

SIMILITUDE IN HEAT EXCHANGE PROCESSES  
AT A WATER SURFACE IN A NON-UNIFORM FLOW

by

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Submitted in partial fulfillment  
of the requirements for the degree of  
Master of Applied Science

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University of Ottawa

Ottawa, Canada

July, 1972

## PREFACE

The complexity of problems such as that of ice formation as the result of heat transfer processes in rivers or streams, particularly the boundary geometry, requires model tests for practical work. The proper method of simulation is the goal of this study.

The purpose, after theoretical studies and derivation of equations, is to relate four of six dimensionless numbers involved, two having been omitted for simplification. More exactly, it is the study of a Nusselt number versus a Froude number for constant value of Reynolds and Reynolds x Prandtl number, which is also called Peclet number. To accomplish this goal a flume thirty-two feet long, six feet wide and fifteen inches in height with an air recirculating device in order to get a measurable drop of the water temperature over the length available was built for the study. From this gradient of water temperature and from the longitudinal dispersion coefficient found by using Fisher's method, it is possible to calculate the surface conductivity and the dimensionless numbers required.

This method is applied to several water flows and to six channel widths in order to cover a range of Froude numbers from 0.050 to 0.160.

### ACKNOWLEDGEMENTS

The author wishes to express sincere gratitude to Dr. R. G. Warnock of the Department of Civil Engineering, University of Ottawa, for his very thoughtful guidance and helpful criticism in this project.

In addition, the author wishes to thank Mr. J. Earl, technician of the Department of Civil Engineering, University of Ottawa for the realization of some equipment and Mr. J.M. Maillard for his help in some of the experiments conducted during the night.

The research reported herein was funded through grants from the Arts Council of Canada and funds held by Dr. R. G. Warnock from the National Research Council of Canada.

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NOTATIONS - apply to Chapter 3 and up

A	- cross section area
$A^{\circ}$	- dimensionless area
B	- dimensionless number
b	- channel width
C	- dimensionless number
$C_p$	- specific heat of water
$C_p^{\circ}$	- dimensionless specific heat of water
$C_{p_0}$	- reference specific heat
D	- longitudinal dispersion coefficient
$D^{\circ}$	- dimensionless longitudinal dispersion coefficient
$D_0$	- reference molecular diffusivity
d	- depth of water
$d_i$	- depth of water at the beginning of the $i^{\text{th}}$ vertical slice
E	- emissivity of water
$E_x$	- turbulent diffusion coefficient in the x direction
$E_y$	- turbulent diffusion coefficient in the y direction
$E_z$	- turbulent diffusion coefficient in the z direction
$E^{\circ}$	- dimensionless emissivity
$E_0$	- reference emissivity
$E_{zi}$	- transfer coefficient between the $i-1^{\text{th}}$ and the $i^{\text{th}}$ vertical slice
$e_a$	- ambient atmospheric vapor pressure
$e_{as}$	- saturated air vapor pressure at the air temperature

$e_{ws}$	- saturated air vapor pressure at the water temperature
$F$	- Froude number
$F'$	- heat exchange term
$F''$	- heat exchange term
$F'^{\circ}$	- dimensionless heat exchange term
$F''^{\circ}$	- dimensionless heat exchange term
$F_o$	- reference energy
$g$	- acceleration of gravity
$H$	- surface conductivity of water
$H^{\circ}$	- dimensionless surface conductivity of water
$H_o$	- reference surface conductivity
$h$	- vertical direction
$h'$	- cloud height
$h^{\circ}$	- dimensionless vertical distance
$K_o$	- reference thermal conductivity
$k$	- Von Karman constant
$k_e$	- exchange coefficient
$L$	- latent heat of vaporization
$L_o$	- reference length
$l$	- channel width
$l^{\circ}$	- dimensionless channel width
$N$	- Nusselt number
$n$	- number of vertical slices
$P$	- Prandtl number
$P_r$	- atmospheric pressure
$p$	- pressure

$p^{\circ}$	- dimensionless pressure
$Q$	- water flow
$Q_a$	- incoming longwave atmospheric radiation
$Q_b$	- back longwave radiation from the water surface
$Q_e$	- evaporative flux
$Q_c$	- convective flux
$Q_s$	- heat stored in water body
$Q_{ar}$	- reflected longwave atmospheric radiation
$Q_{LW}$	- net longwave radiation
$Q_{sw}$	- net shortwave radiation
$q'(z)$	- depth integrated water velocity
$R$	- Reynolds number
$R_b$	- Bowen ratio
$R_H$	- hydraulic radius
$RH$	- relative humidity
$r$	- rate of heat exchanged at water surface
$S$	- slope of the energy gradient
$T$	- temperature
$\bar{T}$	- mean time averaged water temperature at a point
$T''$	- variation in water temperature with distance
$T_a$	- air temperature
$T_d$	- instantaneous water temperature
$T_{\ell}$	- Lagrangian time scale
$T_o$	- reference temperature
$T_w$	- water temperature
$T_a^*$	- air temperature

- $T_a^o$  - dimensionless air temperature
- $T_w^*$  - water temperature
- $T_w^o$  - dimensionless water temperature
- $t$  - time
- $t^o$  - dimensionless time
- $t_o$  - reference time
- $U$  - mean cross sectional water velocity
- $U^*$  - shear velocity
- $U^o$  - dimensionless velocity
- $U_o$  - reference velocity
- $u$  - instantaneous point velocity of a fluid particle  
in the x direction
- $\bar{u}$  - mean time averaged velocity in the x direction
- $\tilde{u}$  - instantaneous velocity of an ensemble average of  
particles
- $\bar{u}'$  - cross sectional mean velocity
- $u'$  - downstream relative water velocity
- $u''$  - variation in water velocity with distance
- $u_i$  - mean water velocity in the  $i^{\text{th}}$  vertical slice
- $V$  - wind velocity
- $v$  - instantaneous point velocity of a particle in the  
y direction
- $\bar{v}$  - mean time averaged velocity in the y direction
- $w$  - instantaneous point velocity of a particle in the  
z direction
- $\bar{w}$  - mean time averaged velocity in the z direction

- X - distance
- $X_c$  - length of convective period
- x - cartesian coordinate
- y - cartesian coordinate
- z - cartesian coordinate
  
- $\alpha$  - thermal conductivity of water
- $\Delta_z$  - width of a vertical slice
- $\nu$  - kinematic molecular viscosity of water
- $\nu^0$  - dimensionless kinematic molecular viscosity of water
- $\nu_o$  - reference-kinematic viscosity
- $\rho$  - density of water
- $\rho^0$  - dimensionless density of water
- $\rho_o$  - reference density
- $\sigma$  - Stephan Boltzmann constant
- $\sigma^0$  - dimensionless Stephan-Boltzmann constant
- $\sigma_o$  - reference constant
- $\tau$  - time

## CHAPTER 1

### INTRODUCTION

Very often engineers are interested in studies dealing with ice formation problems as the result of heat transfer processes. For instance, one among many could be the location of nuclear power stations near a stream so that, using a great deal of water used for cooling it, this warm water could keep a certain length of channel or river free of ice during winter time.

Many thermal pollution problems, either to approach the true conditions or for sake of simplicity, involve stratified fluid conditions because of the lower density of the high-temperature effluent. In many river surveys, it is reported that the mixing is sufficient enough to get a vertical gradient of temperature almost equal to zero; this means that quite a lot of moving water bodies, in nature, are well-mixed and homogeneous.

Similitude is a convenient way of studying something in laboratory instead of going to the field. The chief requirement to be able to do so is to build a model so that both model and prototype have the same dimensionless numbers.

In the case studied here; that is to say thermal pollution, the complexity of the problem must be pointed out,

particularly the boundary geometry in the prototype which requires model tests for practical work. The proper method of simulation is the goal of this study which would apply primarily in rivers or streams where the water is well-mixed and homogeneous.

In this study, the derivation of equations showed that this problem involved six dimensionless numbers. Four of the six were related; the last two, because of their minor importance have been neglected for simplification. Applying Fisher's method of calculation for the longitudinal dispersion coefficient from flow conditions and measuring the gradient of the water temperature, it was possible to calculate the dimensionless numbers involved. This produced graphs: Nusselt number versus Froude number for several constant values of Reynolds and Peclet numbers (Reynolds x Prandtl numbers).

All the equations obtained were of the exponential type. These relations are given in the Discussion of Results. Another relation was worked out including all the data but ignoring the variation with the Peclet number. The resulting equation is also given in the Discussion of Results.

## CHAPTER 2

### PREVIOUS INVESTIGATIONS

#### 2.1 General

Numerous investigations of solar radiation have been carried out based on simple empirical methods or thermodynamic principles for estimates of heat transfer at lake surfaces. Very few such investigations exist for rivers.

In this chapter, these investigations are briefly discussed and summarized.

Heat budgets of several kinds have been devised as a means of expressing aspects of the thermal regimes of water bodies. The older more classic type of heat budget is that devised by Birge, and consequently called the "Birgean heat budget". Birgean heat budgets are primarily concerned with the division of annual heat input between winter and summer. More informative, and much more difficult, are the so-called "analytical heat budgets". These attempts, first, to determine the amounts of heat received from the sun, sky, atmosphere, environment and influent waters, and second, to follow the attrition of this heat by several sources of heat loss until it is possible to arrive at the heat storage in the water.

The analytical heat budgets lead to the well-

known energy-balance equation or energy-budget equation.

One form is

$$Q_{sw} \pm Q_{lw} \pm Q_e \pm Q_c = Q_s$$

where  $Q_{sw}$  = net short-wave radiation  
 $Q_{lw}$  = net long-wave radiation  
 $Q_e$  = heat used in evaporation or heat gained  
because of condensation (heat of  
evaporation)  
 $Q_c$  = heat gained or lost by convection  
 $Q_s$  = heat stored in water body.

Note that all components can be plus or minus, i.e., heat can be added or subtracted to the surface except for short-wave radiation (radiant energy from the sun) which always adds energy to the surface.

Schmidt (46)<sup>1</sup> was the first to apply this method in 1915 and then Sverdrup (47) in 1940 tried to predict oceanic evaporation. Such estimates were prepared on an annual basis, so that changes in heat storage could be assumed zero and because data on radiation were limited.

The main problem is to determine the value of each component.

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<sup>1</sup>Numerals in parentheses refer to corresponding items in the List of References.

2.2 Estimation of Evaporation

Many empirical or semi-empirical equations have been developed to estimate evaporation from free water surfaces. Most of them are based on Dalton's law with modifications for factors affecting evaporation. They all have the common feature that the evaporation is a function of the difference in vapor pressure between water and air.

Dalton (10) stated that evaporation is proportional to the deficit in vapor pressure.

Listed herein are some popular equations:

<u>Name</u>	<u>Date</u>	<u>Equation</u>
Dalton (10)	1802	$E = c(e_w - e_1)$
Fitzgerald (19)	1886	$E = \psi(e_w - e_1) \quad \psi = 0.4 + 0.199 w$
Meyer (34)	1915	$E_3 = c(e_{w3} - e_3)\psi_3 \quad \psi_3 = 1 + 0.1 w_2$
Horton (27)	1917	$E = 0.4(\psi_2 e_w - e_1) \quad \psi_2 = 2 - e^{-0.2 w}$
For large areas, E is multiplied by $(1-P) + P \frac{\psi_2 - 1}{\psi_2 - h}$		
Rohwer (43)	1931	$E = 0.771 (1.465 - 0.0186B)\psi_1 (e_w - e_1)$ with $\psi_1 = 0.44 + 0.118 w$
Harbeck and others (Lake Hefner) (23)	1954	$E = 0.00177 (e_{w2} - e_2) w$
Harbeck (24) Kohler Kober and others (Lake Mead)	1958	$E_2 = 0.001813 w_1 (e_{w2} - e_2)^t [1 - 0.03(T_a - T_w)]$

where:

B = mean barometric reading, in in. Hg at 32°F

- C = coefficient dependent upon various bodies factors affecting evaporation, for example, C = 15 for small, shallow water bodies and C = 11 for large, deep water bodies in Meyer's equation
- e = base of natural logarithms
- $e_3$  = actual vapor pressure in air based on monthly mean air temperature and relative humidity at nearby stations for small bodies of shallow water or based on information about 30 ft above water surface for large bodies of deep water
- $e_2$  = mean vapor pressure of saturated air at temperature of dew point, in mb
- $e_1$  = mean vapor pressure of saturated air at temperature of dew point, in in. Hg.
- $E_3$  = rate of evaporation in in. per 30 days month
- $E_2$  = rate of evaporation in in. per t days
- E = rate of evaporation in in. per 24 hr
- h = relative humidity
- P = fraction of time during which wind is turbulent
- t = number of days in period for evaporation
- $T_a$  = average air temperature,  $^{\circ}\text{C} + 1.9^{\circ}\text{C}$
- $T_w$  = average water-surface temperature,  $^{\circ}\text{C}$
- $w_2$  = monthly mean wind velocity, in mph, at about 30 ft above ground.
- $w_1$  = monthly mean wind velocity near surface of ground in knots

w = monthly mean wind velocity near surface of ground  
in mph

$e_{w3}$  = maximum vapor pressure, in Hg, corresponding to  
monthly mean air temperature observed at nearby  
stations for small bodies of shallow water, or  
corresponding to water temperature for large bodies  
of deep water

$e_{w2}$  = mean vapor pressure at water surface temperature  
in mb

$e_w$  = mean vapor pressure at water surface temperature  
in in. Hg

Penman (39) developed from evaporation-pan studies a formula  
giving the evaporation rate cm/24 hr:  $E_o$

$$E_o = 0.35 \Delta_e \left( 0.5 + \frac{U}{100} \right)$$

where  $\Delta_e$  = vapor pressure difference in mm of Hg

U = average wind speed per day.

In deriving his equation from evaporation pans,  
Penman pointed out the fact that results are different  
according to the location of the pan. These are called  
mid-desert and mid-ocean effects.

Another well accepted formula for calculating daily  
evaporation rates was given by Kohler (31), which, when  
adjusted for use with wind velocities measured at 2 meters  
gives:

$$E = [(0.525)(10^{-2}) + (1.229)(10^{-2})v_a](e_{sw} - e_a)$$

where  $E$  = evaporation rate  
 $v_a$  = average wind velocity at the 2 m level  
 $e_{sw}$  = saturation vapor pressure at the temperature  
of the water surface  
 $e_a$  = vapor pressure of the air at the 2 m level.

### 2.3 Estimation of Convection

Bowen (5) developed an expression relating the convective component to the evaporation component called the Bowen's Ratio:

$$R = \frac{Q_c}{Q_e} = K_1 \frac{\Delta T}{\Delta e}$$

where  $Q_c$  = convective component  
 $Q_e$  = evaporation component  
 $\Delta T$  = difference between water surface temperature  
and air temperature  
 $K_1$  = coefficient depending upon air pressure  
 $\Delta e$  = water vapor pressure difference between the  
air at water surface temperature and at air  
temperature.

This equation is derived on the assumption that the eddy diffusivities of water vapor ( $K_w$ ) and heat ( $K_h$ ) in air are equal. Examination of literature reveals uncertainty about the conditions under which this assumption holds true. Pasquill (38), Veihmeyer (54) and Fritschen (20) all reported

satisfactory results using that assumption when atmospheric conditions are neutral and stable. Anderson (2) also concluded as a result of the Lake Hefner studies that little error appears to be introduced in the estimates as the result of neglecting the effects of atmospheric stability.

Brutsaert (7) cites one report in which  $K = K_h/K_w$  was found equal to unity regardless of atmospheric stability, and another where the ratio was found equal to unity except under highly stable conditions, where it increased to 2 or 3. In unstable conditions, ( $T_w > T_a$ ), Pasquill (38) reported that K could be as large as 3. A similar increase in the value of K with an increase in ( $T_w - T_a$ ) was noted by Roll (44). Yen and Landvatter (56) from an experimental study of the evaporation of warm water into cold air found a ratio up to 2.5. However, Rhimsha and Donchenko (42) successfully used the Bowen ratio to calculate winter time heat losses from calorimeters and open river reaches with important differences in temperatures.

So, though the general validity of this ratio has been the subject of much discussion, and though there appears to be limitations to its accuracy and general use, it is widely used.

Sverdrup developed a formula for daily totals:

$$Q_c = \frac{C_p \cdot \rho \cdot K_o^2 \cdot U (T_{air} - T_{surf}) \times K_1}{\ln\left(\frac{a}{z_o}\right) \ln\left(\frac{b}{z_o}\right)}$$

where  $\rho$  = density of air  
 $z_0$  = roughness of surface  
 $K_0$  = Von Karman coefficient  
 $U$  = daily wind speed  
 $T_{\text{air}}$  = air temperature  
 $K_1$  = coefficient  
 $T_{\text{surf}}$  = surface temperature  
 $a, b,$  = constants.

Budyko (8) gave an empirical formula for monthly or yearly value:

$$Q_c = 14.6 \Delta T^{1.2}$$

where  $\Delta T$  = difference between surface and air temperature.  
Another approach used by Rimsha and Donchenko (42) assumes that the evaporation would be a relation of the form:

$$Q_c = (a + bv_a)(T_w - T_a)$$

where  $a, b$  are constants  
 $v_a$  = air velocity  
 $T_w$  = water temperature  
 $T_a$  = air temperature.

The authors determined the wind function parameters by least squares analyses of data from calorimeters exposed adjacent to natural ice-free reaches. The "constant" in the wind function was found to vary with  $(T_w - T_a)$ . Thus, after adjustment for application to natural reaches (based on actual

comparison of heat losses from rivers and calorimeters), they wrote:

$$Q_c = (K_n + 3.9 v_a)(T_w - T_a)$$

where  $K_n$  is empirically found to be:

$$K_n = 8.0 + 0.35 (T_w - T_a).$$

The parameter  $K_n$  thus represents the portion of heat transfer due to free convection.

#### 2.4 Estimation of Net Shortwave Radiation

Shortwave radiation is that associated with sunlight and is made up of wave-lengths shorter than  $5\mu$ . So, shortwave radiation is received by a surface during daylight hours only. When shortwave radiation strikes a surface, part of it is absorbed and part is reflected upward. The percentage of incoming shortwave radiation reflected is called the "albedo". In literature it is found the range for water albedo given to be from 0.05 to 0.30 [Anderson (2)]. At a point outside the atmosphere of the earth, radiant energy is received from the sun on a surface normal to its rays at a rate of approximately  $430 \text{ Btu/hr/ft}^2$ . This value is called the "solar constant".

The Weather Bureau, United States Department of the Interior (USWR), has collected measurements of solar radiation, and Moon (37) developed standard radiation curves and tables for engineering use. These curves give the intensity

of direct solar radiation on a surface perpendicular to the sun's rays and of the total radiation on a horizontal surface, including reflected radiation from the sky and clouds, at various stations throughout the United States.

Other methods exist to estimate the shortwave radiation magnitude. Usually these consist of tables, graphs, or formulae relating percentage of possible sunshine or cloud cover to the percentage of possible solar radiation that reaches the ground; see for example:

Anderson (2)

List (32), Tables 151, 152

Jensen and Haise (29).

Bolsenga (4) has provided tables of daily sums of cloudless sky shortwave radiation for various altitudes and air mass conditions that can be used as a basis for such estimates.

A well-known empirical formula from Mosby (35) permits the computation of the incoming radiation on a horizontal surface in terms of average solar altitude and average cloudiness:

$$Q_s = k(1 - 0.071 c) \alpha$$

where  $Q_s$  = the solar radiation  
 $k$  = a constant which is function of latitude  
 $c$  = the average cloud cover in tenths of sky covered  
 $\alpha$  = the average altitude of the sun in degrees.

This equation states that minimum insolation is 29% of the maximum and that insolation varies linearly with cloud cover.

Fritz (21) studied the effect of cloud cover. He observed that, although insolation varied linearly with  $s$ , the percentage of sunshine, it did not vary linearly with  $c$ , the cloud cover expressed in tenths. Fritz ascribed much of the difference to the fact that the sunshine recorder generally ignored high cirriform clouds that tend to reduce insolation by relatively small amounts. From limited data that he published on observations at Washington, D.C., Phoenix, Arizona, and Fresno, California, a curvilinear relationship between insolation and cloud cover can be found:

$$Q_i = (1 - 0.0071 c^2)(Q_s - Q_r)$$

where  $Q_i$  = net shortwave insolation including effect of cloud cover in Btu/hr-ft<sup>2</sup>

$c$  = the cloud cover, expressed as tenths of sky covered

$Q_s$  = incoming solar radiation

$Q_r$  = reflected solar radiation.

Using the same idea, Angstrom in List's reference (32) suggested the following relation:

$$Q_{R1} = Q_{cL} [0.35 + 0.061(10-c)]$$

where  $Q_{cL}$  = the incoming shortwave radiation for a cloudless sky

$c$  = cloudiness.

Unfortunately, it is difficult to use Anderson's relation, because the empirical constants must change to correspond to the type of cloud cover. Koberg (30) has suggested a way around this problem by plotting curves for reflected shortwave radiation versus measured shortwave radiation for both clear and cloudy skies. Fitting a polynomial to average values between Koberg's clear and cloudy sky curves leads to the equation:

$$Q_{RR} = 0.108 Q_{RI} - 6.766 \times 10^{-5} Q_{RI}^2$$

Koberg estimates that this should be accurate to within 10%. The amount of energy absorbed by the water is then calculated by:

$$Q_R = Q_{RI} - Q_{RR}$$

where  $Q_{RR}$  = amount of reflected shortwave radiation.

## 2.5 Estimation of Net Longwave Radiation

Net longwave radiation is composed of incoming longwave atmospheric radiation minus the reflected part and minus the back radiation coming from the water body according to the Stephan-Boltzmann law (a body radiates proportional to the fourth power of its absolute temperature).

Atmospheric radiation is principally from water vapor, cloud droplets, carbon dioxide and ozone.

For longwave radiation, water surfaces act very

much like a "black body", they absorb or radiate approximately 90 to 97 percent of the radiation a perfect radiator would when at the same temperature.

The Physical Standards Laboratory of the University of California determined the emissivity of samples of distilled water, Lake Hefner water, Lake Mead water and sea water. It was concluded that the emissivity of water was 0.970, with an estimated accuracy of  $\pm 0.005$ , and that it was independent of water temperature and dissolved solids as reported by Raphael (41).

So, the following equation is found:

$$Q_B = Q_a - Q_{ar} - Q_{bs}$$

where  $Q_b$  = net longwave radiation  
 $Q_a$  = incoming longwave atmospheric radiation  
 $Q_{ar}$  = reflected longwave atmospheric radiation  
 $Q_{bs}$  = back radiation from the water surface

with  $Q_{bs} = 0.970 \sigma T_{\text{water}}^4$

where  $\sigma$  = Stephan-Boltzmann constant.

Duncan et al. (12) studied the atmospheric emissivity constant generally referred to as  $\beta$  and it is dependent on the moisture content of the air and cloud cover. The value of  $\beta$  may be obtained from Burt's study (9) using cloud cover graphs developed by Anderson during Lake Hefner studies. These same studies showed that significant error may be

introduced if reflected atmospheric radiation is ignored. Dunkel et al. fixed this radiation as a constant 3%.

Anderson (2), after reviewing nearly two centuries of research, reached the conclusion that "the atmospheric radiation does not follow any simple law since it is a function of many variables, such as the distribution of moisture, temperature, ozone, carbon dioxide, and perhaps other atmospheric factors as yet unknown". Considering atmospheric radiation with vapor pressure, air temperature, and height and amount of cloud, he proposed the following empirical relationship:

$$\frac{Q_a}{\sigma T_a^4} = a + b e_a = \beta$$

where

$$a = 0.74 + 0.025 c_e^{-0.0584 h}$$
$$b = 0.00490 - 0.00054 c_e^{-0.060 h}$$

$T_a$  = air temperature  
 $Q_a$  = atmospheric radiation  
 $e_a$  = air vapor pressure  
 $h$  = height of clouds  $1600_m < h < \infty$   
 $\epsilon$  = Napierian base  
 $c$  = cloud quantity in tenths of sky covered.

Then, using the relation initially suggested by Brunt (6), he determined a different expression for clear skies and cloudy skies.

For clear skies:  $Q_a = (0.68 + 0.036 \sqrt{e_a}) \sigma T_a^4$

where  $T_a$  = air temperature  
 $e_a$  = air vapor pressure of the air  
 $\sigma$  = Stephan-Boltzmann constant  
 $Q_a$  = atmospheric radiation.

For cloudy skies,  $Q_a$  was estimated by:  $Q_a = (a + b e_a) \sigma T_a^4$

where  $a = 0.740 + 0.025 c. \exp[(-1.92)(10^{-4})H]$ .  
 $b = (4.9)(10^{-3}) - (5.4)(10^{-4}) c. \exp[(-1.97)(10^{-4})H]$ .

and  $500 < H < \infty$

$H$  = cloud height in m.

For cloud heights less than 500 m, the cloud height is taken as a constant at 500 m.

## 2.6 Heat Balance and River Studies

Although heat balance studies of lakes are numerous, very few such investigations exist for rivers. Almost all works which have been done dealing with heat loss disregard flow conditions, only the air temperature and water temperature are taken into account.

Eckel and Reuter (13) who recognized the need for heat balance for rivers were concerned primarily with predicting temperatures in rivers under natural conditions. Therefore, they attempted to develop an equation to estimate the change in temperature of a reach of river or moving parcel of water with time, rather than with distance. They

considered heat budget items associated with short and long wave radiation, evaporation and convection to the atmosphere. Because of the complexity of the total heat balance term, the differential equation they developed could not be solved in closed form, and they used graphical iterative procedures that gave satisfactory agreement between measured and calculated river temperatures, at least for clear summer days.

Pruden et al. (40) studied the heat balance of the St. Lawrence River in an attempt to determine the feasibility of keeping much of the river ice-free for parts of the winter. They considered the meteorological dependence of the various heat balance terms in somewhat more details than did Eckel and Reuter. Because plots of the heat loss rate  $Q^*$  versus air temperature  $T_a$  and versus water temperature  $T_w$  were close to linear (using average monthly values for meteorologic conditions), they wrote:

$$Q^* = 183.6 + 15.5 T_a + 43.1 (T_w - T_a)$$

where  $Q^* = \text{cal cm}^{-2} \text{ day}^{-1}$

$T_w$  and  $T_a = \text{degrees C.}$

This expression of  $Q^*$  was then incorporated in a differential equation for cooling rate with distance. The inverse of their expression can be integrated in closed form, and they found a simple exponential decrease of water temperature with distance. Note that  $Q^*$  was developed for monthly average values at one general location only, and does

not take into account the many meteorologic variables that affect the heat loss rate.

