

TIGHT BINDING

OPTIMUM DESIGN OF STEEL FRAMES INCLUDING  
COMPOSITE ACTION OF STEEL BEAMS AND CONCRETE  
SLABS AT ULTIMATE LOAD

by

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## SYNOPSIS

This thesis presents optimization procedures for the design of steel frames including composite steel beams at ultimate load. The study also includes a theoretical investigation of the effects of providing composite action in a building steel frame at ultimate load and its advantage over the corresponding non-composite steel frame based on the optimum solution.

In this study, a theory of plastic collapse is applied to the design of composite frames in a limited way due to the lack of ductility in the reinforced concrete slab. The optimum solution is found by using linear programming techniques, successively, in order to minimize the weight of the composite steel structures. A computer program generates, automatically, the constraints of the feasible collapse modes for the proposed structure and the corresponding objective function, then produces the optimum solution using linear programming algorithm without violating the safety criteria for the composite beams. The proposed procedure appears to be efficiently applicable when an electronic computer is used.

Furthermore, comparisons of the changes in required plastic moment capacities and relative cost on a weight saving basis of a composite steel frame to that of the corresponding non-composite one are made with several model frames. This investigation indicated that there is a definite advantage of the composite frames over the conventional steel frames from either economy or structural strength-capacity. Since the composite beams have greater shape factors than the normal steel beams, lighter members can be used, and also, it is not necessary to consider the buckling of the compression flange of the steel beams when the composite construction is adopted.

Finally, an alternative optimization procedure, using the nonlinear programming technique, is proposed to eliminate some of the difficulties and arbitrary restrictions which are found in the other method. This approach appears to be powerful in handling the nonlinearity of the minimum weight design problems and also efficient in refining the linear optimum solution.

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NOTATION

The following symbols are used in this paper:

- a = the distance from the top of concrete slab to N. A. of composite section, when N. A. is in slab;
- $a_{ji}$  = the coefficient of the variable  $x_i$  for the j-th constraint of linear programming problem;
- $\bar{a}_{ji}$  =  $a_{ji}/b_j$ ;
- $a_j(X)$  = the current value of the j-th constraint of nonlinear programming problem;
- $A_{ji}$  =  $[A_{ji}]$  = a  $m \times n$  matrix;
- $A_s$  = the total area of steel section;
- b = the effective width of concrete slab;
- $b_j$  = the external work for the j-th collapse mechanism and also the coefficient of the dual variable  $U_j$  for the dual objective function  $G(U)$  defined by  $G(U) = \sum_{j=1}^m b_j^j U_j$ ;
- $\bar{b}_i, \underline{b}_i$  = upper and lower bounds, respectively, for the variable  $x_i$ ;
- B = boundary of the feasible region;
- $C, C'$  = the compressive forces of concrete section and steel section, respectively;
- $C_i$  = the coefficient of the primal variable  $x_i$  for the primal objective function  $F(X)$  defined by  $F(X) = \sum_{i=1}^n C_i x_i$ ;
- d = depth of steel section;
- $d_{kx}$  = the displacement of the load  $P_k$  for the collapse mechanism  $x$ ;
- D = a feasible region;
- $D'$  = a region in  $E_n$  which controls D;
- e, e', e'' = moment arms, Eqs. (1.2) and (1.3);
- $E_n$  = Euclidian n-dimensional space;
- f =  $Z_p/Z_e$  = a shape factor;
- $f_c'$  = the compressive strength of concrete;
- F = an objective function;
- $F_{max}$  = the optimum value of the objective function in nonlinear programming problem;

- $F_w$  = the primal weight function;
- $F(X)$  = -  $W(X)$  = the current value of the objective function in nonlinear programming problem; the primal objective function;
- $F_y$  = the yield strength of steel ; t
- $g(X)$  =  $\nabla F(X)$  = the gradient of the objective function;
- $G$  = the intersection of the hypersurfaces  $G_1, \dots, G_q$ ;
- $G_w$  = the dual weight function;
- $G(U)$  = the dual objective function;
- $G_j(X)$  = the hypersurface in  $E_n$  defined by  $a_j(X) = 0$ ;
- $H_j(X_o)$  = the supporting hyperplane to  $G_j$  at  $X_o$ ;
- $I$  = an identity matrix;
- $L$  = the maximum length of  $D'$ ; also span length;
- $L_i$  = the length of the i-th member;
- $L_s$  = shear span length;
- $m$  = the number of constraints ; t
- $M$  = bending moment;
- $M_a$  = allowable moment;
- $M_p$  = fully plastic moment capacity; also the fully plastic moment capacity of steel section alone;
- $\bar{M}_{p_o}$  = the starting point to obtain  $\bar{M}_{p_{min}}$  ;
- $\bar{M}_{p_{min}}$  =  $X_{max}$  = the nonlinear minimum weight design solution;
- $M_r$  = redundant moment;
- $M_u$  = ultimate moment of composite section;
- $M_x$  = moment anywhere at distance x from the origin of the member;
- $n$  = the number of variables; also the number of shear connectors within  $L_s$ ;
- N. A. = neutral axis;
- $P$  = service load;
- $P_u$  =  $\bullet P$  = ultimate load;

- $P_q(X) = I - U_q(X) V_q(X) U_q^T(X)$  = the symmetric  $n \times n$  projection matrix;
- $q$  = the current number of active constraints;
- $q_u$  = the ultimate strength of one shear connector ;
- $Q$  = the total horizontal shear force in all shear connectors;
- $Q(X_o)$  = the intersection of the supporting hyperplanes  $H_1(X_o), \dots, H_q(X_o)$  at  $X_o$ ;
- $R(X_v) = V_q(X_v) U_q^T(X_v) g$  = a vector of order  $q$ ;
- $s$  = safety factor or load factor;
- $t$  = slab thickness; also the step length;
- $T$  = the tensile force in steel section;
- $u_j(X) = \nabla a_j(X)$  = the gradient of  $a_j(X)$ , a vector of order  $n$ ;
- $U$  = the distance from the original point to the section of the plastic hinge formed due to the distributed loading;
- $U_j$  = the  $j$ -th dual variable;
- $U_q(X) = [u_1(X), \dots, u_q(X)]$  = a  $q \times q$  matrix;
- $v_{ii}$  = the  $i$ -th diagonal element of  $V_q(X)$ ;
- $w_i$  = the unit weight of the  $i$ -th member;
- $W$  = total weight of a frame;
- $W(X)$  = the current weight of the frame;
- $W_q(X) = [a_1(X), \dots, a_q(X)]$  = the error vector of order  $q$ ;
- $|W_q(X)|$  = the length of error vector, a measure of the nearness of the point  $X_v$  to the intersection  $G$ ;
- $x_i$  = the current value of the  $i$ -th design variable; also the  $i$ -th primal variable;
- $x_v$  = a point which lies near the intersection,  $G$ , of  $q$  constraints,  $1 \leq q \leq m$ ;
- $X$  =  $[x_1, \dots, x_n]$  = the current design, a vector of order  $n$ , a point in  $E_n^p$ ;
- $X_{max}$  = the optimum solution;
- $Z_e$  = elastic section modulus;
- $Z_p$  = plastic section modulus;

- $Z(X_v) = P_q(X_v) g / |P_q(X_v) g| =$  a unit vector of order  $n$  which points in the direction of the projected gradient;
- $\alpha = M_u / M_p$ ;
- $\beta(X_v) = \max [ |P_q(X_v) q|, \beta_i ]$ ;
- $\alpha_i(X_v) = v_i(X_v) / 2 v_{ii}$ ;
- $\beta_i(X_v) = \max [ \beta_i(X_v) ]$ ;
- $\sigma_a =$  the allowable stress in bending;
- $\sigma_y =$  the yield strength in bending;
- $\delta =$  a tolerance which controls the nearness of the point  $X_v$  to the intersection  $G$ ;
- $\epsilon =$  a tolerance which controls the nearness of the point  $X_{\max}$  to the true optimum point;
- $\theta =$  the plastic hinge rotation;
- $\alpha_p =$  the plastic hinge rotation capacity;
- $\nabla a_j(X) = u_j(X) =$  the gradient of  $a_j(X)$  at  $X$ , a vector of order  $n$ ;
- $\nabla F(X) = g(X) =$  the gradient of the objective function at  $X$ , a vector of order  $n$ ;

## CHAPTER I

### INTRODUCTION

#### 1.1 Object and Scope of Study

The basic advantages of composite construction compared to non-composite construction are the savings in materials, greater stiffness of a floor system, and also greater overload capacity of a floor. Composite construction, for these reasons, are becoming popular both in bridges and more recently in building frames.

This thesis presents an optimization procedure for the design of steel frames including the composite action of steel beams and concrete slabs at ultimate load. The study also includes a theoretical investigation of the effects of providing composite action in a building steel frame at ultimate load, and its advantages over the corresponding non-composite steel frame based on the optimum solution.

The object of this investigation is to demonstrate the feasibility of using programming techniques for obtaining optimum plastic design of rigid steel frames with composite beams, and also to find the effects of composite action in a building frame, so that the full benefit afforded by composite action is realized.

The nonlinear analysis and design inherent in the plastic design of steel structures or the limit design of concrete structures have the advantage over the methods of analysis based on the normal assumption of elastic structural behavior. The plastic design procedures for steel structures first received extensive acceptance in the AISC Specification of 1961. Limit design for concrete structures has received only limited approval in the ACI Building Code of 1963, due to the uncertain and limited ductility of reinforced concrete members. Both of these nonlinear methods have found acceptance in many countries of Europe and in general throughout

the world. However the application of plastic design method to steel frames with composite beams has not found its way into full acceptance up to this point because of the uncertain and limited ductility of reinforced concrete slab. In this study, a theory of plastic collapse is employed in a limited way due to the lack of ductility in the reinforced concrete slab.

The steel sections used in this study will be rolled sections without cover plates. A non-symmetrical section, with cover plates at the bottom flange of the composite section, in combination with cover plates at the negative moment region, is more economical. In general, further reduction in steel weight can be achieved by transforming steel from the top flange to the bottom flange as it is done in built-up sections. However, the prime concern of this study is to determine the effect of composite action based only on the rolled steel section alone, so that fabrication will also be limited to a minimum.

The minimum weight plastic design method is applied to the design of composite and corresponding non-composite frames. In the plastic design of structures, the main difficulty has been in arriving at the correct mode of failure which occurs at the ultimate load. This collapse mechanism, and the ultimate load at which it occurs, are used as the basis of design. An examination of all possible modes of failure normally requires the testing of a very large number of failure mechanisms. For a structure of even moderate complexity this has made the procedure somewhat impractical to date. The computer program that has been developed overcomes these difficulties by generating only the equilibrium equations of the feasible modes. For the optimization of steel frames with composite beams, the successive linear programming techniques are used.

Finally, the effects of nonlinearity in the plastic optimization problems of both non-composite steel frames and composite steel frames are investigated by employing the nonlinear programming techniques as an optimum tool.

The computer used for this study is an IBM System 360/Model 65 computer.

## 1.2 Organization of Thesis

Chapter 2 illustrates how the optimum design of the steel frames with composite beams can be formulated.

Chapter 3 includes a general discussion and detailed presentation of the linear and nonlinear programming methods, including modifications and revisions which were found to be necessary.

In Chapter 4, the effects of the composite action in the building frames at ultimate load, based on the results of optimum solutions, are investigated. A comparison of the changes in plastic moment capacities and relative cost on a weight saving basis, of a composite steel frame to a corresponding non-composite steel frame, is made, with reference to typical two-story and two-bay rigid frame. (Fig. 4-1)

Several design examples of composite frames, based on the linear programming solutions, are presented in Chapter 5.

In Chapter 6, the nonlinearity in the plastic optimum design problems of both non-composite steel frames and composite steel frames are investigated in a general manner and also the feasibility of using the nonlinear programming techniques are discussed with some examples.

Chapter 7 draws several conclusions, based on the results of this study.

### 1.3 Review of Plastic Theory for Steel Frames

It is shown here the brief description of structural behavior up to collapse. It is followed with the fundamental theorems which are later applied to the minimum weight design problems of steel frames.

The theory of plastic collapse is applicable to the flexural behavior of a ductile material. Such a moment-rotation relationship for steel as shown in Fig. (1-2) is determined by assuming that each fiber will strain according to the idealized stress-strain curve shown in Fig. (1-1). Fig. (1-2) shows that after the maximum moment is reached, the rotation becomes indefinitely large. This moment, termed the plastic moment  $M_p$  is the ultimate moment which this section can maintain under the above assumption.

Upon attainment of the plastic moment at a particular cross section, rotation occurs with no increase in the bending moment, and the plastic hinge is said to have formed there.

Collapse occurs only when a sufficient number of plastic hinges developed to transform part or all of a structure into a mechanism. For structures composed of members with constant sections, the mechanism deformation will develop first at points with peak moment, where plastic hinges form and the intermediate elastic parts do not deform further. Final structural deformation then occurs without change in load.

If the collapse occurs when the frame is subjected to the magnitude of the applied load, it is termed the ultimate load. The ratio between the ultimate load  $P_u$  and service load  $P$  is referred to as the safety factor or load factor. The structure will not transform into a mechanism under  $P < P_u$ .

The ultimate load also can be shown to lie between the loads computed by the so-called upper-bound and lower-bound theorems. The

methods of analysis in the theory of plastic collapse are based on these theorems.

- i) Upper-bound Theorem. A load computed on the basis of an assumed mechanism will always be greater, or at best equal to the true ultimate load.
- ii) Lower-bound Theorem. A load computed on the basis of an assumed moment distribution which is in equilibrium with the applied loading and where no moment exceeds  $M_p$  is less than, or at best equal to the true ultimate load.

At the point of collapse the following conditions must be met:

- 1) The applied loads must be in equilibrium with the internal forces.
- 2) There must be a sufficient number of plastic hinges for the formation of a mechanism.
- 3) The plastic moment must not be exceeded at any point in the structure.

The mechanism method in which the principle of virtual work is used, is based on the upper-bound theorem. The virtual work principle can be stated as follows.

If a system of forces in equilibrium is subjected to a virtual displacement, the work done by the external forces is equal to the work done by the internal forces.

In the plastic analysis, virtual work is generated by subjecting the frame to virtual displacements consistent with the motion of the mechanism. The external work is equal to the sum of the products of all the forces times the displacements through which they move, while the internal work is equal to the sum of the products of the plastic moments times their hinge rotations. No internal work is performed by

the moments between the hinges, since there is no change in the curvature in the elastic parts of the structure.

It is convenient to classify mechanisms as independent or elementary mechanisms and combined mechanisms. Usually the elementary mechanisms are beam mechanisms and sidesway or sway mechanisms. If more than two members meet at a joint, an additional elementary mechanism is the joint mechanism. In addition various combinations of independent mechanisms can be made.

In the above concept in plastic design theory, however, the effects of axial loads and shear forces are not taken into account. It is also assumed that the instability effects can be ignored, and the deflection limitation is secondary. All these concepts and assumptions are used in the following chapters except specially dealt-with cases.

#### 1.4 Review of Plastic Theory for Composite Beams

The plastic design of composite beams presents some interesting problems not found in elastic design as follows:

- 1) There is no difference between the shored and the unshored composite beams in plastic design.
- 2) It is not necessary to locate the shear connectors close to the bearings, as long as the same total number of shear connectors required are used.

The summary of plastic theory of the composite beams are presented in the following parts. For a more detailed presentation of the plastic theory including the experimental results, the papers of (1), (2), (3), and (4) should be consulted.

#### Plastic Behavior of Composite Beams

The concrete slab acts as a cover plate in the composite beam. This is made possible by the use of shear connectors which

ensure that the steel beam and concrete act as a unit. If sufficient shear connectors are provided, the full plastic moment may be reached without the failure of the connectors.

When the moment on the positive composite section is increased beyond the working load, the bottom flange of the steel beam yields first and the neutral axis moves upwards causing tensile cracks at the underside of the slab. Cracking or spalling of the concrete at the compression face occurs when the strain reaches about 0.38 percent. The ultimate strength of the section is then almost reached. As the curvature of the section increases, crushing of the slab extends downwards, while strain hardening begins in the bottom flange of the beam and the load being carried remains approximately constant. As the curvature is increased still further, the carrying capacity of the composite section is eventually reduced by extensive crushing of the concrete.

This general pattern of behavior is exhibited by beams subjected to positive bending and for the case where the neutral axis is near the top steel flange or in the slab. In cases where the number of connectors is limited to such an extent, the failure of the connectors occurs before the maximum moment is developed. The ultimate moment capacity may also not develop if the plastic neutral axis is close to the bottom beam flange, as crushing at the surface of the concrete occurs before full plasticity can develop in the steel section.

#### 1) Shored and Unshored Composite Beams

It may be of interest to compare the differences in the behaviors, up to failure, between the shored and the unshored composite beams. Fig. (1-3) and Fig. (1-4) show the differences in the stress distributions and the deflections, respectively between the shored and the unshored composite beams. At the ultimate load, they reach the same stress distribution.

(a) Stress Distributions

In unshored construction, the steel beams have an initial stress distribution, as shown in Fig. (1-3), (stage 1). This is due to the dead load to be carried before composite action takes place. After composite behavior exists, the beam has stress distributions shown in stage 2.

The stress distributions for the shored composite beam are as shown in stage 2, where the dead and live loads are applied after composite action exists.

Although the bottom flanges reach the yield point as the live load increases, a greater load can be carried in both cases, because parts of the cross sections still remain elastic. The plastic behavior proceeds with increased load producing the stress distributions shown in stage 3.

The stress distribution at the ultimate load stage is the same for both the shored and unshored cases, as shown in stage 4. Therefore, the ultimate strength is independent of the loading process. The resisting moment of the cross section is equal to the external moment produced by the total load.

(b) Deflections

Fig. (1-4) shows the change in the deflection, covering stage 1 to stage 4, in accordance with the change in the stress previously noted. From these results, it is noted that the deflections due to live load, after composite action exists, are the same for the shored and the unshored composite beams. There is no difference between the shored and the unshored composite members in so far as the plastic

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design method is concerned which is based on the failure load. According to the classical method, different sections are designed for the shored and the unshored composite members, respectively, for the same design load. It is based on the working stress design method. Such a design method does not take full advantage of the structural properties.

## 2) Plastic Rotation Capacities of Composite Beams

When applying the plastic design method to composite beams, it is necessary that the composite beams have sufficient plastic rotation capacities. Although the steel is very ductile, the concrete is not, being relatively brittle. The moment curvature diagrams for the composite beams made from these materials, calculated from the stress-strain diagrams that were obtained from material tests of the concrete and the steel have been presented by Spillers <sup>(1)</sup>.

Fig. (1-6) shows the relationship between moment and curvature for the composite girders having the section shown in Fig. (1-5) for various strength of concrete. In Fig. (1-7) the effect of different widths of the cover plate, when the concrete strength is constant, is shown. If a large steel section in comparison to the concrete section is used, the rotation capacity decreases and the girder may fail due to the crushing of the concrete before the load reaches the ultimate load as calculated using the plastic design theory. Thus, when a larger concrete section having a high strength is used, a greater rotation capacity is obtained. These relationships are similar to those for reinforced concrete between the rotation capacity and the ratio of the area of the reinforcing steel to the area of the concrete.

### 3) Shape Factor of Composite Beams

The safety factor for the failure of a simply supported beam, designed by the working stress method, is given by

$$s = \frac{M_p}{M_a} = \frac{\sigma_y}{\sigma_a} \frac{Z_p}{Z_e} = \frac{\sigma_y}{\sigma_a} f \quad (1-1)$$

Where:

$M_p$	= Fully Plastic Moment	$M_a$	= Allowable Moment
$\sigma_y$	= Yield Strength	$\sigma_a$	= Allowable Stress
$Z_p$	= Plastic Section Modulus	$Z_e$	= Elastic Section Modulus
$f$	= Shape Factor = $Z_p/Z_e$	$s$	= Safety Factor

The shape factor in Eq. (1-1) depends on the shape of the section. Structures designed to have a constant allowable stress for the same design loads, obtain a bigger safety factor against failure, when a greater shape factor  $f$  is used. However, in the plastic design method, structures are designed for a given safety factor at failure, the section with the larger shape factor  $f$  is more economical.

The shape factors of shored and unshored composite beams are different. Also the ratio of the load before and after composite action results in different values of the shape factors.

### Ultimate Strength of Composite Beams

The concept of ultimate strength design in reinforced concrete can also be applied to the design of a composite concrete and steel beam. This may be classified into two categories: the first is based solely on the statical equilibrium of internal forces and the second takes into account the distribution of strains in addition to equilibrium of internal forces.

The second theory can theoretically account for the effect of strain hardening which can contribute as much as 25% to the carrying capacity of the beam, but it has not become popular as it is difficult to apply.

The first theory is based on the assumption that at the maximum flexural capacity, each element of the section has reached the plastic state of stress. This method has the advantage of simplicity and represents the lower limit of the bending capacity. Experiments conducted by Slutter and Driscoll<sup>(2)</sup> showed that the theoretical ultimate moment based on this theory can be reached as long as sufficient shear connectors are provided. In this study, the ultimate strength of a composite beam shall be calculated from the fully plastic distribution of stresses.

Normally, a composite beam fails due to the crushing of the concrete slab. At this instant a fully plastic state of stress is assumed for both concrete and steel. The assumed stress distribution is shown in Fig. (1-8). The plastic state of stress is represented by stresses of  $0.85 f_c'$  in concrete and  $F_y$  in the steel. Case I applies when the neutral axis is in the slab or when the concrete slab is adequate to resist the total compressive force at ultimate load. Case II applies when the neutral axis is below the slab or the concrete slab is not large enough to resist the total compressive force.

Referring to Fig. (1-8) the appropriate equations for computing the ultimate moments are:

$$\begin{aligned} \text{Case I} \quad C &= 0.85 f_c' b a \\ T &= A_s F_y \\ C &= T \\ a &= A_s F_y / (0.85 f_c' b) \\ M_u &= T e = T (d/2 + t - a/2) \end{aligned} \quad (1-2)$$

$$\begin{aligned}\text{Case II } C &= 0.85 f_c' b t \\ T &= A_s F_y - C' \\ T &= C + C' \\ M_u &= C e' + C' e''\end{aligned}\tag{1-3}$$

The values of  $e'$  and  $e''$  must be determined from the stress distribution and geometry of the cross section.

The ultimate moment for the negative moment regions is determined by a similar consideration of only the steel portion of the cross section. The longitudinal slab reinforcing steel may be considered if shear connectors are provided in the negative moment regions. However, in this study, the ultimate moment is reduced to the plastic moment of the steel member to neglect the effect of slab steel in the negative moment regions.

#### Ultimate Strength of Stud

The requirement for the shear connectors, when designing composite beams according to the plastic design method, are prescribed as follows:

- 1) The steel beam and the concrete slab to be connected together so that the composite beam can be subjected to its full ultimate capacity.
- 2) Shear connectors to be designed not to fail before the composite beam does.

Therefore, it is important to know the ultimate strength properties of the shear connectors in detail before their use in design. The result of the research done on the frequently used shear connectors are shown herein.

Generally, the ultimate strength of these connectors is investigated by means of the push-out test. Many experimental results are

reported. These reports term studs to be flexible shear connectors which permit some slip as the force increases to the ultimate load. The typical behavior of studs investigated by Thürlimann<sup>(3)</sup> is shown in Fig. (1-9).

The Bernoulli-Euler linear strain assumption is not satisfied when slips occur in the composite section. The AASMO Specification<sup>(5)</sup> defines the ultimate design capacity to be the useful capacity that corresponds to the load producing a slip of 0.003 inch divided by a safety factor.

However, it is reported that slip does not affect greatly the ultimate load-carrying capacity. Hence, the plastic design method can be based on the ultimate load-carrying capacity. The simple beam test is used to investigate the shear connectors under the conditions that are close to their actual behavior. Symmetrical point loads are applied, as shown in Fig. (1-10), to the simply supported beam having shear connectors. The shear connectors are subjected to horizontal shearing forces produced by these loads.

The load-carrying capacity of the shear connector investigated is shown as follows. Assuming that the beam fails when subjected to the ultimate bending moment, the compressive force  $C$  in the slab is balanced by  $\sum q_{cr}$ , the summation of the ultimate loads of the shear connectors within the shear span length  $L_s$ .

$$C = \sum q_u = n q_u \quad (1-4)$$

Where,  $C$  is obtained from the stress distribution at the ultimate bending moment. The ultimate load capacity for a connector is given by:

$$q_u = C/n \quad (1-5)$$

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Slutter and Driscoll<sup>(2)</sup> introduced their experimental formulae for the ultimate loads of shear connectors based on the results of many pushout and beam tests. The AISC Specification<sup>(6)</sup> established the design strength to be this ultimate load divided by the safety factor of 2.5.

### Design of Shear Connectors

Sufficient shear connectors are to be located so that the composite beam, made of concrete slab and steel beam, does not fail prematurely before being subjected to the fully plastic moment. The properties of the shear connectors have already been covered. For convenience, the relationship between load and slip for the shear connectors can be simplified, as shown in Fig. (1-11), and used in the investigation of the behavior of the shear connectors up to the failure. The approach of the plastic design method is tried.

In the case of proportional loading, generally, the horizontal shear force on each connector (connector force) is not the same. With the increase in the forces on the shear connectors some slip occurs, at the most highly stressed connectors, this transfers load to the remaining elastic connectors which finally reach their ultimate loading capacity. When all connectors reach their ultimate strength values, the beam fails by a rupture of the shear connectors.

The total horizontal shear force  $Q$  in all shear connectors is, regardless of the spacing of the connectors, given by

$$Q = \sum q_u = n q_u \quad (1-6)$$

Where  $n$  is the number of shear connectors within the shear span and  $q_u$  is ultimate load for one shear connector.

If the shear connectors fail when the ultimate load capacity of the composite beam is reached, the shear connectors are ideally designed.

From the equilibrium of the stress distribution in the cross section at the ultimate load:

$$C = T = A_s \cdot F_y \quad (\text{Neutral axis remains within slab}) \quad (1-7)$$

Where  $A_s$  is area of the cross section of the steel beam and  $F_y$  is the yield stress of the steel.

Letting  $Q = C$  in Eqs. (1-6) and (1.7), the number of required shear connectors is given by:

$$n = \frac{A_s \cdot F_y}{q_u} \quad (1-8)$$

The paper of (2) reported that the composite beams do not fail prematurely when the number of shear connectors used is greater than that calculated by Eq. (1-8). The beams fail at a lower load when the number of shear connectors used is less than that given by Eq. (1-8). The paper by Kurata and Shodo<sup>(4)</sup> also shows the design of shear connectors in the case of moving loads.

#### 1.5 Previous Applications of Programming Techniques to Optimum Design Problems of Steel Frames

From the view point of structural design, Curbon<sup>(8)</sup> defines the optimization problem as "the optimization of a structure consists of designing and constructing that structure at the lowest cost, with the object of fulfilling a well defined purpose. In particular, the safety factor of the structure must be specified. . . In estimating the cost, consideration must, of course, be given to the service life of the structure, its maintenance, and the possibility of its adaptation to meet any foreseeable changes in the service required."

However the term of optimum design of the frame used in this thesis will be confined to the problem of minimum weight design. Although the weight of a structure is not necessarily the best measure of optimality of civil engineering structures, it may nevertheless be one of the considerations in design, and then a saving in weight can be of importance. In this case, it is assumed that the weight of the structure determines the cost. This problem may be stated as follows. For given member length of plane frame, together with a set of static loads acting upon it, select the member sections that will make the weight of steel used minimum without violating the constraints for the safety criteria.

When the objective function considered for optimization and the constraint inequalities or equations are linear combinations of the variables, the problem is one of linear programming. All programming problems that are not linear but satisfy the general definition for an optimization problem given above are called nonlinear.

#### 1) Application of Linear Programming Techniques to Minimum Weight Design Problems

During the last decade, the application of linear programming techniques to minimum weight plastic design problems has been done by several researchers (12, 13, 14)

An early study by Heyman<sup>(9)</sup> showed that the method of inequalities could readily be adapted to the problem of designing structural frames for minimum weight of steel consumed.

From the theorem of lower bound, the bending moment  $M_{x_i}$  anywhere at distance  $x$  from the origin of the member  $i$  must not exceed the plastic moment capacity  $M_{p_i}$  of the member. Expressed mathematically it takes the form

$$M_{p_i} \geq M_{x_i} \geq -M_{p_i} \quad (i = 1, \dots, n) \quad (1-9)$$

The moment equilibrium equation requires that  $M_{x_i}$  must be expressed in the form of the external load,  $sP_k$ , and a set of independent redundant bending moment,  $M_r$ .  $M_{x_i}$  is implicit in the statement.

$$M_{x_i} = B_{xik} \cdot sP_k + C_{xir} \cdot M_r \quad (1-10)$$

Where B and C are constants, and so (1-10) yields

$$M_{p_i} \geq (B_{xik} \cdot sP_k + C_{xir} \cdot M_r) \geq -M_{p_i} \quad (1-11)$$

Eliminating  $M_r$  by the method due to Dines (1918), (1-11) can be converted into a set of m inequality of the form

$$\sum_{i=1}^n C_{ji} \cdot M_{p_i} \geq B_{ik} \cdot sP_k \quad (1-12)$$

Where D is a constant and  $j = 1, \dots, m$ .

Symond and Neal<sup>(10)</sup> proposed that the mechanism method which employs virtual work equations to find the correct mechanisms of collapse. The virtual work equations correspond to the inequalities of the equilibrium equations (1-12). Foulkes<sup>(11)</sup> later pointed out that these inequalities could readily be derived by considering them in terms of mechanisms. The advantage of this method is that the procedures for eliminating the redundant bending moments are not required, and the inequalities can be handled easily and readily in a systematic way.

Since the mechanism method is based upon the upper bound theorem, the plastic moment capacity  $M_p$  assigned

in the design of the structure will generally be greater than or possibly equal to the theoretically calculated value.

The following summary of the method of minimum weight design proposed by Foulkes has been represented here.

Assuming that each member has constant weight per unit length throughout its length, the total weight of the frame  $W$  is given by

$$W = \sum_{i=1}^n w_i \cdot L_i \quad (1-13)$$

Where  $w_i$  is the  $i$ -th member's weight of the section and  $L_i$  is its length.

The weight  $w_i$  is the function of the fully plastic moment capacity  $M_{p_i}$  and it will be written by

$$w_i = f(M_{p_i}) \quad (1-14)$$

On the assumption that the weight of the section used in appropriate generalization is directly proportional to its plastic moment capacity, Eq.(1-14) may be approximated by

$$w_i \propto M_{p_i} \quad (1-15)$$

Then the objective function to be minimized is expressed as a linear function of plastic moment capacities as follows

$$F_w = \sum_{i=1}^n M_{p_i} \cdot L_i$$

If a collapse mechanism  $x$  for the frame is assumed, the virtual work equation for this mechanism may be written as

$$\sum_k P_k \cdot dx_k = \sum_i M_{p_i} \cdot \theta_{ix}$$

Where  $d_{kx}$  are the displacements of the loads, and  $\theta_{ix}$  are the hinge rotations of the members relevant to the mechanism  $x$ .

In order that the structure is not to transform into a collapse mechanism  $x$ , it is necessary to satisfy the safety criteria,  $s_x \geq 1$ , i. e.

$$\frac{\theta_{ix}}{P_k \cdot d_{kx}} M_{pi} \geq 1$$

Letting

$$\bar{a}_{xi} = \frac{\theta_{ix}}{P_k \cdot d_{kx}}$$

This inequality may be rewritten by

$$\bar{a}_{xi} \cdot M_{pi} \geq 1 \quad (i = 1, \dots, n) \quad (1-16)$$

These inequalities are the same as the set (1-12), which are derived by the inequality method.

Under the above assumption, a problem in plastic minimum weight design method leads to a linear programming problem, which may be stated as follows

$$F_w = \sum_{i=1}^n M_{pi} \cdot L_i \quad (1-17)$$

Subject to the  $m$  constraint functions

$$\sum_{i=1}^n \bar{a}_{ji} \cdot M_{pi} \geq 1 \quad (j = 1, \dots, m) \quad (1-18)$$

and  $M_{pi} \geq 0 \quad (i = 1, \dots, n)$

Foulkes also established the necessary and sufficient condition for minimum weight design procedure. The Simplex method is a well known technique for the numerical solution

of linear programming problems. For a detailed presentation of linear programming techniques used for optimum solution, Section 3.1 shall be referred.

## 2) Application of Nonlinear Programming Techniques to Minimum Weight Design Problems

It should be pointed out that the applications of the nonlinear programming techniques to the minimum weight design problem in the past developments have invariably been based on the normal assumptions of elastic structural behaviors. Schmit<sup>(15)</sup> formulated the minimum weight elastic design of a three-bar plane truss as a nonlinear programming problem.

Moses<sup>(16)</sup> used the cutting plane method which involves solving a succession of linear programming problems to obtain the minimum weight elastic solution to a three bar truss problem and also to obtain the solution to a more practical problem consisting of a single-story single-bay rigid frame.

Using the modified Rosen's Gradient Projection Method of nonlinear programming techniques, Brown and Ang<sup>(17)</sup> developed the computer optimization method for obtaining minimum weight design of elastic structures.

However, the application of the nonlinear programming techniques to the minimum weight plastic design problems has not found its way into the computer application noted to this point.

The paper by Foulkes<sup>(11)</sup> investigated the nonlinearity of the plastic moment capacity - unit weight relations. It

is concluded that the errors introduced by the linear assumption of these relations are usually small.

The equilibrium method by Douglas<sup>(18)</sup> showed that the equilibrium equations in which distributed loadings are a factor and the hinge positions are not predetermined, should take the nonlinear form. Such effects of nonlinearity will be investigated in Chapter 6 in a more general manner.

The nonlinear programming problems may be solved by the gradient projection method, the steepest descent method or other similar methods. Nonlinear programming problems can also be transformed into linear programming problems by using a series of linear programming approximations or a series of unconstrained minimizations. For a detailed presentation of nonlinear programming techniques used for optimum solution, Section 3.2 should be referred.

## CHAPTER 2

### PLASTIC DESIGN OF STEEL BUILDING FRAMES WITH COMPOSITE BEAMS

#### 2.1 General Design Procedures

Plastic composite design has been applied to buildings with success in a limited way. Most of these have been built in England. Johnson, Finlanson and Heyman<sup>(19)</sup>, in the design of a five story building frame, found that plastic composite design gave substantial reduction in cost when compared with the established elastic composite design method. However, the applications of the plastic design method to composite steel frames have been relatively few in number.

In this paper, plastic design of building frames with composite beams is performed in the following way.

- 1) Determine the dimensions of the structure, working loads and ultimate loads.
- 2) Obtain the minimum weight design solution for the proposed composite steel structure.
- 3) Select sections based on the plastic moment capacities of members obtained in step (2).
- 4) Design the beam-columns.
- 5) Check the selected sizes for adequacy against local buckling, lateral buckling, excessive deflection, and shear.
- 6) Design the shear connectors, connections, and lateral bracing systems.
- 7) Consider the problems of fabrication and erection.

## 2.2 Assumption on Composite Beam Design

For design purposes, a composite floor is assumed to be a series of T-beams, each made up of one steel beam and a part of the concrete slab.

The width of concrete slab that is assumed effective as the flange of the T-beam is governed by the codes given in the AISC and the CSA Specifications for composite construction <sup>(6, 7)</sup>. For slabs extending on both sides of a beam, the effective width of the concrete flange must not exceed:

- 1) one-fourth of the beam span
- 2) the flange width of the steel beam plus sixteen times the slab thickness.
- 3) the distance from the center of the steel beam to the centres of adjacent steel beams.

For slabs extending on only one side of a beam, the effective concrete flange width shall not exceed:

- 1) one-twelfth of the beam span
- 2) six times the slab thickness
- 3) one-half of the clear distance to an adjacent steel beam

Furthermore, the ultimate strength is not significantly influenced by shoring, shrinkage or creep, and so except in unusual cases, none of these need be considered in the proportioning of composite beams for buildings.

## 2.3 Loading Conditions

Normally in building design, the following types of loading are considered; dead loads, live loads, lateral loads and deformational loads such as creep and shrinkage.

The dead loads in building frames will include the beams, columns, the floor or concrete slab, the walls, partitions and ceiling plasters. The live loads in building design are normally considered as uniformly distributed over the floor area.

At ultimate loads, all dead loads and live loads other than wind or earthquake are carried by the composite section. Opinions differ on the factor by which design loads should be multiplied. The load factor for plastic design of steel is the same for dead load and live load, while in ultimate strength design of concrete structures the dead-load factor is smaller than the live-load factor. A load factor of 1.70 minimum for plastic design of steel is prescribed in the CSA Codes, in which furthermore the reduction in the load factor is permitted for the loading combination of dead load plus live load plus wind load or earthquake load. However, in this study, it is assumed that the overall load factor of 1.70 for all the prescribed design or working loads, though the larger load factor appears better for the design of composite beams.

The selection of members will be based on the minimum weight design solution. In this study, two types of the loading combinations of dead load plus live load and also wind load, were considered into, simultaneously, through the application of the minimum weight design method to the example frames.

It is assumed that these loads are static and that they remain proportional during the entire loading history. Although this assumption is not fulfilled in many cases, study has shown that the usual load fluctuations existing in buildings will not cause a significant reduction of the ultimate load computed by assuming proportional loading. Plastic design should not be used if load fluctuations are severe enough and repeated often enough to cause fatigue failure.

## 2.4 Application of Plastic Collapse Theory to Steel Frames with Composite Beams

Composite beams are composed of two different materials ; steel possessing ductility and concrete lacking in ductility. Thus the limited ductility of concrete slab makes it difficult to apply the conventional plastic collapse theory to structural design of composite steel frames. On the other hand, the plastic compatibility condition ensures that sections do not fail due to crushing of the concrete slab prior to developing a plastic collapse mechanism. This requires that for any loading level up to collapse the plastic rotation  $\theta_i$  at the section  $i$  should not be larger than the rotation capacity  $\theta_{pi}$ .

$$\theta_i \leq \theta_{pi} \quad (2-1)$$

This condition can be incorporated in the optimization problem as an additional constraint. Alternatively the following approach is taken here.

The previous study<sup>(20)</sup> on the moment capacities of the composite beam section of a rigid frame at working loads indicates that the first plastic hinges in the composite beams are likely to form at the ends or at the negative moment regions and finally the plastic hinges form at the positive moment regions or near the mid-spans when ultimate loads are reached. Such a tendency is similar to the plastic behavior of the fixed-ended beam in which the first hinge will form at the ends and the hinge at the center will be the last to form; therefore rotation capacity is required at the ends and none is required at the center. Thus, though in a limited way, the plastic collapse theory may be applicable to the composite steel frames under the following conditions.

The plastic moment capacity at the negative moment regions shall be reduced to the plastic moment capacity of the steel section alone in order to allow large rotational deformations, and it gives conservative results. However, the plastic moment capacity at the positive moment region

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is assumed to be equal to the ultimate moment capacity of the composite beam (see Sec. 4.2), since the positive hinge likely to form last requires a very small rotation capacity. Furthermore, in order to give a sufficient amount of rotation capacity at the positive hinges against the critical condition and also make the foregoing assumptions more realistic, the following points shall be considered.

- 1) A use of larger concrete section having a high compressive strength gives a greater rotation capacity. (See Sec. 1.4)
- 2) Limit the maximum value of the ratio of the steel beam section to the effective concrete slab section not to cause the failure of composite sections due to crushing of the concrete slab before the ultimate load is reached. (See Sec. 1.4)
- 3) Test the plastic compatibility condition given by Eq. (2-1) after the optimization procedure.

## 2.5 Linear Optimization Procedures

### 1) Outlines of Optimization Procedures

In this section, we will examine how to develop an efficient computer program which employs linear programming techniques as a tool for optimization of steel frames with composite beams.

The formulation of minimum weight design problem consists of the following steps:

- a) Generate all feasible constraints from the elementary mechanisms for the steel frames alone in which the effects of composite action are neglected.
- b) Apply linear programming techniques to obtain the minimum weight design solution for the steel frames alone.

- c) For each composite beam section at which a positive plastic hinge is likely to form, assume the ratio of the ultimate moment capacity of composite section,  $M_u$ , to the plastic moment capacity of the corresponding steel section,  $M_p$ , where both  $M_u$  and  $M_p$  are unknowns, by substituting the value of the plastic moment capacity of the corresponding steel beam alone obtained previously, into  $M_u/M_p$  function which are approximated in terms of  $M_p$  under the given properties of structural members. (See Sec. 4.2).
- d) Modify the elementary mechanisms to assume the variable  $M_u$  approximately equal to the variable  $M_p$  multiplied by the  $M_u/M_p$  ratio which are previously calculated in step (c).
- e) Generate all feasible linear constraints from the newly generated elementary mechanisms in step (d) where the effects of composite action between steel beams and concrete slab are taken into account.
- f) Seek the linear programming solution for new constraints.
- g) Check the convergence of the solution for the composite sections. If all the solutions are satisfactory, then the correct minimum weight design solution is reached. If not, go back to step (c) and repeat the same procedures until the correct minimum weight design solution is found. (See Section 4-2).

These steps are briefly depicted in Fig. (2-1), and the corresponding optimization procedures will be carried out by using computer programs which are composed of a main program and a subroutine. The function of the main program is to generate the equilibrium equations of the feasible modes, to set the linear weight function in terms of variables  $M_{p_i}$ ; to control the whole procedures until the convergence in the solution reached, and also

to read in the input data and print out the minimum weight solution. The function of the subroutine program is to carry out the optimization procedure using the Simplex method in the linear programming. (See Sec. 3.1).

The input to the computer program consists of a description of the elementary failure mechanisms. These characterize the structure for a particular loading, and suitably combined they result in the feasible collapse mechanisms at the ultimate load. The coefficients of the objective function, which is to be minimized, are also required as input.

The programs were written in FORTRAN with desired flexibility. Whole procedures can be done in the computer cores without using any disc or tape units. The program is constructed to be used by other computer with minimum alternation.

A description of input data, a general flow chart for the main program and computer results for the example problems are presented in Appendix 4.

2) Generation of Equilibrium Equations of Feasible Collapse Mechanisms

In the computer program, the equilibrium equations of the collapse mechanisms are expressed in matrix form. This matrix is termed here a hinge rotation matrix, THE TA, which is related to column vector of external work, W<sub>ext</sub>. This will be written by

$$\sum_{i=1}^{NOMP} \sum_{k=1}^{NTHL} \text{THETA}(j, k, i) \cdot M_p(i) \geq W_{\text{ext}}(j) \quad (j = 1, 2, \dots, NELM)$$

..... (2-2)

where: NOMP = The number of plastic moments.  
NTHL = The number of hinge locations.  
NELM = The number of elementary mechanisms.

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The outlines of computer procedure for generating the feasible mechanisms are shown in Fig. (2-2), which will be explained as follows.

Generate all feasible combinations of each elementary mechanisms with a set of combined mechanisms. Such work will be done by checking hinge rotation matrix, and accompanied by series of elimination procedure so as to result in only the feasible combined mechanisms. After this cycle, a set of the combined mechanism No. (i + 1) results in. Repeat the similar procedures until no combined mechanisms can be generated. The constraints to be used for linear programming solution are stored in the matrix form which may be termed in this paper as constraint matrix  $W_{int}$  given by

$$\begin{aligned} & \text{NOMP} \\ & \sum_{i=1} W_{int}(j, i) \cdot M_p(i) \geq W_{ext}(j) \\ & \hspace{15em} (j = 1, 2, \dots, \text{NTMC}) \end{aligned} \quad (2-3)$$

where NTMC is the number of all feasible mechanisms.

## 2.6 Secondary Design Consideration

With the information from the minimum weight solution for the proposed structure, the selection of member sizes and details can begin. However in addition to furnishing the required plastic modulus, each member must meet certain additional requirements which assure its capacity to maintain the plastic moment through large enough rotations to permit all the hinges in the failure mechanism to form. These secondary checks are shown by:

- 1) Adjustment of member size for axial force.
- 2) Design of the shear connectors, connections and lateral bracing system.

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- 3) Local buckling
- 4) Shear force
- 5) Frame stability
- 6) Rotation capacity and deflection

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CHAPTER 3  
MATHEMATICAL PROGRAMMING TECHNIQUES

3.1 Linear Programming Techniques

1) Formulation of Linear Programming Problem

When the optimization problems of structural design are approximated by the linear objective function and the linear constraints, linear programming techniques are used.

In recent years, the availability of computers and computer programs has made the linear programming method applicable to the problems in which large numbers of variables are involved.

A summary of the linear programming problem and some techniques (21, 22) will be presented before its application to the optimization of the structure is discussed. (See Chapters 4 and 5).

The general problem in linear programming is as follows:

To minimize the linear objective function

$$F = \sum_{i=1}^n C_i x_i \quad (3-1)$$

where  $x_1, \dots, x_n$  are the variables of the problem, which are defined by the nature of the problem.

Subject to the  $m$ 's constraint functions

$$\begin{aligned} & x_i \geq 0 \quad (i = 1, \dots, n) \\ \text{and} \quad & \sum_{i=1}^n a_{ji} x_i \geq b_j \quad (j = 1, \dots, m) \end{aligned} \quad (3-2)$$

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One of the special features of all linear programming problems is the discrepancy between the number of constraints and the number of variables.

As the size of a linear programming problem that can be solved on a electronic computer is usually limited by the number of rows  $m$  involved, a problem that is too large when stated in terms of the primal may be of the right dimensions when written as the corresponding dual problem.

From the fundamental theorem of duality,

<p>Primal minimize</p> $F(x_1, \dots, x_n) = \sum_{i=1}^n C_i x_i$ <p>subject to</p> $x_i \geq 0 \quad (i = 1, \dots, n)$ $\sum_{i=1}^n a_{ji} x_i \geq b_j \quad (j = 1, \dots, m)$	<p>Dual maximize</p> $G(U_1, \dots, U_m) = \sum_{j=1}^m U_j b_j$ <p>subject to</p> $U_j \geq 0 \quad (j = 1, \dots, m)$ $\sum_{j=1}^m a_{ij} U_j \leq C_i \quad (i = 1, \dots, n)$ <p style="text-align: right;">..... (3-3)</p>
--	--

In the context of Eqs. (3-1) through (3-3), the minimum weight design problem is as follows:

The variables considered herein are assumed to be the fully plastic moment capacities of the structural members. If the weight of the sections are proportional to their fully plastic moment capacities, the objective weight function can be represented by

$$F_w = \sum_{i=1}^n \frac{M_i}{p_i} L_i \quad (3-4)$$

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Where  $M_{p_i}$  is the plastic moment of the i-th member, and  $L_i$  is its length.

Introducing the column vector  $(C_i)$  whose coefficients are the lengths of the members,  $L_i$ , the objective function can be stated as follows.

$$(C_i)^T (M_{p_i}) - F_w = 0 \quad (3-4)$$

The constraint inequalities are derived from the theory of plastic collapse with virtual work approach as set forth in Section 2.4

The complete system of linear inequality constraints may be written in matrix form,

$$\begin{aligned} \begin{bmatrix} A_{ji} \end{bmatrix} (M_{p_i}) &\geq (b_j) \\ \begin{bmatrix} I_{ii} \end{bmatrix} (M_{p_i}) &\geq (o) \end{aligned} \quad (3-5)$$

where;  $\begin{bmatrix} A_{ji} \end{bmatrix}$  is a matrix of coefficients of  $M_{p_i}$  terms. ( $m \times n$ )  
 $(b_j)$  is a column vector composed of the external work terms of the equilibrium equations. ( $m \times 1$ )  
 $\begin{bmatrix} I_{ii} \end{bmatrix}$  is an identity matrix, whose diagonal elements are one with all other elements zero. ( $n \times n$ )

## 2) Simplex Method and Computational Aspects of Linear Programming Problems

The solution for the specific problem in which there are two variables, can be readily solved by using the graphical method. When the number of constraints and variables are large, this method become useless. In such cases, the Simplex method is used.

Taking the direct expression as a minimization problem, the objective function by Eq. (3-4)' and the constraints by Eq. (3-5) can be arranged in the Simplex Tableau:

	1	n	(n + 1)	
1	$A_{ji}$	$-b_j$		
	(m x n)	(m x 1)		
m	$I_{ii}$	0		
	(n x n)	(n x 1)		
(m + n)	$C_i$	0		
	(1 x n)	(1 x 1)		
(m + n + 1)				

→ minimize  $F_w$  (3-6)

Primal Initial Tableau

where  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, m$ , with  $m > n$

In the minimum weight design problem, the number of constraints usually far exceeds the number of variables. Using the theorem of duality shown in (3.3), the minimization problem will be converted into the maximization problem in order to obtain better solution by computer. Hence the Simplex Tableau in (3.6) will be rearranged as follows:

	1	m	(m + n)	(m + n + 1)	
	$A_{ij}$	$I_{ii}$	$C_i$		
	(n x m)	(n x n)	(n x 1)		
n	$-b_j$	0	0		
	(1 x n)	(1 x n)	(1 x 1)		
(n + 1)					

→ maximize  $G_w$  (3-7)

Dual Initial Tableau

Once initial computational tableau has been constructed, the Simplex procedure calls for the successive application of:

- a) The testing of the elements of the last vector to determine whether a maximum solution has been found, i. e., whether

all elements of last row vector are less than zero, or at best, equal to zero.

- b) The selection of the column  $k$  to be introduced into the basis if some elements of last row vector are positive, i. e., selection of the column  $k$  with maximum element of last row vector.
- c) Test every entry in the pivot column  $k$  to see if it is positive or not. If all the entries in the pivotal column are zero or negative numbers, i. e.,  $a_{ik} = 0$ , then the solution is unbounded. If not, find the pivotal element, i. e.,  $\min (C_i/a_{ik})$ .
- d) The transformation of the tableau by the complete elimination procedure to obtain the new solution and associated elements.

Each such iteration produces a new basic feasible solution. Eventually a minimum solution will be obtained or a bounded solution determined.

An application of the Simplex procedure to the dual initial tableau in (3-7) yields the transformed values of tableau in

(3-8).

1	1	$m$	$(m+n)$	$(m+n+1)$	
$n$					
$(n+1)$			$M_p$ $p_p$		$\max G_w$

(3-8)

From the previous discussion in this section, the maximization of  $G_w$  corresponds to the minimization of the  $F_w$ . Thus the optimum solution is reached.

$$\min F_w = \max G_w \tag{3-9}$$

Of the several computational method available, Dantzig's Simplex method (23) is used.

### 3.2 Nonlinear Programming Techniques

#### 1) Formulation of Nonlinear Programming Problem

When the problem of optimum design can be formulated as nonlinear combinations of variables, nonlinear programming techniques can be used, and the related computational algorithms, provide feasible numerical techniques for the determination of an optimum design.

A general problem in nonlinear programming may be defined as follows:

To maximize the objective function:

$$F(X) = F(x_1, x_2, \dots, x_n) \quad (3-10)$$

Where  $x_1, x_2, \dots, x_n$  are the variables of the problem, which are defined by the nature of the problem.

