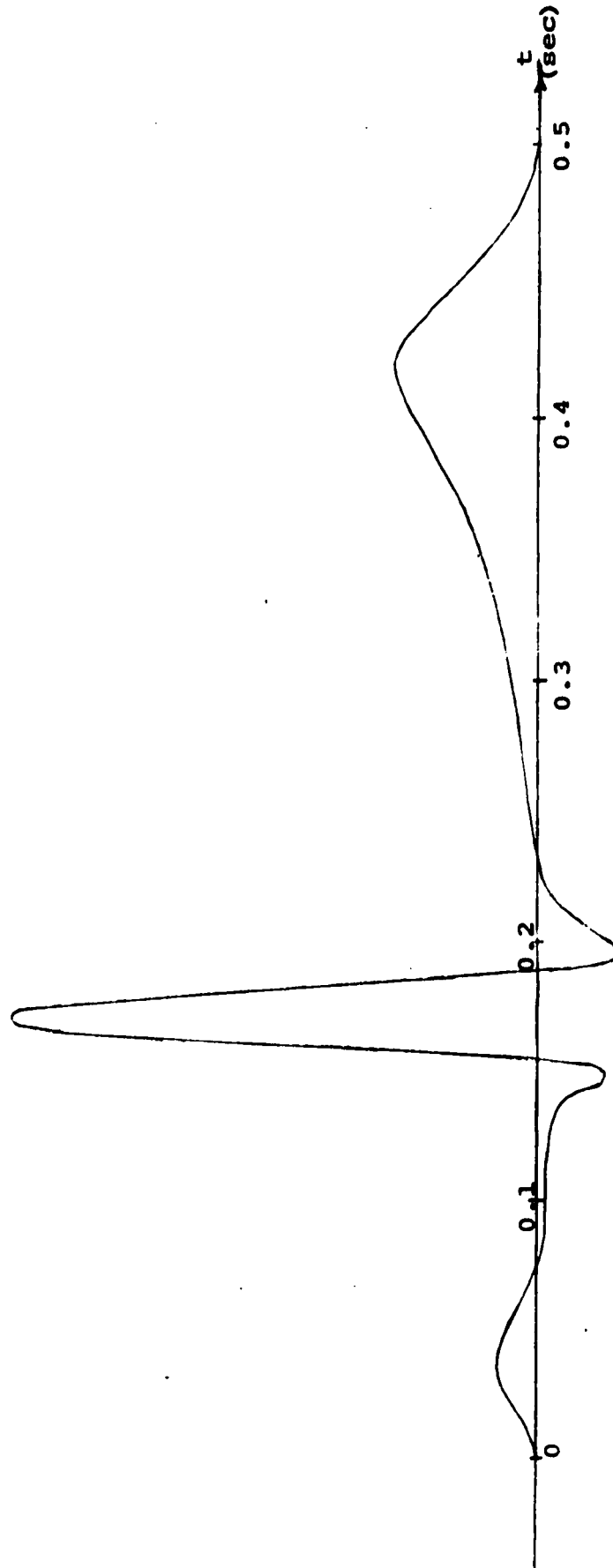
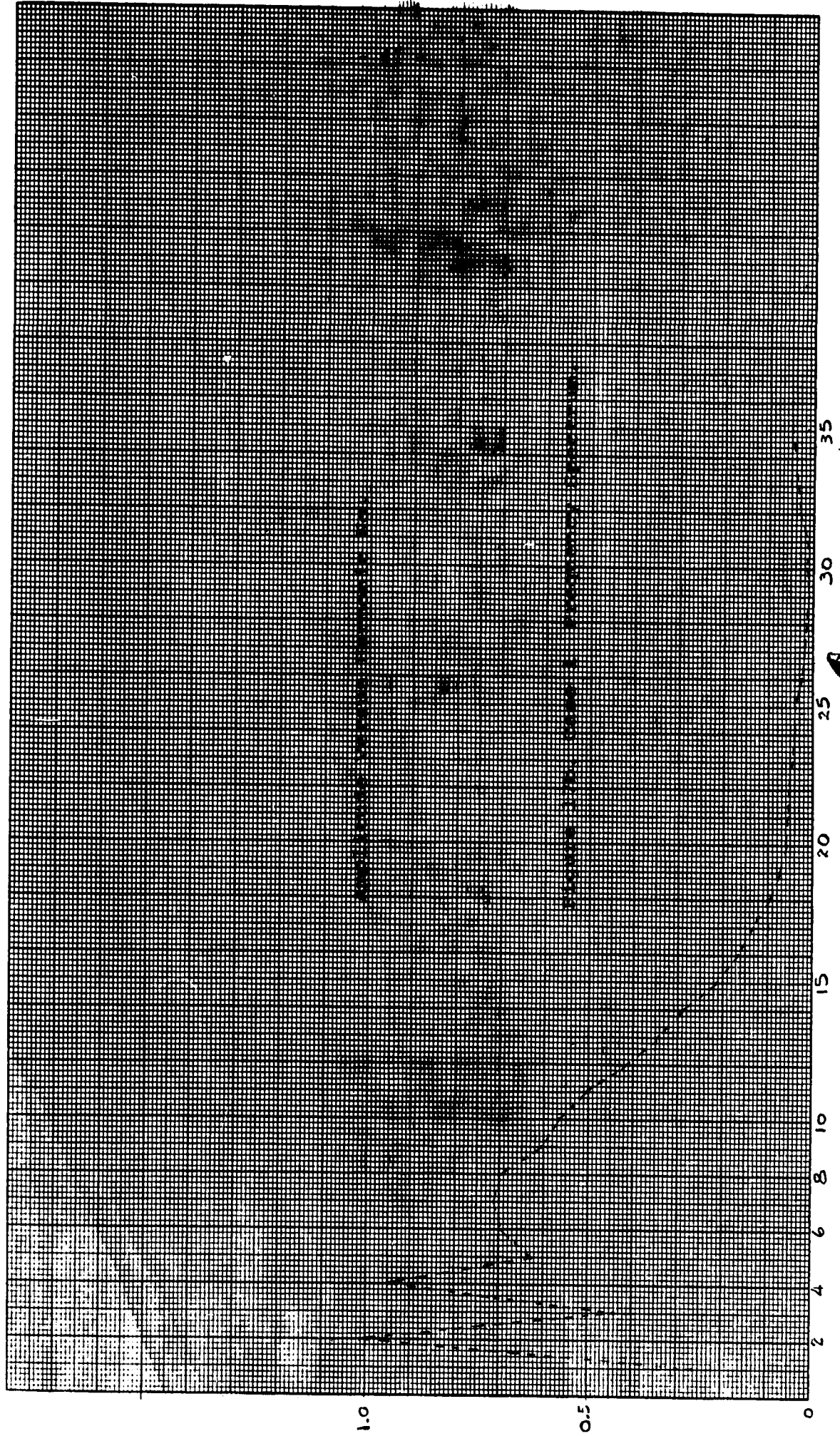
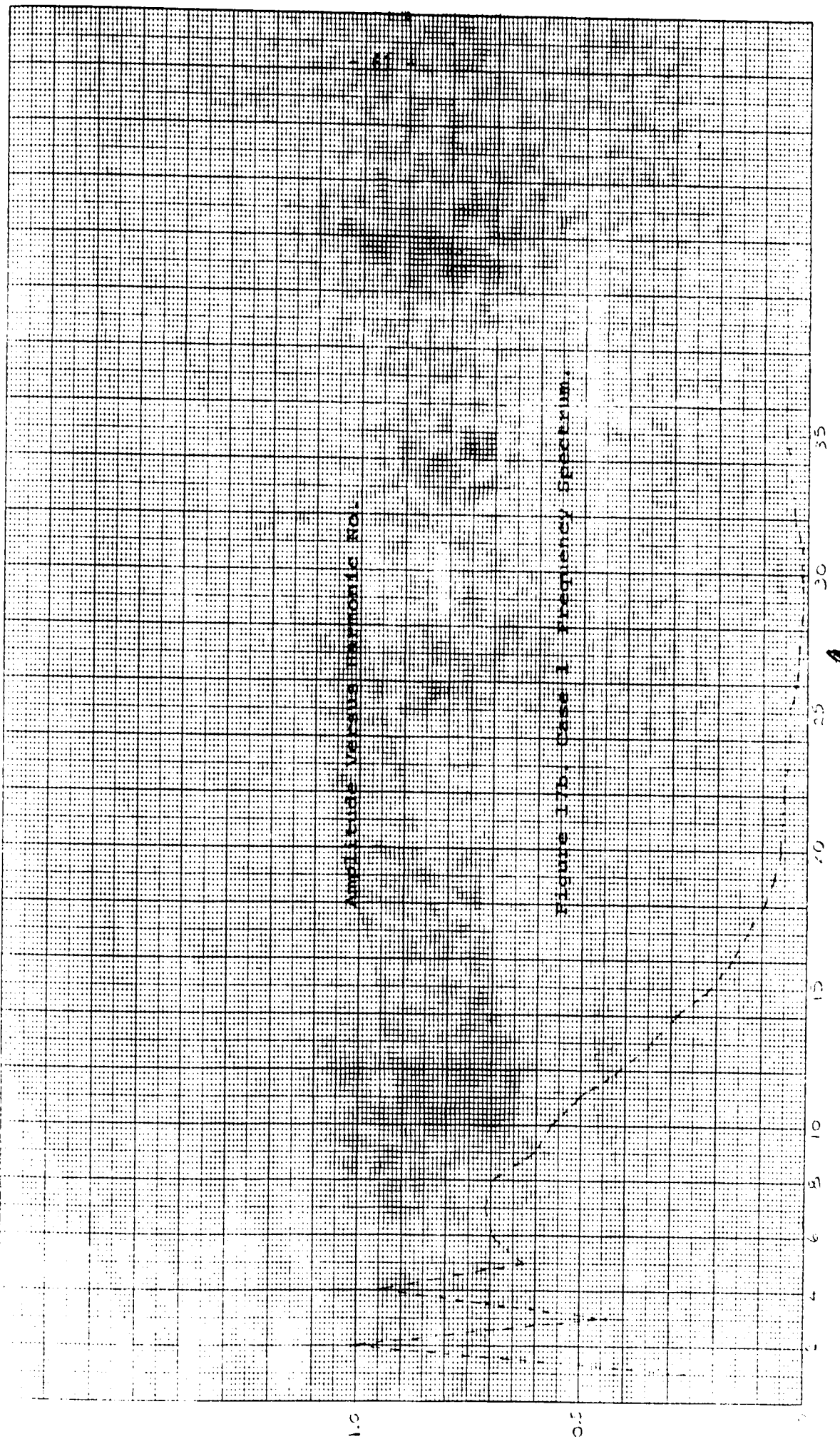


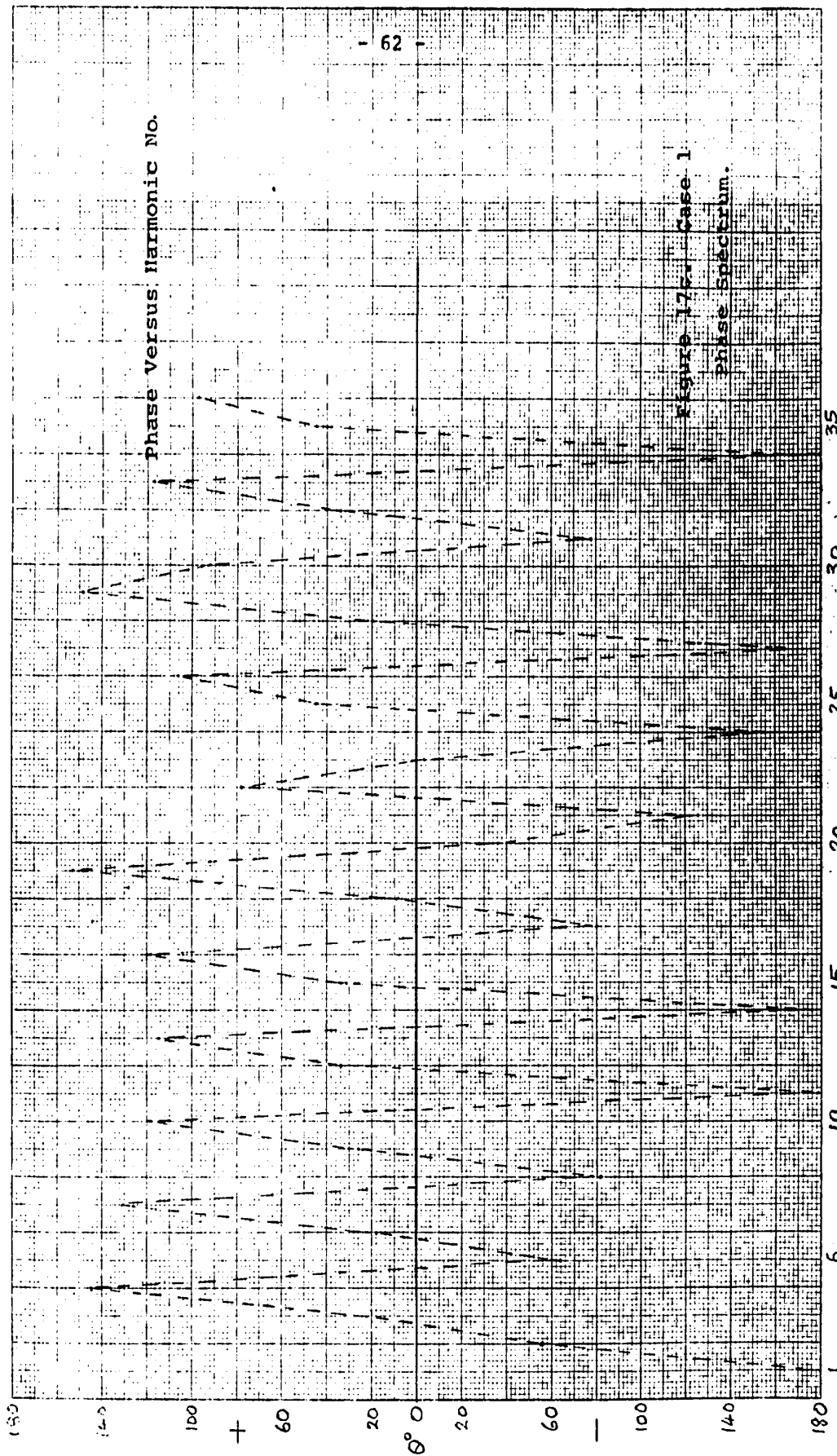
Figure 17a. Case 1 electrocardiogram.

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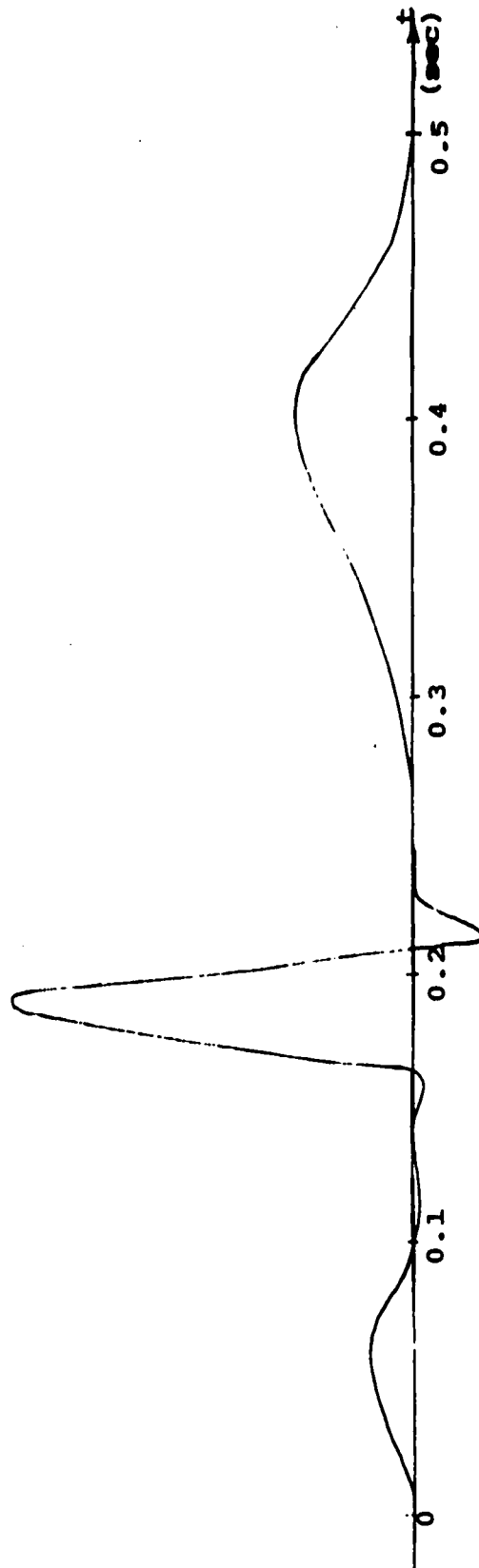
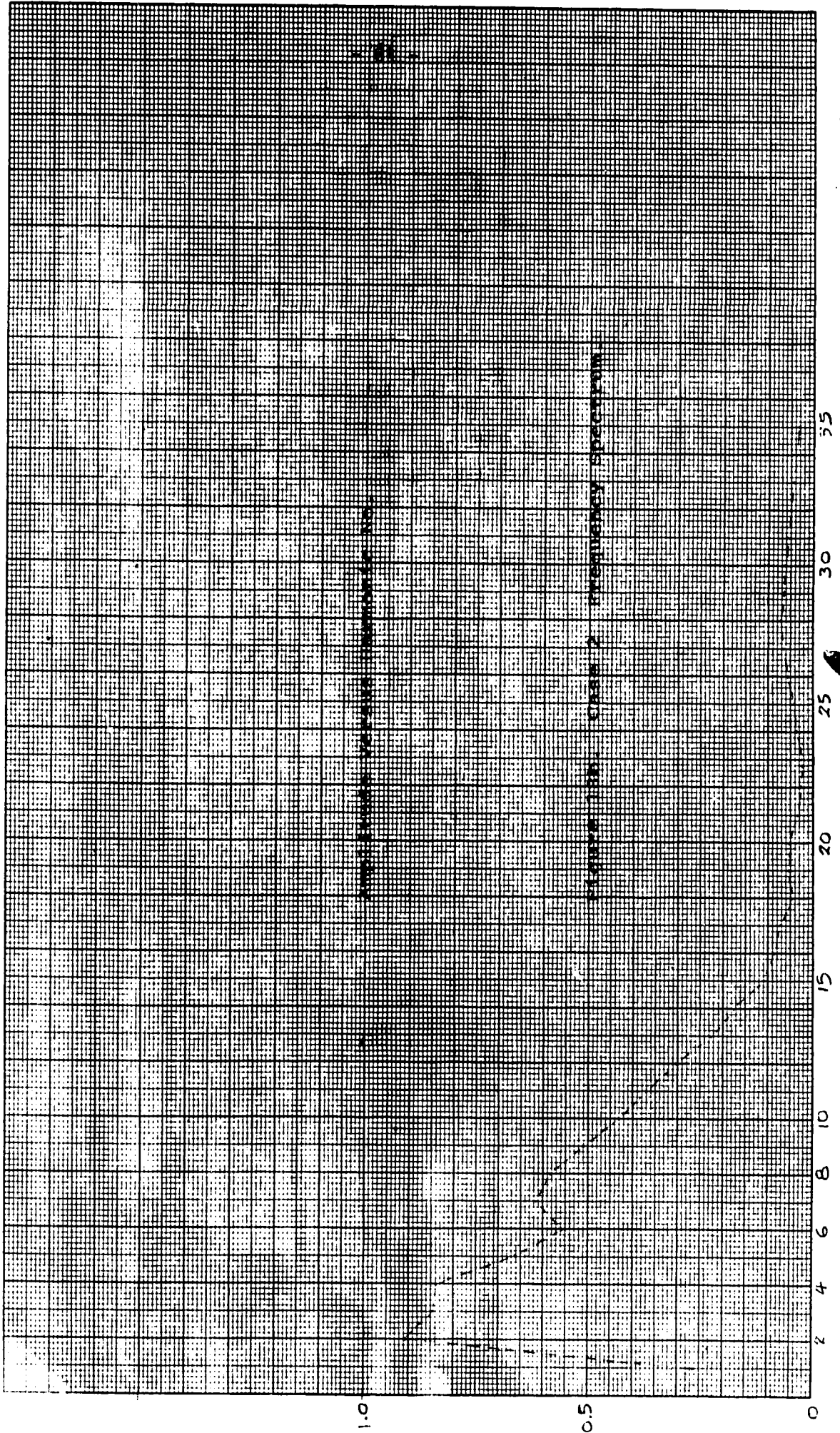
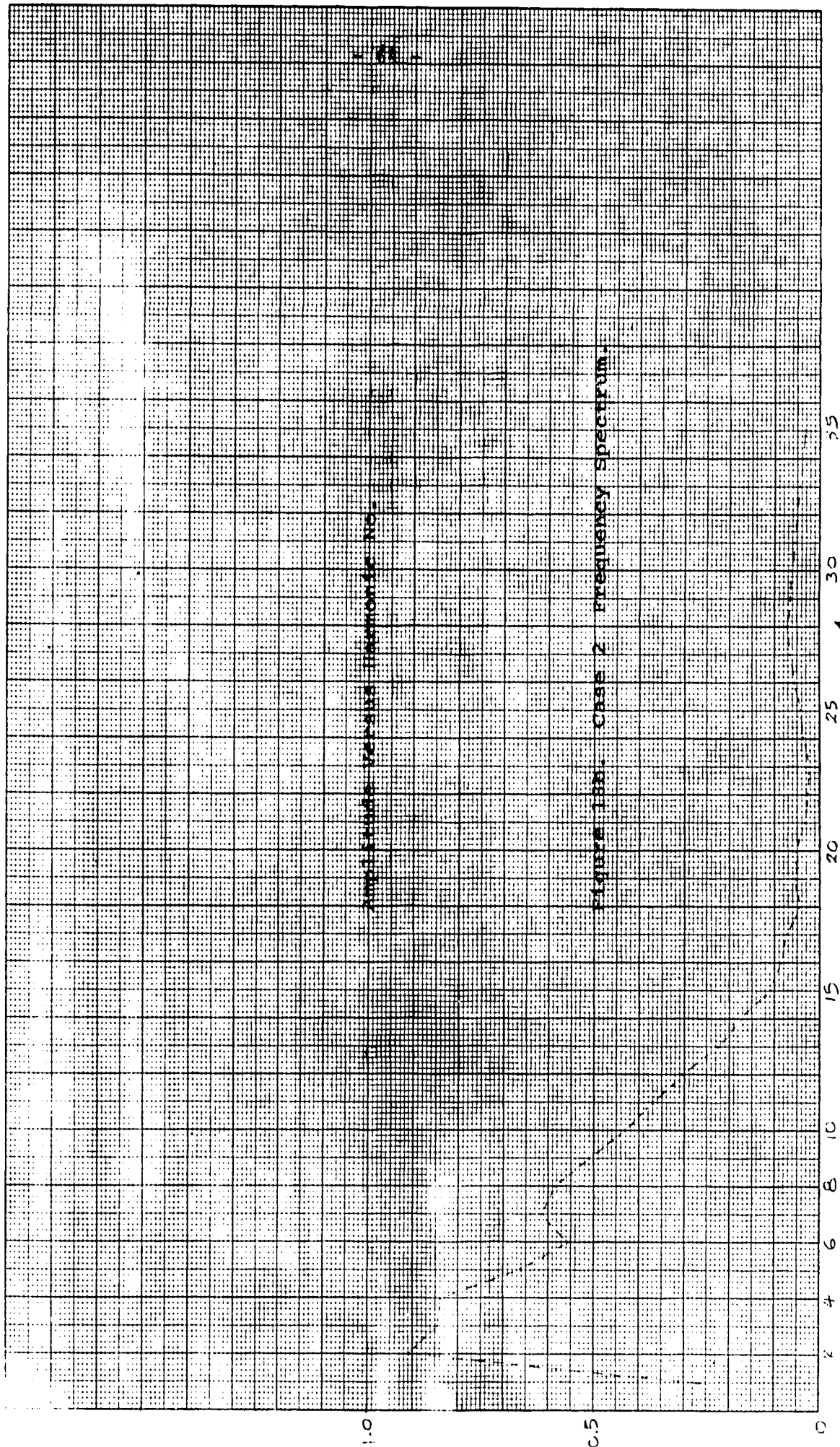


Figure 18a. Case 2 electrocardiogram.

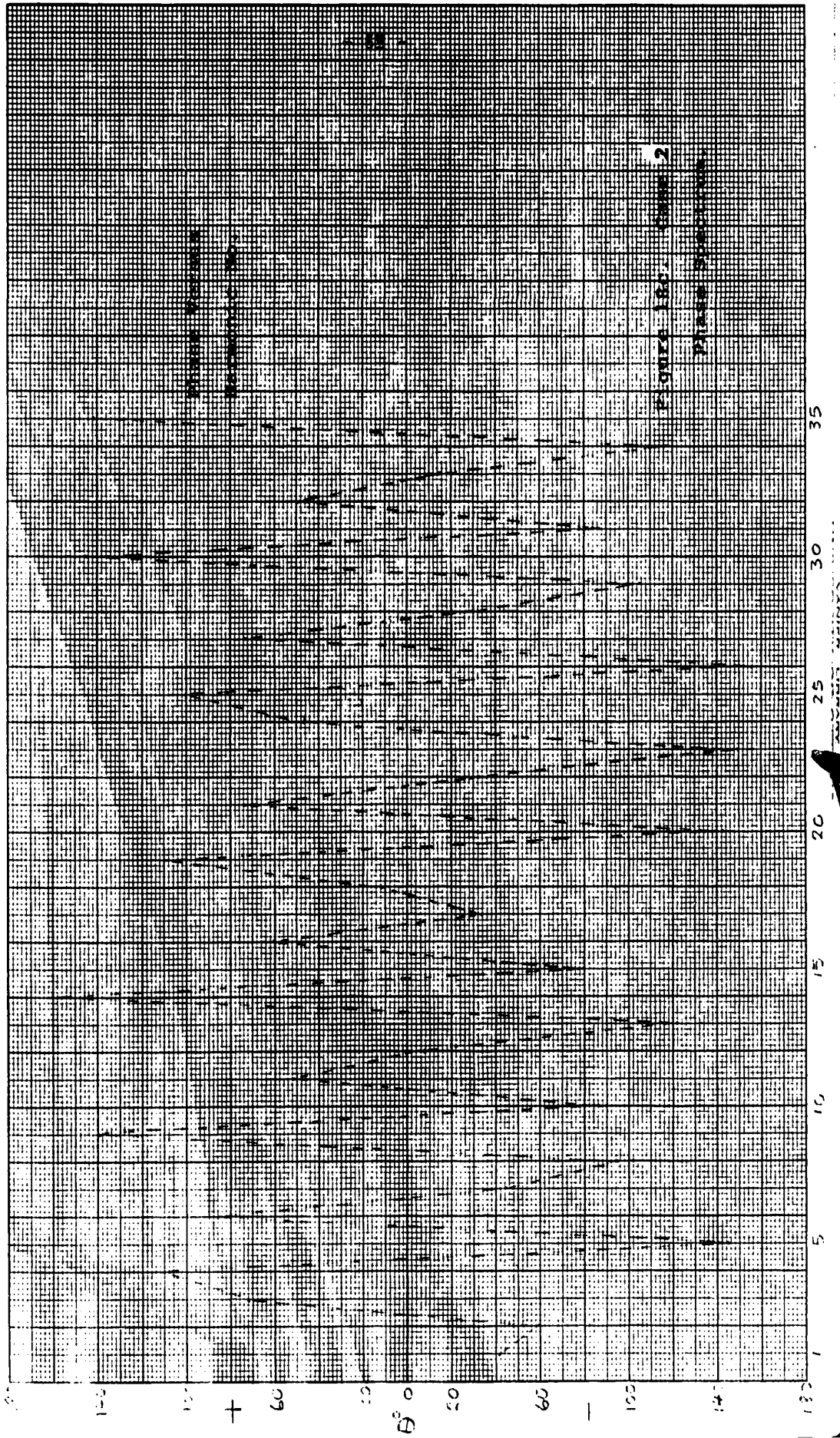
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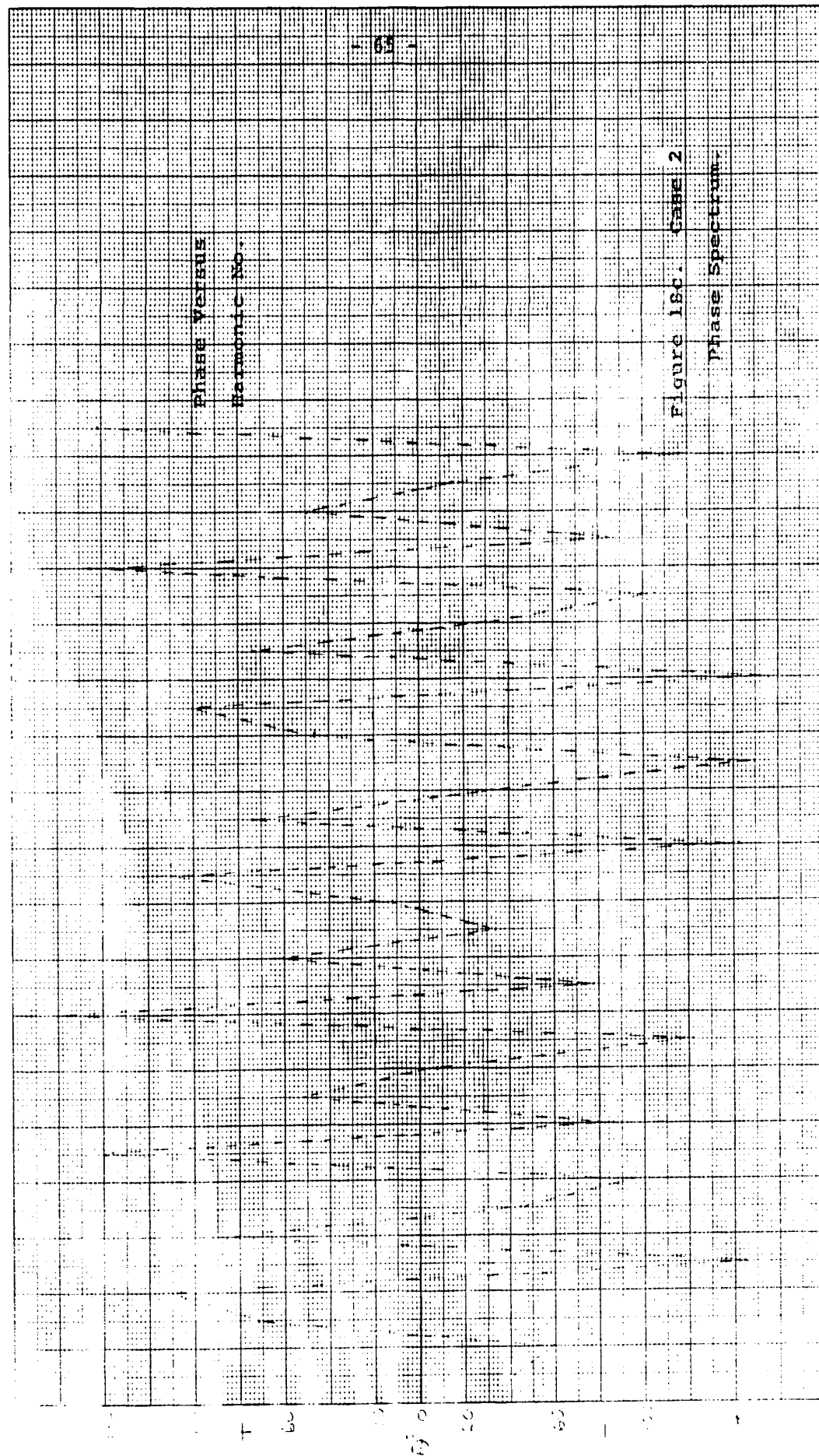


Figure 18c. Case 2
Phase Spectrum

fifth harmonics larger than those in Figure 17, case 1.

7.3. Left Ventricular Hypertrophy (cases 3-6)

The commonest cause of left ventricular hypertrophy [18-21, 24-26] is hypertension.

Many features of left ventricular hypertrophy overlap normal values, that is, many records that meet various criteria of left ventricular hypertrophy may be found in normal individuals and individuals with left ventricular hypertrophy may have reasonable normal electrocardiograms.

The left ventricle under normal conditions contributes approximately three quarters of the potentials which produce the QRS deflection. The enlargement of the left ventricle causes the left ventricle to exert extra effort in order to overcome arterial pressure. With hypertrophy of the left ventricle, this electrical predominance of left over right ventricle becomes all the more accentuated. Thus the mean instant QRS spatial vectors in left ventricular hypertrophy tend to increase in magnitude since the wall of the left ventricle is thicker than normal, the impulse will take longer to traverse it and arrive at the epicardial surface. Generally the left ventricular hypertrophy pattern is not altered very much from the normal.

It is characterized by a tall sharp R-wave and some times when left axis deviation is presented the R-wave will be followed by a depressed S-T segment and a negative T-wave. Left axis deviation means that the main QRS deflection in lead I is upward (positive) while the main deflection in lead II is downward (negative). Q-wave may be just as normal or deeper than normal and the QRS intervals towards upper limit of normal 0.1 second. These evidences can be seen in Figures 19a to 22a.

Figures 19a and 20a are the moderate left ventricular hypertrophy cases. The steep R-wave slope contributes to the high frequency components with amplitudes larger than those in the normal case. The widened deeper S-wave in Figure 19a causes a minimum at the nineteenth harmonic. (see Figure 19b).

In Figures 19b and 20b there are peaks at the second and fourth harmonics as in the normal case, but the seventh harmonic is no longer a peak, and the amplitude in the high frequency range are increased.

In Figures 21a and 22a there is a taller R-wave followed by depressed S-T segment and a negative T-wave. The waveforms produced the following effects on the frequency spectrum: The inverted T-wave was evidenced in the first, second and third harmonics and some effects also produced

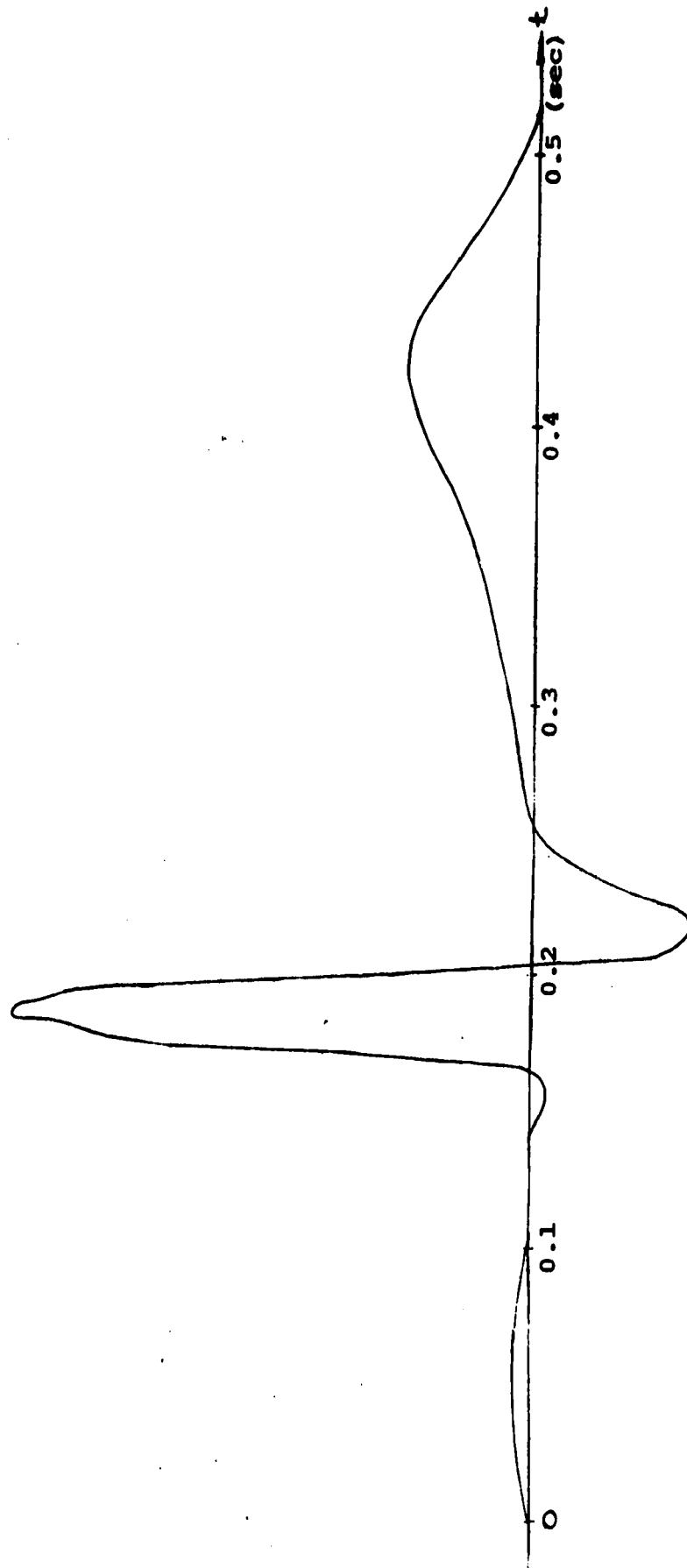
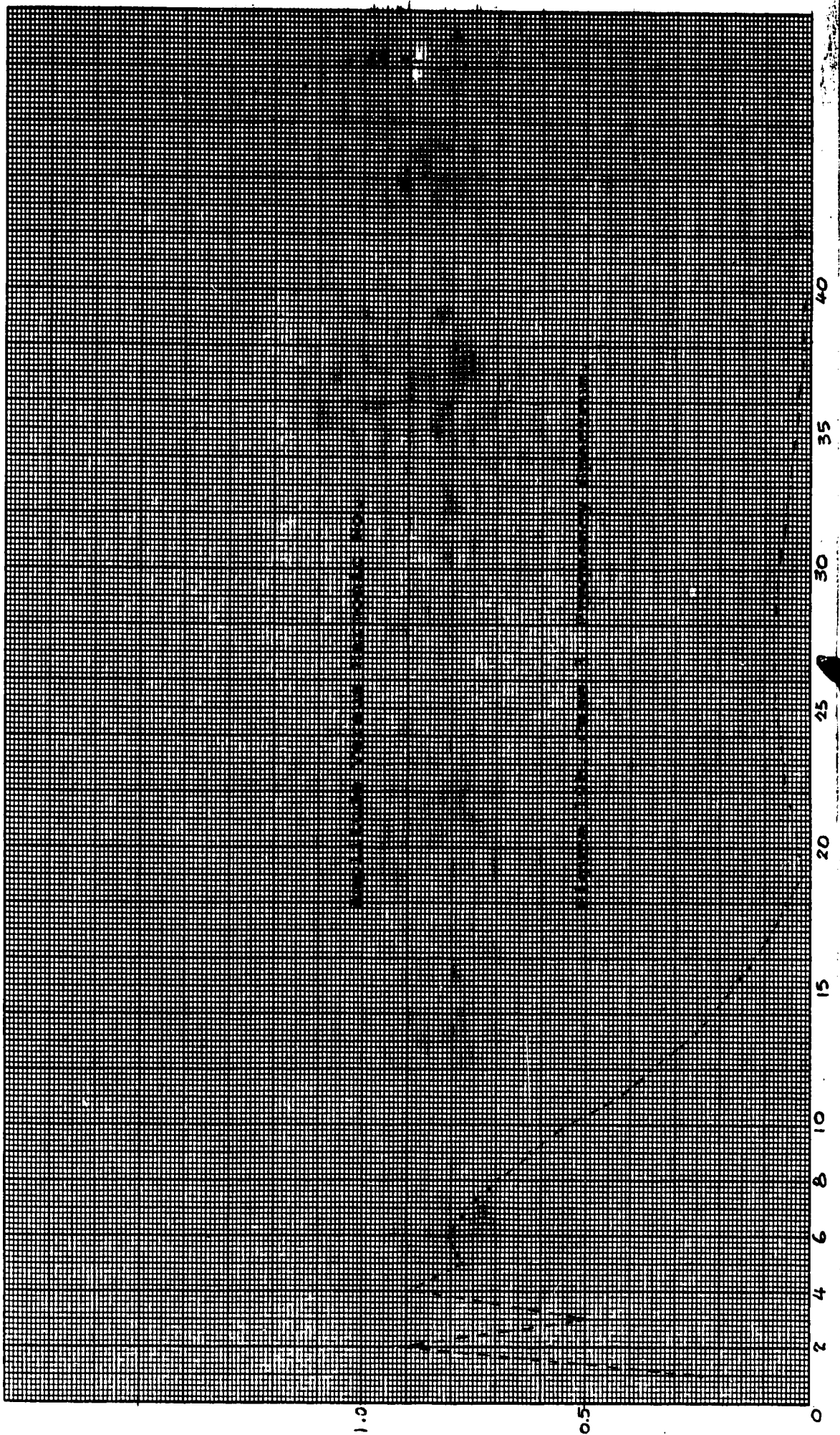


Figure 19a. Case 3 electrocardiogram.

