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Finite-Element Simulation of Ram Pipe Bending

by

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ABSTRACT

The present study is concerned with the simulation of the ram-pipe bending process using the finite element method (FEM). Investigated are springback, ovaliza tion, and growth of plastic zones in the pipe bend. Residual stresses are also calculated for certain parts of the pipe. Simulation is restricted to bends of small angles.

A nonlinear FEM program is developed, utilizing the Updated Lagrangian Jaumann Stress-Rate Formulation. The solution is carried out with the modified Newton-Raphson method and the Profile Solver technique. The formulation of the el-bow element developed earlier in [3] – [5] is extended to model a forming problem (ram pipe bending). To check the accuracy of the program, a number of piping problems are solved, covering both elastic and nonelastic phenomena. Present FEM calculations have been found to be in good agreement with available experimental data and other numerical results.

The above FEM program is then used to obtain the results for the ram-pipe problem. The FEM results are compared with those calculated using an elastic-plastic beam theory. Satisfactory agreements have been obtained between the two sets of results. To further investigate the maximum ovalization of the cross section due to local deformatiion, ADINA shell-element predictions are also given. A number of charts of practical interest are presented for typical sizes of pipe bends formed by ram benders.

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Nomenclature

A_1	half the total area of the plastic zones
A_2	half the total area of the elastic zones
b	distance from one end of a beam to the nearest elastic-plastic boundary
c	width of a ram (in ram benders)
c_i^k	ith ovalization displacement at nodal point k
d	$2r_2$
D	$2R$
e	E/E_T
E	Young's modulus
E_T	strain-hardening modulus
f_γ	flexibility factor
f_d	diameter change factor
h	wall thickness of a pipe
h_k	interpolation function at nodal point k
H^*	strain-hardening constant
I	moment of inertia of a cross section
k	non-dimensional curvature, defined as $\frac{\kappa}{\kappa_y}$
L	length of a straight beam or pipe
m	non-dimensional applied moment, defined as $\frac{M}{M_y}$
M	applied bending moment
r	isoparametric coordinate (along the element length)

r	r_1/r_2
r_m	mean radius of cross section
r_1, r_2	inside radius and outside radius of cross section, respectively
r_1^k	outside radius of cross section at nodal point k
R	radius of curvature
s	isoparametric coordinate (toward the center of curvature)
S_{ij}	components of the deviatoric stress tensor
t	isoparametric coordinate (orthogonal to the r- and s- directions)
u_i	displacement at nodal point k in the x_i direction
v	end vertical displacement
w_ξ, w_ζ	circumferential and radial displacements of the middle surface of pipe wall, respectively
W	applied concentrated load
W_y	applied concentrated load at first yielding
x	coordinate measured from one end of a pipe (or beam)
x_i	Cartesian coordinates, $i = 1, 2, 3$
y	vertical Cartesian coordinate
z	distance from the neutral axis to the elastic-plastic boundary
η	pipe-wall local coordinate (along the center line)
δ	deflection
γ	end rotation
$\gamma_{\xi\eta}$	shear strain in the $\xi\eta$-plane
$\gamma_{\eta\zeta}$	shear strain in the $\eta\zeta$-plane
$\gamma_{\zeta\xi}$	shear strain in the $\zeta\xi$-plane
$\epsilon_{\xi\xi}$	hoop (circumferential) strain
$\epsilon_{\eta\eta}$	longitudinal strain

$\epsilon_{\zeta\zeta}$	radial strain
ϵ_{ij}	components of the strain tensor
κ	curvature
κ_y	curvature at first yielding
λ	pipe geometric parameter
ν	Poisson's ratio
θ	positional angle (along a pipe bend)
θ_i^k	rotation at nodal point k about the x_i-axis
ρ	polar coordinate in the radial direction
σ_y	uniaxial yield stress (proportional limit)
${}^t\sigma_y$	uniaxial yield stress at time t
$\sigma_{\xi\xi}$	hoop (circumferential) stress
$\sigma_{\eta\eta}$	longitudinal stress
σ_{ij}	component of a stress tensor
${}^t\tau_{ij}$	components of the Cauchy stress tensor, referred to time t
ϕ	positional angle (around the cross-section periphery) measured from the intrados
ξ	pipe-wall local coordinate (in the hoop direction)
ζ	pipe-wall local coordinate (in the radial direction)
${}_t^tB_L$	linear strain-displacement transformation matrix at time t
${}_t^tB_{NL}$	nonlinear strain-displacement transformation matrix at time t
$\hat{B}_{OV}$	strain-displacement transformation matrix for ovalization
C^E	linear stress-strain matrix
${}^tC^{EP}$	instantaneous elastic-plastic stress-strain matrix
${}_{t+\Delta t}^{t+\Delta t}F^{(i-1)}$	vector of nodal point forces equivalent to element stresses at time $t+\Delta t$ and in iteration $(i-1)$
J	Jacobian transformation matrix

${}_t^tK_L$	linear strain incremental stiffness matrix
${}_t^tK_{NL}$	nonlinear strain incremental stiffness matrix
$\boldsymbol{P}$, $\boldsymbol{Q}$	transformation matrices for second-order tensors
${}^tq$	vector containing deviatoric stress components at time t
${}^{t+\Delta t}\boldsymbol{R}$	vector of externally applied nodal point loads at time $t + \Delta t$
$\boldsymbol{\Delta U}^{(i)}$	vector of increments in nodal point displacements in iteration i
$\boldsymbol{V}_s^k$, $\boldsymbol{V}_t^k$	unit vectors at nodal point k in the s- and t-directions, respectively

Chapter 1

Introduction

1.1 Finite-Element Modelling in Metal Forming

Metal forming is an essential part of modern industry. It is required in the manufacture of a variety of goods, ranging from customer products to automobiles, aircrafts, ships, etc.. A substantial part of metal forming is still an art, depending on the skills and knowledge of individual artisans and tradesmen. Therefore, it is generally believed that to achieve significant improvements in efficiency, the field will have to be brought onto a scientific basis.

With the development of fast and powerful digital computers in recent years, the Finite Element Method (FEM) has made valuable contributions to metal forming analyses. The method has the capability to overcome complexities arising from geometric and material nonlinearities, as well as friction at the material-tool interface. A number of sheet metal processes have already been simulated by the FEM, thus avoiding much of experimental work. It is possible that the nonlinear FEM will replace most of sheet metal experimentation in the future in the same way that the linear FEM has substituted experimentation in design.

1.2 Overview Of Sheet Metal Forming

Sheet metal forming is one of the two broad categories [1] of metal forming. This category is characterized by significant changes in the shape of the workpiece but not in the cross section. Although sheet metal forming is important in many branches of industry, it has received relatively little attention compared with the other category, bulk forming [2]. Four sheet metal forming processes that have been given extensive FEM research are: deep drawing, stretch forming, hydraulic bulging, and brake bending (Figure 1.1).

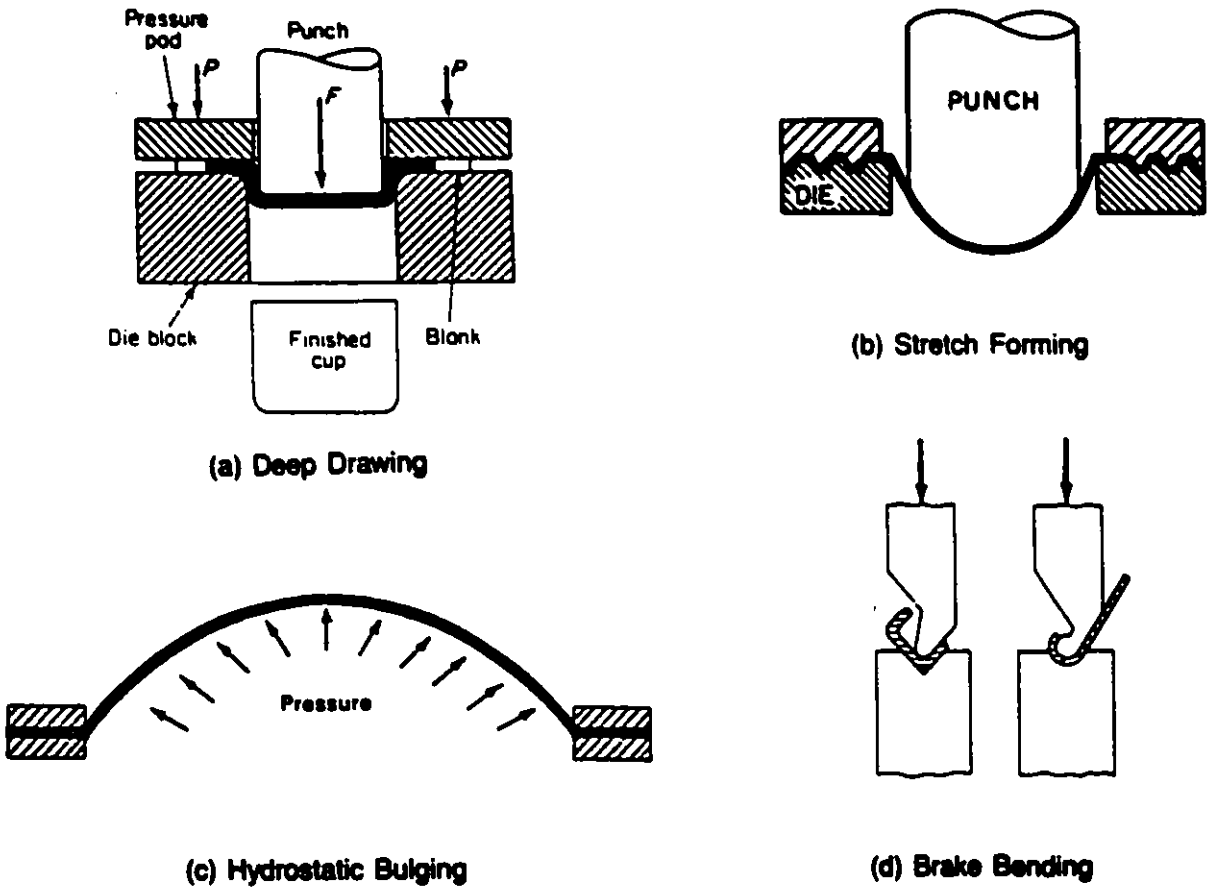


Figure 1.1: Some typical processes of Sheet Metal Forming.

Deep drawing is one of the most common metal working processes. Typical parts produced from this process are beverage cans, containers of all shapes and sizes, sinks, and automobile panels. This process involves a flat sheet of metal being drawn into a cylindrical or box shape by means of a punch pushing the sheet into a die cavity.

Stretch forming is another major process for producing aerospace and automotive parts, e.g. rocket or aircraft-wing skin panels, car-door panels, roofs, or the like. In stretch forming, a sheet of metal is clamped at its edges and a male tool with or without a female backup is pushed into the blank to produce the desired shape. The sheet is stretched just to the elastic limit, and then the stress is slightly raised into the plastic range for permanent set.

Hydraulic bulging is a less common process used for the production of shallow-recessed sheet metal parts as well as for the expansion of tubular parts. Recently, it has been used for testing the ductility of sheet metal. In this test, a blank of metal (of circular shape mostly) is clamped at its periphery and then bulged by hydraulic pressure. Because no friction is involved, the bulging problem is relatively easy to analyze. It is, however, still unclear at present how the results of such a test can be applied to commercial shallow-recessed parts formed by other methods such as punch-and-die forming.

Brake bending is a forming process widely used for the production of linear sections such as angles, channels, or hats from flat sheets and straight plates. There are two types of brake bending; in air bending, the workpiece is supported only at the outer edges; and in press bending, the workpiece is forced into a female die cavity. Air bending is preferred over press bending and usually used for metals with springback problems. Although brake bending is a rather simple process, it also presents technical problems, such as the limitations of bendability, the deviation in the bend angle due to springback, shape accuracies, and so on.

1.3 Techniques of Pipe Bending

Pipe and tube bending is also one of sheet metal forming processes, which has been ignored in some previous classifications. The process is important for many industrial applications such as tubes, pipelines, boilers, heat exchangers, etc.. In recent years, bending technology has become increasingly complex and diversified; equipment for pipe bending now ranges from a simple manual machine for non-critical work to a fully computerized bender for mass production and various design specifications. Relatively little information is as yet available concerning practical and theoretical aspects of the process. Six main types of bending being used today are: ram bending, rotary draw bending, compression bending, coiling, induction bending, and internal roll bending (Figure 1.2).

Ram bending is one of the oldest and simplest methods of pipe bending. In this process, a hydraulic ram pushes against the centre of a pipe supported at each of its ends by a pair of wing dies. These dies play the role of a pivot in constraining and guiding the bend. For ordinary work, ram bending is ideal because it is fast and simple. However, there are limitations to using this method in terms of ovality of the cross section and minimum bend radii achievable.

Rotary draw bending is widely used because of its versality and accuracy, particularly for tight radii and thin-walled tubes. The operation is carried out with the pipe clamped tightly against a form block at the front end of the bend while a pressure bar at the other end guides and constrains the pipe. The form block and clamp rotate together, pulling the pipe around the form block. A mandrel is usually inserted into the pipe to reduce cross-sectional flattening and prevent collapse during the bending action.

Compression bending, on the other hand, has a working principle basically opposite to that of draw bending. The form block and clamp are stationary, rigidly holding the trailing end of the pipe while the pressure bar moves around the periphery of the

form block. One main advantage of this technique is that it can make a series of bends in various planes with almost no straight portion between consecutive bends.

Coiling is a relatively simple process usually carried out with a machine having three rolls. These rolls are often adjustable for various pipe dimensions and wall thicknesses, and their relative positions determine the curvature of the formed coil. Coiling becomes impractical if more than one bend is needed, or if the forming of extremely tight coils with given ovality tolerances is required.

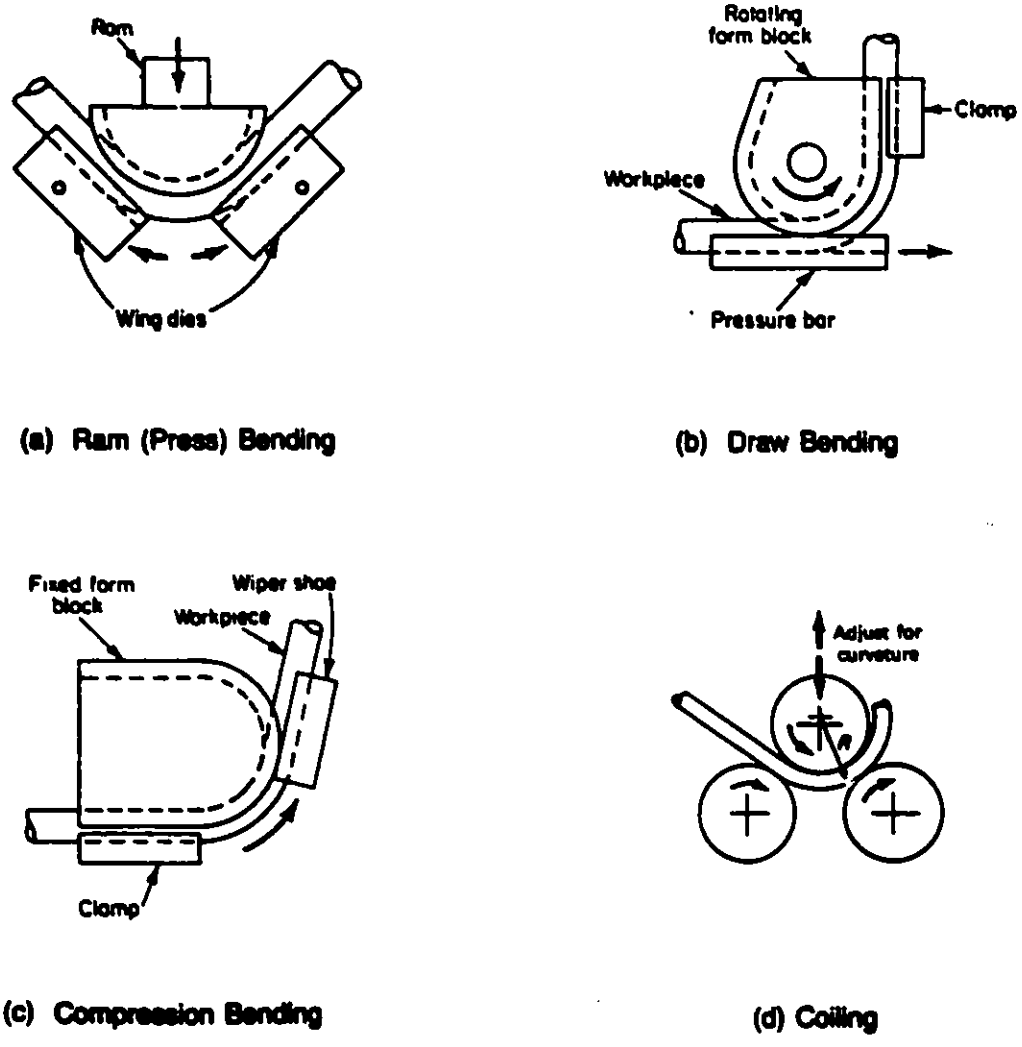


Figure 1.2: Cold pipe bending processes.

In induction bending, the pipe is heated in a narrow annular zone which moves continuously over the length of the bend. Bending only takes place in this zone with the temperature measured and maintained at a pre-determined value by a computer. An advantage of induction bending is that it requires no forming tools and is very flexible with regard to geometric dimensions and desired bend radii. Nevertheless, there are still some limitations on the process; for example, bending of non-ferrous materials or thin-walled pipes cannot as yet be done by this method.

In internal roll bending, pressure is applied to the inside of the pipe by a rolling head rotated within the pipe itself. Benders based on this technique cover an area of pipe bending that is not handled by either cold or induction bending. The main advantage is that almost any desired radius, within a certain minimum-maximum range of the equipment, can be produced by internal roll bending, including thin-walled pipes and non-ferrous materials.

1.4 Outline of Present Investigation

The present study is concerned mainly with ram pipe bending. Attention is given to the simulation of the process using the Finite Element Method (FEM), and the study is carried out with two main objectives:

1. To extend the elbow finite element proposed by Bathe and and Almeida [3] - [6]. In this study, modifications of the formulation of the element are made that allow the element to be used to model structures having continuously changing curvature as a result of plastic deformations. A nonlinear FEM program is developed using the Updated Lagrangian Jaumann Stress-Rate Formulation. The solution is carried out using the Profile Solution Solver, and then iterated with the modified Newton-Raphson technique. A number of elastic and elasto-plastic piping problems are solved to check the accuracy of the program.

2. To obtain practical information on ram bending for different values of the pipe parameters. Investigated are changes in ovality of the cross section of pipe bends of different angles formed from straight pipes, growth of plastic zones in the pipe, possible buckling, and springback due to bending action. The FEM results are compared with those obtained from the elastic-plastic beam theory. The ADINA shell-element results are also obtained for the maximum ovalization of the pipe cross section due to local deformation. A number of charts of practical value are presented.

Chapter 2

Literature Survey

2.1 Background of FEM in Metal Forming

The use of the Finite Element Method (FEM) dates back to the early 1960's when the method, then known as the Structural Stiffness Method, was developed by civil engineers for solving linear elastic structural problems.

As the computer technology progressed in the 1960's, the FEM received greater attention and quickly gained popularity in industry. The number of research papers published on the subject increased dramatically [7], and at the same time the method was extended to many other fields of Science and Engineering, such as Nonlinear Dynamics, Fluid Mechanics, Metal Forming, Electro-Magnetism, ect.. A number of books about the technique were written, covering in detail the various capabilities. Typical ones were those by Zienkiewicz [8] and more recently by Bathe [9].

In the field of metal forming, applications of the FEM began when Yamada et al. [11] introduced the closed form of the plasticity method in the incremental stiffness approach, based on the Prandtl-Reuss equations (Elasto-Plastic analysis). Alternative procedures, such as the "initial stress" method by Zienkiewicz et al. [12] and the weighted stress rate-strain rate technique by Marcal et al. [13] for the elastic-plastic transition elements, were also introduced to reduce computational efforts. Although

these works marked advances in the field, it was realized that the formulations developed were restricted to small-strain problems only. Such a restriction is not realistic since most forming processes, by their nature, generate very large plastic deformations.

The foundation of large-strain analyses had been established in the 1950's by Hill [14,15]. It was not until 1970, however, before Hibbitt et al. [16] introduced the first complete finite element large strain code using a Total Lagrangian Formulation (TLF). In this formulation, the reference state is the original undeformed configuration. One year later, Hofmeister et al. [17] proposed an alternate approach, the Updated Lagrangian Formulation (ULF) in which reference is made to the current state under consideration. The ULF method is more appealing to researchers than the TLF method because of its relative simplicity. Using ULF, there is no need for the so-called "initial displacement matrix" and available small-strain programs can be easily extended to handle large-strain situations. Furthermore, configuration-dependent problems and treatment of contact boundary conditions are easier with ULF.

More recently, a number of other formulations dealing successfully with large plastic deformations have also been presented. The Rigid-Plastic formulation was introduced by Kobayashi and his collaborators. It is based on the idealization that elastic deformations can be ignored in comparison with the large plastic strains, thus simplifying the solution procedure. Zienkiewicz et al. [18] made the first attempt to apply the flow formulation derived from Fluid Mechanics to study the large plastic deformation of sheets of metal, particularly axisymmetric deep drawing and stretch forming. Such problems have also been solved by Wang and Wenner [19], and subsequently by Rebelo and Kobayashi [20] using the Elasto-Viscoplastic and Rigid-Viscoplastic formulations, respectively.

All of the above-mentioned formulations have by no means exhausted the list of FEM techniques used in the field of sheet metal forming. Hughes [7] has given a comprehensive survey on current trends of FEM research with a strong emphasis on practical applications. His work has led to a review by Kobayashi [21] on theoretical

aspects of various FEM approaches, including coupled analyses of heat transfer and metal flow. Meanwhile, the condensed review of Wifi [22] has identified difficulties in the large-strain Elasto-Plastic FEM analysis of forming processes. Recently, Gadala and Oravas [23] have provided a survey on the FEM solution techniques for nonlinear continuum mechanics problems. Cheng and Kikuchi [24] have focussed attention on FEM solutions of large-strain Elasto-Plastic deformations, including those involving unilateral contact and friction.

2.2 Previous FEM Work on Sheet Metal Forming

As briefly mentioned in Art. 1.2, most FEM work that has been done in this field is on deep drawing, stretch forming, bulging and brake bending. Not only is FEM analysis relatively economical, but also the method has successfully been used to solve a number of forming problems of such a complex nature that a theoretical or experimental analysis had long been considered inadequate or impossible.

Research on deep drawing can be traced back to the very early experimental works of Woo [25] – [27]. The topic, however, seemed to be abandoned until Wifi [28] provided complete TLF solutions to deep drawing and stretch forming of a circular blank using a hemispherical punch. His study was focussed on the bending action under tension at the die profile, that had been disregarded in previous works, and on boundary conditions, which are usually unknown beforehand. The problems solved in the study turned out to be computationally time consuming.

Later, Wifi and Yamada [29] introduced a general self-correcting technique to reduce errors due to the performance of finite elements, particularly of plate and shell types, and nonlinearities of large deformations. Andersen [30] analyzed the deep drawing problem with both hemispherical and flat-headed punches using ULF, coupled with a direct Euler procedure to minimize computer time. Because the displacements computed in the thickness direction varied linearly, a shell-theory method was employed for

the analysis, as had been done earlier by Wang and Budianski [31]. Honnor and Wood [32], on the other hand, solved the same problem with a linear Mindlin axisymmetric shell element using the Eulerian formulation which, though less well-known than TLF and ULF, proved to be the most efficient for this highly nonlinear problem.

The first use of the Rigid-Plastic model was by Kobayashi and Kim [33]. They developed a membrane shell theory model from the variational formulation and then verified it by analyzing problems of deep drawing, stretch forming, and hydrostatic bulging. The model was found to handle sheet metal forming problems with efficiency and gave reasonable accuracy although its validity still required further investigation. Following the work of [33], Gotoh and Ishisé [34] – [35] used the Rigid-Plastic approach to predict the deformed shape of the flange region in a deep drawing process with a fourth-degree yield function. The prediction obtained was in good agreement with experimental data, and it was found that numerical analysis, based on the quadratic yield function only, gives inaccurate results, particularly for the earing development.

From another viewpoint, Zienkiewicz et al. [36] reasoned that when plastic deformations are so large that the elastic strains are negligible, the problem becomes that of an incompressible viscous non-Newtonian flow with prescribed boundary friction, strain hardening as well as elastic springback and thermal effects. In addition, when the process involves thin sheets of metal, simplifications on this approach can be made [37] and the deformation of sheets of arbitrary shapes can be analyzed with the general viscous shell element [38]. Using these concepts and taking membrane and bending effects into account, Baynham and Zienkiewicz [39] developed a Rigid-Plastic flow model for the study of multi-stage deep drawing operations with completely general tooling geometry.

Recently, the need for using a large number of elements when bending effects are taken into consideration has been overcome by Tatenami et al. [40], who presented a mixed method between FEM and FDM (Finite Difference Method) that only required a small number of (multi-layered) elements. Good agreement was obtained between

numerical and experimental strains. Gerdeen [41] has developed a FEM program for axisymmetric analyses, that could account for combinations of stretching, bending, and drawing modes of deformation, multiple stages of forming, thickness variations, strain hardening and residual stresses.

Due to the similar nature of the two processes, many FEM analyses for deep drawing have also been devoted to stretch forming, and vice-versa ([28] - [29], [32] - [33], [36,39]). Nevertheless, a number of studies have been done especially for stretch forming, with some emphasizing on the punch stretching test. Mehta and Kobayashi [42] considered a general case of non-axisymmetric stretch forming, in which displacement boundary conditions were assumed for simplicity. Excellent results were obtained for various strain distributions.

Wang and Wenner [19] analyzed axisymmetric and plane-strain stretch forming problems with the Elasto-Viscoplastic approach, that included Coulomb friction, work hardening and normal anisotropy. Their model was based on a rate-sensitive flow theory and the nonlinear membrane theory. Consideration of rate sensitivity was found to increase result accuracy, and was therefore highly recommended. The stretch forming problem was then studied by Robelo and Kobayashi [20]. They used the Rigid-Viscoplastic formulation, which took into account the effects of metal strain-rate sensitivity in conjunction with the assumption that the variational formulation for rate-insensitive materials was also valid for rate-sensitive ones. The strain distributions computed were almost identical to those of [19] and agreed well with experiments.

A more general case of sheet metal stamping with dies and punches of arbitrary shapes was investigated by Wang and Budianski [14], who assumed an Elasto-Plastic material model satisfying a rate-insensitive Mises-flow rule, that handled finite deformation, work hardening and normal anisotropy. Since the results for hemispherical punches were only in fair agreement with experiments, hardening laws based on biaxial stress-strain data were advised to be employed. The case was subsequently extended by Wennerström et al. [43] to include the friction that exists between the tools and

the sheet of metal. Alternatively, Wang [44] analyzed stretch forming of general geometries using a Rigid-Plastic rate-insensitive model with isotropic hardening and flow rules obeying Hill's new field theory.

Axisymmetric hydraulic bulging problems have also been solved in several FEM analyses [22,33,44] in parallel to deep drawing and stretch forming, with generally good solutions. The most general case of non-axisymmetric edge-clamped metal diaphragms bulged by hydraulic pressure was studied by Iseki et al. [45], with an incremental large deformation Elasto-Plastic approach. The formulation developed was then checked for the case of a circular diaphragm before it was applied to calculate the large plastic deformations of thin elliptical and rectangular diaphragms [46]. Reasonable agreements with experimental data were obtained for the central height- and polar thickness-bulge pressure relations, and also for strain distributions on the major and minor axes of the diaphragm.

In another study, Bate [47] compared, in terms of accuracy and computational efficiency, the model proposed in [45] with the ULF based on the small-strain approximation by considering the hydraulic bulging problem for the case of isotropic strain-rate insensitive materials. Little difference could be observed between the two alternatives, with the latter being a bit slower but requiring less storage. Gotoh [48] recommended a polynomial shape function for triangular elements in the analysis of general deformation of sheet metals with or without orthotropic anisotropy. Such elements were expected to be economical when used to solve Elasto-Plastic problems of hydraulically bulged sheet metals.

Unlike deep drawing, stretch forming, and hydrostatic bulging, brake bending problems solved or modelled in the literature are much more industrially oriented, especially to automobile applications, although fewer FEM studies have been undertaken. Oh and Kobayashi [49] treated sheet bending between the dies and the punch as a bulk forming process under plane-strain conditions. Utilized in the study were both the large-strain Elasto-Plastic and incremental Rigid-Plastic formulations. Excellent

agreement was obtained between the two methods in terms of springback, residual stresses in the workpiece, and and detailed mechanics during bending.

A nonlinear shell element proposed by Tang et al. [50] accurately predicted the deformed shapes of automobile body panels by brake bending. With the assumption that two sides of the blank could move freely, the computation was performed by automatically selecting and progressively feeding specific increments of a proposed set of binder surface displacements as boundary conditions. Tang [51] subsequently omitted such an assumption and analyzed the resulting contact problem by means of an approximate method using a FEM program developed at Ford Research. Wang et al. [52] have recently studied the stretch flanging of V-shaped sheet metal blanks, an operation required for various automotive structural members and inner panels, and obtained reasonable agreement between numerical results and test data.

2.3 Previous Analytical Work on Pipe Bending Processes

'Pipe and tube bending' referred to in the present study is a forming process, in which a straight pipe is intentionally bent past the yield point so as to cause permanent plastic deformations. The bent pipe is then called a 'pipe bend' or 'pipe elbow'.

Nowadays, the art of pipe and tube bending is far in advance of any basic understanding of the process in terms of engineering parameters [61]. Technically oriented literature on pipe bending processes includes books by Corpet [53] and Gillanders [54], covering a variety of bending methods associated with their advantages and shortcomings as well as various standards currently adopted by manufacturers producing bends. Some detailed theory on bending has also been given in recent reviews by Winship [55], Addison [56] and Venables [57].

An early experimental study of field bending was carried out by Steffens and Barkow [58]. Its objectives were to investigate changes in wall thickness and in ovality

of the cross section resulting from bending straight pipes into arcs of different angles. Weils et al. [59] then developed formulas to predict such thickness changes, based on the assumption that the meridional natural strain was zero. However, their predictions were in poor agreement with those of [58]. In a recent study of pipe bends for outer coating design purposes, Palynchuk [60] has found that strains caused by field bending operations vary substantially from theoretical predictions.

Inoue and Mellor [61] presented strain distributions in mild steel tubes formed by radial draw bending. It was found that lubrication can have a large effect on strains and that an approximate analysis based on the assumption that bending takes place under uniaxial tension leads to good correlation with experiment. The analysis was then extended [62] to deal with stainless steel tubes and to investigate the effects of friction conditions and a booster force on the pressure die. The results were compared with simple theoretical predictions of strain to find out where improvements in straining pattern are possible. Semenov and Nikitin [63] analyzed the induction bending of thin-walled tubes to a small radius by a profiling device. Blume et al. [64] studied the material properties of pipes after bending and subsequent heat treatment while Vintour et al. [65] investigated the conditions to maintain the original properties of pipes without undergoing a post-bending heat treatment. Asao et al. [66] carried out experiments on pipe bending using high frequency induction heating, from which means were found to avoid buckling at the inside of the pipe and reduction in thickness at the outside.

So far, little work has been published concerning such aspects as final strain distributions, residual stresses due to bending, or applied forces required for bending operations. Furthermore, no studies on numerical modelling of any particular type of pipe or tube bending have appeared in the literature. It is therefore worthwhile to investigate the possibility of simulating the process using the Finite Element Method. Although attention was given to the type of Ram Bending, results obtained from the present study would generally give a better understanding of the process, and any

success achieved would lead to simulations of other types of bending as well.

2.4 Previous Work on Elbow Element

Curved pipes appear frequently and are the most flexible parts in piping systems. In general, the FEM analysis of curved pipes is carried out with three types of elements: 3-D elements, shell elements, and elbow-like elements.

Theoretically, 3-D solid elements such as tetrahedral elements or brick elements are the most general type, and can be used to model any structure. However, to obtain acceptably accurate results a large number of elements is often required. Due to the huge cost of the analysis, such elements have seldom been employed to model curved pipes. On the other hand, in less expensive analyses that usually give excellent predictions a curved pipe is treated as a toroidal shell. It is then modelled with thick-shell elements [67] or thin-shell elements [68] depending on the wall thickness of the pipe. In many cases where there is no demand for high accuracy of the results while the cost of the analysis is more crucial, a FEM analysis with shell elements is still considered to be costly even for linear problems since the analysis involves typically of the order of a thousand of equilibrium equations.

In view of the foregoing, a number of elbow-like elements have been developed for practical use. A curved pipe is considered as a chain of much shorter curved pipes (ring or elbow elements) of the same cross section. The cost of an analysis using these elements is substantially reduced while the elements still provide sufficiently accurate predictions for the overall behaviour of the pipe. In previous linear FEM analyses of pipes, a simple curved beam theory was commonly used in combination with different flexibility and stress intensification factors to account for the ovalization of the pipe cross section and the effects of internal pressure [69], in-plane loading [80], and out-of-plane loading [70]. In some special problems such as elastic or plastic bending of a curved pipe having simple geometry, closed-form element stiffness matrices were

derived for programming convenience and computational efficiency [71] – [73]. Nevertheless, curved pipes modelled by the beam theory showed greater bending flexibility and stress intensification than straight pipes did. Yet, the above analyses assumed that ovalization was constant along the pipe bend; hence, variation in ovality of cross section could not be taken into consideration.

Because of such limitations of the beam theory, different elbow element models based on a shell theory have been proposed. Ohtsubo and Watanabe [74] developed the first ring element from the general thin shell theory including shear strain. The displacement component around and along the pipe were approximated by Fourier and Hermitian polynomials, respectively. Takeda et al. [75] proposed a similar ring element. The ring has two nodal points at its ends on the center line to account for the longitudinal displacement, and the ring wall is modelled by 4-noded doubly-curved quadrilateral elements to account for the circumferential and radial displacements. The circumferential and normal displacement fields were interpolated by 2-D Hermitian functions. The longitudinal displacement field was expanded into a Fourier series in the circumferential direction with each term interpolated by a 1-D cubic function along the longitudinal direction. Later, Takeda et al. [76] modified the formulation of their ring element to take into account transverse shears and the interaction effects between straight and curved pipes. For the new element, the circumferential and radial displacements and rotations were interpolated by 2-D Lagrangian bilinear functions. The longitudinal displacement was expanded into a Fourier series in the circumferential direction, and each term was interpolated by a 1-D linear function along the longitudinal direction. The elements developed in [74] – [76] were used mainly in linear analyses.

So far, few works have been published on the plastic analysis of curved pipes. Hibbitt and Leung [77] formulated a thin-walled elbow element for nonlinear analysis involving arbitrarily large motions which involve small strains. Lazzeri [78] developed an elasto-plastic elbow element based on Vlasov's thin shell theory. The circumferen-

tial displacement was represented by a Fourier series, and the derivatives of Fourier coefficients along the azimuthal direction were evaluated using the finite difference method. A static condensation procedure was then employed to reduce the stiffness matrix to a 12×12 matrix.

Though more cost-effective than shell elements, the elbow-shell elements have a number of drawbacks, especially in nonlinear analysis. Bathe and Almeida [3] developed a new elbow element that is simple, computationally effective, and capable of accurately predicting significant deformations and stress distributions in various curved pipe segments. The elbow is a four-noded displacement-based finite element with axial, torsional, and bending displacements and the von Karman ovalization deformation, all varying cubically along the elbow length. The formulation given is basically an extension and generalization of von Karman's analysis [S0]. The Ritz method is used to take the axial variation of the cross section ovality into account.

The element was then extended to include interaction effects [4] such as those between elbows and rigid flanges, curved pipes of different curvatures. The penalty procedure was used to enforce the continuity of the derivatives in the pipe skin radial displacements. Also covered were stiffening effects due to internal pressure [S1], and nonlinear effects due to material and kinematic nonlinearities [5].

In the present study, the Bathe-Almeida element, that was initially developed for piping analysis, is extended to solve the ram pipe bending forming process problem. The formulation is modified to take into account changes in the curvature of the element. These changes are normally significant due to large plastic deformations. Hence the Updated Lagrangian Jaumann Stress-Rate Formulation was used, which is capable of taking into account the new, deformed configuration of the structure at the end of each time step. This formulation is different from ULF in that it deals directly with stress and strain rates, thus avoiding the transformation of stresses and strains into the new coordinate system.

Chapter 3

Finite Element Modelling

3.1 General

Pipe and tube bending can be carried out using the methods of cold or hot bending as described in Article 1.3. Regardless of the technique applied, all formed bends are subjected to such phenomena as tension at the outside of the bend and compression at the inside. As the material deforms past the yield point, thinning occurs at the outer wall and corresponding thickening occurs at the inner wall; in addition, the cross section of the bend also tends to ovalize. These major effects result from plastic flow during the bending operation.

Ram bending involves the practice and the theory of the simply supported beam, and was one of the first techniques of bending. It is often used with low-precision work, in which the effects of cross-sectional flattening and changes in the wall thickness are not of great importance. Ram bending nevertheless deserves attention because it gives an insight into the mechanics of pipe and tube bending.

Figure 3.1 shows a schematic view of Ram bending and Figure 3.2 shows the modelling using beam theory; i.e. a simply supported beam.

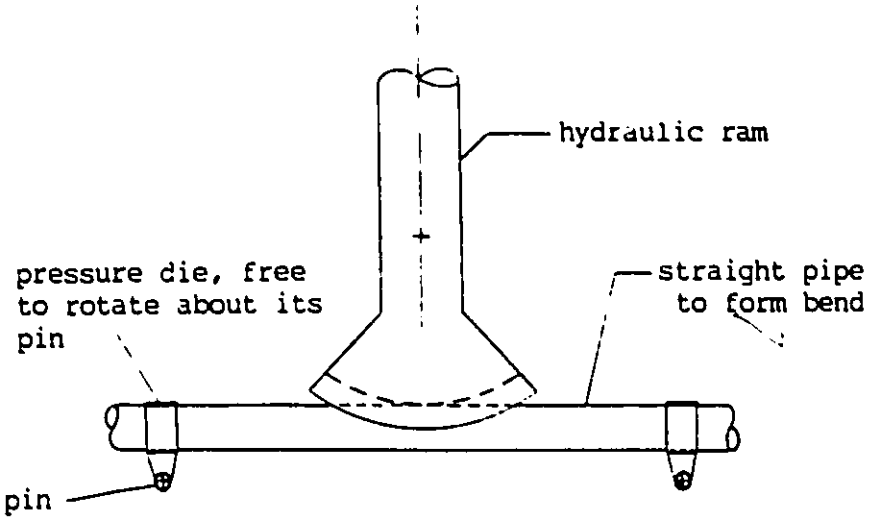


Figure 3.1: Schematic view of ram bending.

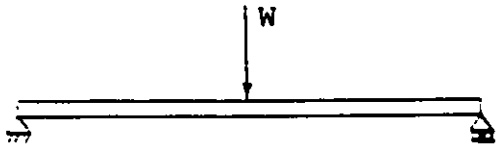


Figure 3.2: Simply supported beam.

3.2 Finite Element Method

After almost two decades of extensive development, the finite element method is now widely accepted as a powerful and general technique for the numerical solution of a variety of problems in engineering.

The basic idea behind this method is that any structure or continuous medium can be regarded as consisting of individual c ponents or "finite elements", whose behaviour is well-defined and readily understood. These elements are interconnected at "nodal points" along their inter-element boundaries, Figure 3.3. Thus such a structure,

no matter how complex it is, can be analyzed with relative ease by studying the behaviour of its building blocks.

The finite element method is based on early work by Ritz (1909), who developed a powerful method to approximate the solution of field problems. It involved the approximation of a potential functional in terms of trial functions containing coefficients of unknown magnitudes. The minimization of the functional with respect to each unknown results in a set of linear equations that are solved for the unknowns. Accuracy could be improved by increasing the number of trial functions.

The finite element method can be applied in one of four different ways: (1) direct approach, (2) variational method, (3) weighted residual method, and (4) energy balance method. Among these, the weighted residual method is the most general approach while the variational method is the most popular one.

The variational method is the extended Ritz method in which the functional of the total potential energy of the system is to be minimized. The concept of minimum potential energy has three main forms corresponding to three variational principles: total potential energy, complementary energy, and Reissner [113].

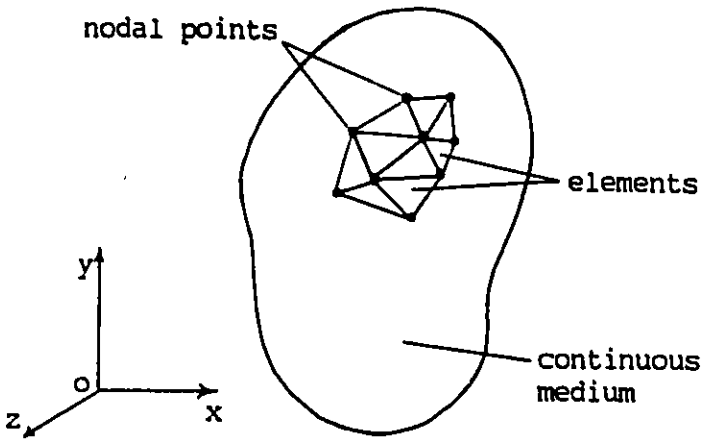


Figure 3.3: Discretization of a 3-D continuous body.

- The total potential energy principle is the cornerstone of the displacement-based finite element method. This is a most important formulation for the solution of many practical problems because of its simplicity, generality, and also numerical stability.

- In the complementary energy principle, stresses are the field variables rather than displacements; thus the use of this principle results in the stress-based equilibrium finite element method.

- A most general variational principle is that of Reissner, the primary field variables being both displacements and stresses, usually referred to as "mixed models".

In summary, the solution to a problem using the finite element method (assumed to be displacement-based from here on) generally consists of six main steps: (1) formulation of the problem, (2) choice of the elements, (3) determination of the boundary conditions, (4) assemblage of element equilibrium equations, (5) solution of the global equations, and (6) calculation of strains, stresses, or any quantities of particular interest.

In linear analyses, the global equations can be represented by:

$$\boldsymbol{K}\boldsymbol{U} = \boldsymbol{R} \tag{3.1}$$

where $\boldsymbol{K}$ is a constant stiffness matrix, $\boldsymbol{U}$ the vector of unknown nodal point displacements and $\boldsymbol{R}$ the vector of externally applied nodal forces.

In nonlinear analysis involving both geometric and material nonlinearities, the solution has to be carried out incrementally and iteratively until the desired accuracy is obtained. If the Updated Lagrangian Jaumann Stress-Rate Formulation and the Modified Newton-Raphson technique are adopted, the above equation becomes more complicated, having the form:

$$\left({}_t^t\boldsymbol{K}_L + {}_t^t\boldsymbol{K}_{NL}\right)\boldsymbol{\Delta U}^{(i)} = {}^{t+\Delta t}\boldsymbol{R} - {}_{t+\Delta t}^{t+\Delta t}\boldsymbol{F}^{(i-1)} \tag{3.2}$$

where

$_{t}^{t}\boldsymbol{K}_{L}$: Linear strain incremental stiffness matrix.

$_{t}^{t}\boldsymbol{K}_{NL}$: Nonlinear strain incremental stiffness matrix.

$^{t+\Delta t}\boldsymbol{R}$: Vector of externally applied nodal point loads at time $t+\Delta t$.

$_{t+\Delta t}^{t+\Delta t}\boldsymbol{F}^{(i-1)}$: Vector of nodal point forces equivalent to element stresses at time $t+\Delta t$ and in iteration $(i-1)$

$\boldsymbol{\Delta U}^{(i)}$: Vector of increments in nodal point displacements in iteration i, given by

$$^{t+\Delta t}\boldsymbol{U}^{(i)} = {}^{t+\Delta t}\boldsymbol{U}^{(i-1)} + \boldsymbol{\Delta U}^{(i)}$$

In the above equation, the left superscript of a quantity represents the configuration of the body in which the quantity is calculated. Meanwhile, the left subscript indicates the configuration with respect to which the quantity is measured. When the left subscript is missing as in the case of $^{t+\Delta t}\boldsymbol{R}$, the quantity occurs in the same configuration (current) with respect to which it is measured. Detailed discussions on the applicability and various theoretical aspects, particularly those related to nonlinearities, of the Finite Element Method are given in [9]. Summaries of the Updated Lagrangian Jaumann Stress-Rate Formulation and the modified Newton-Raphson method are given in Appendix B.

3.3 Review of Elbow Element Formulation

The elbow element is a combined beam and shell finite element. The cross section of the element is assumed to be a circular annulus with constant thickness at all stages of the analysis, linear and nonlinear. The assumption about no cross-sectional change restricts the usefulness of the element to cases where ovalization is minimized

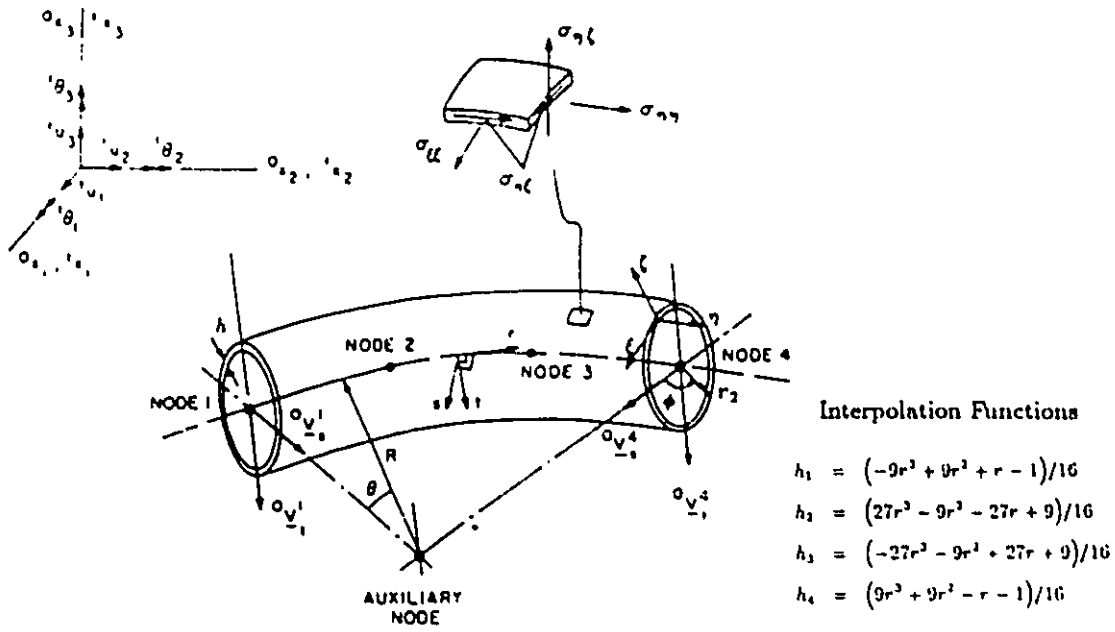


Figure 3.4: Geometry of the pipe elbow element.

using some active means. Small radial displacements are assumed to occur; these are predicted according to von Karman's hypotheses [80].

Detailed description of such an elbow element was given in [9], and the effects of ovalization have been investigated recently in [3] – [5]. For continuity of these works, a brief summary of the formulation of the elbow element is given here using the nomenclature of [3] – [5].

3.3.1 Beam Behaviour

The elbow element is an isoparametric element that has 4 nodal points equally spaced along the centerline of the element. Each nodal point has a total of 6 displacements: 3 translations and 3 rotations $(u_1, u_2, u_3, \theta_1, \theta_2, \theta_3)$ with respect to the referential coordinate axes. This permits consideration of in-plane and out-of-plane loading cases. The current geometry of the element at any time t is approximated by

$$ {}^t x_i(r,s,t) = \sum_{k=1}^{4} h_k {}^t x_i^k + s\sum_{k=1}^{4} r_2^k h_k {}^t V_{ni}^k + t\sum_{k=1}^{4} r_2^k h_k {}^t V_{ti}^k \qquad (3.3) $$

where

i = 1, 2, or 3 representing x, y, or z

k = node number from 1 to 4

r, s, t = isoparametric coordinates, as shown in Figure 3.4

$h_k(r)$ = isoparametric interpolation function for nodal point k, given in Figure 3.4

r_2^k = outer radius of the element at nodal point k

${}^tV_{ti}^k$ = component i of unit vector ${}^tV_t^k$, in direction t

${}^tV_{si}^k$ = component i of unit vector ${}^tV_s^k$, in direction s.

As can be seen from Figure 3.4, Equation (3.3) is only applicable for the values of s and t satisfying

$$\left[t - \frac{h^k}{r_2^k}\right]^2 \leq s^2 + t^2 \leq 1$$

where h^k is the wall thickness of the element at nodal point k, and is assumed constant.

The displacement components, defined by $u_i = {}^{t+\Delta t}x_i - {}^tx_i$, can now be readily calculated from (3.3):

$$u_i(r,s,t) = \sum_{k=1}^{4} h_k u_i^k + t\sum_{k=1}^{4} r_2^k h_k V_{ti}^k + s\sum_{k=1}^{4} r_2^k h_k V_{si}^k \tag{3.4}$$

with

$$V_{ti}^k = {}^{t+\Delta t}V_{ti}^k - {}^tV_{ti}^k$$

$$V_{si}^k = {}^{t+\Delta t}V_{si}^k - {}^tV_{si}^k$$

To evaluate the changes in direction cosines, V_{ti}^k and V_{si}^k, they need to be expressed in terms of nodal point rotations, namely

$$\boldsymbol{V}_t^k = \boldsymbol{\theta}_k \times {}^t\boldsymbol{V}_t^k$$

$$\boldsymbol{V}_s^k = \boldsymbol{\theta}_k \times {}^t\boldsymbol{V}_s^k$$

The strain components can then be determined by calculating the displacement derivatives corresponding to the global coordinate axes x_i $(i = 1, 2, 3)$. Since the

displacements u_i in Equation (3.4) are being expressed in terms of the isoparametric coordinates (r, s, t), it is impractical to differentiate (3.4) directly with respect to x_i. Instead, the computation is carried out in 2 steps.

