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Is There a Principle of Targeting in Environmental Taxation?

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Abstract

We test whether the principle of targeting (alternatively Sandmo's (1975) additivity property and Kopczuk's (2003) decomposition involving the Pigovian rule) has relevance for environmental taxation in a second best world consisting of an exogenous revenue requirement and pre-existing distortionary taxes. In the context of differentiated commodity taxes, we find that Sandmo's additivity property breaks down once one solves explicitly for the marginal cost of public funds (MCPF). Further, in the more realistic setting of a uniform commodity tax and a dedicated emissions tax, we find that the additivity property no longer holds even in the form Sandmo studied it, i.e. without solving explicitly for the MCPF. Finally, we argue that Koczuk's decomposition is not persuasive, as it requires that a second government agency must apply a corrective tax or subsidy to adjust the choice of the Pigovian rule by the environmental agency. In a same-numbers exercise (i.e. the number of tax instruments is not increased), we show that there is no presumption in favour of a direct emissions tax over a uniform commodity tax; rather, the choice depends upon the size of the environmental damages. We conclude that there does not exist a principle of targeting in environmental taxation.

Key words: *environmental taxation; second best; principle of targeting.*

JEL Classification: H23.

Résumé

Nous considérons les implications du «principle of targeting» (soit le principe d'additivité de Sandmo (1975), soit la décomposition de Kopczuk (2003) axés sur la règle de Pigou) pour la fiscalité environnementale au deuxième rang, où le gouvernement fait face à une contrainte de revenu exogène et a recours à des taxes distortionnaires. Étant donné les taxes différenciées sur les biens, nous trouvons que le principe de l'additivité n'est pas soutenu dès qu'on solutionne pour le coût marginal du revenu public (CMRP). En plus, dans la situation plus réaliste qui consiste à une taxe uniforme sur les biens et une taxe directe sur les émissions de pollution, le principe de l'additivité n'est plus soutenu même dans le contexte prôné par Sandmo, à savoir sans solutionner pour le CMRP. Enfin, nous constatons que la décomposition de Koczuk n'est pas convaincante, car elle nécessite qu'une deuxième agence gouvernementale impose une taxe ou subvention corrective pour répond au choix d'une taxe selon la règle de Pigou par l'agence environnementale. Dans une comparaison entre une taxe uniforme sur les biens et une taxe directe sur les émissions de pollution, nous démontrons que le meilleur choix dépend du niveau des dommages. Bref, il n'y a pas de préférence systématique pour une taxe directe sur les émissions. Donc, nous concluons qu'il n'existe pas de «principle of targeting» pour la fiscalité environnementale.

Mots clés: *fiscalité environnementale; politique de deuxième rang; «principle of targeting».*

Classification JEL: H23.

1 Introduction

Like optimal tax theory in general, the literature on environmental taxation makes an important distinction between first-best and second-best. In the first best, as expressed by Pigou (1920), the optimal environmental tax is equal to the marginal social damage of emissions of the pollutant in question. In the second best, the optimal environmental tax usually differs from this Pigouvian level due to other distortions. For example, Buchanan (1969), Barnette (1980), Lavin (1985), and Shaffer (1995) consider the effect of market power in the goods market. Their conclusions all point to a value of the optimal environmental tax that is lower than the marginal social damage of emissions. Similarly, in the context of fixed government expenditures and pre-existing distortionary taxes, Bovenberg and de Mooij (1994) and Goulder (1995) conclude that the optimal emissions tax is lower than the Pigouvian level.

However, several papers have argued that the Pigouvian rule still has a role to play in the second best. Sandmo (1975) shows that, in the presence of differentiated commodity taxes, the pollution externality only appears in the tax formula for the pollution-generating good. Moreover, this tax formula can be decomposed into a weighted average of two parts: the efficiency term, related to the inverse elasticity of demand, resembles the theory of optimal commodity taxation, and the environmental term, based upon the marginal social damage of the pollutants. Sandmo refers to this result as the "additivity property".

Dixit (1985) argues that this additivity property represents a particular case of Bhagwati and Johnson's (1960) principle of targeting, according to which a distortion should be directly addressed by a dedicated tax instrument rather than indirectly addressed through adjustments to other taxes. Recently, Kopczuk (2003) claims to generalize this environmental principle of targeting by establishing that the optimal tax formula for pollution-generating goods can be decomposed into the Pigouvian tax plus a correction tax / subsidy.

The present paper considers the relevance of these claims for environmental tax policy. Of particular note, we observe that Sandmo considers the rather specialized case of differentiated commodity taxes. In contrast, in the real world, most products are subject to a uniform commodity tax, and an emissions tax would then be applied to pollution-generating goods on top of the uniform commodity tax. Given this tax structure, we question whether the appearance of an environmental principle of targeting does not in fact arise simply from the increase in the number of tax instruments, with the addition of the emissions tax. Instead, if the principle of targeting is to be meaningful, it would have to hold in a same-numbers exercise – *i.e.* in a comparison where the number of instruments was not changed.

To study these issues, we consider three perfectly competitive markets, one of which produces a "clean" good without pollution by-product, and the other two produce "dirty" goods with pollution by-product. The government collects tax revenues from all three markets to finance an exogenous public expenditure. The optimal taxation is determined by maximizing social welfare.

In Section 2 of the paper, we re-examine the additivity property in the context of differentiated commodity taxes. Sandmo obtains the results by leaving the marginal cost of public funds (MCPF) unsolved in the tax formula. However, he overlooks the fact that the MCPF also depends on the pollution externality. Once the MCPF is precisely solved, the externality will appear in the tax formulae for both clean and dirty goods. In addition, the externality cannot be additively separable in the tax formula for dirty goods. Therefore, Sandmo's additivity property is not valid even in the presence of differentiated taxes.

In Section 3 of the paper, we study whether the principle of targeting is valid in the case with a uniform commodity tax on all goods, and an emissions tax applied only to the dirty goods on the top of the uniform commodity tax. It is shown that, when the government revenue is funded by both taxes, Sandmo's additivity property is further weakened as the emissions externality appears in the tax formulae for both the commodity tax and the emission tax, even if the MCPF remains unsolved. Furthermore, the emissions tax is unlikely to follow the first-best Pigouvian form (marginal social damage).