Ince and Ashe (28) used observations from the St. Lawrence River to compare two methods of calculating downstream water temperature decreases. Both of these are based on a finite difference approach for a parcel of water moving at the average velocity of the river:

$$T_{w_2} - T_{w_1} = Q\uparrow / (D\rho_w C_p)$$

where

$Q\uparrow$  = heat loss rate (cal/sec)

$D$  = river discharge ( $\text{cm}^3/\text{sec}$ )

$\rho_w$  = mass density of water ( $\text{g}/\text{cm}^3$ )

$C_p$  = specific heat of water ( $\text{cal}/\text{g } ^\circ\text{C}$ )

$(T_{w_2} - T_{w_1})$  = temperature change of the water parcel over some time period ( $^\circ\text{C}$ ).

In the first method, heat exchanges due to radiation, evaporation, convection, and precipitation were accounted for in evaluating  $Q^*$ , using daily averages of the meteorological variables, but no details of the various expressions were presented. This approach was compared by Ince and Ashe to that using the simple formula

$$Q^* = 46.4 (T_w - T_a)$$

recommended by the Canadian Joint Board of Engineers, where  $Q^*$  is heat loss rate in  $\text{cal}/\text{cm}^2$  day and  $(T_w - T_a)$  is based on daily averages of air and water temperatures in  $^\circ\text{C}$ . Both

approaches gave satisfactory estimates of water temperature under certain conditions.

Although the basic approaches of these earlier studies are sound, and led to some success in river temperature prediction, they are all limited in some respect. The principal limitations lie in the formation of the expressions of heat loss rate  $Q^*$ .

Dingman (11) gives a simple but general expression of equilibrium temperature (it is the water temperature at which net energy exchange to the atmosphere is zero) as a function of meteorologic conditions:

$$T_e = T_a + [(Q_R - Q_0)/q]$$

where  $T_e$  = equilibrium temperature in  $^{\circ}\text{C}$   
 $Q_R$  = net incoming solar radiation in  $\text{cal}/\text{cm}^2 \text{ day}$   
 $Q_0$  = heat loss rate when  $T_w = T_a$  -  $\text{cal}/\text{cm}^2 \text{ day}$   
is a function of the wind velocity  
 $q$  = energy exchange coefficient in  $\text{cal}/\text{cm}^2 \text{ day } ^{\circ}\text{C}$   
 $T_a$  = mean air temperature in  $^{\circ}\text{C}$ .

Again, this requires a period of time long enough to calculate averages and disregard of flow conditions.

## 2.7 Convective Diffusion Equation

The basic equations of diffusion were first given by the physiologist, Fick, and have become known as Fick's laws of diffusion and are analogous to Newton's law of viscosity or Fourier's law of heat.

Transport can be done by two mechanisms. The first is convection, the direct process in which a fluid and any of its properties bodily move from place to place through a flow system. The second is conduction or diffusion, the process of movement of mass or heat or momentum in the direction of decreasing concentration of mass, temperature or momentum.

So, the first law is given by:

$$\frac{1}{A} \frac{dm}{dt} = c \frac{d(m/v)}{ds}$$

where  $\frac{dm/dt}{A}$  = time rate of transport of A per unit area normal to transport direction

c = proportionally constant or diffusivity constant

$\frac{d(m/v)}{ds}$  = gradient of m per unit volume of fluid in the transport direction.

Much early work was done with matter instead of heat because of limited laboratory techniques, but all these equations hold true for heat.

Taylor (49,50), deriving a universal distribution of velocity in a pipe from a mean curve using measurements made by Stanton and Pannell (1914) and by Nikuradse (1932) which conform to Von Karman's logarithmic law near the wall, and also assuming Reynold's analogy to be true (that is to say, transfer of matter, heat and momentum by turbulence are exactly analogous), has shown that under certain circumstances,

dispersion in turbulent flow can be described by the one-dimensional diffusion equation:

$$\frac{\partial c}{\partial t} + U \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2}$$

where  $c$  = mean concentration at a distance  $x$  at time  $t$ .  
 $U$  = mean cross-sectional flow velocity  
 $D$  = longitudinal dispersion coefficient.

This longitudinal dispersion coefficient is an apparent value due to the turbulent shear flow which occurs in conduits and channels. The variation of the velocity  $U$  across the section results in an apparent value of the true coefficient of turbulent diffusion which would occur if the velocity was uniform across the section of flow with turbulent fluctuations.

Taylor also neglected time and gave an expression for  $D$  for circular pipes.

Aris (3) extended Taylor's work to a general cross-sectional diffusion coefficient.

Taylor's concept was applied to an infinitely wide, two-dimensional stream with power-law velocity distribution by Thomas (52).

Then, it was applied to the same flow with logarithmic velocity distribution by Elder (14). He used Taylor's method and found the following equation:

$$D = 20.2 R \sqrt{RSg}$$

where  $R$  = hydraulic radius

S = slope of energy gradient.

g = gravity acceleration.

D = longitudinal dispersion coefficient.

Unfortunately, his result is, practically, not applicable.

Holley and Harleman (26), starting with the equation for conservation of mass and making the usual time-averaging procedure for turbulent fluctuations, obtained as the turbulent convective diffusion equation:

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = \frac{\partial}{\partial x} (\epsilon_x \frac{\partial c}{\partial x}) + \frac{\partial}{\partial y} (\epsilon_y \frac{\partial c}{\partial y}) + \frac{\partial}{\partial z} (\epsilon_z \frac{\partial c}{\partial z})$$

where  $x, y, z$  = coordinate directions in the longitudinal, lateral and vertical directions respectively.  
 $\epsilon_x, \epsilon_y, \epsilon_z$  = mass transfer coefficients in the respective directions.

$u, c$  = time-averaged point values of velocity and concentration (where the time average is taken over a period long enough to average the turbulent fluctuations, but short enough to allow description of the decay of  $c$ ).

Fisher (15) using the equation above and neglecting turbulent diffusion in the x-direction as justified by Taylor and Elder, justified the assumption made by Taylor - dropping time out.

About the validity of Taylor's bulk equation,

Fischer (15) showed that this equation should not be applied to a dispersing tracer cloud until after an initial period called the convective period. Within the convective period the tracer distribution becomes sharply skewed, concentration rising steeply at the downstream end and trailing off slowly upstream.

Fisher (17) measured the length of the convective period, and the Lagrangian time scale was evaluated in terms of bulk channel parameters by the relation:

$$T_{\ell} = \alpha \frac{\ell^2}{rU^*}$$

where  $T_{\ell}$  = Lagrangian time scale  
 $\alpha$  = coefficient  
 $\ell$  = characteristic length of the channel  
 $r$  = hydraulic radius  
 $U^*$  = shear velocity.

Dimensionless time was defined by  $\frac{t}{T_{\ell}}$ .

The duration of the convective period was found by laboratory and numerical experiments to be approximately 6 dimensionless time units by Fischer (17); so that he expressed, in terms of distance downstream from injection of a tracer source, the minimum value to get a good approximation of the one-dimensional dispersion model:

$$L > \beta \frac{\ell^2}{r} \frac{\bar{u}}{U^*}$$

where  $L$  = distance downstream from the source

$\bar{u}$  = mean cross sectional velocity

$\beta$  = coefficient,

with the other terms having the same meaning as before.

## 2.8 Methods to Calculate Dispersion Coefficient from Field Data

Several methods have been developed.

### a) From Tracer Study Data

Taylor's (50) method was based on the properties of the Gaussian distribution, and required measuring only the time during which the concentration was greater than one-half the peak value.

Godfrey and Frederick (22) determined variance of the concentration time curves by comparison with properties of the Pearson type III distribution. This method is sensitive to the exact location of the peak of the distribution, but sometimes yields considerable error.

Holley and Harleman (26) gave another method also based on the properties of the Gaussian distribution. It required a semi-log plot of the experimental data to obtain a straight line. Methods based on the properties of the Gaussian distribution will give incorrect results if applied to skewed concentration distributions, as almost always encountered in tests in natural streams, thus, none of these methods should be used with natural stream data, unless sufficient dimensionless time has passed for the time-distribution curves to become nearly Gaussian.

Thackston, Hays and Krenkel (51) have suggested a method based on a mathematical model but did not recognize the convective period.

The basic relation on which all the previous methods are based is that the dispersion coefficient measures the rate of change of the variance of the tracer cloud, i.e.,

$$D = \frac{1}{2} \frac{d}{dt} \sigma_x^2$$

where  $\sigma_x^2$  = variance of the concentration distribution with respect to distance along the stream.

Fischer (18) developed a procedure termed the routing procedure because of similarity to flood routing. The upstream observed curve is used as the initial tracer distribution, and a concentration-time curve for the downstream station is predicted by the one-dimensional dispersion model. The predicted and observed downstream station curves are compared; if the comparison is not adequate, a new dispersion coefficient is selected, and the calculation is repeated until the best possible comparison is obtained.

b) From Flow Conditions

Elder (14) gave a method for predicting the dispersion coefficient in natural streams, by an application of Taylor's analysis. But it was shown that transverse variations in downstream velocity are the primary producers of dispersion in a natural stream, whereas Elder's analysis had only considered vertical variations.

Fischer (18) gave a new analysis, considering only variations in the transverse (z) direction.

$$D = - \frac{1}{A} \int_0^b q'(z) dz \int_0^z \frac{1}{E_z d(z)} dz \int_0^z q'(z) dz$$

in which  $q'(z) = \int_0^{d(z)} u'(y,z) dy$

where  $u'(y,z)$  = velocity at any point on the cross section relative to the mean flow velocity; i.e.,  $u'(y,z) = u(y,z) - \bar{u}$  in which  $u(y,z)$  = the actual velocity at any point in the cross-section

$d(z)$  = depth of flow at any point in the cross section

$E_z$  = transverse turbulent mixing coefficient

$A$  = area of flow cross-section

$b$  = width of channel

$y$  = vertical cartesian coordinate

$z$  = transverse cartesian coordinate.

For his equation, he used as transverse turbulent mixing coefficient the equation derived by several experiments in uniform flow by Elder (14) and verified by Orlob (36), Sayre and Chang (45) and Fischer (16),

that is to say:  $E_z = 0.23 d U^*$

where  $d$  = depth of water

$U^*$  = shear velocity.

Then, Fischer replaced the integrals by summations. This method has been used herein.

CHAPTER 3

DERIVATION OF EQUATIONS

3.1 Heat Exchange at Water Surface

The heat budget will be derived by means of expressing each term of the energy balance equation under the following form:

$$Q_{sw} + Q_{Lw} + Q_e + Q_c = Q_s \quad (3.1.1)$$

where  $Q_{sw}$  = net shortwave radiation.  
 $Q_{Lw}$  = net longwave radiation.  
 $Q_e$  = heat used in evaporation or heat gained because of condensation.  
 $Q_c$  = heat gained or lost by convection.  
 $Q_s$  = heat stored in water body.

All components can be plus or minus, i.e., heat can be added or subtracted to the surface except for shortwave radiation (radiant energy from the sun) which always adds energy to the surface.

In this study, the plus sign means heat goes out from the water body to the atmosphere, and a minus sign means the opposite.

3.1.1 Evaporation Component:  $Q_e$

Heat may be added or removed by condensation or

evaporation at the surface of a stream. The amount of heat exchanged is a function of the latent heat of vaporization and the vapor pressure gradient at the stream-air interface. The Dalton-type equation below assumes horizontal uniformity of all variables.

Marciano and Harbeck (33) gave in Lake Hefner studies:

$$Q_e = k_e L V (e_{ws} - e_a) \quad (3.1.2)$$

where  $Q_e$  = evaporation flux, Btu/ft<sup>2</sup> min  
 $k_e$  = exchange coefficient  
 $L$  = latent heat of vaporization  
 $V$  = wind speed, miles/hour  
 $e_{ws}$  = saturated vapor pressure at the temperature of the stream, inches of mercury  
 $e_a$  = ambient atmospheric vapor pressure, inches of mercury.

Selection of a suitable exchange coefficient is difficult.

Derived at Lake Hefner, Oklahoma, a coefficient using wind speeds at 8 meters level could not be directly transposed to this study.

Tichenor (53) used wind tunnel measurements to compare several methods for computing evaporation. Measured evaporation was consistently twice that predicted using Lake Hefner coefficient. On this basis, an empirical coefficient

twice that derived at Lake Hefner was incorporated in the evaporation equations which follow.

The error induced by using this new coefficient could not be precisely defined. However, the conditions used in Tichenor's models (i.e., wind tunnel) were thought to approximate more closely those above a small sheltered stream than did the conditions under which Lake Hefner coefficient was derived.

Substituting the empirical value obtained from Tichenor's model studies and 1060 Btu/lb for the latent heat of vaporization, Eq. (3.1.2) becomes:

$$Q_e = 0.6140 V (e_{ws} - e_a) \quad (3.1.3)$$

The ambient atmospheric vapor pressure can be expressed as a function of the saturated air vapor pressure at the same temperature, using the relative humidity of air: RH:

$$e_a = e_{as} \text{ RH} \quad (3.1.4)$$

where  $e_a$  = air vapor pressure at temperature  $t$   
 $e_{as}$  = saturated air vapor pressure at temperature  $t$   
RH = relative humidity of air in percent.

The saturated air vapor pressure can be found: either by Hewson and Longley's formula (25) assuming the latent heat of vaporization to be a constant equal to 595 cal/gr, that is to say, for temperatures close to 273<sup>o</sup>K:

$$e_s = 6.11 \times 10^{(8.573 - \frac{2340}{T})}$$

where  $e_s$  = saturation vapor pressure in mb  
 $T$  = temperature  $^{\circ}K$ ,

or by tables given in any chemistry or physics handbook;  
i.e., see reference (1).

So, Eqs. (3.1.3) and 3.1.4) give, correcting for  
 $e$  in mm Hg:

$$Q_e = 0.02417 V [e_{ws} - (e_{as} \cdot RH)] \quad (3.1.5)$$

where  $Q_e$  = evaporation flux, Btu/ft<sup>2</sup> min  
 $V$  = wind speed, mph  
 $e_{ws}$  = saturated air vapor pressure at the surface  
temperature of the stream, in mm Hg  
 $e_{as}$  = saturated air vapor pressure at the ambient  
air temperature, in mm Hg  
 $RH$  = Relative Humidity of air in percent.

It is to be noticed that, because of the experi-  
mental laboratory: air recirculation, at the equilibrium,  
 $Q_e = 0$ .

### 3.1.2 Convection Component: $Q_c$

Convection occurs at the stream surface. It results  
from boundary layer conduction and subsequent transfer of  
this heat through displacement of masses of fluid. Wind speed  
and the temperature gradient between the air and water are the

driving forces for convective heat transfer at the air-water surface. The roughness of the surface also affects the convection process.

The Bowen ratio  $R_B$  has been used to relate convection to evaporation

$$R_B = \frac{Q_c}{Q_E} = 61 \times 10^{-5} P_r \frac{T_w - T_a}{e_{ws} - e_a} \quad (3.1.6)$$

where  $P_r$  = barometric pressure at the altitude of the water-body

$e_{ws}$  = saturation pressure of water vapor at water surface temperature

$e_a$  = observed pressure of water vapor in the air at the same standard height as the observed air temperature

$T_w$  = water surface temperature

$T_a$  = observed air temperature

$P_r$ ,  $e_{ws}$ ,  $e_a$ , being in the same units

$Q_c$  = convection component

$Q_e$  = evaporation component.

This ratio is derived on the assumption that the eddy diffusivities of water vapor ( $K_w$ ) and heat ( $K_h$ ) in air are equal. Examination of the literature reveals uncertainty about the conditions under which this assumption holds true. (For more information, see Chapter 2 (2.3), p.8).

Substituting  $Q_e$  from Eq. (3.1.3) and correcting for temperature in  $^{\circ}\text{K}$ , it is found:

$$Q_c = 45.81 \times 10^{-5} P_r (T_w - T_a) \quad (3.1.7)$$

where

$Q_c$  = convection flux,  $\text{Btu}/\text{ft}^2 \text{ min}$   
 $45.81 \times 10^{-5}$  = exchange coefficient

$V$  = wind speed, mph

$P_r$  = atmospheric pressure, inches of mercury

$T_w$  = water temperature,  $^{\circ}\text{C}$  (or  $^{\circ}\text{K}$ )

$T_a$  = ambient air temperature,  $^{\circ}\text{C}$  (or  $^{\circ}\text{K}$ ).

If the atmospheric pressure is assumed to be 1 atmosphere at  $32^{\circ}\text{F}$  ( $0^{\circ}\text{C}$ ) Eq. (3.1.7) becomes:

$$Q_c = 137.09 \times 10^{-4} \times V (T_w - T_a) \quad (3.1.8)$$

Each term has the same definition and units as above.

Again, because of laboratory conditions,  $Q_c$  cannot be found using the Bowen ratio because  $Q_e$  will become zero. As explained in Chapter 1, these equations were written in order to get a complete picture of the problem.

### 3.1.3 Net Shortwave Component: $Q_{sw}$

Shortwave radiation is that associated with sunlight and is made up of wave lengths shorter than  $5\mu$ . So, shortwave radiation is received by a surface during daylight hours only. When shortwave radiation strikes a surface, part of it

is reflected upward. The balance is called net shortwave radiation.

In this study a wind tunnel is used above the water surface. Because there is no light inside, net short-wave component will be taken as zero. So:

$$Q_{sw} = 0 \quad (3.1.9)$$

#### 3.1.4 Net Longwave Component: $Q_{Lw}$

Net longwave radiation is composed of incoming long-wave atmospheric radiation minus the reflected part and minus the back radiation coming from the water body according to the Stephan-Boltzmann Law: every body radiates with the fourth power of its absolute temperature. If it is a good absorber it is also a good emitter at the same wave length. A perfect absorber and emitter is called a black-body. Long-wave radiation is made up of wave lengths mostly between 5 to 50 $\mu$ .

So, according to the prescribed meaning of the plus sign:

$$Q_{Lw} = -Q_a + Q_{ar} + Q_b \quad (3.1.10)$$

where

$Q_{Lw}$  = net longwave radiation

$Q_a$  = incoming longwave atmospheric radiation

$Q_{ar}$  = reflected longwave atmospheric radiation

$Q_b$  = back longwave radiation from the water surface.

(a) $Q_b$ , back radiation: as reported by Raphael (41) the emissivity of water, E, is  $0.970 \pm 0.005$ , and it is independent of water, temperature and concentration of dissolved solids.

According to the Stephan-Boltzmann law recalled above, the expression of  $Q_b$  can be written as:

$$Q_b = 0.970 \sigma T_w^4 \quad (3.1.11)$$

where  $Q_b$  = back longwave radiation from the water surface in  $\text{cal/cm}^2 \text{ day}$

0.970 = emissivity of water, E

$\sigma$  = Stephan-Boltzmann constant

$\sigma = 1.171 \times 10^{-7} \text{ cal/cm}^2 \text{ day } ^\circ\text{K}^4$

(or  $\sigma = 1.72 \times 10^{-9} \text{ Btu/ft}^2 \text{ hr } ^\circ\text{K}^4$ )

$T_w$  = water temperature in  $^\circ\text{K}$ .

(b) $Q_{ar}$ , reflected radiation: because the emissivity E of water is 0.970, its longwave reflectivity is 0.03, so that:

$$Q_{ar} = 0.03 Q_a \quad (3.1.12)$$

where  $Q_{ar}$  = reflected longwave atmospheric radiation

$Q_a$  = incoming longwave atmospheric radiation.

This reflectivity value was already reported by Dunkel et al. (12).

(c) $Q_a$ , incoming radiation: Brunt's formula with Anderson's constants will be used.

For cloudy skies; that is to say, with the cloudi-

ness expressed in tenths equal 1.0 and the cloud height  $h'$  less than 500 m,

$$h' = 500 \text{ m}$$

So we get,  $Q_a = \beta \sigma T_a^4$  (3.1.13)

where  $Q_a$  = incoming atmospheric radiation  $\text{cal/cm}^2 \text{ day}$

$T_a$  = air temperature in  $^{\circ}\text{K}$

$\sigma$  = Stephan-Boltzmann constant

$$\beta = a + b e_a,$$

where  $e_a$  = air vapor pressure in mb.

$$a = 0.740 + 0.025 C \cdot \exp[(-1.92)(10^{-4})h'] \quad (3.1.14)$$

$$b = (4.9)(10^{-3}) - (5.4)(10^{-4})C \cdot \exp[(-1.97)(10^{-4})h']$$

Applying these equations with  $C = 1.0$  and  $h' = 500 \text{ m}$ ,

it is found:

$$a = 0.7627$$

$$b = (44.106)(10^{-4})$$

According to Eq. (3.1.4), the air vapor pressure can be related to the saturated air vapor pressure at the same temperature.

Now, writing Eq. (3.1.10) according to Eqs. (3.1.4), (3.1.11), (3.1.12), (3.1.13) it is found:

$$Q_{LW} = E \sigma [T_w^4 - (a + b e_a RH) T_a^4] \quad (3.1.15)$$

Substituting  $E$ ,  $\sigma$ ,  $a$ ,  $b$ , from Eqs. (3.1.11), (3.1.14) and correcting for  $e_a$  in mm Hg and  $Q_{LW}$  in  $\text{Btu/ft}^2 \text{ min}$ , it is found:

$$Q_{LW} = (2.908)(10^{-10}) [T_w^4 - [0.763 + (58.793)(10^{-4})e_{as}.RH]T_a^4] \quad (3.1.16)$$

where  $Q_{LW}$  = net long waves radiations going outside,  
 Btu/ft<sup>2</sup> min  
 $T_w$  = water temperature, °K  
 $T_a$  = air temperature, °K  
 RH = relative humidity of air in percent  
 $e_{as}$  = saturated air vapor pressure at air temperature, mm Hg.

Expanding Eq. (3.1.16) in series, it becomes:

$$Q_{LW} = (2.908)(10^{-10}) \left[ [0.237 - [(58.793)(10^{-4})e_{as}.RH]]T_a^4 + 4T_a^3(T_w - T_a) + 6T_a^2(T_w - T_a)^2 + 4T_a(T_w - T_a)^3 + (T_w - T_a)^4 \right] \quad (3.1.17)$$

When temperature difference is not important, the last three terms can be neglected. For example, when the water temperature is 17°C (62.6°F) and the air temperature 0°C (32°F), this introduces an error of around 3%.

So, Eq. (3.1.17) becomes:

$$Q_{LW} = (2.908)(10^{-10}) \left[ [0.237 - [(58.793)(10^{-4})e_{as}.RH]]T_a^4 + 4T_a^3(T_w - T_a) \right] \quad (3.1.18)$$

### 3.1.5 Heat Budget at Water Surface

According to the heat budget equation under the form given by Eq. (3.1.1), it can be written, using Eqs. (3.1.5), (3.1.8), (3.1.9) and (3.1.17):

$$Q_s = H(T_w - T_a) + F'' + F' \quad (3.1.19)$$

where

$Q_s$  = heat storage in water body

$$H = (137.09)(10^{-4}) V + (2.908)(10^{-10}) 4T_a^3$$

$$F'' = (2.908)(10^{-10}) [0.237 - [(58.793)(10^{-4}) e_{as} \cdot RH]] \\ T_a^4 + (241.7)(10^{-4}) V [e_{ws} - (e_{as} \cdot RH)].$$

$$F' = (2.908)(10^{-10}) [6T_a^2 (T_w - T_a)^2 + 4T_a (T_w - T_a)^3 + (T_w - T_a)^4]$$

Again, for a small temperature difference, the

$F'$  term in Eq. (3.1.10) can be neglected so that:

$$Q_s = H(T_w - T_a) + F'' \quad (3.1.20)$$

where, in both Eqs. (3.1.19) and (3.1.20)

$V$  = air velocity, above water surface, mph

$T_a$  = air temperature, above water surface, °K

$T_w$  = water surface temperature, °K

$e_{ws}$  = saturated air vapor pressure at water surface temperature, mm Hg

$e_{as}$  = saturated air vapor pressure at air temperature, mm Hg

$RH$  = Relative Humidity of air in percent

$H$  = surface conductivity, Btu/sq.ft min °K

$F'', F'$  = heat exchange terms, Btu/sq.ft min.

Looking at these equations, it can be seen that:

-  $F'$  is a term present only when temperature difference is important.

-  $F''$  can be assumed to be independent of the air temperature and water temperature difference because the rate of cooling or

warming is very low so the saturated air vapor pressure at water surface temperature can be taken as a constant over a short length of channel or river.

- H is the surface conductivity. As seen in Chapter 1, this formula gives the first way to get an H value, and as mentioned, it is independent of water flow conditions, but is a function of air conditions only.

For a given air temperature, the corresponding surface conductivity coefficient can be calculated and applied for a temperature range function to the accuracy required.

It is possible to express H in a dimensionless expression called the Nusselt number:

$$N = \frac{HL_o}{\alpha} \quad (3.1.21)$$

where H = surface conductivity, Btu/ft<sup>2</sup> min °K

L<sub>o</sub> = reference length, ft.

α = thermal conductivity of water, Btu/min ft °K.

### 3.2 Mixing Length

In the derivation of the thermal convective diffusion equation in the next paragraph, instantaneous dispersion of heat and no gradient of temperature in the vertical direction will be assumed.

This paragraph discusses the validity of these assumptions.

- First, because the flow is turbulent, it is reasonable to

think that, with turbulent mixing in the vertical direction, combined with secondary effects like secondary currents which help the primary mixing process, the gradient of water temperature in the vertical direction will be zero. In the literature, it can be found that in rivers, the water mass is kept at nearly the same temperature by mixing, e.g. Williams (55).

- Second, to insist on this point of view, Fischer's result (17) can be applied to calculate the length of the convective period, that is to say, the length of river before which Taylor's bulk equation should not be applied.

The heat reaching the water surface can be considered as a multitude of hot wires stretched across the channel, these wires being one beside the other.

Let us consider only one wire:

Dealing with a hot wire stretched across the entire river, the convective period will be the length required to mix in the vertical direction. To calculate this length, the Lagrangian time scale can be used.

Lagrangian time scale:

Following an individual particle, for dispersion to be described by a diffusion equation, it is necessary for the motion of each heated particle not to be correlated with its initial velocity. The time required for this to occur is measured by the Lagrangian time scale defined as

$$T_{\ell} = \int_0^{\infty} \psi(\tau) d\tau \quad (3.2.1)$$

in which

$$\psi(\tau) = \frac{\overline{u(t) u(t+\tau)}}{u(t)^2}$$

Herein  $u$  is the instantaneous point velocity of a fluid particle.

the wavy overbar indicates an ensemble average over a large number of particles.

