A NUMERICAL VERSION OF THREE CLASSICAL PAPERS.

MAJOR PAPER[1]

Abdurasul Abdurakhmanov
Department of Economics
University of Ottawa
aabdu072@uottawa.ca

[1] I would like to express my great respect and thanks to Professor Marcel Mérette for his support and guidance in preparing this paper. All errors are mine.

ABSTRACT.

A numerical two-country Diamond type of overlapping-generations model is developed to test the theoretical propositions derived in Buiter (1981), Blanchard (1985) and Assaf R. and Frenkel J. (1992). The paper first derives main propositions of these papers. We show to what extent these three papers are linked. Then we develop a two-country model that can be used to test the theoretical propositions of the three seminal papers. We discuss in particular how lifetime horizon and government policy can affect interest rate, output and capital accumulation.

1. Introduction.

I was inspired by the findings of three seminal papers. Buiter (1981) shows in a two-country overlapping generations model that when both countries are exactly identical except the pure rate of time preference, the more impatient country runs a current account deficit in steady state whereas the other runs a current account surplus

Blanchard (1985) shows that when households have finite horizons, a constant labour income over lifetime decreases capital accumulation, whereas declining labour income over lifetime increases savings for retirement.

Assaf and Frenkel (1992) show that, deficit financing arising from temporary tax reduction, for a given path of government spending, has an impact on the steady state equilibrium, unless horizons are infinite. In other words, Ricardian proposition fails even in the long run.

The main objective of this paper is to numerically replicate the theoretical results of these three papers and compare our simulation results with their theoretical propositions. As an overlapping-generations model is used in all these papers, we develop a Diamond type of overlapping-generations model with five generations.

The paper is organized in following way. Section 2 reviews briefly these three classical papers. Section 3 describes the model. Section 4 presents the calibration procedure and the data of the model. Section 5, describes the simulation experiments. Section 6 discusses simulation results and compares them with the propositions developed in the classical papers. Section 7 concludes. In the annex we have added the numerical program of the model.

2. Related Literature Review.

We briefly discuss the three classical papers over which we develop a numerical model.

In his paper, "Debt, Deficits and Finite Horizons" (1985), Olivier Blanchard described the dependance of steady-state interest rate and the dynamic effects of fiscal policy on the time *horizon of agents.* Blanchard developed a simple analytical model, based on overlapping-generations type of model, where these issues can be examined. In his paper, he focuses on the:

- effects of the time horizon of agents on the economy, particularly the dynamics path and the long-run steady-state of the economy;

- effects of fiscal policy, where he studied roles of government spending, deficits and debt in the determination of the interest rate.

With respect to the time horizon of agents, he assumes first that agents have finite horizons, and throughout the paper he compares results with the case where agents have infinite horizons.

In developing the model, he faced an issue that there was *no simple aggregate consumption* function in an economy with finitely lived agents, as they differ in two aspects; different level and composition of wealth and different propensities to consume out of wealth, where both were affected by the age of agents. The solution adopted by Diamond (1965) and by Buiter (1981), was to assume a simple population age structure, (with two generations, young and old) avoiding in this way the problem of aggregation. The solution chosen by Blanchard was assuming that agents, throughout their life, face constant instantaneous probability of death ρ. Thus, their expected remaining life becomes $1/\rho$, which is constant. He referred this as the time horizon of agents. Particularly, he assumes that: 1) time is continuous, each agent faces a probability of death ρ, which is constant. As a limiting case, when ρ approaches zero, agents have infinite horizons; 2) because of uncertainty about death, agents may leave bequests, and he assumes that there is no bequest, no leaving debt. The latter may cause them to maintain positive wealth position. To deal with this issue, he assumes that there exist life insurance companies which provide riskless insurance. Indeed, given free entry and zero profit condition, also taking into account agents' probability of death, agents accept to have all their wealth in the hands of the life insurance company on their death, in return to receiving a flow of income equal to ρ times their wealth in present value. Therefore, if agents do not die and if their wealth is w, they receive ρw, but pay w if they die.

For simplicity, he also assumes that utility of an agent is logarithmic and that aggregate human wealth is distributed equally across agents. The relaxation of the latter, i.e. the effects of decreasing labour income, is of interest. We discuss it later.

With those assumptions, he can aggregate over all individuals and come up with standard equations for consumption and accumulation of human and non-human wealth.

$$C = (\rho + \theta)(H + W) \tag{1}$$

$$\dot{H} = (r + \rho) H - Y \tag{2}$$

$$\dot{W} = rW + Y - C \tag{3}$$

From the above equations, we see that subjective discount rate of agents is higher in case where agents have finite horizons, which implies that agents discount the future at a higher rate. In steady-state equilibrium, we obtain the following findings:

$$\dot{C} = (r - \theta) C - \rho(\rho + \theta) W \tag{4}$$

$$\dot{W} = rW + Y - C \tag{5}$$

With these two equations, he studies open and closed economy cases. In the open economy case, the interest rate is given and determined by the world market. The only asset available for agents is holdings of foreign assets, F. Note that, if $\bar{r}= \theta$, this implies that the value of F is zero. When $\bar{r}= \theta$, agents do not save or dissave, they have flat labour income and consumption profiles. If r is greater than θ, consumption is increasing, agents are accumulating over their life and the level of F is positive. If r is smaller than θ, agents are decumulating and the level of foreign assets are negative. The country is net debtor in steady-state. Stable steady-state occurs when interest rate is smaller or equal to the subjective discount rate.

In the closed economy case, interest rate is no longer exogeneous, but it is determined by the marginal product of capital, $r(K)$, which is the only asset agents hold. When ρ is equal to 0, which is when agents have infinite horizons, steady-state interest rate is equal to subjective discount factor of agents, $\bar{r}= \theta$, which is defined as the golden rule. As long as agents have finite horizons, steady-state interest rate is above θ, and the steady-state capital accumulation will be lower. Another way to look at it, is that interest rate is an increasing function of ρ. Therefore, the shorter the agents' horizons, the lower the steady-state capital level will be.

Later, he introduces a government into the model. The government spends on goods and finances spending by lump-sum taxes or by debt. Its dynamic budget constraint is thus given by,

$$\dot{D} = rD + G - T \tag{6}$$

where $\dot{D}$ is change in debt, $T - G$ is defined as surplus (or deficit as it may be), and government budget constraint is given by,

$$D_t = \int_t^{\infty} T_s e^{-\int_t^s r_v dv} ds - \int_t^{\infty} G_s e^{-\int_t^s r_v dv} ds \tag{7}$$

where equation (7) implies that present discounted value of future surpluses (right-hand side) is equal to the level of debt. With the introduction of government, we can rewrite equations of (1) to (3)in the following way,

$$C_t = (\rho + \theta)(H_t + W_t); \quad W_t = D_t + K_t \tag{8}$$

$$H_t = \int_t^{\infty} Y_s e^{-\int_t^s (r_v + \rho) dv} ds - \int_t^{\infty} T_s e^{-\int_t^s (r_v + \rho) dv} ds \tag{9}$$

$$\dot{W} = rW_t + Y_t + C_t - T_t \tag{10}$$

Blanchard studies the current tax cut effect on consumption and comes up with the proposition that except for the case where agents have infinite horizons, current budget deficits affect the economy, and interest rate rises with consumption. A decrease in taxes in current period increases human wealth (from equation (9)). He also derives a government fiscal policy index g, that summarizes the effects of government fiscal policy on aggregate demand,

$$\begin{aligned} g_t &= G_t - (\rho+\theta)\int_t^{\infty} G_s e^{-\int_t^s (r_v+\rho)dv} ds \\ &\quad +(\rho+\theta)\left[D_t + \int_t^{\infty} (G_s - T_s) e^{-\int_t^s (r_v+\rho)dv} ds\right] \end{aligned} \tag{11}$$

in which he collected all the terms in aggregate demand that depend directly on fiscal policy. His interest here is to measure the degree to which debt is net wealth and the degree to which it is offset by anticipated future surpluses.

For this purpose, he analyses the fiscal policy of US under the Reagan administration. In that particular case, where there are a sequence of deficits, and as debt accumulates, taxes increase, and surpluses appear eventually. Although the budget goes from deficit to surplus, the increase in debt dominates, leading to increase in g_0 to some g_∞ where $g_\infty > g_0$ and debt accumulates.

Another research in this field was done by Assaf and Frenkel (1992) in the paper entitled "An exposition of the Two-Country overlapping generations model", where they applied the same type of model for a two-country case. Their objective was to analyze the *international* effects of fiscal policies. It is for this reason that the overlapping generations model was extended to a two-country model of the world economy. Domestic fiscal policies are transmitted internationally through world interest rate. The exposition of this is based on the assumption that utility is logarithmic. Another difference from the approach used by Blanchard, is that the lifetime horizons of the agents depends on the survival probability γ. So, if $\gamma = 1$, the extreme case is obtained, where agents have infinite horizons. The object of their interest is finite-lived agents, thus for $\gamma < 1$. For simplification, they divide horizons into two periods: the present (current) $t = 0$, and the future $t = 1, 2...$

They define market for present goods and market for future goods, which is given by,

$$(1 - \gamma\delta) W_o + (1 - \gamma\delta^*) W_o^* = (Y_o - G_o) + (Y_o^* - G_o^*) \tag{12}$$

$$\left[\gamma\delta W_o + \frac{(1-\gamma)}{(R-1)}\frac{R}{(R-\gamma)}(Y - T)\right] + \left[\gamma\delta^* W_o^* + \frac{(1-\gamma)}{(R-1)}\frac{R}{(R-\gamma)}(Y - T)\right]$$
$$= \frac{1}{R-1}\left[(Y - G) + (Y^* - G^*)\right] \tag{13}$$

where variables with asterisk are for foreign country. Equation (12) tells that world demand for goods in period $t = 0$ equals world supply. The left-hand side of equation (12), is the sum of domestic and foreign per-capita private sector consumption, which must be equal to the sum of domestic and foreign outputs net of government spending.

Equation (13) is the requirement that discounted sum of per-capita domestic and foreign demand for future goods ($t = 1, 2..$) is equal to future world outputs net of government spending. It is interesting to note that coefficients of variables, $\gamma\delta W_o$ ($\gamma\delta W_o^*$) is the savings of population at time $t = 0$. This savings will be spent on future goods. The next terms are per-capita wealth of those who will be born in all future periods ($t = 1, 2..$). $(Y - T)$ is the disposable income of an individual and if we multiply it with $R/R - \gamma$, we get the present value of such annuity. Normalization is made, such that the size of each cohort is an individual itself, so $(R/R - \gamma)(Y - T)$ also refers to the value of annuity of a cohort as well. A discounted sum of disposable income of all future cohorts is obtained if we multiply a cohort's wealth by $R/R - 1$. Now, to get the present value of all cohorts' annuity, we divide it by R (constancy-equivalent interest rate). To bring it in per-capita value, we multiply it by $(1 - \delta)$, which converts aggregate to per-capita term.

To calculate the present values of various flows in the future, assuming that output, government spending and taxes do not vary across future periods, they defined an average interest rate r, which is may be thought as of the yield on current investment lasting up to indefinite future, that is actually do change over time and they refer to it as "constancy-equivalent" interest rate. Another requirement for a full equilibrium is that both governments must be solvent in the long run. They use graphical methods to explain the effects changes in government policy. Particularly, they discuss the current tax cut, which eventually must be offset by increase in future taxes, to satisfy the intertemporal budget constraint of government. Except for the case $\gamma = 1$, where agents have infinite horizons, the budget deficit arising from such a tax cut, raises the world interest rate, also raises the equilibrium value of domestic consumption and lowers the corresponding value of foreign consumption, thus the domestic budget deficit is negatively transmitted to the rest of the world. This mechanism is transmitted through interest rate. Further, they discuss the effects of government spending on interest rate, and private-sector spending. They found these effects depend on savings propensities of governments and of the private-sector. They particularly look at effects of past, current and future government spendings on world interest rate and private-sector spending.

In his paper, "Time preference and International Lending and Borrowing in an overlapping generations model" Buiter (1981) uses a different approach. He uses a Samuelson-Diamond type of overlapping generations model, extended for two economies, exactly identical in all respect, except differing in their pure rates of time preference. First, he considers the autarky economy equilibrium. Later, he allows the two countries to join together. Then, he compares capital formation, balance of payments behavior under these two cases. So, the purpose of his paper, is to explain theoretically, international capital movements from differences in pure rate of time preference and evaluate the short-run and long-run welfare implications of a change from a situation of trade and financial autarky to one of openness in trade and finance. To see the effects of pure rate of time parameter on international trade and international lending and borrowing, he ignored all determinants of them, except for the single taste parameter. Therefore, there is no differences in technology, in exogenous factor endowments, in market structure and there are no wedges between buyer and seller price, or between domestic and foreign price.

He describes the autarky equilibrium as follows. Two economies are exactly identical, except in taste. Each one of them is represented by a Diamond (1965) type of model, where there are two overlapping generations. In addition, the model contains identical and well-behaved CRTS (constant returns to scale) production function f with competitive output and factor markets. Individuals are identical within a given country and across generations. In this type of model (Diamond), people live for two periods, work in the first period (young) and retire in the second (old). Labour supply is inelastic.

The capital stock at the beginning of period $t+1$ equals the value of the saving in period t. All savings in period t is performed by the young of generation t. The old dissave previously accumulated wealth.

He describes his findings in his propositions about autarky equilibria.

- the country with high rate of time preference will choose higher value of consumption when young.

- when one starts from a given capital-labour ratio, k, a higher rate of time preference is associated with a lower value of k_{t+1} and higher value of consumption when young.

- when two economies, identical in all respects, except for the pure rate of time preference, which is higher in home country than in the foreign country, begins from any common initial capital-labour ratio at $t = t_o$, the capital-labour ratio of the high rate of time preference country will be below that of the low time preference country for all $t > t_0$.

- under autarky, the country with the higher pure rate of time preference will have a lower steady-state capital-labour ratio.

Then, he links two countries by creating an international commodity market. Again the two countries differ only in their pure rate of time preference, which is higher for home country. Combining the rate of time preference with the finite-lived individuals in a Samuelson-Diamond type of overlapping generations model may be the only analytical way to introduce different pure rates of time preference in models with international trade in goods and financial claims. There he also assumes that there is a single perfect world financial market and a single perfect world market for current production of the homogeneous commodity that can be used as a consumption good or a capital good. His propositions under openness are such that

- The country whose residents have a higher value of consumption when young will run a steady-state current account deficit, but not necessarily outside of it, or it is equal to say, that a country with higher rate of time preference has a steady-state current account deficit.

- the common steady-state open economy capital-labour ratio lies between the two autarky equilibrium.

There is no government in his model, so therefore there are no distortionary taxes or debt. However, the rate of time preference seems to be playing the same role of a country with an overspending government that eventually raise the steady-state interest rate, and hence lower the capital-labour ratio.

Clearly we can see links in these papers. First, they all use an overlapping-generations model, where in Buiter (1981) and Assaf R. and Frenkel J. (1992), it was extended to two-country case to pursuit their purposes. They all discuss the effects of steady-state interest rate on consumption, capital accumulation (capital-labour ratio), with dependence on the time horizon of agents. They all assume finite-lived horizons of agents. Buiter shows that a high rate of time preference yields the same result to a reduction in the horizons of agents. Loosely speaking, a higher pure rate of time preference of agents is the same as shorter horizons of agents. Blanchard, introduced a probability of death of agents that determines the horizon of the agents, and he discusses the role of government in determining the interest rate and the consumption behavior of agents. He describes two opposite phenomena, where the effect of finite horizons of agents is to decrease capital accumulation and the effect of declining labour income on saving is to increase it. The first effects follows from the fact that interest rate is increasing function of ρ, the probability of death, which implies that shorter horizons push the steady-state capital-labour ratio level down. When he assumed that labour income is decreasing over life time, there is incentive to save for retirement, and that increases the capital-labour ratio level. But the net effect is ambiguous and the steady-state can be inefficient as well. In Assaf and Frenkel paper they introduce horizons of agents in opposite way of what Blanchard did, with the introduction of survival probability. They examine current tax cut effects on the world economy and world interest rate and world markets for consumption demand, both present and for future goods. Next, we present a numerical model that will then be used to test the theoretical propositions developed by these authors.

3. The Model.

Here in this paper, we use a Samuelson-Diamond type overlapping generations model. Buiter used such a model, but our model is more sophisticated. First of all, we are working with 5 generations instead of the original two generations in Diamond (1965) and in Buiter (1981). Diamond and Buiter worked with a simple population and age structure to avoid aggregation problem. Applying a numerical approach solves the problem of aggregation in our case. Therefore, in this paper, we introduce data to simulate the model, and compare the numerical results with theoretical results of previously discussed papers. There are two economies in this model, we index them by j, in some cases, we refer them as i implying country of origin. This is necessary when we discuss trade flows between countries and bond issued in one country and owned in another. Note that, if $j = i$, this implies consumption of its own country and bond owned by the residents of the country which issues it. There are five generations at any time in this model (overlapping generations), represented by g. Each period of the model represents twelve years and each agent born at age 17. So each agent lives from age 17 to 77. Both economies are open, as there is trade between each other. Both countries produce one, but differentiated good. The following agents exist in this model. Representative firms hire labour and physical capital to produce one differentiated good. Labour is measured in terms of productivity of labour. Households earn human (labour wage income) and nonhuman income (income from assets, bequests and inheritance) to consume and save. Governments, tax and issue government bonds, and spend on public health and education, as well as other government expenditures. Finally, a National body manages the pension program.

