

## INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

**The quality of this reproduction is dependent upon the quality of the copy submitted.** Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

ProQuest Information and Learning  
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA  
800-521-0600

UMI<sup>®</sup>



## **NOTE TO USERS**

**This reproduction is the best copy available.**

UMI<sup>®</sup>



SC

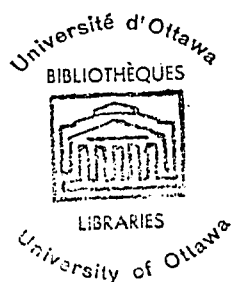
FREE LATERAL VIBRATION ANALYSIS OF  
BEAMS WITH FINITE DISCONTINUITIES

A Thesis  
Submitted to  
the Faculty of Graduate School  
University of Ottawa

In Partial Fulfillment  
of the Requirement for the Degree  
Master of Applied Science

by  
Ong Eng-Kok

May 1974



UMI Number: EC52409

INFORMATION TO USERS

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

UMI<sup>®</sup>

---

UMI Microform EC52409  
Copyright 2007 by ProQuest LLC  
All rights reserved. This microform edition is protected against  
unauthorized copying under Title 17, United States Code.

---

ProQuest LLC  
789 East Eisenhower Parkway  
P.O. Box 1346  
Ann Arbor, MI 48106-1346

FREE LATERAL VIBRATION ANALYSIS OF  
BEAMS WITH FINITE DISCONTINUITIES

---

Advisor

---

Candidate

### ACKNOWLEDGEMENTS

The author is greatly indebted to Dr. Gorman, under whose supervision this research project was carried out, for his valuable suggestions, critical discussions and constant encouragement.

He gratefully acknowledges the financial support in the form of teaching assistantship of the Department of Mechanical Engineering, University of Ottawa. He also wishes to express his gratitude to the personnel of the Computing Centre of the University of Ottawa.

He is indebted, also, to Miss Lucie Dubeau for typing the manuscript.

## PREFACE

Beams are common structural elements of mechanical design. Their importance in structural engineering is attested to by their extremely extensive usage in all sorts of load-carrying applications. In a similar fashion, the vibration of beams is an important aspect of the vibration of continuous system because such beams are very common. It is often necessary to design these elements so that they have either a selected natural frequency or so that their natural frequencies are sufficiently far-removed from excitation frequencies to avoid resonance.

Tables given in this thesis provide a quick procedure for establishing natural frequencies of uniform and non-uniform beams. Linear interpolation may be used for analyzing beams with parameter values falling between those for which eigenvalues are tabulated. For each case, linear interpolation involving any of these parameters has generally been found to give frequencies with maximum error not greater than about 1%. These data enable the designer to assess the effect of changing the dimensions, type of support and material of the structural element. In addition, information required for establishing modal shapes is also provided.

The numerical results were obtained on an IBM 360/65

- iii -

computer. The machine program is appended for the convenience of any user who may wish to carry out computations in a range not covered by the tables given here.

TABLE OF CONTENTS

LIST OF SYMBOLS .....	vi
CHAPTER 1. BEAMS WITH SINGLE DISCONTINUITY IN CROSS SECTION PROPERTIES .....	1
1. Equation of Motion .....	1
2. Method of Analysis .....	7
3. Choice of Parameters .....	12
4. Modal Shapes .....	18
5. Numerical Computations .....	21
6. Check for Correctness .....	23
7. Presentation of Results .....	24
CHAPTER 2. DOUBLE-SPAN BEAMS WITH SINGLE DISCONTINUITY IN CROSS SECTION PROPERTIES .....	37
8. General Consideration .....	37
9. Check for Correctness .....	41
10. Presentation of Results .....	43
CHAPTER 3. BEAMS WITH SYMMETRIC DISCONTINUITIES IN CROSS SECTION PROPERTIES .....	51
11. General Discussion .....	51
A. Beams with Identical End Supports .....	51
B. Beams with Differing End Supports .....	54
12. Check for Correctness .....	60
A. Beams with Identical End Supports .....	60
B. Beams with Differing End Supports .....	61

13.	Presentation of Results .....	62	
A.	Beams with Identical End Supports .....	62	
B.	Beams with Differing End Supports .....	63	
CHAPTER 4. THREE-SPAN BEAMS WITH SYMMETRIC			
DISCONTINUITIES IN CROSS SECTION			
PROPERTIES .....			73
14.	General Consideration .....	73	
A.	Beams with Identical End Supports .....	73	
B.	Beams with Differing End Supports .....	75	
15.	Check for Correctness .....	81	
A.	Beams with Identical End Supports .....	81	
B.	Beams with Differing End Supports .....	82	
16.	Presentation of Results .....	84	
TABLES OF EIGENVALUES $\beta_1$ FOR BEAMS WITH DISCONTINUITIES			
IN CROSS SECTION PROPERTIES .....			97
APPENDIXES .....			147
BIBLIOGRAPHY .....			190

LIST OF SYMBOLS

See context for precise meanings of the symbols.

$A$  = beam cross-sectional area,  $\text{in}^2$

$B_n$  = constants associated with the function  $f(t)$

$C_n = D_n/L$  = constants associated with the function  $X(\xi)$

$D_n$  = constants associated with beam deflection function  $Y(x)$

$E$  = modulus of elasticity in tension and compression,  
 $\text{lb/in}^2$

$f = \omega/2\pi$  = circular frequency, cycles/sec or Hz

$f(t)$  = function of time

$g$  = acceleration of gravity,  $\text{in/sec}^2$

$i$  = an integer in general

$I$  = moment of inertia of area about neutral axis,  $\text{in}^4$

$I_p$  = polar moment of inertia,  $\text{in}^4$

$L$  = length of beam, in

$m = \rho_2 A_2 / \rho_1 A_1$  = mass ratio

$m' = (\rho_2 A_2 / \rho_1 A_1)^{\frac{1}{2}}$

$n$  = an integer in general

$S = E_2 I_2 / E_1 I_1$  = bending stiffness ratio

$S' = (E_2 I_2 / E_1 I_1)^{\frac{1}{2}}$

$t$  = time, sec

$V$  = transverse shear force, lb

$w$  = weight or load per unit of length on beam,  $\text{lb/in}$

$x_i$  = spatial coordinate along beam, in

$X(\xi) = Y(x)/L$

$y(x, t) = Y(x) f(t)$  = total lateral deflection of beam, in

$Y(x)$  = function of  $x$

---

$\alpha = (E_2 I_2 / E_1 I_1)^{\frac{1}{4}}$  = bending stiffness ratio

$\beta_1 = L \sqrt[4]{\frac{\rho_1 A_1}{E_1 I_1}} \omega^2$  = eigenvalue for non-uniform beam

$\beta_2 = \theta \beta_1$

$\gamma$  = dimensionless segment or span length

$\gamma'$  = weight density, lb/in<sup>3</sup>

$\Delta$  = a small increment

$\xi_i = x_i / L$  = dimensionless spatial coordinate along beam

$\theta = \phi / \alpha$

$\lambda_n$  = eigenvalue for uniform beam

$\mu$  = dimensionless segment or span length

$\rho$  = mass density, lb - sec<sup>2</sup>/in<sup>4</sup>

$\phi = (\rho_2 A_2 / \rho_1 A_1)^{\frac{1}{4}}$  = mass ratio

$\omega = 2\pi f$  = natural circular frequency, rad/sec

## INTRODUCTION

While the solution to the linear differential equation governing the lateral vibration of beams has been known for many years, it is frequently found by analysts that computation of frequencies and modes of vibration requires use of a digital computer. In the case of uniform beams with classical boundary conditions the work is greatly diminished since tabulated values of the appropriate eigenvalues exist (1, 5, 6). However, in the more general case of beams with step changes in the mass per unit length  $\rho A$  and the flexural rigidity  $EI$ , the appropriate eigenvalues are not to be found in the literature and the solution must be obtained from the basic differential equation.

There exists a requirement for a set of tables so that the need for much of the time consuming work devoted to obtaining solutions to such problems would be obviated. Furthermore, such a set of tables would bring the analysis of vibration problems much more within the capability of the engineer who is not a vibration specialist.

CHAPTER 1  
BEAMS WITH SINGLE DISCONTINUITY  
IN CROSS SECTION PROPERTIES

(1) Equation of motion

The classical theory of beam vibration, often called the Bernoulli-Euler theory, is based on the same assumption as elementary theory of flexural bending, namely,

- (i) the beam is prismatic and it has an axial plane of symmetry, bending is in the plane of symmetry;
- (ii) the material is homogeneous and it obeys Hooke's law, the modulus of elasticity in tension being the same as that in compression;
- (iii) plane sections remain plane and normal to the longitudinal fibers of the beam after bending;
- (iv) slopes of the deflection curve are small compared to unity;
- (v) deflection due to shear is negligible.

In the various textbooks on strength of materials<sup>1</sup> the differential equation of the static loading of a beam is usually given in the following form:

---

1. Timonshenko, S. Strength of Materials, Part I. Princeton: D. van Nostrand Co., Inc. 1955.

$$\left. \begin{aligned} E I \frac{d^2 y}{dx^2} &= -M \\ \frac{d}{dx} \left( E I \frac{d^2 y}{dx^2} \right) &= -\frac{dM}{dx} = -V \\ \frac{d^2}{dx^2} \left( E I \frac{d^2 y}{dx^2} \right) &= -\frac{dV}{dx} = w \end{aligned} \right\} \quad (1a, b, c)$$

Combining, the following relation is obtained for the intensity of the distributed transverse load  $w(x)$ , the deflection  $y(x)$  of the elastic line of a bar and its flexural rigidity  $E I(x)$

$$\frac{d^2}{dx^2} \left[ E I(x) \frac{d^2 y(x)}{dx^2} \right] = w(x) \quad (2)$$

In this treatment, the only inertia forces considered are those due to the lateral translation of a beam element<sup>1</sup>. To derive the equation of motion for transverse vibration of a bar we use Eq. (2) and apply the D'Alembert principle. To the external force  $w(x)$  we add the "inertia force" per unit length of the bar given by

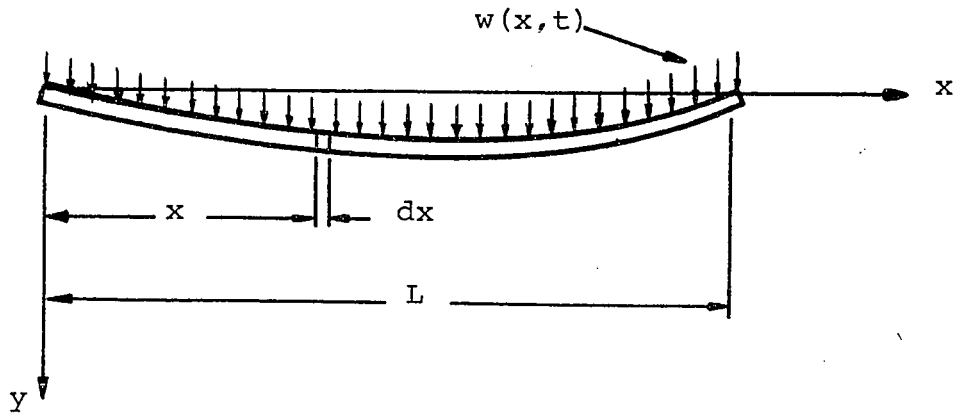
---

1. Actually, during vibration, each element of the beam rotates through an angle  $\partial y / \partial x$ , and there is thus a distributed inertia couple equal to  $-I_p \partial^3 y / \partial x \partial t^2$  per unit length, which is called the rotatory inertia effect. Correction for this rotatory inertia becomes important when the cross-sectional dimensions of the beam are not small compared with the beam length. These assumptions must be kept in mind when determining frequencies from the tables.

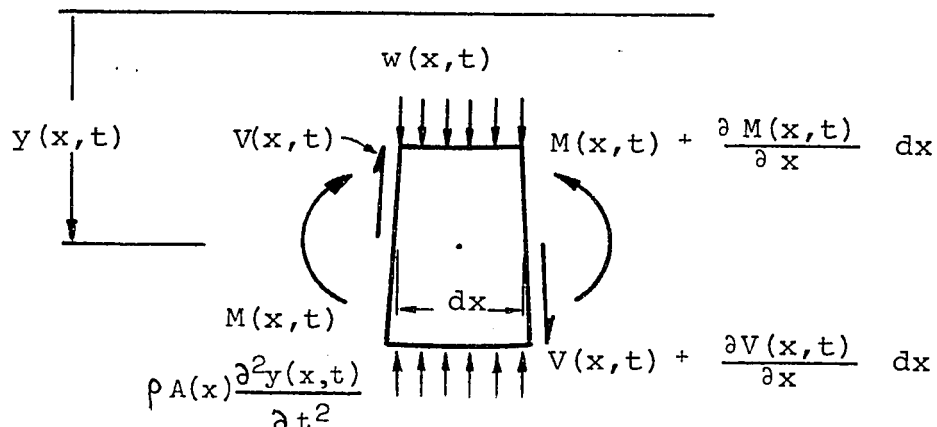
$$-\rho A(x) \frac{\partial^2 y(x,t)}{\partial t^2} \quad (3)$$

In Eq. (3)  $\rho$  is the mass density of the material of the bar and  $A(x)$  is its cross section area. The equation of motion is then [4]

$$\frac{\partial^2}{\partial x^2} \left[ E I(x) \frac{\partial^2 y(x,t)}{\partial x^2} \right] = w(x,t) - \rho A(x) \frac{\partial^2 y(x,t)}{\partial t^2} \quad (4)$$



(a) The non-uniform bar



(b) Free-body diagram of a beam element

Fig. 1.

Eq. (4) represents the general equation of motion for any beam that satisfies the requirements previously set down.

For the case of free vibrations,  $w(x,t) = 0$ ;

Eq. (4) becomes

$$\frac{\partial^2}{\partial x^2} \left[ E I(x) \frac{\partial^2 y(x,t)}{\partial x^2} \right] = - \rho A(x) \frac{\partial^2 y(x,t)}{\partial t^2} \quad (5)$$

To solve Eq. (5) we assume that the solution of Eq. (5) is separable in time and space and of the form

$$y(x,t) = Y(x)f(t) \quad (6)$$

Introducing Eq. (6) in Eq. (5) and then separating the variables, we obtain the equation

$$\frac{1}{\rho A(x) Y(x)} \frac{d^2}{dx^2} \left[ E I(x) \frac{d^2 Y(x)}{dx^2} \right] = - \frac{1}{f(t)} \frac{d^2 f(t)}{dt^2} \quad (7)$$

The left side of Eq. (7) depends on  $x$  only, and the right side depends on  $t$  only. Both  $x$  and  $t$  are independent variables, so Eq. (7) has a solution only if both sides are constant. On the basis of physical consideration, we must choose a positive value,  $\omega^2$ , for this constant.

Eq. (7) leads to two ordinary differential equations

$$\frac{d^2}{dx^2} \left[ E I(x) \frac{d^2 Y(x)}{dx^2} \right] - \omega^2 \rho A(x) Y(x) = 0 \quad (8)$$

$$\frac{d^2 f(t)}{dt^2} + \omega^2 f(t) = 0 \quad (9)$$

Eq. (9) gives a harmonic rather than an exponential solution, which is consistent with the fact that a conservative system has constant total energy.

The general solution of Eq. (9) is

$$f(t) = B_1 \cos \omega t + B_2 \sin \omega t \quad (10)$$

If the beam has constant cross section, then A and I are constant, and Eq. (8) becomes

$$\frac{d^4 Y(x)}{dx^4} - k^4 Y(x) = 0 \quad (11)$$

where

$$k^4 = \frac{\rho A}{E I} \omega^2 \quad (12)$$

The general solution of Eq. (11) is

$$Y(x) = D_1 \sin kx + D_2 \cos kx + D_3 \sinh kx + D_4 \cosh kx \quad (13)$$

where  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$  are arbitrary constants which should be determined in each particular case from the boundary conditions at the ends of the bar. The most common types of boundary conditions encountered in the bending vibration of bars are the following:

- (i) Simply-supported Ends. In this case of simply supported ends, both the deflection and bending moment at an end must vanish. This means that

$$Y = 0, \quad \frac{d^2 Y}{dx^2} = 0 \quad (14)$$

(ii) Clamped Ends. In this case, both the deflection and slope must vanish when it is built into a rigid wall. This condition may be expressed as

$$Y = 0, \quad \frac{dY}{dx} = 0 \quad (15)$$

(iii) Free Ends. In this case of a completely free end, both the bending moment and shearing force acting at that end must vanish. Hence one may write

$$\frac{d^2 Y}{dx^2} = 0, \quad \frac{d^3 Y}{dx^3} = 0 \quad (16)$$

For the two ends of a vibrating bar we always will have a total of four end conditions from which the ratios between the constants of the general solution (13) and the frequency equation can be obtained. In this manner the modes of natural vibration and their frequencies will be established. By superimposing all possible normal vibrations, the general expression for the free lateral vibrations becomes,

$$y(x,t) = \sum_{i=1}^{\infty} Y_i(x) (B_1^{(i)} \cos \omega_i t + B_2^{(i)} \sin \omega_i t) \quad (17)$$

where

$$Y_i(x) = D_1^{(i)} \sin k_i x + D_2^{(i)} \cos k_i x + D_3^{(i)} \sinh k_i x + D_4^{(i)} \cosh k_i x \quad (18)$$

(2) Method of Analysis

Free lateral vibration of beams whose mass and stiffness can vary from segment to segment but are kept constant within the segment is analyzed. We will be interested in step changes in the mass per unit length  $\rho A$ , and in the flexural rigidity  $EI$  of the beam. Such changes might arise from an abrupt change in cross sectional geometry or beam material. Also, they could result from the joining of beam sections of the same cross sectional geometry but of different material properties, or from the addition of uniformly distributed mass.

In order to facilitate presentation of the background underlying the preparation of the tables, it is advantageous to discuss a particular case in detail. We therefore look into the problem of free vibration of a beam with simple support at each end, as shown in Fig. 2. The beam cross sectional properties are shown to be constant over the dimensionless distance  $\mu$  from the left end, and  $\gamma (= 1 - \mu)$ , the remainder of the beam.

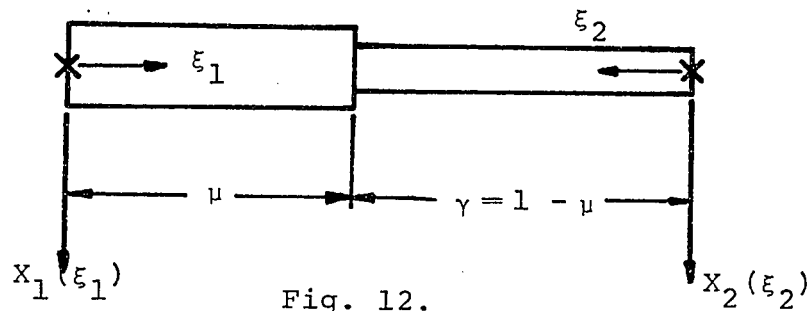


Fig. 12.

For each segment of the beam, the vibration is given by a solution of Eq. (5)

$$\frac{\partial^4 Y_i(x_i, t)}{\partial x_i^4} + \frac{\rho_i A_i}{E_i I_i} \frac{\partial^2 Y_i(x_i, t)}{\partial t^2} = 0 \quad (5')$$

of the type

$$Y_i(x_i, t) = Y_i(x_i) (B_1^{(i)} \cos \omega t + B_2^{(i)} \sin \omega t)$$

where  $Y_i(x_i)$  is the general solution of Eq. (11). In the analysis discussed here it is advantageous to nondimensionalize the beam displacement and the spatial variable  $x_i$ .

This is achieved through dividing them by the beam length  $L$ . It is readily shown that Eq. (13) can be written as

$$X_i(\xi_i) = C_1^{(i)} \sin \beta_i \xi_i + C_2^{(i)} \cos \beta_i \xi_i + C_3^{(i)} \sinh \beta_i \xi_i + C_4^{(i)} \cosh \beta_i \xi_i \quad (19)$$

where

$$\left. \begin{aligned} X_i(\xi_i) &= Y_i(x_i)/L, & C_n^{(i)} &= D_n^{(i)}/L \\ \xi_i &= x_i/L, & \beta_i^4 &= \frac{\rho_i A_i}{E_i I_i} \omega^2 L^4 \end{aligned} \right\} \quad (20a \text{ b, c, d})$$

Let  $\xi_1$  be the dimensionless distance to the right of a generic section from the left support on the first segment, and  $\xi_2$  be the dimensionless distance to the left of a generic section from right support on the second segment (see Fig. 2). It follows that the first and second

segment deflections are

$$x_1(\xi_1) = C_1^{(1)} \sin \beta_1 \xi_1 + C_2^{(1)} \cos \beta_1 \xi_1 + \\ C_3^{(1)} \sinh \beta_1 \xi_1 + C_4^{(1)} \cosh \beta_1 \xi_1$$

and

$$x_2(\xi_2) = C_1^{(2)} \sin \beta_2 \xi_2 + C_2^{(2)} \cos \beta_2 \xi_2 + \\ C_3^{(2)} \sinh \beta_2 \xi_2 + C_4^{(2)} \cosh \beta_2 \xi_2$$

where

$$\beta_1^4 = \frac{\rho_1 A_1}{E_1 I_1} \omega^2 L^4, \quad \beta_2^4 = \frac{\rho_2 A_2}{E_1 I_1} \omega^2 L^4$$

There are evidently eight boundary conditions required. Four are obtained by prescribing conditions at the beam outer ends, the remaining four are obtained by requiring the continuity of displacement, slope, bending moment and shear force across the interface of the two beam segments.

From the conditions that at the left support, the displacement and the bending moment are zero, it is readily shown that

$$C_2^{(1)} = C_4^{(1)} = 0$$

In the same way, from the condition that at the right support, the displacement and the bending moment are zero, we have

$$C_2^{(2)} = C_4^{(2)} = 0$$

Hence

$$X_1(\xi_1) = C_1^{(1)} \sin \beta_1 \xi_1 + C_3^{(1)} \sinh \beta_1 \xi_1 \quad (21)$$

and

$$X_2(\xi_2) = C_1^{(2)} \sin \beta_2 \xi_2 + C_3^{(2)} \sinh \beta_2 \xi_2 \quad (22)$$

The remaining four boundary conditions are written as follows

(i) Continuity of displacement,  $X_1(\mu) = X_2(\gamma)$

$$\begin{aligned} C_1^{(1)} \sin \beta_1 \mu + C_3^{(1)} \sinh \beta_1 \mu + C_1^{(2)} (-\sin \beta_2 \gamma) + \\ C_3^{(2)} (-\sinh \beta_2 \gamma) = 0 \end{aligned} \quad (23)$$

(ii) Continuity of slope,

$$\begin{aligned} \left. \frac{dX_1(\xi_1)}{d\xi_1} \right|_{\xi_1 = \mu} = - \left. \frac{dX_2(\xi_2)}{d\xi_2} \right|_{\xi_2 = \gamma} \\ C_1^{(1)} \cos \beta_1 \mu + C_3^{(1)} \cosh \beta_1 \mu + C_1^{(2)} \left( \frac{\beta_2}{\beta_1} \right) \cos \beta_2 \gamma + \\ C_3^{(2)} \left( \frac{\beta_2}{\beta_1} \right) \cosh \beta_2 \gamma = 0 \end{aligned} \quad (24)$$

(iii) Continuity of bending moment,

$$E_1 I_1 \left. \frac{d^2 X_1(\xi_1)}{d\xi_1^2} \right|_{\xi_1 = \mu} = E_2 I_2 \left. \frac{d^2 X_2(\xi_2)}{d\xi_2^2} \right|_{\xi_2 = \gamma}$$

$$\begin{aligned}
 & C_1^{(1)} (-\sin \beta_1 \mu) + C_3^{(1)} \sinh \beta_1 \mu + C_1^{(2)} \\
 \times & \left\{ \frac{E_2 I_2}{E_1 I_1} \left( \frac{\beta_2}{\beta_1} \right)^2 \right\} \sin \beta_2 \gamma + C_3^{(2)} \left\{ -\frac{E_2 I_2}{E_1 I_1} \left( \frac{\beta_2}{\beta_1} \right)^2 \right\} \\
 \times & \sinh \beta_2 \gamma = 0
 \end{aligned} \tag{25}$$

(iv) Continuity of shear force,

$$\begin{aligned}
 E_1 I_1 \left. \frac{d^3 X_1(\xi_1)}{d\xi_1^3} \right|_{\xi_1 = \mu} &= -E_2 I_2 \left. \frac{d^3 X_2(\xi_2)}{d\xi_2^3} \right|_{\xi_2 = \gamma} \\
 C_1^{(1)} (-\cos \beta_1 \mu) + C_3^{(1)} \cosh \beta_1 \mu + C_1^{(2)} \\
 \times & \left\{ -\frac{E_2 I_2}{E_1 I_1} \left( \frac{\beta_2}{\beta_1} \right)^3 \right\} \cos \beta_2 \gamma + C_3^{(2)} \left\{ \frac{E_2 I_2}{E_1 I_1} \left( \frac{\beta_2}{\beta_1} \right)^3 \right\} \\
 \times & \cosh \beta_2 \gamma = 0
 \end{aligned} \tag{26}$$

From the definition of  $\beta_i$ , we have

$$\frac{\beta_2}{\beta_1} = \left( \frac{\rho_2 A_2}{\rho_1 A_1} \cdot \frac{E_1 I_1}{E_2 I_2} \right)^{\frac{1}{4}}$$

Thus, given any values of  $\mu$  ( $0 < \mu < 1$ ), the mass ratio ( $\rho_2 A_2 / \rho_1 A_1$ ) and the bending stiffness ratio ( $E_2 I_2 / E_1 I_1$ ), we can determine the eigenvalues  $\beta_1$  or  $\beta_2$  of the system by requiring that the determinant of the coefficient matrix of Eqs. (23) to (26) must vanish. Hence

$$|C_{ij}| = 0 \tag{27}$$

is the frequency equation.