$\tau$  is any value of time (which may be different from  $t$ ).

Using Taylor's statistical distribution of particles (48) and the relation between the dispersion coefficient and the variance of this statistical distribution of particles given by Fischer (15) which leads to:

$$D = \overline{u^2} \int_0^t \psi(\tau) d\tau$$

where  $D$  = the dispersion coefficient.

Then, because in open-channel flow the mean turbulence is sufficiently small compared to the deviations within the cross section of time averaged velocity, it may be taken (17) that a good approximation is:

$$\overline{u^2} \approx \overline{u'^2}$$

in which the straight overbar indicates cross sectional mean.

Then, increasing  $\tau$  towards infinity, and assuming the integral converges, he gave the relation:

$$D = \overline{u'^2} T_\ell \quad (3.2.2)$$

For the across-channel hot wire system, the Lagrangian time scale  $T_\ell$  can be calculated using Elder's

results (14) for a two-dimensional flow with logarithmic velocity profile down an infinitely wide plane.

Averaging the velocity distribution given by the logarithmic law gives

$$\overline{u'^2} = \left(\frac{U^*}{k}\right)^2 \quad (3.2.3)$$

where  $k$  = von Karman constant

$U^*$  = shear velocity.

Elder's result, as originally derived including the von Karman constant, is

$$D = \frac{0.404}{k^3} d U^* \quad (3.2.4)$$

where  $d$  = depth.

Inserting Eqs. (3.2.3) and (3.2.4) into Eq. (3.2.2) give the Lagrangian time scale as:

$$T_l = \frac{0.404}{k} \frac{d}{U^*} \quad (3.2.5)$$

From laboratory experiments, Fischer (17) found that the dimensionless time  $\frac{t}{T_l}$  should be greater than 6 for convective diffusion equation to apply.

Replacing for convenience, local depth by hydraulic radius and von Karman constant by 0.41, it is found:

$$x_c \geq 5.9 R_H \frac{U}{U^*}$$

where  $x_c$  = length of convective period.

$R_H$  = hydraulic radius

$U$  = mean cross-sectional velocity.

$U^*$  = shear velocity.

This length is always very short, in general for natural rivers,  $x_c$  is less than the half-width.

If now, we consider the wire system, one being beside the other, because the rate of cooling or heating is very low, each wire is practically at the same temperature as the preceding one. This succession of wires represents the net atmospheric radiation, and because of turbulent flow, it can be assumed that there are instantaneous dispersion of heat and no gradient of water temperature in the vertical direction.

### 3.3 Convective Diffusion Equation in Turbulent Shear Flow

The general convective diffusion equation for laminar flow in cartesian form is:

$$\frac{\partial T_d}{\partial t} + u \frac{\partial T_d}{\partial x} + v \frac{\partial T_d}{\partial y} + w \frac{\partial T_d}{\partial z} = \alpha \left[ \frac{\partial^2 T_d}{\partial x^2} + \frac{\partial^2 T_d}{\partial y^2} + \frac{\partial^2 T_d}{\partial z^2} \right] + r \quad (3.3.1)$$

where  $t$  = time.

$x, y, z$  = cartesian coordinates

$u, v, w$  = instantaneous velocities at a point in respectively  $x, y, z$  directions

$T_d$  = instantaneous water temperature at the same point.

$\alpha$  = thermal conductivity

$r$  = rate of heat exchange at water surface.

From this equation, the turbulent flow counterpart can be obtained by making use of an analogy between molecular and turbulent diffusion.

The instantaneous hydrodynamic velocity components can be represented in terms of the sum of a time-averaged and fluctuating velocity, and by a similar manner for temperature. Furthermore, by analogy with Fick's first law, it is assumed that the turbulent flux is proportional to the gradient of the time-averaged temperature, so, it is found:

$$\frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} + \bar{w} \frac{\partial \bar{T}}{\partial z} = \frac{\partial}{\partial x} (E_x \frac{\partial \bar{T}}{\partial x}) + \frac{\partial}{\partial y} (E_y \frac{\partial \bar{T}}{\partial y}) + \frac{\partial}{\partial z} (E_z \frac{\partial \bar{T}}{\partial z}) + [\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} + \frac{\partial^2 \bar{T}}{\partial z^2}] + r \quad (3.3.2)$$

where  $\bar{T}$  = mean time averaged water temperature at a point.

$\bar{u}, \bar{v}, \bar{w}$  = mean time averaged velocities at the same point, in respectively x, y, z directions.

$E_x, E_y, E_z$  = turbulent diffusion coefficients in respectively x, y, z directions.

In turbulent motion, the turbulent diffusion coefficients are many orders of magnitude larger than the molecular diffusion coefficient (thermal conductivity coefficient), so it is usually permissible to neglect the molecular diffusion entirely.

The general three-dimensional convective-diffusion, Eq. (3.3.2) is extremely difficult to solve, even when the

thermal diffusivity is ignored, due to the variable velocity and diffusion coefficients.

Taylor (50), going through an averaging problem related to the cross-section similar to the preceding, used:

$$\bar{u} = U + u''$$

$$\bar{T} = T_w + T''$$

where  $U, T_w$  = average water velocity and water temperature through the cross section

$u'', T''$  = variation in velocity and in temperature with distance,

and using the same Fick's first law analogy, asserted that, although the primarily mechanism for dispersion in shear flow is the variation in convective velocity within the cross-section, the process could be described by a one-dimensional Fickian diffusion equation:

$$\frac{\partial T_w}{\partial t} + U \frac{\partial T_w}{\partial x} = \frac{\partial}{\partial x} \left( D \frac{\partial T_w}{\partial x} \right) + r \quad (3.3.3)$$

where  $D$  = longitudinal dispersion coefficient

$r$  = heat exchange at water surface.

The difference in the longitudinal convective temperature transfer which is associated with the actual velocity distribution and that which is accounted for by the mean velocity is incorporated into the diffusion term. This effect is known as longitudinal dispersion.

Transposing to this problem, including the heat exchange term at water surface and assuming a steady state,

the definitive result for a turbulent shear flow is;

- For any air temperature and water temperature difference,

Eqs. (3.1.19) and (3.3.3) give:

$$\frac{\partial}{\partial x} \left( D \frac{\partial T_w}{\partial x} \right) - U \frac{\partial T_w}{\partial x} - \mu [(T_w - T_a) + \frac{F''}{H} + \frac{F'}{H}] = 0 \quad (3.3.4)$$

with 
$$\mu = \frac{H}{C_p \rho} \times \frac{\ell}{A}$$

where

- $x$  = longitudinal direction.
- $D$  = longitudinal dispersion coefficient
- $U$  = mean cross-sectional water velocity
- $T_w$  = mean cross-sectional water temperature
- $T_a$  = mean air temperature above water surface
- $F'', F'$  = heat exchange terms.

Same mathematical expressions as in Eqs.

(3.1.19) and (3.1.20).

$H$  = surface conductivity.

$C_p$  = specific heat of water.

$\rho$  = density of water

$\ell$  = channel width

$A$  = cross-section area.

From Eq. (3.3.4) is developed:

$$\frac{\partial}{\partial x} \left( D \frac{\partial T_w}{\partial x} \right) - U \frac{\partial T_w}{\partial x} - \mu [(T_w - T_a) + \frac{F''}{H}] - \frac{\mu}{H} E \sigma [6T_a^2 (T_w - T_a)^2 + 4T_a (T_w - T_a)^3 + (T_w - T_a)^4] = 0 \quad (3.3.5)$$

From Eq. (3.3.4) or (3.3.5) it can be seen that the surface conductivity H can be found, and that this time, it will be related to the water flow conditions.

From Eq. (3.3.4), it is found:

$$H = \frac{1}{T_w - T_a} \times \frac{C_p \rho}{l} \left[ \frac{\partial}{\partial x} \left( AD \frac{\partial T_w}{\partial x} \right) - Q \frac{\partial T_w}{\partial x} - \frac{l F''}{C_p \rho} - \frac{l F'}{C_p \rho} \right] \quad (3.3.6)$$

where Q = water flow.

For several flow conditions, in the laboratory, H values can be calculated by measuring the different terms and gradients required.

From Ref. (52), it is found:

$$\sigma = 0.9987158 \text{ g/cm}^3 \text{ at } 17.5^\circ\text{C (63.5}^\circ\text{F)}$$

$$C_p = 1.000 \text{ cal/g at } 17.5^\circ\text{C (63.5}^\circ\text{F) and 1 atmosphere.}$$

Rearranging, Eq. (3.3.6) gives:

$$H_o = \frac{1}{T_w^* - T_a^*} \times \frac{112.228}{L_o} \left[ 60 \frac{d}{dx} \left( AD \frac{dT_w}{dx} \right) - 60 Q \frac{dT_w}{dx} - \frac{L_o}{112.228} (F'' + F') \right] \quad (3.3.7)$$

where F'' = same mathematical expression as in Eq.

(3.1.19), Btu/sq ft min.

F' = same mathematical expression as in Eq.

(3.1.19), Btu/sq ft min.

$T_w^*, T_a^*$  = mean water temperature and air temperature,  $^\circ\text{R}$

$T_w$  = mean water temperature at one section,  $^\circ\text{K}$

$L_o$  = width of the channel, ft

x = distance downstream, ft

- A = cross-sectional area, sq ft  
D = longitudinal dispersion coefficient, sq ft/sec  
 $H_o$  = surface conductivity, Btu/sq ft min  $^{\circ}$ R  
Q = water flow, cubic ft/sec.

As noticed in this chapter, because of the experimental laboratory: air circulation, at the equilibrium  $Q_e = 0$ , so it was not possible to calculate  $H_o$  from the atmospheric conditions, e.g., Eq. (3.1.19), because  $Q_c$  could not be found using the Bowen ratio.

### 3.4 Calculation of Longitudinal Dispersion Coefficient

The semi-intuitive solution by which Taylor (50) obtained the dispersion coefficient for a long straight pipe may also be applied to natural streams. Only one modification is required; whereas in a pipe [Taylor's solution] dispersion is caused by differences in velocity in the radial direction, and in an infinitely wide two-dimensional flow [Elder's solution (14)], the cause is velocity variations from surface of the flow to the bottom; in a natural stream the primary cause of dispersion is differences in velocity in the lateral (transverse) direction. Almost all natural streams contain one area of relatively high velocity either in the center or in a deep portion near one bank, and other areas of relatively slow velocity in shallow areas along one or both banks. The difference in relative velocities in the lateral and vertical directions is the same (i.e., from maximum to zero), but

because most natural streams have width to depth ratios of 10 or greater, the separation between zones of differing velocity is much greater in the lateral than the vertical direction.

Fischer (17) using Taylor's approach, assuming all motion to be in one direction (i.e., neglecting secondary currents) and considering only variations in the transverse direction (z) produced an equation to calculate the longitudinal dispersion coefficient from flow conditions, that is to say, from the knowledge of the channel geometry (width and depth as function of lateral position), cross-sectional distribution of downstream relative velocity and shear velocity:

$$D = - \frac{1}{A} \int_0^b q'(z) dz \int_0^z \frac{1}{E_z d(z)} dz \int_0^z q'(z) dz \quad (3.4.1)$$

in which  $q'(z) = \int_0^{d(z)} u'(y,z) dy$

where  $D =$  longitudinal dispersion coefficient

$A =$  area of the cross-section

$q'(z) =$  depth integrated velocity at point z

$z =$  lateral direction

$d =$  depth

$b =$  width of the channel

$u' =$  downstream relative velocity

$y =$  vertical direction

$E_z =$  lateral turbulent mixing coefficient.

The lateral turbulent mixing coefficient must be found by experiment. Elder (14) in a flow 1 cm deep down a water table found that:

$$E_z = 0.23 dU^* \quad (3.4.2)$$

where  $U^*$  = shear velocity.

Larger scale experiments by Orlob (36), Sayre and Chang (45) as well as a field experiment by Fischer (16) have all yielded values in a good agreement with Elder's formulation.

Later, Fischer (18) gave a methodology to calculate  $D$ , and apply Eq. (3.4.1). He replaced integrals by summations, and, dividing the cross-section in vertical slices arrived at:

$$D = - \frac{1}{A} \sum_{k=2}^n q'_k \Delta z \left[ \sum_{j=2}^k \frac{\Delta z}{E_{zj} d_j} \left( \sum_{i=1}^{j-1} q'_i \Delta z \right) \right] \quad (3.4.3)$$

in which  $q'_i = \frac{1}{2}(d_i + d_{i+1})u'_i$

where  $u_i$  = mean velocity in the  $i^{\text{th}}$  vertical slice

$u'_i = u_i - U$

$U$  = mean velocity of flow within the entire cross-section.

$d_i$  = depth at beginning of the  $i^{\text{th}}$  vertical slice

$\Delta_z$  = width of a vertical slice

$E_{zi}$  = transfer coefficient between the  $i-1^{\text{th}}$  and the  $i^{\text{th}}$  vertical slice. According to Eq.

$$(3.4.2), E_{zi} = 0.23 d_i U^*$$

$n$  = number of vertical slices.

Note that for  $k = 1$ , the quantity within the brackets is arbitrarily assigned a value of zero, as a beginning point for the integration.

The shear velocity may be calculated by:

$$U^* = \sqrt{g \cdot R_H \cdot S} \quad (3.4.4)$$

where  $R_H$  = the hydraulic radius  
 $g$  = the acceleration of gravity  
 $S$  = the slope of the energy gradient.

With the number of cross-sections studied,  
the number of vertical slices,  
the width of each slice, ft,  
the channel width, ft,  
the bottom slope, ft/ft,  
the distance from left bank to start of slice, ft,  
the depth at each left side of slice, ft,  
the mean water velocity through each slice, ft/sec,  
in input, a Fortran IV program was written which gives in output:  
the flow of the channel, cfs,  
the cross-sectional area, sq ft,  
the mean cross-sectional velocity, ft/sec,  
the hydraulic radius, ft,  
the shear velocity, ft/sec,  
the longitudinal dispersion coefficient, sq ft/sec,  
the product of the cross-sectional area times the  
longitudinal dispersion coefficient,

the dimensionless numbers corresponding to the  
similitude

This program is given in Appendix A.

### 3.5 Equations of Similitude

The study of similar fluid motions forms the  
basis for the theory of models.

Basically, equations of similitude are used to  
study prototype from reduced scale systems known as models.  
This is a procedure for arranging a number of variables into  
one or more dimensionless groups.

This report is dealing with dynamic and thermal  
similitudes leading to three systems of equations characteriz-  
ing this study, namely:

the equation of continuity

the equation of motion

the equation of energy

Convention: In this chapter, any "0" index will refer to  
a dimensionless quantity.

Any "0" subscript will refer to a reference  
quantity.

#### 3.5.1 Equation of Continuity

Because water is an incompressible fluid, the  
equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3.5.1.1)$$

where  $u, v, w$  = components of velocity at a point in  
respectively  $x, y, z$  directions.

Eq. (3.5.1.1) may be written in terms of a set of  
dimensionless quantities defined in the following manner:

$$\begin{aligned}x^o &= \frac{x}{L_o} & u^o &= \frac{u}{U_o} \\y^o &= \frac{y}{L_o} & v^o &= \frac{v}{U_o} \\z^o &= \frac{z}{L_o} & w^o &= \frac{w}{U_o}\end{aligned} \tag{3.5.1.2}$$

In the above equations,  $L_o$  and  $U_o$  are constant  
reference values of length and velocity to be chosen as  
characteristics of the system.

If Eqs. (3.5.1.2) are substituted in Eq. (3.5.1.1),  
it is found:

$$\frac{\partial u^o}{\partial x^o} + \frac{\partial v^o}{\partial y^o} + \frac{\partial w^o}{\partial z^o} = 0 \tag{3.5.1.3}$$

This result introduces no similitude conditions.

### 3.5.2 Equations of Motion

Dense fluids, like liquids, have a large heat  
capacity, therefore, temperature changes due to internal  
friction are small. The density and viscosity in these  
cases are very little affected and may be assumed to be  
constant.

So, Navier-Stokes equations valid for an incom-  
pressible Newtonian fluid in a gravity field, with constant

viscosity and density, z direction being the vertical axis, are;

for the x component:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -g \frac{\partial h}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial x} + \gamma \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

for the y component:

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -g \frac{\partial h}{\partial y} - \frac{1}{\rho} \frac{\partial p}{\partial y} + \gamma \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (3.5.1.4)$$

for the z component:

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -g \frac{\partial h}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial z} + \gamma \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

where

$g$  = acceleration of gravity

$h$  = vertical direction measured positive upward

$\rho$  = density

$p$  = pressure

$\gamma$  = kinematic molecular viscosity

$u, v, w$  = components of velocity at a point in respectively x, y, z direction.

Using Eqs. (3.5.1.2) plus the following ones:

$$h^o = \frac{h}{L_o} \quad t^o = \frac{t}{t_o} \quad t = \frac{U_o}{L_o} \quad p^o = \frac{p}{\rho_o U_o^2}$$

$$\gamma^o = \frac{\gamma}{\gamma_o} \quad \rho^o = \frac{\rho}{\rho_o} \quad g = \text{constant}$$

and substituting in Eqs. (3.5.1.4), the x component becomes:

$$\frac{\partial u^o}{\partial t^o} + u^o \frac{\partial u^o}{\partial x^o} + v^o \frac{\partial u^o}{\partial y^o} + w^o \frac{\partial u^o}{\partial z^o} = - \left[ \frac{1}{F^2} \right] \frac{\partial h^o}{\partial x^o} - \frac{\partial p^o}{\partial x^o} + \left[ \frac{1}{R} \right] \left[ \frac{\partial^2 u^o}{\partial (x^o)^2} + \frac{\partial^2 u^o}{\partial (y^o)^2} + \frac{\partial^2 u^o}{\partial (z^o)^2} \right]$$

(3.5.1.5)

where  $F = \frac{U_o}{\sqrt{g L_o}}$  Froude number (gravity effect)

$R = \frac{U_o L_o}{\gamma_o}$  Reynolds number (viscous effect).

The same dimensionless numbers appear for the y and z components.

It is to be noticed that for an incompressible free-surface flow problem, equality of Reynolds numbers and equality of Froude numbers are both necessary for exact dynamic similitude. The majority of hydraulic model studies of open channels and rivers, because of high turbulence, are cases in which fluid motion is important but molecular viscosity effects are negligible. So, since both model and prototype use the same fluid (water), such models are designed on the basis of Froude number similarity alone.

### 3.5.3 Equation of Energy

Eqs. (3.3.4) and (3.3.5) can be written in terms of dimensionless quantities by the same procedure. Using Eqs. (3.5.1.2) and the following ones:

$$\begin{aligned}
 D^{\circ} &= \frac{D}{D_o} & F''^{\circ} &= \frac{F''}{F_o} & l^{\circ} &= \frac{l}{L_o} \\
 T_w^{\circ} &= \frac{T_w}{T_o} & F'^{\circ} &= \frac{F'}{F_o} & A^{\circ} &= \frac{A}{L_o^2} \\
 T_a^{\circ} &= \frac{T_a}{T_o} & \rho^{\circ} &= \frac{\rho}{\rho_o} & H^{\circ} &= \frac{H}{H_o} \\
 U^{\circ} &= \frac{U}{U_o} & C_p^{\circ} &= \frac{C_p}{C_{p_o}} & \sigma^{\circ} &= \frac{\sigma}{\sigma_o} \\
 E^{\circ} &= \frac{E}{E_o}
 \end{aligned}$$

(3.5.3.1)

where

$D_o = \frac{K_o}{\rho_o C_{p_o}}$  = reference molecular diffusivity  
 $K_o$  = reference thermal conductivity  
 $T_o$  = reference temperature  
 $\rho_o$  = reference density  
 $C_{p_o}$  = reference specific heat  
 $F_o$  = reference heat exchange  
 $H_o$  = reference surface conductivity  
 $\sigma_o$  = Stephan Boltzmann constant.  
 $E_o$  = reference emissivity

it is found:

for any air temperature and water temperature difference:

$$\frac{\partial}{\partial x^{\circ}} \left( D^{\circ} \frac{\partial T_w^{\circ}}{\partial x^{\circ}} \right) - [RP] U^{\circ} \frac{\partial T_w^{\circ}}{\partial x^{\circ}} - [N] \mu^{\circ} (T_w^{\circ} - T_a^{\circ}) - [B] \mu^{\circ} \frac{F''^{\circ}}{H_o} - [C] \mu^{\circ} \frac{F'^{\circ}}{H_o} = 0 \quad (3.5.3.1a)$$

for small air and water temperature difference:

$$\frac{\partial}{\partial x^o} \left( D^o \frac{\partial T_w^o}{\partial x^o} \right) - [RP] U^o \frac{T_w^o}{\partial x^o} - [N] \mu^o (T_w^o - T_a^o) - [B] \mu^o \frac{F''^o}{H^o} = 0 \quad (3.5.3.2)$$

where  $RP = \frac{U_o L_o}{D_o}$  ; product of Reynolds and Prandtl numbers often called Peclet number  $P_e$ .

$N = \frac{H_o L_o}{K_o}$  ; Nusselt number.

$B = \frac{F_o L_o}{K_o T_o}$  ; dimensionless number

$C = \frac{T_o^3 L_o \sigma_o E_o}{K_o}$  ; dimensionless number.

As before, Eq. (3.5.3.1a) can be developed to get the complete equation:

$$\frac{\partial}{\partial x^o} \left( D^o \frac{\partial T_w^o}{\partial x^o} \right) - [RP] U^o \frac{\partial T_w^o}{\partial x^o} - [N] \mu^o (T_w^o - T_a^o) - [B] \mu^o \frac{F''^o}{H^o} - [C] \frac{\mu^o E^o \sigma^o}{H^o} (6T_a^o{}^2 (T_w^o - T_a^o)^2 + 4T_a^o (T_w^o - T_a^o)^3 + (T_w^o - T_a^o)^4) = 0 \quad (3.5.3.1b)$$

The derivation of this dimensionless equation is given in Appendix B.

### 3.5.4 Dimensionless Numbers

Six dimensionless numbers are involved, namely, F, R, RP, N, B, C. To calculate these dimensionless numbers, reference values given herein were used:

$L_o$  = width of the channel, ft

$U_o$  = mean cross-sectional velocity of water in the channel, ft/sec

$H_o$  = surface conductivity of water obtained from experiments, Btu/sq ft min  $^{\circ}R$

$g$  = 32.172 ft/sq sec, acceleration of gravity

$\gamma_o$  =  $1.162 \cdot 10^{-5}$  sq ft/sec, kinematic viscosity of water at  $17.5^{\circ}C$  ( $63.5^{\circ}F$ )

$C_{p_o}$  = 1000 cal/g  $^{\circ}C$ , specific heat of water at  $17.5^{\circ}C$  and 1 atmosphere

$\rho_o$  = 0.9987158 g/cm<sup>3</sup>, density of water at  $17.5^{\circ}C$  ( $63.5^{\circ}F$ )

$K_o$  =  $5.9862 \cdot 10^{-3}$  w/cm  $^{\circ}K$ , thermal conductivity of water at  $17.5^{\circ}C$  ( $63.5^{\circ}F$ )

$E_o$  = 0.97, emissivity of water

$F_o$  = 0.1 Btu/sq ft min, reference energy chosen so that B dimensionless number is small regardless of the experiment and function only of the channel geometry

$\sigma_o$  =  $1.72 \cdot 10^{-9}$  Btu/sq ft hr  $^{\circ}K^4$ , Stephan Boltzmann constant

$T_o$  =  $273.15^{\circ}K$ , temperature difference between Kelvin scale and centigrade scale, reference temperature chosen so that B and C dimensionless numbers are small, regardless of the experiment.

So that, after transformations, the numerical expressions of the dimensionless numbers were found to be:

$$F = \frac{U_o}{\sqrt{32.172 \times L_o}}$$

$$R = \frac{U_o \times L_o}{1.162 \times 10^{-5}}$$

$$RP = U_o \times L_o \times 6.48456 \times 10^5$$

$$N = \frac{H_o \times L_o}{5.7687 \times 10^{-3}}$$

$$B = 3.526 \times 10^{-2} \times L_o$$

$$C = 5.457 \times 10^{-2} \times L_o$$

## CHAPTER 4

### EXPERIMENTAL WORK

#### 4.1 Experimental Model Design

To be able to conduct the experiments required for this study, the main problem consisted of building a channel designed so that it was possible to get a measurable drop of the water temperature over the available length of channel.

In order to obtain such a condition, an insulated flume was built, 6 feet wide and 32 feet long, and with a height of 15 inches. To get a non-uniform flow across it, a step was built with a height of 2 inches and 23 feet long. All the construction has been made with plywood  $\frac{1}{2}$ -inch thick, except for the step in which the plywood thickness was only  $\frac{1}{4}$ -inch. A top using the same material has been insulated as well. The insulation consisted of sheets of 2 inches of styrofoam.

The maximum available inside width for the channel was 5 feet, and a moving side has been designed so that it was possible to study the desired range of channel widths.

To get a measurable drop in the water temperature, a heated air flow was recirculated above the water. To reach the required air temperature (roughly 300°F) the air movement was supplied by a fan with a belt transmission. The power of

the motor was 1/2 H.P. so that an air flow of 852 cfm was obtained. This fan was designed to give a static pressure of 1.5 in. w.g. through a circular inlet of 7 in. diameter, so that the inlet air velocity was 3206 fpm. The air was then circulated through a duct heater - 3 phases - 650 V 40 KW. All the air ducting has been insulated with 2 inches of compressed fiberglass held by a wrapping canvas.

The inflow of water was realized by a 10-inch diameter T pipe with several slots at the bottom coming down into a head box joined to the channel itself by a submerged weir. This was made to avoid any air leakage. A grid was also present to smooth out undesirable velocity variations.

The outflow of water was realized by an inclined side channel return to the sump and an adjustable gate whose opening could vary from 0 to 4 in. was made in the ducting always in the goal of avoiding any air leakage.

Three sections, called 1, 2 and 3, located on the step were studied. In order to have access to the water, the top of the channel, at these particular points, was made with two strips of glass separated by a distance of 1/2 in., distance sufficient enough to insert the equipment. Tracks and a platform moving on them were used to support the equipment which consisted of a pitot tube to measure the local water velocity, a probe connected to a telethermometer to get the water temperature, and a plummet to measure the water depth.

A sketch of the channel is given in Fig. 1, p.63 the approximate scale is 1/24. The distance between section 1 and 2 was: 11.41 ft and between section 2 and 3: 11.36 ft.

In Section 1, a thermostat controlled the operation of the heater and for safety, a high limit thermostat and a high limit manual reset switch were installed immediately after the air duct heater.