In Section 4, we consider a same-numbers exercise where only one tax – *i.e.* either the uniform commodity tax or the emissions tax – finances government spending and corrects for pollution. It is found that the uniform commodity tax will induce higher social welfare than the emissions tax when the marginal social damage of the pollutant is low, and the result is reversed when the marginal social damage is high. In other words, in this same-numbers exercise, it is not true that it is always better to address the pollution externality directly through a dedicated emissions tax. Therefore we conclude that there does not exist an environmental principle of targeting which is distinct from the benefit of adding an additional tax instrument.

The paper is organized as follows. Sections 2 and 3 introduce models with three differentiated output taxes, and a uniform commodity tax with an additional emission tax respectively. Section 4 compares social welfare when only the uniform commodity tax or the emissions tax is available. Section 5 concludes.

2 Three differentiated output taxes $(\theta_1, \theta_2, \theta_3)$

Suppose there are three perfectly competitive industries producing one clean good q_1 and two dirty goods q_2 and q_3, where $q_i, i = 1, 2, 3$ is the quantity. The productions of dirty goods generate pollution while the clean good does not. Each good is sold at the price p_i and produced at the constant marginal cost c_i; furthermore, each good is subject to a unit output tax θ_i. The demand for each good is assumed to be linear: $p_i = a_i - q_i$. Each dirty good industry's emission level is assumed to be in proportion to output: $E_2 = e_2 q_2$, $E_3 = e_3 q_3$, where $e_2 > 0$, $e_3 > 0$ are emission intensities. The total social damage from pollution is defined to be $D = \xi(E_2 + E_3)$, where $\xi > 0$ is the marginal social damage.

Perfect competition requires price equal to marginal cost plus the tax rate:

$$p_i = c_i + \theta_i \tag{1}$$

Then, the equilibrium quantities are found to be:

$$
\begin{aligned}
q_i &= a_i - p_i \\
&= a_i - c_i - \theta_i
\end{aligned} \tag{2}
$$

For simplicity, we define the market size $a_i - c_i$ to be s_i.[1] Rewriting (2), we have

$$q_i = s_i - \theta_i \tag{3}$$

Note that in order to have an interior solution, it must be the case that $\theta_i < s_i$.

The goal of the government is to choose the output taxes θ_i to maximize social welfare, subject to the government budget constraint. Social welfare is composed of consumer surpluses plus firms' profits (which are zero under perfect competition and constant returns to scale) and tax revenues, net of pollution damage.

The consumer surplus and tax revenue in each industry are given by

$$CS_i = \frac{1}{2} q_i^2 \tag{4}$$

$$TR_i = \theta_i q_i \tag{5}$$

Then, social welfare can be expressed as

$$SW = \sum_{i=1}^{3} (CS_i + TR_i) - D \tag{6}$$

[1]The market size is given by q_i, but since the slope of the inverse demand is (-1), we have $q_i = a_i - c_i$ (in the absence of taxation).

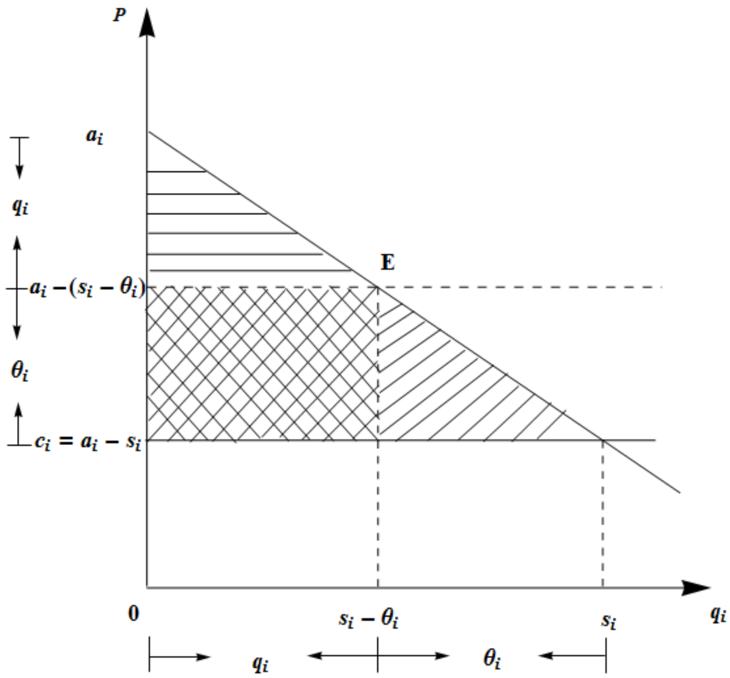

Figure 1: Social welfare for the i^{th} good

Figure 1 illustrates the solution in the market for the i^{th} good. Equilibrium in the presence of the tax occurs at point E on the inverse demand curve. The upper shaded triangle represents consumer surplus derived from the good. The hatched rectangle represents the revenue raised from taxation of the i^{th} good, and the lower shaded triangle represents the corresponding deadweight loss of the tax. Algebraically, the deadweight loss is equal to $\frac{1}{2}\theta_i^2$.

The government needs to generate revenues to cover a budget equal to B:

$$\sum_{i=1}^{3} TR_i = B \tag{7}$$

which (in general) will induce it to choose strictly positive output taxes. In the absence of a budget constraint, the government would impose an output tax on the dirty goods to internalize the environmental externality, but, it would not impose tax on the clean good, since this tax generates deadweight loss which the government wishes to avoid. The total deadweight loss in this problem equals the aggregate across the three markets, *i.e.* $\frac{1}{2}(\theta_1^2 + \theta_2^2 + \theta_3^2)$.

The existence of a solution to (7) depends on the revenue requirement, B, being not too big. The maximum amount of tax revenue that can be raised in market i is $\frac{s_i^2}{4}$.[2] Therefore, the existence of a solution to (7) requires that $B \leqslant \sum_{i=1}^{3} \frac{s_i^2}{4}$.