Firstly, (3.4) is differentiated with respect to r, s, and t to yield

$$\begin{bmatrix} \frac{\partial u_i}{\partial r} \\ \frac{\partial u_i}{\partial s} \\ \frac{\partial u_i}{\partial t} \end{bmatrix} = \sum_{k=1}^{4} \begin{bmatrix} \frac{\partial h_k}{\partial r} & [\ 1 & {}^t(g)^k_{1i} & {}^t(g)^k_{2i} & {}^t(g)^k_{3i}\ ] \\ h_k & [\ 0 & {}^t(\hat{g})^k_{1i} & {}^t(\hat{g})^k_{2i} & {}^t(\hat{g})^k_{3i}\ ] \\ h_k & [\ 0 & {}^t(\bar{g})^k_{1i} & {}^t(\bar{g})^k_{2i} & {}^t(\bar{g})^k_{3i}\ ] \end{bmatrix} \begin{bmatrix} u_i^k \\ \theta_1^k \\ \theta_2^k \\ \theta_3^k \end{bmatrix} \tag{3.5}$$

where

$${}^t(\hat{g})^k = r_2^k \begin{bmatrix} 0 & -{}^tV_{s3}^k & {}^tV_{s2}^k \\ {}^tV_{s3}^k & 0 & -{}^tV_{s1}^k \\ -{}^tV_{s2}^k & {}^tV_{s1}^k & 0 \end{bmatrix}$$

$${}^t(\bar{g})^k = r_2^k \begin{bmatrix} 0 & -{}^tV_{t3}^k & {}^tV_{t2}^k \\ {}^tV_{t3}^k & 0 & -{}^tV_{t1}^k \\ -{}^tV_{t2}^k & {}^tV_{t1}^k & 0 \end{bmatrix}$$

$${}^t(g)^k = s\,{}^t(\hat{g})^k + t\,{}^t(\bar{g})^k$$

Secondly, the above derivatives are transformed into the global coordinate system by means of the Jacobian transformation that has a matrix representation:

$$\begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_3} \end{bmatrix} = \boldsymbol{J}^{-1} \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial s} \\ \frac{\partial}{\partial t} \end{bmatrix} \tag{3.6}$$

with

$$\boldsymbol{J} = \begin{bmatrix} \frac{\partial x_1}{\partial r} & \frac{\partial x_2}{\partial r} & \frac{\partial x_3}{\partial r} \\ \frac{\partial x_1}{\partial s} & \frac{\partial x_2}{\partial s} & \frac{\partial x_3}{\partial s} \\ \frac{\partial x_1}{\partial t} & \frac{\partial x_2}{\partial t} & \frac{\partial x_3}{\partial t} \end{bmatrix}$$

The derivatives in (3.5) can now be rewritten in terms of the global coordinates as

$$\begin{bmatrix} \frac{\partial u_i}{\partial {}^t x_1} \\ \frac{\partial u_i}{\partial {}^t x_2} \\ \frac{\partial u_i}{\partial {}^t x_3} \end{bmatrix} = \sum_{k=1}^{4} \begin{bmatrix} \frac{\partial h_k}{\partial {}^t x_1} & {}^t(G1)^k_{i1} & {}^t(G2)^k_{i1} & {}^t(G3)^k_{i1} \\ \frac{\partial h_k}{\partial {}^t x_2} & {}^t(G1)^k_{i2} & {}^t(G2)^k_{i2} & {}^t(G3)^k_{i2} \\ \frac{\partial h_k}{\partial {}^t x_3} & {}^t(G1)^k_{i3} & {}^t(G2)^k_{i3} & {}^t(G3)^k_{i3} \end{bmatrix} \begin{bmatrix} u^k_i \\ \theta^k_1 \\ \theta^k_2 \\ \theta^k_3 \end{bmatrix} \tag{3.7}$$

where

$${}_t h_{k,i} = \frac{\partial h_k}{\partial {}^t x_i} = {}^t J^{-1}_{i1} \frac{\partial h_k}{\partial r}$$

$${}^t(Gm)^k_{in} = \left[{}^t J^{-1}_{n1}\, {}^t(g)^k_{mi}\right] \frac{\partial h_k}{\partial r} + \left[{}^t J^{-1}_{n2}\, {}^t(\hat{g})^k_{mi} + {}^t J^{-1}_{n3}\, {}^t(\bar{g})^k_{mi}\right] h_k$$

For convenience in incremental nonlinear analyses, increments in strain components are often approximated by their linear parts,

$$\begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} \\ \frac{\partial u_2}{\partial x_2} \\ \frac{\partial u_3}{\partial x_3} \\ \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \\ \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \\ \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} \end{bmatrix} \tag{3.8}$$

With the substitution of (3.7) into (3.8), the linear strain-displacement transformation matrix can now be obtained as

$${}_t^t B_L = \left[\begin{array}{c|ccc|ccc|c} \cdots & {}_t h_{k,1} & 0 & 0 & {}^t(G1)^k_{11} & {}^t(G2)^k_{11} & {}^t(G3)^k_{11} & \cdots \\ & 0 & {}_t h_{k,2} & 0 & {}^t(G1)^k_{22} & {}^t(G2)^k_{22} & {}^t(G3)^k_{22} & \\ & 0 & 0 & {}_t h_{k,3} & {}^t(G1)^k_{33} & {}^t(G2)^k_{33} & {}^t(G3)^k_{33} & \\ & {}_t h_{k,2} & {}_t h_{k,1} & 0 & {}^t(G1)^k_{12} + {}^t(G1)^k_{21} & {}^t(G2)^k_{12} + {}^t(G2)^k_{21} & {}^t(G3)^k_{12} + {}^t(G3)^k_{21} & \\ & 0 & {}_t h_{k,3} & {}_t h_{k,2} & {}^t(G1)^k_{23} + {}^t(G1)^k_{32} & {}^t(G2)^k_{23} + {}^t(G2)^k_{32} & {}^t(G3)^k_{23} + {}^t(G3)^k_{32} & \\ & {}_t h_{k,3} & 0 & {}_t h_{k,1} & {}^t(G1)^k_{31} + {}^t(G1)^k_{13} & {}^t(G2)^k_{31} + {}^t(G2)^k_{13} & {}^t(G3)^k_{31} + {}^t(G3)^k_{13} & \end{array}\right] \tag{3.9}$$

This matrix is required to evaluate the linear stiffness matrix represented by (3.2). Similarly, the calculation of the nonlinear stiffness matrix requires a nonlinear strain-displacement transformation matrix given by:

$$
{}_t^tB_{NL} = \left[\begin{array}{c|ccccccc|c}
 & {}_th_{k,1} & 0 & 0 & {}^t(G1)_{11}^k & {}^t(G2)_{11}^k & {}^t(G3)_{11}^k & \\
 & {}_th_{k,2} & 0 & 0 & {}^t(G1)_{12}^k & {}^t(G2)_{12}^k & {}^t(G3)_{12}^k & \\
 & {}_th_{k,3} & 0 & 0 & {}^t(G1)_{13}^k & {}^t(G2)_{13}^k & {}^t(G3)_{13}^k & \\
 & 0 & {}_th_{k,1} & 0 & {}^t(G1)_{21}^k & {}^t(G2)_{21}^k & {}^t(G3)_{21}^k & \\
\cdots & 0 & {}_th_{k,2} & 0 & {}^t(G1)_{22}^k & {}^t(G2)_{22}^k & {}^t(G3)_{22}^k & \cdots \\
 & 0 & {}_th_{k,3} & 0 & {}^t(G1)_{23}^k & {}^t(G2)_{23}^k & {}^t(G3)_{23}^k & \\
 & 0 & 0 & {}_th_{k,1} & {}^t(G1)_{31}^k & {}^t(G2)_{31}^k & {}^t(G3)_{31}^k & \\
 & 0 & 0 & {}_th_{k,2} & {}^t(G1)_{32}^k & {}^t(G2)_{32}^k & {}^t(G3)_{32}^k & \\
 & 0 & 0 & {}_th_{k,3} & {}^t(G1)_{33}^k & {}^t(G2)_{33}^k & {}^t(G3)_{33}^k &
\end{array}\right] \tag{3.10}
$$

Having computed the strain-displacement matrices, the construction of element stiffness matrices and of the nodal force vector follows the usual finite element procedure, namely

$$
{}_t^tK_L = \int_{{}_tV} {}_t^tB_L^T \, {}_tC^{EP} \, {}_t^tB_L \, {}^tdV \tag{3.11}
$$

$$
{}_t^tK_{NL} = \int_{{}_tV} {}_t^tB_{NL}^T \, {}^t\tau \, {}_t^tB_{NL} \, {}^tdV \tag{3.12}
$$

$$
{}_t^tF = \int_{{}_tV} {}_t^tB_L^T \, {}^t\hat{\tau} \, {}^tdV \tag{3.13}
$$

where

${}_tC^{EP}$ = instantaneous elastic-plastic stress-strain matrix

${}^t\tau$, ${}^t\hat{\tau}$ = matrix and vector representing element residual stresses due to externally applied loads, respectively.

If the von Mises yield condition and the flow theory are utilized with the material assumed to be isothermal and isotropically hardened, ${}_tC^{EP}$ is given [9,10] by:

$$
{}_tC^{EP} = C^E - \frac{C^E \, {}^tq \left(C^E \, {}^tq\right)^T}{(4/9)\, H^* \, {}^t\sigma_y + {}^tq^T \, C^E \, {}^tq} \tag{3.14}
$$

where

$$
\begin{aligned}
{}^{t}q^{T} &= [\,{}^{t}S_{11}\ {}^{t}S_{22}\ {}^{t}S_{33}\ 2{}^{t}S_{12}\ 2{}^{t}S_{23}\ 2{}^{t}S_{31}\,] \\
{}^{t}S_{ij} &= \text{components of the deviatoric stress tensor} \\
{}^{t}\sigma_{y} &= \text{uniaxial yield stress at time } t \\
H^{*} &= \text{strain hardening constant defined by}
\end{aligned}
$$

$$
H^{*} = \frac{E\,E_T}{E - E_T}
$$

$$
\begin{aligned}
E_I &= \text{instantaneous tangential modulus} \\
C^{E} &= \text{elastic stress-strain matrix.}
\end{aligned}
$$

Since all element stiffness matrices are finally assembled in the global coordinate system, C^E has to be recalculated as

$$
C^{E} = Q^{T}\left(\frac{E}{1-\nu^{2}}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2k} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-\nu}{2k} \end{bmatrix}\right) Q \tag{3.15}
$$

where

k : shear factor, normally taken as 1.2

Q : transformation matrix transforming the elastic stress-strain matrix from the local $\xi\eta\zeta$ system to the global $x_1x_2x_3$ system (see Appendix A).

As can be observed from (3.15), the normal stress in the thickness direction (σ_ζ) was assumed to be zero as in shell theory. This assumption simplifies the stress-strain relations and can be justified by the fact that σ_ζ is usually insignificant compared with other stress components except near the loaded areas.

Although closed-form integration can be carried out in some linear analyses, such an exact evaluation of (3.11) – (3.13) is not possible because of the incremental complexity of nonlinearities. Instead, integration is performed numerically. The result accuracy is greatly dependent on the integration schemes adopted. In terms of simplicity and efficiency, the most commonly used scheme is Gauss quadrature, in which the integration limits are 1 and -1. This is not a suitable choice for the current elbow element. To take into consideration full plastic behaviour of the element, especially at its two ends, the following integration scheme is employed in the present study, as suggested earlier in [5]:

- 5-point Newton-Cotes integration along the element length,
- 12- or 24-point integration using the composite trapezoidal rule in the circumferential direction,
- 5- or 7-point Newton-Cotes integration across the wall thickness.

This integration scheme implies that the integration variables are those of the cylindrical coordinates. Hence, (3.11) – (3.13) have to be evaluated appropriately. For example, (3.11) becomes

$$ {}_t^t\boldsymbol{K}_L \;=\; \int_{-1}^{1}\int_{0}^{2\pi}\int_{r_2-h}^{r_2} {}_t^t\boldsymbol{B}_L^T \; {}_t\boldsymbol{C}^{EP} \; {}_t^t\boldsymbol{B}_L \; \det|\boldsymbol{J}| \; \rho \, d\rho \, d\phi \, dr $$

where $\det|\boldsymbol{J}|$ is the determinant of the Jacobian matrix as shown in Equation (3.6).

3.3.2 von Karman's Description of Ovalization [80]

The elbow element is an extension of von Karman's pipe elbow. For in-plane bending, von Karman analyzed the effect of ovalization of the cross section by assuming the displacement patterns:

$$ w_\xi(r,\phi) \;=\; \sum_{m=1}^{N_c}\sum_{k=1}^{4} h_k \, c_m^k \, \sin 2m\phi \qquad (3.16) $$

where c_m^k denotes the unknown generalized ovalization displacements, ϕ is some angular position of the cross section, and N_c the number of nodal point ovalization displacements. The value of N_c ranges from 1 to 3, depending on the pipe geometric parameter $\lambda = Rh/r_m^2$ as specified in Table 3.1.

Table 3.1: Number of ovalization shape functions to be used.

Geometric range	Number of functions N_c
$0.50 \leq \lambda$	1
$0.16 \leq \lambda < 0.50$	2
$0.08 \leq \lambda < 0.16$	3

To evaluate the strains due to ovalization of the cross section, von Karman made a number of assumptions, the five major ones being

1. Plane sections originally plane and normal to the neutral axis continue to be so.
2. Longitudinal strains are of constant magnitudes through the pipe wall thickness.
3. Pipe wall thickness is small in comparison with the pipe outside radius, $h/r_2 \ll 1$.
4. Pipe outside radius is much smaller than the radius of the pipe bend, $r_2/R \ll 1$.
5. Effect of Poisson's ratio is negligible.

Due to ovalization of the cross section, the radial displacement and the circumferential displacement of the middle surface of the pipe wall are related to each other [3] by

$$w_\zeta = -\frac{dw_\xi}{d\phi} \tag{3.17}$$

where ϕ denotes a positional angle of the cross section, and (ξ,η,ζ) represent the natural coordinates in the pipe wall as shown in Figure 3.4.

A study of Novozhilov's shell theory [S2] for a curved tube indicates that in linear analysis the important strain components for the elbow are the usual beam normal and shear strains as well as the strains due to ovalization, namely

$$\epsilon_{\xi\xi} = -\frac{1}{r_m^2}\left[w_\zeta + \frac{d^2 w_\zeta}{d\phi^2}\right]\zeta \tag{3.18}$$

$$\epsilon_{\eta\eta} = \left[\frac{w_\xi \sin\phi + \frac{dw_\xi}{d\phi}\cos\phi}{R - r_m\cos\phi}\right] - \left[\left(\frac{1}{R - r_m\cos\phi}\right)^2 \frac{d^2 w_\zeta}{d\theta^2}\right]\zeta \tag{3.19}$$

$$\gamma_{\eta\xi} = \left[\frac{1}{R - r_m\cos\phi}\right]\frac{dw_\xi}{d\theta} \tag{3.20}$$

where r_m is the mean radius of the cross section, and is assumed constant throughout the analysis.

The strain components due to ovalization displacements can now be calculated using Equations (3.16) - (3.20).

$$\begin{bmatrix} \epsilon_{\xi\xi} \\ \epsilon_{\eta\eta} \\ \epsilon_{\zeta\zeta} \\ \gamma_{\xi\eta} \\ \gamma_{\eta\zeta} \\ \gamma_{\zeta\xi} \end{bmatrix} = \hat{B}_{OV} \begin{bmatrix} \vdots \\ c_1^k \\ c_2^k \\ c_3^k \\ \vdots \end{bmatrix} \tag{3.21}$$

where $\hat{B}_{OV}$ is the strain-displacement transformation matrix for ovalization given by

$$\hat{B}_{OV} = \begin{bmatrix} & | & a_1 & a_2 & a_3 & | & \\ & | & b_1 + \hat{b}_1 & b_2 + \hat{b}_2 & b_3 + \hat{b}_3 & | & \\ \cdots & | & 0 & 0 & 0 & | & \cdots \\ & | & c_1 & c_2 & c_3 & | & \\ & | & 0 & 0 & 0 & | & \\ & | & 0 & 0 & 0 & | & \end{bmatrix}$$

where

$$m = 2l$$

$$
\begin{aligned}
a_l &= \left[\frac{m(1-m^2)}{r_m^2} h_k \cos m\phi\right]\zeta \\
b_l &= \frac{h_k}{(R - r_m \cos\phi)} \left[m \cos m\phi \cos\phi + \sin m\phi \sin\phi\right] \\
\hat{b}_l &= \left[\frac{2}{(R - r_m \cos\phi)\theta}\right] \frac{d^2 h_k}{dr^k} (m \cos m\phi)\zeta \\
c_l &= \left[\frac{2}{(R - r_m \cos\phi)\theta}\right] \frac{dh_k}{dr} \sin m\phi
\end{aligned}
$$

Because $\hat{\boldsymbol{B}}_{OV}$ relates the ovalization displacements to the strain components in the natural coordinates (ξ, η, ζ), it must be transformed into the global system in order to be combined with ${}_t^t\boldsymbol{B}_L$.

$$\boldsymbol{B}_{OV} = \boldsymbol{P}\,\hat{\boldsymbol{B}}_{OV} \tag{3.22}$$

where $\boldsymbol{P}$ is a transformation matrix (given in Appendix A).

Thus the linear strain-displacement transformation matrix in (3.9) should be modified to incorporate (3.22),

$${}_t^t\boldsymbol{B}_L = \left[{}_t^t\boldsymbol{B}_L \mid \boldsymbol{B}_{OV}\right]$$

which corresponds to

$$
\begin{aligned}
\boldsymbol{u}^T &= \left[\cdots \mid \underbrace{u_1^k\ u_2^k\ u_3^k\ \theta_1^k\ \theta_2^k\ \theta_3^k\ c_1^k\ \overbrace{c_2^k\ c_3^k}^{optional}}_{\text{for nodal point } k} \mid \cdots\right] \\
\boldsymbol{\epsilon}^T &= \left[\epsilon_{11}\ \epsilon_{22}\ \epsilon_{33}\ 2\epsilon_{12}\ 2\epsilon_{23}\ 2\epsilon_{31}\right]
\end{aligned}
$$

3.4 Extension of Elbow Element's Capability

In FEM modelling using the elbow element, the initial curvature of an element is the same as that of the curved pipe which the element models. With regard to piping analysis where plastic deformations are usually small, the assumption that the curvatures of all elements remain constant throughout the analysis is usually made

to reduce programming efforts as well as computer time. On the other hand, large plastic deformations are normally expected in forming problems, and as a result the above assumption is no longer appropriate; the curvature of every element has to be updated to give the true picture of the deformed configuration of the pipe as bending progresses.

As far as the current FEM program and in-plane bending are concerned, an elbow element is approximated as a torus having known curvature at the time under consideration. Thus, the radius of curvature of the element at time t is given by

$$^{t}R = \frac{l}{^{t}\theta}$$

where l is the length of the element measured along its center line, $^{t}\theta$ denotes the angle between the two end sections, and $t = 0$ corresponds to the time prior to bending.

At later time $t+\Delta t$, one end of the element undergoes a relative in-plane rotation $\Delta\gamma$ with respect to the other end as shown in Figure 3.5. With the same approximation, the angle between the two end sections then becomes

$$^{t+\Delta t}\theta = {}^{t}\theta + \Delta\gamma$$

which implies that

$$^{t+\Delta t}R = \frac{l}{^{t+\Delta t}\theta} = \frac{l}{^{t}\theta + \Delta\gamma}$$

The updating scheme presented shows that the larger the number of elements modelling the pipe, the better the deformed configuration of the pipe, and therefore the more accurate the analytical predictions. However, a larger number of elements requires a longer computing time. Thus, a compromise has to made between the result accuracy and computer time.

Here, it is important to note that the updatings of the pipe-wall thickness and the cross section are not considered in the computation process. The reasons are that first, the elbow element is formulated based on thin shell theory. One major assumption of this theory is that wall thickness remains constant. Secondly, the strain components

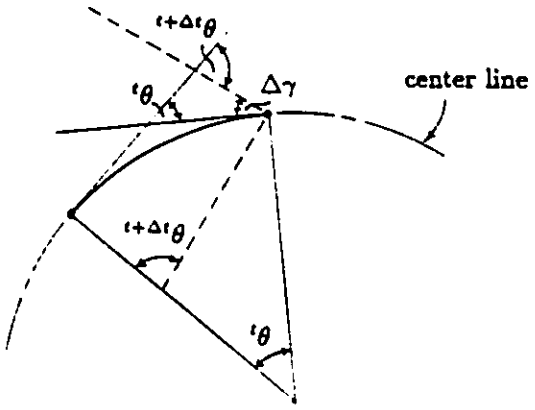


Figure 3.5: Updating of the curvature of an elbow element.

resulting from ovalizations assumed by von Karman are calculated using Novozhilov's shell theory which does not take into account variations in ovality of the cross section.

To facilitate any further reference to the current FEM program and to clarify different sources of results (Chapter 4), the program developed herein will be referred to as 'FENSA' (Finite Element Nonlinear Static Analysis). The program listing of FENSA is shown in Appendix C.3.

Chapter 4

Comparison of FENSA Results with Available Data

4.1 Introduction

In this chapter, the results from the program FENSA for five piping problems are compared with those from other methods and computer programs. The first two problems cover elastic behaviour, while the last three nonelastic behaviour. Efforts are made to assess the accuracy of the elbow element and the reliability of the program FENSA in linear and nonlinear analyses.

4.2 Elastic Bending of a Straight Pipe

The aim of this section is to study the accuracy of the displacement and rotation results produced by FENSA. The problem considered is that of a cantilever beam under a concentrated load, Figure 4.1. The beam has an annular cross section with $r_2/h = 3$ (r_2 : outside radius of the cross section, h : thickness of the pipe wall), and is subjected to bending due to a concentrated load acting on its free end. From elementary beam theory, the theoretical results for the maximum deflection and the maximum rotation

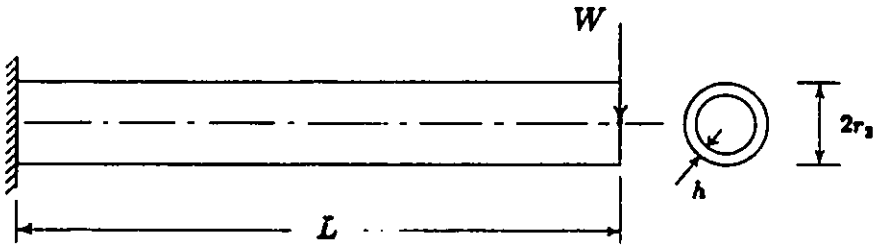


Figure 4.1: Cantilever beam under a concentrated load.

Table 4.1: Comparison of FENSA and beam-theory results for end deflection and rotation.

$\frac{L}{2r_2}$	$\frac{\delta-\delta_{th}}{\delta_{th}}$	$\frac{\gamma-\gamma_{th}}{\gamma_{th}}$
10	.005911	.00000
100	.000057	.00000
1000	.000002	.00000
10000	.000000	.00000

of the beam are

$$\delta_{th} = \frac{WL^3}{3EI}$$
$$\gamma_{th} = \frac{WL^2}{2EI}$$

where E is the Young's modulus of the material, I the second moment of the area of the cross section, L the total length of the beam, and W the concentrated load.

Table 4.1 presents a comparison of the FENSA and theoretical results when the beam is simulated by a one-element model. For $L/2r_2 = 10$, the FENSA prediction for the end deflection is only 0.6% different from the theoretical one while the two predictions for the end rotation are virtually identical. When the ratio $L/2r_2$ becomes greater, the FENSA predictions quickly approach the theoretical values because the Saint Venant effects tend to diminish. With the agreement obtained between the FENSA and beam-theory results for deflection and rotation, the elbow element looks promising in problems with more complex geometries.

4.3 Elastic In-Plane Bending of a Flanged 90^0-Pipe Bend

The problem of a flanged 90^0 bend under a closing bending moment has been solved by Whatham [85] using thin shell theory. Whatham took into account the interaction effects due to the flanged end of the bend, and subsequently supported his theoretical work experimentally. In the present study, the bend is analyzed again as a check problem for FENSA in the linear range. Stress response, end rotations, and ovalizations for various radii of bend curvature are considered.

On considering the plane of symmetry at $\theta = 45^0$ with θ measured from the flanged end, only half of the 90^0 bend needs to be modelled. The resulting 45^0 bend is represented by 6 elements with a total of 19 nodal points as shown in Figure 4.2(b).

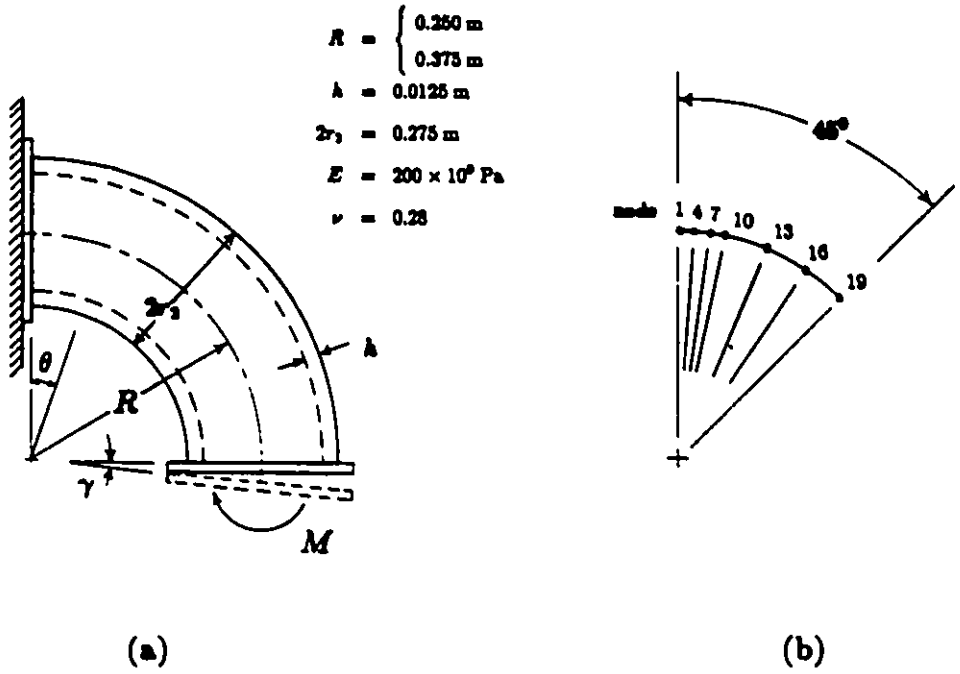


Figure 4.2: Geometry and modelling of a flanged 90^0 elbow.

To effectively represent the behaviour of the bend, 3 elements of 3.8^0 each are used to model the part near the flanged end, and 3 additional elements of 11.2^0 each for the remaining part of the bend. The interaction effects mainly due to the flanged end are accounted for using the penalty procedure as discussed in [4]. All geometric dimensions and material properties of the pipe bend are given in Figure 4.2(a).

The results for local stresses at $\theta = 45^0$ on the outside surface of the bend are given in Figures 4.3 - 4.6 for two different sizes of pipe bends, $R = 0.250\,\mathrm{m}$ and $R = 0.375\,\mathrm{m}$. The circumferential stress $\sigma_{\xi\xi}$ and the longitudinal stress $\sigma_{\eta\eta}$, both being expressed in a non-dimensional form

$$\sigma^*_{\xi\xi} = \frac{\sigma_{\xi\xi}}{M}\pi r_m^2 h$$
$$\sigma^*_{\eta\eta} = \frac{\sigma_{\eta\eta}}{M}\pi r_m^2 h$$

are plotted against the angle of cross section ϕ (r_m : mean radius of the cross section).

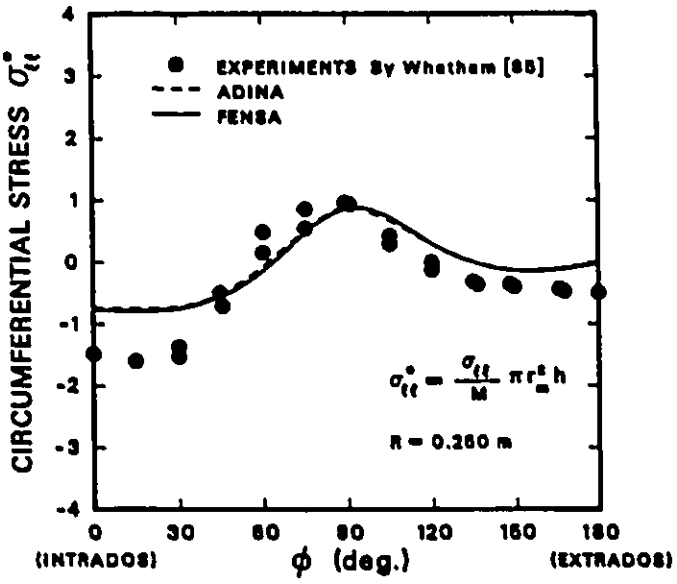


FIG. 4-3 PREDICTED CIRCUMFERENTIAL STRESSES AT θ=45° AND ON OUTSIDE SURFACE FOR R=0.260 m.

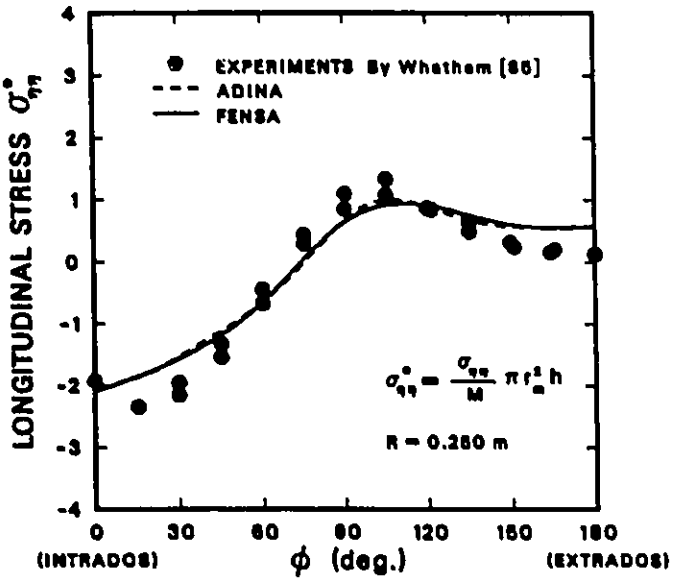


FIG. 4-4 PREDICTED LONGITUDINAL STRESSES AT θ=45° AND ON OUTSIDE SURFACE FOR R=0.260 m.

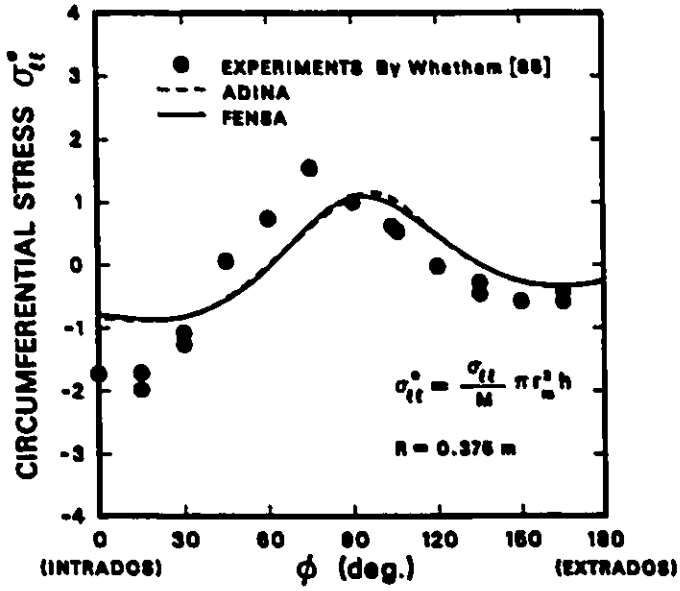


FIG. 4-5 PREDICTED CIRCUMFERENTIAL STRESSES AT θ=45° AND ON OUTSIDE SURFACE FOR R=0.375 m.

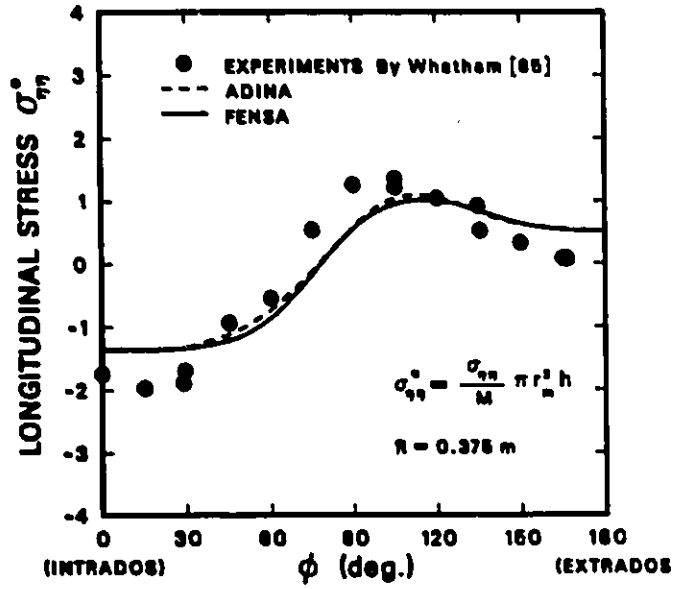


FIG. 4-6 PREDICTED LONGITUDINAL STRESSES AT θ=45° AND ON OUTSIDE SURFACE FOR R=0.375 m.

For $R = 0.250\,\mathrm{m}$ (Figures 4.3 and 4.4), there is good agreement between the FENSA results and experimental data obtained by Whatham for $40^0 \leq \phi < 180^0$. When $0^0 \leq \phi \leq 40^0$, FENSA tends to underestimate $\sigma^*_{\xi\xi}$ by up to 55% at $\phi = 0^0$ (intrados). Discrepancies of such a magnitude in the stress results are also found in the vicinity of $\phi = 180^0$ (extrados). For $R = 0.375\,\mathrm{m}$ (Figures 4.5 and 4.6), only qualitative agreement is obtained between the two sets of results. The FENSA predictions for $\sigma^*_{\xi\xi}$ and $\sigma^*_{\eta\eta}$ are found to deviate noticeably from the measurements when $0^0 \leq \phi \leq 90^0$. A comparison between the stress results for $R = 0.250\,\mathrm{m}$ and $R = 0.375\,\mathrm{m}$ also shows a tendency that the differences between the FENSA predictions and the experimental measurements become larger as R increases.

It should be pointed out from these figures that due to the effects of the flanged end and of the cross-section ovalizations the $\sigma^*_{\eta\eta}$ distribution just obtained is substantially different from the one predicted by the curved-beam theory. $\sigma^*_{\eta\eta}$ is observed to be most negative at $\phi = 0^0$ and most positive at $\phi = 95^0$, approximately. Yet, $\sigma^*_{\xi\xi}$ that was previously considered to be insignificant is in fact comparable in magnitude to $\sigma^*_{\eta\eta}$.

Figure 4.7 shows the results for the flexibility factor defined by

$$f_\gamma = \frac{2\gamma}{M}\frac{r_m^3 Eh}{R}$$

where γ is the end rotation, and M the applied moment. In general, there is good correspondence between the FENSA results and the experimental data. At $R = 0.375\,\mathrm{m}$, the FENSA prediction is found to be only about 10% lower than the measured value.

Presented in Figure 4.8 are the results for the diameter change factor defined by

$$f_d = \frac{2\pi Ehr_m}{\delta_d}$$

with δ_d being the change in the outside diameter of the cross section (at $\theta = 45^0$). The FENSA predictions are generally lower than measurements. The difference ranges from 33% at $R = 0.250\,\mathrm{m}$ to 40% at $R = 0.375\,\mathrm{m}$ for the most contracted parts of the

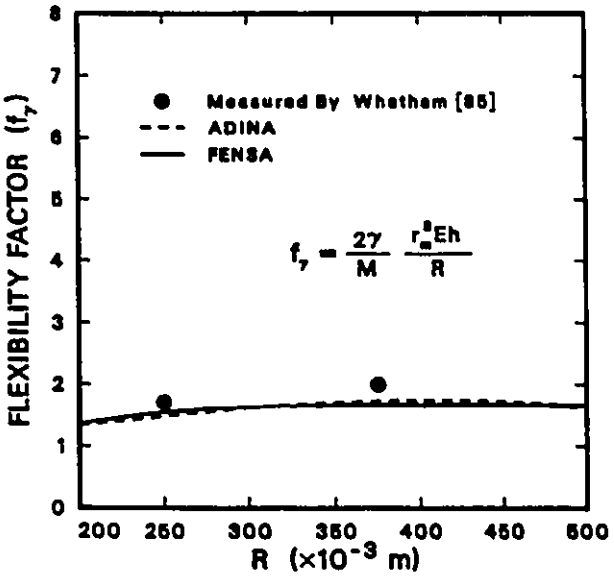


FIG. 4-7 PREDICTED FLEXIBILITY FACTORS OF FLANGED ELBOWS.

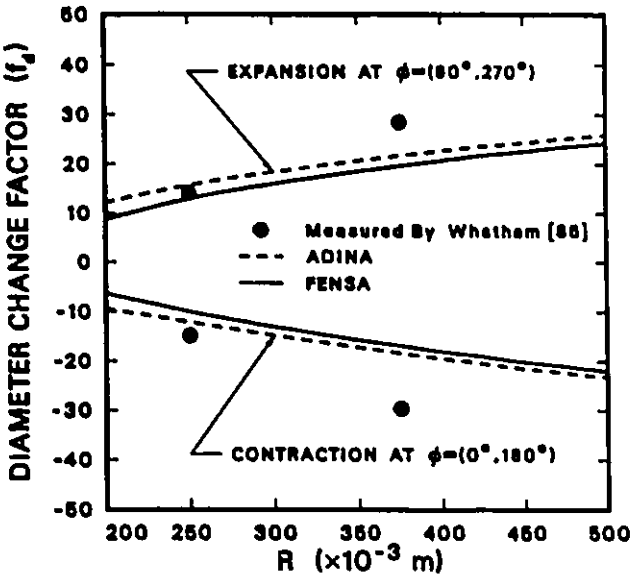


FIG. 4-8 PREDICTED DIAMETER CHANGE FACTORS OF FLANGED ELBOWS ($f_d = 2\pi Ehr_m/\delta_p$).

Table 4.2: Convergence rate of solution.

Flexibility factor (f_γ)				
Experiment	FENSA			
	Number of elements			
	4	6	12	18
2.00	1.66433	1.66608	1.66692	1.67114
Difference (%)	16.78	16.70	16.65	16.44

cross section (intrados and extrados). This range of differences is observed to be a bit smaller for the most expanded parts ($\phi = 90^0$ and $\phi = 270^0$).

It is noted that in Figures 4.3 – 4.8 the results from the commercial finite element program ADINA are also given to indicate the validity of the program FENSA. The two sets of FEM results for $\sigma^*_{\xi\xi}$, $\sigma^*_{\eta\eta}$, f_γ, and f_d are found to be virtually identical, especially in Figures 4.3 – 4.7 where the ADINA and the FENSA curves almost coincide. This excellent agreement further validates the program FENSA. It also implies that there are limitations to the capabilities of the elbow element in predicting stresses and ovalizations.

In Table 4.2, the accuracy of the FENSA prediction for the flexibility factor f_γ with $R = 0.375\,\mathrm{m}$ is monitored as the number of elbow elements is varied. This table shows that the same order of accuracy can be obtained with a mesh having as few as 4 elements. The difference between the FENSA prediction and the measurement is about 16.78% for a mesh of 4 elements, and decreases to 16.44% for a mesh of 18 elements.

4.4 Elastic-Plastic Bending of a Straight Pipe

Thin-walled tubes and pipes subjected to various loading conditions are critical components of power systems and other tubular structures. Basing their work on the Bernoulli-Euler theory, Lau et al. [86] have presented a closed-form solution for the small-deflection analysis of straight thin-walled pipes under the simultaneous actions of bending and twisting moments. Their theoretical results are a useful supplement to finite difference and finite element computer codes for preliminary design and analysis. Nevertheless, since Lau et al. have not taken changes in ovality of the cross section into consideration in the solution, their results are only good in early stages of the solution.

A straight thin-walled pipe subjected to pure bending only is considered in the

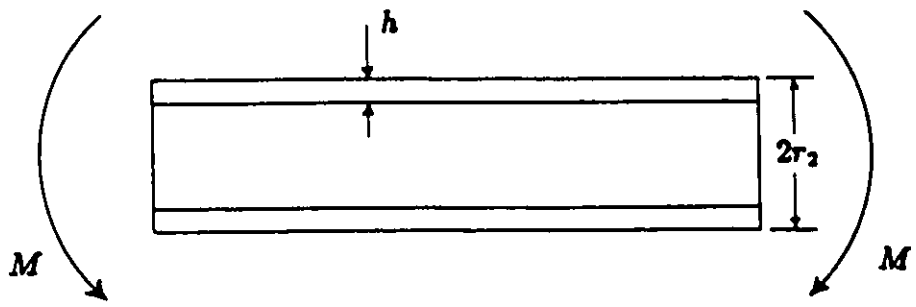


Figure 4.9: Thin-walled straight pipe under pure bending.

present study (Figure 4.9), and the FENSA solution obtained is compared with those given in Reference [86]. Due to its geometric symmetry, the problem can be modelled as a cantilever beam as in Verification Problem 1, except that the cross section at the fixed end of the beam is now allowed to ovalize, and that a bending moment replaces the concentrated load. A total of 7 equal-sized elements and 22 nodal nodal points are used to simulate the beam. All FENSA results presented are those on the plane of symmetry (fixed end).

The material constants of the pipe are shown in Figure 4.10. In order to make use of the results given by Lau et al., the material is assumed to have a bilinear elastic-plastic behaviour. The moment-curvature results in Figure 4.11 show that there is agreement between the FENSA solution and Lau et al.'s. Up to the elastic limit, an identical response is predicted by both solutions, and at $\kappa/\kappa_y = 8$ only a 7% difference is found in M/M_y predictions. However, these predictions are likely to be inaccurate for large plastic bending moments, especially those after collapse, since both solutions are based on beam theory which neglects effects of cross-sectional deformations.

The results for the longitudinal stress $\sigma^*_{\eta\eta}$ is shown in Figure 4.12. The two solutions show good agreement, especially for $M/M_y = 1$, although differences become slightly larger as M/M_y increases beyond 1. Due to a discontinuity in the first deriva-

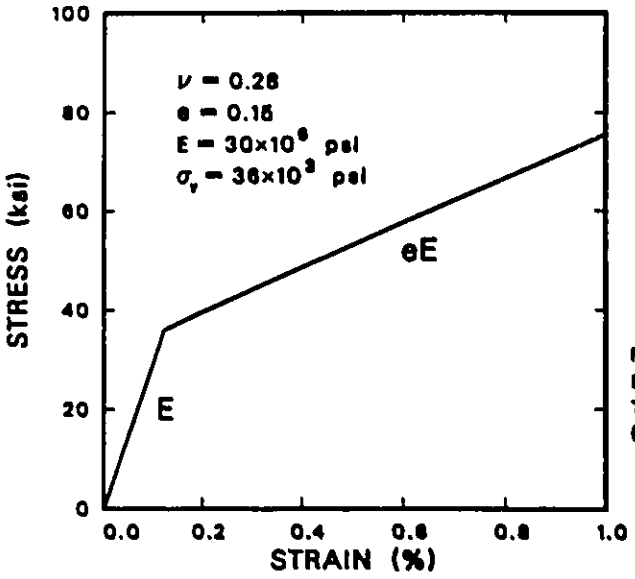


FIG. 4-10
BILINEAR REPRESENTATION OF UNIAXIAL TENSILE STRESS-STRAIN CURVE.
(1 ft = 0.3048 m, 1 psi = 6895 Pa)

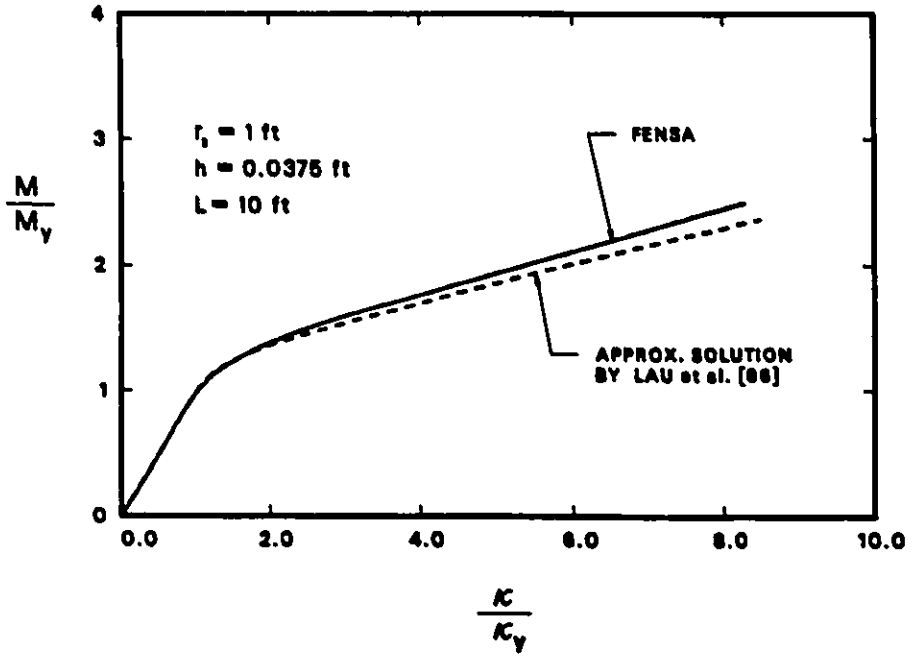


FIG. 4-11 PREDICTED MOMENT-CURVATURE RESULTS FOR STRAIGHT PIPES UNDER PURE BENDING (M_y : LIMIT LOAD FOR YIELD INITIATION).

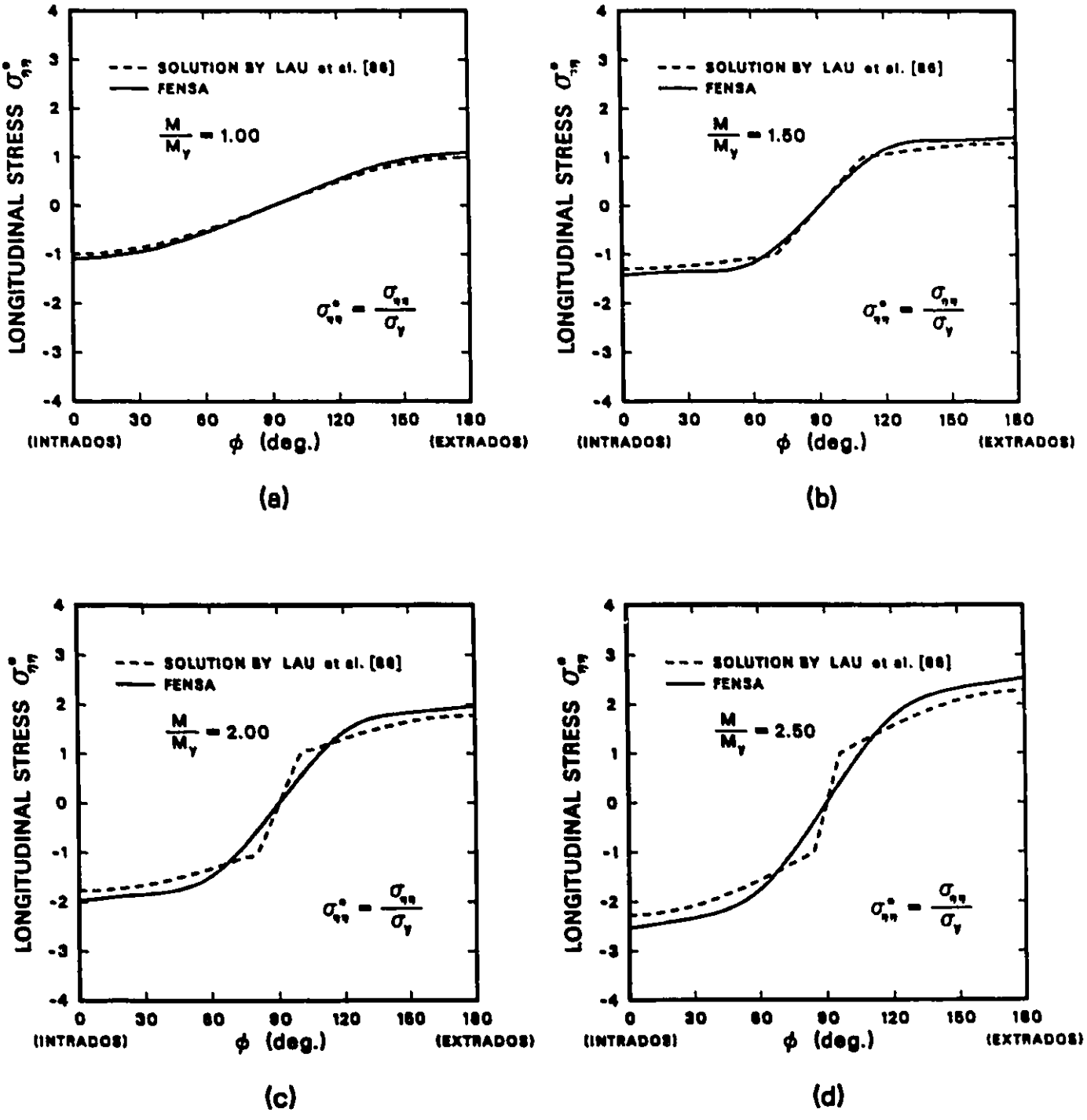


FIG. 4-12 COMPARISON OF FENSA AND THEORETICAL RESULTS FOR THE LONGITUDINAL STRESSES IN THE MIDDLE SURFACE OF THE PIPE WALL AT VARIOUS LOADINGS.

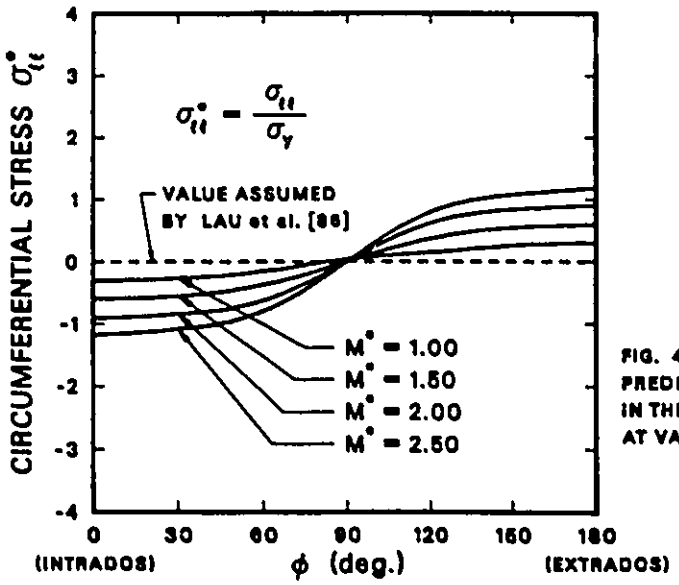


FIG. 4-13
PREDICTED CIRCUMFERENTIAL STRESSES IN THE MIDDLE SURFACE OF PIPE WALL AT VARIOUS LOADINGS ($M^* = M/M_y$).

tive of the theoretical solution for $\sigma^*_{\eta\eta}$, at any value of $M/M_y > 1$ two sharp bends near $\phi = 90^0$ are observed in the theoretical curve, creating large differences between the theoretical and the FENSA curves. At these bends and their vicinities, the solution given by Lau et al. likely overestimates the value of $\sigma^*_{\eta\eta}$. It is also found that the compressive $\sigma^*_{\eta\eta}$ at $\phi = 0^0$ (intrados) and the tensile $\sigma^*_{\eta\eta}$ at $\phi = 180^0$ (extrados) are virtually the same, and their value is proportional to the applied bending moment M/M_y.

The results for the circumferential (hoop) stress $\sigma^*_{\xi\xi}$ are presented in Figure 4.13. The values calculated by FENSA are not small nor zero as predicted by Lau et al. in their analysis. These non-zero hoop stresses are mainly due to the bending of the pipe wall as the cross section ovalizes. Figure 4.13 shows that the maximum value of $\sigma^*_{\xi\xi}$ could range from about 0.3 for $M/M_y = 1.0$ to 1.2 for $M/M_y = 2.5$. Thus, in design of piping structures where the hoop stress in the pipe usually plays a central role in causing pipe cracks, the utility of Lau et al.'s solution is doubtful.

The present comparison indicated that the solutions given by FENSA and Lau et al. for the the moment-curvature relationship and the longitudinal stresses are almost equivalent because both are formulated upon beam theory. Differences between predictions for the circumferential stresses are due to small ovalizations of the cross section according to von Karman's hypotheses.

4.5 Elastic-Plastic Bending of a 180^0 U-Bend

Thin-walled pressurized pipe bends are used extensively in nuclear reactors. These elbows are normally safe if only subjected to bending moments due to the designed working conditions. Under exceptional loadings such as those which arise in earthquakes, the elbows may be under excessive closing or opening moments, or a series of alternate closing and opening moments. Such loadings could lead to failure of the bends and eventually of the whole piping system.

A number of investigations have been carried out on the resistance of pipe bends towards bending moments prior to collapse. Bolt et al. [87] conducted a series of tests to determine the collapse moments for 90^0-pipe elbows of different sizes. Vrillon et al. [88] obtained good agreement between experimental data and numerical results from the FEM program TRICO for different U-bends. They found that FEM analysis using 3-D triangular shell elements is very quick and considerably cheaper than full-scale testing of such structures. Roche et al. [89] proposed a simplified method to carry out elastoplastic design of piping systems, with sufficient accuracy and at low cost. The proposed method was implemented in TEDEL which is also a FEM program based on a simplified nonlinear beam model. Bushnell [90] investigated failures of elbows due to the buckling of pipe wall using a modified version of the computer program BOSSOR5. This program simulates the bending of a straight or curved pipes by means of thermal loading.

In the present study, a 180^0 U-bend earlier tested by Vrillon et al. is analyzed for

end deflections and strains. The elbow is subjected to an in-plane opening moment as shown in Figure 4.14. Owing to symmetry of the U-bend only half of it is considered in the FEM analysis. The resulting 90^0 bend is modelled by a total of 8 equal-sized elements and 25 nodal points, and ovalizations of the cross section at the fixed end are also allowed. To incorporate data from the stress-strain curve of the material obtained from a tensile test into the program FENSA, the original curve is replaced by a multilinear approximation given in Figure 4.15.

In Figure 4.16, the FENSA calculations for the expansion of the bend diameter D as the applied bending moment increases are compared with experimental data and FEM results from TRICO and TEDEL. There is good agreement between all numerical results and measurements for $0 \leq M \leq 16 \times 10^4$ mN while yielding is observed to begin at about $M = 8 \times 10^4$ mN. Both TEDEL and FENSA overestimate the expansion ΔD for $M \geq 16 \times 10^4$ mN. When M is in this range, the accurate predictions given by TRICO for ΔD can be accounted for by the fact that TRICO employed 3-D triangular shell elements with a very fine mesh (shown in [88]) while TEDEL and FENSA used only pipe-like elements with a relatively small number of elements. It is further noticed that TEDEL [89] required twice as many elements as did FENSA to model the same bend; yet, the cross-sectional deformation of the elbow element implemented in FENSA was not accounted for in the computation process though it might be significant. The advantage of TEDEL may explain the observation that the TEDEL curve is closer to the experimental points than is the FENSA curve.