The first four generations supply labour,and save. The last generation is retired and hence dissave. Expectations of households are rational in this model. There are two assets available to households, capital ownership titles and government bonds. Government bonds issued in one country can be owned in another country, whereas capital ownership titles is only for residents of that country where it belongs. They are perfect substitutes to each other. Substitutability of assets imply integrated capital markets. Therefore interest rates in each country are the same and it is the world interest rate. The model describes production sector, household behavior, government sector and equilibrium conditions. Subscript j, t, g under variable or parameter, will imply in country j, at time t, of generation g.

The pension system is a pay-as-you-go type. Hence, to finance it, the workers contribute to the pension program which is the pension benefits of current retired generation. Government collects three kind of taxes in this model, which are consumption tax, levied on price of consumption good, capital tax, which taxes income returns from assets, and wage taxes paid from wage income of and pension benefits.

This is a dynamic Samuelson-Diamond type of overlapping-generations model, extended for two countries. Five generations denoted from $g1$ to $g5$, exist at every period in this model. Individuals of age group $g1$ to $g4$ are working generations and $g5$ the retired generation.

Firms (Producers).

Two countries, populated by five generations, produce one, but differentiated product. Each country has a production function, which has a Cobb-Douglas form:

$$Q_{j,t} = A_{j,t} Kstock_{j,t}^{\alpha_j} L_{j,t}^{1-\alpha_j} \tag{14}$$

Equation (14) expresses the output of country j, at time t, and is related to the production parameter A times the amount of capital available in country j, at time t, also to the effective working population in country j, at time t. Note that $L_{j,t}$ is expressed in terms of effective labour. Therefore,

$$L_{j,t} = \sum_{g1}^{g5} POP_{j,t,g} EP_{j,g} \tag{15}$$

where, POP is population in country j, at time t, of age group g. EP is productivity of a worker in country j of age g. That can be factor of human capital, and of the experience of the worker. EP is assumed to be a quadratic function of age (g):

$$EP_g = \gamma + \lambda g - \phi g^2 \tag{16}$$

The demands for productive factors in country j, at time t, are given by,

$$\left(\frac{Wage}{P}\right)_{j,t} = \frac{(1-\alpha_j)\, A_{j,t} Kstock_{j,t}^{\alpha_j}}{L_{j,t}^{\alpha_j}} \tag{17}$$

and

$$\left(\frac{r}{P}\right)_{j,t} = \frac{\alpha_j A_{j,t} Kstock_{j,t}^{\alpha_j - 1}}{L_{j,t}^{\alpha_j}} \tag{18}$$

where equations (17) and (18) are simply the first order conditions for profit maximization for firms given the Cobb-Douglas production function. The production sector is competitive and firms hire factors (labour and capital) to the point where marginal revenue products (MRP) of factors (right-hand side of (17) and (18)) equal factor payments (left-hand side). Equations (14) to (18) represent the supply side of the economy.

Household.

Households live five periods of twelve years. They maximize utility, which is a Constant Elasticity of Substitution (CES) type. So, households' intertemporal utility function is given by,

$$U = \frac{1}{1-\sigma} \sum_{g=1}^{5} \left(\frac{1}{1+\rho}\right)^{g} \left[\left(Con_{j,g,t+g-1}^{1-\sigma} + Beq_{j,g,t+g-1}^{1-\sigma}\right)\right] \tag{19}$$

where $Con_{j,t,g}$ is the CES function of domestic and foreign consumption by individual living in country i of age group g at time t, ρ is pure rate of time preference, σ is elasticity of substitution, and Beq is bequest.

Derived from the household's optimization problem, equation (20) gives the relationship between future consumption of period $t+1$, with current consumption of period t.

$$Con_{j,t+1,g+1} = \left(\left[\frac{1 + RRET_{j,t-1}(1 - \tau k_{j,t})}{1 + \rho_j} \right]^{\sigma} \frac{Pcon_{j,t}}{Pcon_{j,t+1}} \right) Con_{j,t,g} \qquad (20)$$

where, $Pcon$ is price of consumption, $RRET = 1 + r^k$, is the return on capital, τk is the tax rate on returns on capital. Equation (20) can also be written as,

$$\frac{Con_{j,t+1,g+1}}{Con_{j,t,g}} = \left[\frac{1 + RRET_{j,t-1}(1 - \tau k_{j,t})}{1 + \rho_j} \right]^{\sigma} \frac{Pcon_{j,t}}{Pcon_{j,t+1}} \qquad (21)$$

It can be seen from that equation, that the higher the interest rate as well as the higher the current price, then the lower current consumption is and higher future consumption will be. In the same reasoning, the higher the subjective pure rate of time preference (ρ) of households, the more preferable the current consumption to the future consumption. The magnitude of these substitution between current and future consumption, depends on σ, the elasticity of substitution.

An individual household maximizes his/her utility of (19) subject to budget constraint,

$$\begin{aligned} & (1+\tau c_{j,t})Pcon_{j,t}Con_{j,t,g} + Lend_{j,t+1,g+1} \\ = & (1-\tau w_{j,t} - CTR_t)Wage_{j,t}EP_{j,g} + \\ & \sum_i (1 + RintJ_{i,t-1} - \tau k_{j,t})\frac{P_{i,t}}{P_{i,t-1}}P_{i,t-1}Bij_{i,j,t,g} + \\ & +(1 + RRET_{j,t} - \tau k_{j,t})Pinv_{j,t-1}K_{j,t,g} + \\ & +(1-\tau w_{j,t})Pens_{j,t,g} + Inh_{j,t,g} - Beq_{j,t,g} \end{aligned} \tag{22}$$

where, τc - consumption tax rate (value added tax), $0 \leq \tau c < 1$, $Lend_{j,t+1,g+1}$ are savings of households of country j, of age group $g+1$, of period $t+1$, τw is wage tax rate, CTR is contribution rate to pension program, $0 < CTR < 1$, $RintJ$ is the rate of interest, $Bij_{i,j,t,g}$ is a bond, issued by government in country i, and hold by household of generation g in country j, as of period t, $Pinv$ is price of investment, $K_{j,t,g}$ is capital ownership of age group g, in country j, at time t, capital ownership can be hold only by population of the region where it belongs, $Pens$ is pension income, and Inh is inheritance.

The first line of equation (6) represents expenditure of households, a consumption expenditure and savings to be allocated among the various assets. Consumption expenditure of household consists of domestic good and good produced in other regions, which is then imported to consume. The composition of consumption of households results from assumed CES function and is given by equation (23),

$$ConI_{i,j,t,g} = AlConI_{i,j} \left(\frac{Pcon_{j,t}}{P_{i,t}} \right)^{\sigma_{con_j}} Con_{j,t,g} \tag{23}$$

Equation (23) tells us that consumption from country i to country j (export from i to j, the case where $i = j$ means domestic consumption) depends on consumption preference factor for good from i to j, $AlConI$, the ratio of consumption price over price in country i and on elasticity of substitution of consumption. The coefficient $AlConI$ may differ in each countries, reflecting differences in preferences. As a result, demand for good can be home-biased (large $AlConI$) or foreign consumption good biased (small $AlConI$). The elasticity σ has also crucial interest here, that is, if σ is inelastic, smaller than 1, even large increase in country $i's$ price, will not reduce demand for good produced in country i by much. On the other hand, if σ is greater than 1, then even small increase in the price of country i, will reduce demand for good produced in country i by more than proportional decline in price.

There are two types of assets in this economy, bonds issued by governments of two countries, which can be owned by residents of other country, and ownership of capital stock in the economy, which is owned only by residents of that country. Therefore, individuals make decision at the beginning of each period to allocate their savings on these two assets.

Financial capital market is perfectly integrated, households can easily buy bonds from other country. These two assets are perfect substitutes in a view of households. This can be seen from equation

$$RintJ_{j,t}\frac{P_{j,t+1}}{P_{j,t}} = RRET_{j,t+1} \tag{24}$$

where the rate of return on these assets are equalized. Should there be any difference between the yields, international arbitrage instantaneously equalizes them.

Households income consists of human and nonhuman wealth. Equation (22) shows that households human wealth is from his labour earnings. Households real effective labour wage income is equal to wages minus wage taxes and contribution to pension plan. Government, which will be introduced later into model, collects different taxes such as, taxes on wage income (τw), taxes on return from assets

(τk), also sales taxes, which is embedded in the price of consumption (τc). Households nonhuman wealth comes from real return from holding on assets (remember, there are two assets), as well as pension benefits, and inheritance and bequests.

Bequests and inheritance are introduced in this model in such a way, that only last generation $(g5)$ leaves bequest and it is a fraction from their last year consumption. Inheritance arising from bequests of last generation, is equally distributed among all working generations. These facts are captured in following equations,

$$Beq_{j,t,g} = BeqR_{j,g} Pcon_{j,t} Con_{j,t,g} \tag{25}$$

and

$$POP_{j,t,g} Inh_{j,t,g} = InhR_{j,g} \sum_{g=1}^{4} POP_{j,t,gm} Beq_{j,t,gm} \tag{26}$$

where, $BeqR$ is bequest rate, $0 < BeqR < 1$, $InhR$ is inheritance rate, $0 < InhR < 1$.

As the last generation will not make savings, they dissave as mentioned above as well. With this behavior, the budget constraint of the last generation differ from what we introduced in equation (22) in the following way,

$$
\begin{aligned}
& (1+\tau c_{j,t}) Pcon_{j,t} Con_{j,t,gn} \\
= & (1-\tau w_{j,t} - CTR_t) Wage_{j,t} EP_{j,gn} \\
& + \sum_i (1 + RintJ_{i,t-1} - \tau k_{j,t}) \frac{P_{i,t}}{P_{i,t-1}} P_{i,t-1} Bij_{i,j,t,gn} + \\
& + (1 + RRET_{j,t} - \tau k_{j,t}) Pinv_{j,t-1} K_{j,t,gn} + \\
& + (1 - \tau w_{j,t}) Pens_{j,t,gn} + Inh_{j,t,gn} - Beq_{j,t,gn}
\end{aligned} \tag{27}
$$

As we can see, because t is the last period for last generation, they do not make decision on how to allocate their saving for $t+1$period, they just dissave it, therefore variable *Lend* does not appear in expenditure side of budget constraint.

Households supply their financial resources to the economy. They can buy different government's bonds (debt) or they can invest. We saw that rates of return from these two choices are the same, and households are indifferent between them. The actual holdings of these assets are given by equations (28) and (29),

$$
P_{i,t} Bij_{i,j,t+1,g+1} = \frac{Bij0_{i,j}}{\sum\limits_{ii} Bij_{ii,j} \left(Lend_{j,t+1,g+1} - Pinv_{j,t} K_{j,t+1,g+1} \right)} \tag{28}
$$

and holding of physical capital is given by,

$$K_{j,t+1,g+1} = \frac{Kstock_{j,t+1} Lend_{j,t+1,g+1}}{\sum_{gg} POP_{j,t+1,gg+1} Lend_{j,t+1,gg+1}} \tag{29}$$

According to the equation (28), households can buy bonds of other governments (in equation it says, that bond issued by government i and owned by household in country j), so there can be different bonds in their portfolios issued from different governments. Holdings of bonds at time t follows an ad hoc rule since households are indifferent to the composition of their wealth. The ad hoc rule in this model stipulates that holdings of bonds is equal to an exogenous allocation parameter $(Bij0/\sum Bij0)$ multiplied by available savings to buy these bonds (savings left after buying capital ownership). The ratio $Bij0/\sum Bij0$ is assumed identical for all age groups.

Equation (29) tells that capital ownership in country j, at the beginning of period t, is equal to the fraction of savings spent on buying these titles.

The flow to the capital stock is investment and there is a depreciation of capital stock each period. Capital accumulation (decumulation) occurs if new investment is greater (falls short of) than depreciation of capital stock. It can be see from equation (30),

$$Kstock_{j,t+1} = Inv_{j,t} + (1 - DepR_j)\, Kstock_{j,t} \tag{30}$$

where, $DepR$ - depreciation rate of capital.

The equation (30) states that capital stock available at the beginning of the next period is accumulation (or decumulation) of new capital stock (which is investment) to the existing capital stock. Investment in period t, is savings supplied by households at period t. As we discussed before, households are indifferent between buying bonds (earning interest on it) and investing it, because of equal rate of returns on these assets. The condition for this to hold was given in equation (24). By substituting equation for return on capital ownership, we will get equation for investment,

$$RintJ_{j,t} \frac{P_{j,t+1}}{P_{j,t}} = Rent_{j,t+1} + (1 - DepR_j) \frac{Pinv_{j,t+1}}{Pinv_{j,t}} \tag{31}$$

where, $Rent$ - marginal product of capital.

So, when returns are equalized across regions, household are indifferent between two choices.

We said that consumption expenditure is the consumption of domestic and foreign produced good and we looked at the composition of it in equation (23). So is the case for investment. The physical capital good is a composite of the two final goods. The technology for physical capital is also of the CES type. So investment demands generates an indirect demand for the imported and the domestic final good. The composition of investment is given by,

$$EInv_{i,j,t} = AlInv_{i,j} \left(\frac{Pinv_{j,t}}{P_{i,t}} \right)^{\sigma_{inv_j}} Inv_{j,t} \tag{32}$$

Equation (32) tells that investment demand by country i for goods produced in country j , (the case $i = j$ corresponds to investment demand for domestic resources) depends on investment technology parameter $AlInv$, the ratio of investment price over price of investment in country i and on the elasticity of substitution of investment between countries. The coefficient $AlInvI$ may differ in each country, reflecting difference in the investment technology.

Government.

Government is another agent in this economy. Each country's government collects taxes T, and issues debt (bonds) D, to finance government expenditures,

G. So, each government's budget constraint is given by,

$$\begin{aligned}
& P_{j,t}Bond_{j,t+1} + \sum_g POP_{j,t,g}(\tau w_{j,t}Wage_{j,t}EP_{j,g}) + \\
& +\tau w_{j,t}Pens_{j,t,g} + \tau c_{j,t}Pcon_{j,t}Con_{j,t,g} + \\
& +\tau k_{j,t}\sum_i(RintJ_{i,t-1}\frac{P_{i,t}}{P_{i,t-1}} - 1)P_{i,t-1}Bij_{i,j,t,g} + \\
& +\tau k_{j,t}(RRET_{j,t-1} - 1)Pinv_{j,t-1}K_{j,t,g} \\
= \; & P_{j,t}(Gov_{j,t} + GovH_{j,t} + GovE_{j,t}) + (RintJ_{j,t-1}\frac{P_{j,t}}{P_{j,t-1}})P_{j,t-1}Bond_{j,t} \quad (33)
\end{aligned}$$

The left-hand side of the equation (33), represent government revenues. It consists of debt, issued to the private sector, at the beginning of period $t+1$ and taxes which we discussed in the household problem. The expenditure of governments in each country represents expenditure on health and education, also other government expenditure plus debt servicing, paying interest on previously issued bonds.

Equilibrium in markets.

The equilibrium in goods market is given by following condition,

$$Q_{j,t} = \sum_i(ECon_{j,i,t} + EInv_{j,i,t}) + (Gov_{j,t} + GovH_{j,t} + GovE_{j,t}) \quad (34)$$

where total supply of output in country j, at time t, is equal to total demand of that country at time t. Total demand is total consumption demand plus total investment demand and government expenditures. Equation (34) is known as, the income identity.

Total consumption demand, itself, is given by,

$$ECon_{i,j,t} = \sum_{g} POP_{j,t,g} ConI_{i,j,t,g} \tag{35}$$

The equilibrium in financial asset market is when real interest at time t, is the same in each region,

$$Rint_t = RintJ_{j,t} \frac{P_{j,t+1}}{P_{j,t}} \tag{36}$$

this condition implies integrated asset market condition. The next equation comes from combining of equations (24), (28), (29) and (30), and which is given by,

$$\sum_{j} \sum_{g} POP_{j,t+1,t+1} Lend_{j,t+1,g+1} = \sum_{j} (P_{j,t} Bond_{j,t+1} + Pinv_{j,t} Kstock_{j,t+1}) \tag{37}$$

where it states that total demand for savings (right-hand side) coming from production sector, and governments is equal to total supply of savings, which is savings of population in both countries (left-hand side).

Steady-State.

In steady-state all economic variables are constant. It is assumed, that there is zero economic growth in this model. So, steady-state consumption is

$$Con_{j,t+1,g} = Con_{j,t,g} \tag{38}$$

Equation (38) says that in steady-state, the consumption is constant over periods. The steady-state capital stock is also constant and is given by,

$$Kstock_{j,t} = Kstock_{j,t+1} \tag{39}$$

and with the presence of depreciation in this model, equation (39) becomes,

$$Inv_{j,t} = ((NN_{j,t} - 1) + DepR_j)Kstock_{j,t} \tag{40}$$

where it states that new investment is just equal to depreciation of capital stock and capital stock remains constant over periods. NN is the population growth rate, which is assumed to be simply equal to 1.