The optimal taxes are chosen by the government to maximize social welfare subject to the budget constraint:

$$\max_{\theta_i} \; SW \tag{8}$$
$$s.t. \sum_{i=1}^{3} TR_i = B$$

The Lagrangian is

$$\begin{aligned} L &= \sum_{i=1}^{3}(CS_i + TR_i) - D + \alpha(\sum_{i=1}^{3} TR_i - B) \\ &= \sum_{i=1}^{3}[\frac{1}{2}(s_i - \theta_i)^2 + \theta_i(s_i - \theta_i)] - \xi[e_2(s_2 - \theta_2) + e_3(s_3 - \theta_3)] + \alpha[\sum_{i=1}^{3}\theta_i(s_i - \theta_i) - B] \end{aligned} \tag{9}$$

Focusing on interior solution for θ_i, the four first-order necessary conditions are

$$\frac{\partial L}{\partial \theta_1} = \alpha s_1 - (1 + 2\alpha)\theta_1 = 0 \tag{10}$$

$$\frac{\partial L}{\partial \theta_2} = \alpha s_2 - (1 + 2\alpha)\theta_2 + e_2\xi = 0 \tag{11}$$

$$\frac{\partial L}{\partial \theta_3} = \alpha s_3 - (1 + 2\alpha)\theta_3 + e_3\xi = 0 \tag{12}$$

$$\frac{\partial L}{\partial \alpha} = \sum_{i=1}^{3} TR_i - B = 0 \tag{13}$$

[2] This result is obtained by substituting (3) into (5), maiximizing with respect to θ_i, and then evaluating (5) at the resulting tax rate.

Rewriting (10) to (12), we get

$$\theta_1 = \frac{s_1}{1+2\alpha}\alpha \tag{14}$$

$$\theta_2 = \frac{s_2}{1+2\alpha}\alpha + \frac{e_2\xi}{1+2\alpha} \tag{15}$$

$$\theta_3 = \frac{s_3}{1+2\alpha}\alpha + \frac{e_3\xi}{1+2\alpha} \tag{16}$$

Equations (14) to (16) resemble the tax expressions in Sandmo (1975). In the absence of a pollution externality – *i.e.* with $\xi = 0$ – the commodity taxes for all goods exhibit a similar structure, which only depend on individual market sizes. In the presence of a pollution externality – *i.e.* with $\xi > 0$ – the marginal damage and emission intensities only appear in the tax formulae for dirty goods, and they (in the form of products) are additively separable from the commodity tax portions of the expressions (*i.e.* the first terms). Thus, these results suggest an additivity property, as in Sandmo.

Furthermore, the elasticity of demand for good i can be derived from the demand function $p_i = a_i - q_i$:

$$\varepsilon_i = -\frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i} = \frac{c_i + \theta_i}{s_i - \theta_i} \tag{17}$$

which gives

$$s_i = \frac{c_i + (1+\varepsilon_i)\theta_i}{\varepsilon_i} \tag{18}$$

Substituting (18) into (14) to (16) demonstrates

$$\theta_1 = \frac{c_1}{(1+\alpha)\varepsilon_1 - \alpha}\alpha \tag{19}$$

$$\theta_2 = \frac{c_2}{(1+\alpha)\varepsilon_2 - \alpha}\alpha + \frac{e_2\xi}{1+2\alpha} \tag{20}$$

$$\theta_3 = \frac{c_3}{(1+\alpha)\varepsilon_3 - \alpha}\alpha + \frac{e_3\xi}{1+2\alpha} \tag{21}$$

Clearly, (19) to (21) shows that the differentiated output taxes follow the Ramsey inverse elasticity rule: the optimal tax rates and the elasticity of demand should be inversely related.

Since the price elasticity of linear demand diminishes as we move down the inverse curve, the Ramsey rule entails that the optimal tax rate varies directly with market sizes s_i, as shown in (14) to (16).

However, (14) to (16) do not represent the final solutions for θ_i, as the Lagrangian multiplier α is also an endogenous variable. Substituting (2), (14), (15) and (16) into (13) and solving yields[3]

$$\alpha = \frac{\sqrt{s_1^2 + s_2^2 + s_3^2 + 4\xi[e_2(\xi e_2 - s_2) + e_3(\xi e_3 - s_3)]}}{2\sqrt{s_1^2 + s_2^2 + s_3^2 - 4B}} - \frac{1}{2} \tag{22}$$

Equation (22) illustrates that the marginal cost of public funds (Lagarangian multiplier α) is a function of the pollution externality. Sandmo exaggerates the additivity property by ignoring this connection. Thus, the optimal taxes indeed depend on the externality, even for the clean good, notwithstanding the appearance of additivity in (14) to (16). More precisely, we explicitly solve θ_i by substituting (22) into (14) to (16):

$$\theta_1^* = \frac{s_1}{2}(1 - \frac{\sqrt{s_1^2 + s_2^2 + s_3^2 - 4B}}{\sqrt{s_1^2 + s_2^2 + s_3^2 + 4\xi[e_2(\xi e_2 - s_2) + e_3(\xi e_3 - s_3)]}}) \tag{23}$$

$$\theta_2^* = \frac{s_2}{2} - \frac{(s_2 - 2e_2\xi)\sqrt{s_1^2 + s_2^2 + s_3^2 - 4B}}{2\sqrt{s_1^2 + s_2^2 + s_3^2 + 4\xi[e_2(\xi e_2 - s_2) + e_3(\xi e_3 - s_3)]}} \tag{24}$$

$$\theta_3^* = \frac{s_3}{2} - \frac{(s_3 - 2e_3\xi)\sqrt{s_1^2 + s_2^2 + s_3^2 - 4B}}{2\sqrt{s_1^2 + s_2^2 + s_3^2 + 4\xi[e_2(\xi e_2 - s_2) + e_3(\xi e_3 - s_3)]}} \tag{25}$$

The appearance of (ξ, e_2, e_3) in (23) to (25) lead to the following result.