Subject to  $m$  constraint functions,

$$a_j(X) = a_j(x_1, x_2, \dots, x_n) \geq 0 \quad (3-11)$$

where  $j = 1, \dots, m$ , with  $m \geq n$

The problem of maximizing  $F$  under these conditions is the same as the problem of minimizing  $-F$  under the same conditions. In the above formulae, the function  $F(X)$  and  $a_j(X)$  are assumed to be continuous functions of the variables  $X$ . The optimum design problem of structures may be stated as follows: The objective function  $F$  is considered as the weight of a structure, but as a programming problem must be expressed in a negative value, and the variables are assumed to be the fully plastic moment capacities of the structural members.

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## 2) Gradient Projection Method and Computational Aspects of Programming Problem

Several methods have been proposed for the solution of nonlinear programming problems. Among the available methods the gradient technique seems to have the greatest flexibility. In particular the "gradient projection method" of Rosen has been found to be well suited to the requirements of structural optimization.

The following summary of the gradient projection method of nonlinear programming has been condensed from the papers of Rosen (24, 25)

An Euclidian  $n$ -dimensional vector space  $E_n$  is considered. A point in  $E_n$  is represented by the vector.

$$X = (x_1, x_2, \dots, x_n) \quad (3-12)$$

A region  $D$  within  $E_n$  is defined by a set of  $m$  constraints in the form

$$a_j(X) \geq 0 \quad (j = 1, 2, \dots, m) \quad (3-13)$$

Corresponding to each function  $a_j(X)$  is a hypersurface  $G_j(X)$  defined by

$$G_j(X) = [X \mid a_j(X) = 0, j = 1, 2, \dots, m] \quad (3-14)$$

If a point lies in all of the hypersurface  $G_j, j = 1, \dots, q$ , it is said to lie in the intersection  $G$  of the  $q$ -hypersurfaces.

The feasible region  $D$  and its boundary  $B$  are defined as follows:

$$D = [X \mid a_j(X) \geq 0, j = 1, 2, \dots, m] \quad (3-15)$$

$$B = [X \mid X \text{ in } D, a_j(X) = 0 \text{ for at least one } j] \quad (3-16)$$

Every point in  $D$  is called a feasible point. An interior point  $X$  is one for which  $a_j(X) > 0$ , for all  $j$ . The point  $X$  is said to lie in a  $\delta$ -neighborhood of the intersection  $G$  if  $\sum_{j=1}^q a_j^2(X) \leq \delta^2$ . The region  $D$  can be enclosed in a region  $D'$  determined by an upper and lower bound on each variable,

$$D' = [X \mid -\underline{b}_i \leq x_i \leq \bar{b}_i, i = 1, 2, \dots, n] \quad (3-17)$$

The maximum length  $L$  of the region  $D'$  is defined by

$$L^2 = \sum_{i=1}^n (\underline{b}_i + \bar{b}_i)^2 \quad (3-18)$$

At each point  $X$  in  $D'$  the gradient of each function  $a_j(X)$  is a vector

$$\vec{u}_j(X) = \nabla a_j(X) = \left[ \frac{\partial a_j(X)}{\partial x_1}, \dots, \frac{\partial a_j(X)}{\partial x_n} \right] \quad (3-19)$$

Let  $X_0$  be a point which lies in a  $\delta$ -neighborhood of the intersection  $G$  of  $q$ -hypersurface ( $G_j, j = 1, \dots, q$ ) where  $1 \leq q \leq m$ . It is assumed that  $u_j(X_0) \neq 0$  for all  $j$  and that the  $q$ -vectors  $u_j(X_0)$  are linearly independent. The vector  $u_j(X_0)$  points into  $D$  if  $X$  is in  $B$  and toward  $D$  if  $X$  is outside of  $D$ . The vector  $u_j(X_0)$  is orthogonal to the hypersurface  $G_j$  at  $X_0$ . A hyperplane containing  $X_0$  which is orthogonal to  $u_j(X_0)$  is given by

$$H_j(X_0) = [X \mid (X^T - X_0^T) u_j(X_0) = 0] \quad (3-20)$$

$H_j(X_0)$  is the supporting hyperplane to  $G_j$  at  $X_0$ . Suppose that a point  $X_0$  lies in the intersection  $G$  of  $q$ -hypersurfaces. Corresponding to each hypersurface is a supporting hyperplane  $H_j(X_0)$  at  $X_0$ . These  $q$ -supporting hyperplanes form a linear intersection  $Q(X_0)$  which contains  $X_0$ .

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To compute the vectors  $u_j(X_0)$  requires repeated calculation of the first partial derivatives of the constraint functions. This computation can be done conveniently by a forward difference technique. The objective function  $F$  is defined for every  $X$  in  $D$ . The gradient of the objective function is given by

$$g(X) = \left[ \frac{\partial F(X)}{\partial x_1}, \dots, \frac{\partial F(X)}{\partial x_n} \right] \quad (3-21)$$

The nonlinear programming problem is then to

$$\text{maximize } F(X) \text{ for } X \text{ in } D. \quad (3-22)$$

A matrix  $U_q(X)$  is defined by the normals  $u_j(X)$  to  $H_j(X)$ , where  $j = 1, \dots, q \leq m$ ,

$$U_q(X) = [u_1(X), \dots, u_q(X)], \quad (X \text{ in } D) \quad (3-23)$$

The  $q$ -by- $q$  symmetrical matrix  $U_q^T(X) U_q(X)$  is nonsingular for all  $X$  in  $B$ . Let

$$V_q(X) = [U_q^T(X) U_q(X)]^{-1} \quad (3-24)$$

A symmetric  $n$ -by- $n$  projection matrix  $P_q(X)$  is defined for each  $X$  in  $B$  by

$$P_q(X) = I - U_q(X) V_q(X) U_q^T(X) \quad (3-25)$$

In the course of a typical gradient projection calculation it is necessary to obtain the projection of the gradient vector  $g$  on an intersection  $Q$ . For the purposes of this study  $V_q(X)$  is determined by direct inversion.

The stepwise optimization procedure can be described as follows. It is assumed that a feasible starting point is known. If the starting point is an interior point of  $D$  the constant gradient  $g$

is followed until a boundary point on B is reached. At a boundary point the supporting hyperplanes to the constraint hypersurfaces in which the point lies are determined. The gradient is projected on the intersection of these supporting hyperplanes. A step is taken in the direction of the projected gradient to a new point with an increased value of F. Since the constraints are nonlinear, the new point will generally not be a feasible point. It is then necessary to correct back to the feasible region in such a way that F remains greater than its value prior to taking the step. This correction procedure makes use of the inverse matrix  $V_q$  and is intended to determine a corresponding point on G. One measure of the distance of a point X from G is given by the length of the error vector.

$$|W_q(X)| = |[a_1(X), \dots, a_q(X)]| \quad (3-26)$$

Consider a point  $X_v$  and the corresponding matrices  $U_q(X_v)$ ,  $V_q(X_v)$  and  $P_q(X_v)$  as given by Eqs. (3-23), (3-24) and (3-25).

A q-dimensional vector with components  $\gamma_j(X_v)$  is defined by

$$R(X_v) = [\gamma_1(X_v), \dots, \gamma_q(X_v)] = V_q(X_v) U_q^T(X_v) q \quad (3-27)$$

Let

$$\beta_j(X_v) = \gamma_j(X_v) / 2 \sqrt{v_{jj}} \quad (3-28)$$

where  $v_{jj}$  are the diagonal elements of  $V_q(X_v)$ . Let

$$\beta_t = \max_j [\beta_j(X_v)] \quad (3-29)$$

$$\beta(X_v) = \max [ |P_q(X_v) q|, \beta_t(X_v) ] \quad (3-30)$$

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The unit vector  $Z(X_v)$  is defined by

$$Z(X_v) = P_q(X_v) g / |P_q(X_v) g| \quad (3-31)$$

The necessary and sufficient conditions that  $X_v$  yield a local optimum are  $P_q(X_v) g = 0$  and  $\gamma_j(X_v) \leq 0$ , where  $j = 1, 2, \dots, q$ . This means that the gradient must be orthogonal to the intersection  $G$ , and the component of the gradient in the direction of the normal to any one of the active constraint hypersurfaces  $G_j$  must point away from the feasible region. This is illustrated for the two-dimensional case in Fig. (3-1) In both Fig. (3-1a) and Fig. (3-1b)  $G$  is a vertex of  $R$  and thus  $P_q g = 0$ . In Fig. (3-1a)  $X_v$  is a local optimum because it is not possible to move along either of the curve,  $a_1 = 0$  or  $a_2 = 0$ , such that the objective function  $F$  is improved. But in Fig. (3-1b) it is possible to move along the curve  $a_1 = 0$  and thus improve  $F$ .

Hence, whenever  $|P_q g| > 0$ , it is possible to take a small step of length  $t$  in the direction of the projected gradient such that the objective function is improved. This is accomplished by computing the sequence of points.

$$X^{(0)}(t) = X_v + t Z(X_v) \quad (3-32)$$

$$X^{(j+1)}(t) = X^{(j)}(t) - U_q(X_v) V_q(X_v) W_q[X^{(j)}(t)]$$

$$\text{where } j = 0, 1, 2, \dots \quad (3-33)$$

Eq. (3-32) represents movement along the linear intersection  $Q(X_v)$ , and Eq. (3-33) represents iteration to correct back to feasible region. (See Fig. (3-2)). In taking the step represented by Eq. (3-32), the constant step length  $t = L/100$  in which  $L$  is given by Eq. (3-18), was used for the examples presented herein. This value of  $t$  was selected after other values led to greater computation

times for a typical problem. No attempt has been made to derive an optimum value for  $t$ . Using this constant value for  $t$ , it is possible to overshoot the optimum point  $X_{\max}$ ; this is illustrated in Fig. (3-3). Whenever this occurs, as indicated by a decrease in the objective function after applying Eqs. (3-32) and (3-33), a local optimum can be located by interpolating along the intersection  $G$ .

Whenever  $|P_q g| = 0$  but  $\gamma_j > 0$  for some  $j$ ,  $j = 1, 2, \dots, q$ , i. e., for  $\beta = \beta_t > |P_q g|$ , the objective function can be improved by dropping the constraint for which  $\gamma_j > 0$  and moving along the intersection of the remaining constraints. (see Fig. (3-4)). This is accomplished by using Eqs. (3-32) and (3-33) with  $P_{q-1}$ ,  $U_{q-1}$ ,  $V_{q-1}$  and  $W_{q-1}$  repeating the corresponding quantities in Eq. (3-33). The values  $P_{q-1}$ ,  $U_{q-1}$ ,  $V_{q-1}$  and  $W_{q-1}$  are obtained by omitting  $u_t$  and  $a_t$  from  $U_q$  and  $W_q$ , respectively.

It has been shown that a local optimum is also the global optimum if the feasible region  $D$  is convex. However, the feasible region for a structural optimization problem is not necessarily convex; for convex  $D$ , the gradient projection method may converge to a local optimum which is not global. This difficulty can be overcome, at least intuitively, by starting the algorithm at substantially different points and comparing the resulting solutions.

A tolerance  $\delta$  is provided which measures the nearness of each constrained point  $X_v$  to the corresponding intersection  $G$ , and a tolerance  $\epsilon$  is provided which measures the nearness of the indicated solution to the true optimum. It can be seen that  $\delta$  and  $\epsilon$  are not independent. The further a constrained optimum point

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is from the active constraints, the larger must be the error in the indicated optimum value of the objective function.

A point  $X_v$  is said to be on the intersection  $G$  if  $|W_q(X_v)| \leq \delta$ , where  $W_q(X_v)$  is defined by Eq. (3-26). The tolerance  $\delta$  is limited by the accuracy to which the constraint functions can be computed.

Finally, it is assumed that the feasible region  $D$  contains points  $X$  for which  $x_i > 0$ , where  $i = 1, 2, \dots, n$ . In the course of the calculations one or more variables may approach zero. This can be prevented, if necessary, by defining  $n$  constraints in the form of lower bounds on each variable.

On the basis of the foregoing adaptation of the basic formulation of the gradient projection method of nonlinear programming, the following algorithm is presented. It is assumed that tolerances  $\epsilon > 0$  and  $\delta > 0$  have been specified, and that an initial feasible point is known or can be determined.

a) If  $X_0$  is an interior point of  $D$ , let  $X_1 = X_0 + t g$ , where  $t$  is chosen so that  $X_1$  is in  $B$ . This is a one-dimensional interpolation to determine  $t$  such that  $a_j(X_0 + t g) \geq 0$ ,  $j = 1, 2, \dots, m$ , where the equality holds for at least one  $j$ . (See Fig. (3-5)).

b) Consider a point  $X_v$  which lies in a  $\delta$ -neighborhood of the intersection  $G$  of  $q$  constraints,  $1 \leq q \leq m$ . For convenience let these be  $G_j$ ,  $j = 1, 2, \dots, q$ , so that  $|W_q(X_v)| \leq \delta$ , where  $W_q(X_v)$  is given by Eq. (3-26). Compute  $V_q(X_v) = [U_q^T(X_v) U_q(X_v)]^{-1}$ ,  $R(X_v) = V_q(X_v) U_q^T(X_v) g$ ,  $P_q(X_v) g = g - U_q(X_v) R(X_v)$ , and  $\beta = \beta(X_v)$  as given by Eq. (3-30). If  $\beta \leq \epsilon/2nL$ , then  $X_{\max} = X_v$ , or if  $F(X_v) \leq F(X_{v-1})$ , then  $X_{\max}$  lies along  $G$  between  $X_v$  and  $X_{v-1}$ .

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c) If  $\beta > \epsilon/2 n L$  and  $|P_q g| \geq \beta_t$ , compute  $Z(X_v)$  by Eq. (3-31). (See Fig. (3-2)). A sequence of  $n$  points is computed by Eq. (3-32) and (3-33) with  $t = L/100$ , and  $n$  is the smallest integer such that  $|W_q[X^{(n)}(t)]| \leq \delta$ . If  $a_j[X^{(n)}(t)] \geq 0$ ,  $j = q+1, \dots, m$ , let  $X_{v+1} = X^{(n)}(t)$ . The point  $X_{v+1}$  lies in a  $\delta$ -neighborhood of  $G$ . If  $a_j[X^{(n)}(t)] < 0$  for at least one  $j$ ,  $j = q+1, \dots, m$ , interpolate for the value of  $t' < L/100$  such that  $a_j[X^{(n)}(t')] \geq 0$ ,  $j = q+1, \dots, m$ , with the equality holding for at least one  $j$ . (See Fig. (3-6)). Let  $X_{v+1} = X^{(n)}(t')$ . The point  $X_{v+1}$  lies in a  $\delta$ -neighborhood of a new intersection consisting of  $G$  and at least one additional constraint.

d) If  $\beta > \epsilon/2 n L$  and  $|P_q g| < \beta_t$  drop  $u_t$  from  $U_q$  and obtain  $U_{q-1}$  and  $V_{q-1} = [U_{q-1}^T U_{q-1}]^{-1}$ . Compute  $R = V_{q-1} U_{q-1}^T g$ ,  $P_{q-1} g = g - U_{q-1} R$ , and  $Z = P_{q-1} g / |P_{q-1} g|$ . A sequence of  $n$  points is then computed by Eqs. (3-32) and (3-33) with  $U_{q-1}$ ,  $V_{q-1}$  and  $W_{q-1}$  replacing to corresponding quantities  $U_q$ , etc. The value  $t = L/100$  is used, and  $n$  is the smallest integer such that  $|W_{q-1}[X^{(n)}(t)]| < \delta$ . (See Fig. (3-4)). If  $a_j[X^{(n)}(t)] \geq 0$ ,  $j = q+1, \dots, m$ , let  $X_{v+1} = X^{(n)}(t)$ . The point  $X_{v+1}$  lies in a  $\delta$ -neighborhood of the intersection of the  $q-1$  constraints  $G_j$ ,  $j = 1, \dots, q$ ,  $j \neq t$ . If  $a_j[X^{(n)}(t)] < 0$ , for at least one  $j$ ,  $j = q-1, 2, \dots, m$ , interpolate for  $t'$  such that  $a_j[X^{(n)}(t')] \geq 0$ ,  $j = q+1, \dots, m$ , with the equality holding for at least one  $j$ . Let  $X_{v+1} = X^{(n)}(t')$ . The point  $X_{v+1}$  lies in a  $\delta$ -neighborhood of the intersection of the  $q-1$  constraints  $G_j$ ,  $j = 1, \dots, q$ ,  $j \neq t$ , and at least one other constraint.

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As expected, the determination of the solution to a nonlinear programming problem by the gradient projection method is much more complicated than that of a similar linear programming problem. The cutting plane method of nonlinear programming<sup>(26)</sup> has been proposed as a simpler method of solution. In this latter method a series of linear programming problems are solved to determine the solution to the original nonlinear problem.

A number of advantages of the gradient projection method over the cutting plane method can be noted. With gradient projection, there is no increase in the number of constraints as the solution proceeds, and it is necessary to linearize only those constraints which are active at a particular stage of calculation. In addition, if the optimum solution does not occur at a vertex of the feasible region, the cutting plane method is even less satisfactory. The capabilities of the basic projected gradient approach for problems of structural optimization appear to be extensive.

CHAPTER 4  
INVESTIGATION OF EFFECTS OF COMPOSITE ACTION  
IN BUILDING FRAME AT ULTIMATE LOAD BASED  
ON RESULTS OF OPTIMUM DESIGN SOLUTION

4.1 General Remarks

Generally the type of construction controls the economy of a structure and the method of analysis. For structural steel frames two framing schemes are available.

The first type is usually referred to as simple framing. With this type of framing, the end moments are eliminated but it also results in the use of the largest possible beam or girder. However, elimination of the end moments makes possible the use of the simplest connections. There is also no problem as far as analysis is concerned, as only the simple beam moment is required.

The other type is rigid framing. The effect of rigid frame construction is to reduce the size of the steel beams or girders, but the cost of fabricating connections capable of fully rigid action offset the savings in girder material.

For both simple and rigid framing, a more economical design might be obtained by using composite action in the positive moment regions of the beams by providing shear connectors between concrete and the steel sections. In the following sections, we investigate the composite action for rigid framing.

4.2 Empirical Relationship between Ultimate Moment Capacity of Rolled Wide Flanges and That of Corresponding Composite Beams

As noted in Sec. 2.4, the ultimate moment capacity of a composite beam in the positive moment region,  $M_u$ , shall be calculated as a function

of the structural properties of the corresponding steel wide flange section and concrete slab. In order to make the conventional minimum weight plastic design method applicable to the design of composite steel frames, the ultimate moment capacity of a composite beam in the positive moment region,  $M_u$ , shall be approximated as a function of the plastic moment capacity of a rolled wide flange,  $M_p$ , assuming that all other factors related to the ultimate moment capacity of the composite beam, i. e., the thickness and effective width of concrete slab and strengths of materials are known. It will be written by

$$M_u = f(M_p) \quad (4-1)$$

Since the linear programming techniques are used as an optimum tool, it seems to be convenient to assume that the ultimate moment capacity of a composite beam section,  $M_u$ , is directly proportional to a plastic moment capacity of the wide flange,  $M_p$ , as given by

$$M_u \propto M_p \quad (4-2)$$

Eq. (4-2) may be rewritten by

$$M_u/M_p = \alpha \quad (4-3)$$

where  $\alpha$  is a constant.

However, the actual values of  $M_u/M_p$  will not be the same for all available sections since  $M_u$  depends on the structural properties of the steel members and concrete slab used. Such changes in the values of  $M_u/M_p$  were investigated and simultaneously the value of  $M_u/M_p$  is approximated as a function of the plastic moment capacity of given rolled wide flanges,  $M_p$ , by using the least square method. The results are as shown in Tables (4-2) through (4-4). The rolled wide flanges used for these calculations are identical to the so-called economy sections which provide a given capacity of plastic moment for the least weight as a steel beam alone.

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These sections are tabulated in the handbook of CSA Specification. Table (4-1) lists these sections together with the plastic moment capacities for A36 steel. The thickness of the concrete slab is varied from 3 inches to 6 inches in increments of 1-in. The author assumed the constant yield strength of steel ( $F_y = 36$  Ksi), while various compressive strengths of concrete ( $f'_c = 3, 3.5$  and  $4$  Ksi) are assumed. The effective width of the concrete slab is assumed as the flange width of the steel beam plus sixteen times the slab thickness. (See Sec. 2.2). The ultimate moment capacity of a composite beam section,  $M_u$ , is calculated by Eq. (1-2) and (1-3) which are based solely on the statical equilibrium of internal forces.

These works have been done by using the computer program developed for investigating this problem in a more general manner. It is possible to generate automatically the appropriate value of  $M_u/M_p$  as a function of  $M_p$  for the arbitrary sections of wide flanges under the conditions of the given thickness of concrete slab,  $t$ , yield strength of steel,  $F_y$ , and compressive strength of concrete,  $f'_c$ . The effective width of the concrete flange is defined according to the assumption in Section 2.2.

From the results shown in Tables (4-2) through (4-4), it is found that the ratio of the ultimate moment capacity of a composite beam to the plastic moment capacity of the corresponding wide flange steel section,  $M_u/M_p$ , is likely to lie between 3.0 and 1.0. Also the actual discontinuous relationship between  $M_u/M_p$  and  $M_p$  has been approximated with the various functions. Among these functions, the following functions present good results for the actual discrete relationship.

$$\text{Type A; } M_u/M_p = \epsilon \left( a + b M_p + c M_p^2 + d M_p^3 \right) \quad (M_p \approx 0) \quad (4-4)$$

$$\text{Type B; } M_u/M_p = a + b (\log M_p) + c (\log M_p)^2 + d (\log M_p)^3 \quad (M_p \neq 0) \quad (4-5)$$

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Where the values of coefficients a, b, c, and d corresponding to type A and type B functions are listed in Tables (4-2) through (4-4). Such approximations represent the continuous variation of the ultimate moment capacity of composite beams with plastic moment capacity of steel wide flanges.

Thus Eq. (4-4) and (4-5), compared to Eq. (4-3) will give better results for the approximation of  $M_u/M_p$  values, since the latter is expressed as a function of the variable  $M_p$ .

However, it is necessary to use the successive linear programming techniques for obtaining minimum weight design solution since the effects of composite action in the beams at the positive moment regions are approximated by the nonlinear functions as in Eqs. (4-4), and (4-5). In the following parts of this section, it is discussed how to apply these nonlinear functions, to the minimum weight design problem which employs the linear programming techniques. For the detailed presentation of the linear optimization procedures, refer to Sec. 2.5.

First it is necessary to assume the initial value of plastic moment capacity,  $M_p^{(1)}$ , for each composite beam to calculate  $M_u/M_p$  ratio, since the optimization problem is confined to the application of linear programming techniques. Such initial values of  $M_p^{(1)}$  at composite section which have to be optimized may be estimated from the inspection of the proposed structures, or assumed by the plastic moment capacities of steel beams in which the composite action in the beams is neglected, though the latter ones may be overestimation. It may be started with the average value of  $M_u/M_p$  for the available sections as given in Eq. (4-3). Such average values of  $M_u/M_p$  calculated for various cases are shown in Tables (4-2) through (4.4).

In this study, the non-composite structures were also optimized in order to make comparison with the composite structures under the same

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structural configurations and loading conditions. Hence, for each composite beam section, at which a positive plastic hinge is likely to form, the initial value of  $M_p/M_u$  ratio will be approximated by substituting the value of the plastic moment capacity of the corresponding steel beam alone,  $M_p^{(1)}$ , which obtained as a linear programming solution of the non-composite frame, into either Eq. (4-4) or Eq. (4-5).

Now the elementary mechanisms will be modified at each composite section in which the variable  $M_u$  is assumed as equal to the variable  $M_p$  multiplied by the  $M_u/M_p$  ratio which were calculated previously. Then the new feasible constraints will be generated from these elementary collapse mechanisms where the effects of composite action between steel beams and concrete slab are taken into account.

This yields to a new linear programming solution. Then the convergence of the solution must be checked. This will be done by checking relative errors of the newly obtained ultimate moment capacity of the composite beam,  $M_u^{(n)}$ , to the previously obtained ultimate moment capacity of the same composite beam,  $M_u^{(0)}$  at each composite section considered. It may be written by

$$\text{Relative Error} = (M_{u_i}^{(n)} - M_{u_i}^{(0)}) / M_{u_i}^{(n)} \quad (i = 1, \dots, \text{NOCP}) \quad (4-6)$$

Where NOCP is the number of composite beam sections at which positive plastic hinges are assumed.

If all the relative errors calculated by Eq. (4-6) are satisfactorily small, i. e., within 5%, the correct minimum weight design solution is reached. If not, calculate the new ratio of  $M_p/M_u$  for all the composite sections at which positive plastic hinges are assumed, as done previously, and repeat the same procedure until the correct minimum weight design solution is found.

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For all the example problems studied in this paper, the cycles of linear programming applications required for obtaining the minimum weight design solution through the foregoing procedures, fall within five times while less than 5% of relative error was secured.

#### 4.3 Effects of Composite Action in a Typical Two-Story Two-Bay Rigid Frame

The frame chosen for the theoretical investigation on the effects of composite action in a rigid frame at ultimate loads, is a two-story two-bay frame with fixed foundations shown in Fig. (4-1).

The object is to compare a composite plastic design to a non-composite plastic design by varying the span length  $L$ , loading conditions, the thickness of the concrete slab and the compressive strength of concrete. The results are based on the minimum weight design solutions.

For convenience, it is assumed that the two side-columns are identical. The fully plastic moment capacity of roof-beam, floor-beam, side-columns and center-column are denoted by  $M_{P1}$ ,  $M_{P2}$ ,  $M_{P3}$  and  $M_{P4}$ , respectively. The span length  $L$  is varied from 20 ft to 50 ft in increments of 10 ft, and the column height is kept constant at 12 ft. The loading conditions are varied by having the live load to dead load ratio (LL/DL) equal to approximate 1, 2 and 3. The thickness of the concrete slab is varied from 3 inch to 6 inch in increment of 1 inch. The constant yield strength of steel ( $F_y = 36$  Ksi) is assumed in all cases, while various compressive strengths of concrete ( $f_c' = 3, 3.5$  and  $4$  Ksi) are assumed. The effective width of the concrete slab is assumed as the flange width of the steel beam plus sixteen times the slab thickness.

Based on the slab thickness of 5 inch and a bent spacing of 15 ft, the concrete dead load is about 937 lbs per ft and assuming the beam weight

to be around 100 lbs, the total dead load, DL is assumed to be 1 Kip per ft. Based on this load, the total uniform load ( $w$ ) of 2 K/ft, 3 K/ft and 4 K/ft correspond to the LL/DL ratio of 1, 2 and 3, respectively.

For wind load a conservative value of 50 psf is used, compared to the 20 to 30 psf normally used in multi-story frames. Wind load is replaced by concentrated loads of a 4.5 Kip and 9.0 Kip load at the roof and floor levels, respectively for a column height of 12 ft as shown in Fig. (4-1).

For each plastic design problem, two types of the loading combinations; dead load plus live load and dead load plus live load plus wind load are considered simultaneously through the application of minimum weight design method. According to these loading combinations, working loads are multiplied by the safety factor of 1.7.

In formulating this problem, all the factors used are chosen within practical limits. These works have been carried out systematically, with the aid of a computer.

It is pointed out that in this section mainly the effect of composite action on the beam is investigated since the column remain nearly the same for both the non-composite and the composite design within the range of span length of 20 ft to 30 ft, and in the case of span length of 40 ft or over, the column design is likely to be governed by the axial loads.

One of the most difficult problem in the usual composite design is the determination of the correct beam size. It is desirable to provide composite action in a frame such that it is more useful to be able to obtain a trial section based on the loading conditions and the span length. To do this, an empirical relationship between the plastic moment in a composite beam, the span length and the loading conditions are required. To obtain such a relationship, the example frame of two-story and two-bay building frames are optimized by the foregoing procedures. (See Sections 2.5 and 4.2).

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The results are plotted as curves of plastic moments against span lengths for the various loading conditions in Figs. (4-3) and (4-4). In this case, it is assumed that the thickness of the concrete slab is 5 inches and the compressive strength of the concrete 4.0 Ksi. The plastic moments are given in terms of  $w L^2$ , where  $w$  is the working load (Kip/ft) and  $L$  the span length (ft). The curves given in Figs (4-3) and (4-4) are the plastic moment capacities of the roof beams,  $M_{P1}$ , and also of the floor beams,  $M_{P2}$ , respectively. For roof beams, the values of composite moment range from  $0.0372 wL^2$  to  $0.0432 wL^2$  for the range of loading of 2 to 4 Kips/ft for  $L = 20$  ft. For  $L = 50$  ft the range is  $0.0476 wL^2$  to  $0.0521 wL^2$  for the same range of loading. Values for span lengths between 20 and 50 ft fall within the values given by the above bounds. For floor beams, the changes in the required composite moment capacities are relatively small in comparison with those for roof beams. These curves may be used to estimate a trial steel section for composite design of a rigid frame.

In order to make a comparison, the non-composite design for the corresponding composite case is also carried out. The composite plastic moment capacities of roof and floor beams are expressed as a percentage of the non-composite plastic moment capacities of the corresponding beams, and these are plotted against the span length for the various loading conditions. The relationship obtained are shown in Figs. (4-5) and (4-6). The values of the composite moment of roof beams range from 48.7% to 57.3% of the corresponding non-composite plastic moment for the range of loading of 2 to 4 Kips/ft for  $L = 20$  ft. For  $L = 50$  ft the range is 66.9% to 72.3% for the same range of loading. It can be seen that approximately 30% to 50% reduction for both roof and floor beams in plastic moment capacity or plastic section modulus is possible - larger reduction for the shorter span length of 20 ft and the smaller for the 50 ft span length.

The above investigations have been done for the constant compressive strength of concrete ( $f_c' = 4.0$  Ksi). Further the effect due to the change in

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compressive strength of concrete under the constant loading condition ( $w = 3.0$  Kips/ft) is investigated. The compressive strength of concrete is varied to 3.0, 3.5 and 4.0 Ksi. The results shown in Table (4-5) indicate that when a higher strength of concrete is used, a greater saving in a steel beam is obtained and greater savings for the shorter spans.

It is also studied the effects of the change in the concrete slab thickness upon the required composite plastic moment capacities. The thickness of the concrete slab is varied from 3-in. to 6-in. in increments of 1-in. The results are shown in Figs. (4-7) and (4-8). In each case, the same compressive strength of concrete ( $f'_c = 4.0$  Ksi) is assumed and the total live load of 2.0 Kips/ft is assumed. The total dead load for various slab thicknesses are estimated by the following equations.

$$\begin{aligned} DL &= (t/5) \times 0.9 + 0.1 \text{ (K/ft)} \\ LL &= 2.0 \text{ (K/ft)} \\ w &= DL + LL \end{aligned} \tag{4-7}$$

Where  $t$  is the thickness of the concrete slab (inch). Eq. (4-7) is derived to assume that the dead load of the concrete slab of thickness  $t$  is proportional to that of a 5-in. concrete slab.

The results show that the greater the thickness of the concrete slab is, the larger the possible reduction in the steel beam is, and also the shorter the span-length, the greater the saving.

From the above it can be seen that there is a decided advantage in using the composite construction from the standpoint of savings in steel.

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CHAPTER 5

DESIGN EXAMPLES USING LINEAR PROGRAMMING  
TECHNIQUES

5.1 Symmetrical Two-Story, Three-Bay Steel Frame with Composite Beams

As a first design example a two-story three-bay frame is considered. The overall dimensions, working loads and member codes are shown in Fig. (5-1). The assumed critical sections are shown in Fig. (5-2). The A36 steel wide flanges and the concrete slab of 5-in. thickness with compressive strength of 4.0 Ksi are to be used.

For design purposes, it is assumed that a 1 Klf uniformly distributed dead load and a 2 Klf uniformly distributed live load act on all the beams, and that wind forces of 4.5 Kip and 9.0 Kip act at the roof and floor levels, respectively. The ultimate loads are assumed to be the working loads multiplied by the safety factor of 1.7.

It is also assumed that the final design will be symmetrical since the wind may act from either direction. In this example frame, four plastic moment capacities of  $M_{P1}$  to  $M_{P4}$  are assigned to the members of the beams and columns as shown in Fig. (5-2). Further the effects of the composite action is taken into account for the elementary beam mechanisms according to the procedures described in Sec. 4.2 in which the location of positive plastic hinges is assumed at mid span for all the beams. Based on the elementary mechanisms of 10 beam mechanisms, 3 sway mechanisms and 12 joint mechanisms, the computer program generated 125 equilibrium equations of feasible mechanisms. In addition, the moment capacity of each column member is constrained by the lower bound as shown in Fig. (5-2). Such a plastic moment capacity as a lower bound is approximately set forth under the following assumption. Each column has the sectional capacity to be able to resist such an axial load as calculated by distributing the ultimate gravity loads proportionally over the

corresponding column as for a simple framing building. Thus the column can also accommodate with a certain amount of moment resistant capacity which is assumed as a lower bound.

The result based on the foregoing procedure will be more practical and economical in comparison with those calculated without constraining the column members since smaller capacity for the beams is required for the former case, and even though larger capacity for the columns is obtained, the final refinement on the column members for the axial forces resulting from the critical collapse mechanism usually counteracts such a disadvantage. The linear programming solutions obtained are shown by

$$M_{P_1} = 205.31 \text{ K-ft (334.69 K-ft)}$$

$$M_{P_2} = 184.42 \text{ K-ft (286.88 K-ft)}$$

$$M_{P_3} = 92.21 \text{ K-ft (143.44 K-ft)}$$

$$M_{P_4} = 119.70 \text{ K-ft (119.70 K-ft)}$$

Where ( ) shows the linear programming solution for the corresponding non-composite steel frame, and these solutions are shown in Fig. (5-3).

From the results shown above, it is seen that smaller steel beam capacities for the composite frame in comparison with the corresponding non-composite frame are required. It is also compared in terms of the ratio of the steel beam moment capacity for the composite frame to that for the corresponding non-composite frame. The ratios for the roof and floor beams are given by 61.3% and 64.3%, respectively.

After the simple inspection of the final tableau of the linear programming solution, the critical mechanism is found to be of the beam mechanisms only as shown in Fig. (5-4). For this critical collapse

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mechanism with no sway load acting, the moment distributions are checked by the plastic moment distribution method. The results of moment distributions which are shown in Fig. (5-5) prove that the moment nowhere exceeds its moment capacity. Based on these moments and the corresponding shears, member designs are started. Only the exterior columns are adjusted for the axial forces according to the method presented in CSA Specification. Though the interior columns are designed for the axial loads only, the selected column sections have enough moment capacity to resist the possible moments, i. e. , moments due to erection or side effects beyond the axial loads acting on. The final design sections are shown below.

16WF40 for roof beams  $B_1$  (21WF55)

16WF36 for floor beams  $B_2$  (18WF50)

12WF40 for exterior columns  $C_1$  (14WF48)

12WF40 for interior columns  $C_2$  (12WF40)

Where ( ) shows the final design section for the corresponding non-composite steel frame, and member layout is shown in Fig. (5-6).

The above sections are found to satisfy the requirements for the local buckling. The deflections at working loads are also found to be within the allowable ranges which are proposed in CSA Specification. For the calculation of these deflections, the displacement method is used. The final design for the composite frame leads to a total weight of 10,680 lb. , compared to 13,674 lb. for the non-composite frame. From an economic standpoint, the composite design is more economical than that for the corresponding non-composite design. Based on the weight of the steel beams alone, the savings in steel is 27.7%, and if the saving is based on the total weight of the whole frame, then the saving is reduced to 21.9% .

The computer time required for the linear programming solutions for the composite frame including the corresponding non-composite one is

approximately 15 minutes 53.3 seconds on the IBM 360/ Model 65 computer.

## 5.2 Symmetrical Three-Story Two-Bay Steel Frame with Composite Beams

The second illustrative design is shown in Fig. (5-7). It is a three-story two-bay frame having the overall dimensions, working loads and member codes shown in Fig. (5-7). The assumed critical sections are shown in Fig. (5-8). The A36 steel wide flanges and concrete slab of five inch thickness with compressive strength of 4.0 Ksi are used.

It is assumed that a 1/Klf uniformly distributed dead load and a 2 Klf live load act on all the beams, and that wind forces of 4.5, 9.0 and 9.0 Kip act at the roof, top floor and middle floor levels, respectively. The ultimate loads are assumed to be the working loads multiplied by the safety factor of 1.7. It is also assumed that the final design will be symmetrical since the wind may act from either direction. In this problem, five plastic moment capacities of  $M_{P_1}$  to  $M_{P_5}$  are assigned to the members of the beams and columns as shown in Fig. (5-8).

Further the effects of the composite action is taken into account for the elementary beam mechanisms according to the procedures described in Sec. 4.2 in which the location of positive plastic hinges is assumed at mid span for all the beams.

Based on the elementary mechanisms of 82 beam mechanisms, 4 sway mechanisms and 14 joint mechanisms, the total 405 equilibrium equations of feasible mechanisms are generated by the computer. As in the first example of Sec. 5.1, the moment capacity of each column is constrained by the lower bound which are shown in Fig. (5-8). Linear programming solutions calculated are presented by

$$M_{P_1} = 202.64 \text{ K-ft (331.16 K-ft)}$$

$$\begin{aligned}M_{P_2} &= 200.97 \text{ K-ft (308.03 K-ft)} \\M_{P_3} &= 208.20 \text{ K-ft (308.03 K-ft)} \\M_{P_4} &= 104.10 \text{ K-ft (154.01 K-ft)} \\M_{P_5} &= 179.70 \text{ K-ft (179.70 K-ft)}\end{aligned}$$

Where ( ) shows the linear programming solution for the corresponding non-composite frame, and these solutions are shown in Fig. (5-9).

It is seen that smaller steel beam capacities for the composite frame in comparison with the corresponding non-composite frame are required. The results are also compared in terms of the ratio of the steel beam moment capacity for the composite frame to that for the corresponding non-composite frame. The ratios for the roof, top and middle beams are given by 61.2%, 65.3% and 67.6%, respectively.

The critical mechanism found is composed of the beam and sway mechanisms as shown in Fig. (5-10). The moment distributions for this critical collapse mechanism are checked by the plastic moment distribution method. The results of moment distributions shown in Fig. (5-11) indicate that the moment nowhere exceeds its moment capacity. Based on these moments and the corresponding shears, member designs are started. All the columns are adjusted for the axial forces.

The final design sections are shown below.

16WF40 for roof beams  $B_1$  (21WF55)  
16WF40 for top-floor beams  $B_2$  (21WF55)  
16WF40 for middle-floor beams  $B_3$  (21WF55)  
14WF53 for exterior columns  $C_1$  (16WF58)  
16WF88 for interior columns  $C_2$  (16WF88)

Where ( ) shows the sections selected for the corresponding non-composite steel frame, and the member layout is shown in Fig. (5-12). These sections are found to satisfy the requirements for the local buckling. The deflections at working loads are also found to be within the allowable ranges proposed in CSA Specification. The deflections are calculated by the displacement method. The final design for the composite frame leads to a total weight of 14,184 lb., compared to 17,244 lb. for the non-composite frame. Thus based on the weight of the steel beams alone, the savings in steel is 27.3%, and also the saving based on the total weight of the whole frame is 17.9%.

The computer time required for the linear programming solutions for the example composite frames including non-composite one is approximately 52 minutes 53.3 seconds on the IBM 360/Model 65 computer.

CHAPTER 6

REFINEMENTS OF OPTIMUM DESIGN SOLUTION  
USING NONLINEAR PROGRAMMING TECHNIQUES

6.1 General Remarks

In this chapter, it is investigated the applicability of nonlinear programming techniques to the optimum plastic design of steel frames with composite beams since more general optimization problems can be solved by using nonlinear programming techniques. From among the several available nonlinear programming methods, the gradient projection method is used. (See Sec. 3.2). First the nonlinearity of the constraints and objective function in the optimum design problem will be investigated. Then the general optimization procedures will be shown before entering the example problems.

6.2 Nonlinearity in Plastic Optimum Design Problems

1) Nonlinearity of Plastic Moment Capacity-Unit Weight Relations

For minimum weight design, the objective function is the total weight of a structure,  $W$ , and can be formulated as a function of the weight of each member per foot of length,  $w_i$ , as given by Eq. (1-13).

$$W = \sum_{i=1}^n w_i \cdot L_i \quad (1-13)$$

For solving the minimum weight design problem of steel frames, it is necessary to determine the approximate relationships between the unit weight,  $w_i$ , and the plastic moment capacity,  $M_{P_i}$ , as written by Eq. (1-14).

$$w_i = f(M_{P_i}) \quad (1-14)$$

From the view point of programming problem, the above function can be expressed in any form. For simplicity, however, the unit weight  $w_i$  will be taken to be directly proportional to the plastic moment capacity of the member,  $M_{P_i}$ , as given by

$$w_i = a M_{P_i}^b \quad (6-1)$$

Where  $a$  and  $b$  are coefficients, and using the least square method, the approximate curve representing the above relationship has been constructed for the so-called economy sections which provides given plastic moment capacity for the least weight. These wide flange sections are listed in Table (4-1) of which plastic moment capacities correspond to A36 steel. The values of coefficients  $a$  and  $b$  are given by

$$\begin{aligned} a &= 1.3912 \\ b &= 0.6446 \end{aligned} \quad (6-2)$$

It is noted that Eqs. (6-1) and (6-2) are only the crude approximations of the actual variations of the section properties of selected wide flange members. However, it will be seen that any such approximation is sufficient for the purpose of finding the least weight design from among commercially available sections. The objective function can also take the form of nonlinear cost relationship for the structure. For concrete structures and for steel structures where both rolled and welded sections are used, a cost relationship is more appropriate.

## 2) Nonlinearity of Equilibrium Equations due to Distributed Loading

In the linear programming solution, it is simply assumed that the plastic hinge forms at mid-span of a beam under the uniformly distributed load. Thus the beam and the sway mechanisms were

Expressed as the linear function of plastic moment capacities. However where the combined beam and sway mechanism forms, the hinge positions are not predetermined. For the case of a point load on the beam, the sharp change of slope in the bending moment diagram under such a load would restrict possible hinge positions to directly under the load and to the ends of the beam such that the results in the equilibrium equation take linear function of plastic moment capacities. In the case of a distributed load, it is necessary to treat the problem as that of maximizing fully plastic moment. Hence the resulting equilibrium equation includes the partial derivative of unknown plastic moment capacity, or defined by a nonlinear function of plastic moment capacities.

Equilibrium method by Douglas <sup>(18)</sup> is used to generate the nonlinear constraints for the beam and sway mechanisms under the distributed loads. The application of the equilibrium method is discussed on the example problems in Secs. 6.4 through 6.7.

3) Nonlinearity of Ultimate Moment Capacity of Composite Section-  
Plastic Moment Capacity of Wide Flange Section Relation

As was noted in Sec. 4.2, the ultimate moment capacity of a composite beam in the positive moment region,  $M_u$ , shall be approximated as a function of the plastic moment capacity of a rolled wide flange,  $M_p$ , assuming that all other factors related to the ultimate moment capacity of the composite beam are known.

$$M_u = f(M_p) \quad (4.1)$$

Since nonlinear programming techniques are used as an optimum tool, the ultimate moment capacity of a composite beam section,  $M_u$ , can be formulated as a nonlinear function of the plastic

moment capacity of a rolled wide flange,  $M_p$ .

Using the least square method, the value of  $M_u$  is approximated as a function of  $M_p$  as shown in Tables (6-1) through (6-3). The rolled wide flanges used for these calculations, are identical to so-called economy sections listed in Table (4-1). The thickness of the concrete slab is varied from 3-in. to 6-in. in increments of 1-in. It is assumed the constant yield strength of steel ( $F_y = 36$  Ksi) while various compressive strengths of concrete ( $f'_c = 3, 3.5$  and 4.5 Ksi) are assumed. The effective width of the concrete flange is assumed as the flange width of the steel beam plus sixteen times the slab thickness.

It is noted that the relationship between  $M_u$  and  $M_p$  can be approximated with the various functions. Among these functions, the following function seems to present sufficient approximations for the actual discrete relationships.

$$M_u = a + b M_p + c M_p^2 + d M_p^3 \quad (6-3)$$

Where the value of coefficients a, b, c and d are listed in Tables (6-1) through (6-3). Thus the above relationships are assumed to represent the continuous variation of the ultimate moment capacity of composite beams with plastic moment capacity of steel wide flanges.

### 6.3 Nonlinear Optimization Procedures

The formulation of nonlinear minimum weight design problem consists of the following steps.

- 1) Set the problem such that feasible constraints and objective function can be generated.
- 2) Apply nonlinear programming techniques to obtain the optimum solution.

The nonlinear optimization procedures are carried out by using computer programs which employ nonlinear programming techniques. In order to obtain a computer program which would be applicable to any general type of structure the computation is divided among a main program and several auxiliary subroutines. The function of the main program is to carry out the steps of the gradient projection algorithm as set forth in Sec. 3.2, and also to read in the input data and print out the solution. The functions of the auxiliary subroutines are designed to calculate the values of the constraint functions, to calculate the values of the objective functions and gradient vector, and to calculate the inverse matrix. The main program is applicable to any design problem which has been formulated in terms which correspond to the requirement of the gradient projection method. The auxiliary subroutines may need to be revised for each individual problem, depending on the proposed structure, the constraints and the objective function. A general flow diagram for the main program, and short description of input data are presented in Appendix 4.

#### 6.4 Symmetrical Two-Story Single-Bay Steel Frame

As a first example the structure shown in Fig. (6-1) is considered. It is a two-story single-bay frame having the overall dimensions shown in Fig. (6-1). All the structural members are assumed to be steel alone.

It is assumed that a 2 Klf uniformly distributed load acts on the roof beam and a 3 Klf uniformly distributed load acts on the floor beam, and that wind forces of 4.5 Kip and 10.1 Kip act at the roof and the floor levels, respectively. The ultimate loads are assumed to be the working loads multiplied by the safety factor of 1.7. Since the wind may act from either direction, it is assumed that the final design will be symmetrical. This example involves four plastic moment capacities of  $M_{P1}$  to  $M_{P4}$ .

The idealized structure for optimization is shown in Fig. (6-2). From the 27 feasible collapse mechanisms which are shown in Fig. (6-3), the necessary nonlinear and linear constraints are formulated. In generating such nonlinear constraints the method shown in Sec. 6.2 (2) is employed. For the combined sway and beam mechanisms, the plastic hinges in the roof and floor beams with span length L (30 ft), occur at distance  $U_1$  and  $U_2$ , from the right-hand-side origin of the corresponding beam. The following equilibrium equations are generated.

$$a(1) = 4 M_{P_1} - 0.85 L^2 \geq 0$$

$$a(2) = 2 M_{P_1} + 2 M_{P_3} - 0.85 L^2 \geq 0$$

$$a(3) = 4 M_{P_2} - 1.275 L^2 \geq 0$$

$$a(4) = 2 M_{P_2} + 2 M_{P_3} + 2 M_{P_4} - 1.275 L^2 \geq 0$$

$$a(5) = 4 M_{P_3} - 91.8 \geq 0$$

$$a(6) = 2 M_{P_1} + 2 M_{P_3} - 91.8 \geq 0$$

$$a(7) = 4 M_{P_4} - 372.94 \geq 0$$

$$a(8) = 2 L M_{P_2} + 2 L M_{P_3} + 2 (U_2 + L) M_{P_4} - (372.94 + 2.55 L^2) U_2 \geq 0$$

$$\text{where } U_2 = \sqrt{(M_{P_2} + M_{P_3} + M_{P_4}) / 2.55}$$

$$a(9) = 2 M_{P_2} + 4 M_{P_3} + 2 M_{P_4} - (91.8 + 1.275 L^2) \geq 0$$

$$a(10) = 2 L M_{P_1} + 2 (U_1 + L) M_{P_3} - (91.8 + 1.7 L^2) U_1 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{P_1} + M_{P_3}) / 1.7}$$

$$a(11) = 4 L M_{P_1} + 2 U_1 M_{P_3} - (91.8 + 1.7 L^2) U_1 \geq 0$$

$$\text{where } U_1 = \sqrt{2 M_{P_1}} / 1.7$$

$$a(12) = 2 L M_{P_2} + 2 (U_2 + L) M_{P_3} + 2 (U_2 + L) M_{P_4} - (464.74 + 2.55 L^2)$$

$$U_2 \geq 0$$

$$\text{where } U_2 = \sqrt{(M_{P_2} + M_{P_3} + M_{P_4}) / 2.55}$$

$$a(13) = 2 U_2 M_{P_1} + 2 L M_{P_2} + 2 L M_{P_3} + 2 (U_2 + L) M_{P_4} - (464.74 + 2.55 L^2) U_2 \geq 0$$

$$\text{where } U_2 = \sqrt{(M_{P_2} + M_{P_3} + M_{P_4}) / 2.55}$$

$$a(14) = (4L - U_2) M_{P_2} + U_2 M_{P_3} + 3 U_2 M_{P_4} - (372.94 + 2.55 L^2)$$

$$U_2 \geq 0$$

$$\text{where } U_2 = \sqrt{2 M_{P_2}} / 2.55$$

$$a(15) = 2 L M_{P_1} + 2 U_1 M_{P_2} + 2 (U_1 + L) M_{P_3} + 2 U_1 M_{P_4} - (91.8 + 2.975 L^2)$$

$$U_1 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{P_1} + M_{P_3})} / 1.7$$

$$a(16) = 4 L M_{P_1} + 2 U_1 M_{P_2} + 2 U_1 M_{P_3} + 2 U_1 M_{P_4} - (91.8 + 2.975 L^2)$$

$$U_1 \geq 0$$

$$\text{where } U_1 = \sqrt{2 M_{P_1}} / 1.7$$

$$a(17) = 3 M_{P_2} + 3 M_{P_3} + M_{P_4} - (91.8 + 1.275 L^2) \geq 0$$

$$a(18) = 2 M_{P_1} + 3 M_{P_2} + M_{P_3} + M_{P_4} - (91.8 + 1.275 L^2) \geq 0$$

$$a(19) = 2 L U_2 M_{P_1} + 2 L U_1 M_{P_2} + 2 L (U_1 + U_2) M_{P_3} + 2 (U_1 U_2 + L U_1) M_{P_4} - (464.74 + 4.25 L^2) U_1 U_2 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{P_1} + M_{P_3})/1.7}$$

$$U_2 = \sqrt{(M_{P_2} + M_{P_3} + M_{P_4})/2.55}$$

$$a(20) = 4 L U_2 M_{P_1} + 2 L U_1 M_{P_2} + 2 L U_1 M_{P_3} + 2 (U_1 U_2 + L U_1) M_{P_4} - (464.74 + 4.25 L^2) U_1 U_2 \geq 0$$

$$\text{where } U_1 = \sqrt{2 M_{P_1}/1.7}$$

$$U_2 = \sqrt{(M_{P_2} + M_{P_3} + M_{P_4})/2.55}$$

$$a(21) = 2 U_2 M_{P_1} + 4 L M_{P_2} + 2 U_2 M_{P_4} - (464.74 - 2.55 L^2) U_2 \geq 0$$

$$\text{where } U_2 = \sqrt{2 M_{P_2}/2.55}$$

$$a(22) = 4 L M_{P_2} + 4 U_2 M_{P_4} - (372.94 + 2.55 L^2) U_2 \geq 0$$

$$\text{where } U_2 = \sqrt{2 M_{P_2}/2.55}$$

$$a(23) = 2 L M_{P_1} + 3 U_1 M_{P_2} + (U_1 + 2 L) M_{P_3} + U_1 M_{P_4}$$

$$- (91.8 + 2.975 L^2) U_1 \geq 0.$$

$$\text{where } U_1 = \sqrt{(M_{P_1} + M_{P_2})/1.7}$$

$$a(24) = 4 L M_{P_2} + 2 U_2 M_{P_3} + 2 U_2 M_{P_4} - (464.74 + 2.55 L^2) U_2 \geq 0$$

$$\text{where } U_2 = \sqrt{2 M_{P_2}/2.55}$$

$$a(25) = 4 L M_{P_1} + 3 U_1 M_{P_2} + U_1 M_{P_3} + U_1 M_{P_4} - (91.8 + 2.915 L^2)$$

$$U_1 \geq 0$$

$$\text{where } U_1 = \sqrt{2 M_{P_1} / 1.7}$$

$$a(26) = 2 L U_2 M_{P_1} + 4 L U_1 M_{P_2} + 2 L U_2 M_{P_3} + 2 U_1 U_2 M_{P_4}$$

$$- (464.74 + 4.25 L^2) U_1 U_2 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{P_1} + M_{P_3}) / 1.7}$$

$$U_2 = \sqrt{2 M_{P_2} / 2.55}$$

$$a(27) = 4 L U_2 M_{P_1} + 4 L U_1 M_{P_2} + 2 U_1 U_2 M_{P_4} - (464.74 + 4.25 L^2)$$

$$U_1 U_2 \geq 0$$

$$\text{where } U_1 = \sqrt{2 M_{P_1} / 1.7}$$

$$U_2 = \sqrt{2 M_{P_2} / 2.55}$$

According to the procedure presented in Sec. 6.2 (1) the nonlinear objective function is defined by  $F = -W$ , in which  $W$  is the total weight.