The results for axial strains and hoop strains at the plane of symmetry when $\Delta D = 76$ mm are presented in Figures 4.17 and 4.18, respectively. The experimental measurements were recorded by Bung et al. [91], and the BOSOR5 results were obtained by [90] as mentioned earlier. In general, there is good correlation between FENSA calculations, tests, and BOSOR5 results for both axial and hoop strains. The program FENSA underestimates the outer axial strain when $100^0 \leq \phi \leq 130^0$ while overestimating the outer hoop strain when $80^0 \leq \phi \leq 110^0$, approximately. In both

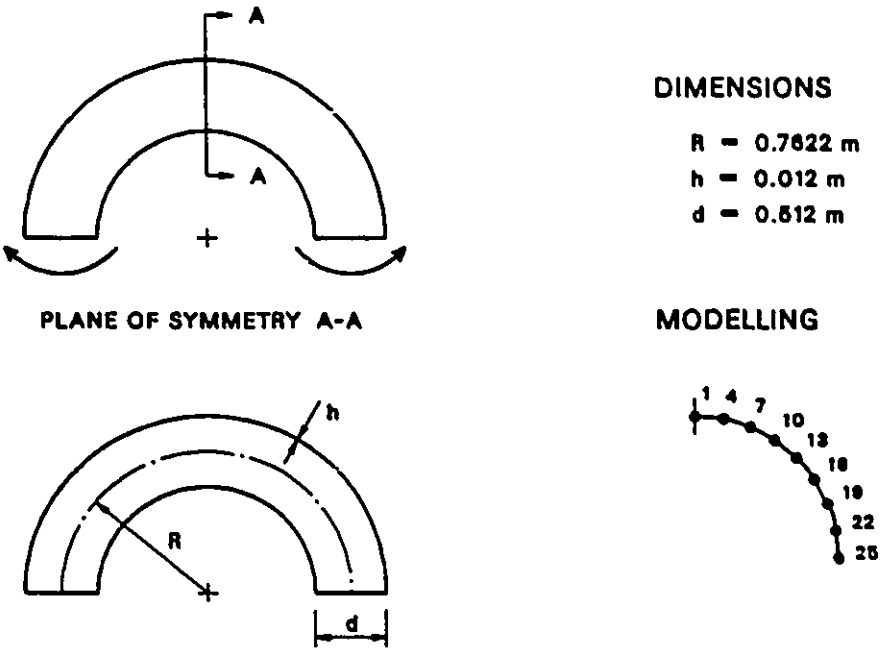


FIG. 4-14 GEOMETRY AND FINITE ELEMENT MODELLING OF A 180° U-BEND UNDER OPENING MOMENT.

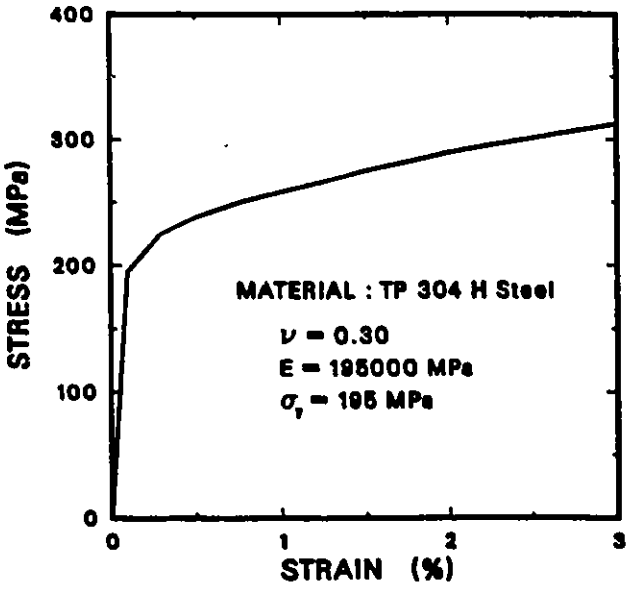


FIG. 4-15
MULTILINEAR APPROXIMATION OF UNIAXIAL TENSILE STRESS-STRAIN CURVE.

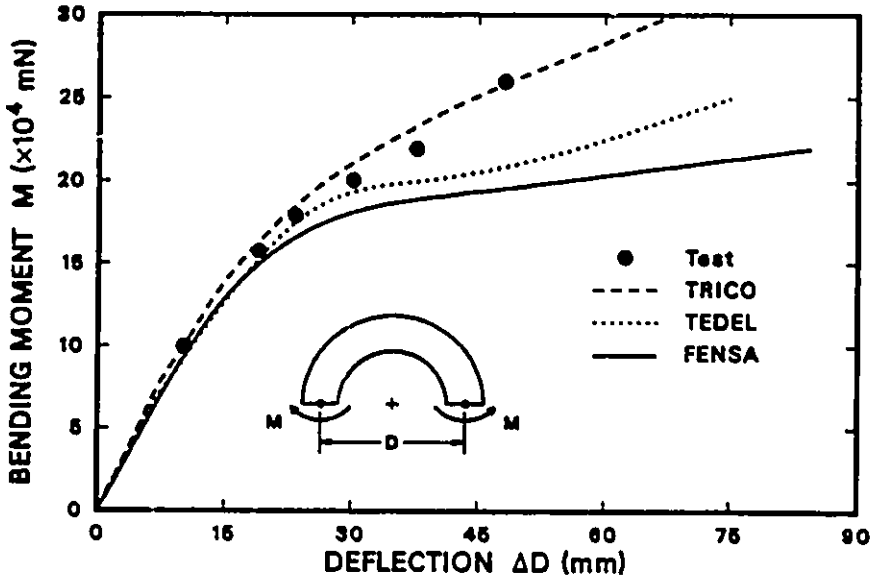


FIG. 4-16 PREDICTED MOMENT-DEFLECTION RESULTS FOR A 180° U-BEND UNDER OPENING MOMENT.

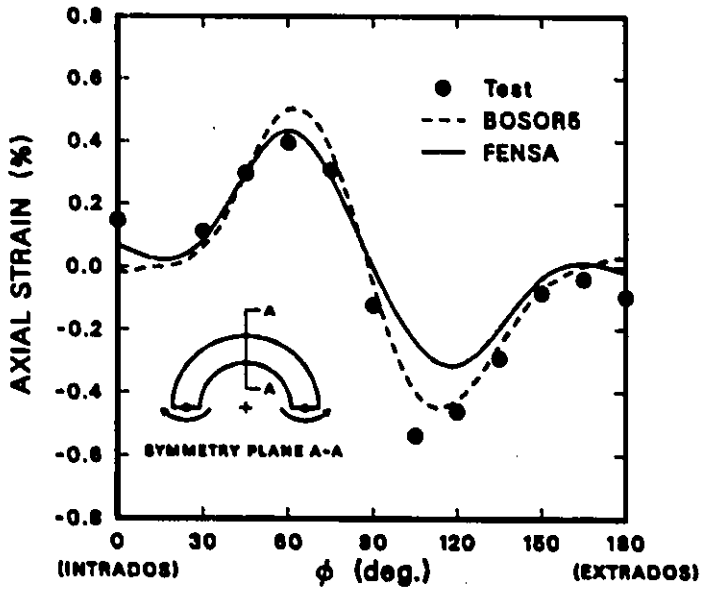


FIG. 4-17 COMPARISON OF FENSA AND EXPERIMENTAL RESULTS FOR OUTER AXIAL STRAINS AT PLANE OF SYMMETRY AND AT ΔD = 76 mm.

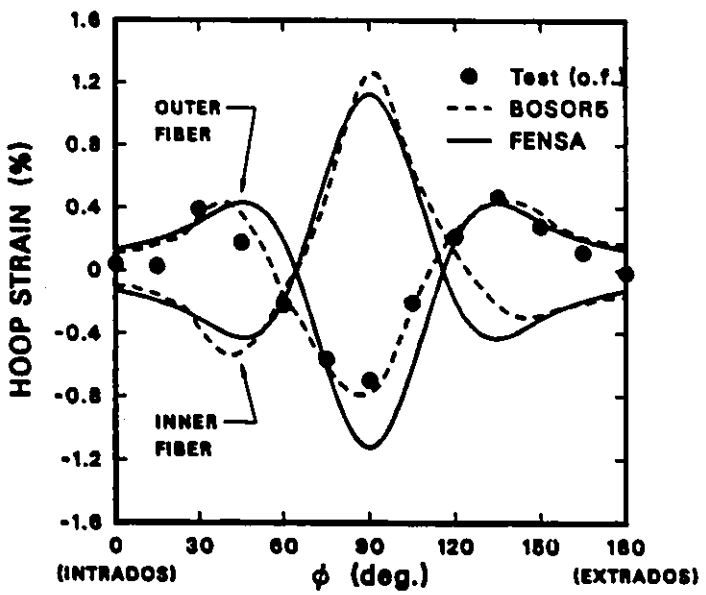


FIG. 4-18 COMPARISON OF FENSA AND EXPERIMENTAL RESULTS FOR HOOP STRAINS AT PLANE OF SYMMETRY AND AT ΔD = 76 mm.

cases, the discrepancies are observed to be as high as 50%. FENSA also predicts identical magnitudes for the hoop strains on the inner and outer surfaces of the pipe as can be seen from Figure 4.18. However, a comparison between the FENSA and BOSOR5 results shows that the hoop strain on the inner surface of the bend is better predicted by FENSA than is the hoop strain on the outer surface.

4.6 Elastic-Plastic Bending of a 90^0-Elbow with Tangent Pipes

Piping systems consist of straight and curved pipes (elbows), integrated in a complex manner in order to function properly. Those essential components were studied individually in previous sections for their elastic and elastoplastic responses resulting from in-plane bending of the structures. To further verify the program FENSA, this section is devoted to an elastic-plastic analysis of a combination of straight and curved pipes.

The structure to be analyzed is a 90^0-pipe elbow, the two ends of which are welded to two straight pipes. These straight tangent pipes are of different lengths and thicknesses as shown in Figure 4.19(a). An in-plane closing moment is applied to the lower end of the structure whereas the upper end is fixed to a rigid wall. Sobel and Newman [92] have tested the structure, and then carried out a numerical analysis using the MARC pipe-bend element 17, with 34 intervals in the hoop direction. The MARC prediction for the collapse load is found to be 10% lower than the experimental value. Bathe et al. [5] have also considered this structure using the program ADINA in order to assess the accuracy of their newly developed elbow element in nonlinear analyses.

The whole structure is modelled with a total of 5 elements using the program FENSA. Three elements each spanning 30^0 are used to simulate the 90^0 elbow, and each of the two straight segments is simulated by one element, Figure 4.19(b). For

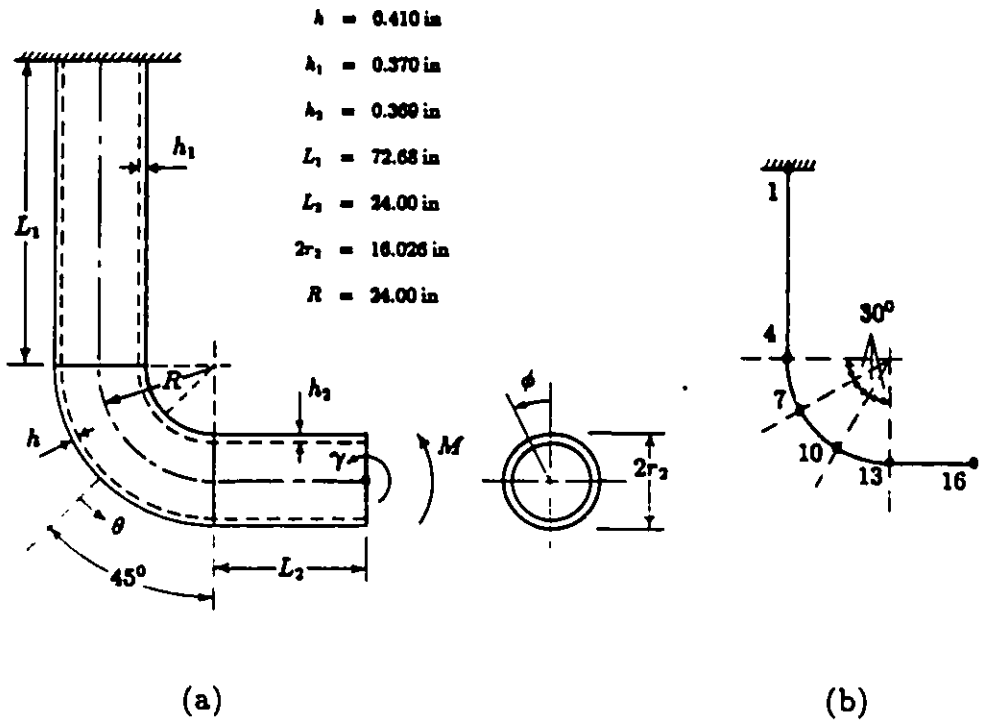


Figure 4.19: Geometry and FEM modelling of a 90^0-elbow structure.

computational purposes, the stress-strain curve obtained from a uniaxial tensile test is represented by a multilinear approximation as given in Figure 4.20.

The results for the vertical displacement at the loaded end, the end rotation, and the hoop strains on the inner surface of the elbow at $\theta = 0^0$ and at $\phi = (90^0, 270^0)$ are presented in Figures 4.21 - 4.23, respectively. It should be noted that in this problem the angle θ is measured from the plane of symmetry of the elbow, that is perpendicular to the center line. Excellent agreement is obtained between the FENSA and the experimental results for the end displacement and the end rotation up to the collapse load ($M = 1.28 \times 10^6$ in-lb). On the other hand, there is only a fair agreement between the FENSA results and experimental data for the hoop strains. As can be seen from Figure 4.23, FENSA overestimates the hoop strains after yielding has begun. Those discrepancies may be due to the fact that the shape of the cross section is not

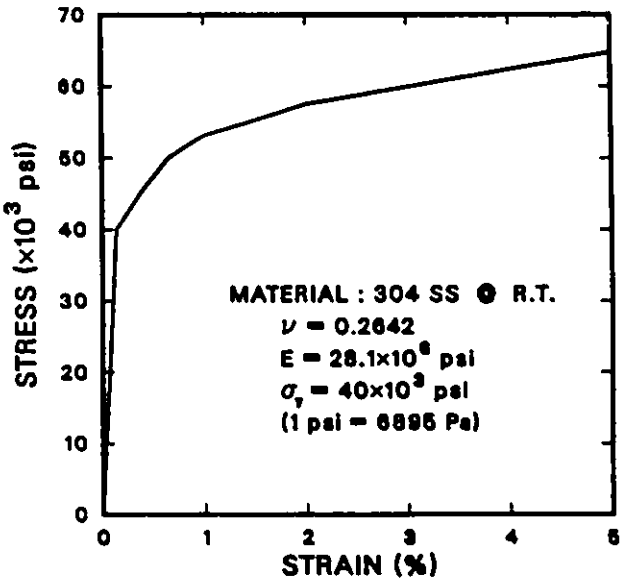


FIG. 4-20 MULTILINEAR APPROXIMATION OF UNIAXIAL TENSILE STRESS-STRAIN CURVE.

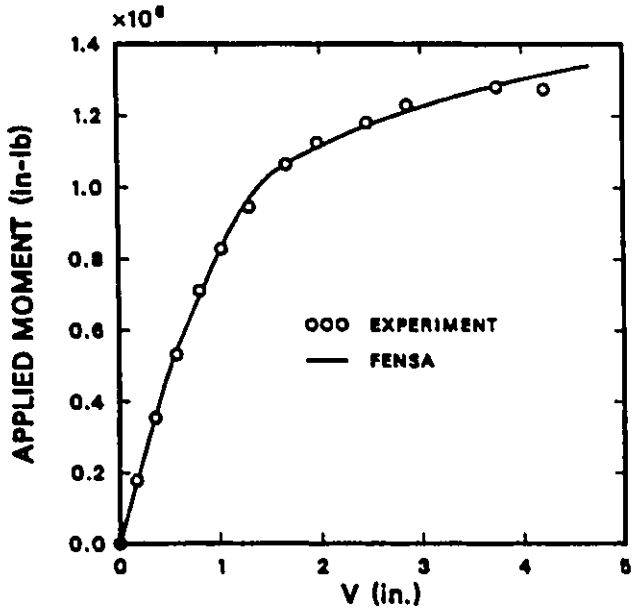


FIG. 4-21 COMPARISON OF FENSA AND EXPERIMENTAL MOMENT-END DISPLACEMENT RESULTS.

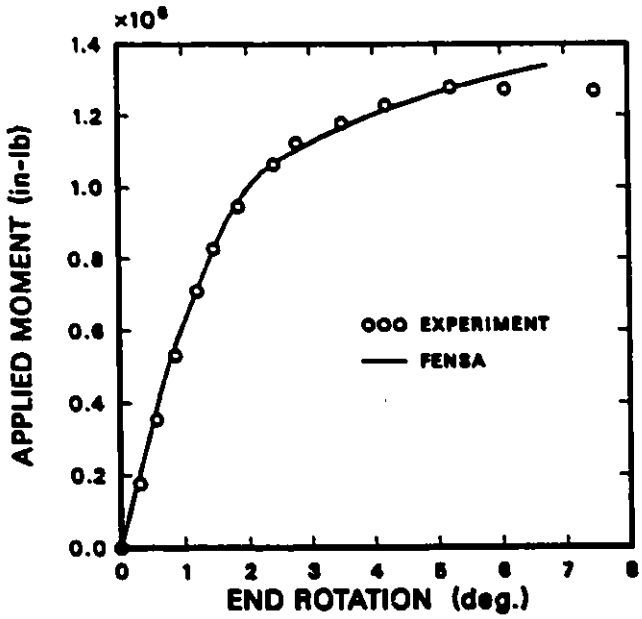


FIG. 4-22 COMPARISON OF FENSA AND EXPERIMENTAL MOMENT-END ROTATION RESULTS.

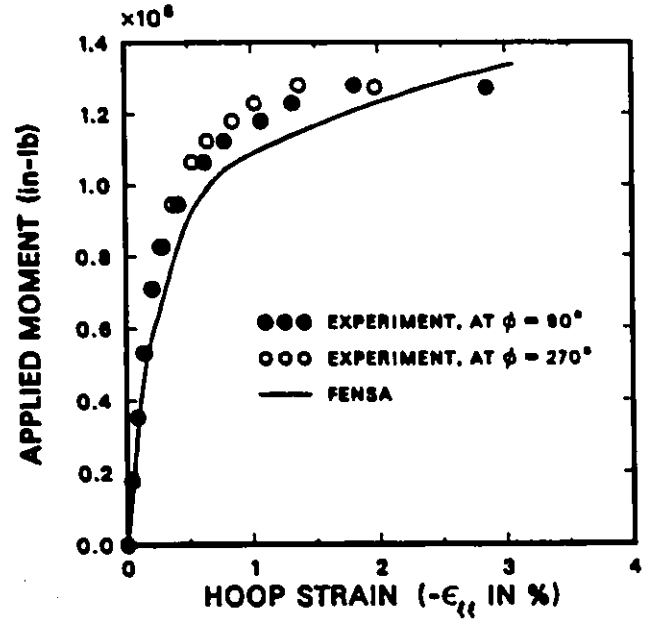


FIG. 4-23 COMPARISON OF FENSA AND EXPERIMENTAL MOMENT-HOOP STRAIN RESULTS (INNER SURFACE)

updated in the computation process.

Figures 4.24 - 4.27 give the results for ovalizations around the cross section and along the elbow length for various values of the bending moment M. Although experimental data for ovalizations are not available for comparison, the results obtained in Article 4.2 suggested that the actual radial displacements of the pipe wall could be about twice as high as the FENSA predictions. It is further noticed that while the FENSA predictions for the expansion and the corresponding contraction of a cross section of the elbow are nearly the same in elastic bending (Figure 4.24), the FENSA predictions for the expansion become much larger than those for the contraction in plastic bending. Figure 4.27 shows that for $M/M_y = 2.40$ the FENSA prediction for the maximum expansion (measured by $\Delta r/r_m$) at $\theta = 0^0$ is about 14% which is roughly twice that for the the maximum contraction of the same cross section. The trend that the expansion of a cross section is about twice the corresponding contraction in plastic bending was also reported by Prinja and Chitkara [93] when they carried out analyses of post-collapse plastic bending of thick pipes.

In Figures 4.28 - 4.30, the FENSA results for elastic-plastic boundaries are plotted on the inner and outer surfaces, and through the wall thickness in the plane of the pipe bend for different values of M. It is noted from these figures that no experimental results are presented for comparison, and that the FENSA predictions are obtained without the shape of the elbow cross section updated. It should also be recalled that along those boundaries the effective stress computed according to the von Mises yield criterion is equal to the yield stress of the material. At $M/M_y = 2.40$ while the FENSA curves in Figures 4.21 - 4.23 have not levelled off, Figures 4.28 - 4.30 show that the cross section at about $\theta = -15^0$ has completely deformed plastically (at $\theta = -15^0$, plastic zones cover both the inside and outside surfaces and through the wall thickness), thereby forming a plastic hinge at this position. Due to the effects of strain hardening of the material, the structure may not collapse at $M/M_y = 2.40$, or equivalently $M = 1.278 \times 10^6$ in-lb. However, the value of M/M_y at which the structure

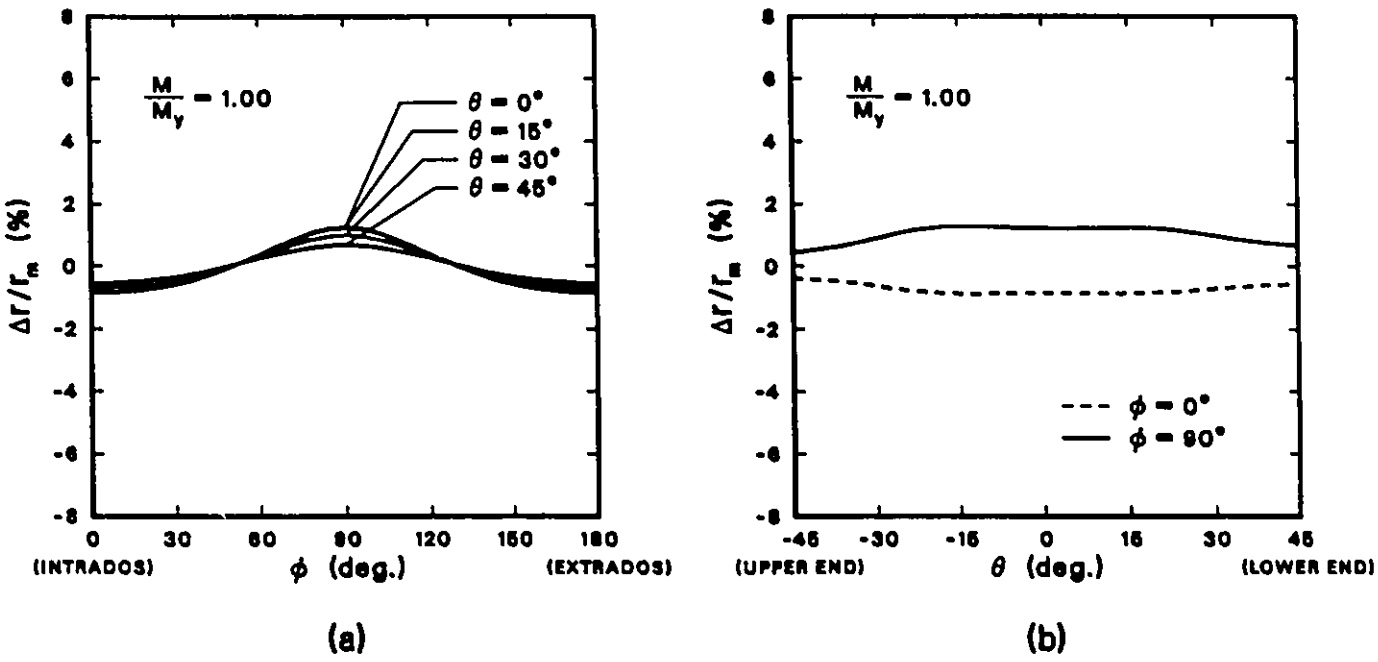


FIG. 4-24 PREDICTED CHANGES IN CROSS-SECTIONAL MEAN RADIUS (a) AROUND THE ELBOW CIRCUMFERENCE AND (b) ALONG THE ELBOW LENGTH AT M/M_y=1.00 (M_y : LIMIT LOAD FOR YIELD INITIATION).

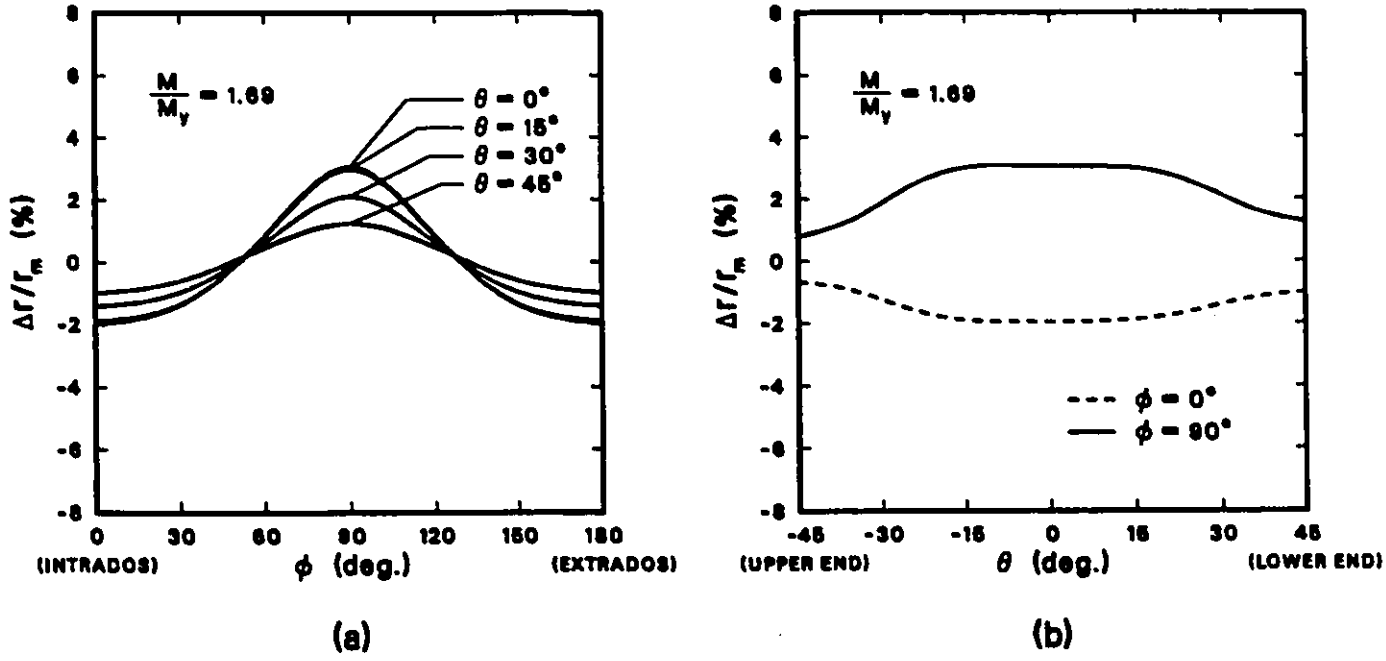


FIG. 4-25 PREDICTED CHANGES IN CROSS-SECTIONAL MEAN RADIUS (a) AROUND THE ELBOW CIRCUMFERENCE AND (b) ALONG THE ELBOW LENGTH AT M/M_y=1.69.

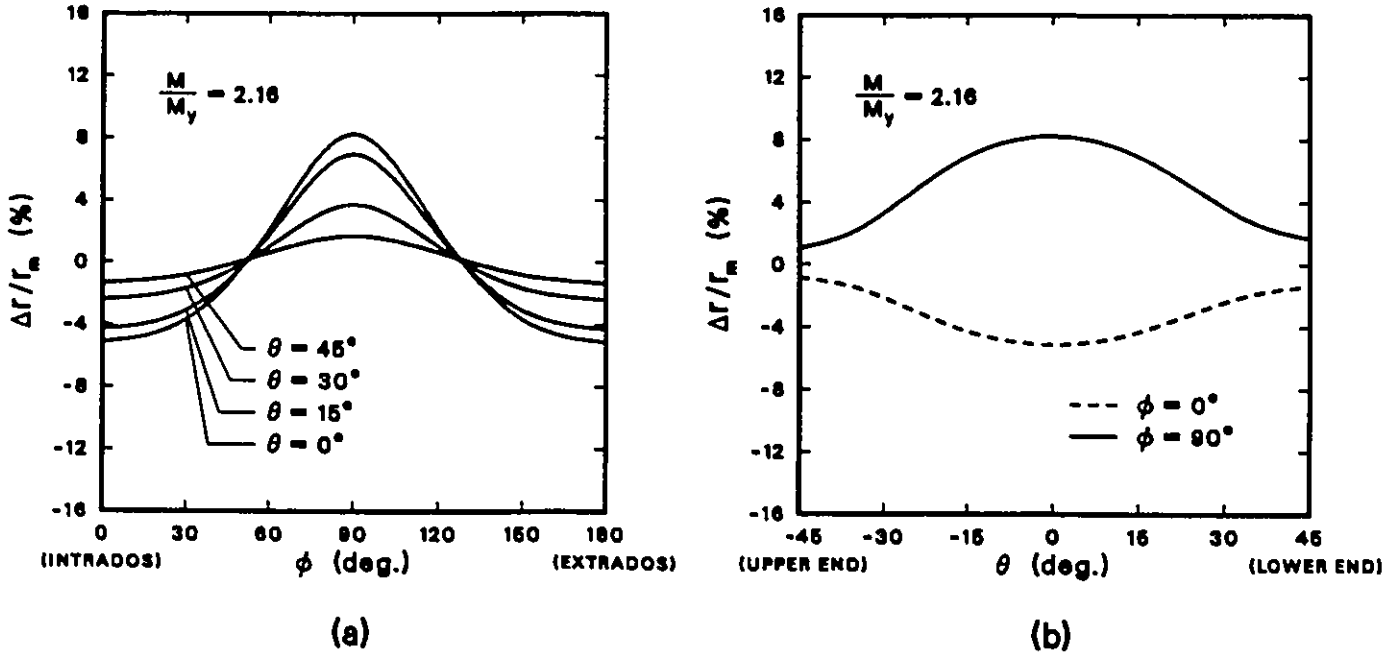


FIG. 4-26 PREDICTED CHANGES IN CROSS-SECTIONAL MEAN RADIUS (a) AROUND THE ELBOW CIRCUMFERENCE AND (b) ALONG THE ELBOW LENGTH AT M/M_y=2.16.

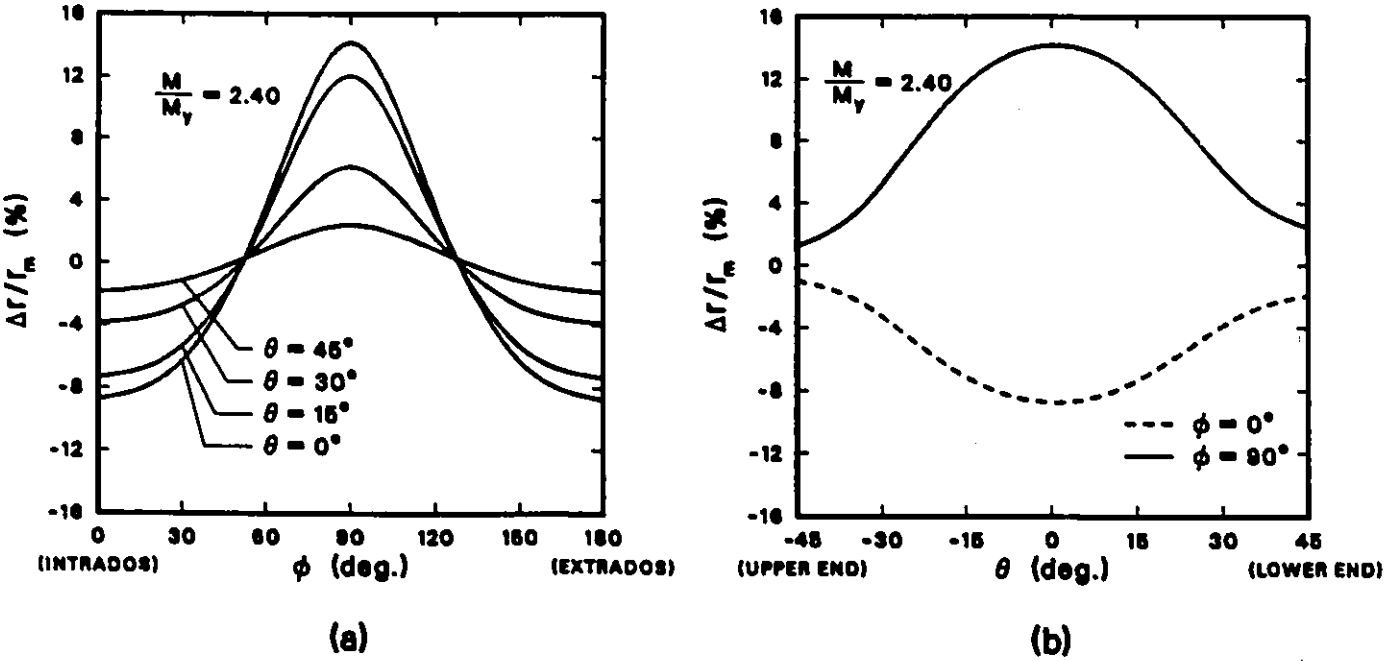
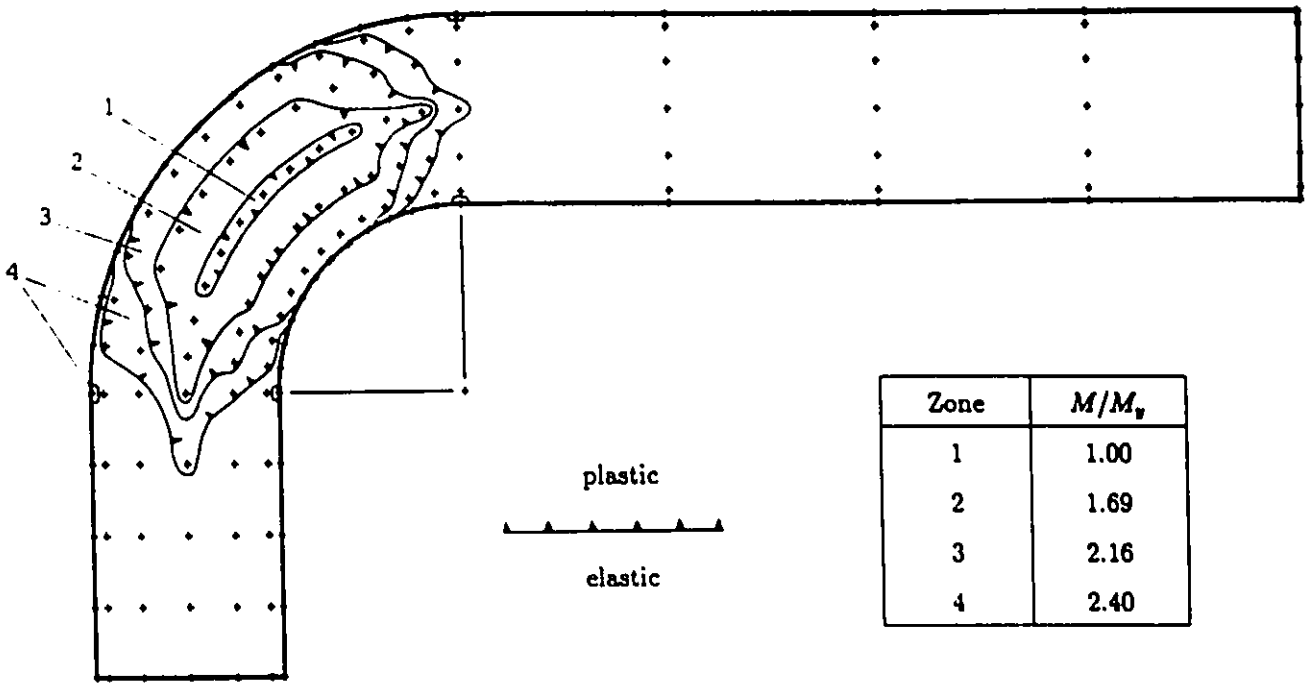
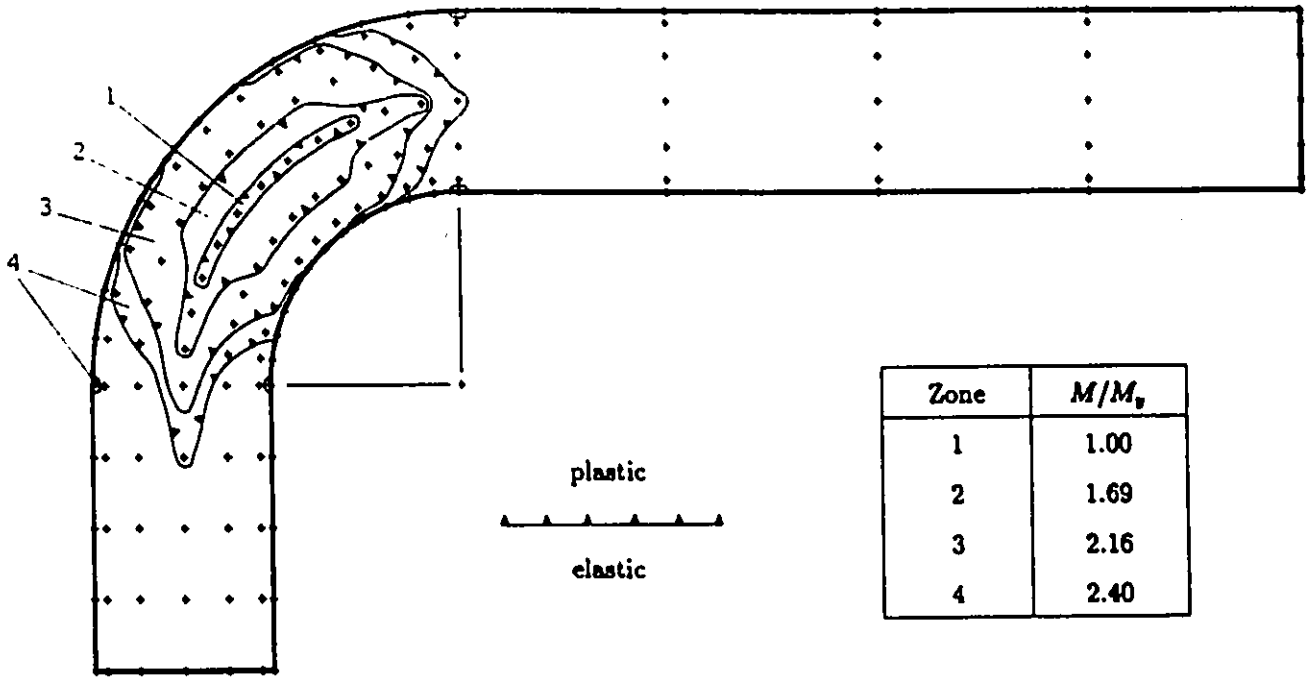


FIG. 4-27 PREDICTED CHANGES IN CROSS-SECTIONAL MEAN RADIUS (a) AROUND THE ELBOW CIRCUMFERENCE AND (b) ALONG THE ELBOW LENGTH AT M/M_y=2.40.



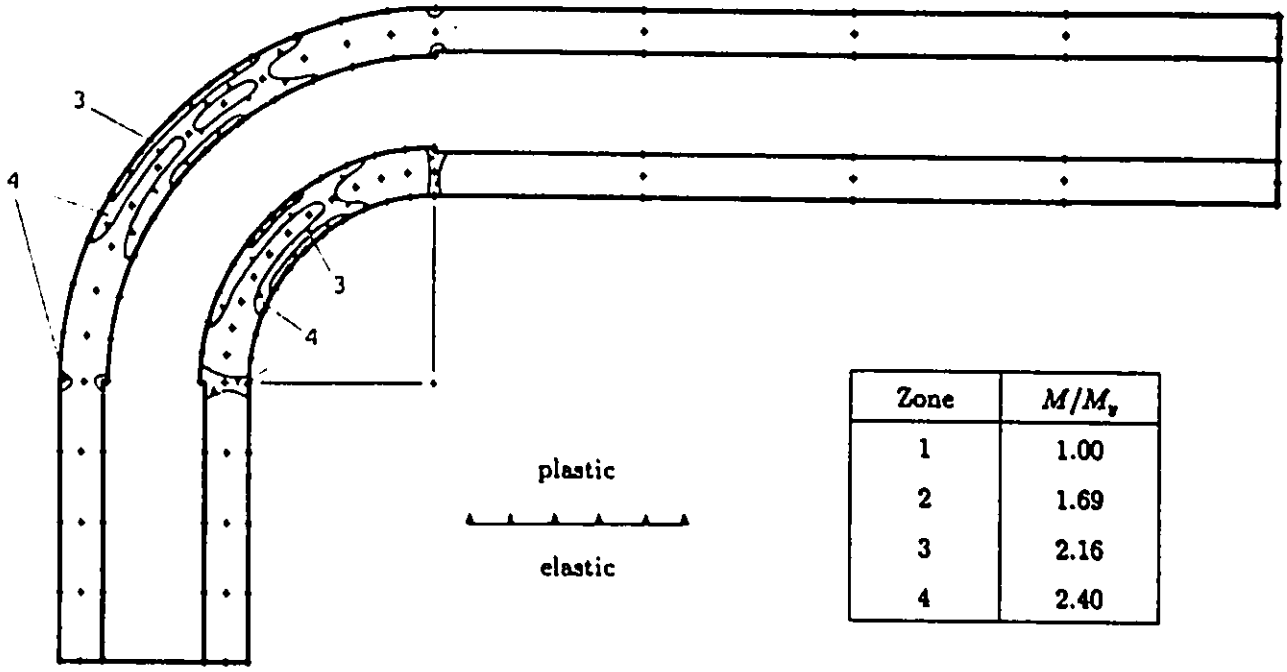
Zone	M/M_y
1	1.00
2	1.69
3	2.16
4	2.40

Figure 4.28: Growth of plastic zones on the outside surface.



Zone	M/M_y
1	1.00
2	1.69
3	2.16
4	2.40

Figure 4.29: Growth of plastic zones on the inside surface.



Zone	M/M_y
1	1.00
2	1.69
3	2.16
4	2.40

Figure 4.30: Growth of plastic zones through the wall thickness (the thickness is magnified 10 times inward).

Table 4.3: Comparison of collapse loads for a 90^0 elbow.

Collapse load ($\times 10^6$ in-lb)		Difference
Measured	FENSA	(%)
1.28	1.278	0.156

collapses is expected not to be far from 2.40. The fact that the collapse load measured by Sobel and Newman is 1.28×10^6 in-lb gives good agreement between the prediction given by FENSA and experimental data concerning the collapse load (Table 4.3).

Observations at $\phi = (90^0, 270^0)$ in Figures 4.28 – 4.29 reveal that yielding occurs simultaneously at those locations on both the inside and outside surfaces. This contradicts an earlier theoretical prediction given by Lang [94] based on the methods of toroidal elasticity. The prediction states that first yielding on the inside surface at $\phi = (90^0, 270^0)$ occurs just before that on the outside surface at the same locations. The contradiction stems from the fact that the hoop strain $\epsilon_{\xi\xi} = 0$ resulting from cross-section ovalizations is assumed to be zero at $\zeta = 0$. In fact, $\epsilon_{\xi\xi} = 0$ only when $\zeta > 0$ as pointed out by Munz and Mattheck [95]. If this adjustment were made in the formulation of the elbow element, such a contradiction would be eliminated.

It is also important to mention that the yielding which first occurs on the elbow at $\phi = (90^0, 270^0)$ is mainly due to circumferential bending stresses. This implies that, assuming the material to be free of irregularities, the first crack will be on the elbow along the line represented by $\phi = (90^0, 270^0)$ as has been observed experimentally by Gross [96].

It is noted that in plasticity the constitutive tensor for stresses and strains is stress-history dependent. For this reason, stresses cannot be directly calculated from experimentally recorded strains. Because of the unavailability of experimental data for stresses, the FENSA stress results in Figures 4.28 – 4.30 are presented merely as supplemental information that may be of general interest.

4.7 Summary of Comparison

In summary, the FENSA program developed herein has been applied to five typical piping problems. The main assumptions in these problems are that the thickness of the pipe (or pipe bend) is constant, and the cross section remains circular annular

throughout the analysis although small radial displacements are allowed.

- For the first two problems that dealt with elastic bending, FENSA calculations for displacements, rotations, ovalizations, and stresses were checked against beam-theory and experimental results. The FENSA predictions were good for displacements and rotations, fair for stresses, and poor for ovalizations.

- For the last three problems that involved plastic bending, the FENSA results for curvatures and stresses were compared with the solution by Lau et al. [S6]. Good correlation was obtained, but both solutions are likely to be inaccurate for large plastic bending since they are based on beam theory. The FENSA results for displacements, rotations, and strains were compared with experimental data. Good agreement was obtained between the FENSA predictions and experimental measurements for pre-collapse loads. So far, FENSA predictions for stresses in the nonlinear range have not been compared with experimental data or with those from a rigorous method.

- The curvature updating scheme presented in Article 3.4 is expected to improve the performance of the elbow element in nonlinear analyses although no comparison has been made with respect to the case where the curvature of the element is assumed to remain constant.

There are still a number of aspects that have not been touched upon in the above testing of FENSA, concerning computational difficulties and other types of loading. For instance, when a pipe is subjected to plastic bending due to a concentrated load, the resulting shear force in the pipe may cause slow convergence or even instability of the incremental solution. It is very important to point out further that due the limitations of the elbow element in modelling boundary conditions, any concentrated load applied to the pipe has to be modelled as if it were applied to the center line of the pipe. Thus, no buckling and no deformations of the pipe wall can be predicted by FENSA. These phenomena can only be studied with shell-type elements or special

elbow-shell elements [74,75]. Nevertheless, the utility of such elements raises the subject of "result accuracy and analysis cost", that was discussed in detail and clarified in Article 2.4. Therefore, investigations of such phenomena and the utility of any shell-like element are beyond the scope of the present study.

Chapter 5

Ram Pipe Bending

5.1 Introduction

The simulation of the ram pipe bending problem is covered in this chapter. The modelling is done using two methods; beam theory, with the elastic-plastic behaviour taken into consideration [97]; and FEM using the program FENSA. It should be noted that the simulation is only possible if in the ram pipe bending process some active means are taken to ensure that cross-sectional ovalizations are minimal so that the cross section can be assumed to remain circular annular. In addition, changes in the pipe-wall thickness are assumed to be small so that the thickness can be considered as constant in the analysis.

5.2 Simulation Using Elastic-Plastic Beam Theory

The elastic-plastic beam theory method (EPBT) to be used herein was first proposed by Roderick [98] to give an estimate to the plastic deflection of rectangular cross-section beams, with the material assumed to be elastic-perfectly plastic. Roderick and Heyman [99] then extended the theory to include the effect of strain hardening, and gave good experimental support to their theoretical work, particularly for mild steels.

The method has also been used in the structural analyses of beams of other symmetric cross sections [100] - [101] and of non-symmetric cross sections [102] - [103]. It should be noted that the above references by no means represent the complete literature survey of the beam theory approach; they only give a brief background of EPBT, and show how extensively EPBT has been used in elastic-plastic beam analyses.

In the present study, a beam of annular cross section (pipe) with strain hardening material is analyzed using EPBT, Figure 5.1(b). Four major assumptions are made in the analysis:

1. There is a linear variation of longitudinal strain $\epsilon_{\eta\eta}$ across the cross section.

2. Under bending, the only stress component that is significant is the longitudinal stress $\sigma_{\eta\eta}$. Thus yielding is presumed to occur when this stress component reaches the yield stress σ_y of the material. This assumption is only valid for thin-walled pipes.

3. The material has identical bilinear elastic-plastic behaviour for both tensile and compressive parts of the cross section, Figure 5.1(a).

4. The beam is considered as straight throughout the analysis.

It is important to mention that Assumption 2 may introduce errors into the solution due to the neglect of circumferential stresses, and the magnitudes of these errors are greatly dependent on the ovalizations of the cross section. Nevertheless, the assumption can be justified by the fact that in ram pipe bending the angle of rotation per unit pipe length is relatively small compared to those in other types of pipe bending. Accordingly, the cross-section ovalizations are also small. This justification becomes more obvious when the FENSA results are presented (Article 5.4).

Taking symmetry into account, only the upper half of the cross section (Figure 5.1(b)) will be referred to in the following calculations.

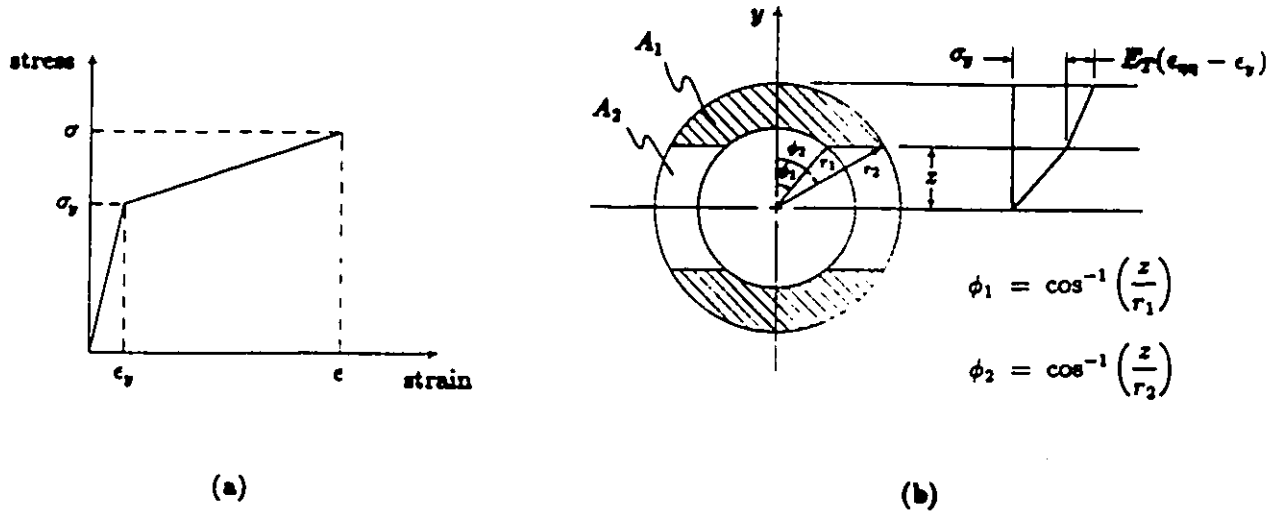


Figure 5.1: Elastic-plastic flexure of beam of annular cross section.

Bending Moment due to Stresses in Elastic Portion

In the elastic portion of the cross section, represented by A_2 of Figure 5.1(b), the longitudinal stress $\sigma_{\eta\eta}$ at any point P (not shown) is simply related to the longitudinal strain $\epsilon_{\eta\eta}$ by Hooke's law. Based on Assumption 2,

$$\sigma_{\eta\eta} = E\epsilon_{\eta\eta}$$

with E being the Young's modulus. According to Assumption 1, $\epsilon_{\eta\eta}$ can be expressed in terms of the yield strain ϵ_y as

$$\epsilon_{\eta\eta} = \epsilon_y \left(\frac{y}{z}\right) \tag{5.1}$$

where y is the vertical coordinate of the point P, and z the distance from the neutral axis to the elastic-plastic boundary.

