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A Unidimensional Representation of Multidimensional Inequality: An Econometric Analysis of Inequalities in the Arab Region

Mohamad A. Khaled*, Paul Makdissi†, D.S. Prasada Rao‡ and Myra Yazbeck§

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* School of Economics, University of Queensland, Australia; Email: m.khaled@uq.edu.au.
† Department of Economics, University of Ottawa, Canada; Email: paul.makdissi@uottawa.ca.
‡ School of Economics, University of Queensland, Australia; Email: d.rao@uq.edu.au.
§ Department of Economics, University of Ottawa, Canada; Email: myazbeck@uottawa.ca.

Abstract

This paper introduces a novel approach for analyzing multidimensional inequality. We use a counting-based approach and a transformed survival function to capture the impact of inequality at the individual level. We then define indices of multidimensional inequality as aggregations of these individual impacts and introduce two new graphical tools. These graphical tools leverage the unidimensional representation to provide an intuitive representation of multidimensional inequality. Through this representation we derive dominance conditions that allow for the identification of robust rankings of multidimensional inequality. This paper also pushes boundaries in stochastic dominance estimation and inference showcasing new estimation and statistical methods related to our new graphical tools and dominance conditions. As a practical demonstration, we apply our method to analyze data from the Harmonized Household Income and Expenditure Surveys for three Arab countries and the Arab Barometer survey for ten Arab countries.

Key words: *Multidimensional inequality, Stochastic Dominance, Multidimensional complaint incidence curve, Cumulative multidimensional complaint curve.*

JEL Classification: D63, I31

1 Introduction

Contemporary socioeconomic research and policy development increasingly recognize the need for a comprehensive perspective on societal well-being that extends beyond traditional income inequality indicators. It is crucial to embrace a multidimensional approach to inequality, as relying solely on income-based measures often overlooks critical dimensions such as education, health, autonomy, and freedom. Furthermore, empirical studies have provided evidence indicating that a multidimensional poverty approach is capable of capturing poverty levels that are more severe and persistent in comparison to traditional income-based measures of poverty (Alkire and Fang, 2019). Thus, understanding that well-being extends beyond just income, a multidimensional perspective considers various aspects of life that contribute to an individual's overall quality of life, surpassing monetary wealth.

This paper is motivated by the significance of adopting a multidimensional approach to inequality and the need for a stronger connection between the different strands of literature exploring this topic. The interest in the multidimensional nature of well-being is not new within economics, with a substantial body of literature focusing on multidimensional poverty measurement and the emerging theoretical research on multidimensional inequality measurement. However, the literature on multidimensional inequality and the counting-based approach to multidimensional poverty have evolved independently. Since the seminal paper by Alkire and Foster (2011), empirical research measuring non-income dimensions of poverty has flourished, particularly because their measurement approach offers advantages when only ordinal data on well-being dimensions are available. Many empirical studies, including white papers, have utilized the multidimensional poverty index developed by Alkire and Foster, including organizations like the United Nations employing it extensively. The widespread use of this index highlights the crucial role of multidimensional approaches in measuring deprivations and the need for a similar measurement framework for inequality.

While the literature on multidimensional inequality is limited, it is not a new area of study. Early work by scholars such as Maasoumi (1986), Dardanoni (1996), and Tsui (1995, 1999) made substantial contributions to conceptualizing the idea of multidimensional inequality. More recently, the focus targeted the ethical principles associated with multidimensional inequality measures and the associated orderings (see, Decancq, 2012; Muller and Trannoy, 2012; Andreoli and Zoli, 2020; Fan, Henry, Pass, and Rivero, 2022). However, to the best of our knowledge, there has yet to be a link between this theoretical literature on multidimensional inequality and the counting-based approach widely used in the literature on multidimensional poverty. This paper aims to address this research gap.

We contribute to the existing literature on multidimensional inequality by introducing a measurement framework based on a counting-based approach. Within this approach, our first contribution is to define inequality as the aggregation of complaints, following the framework proposed by Temkin (1986). Particularly, we define an individual's complaint as the proportion of the population that is *better-off* in all dimensions of well-being.[1] Consequently, in this framework, a multidimensional inequality index aggregates these complaints over all the considered dimensions of well-being. It is important to highlight that the concept of complaint, as defined in this paper, naturally relates to the survival function. This connection between an individual's complaint and the survival function implies that any inequality index within this framework will exhibit an *aversion to positive k-rearrangement*, a fundamental axiom of multidimensional inequality (Decancq, 2012). An additional advantage of defining the multidimensional complaint as the value of the survival function at an individual's attribute realization is that it simplifies the representation of multidimensional inequality in a straightforward and unidimensional form.

[1] Cowell and Ebert (2004) also used Temkin's (1986) definition of inequality as an aggregation of complaints in another context.

Our second contribution consists of proposing two novel graphical tools. The first tool, termed the *multidimensional complaint incidence curve*, serves a similar purpose as the Lorenz curve in the context of unidimensional inequality analysis. It provides an intuitive visualization of inequality based on a multidimensional distribution. The second graphical tool is the *cumulative multidimensional complaint curve*, which, like the *multidimensional complaint incidence curve*, leverages the unidimensional representation of multidimensional inequality. Through this representation we derive the dominance conditions that enables the identification of robust rankings of multidimensional inequality. The dominance conditions derived from *multidimensional complaint incidence curve* provide multidimensional inequality orderings that are applicable to all multidimensional inequality indices considered within the the class of indices presented in this paper. On the other hand, the *cumulative multidimensional complaint curve* identifies robust orderings of multidimensional inequality but focuses on a more restricted set of indices, specifically those that exhibit a higher aversion to inequality than those considered by the *multidimensional complaint incidence curve.*

Lastly, the paper advances the literature on stochastic dominance estimation and inference. It presents the non-parametric estimators and statistical approaches linked to the proposed dominance conditions and graphical tools. Additionally, we provide a comprehensive inference method, which utilizes the bootstrap technique for generating standard errors and confidence bands. The reasoning behind these estimators is rooted in copula theory and Kendall distribution theory.

In our empirical analysis, we place strong emphasis on the significance of data availability in multidimensional inequality measurement. Researchers interested in mapping and monitoring inequality over time often encounter challenges related to data availability, be it in regions like the Arab region, or for specific populations like Indigenous communities. To underscore the importance of conducting inequality analysis even under such circumstances, we examine two data availability scenarios for the Arab region.

In the first application, we utilize the best possible data scenario: a household survey (the Harmonized Household Income and Expenditure Surveys)[2]. Household surveys are widely considered the gold standard data for canonical poverty and inequality analysis due to their provision of continuous income information, making them highly suitable for our analysis. However, gaining access to surveys containing continuous income or expenditure data is not always feasible, leading researchers to explore alternate data sources (see, Atamanov, Tandon, Lopez-Acevedo, and Vergara Bahema, 2020). Since our proposed approach performs well even in such data limitations setting, our second empirical application employs a dataset (the Arab Barometer)[3] which lacks the necessary information to conduct a typical welfare analysis. In fact, our proposed approach, in addition to providing an intuitive way of measuring multidimensional inequality, adapts well to both household surveys and alternative data sources settings, as long as these sources contain ordinal information on wellbeing dimensions.

In our first analysis, we consider three well-being dimensions from the HHIES: households' total expenditure, education, and disability status. While this type of information available in these surveys are ideal for welfare analysis, such surveys are available only for a limited number of Arab countries namely: Egypt, Iraq, and Jordan. In our second analysis, we explore well-being variables from an alternative data source that was not designed to assess inequality and poverty: the Arab Barometer. Some of the Arab Barometer survey waves include an interval income variable, an ordinal education variable, and other ordinal variables that measure the feeling of safety, autonomy, depression, and stress. These variables offer an adequate range of well-being dimensions that can be used to assess mul-

[2]The HHIES (OAMDI, 2012, 2013, and 2015) is provided by the Economic Research Forum for the Arab Countries, Iran and Turkey (ERF)

[3]We use the 2018 wave.

tidimensional inequality using the framework we propose in this paper. Furthermore, the nature of the information collected is comparable across countries allowing for inter country comparisons between Algeria, Egypt, Iraq, Jordan, Lebanon, Libya, Morocco, Palestine, Sudan, and Tunisia. One important aspect of this survey design is such that it is representative of the population.

In our empirical application, we observed consistency in the ranking of countries across both studies, reflecting the robustness and reproducibility of our methodology. When considering the robust rankings for all indices we categorized countries into two groups based on multidimensional inequality magnitudes. The first group, with less multidimensional inequality (i.e., dominating the second group), included Iraq and Palestine whereas the second group, with high inequality included Egypt, Lebanon, Morocco, and Sudan (i.e., does not dominate any group). When considering the robust rankings for a more restricted set of indices, five additional rankings were found. For all these indices, Iraq had less inequality than Algeria, Jordan showed less inequality than Lebanon and Morocco, and Libya exhibited less inequality than Lebanon and Sudan. Our empirical application underscores the importance of adopting dominance in empirical settings. It holds the potential to generate robust rankings in situations where the selected index fails to distinguish statistically significant differences in inequality between two countries. It also holds promise when data on continuous income is not available.

Our results from the two empirical applications reveal that when we factor in uncertainty into our analysis, we encounter intriguing results. We notice that certain results yield statistical significance when we adopt a dominance-based ranking. Yet these same results don't maintain statistical significance under some index-based ranking. Theoretically, in a world in which we have a perfect knowledge on the distributions, an index-based approach delivers more complete rankings compared to its dominance-based counterpart. However, in empirical applications using survey data, this presumption seems to hold true only when we deal with point estimates.

In this paper, we explore untapped potential of data from the Arab region, which has often been overlooked due to data availability challenges. This context serves as a fitting backdrop to underscore the significance our paper's methodological contribution and showcases its empirical applicability. Moreover, the approach we present here can be applied to any context where measuring multidimensional inequality is of interest or where income data availability poses a challenge, such as situation involving marginalized populations and developing countries. Grounded in a determination to address data limitations and the desire to offer meaningful insights to shape societies towards greater inclusivity and equity, our ambition is to develop a framework for measuring multidimensional inequality. This framework is a modest step forward towards unlocking the potential of multidimensional inequality research. We aspire to equip scholars and policymakers with the essential tools to design comprehensive and meaningful policies that effectively address inequality, paving the way for positive societal transformations.

The remainder of the paper is organized as follows. Section 2 proposes a measurement framework for multidimensional inequality. Section 3 proposes dominance conditions for identifying robust orderings of multidimensional inequality. Section 4 introduces a parametric class of indices. Section 5 offers the estimation and statistical inference approach. Section 6 presents findings from our two empirical applications. Finally, Section 7 concludes and presents future areas of investigation.

2 Measurement framework

We assume that we have a population (or a population sample) of n individuals. Also, assume that a m-dimensional vector represents the well-being of each individual. These dimensions may relate to income, education, health, or other variables impacting well-being. We use an $n \times m$ matrix $\Xi = [x_{ij}]$ to represent the information on the well-being for this population. In this matrix, x_{ij} represents the achievement of individual i in dimension j.[4] Each row vector provides the list of individual i's achievements, and every column represents the distribution of attribute j in the population. These attributes can take the form of ordinal, cardinal, or ratio-scale variables.[5] This paper's objective is to develop a measurement framework compatible with all these variables simutaneously.

Let $X_1, \ldots, X_m$ be random variables representing these m dimensions of well-being. Assume that there are d discrete and c continuous random variables with $d + c = m$. Let $\mathcal{D} \subset \{1, \ldots, m\}$ be the set of subscripts corresponding to discrete variables, and let $\mathcal{C} = \{1, \ldots, m\} \setminus \mathcal{D}$. Without loss of generality, we can assume that the first d variables are discrete and the following $c = m - d$ are continuous. The actual data-generating process is fully determined by a joint distribution function $F(X_1, \ldots, X_m)$.