Starting the algorithm at the point  $\bar{M}_{P_0}$  (400.0, 600.0, 400.0, 300.0), the gradient projection procedure leads to the minimum weight solution  $\bar{M}_{P_{\min}}$  (214.89, 325.87, 214.89, 110.98) for this problem. The active constraints at the point  $\bar{M}_{P_{\min}}$  are a(8), a(14), a(19), a(20), a(26) and a(27). The solutions are as follows:

$$M_{P_1} = 214.89 \text{ K-ft (191.25)}$$

$$M_{P_2} = 325.87 \text{ K-ft (332.46)}$$

$$M_{P_3} = 214.89 \text{ K-ft (191.25)}$$

$$M_{P_4} = 110.98 \text{ K-ft (141.21)}$$

Where ( ) shows the corresponding linear programming solution in which all the constraints and the objective function are approximated as the linear function of variables,  $M_{P_i}$ .

The computer time required for this solution was 14.3 seconds on IBM System 360/Model 65 computer.

#### 6.5 Unsymmetrical One-Story Two-Bay Steel Frame

The second example considered is an unsymmetrical one-story two-bay steel frame having the overall dimensions shown in Fig. (6-4). The solutions are sought for the steel frame alone in which the effects of the composite action are neglected.

It is assumed that a 2 Klf uniformly distributed load acts on the beam, and that wind force of 5.63 Kip acts at the roof level. The ultimate loads are assumed as the working loads multiplied by the safety factor of 1.7. In this example the wind forces from both directions are considered since the structure is not symmetrical. The plastic moment capacity of each member is numbered as shown in Fig. (6-5). Thus this example involves three plastic moment capacities of  $M_{P_1}$  to  $M_{P_3}$ . Corresponding to the 12 feasible collapse mechanisms which are shown in Fig. (6-6), the nonlinear and linear constraints are formulated. For the combined sway and beam mechanisms, assuming the plastic hinges in the left-hand-side beam with span length  $L_1$  (35 ft) and the right-hand-side beam with span length  $L_2$  (25 ft), to occur at distance  $U_1$  and  $U_2$ , from the right-hand-side origin of the corresponding beam respectively, the following equilibrium equations are generated.

$$a(1) = 4 M_{P_1} - 0.85 L_1^2 \geq 0$$

$$a(2) = 3 M_{P_1} + M_{P_3} - 0.85 L_1^2 \geq 0$$

$$a(3) = 4 M_{P_2} - 0.85 L_2^2 \geq 0$$

$$a(4) = 3 M_{P_2} + M_{P_3} - 0.85 L_2^2 \geq 0$$

$$a(5) = 6 M_{P_3} - 143.442 \geq 0$$

$$a(6) = (4 L_1 - U_1) M_{P_1} + 5 U_1 M_{P_3} - (143.44 + 1.7 L_1^2) U_1 \geq 0$$

$$\text{where } U_1 = \sqrt{2 M_{P_1} / 1.7}$$

$$a(7) = (4 L_1 - U_1) M_{P_1} + U_1 M_{P_2} + 4 U_1 M_{P_3} - (143.44 + 1.7 L_1^2) U_1 \geq 0$$

$$\text{where } U_1 = \sqrt{2 M_{P_1} / 1.7}$$

$$a(8) = (4 L_2 - U_2) M_{P_2} + 5 U_2 M_{P_3} - (143.44 + 1.7 L_2^2) U_2 \geq 0$$

$$\text{where } U_2 = \sqrt{2 M_{P_2} / 1.7}$$

$$a(9) = 2 M_{P_1} + M_{P_2} + 2 M_{P_3} - 0.85 L_1^2 \geq 0$$

$$a(10) = 2 L_1 M_{P_1} + (2 L_1 - U_1) M_{P_2} + (4 U_1 + 2 L_1) M_{P_3} - (143.44 + 1.7 L_1^2)$$

$$U_1 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{P_1} + M_{P_2} + M_{P_3}) / 1.7}$$

$$a(11) = 2 L_1 M_{P_1} + 2 L_1 M_{P_2} + (3 U_1 + 2 L_1) M_{P_3} - (143.44 + 1.7 L_1^2)$$

$$U_1 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{P_1} + M_{P_2} + M_{P_3}) / 1.7}$$

$$a(12) = 2 L_1 U_2 M_{P_1} + 4 L_2 U_1 M_{P_2} + (3 U_1 U_2 + 2 L_1 U_2) M_{P_3}$$

$$-[143.44 + 1.7 (L_1^2 + L_2^2)] U_1 U_2 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{P_1} + M_{P_3})/1.7}$$

$$U_2 = \sqrt{2 M_{P_2}/1.7}$$

The nonlinear objective function, which is defined as the total weight of the structure, is used as in the first example problem.

Beginning the gradient projection algorithm at the point  $\bar{M} = (800.0, 500.0, 200.0)$ , the solution  $\bar{M}_{P_{min}} = (428.22, 189.44, 25.28)$  is obtained. The active constraints at the point  $\bar{M}_{P_{min}}$  are a(8), a(10) and a(12). Since three constraints are active, and since the problem is three-dimensional, the point  $\bar{M}_{P_{min}}$  corresponds to a unique point which is a vertex of the feasible region. The solution is shown by

$$M_{P_1} = 428.22 \text{ K-ft (390.25)}$$

$$M_{P_2} = 189.44 \text{ K-ft (189.04)}$$

$$M_{P_3} = 25.28 \text{ K-ft (35.86)}$$

Where ( ) shows the corresponding linear programming solution.

The computer time required for this solution was approximately 11.3 seconds on IBM System 360/ Model 65 computer.

## 6.6 Symmetrical Two-Story Two-Bay Steel Frame with Composite Beams

The third example is a two-story two-bay composite steel frame having the overall dimensions shown in Fig. (6-7). In this example, the effects of composite action are considered according to the procedure shown in Sec. 6.2 (3).

It is assumed that a 2 Klf and a 3 Klf uniformly distributed loads act on the roof and the floor beams, respectively, and that wind forces of 4.5 Kip and 10.1 Kip act at the roof and the floor levels, respectively. The ultimate loads are assumed to be the working loads multiplied by the safety factor of 1.7. Since the wind may act from either direction, it is assumed that the final design will be symmetrical. The design variables of  $M_{u_1}$  to  $M_{p_4}$  are assumed. The idealized structure for optimization is shown in Fig. (6-8). From the 32 likely collapse mechanisms, the necessary nonlinear and linear constraints are formulated.

Beginning the gradient projection algorithm at the point  $\bar{M}_p = (300, 500, 200, 200)$ , the solution  $\bar{M}_p^{\min} = (142.13, 259.42, 15.30, 62.16)$  is obtained. The active constraints at the point  $\bar{M}_p^{\min}$  are a(3), a(8), a(11) and a(17). These constraints are expressed by

$$a(3) = 2 L U_3 M_{u_1} + (2 L U_3 - U_1 U_3) M_{p_1} + 2 L U_1 M_{u_2} \\ + (2 L U_1 - U_1 U_3) M_{p_2} + 4 U_1 U_3 M_{p_3} + 5 U_1 U_3 M_{p_4} \\ - (464.74 + 4.25 L^2) U_1 U_3 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{u_1} + M_{p_1})/1.7} \\ U_3 = \sqrt{(M_{u_2} + M_{p_2})/2.55}$$

$$a(8) = 6 M_{p_3} - 91.8 \geq 0$$

$$a(11) = 6 M_{p_4} - 372.94 \geq 0$$

$$a(17) = 2 L (U_1 U_3 + U_2 U_3) M_{u_1} + 2 L U_2 U_3 M_{p_1} + 2 L U_1 U_2 M_{u_2} \\ + (2 L U_1 U_2 - U_1 U_2 U_3) M_{p_2} + 2 (U_1 U_2 U_3 + L U_1 U_3) M_{p_3} \\ + 5 U_1 U_2 U_3 M_{p_4} - (464.74 + 5.95 L^2) U_1 U_2 U_3 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{u_1} + M_{P_1})/1.7}$$

$$U_2 = \sqrt{(M_{u_1} + M_{P_3})/1.7}$$

$$U_3 = \sqrt{(M_{u_2} + M_{P_2})/2.55}$$

and  $L = 30 \text{ ft}$

The ultimate moment capacity of composite section,  $M_{u_1}$ , is given by Eq. (6-3), and the critical mechanisms corresponding to the active constraints are shown in Fig. (6-9). The solution is shown as follows.

$$M_{P_1} = 142.13 \text{ K-ft (130.69)}$$

$$M_{P_2} = 259.42 \text{ K-ft (226.28)}$$

$$M_{P_3} = 15.30 \text{ K-ft (45.90)}$$

$$M_{P_4} = 62.16 \text{ K-ft (65.41)}$$

Where ( ) shows the corresponding linear programming solution. From this example it is noted that the nonlinear programming technique of the gradient method is specially powerful when it is applied to the optimum plastic design of composite frames since the problem is confined to the mathematical programming technique, and it will be well compared to the linear programming method shown in Chapters 4 and 5 in which the convergence checks beyond the mathematical solution are required.

The computer time required for this solution was approximately 18.2 seconds on the IBM System 360/ Model 65 computer.

### 6.7 Unsymmetrical Steel Frame with Composite Beams

The fourth and last example is an unsymmetrical composite steel frame which is shown in Fig. (6-10). In this example, the effects of composite

action are considered as in Sec. 6-6.

The working loads are assumed as shown in Fig. (6-10).

The ultimate loads are assumed to be the working loads multiplied by the safety factor of 1.7. The design variables of  $M_{u1}$  to  $M_{u4}$  are assumed. The idealized structure for optimization is shown in Fig. (6-11).

From the 26 likely collapse mechanisms, the necessary nonlinear and linear constraints are formulated.

Beginning the gradient projection algorithm at the point  $\bar{M}_p = (300, 500, 300, 200)$ , the solution  $\bar{M}_{p \min} = (114.59, 243.97, 114.59, 62.96)$  is obtained. The active constraints at the point  $\bar{M}_{p \min}$  are a(1), a(2), a(3) and a(4). These constraints are expressed by

$$a(1) = 2 M_{u1} + 2 M_{p1} - 0.85 L_1^2 \geq 0$$

$$a(2) = 2 M_{u1} + 2 M_{p3} - 0.85 L_1^2 \geq 0$$

$$a(3) = 6 M_{p4} - 372.94 \geq 0$$

$$a(4) = 2 L_1 U_2 M_{u1} + 2 L_1 U_1 M_{u2} + (2 L_1 U_1 - U_1 U_2) M_{p2} + (U_1 U_2 + 2 L_1 U_2) M_{p3} + 5 U_1 U_2 M_{p4} - (464.74 + 4.25 L_1^2)$$

$$U_1 U_2 \geq 0$$

$$\text{where } U_1 = \sqrt{(M_{u1} + M_{p3})/1.7}$$

$$U_2 = \sqrt{(M_{u2} + M_{p2})/2.55}$$

$$\text{and } L_1 = 30 \text{ ft}$$

$$L_2 = 25 \text{ ft}$$

The ultimate moment capacity of composite section,  $M_{u1}$ , is given by Eq. (6-3), and the critical mechanisms corresponding to the above constraints are shown in Fig. (6-12). The solution is shown by

$$M_{P_1} = 114.59 \text{ K-ft (115.65)}$$

$$M_{P_2} = 243.97 \text{ K-ft (235.08)}$$

$$M_{P_3} = 114.59 \text{ K-ft (115.65)}$$

$$M_{P_4} = 62.16 \text{ K-ft (62.16)}$$

Where ( ) shows the corresponding linear programming solution. In this example, relatively small modification in the solution is performed by employing the nonlinear programming technique since both the linear and nonlinear solutions are controlled by the similar pattern of failure mechanisms. However the refinement by using the nonlinear programming techniques in the minimum weight plastic design appears to be great in such cases as found in the previous examples in which the nonlinear solutions are controlled by the different failure mechanisms from those in the linear programming ones. And also it can be said that the critical failure mechanism found by the application of linear programming technique is not always the same for the nonlinear solution, thus more critical failure mechanism and solution can be realized by employing the nonlinear programming techniques.

The computer time required for this solution was approximately 12.7 seconds on the IBM System 360/ Model 65 computer.

CHAPTER 7  
CONCLUSION

As a result of the application of the programming techniques to the minimum weight design of composite steel frames, the following conclusions can be presented.

1. Problems of the minimum weight plastic design of a steel frame with composite beams can be formulated as both linear and nonlinear programming problems. The simplex method is used from among the available linear programming methods for the linear optimization, because of its simplicity and flexibility. On the other hand, it is noted that the gradient projection method provides an efficient numerical technique for solving nonlinear programming problems in connection with minimum weight plastic design problems.
2. In order to formulate the minimum weight plastic design of the composite steel frames as a programming problems, it is necessary to obtain the approximate relationship between the ultimate moment capacity of a composite beam and the plastic moment capacity of the corresponding steel beam. It is possible to do this for the available wide flange sections.
3. The minimum weight plastic design of the steel frames with composite beams can be carried out by the successive applications of the conventional minimum weight plastic design method for rigid frames. The developed computer program can generate, automatically, the linear constraints of the feasible collapse modes for the proposed structure and the linear objective function, then produces the linear optimum

solution.

4. The application of the nonlinear programming techniques to the minimum weight plastic design of rigid steel frames, including the composite steel frames, made it possible to eliminate some of difficulties and arbitrary restrictions which are found in the previous minimum weight plastic design method which employs linear programming techniques.

From the results of the plastic design of the composite steel frames, the following conclusion can be drawn.

1. Comparisons of the changes in required plastic moment capacities and relative cost on a weight saving basis of a composite steel frame to that of the corresponding non-composite steel frame, made with several model frames, indicate that there is a definite advantage in using composite construction from either economy or structural overload capacity.
2. Two design examples of the composite frames show that the saving in steel, based on the total weight of the structure, is approximately 20%. Thus, from the economic point of view, less steel is required for the same loads and spans when composite construction is adopted.
3. In addition, these exaple designs show that smaller beam-depths for the composite frame, in comparison with the corresponding non-composite frame, are required. It is noted that a further advantage of the composite construction, of the reduction in overall floor-depths, is possible. This aspect is of particular importance in tall buildings.

4. From the investigation carried out on the effects of changing the thickness of the concrete slab, in the typical two-story two-bay composite frames, upon the required plastic moment capacities of the steel beams, it can be seen that the greater the thickness of the concrete slab is, the larger the possible reduction in the steel beams, and also the shorter the span-lengths, the greater the savings in steel.
5. Similarly, the investigation on the effects of changing the compressive strength of the concrete, under the constant loading condition, shows that the higher the strength of the concrete is, the smaller the required steel beam is, and also the shorter the span-lengths, the greater the savings in steel.

APPENDIX I  
FIGURES

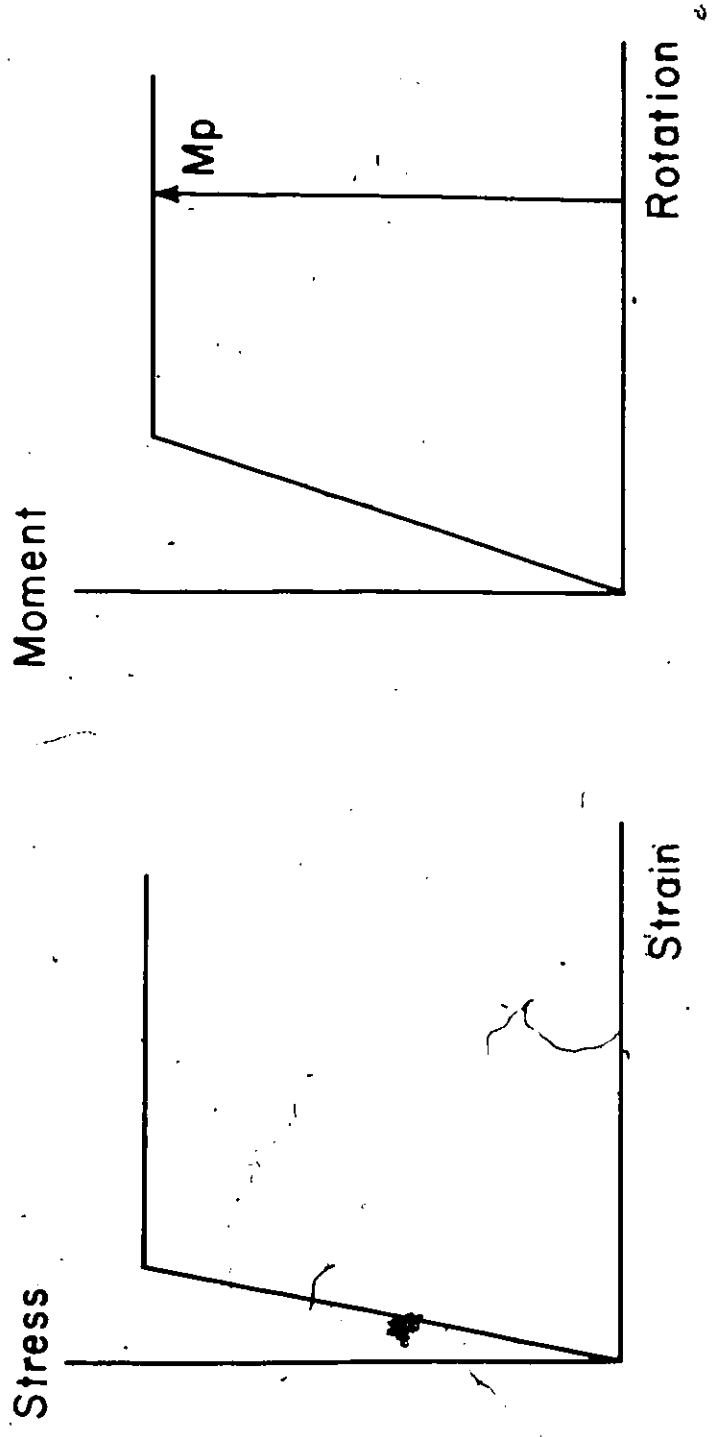


Fig. 1-1 Idealized Stress-Strain Curve  
Fig. 1-2 Moment-Rotation Relationship

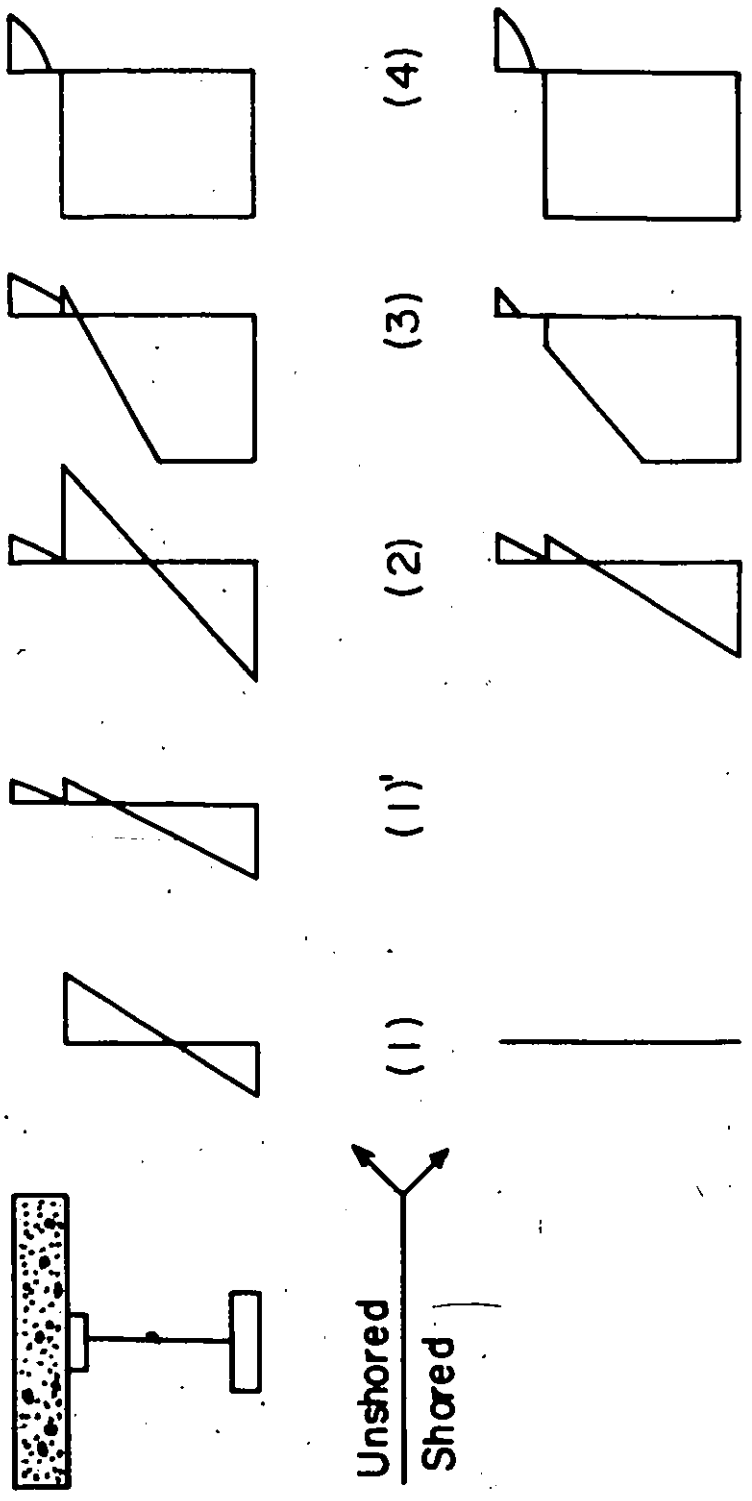


Fig. 1-3 Shored Composite Girders and Unshored Ones

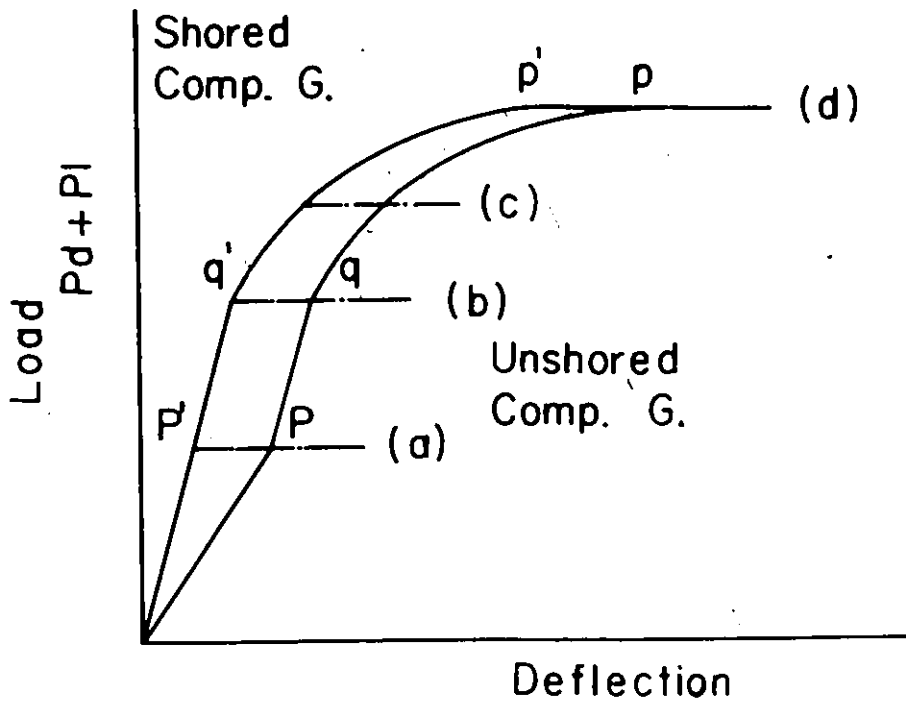


Fig. 1-4 Load-Deflection Curves for Composite Girders

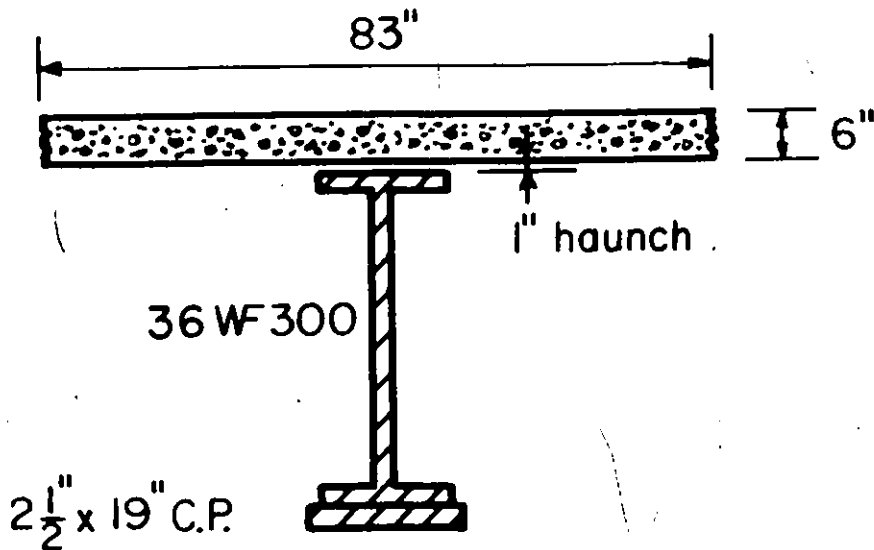


Fig. 1-5 Specimens

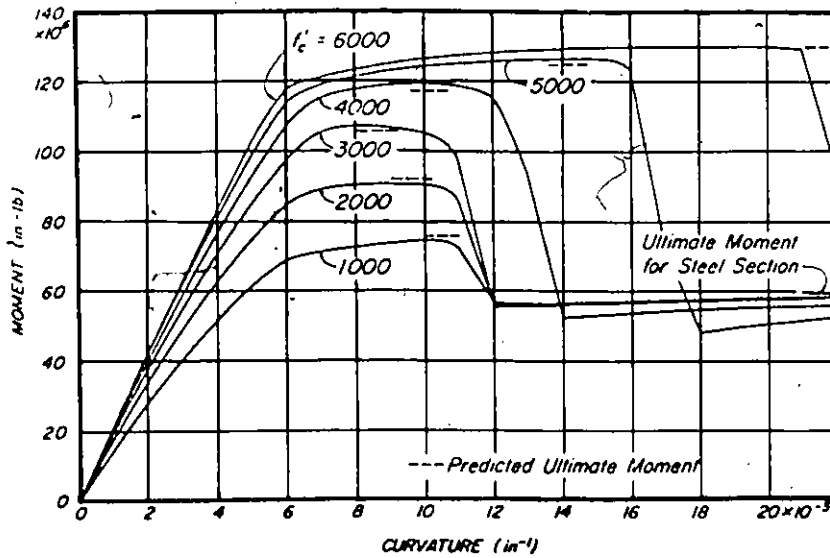


Fig. I-6 Moment-Curvature Curve for Different Strengths of Concrete

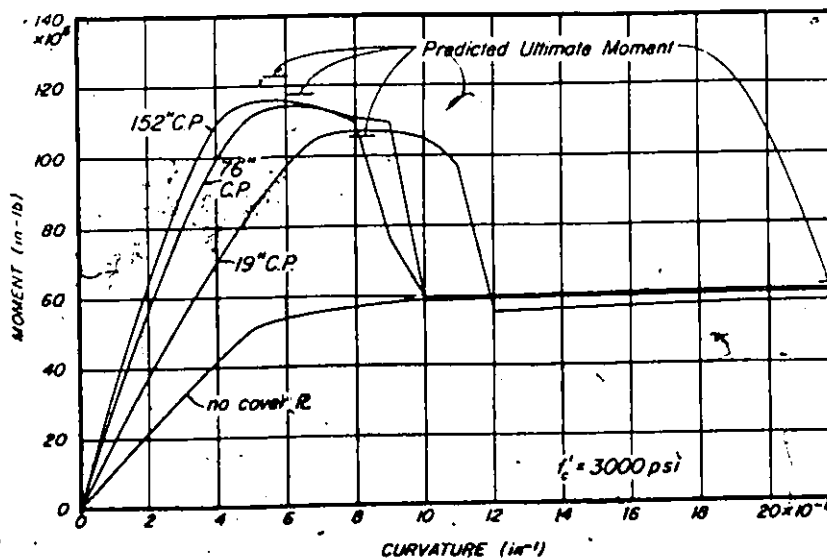


Fig. I-7 Moment-Curvature Curve for Different Widths of Cover Plates

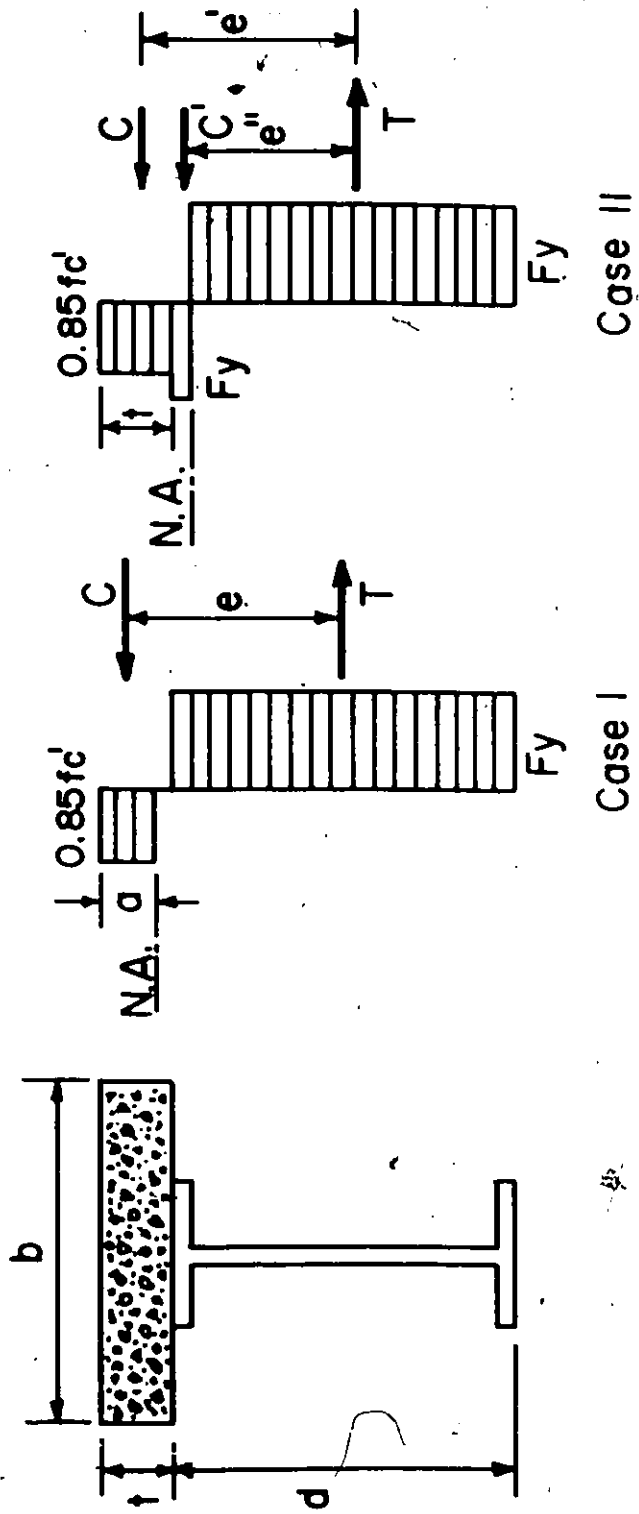


Fig. 1-8 Stress Distribution at Ultimate Moment

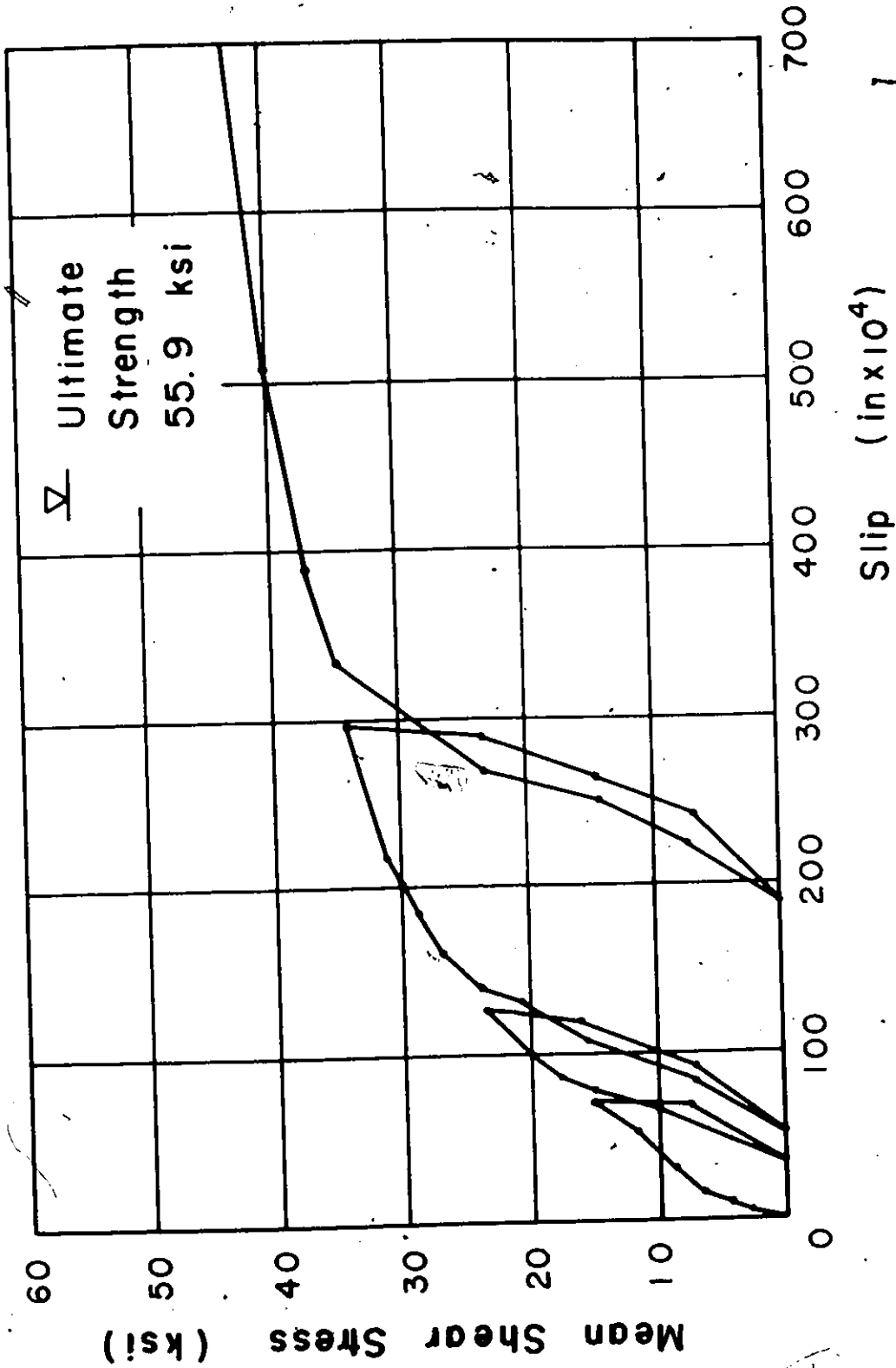


Fig. 1-9 Load versus Slip Curve

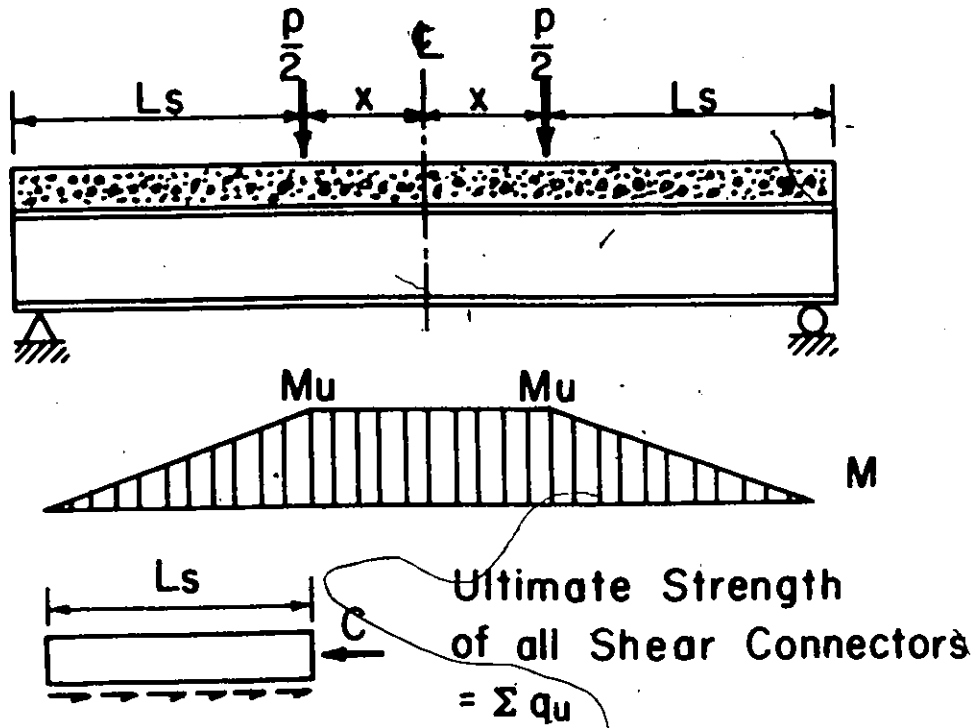


Fig. I-10 Shear Connector Forces at Ultimate Moment

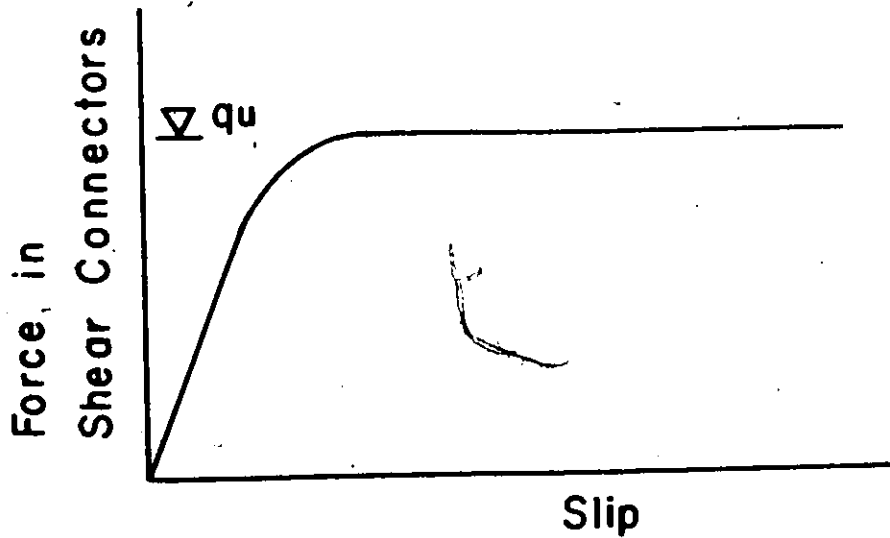
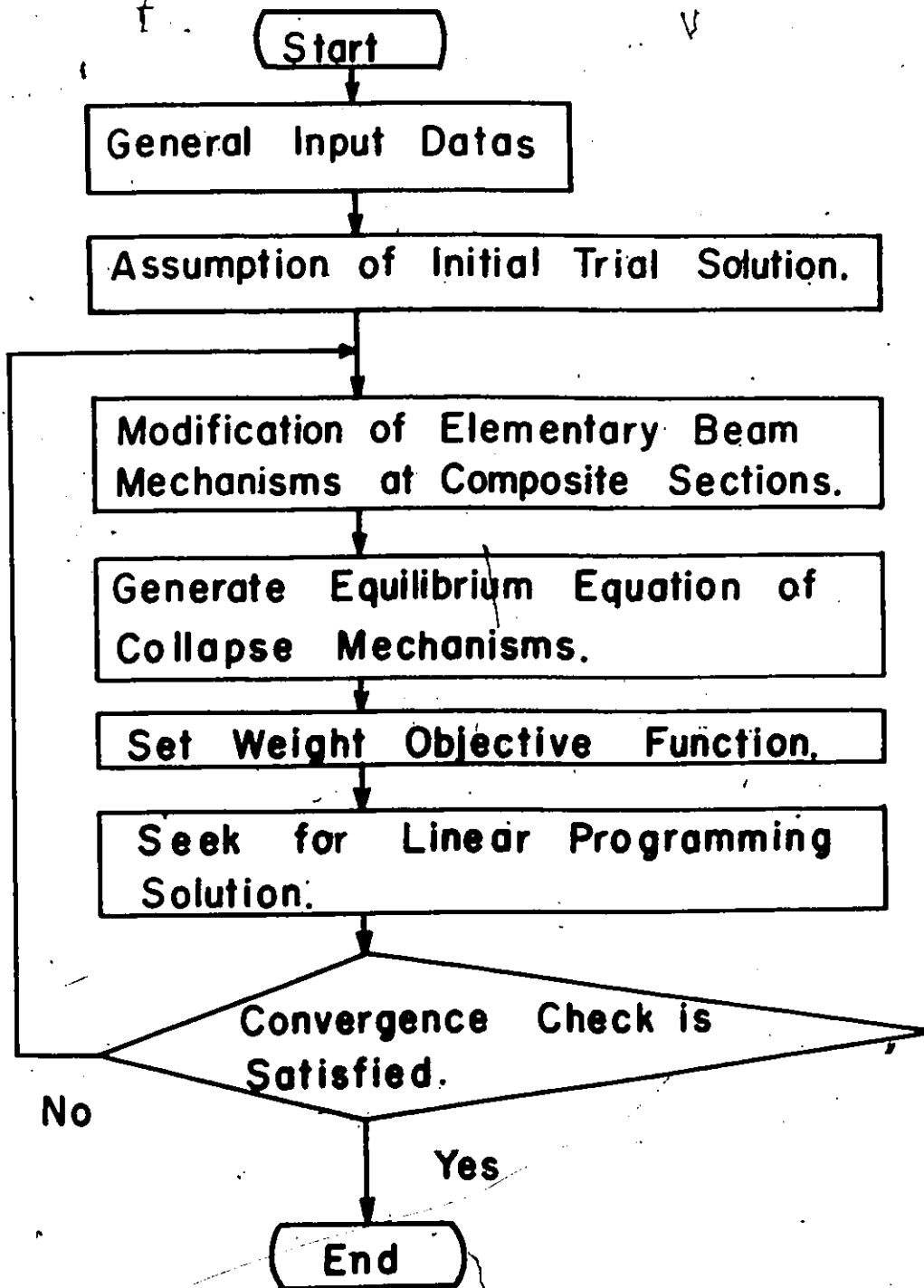
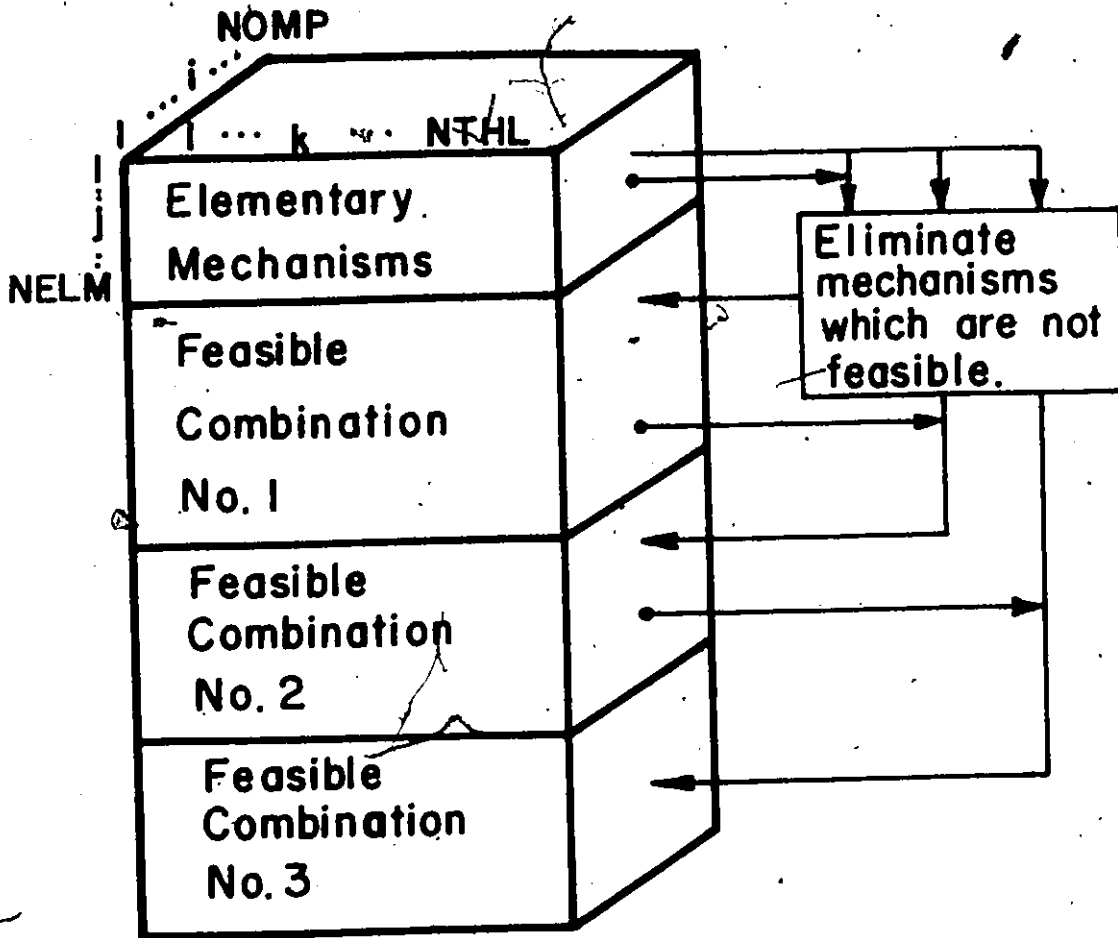


Fig. I-11 Idealized Load-Slip Curve



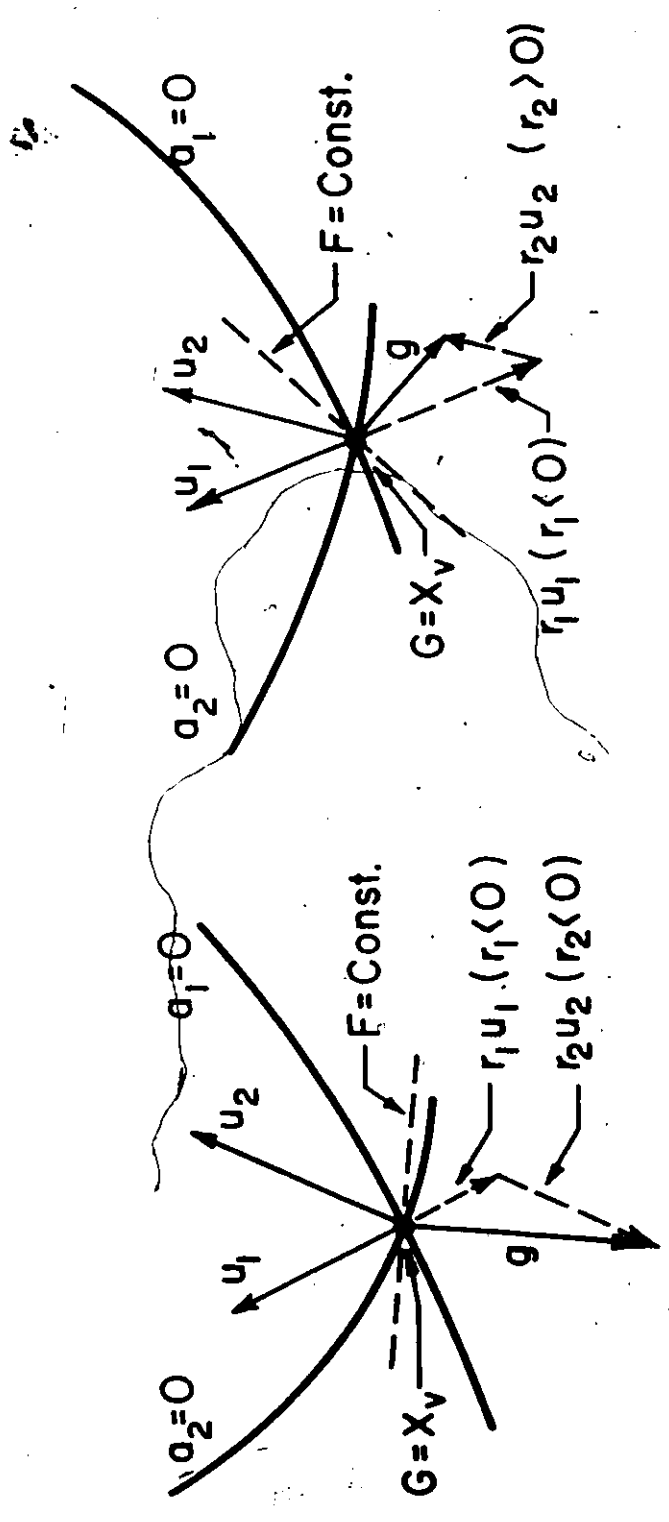
**Fig.2-1 Operations in Optimization of Composite Steel Frames**



$\text{THETA}(j, k, i)$

- Plastic Moment
- Hinge Location
- Mechanism No.