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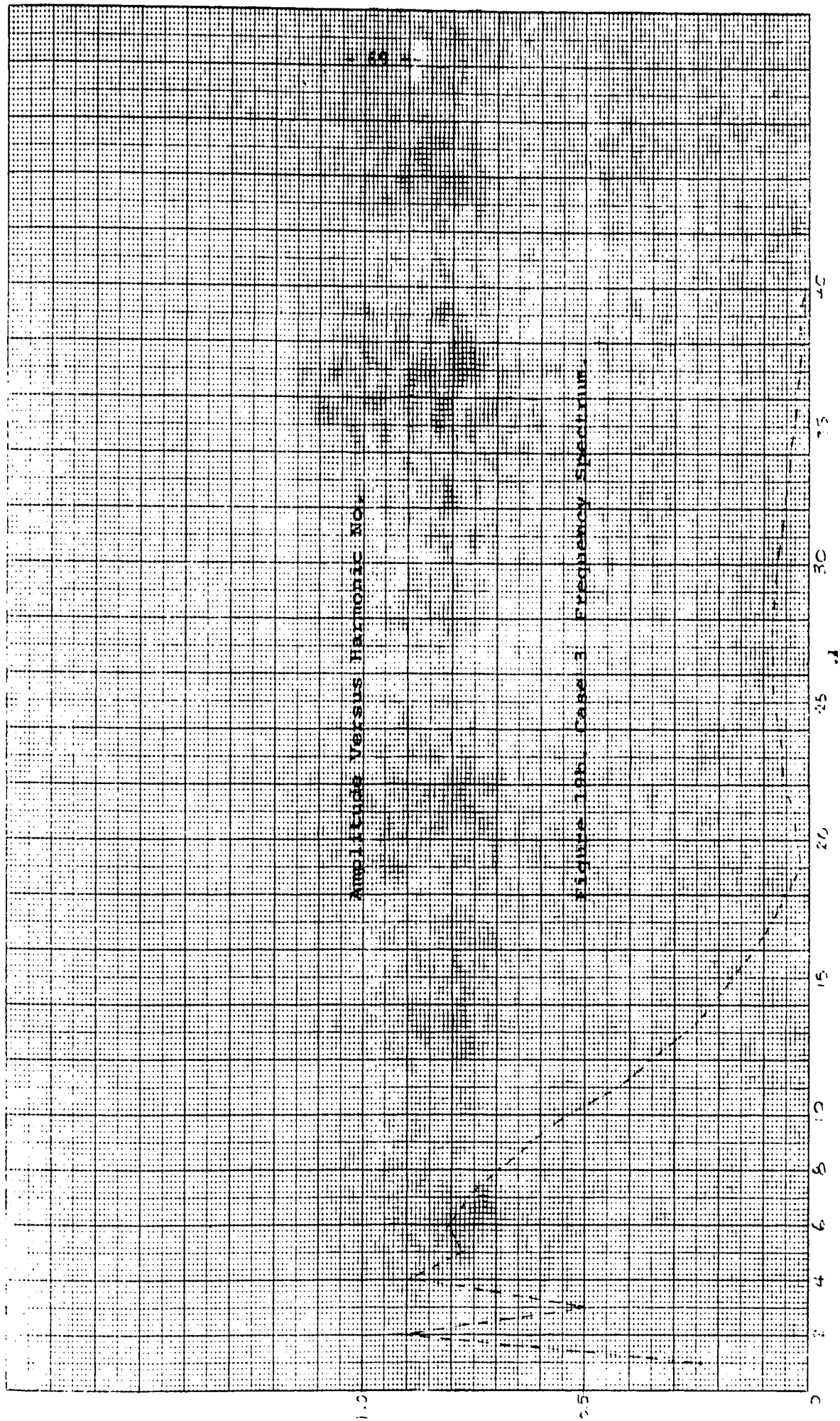
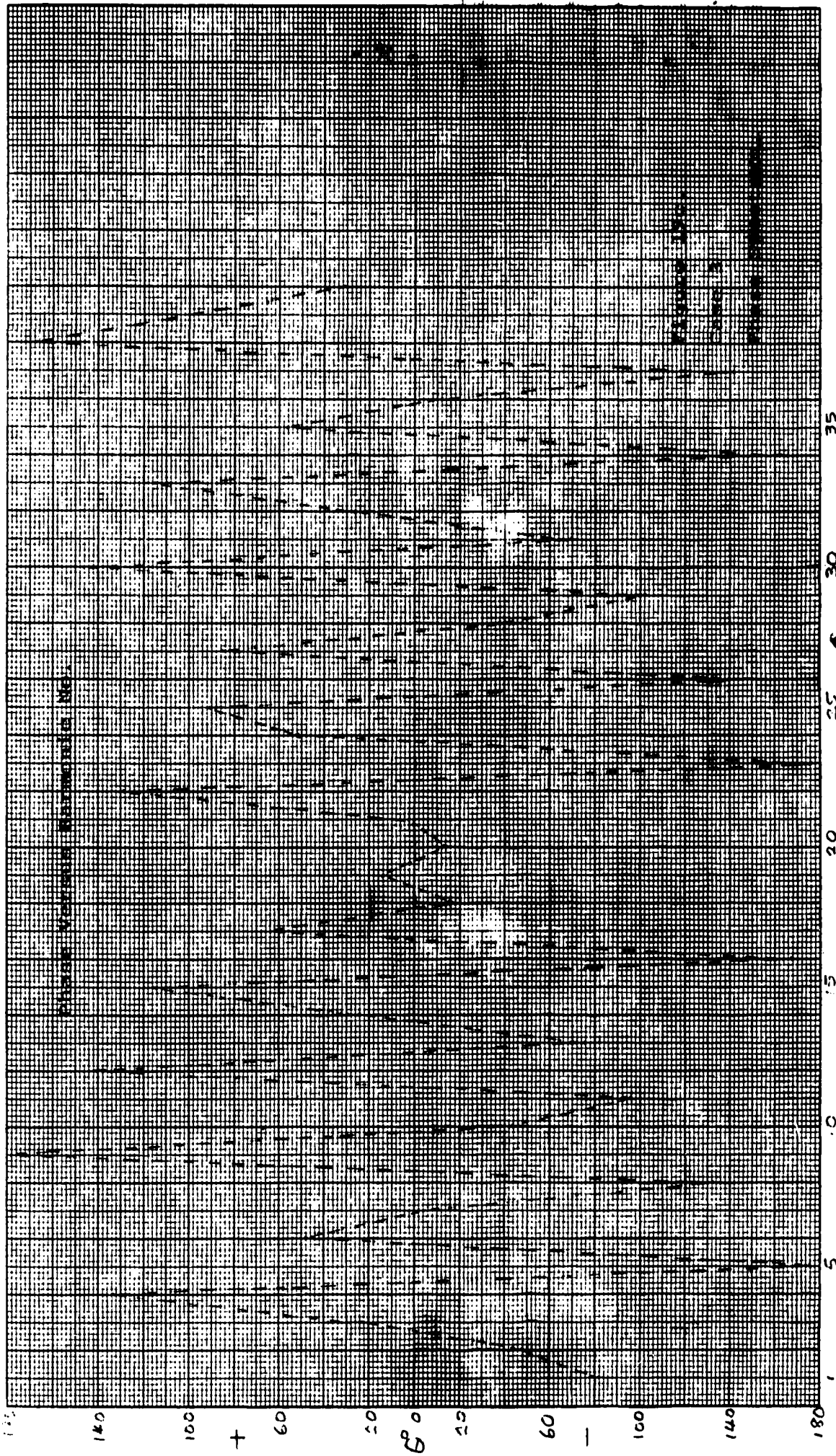
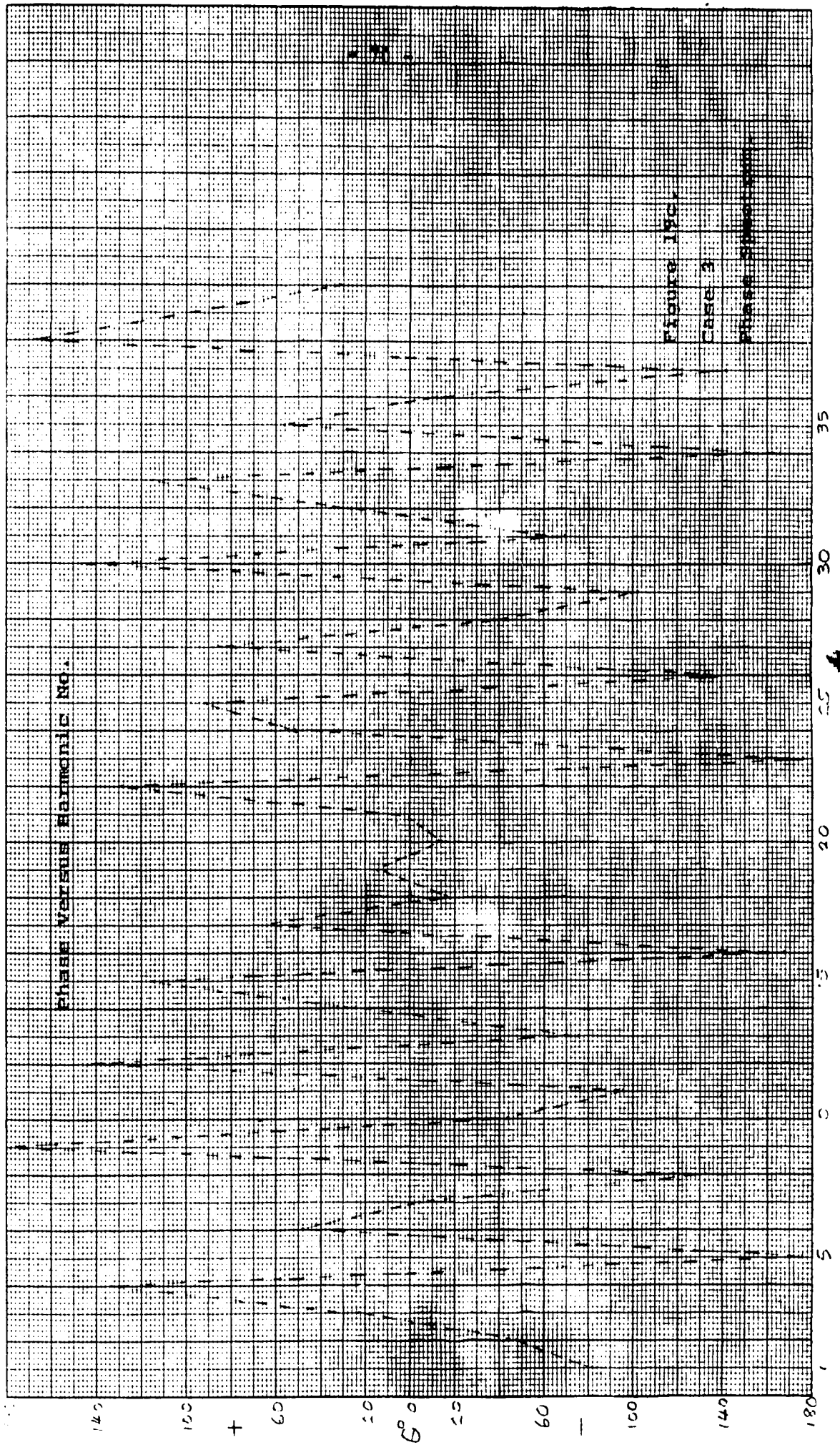


Figure 10b Case 3 Frequency Spectrum

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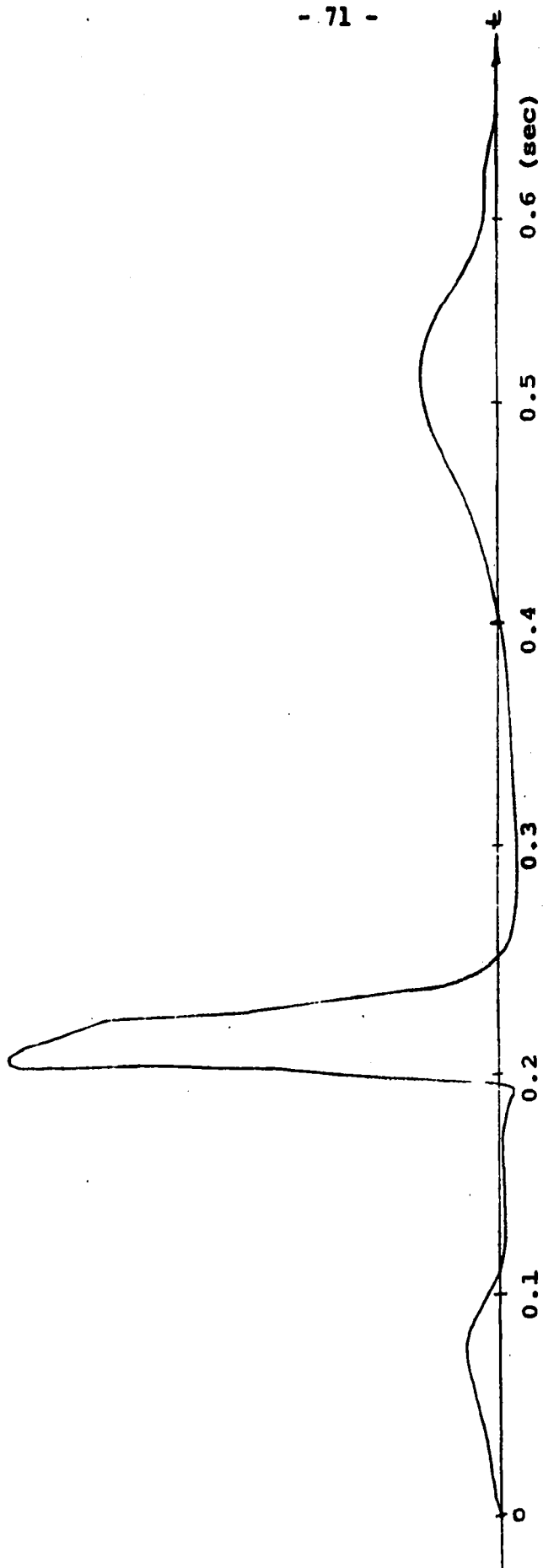
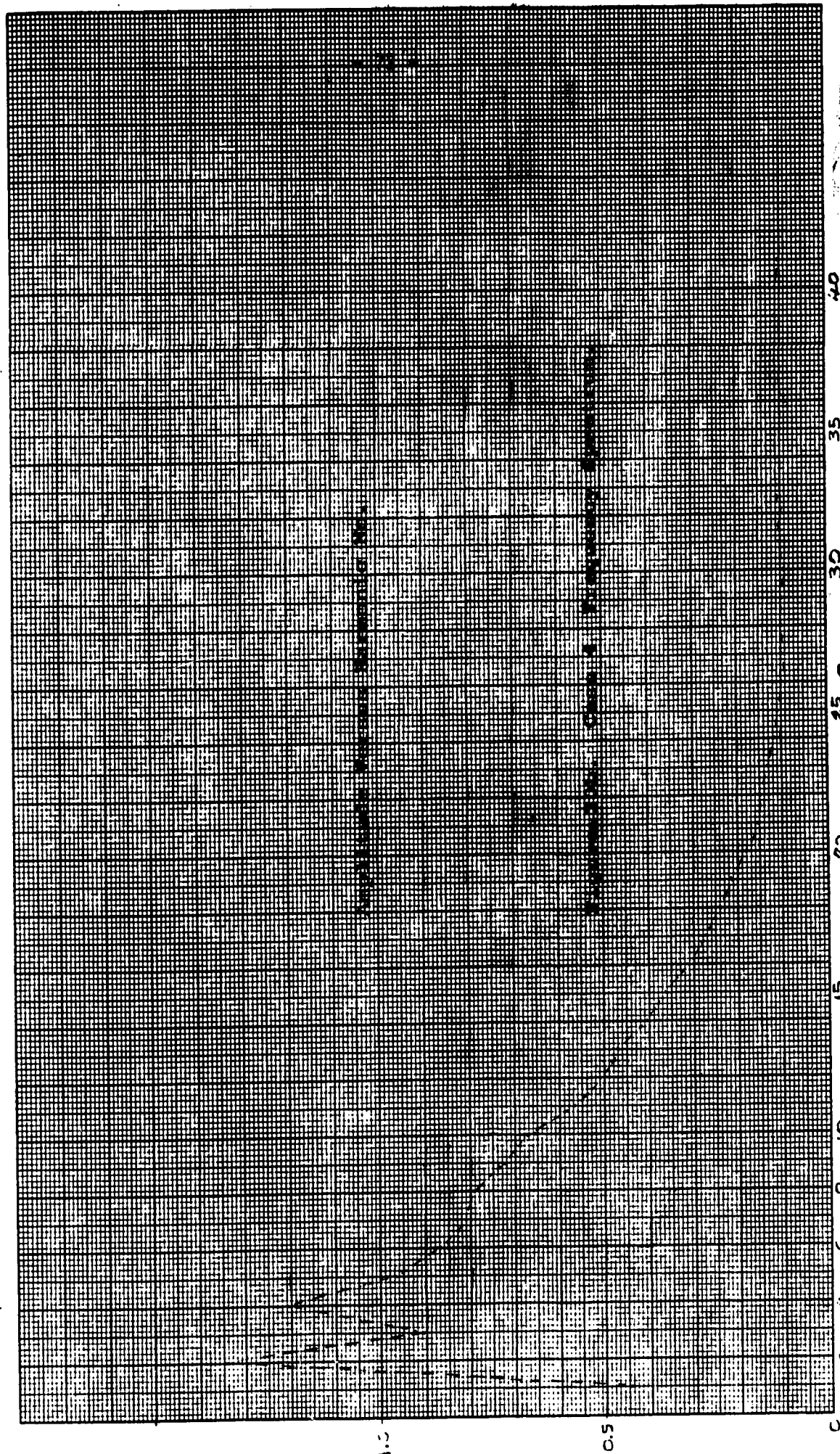
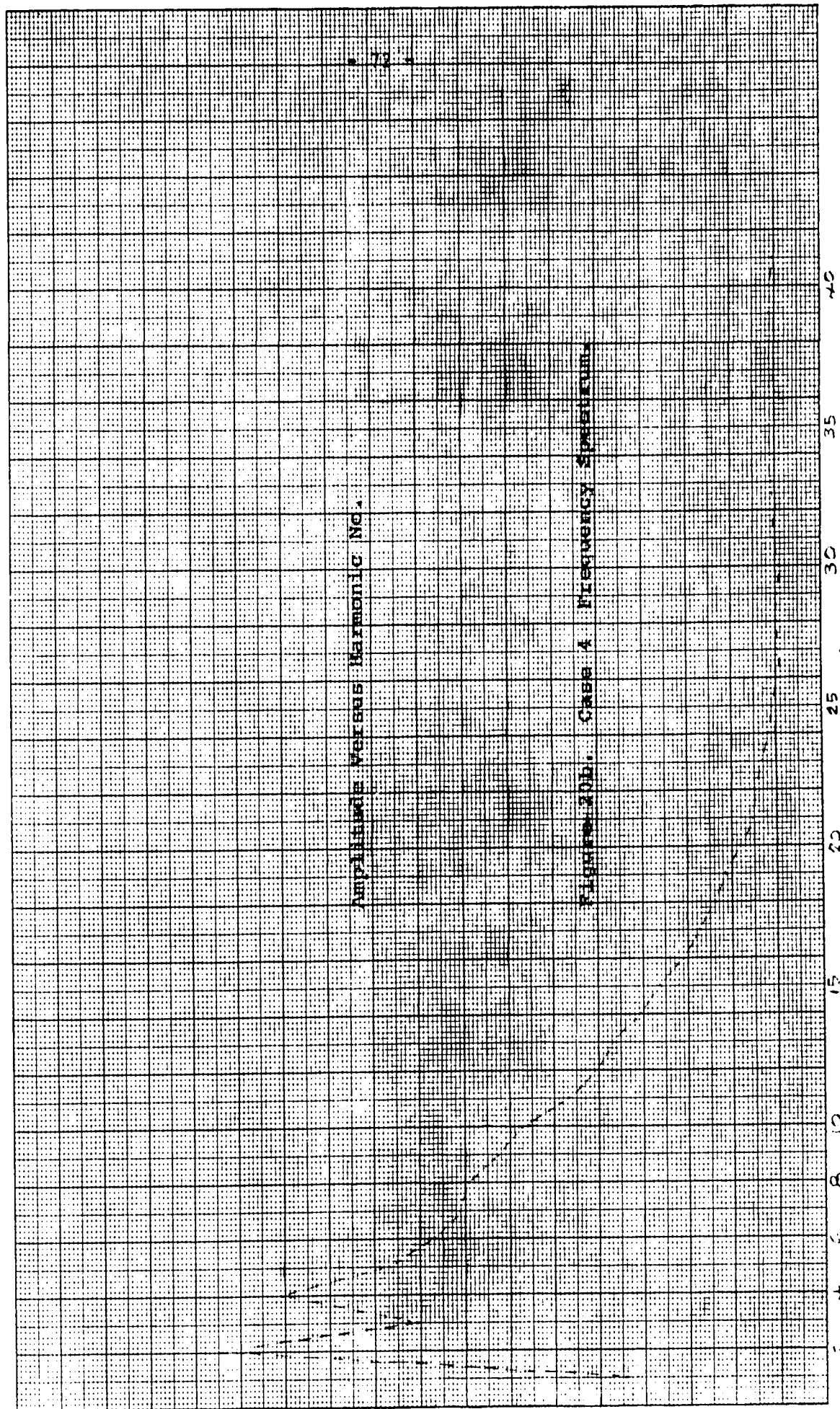


Figure 20a. Case 4 electrocardiogram.

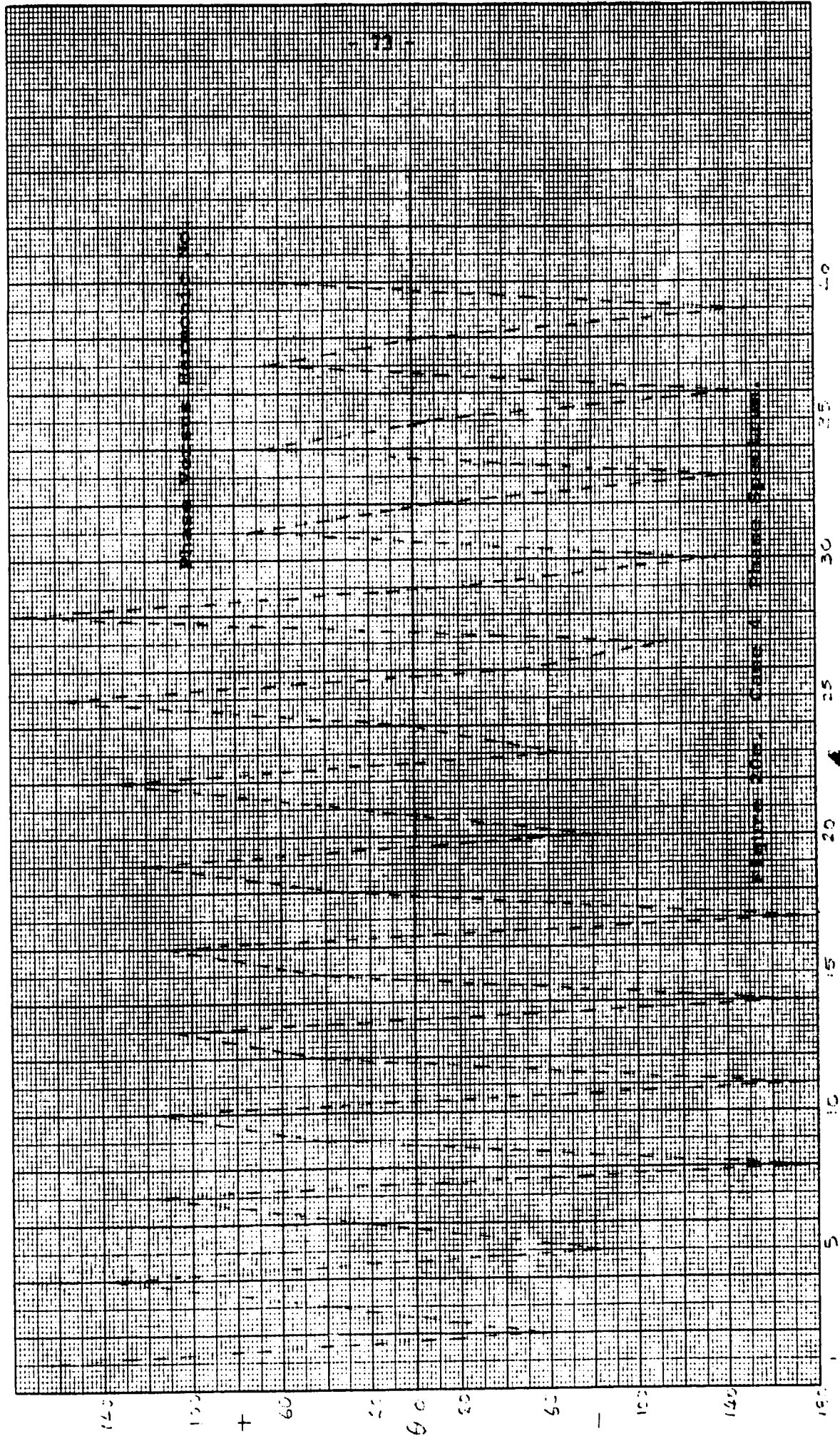
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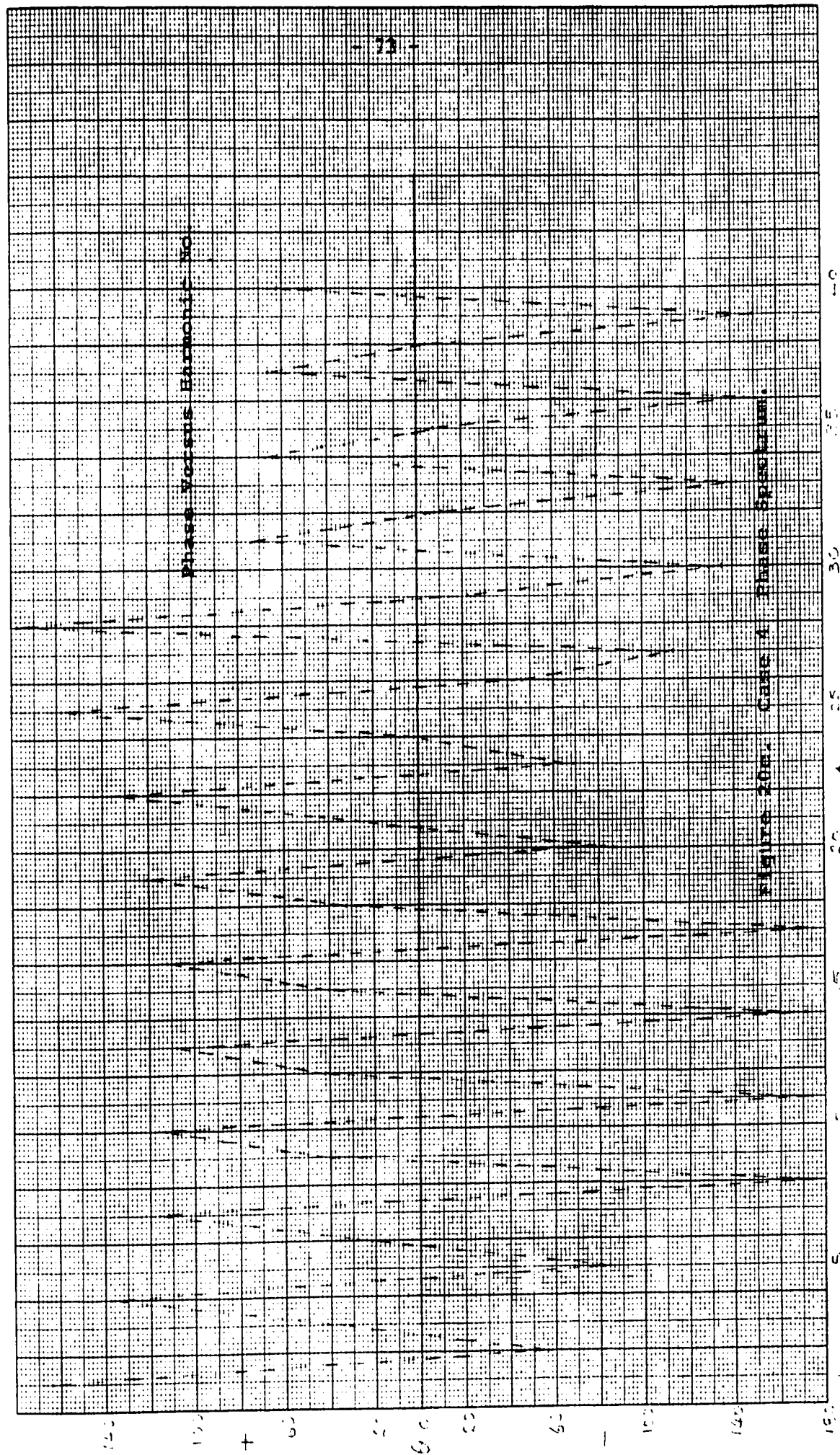
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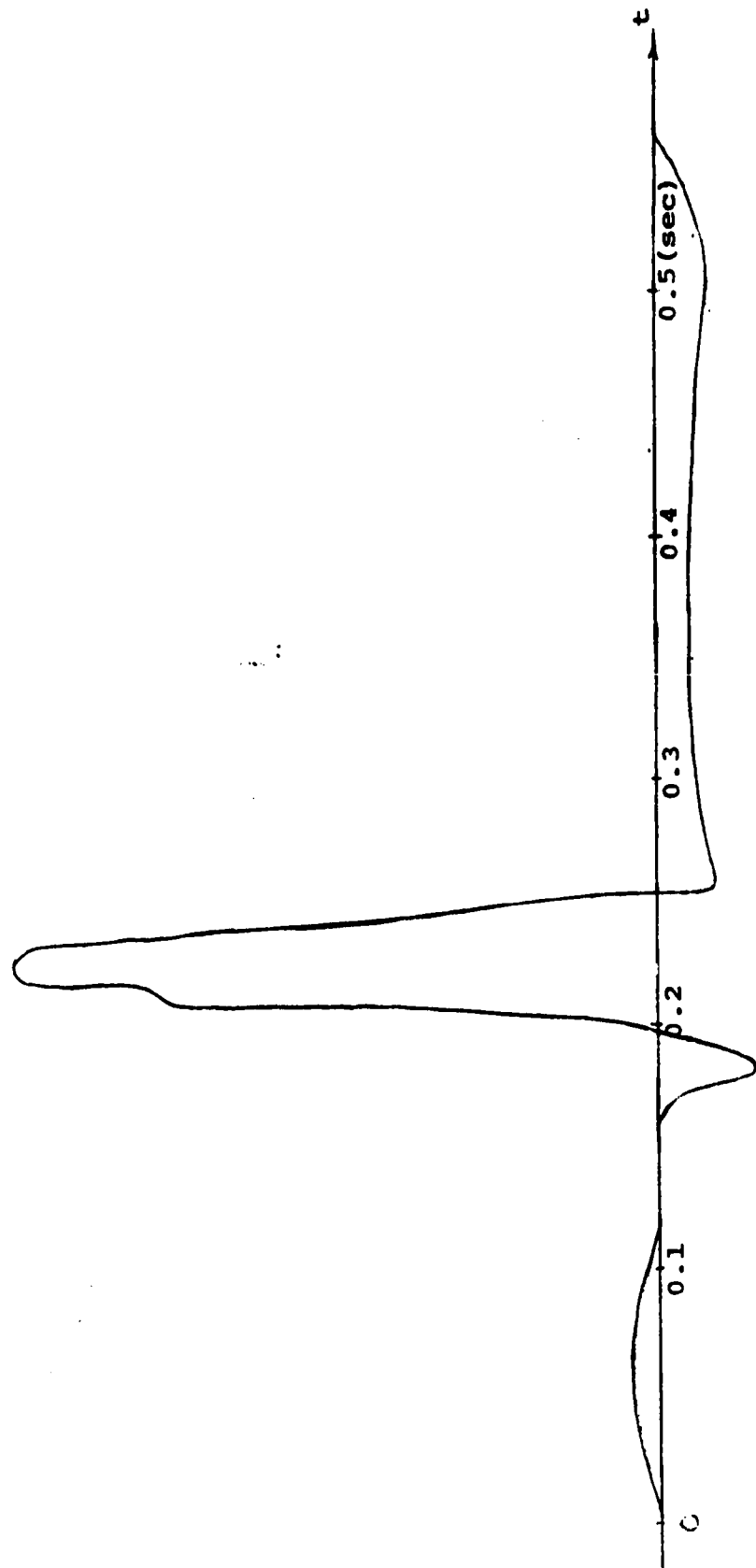
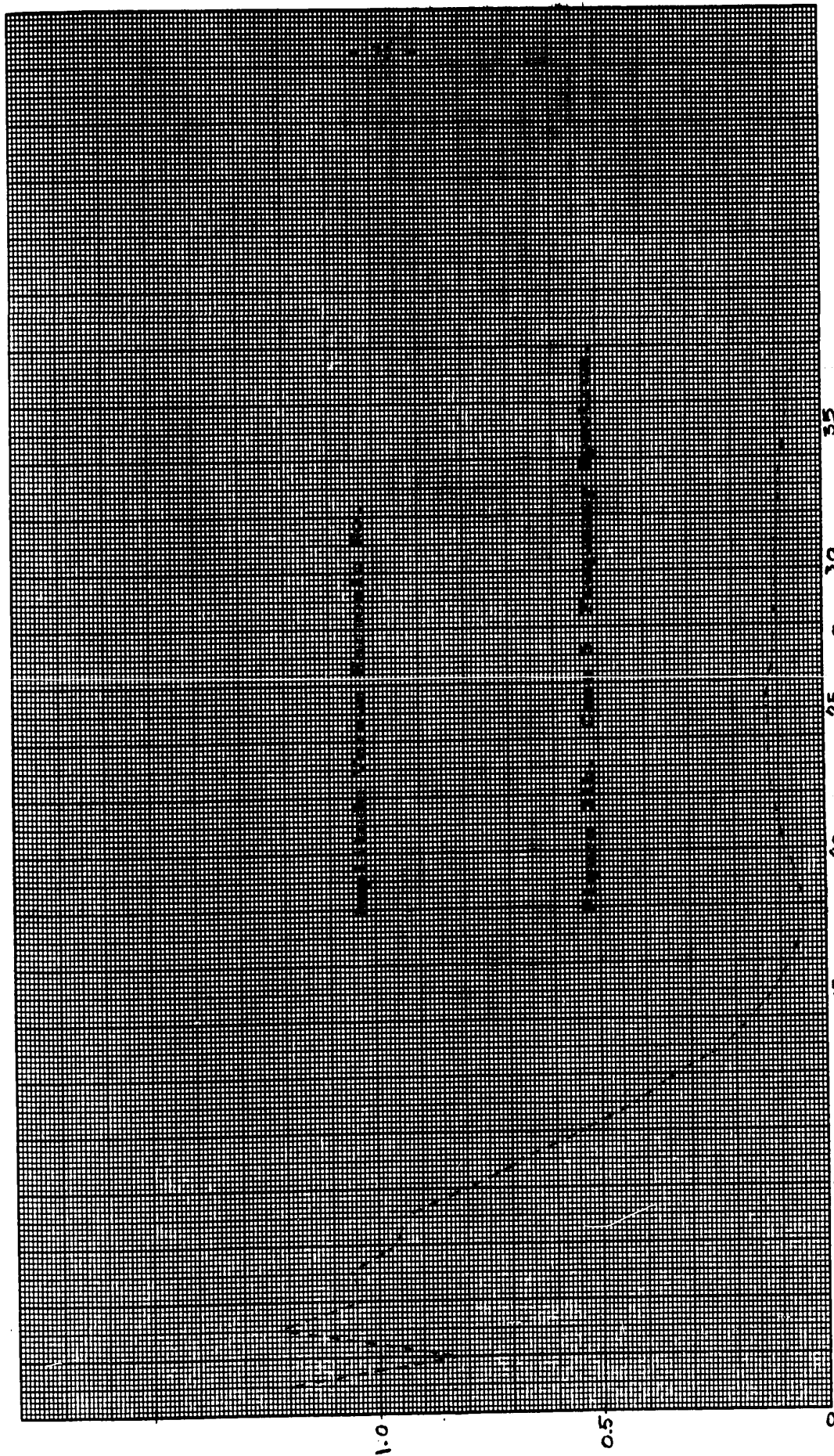


Figure 21a. Case 5 electrocardiogram.

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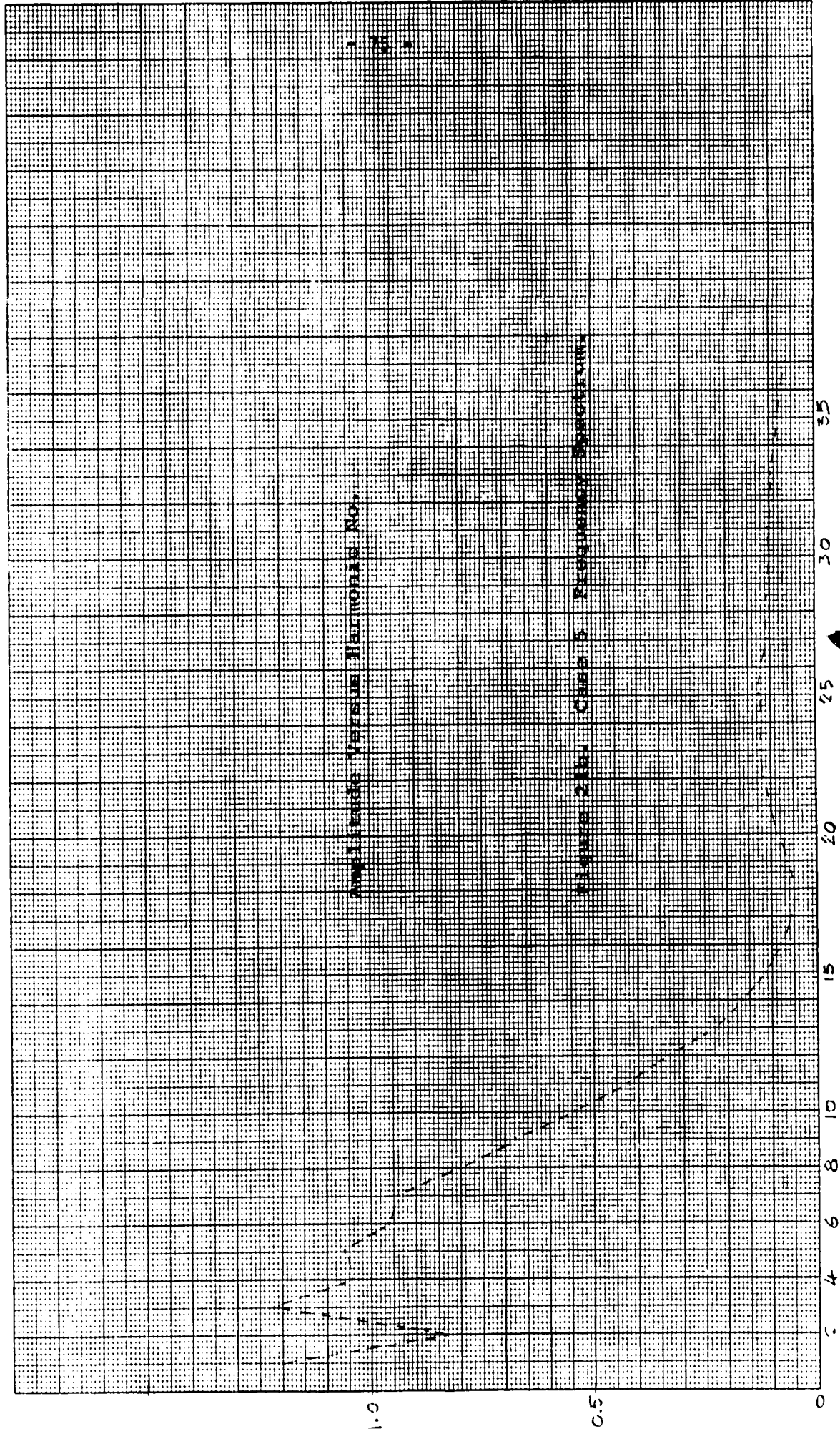


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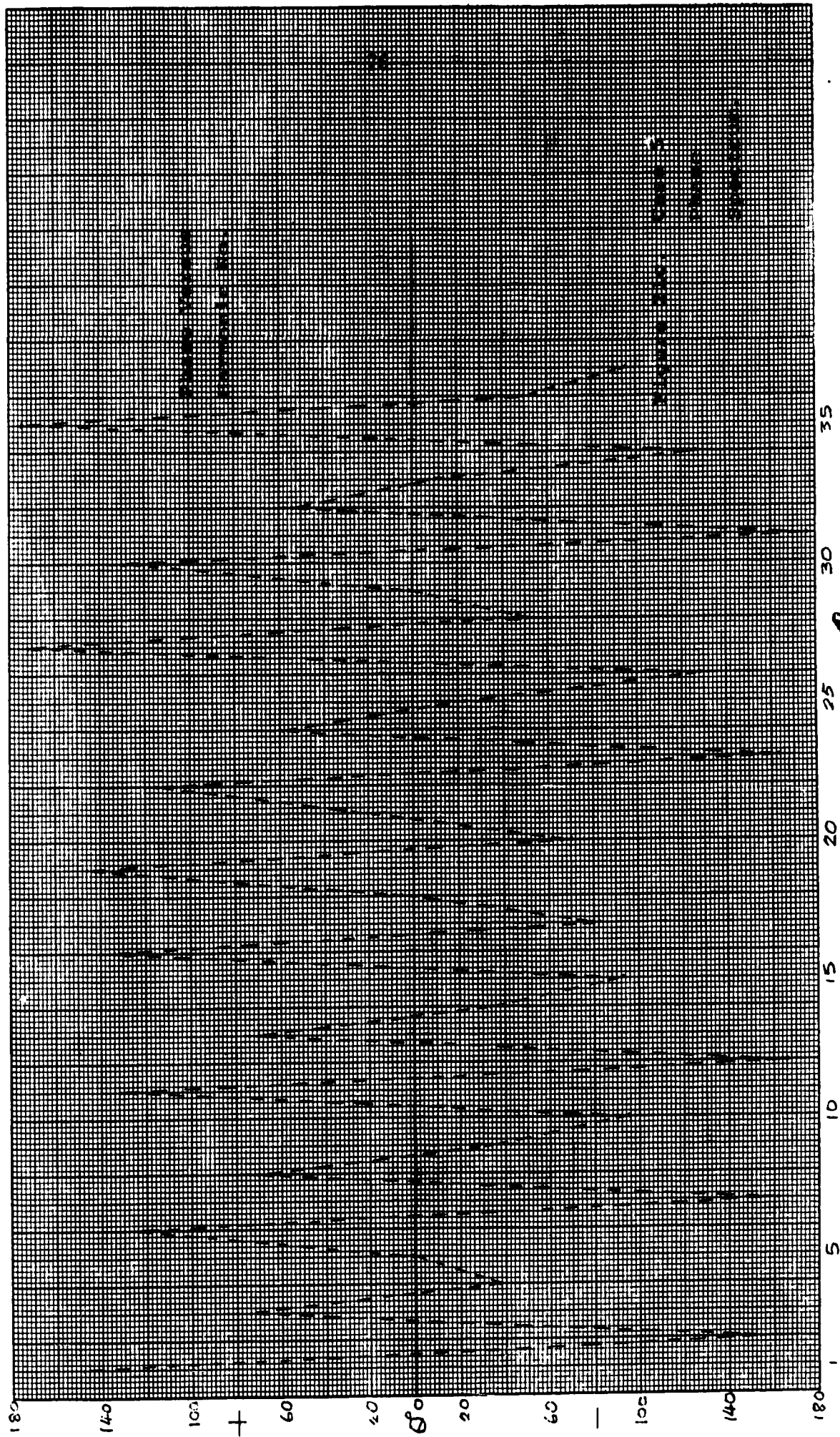
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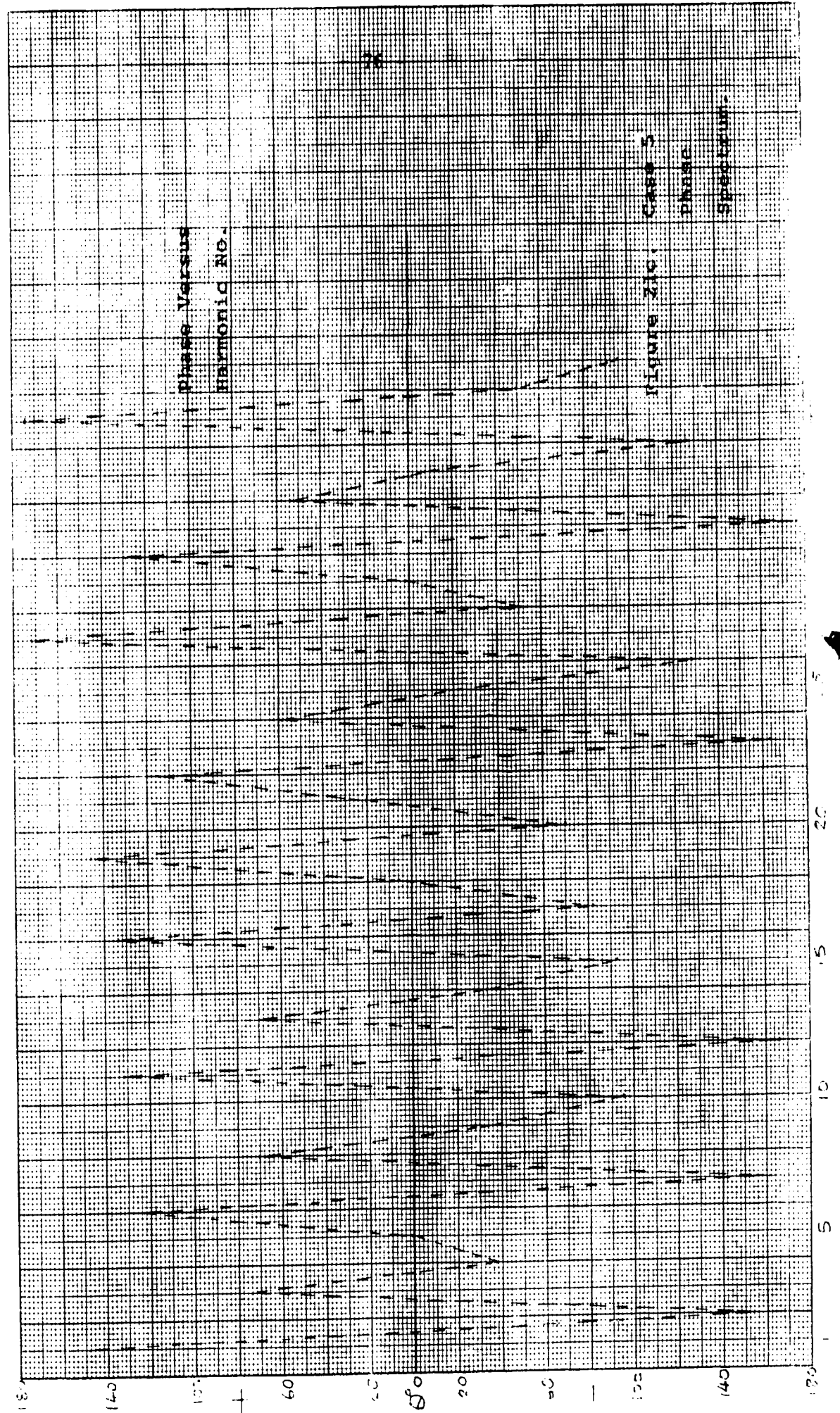
Figure 21b. Case 5 Frequency Spectrum



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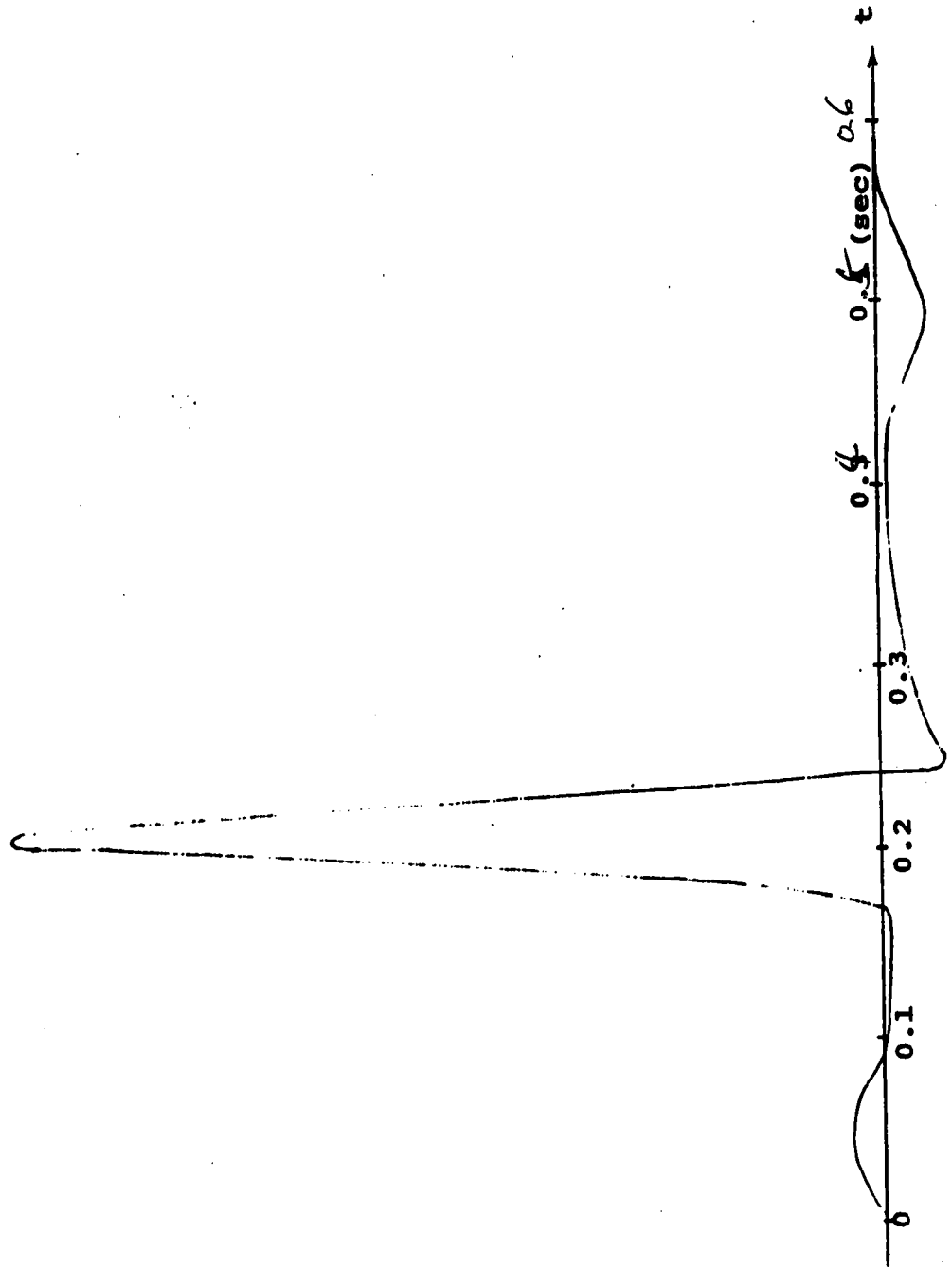
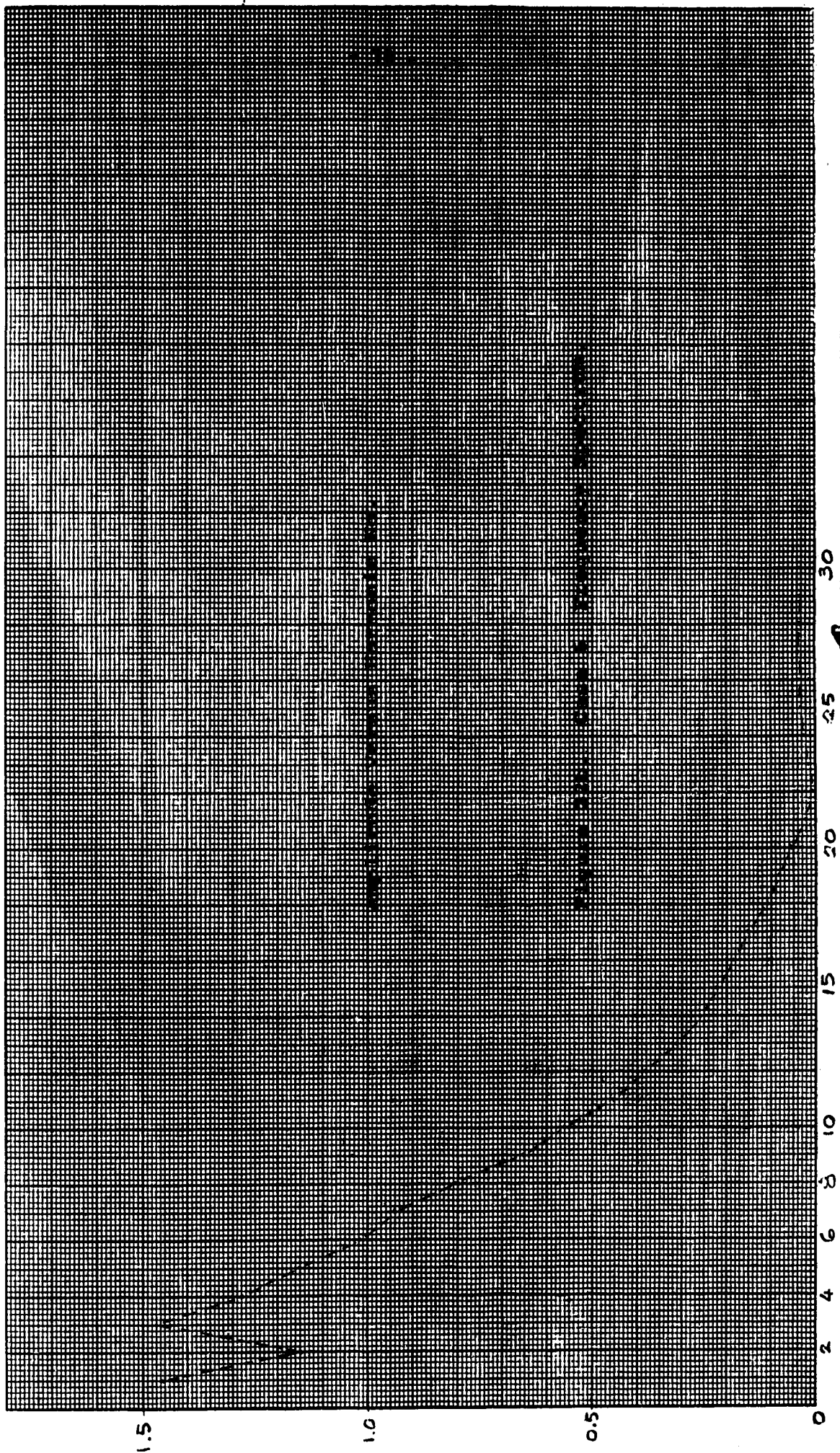


Figure 22a. Case 6 electrocardiogram.