(3) Choice of Parameters.

We see that the frequency equation in the form of a determinant contains array elements that are being multiplied by the factors  $(\beta_2/\beta_1)$ ,  $(E_2 I_2/E_1 I_1)(\beta_2/\beta_1)^2$  or  $(E_2 I_2/E_1 I_1)(\beta_2/\beta_1)^3$ , all of which can be expressed in terms of mass ratio and bending stiffness ratio.

With  $\mu$ ,  $(\rho_2 A_2/\rho_1 A_1)$  and  $(E_2 I_2/E_1 I_1)$  known, we can determine an infinite number of eigenvalues which are related to the natural frequencies of the system, Eq. (20d).

Thus, there exist three parameters for the determination of eigenvalues.

It is natural to divide the beam into ten equal portions, i.e.  $\mu$  ranges from 0 to 1 in increments of 0.1. The values  $\mu = 0$  and  $\mu = 1$  are discarded as the former represents a beam of uniform properties  $\rho_2$ ,  $A_2$ ,  $E_2$ , and  $I_2$ , while the latter represents a beam of uniform properties  $\rho_1$ ,  $A_1$ ,  $E_1$ , and  $I_1$ . Hence  $\mu$  is chosen to range from 0.1 to 0.9 in increments of 0.1.

To start with, we take

$$m = \frac{\rho_2 A_2}{\rho_1 A_1}, \quad s = \frac{E_2 I_2}{E_1 I_1}$$

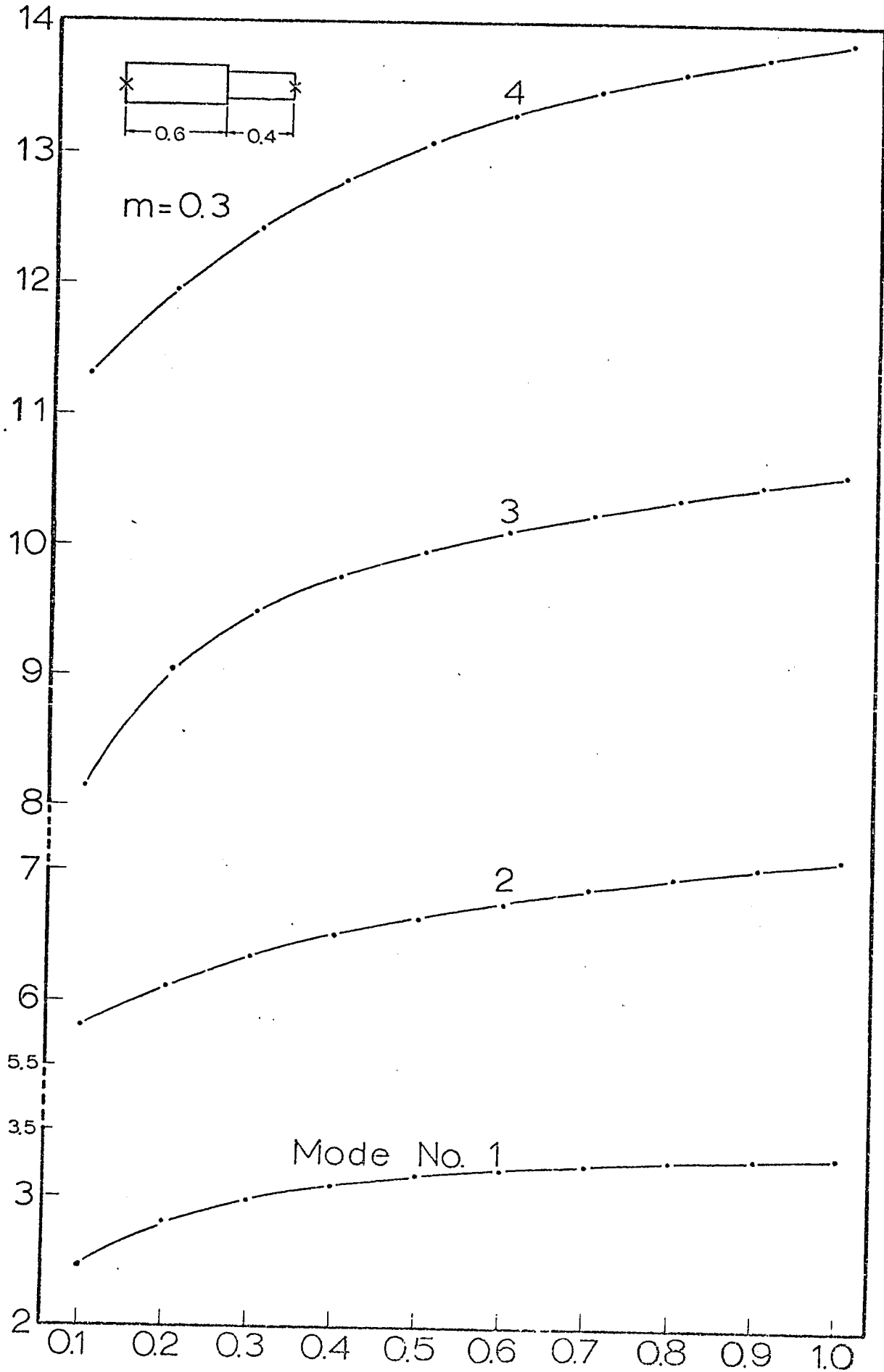
both of which range from 0.1 to 1.0 in increments of 0.1. We look into the error associated with linear interpolation for  $\beta_1$  between two values of a parameter while the other two parameters are kept constant. The errors en-

countered through interpolation involving  $\mu$  are generally greater than 1% for  $m$  and  $s$  smaller than 0.4, while those encountered through interpolation involving  $m$  or  $s$  are generally greater than 1% for  $m$  and  $s$  smaller than 0.2. Also, the number of tables required to cover this range of  $m$  and  $s$  is too large (at least 100 tables for each case). In order to see the dependence of  $\beta_1$  upon  $m$  ( $\mu$  and  $s$  fixed) or  $s$  ( $\mu$  and  $m$  fixed),  $\beta_1$  vs  $m$  and  $\beta_1$  vs  $s$  curves are plotted. We see that both the  $\beta_1$  vs  $m$  and  $\beta_1$  vs  $s$  curves have greater slope when  $m$  or  $s$  is small and flatten out when  $m$  or  $s$  is large (see Figs. 3 and 4). If we take the abscissa of  $\beta_1$  vs  $m$  and  $\beta_1$  vs  $s$  curves to be  $\sqrt{m}$  and  $\sqrt{s}$  respectively, we see that the original curves will be transformed into curves with more uniform slope, thus facilitating linear interpolation and yet with fewer points ( $\sqrt{m}$  and  $\sqrt{s}$  varies from 0.3 to 1.0 in increment of 0.1, see Figs 5 and 6)

Using the same range of  $\mu$  and

$$m' = \left( \frac{\rho_2 A_2}{\rho_1 A_1} \right)^{\frac{1}{2}}, \quad s' = \left( \frac{E_2 I_2}{E_1 I_1} \right)^{\frac{1}{2}}$$

where  $m'$  and  $s'$  range from 0.3 to 1.0 (thus the associated mass ratio and stiffness ratio range from 0.09 to 1.0), the same error analysis is performed. The errors encountered through interpolation involving  $\mu$  are generally greater than 1% for  $m'$  and  $s'$  less than and equal to 0.5.



S  
Fig. 3

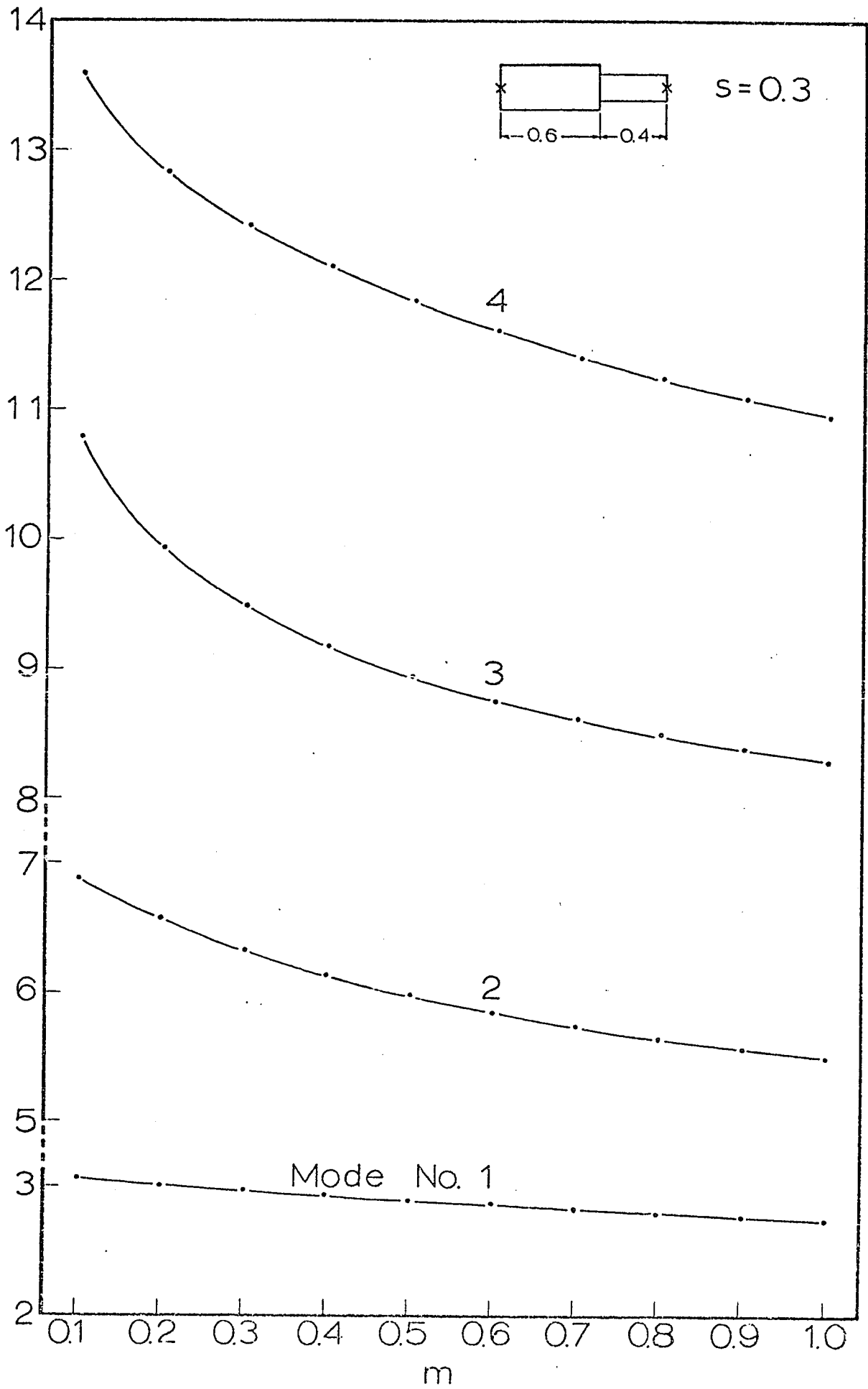


Fig. 1

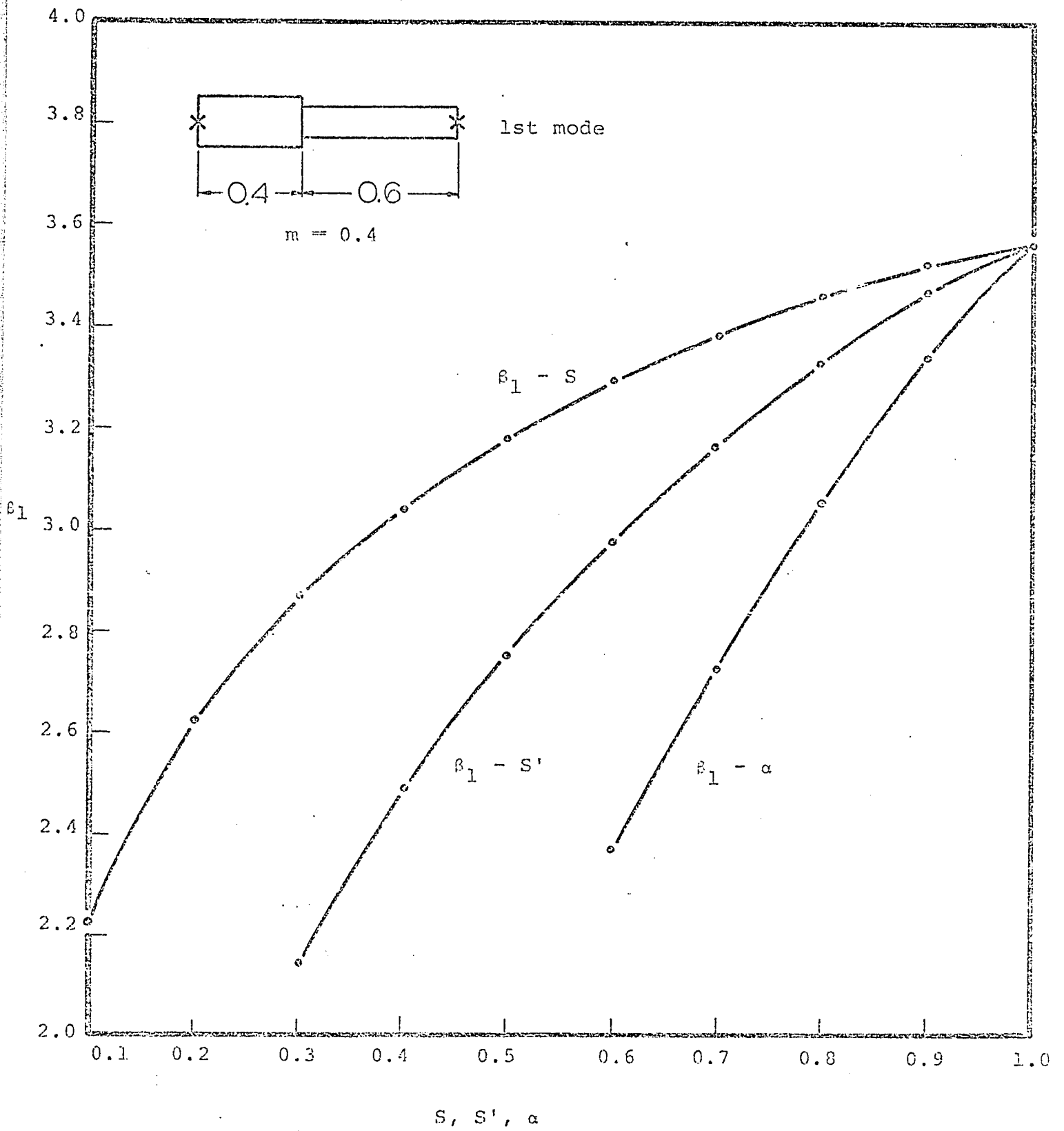
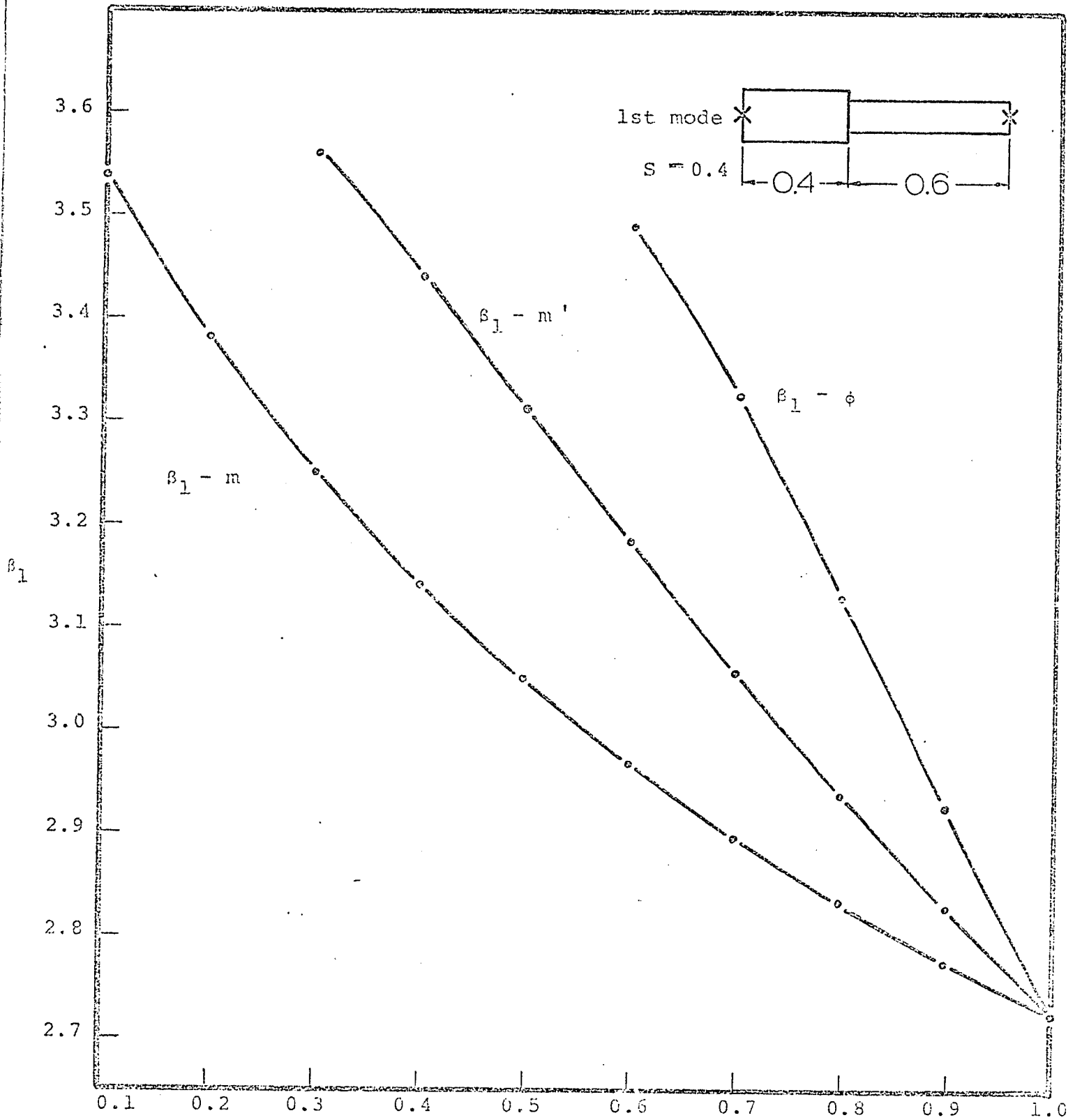


Fig. 5.



$m, m', \phi$

Fig. 6.

Linear interpolation involving  $m'$  and  $s'$  shows that error in frequency is generally less than 2%. The number of tables, though reduced, is large (at least 64 tables for each case).

We see that we could have more linear curves and yet with fewer values of  $m'$  and  $s'$  by choosing

$$\phi = \left( \frac{\rho_2 A_2}{\rho_1 A_1} \right)^{\frac{1}{4}}, \quad \alpha = \left( \frac{E_2 I_2}{E_1 I_1} \right)^{\frac{1}{4}}$$

both of which range from 0.6 to 1.0, at interval of 0.1 (the associated mass ratio and stiffness ratio vary from 0.1296 to 1.0). As concluded from previous choice of  $\mu$ , linear interpolation involving  $\mu$  at intervals of 0.1 is unsatisfactory. We thus reduce the interval to 0.05 and let  $\mu$  vary from 0.15 to 0.85, with the anticipation that this would cover most beams of practical interest. Linear interpolation involving any parameter yields error in frequency that is generally less than 1%. The advantage of raising the mass ratio and stiffness ratio to their one-fourth power is to provide linearity for  $\beta_1 - \phi$  and  $\beta_1 - \alpha$  curves, and also to reduce the number of tables.

#### (4) Modal Shape

Introducing the parameters

$$\phi = \left( \frac{\rho_2 A_2}{\rho_1 A_1} \right)^{\frac{1}{4}} \quad \text{and} \quad \alpha = \left( \frac{E_2 I_2}{E_1 I_1} \right)^{\frac{1}{4}}$$

it is seen, from the definition of  $\beta$ , that  $\beta_2/\beta_1 = \phi/\alpha$ . Let  $\theta = \phi/\alpha$ , then  $\beta_2 = \theta\beta_1$  and Eqs. (23) to (26) are rewritten as follows.

$$\begin{aligned} & C_1^{(1)} \sin \beta_1 \mu + C_3^{(1)} \sinh \beta_1 \mu + C_1^{(2)} (-\sin \beta_1 \theta \gamma) \\ & + C_3^{(2)} (-\sinh \beta_1 \theta \gamma) = 0 \end{aligned} \quad (23')$$

$$\begin{aligned} & C_1^{(1)} \cos \beta_1 \mu + C_3^{(1)} \cosh \beta_1 \mu + C_1^{(2)} (\theta \cos \beta_1 \theta \gamma) \\ & + C_3^{(2)} (\theta \cosh \beta_1 \theta \gamma) = 0 \end{aligned} \quad (24')$$

$$\begin{aligned} & C_1^{(1)} (-\sin \beta_1 \mu) + C_3^{(1)} \sinh \beta_1 \mu + C_1^{(2)} (\alpha^4 \theta^2 \sin \beta_1 \theta \gamma) \\ & + C_3^{(2)} (-\alpha^4 \theta^2 \sinh \beta_1 \theta \gamma) = 0 \end{aligned} \quad (25')$$

$$\begin{aligned} & C_1^{(1)} (-\cos \beta_1 \mu) + C_3^{(1)} \cosh \beta_1 \mu + C_1^{(2)} (-\alpha^4 \theta^3 \cos \beta_1 \theta \gamma) \\ & + C_3^{(2)} (\alpha^4 \theta^3 \cosh \beta_1 \theta \gamma) = 0 \end{aligned} \quad (26')$$

Eqs. (23') to (26') form a set of homogeneous equations in the four unknown constants  $C_1^{(1)}$ ,  $C_3^{(1)}$ ,  $C_1^{(2)}$  and  $C_3^{(2)}$ . By arbitrarily setting one of the non-zero constants  $C_1^{(1)}$  or  $C_3^{(1)}$  to one, Eqs. (23') to (26') are converted into a non-homogeneous set. Only three equations are required to solve for the other three constants, and we choose the first three. Solving these equations by Cramer's rule, the constant  $C_3^{(1)}$ ,  $C_1^{(2)}$  and

$C_3^{(2)}$  may be obtained. Thus the modal shape is determined leaving the amplitude arbitrary. The solution of the frequency equation consists of an infinite sequence of discrete eigenvalues related to the natural frequencies  $\omega_n$  ( $n = 1, 2, 3, \dots$ ). To each of the natural frequencies corresponds an eigenfunction  $A_n Y_n(x)$  ( $n = 1, 2, 3, \dots$ ), where  $A_n$  can be looked upon as the arbitrary amplitude and  $Y_n(x)$  is the unique modal shape associated with the frequency  $\omega_n$ . The function  $A_n Y_n(x)$  are the natural modes, which upon normalization become the normal modes of vibration. The normal modes  $Y_n(x)$  and the associated natural frequencies  $\omega_n$  evidently depend on  $\mu$ , the mass ratio ( $\rho_2 A_2 / \rho_1 A_1$ ), and the bending stiffness ratio ( $E_2 I_2 / E_1 I_1$ ) and, of course, the boundary conditions.