Government issues debt and in steady-state it is also constant, which is equal to say that per capita bond is constant, and is given in equation (41),

$$Bond_{j,t+1} = NNP_{j,t}Bond_{j,t} \tag{41}$$

With this condition for debt issuing, government's steady-state budget constraint becomes,

$$\begin{aligned}
& NN_{j,t} \cdot P_{j,t} \cdot Bond_{j,t} + \sum_g POP_{j,t,g}(\tau w_{j,t}Wage_{j,t}EP_{j,g}) + \\
& +\tau w_{j,t}Pens_{j,t,g} + \tau c_{j,t}Pcon_{j,t}Con_{j,t,g} + \\
& +\tau k_{j,t}\sum_i (RintJ_{i,t-1}\frac{P_{i,t}}{P_{i,t-1}} - 1)P_{i,t-1}Bij_{i,j,t,g} + \\
& +\tau k_{j,t}(RRET_{j,t-1} - 1)Pinv_{j,t-1}K_{j,t,g} \\
= \; & P_{j,t}(Gov_{j,t} + GovH_{j,t} + GovE_{j,t}) + (RintJ_{j,t-1}\frac{P_{j,t}}{P_{j,t-1}})P_{j,t-1}Bond_{j,t} \qquad (42)
\end{aligned}$$

The Steady-state condition for government budget constraint is consistent with equation (41).

In order to numerically analyse Buiter's theoretical results, we assume that both countries are identical in all respects, except for the pure rate of time preference. Blanchard (1985) found that the effect of finite horizons per se is to decrease capital accumulation, which we discussed in reviewing his paper. However, he also showed that the effect of declining labour income on saving for retirement is to increase capital. We will test this latter theoretical result using the effectiveness of labour parameter of the numerical model. We will also refer to the issue on whether deficit financing is relevant in case of finite horizons of agents and its international effects on home and foreign economies. We will thus offer a numerical version of the theoretical findings of Assaf R. and Frenkel J. (1992). To test their theoretical results, we will simulate a temporary decrease in taxes.

4. Data and Calibration.

In order to discuss Buiter's propositions when two autarky economies join together and to analyse the effect of pure rate of time preference on the economy, we impose identical economic features on both countries. In Table 1, the values used for parameters and variables are reported. World GDP is normalised to one and two countries are producing half of it. At the initial steady state productivity and population growth equal 1. Both countries trade, and trade flow is balanced. So there is no current account deficit/surplus in both countries initially. In contrast to Buiter (1981), we include a government sector. We assume that the role of all governments in this model is the same for both countries. For instance, public debt with respect to national GDP is assumed to be the same in each country. Perfect international mobility of capital implies a unique interest rate that can be called the world interest rate. We impose a rate equal 7.2% as in Mercenier and Mérette (2002). This rate is necessary to generate sufficient life-cycles savings to accommodate capital stock in these economies. The share of capital is also assumed equal in both countries. The last generation leaves bequests and the bequest rate is equal to 0.3, which implies that 30 per cent of the value of consumption of last generation is set aside as bequests, such behaviour is assumed similar in home and foreign country. The wage-age profile parameters are set to produce households highest income earnings are between age 29 and 52.

Table 1. Parameters and Exogenous Variables.

	HOME	**FOREIGN**
GDP	0.500	0.500
Government Debt	0.050	0.050
Public Health Care	0.019	0.019
Public Education	0.015	0.015
Trade Balance	0.000	0.000
Wage tax rate	0.362	0.362
Capital tax rate	0.352	0.352
Consumption tax rate	0.216	0.216
Share Value of Capital	0.500	0.500
Intertemporal elasticity of substitution (σ)	0.275	0.275
Rate of interest	0.072	0.072
Bequest rate	0.300	0.300
Age-wage profile parameter (γ)	1	1
Age-wage profile parameter (λ)	0.260	0.260
Age-wage profile parameter (ψ)	0.018	0.018

Composition of public health and education expenditure by generation is given in Table 2. Older age people, g4-g5 or age from 53-77 get almost 65 per cent of each per-capita dollar spent in health care, while more than 65 per cent of each dollar spent on education comes on g1-g2 or in favour of the 17-40 age group. Older age people need more health care, whereas most investment in human capital is directed to young generation. As for other parameters, we assume that the distribution on health care and education is identical in both countries.

Table 2. Public Expenditure per Age Group (dollar share per capita in percentage terms)

Age-Groups/generation	17-28/g1	29-40/g2	41-52/g3	53-64/g4	65-77/g5
Health	0.1096	0.1213	0.1399	0.2043	0.4249
Education	0.330	0.360	0.190	0.10	0.02

In the next table, the parameters and variable values are generated by the calibration procedure. As expected, we see that both countries are similar in all respects. Their production scaling parameter, capital depreciation rate, the pure rate of time preference, pension contribution rate (which is sufficient to generate pension benefits to retired generation) are exactly identical. These values with the data in the above tables solve the model in steady state.

Table 3. Calibrated Variables and Parameters

Parameter	HOME	FOREIGN
Production Scaling parameter (A)	1.825	1.825
Capital depreciation rate (DepR)	0.697	0.697
Rate of time preference (ρ)	0.371	0.371
Pension contribution rate (CTR)	0.084	0.084

The calibration procedure also produced a trade flow matrix between countries. We assume initially, that both countries trade between each other. As there is no current account deficit in both countries, net exports in each country are equal to zero.

Table 4. Trade Flow Matrix between countries.

	HOME	FOREIGN
HOME	0.439	0.061
FOREIGN	0.061	0.439

The numbers in first row are flow of output produced in home country to other countries. Respectively, the second row shows the flow of output produced in foreign country to other countries. We see that home country's demand for its own product equals 0.439 whereas its exports to foreign country equal 0.061. The same numbers apply for the foreign country. Overall, the sum of the numbers of matrix equals 1, the world GDP.

5. Simulation Experiments.

In order to introduce asymmetric preferences, we assume that the pure rate of time preference of home country is 0.426 which is 15% higher that that of foreign country, 0.371. We simulate this and see the results to compare with theoretical propositions of Buiter. For testing Assaf R. and Frenkel J. theoretical result, we will temporarily reduce taxes in home country by 15 per cent, and compare our results with what they came up. For that purpose, we allow governments to have budget deficits. Governments will borrow in order to satisfy balanced budget. In order to numerically analyse Blanchard's theoretical result, the calibration file and data is modified slightly. The only difference is that the wage-age profile of individuals is set such that income earnings are equally distributed over lifetime. To be more precise, ψ and λ parameters of the equation (16) in the model are set equal to zero, whereas γ is equal to 1.

6. Simulations Results.

6.1. Simulation results for Buiter's propositions.

A numerical approach is applied to test theoretical propositions derived by Buiter (1981). As in Buiter (1981), we impose as initial condition that the two economies be exactly identical, except for the pure rate time preference. To perform the Buiter experiment, we calibrate a numerical steady state where the two economies are identical, but as each country produces a differentiated good trade exists between the two countries. Then we assume that the households of home country economy become more impatient with respect to consumption behavior. The results are displayed in Tables 5, 6 and 7. The shock consists of assuming that preferences suddenly changed from period 5. In other words, households in home country become more impatient towards the future consumption. From Table 5 and 6, we see that as a result of this shock, consumption in home country is higher than in foreign country in the short-run and savings decline.

Households now prefer to consume more and hence save less when young. We see also old age (g5) consumption has not really changed and they dissave less.

Table 5. The short-run effects of higher time preference for home country on consumption and savings levels in home country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.200 (0.184)	0.291 (0.268)	0.420 (0.393)	0.597 (0.572)	0.836 (0.836)
Savings level	0.157 (0.171)	0.203 (0.210)	0. 200 (0.205)	0.109 (0.106)	-0.670 (-0.692)

Table 6. The short-run effects of higher time preference for home country on consumption and savings levels in foreign country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.180 (0.184)	0.265 (0.268)	0.390 (0.393)	0.573 (0.572)	0.834 (0.836)
Savings level	0.176 (0.171)	0.209 (0.210)	0.202 (0.205)	0.103 (0.106)	-0.690 (-0.692)

As consumption demand increase in the home country, it pushes interest rate up, which is shown in Table 7. It is not sufficient to accommodate capital stock for the next period. Rise in interest rate discourages firms to make investment and therefore, output and capital stock in home country decline in the next period of the shock. As the interest rate is the world interest rate, it also negatively affects the foreign country's economy in the short-run and foreign output has also decline. Both countries produce one differentiated product. Because of change in preferences in home country, consumption demand increased. That puts pressure on domestic output prices. Part of that consumption comes from abroad, so import demand increases as well. As there are two countries in the world, the home country runs a trade balance deficit in the short-run.

Table 7. The short-run effects of higher time preference for home country. (Numbers in parenthesis are initial values).

	HOME	FOREIGN
GDP	0.493 (0.500)	0.494 (0.500)
Interest rate	0.165 (0.072)	
Capital Stock	0.0121 (0.0125)	0.0122 (0.0125)
Trade Balance	-0.004 (0.000)	0.004 (0.000)
Pure rate time preference	0.426 (0.371)	0.371 (0.371)
Output prices	1.006 (1.000)	1.000 (1.000)

Dynamic adjustment towards the steady state.

Home consumption decline over time, because households prefer to consume more when they are young. Demand decreases as well, and consequently prices fall. Home country's savings decline as well. Foreign country's consumption demand and savings increase. This implies an increase of the foreign demand for home country's output. This offsets somewhat the decline in domestic demand and impedes output to drop further. Given the opposite direction taken by domestic and foreign demands, the foreign country runs a trade balance deficit, whereas the home country runs a trade balance surplus. A lower output price permits home country to run a trade balance surplus, as shown in Charts 1 and 2.

Chart 1. **Dynamics of Output Prices**

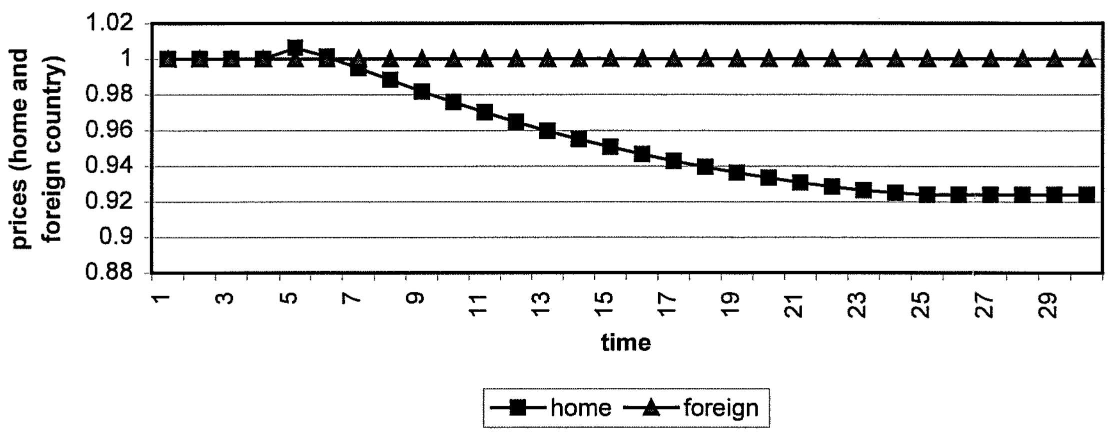
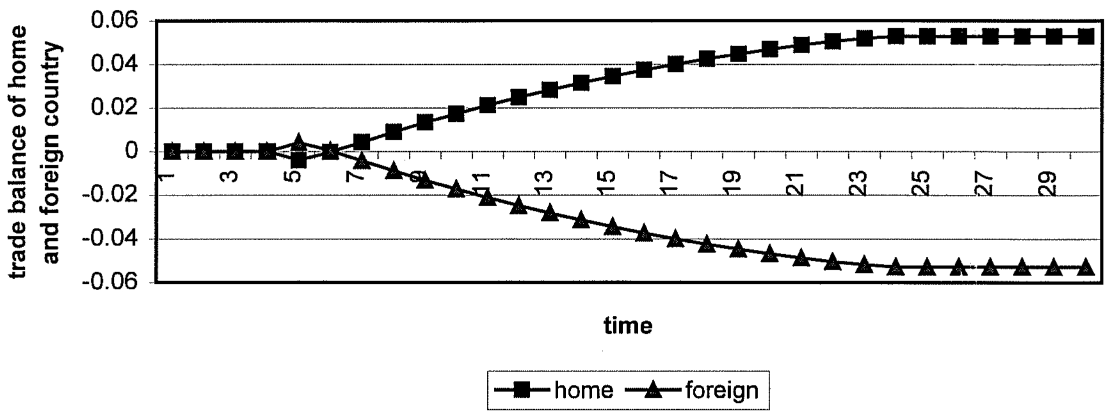
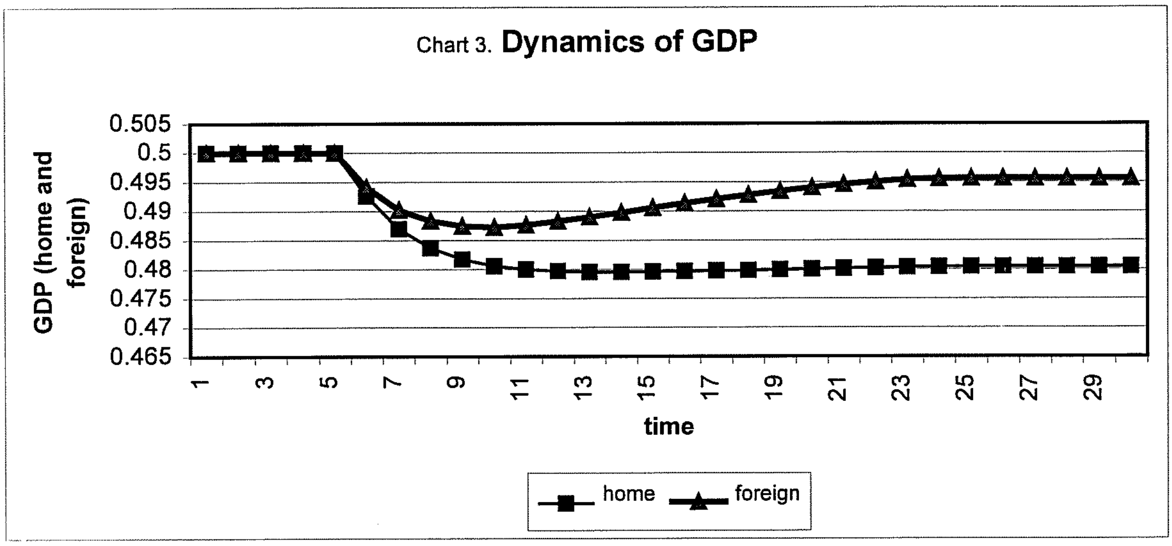
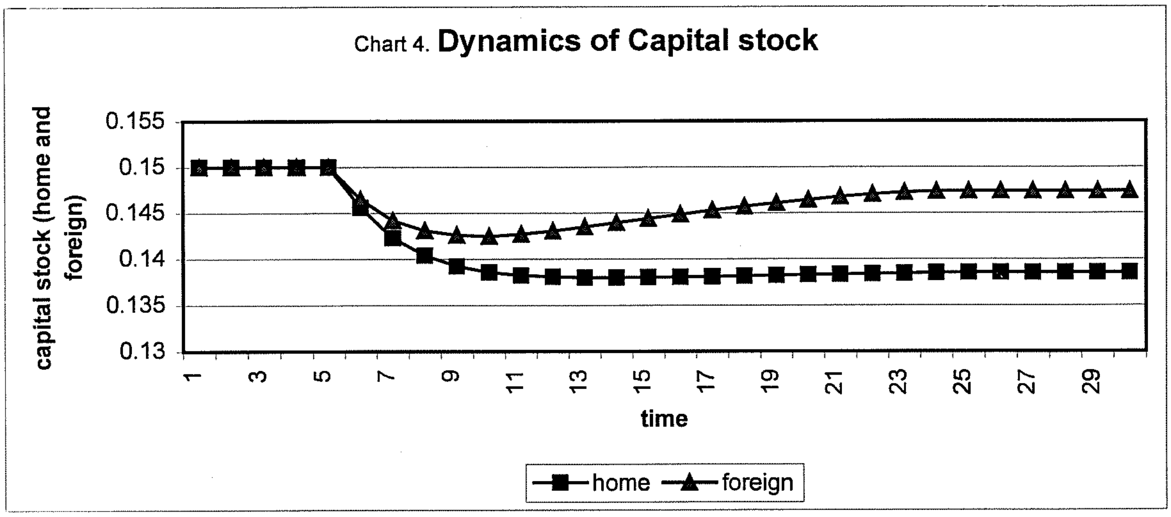


Chart 2. **Dynamics of Trade Balances.**



The rise in interest rate is moderate, since home country's output falls rapidly, but in process it approaches to its long run value, it is relatively stable as a consequent of decrease in consumption and savings and capital stock, hence investment demand declines. The obvious point in dynamic adjustment process is that home country's output, capital stock, price of output as well as consumption and savings level is below of those of foreign country. This is consequent of the higher pure rate of time preference for home country. A lower capital stock in home country implies lower output in home country.





Long Run Steady state.

A higher pure rate time preference for home country pushes the world interest rate up in long run. A higher steady state interest rate discourages investment. Consequently, the capital stock shrinks and as a result, output of the home country is lower in steady state. The higher interest rate encourages savings in foreign country and consumption is also higher in steady state and foreign country runs trade balance deficit in steady state. Table 8, summarizes the effect of higher time preference for home country in long run steady state.

