The Volatility of Bitcoin, Bitcoin Cash, Litecoin, Dogecoin and Ethereum

Khaoula Ghaiti

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Telfer School of Management
University of Ottawa
Thesis Supervisor: Dr. François-Éric Racicot and Dr. Samir Saadi

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Abstract

The purpose of this paper is to select the best GARCH-type model for modelling the volatility of Bitcoin, Bitcoin Cash, Litecoin, Dogecoin and Ethereum. GARCH (1,1), IGARCH(1,1), EGARCH(1,1), TGARCH(1,1) and CGARCH(1,1) are used on the cryptocurrencies closing day return. We select the model with the highest Maximum Likelihood and run an OLS regression on the conditional volatility to measure the day-of-the-week effect. The findings show that EGARCH(1,1) model best suits Bitcoin, Litecoin, Dogecoin and Ethereum data and that the GARCH(1,1) model suits best Bitcoin data. The results show a significant presence of day-of-the-week effects on the conditional volatility of some days for Bitcoin, Bitcoin Cash and Ethereum. Wednesday has a significant negative effect on Bitcoin conditional volatility. Friday, Saturday and Sunday are found to be significant and positive on Bitcoin Cash conditional volatility. Finally, Saturday is found to be significant and positive on Ethereum conditional volatility.

Keywords: Cryptocurrency, Bitcoin, Bitcoin Cash, Litecoin, Dogecoin, Ethereum, GARCH(1,1), IGARCH(1,1), EGARCH(1,1), TGARCH(1,1), CGARCH(1,1), Day-of-the-Week Effect.

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Dedications

To my mother who is my model and whose tenacity pushes me forward.

To my father whose patience and wisdom allowed me to get where I am.

Chapter 1: Introduction

Banking has become an integral part of our lives. This system that is used to control monetary exchanges and assets dates back to the 13th century. Indeed, the Medici family of the Florentine renaissance created a system that dominated transactions in Europe and later different parts of the world. They succeeded in identifying a critical need in society and have responded by creating a revolutionary system that is based on trust. They essentially found a way to mediate between savers and borrowers by contributing their excess capital and redistributing it to those who needed it in return for a fee. Bankers have created a powerful and centralized new system of trust by introducing a myriad of the company's debts and receivables into the central ledger of a single bank. They have succeeded in creating a robust system where money is no longer just a means of physical exchange but a system of organizing and sharing society’s debt and payments. Modern central banks seek to maintain price stability by regulating the supply of money on behalf of governments (Roover, 1946).

“Banking is essential, banks are not.” Bill Gates made this controversial statement in 1994 stipulating that banking is necessary in the capitalist economy but banks are not. Two years after this declaration, companies such as AliPay, We Chat and many others brought cashless comfort to millions of underbanked people living in developing countries. This has shown that when people trust an organization enough to take care of their money, they can approach banking in a non-traditional way.

In addition, the 2008 financial crisis left the world in an unprecedented situation; corporate borrowing and capital expenditures fell sharply. The failure of these institutions resulted in the freeze of global credit markets and required government interventions worldwide. This has created distrust amongst the private banking institutions, the governmental authorities and the public. Until now, the monetary system has been created on a model of centralized ledger-keeping by central banks operating uber-economy-wide ledger. This has created a basis of efficiency and security for communities that did not have any other way of trusting the monitoring of their debts. The problem has always been whether this model delegated too much power and profit to the centralized bank. The emergence of alternatives to the traditional banking system therefore seems to be entirely justified. The challenge then is to find a compromise solution – a reliable and decentralized system to maintain order in society without losing the efficiency and security that centralization had provided. Their decentralized Peer-To-Peer financial system gained immense popularity after society became aware of the limits of the centralized bank. One year after the financial crisis, the first cryptocurrency, also known as the virtual currency or Bitcoin, was created. This alternative, like gold, has a similar finite-supply quality, which supports its value and cannot be disturbed by central banks. With those cryptocurrencies, there is no need for a trusted central agent. Instead, it relies on distributed ledger technology, known as blockchain, in order to conduct a ledger that is maintained across a network.

The success of Bitcoin has since created the explosive appearance of cryptocurrencies, more commonly known as "Altcoins"; however, none of them ever surpassed the pioneer. The phenomenal success of the highly publicized Bitcoin was the primary catalyst for the general interest on the subject.

Due to the uncertainty surrounding this new technology, prices have been highly volatile.

While the models depicting the return volatility of some cryptocurrencies has been examined in recent literature, minimal research elaborates on the presence of day-of-the-week effect in regards to the conditional volatility of cryptocurrencies. The nascency of the cryptocurrency market can contribute to a lack of data, making such research difficult.

The day-of-the-week effect is typically examined in stock markets, bond markets, currencies, and a number of commodities including interest rates. This thesis contributes to previously acquired models used to describe cryptocurrency volatility and the day of the week effect.

In the first section, we will detail the characteristics of the candidate cryptocurrencies (Bitcoin, Bitcoin Cash, Litecoin, Dogecoin, Ethereum). Afterwards, we will observe their classification and the mechanism behind it. We will list the advantages and disadvantages of this new technology and the literature review around cryptocurrencies.

In the second section, we will be running 5-GARCH type models to determine the best model for forecasting the volatility of Bitcoin, Bitcoin Cash, Litecoin, Dogecoin and Ethereum. The AIC and Log-Likelihood will determine the best GARCH model. We will add dummy variables to see if there is a day-of-the week effect on the regression equation.

Chapter 2: Leading cryptocurrencies

This section will look at the cryptocurrencies that have been selected for this research paper. The five chosen cryptocurrencies are among the most common in the world as of 2018: Bitcoin, Bitcoin Cash, Litecoin, Dogecoin and Ethereum (The Telegraph, 2018).

The concept of cryptocurrency has gained high interest and increased popularity in the last few years, notably because of the rise of Bitcoin. The general concept of decentralized and encrypted currency has become more famous and with it the altcoins were born. Altcoin is the term used to refer to the cryptocurrencies that came after Bitcoin, as a result of its popularity. They are also based on a blockchain system but market themselves as a better, quicker and more efficient alternative to Bitcoin.

2.1 Bitcoin

Bitcoin is a digital currency created by an unknown person or a group of developers using the alias Satoshi Nakamoto. In 2008, Nakamoto released a whitepaper that explains the foundation for cryptocurrencies. The main features presented in his whitepaper are mostly the same for the other cryptocurrencies that are referred to as alt-coins (Alternative coins). Its features are defined by a decentralized authority, the use of a peer-to-peer connection and the requirement of an internet access with a form of cryptology incorporated in their account known as wallets (Nakamoto, 2008). Bitcoin is the original cryptocurrency and remains the most widely used since 2009. Compared to conventional online payment systems, it provides lower transaction fees and is run by a decentralized authority.

Bitcoin was revolutionary when it came out since it was the first digital currency system that did not need a centralized intermediary. Instead, it relied on distributed ledgers, commonly known as blockchains, that manage books and records. Communication is done through each node of the network that receives and communicates each user transaction. The ledgers remain up to date with the transactions thanks to the node's incentives and code-based controls. The uniqueness of this system lies in the fact that it relies not on the traditional authority associated with banking institutions, but on the trust of third-parties facilitating transactions.

2.2 Bitcoin Cash

Created in 2017, Bitcoin Cash arose from a fork of Bitcoin Classic. A group of developers forked Bitcoin to create a new version that increases the size of blocks in its blockchain, which allows more transactions to be processed per second.

The main differences are listed below:

- Transactions with Bitcoin Cash are cheaper than with Bitcoin due to lower fees ($0.20 per transaction vs $1 per transaction for Bitcoin).
- Shorter transfer period – something that Bitcoin lacks notably
- Bitcoin Cash can handle more transactions per second. This means that more users can use it at the same time than they can with Bitcoin. (Bitcoin Cash, 2018)

This is possible because Bitcoin Cash blocks are eight times bigger than regular Bitcoin blocks. It is more scalable and more people can therefore perform transactions on the blockchain system.

However, Bitcoin Cash lacks the first mover advantage that Bitcoin enjoys: being the newer one, it does not have the same market penetration or investor confidence that Bitcoin has. From a mining perspective, Bitcoin Cash and Bitcoin miners use the same equipment, but Bitcoin Cash miners make around 50-60% less profit (Javarone & Wright, 2018).

2.3 Litecoin

Famous for being the "Bitcoin's little brother," a former Google engineer created this virtual currency in 2011. The motivation behind it was to improve on Bitcoin. It resembles the original for being a peer-to-peer cryptocurrency that runs under a decentralized system by anyone running a computer with internet.

Compared to Bitcoin, Litecoin transactions are processed quicker and have increased storage efficiency. Its rate is four times that of Bitcoin. Litecoin also has a token production rate that quadruples that of Bitcoin (10min vs 2.5min) and an overall token capacity that goes beyond 84 million (Andrews, 2018).