Fig. 2-2 Generation of Feasible Mechanisms



(a)  $X_v = X_{\text{max}}$

(b)  $X_v \neq X_{\text{max}}$

Fig. 3-1 Conditions for Local Optimum

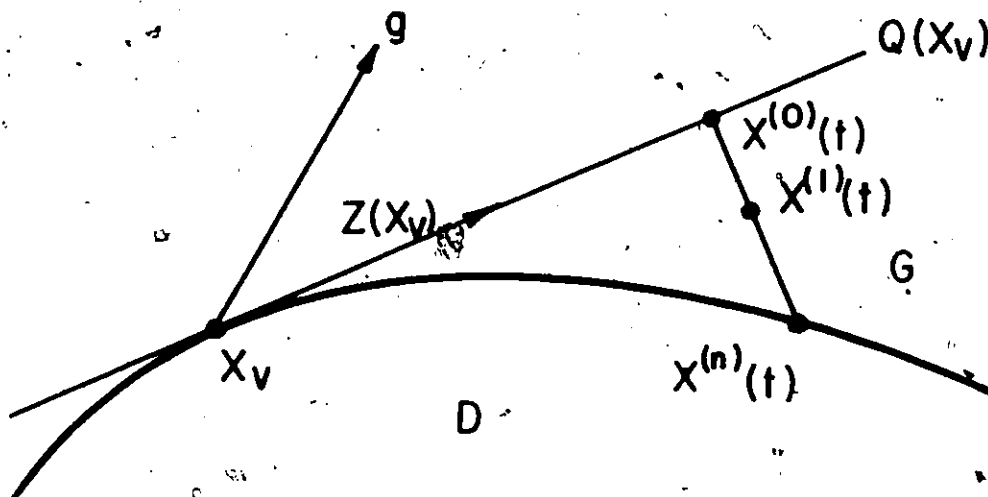


Fig. 3-2 Iteration to Obtain Feasible Point  $X_{v+1}$  in Intersection  $G$

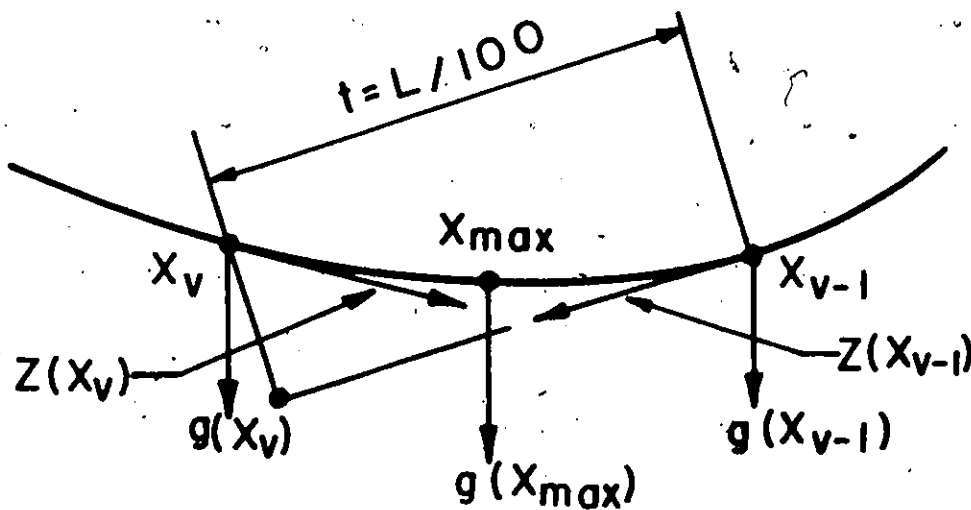


Fig. 3-3 Solution Point not at Vertex of Feasible Region

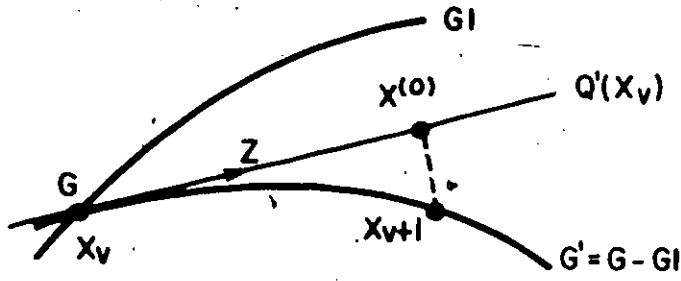


Fig. 3-4 Determination of Point  $X_{v+1}$  in Intersection  $G'$  but not in  $G_1$

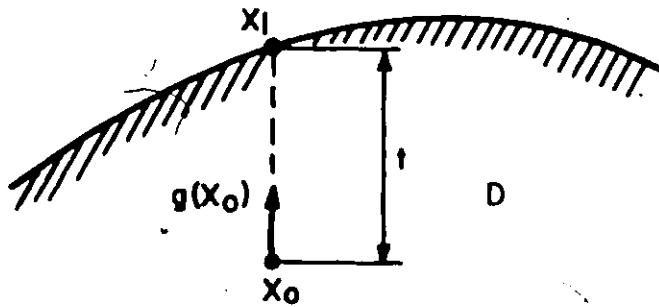


Fig. 3-5 Interpolation to Obtain Point  $X_1$  on B

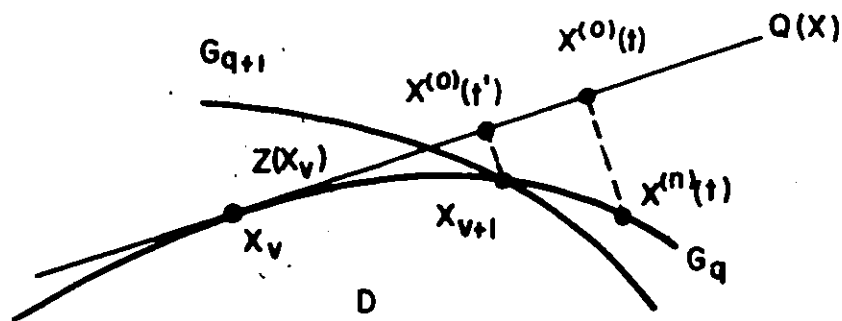


Fig. 3-6 Interpolation to Obtain Feasible Point  $X_{v+1}$

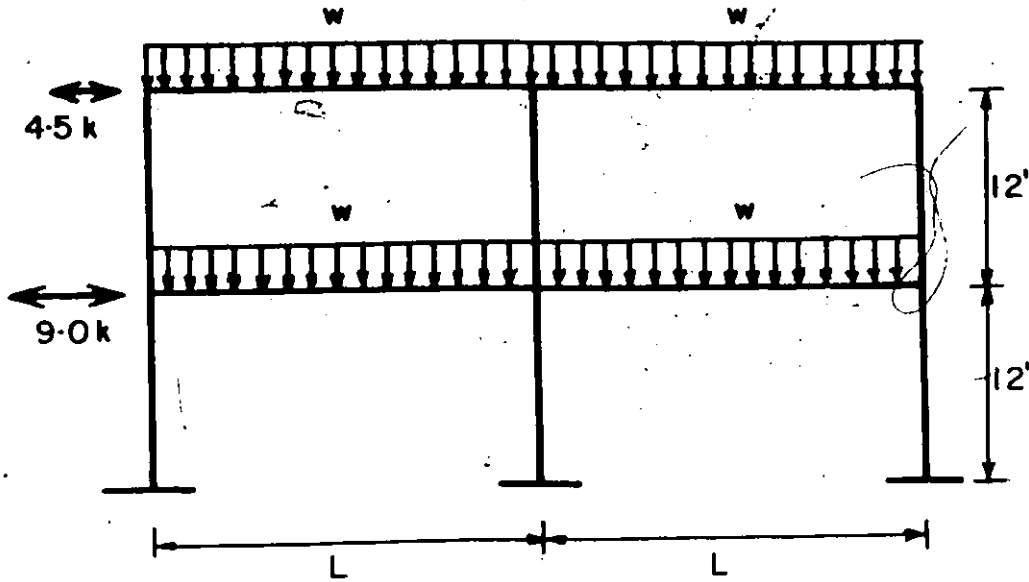


Fig. 4-1 Symmetrical Two-Story Two-Bay Frame Showing Working Loads

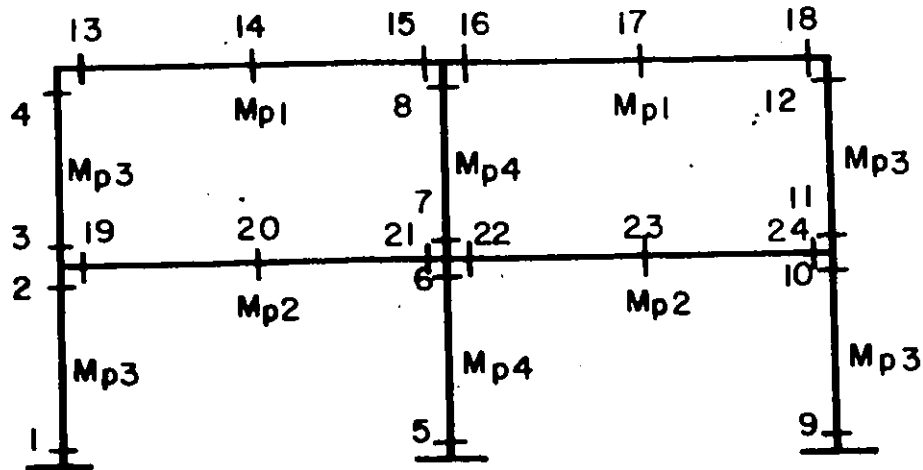


Fig. 4-2 Idealized Symmetrical Two-Story and Two-Bay Frame

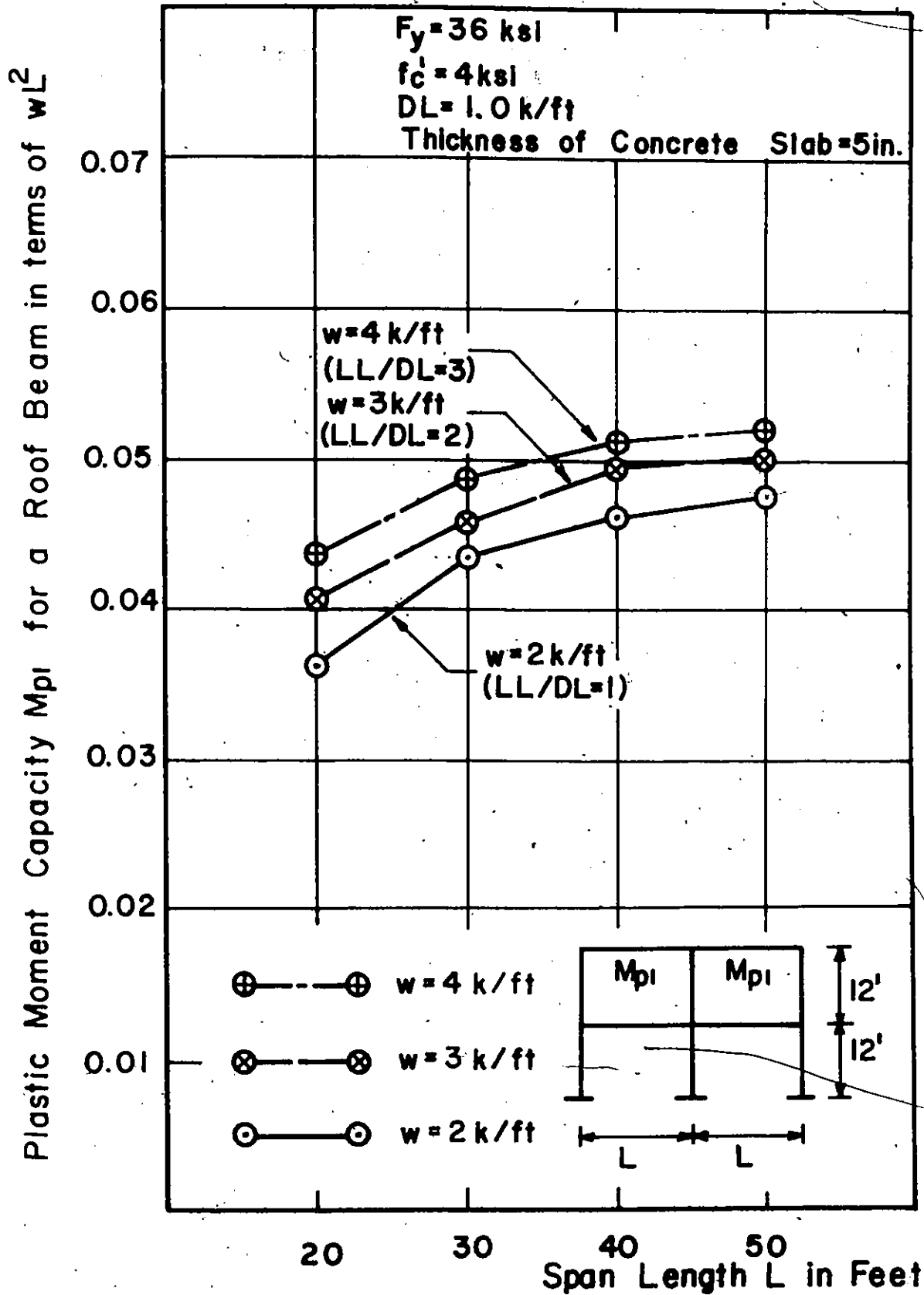


Fig. 4-3 Empirical Relationship between Plastic Moment Capacity, Loading and Span Length for a Roof Beam of Two-Story Two-Bay Example Frame

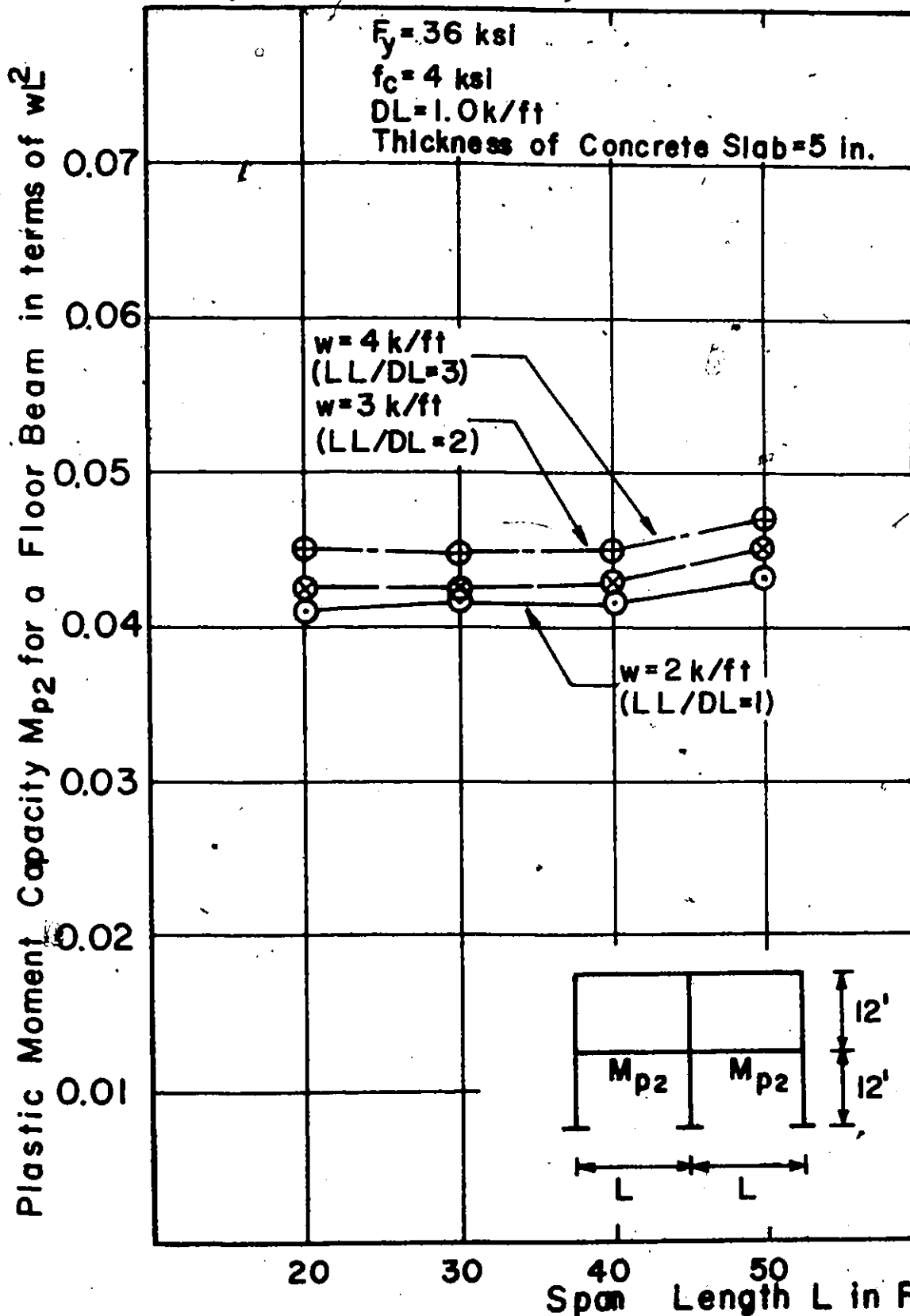
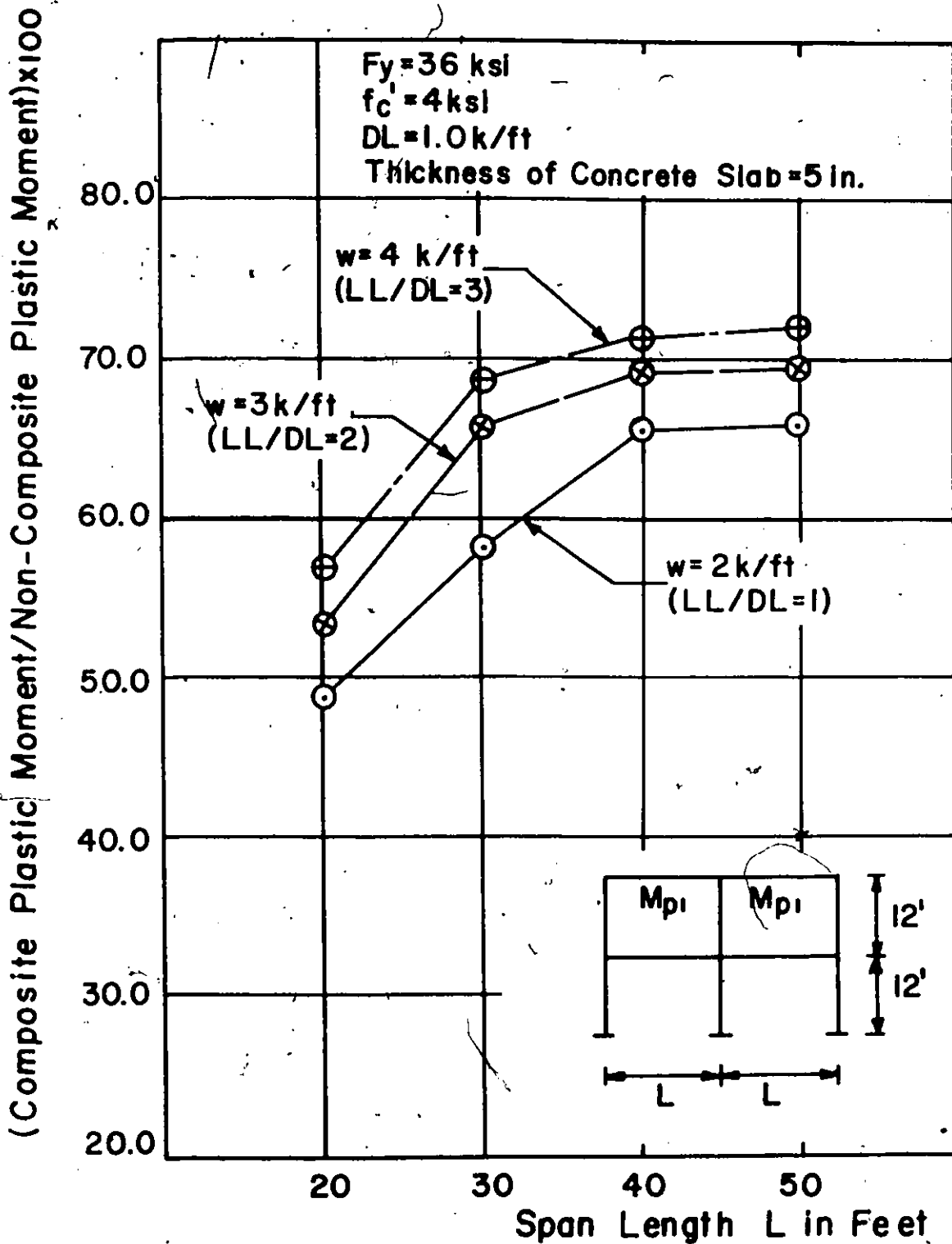


Fig.4-4 Empirical Relationship between Plastic Moment Capacity, Loading and Span Length for a Floor Beam of Two-Story Two-Bay Example Frame



**Fig.4-5 Empirical Relationship between Composite Plastic Moment/Non-Composite Plastic Moment, Loading and Span Length for a Roof Beam of Two-Story Two-Bay Example Frame**

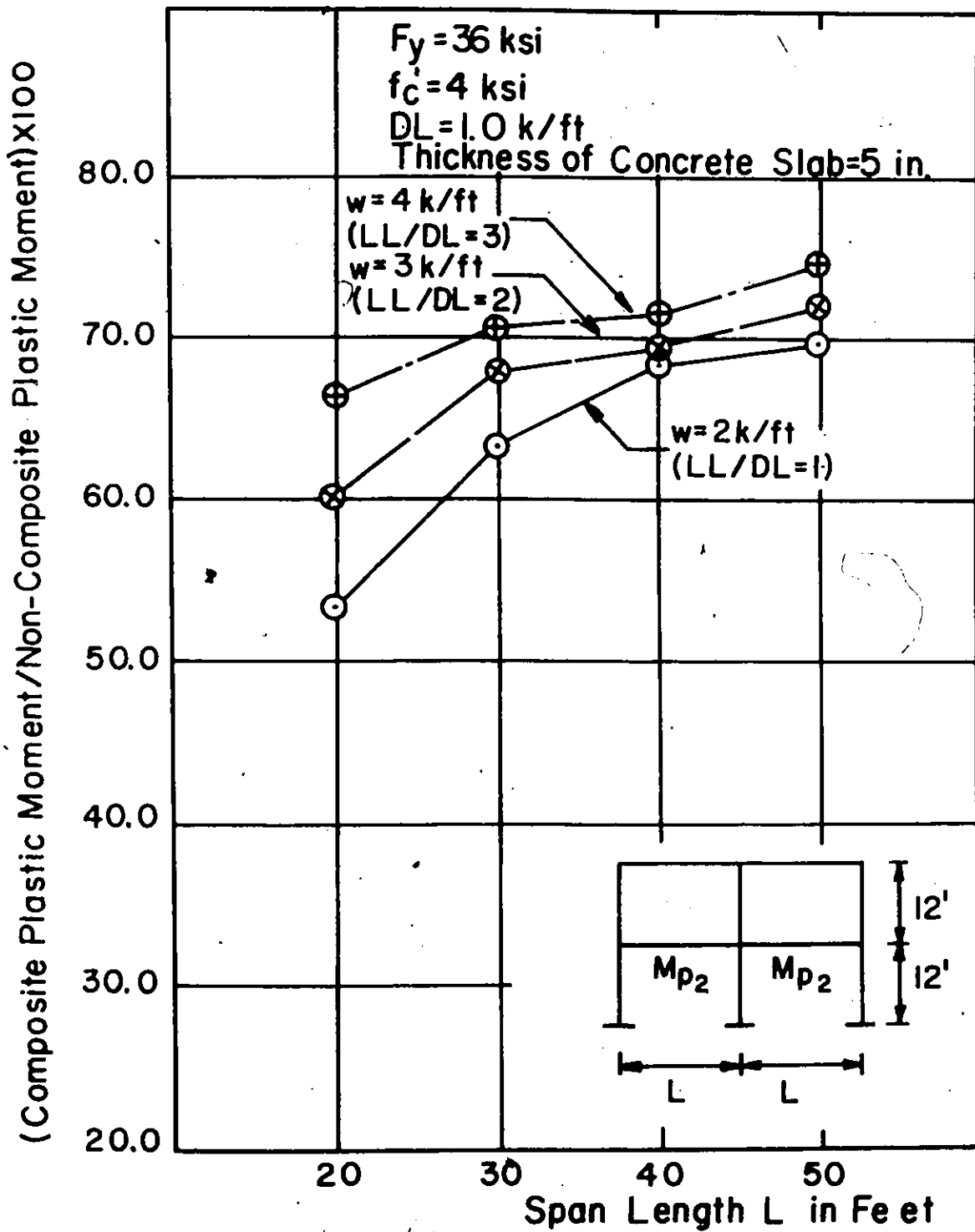
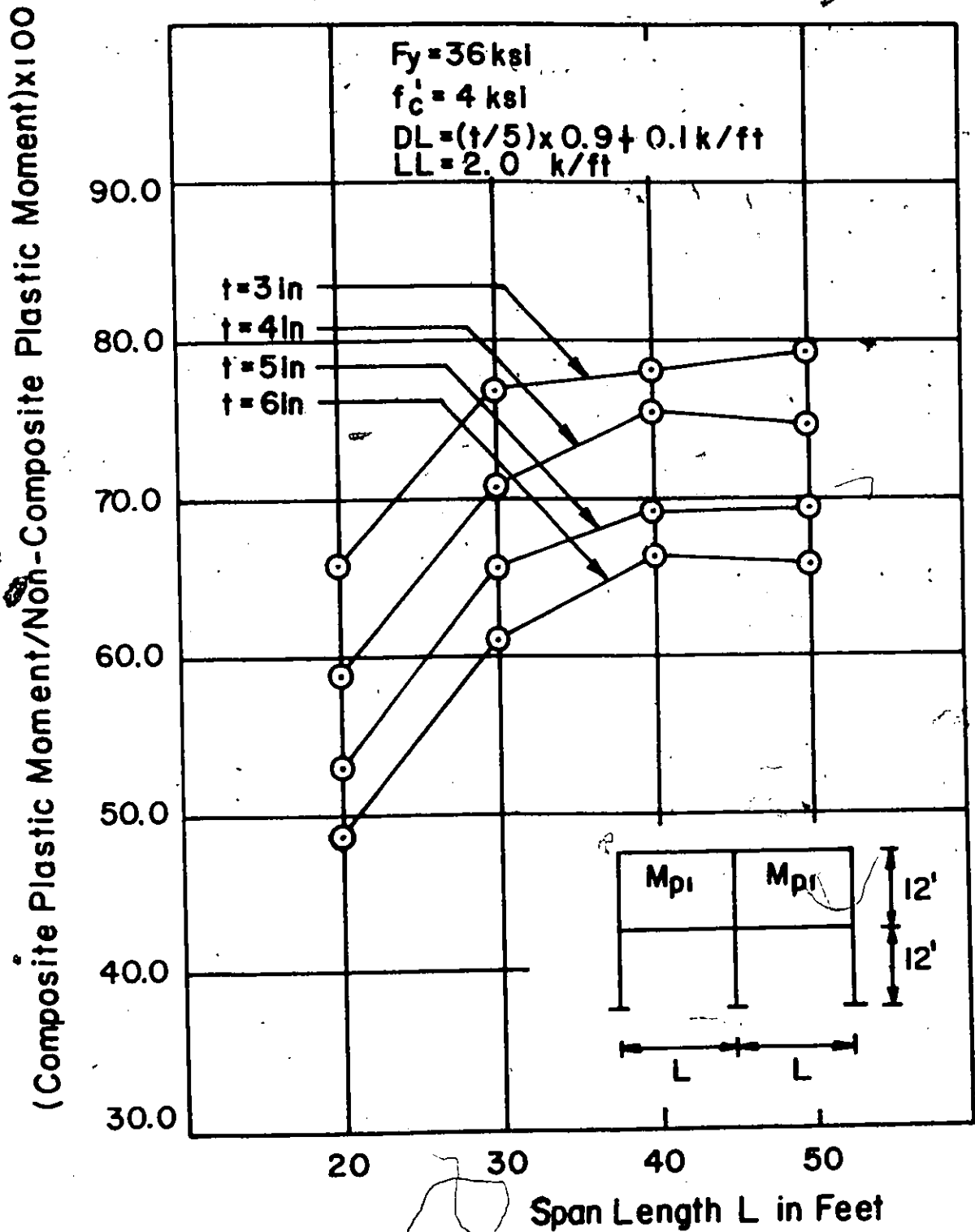


Fig. 4-6 Empirical Relationship between Composite Plastic Moment/Non-Composite Plastic Moment, Loading and Span Length for a Floor Beam of Two-Story Two-Bay Example Frame



**Fig.4-7 Empirical Relationship between Composite Plastic Moment/Non-Composite Plastic Moment, Slab Thickness and Span Length for a Roof Beam of Two-Story Two-Bay Example Frame**

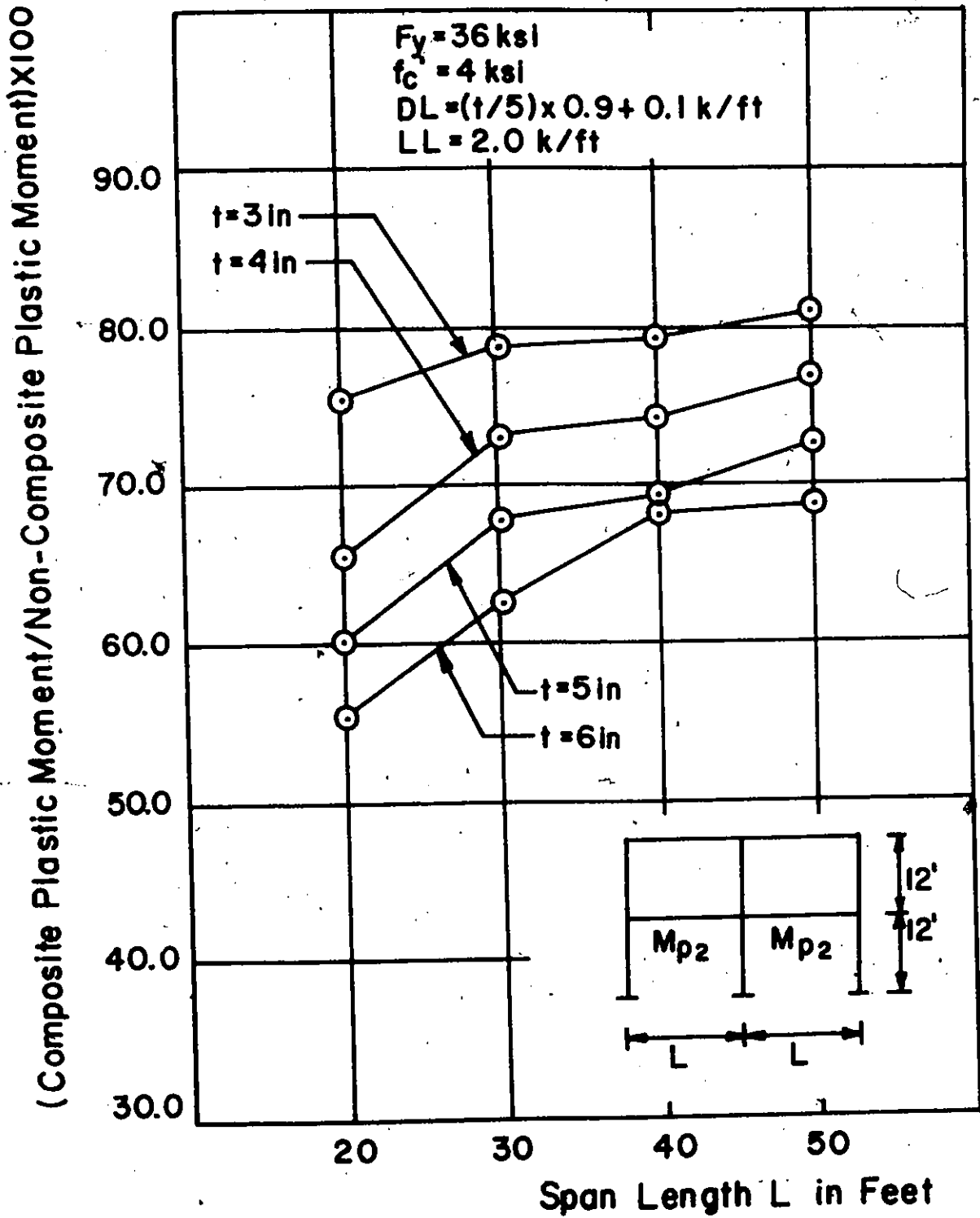


Fig. 4-8 Empirical Relationship between Composite Plastic Moment/Non-Composite Plastic Moment, Slab Thickness and Span Length for a Floor Beam of Two-Story Two-Bay Example Frame

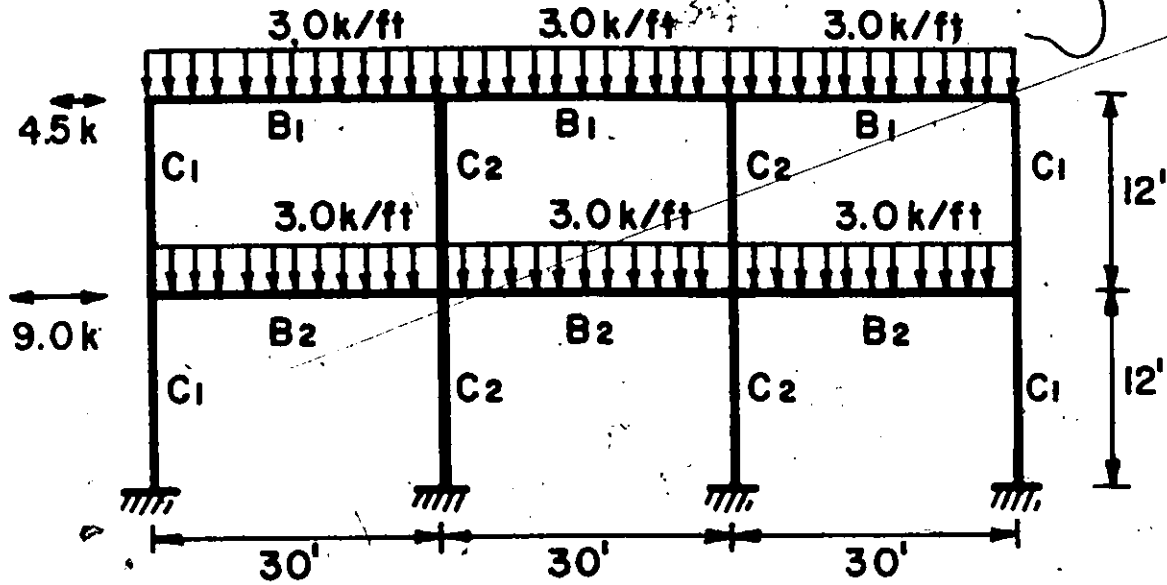
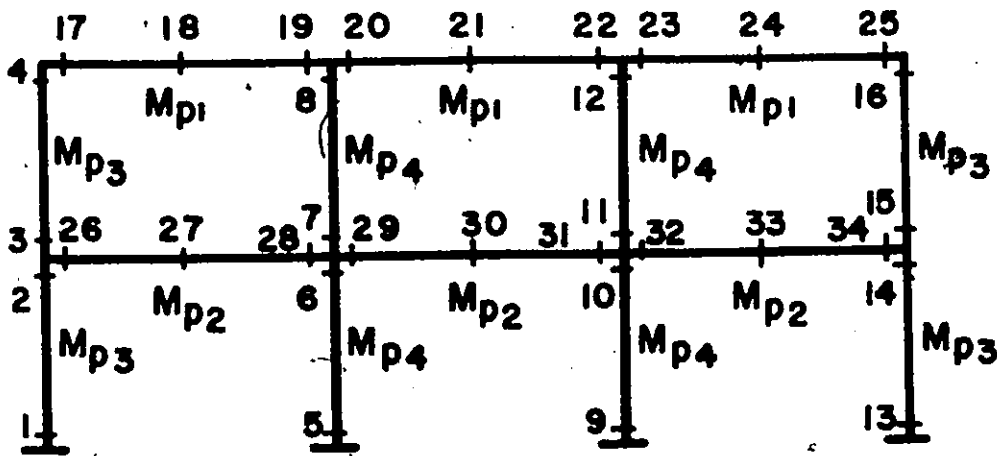
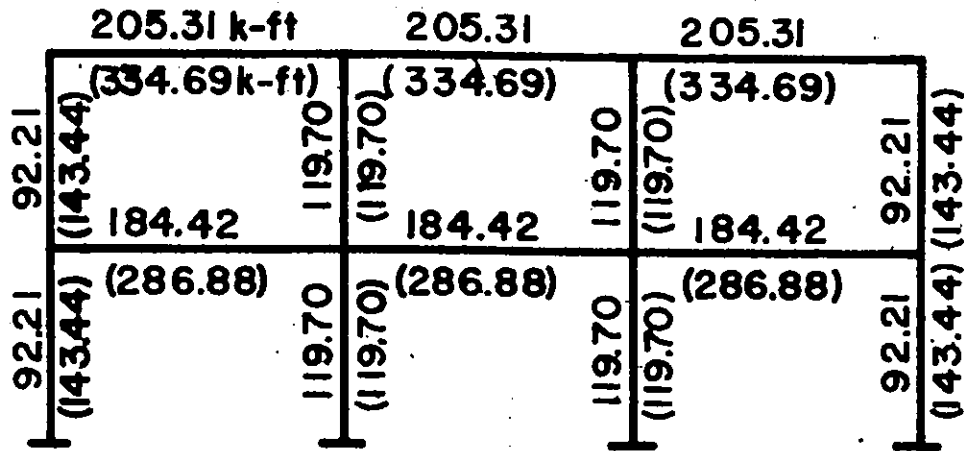


Fig.5-1 Symmetrical Two-Story Three-Bay Composite Frame Showing Working Loads



Where  $M_{p3} \geq 57.0$  k-ft and  $M_{p4} \geq 119.7$  k-ft.

Fig. 5-2 Idealized Symmetrical Two-Story Three-Bay Composite Frame



Where ( ) shows linear programming solution for non-composite case.

Fig. 5-3 Linear Programming Solution for Two-Story Three-Bay Composite Frame

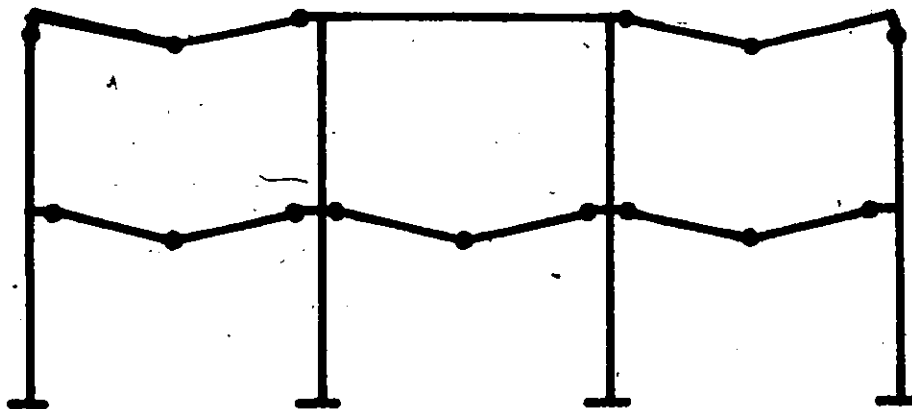
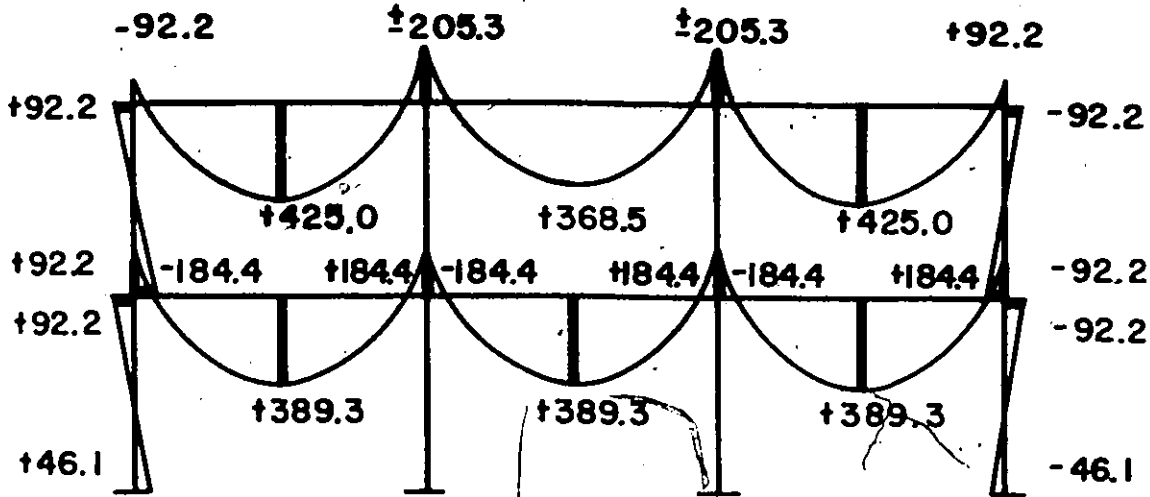
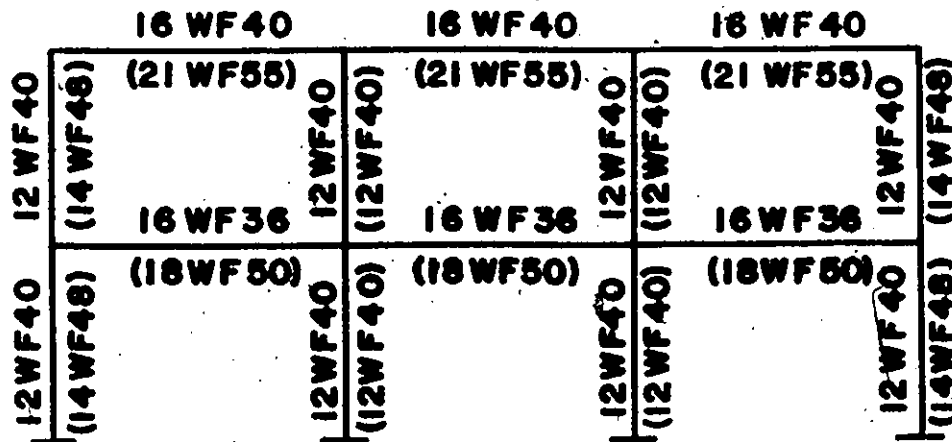


Fig. 5-4 Critical Collapse Mechanism for Symmetrical Two-Story Three-Bay Composite Frame



Where unit is k-ft.

Fig. 5-5 Moment Distributions at Critical Collapse of Symmetrical Two-Story Three-Bay Composite Frame



Where ( ) shows member layout for non-composite case.

Fig. 5-6 Member Layout for Two-Story Three-Bay Composite Frame

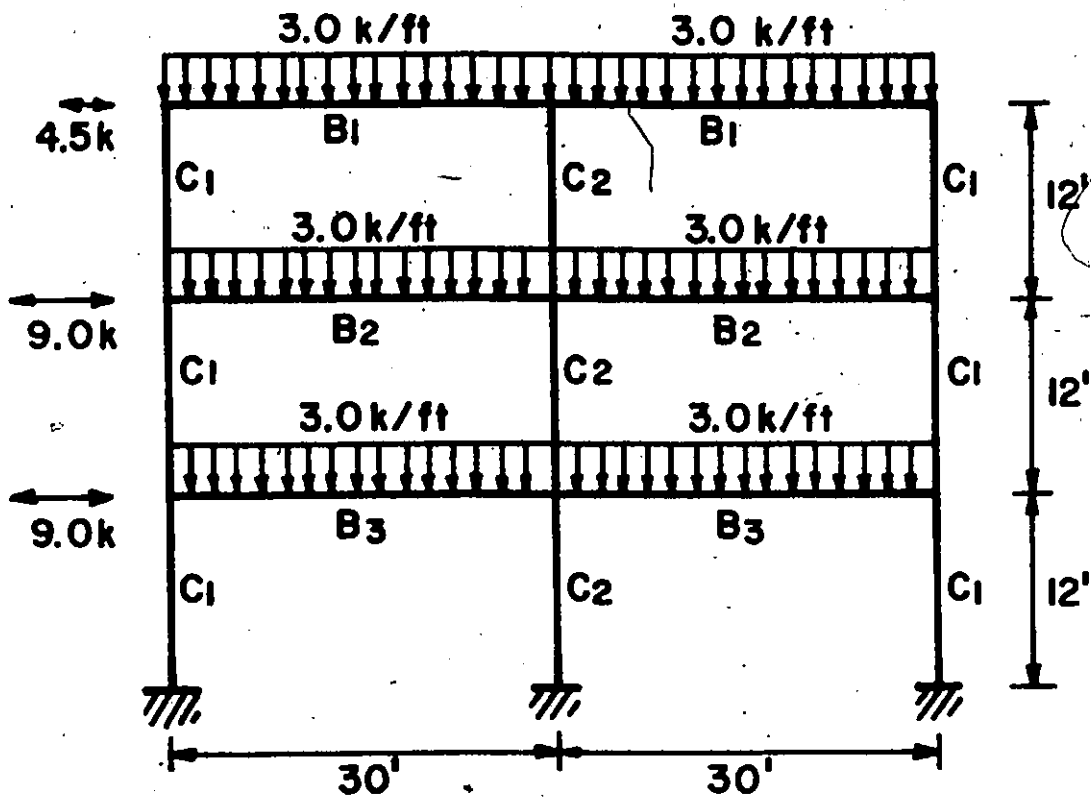
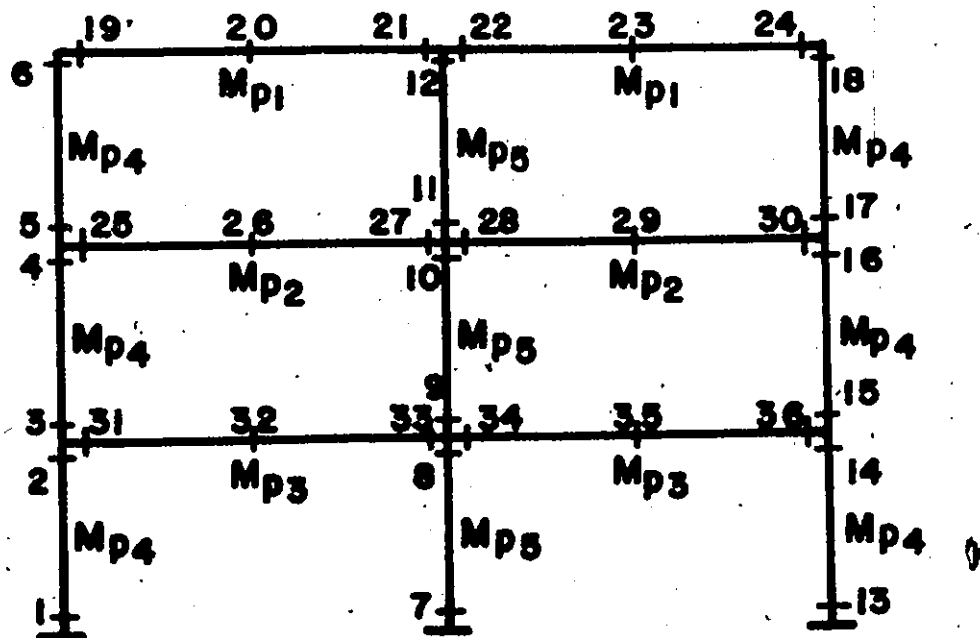


Fig. 5-7 Symmetrical Three-Story Two-Bay Composite Frame Showing Working Loads



Where  $M_{p4} \geq 104.1$  and  $M_{p5} \geq 179.7$  (k-ft).

Fig. 5-8 Idealized Symmetrical Three-Story Two-Bay Composite Frame

	202.64 k-ft (331.16 k-ft)		202.64 (331.16)	
104.10 (154.01)	200.97 (308.03)	179.70 (179.70)	200.97 (308.03)	104.10 (154.01)
104.10 (154.01)	208.20 (308.03)	179.70 (179.70)	208.20 (308.03)	104.10 (154.01)
104.10 (154.01)		179.70 (179.70)		104.10 (154.01)

Where ( ) shows linear programming solution for non-composite case.

Fig. 5-9 Linear Programming Solution for Three-Story Two-Bay Composite Frame

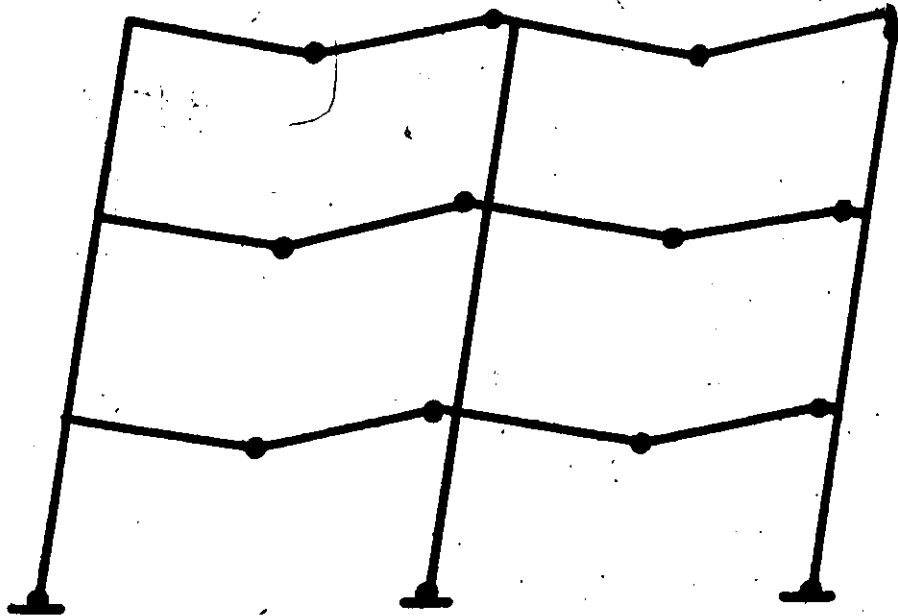
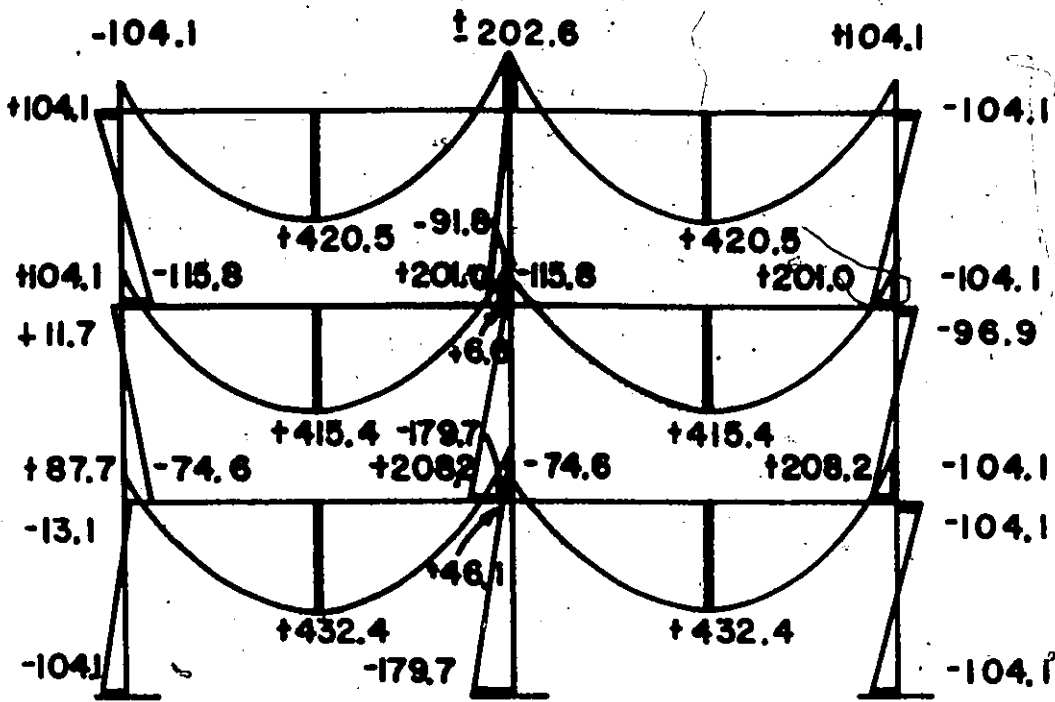


Fig. 5-10 Critical Collapse Mechanism for Symmetrical Three-Story Two-Bay Frame



Where unit is k-ft.