Table 8. Steady state effects of higher time preference for home country. (Numbers in parenthesis are initial values).

	HOME	FOREIGN
GDP	0.480 (0.500)	0.496 (0.500)
Interest rate	0.168 (0.072)	
Capital Stock	0.0115 (0.0125)	0.0123 (0.0125)
Trade Balance	0.052 (0.000)	-0.052 (0.000)
Pure rate time preference	0.426 (0.371)	0.371 (0.371)

Home country has lower consumption and savings level in long run steady state. It is a consequence of that that household prefers to consume more when young. With higher consumption and savings level, foreign country worsens its trade balance.

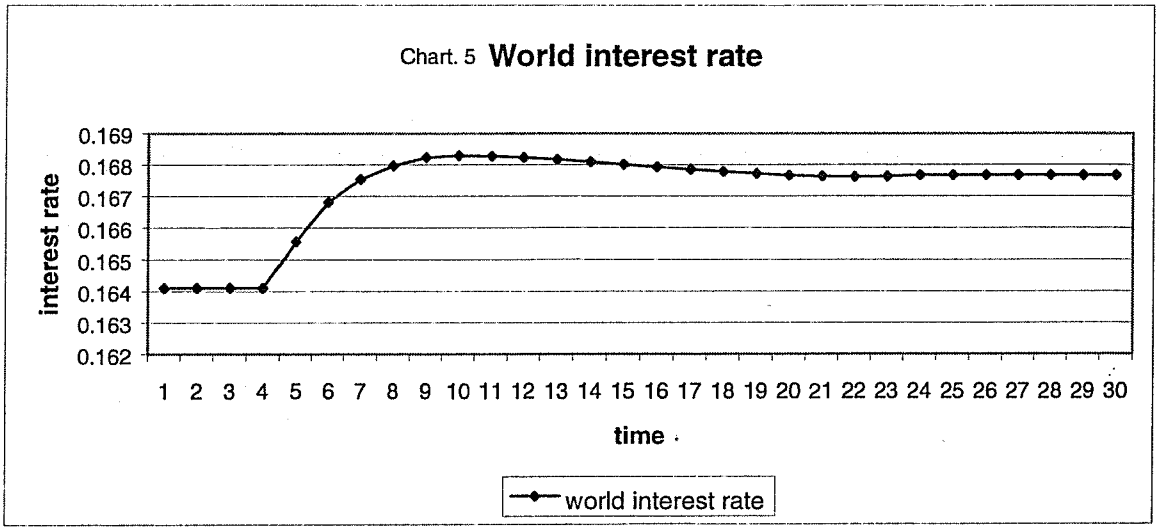
Table 9. The steady state of higher time preference for home country on consumption and savings levels in home country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.139 (0.184)	0.193 (0.268)	0.268 (0.393)	0.373 (0.572)	0.519 (0.836)
Savings level	0.088 (0.171)	0.110 (0.210)	0.113 (0.205)	0.079 (0.106)	-0.389 (-0.692)

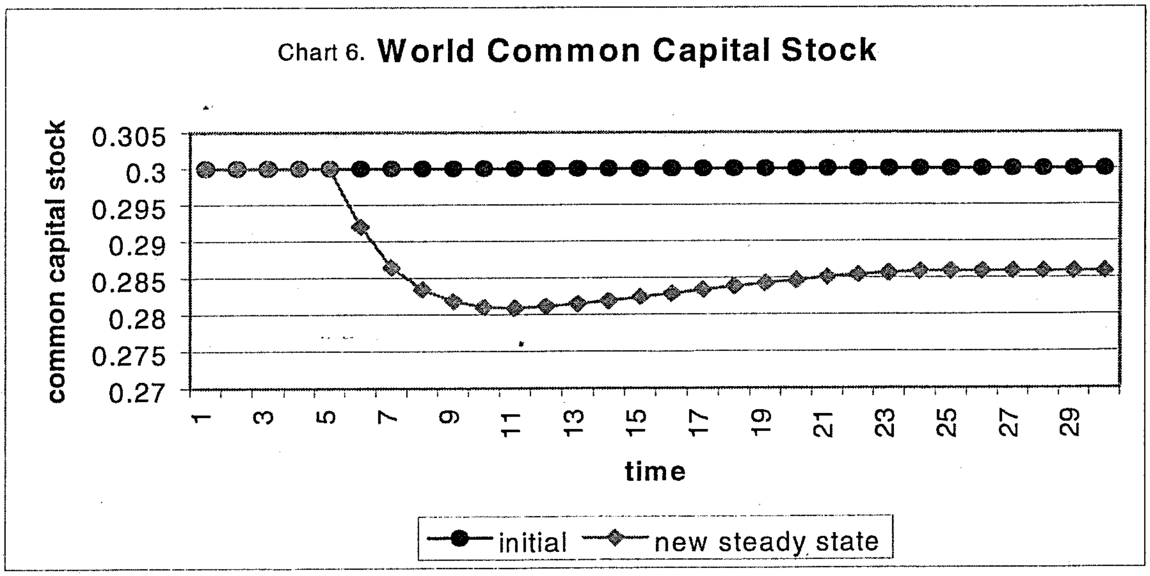
Table 10. The steady state of higher time preference for home country on consumption and savings levels in foreign country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.210 (0.184)	0.318 (0.268)	0.482 (0.393)	0.731 (0.572)	1.108 (0.836)
Savings level	0.226 (0.171)	0.274 (0.210)	0.271 (0.205)	0.124 (0.106)	-0.899 (-0.692)

Overall, as a conclusion for this experiment, as a result of change in time preference of households in home country, as shown in Charts 5 and 6, the world interest has risen and discourages the accumulation of capital stock in each country.



World's level of capital stock thus declines. The higher the world interest rate, the lower the world capital stock is. As world capital stock is lower in steady state, world output is consequently also lower in steady state.



Comparison with Buiter's propositions.

Buiter (1981) derived a number of propositions from his model. We now compare our simulation results with Buiter's theoretical propositions.

First proposition: the country with high rate of time preference will choose higher value of consumption when young; As shown in Tables 5 and 6, home country's consumption in our model, which has a higher rate of time preference, is higher than that of foreign country in the short-run.

Second proposition: when one starts from a given capital-labour ratio, *k,* a higher rate of time preference is associated with a lower value of *k (t+1)* and higher value of consumption when young; As shown in Tables 5, 6 and 7, the two economies start from exactly identical value for capital stock, but home country has higher rate of time preference, hence lower *k (t+1)* and higher consumption when young in home country than in foreign country. That is, households in home country prefer to consume more when young and save less, which implies lower capital stock in consequent periods.

Third proposition: when two economies, identical in all respects, except the pure rate of time preference (which is higher for home country than for the foreign country), begins from any common initial capital-labour ratio at *t=t (0)*, the capital-labour ratio of high rate of time preference country will be below that of the lower time preference country for all *t>t (0)*. This directly follows from proposition two, lower savings implies lower capital stock in subsequent periods. From Chart 2, it can be seen that home country's capital stock is below of that of foreign country in any period after preferences change for home country.

Fourth proposition: The country whose residents have a higher value of consumption when young will run a steady-state current account deficit, but not necessarily outside of it, or it is equal to say, that a country with higher rate of time preference has a steady-state current account deficit. As we mentioned, our model is more sophisticated than that of Buiter (1981) used. Here each country produces one, but differentiated good. This implies output prices can be different. And as these economies allowed trading each other, the results showed that actually foreign country, which has lower rate of time preference, runs a trade balance deficit. Foreign country's consumption increase over time and in steady state its consumption level is above of home country's level. Part of that consumption comes from home country, and increase import demand worsens trade balance of foreign country.

Fifth proposition: the common steady-state open economy capital-labour ratio lies between the two autarky equilibrium; from Chart 5 and 6, we see that higher steady state interest rate discourages investment, which result in a lower steady state capital stock in the world economy.

6.2. Simulation results for Blanchard propositions.

Blanchard (1985) argues that when households' wage earnings profiles are constant over lifetime, shorter horizon of households decreases savings and implies a lower steady state capital stock. We increase impatient rate of agents using the pure rate of time preference parameter. In order to carry out the Blanchard experiment, we calibrate a numerical steady state where earning profile of households is constant over lifetime. As there are two countries in this model, it is easier to compare results with Blanchard's (1985) propositions. A higher rate of time preference is introduced only to home country, as if preferences were suddenly changing. Simulation of the model gives the following results.

Finite horizons were combined with constant labour earnings over time implies a lower capital stock in steady state. As shown in Table 11 and 12, an increase in horizons of agents pushes savings down in home country that result in smaller accumulation of capital. As a consequence of this unanticipated shock, consumption increases in home country in the short-run.

Table 11. The short-run effects of shorter horizons for home country with constant labour income over lifetime on consumption and savings levels in home country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.167 (0.150)	0.238 (0.214)	0.334 (0.305)	0.461 (0.435)	0.622 (0.621)
Savings level	0.129 (0.143)	0.149 (0.156)	0.138 (0.143)	0.078 (0.074)	-0.493 (-0.515)

Table 12. The short-run effects of shorter horizons for home country with constant labour income over lifetime on consumption and savings levels in foreign country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.145 (0.150)	0.210 (0.214)	0.303 (0.305)	0.436 (0.435)	0.620 (0.621)
Savings level	0.149 (0.143)	0.154 (0.156)	0.140 (0.143)	0.071 (0.074)	-0.514 (-0.515)

Capital stock is reduced in both countries in the short-run, but in home country it is smaller than in foreign country, as given in Table 13. More impatience towards the consumption increase it and as a result interest rate increases. Firm's investment decisions are discouraged by increase in interest rate, therefore less investment is made. As a consequence, output is lower in both countries in the short-run. The increase in consumption demand in home country combined with the increases in output price, results in a rise in home country's import of goods. Consequently, the home country runs trade deficit in the short-run.

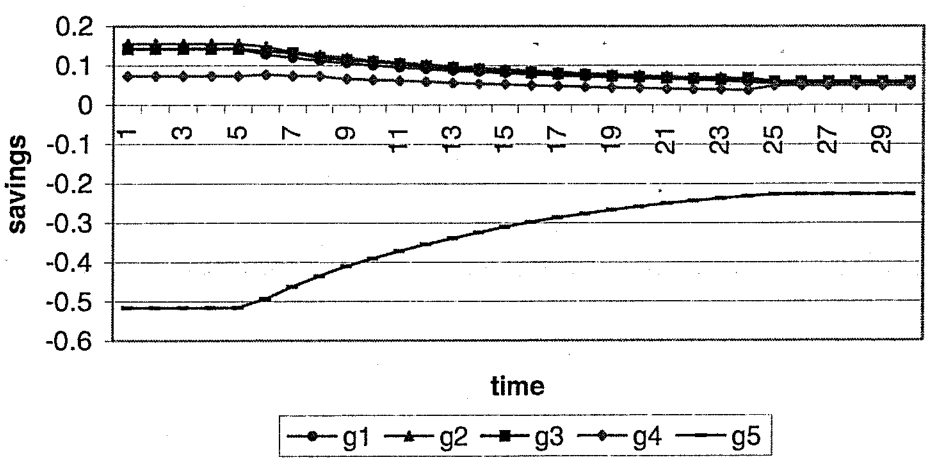
Table 13. The short-run effects of shorter horizons for home country with constant labour income over lifetime. (Numbers in parenthesis are initial values).

	HOME	FOREIGN
GDP	0.490 (0.500)	0.492 (0.500)
Interest rate	0.1661 (0.072)	
Capital Stock	0.012 (0.0125)	0.0121 (0.0125)
Trade Balance	-0.005 (0.000)	0.005 (0.000)
Pure rate time preference	0.380 (0.371)	0.371 (0.371)
Output prices	1.008 (1.000)	1.000 (1.000)

Dynamic adjustment towards the steady state.

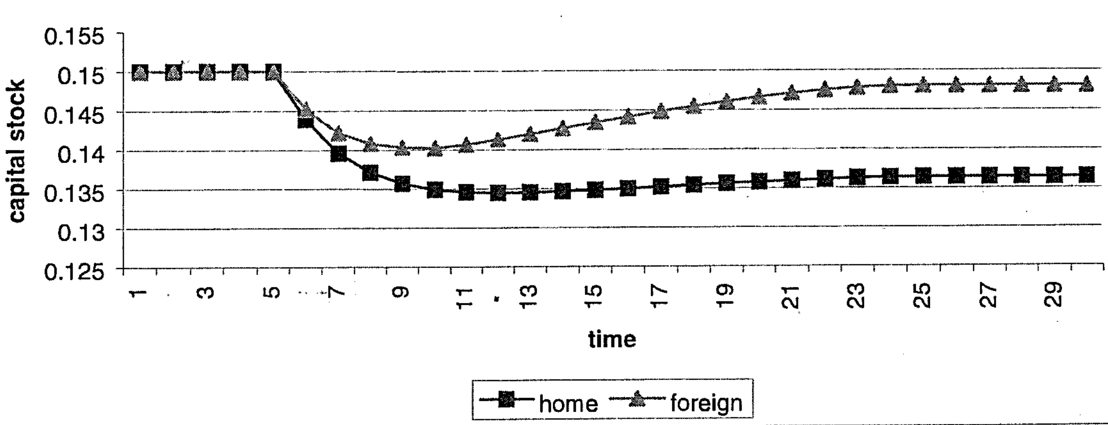
The main result, which reflects Blanchard's (1985) proposition, is that savings in home country declines over time as shown in Chart 7. That is, if households' labour income earnings are constant in his lifetime, a higher discount rate, implies a smaller amount of savings. Consumption also decreases over time. The opposite is true for foreign country.

chart 7. **Dynamics of savings by age group in home country**

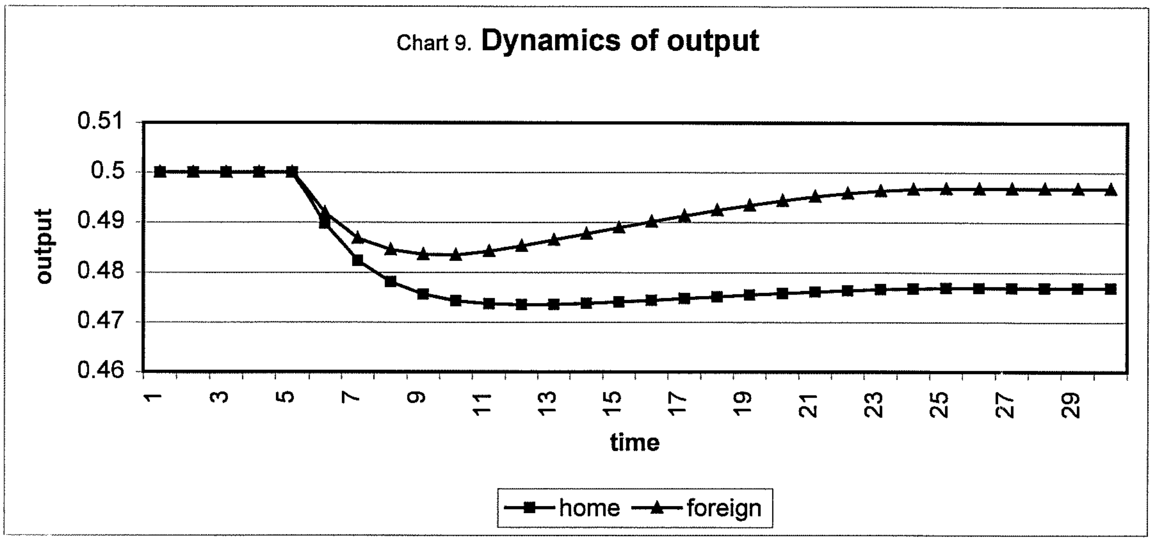


Higher interest rate discourages investment by firms and output declines in home country. Lower savings is followed by a lower capital stock. Chart 8 describes this phenomenon. Comparison is made with the evolution of the capital stock in foreign country. Recall that the horizons of households are longer in the foreign than in the home country. Overtime, the capital stock recovers even if interest rate remains high. To respond to increased demand for its goods, foreign firms invest to expand output.

Chart 8. **Dynamics of capital stock**



As the foreign country runs a trade deficit as a result of an increase in its consumption demand, home output rebounds slightly in the medium and long run as illustrated in Chart 9.



Long Run Steady state.

Table 14 shows that a shorter horizon for households in home country results in higher steady state world interest rate. This discourages investment in both countries, and both have lower steady state capital stock and output. As the home country has the shortest horizon, both capital stock and output are lower in home than in the foreign country.

Table 14. The Long run steady state values of shorter horizons for home country with constant labour income over lifetime. (Numbers in parenthesis are initial values).

	HOME	FOREIGN
GDP	0.477 (0.500)	0.497 (0.500)
Interest rate	0.1682 (0.072)	
Capital Stock	0.0113 (0.0125)	0.0123 (0.0125)

Trade Balance	0.069 (0.000)	-0.069 (0.000)
Pure rate time preference	0.380 (0.371)	0.371 (0.371)
Output prices	0.901 (1.000)	1.000 (1.000)

Foreign country has higher steady state savings. That makes households richer in the long run and results in higher steady state consumption. Foreign country runs steady state trade deficit because price of output in home country is lower, boosting foreign demand for home output.