Proposition 1 *Sandmo's additivity property does not hold under differentiated commodity taxes, as the pollution externality appears in the tax formulae for both the clean and the dirty goods, and it is not additively separable in the tax formulae for the dirty goods.*

It is possible to decompose the differentiated taxes on the dirty goods in order to isolate a role for the first-best pollution tax on the dirty goods. First, define $\widehat{\theta}_2 \equiv \theta_2^* - e_2\xi$, $\widehat{\theta}_3 \equiv \theta_3^* - e_3\xi$.

[3] (22) is the positive root of a quadratic expression. Since α represents the marginal cost of public funds (in welfare units), it must be positive.

Then, θ_2^* and θ_3^* can be decomposed as follows:

$$\theta_2^* = \widehat{\theta}_2 + e_2\xi \tag{26}$$

$$\theta_3^* = \widehat{\theta}_3 + e_3\xi \tag{27}$$

These expressions indicate that one government agency (*e.g.* the environment ministry) could apply the first-best pollution tax on the dirty goods (*i.e.* the Pigouvian tax ξ multiplied by e_2 and e_3) without jeopardizing the optimality of the tax system, provided there was another agency (*e.g.* the finance ministry) who could apply a corrective tax or subsidy, $\widehat{\theta}_2$ and $\widehat{\theta}_3$. This result echoes Kopczuk (2003) and follows directly from the additivity of taxes on the dirty goods.

However, there is less to this result than it appears. First, any linear decomposition of θ_2^* and θ_3^* is possible, not just one based on the Pigouvian tax. Second, in practical terms, this result amounts to little more than saying that one department can choose any dirty-goods tax it wants, including one based on the Pigouvian rule, as long as there is a second department which will apply the necessary correction. This is hardly a serious recommendation for policy.

3 A uniform commodity tax τ with an additional emissions tax t

We now turn to the model where a uniform commodity tax is applicable. The model is under the same setting except that all goods face a uniform per-unit output tax τ, and emissions from the two dirty goods are charged a tax t.[4]

Perfect competition requires price to be equal to marginal cost plus the tax burden(s)

$$p_1 = c_1 + \tau \tag{28}$$

[4]With 3 goods (1 clean and 2 dirty goods) and 2 taxes, we can see the difference between the 2 tax systems. If there are only two goods (1 clean and 1 dirty goods) and 2 taxes, then the two tax systems are identical.

$$p_j = c_j + \tau + te_j,\ j = 2, 3 \tag{29}$$

Consequently, the equilibrium quantities are

$$q_1 = s_1 - \tau \tag{30}$$

$$q_j = s_j - \tau - te_j \tag{31}$$

Furthermore, for an interior solution, (30) and (31) indicate that $s_1 > \tau$ and $s_j > \tau + te_j$. It then follows that $s_j > \tau$ and $s_j > te_j$.

Define the output tax revenue to be TR_q and emissions tax revenue to be TR_e:

$$TR_q = \tau \sum_{i=1}^{3} q_i \tag{32}$$

$$TR_e = t \sum_{j=2}^{3} E_j \tag{33}$$

Note that the commodity tax is applied in all three markets, while the emissions tax is applied only in the dirty goods markets. The social welfare function is defined as

$$SW(\tau, t) = \sum_{i=1}^{3} CS_i + TR_q + TR_e - D \tag{34}$$

The optimal taxes are chosen to maximize social welfare. We consider both the constrained optimization, where the total tax revenue must equal the budget requirement B, as well as unconstrained optimization. We also consider the possibility of corner solutions for the tax rates, *i.e.* $t = 0$ or $\tau = 0$. Formally, in the case of the constrained optimization, the problem is

$$\max_{\tau, t}\ SW(\tau, t) \tag{35}$$

$$s.t.(i)\ TR_q + TR_e = B, (ii)\ \tau \geqslant 0,\ \text{and}\ (iii)\ t \geqslant 0$$

Then, the Lagrangian function for this problem is

$$\begin{aligned} L &= \sum_{i=1}^{3} CS_i + TR_q + TR_e - D + \lambda(TR_q + TR_e - B) \qquad (36) \\ &= \frac{1}{2}[(s_1-\tau)^2 + \sum_{j=2}^{3}(s_j-\tau-te_j)^2] + \tau[(s_1-\tau) + \sum_{j=2}^{3}(s_j-\tau-te_j)] + (t-\xi)\sum_{j=2}^{3}[e_j(s_j-\tau-te_j)] \\ &\quad +\lambda\{\tau[(s_1-\tau) + \sum_{j=2}^{3}(s_j-\tau-te_j)] + t\sum_{j=2}^{3}[e_j(s_j-\tau-te_j)] - B\} \end{aligned}$$

The unconstrained optimization (i.e. no revenue constraint) is a special case of this problem for which $\lambda = 0$.

The possibility of zero and non-zero values for all three variables λ, τ and t yields eight different cases, of which only four are of practical interest. In particular, we consider (i) $\lambda = 0, \tau = 0$ and $t > 0$, (ii) $\lambda > 0, \tau > 0$ and $t > 0$, (iii) $\lambda > 0, \tau > 0$ and $t = 0$, and (iv) $\lambda > 0, \tau = 0$ and $t > 0$. The Kuhn-Tucker conditions for the problem are

$$\frac{\partial L}{\partial \tau} = -3\tau + (-6\tau + s_1 + s_2 + s_3)\lambda + (e_2 + e_3)(t - \xi + 2t\lambda) \leqslant 0 \qquad (37)$$
$$\text{and } \frac{\partial L}{\partial \tau} * \tau = 0$$

$$\frac{\partial L}{\partial t} = (e_2^2 + e_3^2)(-t + \xi - 2t\lambda) - e_2[\tau + (2\tau - s_2)\lambda] - e_3[\tau + (2\tau - s_3)\lambda] \leqslant 0 \qquad (38)$$
$$\text{and } \frac{\partial L}{\partial t} * t = 0$$

$$\frac{\partial L}{\partial \lambda} = \tau[(s_1-\tau) + \sum_{j=2}^{3}(s_j-\tau-te_j)] + t\sum_{j=2}^{3}[e_j(s_j-\tau-te_j)] - B = 0 \qquad (39)$$

We consider now the four cases.[5]

[5] The condition (39) is an equality since the budget constraint, when there is one, must hold with equality.