2.4 Dogecoin

Launched in December 2015, Dogecoin started as a joke with a Shibu Inu (Japanese dog) as a logo. Its funny and accessible side played in its favor and made it progress into a full-fledged cryptocurrency based on the blockchain system. Like the other cryptocurrencies, it is defined as a decentralized virtual currency that uses peer-to-peer technology to perform out its operations. Its most notable characteristic is that it allows very fast payments and transfers compared to the others. It allows fast payments to all its users around the world.

The reasons that allowed this cryptocurrency to be so accepted and used are:

- The transaction fees are only $0.01.

- Transaction speed is very quick and efficient with an average time of 1 minute for the transaction to be completed and confirmed. This improvement is significantly better compared to other competitors.
- There is no limit on the quantity of Dogecoins a user can mine and collect. The Dogecoin supply continues to expand as long as miners continue to operate.

2.5 Ethereum

Launched in 2015, Ethereum is one of the most recent cryptocurrencies but it has enjoyed tremendous success. At the start of the year 2017, the Ethereum token was worth just 10 dollars, and the value recorded a 9900 percent increase where it was valued at over 1000 dollars by January 5 2018 (*Ethereum Price Index — CoinDesk 20*, 2018). It is also based on the fundamental concepts of blockchain and is particularly known for its implementation of Distributed Applications (DApps) combined with SmartContracts.

Ethereum is not only a currency but also a ledger technology that companies rely on to build new programs. The essence of Ethereum differs in purpose from that of Bitcoin. Bitcoin was created as a new kind of currency whereas Ethereum was created through its own currency engine, Ether, as a network that significantly eases the use of peer-to-peer contracts and applications.

Chapter 3: The classification of cryptocurrencies

Since the appearance of virtual currencies in the market, several questions have arisen about their classification. Their particularity and uniqueness have left many perplexed. They do, however, share similarities with currencies and commodities and it can be argued that they belong to a category of their own.

Digital currencies are classified as digital assets that can be used as a medium of exchange, as an account unit or as a value store. Unlike fiat currencies, virtual currencies are not supported by a sovereign nation or commodity but can be converted into real money (Fordham, 2018).

Grinberg (2012) states the similarities and differences between virtual currencies and traditional currencies and commodities. The similarity lies in the fact that users can use them as they wish, even by simply writing contracts that involve them. Furthermore, they are "liquid, digital, easy for end users to exchange with one another, generally anonymous, and popular among government-distrusting gold bugs." Since their amount is limited, this gives them the criteria of rarity such as fiat currencies. Their differences can be summarized as follows: There are no centralized authorities administering new currencies, it tends to be more like fiat currency than commodity currency, and regulation is not evident since the control is decentralized.

The International Monetary Fund (IMF) has determined that virtual currencies combine the characteristics of traditional currencies, commodities and payment systems. This is very important as it will influence their future of virtual currencies in terms of legal status and in determining which agencies will be trusted for regulating them. This makes quite a difference since, within the same jurisdiction, different agencies have different classifications and thus totally different outcomes for the virtual currency market. For instance, virtual currency has been classified as value for AML/CFT purpose (source IMF, 2016) by the treasure Department FinCen, but has also been categorized as property by the US Tax Authority. This illustrates that the context and role of each agency can drastically affect the future of Bitcoin and all other virtual currencies.

To this day, virtual currency still has not managed to fully fill the three main roles associated with traditional currency.

The more obvious problem is that, although there has been tremendous progress in the last few years, the limited acceptance network and the relatively smaller audience are not significant enough to give it a comparable medium of exchange to that of the traditional currencies. Without the role of legal tender, this market is limited to being accepted solely when both transacting parties are already in compliance with it. There is much growth that is still needed if the weight and volume of virtual currencies is to attempt to compete with traditional currency.

The second problem is its elevated volatility that considerably limits its reliability and reputation as a store of value. For comparison, it has not been uncommon for cryptocurrency volatilities to reach levels higher than those of fiat currencies. The study of volatility is outlined with much more detail in the later sections of this paper.

The third and last problem is the lack of evidence that these virtual currencies act as independent unit of account or act as a direct representation of the value of goods and services such as gold. Instead, their value is measured with respect to other currencies' exchange rate.

Moreover, because it’s a risky volatile market, the majority of users tend to change their payments into traditional currency as soon as possible.

Chapter 4: The mechanism behind cryptocurrencies

Unlike fiat currencies, cryptocurrencies are not issued by the government but are the result of a decentralized mechanism based on a cryptography system. Cryptocurrencies function differently from fiat currency. This section will take a look at the process behind it.

The cryptocurrency protocol runs on a network of computers belonging to miners, who are responsible for maintaining blockchain systems. These cryptocurrencies are created through a process known as *mining,* who help in the validation of the nodal blockchain network.

Cryptocurrency mining or cryptomining is the process in which transactions for various forms of cryptocurrency are verified and added to the blockchain digital ledger. The Miners will essentially check the ledger's validity and authenticity. To run the program, they need a specialized hardware that can be downloaded on the multiple cryptocurrencies' platforms online. They got rewarded with newly minted cryptocurrency (CoinDesk, 2018).

The blockchain is a public ledger of all transactions that occurred on the network. It's a technology for storing and transmitting information that is transparent, secure, and operating without a central control body. Transactions between network users are grouped in blocks. Each block is validated by the network nodes, according to techniques that depend on the type of blockchain. Once the block has been validated, it is time-stamped and added to the block chain. The transaction is then visible to the receiver as well as to the whole network. The information stays unchanged for all the users (Crosby et al., 2016).

In order for the transaction to take place on the network, users need a digital wallet, including a private key and a public ledger (Fink & Johann, 2014) . The wallet is composed by an address using random letters and numbers that is unique for each user. The identity is private since users can either choose a nickname or an address to make transactions. Furthermore, they are not limited to one address.(CoinDesk, 2018).

During a transaction, when the sender provides his input address to the receiver, the transaction becomes a public record in a blockchain ledger. When it takes place, any user keeping a copy of the ledger will be informed that an income transaction is taking place on the virtual network. When the sender makes a transaction with the receiver it needs to be signed by the sender's private key. This key represents the digital signature, the personal identifier. Only a private key creates a new message and, once it is signed, in order to send it to the receiver, it is sent to the network with a public key. This public key acts as a verifying mechanism and mostly confirms that the sender's message was marked by his personal signature and was associated with a public key sent along with the transaction. Once the transaction is on the public network, it is then announced to all the people who are maintaining the network and watching the ledger: the miners. The miners are using algorithms in order to verify the validity of individual transactions. This process occurs to mainly ensure that no fraudulent transactions are taking place by using hash function (Böhme et al., 2015). This prevents the double spending; spending a virtual currency twice. The miner will be able to confirm a transaction that will be added to the blockchain ledger. The process of verifying is time- and money-consuming but required to build the necessary equipment. Every time a miner verifies a transaction, he gets rewarded by a cryptocurrency.

Chapter 5: Benefits and risks of cryptocurrencies

Bitcoin, Bitcoin Cash, and a dozen others virtual currencies have become the new Eldorado for some and the best way to make a transaction for others. They have advantages and disadvantages to take into account.

It cannot be denied that virtual currencies are a highly disruptive technology in the financial sector. This new technology announces changes not only in the political aspect, but also in the economic and social aspects. By reducing operating costs and eliminating the financial middlemen, it removes government control and creates a potential conflict of interest.

Whether for companies or individuals, cryptocurrencies are easy to handle because they can be used everywhere, as long there is access to Internet. In fact, they can be sent everywhere, regardless of the location; it is a currency without borders. All those transactions are done from an account to another without an intermediary. The transfer is very fast regardless of the country of origin and arrival. This is not the case for fiat currencies that have to be transferred from one country to another by the intermediary of the bank (DeVries, 2016).

There are many countries where banks are not efficient and do not respond to people's needs. Indeed, for many reasons, some people are forced to limit their financial privileges due to incompetent financial institutions and their countries' laws. Some women need their guardian's consent to make certain transactions, or some people simply do not have the profile to access financial services.

Contrarily, cryptocurrencies are accessible and serve everyone without the need for proof or identification. In addition, financial access comes at a lower cost. The opportunity this creates for the countries involved is remarkable and promises great changes at the global level.

Some African countries have seen the potential that cryptocurrency can have on their economies, such as Nigeria or South Africa. About ten Bitcoin exchanges have seen the emergence in African markets and some exchanges have adapted their services to meet this new demand (How We Made It In Africa, 2017).

Nevertheless, cryptocurrencies come with some <u>disadvantages</u> to consider.

The issue with this technology is that it is slow in comparison to banks that deal with credit card transaction. Visa, for instance, processes around 2,000 transactions per second on average, amounting to 150 million transactions per day. The Bitcoin network, however, only processes 4.6 transactions per second and it takes 10 minutes to process. Due to Bitcoin's popularity, we can only expect that the waiting time will not be long since the number of transactions will considerably increase in time.

