NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30, and subsequent amendments.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30, et ses amendements subséquents.

Distributed Algorithms for File Sorting and Shortest Path Finding

by

Buddy Kin-Hung Leung

A thesis
submitted to the school of graduate studies
in partial fulfillment of the requirements for the
Master of Computer Science Degree

at the
University of Ottawa

ISBN 0-315-62297-0

UNIVERSITÉ D'OTTAWA
UNIVERSITY OF OTTAWA

Acknowledgements

I would like to thank my thesis supervisor, Dr. Jorge Urrutia, for his guidance and advice during the development of this work.

I am very grateful for the encouragement, love and support provided by my family.

Above all, my deepest thanks must go to my parents for their endless love, trust and support during these years.

Financial assistance in the form of teaching and research assistantships from the Department of Computer Science at the University of Ottawa are acknowledged with appreciation.

Preface

During the 1980s the marriage of computers and communications has played an increasingly important role in the development of automation technology. This new technology has allowed us to build more sophisticated and advanced systems ranging from banking to military defense systems. A common characteristic among these systems is to serve large networks with geographically distributed resources (e.g. computers and database). This property along with the two basic design objectives, cost-effective computing and system fault tolerance, form major concerns for these systems now generally called distributed systems.

In order to use these systems effectively and efficiently, some distributed algorithms are designed to solve various problems occurring in them. When we design a distributed algorithm, certain assumptions about the network (or computation) model must be made. These assumptions often create some tradeoffs which occur between the local memory and computation complexity, between the message capacity and computation complexity, and between the time and number of messages used by the algorithms. In this thesis we focus on design and analysis, in particular the tradeoffs, of our new distributed algorithms.

The thesis consists of four chapters. Chapter one lays out the foundations of distributed computing. Chapter two presents new decentralized algorithms for sorting files of integers stored in a distributed and asynchronous network. An optimal communication complexity $O(n \log_2 d)$ is obtained under the assumption that extra memory is available at each site in an almost balanced binary tree network, where n is the file size and d is the number of nodes in the network. When extra memory is removed from the nodes, our algorithm still manages to have a very good complexity of $O(n \log_2^2 d)$. Chapter three presents a new decentralized algorithm in shortest path finding in a distributed and synchronous network, in particular we focus on the termination detection problem. The analysis of this algorithm shows an improved complexity over previous methods. Moreover, in chapters two and three we discuss some tradeoffs in the design of these two algorithms. Finally in chapter four we summarize our results and present some unsolved problems arising from this work. Contents of different chapters are intended to be self-contained so that they can be studied independently of each other. This should ease the approach to the different problems.

Table of Contents

Acknowledgements
Preface

Chapter 1 Background Knowledge
1.1 Models of Distributed Computations 1
1.2 Distributed Computing and Parallel Computing 3
1.3 Important Issues of Distributed Algorithms 4
1.3.1 Differences between Distributed and Non-Distributed Algorithms 4
1.3.2 Types of Messages 5
1.3.3 Issue of Synchronous and Asynchronous Networks 5
1.3.4 Issue of Centralized and Decentralized Controls 5
1.3.5 Issue of Parallelism 6
1.3.6 Issue of Termination 7
1.3.7 Performance Measurement 7
1.4 Tradeoffs in Distributed Algorithms Design 8
1.5 Basic Computation Model and Notations Used in this Thesis 9

Chapter 2 Tradeoffs in Distributed File Sorting
2.1 Introduction 12
2.2 Previous Results 14
2.3 Model and Notations 16
2.4 Sorting With Extra Memory 18
2.4.1 Communication Complexity Analysis of the SDF Algorithm 24
2.4.2 Time Complexity in the Synchronous Case 24
2.5 Sorting Without Extra Memory 26
2.5.1 Communication Complexity Analysis of the SDFW Algorithm 50
2.5.2 Time Complexity in the Synchronous Case 50
2.6 Tradeoffs between Extra Memory and Complexity 52
2.7 Impacts of Local Computation Cost 53
2.8 Closing Remarks 56

Table Of Contents

Chapter 3 Tradeoffs in Distributed Shortest Path Finding

3.1 Introduction 58
3.2 Model and Notations 60
3.3 Finding the Shortest Path in Synchronous Networks 62
3.4 Complexity Analysis of the FSP Algorithm with the Longest Delay 64
3.4.1 For Arbitrary Tree Networks 64
3.4.2 For Linear Networks 66
3.4.3 For Ring Networks 67
3.5 Tradeoffs between Delay Function and Complexity 69
3.6 Closing Remarks 70

Chapter 4 Conclusions 71

Appendix

A. A complete Algorithm for Sorting a Distributed File in Asynchronous Almost Balanced Q-ary Tree Networks with Extra Memory 73
B. A complete Algorithm for Sorting a Distributed File in Asynchronous Almost Balanced Q-ary Tree Networks Without Extra Memory 77
C. A complete Algorithm for Finding the Shortest Path in Arbitrary Synchronous Networks 85

References 87

Chapter 1

Background Knowledge

While distributed computing methodology is becoming an important research area, the concepts of distributed computing have never been clearly and sufficiently defined. Although the main objective of this thesis is to develop new algorithms, it seems necessary to provide some foundations of distributed computing before we describe them. In order to make our explanation as concise and precise as possible, we will use point-form in our description throughout this chapter.

In the following discussion we consider the *process* to be the execution of a program and the *processor* to be a functional unit by which a process can be carried out. A set of processors communicating to each other via a communication network is called a *distributed system* (or a multicomputer). Algorithms for distributed systems are known as *distributed algorithms*.

1.1 Models of Distributed Computations

Although many different models for distributed computations have been suggested in the literature, the practicality of most of these models still remains in question. To model a distributed system, the usual approach is to embed a communication network into a graph in such a way that the vertices of the graph represent the nodes (also called sites, stations or processors through the entire thesis) of the network and the arcs of the graph represent communication links (or channels). Different assumptions may be used in the model; they include:

a) Static or dynamic topology:
   Some distributed system models assume a static topology which disallows node/link failures, insertions or recoveries during the course of a distributed computation. Obviously this restriction is not a fair assumption since a node/link in the network can go down at any arbitrary time. Systems allowing topological changes are called systems with dynamic topology. The dynamic environment causes difficulties in updating topological information, routing tables, etc. Currently most distributed algorithms assume a static topology, e.g. the ring, tree, or complete network. In this case, the

algorithm must be rerun whenever a topological change is detected.

b) Different types of communication channels:
Communication channels may be directed (i.e. messages can be transmitted in only one direction) or undirected (i.e. messages can be transmitted in both directions). Message transmission through a communication channel is either in fixed order or random order. In the case of fixed order, various strategies may be used. Two popular ones are:

i) First-In-First-Out (FIFO) - message transmission along a communication channel is serial; this implies that messages will arrive at the destination in the same order that they were sent from the originating processor.

ii) Priority queue - messages along a communication channel are transmitted according to their priorities, regardless of which one arrives first.

c) Synchronous or asynchronous networks:
In distributed systems, information exchanges are accomplished by sending messages from one node to another via communication channels. By message transmission delays, we mean the time needed to transmit a message from one node to a neighbouring node. In synchronous networks, the message transmission delay is uniform among the nodes and links (i.e. messages sent out by a node will arrive at its neighbor nodes at the same time). In asynchronous networks, the delay time is variable but finite.

d) Centralized or decentralized controls:
By the nature of distributed algorithms, an algorithm is executed in all sites concurrently. Each participant processor has a copy of the algorithm. Information updates are accomplished by exchanging messages among processors. If the process is controlled by a single processor, other processors will just follow instructions given by this controller. Such approach is called the centralized control. If the process is controlled jointly by a collection of processors, each processor will play the same role. Processors will take turns in controlling the process. This is called the decentralized control approach.

e) Local and global control information:

According to [Che83] control information is the combination of logical, topological, quantitative, and chronological information derived from the problem, the graph and the processing information. In general, no or limited global control information, i.e. knowledge about the network or graph, is available to each processor. What is available to each processor is the information related to its neighbor nodes, called local control information.

f) Processors' identification:
The most common assumption is that processors have unique identities and do not share common memory.

1.2 Distributed Computing and Parallel Computing

There has been some confusion in understanding distributed computing and parallel computing, ever though they are different concepts and used for different purposes. First of all, let us define some terminology using in parallel computing. We call a set of processors which share a common memory *parallel computers* (or multiprocessors). Algorithms for parallel computers are known as *parallel algorithms*. The confusion between distributed and parallel computing is mainly caused by a few similar ideas used in these two types of computing models, namely

a) Ideas in centralized distributed systems and single-instruction multiple-data stream (SIMD) parallel computers:
With a SIMD, the central unit issues one stream of instructions which controls all the processors, each operating upon its own memory synchronously.

b) Ideas in decentralized distributed systems and multiple-instruction multiple-data stream (MIMD) parallel computers:
With a MIMD, the processors have independent instruction counters and operate asynchronously on shared memory.

Major differences between parallel computing and distributed computing are outlined below:

a) Processors of a distributed system are geographically distributed and connected by a

communication network, while in parallel computing, processors are usually installed in one physical site.

b) Only local data is allowed in distributed systems, whereas global data is allowed in classical parallel cases.

c) In distributed algorithms the problem is solved under the assumption that processors have **no or limited knowledge** about each other. Each processor executes the same algorithm and exchanges data and results with other processors by sending messages. In parallel algorithms the problem is divided into subsessions which will be computed in different processors separately. The final answer is obtained by combining the results of these subsessions once they are reported to the shared memory.

d) Due to the direct relation between parallel algorithms and parallel architectures (e.g. parallel sorting machines), the size of **hardware** is an important complexity measure for parallel algorithms. However this factor is irrelevant to distributed algorithms.

1.3 Important Issues of Distributed Algorithms

1.3.1 Differences between Distributed and Non-Distributed Algorithms

We will summarize the important differences between distributed algorithms and non-distributed algorithms (i.e. serial and parallel algorithms) as follows:

a) For non-distributed algorithms, complete control information is available at one site. For distributed algorithms, no single node in the network has complete information.

b) For non-distributed algorithms, the control of the process is carried out at one site. For typical distributed algorithms, this responsibility is shared by all processors.

c) For non-distributed algorithms, the CPU cost is the criterion for performance measurement. For distributed algorithms, the number of message transmissions is the

most important factor, due mainly to the transmission delays which are relatively large compared to the time spent in local computations. (More of this will be discussed later).

d) For parallel algorithms, processors share a common memory, which is not true for distributed algorithms.

1.3.2 Types of Messages

There are two types of messages: control messages and data messages. Control messages contain control information and serve purposes such as initiating a process, locating a network's component, requesting data, terminating a process, etc. Data messages contain data and serve for the purpose of data exchanges.

1.3.3 Issue of Synchronous and Asynchronous Networks

In reality, the time to execute the steps of a process is unpredictable due to the variations in computation time caused by different inputs, system scheduling policies, variations in the individual process speeds, etc. For these reasons, transmission delays are not uniform. This makes the assumption of asynchronous networks a natural choice. Unfortunately asynchronous behavior leads to a serious issue regarding the correctness of the distributed algorithm. The correctness issue arises because during the execution of an algorithm, operations from different processes may interleave in an unpredictable manner.

Nowadays more and more efforts are directed to the development of computation methods for asynchronous networks. The traditional approach is to simulate synchronous behavior in an asynchronous environment so that the synchronous algorithms can be used. Another approach is to design new algorithms to directly fit in an asynchronous environment. The first approach is the more popular one but the second approach may lead to a better solution.

1.3.4 Issue of Centralized and Decentralized Controls

Algorithms with centralized control are easier to design and involve fewer message transmissions, since useful information can be made available at the controller station. However, algorithms with decentralized control are preferable because of the following:

a) They avoid the 'bottleneck' situations occurring at the controller station, which appear in the centralized case when most of the control message transmissions are concentrated in one single node.

b) They are more reliable, since the whole process does not depend on any single mechanism or any single node, e.g. the controller.

c) They are more flexible since insertions/deletions of new nodes/links do not affect the overall view of the algorithm.

d) They reduce on-site technical expertise since the control task is shared by all processors.

In order to design a decentralized algorithm with a reasonable number of message transmissions, it is justifiable to use centralized techniques in some parts of the algorithm.

1.3.5 Issue of Parallelism

One of the advantages of distributed systems is their parallel computing capability. In order to reduce the computation time of a distributed algorithm, it is important to design the algorithm so as to maximize parallelism. Some interesting observations regarding this issue are listed below:

a) Algorithms with centralized control, in many cases, can normalize the sequence of operations taking place at different processors. This improves the parallelism of the process. A major drawback is the introduction of a longer waiting period from the time a processor finishes the current instruction to the time it receives the next instruction from the controller.

b) Algorithms designed for synchronous networks provide greater parallelism.

c) The network topology affects the parallelism of the computation. For example, finding the maximum of n integers distributed in n nodes takes O(n) time in a linear topology but only $O(\log_2 n)$ time in a binary tree topology.

1.3.6 Issue of Termination

Consider a distributed system of n processors, each of which is either active or passive. An active processor is computing, sending or waiting for a message, and becomes inactive afterwards. A passive processor can become active only by receiving a message. A distributed computation is said to have terminated if all processors are passive, that is, no more computing and no more messages are in transit. Termination detection of a distributed computation is a hard and non-trivial problem because processors are aware of their local states (i.e. local information) but have only limited information about the states of other processes in the network (i.e. global information).

Many termination detection algorithms with various assumptions about the underlying model of distributed computation have been proposed in recent years ([Ma82],[To84],[Fr80],[MiCh82], [DiFeGa83]). The basic idea of these methods is to circulate one or more so-called tokens (i.e. a signal) around the network so that one of the processors (usually the token initiator) can realize if all processors are passive.

1.3.7 Performance Measurement

For serial and parallel computations, the CPU processing cost is the main factor for measuring algorithm performance. However, this is no longer the case in distributed computing since the message transmission delay is variable and lengthy. As a result of this, message (or communication) complexity and traffic complexity are more important factors in the complexity evaluation of distributed algorithms. Message complexity is the number of messages (for both control and data) exchanged during the process. Traffic complexity not only concerns the number of messages, it also takes into the consideration the size of each message (e.g. number of bits). This often leads to local storage problems, i.e. is there enough space in each processor to store the incoming messages?

Finally, time complexity is the total execution time ,which is calculated under the assumption that message transmission delay takes one unit of time. In other words when we calculate the time complexity, an asynchronous network is always treated as a 'synchronous' network. Thus time complexity is a less important factor in evaluating a distributed algorithm.

1.4 Tradeoffs in Distributed Algorithms Design

To design a distributed algorithm, we have to make certain assumptions about the computational model. These may include message capacity, memory storage at each node, etc. It is important for the designer to recognize the tradeoffs generated by these assumptions. The following example will illustrate how these assumptions could affect the algorithm performance.

Assume we have a distributed system of three nodes, S_1, S_2 and S_3. Any two nodes are connected by a communication link and exchange information by sending messages. We also assume that each message can carry up to x integers and that each site can store at most c integers in its local non-shared memory. Initially there are two integers in each node. Our goal is to rearrange these integers so that integers in S_1 are greater than integers in S_2 and S_3.

An easy solution is that S_1 will collect two integers from each of S_2 and S_3, compare them against its own integers, keep the two largest values, and return the smaller integers to S_2 and S_3. To achieve this, four messages are enough. However one should not overlook the required conditions hidden behind this solution, namely, the message capacity x to be two integers and the local storage c to be 6 integers in S_1. With different conditions, the complexity for the same algorithm will vary dramatically. For the case of x=1 and c=4, the number of messages needed is 8.

In fact, tradeoffs do not only occur between the capacities and the message complexity, they also occur between the capacities and the time complexity. For the case of $c \geq 4$ in our example, S_1 may request data from S_2 and S_3 at the same time. However, if c=3, then S_1 can only send data requests to S_2 and S_3 one at a time. The given condition in the latter case allows lower parallelism

of the algorithm and thus results in a longer process time.

The above example shows the effect that different assumptions can create in distributed algorithms design. Other types of tradeoffs exist. In particular, when local computation cost is taken into consideration (as we will show 'why' in Section 2.1), tradeoffs occur between the topological choice and time complexity. A good example of this is the file sorting problem which will be discussed in detail in Section 2.8. We should also point out that in most algorithms, these tradeoffs are often ignored by designers. As a result, they sometimes fail to take advantage of existing assumptions.

1.5 Basic Computational Model and Notation

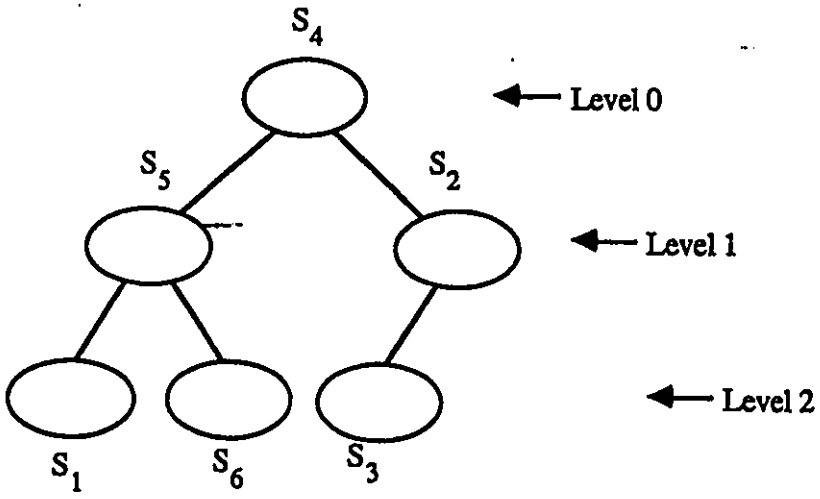
In this section we will describe the basic computational model and notation used throughout this thesis. Additional assumptions for the model and notation for the sorting problem and shortest path problem will be given in Chapters 2 and 3.

A distributed communication network of size d and capacity c is a set $T=\{S_1, ..., S_d\}$ of d sites where each site can store c records in its local non-shared memory. (Recall that site, station, node and processor are used interchangingly in this thesis.) We assume that each site has a unique identification number, id. The underlying graph of T is an undirected and connected graph G(S,E) where S represents a set of nodes and E a set of edges. The distance between two adjacent nodes is one. Two sites S_i and S_j are neighbors if there is a communication link (S_i,S_j) connecting them. The communication links are undirected and FIFO. Two neighbor stations in the network communicate by sending messages via communication links. We will also assume that each message has capacity x, that is, each message carries a counter, an id or up to x integers. The diameter of the network T, called DIAM(T), is the maximum of the distance over any pair of sites of the network.

An almost balanced binary tree network T (for short an AB binary tree network) is a distributed communication network whose underlying graph is an almost balanced binary tree (see Figure 1.1). In an AB binary tree network, each station S_j in T is aware of the id of its father F_j as well as the id's of left and right children. For every element S_j in T we denote T_j the subtree of T

induced by S_j together with all its descendants. We will also denote by LN_j the level number of the site S_j. For instance in the network shown in Figure 1.1, T_2 is the subtree with nodes S_2, S_3 and S_5, and $LN_4=0$, $LN_5=LN_2=1$, $LN_1=LN_6=LN_3=2$, etc.

We define AB q-ary tree networks, arbitrary tree networks, and any other rooted tree networks in the same way as described above.



An almost balanced tree network

Figure 1.1

Chapter 2

Tradeoffs in Distributed File Sorting

In this chapter we study the problem of sorting a file of n integers distributed over d stations in a communication network. We present new decentralized algorithms for almost balanced q-ary tree networks. These algorithms can easily be adapted to other topologies. The first algorithm requires extra memory space available at each node to store some temporary values. The second algorithm removes such a requirement but only works on tree networks with postorder labeling. Their message complexities are $O(n/x \log_q d)$ and $O(n/x \log_q^2 d)$ respectively, where x is the message capacity. Tradeoffs between memory requirements and communication complexity are then discussed. Under similar conditions, our algorithms improve existing methods. Finally, we analyze the impact of local processing on time complexity for synchronous networks.

2.1 Introduction

Let $F=\{a_1< a_2<...< a_n\}$ a large file of n integers distributed over a communication network of d sites $T=\{S_1,S_2,...,S_d\}$. The file sorting problem on T consists of relocating the elements in F in such a way that at the end, any element of F stored in S_i is smaller than any other element of F stored in station S_j, i<j.

In this chapter we present two new distributed sorting algorithms based on almost balanced binary tree networks. In the first one, we assume that we have extra memory available at each node of our tree. Under this assumption, a simple Sorting Distributed File algorithm (from now on called SDF) with communication complexity $O(n \log_2 d)$ is obtained. We will show that this result is optimal. (It is interesting to notice that most existing distributed sorting algorithms implicitly assume extra space available at each site, and yet they fail to take advantage of this fact). The SDF algorithm is similar to Zak's sorting algorithm [Za84], however his algorithm was designed under the assumption that only one integer is stored in each node.

Our second algorithm, called Sorting Distributed File Without extra memory space (SDFW), eliminates the need for extra storage space, and still manages to obtain a very good communication complexity $O(n \log_2{}^2 d)$.

The two algorithms presented in this chapter can be easily adapted to work in an arbitrary tree network T in such a way that if its diameter DIAM(T) is k, our SDF and SDFW algorithms have an O(k n/x) and $O(k^2 n/x)$ communication complexity respectively, where x is the message capacity. Notice that in the average k is approximately equal to $\log_2 d$. In particular, for almost balanced q-ary trees, the communication complexities for the SDF and SDFW algorithms are $O(\log_q d\ n/x)$ and $O(\log_q{}^2 d\ n/x)$ respectively.

It is customary in distributed computing to ignore the cost of local computations at each site in the network, due mainly to the relatively cheap cost of local computations with respect to the cost of communication activities. Nevertheless in some cases this could lead to misleading conclusions; for instance the communication complexity of an NP-complete problem is polynomial if enough storage space is available at each site.

In the last section of this chapter the impact of local computations in the time complexity of synchronous networks is studied. We will show that in some cases, we can not ignore the cost of local computations. A specific example concerning sorting in complete synchronous networks is studied in which the cost of local processing has a strong effect in the time complexity of our algorithms. In the algorithms presented in this chapter for almost balanced q-ary trees, when the impact of local computations is taken into consideration, the total complexities of our algorithms (including local and communication costs) will be increased by a factor of $\log_2 q$.

2.2 Previous Results

In the literature, several algorithms have been proposed to solve the distributed file sorting problem using different assumptions and approaches. In this section we assume that each station originally holds an equal number n/d of integers. The file sorting problem is to relocate these elements according to some distribution order P={1,2,...,d} so that at the end, any element stored in $S_{p(i)}$ is smaller than any other element stored in station $S_{p(i+1)}$.

Wegner first presented an algorithm [We84] which uses centralized control and works only on linear networks. His algorithm is based on quicksort. To simplify its description, we assume that stations in the linear network as well as the distribution order are labeled in ascending order and S_d is the predetermined controller station.

In the first iteration, the medium of the file elements, called pivot value z, is calculated by S_d and distributed to other stations. Upon received z, station S_i will partition its elements into two sets LOW_i and $HIGH_i$ so that elements of LOW_i are smaller than or equal to z and elements of $HIGH_i$ are greater than z. When the local partition is finished, station S_i will report the number of elements in LOW_i to the controller S_d. Thus S_d can determine the split point S_p in such a way that the total number of elements in $\{S_1,...,S_p\}$ is equal to $\Sigma_{i=1,d-1} |LOW_i|$. Lets denote $FL=\{S_1,...,S_p\}$ and $FR=\{S_{p+1},...,S_{d-1}\}$. Using S_p as a reference point, any two adjacent stations can exchange their elements in order to achieve the following: FL and FR will contain all LOW's and all HIGH's respectively. In other words at this point the file is partitioned into two halves, one half in FL and another in FR. This ends the first cycle. Elements in FL and FR can be sorted in a recursive way using the same method.

A careful study of Wegner's algorithm shows that his method assumes 2d extra memory space at each station and n/d message capacity. The traffic complexity is $O(n \log_2 d)$ in the average and $O(n^2)$ in the worst.

Rotem, Santoro and Sidney have designed a decentralized algorithm [RoSaSi85] for completely connected networks based on the k^{th} selection method. Their algorithm consists of two steps: (1) each station S_i finds its partition points $e_{p(i)}$ which in fact is the $(i+n/d)^{th}$ element and broadcasts this information to other stations, and (2) each station inserts elements to their destinations according to the partition points, i.e. elements fall in between $e_{p(i-1)}$ (for S_{i-1}) and $e_{p(i)}$

will go into S_i. Step one is itself a selection problem which can be solved by any of the existing selection algorithms [Fr83], [Ma83] and [RoSaSi83$_{1,2}$]. The element insertion in step two can be done in a straight forward manner.

This algorithm implicitly assumes an extra memory of n/d and a message capacity of 1. The algorithm has the traffic complexity of $O(d^2 \log_2 d)$ in the average and $O(d^2 \log_2 n) + O(n)$ in the worst.

Cheung has developed a decentralized algorithm [Che88] for completely connected networks. His algorithm is an iterative algorithm in which each station takes turn as the controller according to P. Thus in the i^{th} cycle of operation, processor $S_{p(i)}$ is the controller. Within one cycle there are a number of iterations. In each iteration, the controller $S_{p(i)}$ uses a mathematical formula to calculate the number of expected elements u_i which has a least value of n/d minus the number of elements in $S_{p(i)}$. Processor $S_{p(i)}$ then sends messages containing its largest element value $r_{p(i)}$ to each of other stations to request for $u_i/(d-1)$ elements whose values are below $r_{p(i)}$. If sufficient elements are collected, $S_{p(i)}$ will transfer the control to the next station $S_{p(i+1)}$ who subsequently starts the $(i+1)^{th}$ cycle's operation. Otherwise, $S_{p(i)}$ will recalculate the number of elements u_i and repeat the iteration again. The algorithm terminates when processor $S_{p(n)}$ has collected its expected elements at the end of the n^{th} cycle.