3.1 Case 1: $\lambda = 0, \tau = 0$ and $t > 0$

In this case where only the emissions tax corrects the externality and there is no revenue requirement, the only first-order necessary condition (derived from 38) is

$$\frac{\partial L}{\partial t} = (e_2^2 + e_3^2)(-t + \xi) = 0 \tag{40}$$

Given e_2, $e_3 > 0$, (40) yields

$$t = \xi \tag{41}$$

Equation (41) is just the standard first-best solution, where the optimal emissions tax equals the Pigouvian tax (marginal social damage rate).

3.2 Case 2: $\lambda > 0, \tau > 0$ and $t > 0$

In this case, both uniform commodity tax and emissions tax contribute to government expenditure. The existence of a solution to $TR_q + TR_e = B$ depends on the revenue requirement, B, being not too big. The maximum amount of tax revenue that can be raised under present assumptions is $\frac{1}{36}[9s_1^2 - \sum\limits_{j=2}^{3} \frac{4(1+(-3+e_j)e_j)(1+e_j^2)s_j^2}{e_j^2}]$.[6] Therefore, it is necessary that $B \leqslant \frac{1}{36}[9s_1^2 - \sum\limits_{j=2}^{3} \frac{4(1+(-3+e_j)e_j)(1+e_j^2)s_j^2}{e_j^2}]$.

From (37) and (38), we get

$$\tau^* = \frac{(s_1 + s_3)e_2^2 - (s_2 + s_3)e_2e_3 + (s_1 + s_2)e_3^2}{2(1 + 2\lambda)(e_2^2 - e_2e_3 + e_3^2)}\lambda \tag{42}$$

$$t^* = \frac{(2s_2 - s_1 - s_3)e_2 + (2s_3 - s_1 - s_2)e_3}{2(1 + 2\lambda)(e_2^2 - e_2e_3 + e_3^2)}\lambda + \frac{\xi}{1 + 2\lambda} \tag{43}$$

(42) and (43) establish that Sandmo's additivity property is even further weakened in the presence of the uniform commodity tax even without solving for the marginal cost of public funds λ. Different from differentiated taxes, even in the absence of marginal social damage –

[6] This result is obtained by substituting (30) and (31) into (32) and (33), maximizing with respect to τ and t, and then evaluating (32) and (33) at the resulting tax rate.

i.e. $\xi = 0$, the emissions intensities e_2 and e_3 emerge in the expressions of the commodity tax and the emissions tax. Therefore, the externality affects both optimal taxes. Additionally, (43) illustrates that the externality is not additively separable in that the emissions intensities appear in the first term and the social damage ξ appears in the second term (as in (15) and (16)). Moreover, λ can be obtained by substituting (42) and (43) into (39)[7]

$$\lambda = \frac{1}{2}(-1 + \frac{\sqrt{A+C}}{2\sqrt{C}}) \tag{44}$$

where $A = 8(e_2^2 - e_2e_3 + e_3^2)\{B + \xi[(\xi e_2 - s_2)e_2 + (\xi e_3 - s_3)e_3]\}$, $B = [(s_1 + s_3)^2 + 2(s_2^2 - 4B)]e_2^2 - 2[s_2^2 - s_2s_3 + s_3^2 + s_1(s_2 + s_3) - 4B]e_2e_3 + [(s_1 + s_2)^2 + 2(s_3^2 - 4B)]e_3^2$. Obviously, the marginal cost of public funds is a function of the externality, as in the previous case.

Moreover, (43) and (44) demonstrate that in general the optimal emissions tax does not follow the Pigouvian tax (tax equal to marginal social damage). The only special case where it does follow the Pigouvian tax is when

$$\xi = \frac{(2s_2 - s_1 - s_3)e_2 + (2s_3 - s_1 - s_2)e_3}{4(e_2^2 - e_2e_3 + e_3^2)} \tag{45}$$

by substituting (44) into (43), and equating t^* with ξ.

(42) to (45) lead to the following result.

Proposition 2 (*i*) *Sandmo's additivity property is further weakened under the combination of a uniform commodity tax and an emissions tax, as the emissions intensities appear in the tax formulae for both the commodity tax and the emission tax.* (*ii*) *The Pigouvian tax is unlikely to apply on the dirty goods.*

The next two cases involve the use of a single tax instrument – either the commodity tax, τ or the emissions tax, t – to meet the revenue requirement. The discussion of these cases will be taken up in the subsequent section.

[7] Again, we drop the negative root.

3.3 case 3: $\lambda > 0, \tau > 0$ and $t = 0$

In this case, the regulator only uses the uniform commodity tax to collect revenue and control pollution. Once again, the existence of a solution to $TR_q = B$ depends on the revenue requirement, B, being not too big. The maximum amount of tax revenue that can be raised under present assumptions is $\frac{(s_1+s_2+s_3)^2}{12}$.[8] Therefore, the existence of a solution for $TR_q = B$ requires that $B \leqslant \frac{(s_1+s_2+s_3)^2}{12}$.

The two first-order necessary conditions from (37) and (39) become

$$\frac{\partial L}{\partial \tau} = -3\tau + \xi(e_2 + e_3) + (-6\tau + s_1 + s_2 + s_3)\lambda = 0 \tag{46}$$

$$\frac{\partial L}{\partial \lambda} = \tau[(s_1 - \tau) + \sum_{j=2}^{3}(s_j - \tau)] - B = 0 \tag{47}$$

The revenue constraint (47) determines τ and condition (46) then determines λ. Hence τ is obtained directly from (47)

$$\tau = \frac{1}{6}(s_1 + s_2 + s_3 - \sqrt{(s_1 + s_2 + s_3)^2 - 12B}) \tag{48}$$

and thus λ from (46)

$$\lambda = \frac{1}{2}[-1 + \frac{\sqrt{[-2\xi(e_2 + e_3) + s_1 + s_2 + s_3]^2}}{\sqrt{(s_1 + s_2 + s_3)^2 - 12B}}] \tag{49}$$

We note that $(s_1 + s_2 + s_3)^2 - 12B \geqslant 0$ in both these expressions by virtue of the assumption that $B \leqslant \frac{(s_1+s_2+s_3)^2}{12}$.