Table 15. The Long run steady state values shorter horizons for home country with constant labour income over lifetime on consumption and savings levels in home country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.101 (0.150)	0.134 (0.214)	0.178 (0.305)	0.236 (0.435)	0.313 (0.621)
Savings level	0.056 (0.143)	0.061 (0.156)	0.060 (0.143)	0.050 (0.074)	-0.227 (-0.515)

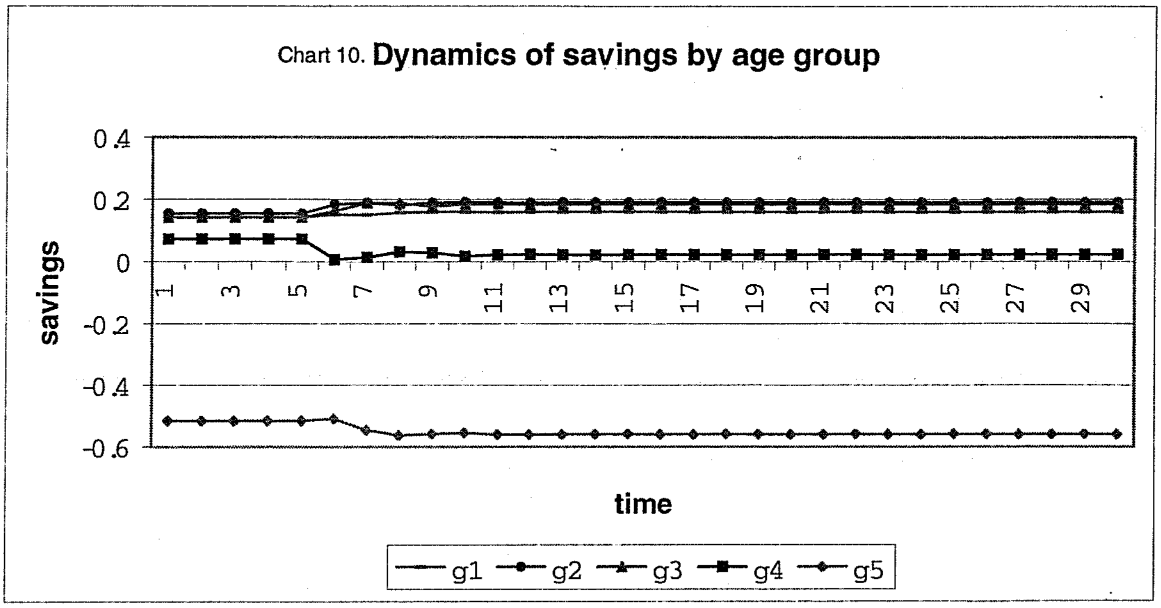
Table 16. The Long run steady state values of shorter horizons for home country with constant labour income over lifetime on consumption and savings levels in foreign country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.181 (0.150)	0.269 (0.214)	0.402 (0.305)	0.599 (0.435)	0.884 (0.621)
Savings level	0.201 (0.143)	0.223 (0.156)	0.205 (0.143)	0.090 (0.074)	-0.720 (-0.515)

The next numerical result assumes that labour income is declining over time. Therefore, we calibrate a numerical steady state where earnings of households are constant over time. The purpose of this is to capture the effect of this income on saving for retirement. The short-run and the long-run movements of variables behave in a similar way. An unanticipated shock introduced on the earnings profile that is not constant, but declining over time, induces households to save more. Consequently, households' marginal labour productivity also increases. As the shock is symmetric, the results can be described for one country only. Increase in savings makes eventually households richer, and demand increases in both countries. In the long run steady state countries benefit from a positive level effort in saving and consumption. If we look at Table 18 and Chart 10, from assumption that labour earning are declining over time, the highest earnings of households come to middle age group people. Households are rational. Therefore we see savings increase in g2 and g3 where g4 and g5 are dissaving. This implies they are saving for retirement. The explanation for that is that if we consider consumption behavior, it increases more in old ages (g4 and g5) than in g2 and g3. This shows the effect of declining labour income on savings, middle age people save to consume when they are old.

Table 17. The Long run steady state values with declining labour income over lifetime. Consumption and savings levels in home and foreign country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.185 (0.150)	0.277 (0.214)	0.414 (0.305)	0.620 (0.435)	0.927 (0.621)
Savings level	0.161 (0.143)	0.191 (0.156)	0.185 (0.143)	0.023 (0.074)	-0.560 (-0.515)



Increased productivity of workers makes stimulate investment, so capital stock also increases as a result of increased savings. Output increases as a result of increase in capital stock and labour productivity. As the shock is symmetric, there is no trade deficit in any of these countries. In steady state both countries enjoy higher levels of output and capital stock. The interest rate rises as investment increases by more than savings.

Table 18. The steady state values with declining labour income over lifetime. (Numbers in parenthesis are initial values).

	HOME	FOREIGN
GDP	0.605 (0.500)	0.605 (0.500)
Interest rate	0.1686 (0.072)	
Capital Stock	0.0147 (0.0125)	0.0147 (0.0125)
Trade Balance	0.000 (0.000)	0.000 (0.000)
Output prices	1.000 (1.000)	1.000 (1.000)

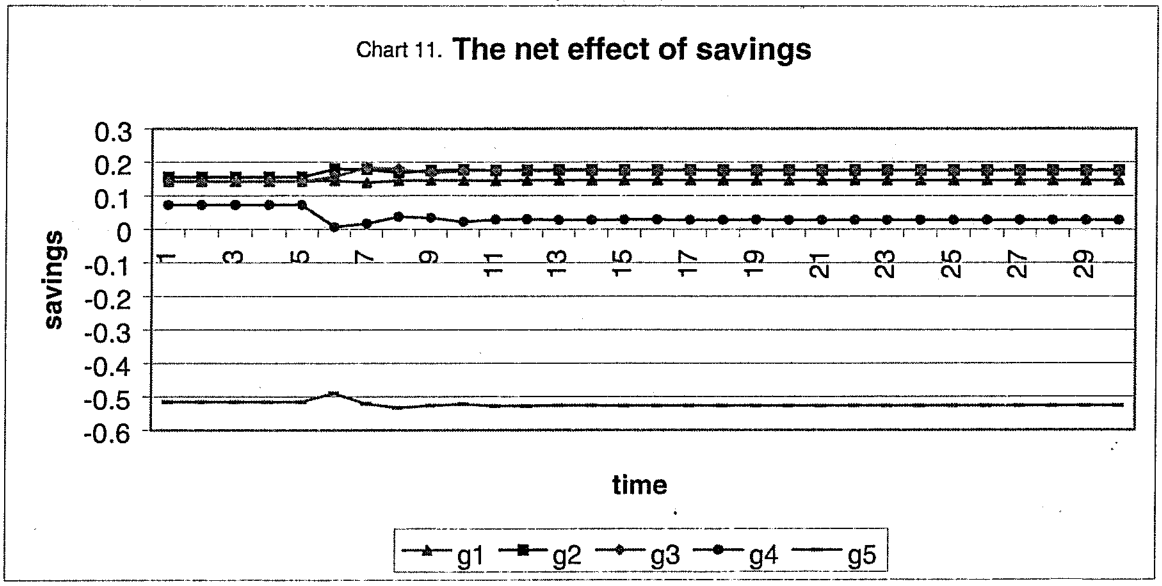
Comparison with Blanchard results.

We now compare our numerical result with Blanchard's theoretical result. He describes two opposite phenomena.

First phenomena: the effect of finite horizons of agents is to decrease capital accumulation; Chart 7 tells us, where households in home country have higher discount rate, savings decline. Households consume more when young and reduce savings. Here the assumption is that labour income is constant over time. The decline in savings follows from the fact that interest rate is an increasing function of probability of death. So an increase implies shorter horizons and pushes the steady-state capital-labour ratio level down.

Second phenomena: the effect of declining labour income is to increase saving for retirement, and that increases the capital-labour ratio. From Chart 10, we see how savings are increasing over time. Savings of old generation stays almost at the same level, but all other generations, who are working actually, begin to save more after the unanticipated shock. Hence, the working age group is saving for retirement. Middle age group people, whose income is the highest during lifetime, react the most with respect to their saving behavior. An assumption of declining labour income hence has positive effect on saving.

But the net effect is ambiguous and the steady state can be inefficient as well. We now simulate the combination of higher discount rate of households for home country with declining labour income over time. In this example, the effect of declining labour on savings is higher, so this effect induces savings to increase, but we see how inclusion of higher discount rate pushes savings down (compare Charts 10 and 11). It also true that if discount rate is high enough, then its effect may overcome, so savings may actually decline.



6.3. Simulation results for Assaf R. and Frenkel J. propositions.

To analyze the effects of budget deficits, we take government spending as given. Thus, deficits result from changes in taxes. We impose a condition that only home country decrease taxes. Dynamic budget constraint of government requires that current changes in taxes be accompanied by offsetting changes in future taxes. This shock is introduced such that home country's government reduces consumption and capital taxes (exogenous taxes) by 15 percent in period 5 for one period of twelve years. Households' expectations are forward looking. They realize that this current reduction in taxes must be followed by future increase. Households try to smooth their consumption over lifetime. Current reduction in taxes makes them wealthier, but because there will be burden of taxes in future periods, they actually save that additional wealth, but not everyone does this. As Table 19 shows, consumption of first four generations do not increase, instead savings increase in the year of shock, which means they anticipate future increase in taxes and save. Regarding to last generation, whose consumption increases as a consequence of reduction in taxes. They benefit from this tax reduction, as they will not be there when taxes will increase again and thus they increase consumption.

Table 19. **The short run consumption and savings levels in home country. (Numbers in parenthesis are initial values).**

	G1	G2	G3	G4	G5
Consumption level	0.138 (0.150)	0.203 (0.214)	0.296 (0.305)	0.433 (0.435)	0.652 (0.621)
Savings level	0.165 (0.144)	0.164 (0.156)	0.150 (0.143)	0.077 (0.074)	-0.557 (-0.515)

As Table 20 shows, the interesting result came out when we prolong the period in the reduction in taxes. Tax reduction now is not for period one, but two or three periods. Therefore the longer the period in reduction for taxes, the less savings of those generations that anticipate that they will not have to bear the tax burden. We see that if the tax reduction is for one period, g4 people will have to bear the tax burden, so, they increase savings. But as soon tax reduction is longer, there is large decline in the savings rate of the g4 age group.

Table 20. Percentage changes in savings for various tax reduction periods.

Reduction for	G1	G2	G3	G4	G5
One year	5.12	5.12	4.89	4.05	-7.96
Two years	19.58	5.12	4.89	0.12	-5.82
Three years	21.67	5.12	0.01	0.13	-5.82

Decrease in taxes affect consumption. Increase in consumption demand raises output price in home country. These all push world interest rate go up. Increased wealth, hence consumption and savings induces firms to invest more, as demand is strong. So the short run capital stock and output is above its initial steady state value. But the change is very small, as households save large amount of increase in wealth, as a result of the reduction in taxes. The increase in consumption and world interest rate tells us that Ricardian Equivalence Theorem fails: deficit financing has effect on real variables in the model.

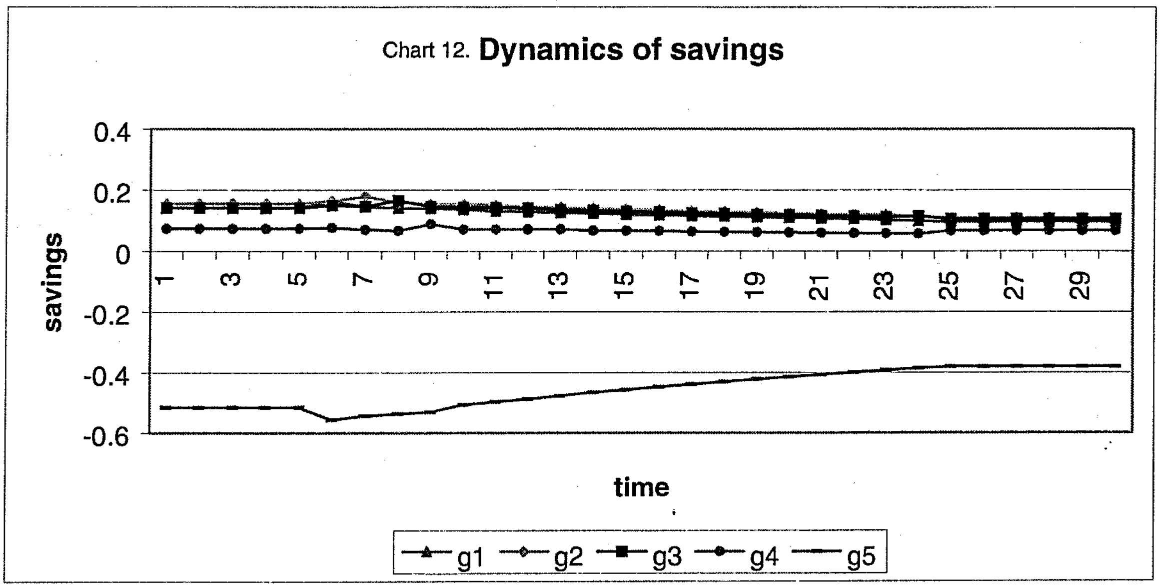
Increase in world interest rate lowers foreign wealth and in the short run foreign country's output is lower, as a consequent of fall in demand. As there is an increase in home demand for foreign output, home country runs short run trade balance deficit. Home country's economy is in temporary boom as a consequence of this tax reduction, whereas foreign country is in recession as a result of rise in world interest rate. The effect of deficit financing in home country is negatively transmitted through world interest to the foreign country.

Table 21. The short-run effects of budget deficits resulting from reduction in home country taxes. (Numbers in parenthesis are initial values).

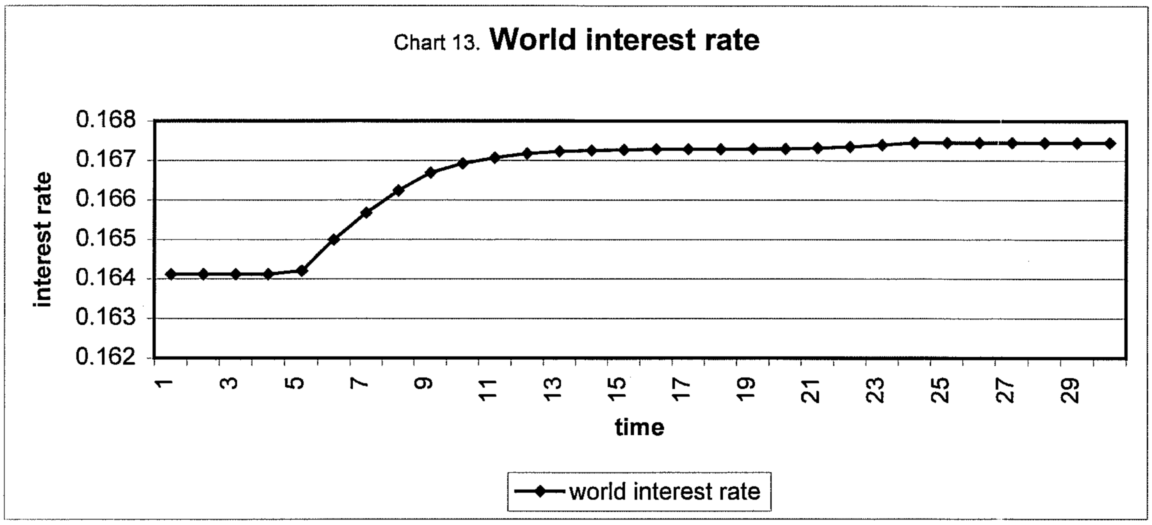
	HOME	FOREIGN
GDP	0.501 (0.500)	0.499 (0.500)
Interest rate	0.1641 (0.072)	
Capital Stock	0.01254 (0.0125)	0.0124 (0.0125)
Trade Balance	-0.0003 (0.000)	0.0003 (0.000)
Output prices	1.0004 (1.000)	1.000 (1.000)

Dynamic adjustment towards the steady state.

Necessary increase in taxes in future periods induces a fall in domestic demand. Consumption and savings, as shown in Chart 12, decline over time as a consequence of the reduction in wealth. Here we also see how savings increase in the year of the shock, as discussed in short run results section.



As demand falls, output decline as well. The fall in demand and the rise in world interest rate discourage firms, hence the capital stock declines. The fall in demand implies lower output prices. Opposite effects occur in foreign country. Home country's increased demand for foreign output in the short run boosts foreign output and investment. This increases foreign wealth, and that boosts consumption and savings in foreign country. All this implies that the foreign country runs trade balance deficit.



Long Run steady state.

Budget deficit arising from one period increase in taxes in home country result in higher world interest rate, shown in Chart 13. That discourages capital accumulation, lower long run steady state capital stock, and hence lower output. But foreign country has higher steady state output as a result of increased demand and savings. And foreign country runs steady state trade balance deficit.

Table 22. The steady state. (Numbers in parenthesis are initial values).

	HOME	FOREIGN
GDP	0.483 (0.500)	0.494 (0.500)
Interest rate	0.1674(0.072)	
Capital Stock	0.0116 (0.0125)	0.0122 (0.0125)
Trade Balance	0.040 (0.000)	-0.040 (0.000)
Output prices	0.942 (1.000)	1.000 (1.000)

Home country ends up with lower long run steady state consumption and savings, whereas foreign country has opposite situation.