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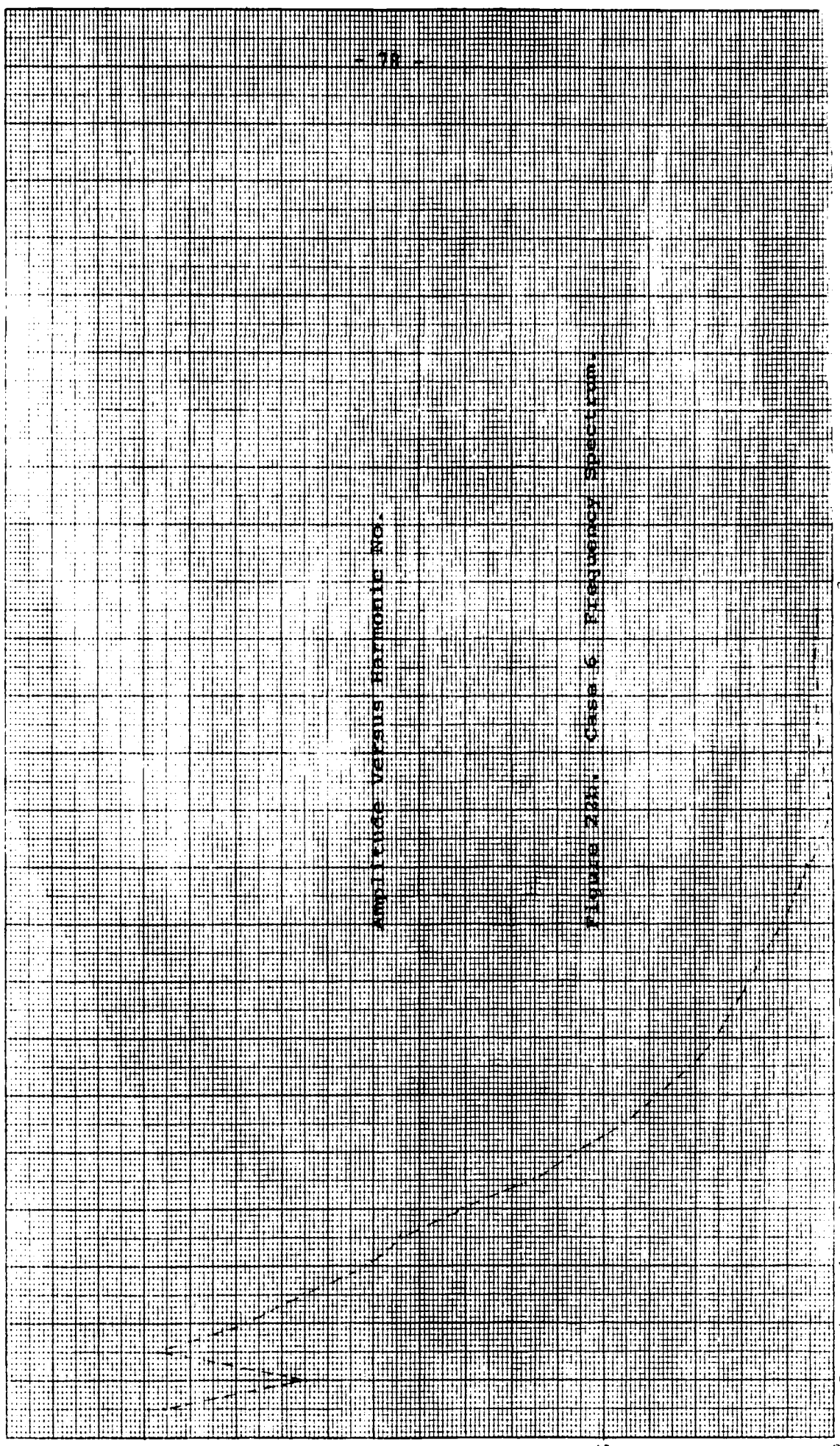


Figure 22H. Case 6. Frequency Spectrum.

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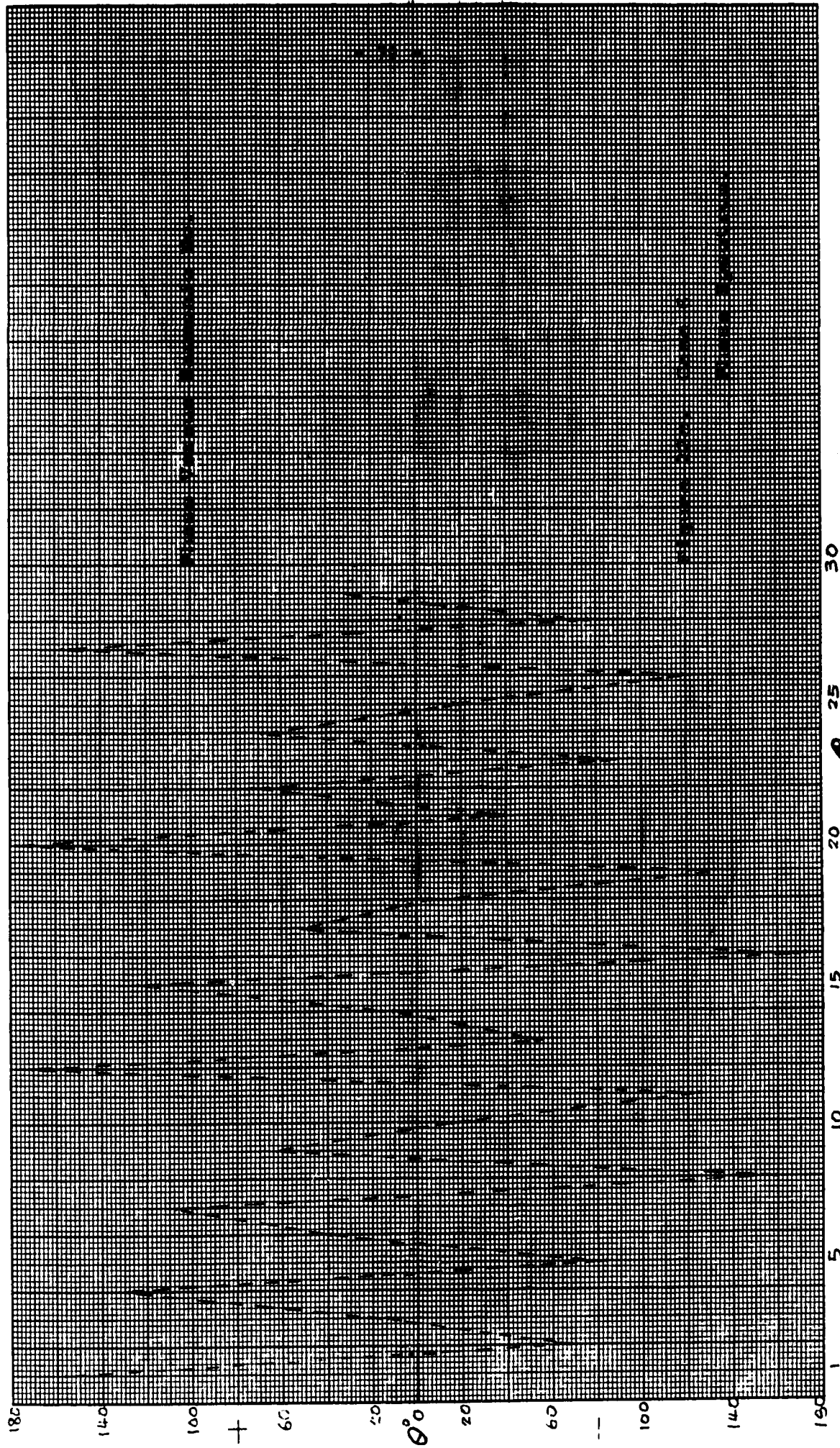
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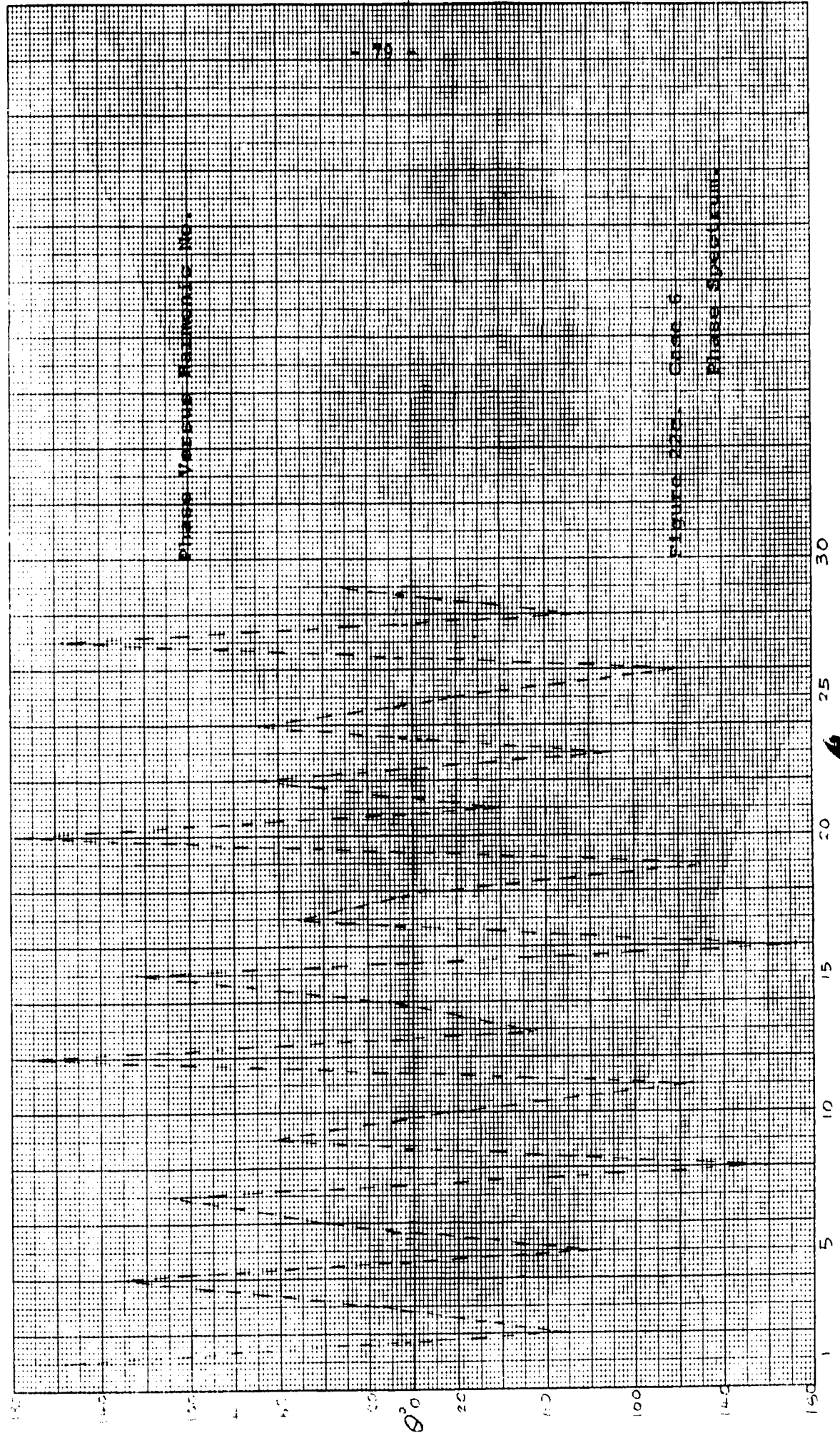
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on the fifth harmonic. The most marked effect was the variation in the relative magnitude among the first, second, third and fourth harmonics from the normal case with a dramatic increase in the first harmonics. The peaks appear at the first and third harmonics. The depressed S-T segment and the inverted T-wave have the effect of canceling some of the contributions in higher frequency components produced by the tall R-wave. This can be seen in Figure 22b. In Figure 21b the remaining higher frequency components are produced by the notching in the R-wave (see Figure 21a).

7.4. Right Ventricular Hypertrophy (cases 7-9)

The main causes of right ventricular hypertrophy [18-21] are congenital lesions. The normal electrocardiogram is a levocardigram, the reason being that the electrical forces produced by the left ventricle determines for the most part of the characteristics of QRS and T deflections in the normal electrocardiogram. Left ventricular hypertrophy simply exaggerates the normal electrical preponderance of left over right ventricle. The general form of the QRS deflection is not strikingly altered. Right ventricular hypertrophy, on the other hand, tends to cause a marked change in the balance of cardiac forces.

This change is accompanied by characteristic alterations in the appearance of the QRS deflections. The increment in the magnitude of the right ventricular forces must be such that the normal electrical predominance of the left ventricle is abolished or at least somewhat reduced. The specific changes in QRS configuration or voltage appear in the electrocardiogram. When the right ventricle shows extreme hypertrophy it becomes the major ventricle and the RS pattern appears over the now mirror left ventricle. The salient features of right ventricular hypertrophy in lead I are that the QRS interval is within normal limits, small R-wave and a prominent S-wave of long duration. Due to the small R-wave and the deepened broad S-wave the frequency spectrum corresponding to right ventricular hypertrophy pattern demonstrates most variation in the low-frequency components.

In Figures 23b, 24b and 25b there is a first harmonic with a prominent large amplitude. In Figures 23b and 25b the peaks appear at the first, third and fifth harmonics and the valleys at the second and fourth harmonics. Figure 24a has a widened short S-wave compared with those in Figures 23a and 25b. This small RS-wave pattern contributes the large second and fourth harmonics and a very small third harmonic compared with those shown in Figures 23b and 25b. In Figure 24b the peaks

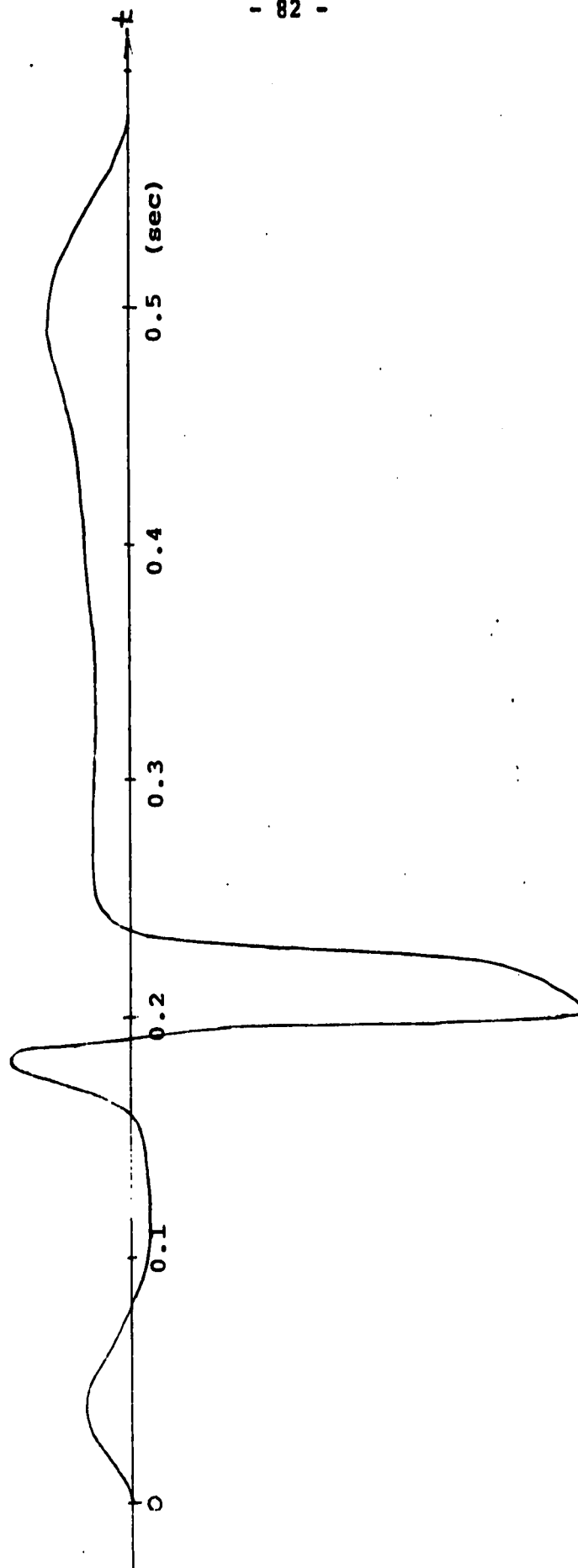
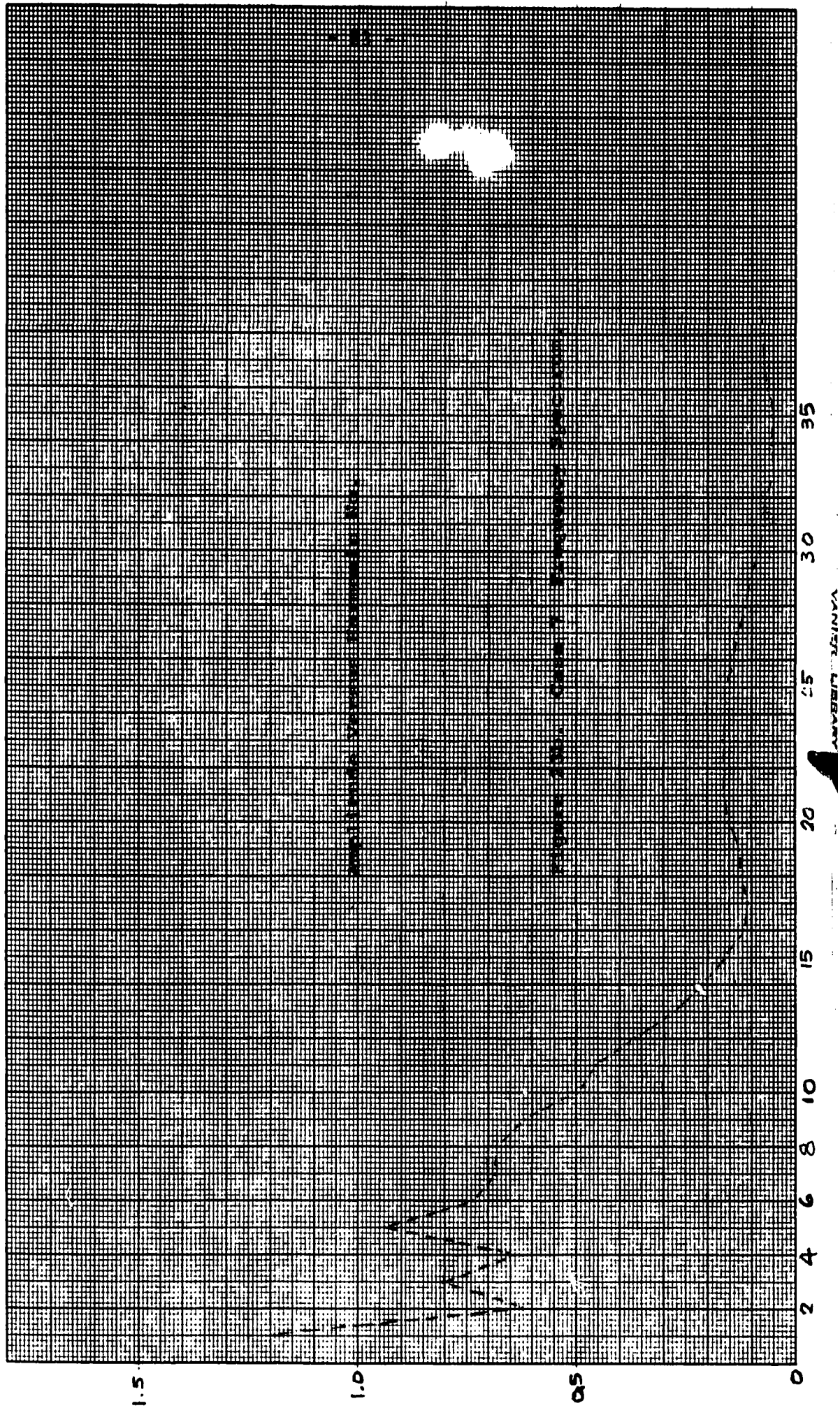
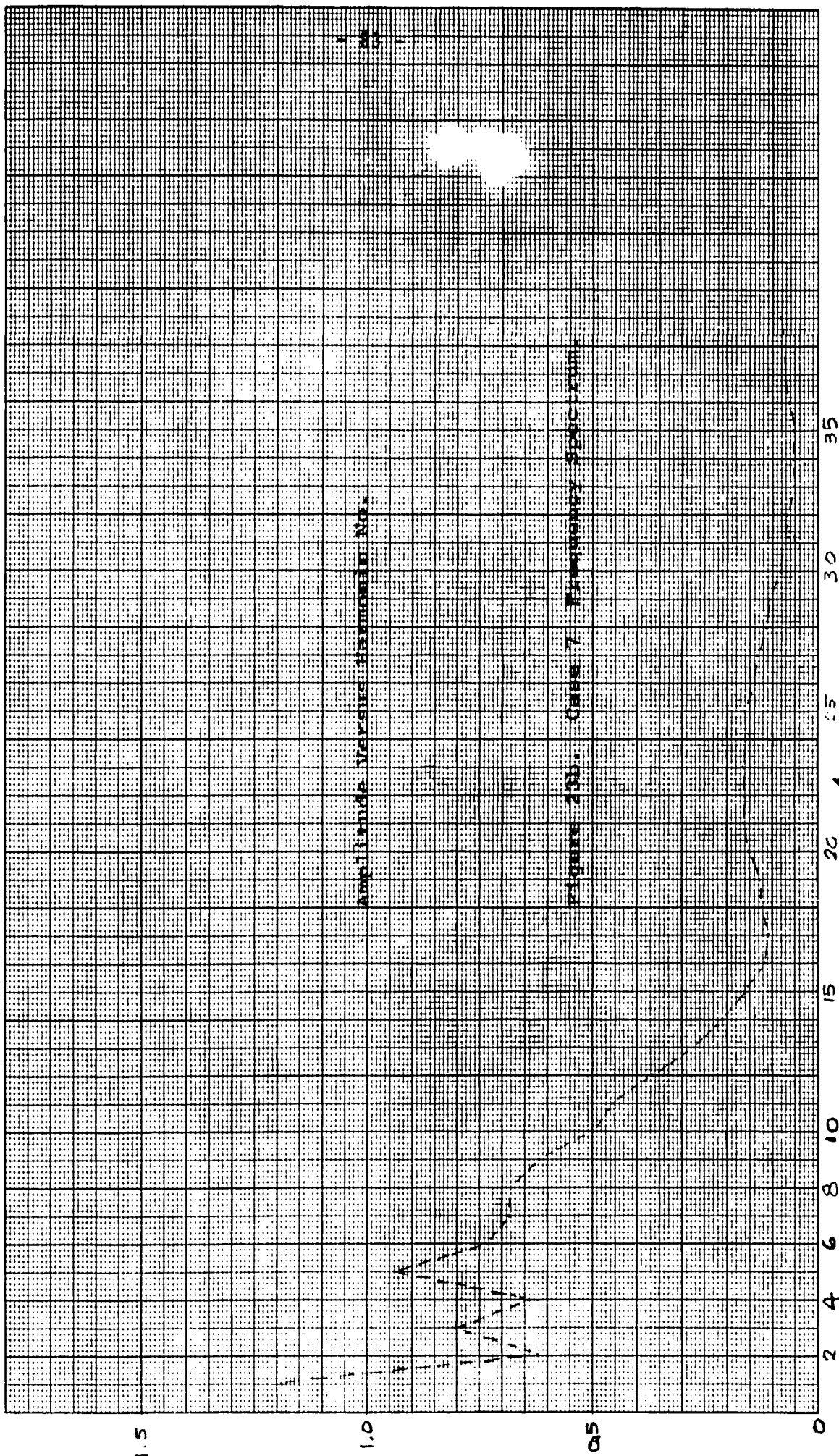


Figure 23a. Case 7 electrocardiogram.

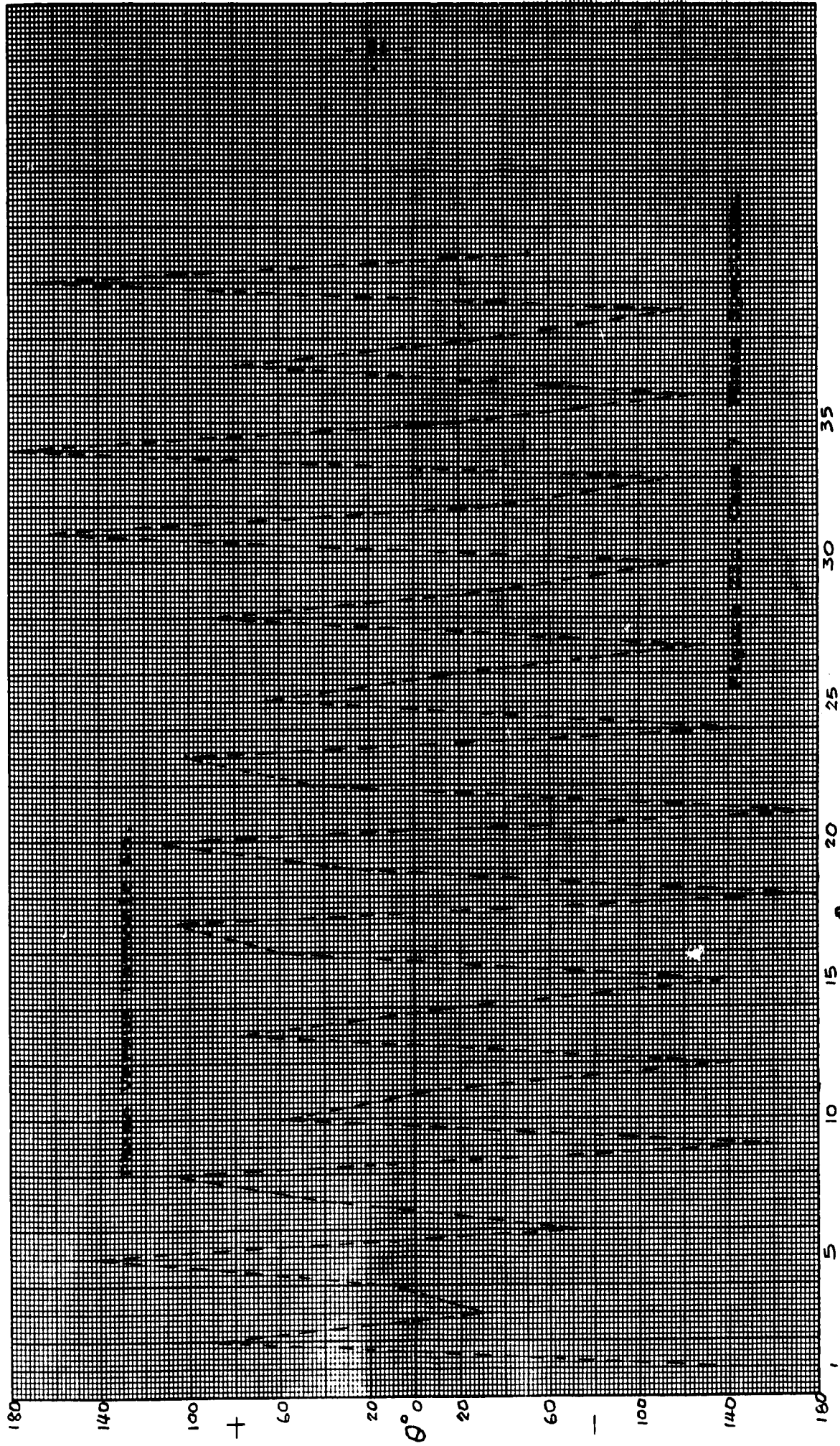
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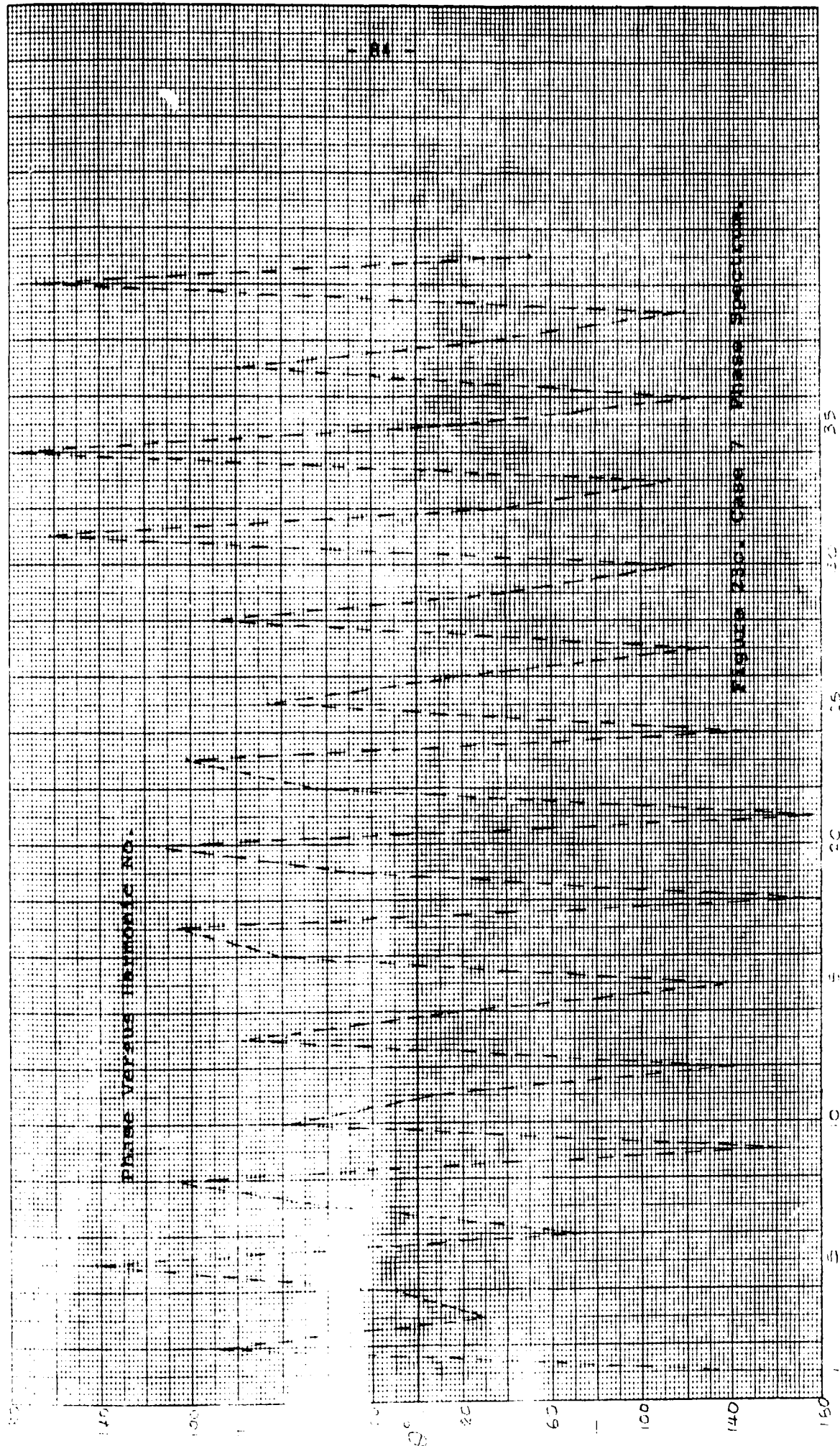


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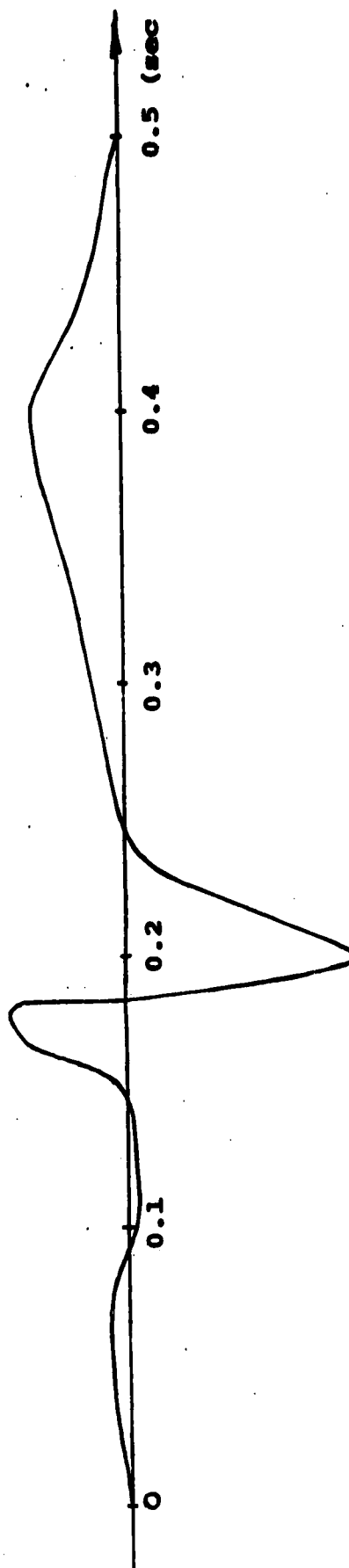
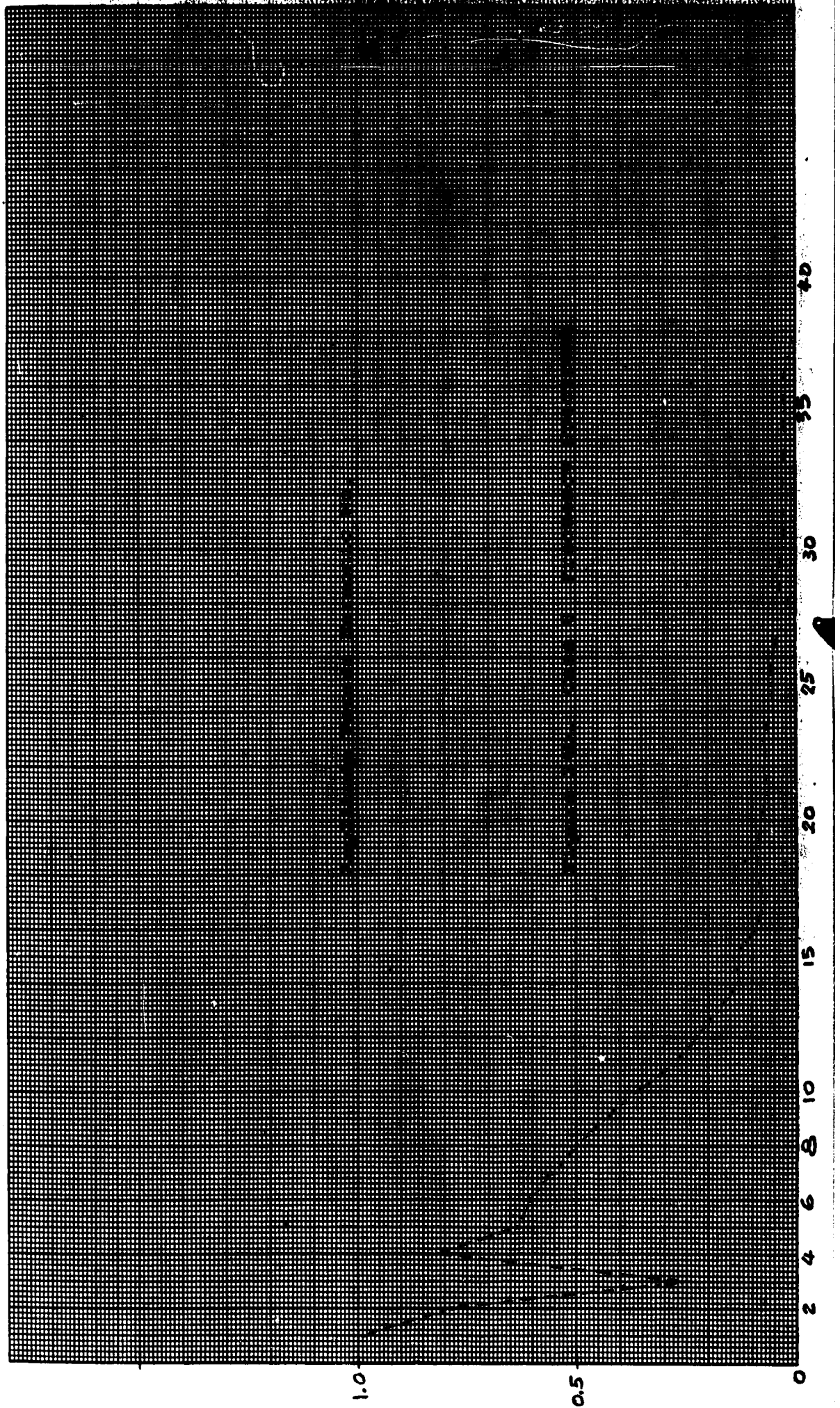
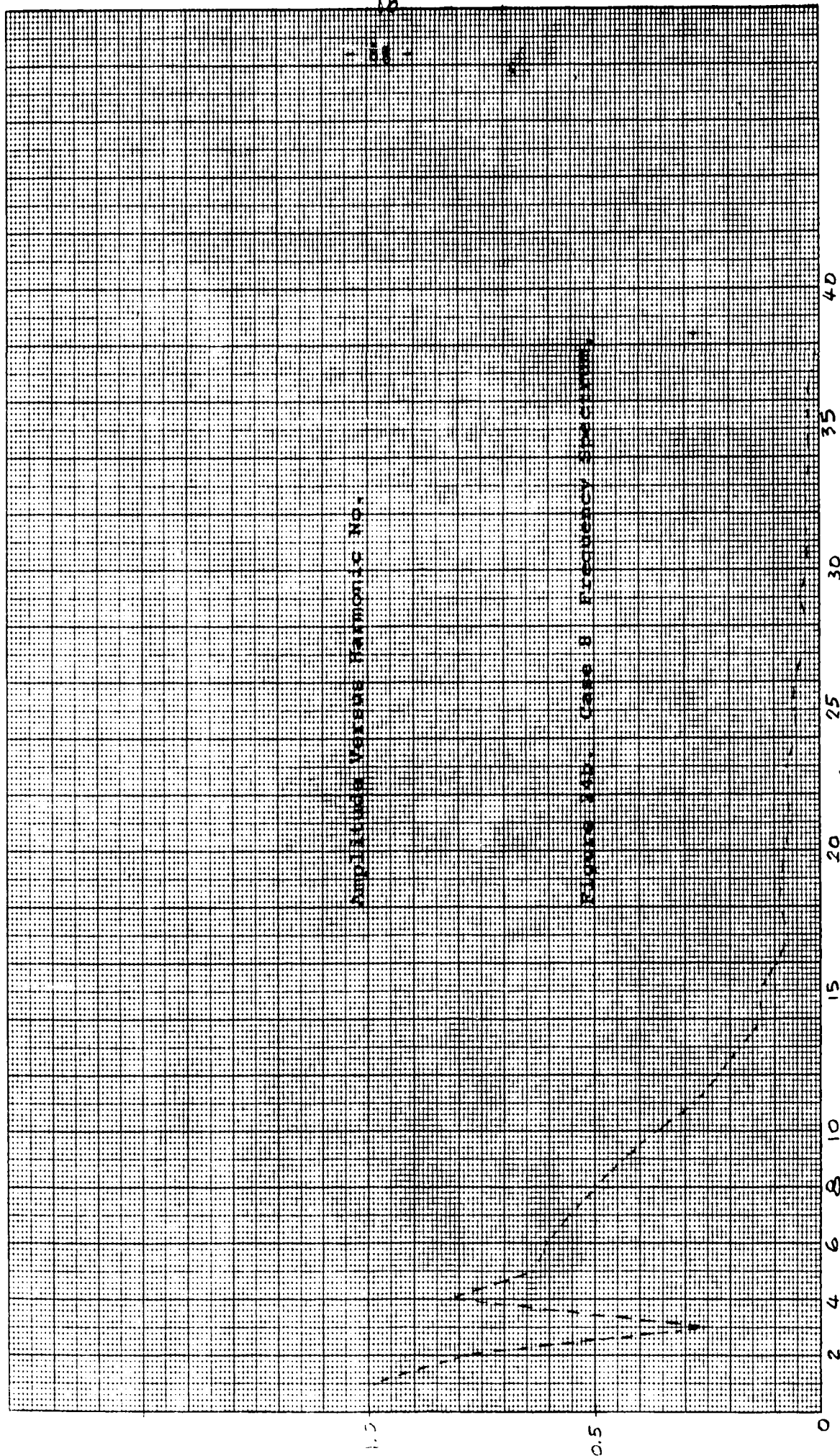


Figure 24a. Case 8 electrocardiogram.