2.1 Measuring inequality

In our approach, inspired by Temkin (1986), we view inequality as an aggregation of "complaints". Temkin's philosophical exploration of welfare and inequality raises a fundamental question: "how bad is inequality in a situation from the standpoint of particular individuals in that situation?" (Temkin, 1986, p. 102). Consequently, as analysts, we can consider that an individual has a complaint if their welfare falls short of that another equally-deserving reference person. When defining an individual's sense of complaint, Temkin's notion of the reference person allows for various perspectives such as relative to the average welfare, the best-off person, or all those *better-off*.

We adopt a definition of the individual's complaint based on the comparison with all those who are *better-off*. Our rational for choosing the reference person stems from the

[4]It is helpful to consider Ξ as the realization of n draws from an actual data generating process.

[5]A ratio-scale variable is a cardinal variable with a well-defined 0. Income is an obvious example.

multidimensional nature of our context. Indeed, in a multidimensional setting, linking the complaint's size to cardinal differences may raise measurement issues, especially when some dimensions may be ordinal. As a result, using the average welfare to measure complaints is not suitable, as the average lacks a well-defined statistic for ordinal variables. Moreover, the cardinal size of the distance in welfare is difficult to assess. Thus, using standard measures of dispersion, such as the variance, and inequality indices, and deriving tests, such as Lorenz dominance tests or other off-the-shelf stochastic dominance tests, can lead to arbitrary rankings of distributions (Zheng, 2008).

Many prior works on poverty and health inequality, when faced with multiple categorical variables, have resorted to a counting-based approach and a dichotomization of the ordinal variables (Alkire and Foster, 2011; Makdissi and Yazbeck, 2014). While counting dichotomized variables allows for computing inequalities using multiple categorical variables, it comes at the cost of sacrificing the variation within each dimension. Additionally, the analyst must select a threshold in each dimension for this dichotomization, which can introduce subjectivity. An alternative approach commonly used in the context of unidimensional inequality, involves using population shares in each category (see Allison and Foster, 2004; Abul Naga and Yalcin, 2008; Cowell and Flachaire, 2017; and Makdissi and Yazbeck, 2017). Specifically, Cowell and Flachaire (2017) build their measurement framework on the status of a person in a society, which they define as the position of the person within the unidimensional distribution of an ordinal attribute. In this paper, we extend this population share approach to a multidimensional framework. We define an individual's complaint as the proportion of the population that is *better-off*. However, with multiple dimensions, some people may be better-off in some dimensions but worse-off in others. To address this, we adopt an intersection approach, and define the individual's complaint as the proportion of individuals in a population who are *better-off* than the individual in all the dimensions.

Formally, we define the survival function, denoted as $S(x)$, which represents the proportion of individuals with welfare greater than or equal to x in all dimensions

$$S(x) = \Pr\left[X_1 \geqslant x_1, X_2 \geqslant x_2, \ldots, X_m \geqslant x_m\right] \tag{1}$$

It is important to note that we are using a definition of the survival function definition that allows for equality rather than just strict inequality. The survival function as defined in equation (1) allows us to define complaints as follows.

Definition 1 COMPLAINT: *A complaint associated with an attribute vector x is given by $S(x)$.*

Note that technically, Definition 1 defines all those who are *better-off* as those who are at least as well-off in all dimensions. Despite this, we will maintain the terminology "*better-off*" for the sake of simplicity and consistency throughout the paper.[6]

Let μ be the measure induced by S. The Kendall survival function, is then be given by

$$K_S(p) = \Pr[S(X) \geqslant p] = \mu\left(\{x \in \Re^m | S(x) \geqslant p\}\right) \tag{2}$$

A useful tool to measure multidimensional complaints is the inverse of this Kendall survival function

$$\psi_S(p) = \sup\left\{q \,|K_S(q) \geqslant p\right\}. \tag{3}$$

The inverse of this Kendall survival function,$\psi_S(p)$, gives the value of the survival function associated with the $(100-p)$th quantile in the distribution of complaints. In other words, it gives the level of complaints at the $(100-p)$th quantile in the distribution of complaints. In this context, one can interpret p as social ranks. For this reason, we call this function $\psi_S(p)$,[7]

[6] In cases where there is at least one continuous variable, $\Pr[X = x] = 0$ unless we assume a degenerate density function. The mathematical definition is thus consistent with the literal concept of all those who are *better-off*.

[7] We chose ψ for the multidimensional complaint curve because ψόγος (psogos) means criticism in ancient Greek.

the *multidimensional complaint incidence curve* since it gives the "expected complaint" for each social rank p.

The empirical intuition underlying the *multidimensional complaint incidence curve* is relatively straightforward, particularly from a counting-based approach perspective. Let us assume we have a survey with N observations. For each observation, $i \in \{1, 2, \ldots, N\}$, $S(x_i)$ represents the proportion of the population that is *better-off* in all the dimensions. We can then rank the observations based on the value of their $S(x_i)$, from the highest to the lowest. Let r_i denote the rank of observation i, with no ties between the values of $S(x)$. The socioeconomic rank of i is then $p_i = r_i/N$. To construct the *multidimensional complaint incidence curve*, $\psi_S(p)$, we plot the values of $S(x_i)$ on the x-axis and the corresponding socioeconomic rank p_i on the y-axis. This mapping of $S(x_i)$ with respect to p_i provides us with the *multidimensional complaint incidence curve.*

In several empirical scenarios, all attributes may be discrete. In such cases, applying Definition 1 will not generate enough variability between individuals. To address this limitation, we adopt an alternative definition.[8]

Remark 1 COMPLAINT IN DISCRETE CASES: *When all the attributes are discrete,* $S_D(x) = S(x) - \Pr[X = x]$ *is the complaint associated with an attribute vector* x*.*

The *multidimensional complaint incidence curve*, $\psi_S(p)$, offers an interesting unidimensional graphical representation of multidimensional inequality. To illustrate this concept, let us consider the "smooth" version of the *multidimensional complaint curve*, where all variables are continuous (see Figure 1). There are two possible extreme cases for the function $\psi_S(p)$. The first extreme case occurs when all dimensions are perfectly correlated. For example, let us consider attributes like income and health, both of which are continuous variables. If the income gradient in health is everywhere positive, then we encounter

[8] If all variables are discrete, $\Pr[X = x]$ may be different than 0. In this case it is important to subtract $\Pr[X = x]$ to match the mathematical definition with the literal concept of all those *better-off*.

maximum multidimensional inequality. This maximum multidimensional inequality case is represented by the straight line $\psi_S^{\max}(p)$ in Figure 1.

On the other hand, the second extreme case is characterized by the absence of multidimensional inequality. The absence of inequality happens if the income gradient in health is everywhere negative. This scenario arises when there is perfect negative correlation between attributes. Thus, even if there is inequality in the unidimensional distribution of each attribute, the perfect negative correlation implies that there is no multidimensional inequality. In Figure 1, this situation depicted by the straight line $\psi_S^{\min}(p)$.

The curve $\psi_S(p)$ shown in Figure 1 represents an intermediate case with some multidimensional inequality. It displays the expected complaint at each social rank p. The value of $p_I := \inf\{p|\psi_S(p) = 0\}$ corresponds to the proportion of the population affected by multidimensional inequality, which represents the intersection of the curve $\psi_S(p)$ with the horizontal axis. The remaining $1 - p_I$ of the population is not affected by multidimensional inequality (they have no complaints).

Once complaints are defined, the analyst faces the task of selecting a method to aggregate these complaints at the social level. As mentioned earlier, Temkin (1986) proposes three aggregation methods. The first consists of using the maximin principle, focusing on the worst-off individuals. The second approach is an additive, summing up the complaints. Finally, a third approach, which generalizes and incorporates the first two approaches, adopts a weighted additive approach. To encompass all three approaches, we define an aggregation framework that accommodates each perspective. In this framework we assume a sympathetic ethical observer, who evaluates multidimensional inequality and aggregates complaints. The observer assigns each complaint a weight using a function $\omega(p)$ where $\omega(p) \geqslant 0$ for all $p \in [0, 1]$ and $\int_0^1 \omega(p)\mathrm{d}p > 0$. Within this context, multidimensional inequality emerges as a weighted average of complaints (potentially multiplied by some normalizing constant):[9]

$$I(S) = \int_0^1 \omega(p)\psi_S(p)\mathrm{d}p. \tag{4}$$

Equation (4) is compatible with the three aggregation methods proposed in Temkin (1986). If one opts for the maximin principle and defines the worst-off group as those individuals at social ranks $p \in [0, z]$, then the weight function would be such that $\omega(p) \geqslant 0$ if $p \in [0, z]$, and $\omega(p) = 0$ elsewhere. If one prefers the additive approach, then $\omega(p) = 1$ (or any other constant) for all $p \in [0, 1]$. As for the weighted additive approach, it is compatible with weight functions such as $\omega(p) \geqslant 0$ for all $p \in [0, 1]$.

Let $\mathscr{C}$ represent the set of all continuous functions on $[0, 1]$. Then, we can formally define the set of rank-dependent multidimensional inequality indices.

Definition 2 Set of rank dependent multidimensional inequality indices:

$$\Upsilon := \left\{ I(S) \,\middle|\, \omega(p) \in \mathscr{C} \ \wedge \ \omega(p) \geqslant 0 \ \wedge \ \int_0^1 \omega(p)\mathrm{d}p > 0 \right\}.$$

The rank-dependent multidimensional inequality indices described in Definition 2 rely on the population shares that are *better-off* than x. Moreover, these population shares serve as robust statistics applicable to ratio-scale, cardinal, and ordinal variables. By adopting this approach, the analyst effectively overcomes the potential for arbitrary rankings raised by Zheng (2008).

2.2 Properties

We propose an axiomatic framework for multidimensional inequality measurement. In order to describe these axioms, we consistently compare an initial matrix of achievement Ξ^0 and the survival function of its associated data-generating process, S_0, with an alternative

[9] One may consider restricting $\int_0^1 \omega(p)\mathrm{d}p = 1$. However, this would be less general and may preclude the analyst from rescaling the index to make it lie in some desirable interval, like between 0 and 1. A quick inspection of Figure 1 indicates that the maximum average complaint equals 1/2. In this context, if one wants an index that takes values between 0 and 1, it would be necessary to multiply this average by 2.

matrix of achievement Ξ^1 and the survival function of its associated data-generating process S_1.

Definition 3 Replication: *A matrix of attributes Ξ^1 is said to be a replication of Ξ^0 if*

$$\Xi^1 = \begin{pmatrix} \Xi^0 \\ \Xi^0 \\ \vdots \\ \Xi^0 \end{pmatrix}$$

When Ξ^1 is a replication of Ξ^0, and the only difference between the two populations is their size, then it is reasonable to assume that the data-generating processes are the same (i.e., $S_1 = S_0$). This is the case because Ξ^1 has the same underlying relative structure as Ξ^0 except for the number of observations. Hence, the index of multidimensional inequality should take a similar value in these two population.

Axiom 1 Replication invariance: *If Ξ^1 is obtained from Ξ^0 by a replication, then $I(S_1) = I(S_0)$*

This first axiom allows meaningful comparisons of populations of different sizes. Moreover, since our multidimensional inequality measure is based on the multivariate survival function's data-generating process, this axiom is automatically verified.

To ensure index's invariance to the person's identity and its dependence solely on the list of attributes allocated to this person,we impose an additional property called symmetry.

Definition 4 Permutation: *A matrix of attribute Ξ^1 is obtained from Ξ^0 by a permutation if $\Xi^1 = \Pi\Xi^0$, where Π is some $n \times n$ permutation matrix.*[10]

When Ξ^1 is a permutation of Ξ^0, the only difference is the order in which each observation appears. In this case, it is reasonable to assume that the data-generating processes are the same (i.e., $S_1 = S_0$) since Ξ^1 has the same underlying relative structure as Ξ^0 except for the

[10] A permutation matrix is a square matrix with a single 1 in each row and each column and 0 elsewhere.

arbitrary order of the observations. Once again, the index of multidimensional inequality should take a similar value in these two populations.