There may arise cases for which  $C_1^{(1)}$  is zero. This situation will manifest itself in the obtaining of infinite or indeterminate values for the other constants. In such a case, since both  $C_1^{(1)}$  and  $C_3^{(1)}$  cannot both equal zero, it is evident that  $C_3^{(1)}$  should be set equal to one. Solutions for  $C_1^{(2)}$  and  $C_3^{(2)}$  can be obtained by neglecting the right hand side of the non-homogeneous equations provided, setting  $C_3^{(1)}$  equal to one, and solving for  $C_1^{(2)}$  and  $C_3^{(2)}$  from the first two equations of the new non-homogeneous set.

(5) Numerical Computations

The eigenvalues  $\beta_1$  associated with the left segment of the beam are obtained by solving the frequency equation

$$|C_{ij}| = 0 \quad (27)$$

using the method of false position (regula falsi). The first four roots of the frequency equations were computed for various values of  $\mu$ ,  $\phi$  and  $\alpha$ . The numerical computations were done on the IBM 360/65 computer of the Computing Centre, University of Ottawa.

For fixed values of  $\mu$ ,  $\phi$  and  $\alpha$ , the determinant (27) is a function of  $\beta_1$  alone. It is evaluated starting from an initial value of  $\beta_1$  and proceeding at discrete intervals  $\Delta\beta_1$  until the first four roots are computed. A change of sign of the determinant across a  $\Delta\beta_1$  interval indicates a root in that interval. An approximation to a root of the equation  $|C_{ij}| = 0$  is

$$\beta = \beta_0 + \frac{(\Delta\beta_1) |f(\beta_0)|}{|f(\beta_0)| + |f(\beta_1)|} \quad (28)$$

The process is repeated until a root is obtained within a prescribed accuracy. Additional roots are obtained by re-entering successively into the iteration with a new initial value of  $\beta_1$  just above the last computed root.

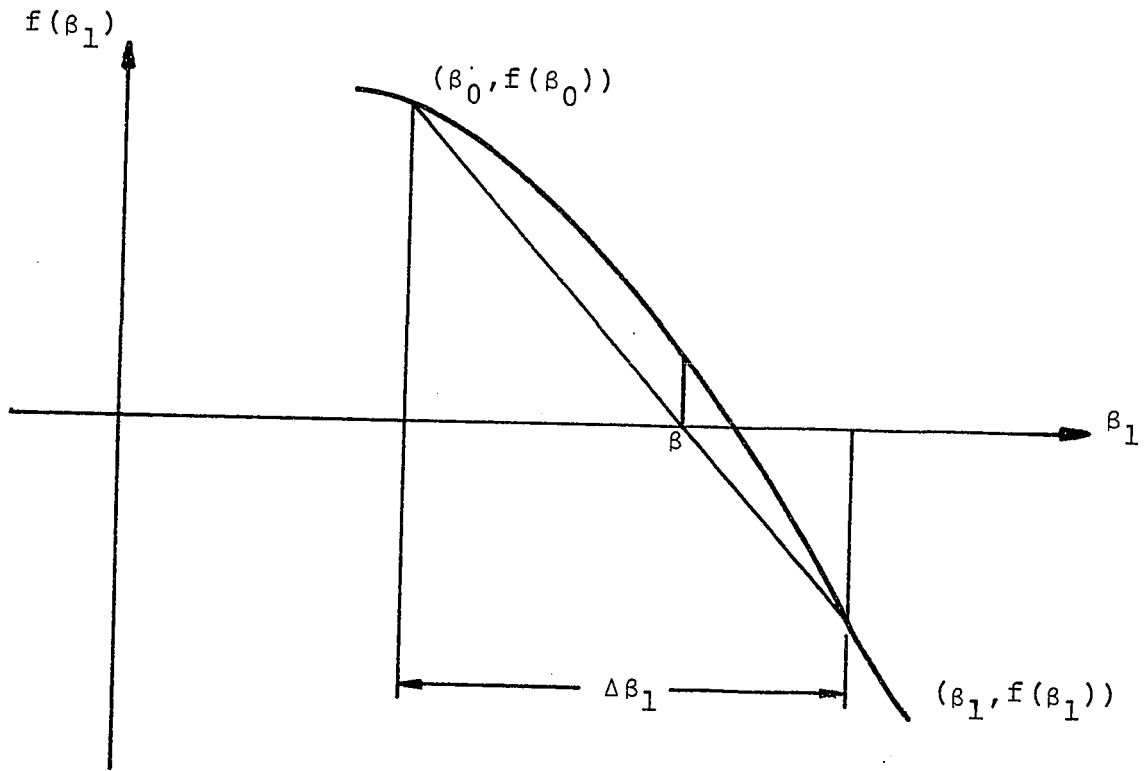


Fig. 7.

The scanning interval cannot be too large, if it is so large that two roots may exist between two successive evaluation points, no change of sign of the determinant will be observed, and consequently the two roots will be missed.

The following range of parameters was used in the computation:

$$0.15 \leq \mu \leq 0.85, \quad 0.6 \leq \phi \leq 1.0, \quad 0.6 \leq \alpha \leq 1.0,$$

$$0.15 \leq \mu \leq 0.85, \quad 0.6 \leq 1/\phi \leq 1.0, \quad 0.6 \leq 1/\alpha \leq 1.0.$$

The accuracy of the computation, i.e. the difference of

the two successive approximations, is  $10^{-6}$ .

(6) Check for Correctness

It is important that the output be examined for its correctness before it is accepted for use.

When  $\phi = 1$  and  $\alpha = 1$ , for any value of  $\mu$ , the eigenvalues  $\beta_1$  of a beam with simple support at each end are given by  $n\pi$  ( $n = 1, 2, 3, 4$ ), the classical values. This is the case, as the values  $\phi = 1$  and  $\alpha = 1$  represent a uniform beam of constant geometry and material property throughout its length.

For any combination of values of  $\phi$  and  $\alpha$ , when  $\mu = 1$ , the eigenvalues are given by  $n\pi$  ( $n = 1, 2, 3, 4$ ), which is the case as the value  $\mu = 1$  represents a uniform beam of properties  $\rho_1, A_1, E_1$ , and  $I_1$ . Whereas for any value of  $\phi$  and  $\alpha$ , when  $\mu = 0$ , the eigenvalues are given by  $(\phi/\alpha)\beta_1$ . The value  $\mu = 0$  represents a uniform beam of properties  $\rho_2, A_2, E_2$ , and  $I_2$ , whose eigenvalues are  $\beta_2$ . From the definition of  $\beta$ , we have  $\beta_2/\beta_1 = \phi/\alpha$ , whence  $\beta_2 = (\phi/\alpha)\beta_1$ . For example, when  $\phi = 0.6, \alpha = 0.8, \mu = 0.0, {}^1\beta_1 = 4.1888$ , then  ${}^1\beta_2 = (0.6/0.8)(4.1888) = 3.1416 = \pi$ , where the superscript denotes the mode number. As another example, when  $\phi = 0.8, \alpha = 0.6, \mu = 0.0, {}^2\beta_1 = 4.7124$ , hence  ${}^2\beta_2 = (0.8/0.6)(4.7124) = 6.2832 = 2\pi$ .

As another check, we must have, for any value of  $\mu$ ,

the eigenvalues approaching the classical values of  $n\pi$  ( $n = 1, 2, 3, 4$ ) when (i)  $\phi = 1$  and  $\alpha \rightarrow 1$ , and (ii)  $\alpha = 1$  and  $\phi \rightarrow 1$ . Curves are plotted for verifying the above mentioned limits (see Figs. 8 and 9).

Similarly, the outputs of the other three cases are checked for their correctness.

#### (7) Presentation of Results

One should always bear in mind that the elementary theory used is valid for the lower modes only, as the mode number increases the effects of shearing deformation and rotatory inertia become significant.

The first four eigenvalues are presented in the tables. Based on the definition of  $\beta_1$ , we may write

$$\omega = \frac{\beta_1^2}{L^2} \sqrt{\frac{E_1 I_1}{\rho_1 A_1}}$$

and, having obtained a value for  $\beta_1$  from the tables we may obtain the beam frequency from the expression

$$f = \frac{1}{2\pi} \frac{\beta_1^2}{L^2} \sqrt{\frac{E_1 I_1}{\rho_1 A_1}} \quad (29)$$

In order to minimize storage requirements it has been decided to prepare the tables on the assumption that the stiffness ratio and mass ratio for any beam will both be either greater or less than 1.0. Combinations where one parameter is greater than 1.0 and the other is

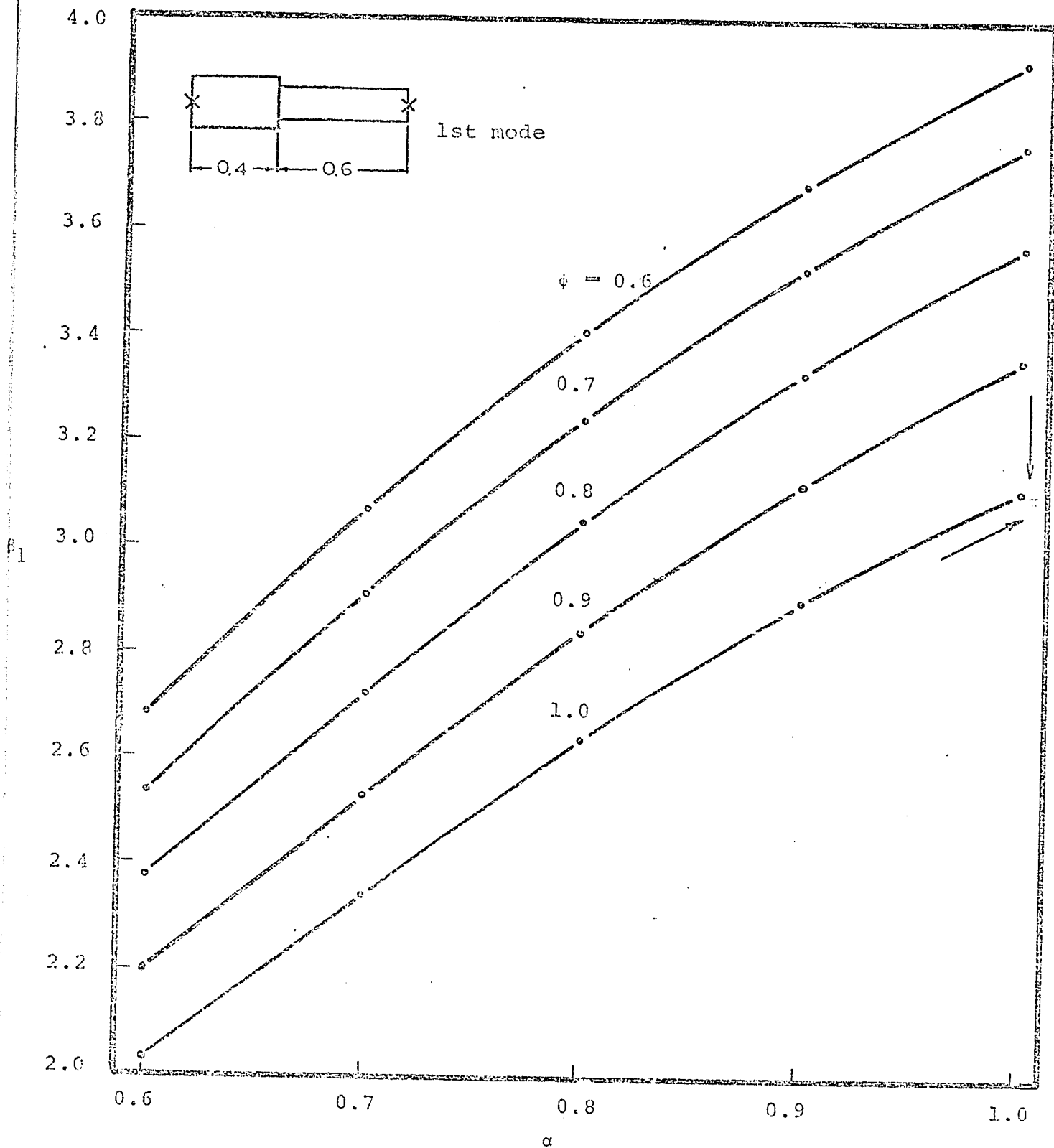


Fig. 8.

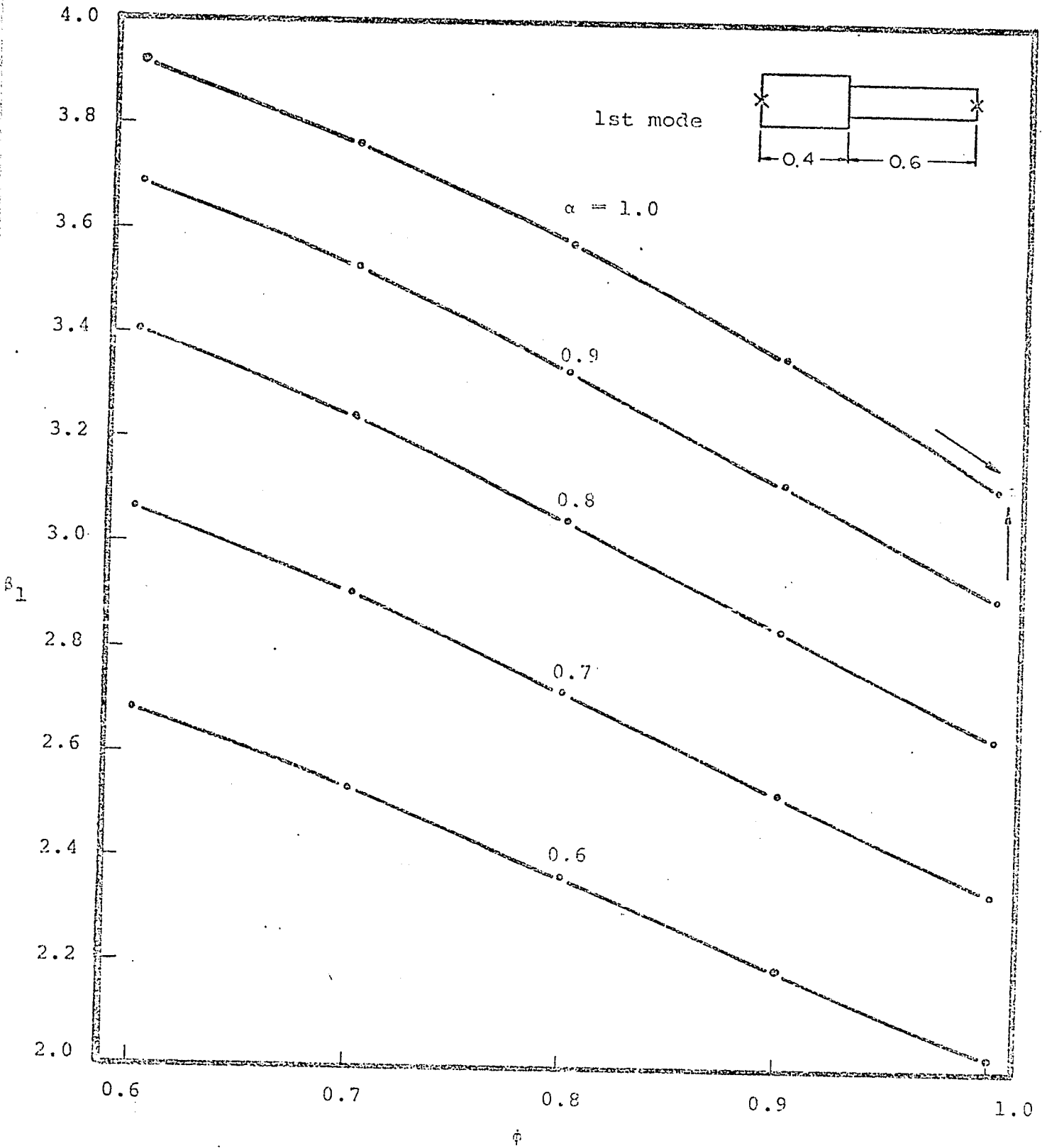


Fig. 9.

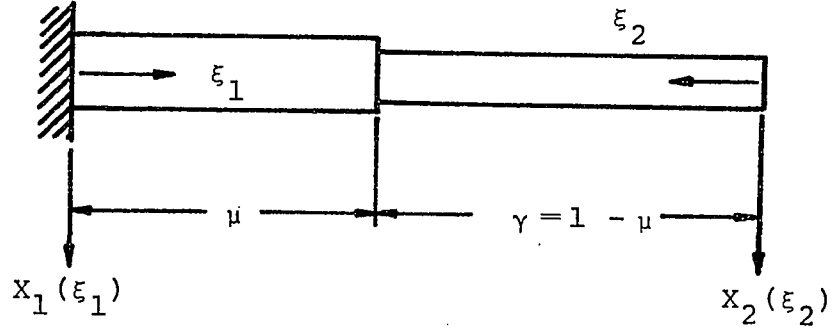
less than 1.0 are expected to be rare and are not given in the tables.

In the tables the three parameters of interest are  $\mu$ ,  $\phi$ , and  $\alpha$ . The tables are provided with values of  $\mu$  varying from 0.15 to 0.85 in increments of 0.05, while  $\phi$  and  $\alpha$  range from 0.6 to 1.0 in intervals of 0.1. The same is true for the inverse of the parameters  $1/\phi$  and  $1/\alpha$ . Linear interpolation may be used for analyzing beams with parameter values falling between those for which eigenvalues are tabulated. For each case, linear interpolation involving  $\mu$  has generally been found to give frequencies with maximum error not greater than about 1%. On the other hand, maximum errors encountered through interpolation involving  $\phi$  or  $\alpha$  (also  $1/\phi$  or  $1/\alpha$ ) will be less than about 1%.

Note that for beams that possess symmetry in boundary conditions, no inverse of the parameter  $1/\phi$  and  $1/\alpha$  is needed. Where this symmetry does not exist we will be interested in eigenvalues for the full range of  $\phi$  and  $\alpha$ , also that of  $1/\phi$  and  $1/\alpha$ .

In presenting the data, each case is given a case number. The same number is utilized in the corresponding table. For each case the modal shape expressions and the three non-homogeneous equations which must be solved to obtain the constants appearing in the expressions are given.

Case 1. Beam clamped at one end and free at the other.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 + C_2^{(1)} (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

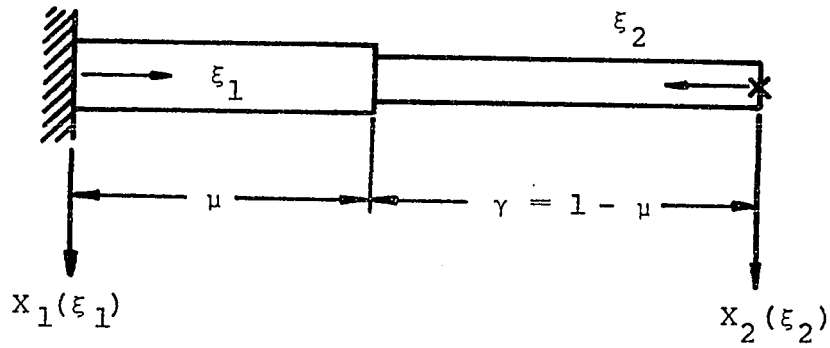
$$X_2(\xi_2) = C_1^{(2)} (\sin \beta_1 \theta \xi_2 + \sinh \beta_1 \theta \xi_2) + C_2^{(2)} (\cos \beta_1 \theta \xi_2 + \cosh \beta_1 \theta \xi_2)$$

$$C_2^{(1)} (\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_1^{(2)} (-\sin \beta_1 \theta \gamma - \sinh \beta_1 \theta \gamma) + C_2^{(2)} (-\cos \beta_1 \theta \gamma - \cosh \beta_1 \theta \gamma) = \sinh \beta_1 \mu - \sin \beta_1 \mu$$

$$C_2^{(1)} (-\sin \beta_1 \mu - \sinh \beta_1 \mu) + C_1^{(2)} \theta (\cos \beta_1 \theta \gamma + \cosh \beta_1 \theta \gamma) + C_2^{(2)} \theta (-\sin \beta_1 \theta \gamma + \sinh \beta_1 \theta \gamma) = \cosh \beta_1 \mu - \cos \beta_1 \mu$$

$$C_2^{(1)} (-\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_1^{(2)} \alpha^4 \theta^2 (\sin \beta_1 \theta \gamma - \sinh \beta_1 \theta \gamma) + C_2^{(2)} \alpha^4 \theta^2 (\cos \beta_1 \theta \gamma - \cosh \beta_1 \theta \gamma) = \sin \beta_1 \mu + \sinh \beta_1 \mu$$

Case 2. Beam clamped at one end and with simple support at the other.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 + C_2^{(1)} (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = C_1^{(2)} \sin \beta_1 \theta \xi_2 + C_3^{(2)} \sinh \beta_1 \theta \xi_2$$

$$C_2^{(1)} (\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_1^{(2)} (-\sin \beta_1 \theta \gamma) +$$

$$C_3^{(2)} (-\sinh \beta_1 \theta \gamma) = \sinh \beta_1 \mu - \sin \beta_1 \mu$$

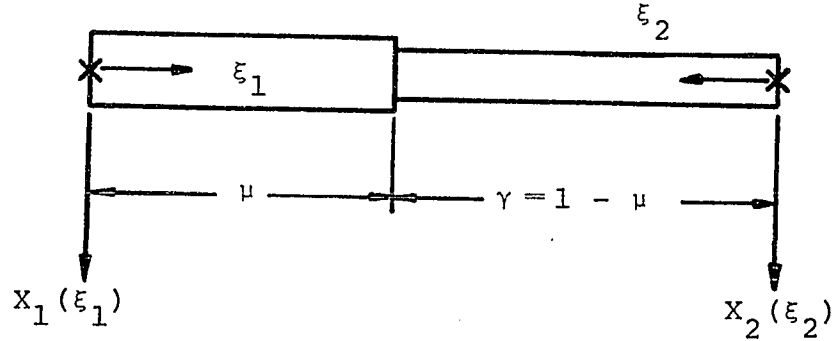
$$C_2^{(1)} (-\sin \beta_1 \mu - \sinh \beta_1 \mu) + C_1^{(2)} \theta (\cos \beta_1 \theta \gamma) +$$

$$C_3^{(2)} \theta (\cosh \beta_1 \theta \gamma) = \cosh \beta_1 \mu - \cos \beta_1 \mu$$

$$C_2^{(1)} (-\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_1^{(2)} \alpha^4 \theta^2 (\sin \beta_1 \theta \gamma) +$$

$$C_3^{(1)} \alpha^4 \theta^2 (-\sinh \beta_1 \theta \gamma) = \sinh \beta_1 \mu + \sin \beta_1 \mu$$

Case 3. Beam with simple support at each end.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 + C_3^{(1)} \sinh \beta_1 \xi_1$$

$$X_2(\xi_2) = C_1^{(2)} \sin \beta_1 \theta \xi_2 + C_3^{(2)} \sinh \beta_1 \theta \xi_2$$

$$C_3^{(1)} \sinh \beta_1 \mu + C_1^{(2)} (-\sin \beta_1 \theta \gamma) + C_3^{(2)} (-\sinh \beta_1 \theta \gamma)$$

$$= -\sin \beta_1 \mu$$

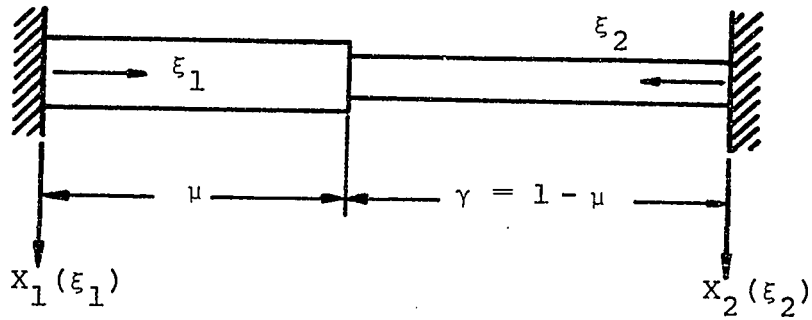
$$C_3^{(1)} \cosh \beta_1 \mu + C_1^{(2)} \theta (\cos \beta_1 \theta \gamma) + C_3^{(2)} \theta (\cosh \beta_1 \theta \gamma)$$

$$= -\cos \beta_1 \mu$$

$$C_3^{(1)} \sinh \beta_1 \mu + C_1^{(2)} \alpha^4 \theta^2 (\sin \beta_1 \theta \gamma) + C_3^{(2)} \alpha^4 \theta^2 (-\sinh \beta_1 \theta \gamma)$$

$$= \sin \beta_1 \mu$$

Case 4. Beam clamped at each end.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 + C_2^{(1)} (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = C_1^{(2)} (\sin \beta_1 \theta \xi_2 - \sinh \beta_1 \theta \xi_2) + C_2^{(2)} (\cos \beta_1 \theta \xi_2 - \cosh \beta_1 \theta \xi_2)$$

$$C_2^{(1)} (\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_1^{(2)} (-\sin \beta_1 \theta \gamma + \sinh \beta_1 \theta \gamma) +$$

$$C_2^{(2)} (-\cos \beta_1 \theta \gamma + \cosh \beta_1 \theta \gamma) = \sinh \beta_1 \mu - \sin \beta_1 \mu$$

$$C_2^{(1)} (-\sin \beta_1 \mu - \sinh \beta_1 \mu) + C_1^{(2)} \theta (\cos \beta_1 \theta \gamma - \cosh \beta_1 \theta \gamma) +$$

$$C_2^{(2)} \theta (-\sin \beta_1 \theta \gamma - \sinh \beta_1 \theta \gamma) = \cosh \beta_1 \mu - \cos \beta_1 \mu$$

$$C_2^{(2)} (-\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_1^{(2)} \alpha^4 \theta^2 (\sin \beta_1 \theta \gamma + \sinh \beta_1 \theta \gamma) +$$

$$C_2^{(2)} \alpha^4 \theta^2 (\cos \beta_1 \theta \gamma + \cosh \beta_1 \theta \gamma) = \sin \beta_1 \mu + \sinh \beta_1 \mu$$

Examples are given to illustrate the method to use the tables for obtaining natural frequencies of a beam.