Cheung's algorithm is strictly decentralized in a sense that each processor shares the control responsibility and has an equal amount of workload. It has a traffic complexity of $O(n+d^2 \log_2 n)$ in the average and $O(d^2 n)$ in the worst. However in some cases his algorithm requires $n-d^2/2$ extra memory space at each station and n/d message capacity. This happens, for instance, when elements of the file F are initially stored in stations in the reverse order of P such that any element in $S_{p(i)}$ is larger than any other element in station $S_{p(i+1)}$.

2.3 Model and Notations

In addition to the assumptions described in section 1.5, the following requirements will be used in our sorting algorithms.

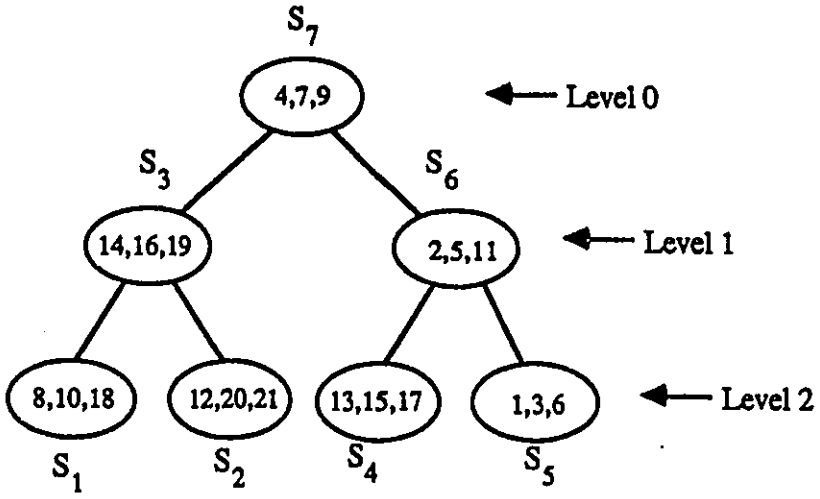
a) The networks under consideration are asynchronous networks.

b) When we describe our first algorithm in sections 2.4, we will assume that each message has capacity one (x=1), that is each message contains a single value which is a counter, an id, or an integer.
Our second algorithm, as described in sections 2.5, requires the message capacity to be two (x=2).
However, the above assumptions will be revised in section 2.6 when we discuss the tradeoffs in these algorithms. In this discussion, if we increase the message capacity to $1 \le x \le n/d$, the communication complexity will be substantially decreased.

c) For a rooted tree network T, we will assume that the elements $\{S_1, ..., S_d\}$ of T have a postorder labeling (see Figure 2.1). Under this assumption, S_d will always correspond to the root site of T.

An α distribution of F over the network T is a partition of F into d disjoint subsets $F_1,...,F_d$ such that F_i is stored in S_i and $|F_i| \le \alpha \le c$, i=1,...,d and c is the local storage at each site. The distributed file F is said to be sorted in T if for $a \in F_i$ and $a' \in F_j$, $a < a'$ iff $i < j$.

Our main objective in this chapter is to obtain distributed sorting algorithms for AB binary tree networks which produce postorder sorting of α distributed files. Two main cases are considered, in the first case $\alpha \le c/2$ (i.e. extra space is required at each site), in the second case $\alpha = c-2x$, recall that x is the message capacity. Although our algorithms will be designed for AB binary tree networks, they can easily be modified to work for general rooted tree networks and almost balanced q-ary tree networks.



A file F={1,...,21} stored in a balanced binary tree network with sites labelled in post order form.

Figure 2.1

2.4 Sorting With Extra Memory

In this section, we present our first sorting algorithm (SDF algorithm) for sorting files in AB binary tree networks T of d sites each with capacity c. We will make the following assumption: at each site S_i of our AB binary network $|F_i|=\alpha \leq c/2$; that is at each site we have at least twice as much memory available as records stored in S_i, i=1,...,d. Notice that this is a very realistic assumption since in most real life problems extra space is usually available.

The SDF algorithm consists of two phases:

Phase 1: Wake up and initialize.

Phase 2: Sort the elements.

A brief description of each of these phases is now given:

Phase 1: Wake up and initialization

Initially, all stations in the network are sleeping. One or more stations wakes up. As soon as a station wakes up or is woken up by a WAKE_UP message, it sends a WAKE_UP message to its children and father.

Phase 2: Sort the elements

Each station will proceed as follows, depending on whether it is a leaf station, an intermediate station (i.e. neither a leaf or the root), or the root station.

Leaf stations:

As soon as a leaf station S_i wakes up, it sends a message SEND_MIN(a_k) to its father containing the minimum value a_k of F stored in S_i.

Intermediate stations:

Once a station S_j (not a leaf station or the root station S_d) receives an SEND_MIN message from each of its children, S_j will know the value of the smallest element (say a_u) stored in the subtree T_j rooted at S_j. At this point, S_j will send a_u to its father F_i.

Root station S_d:

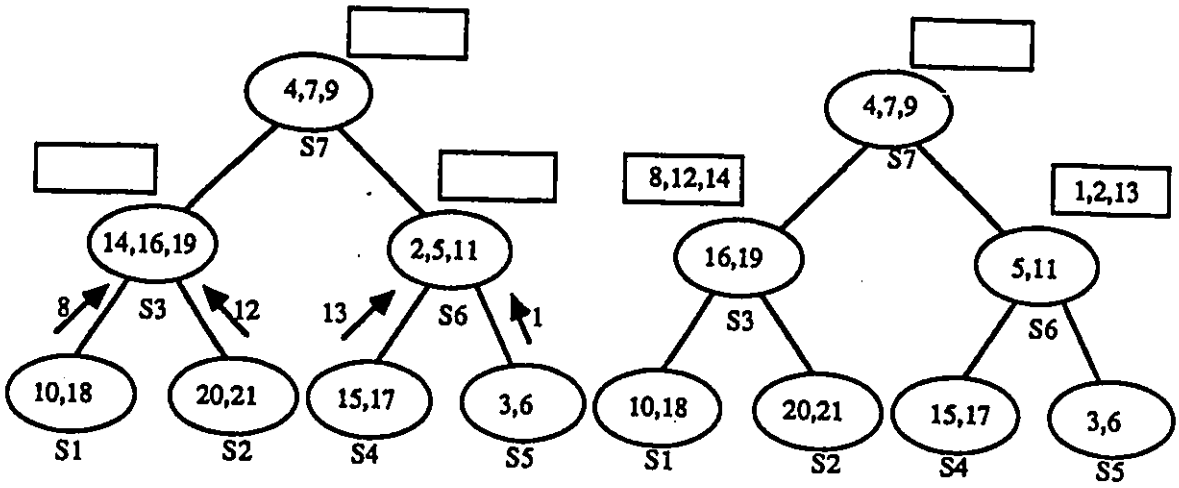
When the root station S_d receives an SEND_MIN message from each of its children, it will know the value a_1 of the minimum element of F stored in our communication network T. At this point S_d will send a_1 down to the leftmost leaf station S_1 where a_1 will be permanently stored.

Updating the minimum element in an arbitrary station S_j:

As soon as a station S_j sends SEND_MIN(a_u) to its father F_j, it can proceed to calculate the new minimum element stored in T_j (given that S_j is not a leaf station) as follows:

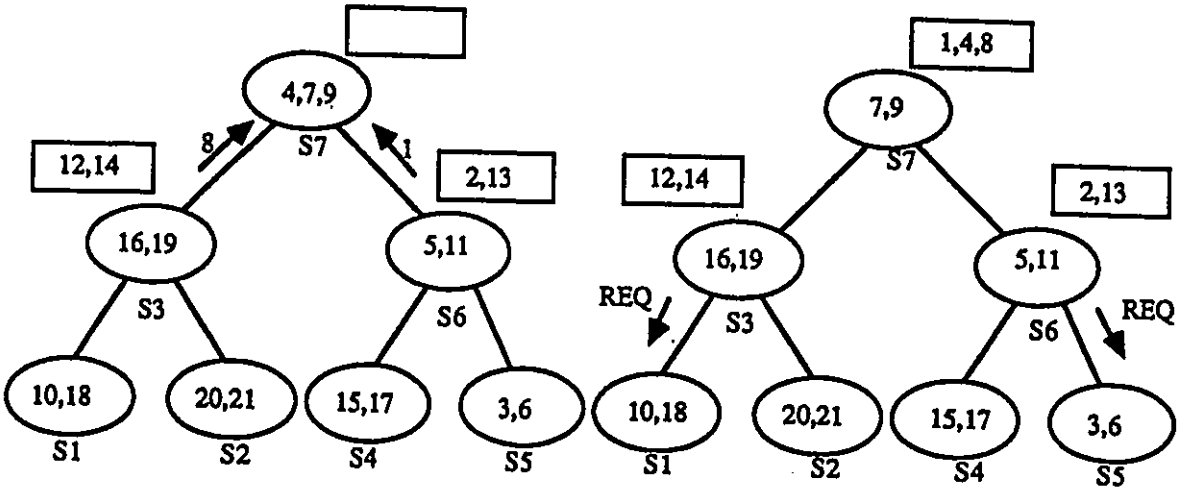
Immediately after S_j sends SEND_MIN(a_u) to its father F_j, S_j will issue a message REQ_MIN to request the next minimum value from the child S_c where SEND_MIN(a_u) came from. If all integers originally stored at S_c have previously been sent out, S_j will request such a minimum value from another child. Of course, no message exchange is involved if SEND_MIN(a_u) originated from S_j itself. As a result, the processes in all stations can be kept running in parallel.

Using the same strategy, the values of a_2, a_3, ... can be determined at the root station. As soon as the root station determines the i^{th} element a_i, it directs it towards the node $S_{\lfloor i/d \rfloor}$, where a_i will be stored in the extra memory available at $S_{\lfloor i/d \rfloor}$.



(a) Leaf stations send min. to their fathers

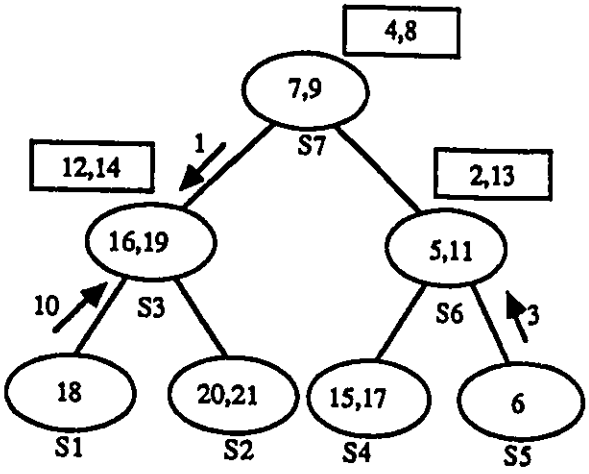
(b) Intermediate stations S_3 & S_6 can now calculate minimums of their subtrees



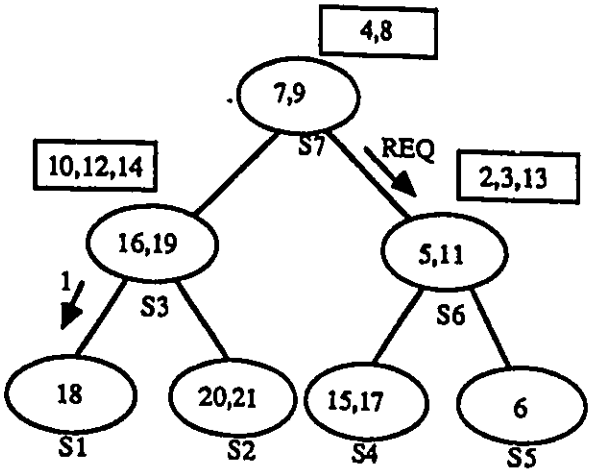
(c) Sending min. of left subtree and min. of right subtree to the root

(d) Requesting the next min. from S_1 & S_5 where previous minimums came from

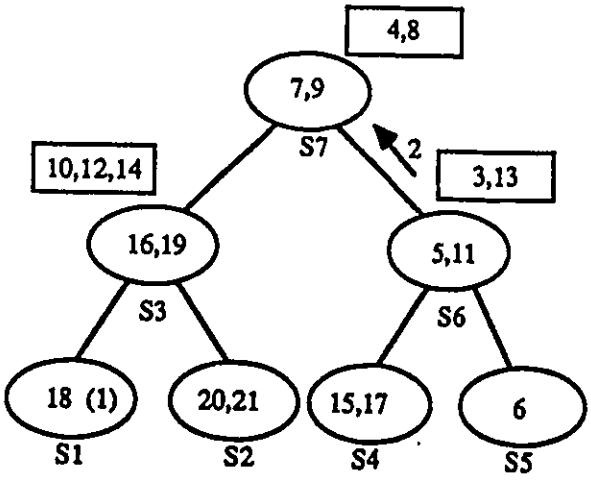
Figure 2.2 (a)-(d) Snapshots of the sorting algorithm SDF (extra memory is assumed)



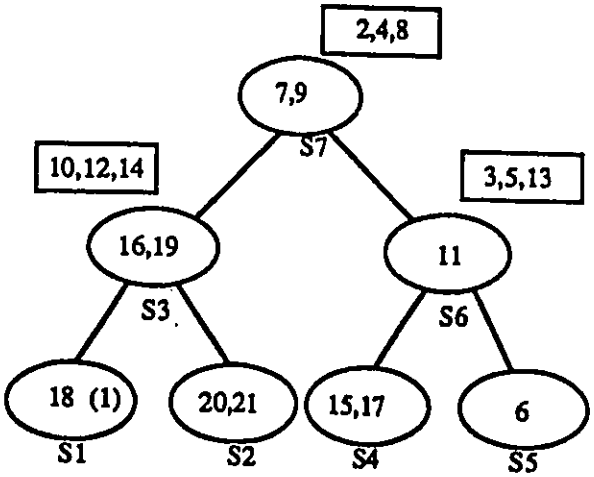
(e) Root sends the smallest integer a_1 down



(f) Root requests next min. from its child S_6 while a_1 is approaching its destination S_1

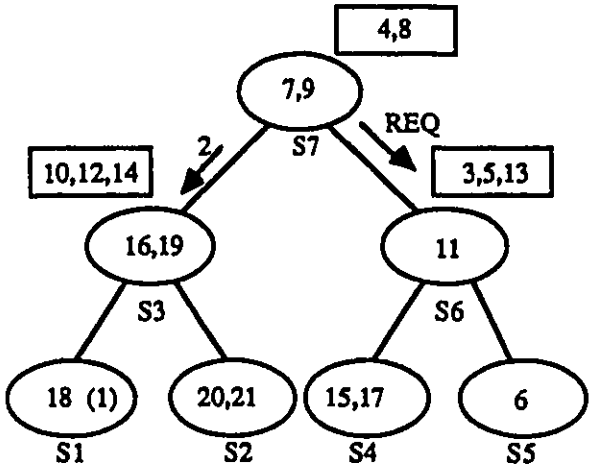


(g) The next min. is being sent to the root and a_1 arrives at its destination S_1

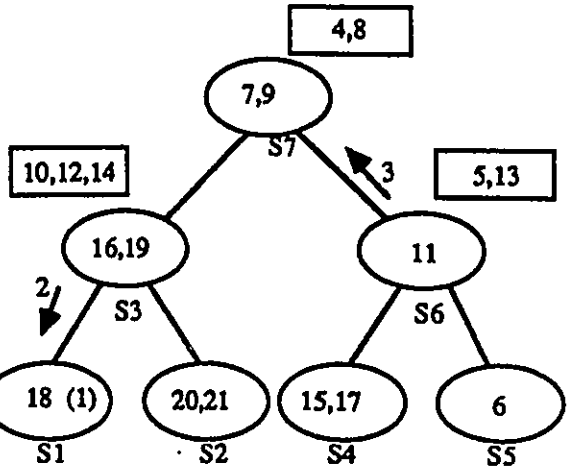


(h) Determining the next smallest integer a_2 at the root

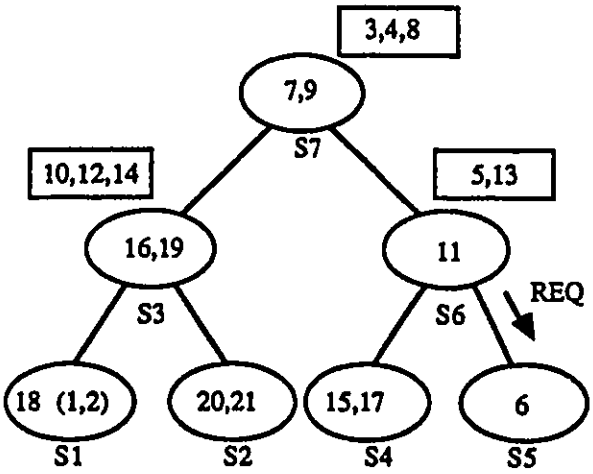
Figure 2.2 (e)-(h) Snapshots of the sorting algorithm SDF (extra memory is assumed)



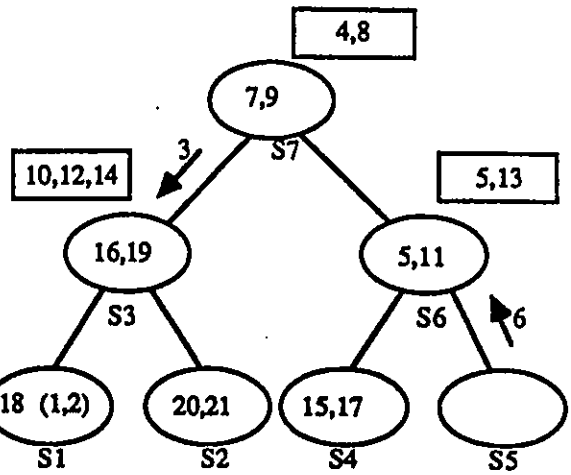
(i) Root sends a_2 down and then requests the next min. from its child $\overline{S_6}$



(j) The next min. is sent to the root while a_2 is approaching its destination S_1

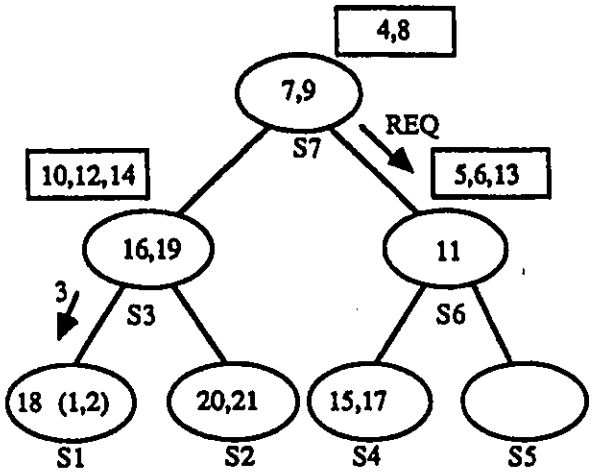


(k) a_2 arrives at its destination while update of min. at each level is in process

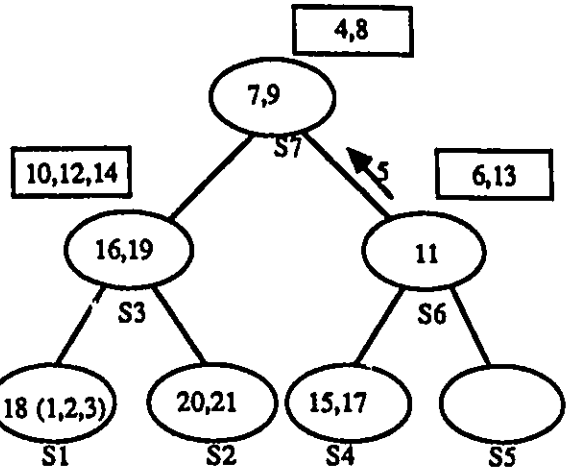


(l) Root sends a_3 down and updating is still in process

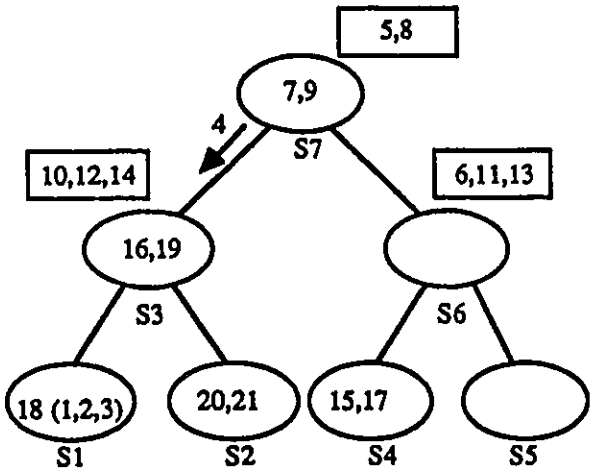
Figure 2.2 (i)-(l) Snapshots of the sorting algorithm SDF (extra memory is assumed)



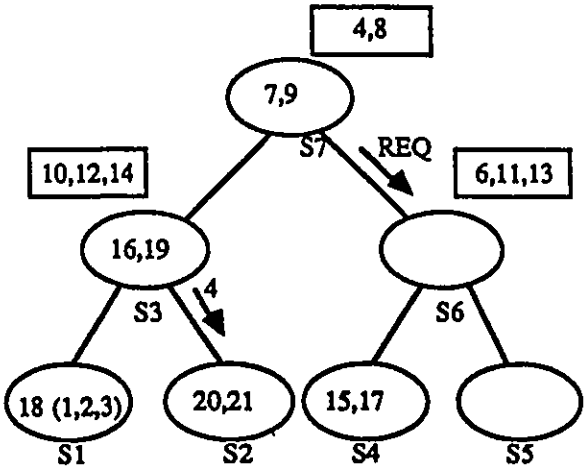
(m) Root requests the next min. from S_6 while a_3 is approaching $\overline{S_1}$



(n) a_3 arrives at its destination S_1 and the 4^{th} smallest a_4 is updated at the root



(o) The root sends a_4 down



(p) a_4 approaches its destination, i.e. station S_2, since station S_1 is full

Figure 2.2 (m)-(p) Snapshots of the sorting algorithm SDF (extra memory is assumed)

Termination conditions:

The termination of phase 2 is not hard to determine. We know that a station will terminate the local process after a) it has received all desired integers, and b) it has sent out all the integers which do not belong to itself. Globally, we can determine if the entire process is completed as follow: When a leaf station terminates its local process, it sends a termination message to its father. When a station has completed all sorting tasks and also collected termination notices from all its children, it sends a termination notice to its father. Therefore, once the root has received such notices from all its children, it declares the end of the computation.

2.4.1 Communication Complexity Analysis of the SDF Algorithm

In phase 1 each edge connecting a site S_i to its father F_i transmits at most 2 wake up messages. Thus the communication complexity of phase 1 is 2d. The communication complexity in phase 2 is at most $3n \log_2 d$. This follows from the observation that before an element of F can reach its destination say S_j, it has to go up from its original site S_i to the root site S_d and then come down to S_j. In the worst case, a value originating from the bottom level will have to go to the root station S_d and down to the bottom level again. This requires $2 \log_2 d$ messages per element of F. Thus, for a file of n records, the message complexity is $2n \log_2 d$. On top of this, before an element moves up one level from a station S_i to its father station F_i, a request message has to be sent from F_i to S_i. Thus the total communication complexity of the SDF algorithm is $2d + 3n \log_2 d$.

Since a message carries one value, the traffic complexity for SDF is also $2d+ 3n \log_2 d$.

To prove optimality of our algorithm, suppose we have a balance tree with d elements with a post order labeling. Assume that the value stored in each site is the label of its mirror image with respect to the root of our tree. It is easy to see in this example that since any value has to move from its original position to the mirror image position, the number of messages required is $O(n \log_2 d)$.

2.4.2 Time Complexity in the Synchronous Case

In order to calculate the time complexity of the SDF algorithm for synchronous networks, we must assume that a message takes exactly one unit of time to transmit from one station to another adjacent station. Trivially phase 1 takes at most 2 $\log_2 d$ units of time.

The time complexity of phase 2 is a bit harder to calculate. First we notice that the root site S_d of our network will receive a_1 in exactly $\log_2 d$ time. However, the time needed to calculate each of a_2, a_3,... is constant, since once a site S_j sends a message SEND_MIN(a_u) to its father it will immediately request a new minimum from one of its children (as explained in the updating minimum values section of phase 2). It is now an easy matter to verify that after the integer a_1 has reached S_d, the amount of time needed for each of a_2, a_3,... to reach S_d is at most 2 time units. Thus the time complexity of phase 2 is $\log_2 d+2n$ and the total time complexity of SDF is $3\log_2 d+2n$.

The SDF algorithm as presented here works in binary tree networks. In fact, the SDF algorithm can be easily modified to work in any AB q-ary tree network and any labeling of its nodes (not necessarily the post order labeling used in this section) using similar ideas to those presented in [Za84]'s paper. Thus we can obtain the following result:

Theorem 2.1 Distributed sorting in AB q-ary tree networks with message capacity of 1 can be accomplished in $2d+3n \log_q d$ communication and traffic complexity and in $3\log_q d+2n$ time complexity (in synchronous networks).