Axiom 2 ANONIMITY: *If Ξ^1 is obtained from Ξ^0 by a permutation, then $I(S_1) = I(S_0)$*

The anonymity axiom, also known as the symmetry axiom, implies that the inequality index remains unchanged when there is a switch of all attributes between two individuals. Once again, given that we are using the multivariate survival function, this axiom is automatically satisfied.

One desirable property of a multidimensional inequality index is that it attains its minimum value when the set of all row vectors x_i is an antichain, indicating an absence of multidimensional inequality, and its maximum value when it is a chain, representing a perfect correlation case.[11] To have this property, the index should be capable of taking at least two different values.

Axiom 3 NONTRIVIALITY: *$I(S)$ takes at least two distinct values.*

Since we assume that $\int_0^1 \omega(p)\mathrm{d}p > 0$, this implies $\omega(p) > 0$ for some measurable space on $[0, 1]$. In turn, this guarantees that the index obeys the axiom of nontriviality.

In addition to the Axiom of *Nontriviality*, it is essential to define the appropriate concept of inequality aversion for the index to possess the mentioned desirable properties. In the context of unidimensional inequality, additional ethical axioms often focus on principles of transfers between individuals. However, in a multidimensional framework, these axioms may lead to counterintuitive results. To illustrate this, let us consider two individuals with vectors of attribute $x_i^0 = (x_1^\ell, x_2^h)$ and $x_{i'}^0 = (x_1^h, x_2^\ell)$, where $x_1^\ell < x_1^h$ and $x_2^\ell < x_2^h$. Now, if we perform a *progressive* transfer in the first dimension alone, equalizing the two individuals at $\overline{x}_1$ in that dimension, we end up with $x_i^1 = (\overline{x}_1, x_2^h)$ and $x_{i'}^1 = (\overline{x}_1, x_2^\ell)$. Intuitively, many

[11] A chain is a set in which each pair of vectors is comparable, and an antichain is a set in which each pair is incomparable.

would consider starting from a situation in which neither individual is *better-off* than the other in both dimensions and ending with a situation in which one individual, i, is strictly better than the other, as an increase in inequality. However, when considering the unidimensional concept of *progressive* transfer in isolation, this change would be seen a strict social improvement. Therefore it is important that we properly address such issues when extending concepts of transfers in inequality from the unidimensional to the multidimensional setting.

The literature on multidimensional inequality has introduced a concept that encapsulates the fundamental idea of aversion to multidimensional inequality. This concept is based on elementary rearrangement to address the multidimensional nature of inequality.[12] In the 2-dimension case, an elementary rearrangement involves considering the same individuals as before, i and i', with vectors of attribute $x_i^0 = (x_1^\ell, x_2^h)$ and $x_{i'}^0 = (x_1^h, x_2^\ell)$. If we switch the second attribute between the two individuals to obtain $x_i^1 = (x_1^\ell, x_2^\ell)$ and $x_{i'}^1 = (x_1^h, x_2^h)$, inequality should not decrease. To frame the analysis in terms of the underlying data-generating process, let us denote by f, the probability density (with respect to a mixture of Lebesgue and counting measure).[13] A positive rearrangement involves marginally changing the probability density, $f_0(x)$, at $x_i^0, x_{i'}^0, x_i^1, x_{i'}^1$ such that $f_1(x) = f_0(x) - \delta$ for $x \in \{x_i^0, x_{i'}^0\}$, $f_1(x) = f_0(x) + \delta$ for $x \in \{x_i^1, x_{i'}^1\}$, and $f_1(x) = f_0(x)$ for $x \notin \{x_i^0, x_{i'}^0, x_i^1, x_{i'}^1\}$. This rearrangement ensures that $S_1(x) = S_0(x)$ everywhere except within the rectangle formed by $x_i^0, x_{i'}^0, x_i^1$, and $x_{i'}^1$ where $S_1(x) \geqslant S_0(x)$.[14]

Decancq (2012) generalizes the concept of positive rearrangement to the multivariate case. Following Decancq (2012) we denote by $x <_k y$ the relation in which $x_j \leqslant y_j$

[12] See, Atkinson and Bourguignon, 1982; Tsui, 1999; Bourguignon and Chakravarty, 2003; D'Agostino and Dardanoni, 2009; Alkire and Foster, 2011; Decancq, 2012; Gravel and Moyes, 2012.

[13] If all the variables are continuous, the f is the usual density function with respect to Lebesgue's measure. If all the variables are discrete, then f is a probability mass function.

[14] $S_1(x) = S_0(x) + \delta$ for all x in the rectangle except on the line segments $(x_i^1, x_i^0]$ and $(x_i^1, x_{i'}^0]$ where $S_1(x) = S_0(x)$

for all $j \in \{1, \ldots, m\}$ and $x_j < y_j$ for k values in $\{1, \ldots, m\}$. Let us assume that we have two different vectors $x^\ell = (x_1^\ell, \ldots, x_m^\ell)$ and $x^h = (x_1^h \ldots, x_m^h)$ such that $x^\ell <_k x^h$ and $2 \leqslant k \leqslant m$. Let $B^k(x^\ell, x^h) = \prod_{j=1}^m \left[x_j^\ell, x_j^h\right]$ be a k-dimensional hyperbox with 2^k vertices. Let $\mathscr{B}^k(x^\ell, x^h)$ be the set of all vertices, $\mathscr{B}_o^k(x^\ell, x^h) \subset \mathscr{B}^k(x^\ell, x^h)$, the set of all odd-numbered vertices such that $x_j = x_j^\ell \neq x_j^h$ and $\mathscr{B}_e^k(x^\ell, x^h) \subset \mathscr{B}^k(x^\ell, x^h)$, the set of all vertices with an even number of entries such that $x_j = x_j^\ell \neq x_j^h$. Decancq proposes a generalization of the positive rearrangement, which rearranges probability mass on the vertices of the hyperbox.

Definition 5 Positive k-rearrangement: *Let $f_1(x)$ and $f_0(x)$ be the probability distribution underlying Ξ^1 and Ξ^0. Ξ^1 is obtained from Ξ^0 by a positive k-rearrangement if there exists x^ℓ and x^h with $x^\ell <_k x^h$ and $\delta > 0$ such that*

$$f_1(x) = \begin{cases} f_0(x) + \delta & \text{if } x \in \mathscr{B}_e^k(x^\ell, x^h) \\ f_0(x) - \delta & \text{if } x \in \mathscr{B}_e^k(x^\ell, x^h) \\ f_0(x) & \text{if } x \notin \mathscr{B}^k(x^\ell, x^h) \end{cases}$$

A positive k-rearrangement plays the same role as the positive rearrangement in the bivariate case. This rearrangement is such that $S_1(x) = S_0(x)$ everywhere except within the hyperbox $B^k(x^\ell, x^h)$ where $S_1(x) \geqslant S_0(x)$.

Axiom 4 Weak rearrangement: *If Ξ^1 is obtained from Ξ^0 by a positive k-rearrangement, then $I(S_1) \geqslant I(S_0)$.*

Axiom 4 guarantees that the index takes its minimum value when the set of all row vectors x_i is an antichain and its maximum value when it is a chain.

In the context of income inequality, Kolm (1976) introduced the principle of transfer sensitivity (for a unidimensional setting). The essence of this principle lies in the notion that a regressive transfer can be offset by an equivalent progressive transfer occurring in a lower part of the income distribution. This principle encapsulates the notion of higher aversion to inequality. In the context of multidimensional inequality, we can extend a similar concept to capture aversion to multidimensional inequality.

Definition 6 Composite rearrangement: *Let* $f_0(x)$, $f_{1_a}(x)$, $f_{1_b}(x)$, *and* $f_1(x) = 0.5f_{1_a} + 0.5f_{1_b}(x)$ *be the probability distribution underlying* Ξ^0, Ξ^{1_a}, Ξ^{1_b}, *and* Ξ^1. *Let* $x^{\ell_b} <_k x^{h_b} <_k x^{\ell_a} <_k x^{h_a}$. *If* Ξ^{1_a} *is obtained from* Ξ^0 *by a positive k-rearrangement on the* $\mathscr{B}^k(x^{\ell_a}, x^{h_a})$, Ξ^0 *is obtained from* Ξ^{1_b} *by a positive rearrangement on the* $\mathscr{B}^k(x^{\ell_b}, x^{h_b})$, *and* $\int_0^1 \left[\psi_{S_{1_a}}(p) - \psi_{S_0}(p)\right] \mathrm{d}p = \int_0^1 \left[\psi_{S_0}(p) - \psi_{S_{1_b}}(p)\right] \mathrm{d}p$, *we say that* Ξ^1 *is obtained from* Ξ^0 *by a composite rearrangement.*

Axiom 5 Weak rearrangement sensitivity: *If* Ξ^1 *is obtained from* Ξ^0 *by a composite rearrangement, then* $I(S_1) \leqslant I(S_0)$.

Theorem 1 *A rank dependent multidimensional inequality index* $I(S)$ *satisfies replication invariance, anonimity, nontriviality and weak rearrangement. In addition, if* $\mathrm{d}\omega(p)/\mathrm{d}p \leqslant 0$, *it also satisfies weak rearrangement sensitivity.*

Proof. See appendix. ∎

By imposing weak rearrangement sensitivity, we present a formalized version of the Temkin's (1986) weighted average approach. Indeed, Axiom 5 aligns with Temkin's philosophical perspective that "the larger someone's complaint is, the more weight is attached to it" (Temkin, 1986, p. 113).

3 Robust orderings

The *multidimensional complaint incidence curve*, $\psi_S(p)$, not only offers a clear visual representation of multidimensional inequality, but it also allows us to establish partial ordering for all rank-dependent multidimensional inequality indices $\in \Upsilon$.

Theorem 2 *$I(S_1) \leqslant I(S_0)$ for all $I(S) \in \Upsilon$ if and only if*

$$\psi_{S_1}(p) \leqslant \psi_{S_0}(p) \text{ for all } p \in [0,1].$$

Proof. See appendix. ■

Theorem 2 provides a simple visual criterion to detect robust multidimensional inequality orderings. The theorem implies that when two *multidimensional complaint incidence curves* do not intersect, the curve located beneath the other displays less multidimensional inequality for any index within the set Υ.

However, the orderings obtained from Theorem 2 may be incomplete. It possible to enhance the ordering capacity by restricting the analysis to rank-dependent multidimensional inequality indices obeying weak rearrangement sensitivity.

Definition 7 SET OF RANK DEPENDENT MULTIDIMENSIONAL INEQUALITY INDICES OBEYING WEAK REARRANGEMENT SENSITIVITY:

$$\Upsilon_R := \left\{ I(S) \in \Upsilon \,\middle|\, \frac{\mathrm{d}\omega(p)}{\mathrm{d}p} \leqslant 0 \right\}.$$

To identify orderings of multidimensional inequality that are consistent for all indices belonging to Υ_R, we utilize another construct, the *cumulative multidimensional complaint curve*, $\Psi_S(p)$.

$$\Psi_S(p) = \int_0^p \psi_S(u)\mathrm{d}u. \tag{5}$$

Figure 2 visually represents the concept of the *cumulative multidimensional complaint curve*. Since we rank individuals from the highest to the lowest level of complaints, $\Psi_S(p)$ has a concave shape. In the extreme case of the absence of multidimensional inequality, *cumulative multidimensional complaint curve* is equal to the horizontal axis:

$$\Psi_S^{\min}(p) = 0 \text{ for all } p \in [0,1]. \tag{6}$$

For the other extreme case with maximum multidimensional inequality, the *cumulative multidimensional complaint curve* is

$$\Psi_S^{\max}(p) = \frac{p(2-p)}{2}. \tag{7}$$

Note that, in general, the value of the *cumulative multidimensional complaint curve* at $p = 1$, $\Psi_S(1)$ represents the unweighted average of complaints. This value also corresponds to the measure of inequality if the analyst uses the ethical weight function $\omega(p) = 1$ for all $p \in [0, 1]$:

$$\overline{\psi}_S = \int_0^1 \psi_S(p)\mathrm{d}p \tag{8}$$

When encountering non-intersecting *cumulative multidimensional complaint curves* of complaints, it indicates robust partial orderings of multidimensional inequality for indices that adhere to weak rearrangement sensitivity. Formally, we can express this as follows:

Theorem 3 *$I(S_1) \leqslant I(S_0)$ for all $I(S) \in \Upsilon_R$ if and only if*

$$\Psi_{S_1}(p) \leqslant \Psi_{S_0}(p) \text{ for all } p \in [0, 1].$$

Proof. See appendix. ■

4 A parametric class of rank-dependent multidimensional inequality indices

Theorems 2 and 3 in the previous section generate partial orderings. To achieve a complete ordering, one must choose a particular index. Nevertheless, it is important to note that the chosen index might influence the ordering based on its specific form. Despite this limitation, estimating indices can be highly beneficial as it provides an analysis that is more easily comprehensible for non-specialized audiences.