Table 23. The Long Run steady state values on consumption and savings levels in home country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.100 (0.150)	0.148 (0.214)	0.220 (0.305)	0.326 (0.435)	0.483 (0.621)
Savings level	0.097 (0.143)	0.109 (0.156)	0.106 (0.143)	0.067 (0.074)	-0.380 (-0.515)

Table 24. The Long run steady state values on consumption and savings levels in foreign country. (Numbers in parenthesis are initial values).

	G1	G2	G3	G4	G5
Consumption level	0.162 (0.150)	0.240 (0.214)	0.355 (0.305)	0.524 (0.435)	0.776 (0.621)
Savings level	0.178 (0.143)	0.197 (0.156)	0.179 (0.143)	0.078 (0.074)	-0.632 (-0.515)

Overall, deficit financing rises interest rate and results in a lower world capital stock and output. Thus the Ricardian Equivalence Theorem fails. Deficit has real impact in the long run.

Comparison with Assaf R. and Frenkel J. results.

If agents' probability of survival is unity (infinite horizons), then budget deficits do not alter interest rate and consumption. That yields Ricardian proposition according to which timing of deficit do not influence real equilibrium of the system. If that probability is less than unity budget deficits exert real effects.

In this numerical approach, horizons of agents is not infinite, therefore from Chart 13, we see increase in world interest rate. The consumption in home country increases at least for some generations. In long run steady state, interest rate is higher and world output is lower, as a result of lower capital accumulation.

7. Conclusion.

A numerical version of overlapping generations model is used to test theoretical hypothesis of three classical papers. The comparison is made to propositions of Buiter (1981), Blanchard (1985) and Assaf and Frenkel (1992). Result show that when two countries are identical except for pure rate of time preference, the one with higher preference end up with lower capital accumulation level and output than the other country. When households' earnings are constant over lifetime, higher discount rate implies less capital accumulation, whereas earnings are decreasing over time that induces households to increase savings for retirement purpose. Also when a government deficit financing occurred as a result of temporary drop in taxes, short horizons of households have impact on the long run steady state equilibrium. When horizons are not infinite, households respond to the decline in taxes, by increasing consumption and that pushes interest rate up. In the long run output declines. But, when households expect the burden of tax fall on them, they actually save more as to repay that debt in the future. In this paper we have developed a computable two-country overlapping generations that is able to simulate the theoretical experiments of Buiter, Blanchard, Assaf and Frenkel. The numerical experiment confirmed the theoretical findings of these papers.

We believe that our modeling efforts are useful for further experiments on these important issues. The numerical model can also be used for pedagogical purposes.

References:

Auerbach, Alan J. and Lawrence J. Kotlikoff (1987), *Dynamic Fiscal Policy,* Cambridge: Cambridge University Press.

Jacob A. Frenkel and Assaf Razin (1992), *Fiscal Policies and the World Economy*, Second edition, Cambridge, Massachusetts, MIT Press.

Mérette M. and Mercenier J. (2002) "Will population ageing increase inequality across regions in Canada?" Report for HRDC (Human Resource Development Canada).

Olivier J. Blanchard (1985), "Debt, Deficits and Finite Horizons" The Journal of Political Economy, 1985, Vol. 93, no. 2, pp 223-247.

Willem H. Buiter (1981), "Time preference and international lending and borrowing in an overlapping-generations model", The Journal of Political Economy, 1981, Vol. 89, no. 41, pp 769-797.

ANNEX

```
*****************************
* accompagne la
* -------------
* Version corrigee le 9-8-2001 (introduction d'une structure fictive de portefeuille d'actifs)
*
*          et reintroduction des pensions, bequests, inheri
* 9-8-2001 17h
*
* et introduction des taxes sur le capital  ( 10-8-2001, 11H20 )
* Introduction des depense publiques de sante et education  (14-08-2001)
* Introduction du programme de pensions finance par repartition (15-08-2001)
* Version identique a la version cal-6g3 mais pour 15 generations
*
*****************************
$OFFSYMXREF OFFSYMLIST OFFUELLIST OFFUELXREF
OPTION DECIMALS=6;
 SETS     G     GENERATIONS          / G1 * G5 /
        GI(G)    FIRST GENERATION
       GJ(G)    GENERATIONS WORKING  / G1 * G4 /
       GM(G)    RETIRED GENERATIONS  /  G5 /
       GN(G)    LAST GENERATION
;
 GN(G) = YES$(ORD(G) EQ CARD(G));
 GI(G) = YES$(ORD(G) EQ(1));
 SET J / home , foreign  /;
 ALIAS (I,J), (I,II), (J,JJ), (G,GG);
Scalars
 SIGC    inter-temporal rate of substitution
 RR      final interest rate
;
 SIGC = 1.1*2;
 RR = 0.3231*3 ;
*RR represents 7.25% rate per year
 PARAMETERS
 ALFA(J)    production function parameter
 B (J)      net government debt
 WW(J)      final wage rate
 WTAX(J)    wage-income tax rate
 KTAX(J)    capital tax
 CTAX(J)    consumption tax
 NC1(J,G)   population
 HAC(J,G)   financial assets accumulated
```

```
 BEC(J,G)    bequest parameter
 IDC(G)      inheritance distribution
 EP9(G)      human capital profile
 BR          benefit ratio
 NGR(J)      population growth rate (is determined in the demog.gms file when the
*             latter is used; otherwise is fixed at 1 or 1.156)
 FA(J)       foreign assets (must sum to zero)
 GHEA(J)
 GEDU(J)

;

  ALFA("home") = 0.50; ALFA("foreign") = 0.50;
* ALFA("WE") = 0.302;
  WTAX("home") = 0.362; WTAX("foreign") =0.362;
*WTAX("WE")= 0.308;
  KTAX("home") = 0.352; KTAX("foreign") = 0.352;
*KTAX("WE") = 0.412;
  CTAX("home") = 0.216; CTAX("foreign") = 0.216;
*CTAX("WE") = 0.177;
  BEC(J,G) = 0;  BEC(J,"G5")=.3;
  IDC(G) =1/4; IDC(GM)=0;
  EP9(G)  =1+.237*ORD(G)-.038*(ORD(G)**2);
  BR      =.3216*EP9("G3");
*  NC(J,G) = Pop(J,"T6",G) ;
  NGR(J) =1.156;
  FA("home") = .0000; FA("foreign") = .0000;
*FA("WE") = .0604;
*These numbers imply Net Exports equal to -0.0112 for EA, .03072 for CN, and -.
01952 for WE

*  KTAX(J) = 0;
*  BEC(J,G) = 0;
*  IDC(G)=0;
*  BR =0;
  NGR(J) =1.0;
*  FA(J) =0;
   GHEA("home")=.0191; GHEA("foreign")=.0191;
*GHEA("WE")=.0178;
   GEDU("home")=.0150; GEDU("foreign")=.0150;
*GEDU("WE")=.0129;
*    GHEA(J)=0;GEDU(J)=0;


PARAMETER DH(g) distribution parameter of health care by age;
    DH(GI)=.1096; DH("G2")=.1213;DH("G3")=.1399;DH("G4")=.2043;DH("G5")=.4249;

PARAMETER DE(G) distribution parameter of health care by age;
    DE(GI)=.330; DE("G2")=.360;DE("G3")=.190;DE("G4")=.10;DE("G5")=.065;


 POSITIVE VARIABLES

 RC(J)      rental cost of capital
 LL(J)      labour supply
 KK(J)      capital stock
 YY(J)      output
```

```
 CC(J,G)    consumption level
 WC(J)      initial wage rate
 BEQC(J,G)  bequest
 INHC(J,G)  inheritance
 TT(J)      tax income
 AA(J)      scalar in the production function
 GEXP(J)    government expenditure
 NNJ(J)     population level adjustment (useful to normalize popul.
*           when demog.gms is used; otherwise fix to 1)
 NC(J,G)    population
 HEAC(J,G)
 EDUC(J,G)
 CTRC

Free variables

 DEP(J)        physical depreciation rare of capital
 DELTAC(J)     consumers rate of time preference
 SC(J,G)       private savings
 HA(J,G)       households assets
 DIC(J,G)      after tax income
 WAL0(J)       walras variable
 OBJ

EQUATIONS

 EL1(J)
 EL2(J,G)

 E1(J)
 E2(J)
 E3(J)

 EC1(J,G)
 EC2(J,G)
 EC3(J,G)
 EC4(J,G)
 EC5(J,G)
 EC6(J,G)
 EC7(J,G)
 EC8(J,G)

 EE1(J)
 EE2(J)

 EDH1(J,G)
 EDH2(J)
 EDE1(J,G)
 EDE2(J)
 EG1(J)
 EG2(J)
 EG3

 OBJeq
;

*     POPULATION CALIBRATION
```

```

 EL1(J)..          LL(J)       =E= SUM(GJ, NNJ(J)*NC(J,GJ)*EP9(GJ) )
;
 EL2(J,G+1)..    NC(J,G+1) =E= NC(J,G) / NGR(J)
;

*      FIRM PROBLEM

 E1(J)..           WC(J) =E= (1-ALFA(J))*AA(J)*(KK(J)/LL(J))**ALFA(J)
;
 E2(J)..           RC(J) =E=    ALFA(J) *AA(J)*(LL(J)/KK(J))**(1-ALFA(J))
;
 E3(J)..           YY(J) =E=   AA(J)* KK(J)**ALFA(J)* LL(J)**(1-ALFA(J))
;

*      CONSUMER PROBLEM

 EC1(J,G+1)..    CC(J,G+1) =E= ( (1+RR*(1-KTAX(J))) / (1+DELTAC(J)) )**SIGC *CC(J,G)
;
 EC2(J,GJ)..     DIC(J,GJ) =E= WW(J)*EP9(GJ)*(1-WTAX(J)-CTRC) + RR*HA(J,GJ)*(1-KTAX(J)) - CTAX(J)*CC(J,GJ)
;
 EC3(J,GM)..     DIC(J,GM) =E= WW(J)*BR*(1-WTAX(J))      + RR*HA(J,GM)*(1-KTAX(J)) - CTAX(J)*CC(J,GM)
;
 EC4(J,G)..      SC(J,G)    =E= DIC(J,G) + INHC(J,G) - CC(J,G) - BEQC(J,G)
;
 EC5(J,GN)..     SC(J,GN)   =E= -HA(J,GN)
;
 EC6(J,G+1)..    HA(J,G+1) =E= HA(J,G) + SC(J,G)
;
 EC7(J,G)..      BEQC(J,G) =E= BEC(J,G)*CC(J,G)
;
 EC8(J,G)..      INHC(J,G) =E= IDC(G)*SUM(GM, NC1(J,GM)*BEQC(J,GM) ) / NC1(J,G)
;

* AGGREGATION AND EQUILIBRIUM CONDITIONS

 EE1(J)..        KK(J)+ FA(J)    =E= SUM(G, NC1(J,G)*HA(J,G) ) - B(J)
;
 EE2(J)..        WAL0(J) =E= YY(J) -(NGR(J)-1+DEP(J))*KK(J)-(GEXP(J)+GHEA(J)+GEDU(J))
         - SUM(G, NC1(J,G)*CC(J,G) )- RR* (B(J)+KK(J)-SUM(G,NC1(J,G)*HA(J,G)))

;

* GOVERNMENT

 EDH1(J,G).. HEAC(J,G+1) =e= (DH(G+1)/DH(G))*HEAC(J,G)
;
 EDH2(J)..     SUM(G,NC1(J,G)*HEAC(J,G)) =e= GHEA(J)
;
 EDE1(J,G).. EDUC(J,G+1) =e= (DE(G+1)/DE(G))*EDUC(J,G)
;
 EDE2(J)..     SUM(G,NC1(J,G)*EDUC(J,G)) =e= GEDU(J)
```

```
;
 EG1(J)..       TT(J) =E= SUM(GJ, NC1(J,GJ)*EP9(GJ)*WW(J)*WTAX(J) )+
                          SUM(GM, NC1(J,GM)*BR    *WW(J)*WTAX(J) )+
                          SUM(G,  NC1(J,G) *RR    *HA(J,G)*KTAX(J) )+
                          SUM(G,  NC1(J,G) *CC(J,G) *CTAX(J) )
;
 EG2(J)..      GEXP(J)+GHEA(J)+GEDU(J) =E= TT(J) - (RR-(NGR(J)-1))*B(J)
;
 EG3..        SUM((J,GM),NC1(J,GM)*BR*WW(J)) =E= CTRC*SUM(J,LL(J)*WW(J))
;
 OBJeq..        OBJ =E= 0
;
*  Restrictions
HA.FX(J,GI)=  0.0;
 YY.FX("home")=0.500; YY.FX("foreign")=0.500;
* YY.FX("WE")=0.302;
 YY.FX(J) =YY.L(J);
 KK.FX(J) = YY.L(J)*0.3;
* KK.FX("EA")=YY.L("EA")*0.75; KK.FX("CN")=YY.L("CN")*0.9; KK.FX("WE")=YY.L("WE")*0.5;
 LL.FX(J)=   YY.L(J)*1;
 B("home")=YY.L("foreign")*0.05; B("foreign")=YY.L("foreign")*0.05;
*B("WE")=YY.L("WE")*0.2069;
*Federal debt is assumed at 74% GDP and has been distributed in proportion to regional GDP
*This will permit to simulate the recent debt reduction policy
*  Initial guesses
 NNJ.FX(J)=1;
* NGR.l(J)  =  NNGR(J,"T6") ;
 NC.L(J,G)  = 1;
 RC.L(J) = .12;
 LOOP(GJ, HA.L(J,GJ+1) = HA.L(J,GJ)+.4);  HA.L(J,"G3") = .5*HA.L(J,"G2");
 LOOP(GM, HA.L(J,GM+1) =.5*HA.L(J,GM));
 DELTAC.L(J) = .11;
 WC.L(J)     = .7;
 CC.L(J,GI) = .1; LOOP(G,CC.L(J,G+1)= ((1+RR)/(1+DELTAC.L(J)))**SIGC*CC.L(J,G));
 BEQC.L(J,G) = 0; BEQC.L(J,GN)= .5*CC.L(J,GN);
 INHC.L(J,G) = IDC(G)*NC.L(J,"G5")*BEQC.L(J,"G5")/NC.L(J,G);
 DIC.L(J,GJ) =    WC.L(J)*(1-WTAX(J))+RR*HA.L(J,GJ)*(1-KTAX(J)) -
                  CC.L(J,GJ)*CTAX(J) + INHC.L(J,GJ);
 DIC.L(J,GM) = BR*WC.L(J)*(1-WTAX(J))+RR*HA.L(J,GM)*KTAX(J) - CC.L(J,GM)*CTAX(J)
             - BEQC.L(J,GM);
 SC.L(J,G)   = DIC.L(J,G)-CC.L(J,G);
 TT.L(J)      = 0.17;
 GEXP.L(J)    = 0.17;
 AA.L(J)      = 1.25;
 WAL0.FX(J)     = 0;
 CTRC.L       = .10;
*The model CA1 serves to determines demographic and production parameters and prices
 MODEL CA1 /
```

```
               E1,E2,E3,EL1,EL2, OBJeq /
;
 OPTIONS SOLPRINT=OFF, LIMCOL=0, LIMROW=0;
 SOLVE CA1 USING NLP MINIMIZING OBJ;

* THROW FACTOR PRICES DETERMINED BY CA1 INTO CONSUMER PROBLEM
* FIX POPULATION STRUCTURE AS DETERMINED BY CA1

 NC1(J,G) = NNJ.L(J)*NC.L(J,G);
 WW(J)     = WC.L(J);
 DEP.FX(J) = RC.L(J)-RR ;

 DISPLAY WW,NC1,DEP.L,AA.L;

 MODEL CA2 /
               EC1,EC2,EC3,EC4,EC5,EC6,EC7,EC8,EE1,EG1,EG2,EG3,EE2
     OBJeq /
;

 MODEL CA3 / EDH1,EDH2,EDE1,EDE2,OBJeq/
;

  CA2.HOLDFIXED = 1;
 OPTIONS SOLPRINT=OFF, LIMCOL=0, LIMROW=0, DECIMALS=6;
 SOLVE CA2 USING NLP MINIMIZING OBJ;

 CA3.HOLDFIXED = 1;
 OPTIONS SOLPRINT=OFF, LIMCOL=0, LIMROW=0, DECIMALS=6;
 SOLVE CA3 USING NLP MINIMIZING OBJ;

 PARAMETER
 DEF(J)       DEFICIT-GDP RATIO
 TR(J)        TRANSFER TO THE OLD
 BX(J)        BEQUEST CHECK
 PSR(J)       PRIVATE SAVING RATE
 NS(J)        NATIONAL SAVING RATE
 SA(J,G)      SAVING BY AGE
 CCI(I,J,G)   INTER-REGIONAL CONSUMPTION
 INVI(I,J)    INTER-REGIONAL INVESTMENT
 NX(J)        NET EXPORTS
 WALOX(J)     GOODS MARKET BALANCE
;

 DEF(J)   = (NGR(J)-1)*B(J);
 TR(J)    = SUM(GM,NC1(J,GM))*WW(J)*BR;
 BX(J)    = SUM(G,BEQC.L(J,G)*NC1(J,G))-SUM(G,INHC.L(J,G)*NC1(J,G));
 PSR(J)   = (SUM(G,NC1(J,G)*SC.L(J,G))+DEP.L(J)*KK.L(J))/YY.L(J);
 NS(J)    = (YY.L(J)-SUM(G,CC.L(J,G)*NC1(J,G))-(GEXP.L(J)+GHEA(J)+GEDU(J))
                          +RR*FA(J))/YY.L(J);
 SA(J,G)= (SC.L(J,G)+BEQC.L(J,G))/(DIC.L(J,G)+INHC.L(J,G));

*SMALL MODEL TO CALIBRATE TRADE FLOWS MATRIX BETWEEN REGIONS

VARIABLE
E9(I,J)  TRADE FLOW MATRIX BETWEEN REGION I AND J
;