Illustrative Example 1. A solid shaft has an abrupt change in cross section as shown in Fig. 10. Determine the fundamental mode of vibration. Take  $\gamma' = 0.283 \text{ lb/in}^3$  and  $E = 30 \times 10^6 \text{ psi}$ .

Solution:

$$\mu = \frac{45.5}{70} = 0.65$$

$$\phi = \left( \frac{\rho_2 A_2}{\rho_1 A_1} \right)^{\frac{1}{4}} = \left[ \frac{\left( \frac{\pi}{4} \right) \left( \frac{13}{4} \right)^2}{\left( \frac{\pi}{4} \right) (4)^2} \right]^{\frac{1}{4}}$$
$$= \sqrt{\frac{13}{16}} = 0.9015$$

$$\alpha = \left( \frac{E_2 I_2}{E_1 I_1} \right)^{\frac{1}{4}} = \left[ \frac{\left( \frac{\pi}{64} \right) \left( \frac{13}{4} \right)^4}{\left( \frac{\pi}{64} \right) (4)^4} \right]^{\frac{1}{4}}$$
$$= \frac{13}{16} = 0.8125$$

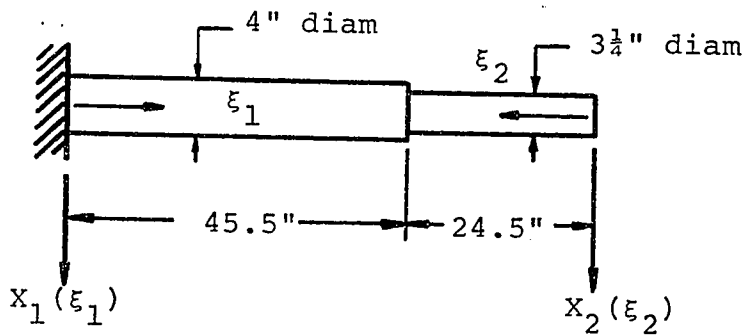


Fig. 10.

As an approximation to the value of  ${}^1\beta_1$  without linear interpolation, we take  $\mu = 0.65$ ,  $\phi = 0.9$ ,  $\alpha = 0.8$ .

From Table 8.1, we have

$${}^1\beta_1 = 2.035$$

$$f = \frac{1}{2\pi} \frac{{}^1\beta_1^2}{L^2} \sqrt{\frac{E_1 I_1}{\rho_1 A_1}} = \frac{1}{2\pi} \left(\frac{2.035}{70}\right)^2 \sqrt{\frac{(30 \times 10^6) \left(\frac{\pi}{64}\right) (4)^4}{\left(\frac{0.283}{386}\right) \left(\frac{\pi}{4}\right) (4)^2}}$$

$$= 27.2, \text{ Hz}$$

Substitution of the values of  ${}^1\beta_1$ ,  $\mu$ ,  $\gamma$ , and  $\theta$  into the three equations given yields the following equations,

$$-1.7640C_2^{(1)} - 1.5854C_1^{(2)} - 2.0324C_2^{(2)} = 0.7739$$

$$-2.7126C_2^{(1)} + 2.6881C_1^{(2)} + 0.2176C_2^{(2)} = 1.7640$$

$$-2.2554C_2^{(1)} - 0.0883C_1^{(2)} - 0.3353C_2^{(2)} = 2.7126$$

Solving,

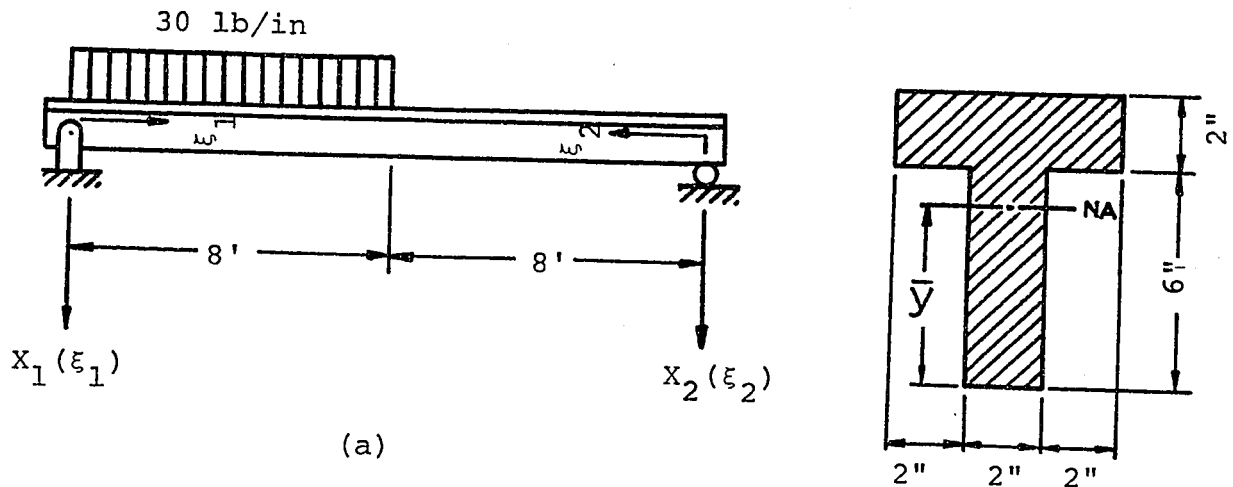
$$C_2^{(1)} = 1.3927, \quad C_1^{(2)} = -0.8713, \quad C_2^{(2)} = 1.5076$$

The modal shapes are

$$X_1(\xi_1) = \sin(2.035 \xi_1) - \sinh(2.035 \xi_1) + (1.3927) \\ \times \left[ \cos(2.035 \xi_1) - \cosh(2.035 \xi_1) \right]$$

$$X_2(\xi_2) = (-0.8713) \left[ \sin(2.258 \xi_2) + \sinh(2.258 \xi_2) \right] + \\ (1.5076) \left[ \cos(2.258 \xi_2) + \cosh(2.258 \xi_2) \right]$$

Illustrative Example 2. The simply supported beam of Fig. 11(a) has the cross section shown in Fig. 11(b). Determine the fundamental frequency and modal shape of vibration. Assume mass does not part from beam. Take  $\gamma' = 0.283 \text{ lb/in}^3$  and  $E = 30 \times 10^6 \text{ psi}$ :



Solution:

Fig. 11.

(b)

$$\mu = 0.5$$

$$\alpha = \left( \frac{E_2 I_2}{E_1 I_1} \right)^{\frac{1}{4}} = 1.0$$

$$A_1 = A_2 = (2 \times 6) + (2 \times 6) = 24, \text{ in}^2$$

$$g \rho_1 A_1 = (0.283)(24) + 30 = 6.792 + 30 = 36.792, \text{ lb/in}$$

$$g \rho_2 A_2 = (0.283)(24) = 6.792, \text{ lb/in}$$

$$\phi = \left( \frac{\rho_2 A_2}{\rho_1 A_1} \right)^{\frac{1}{4}} = \left( \frac{6.792}{36.792} \right)^{\frac{1}{4}} = (0.1846)^{\frac{1}{4}} = 0.6555$$

From Table 8.3:

$\phi$	$\alpha$	$\mu$	${}^1\beta_1$
0.6	1.0	0.5	3.599
0.7	1.0	0.5	3.525
0.6555	1.0	0.5	3.558

Location of centroidal axis:

$$24\bar{y} = (12)(7) + (12)(3)$$

$$\bar{y} = 5, \text{ in. from bottom fibre}$$

Moment of inertia:

$$I_1 = I_2 = \left( \frac{1}{12} \times 6 \times 2^3 + 12 \times 2^2 \right) + \left( \frac{1}{12} \times 2 \times 6^3 + 12 \times 2^2 \right)$$

$$= (4 + 48) + (36 + 48) = 136, \text{ in}^4$$

$$f = \frac{1}{2\pi} \frac{{}^1\beta_1^2}{L^2} \sqrt{\frac{E_1 I_1}{\rho_1 A_1}} = \frac{1}{2\pi} \left( \frac{3.558}{16 \times 12} \right)^2 \sqrt{\frac{(30 \times 10^6)(136)}{\left( \frac{36.792}{386} \right)}}$$

$$= 11.31, \text{ Hz}$$

The following equations are obtained by substituting the values of  ${}^1\beta_1$ ,  $\mu$ ,  $\gamma$ , and  $\theta$  into the equations provided.

$$2.8773C_3^{(1)} - 0.9192C_1^{(2)} - 1.4489C_3^{(2)} = -0.9784$$

$$3.0461C_3^{(1)} + 0.2581C_1^{(2)} + 1.1540C_3^{(2)} = 0.2066$$

$$2.8773C_3^{(1)} + 0.3950C_1^{(2)} - 0.6226C_3^{(2)} = 0.9784$$

Solving, we have

$$c_3^{(1)} = 0.04219, c_1^{(2)} = 1.6832, c_3^{(2)} = -0.3088$$

Therefore, the fundamental modal shape is

$$X_1(\xi_1) = \sin(3.558 \xi_1) + (0.04219) \sinh(3.558 \xi_1)$$

$$X_2(\xi_2) = (1.6832) \sin(2.332 \xi_2) + \\ (-0.3086) \sinh(2.332 \xi_2)$$

CHAPTER 2  
DOUBLE SPAN BEAM  
WITH SINGLE DISCONTINUITY  
IN CROSS SECTION PROPERTIES

(8) General Consideration

The family of beams under discussion is that whose mass and stiffness can vary from span to span but are kept constant within the span. Also there is an intermediate simple support located at the discontinuity.

The method of analysis for this family of beams is the same as that for the family of beams discussed in Chapter 1. For this family of beams, we see that it is possible to eliminate two of the constants appearing in the modal shape expressions.

For illustrative purposes, the case of a beam with simple support at each end will be considered in detail (see Fig. 12). The non-dimensional length of the first span is designated as  $\mu$  (ratio of span to beam length), where in general,  $0 < \mu < 1$ . The remaining span has length  $\gamma = 1 - \mu$ . The definition of  $X_i(\xi_i)$  and  $\xi_i$  are as given in Chapter 1.

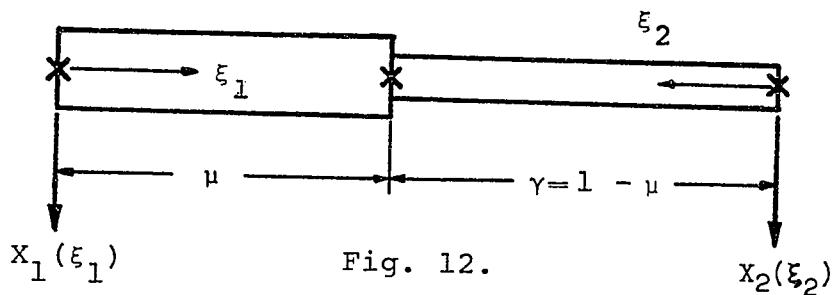


Fig. 12.

In order to simplify the problem of enforcing the boundary conditions,  $\xi_1$  and  $\xi_2$  are measured in each span from the beam outer end. As derived in Chapter 1, the first and second span deflections are

$$X_1(\xi_1) = C_1^{(1)} \sin \beta_1 \xi_1 + C_3^{(1)} \sinh \beta_1 \xi_1 \quad (21)$$

and

$$X_2(\xi_2) = C_1^{(2)} \sin \beta_2 \xi_2 + C_3^{(2)} \sinh \beta_2 \xi_2 \quad (22)$$

respectively, where

$$\beta_1^4 = \frac{\rho_1 A_1}{E_1 I_1} \omega^2 L^4, \quad \beta_2^4 = \frac{\rho_2 A_2}{E_2 I_2} \omega^2 L^4$$

There are evidently four boundary conditions required. Two are obtained by requiring zero displacement at the intermediate support, and two more are obtained by requiring continuity of beam slope and bending moment across the intermediate support, i.e.

$$(i) \quad X_1(\mu) = 0 \qquad (ii) \quad X_2(\gamma) = 0$$

$$(iii) \quad \left. \frac{dX_1(\xi_1)}{d\xi_1} \right|_{\xi_1=\mu} = - \left. \frac{dX_2(\xi_2)}{d\xi_2} \right|_{\xi_2=\gamma}$$

$$(iv) \quad E_1 I_1 \left. \frac{d^2 X_1(\xi_1)}{d\xi_1^2} \right|_{\xi_1=\mu} = E_2 I_2 \left. \frac{d^2 X_2(\xi_2)}{d\xi_2^2} \right|_{\xi_2=\gamma}$$

From boundary condition (i), we have

$$C_3^{(1)} = - \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} C_1^{(1)}$$

while boundary condition (ii) yields

$$C_3^{(2)} = - \frac{\sin \beta_2 \gamma}{\sinh \beta_2 \gamma} C_1^{(2)}$$

Thus Eqs. (21) and (22) become

$$X_1(\xi_1) = C_1^{(1)} \left( \sin \beta_1 \xi_1 - \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \sinh \beta_1 \xi_1 \right) \quad (21')$$

and

$$X_2(\xi_2) = C_1^{(2)} \left( \sin \beta_2 \xi_2 - \frac{\sin \beta_2 \gamma}{\sinh \beta_2 \gamma} \sinh \beta_2 \xi_2 \right) \quad (22')$$

Boundary conditions (iii) and (iv) imply that

$$C_1^{(1)} \left( \cos \beta_1 \mu - \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \cosh \beta_1 \mu \right) + C_1^{(2)} \left( \frac{\beta_2}{\beta_1} \right) \\ \times \left( \cos \beta_2 \gamma - \frac{\sin \beta_2 \gamma}{\sinh \beta_2 \gamma} \cosh \beta_2 \gamma \right) = 0 \quad (30)$$

and

$$C_1^{(1)} \sin \beta_1 \mu + C_1^{(2)} \left[ - \left( \frac{E_2 I_2}{E_1 I_1} \right) \left( \frac{\beta_2}{\beta_1} \right)^2 \sin \beta_2 \gamma \right] = 0 \quad (31)$$

The determinant formed by the coefficients of Eqs. (30) and (31), when set equal to zero, constitutes the

frequency equation for this particular beam. From the definition of  $\beta_1$  and  $\beta_2$ , we have

$$\beta_2/\beta_1 = \left( \frac{\rho_2 A_2}{\rho_1 A_1} \cdot \frac{E_1 I_1}{E_2 I_2} \right)^{\frac{1}{4}}$$

Thus, given any values of  $\mu$  ( $0 < \mu < 1$ ), the mass ratio ( $\rho_2 A_2 / \rho_1 A_1$ ) and the bending stiffness ratio ( $E_2 I_2 / E_1 I_1$ ), we can determine the eigenvalues  $\beta_1$  or  $\beta_2$  of the system by requiring that the determinant of the coefficient matrix for Eqs. (30) and (31) must vanish.

The same set of parameters is used for evaluating the eigenvalues  $\beta_1$ , i.e.

$$\phi \equiv \left( \frac{\rho_2 A_2}{\rho_1 A_1} \right)^{\frac{1}{4}} \quad \alpha \equiv \left( \frac{E_2 I_2}{E_1 I_1} \right)^{\frac{1}{4}}$$

both of which range from 0.6 to 1.0 at interval of 0.1, and  $\mu$  varies from 0.15 to 0.85 at interval of 0.05. Linear interpolation involving  $\mu$ ,  $\phi$  or  $\alpha$  shows that associated error in frequency is generally below 1%.

In determining expressions for modal shapes, one of the non-zero coefficients may be arbitrarily set equal to unity. We choose to set  $C_1^{(1)}$  equal to unity. From Eq. (21') we may write

$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \frac{\sin \beta_1 \mu}{\sinh \beta \mu} \sinh \beta_1 \xi_1 \quad (32)$$

Setting  $C_1^{(1)}$  equal to unity is equivalent to dividing both Eqs. (21') and (22') by  $C_1^{(1)}$ . Eq. (22') may be written as

$$X_2(\xi_2) = \frac{C_1^{(2)}}{C_1^{(1)}} \left( \sin \beta_2 \xi_2 - \frac{\sin \beta_2 \gamma}{\sinh \beta_2 \gamma} \sinh \beta_2 \xi_2 \right) \quad (33)$$

where, from Eq. (30),

$$\frac{C_1^{(2)}}{C_1^{(1)}} = - \frac{\beta_1}{\beta_2} \cdot \frac{\sinh \beta_2 \gamma}{\sinh \beta_1 \mu} \cdot \frac{\cos \beta_1 \mu \sinh \beta_1 \mu - \sin \beta_1 \mu \cosh \beta_1 \mu}{\cos \beta_2 \gamma \sinh \beta_2 \gamma - \sin \beta_2 \gamma \cosh \beta_2 \gamma}$$

Introducing the parameters,  $\beta_2/\beta_1 = \phi/\alpha \equiv \theta$ . Hence

$$\frac{C_1^{(2)}}{C_1^{(1)}} = \frac{1}{\theta} \cdot \frac{\sinh \beta_1 \theta \gamma}{\sinh \beta_1 \mu} \cdot \frac{\sin \beta_1 \mu \cosh \beta_1 \mu - \cos \beta_1 \mu \sinh \beta_1 \mu}{\cos \beta_1 \theta \gamma \sinh \beta_1 \theta \gamma - \sin \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma} \quad (34)$$

Following the same procedure, the frequency equation and expressions for the modal shapes may be obtained for the remaining choice of boundary conditions.

(9) Check for Correctness

It is important that the output be examined for its correctness before it is accepted for use.

For arbitrary value of  $\mu$ , when  $\phi = 1$  and  $\alpha = 1$ , we have uniform double span beams of constant geometry and material property throughout its length. The intermediate simple support is located at a distance of  $\mu L$  from the left support. The eigenvalues  $\beta_1$  are checked against those given in [6].

As another check, when  $\phi = 1$  and  $\alpha = 1$ , we must have the eigenvalues approaching the well-known values for single span beam as the intermediate support approaches the beam ends. For example, in Case (7), the eigenvalues approach those of a clamped-hinged beam and hinged-clamped beam as  $\mu$  approaches zero and one, respectively.

For any combination of values of  $\phi$  and  $\alpha$ , when  $\mu$  approaches 1, the eigenvalues approach the classical values as given in (3), Appendix A. This is the case as the value  $\mu = 1$  represents a uniform beam of properties  $\rho_1, A_1, E_1$  and  $I_1$ . Whereas for any value of  $\phi$  and  $\alpha$ , when  $\mu$  approaches zero, the eigenvalues approach those given in (3), Appendix A. The value  $\mu = 0$  represents a uniform beam of properties  $\rho_2, A_2, E_2$ , and  $I_2$ , whose eigenvalues are  $\beta_2 = (\phi/\alpha)\beta_1$ .

Finally, for any value of  $\mu$ , we must have the eigenvalues approaching those given in [6], when (i)  $\phi = 1$  and  $\alpha \rightarrow 1$ , and (ii)  $\alpha = 1$  and  $\phi \rightarrow 1$ .

Curves are plotted for verifying the above mentioned limits.

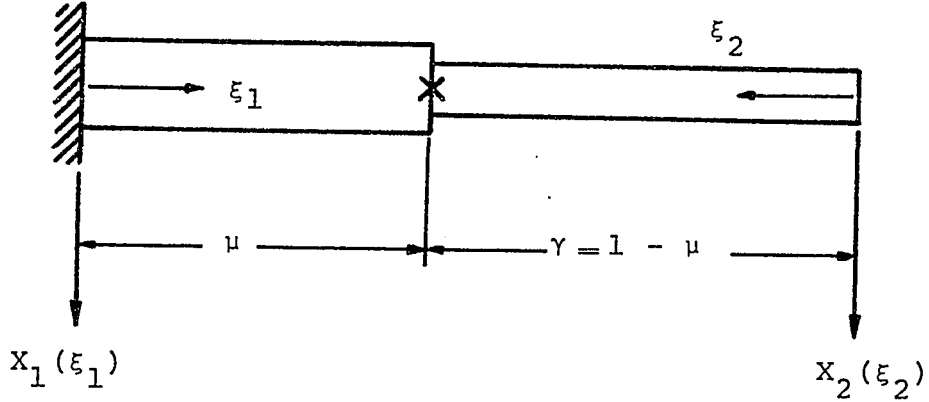
In a similar way, the outputs of the other cases are checked for their correctness.

(10) Presentation of Results.

It is readily shown that there exists five practical cases for double span beams of the type under consideration. Each case is given a case number. The same number is utilized in the corresponding tables. For each case the modal shape expressions are given.

The discussion given in Section 7, Chapter 1, applies to this family of beams.