For convenience, water flow direction and air flow direction were opposite, but because of the low air velocity through the flume (less than 3 mph), this had no influence on this study. To avoid water leakage, navy products and polyester resin were used to seal the channel.

Elevation measurements in the dry flume gave the following, looking downstream:

at Section 1, a lateral slope directed up to the right side of 0.01700

at Section 2, a lateral slope directed up to the right side of 0.03240

at Section 3, a lateral slope directed up to the right side of 0.02040.

Furthermore, on the longitudinal axis of the flume, there was:

from Section 1 to 2 (2 higher than 1), an average slope of 0.01641

from Section 2 to 3 (3 lower than 2), an average slope of 0.01043.

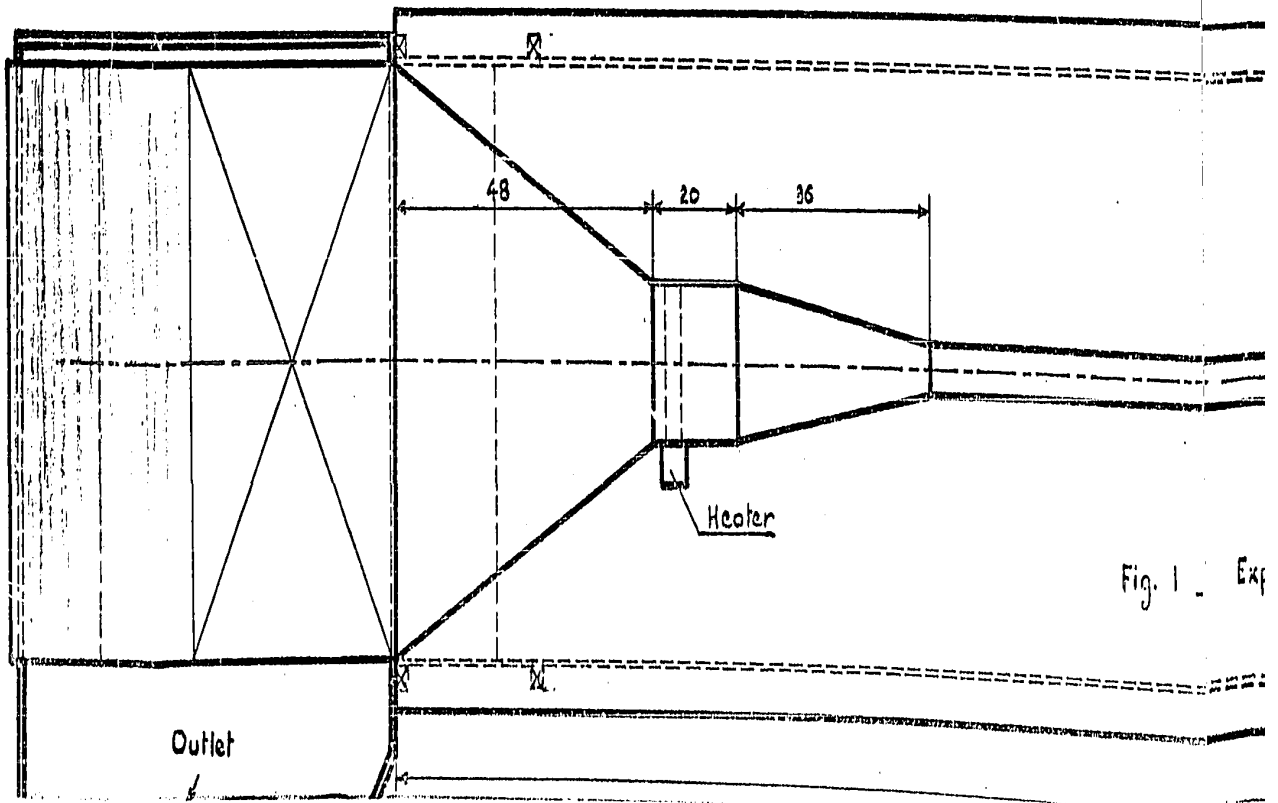
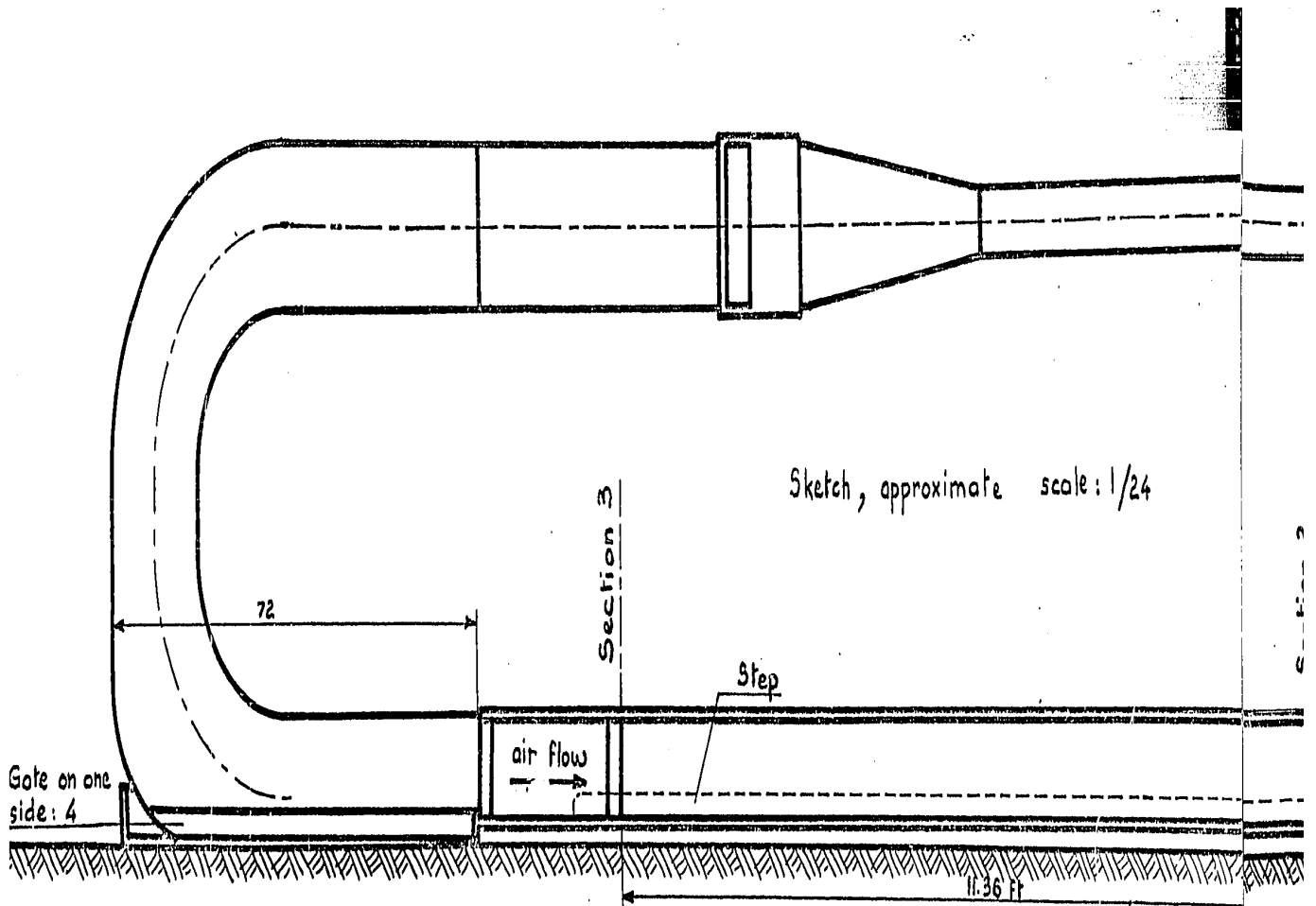
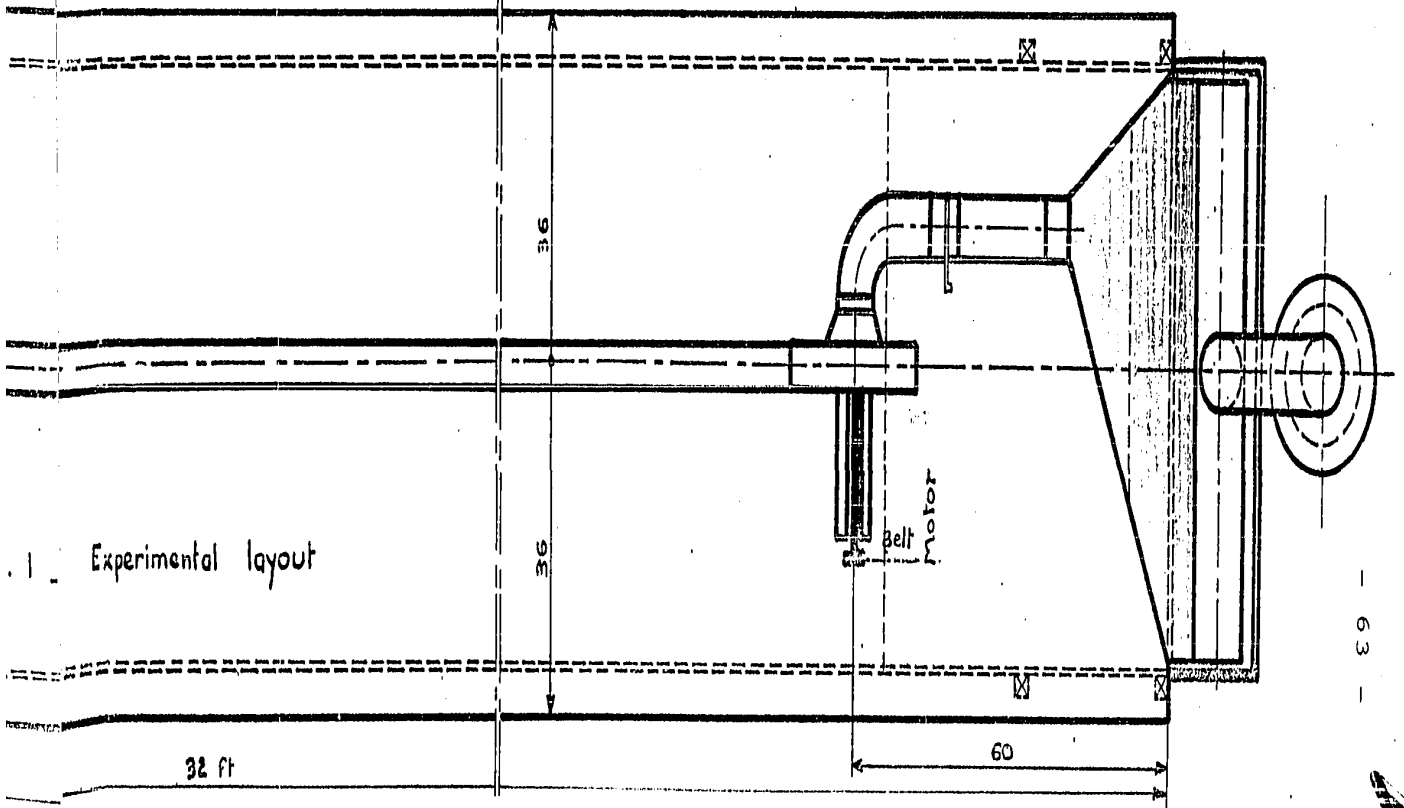
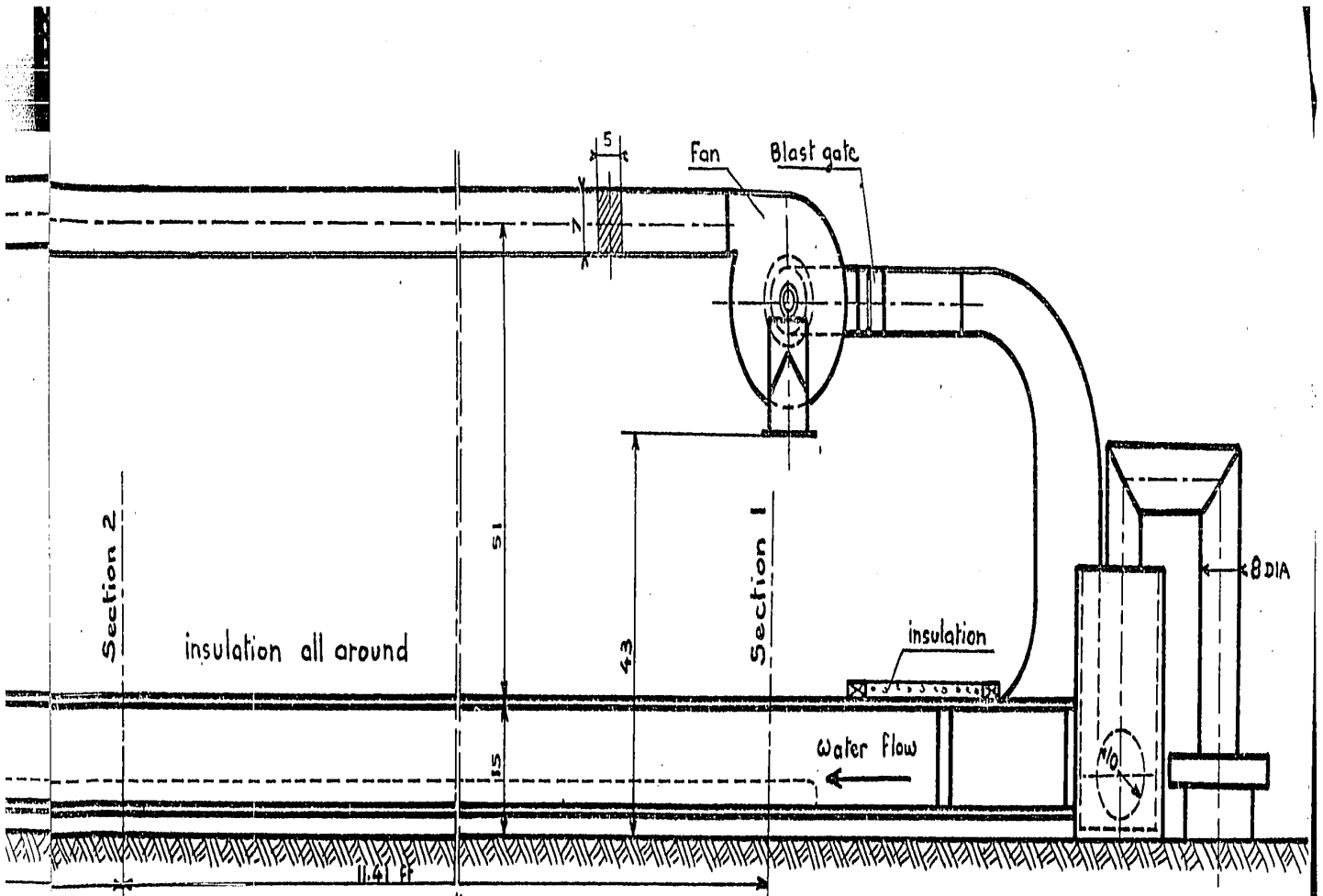


Fig. 1 - Exp



So that there was:

from Section 1 to 3 (3 higher than 1), an average slope of 0.002486.

In Fig. 2, p.65, the first picture (a) shows the channel alone with the step across, the second one (b) shows a part of the completely insulated flume.

In Fig. 3, p.66, the first picture (a) shows the glass part of Section 2 with the platform, and the second one (b) shows more in detail part of the top being removed, the Section 3. One can see the platform again, on the tracks, and the equipment including the telethermometer, the end of the step and the moving side in one study location.

The air temperature was measured at Sections 1, 2 and 3. A dew-probe sensor, located between Sections 1 and 2 has been used to check the assumption that the evaporation rate was zero due to the heated air recirculating system adopted. It is an electrically heated, self-regulating lithium chloride dew point hygrometer. It consists of two wire electrodes wound side by side on a cloth sleeve which covers a hollow tube. The cloth sleeve is impregnated with lithium chloride which absorbs water from the air and becomes conductive. This allows current to flow from one electrode through the cloth sleeve to the other electrode producing heat at the bobbin. Moisture is thereby evaporated from the lithium chloride until a heat moisture equilibrium is obtained. The equilibrium bobbin temperature is related to the dew

a



b

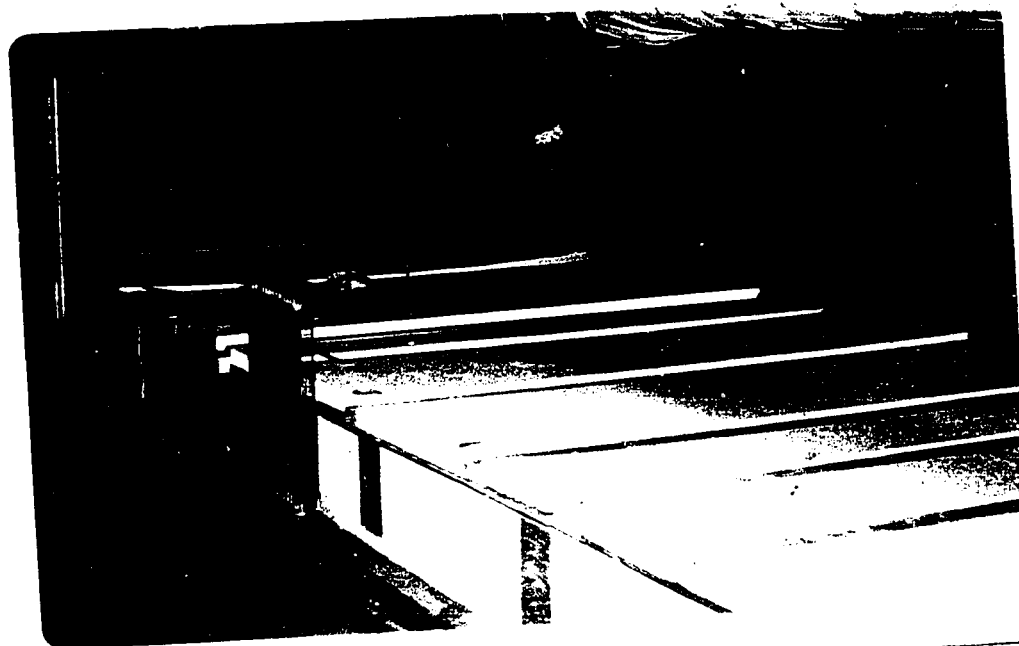
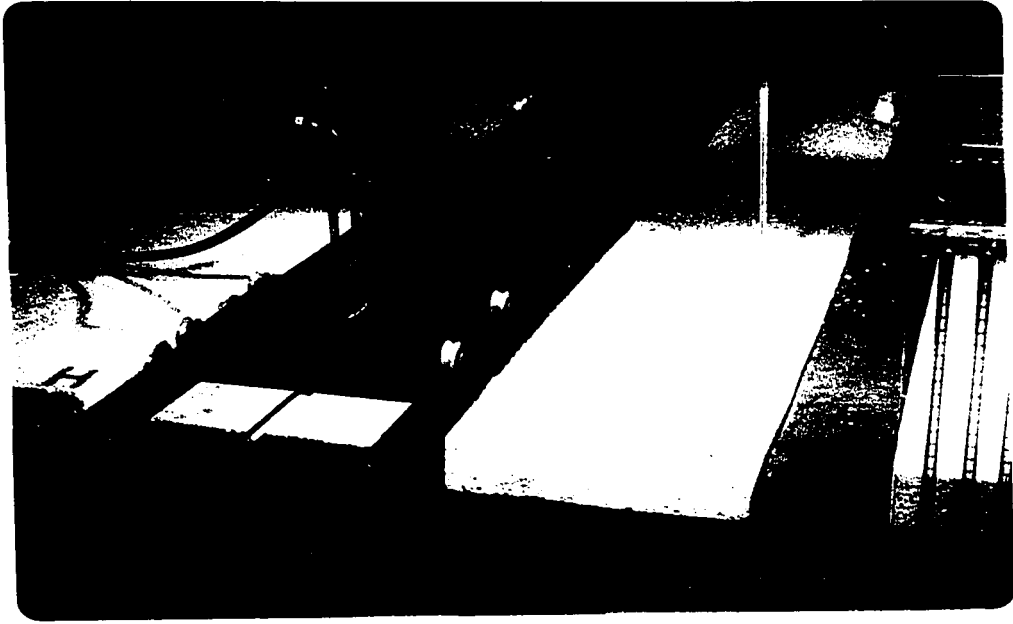


Fig. 2 — Part of the experimental design

a



b

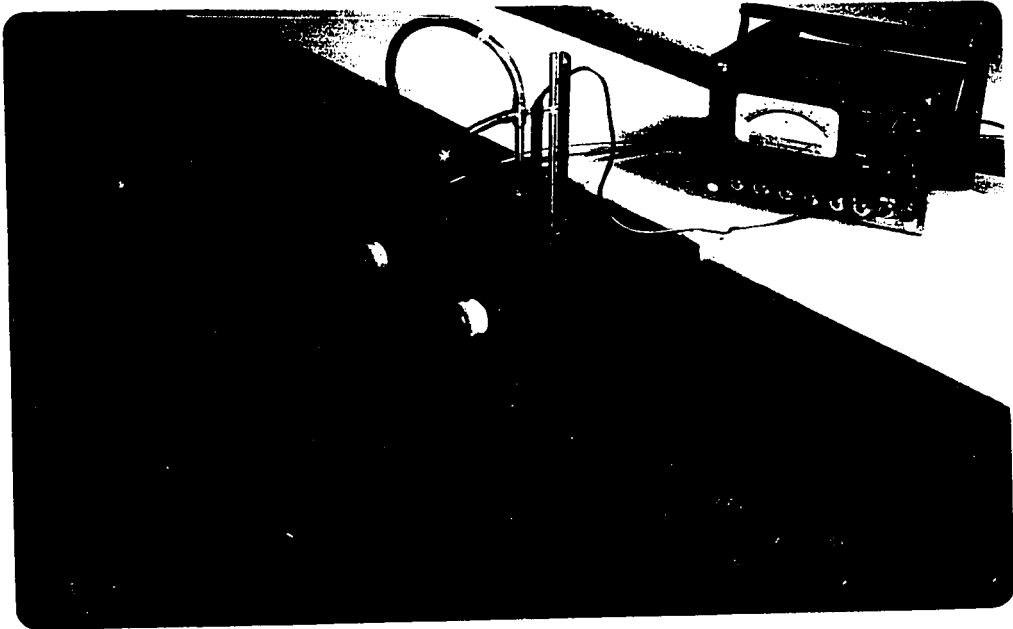


Fig. 3 — Part of the experimental design: details

point of the gas sample. By measuring the temperature in the cavity of the bobbin, the dew point can be determined, then the relative humidity. The range of the apparatus was -40 to +160°F.

#### 4.2 Operational and Analytical Methods

After a steady state of water flow has been reached in the channel, data for the longitudinal dispersion coefficients were found using Fischer's method, dividing each cross section in 15 to 25 vertical slices as described in 3.4. The water rate of flow was found using the same data. Each slice width was 0.2 ft.

The water temperatures were measured at each vertical slice and the air temperature at Sections 1, 2 and 3.

Before any temperature measurements were made, a steady state was obtained for and during these measurements; because the air temperature fluctuated, each reading was done by the time the air temperature was maximum.

For each channel width, four rates of water flow were studied. Then the channel width was changed and an initial rate of water flow was estimated from the preceding set of experiments so that it was tried to keep the product RP equal to a constant.

After calculations from the first result of this new channel width, a correction was applied to the other following estimated rates of flow, always in the goal of

trying to keep RP a constant. This method was then applied to another channel width.

Thus, six different channel widths have been studied: 3.0 ft, 3.4 ft, 3.8 ft, 4.2 ft, 4.6 ft and 5.0 ft; using four rates of water flow for the first four widths and only three for the last two because of the impossibility of doing any measurement due to the shallow depth of water.

## CHAPTER 5

### RESULTS

#### 5.1 Non-uniformity of Water Flows

The basic idea of setting and studying a cross-section number 2 was the possibility of studying a non-uniform flow over the channel length corresponding to the distance between Section 1 and Section 2, and a uniform flow over the channel length corresponding to the distance between Sections 2 and 3, at least for the water rates of flow which would allow the measurement of any drop in the water temperature over these 11.36 ft.

Unfortunately, this hope could not be satisfied as there was a non-uniform flow over the full length of the flume as can be seen in Fig. 4, p.70, which shows the depth of water versus the water velocity for the three sections for the same rate of flow and the same vertical slice (the fifth one from the left side, looking downstream).

The depth ratio giving the mean velocity location for each section has been calculated by an iteration method. These values have been used during the experiments whenever it was possible. However, whenever the required distance from the bottom was less than half the diameter of the pitot tube (8 mm diameter), this last value was used so that it was as close to the bottom as possible.

There is a velocity shift when using a pitot tube near a wall; however, this effect is believed to be negligible in this study. In Fig. 4b, p.70b, one can see the method of sectioning, the various bottom slopes, and a typical lateral velocity distribution. In Fig. 4, p.70a, the areas under the velocity distribution curves are not constant because of the channel geometry. These have been obtained with the same flow, at the same distance from the walls, but the bottom geometry being somewhat different at each section explains the differences:  $u$  is not the cross sectional mean velocity but a local velocity.

A uniform flow between sections 2 and 3 has not been achieved because the length required to get a measurable drop of water temperature was such that section 3 was too close to the end of the step, so the water flow was affected by the free overfall.