Fig. 5-11 Moment Distributions at Critical Collapse of Symmetrical Three-Story Two-Bay Composite Frame

		16WF40		16WF40	
		(21WF55)		(21WF55)	
14WF53	14WF53	16WF40	16WF88	16WF40	14WF53
(16WF58)	(16WF58)	(21WF55)	(16WF88)	(21WF55)	(16WF58)
14WF53	14WF53	16WF40	16WF88	16WF40	14WF53
(16WF58)	(16WF58)	(21WF55)	(16WF88)	(21WF55)	(16WF58)
14WF53	14WF53	16WF40	16WF88	16WF40	14WF53
(16WF58)	(16WF58)	(21WF55)	(16WF88)	(21WF55)	(16WF58)

Where ( ) shows member layout for non-composite case.

Fig. 5-12 Member Layout for Three-Story Two-Bay Composite Frame

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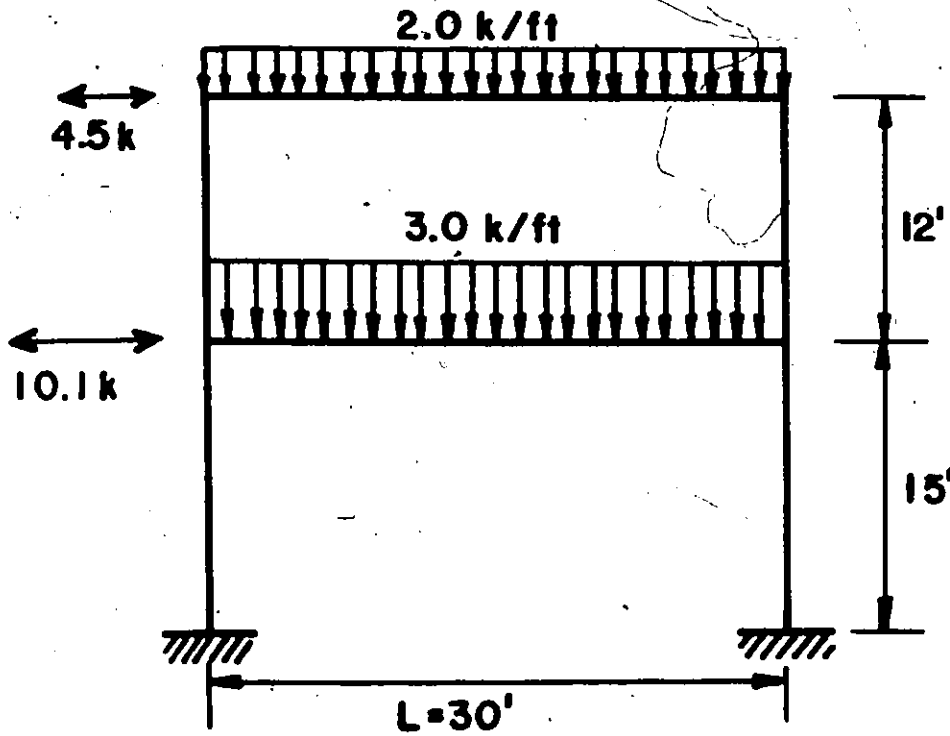


Fig. 6-1 Symmetrical Two-Story Single-Bay Steel Frame Showing Working Loads

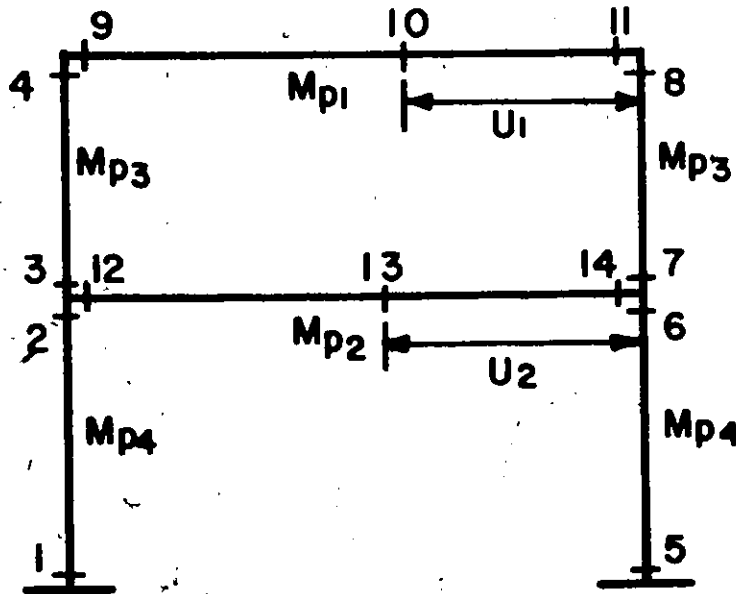


Fig. 6-2 Idealized Symmetrical Two-Story Single-Bay Steel Frame

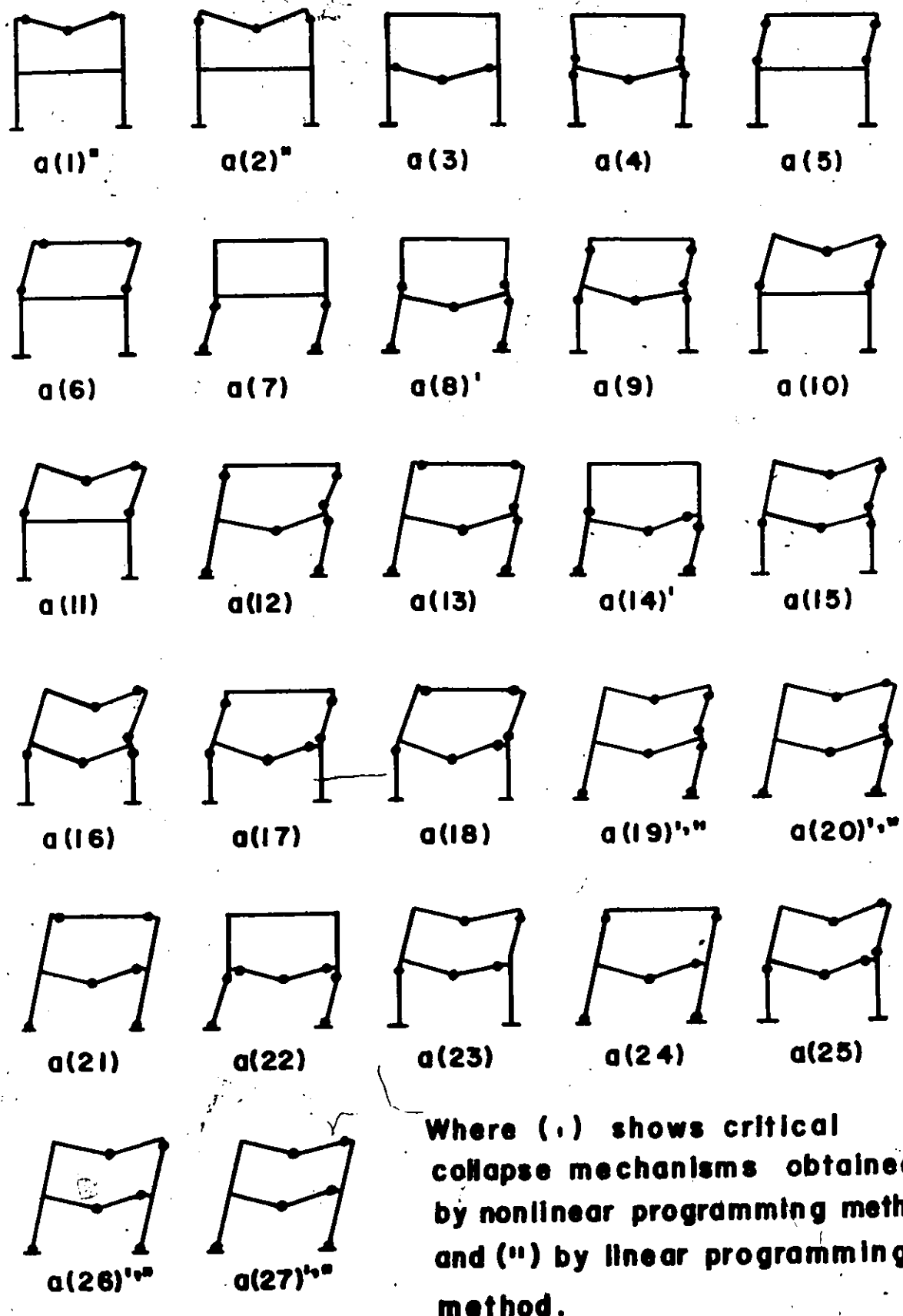


Fig. 6-3 Feasible Collapse Mechanisms for Symmetrical Two-Story Single-Bay Steel Frame

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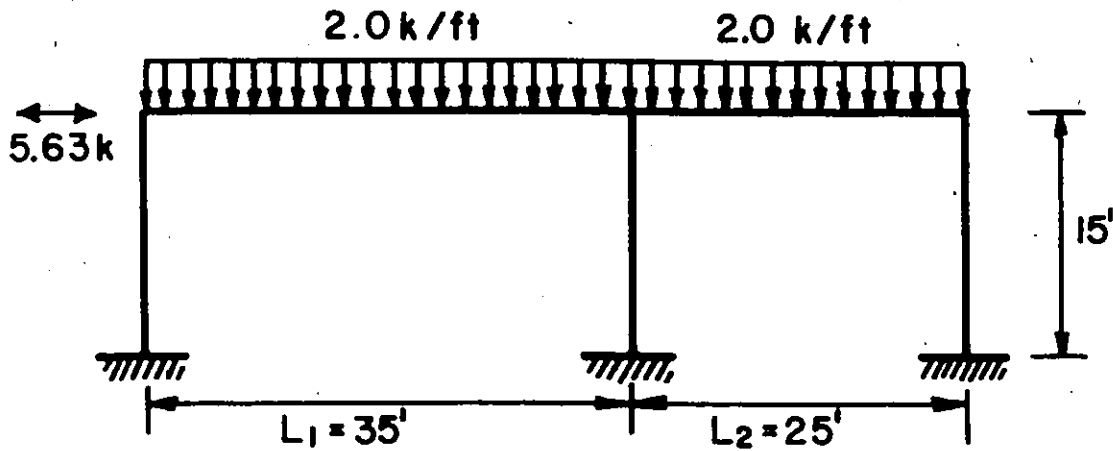


Fig. 6-4 Unsymmetrical One-Story Two-Bay Steel Frame Showing Working Loads

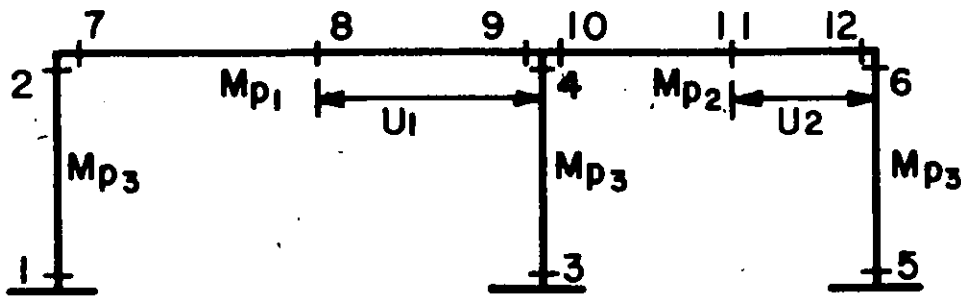
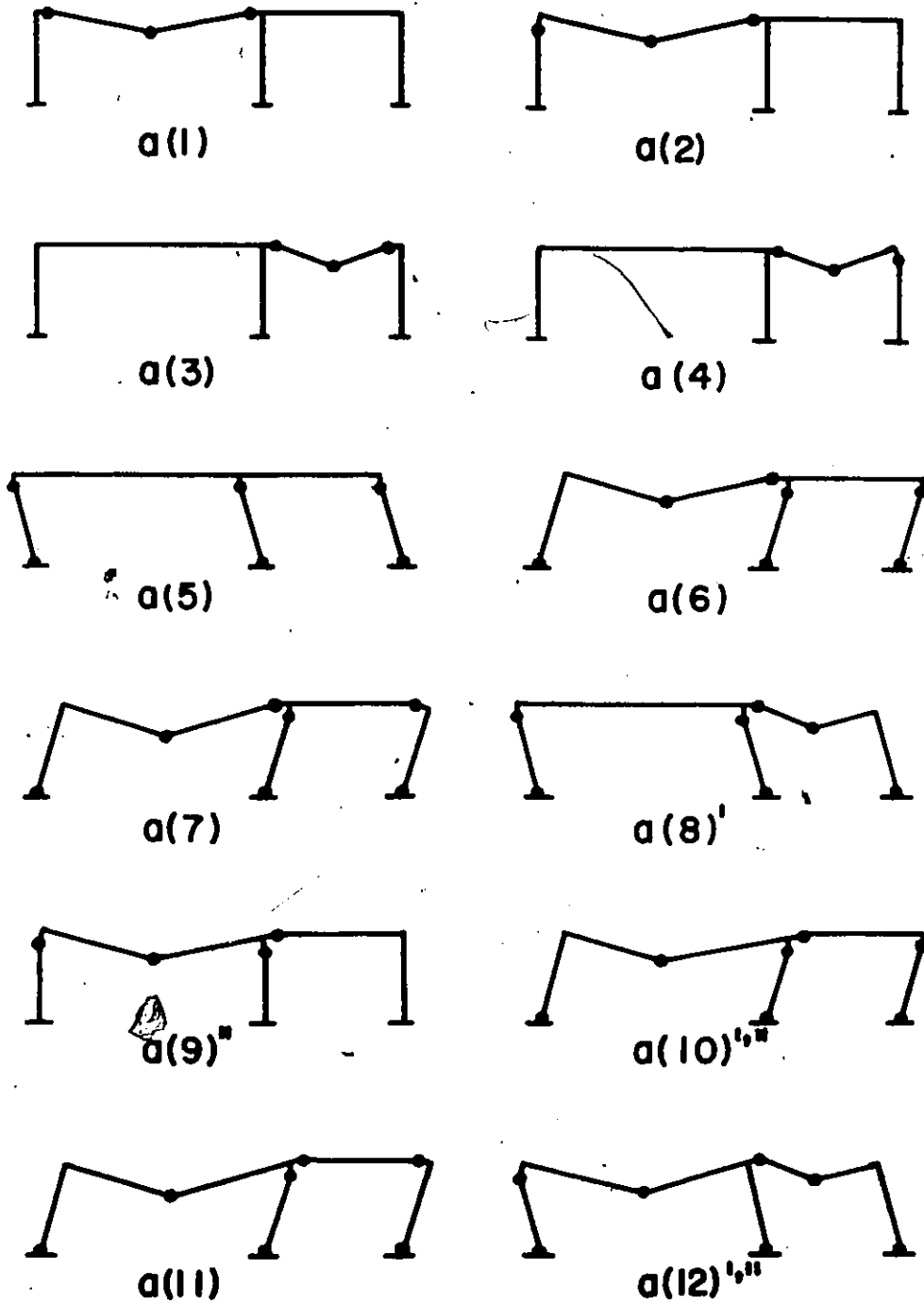


Fig. 6-5 Idealized Unsymmetrical One-Story Two-Bay Steel Frame



Where (·) shows critical collapse mechanisms obtained by nonlinear programming method and (") by linear programming method.

Fig. 6-6 Feasible Collapse Mechanisms for Unsymmetrical One-Story Two-Bay Steel Frame

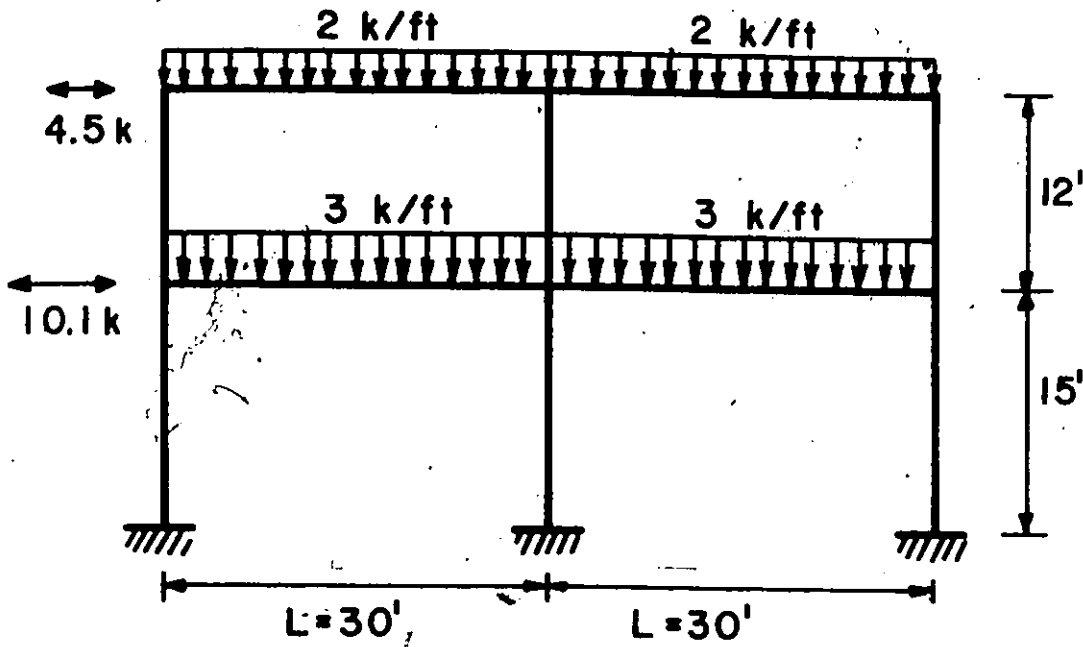


Fig. 6-7 Symmetrical Two-Story Two-Bay Composite Frame Showing Working Loads

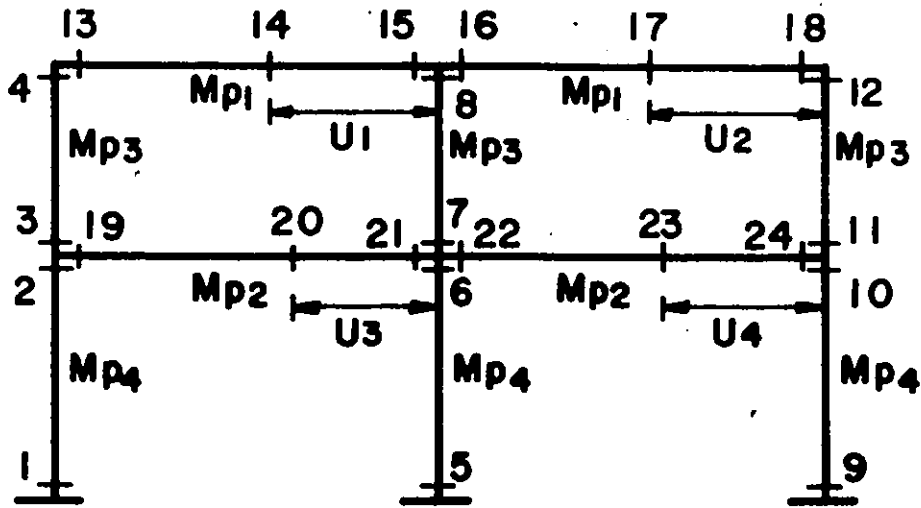
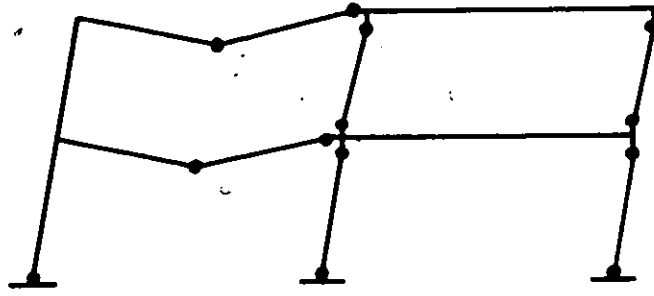
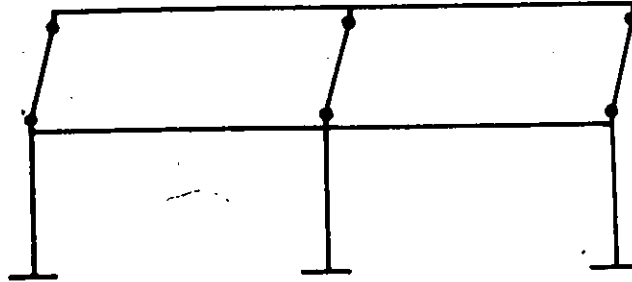


Fig. 6-8 Idealized Symmetrical Two-Story Two-Bay Composite Frame

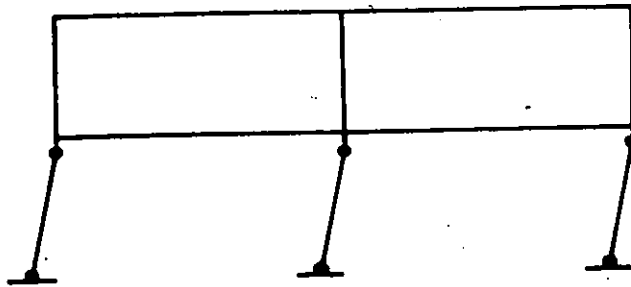
a(3);



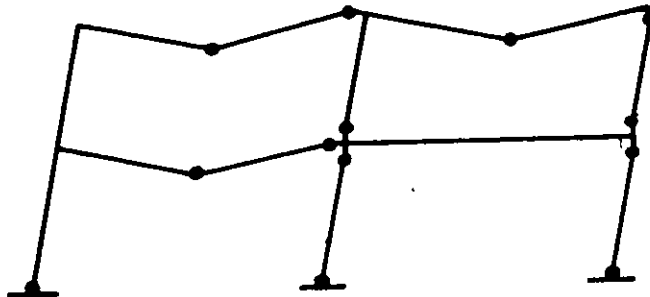
a(8);



a(11);



a(17);



**Fig. 6-9 Critical Collapse Mechanisms corresponding to Active Constraints for Symmetrical Two-Story Two-Bay Composite Frame**

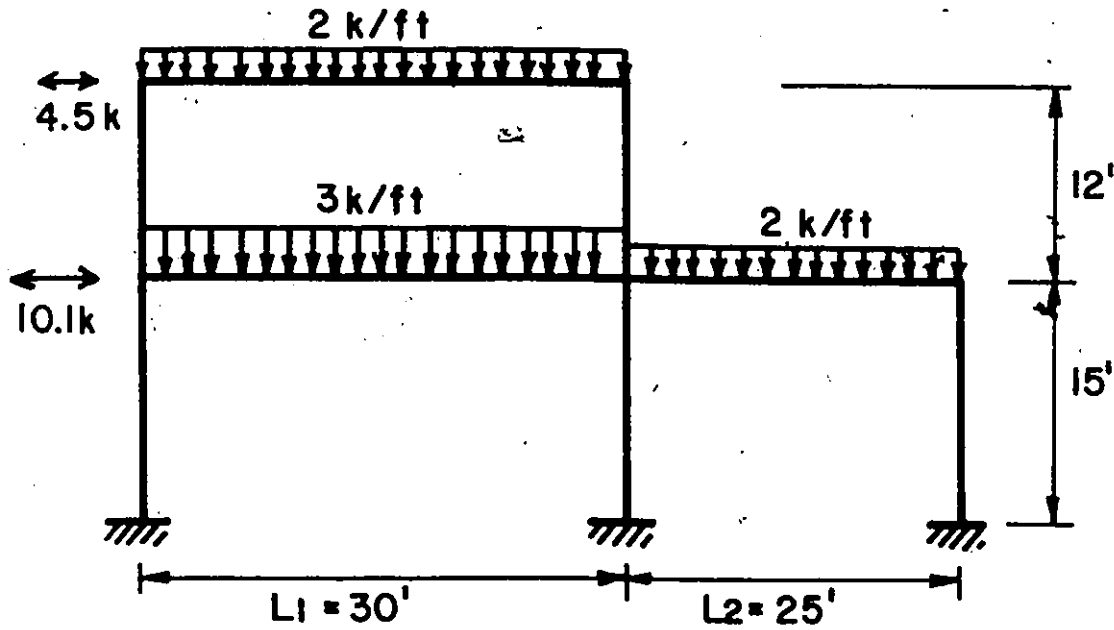


Fig. 6-10 Unsymmetrical Composite Frame Showing Working Loads

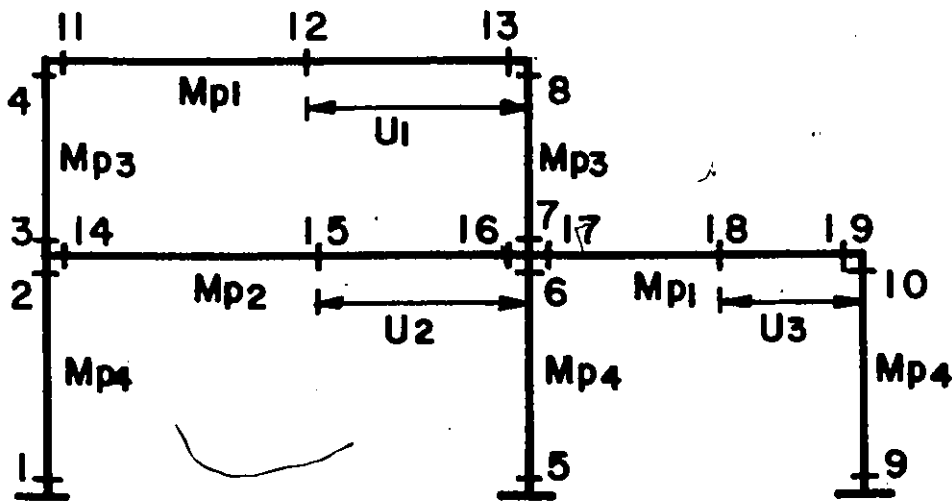
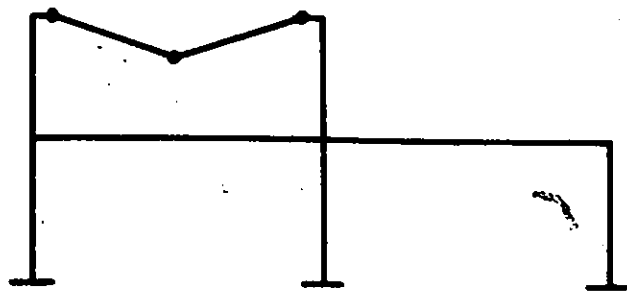
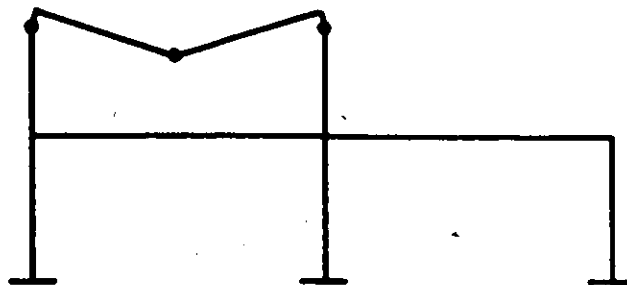


Fig. 6-11 Idealized Unsymmetrical Composite Frame

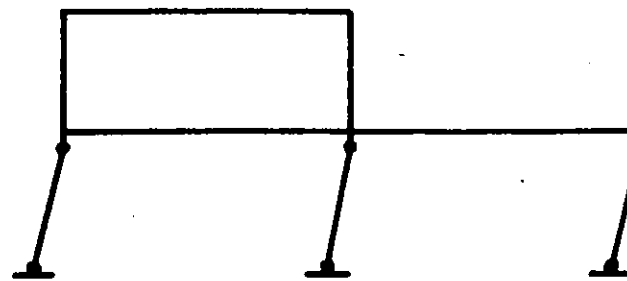
a(1);



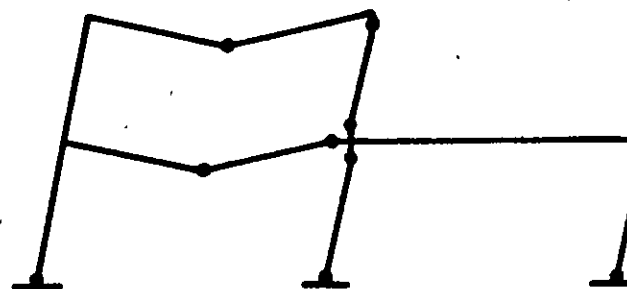
a(2);



a(3);



a(4);



**Fig. 6-12 Critical Collapse Mechanisms corresponding to Active Constraints for Unsymmetrical Composite Frame**

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APPENDIX 2

TABLES

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TABLE 4-1

Available Economical Rolled Wide Flanges and Ultimate  
Moment Capacities for A36 Steel

No	Shapes	$M_u$ (K-ft)	No	Shapes	$M_u$ (K-ft)
1	8WF20	57.3	26	27WF94	834.0
2	8WF24	69.3	27	27WF102	912.0
3	8WF28	81.3	28	30WF99	936.0
4	10WF25	88.5	29	21WF127	954.0
5	12WF27	114.0	30	21WF142	1071.0
6	8WF40	119.7	31	30WF116	1134.0
7	12WF31	132.0	32	33WF118	1242.0
8	12WF36	154.2	33	24WF145	1248.0
9	14WF34	163.5	34	27WF145	1356.0
10	12WF45	194.7	35	33WF130	1398.0
11	16WF40	218.1	36	33WF141	1539.0
12	10WF60	225.3	37	33WF152	1674.0
13	16WF50	278.1	38	36WF150	1740.0
14	16WF58	318.0	39	36WF160	1869.0
15	18WF55	336.0	40	36WF170	2001.0
16	21WF55	375.0	41	36WF194	2301.0
17	18WF64	396.0	42	33WF220	2508.0
18	12WF99	458.0	43	36WF230	2829.0
19	21WF68	480.0	44	36WF245	3024.0
20	24WF68	595.0	45	36WF260	3228.0
21	24WF76	600.0	46	36WF280	3501.0
22	12WF133	630.0	47	36WF300	3765.0
23	24WF84	672.0			
24	27WF84	729.0			
25	12WF161	770.0			

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TABLE 4-2

Approximate Functions for  $M_u/M_p$  ( $F_y = 36.0$  Ksi and  $f'_c = 4.0$  Ksi)

Thickness of Coeff. of Functions	3.0		4.0		5.0		6.0																	
	Type A	a	0.5969	0.7644	0.8954	0.1006 x 10	b	$-0.4972 \times 10^{-3}$	$-0.6357 \times 10^{-3}$	$-0.6973 \times 10^{-3}$	c	$0.2489 \times 10^{-6}$	$0.3054 \times 10^{-6}$	$0.3215 \times 10^{-6}$	d	$-0.3953 \times 10^{-10}$	$-0.4679 \times 10^{-10}$	$-0.4833 \times 10^{-10}$	$-0.4955 \times 10^{-10}$	Standard Deviation	0.076	0.087	0.105	0.130
Type B	a	$0.6212 \times 10$	$0.6404 \times 10$	$0.7533 \times 10$	$0.8740 \times 10$	b	$-0.1863 \times 10$	$-0.1631 \times 10$	$-0.1947 \times 10$	$-0.2306 \times 10$	c	0.2469	0.1863	0.2233	d	$-0.1156 \times 10^{-1}$	$-0.7665 \times 10^{-2}$	$-0.9424 \times 10^{-2}$	$-0.1139 \times 10^{-1}$	Standard Deviation	0.043	0.042	0.051	0.071
	Mean ( $M_u/M_p$ )	1.475	1.639	1.804	1.969	Standard Deviation	0.187	0.267	0.341	0.410	Max ( $M_u/M_p$ )	1.997	2.346	2.680	3.005	Min ( $M_u/M_p$ )	1.225	1.308	1.369	1.445				

Where Type A:  $M_u/M_p = e^a + bM_p + cM_p^2 + dM_p^3$  (if  $M_p \approx 0$ )

Type B:  $M_u/M_p = a + b(\log M_p) + c(\log M_p)^2 + d(\log M_p)^3$  (if  $M_p \neq 0$ )

TABLE 4-3

Approximate Function for  $M_u/M_p$  ( $F_y = 36.0$  Ksi and  $f_c' = 3.5$  Ksi)

Thickness of Slab Conc. (in.)	Coeff. of Functions				
	3.0	4.0	5.0	6.0	
Type A	a	0.5771	0.7531	0.8884	$0.1001 \times 10$
	b	$-0.4839 \times 10^{-3}$	$-0.6505 \times 10^{-3}$	$-0.7171 \times 10^{-3}$	$-0.7572 \times 10^{-3}$
	c	$0.2403 \times 10^{-6}$	$0.3169 \times 10^{-6}$	$0.3331 \times 10^{-6}$	$0.3419 \times 10^{-6}$
	d	$-0.3808 \times 10^{-10}$	$-0.4888 \times 10^{-10}$	$-0.5018 \times 10^{-10}$	$-0.5084 \times 10^{-10}$
	Standard Deviation	0.077	0.089	0.104	0.128
Type B	a	$0.6385 \times 10$	$0.6366 \times 10$	$0.7347 \times 10$	$0.8608 \times 10$
	b	$-0.1967 \times 10$	$-0.1613 \times 10$	$-0.1847 \times 10$	$-0.2237 \times 10$
	c	0.2674	0.1811	0.2040	0.2531
	d	$-0.1271 \times 10^{-1}$	$-0.7251 \times 10^{-2}$	$-0.8246 \times 10^{-2}$	$-0.1061 \times 10^{-1}$
	Standard Deviation	0.043	0.044	0.049	0.067
Mean ( $M_u/M_p$ )	1.451	1.615	1.780	1.946	
Standard Deviation	0.184	0.265	0.342	0.413	
Max ( $M_u/M_p$ )	1.971	2.326	2.664	2.992	
Min ( $M_u/M_p$ )	1.205	1.291	1.351	1.421	

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TABLE 4-4

Approximate Function for  $M_u/M_p$  ( $F_y = 36.0$  Ksi and  $f'_c = 3.0$  Ksi)

Type	Thickness of Conc. Slab (in.)	Coeff. of Functions			
		3.0	4.0	5.0	6.0
Type A	a	0.5531	0.7366	0.8790	0.9948
	b	$-0.4676 \times 10^{-3}$	$-0.6630 \times 10^{-3}$	$-0.7438 \times 10^{-3}$	$-0.7831 \times 10^{-3}$
	c	$0.2274 \times 10^{-6}$	$0.3285 \times 10^{-6}$	$0.3498 \times 10^{-6}$	$0.3557 \times 10^{-6}$
	d	$-0.3578 \times 10^{-10}$	$-0.5118 \times 10^{-10}$	$-0.5286 \times 10^{-10}$	$-0.5295 \times 10^{-10}$
	Standard Deviation	0.079	0.091	0.105	0.125
Type B	a	$0.6388 \times 10$	$0.6710 \times 10$	$0.7261 \times 10$	$0.8370 \times 10$
	b	$-0.2007 \times 10$	$-0.1795 \times 10$	$-0.1794 \times 10$	$-0.2108 \times 10$
	c	0.2770	0.2090	0.1912	0.2288
	d	$-0.1334 \times 10^{-1}$	$-0.8611 \times 10^{-2}$	$-0.7348 \times 10^{-2}$	$-0.9169 \times 10^{-2}$
	Standard Deviation	0.043	0.045	0.048	0.063
	Mean ( $M_u/M_p$ )	1.423	1.587	1.751	1.916
	Standard Deviation	0.179	0.262	0.342	0.415
	Max ( $M_u/M_p$ )	1.937	2.300	2.642	2.974
	Min ( $M_u/M_p$ )	1.182	1.268	1.333	1.395

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TABLE 4-5

Maximum Plastic Moment for a Symmetrical Two-Story Two-Bay  
 Frame Example (w = 3 K/ft, LL/DL = 2)

f' (Ksi)	Span length (ft)		20.0	30.0	40.0	50.0
	M <sub>CB</sub> (K-ft)	M <sub>CB</sub> (K-ft)				
3.0	Composite Nog	M <sup>P1</sup>	155.83	219.39	579.70	914.39
		M <sup>P2</sup>	143.93	286.88	510.00	796.88
		M <sup>P3</sup>	71.97	189.34	300.90	444.34
		M <sup>P4</sup>	32.87	0.00	0.00	0.00
3.5	Composite	M <sup>P1</sup>	83.88 (53.8%)	215.90 (67.6%)	415.35 (71.6%)	657.90 (72.0%)
		M <sup>P2</sup>	88.10 (61.2%)	198.06 (69.0%)	362.34 (71.0%)	592.94 (74.4%)
		M <sup>P3</sup>	46.83 (65.1%)	99.03 (52.3%)	181.17 (60.2%)	342.37 (77.1%)
		M <sup>P4</sup>	44.05 (134%)	24.16 (-----)	30.60 (-----)	0.00 (0.0%)
4.0	Composite	M <sup>P1</sup>	83.17 (53.4%)	213.10 (66.7%)	408.93 (70.5%)	647.21 (70.8%)
		M <sup>P2</sup>	87.19 (60.6%)	195.91 (68.3%)	357.54 (70.1%)	584.55 (73.4%)
		M <sup>P3</sup>	47.05 (65.4%)	97.95 (51.7%)	178.77 (59.1%)	338.17 (76.1%)
		M <sup>P4</sup>	43.60 (132.6%)	24.47 (-----)	30.60 (-----)	0.00 (0.0%)
4.0	Composite	M <sup>P1</sup>	82.64 (53.0%)	211.12 (66.1%)	404.23 (69.7%)	639.09 (69.9%)
		M <sup>P2</sup>	86.53 (60.1%)	194.39 (67.8%)	354.04 (69.4%)	578.22 (72.6%)
		M <sup>P3</sup>	47.22 (65.6%)	97.19 (51.3%)	177.92 (58.8%)	335.01 (75.4%)
		M <sup>P4</sup>	43.26 (131.6%)	24.69 (-----)	30.60 (-----)	0.00 (0.0%)

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TABLE 6-1

Approximate Function for  $M_u - M_p$  Relation ( $F_y = 36.0$  Ksi and  $f_c' = 4.0$  Ksi)

Thickness of Conc. Slab of Function (in)	3.0	4.0	5.0	6.0
a	$0.3902 \times 10^2$	$0.6186 \times 10^2$	$0.7265 \times 10^2$	$0.8971 \times 10^2$
b	$0.1390 \times 10$	$0.1519 \times 10$	$0.1688 \times 10$	$0.1850 \times 10$
c	$-0.2970 \times 10^{-4}$	$-0.7451 \times 10^{-4}$	$-0.1348 \times 10^{-3}$	$-0.1823 \times 10^{-3}$
d	$-0.5020 \times 10^{-8}$	$0.3258 \times 10^{-8}$	$0.1148 \times 10^{-7}$	$0.1803 \times 10^{-7}$
Standard Deviation	40.30	39.13	34.81	33.39
Mean ( $M_u$ )	1407.00	1515.82	1626.48	1746.13
Standard Deviation	1232.53	1300.01	1358.11	1433.07
Max ( $M_u$ )	4611.88	4923.90	5154.08	5441.07
Min ( $M_u$ )	114.41	134.43	153.56	172.21

Where Type of Function;  $M_u = a + b M_p + c M_p^2 + d M_p^3$  (K-ft)

TABLE 6-2

Approximate Function for  $M_u - M_p$  Relation ( $F_y = 36.0$  Ksi and  $f'_c = 3.5$  Ksi)

Thickness of Conc. Slab (in)	3.0	4.0	5.0	6.0
a	$0.3513 \times 10^2$	$0.6377 \times 10^2$	$0.7742 \times 10^2$	$0.9074 \times 10^2$
b	$0.1385 \times 10$	$0.1477 \times 10$	$0.1655 \times 10$	$0.1823 \times 10$
c	$-0.4202 \times 10^{-4}$	$-0.5550 \times 10^{-4}$	$-0.1283 \times 10^{-3}$	$-0.1821 \times 10^{-3}$
d	$-0.2677 \times 10^{-8}$	$-0.7278 \times 10^{-10}$	$0.1080 \times 10^{-7}$	$0.1807 \times 10^{-7}$
Standard Deviation	38.56	48.36	34.89	33.28
Mean ( $M_u$ )	1384.72	1496.14	1602.99	1719.62
Standard Deviation	1211.67	1286.36	1339.36	1407.51
Max ( $M_u$ )	4535.45	4859.92	5087.97	5348.90
Min ( $M_u$ )	112.93	133.30	152.64	171.44

Where the units for  $M_u$  and standard deviations are K-ft.



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TABLE 6-3

Approximate Function for  $M_u - M_p$  Relation ( $F_y = 36$  Ksi and  $f_c' = 3.0$  Ksi)

Thickness of Conc. Slab (in) Coeff of Function	3.0	4.0	5.0	6.0
a	$0.3152 \times 10^2$	$0.6389 \times 10^2$	$0.8006 \times 10^2$	$0.9324 \times 10^2$
b	$0.1377 \times 10$	$0.1435 \times 10$	$0.1606 \times 10$	$0.1781 \times 10$
c	$-0.5726 \times 10^{-4}$	$-0.3596 \times 10^{-4}$	$-0.1134 \times 10^{-3}$	$-0.1751 \times 10^{-3}$
d	$0.5074 \times 10^{-9}$	$-0.3950 \times 10^{-8}$	$0.8952 \times 10^{-8}$	$-0.1729 \times 10^{-7}$
Standard Deviation	35.54	40.93	37.38	34.24
Mean ( $M_u$ )	1358.37	1471.54	1575.15	1687.84
Standard Deviation	1187.26	1267.69	1319.66	1379.73
Max ( $M_u$ )	4450.70	4772.92	5018.94	5250.99
Min ( $M_u$ )	110.97	131.79	151.41	170.41

Where the units for  $M_u$  and standard deviations are K-ft

APPENDIX 3

11/11/11

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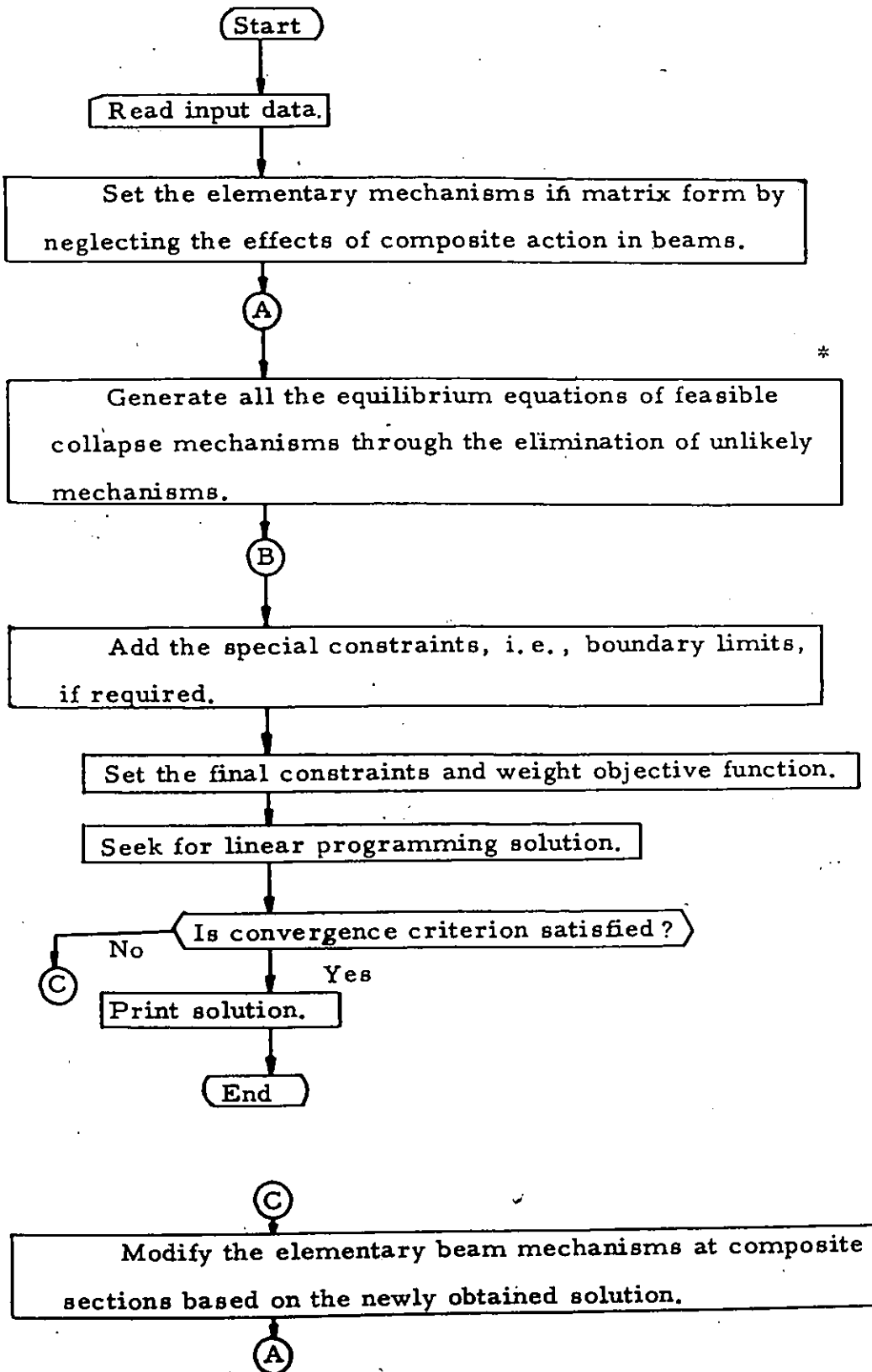
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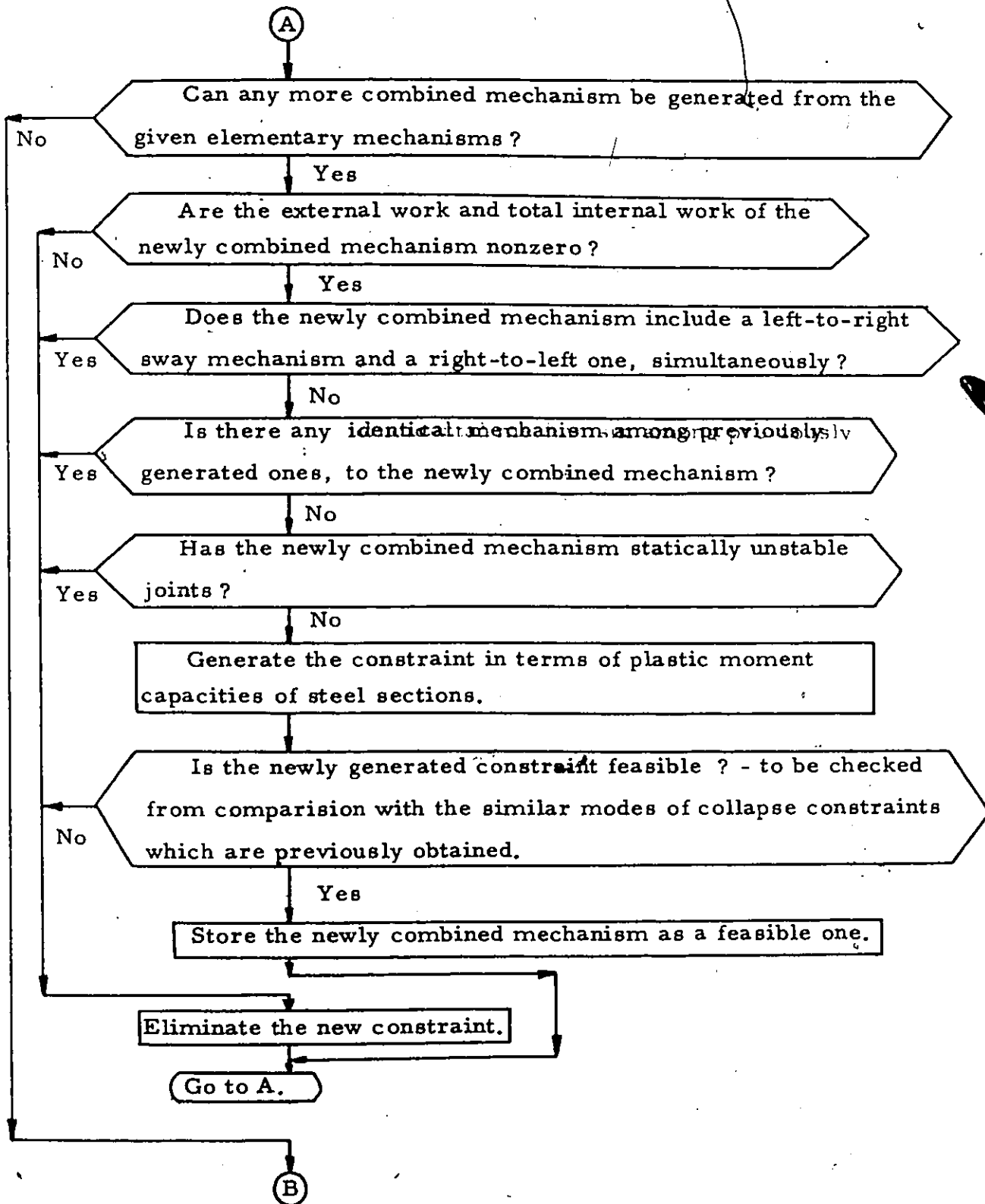
APPENDIX 4

DESCRIPTION AND LIST OF COMPUTER  
PROGRAMS

A. 4-1 Flow Chart of Main Program for Linear Optimization of Composite Steel Frames



(\*) Detailed Block Chart for Part A to B



A. 4-2 Descriptions of Input Data for Linear Optimization of Composite Steel Frames

1) First Set of Data

NTELM, NOMP, NTHL, NBELM

Where NTELM, NOMP, NTHL and NBELM denote the number of all the elementary mechanisms, number of all the plastic moments defined as design variables, number of all the assumed critical sections, and number of the beam elementary mechanisms, respectively.