In this case, the commodity tax is used to indirectly control pollution.

[8] This result is obtained by substituting (30) and (31) into (32), maiximizing with respect to τ, and then evaluating (32) at the resulting tax rate.

3.4 Case 4: $\lambda > 0, \tau = 0$ and $t > 0$

In this case, only the emissions tax contributes to the government revenue and corrects the externality. The maximum amount of tax revenue that can be raised under present assumptions is $\frac{(e_2 s_2 + e_3 s_3)^2}{4(e_2^2+e_3^2)^2}$.[9] Therefore, the existence of a solution for $TR_e = B$ requires that $B \leqslant \frac{(e_2 s_2 + e_3 s_3)^2}{4(e_2^2+e_3^2)^2}$.

The two first-order necessary conditions developed from (38) and (39) become

$$\frac{\partial L}{\partial t} = (e_2^2 + e_3^2)(-t + \xi) + [e_2 s_2 + e_3 s_3 - 2t(e_2^2 + e_3^2)]\lambda = 0 \tag{50}$$

$$\frac{\partial L}{\partial \lambda} = t \sum_{j=2}^{3} [e_j(s_j - te_j)] - B = 0 \tag{51}$$

As in the previous case, the revenue constraint (51) determines the optimal tax value, t. Condition (50) then determines λ. Hence, t is obtained directly from (51)

$$t = \frac{e_2 s_2 + e_3 s_3 - \sqrt{(e_2 s_2 + e_3 s_3)^2 - 4B(e_2^2 + e_3^2)}}{2(e_2^2 + e_3^2)} \tag{52}$$

and thus λ from (50)

$$\lambda = \frac{1}{2}\left(\frac{\sqrt{|2\xi(e_2^2 + e_3^2) - (e_2 s_2 + e_3 s_3)|}}{\sqrt{(e_2 s_2 + e_3 s_3)^2 - 4B(e_2^2 + e_3^2)}} - 1\right) \tag{53}$$

We note that $(e_2 s_2 + e_3 s_3)^2 - 4B(e_2^2 + e_3^2) > 0$ in both these expressions by virtue of the assumption that $B \leqslant \frac{(e_2 s_2 + e_3 s_3)^2}{4(e_2^2+e_3^2)^2}$.

(52) illustrates that the second-best emissions tax depends upon the emissions intensities but not on pollution damage (*i.e.* ξ does not appear). This follows from the budget constraint, since the government still has to finance its expenditure target B.

[9] This result is obtained by substituting (31) into (33), maiximizing with respect to t, and then evaluating (33) at the resulting tax rate.

4 Comparison between $SW(\tau, 0)$ and $SW(0, t)$

Proposition 2 indicates that there does not exist a principle of targeting in environmental taxation. Nonetheless, the idea of using an emissions tax to directly address a pollution externality remains appealing. There are at least two reasons which may explain this appeal. First, the logic of the first best solution (the Pigouvian tax) is so compelling and transparent that we tend to transform it into a rule of thumb, which we then apply universally, even in cases when it is not appropriate.

Second, and perhaps more important, is the observation that two tax instruments can never be worse than one and in many cases will prove to be better, in terms of social welfare. Since a uniform commodity tax already exists in most jurisdictions, the choice is whether to adjust the commodity tax to reflect pollution damages or to add an emissions tax to address pollution emissions directly, while maintaining the commodity tax. Since two instruments are better than one, the second choice is preferable. Thus, we arrive at a result which looks rather like a principle of targeting but which in fact is due to the additional degree of freedom which follows from adding another tax instrument.

It follows from this discussion that an environmental principle of targeting would only be meaningful when (*i*) we are in a second best setting, and (*ii*) we must choose between the same number of different instruments rather than between say n instruments and $n+1$ instruments, where the $n+1^{th}$ instrument is an emissions tax. To illustrate, we consider the same distortions as before, *i.e.* the pollution externality and the lack of a lump-sum tax to meet the revenue requirement. In terms of the instruments, we consider a choice between using the commodity tax on its own and the emissions tax on its own (*i.e.* the same number of different instruments). These choices correspond with Case 3 and Case 4 respectively, in the previous section.

To demonstrate the absence of a principle of targeting, we must be able to show that, for some parameter values, the commodity tax will be preferred to the emissions tax, despite the presence of the pollution externality. Substituting (48) into (34) and setting $t = 0$ yields the value of social welfare using the commodity tax only, *i.e.*

$$\begin{aligned} & SW(\tau, 0) \\ = \ & \frac{1}{2}(s_1 - \tau^*)^2 + \frac{1}{2}(s_2 - \tau^*)^2 + \frac{1}{2}(s_3 - \tau^*)^2 + B - \xi[e_2(s_2 - \tau^*) + e_3(s_3 - \tau^*)] \end{aligned} \tag{54}$$

Then substituting (52) into (34) and setting $\tau = 0$ yields the value of social welfare using the emissions tax only, *i.e.*

$$\begin{aligned} & SW(0, t) \\ = \ & \frac{1}{2}(s_1)^2 + \frac{1}{2}(s_2 - \widehat{t}_2)^2 + \frac{1}{2}(s_3 - \widehat{t}_3)^2 + B - \xi[e_2(s_2 - \widehat{t}_2) + e_3(s_3 - \widehat{t}_3)] \end{aligned} \tag{55}$$

where $\widehat{t}_2 \equiv te_2$ and $\widehat{t}_3 \equiv te_3$. Taking the difference between (54) and (55) yields

$$\begin{aligned} & SW(\tau, 0) - SW(0, t) \\ = \ & \frac{1}{2}(\widehat{t}_2^2 + \widehat{t}_3^2) - \frac{3}{2}\tau^2 - \xi[e_2(\widehat{t}_2 - \tau) + e_3(\widehat{t}_3 - \tau)] \end{aligned} \tag{56}$$

where we have exploited the fact that $TR_q = TR_e = B$.