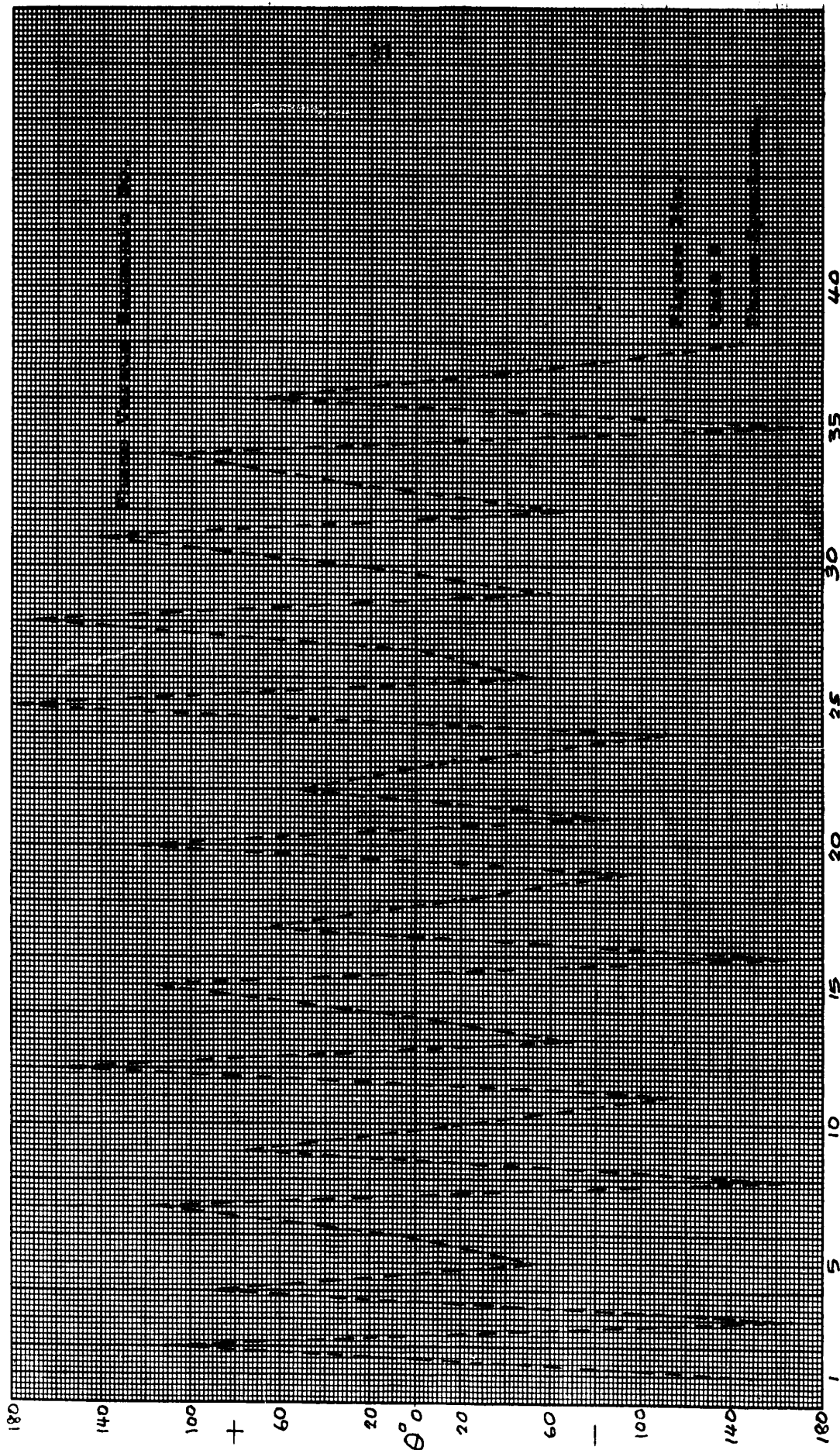
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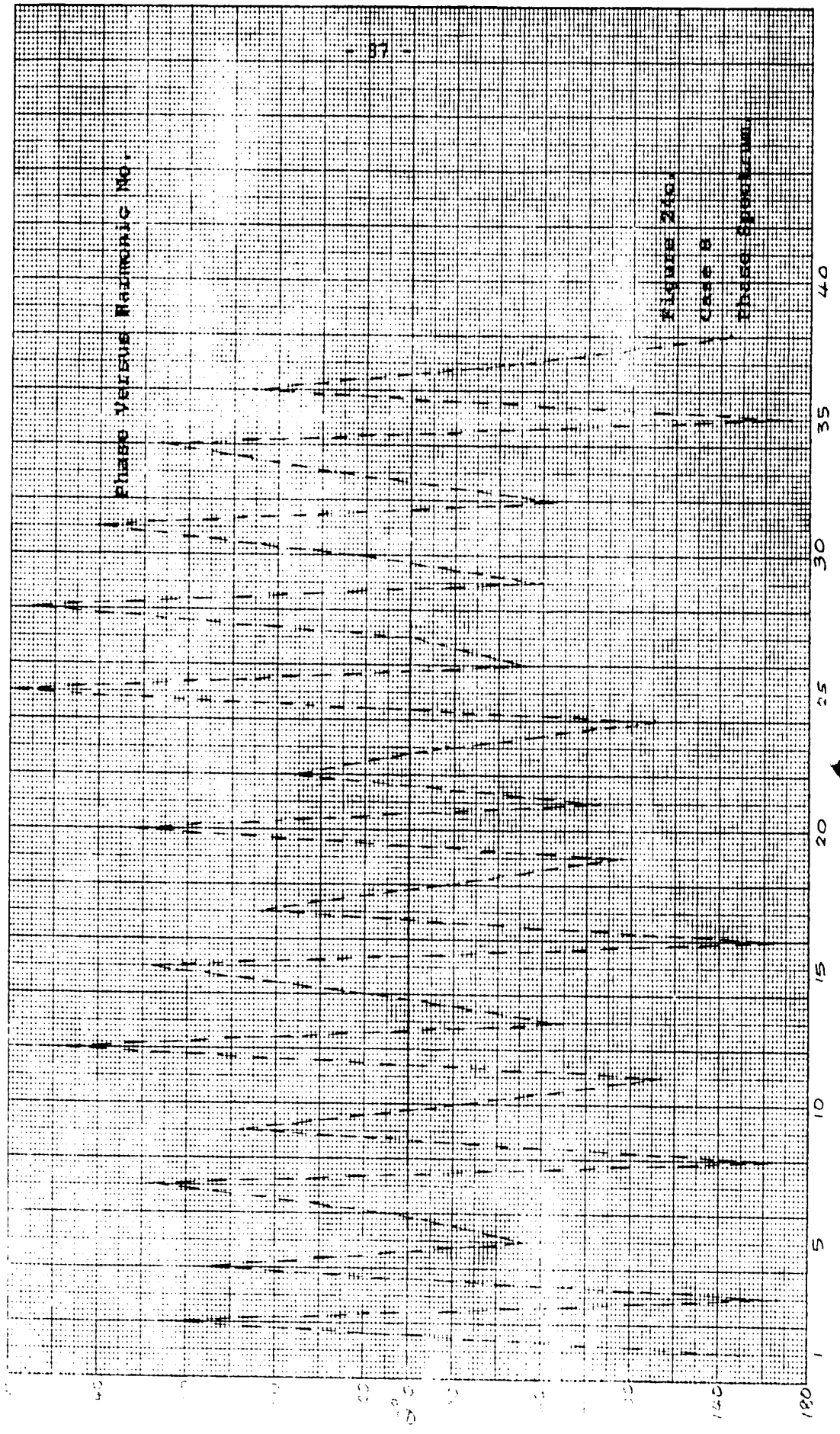


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Case B

Phase Spectrum



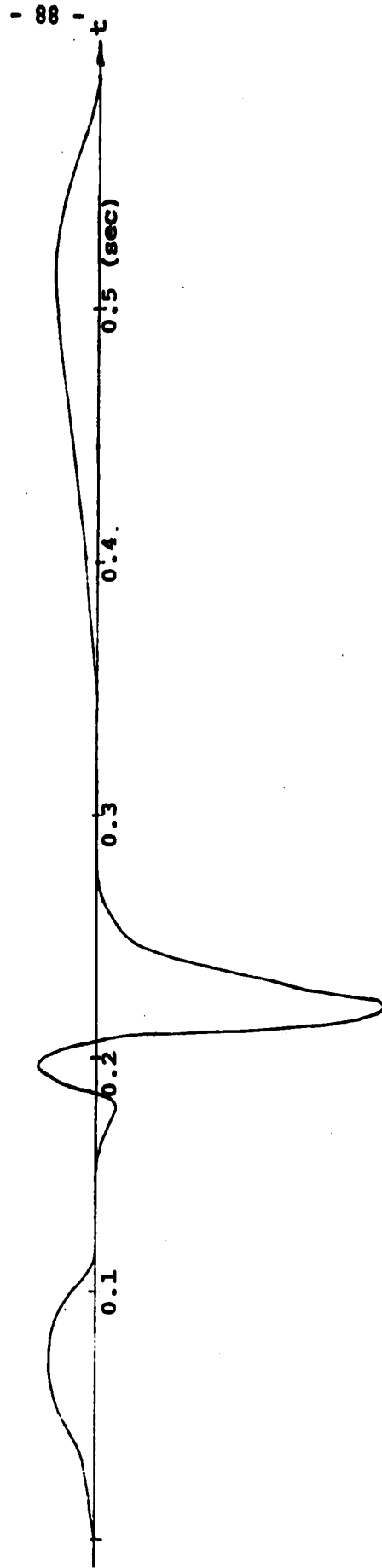
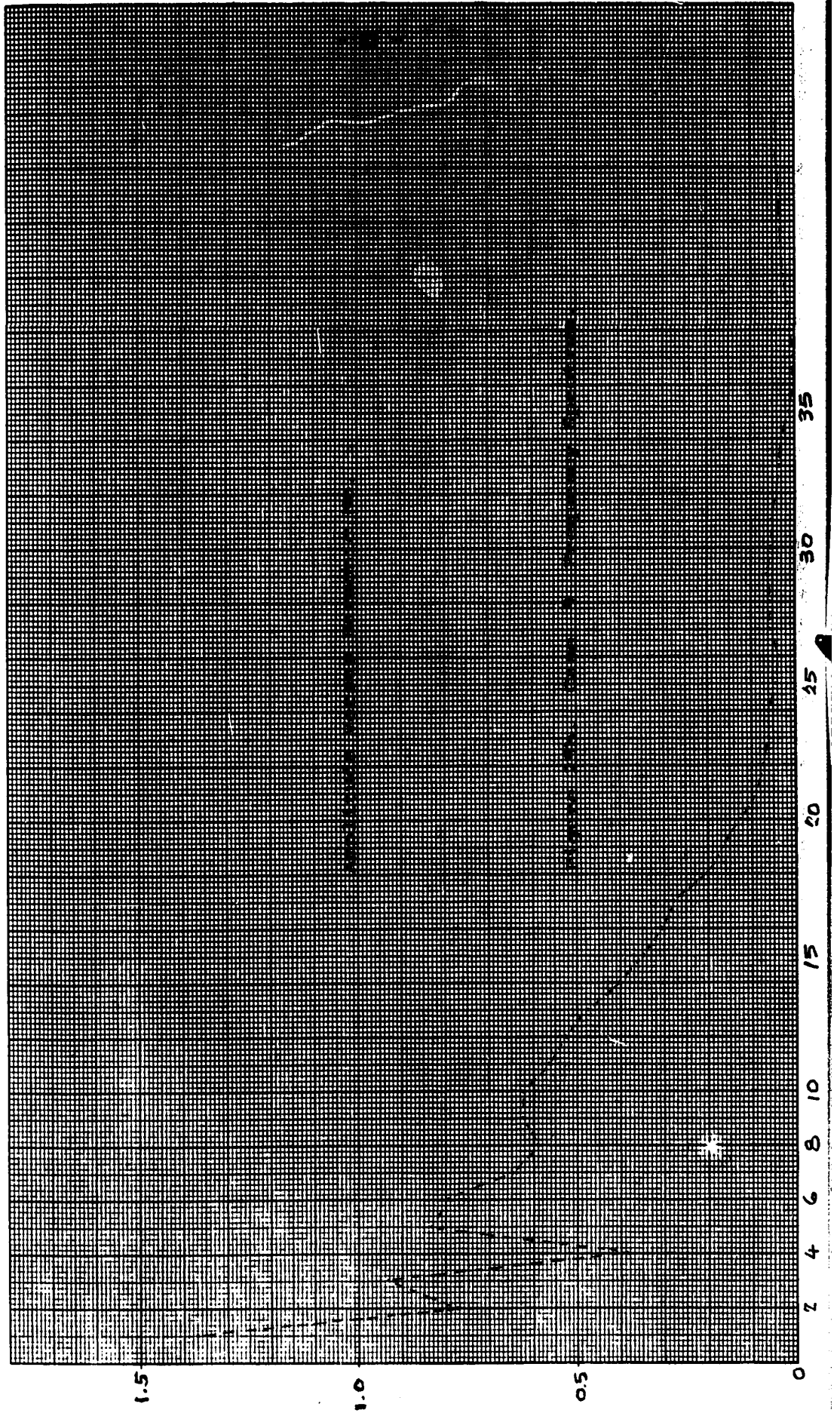


Figure 25a. Case 9 electrocardiogram.

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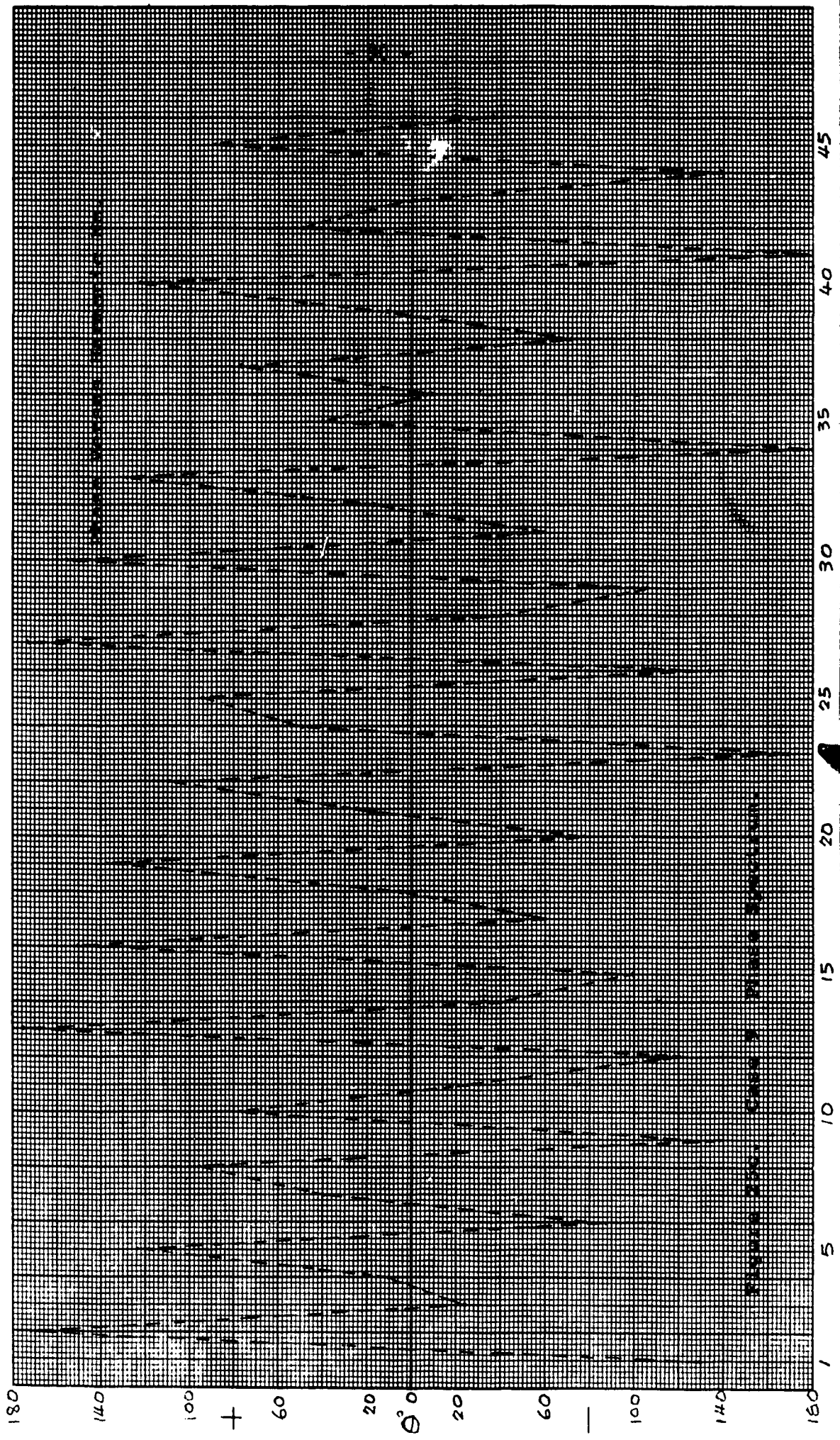


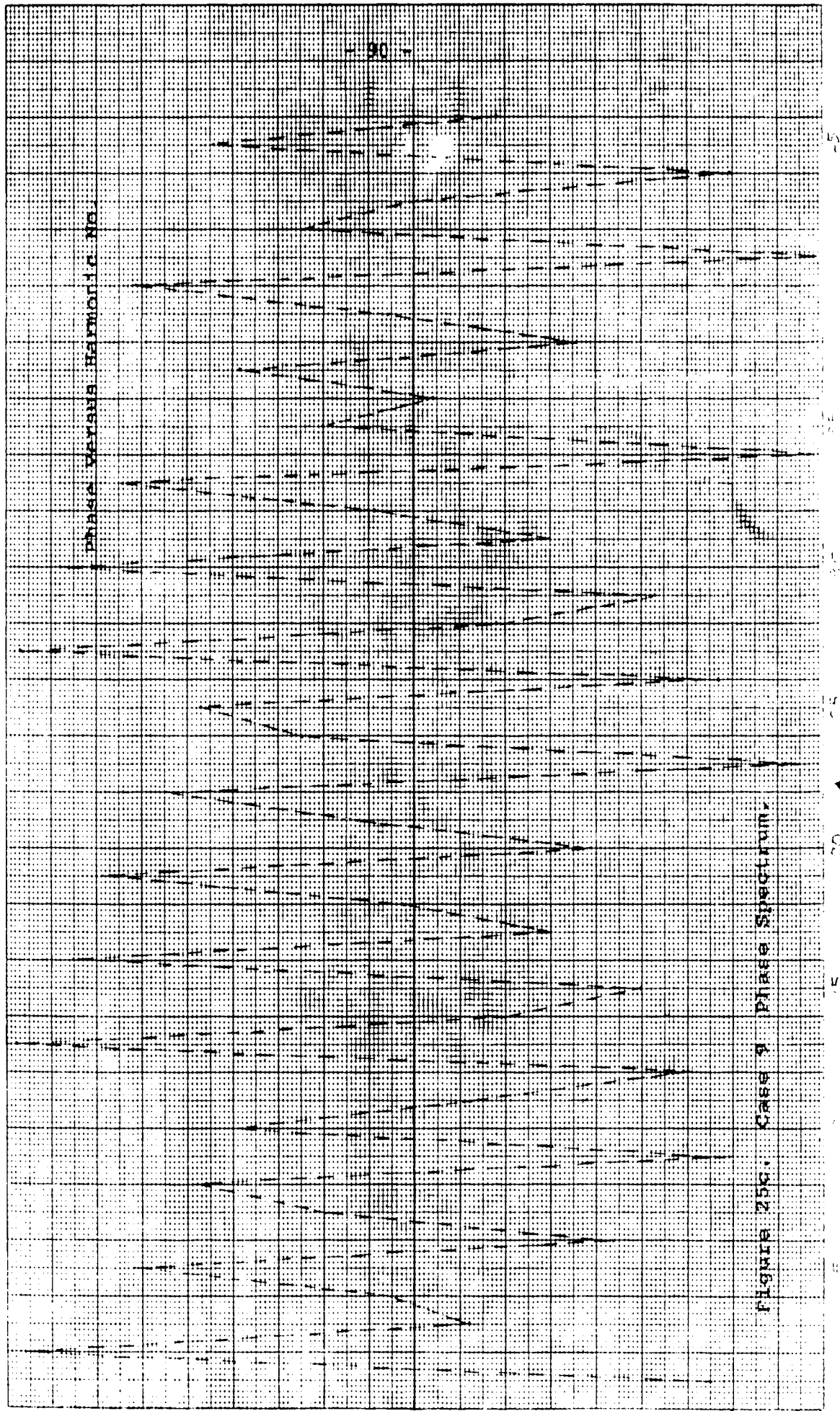
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Figure 25b. Case 9 Frequency Spectrum.



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appear at the first and fourth harmonics and a deep valley appears at the third harmonic.

7.5. Bundle Branch Block

Bundle branch block [18-21, 24-26] occurs most commonly when there is a pathological or physiological lesion, involving either right or left bundle branch.

The bundle branch block is a defect in the conduction path from the atrioventricular node through the bundle of His to the Purkenje fibers and the ventricular walls. The waveform pattern of the bundle branch block in many cases is simply an exaggeration of the corresponding ventricular hypertrophy, because the pattern may be produced not only by interruption of one of the main bundle branches, but also by diffuse slowing of the impulse throughout the conduction system of one ventricle or by conduction disturbances in the ventricular wall. If one of the branches of the bundle of His is blocked by disease, then the impulse will travel down the branch to the other ventricle first. Having activated this ventricle, the impulse will spread through the septum to the ventricle on the side of the block and in turn will activate it. In other words, the ventricles will be activated one after the other instead of simultaneously, in series instead of

in parallel. There is prolongation of the QRS interval. The QRS deflections in block are of lower voltage. In left bundle branch block the normal Q-wave is absent because, as the left branch is blocked, no impulse reaches this left bundle.

Bundle Branch Block is classified into complete bundle branch block and incomplete bundle branch block. The electrocardiographic criterion is that for complete bundle branch block there is a delay of excitation and abnormal spread of excitation through the ventricle whose bundle is blocked, resulting in an abnormal QRS configuration, prolongation of interval to 0.12 seconds or longer and prolonged ventricular. Activation time; for incomplete bundle branch block the same abnormalities are present except that the QRS interval is less than 0.12 seconds and the ventricular activation time is less than that seen in a complete bundle branch block.

7.5.1. Right Bundle Branch Block (Case 10, 11)

The characteristic feature of a right bundle branch block is the delay of the right ventricular depolarization. This delay produces the wide S-wave and small Q-wave. This waveform concentrates the spectral components toward the low frequency end of the spectrum. The first harmonic

is larger in magnitude than the third harmonic, as shown in Figures 26b and 27b. In Figure 26b the fourth harmonic becomes prominent. In Figure 27b the fifth harmonic is no longer a peak in the spectrum.

7.5.2. Left Bundle Branch Block (cases 12,13)

Since the left bundle is "blocked" the excitation wave will spread down the right bundle and through the right ventricular myocardium. The impulse will then pass around the "blocked" left bundle to the left ventricular myocardium, producing a wide and slurred R-wave. The ventricular activation time is prolonged to 0.09 second or longer; the QRS interval is 0.12 seconds or longer. Usually the S-T segments are depressed and the T-waves sometimes inverted. The electrocardiogram will vary with the heart position. The wide and slurred R-wave tends to concentrate the frequency components toward the low frequency end. The deformed T-wave should be evidenced in the first, second and third harmonics. In Figures 28b and 29b, there are much larger first and third harmonics than those in the normal case, Figure 17b. The spectral envelope drops rapidly to the very small sixth harmonic in Figure 28b, and to the very small seventh harmonic in Figure 29b. In Figure 28b the shape of the spectral

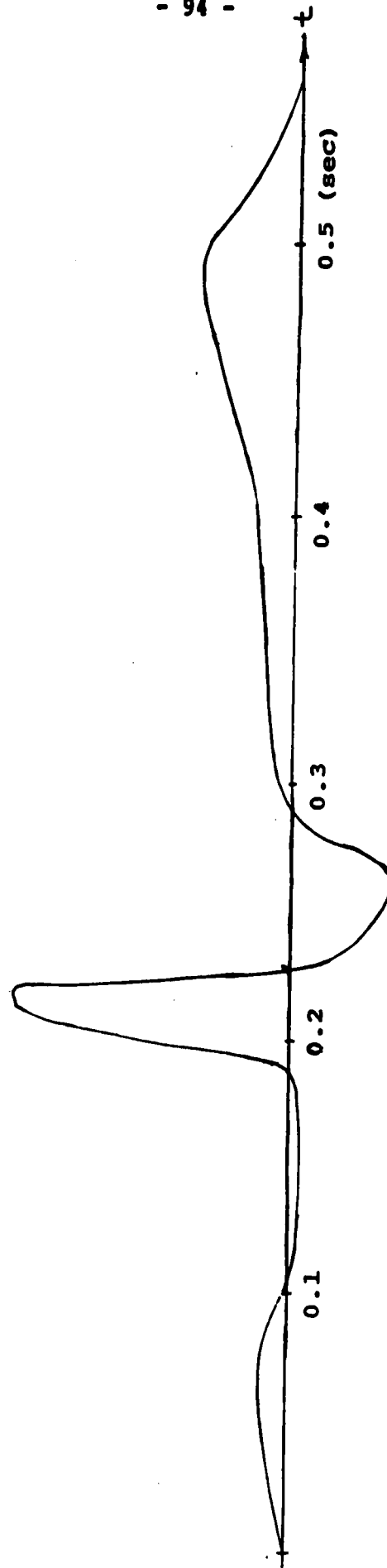
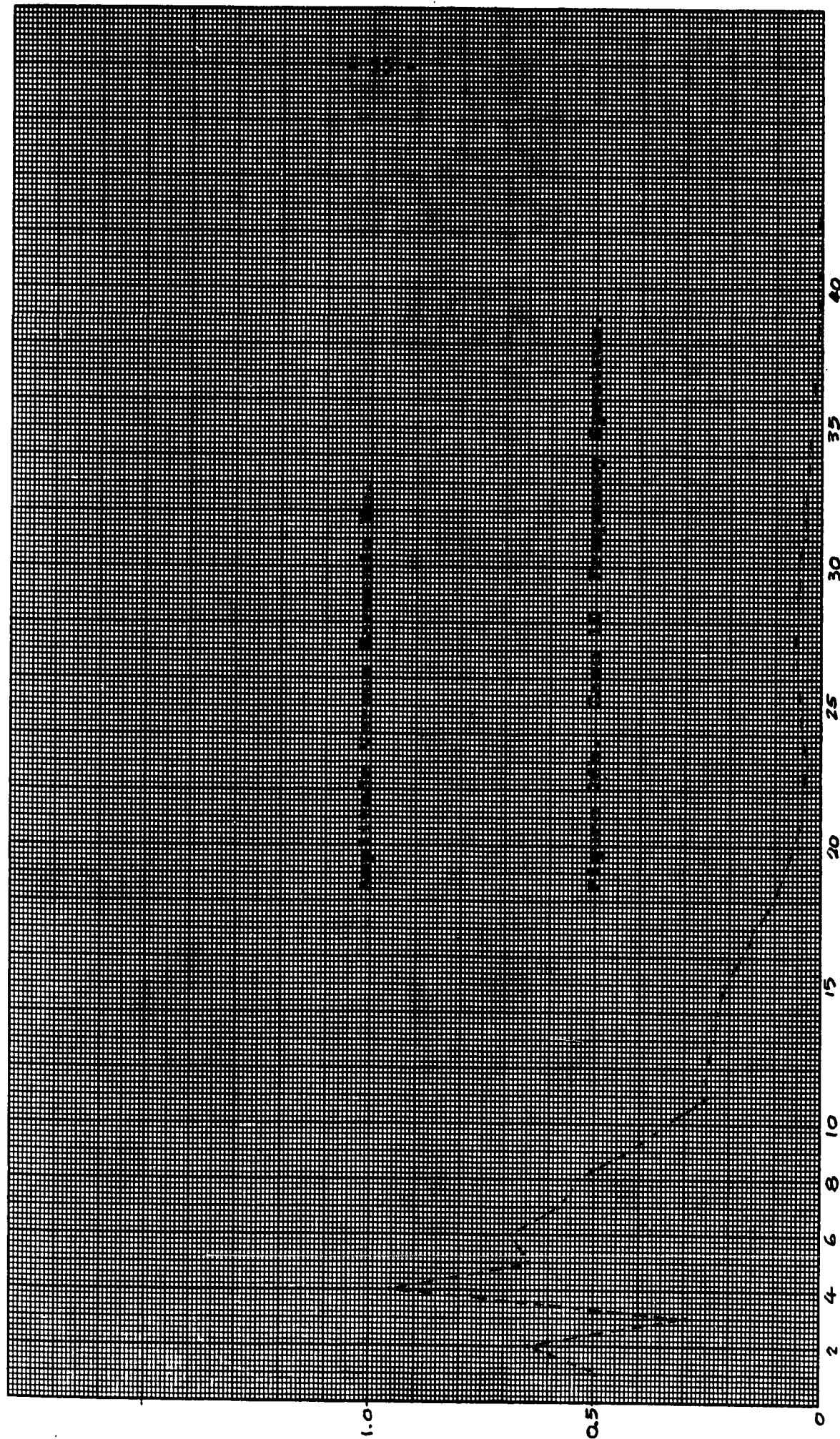
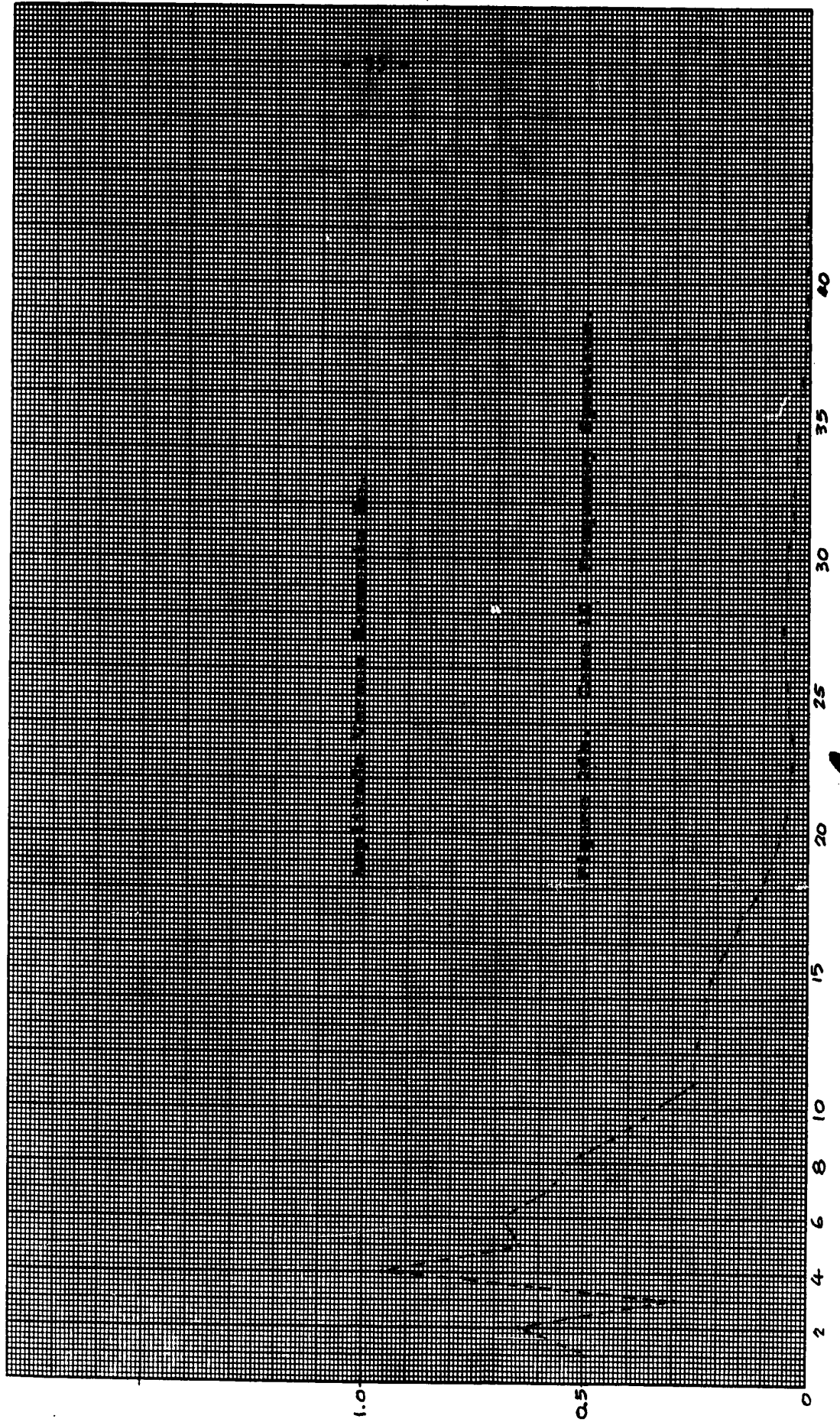


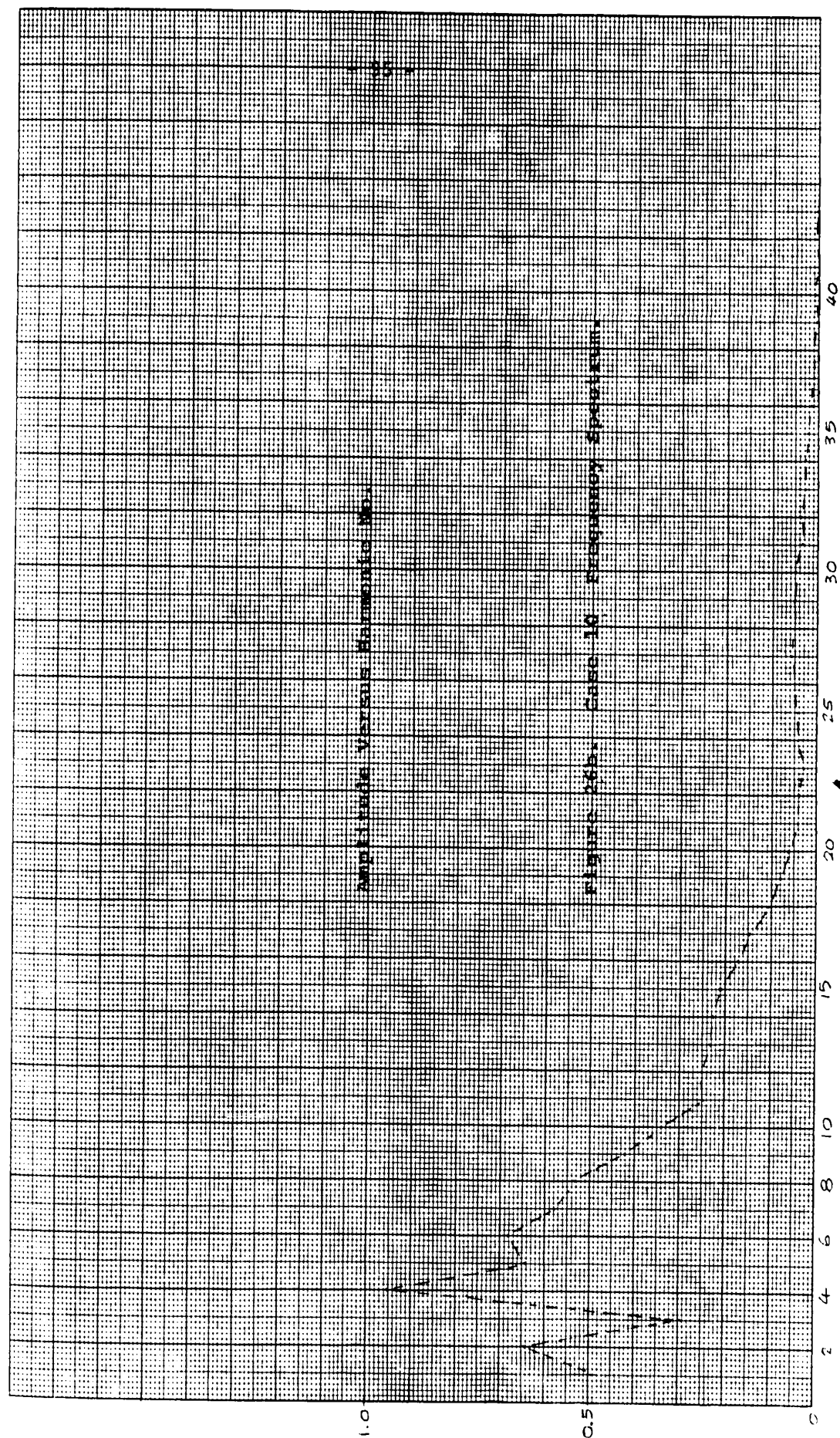
Figure 26a. Case 10 electrocardiogram.

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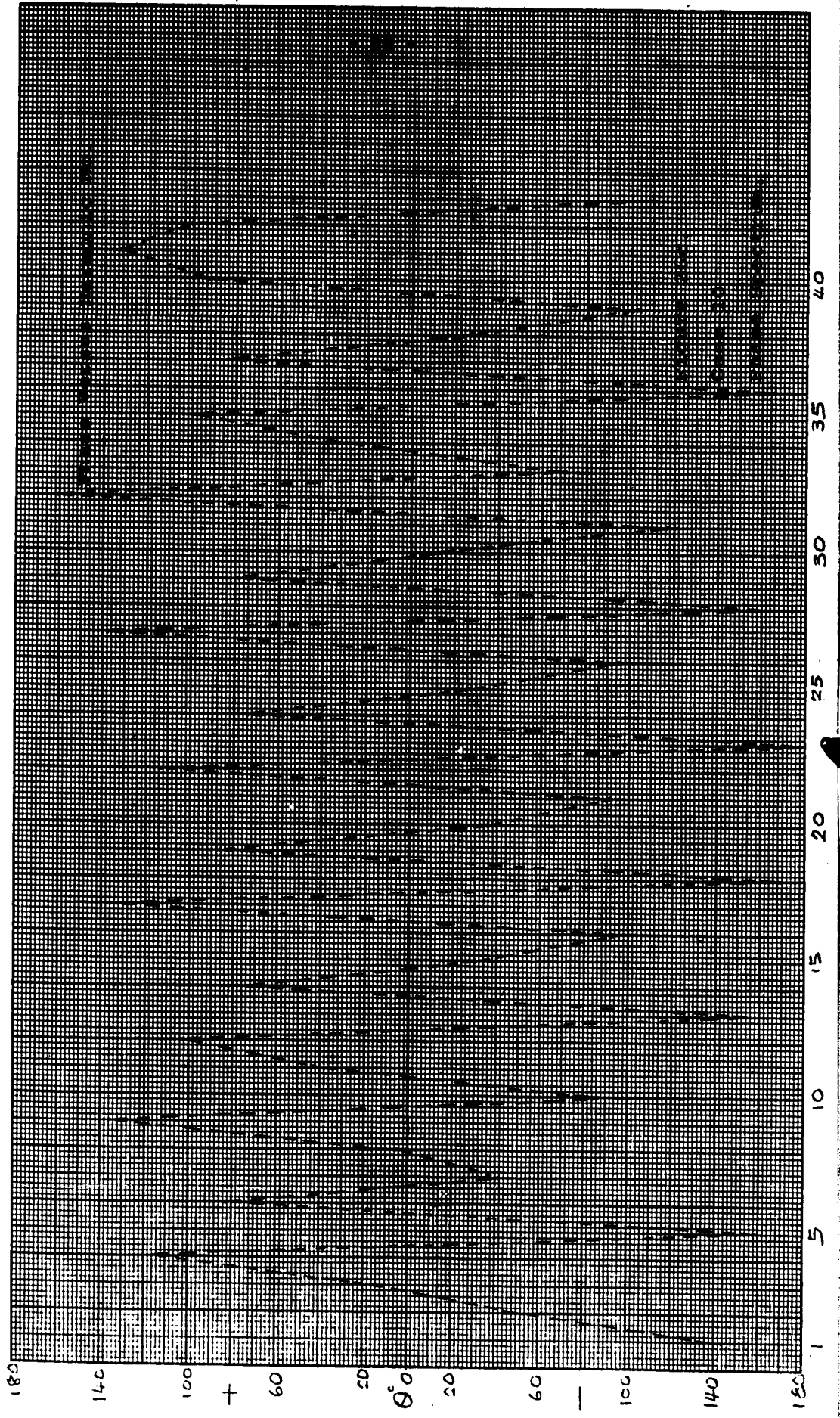


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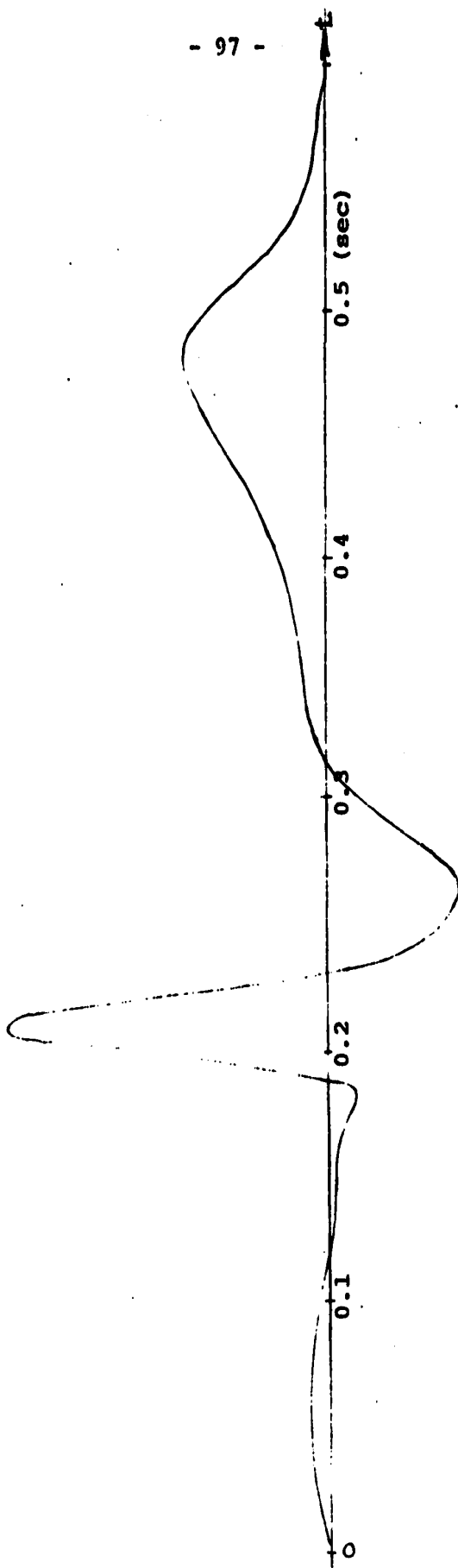
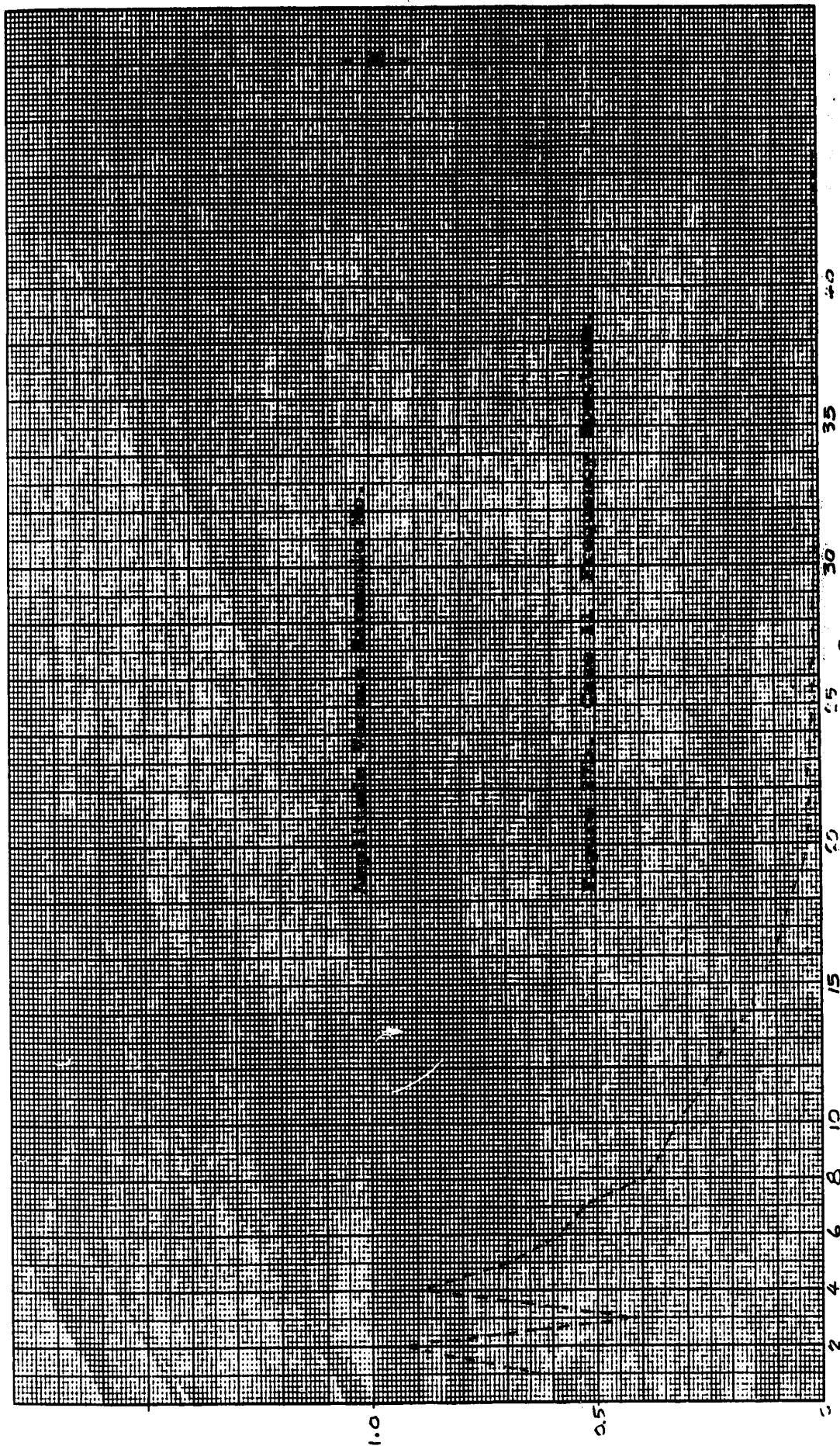
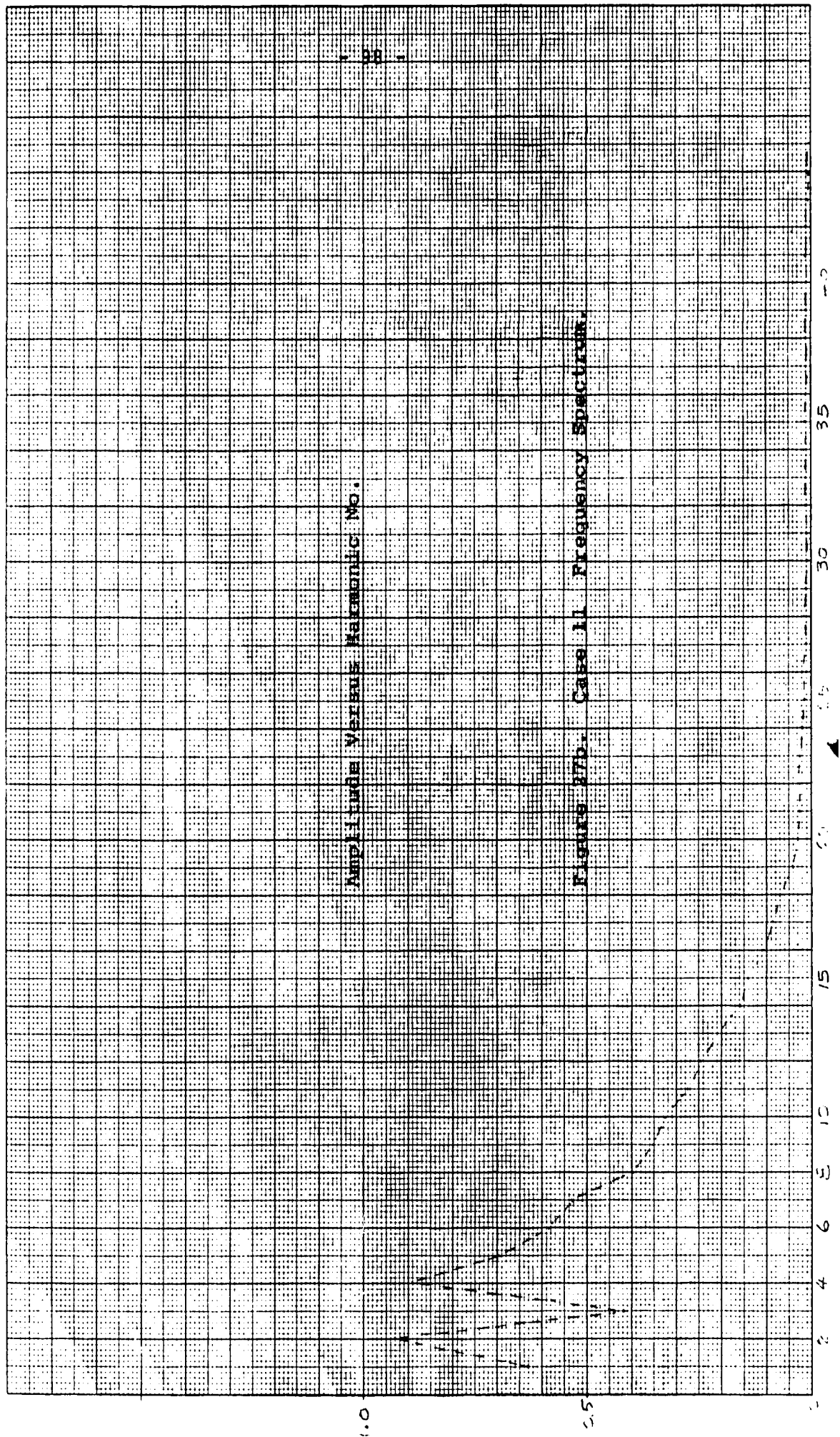