2) Second Groups of Data

NOH, WEXTN, WEXTD, MECO (i)

for  $i = 1, \text{NTELM}$

Where NOH is the number of plastic hinges which occurs at the  $i$ -th elementary mechanism. WEXTN and WEXTD are the numerator and denominator of the coefficient of the corresponding external work WEXTO (i), respectively. MECO (i) is the  $i$ -th elementary mechanism's code number which is used in classifying the mechanism according to its way of collapse, and also to be used for eliminating the unlikely combined mechanisms composed of the right-to-left and left-to-right elementary sway mechanisms simultaneously. Assign integer number of -1, 0 and +1 to MECO (i) corresponding to right-to-left, non-lateral and left-to-right mechanisms if all of three types of mechanisms have to be considered, i. e., in the case of the unsymmetrical structures or loadings, otherwise leave blank. These input datas for elementary mechanisms have to be read in the order of beam and sway or joint or other mechanisms in order to identify beam mechanisms from another types of elementary mechanisms.

3) Third Groups of Data

$J, MP, THTAN, THTAD$

for  $i = 1, NTELM$  and  $k = 1, NOH$

Where  $J$  means the  $J$ -th section at which the plastic hinge is assumed for the  $i$ -th elementary mechanism, and it must be given in ~~negative~~ integer if the corresponding hinge is assumed at composite section, otherwise given in positive integer.

$MP$  means the  $MP$ -th plastic moment which corresponds to the  $J$ -th section.  $THTAN$  and  $THTAD$  are the numerator and denominator for the value of the hinge rotation at the  $J$ -th section. These data represent the  $i$ -th elementary mechanism at  $J$ -th section.

4) Fourth Set of Data

$JNT(j), j = 1, NTHL$

Where  $JNT(j)$  is assigned to the smallest integer number among those assigned to the critical sections around a joint of near the  $j$ -th section, including the number  $j$ . In addition,  $JNT(j)$  is the  $j$ -th section's code number to be used for eliminating the unlikely combined mechanisms with the elementary joint mechanism separately near the  $j$ -th section. If the number of the assumed critical sections near the  $j$ -section is only one, i. e., at supports or mid-span,  $JNT(j)$  must be identical to a negative integer  $-j$  for a convenience in the elimination procedures.

5) Fifth Set of Data

$MINHL, MXNHL, MINCM, IPRT, NP$

Where  $MINHL$ ,  $MXNHL$  and  $MINCM$  denote the minimum number of the required hinges for all through the feasible mechanisms, maximum number of the required hinges for and the minimum number of the combined feasible mechanisms to be generated,

respectively. MINHL, MXNHL and MINCM, however, may be assumed only for obtaining an approximate solution, otherwise for the generation of all feasible mechanisms must be left blank.

If the detailed process during optimization by computer is to be known, IPRT must be given in any positive integer, otherwise left blank. NP is the total number of the linear programming solutions of the original one and additional particular ones in which some of the original variables are considered to depend on the rest ones, and its results in the certain number of the different combinations of the particular variables. If such a particular solution is not needed as usual, NP is equal to 1.

6) Six Set of Data

$\boxed{\text{FLENG, WL, WD, T}}$

Where FLENG, WL and WD denote the standard span length (ft) for the proposed frame, the standard working live load and dead load acting on the typical beam including its own weight (K/ft), respectively. T denotes the thickness of the concrete slab (inch).

7) Seventh Set of Data

$\boxed{\text{PMMIN (MP), MP = 1, NOMP}}$

Where PMMIN (MP) denotes the value of lower limit (K-ft) for the MP-th variable.

8) Eighth Groups of Data

$\boxed{\text{CO (l, k), l = 1, \dots, 4}}$

... for  $k = 1, 2$

Where CO (l, 1) and CO (l, 2),  $l = 1, \dots, 4$  denote the coefficients of a, b, c and d for the type A and the type B functions representing

the continuous variations of  $M_u/M_p$ , respectively.

(See Tables, 4-2 through 4-4)

9) Ninth and Last Groups of Data

$NMP(j), j = 1, NOMP$

... for  $k = 1, NP$

Where  $NMP(j)$  denotes the new variable number corresponding to the  $j$ -th original variable to be used for obtaining the  $k$ -th particular solution. If  $NP$  is equal to 1, these data are not required.

A. 4-3 List of Programs for Linear Optimization of  
Composite Steel Frames

//UOCC04C7 JOB ( )  
// JCL AMAC  
// CLASS=C, TYPRUN=HJLL  
// EXEC FORTGCLG, REGION, FORT=150K, FORT=150K, TIME=5.000

//FCRT.SYSIN DD \*  
IEF3731 STEP /FORT / START 72301.2304 / CPU 32.80SEC MAIN 157K LCS 3K  
IEF3741 STEP /FORT / STOP 72301.2305 / CPU 52MIN 28.79SEC MAIN 427K LCS 3K  
IEF3731 STEP /LKED / START 72301.2305 / CPU 52MIN 28.79SEC MAIN 427K LCS 3K  
IEF3741 STEP /LKED / STOP 72301.2305 / CPU 53MIN 10.43SEC

FORTAN IV G LEVEL 20 MAIN DATE = 72301

\*OPTIONS IN EFFECT\* IO, EBCDIC, SOURCE, FULLIST, NODECK, LOAD, MAP  
\*OPTIONS IN EFFECT\* NAME = MAIN, LINESCT = 45  
\*STATISTICS\* SOURCE STATEMENTS = 641, PROGRAM SIZE = 404514  
\*STATISTICS\* NO DIAGNOSTICS GENERATED

FORTAN IV G LEVEL 20 LINEAR DATE = 72301

\*OPTIONS IN EFFECT\* IO, EBCDIC, SOURCE, FULLIST, NODECK, LOAD, MAP  
\*OPTIONS IN EFFECT\* NAME = LINEAR, LINESCT = 45  
\*STATISTICS\* SOURCE STATEMENTS = 120, PROGRAM SIZE = 4800  
\*STATISTICS\* NO DIAGNOSTICS GENERATED

\*STATISTICS\* NO DIAGNOSTICS THIS STEP 11

BY LINDA M. ...  
WRITTEN BY S. AFAC, DEPARTMENT OF CIVIL ENGINEERING,  
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LIMITATIONS FOR THE PROGRAM ARE THE FOLLOWINGS

- C THE MAXIMUM NUMBER OF HINGE LOCATIONS IS LIMITED TO 36
- C THE MAXIMUM NUMBER OF PLASTIC MOMENTS APPLICABLE IS 10
- C THE MAXIMUM NUMBER OF COMBINED MECHANISMS IS 800
- C THE MAXIMUM NUMBER OF EACH COMBINED MECHANISMS IS 400
- C THE MAXIMUM NUMBER OF ELEMENTARY MECHANISMS IS 30

DOUBLE PRECISION THETA(400,36,2),A(10000),WNT(800,10)  
DIMENSION THETA(36,36),THETA1(36),WEXT(800),WX(400,2)  
DIMENSION WINT(800,10),NO(400),WTF(10),FL(10)  
DIMENSION MPT(36),JNT(36),MECE(30),MEC(400,2),MELM(400,30,2)  
DIMENSION NMP(10),NPEC(10),PM(10,2),IPM(10),IPMM(10),PMWIN(10)  
DIMENSION THETA0(30,36),WEXT0(30),ALP(36),MPT1(36)  
DIMENSION GFN(2),C(4,2)  
EQUIVALENCE (THETA(1),WNT(1)),(THETA(800),A(1))

NREAD=1  
NWRITE=3  
MXELM=800  
MXMELM=400  
MXHL=36  
MXMP=10  
DO 5 J=1,MXHL  
MPT(J)=J.C  
5 CONTINUE

READ IN THE NUMBER OF THE TOTAL ELEMENTARY MECHANISMS  
READ IN THE NUMBER OF THE TOTAL PLASTIC MOMENTS  
READ IN THE NUMBER OF THE TOTAL HINGE LOCATIONS  
READ IN THE NUMBER OF THE SEAN ELEMENTARY MECHANISMS

READ(INREAD,10) NTELM,NOMP,NTHL,NSELM  
10 FORMAT(20I4)  
15 FORMAT(6F10.0)

ASSIGN THE INITIAL VALUES TO THE ELEMENTARY HINGE ROTATION MATRIX

0018  
0019  
0020

0021  
0022  
0023

IN THE TA(I,J)=.

READ IN THE PROPERTIES OF EACH ELEMENTARY MECHANISMS  
AND ARRANGE ALL ELEMENTARY MECHANISMS IN THE FOLLOWING ORDER  
-BEAM, SWAY OR JOINT OR OTHER MECHANISMS

0024  
0025  
0026  
0027

MXNCM=NTELM  
DO 49 I=1,NTELM  
WRITE(NWRIT,20) I  
20 FORMAT (//, ' THIS IS ELEMENTARY MECHANISM NO. =', I4//)

PUNCH A NEGATIVE, ZERO AND POSITIVE INTEGER FOR THE MECHANISM CODE  
'MECE(I)', CORRESPONDING TO NEGATIVE, NON-HORIZONTAL AND POSITIVE  
HORIZONTALLY DISPLACING MECHANISMS RESPECTIVELY, REQUIRED FOR ELIMI.

0028  
0029  
0030  
0031  
0032  
0033  
0034

READ(NREAD,22) NOP,WEXTN,WEXTD,MECO(I)  
22 FORMAT(I4,2F10.4,I4)  
IF (WEXTD) 24,23,24  
23 WEXTD=1.0  
24 WEXTD(I)=WEXTN/WEXTD  
IF (MECO(I)) 25,27,26  
25 MECO(I)=-1

FIX THE NUMBER OF ACTUAL ELEMENTARY MECHANISMS  
EXCLUDING THE DUPLICATED LATERAL MECHANISMS

0035  
0036  
0037  
0038  
0039  
0040

MXNCM=MXNCM-1  
GO TO 30  
26 MECO(I)=1  
27 IF (I-NBELM) 23,28,30  
28 WRITE(NWRIT,29) NOP,WEXTD(I),MECO(I)  
29 FORMAT(' THIS MECHANISM HAS', I4, ' HINGES  
IF I0.4, (SF\*W#L2) MECHANISM CODE =', I3//)

0041  
0042  
0043  
0044

GO TO 32  
30 WRITE(NWRIT,31) NOP,WEXTD(I),MECO(I)  
31 FORMAT(' THIS MECHANISM HAS', I4, ' HINGES  
IF I0.4, (SF) MECHANISM CODE =', I3//)  
22 DO 49 K=1,NCH  
PUNCH A NEGATIVE SIGN ON THE SECTION NUMBER 'J' WHICH CORRESPONDS  
TO THE COMPOSITE SECTION

```

0045 READ(NREAD,10) J,THETA(I,J),MPT(I,J)
0046 FORMAT(2I4,2F4,4)
0047 JJ=J
0048 IF (J) 24,25,25
0049 J=-J
0050 IF (THTAD) 38,37,38
0051 THTAD=1.
0052 IF (MXHL-J) 39,41,41
0053 WRITE(NWRITE,40) J
0054 FORMAT(1H1//) THE MAXIMUM NUMBER OF HINGE LOCATIONS APPLICABLE IS
1 EXCEEDED BY ND,I2)
0055 GO TO 9999
0056 IF (MXMP-MP) 42,44,44
0057 WRITE(NWRITE,43) MP
0058 FORMAT(1H1//) THE MAXIMUM NUMBER OF PLASTIC MOMENTS APPLICABLE IS
1 EXCEEDED BY MP,I2)
0059 GO TO 9999
0060 THETAC(I,J)=THAN/THTAD
0061 MPTI(J)=JJ
0062 MPT(J)=MP
0063 IF (JJ) 47,45,45
0064 WRITE(NWRITE,46) J,THETAC(I,J),MPT(J)
0065 FORMAT(1) THIS HINGE IS AT SECTION,I4,1)
ANGLE ROTATION =,F3.4)
1 AND PLASTIC MOMENT CAPACITY IS MP,I2)
0066 GO TO 49
0067 WRITE(NWRITE,48) J,THETAC(I,J),MPT(J)
0068 FORMAT(1) THIS HINGE IS AT SECTION,I4,1)
ANGLE ROTATION =,F3.4)
1 AND PLASTIC MOMENT CAPACITY IS MP,I2,1)
(COMP. SECTION)
0069 CONTINUE
0070 WRITE(NWRITE,50)
0071 FORMAT(//10X,1) WHERE THE NEGATIVE MECHANISM CIDE NUMBER SHOWS A PIG
1 HT-TO-LEFT LATERAL MECHANISMS//)
C
C READ IN THE JOINT DATA FOR ELIMINATING MECHANISMS WITH STATISTICALLY
C UNSTABLE JOINTS OR INDEPENDENT JOINT MECHANISMS
C
0072 READ(NREAD,10) (JNT(J),J=1,NTHT)
0073 DO 54 J=1,NTHTL
0074 IF (JNT(J)) 54,53,54
0075 JNT(J)=J
0076 CONTINUE
C

```



0102 READ(MP,13) (MP1(MP),MP2(MP))  
C  
C READ IN THE COEFFICIENTS C1,C2,C3,C4 FOR THE FUNCTION MU/MP  
C

0103 DO 70 K=1,2  
0104 70 READ(NREAD,13) (CC(L,K),L=1,4)  
0105 WRITE(NWRIT,71)  
0106 71 FORMAT(1H1//20X,'APPROXIMATED FUNCTION FOR MU/MP://'  
L10X,'CASE 1 : MU/MP=EXP(C1+C2\*MP+C3\*MP\*\*2+C4\*MP\*\*3)',15X,  
2,'CASE 2 : MU/MP=C1+C2\*LOGMP+C3\*(LOGMP)\*\*2+C4\*(LOGMP)\*\*3'//)

0107 DO 72 L=1,4  
0108 72 WRITE(NWRIT,73) L,CC(L,1),L,CC(L,2)  
0109 73 FORMAT(16X,'C',I2,'=',F10.6,30X,'C',I2,'=',F10.6)

C SET THE MEMBER CODE TO IDENTIFY WHETHER THE MEMBER IS A BEAM  
C  
C OR A COLUMN  
C

0110 DO 75 MP=1,NOMP  
0111 IPMM(MP)=1  
0112 IF (WTF(MP)) 74,75,75  
0113 74 WTF(MP)=-WTF(MP)  
0114 IPMM(MP)=-1

0115 75 CONTINUE  
0116 WRITE(NWRIT,76) FLNG,I,WT,WD,HL  
0117 76 FORMAT(1H1//10X,'SPAN LENGTH IS',F7.2,'FT'  
1//10X,'THICKNESS OF CONCRETE SLAB IS',F5.1,'INCH'  
2//10X,'TOTAL WORKING LOAD WT IS',F6.2,'K/FT',5X,'(DL=',F5.2,  
3,'K/FT LL=',F6.2,'K/FT)',  
4//10X,'COEFF. FOR OBJECTIVE FUN. LOWER LIMITS//')

0118 DO 77 MP=1,NOMP  
0119 77 WRITE(NWRIT,78) MP,WTF(MP),PMM(MP)  
0120 78 FORMAT(10X,'FOR MP',I2,F17.3,F17.3)  
0121 IPPRT=IPRT

C  
C ASSUME THE SAFETY FACTOR SF AS FOLLOWS  
C  
C SF=1.7  
0122 NELM=NTELM  
0123 PU=SF\*WT\*FLNG  
0124

```

0125 LOOP=0
0126 DO 9000 IST=J,2
0127 LOOP=LOOP+1
0128 IF (LOOP-1) 79,79,9000
0129 IPRI=IPRPT
0130 GO TO 81
0131 80 IPRI=0
0132 81 IF (LOOP-5) 84,84,82
0133 82 WRITE(NWRIT,83)
0134 83 FSRMAX(//10X,'CALCULATION IS STOPPED AFTER FIVE CIRCLES OF TRIAL')
0135 GO TO 9000

```

```

C
C
C

```

```

0136 84 DO 96 I=1,NELM
0137 IF (I-NBELM) 85,85,86
0138 85 TM=WEXTO(I)*PU*FLENG
0139 GO TO 87
0140 86 TM=WEXTO(I)*SF
0141 87 WEXT(I)=TM
0142 DO 96 J=1,NTHL
0143 ALP(J)=1.
0144 IF (MPTI(J)) 98,89,89
0145 88 GO TO (89,90),IST
0146 89 THETA(I,J)=THETAO(I,J)
0147 GO TO 96

```

```

C
C
C

```

```

0148 50 K=MPT(J)
0149 X=PM(K,2)
0150 IF (ABS(X)-0.00001) 91,91,92
0151 91 ALP(J)=CO(1,1)+CO(2,1)*X**2+CO(3,1)*X**3
0152 ALP(J)=EXP(ALP(J))
0153 GO TO 93
0154 92 X=ALOG(X)
0155 ALP(J)=CO(1,2)+CO(2,2)*X+CO(3,2)*X**2+CO(4,2)*X**3
0156 93 THETA(I,J)=ALP(J)*THETAO(I,J)
0157 96 CONTINUE
0158 WRITE(NWRIT,119)
0159 GO TO (102,97),IST

```

```

C
C
C

```

```

0161 97 WRITE(NWRITE,1)
0162 98 FORMAT(//10X,'COMPOSITE STEEL FRAME')
0163 110X,'CYCLE NO.',I3)
0164 WRITE(NWRITE,99)
0165 99 FORMAT(//10X,'ASSUMED RATIO OF MU/MP(//
0166 DO 100 J=1,NTHL
0167 WRITE(NWRITE,101) J,ALP(J)
0168 100 FORMAT(10X,'MU/MP(',I3,')=',F7.3)
0169 GO TO 104
0170 102 WRITE(NWRITE,103)
0171 103 FORMAT(//10X,'NONCOMPOSITE STEEL FRAMES')
0172 104 WRITE(NWRITE,105) NELM,NTHL,(MPT(J),J=1,NTHL)
0173 105 FORMAT (/ ' ELEMENTARY HINGE ROTATION MATRIX IS '
1 14,' BY',I4//14(3X,'MP',I2,2X))
0174 DO 106 I=1,NELM
0175 WRITE (NWRITE,68) I,I,(THETAE(I,J),J=1,NTHL),WEXT(I)
C
C GENERATE THE INITIAL INEQUALITIES
C
0176 DO 108 MP=1,NOMP
0177 DO 108 I=1,NELM
0178 DO 108 J=1,NTHL
0179 IF (MPT(J)-MP) 108,107,108
0180 WINT(I,MP)=WINT(I,MP)+ABS(THETAE(I,J))
108 CONTINUE
C
C SET THE INITIAL HINGE ROTATION MATRIX AND MECHANISM CODE DATA
C
0181 DO 109 I=1,NELM
0182 MECE(I)=MECO(I)
0183 WX(I,1)=WEXT(I)
0184 MEC(I,1)=MECE(I)
0185 DO 109 J=1,NTHL
0186 THETA(I,J,1)=THETA(I,J)
C
C DETERMINE THE ERROR CONTROL DATA 'EPS' FOR THE ELIMINATION
C PROCEDURE
C
0187 EPS=999999.
0188 DO 115 I=1,NELM
0189 IF (WEXT(I)) 115,115,110
C

```

```

0190      110 WIMIN=999999.
0191      DO 113 MP=1,NOMP
0192      IF (WINT(I,MP)) 113,113,111
0193      111 IF (WINT(I,MP)-WIMIN) 112,113,113
0194      112 WIMIN=WINT(I,MP)
0195      113 CONTINUE
0196      EPSI=WIMIN/WEXT(I)
0197      IF (EPSI-EPS) 114,115,115
0198      114 EPS=EPSI
0199      115 CONTINUE
0200      EPS=EPS*0.0001
0201      WRITE(NWRIT,116) NFLM,NOMP
0202      116 FORMAT (1H1// ' MATRIX OF COLLAPSE MECHANISM INEQUALITIES',I4,' BY
          1 ',I4//)
0203      DO 117 I=1,NELM
0204      117 WRITE(NWRIT,118) I,(WINT(I,MP),MP=1,NOMP),WEXT(I)
0205      118 FORMAT(' NO.',I4/12(F11.4))
0206      119 FORMAT (1H1)
          C
          C      SET THE INITIAL VALUES FOR THE MECHANISM SORTING MATRIX AND E.T.C.
          C
0207      124 DO 123 I=1,NELM
0208      DO 122 L=1,NELM
0209      122 MELM(I,L,1)=0
0210      123 MELM(I,L,1)=1
0211      IF (MINCM-1) 124,125,125
0212      124 KKK=1
0213      125 MINCM=MINCM+1
0214      IF (MXNHL) 126,126,127
0215      126 MXNHL=NTHL
          C
          C      GENERATE THE MATRIX OF HINGE ROTATIONS FOR COMBINED MECHANISMS
          C
0216      127 ITCP=1
0217      IBOT=NELM
0218      K2=NELM
0219      K2P=NELM
0220      I12P=NELM
0221      NCNT=1
0222      KK=IBOT
0223      I12=0

```

```

0224 C 275 J=1,NTHL
0225 C 275 I=1,NTHL
0226 I11=0
0227 IF (THETA(I,J)) 128,129,128
0228 128 DO 270 IK=ITOP,I80T
0229 I11=I11+1
0230 IF (MECE(I)*MEC(I11,1)) 270,129,129
0231 129 IF (MELM(I11,I,1)) 130,130,270
0232 130 RHTA=THETA(I11,J,1)/THETA(I,J)
0233 IF (RHTA) 131,270,270
C
C COMBINE TWO MECHANISMS WITH OPPOSITE HINGE ROTATION EACH OTHER
C
0234 131 RHTA=ABS(RHTA)
C
C GENERATE THE NEW MECHANISM BY ADDING TWO EQUATIONS AFTER MODIFYING
C THE EQUATION WITH LARGER COEFFICIENTS
C
0235 IT=I
0236 JT=IK
0237 IF (RHTA-1.0) 145,145,132
0238 132 WEXTI=WEXT(I)+WX(I11,1)/RHTA
0239 TI=1.
0240 TJ=1./RHTA
C
C ELIMINATE THIS COMBINED MECHANISM IF THE CORRESPONDING EXTERNAL
C WORK OR TOTAL INTERNAL WORK IS ZERO.
C
0241 IF (WEXTI-EPS) 270,270,133
0242 133 DO 134 JJ=1,NTHL
0243 THETA(I,JJ)=THETA(I11,JJ,1)/RHTA
0244 134 CONTINUE
0245 KK=KK+1
0246 K2=K2+1
0247 I12=I12+1
0248 (IF (K2-MXEL4) 135,4444,4444
0249 135 SUM=0.
C
C DO 140 JJ=1,NTHL
0250 THETA(I12,JJ,2)=THETA(I11,JJ)+THETA(I,JJ)
0251 SUM=SUM+DABS(THETA(I12,JJ,2))
0252 140 CONTINUE
0253 GO TO 161
0254 145 WEXTI=WEXT(I)*RHTI+WX(I11,1)
0255

```

0257

IF (WEXT1-EPS) 27C,270,147

ELIMINATE THE COMBINED MECHANISM IF THE CORRESPONDING EXTERNAL WORK CR TOTAL INTERNAL WORK IS ZERO.

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0279

0280

0281

0282

IF (WEXT1-EPS) 27C,270,147

147 DO 150 JJ=1,NTHL

THE TA1(JJ)=THETA(I,JJ)\*RHTA

150 CONTINUE

KK=KK+1

K2=K2+1

II2=II2+1

IF (K2-MXELM) 155,4444,4444

155 SUM=0.

DO 160 JJ=1,NTHL

THETA(II2,JJ,2)=THETA(JJ)+THETA(II2,JJ,1)

SUM=SUM+DABS(THETA(II2,JJ,2))

160 CONTINUE

161 KK1=KK-1

II21=II2-1

K21=K2-1

WX(II2,2)=WEXT1

K=MECE(I)+MEC(II1,1)

IF (K) 162,163,164

162 MEC(II2,2)=-1

GO TO 165

163 MEC(II2,2)=0

GO TO 165

164 MEC(II2,2)=1

165 IF (SUM-EPS) 255,255,166

ELIMINATE THE DUPLICATED COMBINED MECHANISM

0283

0284

0285

0286

0287

0288

0289

0290

0291

166 DO 170 K=1,II2P

IF (WX(K,1)-EPS) 17C,170,167

167 DO 168 JJ=1,NTHL

TM1=DABS(THETA(II2,JJ,2))/WX(II2,2)

TM2=DABS(THETA(K,JJ,1))/WX(K,1)

IF (ABS(TM1-TM2)-EPS) 168,168,170

168 CONTINUE

GO TO 255

170 CONTINUE

```

0292 IF (II2=1) 174,176,177,178,179,180,181,182
0293 172 (DO 176 K=1,115)
0294 IF (MX(K,2)-(EPS) 176,177,178,179,180,181,182)
0295 173 DO 174 JJ=1,NTHL
0296 TM1=DABS(THETA(II2, JJ, 2))/MX(II2, 2)
0297 TM2=DABS(THETA(K, JJ, 2))/MX(K, 2)
0298 IF (ABS(TM1-TM2)-(EPS) 174,174,176)
0299 174 CONTINUE
0300 GO TO 255
0301 176 CONTINUE
C
C ELIMINATION OF MECHANISMS WITH LESS HINGES THAN THE MINIMUM
C REQUIRED HINGES OR WITH MORE HINGES THAN THE MAXIMUM POSSIBLE HING
C
0302 178 NTHG=0
0303 DO 196 JJ=1,NTHL
0304 IF (THETA(II2, JJ, 2)) 195,196,195
0305 195 NTHG=NTHG+1
0306 196 CONTINUE
0307 IF (NTHG-MXNHL) 198,198,255
0308 198 IF (NTHG-MINHL) 255,201,201
C
C ELIMINATE THE COLLAPSE MECHANISM WITH STATICALLY UNSTABLE JOINT
C ..CHECK BY JOINT LATAS
0309 201 DO 209 JJ=1,NTHL
C
0310 IF (JNT(JJ)-JJ) 209,202,209
0311 202 JNTL=1
0312 IF (DABS(THETA(II2, JJ, 2))-(EPS) 209,209,203
0313 203 JNTL=1
0314 JJI=JJ+1
0315 IF (NTHL-JJI) 209,204,204
0316 204 DO 208 JJJ=JJI,NTHL
0317 IF (JNT(JJ)-JNT(JJJ)) 208,205,208
0318 205 JNTL=JNTL+1
0319 IF (DABS(THETA(II2, JJJ, 2))-(EPS) 209,209,206
0320 206 JNTL=JNTL+1
0321 208 CONTINUE
0322 IF (JNTL-JNTL1) 255,255,209
0323 209 CONTINUE
C
C GENERATE THE COMBINED COLLAPSE INEQUALITIES IN TERMS OF
C PLASTIC MOMENTS

```

0324 DO 215 MP=1, K2P  
 0325 WINT(K2,MP)=0.  
 0326 DO 215 JJ=1, NTHL  
 0327 IF (MPT(JJ)-MP) 215, 215, 215  
 0328 210 WINT(K2,MP)=WINT(K2,MP)+CADS(THETA(II2, JJ, P))  
 0329 215 CONTINUE  
 0330 WEXT(K2)=WX(II2, 2)  
 0331 NO(II2)=K2

C  
 C ELIMINATE THE NEWLY COMBINED MECHANISM PRESENTING BIGGER VALUES OF  
 C NORMALIZED INTERNAL WORK THAN THE PREVIOUS ONES  
 C BY FORWARD ELIMINATION

0332 DO 230 K=1, K2P  
 0333 SUM=0.  
 0334 IF (WEXT(K)-EPS) 230, 230, 220  
 0335 220 DO 227 MP=1, NOMP  
 0336 IF (WINT(K2,MP)-EPS) 224, 221, 221  
 0337 221 DNRWI=WINT(K2,MP)/WEXT(K2)-WINT(K,MP)/WEXT(K)  
 0338 IF (ABS(DNRWI)-EPS) 227, 222, 222  
 0339 222 IF (DNRWI) 230, 227, 223  
 0340 223 SUM=SUM+DNRWI  
 0341 GO TO 227  
 0342 224 IF (WINT(K,MP)-EPS) 227, 227, 230  
 0343 227 CONTINUE  
 0344 IF (SUM) 228, 228, 255  
 0345 228 DO 229 MP=1, NOMP  
 0346 229 WINT(K2,MP)=0.  
 0347 WEXT(K2)=0.  
 0348 K2=K21

0349 NO(II2)=999999  
 0350 GO TO 236  
 0351 230 CONTINUE  
 0352 236 K2P=K2  
 0353 IF (II2-MXMFELM) 238, 238, 444  
 0354 238 DO 240 L=1, NELM  
 0355 240 MELM(II2, L, 2)=MELM(II2, L, 1)  
 0356 MELM(II2, 1, 2)=1  
 0357 IF (IPRT) 270, 270, 245  
 0358 245 WRITE (NWRIT, 246) KK, II, IT, TJ, JT, MEC(II2, 2)  
 0359 246 FORMAT(/' COLLAPSE MECHANISM NO., I4, ' IS COMBINED (' ,  
 1 F8.3, '# NO., I4, ' WITH (' , F8.3, '# NO., I4, ' ) MECHANISM NO

```

0360
0361 250 AX(I12,1)=
0362 00 290 JJ=1,NTH
0363 260 THETA(I12,JJ,2)=0.
0364 00 265 MP=1,NOMP
0365 265 MINT(K2,RP)=0.
0366 WEXT(K2)=0.
0367 KK=KK1
0368 I12=I121
0369 K2=K21
0370 270 CONTINUE
0371 275 CONTINUE
0372 I12P=I12
0373 KKK=KK
0374 IF (KKK.EQ.IBOT) GO TO 3233
0375 IBOT1=IBOT+1
0376 I1TOP=IBOT1
0377 IBOT=KKK
0378 ICOMB=IBOT-I1TOP+1
0379 I12=0
0380 00 295 I=I1TOP,IBOT
0381 I12=I12+1
0382 WX(I12,1)=WX(I12,2)
0383 MEC(I12,1)=MEC(I12,2)
0384 00 293 J=1,NTHL
0385 293 THETA(I12,J,1)=THETA(I12,J,2)
0386 00 294 L=1,NELM
0387 294 MELM(I12,L,1)=MELM(I12,L,2)
0388 295 CONTINUE
0389 NCNT=NCNT+1
0390 IF (ICNT-MIN(CM)) 300,297,300
0391 297 KKK=I12P
0392 300 IF (NCNT-MX(CM)) 2222,3233,3333
0393 3333 CONTINUE
0394 WRITE (NWRT,19)
0395 WRITE (NWRT,205)
0396 305 FORMAT (// ALL POSSIBLE COMBINATIONS INVOLVED IN GENERATING)
0397 GO TO 320
0398 4444 WRITE (NWRT,119)
0399 315 WRITE (NWRT,315)
0400 315 FORMAT (// STOP GENERATING THE MATRIX BECAUSE STATISTICS FOR COMBI
        )
        NED MECHANISM 50 AS NOT TO EXCEED LIMIT OF SIZE OF DIMENSION 1000)

```

```

0402      IJ=NP
C
C      INTRODUCE THE LOWER BOUNDARY CONSTRAINT TO CONSTRAINTS
C
0403      DO 350 MP=1,NCMP
0404      IF (PMMIN(MP)) 350,350,340
0405      340 IJ=IJ+1
0406      DO 345 MPI=1,NCMP
0407      WINT(IJ,MPI)=0.
0408      WINT(IJ,MP)=1.
0409      WEXT(IJ)=PMMIN(MP)
0410      350 CONTINUE
0411      WRITE(NWRITE,365)IJ
0412      365 FORMAT(' THE NUMBER OF POSSIBLE MECHANISMS =',I4)
C
C      NORMALIZE THE EXTERNAL WORK TO UNIT 1
C
0413      DO 390 I=1,IJ
0414      IF (WEXT(I)-EPS) 350,350,387
0415      DO 388 MP=1,NCMP
0416      WINT(I,MP)=WINT(I,MP)/WEXT(I)
0417      388 CONTINUE
0418      WEXT(I)=1.
0419      390 CONTINUE
0420      IF (IPRT) 400,400,391
0421      WRITE(NWRITE,116) IJ,TEMP
0422      DO 392 I=1,IJ
0423      WRITE(NWRITE,116) I,(WINT(I,MP),MP=1,NCMP),WEXT(I)
0424      WRITE(NWRITE,394)
0425      394 FORMAT('/\Ox, THESE ARE POSSIBLE COLLAPSE INEQUALITIES NORMALIZED
1 AFTER BACKWARD ELIMINATION*/
210X, TO BE USED FOR GENERATING FEASIBLE COLLAPSE INEQUALITIES-34)
C
C      GENERATE THE MATRIX OF FINAL COLLAPSE INEQUALITIES
C      FOR BOTH ORIGINAL AND PARTICULAR SOLUTIONS
C      AND PROCEED TO THE FINAL CALCULATIONS
C
0426      400 NCOMP=NCMP
0427      DO 388R LLL=1,NP
0428      DO 405 J=1,NCMP
0429      WTF(J)=WTF(J)
0430      IPM(J)=IPM(J)

```

```

0421 WNT(I,J)=WNT(I,J)
0422 DO 430 I=1,IJ
0423 WNT(I,J)=WNT(I,J)
0424 DO 430 I=1,IJ
0425 WNT(I,J)=WNT(I,J)
0426 IF (NOMP.EQ.1) GO TO 440
0427 IF (LLL.EQ.1) GO TO 470

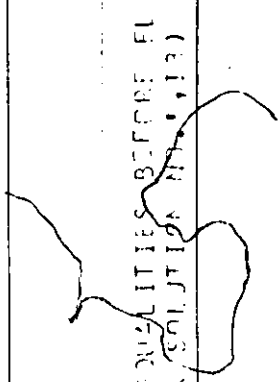
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PARTICULAR SOLUTIONS

```

0428 NOMP)=NOMP
0429 READ (NREAD,10) (NMP(J),J=1,NOMP)
0440 WRITE(NWRITE,415) LLL,(J,NMP(J),J=1,NOMP)
0441 415 FORMAT(' PARTICULAR SOLUTION ',I3//9(' ',MP1,I?,I=VPI,I',2X))
0442 DO 425 J=1,NOMP
0443 NREC(J)=0
0444 DO 440 J=1,NOMP
0445 J1=J+1
0446 IF (J1.GT.NOMP) GO TO 440
0447 DO 440 JJ=J1,NOMP
0448 IF (NMP(J)-NMP(JJ)) 440,430,440
0449 NOMP1=NOMP1-1
0450 WTF(J)=WTF(J)+WTF(JJ)
0451 WTF(JJ)=0
0452 DO 435 I=1,IJ
0453 WNT(I,J)=WNT(I,J)+WNT(I,JJ)
0454 WNT(I,JJ)=0
0455 CONTINUE
0456 NREC(JJ)=JJ
0457 CONTINUE
0458 IF (IPET) 448,448,442
0459 WRITE(NWRITE,110) IJ,NOMP
0460 DO 445 I=1,IJ
0461 WRITE(NWRITE,118) I,(WNT(I,J),J=1,NOMP),WNT(I)
0462 WRITE(NWRITE,447) LLL
0463 FORMAT(/10X, ' THESE ARE NORMALIZED DISCALF INEQUALITIES BEFORE ELIMINATING DUPLICATED PLASTIC MOMENT TERMS FOR SOLUTION N= ',I3)
0464 JJ=0
0465 DO 457 J=1,NOMP
0466 IF (NREC(J)) 457,45(,457
0467 JJ=JJ+1
0468 WTF(JJ)=WTF(J)
0469 IPM(JJ)=IPM(J)

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```

0470      WNT(I,J)=WNT(I,J)
0471      CONTINUE
0472      NOMP=JJ
0473      IF (IPFT) GO TO 476
0474      WRITE (NWRIT,116) IJ,NOMP
0475      DO 460 I=1,IJ
0476      WRITE (NWRIT,119) I,(WNT(I,J),J=1,I*MD),WXT(I)
0477      WRITE (NWRIT,461) LLL
0478      FORMAT(/)OX, 'THESE ARE NORMALIZED POSSIBLE INEQUALITIES SETS FI
0479      LIMINATING DUPLICATED PLASTIC MOMENT TERMS FOR SOLUTION NO. (13)
0480      IF (NOMP-1) 464,464,470
0481      PM(I,IST)=WXT(I)/WNT(I,I)
0482      DO 466 I=2,IJ
0483      TM=WXT(I)/WNT(I,I)
0484      IF (PM(I,IST)-TM) 465,465,466
0485      PM(I,IST)=TM
0486      CONTINUE
0487      OBFN(IST)=WTF(I)*PM(I,IST)
0488      WRITE(NWRIT,467) OBFN(IST)
0489      FORMAT(1H1// 'OBJ. FUNCTION',F20.5//)
0490      WRITE(NWRIT,468)
0491      FORMAT(' VARIABLE VALUE')
0492      WRITE(NWRIT,469) NOMP,PM(I,IST)
0493      FORMAT(15,F25.5)
0494      FL(I)=WTF(I)
0495      GO TO 8888
C
C      ELIMINATE IDENTICAL EQUATIONS
C
0496      IJ1=IJ
0497      DO 500 I=1,IJ
0498      IF (WXT(I)-EPS) 500,500,471
0499      I1=I+1
0500      IF (I1-GT.IJ) GO TO 500
0501      DO 495 I1=I1,IJ
0502      IF (WXT(I1)-EPS) 495,495,472
C
C      ELIMINATE THE COMBINED INEQUALITIES WITH BIGGER VALUES OF
C      NORMALIZED INTERNAL WORK THAN THE OTHERS.
C      BY FORWARD ELIMINATIONS
C
0503      DO 472 DO 476 J=1,NOMP

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```

0505 IF (WNT(I,J)-EPS) .GT. 0.01
0506 WNT(I,J)=WNT(I,J)-WNT(I,I,J)
0507 IF (DNRWI+EPS) .GT. 475,476,477
0508 IF (WNT(II,J)-EPS) .GT. 475,476,477
0509 CONTINUE
0510 WXT(I)=0.
0511 DO 478 J=1,NOMP

```

```

0511 WNT(I,J)=0.
0512 GO TO 500

```

```

C
C BACKWARD ELIMINATION
C

```

```

0513 DO 484 J=1,NOMP
0514 IF (WNT(II,J)-EPS) 492,482,482
0515 482 DNRWI=WNT(II,J)-WNT(I,J)
0516 IF (DNRWI+EPS) 495,484,484
0517 483 IF (WNT(I,J)-EPS) 484,484,495
0518 484 CONTINUE
0519 WXT(II)=C.

```

```

0520 DO 490 J=1,NOMP
0521 WNT(II,J)=C.
0522 495 CONTINUE
0523 500 CONTINUE
0524 IF (IPKT) 502,508,502
0525 502 WRITE(NWRITE,116) I,J,NOMP

```

```

0526 DO 505 I=1,IJ
0527 505 WRITE (NWRITE,118) I,(WNT(I,J),J=1,NOMP),WEXT(I)
0528 WRITE(NWRITE,507) LLI
0529 507 FOPMAT(//IOX,THESI) ARE NORMALIZED FEASIBLE COLLAPSE INEQUALITIES

```

```

I AFTER OVERALL ELIMINATION FOR SOLUTION NO.,I3)

```

```

C
C GENERATE THE FEASIBLE COLLAPSE INEQUALITIES

```

```

C ELIMINATION FOR THE JOINT ELEMENTARY MECHANISMS ARE EXCLUDED ,TOP.
C

```

```

0530 508 II=0
0531 GO 525 I=1,IJ
0532 IF (WXT(II) 510,525,510)

```

```

0533 510 II=II+1
0534 WXT(II)=WXT(I)
0535 DO 515 J=1,NOMP
0536 515 WNT(II,J)=WNT(I,J)
0537 525 CONTINUE
0538 IJ=II

```

```

C543 DO 550 J=1,NMPP
C544   WNT(I,553)=WNT(I,J)
C545   WNT(I,553)=WNT(I,J)
C546   WNT(I,553)=WNT(I,J)
C547   WNT(I,553)=WNT(I,J)
C548   WNT(I,553)=WNT(I,J)
C549   WNT(I,553)=WNT(I,J)
C550   WNT(I,553)=WNT(I,J)
C551   WNT(I,553)=WNT(I,J)
C552   WNT(I,553)=WNT(I,J)
C553   WNT(I,553)=WNT(I,J)
C554   WNT(I,553)=WNT(I,J)
C555   WNT(I,553)=WNT(I,J)
C556   WNT(I,553)=WNT(I,J)
C557   WNT(I,553)=WNT(I,J)
C558   WNT(I,553)=WNT(I,J)
C559   WNT(I,553)=WNT(I,J)
C560   WNT(I,553)=WNT(I,J)
C561   WNT(I,553)=WNT(I,J)
C562   WNT(I,553)=WNT(I,J)
C563   WNT(I,553)=WNT(I,J)
C564   WNT(I,553)=WNT(I,J)
C565   WNT(I,553)=WNT(I,J)
C566   WNT(I,553)=WNT(I,J)
C567   WNT(I,553)=WNT(I,J)
C568   WNT(I,553)=WNT(I,J)
C569   WNT(I,553)=WNT(I,J)
C570   WNT(I,553)=WNT(I,J)
C571   WNT(I,553)=WNT(I,J)
C572   WNT(I,553)=WNT(I,J)
C573   WNT(I,553)=WNT(I,J)
C574   WNT(I,553)=WNT(I,J)
C575   WNT(I,553)=WNT(I,J)
C576   WNT(I,553)=WNT(I,J)
C577   WNT(I,553)=WNT(I,J)
C578   WNT(I,553)=WNT(I,J)
C579   WNT(I,553)=WNT(I,J)
C580   WNT(I,553)=WNT(I,J)
C581   WNT(I,553)=WNT(I,J)
C582   WNT(I,553)=WNT(I,J)
C583   WNT(I,553)=WNT(I,J)
C584   WNT(I,553)=WNT(I,J)
C585   WNT(I,553)=WNT(I,J)
C586   WNT(I,553)=WNT(I,J)
C587   WNT(I,553)=WNT(I,J)
C588   WNT(I,553)=WNT(I,J)
C589   WNT(I,553)=WNT(I,J)
C590   WNT(I,553)=WNT(I,J)
C591   WNT(I,553)=WNT(I,J)
C592   WNT(I,553)=WNT(I,J)
C593   WNT(I,553)=WNT(I,J)
C594   WNT(I,553)=WNT(I,J)
C595   WNT(I,553)=WNT(I,J)
C596   WNT(I,553)=WNT(I,J)
C597   WNT(I,553)=WNT(I,J)
C598   WNT(I,553)=WNT(I,J)
C599   WNT(I,553)=WNT(I,J)
C600   WNT(I,553)=WNT(I,J)

```

```

2 * TO BE USED IN LINEAR PROGRAMMING *)
C STORE THE LENGTH OF EACH MEMBER FOR CALCULATING THE TOTAL WEIGHT
C
C
545 DO 550 J=1,NMPP
550 FL(J)=WTF(J)

```

```

C CHECK IF THERE IS AN EQUATION PARALLEL TO THE OBJECTIVE FUNCTION
C
C
NMPP1=NMPP-1
DO 555 I=1,IJI
DO 552 J=1,NMPP1

```

```

JI=J+1
IF (WNT(I,JI)-EPS) 555,555,551
551 TM1=WNT(I,J)/WNT(I,JI)
552 TM2=WTF(J)/WTF(JI)
IF (ABS(TM1-TM2)-G.OUL)*TM1) 552,552,555
552 CONTINUE
WRITE(NWRITE,553) I
553 FORMAT(/10X,'IT IS NECESSARY TO INVESTIGATE THE POSSIBILITY OF IN
DEFINITE SOLUTIONS',/10X,'SINCE EQUATION NO.',I4,' IS PARALLEL TO
THE OBJECTIVE FUNCTION',/7)
JUN=NMPP+1
GO TO 557

```

```

555 CONTINUE
556 JJJ=1
C IF THERE IS AN EQUATION PARALLEL TO OBJECTIVE FUNCTION,
C IT IS POSSIBLE FOR THE SOLUTION TO BE UNDEFINITE.
C HENCE INVESTIGATE SEVERAL SOLUTIONS AT BOUNDARY POINTS
C BY MULTIPLYING EACH COEFFICIENTS OF THE OBJECTIVE FUNCTION
C BY 1.0001
C
557 DO 7777 JJJ=1,JJN

```

```

C GENERATE THE DATA FOR SUBROUTINE 'LINEAR'
C

```

```

0563      (A, B) = (C, D)
0564      L=0
0565      DO 565 I=1, IJ1
0566      L=L+1
0567      A(L)=-WXT(I)
0568      DO 559 J=1, (NCOMP,
0569      L=L+1
0570      A(L)=WXT(I, J)
0571      559 CONTINUE
0572      L=L+1
0573      A(L)=0.
0574      560 CONTINUE
0575      DO 570 J=1, NCOMP
0576      L=L+1
0577      A(L)=0.
0578      LJ=L+J
0579      DO 565 JJ=1, NCOMP
0580      L=L+1
0581      565 A(L)=0.
0582      A(LJ)=1.
0583      L=L+1
0584      570 A(L)=0.
0585      L=L+1
0586      A(L)=0.
0587      DO 585 J=1, NCOMP
0588      L=L+1
0589      IF (JJJ-1) 586, 580, 575
0590      575 IF (J-JJJ+1) 580, 578, 580
0591      578 A(L)=WTF(J)*1.0001
0592      GO TO 585
0593      580 A(L)=WTF(J)
0594      585 CONTINUE
0595      L=L+1
0596      A(L)=0.
0597      III=NCOMP+2
0598      JJ=IJ1+NCOMP+1
0599      CALL LINEAR (A, III, JJ, JJJ)
0600      587 IF (LLL-1) 590, 590, 7777
0601      590 IF (JJJ-1) 595, 595, 7777
0602      595 I=III*(JJ-1)+1
0603      ORFN(IST)=A(I)
0604      LOOP1=0

```

```

0607 K=I*I*(J-J-1)*MP*(J-2)+
0608 GO TO (830,835),IST
0609 IF (A(K)-0.00001) (12,615,616)
0610 A(K)=0.00001
0611 TM=A(K)-PM(J,IST)
0612 TM=TM/A(K)
0613 IF (ABS(TM)-0.05) 630,620,620
0614 LOOP1=1
0615 PM(J,IST)=A(K)
0616 640 CONTINUE
0617 777 CONTINUE
0618 888 NOMP=NOMP
0619 GO TO (650,670),IST
0620 DO 660 J=1,NOMP
0621 PM(J,2)=PM(J,1)
0622 GO TO 9000
0623 IF (LOOP1) 9000,9000,1,1,1
0624 9000 CONTINUE
0625 WRITE(NWRIT,820)
0626 820 FORMAT(1H1//16X,'MP (NOMP-COMP.)',16X,'MP (COMP.)',/
16X,'(K.FT) (WT2.L)',9X,'(K.FT) (WT2.L)')
0627 DO 850 I=1,NOMP
0628 IF (PM(I,1)-0.00001) 825,827,827
0629 PM(I,1)=0.00001
0630 TM=PM(I,2)/PM(I,1)
0631 WTL2=(WT*SF)*FLENG**2
0632 TM1=PM(I,1)/WTL2
0633 TM2=PM(I,2)/WTL2
0634 IF (IPMM(I)) 840,830,830
0635 WRITE(NWRIT,835) I,PM(I,1),TM1,PM(I,2),TM2,TM
0636 835 FORMAT(14,2(F15.3,F10.5),F5.5,'...REAR MEMBER')
0637 GO TO 850
0638 840 WRITE(NWRIT,845) I,PM(I,1),TM1,PM(I,2),TM2,TM
0639 845 FORMAT(14,2(F15.3,F10.5),F5.5,'...COLUMN MEMBER')
0640 850 CONTINUE
0641 9999 STOP
0642 END

```

```

0001      SUBROUTINE LINPROG (I,II,III,IIII,JJ,JJJ)
0002      C
0003      C SIMPLEX METHOD FOR SOLVING LINEAR PROGRAMS
0004      C III=TOTAL NUMBER OF ROWS IN THE SIMPLEX TABLEAU
0005      C II=IIII-I
0006      C JJ=TOTAL NUMBER OF COLUMNS OF THE GIVEN ARGUMENTED TABLEAU
0007      C JJJ=CONTROL DATA FOR THE INDEFINITE PROBLEM
0008      C
0009      C NEXT STATEMENT FOR INITIALIZATION
0010      C
0011      C DOUBLE PRECISION A(III,JJ),W(50),X,XMIV,OBJF(50)
0012      C DIMENSION L(50)
0013      C NREAD=1
0014      C NWRITE=3
0015      C II=IIII-I
0016      C DO 3 I=2,II
0017      C   K=I-1
0018      C   3 OBJF(K)=A(I,JJ)
0019      C   5 FORMAT(10X,9F14.5)
0020      C   DO 10 I=1,III
0021      C     W(I)=0.
0022      C     10 L(I)=0
0023      C     DO 15 I=2,II
0024      C       15 L(I)=JJ-III+I
0025      C       I=II-1
0026      C       IF (JJJ-I) 18,18,3)
0027      C         18 WRITE(NWRITE,20) I,III,JJ
0028      C         20 FORMAT(1H1//1X,'NUMBER OF CONSTRAINTS IS ',I3//1X,
0029      C           1,'NUMBER OF ROWS IN MATRIX A(III,JJ) IS ',I3//1X,
0030      C           2,'NUMBER OF COLUMNS IN MATRIX A(III,JJ) IS ',I3//1)
0031      C         31 CONTINUE
0032      C         32 FORMAT(1J1)
0033      C         . KKK=0
0034      C
0035      C THE NEXT STATEMENTS ARE TO LOOK FOR THE ROW AT WHICH THERE IS
0036      C NO SLACK VARIABLE (NOT INCLUDING THE FIRST ROW).
0037      C
0038      C   35 I=1
0039      C   40 I=I+1
0040      C   IF (I-III) 45,65,65
0041      C   45 IF (L(I)) 40,50,40
0042      C
0043      C CALCULATE
0044      C NEW LAST ROW=LAST ROW- THE ROW WITHOUT SLACK VARIABLE

```

0027 DO J=1, JJ  
 0028 IF (A(I, J)) 95, 60, 95  
 0029 A(III, J)=A(III, J)-A(I, J)  
 0030 CONTINUE  
 0031 GO TO 40

C  
 C NEXT STATEMENTS FOR SEARCHING FOR THE COLUMN AT WHICH THE MOST  
 C NEGATIVE ENTRY APPEARS EITHER IN THE FIRST (OBJECTIVE FUNCTION)  
 C OR LAST (FORM P) ROW.  
 C

0032 65 K=III  
 0033 70 J=0  
 0034 W(K)=0.  
 0035 L(K)=0  
 0036 75 J=J+1  
 0037 IF (J-JJ) 80, 95, 95  
 0038 80 IF (A(K, J)) 85, 75, 75  
 0039 85 IF (W(K)-A(K, J)) 75, 75, 90  
 0040 90 W(K)=A(K, J)  
 0041 L(K)=J  
 0042 GO TO 75

C  
 C TEST FOR L(K). IF L(K) IS EQUAL TO ZERO, THAT IS, ALL THE  
 C ENTRIES EXCEPT THE EXTREME RIGHT ONE EITHER IN THE FIRST OR LAST  
 C ROW ARE POSITIVE, GO TO ST. 62 FOR FURTHER EXAMINATION.  
 C

0043 95 IF (L(K)) 100, 195, 100  
 C  
 C FIND OUT THE PIVOT COLUMN  
 C

0044 100 KJ=L(K)  
 C  
 C TEST EVERY ENTRY IN THE PIVOT COLUMN TO SEE IF IT IS POSITIVE  
 C OR NOT. IF IT IS, GO TO ST. 121 TO COMPUTE THE RATIO DEFINED IN  
 C THE LAST SECTION  
 C

0045 DO 105 I=2, II  
 0046 IF (A(I, KJ)) 105, 105, 115  
 0047 105 CONTINUE  
 C

C  
 C IF ALL THE ENTRIES IN THE PIVOT COLUMN ARE ZERO (OR NEGATIVE  
 C NUMBERS, 'UNBOUNDED' IS GOING TO BE TYPED.  
 C