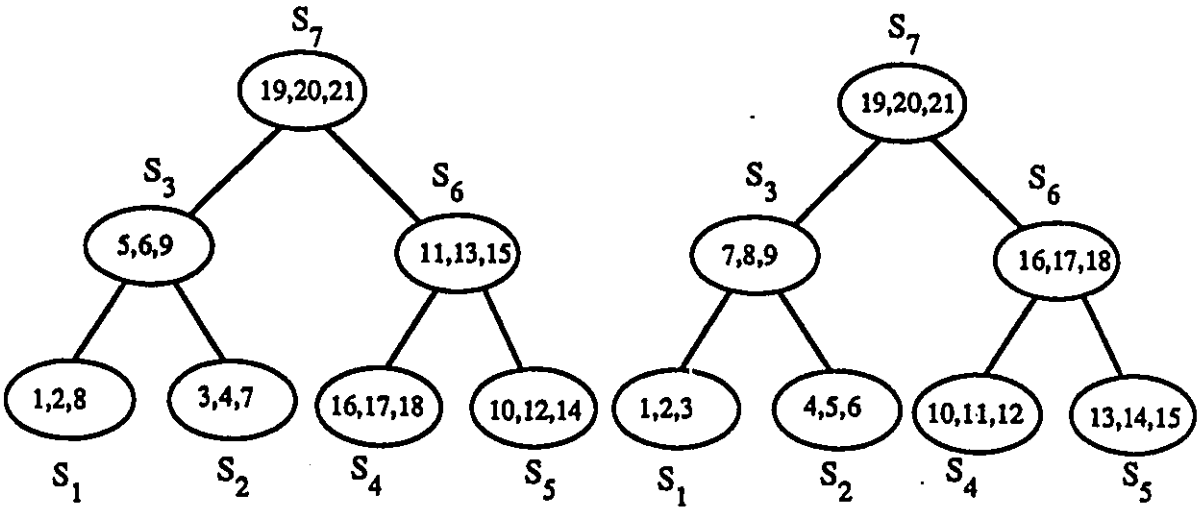
2.5 Sorting Without Extra Memory

In this section, we study the distributed sorting problem in a AB binary tree network in which **the assumption of extra space is removed.** More formally, we assume that each site S_i has capacity c and holds the same number α=n/d of elements of our distributed file F where α=c-2x, x is the message capacity. For the basic discussion of our method we consider x to be 2, that is at each site the set of records R_i stored in S_i occupy most of the memory space available at S_i.

The algorithm presented in this section (called the SDFW algorithm) can be thought of as a recursive algorithm. During the first iteration of our algorithm, we will exchange elements of F among the sites of the network in such a way that, at the end of the first iteration:

a) The α largest elements are stored in S_d.
b) All the elements of F stored at sites in the left subtree of the root station are smaller than those elements stored in the right subtree. (See Figure 2.4(a)).

The procedure will then be repeated with the subtrees T_i of T with root stations S_i at level 1, 2,... until we reach the leaf stations of T. At this point our algorithm terminates. (See Figure 2.4(b)).



(a) Configuration of Fig.2.1 after the first pass (b) After the second pass the file is sorted

Figure 2.4

Our algorithm again consists of two phases:

Phase 1: Wake up and initialize

Phase 2: Sort the elements.

We proceed now to give description of Phases 1 and 2:

Phase 1: Wake up and initialize

Initially, all stations in the network are sleeping. One or more stations wake up. As soon as a station wakes up or is woken up by a WAKE_UP message, it sends a WAKE_UP message to its children and father. Thus all stations in the network will eventually become active.The root station S_d, when it wakes up, will set a counter Iternum to 1 indicating the first iteration of the algorithm. This counter will be incremented by 1 in each iteration of computation.

Phase 2: Sort the elements

Each station, except the leaf stations, will issue a message REQ_MAX to its children. After this, each station will proceed as follows, depending on whether it is an intermediate station, a leaf station, or the root station.

Intermediate stations:

i) If S_j receives a request message REQ_MAX from its father F_j, two possibilities arise:

a) S_j is waiting for an answer to a request message from one of its children. In this case, S_j has to wait until its own request is answered. Once it receives an answer to its request it will proceed as in b).

b) S_j has received a message SEND_MAX(a_l, S_u) from its left child and a message SEND_MAX(a_r, S_v) from its right child (or in the case that it has only one child, a message from its unique child). At this point two cases may occur:

1) If a message PUSH_REQ has arrived at S_j, S_j will proceed as in ii) b).

2) If no PUSH_REQ has arrived at S_j , S_j will proceed as in iii).

ii) A message PUSH_REQ(a_b,S_b) from F_j to S_j carries an element a_b which is heading to its destination S_b. S_j must also send the maximum available element stored in T_j to F_j when S_j receives this message.

If S_j receives a message PUSH_REQ, two possibilities arise:

a) S_j is waiting for an answer for a request message from one of its children. In this case, S_j has to wait until its own request is answered. Once it receives an answer to its request it will proceed as in b).

b) If S_j has got the maximum elements from its children, two cases may occur:

1) If S_j is the destination for element a_b, i.e. $S_b=S_j$, a_b will be stored in S_j and then proceed as in iii).

2) If S_j is not the destination for element a_b, i.e. $S_b \neq S_j$, S_j will determine if the element a_b should be transferred to the left child C_{j1} or right child C_{j2} in order to get to its destination S_b. When left route is taken, S_j compares the left maximum element (a_l,S_u) with (a_b,S_b). If a_b is greater than a_l, then a_b should really be the new left maximum. Thus we must update the left maximum by **swapping** (a_l,S_u) and (a_b,S_b). After this, S_j will proceed as in iv). The same principle applies to the case in which the right route is taken.

iii) S_j will determine $a'=\max\{a_r, a_l, a_j\}$, where a_j is the maximum available element stored in S_j. Notice that a' was first originated from the station S_w=(either S_u, S_v or S_j) and is the largest available element of F stored in T_j. S_j will then proceed as in vi). If all elements of T_j have already been processed, S_j notifies F_j that there are no more elements to be processed.

iv) S_j will determine $a'=\max\{a_r, a_l, a_j\}$, where a_j is the maximum available element stored in S_j. Notice that a' was first originated from the station S_w=(either S_u, S_v or S_j) and is the largest available element of F stored in T_j. If left route was taken in ii) b) 2), two cases may occur:

a) If $S_w=S_l$, we will like to push the element a_b down and request the maximum from C_{jl}. To do this, S_j will send a message PUSH_REQ(a_b, S_b) to C_{jl}.

b) If $S_w \neq S_l$, we will just want to push element a_b down without requesting an update maximum from C_{jl}. To accomplish this, S_j will send a message PUSH(a_l, S_b) to C_{jl}.

At this point, S_j will proceed as in vi). If all elements of T_j have already been processed, S_j notifies F_j that there are no more elements to be processed. The same principle applies to the case in which the right route was taken in ii) b) 2).

v) A message PUSH(a_b,S_b) from F_j to S_j carries an element a_b which is heading to its destination S_b. If S_j receives a message PUSH_REQ, two possibilities arise:

a) If S_j is the destination for element a_b, i.e. $S_b=S_j$, a_b will be stored in S_j.

b) If S_j is not the destination for element a_b, i.e. $S_b \neq S_j$, S_j will wait until it has got the maximum elements from both of its children. At this point, S_j will determine if the element a_b should be transferred to the left child C_{jl} or right child C_{j2} in order to get to its destination S_b. When the left route is taken, S_j compares the left maximum element (a_l,S_u) with (a_b,S_b). Two cases arise:

1) If a_b is not greater than a_l, S_j will retransmit message PUSH(a_b, S_b) to C_{jl}.

2) If a_b is greater than a_l, then a_b should really be the new left maximum. Thus, S_j should push a_l down instead of to push a_b down. To do this, S_j will transmit a message PUSH(a_l, S_b) to C_{jl}. We must also update the left maximum by swapping (a_l,S_u) and (a_b,S_b).

vi) S_j will send a message SEND_MAX(a', S_w) to its father. Immediately after S_j sends a' to its father, S_j will proceed to find the next largest element (if any) stored in T_j by issuing a message REQ_MAX to request the next maximum element (if it exists) from the child where a' came from. At this point S_j will wait until an answer (containing the next largest element stored in T_w or a notification that all elements of T_w have been already processed) from this child arrives. Once S_j gets the answer it will be able to calculate the next largest element a'

(if any) stored in T_j. At this point, S_j will have to wait until it receives a request message REQ_MAX from its father and then proceed as in i).

vii) When S_j receives a message ASSIGN(a_p,S_p), S_j will store this element in itself **permanently** if S_j is the destination of a_p, i.e. S_j=S_p. Otherwise, S_j will retransmit this message ASSIGN(a_p,S_p) to its left child C_{jl} or right child C_{jr}, depending on which route the element a_p should take in order to get to S_p.

Leaf station:

viii) As soon as a leaf station S_i receives a request message REQ_MAX from its father F_i, it will send SEND_MAX(a', S_i) containing its next largest element (if any) a' and its own id S_i to F_i.

ix) When receiving a message ASSIGN(a_p,S_p), a leaf station S_p will keep the element a_p **permanently**. When receiving a message ASSIGN(a_p,S_p), a leaf station S_p will store the element a_p temporarily.

Root station S_d:

x) The root station S_d waits until it receives the largest element (a_l S_u) stored in its left subtree and the largest element (a_r, S_v) stored in its right subtree together with the id's of the stations where they were originally stored. At this point S_d will be able to determine the largest value a_n of F (it is either one of a_l, a_r or the largest element say a_m originally stored in S_d) (see Figure 2.5(d)). As in the SDF algorithm, this procedure will enable S_d to determine the largest, second largest, ... elements of F.

xi) For k=1,2,..,n, once S_d has determined the k^{th} largest element a_{n-k+1}, it will process as follows depending whether such element is an α element:

a) For the α elements a_{n-k+1}, $1 \le k \le \alpha$:
Since the α largest elements of F have to be stored in S_d, as soon as S_d determines the k^{th} largest element a_{n-k+1} of F, $1 \le k \le \alpha$, and the site S_t where it came from, $S_t \neq S_d$, it will keep a_{n-k+1} **permanently** and direct the k^{th} smallest element a_i stored in S_d

towards S_t using a message PUSH_REQ(a_l, S_t) (see Figure 2.5(e)).

b) For the non-α elements a_{n-k+1}, $\alpha<k\leq n$:

1) If $a_l>a_r$ then we will exchange the positions of a_l and a_r, that is, a_l will be directed towards S_v where it will reside for the rest of this iteration (using a message ASSIGN(a_l, S_v) as in Figure 2.5(h.2)) while a_r will be moved into S_u (using a message PUSH_REQ(a_r, S_u)). Notice that the location of a_r may change again during the current iteration (see Figure 2.5(h.1-4)).

2) If $a_l<a_r$ then a_l and a_r will return to their original positions (using a message ASSIGN(a_r, S_v) and a_r will remain in S_v for the rest of the current iteration. S_d will then proceed as in xii).

xii) S_d requests the next largest element available (if any) at the child of S_d where the k^{th} largest element a_{n-k+1} came from (using a message REQ_MAX). Once S_d receives an answer to this request, it will be able to determine the $k+1^{th}$ largest element and the process x) will now be repeated again for k+1.

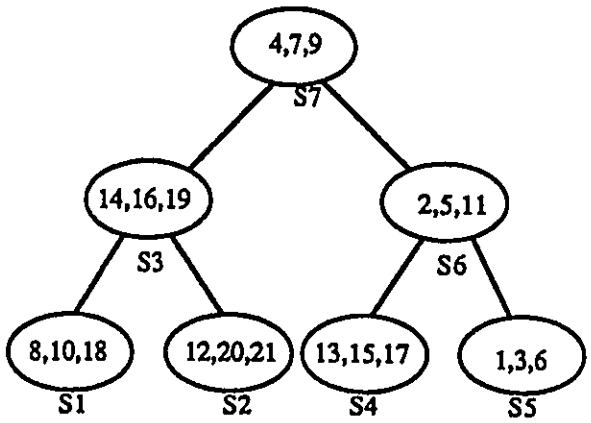
The above steps i) to xii) will be repeated until all elements stored in the right subtree are smaller than those stored in the left subtree, at which point the current iteration will be finished.

To determine the next largest element:

The root station gets two values, one a_l from its left son and one a_r from its right son. Notice that in both the left and right subtrees a_l and a_r determine each a winning path WP_l and WP_r (i.e. the paths from the original sites of a_l and a_r to the root station S_d). Using these paths we can now exchange a_l and a_r (if $a_l>a_r$) as described before.

Notice that to determine the second largest, third largest,..., element in the right and left subtrees, we need only to update values along the winning paths WP_l and WP_r of a_l and a_r in the left and right subtrees, thus the extra amount of work needed to obtain next largest element will be at most $2\log_2 d$.

j	(left) Dj1	(right) Dj2	(left) Pj1	(right) Pj2
1	nil	nil	(−∞,)	(−∞,)
2	nil	nil	(−∞,)	(−∞,)
3	nil	.nil	nil	nil
4	nil	nil	(−∞,)	(−∞,)
5	nil	nil	(−∞,)	(−∞,)
6	nil	nil	nil	nil
7	nil	nil	nil	nil

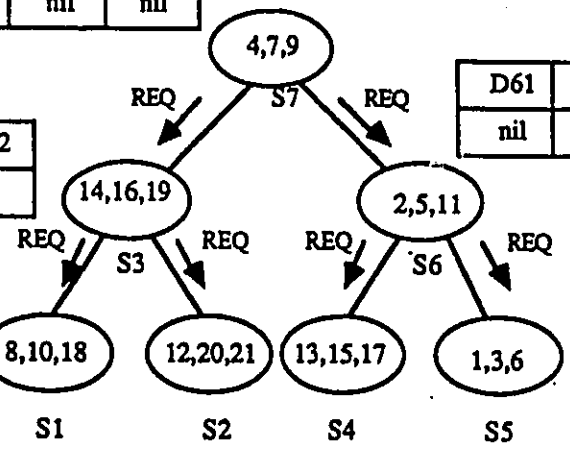


(a) Initial set up of the SDFW algorithm for the network shown in Figure 2.1

D71	D72	P71	P72
nil	nil	nil	nil

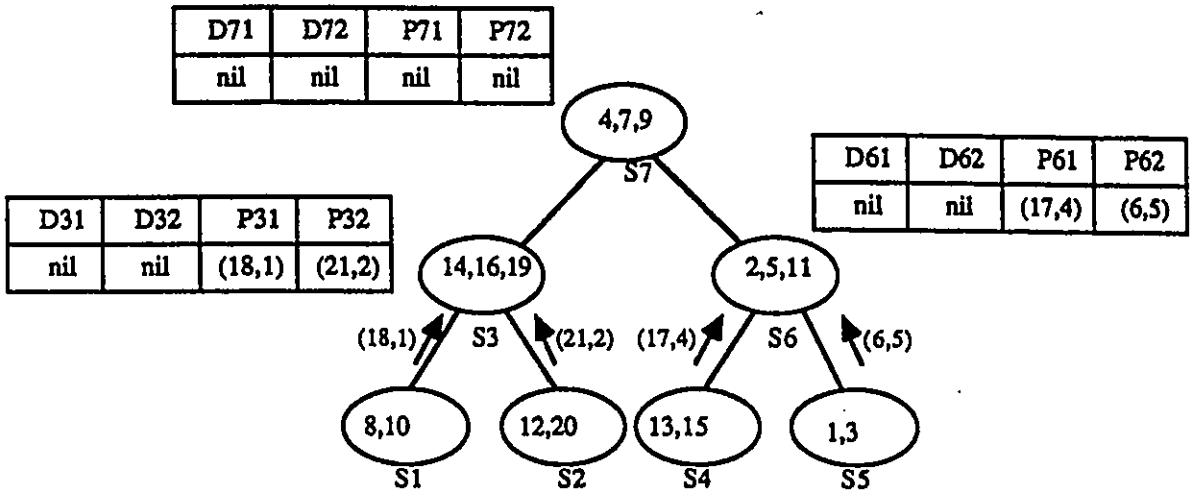
D31	D32	P31	P32
nil	nil	nil	nil

D61	D62	P61	P62
nil	nil	nil	nil

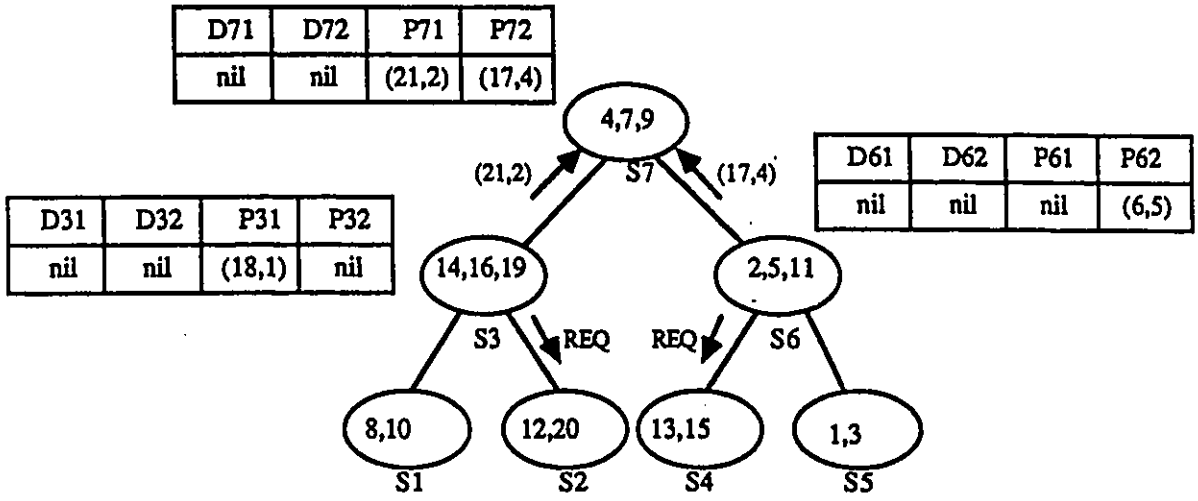


(b) When the condition P_{ji}=D_{ji}=nil is fulfilled, station S_j sends message REQ to its child C_{ji} to request the max. stored at C_{ji}

Figure 2.5 (a)-(b) Snapshots of the sorting algorithm SDFW (NO extra memory)

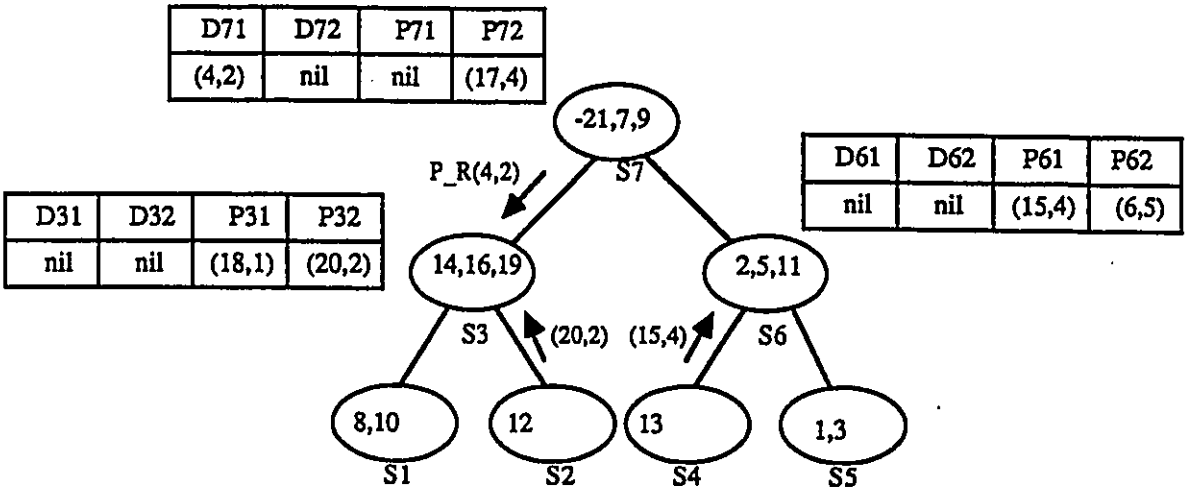


(c) After station S_3 has received max. from all its children (P_{31}≠nil & P_{32}≠nil), S_3 is now in the local process to determine its own max. The same thing occurs in station S_6.

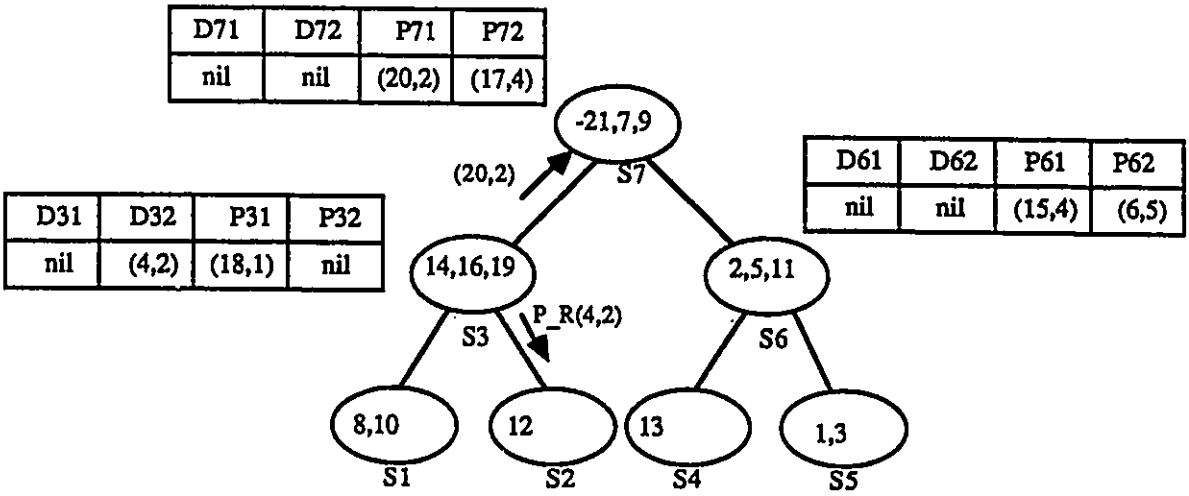


(d) The root S_7 is in the local process to determine its max. while S_3 and S_6 are updating their maximums (Notice that no REQ is sent from S_3 to S_1 since P_{31}≠nil)

Figure 2.5 (c)-(d) Snapshots of the sorting algorithm SDFW (NO extra memory)

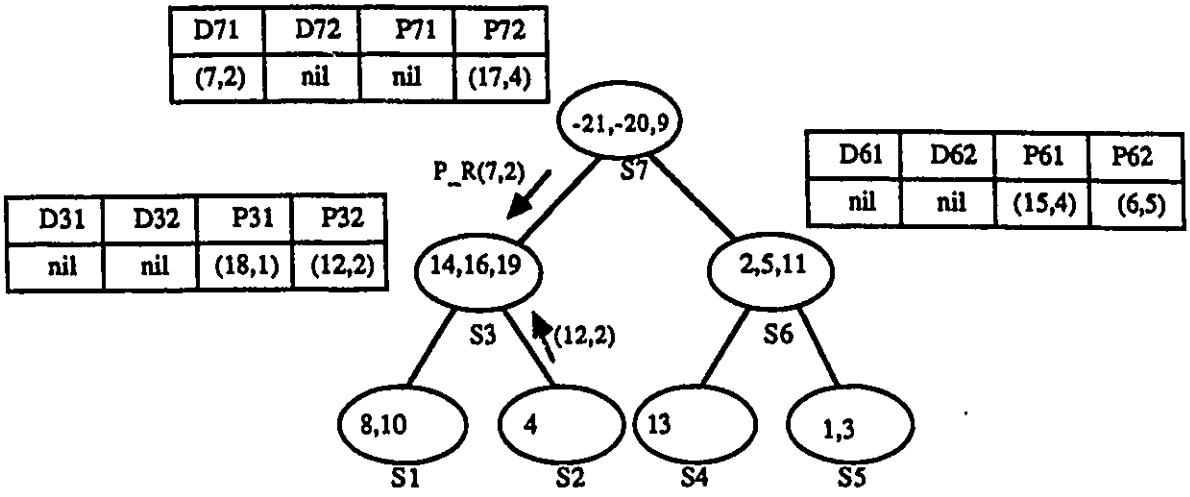


(e.1) After keeping the α-integer 21, root S_7 sends P_R to its left child S_3 to **push** its smallest integer 4 towards destination S_2 (where 21 came from) and also **request** next max. from S_3

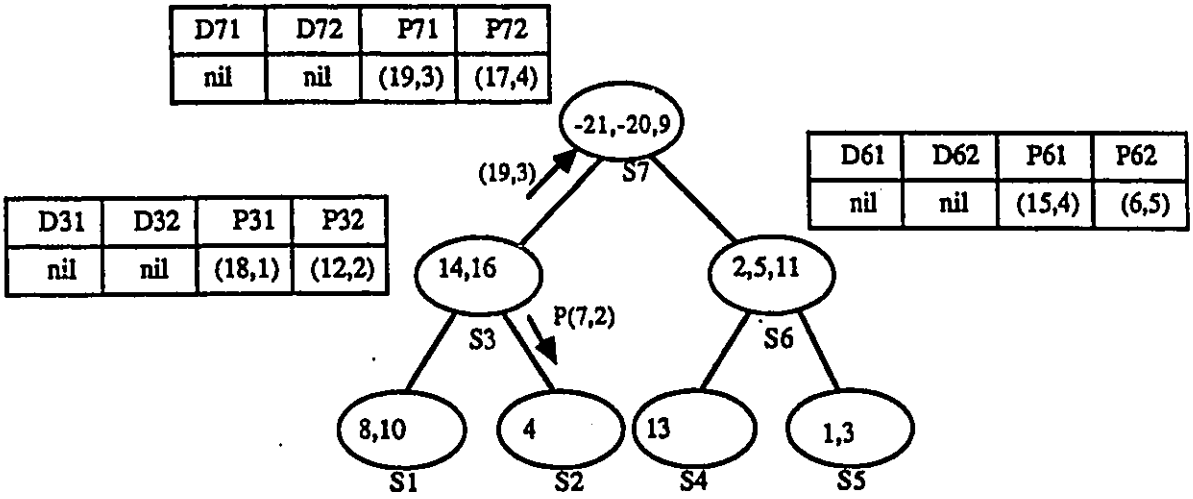


(e.2) S_3 pushes 4 one level down towards its destination (Message P_R is used here since P_{32}=nil, i.e. next max. required). Upon received next max. 20 from left child, S_7 changes D_{71} to be nil.