Case 5. Beam clamped at one end, free at the other,  
and with simple support at the discontinuity.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 +$$

$$\left( \frac{\sinh \beta_1 \mu - \sin \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \right) (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

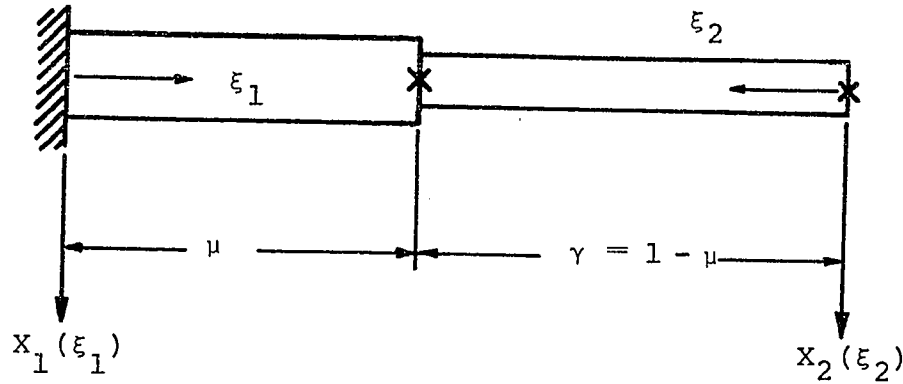
$$X_2(\xi_2) = \gamma_1 \left[ \sin \beta_1 \theta \xi_2 + \sinh \beta_1 \theta \xi_2 -$$

$$\left( \frac{\sin \beta_1 \theta \gamma + \sinh \beta_1 \theta \gamma}{\cos \beta_1 \theta \gamma + \cosh \beta_1 \theta \gamma} \right) (\cos \beta_1 \theta \xi_2 + \cosh \beta_1 \theta \xi_2) \right]$$

where

$$\gamma_1 = \frac{1}{\theta} \cdot \frac{\cos \beta_1 \theta \gamma + \cosh \beta_1 \theta \gamma}{\cosh \beta_1 \mu - \cos \beta_1 \mu} \cdot \frac{1 - \cos \beta_1 \mu \cosh \beta_1 \mu}{1 + \cos \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma}$$

Case 6. Beam clamped at one end, simply supported at the other, and with simple support at the discontinuity.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 + \left( \frac{\sinh \beta_1 \mu - \sin \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \right) \\ \times (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

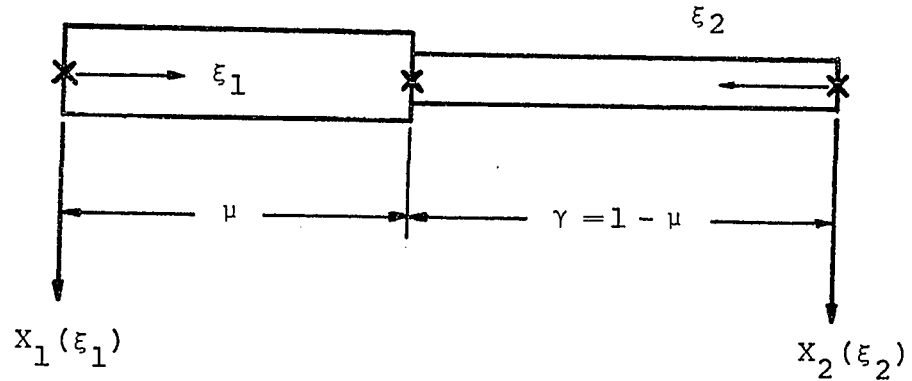
$$X_2(\xi_2) = \gamma_1 \left( \sin \beta_1 \theta \xi_2 - \left( \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \sinh \beta_1 \theta \xi_2 \right)$$

where

$$\gamma_1 = \frac{1}{\theta} \cdot \frac{2(1 - \cos \beta_1 \mu \cosh \beta_1 \mu)}{\cosh \beta_1 \mu - \cos \beta_1 \mu} \cdot$$

$$\times \frac{\sinh \beta_1 \theta \gamma}{\cos \beta_1 \theta \gamma \sinh \beta_1 \theta \gamma - \sin \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma}$$

Case 7. Beam with simple support at each end and with simple support at the discontinuity.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \sinh \beta_1 \xi_1$$

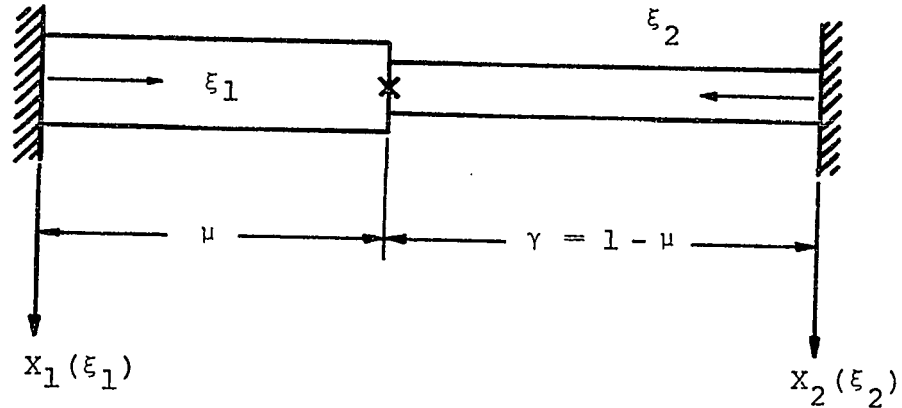
$$X_2(\xi_2) = \gamma_1 \left( \sin \beta_1 \theta \xi_2 - \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \sinh \beta_1 \theta \xi_2 \right)$$

where

$$\gamma_1 = \frac{1}{\theta} \cdot \frac{\sinh \beta_1 \theta \gamma}{\sinh \beta_1 \mu} \cdot$$

$$\times \frac{\sin \beta_1 \mu \cosh \beta_1 \mu - \cos \beta_1 \mu \sinh \beta_1 \mu}{\cos \beta_1 \theta \gamma \sinh \beta_1 \theta \gamma - \sin \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma}$$

Case 8. Beam clamped at each end and with simple support at the discontinuity.



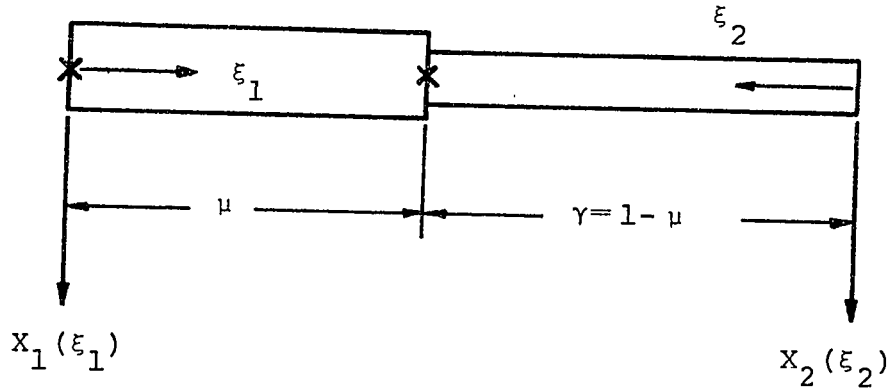
$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 + \left( \frac{\sinh \beta_1 \mu - \sin \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \right) \\ \times (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = \gamma_1 \left\{ \sin \beta_1 \theta \xi_2 - \sinh \beta_1 \theta \xi_2 + \left( \frac{\sinh \beta_1 \theta \gamma - \sin \beta_1 \theta \gamma}{\cos \beta_1 \theta \gamma - \cosh \beta_1 \theta \gamma} \right) (\cos \beta_1 \theta \xi_2 - \cosh \beta_1 \theta \xi_2) \right\}$$

where

$$\gamma_1 = \frac{1}{\theta} \cdot \frac{\cos \beta_1 \theta \gamma - \cosh \beta_1 \theta \gamma}{\cosh \beta_1 \mu - \cos \beta_1 \mu} \cdot \\ \times \frac{1 - \cos \beta_1 \mu \cosh \beta_1 \mu}{1 - \cos \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma}$$

Case 9. Beam with simple support at one end, free at the other, and with simple support at the discontinuity.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \sinh \beta_1 \xi_1$$

$$X_2(\xi_2) = \gamma_1 \left( \sin \beta_1 \theta \xi_2 + \sinh \beta_1 \theta \xi_2 - \left( \frac{\sin \beta_1 \theta \gamma + \sinh \beta_1 \theta \gamma}{\cos \beta_1 \theta \gamma + \cosh \beta_1 \theta \gamma} \right) \cdot (\cos \beta_1 \theta \xi_2 + \cosh \beta_1 \theta \xi_2) \right)$$

where

$$\gamma_1 = \frac{1}{\theta} \cdot \frac{\cos \beta_1 \theta \gamma + \cosh \beta_1 \theta \gamma}{2(1 + \cos \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma)}$$

$$\times \frac{\sin \beta_1 \mu \cosh \beta_1 \mu - \cos \beta_1 \mu \sinh \beta_1 \mu}{\sinh \beta_1 \mu}$$

Illustrative Example 3. The simply supported beam of Illustrative Example 2 has simple support at the middle of the beam. Determine the fundamental frequency and modal shape of vibration.

Solution:

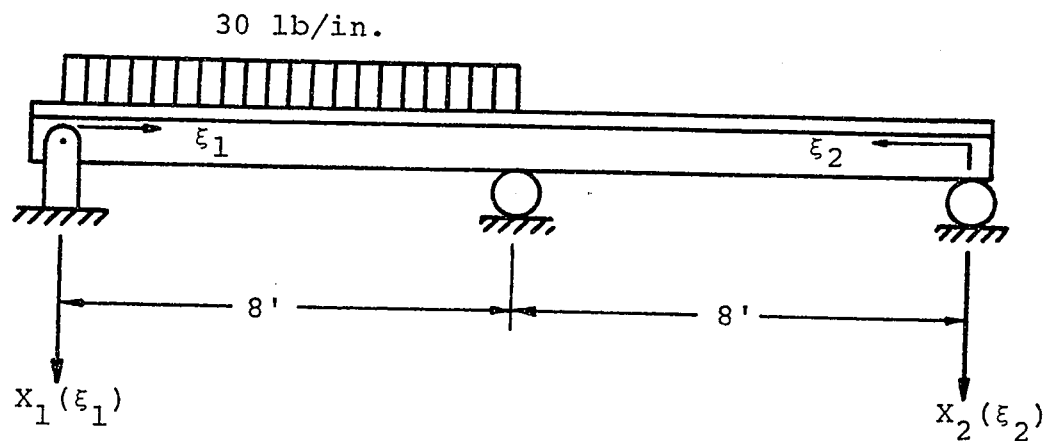
$$\mu = 0.5$$

$$\alpha = 1.0$$

$$\phi = 0.6555$$

From Table 8.7:

$\phi$	$\alpha$	$\mu$	${}^1_{\beta_1}$
0.6	1.0	0.5	6.827
0.7	1.0	0.5	6.781
0.6555	1.0	0.5	6.801



$$f = \left( \frac{6.801}{3.558} \right)^2 (11.31) = 41.32, \text{ Hz}$$

$$\gamma_1 = -0.7619$$

Substituting the values of  $\beta_1$ ,  $\mu$ ,  $\gamma$ ,  $\gamma_1$ , and  $\theta$  into the modal shape expression given, we have

$$X_1(\xi_1) = \sin (6.801\xi_1) + (0.0171) \sinh (6.801 \xi_1)$$

$$X_2(\xi_2) = (-0.7619) \left[ \sin (4.458\xi_2) - (0.1722) \right. \\ \left. \times \sinh (4.458\xi_2) \right]$$

which is the required fundamental modal shape of vibration.

CHAPTER 3  
BEAMS WITH SYMMETRIC DISCONTINUITIES  
IN CROSS SECTION PROPERTIES

(11) General Discussion

This family of beams has a discontinuity in cross section properties distributed symmetrically about the center of the beam as shown in Fig. 12. There exists three cases of practical interest for this type of beam, namely (i) S--S, (ii) C--C, and (iii) C--S, where S stands for simply supported end, C for clamped end and F for free end.

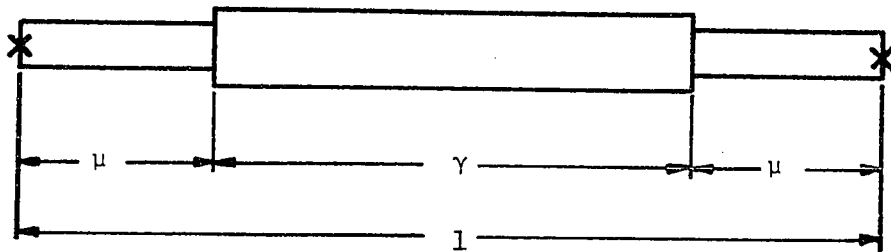


Fig. 12. S--S beam.

A. Beams with Identical End Supports.

It is noted that the boundary conditions of the first two cases also present symmetry with respect to the center of the beam. It is known that beams of this type will vibrate in modal shapes that are alternatively symmetric and antisymmetric with respect to the center.

of the beam, i.e. the first mode will be symmetric, the second antisymmetric and the third symmetric, etc.

It is of utmost importance to observe that the antisymmetric modes of vibration can be determined by focussing our attention on half the beam only, say the left half. Since for an antisymmetric mode, the displacement and bending moment must be zero at the center of the beam, we may consider the left half to be severed from the right half and to be given simple support at the end where it is severed, as shown in Fig. 13.

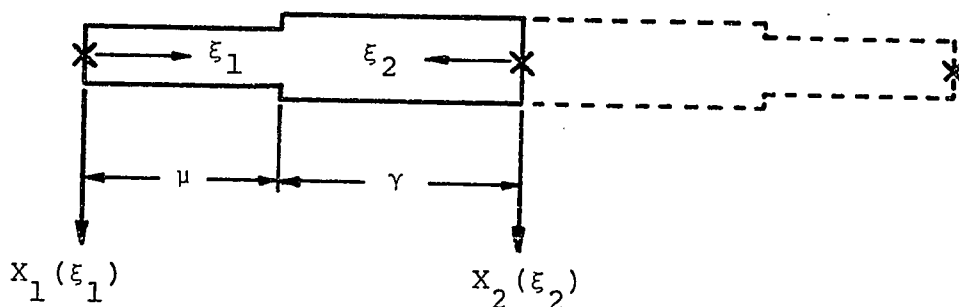
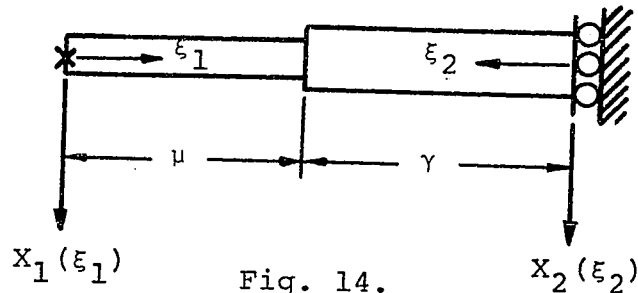


Fig. 13.

Since the geometry and material properties of the original beam of Fig. 12 are known, we also know the geometry and material properties of the beam of Fig. 13 which has half the length of the original beam. We can determine the first and second modes of vibration from Tables 8.3 of Chapter 1. The first and second modes of the half beam correspond to the second and fourth modes

of the original beam. When the modal shape of the left half of the beam is determined that of the right half of the beam can be immediately inferred from the anti-symmetry. Therefore the information necessary for establishing the antisymmetric (second and fourth) modes of vibration of the first two cases (S--S, C--C) is available in Chapter 1.

Next, we turn to the first and third modes of vibration of the beam, i.e. the symmetric modes. Again, we need only focus our attention on half the beam by taking advantage of the symmetry. For symmetric modes of vibration of the original beam, no slope or shear force can exist at the center of the beam. In order to analyze the left half of the beam only, we must enforce the condition of zero slope and zero shear force at the right hand end. The half beam of interest is shown schematically in Fig. 14 where the guided end of the half beam forbids the existence of slope and shear.



The method of analysis of the half beam above is similar to that described for the beams of Chapter 1,

Section 2. The boundary conditions at the guided end are

$$\left. \frac{dx_2(\xi_2)}{d\xi_2} \right|_{\xi_2=0} = 0$$

$$\left. \frac{d^3x_2(\xi_2)}{d\xi_2^3} \right|_{\xi_2=0} = 0$$

The first and second modes of vibration of the half beam correspond to the first and third (symmetric) modes of vibration of the original beam. Modal shapes can be obtained in a manner identical to that described in Section 4, Chapter 1.

#### B. Beams with Differing End Supports

In cases where the end supports are not the same, it is necessary to introduce three displacement functions as shown in Fig. 15. Initially there are a total of 12 unknown constants in the functions. It is always possible to eliminate four of them by enforcing the boundary conditions at each end of the beam. There are evidently eight boundary conditions required which are obtained by requiring continuity of displacement, slope, bending moment and shear force at the cross section discontinuities.

The beam with one end clamped and the other simply supported is considered in detail. The first and last

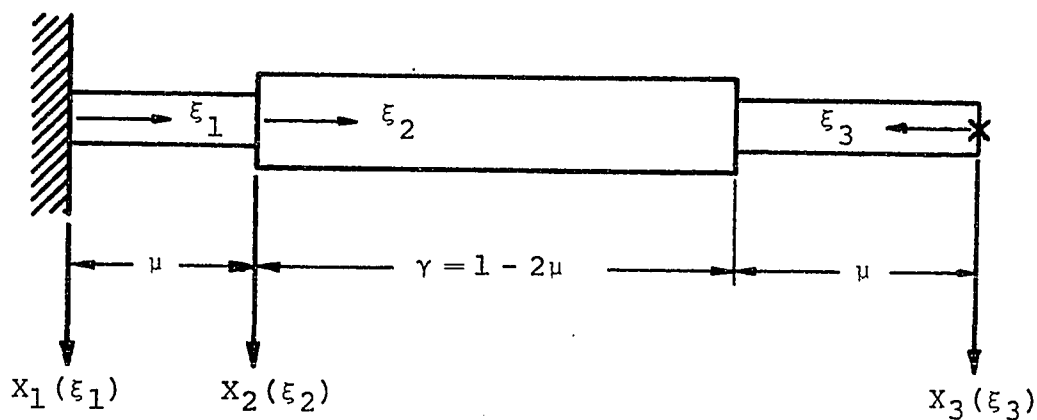


Fig. 15. C--S beam.

segments are of dimensionless length  $\mu$ , where, in general,  $0 < \mu < 0.5$ . The central portion has dimensionless length  $\gamma = 1 - 2\mu$ .

The three displacement functions are

$$X_1(\xi_1) = C_1^{(1)} \sin \beta_1 \xi_1 + C_2^{(1)} \cos \beta_1 \xi_1 + C_3^{(1)} \sinh \beta_1 \xi_1 + C_4^{(1)} \cosh \beta_1 \xi_1$$

$$X_2(\xi_2) = C_1^{(2)} \sin \beta_2 \xi_2 + C_2^{(2)} \cos \beta_2 \xi_2 + C_3^{(2)} \sinh \beta_2 \xi_2 + C_4^{(2)} \cosh \beta_2 \xi_2$$

$$X_3(\xi_3) = C_1^{(3)} \sin \beta_3 \xi_3 + C_2^{(3)} \cos \beta_3 \xi_3 + C_3^{(3)} \sinh \beta_3 \xi_3 + C_4^{(3)} \cosh \beta_3 \xi_3$$

From the conditions that the deflection and slope at the left support are zero, it is readily shown that

$$C_2^{(1)} = - C_4^{(1)}$$

$$C_1^{(1)} = - C_3^{(1)}$$

In the same way, from the condition that at the right support the displacement and bending moment are zero, we have

$$C_2^{(3)} = C_4^{(3)} = 0$$

From the definition of  $\beta_1$  and  $\beta_3$ , we see that  $\beta_1 = \beta_3$ . Thus, the deflections of the three segments are

$$X_1(\xi_1) = C_1^{(1)} (\sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1) + C_2^{(1)} (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = C_1^{(2)} \sin \beta_2 \xi_2 + C_2^{(2)} \cos \beta_2 \xi_2 + C_3^{(2)} \sinh \beta_2 \xi_2 + C_4^{(2)} \cosh \beta_2 \xi_2$$

$$X_3(\xi_3) = C_1^{(3)} \sin \beta_1 \xi_3 + C_3^{(3)} \sinh \beta_1 \xi_3$$

The remaining eight boundary conditions are written as follows:

$$(i) \quad X_1(\mu) = X_2(0)$$

$$C_1^{(1)} (\sin \beta_1 \mu - \sinh \beta_1 \mu) + C_2^{(1)} (\cos \beta_1 \mu - \cosh \beta_1 \mu) - C_2^{(2)} - C_4^{(2)} = 0$$

(34)

$$(ii) \left. \frac{dX_1(\xi_1)}{d\xi_1} \right|_{\xi_1=\mu} = \left. \frac{dX_2(\xi_2)}{d\xi_2} \right|_{\xi_2=0}$$

$$C_1^{(1)} (\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_2^{(1)} (-\sin \beta_1 \mu - \sinh \beta_1 \mu) + C_1^{(2)} (-\theta) + C_3^{(2)} (-\theta) = 0 \quad (35)$$

$$(iii) E_1 I_1 \left. \frac{d^2 X_1(\xi_1)}{d\xi_1^2} \right|_{\xi_1=\mu} = E_2 I_2 \left. \frac{d^2 X_2(\xi_2)}{d\xi_2^2} \right|_{\xi_2=0}$$

$$C_1^{(1)} (-\sin \beta_1 \mu - \sinh \beta_1 \mu) + C_2^{(1)} (-\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_2^{(2)} (\alpha^4 \theta^2) + C_4^{(2)} (-\alpha^4 \theta^2) = 0 \quad (36)$$

$$(iv) E_1 I_1 \left. \frac{d^3 X_1(\xi_1)}{d\xi_1^3} \right|_{\xi_1=\mu} = E_2 I_2 \left. \frac{d^3 X_2(\xi_2)}{d\xi_2^3} \right|_{\xi_2=0}$$

$$C_1^{(1)} (-\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_2^{(1)} (\sin \beta_1 \mu - \sinh \beta_1 \mu) + C_1^{(2)} (\alpha^4 \theta^3) + C_3^{(2)} (-\alpha^4 \theta^3) = 0 \quad (37)$$

$$(v) X_2(\gamma) = X_3(\mu)$$

$$C_1^{(2)} \sin \beta_1 \theta \gamma + C_2^{(2)} \cos \beta_1 \theta \gamma + C_3^{(2)} \sinh \beta_1 \theta \gamma + C_4^{(2)} \cosh \beta_1 \theta \gamma + C_1^{(3)} (-\sin \beta_1 \mu) + C_3^{(3)} (-\sinh \beta_1 \mu) = 0 \quad (38)$$

$$\begin{aligned}
 \text{(vi)} \quad \left. \frac{dX_2(\xi_2)}{d\xi_2} \right|_{\xi_2=\gamma} &= - \left. \frac{dX_3(\xi_3)}{d\xi_3} \right|_{\xi_3=\mu} \\
 C_1^{(2)} (\theta \cos \beta_1 \theta \gamma) + C_2^{(2)} (-\theta \sin \beta_1 \theta \gamma) + \\
 C_3^{(2)} (\theta \cosh \beta_1 \theta \gamma) + C_4^{(2)} (\theta \sinh \beta_1 \theta \gamma) + \\
 C_1^{(3)} \cos \beta_1 \mu + C_3^{(3)} \cosh \beta_1 \mu &= 0 \tag{39}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad E_2 I_2 \left. \frac{d^2 X_2(\xi_2)}{d\xi_2^2} \right|_{\xi_2=\gamma} &= E_1 I_1 \left. \frac{d^2 X_3(\xi_3)}{d\xi_3^2} \right|_{\xi_3=\mu} \\
 C_1^{(2)} (-\alpha^4 \theta^2 \sin \beta_1 \theta \gamma) + C_2^{(2)} (-\alpha^4 \theta^2 \cos \beta_1 \theta \gamma) + \\
 C_3^{(2)} (\alpha^4 \theta^2 \sinh \beta_1 \theta \gamma) + C_4^{(2)} (\alpha^4 \theta^2 \cosh \beta_1 \theta \gamma) + \\
 C_1^{(3)} \sin \beta_1 \mu + C_3^{(3)} (-\sinh \beta_1 \mu) &= 0 \tag{40}
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad E_2 I_2 \left. \frac{d^3 X_2(\xi_2)}{d\xi_2^3} \right|_{\xi_2=\gamma} &= - E_1 I_1 \left. \frac{d^3 X_3(\xi_3)}{d\xi_3^3} \right|_{\xi_3=\mu} \\
 C_1^{(2)} (-\alpha^4 \theta^3 \cos \beta_1 \theta \gamma) + C_2^{(2)} (\alpha^4 \theta^3 \sin \beta_1 \theta \gamma) + \\
 C_3^{(2)} (\alpha^4 \theta^3 \cosh \beta_1 \theta \gamma) + C_4^{(2)} (\alpha^4 \theta^3 \sinh \beta_1 \theta \gamma) + \\
 C_1^{(3)} (-\cos \beta_1 \mu) + C_3^{(3)} \cosh \beta_1 \mu &= 0 \tag{41}
 \end{aligned}$$

where

$$\phi \equiv \left( \frac{\rho_2 A_2}{\rho_1 A_1} \right)^{\frac{1}{4}}, \quad \alpha \equiv \left( \frac{E_2 I_2}{E_1 I_1} \right)^{\frac{1}{4}}, \quad \frac{\beta_2}{\beta_1} = \frac{\phi}{\alpha} \equiv \theta$$

The determinant formed by the coefficients of the above eight homogeneous equations, when set equal to zero, constitutes the frequency equation for this particular beam. Given any value of  $\mu$  ( $0 < \mu < 0.5$ ), the mass ratio ( $\rho_2 A_2 / \rho_1 A_1$ ) and the bending stiffness ratio ( $E_2 I_2 / E_1 I_1$ ), we can determine the eigenvalues  $\beta_1$  or  $\beta_2$  of the system by requiring that the determinant of the coefficient matrix must vanish. With both  $\phi$  and  $\alpha$  ranging from 0.6 to 1.0 at interval of 0.1, and  $\mu$  varying from 0.075 to 0.425 in increment of 0.025, linear interpolation involving  $\mu$ ,  $\phi$  or  $\alpha$  shows that associated error in frequency is generally less than 1%.