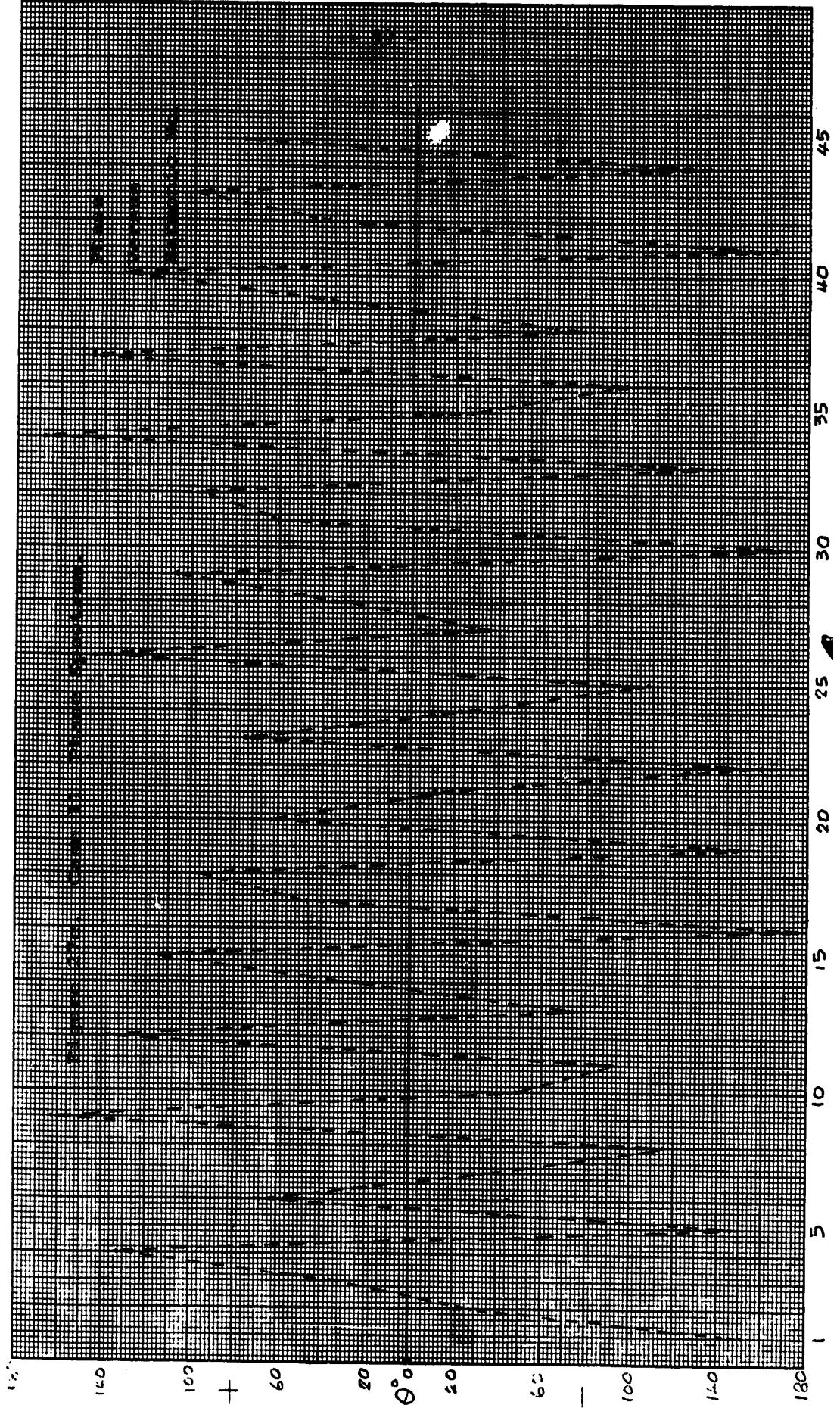
Figure 27a. Case 11 electrocardiogram.

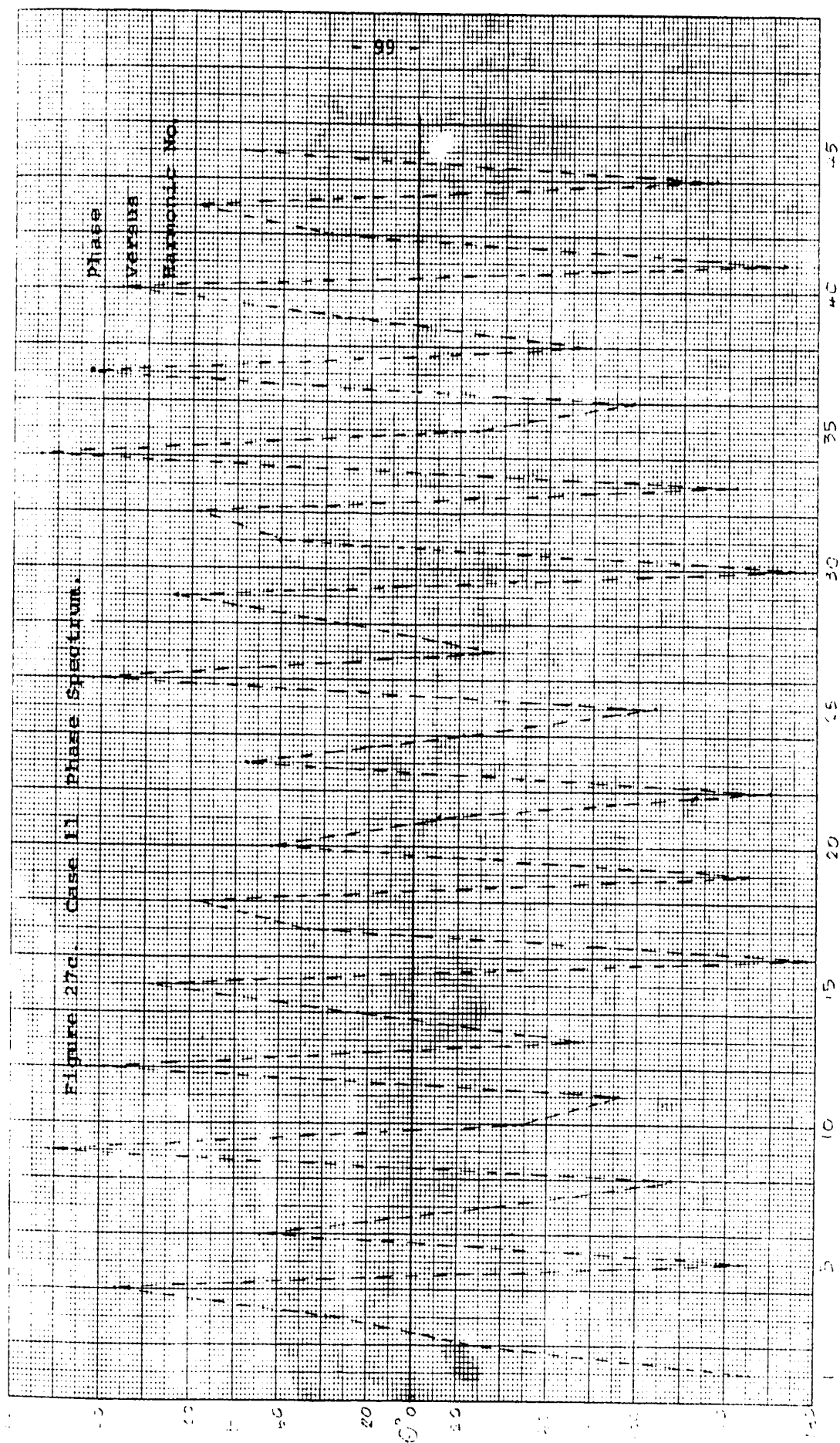
K-E 10 X 10 TO THE CENTIMETER 46 1513
MADE IN U.S.A.
KEUFFEL & ESSER CO.





K-E 10 X 10 TO THE CENTIMETER 461513
19 X 25 CM
MADE IN U.S.A.
KEUFFEL & ESSER CO.





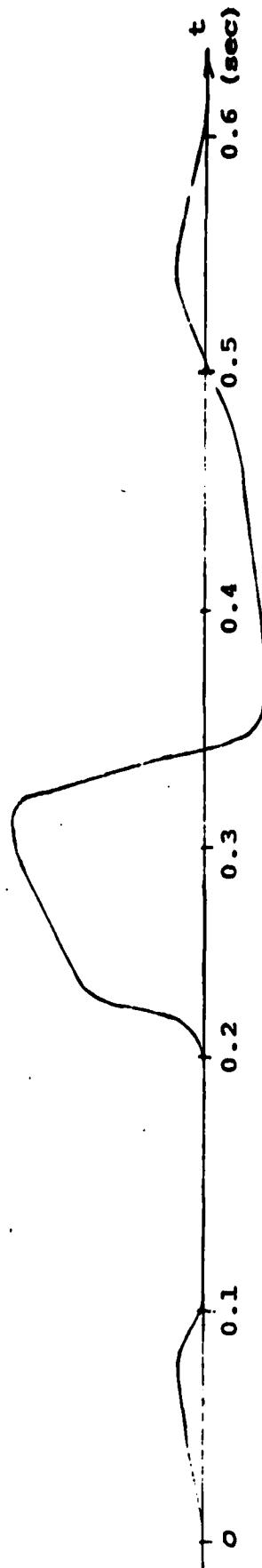
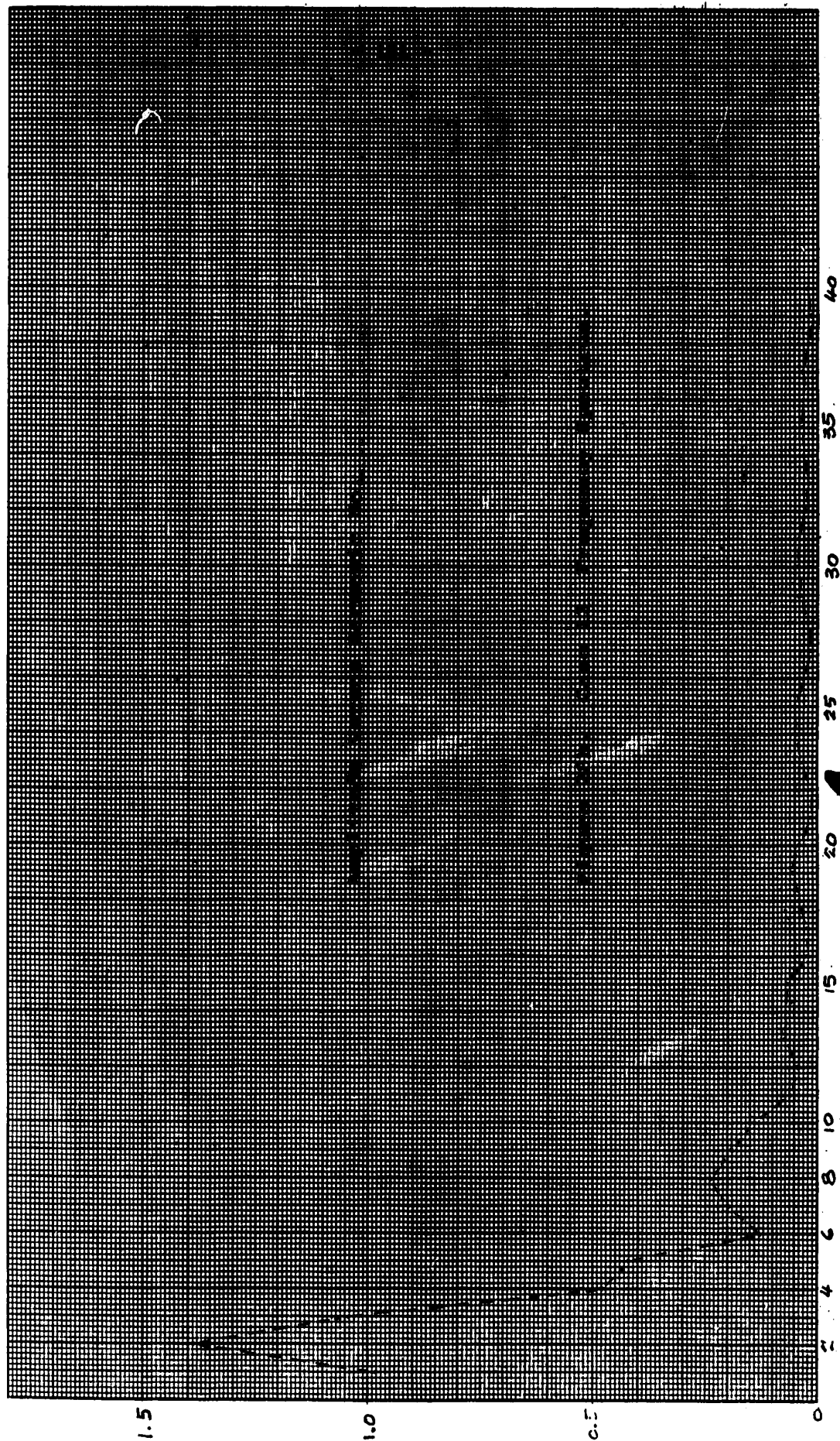
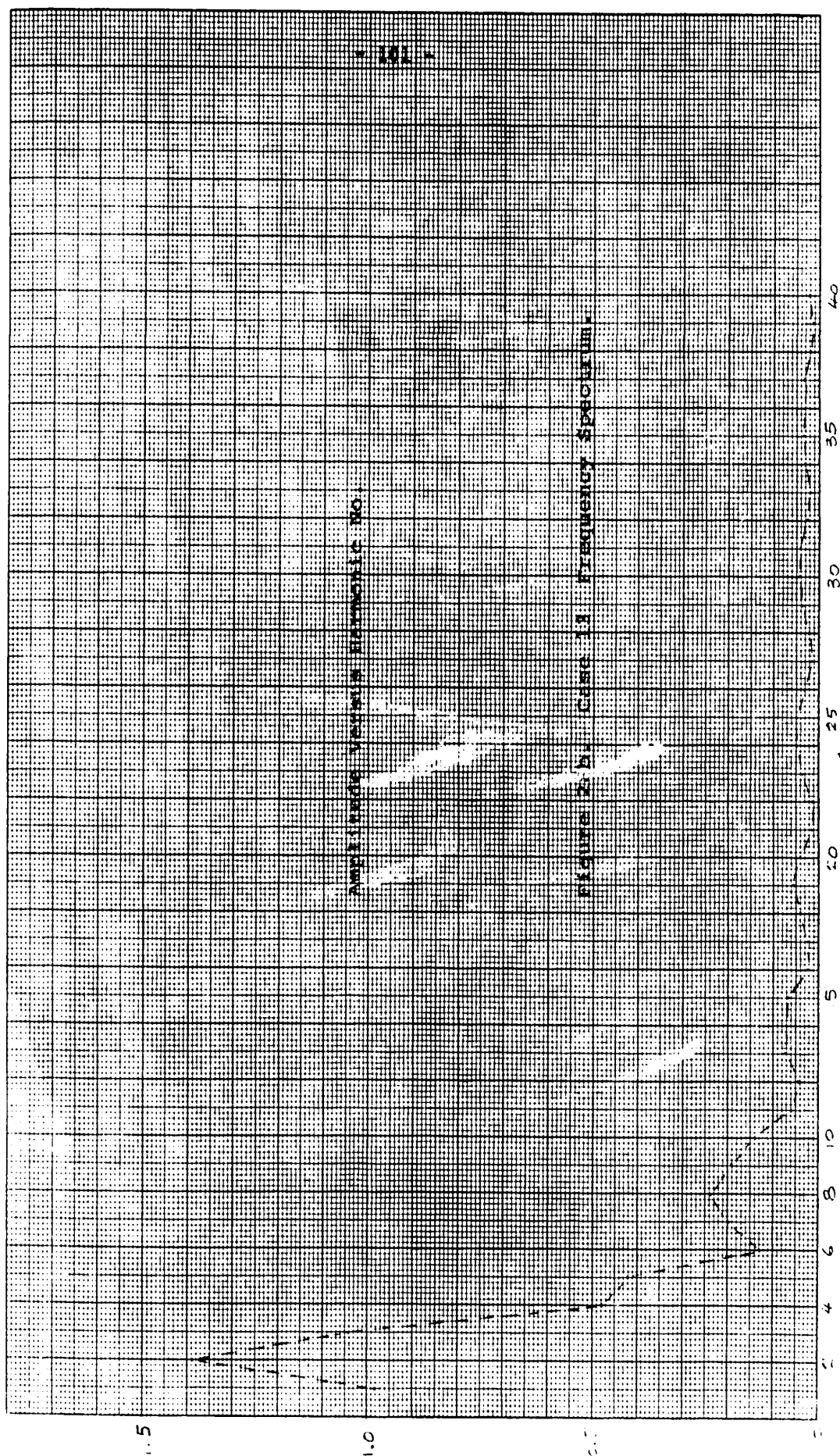


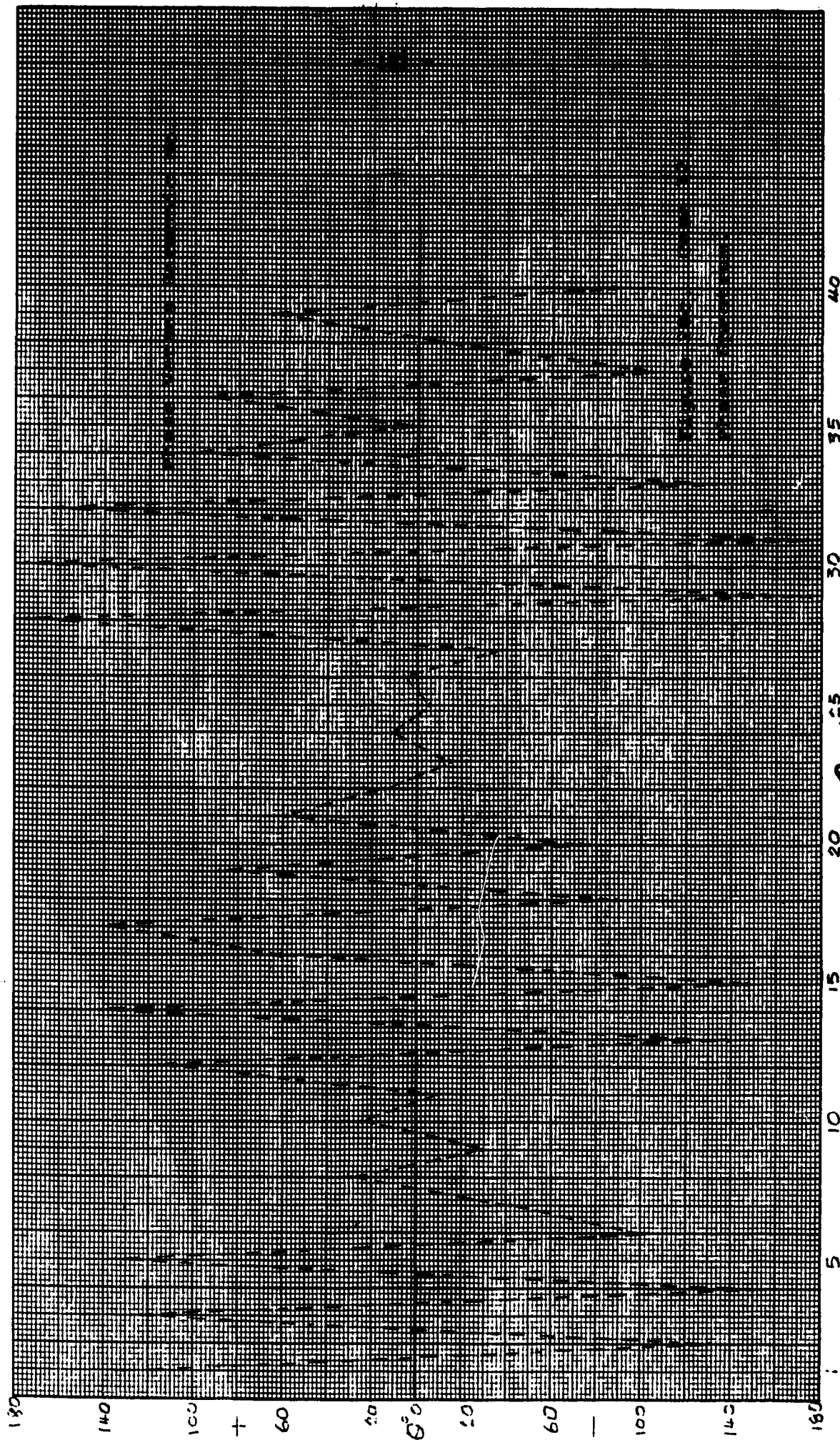
Figure 28a. Case 12 electrocardiogram.

K-E 10 X 10 TO THE CENTIMETER 46 1513
16 X 25 CM.
MADE IN U.S.A.
KEUPPEL & ESSER CO.

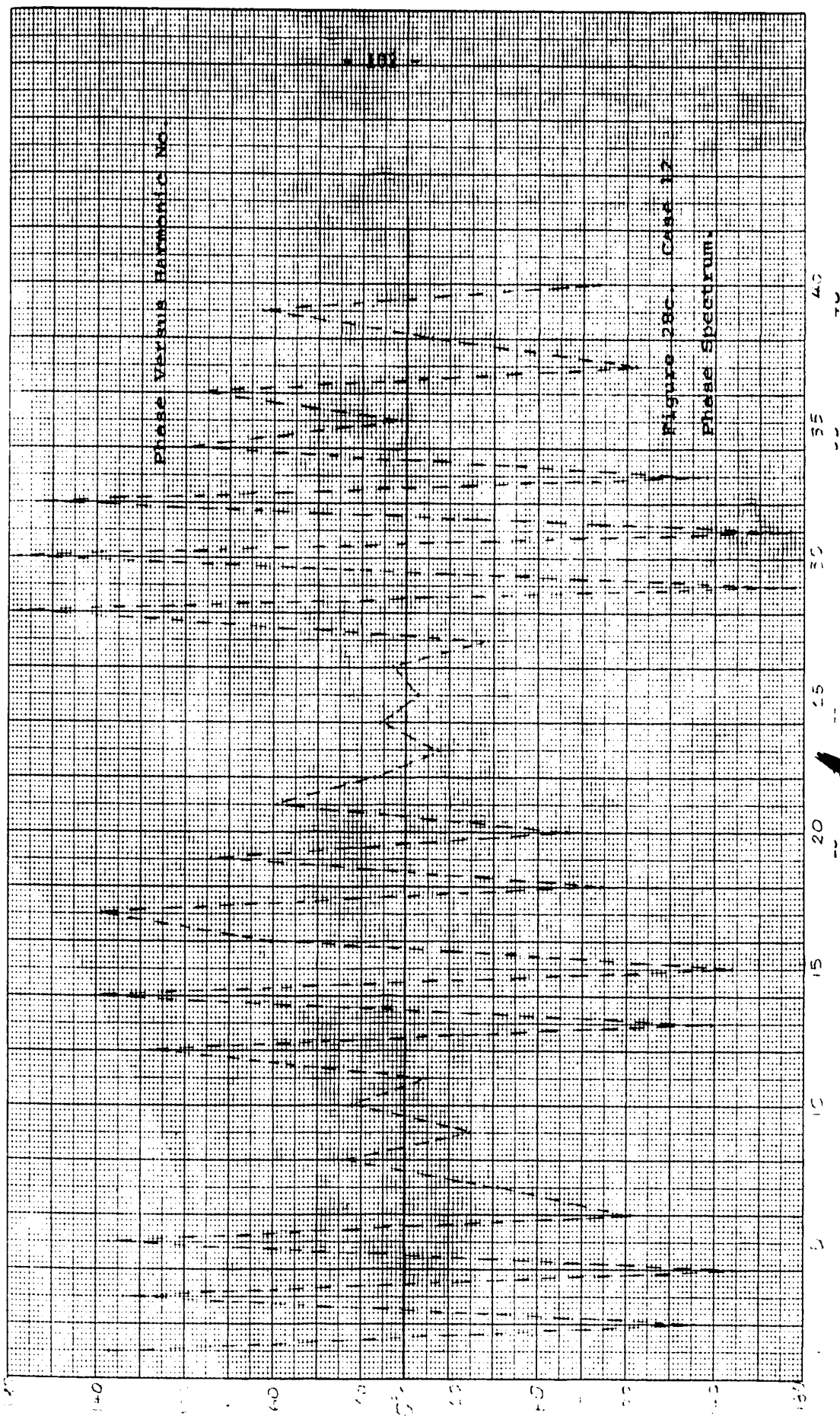




K-E 10 X 10 TO THE CENTIMETER 46 1513
 10 X 25 CM. MADE IN U.S.A.
 KEUFFEL & ESSER CO.



K-E 10 X 10 TO THE CENTIMETER 46 151
 REUPPE & ESSER CO.



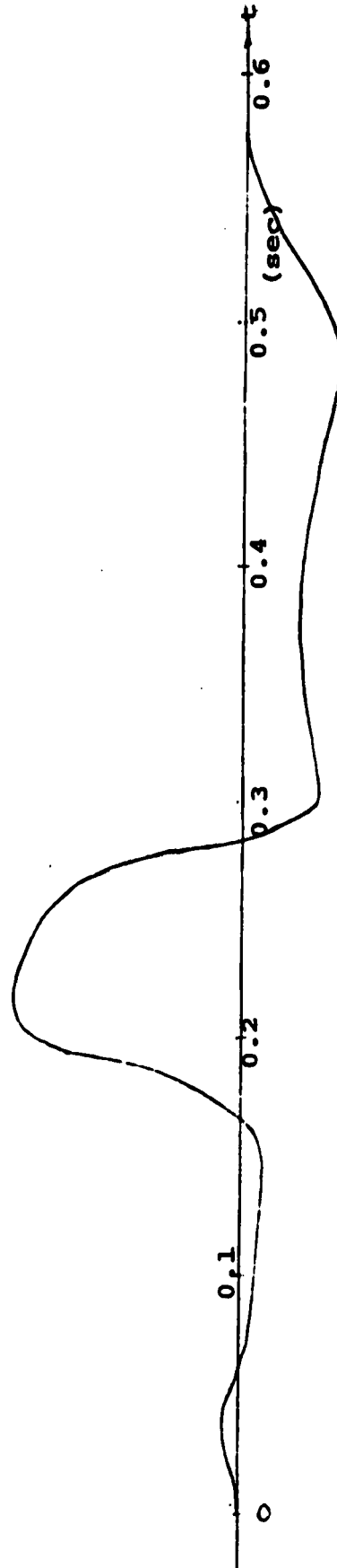
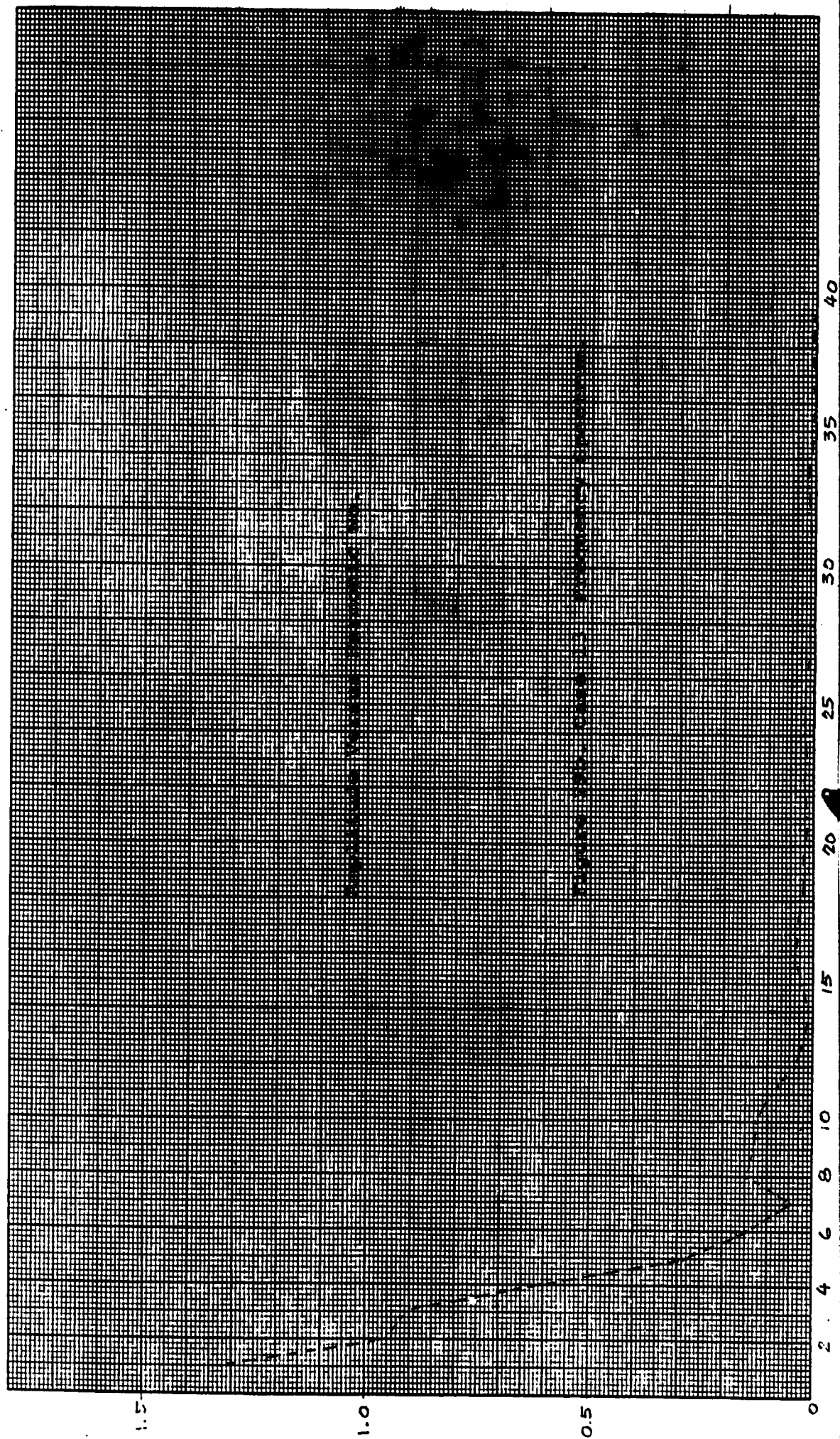
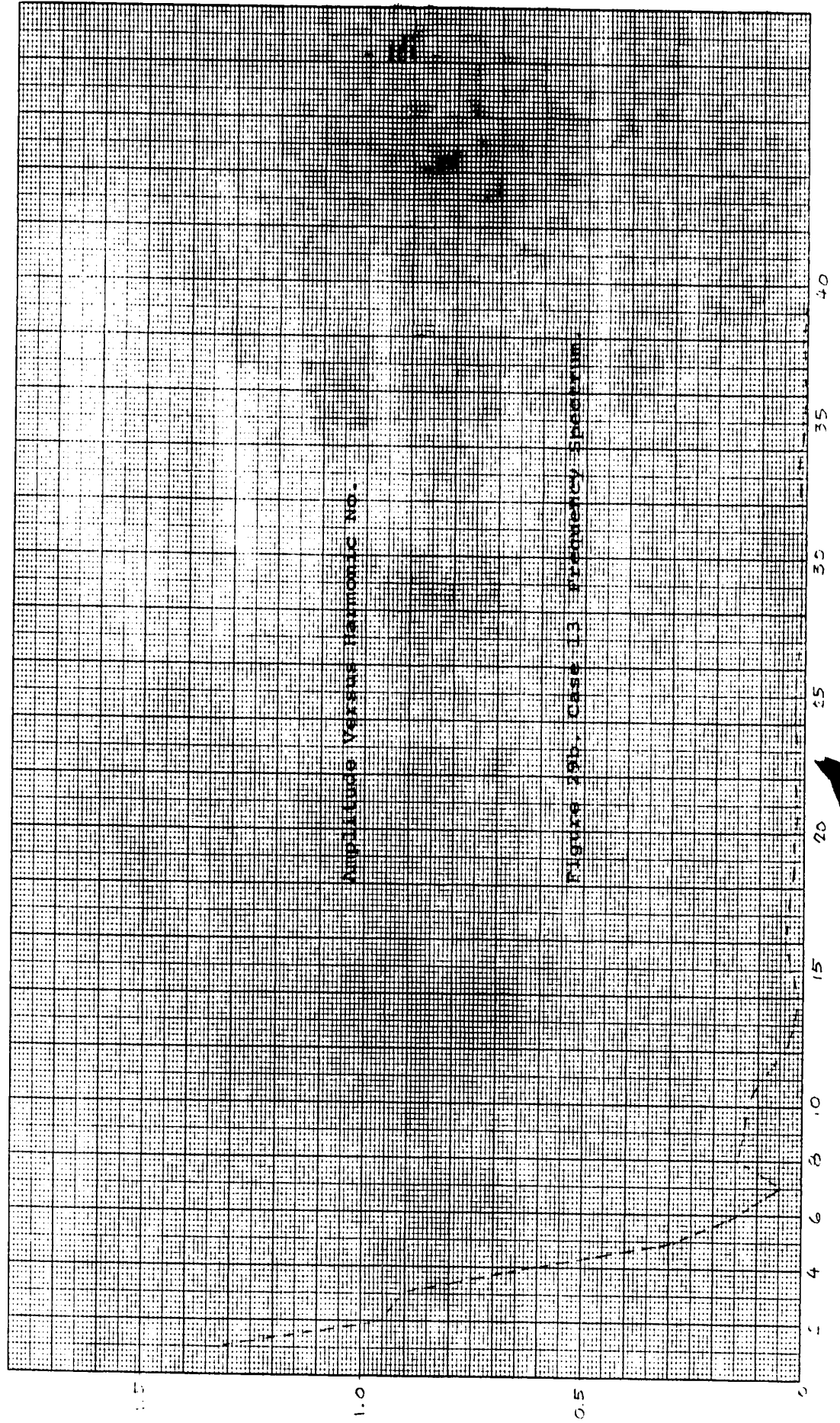


Figure 29a. Case 13 electrocardiogram.

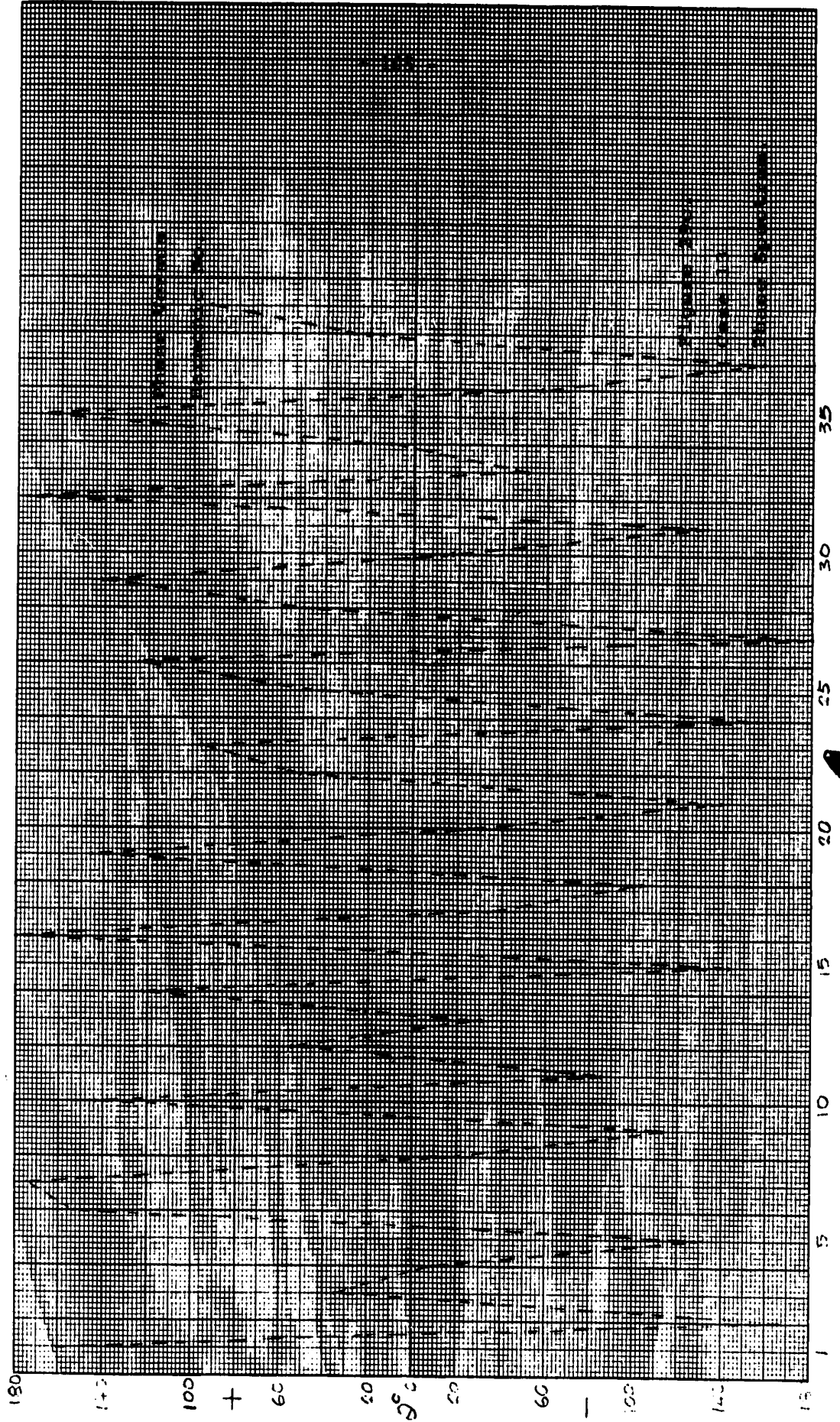
K-E 10 X 10 TO THE CENTIMETER 46-1513
10 X 25 CM.
MADE IN U.S.A.
KEUFFEL & ESSER CO.

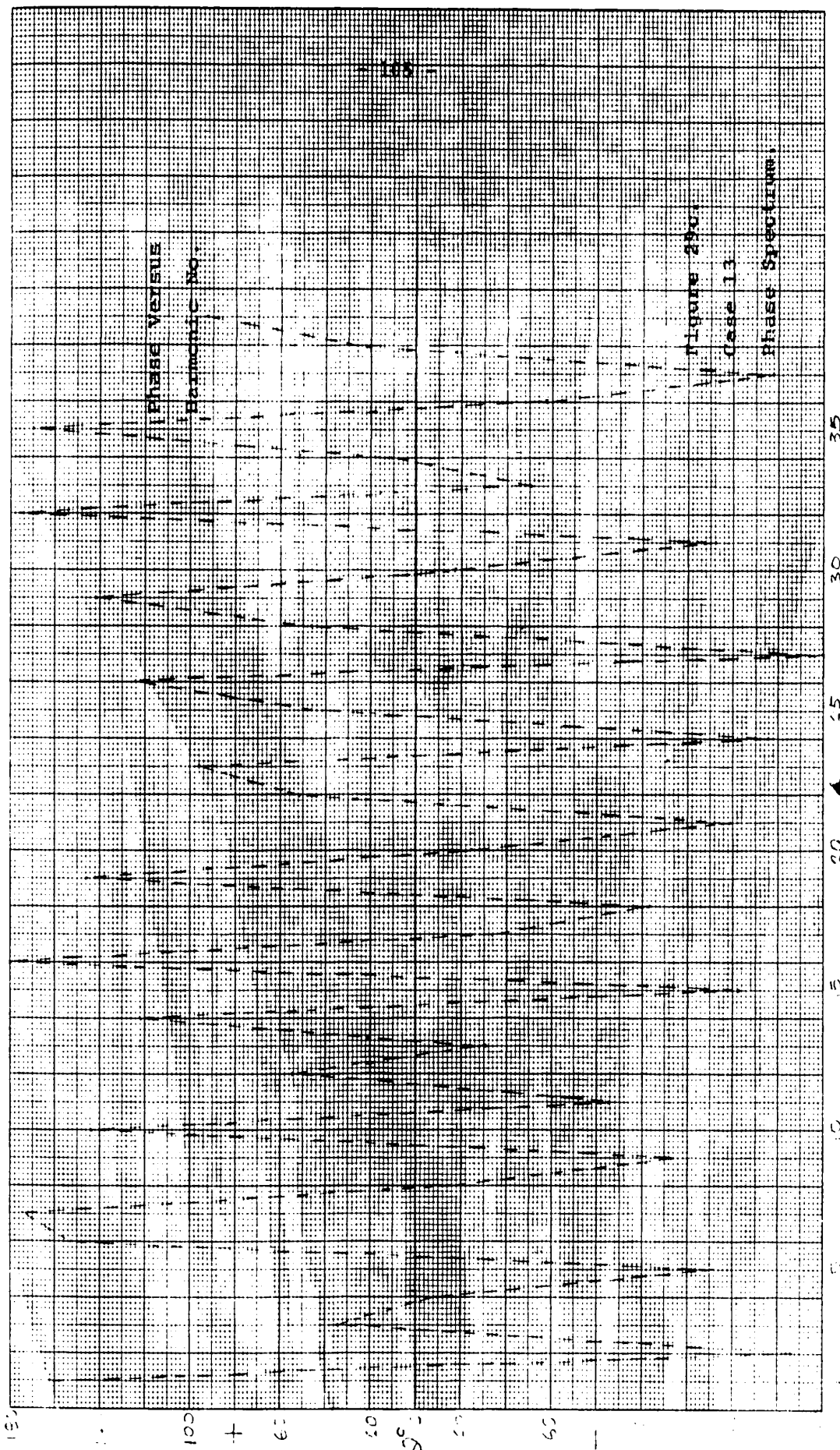


K-2 ELECTRODYNAMIC COMPENSATOR 48 1512
PEPPER & ASSOC.
PEPPER & ASSOC.



K-E 10 X 10 TO THE CENTIMETER 40 1513
10 X 25 CM
KEUFFEL & ESSER CO.





NO. 1
REF. PELL A ESSER CO

envelope after the tenth harmonic appears approximately as a sinusoid. In Figure 29b the inverted T-wave contributes an extremely large first harmonic and a smaller second harmonic compared with those in Figure 28b.