Many countries and international organizations are trying to intervene and exert control over the quick expansion and growing direction of cryptocurrency in the world. While some goverments are looking out for the interest of investors, other governments and organizations are focusing on the digital assets they represent. There is thus a lack of uniformity in priorities, making it more difficult to stay up to speed with the quickly expanding and evolving reality of cryptocurrency (Brandvold et al., 2015). A lack of appropriate framework and structure around new markets opens the door to potential market abuse and manipulation. This usually demotivates investor participation and particularly that of instutions whose participation is usually critical. The presence of institutions in investments usually leads to better stability and a positive overal outcome of that investment market. In the meantime, the cryptocurrency market is bound to be very volatile and responsive to any sort of news or even rumors. This again makes it less interesting for new investors to come in, thus creating a vicious cycle that can be difficult to break.

Another concern regarding cryptocurrencies is their susceptibility to fraud, money laundering, tax evasion and terrorism financing mainly because of its anonymous nature. It is therefore important to have intermediate checks validating the identity of both parties before approving them. It is worth mentioning here that the presence of cryptocurrency further complicates policy making for institutions such as central banks that want to have control over how much of their currency is circulating to put certain policies in place (Brown, 2016) .

Chapter 6: Volatility of cryptocurrencies

Cryptocurrencies have seen a surge in interest in recent years and have attracted attention from media, financial and government institutions and venture capitalists. Their promises and dazzling successes led to more interest in the possibilities they could offer. We are going to list here some relevant papers to understand the evolution and the methods used to determine the factors influencing volatility.

Griberg (2012) was the first to publish an academic article on Bitcoin in which he addresses gold-backed currencies and virtual world and game-related commerce. Since then, the pace of cryptocurrencies research has accelerated.

Some articles are organized around the different factors that influence and explain the determination of Bitcoin price.

Kristoufek (2013) finds that there is a direct relationship between Bitcoin and search queries on Google and Wikipedia. He determined that relating the interest on Bitcoin and the trend value connects search queries and prices.

Shortly after, Kristoufek (2015) examines the potential drivers of Bitcoin prices and finds that Bitcoin is a unique asset with properties of both a standard financial asset and a speculative asset. Furthermore, he finds that fundamental factors as usage in trade, money supply and price level have an impact on Bitcoin price in the long run.

On the other hand, Bouoiyour and Selmi (2015) discover that Bitcoin price is not influenced by macroeconomic fundamentals but acts as a “speculative bubble”.

Other papers look in depth at the attractiveness of cryptocurrencies to investors.

Yermack (2014), argues that Bitcoin prices and volatility seem unrelated to economic or financial causes, making them impossible to hedge or predict. On the other hand, Ciaian, Rajcaniova, and Kancs (2016) findings shows that Bitcoin prices are affected by the market fundamentals and the attractiveness.

Bouri and al.(2017), assert that Bitcoin serves better as a diversifier than as a hedging instrument against equity indices, bonds, oil and gold.

Corbet et al.(Corbet et al., 2018) inspect the connection between Bitcoin, Ripple and Litecoin and observe that diversification benefits can emerge for investors oriented towards short-term horizons.

A number of academic studies use derivatives of the GARCH model to explain the prediction of volatility.

For instance, Guesmi et al.(2019) explores the conditional cross effects and volatility spillover between Bitcoin and financial indicators using different multivariate GARCH specifications. They find that there are significant returns and volatility spillovers.

Beneki et al. (2019) uses multivariate BEKK-GARCH to evaluate whether there are any volatility spillovers and hedging possibilities when looking at Bitcoin and Ethereum. Their study reveals that there are some correlations that are dependent on time. It is brought to the reader's attention that Ethereum has a delayed impact that responds to Bitcoin volatility.

Kumar and Anandaro (2019) also examine volatility spillovers, but they base their study on returns of Bitcoin, Ethereum, Ripple and Litecoin. Their results show a short-term correlation, which can be interpreted as evidence of turbulence as well as herding behavior in this market.

Chapter 7: Data and Methodology

The data used in this paper was obtained from CoinMarketCap's website and includes the earliest date available to December 31, 2018. The data set consists of all daily data for Bitcoin, Bitcoin Cash, Litecoin, Dogecoin and Ethereum.

- Bitcoin closing price from Septembre 18, 2014 to December 31, 2018.
- Bitcoin Cash closing price from August 20, 2017 to December 31, 2018.
- Litecoin closing price from September 18, 2014 to December 31, 2018.
- Dogecoin closing price from September 15, 2015 to December 31, 2018.
- Ethereum closing price from August 08, 2015 to December 31, 2018.

Assuming the price change of the five cryptocurrencies is continuous, the daily rate of return $r_{t,}$ of the closing prices is computed using the log return formula:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right),$$

where P_t is the current price and P_{t-1} is the previous price. Five different GARCH-type models (GARCH, IGARCH , EGARCH, TGARCH, CGARCH) were applied to model Bitcoin, Bitcoin Cash, Litecoin and Ethereum volatility with the assumption that p = 1 and q = 1.

Using Eviews (11) to find the model that best fits the data, the models are analyzed with a Student's t distribution.[1] The optimal models are selected according to the Log-likelihood and

[1] By performing the data analysis using multiple distribution models, we found that the Student's t -distribution is better than the others and robust with the nature of the data. Indeed, the Student's t-distribution outperforms the

Akaike Information Criterion (AIC) and then used to measure the day-of-the-week effect on the cryptocurrency volatility.

Chapter 8: Analysis and Discussion

8.1: Modelling volatility using GARCH(1,1)-type models

8.1.1 GARCH (1,1)

The GARCH(1,1) model stands for Generalized Autoregressive Conditional Heteroscedasticity. It is a statistical model that is mostly used to estimate the volatility of returns. Created by Bollerslev in 1986, it predicts the conditional variance of an independent variable based on the past volatility σ_{t-1}^2, and the past error term, e_{t-1}^2. The model is described in the equation below:

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 ,$$

Where σ_t^2 is the conditional volatility and $e_{t-1}^2$is the squared unexpected returns for the previous period. The constant α_0 is the coefficient of the average volatility of the cryptocurrencies. The coefficient α_1 indicates the reaction of volatility to unexpected returns, whereas, the coefficient β_1 shows the persistence of the volatility. When past volatility, σ_{t-1}^2, increases by one unit, future volatility will either increase or decrease by β_1 units. When the past error term, e_{t-1}^2, increases by one unit, the future volatility will either increase or decrease by α_1 units.

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normal distribution as well as the GED and DED. For this reason, this distribution will be used throughout this study.

a. Volatility of Bitcoin using GARCH(1,1), Student's t error distribution

Running the Bitcoin data on a GARCH (1,1) model with a Student's t-distribution yields the following equation:

$$\sigma_t^2 = 0.000011 + 0.271434 e_{t-1}^2 + 0.851244 \sigma_{t-1}^2$$

The coefficients of the constant variance term are positive and statistically significant at the 1% level. The p-values for the coefficients are p = .0589, p = .0004 and p = .000. The p-value of the constant term α_0 is not statistically significant at 1% level. The constant term represents the average volatility of the past Bitcoin prices and is estimated at 0.000011. Furthermore, the past volatility and error term of Bitcoin predict future volatility. When today's volatility increases by 1 unit, tomorrow's volatility will increase by 0.851244 units. If today's past error increases by 1 unit, tomorrow's volatility will increase by 0.271434 unit.

The Log likelihood is 3272.381 and the AIC -4.17290 (see Table 1).

b. Volatility of Bitcoin Cash using GARCH (1,1), Student's t error

Running the Bitcoin Cash data on a GARCH (1,1) model with a Student's t-distribution yields the following equation:

$$\sigma_t^2 = 0.0000453 + 0.055166 e_{t-1}^2 + 0.955975 \sigma_{t-1}^2$$

The p-values for the coefficients are p = .4575, p = .0370 and p = .000. All the coefficient are significant at a 5% level ($p < .05$) except for the constant term. This implies that the constant has no significant effect in predicting the volatility of future Bitcoin Cash prices. When today's volatility increases by 1 unit, tomorrow's volatility will increase by 0.955975 unit. If today's past error increases by 1 unit, tomorrow's volatility will increase by 0.055166 unit.

The Log likelihood is 598.9400 and AIC -2.380521(Table 2).

c. Volatility of Litecoin using GARCH (1,1), Student's t error distribution

Running the Litecoin data on a GARCH (1,1) model with a Student's t-distribution yields the following equation:

$$\sigma_t^2 = 0.0000482 + 0.730393\, e_{t-1}^2 + 0.860343\sigma_{t-1}^2$$

The *p*-values for the coefficients are p = .2995, p = .2836 and p = .000. This implies that the constant and the past error term have no significant effect in predicting the volatility of future Litecoin prices. When today's volatility increases by 1 unit, tomorrow's volatility will increase by 0.860343 unit.