Prior to defining a specific a particular class of indices, it is essential to introduce an additional desirable axiom that normalizes the index values between 0 and 1.

Axiom 6 Normalization: *$I(S)$ takes a minimum value at 0 and a maximum value at 1.*

Expanding upon Yitzhaki (1983), we put forth the following class of rank-dependent multidimensional inequality indices:

$$I_\nu(S) = \int_0^1 \nu(\nu-1)(1-p)^{\nu-2}\psi_S(p)\mathrm{d}p, \tag{9}$$

where $\nu > 1$ represents a parameter of aversion to multidimensional inequality. When $\nu = 2$, all the complaints are equally weighted, and the index is equal to twice the distance below $\psi_S(p)$ or twice the average complaint. When $\nu > 2$, the index puts more weight on complaints at lower values of p. When $\nu < 2$, the index puts more weight on complaints at higher values of p.

The weight function $\nu(\nu-1)(1-p)^{\nu-2}$ in equation (9) bears similarities to the weights used in the extended Gini indices function proposed by Yitzhaki (1983). However, in the context of the paper, we integrate the distance between the multidimensional complaint incidence curve and the horizontal axis (no multidimensional inequality) instead of integrating the distance between the 45-degree line (perfect income equality) and the Lorenz curve. Despite this difference in integration approach, the shape of the social preferences for social ranks is the same in both cases.

Theorem 4 *$I_\nu(S)$ satisfies replication invariance, symmetry, nontriviality, normalization, and weak rearrangement. In addition, if $\nu > 2$, it satisfies weak rearrangement sensitivity.*

5 Estimation and inference

If we denote by $X_1, \ldots, X_n$ an i.i.d. sample from a m-dimensional discrete attribute vectors where $X_i = (X_{1,i}, \ldots, X_{m,i})$, then an empirical counterpart of the complaint, $\widehat{S}_D(x)$, at the point $(x_1, \ldots, x_m)$, is given by

$$\widehat{S}_D(x) = \frac{1}{n}\sum_{i=1}^{n}\prod_{k=1}^{m}\mathbb{1}\{X_{k,i} \geqslant x_k\} - \frac{1}{n}\sum_{i=1}^{n}\prod_{k=1}^{m}\mathbb{1}\{X_{k,i} = x_k\}. \tag{10}$$

An estimator of $K_S(p)$, as defined in equation (2), is given by

$$\widehat{K}_S(p) = \frac{1}{n}\sum_{i=1}^{n}\mathbb{1}\{\widehat{S}_D(X_i) \geqslant p\}. \tag{11}$$

The generalized inverse of $\widehat{K}_S$ gives us a natural estimator of ψ_S:

$$\widehat{\psi}_S = \sup\left\{q \,\middle|\, \widehat{K}_S(q) \geqslant p\right\}. \tag{12}$$

A numerical integration can easily yield an estimator of $\Psi_S(p)$.

Assume we have two i.i.d. samples $X_1, \ldots, X_{n_1}$ and $Y_1, \ldots, Y_{n_2}$ with S_1 and S_2 being the complaints for both variables X and Y. If ψ_{S_1}, Ψ_{S_1} (respectively ψ_{S_2}, Ψ_{S_2}) are the functions corresponding to S_1 (and S_2 respectively), then stochastic dominance tests associated with the conditions in Theorems 2 and 3 can be conducted as follows. For Theorem 2, we aim to test $H_0 : \psi_{S_1}(p) \leqslant \psi_{S_2}(p)\ \forall p \in [0,1]$ v.s $H_1 : \psi_{S_1}(p) > \psi_{S_2}(p)$ for some $p \in [0,1]$. To do so, the following statistic could be computed

$$\widehat{\tau} = \sqrt{\frac{n_1 n_2}{n_1 + n_2}}\sup_p[\widehat{\psi}_{S_1}(p) - \widehat{\psi}_{S_2}(p)] \tag{13}$$

The bootstrap can then be used by creating B samples of size n_1 and n_2 by independent sampling with replacement of the indices of the observations in the original samples and then computing bootstrap estimators $\widehat{\psi}^b_{S_1}$ and $\widehat{\psi}^b_{S_2}$ for $b = 1, \ldots, B$. The distribution of the re-centered bootstrap statistic

$$\widehat{\tau}_b = \sqrt{\frac{n_1 n_2}{n_1 + n_2}}\sup_p[\widehat{\psi}^b_{S_1}(p) - \widehat{\psi}^b_{S_2}(p) - \widehat{\psi}_{S_1}(p) + \widehat{\psi}_{S_2}(p)] \tag{14}$$

could be used to determine the critical values for the test. The p-values are then obtained using

$$p = \frac{1}{B}\sum_{b=1}^{B}\mathbb{1}\left[\tau_b > \tau\right] \tag{15}$$

It is important to note that in the test above, the null hypothesis is a null of dominance, and the alternative is non-dominance. The obvious question is, why not test for a null of non-dominance to establish strict dominance? There are two reasons for not doing this as it is done in most empirical settings. First, the null of non-dominance over all the support of a curve is rarely rejected (see Davidson and Duclos, 2013). One can easily understand this issue in our context. If we refer to Figure 1, at $p = 1$, $\psi_S^{\min}(1) = \psi_S^{\max}(1) = 0$. This implies that at $p = 1$, all $\psi_S(1) = 0$. This mathematical result alone makes it theoretically impossible to reject a null of non-dominance $H_0 : \exists p \in [0, 1]$ such that $\psi_{S_1}(p) \geqslant \psi_{S_2}(p)$ v.s $H_1 : \psi_{S_1}(p) < \psi_{S_2}(p)\ \forall p \in [0, 1]$. This is a direct consequence of the fact that $\psi_{S_1}(1) = \psi_{S_2}(1) = 0$ for any pair of multidimensional complaint incidence curves. Another reason for not using a null of non-dominance is that, if one refers to Theorem 2, only non-strict dominance is required to establish robust orderings of the distributions. For these reasons, empirical researchers usually test for both

$$H_0^A: \ \psi_{S_1}(p) \leqslant \psi_{S_2}(p) \ \ \forall p \in [0, 1]$$

$$H_0^B: \ \psi_{S_2}(p) \leqslant \psi_{S_1}(p) \ \ \forall p \in [0, 1]$$

against their respective alternatives and use the decision rules depicted in Table 1. One can adapt a similar procedure for Ψ_S in testing the dominance condition in Theorem 3.

The asymptotic distributions for $\widehat{\psi}_S$ and $\widehat{\Psi}_S$ follow by basic finite-dimensional law of large numbers and central limit theorems as long as the support of X is finite. We can compute the asymptotic variances using the bootstrap. We assume finite support for this paper. This assumption is relevant if one is ready to consider all variables as discrete. For example, one can assume that total expenditure is a discrete random variable with a large finite support. In the case where X is absolutely continuous, empirical process methods derived for Kendall processes (as in Barbe, Genest, Ghoudi, and Rémillard, 1996; and van der Vaart and Wellner, 2007) can be used to derive the asymptotic distribution of ψ_S and

Ψ_S under additional smoothness conditions. In the case where X is absolutely continuous, it is immediate that ψ_S is a function of the copula of X and does not depend on its margins. The literature has not yet studied the mixed case where X has a subvector whose distribution is absolutely continuous with respect to Lebesgue's measure and another subvector whose distribution is discrete. However, results in Genest, Nešlehová, and Rémillard (2017) on multilinear empirical copula processes constitute a promising avenue for deriving the asymptotic distributions of the nonparametric estimators $\widehat{\psi}_S$ and $\widehat{\Psi}_S$. Note that alternative semiparametric estimators of $\widehat{\psi}_S$ and $\widehat{\Psi}_S$ (where a parametric copula family is combined with nonparametric marginal distribution estimators) could be derived using the methods in Gunawan, Khaled, and Kohn (2020).

6 Empirical application: Multidimensional inequality in the Arab World

6.1 Data

This section presents data for the two empirical applications mentioned earlier. For the first empirical application we use household survey data from harmonized Household Income and Expenditure Surveys (HHIES) compiled by the ERF. The data covers three countries: Egypt, Iraq, and Jordan. Specifically, we use: (1) the Income, Expenditure and Consumption Survey, Egypt (2015);[15] (2) the Household Socio-Economic Survey, Iraq (2012); and (3) the Household Expenditure and Income Survey, Jordan (2013). Household surveys are widely considered the gold standard data for canonical poverty and inequality analysis due to their provision of continuous income data (total expenditures), making them suitable for our analysis. For our multidimensional inequality analysis, we use three well-being dimensions: household's total expenditure, education, and disability status. We exclude individuals with essential missing information under the assumption that these

[15]There is a 2017/2018 version, but we are using a year that is closer to the survey years for Iraq and Jordan.

data points are missing at random. The final sample sizes are 11,988 for Egypt, 25,105 for Iraq, and 4,850 for Jordan. Table 2 provides information about the type variables and their respective categories, where applicable. However, it is important to note that the data is only available for a limited number of countries. Moreover, the frequency of these surveys is relatively low in the Arab region. Since our proposed approach adapts well to households surveys and alternative data settings as long as these sources contain ordinal information on well-being dimensions, we address this limitation by presenting an additional empirical application alongside the first one.

For the second empirical application we use data from 10 Arab countries from Wave-V of the Arab Barometer that covers 2018.[16] These ten countries are Algeria, Egypt, Iraq, Jordan, Lebanon, Libya, Morocco, Palestine, Sudan, and Tunisia.[17] Our multidimensional inequality analysis is built based on six discrete variables, as outlined in Table 5. We exclude individuals with essential missing information under the assumption that these data points are missing at random. The final sample sizes are 750 for Algeria, 1,008 for Egypt, 1,200 for Iraq, 1,172 for Jordan, 1,149 for Lebanon, 892 for Libya, 663 for Morocco, 1,176 for Palestine, 644 for Sudan, and 1,045 for Tunisia. Despite the Arab Barometer surveys having relatively smaller sample sizes compared to the HHIES, the method proposed in this paper enables us to rank multidimensional inequality effectively in numerous cases.

[16]There is more recent waves, but it does not include as much information on the wellbeing dimensions we use in this illustration.

[17]This survey includes data for Yemen. However, the income in Yemen is continuous, and the variable consists of discrete categories for all other countries. We have decided not to use Yemen for two reasons. First, for comparability with other countries, Second, the proportion of non-response to this continuous income question is much higher than for the discrete version in the other countries. The 2021/22 wave of the survey also contains continuous income information for other countries. Unfortunately the non-response rate is also substantial in this case.

6.2 An empirical application using the Harmonized Household Income and Expenditure Surveys

We estimate the multidimensional graphical tools and perform the associated dominance tests associated with Theorems 2 and 3. We perfom 299 bootstrap replications to generate the confidence bands on the figures and estimate the test statistics in equations (13) and (14). Figure 4 presents the comparisons of *multidimensional complaint incidence curves*, $\psi_S(p)$, for Egypt, Iraq, and Jordan. We conduct visual checks testing Theorems 2 and 3. A visual inspection of these figures indicates that Iraq exhibits lower multidimensional inequality compared to Egypt and Jordan. To corroborate these visual assessments, we perform a formal test using p-values as show in Table 3. The formal test confirms that Iraq's *multidimensional complaint incidence curve* is consistently below both Egypt's and Jordan's curves.[18] However, for the comparison between Egypt and Jordan, there is no clear dominance, as their curves for Egypt and Jordan intersect in Figure 4.