7.6. Myocardial Infraction (cases 14-19)

The disease of myocardial infraction [18-21,24-26] is characterized by the necrosis of a portion of the myocardium resulting from a lack of sufficient blood supply to keep the muscle viable. The most common cause is complete occlusion of a coronary artery by arteriosclerotic coronary thrombosis. Infraction may also result from incomplete occlusion of a coronary artery if this latter process sufficiently reduces the blood supply to that portion of the myocardium, and if collateral circulation is insufficient. The infractioned muscle may be considered electrically inert. The left ventricle is involved in practically all instances of myocardial infraction. The electrocardiogram may actually be normal in myocardial infraction or may show only nonspecific changes which are not diagnostic of infraction. The most characteristic electrocardiographic finding in infraction is an abnormal Q-wave and T-wave inversion. Sometimes abnormal Q-wave may never appear, instead there may be a reduced R-wave.

In Figure 30a the postero-inferio myocardial infraction and in Figure 31a the high positerior myocardial infraction cardiac waveforms show a small R-wave. In Figures 30b and 31b the second and fourth harmonics are maximum points whereas the first and the third harmonics are minimum points. This situation is similar to the normal case shown in Figure 17b. However in Figure 30b there are peaks at the sixth and ninth harmonics and in Figure 31b there is a peak at the sixth harmonic with some variations in the high frequency range. These are not present in the normal case spectrum (Figure 17b).

All of the Figures 32b to 35b have a large first harmonic and the relative magnitudes of the first six harmonics are quite different from the normal case spectrum (Figure 17b). These spectral features are due to the deformed T-wave and S-T segment. In Figure 32a the notching in the R-wave contributes some high frequency amplitudes larger than those in the normal case (Figure 17b). In Figure 33b the shape of the spectral envelop after the fifteenth harmonic is a consequence of the smooth cardiac waveform in Figure 33a.

7.7. Chronic Coronary Insufficiency (case 20)

Figure 36a shows the abnormalities of S-T segment and T-wave

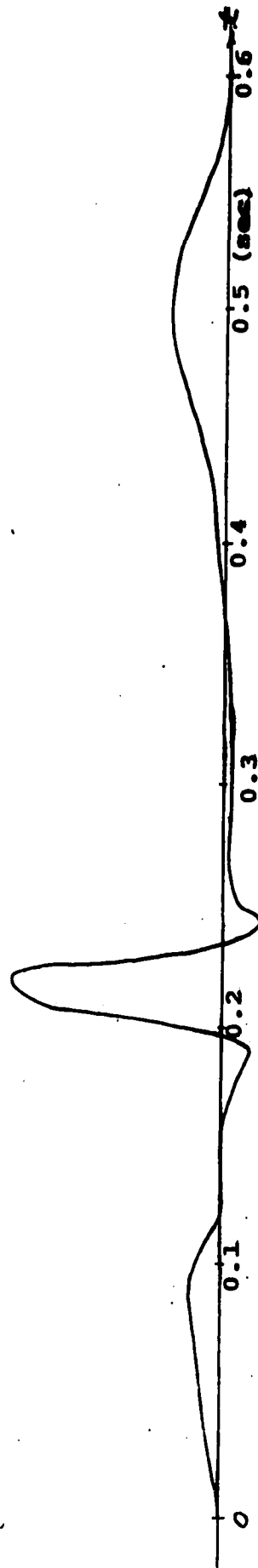
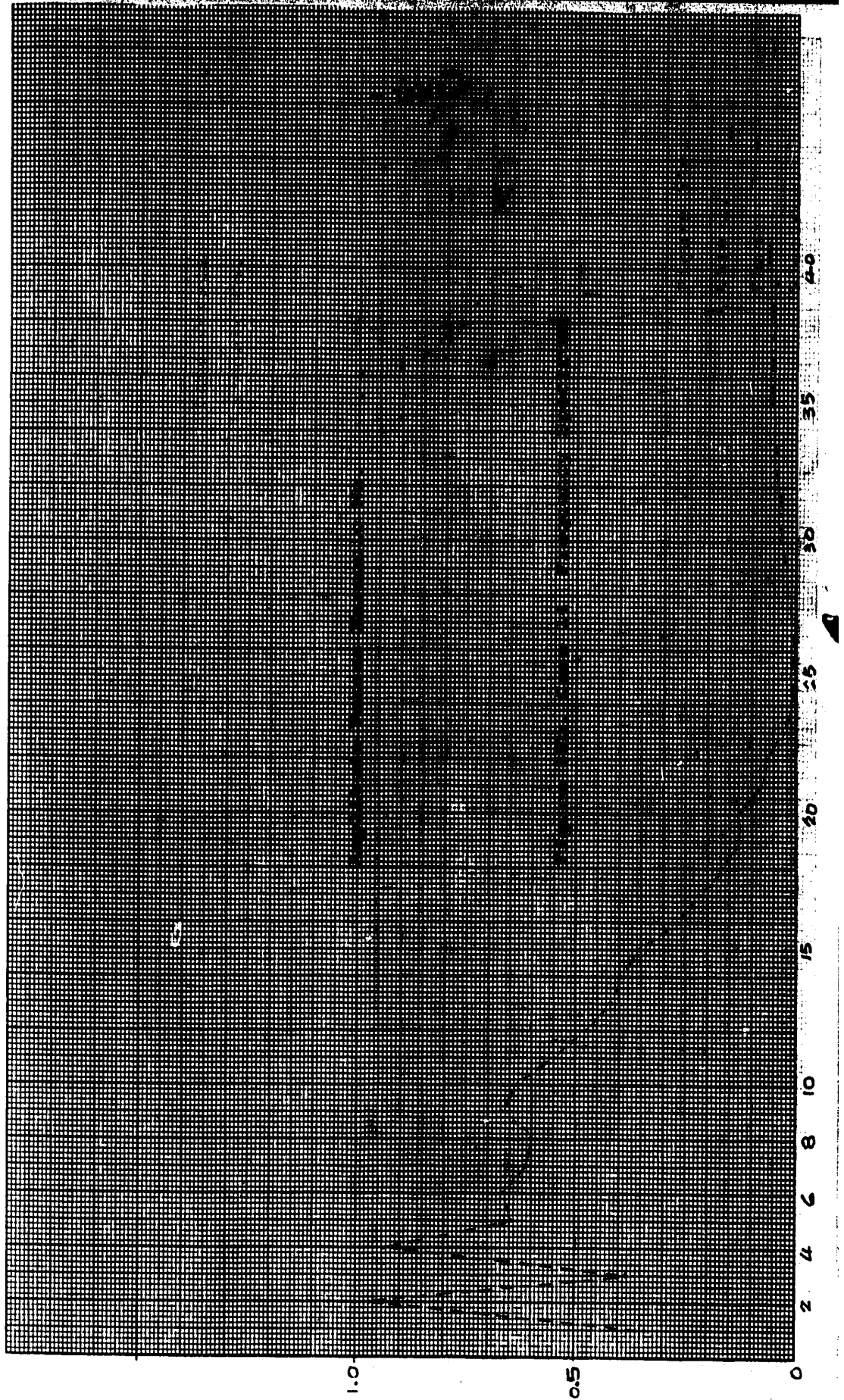
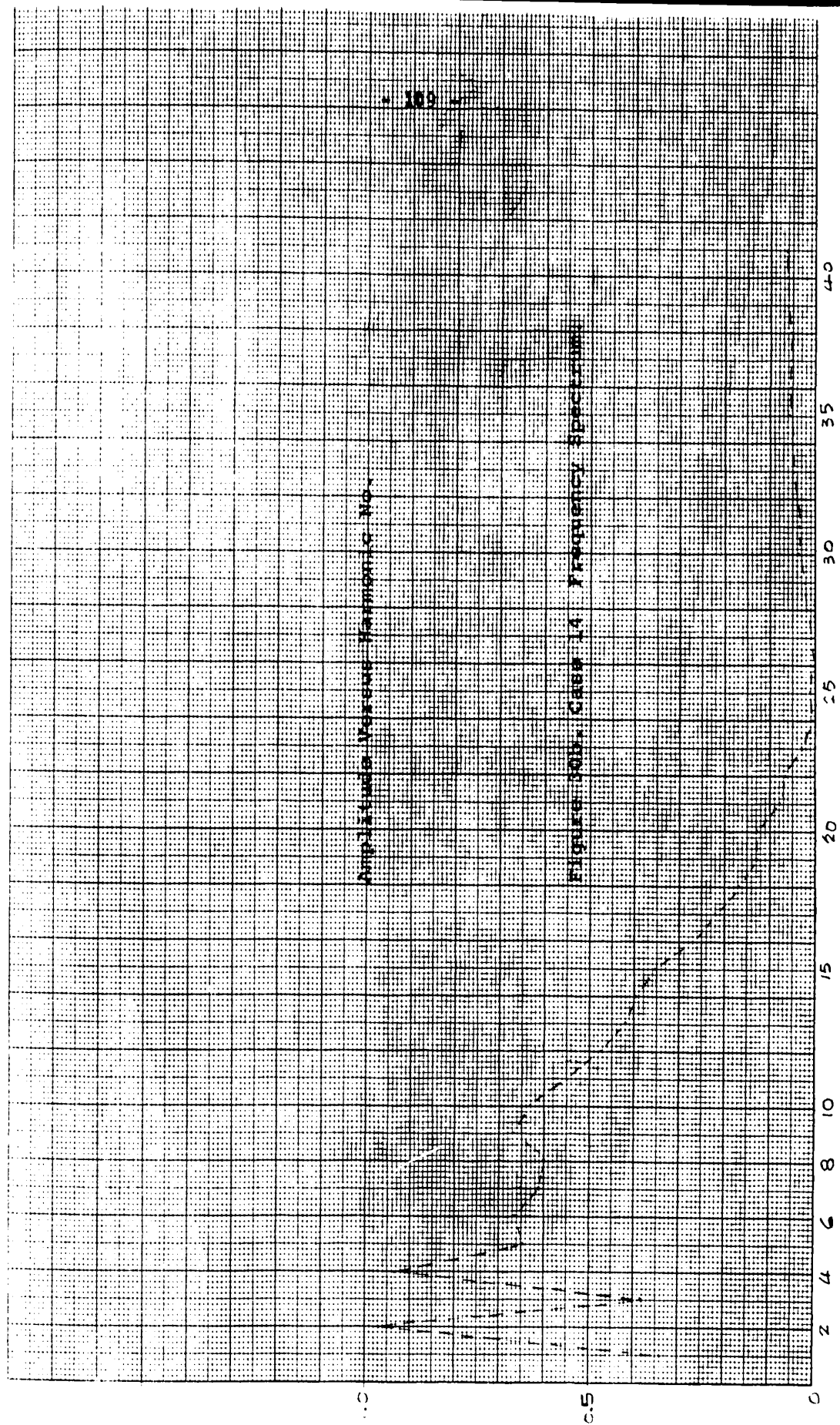


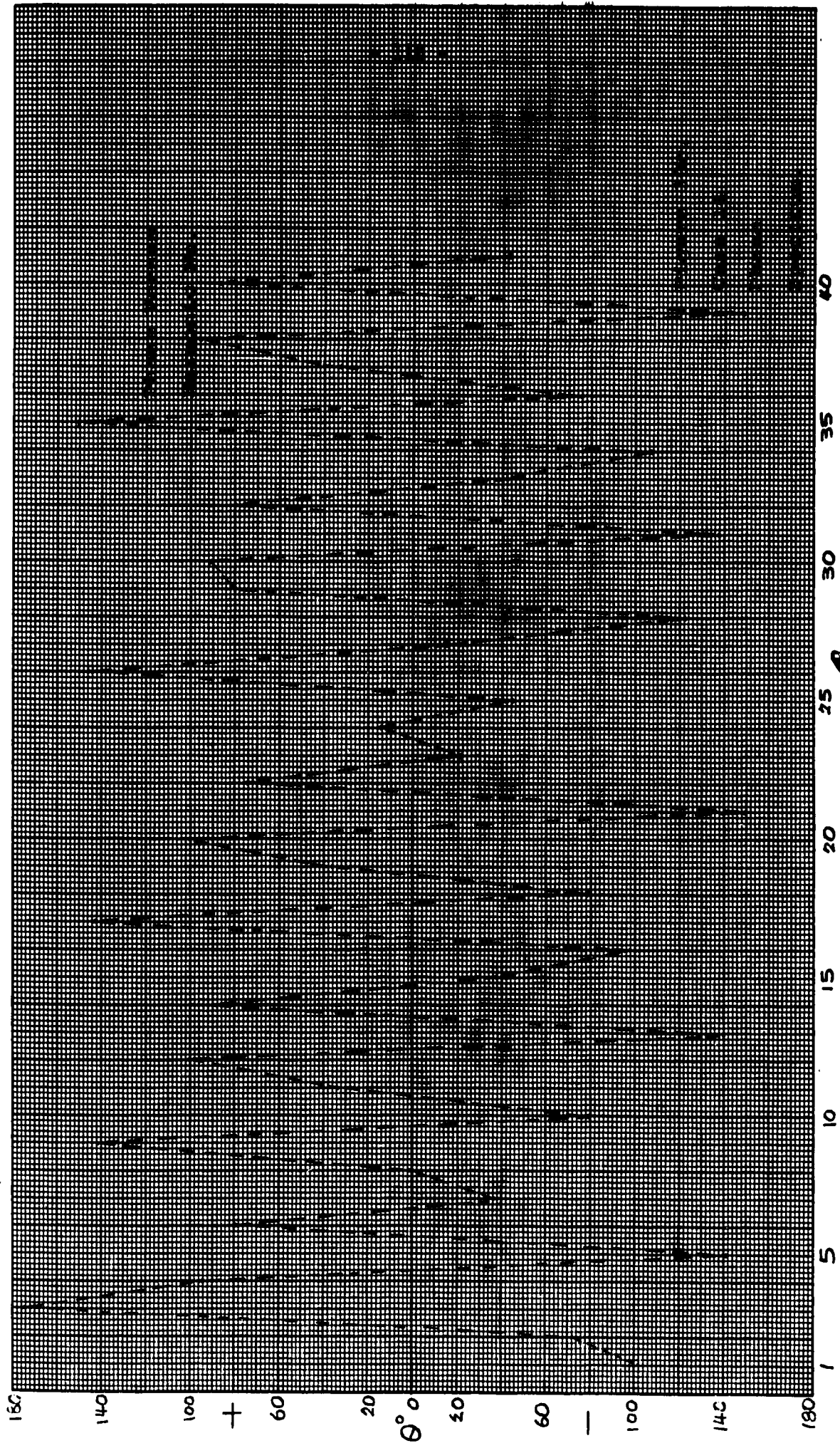
Figure 30a. Case 14 electrocardiogram.

K-E 10 X 10 TO THE CENTIMETER 20 513
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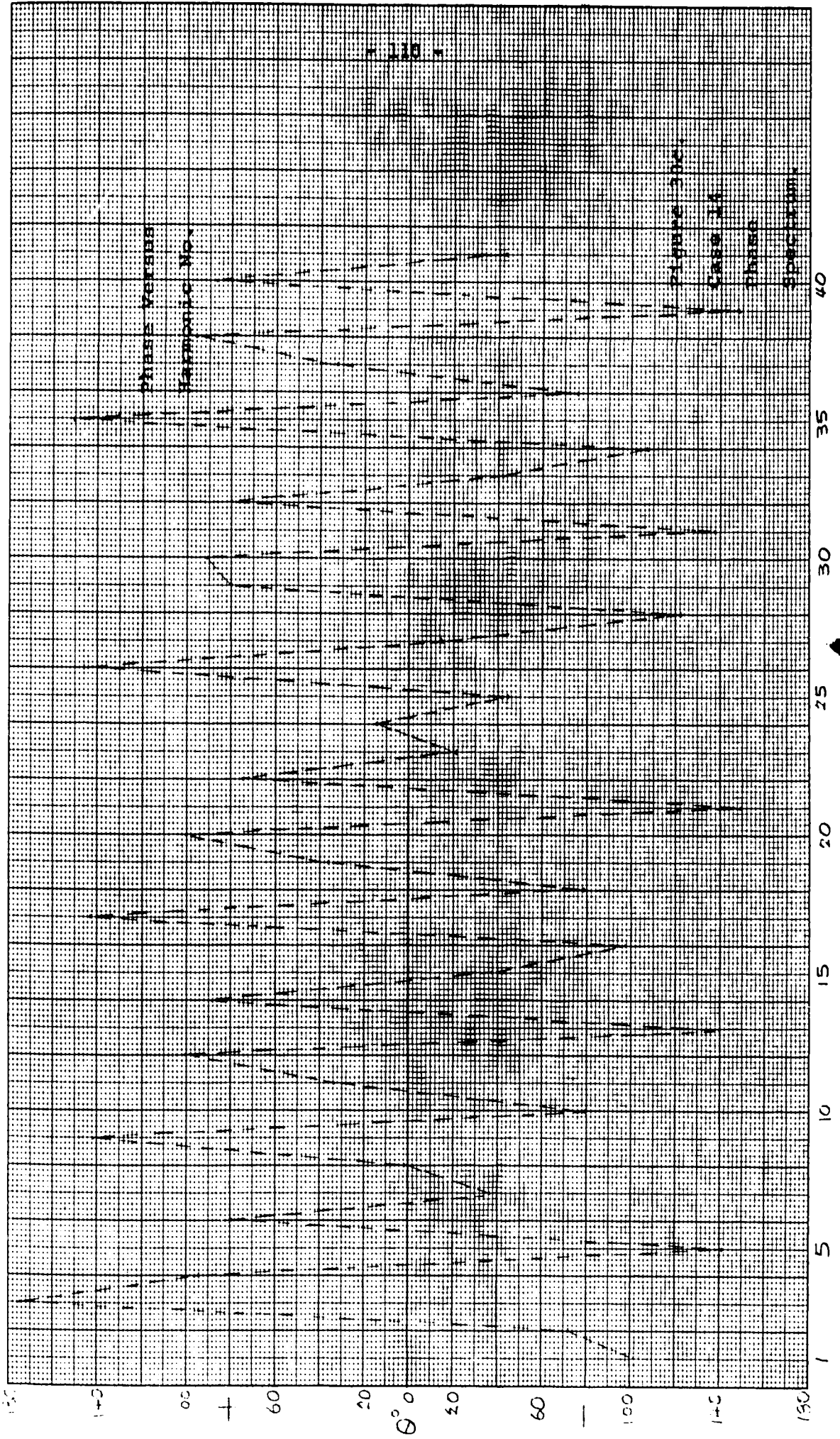




R-2E 10 X 10 TO THE CENTIMETER 10-1010
MADE IN U.S.A.
KEUPPEL & EBBER CO.



K-E 10 X 10 TO THE CENTIMETER 40-1813
IN X 25 CM
REUPPEL & ESSEN CO



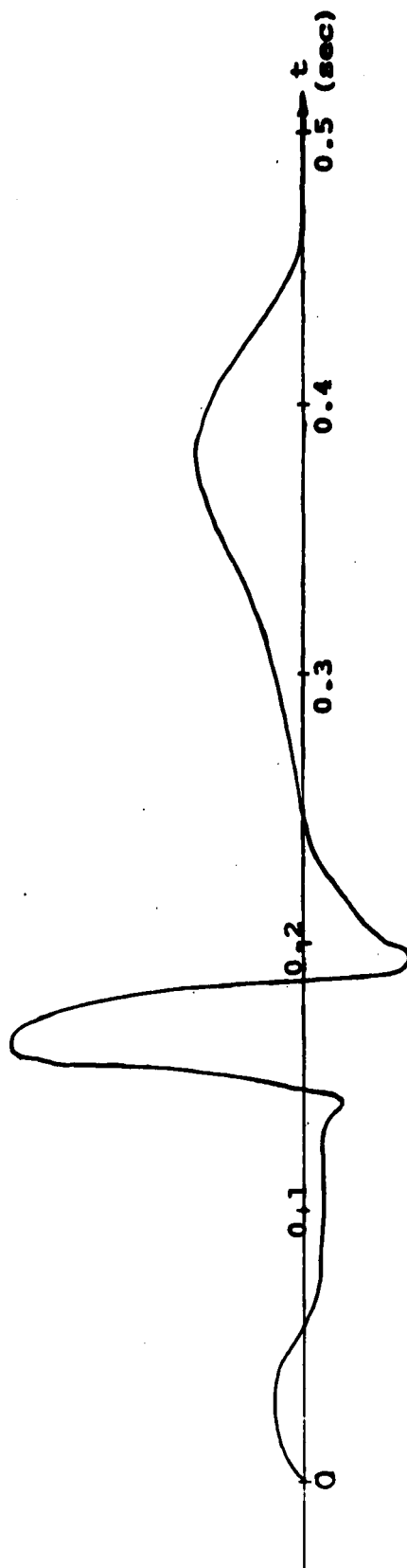
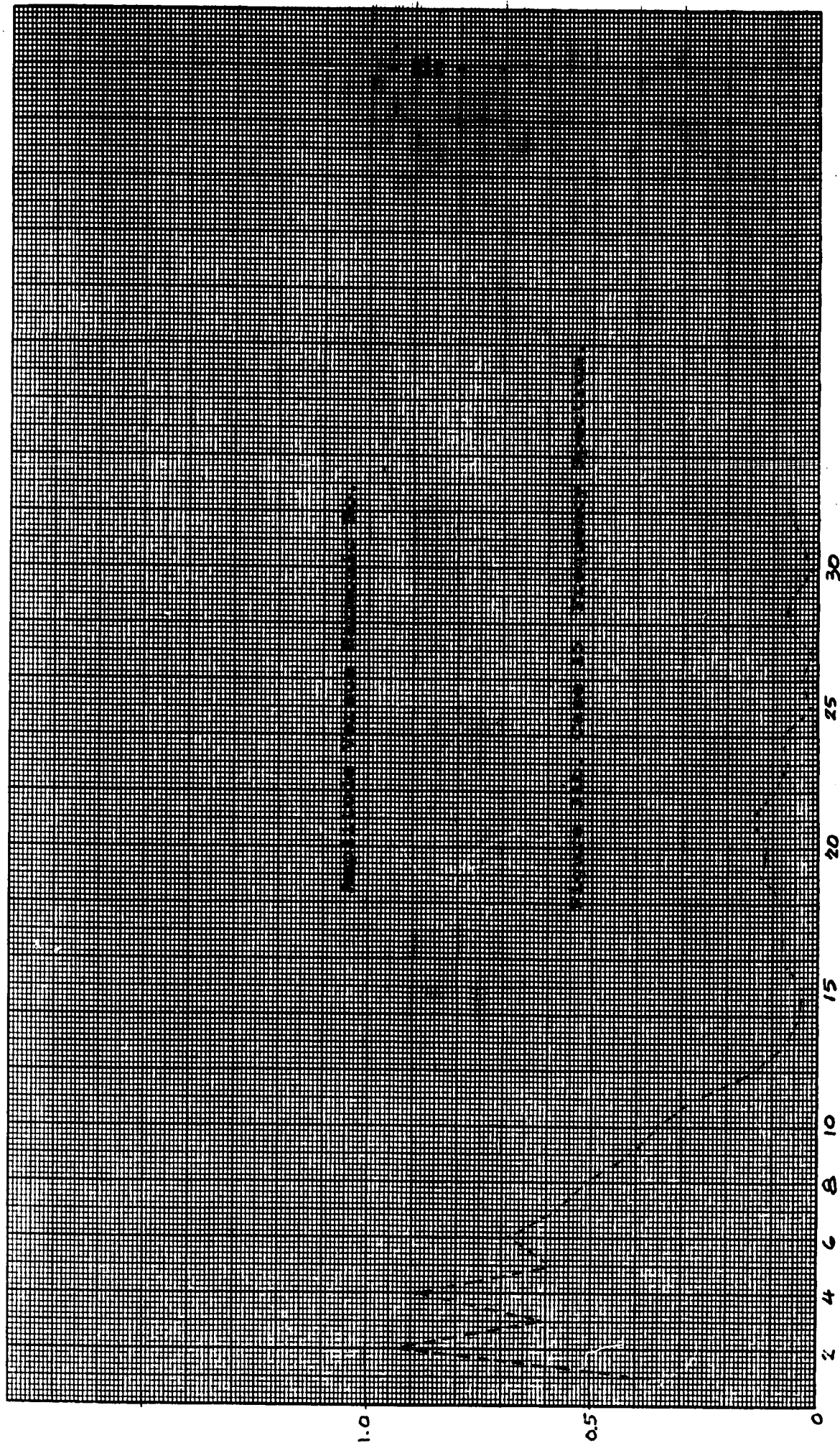
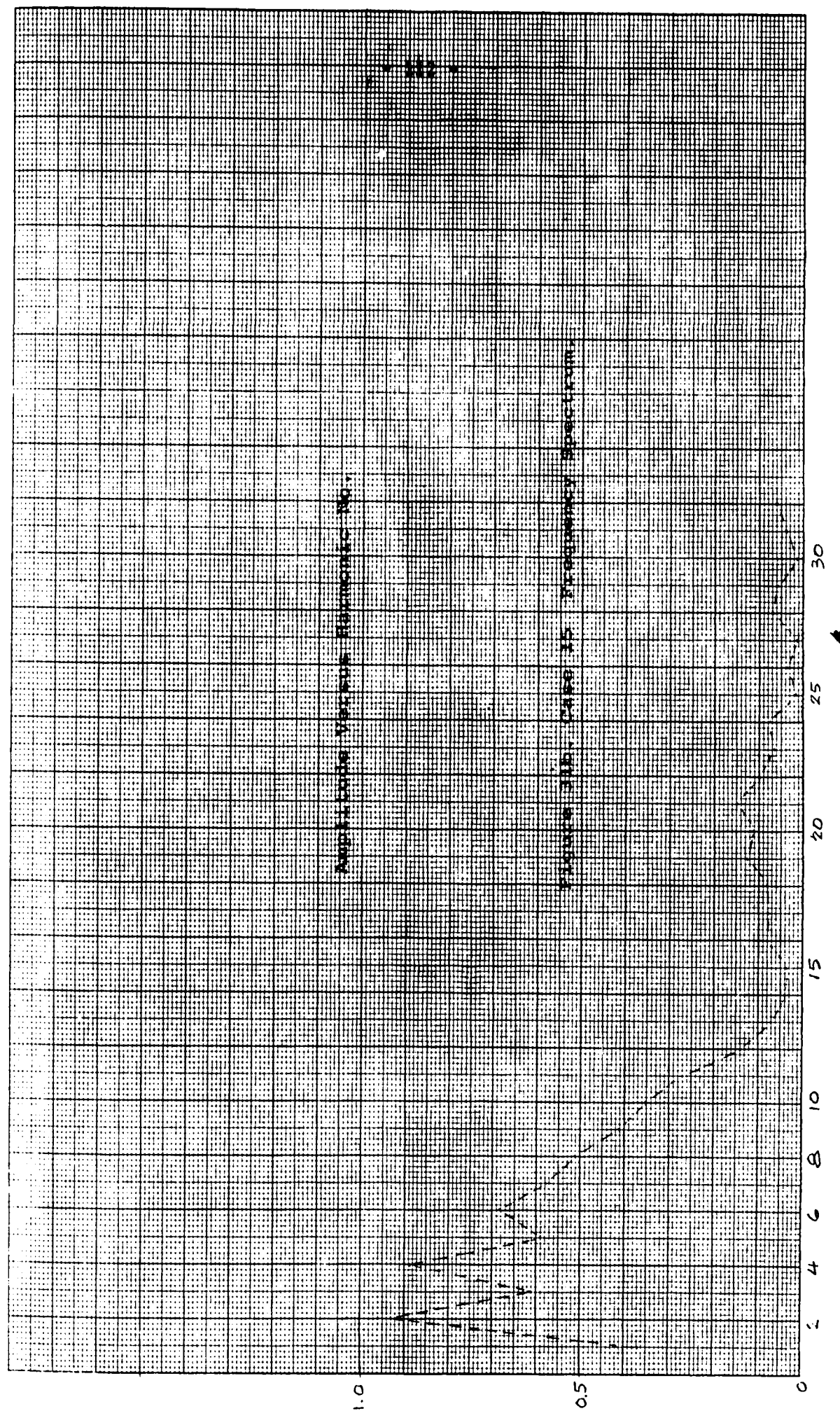


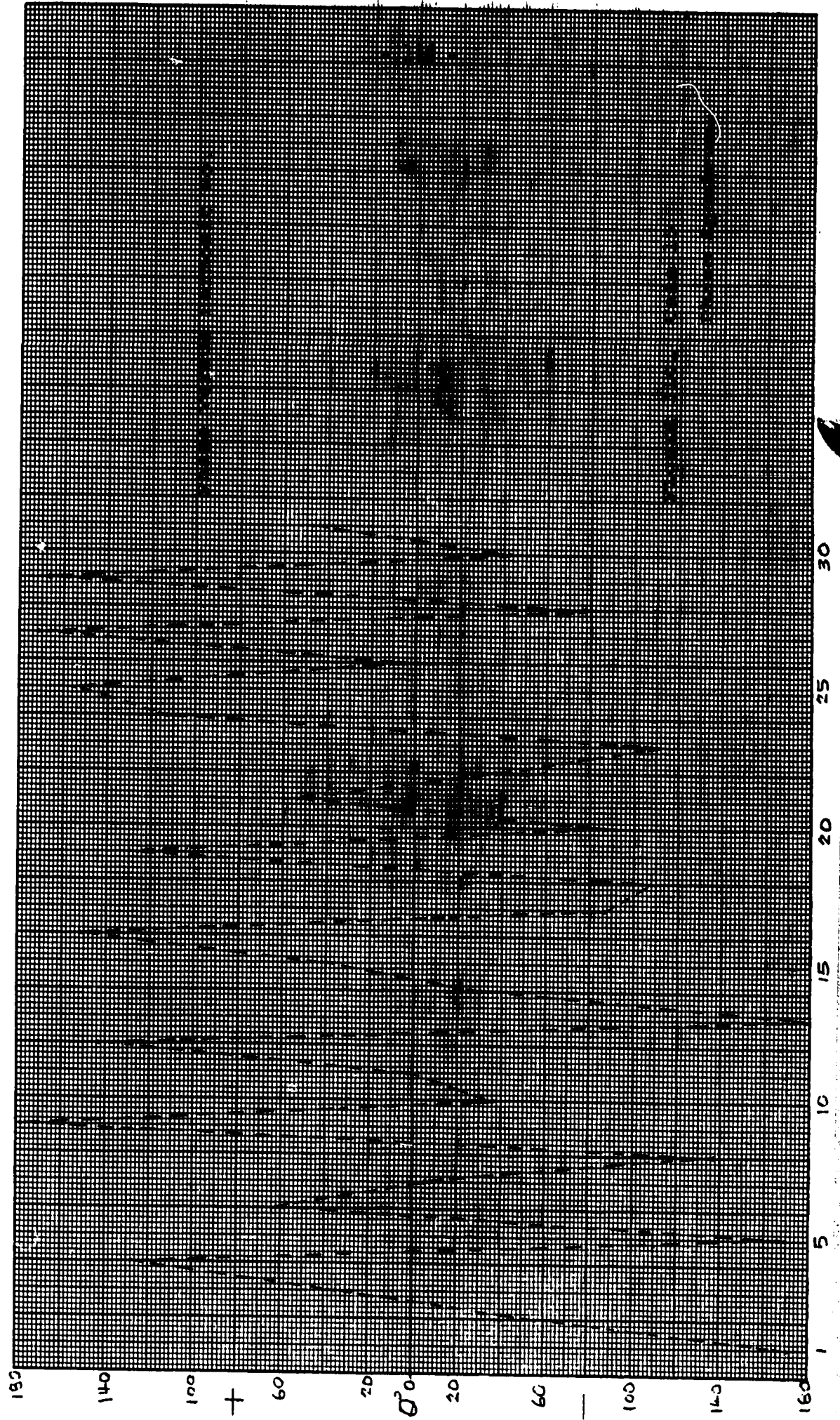
Figure 31a. Case 15 electrocardiogram.

K-E 10 X 10 TO THE CENTIMETER 46-1513
MADE IN U.S.A.
KEUFFEL & ESSER CO.

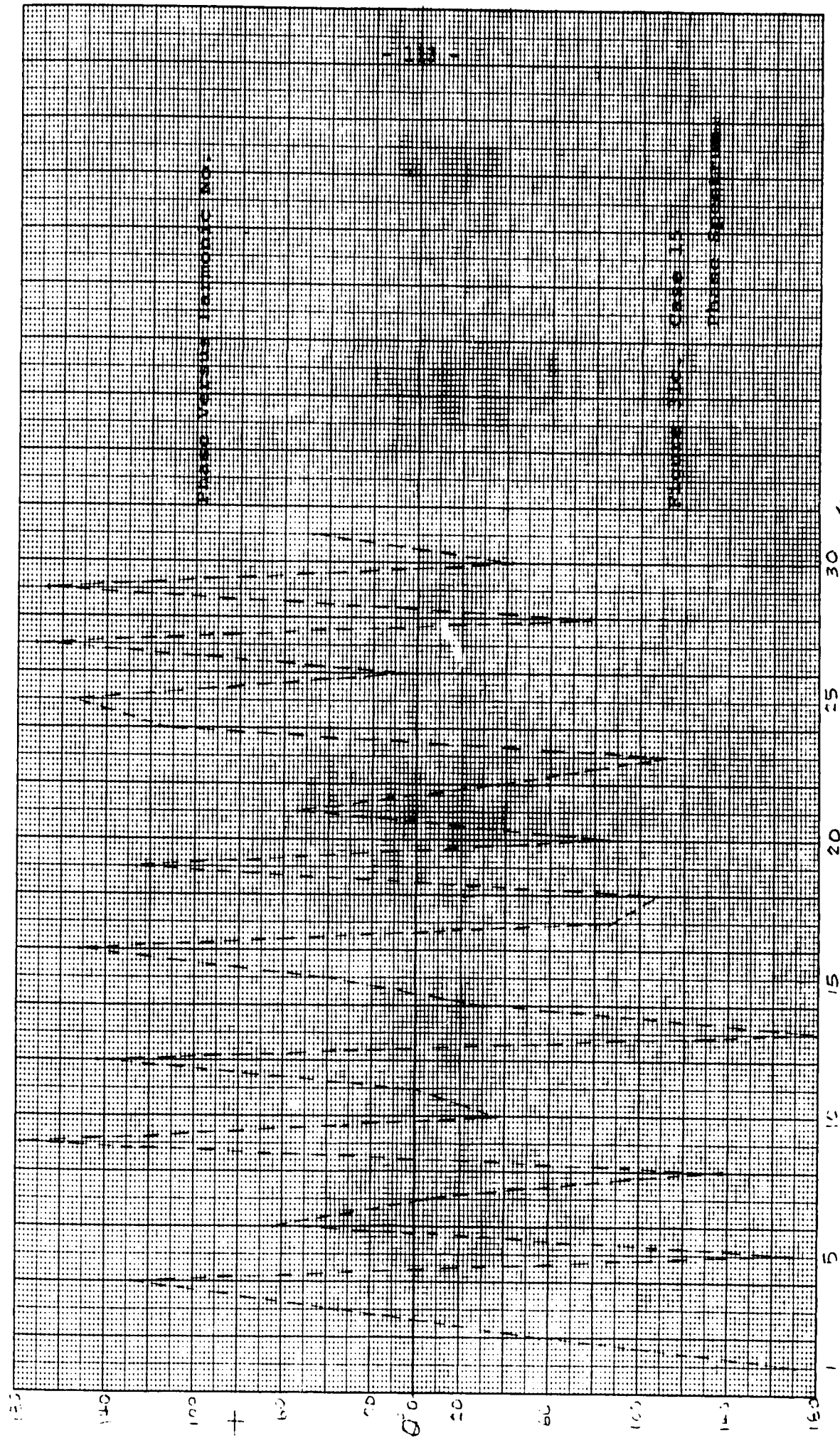




K-E 10 X 10 TO THE CENTIMETER 46 1513
MADE IN U.S.A.
KEUPPEL & ESSER CO.



K-E 10 X 10 TO THE CENTIMETER 46 1513
MADE IN U.S.A.
KEUFFEL & ESSER CO.



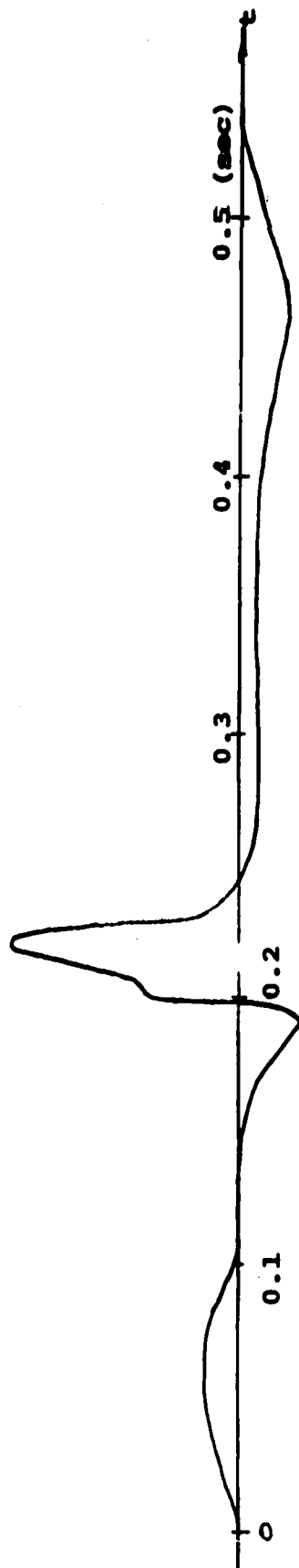
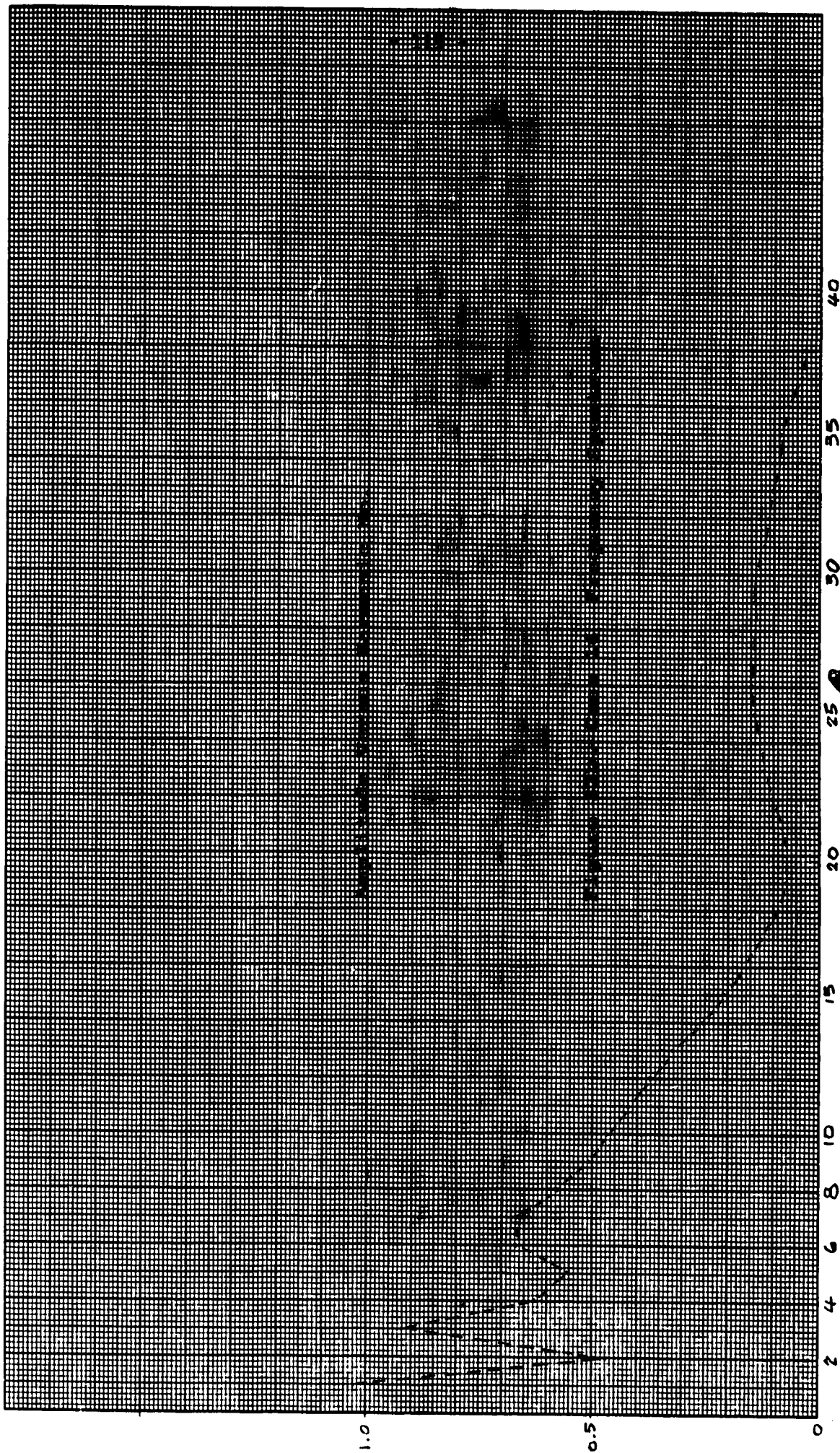
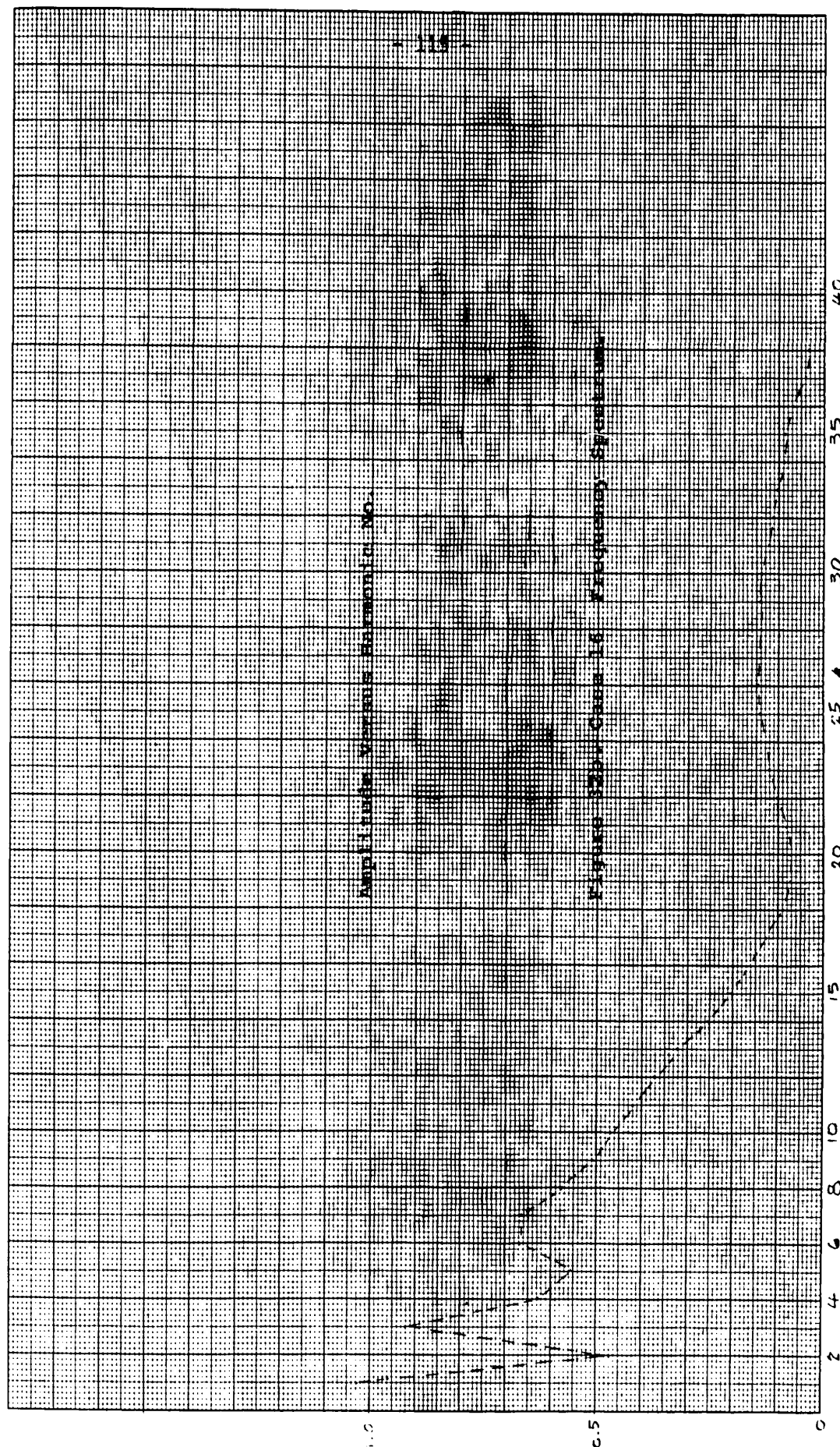


Figure 32a. Case 16 electrocardiogram.

K-E 10 X 10 TO THE CENTIMETER 46 1513
18 X 23 CM. MADE IN U.S.A.
KEUFFEL & ESSER CO.