```

```
EQUATIONS
EXeq1(J)
EXeq2(J)
;
EXeq1(I)..
          SUM(J,E9(I,J)) =E= YY.L(I)
;
EXeq2(I)..
          SUM(J,E9(I,J)) - SUM (J,E9(J,I)) =E= -RR*FA(I)
;
MODEL EXMOD /EXeq1, EXeq2, OBJeq/;
*INITIALISATION
E9.L(I,J) = .1*( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) ) ;
*E9.LO(I,J) = .02*( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) ) ;
E9.L(J,J) = ( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) +
                 (GEXP.L(J)+GHEA(J)+GEDU(J)) - RR*FA(J) ) ;
E9.FX("home","home")=.878*E9.L("home","home");
E9.L("foreign","foreign")=.915*E9.L("foreign","foreign");
*E9.FX("WE","WE")=.902*E9.L("WE","WE");
E9.L("home","foreign")=.0737*E9.L("home","home");
*E9.FX("WE","EA")=.0243*E9.L("EA","EA");
*E9.FX("CN","WE")=.080*E9.L("WE","WE");
*E9.FX("WE","CN")=.043*E9.L("WE","WE");
OPTION ITERLIM=1000;
SOLVE EXMOD MAXIMIZING OBJ USING NLP ;
DISPLAY E9.L;
E9.L(I,I)=E9.L(I,I)-(GEXP.L(I)+GHEA(I)+GEDU(I)); DISPLAY E9.L;
CCI(I,J,G) = E9.L(I,J)*CC.L(J,G)/
          ( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) );
INVI(I,J) = E9.L(I,J)*(NGR(J)-1+DEP.L(J))*KK.L(J)/
          ( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) );
E9.L(I,J) = SUM(G,NC1(J,G)*CCI(I,J,G))+INVI(I,J); DISPLAY E9.L;
E9.L(I,I)=E9.L(I,I)+(GEXP.L(I)+GHEA(I)+GEDU(I)); DISPLAY E9.L;
 NX(J)    = SUM((G,I), NC1(I,G)*CCI(J,I,G))+SUM(I,INVI(J,I)) -
            SUM((G,I), NC1(J,G)*CCI(I,J,G))-SUM(I,INVI(I,J))
;
 WALOX(J)= YY.L(J)-SUM((I,G), NC1(I,G)*CCI(J,I,G) ) -(GEXP.L(J)+GHEA(J)+GEDU(J))
              -  SUM(I, INVI(J,I) ) ;
DISPLAY DEF,TR,BX,PSR,NS,SA,NX,WALOX,CCI,DELTAC.L,EP9,CTRC.L,GEXP.L,GHEA,GEDU ;
```

```
*****************************
*
* Version corrigee le 9-8-2001 (introduction d'une structure fictive de portefeuille d'actifs)
*
*          et reintroduction des pensions, bequests, inheri
* 9-8-2001 17h
*
* et introduction des taxes sur le capital  ( 10-8-2001, 11H20 )
* Introduction des depenses publiques de sante et d'education (14-08-2001)
* Introduction du programme de pension finance par repartition (15-08-2001)
* Version testee avec chocs de productivites
*
*****************************

*This file is used to generate the initial equilibrium over the entire horizon
*Can also be used to execute the test of the numeraire and productivity shocks

*$include cal-6g.gms

*$include cal-6g.gms or cal-6gS for symmetric region
*Use in the initialization below DiscR(J)= DELTAC.L(J) for asymmetric region version or
*DiscR(J)= DELTAC.L for the ymmetric version of the model ;

 SET TTP       TOTAL TIME HORIZON                           /T1  * T30 /
     TP(TTP)   PERIODS OF PREVIOUSLY BORN                   /T1  * T4  /
     TP1(TTP)  PERIODS OF POST STEADY-STATE BIRTH           /T26 * T30 /

     T(TTP)     PERIODS OF ENDOGENOUS BIRTH                 /T5 * T25 /
     TI(T)      FIRST PERIOD OF ENDOGENOUS BIRTH
     TN(T)      LAST TWO PERIODS OF ENDOGENOUS BIRTH        /T24 , T25 /

;
   TI(T) = YES$(ORD(T) EQ (1));

*+++++++++++
 PARAMETERS
*+++++++++++

 RkG(G)
*--->Producers J
 AlQ(J,TTP)
 AlK(J)
*--->Households J
 Sig(J)
 DiscR(J)
 AlConI(I,J)
 SigCon(J)
 BeqR(J,G)
 InhR(J,G)
 NN(J,TTP)
 TPOP(J,TTP)
 NNP(J,TTP)
 Pop(J,TTP,G)
 EP(J,G)
 BSh
```

```
 KSh(I)
 Bij0(I,J)
 DepR(J)
 AlInv(I,J)
 SigInv(J)
*--->Government J
 KTxR(J,TTP)
 ConTxR(J,TTP)
 PensR(J)
 ExoBonds(J,TTP)       Choose tax (1) vs Bond (0) financed deficits
 GovH(J,TTP)
 GovE(J,TTP)
;


*++++++++++
 VARIABLES
*++++++++++
*--->Producers J
 Ldem(J,TTP)
 Kdem(J,TTP)
 Q(J,TTP)
*--->Households J
 Lend(J,TTP,G)
 K(J,TTP,G)
 BondD(J,TTP,G)
 Bij(I,J,TTP,G)
 Con(J,TTP,G)
 Pcon(J,TTP)
 ConI(I,J,TTP,G)
 Beq(J,TTP,G)
 Inh(J,TTP,G)
 Pens(J,TTP,G)
 ECon(I,J,TTP)
 Investor J
 Inv(J,TTP)
 PInv(J,TTP)
 EInv(I,J,TTP)
 RRET(J,TTP)
*--->Government J
 Bond(J,TTP)
 Gov(J,TTP)
 WTxR(J,TTP)
 CTR(TTP)
*--->Markets
 Wage(J,TTP)
 Kstock(J,TTP)
 Rent(J,TTP)
 P(J,TTP)
 Rint(TTP)
 RintJ(J,TTP)

 EXC(J,TTP)  EXCESS WALRAS VARIABLE FOR THE CASE IN WHICH POPUL. GROWTH IS
*            DIFFERENT ACROSS REGION AT THE INITIAL EQUILIBRIUM
;
 RkG(G) = ORD(G);