The Log likelihood is 2894.530 and AIC -3.690332 (Table 3).

d. Volatility of Dogecoin using GARCH (1,1), Student's t error distribution

Running the Dogecoin data on a GARCH (1,1) model with a Student's t-distribution yields the following equation:

$$\sigma_t^2 = 0.0000898 + 0.391798\, e_{t-1}^2 + 0.749063\sigma_{t-1}^2$$

The *p*-values for the coefficients are p = .0046, p = .0001 and p = .000. The coefficients are significant at the 1% level. We can predict the rate of change of future volatility based on these estimates. If the volatility of today's Dogecoin price increases by 1 unit then the volatility will increase by 0.749063 the next day. Similarly, if today's error term increases by 1 unit then the volatility will increase by 0.391798 the following day.

The Log likelihood is 1996.924 and AIC -3.308844 (Table 4).

e. <u>Volatility of Ethereum using GARCH(1,1), Student's t error distribution</u>

Running the Ethereum data on a GARCH (1,1) model with a Student's t-distribution yields the following equation:

$$\sigma_t^2 = 0.0000326 + 0.369338\, e_{t-1}^2 + 0.3697041\, \sigma_{t-1}^2 \text{ (see Table 5).}$$

The *p*-values for the coefficients are p = .0005, p = .0001 and p = .000. As we can see, all the coefficients are less than 1% significance level and therefore statistically significant. This means that if today's Ethereum price volatility increases by 1 unit then the volatility will increase by 0.3697041 the next day. In the same vein, if today's error term increases by 1 unit then the volatility will increase by 0.3697041 the next day.

The Log likelihood is 1796.463 and AIC -2.884803.

8.1.2 IGARCH (1,1)

The IGARCH (1,1) model stands for the Integrated Generalized Autoregressive Conditional Heteroscedasticity and is a restricted version of the GARCH model. The difference is that the sum of the coefficients is restricted to 1. The model is described by the following equation:

$$\sigma_t^2 = \alpha_0 + \beta_1 e_{t-1}^2 + (1 - \beta_1)\sigma_{t-1}^2,$$

where σ_t^2 is the conditional volatility, where $e_{t-1}^2$is the squared unexpected returns for the previous period and where the sum of coefficients is restricted to 1. The coefficient α_1 indicates the reaction of volatility to unexpected returns whereas the coefficient β_1 shows the persistence of the volatility. When past volatility, σ_{t-1}^2, increases by one-unit, future volatility will either increase or decrease by β_1 units. When the past error term, e_{t-1}^2, increases by one unit, the future volatility will either increase or decrease by α_1 units.

When the past error increases by 1%, the future volatility will also increase by α_1%.

a. <u>Volatility of Bitcoin using IGARCH (1,1), Student's t error distribution</u>

Running the Bitcoin historical data on a GARCH (1,1) model with a Student's t-distribution yields the following equation:

$$\sigma_t^2 = 0.167763 + 0.832237 e_{t-1}^2 + 0.167763\sigma_{t-1}^2.$$

The p-values for the coefficients are p = .000 and p = .000 and they are therefore all statistically significant. The past volatility and error term of Bitcoin predict future volatility. When today's volatility increases by 1 unit, tomorrow's volatility will increase by 0.167763 unit. If today's past error increases by 1 unit, tomorrow's volatility will increase by 0.832237unit.

The AIC is -4.153711 and the log likelihood is 3255.355 (see Table 1).

b. Volatility of Bitcoin Cash using IGARCH (1,1), Student's t error

The estimated model is written as follows:

$$\sigma_t^2 = 0.031136 + 0.968864 e_{t-1}^2 + 0.0311136 \sigma_{t-1}^2.$$

The *p*-values for the coefficients are p = .000 and p = .000 and they are therefore all statistically significant. The past volatility and error term of Bitcoin predict future volatility. When today's volatility increases by 1 unit, tomorrow's volatility will increase by unit 0.0311136. If today's past error increases by 1 unit, tomorrow's volatility will increase by 0.968864 unit.

The AIC is -2.380462, and the log likelihood is 596.9254 (Table 2).

c. Volatility of Litecoin using IGARCH (1,1), Student's t error distribution

The estimated model is written as follow:

$$\sigma_t^2 = 0.095507 + 0.904493 e_{t-1}^2 + 0.095507 \sigma_{t-1}^2.$$

The *p*-values for the coefficients are p = .000 and p = .000 and they are therefore all statistically significant. When today's volatility increases by 1 unit, tomorrow's volatility will increase by unit 0.095507 . If today's past error increases by 1 unit, tomorrow's volatility will increase by 0.904493 unit.

The Log likelihood is 2721.191 and AIC is -3.472785 (Table 3).

d. Volatility of Dogecoin using IGARCH (1,1), Student's t error distribution

The estimated model is written as follow:

$$\sigma_t^2 = 0.154609 + 0.845392 e_{t-1}^2 + 0.154608 \sigma_{t-1}^2.$$

The *p*-values for the coefficients are p = .000 and p = .000 and they are therefore all statistically significant. When today's volatility increases by 1 unit, tomorrow's volatility will increase by 0.154608 unit . If today's past error increases by 1 unit, tomorrow's volatility will increase by 0.845392 unit.

The Log likelihood is 1965.754 and AIC is -3.260388 (Table 4).

e. Volatility of Ethereum using IGARCH (1,1), Student's t error distribution

The estimated model is written as follows:

$$\sigma_t^2 = 0.167763 + 0.832237 e_{t-1}^2 + 0.167763 \sigma_{t-1}^2.$$

The *p*-values for the coefficients are p = .000 and p = .000 and they are therefore all statistically significant. When today's volatility increases by 1 unit, tomorrow's volatility will increase by 0.167763 unit . If today's past error increases by 1 unit, tomorrow's volatility will increase by 0.832237 unit.

The Log likelihood is 1759.632 and AIC is -2.816337 (Table 5).

8.1.3 EGARCH (1,1)

Similar to the TGARCH (1,1), the EGARCH (1,1) model or Exponential Generalized Autoregressive Conditional Heteroscedastic is developed by Nelson in 1991. It captures the leverage effects of shocks on the financial market. This model allows testing of asymmetric effects between negative and positive asset returns. It also accounts for leverage effect when examining volatility models with financial time series. The general equation is written as

$$\log(\sigma_t^2) = \alpha_0 + \alpha_1 e_{t-1} + \alpha_2(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + \beta_1 \log(\sigma_{t-1}^2)$$

where α_0 (the constant) is the average volatility of the cryptocurrencies, α_1 is the previous error term and α_2 the leverage term that determine the asymmetric impact of the news, and β_1 the persistence of the past volatility. , σ_{t-1}^2, and current error term, e_t. $|e_{t-1}|$ is the absolute value of the error term of the volatility for the previous period. When the past volatility increases by 1%, the future volatility will also increase by β_1%.

a. Volatility of Bitcoin using EGARCH(1,1), Student's t error distribution

Using EGARCH (1,1) model with a Student's t error distribution yields the following equation:

$$\log(\sigma_t^2) = -0.276731 + 0.358849 e_{t-1} + 0.05113(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + 0.9888964 \log(\sigma_{t-1}^2)$$

The p-values for the coefficients are p = .000, p = .000, p = .00275, and p = .000. The coefficients are all statistically significant at the 1% level. The previous error term is positive which shows there is positive relation between the past variance and current variance in absolute value. In other words, a bigger the magnitude shock to the variance means a higher volatility. The leverage term is positive and significant. That indicates that good news has more impact on the volatility than bad news. The findings indicate that when the volatility increase by 1%, the future volatility will increase by 0.98%.

The Log likelihood is 3280.595 and the AIC is -4.1812114 (see Table 1).

b. Volatility of Bitcoin Cash using EGARCH (1,1), Student's t error

I perform the EGARCH (1,1) model using the Student's error distribution on the Bitcoin Cash data and I obtain the following equation:

$$\log(\sigma_t^2) = -0.178140 + 0.177080 e_{t-1} + 0.018764(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + 0.984746\log(\sigma_{t-1}^2)$$

The p-values for the coefficients are p = .0297, p = .0033, p = .5350, and p = .000. The coefficients are all statistically significant at the 5% level, except the leverage term. The leverage term is not statistically significant and therefore does not have an effect in predicting the volatility of Bitcoin Cash future price. The findings indicate that if the past volatility increases by 1%, the future volatility will increase by 0.98%.