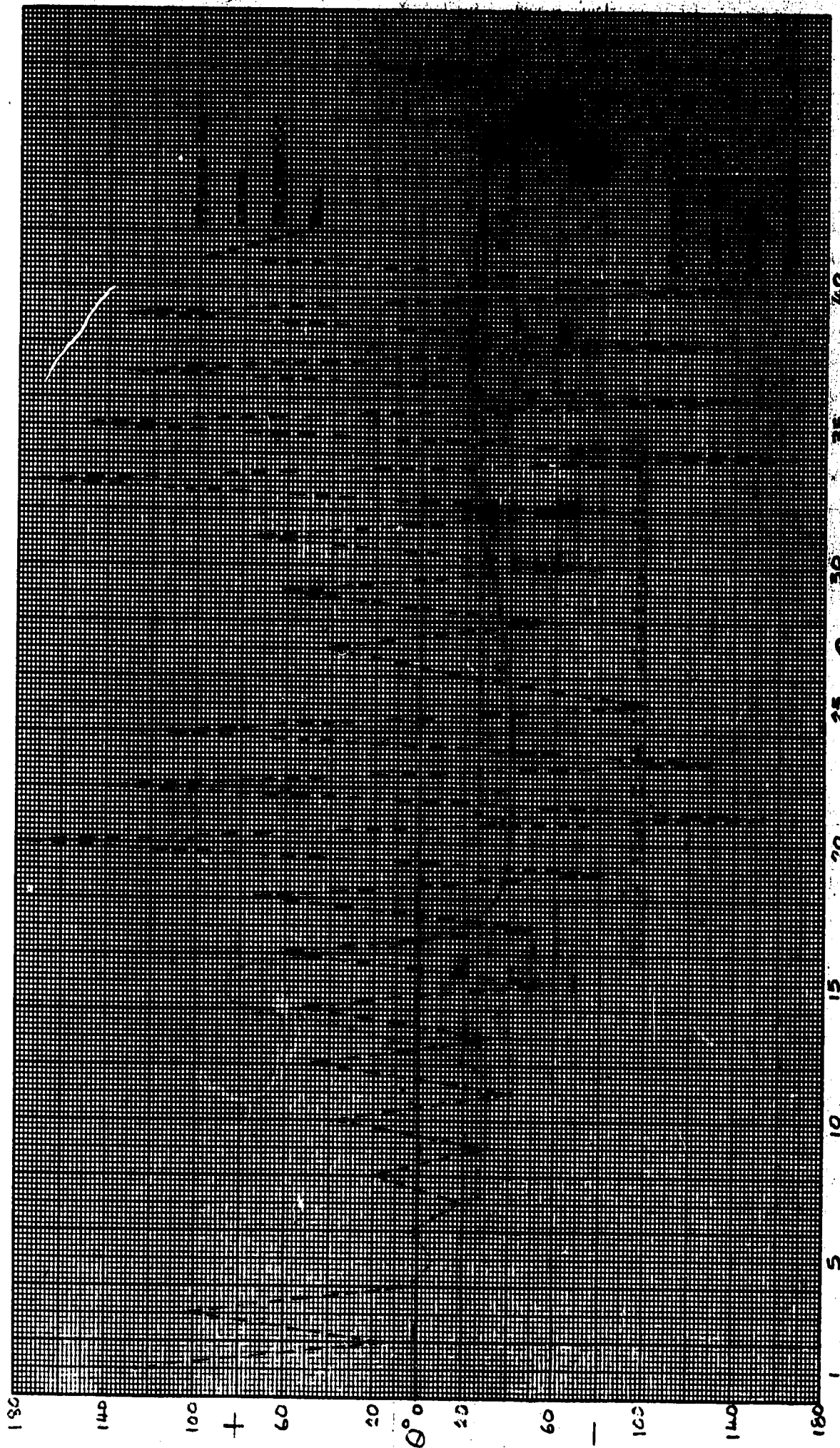


K-E 10 X 10 TO THE CENTIMETER 46 1513
 10 X 2.5 CM MADE IN U.S.A.
 KEUFFEL & ESSER CO

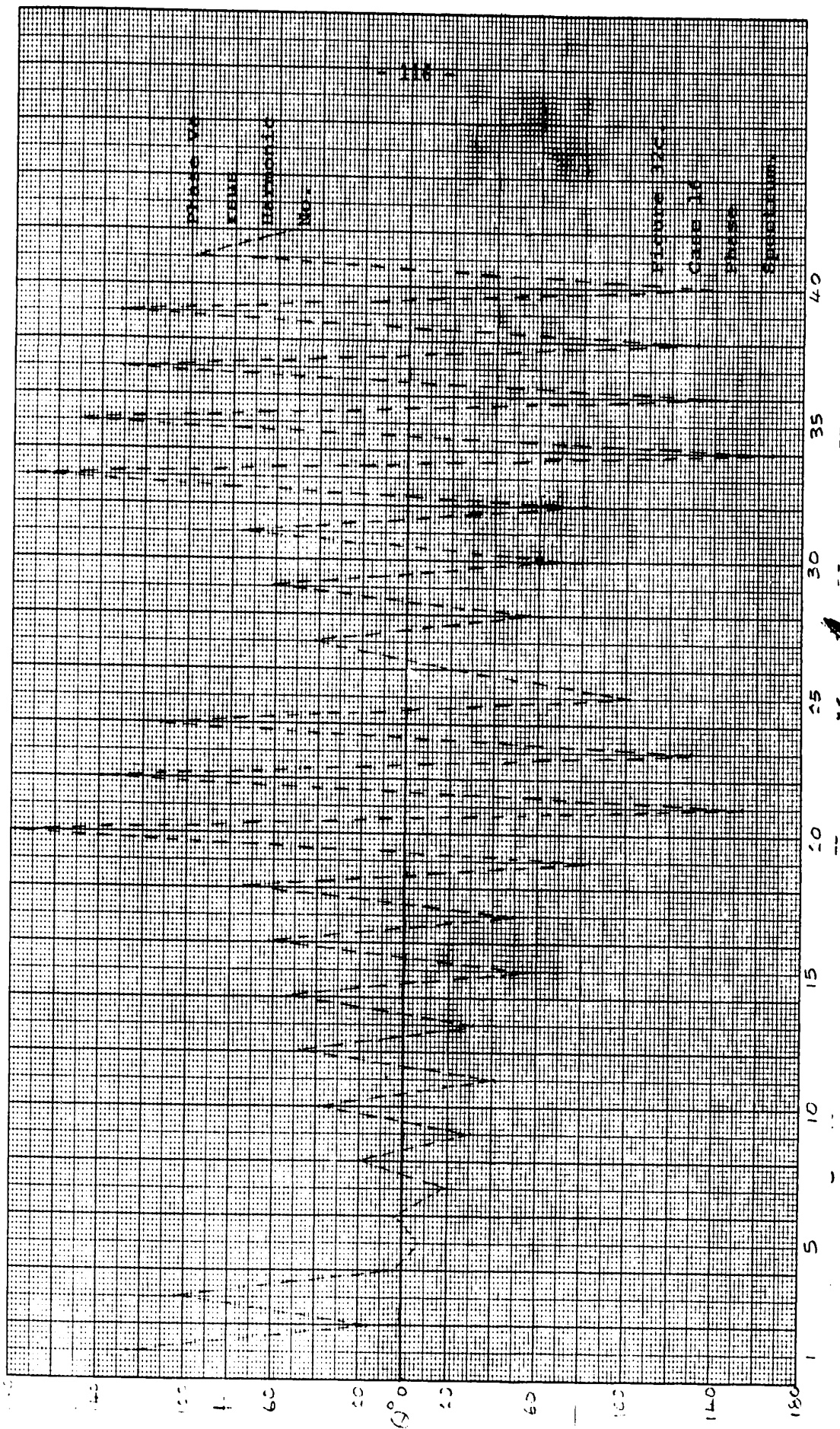


RETRACT & EXTEND
10 X 10 TO THE CENTIMETER 46 1513
10 X 25 CM.
MADE IN U.S.A.
KEUPPEL & ESSER CO.

180 140 100 60 20 0° 20 60 100 140 180



K-E 10 X 10 TO THE CENTIMETER 46 1513
 MADE IN U.S.A.
 KEUFFEL & ESSER CO.



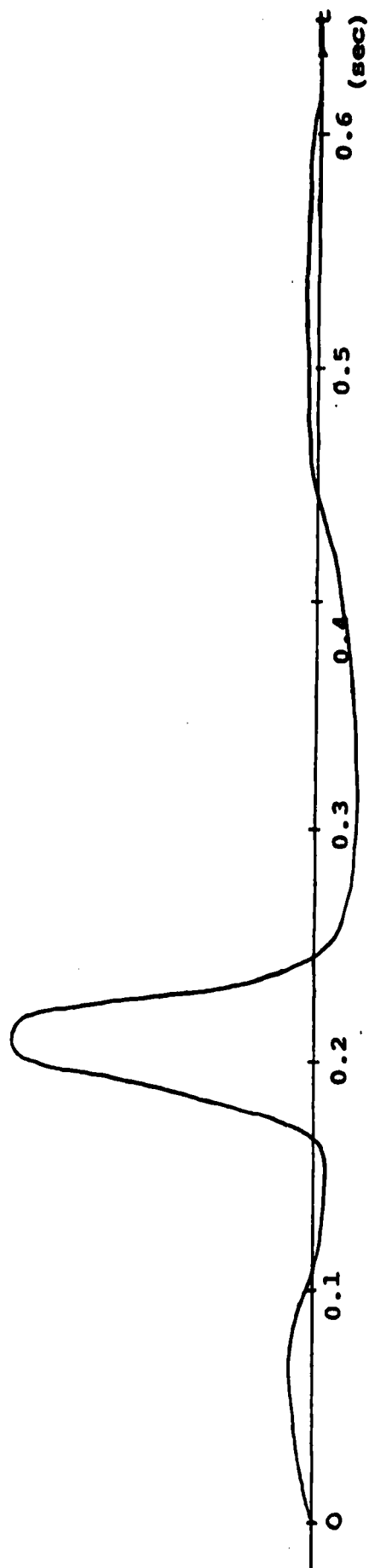
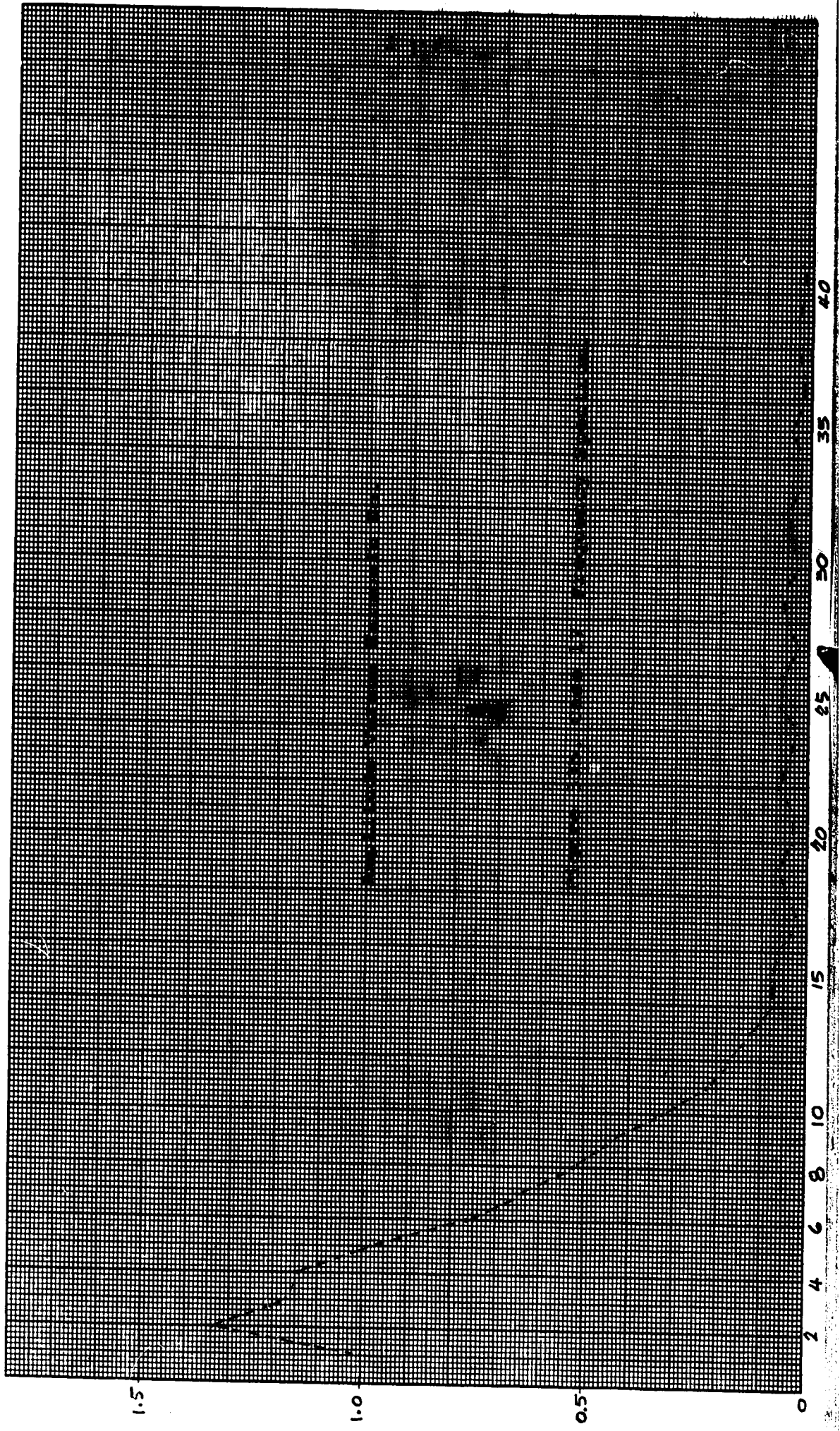


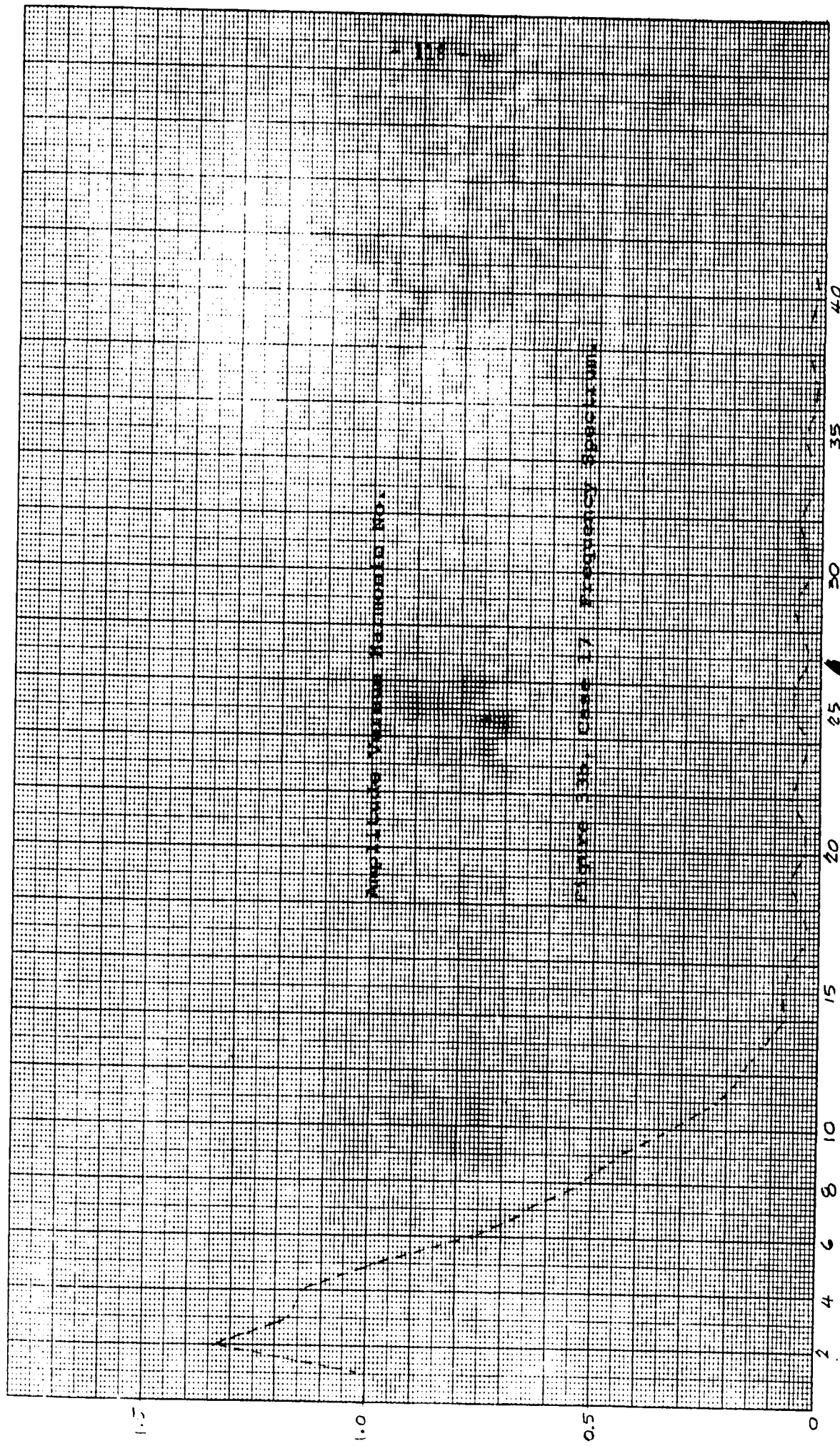
Figure 33a. Case 17 electrocardiogram.

K-E 10 X 10 TO THE CENTIMETER 48 1513
16 X 25 CM. "NOT IN U.S.A."
KEUFFEL & ESSER CO.

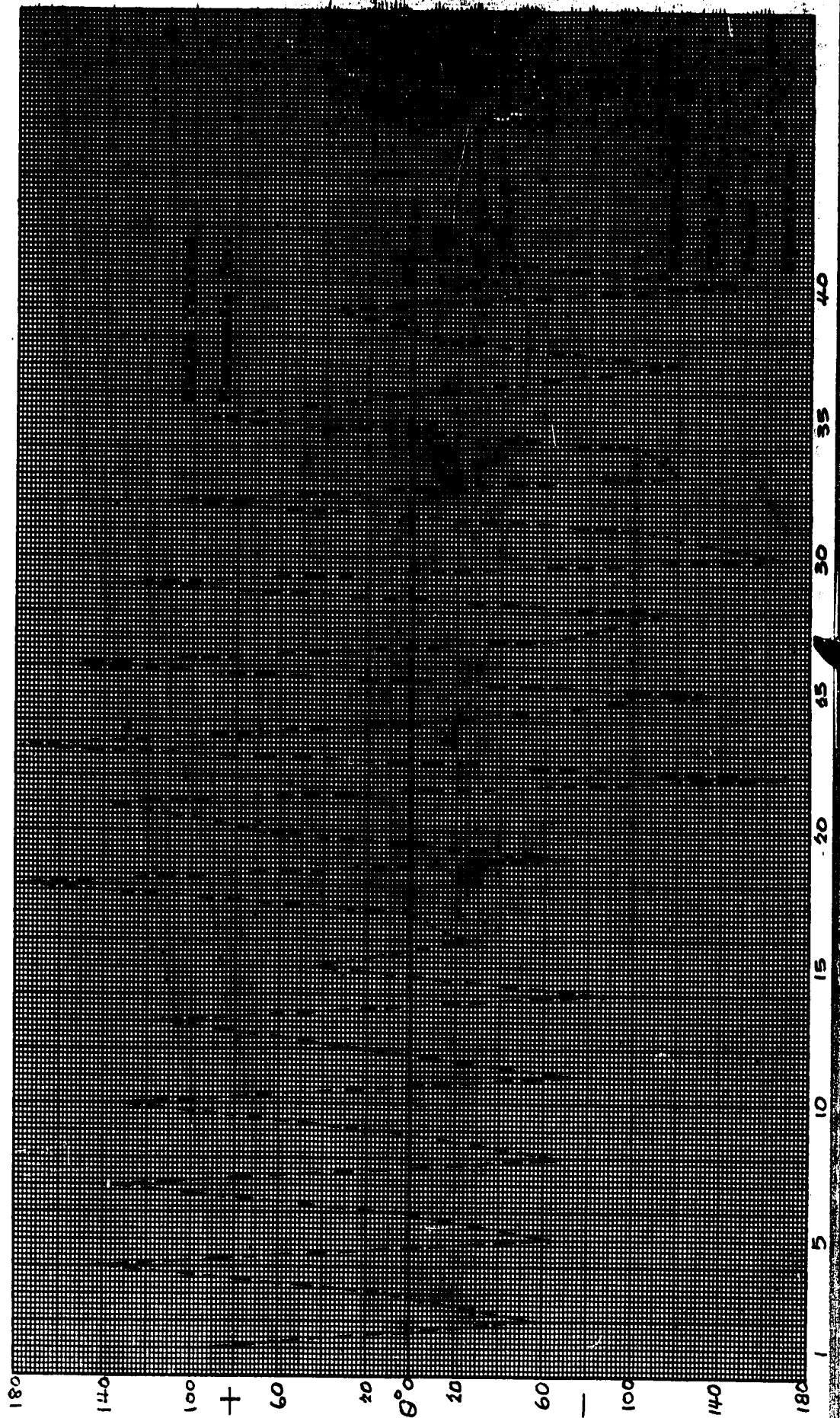


RAY

100-48-1-16-1-1-1
100-48-1-16-1-1-1



K-E 10 X 10 TO THE CENTIMETER 46 1513
10 X 25 CM. MADE IN U. S. A.
KEUPPEL & ESSER CO.



Phase Velocity

Harmonic No.

Case 11

Phase Velocity

Harmonic No.

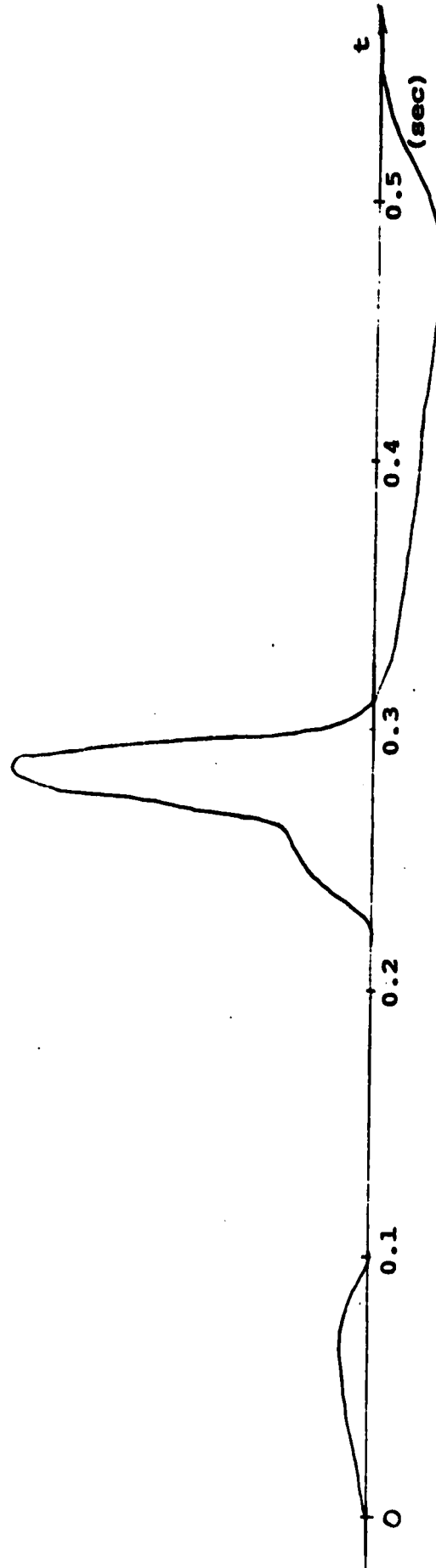
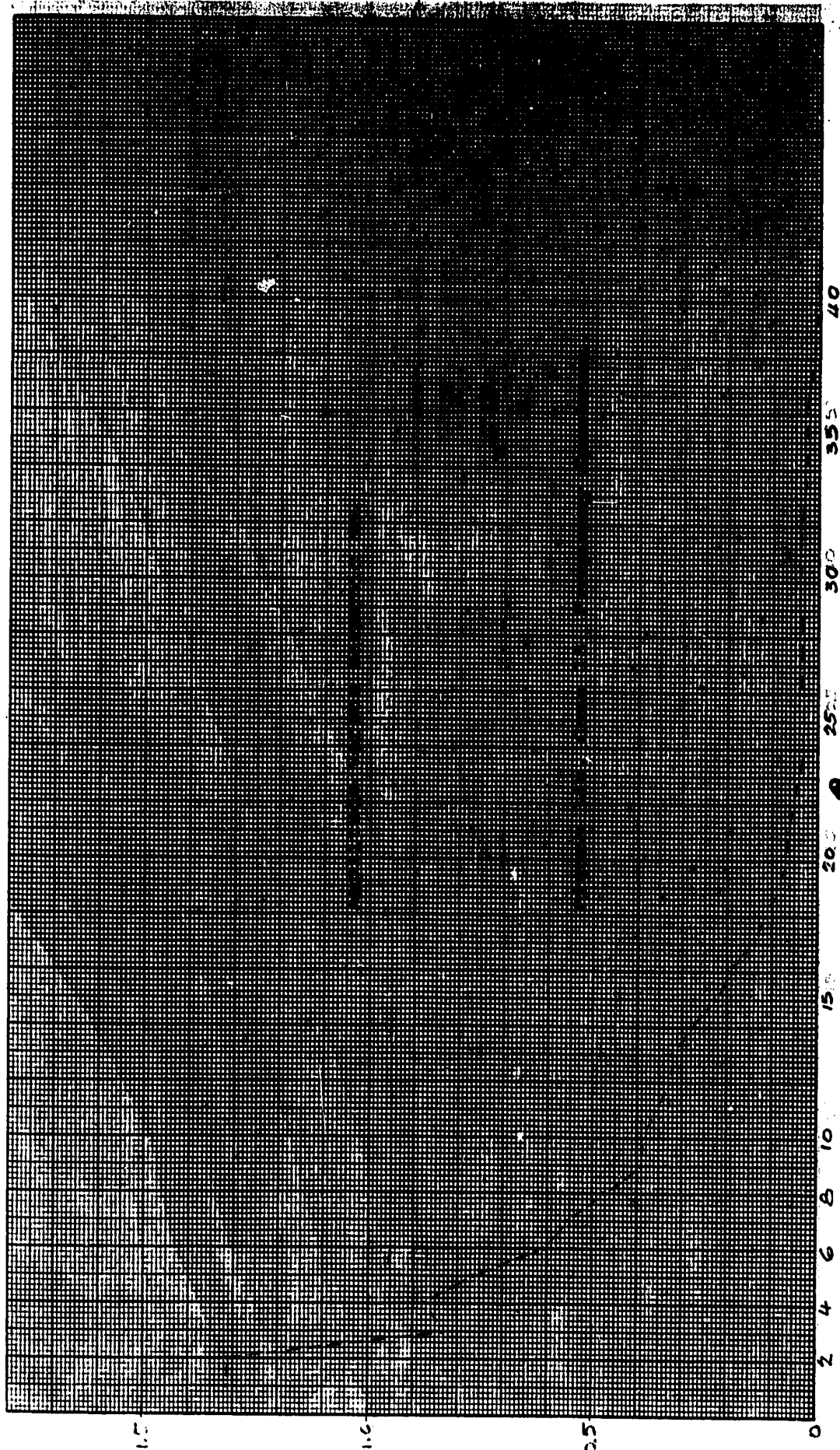
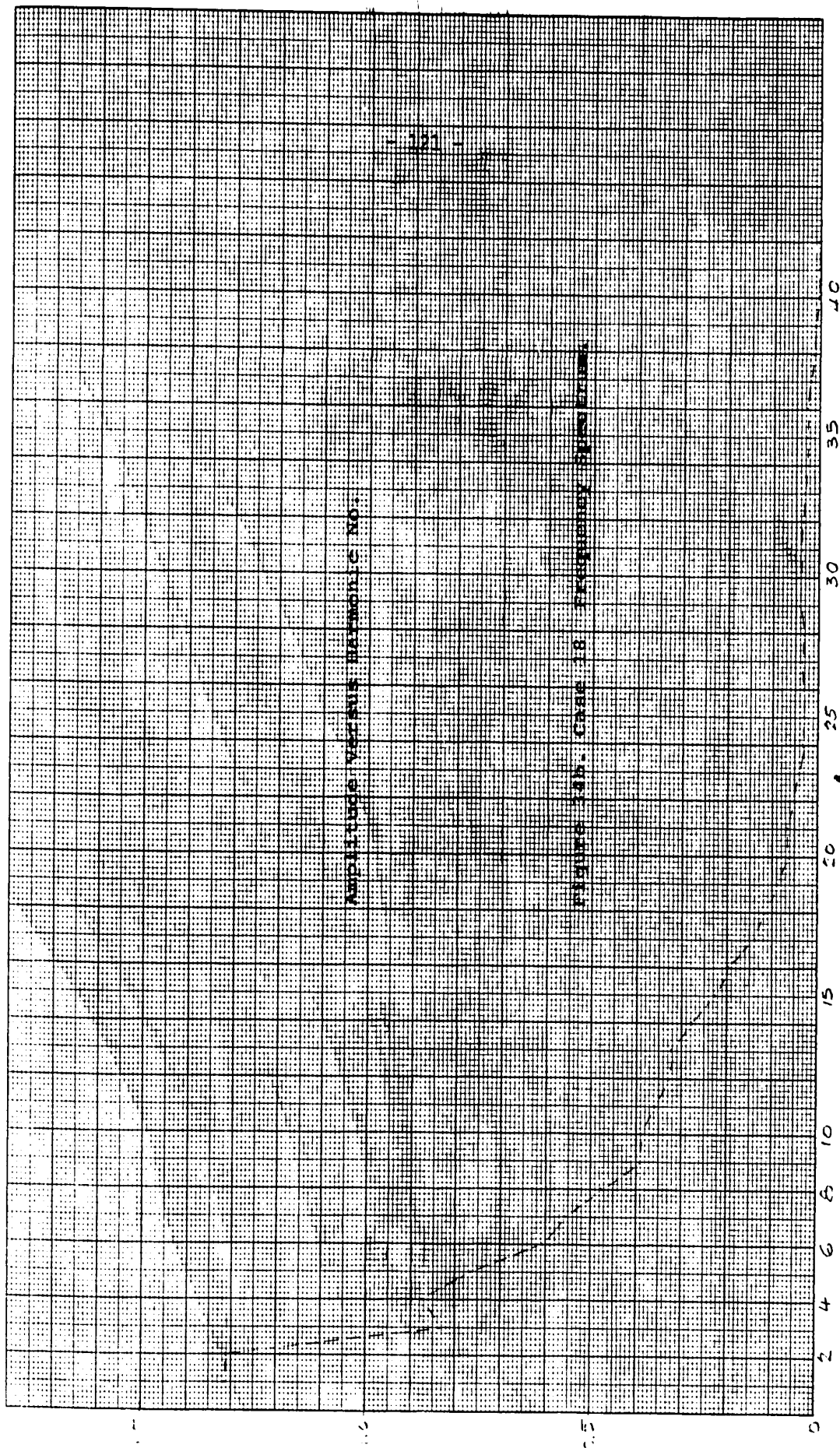


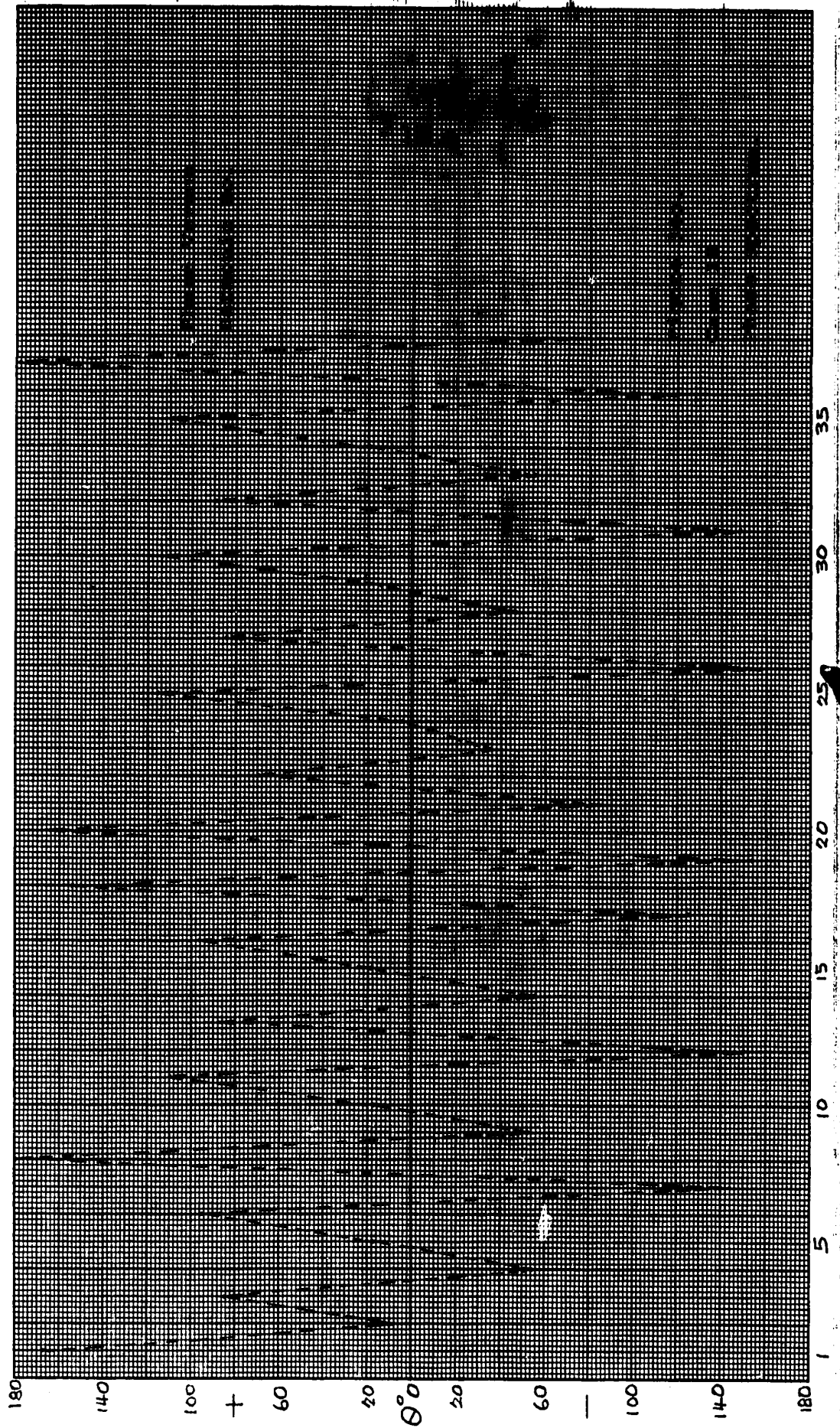
Figure 34a. Case 18 electrocardiogram.

K&W 10 X 10 TO THE CENTIMETER 46 1513
MADE IN U.S.A.
KEUFFEL & ESSER CO.

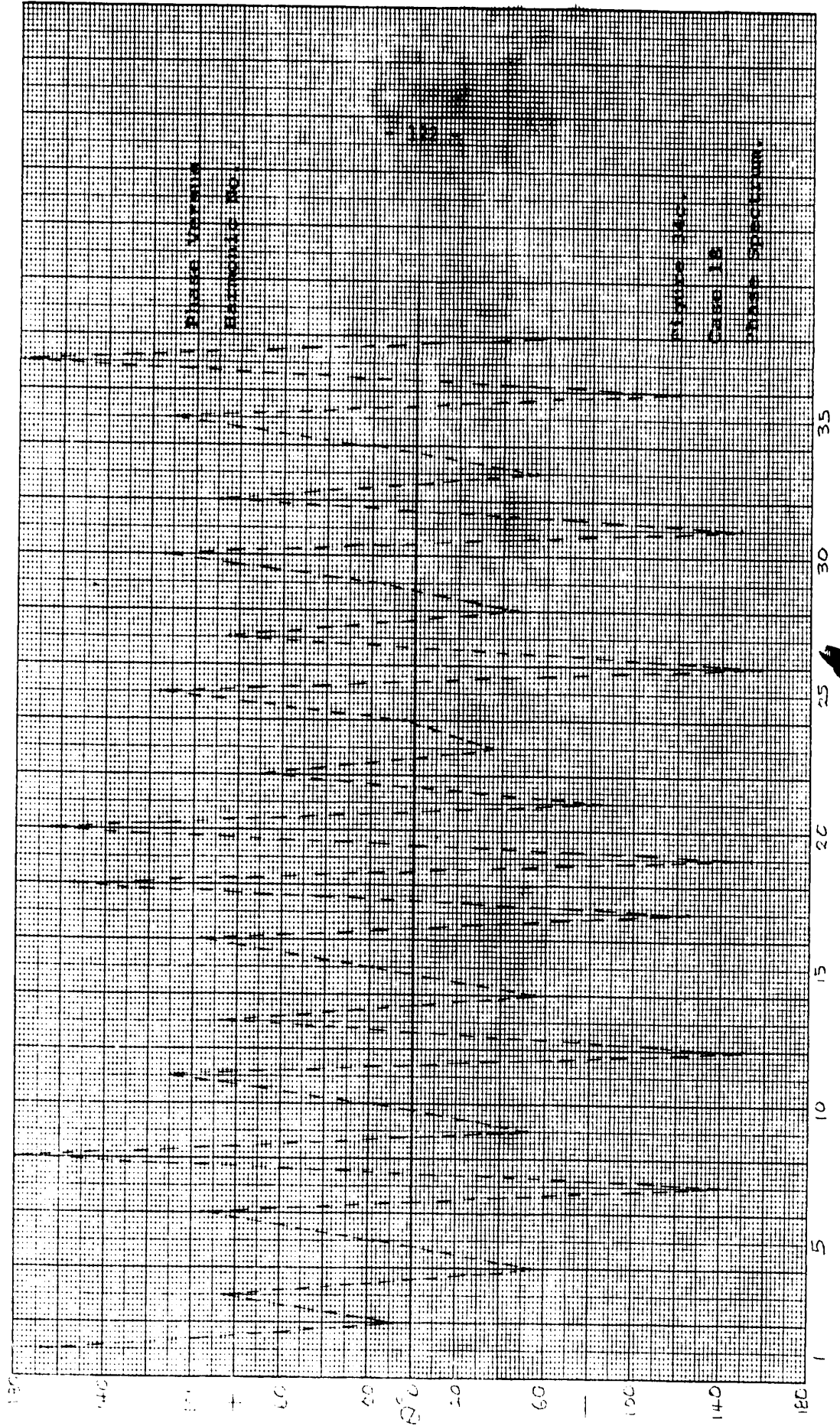




K-E 10 X 10 TO THE CENTIMETER 48 1513
10 X 25 CM. MADE IN U.S.A.
KEUFFEL & ESSER CO.



K-E 10.2 IN TO THE CENTIMETER 46 1513
NEUFEL & ESSER CO.



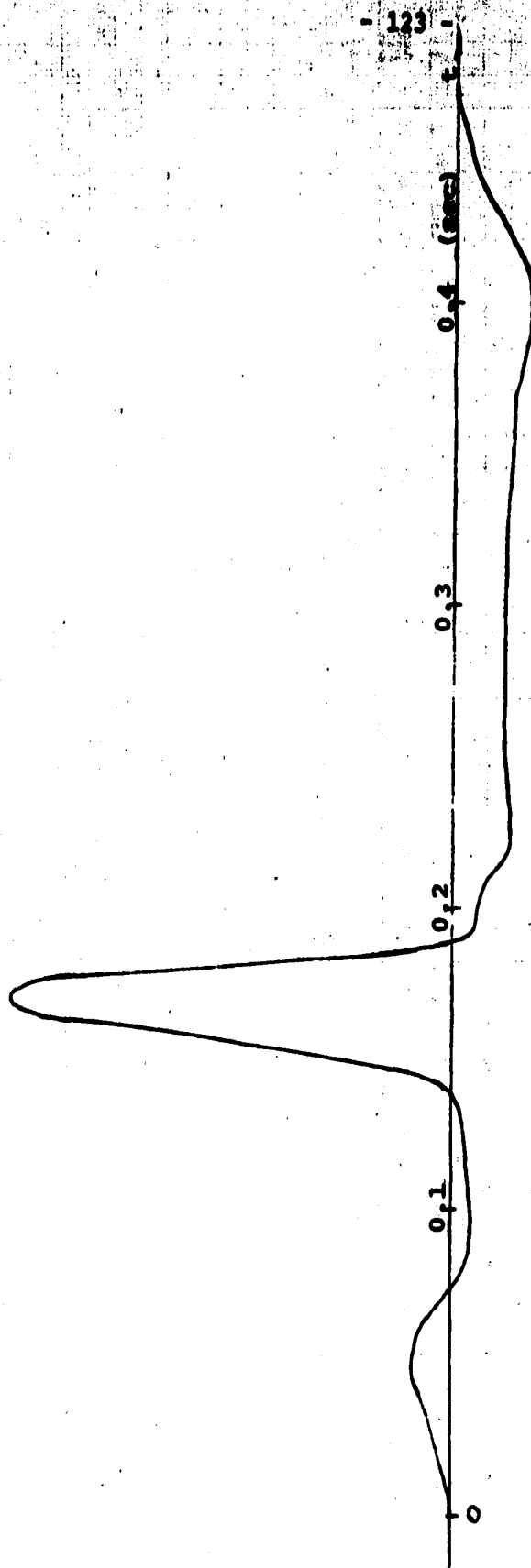
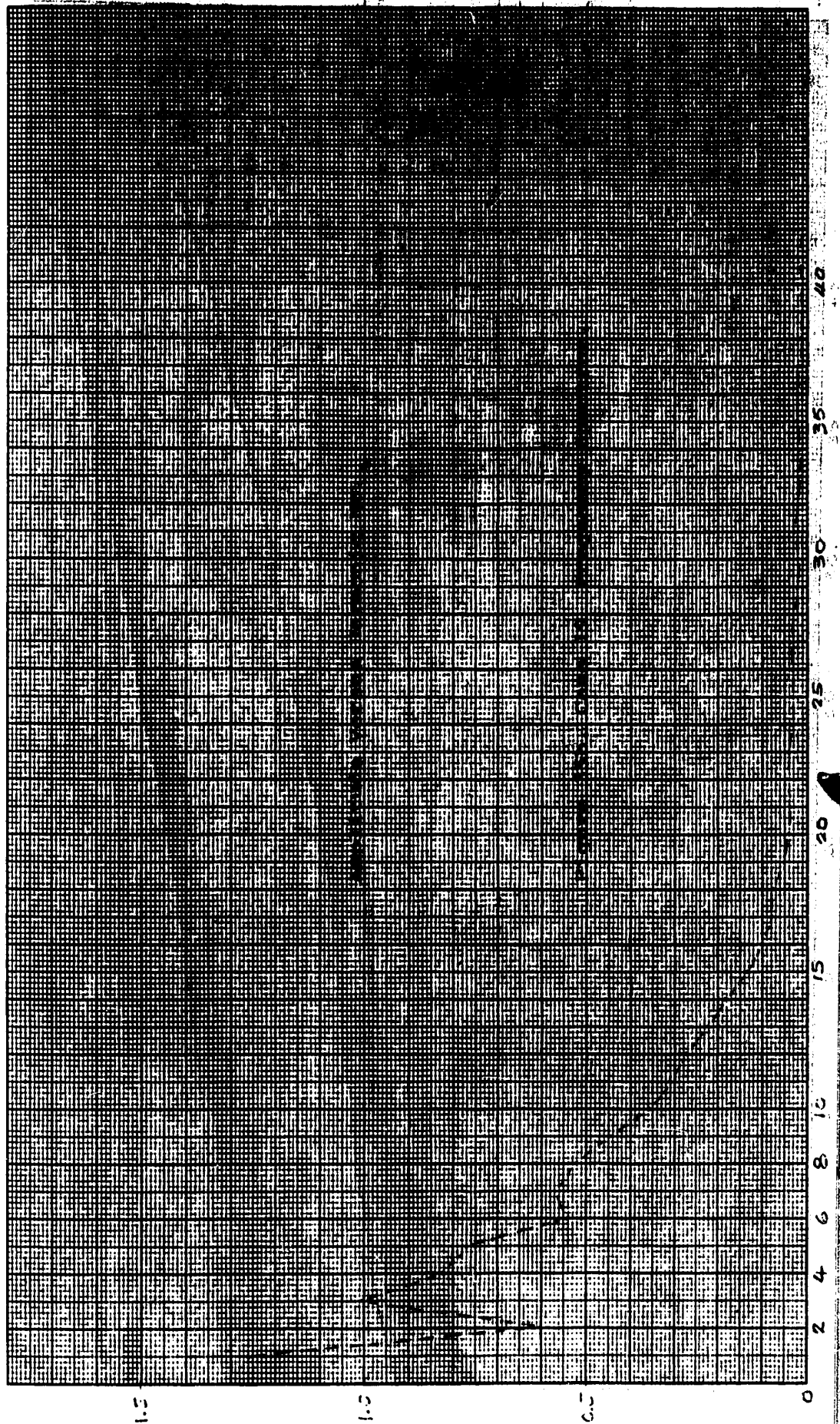
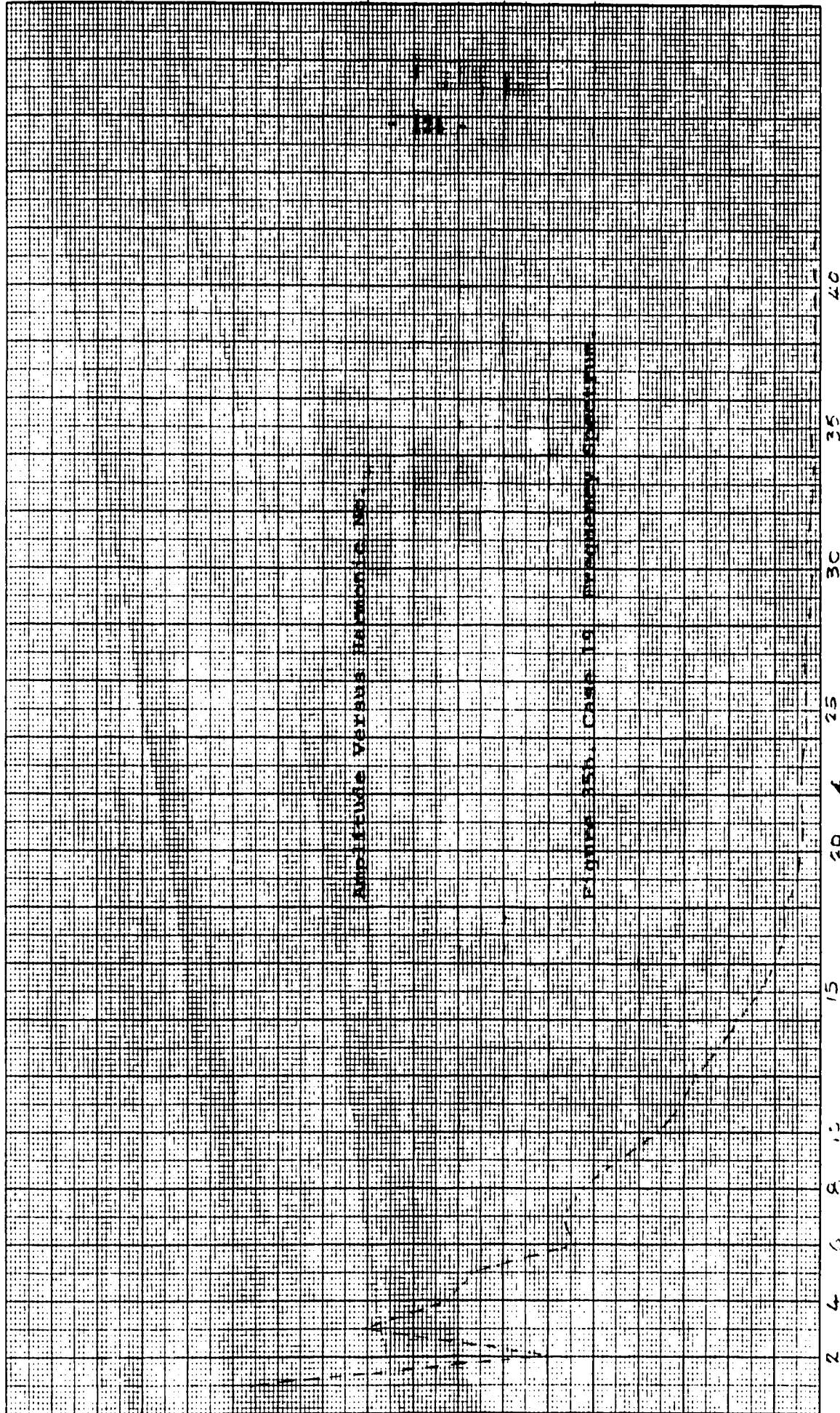


Figure 35a. Case 19 electrocardiogram.