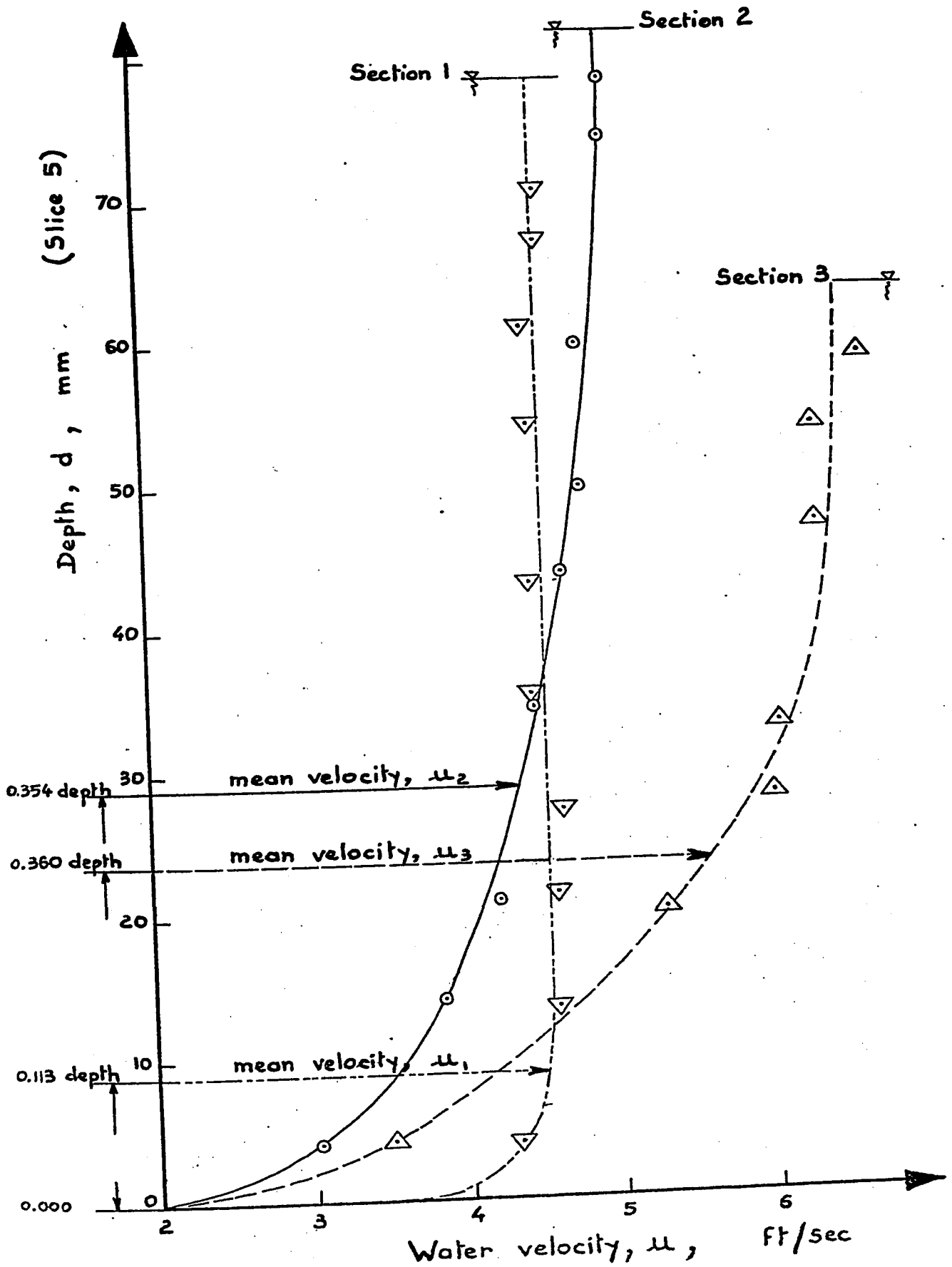
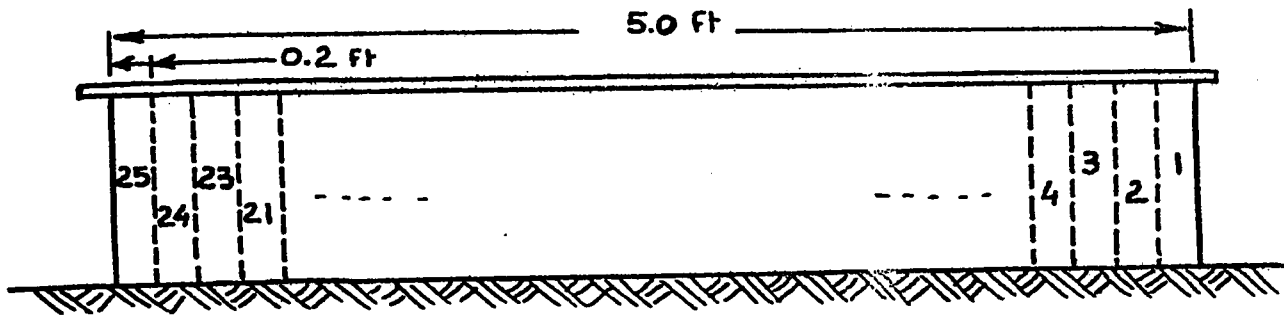
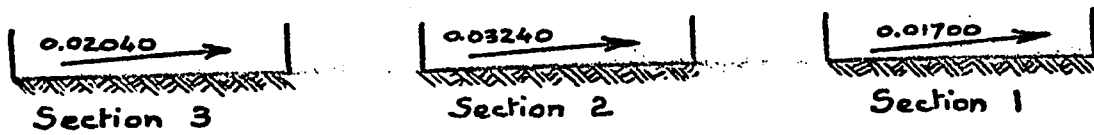


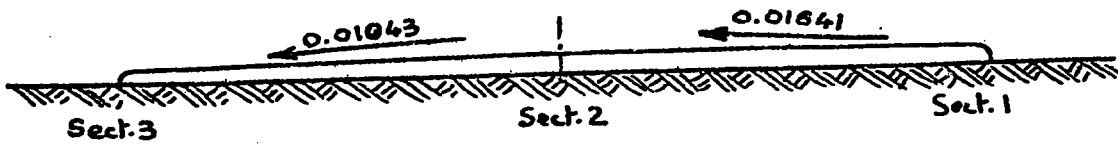
Fig. 4 — Depth vs water velocity



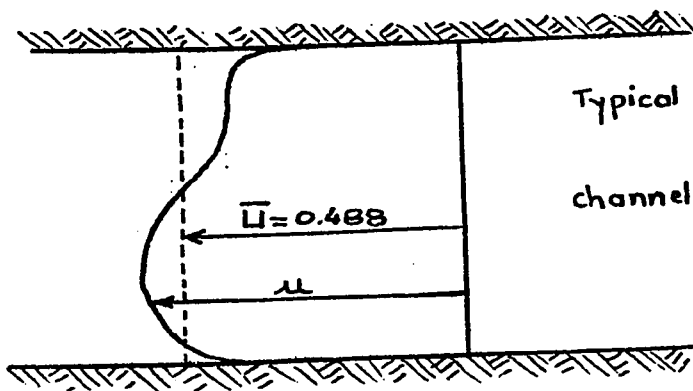
Cross-section of one study section, looking downstream



Transversal bottom slopes, looking downstream



Longitudinal bottom slopes



Typical lateral velocity distribution

channel width : 5.0 ft    Section : 1  
Mean veloc : 0.488 ft/sec

Fig. 4b

## 5.2 Evaporation Rate

According to the theory (see Chapter 3) there should be zero evaporation rate once the equilibrium state has been reached. To check this, the dew-point sensor was used. The maximum temperature allowable was  $160^{\circ}\text{F}$ , as compared to the working air temperature of roughly  $300^{\circ}\text{F}$ . Furthermore, for this apparatus, the characteristics were such that very good accuracy was obtained from 100 to 12 percent relative humidity, but under this last value, estimations were required. So, working with  $160^{\circ}\text{F}$ , at the equilibrium stated, although it was not possible to say exactly what was the relative humidity, this latter value tended to be zero. This fact was enough to confirm the assumption that the evaporation rate was zero, because the water temperature was  $22.05^{\circ}\text{C}$ . That means that in Eq. (3.1.19)  $e_{ws} = e_{as} \text{ RH}$ . Furthermore, to reinforce this conclusion, the air velocity was always very low (less than 3 mph) and the multiplier coefficient is small. So, for the study, the saturated air vapor pressure at water surface temperature  $e_{ws}$  was considered equal to the saturated air vapor pressure at air temperature  $e_{as}$  times the relative humidity RH.

Because of these special laboratory conditions, the surface conductivity could not be calculated from atmospheric factors, i.e., Eq. (3.1.19), because the Bowen ratio could not be used.

The surface conductivity  $H_0$  has been calculated applying Eq. (3.3.7), that is to say from water flow conditions only, without any possible comparison with the usual way of obtaining its value in natural conditions.

### 5.3 Water Temperature

In Chapter 3 (3.2), it was assumed and demonstrated to be reasonable that there was instantaneous dispersion of heat and no gradient of water temperature in the vertical direction.

During pre-experiments, and because of the turbulent flow, no gradient of water temperature in the vertical direction has been observed, and in the lateral direction, even with the lateral channel slope, no gradient of water temperature has been observed or at least, if there was one, it was not measurable. This confirms the previous assumption.

The difficulty experienced in measuring the water temperature for shallow depths of water must be mentioned here. This was due to the probe which, even when it was immersed, was influenced by the heated air flow which the vertical support had to cross, coming down from the platform. No difficulty was encountered for greater depths of water.

### 5.4 Air Temperature

Because of the thermostat controlling the power of the heater and because of some air leakage, the air temperature

fluctuated throughout the flume. The order of magnitude of these fluctuations was 22°C at Section 1,  
22°C at Section 2,  
28°C at Section 3.

The time required for the air temperature at one point, to rise from the minimum value to the maximum was around 75 sec, and the time required to drop down was around 85 sec.

Because of the air velocity over the 22.77 ft length, no significant lag time was noticed between the moment where the air temperature was maximum at Section 3 and the moment it was maximum at Section 1.

During the experiments, the maximum air temperature value at the study section was awaited before taking any water temperature measurements. The consequence was that between all the experiments, the maximum air temperature difference at the time the readings were taken was only 6°C.

A typical air temperature profile along the channel is given in Fig. 5, p.74, this has been drawn with the average values taken from the data.

In the calculations,  $T_a$  has been taken as the average value of the air temperature maxima at Sections 1 and 3.

## 5.5 Data

Because of the number of rates of water flow studied (22) the data are given in Appendix C.

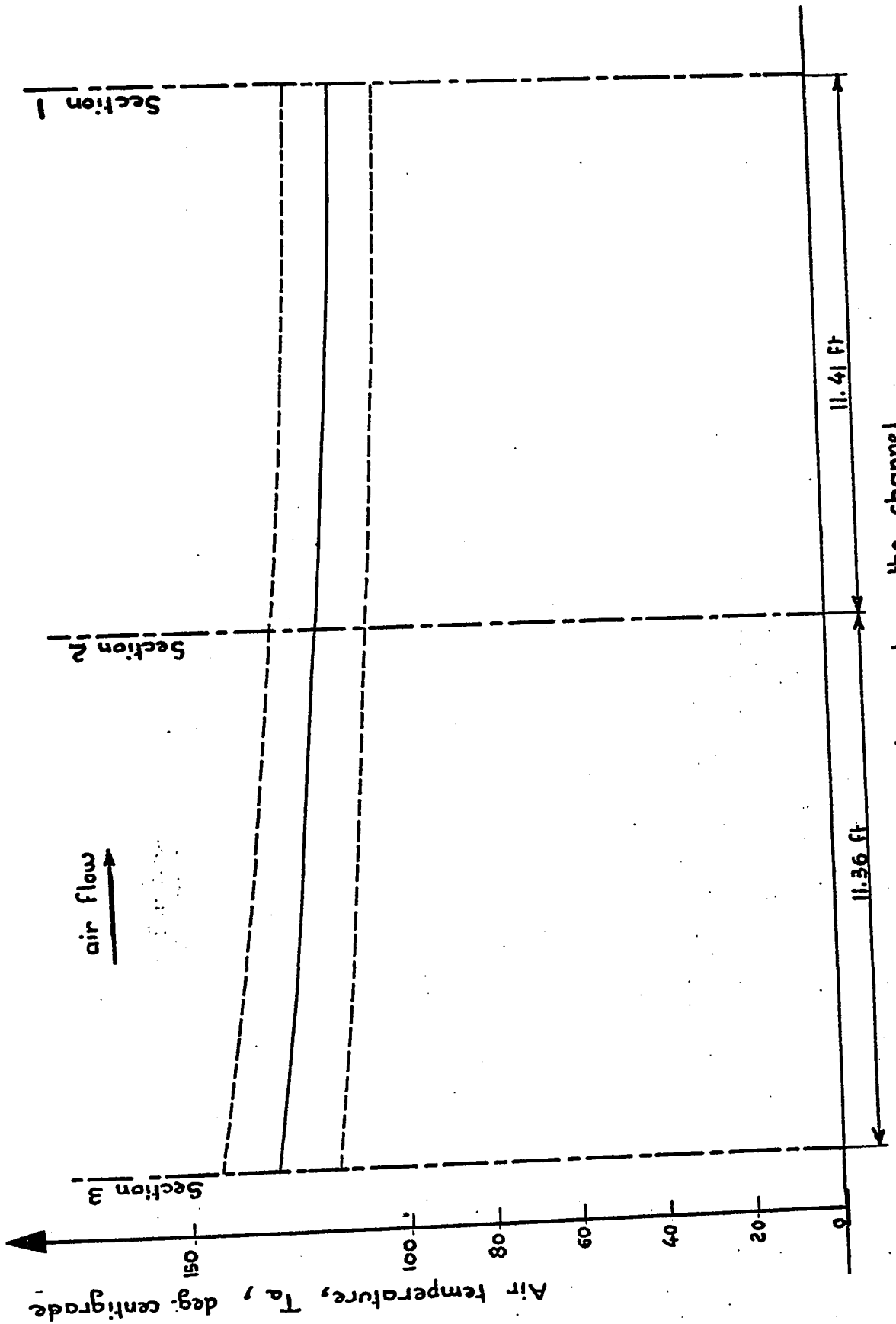


Fig. 5 — Air temperature along the channel

## CHAPTER 6

### DISCUSSION OF RESULTS

As mentioned in Chapter 4 (4.2), the first set of experiments was used (i.e., channel width 3.0 ft) to estimate what should be, for the next channel width studied, the first average cross-sectional depth in order to keep the Reynolds number and the product of the Reynolds and Prandtl numbers constants. Then, after conducting the first experiment of the second set, a correction was made for the second rate of water flow to study and so forth. Fig. 6, p.76 shows the first mean depth water flow relation obtained, that is to say, at channel width 3.0 ft, and at Section 2.

Even working with great care, it was not possible, of course, to get exactly the same R and RP values from one experiment to the next. Another problem arose because of the non-uniformity of the water flow. It was possible to calculate, for the same rate of water flow, as many values of R, RP and F as there were cross sections. There was no special reason to work with one set of values, so, arbitrarily, the average between Sections 1 and 3 was chosen. These results appear in the data sheets given in Appendix C. Order of magnitude of the variations of R and RP was 2 to 8%. The maximum deviation was 14%.

Sample calculations of dimensionless numbers for a typical  
experimental run: channel width 5.0 ft

water flow 0.148 ft/sec

- mean cross sectional water velocity: Section 1: 0.488 ft/sec  
Section 3: 0.692 ft/sec

- mean water velocity used in calculations:

$$1/2(0.488 + 0.692) = 0.590 \text{ ft/sec}$$

- dimensionless numbers:

$$F = \frac{U_o}{\sqrt{32.172 \times L_o}} = \frac{0.590}{\sqrt{32.172 \times 5}} = 0.0465$$

$$R = \frac{10^5}{1.162} \times U_o \times L_o = \frac{10^5}{1.162} \times 0.590 \times 5 = 254,000$$

$$P_e = 6.484 \times 10^5 \times U_o \times L_o = 6.484 \times 10^5 \times 0.590 \times 5 \\ = 1,914,000$$

$$B = 3.526 \times 10^{-2} \times L_o = 3.526 \times 10^{-2} \times 5 = 0.17630$$

$$C = 5.457 \times 10^{-2} \times L_o = 5.457 \times 10^{-2} \times 5 = 0.27285$$

$$N = 5.768 \times 10^3 \times H_o \times L_o = 5.768 \times 10^3 \times 29.989 \times 5 \\ = 865$$

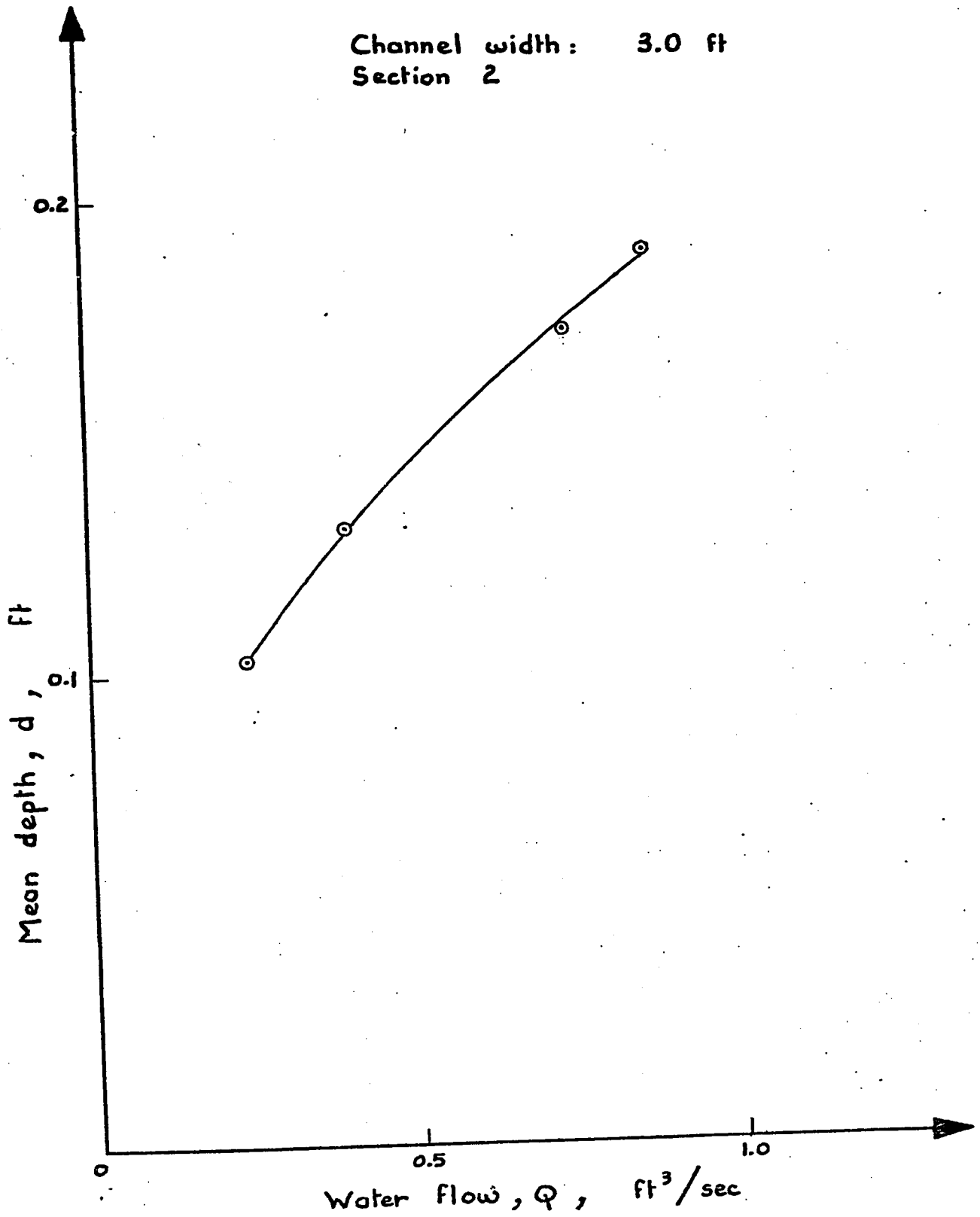


Fig. 6 — Mean depth vs water flow

With the remarks stated in Chapter 5 (5.3) on the difficulty in making some of the water temperature measurements, it was plotted, for each channel width, the drop in water temperature from Section 1 to Section 3. These graphs are given in Fig. 7, p.78 and Fig. 8, p.79. Similar curvature for all the curves is to be noted.

Fig. 9, p.80 shows, on a semi-logarithmic scale, Nusselt numbers versus Froude numbers for four constant values of R and RP. This has been drawn using the least square method. Keeping in mind that each set of points has slightly different R and RP values, it has not been possible to estimate where was the theoretical point because of the complexity of Eq. (3.3.7). As noted above, all these points are approximately within 8% of each other. The constant values of R and RP are simply the averaged values of the corresponding set of results obtained.

Fig. 10, p.81 shows the same relation on a normal scale and the equations obtained are the following:

$$N=1461.187 e^{-6.544F} \text{ for } R=3.90 \cdot 10^5 \text{ and } RP=2.94 \cdot 10^6$$

$$N=1712.431 e^{-6.357F} \text{ for } R=3.52 \cdot 10^5 \text{ and } RP=2.65 \cdot 10^6$$

$$N=1692.177 e^{-7.008F} \text{ for } R=2.84 \cdot 10^5 \text{ and } RP=2.14 \cdot 10^6$$

$$N=1514.099 e^{-5.273F} \text{ for } R=2.35 \cdot 10^5 \text{ and } RP=1.78 \cdot 10^6$$

These results are valid for F in the range studied, 0.050 to 0.160. One can see that the last one is a little bit different from the others, but only 4 points were used