As mentioned, in cases where no clear dominance is indicated by Theorem 2, we employ *weak rearrangement sensitivity principle* and conduct tests based on Theorem 3. Table 3 displays the p-values associated with testing the condition in Theorem 3 for the Egypt vs. Jordan comparison confirming the absence of clear dominance. Table 4 summarizes the results for the rankings obtained using the aforementioned dominance tests. On one hand, Table 4 reveals that Iraq has less multidimensional inequality than Egypt and Jordan across all indices in Υ. On the other hand it shows that comparisons of multidimensional inequality between Egypt and Jordan did not yield robust results, as neither Theorem 2 nor in Theorem 3 demonstrates clear dominance.

In this study, we not only explore dominance tests but also leverage the rich data from the HHIES to estimate the multidimensional index of inequality, as defined in equation (9) using the HHIES data. The index-based approach has long been favoured by policy-

[18] We use a critical level of 0.05 for all the dominance tests.

makers due to its ability to provide a single, concise numerical measure of inequality while allowing for complete rankings of inequality across various dimensions. White papers and policy reports frequently highlight such index-based measures for their simplicity and clarity. However, our empirical investigation challenges this conventional view by introducing a novel perspective: the consideration of uncertainty. We reveal that when uncertainty is taken into account, the efficacy of the index-based approach may not always be as robust as previously believed. Instead, our findings underscore the significant advantages of employing a dominance-based approach, particularly in the presence of uncertainty.

In Section 4, we elaborated on how the index values are calculated, which hinges on varying levels of the inequality aversion parameter. This parameter quantitatively represents the extent of inequality aversion – a greater value signifies more pronounced aversion. For our calculations, we have chosen the values of 1.25, 2, and 3, which are most frequently used in this field. Unfortunately, the three values of the parameter chosen did not produce any additional ranking. If required, we could investigate other aversion degrees. We will revisit this matter in the final subsection, where we have more comparisons at hand to illustrate the importance of using a dominance-based approach in addition to an index-based approach, if the goal is to identify as many rankings as possible under uncertainty.

6.3 An empirical application using the Arab Barometer illustration

We enhance the scope of our initial analysis by incorporating a more extensive set of Arab countries. This expansion not only enriches our analysis but also underscores the feasibility of our proposed method, demonstrating its effectiveness even in situations where income data is presented categorically in surveys. As with our earlier process, we construct the figures and execute the dominance tests associated with Theorems 2 and 3, utilizing 299 bootstrap replications for this purpose. We first test Theorems 2 and 3 through visual checks, we then cross-compare them with the visual inspections from the previous empirical implementation. Figure 6 presents comparative data of *multidimensional complaint incidence curves*, $\psi_S(p)$, for the countries that feature in both surveys, namely Egypt, Iraq, and Jordan. On visual inspection of these figures, it appears that Iraq exhibits less multidimensional inequality than Egypt and Jordan. We further validate these visual insights through formal testing of the theorems presented in Table 3, using the p-values linked with testing the condition in Theorem 2. Table 6 displays all the p-values for the tests related to Theorem 2. For the countries examined in both surveys, the p-values corroborate the preliminary findings derived from the visual checks. As noted earlier, for cases in which Theorems 2 does not show dominance, we run an additional test based on Theorem 3. This restricts our testing process to include only indices that align with *weak rearrangement sensitivity*, allowing us to obtain additional rankings. Table 7 exhibits the p-values linked to these additional tests. As we will discuss subsequently, the results presented in Table 7 clearly highlight the practical utility of the dominance condition of Theorem 3. Indeed, in an empirical setting, the practitioner can obtain five additional multidimensional inequality rankings by restricting indices to those falling within the Υ_R.

Within the array of multi-country comparisons we're executing, the comparison of multidimensional inequality between Lebanon and Libya emerges as a particularly noteworthy instance from a methodological standpoint. An initial visual assessment of the *multidimensional complaint incidence curves*, depicted in the left panel of Figure 7, may lead one to preliminary conclude that Libya exhibits less multidimensional inequality than Lebanon across all indices in Υ. To verify this initial visual interpretation, it is essential to perform two statistical tests:

$$H_0^A : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$$

and

$$H_0^B : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1].$$

The p-value associated with testing H_0^A is 0.0502, and the one associated with testing H_0^B is 0.9766 (see, Table 6). An analyst adhering to the decision rules in Table 1 would infer that $\psi_{S_{\text{Lebanon}}}(p) = \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0, 1]$. Put differently, the analyst will determine that at the critical level of 0.05, a robust ranking is unattainable because the statistical tests cannot distinguish the two *multidimensional complaint incidence curves* at this degree of significance (since $0.0502 > 0.05$). Nevertheless, the visual check of the right panel suggests that Libya's cumulative multidimensional complaint curve is everywhere below Lebanon's curve. This example highlights the importance of combining information from the visual checks and the statistical tests when deciding whether to test for a more restrictive set of inequality indices. The p-values of the statistical tests the associated with Theorem 3, as stated in Table 7, reinforce the intuition derived from the visual inspection. Consequently, we can conclude that Lebanon experiences more multidimensional inequality compared to Libya for all indices included in Υ_R.

Table 8 summarizes the rankings based on the conditions stipulated in Theorems 2 and 3 and cover the comparisons mentioned earlier, as well as those for all other countries included in the Arab Barometer survey. Numerous robust rankings are obtained for all indices in Υ at the 0.05 level, with many even at the more stringent 0.01 level.

We divide the resulting robust rankings for all indices in Υ, in two groups based on the extent of magnitude of their multidimensional inequality. The first group contains, Iraq and Palestine which display lower multidimensional inequality compared to Egypt, Jordan, Lebanon, Morocco, Sudan, and Tunisia. Conversely, the second group positioned on the higher end of the distribution consists of Egypt, Lebanon, Morocco, and Sudan. These are the countries exhibiting the highest multidimensional inequality. Notably, none of these countries outrank any other country surveyed by the Arab Barometer.

As for the remaining robust comparisons, which include Algeria, Jordan, Libya, and

Tunisia, the findings suggest that Algeria has less inequality than Egypt, Jordan, Lebanon, Morocco, and Sudan. Libya fares better in terms of inequality than Egypt, Jordan, Morocco, and Tunisia. Tunisia shows less inequality than Egypt, Morocco, and Sudan. Finally, Jordan outperforms Tunisia in terms of lower multidimensional inequality.

When we delve into the robust rankings for all indices in Υ_R, we can add five additional rankings. For these indices, Iraq holds a better position than Algeria, Jordan outperforms Lebanon and Morocco, and Libya exhibits less inequality than Lebanon and Sudan.

It is noteworthy that both empirical applications deliver consistent conclusions. The three countries featured in both applications maintain the same rankings in terms of inequality. However, it's crucial to bear in mind that this result is not universally applicable. Indeed, the rankings might vary when the number of dimensions considered for the multidimensional inequality index changes.

6.4 Dominance-Based Approach vs. Index Based Approach

Examination of Figures 3 and 5 reveals intriguing insights into the sensitivity of index-based rankings to the aversion parameter, ν. In some cases, the rankings remain unchanged despite alterations in the aversion parameter. However, in others, the rankings exhibit substantial variability. In instances where rankings fail to maintain consistency, we typically resort to a dominance-based approach. This approach enables the establishment of robust rankings, unaffected by varying levels of the inequality aversion parameter. It is interesting to observe that when we compare these results with those obtained using the dominance-based approach, the latter stands out. We note that a dominance-based approach offers a wider scope for ranking possibilities exceeding the capabilities of the index-based approach.

Particularly, when we introduce the factor of uncertainty into our analysis, we encounter intriguing results. We notice that certain results yield statistical significance when we adopt a dominance-based ranking. Yet these same results don't maintain statistical significance under index-based ranking. It is generally assumed that the index-based approach delivers more complete rankings compared to its dominance-based counterpart. However, this presumption seems to hold true only when we deal with point estimates. The reason behind this could be that dominance tests have the ability to identify situations where multiple indices might rank two distributions differently, without conflicting with other indices. This comes into play when confidence intervals for two indices overlap.

Take for example our comparison between Algeria and Morocco shown in Figure 8. When $\nu = 2$, Algeria shows less inequality than Morocco, despite the intersecting confidence intervals. Yet, as illustrated in Figure 9 and Table 8, Algeria consistently exhibits less (or at the very least, not more) inequality than Morocco across all indices. This result hold significance at the 1% level. While initially, this may seem contradictory, a close look at Figure 9 reveals that the curves appear to be statistically different only above $p > 0.45$. This suggests that if we employ an index that displays less inequality aversion than the index I_ν with $\nu = 2$, we could obtain significant results. One such index is I_ν with $\nu = 1.25$

Returning to Figure 8, when we compare the values for $\nu = 1.25$, we find that the two confidence intervals no longer intersect, aligning with our earlier deduction. This empirical example underscores the importance of adopting dominance in empirical settings. It holds the potential to generate robust rankings in situations where the selected index fails to distinguish statistically significant differences in inequality between two countries.

7 Conclusion

In this paper, we bridge the gap between the literature on multidimensional inequality and Alkire and Foster's (2011) counting-based method for measuring multidimensional poverty. Our goal is to craft a framework for multidimensional inequality suitable for data commonly used in multidimensional poverty studies. We introduce two new graphical tools to achieve this: the *multidimensional complaint incidence curve* and the *cumulative multidimensional complaint incidence curve*. These tools effectively simplify the complexities involved in measuring of multidimensional inequality and provide a unidimensional representation of multidimensional inequality, offering a clear depiction of the subject. Further, we derive the dominance conditions linked with these tools, which are essential for determining the robust orderings of multidimensional inequality comparisons. As our measurement framework is new, we introduce the estimation and the statistical testing procedure associated with the aforementioned dominance conditions. To demonstrate the empirical relevance of the proposed approach, we apply the theoretical conditions, estimation, and testing procedures developed in the paper in analyzing two survey datasets: one with continuous income another with categorical income information. Our findings underscore the robustness of our proposed method, especially when studying multidimensional inequality in developing nations with limited statistical information.

Promising avenues for future exploration encompass the development of a multidimensional poverty measurement, along with the formulation of a social welfare framework grounded in this conception of multidimensional inequality. Furthermore, in the realm of statistical theory, a promising avenue lies in the development of asymptotic theory linked with the multidimensional complaint incidence curve in cases where one variable is assumed to have a continuous support while another has a discrete one.

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A Proofs

Proof of Theorem 1.

- The index being constructed from the survival function insures that it obeys replication invariance and symmetry.
- Since $\int_0^1 \omega(p)\mathrm{d}p > 0$, this implies that $\omega(p) > 0$ over at least some interval on $[0, 1]$. This insures nontriviality.
- Weak rearrangement is then a consequence of Proposition 3 in Decancq (2012). This proposition implies that the survival function $S_1(x) \leqslant S_0(x)$ for all $x \in \Re^m$. This implies that $K_{S_1}(p) \leqslant K_{S_0}(p)$ for all $p \in [0, 1]$, $\psi_{S_1}(p) \leq \psi_{S_0}(p)$ for all $p \in [0, 1]$ and $I(S_1) \leqslant I(S_0)$.
- $\int_0^1 [\psi_{S_{12}}(p) - \psi_{S_0}(p)]\,\mathrm{d}p = \int_0^1 [\psi_{S_0}(p) - \psi_{S_{11}}(p)]\,\mathrm{d}p$ and $\mathrm{d}\omega(p)/\mathrm{d}p \leqslant 0$ implies weak rearrangement sensitivity.