We next focus our attention on the modal shapes. Eqs. (34) to (41) form a set of homogeneous equations in the eight unknown constants  $C_1^{(1)}$ ,  $C_2^{(1)}$ ,  $C_1^{(2)}$ ,  $C_2^{(2)}$ ,  $C_3^{(2)}$ ,  $C_4^{(2)}$ ,  $C_1^{(3)}$  and  $C_3^{(3)}$ . By arbitrarily setting one of the nonzero constants  $C_1^{(1)}$  or  $C_2^{(1)}$  equal to one, Eqs. (34) to (41) are converted into a non-homogeneous set. Only seven equations are required to solve for the other seven constants, and we choose the first seven. Solving these equations, the constants  $C_2^{(1)}$ ,  $C_1^{(2)}$ ,  $C_2^{(2)}$ ,  $C_3^{(2)}$ ,

$C_4^{(2)}$ ,  $C_1^{(3)}$  and  $C_3^{(3)}$  may be obtained. Thus the modal shape is determined leaving the amplitude arbitrary.

There may arise cases for which  $C_1^{(1)}$  is zero. In such a case, since both  $C_1^{(1)}$  and  $C_2^{(1)}$  cannot both equal zero, it is evident that  $C_2^{(1)}$  should be set equal to one. Solutions for  $C_1^{(2)}$ ,  $C_2^{(2)}$ ,  $C_3^{(2)}$ ,  $C_4^{(2)}$ ,  $C_1^{(3)}$  and  $C_3^{(3)}$  can be obtained by neglecting the right hand side of the non-homogeneous equations provided, setting  $C_2^{(1)}$  equal to one, and solving for the remaining six unknowns from the first six equations of the new non-homogeneous set.

#### (12) Check for Correctness

It is important the the output be examined for its correctness before it is accepted for use.

##### A. Beams with Identical End Supports

As concluded from the previous section, we need only to evaluate the eigenvalues of the symmetric (first and third) modes of vibration. Thus we need to conduct checking of these two modes only.

When  $\phi = 1$  and  $\alpha = 1$ , for any value of  $\mu$ , the eigenvalues  $\beta_1$  of case (i) are given by  $(2n - 1)\pi/2$  ( $n = 1, 2$ ), the classical values. This is case, as the values  $\phi = 1$  and  $\alpha = 1$  represents a uniform beam of constant geometry and material property throughout its length.

For any combination of values of  $\phi$  and  $\alpha$ , when  $\mu = 1$ , the eigenvalues are given by  $(2n - 1)\pi/2$  which is the case as the value  $\mu = 1$  represents a uniform beam of properties  $\rho_1, A_1, E_1$  and  $I_1$ . Whereas for any value of  $\phi$  and  $\alpha$ , when  $\mu = 0$ , the eigenvalues are given by  $(\phi/\alpha)\beta_1$ . The value  $\mu = 0$  represents a uniform beam of properties  $\rho_2, A_2, E_2$  and  $I_2$ , whose eigenvalues are  $\beta_2$ .

As another check, we must have, for any value of  $\mu$ , the eigenvalues approaching the classical values of  $(2n - 1)\pi/2$  when (i)  $\phi = 1$  and  $\alpha \rightarrow 1$ , and (ii)  $\alpha = 1$  and  $\phi \rightarrow 1$ . Curves are plotted for verifying the above mentioned limits.

#### B. Beams with Differing End Supports

In contrast to cases (i) and (ii), we need to perform checking of all the four modes for case (iii).

When  $\phi = 1$  and  $\alpha = 1$ , for any value of  $\mu$ , the eigenvalues  $\beta_1$  of case (iii) are given by those in (3), Appendix A. This is the case, as the values  $\phi = 1$  and  $\alpha = 1$  represent a uniform beam of constant geometry and material property throughout its length.

For any combination of values of  $\phi$  and  $\alpha$ , when  $\mu = 0.5$ , the eigenvalues are again given by (3), Appendix A, which is the case as the value  $\mu = 0.5$  represents a uniform beam of properties  $\rho_1, A_1, E_1,$

and  $I_1$ . Whereas for any value of  $\phi$  and  $\alpha$ , when  $\mu = 0$  the eigenvalues are given by  $(\phi/\alpha)\beta_1$ . The value  $\mu = 0$  represents a uniform beam of properties  $\rho_2$ ,  $A_2$ ,  $E_2$ , and  $I_2$ , whose eigenvalues are  $\beta_2$ .

We must have, for any value of  $\mu$ , the eigenvalues approaching the classical value of (3), Appendix A, when (i)  $\phi = 1$  and  $\alpha \rightarrow 1$ , and (ii)  $\alpha = 1$  and  $\phi \rightarrow 1$ . Curves are plotted for verifying these limits.

### (13) Presentation of Results

All of the beams are given a reference number. Beside each reference number is a sketch of the beam and a brief description as in the previous chapters.

In order to minimize storage requirements it has been decided to prepare the tables on the assumption that the stiffness ratio and mass ratio for any beam will both be either greater or less than 1.0. Combinations where one parameter is greater than 1.0 and the other is less than 1.0 are expected to be rare and are not provided in the tables.

The objective of the tables is to permit beam vibration analysis, with the length of the middle section of the original beam varying from 0.15 to 0.85 of the beam overall length.

#### A. Beams with Identical End Supports.

As indicated above, the antisymmetric modes can be

analyzed using existing cases of Chapter 1. These will be referred to by case number for each of the beam. In addition, information for the analysis of the half beams with zero shear and slope are given. Tables have numbers corresponding to the case numbers. In the tables for the half beams,  $\mu$  varies from 0.15 to 0.85 in increments of 0.05, while  $\phi$  and  $\alpha$  range from 0.6 to 1.0 in intervals of 0.1. The same is true for the inverse of the parameters  $1/\phi$  and  $1/\alpha$ .

Non-homogeneous equations for evaluating the constants are given. These equations are to be solved in a manner similar to that described in Chapter 1.

#### B. Beams with Differing End Supports

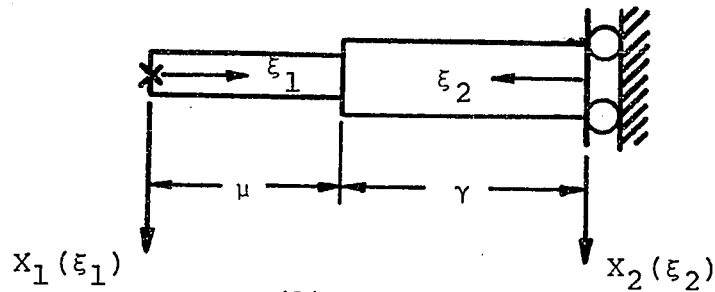
In the tables,  $\mu$  varies from 0.075 to 0.425 in increments of 0.025 (the corresponding dimensionless length of the central segment  $\gamma$  varies from 0.85 to 0.15) while  $\phi$  and  $\alpha$  range from 0.6 to 1.0 in intervals of 0.1. The same is true for the inverse of the parameters  $1/\phi$  and  $1/\alpha$ .

Seven non-homogeneous equations are provided. The unknown constants in modal shape expressions can be obtained by solving these seven equations.

Case 10. Beam with simple support at each end and with symmetric discontinuity in cross section.



Antisymmetric modes and frequencies (2nd and 4th modes): utilize first and second mode data of Case 8.3. Symmetric modes and frequencies (1st and 3rd modes): Utilize first and second mode data, Tables 8.10, and modal shape information given below.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 + C_3^{(1)} \sinh \beta_1 \xi_1$$

$$X_2(\xi_2) = C_2^{(2)} \cos \beta_1 \theta \xi_2 + C_4^{(2)} \cosh \beta_1 \theta \xi_2$$

$$C_3^{(1)} \sinh \beta_1 \mu + C_2^{(2)} (-\cos \beta_1 \theta \gamma) + C_4^{(2)} (-\cosh \beta_1 \theta \gamma) = -\sin \beta_1 \mu$$

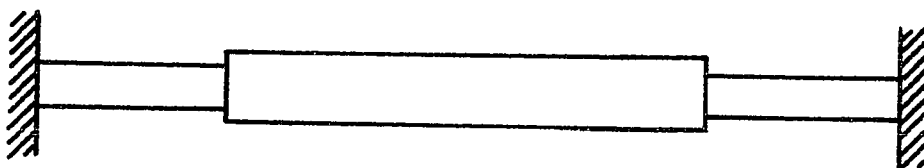
$$C_3^{(1)} \cosh \beta_1 \mu + C_2^{(2)} \theta (-\sin \beta_1 \theta \gamma) + C_4^{(2)} \theta (\sinh \beta_1 \theta \gamma)$$

$$= -\cos \beta_1 \mu$$

$$C_3^{(1)} \sinh \beta_1 \mu + C_2^{(2)} \alpha^4 \theta^2 (\cos \beta_1 \theta \gamma) + C_4^{(2)} \alpha^4 \theta^2 (-\cosh \beta_1 \theta \gamma)$$

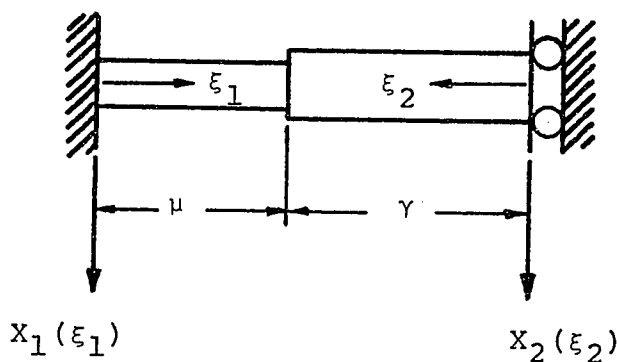
$$= \sin \beta_1 \mu$$

Case 11. Beam clamped at each end and with symmetric discontinuity in cross section.



Antisymmetric modes and frequencies (2nd and 4th modes):  
utilize first and second mode data of Case 8.2.

Symmetric modes and frequencies (1st and 3rd modes):  
utilize first and second mode data, Tables 8.11, and  
modal shape information given below.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 + C_2^{(1)} (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = C_2^{(2)} \cos \beta_1 \theta \xi_2 + C_4^{(2)} \cosh \beta_1 \theta \xi_2$$

$$C_2^{(1)} (\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_2^{(2)} (-\cos \beta_1 \theta \gamma) +$$

$$C_4^{(2)} (-\cosh \beta_1 \theta \gamma) = \sinh \beta_1 \mu - \sin \beta_1 \mu$$

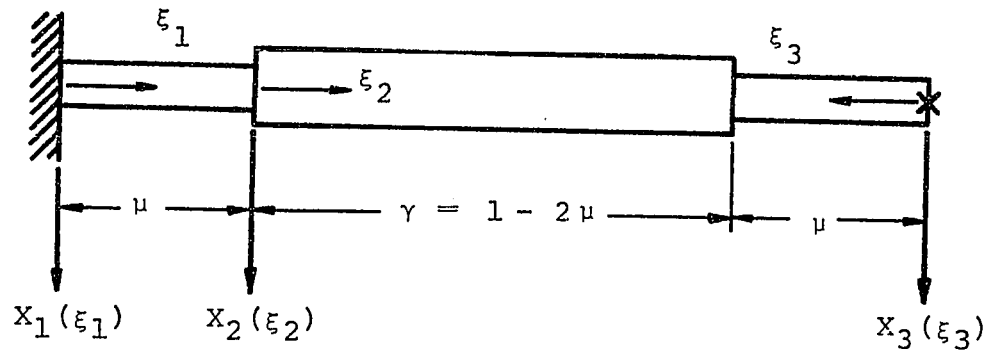
$$C_2^{(1)} (-\sin \beta_1 \mu - \sinh \beta_1 \mu) + C_2^{(2)} \theta (-\sin \beta_1 \theta \gamma) +$$

$$C_4^{(2)} \theta (\sinh \beta_1 \theta \gamma) = \cosh \beta_1 \mu - \cos \beta_1 \mu$$

$$C_2^{(1)} (-\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_2^{(2)} \alpha^4 \theta^2 (\cos \beta_1 \theta \gamma) +$$

$$C_4^{(2)} \alpha^4 \theta^2 (-\cosh \beta_1 \theta \gamma) = \sinh \beta_1 \mu + \sin \beta_1 \mu$$

Case 12. Beam clamped at one end, with simple support at the other, and with symmetric discontinuity in cross section. Utilize Tables 8.15



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 + C_2^{(1)} (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = C_1^{(2)} \sin \beta_1 \theta \xi_2 + C_2^{(2)} \cos \beta_1 \theta \xi_2 + C_3^{(2)} \sinh \beta_1 \theta \xi_2 + C_4^{(2)} \cosh \beta_1 \theta \xi_2$$

$$X_3(\xi_3) = C_1^{(3)} \sin \beta_1 \xi_3 + C_3^{(3)} \sinh \beta_1 \xi_3$$

$$C_2^{(1)} (\cos \beta_1 \mu - \cosh \beta_1 \mu) + C_1^{(2)} (0) + C_2^{(2)} (-1) + C_3^{(2)} (0) + C_4^{(2)} (-1) = \sinh \beta_1 \mu - \sin \beta_1 \mu$$

$$C_2^{(1)} (\sin \beta_1 \mu + \sinh \beta_1 \mu) + C_1^{(2)} (\theta) + C_2^{(2)} (0) + C_3^{(2)} (\theta) + C_4^{(2)} (0) = \cos \beta_1 \mu - \cosh \beta_1 \mu$$

$$C_2^{(1)} (\cos \beta_1 \mu + \cosh \beta_1 \mu) + C_1^{(2)} (0) + C_2^{(2)} (-\alpha^4 \theta^2) +$$

$$C_3^{(2)}(0) + C_4^{(2)}(\alpha^4 \theta^2) = -\sin \beta_1 \mu - \sinh \beta_1 \mu$$

$$C_2^{(1)}(\sin \beta_1 \mu - \sinh \beta_1 \mu) + C_1^{(2)}(\alpha^4 \theta^3) + C_2^{(2)}(0) +$$

$$C_3^{(2)}(-\alpha^4 \theta^3) + C_4^{(2)}(0) = \cos \beta_1 \mu + \cosh \beta_1 \mu$$

$$C_1^{(2)} \sin \beta_1 \theta \gamma + C_2^{(2)} \cos \beta_1 \theta \gamma + C_3^{(2)} \sinh \beta_1 \theta \gamma + C_4^{(2)} \cosh \beta_1 \theta \gamma +$$

$$C_1^{(3)}(-\sin \beta_1 \mu) + C_3^{(3)}(-\sinh \beta_1 \mu) = 0$$

$$C_1^{(2)} \theta (\cos \beta_1 \theta \gamma) + C_2^{(2)} \theta (-\sin \beta_1 \theta \gamma) + C_3^{(2)} \theta (\cosh \beta_1 \theta \gamma) +$$

$$C_4^{(2)} \theta (\sinh \beta_1 \theta \gamma) + C_1^{(3)} (\cos \beta_1 \mu) +$$

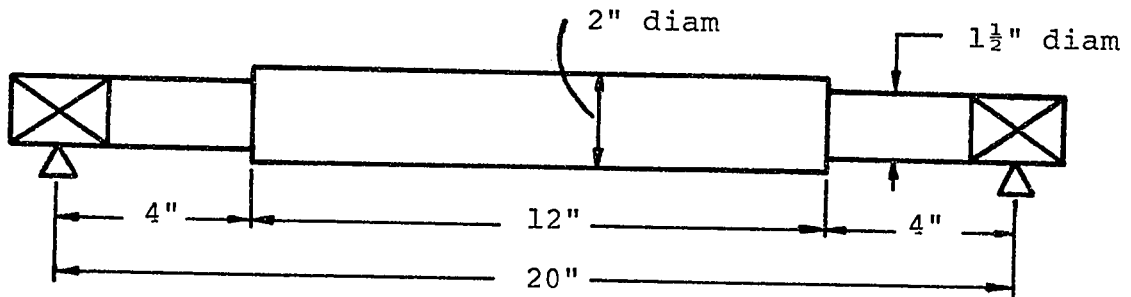
$$C_3^{(3)} (\cosh \beta_1 \mu) = 0$$

$$C_1^{(2)} \alpha^4 \theta^2 (-\sin \beta_1 \theta \gamma) + C_2^{(2)} \alpha^4 \theta^2 (-\cos \beta_1 \theta \gamma) +$$

$$C_3^{(2)} \alpha^4 \theta^2 (\sinh \beta_1 \theta \gamma) + C_4^{(2)} \alpha^4 \theta^2 (\cosh \beta_1 \theta \gamma) +$$

$$C_1^{(3)} (\sin \beta_1 \mu) + C_3^{(3)} (-\sinh \beta_1 \mu) = 0$$

Illustrative Example 4. A shaft with symmetric discontinuity in cross section is mounted on self-aligning bearings on rigid foundation. Determine the first and second modes of vibration. Take  $\gamma' = 0.283 \text{ lb/in}^3$ ,  $E = 30 \times 10^6 \text{ psi}$ .

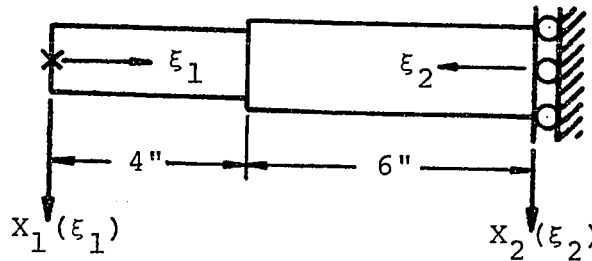


Solution: For the original beam

$$\mu = \frac{4}{20} = 0.2$$

$\therefore$  for the half beam,  $\mu = 0.4$ .

1st mode



$$\phi = \left( \frac{\rho_2 A_2}{\rho_1 A_1} \right)^{\frac{1}{4}} = \left[ \frac{\frac{\pi}{4} (2)^2}{\frac{\pi}{4} \left(\frac{3}{2}\right)^2} \right]^{\frac{1}{4}} = \sqrt{\frac{4}{3}}$$

$$\therefore \frac{1}{\phi} = 0.866$$

$$\alpha = \left( \frac{E_2 I_2}{E_1 I_1} \right)^{\frac{1}{4}} = \left[ \frac{\frac{\pi}{64} (2)^4}{\frac{\pi}{64} \left(\frac{3}{2}\right)^4} \right]^{\frac{1}{2}} = \frac{4}{3}$$

$$\therefore \frac{1}{\alpha} = 0.75$$

From Taylor's series

$$\begin{aligned} f(x + h, y + k) &\approx f(x, y) + [h f_x(x, y) + k f_y(x, y)] \\ &\approx f(x, y) + h \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x} \\ &\quad + k \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y} \end{aligned}$$

Using first mode data of Tables 8.10,

$$\begin{aligned} {}^1\beta_1\left(\frac{1}{\phi}, \frac{1}{\alpha}\right) &= {}^1\beta_1(0.866, 0.75) \\ &= {}^1\beta_1(0.8, 0.7) + \frac{0.066}{0.1} [{}^1\beta_1(0.9, 0.7) - \\ &\quad {}^1\beta_1(0.8, 0.7)] + \frac{0.05}{0.1} [{}^1\beta_1(0.8, 0.8) - \\ &\quad {}^1\beta_1(0.8, 0.7)] \\ &= 1.710 + (0.66)(1.908 - 1.710) + \\ &\quad (0.5)(1.546 - 1.710) \\ &= 1.759 \end{aligned}$$

$$f = \frac{{}^1\beta_1^2}{2\pi L^2} \sqrt{\frac{E_1 I_1}{\rho_1 I_1}} = \frac{1}{2\pi} \left(\frac{1.759}{10}\right)^2 \sqrt{\frac{(30 \times 10^6) \left(\frac{\pi}{64}\right) \left(\frac{3}{2}\right)^4}{\left(\frac{0.283}{386}\right) \left(\frac{\pi}{4}\right) \left(\frac{3}{2}\right)^2}}$$

$$= 373.55, \text{ Hz}$$

Solving the following equations,

$$0.7631C_3^{(1)} - 0.6106C_2^{(2)} - 1.4476C_4^{(2)} = - 0.6469$$

$$1.2579C_3^{(1)} - 0.6859C_2^{(2)} + 0.9065C_4^{(2)} = - 0.7625$$

$$0.7631C_3^{(1)} + 1.4474C_2^{(2)} - 3.4315C_4^{(2)} = 0.6469$$

we have

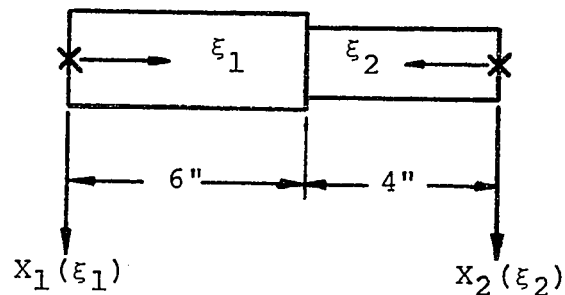
$$C_3^{(1)} = - 0.2689, \quad C_2^{(2)} = 0.6561, \quad C_4^{(2)} = 0.0284$$

Thus, the modal shape is

$$X_1(\xi_1) = \sin(1.759 \xi_1) - (0.2689) \sinh(1.759 \xi_1)$$

$$X_2(\xi_2) = (0.6561) \cos(1.523 \xi_2) + (0.0284) \cosh(1.523 \xi_2)$$

2nd mode



$$\mu = 0.6, \quad \phi = 0.866, \quad \alpha = 0.75.$$

Similarly, using Taylor's series and first mode data of Tables 8.3,

$$\begin{aligned} \beta_1^2 &= 2.837 + (0.66)(2.745 - 2.837) \\ &\quad + (0.5)(3.048 - 2.837) \\ &= 2.882 \end{aligned}$$

$$^2f = \frac{1}{2\pi} \left(\frac{2.882}{10}\right)^2 \sqrt{\frac{(30 \times 10^6) \left(\frac{\pi}{64}\right) (2)^4}{\left(\frac{0.283}{386}\right) \left(\frac{\pi}{4}\right) (2)^2}}$$
$$= 1,337.06, \text{ Hz}$$

Solving the equations

$$2.7291C_3^{(1)} - 0.9714C_1^{(2)} - 1.7604C_3^{(2)} = -0.9875$$

$$2.9065C_3^{(1)} + 0.2742C_1^{(2)} + 2.3377C_3^{(2)} = 0.1576$$

$$2.7291C_3^{(1)} + 0.4098C_1^{(2)} - 0.7426C_3^{(2)} = 0.9875$$

we have

$$C_3^{(1)} = 0.0691, \quad C_1^{(2)} = 1.5801, \quad C_3^{(2)} = -0.2038$$

Thus, the modal shape is

$$X_1(\xi_1) = \sin(2.882 \xi_1) + (0.0691) \sinh(2.882 \xi_1)$$

$$X_2(\xi_2) = (1.5801) \sin(3.328 \xi_2) - (0.2038)$$

$$\sinh(3.328 \xi_2)$$

CHAPTER 4  
THREE-SPAN BEAMS  
WITH SYMMETRIC DISCONTINUITIES  
IN CROSS SECTION PROPERTIES

(14) General Consideration

The family of beams under discussion has a discontinuity in cross section properties distributed symmetrically about the center of the beam. Also there are intermediate supports located at the discontinuities, as shown in Fig. 16. There exists six cases of practical interest for this type of beam, namely, (i) SSSS, (ii) CSSC, (iii) FSSF, (iv) CSSS, (v) CSSF, and (vi) SSSF, where S stands for simply supported condition, C for clamped condition, and F for free condition.