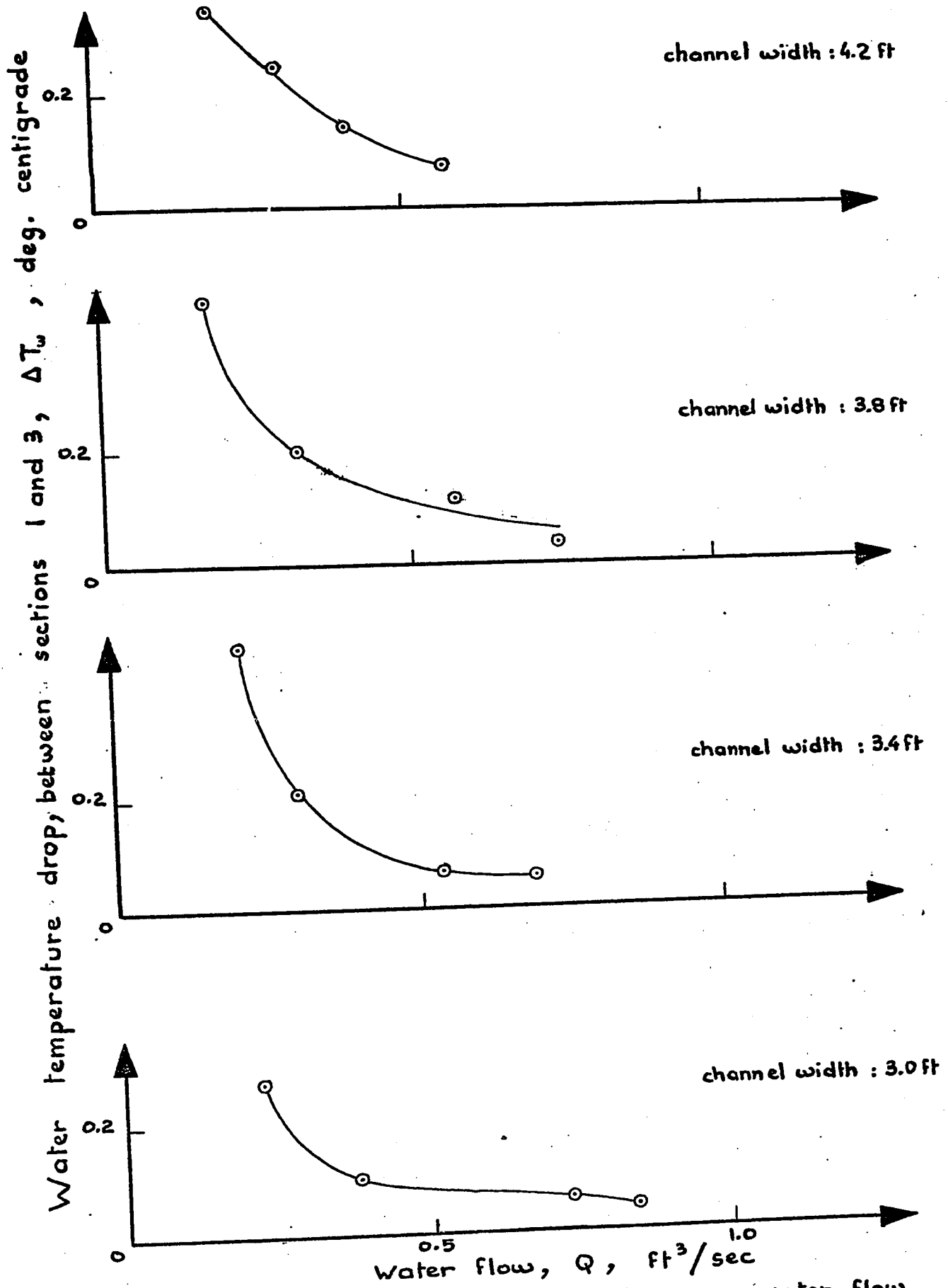


Fig. 7 — Water temperature drop vs water flow

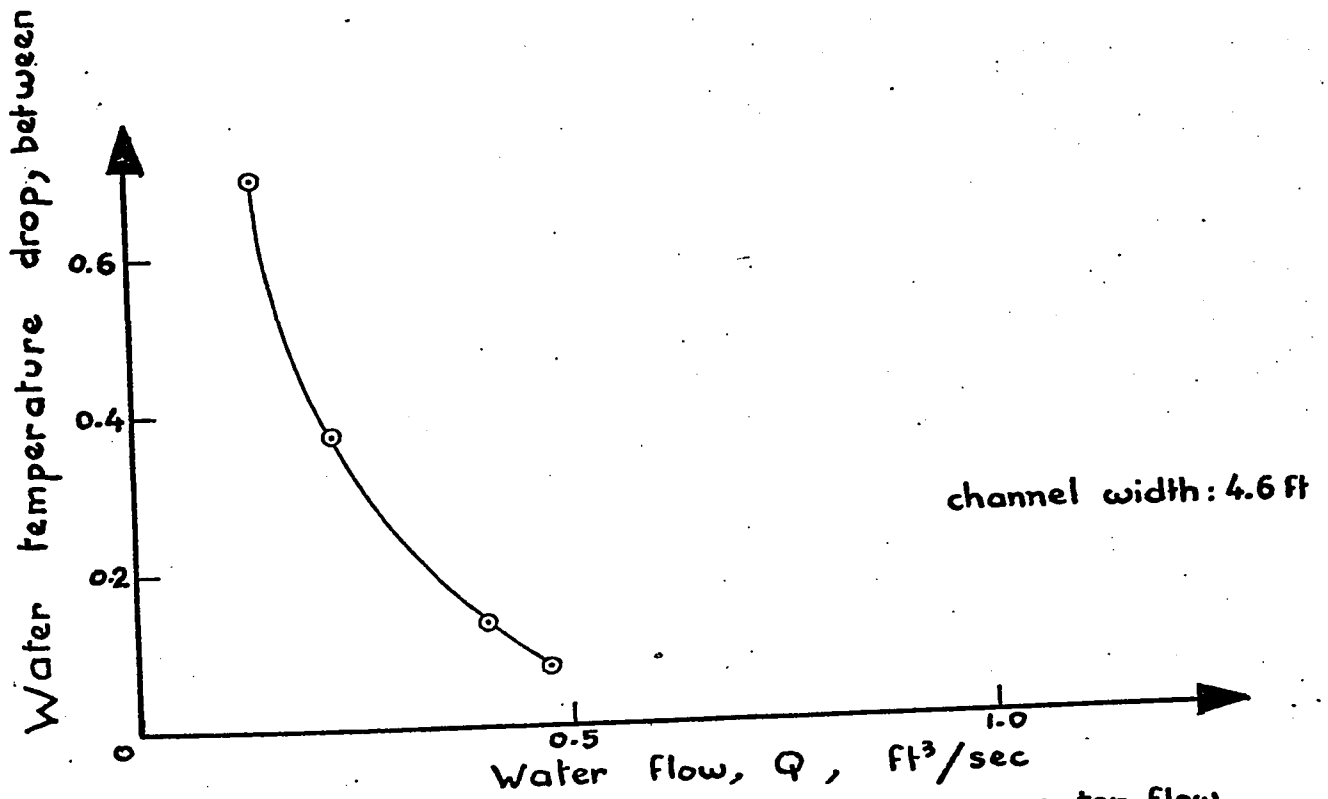
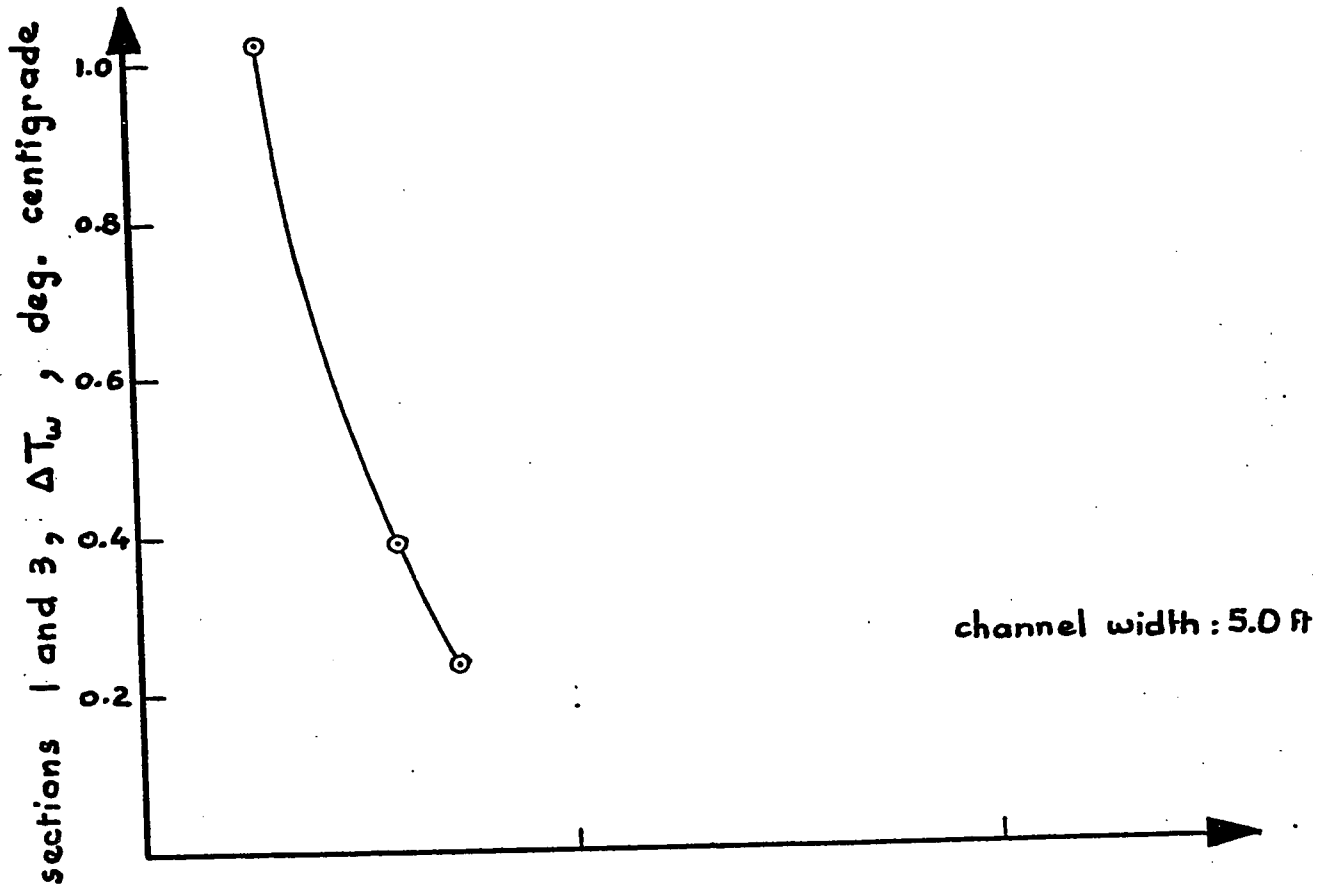


Fig. 8 — Water temperature drop vs water flow

- ▽—  $R = 3.90 \cdot 10^5$   $RP = 2.94 \cdot 10^6$
- $R = 3.52 \cdot 10^5$   $RP = 2.65 \cdot 10^6$
- △-  $R = 2.84 \cdot 10^5$   $RP = 2.14 \cdot 10^6$
- $R = 2.36 \cdot 10^5$   $RP = 1.78 \cdot 10^6$

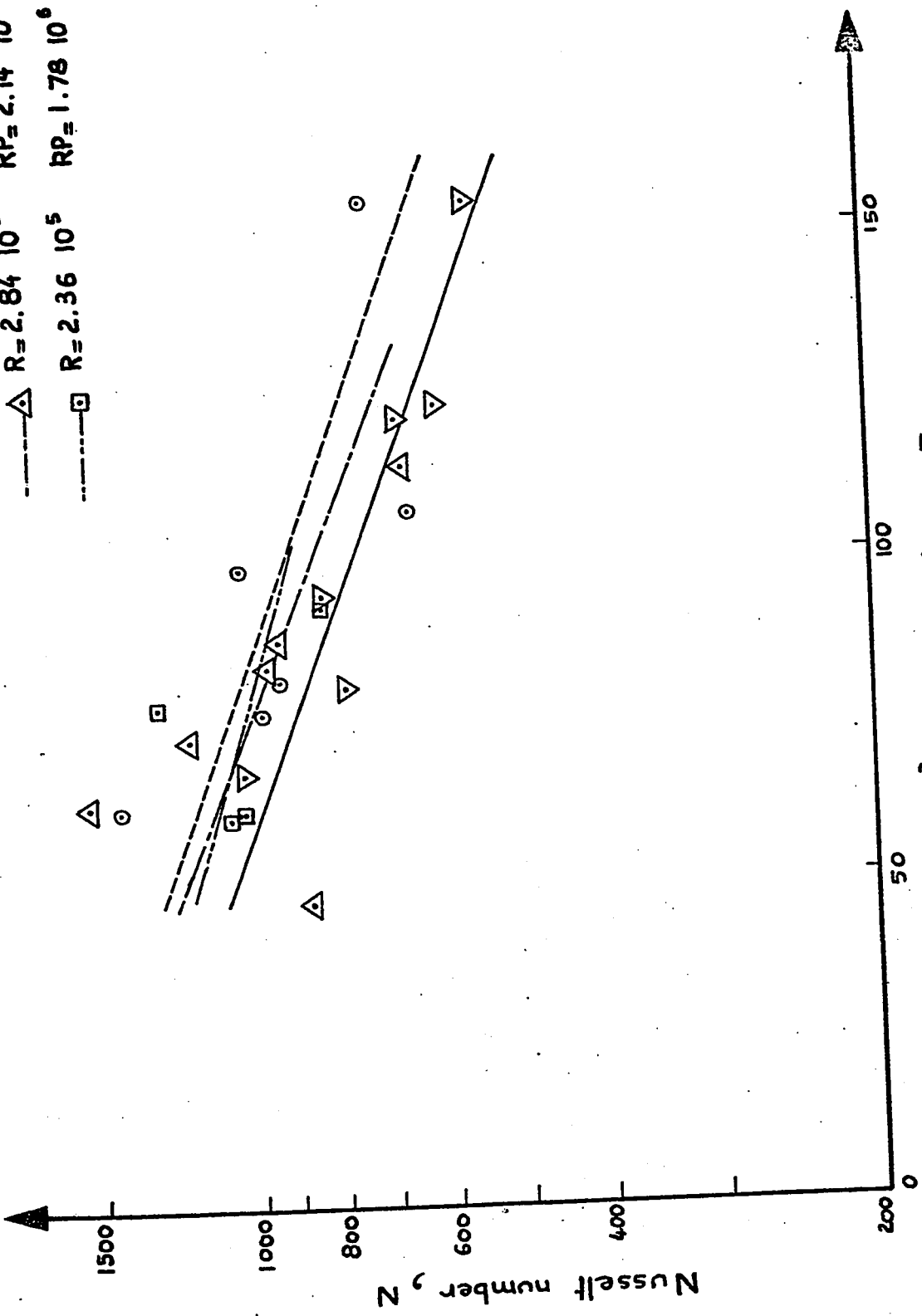


Fig. 9 — Nusselt number vs Froude number (semi-logarithmic scale)

- ▽—  $R = 3.90 \cdot 10^5$   $RP = 2.94 \cdot 10^6$
- $R = 3.52 \cdot 10^5$   $RP = 2.65 \cdot 10^6$
- △—  $R = 2.84 \cdot 10^5$   $RP = 2.14 \cdot 10^6$
- $R = 2.36 \cdot 10^5$   $RP = 1.78 \cdot 10^6$

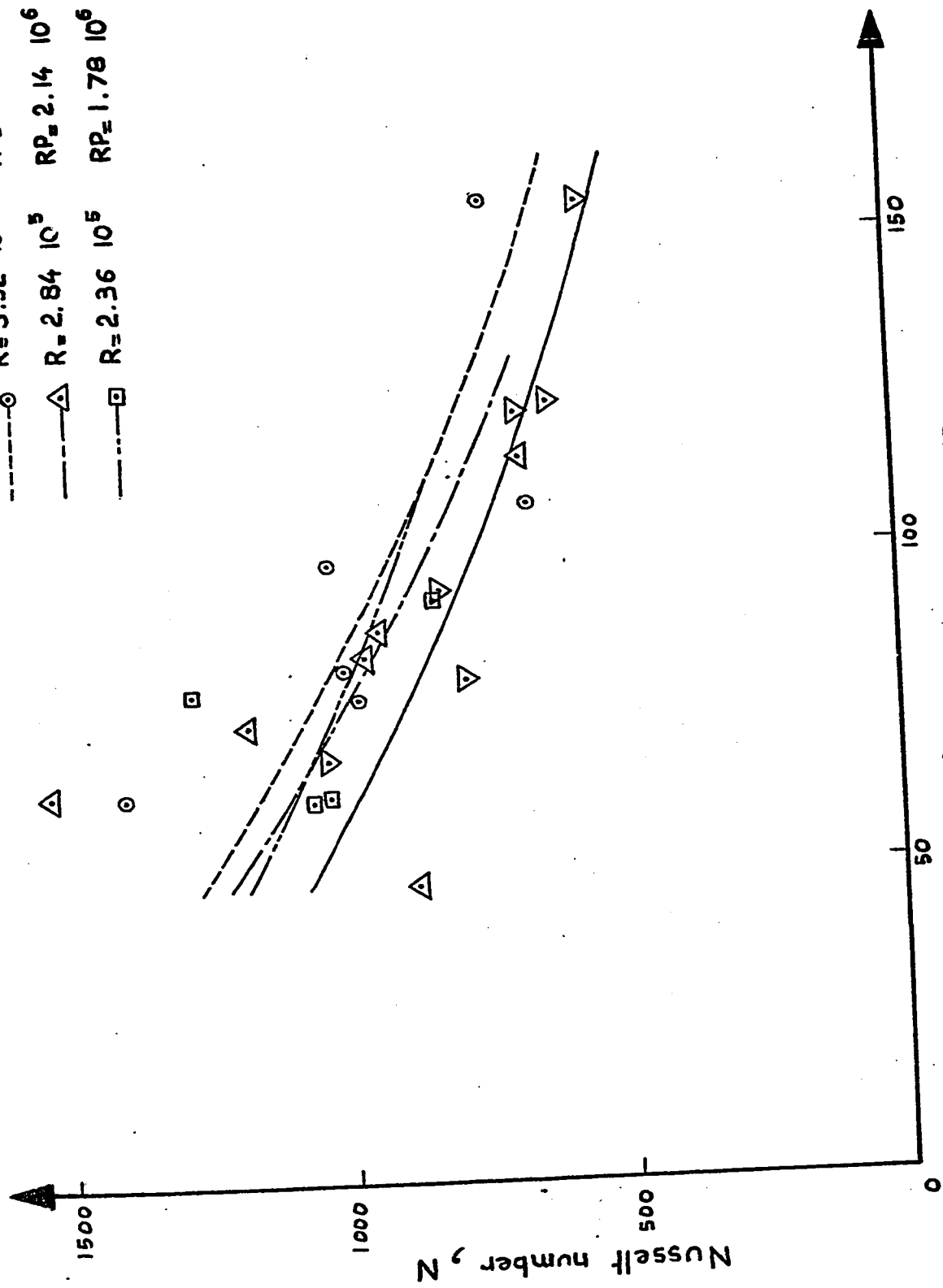


Fig. 10 — Nusselt number vs Froude number

instead of 6 as for the others because of the impossibility of making measurements for the 4.6 ft channel and the 5.0 ft channel, the depth of water being too small. For this reason, the corresponding experiments were always the most difficult to conduct. One can notice also that Froude numbers were small. This is due to the limitations of the experimental apparatus and was the maximum range obtainable. However, this is a range of great practical interest, being the same as that which occurs in many rivers. Looking more in detail at these two figures, we see that the curvature is quite similar for three of the four curves considering the above remarks.

Going from large to small Nusselt numbers, at a value of  $F = 0.110$  for example, we find the curves corresponding to:

first	$R=3.52 \cdot 10^5$	$RP=2.65 \cdot 10^6$
then	$R=2.84 \cdot 10^5$	$RP=2.14 \cdot 10^6$
then	$R=3.90 \cdot 10^5$	$RP=2.94 \cdot 10^6$

This may suggest that the difference between all R and RP numbers was not particularly significant. Following this idea, all the 22 points were included together and in Fig. 11, p.83 on semi-logarithmic scale the best fit was found with the least square method. Fig. 12, p.84 shows the result on a normal scale. The corresponding equation is:  
 $N=1691.351 e^{-7.043F}$  for  $R=3.23 \cdot 10^5$  and  $RP=2.43 \cdot 10^6$   
R and RP values being this time the average values of all points.

—  $R = 3.23 \cdot 10^5$   $RP = 2.43 \cdot 10^6$

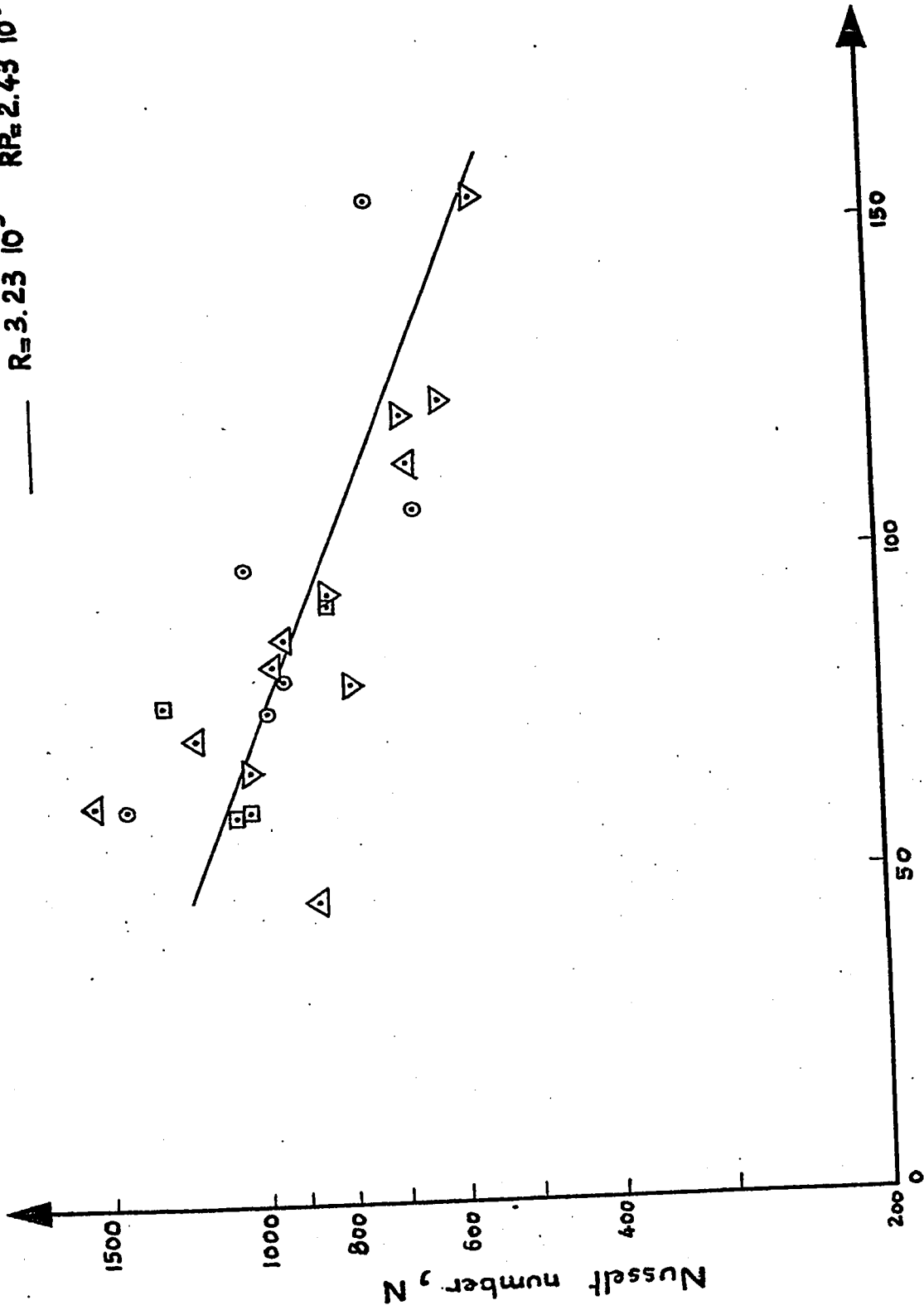


Fig. 11 — Nusselt number vs Froude number (semi-logarithmic scale)

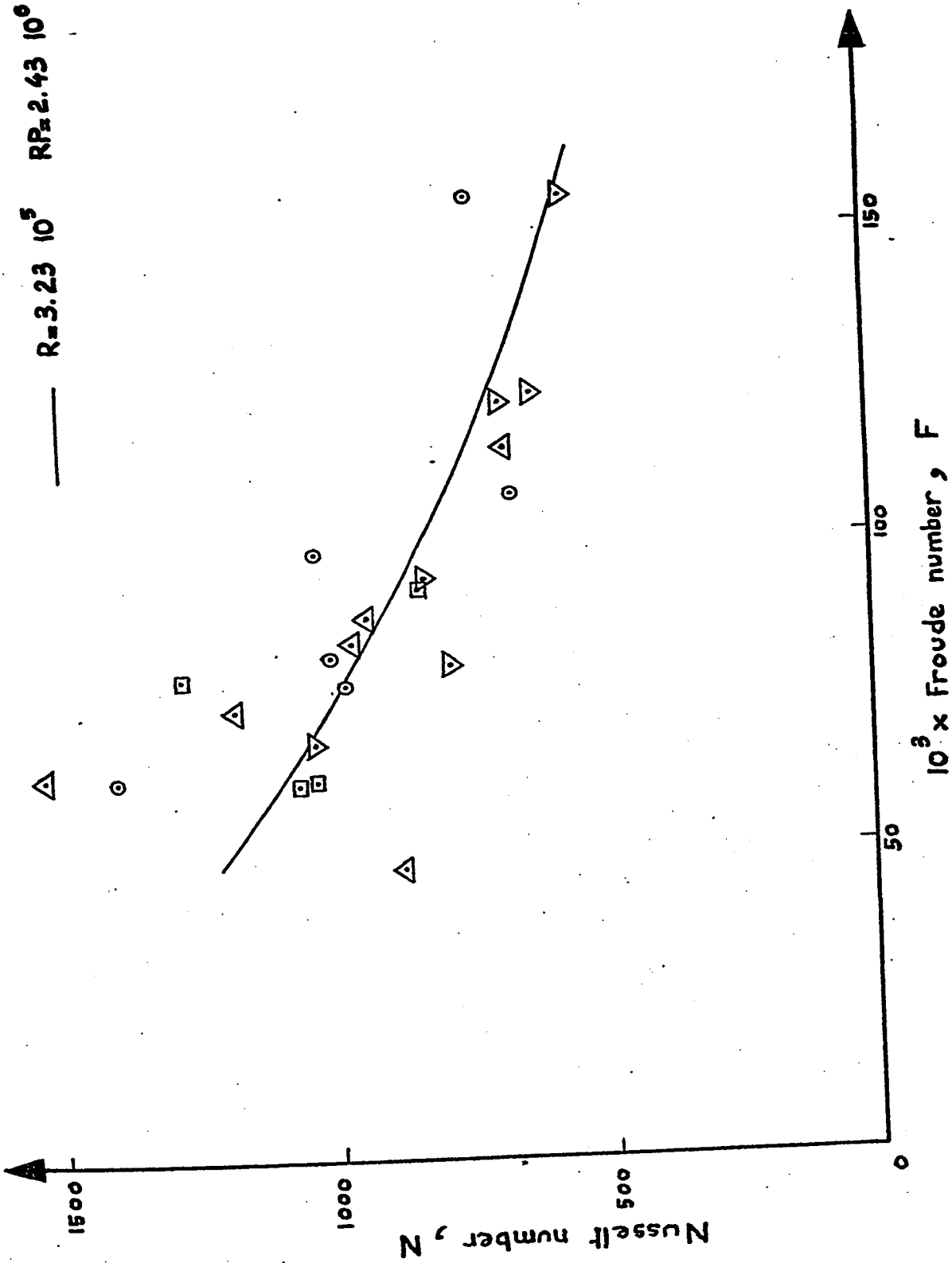


Fig. 12 — Nusselt number vs Froude number

So, summarizing the results of the figures, it can be seen that the water temperature drop vs water flow is not a linear relation, but it shows the same curvature, whatever may be the channel width.

The graph of Nusselt number vs Froude number shows, on normal scales, the same exponential type of curve whatever may be the values of  $R$  and  $RP = P_e$ . That means that in modelling a river or stream, the Froude number having been determined, the Nusselt number corresponding to the prototype water flow must be calculated and applied to the laboratory conditions such that it becomes possible to study the heat exchange processes of the prototype.

It must also be mentioned again that results do not apply to two dimensional problems, and that an air velocity of 3 mph may not be small if a comparison is made using a realistic profile for atmospheric air movement, but that is of no influence in this study because the other term of the expression of evaporation component was zero. It should also be mentioned that in a zone of developing flow, the turbulence intensity is also an important parameter for similarity.

## CHAPTER 7

### CONCLUSIONS AND RECOMMENDATIONS

#### Conclusions

1. In any model study of heat transfer to a water body, such as that of ice formation as the result of a heat loss, for example, variations of Nusselt number versus Froude number should be considered for satisfactory results.
2.  $N = N(F)$  for  $RP = P_e = \text{constant}$  applies primarily to modelling sections of non-uniform flow in rivers and streams where the water is well mixed and homogeneous.
3. In most natural conditions, the dimensionless number,  $C$ , can be neglected because of the small differences in water and air temperature differences compared with the laboratory conditions.
4. Technical problems and difficulties were encountered in this work, especially the insulation for an air flow of almost  $300^{\circ}\text{F}$ . Also, because of such an air temperature, when the water flow was shallow, it was difficult to measure the water temperature, because the probe connected to the telethermometer had to cross the heated air flow and was influenced by it. Another difficulty came from the exterior diameter of the pitot tube used; for some shallow depths of water it has not been possible to put the axis of the pitot tube at the required distance from the bottom.

## Recommendations

Recommendations in order to increase the eventual benefits of this study would be to increase the Froude number range, keeping in mind, as has been mentioned, that the study range is of great practical interest, being the same as that which occurs in many rivers. So, in order to conduct new experiments in the goal of increasing the range of the Froude number, it is suggested that the air temperature should be increased. This can be done without difficulty provided that a few sheets of styrofoam insulation are changed because, being in direct contact with the air flow, they will not resist a higher air temperature, especially at Section 3. This, combined with a thinner pitot tube should permit the study of both deeper and shallower water flows. A more sensitive telethermometer could also be used although it is not believed that the benefits will be so great. Independently of that, the probe should be insulated in order to reduce the influence of the heated air flow its vertical support had to cross. A last way of increasing the range of the Froude number would be to reduce the channel width. In that case, however, difficulties in getting more accuracy on water flow rates will arise, as well as giving width-over-depth ratios which will be far away from reality. It is to be noted that in this study this ratio was in the range 9.6 to 182, and in nature, it is usually of 10 or greater.

It is thought that whatever might be done in addition, the range of the Froude number should not be extended over that which is found in natural conditions in order to keep the results of practical interest.