The Log likelihood is 598.46 and the AIC is -2.374589 (Table 2).

c. Volatility of Litecoin using EGARCH (1,1), Student's t error distribution

Running the Litecoin data on a EGARCH (1,1) model with a Student's t-distribution yields the following equation:

$$\log(\sigma_t^2) = -0.02027 + 0.398412 e_{t-1} + 0.021221(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + 0.992379\log(\sigma_{t-1}^2)$$

The p-values for the coefficients are p = .000, p = .0001, p = .4659 and p = .000. The leverage term is not significant and therefore does not have an effect in predicting the volatility of Litecoin future price. The findings indicate that when the past volatility increase by 1%, the future volatility will increase by 0.98%.

The Log likelihood is 2901.502 and the AIC is -3.677436 (Table 3).

d. Volatility of Dogecoin using EGARCH(1,1), Student's t error distribution

Running the Dogecoin data on a EGARCH(1,1) model with a Student's t-distribution yields the following equation:

$$\log(\sigma_t^2) = -0.524700 + 0.462280 e_{t-1} + 0.053535(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + 0.959366 \log(\sigma_{t-1}^2)$$

The p-values for the coefficients are p = .000, p = .000, p = .1322, and p = .000. The leverage term is not significant and therefore does not have an effect in predicting the volatility of Dogecoin future price. The findings indicate that when the past volatility increase by 1% the future volatility will increase by 0.96%.

The Log likelihood is 1998.138 and the AIC is -3.309200 (Table 4).

e. Volatility of Ethereum using EGARCH(1,1), Student's t error distribution

Running the Ethereum data on a EGARCH(1,1) model with a Student's t-distribution yields the following equation:

$$\log(\sigma_t^2) = -0.697503 + 0.454513 e_{t-1} - 0.001507(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + 0.925584 \log(\sigma_{t-1}^2)$$

The p-values for the coefficients are p = .000, p = .000, p = .9638, and p = .000. Once again, the leverage term is not significant and therefore, doesn't have an effect in predicting the volatility of Litecoin future price. The findings indicate that when the past volatility increase by 1% the future volatility will increase by 0.925%.

The Log likelihood is 1801.385 and the AIC is -2.891118 (Table 5).

8.1.4 TGARCH (1,1)

The TGARCH(1,1) or threshold GARCH(1,1) is a commonly used model that captures asymmetry and accounts for handling leverage effect and is attributed to Zakoian (1990). It's a particular case of a nonlinear ARCH model and it models the conditional standard deviation instead of the conditional variance and predicts the square root of the variance based on the sign of the past error term (the leverage effect).

$$(\sigma_t) = \alpha_0 + \alpha_1(e^+_{t-1}) + \alpha_2(e^-_{t-1}) + \beta_1(\sigma_{t-1}).$$

where α_0 (the constant) is the average volatility of the cryptocurrencies, α_1 is the positive error term , α_2 is the leverage effect term and β_1 the persistence of the past volatility.

The error terms e^+_{t-1} and e^-_{t-1} are variables that refer to the value of the past error, e_{t-1}, depending on the sign of the error. If the error is positive (good news), then $e^+_{t-1} = e_{t-1}$ and 0 otherwise, and if the error is negative (bad news), then $e^-_{t-1} = e_{t-1}$ and 0 otherwise.

a. <u>Volatility of Bitcoin using TGARCH(1,1), Student's t error distribution</u>

The following equation is obtained from analyzing the TGARCH(1,1) model for Bitcoin:

$$(\sigma_t) = 0.00000798 + 0.0323525(e^+_{t-1}) - 0.136202\ (e^-_{t-1}) + 0.86198(\sigma_{t-1}).$$

The p-values for the coefficients are, p = .1035, p = .0007, p = .04441, and p = .000. The coefficients are all statistically significant at the 5% level, except the constant term . This means that the average volatility of Bitcoin has no effect on predicting the volatility. The findings indicate that positive error term and the leverage effect term of Bitcoin affect future volatility.

Furthermore, the negative sign of the leverage effect shows that bad news affects volatility more than good news. When we increase today's standard deviation of the price by 1%, then tomorrow's standard deviation will increase by 0.86198.

The AIC is -4.175818, and log likelihood is 3275.665 (see Table A1).

b. Volatility of Bitcoin Cash using TGARCH(1,1), Student's t error

The following equation is obtained from analyzing the TGARCH(1,1) model for Bitcoin Cash:

$$(\sigma_t) = 0.000234 + 0.107554(e_{t-1}^{+}) - 0.041741(e_{t-1}^{-}) + 0.910079(\sigma_{t-1}).$$

The p-values for the coefficients are p = .0835, p = .0433, p = .3803, and p = .000). The coefficients are all statistically significant at the 5% level, except for the constant and the leverage effect term. The findings indicate that the future of Bitcoin Cash price is not affected by the leverage term but is affected by the positive error term. When we increase today's standard deviation of the price by 1%, then tomorrow's standard deviation will increase by 0.910079.

AIC is -4.175818, and log likelihood is 3275.665 (Table A2).

c. Volatility of Litecoin using TGARCH(1,1), Student's t error distribution

The following equation is obtained from analyzing the TGARCH(1,1) model for Litecoin data:

$$(\sigma_t) = 0.0000415 + 0.845155(e_{t-1}^{+}) - 0.333957(e_{t-1}^{-}) + 0.867151(\sigma_{t-1})$$

The p-values for the coefficients are p = .3035, p = .02781, p = .3300, and p = .000). The coefficients are all statistically significant at the 5% level, except for the constant and the leverage effect term. The results indicate that the future of Litecoin price is not affected by the leverage effect term but is affected by the positive error term. When we increase today's standard deviation by 1%, then tomorrow's standard deviation increases by 0.867151.

The quality criteria of the model are AIC -3.691917, and log likelihood 2896.771 (Table 3).

d. Volatility of Dogecoin using TGARCH(1,1), Student's t error distribution

The following equation is obtained after analyzing the TGARCH(1,1) for Dogecoin data:

$$(\sigma_t) = 0.0000907 + 0.435534(e_{t-1}^{+}) - 0.0953354(e_{t-1}^{-}) + 0.747855(\sigma_{t-1}).$$

The p-values for the coefficients are p = .00044, p = .00001, p = .3303, and p = .000). The coefficients are all statistically significant at the 1% level, except for the negative past error term. The results indicate that the future of Dogecoin price is not affected by the leverage effect term but is affected by the positive past error term. When today's standard deviation increases by 1%, tomorrow's standard deviation increases by 0.747855.

The quality criteria of the model are AIC -3.308127, and log likelihood 1997.493 (Table 4).

e. Volatility of Ethereum using TGARCH(1,1), Student's t error distribution

The following equation is obtained after analyzing the TGARCH (1,1) for Ethereum data:

$$(\sigma_t) = 0.000327 + 0.3514676(e_{t-1}^{+}) + 0.055126(e_{t-1}^{-}) + 0.694759(\sigma_{t-1})$$

The p-values for the coefficients are p = .00006, p = .00002, p = .5562, and p = .000. Once again, the coefficients are all statistically significant at the 1% level, except for the leverage effect term. Since the leverage effect term is insignificant, there is a symmetric effect of the news on the Ethereum volatility. The results indicate that the future of Ethereum price is affected by the positive error term. When we increase today's standard deviation of the price by 1%, then tomorrow's standard deviation will increase by 0.694759.

The quality criteria of the model are AIC -2.883504, and log likelihood 1796.656 (Table 5).

8.1.5 CGARCH (1,1)

Originating from Engle and Lee, the CGARCH model take into consideration the time-varying aspect of the mean reversion:

$$h_t = q_t + \alpha(\varepsilon_{t-1}^2 - q_{t-1}) + \beta(h_{t-1} - q_{t-1})$$

where h is the variance, epsilon is the residual, h_{t-1} is the variance of the previous period, q is a time-varying permanent component towards which the variance converges on the long-run. Alpha and beta are the linear coefficients for previous variance and residual components of the equation respectively. As one component increases by one unit, the overall equation increases by the coefficient's value (e.g. for each increase in $(\varepsilon_t^2 - \nu_t^2)$ there is an overall increase of alpha units).

The next equation below shows how the time-dependent long-run volatility q_t is obtained:

$$q_t = \omega + \rho(q_{t-1} - \omega) + \varphi(\varepsilon_{t-1}^2 - h_{t-1})$$

where the new terms are omega, h(t-1), rho and phi and respectively represent the mean reversion constant, the previous volatility and two linear coefficients. The other variables are all defined as in the previous equation. The constant term omega is the permanent value of qt if it were constant and qt converges towards it on the long-run. In practice, the rho coefficient is very close but below one which means that q_t converges slowly towards the permanent constant value of omega.

a. Volatility of Bitcoin using CGARCH(1,1), Student's t-distribution

The following equations are obtained from CGARCH(1,1) model analysis for Bitcoin with Student's t-distribution:

$$h_{\square} = q_t + 0.145492\ (\varepsilon_{t-1}^2 - q_{t-1}) - 0.304727\ (h_{t-1} - q_{t-1})$$

$$q_t = -0.0000754 + 1.118164\ (q_{t-1} + 0.0000754) + 0.280562\ (\varepsilon_{t-1}^2 - h_{t-1})$$

The p-values for $\omega, \rho, \varphi, \alpha, \beta$ are p = .1804, p = .0000, p = .00034, p = .0216 and p = .3315 respectively. Coefficients ρ, φ, α are all statistically significant at the 5% level meaning that Bitcoin is unaffected by the corresponding components.