Note that the first two terms in (56), *i.e.* $\frac{1}{2}(\widehat{t}_2^2 + \widehat{t}_3^2) - \frac{3}{2}\tau^2$, represent the difference in deadweight loss between the emissions tax and the commodity tax (see the discussion of the calculation of deadweight loss in section 2). Conventional wisdom suggests that the deadweight loss under the emissions tax will be larger than under the commodity tax, since the emissions tax raises the same revenue from a narrower tax base, *e.g.* two markets rather than three, under present assumptions. For the same reason, we should expect the emissions tax rate to be higher than the commodity tax rate, *i.e.* $\widehat{t}_2 > \tau$ and $\widehat{t}_3 > \tau$.

If true, then inspection of (56) indicates that $SW(\tau, 0) - SW(0, t) > 0$ and the commodity tax will be preferred, for a sufficiently small value of pollution damage, ξ. In contrast, for a high value of ξ, $SW(\tau, 0) - SW(0, t) < 0$ and the pollution tax will be preferred. We summarize as follows.

Proposition 3 *If* $\frac{1}{2}(\widehat{t}_2^2 + \widehat{t}_3^2) > \frac{3}{2}\tau^2$ *and* $\widehat{t}_2, \widehat{t}_3 > \tau$*, then there exists a value of pollution damage* $\overline{\xi}$ *such that* $SW(\tau, 0) - SW(0, t) > 0$ *for* $0 < \xi < \overline{\xi}$ *and* $SW(\tau, 0) - SW(0, t) < 0$ *for* $\xi > \overline{\xi}$*.*

The practical significance of this result is that there is no presumption in favour of the emissions tax, notwithstanding the existence of a pollution externality. Rather, when faced with a choice between the emissions tax alone and the commodity tax alone, the preferred choice depends upon parameter values. Stated differently, there does not exist any principle of targeting in environmental taxation which holds that we should always prefer to target the pollution externality directly, when choosing between the same number of different instruments.

Proving that the deadweight loss is greater under the emission tax ($i.e.$ $\frac{1}{2}(\widehat{t}_2^2 + \widehat{t}_3^2) > \frac{3}{2}\tau^2$) and that emission tax rates are greater than the commodity tax rate ($i.e.$ $\widehat{t}_2, \widehat{t}_3 > \tau$) is complicated by the fact that, for a uniform emissions tax, t, the corresponding dirty-good taxes, $\widehat{t}_2$ and $\widehat{t}_3$, are differentiated by exogenous differences in the emission intensities, e_2 and e_3. Nonetheless, in order to show the failure of the principle of targeting, we need to find only one case where the commodity tax is preferable to the emissions tax.

The simplest case which can be proven analytically involves only one dirty good and markets of equal size. Consider then the case where goods 1 and 2 are clean goods and good 3 is the only dirty good ($i.e.$ $e_2 = 0, e_3 > 0$), and where all three markets are of equal size ($s_1 = s_2 = s_3 = s$). The revenue requirement under the emissions tax is then

$$\widehat{t}_3(s - \widehat{t}_3) = B \tag{57}$$

and under the commodity tax it is

$$3\tau(s - \tau) = B \tag{58}$$

The deadweight loss of the emissions tax is now $\frac{1}{2}\widehat{t}_3^2$, and the welfare differential is

$$\begin{aligned} & SW(\tau,0) - SW(0,t) \\ = \quad & \frac{1}{2}\widehat{t}_3^2 - \frac{3}{2}\tau^2 - \xi e_3(\widehat{t}_3 - \tau) \end{aligned} \tag{59}$$

The existence of solutions to (57) and (58) depends on B being not too big. The maximum amount of tax revenue that can be raised by the emission tax is $\frac{s^2}{4}$, while the maximum amount of revenue that can be raised by the commodity tax is $\frac{3s^2}{4}$.[10] Since the first amount is lower, it provides the upper bound on the permissible value of B, *i.e.* $B \leqslant \frac{s^2}{4}$.

First, we will prove that the emission tax rate $\widehat{t}_3$ exceeds the commodity tax, *i.e.* $\widehat{t}_3 > \tau$. Rearranging (57) and (58) yields quadratic equations

$$\widehat{t}_3^2 - s\widehat{t}_3 + B = 0 \tag{60}$$

and

$$\tau^2 - s\tau + \frac{B}{3} = 0 \tag{61}$$

Solving for the smallest roots (*i.e.* the left-hand side of the Laffer curve), we obtain

$$\widehat{t}_3 = \frac{s - \sqrt{s^2 - 4B}}{2} \tag{62}$$

and

$$\tau = \frac{s - \sqrt{s^2 - \frac{4B}{3}}}{2}$$

These expressions are well defined given the assumed upper-bound on B (*i.e.* $B \leqslant \frac{s^2}{4}$). It follows that $\widehat{t}_3 > \tau$, since $s^2 - 4B < s^2 - \frac{4B}{3}$.

To prove that the deadweight loss is greater under the emissions tax in this case, we consider a thought experiment in which an emissions tax, $\widehat{t}$, is imposed on the one polluting good, while at the same time a uniform commodity tax, $\widehat{\tau}$, is imposed on the two clean goods

[10] See above for the details of caculating the maximum obtainable tax revenue.

but not on the dirty good. In this case, the revenue constraint is

$$2\widehat{\tau}(s-\widehat{\tau})+\widehat{t}(s-\widehat{t})=B \tag{63}$$

The case of an emissions tax alone which we have studied above corresponds with values $\widehat{\tau}=0$ and $\widehat{t}=\widehat{t}_3$.