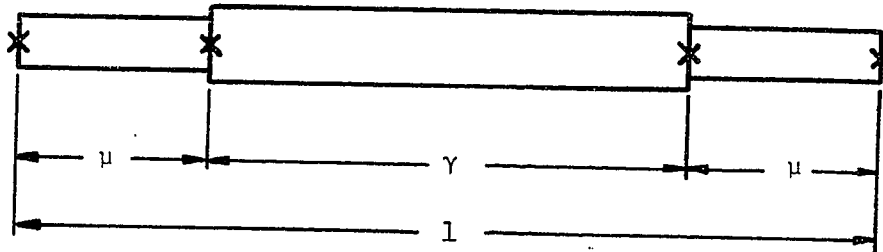


Fig. 16. SSSS beam.

A. Beams with Identical End Supports.

It is noted that the end supports of the first three cases also present symmetry with respect to the center of the beam. It is known that beams of this type will vibrate in modal shapes that are alternately

symmetric and antisymmetric with respect to the center of the beam, i.e. the first mode will be symmetric, the second antisymmetric, and the third symmetric, etc.

As discussed in Section 11, Chapter 3, both the symmetric and antisymmetric modes can be determined by analyzing half the beam only, say the left half. For antisymmetric modes, we can consider the left half to be severed from the right half and to be given simple support at the end where it is severed, as shown in Fig. 17(a). Whereas for symmetric modes, we can consider the left half to be severed from the right half and to be given a guided end at the end where it is severed, as shown in Fig. 17(b).

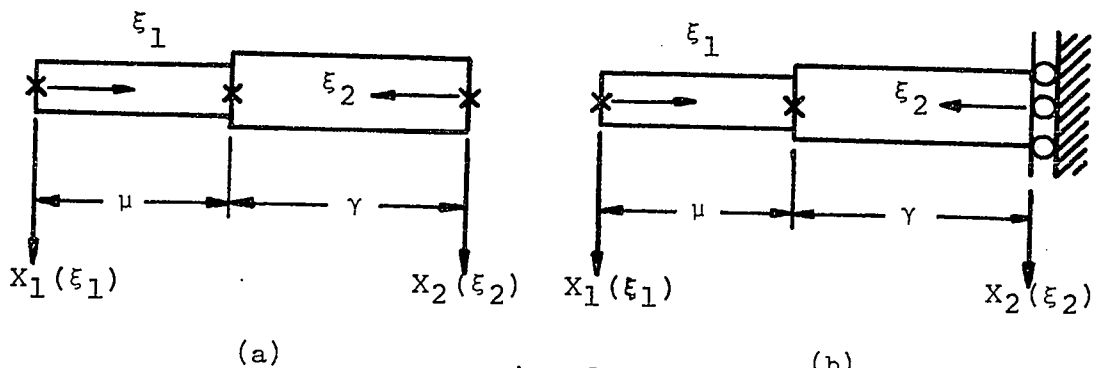


Fig. 17.

The first and the second modes of vibration of the half beam, Fig. 17(a), correspond to the second and fourth modes of vibration of the original beam, Fig. 16. The modes of vibration can be determined from Table 8.7 of Chapter 2. When the modal shape of the left half of

the beam is determined, that of the right half of the beam can be immediately inferred from the antisymmetry. The information necessary for establishing the anti-symmetric (second and fourth) modes of vibration of the first three cases is available in Chapter 2.

The first and second modes of vibration of the half beam, Fig. 17(b), correspond to the first and third modes of vibration of the original beam, Fig. 16. The method of analysis of the half beam is similar to that described for the beams of Chapter 2 (see Section 8). The boundary conditions at the guided end are

$$\left. \frac{dX_2(\xi_2)}{d\xi_2} \right|_{\xi_2=0} = 0$$

$$\left. \frac{d^3X_2(\xi_2)}{d\xi_2^3} \right|_{\xi_2=0} = 0$$

Following the same procedure as described in Section 8, the expressions for modal shapes may be obtained.

#### B. Beams with Differing End Supports.

For cases (iv), (v), and (vi) where the end supports are not the same, it is necessary to introduce three displacement functions as shown in Fig. 18.

Initially there are a total of 12 unknown constants in the functions. It is always possible to eliminate four

of them by enforcing the boundary conditions at each end of the beam. There are evidently eight boundary conditions required which are obtained by requiring continuity of displacement, slope, and bending moment at the cross section discontinuities. Four more unknown constants can be eliminated during the course of enforcing continuity conditions at the intermediate simple supports.

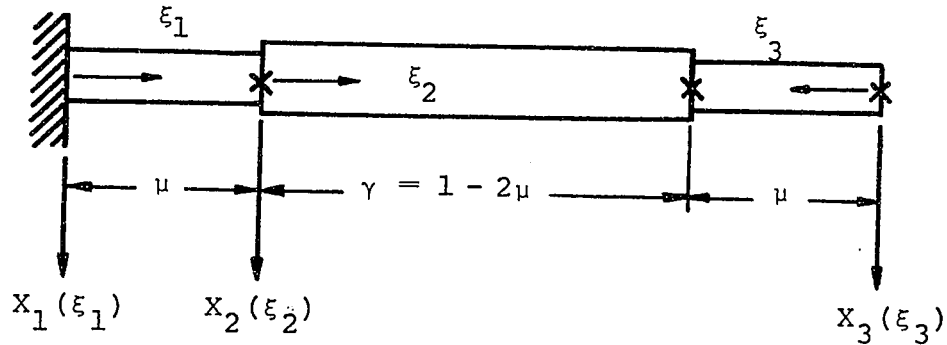


Fig. 18.

For illustrative purposes, the case of a beam with one end clamped and the other simply supported is considered in detail. The first and third segment are of dimensionless length  $\mu$  (ratio of span length to beam length) where, in general,  $0 < \mu < 0.5$ . The central portion has dimensionless length  $\gamma = 1 - 2\mu$ .

As derived in Section 11, the first and third span deflections are

$$X_1(\xi_1) = C_1^{(1)} (\sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1) + C_2^{(1)} (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1) \quad (42)$$

$$x_3(\xi_3) = C_1^{(3)} \sin \beta_1 \xi_3 + C_3^{(3)} \sinh \beta_1 \xi_3 \quad (43)$$

Considering the second span and observing that the deflection at  $\xi_2 = 0$  is equal to zero, the displacement function for the second span is

$$x_2(\xi_2) = C_1^{(2)} \sin \beta_2 \xi_2 + C_2^{(2)} (\cos \beta_2 \xi_2 - \cosh \beta_2 \xi_2) + C_3^{(2)} \sinh \beta_2 \xi_2 \quad (44)$$

The remaining seven boundary conditions are written as follows:

(i)  $x_1(\mu) = 0$

$$C_2^{(1)} = - \left( \frac{\sin \beta_1 \mu - \sinh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \right) C_1^{(1)} \quad (45)$$

(ii)  $\left. \frac{dx_1(\xi_1)}{d\xi_1} \right|_{\xi_1=\mu} = \left. \frac{dx_2(\xi_2)}{d\xi_2} \right|_{\xi_2=0}$

$$C_1^{(1)} \left\{ \frac{2 - 2 \cos \beta_1 \mu \cosh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \right\} + C_1^{(2)} (-\theta) + C_3^{(2)} (-\theta) = 0 \quad (46)$$

(iii)  $E_1 I_1 \left. \frac{d^2 x_1(\xi_1)}{d\xi_1^2} \right|_{\xi_1=\mu} = E_2 I_2 \left. \frac{d^2 x_2(\xi_2)}{d\xi_2^2} \right|_{\xi_2=0}$

$$C_1^{(1)} \left\{ \frac{\sin \beta_1 \mu \cosh \beta_1 \mu - \cos \beta_1 \mu \sinh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \right\} +$$

$$C_2^{(2)} (\alpha^4 \theta^2) = 0 \quad (47)$$

(iv)  $X_2(\gamma) = 0$

$$C_1^{(2)} \sin \beta_1 \theta \gamma + C_2^{(2)} (\cos \beta_1 \theta \gamma - \cosh \beta_1 \theta \gamma) + C_3^{(2)} \sinh \beta_1 \theta \gamma = 0 \quad (48)$$

(v)  $X_3(\mu) = 0$

$$C_3^{(3)} = - \left( \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \right) C_1^{(3)} \quad (49)$$

$$(vi) \left. \frac{dX_2(\xi_2)}{d\xi_2} \right|_{\xi_2=\gamma} = - \left. \frac{dX_3(\xi_3)}{d\xi_3} \right|_{\xi_3=\mu}$$

$$C_1^{(2)} \theta (\cos \beta_1 \theta \gamma) +$$

$$C_2^{(2)} \theta (-\sin \beta_1 \theta \gamma - \sinh \beta_1 \theta \gamma) + C_3^{(2)} \theta (\cosh \beta_1 \theta \gamma) +$$

$$C_1^{(3)} \left\{ \frac{\cos \beta_1 \mu \sinh \beta_1 \mu - \sin \beta_1 \mu \cosh \beta_1 \mu}{\sinh \beta_1 \mu} \right\}$$

$$= 0$$

(50)

$$(vii) E_2 I_2 \left. \frac{d^2 X_2(\xi_2)}{d\xi_2^2} \right|_{\xi_2=\gamma} = E_1 I_1 \left. \frac{d^2 X_3(\xi_3)}{d\xi_3^2} \right|_{\xi_3=\mu}$$

$$C_1^{(2)} \alpha^4 \theta^2 (-\sin \beta_1 \theta \gamma) + C_2^{(2)} \alpha^4 \theta^2 (-\cos \beta_1 \theta \gamma -$$

$$\cosh \beta_1 \theta \gamma) + C_3^{(2)} \alpha^4 \theta^2 (\sinh \beta_1 \theta \gamma) +$$

$$C_1^{(3)} (2 \sin \beta_1 \mu) = 0 \quad (51)$$

where

$$\phi \equiv (\rho_2 A_2 / \rho_1 A_1)^{\frac{1}{2}}, \quad \alpha \equiv (E_2 I_2 / E_1 I_1)^{\frac{1}{4}}, \quad \beta_2 / \beta_1 = \phi / \alpha \equiv \theta.$$

The eigenvalues  $\beta_1$  can then be found, as was done in Chapter 1, by obtaining those values of  $\beta_1$  which make the determinant of the coefficient matrix of the unknowns, Eqs. (46), (47), (48), (50), and (51), vanish.

Next, we focus our attention on the modal shape. Only one unknown,  $C_1^{(1)}$ , is associated with the first span deflection. Since it can never be equal to zero, it is set equal to unity. Substituting Eqs. (45) and (49) in Eqs. (42) and (43) respectively, we have

$$\begin{aligned} x_1(\xi_1) = & \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 - \left( \frac{\sin \beta_1 \mu - \sinh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \right) \\ & \times (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1) \end{aligned} \quad (42')$$

$$x_3(\xi_3) = C_1^{(3)} \left[ \sin \beta_1 \xi_3 - \left( \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \right) \sinh \beta_1 \xi_3 \right] \quad (43')$$

From Eq. (48), we have

$$\begin{aligned} C_3^{(2)} = & - \left( \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) C_1^{(2)} + \\ & \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) C_2^{(2)} \end{aligned} \quad (52)$$

Substitution of the above equation into Eq. (44), we obtain

$$\begin{aligned}
 X_2(\xi_2) = & C_1^{(2)} \left[ \sin \beta_1 \theta \xi_2 - \left( \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \right. \\
 & \times \sinh \beta_1 \theta \xi_2 \left. \right] + C_2^{(2)} \left[ \cos \beta_1 \theta \xi_2 - \cosh \beta_1 \theta \xi_2 + \right. \\
 & \left. \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \sinh \beta_1 \theta \xi_2 \right] \quad (44')
 \end{aligned}$$

where, from Eq. (47),

$$C_2^{(2)} = \frac{1}{\alpha^4 \theta^2} \cdot \frac{\cos \beta_1 \mu \sinh \beta_1 \mu - \sin \beta_1 \mu \cosh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \quad (47')$$

after setting  $C_1^{(1)}$  equal to unity.

The other two unknown constants  $C_1^{(2)}$  and  $C_1^{(3)}$  can be obtained by solving the following equations which are obtained by substituting Eq. (52) into Eqs. (46) and (50) respectively.

$$\begin{aligned}
 C_1^{(2)} \left( \frac{\sinh \beta_1 \theta \gamma - \sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) + C_2^{(2)} \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \\
 = \frac{2}{\theta} \cdot \frac{1 - \cos \beta_1 \mu \cosh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \quad (53)
 \end{aligned}$$

$$C_1^{(2)} \theta \left[ \cos \beta_1 \theta \gamma - \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \cosh \beta_1 \theta \gamma \right] +$$

$$C_2^{(2)} \theta \left[ -\sin \beta_1 \theta \gamma - \sinh \beta_1 \theta \gamma + \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \right]$$

$$\cosh \beta_1 \theta \gamma ] + C_1^{(3)} \left[ \cos \beta_1 \mu - \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \cosh \beta_1 \mu \right] = 0 \quad (54)$$

Following the same procedure, the frequency equation and expressions for modal shape may be obtained for the remaining two cases.

(15) Check for Correctness

It is important that the output be examined for its correctness before it is accepted for use.

A. Beams with Identical End Supports

As concluded from the previous sections we need only to evaluate the eigenvalues of the symmetric (first and third) modes of vibration. Thus we need to conduct checking of these two modes only.

For arbitrary value of  $\mu$ , when  $\phi = 1$  and  $\alpha = 1$ , we have double span beams of constant geometry and material property throughout its length. The eigenvalues  $\beta_1$  are checked against those given in Mr. R.K. Sharma's work<sup>1</sup>.

As another check, when  $\phi = 1$  and  $\alpha = 1$ , we must have the eigenvalues approaching the well known values for single span beam as the intermediate supports approach the beam ends. For example, in case (i), the first mode eigenvalue approaches that of a clamped-guided beam and hinged-clamped beam as  $\mu$  approaches zero

---

1. R.K. Sharma: Ph.D. Dissertation work (in progress)  
Dept. of Mechanical Engineering, Univ. of Ottawa, 1974.

and one, respectively.

For any combination of values of  $\phi$  and  $\alpha$ , when  $\mu \rightarrow 1$ , the eigenvalues approach the classical values as given in (3), Appendix A. This is the case as the value  $\mu = 1$  represents a uniform beam of properties  $\rho_1$ ,  $A_1$ ,  $E_1$ , and  $I_1$ . Whereas for any value of  $\phi$  and  $\alpha$ , when  $\mu = 0$ , the eigenvalues are given by  $(\phi/\alpha)\beta_1$ . The value  $\mu = 0$  represents a uniform beam of properties  $\rho_2$ ,  $A_2$ ,  $E_2$ , and  $I_2$ , whose eigenvalues are  $\beta_2$ .

Finally, for any value of  $\mu$ , we must have the eigenvalues approaching those obtained by Mr. R.K. Sharma when (i)  $\phi = 1$  and  $\alpha \rightarrow 1$ , and (ii)  $\alpha = 1$  and  $\phi \rightarrow 1$ . Curves are plotted for verifying the above mentioned limits.

#### B. Beams with Differing End Supports

In contrast to cases (i), (ii), and (iii), we need to perform checking of all the four modes for cases (iv), (v), and (vi).

When  $\phi = 1$  and  $\alpha = 1$ , for any value of  $\mu$ , we have three span beams of uniform geometric and material properties throughout its length, with intermediate simple supports located at distances  $\mu L$  from the left and right ends of the beam, respectively. The eigenvalues  $\beta_1$  are checked against those given in Mr. R.K. Sharma's work.

As another check, when  $\phi = 1$  and  $\alpha = 1$ , we must have the eigenvalues approaching the well known values for single span beams as the intermediate supports approach the beam ends. For example, in case (iv), the first mode eigenvalue approaches that of a clamped-clamped beam as  $\mu$  approaches zero. When  $\mu$  approaches 0.5, the first mode eigenvalue approaches twice the first mode eigenvalue of a clamped-hinged beam; the second mode eigenvalue approaches twice the first mode eigenvalue of a clamped-clamped beam, etc. This is in accordance with the work of Mr. R.K. Sharma.

For any combination of  $\phi$  and  $\alpha$ , when  $\mu \rightarrow 0.5$ , the eigenvalues approach those of clamped-hinged and clamped-clamped beams alternately, as mentioned in the preceding paragraph. This is the case as the value  $\mu = 0.5$  represents a uniform beam of properties  $\rho_1, A_1, E_1$  and  $I_1$ . Whereas for any value of  $\phi$  and  $\alpha$ , when  $\mu \rightarrow 0$ , the eigenvalues of, say, case (iv) approach those of a uniform clamped-clamped beam, and are given by  $(\phi/\alpha)\beta_1$ . The value  $\mu = 0$  represents a uniform beam of properties  $\rho_2, A_2, E_2$ , and  $I_2$ , whose eigenvalues are  $\beta_2$ .

Finally, for any value of  $\mu$ , we must have the eigenvalues approaching those obtained by Mr. R.K. Sharma when (i)  $\phi = 1$  and  $\alpha \rightarrow 1$  and (ii)  $\alpha = 1$  and  $\phi \rightarrow 1$ . Curves are plotted for verifying the above mentioned limits.

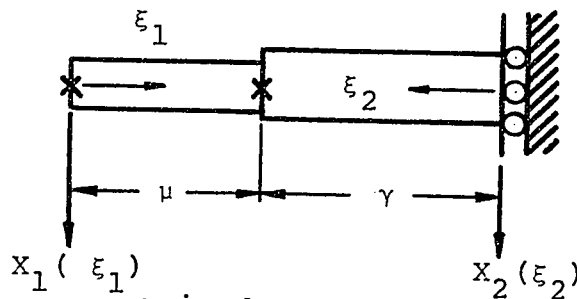
(16) Presentation of Results

The discussion given in Section (13), Chapter 3, applies to this family of beams, except that for beams with identical end supports, there is only one unknown constant, while for beams with differing end supports, there are three unknown constants.

Case 13. Three span beam with simple support at each end, and with symmetric discontinuity in cross section.



Antisymmetric modes and frequencies (2nd and 4th modes): Utilize first and second mode data of case 8.7. Symmetric modes and frequencies (1st and 3rd modes): utilize first and second mode data, Tables 8.12, and modal shape information given below.



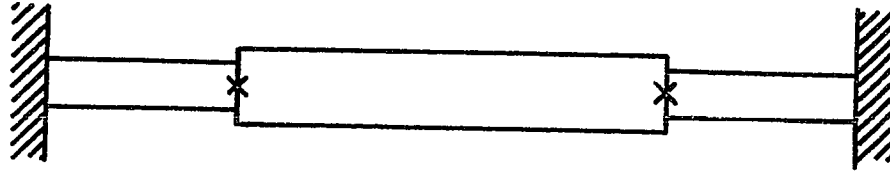
$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \left( \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \right) \sinh \beta_1 \xi_1$$

$$X_2(\xi_2) = \gamma_1 \left[ \cos \beta_1 \theta \xi_2 - \left( \frac{\cos \beta_1 \theta \gamma}{\cosh \beta_1 \theta \gamma} \right) \cosh \beta_1 \theta \xi_2 \right]$$

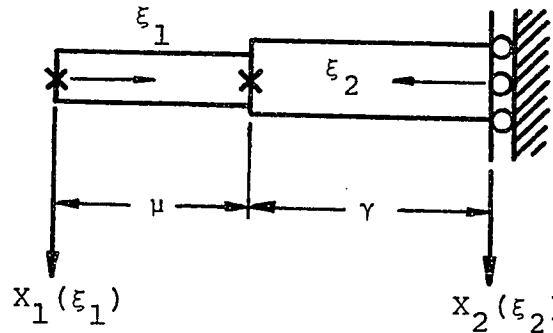
where

$$\gamma_1 = \frac{1}{\theta} \cdot \frac{\cosh \beta_1 \theta \gamma}{\sinh \beta_1 \mu} \cdot \frac{\cos \beta_1 \mu \sinh \beta_1 \mu - \sin \beta_1 \mu \cosh \beta_1 \mu}{\sin \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma + \cos \beta_1 \theta \gamma \sinh \beta_1 \theta \gamma}$$

Case 14. Three span beam clamped at each end and with symmetric discontinuity in cross section.



Antisymmetric modes and frequencies (2nd and 4th modes): Utilize first and second mode data of Case 8.6. Symmetric modes and frequencies (1st and 3rd modes): Utilize first and second mode data, Tables 8.13, and modal shape information given below.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 + \left( \frac{\sin \beta_1 \mu - \sinh \beta_1 \mu}{\cosh \beta_1 \mu - \cos \beta_1 \mu} \right) \times (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = \gamma_1 \left[ \cos \beta_1 \theta \xi_2 - \left( \frac{\cos \beta_1 \theta \gamma}{\cosh \beta_1 \theta \gamma} \right) \cosh \beta_1 \theta \xi_2 \right]$$

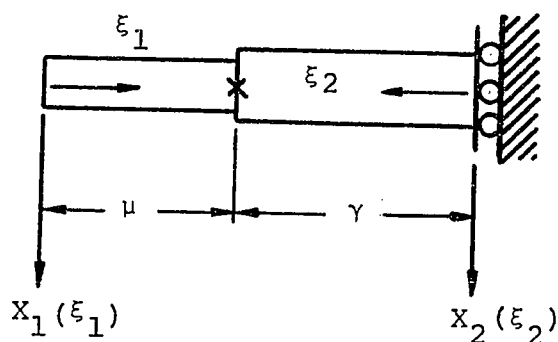
where

$$\gamma_1 = \frac{2}{\theta} \cdot \frac{\cosh \beta_1 \theta \gamma}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \left( \frac{1 - \cos \beta_1 \mu \cosh \beta_1 \mu}{\sin \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma + \cos \beta_1 \theta \gamma \sinh \beta_1 \theta \gamma} \right)$$

Case 15. Three span beam free at each end and with symmetric discontinuity in cross section.



Antisymmetric modes and frequencies (2nd and 4th modes): Utilize first and second mode data of Case 8.9. Symmetric modes and frequencies (1st and 3rd modes): Utilize first and second mode data, Tables 8.14, and modal shape information given below.