Substituting the value of $\epsilon_{\eta\eta}$ into the expression of $\sigma_{\eta\eta}$ gives

$$\sigma_{\eta\eta} = E\epsilon_y \left(\frac{y}{z}\right) = \sigma_y \left(\frac{y}{z}\right)$$

The bending moment resulting from stresses in A_2 about the neutral axis is

derived from

$$
\begin{aligned}
M_2 &= \int_{A_2} y\sigma_{\eta\eta}\, dA \qquad \text{with} \quad y = \rho\cos\phi \\
&= 2\int_{\phi_1}^{\phi_2}\int_{r_1}^{z/\cos\phi} \sigma_y\left(\frac{y^2}{z}\right)\rho\, d\rho\, d\phi \;+\; 2\int_{\phi_2}^{\pi/2}\int_{r_1}^{r_2} \sigma_y\left(\frac{y^2}{z}\right)\rho\, d\rho\, d\phi \\
&= \frac{2\sigma_y}{z}\int_{\phi_1}^{\phi_2}\int_{r_1}^{z/\cos\phi} \rho^3\cos^2\phi\, d\rho\, d\phi \;+\; \frac{2\sigma_y}{z}\int_{\phi_2}^{\pi/2}\int_{r_1}^{r_2} \rho^3\cos^2\phi\, d\rho\, d\phi \\
&= \frac{2\sigma_y}{z}\int_{\phi_1}^{\phi_2}\left[\frac{\rho^4}{4}\cos^2\phi\right]_{r_1}^{z/\cos\phi} d\phi \;+\; \frac{2\sigma_y}{z}\int_{\phi_2}^{\pi/2}\left[\frac{\rho^4}{4}\cos^2\phi\right]_{r_1}^{r_2} d\phi \\
&= \frac{\sigma_y}{2z}\int_{\phi_1}^{\phi_2}\left[\frac{z^4}{\cos^2\phi} - r_1^4\cos^2\phi\right] d\phi \;+\; \frac{\sigma_y}{2z}\left(r_2^4 - r_1^4\right)\int_{\phi_2}^{\pi/2}\cos^2\phi\, d\phi \\
&= \frac{\sigma_y}{2z}\left[z^4\tan\phi - \frac{r_1^4}{2}\left(\phi + \frac{\sin 2\phi}{2}\right)\right]_{\phi_1}^{\phi_2} \;+\; \frac{\sigma_y}{4z}\left[\left(r_2^4 - r_1^4\right)\left(\phi + \frac{\sin 2\phi}{2}\right)\right]_{\phi_2}^{\pi/2} \\
M_2 &= \frac{\sigma_y}{8z}\left\{4z^4\left(\tan\phi_2 - \tan\phi_1\right) + \pi\left(r_2^4 - r_1^4\right) + r_1^4\left(2\phi_1 + \sin 2\phi_1\right) - r_2^4\left(2\phi_2 + \sin 2\phi_2\right)\right\}
\end{aligned}
$$

Here ϕ_1 and ϕ_2 denote the positional angles of the elastic-plastic boundaries on the inside and outside surfaces of the pipe, and r_1 and r_2 are the inside and outside radii of the cross section, respectively. It should be noted that since the pipe wall is thin, ϕ_1 is very close to ϕ_2 and therefore both ϕ_1 and ϕ_2 could have been replaced by a single ϕ. The reason that ϕ_1 and ϕ_2 are used is to clarify the mathematical derivation of the moment-curvature relationship.

Bending Moment due to Stresses in Plastic Portion

Due to the effect of strain hardening, once the material deforms past the yield point, $\sigma_{\eta\eta}$ is given by

$$\sigma_{\eta\eta} = \sigma_y + E_T\left(\epsilon_{\eta\eta} - \epsilon_y\right)$$

with E_T being the strain-hardening modulus. If $\epsilon_{\eta\eta}$ is again replaced by means of (5.1), then

$$\sigma_{\eta\eta} = \sigma_y + E_T\left[\epsilon_y\left(\frac{y}{z}\right) - \epsilon_y\right]$$

$$= \sigma_y + E_T\epsilon_y\left(\frac{y}{z}-1\right)$$

Like M_2, the bending moment M_1 from stresses in the plastic portion (A_1 of Figure 5.1(b)) is determined from

$$\begin{aligned}
M_1 &= \int_{A_1} y\sigma_{\eta\eta}\,dA \\
&= 2\int_0^{\phi_1}\int_{r_1}^{r_2} y\left[\sigma_y + E_T\epsilon_y\left(\frac{y}{z}-1\right)\right]\rho\,d\rho\,d\phi \\
&\qquad + 2\int_{\phi_1}^{\phi_1}\int_{z/\cos\phi}^{r_2} y\left[\sigma_y + E_T\epsilon_y\left(\frac{y}{z}-1\right)\right]\rho\,d\rho\,d\phi \\
&= 2\int_0^{\phi_1}\int_{r_1}^{r_2}\left[(\sigma_y - E_T\epsilon_y)\,\rho^2\cos\phi + \frac{E_T\epsilon_y}{z}\rho^3\cos^2\phi\right]d\rho\,d\phi \\
&\qquad + 2\int_{\phi_1}^{\phi_1}\int_{z/\cos\phi}^{r_2}\left[(\sigma_y - E_T\epsilon_y)\,\rho^2\cos\phi + \frac{E_T\epsilon_y}{z}\rho^3\cos^2\phi\right]d\rho\,d\phi \\
&= 2\int_0^{\phi_1}\left[(\sigma_y - E_T\epsilon_y)\frac{\rho^3}{3}\cos\phi + \frac{E_T\epsilon_y}{4z}\rho^4\cos^2\phi\right]_{r_1}^{r_2} d\phi \\
&\qquad + 2\int_{\phi_1}^{\phi_2}\left[(\sigma_y - E_T\epsilon_y)\frac{\rho^3}{3}\cos\phi + \frac{E_T\epsilon_y}{4z}\rho^4\cos^2\phi\right]_{z/\cos\phi}^{r_2} d\phi \\
&= 2\int_0^{\phi_1}\left[\frac{(\sigma_y - E_T\epsilon_y)}{3}\left(r_2^3 - r_1^3\right)\cos\phi + \frac{E_T\epsilon_y}{4z}\left(r_2^4 - r_1^4\right)\cos^2\phi\right]d\phi \\
&\quad + 2\int_{\phi_1}^{\phi_2}\left[\frac{(\sigma_y - E_T\epsilon_y)}{3}\left(r_2^3\cos\phi - \frac{z^3}{\cos^2\phi}\right) + \frac{E_T\epsilon_y}{4z}\left(r_2^4\cos^2\phi - \frac{z^4}{\cos^2\phi}\right)\right]d\phi \\
&= 2\left[\frac{(\sigma_y - E_T\epsilon_y)}{3}\left(r_2^3 - r_1^3\right) + \frac{E_T\epsilon_y}{8z}\left(r_2^4 - r_1^4\right)\left(\phi + \frac{\sin 2\phi}{2}\right)\right]_0^{\phi_1} \\
&\quad + 2\left[\frac{(\sigma_y - E_T\epsilon_y)}{3}\left(r_2^3\sin\phi - z^3\tan\phi\right) + \frac{E_T\epsilon_y}{8z}\left\{r_2^4\left(\phi + \frac{\sin 2\phi}{2}\right) - 2z^4\tan\phi\right\}\right]_{\phi_1}^{\phi_2} \\
M_1 &= \frac{2(\sigma_y - E_T\epsilon_y)}{3}\left[\left(r_2^3\sin\phi_2 - r_1^3\sin\phi_1\right) - z^3\left(\tan\phi_2 - \tan\phi_1\right)\right] \\
&\quad + \frac{E_T\epsilon_y}{8z}\left[r_2^4\left(2\phi_2 + \sin 2\phi_2\right) - r_1^4\left(2\phi_1 + \sin 2\phi_1\right) - 4z^4\left(\tan\phi_2 - \tan\phi_1\right)\right]
\end{aligned}$$

Total Bending Moment

Since M_1 and M_2 have been calculated for only one half of the cross section, the total bending moment M about the neutral axis will be twice the sum of M_1 and M_2, i.e.

$$\begin{aligned} M &= \frac{4(\sigma_y - E_T\epsilon_y)}{3}\left[\left(r_2^3\sin\phi_2 - r_1^3\sin\phi_1\right) - z^3(\tan\phi_2 - \tan\phi_1)\right] \\ &+ \frac{E_T\epsilon_y}{4z}\left[r_2^4(2\phi_2 + \sin 2\phi_2) - r_1^4(2\phi_1 + \sin 2\phi_1) - 4z^4(\tan\phi_2 - \phi_1)\right] \\ &+ \frac{\sigma_y}{4z}\left[4z^4(\tan\phi_2 - \tan\phi_1) + \pi\left(r_2^4 - r_1^4\right) + r_1^4(2\phi_1 + \sin\phi_1) - r_2^4(2\phi_2 + \sin 2\phi_2)\right] \end{aligned} \tag{5.2}$$

Figure 5.1(b) indicates that yielding first occurs at $z = r_2$, $\phi_1 = \phi_2 = 0$, and for this point Equation (5.2) reduces to

$$M = M_y = \frac{\pi\sigma_y}{4r_2}\left(r_2^4 - r_1^4\right) \tag{5.3}$$

which can also be found from the familiar formula $\sigma = Mc/I$ for bending stress.

It is shown in elementary texts on the Strength of Materials that the curvature $\kappa = 1/R$ of the beam at the cross section under consideration is given by

$$\kappa = \frac{\epsilon_{\eta\eta}}{y}$$

which is valid for $0 \leq \epsilon_{\eta\eta} \leq \epsilon_y$. In extreme cases with y denoting the distance from the neutral axis to the elastic-plastic boundary,

when $y = z$

$$\kappa = \frac{\epsilon_y}{z} \tag{5.4}$$

when $y = r_2$

$$\kappa_y = \frac{\epsilon_y}{r_2} \tag{5.5}$$

where κ_y is the curvature at initial yield in the cross section.

Dividing (5.4) by (5.5), the ratio of the two curvatures is found to be

$$\frac{\kappa}{\kappa_y} = \frac{r_2}{z} \tag{5.6}$$

By introducing the non-dimensional parameters

$$r = \frac{r_1}{r_2} \quad e = \frac{E_T}{E} \quad k = \frac{\kappa}{\kappa_y} \quad m = \frac{M}{M_y} \tag{5.7}$$

the moment-curvature relationship can now be represented non-dimensionally with the substitution of (5.3), (5.6), and (5.7) into (5.2):

$$m = k + \frac{1-e}{\pi(1-r^4)}\left\{\frac{16}{3}\left[\sin\phi_2 - r^3\sin\phi_1\right] - k\left[(2\phi_2 + \sin 2\phi_2) - r^4(2\phi_1 + \sin 2\phi_1)\right] - \frac{4}{3k^3}\left[\tan\phi_2 - \tan\phi_1\right]\right\} \tag{5.8}$$

where ϕ_1 and ϕ_2 become

$$\phi_1 = \cos^{-1}\left(\frac{z}{r_1}\right) = \cos^{-1}\left(\frac{r_2}{r_1}\frac{z}{r_2}\right) = \cos^{-1}\left(\frac{1}{rk}\right)$$

$$\phi_2 = \cos^{-1}\left(\frac{z}{r_2}\right) = \cos^{-1}\left(\frac{1}{k}\right)$$

For thin pipes with wall thickness $h \ll r_2$, the following simplifications can be made

$$r^n = \left(\frac{r_1}{r_2}\right)^n = \left(\frac{r_2 - h}{r_2}\right)^n = \left(1 - \frac{h}{r_2}\right)^n \simeq 1 - n\left(\frac{h}{r_2}\right)$$

$$\phi_2 \simeq \phi_1 = \phi$$

which lead to

$$m = k\left\{1 + \frac{1-e}{\pi}(\sin 2\phi - 2\phi)\right\} \tag{5.9}$$

To visualize the moment-curvature relationship, Equation (5.8) is plotted for $r_m/h = 25$ with various values of $e = E_T/E$, and is shown in Figure 5.5. As can be seen from this figure, all curves originate from the same line for $0 \leq k \leq 1$ and then branch out to different paths because of the effect of strain hardening. For $e = 0$ that corresponds to the elastic-perfectly plastic material idealization, the curve appears as a horizontal line for $1 \leq k$ as would normally be expected.

It is also worthwhile to note that these results are almost identical to those obtained earlier by Lau et al. [86] for thin-walled pipes using an approximate method

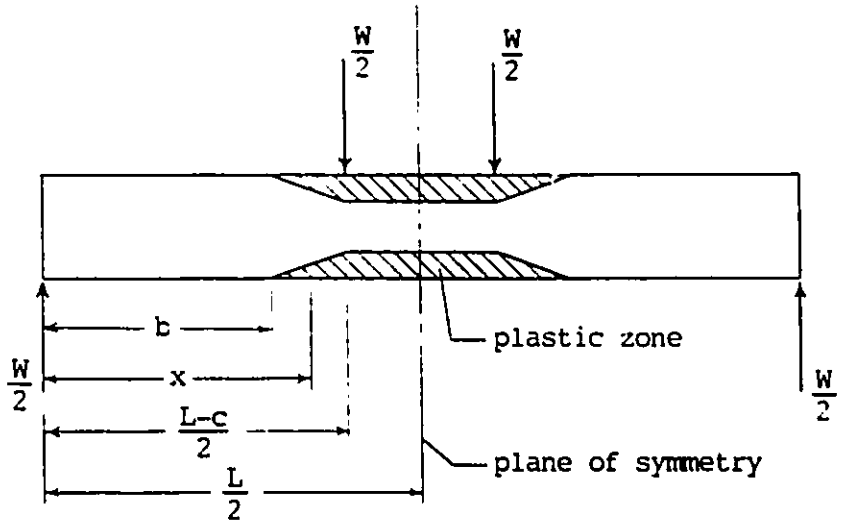


Figure 5.2: Simply supported beam under a symmetrical two-point loading.

based on the beam theory. On the other hand, larger deviations between the two results would be expected for $r_m/h < 10$ since the solution given in [86] is only good for thin pipes ($10 \leq r_m/h$). Nevertheless, the agreement just mentioned already gives some confidence in EPBT.

The moment-curvature relationship represented by (5.8) will serve as the basic equation. From it such important information as the slope and the deflection of the beam as well as the growth of the plastic zones in a particular cross section and along the length of the beam will be found, given the loading conditions. Ram bending can be modelled as a simply supported straight beam under a symmetrical two-point loading as shown in Figure 5.2.

In this figure, the bending moment in the beam is maximum and remains so at locations between the two concentrated loads. At first yielding (Figure 5.3), the bending moment is simply calculated from

$$M_y = \frac{1}{4} W_y (L - c) \tag{5.10}$$

where W_y is the yielding load, L the total length of the beam, and c the distance between the two concentrated loads. Physically, c represents the width of the ram of the bender.

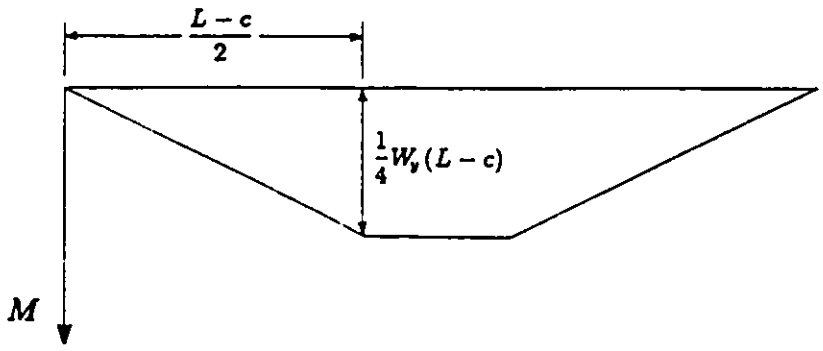


Figure 5.3: Bending moment diagram of the beam for $W = W_y$.

As the plastic zones spread sideways to the two ends of the beam, the yielding moment M_y at the cross sections separating the purely elastic and elastic-plastic parts of the beam is similarly determined by

$$M_y = \frac{1}{2}Wb \tag{5.11}$$

which infers from (5.10) that

$$b = \left(\frac{L-c}{2}\right)\frac{W_y}{W}$$

with b representing the length of the purely elastic parts of the beam. Also, the bending moment at some cross section that is partially plastic is equal to

$$M = \frac{1}{2}Wx$$

where x is the distance from the cross section to the nearer end of the beam. The above expression is combined with (5.11) to yield

$$\frac{M}{M_y} = \frac{x}{b} = \left(\frac{2x}{L-c}\right)\frac{W}{W_y} \tag{5.12}$$

By replacing k and m in (5.8) using Equations (5.6) and (5.12), respectively, and by denoting the right-hand side of (5.8) as $f(k)$, Equation (5.8) can now be rewritten in the shorter form

$$\left(\frac{2x}{L-c}\right)\frac{W}{W_y} = f\left(\frac{r_2}{z}\right)$$

or equivalently,

$$z = \frac{r_2}{f^{-1}\left\{\left(\frac{2x}{L-c}\right)\frac{W}{W_y}\right\}} \tag{5.13}$$

This last equation dictates the shape of the plastic zones for a particular value of W/W_y. Unfortunately, z cannot be explicitly expressed in terms of W/W_y as finding a closed form for $f^{-1}(k)$ in Equation (5.13) is not possible. Therefore, the determination of $f^{-1}(k)$ will be carried out numerically with a sufficiently large number of points. Once z is known, the curvature of the beam at any cross section of the beam can be readily computed from Equation (5.4).

It is well established in the Strength of Materials that the slope and the deflection of an initially straight beam may be determined from its curvature using the basic differential equation

$$k = -\frac{d^2\delta}{dx^2} \tag{5.14}$$

with δ being the deflection of the beam. Again due to the fact that no closed form of $f^{-1}(k)$ could be found, indefinite integration of (5.14) for $W/W_y \geq 1$ would have to be carried out numerically with appropriate boundary conditions from the problem. A small computer program (Appendix C) has been written to find the inverse of $f(k)$, and the slope and the deflection of the beam based on (5.14). On the other hand, for $W/W_y \leq 1$ Equation (5.14) can be integrated exactly using some elementary method [104]; thus, closed-form expressions for the slope and the deflection can be obtained. In the limiting case $W/W_y = 1$, numerical results are found to be nearly identical to the those from the closed-form solution.

Although the beam deflection is only a quantity of minor interest, it is is also calculated for comparative purposes. The present study is more concerned with factors that have more practical value, such as the growth of the plastic zones in the beam with increasing loading and the final pipe rotation achievable after the removal of applied loads. These results are further discussed in Article 5.4, following the description of the FEM simulation.

5.3 Simulation Using Program FENSA

Owing to the symmetry of the ram bending problem, only half of the pipe needs to be modelled. Specifically, the problem simplifies to the analysis of a cantilever beam. Two possibilities are considered for the modelling of the loading.

One possible choice for the loading is a set of two opposite concentrated forces, one at the free end of the beam and the other near the fixed end (Figure 5.4(a)). For this choice the beam is subjected to shearing, that has been found in this study to cause slow convergence or instability in the nonlinear numerical solution. This phenomenon has been referred to as "element locking" [105,106]. The effect of "element locking" can be avoided by not integrating the shear strain 'exactly' [9], but this treatment would render the element unreliable. This computational difficulty with the elbow-type element has not been noted previously in the literature. Since the pipe is normally quite long compared with the other two dimensions, the deflection or rotation of the beam due to shear force is very much smaller than that due to bending moment. Using the energy method, the central deflection of the beam is found to be of the order of 10^{-10} the value due to bending moment for $L/r_m = 10$ and $r_m/h = 10$ (in the linear range). As a result, another model is considered for the FEM simulation.

The second choice for the loading is a set of concentrated moments along the beam, Figure 5.4(b). This choice is equivalent to the first in most respects because the two concentrated loads have been replaced by a series of equivalent bending moments at every nodal point to the right of the concentrated load near the fixed end. The values of the moments M'' are taken as twice those of the moments M'. Such a replacement has completely eliminated the shear effects due to the concentrated loads in the first choice.

In the numerical solution, 10 elbow elements with a total of 31 nodal points, all equally spaced, are used. In actual ram bending, the central part of the pipe is restrained by the ram, and the two ends are restrained by ring supports. Therefore, it

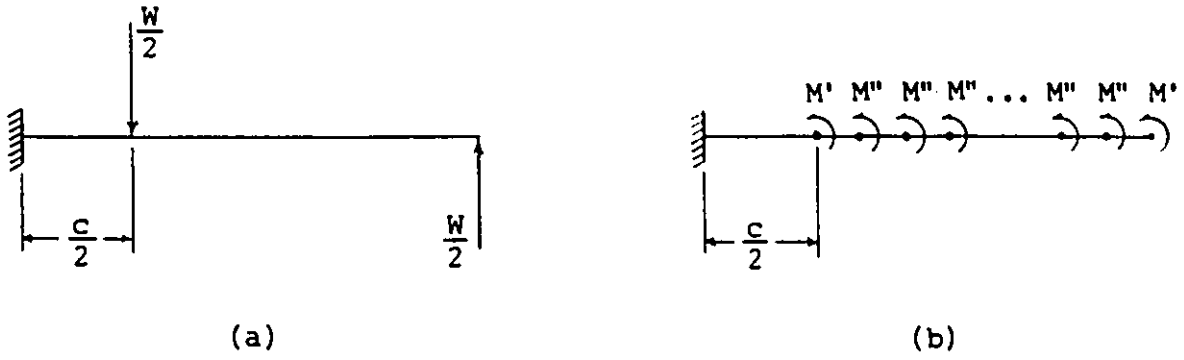


Figure 5.4: Models of ram pipe bending.

is assumed that there is no change in ovality of the cross section at these locations.

5.4 Comparison of EPBT and FENSA Results

Figures 5.6 – 5.8 show the FENSA results together with those from EPBT in the previous section for $L/r_m = 20$, $r_m/h = 25$, and $c/r_m = 2$.

Figure 5.6 gives the load-central deflection results for the pipe both in the linear range and beyond the elastic limit. Generally speaking, there is good correlation between EPBT and FENSA predictions on loading and unloading paths although some slight deviations have been observed in the unrecoverable displacements after springback. On the unloading path starting from $W/W_y = 1.5$, the permanent deflections of the central section predicted by the two methods differ by 10%. However, such a difference is quite likely to occur, and could be considered to be acceptable in view of the fact that springback accounts for about 74% of the total deflection.

The results for ovalization of the cross section for various values of W/W_y are presented in Figure 5.7. The radial displacements of the middle surface of the pipe wall are generally of the order of 10^{-5} the cross-sectional mean radius. Maximum changes in ovality of the cross section are found to be at $x/L = 0.3$ from either end of the pipe. For $0.45 \leq x/L \leq 0.5$ which represents the part of the pipe restrained by the ram, there is no ovalization ($\Delta r/r_m = 0$) as previously assumed in the FEM modelling. For

$W/W_y = 1.5$, $\Delta r/r_m$ has a maximum value of 5×10^{-5} (or $5 \times 10^{-3}\%$) which corresponds to a final beam rotation of 1.5^0 (or 0.25^0 per meter of pipe length). Assuming there is a linear variation between the pipe rotation and the radial displacement of the pipe wall, a final beam rotation of 15^0 would imply that $\Delta r/r_m = 5 \times 10^{-4}$ (or 0.05%) which is equivalent to 0.1% if the change in ovality is measured by $(r_m^+ - r_m^-)/r_m$. Since FENSA results are low for ovalization, the actual change in ovality which may be up to twice as much as the FENSA prediction is still well below the maximum allowed ovality of 1% according to "VGB-Dampftechnik GmbH, Essen" [54]. The magnitude of the maximum ovalization also gives support to Assumption 2 made in Article 5.2, and further implies that the pipe would be free of cracks caused by hoop stresses.

Figure 5.8 shows the shape of the plastic zones plotted for the lower left quarter of the pipe. Initially, the plastic zones develop at the plane of symmetry where the maximum bending moment takes place, and spread outward to the two ends of the pipe. Also seen from this figure is that the plastic zones predicted by FENSA are somewhat smaller than those by EPBT. The elastic-plastic boundaries for $W/W_y = 1.3$ and $W/W_y = 1.5$ given by FENSA are very close to the ones for $W/W_y = 1.2$ and $W/W_y = 1.4$ given by EPBT, respectively.

One obvious explanation for such differences is that in the FENSA solution the von Mises yield criterion was used, which takes into account all stress components. As a result, the effective stress is often smaller than the largest stress component while EPBT assumes the longitudinal stress $\sigma_{\eta\eta}$ to be the only significant stress, and uses it as the effective stress. Therefore, FENSA is more likely to give the true picture of the plastic zones than EPBT. On the other hand, predictions from EPBT tend to be on the safer side if the growth of the plastic zones is of major concern in the forming process.

It should be mentioned further that as W/W_y increases the plastic zones grow toward the neutral axis as well. It is expected that a plastic hinge would eventually form at the intersection of the neutral axis and the plane of symmetry, and then the

pipe would collapse. As implied from Equation (5.4), such a plastic hinge would require that the curvature κ approach to infinity, which is not possible from a practical point of view. Thus if the pipe fails, the failure will be due to buckling of the pipe wall (wrinkles).

Wall buckling of tubes and thin pipes subjected to bending has been studied by many researchers, for example [107] - [110]. In the present study, the program FENSA developed adopted the critical stress proposed by Brazier [107]

$$\sigma_{cr} = 0.320\left(\frac{Eh}{r_m}\right)$$

as a means to detect buckling. This value is lower than the ones given later in [108] and [109]. In the computation process of the solution, stresses at all integration points on the middle surface of the pipe wall are checked for possible buckling. It was found that at $W/W_y = 1.5$ the maximum compressive stress calculated for these points is about 1/7 the critical value σ_{cr} determined from the above formula. It is therefore expected that no buckling would occur if W/W_y were increased to 4 or 5 times as large. These are the highest values selected to appear in Figures 5.11 - 5.22.

For the set of data under consideration, buckling of the pipe wall is possible if $r_m/h = 25$ and $W/W_y \geq 10.5$, or $r_m/h \geq 25$ and $W/W_y = 10.5$. It should be noted that $r_m/h = 25$ chosen herein is considered as an extreme value for checking buckling. Typical values of r_m/h of pipe bends produced by ram bending or by other methods are about or below 20, thus allowing the value of W/W_y to be slightly higher than 10.5. Therefore, as far as buckling is concerned, it would theoretically be safe if Figures 5.11 - 5.22 were used because therein $r_m/h \leq 20$ and $W/W_y \leq 10$.

Presented in Figures 5.9 - 5.10 are the FENSA predictions for residual stresses in the pipe at the plane of symmetry. On both inner and outer surfaces of the pipe, the residual circumferential stress $(\sigma^*_{\xi\xi})_{res}$ is observed to be maximum at $\phi = (0^0, 180^0)$, and is about 1/6 the corresponding $\sigma^*_{\xi\xi}$ obtained at $W//W_y = 1.5$ on the loading path. Meanwhile, not only is the maximum longitudinal stress $\sigma_{\eta\eta}$ at $\phi = (0^0, 180^0)$ reduced in magnitude after springback, its sign is also reversed, leaving the maximum residual

$(\sigma^*_{\eta\eta})_{res}$ at $\phi = (60^0, 120^0)$ which is found to be about 1/5 the corresponding $\sigma^*_{\eta\eta}$ at the same locations. The sign reversal of $\sigma^*_{\eta\eta}$ in the vicinities of $\phi = (0^0, 180^0)$ is mainly due to relatively large springback of the pipe, which induces an opposite bending moment even after the removal of the applied loading. As W/W_y is increased further, springback becomes relatively smaller; nence, the $\sigma^*_{\eta\eta}$ and $(\sigma^*_{\eta\eta})_{res}$ distributions would be expected to have similar patterns.

5.5 Computed Results for Practical Cases

In ram bending and a few other bending processes, a major concern is how to control and predict springback. The finite element program FENSA developed in the study can give an estimate of springback in addition to other aspects of pipe bends. On the other hand, with the agreement obtained in Figure 5.6 the final bend angle after springback can be calculated much less expensively using EPBT. This alternative by no means implies that the use of FENSA in the bending process has been taken over by EPBT. Rather EPBT is used as a secondary tool of analysis to save time and resources whenever possible.

In Figures 5.11 – 5.22, the applied load W/W_y has been plotted against the 'specific' final pipe rotation (rotation per unit pipe length) for different ratios of L/r_m and r_m/h. The ratios used represent typical values of pipe bends handled by most bend manufacturers [54], and the curves tabulated are expected to be of practical value. With these charts, the ram force required to form a certain bend (after springback) from a straight pipe can be determined in advance from the geometric dimensions of the pipe.

Figures 5.11 – 5.16 represent the case in which the ram width c is proportional to the pipe mean radius r_m ($c/r_m = 2$). For any particular value of L/r_m, the specific final rotation increases as the ratio r_m/h decreases. Physically, smaller values of r_m/h mean thicker pipe walls, thus allowing more plastic flow and larger plastic deformation but

less springback. As L/r_m increases, the curves representing different values of r_m/h are slowly shifted to the left, indicating a decrease in the specific final rotation. It is also noted from these figures that the specific final rotation is zero at $W/W_y \leq 1$, which represents the range of elastic loading with any deformation of the pipe completely recoverable.

Presented in the Figures 5.17 – 5.22 are the results for the case in which the ram width c is a constant, regardless of the size of the pipe cross section. The trends of the curves in these figures are observed to be the same as those in Figures 5.11 – 5.16. A comparison of the two sets of figures indicates that the specific final rotation increases with the ram width c. This observation can be explained by the fact that as c is increased, the part of the pipe with the maximum bending moment is widened and the plastic zones are enlarged, thereby permitting larger plastic deformation. From a practical viewpoint, the case with c being a constant does not seem realistic; in practice, the size of the ram chosen normally depends on the cross-section diameter of the pipe as in the case of Figures 5.11 – 5.16. The results in Figures 5.17 – 5.22 are calculated mainly to study the effect of the parameter c on the specific final rotation.

It should be mentioned that the deviation of the results presented in Figures 5.11 – 5.22 from experimental values is expected to be less than 10% as W/W_y gets higher. This expectation is based on the comparison of the EPBT and FENSA results as shown in Figure 5.6, and on the experimental work of Roderick and Heyman [99]. They showed that for $E_T/E \geq 0.08$ the deflection results calculated for a rectangular cross-section beam using EPBT are almost identical to the experimental measurements. The value of $e = E_T/E$ used in Figures 5.11 – 5.22 is 0.15 while typical values of e are expected to be slightly higher than 0.15. In the present study, the result accuracy due to higher values of e is expected to offset the error caused by the ovalizations of a bent pipe.

5.6 Remarks

The use of Brazier's formula in Article 5.4 for checking buckling of the pipe wall implicitly assumes that the pipe is free of local buckling, that may occur in the highly compressive area of contact between the ram and the pipe. In the analysis of a pipe bending process where investigation of such a buckling type is required, the present elbow element is no longer suitable because of its limitations in modelling applied loading. Therefore, other types of elements must be considered instead. In the ram pipe bending with typical values of r_m/h, the pipe cross section at the part restrained by the ram may ovalize non-symmetrically, but the local buckling is very unlikely. Hence, the elbow element developed by Bathe et al. [3] - [5] is still considered to be applicable.

The results given in Figures 5.11 – 5.22 may be greatly affected by the use of a different set of material constants (yield stress, Young's modulus, and strain-hardening modulus). In addition, for materials that are not represented by a simple bilinear elastic-plastic stress-strain curve, the program FENSA must be used instead of EPBT. The actual stress-strain curve of the material will then be replaced by a multilinear approximation as was previously done in the verification problems.

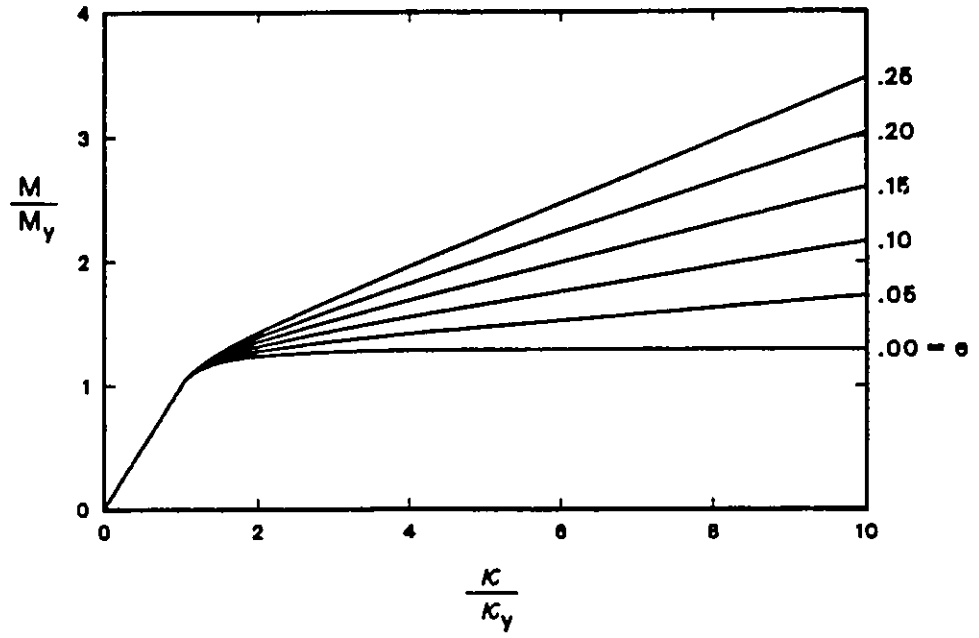


FIG. 5-5 MOMENT-CURVATURE RESULTS CALCULATED BY A BEAM-THEORY METHOD (M_y : LIMIT LOAD FOR YIELD INITIATION).

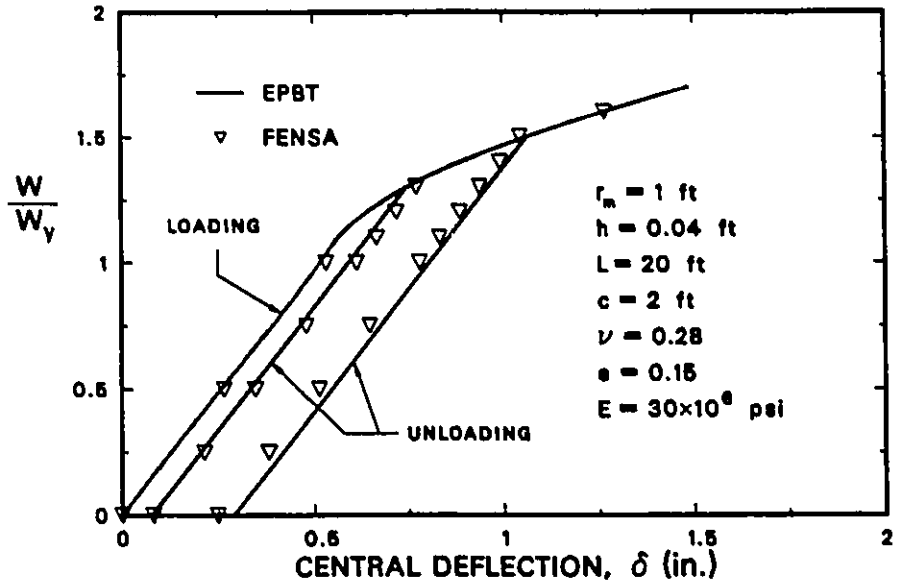


FIG. 5-6 COMPARISON OF THEORETICAL AND NUMERICAL LOAD-DEFLECTION RESULTS FOR RAM PIPE BENDING (1ft = 0.3048 m, 1psi = 6895 Pa).

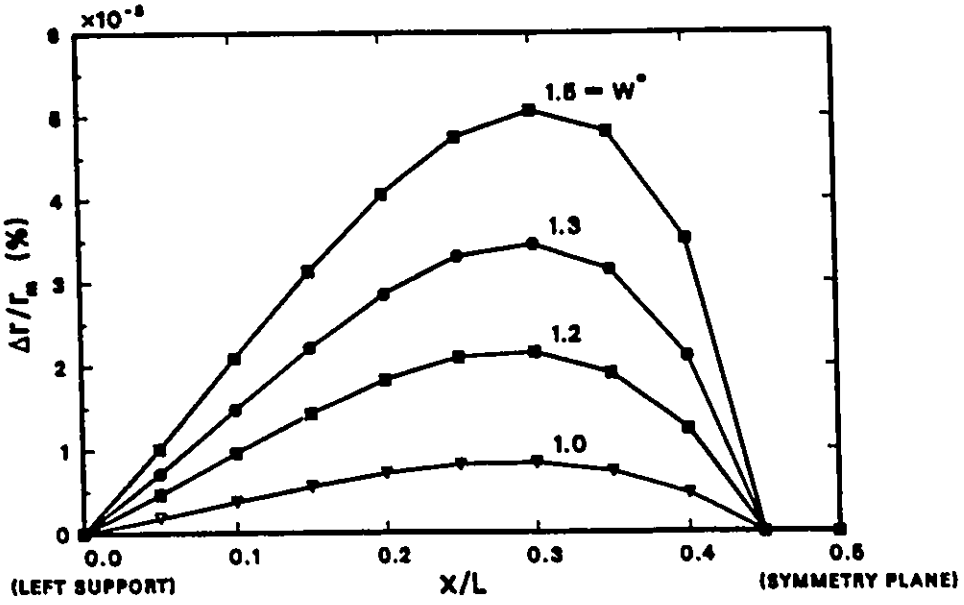


FIG. 5-7 VARIATION OF MAXIMUM CHANGES IN CROSS-SECTIONAL MEAN RADIUS ALONG PIPE LENGTH PREDICTED BY FENSA ($W^\circ = W/W_y$).

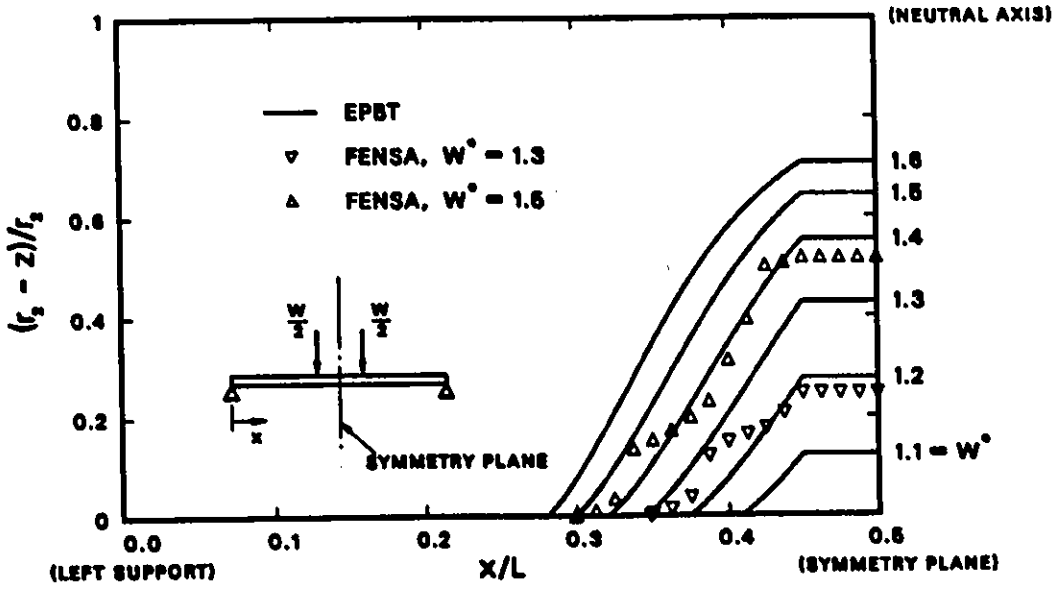


FIG. 5-8 PREDICTED GROWTH OF PLASTIC ZONES TOWARDS THE CENTRE OF PIPE DUE TO INCREASING LOADING.

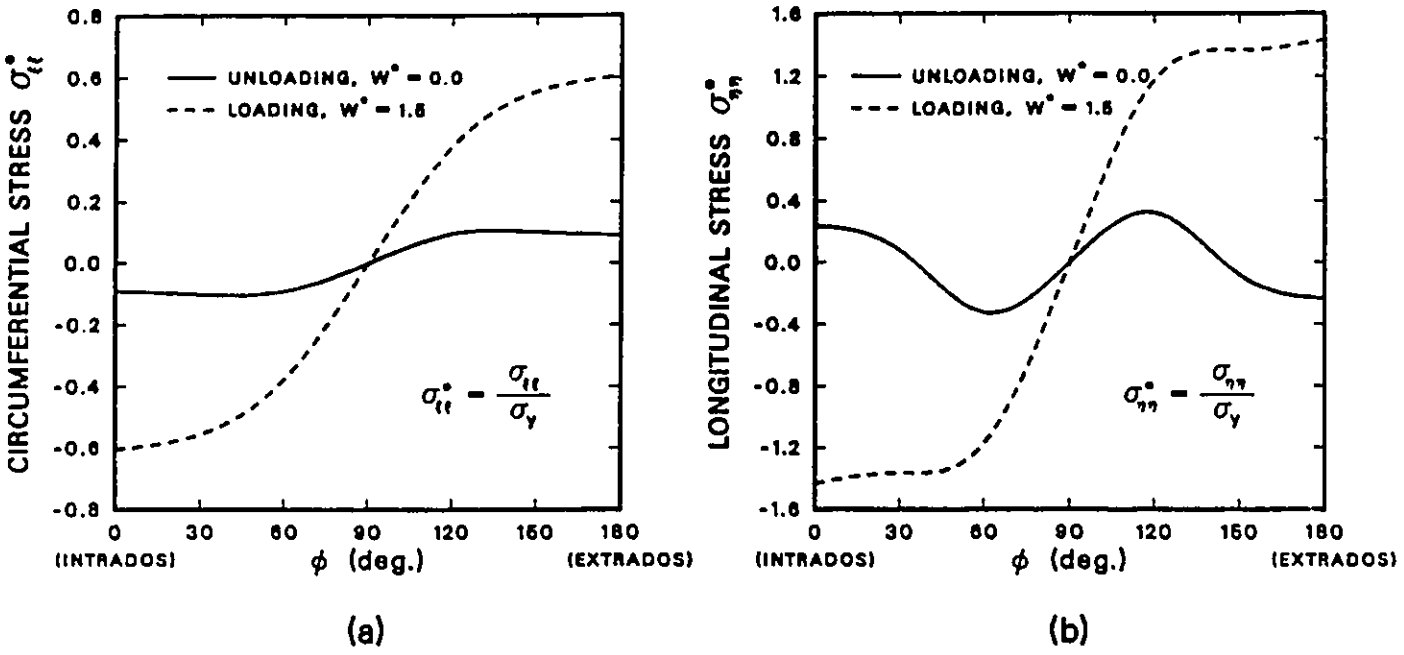


FIG. 5-9 PREDICTED RESIDUAL (a) CIRCUMFERENTIAL AND (b) LONGITUDINAL STRESSES AT THE PLANE OF SYMMETRY AND ON THE OUTSIDE SURFACE OF THE PIPE.

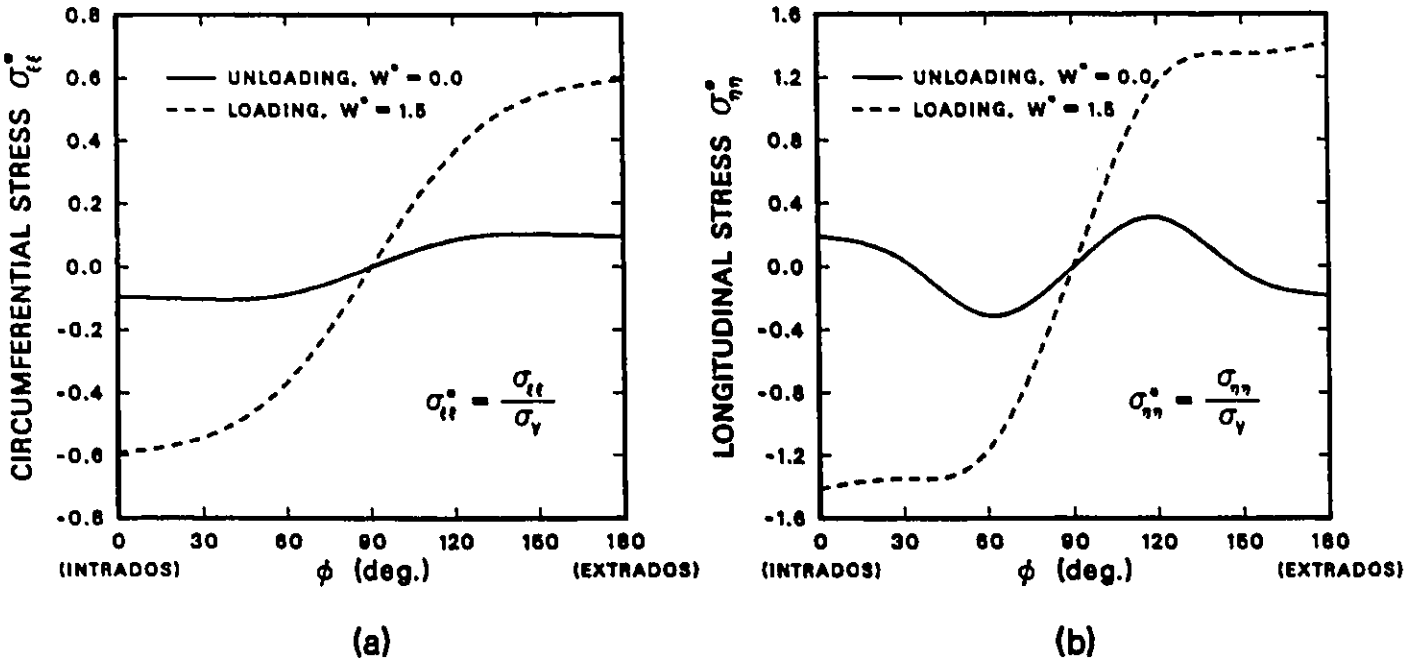


FIG. 5-10 PREDICTED RESIDUAL (a) CIRCUMFERENTIAL AND (b) LONGITUDINAL STRESSES AT THE PLANE OF SYMMETRY AND ON THE INSIDE SURFACE OF THE PIPE.

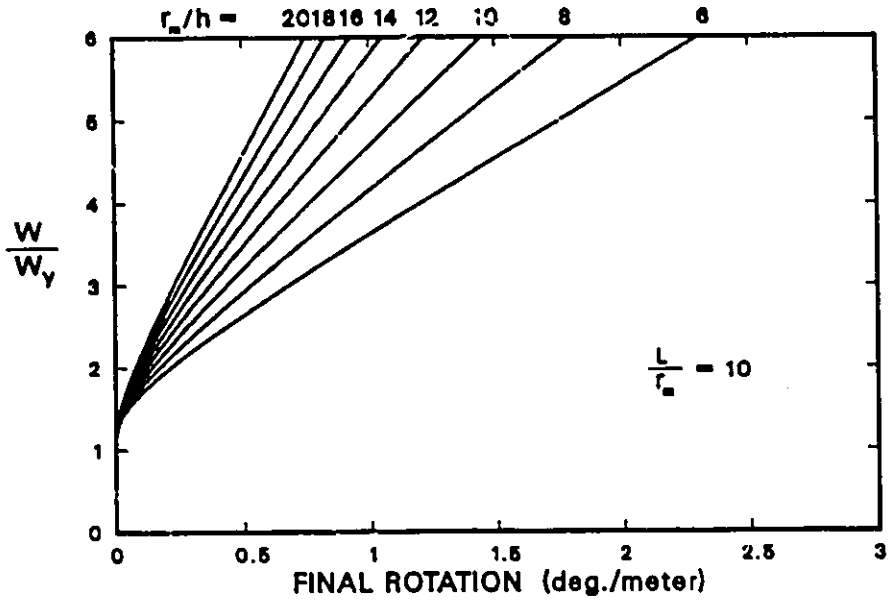


FIG. 5-11 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 10$ AND $c/r_m = 2$.

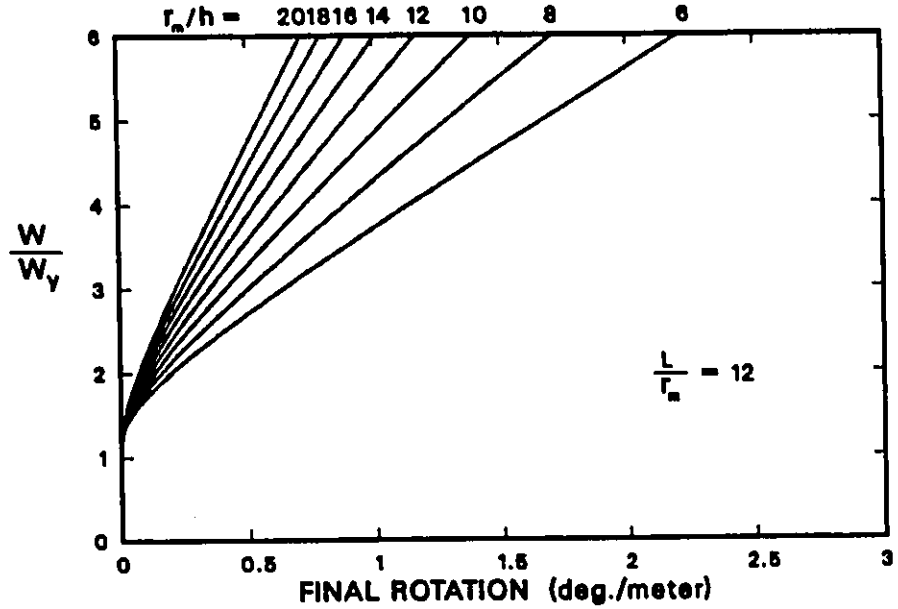


FIG. 5-12 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 12$ AND $c/r_m = 2$.

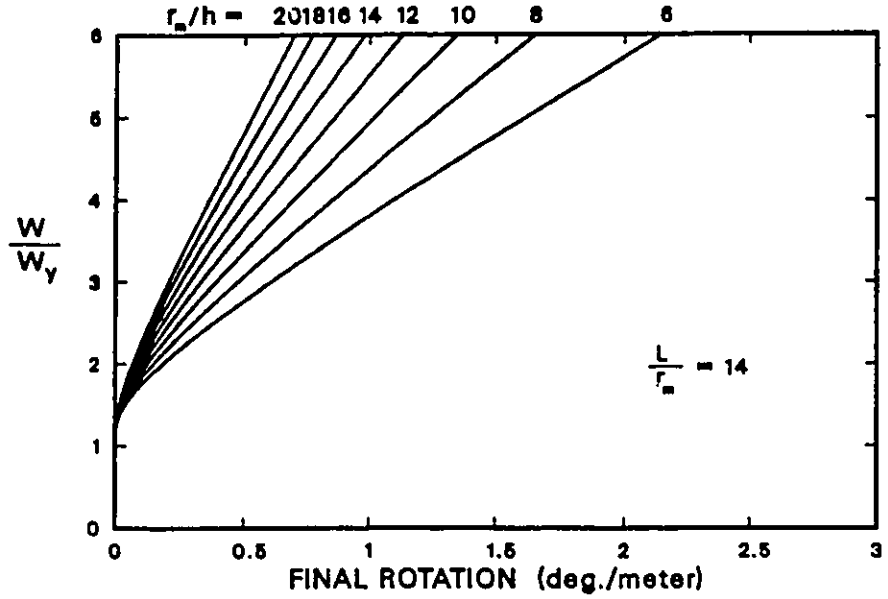


FIG. 5-13 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 14$ AND $c/r_m = 2$.

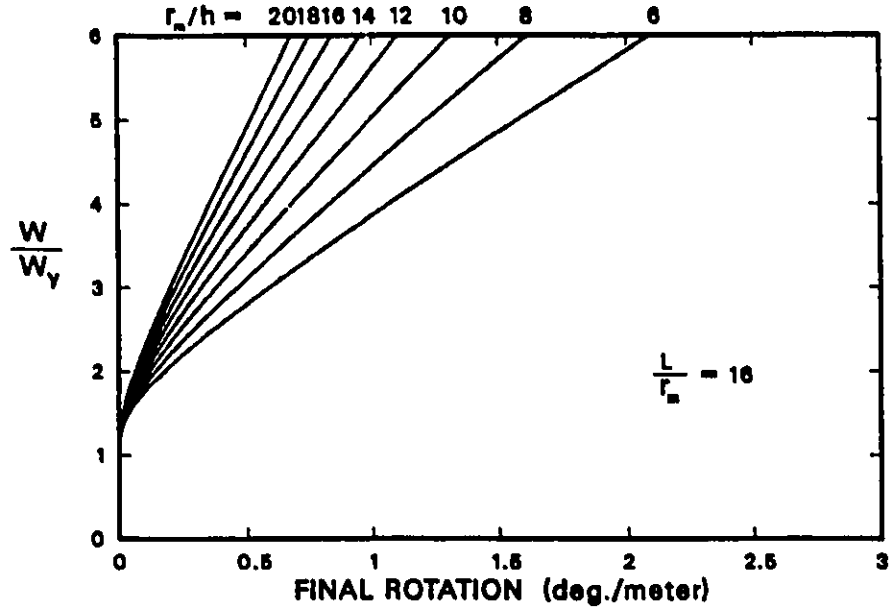


FIG. 5-14 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 16$ AND $c/r_m = 2$.

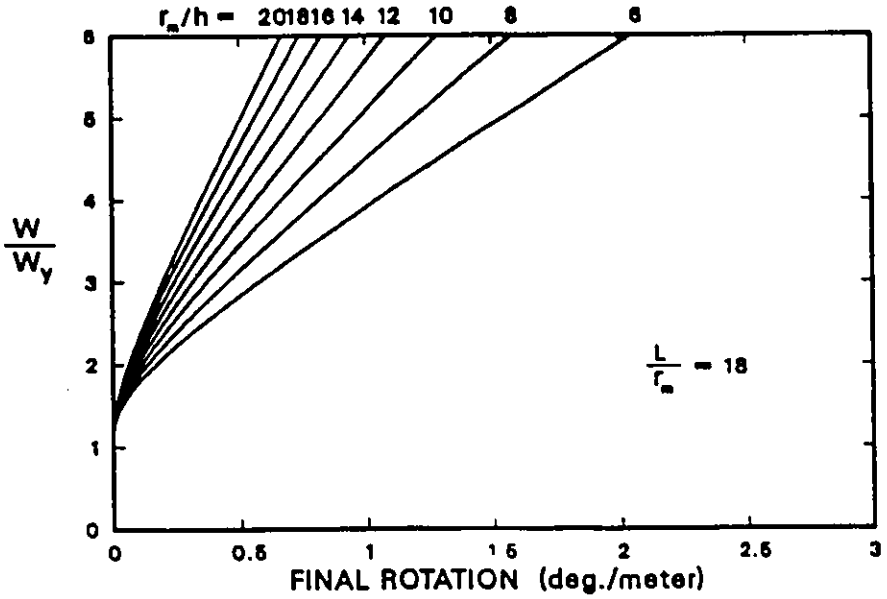


FIG. 5-15 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 18$ AND $c/r_m = 2$.