Proof of Theorem 2. Sufficiency is a direct implication of $\omega(p) \geq 0$. In order to prove for necessity, consider the following weight function that assigns all the social weight on the social positions $p \in [p_0 + p_0 + \varepsilon]$:

$$\omega_{\varepsilon 1}(p) = \begin{cases} 0 & \text{if } p \in [0, p_0] \\ \frac{1}{\varepsilon} & \text{if } p \in [p_0, p_0 + \varepsilon] \\ 0 & \text{if } p \in [p_0 + \varepsilon, 1] \end{cases} \tag{16}$$

If $\psi_{S_1}(p) > \psi_{S_2}(p)$ on the interval $[p_0 + p_0 + \varepsilon]$ then $I(S_1) > I(S_0)$ for $\omega_{\varepsilon 1}(p)$. Hence it cannot be that $\psi_{S_1}(p) > \psi_{S_2}(p)$ for $p \in [p_0, p_0 + \varepsilon]$, for any p_0 and any $\varepsilon > 0$. This proves the necessity of the condition.

Proof of Theorem 3. In order to prove Theorem 3, we first need to integrate equation (2) by parts:

$$I(S) = \omega(p)\Psi_S(p)|_0^1 - \int_0^1 \frac{\mathrm{d}\omega(p)}{\mathrm{d}p}\Psi_S(p)\mathrm{d}p \tag{17}$$

Since by definition $GL(0) = 0$, the difference $I(S_1) - I(S_0)$ can be written as

$$I(S_1) - I(S_0) = \omega(1) \left[\Psi_{S_1}(1) - \Psi_{S_0}(1)\right] - \int_0^1 \frac{\mathrm{d}\omega(p)}{\mathrm{d}p} \left[\Psi_{S_1}(p) - \Psi_{S_0}(p)\right] \mathrm{d}p \tag{18}$$

To prove sufficiency, we only need to remember that $\omega(p) \geq 0$ and $\frac{\mathrm{d}\omega(p)}{\mathrm{d}p} \leq 0$. We can then conclude that, if we have $\Psi_{S_1}(p) - \Psi_{S_0}(p) \leqslant 0$ for all $p \in [0,1]$, we must have $I(S_1) - I(S_0) \leq 0$.

In order to establish necessity, we need to consider two particular cases. First consider the ethical weight function $\omega(p) = 1$ for all $p \in [0,1]$. Since, for this particular index, $\frac{\mathrm{d}\omega(p)}{\mathrm{d}p} = 0$, equation (18) becomes:

$$I(S_1) - I(S_0) = \omega(1) \left[\Psi_{S_1}(1) - \Psi_{S_0}(1)\right]. \tag{19}$$

Hence it cannot be that $\Psi_{S_1}(1) > \Psi_{S_0}(1)$. Now consider the following weight function:

$$\omega_{\varepsilon 2}(p) = \begin{cases} \frac{1}{p_0 + 0.5\varepsilon} & \text{if } p \in [0, p_0] \\ \frac{p_0 + \varepsilon - p}{p_0 + 0.5\varepsilon} & \text{if } p \in [p_0, p_0 + \varepsilon] \\ 0 & \text{if } p \in [p_0 + \varepsilon, 1] \end{cases} \tag{20}$$

For this particular index, equation (18) becomes:

$$I(S_1) - I(S_0) = \int_{p_0}^{p_0 + \varepsilon} \frac{1}{p_0 + 0.5\varepsilon} \left[\Psi_{S_1}(p) - \Psi_{S_0}(p)\right] \mathrm{d}p. \tag{21}$$

If $\Psi_{S_1}(p) > \Psi_{S_0}(p)$ for $p \in [p_0, p_0 + \varepsilon]$, then $I(S_1) > I(S_0)$ for this index. Since the ethical weight function in (20) is not everywhere differentiable, it does not belong to Υ. However, there is a sequence of ethical functions $\{\omega_n(p)\}$ belonging to Υ such that $\lim_{n\to\infty} \omega_n(p) = \omega_{\varepsilon 2}(p)$. Applying Lebesgue's dominated convergence theorem leads to

$$\lim_{n\to\infty} \int_0^1 \omega_n(p)[\psi_{S_1}(p) - \psi_{S_2}(p)]\mathrm{d}p = \int_{p_0}^{p_0 + \varepsilon} \frac{1}{p_0 + 0.5\varepsilon} \left[\Psi_{S_1}(p) - \Psi_{S_0}(p)\right] \mathrm{d}p. \tag{22}$$

Hence it cannot be that $\Psi_{S_1}(p) > \Psi_{S_0}(p)$ for $p \in [p_0, p_0 + \varepsilon]$, for any p_0 and any $\varepsilon > 0$. This proves the necessity of the condition.