Figure 2.5 (e) Snapshots of the sorting algorithm SDFW (NO extra memory)

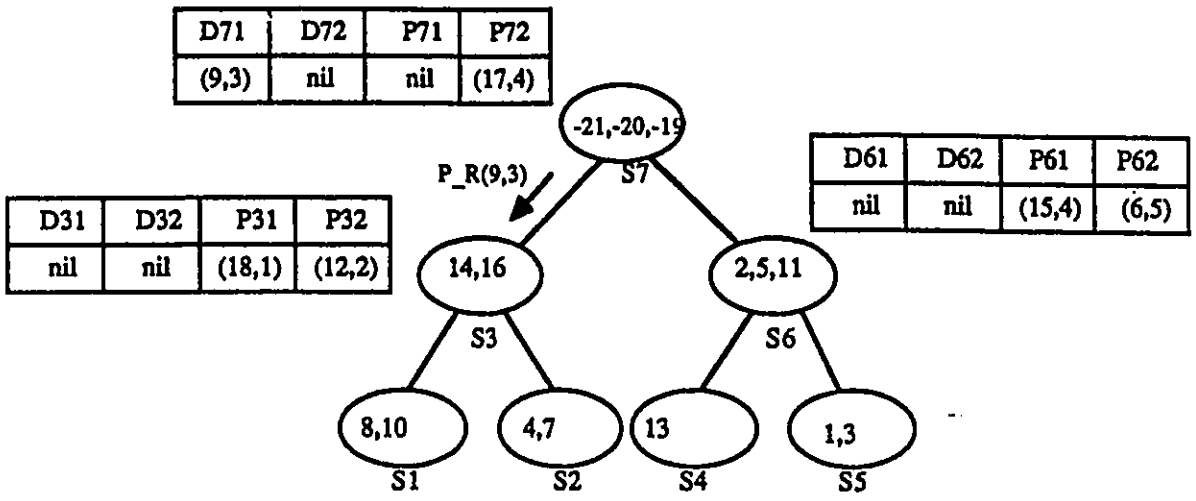


(f.1) Root S_7 keeps the second α-integer 20 and then sends the smallest integer 7 toward its destination S_2 (where 20 came from)

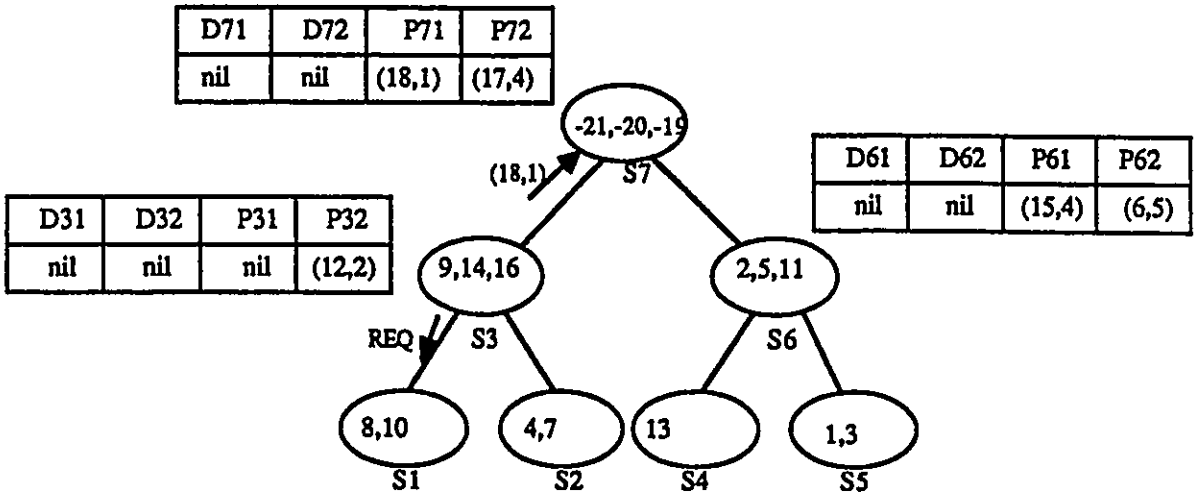


(f.2) S_3 pushes integer 7 towards its destination S_2 (Message P is used here because $P_{32} \neq$ nil)

Figure 2.5 (f) Snapshots of the sorting algorithm SDFW (NO extra memory)

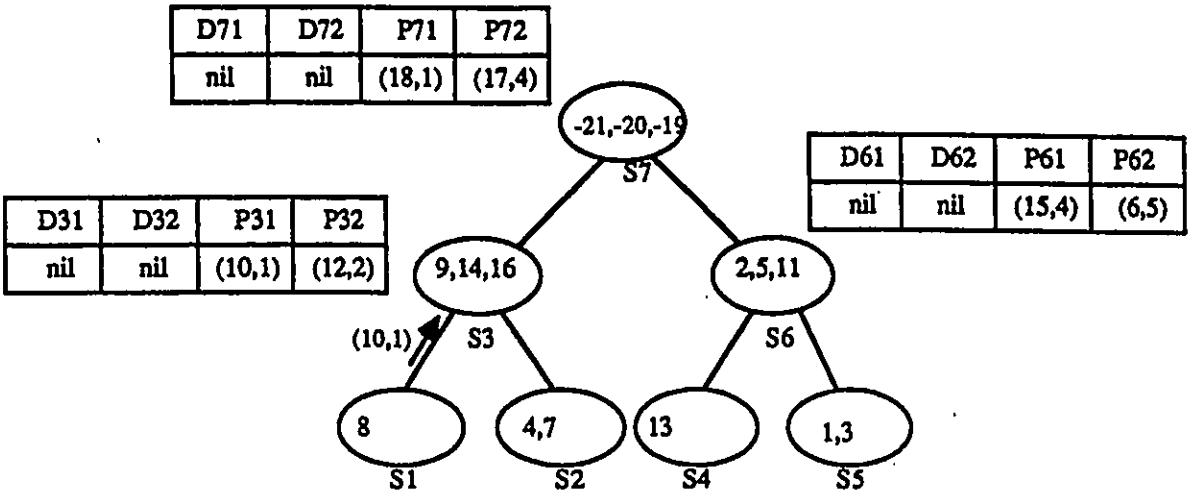


(g.1) Root S_7 keeps the last α-integer 19 and then sends integer 9 to station S_3

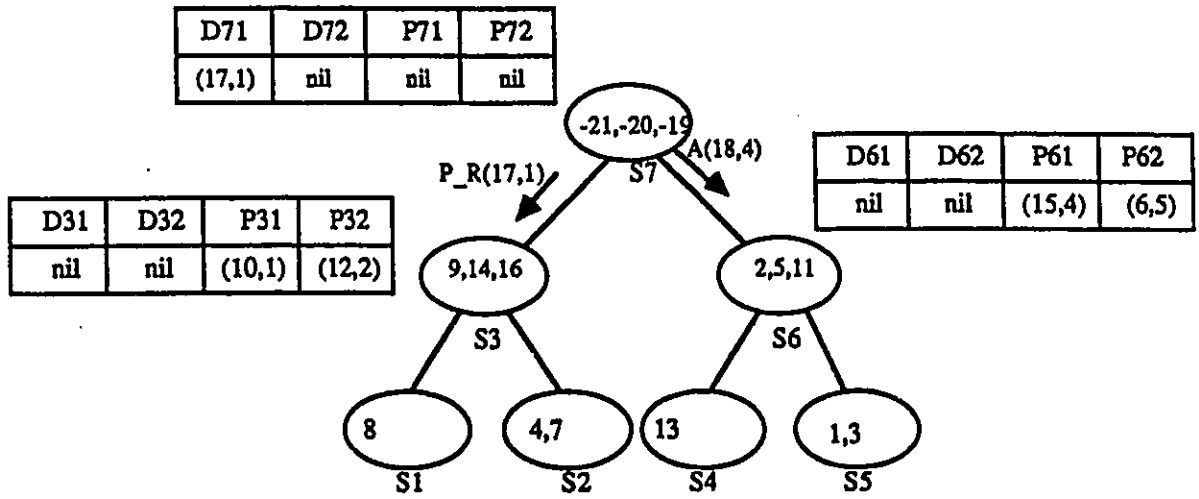


(g.2) S_3 keeps 9, sends current max. 18 up, and requests next max. from S_1. (since D_{31}=P_{31}=nil)

Figure 2.5 (g) Snapshots of the sorting algorithm SDFW (NO extra memory)

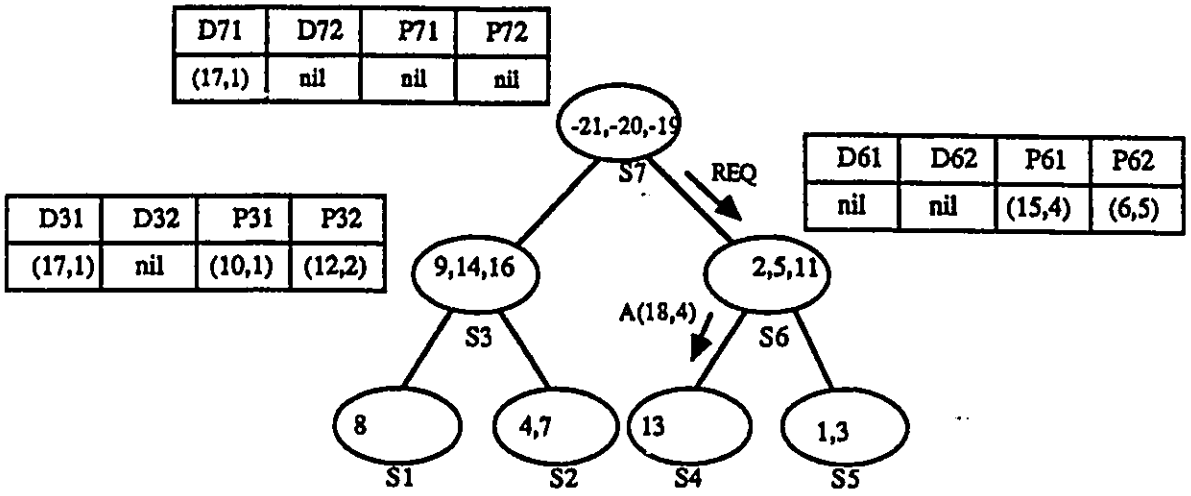


(h.1) S_1 returns the updated max. to S_3 while the root S_7, which has collected all α-integers, is comparing P_{71} and P_{72} to determine the $(\alpha+1)^{th}$ largest integer

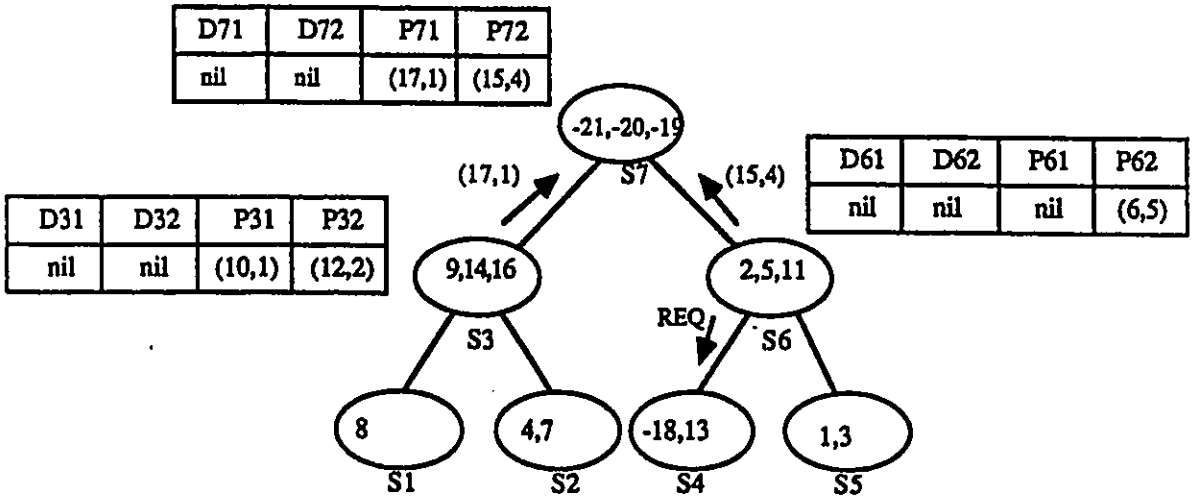


(h.2) In root S_7, since left max. P_{71}=(18,1) is greater than right max. P_{72}=(17,4), these two integers are exchanged so that 18 is sent (in message A) to S_4 and 17 is pushed (in message P_R) towards S_1. (Note that 17 will be compared to upcoming max. along the path from S_7 to S_1 in order to determine next left max.)

Figure 2.5 (h.1-2) Snapshots of the sorting algorithm SDFW (NO extra memory)

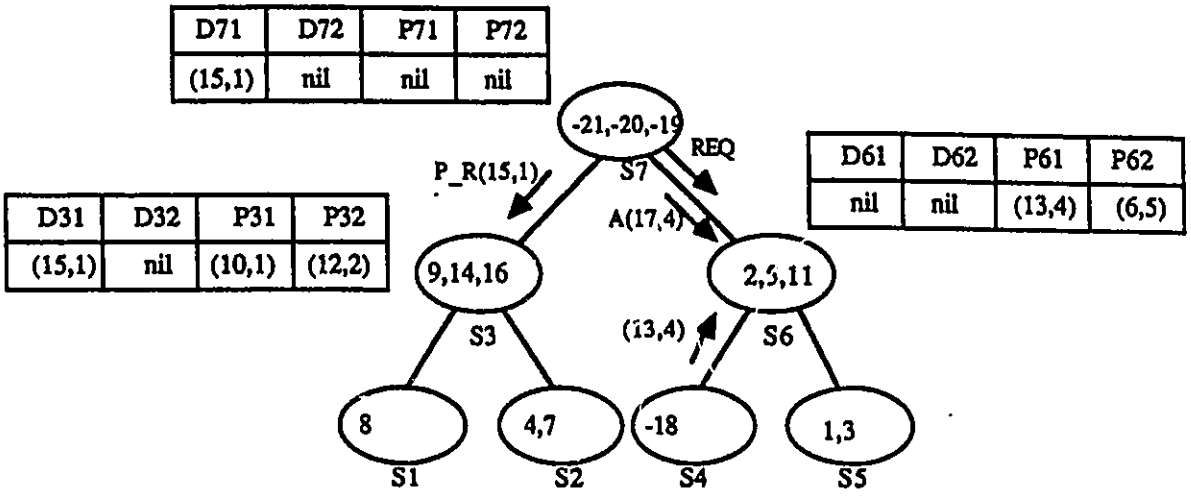


(h.3) The 4th largest integer 18 is approaching its destination S_4 while S_7 is requesting from its right child S_6 the next max. because $D_{72}=P_{72}=nil$

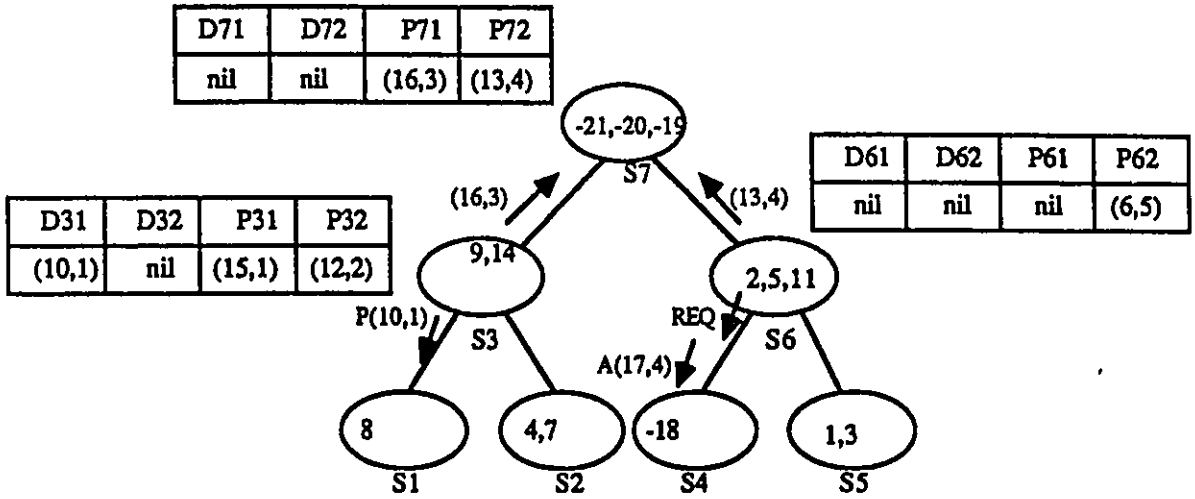


(h.4) Integer 17, which stopped at S_3 on the way to S_1, rebounds back because it is greater than the upcoming max., i.e. $D_{31}>(P_{31}$ & $P_{32})$. Now root S_7 has 17 from the left and 15 from the right. On the other hand, integer 18 has reached S_4 and will stay there permanently.

Figure 2.5 (h.3-4) Snapshots of the sorting algorithm SDFW (NO extra memory)

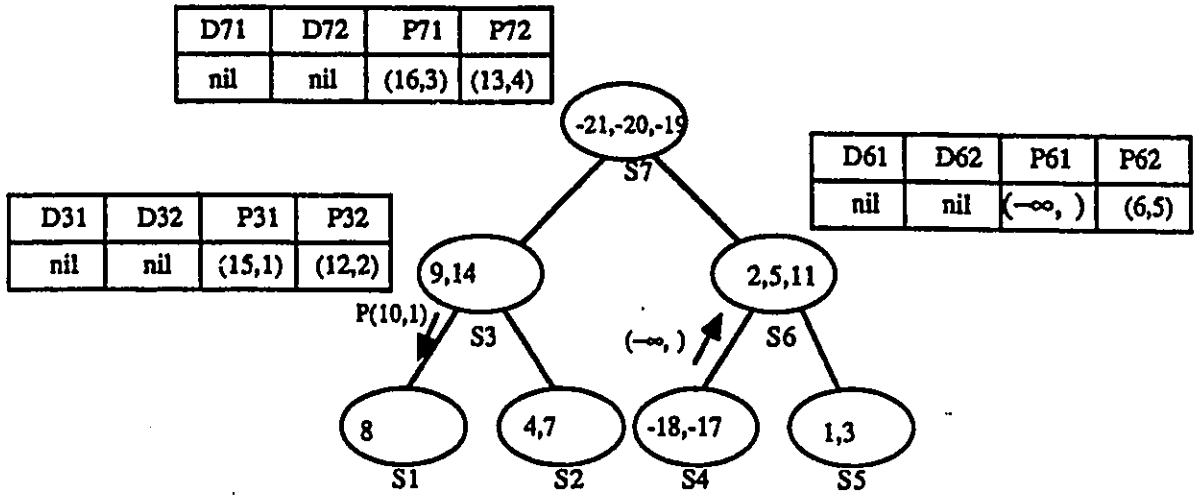


(i.1) In root S_7, since P_{71}=(17,1) > P_{72}=(15,4), these two integers are exchanged so that 17 is sent (in message A) to S_4 and 15 is pushed (in message P_R) towards S_1

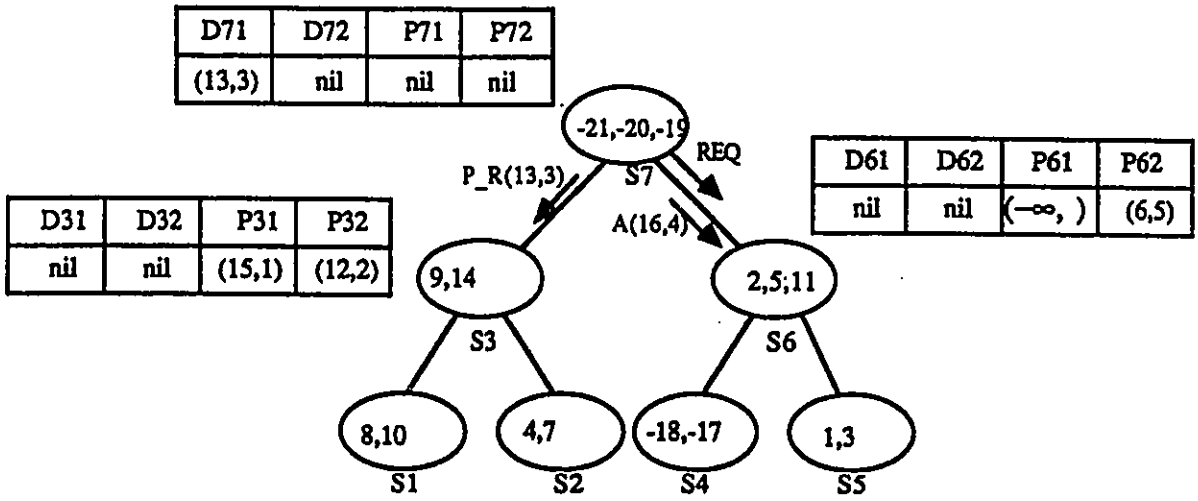


(i.2) Integer 15, which stopped at S_3 on the way to S_1, replaces 10 as the max. of S_1 because D_{31}>P_{31} and D_{31}<P_{32}. Integer 10 (instead of 15) will subsequently be pushed towards S_1. Meanwhile, the updated max. 16 of S_3 is sent to S_7.

Figure 2.5 (i.1-2) Snapshots of the sorting algorithm SDFW (NO extra memory)

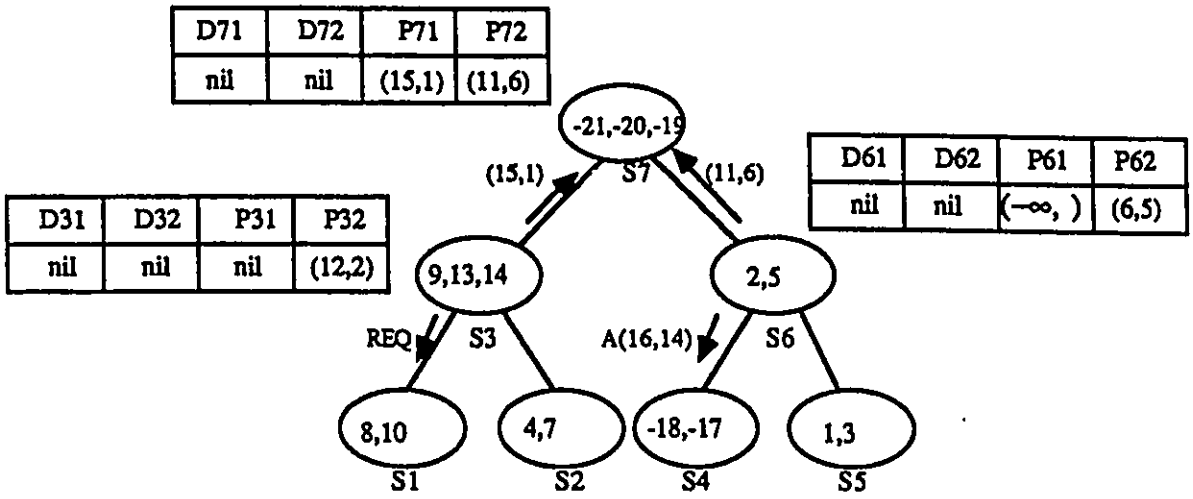


(j.1) Integer 10 is pushed to S_1.(Message P is used because P31 has filled up so that no request for next max. from S1 is needed). (-∞,) is sent from S_4 to S_6 since no more integers are available in S_4.