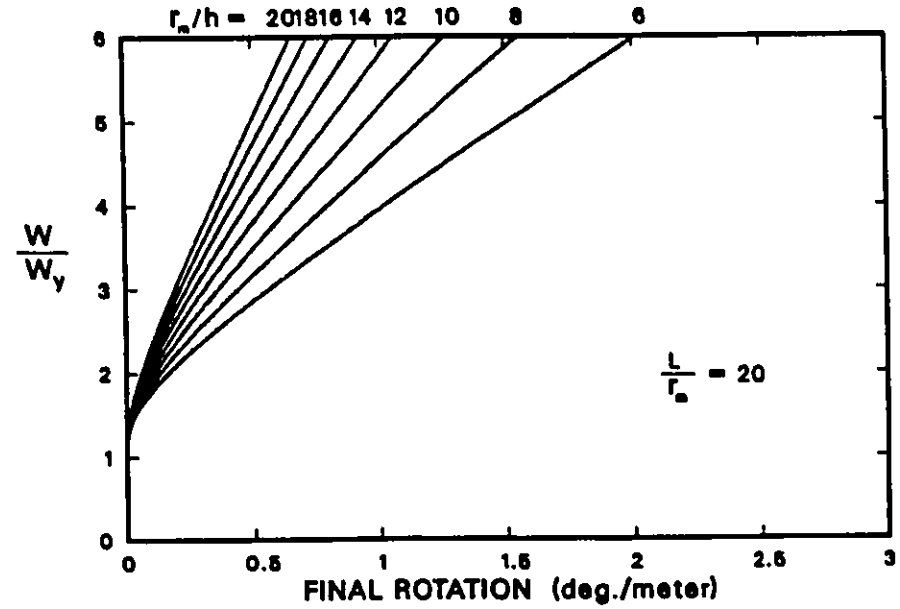


FIG. 5-16 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 20$ AND $c/r_m = 2$.

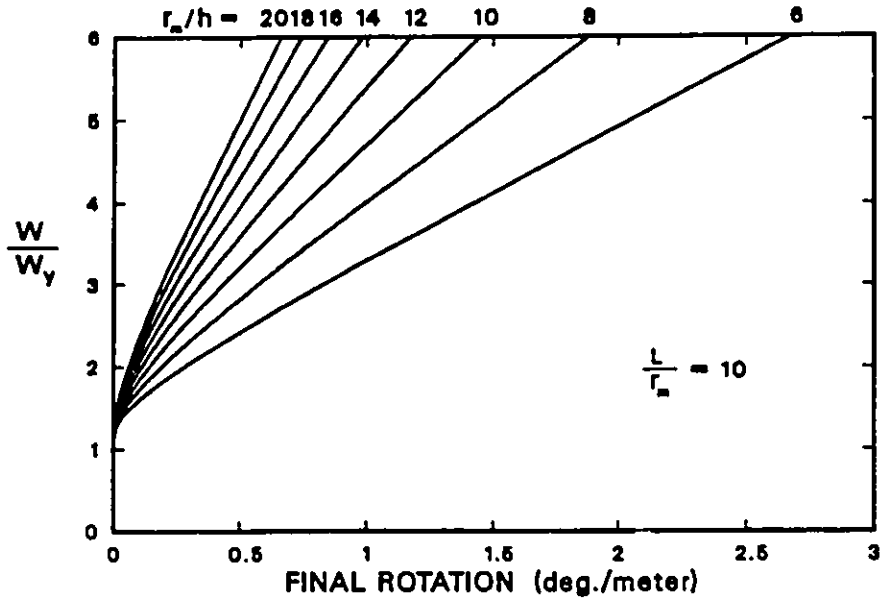


FIG. 6-17 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 10$ AND $c/h = 20$.

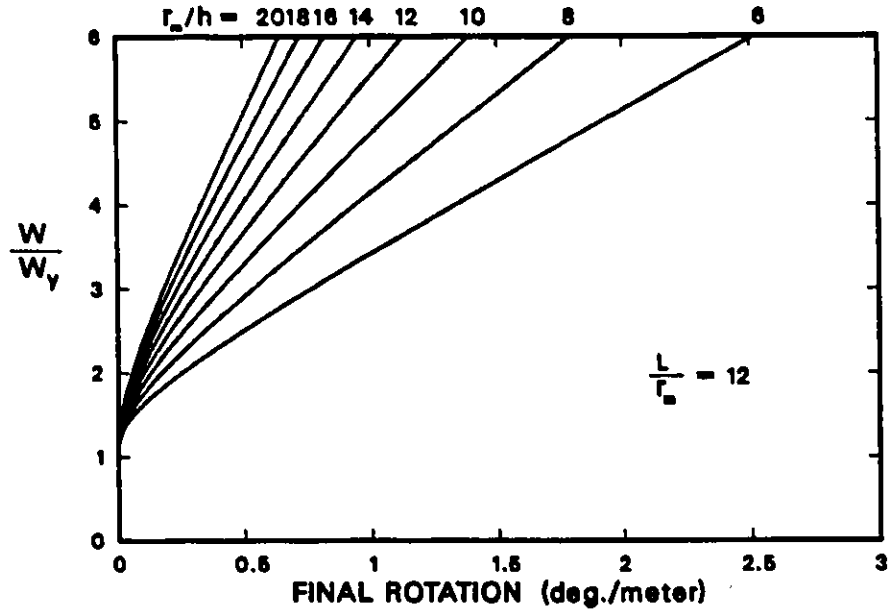


FIG. 6-18 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 12$ AND $c/h = 20$.

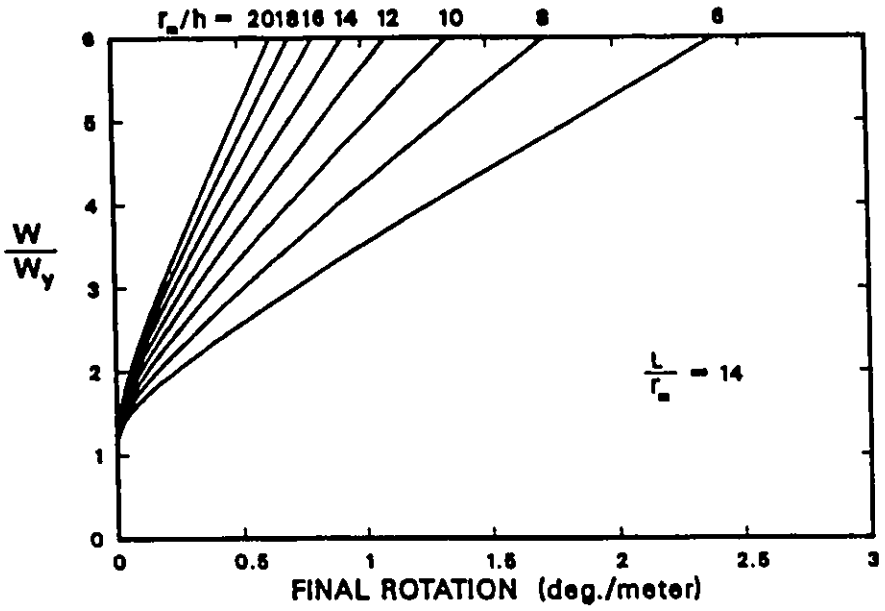


FIG. 8-18 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 14$ AND $c/h = 20$.

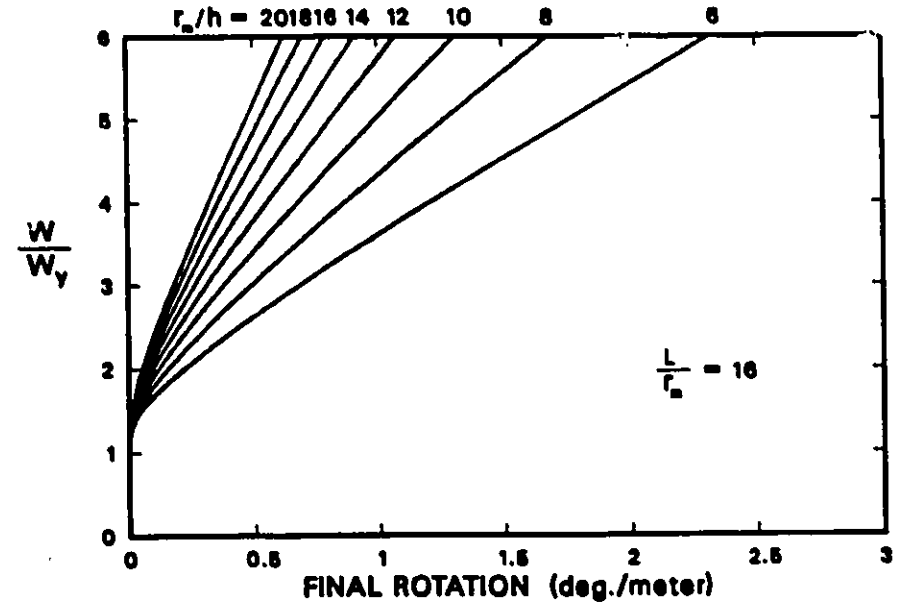


FIG. 8-20 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 16$ AND $c/h = 20$.

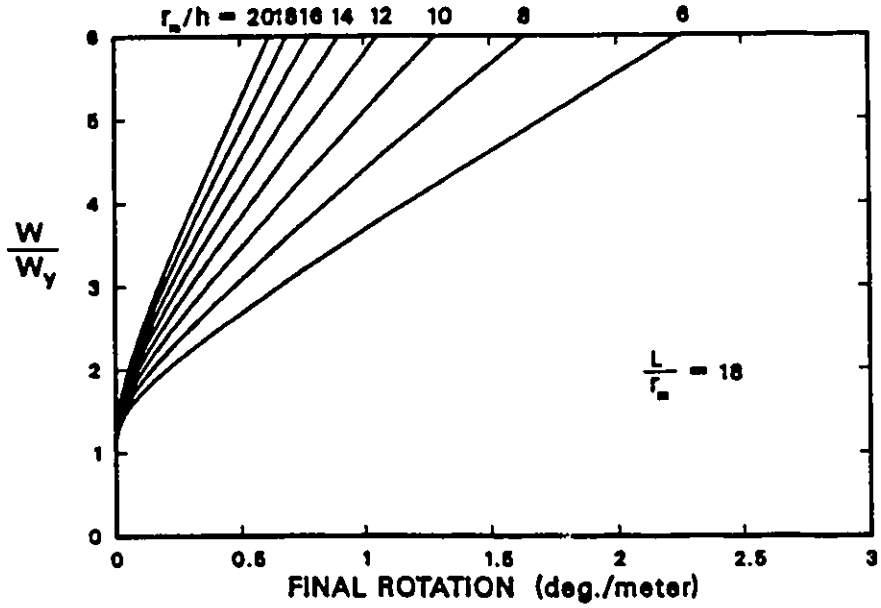


FIG. 5-21 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 18$ AND $c/h = 20$.

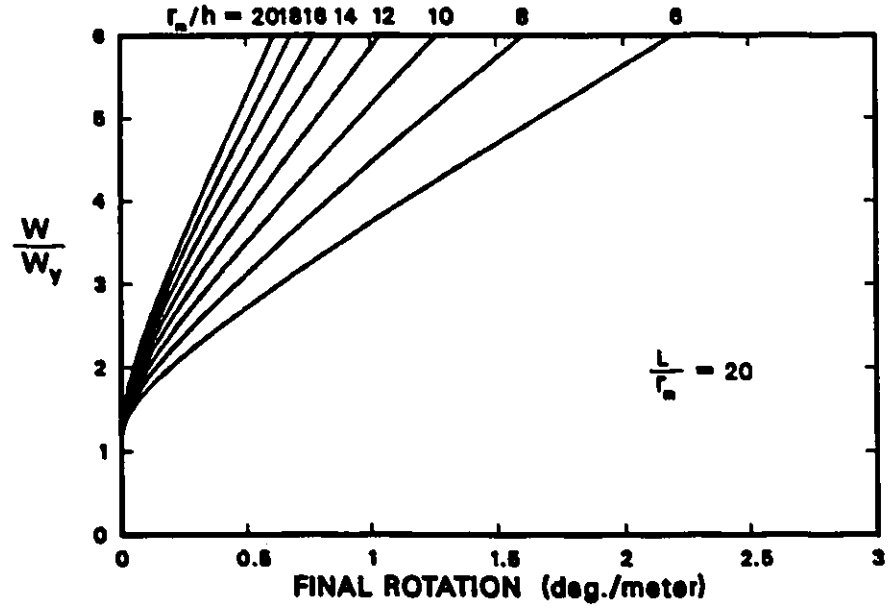


FIG. 5-22 PREDICTED FINAL ROTATION PER UNIT LENGTH OF BENT PIPE FOR $L/r_m = 20$ AND $c/h = 20$.

Chapter 6

Conclusions

From the present study, the following conclusions can be drawn regarding the use of the program FENSA to model piping problems and to simulate the ram-pipe bending process.

- Piping problems

  1. In linear analyses, displacements and rotations are predicted accurately within 0.6% from the theoretical values. Due to the high order of displacement interpolation, relatively few elements are required for every day piping problems.

  2. The elbow element developed in [3] – [5] underestimates the cross-section ovalizations of curved pipes. As the curvature of a pipe bend becomes larger, the predictions get closer to the experimental values. For pipe bends of $R = 0.250\,\mathrm{m}$ and $R = 0.375\,\mathrm{m}$, the measurements for the maximum radial displacements of the pipe wall are found to be 1.4 and 1.7 times the FENSA predictions, respectively.

  3. Although the expansion and the corresponding contraction of a cross section are nearly the same in elastic bending, the expansion is roughly twice the contraction in plastic bending. In nonlinear analyses, the use of the elbow

element should be restricted to cases where cross-sectional ovalizations are small since any change in the shape of the cross section of the element is not taken into account in the computation process.

4. Stresses and strains in straight and curved pipes are predicted closely in the linear range by the program FENSA. To determine completely their accuracy, FENSA stresses need to be checked further against other solutions.

5. The program FENSA is currently not capable of predicting exactly where first yielding occurs. However, this shortcoming could be eliminated with slight modifications in the present formulation in accordance with the theoretical work of [95].

- Ram-pipe bending process

1. For angles of bends dictated by Figures 5.11 – 5.22, the maximum value of FENSA predictions for cross-sectional ovalizations (measured by $\Delta r/r_m$) is about 0.2% in typical pipe bends produced by ram benders.

2. The pipe wall is predicted to be safe from buckling because the maximum compressive stress is below the critical value given by Brazier [107]. In the process of bending, pipe bends are free from collapse arising from the formation of a plastic hinge because plastic zones will not cover any cross section completely.

3. Predictions of final bend angles can be obtained inexpensively using EPBT while predictions for ovalizations and checking against buckling [107] are carried out with the program FENSA. The use of applied load-final pipe rotation results can save time and energy in shops producing pipe bends.

4. The present elbow element can provide preliminary information for the ram pipe bending process. For accurate simulation of the cross-section ovalization, other types of elements should be considered and more computing

time will certainly be required. Results for the maximum ovalization from a shell element using ADINA are presented in Appendix D. Friction and contact effects are ignored in the present model. The results presented thus are not applicable to practical cases where those effects are significant.

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Appendix A

Transformation

Physical stress and strain components belong to second-order tensors. In the finite element method, such components are converted from one system of (orthogonal) axes to another for computational purposes, and then converted back for result interpretations. These conversions are achieved with appropriate transformation matrices.

The matrices are derived from basic relations in Continuum Mechanics (for example, see [83]). If T_{pq} represents components of a second-order tensor in a first (unbarred) system of axes and $\bar{T}_{rs}$ is its counterpart in a second (barred) system, then the following relations hold

$$T_{pq} = a_r^p a_s^q \bar{T}_{rs}$$

$$\bar{T}_{rs} = a_r^p a_s^q T_{pq}$$

where

a_r^p, a_s^q : direction cosines defined as

$$a_r^p = \cos(\bar{x}_r, x_p)$$

$$a_s^q = \cos(\bar{x}_s, x_q)$$

p, q, r, s : indices (equal to 1, 2, or 3).

With the same argument for stress and strain components, similar relations can

be obtained

$$\text{Stresses} \qquad \sigma_{pq} = a_r^p a_s^q \bar{\sigma}_{rs} \qquad \bar{\sigma}_{rs} = a_r^p a_s^q \sigma_{pq} \qquad (a)$$

$$\text{Strains} \qquad \epsilon_{pq} = a_r^p a_s^q \bar{\epsilon}_{rs} \qquad \bar{\epsilon}_{rs} = a_r^p a_s^q \epsilon_{pq} \qquad (b)$$

By letting

$$\begin{aligned} l_1 &= \cos(e_\xi, e_{x_1}) & m_1 &= \cos(e_\eta, e_{x_1}) & n_1 &= \cos(e_\zeta, e_{x_1}) \\ l_2 &= \cos(e_\xi, e_{x_2}) & m_2 &= \cos(e_\eta, e_{x_2}) & n_2 &= \cos(e_\zeta, e_{x_2}) \\ l_3 &= \cos(e_\xi, e_{x_3}) & m_3 &= \cos(e_\eta, e_{x_3}) & n_3 &= \cos(e_\zeta, e_{x_3}) \end{aligned}$$

where e_ξ, e_η, e_ζ, and $e_{x_1}, e_{x_2}, e_{x_3}$ are the unit vectors in the natural coordinate system and the global system, respectively. (a) and (b) can now be represented in a matrix form as

$$\boldsymbol{\sigma} = \boldsymbol{Q}^T \bar{\boldsymbol{\sigma}} \qquad \bar{\boldsymbol{\sigma}} = \boldsymbol{P}^T \boldsymbol{\sigma}$$

$$\boldsymbol{\epsilon} = \boldsymbol{P} \bar{\boldsymbol{\epsilon}} \qquad \bar{\boldsymbol{\epsilon}} = \boldsymbol{Q} \boldsymbol{\epsilon}$$

where

$$\boldsymbol{Q} = \begin{bmatrix} (l_1)^2 & (l_2)^2 & (l_3)^2 & l_1 l_2 & l_2 l_3 & l_3 l_1 \\ (m_1)^2 & (m_2)^2 & (m_3)^2 & m_1 m_2 & m_2 m_3 & m_3 m_1 \\ (n_1)^2 & (n_2)^2 & (n_3)^2 & n_1 n_2 & n_2 n_3 & n_3 n_1 \\ 2l_1 m_1 & 2l_2 m_2 & 2l_3 m_3 & (l_1 m_2 + l_2 m_1) & (l_2 m_3 + l_3 m_2) & (l_3 m_1 + l_1 m_3) \\ 2m_1 n_1 & 2m_2 n_2 & 2m_3 n_3 & (m_1 n_2 + m_2 n_1) & (m_2 n_3 + m_3 n_2) & (m_3 n_1 + m_1 n_3) \\ 2n_1 l_1 & 2n_2 l_2 & 2n_3 l_3 & (n_1 l_2 + n_2 l_1) & (n_2 l_3 + n_3 l_2) & (n_3 l_1 + n_1 l_3) \end{bmatrix}$$

$$\boldsymbol{P} = \begin{bmatrix} (l_1)^2 & (m_1)^2 & (n_1)^2 & l_1 m_1 & m_1 n_1 & n_1 l_1 \\ (l_2)^2 & (m_2)^2 & (n_2)^2 & l_2 m_2 & m_2 n_2 & n_2 l_2 \\ (l_3)^2 & (m_3)^2 & (n_3)^2 & l_3 m_3 & m_3 n_3 & n_3 l_3 \\ 2l_1 l_2 & 2m_1 m_2 & 2n_1 n_2 & (l_1 m_2 + l_2 m_1) & (m_1 n_2 + m_2 n_1) & (n_1 l_2 + n_2 l_1) \\ 2l_2 l_3 & 2m_2 m_3 & 2n_2 n_3 & (l_2 m_3 + l_3 m_2) & (m_2 n_3 + m_3 n_2) & (n_2 l_3 + n_3 l_2) \\ 2l_3 l_1 & 2m_3 m_1 & 2n_3 n_1 & (l_3 m_1 + l_1 m_3) & (m_3 n_1 + m_1 n_3) & (n_3 l_1 + n_1 l_3) \end{bmatrix}$$

$$\boldsymbol{\sigma} = [\sigma_{11} \ \sigma_{22} \ \sigma_{33} \ \sigma_{12} \ \sigma_{23} \ \sigma_{31}]$$

$$\bar{\boldsymbol{\sigma}} = [\bar{\sigma}_{11} \ \bar{\sigma}_{22} \ \bar{\sigma}_{33} \ \bar{\sigma}_{12} \ \bar{\sigma}_{23} \ \bar{\sigma}_{31}]$$

$$\epsilon = [\epsilon_{11}\ \epsilon_{22}\ \epsilon_{33}\ 2\epsilon_{12}\ 2\epsilon_{23}\ 2\epsilon_{31}]$$

$$\bar{\epsilon} = [\bar{\sigma}_{11}\ \bar{\epsilon}_{22}\ \bar{\epsilon}_{33}\ 2\bar{\epsilon}_{12}\ 2\bar{\epsilon}_{23}\ 2\bar{\epsilon}_{31}]$$

The transformation matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ have been presented for the 3-dimensional case with all stress and strain components present. If some of these are absent, however, $\boldsymbol{P}$ and $\boldsymbol{Q}$ can be simplified by eliminating appropriate row(s) and column(s).

Appendix B

Summaries

B.1 Updated Lagrangian Formulation with Jaumann Stress Rate

The Updated Lagrangian Formulation (ULF) and the Total Lagrangian Formulation (TLF) are the two approaches developed for nonlinear analyses with the elastic-plastic material model. In small-strain small-displacement problems, there is no distinction between the two formulations because the deformed configuration of a body at any time t is not much different from the undeformed configuration at time $t = 0$. For analyses involving large displacements and/or large strains (displacement gradients), however, there is a significant difference between the two formulations. The FEM literature shows that the ULF formulation is the most appropriate for sheet metal forming. Furthermore, the most effective stress measure in the ULF formulation for these large-strain cases is the Jaumann stress rate [9,84].

The development of the Updated Lagrangian Jaumann Stress-Rate Formulation begins with Equation (3.2)

$$\left({}_t^tK_L + {}_t^tK_{NL}\right)\Delta U^{(i)} = {}^{t+\Delta t}R - {}_{t+\Delta t}^{t+\Delta t}F^{(i-1)}$$

In the above equation, all quantities are evaluated with respect to the current con-

figuration, that is, the updated configuration of the material at time t. The key ingredients in the evaluation of ${}_t^tK_L$, ${}_t^tK_{NL}$, and ${}_{t+\Delta t}^{t+\Delta t}F^{(i-1)}$ are the Cauchy stresses (physical stresses).

It is known that the most crucial part in a nonlinear FEM analysis is the calculation of stresses because the stress increments from time t to time $t+\Delta t$ are determined by the constitutive relation ${}_tC^{EP}$ (or the instantaneous stress-strain matrix) which is stress-history dependent. If it is assumed that all Cauchy stress components ${}^t\tau_{ij}$ at time t are known, then the stress components at time $t+\Delta t$ are determined as follows

$$ {}^{t+\Delta t}\tau \;=\; {}^t\tau \;+\; \int_{t_\epsilon}^{t+\Delta t_\epsilon} {}_tC^{EP}\, d\epsilon \;+\; \Delta^t\tau \qquad \text{(B.1)} $$

The time derivative of the second term on the right-hand side of Equation (B.1) is known as the Jaumann stress rate which is solely due to straining of the material, and is therefore invariant with respect to rigid-body rotations. The third term on the right-hand side can be physically interpreted as stress corrections to be added to ${}^t\tau$ to account for rigid-body rotations (large displacements) and material distortions (large strains). Individual components of $\Delta^t\tau$ are determined from

$$ \Delta^t\tau_{ij} \;=\; \int_t^{t+\Delta t} \left\{ {}^t\tau_{ip}\Omega_{pj} \;-\; \Omega_{ip}\,{}^t\tau_{pj} \right\} dt \qquad \text{(B.2)} $$

with Ω_{ij} being the components of the spin tensor defined by

$$ \Omega_{ij} \;=\; \frac{1}{2}\left\{ \frac{\partial \dot{u}_j}{\partial {}^t x_i} \;-\; \frac{\partial \dot{u}_i}{\partial {}^t x_j} \right\} $$

The integrations in Equations (B.1) and (B.2) are usually carried out using some direct integration technique such as the Euler forward integration method with a large number of steps so as to obtain sufficient accuracy for the solution. Although the magnitude of such a number can be arbitrarily chosen, the specific value is mainly determined by the plastic-strain increments according to some rule to minimize computing time.

The accumulated strains (elastic and plastic) are also calculated in the same manner, namely

$$ {}^{t+\Delta t}\epsilon \;=\; {}^{t}\epsilon \;+\; \int_{t}^{t+\Delta t} {}_{t}\dot{e}\, dt \;+\; \Delta {}^{t}\epsilon \qquad \text{(B.3)} $$

The components of the second term on the right-hand side of Equation (B.3) are nothing but the strain increments, and can be calculated directly from the corresponding displacement increments.

$$ {}_{t}e_{ij} \;=\; \frac{1}{2}\left\{ \frac{\partial u_i}{\partial {}^{t}x_j} + \frac{\partial u_j}{\partial {}^{t}x_i} \right\} $$

Detailed discussion of the Updated Lagrangian Formulation with Jaumann stress rate can be found in Reference [9].

B.2 Modified Newton-Raphson Method

As mentioned in Chapter 1, a nonlinear FEM analysis is most effectively carried out incrementally and iteratively using some iterative technique. The most common one is the Newton-Raphson method in which to solve a set of nonlinear equations such as Equation (3.2),

$$ {}^{t}\boldsymbol{K}\,\Delta\boldsymbol{U}^{(i)} = {}^{t+\Delta t}\boldsymbol{R} - {}^{t+\Delta t}\boldsymbol{F}^{(i-1)} $$

(${}^{t}\boldsymbol{K}$ represents the total global system stiffness matrix) the coefficient matrix ${}^{t}\boldsymbol{K}$ is recalculated and factorized in every iteration until the desired accuracy of the solution is obtained, Figure B.1. Although convergence is fast and the procedure is effective in some special case, the method does not have an overall economy for general elastic-plastic analyses. It is even prohibitively expensive to use in highly nonlinear problems with all kinematic and material nonlinearities present since a large number of small-sized time steps is usually required.

An alternative to the Newton-Raphson method is the initial stiffness method. The system stiffness matrix is now calculated and factorized only once at the beginning of the analysis ($t = 0$), and then reused over and over for all subsequent time steps and iterations. This method proves to be slightly more effective than the Newton-Raphson method, but there is an extremely slow convergence because no further reformation of the system stiffness matrix is made in the analysis. Also associated with the initial stiffness method are a few other drawbacks. For example, the slow convergence often terminates the computation too early because the maximum allowable number of iterations have been met while the prescribed tolerances have not, or the iteration process may even diverge as pointed out in [8,9,111]

In the modified Newton-Raphson method (Figure B.2), the system stiffness matrix is recalculated at the beginning of every time step, and then reused for all iterations within that time step. Convergence is now faster than in the initial stiffness method but slower than in the full Newton-Raphson method. Overall, the modified Newton-

Raphson method is more economical than the other two. In cases of slow convergence, the method can be made more effective by using such an acceleration scheme as the Aitken method [112].

Experience from the present study shows that caution should be exercised when using the Aitken acceleration because of possible negative effects. For instance, instead of speeding up the convergence, it could slow it down or even lead the solution to divergence because the solutions for individual unknowns being solved for neither converge at the same rate nor all converge monotonically. For that reason, the Aitken acceleration was not implemented in the program FENSA. However, a variation of the modified Newton-Raphson was introduced; within a time step, the system stiffness matrix is allowed to be reformed a second time after a number of iterations prescribed by the user. This option is represented by the broken line in the flowchart of the program FENSA shown in Appendix C.2.

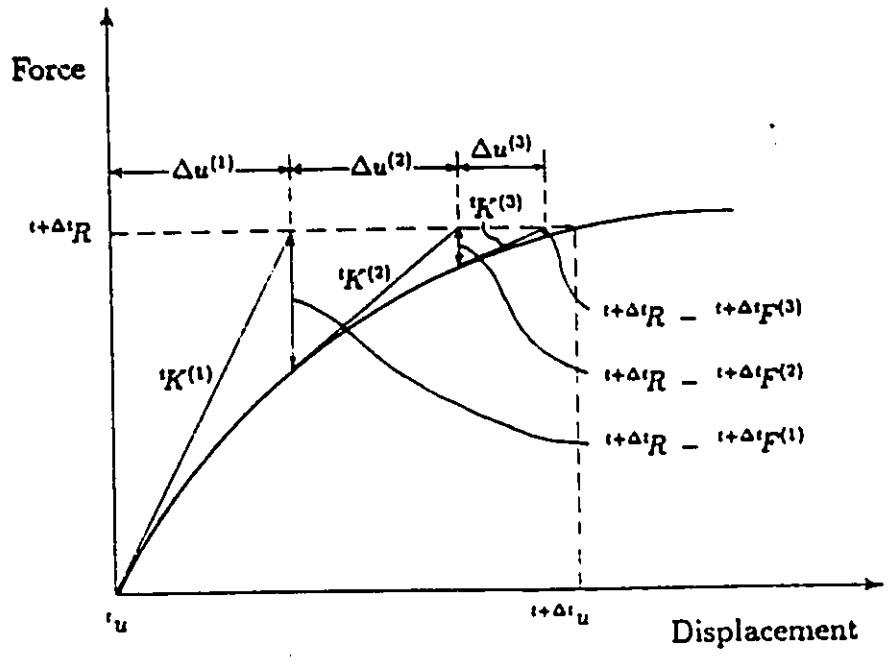


Figure B.1: Full Newton-Raphson method.

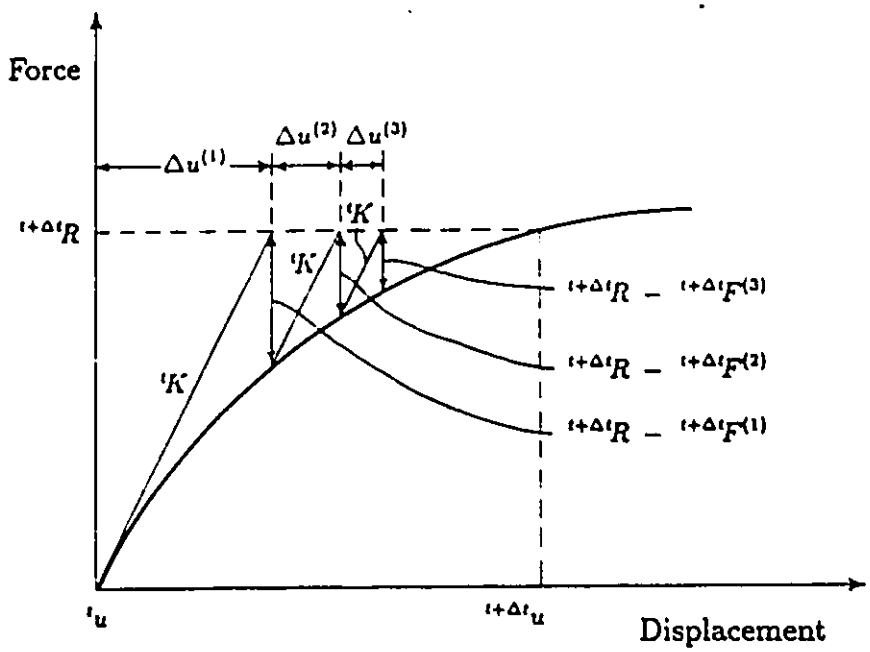


Figure B.2: Modified Newton-Raphson method.

Appendix C

Computer Software

C.1 Computer Program for EPBT

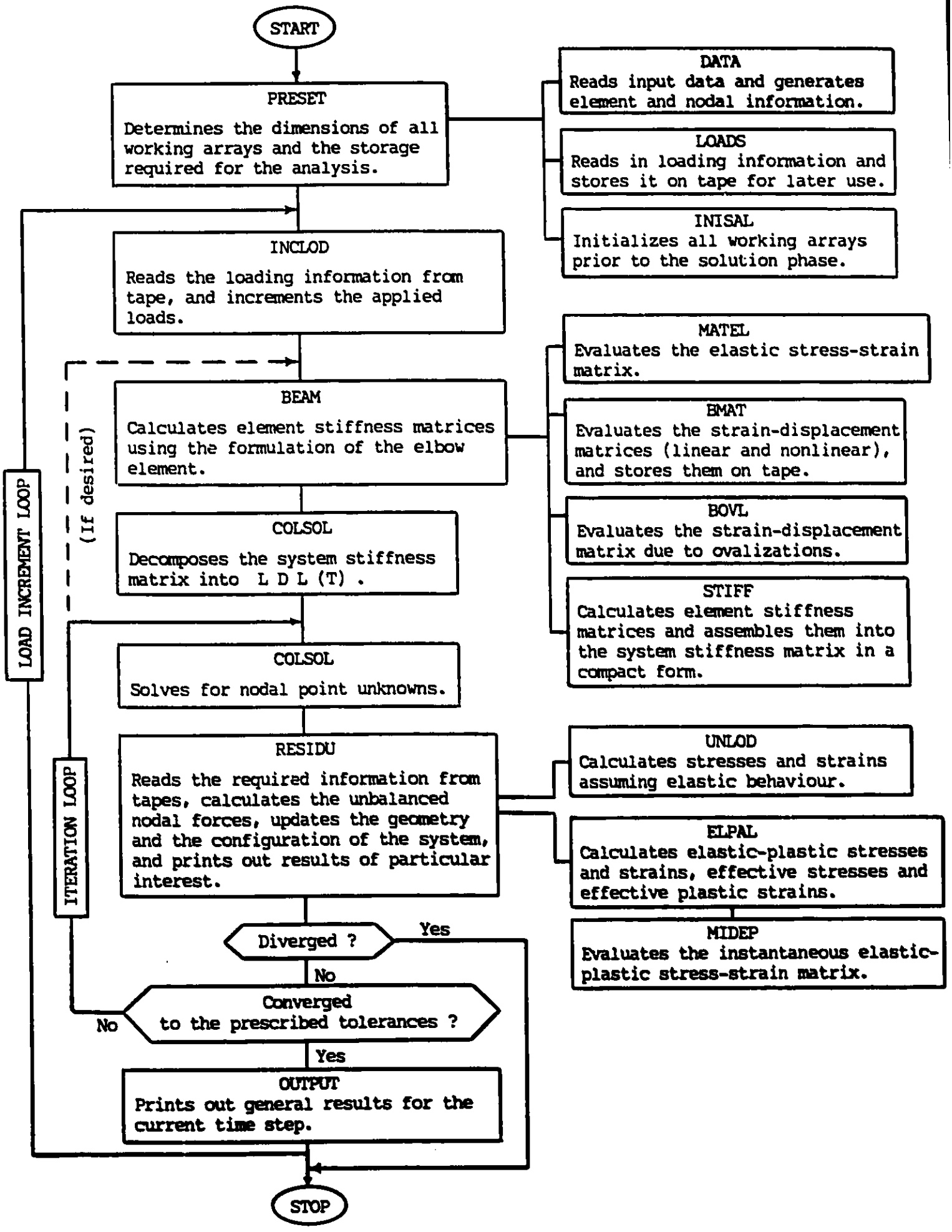
```
      IMPLICIT  REAL*8(A-H,O-Z)                                                 DEF00010
      DIMENSION  X(40000),Y(40000),X1(5000),Y1(5000),X2(5000),Y2(5000)          DEF00020
      DIMENSION  WRAT(26)                                                       DEF00030
      DATA  WRAT/ 1.0D0, 1.2D0, 1.4D0, 1.6D0, 1.8D0,                            DEF00040
     1            2.0D0, 2.2D0, 2.4D0, 2.6D0, 2.8D0, 3.0D0, 3.2D0,            DEF00050
     2            3.4D0, 3.6D0, 3.8D0, 4.0D0, 4.2D0, 4.4D0, 4.6D0,            DEF00060
     3            4.8D0, 5.0D0, 5.2D0, 5.4D0, 5.6D0, 5.8D0, 6.0D0  /          DEF00070
      KKK= 5                                                                    DEF00080
      DO 600  IS=1,48                                                           DEF00090
      READ (5,10)  BI,BO,TLENG,DIST,EYLD,ELAM                                   DEF00100
      WRITE (6,15)  BI,BO,TLENG,DIST,EYLD,ELAM                                  DEF00110
   10 FORMAT (7D10.3)                                                           DEF00120
   15 FORMAT (7D15.6)                                                           DEF00130
      B= BI/BO                                                                  DEF00140
      B2= B*B                                                                   DEF00150
      B3= B2*B                                                                  DEF00160
      B4= B2*B2                                                                 DEF00170
      BOO= BO/BI                                                                DEF00180
      TD= 0.5*(TLENG - DIST)                                                    DEF00190
      N1= 51                                                                    DEF00200
      N2= 39940                                                                 DEF00210
      PI= 3.14159265358979300                                                   DEF00220
      THICK= BO - BI                                                            DEF00230
      BMEAN= (BO + BI)/2.                                                       DEF00240
      RAT1= BMEAN/THICK                                                         DEF00250
      RAT2= TLENG/BMEAN                                                         DEF00260
      WRITE (6,20)  RAT1,RAT2                                                   DEF00270
   20 FORMAT ('////'  RESULTS FOR   RAT1 =',D15.6,5X,'RAT2 = ',D15.6)          DEF00280
C ... CONSTRUCT THE MOMENT-CURVATURE CURVE                                      DEF00290
      DX= 0.02D0                                                                DEF00300
      J= 0                                                                      DEF00310
      DO 30  I=1,N1                                                             DEF00320
      J= J + 1                                                                  DEF00330
      X(J)= DX*FLOAT(I-1)                                                       DEF00340
      Y(J)= X(J)                                                                DEF00350
   30 CONTINUE                                                                  DEF00360
      DX= 0.01D0                                                                DEF00370
      DO 40  I=1,N2                                                             DEF00380
      J= J + 1                                                                  DEF00390
      X(J)= X(J-1) + DX                                                         DEF00400
      XK= X(J)                                                                  DEF00410
      TH1= 0.                                                                   DEF00420
      TH2= DACOS(1./XK)                                                         DEF00430
      IF (XK.GT.BOO)  TH1= DACOS(BOO/XK)                                        DEF00440
      CON1= DSIN(TH2) - B3*DSIN(TH1)                                            DEF00450
      CON2= (2.*TH2 + DSIN(2.*TH2)) - B4*(2.*TH1 + DSIN(2.*TH1))                DEF00460
      CON3= DTAN(TH2) - DTAN(TH1)                                               DEF00470
      CON4= (1. - ELAM)/(PI*(1. - B4))                                          DEF00480
      Y(J)= XK + CON4*(16.*CON1/3. - XK*CON2 - 4.*CON3/(3.*XK*XK*XK) )          DEF00490
   40 CONTINUE                                                                  DEF00500
      JPERM= J                                                                  DEF00510
C ... PLOT THE GROWTH OF PLASTIC ZONES                                          DEF00520
      DO 500  ITIME=1,26                                                        DEF00530
      W= WRAT(ITIME)                                                            DEF00540
      K= 0                                                                      DEF00550
```

```
      DX= 0.01D0                                                                DEF00560
      DO 100  I=N1,JPERM                                                        DEF00570
      K= K + 1                                                                  DEF00580
      X1(K)= Y(I)*TC/W                                                          DEF00590
      Y1(K)= BO/X(I)                                                            DEF00600
      IF (X1(K).LE.TD)  GO TO 100                                               DEF00610
      TX= X1(K)                                                                 DEF00620
      TZ= Y1(K)                                                                 DEF00630
      X1(K)= TD                                                                 DEF00640
      Y1(K)= Y1(K-1) + (TD-X1(K-1))*(TZ-Y1(K-1))/(TX-X1(K-1))                   DEF00650
      GO TO 120                                                                 DEF00660
  100 CONTINUE                                                                  DEF00670
      WRITE (6,110) JPERM                                                       DEF00680
  110 FORMAT (' **** ERROR, MAXIMUM VALUES OF JPERM EXCEEDED ... =',I5)         DEF00690
      STOP                                                                      DEF00700
  120 N3= (0.5*TLENG - TD)/DX + 1                                               DEF00710
      DX= (0.5*TLENG - TD)/FLOAT(N3)                                            DEF00720
      DO 140  I=1,N3                                                            DEF00730
      K= K + 1                                                                  DEF00740
      X1(K)= X1(K-1) + DX                                                       DEF00750
      Y1(K)= Y1(K-1)                                                            DEF00760
  140 CONTINUE                                                                  DEF00770
C ... CONSTRUCT THE SLOPE CURVE                                                 DEF00780
      J= 1                                                                      DEF00790
      X2(J)= X1(K)                                                              DEF00800
      Y2(J)= 0.                                                                 DEF00810
      K1= K - 1                                                                 DEF00820
      DO 200  I=K1,1,-1                                                         DEF00830
      J= J + 1                                                                  DEF00840
      X2(J)= X1(I)                                                              DEF00850
      SLOPE= EYLD/Y1(I)                                                         DEF00860
      Y2(J)= Y2(J-1) - SLOPE*(X2(J-1) - X2(J))                                  DEF00870
  200 CONTINUE                                                                  DEF00880
      AA= X2(J)                                                                 DEF00890
      N4= AA/DX + 1                                                             DEF00900
      DX= AA/FLOAT(N4)                                                          DEF00910
      DO 220  I=1,N4                                                            DEF00920
      J= J + 1                                                                  DEF00930
      X2(J)= X2(J-1) - DX                                                       DEF00940
      SLOPE= X2(J)*EYLD/(AA*BO)                                                 DEF00950
      Y2(J)= Y2(J-1) - SLOPE*(X2(J-1) - X2(J))                                  DEF00960
  220 CONTINUE                                                                  DEF00970
      IF (ITIME.GT.1)  GO TO 240                                                DEF00980
      REF1= -(180./PI)*DATAN(Y2(J))                                             DEF00990
  240 REF2= -(180./PI)*DATAN(Y2(J))                                             DEF01000
      REF3= (REF2 - W*REF1)/TLENG                                               DEF01010
      WRITE (6,280)  ITIME,W,X2(J),Y2(J),REF1,REF2,REF3                         DEF01020
  280 FORMAT (I5,6D15.6)                                                        DEF01030
      IF (KKK.EQ.5)  GO TO 500                                                  DEF01040
C ... CONSTRUCT THE DEFLECTION CURVE                                            DEF01050
      N5= J                                                                     DEF01060
      X1(1)= 0.                                                                 DEF01070
      Y1(1)= 0.                                                                 DEF01080
      DO 300  I=2,N5                                                            DEF01090
      K= N5 - I + 2                                                             DEF01100
```

```
      X1(I)= X2(K-1)                                                                    DEF01110
      Y1(I)= Y2(K)*(X1(I) - X1(I-1)) + Y1(I-1)                                          DEF01120
  300 CONTINUE                                                                          DEF01130
      DO 320  I=1,NS                                                                    DEF01140
  320 WRITE (6,340)  I,X1(I),Y1(I)                                                      DEF01150
  340 FORMAT (' I X1 Y1 ',I5,2D20.10)                                                   DEF01160
  500 CONTINUE                                                                          DEF01170
  600 CONTINUE                                                                          DEF01180
      STOP                                                                              DEF01190
      END                                                                               DEF01200
C$DATA                                                                                  DEF01210
  0.55D+00   0.65D+00    6.0D+00    1.2D+00   1.2D-03   0.15D+00                        DEF01220
  0.75D+00   0.85D+00    8.0D+00    1.6D+00   1.2D-03   0.15D+00                        DEF01230
  0.95D+00   1.05D+00   10.0D+00    2.0D+00   1.2D-03   0.15D+00                        DEF01240
  1.15D+00   1.25D+00   12.0D+00    2.4D+00   1.2D-03   0.15D+00                        DEF01250
  1.35D+00   1.45D+00   14.0D+00    2.8D+00   1.2D-03   0.15D+00                        DEF01260
  1.55D+00   1.65D+00   16.0D+00    3.2D+00   1.2D-03   0.15D+00                        DEF01270
  1.75D+00   1.85D+00   18.0D+00    3.6D+00   1.2D-03   0.15D+00                        DEF01280
  1.95D+00   2.05D+00   20.0D+00    4.0D+00   1.2D-03   0.15D+00                        DEF01290
  0.55D+00   0.65D+00    7.2D+00    1.2D+00   1.2D-03   0.15D+00                        DEF01300
  0.75D+00   0.85D+00    9.6D+00    1.6D+00   1.2D-03   0.15D+00                        DEF01310
  0.95D+00   1.05D+00   12.0D+00    2.0D+00   1.2D-03   0.15D+00                        DEF01320
  1.15D+00   1.25D+00   14.4D+00    2.4D+00   1.2D-03   0.15D+00                        DEF01330
  1.35D+00   1.45D+00   16.8D+00    2.8D+00   1.2D-03   0.15D+00                        DEF01340
  1.55D+00   1.65D+00   19.2D+00    3.2D+00   1.2D-03   0.15D+00                        DEF01350
  1.75D+00   1.85D+00   21.6D+00    3.6D+00   1.2D-03   0.15D+00                        DEF01360
  1.95D+00   2.05D+00   24.0D+00    4.0D+00   1.2D-03   0.15D+00                        DEF01370
  0.55D+00   0.65D+00    8.4D+00    1.2D+00   1.2D-03   0.15D+00                        DEF01380
  0.75D+00   0.85D+00   11.2D+00    1.6D+00   1.2D-03   0.15D+00                        DEF01390
  0.95D+00   1.05D+00   14.0D+00    2.0D+00   1.2D-03   0.15D+00                        DEF01400
  1.15D+00   1.25D+00   16.8D+00    2.4D+00   1.2D-03   0.15D+00                        DEF01410
  1.35D+00   1.45D+00   19.6D+00    2.8D+00   1.2D-03   0.15D+00                        DEF01420
  1.55D+00   1.65D+00   22.4D+00    3.2D+00   1.2D-03   0.15D+00                        DEF01430
  1.75D+00   1.85D+00   25.2D+00    3.6D+00   1.2D-03   0.15D+00                        DEF01440
  1.95D+00   2.05D+00   28.0D+00    4.0D+00   1.2D-03   0.15D+00                        DEF01450
  0.55D+00   0.65D+00    9.6D+00    1.2D+00   1.2D-03   0.15D+00                        DEF01460
  0.75D+00   0.85D+00   12.8D+00    1.6D+00   1.2D-03   0.15D+00                        DEF01470
  0.95D+00   1.05D+00   16.0D+00    2.0D+00   1.2D-03   0.15D+00                        DEF01480
  1.15D+00   1.25D+00   19.2D+00    2.4D+00   1.2D-03   0.15D+00                        DEF01490
  1.35D+00   1.45D+00   22.4D+00    2.8D+00   1.2D-03   0.15D+00                        DEF01500
  1.55D+00   1.65D+00   25.6D+00    3.2D+00   1.2D-03   0.15D+00                        DEF01510
  1.75D+00   1.85D+00   28.8D+00    3.6D+00   1.2D-03   0.15D+00                        DEF01520
  1.95D+00   2.05D+00   32.0D+00    4.0D+00   1.2D-03   0.15D+00                        DEF01530
  0.55D+00   0.65D+00   10.8D+00    1.2D+00   1.2D-03   0.15D+00                        DEF01540
  0.75D+00   0.85D+00   14.4D+00    1.6D+00   1.2D-03   0.15D+00                        DEF01550
  0.95D+00   1.05D+00   18.0D+00    2.0D+00   1.2D-03   0.15D+00                        DEF01560
  1.15D+00   1.25D+00   21.6D+00    2.4D+00   1.2D-03   0.15D+00                        DEF01570
  1.35D+00   1.45D+00   25.2D+00    2.8D+00   1.2D-03   0.15D+00                        DEF01580
  1.55D+00   1.65D+00   28.8D+00    3.2D+00   1.2D-03   0.15D+00                        DEF01590
  1.75D+00   1.85D+00   32.4D+00    3.6D+00   1.2D-03   0.15D+00                        DEF01600
  1.95D+00   2.05D+00   36.0D+00    4.0D+00   1.2D-03   0.15D+00                        DEF01610
  0.55D+00   0.65D+00   12.0D+00    1.2D+00   1.2D-03   0.15D+00                        DEF01620
  0.75D+00   0.85D+00   16.0D+00    1.6D+00   1.2D-03   0.15D+00                        DEF01630
  0.95D+00   1.05D+00   20.0D+00    2.0D+00   1.2D-03   0.15D+00                        DEF01640
  1.15D+00   1.25D+00   24.0D+00    2.4D+00   1.2D-03   0.15D+00                        DEF01650
```

```
1.350+00  1.450+00  28.00+00  2.80+00  1.20-03  0.150+00   DEF01660
1.550+00  1.650+00  32.00+00  3.20+00  1.20-03  0.150+00   DEF01670
1.750+00  1.850+00  36.00+00  3.60+00  1.20-03  0.150+00   DEF01680
1.950+00  2.050+00  40.00+00  4.00+00  1.20-03  0.150+00   DEF01690
```

C.2 Flowchart of Program FENSA


START
PRESET
Determines the dimensions of all working arrays and the storage required for the analysis.
DATA
Reads input data and generates element and nodal information.
LOADS
Reads in loading information and stores it on tape for later use.
INISAL
Initializes all working arrays prior to the solution phase.
INCLOD
Reads the loading information from tape, and increments the applied loads.
BEAM
Calculates element stiffness matrices using the formulation of the elbow element.
MATEL
Evaluates the elastic stress-strain matrix.
BMAT
Evaluates the strain-displacement matrices (linear and nonlinear), and stores them on tape.
BOVL
Evaluates the strain-displacement matrix due to ovalizations.
STIFF
Calculates element stiffness matrices and assembles them into the system stiffness matrix in a compact form.
(If desired)
COLSOL
Decomposes the system stiffness matrix into L D L (T) .
COLSOL
Solves for nodal point unknowns.
RESIDU
Reads the required information from tapes, calculates the unbalanced nodal forces, updates the geometry and the configuration of the system, and prints out results of particular interest.
UNLOD
Calculates stresses and strains assuming elastic behaviour.
ELPAL
Calculates elastic-plastic stresses and strains, effective stresses and effective plastic strains.
MIDEP
Evaluates the instantaneous elastic-plastic stress-strain matrix.
LOAD INCREMENT LOOP
ITERATION LOOP
Diverged ?
Yes
No
Converged to the prescribed tolerances ?
No
Yes
OUTPUT
Prints out general results for the current time step.
STOP