The quality criteria of the model are AIC -4.174054 and log likelihood 3275.284 (Table 1).

b. Volatility of Bitcoin Cash using CGARCH(1,1), Student's t-distribution

The following equations are obtained from CGARCH(1,1) model analysis for Bitcoin Cash with Student's t-distribution:

$$h_t = q_t + 0.226961\ (\varepsilon_{t-1}^2 - q_{t-1}) - 0.026949\ (h_{t-1} - q_{t-1})$$

$$q_t = 0.099861 + 0.999359\ (q_{t-1} - 0.099861) + 0.044854\ (\varepsilon_{t-1}^2 - h_{t-1})$$

The p-values for $\omega, \rho, \varphi, \alpha, \beta$ are respectively p = .9540, p = .0000, p = .0758, p = .0252 and p = .8916 respectively. The only significant values are those of ρ and φ meaning that the corresponding terms don't have an effect for Bitcoin Cash.

The quality criteria of the model are AIC -2.39625 and log likelihood 585.6931 (Table 2).

c. <u>Volatility of Litecoin unsing CGARCH(1,1), Student's t-distribution</u>

The following equations are obtained from CGARCH(1,1) model analysis for Litecoin with Student's t-distribution:

$$h_t = q_t + 0.078213\ (\varepsilon_{t-1}^2 - q_{t-1}) - 0.424629\ (h_{t-1} - q_{t-1})$$

$$q_t = 0.0000613 + 2.765730\ (q_{t-1} - 0.0000613) + 2.267142\ (\varepsilon_{t-1}^2 - h_{t-1})$$

The p-values for $\omega, \rho, \varphi, \alpha, \beta$ are respectively p = .1583, p = .4669, p = .5775, p = .5748 and p = .2443 respectively. All results here are insignificant and thus Litecoin is unaffected by them.

The quality criteria of the model are AIC -3.69772 and log likelihood 2898.634 (Table 3).

d. <u>Volatility of Dogecoin using CGARCH(1,1), Student's t-distribution</u>

The following equations are obtained from CGARCH(1,1) model analysis for Dogecoin with Student's t-distribution:

$$h_t = q_t + 0.168522\ (\varepsilon_{t-1}^2 - q_{t-1}) - 0.120246(h_{t-1} - q_{t-1})$$

$$q_t = 0.066687 + 0.998980\ (q_{t-1} - 0.066687) + 0.213862\ (\varepsilon_{t-1}^2 - h_{t-1})$$

The p-values for $\omega, \rho, \varphi, \alpha, \beta$ are respectively p = .8655, p = .0000, p = .0000, p = .0013 and p = .5945 respectively. Coefficients ρ, φ, α are all statistically significant at the 5% level meaning it is unaffected by these components.

The quality criteria of the model are AIC -3.30701 and log likelihood 1997.82 (Table 4).

e. Volatility of Ethereum using CGARCH(1,1), Student's t-distribution

The following equations are obtained from CGARCH(1,1) model analysis for Ethereum with Student's t-distribution:

$$h_t = q_t + 0.020763\ (\varepsilon_{t-1}^2 - q_{t-1}) - 0.622221\ (h_{t-1} - q_{t-1})$$

$$q_t = 0.252884 + 0.998697\ (q_{t-1} - 0.252884) + 0.302863\ (\varepsilon_{t-1}^2 - h_{t-1})$$

The p-values for ω, ρ, φ, α, β are respectively p = .9220, p = .0000, p = .0000, p = .6391 and p = .4442 respectively. Coefficients ρ and φ are all statistically significant at the 5% level meaning that is unaffected by those components.

The quality criteria of the model are AIC -2.880756 and log likelihood 1795.95 (Table 5).

8.2 Results

The GARCH models that are best suited to represent the cryptocurrency data are determined on the basis of the AIC quality criteria and the maximum likelihood method. It can be seen that the EGARCH(1,1) model with a Student's t-distribution fits the Bitcoin, Litecoin, Dogecoin and Ethereum volatilities better than the other models. On the other hand, it is the GARCH(1,1) that seems to better represent Bitcoin Cash's volatility.

<u>Bitcoin:</u>

The EGARCH(1,1) model with Student's t-distribution better represents the Bitcoin volatility with a Log likelihood of 3280.595 and an AIC of -4.1812114.

$$\log(\sigma_t^2) = -0.276731 + 0.358849 e_{t-1} + 0.05113(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + 0.9888964 \log(\sigma_{t-1}^2)$$

The coefficients are all statistically significant at the 1% level. The previous error term is positive which shows that there is a positive relation between the past variance and current variance in absolute value. In other words, the bigger the magnitudes shock to the variance, the higher the volatility. The leverage term is positive and significant. This indicates that good news has more impact on the volatility than bad news. The findings indicate that when the volatility increase by 1%, the future volatility will increase by 0.98%.

Bitcoin Cash:

On the other hand, the results shows that the GARCH(1,1) model with Student's t-distribution fits better the Bitcoin Cash volatility with a the log likelihood representing 598.9400 and the AIC -2.380521.

By runing the model we obtain the following equation:

$$\sigma_t^2 = 0.0000453 + 0.055166 e_{t-1}^2 + 0.955975 \sigma_{t-1}^2$$

The *p*-values for the coefficients are p = .4575, p = .0370 and p = .000. All the coefficient are significant at a 5% level ($p < .05$), expect for the constant term. This implies that the constant has no significant effect in predicting the volatility of future Bitcoin Cash prices. When today's volatility increases by 1 unit, tomorrow's volatility will increase by 0.955975 unit. If today's past error increases by 1 unit, tomorrow's volatility will increase by 0.055166 unit.

Litecoin:

As for the litecoin, the EGARCH(1,1) model is more relevant to determine the volatility based on the historical data sample with a log likelihood of 2901.502 and an AIC of -3.677436 .

By runing the model I obtain the following equation:

$$\log(\sigma_t^2) = -0.02027 + 0.398412\, e_{t-1} + 0.021221(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + 0.992379 \log(\sigma_{t-1}^2)$$

The *p*-values for the coefficients are p = .000, p = .0001, p = .4659 and p = .000. The leverage term is not significant and therefore, does not have an effect in predicting the volatility of litecoin future price. The findings indicate that when the past volatility increases by 1%, the future volatility will increase by 0.98%.

Dogecoin:

The EGARCH(1,1) model is also more relevant to determine the volatility based on the Dogecoin historical data sample with a log likelihood of 1998.138 and an AIC of -3.309200.

By runing the model we obtain the following equation:

$$\log(\sigma_t^2) = -0.524700 + 0.462280 e_{t-1} + 0.053535(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + 0.959366\log(\sigma_{t-1}^2)$$

The *p*-values for the coefficients are p = .000, p = .000, p = .1322, and p = .000. The leverage term is not significant and therefore, does not have an effect on predicting the volatility of Dogecoin future price. The findings indicate that when the past volatility increase by 1% the future volatility will increase by 0.96%.

Ethereum:

Lastly, the EGARCH(1,1) model with Student's t-distribution represents better the Ethereum volatility with a Log likelihood of 1801.385 and an AIC of -2.891118.

By running the model on the ethereum data, we obtained the following equation:

$$\log(\sigma_t^2) = -0.697503 + 0.454513 e_{t-1} - 0.001507(|e_{t-1}| - \sqrt{\frac{2}{\pi}}) + 0.925584\log(\sigma_{t-1}^2)$$

The *p*-values for the coefficients are $p = .000$, $p = .000$, $p = .9638$, and $p = .000$. Once again, the leverage term is not significant and therefore, does not have an effect on predicting the volatility of litecoin future price. The findings indicate that when the past volatility increases by 1% the future volatility will increase by 0.925%.

8.3 Day-of-the week effect on cryptocurrency volatility

After determining which model represents best the different data, of the cryptocurrencies; We integrated variable dummies to have the week-day effect on the prices. The volatility time series is used as the dependant variable and days of the week are the independent variables. We input the week-day effect into the conditional variance equation of the chosen model to determine whether volatility changes across the week-day. We use Monday as the reference and the coefficient will be compared to Monday's volatility.

The following equation is created by inputting the week-day effect:

$$Volatility = a_1T + a_2W + a_3Th + a_4F + a_5S + a_6\,Su$$

The dummy variables a_t represent the days of the week effect on the cryptocurrency returns. In other words, coefficients $a_{1,}$ $a_{2,}$ $a_{3,}$ $a_{4,}$ $a_{5,}$ a_6 represent Tuesday, Wednesday,…and Sunday effect on cryptocurrency returns respectively. If today is Monday, we expect the volatility to increase or decrease by a_1 units on Tuesday, a_2 units on Wednesday,…and a_6 on Sunday repectively.