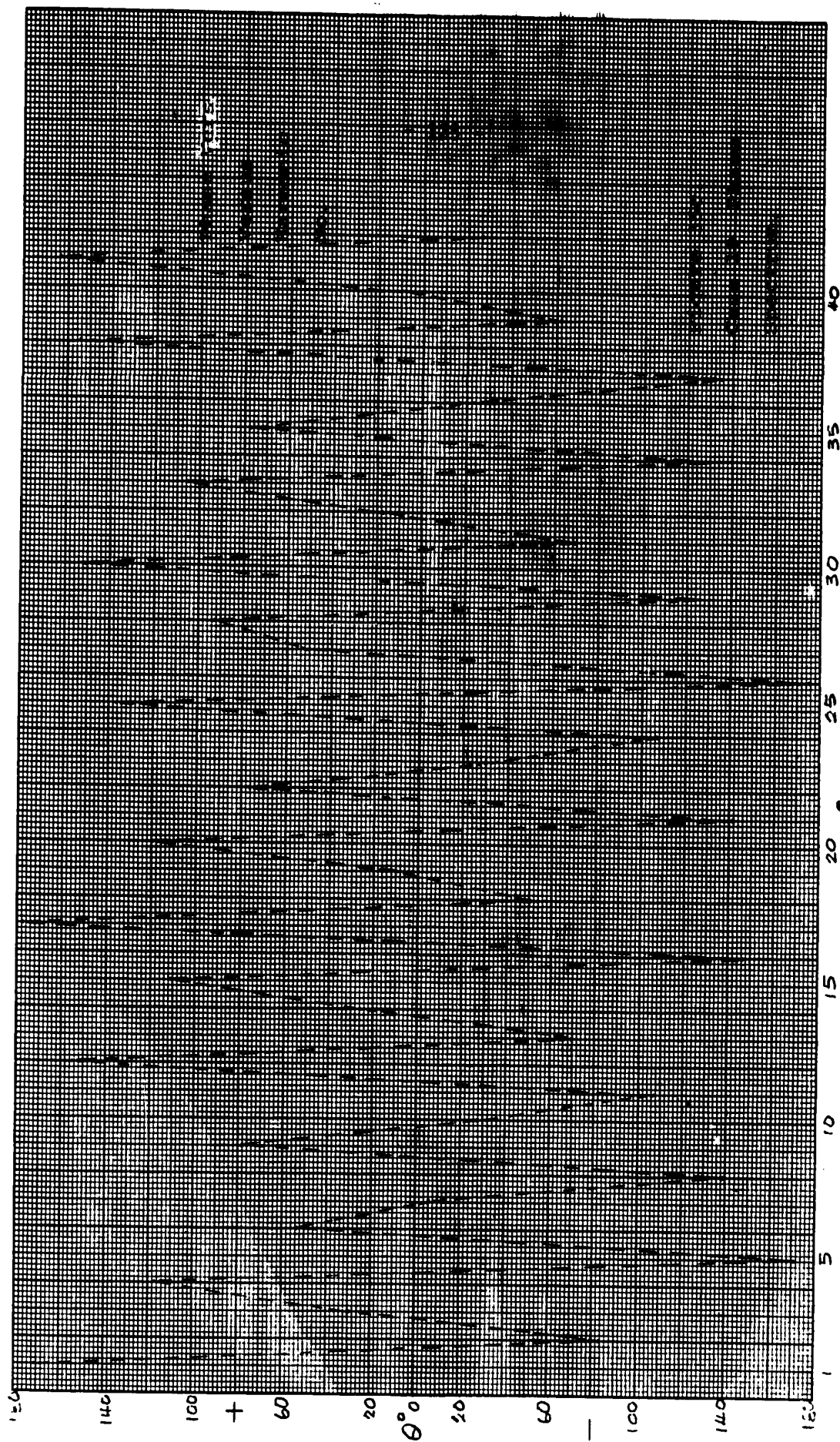
K-E 10 X 10 TO THE CENTIMETER 46 1513
10 X 25 CM.
MADE IN U.S.A.
NEUFFEL & ESSER CO.



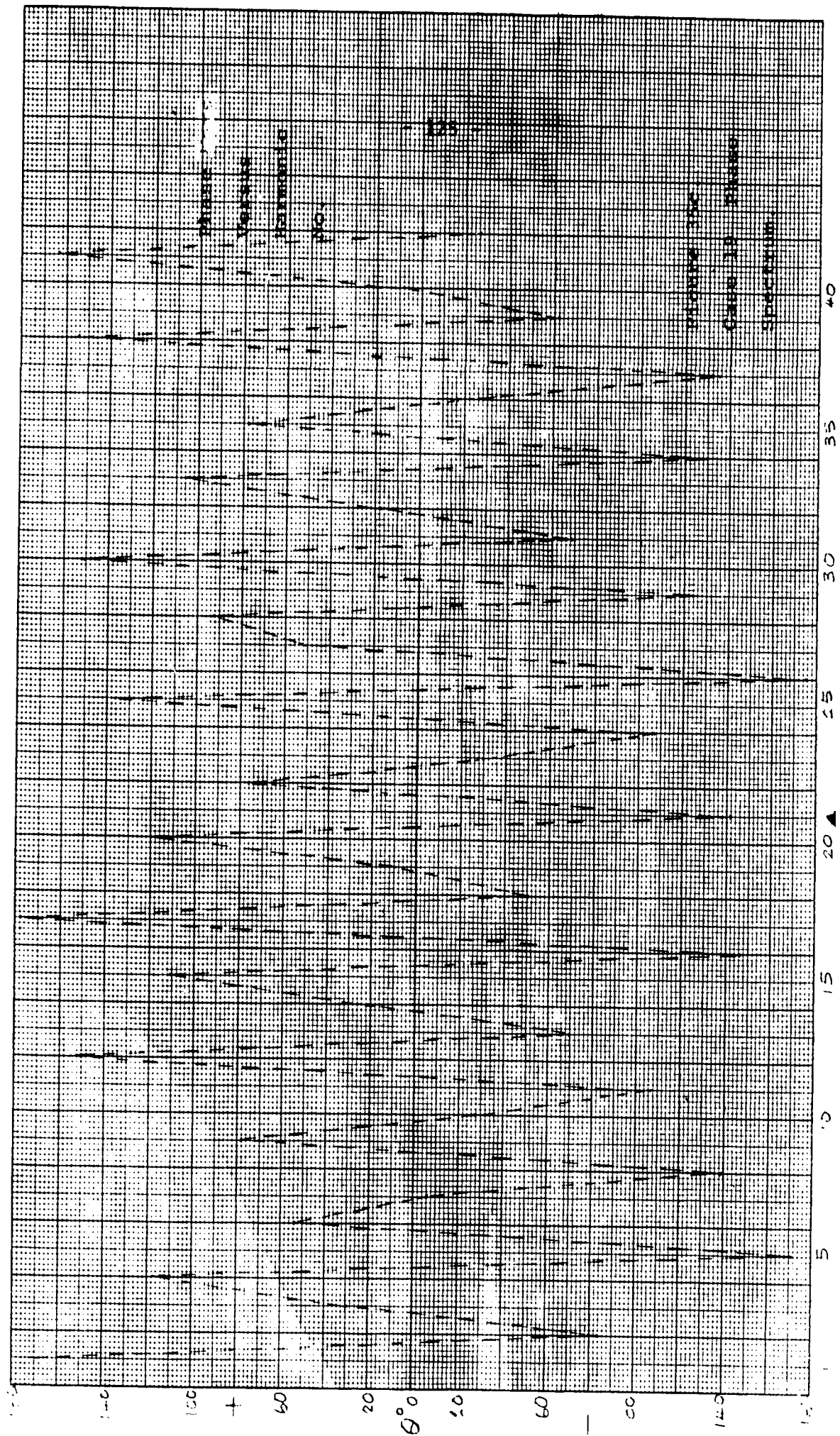
K-E 10 X 10 TO THE CENTIMETER 46 1513
10 X 25 CM
MADE IN U.S.A.
KEIFFEL & ESSER CO.



K-E 10 X 10 TO THE CENTIMETER 46 1513
18 X 25 CM.
MADE IN U.S.A.
KEUFFEL & ESSER CO.



KELUPPEL & EGGEN CO



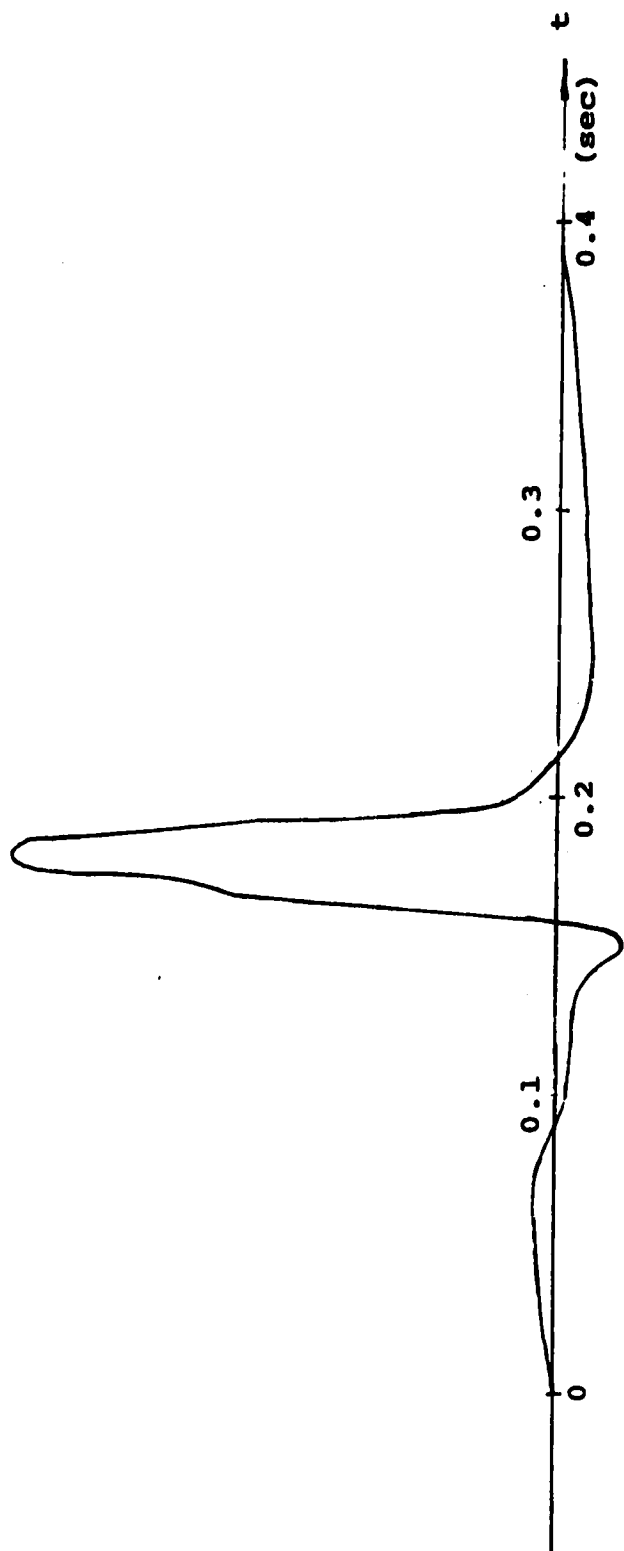
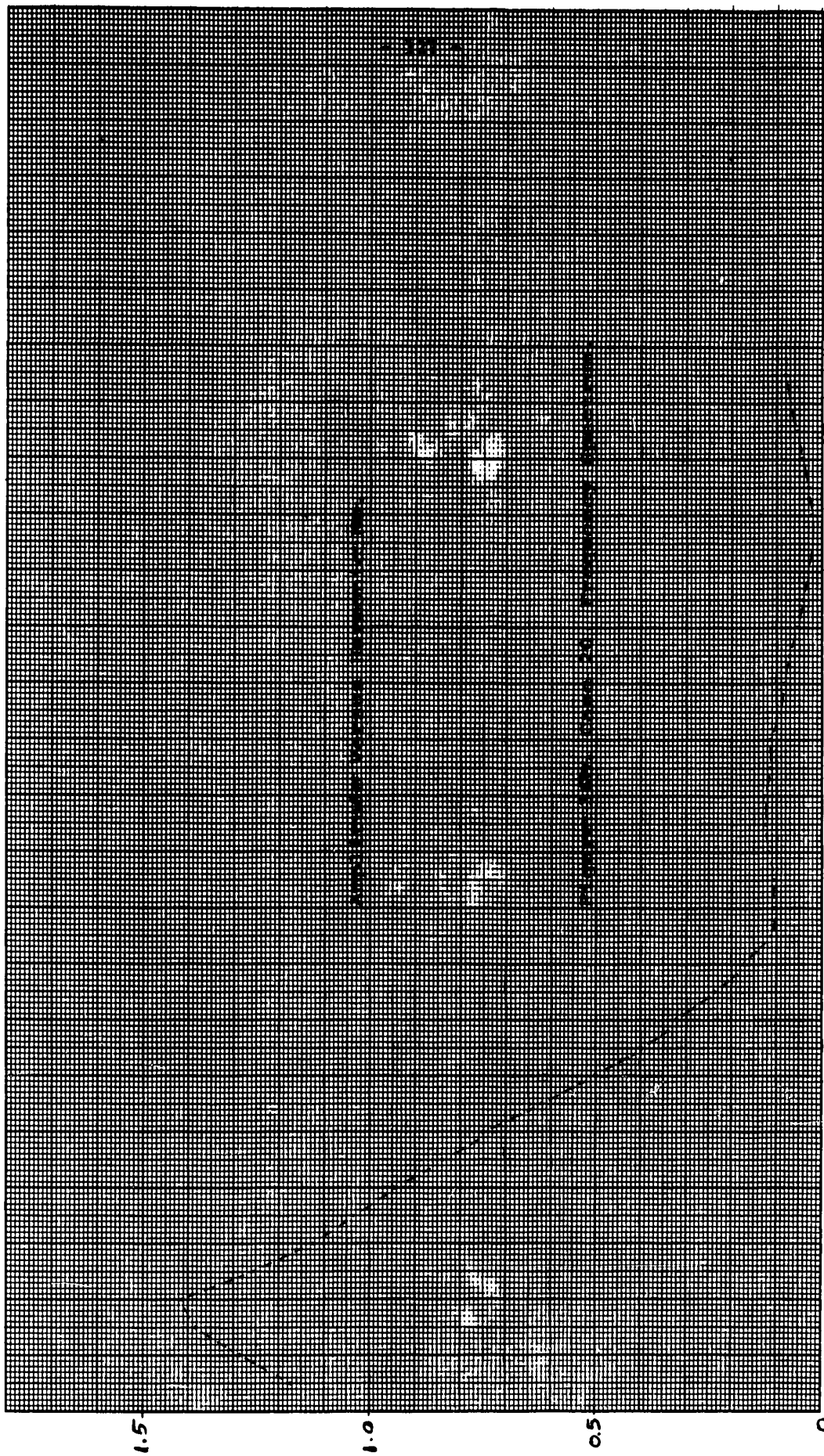
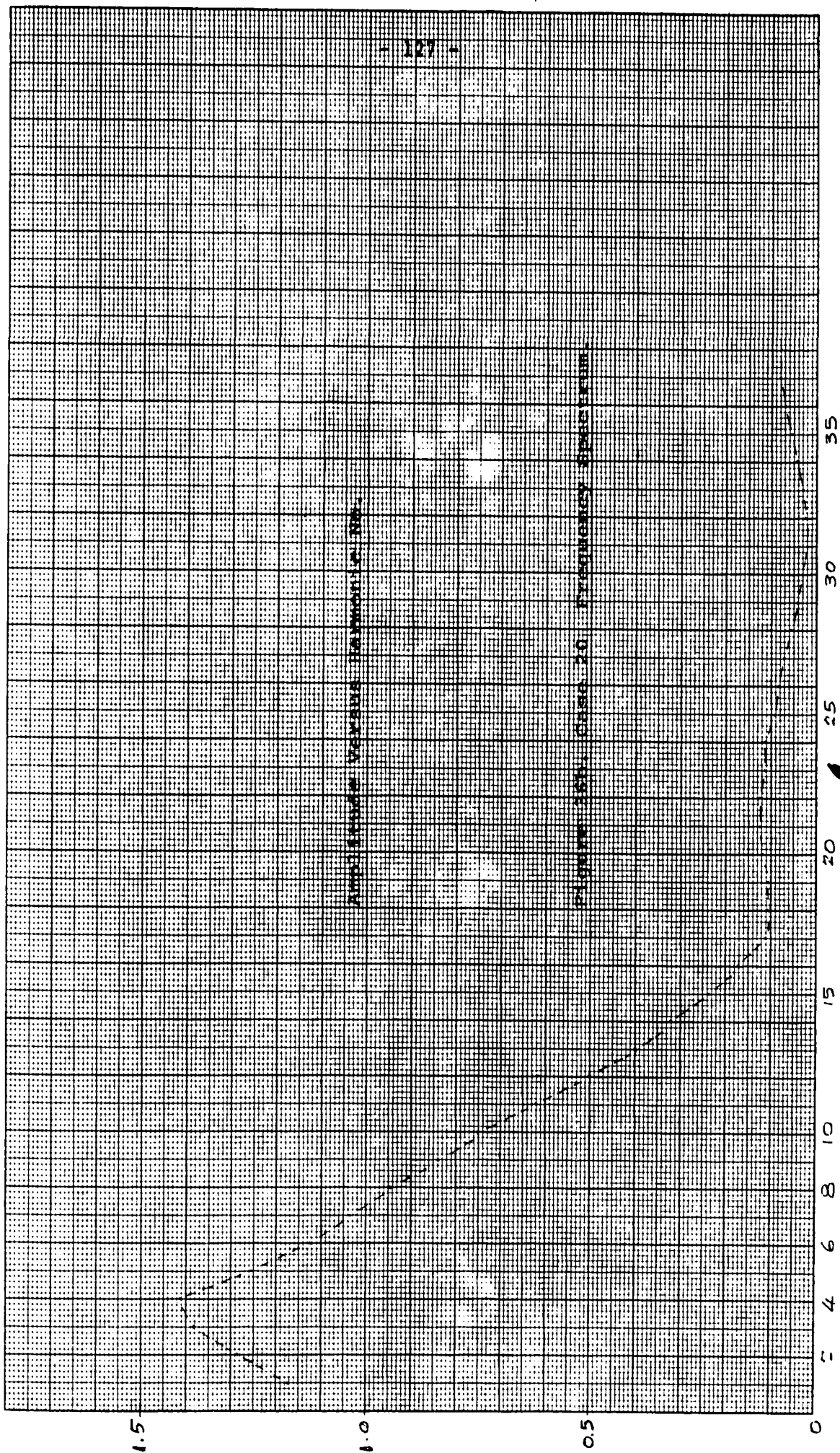


Figure 36a. Case 20 electrocardiogram.

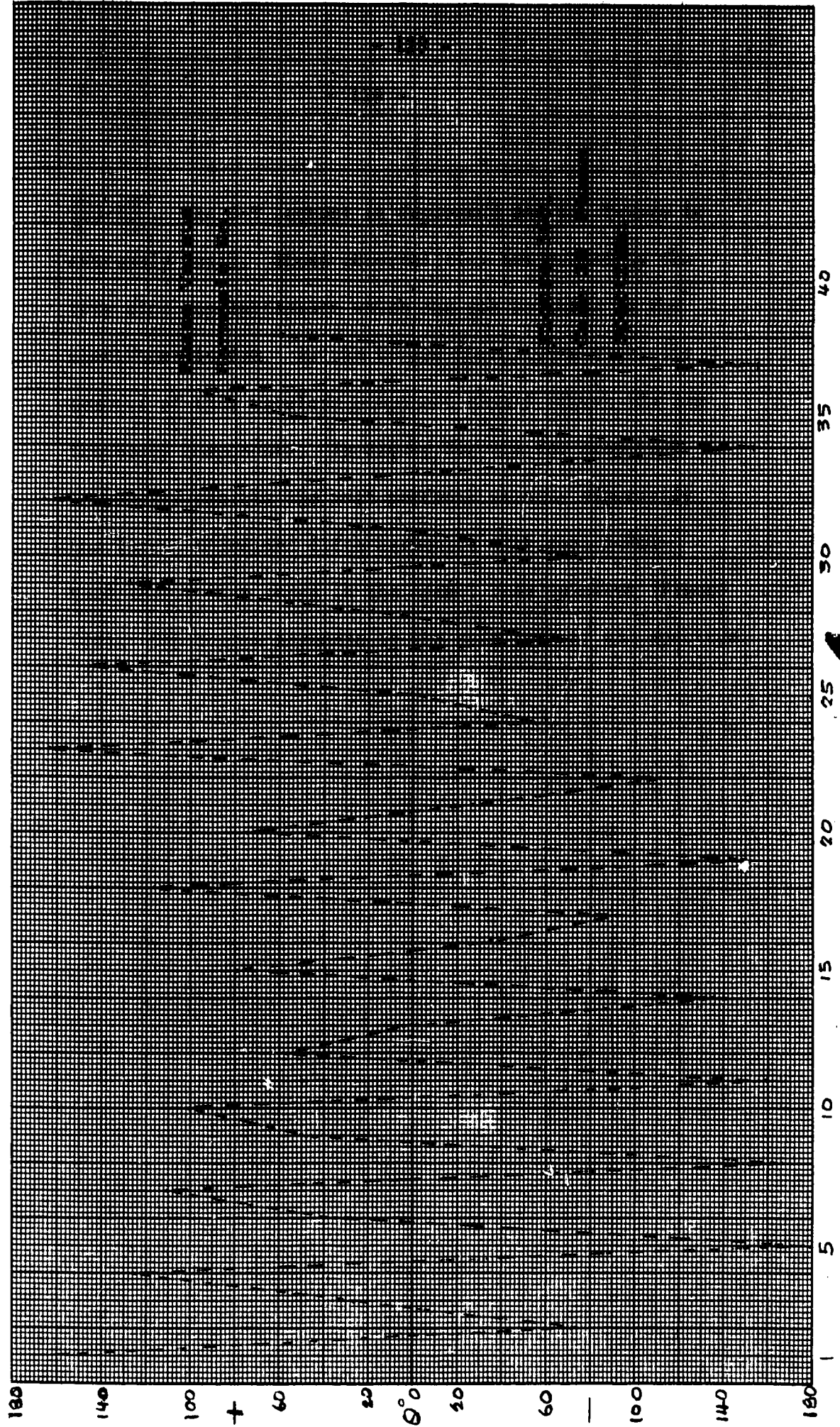
K+E 10 X 10 TO THE CENTIMETER 46 1513
MADE IN U.S.A.
KEUFFEL & ESSER CO.

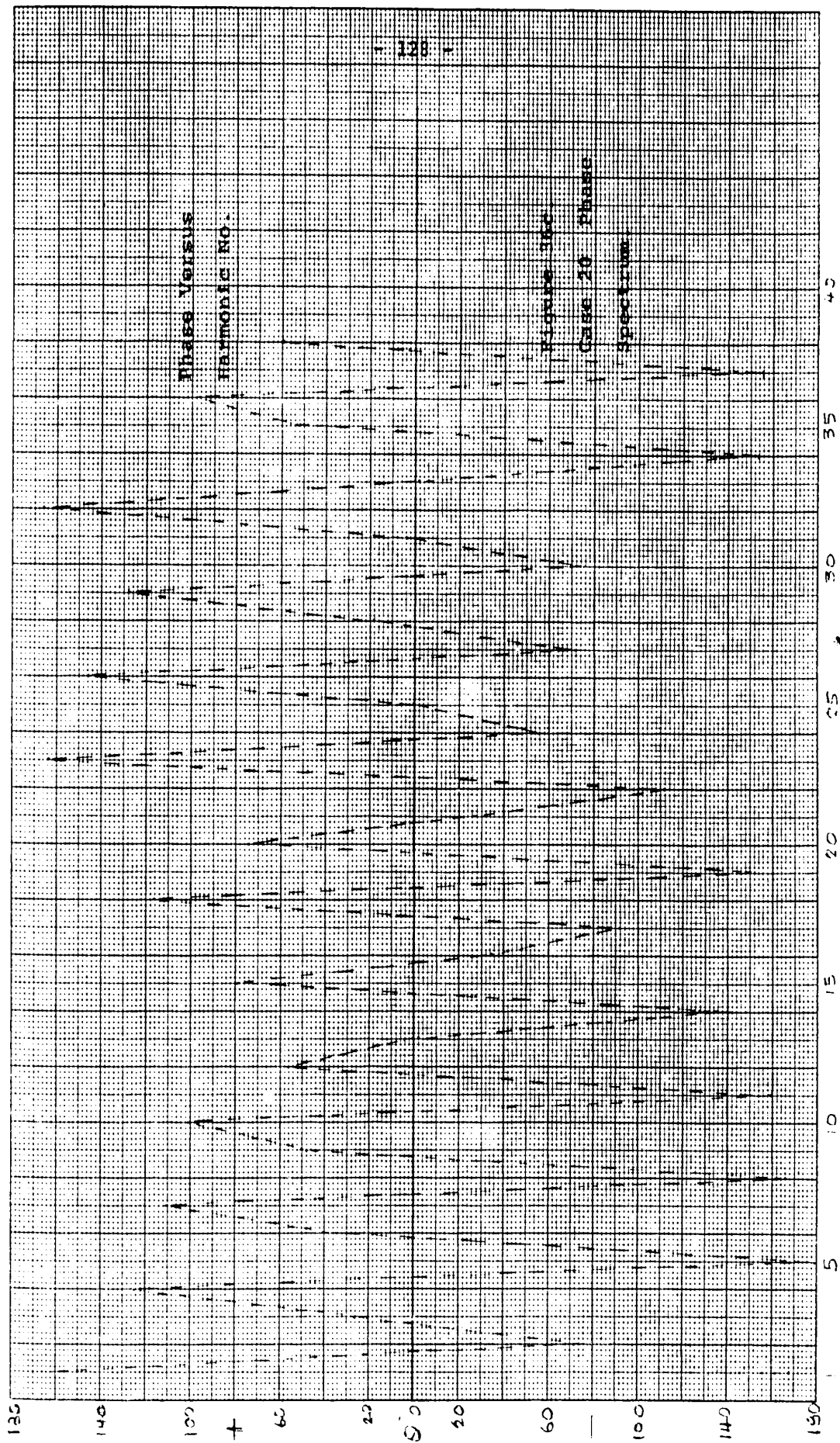


K-E 10A CUTO THE CENTIMETER 411110
 IN VACUUM
 NEUPILA ESEERCO



ROE 10 X 10 TO THE CENTIMETER 26 1513
MADE IN U.S.A.
KEUFFEL & ESSER CO.





[18-21]. The S-T segment is depressed and the T-wave is extremely small. These abnormalities are caused by the very slow repolarization which can not be shown in the electrocardiogram. As the T-wave normally is slow, the removal of this waveform component should affect the low frequency components. In Figure 36b the low frequency components exhibit the most profound changes especially the first, third and fifth harmonics with amplitudes much larger than those in the normal case spectrum (Figure 17b). The fourth harmonic is the only peak in the spectrum.

CONCLUSION

The normal and abnormal electrocardiograms associated with certain diseases vary with the individual case according to the age, sex, size of the body, the heart size and its position etc.

The Fourier analysis gives quantitative measures of the characteristics of the various periodic and random components present. It was not the purpose of this thesis to derive specific frequency domain criteria for the interpretation of actual electrocardiograms. This could be achieved by the application of Fourier analysis technique to a large amount of clinical records. If a statistically significant large sample of records were analyzed, then it would be possible to establish a relation between the variation of certain harmonics and various diseases.

The frequency amplitude spectrum of each available cardiac waveform has been found. The deviation of spectrum from the normal as related to the waveform characteristics, such as width, depth and duration of individual wave portion has been discussed.

Generally, the P-wave, when compared with the QRS complex and T-wave, is a small portion of the total waveforms, therefore small variation in the frequency amplitude

spectrum would not be so evident from this wave. If any variation does occur in the P-wave, it would most likely appear in the fourth harmonic since it is largely this harmonic which contributes to the P-wave. The T-wave contributes energy mostly in the first three harmonics and possibly the fifth harmonic. Any variation in T-wave would primarily affect these harmonics. The high frequency range components are mostly contributed by the steep and tall R-wave. If the R-wave is sharper and taller, the spectral envelope would decay more gradually, since it needs more power in the high frequency components to form such R-wave.

The significant maximal, minimal, and the largest harmonics are listed in Table 1. From this table one can see the approximate spectral variations in all the cases discussed. Table 2 lists the highest harmonic for half power and ninety percent power and fundamental frequency of the frequency spectrum. The first ten harmonics contribute between 78% to 99% of the total power and therefore are the most significant harmonics. From our limited study of samples of electrocardiogram records it appears that the harmonic analysis of an electrocardiogram may give some additional information about the disease involved. Hence we can say that harmonic analysis of the electrocardiogram provides an extra tool in the diagnosis

of heart diseases.

The phase frequency spectra of the cardiac waveforms were also computed. The phase relations need further investigation and a statistical study of the phase relation might yield useful information in detecting heart diseases.

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- 134 -

APPENDICES

Appendix 1: List of Cases Under Study

Case 1: Normal male, 26 years.

Case 2: Normal male, 30 years.

Case 3: Moderate left ventricular hypertrophy. Male
33 years. Renal hypertension.

Case 4: Moderate left ventricular hypertrophy. Male
45 years. Valvular aortic stenosis.

Case 5: Marked left ventricular hypertrophy. Female
43 years. Stenotic left ventricular subvalvular
myopathy.

Case 6: Marked left ventricular hypertrophy. Female 37
years. Idiopathic hypertrophy.

Case 7: marked right ventricular hypertrophy. Female
22 years. Pure valvular pulmonary stenosis.

Case 8: Marked right ventricular hypertrophy and dila-
tation. Female 37 years. Auricular septal de-
fect II type.

Case 9: Marked right ventricular hypertrophy. Male 62 years. Progressive occlusive pulmonary thrombosis.

Case 10: Right bundle branch block, wilson type. Male 53 years. Asymtomatic.

Case 11: Complete right bundle branch block. Male 56 years. Asymptomatic.

Case 12: Complete left bundle branch block. Male 57 years. No definite heart trouble.

Case 13: Complete left bundle branch block plus myocardial infraction. Male 76 years. Long standing history of anginal pain with several acute episodes.

Case 14: Postero-inferior myocardial infraction. Male 56 years.

Case 15: High posterior myocardial infraction. Male 40 years.

Case 16: Antero-lateral myocardial infraction. Male
43 years.

Case 17: Antero-inferior myocardial infraction. Male
60 years.

Case 18: Left ventricular wall infraction. Male 49
years.

Case 19: Anterior wall myocardial infraction. Male
63 years.

Case 20: Chronic coronary insufficiency. Male 63
years.

Appendix 2:

Table 1: A list of significant Maximal, Minimal and
Largest Amplitude Harmonics obtained from the
frequency spectra.

Case No.	Maximal Harmonics	Minimal Harmonics	Largest Amplitude Harmonics
1	2,4,7	1,3,5	2
2	2,7	1,5	2
3	2,4,6,27	1,3,5,19	2
4	2,4	1,3	2
5	1,3,5,23	2,4,18	3
6	1,3	2,22	3
7	1,3,5	2,4,17	1
8	1,4	3,15	1
9	1,3,5,10	2,4,8	1
10	2,4,6	1,3,5	4
11	2,4	1,3	2
12	2,8	1,6,12	2
13	1,8,16	7,14	1
14	2,4,6,9	1,3,5,8	2
15	2,4,6,21	1,3,5,15	2
16	1,3,6	2,5,20	1
17	2	1,17	2
18	1,4	3	1
19	1,3,7	2,6	1
20	4	1	4

Appendix 3:

Table 2: A list of Highest Harmonic for Fifty Percent Power and Ninety Percent Power, and Fundamental Frequency of the Frequency spectrum.

Case No.	Highest Harmonic For		Fundamental Frequency (H_z)
	Fifty Percent Power	Ninety Percent Power	
1	5	11	2.22
2	4	10	2.03
3	5	10	1.96
4	5	13	1.58
5	4	9	1.74
6	3	8	1.65
7	5	11	1.72
8	4	9	1.99
9	5	12	1.69
10	5	10	1.80
11	4	9	1.70
12	2	4	1.61
13	1	3	1.65
14	6	13	1.72
15	4	9	2.15
16	5	12	1.85
17	3	7	1.61
18	2	9	1.81
19	3	8	2.12
20	4	10	1.91

Appendix 4: Computer Program

FORTRAN IV G LEVEL 1, MOD 3

MAIN

DATE = 69083

		C	FREQUENCY DOMAIN ANALYSIS IN ELECTROCARDIOGRAMS
		0001	DIMENSION X(150),ANG(100),T(100),S(150), IC(150),HC(100),POW(100),PPOW(100),APPOW(100)
		0002	200 READ(1,10) NCASE,N,H
		0003	10 FORMAT(215,F12.10)
		0004	IF(N.EQ.999) GO TO 100
		0005	44 READ(1,20) (X(I),I=1,N)
		0006	20 FORMAT(10(F7.2))
		0007	A=N
		0008	FF=1./A*H
		0009	SX=0
		0010	DO 444 I=1,N
		0011	444 SX=SX+X(I)
		0012	ASX=SX/A
		0013	SAX=0
		0014	DO 441 I=1,N
		0015	441 SAX=SAX+ABS(X(I))
		0016	DO 555 I=1,N
		0017	X(I)=X(I)-SAX
		0018	555 X(I)=X(I)/SAX
		0019	L=(N/2)-1
		0020	DO 50 J=1,L
		0021	SUMS=0
		0022	SUMC=0
		0023	DO 40 I=1,N
		0024	AI=I
		0025	AJ=J
		0026	S(I)=X(I)*SIN(6.2832*AI*AJ/A)
		0027	C(I)=X(I)*COS(6.2832*AI*AJ/A)
		0028	SUMS=SUMS+S(I)
		0029	40 SUMC=SUMC+C(I)
		0030	POW(J)=(A*H/2.)*(SUMS*SUMS+SUMC*SUMC)
		0031	HC(J)=SQRT(POW(J)*2./(A*H))
		0032	T(J)=(SUMS)/(SUMC)
		0033	ANG(J)=(ATAN(T(J)))*(180.0/3.1416)
		0034	IF(SUMS) 13,14,14
		0035	13 IF(SUMC) 15,16,16
		0036	15 ANG(J)=-ABS(ANG(J))
		0037	GO TO 50
		0038	16 ANG(J)=-90-ABS(ANG(J))
		0039	GO TO 50
		0040	14 IF(SUMC) 17,50,50
		0041	17 ANG(J)=90+ABS(ANG(J))

FORTTRAN IV G LEVEL 1, MOD 3

MAIN

DATE = 69083

```
0042      GO TO 50
0043      50  CONTINUE
0044      SPOW=0
0045      DO 111 J=1,L
0046      111  SPOW=SPOW+POW(J)
0047      DO 222 J=1,L
0048      222  PPOW(J)=POW(J)*100.0/(SPOW)
0049      APPOW(1)=PPOW(1)
0050      DO 221 J=2,L
0051      221  APPOW(J)=APPOW(J-1)+PPOW(J)
0052      WRITE(3,60) NCASE,FF
0053      60  FORMAT(1H0,'CASE NO. '/(15,10X,F6.4))
0054      WRITE(3,80) (HC(J),J=1,L)
0055      80  FORMAT(1H0,'HARMONICS'/(5(2X,F6.3))/)
0056      WRITE(3,99) (APPOW(J),J=1,L)
0057      99  FORMAT(1H0,'A. % POWER'/(5(2X,F6.2))/)
0058      WRITE(3,90) (PPOW(J),J=1,L)
0059      90  FORMAT(1H0,'% OF TOTAL POWER'/(5(2X,F6.3))/)
0060      WRITE(3,70) (ANG(J),J=1,L)
0061      70  FORMAT(1H0,'ANGLE'/(5(2X,F7.2))/)
0062      GO TO 200
0063      100  RETURN
0064      END
```

REFERENCES

1. T.R. Taylor: "The Principles of Medical Computation", A.R. Mowbray and Co. Ltd., 1967.
2. H.V. Pipberger: "Automatic Analysis of the P-QRS-T Complex of the Electrocardiogram by Digital Computers", Annals of Internal Medicine, Vol. 57, PP. 776-787, 1962.
3. L.F. Staples, J.E. Gustafson, G.J. Baln and A. Tate: "Computer Interpretation of Electrocardiogram", American Heart Jurnal Vol. 72, PP. 351-358, 1966.
4. R.J. Dobrow and P.A. Gorman: "Accuracy of Electrocardiographic Measurements by Computer", American Jurnal of Med. Electronic Vol. 4, 1965, PP. 121-126.
5. H.V. Pipberger: "Use of computer in ECG Interpretation", American Heart Jurnal, Vol. 64, PP. 285-286, 1962.
6. P.W. Duchosal: "Thirty Synoptic Electro-Vectorcardiograms", Symposium on Advanced Electrocardiography and Vector-cardiography, March 1964.

7. C.A. Caceres: " Computer Extraction of Electrocardiographic Parameters", Circulation Vol. 25, PP. 356-362, 1962.
8. H. V. Pipberger: " Automatic Screening of Normal and Abnormal Electrocardiograms by Means of a Digital Electronic Computer", Society for Experimental Biology and Medicine Proceedings" Vol. 106, PP. 130-132, 1961.
9. F. W. Stallmaum: "Automatic Recognition of Electrocardiographic Waves by Digital Computer", Circulation Research, Vol.9, PP. 1135-1143, 1961.
10. H. V. Pipberger: "Use of Computers in Interpretation of Electrocardiograms", Circulation Research Vol.2 PP. 555-562, 1962.
11. J. V. der Groeben: "The Spatial Frequency Distribution of the QRS Loop as Studied on 154 Normal Individuals", American Heart Journal Vol.59, PP. 875-891, 1960.

12. L. D. Cady and L.J. Tick: "A Method for Electrocardiogram Wave-Pattern Estimation", Circulation Research Vol.9, PP. 1078-1082, 1961.
13. P. H. Langer: "Characteristics of the Frequency Spectrum in the Normal Electrocardiogram and in Subjects following Myocardial Infraction", Research Circulation Vol.8, PP. 577-584, 1960.
14. P. H. Langer: "Further Studies In High Fidelity Electrocardiography: Myocardial Infraction", Circulation Vol.8, PP. 905-913, 1953.
15. P.H.Langer: "The Value of High-frequency Electrocardiography", Medical Dept. of the Provident Mutual Life Insurance Co. of Philadelphia, 1963.
16. A. M. Scher: "Frequency Analysis of the Electrocardiogram", Circulation Research Vol.8, PP. 344-346, 1960.
17. N. P. Thompson: "Fourier Analysis of the Electrocardiographic Function", American Journal of Medical Electronic, Vol.1, PP. 299-307, 1961-1962.

18. J. E. F. Riseman: "A Guide to Electrocardiogram Interpretation". The McMillan Co., 1960.
19. M. J. Goldman: "Principles of Clinical Electrocardiography" Lange Medical Publications, 1962.
20. L. E. Lamb: "Electrocardiography and Vectrocardiography", W. B. Saunder Co., 1965.
21. J. J. L. Marriott: "Practical Electrocardiography" The Williams and Wilkins Co., 1954.
22. E. Massie: "Clinical Vectrocardiography and Electrocardiography". Year Book Publishers, 1960.
23. D. Sodi-Pollares: "New Bases of Electrocardiography". The C.V. Mosby Co., 1956.
24. H. E. B. Pardee: "Clinical Aspects of the Electrocardiogram" Paul B. Hoeber Inc., 1948.
25. E. Lipeschkin: "Modern Electrocardiography". The Williams & Wilkins Co., 1965.
26. G. S. Glinski, F. Berkman, N. S. Bobey:
"Computers in Electrocardiogram Analysis and Diagnosis" Technical Report No. 67-6, Electrical Engineering Department, University of Ottawa, May 1967.
27. G. S. Glinski, F. Berkman and S. I. Ahmad:
"Direct and Inverse Solutions for Electrical Activity of the Heart with multiple Dipole

Model". Digest of the 7th International Conference on Medical and Biological Engineering, P409, 1967.

28. R. W. D. Turner: "Electrocardiography" E. S. S. Livingstone Ltd., 1963.
29. R. M. Bracwell: "The Fourier Transform and the Applications". McGraw Hill, 1965.
30. M. E. V. Valkenburg: "Network Analysis", Printic Hall Inc., 1964.
31. Ruel V. Churchill: "Fourier Series and Boundary Value Problem" McGraw-Hill Book Co. Inc., 1941.
32. H. S. Garslaw: "Fourier Series and Integrals" MacMillan and Co. Ltd., 1930.
33. J. S. Bendat: "Measurement and Analysis of Random Data", John Wiley and Sons Inc., 1967.
34. A. Papoulis: "Probability, Random Variables, and Stochastic Processes", McGraw-Hill Book Co., 1965.
35. R. B. Blackman: "The Measurement of Power Spectra". Dover Publication, Inc., 1958.
36. E. de Boer: "Triggerd Correlation" IEEE Transaction, Vol. BME-15, No. 3 PP169-179, July, 1968.
37. W. W. Wierwille: "Off Line Correlation Analysis of Nonstatinary Signals" IEEE Transaction, Vol. C-17, No. 6 PP525-536, June, 1968.

38. P. Raoult: "Correlation Function and Fourier Analysis" Digest of the 1961 International Conference on Medical Electronics, Page 237, July 1961.
39. Harry Andrews: "A high-speed Algorithm for the Computer Generation of Fourier Transforms". IEEE transactions on Computers, April 1968.
40. W. T. Cochran, J. W. Cooley, D. L. Favin, H. D. Helms, R. A. Kaenel, W. W. Lang, G. C. Maling, D. E. Nelson, C. M. Rader, and P. D. Welch:
"What is the Fast Fourier Transform?" IEEE Transactions on Audio and Electroacoustics Vol. Au-15, No. 2, June 1967.
41. J. W. Cooley, P. A. W. Lewis and P. D. Welch:
"Application of the Fast Fourier Transform to Computation of Fourier Integrals, Fourier Series, and Convolution Integrals". IEEE Transactions on Audio and Electroacoustics, Vol. Au-15, No. 2, June 1967.
42. E. O. Brigham and R. E. Morrow: "The fast Fourier transform" IEEE Spectrum December, 1967.
43. J. W. Cooley, P. A. W. Lewis, and P. D. Welch:
"Historical Notes on the Fast Fourier Transform". IEEE Transaction on Audio and Electroacoustics, Vol. Au-15, No. 2, June, 1967.

44. R. C. Singleton: "A Method for Computing the Fast Fourier Transform with Auxiliary Memory and Limited High-Speed Storage." IEEE Transactions on Audio and Electroacoustics, Vol. Au-15, No. 2, June, 1967.
45. M. Schwartz: "Information Transmission Modulation, and noise". McGraw-Hill Book Co., 1959.
46. B. P. Lathi: "Signals, Systems and Communication John Wiley & Sons Inc., 1967.
47. Syed I. Akmad: "Mathematical Models for Direct and Inverse Solutions of the Electrical Activity of Heart". Technical Report No. 67-8, Electrical Engineering Department, University of Ottawa, July, 1967.
48. E. K. Franke: "Study of High Frequency Components in Electrocardiogram by Power Spectrum Analysis". Circulation Research, Vol. 10, PP 870-879m 1962.
49. A. J. Kerwin: "The Effect of the Frequency Response of Electrocardiographs on the Form of Electrocardiograms and Vectrocardiograms".

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