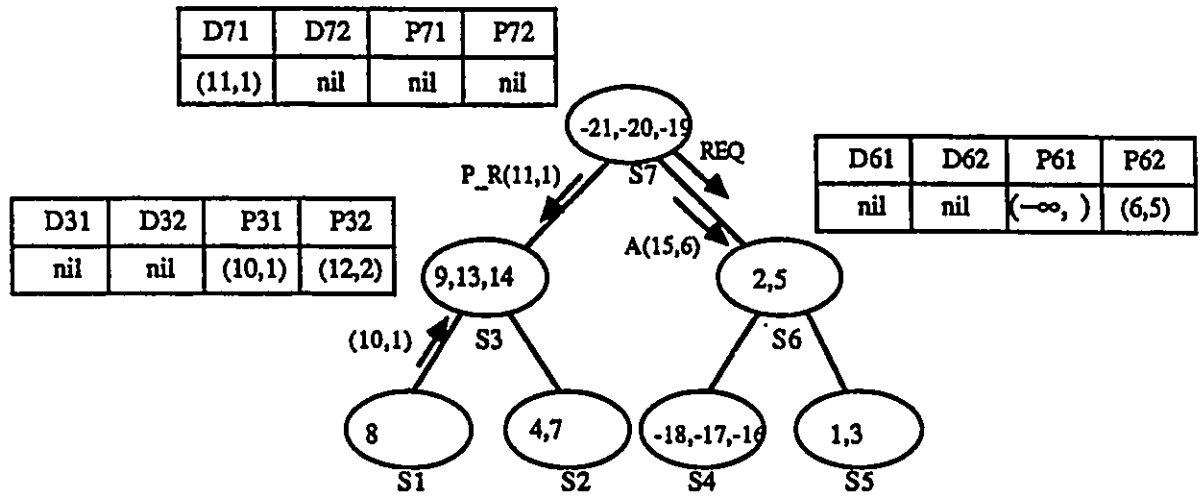


(j.2) The 6th largest integer 16 is directed to S_4 and integer 13 is pushed towards S_3

Figure 2.5 (j.1-2) Snapshots of the sorting algorithm SDFW (NO extra memory)

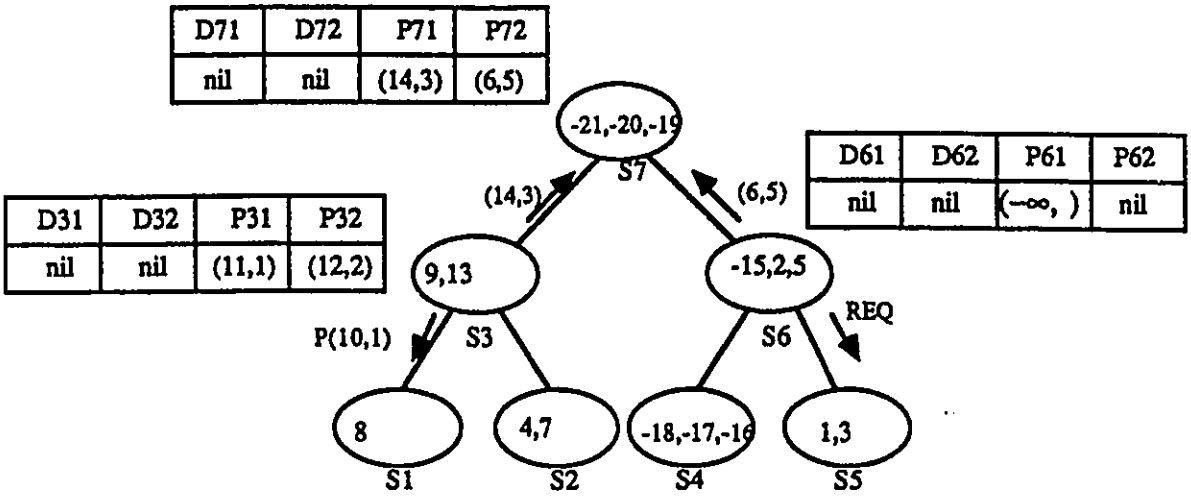


(k.1) The 7th largest integer is being determined in S_7

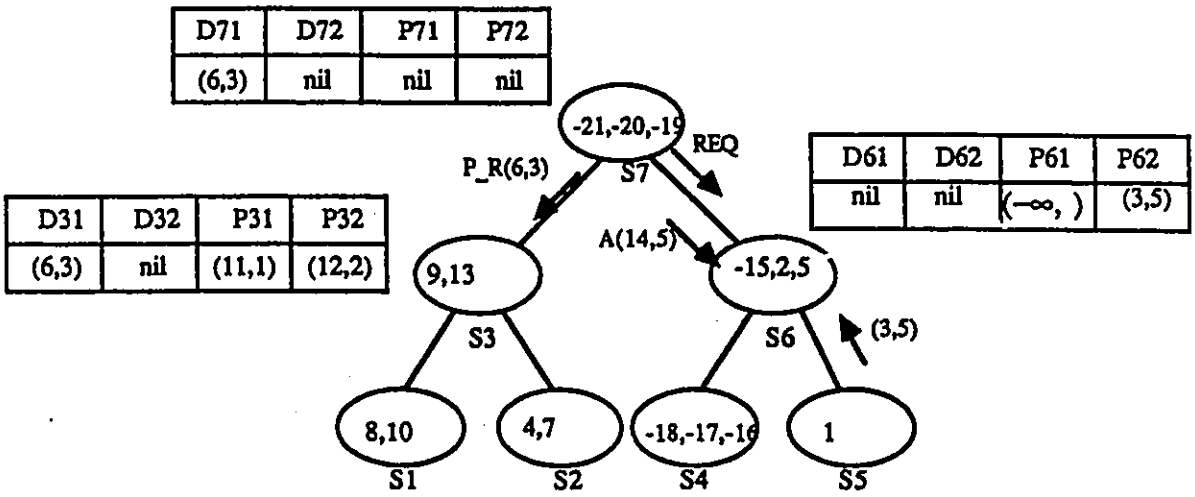


(k.2) The 7th largest integer 15 is directed to S_6

Figure 2.5 (k.1-2) Snapshots of the sorting algorithm SDFW (NO extra memory)

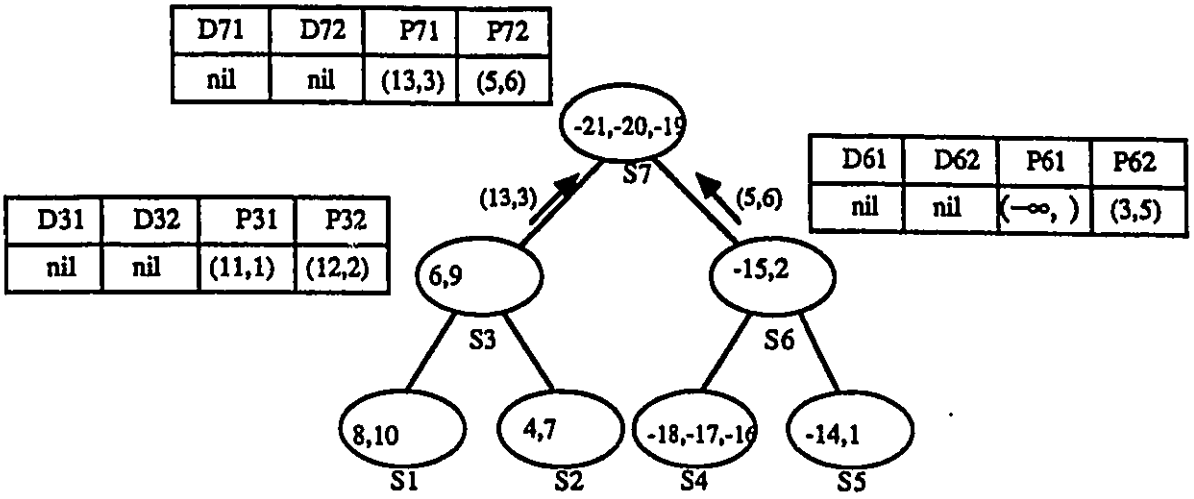


(1.1) The 8th largest integer is being determined in S_7

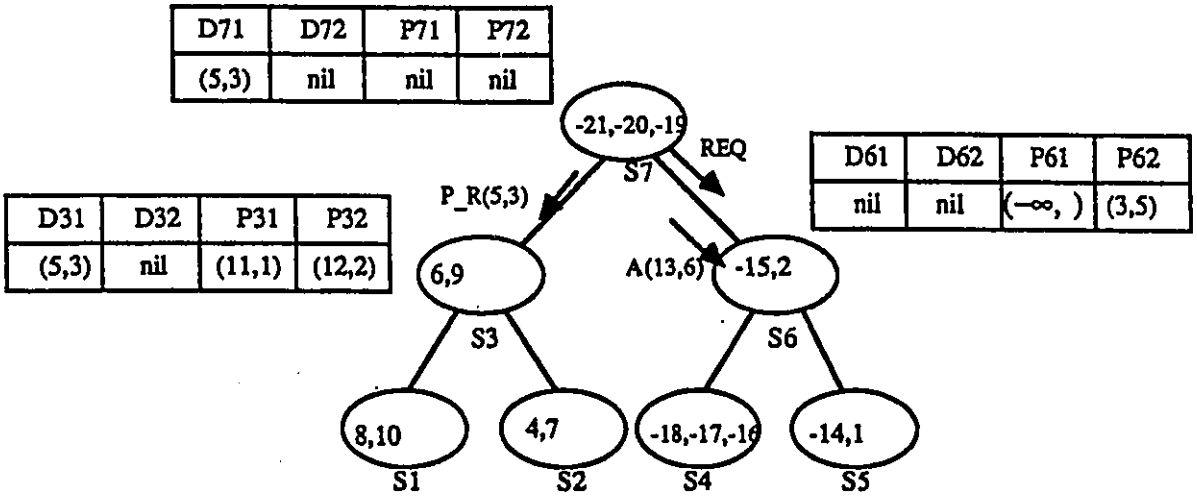


(1.2) The 8th largest integer 14 is directed to S_5

Figure 2.5 (1.1-2) Snapshots of the sorting algorithm SDFW (NO extra memory)

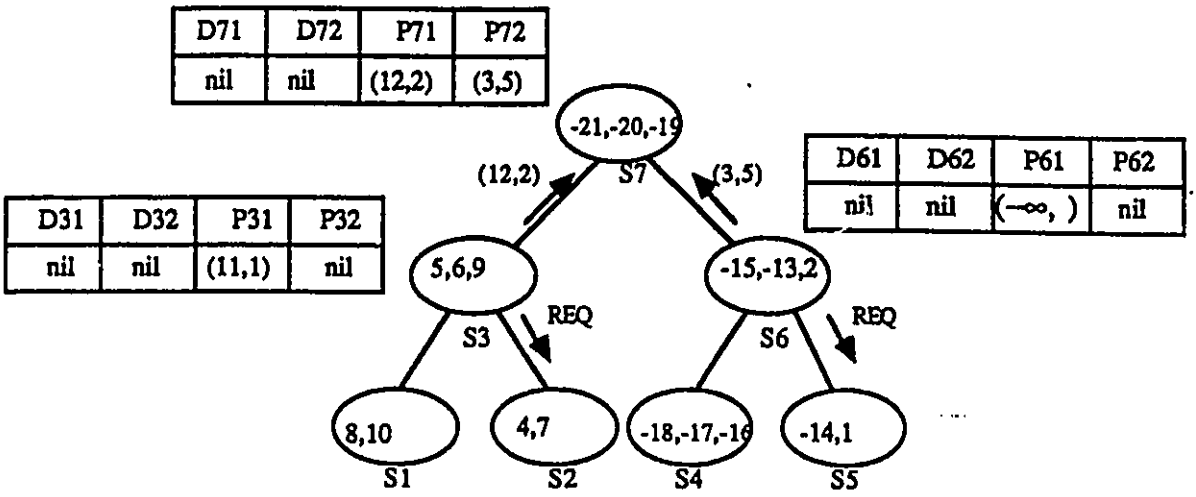


(m.1) The 9th largest integer is being determined in S_7

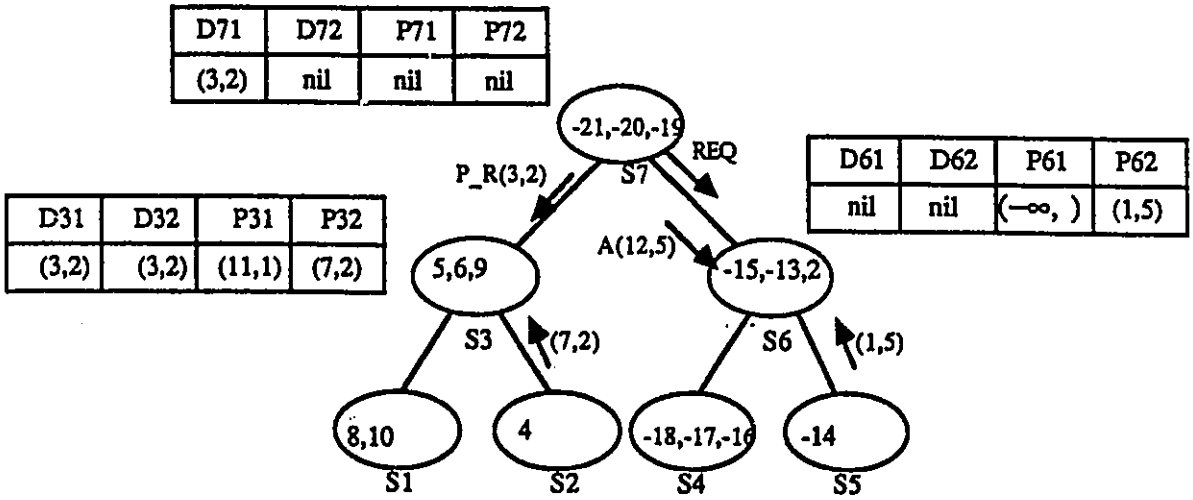


(m.2) The 9th largest integer 13 is directed to S_6

Figure 2.5 (m.1-2) Snapshots of the sorting algorithm SDFW (NO extra memory)

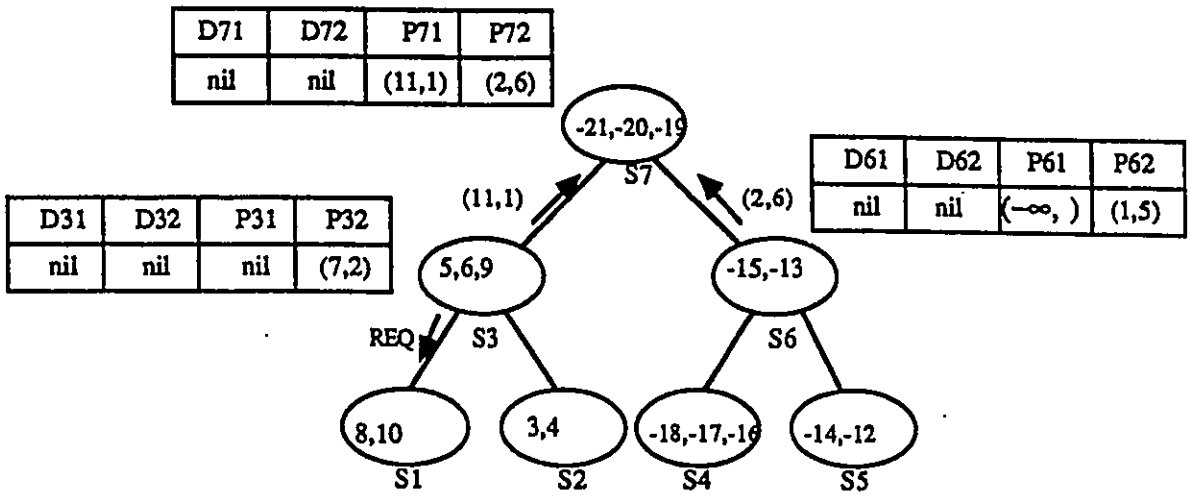


(n.1) The 10th largest integer is being determined in S_7

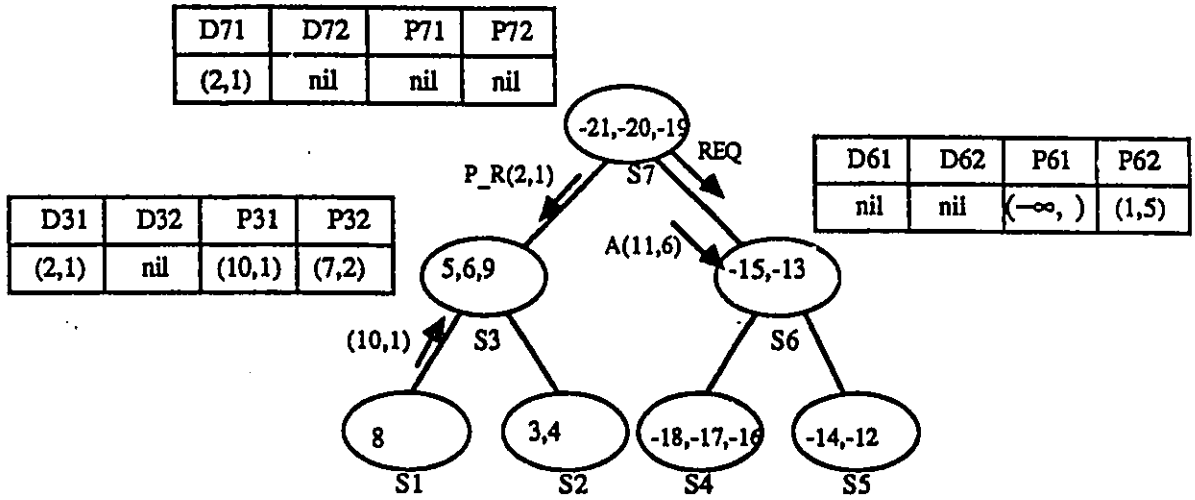


(n.2) The 10th largest integer 12 is directed to S_5

Figure 2.5 (n.1-2) Snapshots of the sorting algorithm SDFW (NO extra memory)

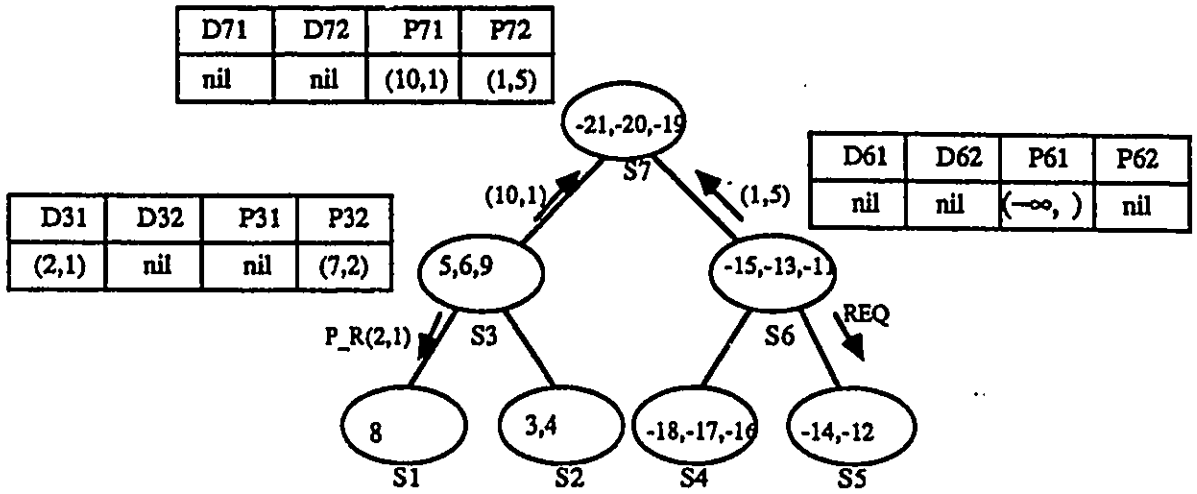


(o.1) The 11^{th} largest integer is being determined in S_7

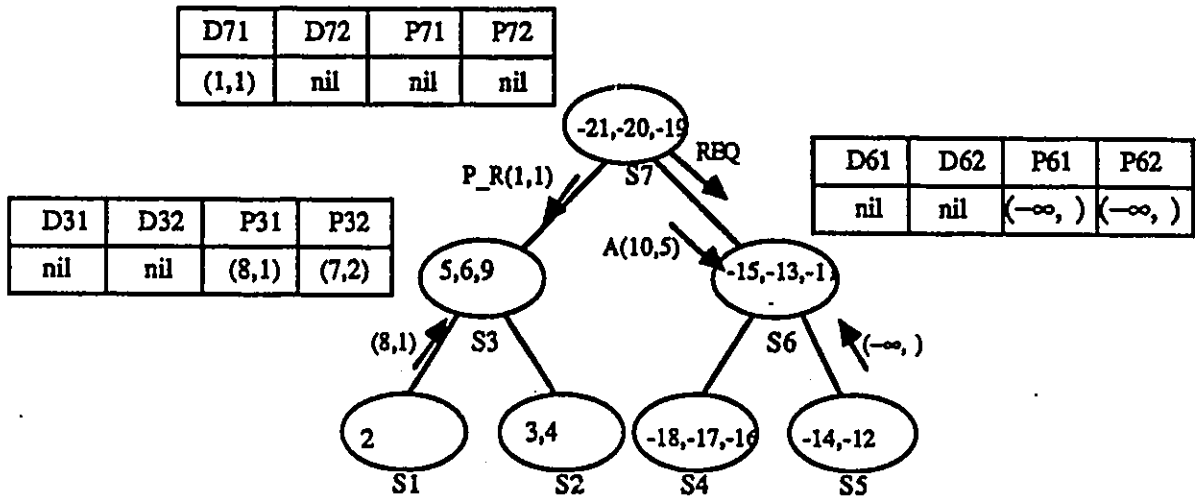


(o.2) The 11^{th} largest integer 11 is directed to S_6

Figure 2.5 (o.1-2) Snapshots of the sorting algorithm SDFW (NO extra memory)

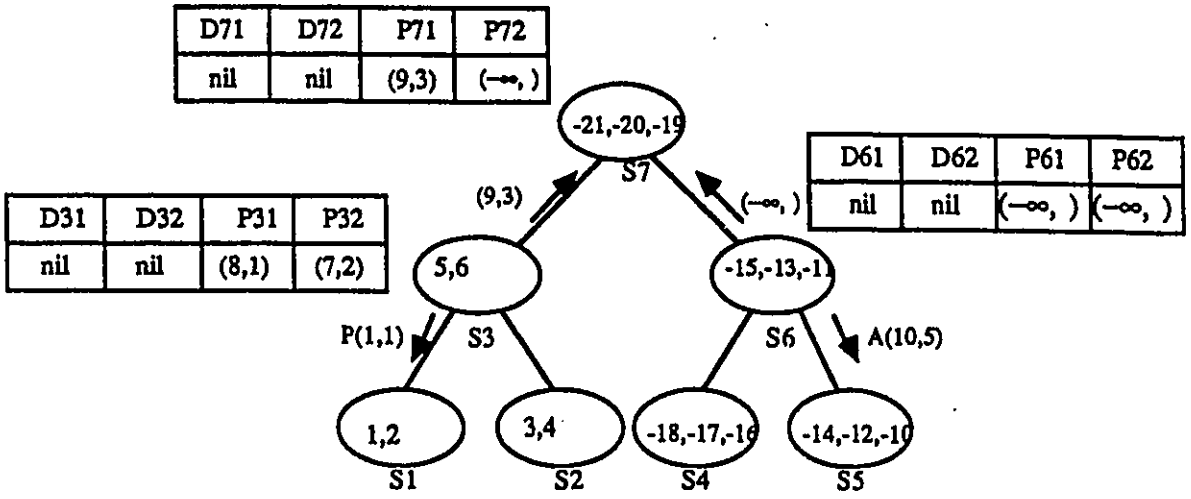


(p.1) The 12th largest integer is being determined in S_7

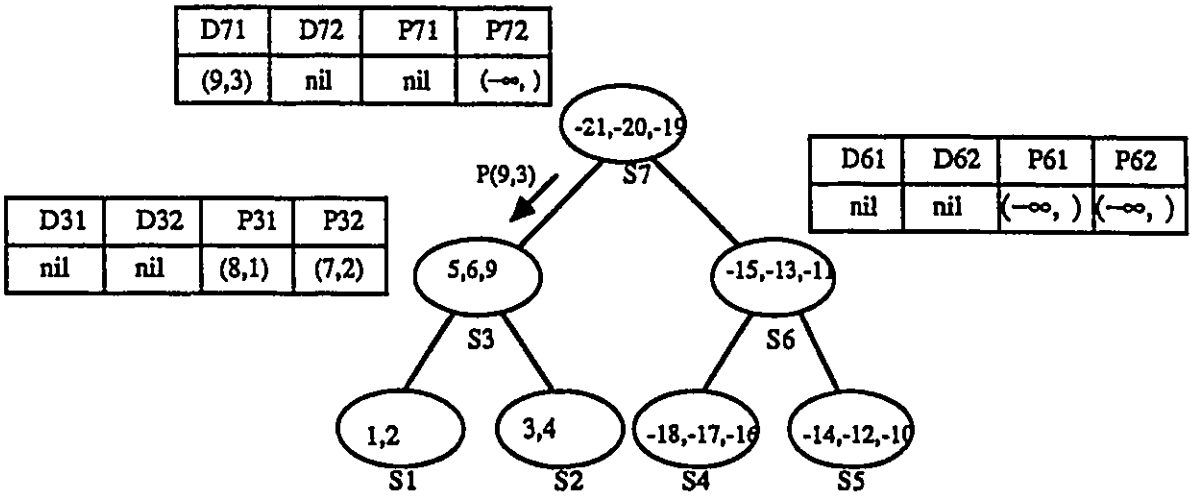


(p.2) The 12th largest integer 10 is directed to S_5

Figure 2.5 (p.1-2) Snapshots of the sorting algorithm SDFW (NO extra memory)

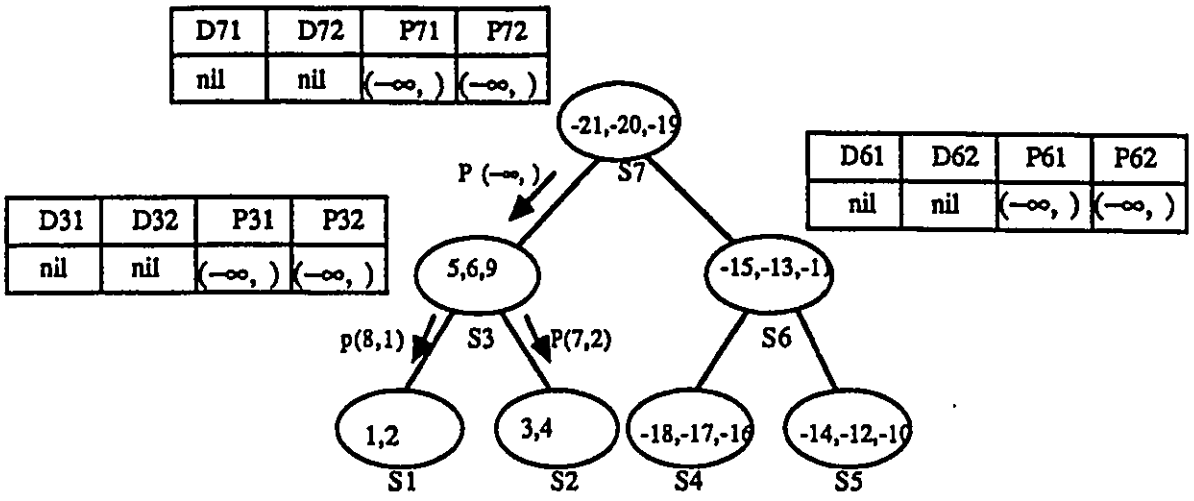


(q) The end of exchanging integers when all integers in S_7's left subtree are smaller than integers stored in S_7's right subtree, thus P_{72} is set to (-∞,).