We now consider tradeoffs between $\widehat{\tau}$ and $\widehat{t}$ which keep the revenue constraint satisfied. Taking the differential of (63) yields

$$2(s-2\widehat{\tau})\ d\widehat{\tau}+(s-2\widehat{t})\ d\widehat{t}=0 \tag{64}$$

which, upon rearranging, gives the tradeoff

$$d\widehat{\tau}=-\frac{s-2\widehat{t}}{2(s-2\widehat{\tau})}d\widehat{t} \tag{65}$$

The deadweight loss of taxation in this case is

$$DWL=\widehat{\tau}^2+\frac{1}{2}\widehat{t}^2 \tag{66}$$

The change in deadweight loss which can obtained by shifting the tax burdens between $\widehat{\tau}$ and $\widehat{t}$ is given by

$$dDWL=2\widehat{\tau}\ d\widehat{\tau}+\widehat{t}\ d\widehat{t} \tag{67}$$

Starting with $\widehat{\tau}=0$ and $\widehat{t}=\widehat{t}_3$, we have

$$dDWL=\widehat{t}\ d\widehat{t} \tag{68}$$

which is negative for a tax shift from $\widehat{t}$ to $\widehat{\tau}$, *i.e.* for $d\widehat{t}<0$ and $d\widehat{\tau}>0$. This result verifies the conventional wisdom that expanding from a narrow tax base to a broader tax base, given B, will decrease aggregated deadweight loss, since the measure of deadweight loss is quadratic in the tax rate.

We wish to characterize the combination of $\widehat{t}$ and $\widehat{\tau}$ which minimize deadweight loss. Setting $dDWL = 0$ in (67) and substituting for $d\widehat{\tau}$ from (65), we obtain

$$\widehat{t}\, d\widehat{t} = 2\widehat{\tau}\frac{s - 2\widehat{t}}{2(s - 2\widehat{\tau})}d\widehat{t} \tag{69}$$

which reduces to $\widehat{t} = \widehat{\tau}$. In other words, starting from a position of the emissions tax alone ($\widehat{t} = \widehat{t}_3$ and $\widehat{\tau} = 0$), we can reduce the deadweight loss by reducing $\widehat{t}$ and increasing $\widehat{\tau}$, following (65), until we reach a position of equality, $\widehat{t} = \widehat{\tau}$, at which point the deadweight loss is minimized. But this outcome is none other than the uniform commodity tax on all three goods, *i.e.* $\widehat{t} = \widehat{\tau} = \tau$. It follows that the deadweight loss of the tax system is smaller under the uniform commodity tax than under the emissions tax alone. This result, combined with $\widehat{t}_3 > \tau$, verifies the assumption required for Proposition 3 in the simple case of one dirty good and markets of equal size.

This exercise quickly becomes intractable for more complex cases involving differentiated market sizes and emissions intensities. Nonetheless, we can easily test numerical parameters values to verify the applicability of Proposition 3. For example, if we assume the parameters are uniformly distributed over the intervals $100 \leqslant s_1 \leqslant 400$, $200 \leqslant s_2 \leqslant 500$, $50 \leqslant s_3 \leqslant 300$, $0 \leqslant e_2 \leqslant 10$, $0 \leqslant e_3 \leqslant 5$ and let ξ vary, then the results from running Monte Carlo experiments 1000 times by randomly choosing the values are shown in Table 1.[11] Obviously, the higher the value of ξ, the more likely that $SW(\tau, 0) - SW(0, t) < 0$, which echoes Proposition 3.

Table 1: Monte Carlo Experiment Results

	$\xi = 1$		$\xi = 3$		$\xi = 5$		$\xi = 10$		$\xi = 20$		$\xi = 50$	
$SW(\tau, 0) - SW(0, t)$	P	N	P	N	P	N	P	N	P	N	P	N
Number of results	801	199	646	354	588	412	408	592	244	756	92	908

[11] P denotes $SW(\tau, 0) - SW(0, t) > 0$ and N represents $SW(\tau, 0) - SW(0, t) < 0$.

5 Conclusion

The "principle of targeting" in environmental taxation provides policy makers an easily implemented rule to curb pollution. Dixit (1985) first refers to this principle, based on Bhagwati and Johnson (1960), that externality-generating sources should be directly targeted, and Sandmo (1975) that the externality only appears in the tax formulae for polluting goods and it is additively separable. Kopczuk (2003) further generalizes this principle to state that the first-best Pigouvian tax should be directly imposed on polluting sources in the second best, provided a corrective tax or subsidy is also applied. However, Sandmo's additivity property is based upon differentiated taxes. In reality, governments do not always have enough instruments to correct each distortion. More precisely, consumption goods are usually subject to a uniform commodity tax, and an emissions tax would be charged on top of that. Thus, it would be important to investigate whether the principle of targeting still remains valid under such a tax system.

The current paper establishes that the principle of targeting is unlikely to be valid under either differentiated taxes or a uniform commodity tax with an additional emissions tax. We first reexamine Sandmo's additivity property under differentiated taxes. It is found that Sandmo exaggerates such property by overlooking the explicit value of the marginal cost of public funds (MCPF). Given that the MCPF depends on pollution damage, the externality also appears in the tax formulae for the non-polluting goods. Furthermore, the externality is no long additively separable in the tax formulae for the polluting goods. The additivity property is further weakened under a uniform commodity tax since the externality appears in the tax formulae for the non-polluting goods even without solving for the MCPF, since the uniform tax is jointly chosen from all three markets.

We also compare the social welfare when only one tax instrument – either the uniform commodity tax or the emissions tax – is available to the regulator. It is demonstrated that the emissions tax should only be employed when the marginal social damage is relatively high; otherwise, the uniform commodity tax needs to be imposed to generate higher social welfare. Therefore, it is not always true that a direct approach to controlling emissions is preferable when the same number of different instruments is compared.

Finally, linear decomposition of the optimal tax on dirty goods and the optimal tax on emissions are possible, as argued by Kopczuk (2003), but they hardly present a meaningful role for the first-best Pigouvian rule in the second best. We conclude that a principle of targeting does not exist in environmental taxation.

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