C.3 Listing of Program FENSA

```
FILE: FENSA    FORTRAN  *   UNIV. O'/OF OTTAWA CMS RELEASE 4

C ... MAIN PROGRAM                                                              FEN00010
      IMPLICIT REAL*8(A-H,O-Z)                                                  FEN00020
      COMMON/SOL/ NEQ,NEQ1,NWK,MK,NES                                           FEN00030
      COMMON/MAX/ NPODS,NINCS,MITER,MODEX,INTUP                                 FEN00040
      COMMON/VAR/ IINCS,IITER,IPEL,IREM,IND,IPT,ICHEK                           FEN00050
      COMMON/DIM/ N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15,           FEN00060
     1            N16,N17,N18,N19,N20,N21,N22,N23,N24,N25                       FEN00070
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                              FEN00080
      COMMON  A(60000)                                                          FEN00090
      MTOT= 60000                                                               FEN00100
C ... SET UP STORAGE AND READ IN DATA                                           FEN00110
      CALL  PRESET (MTOT)                                                       FEN00120
      IF (MODEX.EQ.0)  STOP                                                     FEN00130
C ... ENTER SOLUTION PHASE                                                      FEN00140
      ACLOD= 0.                                                                 FEN00150
      DO 200  IINCS=1,NINCS                                                     FEN00160
      CALL  INCLOD (A(N17),A(N20),A(N21),ACLOD)                                 FEN00170
      KTR= 1                                                                    FEN00180
      WRITE (IT6,2000)  IINCS                                                   FEN00190
      WRITE (IT7,2000)  IINCS                                                   FEN00200
      DO 100  IITER=1,MITER                                                     FEN00210
      IND= 1                                                                    FEN00220
      IF (IINCS.EQ.1.AND.IITER.EQ.1)  IND= 0                                    FEN00230
      IF (KTR-1)  20,20,40                                                      FEN00240
   20 CALL  BEAM (A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N9),              FEN00250
     1       A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N23),A(N25))          FEN00260
      CALL  COLSOL (A(N14),A(N17),A(N9),NEQ,NWK,NEQ1,KTR)                       FEN00270
      KTR= 2                                                                    FEN00280
   40 CALL  COLSOL (A(N14),A(N17),A(N9),NEQ,NWK,NEQ1,KTR)                       FEN00290
C ... COMPUTE RESIDUAL STRESSES                                                 FEN00300
      CALL  RESIDU (A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N10),           FEN00310
     1 A(N11),A(N12),A(N13),A(N15),A(N16),A(N17),A(N18),A(N19),A(N20),          FEN00320
     2 A(N21),A(N22),A(N23),A(N24),A(N25))                                      FEN00330
C ... IF SOLUTION HAS CONVERGED, STOP ITERATING AND OUTPUT RESULTS              FEN00340
      IF (ICHEK)  250,80,120                                                    FEN00350
   80 IF (IITER.EQ.MITER)  GO TO 250                                            FEN00360
C ... IF STIFFNESS IS UPDATED BEFORE CONVERGENCE, SET  KTR= 1                   FEN00370
      IF (IITER.NE.INTUP)  GO TO 100                                            FEN00380
      KTR= 1                                                                    FEN00390
      WRITE (IT6,2020)  IITER                                                   FEN00400
  100 CONTINUE                                                                  FEN00410
  120 CALL  OUTPUT (A(N1),A(N6),A(N19),ACLOD)                                   FEN00420
  200 CONTINUE                                                                  FEN00430
      GO TO 300                                                                 FEN00440
  250 CALL  OUTPUT (A(N1),A(N6),A(N19),ACLOD)                                   FEN00450
  300 STOP                                                                      FEN00460
 2000 FORMAT (////50H *>*>*>*>  C U R R E N T    O U T P U T S    F O R,FEN00470
     1 3X,38H L O A D    I N C R E M E N T    N O. ,I4//)                       FEN00480
 2020 FORMAT (///1CX,46H =====>  SYSTEM CONFIGURATION HAS BEEN UPDATED,         FEN00490
     1 6H AFTER,I5,14H   ITERATIONS /)                                          FEN00500
      END                                                                       FEN00510
C                                                                               FEN00520
      SUBROUTINE ACCBAN (A,MAXA,S,LM,ND)                                        FEN00530
C . . . . . .                                                                   FEN00540
C .                                                                             FEN00550
```

```
C . . P R O G R A M                                                    . FEN00560
C .     TO ASSEMBLE UPPER TRIANGULAR ELEMENT STIFFNESS INTO            . FEN00570
C .     COMPACTED GLOBAL STIFFNESS                                     . FEN00580
C .                                                                    . FEN00590
C .       A = GLOBAL  STIFFNESS                                        . FEN00600
C .       S = ELEMENT STIFFNESS                                        . FEN00610
C .       ND = DEGREES OF FREEDOM IN ELEMENT STIFFNESS                 . FEN00620
C .                                                                    . FEN00630
C .                 S(1)        S(2)        S(3)          . . .      . FEN00640
C .       S   =                 S(ND+1)     S(ND*2)       . . .      . FEN00650
C .                                         S(2*ND)       . . .      . FEN00660
C .                                                       . . .      . FEN00670
C .                                                                    . FEN00680
C .                                                                    . FEN00690
C .                 A(1)        A(3)        A(6)          . . .      . FEN00700
C .       A   =                 A(2)        A(5)          . . .      . FEN00710
C .                                         A(4)          . . .      . FEN00720
C .                                                       . . .      . FEN00730
C .                                                                    . FEN00740
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . FEN00750
      IMPLICIT REAL*8(A-H,O-Z)                                         FEN00760
      DIMENSION A(1),MAXA(1),S(1),LM(1)                                FEN00770
C                                                                      FEN00780
      NDI=0                                                            FEN00790
      DO 200 I=1,ND                                                    FEN00800
      II= LM(I)                                                        FEN00810
      IF (II) 200,200,100                                              FEN00820
  100 MI= MAXA(II)                                                     FEN00830
      KS= I                                                            FEN00840
      DO 220 J=1,ND                                                    FEN00850
      JJ= LM(J)                                                        FEN00860
      IF (JJ) 220,220,110                                              FEN00870
  110 IJ= II - JJ                                                      FEN00880
      IF (IJ) 220,210,210                                              FEN00890
  210 KK= MI + IJ                                                      FEN00900
      KSS= KS                                                          FEN00910
      IF (J.GE.I) KSS= J + NDI                                         FEN00920
      A(KK)= A(KK) + S(KSS)                                            FEN00930
  220 KS= KS + ND - J                                                  FEN00940
  200 NDI= NDI + ND - I                                                FEN00950
      RETURN                                                           FEN00960
      END                                                              FEN00970
C                                                                      FEN00980
      SUBROUTINE ACCRES (MAXA,MHT)                                     FEN00990
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . FEN01000
C .                                                                    . FEN01010
C .   P R O G R A M                                                    . FEN01020
C .     TO CALCULATE ADDRESSES OF DIAGONAL ELEMENTS IN BANDED          . FEN01030
C .     MATRIX WHOSE COLUMN HEIGHTS ARE KNOWN                          . FEN01040
C .       MHT  = ACTIVE COLUMN HEIGHTS                                 . FEN01050
C .       MAXA = ADDRESSES OF DIAGONAL ELEMENTS                        . FEN01060
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . FEN01070
      IMPLICIT REAL*8(A-H,O-Z)                                         FEN01080
      COMMON/SOL/ NEQ,NEQ1,NWK,MK,NES                                  FEN01090
      DIMENSION  MAXA(1),MHT(1)                                        FEN01100
```

```
C ... CLEAR ARRAY MAXA                                                  FEN01110
      DO 20  I=1,NEQ1                                                   FEN01120
   20 MAXA(I)= 0.                                                       FEN01130
      MAXA(1)= 1                                                        FEN01140
      MAXA(2)= 2                                                        FEN01150
      MK= 0                                                             FEN01160
      IF (NEQ.EQ.1)  GO TO 100                                          FEN01170
      DO 10  I=2,NEQ                                                    FEN01180
      IF (MHT(I).GT.MK)  MK= MHT(I)                                     FEN01190
   10 MAXA(I+1)= MAXA(I) + MHT(I) + 1                                   FEN01200
  100 MK= MK + 1                                                        FEN01210
      NWK= MAXA(NEQ+1) - MAXA(1)                                        FEN01220
      RETURN                                                            FEN01230
      END                                                               FEN01240
C                                                                       FEN01250
      BLOCK DATA                                                        FEN01260
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN01270
      COMMON/INTG/ ELXY(3,4),VS(3,4),VT(3,4),WGR(5),WGP(2),WGW(5)       FEN01280
      DATA  WGR/ 7.D0, 32.D0, 12.D0, 32.D0, 7.D0 /                      FEN01290
      DATA  WGP/ 1.D0, 1.D0 /                                           FEN01300
      DATA  WGW/ 7.D0, 32.D0, 12.D0, 32.D0, 7.D0 /                      FEN01310
      END                                                               FEN01320
C                                                                       FEN01330
      SUBROUTINE  EEAM (ID,NEL,XYZ,VSP,VTP,RDS,ELG,MAXA,LM,BL,BN,SE,AG, FEN01340
     1                  ASIG,IUP,ISTATE)                                FEN01350
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN01360
      COMMON/GEN/  NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH     FEN01370
      COMMON/MAT/  STRESS(6),STRAIN(6),C(6,6),YLD,HPRIME,IST,ISR        FEN01380
      COMMON/PRO/  GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)          FEN01390
      COMMON/VAR/  IINCS,ITER,IPEL,IREM,IND,IPT,ICHEK                   FEN01400
      COMMON/SOL/  NEQ,NEQ1,NWK,MK,NES                                  FEN01410
      COMMON/LOCA/ B,S,T,TH,THIC,ZT,PI,CS,SN,RD,EG,IP1,IP2,N,NC,NTC     FEN01420
      COMMON/INTG/ ELXY(3,4),VS(3,4),VT(3,4),WGR(5),WGP(2),WGW(5)       FEN01430
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                      FEN01440
      DIMENSION   ID(NDF,1),NEL(NPE,1),XYZ(3,1),VSP(3,1),VTP(3,1),RDS(1),FEN01450
     1            ELG(1),MAXA(1),LM(NDE,1),BL(6,NDE),BN(9,NDE),SE(1),  FEN01460
     2            AG(1),ASIG(ISI,1),IUP(IPH,1),ISTATE(1)               FEN01470
      DIMENSION   WW(7),Q1(6,6),Q2(6,6),EXA(324)                        FEN01480
      EQUIVALENCE  (GEOM(2),RA),(NPAR(3),INTEF),(NPAR(4),IFIX),         FEN01490
     1             (EXA(1),Q1(1,1)),(EXA(37),Q2(1,1))                   FEN01500
      REWIND IT2                                                        FEN01510
      REWIND IT3                                                        FEN01520
C ... INITIALIZE GLOBAL STIFFNESS ARRAY                                 FEN01530
      CALL CLEAR (AG,NWK)                                               FEN01540
      NC= NDF - 6                                                       FEN01550
      NTC= NC*NPE                                                       FEN01560
      PI= 3.14159265358979300                                           FEN01570
      KP= IPP/6                                                         FEN01580
      IP1= 1 + IPP/4                                                    FEN01590
      IP2= IP1 + IPP/4                                                  FEN01600
      IMW= (IPW + 1)/2                                                  FEN01610
      DTA=  2.*PI/DFLOAT(IPP)                                           FEN01620
      DT= DFLOAT(IPW-1)                                                 FEN01630
      IF (INC.GT.0)  GO TO 40                                           FEN01640
      DO 10  I=1,IPL                                                    FEN01650
```

```
   10 WGR(I)= WGR(I)/45.                                                  FEN01660
      DO 20  I=1,KP                                                       FEN01670
   20 WGP(I)= PI*WGP(I)/12.                                               FEN01680
   40 IPT= 0                                                              FEN01690
      CALL MATEL (PROP,C)                                                 FEN01700
      DO 200  N=1,NUMEL                                                   FEN01710
      DO 60  M=1,NPE                                                      FEN01720
      M1= NEL(M,N)                                                        FEN01730
      DO 60  L=1,3                                                        FEN01740
      ELXY(L,M)= XYZ(L,M1)                                                FEN01750
      VS(L,M)= VSP(L,M1)                                                  FEN01760
   60 VT(L,M)= VTP(L,M1)                                                  FEN01770
      RD= RDS(N)                                                          FEN01780
      EG= ELG(N)                                                          FEN01790
      CALL CLEAR (SE,NES)                                                 FEN01800
C ... ENTER INTEGRATION LOOP WITH NEWTON-COTES FORMULAS                   FEN01810
      DO 160  IR=1,IPL                                                    FEN01820
      R= -1.5 + 0.5*DFLOAT(IR)                                            FEN01830
      ITHIC= (N-1)*IPL + IR                                               FEN01840
      DO 160  IA=1,IPH                                                    FEN01850
      FAC1= 2.                                                            FEN01860
      FAC2= 2.                                                            FEN01870
      IF (IA.EQ.1.OR.IA.EQ.IPH)  FAC2= 1.                                 FEN01880
      FI= DTA*DFLOAT(IA-1)                                                FEN01890
      CS= DCOS(FI)                                                        FEN01900
      SN= DSIN(FI)                                                        FEN01910
      IF (IA.EQ.IP1)  CS= 0.                                              FEN01920
      IF (IA.EQ.IP2)  SN= 0.                                              FEN01930
      THIC= TUP(IA,ITHIC)                                                 FEN01940
      RAT= THIC/RA                                                        FEN01950
      ZIN= THIC/DT                                                        FEN01960
      DO 80  I=1,IPW                                                      FEN01970
   80 WW(I)= RAT*WGW(I)/90.                                               FEN01980
      DO 160  IT=1,IPW                                                    FEN01990
      TH= 1. - RAT*DFLOAT(IPW-IT)/DT                                      FEN02000
      ZI= -ZIN*DFLOAT(IPW-IT)                                             FEN02010
      S= TH*CS                                                            FEN02020
      T= TH*SN                                                            FEN02030
      IPT= IPT + 1                                                        FEN02040
      IPEL= ISTATE(IPT)                                                   FEN02050
C ... CALCULATE STRAIN-DISPLACEMENT TRANSFORMATION MATRICES               FEN02060
      IF (INC.EQ.-1)  CALL LOCAT (ELXY,VS,VT,R,S,T,RA,N,IR,IA,IT,IPT)     FEN02070
      CALL SHAPE (ELXY,VS,VT,Q1,Q2,BL,BN,DET)                             FEN02080
      WT= FAC1*FAC2*TH*DET*WGR(IR)*WGP(1)*WW(IT)                          FEN02090
      IF (INC.EQ.0.OR.IREM.EQ.1)  GO TO 120                               FEN02100
C ... DETERMINE CURRENT STRESSES                                          FEN02110
      DO 100  I=1,IST                                                     FEN02120
  100 STRESS(I)= ASIG(I,IPT)                                              FEN02130
  120 CALL STIFF (BL,BN,Q1,Q2,C,SE,STRESS,EXA,EXA,WT)                     FEN02140
  160 CONTINUE                                                            FEN02150
      CALL ADDBAN (AG,MAXA,SE,LM(1,N),NDE)                                FEN02160
  200 CONTINUE                                                            FEN02170
      IF (NC.EQ.0.OR.(INDEF.EQ.0.AND.IFIX.EQ.0))  RETURN                  FEN02180
      CALL PENAL (ID,AG,MAXA,NUMNP,NDF,ALFA)                              FEN02190
      CALL BPCII (GDS,ELG,LM,SE,ALFA)                                     FEN02200
```

```
      RETURN                                                            FEN02210
      END                                                               FEN02220
C                                                                       FEN02230
      SUBROUTINE  BMAT (BL,BN,VS,VT,XJI,H,DH,S,T,THIC,RA,NPE,NDF)       FEN02240
C                                                                       FEN02250
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN02260
      DIMENSION  BL(6,1),BN(9,1),VS(3,1),VT(3,1),XJI(3,1),H(1),DH(1)    FEN02270
      DIMENSION  G1(3,3),G2(3,3),G3(3,3),GS1(3,3),GS2(3,3),GS3(3,3),    FEN02280
     1           DHXY(3)                                                FEN02290
      DO 120  K=1,NPE                                                   FEN02300
      DO 10  I=1,3                                                      FEN02310
      GS1(I,I)= 0.                                                      FEN02320
   10 GS2(I,I)= 0.                                                      FEN02330
      GS1(1,2)= -VS(3,K)*RA                                             FEN02340
      GS1(1,3)=  VS(2,K)*RA                                             FEN02350
      GS1(2,3)= -VS(1,K)*RA                                             FEN02360
      GS2(1,2)= -VT(3,K)*RA                                             FEN02370
      GS2(1,3)=  VT(2,K)*RA                                             FEN02380
      GS2(2,3)= -VT(1,K)*RA                                             FEN02390
      DO 20  I=2,3                                                      FEN02400
      I1= I - 1                                                         FEN02410
      DO 20  J=1,I1                                                     FEN02420
      GS1(I,J)= -GS1(J,I)                                               FEN02430
   20 GS2(I,J)= -GS2(J,I)                                               FEN02440
      DO 30  I=1,3                                                      FEN02450
      DO 30  J=1,3                                                      FEN02460
   30 GS3(I,J)= S*GS1(I,J) + T*GS2(I,J)                                 FEN02470
      DO 40  I=1,3                                                      FEN02480
      DHXY(I)= XJI(I,1)*DH(K)                                           FEN02490
      DO 40  N=1,3                                                      FEN02500
      G1(I,N)= XJI(N,1)*GS3(1,I)*DH(K) + H(K)*(XJI(N,2)*GS1(1,I)        FEN02510
     1                                       + XJI(N,3)*GS2(1,I))       FEN02520
      G2(I,N)= XJI(N,1)*GS3(2,I)*DH(K) + H(K)*(XJI(N,2)*GS1(2,I)        FEN02530
     1                                       + XJI(N,3)*GS2(2,I))       FEN02540
      G3(I,N)= XJI(N,1)*GS3(3,I)*DH(K) + H(K)*(XJI(N,2)*GS1(3,I)        FEN02550
     1                                       + XJI(N,3)*GS2(3,I))       FEN02560
      IF (DABS(G1(I,N)).LT.1.D-35)  G1(I,N)= 0.                         FEN02570
      IF (DABS(G2(I,N)).LT.1.D-35)  G2(I,N)= 0.                         FEN02580
      IF (DABS(G3(I,N)).LT.1.D-35)  G3(I,N)= 0.                         FEN02590
   40 CONTINUE                                                          FEN02600
      KM= (K-1)*NDF                                                     FEN02610
      KM1= KM + 1                                                       FEN02620
      KM2= KM + NDF                                                     FEN02630
C ... CALCULATE LINEAR  STRAIN-DISPLACEMENT TRANSFORMATION MATRIX       FEN02640
      DO 50  I=1,6                                                      FEN02650
      DO 50  J=KM1,KM2                                                  FEN02660
   50 BL(I,J)= 0.                                                       FEN02670
      DO 60  I=1,3                                                      FEN02680
      BL(I,KM+I)= DHXY(I)                                               FEN02690
      BL(I,KM+4)= G1(I,I)                                               FEN02700
      BL(I,KM+5)= G2(I,I)                                               FEN02710
      BL(I,KM+6)= G3(I,I)                                               FEN02720
      M= 1 + MOD(I,3)                                                   FEN02730
      BL(I+3,KM+I)= DHXY(M)                                             FEN02740
      BL(I+3,KM+M)= DHXY(I)                                             FEN02750
```

```
      BL( I+3,KM+4 )= G1( I,M ) * G1(M,I)                                    FEN02760
      BL( I+3,KM+5 )= G2( I,M ) * G2(M,I)                                    FEN02770
   60 BL( I+3,KM+6 )= G3( I,M ) * G3(M,I)                                    FEN02780
C ... CALCULATE NON-LINEAR  STRAIN-DISPLACEMENT TRANSFORMATION MATRIX        FEN02790
      DO 70  I=1,9                                                           FEN02800
      DO 70  J=KM1,KM2                                                       FEN02810
   70 BN( I,J )= 0.                                                          FEN02820
      DO 80  I=1,3                                                           FEN02830
      DO 80  J=1,3                                                           FEN02840
      IJ= (I-1)*3 + J                                                        FEN02850
      BN( IJ,KM+J )= DHXY(I)                                                 FEN02860
      BN( IJ,KM+4 )= G1(J,I)                                                 FEN02870
      BN( IJ,KM+5 )= G2(J,I)                                                 FEN02880
   80 BN( IJ,KM+6 )= G3(J,I)                                                 FEN02890
  120 CONTINUE                                                               FEN02900
      RETURN                                                                 FEN02910
      END                                                                    FEN02920
C                                                                            FEN02930
      SUBROUTINE BCVL ( H,DH,Q2,BL,IB,IA,IT)                                 FEN02940
C                                                                            FEN02950
      IMPLICIT REAL*8(A-H,O-Z)                                               FEN02960
      COMMON/PRO/ GEOM( 8),PROP( 6),EMOD( 3,10),FLOD( 30),NPAR( 6)           FEN02970
      COMMON/GEN/ NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPH,IPH           FEN02980
      COMMON/LOCA/ R,S,T,TH,THLC,ZT,FI,CS,SN,RD,EG,IP1,IP2,N,NC,NTC          FEN02990
      COMMON/VAR/ IINCS,IITER,IPEL,IREM,IND,IPT,ICHEK                        FEN03000
      DIMENSION  H(1),DH(4,1),Q2(6,1),BL( 6,1),BO(6,12),DB(6,12)             FEN03010
      EQUIVALENCE  (GEOM(1),TAVG),( GEOM(2),BA),(NPAR(3),INTEF)              FEN03020
C                                                                            FEN03030
      A= BA  - 0.5*TAVG                                                      FEN03040
      DO 30  K=1,NPE                                                         FEN03050
      KK= (K-1)*NC                                                           FEN03060
      DO 30  J=1,NC                                                          FEN03070
      DO 10  I=1,6                                                           FEN03080
   10 BO( I,KK+J)= 0.                                                        FEN03090
      FM= DFLOAT( 2*J)                                                       FEN03100
      C1= DCOS(FM*FI)                                                        FEN03110
      C2= CS                                                                 FEN03120
      C3= DSIN(FM*FI)                                                        FEN03130
      C4= SN                                                                 FEN03140
      VB1= FM*(1. - FM*FM)*H(K)*C1*ZT/(A*A)                                  FEN03150
      VB2= H(K)*(FM*C1*C2 + C3*C4)/(RD - A*C2)                               FEN03160
      IF (INTEF.EQ.0)  GO TO 20                                              FEN03170
      TET= EG/RD                                                             FEN03180
      C5= 2./((RD-A*C2)*TET)                                                 FEN03190
      VB2= VB2 + C5*C5*DH(K,2)*FM*C1*ZT                                      FEN03200
      VB3= C5*C3*DH(K,1)                                                     FEN03210
      IF (DABS(VB3).GE.1.D-35)  BO( 4,KK+J )= VB3                            FEN03220
   20 IF (DABS(VB2).GE.1.D-35)  BO( 2,KK+J )= VB2                            FEN03230
      IF (DABS(VB1).GE.1.D-35)  BO( 1,KK+J )= VB1                            FEN03240
   30 CONTINUE                                                               FEN03250
C                                                                            FEN03260
      DO 50  I=1,6                                                           FEN03270
      DO 50  J=1,NIC                                                         FEN03280
      SUM= 0.                                                                FEN03290
      DO 40  K=1,6                                                           FEN03300
```

```
   40 SUM= SUM + C2(I,K)*BO(K,J)                                        FEN03310
      IF (DABS(SUM).LT.1.D-35)  SUM= 0.                                 FEN03320
   50 DB(I,J)= SUM                                                      FEN03330
      DO 60  K=1,NPE                                                    FEN03340
      K1= (K-1)*NDF + 6                                                 FEN03350
      K2= (K-1)*NC                                                      FEN03360
      DO 60  I=1,6                                                      FEN03370
      DO 60  J=1,NC                                                     FEN03380
   60 BL(I,K1+J)=  DB(I,K2+J)                                           FEN03390
      RETURN                                                            FEN03400
      END                                                               FEN03410
C                                                                       FEN03420
      SUBROUTINE CLEAR (Z,LZ)                                           FEN03430
C                                                                       FEN03440
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN03450
      DIMENSION Z(LZ)                                                   FEN03460
      DO 10  I=1,LZ                                                     FEN03470
   10 Z(I)= 0.                                                          FEN03480
      RETURN                                                            FEN03490
      END                                                               FEN03500
C                                                                       FEN03510
      SUBROUTINE  CCLHT (MHT,LM,ND)                                     FEN03520
C                                                                       FEN03530
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN03540
      DIMENSION  MHT(1),LM(1)                                           FEN03550
      LS= 100000                                                        FEN03560
      DO 100 I=1,NC                                                     FEN03570
      IF (LM(I)) 110,100,110                                            FEN03580
  110 IF (LM(I)-LS) 120,100,100                                         FEN03590
  120 LS= LM(I)                                                         FEN03600
  100 CONTINUE                                                          FEN03610
C                                                                       FEN03620
      DO 200 I=1,NC                                                     FEN03630
      II= LM(I)                                                         FEN03640
      IF (II.EQ.0) GO TO 200                                            FEN03650
      ME= II - LS                                                       FEN03660
      IF (ME.GT.MHT(II)) MHT(II)= ME                                    FEN03670
  200 CONTINUE                                                          FEN03680
      RETURN                                                            FEN03690
      END                                                               FEN03700
C                                                                       FEN03710
      SUBROUTINE CCLSOL (A,V,MAXA,NN,NWK,NNM,KKK)                       FEN03720
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . FEN03730
C .                                                                     . FEN03740
C .   P R O G R A M                                                     . FEN03750
C .      TO SOLVE FINITE ELEMENT STATIC EQUILIBRIUM EQUATIONS IN        . FEN03760
C .      CORE, USING COMPACTED STORAGE AND COLUMN REDUCTION SCHEME      . FEN03770
C .                                                                     . FEN03780
C .   ** INPUT VARIABLES                                                . FEN03790
C .         A(NWK)     = STIFFNESS MATRIX STORED IN COMPACTED FORM      . FEN03800
C .         V(NN)      = RIGHT-HAND-SIDE LOAD VECTOR                    . FEN03810
C .         MAXA(NNM)  = VECTOR CONTAINING ADDRESSES OF DIAGONAL        . FEN03820
C .                      ELEMENTS OF STIFFNESS MATRIX IN  A             . FEN03830
C .         NN         = NUMBER OF EQUATIONS                            . FEN03840
C .         NWK        = NUMBER OF ELEMENTS BELOW SKYLINE OF MATRIX     . FEN03850
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C .          NNM        = NN + 1                                                  . FEN03860
C .          MKK        = INPUT FLAG                                              . FEN03870
C .              EQ. 1 TRIANGULARIZATION OF STIFFNESS MATRIX                       . FEN03880
C .              EQ. 2 REDUCTION AND BACK-SUBSTITUTION OF LOAD VECTOR              . FEN03890
C .          IT6        = NUMBER OF OUTPUT DEVICE                                 . FEN03900
C .                                                                               . FEN03910
C .  ** OUTPUT                                                                    . FEN03920
C .          A(NWK)     = D AND L : FACTORS OF STIFFNESS MATRIX                   . FEN03930
C .          V(NN)      = DISPLACEMENT VECTOR                                     . FEN03940
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . FEN03950
      IMPLICIT REAL*8(A-H,O-Z)                                                    FEN03960
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                                FEN03970
      DIMENSION  A(NWK),V(1),MAXA(1)                                              FEN03980
C ... PERFORM L*D*L(T) FACTORIZATION OF STIFFNESS MATRIX                          FEN03990
      IF (KKK-2) 40,150,150                                                       FEN04000
   40 DO 140  N=1,NN                                                              FEN04010
      KN= MAXA(N)                                                                 FEN04020
      KL= KN + 1                                                                  FEN04030
      KU= MAXA(N+1) - 1                                                           FEN04040
      KH= KU - KL                                                                 FEN04050
      IF (KH) 110,90,50                                                           FEN04060
   50 K= N - KH                                                                   FEN04070
      IC= 0                                                                       FEN04080
      KLT= KU                                                                     FEN04090
      DO 80  J=1,KH                                                               FEN04100
      IC= IC + 1                                                                  FEN04110
      KLT= KLT - 1                                                                FEN04120
      KI= MAXA(K)                                                                 FEN04130
      ND= MAXA(K+1) - KI - 1                                                      FEN04140
      IF (ND) 80,80,60                                                            FEN04150
   60 KK= MIN0(IC,ND)                                                             FEN04160
      C= 0.                                                                       FEN04170
      DO 70  L=1,KK                                                               FEN04180
   70 C= C + A(KI+L)*A(KLT+L)                                                     FEN04190
      A(KLT)= A(KLT) - C                                                          FEN04200
   80 K= K + 1                                                                    FEN04210
   90 K= N                                                                        FEN04220
      B= 0.                                                                       FEN04230
      DO 100  KK=KL,KU                                                            FEN04240
      K= K - 1                                                                    FEN04250
      KI= MAXA(K)                                                                 FEN04260
      C= A(KK)/A(KI)                                                              FEN04270
      B= B + C*A(KK)                                                              FEN04280
  100 A(KK)= C                                                                    FEN04290
      A(KN)= A(KN) - B                                                            FEN04300
  110 IF (A(KN))  120,120,140                                                     FEN04310
  120 WRITE(IT6,2000) N,A(KN)                                                     FEN04320
      STOP                                                                        FEN04330
  140 CONTINUE                                                                    FEN04340
      RETURN                                                                      FEN04350
C ... REDUCE RIGHT-HAND-SIDE LOAD VECTOR                                          FEN04360
  150 DO 180  N=1,NN                                                              FEN04370
      KL= MAXA(N) + 1                                                             FEN04380
      KU= MAXA(N+1) - 1                                                           FEN04390
      IF (KU-KL) 180,160,160                                                      FEN04400
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  160 K= N                                                                  FEN04410
      C= 0.                                                                 FEN04420
      DO 170  KK=KL,KU                                                      FEN04430
      K= K - 1                                                              FEN04440
  170 C= C + A(KK)*V(K)                                                     FEN04450
      V(N)= V(N) - C                                                        FEN04460
  180 CONTINUE                                                              FEN04470
C ... BACK-SUBSTITUTE                                                       FEN04480
      DO 200  N=1,NN                                                        FEN04490
      K= MAXA(N)                                                            FEN04500
  200 V(N)= V(N)/A(K)                                                       FEN04510
      IF (NN.EQ.1) RETURN                                                   FEN04520
      N= NN                                                                 FEN04530
      DO 230  L=2,NN                                                        FEN04540
      KL= MAXA(N) + 1                                                       FEN04550
      KU= MAXA(N+1) - 1                                                     FEN04560
      IF (KU-KL) 230,210,210                                                FEN04570
  210 K= N                                                                  FEN04580
      DO 220  KK=KL,KU                                                      FEN04590
      K= K - 1                                                              FEN04600
  220 V(K)= V(K) - A(KK)*V(N)                                               FEN04610
  230 N= N - 1                                                              FEN04620
      RETURN                                                                FEN04630
 2000 FORMAT(//48H STOP - STIFFNESS MATRIX NOT POSITIVE DEFINITE  //        FEN04640
     1          32H NONPOSITIVE PIVOT FOR EQUATION  ,I4//                  FEN04650
     2          10H PIVOT =  ,D20.12 )                                     FEN04660
      END                                                                   FEN04670
C                                                                           FEN04680
      SUBROUTINE  CCSET (ID,NEL,MHT,MAXA,LM)                                FEN04690
C                                                                           FEN04700
      IMPLICIT REAL*8(A-H,O-Z)                                              FEN04710
      COMMON/GEN/ NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH          FEN04720
      COMMON/PRO/ GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)               FEN04730
      COMMON/SOL/ NEQ,NEQ1,NWK,MK,NES                                       FEN04740
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                          FEN04750
      DIMENSION  ID(NDF,1),NEL(NPE,1),MHT(1),MAXA(1),LM(NDE,1)              FEN04760
      EQUIVALENCE  (NPAR(3),INTEF)                                          FEN04770
C ... CALCULATE COLUMN HEIGHTS                                              FEN04780
      DO 10  I=1,NEQ                                                        FEN04790
   10 MHT(I)= 0.                                                            FEN04800
      DO 40  I=1,NUMEL                                                      FEN04810
      IJ= 0                                                                 FEN04820
      DO 20  J=1,NPE                                                        FEN04830
      IK= NEL(J,I)                                                          FEN04840
      DO 20  K=1,NDF                                                        FEN04850
      IJ= IJ + 1                                                            FEN04860
   20 LM(IJ,I)= ID(K,IK)                                                    FEN04870
      CALL CCLHT (MHT,LM(1,I),NDE)                                          FEN04880
   40 CONTINUE                                                              FEN04890
      IF (INTEF.EQ.0.OR.NUMEL.EQ.1.OR.NDF.EQ.6)  GO TO 150                  FEN04900
C ... RECALCULATE COLUMN HEIGHTS IF INTEF'S ARE TAKEN INTO ACCOUNT          FEN04910
      NPE1= NPE - 1                                                         FEN04920
      NUM= NUMEL - 1                                                        FEN04930
      DO 120  I=1,NUM                                                       FEN04940
      LOW= 100000                                                           FEN04950
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      DO 80  J=1,NPE1                                                   FEN04960
      J1= (J-1)*NDF                                                     FEN04970
      DO 80  K=7,NCF                                                    FEN04980
      NEW= LM(J1+K,I)                                                   FEN04990
      IF (NEW)  80,80,60                                                FEN05000
   60 IF (NEW.LT.LOW)  LOW= NEW                                         FEN05010
   80 CONTINUE                                                          FEN05020
      DO 100  J=2,NPE                                                   FEN05030
      J1= (J-1)*NDF                                                     FEN05040
      DO 100  K=7,NDF                                                   FEN05050
      KK= LM(J1+K,I+1)                                                  FEN05060
      IF (KK.EQ.0)  GO TO 100                                           FEN05070
      KW= KK - LOW                                                      FEN05080
      IF (KW.GT.MHT(KK))  MHT(KK)= KW                                   FEN05090
  100 CONTINUE                                                          FEN05100
  120 CONTINUE                                                          FEN05110
C ... CALCULATE ADDRESS OF DIAGONAL ELEMENTS OF STIFFNESS MATRIX        FEN05120
  150 CALL ADDRES (MAXA,MHT)                                            FEN05130
      MM= NWK/NEQ                                                       FEN05140
      WRITE (IT6,2000)  NEQ,NWK,MK,MM                                   FEN05150
      RETURN                                                            FEN05160
 2000 FORMAT (// 40H T O T A L   S Y S T E M   D A T A      //5X,       FEN05170
     1 40H NUMBER OF EQUATIONS                (NEQ) =,I6/5X,           FEN05180
     2 40H NUMBER OF MATRIX ELEMENTS          (NWK) =,I6/5X,           FEN05190
     3 40H MAXIMUM HALF BANDWIDTH              (MK) =,I6/5X,           FEN05200
     4 40H MEAN HALF BANDWIDTH                 (MM) =,I6)              FEN05210
      END                                                               FEN05220
C                                                                       FEN05230
      SUBROUTINE  DATA (ID,NEL,XYZ,VSP,VTP,RDS,ELG,NRN)                 FEN05240
C                                                                       FEN05250
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN05260
      COMMON/GEN/  NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH     FEN05270
      COMMON/MAT/  STRESS(6),STRAIN(6),C(6,6),YLD,HPRIME,IST,ISR        FEN05280
      COMMON/PRO/  GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)          FEN05290
      COMMON/MAX/  NMODS,NINCS,MITER,MODEX,INTUP                        FEN05300
      COMMON/CON/  FTOL,ETOL,TOLFO,TOLEN,CURVY                          FEN05310
      COMMON/SOL/  NEQ,NE)1,NWK,MK,NES                                  FEN05320
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                      FEN05330
      DIMENSION   ID(NDF,1),NEL(NPE,1),XYZ(3,1),VSP(3,1),VTP(3,1),RDS(1),FEN05340
     1           ELG(1)                                                 FEN05350
      EQUIVALENCE  (GEOM(3),RMAX),(PROP(1),YM),(NPAR(1),ITYPE)          FEN05360
      WRITE (IT6,2000)                                                  FEN05370
      READ (IT5,1000)  (GEOM(I),I=1,6),(PROP(I),I=1,6),(NPAR(I),I=1,6)  FEN05380
      WRITE (IT6,2020) (GEOM(I),I=1,4),(PROP(I),I=1,4),(NPAR(I),I=1,4)  FEN05390
      NDE= NPE*NDF                                                      FEN05400
      IPH= 1 + IPP/2                                                    FEN05410
      NIPS= NUMEL*IPL*IPH*IPW                                           FEN05420
      NES= NCE*(NDE + 1)/2                                              FEN05430
      IST= 6                                                            FEN05440
      ISR= 6                                                            FEN05450
C ... DATA ARE READ IN                                                  FEN05460
      DO 20  I=1,NUMEL                                                  FEN05470
      RDS(I)= RMAX                                                      FEN05480
   20 READ (IT5,1010)  J,ELG(J)                                         FEN05490
C ... ELEMENT CONNECTIVITY IS GENERATED                                 FEN05500
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      I= 1                                                                  FEN05510
      DO 40  N=1,NUMEL                                                      FEN05520
      I= I - 1                                                              FEN05530
      DO 40  K=1,NPE                                                        FEN05540
      I= I + 1                                                              FEN05550
   40 NEL(K,N)= I                                                           FEN05560
C ... COORDINATES AND UNIT VECTORS ARE GENERATED                            FEN05570
      DO 60  I=1,NUMNP                                                      FEN05580
      XYZ(2,I)=  0.                                                         FEN05590
      XYZ(3,I)=  0.                                                         FEN05600
      VSP(1,I)=  0.                                                         FEN05610
      VSP(2,I)=  1.                                                         FEN05620
      VSP(3,I)=  0.                                                         FEN05630
      VIP(1,I)=  0.                                                         FEN05640
      VIP(2,I)=  0.                                                         FEN05650
      VIP(3,I)=  1.                                                         FEN05660
   60 CONTINUE                                                              FEN05670
      XYZ(1,1)= 0.                                                          FEN05680
      JJ= 1                                                                 FEN05690
      DO 80  I=1,NUMEL                                                      FEN05700
      XINC= ELG(I)/3.                                                       FEN05710
      DO 70  J=2,NPE                                                        FEN05720
      JI= NEL(J,I)                                                          FEN05730
   70 XYZ(1,JI)= XYZ(1,JJ) + XINC*DFLOAT(J-1)                               FEN05740
   80 JJ= JI                                                                FEN05750
      WRITE (IT6,2040)                                                      FEN05760
      DO 120  N=1,NUMEL                                                     FEN05770
  120 WRITE (IT6,2060)  N,(NEL(J,N),J=1,NPE),RDS(N),ELG(N)                  FEN05780
C ... READ IN RESTRAINT NODE INFORMATION                                    FEN05790
      WRITE (IT6,2080)                                                      FEN05800
      DO 140  I=1,NUMNP                                                     FEN05810
      DO 140  J=1,NDF                                                       FEN05820
  140 ID(J,I)= 0.                                                           FEN05830
      DO 160  IRN=1,NRN                                                     FEN05840
      READ (IT5,1020)  NORN,(ID(IDF,NORN),IDF=1,NDF)                        FEN05850
  160 WRITE (IT6,2100) NORN,(ID(IDF,NORN),IDF=1,NDF)                        FEN05860
      CALL NUMUN (ID,NDF,NEQ,NUMNP)                                         FEN05870
      NEQ1= NEQ + 1                                                         FEN05880
C ... READ IN LOAD FACTORS                                                  FEN05890
      READ (IT5,1040)  (FLOD(I),I=1,NINCS)                                 FEN05900
      WRITE (IT6,2120)                                                      FEN05910
      WRITE (IT6,2140) (FLOD(I),I=1,NINCS)                                  FEN05920
      IF (ITYPE.EQ.0)  RETURN                                               FEN05930
      WRITE (IT6,2160)                                                      FEN05940
C ... READ IN DATA FROM TRUE STRESS-STRAIN CURVE OF MATERIAL - A MAXIMUM    FEN05950
C     OF 10 SETS OF SIG-EPS COORDINATES COULD BE INPUT                      FEN05960
      DO 180  IMODS=1,NMODS                                                 FEN05970
  180 READ (IT5,1060)  I,(EMOD(J,I),J=1,2)                                  FEN05980
      DO 220  I=1,NMODS                                                     FEN05990
      IF (I.EQ.1)  GO TO 200                                                FEN06000
      ET= (EMOD(2,I)-EMOD(2,I-1))/(EMOD(1,I)-EMOD(1,I-1))                   FEN06010
      EMOD(3,I)= YM*ET/(YM - ET)                                            FEN06020
      GO TO 220                                                             FEN06030
  200 EMOD(3,I)= 0.                                                         FEN06040
  220 CONTINUE                                                              FEN06050
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C ... CALCULATE THE ACCUMULATIVE EFFECTIVE PLASTIC STRAIN FOR EACH       FEN06060
C     LINEAR RANGE                                                       FEN06070
      DO 260  I=1,NMODS                                                  FEN06080
      IF (I.EQ.1)  GO TO 240                                             FEN06090
      EMOD(1,I)= EMOD(1,I) - EMOD(2,I)/YM                                FEN06100
      GO TO 260                                                          FEN06110
  240 EMOD(1,I)= 0.                                                      FEN06120
  260 WRITE (IT6,2180)  I,(EMOD(J,I),J=1,3)                              FEN06130
C ... READ IN CONVERGENCE CRITERIA FOR NON-LINEAR ANALYSIS               FEN06140
      READ (IT5,1080)  FTOL,ETOL                                         FEN06150
      WRITE (IT6,2200) FTOL,ETOL                                         FEN06160
      TOLFO= 0.                                                          FEN06170
      RETURN                                                             FEN06180
 1000 FORMAT (6D10.4/,6D10.4/,6I5)                                       FEN06190
 1010 FORMAT (I5,D15.5)                                                  FEN06200
 1020 FORMAT (I5,5X,9I5)                                                 FEN06210
 1040 FORMAT (7F10.5)                                                    FEN06220
 1060 FORMAT (I5,2D15.5)                                                 FEN06230
 1080 FORMAT (2D10.4)                                                    FEN06240
 2000 FORMAT (//48H I N F O R M A T I O N   O F   S T R U C T U R E )    FEN06250
 2020 FORMAT (//5X,40H * *  G E O M T R I C   D A T A  * *          //5X, FEN06260
     1 50H PIPE WALL THICKNESS                          (GEOM(1)) =,D15.5/5X, FEN06270
     2 50H INITIAL OUTSIDE RADIUS OF X-SECTION          (GEOM(2)) =,D15.5/5X, FEN06280
     3 50H INITIAL RADIUS OF CURVATURE                  (GEOM(3)) =,D15.5/5X, FEN06290
     4 50H TOTAL LENGTH OF THE PIPE                     (GEOM(4)) =,D15.5//5X, FEN06300
     1 50H * *  M A T E R I A L   D A T A  * *                      //5X, FEN06310
     2 50H YOUNG'S MODULUS  ( YM )                      (PROP(1)) =,D15.5/5X, FEN06320
     3 50H POISSON RATIO    ( NU )                      (PROP(2)) =,D15.5/5X, FEN06330
     4 50H SHEAR FACTOR     ( K )                       (PROP(3)) =,D15.5/5X, FEN06340
     5 50H UNIAXIAL YIELD STRENGTH  ( UNIAX )           (PROP(4)) =,D15.5//5X, FEN06350
     1 50H * *  O P T I O N S   O F   A N A L Y S I S  * *  //5X,       FEN06360
     2 50H TYPE OF ANALYSIS                             (NPAR(1)) =,I5/5X,  FEN06370
     3 50H       EQ.0 : LINEAR,    EQ.1 : NON-LINEAR                 /5X,  FEN06380
     4 50H TYPE OF DATA INPUT                           (NPAR(2)) =,I5/5X,  FEN06390
     5 50H       EQ.0 : GENERATED, EQ.1 : READ-IN                    /5X,  FEN06400
     6 50H OPTION OF INTERACTION EFFECTS                (NPAR(3)) =,I5/5X,  FEN06410
     7 50H       EQ.0 : IGNORED,   EQ.1 : CONSIDERED                 /5X,  FEN06420
     8 50H OPTION OF FIXITY CONDITION                   (NPAR(4)) =,I5/5X,  FEN06430
     9 52H       EQ.0 : IGNORED,   EQ.J : CONSIDERED AT J                )  FEN06440
 2040 FORMAT (//40H E L E M E N T   I N F O R M A T I O N   //          FEN06450
     1 6X,7HELEMENT,7X,12HCONNECTIVITY,9X,8HRADIUS-C,7X,6HLENGTH)        FEN06460
 2060 FORMAT (/I10,9X,4I3,10X,D10.4,5X,D10.4)                            FEN06470
 2080 FORMAT (//35H * * *  RESTRAINT NODE INFORMATION ,//10X,4HNODE,13X,FEN06480
     1 18HDEGREES OF FREEDOM )                                           FEN06490
 2100 FORMAT (/8X,I5,10X,9I5)                                            FEN06500
 2120 FORMAT (//50H * * *  ORDER OF INCREMENTAL LOAD FACTORS APPLIED  ) FEN06510
 2140 FORMAT (/10F10.4)                                                  FEN06520
 2160 FORMAT (//50H * * *  DATA FROM TRUE STRESS-STRAIN CURVE FOR NON,  FEN06530
     1 16H-LINEAR ANALYSIS //16X,3HSET,12X,9HP-EPSILON,10X,5HSIGMA,13X,  FEN06540
     2 2HET )                                                            FEN06550
 2180 FORMAT (/16X,I2,7X,3D15.5)                                         FEN06560
 2200 FORMAT (//53H * * *  CONVERGENCE CONDITIONS FOR NONLINEAR ANALYSISFEN06570
     1 //5X,40H FORCE TOLERANCE                    (FTOL) =,D12.3        FEN06580
     2 //5X,40H ENERGY TOLERANCE                   (ETOL) =,D12.3)       FEN06590
      END                                                                FEN06600
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C                                                                               FEN06610
      SUBROUTINE  ELPAL (SIG,EPS,OHM,EFSI)                                      FEN06620
C                                                                               FEN06630
      IMPLICIT REAL*8(A-H,O-Z)                                                  FEN06640
      COMMON/MAT/ STRESS(6),STRAIN(6),C(6,6),YLD,HPRIME,IST,ISR                 FEN06650
      COMMON/MAX/ NMODS,NINCS,MITER,MODEX,INTUP                                 FEN06660
      COMMON/PRO/ GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)                   FEN06670
      COMMON/VAR/ IINCS,IITER,IPEL,IREM,IND,IPT,ICHEK                           FEN06680
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                              FEN06690
      DIMENSION   SIG(1),EPS(1),OHM(3,1)                                        FEN06700
      DIMENSION   TAU(6),DELSIG(6),DELEPS(6),DEPS(6),CEP(6,6),RS(3,6),          FEN06710
     1            RN(3,6)                                                      FEN06720
      EQUIVALENCE  (CEP(1,1),RS(1,1)),(CEP(1,4),RN(1,1)),(DELEPS,DEPS)          FEN06730
      YM= PROP(1)                                                               FEN06740
      UNIAX= PROP(4)                                                            FEN06750
      IMODS= 2                                                                  FEN06760
      IF (IPEL.EQ.1)  GO TO 60                                                  FEN06770
      DO 20  IMODS=2,NMODS                                                      FEN06780
      IF (EFSI.LT.EMOD(1,IMODS))  GO TO 40                                      FEN06790
   20 CONTINUE                                                                  FEN06800
      WRITE (IT6,2000)  EFSI                                                    FEN06810
      STOP                                                                      FEN06820
   40 H= (EMOD(2,IMODS)-EMOD(2,IMODS-1))/(EMOD(1,IMODS)-EMOD(1,IMODS-1))        FEN06830
      YLD= EMOD(2,IMODS-1) + H*(EFSI - EMOD(1,IMODS-1))                         FEN06840
      YLDMAX= EMOD(2,IMODS)                                                     FEN06850
      NEULER= 20                                                                FEN06860
      GO TO 80                                                                  FEN06870
   60 YLD= UNIAX                                                                FEN06880
      YLDMAX= EMOD(2,2)                                                         FEN06890
      NEULER= 5                                                                 FEN06900
      IF (INC.EQ.0)  GO TO 240                                                  FEN06910
   80 XNEU= DFLOAT(NEULER)                                                      FEN06920
      OHM(1,2)= OHM(1,2)/XNEU                                                   FEN06930
      OHM(2,3)= OHM(2,3)/XNEU                                                   FEN06940
      OHM(1,3)= OHM(1,3)/XNEU                                                   FEN06950
      OHM(2,1)= -OHM(1,2)                                                       FEN06960
      OHM(3,2)= -OHM(2,3)                                                       FEN06970
      OHM(3,1)= -OHM(1,3)                                                       FEN06980
C ... MAKE CORRECTIONS FOR STRESSES AND STRAINS DUE TO  H.B.  ROTATIONS         FEN06990
      DO 100  I=1,3                                                             FEN07000
      RS(I,I)= SIG(I)                                                           FEN07010
  100 RN(I,I)= EPS(I)                                                           FEN07020
      RS(1,2)= SIG(4)                                                           FEN07030
      RS(2,3)= SIG(5)                                                           FEN07040
      RS(1,3)= SIG(6)                                                           FEN07050
      RN(1,2)= 0.5*EPS(4)                                                       FEN07060
      RN(2,3)= 0.5*EPS(5)                                                       FEN07070
      RN(1,3)= 0.5*EPS(6)                                                       FEN07080
      DO 120  I=2,3                                                             FEN07090
      I1= I-1                                                                   FEN07100
      DO 120  J=1,I1                                                            FEN07110
      RS(I,J)= RS(J,I)                                                          FEN07120
  120 RN(I,J)= RN(J,I)                                                          FEN07130
C ... START PERFORMING EULER FORWARD INTEGRATION WITH NEULER INTERVALS          FEN07140
      DO 140  I=1,3                                                             FEN07150
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      DO 140  J=1,3                                                     FEN07160
      BS(I,J+3)= BS(I,J)                                                FEN07170
  140 BN(I,J+3)= BN(I,J)                                                FEN07180
      DO 200  IE=1,NEULER                                               FEN07190
      DO 160  I=1,3                                                     FEN07200
      DO 160  J=1,3                                                     FEN07210
      DO 160  K=1,3                                                     FEN07220
      BS(I,J+3)= BS(I,J+3) + BS(I,K)*OHM(K,J) - OHM(I,K)*BS(K,J)        FEN07230
  160 BN(I,J+3)= BN(I,J+3) + BN(I,K)*OHM(K,J) - OHM(I,K)*BN(K,J)        FEN07240
      DO 180  I=1,3                                                     FEN07250
      DO 180  J=1,3                                                     FEN07260
      BS(I,J)= BS(I,J+3)                                                FEN07270
  180 BN(I,J)= BN(I,J+3)                                                FEN07280
  200 CONTINUE                                                          FEN07290
      DO 220  I=1,3                                                     FEN07300
      SIG(I)= BS(I,I)                                                   FEN07310
  220 EPS(I)= BN(I,I)                                                   FEN07320
      SIG(4)= BS(1,2)                                                   FEN07330
      SIG(5)= BS(2,3)                                                   FEN07340
      SIG(6)= BS(1,3)                                                   FEN07350
      EPS(4)= 2.*BN(1,2)                                                FEN07360
      EPS(5)= 2.*BN(2,3)                                                FEN07370
      EPS(6)= 2.*BN(1,3)                                                FEN07380
C ... 1. CALCULATE INCREMENTAL STRAINS                                  FEN07390
C ... UPDATE STRAIN COMPONENTS AND TOTAL EFFECTIVE STRAIN               FEN07400
  240 DO 260  I=1,ISR                                                   FEN07410
      EPS(I)= EPS(I) + STRAIN(I)                                        FEN07420
  260 DELEPS(I)= STRAIN(I)                                              FEN07430
C ... 2. CALCULATE STRESS INCREMENT, ASSUMING ELASTIC BEHAVIOUR         FEN07440
      DO 280  IS=1,IST                                                  FEN07450
      DELSIG(IS)= 0.                                                    FEN07460
      DO 280  IR=1,ISR                                                  FEN07470
  280 DELSIG(IS)= DELSIG(IS) + C(IS,IR)*DELEPS(IR)                      FEN07480
C ... 3. CALCULATE TOTAL STRESSES, ASSUMING ELASTIC BEHAVIOUR           FEN07490
      DO 300  I=1,IST                                                   FEN07500
      SUM= SIG(I) + DELSIG(I)                                           FEN07510
      IF (DABS(SUM).LT.1.D-35)  SUM= 0.                                 FEN07520
  300 TAU(I)= SUM                                                       FEN07530
C ... 4. CHECK WHETHER "TAU" STATE OF STRESS FALLS OUTSIDE THE LOADING  FEN07540
C          SURFACE                                                      FEN07550
      SM= (TAU(1)+TAU(2)+TAU(3))/3.                                     FEN07560
      SX= TAU(1) - SM                                                   FEN07570
      SY= TAU(2) - SM                                                   FEN07580
      SZ= TAU(3) - SM                                                   FEN07590
      SXY= TAU(4)                                                       FEN07600
      SYZ= TAU(5)                                                       FEN07610
      SZX= TAU(6)                                                       FEN07620
      FI= 0.5*(SX*SX+SY*SY+SZ*SZ)+(SXY*SXY+SYZ*SYZ+SZX*SZX)-YLD*YLD/3.  FEN07630
      IF (FI)  620,620,340                                              FEN07640
C ... STATE OF STRESS WITHIN LOADING SURFACE - ELASTIC BEHAVIOUR        FEN07650
C 620 IPEL= 1                                                           FEN07660
C ... STATE OF STRESS OUTSIDE LOADING SURFACE - PLASTIC BEHAVIOUR       FEN07670
  340 IF (IPEL.EQ.1)  GO TO 380                                         FEN07680
C ... MATERIAL WAS PREVIOUSLY PLASTIC                                   FEN07690
      RATIO= 0.                                                         FEN07700
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      DO 360  I=1,IST                                                    FEN07710
  360 TAU(I)= SIG(I)                                                     FEN07720
      GO TO 420                                                          FEN07730
C ... MATERIAL WAS PREVIOUSLY ELASTIC                                    FEN07740
  380 IPEL= 2                                                            FEN07750
      SM= (SIG(1) + SIG(2) + SIG(3))/3.                                  FEN07760
      SX= SIG(1) - SM                                                    FEN07770
      SY= SIG(2) - SM                                                    FEN07780
      SZ= SIG(3) - SM                                                    FEN07790
      SXY= SIG(4)                                                        FEN07800
      SYZ= SIG(5)                                                        FEN07810
      SZX= SIG(6)                                                        FEN07820
      DM= (DELSIG(1) + DELSIG(2) + DELSIG(3))/3.                         FEN07830
      DX= DELSIG(1) - DM                                                 FEN07840
      DY= DELSIG(2) - DM                                                 FEN07850
      DZ= DELSIG(3) - DM                                                 FEN07860
      DXY= DELSIG(4)                                                     FEN07870
      DYZ= DELSIG(5)                                                     FEN07880
      DZX= DELSIG(6)                                                     FEN07890
      A= (DX*DX + DY*DY + DZ*DZ) + 2.*(DXY*DXY + DYZ*DYZ + DZX*DZX)      FEN07900
      B= (SX*DX + SY*DY + SZ*DZ) + 2.*(SXY*DXY + SYZ*DYZ + SZX*DZX)      FEN07910
      E= (SX*SX+SY*SY+SZ*SZ)+2.*(SXY*SXY+SYZ*SYZ+SZX*SZX)-2.*YLD*YLD/3.  FEN07920
      RATIO= (-B + DSQRT(B*B-A*E))/A                                     FEN07930
      DO 400  I=1,IST                                                    FEN07940
  400 TAU(I)= SIG(I) + RATIO*DELSIG(I)                                   FEN07950
C ... "TAU" NOW CONTAINS (PREVIOUS STRESSES + STRESSES DUE TO ELASTIC    FEN07960
C                                                    STRAIN INCREMENTS)  FEN07970
C                                                                        FEN07980
C     5. CALCULATE PLASTIC STRESSES                                      FEN07990
C ... DETERMINE INCREMENT INTERVAL                                       FEN08000
  420 M= 30.*DSQRT(FT)/YLD + 1.                                          FEN08010
      IF (M.GT.30)  M= 30                                                FEN08020
      XM= (1. - RATIO)/DFLOAT(M)                                         FEN08030
      DO 440  I=1,ISR                                                    FEN08040
  440 DEPS(I)= XM*DELEPS(I)                                              FEN08050
      DO 460  I=1,IST                                                    FEN08060
  460 STRESS(I)= XM*DELSIG(I)                                            FEN08070
C ... DETERMINE CURRENT VALUE OF STRAIN HARDENING                        FEN08080
      HPRIME= EMOD(3,IMODS)                                              FEN08090
C ... CALCULATION OF ELASTO-PLASTIC STRESSES (START...)                  FEN08100
      EBAR= 0.                                                           FEN08110
      DO 600  IM=1,M                                                     FEN08120
      CALL MIDEP (TAU,CEP,DEBAR,1)                                       FEN08130
      DO 520  I=1,IST                                                    FEN08140
      DO 520  J=1,ISR                                                    FEN08150
  520 TAU(I)= TAU(I) + CEP(I,J)*DEPS(J)                                  FEN08160
C ... SCALE DOWN ICTAL EFFECTIVE STRESS TO YIELD SURFACE IF REQUIRED     FEN08170
      EBAR= EBAR + DEBAR                                                 FEN08180
      YLD= YLC + HPRIME*DEBAR                                            FEN08190
      IF (YLC.LE.YLCMAX)  GO TO 560                                      FEN08200
C ... RECALCULATE 'HPRIME' AND 'YLD' IF 'YLD' ENTERS A NEW LINEAR RANGE  FEN08210
  540 IMODS= IMODS + 1                                                   FEN08220
      IF (IMODS.LE.NMODS)  GO TO 530                                     FEN08230
      WRITE (IT6,2000)                                                   FEN08240
      STOP                                                               FEN08250
  530 EPEX= (YLD - YLDMAX)/HPRIME
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      HPRIME= EMOD(3,IMCOS)                                                  FEN08260
      YLD= YLDMAX + EPEX*HPRIME                                              FEN08270
      YLDMAX= EMOD(2,IMCOS)                                                  FEN08280
      IF (YLD.GT.YLDMAX)  GO TO 540                                          FEN08290
  560 DM= (TAU(1) + TAU(2) + TAU(3))/3.                                      FEN08300
      DX= TAU(1) - DM                                                        FEN08310
      DY= TAU(2) - DM                                                        FEN08320
      DZ= TAU(3) - DM                                                        FEN08330
      DXY= TAU(4)                                                            FEN08340
      DYZ= TAU(5)                                                            FEN08350
      DZX= TAU(6)                                                            FEN08360
      SBAR= DSQRT(1.5*(DX*DX+DY*DY+DZ*DZ)+3.*(DXY*DXY+DYZ*DYZ+DZX*DZX))      FEN08370
      IF (YLD.GT.SBAR) GO TO 600                                             FEN08380
      SCALE= YLD/SBAR                                                        FEN08390
      DO 580  I=1,IST                                                        FEN08400
  580 TAU(I)= SCALE*TAU(I)                                                   FEN08410
  600 CONTINUE                                                               FEN08420
C ... CALCULATION OF ELASTO-PLASTIC STRESSES (END...)                        FEN08430
C ... UPDATE EFFECTIVE PLASTIC STRAIN AND FORM NEW MATERIAL LAW              FEN08440
      EPST= EPST + EBAR                                                      FEN08450
  620 DO 640  I=1,IST                                                        FEN08460
  640 SIG(I)= TAU(I)                                                         FEN08470
      IF (IPEL.GT.1.AND.ICHEK.GT.0)  GO TO 660                               FEN08480
      RETURN                                                                 FEN08490
  660 CALL MIDEP (TAU,CEP,DEBAR,0)                                           FEN08500
      DO 680  I=1,6                                                          FEN08510
      DO 680  J=1,6                                                          FEN08520
  680 C(I,J)= CEP(I,J)                                                       FEN08530
      RETURN                                                                 FEN08540
 2000 FORMAT (////50H ***** CALCULATED PLASTIC STRAIN HAS EXCEEDED DATA,     FEN08550
     1 30H PROVIDED,  PROGRAM STOPPED...,5X,D15.6)                          FEN08560
      END                                                                    FEN08570
C                                                                            FEN08580
      SUBROUTINE  EPSLON (BL,BN,EDIS,STRAIN,OHM,NDE)                         FEN08590
C                                                                            FEN08600
      IMPLICIT REAL*8(A-H,O-Z)                                               FEN08610
      DIMENSION  BL(6,1),BN(9,1),EDIS(1),STRAIN(1),OHM(3,1),STR(9)           FEN08620
C ... CALCULATE INCREMENTAL LINEAR STRAINS                                   FEN08630
      DO 20  I=1,6                                                           FEN08640
      SUM= 0.                                                                FEN08650
      DO 10  J=1,NDE                                                         FEN08660
   10 SUM= SUM + BL(I,J)*EDIS(J)                                             FEN08670
      IF (DABS(SUM).LT.1.D-35)  SUM= 0.                                      FEN08680
   20 STRAIN(I)= SUM                                                         FEN08690
C ... CALCULATE INCREMENTAL NON-LINEAR STRAINS                               FEN08700
      CALL  CLEAR (STR,9)                                                    FEN08710
      DO 60 I=1,9                                                            FEN08720
      SUM= 0.                                                                FEN08730
      DO 40 J=1,NDE                                                          FEN08740
   40 SUM= SUM + BN(I,J)*EDIS(J)                                             FEN08750
      IF (DABS(SUM).LT.1.D-35)  SUM= 0.                                      FEN08760
   60 STR(I)= SUM                                                            FEN08770
C ... CALCULATE COMPONENTS OF SPIN TENSOR                                    FEN08780
      DO 160  I=1,3                                                          FEN08790
  160 OHM(I,I)= 0.                                                           FEN08800
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      OHM(1,2)= 0.5*(STR(2) - STR(4))                                   FEN08810
      OHM(1,3)= 0.5*(STR(3) - STR(7))                                   FEN08820
      OHM(2,3)= 0.5*(STR(6) - STR(8))                                   FEN08830
      RETURN                                                            FEN08840
      END                                                               FEN08850
C                                                                       FEN08860
      SUBROUTINE ERROR (M,I)                                            FEN08870
C                                                                       FEN08880
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                      FEN08890
      GO TO (1,2,3,4), I                                                FEN08900
    1 WRITE (IT6,2000)                                                  FEN08910
      GO TO 10                                                          FEN08920
    2 WRITE (IT6,2010)                                                  FEN08930
      GO TO 10                                                          FEN08940
    3 WRITE (IT6,2020)                                                  FEN08950
      GO TO 10                                                          FEN08960
    4 WRITE (IT6,2030)                                                  FEN08970
C                                                                       FEN08980
   10 WRITE (IT6,2090)  M                                               FEN08990
      STOP                                                              FEN09000
 2000 FORMAT (//47H NOT ENOUGH STORAGE FOR READ-IN OF ID ARRAY AND,     FEN09010
     1 24H NODAL POINT COORDINATES)                                     FEN09020
 2010 FORMAT (//50H NOT ENOUGH STORAGE FOR DEFINITION OF LOAD VECTORS)  FEN09030
 2020 FORMAT (//42H NOT ENOUGH STORAGE FOR ELEMENT DATA INPUT  )        FEN09040
 2030 FORMAT (//44H NOT ENOUGH STORAGE FOR ASSEMBLAGE OF GLOBAL,        FEN09050
     1 58H STRUCTURE STIFFNESS, AND DISPLACEMENT AND STRESS SOLUTION,   FEN09060
     2 10H PHASE    )                                                  FEN09070
 2090 FORMAT (//35H **** ERROR.... STORAGE EXCEEDED BY,I10)             FEN09080
      END                                                               FEN09090
C                                                                       FEN09100
      SUBROUTINE  INCLOD (U,ALOD,SLOD,ACLOD)                            FEN09110
C                                                                       FEN09120
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN09130
      COMMON/PRO/ GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)           FEN09140
      COMMON/VAR/ IINCS,IITER,IPEL,IREM,IND,IPT,ICHEK                   FEN09150
      COMMON/SOL/ NEQ,NEQ1,NWK,MK,NES                                   FEN09160
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                      FEN09170
      DIMENSION  U(NEQ),ALOD(NEQ),SLOD(NEQ)                             FEN09180
      CALL CLEAR (ALOD,NEQ)                                             FEN09190
      REWIND IT1                                                        FEN09200
      ACNEW= ACLOD + FLOD(IINCS)                                        FEN09210
      IF (ACNEW-ACLOD)  20,20,40                                        FEN09220
C ... SET  IREM = 1  UPON ENCOUNTERING UNLOADING                        FEN09230
   20 IF (IREM.EQ.0)  WRITE (IT6,2000)  IREM                            FEN09240
      IREM= 1                                                           FEN09250
      GO TO 60                                                          FEN09260
   40 IREM= 0                                                           FEN09270
   60 ACLOD= ACNEW                                                      FEN09280
      READ (IT1) ALOD                                                   FEN09290
      DO 80  I=1,NEQ                                                    FEN09300
      ALOD(I)= ALOD(I)*ACLOD                                            FEN09310
   80 U(I)= ALOD(I) - SLOD(I)                                           FEN09320
      RETURN                                                            FEN09330
 2000 FORMAT (/////49H >>  >>  >>  U N L O A D I N G  H A S  B E E N,    FEN09340
     1 40H  E N C O U N T E R E D . . .  ( IREM=,I2,2H ) )              FEN09350
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      END                                                                       FEN09360
C                                                                               FEN09370
      SUBROUTINE  INISAL (ASIG,AEPS,ADIS,SLOD,PLOD,IUP,EFSTN,ISTATE)            FEN09380
C                                                                               FEN09390
      IMPLICIT REAL*8(A-H,O-Z)                                                  FEN09400
      COMMON/GEN/ NUMNP,NUMEL,NPS,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH              FEN09410
      COMMON/MAT/ STRESS(6),STRAIN(6),C(6,6),YLD,HPRIME,IST,ISR                 FEN09420
      COMMON/PRO/ GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)                   FEN09430
      COMMON/SOL/ NEQ,NEQ1,NWK,MK,NES                                           FEN09440
      DIMENSION    ASIG(1),AEPS(1),ADIS(1),SLOD(1),PLOD(1),IUP(1),              FEN09450
     1             EFSTN(1),ISTATE(1)                                           FEN09460
      EQUIVALENCE (GEOM(1),THIC)                                                FEN09470
      IJ= IST*NIPS                                                              FEN09480
      IK= ISR*NIPS                                                              FEN09490
      CALL CLEAR (ASIG,IJ)                                                      FEN09500
      CALL CLEAR (AEPS,IK)                                                      FEN09510
      CALL CLEAR (ADIS,NEQ)                                                     FEN09520
      CALL CLEAR (SLOD,NEQ)                                                     FEN09530
      CALL CLEAR (PLOD,NEQ)                                                     FEN09540
      CALL CLEAR (EFSTN,NIPS)                                                   FEN09550
      NTHIC= NUMEL*IPL*IPH                                                      FEN09560
      DO 20   I=1,NTHIC                                                         FEN09570
   20 IUP(I)= THIC                                                              FEN09580
      DO 100  I=1,NIPS                                                          FEN09590
  100 ISTATE(I)= 1                                                              FEN09600
      RETURN                                                                    FEN09610
      END                                                                       FEN09620
C                                                                               FEN09630
      SUBROUTINE JACOB (XJ,XJI,DET,N)                                           FEN09640
C                                                                               FEN09650
      IMPLICIT REAL*8(A-H,O-Z)                                                  FEN09660
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                              FEN09670
      DIMENSION  XJ(3,3),XJI(3,3)                                               FEN09680
C ... DETERMINANT OF THE JACOBIAN MATRIX AT POINT (R,S,T)                       FEN09690
      DET =   XJ(1,1)*(XJ(2,2)*XJ(3,3) - XJ(2,3)*XJ(3,2))                      FEN09700
     1      -XJ(1,2)*(XJ(2,1)*XJ(3,3) - XJ(2,3)*XJ(3,1))                       FEN09710
     2      +XJ(1,3)*(XJ(2,1)*XJ(3,2) - XJ(2,2)*XJ(3,1))                       FEN09720
      IF (DET.GT.1.D-08) GO TO 10                                               FEN09730
      WRITE (IT6,2000)  N                                                       FEN09740
      STOP                                                                      FEN09750
C ... INVERSE OF THE JACOBIAN MATRIX AT POINT (R,S,T)                           FEN09760
   10 AA = 1./DET                                                               FEN09770
      XJI(1,1)= (XJ(2,2)*XJ(3,3) - XJ(3,2)*XJ(2,3))*AA                          FEN09780
      XJI(1,2)= (XJ(3,2)*XJ(1,3) - XJ(1,2)*XJ(3,3))*AA                          FEN09790
      XJI(1,3)= (XJ(1,2)*XJ(2,3) - XJ(2,2)*XJ(1,3))*AA                          FEN09800
      XJI(2,1)= (XJ(3,1)*XJ(2,3) - XJ(2,1)*XJ(3,3))*AA                          FEN09810
      XJI(2,2)= (XJ(1,1)*XJ(3,3) - XJ(3,1)*XJ(1,3))*AA                          FEN09820
      XJI(2,3)= (XJ(2,1)*XJ(1,3) - XJ(1,1)*XJ(2,3))*AA                          FEN09830
      XJI(3,1)= (XJ(2,1)*XJ(3,2) - XJ(3,1)*XJ(2,2))*AA                          FEN09840
      XJI(3,2)= (XJ(3,1)*XJ(1,2) - XJ(1,1)*XJ(3,2))*AA                          FEN09850
      XJI(3,3)= (XJ(1,1)*XJ(2,2) - XJ(2,1)*XJ(1,2))*AA                          FEN09860
      RETURN                                                                    FEN09870
 2000 FORMAT(11H0*** ERROR-,                                                    FEN09880
     1    53HZERO OR NEGATIVE JACOBIAN DETERMINANT FOR ELEMENT NO.,I3)         FEN09890
      END                                                                       FEN09900
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C                                                                               FEN09910
      SUBROUTINE LOADS (ID,R,NOD,IDIRN,FLOAD,NLOAD)                             FEN09920
C                                                                               FEN09930
      IMPLICIT REAL*8(A-H,O-Z)                                                  FEN09940
      COMMON/GEN/  NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH             FEN09950
      COMMON/MAX/  NMODS,NINCS,MITER,MODEX,INTUP                                FEN09960
      COMMON/SOL/  NEQ,NEQ1,NWK,MK,NES                                          FEN09970
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                              FEN09980
      DIMENSION  ID(NDF,1),R(NEQ),NOD(1),IDIRN(1),FLOAD(1)                      FEN09990
      WRITE (IT6,2000)                                                          FEN10000
      DO 20  I=1,NLCAD                                                          FEN10010
      READ  (IT5,1000)    NOD(I),IDIRN(I),FLOAD(I)                              FEN10020
      WRITE (IT6,2010)    NOD(I),IDIRN(I),FLOAD(I)                              FEN10030
   20 CONTINUE                                                                  FEN10040
      IF (MODEX.EQ.0)  RETURN                                                   FEN10050
      REWIND IT1                                                                FEN10060
      CALL CLEAR (R,NEQ)                                                        FEN10070
      DO 60  L=1,NLCAD                                                          FEN10080
      LN= NOD(L)                                                                FEN10090
      LI= IDIRN(L)                                                              FEN10100
      II= ID(LI,LN)                                                             FEN10110
      IF (II)  60,60,40                                                         FEN10120
   40 R(II)= R(II) + FLOAD(L)                                                   FEN10130
   60 CONTINUE                                                                  FEN10140
      WRITE (IT1)  R                                                            FEN10150
      RETURN                                                                    FEN10160
 1000 FORMAT (2I5,3D15.8)                                                       FEN10170
 2000 FORMAT (//47H * * *  LOCATIONS OF CONCENTRATED LOADS APPLIED //4X,FEN10180
     1 4HNODE,7X,9HDIRECTION,9X,4HLOAD /3X,6HNUMBER,21X,9HMAGNITUDE )           FEN10190
 2010 FORMAT (/I7,9X,I4,3X,D12.5)                                               FEN10200
      END                                                                       FEN10210
C                                                                               FEN10220
      SUBROUTINE  LOCAT (ELXY,VS,VT,R,S,T,RA,N,IR,IA,IT,IPT)                    FEN10230
C                                                                               FEN10240
      IMPLICIT REAL*8(A-H,O-Z)                                                  FEN10250
      COMMON/GEN/  NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH             FEN10260
      DIMENSION  ELXY(3,1),VS(3,1),VT(3,1),H(4),XYZ(3)                          FEN10270
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                              FEN10280
      JA= MOD(IA-1,IPH-1)                                                       FEN10290
      JT= MOD(IT-1,IPW-1)                                                       FEN10300
      IF (JA)  10,30,10                                                         FEN10310
   10 IF (JT)  20,30,20                                                         FEN10320
   20 RETURN                                                                    FEN10330
   30 R2= R**2                                                                  FEN10340
      R3= R*R2                                                                  FEN10350
      H(1)= 0.0625*(-9.*R3 + 9.*R2 + R - 1.)                                    FEN10360
      H(2)= 0.0625*( 27.*R3 - 9.*R2 - 27.*R + 9.)                               FEN10370
      H(3)= 0.0625*(-27.*R3 - 9.*R2 + 27.*R + 9.)                               FEN10380
      H(4)= 0.0625*( 9.*R3 + 9.*R2 - R - 1.)                                    FEN10390
      DO 60  I=1,3                                                              FEN10400
      SUM= 0.                                                                   FEN10410
      DO 40  J=1,NPE                                                            FEN10420
   40 SUM= SUM + H(J)*(ELXY(I,J) + RA*(S*VS(I,J) + T*VT(I,J)))                  FEN10430
   60 XYZ(I)= SUM                                                               FEN10440
      WRITE (IT8,2000)  N,IR,IA,IT,IPT,(XYZ(I),I=1,3)                           FEN10450
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      RETURN                                                            FEN10460
 2000 FORMAT (I5,5X,3I5,5X,I5,5X,3D15.6)                                FEN10470
      END                                                               FEN10480
C                                                                       FEN10490
      SUBROUTINE  MATEL (PROP,C)                                        FEN10500
C                                                                       FEN10510
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN10520
      DIMENSION  PROP(1),C(6,1)                                         FEN10530
      YM= PROP(1)                                                       FEN10540
      PV= PROP(2)                                                       FEN10550
      FI= PROP(3)                                                       FEN10560
      A1= YM/(1. - PV*PV)                                               FEN10570
      A2= A1*PV                                                         FEN10580
      A3= 0.5*A1*(1. - PV)                                              FEN10590
      A4= A3/FI                                                         FEN10600
      CALL CLEAR (C,36)                                                 FEN10610
      C(1,1)= A1                                                        FEN10620
      C(1,2)= A2                                                        FEN10630
      C(2,1)= A2                                                        FEN10640
      C(2,2)= A1                                                        FEN10650
      C(4,4)= A3                                                        FEN10660
      C(5,5)= A4                                                        FEN10670
      RETURN                                                            FEN10680
      END                                                               FEN10690
C                                                                       FEN10700
      SUBROUTINE  MIDEP (TAU,CEP,DEBAR,IFLAG)                          FEN10710
C                                                                       FEN10720
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN10730
      COMMON/MAT/  STRESS(6),STRAIN(6),C(6,6),YLD,HPRIME,IST,ISR        FEN10740
      DIMENSION  TAU(1),CEP(6,1),CQ(6),Q(6)                             FEN10750
      SM= (TAU(1)+TAU(2)+TAU(3))/3.                                     FEN10760
      DO 20  I=1,3                                                      FEN10770
      Q(I)= TAU(I) - SM                                                 FEN10780
   20 Q(I+3)= 2.*TAU(I+3)                                               FEN10790
C ... CALCULATE THE INSTANTANEOUS ELASTIC-PLASTIC STRESS-STRAIN MATRIX  FEN10800
      CON1= 4.*HPRIME*YLD*YLD/3.                                        FEN10810
      DO 60  I=1,6                                                      FEN10820
      CQ(I)= 0.                                                         FEN10830
      DO 60  J=1,6                                                      FEN10840
   60 CQ(I)= CQ(I) + C(I,J)*Q(J)                                        FEN10850
      CON2= 0.                                                          FEN10860
      DO 80  I=1,6                                                      FEN10870
   80 CON2= CON2 + Q(I)*CQ(I)                                           FEN10880
      CON3= 1./(CON1 + CON2)                                            FEN10890
      CALL CLEAR (CEP,36)                                               FEN10900
      DO 120  K=1,6                                                     FEN10910
      DO 120  J=K,6                                                     FEN10920
  120 CEP(K,J)= C(K,J) - CON3*CQ(K)*CQ(J)                               FEN10930
      DO 140  I=2,6                                                     FEN10940
      K= I-1                                                            FEN10950
      DO 140  J=1,K                                                     FEN10960
  140 CEP(I,J)= CEP(J,I)                                                FEN10970
      IF (IFLAG.EQ.0)  RETURN                                           FEN10980
C ... CALCULATE EFFECTIVE PLASTIC-STRAIN INCREMENT                      FEN10990
      SUM= 0.                                                           FEN11000
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      DO 200  I=1,6                                                     FEN11010
  200 SUM= SUM + Q(I)*STRESS(I)                                         FEN11020
      DLAM= CON3*SUM                                                    FEN11030
      IF (DLAM.LT.0.)  DLAM= 0.                                         FEN11040
      DEBAR= 2.*DLAM*YLD/3.                                             FEN11050
      RETURN                                                            FEN11060
      END                                                               FEN11070
C                                                                       FEN11080
      SUBROUTINE NUMON (ID,NDF,NEQ,NUMNP)                               FEN11090
C                                                                       FEN11100
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN11110
      DIMENSION ID(NDF,1)                                               FEN11120
C ... CALCULATE THE NUMBER OF UNKNOWNS                                  FEN11130
      NEQ= 0                                                            FEN11140
      DO 40  N=1,NUMNP                                                  FEN11150
      DO 40  I=1,NDF                                                    FEN11160
      IF (ID(I,N)) 30,20,30                                             FEN11170
   20 NEQ= NEQ + 1                                                      FEN11180
      ID(I,N)= NEQ                                                      FEN11190
      GO TO 40                                                          FEN11200
   30 ID(I,N)= 0.                                                       FEN11210
   40 CONTINUE                                                          FEN11220
      RETURN                                                            FEN11230
      END                                                               FEN11240
C                                                                       FEN11250
      SUBROUTINE OUTPUT (ID,RDS,ADIS,ACLOD)                             FEN11260
C                                                                       FEN11270
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN11280
      COMMON/GEN/ NUMNP,NUMEL,NPS,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH      FEN11290
      COMMON/PRO/ GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)           FEN11300
      COMMON/VAR/ IINCS,ITER,IPEL,IHEM,IND,IPT,ICHEK                    FEN11310
      COMMON/MAX/ NMODS,MINCS,MITER,MODEX,INTUP                         FEN11320
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                      FEN11330
      DIMENSION  ID(NDF,1),RDS(1),ADIS(1),D(9),O(9)                     FEN11340
      EQUIVALENCE  (GEOM(1),THIK),(GEOM(2),RA),(PROP(1),YM)             FEN11350
      WRITE (IT6,2000)  ICHEK                                           FEN11360
      IF (ICHEK)  1,2,3                                                 FEN11370
    1 WRITE (IT6,2020)  ITER                                            FEN11380
      GO TO 10                                                          FEN11390
    2 WRITE (IT6,2040)  MITER                                           FEN11400
      GO TO 10                                                          FEN11410
    3 WRITE (IT6,2060)  ITER                                            FEN11420
   10 WRITE (IT6,2080)  ACLOD                                           FEN11430
      DO 60  N=1,NUMNP                                                  FEN11440
      DO 20  M=1,NDF                                                    FEN11450
      D(M)= 0.                                                          FEN11460
      II= ID(M,N)                                                       FEN11470
      IF (II.NE.0)  D(M)= ADIS(II)                                      FEN11480
   20 CONTINUE                                                          FEN11490
   60 WRITE (IT6,2100) N,(D(I),I=1,NDF)                                 FEN11500
      WRITE (IT6,2120)                                                  FEN11510
      DO 100  N=1,NUMEL                                                 FEN11520
  100 WRITE (IT6,2140)  N,RDS(N)                                        FEN11530
      RETURN                                                            FEN11540
 2000 FORMAT (//5X,36H * * *  CONVERGENCE MODE OF SOLUTION, I10)        FEN11550
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 2020 FORMAT (//5X,49H * * *  PROGRAM STOPPED...  SOLUTION DIVERGING AT,FEN11560
     1 10H   IITER =,I4 )                                                FEN11570
 2040 FORMAT (//5X,45H * * *  NO CONVERGENCE ACHIEVED  WITH MAXIMUM,     FEN11580
     1 25H NUMBER OF ITERATIONS   (,I3,2H ) )                            FEN11590
 2060 FORMAT (//5X,41H * * *  CONVERGENCE ACHIEVED AT   IITER =,I4 )     FEN11600
 2080 FORMAT (//5X,43H * * *  DISPLACEMENTS :   TRANSLATIONS  &  ,       FEN11610
     1 50HROTATIONS  &  OVALIZATIONS   ( LOAD ACCUMULATION =,F8.4,2H )//FEN11620
     2 6H  NODE,7X,7HX-DISPL,6X,7HY-DISPL,6X,7HZ-DISPL,6X,5HX-ROT,8X,    FEN11630
     3 5HY-ROT,8X,5HZ-ROT,8X,8H1ST-OVAL,5X,8H2ND-OVAL,5X,8H3RD-OVAL /)   FEN11640
 2100 FORMAT (2X,I3,6X,9D13.5)                                           FEN11650
 2120 FORMAT (//5X,45H * * *  UPDATED RADII OF CURVATURE OF ELEMENTS,//  FEN11660
     1 10X,8H ELEMENT,17X,9H RADIUS-2/ )                                 FEN11670
 2140 FORMAT (13X,I2,16X,D15.6)                                          FEN11680
      END                                                                FEN11690
C                                                                        FEN11700
      SUBROUTINE PENAL (ID,AG,MAXA,NUMNP,NDF,ALPHA)                      FEN11710
C                                                                        FEN11720
      IMPLICIT REAL*8(A-H,O-Z)                                           FEN11730
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                       FEN11740
      DIMENSION  ID(NDF,1),AG(1),MAXA(1)                                 FEN11750
      ALPHA= 0.                                                          FEN11760
      DO 20  N=1,NUMNP                                                   FEN11770
      DO 20  M=7,NDF                                                     FEN11780
      I= ID(M,N)                                                         FEN11790
      IF (I.EQ.0)  GO TO 20                                              FEN11800
      K= MAXA(I)                                                         FEN11810
      IF (AG(K).GT.ALPHA)  ALPHA= AG(K)                                  FEN11820
   20 CONTINUE                                                           FEN11830
      WRITE (IT6,2000)  ALPHA                                            FEN11840
      RETURN                                                             FEN11850
 2000 FORMAT (42H * * *  FOR INTERACTION EFFECTS     ALPHA =,D15.5)      FEN11860
      END                                                                FEN11870
C                                                                        FEN11880
      SUBROUTINE  PRESET (MTOT)                                          FEN11890
C                                                                        FEN11900
      IMPLICIT REAL*8(A-H,O-Z)                                           FEN11910
      COMMON/GEN/ NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH       FEN11920
      COMMON/MAX/ NMODS,NINCS,MITER,MODEX,INTUP                          FEN11930
      COMMON/SOL/ NEQ,NEQ1,NWK,MK,NES                                    FEN11940
      COMMON/MAT/ STRESS(6),STRAIN(6),C(6,6),YLD,HPRIME,IST,ISR          FEN11950
      COMMON/DIM/ N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15,    FEN11960
     1            N16,N17,N18,N19,N20,N21,N22,N23,N24,N25                FEN11970
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                       FEN11980
      COMMON  A(1)                                                       FEN11990
      DIMENSION  HED(18)                                                 FEN12000
      IT1= 1                                                             FEN12010
      IT2= 2                                                             FEN12020
      IT3= 3                                                             FEN12030
      IT4= 4                                                             FEN12040
      IT5= 5                                                             FEN12050
      IT6= 6                                                             FEN12060
      IT7= 7                                                             FEN12070
      IT8= 8                                                             FEN12080
      READ (IT5,1000)  HED,NUMNP,NUMEL,NDF,NPE,IPL,IPP,IPW,NRN,NLOAD,   FEN12090
     1                 NMODS,NINCS,MITER,INTUP,MODEX                     FEN12100
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      WRITE (IT6,2000) HED,NUMNP,NUMEL,NDF,NPE,IPL,IPP,IPW,NRN,NLOAD,     FEN12110
     1                 NMODS,NINCS,MITER,INTUP,MODEX                      FEN12120
                                                                          FEN12130
      N1= 1                                                               FEN12140
      N2= N1  + NDF*NUMNP                                                 FEN12150
      N3= N2  + NPE*NUMEL                                                 FEN12160
      N4= N3  + 3*NUMNP                                                   FEN12170
      N5= N4  + 3*NUMNP                                                   FEN12180
      N6= N5  + 3*NUMNP                                                   FEN12190
      N7= N6  + NUMEL                                                     FEN12200
      N8= N7  + NUMEL                                                     FEN12210
      IF (N8.GT.MTOT)  CALL ERROR (N8-MTOT,1)                             FEN12220
      CALL  DATA (A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),NRN)          FEN12230
      N9= N8  + NEQ                                                       FEN12240
      N10=  N9  + NLOAD                                                   FEN12250
      N11= N10 + NLOAD                                                    FEN12260
      N12= N11 + NLOAD                                                    FEN12270
      IF (N12.GT.MTOT)  CALL ERROR (N12-MTOT,2)                           FEN12280
      CALL  LOADS (A(N1),A(N8),A(N9),A(N10),A(N11),NLOAD)                 FEN12290
      N10= N9  + NEQ  + 1                                                 FEN12300
      N11= N10 + NLE*NUMEL                                                FEN12310
      IF (N11.GT.MTOT)  CALL ERROR (N11-MTOT,3)                           FEN12320
      CALL  COSET (A(N1),A(N2),A(N8),A(N9),A(N10))                        FEN12330
      N12= N11 + 6*NDE                                                    FEN12340
      N13= N12 + 9*NDE                                                    FEN12350
      N14= N13 + NES                                                      FEN12360
      N15= N14 + NWK                                                      FEN12370
      N16= N15 + IST*NIPS                                                 FEN12380
      N17= N16 + ISR*NIPS                                                 FEN12390
      N18= N17 + NEQ                                                      FEN12400
      N19= N18 + NEQ                                                      FEN12410
      N20= N19 + NEQ                                                      FEN12420
      N21= N20 + NEQ                                                      FEN12430
      N22= N21 + NEQ                                                      FEN12440
      N23= N22 + NEQ                                                      FEN12450
      N24= N23 + NUMEL*IPL*IPW                                            FEN12460
      N25= N24 + NIPS                                                     FEN12470
      N26= N25 + NIPS                                                     FEN12480
      NLAST= N26                                                          FEN12490
      WRITE (IT6,2020)  NLAST                                             FEN12500
      IF (NLAST.GT.MTOT)  CALL ERROR (NLAST-MTOT,4)                       FEN12510
      IF (MODEX.EQ.0)  RETURN                                             FEN12520
      CALL  INISAL (A(N15),A(N16),A(N19),A(N21),A(N22),A(N23),A(N24),     FEN12530
     1              A(N25))                                               FEN12540
      RETURN                                                              FEN12550
 1000 FORMAT (18A4/14I5)                                                  FEN12560
 2000 FORMAT (18A4//                                                      FEN12570
     1 50H C O N T R O L    I N F O R M A T I O N                  //5X,    FEN12580
     2 50H TOTAL NUMBER OF NODES                        (NUMNP) =,I5/5X,  FEN12590
     3 50H TOTAL NUMBER OF ELEMENTS                     (NUMEL) =,I5/5X,  FEN12600
     4 50H DEGREES OF FREEDOM PER NODE                    (NDF) =,I5/5X,  FEN12610
     5 50H NUMBER OF NODES PER ELEMENT                    (NPE) =,I5/5X,  FEN12620
     6 50H INTEGRATION POINTS ALONG ELEMENT LENGTH        (IPL) =,I5/5X,  FEN12630
     7 50H INTEGRATION POINTS AROUND PERIMETER            (IPP) =,I5/5X,  FEN12640
     8 50H INTEGRATION POINTS ACROSS WALL THICKNESS       (IPW) =,I5/5X,  FEN12650
     9 50H NUMBER OF RESTRAINED NODES                     (NRN) =,I5/5X,
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     1 5OH NUMBER OF CONCENTRATED LOADS                    (NLOAD) =,I5/5X,      FEN12660
     2 5OH NUMBER OF STRESS-STRAIN POINTS                  (NMCDS) =,I5/5X,      FEN12670
     3 5OH NUMBER OF LOAD INCREMENTS                       (NINCS) =,I5/5X,      FEN12680
     4 5OH MAXIMUM NUMBER OF ITERATIONS                    (MITER) =,I5/5X,      FEN12690
     5 5OH STIFFNESS UPDATING OPTION                       (INTUP) =,I5/5X,      FEN12700
     6 5OH     EQ. N : UPDATED EVERY  N  ITERATIONS                  /5X,      FEN12710
     7 5OH     EQ. 0 : UPDATING NOT REQUIRED                         /5X,      FEN12720
     8 5OH SOLUTION MODE                                   (MODEX) =,I5/5X,      FEN12730
     9 5OH                  EQ. 0 :  DATA CHECK                      /5X,      FEN12740
     1 5OH                  EQ. 1 :  EXECUTION                        )        FEN12750
 2020 FORMAT (5X,40H STORAGE OF COMMON ARRAY OCCUPIED      =,I6)               FEN12760
      END                                                                      FEN12770
C                                                                              FEN12780
      SUBROUTINE RADII (RDS,ELG,LM,SE,ALPHA)                                   FEN12790
C                                                                              FEN12800
      IMPLICIT REAL*8(A-H,O-Z)                                                 FEN12810
      COMMON/GEN/ NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH             FEN12820
      COMMON/PRO/ GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)                  FEN12830
      COMMON/INTG/ ELXY(3,4),VS(3,4),VT(3,4),WGR(5),WGP(2),WGW(5)              FEN12840
      COMMON/LOCA/ R,S,T,TH,THIC,ZT,PI,CS,SN,RD,EG,IP1,IP2,N,NC,NTC            FEN12850
      COMMON/DIM/ N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15,          FEN12860
     1            N16,N17,N18,N19,N20,N21,N22,N23,N24,N25                      FEN12870
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                             FEN12880
      COMMON  A(1)                                                             FEN12890
      DIMENSION  RDS(1),ELG(1),LM(NDE,1),SE(1),DH(4,2),G(21),LL(21)            FEN12900
      EQUIVALENCE  (GEOM(1),TAVG),(GEOM(2),RA),(NPAR(3),INTEP),                FEN12910
     1             (NPAR(4),IFIX)                                            FEN12920
C ... VALUES FOR FIRST DERIVATIVES OF EL#I AT R=+1 AND EL#II AT R=-1           FEN12930
      DATA  LH/-0.5D0,2.25D0,-4.5D0,2.75D0,-2.75D0,4.5D0,-2.25D0,0.5D0/        FEN12940
C ... ENFORCE CONTINUITY BETWEEN ADJACENT ELEMENTS                             FEN12950
      AMEAN = RA - TAVG/2.                                                     FEN12960
      PI= 3.1415926535897900                                                   FEN12970
      DIA= 2.*PI/DFLOAT(IPP)                                                   FEN12980
      IF (INTEP.EQ.0)  GO TO 180                                               FEN12990
      NPE1= 2*NPE - 1                                                          FEN13000
      KI= NPE1*NC                                                              FEN13010
      KS= KI*(KI + 1)/2                                                        FEN13020
      NEM= NUMEL - 1                                                           FEN13030
      DO 160  N=1,NEM                                                          FEN13040
      RD1= RDS(N)                                                              FEN13050
      RD2= RDS(N+1)                                                            FEN13060
      CALL CLEAR (SE,KS)                                                       FEN13070
      DO 120  KI=1,IPH                                                         FEN13080
      FAC1= 2.                                                                 FEN13090
      FAC2= 2.                                                                 FEN13100
      IF (KI.EQ.1.OR.KI.EQ.IPH)  FAC2= 1.                                      FEN13110
      PHI= DIA*DFLOAT(KI-1)                                                    FEN13120
      AC= AMEAN*DCOS(PHI)                                                      FEN13130
      IF (KI.EQ.IP1)  AC= 0.                                                   FEN13140
      TET= ELG(N)/RD1                                                          FEN13150
      C1= 2./((RD1-AC)*TET)                                                    FEN13160
      TET= ELG(N+1)/RD2                                                        FEN13170
      C2= 2./((RD2-AC)*TET)                                                    FEN13180
      DO 100  I=1,NPE1                                                         FEN13190
      II= (I-1)*NC                                                             FEN13200
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      DO 100  J=1,NC                                                    FEN13210
      FM= DFLOAT(2*J)                                                   FEN13220
      C3= FM*DCOS(FM*PHI)                                               FEN13230
      IF (I-4)  70,80,90                                                FEN13240
   70 G(II+J)= -C1*C3*DH(I,1)                                           FEN13250
      GO TO 100                                                         FEN13260
   80 G(II+J)= (C2*DH(1,2) - C1*DH(4,1))*C3                             FEN13270
      GO TO 100                                                         FEN13280
   90 G(II+J)=  C2*C3*DH(I-3,2)                                         FEN13290
  100 CONTINUE                                                          FEN13300
      IK= 0                                                             FEN13310
      DO 110  I=1,KT                                                    FEN13320
      DO 110  J=I,KT                                                    FEN13330
      IK= IK + 1                                                        FEN13340
  110 SE(IK)= SE(IK) + FAC1*FAC2*WGP(1)*G(I)*G(J)                       FEN13350
  120 CONTINUE                                                          FEN13360
      DO 130  I=1,KS                                                    FEN13370
  130 SE(I)= ALPHA*SE(I)                                                FEN13380
      IK= 0                                                             FEN13390
      DO 140  I=1,NPE                                                   FEN13400
      II= (I-1)*NDF + 6                                                 FEN13410
      DO 140  J=1,NC                                                    FEN13420
      IK= IK + 1                                                        FEN13430
  140 LL(IK)= LM(II+J,N)                                                FEN13440
      DO 150  I=2,NPE                                                   FEN13450
      II= (I-1)*NDF + 6                                                 FEN13460
      DO 150  J=1,NC                                                    FEN13470
      IK= IK + 1                                                        FEN13480
  150 LL(IK)= LM(II+J,N+1)                                              FEN13490
      WRITE (IT2)  (SE(I),I=1,KS),(LL(I),I=1,KT)                        FEN13500
      CALL ACDBAN (A(N14),A(N7),SE,LL,KT)                               FEN13510
  160 CONTINUE                                                          FEN13520
  180 IF (IFIX.EQ.0)  RETURN                                            FEN13530
C ... ENFORCE FIXITY CONDITION AT FIXED END                             FEN13540
      KS= NTC*(NTC+1)/2                                                 FEN13550
      CALL CLEAR (SE,KS)                                                FEN13560
      RD= RDS(IFIX)                                                     FEN13570
      EG= ELG(IFIX)                                                     FEN13580
      DO 300  KI=1,IPH                                                  FEN13590
      FAC1= 2.                                                          FEN13600
      FAC2= 2.                                                          FEN13610
      IF (KI.EQ.1.OR.KI.EQ.IPH)  FAC2= 1.                               FEN13620
      PHI= DIA*DFLOAT(KI-1)                                             FEN13630
      AC= AMEAN*DCOS(PHI)                                               FEN13640
      IF (KI.EQ.IP1)  AC= 0.                                            FEN13650
      TET= EG/RD                                                        FEN13660
      C1= 2./((RD-AC)*TET)                                              FEN13670
      DO 260  I=1,NPE                                                   FEN13680
      II= (I-1)*NC                                                      FEN13690
      DO 260  J=1,NC                                                    FEN13700
      FM= DFLOAT(2*J)                                                   FEN13710
      C2= FM*DCOS(FM*PHI)                                               FEN13720
      G(II+J)= -C1*C2*DH(I,2)                                           FEN13730
  260 CONTINUE                                                          FEN13740
      IK= 0                                                             FEN13750
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      DO 280  I=1,NIC                                                    FEN13760
      DO 280  J=I,NIC                                                    FEN13770
      IK= IK + 1                                                         FEN13780
  280 SE(IK)= SE(IK) + FAC1*FAC2*WGP(1)*J(I)*G(J)                        FEN13790
  300 CONTINUE                                                           FEN13800
      DO 320  I=1,KS                                                     FEN13810
  320 SE(I)= ALPHA*SE(I)                                                 FEN13820
      IK= 0                                                              FEN13830
      DO 340  I=1,NPE                                                    FEN13840
      II= (I-1)*NDF + 6                                                  FEN13850
      DO 340  J=1,NC                                                     FEN13860
      IK= IK + 1                                                         FEN13870
  340 LL(IK)= LM(II+J,IFIX)                                              FEN13880
      WRITE (IT2)  (SE(I),I=1,KS),(LL(I),I=1,NTC)                        FEN13890
      CALL ACOBAN (A(N14),A(N9),SE,LL,NIC)                               FEN13900
C                                                                        FEN13910
      CALL CLEAR (SE,KS)                                                 FEN13920
      RD= RDS(NUMEL)                                                     FEN13930
      EG= ELG(NUMEL)                                                     FEN13940
      DO 500  KI=1,IPH                                                   FEN13950
      FAC1= 2.                                                           FEN13960
      FAC2= 2.                                                           FEN13970
      IF (KI.EQ.1.OR.KI.EQ.IPH)  FAC2= 1.                                FEN13980
      PHI= DTA*DFLCAI(KI-1)                                              FEN13990
      AC= AMEAN*DCOS(PHI)                                                FEN14000
      IF (KI.EQ.IP1)  AC= 0.                                             FEN14010
      TET= EG/RD                                                         FEN14020
      C1= 2./((RD-AC)*TET)                                               FEN14030
      DO 460  I=1,NPE                                                    FEN14040
      II= (I-1)*NC                                                       FEN14050
      DO 460  J=1,NC                                                     FEN14060
      FM= DFLCAI(2*J)                                                    FEN14070
      C2= FM*DCOS(FM*PHI)                                                FEN14080
      G(II+J)= -C1*C2*DH(I,1)                                            FEN14090
  460 CONTINUE                                                           FEN14100
      IK= 0                                                              FEN14110
      DO 480  I=1,NIC                                                    FEN14120
      DO 480  J=I,NIC                                                    FEN14130
      IK= IK + 1                                                         FEN14140
  480 SE(IK)= SE(IK) + FAC1*FAC2*WGP(1)*J(I)*G(J)                        FEN14150
  500 CONTINUE                                                           FEN14160
      DO 520  I=1,KS                                                     FEN14170
  520 SE(I)= ALPHA*SE(I)                                                 FEN14180
      IK= 0                                                              FEN14190
      DO 540  I=1,NPE                                                    FEN14200
      II= (I-1)*NDF + 6                                                  FEN14210
      DO 540  J=1,NC                                                     FEN14220
      IK= IK + 1                                                         FEN14230
  540 LL(IK)= LM(II+J,NUMEL)                                             FEN14240
      WRITE (IT2)  (SE(I),I=1,KS),(LL(I),I=1,NTC)                        FEN14250
      CALL ACOBAN (A(N14),A(N9),SE,LL,NIC)                               FEN14260
      RETURN                                                             FEN14270
      END                                                                FEN14280
C                                                                        FEN14290
      SUBROUTINE  RESIDU (ID,NEL,XYZ,VSP,VIP,RDS,ELG,LM,BL,BN,SE,ASIG,   FEN14300
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     1            AEPS,U,UDIS,ADIS,ALOD,SLOD,PLOD,TUP,EPSTN,ISTATE)      FEN14310
C                                                                        FEN14320
      IMPLICIT REAL*8(A-H,O-Z)                                           FEN14330
      COMMON/GEN/  NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPH,IPH     FEN14340
      COMMON/MAT/  STRESS(6),STRAIN(6),C(6,6),YLD,HPRIME,IST,ISR         FEN14350
      COMMON/PRO/  GEOM(8),PROP(6),EMOD(3,10),FLCD(30),NPAR(6)           FEN14360
      COMMON/VAR/  IINCS,ITER,IPEL,IRCM,IND,IPT,ICHEK                    FEN14370
      COMMON/MAX/  NMODS,MINCS,MITER,MODEX,INTUP                         FEN14380
      COMMON/CON/  FTOL,ETOL,TOLFO,TOLEN,CURVY                           FEN14390
      COMMON/SOL/  NEQ,NEQ1,NWK,MK,NES                                   FEN14400
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                       FEN14410
      DIMENSION   ID(NDF,1),NCL(NPE,1),XYZ(3,1),VSP(3,1),VTP(3,1),RDS(1),FEN14420
     1            ELG(1),LM(NDE,1),BL(6,NDE),BN(9,NDE),SE(1),ASIG(IST,1),FEN14430
     2            AEPS(ISR,1),U(1),UDIS(1),ADIS(1),ALOD(1),SLOD(1),      FEN14440
     3            PLOD(1),TUP(IPH,1),EPSTN(1),ISTATE(1)                  FEN14450
      DIMENSION   DISP(36),ELOD(36),Q1(6,6),Q2(6,6),OHM(3,3),SIG(6),     FEN14460
     1            EPS(6),AA(3),VV(3)                                     FEN14470
      EQUIVALENCE  (GEOM(1),THICK),(GEOM(2),RA),(PROP(1),YM),            FEN14480
     1             (PROP(4),UNIAX)                                       FEN14490
      REWIND IT2                                                         FEN14500
      REWIND IT3                                                         FEN14510
C ... PREPARE FOR CONVERGENCE CHECKS                                     FEN14520
      IF (ITER.NE.1)  GO TO 40                                           FEN14530
      CALL CLEAR (UDIS,NEQ)                                              FEN14540
      DO 10   I=1,NEQ                                                   FEN14550
   10 SLOD(I)= ALOD(I) - SLOD(I)                                         FEN14560
C ... CALCULATE NORM OF INITIAL LOAD INCREMENT                           FEN14570
      SUM= 0.                                                            FEN14580
      DO 20   I=1,NEQ                                                   FEN14590
   20 SUM= SUM + SLOD(I)*SLOD(I)                                         FEN14600
      TOLFO= DSQRT(SUM)                                                  FEN14610
C ... CALCULATE INITIAL ENERGY INCREMENT                                 FEN14620
      SUM= 0.                                                            FEN14630
      DO 30   I=1,NEQ                                                   FEN14640
   30 SUM= SUM + U(I)*SLOD(I)                                            FEN14650
      TOLEN= SUM                                                         FEN14660
      ICHEK= 0                                                           FEN14670
      GO TO 80                                                           FEN14680
C ... CALCULATE FORCE AND ENERGY FOR CONVERGENCE CHECKS                  FEN14690
   40 DO 50   I=1,NEQ                                                   FEN14700
   50 SLOD(I)= ALOD(I) - SLOD(I)                                         FEN14710
      SUM= 0.                                                            FEN14720
      DO 60   I=1,NEQ                                                   FEN14730
   60 SUM= SUM + SLOD(I)*SLOD(I)                                         FEN14740
      FORCE= DSQRT(SUM)                                                  FEN14750
      FRAT= FORCE/TOLFO                                                  FEN14760
      ENERGY= 0.                                                         FEN14770
      DO 70   I=1,NEQ                                                   FEN14780
   70 ENERGY= ENERGY + U(I)*SLOD(I)                                      FEN14790
      ERAT= ENERGY/TOLEN                                                 FEN14800
      WRITE (IT6,2000) ITER,FRAT,ERAT                                    FEN14810
      IF (FRAT.LE.FTOL.AND.ERAT.LE.ETOL)  ICHEK= 999                     FEN14820
      IF (ERAT.GT.1.)  ICHEK= -999                                       FEN14830
   80 DO 90   I=1,NEQ                                                   FEN14840
      UDIS(I)= UDIS(I) + U(I)                                            FEN14850
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   90 ADIS(I)= ADIS(I) + U(I)                                              FEN14860
C ... DETERMINE LOCATIONS OF TOP AND BOTTOM OF PLANE OF SYMMETRY           FEN14870
      LTOP= IPW                                                            FEN14880
      LBOT= IPH*IPW                                                        FEN14890
C ... DETERMINE ACCUMULATIVE LOAD FACTOR                                   FEN14900
      APLOD= 0.                                                            FEN14910
      DO 100  I=1,IINCS                                                    FEN14920
      APLOD= APLOD + FLOD(I)                                               FEN14930
  100 CONTINUE                                                             FEN14940
      IF (APLOD.LT.1.)  CONVY= 1.                                          FEN14950
C ... CALCULATE THE CRITICAL VALUE OF BUCKLING STRESS                      FEN14960
      SCRIT= -0.32*YM*THICK/RA                                             FEN14970
      IF (INC.EQ.0)  WRITE (IT6,2010)  SCRIT                               FEN14980
C ... CALCULATION OF STRESSES, STRAINS AND NODAL FORCES  (START...)        FEN14990
      CALL CLEAR (SLOD,NEQ)                                                FEN15000
      IPT= 0                                                               FEN15010
      DO 400  N=1,NUMEL                                                    FEN15020
C ... CALCULATE LOCAL DISPLACEMENTS                                        FEN15030
      DO 140  M=1,NDE                                                      FEN15040
      M1= LM(M,N)                                                          FEN15050
      DISP(M)= 0.                                                          FEN15060
      IF (M1.NE.0)  DISP(M)= U(M1)                                         FEN15070
  140 CONTINUE                                                             FEN15080
      CALL CLEAR (ELOD,NDE)                                                FEN15090
C ... ENTER INTEGRATION LOOP                                               FEN15100
      DO 360  IR=1,IPL                                                     FEN15110
      ITHIC= (N-1)*IPL + IR                                                FEN15120
      DO 360  IA=1,IPH                                                     FEN15130
      THIC= IUP(IA,ITHIC)                                                  FEN15140
      ESUM= 0.                                                             FEN15150
      DO 350  IT=1,IPW                                                     FEN15160
      IPT= IPT + 1                                                         FEN15170
      READ (IT2)  BL,BN,J1,J2,C,WT                                         FEN15180
C ... CALCULATE INCREMENTAL STRAINS                                        FEN15190
      CALL EPSLON (BL,BN,DISP,STRAIN,OHM,NDE)                              FEN15200
      DO 150  I=1,IST                                                      FEN15210
  150 SIG(I)= ASIG(I,IPT)                                                  FEN15220
      DO 160  I=1,ISR                                                      FEN15230
  160 EPS(I)= AEPS(I,IPT)                                                  FEN15240
      EFST= EFSTN(IPT)                                                     FEN15250
      IPEL= ISTATE(IPT)                                                    FEN15260
C ... CALCULATE INCREMENTAL STRESSES                                       FEN15270
      IF (IBEM)  1,1,2                                                     FEN15280
    1 CALL ELPAL (SIG,EPS,OHM,EFST)                                        FEN15290
C ... WRITE 'NONLINEAR' MATERIAL LAW ON TAPE IF YIELDING HAS OCCURRED      FEN15300
      IF (IPEL.GT.1.AND.ICHEK.GT.0)  WRITE (IT3)  C                        FEN15310
      IF (ICHEK.NE.0.OR.ITER.EQ.MITER)  GO TO 170                          FEN15320
      GO TO 190                                                            FEN15330
  170 DM= (SIG(1) + SIG(2) + SIG(3))/3.                                    FEN15340
      DX= SIG(1) - DM                                                      FEN15350
      DY= SIG(2) - DM                                                      FEN15360
      DZ= SIG(3) - DM                                                      FEN15370
      DXY= SIG(4)                                                          FEN15380
      DYZ= SIG(5)                                                          FEN15390
      DZX= SIG(6)                                                          FEN15400
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      SBAR= DSQRT(1.5*(DX*DX+DY*DY+DZ*DZ)+3.*(DXY*DXY+DYZ*DYZ+DZX*DZX))  FEN15410
      WRITE (IT7,2020)  N,IR,IA,IT,IPT,SBAR,EFST                         FEN15420
  190 ISTATE(IPT)= IPEL                                                  FEN15430
      EFSTN(IPT)= EFST                                                   FEN15440
      GO TO 200                                                          FEN15450
    2 CALL UNLOD (SIG,EPS,DUM)                                           FEN15460
C ... UPDATE TOTAL STRESSES AND STRAINS                                  FEN15470
  200 DO 210  I=1,IST                                                    FEN15480
  210 ASIG(I,IPT)= SIG(I)                                                FEN15490
      DO 220  I=1,ISR                                                    FEN15500
  220 AEPS(I,IPT)= EPS(I)                                                FEN15510
C ... UPDATE THICKNESS USING AVERAGE STRAIN FROM INTEGRATION POINTS      FEN15520
      ETHIC= 0.                                                          FEN15530
      DO 230  J=1,ISR                                                    FEN15540
  230 ETHIC= ETHIC + Q1(3,J)*STRAIN(J)                                   FEN15550
      ESUM= ESUM + ETHIC                                                 FEN15560
C ... CALCULATE ELEMENT NODAL FORCES                                     FEN15570
      DO 260  I=1,NDE                                                    FEN15580
      SUM= 0.                                                            FEN15590
      DO 240  J=1,IST                                                    FEN15600
  240 SUM= SUM + BL(J,I)*SIG(J)                                          FEN15610
  260 ELOD(I)= ELOD(I) + WT*SUM                                          FEN15620
      IF (ICHEK.EQ.0.AND.ITER.NE.MITER)  GO TO 300                       FEN15630
      DO 11  I=1,6                                                       FEN15640
      SS= 0.                                                             FEN15650
      DO 22  J=1,6                                                       FEN15660
   22 SS= SS + Q2(J,I)*SIG(J)                                            FEN15670
   11 STRESS(I)= SS/UNIAX                                                FEN15680
      WRITE (IT8,33)  N,IR,IA,IT,IPT,(STRESS(K),K=1,6)                   FEN15690
   33 FORMAT (I5,5X,3I5,I5,5X,6D15.6)                                    FEN15700
C ... CALCULATE NON-DIMENSIONAL LOCAL STRESSES AT PLANE OF SYMMETRY      FEN15710
      IF (N.NE.1.OR.IR.NE.1.OR.(IT.NE.1.AND.IT.NE.IPW)) GO TO 300        FEN15720
      DO 270  I=1,3                                                      FEN15730
      SUM= 0.                                                            FEN15740
      DO 265  J=1,IST                                                    FEN15750
  265 SUM= SUM + Q2(J,I)*SIG(J)                                          FEN15760
  270 STRESS(I)= SUM/UNIAX                                               FEN15770
      WRITE (IT6,2080) IA,IT,(STRESS(I),I=1,3)                           FEN15780
C ... CALCULATE CURVATURE AT PLANE OF SYMMETRY                           FEN15790
      IF (IPT.NE.LBOT.AND.IPT.NE.LTOP)  GO TO 300                        FEN15800
      EPHI= 0.                                                           FEN15810
      DO 280  I=1,ISR                                                    FEN15820
  280 EPHI= EPHI + Q1(2,I)*EPS(I)                                        FEN15830
      IF (IPT-LTOP)  3,3,4                                               FEN15840
    3 ETOP= EPHI                                                         FEN15850
      GO TO 300                                                          FEN15860
    4 EBOT= EPHI                                                         FEN15870
      CURVA= 0.5*DABS(EBOT-ETOP)/RA                                      FEN15880
      IF (APLOD.EQ.1.)  CURVY= CURVA                                     FEN15890
      CURRAT= CURVA/CURVY                                                FEN15900
      WRITE (IT6,2040)  CURVY,CURRAT,APLOD                               FEN15910
C ... CHECK NORMAL STRESSES FOR POSSIBLE BUCKLING IN ELEMENTS 1 AND 2    FEN15920
  300 IF (N.GT.2)  GO TO 350                                             FEN15930
      DO 320  I=1,3                                                      FEN15940
      SUM= 0.                                                            FEN15950
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      DO 310  J=1,6                                                          FEN15960
  310 SUM= SUM  + C2(J,I)*SIG(J)                                             FEN15970
      IF (SUM.LT.SCRIT)  WRITE (IT6,2060)  IPT,SUM                           FEN15980
  320 CONTINUE                                                               FEN15990
  350 CONTINUE                                                               FEN16000
      EAVG= ESUM/DFLOAT(IPT)                                                 FEN16010
      TUP(IA,ITHIC)= (1. + EAVG)*THIC                                        FEN16020
  360 CONTINUE                                                               FEN16030
C ... ASSEMBLE ELEMENT NODAL FORCES INTO GLOBAL NODAL FORCE VECTOR           FEN16040
      DO 380  I=1,NDE                                                        FEN16050
      J= LM(I,N)                                                             FEN16060
      IF (J)  380,380,370                                                    FEN16070
  370 SLOD(J)=  SLOD(J) + ELOD(I)                                            FEN16080
  380 CONTINUE                                                               FEN16090
  400 CONTINUE                                                               FEN16100
C ... CALCULATE NODAL FORCES CORRESPONDING TO PENALTY MATRICES               FEN16110
      IF (NDF.GT.6)  CALL RESINT (SLOD,PLOD,ELOD,U,DISP,SE,NEQ)             FEN16120
C ... UPDATE NODAL COORDINATES AND NODAL VECTOR COMPONENTS                   FEN16130
      DO 520  I=1,NUMNP                                                      FEN16140
      DO 420  J=1,3                                                          FEN16150
      IJ= ID(J,I)                                                            FEN16160
      IF (IJ.NE.0)  XYZ(J,I)= XYZ(J,I) + U(IJ)                               FEN16170
      IJ= ID(J+3,I)                                                          FEN16180
      AA(J)= 0.                                                              FEN16190
      IF (IJ.NE.0)  AA(J)= U(IJ)                                             FEN16200
  420 CONTINUE                                                               FEN16210
C ... PERFORM EULER FORWARD INTEGRATION WITH NEULER INTERVALS                FEN16220
      COM1= CABS(AA(1))                                                      FEN16230
      COM2= CABS(AA(2))                                                      FEN16240
      COM3= CABS(AA(3))                                                      FEN16250
      BIGGY= DMAX1(COM1,COM2,COM3)                                           FEN16260
      NEULER= 1. + 1000.*BIGGY                                               FEN16270
      DO 430  K=1,3                                                          FEN16280
  430 AA(K)= AA(K)/DFLOAT(NEULER)                                            FEN16290
      DO 500  IE=1,NEULER                                                    FEN16300
      VV(1)= AA(2)*VSP(3,I) - AA(3)*VSP(2,I)                                 FEN16310
      VV(2)= AA(3)*VSP(1,I) - AA(1)*VSP(3,I)                                 FEN16320
      VV(3)= AA(1)*VSP(2,I) - AA(2)*VSP(1,I)                                 FEN16330
      DO 440  K=1,3                                                          FEN16340
  440 VSP(K,I)= VSP(K,I) + VV(K)                                             FEN16350
      VV(1)= AA(2)*VTP(3,I) - AA(3)*VTP(2,I)                                 FEN16360
      VV(2)= AA(3)*VTP(1,I) - AA(1)*VTP(3,I)                                 FEN16370
      VV(3)= AA(1)*VTP(2,I) - AA(2)*VTP(1,I)                                 FEN16380
      DO 450  K=1,3                                                          FEN16390
  450 VTP(K,I)= VTP(K,I) + VV(K)                                             FEN16400
      W= 1./DSQRT(VSP(1,I)*VSP(1,I)+VSP(2,I)*VSP(2,I)+VSP(3,I)*VSP(3,I))     FEN16410
      Z= 1./DSQRT(VTP(1,I)*VTP(1,I)+VTP(2,I)*VTP(2,I)+VTP(3,I)*VTP(3,I))     FEN16420
      DO 460  K=1,3                                                          FEN16430
      VSP(K,I)= W*VSP(K,I)                                                   FEN16440
  460 VTP(K,I)= Z*VTP(K,I)                                                   FEN16450
  500 CONTINUE                                                               FEN16460
  520 CONTINUE                                                               FEN16470
      IF (ICHEK.LT.999.AND.IITER.NE.MITER.AND.IITER.NE.INTUP) GO TO 580      FEN16480
C ... UPDATE ELEMENT RADII OF CURVATURE ASSUMING Z-ROTATIONS                 FEN16490
      DO 560  N=1,NUMEL                                                      FEN16500
```