B Figures

Figure 1: Multidimensional complaint incidence curve, $\psi_S(p)$

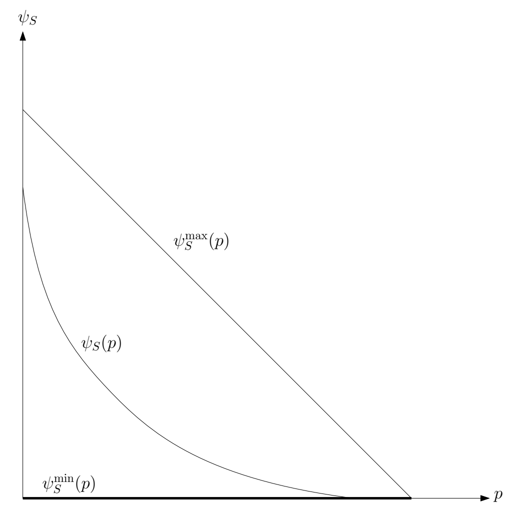


Figure 2: Cumulative multidimensional complaint curve, $\Psi_S(p)$

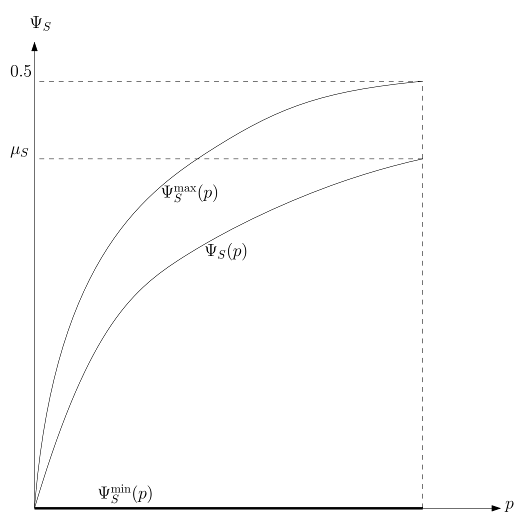


Figure 3: $I_\nu(S)$, HHIES

Figure 4: Comparisons of complaint incidence curve, HHIES

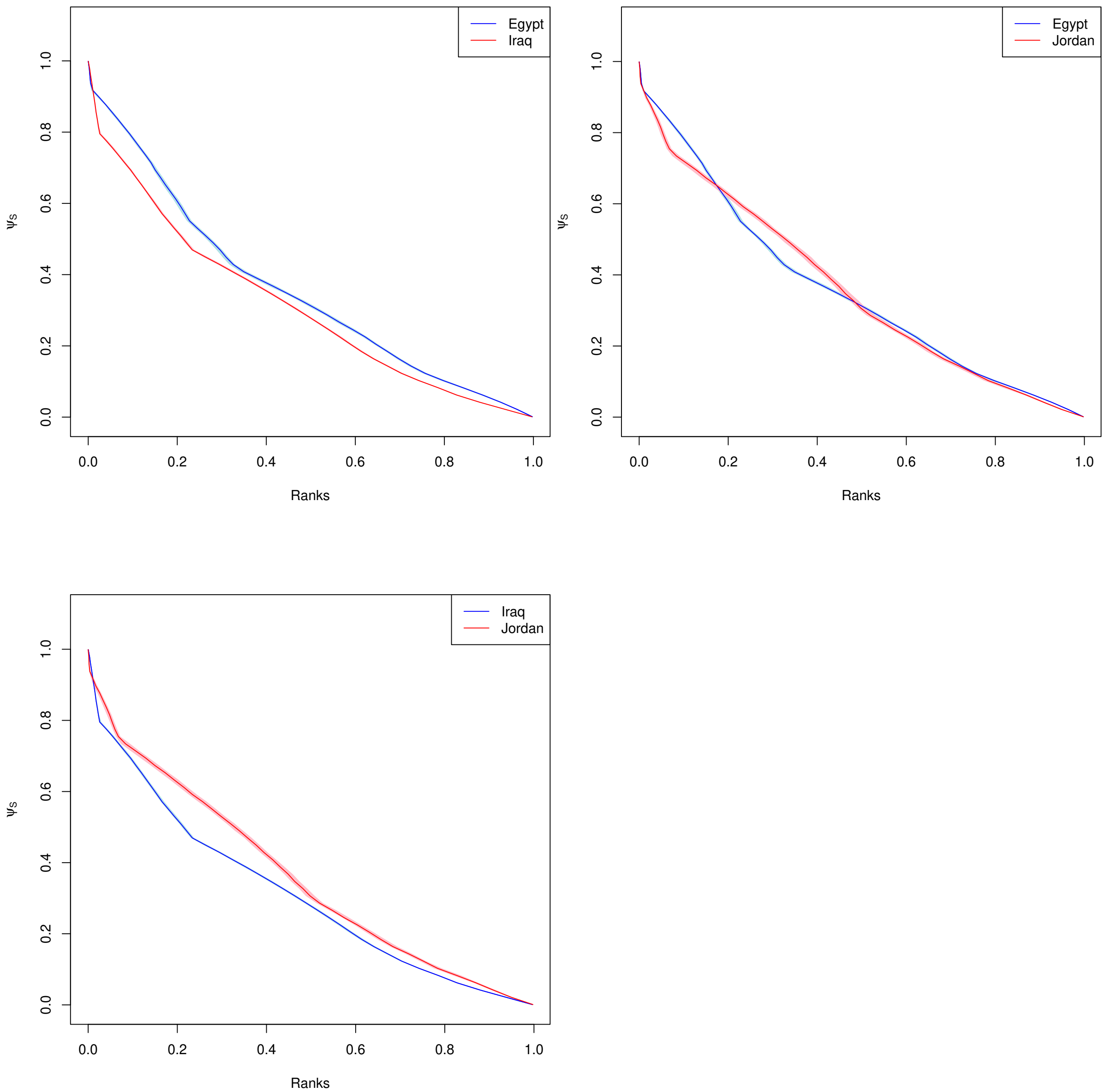


Figure 5: $I_\nu(S)$, Arab Barometer

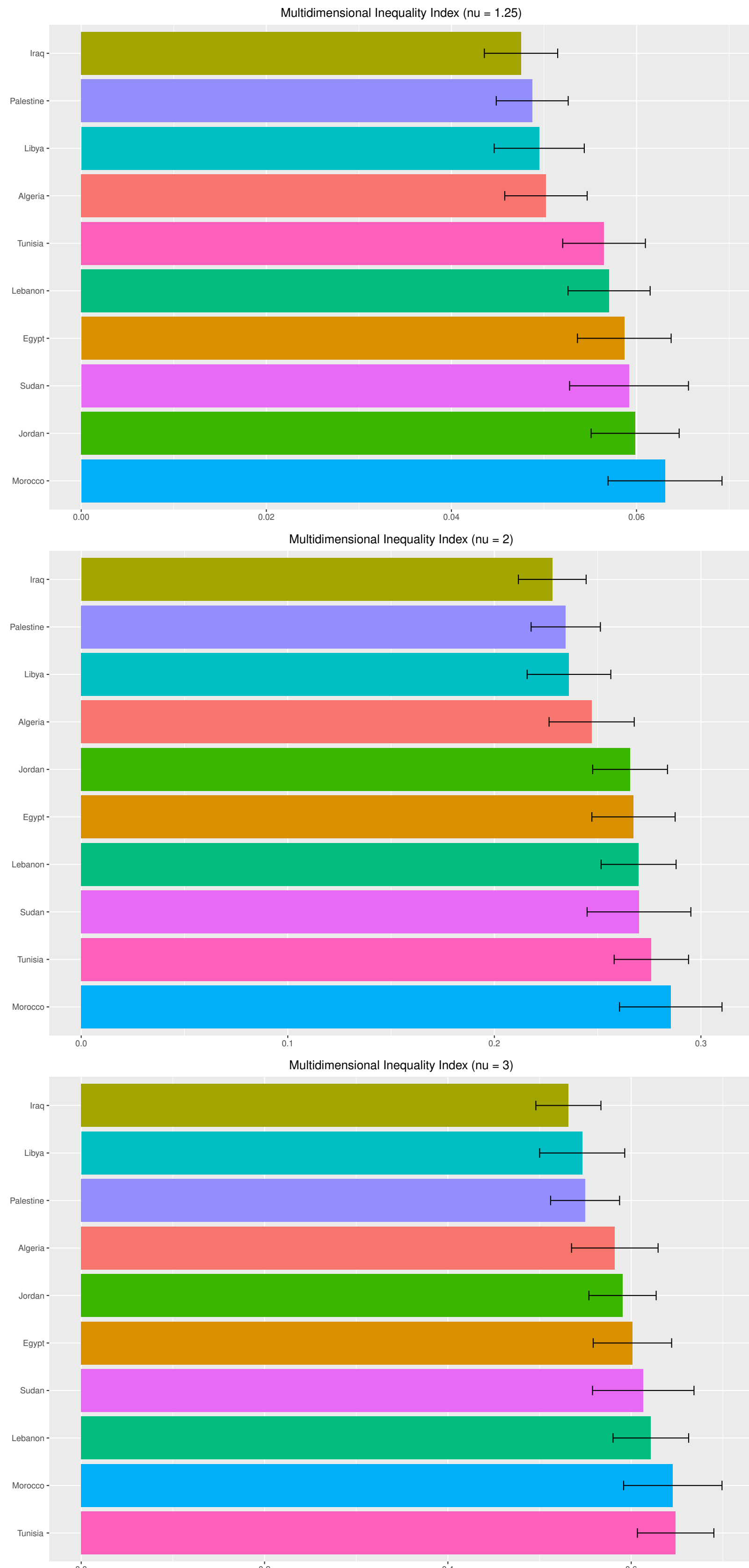


Figure 6: Comparisons of complaint incidence curve, Arab Barometer

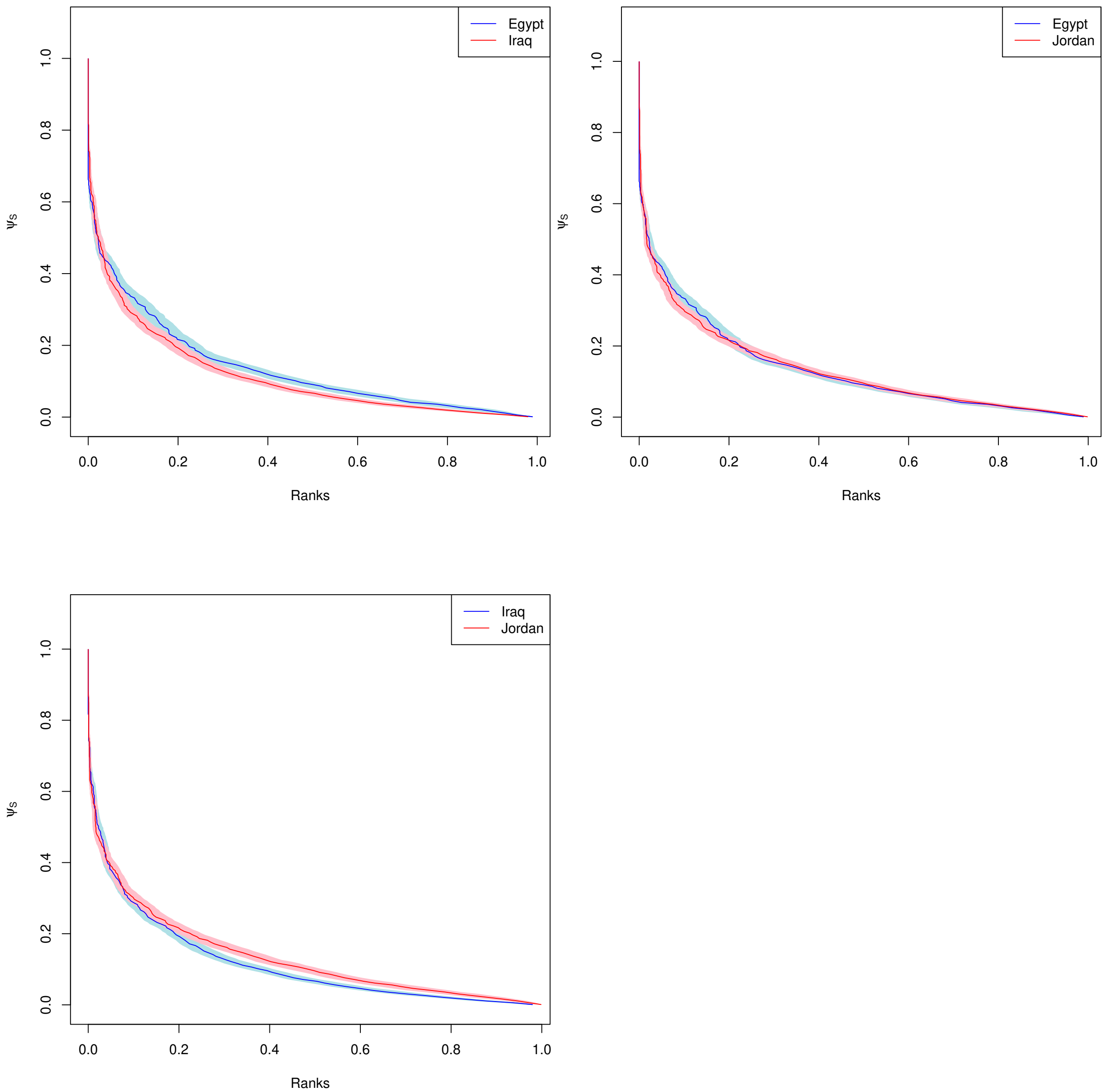


Figure 7: Comparisons of Lebanon and Libya, Arab Barometer

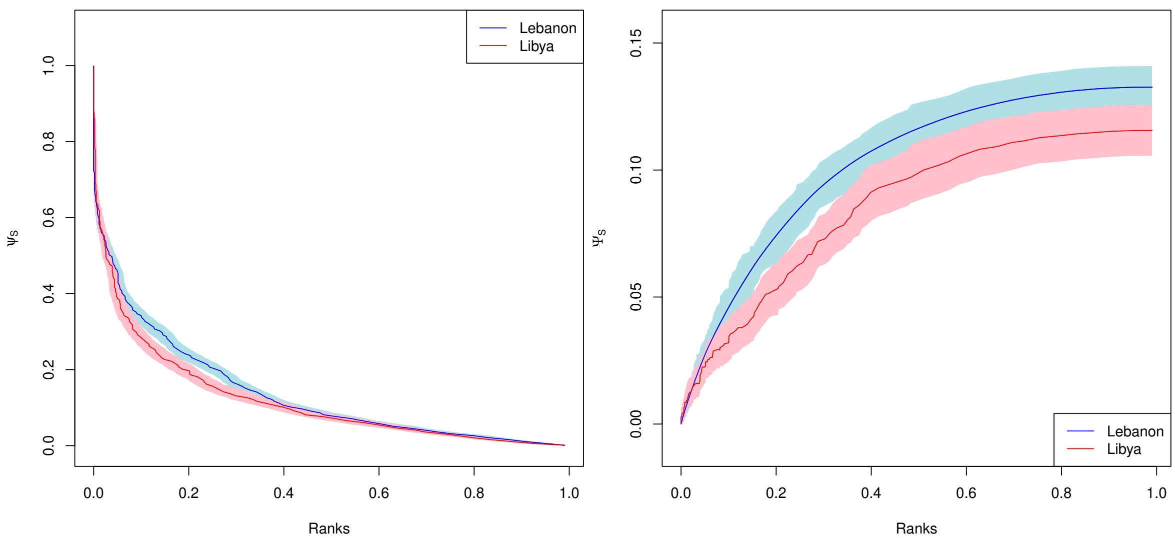
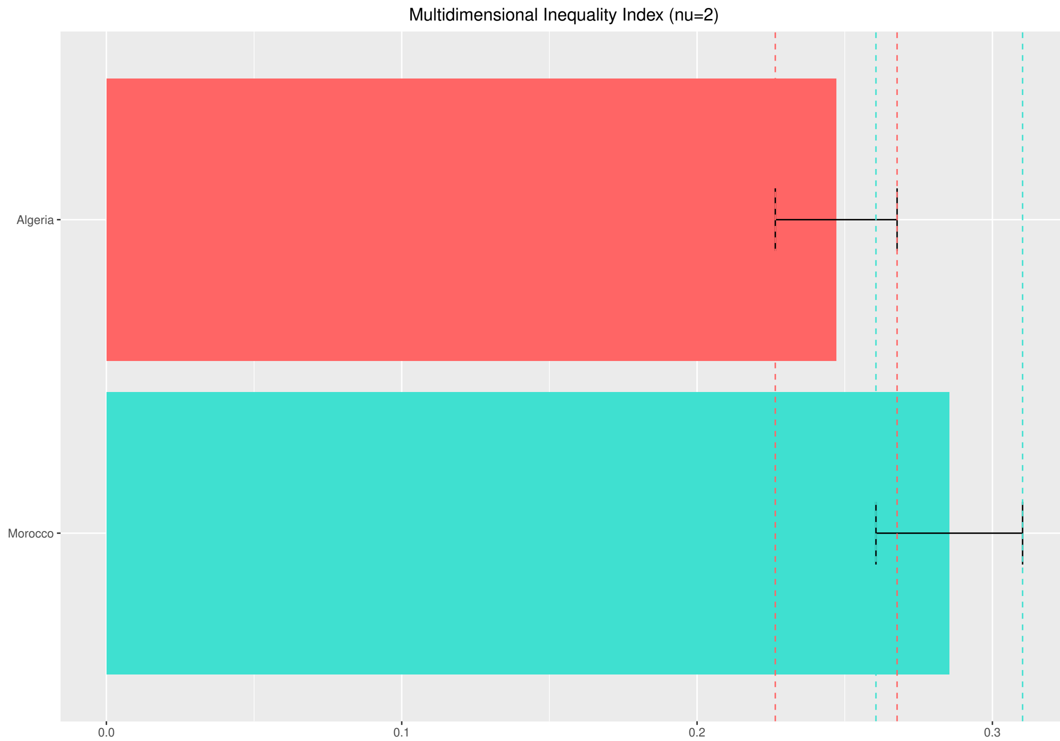