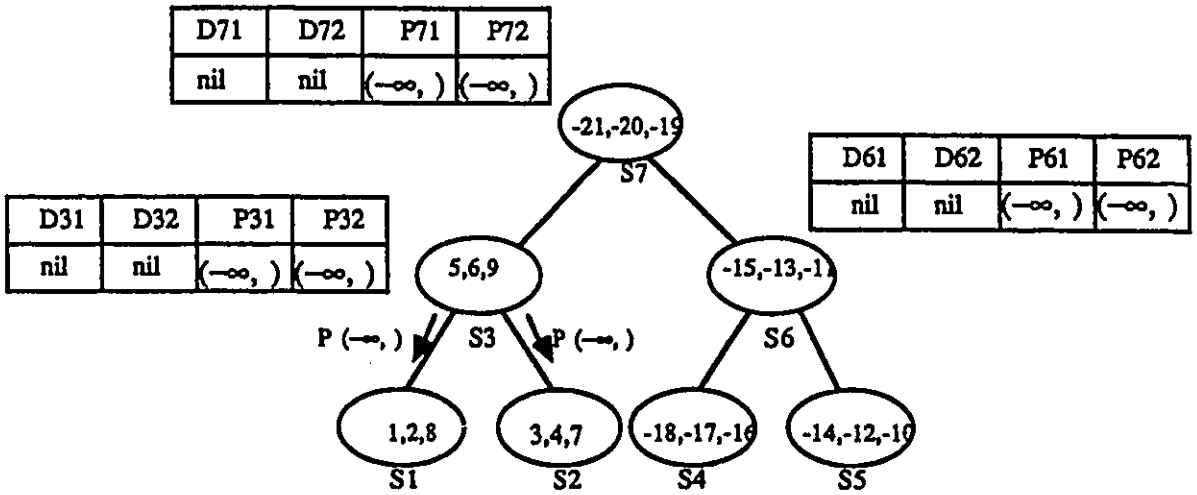


(r) At this point, a station S_j on the left side of S_7 will in turn send whatever is stored in P_{ji} back to where it was sent from. Here S_7 sends P_{71} (integer 9) back to S_3.

Figure 2.5 (q)-(r) Snapshots of the sorting algorithm SDFW (NO extra memory)



(s) S_3 sends P_{31} (integer 8) back to S_1 and P_{32} (integer 7) back to S_2



(t) End of the first iteration

Figure 2.5 (s)-(t) Snapshots of the sorting algorithm SDFW (NO extra memory)

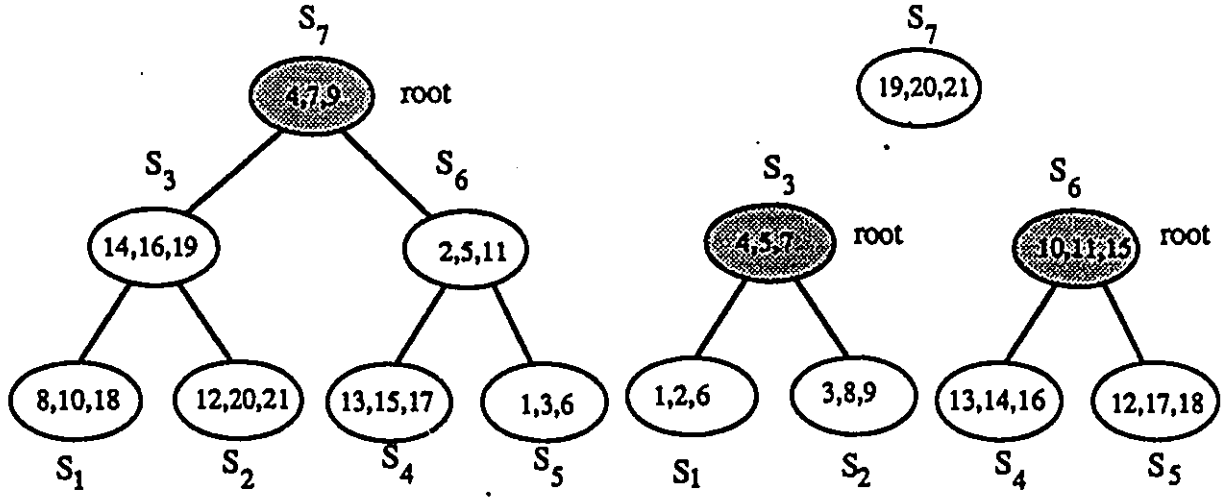
To sort the whole file:

As we have seen, the first n/2 largest elements can be determined and placed in the right subtree by the end of the first iteration. Since the root station S_d now contains the α largest elements, it can retire from further operations. Although the elements stored at sites in its left subtree are smaller than those elements stored in its right subtree, they still may not be in their final destinations yet.

We now proceed as follow until the counter Iternum $\geq$ the diameter of the tree $(\log_2 d) - 1$:

We first increment the counter Iternum by 1. For the Iternumth iteration we repeat the same procedure described above on the subtrees whose root nodes are located on the Iternum-1th level. It is obvious that in the Iternumth iteration there are $2^{\text{Iternum-1}}$ subtrees (see Figure 2.6). Thus for these subtrees, when the current iteration ends the elements stored in their left sub-subtrees are smaller than those stored in their right sub-subtrees. In addition, the roots of the current iteration will retire from further operations.

If we repeat such an operation on all subtrees, level by level, the elements will eventually be sorted in the final order. The termination of SDFW algorithm can be determined in the same way as the termination of SDF algorithm described in previous section.



(a) There is one tree at the beginning of the first iteration

(b) There are two subtrees at the beginning of the second iteration

Figure 2.6

2.5.1 Communication Complexity Analysis of the SDFW Algorithm

In Phase 1 each edge connecting a site S_i to its father F_i transmits at most two wake up messages. This requires 2d messages. The operation of each leaf station sending its maximum element to its father involves d/2 messages. Therefore the communication complexity of Phase 1 is $2^1/2$ d.

Now let us look at the most important Phase 2 - 'Sorting the elements'. It is interesting to see that its message complexity is independent of the number of subtrees in the graph. In fact, the message complexity is determined by the number of elements being moved from one subtree to another subtree during the process.

In the first iteration, there are two subtrees. Each involves at most n/2 element movements so that the total element movements for the first iteration is n. In the second iteration, there are four subtrees. Each involves at most n/4 element movements so that the total element movements for the second iteration is still n. In general, in the i^{th} iteration, there are 2^i subtrees with each involving at most $n/2^i$ element movements. Thus the total element movements for one iteration is always at most n. In reality, the element movement means to find the next largest element. Since finding one next largest element needs $2\log_2 d$ messages (as explained previously), it takes a total of $2n \log_2 d$ messages to complete n element movements, i.e. one iteration. Since there are $\log_2 d$ iterations (levels), the total message complexity for phase 2 is $2n \log_2^2 d$.

A detailed mathematical calculation for the complexity of Phase 2 is

$$\Sigma_{j=1,\log d}\Sigma_{i=1,\log(d/2^j)} ((n/2^j) \log(d/2^{j+i-1})).$$

Thus, the overall communication complexity of the SDFW is $2^1/2d + 2n \log_2^2 d$. Since a message carries one value, the traffic complexity for SDFW is also $2^1/2d + 2n \log_2^2 d$.

2.5.2 Time Complexity in the Synchronous Case

In synchronous networks a message takes exactly one unit of time to transmit from one station to another adjacent station. Trivially Phase 1 takes at most $2 \log_2 d + 1$ units of time.

We will now analyze the time complexity for Phase 2. As we mentioned before, Phase 2 can be thought of as a recursive procedure. When Phase 2 is executed, this procedure will be in turn applied on the subtrees rooted at the level 0, then on the subtrees rooted at the level 1, ..., and finally on the subtrees rooted at the highest level. During the execution, each level is considered as one iteration. Since the subtrees with roots located at the same level execute this procedure simultaneously, they take the same amount of time to complete this procedure. Thus, although there are d/2 subtrees in an AB binary tree network, the number of iterations is only $\log_2 d$ (which equals the number of levels). Recall that the time complexity for one iteration in Phase 2 is $3\log_2 d + 2n$ (as previously explained in the time complexity for the SDF algorithm). Consequently, the time complexity for Phase 2 of SDFW is $3\log_2^2 d + 2n \log_2 d$ and the total time complexity for the SDFW algorithm is $3\log_2^2 d + 2(n+1) \log_2 d + 1$.

The SDFW algorithm as presented here works in binary tree networks. In fact, the SDFW algorithm can be easily modified to work in any AB q-ary tree network with nodes in postorder labeling. Thus we can obtain the following result:

Theorem 2.2 Distributed sorting in AB q-ary tree networks without extra memory available at each site and with message capacity of 1 can be accomplished in $2^1/2\, d + 2n \log_q^2 d$ communication and traffic complexity and in $3\log_q^2 d + 2(n+1) \log_q d + 1$ time complexity (in synchronous networks).

2.6 Tradeoffs between Extra Memory and Complexity

As we have seen in the previous sections, the SDF algorithm requires extra local memory at each site to store some temporary values, but in return it is $\log_2 d$ times faster and needs $\log_2 d$ times fewer messages than the SDFW algorithm. It is not hard to verify that, if one is willing to sacrifice more local storage and increases the message capacity, the performance of the SDF algorithm can be further improved. The tradeoffs between the memory storage and complexity, as we apply the SDF algorithm in an AB binary tree network, is summarized as follow:

	Message Capacity	Extra storage	Message complexity	Time complexity
*	1 element	n/d+2 elements	$O(n \log_2 d)$	$O(n + \log_2 d)$
	2 elements	n/d+4 elements	$O(n/2 \log_2 d)$	$O(n/2 + \log_2 d)$
	...			
	x elements	n/d+2x elements	$O(n/x \log_2 d)$	$O(n/x + \log_2 d)$
	...			
**	n/d elements	2n/d elements	$O(d \log_2 d)$	$O(d + \log_2 d)$

This summary also shows that under the similar conditions our method manages to improve the existing algorithms. (Compared * to [RoSaSi85] and ** to [Che86].) Extra local memory storage is proved to be a useful strategic tool in distributed algorithms design.

2.7 Impacts of Local Computation Cost

Given a completely connected network, the distributed file sorting problem can be solved in three steps:

i) Elect a leader.

ii) Use the leader as the root to construct a q-ary tree.

iii) Apply the algorithm SDF or SDFW.

Since our algorithms SDF and SDFW work on AB q-ary tree networks, we are interested in knowing how the choice of q affects the computing performance. Let us take the two extreme cases for comparison.

Case 1: $q = d-1$

In this case our q-ary network is a star tree. For the SDF algorithm, this configuration has the advantage of minimizing the message complexity. The message complexity in this case is O(n). Furthermore, if we ignore the time spent performing local calculations (as customarily done in distributed computing) we will also get a linear time complexity for synchronous networks.

Case 2: $q = 2$

In this case we obtain binary trees which have been fully discussed in previous sections. In this case the message complexity is $O(n \log_2 d)$ and time complexity is O(n). Evidently the communication complexity is worse than that of the star network and the time complexity in this case is also linear.

This leads us to correctly believe that the best strategy to optimize the communication complexity of distributed sorting in complete networks will be achieved by choosing a star network. For the time complexity of synchronous networks, however, a subtle mistake in the previous conclusion would lead us to believe that both of the star networks and binary tree networks are equally fast.

A more careful analysis of the time complexity for synchronous networks will reveal an

obvious mistake. When the root station S_d determines the largest, second largest, ... element, it has to select it from among q candidates (one from each child). Clearly this cannot be achieved in constant time. Thus the real time complexity in this case is $O(n \log_2 d)$ as opposed to the linear time complexity which would result from the typical analysis for synchronous networks.

One fact hidden behind the file sorting problem and never before considered is the local processing time. This local processing time becomes a noticeable factor in our study of sorting file in the tree networks. In the above example, the root processor in the star network could have much longer local processing time (i.e. the queuing time at the message arrival port plus the local sorting time) than the one in the binary network if we are dealing with a very large network and/or a very large file. In more specific, when the SDF is applied, this long local processing time for the star network requires $\log_2(d-1)$ unit of local time to find its minimum while for the binary tree network it requires exactly two comparisons.

We are going to investigate the impact of local processing on the total cost of time in the case of SDF algorithm. Let the symbol g represent a very tiny value such that 1 unit of local processing time is equal to 1 unit of communication time multiplied by g. Also let t_1 and t_2 be the total time cost for star and binary tree networks, respectively. We obtain the following results:

$q = d-1 \quad \Rightarrow \quad t_1 = g(n - d) \log_2 d + n - d$

$q = 2 \quad \Rightarrow \quad t_2 = (g + 1) \log_2 d + (n - 3)(g + 1) + \log_2 d$

Thus, $t_1 < t_2$ iff $g < (2\log_2 d + d - 3) / ((n - d - 1) \log_2 d - n - 3)$.

This result indicates that it needs shorter time to sort a file in a star network than in a binary tree network if and only if the constant g is smaller than some value. However, as the value of n (the number of elements) and/or the value of d (the number of stations) gets larger and larger, this condition becomes harder and harder to be satisfied. This shows that for sorting a very large file distributed over a very large network, the local processing could hurt the time performance.

In the general case of applying the SDF in the AB q-ary tree networks, the first element takes $\log_q d$ communication time and $\log_q d * \log_2 q$ local processing time (for selecting the local minimum) to arrive at the top, while each of the remaining (n-q-1) elements takes 1 unit of

communication time and $\log_2 q$ local processing time. The last element to come down from the root station to its destination requires $\log_q d$ communication time but no local processing time. Thus, the total time complexity is $g(\log_q d+n-q-1)*\log_2 q \div (n-q-1+2\log_q d)$. Using this general formula, we can actually calculate the total time complexity, including the external and internal processing, for different values of q. From these results we can choose a value of q which meets the design requirement most closely.

2.8 Closing Remarks

Before closing the chapter, several final remarks should be made.

First, do our algorithms cause a bottleneck at the root node? Not really! The computation load is evenly distributed at each participating node in each cycle, i.e. no one specific node has heavier work load than the others, in spite of the fact that the root has to participate in all cycles during the entire process. Besides, our algorithms provide a great deal of parallelism. This compensates the possibly longer waiting time at the root node.

Second, if the message capacity x is assumed to be greater than 1, our algorithms SDF and SDFW will be x times faster and need x times fewer messages. This has been shown in Section 2.6.

Third, our SDF algorithm (for sorting with extra memory) can be modified so that it will work on tree networks of any labeling. This modification requires that the root station S_d has knowledge of the distribution order P. Therefore S_d can send assigned elements directly to their destinations.

Fourth, the proposed methods use some blind operations which cannot recognize the already-sorted inputs. However, this shortcoming could be improved by sending both of the minimum and maximum elements in a station to its father. Thus, at the end, the root station can compare the maximum element of the left subtree and the minimum element of the right subtree. If the former is smaller, then all elements in the left subtree are smaller than those in the right subtree. The root keeps the largest elements and will retire from the further actions. The original tree is then divided into two subtrees, each being operated with the same algorithm. Applying this technique repeatedly in AB binary tree networks, already-sorted inputs can be detected in $O(\log_2^2 d)$ time with $O(d \log_2 d)$ messages.

Chapter 3

Tradeoffs in Distributed Shortest Path Finding

In recent years there has been a great deal of studies on the tradeoffs between the message and time complexities in distributed algorithm design. Research shows that the higher the number of messages used, the shorter the time complexity will be, and vise versa. For instance, election in a ring can be accomplished with O(n) messages with a long waiting time. However, if we want to complete it in O(n) time, $O(n^2)$ messages will be needed. On the other hand, one could also solve the election problem with O(n log n) messages and O(n log n) time complexity [Pe82].

In this chapter we study the tradeoffs between time and message complexities for the problem of finding a shortest path between two arbitrary stations S_m (source) and S_n (sink) in distributed and *synchronous communication networks*. In particular we focus on the termination detection problem. Based on Dijkstra's method our algorithm performs search and trail operations. During the search process search messages fan out from the source and start building a spanning tree until S_n is reached. Once the sink has received the search message it starts the trail process, in which the sink sends a trail message backwards from S_n to S_m in order to stop all outgoing search messages as soon as possible. To accomplish this, before a full spanning tree is built, a delay is introduced at each node which has recieved a search message, that is after receiving a search message a node will retain the search message for some specified amount of time before sending it to its neighbours. Thus the search message does not always traverse the entire network like a straightforward implementation of Dijkstra's algorithm would do.

Our algorithm works in arbitrary synchronous networks. However, in this paper we will analyze its message and time complexities only on the tree, linear and ring networks. Further studies are needed to determine the complexities of our algorithm on other network topologies.

3.1 Introduction

Shortest-path problems are some of the most fundamental and commonly encountered problems in the study of communication networks. While there is a large class of shortest-path problems, in this chapter we will confine ourselves to the basic problem of how to determine a shortest path $SP_{m,n}$ between two specific nodes S_m (source) and S_n (sink) in a communication tree network T.

One of the solutions for the above problem in the **non-distributed environment** was due to Dijkstra in 1959. The basic idea behind Dijkstra's algorithm is to fan out from S_m and proceed towards S_n to construct a tree. The algorithm works as follows: All nodes are initially unlabeled. We start by labeling the source S_m. Then we label each immediate successor S_v of source S_m. If none of these S_v's is the sink S_n, the algorithm will proceed to label the immediate successor(s) of each S_v. However the labeling request to a node say S_w will be rejected if S_w has already been labeled (or visited). We continue processing in this fashion until the sink S_n is finally reached. The actual shortest path $SP_{m,n}$ can be obtained by tracing backward from S_n to S_m. It is quite easy to see that Dijkstra's algorithm generates a spanning tree rooted at the source S_m, lets call it source-rooted tree. Since the process is stopped as soon as the sink S_n is reached, the tree constructed by Dijkstra's algorithm is not necessarily a spanning tree of the network.

Dijkstra's method can be adapted in a straight forward manner to solve the same problem in **distributed networks** in the follwoing way: A labeling request here means a search message which carries the id of the sink S_n. The source S_m broadcasts a search message. Upon receiving a search message, a node determines whether to broadcast this message or not, depending on if this node has already been labelled (or visited). Since in distributed networks there is not a central controller, no node can know what happens to other nodes. Thus the algorithm will not stop until all the nodes have been visited, i.e. a spanning tree of the network is generated.

Unfortunately, there is a major drawback of this approach: it always takes the same (maximum) amount of messages regardless of where the sink is located. This is caused by the fact that (in distributed environment) according to the algorithm a **complete** source-rooted tree of the network must be generated no matter how soon the shortest path is obtained. Such drawback is highly undesirable for the communication complexity (i.e. the number of messages exchanged) is our major concern.

Dijkstra's method for finding the shortest path between two nodes in distributed networks requires O(d) units of time in the worst case (i.e. in linear topology) and $O(d^{1/2})$ units of time in the average case (i.e. in random tree topology, see [MeMo70]), where d=|G| is the number of nodes. And it takes O(|E|) number of messages.

In this chapter we will develop a new distributed algorithm to solve the shortest path problem. Our algorithm, called Finding Shortest Path (FSP), is based on Dijkstra's method but eliminates the burden of having maximum communication complexity at all times. The basic idea of the FSP algorithm is **delay and trail**. Once the sink node S_n is reached, S_n will issue a trail message tracing back from S_n to S_m to stop the outgoing search messages. Meanwhile, after receiving a search message, a node $S_w \neq S_n$ will wait for a certain amount of time (determined by a predefined **delay** function $W(S_w)$). When the delay time is up, S_w will send the search message to its successors and then remain idle. If an idle node S_w receives a trail message from another node say S_v, S_w will immediately transmit the trail message to all its neighbours in the tree except S_v and then terminate its own process. If S_w receives a trail message during the delay time, it will terminate the process of S_w at once.

Clearly, if the delay time is long enough, the entire FSP process could be terminated at the level where the sink S_n is located. We will analyze this situation in section 3.4. This also leads us to a new observation: altering the delay function W can change the amount of messages required in the process. Thus, the greater the delay time is, the smaller the amount of messages neededwill be. In sections 3.5, the tradeoffs between the time and communication complexities of our algorithm will be studied.

3.2 Model and Notations

In addition to those as described in Section 1.6, the following model assumptions will be used in this chapter.

a) The networks under consideration are synchronous networks.
b) Each site has a timer which can be reset to some specific value at any arbitrary time. There is no mutual drift between timers in different sites. In synchronous networks, one unit of local time is equivalent to one unit of global time.
c) The network size d is made public to all stations.
d) Each search message has fixed capacity for containing the id of the sink node. Each trail message has capacity of one bit for signaling that the sink is reached.
e) The stations $\{S_1, ..., S_d\}$ of T have a random ordering.

During the execution of the FSP algorithm, S_j will also keep track of two sets of information: the predecessor PRE_j from where S_j receives the search message, and a set of successors SUC_j to where S_j sends the search messages.

A source-rooted tree network of the network T (for short called SR-T network) is an ordinary tree which is rooted at the source node S_m and orients each edge away from the root (see Figure 3.2). As described in the introduction, this SR-T can be generated by an exhaustive execution of Dijkstra's algorithm (i.e. until every node in the network has been visited). For the sake of simplicity, we will conduct the analysis of our FSP algorithm directly on the SR-T network. Note that this approach is a very reasonable one since the SR-T not only represents the network T but is actually generated during the execution of the algorithm.

For every element S_j in the SR-T the father of S_j is its predecessor PRE_j and the children of S_j are its successors SUC_j. In our algorithm, the $W(LN_j)$ function represents the amount of time for a search message to be delayed at a site S_j located at the LN_j^{th} level, before being sent to the adjacent sites. In this chapter we will choose $W(LN_j)=\omega(i)$ and assume the predetermined constant ω to be publicly known.

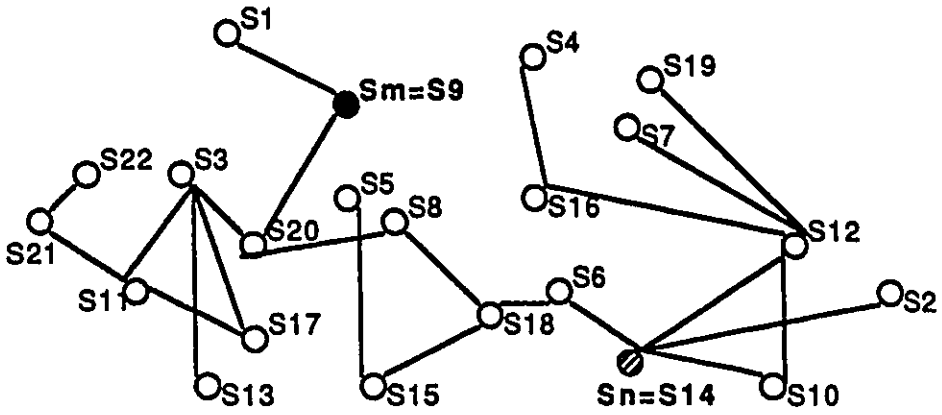
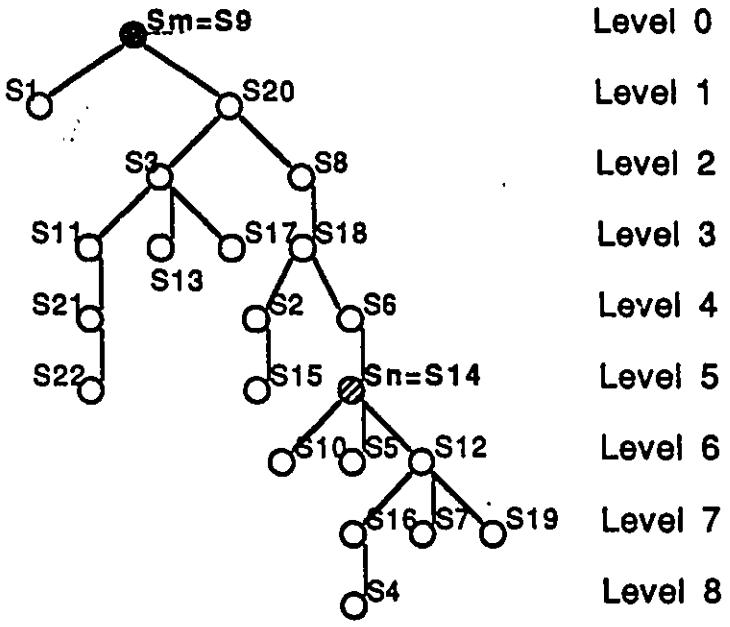


Figure 3.1 A connected network T



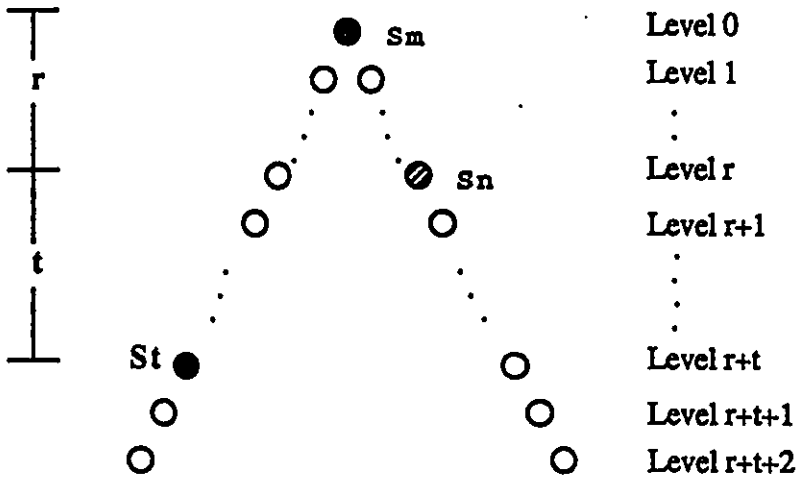
The source-oriented tree network SR-T of the network shown in Figure 3.1

Figure 3.2

3.3 Finding the Shortest Path in Synchronous Networks

In this section we present our search algorithm (FSP algorithm) for finding a shortest path $SP_{m,n}$ between a source station S_m and a sink station S_n in a synchronous network T of d sites.