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      N1= NEL(1,N)                                                      FEN16510
      N2= NEL(NPE,N)                                                    FEN16520
      I1= ID(6,N1)                                                      FEN16530
      I2= ID(6,N2)                                                      FEN16540
      A1= 0.                                                            FEN16550
      A2= 0.                                                            FEN16560
      IF (I1.NE.0)  A1= UDIS(I1)                                        FEN16570
      IF (I2.NE.0)  A2= UDIS(I2)                                        FEN16580
C ... ASSUME ELEMENTS OF CIRCULAR ARC, UNDERGOING CLOSING MOMENTS       FEN16590
      TETOLD= ELG(N)/RDS(N)                                             FEN16600
      TETUPD= TETOLD + DABS(A2 - A1)                                    FEN16610
      RDS(N)= ELG(N)/TETUPD                                             FEN16620
  560 CONTINUE                                                          FEN16630
      IF (IITER.EQ.INTUP)  CALL CLEAR (UDIS,NEQ)                        FEN16640
C ... CALCULATE UNBALANCED FORCES FOR NEXT INTERATION                   FEN16650
  580 DO 600  I=1,NEQ                                                   FEN16660
  600 U(I)= ALOD(I) - SLOD(I)                                           FEN16670
      RETURN                                                            FEN16680
 2000 FORMAT (//39H * * *  CONVERGENCE CHECKS AT   IITER =,I4,4H   :,5X,FEN16690
     1  8H  FRAT =,D10.3,5X,8H  ERAT =,D10.3 )                         FEN16700
 2010 FORMAT (//40H *** CRITICAL VALUE OF BUCKLING STRESS =,D15.6)      FEN16710
 2020 FORMAT (I5,5X,3I5,5X,I5,5X,2D15.6)                                FEN16720
 2040 FORMAT (/53H *** CURVATURE AT PLANE OF SYMMETRY :  KY  KRAT  MRAT,FEN16730
     1  3D15.6)                                                        FEN16740
 2060 FORMAT (//52H  **** POSSIBLE BUCKLING HAS BEEN DETECTED AT  IPT =,FEN16750
     1  I5,28H   WITH COMPRESSIVE STRESS =,D15.6)                      FEN16760
 2080 FORMAT (41H  >>> LOCAL NORMAL STRESSES AT   IA  IT =,2I4,2H :,    FEN16770
     1  3D15.6)                                                        FEN16780
      END                                                               FEN16790
C                                                                       FEN16800
      SUBROUTINE  RESINT (SLOD,PLOD,ELOD,U,DISP,SE,NEQ)                 FEN16810
C                                                                       FEN16820
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN16830
      COMMON/GEN/ NUMNP,NUMEL,NPE,NDF,NDC,NDS,NIPS,IPL,IPP,IPW,IPH      FEN16840
      COMMON/PRO/ GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)           FEN16850
      COMMON/LOCA/ B,S,T,TH,THIC,ZT,FI,CS,SN,RD,EG,IP1,IP2,N,NC,NTC     FEN16860
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                      FEN16870
      DIMENSION  SLOD(1),PLOD(1),ELOD(1),U(1),DISP(1),SE(21,1),LL(21)   FEN16880
      EQUIVALENCE  (NPAR(3),INTEF),(NPAR(4),IFIX)                       FEN16890
      IF (INTEF.EQ.0.AND.IFIX.EQ.0)  RETURN                             FEN16900
      IF (INTEF.EQ.0)  GO TO 200                                        FEN16910
      KT= (2*NPE - 1)*NC                                                FEN16920
      KS= KT*(KT + 1)/2                                                 FEN16930
      NEM= NUMEL - 1                                                    FEN16940
C ... READ PENALTY STIFFNESS MATRIX AND DOF ADDRESSES FROM TAPE         FEN16950
      DO 140  N=1,NEM                                                   FEN16960
      READ (IT2)  ((SE(I,J),J=I,KT),I=1,KT),(LL(I),I=1,KT)              FEN16970
      DO 20  I=2,KT                                                     FEN16980
      II= I - 1                                                         FEN16990
      DO 20  J=1,II                                                     FEN17000
   20 SE(I,J)= SE(J,I)                                                  FEN17010
      DO 40  I=1,KT                                                     FEN17020
      J= LL(I)                                                          FEN17030
      DISP(I)= 0.                                                       FEN17040
      IF (J.NE.0)  DISP(I)= U(J)                                        FEN17050
```