```

0048 WRITE (A,11,I,1)
0049 FORMAT(' FEASIBLE')
0050 GO TO 240
C
C THE FOLLOWING STATEMENTS ARE FOR COMPUTING THE RATIO DEFINED
C AND FOR DETERMINING THE LOCATION OF THE PIVOT.
C
0051 I=1
0052 JK=0
0053
0054 I=I+1
0055 IF (I-11) 125,125,145
0056 IF (A(I,KJ)) 120,120,130
0057 X=A(I,JJ)/A(I,KJ)
0058 IF (JK) 135,140,135
0059 IF (X-XMIN) 140,120,120
0060 XMIN=X
0061 JK=I
0062 GO TO 120
C
C THE NEXT STATEMENT INDICATES THE PIVOT ELEMENT BEFORE NORMALIZATION
C
0062 X=A(JK,KJ)
0063 L(JK)=KJ
C
C THE NEXT STATEMENT FOR CALCULATING THE NEW ROWS ABOVE THE PIVOT ROW
C
0064 DO 150 I=1,III
0065 W(I)=A(I,KJ)
0066 IJ=JK-1
0067 DO 165 I=1,IJ
0068 DO 165 J=1,JJ
0069 IF (A(JK,J)) 155,165,165
0070 IF (W(I)) 160,165,160
0071 A(I,J)=A(I,J)-W(I)*A(JK,J)/X
0072 CONTINUE
C
C NEXT STATEMENTS FOR CALCULATING THE NEW ROWS BELOW THE PIVOT ROW
C
0073 IJ=JK+1
0074 DO 180 I=IJ,III
0075 DO 180 J=1,JJ
0076 IF (A(JK,J)) 170,160,170

```

173 IF (A(I,J)) THEN A(I,J)=A(I,J)/C  
175 A(I,J)=A(I,J)+C\*(C(I,J)/C)  
180 CONTINUE

C NEXT STATEMENTS FOR NORMALIZATION

0080 DO 185 J=1,JJ  
0081 185 A(JK,J)=A(JK,J)/X  
0082 KKK=KKK+1  
0083 WRITE(NWRITE,190) KKK,(K, JJ),L(JK)  
0084 190 FORMAT(2X,I4,6X,F15.2,10X,I4)  
0085 GO TO 70

C NEXT STATEMENT FOR TESTING TO SEE IF IT IS THE FIRST ROW ON  
C WHICH ALL THE ENTRIES ARE POSITIVE EXCEPT THE EXTREME RIGHT ONE  
C IF IT IS, THAT MEANS, NO FURTHER IMPROVEMENT ON THE SOLUTION  
C CAN BE MADE, GO TO ST.70 AND THE ANSWER WILL BE TYPED OUT.

0086 195 IF (K-1) 240,240,200  
0087 200 IJ=JJ-1

C TEST TO SEE WHETHER ALL THE ELEMENTS ON THE LAST ROW (NOT  
C INCLUDING THE EXTREME RIGHT ONE) ARE CLOSE TO ZERO.  
C IT IS DEFINED IN THE NEXT STATEMENTS THAT THE PROBLEM IS  
C INFEASIBLE IF ONE (OR MORE) OF THEM IS LARGER THAN 0.0001.

0088 DO 205 J=1,IJ  
0089 IF (A(K,J)-0.00001) 205,205,225  
0090 205 CONTINUE  
0091 WRITE(NWRITE,210)  
0092 210 FORMAT(//' UNBOUNDED')  
0093 WRITE(NWRITE,215)  
0094 215 FORMAT(' ITERATION ' CRJ, FUNCTION NEW BASIC V/R.')

C IF, AFTER ITERATIONS, ALL THE ELEMENTS IN THE LAST ROW HAVE  
C BECOME POSITIVE BUT NEAR ZERO, DEFINE ALL OF THEM TO BE ZERO.

0095 DO 220 J=1,JJ  
0096 220 A(IJJ,J)=0.

C IN CASE OF NONARTIFICIAL PROBLEM, DEFINE K=1, AND GO TO ST.44  
C FOR SEARCHING FOR THE PIVOT COLUMN.

0097  
CC58  
0099

NEJ  
KKKE  
50 J 70

C TYPE OUT THE SOLUTION  
C  
C

```
0100 225 CONTINUE
0101 WRITE(NWRIT,230)
0102 230 FORMAT(' INFEASIBLE')
0103 240 IF (JJJ-1) 242,242,247
0104 242 WRITE(NWRIT,245) A(1, JJJ)
0105 245 FORMAT(' CBJ. FUNCTION', F20.5)
0106 GO TO 253
0107 247 WRITE(NWRIT,248)
0108 248 FORMAT(' GOVERNING INEQUALITIES')
0109 K=JJJ-III+1
0110 DO 251 J=1, K
0111 IF (A(1, J)) 251, 249, 251
0112 249 WRITE(NWRIT, 250) J
0113 250 FORMAT(2X, 'EQ. NO.', I4)
0114 251 CONTINUE
0115 K=JJJ-1
0116 WRITE(NWRIT, 252) K, A(1, JJJ)
0117 252 FORMAT(' PART. CBJ. FUNCTION (NO.', I3, '), F20.5')
0118 253 WRITE(NWRIT, 254)
0119 254 FORMAT(' VARIABLE VALUE INPUT COST')
0120 NJMP=III-2
0121 DO 255 I=1, NJMP
0122 K=JJJ-NOMP+I-1
0123 255 WRITE(NWRIT, 260) I, A(1, K), DBF(I)
0124 260 FORMAT(I4, 2F20.5)
0125 9999 RETURN
0126 END
```

F88-LEVEL LINKAGE-EDITOR OPTIONS SPECIFIED M/P,LIST,LET  
 DEFAULT OPTION(S) USED - SIZE=(102400,17289)

MODULE MAP

CONTROL SECTION		ENTRY		NAME	LOCATION	NAME	LOCATION	NAME	LOCATION
NAME	ORIGIN	LENGTH	NAME	LOCATION	NAME	LOCATION	NAME	LOCATION	NAME
MAIN	00	62022							
LINEAR	62025	12FC							
INCSLOG	* 53F29	196							
INCEXP	* 840F0	192	ALOG10	63F28	ALOG	63F4D			
INCECCMH	* 64278	F4J	FXP	64050			INSTRCH	64234	
INCCDMPZ	* 651C0	65D	INCPM#	64278					
INCFVTH	* 65820	119D	SFOUASD	45538					
INCFVTH	* 669C0	512	ADCON#	65820			FCVLOUTP	659CA	FCVZOUTP 65AAA
INCFVTH	* 669C0	512	FCVLOUTP	65858			FCVLOUTP	6595A	INTSSWCH 6595B
INCFVTH	* 669C0	512	FRITH#	669C0			ADJUSWCH	6692C	
INCFVTH	* 669C0	512	FLCS#	66E08			PTOCSEB#	66E0E	
INCFVTH	* 68250	58C	EXRMGN	68250			INCEPSE	68268	
INCUOPT	* 68910	30C							
INCEPCH	* 68910	28E	INCTPCH	68B10			ERRTR	68P18	
INCUATBL	* 68DA0	146							

ENTRY ADDRESS 00  
 TOTAL LENGTH 68EE8

\*\*\*MAIN DOES NOT EXIST BUT HAS BEEN ADDED TO DATA SET

A.4-4 Output Example for the Linear Optimization of  
the Composite Frame in Sec. 5-2

TOTAL WORKING TIME (MIN) = 1000

COEFF. FOR OBJECTIVE FUN. TIME LIMITS

FOR MP 1	60.000	0.0
FOR MP 2	60.000	0.0
FOR MP 3	60.000	0.0
FOR MP 4	72.000	104.100
FOR MP 5	30.000	174.700

APPROXIMATED FUNCTION FOR MU/MP

CASE 1 : MU/MP = EXP (C1 + C2 \* MP + C3 \* MP \*\* 2 + C4 \* MP \*\* 3)

- C 1 = 0.835428
- C 2 = -0.000697
- C 3 = 0.000000
- C 4 = 0.0

CASE 2 : MU/MP = C1 + C2 \* L (MP) + C3 \* (L (MP)) \*\* 2 + C4 \* (L (MP)) \*\* 3

- C 1 = 7.532917
- C 2 = -1.967446
- C 3 = 0.233332
- C 4 = -0.009424

JOINT NO. 601

- 1- 1
- 2- 2
- 3- 2
- 4- 4
- 5- 4
- 6- 6
- 7- 7
- 8- 8
- 9- 8
- 10- 10
- 11- 10
- 12- 12
- 13- 13
- 14- 14
- 15- 14
- 16- 16
- 17- 16
- 18- 18
- 19- 9
- 20- 20
- 21- 12
- 22- 12
- 23- 23
- 24- 18
- 25- 4
- 26- 26
- 27- 10
- 28- 10
- 29- 29
- 30- 10
- 31- 2
- 32- 32
- 33- 3
- 34- 8
- 35- 35
- 36- 14

WHERE THE NEGATIVE JOINT CORP. NUMBERS SHOWN A SINGLE JOINT.

THIS IS ELEMENTARY MECHANISM NO. = 1

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.2500(SF\*W\*L2) MECHANISM CODE = 0  
THIS HINGE IS AT SECTION 19 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 1  
THIS HINGE IS AT SECTION 20 ANGLE ROTATION = 2.000000 AND PLASTIC MOMENT CAPACITY IS MP 1  
THIS HINGE IS AT SECTION 21 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 1

THIS IS ELEMENTARY MECHANISM NO. = 2

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.2500(SF\*W\*L2) MECHANISM CODE = 0  
THIS HINGE IS AT SECTION 6 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 20 ANGLE ROTATION = 2.000000 AND PLASTIC MOMENT CAPACITY IS MP 1  
THIS HINGE IS AT SECTION 21 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 1

THIS IS ELEMENTARY MECHANISM NO. = 3

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.2500(SF\*W\*L2) MECHANISM CODE = 0  
THIS HINGE IS AT SECTION 22 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 1  
THIS HINGE IS AT SECTION 23 ANGLE ROTATION = 2.000000 AND PLASTIC MOMENT CAPACITY IS MP 1  
THIS HINGE IS AT SECTION 24 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 1

THIS IS ELEMENTARY MECHANISM NO. = 4

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.2500(SF\*W\*L2) MECHANISM CODE = 0  
THIS HINGE IS AT SECTION 22 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 1  
THIS HINGE IS AT SECTION 23 ANGLE ROTATION = 2.000000 AND PLASTIC MOMENT CAPACITY IS MP 1  
THIS HINGE IS AT SECTION 24 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 1

THIS IS ELEMENTARY MECHANISM NO. = 5

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.2500(SF\*W\*L?) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 25 ANGLE ROTATION = -1.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS HINGE IS AT SECTION 26 ANGLE ROTATION = 2.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS HINGE IS AT SECTION 27 ANGLE ROTATION = -1.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 6

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.2500(SF\*W\*L?) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 28 ANGLE ROTATION = -1.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS HINGE IS AT SECTION 29 ANGLE ROTATION = 2.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS HINGE IS AT SECTION 30 ANGLE ROTATION = -1.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 7

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.2500(SF\*W\*L?) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 31 ANGLE ROTATION = -1.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS HINGE IS AT SECTION 32 ANGLE ROTATION = 2.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS HINGE IS AT SECTION 33 ANGLE ROTATION = -1.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 8

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.2500(SF\*W\*L?) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 34 ANGLE ROTATION = -1.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS HINGE IS AT SECTION 35 ANGLE ROTATION = 2.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS HINGE IS AT SECTION 36 ANGLE ROTATION = -1.00000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 9  
THIS MECHANISM HAS 4 HINGES EXTERNAL WORK = 0.2500(SF\*W\*LZ) MECHANISM CODE = 1  
THIS HINGE IS AT SECTION 4 ANGLE ROTATION = -1.0000AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 5 ANGLE ROTATION = 1.0000AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 26 ANGLE ROTATION = 2.0000AND PLASTIC MOMENT CAPACITY IS MP 2  
THIS HINGE IS AT SECTION 27 ANGLE ROTATION = -1.0000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 10  
THIS MECHANISM HAS 4 HINGES EXTERNAL WORK = 0.2500(SF\*W\*LZ) MECHANISM CODE = 1  
THIS HINGE IS AT SECTION 28 ANGLE ROTATION = -1.0000AND PLASTIC MOMENT CAPACITY IS MP 2  
THIS HINGE IS AT SECTION 29 ANGLE ROTATION = 2.0000AND PLASTIC MOMENT CAPACITY IS MP 2  
THIS HINGE IS AT SECTION 16 ANGLE ROTATION = -1.0000AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 17 ANGLE ROTATION = 1.0000AND PLASTIC MOMENT CAPACITY IS MP 4

THIS IS ELEMENTARY MECHANISM NO. = 11  
THIS MECHANISM HAS 4 HINGES EXTERNAL WORK = 0.2500(SF\*W\*LZ) MECHANISM CODE = 1  
THIS HINGE IS AT SECTION 2 ANGLE ROTATION = -1.0000AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 3 ANGLE ROTATION = 1.0000AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 32 ANGLE ROTATION = 2.0000AND PLASTIC MOMENT CAPACITY IS MP 2  
THIS HINGE IS AT SECTION 33 ANGLE ROTATION = -1.0000AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 12  
THIS MECHANISM HAS 4 HINGES EXTERNAL WORK = 0.2500(SF\*W\*LZ) MECHANISM CODE = 1  
THIS HINGE IS AT SECTION 34 ANGLE ROTATION = -1.0000AND PLASTIC MOMENT CAPACITY IS MP 2  
THIS HINGE IS AT SECTION 35 ANGLE ROTATION = 2.0000AND PLASTIC MOMENT CAPACITY IS MP 2  
THIS HINGE IS AT SECTION 14 ANGLE ROTATION = -1.0000AND PLASTIC MOMENT CAPACITY IS MP 2  
THIS HINGE IS AT SECTION 15 ANGLE ROTATION = 1.0000AND PLASTIC MOMENT CAPACITY IS MP 4

THIS IS ELEMENTARY MECHANISM NO. = 13

THIS MECHANISM HAS 6 HINGES EXTERNAL WORK = 54.0000(SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 5	ANGLE ROTATION =	-1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 6	ANGLE ROTATION =	1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 11	ANGLE ROTATION =	-1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 5
THIS HINGE IS AT SECTION 12	ANGLE ROTATION =	1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 5
THIS HINGE IS AT SECTION 17	ANGLE ROTATION =	1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 18	ANGLE ROTATION =	-1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4

THIS IS ELEMENTARY MECHANISM NO. = 14

THIS MECHANISM HAS 6 HINGES EXTERNAL WORK = 54.0000(SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 5	ANGLE ROTATION =	-1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 11	ANGLE ROTATION =	-1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 5
THIS HINGE IS AT SECTION 12	ANGLE ROTATION =	1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 5
THIS HINGE IS AT SECTION 17	ANGLE ROTATION =	1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 19	ANGLE ROTATION =	1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 1
THIS HINGE IS AT SECTION 24	ANGLE ROTATION =	-1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 1

THIS IS ELEMENTARY MECHANISM NO. = 15

THIS MECHANISM HAS 9 HINGES EXTERNAL WORK = 172.0000(SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 3	ANGLE ROTATION =	-1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 4	ANGLE ROTATION =	1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 9	ANGLE ROTATION =	-1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 5
THIS HINGE IS AT SECTION 10	ANGLE ROTATION =	1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 5
THIS HINGE IS AT SECTION 15	ANGLE ROTATION =	1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 16	ANGLE ROTATION =	-1.0000	AND	PLASTIC MOMENT CAPACITY IS MP 4

THIS IS ELEMENTARY MECHANISM NO. = 16

THIS MECHANISM HAS 6 HINGES EXTERNAL WORK = 270.0000 (SF) MECHANISM CODE =

THIS HINGE IS AT SECTION 1	ANGLE ROTATION =	-1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 4
THIS HINGE IS AT SECTION 2	ANGLE ROTATION =	1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 4
THIS HINGE IS AT SECTION 7	ANGLE ROTATION =	-1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 5
THIS HINGE IS AT SECTION 8	ANGLE ROTATION =	1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 5
THIS HINGE IS AT SECTION 13	ANGLE ROTATION =	1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 4
THIS HINGE IS AT SECTION 14	ANGLE ROTATION =	-1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 4

THIS IS ELEMENTARY MECHANISM NO. = 17

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 12	ANGLE ROTATION =	-1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 5
THIS HINGE IS AT SECTION 21	ANGLE ROTATION =	-1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 1
THIS HINGE IS AT SECTION 22	ANGLE ROTATION =	1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 1

THIS IS ELEMENTARY MECHANISM NO. = 18

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE =

THIS HINGE IS AT SECTION 4	ANGLE ROTATION =	-1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 4
THIS HINGE IS AT SECTION 5	ANGLE ROTATION =	1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 7
THIS HINGE IS AT SECTION 25	ANGLE ROTATION =	1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 2

THIS IS ELEMENTARY MECHANISM NO. = 19

THIS MECHANISM HAS 4 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 10	ANGLE ROTATION =	-1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 5
THIS HINGE IS AT SECTION 11	ANGLE ROTATION =	1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 5
THIS HINGE IS AT SECTION 27	ANGLE ROTATION =	-1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 2
THIS HINGE IS AT SECTION 28	ANGLE ROTATION =	1.0000	AND PLASTIC MOMENT CAPACITY IS	MP 2

THIS IS ELEMENTARY MECHANISM NO. = 20

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 16 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 17 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 30 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 21

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 2 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 3 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 31 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 3

THIS IS ELEMENTARY MECHANISM NO. = 22

THIS MECHANISM HAS 4 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 8 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 9 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 5  
THIS HINGE IS AT SECTION 33 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 3  
THIS HINGE IS AT SECTION 34 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 3

THIS IS ELEMENTARY MECHANISM NO. = 23

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 14 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 15 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 36 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 3

THIS IS ELEMENTARY MECHANISM NO. = 25

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE =

THIS HINGE IS AT SECTION 12 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 5  
THIS HINGE IS AT SECTION 21 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 1  
THIS HINGE IS AT SECTION 22 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 1

THIS IS ELEMENTARY MECHANISM NO. = 25

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE =

THIS HINGE IS AT SECTION 4 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 5 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 25 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 26

THIS MECHANISM HAS 4 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 10 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 5  
THIS HINGE IS AT SECTION 11 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 5  
THIS HINGE IS AT SECTION 27 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 2  
THIS HINGE IS AT SECTION 28 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 27

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 16 ANGLE ROTATION = -1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 17 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 4  
THIS HINGE IS AT SECTION 30 ANGLE ROTATION = 1.000000 AND PLASTIC MOMENT CAPACITY IS MP 2

THIS IS ELEMENTARY MECHANISM NO. = 28

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 2	ANGLE ROTATION = 1.000000	AND PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 3	ANGLE ROTATION = -1.000000	AND PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 31	ANGLE ROTATION = -1.000000	AND PLASTIC MOMENT CAPACITY IS MP 3

THIS IS ELEMENTARY MECHANISM NO. = 29

THIS MECHANISM HAS 4 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 8	ANGLE ROTATION = 1.000000	AND PLASTIC MOMENT CAPACITY IS MP 5
THIS HINGE IS AT SECTION 9	ANGLE ROTATION = -1.000000	AND PLASTIC MOMENT CAPACITY IS MP 5
THIS HINGE IS AT SECTION 33	ANGLE ROTATION = 1.000000	AND PLASTIC MOMENT CAPACITY IS MP 3
THIS HINGE IS AT SECTION 34	ANGLE ROTATION = -1.000000	AND PLASTIC MOMENT CAPACITY IS MP 3

THIS IS ELEMENTARY MECHANISM NO. = 30

THIS MECHANISM HAS 3 HINGES EXTERNAL WORK = 0.0 (SF) MECHANISM CODE = 0

THIS HINGE IS AT SECTION 14	ANGLE ROTATION = -1.000000	AND PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 15	ANGLE ROTATION = 1.000000	AND PLASTIC MOMENT CAPACITY IS MP 4
THIS HINGE IS AT SECTION 36	ANGLE ROTATION = 1.000000	AND PLASTIC MOMENT CAPACITY IS MP 4

WHERE THE NEGATIVE MECHANISM CODE NUMBER SHOWS A RIGHT-TO-LEFT LATERAL MECHANISMS

COMPOSITE STEEL BEAMS

NUMBER OF CONSTRAINTS IS 5

NUMBER OF ROWS IN MATRIX A(III,JJ) IS 7

NUMBER OF COLUMNS IN MATRIX A(III,JJ) IS 282

UNBOUNDED ITERATION	OBJ. FUNCTION	NEW BASIC VAR.
1	17212.49	1
2	34424.95	3
3	51637.48	4
4	59899.48	7
5	59899.48	276
6	65601.89	275
7	69257.36	2
8	73172.89	245
9	74391.34	272
10	74391.34	265
11	74391.35	266

OBJ. FUNCTION 74391.35370

VARIABLE	VALUE	INPLT COST
1	331.16195	60.00000
2	308.02872	60.00000
3	308.02872	60.00000
4	154.01371	72.00000
5	179.70009	36.00000

COMPOSITE STATE FRAMES

CYCLE NO. 2

ASSUMED RATIO OF MU/MP

MU/MP( 1)= 1.000  
 MU/MP( 2)= 1.000  
 MU/MP( 3)= 1.000  
 MU/MP( 4)= 1.000  
 MU/MP( 5)= 1.000  
 MU/MP( 6)= 1.000  
 MU/MP( 7)= 1.000  
 MU/MP( 8)= 1.000  
 MU/MP( 9)= 1.000  
 MU/MP( 10)= 1.000  
 MU/MP( 11)= 1.000  
 MU/MP( 12)= 1.000  
 MU/MP( 13)= 1.000  
 MU/MP( 14)= 1.000  
 MU/MP( 15)= 1.000  
 MU/MP( 16)= 1.000  
 MU/MP( 17)= 1.000  
 MU/MP( 18)= 1.000  
 MU/MP( 19)= 1.000  
 MU/MP( 20)= 1.911  
 MU/MP( 21)= 1.000  
 MU/MP( 22)= 1.000  
 MU/MP( 23)= 1.911  
 MU/MP( 24)= 1.000  
 MU/MP( 25)= 1.000  
 MU/MP( 26)= 1.934  
 MU/MP( 27)= 1.000  
 MU/MP( 28)= 1.000  
 MU/MP( 29)= 1.934  
 MU/MP( 30)= 1.000  
 MU/MP( 31)= 1.000  
 MU/MP( 32)= 1.934  
 MU/MP( 33)= 1.000  
 MU/MP( 34)= 1.000  
 MU/MP( 35)= 1.934  
 MU/MP( 36)= 1.000

NUMBER OF CONSTRAINTS IS 5

NUMBER OF ROWS IN MATRIX A(III, JJ) IS 7

NUMBER OF COLUMNS IN MATRIX A(III, JJ) IS 387

UNBOUNDED ITERATION

NEW BASIC VAR.

1 11825.82 1  
 2 23560.59 3  
 3 35295.36 4  
 4 43557.36 7  
 5 43557.36 381  
 6 49259.77 380  
 7 50416.93 2  
 8 52657.97 378  
 9 52657.97 370  
 10 52725.65 371  
 11 52801.10 347

OBJ. FUNCTION 52801.10033

VARIABLE	VALUE	INPUT COST
1	215.73095	40.00000
2	214.00759	60.00000
3	214.00759	60.00000
4	107.00305	72.00000
5	179.70000	36.00000

COMPOSITE STEEL FRAMES

CYCLE NO. 3

ASSUMED RATIO OF MU/MP

- MU/MP( 1) = 1.000
- MU/MP( 2) = 1.000
- MU/MP( 3) = 1.000
- MU/MP( 4) = 1.000
- MU/MP( 5) = 1.000
- MU/MP( 6) = 1.000
- MU/MP( 7) = 1.000
- MU/MP( 8) = 1.000
- MU/MP( 9) = 1.000
- MU/MP( 10) = 1.000
- MU/MP( 11) = 1.000
- MU/MP( 12) = 1.000
- MU/MP( 13) = 1.000
- MU/MP( 14) = 1.000
- MU/MP( 15) = 1.000
- MU/MP( 16) = 1.000
- MU/MP( 17) = 1.000
- MU/MP( 18) = 1.000
- MU/MP( 19) = 1.000
- MU/MP( 20) = 2.054
- MU/MP( 21) = 1.000
- MU/MP( 22) = 1.000
- MU/MP( 23) = 2.054
- MU/MP( 24) = 1.000
- MU/MP( 25) = 1.000
- MU/MP( 26) = 2.057
- MU/MP( 27) = 1.000
- MU/MP( 28) = 1.000
- MU/MP( 29) = 2.057
- MU/MP( 30) = 1.000
- MU/MP( 31) = 1.000
- MU/MP( 32) = 2.057
- MU/MP( 33) = 1.000
- MU/MP( 34) = 1.000
- MU/MP( 35) = 2.057
- MU/MP( 36) = 1.000

NUMBER OF CONSTRAINTS IS 5

NUMBER OF ROWS IN MATRIX A(III, JJ) IS 7

NUMBER OF COLUMNS IN MATRIX A(III, JJ) IS 413

UNSCALED ITERATION OBJ. FUNCTION NEW BASIC VAR.

ITERATION	OBJ. FUNCTION	NEW BASIC VAR.
1	11270.79	1
2	22530.51	3
3	33790.23	4
4	42052.22	7
5	42052.22	407
6	47754.63	406
7	50726.70	404
8	51888.52	2
9	50868.52	396

OBJ. FUNCTION 50868.51832

VARIABLE	VALUE	INPUT COST
1	204.23942	60.00000
2	208.20006	60.00000
3	202.96241	50.00000
4	104.10003	72.00000
5	179.70009	26.00000

COMPOSITE SHEET HEADS

CYCLE NO. 4

ASSUMED RATIO OF MU/MP

MU/MP( 1)= 1.000  
 MU/MP( 2)= 1.000  
 MU/MP( 3)= 1.000  
 MU/MP( 4)= 1.000  
 MU/MP( 5)= 1.000  
 MU/MP( 6)= 1.000  
 MU/MP( 7)= 1.000  
 MU/MP( 8)= 1.000  
 MU/MP( 9)= 1.000  
 MU/MP( 10)= 1.000

NUMBER OF CONSTRAINTS IS 5

NUMBER OF ROWS IN MATRIX A(III,JJI) IS 7

NUMBER OF COLUMNS IN MATRIX A(III,JJI) IS 413

UNBOUNDED ITERATION	OBJ. FUNCTION	NEW BASIC VAR.
1	11190.74	1
2	22419.43	2
3	33607.58	4
4	41869.67	7
5	41869.67	407
6	47572.08	305
7	50513.25	405
8	50770.22	1
9	50772.78	397

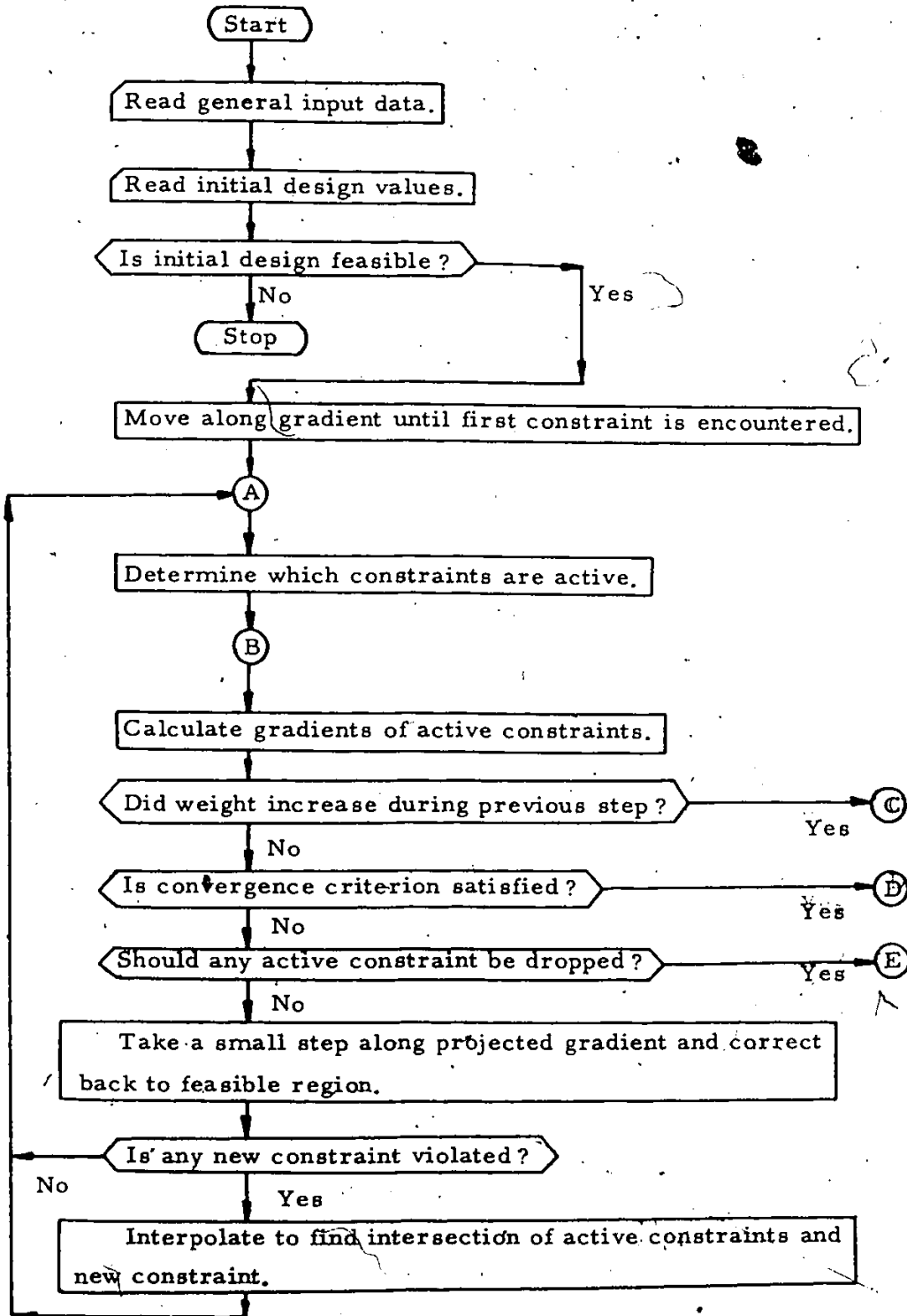
OBJ. FUNCTION 50077.78300

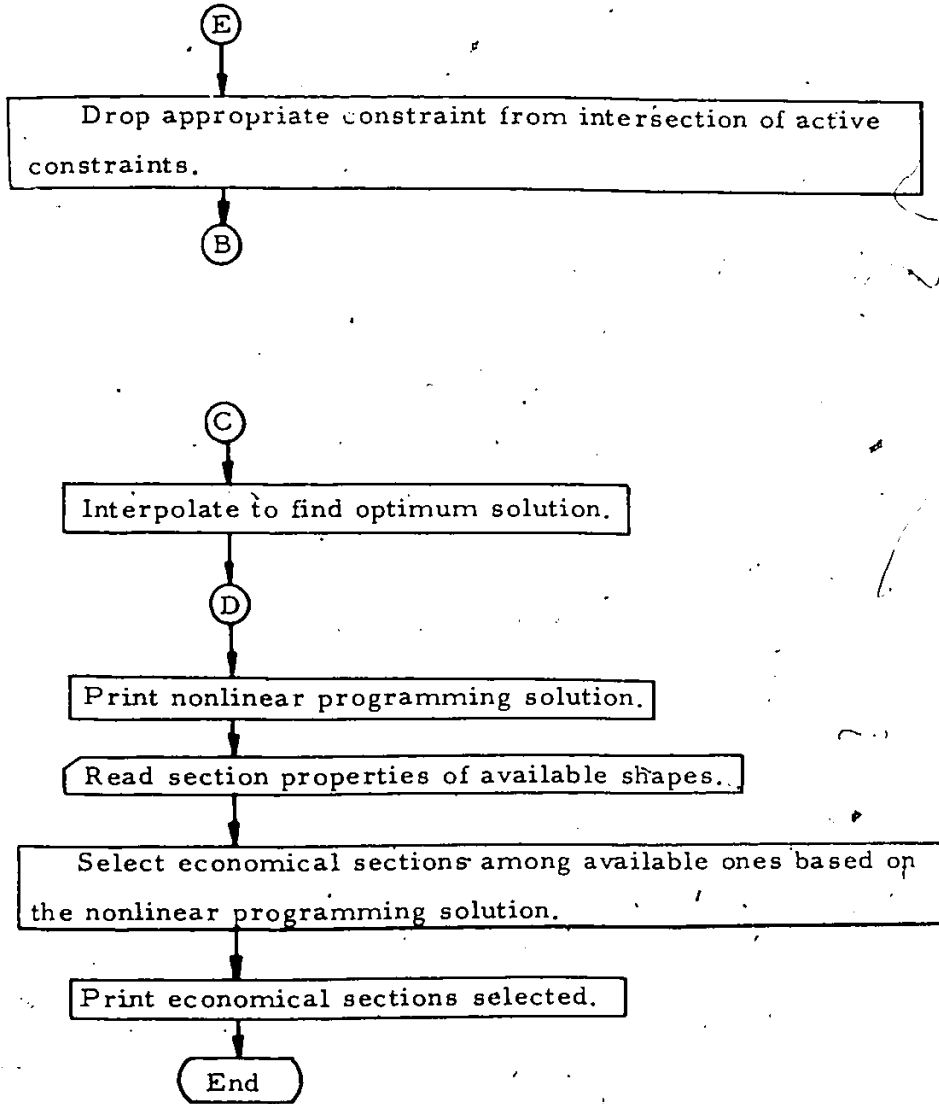
VARIABLE	VALUE	INPUT COST
1	202.68692	60.00000
2	230.95732	60.00000
3	238.23006	60.00000
4	104.10000	72.00000
5	179.70000	75.00000

MU/MP( 23)= 2.075  
 MU/MP( 24)= 1.000  
 MU/MP( 25)= 1.000  
 MU/MP( 26)= 2.067  
 MU/MP( 27)= 1.000  
 MU/MP( 28)= 1.000  
 MU/MP( 29)= 2.067  
 MU/MP( 30)= 1.000  
 MU/MP( 31)= 1.000  
 MU/MP( 32)= 2.077  
 MU/MP( 33)= 1.000  
 MU/MP( 34)= 1.000  
 MU/MP( 35)= 2.077  
 MU/MP( 36)= 1.000

	MP(NON-COMP.)		MP(COMP.)		
	(K.FT)	(WT.L)	(K.FT)	(WT.L)	
1	331.162	0.07215	202.637	0.04415	0.61190...BEAM MEMBER
2	308.029	0.06711	200.969	0.04378	0.65244...BEAM MEMBER
3	308.029	0.06711	208.200	0.04536	0.67591...BEAM MEMBER
4	154.014	0.03355	104.100	0.02268	0.67591...COLUMN MEMBER
5	179.700	0.03915	179.700	0.03915	1.00000...COLUMN MEMBER

A. 4-5 Flow Chart of Main Program for Nonlinear Optimization of Steel Frames including Composite Ones





A. 4-6 Descriptions of Auxiliary Subroutines for Nonlinear Optimization of Steel Frames including Composite Ones

**INVERT :** A subroutine which calculates the inversion of a symmetric matrix  $[U_q^T(X) U_q(X)]$  which are composed of the gradient of the constraint function  $a_j, j = 1, \dots, m$  at the current design variables.

**CONST:** A subroutine which calculates the value of the constraint functions on the basis of the current design variables.

**OBJECT:** A subroutine which computes the values of the objective function and the gradient/vector on the basis of the current design variables.

A. 4-7 Descriptions of Input Data for Nonlinear Optimization of Steel Frames including Composite Ones

1) First Set of Data

**M, N, NOAV**

Where, M, N and NOAV denote the number of constraints, number of variables and number of available rolled sections, if economical selection of sections on the basis of the nonlinear optimum solution, is required, otherwise leave blank.

2) Second Set of Data

**EPS, DELTA, XOO, WOO**

Where EPS and DELTA denote the tolerance  $\epsilon$  and  $\delta$ . XOO and WOO are the reference values to be used for defining the non-dimensional design variables and objective function, respectively.

3) Third Groups of Data

$BU(i), i = 1, N$

$BL(i), i = 1, N$

Where  $BU(i)$  and  $BL(i)$  denote the upper and lower bounds for the  $i$ -th variable, respectively.

4) Fourth Set of Data

$FL(i), i = 1, N$

Where  $FL(i)$  denotes the  $i$ -th member's length. If an economical selection of members from available rolled sections is required, a member length for each column must be given in negative value for a convenience in identifying a column member from beam members.

5) Fifth Set of Data

$X(i), i = 1, N$

In which the initial value of the  $i$ -th design variable  $X(i)$  must be assumed.

6) Sixth and Last Groups of Data

$IAVDP(i), AVWT(i), AVPM(i)$

... for  $i = 1, NOAV$

Where  $IAVDP(i)$ ,  $AVWT(i)$  and  $AVPM(i)$  denote the nominal depth, weight and plastic moment capacity of the  $i$ -th available shape of a rolled wide flange section or a light beam or a standard beam. If  $IAVDP(i)$  corresponds to a rolled wide flange, it must be given in positive integer, otherwise must be in negative integer in order to identify wide flanges from other shapes since for column members only wide flanges are to be used.

**A. 4-8 List of Programs for Nonlinear Optimization of  
Steel Frames including Composite Ones**



DIMENSION X(10), X0(10), P(40), Z(40), PA(40), P1(10), P2(10), C1(40),  
 C2(40), U(10,40), UQ(10,40), XQ(40), VS(40,40), Z(40), Z1(40),  
 R(40), PQC(40), FL(10), Z(10), CCEB(40), COEF(10),  
 WT(10), PM(10), IPM(10),  
 PM1(10), WT1(10), ID1(10), PM2(10), WT2(10), JCC(10),  
 TAVDP(100), AVMT(100), AVPM(100)

0001

NREAD=1  
 NWRITE=2

READ IN THE INPUT DATA REQUIRED IN MAINPROGRAM  
 READ IN THE NUMBER OF CONSTRAINTS M  
 READ IN THE NUMBER OF VARIABLES N  
 READ IN THE NUMBER OF AVAILABLE SECTIONS, IF AVAILABLE SOL,N,PRO,D

READ(NREAD,10) M,N,NMAX  
 10 FORMAT(20I4)

READ IN THE TOLERANCES OF EPS AND DELTA  
 READ IN THE REFERENCE VALUES OF X00 AND W00

READ(NREAD,15) EPS,DELTA,X00,W00  
 15 FORMAT(RF10.0)

READ IN THE UPPER AND LOWER BOUNDS

READ(NREAD,15) (PM(I),I=1,N)  
 READ(NREAD,15) (RL(I),I=1,N)

READ IN THE MEMBER LENGTHS  
 IF AVAILABLE SOL. REQUIRED, BUT NEGATIVE SIGN FOR COLUMN MEMBERS  
 FOR IDENTIFYING AS A COLUMN MEMBERS IN SORTING AVAILABLE COLUMN SEC.

READ(NREAD,15) (FL(I),I=1,N)

PRINT INPUT DATA

WRITE(NWRITE,20) M,N,EPS,DELTA,X00,W00  
 20 FORMAT(1H1//10X,1 THE NUMBER OF CONSTRAINT FUNCTION ME1,14

0011  
0012





C ALL CONSTRAINTS ARE OF THE TYPE  $f(x) \leq 0$  OR  $f(x) = 0$   
 C OBJECTIVE FUNCTION IS OF THE TYPE  $f(x) \leq 0$   
 C USE GRADIENT METHOD FOR OPTIMIZATION

C COMPUTE THE VALUES OF THE OBJECTIVE FUNCTION AND THE GRADIENT  
 C VECTOR ON THE BASIS OF THE CURRENT DESIGN VARIABLES

```

0060 C
0061 C 100 CALL OBJECT (F,G,X,EI,W,T,N,X(7),W(7))
0062 C TM=-F
0063 C W=-F*WCC
0064 C WRITE(NWPIT,105) TM
0065 C FORMAT (//10X,'W/WCC=',F15.5)
0066 C SUM=0.
0067 C DO 110 I=1,N
0068 C SUM=SUM+G(I)*G(I)
0069 C DO 115 I=1,N
0070 C G1(I)=G(I)/SQRT(SUM)
0071 C T=STEP
0072 C PMIN=P(I)
0073 C DO 125 I=2,M
0074 C IF (PMIN-P(I)) 125,125,120
0075 C 120 PMIN=P(I)
0076 C 125 CONTINUE
0077 C DO 130 I=1,N
0078 C YD(I)=X(I)
0079 C FMIND=PMIN
0080 C 140 TD=T
0081 C DO 145 I=1,N
0082 C X(I)=X(I)+T*G1(I)
0083 C DO 160 I=1,N
0084 C IF (X(I)) 150,150,140
0085 C 150 CC 155 J=1,N
0086 C 155 X(J)=X(J)
0087 C T=T/2.
0088 C GO TO 140
0089 C 160 CONTINUE
0090 C DO 165 I=1,N
0091 C WRITE(NWPIT,170) I,X(I)
0092 C FORMAT (10X,'X(',I4,')=',F15.7)
0093 C CALL CONST(P,X,M,N,X(7))
0094 C 172 I=1,M
0095 C WRITE(NWPIT,173) I,P(I)
  
```

```

0085 173 FC=PMIN-D(I) 180,190,195
0086 IF (P(I)) 180,190,195
0087 175 CONTINUE
0088 GO TO 100
0089 PMIN=P(I)
0090 IF (PMIN-D(I)) 180,190,195
0091 185 PMIN=P(I)
0092 190 CONTINUE
0093 TM=PMIN-PMIN
0094 IF (ABS(TM)-1./10.**20) 202,202,204
0095 202 DELTA=ABS(PMIN)
0096 DELTA2=DELTA*DELTA
0097 WRITE(NWRITE,203) DELTA
0098 203 FORMAT(10X,'UNABLE TO SATISFY THE TOLERANCE DELTA, HENCE RESET DELTA TO ',F15.7)
0099 GO TO 250
0100 204 T=TM*(PMIN/(PMIN-PMIN))
0101 DO 205 I=1,N
0102 XC(I)=X(I)
0103 PMIN=PMIN
0104 T=1/2
0105 DO 215 I=1,N
0106 IF (X(I)) 220,220,230
0107 DO 225 J=1,N
0108 X(J)=X(J)
0109 T=1/2
0110 GO TO 210
0111 230 CONTINUE
0112 DO 235 I=1,N
0113 WRITE(NWRITE,170) I,X(I)
0114 CALL CONST(P,X,M,N,XCO)
0115 DO 237 I=1,M
0116 WRITE(NWRITE,173) I,P(I)
0117 PMIN=P(I)
0118 DO 245 I=2,M
0119 IF (PMIN-P(I)) 245,245,240
0120 240 PMIN=P(I)
0121 245 CONTINUE
0122 IF (PMIN*PMIN-DELTA2) 250,250,200

```





```

C212 WRITE(NWRIT,105) TM
C213 CALL FUNCT (X0,X1,X2,X3) (X0(I),J),J=1,N
C214 CALL FUNCT (X0,X1,X2,X3) (X0(I),J),J=1,N
C215 WRITE(NWRIT,105)
C216 405 FORMAT(/,10X,10)
C217 DO 410 I=1,IC
C218 WRITE(NWRIT,107) I
C219 410 WRITE(NWRIT,105) (V0(I),J),J=1,IO
C220 IF 420 I=1,IC
C221 420 WRITE(NWRIT,105) I,W0(I)
C222 425 FORMAT (10X,10) (I,14,1) =1,15.7)
C223 TM=I
C224 W=-F+W0C
C225 WRITE(NWRIT,105) TM
C
C C10 WEIGHT INCREASE DURING PREVIOUS STEP
C
C THIS THE LOCAL MINIMUM VALUES ARE ASSUMED TO BE GLOBAL OPTIMUM
C VALUES, AND SO REPEAT THE WHOLE CALCULATIONS FOR DIFFERENT INITIAL
C VARIABLES TO GET THE BEST RESULTS.
C
C
C TM=W0-W
C227 WRITE(NWRIT,105) TM
C228 440 FORMAT(10X,10) W0C-W=1,15.7)
C229 IF (W0-W) GO TO 450,450
C230 W0C=W
C231 IF (IO-1) GO TO 452,452,453
C232 W0(I,1)=1./V0(I,1)
C233 GO TO 454
C234 452 CALL INVERT(V0,IO)
C235 454 GO TO 460 I=1,IO
C236 P(I)=0
C237 DO 455 J=1,N
C238 P(I)=P(I)+V0(I,J)*W0(J)
C239 CONTINUE
C240 P(I)=1
C241 P(I)=1
C242 DO 465 J=1,IC
C243 P(I)=P(I)+V0(I,J)*W0(J)
C244 CONTINUE
C245 DO 470 I=1,N
C246 P0(I)=P(I)
C247 DO 475 J=1,IC

```

```

C248 C(C(I)*C(J))-1.0/(I+J)*C(J)
C249 C=C+1
C250 CONTINUE
C251 C=C+1
C252 R(I)=C(I)/(2.*SQRT(VC(I,1)))
C253 PMAX=R(I)
C254 IPRD=1
C255 IF (IQ-1) 497,497,497
C256 C 495 I=3,10
C257 IF (RMAX-R(I)) 490,495,495
C258 PMAX=R(I)
C259 IPRD=I
C260 495 CONTINUE
C261 497 SUM=0.
C262 C 500 I=1,N
C263 SUM=SUM+PQG(I)*PQC(I)
C264 ABPQG=SQRT(SUM)
C265 WRITE(NWRIT,502) ABPQG,R(IORJIP)
C266 502 FORMAT(/10X,'ABS(PQG)=' ,F15.7)

```

R(TDRCP)=' , F15.7)

C SET BET TO THE BIGGER VALUE OF ABS(PQG) AND B(IORJIP)

```

C267 IF (ABPQG-R(IORJIP)) 505,510,510
C268 505 BET=B(IORJIP)
C269 GO TO 515
C270 510 BET=ABPQG
C271 515 WRITE(NWRIT,520) BET
C272 520 FORMAT(/10X,'BET=' ,F15.7)

```

C IS CONVERGENCE CRITERION SATISFIED

```

C273 IF (BET-EDS/(2.*EN+1)) 510,510,515
C274 525 IF (ABPQG-R(IORJIP)) 520,520,520

```

C GOOD APPROXIMATE CONSISTENT BOUND INTERSECTION OF ACTIVE CONSTRAINTS

```

C275 530 J=1
C276 C 555 I=1,M
C277 IF (PA(I)) 555,555,535
C278 535 IF (J-IPROD) 550,540,550
C279 540 PA(I)=C.
C280 IC=IC+1
C281 WRITE(NWRIT,545) IPRD,IO

```

```

C282 C 575 FORMAT(/,10X,'ITERATION 7 (I=7), X(I)=',10F10.2)
C283 C
C284 C 580 J=J+1
C285 C 585 CONTINUE
C286 C 590 WRITE(NWRITE,580)
C287 C 595 FORMAT(/,10X,'ITERATION 8 (I=8), X(I)=',10F10.2)
C288 C 600 TO 5850
C289 C 585 DO 588 I=1,M
C290 C 589 WRITE(NWRITE,315) I,PA(I),I,P(I)
C291 C 605 TO 580
C292 C 570 CONTINUE

```

C TAKE A SMALL STEP ALONG PROJECTED GRADIENT AND CHECK BACK TO  
 C FEASIBLE REGION

C ITERATION TO OBTAIN FEASIBLE POINT X(V+1) IN INTERSECTION G

```

C293 C 605 WRITE(NWRITE,570)
C294 C 610 FORMAT(/,10X,'ITERATION PROCEDURE IS REQUESTED')
C295 C 615 I=1,N
C296 C 620 Z(I)=PQ(I)/ABQ(I)
C297 C 625 X(I)=Y(I)
C298 C 630 I=I+1
C299 C 635 P(I)=P(I)

```

605 REQUESTED

```

C300 C 640 I=1,N
C301 C 645 X(I)=X(I)+TAL*Z(I)
C302 C 650 I=I+1
C303 C 655 Y(I)=Y(I)
C304 C 660 X(I)=X(I)+TAL*Z(I)
C305 C 665 J=J+1
C306 C 670 Y(J)=Y(J)

```

670 TAL=TAL/2

```

C307 C 675 TO 585
C308 C 680 CONTINUE

```

```

C309 C 685 DO 687 I=1,M
C310 C 690 WRITE(NWRITE,605) I,Z(I),P(I)
C311 C 695 FORMAT(/,10X,'ITERATION 7 (I=7), X(I)=',10F10.2)
C312 C 700 TO 585

```

```

C313 C 705 TO 585
C314 C 710 WRITE(NWRITE,310) I,X(I)
C315 C 715 CONTINUE

```

```

0314      CONTINUE
0315      J=1
0316      DO 430 I=1,N
0317      IF (DA(I)) GO TO 435
0318      W(I)=0
0319      IF (DA(I)) GO TO 435
0320      W(I)=0
0321      JEJAI
0322      CONTINUE
0323      FDEQ=0
0324      DO 435 I=1,N
0325      FDEQ=FDEQ+W(I)*W(I)
0326      IF (FDEQ-DELTAB) GO TO 440,440,440
0327      DO 450 I=1,N
0328      CDEQ(I)=0
0329      [C 455 JEJAI]
0330      CDEQ(I)=CDEQ(I)+W(I)*W(I)
0331      CONTINUE
0332      DO 460 I=1,N
0333      CDEQ(I)=0
0334      [C 465 JEJAI]
0335      CDEQ(I)=CDEQ(I)+W(I)*W(I)
0336      CONTINUE
0337      [C 465 I=1,N]
0338      X(I)=CDEQ(I)
0339      DO 477 I=1,N
0340      WRITE(UNIT,200) I,W(I)
0341      DO 477 I=1,N
0342      WRITE(UNIT,200) I,X(I)
0343      FORMAT (I3,X12.1,1X,12.1,5X,1000)
0344      GO TO 480
0345      DO 480 I=1,N
0346      IF (DA(I)) 485,485,480
0347      IS ANY NEW PERSISTENT VIOLATION
0348      CONTINUE
0349      DO 485 I=1,N
0350      WRITE(UNIT,170) I,X(I)
0351      GO TO 250

```

INTERPOLATE TO FIND INTERSECTION OF ACTIVE COMPONENT