Our algorithm starts by sending a search message, which contains the id of the sink node S_n, to each immediate successor S_v of source S_m. A node which has received a search message is said to be 'visited'. If none of these S_v's is the sink, the algorithm will proceed to search the immediate successor(s) of each S_v. However the search request to a node, say S_w, will be rejected if S_w has already been visited. We continue processing in this fashion until the sink S_n is finally reached. During the search process, message transmission delay occurs at visited nodes. Let r be the distance between the source (root) S_m and sink S_n; r is called the target distance (see Figure 3.3). Once the sink S_n is found, S_n will issue a trail message tracing backward from S_n to S_m and propagating along the tree currently traversed by the algorithm. The trail message is intended to reach the leaves before more search messages are being sent out from other nodes. Let t be the maximum trail distance, i.e. the distance between the level of the sink node and the level at which a search message is stopped. We will discuss in Section 3.5 how to define such t in terms of the predetermined delay function W and the level number of the sink LN_n. If we look at the FSP algorithm more carefully, we will find that for r ≥DIAM(T)-t search messages must traverse the whole network. In this case the trail process will be unnecessary.



Display of the target distance r and trail distance t

Figure 3.3

A formal description of our FSP algorithm now follows:

i) Initially, all stations in the network T are sleeping. The source station S_m wakes up and sends a search message SEARCH(LN_m,S_n) containing its level number LN_m=0 and the id of the sink station S_n to its adjacent stations. Upon receiving a SEARCH(e,S_n) message from its adjacent station S_i, a sleeping station S_j will wake up and proceed as in ii).

ii) If S_j is a leaf it will terminate its own process. Otherwise, S_j will obtain the information of its predecessor by PRE_j=S_i, its successors by SUC_j={all S_j's adjacent nodes excluding S_i} and its level number LN_j=e+1. Then S_j will compare its own id to S_n and act according to the following:

   a) If S_j is not the sink node it will proceed as in iii).
   b) If S_j is the sink node but LN_j<DIAM(T)-t it will proceed as in iv).
   c) If S_j is the sink node and LN_j≥DIAM(T)-t it will terminate its own process.

iii) S_j will wait for W(LN_j) units of time. During the delay if S_j receives a trail message, S_j will terminate its own process and perform no further action. Otherwise, when the delay timer expires, S_j will perform the following:

   a) S_j will send SEARCH(LN_j,S_n) to its successors SUC_j and then remain idle. Upon receiving SEARCH(a,S_n) each of SUC_j will proceed as in ii).
   b) Once the idle S_j receives a trail message it will become active again and proceed as in iv).

iv) The trail message TRAIL is used to inform other stations that the sink station has been reached and thus stop the outgoing search messages. Thus station S_j will send a TRAIL(1) message with a bit 1 to its predecessor PRE_j and TRAIL(0) messages with a bit 0 to its successors SUC_j. After sending message(s) S_j will terminate its own process. Note that the path on which all nodes have received TRAIL(1) is actually the shortest path $SP_{m,n}$.

The entire FSP algorithm will terminate when all participating sites in the network have terminated their processes. The termination detection can be done as described in Chapter 2.

3.4 Complexity Analysis of the FSP Algorithm with the Longest Delay

In this section we will analyze the message and time complexities of our FSP algorithm in particular for **the delay function $W(LN_i)=2i+1$.** Since in the LN_i iteration of our algorithm, we build a tree T of depth LN_i, the distance between two nodes in T is at most 2i; thus any node in T can be reached from any other nodes in T in at most 2i time using only edges in T. Then if we set our delay function $W(LN_i)=2i+1$, S_n will have enough time to inform any other nodes in T that that a shortest path has been established, and no further search message will be needed. In other words the trail distance t is zero.

For every S_i in T, let $T(S_i)$ be the number of messages (called *total message*) used to calculate the shortest path between S_m and S_i, i=1,2,...,d, i≠m. The average number of messages (called *average message*) needed to compute the shortest path between S_m and a randomly chosen node S_i in T is $\Sigma_{i=1,d\ \&\ i\neq m}T(S_i)/(d-1)$. We define *total time* and the *average time* in the similar manner.

Clearly as the delay function W decreases, the time complexity of our algorithm will decrease while the message complexity increases. This relationship will be studied in Section 3.5.

3.4.1 For Random Tree Networks

In this section, we analyze our algorithm in the case when our communication network is a random tree.

According to [MeMo70] the expected average distance between two arbitrary points in a random tree T is $E(H,T)=(\pi d/2)^{1/2}$ and the expected average degree of the nodes is $E(D,T)=2(1-1/d)$. It is clear that, for the delay function $W(LN_i)=2i+1$, the message complexity of our FSP algorithm is the number of nodes in T at distance at most E(H,T) from S_m. Since the number of edges between the level 0 and level 1 of the tree is E(D,T) and that between levels i and (i+1) is $E(D,T)[E(D,T)-1]^i$, i=1,2,...,E(H,T), we can calculate the average message complexity as follows:

$$\text{Search Messages}_{random} = b\,[q + \Sigma_{i=1,e}\, q(q-1)^i]$$
$$= b\,[q\,(1 + \Sigma_{i=1,e}\,(q-1)^i)]$$

where $e = E(H,T) = (\pi d/2)^{1/2}$ and $q = E(D,T) = 2(1-1/d)$ and

b = number of bits required to send a node id.

We will calculate the summation by substituting $q=2(1-1/d)$:

$$\Sigma_{i=1,e}\,(q-1)^i = \Sigma_{i=1,e}\,(2-2/d-1)^i$$
$$= \Sigma_{i=1,e}\,(1-2/d)^i$$
$$< \Sigma_{i=1,e}\,(1) = e = (\pi d/2)^{1/2}$$

Thus the search message complexity is:

$$\text{Search Messages}_{random} < b\,[2(1-1/d)\,(1+(\pi d/2)^{1/2})] = b\,[2+(2\pi d)^{1/2}-2/d-(2\pi/d)^{1/2}] \text{ bits}$$

Since the number of Trail messages is equal to the number of Search messages and each Trail message is of constant size, then the total communication complexity of the trail process is

$$\text{Trail Messages}_{random} < 2+(2\pi d)^{1/2}-2/d-(2\pi/d)^{1/2} \text{ bits}$$

$$\therefore \text{ Average Messages}_{random} = \text{Search Messages} + \text{Trail Messages}$$
$$< (b+1)\,[2+(2\pi d)^{1/2}-2/d-(2\pi/d)^{1/2}] \text{ bits}$$

Then the total time taken by our algorithm equals the message transmission time plus the message delay time. Let us take an instance from Figure 3.2: the source S_9 sent a search message to the sink S_{18} through node S_{20} and S_8. In this case the total time includes 3 time units for transmission plus the delay time in nodes S_{20}, S_8 and S_{18}. In general, the time complexity of our algorithm is the height E(D,T) plus the delay time which is $\Sigma_{i=1,E(H,T)}(2i+1)$.

$$\therefore \quad \text{Average Time}_{random} = e + \Sigma_{i=1,e}(2i+1) \qquad \text{where } e = E(H,T) = (\pi d/2)^{1/2}$$
$$= 2e + 2\,\Sigma_{i=1,e} i$$
$$= 2e + e(e+1)$$
$$= e^2 + 3e$$
$$= \pi d/2 + 3(\pi d/2)^{1/2}$$

Theorem 3.1 The average communication and time complexities of finding a shortest path between two arbitrary points in distributed and synchronous random tree networks are (b+1) $[2+(2\pi d)^{1/2}-2/d-(2\pi/d)^{1/2}]$ bits and $\pi d/2 + 3(\pi d/2)^{1/2}$ units of time respectively, with the delay function $W(LN_i)=2i+1$, where b is the number of bits required to send a node id.

3.4.2 For Linear Networks

The linear network is the simplest topology for analysis. Since there is only one node on each level of the linear network, the number of messages needed to go from the source S_m to the sink S_i is LEVEL(i), $i \neq m$; and it requires a time complexity of $i+\Sigma_{i=1,LEVEL(i)}(2i+1)$ for the delay function $W(LN_i)=2i+1$. In the following, we will calculate the average message and time complexities for our FSP algorithm on linear networks:

$$\begin{aligned}\text{Total Search Messages}_{linear} &= b\,\Sigma_{i=1,d-1} i \\ &= b\,(1+2+3+\ldots+d-1) \\ &= b*d(d-1)/2 \text{ bits}\end{aligned}$$

where b = the number of bits required to send a node id

$$\begin{aligned}\text{Total Trail Messages}_{linear} &= \Sigma_{i=1,d-1} i \\ &= d(d-1)/2 \text{ bits}\end{aligned}$$

$$\text{Total Messages}_{linear} = (b+1)\,[d(d-1)/2] \text{ bits}$$

$$\begin{aligned}\therefore \text{ Average Messages}_{linear} &= (1/(d-1))\ \text{Total Messages}_{linear} \\ &= (b+1)\ d/2 \text{ bits}\end{aligned}$$

$$\begin{aligned}\text{Total Time}_{linear} &= \Sigma_{i=1,d-1}[\Sigma_{j=1,i}\,(2j+1) + i] \\ &= \Sigma_{i=1,d-1}[i^2+3i] \\ &= d(d-1)(2d-1)/6+3d(d-1)/2 \\ &= d(d-1)(d+4)/3\end{aligned}$$

$$\therefore \text{ Average Time}_{linear} = (1/(d-1))\ \text{Total Time}_{linear} = d(d+4)/3$$

Theorem 3.2 The average communication and time complexities of finding a shortest path between two arbitrary points in distributed and synchronous linear networks are (b+1)*d/2 bits and d(d+4)/3 units of time respectively, with the delay function $W(LN_i)=2i+1$, where b is the number of bits required to send a node id.

3.4.3 For Ring Networks

Ring networks have been the most well-studied topology in the literature of distributed computing. They are very similar to the linear networks except that in linear networks the source node and the last node on line are connected to only one other node, while in ring networks every node is connected to two other nodes. In other words, the diameter of the spanning tree generated from the ring networks is half of that generated from the linear networks. Because of this difference, when we apply the FSP algorithm on ring networks, we can obtain better message and time complexities than those for linear networks.

$$\begin{aligned}\text{Total Search Messages}_{ring} &= b\,\Sigma_{i=1,d/2}2i \\ &= b\,[\,2(1+2+3+...+d/2)\,] \\ &= b\,[d^2/4+d/2] \text{ bits}\end{aligned}$$

where b = number of bits required to send a node id

$$\begin{aligned}\text{Total Trail Messages}_{ring} &= 2\Sigma_{i=1,d/2}i \\ &= d^2/4+d/2 \text{ bits}\end{aligned}$$

$$\text{Total Messages}_{ring} = (b+1)[d^2/4+d/2] \text{ bits}$$

$$\begin{aligned}\therefore \text{ Average Messages}_{ring} &= (1/(d-1))\ \text{Total Messages}_{ring} \\ &= [(b+1)/2(d-1)]\ (d^2/2+d) \text{ bits}\end{aligned}$$

$$\begin{aligned}\text{Total Time}_{ring} &= \Sigma_{i=1,d/2}[\Sigma_{j=1,i}\,(2j+1) + i] \\ &= \Sigma_{i=1,d/2}[i^2+3i] \\ &= (d/2)(d/2+1)(d+1)/6+3(d/2)(d/2+1)/2 \\ &= (d/2)(d/2+1)(d/6+5/3)\end{aligned}$$

$$= d^3/24+d^2/2+5d/6$$

$$\therefore \text{ Average Time}_{ring} = (1/(d-1))\ \text{Total Time}_{ring}$$
$$= (d/2)(d/2+1)(d/6+5/3) / (d-1)$$
$$= d^3/24(d-1)+d^2/2(d-1)+5d/6(d-1)$$

Theorem 3.3 The average communication and time complexities of finding a shortest path between two arbitrary points in distributed and synchronous ring networks are $[(b+1)/2(d-1)]$ $(d^2/2+d)$ bits and $\pi d/2 + d^3/24(d-1)+d^2/2(d-1)+5d/6(d-1)$ units of time respectively, with the delay function $W(LN_i)=2i+1$, where b is the number of bits required to send a node id.

3.5 Tradeoffs Between Delay Function and Complexity

If we decrease the delay time, the time complexity of our algorithm FSP will decrease but its message complexity will increase. In this section we will analyze their tradeoffs.

Recall that, in Section 3.3, we called the target distance r and the trail distance t. In Section 3.4 we chose the delay function $W(LN_i)=2i+1$ in order to enforce the trail distance t to be zero (i.e. all outgoing search messages are stopped at the level of the sink). Thus, in this section we will examine the situation when t is greater than zero.

Assume the delay function $W(LN_j) = \omega(i)$ where ω is a predetermined constant and $0<\omega<2$. Once the sink node S_n has been reached, the trail message is sent back, from level r to level 0 and then from level 0 to level r+t, to stop the search messages (refer to tFigure 3.3). Thus the total length the trail message traveled is 2r+t, which takes 2r+t time units as well. Meanwhile, the search message is being delayed and forwarded toward the level r+t. Such a delay and forward process takes $\Sigma_{i=r,r+t}F(LN_i)+t$ time units, in which the first term is for the delay time and the second term is for the transmission time. Using the following formula we can express the term ω in r and t.

$$
\begin{aligned}
& 2r+t \le \Sigma_{i=r,r+t}F(LN_i) + t \\
& 2r \quad \le \Sigma_{i=r,r+t}F(LN_i) = \Sigma_{i=0,r+t}\,\omega(i) - \Sigma_{i=0,r-1}\,\omega(i) \\
& \qquad \le (\omega/2)\,[(r+t)(r+t+1) - r(r-1)] = (\omega/2)\,(t+2r)(t+1) \\
\therefore \quad & \omega \ge 4r / (t+2r)(t+1)
\end{aligned}
$$

The function obtained shows that if the trail distance t increases, the delay time will decrease substantially. As the delay time gets shorter, more and more search messages cannot be stopped before they are retransmitted from one node to others. Thus the number of messages needed will increase.

3.6 Closing Remarks

Before closing the chapter, several final remarks should be made.

First, our FSP algorithm works on arbitrary synchronous networks. In this chapter we have analyzed its complexities on arbitrary tree, linear and ring networks in the case that the maximum delay time is chosen. Further study is needed in order to determine its complexities on other network topologies, in particular the random graphs.

Second, in Section 3.5 we stated that if the trail distance increases, the delay time will decrease substantially but the number of messages needed will increase. We have also shown this relationship in mathematical terms. Further study may help develop an exact function for this relation and thus enable us to determine the average complexities of our FSP algorithm in the case that an arbitrary delay time is chosen.

Chapter 4

Conclusions

In this thesis we study distributed algorithms used in communication networks. In particular we have designed and analyzed new algorithms for the file sorting and shortest path finding problems.

The file sorting problem is to sort a file of n integers distributed over d stations in a communication network. We have presented new decentralized algorithms for almost balanced q-ary tree networks. These algorithms can easily be adapted to other topologies. The first algorithm, called SDF algorithm, requires extra memory space available at each node to store some temporary values. The second algorithm, called SDFW algorithm, removes such a requirement but only works on tree networks with postorder labeling. Their message complexities are $O(n/x \log_q d)$ and $O(n/x \log_q^2 d)$ respectively, where x is the message capacity.

Tradeoffs between memory requirements and communication complexity of the SDF algorithm have been discussed. If one is willing to sacrifice more local storage and larger message capacity, the performance of the SDF algorithm can be further improved. We have shown that, under similar conditions, our algorithm can achieve a better performance than existing file sorting algorithms. On the other hand, the impact of local processing on time complexity for synchronous networks is analyzed. We showed that the local processing time became a noticeable factor in our study of sorting file in the tree networks.

The two new algorithms have managed to evenly distribute the computation workload at each participating node in each cycle. They also provide a great deal of parallelism. If the message capacity x is assumed to be greater than 1, these algorithms will be x times faster and need x times fewer messages. Moreover, the SDF algorithm (for sorting with extra memory) can be modified so that it will work on tree networks of any labeling. This modification requires that the root station S_d has knowledge of the distribution order P. Therefore S_d can send assigned elements directly to their destinations.

Although the proposed file sorting methods use some blind operations which cannot recognize the already-sorted inputs, this shortcoming could be improved by sending both of the

minimum and maximum elements in a station to its father. Thus, at the end, the root station can compare the maximum element of the left subtree and the minimum element of the right subtree. If the former is smaller, then all elements in the left subtree are smaller than those in the right subtree. The root keeps the largest elements and will retire from further actions. The original tree is then divided into two subtrees, each being operated on with the same algorithm. Applying this technique repeatedly in AB binary tree networks, an already-sorted inputs can be detected in $O(\log_2^2 d)$ time with $O(d \log_2 d)$ messages.

In this thesis we have also studied the problem of finding a shortest path between two arbitrary stations S_m (source) and S_n (sink) in distributed and synchronous communication networks. Based on Dijkstra's method our algorithm performs search and trail operations. During the search process search messages fan out from the source and start building a spanning tree until S_n is reached. Once the sink has received the search message it starts the trail process, in which the sink sends a trail message backwards from S_n to S_m in order to stop the outgoing search messages as soon as possible. To accomplish this, before a full spanning tree is built, a delay is introduced at each node which has recieved a search message; that is after receiving a search message a node will retain the search message for some specified amount of time before sending it to its neighbours. Thus the search message does not always traverse the entire network as a straightforward implementation of Dijkstra's algorithm would do.

The average communication and time complexities of finding a shortest path between two arbitrary points in distributed and synchronous random tree networks are $(b+1)[2+(2\pi d)^{1/2}-2/d-(2\pi/d)^{1/2}]$ bits and $\pi d/2 + 3(\pi d/2)^{1/2}$ units of time respectively, with the delay function $W(LN_i)=2i+1$, where b is the number of bits required to send a node id.

Complexities of the FSP algorithm have been analyzed for arbitrary tree, linear and ring networks for the case that the maximum delay time is chosen. Further study is needed in order to determine its complexities on other network topologies. We have also mathematically shown that if the trial distance increases, the delay time will decrease substantially but the number of messages needed will increase. Further study is needed to develop an exact function for this relation and thus enable us to determine the average complexities of our FSP algorithm in the case that an arbitrary delay time is chosen.

Appendix A

The Complete SDF Algorithm for AB q-ary Tree Networks

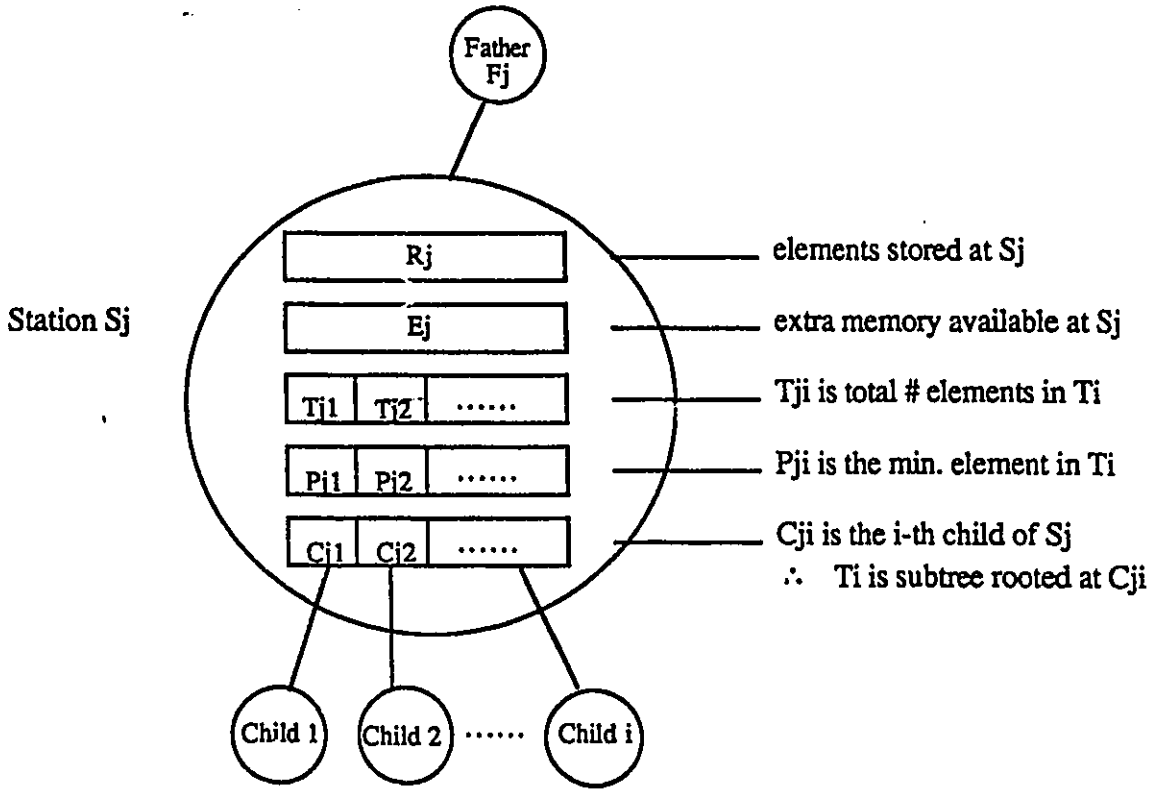


Figure A Local variables at a station S_j

During the execution of the SDF algorithm, the processor in a station S_j changes its state from time to time. There are four states, $STATE_j$ = {'Sleep', 'Initialize', 'Sort_elements', 'Final'}. The algorithm begins with all the stations in the state 'Sleep' and ends when all the stations reach the state 'Final'. Since processors communicate through exchange of messages, six types of messages are needed in our algorithms. They are:

WAKE_UP - a message from S_j to its children set C_j to wake up each of C_j.

NUM(h) - a message from S_j to its father F_j which contains the total number of

integers stored in the subtree T_j rooted at S_j , i.e. $h = |R_j|+S_t$ where $t \in T_j$.

REQ_MIN - a message from S_j to its child C_{ji} to request the minimum integer available in C_{ji}; here C_{ji} denotes the ith child of C_j .

SEND_MIN(k) - a message from S_j to its father F_j which carries its minimum integer k.

ASSIGN(k) - a message from S_j to its child C_{ji} which carries the assigned element k; here C_{ji} is one of the intermediate stations through which the assigned element k could go to its destination. In our algorithm, C_{ji} is determined by taking the leftmost child of S_j whose T_{ji} is not equal to zero.

TERMINATE - a message from S_j to its father F_j to indicate termination of the process.

The complete SDF algorithm on AB q-ary tree networks is presented as following:

In $STATE_j$ = 'Sleep'

While In $STATE_j$ = 'Sleep' Do
A sleeping station can be woken up by: (a) itself, or (b) receiving a WAKE_UP message;
set $STATE_j$ = 'Initialize' after being woken up;
Endwhile;

In $STATE_j$ = 'Initialize'

If (the father position of S_j is not connected to another station) Then
set F_j = nil;
Endif;

If (the ith child position of S_j is not connected to another station) Then
set C_{ji} = nil and P_{ji} = •;
Else
set P_{ji} = nil;
Endif;

If (all C_{ji}'s = nil (where i=1,2,...,q)) Then { a leaf station }

Send message NUM($|R_j|$) to F_j ;
set all T_{ji} 's = 0 (where i=1,2,...,q);
Else
Send message WAKE_UP to F_j and all C_{ji} 's (where i=1,2,...,q);
Endif;

Wait Until (received message NUM(h_i) from every child C_{ji} (where i=1,2,...,q)) Do
set $T_{ji} = h_i$;
Send message NUM($|R_j|+\Sigma_{g=1,i} h_g$) to F_j;
Enddo;

set E_j = {};
set $STATE_j$ = 'Sort_elements';

In $STATE_j$ = 'Sort_elements'

While In $STATE_j$ = 'Sort_elements' Do
If (P_{ji} = nil) Then
Send message REQ_MIN to C_{ji} to request for the min.;
Endif;

If (message REQ_MIN from F_j has arrived) And (no P_{ji} = nil) Then
Send message SEND_MIN(k=Small(R_j,P_j)) to F_j ;
remove (k) from either R_j or P_j accordingly;
Endif;

If (received message SEND_MIN(k) from C_{ji}) Then
store (k) in P_{ji};
Endif;

If (F_j = nil) And (no P_{ji} = nil) Then
Send message ASSIGN(k=Small(R_j,P_j)) to the leftmost child C_{ji} whose $T_{ji} \neq 0$;
remove (k) from either R_j or P_j accordingly;
$T_{ji} = T_{ji} - 1$;
Endif;

If (received message ASSIGN(k)) Then
    If (all T_{ji} 's = 0 (where i=1,2,...,q)) Then
        add (k) into E_j;
    Else
        Retransmit this message ASSIGN(k) to the leftmost child C_{ji} whose $T_{ji} \neq 0$;
    Endif;
  $T_{ji} = T_{ji} - 1$;
  Endif;

If (E_j is full) Or (all T_{ji} 's = 0 (where i=1,2,...,q)) Then
    set $P_{ji} = \bullet$;
    If ($|R_j| = 0$) Then
        set $STATE_j$ = 'Final';
    Endif;
  Endif;
Endwhile;

In $STATE_j$ = 'Final'

While In $STATE_j$ = 'Final' Do
If (C_{ji} = nil) Or (received message TERMINATE from C_{ji}) Then
    If ($F_j \neq$ nil) Then
        send message TERMINATE to F_j and then terminate the SDF process in S_j;
  Else
        declare the END OF ENTIRE PROCESS;
    Endif;
Endif;
Endwhile;

Appendix B

The Complete SDFW Algorithm for AB q-ary Tree Networks

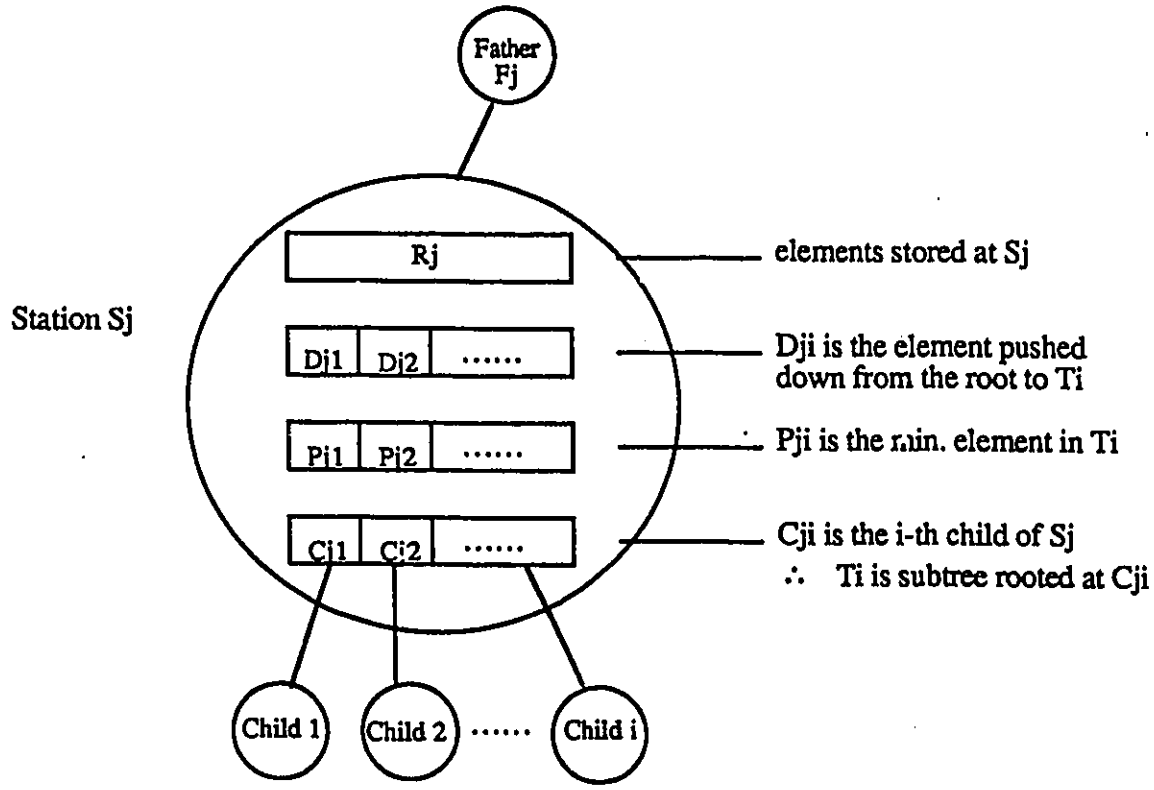


Figure B Local variables at a station S_j

During the execution of the SDF algorithm, the processor in a station S_j changes its state from time to time. There are four states, **STATE$_j$ = {'Sleep', 'Initialize', 'Sort_elements', 'Final'}**. The algorithm begins with all the stations in the 'Sleep' state and ends when all the stations reach the state 'Final'.