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APPENDIX A

16/33/49

DATE = 72146

MAIN

20

FORTRAN IV G LEVEL

RECTANG CHANNEL

```

C COMPUTATION OF LONG DISP COEF IN FT*FT/SEC FOR RECTANG CHANNEL
C ISEC:NB OF STUDIED SECTIONS
C KK:SECTION LOCATION.....1,2 OR 3
C N:NB OF VERT SLICES
C DELTAZ:WIDTH OF EACH SLICE IN FT
C WIDTH:CHANNEL WIDTH IN FT
C SE:BOTTOM SLOPE
C DISP(1,1):VERTICAL SLICE BANK TO START OF SLICE IN FT
C DISP(1,2):DIST FROM LEFT BANK TO START OF SLICE IN FT
C DISP(1,3):DEPTH AT LEFT SIDE OF SLICE IN FT
C DISP(1,4):MEAN VELOCITY IN SLICE IN FT/SEC
C DEBIT:TOTAL DEBIT IN FT*FT*FT*FT/SEC
C FO:TOTAL CROSSECTIONAL AREA IN FTFT
C RH:HYDRAULIC RADIUS IN FT
C SHEARU: SHEAR VELOCITY IN FT/SEC
C UMEAN:MEAN FLOW VEL. TH THE ENTIRE CROSS*SECTION IN FT/SEC
C REY:REYNOLDS NUMBER
C RP:PRODUCT REYNOLDS AND PRANDTL NUMBERS
C FR:FROUDE NUMBER
C B: DIMENSIONLESS B NUMBER
C C: DIMENSIONLESS C NUMBER
C D: DISP(N,3)
C E: DISP(N,3)
C F: DISP(N,4)
C S: UMEAN
C T: WIDTH
C DIMENSION DISP(240,4)
C FONC(D,E,F)=0.5*(D+E)*(F-UMEAN)*DELTAZ
C PR(S,T)=S/((32.172*T)**0.5)
C REY(S,T)=S*T/1.162E*5
C RP(S,T)=S*T*6.48456E5
C B(T)=3.526E*2*T
C C(T)=5.457E*2*T

```

0001  
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0003  
0004  
0005  
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0007

WRITE(3,1000)  
FORMAT(11,40X,'COMPUTATION OF LONG DISP COEF IN FT-FT/SEC')

1000  
READ(1,3)ISEC  
FORMAT(I1)  
WRITE(3,4)ISEC  
FORMAT(///,75X,I1)

3  
4  
5  
KSEC=1  
WRITE(3,6)

6  
FORMAT(11,1X,'SECTION1')  
READ(1,10)KK,N,DELTAZ,WIDTH  
FORMAT(I1,3X,I2,8X,F3.1,7X,F3.1)

10  
5100  
READ(1,5100)SE  
FORMAT(F8,6)  
WRITE(3,715)KK  
FORMAT(1+1,9X,I1)

715  
DO 30 M=1,N  
READ(1,20)(DISP(M,L),L=1,4)  
FORMAT(2X,F4.1,7X,F3.1,6X,F7.5,3X,F7.5)

20  
30  
CONTINUE  
FUNA=DELTAZ\*(DISP(1,3)+DISP(2,3))/2.  
FUNB=DELTAZ\*DISP(N,3)  
DEBIT=PUNA\*DISP(1,4)+FUNB\*DISP(N,4)  
FUN=FUNA+FUNB  
NN=N+1

40  
DO 40 M=2,NN  
DEBIT=DEBIT+(DELTAZ\*(DISP(M,3)+DISP(M+1,3))\*DISP(M,4)/2.)  
CONTINUE  
FO=FUN

50  
DO 50 M=2,NN  
FO=FO+(DELTAZ\*(DISP(M,3)+DISP(M+1,3))/2.)  
CONTINUE  
RH=FO/(WIDTH+DISP(1,3)+DISP(N,3))  
SHEARU=(RH\*SE\*32,172)\*\*0.5  
UMEAN=DEBIT/FO

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0043

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0044 ENQ=0.
0045 TNT=0.
0046 BNB=FUNC(DISP(1,3),DISP(2,3),DISP(1,4))
0047 SUM=BNB
0048 RIZ=0.
0049 WRITE(3,300)
0050 FORMAT(/,1X,1VERTICAL SLICE1,5X,1DISTANCE1,5X,1DEPTH1,5X,1MEAN VE
300 ILOCALITY1,5X,1DISCHARGE1,5X,1CUMULATIVE RELATIVE DISCHARGE1,5X,1SUM
I IN BRACKETS1)
0051 WRITE(3,541)DISP(1,1),DISP(1,2),DISP(1,3),DISP(1,4),BNB,SUM,RIZ
541 FORMAT(/,F9.1,F16.1,F14.5,F14.5,F16.5,F25.5,F27.5)
DO 60 M=2,NN
0052 ONO=FUNC(DISP(M-1,3),DISP(M,3),DISP(M+1,4))
0053 TNT=TNT+ONO
0054 ANA=FUNC(DISP(M,3),DISP(M+1,3),DISP(M,4))
0055 SUM=SUM+ANA
0056 TRA=0.23*DISP(M,3)*SHEARU
0057 TOL=DELTAZ*TNT/(TRA*DISP(M,3))
0058 RIZ=RIZ+TOL
0059 DEN=ANA*RIZ
0060 ENO=ENO+DEN
0061 WRITE(3,540)DISP(M,1),DISP(M,2),DISP(M,3),DISP(M,4),ANA,SUM,RIZ
540 FORMAT(F9.1,F16.1,F14.5,F14.5,F16.5,F25.5,F27.5)
60 CONTINUE
ANA=DISP(N,3)*(DISP(N,4)-UMEAN)*DELTAZ
SUM=SUM+ANA
TRA=0.23*DISP(N,3)*SHEARU
TOL=DELTAZ*TNT/(TRA*DISP(N,3))
RIZ=RIZ+TOL
WRITE(3,540)DISP(N,1),DISP(N,2),DISP(N,3),DISP(N,4),ANA,SUM,RIZ
DEN=ANA*RIZ
ENO=ENO+DEN
COEFD=1.*ENO/FO
KSEC=KSEC+1
WRITE(3,600)
0071
0072
0073
0074
0075
0076

```

```

0077 600  FORMAT(/,4X,'DEBIT',10X,'AREA',10X,'MEAN CROSS VELOCITY',10X,'HYD
0078 610  1RADIUS',10X,'LONG DISP COEF',10X,'SHEAR VEL')
0079 620  WRITE(3,610)DEBIT,FO,UMEAN,RH,COEFD,SHEARU
0080 630  FORMAT(F10.5,F14.5,F22.5,F24.5,F23.5,F21.5)
0081 640  WRITE(3,620)
0082 650  FORMAT(/,4X,'AREA*LONG DISP COEF')
0083 660  FAT#FO*COEFD
0084 670  WRITE(3,630)FAT
0085 680  FORMAT(F17.5)
0086 690  WRITE(3,640)
0087 700  FORMAT(/,8X,'FRI',10X,'REY',10X,'RPI',15X,'B',10X,'C')
0088 710  GA#FR(UMEAN,WIDTH)
0089 720  GE#REY(UMEAN,WIDTH)
0090 730  GI#RP(UMEAN,WIDTH)
0091 740  GR#B(WIDTH)
0092 750  GU#C(WIDTH)
0093 760  WRITE(3,650)GA,GE,GI,GR,GU
0094 770  FORMAT(F12.5,F15.3,F14.3,F14.6,F11.6)
0095 780  IF(KSEC,EO,3)GO TO 652
0096 790  IF(KSEC,EO,4)GO TO 712
0097 800  IF(ISEC,GT,1)GO TO 700
0098 810  IF(ISEC,EO,1)GO TO 774
0099 820  DEBIT=(DEBIT+TEBIT+BEDIT)/3,
0100 830  UMEAN#(UMEAN+RAR+TAR)/3,
0101 840  GO TO 713
0102 850  BEDIT#DEBIT
0103 860  TAR#UMEAN
0104 870  GO TO 1
0105 880  TEBIT#DEBIT
0106 890  RAR#UMEAN
0107 900  IF(ISEC,EO,3)GO TO 1
0108 910  DEBIT=(DEBIT+BEDIT)*0.5
0109 920  UMEAN#(UMEAN+TAR)*0.5
0110 930  WRITE(3,653)
0111 940  FORMAT(11,1X,'MEAN VALUES')
0112 950  WRITE(3,654)

```

```

FORMAT(//,4X,'DEBIT',10X,'MEAN CROSS SECTIONAL VELOCITY')
WRITE(3,655)DEBIT,UMEAN
FORMAT(F10,5,F26,5)
WRITE(3,640)
GA=FR(UMEAN,WIDTH)
GE=REY(UMEAN,WIDTH)
GI=RP(UMEAN,WIDTH)
GR=B(WIDTH)
GU=C(WIDTH)
WRITE(3,650)GA,GE,GI,GR,GU
RETURN
END

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654

655

774

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APPENDIX B

Development of Eq. (3.5.3.1b)

Recall: Any "o" in exponent refers to a dimensionless quantity.

Any "o" in subscript refers to a reference quantity.

Starting with the equation of energy as given by Eq. (3.3.4)

$$\frac{\partial}{\partial z} \left( D \frac{\partial T_w}{\partial x} \right) - U \frac{\partial T_w}{\partial z} - \mu \left[ (T_w - T_a) + \frac{F''}{H} + \frac{F'}{H} \right] = 0 \quad (1)$$

with  $\mu = \frac{H}{C_p \rho} \times \frac{\ell}{A}$

and using Eqs. (3.5.1.2) and (3.5.3.1) it is found:

$$\frac{\partial}{\partial (x^o L_o)} \left[ D^o D_o \frac{\partial (T_w^o T_o)}{\partial (x^o L_o)} \right] - U^o U_o \frac{\partial (T_w^o T_o)}{\partial (x^o L_o)} - \frac{H^o H_o}{C_p^o C_{p_o} \rho_o} \frac{1}{\rho_o \rho_o} \frac{\ell^o L_o}{A^o L_o^2} \left[ (T_w^o T_o - T_a^o T_o) + \frac{F''^o F_o}{H^o H_o} + \frac{F'^o F_o}{H^o H_o} \right] = 0$$

Rearranging, it is found:

$$\frac{1}{L_o} \frac{\partial}{\partial x^o} \frac{D_o T_o}{L_o} \left( D^o \frac{\partial T_w^o}{\partial x^o} \right) - \left( \frac{U_o T_o}{L_o} \right) U^o \frac{\partial T_w^o}{\partial x^o} - \left( \frac{H_o}{C_p^o \rho_o} \frac{1}{L_o} \right) \frac{H^o \ell^o}{C_p^o \rho_o^o A^o} \left[ T_o (T_w^o - T_a^o) + \frac{F_o}{H_o} \left( \frac{F''^o}{H^o} + \frac{F'^o}{H^o} \right) \right] = 0$$

Multiplying by  $\frac{L_o^2}{D_o T_o}$  and rearranging, it is found:

$$\frac{\partial}{\partial x^o} \left( D^o \frac{\partial T_w^o}{\partial x^o} \right) - \left[ \frac{U_o L_o}{D_o} \right] U^o \frac{\partial T_w^o}{\partial x^o} - \left[ \frac{H_o L_o}{C_{p_o} \rho_o D_o} \right] \mu^o (T_w^o - T_a^o) - \left[ \frac{F_o L_o}{C_{p_o} \rho_o D_o T_o} \right] \mu^o \left( \frac{F''^o}{H_o} + \frac{F'^o}{H_o} \right) = 0 \quad (2)$$

which is the dimension less form of equation (1)

Because

$$D_o = \frac{K_o}{\rho_o C_{p_o}} ; \text{molecular diffusivity of water}$$

$$K_o = \text{thermal conductivity of water}$$

it can be seen that:

$$\frac{U_o L_o}{D_o} = RP ; \text{product of Reynolds and Prandtl numbers}$$

$$\frac{H_o L_o}{C_{p_o} \rho_o D_o} = \frac{H_o L_o}{K_o} = N ; \text{Nusselt number}$$

$$\frac{F_o L_o}{C_{p_o} \rho_o D_o T_o} = \frac{F_o L_o}{K_o T_o} , \text{let B, a dimensionless number}$$

From Eq. (1), it can be seen that the expression in brackets is the energy storage, as a matter of fact:

$$\mu \left[ (T_w - T_a) + \frac{F''}{H} + \frac{F'}{H'} \right] = \left( \frac{\ell}{A C_p \rho} \right) [H(T_w - T_a) + F'' + F']$$

According to Eq. (3.1.19), H and F'' are independent of the difference in temperature between air and water.

According to the same equation,

$$F' = E\sigma \left[ 6T_a^2 (T_w - T_a)^2 + 4T_a (T_w - T_a)^3 + (T_w - T_a)^4 \right]$$

By the same procedure as above, this equation can be written under a dimensionless form:

$$F'{}^o = \left[ \frac{E_o \sigma_o T_o^4}{F_o} \right] E_o \sigma_o [6T_a^o{}^2 (T_w^o - T_a^o)^2 + 4T_a^o (T_w^o - T_a^o)^3 + (T_w^o - T_a^o)^4] \quad (3)$$

Eqs. (2) and (3) give the final result:

$$0 = \frac{\partial}{\partial x^o} \left( D^o \frac{\partial T_w^o}{\partial x^o} \right) - [RP] U^o \frac{\partial T_w^o}{\partial x^o} - [N] \mu^o (T_w^o - T_a^o) - [B] \mu^o \frac{F''^o}{H^o} - [C] \frac{\mu^o E_o \sigma_o}{H^o} (6T_a^o{}^2 (T_w^o - T_a^o)^2 + 4T_a^o (T_w^o - T_a^o)^3 + (T_w^o - T_a^o)^4)$$

where

$$C = \frac{T_o^3 L_o \sigma_o E_o}{K_o} \quad \text{a dimensionless number.}$$

APPENDIX C

In all these data sheets:

DIST = Distance from the left bank of the channel  
to the left side of the study vertical slice,  
looking downstream, ft.

DEPTH = Water depth at the left side of the vertical  
slice, looking downstream, ft.

VELOCITY = Mean velocity through the vertical slice,  
ft/sec.

DEBIT = Water flow in the study channel, ft<sup>3</sup>/sec.

APPENDIX C

CHANNEL WIDTH : 3.0 FT

SLICE	DIST.	SECTION 3		SECTION 2		SECTION 1	
		DEPTH	VELOCITY	DEPTH	VELOCITY	DEPTH	VELOCITY
1.0	0.0	0.14764	1.47098	0.21489	1.48881	0.20340	1.52385
2.0	0.2	0.15748	1.52385	0.22145	1.48881	0.20177	1.52385
3.0	0.4	0.15912	1.62442	0.21653	1.48881	0.20341	1.59160
4.0	0.6	0.17388	1.48881	0.21981	1.48881	0.20505	1.52385
5.0	0.8	0.17716	1.55809	0.21981	1.48881	0.20997	1.48881
6.0	1.0	0.17716	1.52384	0.21981	1.48881	0.20997	1.52385
7.0	1.2	0.16732	1.52384	0.21981	1.48881	0.20997	1.52385
8.0	1.4	0.16404	1.48881	0.21981	1.55809	0.21489	1.52385
9.0	1.6	0.15420	1.45293	0.21981	1.55809	0.21161	1.55809
10.0	1.8	0.15748	1.48881	0.21653	1.55809	0.21161	1.52385
11.0	2.0	0.14764	1.52384	0.21653	1.55809	0.21161	1.52385
12.0	2.2	0.14272	1.65659	0.21489	1.52384	0.21161	1.52385
13.0	2.4	0.14764	1.55809	0.21653	1.37837	0.21817	1.52385
14.0	2.6	0.15256	1.55809	0.20997	1.21561	0.21489	1.45293
15.0	2.8	0.15092	1.55809	0.20669	1.21561	0.21653	1.17139

AIR VELOCITY : 2.96 MPH  
 AIR TEMP DEG C : 142  
 116

127  
 106  
 118  
 99

WATER TEMP DEG C : 22.08

22.05  
 22.05

DEBIT	F	R	RP	N	B	C
0.83852	0.15431	391400	2949225	551	0.10578	0.16371

CHANNEL WIDTH : 3.0 FT

SLICE	DIST.	SECTION 3		SECTION 2		SECTION 1	
		DEPTH	VELOCITY	DEPTH	VELOCITY	DEPTH	VELOCITY
1.0	0.0	0.12303	1.65660	0.18865	1.33953	0.15584	1.48881
2.0	0.2	0.13123	1.65660	0.19521	1.33953	0.16896	1.45293
3.0	0.4	0.13123	1.74956	0.19357	1.37837	0.17388	1.41614
4.0	0.6	0.13451	1.62442	0.19685	1.37837	0.17716	1.41614
5.0	0.8	0.14272	1.65660	0.19521	1.37837	0.17716	1.41614
6.0	1.0	0.14600	1.68815	0.19685	1.37837	0.17716	1.37837
7.0	1.2	0.14764	1.68815	0.20013	1.37837	0.18044	1.37837
8.0	1.4	0.15912	1.71913	0.19849	1.41614	0.18209	1.45293
9.0	1.6	0.15912	1.68815	0.19521	1.37837	0.18044	1.41614
10.0	1.8	0.15912	1.70061	0.19029	1.43097	0.17880	1.41614
11.0	2.0	0.16240	1.68815	0.19357	1.41614	0.18209	1.41614
12.0	2.2	0.14928	1.59160	0.19357	1.35520	0.18209	1.37837
13.0	2.4	0.13780	1.62442	0.19029	1.29954	0.18537	1.37837
14.0	2.6	0.13287	1.55809	0.18701	1.17139	0.18865	1.29954
15.0	2.8	0.13123	1.62442	0.18209	1.21561	0.18701	0.91891

AIR VELOCITY : 2.77 MPH  
 AIR TEMP DEG C : 150  
 121

131  
 110

122  
 98

WATER TEMP DEG C : 22.58

22.54

22.52

DEBIT	F	R	RP	N	B	C
0.72730	0.15448	391814	2952342	722	0.10578	0.16371

CHANNEL WIDTH : 3.0 FT

SLICE	DIST.	SECTION 3		SECTION 2		SECTION 1	
		DEPTH	VELOCITY	DEPTH	VELOCITY	DEPTH	VELOCITY
1.0	0.0	0.08858	1.17139	0.13779	1.02737	0.11155	0.95275
2.0	0.2	0.09350	1.24138	0.14435	1.07752	0.11975	1.07752
3.0	0.4	0.09350	1.21561	0.14435	1.02737	0.12467	1.07752
4.0	0.6	0.09186	1.21561	0.14272	1.07752	0.12959	1.05775
5.0	0.8	0.10170	1.17139	0.14337	1.07752	0.12959	1.05775
6.0	1.0	0.10499	1.17139	0.14435	1.07752	0.13123	1.05775
7.0	1.2	0.10499	1.21561	0.14764	1.02737	0.13615	1.05775
8.0	1.4	0.10662	1.21561	0.14600	1.02737	0.13615	1.05775
9.0	1.6	0.10499	1.25827	0.14829	1.07752	0.13451	1.05775
10.0	1.8	0.10499	1.25827	0.14600	1.07752	0.13451	1.05775
11.0	2.0	0.09842	1.21561	0.14107	1.02737	0.13615	1.02738
12.0	2.2	0.09350	1.21561	0.14107	1.02737	0.13615	1.02738
13.0	2.4	0.09350	1.21561	0.13845	0.97465	0.13943	1.00662
14.0	2.6	0.09514	1.17139	0.13779	0.72646	0.14107	1.00662
15.0	2.8	0.09350	1.21561	0.13287	0.72646	0.14107	0.83464

AIR VELOCITY : 2.62 MPH  
 AIR TEMP DEG C : 148  
 116

130  
 105

122  
 98

WATER TEMP DEG C : 23.05

22.98

22.95

DEBIT	F	R	RP	N	B	C
0.38337	0.11392	288951	2177260	667	0.10578	0.16371

CHANNEL WIDTH : 3.0 FT

SLICE	DIST.	SECTION 3		SECTION 2		SECTION 1	
		DEPTH	VELOCITY	DEPTH	VELOCITY	DEPTH	VELOCITY
1.0	0.0	0.06233	1.04772	0.10827	0.82190	0.07874	0.85956
2.0	0.2	0.07054	1.07752	0.11483	0.82190	0.08530	0.79580
3.0	0.4	0.07382	1.07752	0.10991	0.82190	0.08858	0.89564
4.0	0.6	0.07218	0.91891	0.11155	0.82190	0.09514	0.89564
5.0	0.8	0.07677	1.02737	0.11155	0.82190	0.09514	0.89564
6.0	1.0	0.08038	0.97465	0.11155	0.82190	0.09842	0.89564
7.0	1.2	0.07874	0.97465	0.11483	0.82190	0.09842	0.89564
8.0	1.4	0.07546	0.97465	0.11483	0.82190	0.10006	0.83464
9.0	1.6	0.07874	0.95275	0.11155	0.82190	0.10006	0.83464
10.0	1.8	0.07710	0.95275	0.10991	0.82190	0.09842	0.83464
11.0	2.0	0.07546	0.91891	0.10991	0.82190	0.09842	0.83464
12.0	2.2	0.07054	0.91891	0.10991	0.82190	0.09842	0.83464
13.0	2.4	0.07054	0.91891	0.10335	0.83464	0.10170	0.83464
14.0	2.6	0.06726	0.91891	0.10335	0.69680	0.10498	0.83464
15.0	2.8	0.06233	0.79580	0.10170	0.64977	0.10335	0.76882

AIR VELOCITY : 2.52 MPH  
 AIR TEMP DEG C : 144  
 117

128  
 106

121  
 98

WATER TEMP DEG C : 23.65

23.41

23.38

DEBIT	F	R	RP	N	B	C
0.22920	0.09235	234232	1764955	825	0.10580	0.16371

CHANNEL WIDTH : 3.4 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.12795	1.41614
2.0	0.2	0.13287	1.52384
3.0	0.4	0.13779	1.48881
4.0	0.6	0.14435	1.25827
5.0	0.8	0.14764	1.49588
6.0	1.0	0.15256	1.50993
7.0	1.2	0.14928	1.52384
8.0	1.4	0.15584	1.54448
9.0	1.6	0.16240	1.56486
10.0	1.8	0.16404	1.52384
11.0	2.0	0.15748	1.52384
12.0	2.2	0.14764	1.52384
13.0	2.4	0.14927	1.52384
14.0	2.6	0.14271	1.48881
15.0	2.8	0.13780	1.48881
16.0	3.0	0.12959	1.37837
17.0	3.2	0.12467	1.41614

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.16404	1.12534
0.16404	1.12534
0.17224	1.11602
0.17552	1.10652
0.17716	1.07752
0.18044	1.05775
0.18044	1.06768
0.18208	1.06768
0.17716	1.06768
0.18044	1.06768
0.18208	1.06768
0.18700	1.06768
0.19028	1.04772
0.19028	0.99608
0.18865	0.99608
0.18700	0.99608
0.18372	0.79580

AIR VELOCITY : 2.77 MPH  
 AIR TEMP DEG C : 147  
 119

130  
109

121  
100

WATER TEMP DEG C : 24.13

24.09

24.08

DEBIT F R  
 0.68771 0.12111 370617

RP  
2792621

N  
675

B C  
 0.11988 0.18553

CHANNEL WIDTH : 3.4 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.11483	1.25827
2.0	0.2	0.12139	1.35520
3.0	0.4	0.12795	1.41614
4.0	0.6	0.14271	0.91891
5.0	0.8	0.12795	1.25827
6.0	1.0	0.12959	1.33953
7.0	1.2	0.12795	1.25583
8.0	1.4	0.12139	1.21561
9.0	1.6	0.11811	1.21561
10.0	1.8	0.11975	1.25827
11.0	2.0	0.11811	1.21561
12.0	2.2	0.11647	1.21561
13.0	2.4	0.11155	1.25827
14.0	2.6	0.11155	1.29954
15.0	2.8	0.11647	1.37837
16.0	3.0	0.10663	1.21561
17.0	3.2	0.11483	1.21561

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.14107	1.07752
0.14927	1.01705
0.15420	1.02738
0.16076	1.01705
0.16404	1.01705
0.16404	1.00662
0.16732	1.00662
0.16568	0.98542
0.16404	0.99608
0.16404	0.99608
0.16658	0.99608
0.16896	1.02738
0.17224	1.02738
0.17552	0.97465
0.17060	0.91891
0.16732	0.97465
0.16896	0.64977

AIR VELOCITY : 2.71 MPH  
 AIR TEMP DEG C : 145  
 118

128  
108

120  
100

WATER TEMP DEG C : 24.19

24.15

24.13

DEBIT F R  
 0.53101 0.10677 326750

RP  
2462077

N  
653

B C  
 0.11988 0.18553

CHANNEL WIDTH : 3.4 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.07710	1.07752
2.0	0.2	0.08366	1.10652
3.0	0.4	0.08530	1.15323
4.0	0.6	0.09022	0.83464
5.0	0.8	0.08694	1.15323
6.0	1.0	0.08530	1.15323
7.0	1.2	0.09022	1.15323
8.0	1.4	0.08858	1.15323
9.0	1.6	0.09022	1.15323
10.0	1.8	0.09022	1.13477
11.0	2.0	0.08694	1.10652
12.0	2.2	0.08366	1.05775
13.0	2.4	0.08366	1.05775
14.0	2.6	0.08038	1.09694
15.0	2.8	0.07546	1.07752
16.0	3.0	0.07458	1.07752
17.0	3.2	0.07546	0.91891

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.09514	0.85956
0.10170	0.83464
0.10334	0.85956
0.10991	0.83464
0.10991	0.72646
0.11155	0.72646
0.11155	0.72646
0.11319	0.72646
0.11319	0.72646
0.11319	0.72646
0.11319	0.72646
0.11483	0.72646
0.11647	0.72646
0.11975	0.72646
0.12139	0.72646
0.12139	0.64977
0.11811	0.75496
0.11647	0.45946

AIR VELOCITY : 2.57 MPH  
 AIR TEMP DEG C : 146  
 117

128  
106

122  
100

WATER TEMP DEG C : 24.58

24.40

24.38

DEBIT	F	R	RP	N	B	C
0.29620	0.08707	266466	2007837	924	0.11988	0.18553

CHANNEL WIDTH : 3.4 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.05741	0.97465
2.0	0.2	0.06397	0.97465
3.0	0.4	0.06397	0.94161
4.0	0.6	0.06726	0.64977
5.0	0.8	0.06980	0.97465
6.0	1.0	0.07054	0.97465
7.0	1.2	0.07054	0.97465
8.0	1.4	0.07054	0.91891
9.0	1.6	0.07218	0.91891
10.0	1.8	0.07382	0.91891
11.0	2.0	0.07054	0.91891
12.0	2.2	0.06890	0.91891
13.0	2.4	0.06562	0.91891
14.0	2.6	0.06233	0.91891
15.0	2.8	0.05741	0.91891
16.0	3.0	0.05905	0.97465
17.0	3.2	0.05741	0.78243

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.07054	0.64977
0.07874	0.64977
0.08038	0.64977
0.08530	0.64977
0.08694	0.64977
0.09022	0.72646
0.09022	0.75496
0.09186	0.64977
0.09022	0.64977
0.09186	0.64977
0.09186	0.64977
0.09186	0.64977
0.09186	0.64977
0.09842	0.64977
0.10006	0.88378
0.09678	0.88378
0.09514	0.79580
0.09350	0.59906