```

0362 700 CONTINUE
0363 WRITE(NWRITE,703)
0364 WRITE(NWRITE,703)
0365 FORMAT(11Y,1P10(11X))
0366 CC TO 000
0367 705 CONTINUE
0368 DO 710 I=1,N
0369 X(I)=X(I)+T+Z(I)
0370 DO 715 I=1,N
0371 WRITE(NWRITE,170) I,X(I)
0372
0373
0374
0375
0376
0377
0378
0379
0380
0381
0382
0383
0384
0385
0386
0387
0388
0389

```

C TAKE CORRECTION PROCEDURE

```

0362 720 CONTINUE
0363 CALL CONST(P,X,M,N,X00)
0364 J=1
0365 DO 735 I=1,M
0366 IF (PA(I)) 735,735,730
0367 W0(J)=R(I)
0368 J=J+1
0369 735 CONTINUE
0370 F0=0.
0371 DO 740 I=1,I0
0372 F0=F0+W0(I)+W0(I)
0373 IF (F0-DELTA) 000,000,745
0374 DO 755 I=1,I0
0375 CORR(I)=0.
0376 DO 750 J=1,J0
0377 CORR(I)=CORR(I)+V(I,J)+W0(J)
0378
0379 DO 765 I=1,N
0380 CORR(I)=0.
0381 DO 760 J=1,J0
0382 CORR(I)=CORR(I)+W0(I,J)+CORR(J)
0383
0384 DO 770 I=1,N
0385 X(I)=X(I)-CORR(I)
0386 DO 775 I=1,I0
0387 WRITE(NWRITE,405) I,W0(I)
0388 DO 775 I=1,N
0389 WRITE(NWRITE,475) I,X(I),CORR(I)

```



```

0428
0429 X0429
0430 TW=X(I)*Y
0431 012 WRITE(NRPT,C13) I,TW
0432 013 FORMAT(/10X,10D,12,1,=,5,20,5)
0433 TATE=F*WC1
0434 WRITE(NRPT,C15) TAT
0435 015 FORMAT(/10X,10D,12,1,=,5,20,5)
0436 IF (NOAV) GO TO 018
C
C SELECT AVAILABLE SECTIONS OF GIVEN WIDE FLANGES
C
0436 018 ORFN=F
0437 CC 019 I=1,N
0438 019 FM(I)=X(I)*XC0
C
C READ SECTION PROPERTIES FOR AVAILABLE WF AND P-SHAPES
C WHERE PUT A NEGATIVE SIGN FIRST FOR THE BEAM SHAPES FOR IDENTIFYING
C FROM WIDE FLANGES
C
0439 IF (LLL-1) GO TO 020,055
0440 020 WRITE (NRPT,021)
0441 021 FORMAT (1H1//10X,'AVAILABLE SECTION PROPERTIES'//10X,
1// '* AVAILABLE ONLY FOR BEAM MEMBERS'//)
2PX,1ND P-SHAPES MP(FT,KIPS)///
CC 050 I=1,NOAV
C
0442 READ (NREAD,025) IAVDP(I),AVWT(I),AVPM(I)
0443 025 FORMAT (I3,2X,F4.0,F6.1)
0444 IF (IAVDP(I)) GO TO 040,040
0445 030 J=-IAVDP(I)
0446 WRITE(NRPT,045) I,J,AVWT(I),AVPM(I)
0447 045 FORMAT (I10,I4,1X,8,F5.1,F12.1)
0448 GO TO 050
C
0449 040 WRITE(NRPT,045) I,IAVDP(I),AVWT(I),AVPM(I)
0450 045 FORMAT (I10,I4,1X,8,F5.1,F12.1)
0451 GO TO 050
C
C SELECT THE BIGGER BUT THE CLOSEST SECTION TO THE CONT. SOLUTION
C
0452 055 CC 045 I=1,N
0453 045 045 045 045
0454 045 045 045 045
0455 045 045 045 045
0456 045 045 045 045
0457 045 045 045 045

```

```

C465 IF (AVPM(J)-SM(I)) GO TO 500
C466 IF (AVWT(J)-WT(I)) GO TO 500
C467 FM(I)=AVPM(J)
C468 WT(J)=AVWT(J)
C469 IC(I)=J
C470 CONTINUE
C471 DO 1030 J=1,N

```

C CHECK CONSTRAINTS FOR THE INITIAL VALUES

```

C472 DO 1000 I=1,N
C473 X(I)=PM(I)/X00
C474 CALL CONST(P,X,M,N,X00)
C475 DO 1035 I=1,M
C476 IF (P(I) GO 1035,1035,1035
C477 DO 1030 J=1,N
C478 K=ID1(J)
C479 1000 K=K+1
C480 IF (NDAY-K) 1005,1015,1015
C481 WRITE(NWRIT,1010)
C482 1010 FORMAT('HI/IOX, THERE IS NO AVAILABLE SECTION')
C483 GO TO 60
C484 1015 IF (IPM(J)) 1020,1025,1025
C485 1020 IF (IAVDP(K)) 1000,1025,1025
C486 1025 IC(J)=K
C487 FM(J)=AVPM(K)
C488 WT(J)=AVWT(K)
C489 CONTINUE
C490 GO TO 500
C491 1035 CONTINUE

```

C CALCULATE THE TOTAL WEIGHT FOR THE INITIAL VALUE

```

C492 TWT=0
C493 DO 1040 J=1,N
C494 TWT=TWT+WT1(J)*FL(J)
C495 1040 TWT1=TWT+WT1(J)*FL(J)
C496 DECREASE THE WEIGHT AS MUCH AS POSSIBLE & SELECT THE ECONOMICAL
C497 SECTIONS
C498
C499 1045 IND=0
C500 DO 1050 J=1,N

```

```

C491      PM2(J)=PM1(J)
C492      WT2(J)=WT1(J)
C493      ID2(J)=ID1(J)
C494      1050 CONTINUE
C495      DO 1105 L=1,N
C496      K=102(L)
C497      1055 K=K-1
C498      IF (K) 1105,1105,1060
C499      1060 IF (AVWT(K)-WT2(L)) 1065,1105,1105
C500      1065 IF (IPM(L)) 1070,1075,1075
C501      1070 IF (IAVDP(K)) 1055,1075,1075
C502      1075 PM2(L)=AVPM(K)
C503      WT2(L)=AVWT(K)
C504      ID2(L)=K
C          CHECK CCNSTRANS
C
C505      DO 1076 I=1,N
C506      1076 X(I)=PM2(I)/X00
C507      CALL CCNST(P,X,M,N,XOFF)
C508      DO 1078 I=1,M
C509      IF (P(I)) 1100,1100,1078
C510      1078 CONTINUE
C511      1080 CONTINUE
C          CALCULATE THE TOTAL WEIGHT FOR THE TRIAL VALUES
C          TWT2=0
C512      TWT2=0
C513      DO 1095 J=1,N
C514      1095 TW2=TW2+WT2(J)*EL(J)
C515      IF (TWT2-TWT1) 1090,1100,1100
C516      1090 INC=1
C517      PM11=PM1(L)
C518      WT11=WT1(L)
C519      ID11=ID1(L)
C520      DO 1095 J=1,N
C521      PM1(J)=PM11
C522      WT1(J)=WT11
C523      ID1(J)=ID11
C524      1095 CONTINUE
C525      TW11=TW2
C526      PM2(L)=PM11
C527      WT2(L)=WT11

```

```

0528 IF (L)=100
0529 GO TO 1105
0530 PM3(L)=PM1(I)
0531 WT3(L)=WT1(I)
0532 IP2(L)=IP1(I)
0533 CONTINUE
0534 IF (IND) 1110,1110,1045
0535 CONTINUE
0536 DO 1112 I=1,N
0537 X(I)=PM1(I)/XCO
0538 CALL OBJECT (F,G,X,FL,WT,N,XCO,WCO)
0539 OBFN1=-F
0540 WRITE (NWPIT,1115)
0541 FORMAT (1F17//17X,'CONTINUOUS SOLUTION AVAILABLE SOLUTION'//)
0542 DO 1155 J=1,N
0543 K=ID1(J)
0544 K=IAVDP(K)
0545 IF (IPM(J)) 1143,1120,1120
0546 IF (K) 1125,1125,1135
0547 K=-K
0548 WRITE (NWPIT,1130) J, PM(J), PM1(J), K, WT1(J)
0549 FORMAT (10X, 'MP', I2, '(FT.KIPS)', F12.3, F12.3, '(', I4, ' B', F5.1,
1 '...', SELECTED FROM AVAILABLE WF AND B SHAPES AS A BEAM MEMBER )' )
0550 GO TO 1155
0551 WRITE (NWPIT,1140) J, PM(J), PM1(J), K, WT1(J)
0552 FORMAT (10X, 'MP', I2, '(FT.KIPS)', F12.3, F12.3, '(', I4, ' WF', F5.1,
1 '...', SELECTED FROM AVAILABLE WF AND R SHAPES AS A BEAM MEMBER )' )
0553 GO TO 1155
0554 WRITE (NWPIT,1150) J, PM(J), PM1(J), K, WT1(J)
0555 FORMAT (10X, 'MP', I2, '(FT.KIPS)', F12.3, F12.3, '(', I4, ' WF', F5.1,
1 '...', SELECTED FROM AVAILABLE WF SHAPES AS A COLUMN MEMBER )' )
0556 CONTINUE
0557 TWT=TW1/1000.
0558 TWT1=TW11/1000.
0559 WRITE (NWPIT,1160) OBFN, OBFN1, TWT, TWT1
0560 FORMAT (17//10X, 'OBJ. FUNCTION', 2F12.3//10X,
1 'TOTAL WEIGHT', 2F12.3/15X, '(KIPS)' )
0561 GO TO 60
0562 3000 STND
0563 END

```

```

C001      C
C002      DIMENSION A(40,40)
C003      NWRITE=2
C004      N1=N-1
C005      I=00 INDE=1
C006      C
C007      DECOMPOSING A MATRIX INTO (U)**(U)
C008      DC 245 I=1,N
C009      DO 240 J=1,N
C010      SUM=A(I,J)
C011      GC TO (21C,200),IND
C012      200 I=I-1
C013      DC 205 K=1,I
C014      205 SUM=SUM-A(K,I)*A(K,J)
C015      210 IF (J-I) 215,220,215
C016      215 A(I,J)=SUM*TFMP
C017      GC TO 240
C018      220 IF (SUM) 225,225,235
C019      225 WRITE(NWRITE,230)
C020      230 FORMAT(/5X,'DIAGONAL ELEMENT OF (U*) LESS THAN ZERO//)
C021      GC TO 9090
C022      235 TEMP=1./SQRT(SUM)
C023      A(I,J)=TEMP
C024      240 CONTINUE
C025      IND=2
C026      245 CONTINUE
C027      C
C028      INVERT MATRIX V OF U
C029      DO 255 I=1,N1
C030      I1=I+1
C031      DO 255 J=I1,N
C032      SUM=0.
C033      J1=J-1
C034      DO 250 K=I,J1
C035      250 SUM=SUM-A(K,I)*A(K,J)
C036      255 A(J,I)=SUM*A(J,J)
C037      DO 265 I=1,N
C038      CC 265 J=I,N
C039      SUM=0.
C040      DO 260 K=J,N
C041      260 SUM=SUM+A(K,I)*A(K,J)
C042      A(J,I)=SUM
C043      A(I,J)=A(J,I)
C044      265 CC CONTINUE
C045      RETURN

```

CC42  
0043

COOD SIGN  
ENRY



0007  
 0008  
 0009  
 0010

0002  
 0003  
 0004  
 SIMULATION OF (M), X(1)  
 FL=2.  
 FL2=FL\*FL

0005  
 0006  
 MIL-IP RELAXATION IS APPROXIMATED BY  $\sum_{i,j} u_{ij} = A + u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 + u_8 + u_9 + u_{10}$

0005  
 0006  
 0007  
 0008  
 0009  
 0010  
 $A = 0.7624525 \text{ } C_1$   
 $P = 0.1697975 \text{ } C_1$   
 $C = 0.1247005 - 0.2$   
 $D = 0.1147875 - 0.7$   
 $X(1) = A + P * X(1) + X(2) + C * X(3) + D * X(4) + E * X(5) + F * X(6) + G * X(7) + H * X(8) + I * X(9) + J * X(10)$   
 $X(2) = A + P * X(2) + X(3) + C * X(4) + E * X(5) + F * X(6) + G * X(7) + H * X(8) + I * X(9) + J * X(10)$

0011  
 0012  
 0013  
 THE CONSTRAINTS WHICH GUBERN THE LINEAR PROGRAMMING SOLUTION.

0011  
 0012  
 0013  
 0014  
 0015  
 0016  
 0017  
 0018  
 0019  
 0020  
 $P(1) = 2.0 * X(1) + X(2) + X(3) + X(4) - 0.95 * FL$   
 $UR = \text{SQRT}((X(1) + X(2) + X(3) + X(4)) / 2.55)$   
 $P(2) = 2.0 * FL + X(1) + 2.0 * FL - (P(1) + X(2) * X(3) + X(4) * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11))$   
 $1 - (2.72 * 54 + 2.55 * FL) + UR$   
 $U1 = \text{SQRT}((X(1) + X(2) * X(3) + X(4) * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11)) / 1.7)$   
 $U2 = \text{SQRT}((X(1) + X(2) * X(3) + X(4) * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11)) / 2.55)$   
 $P(3) = 2.0 * FL + U1 * X(1) + (2.0 * FL * U2 - U1 * U3) * X(2) + X(3) * X(4) + 2.0 * FL + U1 * X(5)$   
 $1 + (2.0 * FL + U1 - U2 * U3) * X(2) + X(3) * X(4) + 4.0 * U1 * U3 * X(3) + X(4) * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11)$   
 $2 - (464.74 + 4.0 * 2.55 * FL) * U1 + U2$   
 $U1 = \text{SQRT}((X(1) + X(2) * X(3) + X(4) * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11)) / 1.7)$   
 $U2 = \text{SQRT}((X(1) + X(2) * X(3) + X(4) * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11)) / 2.55)$   
 $U3 = \text{SQRT}((X(1) + X(2) * X(3) + X(4) * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11)) / 2.55)$   
 $P(4) = 2.0 * FL + U1 * U2 * X(1) + (2.0 * FL * U1 * U2 - U1 * U3 * X(2) + X(3) * X(4) + 2.0 * FL + U1 * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11))$   
 $1 + 2.0 * FL * (U1 * U2 + U1 * U3) * X(2) + 2.0 * FL * U1 * U2 * X(3) + X(4) * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11)$   
 $2 + 2.0 * (U1 * U3 + U1 * U2 * X(2) + X(3) * X(4) + 4.0 * U1 * U3 * X(3) + X(4) * X(5) + X(6) * X(7) + X(8) * X(9) + X(10) * X(11))$   
 $3 * X(4) * X(5) - (464.74 + 5.0 * FL) * U1 + U2 + U3$

0021  
 0022  
 0023  
 0024  
 CONSTRAINTS CORRESPONDING TO ELEMENTARY REQUIREMENTS

0021  
 0022  
 0023  
 0024  
 $P(5) = 2.0 * X(1) + 2.0 * X(2) + X(3) - 0.95 * FL$   
 $P(6) = 2.0 * X(1) + 2.0 * X(2) + X(3) - 1.0 * 75 * FL$   
 $P(7) = 2.0 * X(1) + X(2) + X(3) + X(4) + X(5) - 1.0 * 275 * FL$   
 $P(8) = 2.0 * X(1) + X(2) + X(3) + X(4) - 0.1 * 8$

CC26  
CC27  
CC28

$f(x) = x^2 + 2x + 1$   
 $f'(x) = 2x + 2$   
 $f''(x) = 2$

CC29  $U_1 = \text{SQRD}((XU1 + X(1)) * X(1), 1.7)$   
CC30  $P(1,2) = 7 * FL * X(1) + 2 * X(2) + X(3) + X(4) + X(5)$   
 $1 + (2 * FL * X(1) + X(2) + X(3) + X(4) + X(5))$   
 $2 - (464.74 + 4 * FL * X(1) + X(2) + X(3) + X(4) + X(5))$

CC31  
CC32  
CC33

$U_1 = \text{SQRD}((XU1 + X(1)) * X(1), 1.7)$   
 $1 + 2 * FL * X(1) + X(2) + X(3) + X(4) + X(5)$   
 $2 - (464.74 + 4 * FL * X(1) + X(2) + X(3) + X(4) + X(5))$

CC34  
CC35  
CC36

$U_1 = \text{SQRD}((XU1 + X(1)) * X(1), 1.7)$   
 $U_2 = \text{SQRD}((XU2 + X(2)) * X(2), 2.55)$   
 $P(1,4) = 2 * FL * X(1) + X(2) + X(3) + X(4) + X(5)$   
 $1 + 2 * FL * X(1) + X(2) + X(3) + X(4) + X(5)$   
 $2 - (464.74 + 4 * FL * X(1) + X(2) + X(3) + X(4) + X(5))$

CC37  
CC38  
CC39

$U_1 = \text{SQRD}((XU1 + X(1)) * X(1), 1.7)$   
 $U_2 = \text{SQRD}((XU2 + X(2)) * X(2), 2.55)$   
 $P(1,5) = 2 * FL * X(1) + X(2) + X(3) + X(4) + X(5)$   
 $1 + (2 * FL * X(1) + X(2) + X(3) + X(4) + X(5))$   
 $2 - (370.56 + 5 * FL * X(1) + X(2) + X(3) + X(4) + X(5))$

CC40  
CC41  
CC42  
CC43

$U_1 = \text{SQRD}((XU1 + X(1)) * X(1), 1.7)$   
 $U_2 = \text{SQRD}((XU2 + X(2)) * X(2), 2.55)$   
 $U_3 = \text{SQRD}((XU3 + X(3)) * X(3), 2.55)$   
 $P(1,6) = 2 * FL * X(1) + X(2) + X(3) + X(4) + X(5)$   
 $1 + 2 * FL * X(1) + X(2) + X(3) + X(4) + X(5)$   
 $2 + 2 * FL * X(1) + X(2) + X(3) + X(4) + X(5)$

CC44  
CC45  
CC46  
CC47

$U_1 = \text{SQRD}((XU1 + X(1)) * X(1), 1.7)$   
 $U_2 = \text{SQRD}((XU2 + X(2)) * X(2), 2.55)$   
 $U_3 = \text{SQRD}((XU3 + X(3)) * X(3), 2.55)$   
 $U_4 = \text{SQRD}((XU4 + X(4)) * X(4), 2.55)$   
 $1 + (2 * FL * X(1) + X(2) + X(3) + X(4) + X(5))$   
 $2 + 2 * FL * X(1) + X(2) + X(3) + X(4) + X(5)$

CC48  
CC49  
CC50

$U_1 = \text{SQRD}((XU1 + X(1)) * X(1), 1.7)$   
 $U_2 = \text{SQRD}((XU2 + X(2)) * X(2), 2.55)$   
 $U_3 = \text{SQRD}((XU3 + X(3)) * X(3), 2.55)$

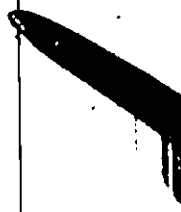


CC76  $U = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC77  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC78  $U2 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 1+2.  $U3 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 2+2.  $U4 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 3+2.  $U5 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 4-  $(464.74 + 8.50 * FL * U1 + U2 + U3 + U4) * X(4) * X(1)$   
 CC79  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC80  $U2 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC81  $U3 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC82  $U4 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC83  $P(27) = 2. * FL * U1 * U2 * U3 * U4 * X(1) + 2. * FL * U2 * U3 * U4 * X(1) + X(1) * X(2) * X(3) * X(4)$   
 1+2.  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 2+2.  $U2 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 3+2.  $U3 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 4-  $(464.74 + 8.50 * FL * U1 + U2 + U3 + U4) * X(4) * X(1)$   
 CC84  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC85  $U2 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC86  $U3 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC87  $U4 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC88  $P(28) = 2. * FL * U1 * U2 * U3 * U4 * X(1) + 2. * FL * U2 * U3 * U4 * X(1) + X(1) * X(2) * X(3) * X(4)$   
 1+2.  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 2+2.  $U2 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 3+2.  $U3 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 4-  $(464.74 + 8.50 * FL * U1 + U2 + U3 + U4) * X(4) * X(1)$   
 CC89  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC90  $U2 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC91  $U3 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC92  $U4 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC93  $F(29) = 2. * FL * U1 * U2 * U3 * U4 * X(1) + 2. * FL * U2 * U3 * U4 * X(1) + X(1) * X(2) * X(3) * X(4)$   
 1+2.  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 2+2.  $U2 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 3+2.  $U3 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 4-  $(464.74 + 8.50 * FL * U1 + U2 + U3 + U4) * X(4) * X(1)$   
 CC94  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC95  $U2 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC96  $U3 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 CC97  $P(30) = 2. * FL * U1 * U2 * U3 * U4 * X(1) + 2. * FL * U2 * U3 * U4 * X(1) + X(1) * X(2) * X(3) * X(4)$   
 1+2.  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 2+2.  $U2 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 3+2.  $U3 = SQRT((X1)^2 + X(2)^2) / 2.55$   
 4-  $(464.74 + 8.50 * FL * U1 + U2 + U3 + U4) * X(4) * X(1)$   
 CC98  $U1 = SQRT((X1)^2 + X(2)^2) / 2.55$

52

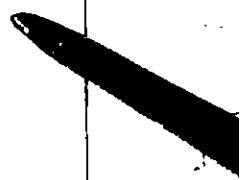
2

1



C100 (כ) \* X (כ) / 72 + 5  
 C100 (כ) \* X (כ) / 72 + 5  
 C101 (כ) \* X (כ) / 72 + 5  
 C101 (כ) \* X (כ) / 72 + 5  
 C102 (כ) \* X (כ) / 72 + 5  
 C103 (כ) \* X (כ) / 72 + 5  
 C104 (כ) \* X (כ) / 72 + 5  
 C105 (כ) \* X (כ) / 72 + 5  
 C106 (כ) \* X (כ) / 72 + 5  
 C107 (כ) \* X (כ) / 72 + 5

80  
 70



0001

0002

0003

0004

0005

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0007

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0009

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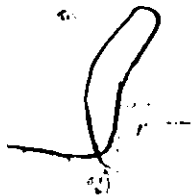
0016

$W(I) = 1.0 / W(I) + W(I) * (X(I) - Y(I))$   
 $WT(I) = W(I) * (X(I) - Y(I))$   
 DO 20 CONTINUE  
 $W = 0.$   
 $DC 20 I = 1, N$   
 $W = W + WT(I) * FI(I)$

CONTINUE  
 $F = W / W00$   
 RETURN  
 END



**A. 4-9 Output Example for the Nonlinear Optimization of the  
Composite Frame in Sec. 6-6**



THE NUMBER OF CONSTRAINT EQUATION IS 32

THE NUMBER OF DESIGN VARIABLES IS 4

THE TOLERANCE EPS= 0.000000E-07 ( WHICH CONTROLS THE NEARNESS OF THE POINT X MAX TO THE TRUF OPTIMUM POINT )

THE TOLERANCE DELTA= 0.000000E-06 ( WHICH CONTROLS THE NEARNESS OF THE POINT X TO THE INTERSECTION G )

THE PREFERENCE VALUE X00= 1000. ( USED FOR DEFINING THE NON-DIMENSIONAL DESIGN VARIABLES )

THE PREFERENCE VALUE X000= 1000. ( USED FOR DEFINING THE NON-DIMENSIONAL OBJECTIVE FUNCTION )

THE MEMBERS LENGTH NUMBER CODE

L ( WHERE NEGATIVE AND POSITIVE CODES SHOW COLUMN AND ROW MEMBERS RESPECTIVELY )

- 1 50.00000 1
- 2 50.00000 1
- 3 35.00000 -1
- 4 45.00000 -1

THE INITIAL DESIGN VARIABLES X\*, NONDIMENSIONALIZED VARIABLES X, UPPER BOUNDS PU AND LOWER BOUNDS PL ARE AS FOLLOWS

X*	X	PU	PL
1	200.00000	0.00000	0.0
2	500.00000	0.50000	0.0
3	200.00000	0.00000	0.0
4	200.00000	0.00000	0.0

W/W00= 11.31203  
 X( 1)= 0.2904256E 00  
 X( 2)= 0.4920152E 00  
 X( 3)= 0.1889416E 00  
 X( 4)= 0.1889416E 00  
 P ( 1)= 0.8251282E 03  
 P ( 2)= 0.3512560E 05  
 P ( 3)= 0.1263759E 07  
 P ( 4)= 0.3756046E 08  
 P ( 5)= 0.9266223E 03  
 P ( 6)= 0.1587503E 04  
 P ( 7)= 0.1473271E 04  
 P ( 8)= 0.1041849E 04  
 P ( 9)= 0.1244817E 04  
 P ( 10)= 0.1636727E 04  
 P ( 11)= 0.7607087E 03  
 P ( 12)= 0.1316179E 07  
 P ( 13)= 0.1322712E 07  
 P ( 14)= 0.1375133E 07  
 P ( 15)= 0.1497576E 07  
 P ( 16)= 0.3576670E 08  
 P ( 17)= 0.320306E 08  
 P ( 18)= 0.2708170E 08  
 P ( 19)= 0.3326365E 08  
 P ( 20)= 0.1497576E 07  
 P ( 21)= 0.1482763E 07  
 P ( 22)= 0.1547449E 07  
 P ( 23)= 0.1667288E 07  
 P ( 24)= 0.1651654E 07  
 P ( 25)= 0.1806234E 07  
 P ( 26)= 0.1005807E 10  
 P ( 27)= 0.9097864E 09  
 P ( 28)= 0.1090513E 10  
 P ( 29)= 0.5881203E 09  
 P ( 30)= 0.3872264E 08  
 P ( 31)= 0.4104051E 08  
 P ( 32)= 0.4225446E 08

W/W00= 11.07302  
 X( 1)= 0.2803934E 00  
 X( 2)= 0.4841115E 00  
 X( 3)= 0.1778355E 00  
 X( 4)= 0.1778359E 00  
 W/W00= 10.58444  
 X( 1)= 0.2714093E 00  
 X( 2)= 0.4762956E 00  
 X( 3)= 0.1666787E 00  
 X( 4)= 0.1666787E 00  
 W/W00= 10.58444  
 X( 1)= 0.2619709E 00  
 X( 2)= 0.4685746E 00  
 X( 3)= 0.1554654E 00  
 X( 4)= 0.1554654E 00  
 W/W00= 8.12769  
 X( 1)= 0.1818498E 00  
 X( 2)= 0.4053902E 00  
 X( 3)= 0.5071071E 01  
 X( 4)= 0.5071071E 01

X( 1 ) = 0.185573CF 00  
 X( 2 ) = 0.4112593E 00  
 X( 3 ) = 0.6215667E-01  
 X( 4 ) = 0.6215667E-01

C = 1

PHI-ACTIVE ( 1 ) = 0.	PHI ( 1 ) =	0.2696733E 03
PHI-ACTIVE ( 2 ) = 0.	PHI ( 2 ) =	0.1186906E 05
PHI-ACTIVE ( 3 ) = 0.	PHI ( 3 ) =	0.3222650E 06
PHI-ACTIVE ( 4 ) = 0.	PHI ( 4 ) =	0.1105419E 08
PHI-ACTIVE ( 5 ) = C.	PHI ( 5 ) =	0.3970898E 03
PHI-ACTIVE ( 6 ) = 0.	PHI ( 6 ) =	0.1171853E 04
PHI-ACTIVE ( 7 ) = 0.	PHI ( 7 ) =	0.8849468E 03
PHI-ACTIVE ( 8 ) = C.	PHI ( 8 ) =	0.2811399E 03
PHI-ACTIVE ( 9 ) = 0.	PHI ( 9 ) =	0.5359722E 03
PHI-ACTIVE ( 10 ) = 0.	PHI ( 10 ) =	0.8529617E 03
PHI-ACTIVE ( 11 ) = 1.	PHI ( 11 ) =	0.0
PHI-ACTIVE ( 12 ) = 0.	PHI ( 12 ) =	0.3724990E 04
PHI-ACTIVE ( 13 ) = 0.	PHI ( 13 ) =	0.4352960E 04
PHI-ACTIVE ( 14 ) = C.	PHI ( 14 ) =	0.4855300E 06
PHI-ACTIVE ( 15 ) = 0.	PHI ( 15 ) =	0.5685880E 06
PHI-ACTIVE ( 16 ) = 0.	PHI ( 16 ) =	0.9026512E 07
PHI-ACTIVE ( 17 ) = 0.	PHI ( 17 ) =	0.6561728E 07
PHI-ACTIVE ( 18 ) = 0.	PHI ( 18 ) =	0.1111803E 08
PHI-ACTIVE ( 19 ) = C.	PHI ( 19 ) =	0.8409760E 07
PHI-ACTIVE ( 20 ) = 0.	PHI ( 20 ) =	0.4855300E 06
PHI-ACTIVE ( 21 ) = 0.	PHI ( 21 ) =	0.6932670E 06
PHI-ACTIVE ( 22 ) = 0.	PHI ( 22 ) =	0.5568930E 06
PHI-ACTIVE ( 23 ) = 0.	PHI ( 23 ) =	0.8385510E 06
PHI-ACTIVE ( 24 ) = 0.	PHI ( 24 ) =	0.6574310E 06
PHI-ACTIVE ( 25 ) = C.	PHI ( 25 ) =	0.9544440E 06
PHI-ACTIVE ( 26 ) = 0.	PHI ( 26 ) =	0.2611343E 09
PHI-ACTIVE ( 27 ) = C.	PHI ( 27 ) =	0.2045750E 09
PHI-ACTIVE ( 28 ) = 0.	PHI ( 28 ) =	0.3682143E 09
PHI-ACTIVE ( 29 ) = 0.	PHI ( 29 ) =	0.2951941E 09
PHI-ACTIVE ( 30 ) = 0.	PHI ( 30 ) =	0.1198355E 09
PHI-ACTIVE ( 31 ) = 0.	PHI ( 31 ) =	0.1637722E 09
PHI-ACTIVE ( 32 ) = 0.	PHI ( 32 ) =	0.1744851E 09

W/WOC= 0.11016

WOLD-W= 0.999997E 20

ABS(PQG)= 0.2705595E-03     R(IDRNP)= -0.1033394E-03

BET = 0.2705595E-03

ITERATION PROCEDURE IS REQUIRED

W/WOC= 7.85464

WOLD-W= 0.2555195E 03

ABS(PQG)= 0.2891729E-03     R(IDRNP)= -0.1033394E-03

BET = 0.2891729E-03

ITERATION PROCEDURE IS REQUIRED

W ( 1 )= 0.0

W/WOC= 7.59402

WOLD-W= 0.2706211E 03

ABS(PQG)= 0.3198532E-03     R(IDRNP)= -0.1033393E-03

BET = -0.3198532E-03

ITERATION PROCEDURE IS REQUIRED

UNABLE TO ADD NEXT CONSTRAINT

0.4577637E-04

HENCE RESET DELTA TO

- X( 1) = 0.1608502E 00
- X( 2) = 0.3857150E 00
- X( 3) = 0.1530001E-01
- X( 4) = 0.6215665E-01

Q = 2

PHI-ACTIVE ( 1 ) = 0.	PHI ( 1 ) = 0.9978052E 02
PHI-ACTIVE ( 2 ) = 0.	PHI ( 2 ) = 0.9403297E 04
PHI-ACTIVE ( 3 ) = 0.	PHI ( 3 ) = 0.1698250E 06
PHI-ACTIVE ( 4 ) = 0.	PHI ( 4 ) = 0.7010416E 07
PHI-ACTIVE ( 5 ) = 0.	PHI ( 5 ) = 0.2453311E 03
PHI-ACTIVE ( 6 ) = 0.	PHI ( 6 ) = 0.1060486E 04
PHI-ACTIVE ( 7 ) = 0.	PHI ( 7 ) = 0.7482273E 03
PHI-ACTIVE ( 8 ) = 1.	PHI ( 8 ) = 0.4577637E-04
PHI-ACTIVE ( 9 ) = 0.	PHI ( 9 ) = 0.2911003E 03
PHI-ACTIVE ( 10 ) = 0.	PHI ( 10 ) = 0.5975010E 03
PHI-ACTIVE ( 11 ) = 1.	PHI ( 11 ) = 0.0
PHI-ACTIVE ( 12 ) = 0.	PHI ( 12 ) = 0.2220300E 06
PHI-ACTIVE ( 13 ) = 0.	PHI ( 13 ) = 0.2818250E 06
PHI-ACTIVE ( 14 ) = 0.	PHI ( 14 ) = 0.3340300E 06
PHI-ACTIVE ( 15 ) = 0.	PHI ( 15 ) = 0.4327860E 06
PHI-ACTIVE ( 16 ) = 0.	PHI ( 16 ) = 0.5144368E 07
PHI-ACTIVE ( 17 ) = 0.	PHI ( 17 ) = 0.2907040E 07
PHI-ACTIVE ( 18 ) = 0.	PHI ( 18 ) = 0.7075072E 07
PHI-ACTIVE ( 19 ) = 0.	PHI ( 19 ) = 0.4536016E 07
PHI-ACTIVE ( 20 ) = 0.	PHI ( 20 ) = 0.4158060E 06
PHI-ACTIVE ( 21 ) = 0.	PHI ( 21 ) = 0.5600350E 06
PHI-ACTIVE ( 22 ) = 0.	PHI ( 22 ) = 0.3875600E 06
PHI-ACTIVE ( 23 ) = 0.	PHI ( 23 ) = 0.6687280E 06
PHI-ACTIVE ( 24 ) = 0.	PHI ( 24 ) = 0.4942870E 06
PHI-ACTIVE ( 25 ) = 0.	PHI ( 25 ) = 0.7947560E 06
PHI-ACTIVE ( 26 ) = 0.	PHI ( 26 ) = 0.1500091E 09
PHI-ACTIVE ( 27 ) = 0.	PHI ( 27 ) = 0.1095620E 09
PHI-ACTIVE ( 28 ) = 0.	PHI ( 28 ) = 0.2515517E 09
PHI-ACTIVE ( 29 ) = 0.	PHI ( 29 ) = 0.1824156E 09
PHI-ACTIVE ( 30 ) = 0.	PHI ( 30 ) = 0.7930304E 07
PHI-ACTIVE ( 31 ) = 0.	PHI ( 31 ) = 0.1191686E 08
PHI-ACTIVE ( 32 ) = 0.	PHI ( 32 ) = 0.1700310E 08

UNIN

W/WOO= 7.20907

WOLD-W= 0.2850509E C3

ABS(POG)= C.1825276E-03

B(IDROP)= -0.1033303E-03

BET = 0.1825276E-03

ITERATION PROCEDURE IS REQUIRED

- X( 1)= 0.1446577E 00
- X( 2)= 0.3779212E 00
- X( 3)= 0.1530001E-01
- X( 4)= 0.6215667E-01

Q = 2

W/WOO= 7.0769E

WOLD-W= 0.2221172E C3

ABS(POG)= C.1878026E-03

B(IDROP)= -0.1033303E-03

BET = 0.1878026E-03

ITERATION PROCEDURE IS REQUIRED

- X( 1)= C.1283970E 00
- X( 2)= 0.3663327E 00
- X( 3)= 0.1530000E-01
- X( 4)= 0.6215666E-01

INTERPOLATION PROCEDURE IS REQUIRED

X( 1) = 0.1376292E 00  
 X( 2) = 0.3728961E 00  
 X( 3) = 0.1530001E-01  
 X( 4) = 0.6215667E-01

C = 2

PHI-ACTIVE ( 1 ) = 1.0	PHI ( 1 ) = 0.02
PHI-ACTIVE ( 2 ) = 0.	PHI ( 2 ) = 0.8265117E 04
PHI-ACTIVE ( 3 ) = 0.	PHI ( 3 ) = 0.1118660E 04
PHI-ACTIVE ( 4 ) = 0.	PHI ( 4 ) = 0.5207136E 07
PHI-ACTIVE ( 5 ) = 0.	PHI ( 5 ) = 0.1223256E 02
PHI-ACTIVE ( 6 ) = 0.	PHI ( 6 ) = 0.9733586E 03
PHI-ACTIVE ( 7 ) = 0.	PHI ( 7 ) = 0.6779187E 03
PHI-ACTIVE ( 8 ) = 1.	PHI ( 8 ) = 0.4577637E-04
PHI-ACTIVE ( 9 ) = 0.	PHI ( 9 ) = 0.2644585E 03
PHI-ACTIVE ( 10 ) = 0.	PHI ( 10 ) = 0.5046169E 02
PHI-ACTIVE ( 11 ) = 1.	PHI ( 11 ) = 0.0
PHI-ACTIVE ( 12 ) = 0.	PHI ( 12 ) = 0.1521660E 04
PHI-ACTIVE ( 13 ) = 0.	PHI ( 13 ) = 0.2091960E 06
PHI-ACTIVE ( 14 ) = 0.	PHI ( 14 ) = 0.2494950E 04
PHI-ACTIVE ( 15 ) = 0.	PHI ( 15 ) = 0.3733060E 04
PHI-ACTIVE ( 16 ) = 0.	PHI ( 16 ) = 0.3003656E 07
PHI-ACTIVE ( 17 ) = 0.	PHI ( 17 ) = 0.1444459E 07
PHI-ACTIVE ( 18 ) = 0.	PHI ( 18 ) = 0.4576032E 07
PHI-ACTIVE ( 19 ) = 0.	PHI ( 19 ) = 0.2782720E 07
PHI-ACTIVE ( 20 ) = 0.	PHI ( 20 ) = 0.2617560E 04
PHI-ACTIVE ( 21 ) = 0.	PHI ( 21 ) = 0.4921650E 04
PHI-ACTIVE ( 22 ) = 0.	PHI ( 22 ) = 0.3347350E 04
PHI-ACTIVE ( 23 ) = 0.	PHI ( 23 ) = 0.5895730E 06
PHI-ACTIVE ( 24 ) = 0.	PHI ( 24 ) = 0.4211500E 04
PHI-ACTIVE ( 25 ) = 0.	PHI ( 25 ) = 0.6913150E 06
PHI-ACTIVE ( 26 ) = 0.	PHI ( 26 ) = 0.1048440E 09
PHI-ACTIVE ( 27 ) = 0.	PHI ( 27 ) = 0.6997171E 08
PHI-ACTIVE ( 28 ) = 0.	PHI ( 28 ) = 0.1744522E 09
PHI-ACTIVE ( 29 ) = 0.	PHI ( 29 ) = 0.1257961E 09
PHI-ACTIVE ( 30 ) = 0.	PHI ( 30 ) = 0.5005136E 07
PHI-ACTIVE ( 31 ) = 0.	PHI ( 31 ) = 0.9288432E 07
PHI-ACTIVE ( 32 ) = 0.	PHI ( 32 ) = 0.1011026E 08

W/WOC= 6.07848

WCLD-W= 0.0822201E 02

ABS(PQG)= 0.1003356E-03 R(IPROP)= -0.7700662E-04

BET = 0.1003356E-02

ITERATION PROCEDURE IS REQUIRED

W/WOC= 6.71081

WCLD-W= 0.1351836E 02

ABS(PQG)= 0.1178240E-03 R(IPROP)= -0.7790668E-04

BET = 0.1178240E-02

ITERATION PROCEDURE IS REQUIRED

W/WOC= 6.57271

WCLD-W= 0.1381116E 02

ABS(PQG)= 0.1162456E-03 R(IPROP)= -0.7700669E-04

BET = 0.1162456E-02

ITERATION PROCEDURE IS REQUIRED

W/WOC= 6.28440

WCLD-W= 0.1447500E 02

ABS(PQG)= 0.1221650E-03 R(IPROP)= -0.7700669E-04

BET = 0.1221650E-02

ITERATION PROCEDURE IS REQUIRED

UNIVERSITY

X( 1 ) = 0.1774000000  
 X( 2 ) = 0.2700000000  
 X( 3 ) = 0.1500000000  
 X( 4 ) = 0.6215667500

C = 4

PHI-ACTIVE ( 1 ) = 1.0	PHI ( 1 ) = 0.0
PHI-ACTIVE ( 2 ) = 0.0	PHI ( 2 ) = 0.1520176E 04
PHI-ACTIVE ( 3 ) = 0.0	PHI ( 3 ) = 0.5878000E 04
PHI-ACTIVE ( 4 ) = 0.0	PHI ( 4 ) = 0.8251040E 06
PHI-ACTIVE ( 5 ) = 0.0	PHI ( 5 ) = 0.1220200E 02
PHI-ACTIVE ( 6 ) = 0.0	PHI ( 6 ) = 0.6000000E 02
PHI-ACTIVE ( 7 ) = 0.0	PHI ( 7 ) = 0.2461520E 02
PHI-ACTIVE ( 8 ) = 1.0	PHI ( 8 ) = 0.4577600E -04
PHI-ACTIVE ( 9 ) = 0.0	PHI ( 9 ) = 0.2646580E 02
PHI-ACTIVE ( 10 ) = 0.0	PHI ( 10 ) = 0.5046167E 02
PHI-ACTIVE ( 11 ) = 1.0	PHI ( 11 ) = 0.0
PHI-ACTIVE ( 12 ) = 0.0	PHI ( 12 ) = 0.4073400E 05
PHI-ACTIVE ( 13 ) = 0.0	PHI ( 13 ) = 0.6080000E 05
PHI-ACTIVE ( 14 ) = 0.0	PHI ( 14 ) = 0.9568900E 05
PHI-ACTIVE ( 15 ) = 0.0	PHI ( 15 ) = 0.9306800E 05
PHI-ACTIVE ( 16 ) = 0.0	PHI ( 16 ) = 0.1129824E 07
PHI-ACTIVE ( 17 ) = 1.0	PHI ( 17 ) = 0.0
PHI-ACTIVE ( 18 ) = 0.0	PHI ( 18 ) = 0.2017616E 07
PHI-ACTIVE ( 19 ) = 0.0	PHI ( 19 ) = 0.7555000E 06
PHI-ACTIVE ( 20 ) = 0.0	PHI ( 20 ) = 0.8128600E 05
PHI-ACTIVE ( 21 ) = 0.0	PHI ( 21 ) = 0.1378270E 06
PHI-ACTIVE ( 22 ) = 0.0	PHI ( 22 ) = 0.6050200E 05
PHI-ACTIVE ( 23 ) = 0.0	PHI ( 23 ) = 0.1787000E 06
PHI-ACTIVE ( 24 ) = 0.0	PHI ( 24 ) = 0.1267960E 06
PHI-ACTIVE ( 25 ) = 0.0	PHI ( 25 ) = 0.2548980E 06
PHI-ACTIVE ( 26 ) = 0.0	PHI ( 26 ) = 0.2020800E 08
PHI-ACTIVE ( 27 ) = 0.0	PHI ( 27 ) = 0.1010000E 08
PHI-ACTIVE ( 28 ) = 0.0	PHI ( 28 ) = 0.6206000E 08
PHI-ACTIVE ( 29 ) = 0.0	PHI ( 29 ) = 0.2000000E 08
PHI-ACTIVE ( 30 ) = 0.0	PHI ( 30 ) = 0.1000000E 07
PHI-ACTIVE ( 31 ) = 0.0	PHI ( 31 ) = 0.2711744E 07
PHI-ACTIVE ( 32 ) = 0.0	PHI ( 32 ) = 0.3000000E 07

UNIVERSITY

W/WOC = 6.0678E

W/CLO-W = 0.1807422E 00

ABS(PQG) = (0.1405146E-08) B(IPROD) = 0.2766475E-04

RFT = 0.2766475E-04

IPROD = 1 0 = 3

W/WOC = 5.02678E

W/CLO-W = 0.0

ABS(PQG) = (0.5530933E-04) B(IPROD) = -0.4561205E-04

RFT = 0.5530933E-04

ITERATION PROCEDURE TO BE USED

X( 1) =	0.1453452E 00	CORR ( 1 ) =	0.4105805E-05
X( 2) =	0.2618784E 00	CORR ( 2 ) =	0.1656190E-06
X( 3) =	0.1530000E-01	CORR ( 3 ) =	0.7706205E-08
X( 4) =	0.6215466E-01	CORR ( 4 ) =	0.2842171E-12

INTERPOLATION PROCEDURE TO BE USED

X( 1 ) = 0.1421309E 00  
 X( 2 ) = 0.2564205E 00  
 X( 3 ) = 0.1530001E -01  
 X( 4 ) = 0.6215668E -01

C = 4

PHI-ACTIVE ( 1 ) = 0.	PHI ( 1 ) =	0.1026548E 02
PHI-ACTIVE ( 2 ) = 0.	PHI ( 2 ) =	0.8350039E 03
PHI-ACTIVE ( 3 ) = 1.	PHI ( 3 ) =	0.0
PHI-ACTIVE ( 4 ) = 0.	PHI ( 4 ) =	0.5244960E 06
PHI-ACTIVE ( 5 ) = 0.	PHI ( 5 ) =	0.1461965E 03
PHI-ACTIVE ( 6 ) = 0.	PHI ( 6 ) =	0.3818750E 03
PHI-ACTIVE ( 7 ) = 0.	PHI ( 7 ) =	0.1999109E 03
PHI-ACTIVE ( 8 ) = 1.	PHI ( 8 ) =	0.4577637E -04
PHI-ACTIVE ( 9 ) = 0.	PHI ( 9 ) =	0.2536617E 02
PHI-ACTIVE ( 10 ) = 0.	PHI ( 10 ) =	0.5226233E 03
PHI-ACTIVE ( 11 ) = 1.	PHI ( 11 ) =	0.0
PHI-ACTIVE ( 12 ) = 0.	PHI ( 12 ) =	0.2595500E 05
PHI-ACTIVE ( 13 ) = 0.	PHI ( 13 ) =	0.5159500E 05
PHI-ACTIVE ( 14 ) = 0.	PHI ( 14 ) =	0.8754000E 05
PHI-ACTIVE ( 15 ) = 0.	PHI ( 15 ) =	0.4894300E 05
PHI-ACTIVE ( 16 ) = 0.	PHI ( 16 ) =	0.1166448E 07
PHI-ACTIVE ( 17 ) = 1.	PHI ( 17 ) =	0.0
PHI-ACTIVE ( 18 ) = 0.	PHI ( 18 ) =	0.2010928E 07
PHI-ACTIVE ( 19 ) = 0.	PHI ( 19 ) =	0.7172769E 04
PHI-ACTIVE ( 20 ) = 0.	PHI ( 20 ) =	0.5667700E 02
PHI-ACTIVE ( 21 ) = 0.	PHI ( 21 ) =	0.1067140E 04
PHI-ACTIVE ( 22 ) = 0.	PHI ( 22 ) =	0.2665000E 02
PHI-ACTIVE ( 23 ) = 0.	PHI ( 23 ) =	0.1420280E 04
PHI-ACTIVE ( 24 ) = 0.	PHI ( 24 ) =	0.1030520E 04
PHI-ACTIVE ( 25 ) = 0.	PHI ( 25 ) =	0.2190050E 04
PHI-ACTIVE ( 26 ) = 0.	PHI ( 26 ) =	0.2621875E 09
PHI-ACTIVE ( 27 ) = 0.	PHI ( 27 ) =	0.7293584E 07
PHI-ACTIVE ( 28 ) = 0.	PHI ( 28 ) =	0.5708722E 09
PHI-ACTIVE ( 29 ) = 0.	PHI ( 29 ) =	0.2133410E 07
PHI-ACTIVE ( 30 ) = 0.	PHI ( 30 ) =	0.1069017E 07
PHI-ACTIVE ( 31 ) = 0.	PHI ( 31 ) =	0.2253328E 07
PHI-ACTIVE ( 32 ) = 0.	PHI ( 32 ) =	0.2875048E 07



AVAILABLE SECTION FEES

(\* AVAILABLE ONLY FOR REG. MEMBERS)

NO	SHADES	MP (PT. KIPS)				
1	8 WF 20.C	57.3	25	12	WF161.C	777.0
2	8 WF 24.C	45.3	26	27	WF 64.C	834.0
3	8 WF 28.C	81.3	27	27	WF102.C	912.0
4	10 WF 25.C	89.5	28	20	WF 08.C	926.0
5	12 WF 27.C	114.0	29	21	WF127.C	954.0
6	8 WF 40.C	119.7	30	21	WF142.C	1071.0
7	12 WF 31.C	132.0	31	20	WF116.C	1134.0
8	12 WF 36.C	154.2	32	22	WF118.C	1242.0
9	14 WF 34.C	163.5	33	24	WF145.C	1243.0
10	12 WF 45.C	194.7	34	27	WF145.C	1254.0
11	16 WF 40.C	219.1	35	23	WF130.C	1398.0
12	10 WF 60.C	225.3	36	22	WF141.C	1529.0
13	16 WF 50.C	278.1	37	23	WF152.C	1674.0
14	16 WF 58.C	319.0	38	26	WF150.C	1740.0
15	18 WF 55.C	326.0	39	26	WF140.C	1869.0
16	21 WF 55.C	375.0	40	26	WF170.C	2001.0
17	18 WF 64.C	396.0	41	26	WF194.C	2201.0
18	12 WF 99.C	455.0	42	23	WF220.C	2509.0
19	21 WF 48.C	480.0	43	26	WF220.C	2429.0
20	24 WF 48.C	525.0	44	26	WF245.C	3024.0
21	24 WF 76.C	600.0	45	26	WF260.C	3228.0
22	12 WF133.C	630.0	46	26	WF280.C	3501.0
23	24 WF 84.C	672.0	47	26	WF300.C	3765.0
24	27 WF 84.C	729.0				

CONTINUOUS SOLUTION AVAILABLE SOLUTION

WF 1 (PT. W/DR) 142.131 152.5101 16 WF 16.0000 SELECTED FROM AVAILABLE WF AND R SHIPPS AS A BEAM MEMBER I  
 WF 2 (PT. W/DR) 188.622 226.1001 15 WF 15.0000 SELECTED FROM AVAILABLE WF AND R SHIPPS AS A BEAM MEMBER I  
 WF 3 (PT. W/DR) 18.700 57.3001 9 WF 20.0000 SELECTED FROM AVAILABLE WF SHIPPS AS A COLUMN MEMBER I  
 WF 4 (PT. W/DR) 62.107 49.3001 8 WF 16.0000 SELECTED FROM AVAILABLE WF SHIPPS AS A COLUMN MEMBER I

OBJ. FUNCTION 6.226 7.014

TOTAL WEIGHT 5.226 0.440