In this algorithm, we use a function called **DIRECTION(j,l)** to solve the following problem: In an q-ary tree network with postorder iabeling, a station S_j which has children $(C_{j1},C_{j2},...,C_{jq})$ wants to send a message to station S_l. The function DIRECTION determines to which child S_j should transmit the message so that this message will be in the correct route towards

its final destination S_t . Since the network is postorder labeled, this function is not hard to implement.

In a communication network, processors communicate through exchange of messages, six types of messages are needed in our algorithms. They are:

WAKE_UP - a message from S_j to its children set C_j to wake up each of C_j.

REQ_MAX - a message from S_j to its child C_{ji} to request for the maximum integer available in C_{ji}, here C_{ji} denotes the i^{th} child of C_j.

SEND_MAX(k,t) - a message from S_j to its father F_j to carry the maximum integer k which is currently available at S_j but originally from S_t.

ASSIGN(k,u) - a message from S_j to its child C_{ji} to carry the assigned element k that is heading to its destination S_u.

PUSH(k,v) - a message from S_j to its child C_{ji} to push the element k to its destination S_v, here C_{ji} is the i^{th} child of S_j and the value of i is determined according to the value of v and the postorder labelling of the tree network.

PUSH_REQ(k,v) - a message from S_j to its child C_{ji} to push the element k to its destination S_v and to request for the maximum integer available in C_{ji}.

TERMINATE - a message from S_j to its children C_j to indicate the end of the current iteration. Station S_j will retire from the SDFW process.

The **complete SDFW algorithm** on AB q-ary tree networks is presented now:

While In $STATE_j$ = 'Sleep' Do

A sleeping station can be woken up by: (a) itself, or (b) a WAKE_UP message;

If (the father position of S_j is not connected to another station) Then

set ITERNUM = 0 and F_j = nil;
Endif;
set $STATE_j$ = 'Initialize' after being woken up;
Endwhile;

In $STATE_j$ = 'Initialize'

If (F_j = nil) Then
ITERNUM = ITERNUM+1;
Endif;

If (the i^{th} child position of S_j is not connected to another station) Then
set C_{ji} = nil and P_{ji} = ($-\infty$,) and D_{ji} = (nil,nil);
Else
set P_{ji} = (nil,nil) and D_{ji} = (nil,nil);
Endif;

set $R_j = |R_j|$; {normalize the integer elements stored in S_j}
Send message WAKE_UP to F_j and all C_{ji} 's (where i=1,2,...,q) ;
set $STATE_j$ = 'Sort_elements';

While **In $STATE_j$ = 'Sort_elements'** Do
If ($P_{ji} = D_{ji}$ = (nil,nil)) Then
Send message REQ_MAX to C_{ji} to request for the max.;
Endif;

If (message REQ_MAX from F_j has arrived) And (no P_{ji} = (nil,nil)) Then
let R_j' = {all positive integer elements in R_j};
let (b' , v') = Large($R_j' \cup P_j$); {where v = j for set R_j}

```
   Send message SEND_MAX(b', v') to F_j ;
   remove (b', v') from either R_j or P_j accordingly;
Endif;

If (received message SEND_MAX(b;v) from C_ji) Then
   P_ji = (b, v);
   D_ji = (nil,nil);
Endif;

{if all P_ji's are (-∞, ) then F_j will retire from further operations, otherwise do the following}

If (F_j = nil) And (no P_ji = (nil,nil)) And (more than one P_jl ≠ (-∞, )) Then
   let R_j' = {all positive integer elements in R_j};
   If R_j' ≠ {} Then                    {the root station is collecting the α-integers}
      let (b,v) = Large(R_j∪P_j);       {where v = j for set R_j}
      If (any P_ji = (b,v)) Then
         remove k = Small(R_j') from R_j ;
         l = DIRECTION(j,v);
         Send message PUSH_REQ(k,v) to S_l (towards final destination S_v) ;
         P_ji = (nil,nil);
         D_ji = (k,v);
      Else
         remove b from R_j;
      Endif;
      add (-b) into R_j;
   Else                                 {the root will compare left and right max.}
      For i = 1,q Do
         let (b_i , v_i) = P_ji;
      Endfor;
      let l = q;
```

```
      While (P_jl = (-∞, )) Do
         l = l-1;
      Endwhile;
      remove (b' , v') = Large(b_i , v_i) from P_j ;
      Send message ASSIGN(b' , v') to C_jl (towards final destination S_vi);
      For l' = 1,l-1 Do
         remove (b' , v') = the l'-th Small(b_i, v_i) from P_j ;
         Send message PUSH_REQ(b', v') to C_jl' (towards destination S_vi);
         D_jl' = (b_i, v_i);
      Endfor;
   Endif;
Endif;

If (message PUSH_REQ(b,v) has arrived) And (no P_ji = (nil,nil)) And (v ≠ j) Then
   l = DIRECTION(j,v);
   D_jl = (b,v);
   let (b_l , v_l) = P_jl ;
   If (b > b_l) Then                    {swap D_jl and P_jl}
      D_jl = (b_l , v_l);
      P_jl = (b,v);
   Endif;
   let R_j' = {all positive integer elements in R_j};
   remove (b' , v')=Large(R_j' ∪ P_j) from either R_j or P_j according;  {where v=j for set R_j}
   Send message SEND_MAX(b', v') to F_j ;
   If (v' = l) Then
      l = DIRECTION(j,l);
      Send message PUSH_REQ(D_jl) to C_jl ;
   Else
      Send message PUSH(D_jl) to C_jl ;
      If (v' ≠ j) Then
         l = DIRECTION(j,v');
```

```
        Send message REQ_MAX to C_jl ;   {towards final destination S_v'}
      Endif;
    Endif;
    D_jl = (nil,nil);
Endif;

If (message PUSH_REQ(b,v) has arrived) And (no P_ji = (nil,nil)) And (v = j) Then
    add (b) into R_j ;
    let R_j' = {all positive integer elements in R_j};
    remove (b' , v')=Large(R_j' ∪ P_j) from either R_j or P_j according;  {where v=j for set R_j}
    Send message SEND_MAX(b' , v') to F_j ;
    If (v' ≠ j) Then
      l = DIRECTION(j,v');
      Send message REQ_MAX to C_jl;
    Endif;
Endif;

If (message PUSH(b,v) has arrived) Then
    l = DIRECTION(j,v);
    D_jl = (b,v);
    If (P_jl ≠ (nil,nil)) Then
      let  (b' , v') = P_jl ;
      If (b > b') Then                {swap D_jl and P_jl}
          D_jl = (b', v');
          P_jl = (b,v);
          l' = DIRECTION(j,v);
          Send message PUSH(D_jl) to C_jl' ;
      Else
          l' = DIRECTION(j,v');
          Send message PUSH(D_jl) to C_jl' ;
    Endif;
```

D_{jl} = (nil,nil);
Endif;

If (received message ASSIGN(b,v)) Then
  If (j = v) Then
    remove (b) from R_j;
    add (-b) into R_j;
  Else
    l = DIRECTION(j,v);
    Retransmit this message ASSIGN(b,v) to the child C_{jl};{towards final destination S_v}
  Endif;
Endif;

If (F_j = nil) And (no P_{ji} = (nil,nil)) And (exactly one $P_{jl} \neq (-\infty,)$) Then
  For each (b,v) in {$P_j \cup D_j$) Do
    l = DIRECTION(j,v);
    Send message PUSH(b,v) to C_{jl};
  Endfor;
  Send message TERMINATE to each C_{jl};
  set $STATE_j$ = 'Final';
Endif;

If (received message TERMINATE) Then
  If (all C_{ji} 's = nil) Then {the leaf station}
    set $STATE_j$ = 'Final';
  Else
    While ({$P_j \cup D_j$) $\neq$ {}) Do
      remove an element (b,v) from {$P_j \cup D_j$);
      If ((b,v) $\neq$ (nil,nil)) And (b,v) $\neq (-\infty,))$) Then
        l = DIRECTION(j,v);

Send message PUSH(b,v) to C_{jl};
Endif;
Endwhile;
set F_j = nil;
set $STATE_j$ = 'Initialize';
Endif;
Endif;
Endwhile;

In STATEj = 'Final'

declare the end of the ITERNUMth iteration;
terminate the SDFW process in S_j;

Appendix C

The Complete FSP Algorithm on Synchronous Networks

The FSP algorithm begins with all stations in the 'Sleep' state. The execution for a station S_j ends in the 'Find' state when it receives (and retransmits in some cases) TRAIL message(s). In the algorithm described below PRE_j, SUC_j, LN_j and $TIMER_j$ are four local variables in S_j. The PRE_j and SUC_j contain the predecessor and successors of S_j. The LN_j is the level number of the source-rooted tree of T at which S_j is located. The $TIMER_j$ monitors the delay time in S_j. (Note: one unit of local time is equal to one unit of global time in synchronous networks.) Let r be the target distance from the source to sink and t be the predetermined trail distance from the sink to where all outgoing search messages are stopped. Thus the delay function is equal to $W(LN_j)=\omega(j)$ where ω is a constant to satisfy $\omega \geq 4r/(t+2r)(t+1)$. Three types of messages are:

SEARCH(e,S_n) - from S_j to its successors SUC_j carries an integer e which is the level number LN_j and id of the sink station S_n.

TRAIL(bit 0) - from S_j to its successors SUC_j stops the outgoing SEARCH message.

TRAIL(bit 1) - from S_j to its predecessor PRE_j stops the outgoing SEARCH message; it also marks the receiving node PRE_j to be a node on the selected shortest path $SP_{m,n}$.

The complete FSP algorithm on AB q-ary tee networks is presented as follows:

In $STATE_j$ = 'Sleep'

A sleeping station can be woken up by:

(a) itself for it is the source station S_m, or

(b) receiving a SEARCH(e,S_n);

After the source station S_m wakes up, it does the following:

(a) set its predecessor PRE_m = {} and successors SUC_m = {all its adjacent stations};

(b) set $LN_m = 0$;

(c) send SEARCH(LN_m,S_n) to its successors SUC_j;

Set $STATE_j$ = 'Find';

In $STATE_j$ = 'Find'

If (S_j received SEARCH(e,S_n) from S_k) Then
  Set S_j's predecessor $PRE_j = S_k$ and
    S_j's successors SUC_j = {all adjacent stations of S_j excluding S_k};
  Set LN_j = e+1;

  If (S_j is the sink station) Then
    r = LN_j;
    declare the SINK STATION HAS BEEN REACHED;
    send TRAIL(1) to its predecessor PRE_j;
    declares the END OF EXECUTION FOR STATION S_j;
  Else
    set $TIMER_j$ = W(LN_j) units of time;
    wait until either $TIMER_j$ is expired or receiving TRAIL or TRAIL+;

    If (received TRAIL(0) or TRAIL(1) during waiting) Then
      declares the END OF EXECUTION FOR STATION S_j;
    Else
      send SEARCH(LN_j,S_n) to its successors SUC_j;
    Endif;
  Endif;
Endif;

If (S_j received TRAIL(1) from S_k) Then
  S_j marks itself as a node on the resulting shortest path $SP_{m,n}$;
  send TRAIL(1) to its predecessor PRE_j;
  send TRAIL(0) to each of its successors SUC_j;
  declares the END OF EXECUTION FOR STATION S_j;
Endif;

References

[AbDa80] Steven M.Abraham and Yogen K.Dalal, "Techniques for Decentralized Management of Distributed Systems", *Proceedings of the IEEE*, 1980, pp.430-437.

[AhHoUl83] A.V.Aho, J.E.Hopcroft and Jeffrey D.Ullman, *Data Structures and Algorithms*, Addison-Wesley.

[Ak85] Selim G.Akl, *Parallel Sorting Algorithms*, Academic Press, Inc.

[Am82] H.H.Abu-Amara, "Fault-Tolerant Distributed Algorithm for Election in Complete Networks, *Proceedings of the 2nd International Workshop on Distributed Algorithms*, The Netherlands.

[Aw85] B.Awerbuch, "Communication-Time Trade-offs in Network Synchronization", *Proceedings of the Fourth Annual ACM Symposium on Principles of Distributed Computing*, Minaki, Ontario, Canada, pp.272-276.

[BeGoMaSa86] A.Bertoni, M.Goldwurm, G.Mauri and N.Sabadine, "Parallel Algorithms and the Classification of Problems", *Lecture Notes in Computer Science*, Vol.253, pp.206-225, North-Holland.

[BhRu86] Bharat Bhargava and Zuwabg Ruan, "Site Recovery in Replicated Distributed Database Systems", *Proceeding of the 6th International Conference on Distributed Computing Systems*, Cambridge, Massachusetts, pp.621-627, Computer Society Press.

[BoPa82] M.Bozyigit and Y.Paker, "A Topology Reconfiguration Mechanism for Distributed Computer Systems", *The Computer Journal*, Vol.25, No.1, pp.87-92, Heyden & Son Ltd.

[Cha79] Ernest Chang, "An Introduction To Echo Algorithms", *IEEE*, pp.193-198.

[Che83] To-Yat Cheung, "Graph Traversal Techniques and the Maximum Flow Problem in Distributed Computation" *IEEE Transactions on Software Engineering*, Vol.SE-9, No. [illegible] 1983, pp.504-512.

[Che86] To-Yat Cheung, "An Algorithm with Decentralized Control for Sorting Files in a Network", *TR-86-10*, Dept. Comp. Sci., University of Ottawa, Ottawa, Canada.

[ChCiGoZa82] C.T.Chou, I.Cidon, I.S.Gopal and S.Zaks, "Synchronizing Asynchronous Bounded Delay Networks", *Proceedings of the 2nd International Workshop on Distributed Algorithms*, The Netherlands.

[ChMi82] K.M.Chandy and J.Misra, "Distributed Computation on Graphs: Shortest Path Algorithms", *Communications of the ACM*, Vol.25, pp.833-837.

[ChRo79] Ernest Chang and Rosemary Roberts, "An Improved Algorithm for Extrema-Finding in Circular Configurations of Processes", *Communications of the ACM*, Vol.22, No.5, May 1979, pp.281-283.

[DiFeGa83] W.Dijkstra, W.H.J.Feijen and A.J.M.Gasteren, "Derivation of a Termination Detection Algorithm *for Distributed Computations*", *Information Processing Letters*, **16**, pp.217-219, North-Holland.

[DrBa85] Z.Drezner and A.Barak, "Efficient Algorithms for Routing Information in a Multicomputer System", Distributed Algorithms on Graphs, *Proceedings of the 1st International Workshop on Distributed Algorithms*, Ottawa, Canada, pp.41-48, Carleton University Press.

[Fr80] Nissim Francez, "Distributed Termination", *ACM Transactions on Programming Languages and Systems*, Vol.2, No.1, January 1980, pp.42-55.

[Fr83] G.N.Frederickson, "Tradeoffs for Selection in Distributed Networks", *Proceedings of the 2nd ACM Symposium on Principles of Distributed Computing*, August 1983, pp.154-160.

[Ga82] E.Gafni, "Generalized Scheme for Topology Update in Dynamic Networks", *Proceedings of the 2nd International Workshop on Distributed Algorithms*, The Netherlands.

[GhBaGh86] A.Ghafoor, T.R.Bashkow and I.Ghafoor, "Fault-Tolerance and Diagnosability of Bisectional Interconnection Networks", *Proceeding of the 6th International Conference on Distributed Computing Systems*, Cambridge, Massachusetts, pp.62-69, Computer Society Press.

[HaLi86] Satoshi Hasegawa and Jane W.S.Liu, "A Reliable Token-Driven Process Synchronization Algorithm", *Proceeding of the 6th International Conference on Distributed Computing Systems*, Cambridge, Massachusetts, pp.598-604, Computer Society Press.

[HaSiStDo83] J.Y.Halpern, B.Simons, R.Strong and D.Dolev, "Fault-Tolerant Clock Synchronization", *Proceeding of the Third Annual Symposium on Principles of Distributed Computing*, Vancouver, Canada, pp.89-102.

[HoSh86] S.Horiguchi and Y.Shigei, "A Parallel Sorting Algorithm for a Linearly Connected Multiprocessor System", *Proceeding of the 6th International Conference on Distributed Computing Systems*, Cambridge, Massachusetts,

pp.111-118, Computer Society Press.

[JoNiSk87] K.Johansen, U.Jorgensen, S.Nielsen, S.E.Nielsen, S.Skyum, "A Distributed Spanning Tree Algorithm", *2nd International Workshop on Distributed Algorithms*, the Netherlands.

[Ko81] Walter H.Kohler, "A Survey of Techniques for Synchronization and Recovery", *Computing Surveys*, Vol.13, No.2.

[Kn82] Donald E. Knuth, *The Art of Computer Programming*, Vol.1-3, Addison-Wesley.

[Ku80] H.T.Kung, "The Structure of Parallel Algorithms", *Advances in Computers*, Vol.19, pp.65-112, Academic Press Inc.

[Kum84] Devendra Kumar, "A Class of Termination Detection Algorithms for Distributed Computations", *Lecture Notes in Computer Science 206*, pp.73-110.

[La77] G.LeLann,"Distributed Systems - Towards A Formal Approach", *IFIP Congress Proceedings*, pp.155-160.

[LaLa85] I.Lavallee and C.Lavault, "Scheme for Efficiency-Performance Measures of Distributed and Parallel Algorithms", *Distributed Algorithms on Graphs, Proceedings of the 1st International Workshop on Distributed Algorithms*, Ottawa, Canada, pp.69-102, Caleton University Press.

[LaDhMi84] S.Lakshmivarhan, Sudarshan K.Dhall and Leslie L.Miller, "Parallel Sorting Algorithms", *Advances in Computers*, Vol.23, pp.295-354, Academic Press Inc.

[LaTh82] B.Lakshmanan and K.Thulasiraman, "On the Use of Synchronizers for Asynchronous Communication Networks", *Proceedings of the 2nd International Workshop on Distributed Algorithms*, The Netherlands.

[LeSaUr87] Van Leeuwen, Nicola Santoro and Jorge Urrutia, "Guessing Games and Distributed Computations in Synchronous Networks", *Lecture Notes in Computer Science 267*, ICALP, pp.347-356.

[Lo84] Michael C.Loui, "The Complexity of Sorting on Distributed Systems", *Information and Control 60*, pp.70-85.

[Ma83] T.A.Matsushita, "Distributed Algorithms for Selection", *Rep.T-127*, Coord. Sci. Lab. Univ. Illinois, Urbana-Champaign, July 1983.

[Ma82] F.Mattern, "Experience with a New Distributed Termination Detection Algorithm", *Proceedings of the 2nd International Workshop on Distributed Algorithms*, The Netherlands.

[McSc86] Robert McCurley and Fred B.Schneider, "Derivation of a Distributed Algorithm

for Finding Paths in Directed Networks", *Science of Computer Programming 6*, pp.1-9, North-Holland.

[Me83] M.Merritt, "Elections in the Presence of Faults", *Proceedings of the Third Annual ACM Symposium on Principles of Distributed Computing*, Vancouver, Canada, pp.134-142.

[MeMo70] A.Meir and J.W.Moon, "The Distance between Points in Random Trees", *Journal of Combinatorial Theory 8*, pp.99-103.

[MiCh82] Jayadev Misra and K.M.Chandy, "Termination Detection of Diffusing Computations in Communicating Sequential Processes", *ACM Transactions on Programming Languages and Systems*, Vol.4, No1, January 1982, pp.37-43.

[Mo70] J.W.Moon, *Counting Labelled Trees*, Canadian Mathematical Monographs.

[NeSaUr87] A.Negro, N.Santoro and J.Urrutia, "On the Packet Complexity of Distributed Selection", *Proceedings of the Second International Workshop on Distributed Algorithms*, Amsterdam, Holland, 1987, to also appear in Lecture Notes in Computer Science, Springer-Verlag.

[OrRo86] A.Orda and R.Rom, "Packet Duplication and Elimination in Distributed Networks", *Proceeding of the 6th International Conference on Distributed Computing Systems*, Cambridge, Massachusetts, pp.401-404, Computer Society Press.

[PaSa81] D.Stott Parker, Jr. and Behrokh Samadi, "Distributed Minimal Spanning Tree Algorithms", *Performance of Data Communication Systems and Their Applications*, pp.45-54, North-Holland.

[Pe82] Gary L.Peterson, "An O(n $\log_2$n) Unidirectional Algorithm for the Cicular Extrema Problem, *ACM Transactions on Programming Languages and Systems*, Vol.4, No.4, October 1982, pp.758-762.

[PeMaWo80] J. Kent Peacock, Eric Manning and J.W.Wong, "Synchronization of Distributed Simulation Using Broadcast Algorithms", *Computer Networks 4*, pp.3-10, North-Holland Publishing Company.

[RoSaSi83$_1$] Doron Rotem, Nicola Santoro, Jeffrey Sidney, "A Shout-Echo Selection in Distributed Files", *Proceedings of the 14th Software Engineering Conference on Combinatorics, Graph Theory and Computing*.

[RoSaSi83$_2$] "A Reduction Technique for Distributed Selection: I", *SCS-TR-35*, School Comp. Sci., Carleton University, Ottawa, Canada.

[RoSaSi85] "Distributed Sorting", *IEEE Transactions on Computers*, Vol.c-34, No.4,

pp.372-376.

[SaUrZa86] Nicola Santoro, Jorge Urrutia and Shmuel Zaks, "Sense of Direction and Communication Complexity in Distributed Algorithms", *Distributed Algorithms on Graphs - Proceedings of the First International Workshop on Distributed Algorithms*, Ottawa, Canada, August 1985, pp.123-132, Carleton University Press.

[SiUr86] J.B.Sidney and J.Urrutia, "The Communication Complexity Hierarchy in Distributed Computing", *Distributed Algorithms on Graphs - Proceedings of the First International Workshop on Distributed Algorithms*, Ottawa, Canada, August 1985, pp.133-142, Carleton University Press.

[StTo86] J.A.Stankovic and D.Towsley, "Dynamic Reallocation in a Highly Integrated Real-Time Distributed System", *Proceeding of the 6th International Conference on Distributed Computing Systems*, Cambridge, Massachusetts, pp.374-381, Computer Society Press.

[SyDeKo83] Maciej M.Syslo, Narsingh Deo and Janusz S.Kowalik, *Discrete Optimization Algorithms with Pascal Programs*, Prentice Hall.

[T084] Rodney W.Topor, "Termination Detection for Distributed Computations", *Information Processing Letters* 18, pp.33-36, North-Holland.

[WaOw83] David W.Wall and Susan Owicki, "Construction of Centered Shortest-Path Trees in Networks", *Networks*, Vol.13, pp.207-231, John Wiley & Sons, Inc.

[We84] Lutz Wegner, "Sorting a Distributed File in a Network", *Computer Network 8*, pp.451-461, North-Holland.

[Za84] Shmuel Zaks, "Optimal Distributed Algorithms for Sorting and Ranking", *M.I.T.Technical Report*..