AIR VELOCITY : 2.54 MPH  
 AIR TEMP DEG C : 142  
 114

131  
 105

122  
 100

WATER TEMP DEG C : 24.95

24.52

24.48

DEBIT F R  
 0.20962 0.07708 235895

RP N  
 1777487 1271

B C  
 0.11988 0.18553

CHANNEL WIDTH : 3.8 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.11155	1.46739
2.0	0.2	0.12139	1.59160
3.0	0.4	0.12139	1.59160
4.0	0.6	0.12631	1.48881
5.0	0.8	0.12795	1.55809
6.0	1.0	0.12959	1.55809
7.0	1.2	0.13451	1.55809
8.0	1.4	0.13615	1.52384
9.0	1.6	0.13615	1.55809
10.0	1.8	0.14435	1.55809
11.0	2.0	0.14107	1.57158
12.0	2.2	0.13615	1.55809
13.0	2.4	0.13615	1.55809
14.0	2.6	0.13451	1.59160
15.0	2.8	0.13287	1.57158
16.0	3.0	0.12795	1.52385
17.0	3.2	0.12139	1.52385
18.0	3.4	0.12139	1.33953
19.0	3.6	0.11319	1.29954

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.14107	1.24138
0.14928	1.24138
0.14928	1.21561
0.15584	1.22426
0.15912	1.22426
0.16240	1.21561
0.16404	1.17139
0.16404	1.17139
0.16502	1.17139
0.16240	1.17139
0.16404	1.24138
0.16568	1.24138
0.16896	1.19812
0.17060	1.19812
0.17060	1.18036
0.16896	1.18927
0.16568	1.14404
0.16404	1.14404
0.16404	1.09694

AIR VELOCITY : 2.72 MPH  
 AIR TEMP DEG C : 147  
 120

130  
109

122  
100

WATER TEMP DEG C : 23.41

23.38

23.38

DEBIT F R  
 0.74364 0.12310 445121

RP N  
 3354010 611

B C  
 0.13398 0.20736

CHANNEL WIDTH : 3.8 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.09678	1.18927
2.0	0.2	0.10006	1.29954
3.0	0.4	0.10006	1.29954
4.0	0.6	0.09678	0.91891
5.0	0.8	0.10663	1.33953
6.0	1.0	0.10334	1.25827
7.0	1.2	0.10827	1.25827
8.0	1.4	0.10991	1.24986
9.0	1.6	0.11319	1.21561
10.0	1.8	0.11319	1.18927
11.0	2.0	0.10827	1.25827
12.0	2.2	0.10663	1.25827
13.0	2.4	0.10006	1.18927
14.0	2.6	0.10498	1.18927
15.0	2.8	0.10006	1.18927
16.0	3.0	0.09514	1.17139
17.0	3.2	0.09186	1.17139
18.0	3.4	0.09350	1.17139
19.0	3.6	0.09350	1.12543

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.11483	1.05774
0.12467	1.02738
0.12631	1.02738
0.13287	0.99608
0.13451	0.98542
0.13779	0.97465
0.13943	0.97465
0.13943	0.96376
0.13943	0.94161
1.13779	0.94161
0.13943	0.95275
0.14107	0.95275
0.14599	0.95275
0.14599	0.94161
0.14271	0.94161
0.14271	0.94161
0.13943	0.93033
0.13615	0.89565
0.13779	0.83464

AIR VELOCITY : 2.71 MPH  
 AIR TEMP DEG C : 147  
 117

129  
109

122  
100

WATER TEMP DEG C : 23.53

23.44

23.41

DEBIT F R  
 0.57887 0.09780 353621

RP N  
 2664552 1021

B C  
 0.13398 0.20736

CHANNEL WIDTH : 3.8 FT

		SECTION 3	
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.06726	1.02738
2.0	0.2	0.07546	1.04772
3.0	0.4	0.07546	1.01705
4.0	0.6	0.07546	1.01705
5.0	0.8	0.08202	0.99608
6.0	1.0	0.08202	0.99608
7.0	1.2	0.08202	0.99608
8.0	1.4	0.08202	0.99608
9.0	1.6	0.08366	0.99608
10.0	1.8	0.08530	0.99608
11.0	2.0	0.08038	0.99608
12.0	2.2	0.07874	0.99608
13.0	2.4	0.08034	0.97465
14.0	2.6	0.07874	0.94160
15.0	2.8	0.07546	0.94160
16.0	3.0	0.07546	0.98542
17.0	3.2	0.07382	1.02738
18.0	3.4	0.07218	0.97465
19.0	3.6	0.06890	0.89565

SECTION 2

		SECTION 1	
DEPTH	VELOCITY	DEPTH	VELOCITY
0.08202	0.85956		
0.09186	0.89565		
0.09514	0.91891		
0.09842	0.88378		
0.10334	0.85956		
0.10170	0.85956		
0.10663	0.79580		
0.10663	0.79580		
0.10498	0.82190		
0.10663	0.82190		
0.10827	0.82190		
0.10827	0.82190		
0.11319	0.82190		
0.11483	0.82190		
0.11319	0.82190		
0.10991	0.85956		
0.10991	0.85956		
0.10663	0.85956		
0.10663	0.79580		

AIR VELOCITY : 2.55 MPH  
 AIR TEMP DEG C : 145  
 117

128  
 108

120  
 100

WATER TEMP DEG C : 23.92

23.76

23.72

DEBIT F R RP N  
 0.31438 0.08283 299483 2256619 955

B C  
 0.13398 0.20736

CHANNEL WIDTH : 3.8 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.05085	0.85956
2.0	0.2	0.05741	0.89565
3.0	0.4	0.05577	0.91891
4.0	0.6	0.05741	0.64977
5.0	0.8	0.05741	0.89565
6.0	1.0	0.06069	0.89565
7.0	1.2	0.06069	0.89565
8.0	1.4	0.06069	0.89565
9.0	1.6	0.06069	0.89565
10.0	1.8	0.06397	0.82190
11.0	2.0	0.06069	0.82190
12.0	2.2	0.05905	0.82190
13.0	2.4	0.05741	0.82190
14.0	2.6	0.05413	0.79580
15.0	2.8	0.05085	0.79580
16.0	3.0	0.04921	0.76882
17.0	3.2	0.05249	0.78243
18.0	3.4	0.05246	0.75496
19.0	3.6	0.04921	0.75496

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.06069	0.41095
0.06693	0.45956
0.06988	0.50331
0.07382	0.52386
0.07710	0.52386
0.07710	0.52386
0.08038	0.52386
0.08038	0.52386
0.08038	0.52386
0.08038	0.52386
0.07874	0.52386
0.08038	0.52386
0.08202	0.45946
0.08530	0.45946
0.08694	0.52386
0.08530	0.52386
0.08366	0.52386
0.08038	0.52386
0.08038	0.56272
0.08202	0.56272

AIR VELOCITY : 2.47 MPH  
 AIR TEMP DEG C : 143  
 114

126  
105

121  
109

WATER TEMP DEG C : 24.32

23.90

23.85

DEBIT F R  
 0.16591 0.06075 219660

RP N  
 1655150 1026

B C  
 0.13398 0.20736

CHANNEL WIDTH : 4.2 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.10170	1.17138
2.0	0.2	0.10499	1.24138
3.0	0.4	0.10499	1.25827
4.0	0.6	0.10663	0.79580
5.0	0.8	0.10499	1.21561
6.0	1.0	0.10663	1.25827
7.0	1.2	0.11811	1.29954
8.0	1.4	0.11975	1.33953
9.0	1.6	0.12795	1.33953
10.0	1.8	0.12533	1.33953
11.0	2.0	0.12631	1.25827
12.0	2.2	0.12467	1.28319
13.0	2.4	0.12139	1.29954
14.0	2.6	0.11319	1.27907
15.0	2.8	0.10827	1.25827
16.0	3.0	0.10006	1.25827
17.0	3.2	0.09350	1.25827
18.0	3.4	0.09186	1.17139
19.0	3.6	0.08694	1.15323
20.0	3.8	0.08366	1.17139
21.0	4.0	0.08694	1.02738

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.12631	1.02738
0.13287	1.02738
0.13943	0.97465
0.14436	0.99608
0.14764	0.99608
0.14928	0.99608
0.15092	0.98542
0.14928	0.98542
0.15092	0.97465
0.14928	0.97465
0.15092	0.99608
0.15420	0.99608
0.15584	0.99608
0.15912	0.98542
0.15584	0.91891
0.15420	0.91891
0.15092	0.91891
0.15256	0.91891
0.14928	0.91891
0.14928	0.91891
0.14928	0.45946

AIR VELOCITY : 2.67 MPH  
 AIR TEMP DEG C : 145  
 118

131  
107

122  
100

WATER TEMP DEG C : 24.28

24.24

24.21

DEBIT	F	R	RP	N	B	C
0.57336	0.09362	393362	2964008	823	0.14809	0.22919

CHANNEL WIDTH : 4.2 FT

		SECTION 3	
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.08366	1.02738
2.0	0.2	0.08858	1.17139
3.0	0.4	0.09022	0.95275
4.0	0.6	0.08694	0.61642
5.0	0.8	0.10663	1.15323
6.0	1.0	0.10335	1.17139
7.0	1.2	0.09842	1.17139
8.0	1.4	0.09842	1.17139
9.0	1.6	0.09678	1.17139
10.0	1.8	0.09842	1.17139
11.0	2.0	0.09350	1.12543
12.0	2.2	0.08694	1.12543
13.0	2.4	0.08858	1.09694
14.0	2.6	0.08530	1.12543
15.0	2.8	0.08136	1.12543
16.0	3.0	0.08448	1.12543
17.0	3.2	0.08530	1.17139
18.0	3.4	0.08530	1.18927
19.0	3.6	0.08694	1.21561
20.0	3.8	0.09186	1.07752
21.0	4.0	0.07710	0.97465

SECTION 2

		SECTION 1	
DEPTH	VELOCITY	DEPTH	VELOCITY
0.10663	0.88378	0.10663	0.88378
0.11483	0.88378	0.11483	0.88378
0.11647	0.85956	0.11647	0.85956
0.12139	0.85956	0.12139	0.85956
0.12467	0.79580	0.12467	0.79580
0.12467	0.79580	0.12467	0.79580
0.12631	0.79580	0.12631	0.79580
0.12795	0.79580	0.12795	0.79580
0.12631	0.79580	0.12631	0.79580
0.12795	0.79580	0.12795	0.79580
0.12959	0.79580	0.12959	0.79580
0.12959	0.79580	0.12959	0.79580
0.13451	0.79580	0.13451	0.79580
0.13451	0.79580	0.13451	0.79580
0.13451	0.79580	0.13451	0.79580
0.12959	0.72646	0.12959	0.72646
0.12959	0.72646	0.12959	0.72646
0.12959	0.72646	0.12959	0.72646
0.12795	0.72646	0.12795	0.72646
0.12795	0.72646	0.12795	0.72646
0.13287	0.45946	0.13287	0.45946

AIR VELOCITY : 2.60 MPH  
 AIR TEMP DEG C : 146  
 118

128  
 107

124  
 101

WATER TEMP DEG C : 24.34

24.24

24.20

DEBIT F R  
 0.41594 0.08077 339338

RP  
 2556932

N  
 993

B C  
 0.14809 0.22919

CHANNEL WIDTH : 4.2 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.06562	0.97465
2.0	0.2	0.07382	0.97465
3.0	0.4	0.07382	1.02737
4.0	0.6	0.07874	0.72646
5.0	0.8	0.08120	1.02737
6.0	1.0	0.08202	1.03760
7.0	1.2	0.08038	1.03760
8.0	1.4	0.08202	1.03760
9.0	1.6	0.08366	1.04772
10.0	1.8	0.08366	1.04772
11.0	2.0	0.08038	1.00662
12.0	2.2	0.07546	1.00662
13.0	2.4	0.07382	0.99610
14.0	2.6	0.07213	0.99610
15.0	2.8	0.06890	0.94161
16.0	3.0	0.06562	0.94161
17.0	3.2	0.06562	0.97465
18.0	3.4	0.06562	0.95275
19.0	3.6	0.06562	0.93033
20.0	3.8	0.06398	0.89565
21.0	4.0	0.06398	0.85956

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.08694	0.72646
0.09186	0.75496
0.09514	0.74085
0.10170	0.72646
0.10007	0.72646
0.10663	0.72646
0.10499	0.74085
0.10499	0.74085
0.10499	0.74085
0.10499	0.74085
0.10499	0.74085
0.10827	0.74085
0.11155	0.72646
0.11319	0.72646
0.11155	0.72646
0.10991	0.64977
0.10663	0.64977
0.10663	0.64977
0.10499	0.68148
0.10499	0.69680
0.10335	0.32488

AIR VELOCITY : 2.54 MPH  
 AIR TEMP DEG C : 144  
 116

127  
 106

120  
 100

WATER TEMP DEG C : 24.78

24.57

24.53

DEBIT F R  
 0.30440 0.07206 302772

RP N  
 2281404 1170

B C  
 0.14809 0.22919

CHANNEL WIDTH : 4.2 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.04921	0.97465
2.0	0.2	0.05577	0.97465
3.0	0.4	0.05577	0.85956
4.0	0.6	0.05741	0.45945
5.0	0.8	0.06069	0.83464
6.0	1.0	0.06069	0.85956
7.0	1.2	0.05905	0.85956
8.0	1.4	0.06069	0.85956
9.0	1.6	0.06233	0.82190
10.0	1.8	0.06233	0.79580
11.0	2.0	0.05741	0.79580
12.0	2.2	0.05577	0.79580
13.0	2.4	0.05413	0.79580
14.0	2.6	0.05249	0.79580
15.0	2.8	0.04921	0.79580
16.0	3.0	0.04593	0.83464
17.0	3.2	0.04593	0.80896
18.0	3.4	0.04757	0.79580
19.0	3.6	0.04265	0.82190
20.0	3.8	0.03937	0.85956
21.0	4.0	0.03937	0.79580

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.05905	0.52386
0.06562	0.41095
0.06890	0.56272
0.07218	0.52386
0.07382	0.52386
0.07546	0.56272
0.07710	0.56272
0.07874	0.63332
0.07874	0.63332
0.07710	0.63332
0.07874	0.63332
0.08038	0.63332
0.08366	0.63332
0.08858	0.63332
0.08366	0.63332
0.08202	0.63332
0.08038	0.64977
0.07710	0.58117
0.07710	0.52386
0.07874	0.61642
0.08202	0.45945

AIR VELOCITY : 2.47 MPH  
 AIR TEMP DEG C : 140  
 114

128  
105

120  
100

WATER TEMP DEG C : 25.00

24.71

24.65

DEBIT F R  
 0.18590 0.06030 253346

RP N  
 1908980 1056

B C  
 0.14809 0.22919

CHANNEL WIDTH : 4.6 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.09021	1.05775
2.0	0.2	0.09677	1.05775
3.0	0.4	0.09677	1.11602
4.0	0.6	0.09677	0.63332
5.0	0.8	0.10169	1.19811
6.0	1.0	0.10825	1.15323
7.0	1.2	0.10005	1.17139
8.0	1.4	0.10005	1.18927
9.0	1.6	0.10333	1.18036
10.0	1.8	0.10497	1.17139
11.0	2.0	0.10333	1.14404
12.0	2.2	0.10005	1.12543
13.0	2.4	0.10169	1.08727
14.0	2.6	0.09513	1.04772
15.0	2.8	0.09185	1.04772
16.0	3.0	0.08365	1.04772
17.0	3.2	0.08529	1.12543
18.0	3.4	0.08201	1.15323
19.0	3.6	0.07873	1.23285
20.0	3.8	0.07544	1.17139
21.0	4.0	0.07381	1.21561
22.0	4.2	0.07544	1.21561
23.0	4.4	0.07709	0.85956

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.10825	0.85956
0.11481	0.88378
0.11809	0.85956
0.12301	0.85956
0.12465	0.85956
0.12793	0.83464
0.12793	0.83464
0.12793	0.83464
0.12793	0.83464
0.12793	0.83464
0.12957	0.83464
0.13285	0.83464
0.13613	0.83464
0.13777	0.79580
0.13449	0.76882
0.13121	0.79580
0.13285	0.85956
0.12793	0.85956
0.12957	0.85956
0.12629	0.82190
0.12957	0.79580
0.12957	0.82190
0.12793	0.45945

AIR VELOCITY : 2.61 MPH  
 AIR TEMP DEG C : 141  
 112

124  
 104

119  
 98

WATER TEMP DEG C : 24.80

24.75

24.73

DEBIT F R RP N  
 0.47444 0.07900 380434 2866594 778

B C  
 0.16219 0.25102

CHANNEL WIDTH : 4.6 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.07546	1.07752
2.0	0.2	0.08202	1.12543
3.0	0.4	0.08366	1.14404
4.0	0.6	0.08858	0.85956
5.0	0.8	0.08858	1.17139
6.0	1.0	0.08858	1.18927
7.0	1.2	0.09350	1.17139
8.0	1.4	0.09186	1.12543
9.0	1.6	0.09022	1.07752
10.0	1.8	0.09022	1.06768
11.0	2.0	0.08858	1.05775
12.0	2.2	0.08858	1.07752
13.0	2.4	0.08694	1.00662
14.0	2.6	0.08202	1.02738
15.0	2.8	0.07710	1.05775
16.0	3.0	0.07382	1.14404
17.0	3.2	0.07054	1.04772
18.0	3.4	0.06890	1.19812
19.0	3.6	0.06890	1.18036
20.0	3.8	0.06726	1.18927
21.0	4.0	0.06562	1.18927
22.0	4.2	0.06726	1.17139
23.0	4.4	0.06726	0.94161

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.09678	0.79580
0.10170	0.85956
0.10663	0.85956
0.10991	0.82190
0.11319	0.76882
0.11483	0.76882
0.11647	0.75496
0.11483	0.75496
0.11319	0.75496
0.11319	0.75496
0.11647	0.75496
0.11975	0.75496
0.12467	0.74085
0.12631	0.72646
0.12139	0.72646
0.11811	0.72646
0.11975	0.72646
0.11647	0.74085
0.11647	0.72646
0.11483	0.72646
0.11811	0.75496
0.11811	0.75496
0.11975	0.56272

AIR VELOCITY : 2.57 MPH  
 AIR TEMP DEG C : 140  
 112

123  
 102

118  
 96

WATER TEMP DEG C : 24.92

24.81

24.79

DEBIT	F	R	RP	N	B	C
0.40222	0.07601	366054	2758238	971	0.16219	0.25102

CHANNEL WIDTH : 4.6 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.05085	0.85956
2.0	0.2	0.05413	0.91891
3.0	0.4	0.05905	0.91891
4.0	0.6	0.06069	0.41095
5.0	0.8	0.06233	0.93033
6.0	1.0	0.06398	0.91891
7.0	1.2	0.06398	0.90152
8.0	1.4	0.06069	0.88378
9.0	1.6	0.06398	0.88378
10.0	1.8	0.06726	0.85340
11.0	2.0	0.06233	0.88378
12.0	2.2	0.06069	0.85340
13.0	2.4	0.05905	0.83464
14.0	2.6	0.05577	0.83464
15.0	2.8	0.05249	0.79580
16.0	3.0	0.04921	0.84720
17.0	3.2	0.05085	0.84720
18.0	3.4	0.05085	0.83464
19.0	3.6	0.04593	0.83464
20.0	3.8	0.04429	0.75496
21.0	4.0	0.04265	0.82190
22.0	4.2	0.04265	0.82190
23.0	4.4	0.04429	0.68148

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.06726	0.45946
0.07218	0.56272
0.07546	0.56272
0.08038	0.56272
0.08202	0.56272
0.08366	0.56272
0.08366	0.56272
0.08366	0.64977
0.08530	0.64977
0.08366	0.64977
0.08366	0.76882
0.08530	0.76882
0.08858	0.76882
0.09186	0.79580
0.09022	0.56272
0.08694	0.79580
0.08694	0.72646
0.08530	0.64977
0.08366	0.74085
0.08366	0.82190
0.08202	0.72646
0.08858	0.72646
0.08858	0.56272

AIR VELOCITY : 2.48 MPH  
 AIR TEMP DEG C : 140  
 111

124  
 105

119  
 98

WATER TEMP DEG C : 25.60

25.26

25.23

DEBIT F R RP N B C  
 0.23402 0.06162 296745 2235993 1525 0.16219 0.25102

CHANNEL WIDTH : 5.0 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.07054	1.02738
2.0	0.2	0.07710	1.02738
3.0	0.4	0.07546	1.05775
4.0	0.6	0.07874	0.83464
5.0	0.8	0.08038	1.07752
6.0	1.0	0.08366	1.09694
7.0	1.2	0.08366	1.10652
8.0	1.4	0.08202	1.10652
9.0	1.6	0.08366	1.09694
10.0	1.8	0.08038	1.07752
11.0	2.0	0.07874	1.02738
12.0	2.2	0.07546	1.02738
13.0	2.4	0.07546	0.97465
14.0	2.6	0.07382	0.91892
15.0	2.8	0.07054	0.97465
16.0	3.0	0.06562	0.98542
17.0	3.2	0.06562	1.00662
18.0	3.4	0.06397	1.02738
19.0	3.6	0.06562	0.95275
20.0	3.8	0.06397	0.94161
21.0	4.0	0.06233	1.07752
22.0	4.2	0.06069	1.07752
23.0	4.4	0.06069	1.07752
24.0	4.6	0.06069	1.07752
25.0	4.8	0.06233	0.56272

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.09022	0.72646
0.09514	0.72646
0.09842	0.72646
0.10170	0.72646
0.10335	0.68148
0.10663	0.68148
0.10827	0.68148
0.10827	0.68148
0.10827	0.68148
0.10991	0.68148
0.10991	0.68148
0.11319	0.68148
0.11319	0.68148
0.11647	0.68148
0.11319	0.56272
0.11483	0.64977
0.11155	0.64977
0.10827	0.64977
0.10991	0.64977
0.10991	0.64977
0.10991	0.64977
0.11319	0.64977
0.11319	0.64977
0.11483	0.64977
0.11811	0.64977

AIR VELOCITY : 2.55 MPH  
 AIR TEMP DEG C : 147  
 118

130  
106

122  
100

WATER TEMP DEG C : 25.70

25.55

25.53

DEBIT	F	R	RP	N	B	C
0.36510	0.06633	361976	2727508	1032	0.17630	0.27285

CHANNEL WIDTH : 5.0 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.05741	0.96376
2.0	0.2	0.06398	0.89565
3.0	0.4	0.06562	0.99608
4.0	0.6	0.06890	0.75496
5.0	0.8	0.07382	0.97465
6.0	1.0	0.07382	0.95275
7.0	1.2	0.07382	0.94161
8.0	1.4	0.07382	0.95275
9.0	1.6	0.07382	0.94161
10.0	1.8	0.07546	0.91891
11.0	2.0	0.07218	0.85956
12.0	2.2	0.06890	0.87176
13.0	2.4	0.06562	0.89565
14.0	2.6	0.06398	0.85956
15.0	2.8	0.06233	0.85956
16.0	3.0	0.05741	0.91891
17.0	3.2	0.05905	0.89565
18.0	3.4	0.05905	0.89565
19.0	3.6	0.05577	0.91891
20.0	3.8	0.05413	0.88378
21.0	4.0	0.05085	0.85956
22.0	4.2	0.05249	0.85956
23.0	4.4	0.05086	0.85956
24.0	4.6	0.05249	0.89565
25.0	4.8	0.05577	0.43588

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.07546	0.56272
0.08202	0.61642
0.08530	0.59906
0.09022	0.61642
0.09186	0.64977
0.09350	0.64977
0.09514	0.64977
0.09350	0.59906
0.09514	0.59906
0.09514	0.66582
0.09678	0.68148
0.10006	0.69680
0.10335	0.66582
0.10499	0.64977
0.10335	0.64977
0.09842	0.69680
0.09842	0.69680
0.10006	0.72646
0.09514	0.64977
0.09350	0.64977
0.09514	0.72646
0.10006	0.75496
0.09842	0.72646
0.10006	0.72646
0.10170	0.64977

AIR VELOCITY : 2.51 MPH  
 AIR TEMP DEG C : 144  
 115

128  
 105

122  
 100

WATER TEMP DEG C : 25.97

25.62

25.58

DEBIT F R  
 0.29952 0.06114 333649

RP N  
 2514067 1387

B C  
 0.17630 0.27285

CHANNEL WIDTH : 5.0 FT

SECTION 3			
SLICE	DIST.	DEPTH	VELOCITY
1.0	0.0	0.03773	0.79580
2.0	0.2	0.04429	0.79580
3.0	0.4	0.04429	0.82190
4.0	0.6	0.04593	0.56272
5.0	0.8	0.04921	0.79580
6.0	1.0	0.05085	0.79580
7.0	1.2	0.04757	0.79580
8.0	1.4	0.04921	0.72646
9.0	1.6	0.04921	0.69680
10.0	1.8	0.05085	0.69680
11.0	2.0	0.04757	0.69680
12.0	2.2	0.04593	0.74085
13.0	2.4	0.04265	0.72646
14.0	2.6	0.03937	0.64977
15.0	2.8	0.03609	0.63332
16.0	3.0	0.03281	0.63332
17.0	3.2	0.03609	0.64977
18.0	3.4	0.03609	0.72646
19.0	3.6	0.03117	0.61642
20.0	3.8	0.02953	0.56272
21.0	4.0	0.02953	0.56272
22.0	4.2	0.02953	0.61642
23.0	4.4	0.02953	0.64977
24.0	4.6	0.03117	0.64977
25.0	4.8	0.03117	0.41095

SECTION 2

SECTION 1	
DEPTH	VELOCITY
0.04757	0.43588
0.05249	0.45946
0.05413	0.41095
0.05905	0.41095
0.06069	0.41095
0.06233	0.41095
0.06398	0.41095
0.06398	0.41095
0.06562	0.41095
0.06233	0.41095
0.06562	0.50331
0.06726	0.50331
0.07054	0.50331
0.07054	0.50331
0.06726	0.56272
0.06726	0.56272
0.06398	0.56272
0.06562	0.56272
0.06233	0.56272
0.06562	0.56272
0.06726	0.54364
0.06726	0.52386
0.06890	0.50331
0.07054	0.50331
0.07218	0.50331

AIR VELOCITY : 2.43 MPH  
 AIR TEMP DEG C : 145  
 116

129  
 107

122  
 100  
 25.95

WATER TEMP DEG C : 26.97

26.02

DEBIT F R  
 0.14776 0.04654 254007

RP  
 1913959

N  
 865

B C  
 0.17630 0.27285