```
   40 CONTINUE                                                          FEN17060
C ... CALCULATE NODAL FORCES                                            FEN17070
      DO 80  I=1,KI                                                     FEN17080
      SUM= 0.                                                           FEN17090
      DO 60  J=1,KI                                                     FEN17100
   60 SUM= SUM + SE(I,J)*DISP(J)                                        FEN17110
   80 ELOD(I)= SUM                                                      FEN17120
      DO 120  I=1,KI                                                    FEN17130
      J= LL(I)                                                          FEN17140
      IF (J)  120,120,100                                               FEN17150
  100 PLOD(J)= PLOD(J) + ELOD(I)                                        FEN17160
  120 CONTINUE                                                          FEN17170
  140 CONTINUE                                                          FEN17180
      IF (IFIX.EQ.0)  GO TO 400                                         FEN17190
C                                                                       FEN17200
  200 KS= NIC*(NIC + 1)/2                                               FEN17210
      DO 340  JFIX=1,2                                                  FEN17220
      READ (IT2)  ((SE(I,J),J=I,NIC),I=1,NIC),(LL(I),I=1,NIC)          FEN17230
      DO 220  I=2,NIC                                                   FEN17240
      II= I - 1                                                         FEN17250
      DO 220  J=1,II                                                    FEN17260
  220 SE(I,J)= SE(J,I)                                                  FEN17270
      DO 240  I=1,NIC                                                   FEN17280
      J= LL(I)                                                          FEN17290
      DISP(I)= 0.                                                       FEN17300
      IF (J.NE.0)  DISP(I)= U(J)                                        FEN17310
  240 CONTINUE                                                          FEN17320
      DO 280  I=1,NIC                                                   FEN17330
      SUM= 0.                                                           FEN17340
      DO 260  J=1,NIC                                                   FEN17350
  260 SUM= SUM + SE(I,J)*DISP(J)                                        FEN17360
  280 ELOD(I)= SUM                                                      FEN17370
      DO 320  I=1,NIC                                                   FEN17380
      J= LL(I)                                                          FEN17390
      IF (J)  320,320,300                                               FEN17400
  300 PLOD(J)= PLOD(J) + ELOD(I)                                        FEN17410
  320 CONTINUE                                                          FEN17420
  340 CONTINUE                                                          FEN17430
C                                                                       FEN17440
  400 DO 420  I=1,NEQ                                                   FEN17450
  420 SLOD(I)= SLOD(I) + PLOD(I)                                        FEN17460
      RETURN                                                            FEN17470
      END                                                               FEN17480
C                                                                       FEN17490
      SUBROUTINE  SHAPE (ELXY,VS,VT,Q1,Q2,BL,BN,DET)                    FEN17500
C                                                                       FEN17510
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN17520
      COMMON/PRO/ GEOM(8),PROP(6),EMOD(3,10),FLOD(30),NPAR(6)           FEN17530
      COMMON/GEN/ NUMNP,NUMEL,NPE,NUF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH      FEN17540
      COMMON/LOCA/ R,S,T,TH,THIC,ZT,FI,CS,SN,RD,EG,IP1,IP2,N,NC,NTC     FEN17550
      DIMENSION  ELXY(3,1),VS(3,1),VT(3,1),Q1(1),Q2(1),BL(1),BN(1)      FEN17560
      DIMENSION  H(4),DH(4,2),XJ(3,3),XJI(3,3)                          FEN17570
      EQUIVALENCE (GEOM(2),RA)                                          FEN17580
C                                                                       FEN17590
      A2= R*R                                                           FEN17600
```

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      R3= R*R2                                                          FEN17610
C ... INTERPOLATION FUNCTIONS                                           FEN17620
      H(1)= 0.0625*(-9.*R3 + 9.*R2 + R - 1.)                            FEN17630
      H(2)= 0.0625*( 27.*R3 - 9.*R2 - 27.*R + 9.)                       FEN17640
      H(3)= 0.0625*(-27.*R3 - 9.*R2 + 27.*R + 9.)                       FEN17650
      H(4)= 0.0625*( 9.*R3 + 9.*R2 - R - 1.)                            FEN17660
C ... THEIR DERIVATIVES  W.R.T.  R                                      FEN17670
      DH( 1,1)= 0.0625*( -27.*R2 + 18.*R + 1.)                          FEN17680
      DH( 2,1)= 0.0625*(  81.*R2 - 18.*R - 27.)                         FEN17690
      DH( 3,1)= 0.0625*( -81.*R2 - 18.*R + 27.)                         FEN17700
      DH( 4,1)= 0.0625*(  27.*R2 + 18.*R - 1.)                          FEN17710
      DH( 1,2)= 0.0625*( -54.*R + 18.)                                  FEN17720
      DH( 2,2)= 0.0625*( 162.*R - 18.)                                  FEN17730
      DH( 3,2)= 0.0625*(-162.*R - 18.)                                  FEN17740
      DH( 4,2)= 0.0625*(  54.*R + 18.)                                  FEN17750
C ... JACOBIAN MATRIX AT NODE (R,S,T)                                   FEN17760
      DO 40  I=1,3                                                      FEN17770
      DO 40  J=1,3                                                      FEN17780
      XJ(I,J)= 0.                                                       FEN17790
      GO TO (1,2,3), I                                                  FEN17800
    1 DO 10  K=1,NPE                                                    FEN17810
   10 XJ(I,J)= XJ(I,J) + DH(K,1)*(ELXY(J,K)+RA*(S*VS(J,K)+T*VT(J,K)))   FEN17820
      GO TO 40                                                          FEN17830
    2 DO 20  K=1,NPE                                                    FEN17840
   20 XJ(I,J)= XJ(I,J) + RA*H(K)*VS(J,K)                                FEN17850
      GO TO 40                                                          FEN17860
    3 DO 30  K=1,NPE                                                    FEN17870
   30 XJ(I,J)= XJ(I,J) + RA*H(K)*VT(J,K)                                FEN17880
   40 CONTINUE                                                          FEN17890
C                                                                       FEN17900
      CALL JACOB (XJ,XJI,DET,N)                                         FEN17910
      CALL TRANS (XJ,Q1,Q2)                                             FEN17920
      CALL BMAT (BL,BN,VS,VT,XJI,H,DH,S,T,THIC,RA,NPE,NDF)              FEN17930
      IF (NC.GT.0)  CALL BDVL (H,DH,Q2,BL)                              FEN17940
      RETURN                                                            FEN17950
      END                                                               FEN17960
C                                                                       FEN17970
      SUBROUTINE  STIFF (BL,BN,Q1,Q2,C,S,SIG,DBN,DBL,WT)                FEN17980
C                                                                       FEN17990
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN18000
      COMMON/GEN/ NUMNP,NUMEL,NPE,NDF,NDE,NDS,NIPS,IPL,IPP,IPW,IPH      FEN18010
      COMMON/VAR/ IINCS,IITER,IPEL,IREM,IND,IPT,ICHEK                   FEN18020
      COMMON/TAPE/ IT1,IT2,IT3,IT4,IT5,IT6,IT7,IT8                      FEN18030
      DIMENSION  BL(6,NDS),BN(3,NDE),Q1(6,6),Q2(6,6),C(6,1),S(1),SIG(1),FEN18040
     1           DEN(9,1),DBL(6,1),TAU(3,3),SBAR(9,9),CQ(6,6),D(6,6)    FEN18050
      EQUIVALENCE  (SBAR(1,1),CQ(1,1)),(SBAR(1,5),D(1,1))               FEN18060
C ... FORM LINEAR MATERIAL LAW IN GLOBAL COORDINATES                    FEN18070
      DO 20  I=1,6                                                      FEN18080
      DO 20  J=1,6                                                      FEN18090
      CQ(I,J)= 0.                                                       FEN18100
      DO 20  K=1,6                                                      FEN18110
   20 CQ(I,J)= CQ(I,J) + C(I,K)*Q1(K,J)                                 FEN18120
      DO 50  I=1,6                                                      FEN18130
      DO 50  J=I,6                                                      FEN18140
      SUM= 0.                                                           FEN18150
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      DO 40  K=1,6                                                      FEN18160
   40 SUM= SUM + C1(K,I)*CJ(K,J)                                        FEN18170
   50 D(I,J)= SUM                                                       FEN18180
      DO 60  I=2,6                                                      FEN18190
      I1= I - 1                                                         FEN18200
      DO 60  J=1,I1                                                     FEN18210
   60 D(I,J)= D(J,I)                                                    FEN18220
C ... WRITE TRANSFORMATION MATRICES ON TAPE IT2                         FEN18230
      WRITE (IT2) BL,BN,J1,J2,D,WT                                      FEN18240
      IF (IPEL.EQ.1.OR.IREM.EQ.1)  GO TO 80                             FEN18250
C ... RETRIEVE 'NONLINEAR' MATERIAL LAW FROM TAPE IT3                   FEN18260
      CALL CLEAR (D,36)                                                 FEN18270
      READ (IT3) D                                                      FEN18280
C ... CALCULATE LINEAR STIFFNESS CONTRIBUTION                           FEN18290
   80 DO 110  I=1,6                                                     FEN18300
      DO 110  J=1,NDE                                                   FEN18310
      DUM= 0.                                                           FEN18320
      DO 100  K=1,6                                                     FEN18330
  100 DUM= DUM + D(I,K)*BL(K,J)                                         FEN18340
      IF (DABS(DUM).LT.1.D-35)  DUM= 0.                                 FEN18350
  110 DBL(I,J)= DUM                                                     FEN18360
      IJ= 0                                                             FEN18370
      DO 140  I=1,NDE                                                   FEN18380
      DO 140  J=I,NDE                                                   FEN18390
      DUM= 0.                                                           FEN18400
      DO 120  K=1,6                                                     FEN18410
  120 DUM= DUM + BL(K,I)*DBL(K,J)                                       FEN18420
      IJ= IJ + 1                                                        FEN18430
  140 S(IJ)= S(IJ) + DUM*WT                                             FEN18440
      IF (INC.EQ.0.OR.IREM.EQ.1)  RETURN                                FEN18450
C ... FORM STRESS TENSOR                                                FEN18460
      DO 160  I=1,3                                                     FEN18470
  160 TAU(I,I)= SIG(I)                                                  FEN18480
      TAU(1,2)= SIG(4)                                                  FEN18490
      TAU(2,1)= SIG(4)                                                  FEN18500
      TAU(2,3)= SIG(5)                                                  FEN18510
      TAU(3,2)= SIG(5)                                                  FEN18520
      TAU(1,3)= SIG(6)                                                  FEN18530
      TAU(3,1)= SIG(6)                                                  FEN18540
      CALL CLEAR (SBAR,81)                                              FEN18550
      DO 180  I=1,3                                                     FEN18560
      I1= (I-1)*3                                                       FEN18570
      DO 180  J=1,3                                                     FEN18580
      J1= (J-1)*3                                                       FEN18590
      DO 180  K=1,3                                                     FEN18600
  180 SBAR(I1+K,J1+K)= TAU(I,J)                                         FEN18610
C ... CALCULATE NONLINEAR STIFFNESS CONTRIBUTION                        FEN18620
      DO 200  I=1,9                                                     FEN18630
      DO 200  J=1,NDE                                                   FEN18640
      DUM= 0.                                                           FEN18650
      DO 190  K=1,9                                                     FEN18660
  190 DUM= DUM + SBAR(I,K)*BN(K,J)                                      FEN18670
      IF (DABS(DUM).LT.1.D-35)  DUM= 0.                                 FEN18680
  200 DBN(I,J)= DUM                                                     FEN18690
      IJ= 0                                                             FEN18700
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      DO 240  I=1,NDE                                                    FEN18710
      DO 240  J=I,NDE                                                    FEN18720
      DUM= 0.                                                            FEN18730
      DO 220  K=1,9                                                      FEN18740
  220 DUM= DUM + BK(K,I)*DBK(K,J)                                        FEN18750
      IJ= IJ + 1                                                         FEN18760
  240 S(IJ)= S(IJ) + DUM*WT                                              FEN18770
      RETURN                                                             FEN18780
      END                                                                FEN18790
C                                                                        FEN18800
      SUBROUTINE TRANS (XJ,J1,J2)                                        FEN18810
C                                                                        FEN18820
      IMPLICIT REAL*8(A-H,O-Z)                                           FEN18830
      COMMON/LOCA/ R,S,T,TH,THIC,ZT,FI,CS,SN,RD,EG,IP1,IP2,N,NC,NTC      FEN18840
      DIMENSION  XJ(3,3),J1(6,6),J2(6,6),E1(3),E2(3),E3(3)               FEN18850
C ... CALCULATE ORTHOGONAL VECTORS AT INTEGRATION POINT (R,S,T)          FEN18860
      DUMS= 1./DSQRT(XJ(2,1)*XJ(2,1)+XJ(2,2)*XJ(2,2)+XJ(2,3)*XJ(2,3))   FEN18870
      DUMT= 1./DSQRT(XJ(3,1)*XJ(3,1)+XJ(3,2)*XJ(3,2)+XJ(3,3)*XJ(3,3))   FEN18880
      DO 20   I=1,3                                                      FEN18890
      E2(I)= XJ(2,I)*DUMS                                                FEN18900
   20 E3(I)= XJ(3,I)*DUMT                                                FEN18910
      CALL VEX (E2,E3,E1)                                                FEN18920
      DO 30   I=1,3                                                      FEN18930
      J1(I,1)= E1(I)                                                     FEN18940
      J1(I,2)= E2(I)                                                     FEN18950
      J1(I,3)= E3(I)                                                     FEN18960
      DO 30   J=1,3                                                      FEN18970
   30 J1(I+3,J+3)= 0.                                                    FEN18980
      J1(5,4)= -SN                                                       FEN18990
      J1(6,4)=  CS                                                       FEN19000
      J1(4,5)=  1.                                                       FEN19010
      J1(5,6)=  CS                                                       FEN19020
      J1(6,5)=  SN                                                       FEN19030
      DO 40   I=1,3                                                      FEN19040
      DO 40   J=1,3                                                      FEN19050
      J1(I,J+3)= 0.                                                      FEN19060
      DO 40   K=1,3                                                      FEN19070
   40 J1(I,J+3)= J1(I,J+3) + J1(I,K)*J1(K+3,J+3)                         FEN19080
      DO 60   I=1,3                                                      FEN19090
      E1(I)= J1(I,4)                                                     FEN19100
      E2(I)= J1(I,5)                                                     FEN19110
   60 E3(I)= J1(I,6)                                                     FEN19120
C ... ELEMENTS OF MATRIX  Q  FOR   EPSBAR = Q*EPS                        FEN19130
      J1(1,1)= E1(1)*E1(1)                                               FEN19140
      J1(2,1)= E2(1)*E2(1)                                               FEN19150
      J1(3,1)= E3(1)*E3(1)                                               FEN19160
      J1(4,1)= 2.*E1(1)*E2(1)                                            FEN19170
      J1(5,1)= 2.*E2(1)*E3(1)                                            FEN19180
      J1(6,1)= 2.*E3(1)*E1(1)                                            FEN19190
C                                                                        FEN19200
      J1(1,2)= E1(2)*E1(2)                                               FEN19210
      J1(2,2)= E2(2)*E2(2)                                               FEN19220
      J1(3,2)= E3(2)*E3(2)                                               FEN19230
      J1(4,2)= 2.*E1(2)*E2(2)                                            FEN19240
      J1(5,2)= 2.*E2(2)*E3(2)                                            FEN19250
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      Q1(6,2)= 2.*E3(2)*E1(2)                                                  FEN19260
C                                                                              FEN19270
      Q1(1,3)= E1(3)*E1(3)                                                     FEN19280
      Q1(2,3)= E2(3)*E2(3)                                                     FEN19290
      Q1(3,3)= E3(3)*E3(3)                                                     FEN19300
      Q1(4,3)= 2.*E1(3)*E2(3)                                                  FEN19310
      Q1(5,3)= 2.*E2(3)*E3(3)                                                  FEN19320
      Q1(6,3)= 2.*E3(3)*E1(3)                                                  FEN19330
C                                                                              FEN19340
      Q1(1,4)= E1(1)*E1(2)                                                     FEN19350
      Q1(2,4)= E2(1)*E2(2)                                                     FEN19360
      Q1(3,4)= E3(1)*E3(2)                                                     FEN19370
      Q1(4,4)= E1(1)*E2(2) + E2(1)*E1(2)                                       FEN19380
      Q1(5,4)= E2(1)*E3(2) + E3(1)*E2(2)                                       FEN19390
      Q1(6,4)= E3(1)*E1(2) + E1(1)*E3(2)                                       FEN19400
C                                                                              FEN19410
      Q1(1,5)= E1(2)*E1(3)                                                     FEN19420
      Q1(2,5)= E2(2)*E2(3)                                                     FEN19430
      Q1(3,5)= E3(2)*E3(3)                                                     FEN19440
      Q1(4,5)= E1(2)*E2(3) + E2(2)*E1(3)                                       FEN19450
      Q1(5,5)= E2(2)*E3(3) + E3(2)*E2(3)                                       FEN19460
      Q1(6,5)= E3(2)*E1(3) + E1(2)*E3(3)                                       FEN19470
C                                                                              FEN19480
      Q1(1,6)= E1(3)*E1(1)                                                     FEN19490
      Q1(2,6)= E2(3)*E2(1)                                                     FEN19500
      Q1(3,6)= E3(3)*E3(1)                                                     FEN19510
      Q1(4,6)= E1(3)*E2(1) + E2(3)*E1(1)                                       FEN19520
      Q1(5,6)= E2(3)*E3(1) + E3(3)*E2(1)                                       FEN19530
      Q1(6,6)= E3(3)*E1(1) + E1(3)*E3(1)                                       FEN19540
C ... ELEMENTS OF MATRIX  Q  FOR   EPS = Q*EPS3AR                              FEN19550
      Q2(1,1)= E1(1)*E1(1)                                                     FEN19560
      Q2(2,1)= E1(2)*E1(2)                                                     FEN19570
      Q2(3,1)= E1(3)*E1(3)                                                     FEN19580
      Q2(4,1)= 2.*E1(1)*E1(2)                                                  FEN19590
      Q2(5,1)= 2.*E1(2)*E1(3)                                                  FEN19600
      Q2(6,1)= 2.*E1(3)*E1(1)                                                  FEN19610
C                                                                              FEN19620
      Q2(1,2)= E2(1)*E2(1)                                                     FEN19630
      Q2(2,2)= E2(2)*E2(2)                                                     FEN19640
      Q2(3,2)= E2(3)*E2(3)                                                     FEN19650
      Q2(4,2)= 2.*E2(1)*E2(2)                                                  FEN19660
      Q2(5,2)= 2.*E2(2)*E2(3)                                                  FEN19670
      Q2(6,2)= 2.*E2(3)*E2(1)                                                  FEN19680
C                                                                              FEN19690
      Q2(1,3)= E3(1)*E3(1)                                                     FEN19700
      Q2(2,3)= E3(2)*E3(2)                                                     FEN19710
      Q2(3,3)= E3(3)*E3(3)                                                     FEN19720
      Q2(4,3)= 2.*E3(1)*E3(2)                                                  FEN19730
      Q2(5,3)= 2.*E3(2)*E3(3)                                                  FEN19740
      Q2(6,3)= 2.*E3(3)*E3(1)                                                  FEN19750
C                                                                              FEN19760
      Q2(1,4)= E1(1)*E2(1)                                                     FEN19770
      Q2(2,4)= E1(2)*E2(2)                                                     FEN19780
      Q2(3,4)= E1(3)*E2(3)                                                     FEN19790
      Q2(4,4)= E1(1)*E2(2) + E2(1)*E1(2)                                       FEN19800
```

```
      Q2( 5,4 )= E1( 2)*E2( 3) + E2( 2)*E1( 3)                          FEN19810
      Q2( 6,4 )= E1( 3)*E2( 1) + E2( 3)*E1( 1)                          FEN19820
C                                                                       FEN19830
      Q2( 1,5 )= E2( 1)*E3( 1)                                          FEN19840
      Q2( 2,5 )= E2( 2)*E3( 2)                                          FEN19850
      Q2( 3,5 )= E2( 3)*E3( 3)                                          FEN19860
      Q2( 4,5 )= E2( 1)*E3( 2) + E3( 1)*E2( 2)                          FEN19870
      Q2( 5,5 )= E2( 2)*E3( 3) + E3( 2)*E2( 3)                          FEN19880
      Q2( 6,5 )= E2( 3)*E3( 1) + E3( 3)*E2( 1)                          FEN19890
C                                                                       FEN19900
      Q2( 1,6 )= E3( 1)*E1( 1)                                          FEN19910
      Q2( 2,6 )= E3( 2)*E1( 2)                                          FEN19920
      Q2( 3,6 )= E3( 3)*E1( 3)                                          FEN19930
      Q2( 4,6 )= E3( 1)*E1( 2) + E1( 1)*E3( 2)                          FEN19940
      Q2( 5,6 )= E3( 2)*E1( 3) + E1( 2)*E3( 3)                          FEN19950
      Q2( 6,6 )= E3( 3)*E1( 1) + E1( 3)*E3( 1)                          FEN19960
      DO 80   I=1,6                                                     FEN19970
      DO 80   J=1,6                                                     FEN19980
      IF (DABS(Q1(I,J)).LT.1.D-35)  Q1(I,J)= 0.                         FEN19990
      IF (DABS(Q2(I,J)).LT.1.D-35)  Q2(I,J)= 0.                         FEN20000
   80 CONTINUE                                                          FEN20010
      RETURN                                                            FEN20020
      END                                                               FEN20030
C                                                                       FEN20040
      SUBROUTINE  UNLOD ( SIG,EPS,OHM)                                  FEN20050
C                                                                       FEN20060
      IMPLICIT REAL*8(A-H,O-Z)                                          FEN20070
      COMMON/MAT/ STRESS(6),STRAIN(6),C(6,6),YLD,HPRIME,IST,ISR         FEN20080
      DIMENSION  SIG(1),EPS(1),OHM(3,1),RS(3,6),RN(3,6)                 FEN20090
      NEULER= 10                                                        FEN20100
      DO 20   I=1,3                                                     FEN20110
      DO 20   J=1,3                                                     FEN20120
   20 OHM(I,J)= OHM(I,J)/DFLOAT(NEULER)                                 FEN20130
C ... MAKE CORRECTIONS FOR STRESSES AND STRAINS DUE TO  R.B.  ROTATIONS  FEN20140
      DO 40   I=1,3                                                     FEN20150
      RS(I,I)= SIG(I)                                                   FEN20160
   40 RN(I,I)= EPS(I)                                                   FEN20170
      RS(1,2)= SIG(4)                                                   FEN20180
      RS(2,3)= SIG(5)                                                   FEN20190
      RS(1,3)= SIG(6)                                                   FEN20200
      RN(1,2)= 0.5*EPS(4)                                               FEN20210
      RN(2,3)= 0.5*EPS(5)                                               FEN20220
      RN(1,3)= 0.5*EPS(6)                                               FEN20230
      DO 60   I=2,3                                                     FEN20240
      I1= I-1                                                           FEN20250
      DO 60   J=1,I1                                                    FEN20260
      RS(I,J)= RS(J,I)                                                  FEN20270
   60 RN(I,J)= RN(J,I)                                                  FEN20280
C ... START PERFORMING EULER FORWARD INTEGRATION WITH NEULER INTERVALS  FEN20290
      DO 80   I=1,3                                                     FEN20300
      DO 80   J=1,3                                                     FEN20310
      RS(I,J+3)= RS(I,J)                                                FEN20320
   80 RN(I,J+3)= RN(I,J)                                                FEN20330
      DO 120   IE=1,NEULER                                              FEN20340
      DO 100   I=1,3                                                    FEN20350
```

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      DO 100  J=1,3                                                          FEN20360
      DO 100  K=1,3                                                          FEN20370
      RS(I,J+3)= RS(I,J+3) + RS(I,K)*OHM(K,J) - OHM(I,K)*RS(K,J)             FEN20380
  100 RN(I,J+3)= RN(I,J+3) + RN(I,K)*OHM(K,J) - OHM(I,K)*RN(K,J)             FEN20390
      DO 110  I=1,3                                                          FEN20400
      DO 110  J=1,3                                                          FEN20410
      RS(I,J)= RS(I,J+3)                                                     FEN20420
  110 RN(I,J)= RN(I,J+3)                                                     FEN20430
  120 CONTINUE                                                               FEN20440
      DO 140  I=1,3                                                          FEN20450
      SIG(I)= RS(I,I)                                                        FEN20460
  140 EPS(I)= RN(I,I)                                                        FEN20470
      SIG(4)= RS(1,2)                                                        FEN20480
      SIG(5)= RS(2,3)                                                        FEN20490
      SIG(6)= RS(1,3)                                                        FEN20500
      EPS(4)= 2.*RN(1,2)                                                     FEN20510
      EPS(5)= 2.*RN(2,3)                                                     FEN20520
      EPS(6)= 2.*RN(1,3)                                                     FEN20530
C ... CALCULATE TOTAL STRESSES, ASSUMING PURELY ELASTIC BEHAVIOUR            FEN20540
      DO 180  IS=1,IST                                                       FEN20550
      DELSIG= 0.                                                             FEN20560
      DO 160  IR=1,ISR                                                       FEN20570
  160 DELSIG= DELSIG + C(IS,IR)*STRAIN(IR)                                   FEN20580
  180 SIG(IS)= SIG(IS) + DELSIG                                              FEN20590
      DO 200  IR=1,ISR                                                       FEN20600
  200 EPS(IR)= EPS(IR) + STRAIN(IR)                                          FEN20610
      RETURN                                                                 FEN20620
      END                                                                    FEN20630
C                                                                            FEN20640
      SUBROUTINE VEK (W1,W2,W3)                                              FEN20650
C                                                                            FEN20660
      IMPLICIT REAL*8(A-H,O-Z)                                               FEN20670
      DIMENSION  W1(1),W2(1),W3(1)                                           FEN20680
C ... GENERATE A UNIT VECTOR ORTHOGONAL TO TWO GIVEN VECTORS                 FEN20690
      W3(1)= W1(2)*W2(3) - W1(3)*W2(2)                                       FEN20700
      W3(2)= W1(3)*W2(1) - W1(1)*W2(3)                                       FEN20710
      W3(3)= W1(1)*W2(2) - W1(2)*W2(1)                                       FEN20720
      WW= 1./DSQRT(W3(1)*W3(1)+W3(2)*W3(2)+W3(3)*W3(3))                      FEN20730
      DO 30   I=1,3                                                          FEN20740
   30 W3(I)= W3(I)*WW                                                        FEN20750
      RETURN                                                                 FEN20760
      END                                                                    FEN20770
C$DATA                                                                       FEN20780
 PROBLEM :   SIMULATION OF PIPE BENDING                                      FEN20790
   31    10      7      4      3     12      5     31     28      2      9    25      0      1  FEN20800
 0.48D+00 12.24D+00   1.00D+31   1.20D+02                                    FEN20810
 30.0D+06  0.28D+00   1.20D+00  36.00D+03                                    FEN20820
    1     0      1      1                                                    FEN20830
    1          0.12D+02                                                     FEN20840
    2          0.12D+02                                                     FEN20850
    3          0.12D+02                                                     FEN20860
    4          0.12D+02                                                     FEN20870
    5          0.12D+02                                                     FEN20880
    6          0.12D+02                                                     FEN20890
    7          0.12D+02                                                     FEN20900
```

```
   8        0.12D+02                                                                FEN20910
   9        0.12D+02                                                                FEN20920
  10        0.12D+02                                                                FEN20930
   1          1    1    1    1    1    1    1                                       FEN20940
   2          0    0    1    1    1    0    1                                       FEN20950
   3          0    0    1    1    1    0    1                                       FEN20960
   4          0    0    1    1    1    0    1                                       FEN20970
   5          0    0    1    1    1    0    0                                       FEN20980
   6          0    0    1    1    1    0    0                                       FEN20990
   7          0    0    1    1    1    0    0                                       FEN21000
   8          0    0    1    1    1    0    0                                       FEN21010
   9          0    0    1    1    1    0    0                                       FEN21020
  10          0    0    1    1    1    0    0                                       FEN21030
  11          0    0    1    1    1    0    0                                       FEN21040
  12          0    0    1    1    1    0    0                                       FEN21050
  13          0    0    1    1    1    0    0                                       FEN21060
  14          0    0    1    1    1    0    0                                       FEN21070
  15          0    0    1    1    1    0    0                                       FEN21080
  16          0    0    1    1    1    0    0                                       FEN21090
  17          0    0    1    1    1    0    0                                       FEN21100
  18          0    0    1    1    1    0    0                                       FEN21110
  19          0    0    1    1    1    0    0                                       FEN21120
  20          0    0    1    1    1    0    0                                       FEN21130
  21          0    0    1    1    1    0    0                                       FEN21140
  22          0    0    1    1    1    0    0                                       FEN21150
  23          0    0    1    1    1    0    0                                       FEN21160
  24          0    0    1    1    1    0    0                                       FEN21170
  25          0    0    1    1    1    0    0                                       FEN21180
  26          0    0    1    1    1    0    0                                       FEN21190
  27          0    0    1    1    1    0    0                                       FEN21200
  28          0    0    1    1    1    0    0                                       FEN21210
  29          0    0    1    1    1    0    0                                       FEN21220
  30          0    0    1    1    1    0    0                                       FEN21230
  31          0    0    1    1    1    0    1                                       FEN21240
    0.500     0.500   0.300   0.100   0.100   -0.200   -0.300                       FEN21250
   -0.500    -0.500                                                                 FEN21260
   1        0.1200D-02        36.0D+03                                              FEN21270
   2        5.000D-02        255.6D+03                                              FEN21280
 0.1D-01     0.1D-02                                                                FEN21290
   4    6   158901.11D+00                                                           FEN21300
   5    6   317802.22D+00                                                           FEN21310
   6    6   317802.22D+00                                                           FEN21320
   7    6   317802.22D+00                                                           FEN21330
   8    6   317802.22D+00                                                           FEN21340
   9    6   317802.22D+00                                                           FEN21350
  10    6   317802.22D+00                                                           FEN21360
  11    6   317802.22D+00                                                           FEN21370
  12    6   317802.22D+00                                                           FEN21380
  13    6   317802.22D+00                                                           FEN21390
  14    6   317802.22D+00                                                           FEN21400
  15    6   317802.22D+00                                                           FEN21410
  16    6   317802.22D+00                                                           FEN21420
  17    6   317802.22D+00                                                           FEN21430
  18    6   317802.22D+00                                                           FEN21440
  19    6   317802.22D+00                                                           FEN21450
```

```
      20     6   317802.22D+00                                          FEN21460
      21     6   317802.22D+00                                          FEN21470
      22     6   317802.22D+00                                          FEN21480
      23     6   317802.22D+00                                          FEN21490
      24     6   317802.22D+00                                          FEN21500
      25     6   317802.22D+00                                          FEN21510
      26     6   317802.22D+00                                          FEN21520
      27     6   317802.22D+00                                          FEN21530
      28     6   317802.22D+00                                          FEN21540
      29     6   317802.22D+00                                          FEN21550
      30     6   317802.22D+00                                          FEN21560
      31     6   158901.11D+00                                          FEN21570
```

Appendix D

ADINA Shell-Element Results

In this Appendix, cross-section ovalization of a pipe bent by a ram bender will be calculated using an ADINA shell element, and the results obtained will be compared to those by FENSA as were presented in Chapter 5.

Taking into account the symmetry of the pipe (Figure 5.2), only a quarter of it needs to be considered. This portion is modelled by a mesh of 48 unequal-sized 9-noded shell elements and a total of 221 nodal points, Figure D.1. However, all elements each span 15^0 in the circumferential direction of the cross section. The dimensions of the pipe and the size of the ram used are those shown in Figure 5.6. To simulate the ram force exerting on the pipe wall, a concentrated load is applied to each nodal point that happens to be in the area of contact between the ram and the pipe wall. The ratios of the magnitudes of these concentrated loads are the same as those of the vertical distances from the corresponding nodes to the horizontal plane (zx-plane). The Gaussian integration scheme employed is $3 \times 3 \times 2$ (r-s-t), and a total of 1011 equilibrium equations are solved in the numerical solution.

Figure D-2 shows the ovalization results obtained for the cross section in the plane of symmetry. A number of important observations have been made from this figure.

- The fact that the curves representing expansion and contraction are almost

straight lines supports the assumption made earlier that ovalization increases linearly with the applied load (even beyond the yielding point).

- At the plane of symmetry, the cross-sectional expansion is smaller than the corresponding contraction. The shell prediction is attributed to the local deformation of the pipe wall at the ram. Thus the prediction by no means contradicts the fact that the expansion is larger than the contraction in most pipe bending problems with no local deformations.

- At $W/W_y = 1.5$, the maximum change (contraction) in the mean diameter of the cross section (measured by $\Delta r/r_m$) is observed to be about 1%. This value is of the order of 200 times that given by FENSA using the elbow element (Figure 5.7). The prediction also implies that local cross-sectional deformation at the ram might be much larger than that predicted by the elbow element although the magnitude of the shell prediction may vary somewhat, depending on how the ram force exerting on the pipe wall is modelled.

Thus, as far as local deformation and cross-section ovalization are concerned, details of the pipe can be obtained using a shell-element approach.

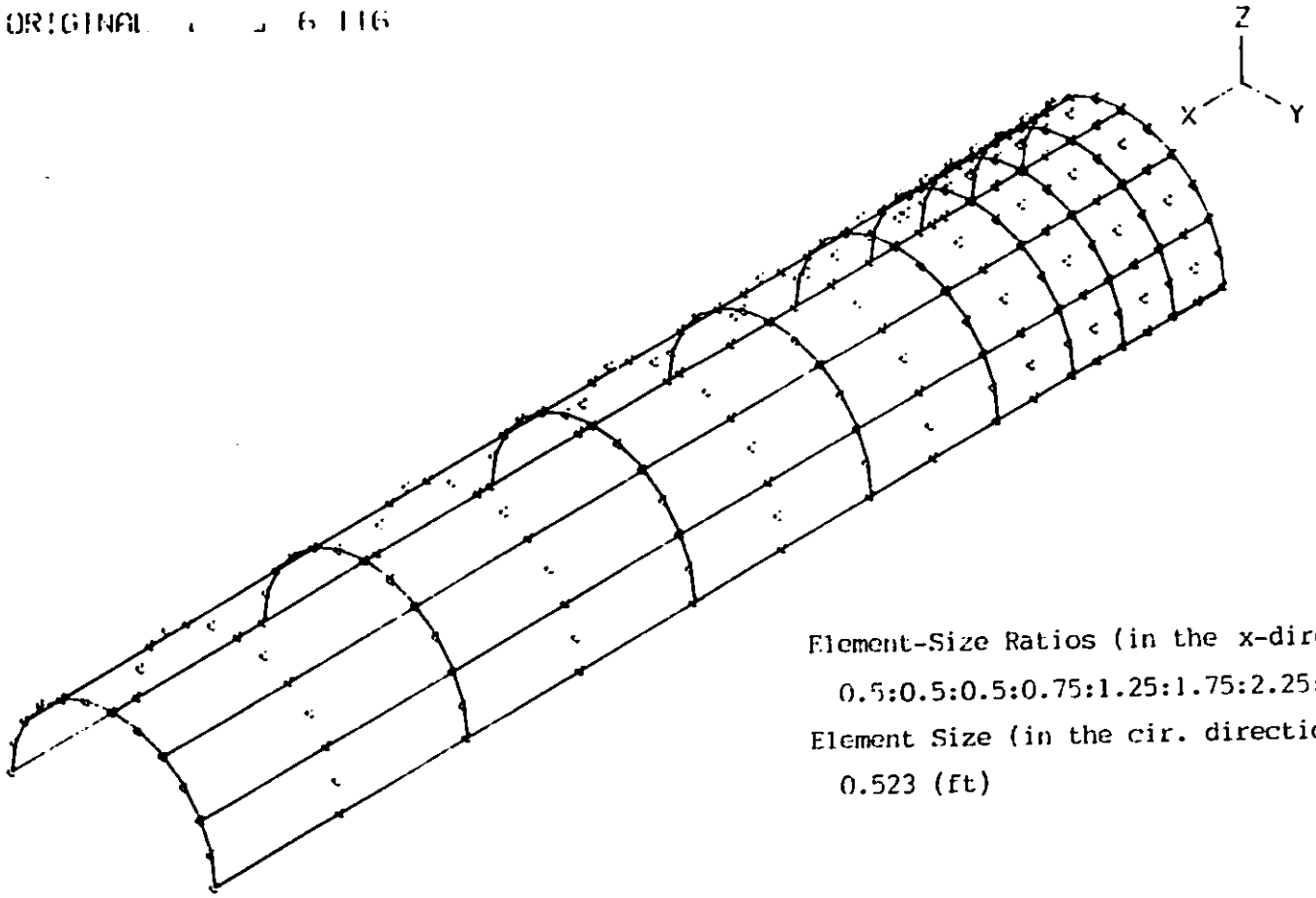

Fig. D-1 Undeformed mesh of 9-noded shell elements generated by ADINA.

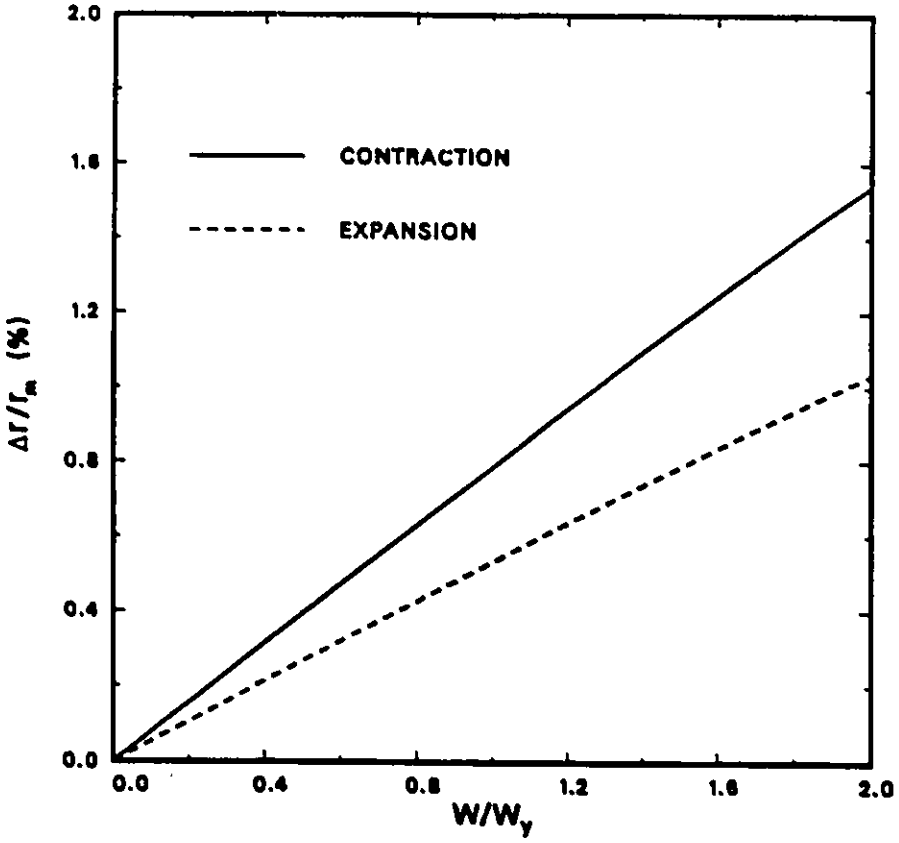

FIG. D-2 MAXIMUM CHANGES IN CROSS-SECTIONAL MEAN RADIUS PREDICTED BY ADINA.