```

```
*++++++++++
 EQUATIONS
*++++++++++
*--->Producers J
 WageEq(J,TTP)
 RentEq(J,TTP)
 QEq(J,TTP)
*--->Household J
 HBudgEq1(J,TTP,G)
 HBudgEq2(J,TTP,G)
 BeqEq(J,TTP,G)
 InhEq(J,TTP,G)
 PensEq(J,TTP,G)
 ConEq(J,TTP,G)
 ConSSEq(J,TTP,G)
 PConEq(J,TTP)
 ConIEq(I,J,TTP,G)
 EConEq(I,J,TTP)
 BijEQ(I,J,TTP,G)
 KEq(J,TTP,G)
 RRetKeq(J,TTP)
 InvEq(J,TTP)
 PInvEq(J,TTP)
 EInvEq(I,J,TTP)
 KstockEq(J,TTP)
 KstockSSEq(J,TTP)
*--->Government J
 GBudgEq1(J,TTP)
 GBudgSSEq(J,TTP)
 GBudgEq2(J,TTP)
 GBudgEq3(J,TTP)
 GPENS(TTP)
*--->Markets
 PEq(J,TTP)
 RintEq(J,TTP)
 RintJIntEq(TTP)
;

*++++++++
* MODEL
*++++++++

*============
* Producers J
*============

* Labor
 WageEq(J,TTP)      $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 Wage(J,TTP)/P(J,TTP) =E= (1-AlK(J))*AlQ(J,TTP)*(Kstock(J,TTP)/SUM(G,Pop(J,TTP,G)*EP(J,G)))**AlK(J)
;
* Capital
 RentEq(J,TTP)      $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 Rent(J,TTP)/P(J,TTP) =E= AlK(J)*AlQ(J,TTP)*(Kstock(J,TTP)/SUM(G,Pop(J,TTP,G)*EP(J,G)))**(AlK(J)-1)
;
* Output
```

```
QEq(J,TTP)          $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
Q(J,TTP) =E= AlQ(J,TTP)*Kstock(J,TTP)**AlK(J)*SUM(G,Pop(J,TTP,G)*EP(J,G))**(1-AlK(J))
;
*=============
* Household J
*=============

* Budget constraint
HBudgEq1(J,TTP+1,G+1)   $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
(1+ConTxR(J,TTP))*Pcon(J,TTP)*Con(J,TTP,G)+Lend(J,TTP+1,G+1)
=E=
  (1-WTxR(J,TTP)-CTR(TTP))*Wage(J,TTP)*EP(J,G) + (
                  SUM(I,  (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)   )*P(I,TTP-1)*Bij(I,J,TTP,G) )
  - KTxR(J,TTP)*SUM(I,  (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)-1)*P(I,TTP-1)*Bij(I,J,TTP,G) )
  +                RRET(J,TTP)    *PInv(J,TTP-1)*K(J,TTP,G)
  - KTxR(J,TTP)*(RRET(J,TTP)-1)*PInv(J,TTP-1)*K(J,TTP,G)
                                                 )$(ORD(G) GT 1)
 +
  (1-WTxR(J,TTP))*Pens(J,TTP,G)+Inh(J,TTP,G)-Beq(J,TTP,G)
;
HBudgEq2(J,TTP,GN) $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..

(1+ConTxR(J,TTP))*Pcon(J,TTP)*Con(J,TTP,GN)
=E=
  (1-WTxR(J,TTP)-CTR(TTP))*Wage(J,TTP)*EP(J,GN) +
                 SUM(I,   (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)   )* P(I,TTP-1)*Bij(I,J,TTP,GN) )
 - KTxR(J,TTP)*SUM(I,   (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)-1)* P(I,TTP-1)*Bij(I,J,TTP,GN) )
 +                RRET(J,TTP)    *PInv(J,TTP-1)*K(J,TTP,GN)
 - KTxR(J,TTP)*(RRET(J,TTP)-1)*PInv(J,TTP-1)*K(J,TTP,GN)
 +
  (1-WTxR(J,TTP))*Pens(J,TTP,GN)+Inh(J,TTP,GN)-Beq(J,TTP,GN)
;
* Bequests
BeqEq(J,TTP,G)          $(( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                            CARD(T) ) AND BeqR(J,G) NE 0)..
Beq(J,TTP,G) =E= BeqR(J,G)*Pcon(J,TTP)*Con(J,TTP,G)
;
* Inheritance
InhEq(J,TTP,G)          $(( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                            CARD(T) ) AND InhR(J,G) NE 0)..
Pop(J,TTP,G)*Inh(J,TTP,G) =E= InhR(J,G)*SUM(GM,Pop(J,TTP,GM)*Beq(J,TTP,GM))
;
* Pensions
PensEq(J,TTP,GM)        $(( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                            CARD(T) ) AND PensR(J) NE 0)..
Pens(J,TTP,GM) =E= PensR(J)*SUM(GJ, Wage(J,TTP-(ORD(GM)-1+ORD(GJ)))   )/CARD(GJ)
;
* Consumption
ConEq(J,TTP+1,G+1) $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
Con(J,TTP+1,G+1)/Con(J,TTP,G) =E=
```

```
(
( ( 1 + (1 - KTxR(J,TTP)) * (RRET(J,TTP)-1) ) * PCon(J,TTP)    ) /
( ( 1 + DiscR(J)                                ) * PCon(J,TTP+1) )
)**Sig(J)
;
ConSSEq(J,TTP+1,G)    $( ORD(TTP) EQ CARD(TP)+CARD(T)-1 AND ORD(G) LT CARD(G))..
Con(J,TTP+1,G) =E= Con(J,TTP,G)
;
* Price of consumption
PConEq(J,TTP)        $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
PCon(J,TTP)**(1-SigCon(J)) =E= SUM(I$(AlConI(I,J) GT 1.E-13),
                               AlConI(I,J)*P(I,TTP)**(1-SigCon(J)))
;
* Composition of consumption (countries I of origin)
ConIEq(I,J,TTP,G)     $( (ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T))
                         AND AlConI(I,J) GT 1.E-13)..
ConI(I,J,TTP,G) =E= AlConI(I,J)*(PCon(J,TTP)/P(I,TTP))**SigCon(J)*Con(J,TTP,G)
;
EConEq(I,J,TTP)      $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
ECon(I,J,TTP) =E= SUM(G,Pop(J,TTP,G)*ConI(I,J,TTP,G))
;
* Holding of bonds
BijEq(I,J,TTP+1,G+1) $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
P(I,TTP)*Bij(I,J,TTP+1,G+1) =E= Bij0(I,J)/SUM(II,Bij0(II,J))*
                                (Lend(J,TTP+1,G+1)-PInv(J,TTP)*K(J,TTP+1,G+1))
;
* Holding of physical capital
KEq(J,TTP+1,G+1)        $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT
                           CARD(TP)+CARD(T) )..
K(J,TTP+1,G+1) =E= ( Kstock(J,TTP+1)*        Lend(J,TTP+1,G+1))/
                   SUM(GG,Pop(J,TTP+1,GG+1)*Lend(J,TTP+1,GG+1))
;
* Rate of return on capital
RRETKeq(J,TTP)       $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
RRET(J,TTP) =E= (Rent(J,TTP)+(1-DepR(J))*PInv(J,TTP))/PInv(J,TTP-1)
;
* Investment
InvEq(J,TTP+1)       $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
RintJ(J,TTP)*P(J,TTP+1)/P(J,TTP) =E= RRET(J,TTP+1)
;
* Price of investment
PInvEq(J,TTP)       $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
PInv(J,TTP)**(1-SigInv(J)) =E= SUM(I$(AlInv(I,J) GT 1.E-13),AlInv(I,J)*P(I,TTP)*
*(1-SigInv(J)))
;
* Composition of investment (countries I of origin)
EInvEq(I,J,TTP)   $( (ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T)) AND
                     AlInv(I,J) GT 1.E-13 )..
EInv(I,J,TTP) =E= AlInv(I,J)*(PInv(J,TTP)/P(I,TTP))**SigInv(J)*Inv(J,TTP)
;
KstockEq(J,TTP+1)   $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
Kstock(J,TTP+1) =E= Inv(J,TTP)+(1-DepR(J))*Kstock(J,TTP)
;
KstockSSEq(J,TTP)        $( ORD(TTP) EQ CARD(TP)+CARD(T) )..
Inv(J,TTP) =E=  ((NN(J,TTP)-1)+DepR(J))*Kstock(J,TTP)
;
*=============
```

```
* Government J
*==============

* Budget constraint
 GBudgEq1(J,TTP+1)  $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) ) ..
 P(J,TTP)*Bond(J,TTP+1) + SUM(G,POP(J,TTP,G)*(
     WTxR(J,TTP)*Wage(J,TTP)*EP(J,G)
  +  WTxR(J,TTP)*Pens(J,TTP,G) + ConTxR(J,TTP)*Pcon(J,TTP)*Con(J,TTP,G)
  + KTxR(J,TTP)*SUM(I, (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)-1)*P(I,TTP-1)*Bij(I,J,
TTP,G) )
  + KTxR(J,TTP)*(RRET(J,TTP)-1)*PInv(J,TTP-1)*K(J,TTP,G)
                                                  ))
 =E=
 P(J,TTP)*( Gov(J,TTP) + GovH(J,TTP) + GovE(J,TTP) )
  +(RintJ(J,TTP-1)*P(J,TTP)/P(J,TTP-1))*P(J,TTP-1)*Bond(J,TTP)
;
 GBudgSSEq(J,TTP)         $( ORD(TTP) EQ CARD(TP)+CARD(T) ) ..
  NN(J,TTP)*P(J,TTP)*Bond(J,TTP)  + SUM(G,POP(J,TTP,G)*(
    WTxR(J,TTP)*Wage(J,TTP)*EP(J,G)
  + WTxR(J,TTP)*Pens(J,TTP,G) + ConTxR(J,TTP)*Pcon(J,TTP)*Con(J,TTP,G)
  + KTxR(J,TTP)*SUM(I, (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)-1)*P(I,TTP-1)*Bij(I,J,
TTP,G) )
  + KTxR(J,TTP)*(RRET(J,TTP)-1)*PInv(J,TTP-1)*K(J,TTP,G)


))
 =E=
 P(J,TTP)*( Gov(J,TTP) + GovH(J,TTP) + GovE(J,TTP) )
 +(RintJ(J,TTP-1)*
                     P(J,TTP)/P(J,TTP-1))*P(J,TTP-1)*Bond(J,TTP)
;
 GBudgEq2(J,TTP+1)        $( (ORD(TTP) GT CARD(TP) AND ORD(TTP) LT
                           CARD(TP)+CARD(T)) AND ExoBonds(J,TTP+1) EQ 1 )..
 Bond(J,TTP+1) =E= NNP(J,TTP)*Bond(J,TTP)
;
 GBudgEq3(J,TTP)          $( (ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                            CARD(T)) AND ExoBonds(J,TTP+1) EQ 0 )..
 WTxR(J,TTP) =E= WTxR(J,TTP-1)
;
 GPENS(TTP)   $ ( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                            CARD(T))..
 SUM((J,GM),Pop(J,TTP,GM)*Pens(J,TTP,GM)) =E=
                      CTR(TTP)*SUM((J,G),Pop(J,TTP,G)*EP(J,G)*Wage(J,TTP))
;
*=========
* Markets
*=========
* Goods
 PEq(J,TTP)          $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 Q(J,TTP) =E= SUM(I, ECon(J,I,TTP) + EInv(J,I,TTP) ) +
       ( Gov(J,TTP) + GovH(J,TTP) + GovE(J,TTP) ) + EXC(J,TTP)
;

* Integrated asset markets
 RintEq(J,TTP+1)    $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
 Rint(TTP) =E= RintJ(J,TTP)*P(J,TTP+1)/P(J,TTP)
```

```
;
 RintJIntEq(TTP+1)  $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
 SUM((J,G),Pop(J,TTP+1,G+1)*LEND(J,TTP+1,G+1)) =E=
 SUM(J,P(J,TTP)*Bond(J,TTP+1)+PInv(J,TTP)*Kstock(J,TTP+1))
;

 MODEL OLGMultiR /

*---> Producers J
 WageEq, RentEq, QEq
*--->Household J
 HBudgEq1, HBudgEq2,
 BeqEq, InhEq, PensEq
 ConEq,
 ConSSEq,
 PConEq,
 ConIEq,
 EConEq
 BijEQ
  KEq,
  RRetKeq, InvEq,PInvEq,EInvEq
  KstockEq,
  KstockSSEq
*--->Government J
  GBudgEq1, GBudgSSEq,GBudgEq2,GBudgEq3,GPENS
*--->Markets
  PEq,
  RintEq
* RintJIntEq
 OBJEQ          /
;
 OLGMultiR.HOLDFIXED = 1;
 OLGMultiR.OPTFILE   = 1;

 FILE MAR /Margins.chk/;

*++++++++++++++++++++++++++++++++++++++++++++++
* Initialisation (from Marcel's single country)
*++++++++++++++++++++++++++++++++++++++++++++++

*===Parameters
*--->Producers
 AlQ(J,TTP)    = AA.L(J);
 AlK(J)        = Alfa(J);
*--->Households J
 Sig(J)        = SIGC;
 DiscR(J)      = DELTAC.L(J);
* DiscR(J)      = DELTAC.L;
 SigCon(J)     = 4.5;
 BeqR(J,G)     = BEC(J,G);
 InhR(J,G)     = IDC(G);
*Population is assumed to be constant from period T1 to T6
 NN(J,TTP)     = NGR(J) ;
 Pop(J,"T5",G)= NC1(J,G);
 LOOP(TTP$(ORD(TTP) GT CARD(TP)) , Pop(J,TTP+1,G) = Pop(J,TTP,G)*NN(J,TTP) ) ;
 POP(J,TTP,G)$(ORD(TTP) LE CARD(TP))=POP(J,"T5",G);
 TPop(J,TTP)= SUM(G,Pop(J,TTP,G));
```

```
LOOP(TTP, NNP(J,TTP) = TPop(J,TTP+1)/TPop(J,TTP)) ;
DISPLAY TPop,NNP;
EP9(GM) = 0;
EP(J,G)       = EP9(G);
DepR(J)       = DEP.L(J);
SigInv(J)     = 4.5;
*--->Government J
KTxR(J,TTP)       = KTAX(J);
ConTxR(J,TTP)     = CTAX(J);
PensR(J)          = BR;
ExoBonds(J,TTP) = 1;
GovH(J,TTP)       = SUM(G,POP(J,TTP,G)*HEAC.L(J,G));
GovE(J,TTP)       = SUM(G,POP(J,TTP,G)*EDUC.L(J,G));
DISPLAY GovH, GovE;


*===Variables
*--->Producers
Q.L(J,TTP)          = YY.L(J);
LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Q.L(J,TTP+1)=NNP(J,TTP)*Q.L(J,TTP));
*--->Households J
Con.L(J,TTP,G)    = CC.L(J,G);
Lend.L(J,TTP,G)   = HA.L(J,G);
Pcon.L(J,TTP)     = 1;
ConI.L(I,J,TTP,G)= 0;  ConI.L(I,J,TTP,G) = CCI(I,J,G);
Beq.L(J,TTP,G)    = BEQC.L(J,G);
Inh.L(J,TTP,G)    = INHC.L(J,G);
Pens.L(J,TTP,GM) = WW(J)*BR;  Pens.FX(J,TTP,GJ) = 0;

ECon.L(I,J,TTP)   = SUM(G,Pop(J,TTP,G)*ConI.L(I,J,TTP,G));
Inv.L(J,TTP)      = ((NGR(J)-1)+DEP.L(J))*KK.L(J);
LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Inv.L(J,TTP+1)=NNP(J,TTP)*Inv.L(J,TTP));
PInv.L(J,TTP)     = 1;

*--->Government J
Bond.L(J,TTP)     = B(J);
LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Bond.L(J,TTP+1)=NNP(J,TTP)*Bond.L(J,TTP));
Gov.FX(J,TTP)      = GEXP.L(J);
LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Gov.FX(J,TTP+1)=NNP(J,TTP)*Gov.L(J,TTP));
WTxR.L(J,TTP)     = WTAX(J);
EInv.L(I,J,TTP)= 0;        EInv.L(I,J,TTP) = INVI(I,J);
LOOP(TTP $(ORD(TTP) GT CARD(TP) ),EInv.L(I,J,TTP+1)=NNP(J,TTP)*EInv.L(I,J,TTP));
CTR.L(TTP) = CTRC.L;

*--->Markets
Wage.L(J,TTP)     = WW(J);
Kstock.L(J,TTP)   = KK.L(J);
LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Kstock.L(J,TTP+1)=NNP(J,TTP)*Kstock.L(J,TTP));
Rent.L(J,TTP)     = RR+DEP.L(J);
P.L(J,TTP)        = 1;
Rint.L(TTP)       = 1+RR;
K.L(J,TTP+1,G+1)$SUM(GG,Pop(J,TTP+1,GG+1)) = (Kstock.L(J,TTP+1)*
    Lend.L(J,TTP+1,G+1))/SUM(GG,Pop(J,TTP+1,GG+1)*Lend.L(J,TTP+1,GG+1)) ;
BondD.fx(J,TTP,G) = 0;

*-------------> Choosing an (arbitrary) matrix of bilateral bond holding consistent with data
```

```
    PARAMETER Bij0(I,J);
    VARIABLES Bij0v(I,J);   EQUATIONS PortEQ1(I,T),PortEQ2(I,T),OBJPortEQ;
    Bij0v.L(I,I)= .8*B(I); Bij0v.L(I,J)$(ORD(I) NE ORD(J)) = (B(I)-Bij0v.L(I,I))/(CARD(J)-1);
    PortEQ1(I,TI)..           Bond.L(I,TI) =E= SUM(J, Bij0v(I,J) );
    PortEQ2(J,TI)..     SUM(I, Bij0v(I,J) ) =E= SUM(G,Pop(J,TI,G)*(Lend.L(J,TI,G)-PInv.L(J,TI)*K.L(J,TI,G)));
    OBJPortEQ..                        OBJ =E= SUM(I, (Bij0v(I,I)-.8*B(I))*(Bij0v(I,I)-.8*B(I)) );
    MODEL PORTF0 / PortEQ1, PortEQ2, OBJPortEQ / ;
    PORTF0.OPTFILE   = 1;
*    Bij0v.LO(I,J) = .02*B(J);Bij0v.UP(I,J) = .99*B(J);
    OPTION NLP=MINOS5;
    OPTION SOLPRINT=ON, LIMROW=0, LIMCOL=0, ITERLIM=5000;
    SOLVE PORTF0 MINIMIZING OBJ USING NLP;
    Bij0(I,J)              = Bij0v.L(I,J);
    Bij.L(I,J,TTP+1,G+1) = Bij0(I,J)/SUM(II,Bij0(II,J))* (Lend.L(J,TTP+1,G+1)-PInv.L(J,TTP)*K.L(J,TTP+1,G+1));
    Bij.L(I,J,TTP+1,G+1) = Bij.L(I,J,TTP+1,G+1)/P.L(I,TTP);
DISPLAY Bij.L, Bij0;
*-------------<


 Lend.FX(J,TTP,GI)  = 0;
 K.FX(J,TTP,GI)     = 0;
 Bij.FX(I,J,TTP,GI) = 0;

*===OTHER PARAMETERS
  AlConI(I,J)   = SUM((TI,GI), ConI.L(I,J,TI,GI)/Con.L(J,TI,GI) );
  AlInv(I,J)    = SUM(TI, EInv.L(I,J,TI)/Inv.L(J,TI) );


 LOOP(TTP $(ORD(TTP) GE CARD(TP)),
  RRET.L(I,TTP) = (Rent.L(I,TTP)+(1-DepR(I))*PInv.L(I,TTP))/PInv.L(I,TTP-1) ;
     );

 Lend.FX(J,TTP,G) $(ORD(TTP) LE CARD(TP)+1) = Lend.L(J,TTP,G);
 K.FX(J,TTP,G)    $(ORD(TTP) LE CARD(TP)+1) = K.L(J,TTP,G);
 Bij.FX(I,J,TTP,G)$(ORD(TTP) LE CARD(TP)+1) = Bij.L(I,J,TTP,G);
 Beq.FX(J,TTP,G)  $(BeqR(J,G) EQ 0) = 0;
 Inh.FX(J,TTP,G)  $(InhR(J,G) EQ 0) = 0;
 Pens.FX(J,TTP,G) $(PensR(J)  EQ 0) = 0;

* Gov.FX(J,TTP) = Gov.L(J,TTP);
 BOND.FX(J,TTP)$(ORD(TTP) LE CARD(TP)+1) = BOND.L(J,TTP);
 WTxR.FX(J,TTP)$(ORD(TTP) LE CARD(TP))   = WTAX(J);

 Wage.FX(J,TTP)$(ORD(TTP) LE CARD(TP))   = Wage.L(J,TTP);
 Kstock.FX(J,TI)     = Kstock.L(J,TI);

 Rint.FX(TTP)$(ORD(TTP) EQ CARD(TP))      = Rint.L(TTP);

 ConI.FX(I,J,TTP,G) $(AlConI(I,J) LT 1.E-13) = 0;
 P.FX("foreign",TTP)                            = 1;
 P.FX(J,TTP)$(ORD(TTP) LE CARD(TP))          = 1;
```

```
RintJ.L(J,TTP)     = Rint.L(TTP);
RintJ.FX(J,TTP)$(ORD(TTP) EQ CARD(TP))   = Rint.L(TTP);

Con.LO(J,TTP,G) = .50*Con.L(J,TTP,G);  Con.UP(J,TTP,G) = 1.50*Con.L(J,TTP,G);
EInv.FX(I,J,TTP) $(AlInv(I,J) LT 1.E-13)     = 0;
EInv.LO(I,J,TTP)    = .10*EInv.L(I,J,TTP)   ; EInv.UP(I,J,TTP)    = 2.00*EInv.L(I,J,TTP)  ;

Inv.LO(J,TTP)    = .50*Inv.L(J,TTP);     Inv.UP(J,TTP)    = 1.50*Inv.L(J,TTP);
PInv.FX(J,TTP)$(ORD(TTP) LE CARD(TP))        = PInv.L(J,TTP);

EXC.FX(J,TTP)$(ORD(TTP) LE CARD(TP)) = 0;
EXC.FX(J,TTP)$(ORD(TTP) GE 12) = 0;
EXC.FX(J,TTP) = 0;

OPTIONS SOLPRINT=On, LIMROW=9999, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
OPTION NLP=CONOPT2 ;
SOLVE OLGMultiR USING NLP MINIMIZING OBJ;

*++++++++++++++++++
*TEST of NUMERAIRE
*++++++++++++++++++
*$ONTEXT
P.FX("foreign",TTP)                                  = 1.2;
P.FX(J,TTP)$(ORD(TTP) LE CARD(TP))           = 1.2;
Lend.FX(J,TTP,G)   $(ORD(TTP) LE CARD(TP)+1) = 1.2*Lend.L(J,TTP,G);
Wage.FX(J,TTP)$(ORD(TTP) LE CARD(TP))        = 1.2*Wage.L(J,TTP);
PInv.FX(J,TTP)$(ORD(TTP) LE CARD(TP))        = 1.2*PInv.L(J,TTP);
Rent.FX(J,TTP)$(ORD(TTP) LE CARD(TP))        = 1.2*Rent.L(J,TTP);


OPTIONS SOLPRINT=OFF, LIMROW=0, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
OPTION NLP=CONOPT2;
SOLVE OLGMultiR USING NLP MINIMIZING OBJ;

OPTIONS SOLPRINT=Off, LIMROW=0, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
OPTION NLP=MINOS5;
SOLVE OLGMultiR USING NLP MINIMIZING OBJ;
*$OFFTEXT

*+++++++
* SHOCKS
*+++++++

*$ONTEXT

* (temporary unexpected) Productivity shock on all countries:
AlQ(J,TTP)    $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+3) = 1.01*AlQ(J,TTP);
AlQ("home",TTP)$( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+5) = 1.03*AlQ("home",TTP);
DiscR("home")=1.15*DiscR("home");
EP(J,TTP,G)$( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+6) = 1.12*EP(J,TTP,G);
EP(J,TTP,"G1")=.9;
EP(J,TTP,"G2")=1;
EP(J,TTP,"G3")=1.1;
```

```
 EP(J,TTP,"G4")=1;
 EP(J,TTP,"G5")=1;
 KTxR("home",TTP)$( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+1) = 0.85*KTxR("home",TTP);
 ConTxR("home",TTP) $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+1) = 0.85*ConTxR("home",TTP);
*++++++++++++++++
* Policy options
*++++++++++++++++

* Choose periods of BOND-financed government deficits (default is tax financing)
* ExoBonds(J,TTP)$( (ORD(TTP) GT CARD(TP)+1) AND (ORD(TTP) LE CARD(TP)+5)) = 0;

*++++++++++
* Solving
*++++++++++

 OPTIONS SOLPRINT=OFF, LIMROW=0, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
 OPTION NLP=CONOPT2;
 SOLVE OLGMultiR USING NLP MINIMIZING OBJ;

 OPTIONS SOLPRINT=OFF, LIMROW=0, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
 OPTION NLP=MINOS5;
 SOLVE OLGMultiR USING NLP MINIMIZING OBJ;

*$OFFTEXT


*+++++++++++++++++++++++++++
* VARIOUS EQUILIBRIUM TESTS
*+++++++++++++++++++++++++++
*$ONTEXT
 PARAMETER TRADEBAL(I,TTP);
 TRADEBAL(I,TTP)$(ORD(TTP) GE CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T)) =
                     SUM(J$(ORD(J) NE ORD(I)), P.L(I,TTP)*(ECon.L(I,J,TTP)+EInv.L(I,J,TTP)))-
                     SUM(J$(ORD(J) NE ORD(I)), P.L(J,TTP)*(ECon.L(J,I,TTP)+EInv.L(J,I,TTP)));
 TRADEBAL(J,T)$(ABS(TRADEBAL(J,T)) LT 1.E-7) = 0; DISPLAY TRADEBAL;

 PARAMETER WTRADEBAL(TTP);
 WTRADEBAL(TTP)   = SUM(I,TRADEBAL(I,TTP));
 WTRADEBAL(TTP)$(ABS(WTRADEBAL(TTP)) LT 1.E-7) = 0; DISPLAY WTRADEBAL;

 PARAMETER WALRASJ(J,TTP);
 WALRASJ(J,T)     = Q.L(J,T)-SUM(I,ECon.L(J,I,T)+EInv.L(J,I,T))-Gov.L(J,T);
 WALRASJ(J,T)$(ABS(WALRASJ(J,T)) LT 1.E-7) = 0; DISPLAY WALRASJ;

 PARAMETER WASSETBAL(TTP);
 WASSETBAL(TTP+1)$(ORD(TTP) GE CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T)) =
     SUM((J,G),Pop(J,TTP+1,G+1)*LEND.L(J,TTP+1,G+1))-
     SUM( J   ,P.L(J,TTP)*Bond.L(J,TTP+1)+PInv.L(J,TTP)*Kstock.L(J,TTP+1));
 WASSETBAL(TTP)$(ABS(WASSETBAL(TTP)) LT 1.E-7)  = 0; DISPLAY WASSETBAL;

 PARAMETER WASSETBAL(TTP);
 WASSETBAL(TTP+1)$(ORD(TTP) GE CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T)) =
     SUM((I,J,G),Pop(J,TTP+1,G+1)*P.L(I,TTP)*Bij.L(I,J,TTP+1,G+1))-
```

```
     SUM( J   ,P.L(J,TTP)*Bond.L(J,TTP+1));
 DISPLAY WASSETBAL;

*$OFFTEXT

*$INCLUDE OLG6-RCan\0801\Pprint-OLG2.INC
```