Bitcoin:

We run the regression model on the Bitcoin data using the EGARCH(1,1) model. We use Monday as the base to compare Monday's volatility to the rest of the day's volatility.

Table 6 Day-of-the-week effect in volatility equation of Bitcoin using EGARCH(1,1) model

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.0027076	0.001175	1.765775	0.0774
Tuesday	0.000708	0.001582	0.447319	0.6546
Wednesday	(-0.003016)	0.001660	-1.816713	0.0693*
Thursday	0.000135	0.001670	0.080885	0.9355
Friday	0.001244	0.001706	0.729167	0.4659
Saturday	-0.000899	0.079167	-0.526240	0.5987
Sunday	-0.000997	0.001635	-0.610191	0.5417

This table reports the results of introducing dummy variables into the conditional variance equation for the Student's t distribution. The table presents the week-of the-day dummy variables, the EGARCH(1,1) coefficients and descriptive statistics of the daily Bitcoin Cash volatility between August 20, 2017 and December 31, 2018. Data is drawn from CoinMarketCap.
*The values that are statistically significant are reported in parentheses. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively*

We then obtain the following equation:

$$Volatility = 0.000708\,T - 0.0003016\,W + 0.000135\,Th + 0.001244F - 0.000899\,S - 0.000997Su.$$

The p-values for the coefficient are respectively p = .6546, p = .0693, p = .9355, p = .4659, p = .5987 and p = .5417. As we can see, Wednesday is the only cofficient with a significant p-value (p = .0693, p > .1). Since the coefficient is negative, Monday has the highest level of volatiliy compared to Wednesday. The change in volatility that happens from Monday to Wednesday is a decrease of 0.03%.

Bitcoin Cash:

We run the regression model on the Bitcoin Cash data using the GARCH(1,1) model. We use Monday as the base to compare Monday's volatility to the rest of the day's volatility.

Table 7 Day-of-the-week effect in volatility equation of Bitcoin Cash using GARCH(1,1) model

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.020671	0.006317	-3.272373	0.0011
Tuesday	0.013646	0.008732	1.562807	0.1181
Wednesday	0.006282	0.009677	0.649117	0.5163
Thursday	0.010307	0.009014	1.143432	0.2529
Friday	(-0.019184)	0.009215	2.081740	0.0374**
Saturday	(-0.021267)	0.009709	2.190382	0.0285**
Sunday	(0.024054)	0.009805	2.453149	0.0142**

This table reports the results of introducing dummy variables into the conditional variance equation for the Student's t distribution. The table presents the week-of the-day dummy variables, the GARCH(1,1) coefficients and descriptive statistics of the daily Bitcoin Cash volatility between August 20, 2017 and December 31, 2018. Data is drawn from CoinMarketCap.
*The values that are statistically significant are reported in parentheses. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively*

We then obtain the following equation:

$$Volatility = 0.013646\ T + 0.006282\ W + 0.010307\ T + 0.019184F + 0.021267\ S + 0.024054\ Su$$

The p-values for the coefficient are respectively p = .1181 p = .5163, p = .2529, p = .0374, p = .0285 and p = .0142.

The results show that Friday, Saturday and Sunday are all significant with a p > .05. Since Friday coefficient is positive, there is an increase of 1.92% volatility from Monday to Friday. Saturday coefficient is also positive and there is an increase of 2,23% volatility from Monday to Saturday. At last, Sunday coefficient is positive and therefore there is 2.41% increase of volatility from Monday to Sunday.

Litecoin:

We run the regression model on the Litecoin data using the EGARCH(1,1) model. I use Monday as the base to compare Monday's volatility to the rest of the day's volatility.

Table 8 Day-of-the week-effect in volatility equation of Litecoin using EGARCH(1,1) model

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.001050	0.001332	-0.788368	0.4305
Tuesday	0.001718	0.001892	0.908063	0.3638
Wednesday	-0.001805	0.001900	-0.579717	0.5621
Thursday	0.001805	0.001925	0.937617	0.3484
Friday	0.002150	0.001918	1.121032	0.2623
Saturday	-0.000840	0.001965	-0.427640	0.6689
Sunday	0.000734	0.001895	0.387544	0.6984

This table reports the results of introducing dummy variables into the conditional variance equation for the Student's t distribution. The table presents the week-of the-day dummy variables, the EGARCH(1,1) coefficients and descriptive statistics of the daily Litecoin volatility between Septembre 18, 2015 and December 31, 2018. Data is drawn from CoinMarketCap.
*The values that are statistically significant are reported in parentheses. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively*

I then obtain the following equation:

Volatility = 0.001718 T -0.001101 W +0.0001805 Th +0.002150 F -0.000840 S +0.000734 Su.

The p-values for the coefficient are respectively p = .3638, p = .5621, p = .3484, p = .2623, p = .6689 and p = .6984.

In this case, since all the coefficients are not significant, it can be argued that there is no day of the week effect taking Monday as a reference.

Dogecoin:

We run the regression model on the Dogecoin data using the EGARCH(1,1) model. I use Monday as the base to compare Monday's volatility to the rest of the day's volatility.

Table 9 Day-of-the-week effect in volatility equation of Dogecoin using EGARCH(1,1) model

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000602	0.002207	0.272905	0.7849
Tuesday	-0.003915	0.002943	-1.330276	0.1834
Wednesday	-0.002590	0.002988	-0.866752	0.3861
Thursday	-0.001853	0.002994	-0.618993	0.5359
Friday	0.001986	0.003157	0.629175	0.5292
Saturday	-0.001416	0.003053	-0.463897	0.6427
Sunday	-0.003533	0.003254	-1.085587	0.2777

This table reports the results of introducing dummy variables into the conditional variance equation for the Student's t distribution. The table presents the week-of the-day dummy variables, the EGARCH(1,1) coefficients and descriptive statistics of the daily Dogecoin volatility and return between Septembre 15, 2015 and December 31, 2018. Data is drawn from CoinMarketCap.

*Note: The values that are statistically significant are reported in parentheses. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively*

The following equation is then obtained:

$$Volatility = -0.003915\,T - 0.002590\,W - 0.001853\,Th + 0.001986\,F - 0.001416\,S - 0.003533\,Su$$

The p-values for the coefficient are respectively p = .1834 p = .3861, p = .5359, p = .5292, p = .6427 and p = .2777. In this case, since all the coefficients are not significant, it can be argued that there is no day of the week effect taking Monday as a reference.

Ethereum:

Finally, we run the regression model on the Ethereum data using the EGARCH(1,1) model. We use Monday as the base to compare Monday's volatility to the rest of the day's volatility.

Table 10 Day-of-the-week effect in volatility equation of Ethereum using EGARCH(1,1) model

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.003872	0.003047	-1.270727	0.2038
Tuesday	0.000643	0.004010	1.603510	0.1088
Wednesday	-0.000276	0.004367	-0.063313036	0.9495
Thursday	0.001622	0.004439	0.361898	0.7148
Friday	0.005936	0.004458	1.361898	0.1732
Saturday	(0.008004)	0.004464	1.792786	0.0730*
Sunday	0.002942	0.004437	0.662952	0.5074

This table reports the results of introducing dummy variables into the conditional variance equation for the Student's t distribution. The table presents the week-of-the-day dummy variables, the EGARCH(1,1) coefficients and descriptive statistics of the daily Ethereum price between August 8, 2015 and December 31, 2018. Data is drawn from CoinMarketCap.
*The values that are statistically significant are reported in parentheses. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively*

The following equation is then obtained:

$$Volatility = 0.006430\ T - 0.000276\ W + 0.001622\ Th + 0.005936\ F + 0.0089004\ S + 0.002942\ Su$$

The p-values for the coefficient are respectively p = .1088, p = .9495, p = .7148, p = .1732, p = .0730 and p = .5074. The results show that Saturday is significant at 10% level with a = .0730 and is positive. This means that there is an increase of 2.3% in volatility from Monday to Saturday.

Results indicate significant presence of day-of-the-week effect in the conditional volatility of Bitcoin, Bitcoin Cash and Ethereum returns.

The day-of-the-week effect patterns in return and volatility might enable investors to take advantage of relatively regular shifts in the market by designing trading strategies, which aim to capitalize on predictable market patterns.

For a rational financial decision maker, returns constitute only a single part of the decision-making process. A key aspect in investment decisions is the risk or volatility of returns.
It is important to both understand and predict the volatility of stock returns by the day of the week. This can help financial analysts determine if a high (low) return is associated with a correspondingly high (low) volatility for a given day.

By identifying certain patterns in volatility, investors will be able to make data driven investment strategies based on risk and return.