Figure 8: Comparisons of Algeria and Morocco using indices



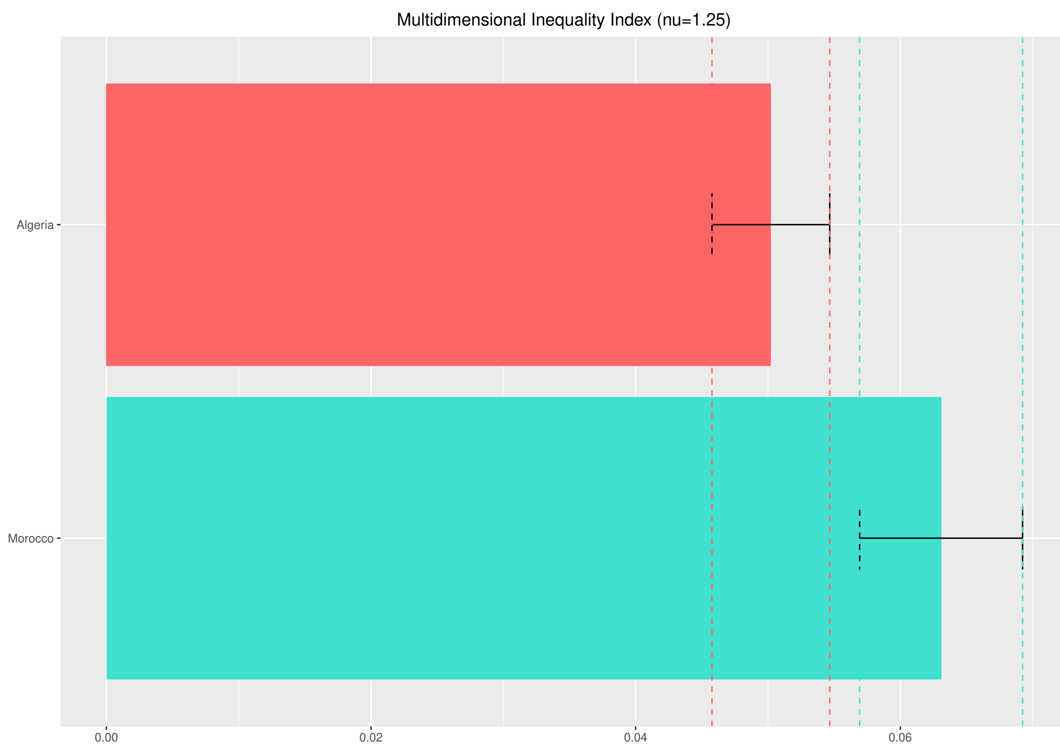
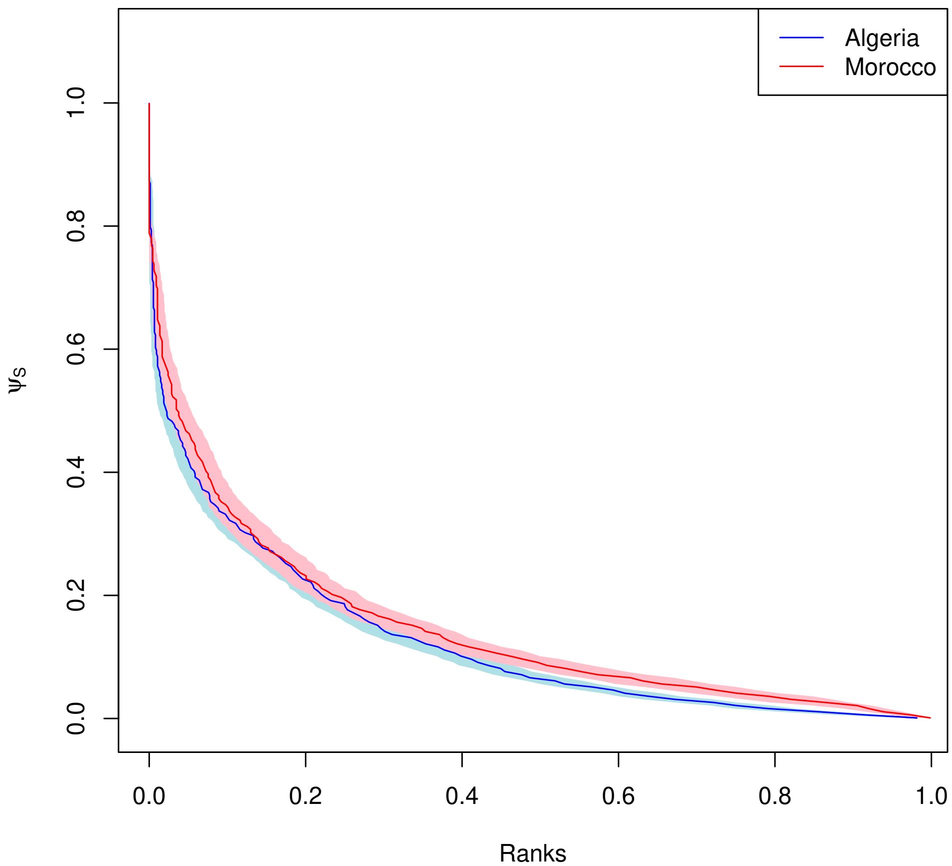


Figure 9: Comparisons of Algeria and Morocco using dominance, Arab Barometer



C Tables

Table 1: Dominance tests

Tests	Interpretation
H_0^A not rejected and H_0^B not rejected	$\psi_{S_1}(p) = \psi_{S_2}(p)\ \forall p \in [0,1]$
H_0^A rejected and H_0^B not rejected	$\psi_{S_2}(p) \leqslant \psi_{S_1}(p)\ \forall p \in [0,1]$
H_0^A not rejected and H_0^B rejected	$\psi_{S_1}(p) \leqslant \psi_{S_2}(p)\ \forall p \in [0,1]$
H_0^A rejected and H_0^B rejected	$\psi_{S_1}(p)$ and $\psi_{S_2}(p)$ intersect

Table 2: Variables in the Harmonized Household Income and Expenditure Surveys (HHIES)

Total expenditure	*Education*	*Disability status*
Continuous variable	None	Disabled
	Primary/Lower secondary	No disability
	Secondary	
	Post secondary or equivalent	
	University	
	Postgraduate	

Table 3: Dominance tests, HHIES

Theorem 2	p-value
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p)\quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p)\quad \forall p \in [0,1]$	0.7960
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p)\quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p)\quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p)\quad \forall p \in [0,1]$	0.7692
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p)\quad \forall p \in [0,1]$	0.0000
Theorem 3	**p-value**
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p)\quad \forall p \in [0,1]$	0.0000
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p)\quad \forall p \in [0,1]$	0.0000

Table 4: Inequality orderings, HHIES

	Egypt	Iraq	Jordan
Egypt	-		ND
Iraq	Υ ***	-	Υ ***
Jordan			-

$**$: Dominance at the 0.05 level for the indicated set of indices
$***$: Dominance at the 0.01 level for the indicated set of indices
ND: No dominance

Note: Table 5 is displayed on the next page.

Table 5: Variables in the Arab Barometer

Income	*Education*	*Feeling that own personal/family's safety and security are ensured of not*	Free to make decision for myself	Feel depressed	Feel stressed
Lebanon 9 intervals	No formal education	Not at all ensured	I strongly disagree	Most of the time	Most of the time
Others 12 intervals	Elementary	Not ensured	I disagree	Often	Often
	Preparatory/basic	Ensured	I agree	Sometimes	Sometimes
	Secondary	Fully ensured	I strongly agree	Never	Never
	Mid-level diploma/ professional or technical				
	BA				
	MA and above				

Table 6: Dominance tests (Theorem 2), Arab Barometer

Beginning of Table 6	
	p-value
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.8930
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.2207
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.3645
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.5484
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.9866
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.2609
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.1037
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9833
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.2341
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.2943
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.9465
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Algeria}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.9967
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.2575
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.9197
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.5117
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.3746
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.2676
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.3077
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0201
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.8361
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9365
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.5084
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.8763
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.3512
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.7826
$H_0 : \psi_{S_{\text{Egypt}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.0033
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.5452
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.8662
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.9866
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0201
Table 6 continues on the next page	

Continuation of Table 6	
	p-value
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.8495
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.1472
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9967
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.8896
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.6187
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.9833
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Iraq}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.8462
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0167
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.0970
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.2943
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0033
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.7726
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.7124
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.5017
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.8027
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.1204
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.4749
$H_0 : \psi_{S_{\text{Jordan}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.0033
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.1906
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0502
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.9766
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.7157
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.0669
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0134
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.9666
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.6388
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.2207
$H_0 : \psi_{S_{\text{Lebanon}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.1572
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.7324
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9933
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0033
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.2876
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.6823
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.9833
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0602
$H_0 : \psi_{S_{\text{Libya}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.6823
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0435
Table 6 continues on the next page	

Continuation of Table 6	
	p-value
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	1.0000
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.3980
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9933
$H_0 : \psi_{S_{\text{Morocco}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.8261
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.9565
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0000
$H_0 : \psi_{S_{\text{Palestine}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.9632
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0067
$H_0 : \psi_{S_{\text{Sudan}}}(p) \leqslant \psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	0.0134
$H_0 : \psi_{S_{\text{Tunisia}}}(p) \leqslant \psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.5753
End of Table 6	

Note: Table 7 is displayed on the next page.

Table 7: Dominance tests, Arab Barometer

	p-value
$H_0 : \Psi_{S_{\text{Algeria}}}(p) \leqslant \Psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.0468
$H_0 : \Psi_{S_{\text{Iraq}}}(p) \leqslant \Psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.9030
$H_0 : \Psi_{S_{\text{Algeria}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0635
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.6856
$H_0 : \Psi_{S_{\text{Algeria}}}(p) \leqslant \Psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.1416
$H_0 : \Psi_{S_{\text{Palestine}}}(p) \leqslant \Psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.7191
$H_0 : \Psi_{S_{\text{Algeria}}}(p) \leqslant \Psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	1.0000
$H_0 : \Psi_{S_{\text{Tunisia}}}(p) \leqslant \Psi_{S_{\text{Algeria}}}(p) \quad \forall p \in [0,1]$	0.1137
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.1806
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.7492
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	1.0000
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.1304
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0368
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.2943
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	1.0000
$H_0 : \Psi_{S_{\text{Morocco}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.0970
$H_0 : \Psi_{S_{\text{Egypt}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.8328
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Egypt}}}(p) \quad \forall p \in [0,1]$	0.2776
$H_0 : \Psi_{S_{\text{Iraq}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.8629
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.5552
$H_0 : \Psi_{S_{\text{Iraq}}}(p) \leqslant \Psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.0508
$H_0 : \Psi_{S_{\text{Palestine}}}(p) \leqslant \Psi_{S_{\text{Iraq}}}(p) \quad \forall p \in [0,1]$	0.2074
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.8395
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.0167
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9866
$H_0 : \Psi_{S_{\text{Morocco}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.0368
$H_0 : \Psi_{S_{\text{Jordan}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.8328
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Jordan}}}(p) \quad \forall p \in [0,1]$	0.1137
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0067
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.7425
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.8094
$H_0 : \Psi_{S_{\text{Morocco}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.4047
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.4682
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.7458
$H_0 : \Psi_{S_{\text{Lebanon}}}(p) \leqslant \Psi_{S_{\text{Tunisia}}}(p) \quad \forall p \in [0,1]$	1.000
$H_0 : \Psi_{S_{\text{Tunisia}}}(p) \leqslant \Psi_{S_{\text{Lebanon}}}(p) \quad \forall p \in [0,1]$	0.1739
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Palestine}}}(p) \quad \forall p \in [0,1]$	0.6656
$H_0 : \Psi_{S_{\text{Palestine}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.3880
$H_0 : \Psi_{S_{\text{Libya}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.8127
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Libya}}}(p) \quad \forall p \in [0,1]$	0.0468
$H_0 : \Psi_{S_{\text{Morocco}}}(p) \leqslant \Psi_{S_{\text{Sudan}}}(p) \quad \forall p \in [0,1]$	0.3579
$H_0 : \Psi_{S_{\text{Sudan}}}(p) \leqslant \Psi_{S_{\text{Morocco}}}(p) \quad \forall p \in [0,1]$	0.9331

Table 8: Inequality orderings, Arab Barometer

	Algeria	Egypt	Iraq	Jordan	Lebanon	Libya	Morocco	Palestine	Sudan	Tunisia
Algeria	-	Υ^{***}		Υ^{***}	Υ^{***}	ND	Υ^{***}	ND	Υ^{***}	ND
Egypt		-		ND	ND		ND		ND	
Iraq	Υ_R^{**}	Υ^{***}	-	Υ^{***}	Υ^{**}	ND	Υ^{***}	ND	Υ^{***}	Υ^{**}
Jordan				-	Υ_R^{**}		Υ_R^{**}		ND	Υ^{***}
Lebanon					-		ND		ND	ND
Libya		Υ^{**}		Υ^{***}	Υ_R^{***}	-	Υ^{***}	ND	Υ_R^{**}	Υ^{**}
Morocco							-		ND	
Palestine		Υ^{***}		Υ^{***}	Υ^{**}		Υ^{***}	-	Υ^{***}	Υ^{***}
Sudan									-	
Tunisia		Υ^{***}					Υ^{***}		Υ^{**}	-

$**$: Dominance at the 0.05 level for the indicated set of indices
$***$: Dominance at the 0.01 level for the indicated set of indices
ND: No dominance