For example, Engle (1993) argues that investors who dislike risk may adjust their portfolios by reducing their investments in assets whose volatility is expected to increase.
Uncovering certain volatility patterns in returns might also benefit investors in valuation, portfolio optimization, option pricing, and risk management.

For example, Wednesday has a significant negative effect on Bitcoin return. Monday has a higher volatility compared to Wednesday. Inventors can take a short position on Wednesday for trading strategy.

In the Bitcoin Cash market, Friday, Saturday and Sunday returns are found to be significant and the effect is positive unlike the effect found on the Bitcoin market. Investors can take a long position on Friday and close it at the end of Sunday to make profit.

As for the Ethereum market, Saturday return is found to be significant and positive. It is better to take a long position on Saturday to make profit.

We can say that Bitcoin Cash market seems to be relatively more efficient than Bitcoin and Ethereum market respctively.

Chapter 9: Conclusion

This thesis attempted to find the best model to forecast the volatility of Bitcoin, Bitcoin Cash, Litecoin, Dogecoin and Ethereum by running five GARCH-type models using the Student t distribution and applying them to the selected cryptocurrencies' closing prices. The AIC and the Log-Likelihood are the criteria that allowed us to choose which GARCH model is the most suitable. We selected the EGARCH(1,1) model for the Bitcoin, Litecoin, Dogecoin and Ethereum and the GARCH(1,1) model for Bitcoin Cash. Subsequently, day-of-the-week dummy variables were introduced to determine the change of volatility during the week with Monday as the reference. The results show that there is a significant presence of weekday effects on the return of some days for Bitcoin, Bitcoin Cash and Ethereum. Wednesday has a significant negative effect on Bitcoin return. Friday, Saturday and Sunday returns are found to be significant and positive on Bitcoin Cash Return. Finally, Saturday is found to be significant and positive on Ethereum return. Other studies show some consistent results with my findings. Latif and Azrin (2017) use the GARCH (1,1) model to analyze Bitcoin and Litecoin and find that the market efficiency of Bitcoin and Litecoin is inconsistent due to a higher predictability power on the cryptocurrency market. Kurihara and Fukushima (2017) also show that the Bitcoin market is not efficient, but might be in the future.

This research paper contributes to the extended literature on the models used to describe the volatility of cryptocurrencies as well as the day-of-the-week effect.

Introducing widely used conditional volatility models like the FIEGARCH, DCC- FIGARCH and DCC-FIAPARCH could expand this study further.

Appendix

Table 1 Estimation results of GARCH-type models for Bitcoin returns

		GARCH(1,1)	IGARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)	CGARCH(1,1)
coefficients	a0	(0.000011)	(0.167763)	(-0.276731)	0.000008	-0.000075
	a1	(0.271434)	(0.832237)	(0.358849)	(0.323525)	(1.118164)
	a2	-	-	(0.057113)	(-0.136202)	(0.28056)2
	a3	-	-	-	-	(0.014549)
	b1	(0.851244)	(0.167763)	(0.988964)	(0.861891)	-0.304727
p value	a0	0.0589 *	0.0000 ***	0.000 ***	0.103500	0.180400
	a1	0.004 ***	0.0000 ***	0.000 ***	0.0007 ***	0.0000 ***
	a2	-	-	0.0275 **	0.0441 **	0.0034 ***
	a3	-	-	-	-	0.0216 **
	b1	0.0000 ***	0.0000 ***	0.0000 ***	0.0000 ***	0.331500
Log Likelihood		3 272.38	3 255.36	3 280.60	3 275.67	3 275.28
AIC		-4.172900	-4.153711	-4.182114	-4.175818	-4.174054

This table presents the estimation results for Bitcoin returns using the GARCH-type models from Septembre 18, 2014 to December 31, 2018. The data is drawn from CoinMarketCap.
*The values that are statistically significant are reported in parentheses. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively.*

Table 2 Estimation results of GARCH-type models for Bitcoin Cash returns

		GARCH(1,1)	IGARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)	CGARCH(1,1)
coefficients	□ □	0.000045	(0.031136)	(-0.178140)	(0.000234)	0.099861
	□1	(0.055166)	(0.968864)	(0.177080)	(0.107554)	(0.999359)
	□2	-	-	0.018767	-0.041741	(0.044854)
	□3	-	-	-	-	(0.226961)
	□1	(0.955975)	(0.031114)	(0.984746)	(0.910079)	-0.026949
p value	□0	0.457500	0.0000***	0.0297**	0.08350*	0.954000
	□1	0.0370**	0.0000***	0.0033***	0.00433***	0.0000***
	□2	-	-	0.535000	0.380300	0.0758*
	□3	-	-	-	-	0.025200**
	□1	0.0000***	0.0000***	0.0000***	0.0000***	0.891600
Log Likelihood		598.46	596.93	598.46	598.05	598.69
AIC		-2.323937	-2.380462	-2.323937	-2.285908	-2.396245

This table presents the estimation results for Bitcoin returns using the GARCH-type models from August 20, 2017 to December 31, 2018. The data is drawn from CoinMarketCap.
*The values that are statistically significant are reported in parentheses. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively.*

Table 3 Estimation results of GARCH-type models for Litecoin returns

		GARCH(1,1)	IGARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)	CGARCH(1,1)
coefficients	□□	0.000048	(0.095507)	(-0.202027)	0.000042	-0.000061
	□1	0.730393	(0.904493)	(0.398412)	0.845155	2.765730
	□2	-	-	0.021221	-0.333957	2.267142
	□3	-	-	-	-	0.078213
	□1	(0.860343)	(0.095507)	(0.992379)	(0.867151)	(0.424629)
p value	□0	0.299500	0.000***	0.000***	0.303500	0.158300
	□1	0.283600	0.000***	0.001***	0.278100	0.466900
	□2	-	-	0.465900	0.330000	0.577500
	□3	-	-	-	-	0.574800
	□1	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
Log Likelihood		2 894.53	2 721.19	2 901.50	2 896.77	2 898.63
AIC		-3.690332	-3.472785	-3.697959	-3.691917	-3.697742

This table presents the estimation results for Litecoin returns using the GARCH-type models from September 18, 2014 to December 31, 2018. The data is drawn from CoinMarketCap.
*The values that are statistically significant are reported in parentheses. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively*

Table 4 Estimation results of GARCH-type models for Dogecoin returns

		GARCH(1,1)	IGARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)	CGARCH(1,1)
coefficients	□□	(0.000090)	(0.154609)	(-0.524700)	(0.000091)	0.066687
	□1	(0.391798)	(0.845392)	(0.462280)	(0.435534)	(0.998980)
	□2	-	-	0.053535	-0.095354	(0.213862)
	□3	-	-	-	-	(0.168522)
	□1	(0.749063)	(0.154608)	(0.959366)	(0.747885)	0.120246
p value	□0	0.0046***	0.0000***	0.0000***	0.0044***	0.865500
	□1	0.0001***	0.0000***	0.0000***	0.00001***	0.0000 ***
	□2	-	-	0.132200	0.330300	0.0000***
	□3	-	-	-	-	0.0013***
	□1	0.0000***	0.0000***	0.0000***	0.0000***	0.594500
Log Likelihood		1 996.92	1 759.63	1 998.14	1 997.49	1 997.82
AIC		-3.308844	-3.260388	-3.309200	-3.308127	-3.307010

This table presents the estimation results for Dogecoin returns using the GARCH-type models from September 15, 2015 to December 31, 2018. The data is drawn from CoinMarketCap.

*The values that are statistically significant are reported in parentheses. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively*

Table 5 Estimation results of GARCH-type models for Ethereum returns

		GARCH(1,1)	IGARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)	CGARCH(1,1)
coefficients	□□	(0.000326)	(0.167763)	(-0.697503)	(0.000327)	0.252884
	□1	(0.369338)	(0.832237)	(0.454513)	(0.351476)	(0.998697)
	□2	-	-	-0.001507	0.055126	(0.302863)
	□3	-	-	-	-	(0.020763)
	□1	(0.369704)	(0.167763)	(0.925584)	0.694759	-0.622210
p value	□0	0.0005***	0.0000***	0.0000***	0.0006***	0.922000
	□1	0.0001***	0.0000***	0.0000***	0.0002***	0.0000***
	□2	-	-	0.963800	0.556200	0.0000***
	□3	-	-	-	-	0.6391***
	□1	0.0000***	0.0000***	0.0000***	0.0000***	0.444200
Log Likelihood		1 796.46	1 759.63	1 801.39	1 769.66	1 795.95
AIC		-2.884803	-2.816337	-2.891180	-2.883504	-2.880756

This table presents the estimation results for Ethereum returns using the GARCH-type models from August 08, 2015 to December 31, 2018. The data is drawn from CoinMarketCap. *The values that are statistically significant are reported in parentheses. *, ** and ****
represent the significance at the 10%, 5% and 1% levels, respectively

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