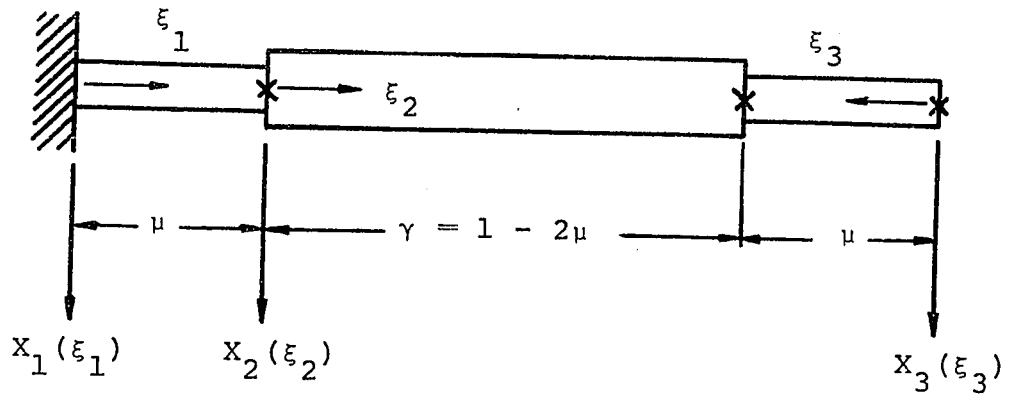
$$X_1(\xi_1) = \sin \beta_1 \xi_1 + \sinh \beta_1 \xi_1 - \left( \frac{\sin \beta_1 \mu + \sinh \beta_1 \mu}{\cos \beta_1 \mu + \cosh \beta_1 \mu} \right) \times (\cos \beta_1 \xi_1 + \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = \gamma_1 \left[ \cos \beta_1 \theta \xi_2 - \left( \frac{\cos \beta_1 \theta \gamma}{\cosh \beta_1 \theta \gamma} \right) \cosh \beta_1 \theta \xi_2 \right]$$

where

$$\gamma_1 = \frac{2}{\theta} \cdot \frac{\cosh \beta_1 \theta \gamma}{\cos \beta_1 \mu + \cosh \beta_1 \mu} \left( \frac{1 + \cos \beta_1 \mu \cosh \beta_1 \mu}{\sin \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma + \cos \beta_1 \theta \gamma \sinh \beta_1 \theta \gamma} \right)$$

Case 16. Three span beam clamped at one end, with simple support at the other, and with symmetric discontinuity in cross section. Utilize Tables 8.16.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 - \left( \frac{\sin \beta_1 \mu - \sinh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \right) \\ \times (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = C_1^{(2)} \left[ \sin \beta_1 \theta \xi_2 - \left( \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \sinh \beta_1 \theta \xi_2 \right] + \\ C_2^{(2)} \left[ \cos \beta_1 \theta \xi_2 - \cosh \beta_1 \theta \xi_2 + \right. \\ \left. \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \sinh \beta_1 \theta \xi_2 \right]$$

$$X_3(\xi_3) = C_1^{(3)} \left[ \sin \beta_1 \xi_3 - \left( \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \right) \sinh \beta_1 \xi_3 \right]$$

$$C_1^{(2)} \left( \frac{\sinh \beta_1 \theta \gamma - \sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) + C_2^{(2)} \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right)$$

$$= \frac{2}{\theta} \cdot \frac{1 - \cos \beta_1 \mu \cosh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu}$$

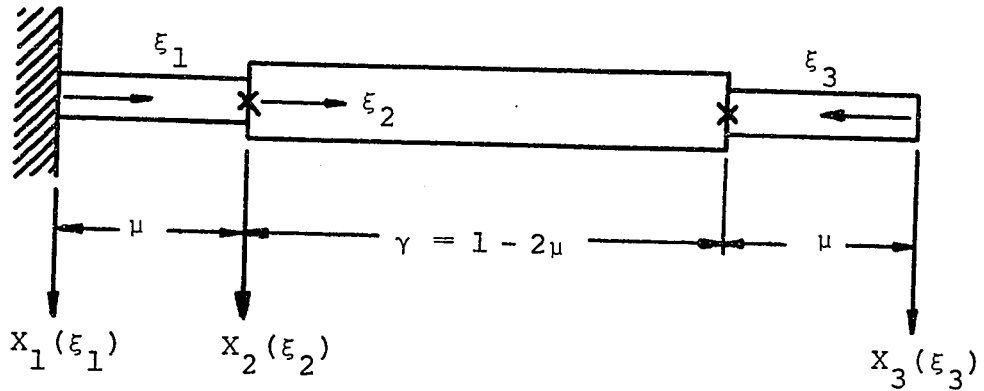
$$C_2^{(2)} = \frac{1}{\alpha^4 \theta^2} \cdot \frac{\cos \beta_1 \mu \sinh \beta_1 \mu - \sin \beta_1 \mu \cosh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu}$$

$$C_1^{(2)} \theta \left\{ \cos \beta_1 \theta \gamma - \left( \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \cosh \beta_1 \theta \gamma \right\} + C_2^{(2)}$$

$$\times \theta \left\{ \frac{1 - \sin \beta_1 \theta \gamma \sinh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right\} + C_1^{(3)}$$

$$\times \left\{ \cos \beta_1 \mu - \left( \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \right) \cosh \beta_1 \mu \right\} = 0$$

Case 17. Three span beam clamped at one end, free at the other, and with symmetric discontinuity in cross section. Utilize Tables 8.17.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \sinh \beta_1 \xi_1 - \left( \frac{\sin \beta_1 \mu - \sinh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu} \right) \\ \times (\cos \beta_1 \xi_1 - \cosh \beta_1 \xi_1)$$

$$X_2(\xi_2) = C_1^{(2)} \left[ \sin \beta_1 \theta \xi_2 - \left( \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \sinh \beta_1 \theta \xi_2 \right] + C_2^{(2)} \\ \times \left[ \cos \beta_1 \theta \xi_2 - \cosh \beta_1 \theta \xi_2 + \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \right. \\ \left. \times \sinh \beta_1 \theta \xi_2 \right]$$

$$X_3(\xi_3) = C_1^{(3)} \left[ \sin \beta_1 \xi_3 + \sinh \beta_1 \xi_3 - \left( \frac{\sin \beta_1 \mu + \sinh \beta_1 \mu}{\cos \beta_1 \mu + \cosh \beta_1 \mu} \right) \right. \\ \left. \times (\cos \beta_1 \xi_3 + \cosh \beta_1 \xi_3) \right]$$

$$C_1^{(2)} \left( \frac{\sinh \beta_1 \theta \gamma - \sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) + C_2^{(2)} \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right)$$

$$= \frac{2}{\theta} \cdot \frac{1 - \cos \beta_1 \mu \cosh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu}$$

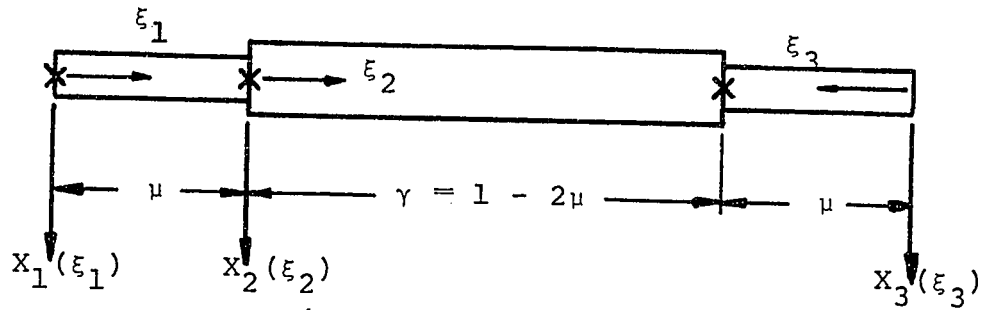
$$C_2^{(2)} = \frac{1}{\alpha^4 \theta^2} \cdot \frac{\cos \beta_1 \mu \sinh \beta_1 \mu - \sin \beta_1 \mu \cosh \beta_1 \mu}{\cos \beta_1 \mu - \cosh \beta_1 \mu}$$

$$C_1^{(2)} \theta \left( \cos \beta_1 \theta \gamma - \left( \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \cosh \beta_1 \theta \gamma \right) + C_2^{(2)}$$

$$\times \theta \left( \frac{1 - \sin \beta_1 \theta \gamma \sinh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) + C_1^{(3)}$$

$$\times \left( \frac{2(1 + \cos \beta_1 \mu \cosh \beta_1 \mu)}{\cos \beta_1 \mu + \cosh \beta_1 \mu} \right) = 0$$

Case 18. Three span beam with simple support at one end, free at the other, and with symmetric discontinuity in cross section. Utilize Tables 8.18.



$$X_1(\xi_1) = \sin \beta_1 \xi_1 - \left( \frac{\sin \beta_1 \mu}{\sinh \beta_1 \mu} \right) \sinh \beta_1 \xi_1$$

$$X_2(\xi_2) = C_1^{(2)} \left[ \sin \beta_1 \theta \xi_2 - \left( \frac{\sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \sinh \beta_1 \theta \xi_2 \right] + C_2^{(2)} \\ \times \left[ \cos \beta_1 \theta \xi_2 - \cosh \beta_1 \theta \xi_2 + \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \sinh \beta_1 \theta \xi_2 \right]$$

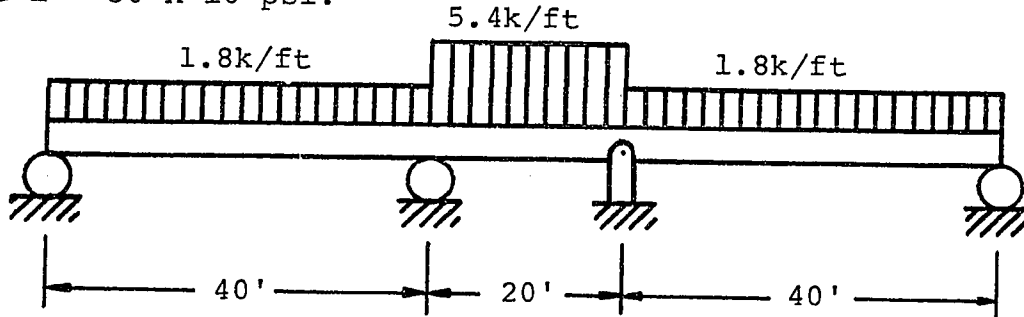
$$X_3(\xi_3) = C_1^{(3)} \left[ \sin \beta_1 \xi_3 + \sinh \beta_1 \xi_3 - \left( \frac{\sin \beta_1 \mu + \sinh \beta_1 \mu}{\cos \beta_1 \mu + \cosh \beta_1 \mu} \right) (\cos \beta_1 \xi_3 + \cosh \beta_1 \xi_3) \right]$$

$$C_1^{(2)} \theta \left( \frac{\sinh \beta_1 \theta \gamma - \sin \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) + C_2^{(2)} \theta \left( \frac{\cosh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) \\ = \cos \beta_1 \mu - \frac{\sin \beta_1 \mu \cosh \beta_1 \mu}{\sinh \beta_1 \mu}$$

$$C_2^{(2)} = \frac{1}{\alpha^4 \theta^2} \cdot \sin \beta_1 \mu$$

$$C_1^{(2)} \theta \left( \cos \beta_1 \theta \gamma - \frac{\sin \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) + C_2^{(2)}$$
$$\times \theta \left( \frac{1 - \sin \beta_1 \theta \gamma \sinh \beta_1 \theta \gamma - \cos \beta_1 \theta \gamma \cosh \beta_1 \theta \gamma}{\sinh \beta_1 \theta \gamma} \right) +$$
$$C_1^{(3)} \left\{ \frac{2(1 + \cos \beta_1 \mu \cosh \beta_1 \mu)}{\cos \beta_1 \mu + \cosh \beta_1 \mu} \right\} = 0$$

Illustrative Example 5. The continuous beam is a 12WF27 and is loaded as shown. Determine the first and second modes of vibration. Assume mass does not part from beam. Take  $E = 30 \times 10^6$  psi.



Solution: From tables on properties of rolled steel shapes, for a 12WF27 section,

weight per foot = 27 lb

Area of cross section =  $7.97 \text{ in}^2$

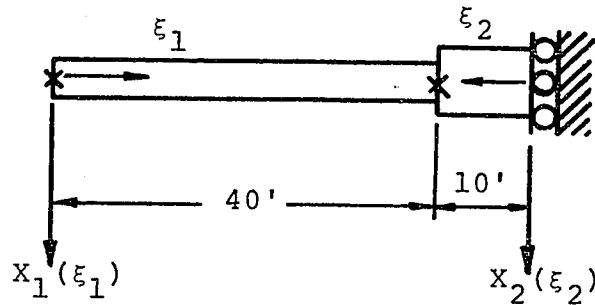
$I = 204.1 \text{ in}^4$

For the original beam,

$$\mu = \frac{40}{100} = 0.4$$

$\therefore$  for the half beam,  $\mu = 0.8$ .

1st modes



$$1.8 \text{ k/ft} = 1800/12 = 150 \text{ lb/in}$$

$$5.4 \text{ k/ft} = 450 \text{ lb/in}$$

$$g\rho_1 A_1 = \frac{27}{12} + 150 = 152.25, \text{ lb/in}$$

$$g\rho_2 A_2 = \frac{27}{12} + 450 = 452.25, \text{ lb/in}$$

$$\phi = \left( \frac{\rho_2 A_2}{\rho_1 A_1} \right)^{\frac{1}{4}} = \left( \frac{452.25}{152.25} \right)^{\frac{1}{4}} = 1.313$$

$$\therefore \frac{1}{\phi} = 0.7616$$

$$\frac{1}{\alpha} = 1.0$$

Taking  $\frac{1}{\phi}$  as the only independent variable, we have, from Tables 8.12,

$$\begin{aligned} {}^1\beta_1(0.7616) &= {}^1\beta_1(0.7) + (0.0616) \left[ \frac{{}^1\beta_1(0.8) - {}^1\beta_1(0.7)}{0.1} \right] \\ &= 4.274 + (0.616)(4.314 - 4.274) \\ &= 4.299 \end{aligned}$$

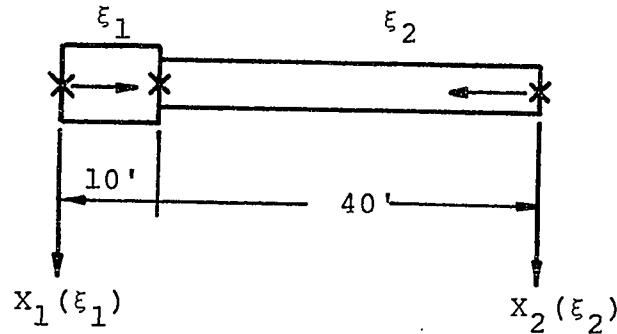
$$\begin{aligned} {}^1f &= \frac{1}{2\pi} \left( \frac{{}^1\beta_1}{L} \right)^2 \sqrt{\frac{E_1 I_1}{\rho_1 A_1}} = \frac{1}{2\pi} \left( \frac{4.299}{50} \right)^2 \sqrt{\frac{(30 \times 10^6)(204.1)}{\left(\frac{152.25}{386}\right)(7.97)}} \\ &= 51.92, \text{ Hz} \end{aligned}$$

The modal shape is

$$X_1(\xi_1) = \sin(4.299 \xi_1) + (0.0188) \sinh(4.299 \xi_1)$$

$$X_2(\xi_2) = (-0.4036) [\cos(5.645 \xi_2) - (0.2505) \cosh(5.645 \xi_2)]$$

2nd mode



$$\mu = 0.2, \quad \phi = 0.7616, \quad \alpha = 1.0$$

Similarly, from Tables 8.7,

$$\begin{aligned} 2_{\beta_1}(0.7616) &= 2_{\beta_1}(0.7) + (0.0616) \left[ \frac{2_{\beta_1}(0.8) - 2_{\beta_1}(0.7)}{0.1} \right] \\ &= 6.593 + (0.616) (5.771 - 6.593) \\ &= 6.087 \end{aligned}$$

$$\begin{aligned} 2_f &= \frac{1}{2\pi} \left( \frac{2_{\beta_1}}{L} \right)^2 \sqrt{\frac{E_1 I_1}{\rho_1 A_1}} = \frac{1}{2\pi} \left( \frac{6.087}{50} \right)^2 \sqrt{\frac{(30 \times 10^6) (204.1)}{\left( \frac{452.25}{386} \right) (7.97)}} \\ &= 60.41, \text{ Hz} \end{aligned}$$

The modal shape is

$$X_1(\xi_1) = \sin(6.087 \xi_1) - (0.6088) \sinh(6.087 \xi_1)$$

$$X_2(\xi_2) = (-3.3159) \left[ \sin(4.636 \xi_2) + (0.0263) \sinh(4.636 \xi_2) \right]$$

## CONCLUSION

The main factor taken into consideration in selecting a format for the vibration tables described herein has been to prepare tables which can be easily utilized by the design engineer with a minimum of effort. Once a set of specified values of the parameters is known, one can quickly determine the natural frequencies and establish the corresponding normal modes of vibration. The simplicity of the presentation makes it particularly useful to the designer who is not a vibration specialist.

The decision to limit the tables to the first four modes was made after taking several factors into consideration. In practice, it often is necessary to know only a few of the natural frequencies. Sometimes only one, usually the lowest frequency is the most important. As the mode number increases the effects of shearing deformation and rotatory inertia become significant and the Bernoulli-Euler assumptions are no longer applicable.

As pointed out earlier, the tables can be used to obtain frequencies with an accuracy generally within one percent. Exceptions are some fourth mode frequencies. It is very important to note that, in general, error in computed frequencies arising from uncertainty in beam properties will be greater than one percent and therefore nothing is to be gained by spacing the tabulated values at closer intervals.

TABLES<sup>1</sup> OF EIGENVALUES  $\beta_1$  FOR BEAMS WITH DISCONTINUITIES  
IN CROSS SECTION PROPERTIES

Beams with single discontinuities.

- 8.1 - Beam clamped at one end and free at the other.
- 8.2 - Beam clamped at one end and with simple support at the other.
- 8.3 - Beam with simple support at each end.
- 8.4 - Beam clamped at each end.
- 8.5 - Beam clamped at one end, free at the other, and with simple support at the discontinuity.
- 8.6 - Beam clamped at one end, with simple support at the other, and with simple support at the discontinuity.
- 8.7 - Beam with simple support at each end and with simple support at the discontinuity.
- 8.8 - Beam clamped at each end and with simple support at the discontinuity.
- 8.9 - Beam with simple support at one end, free at the other, and with simple support at the discontinuity.

Beams with symmetric discontinuities in cross section.

- 8.10 - Beam with simple support at each end.
- 8.11 - Beam clamped at each end.

---

1. As the tables are prepared for another publication, they are preceded by the numeral 8.

- 8.12 - Beam with simple support at each end, and with simple supports at the discontinuities.
- 8.13 - Beam clamped at each end, and with simple supports at the discontinuities.
- 8.14 - Beam free at each end, and with simple supports at the discontinuities.
- 8.15 - Beam clamped at one end and with simple support at the other.
- 8.16 - Beam clamped at one end, with simple support at the other, and with simple supports at the discontinuities.
- 8.17 - Beam clamped at one end, free at the other, and with simple supports at the discontinuities.
- 8.18 - Beam with simple support at one end, free at the other, and with simple supports at the discontinuities.

TABLE 8.1 : f1 (a = -7, a = -8)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.300 2.388 2.442 2.489 2.526 2.559 2.586 2.614 2.633 2.645 2.649 2.643 2.629 2.607 2.579 2.545

(2) 5.696 5.203 5.630 5.489 5.315 5.139 5.048 4.995 4.999 5.033 5.136 5.215 5.253 5.274 5.274 5.252

(3) 9.256 8.973 8.631 8.468 8.440 8.607 8.532 8.527 8.607 8.250 8.139 8.106 8.199 8.352 8.475

(4) 12.49 12.10 12.04 12.14 12.13 11.92 11.66 11.55 11.69 11.71 11.64 11.44 11.44 11.27 11.26 11.47

TABLE 8.1 : f1 (a = -7, a = -9)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.523 2.533 2.518 2.597 2.607 2.606 2.593 2.568 2.534 2.470 2.405 2.334 2.258 2.179 2.101

(2) 6.198 6.160 6.031 5.806 5.683 5.500 5.355 5.282 5.219 5.219 5.266 5.273 5.291 5.282 5.254

(3) 10.11 9.758 9.372 9.135 9.054 9.039 9.061 9.387 9.828 9.633 9.468 9.300 9.282 9.316 9.445

(4) 13.69 13.22 13.10 13.19 13.01 12.70 12.37 12.21 12.21 12.21 12.06 11.81 11.51 11.37 11.57

TABLE 8.1 : f1 (a = -7, a = -10)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.678 2.637 2.625 2.629 2.658 2.684 2.684 2.708 2.730 2.747 2.747 2.747 2.734 2.739 2.719 2.701

(2) 6.600 6.614 6.616 6.621 6.623 6.623 6.623 6.623 6.623 6.623 6.623 6.623 6.623 6.623 6.623

(3) 10.97 10.51 10.02 9.715 9.603 9.603 9.586 9.586 9.586 9.586 9.586 9.586 9.586 9.586 9.586

(4) 14.04 14.24 14.09 14.07 13.85 13.39 12.97 12.77 12.74 12.68 12.45 12.12 11.82 11.65 11.62

TABLE 8.1 : f1 (a = -8, a = -6)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.610 1.646 1.767 1.840 1.934 2.013 2.089 2.149 2.189 2.204 2.198 2.173 2.135 2.089 2.039

(2) 3.973 4.094 4.175 4.193 4.150 4.076 4.010 3.985 4.024 4.137 4.272 4.359 4.394 4.382 4.373

(3) 6.533 6.567 6.600 6.645 6.640 6.640 6.645 6.685 6.733 6.846 6.980 6.950 7.065 7.476 7.962

(4) 8.918 8.753 8.691 8.654 9.130 9.371 9.416 9.332 9.381 9.384 10.21 10.15 10.15 10.11 10.48

TABLE 8.1 : f1 (a = -8, a = -7)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.818 1.907 1.975 2.041 2.103 2.156 2.199 2.236 2.237 2.231 2.210 2.178 2.137 2.090 2.039

(2) 4.508 4.580 4.599 4.643 4.646 4.642 4.634 4.634 4.634 4.634 4.634 4.634 4.634 4.634 4.634

(3) 7.372 7.308 7.174 7.082 7.114 7.219 7.356 7.460 7.482 7.435 7.385 7.410 7.569 7.832 8.099

(4) 10.04 9.867 9.862 10.01 10.19 10.25 10.17 10.11 10.21 10.43 10.42 10.42 10.42 10.42 10.42

TABLE 8.1 : f1 (a = -8, a = -8)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.019 2.090 2.138 2.180 2.216 2.244 2.261 2.267 2.261 2.244 2.216 2.180 2.138 2.090 2.039

(2) 4.989 5.012 4.990 4.931 4.854 4.783 4.733 4.716 4.733 4.783 4.854 4.931 4.989 5.012 4.989

(3) 8.140 8.009 7.906 7.817 7.804 7.847 7.903 7.978 7.983 7.887 7.804 7.817 7.906 8.049 8.160

(4) 11.16 10.99 10.95 11.02 11.08 11.04 10.95 10.89 10.85 11.04 11.05 11.02 10.95 10.99 11.16

TABLE 8.1 : f1 (a = -8, a = -9)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.208 2.234 2.237 2.276 2.289 2.286 2.287 2.289 2.274 2.250 2.219 2.180 2.138 2.090 2.039

(2) 5.435 5.422 5.399 5.308 5.219 5.129 5.050 4.994 4.964 4.964 4.964 4.964 4.964 4.964 4.964

(3) 8.939 8.796 8.615 8.468 8.391 8.316 8.304 8.316 8.316 8.316 8.316 8.316 8.316 8.316 8.316

(4) 12.29 12.05 11.93 11.92 11.93 11.93 11.93 11.93 11.93 11.93 11.93 11.93 11.93 11.93 11.93

TABLE 8.1 : f1 (a = -6, a = -6)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.146 2.204 2.248 2.285 2.314 2.331 2.337 2.337 2.337 2.337 2.337 2.337 2.337 2.337 2.337

(2) 5.437 5.437 5.437 5.437 5.437 5.437 5.437 5.437 5.437 5.437 5.437 5.437 5.437 5.437 5.437

(3) 8.465 8.392 8.279 8.145 7.989 7.837 7.687 7.543 7.413 7.308 7.225 7.165 7.129 7.102 7.082

(4) 11.45 10.75 10.76 11.10 11.40 11.24 10.78 10.55 10.28 11.24 11.40 11.10 10.76 10.75 11.45

TABLE 8.1 : f1 (a = -6, a = -7)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.543 2.512 2.478 2.439 2.395 2.346 2.293 2.237 2.177 2.113 2.048 1.984 1.921 1.858 1.795

(2) 6.091 6.014 5.937 5.860 5.783 5.706 5.629 5.552 5.475 5.398 5.321 5.244 5.167 5.090 4.990

(3) 9.615 9.144 8.631 8.443 8.521 8.701 8.822 8.742 8.489 8.213 8.018 8.044 8.327 8.642 8.990

(4) 12.69 12.13 12.21 12.43 12.44 12.00 11.59 11.53 11.40 11.99 11.44 11.16 11.10 11.30 11.76

TABLE 8.1 : f1 (a = -6, a = -8)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.714 2.704 2.687 2.697 2.700 2.719 2.748 2.783 2.827 2.878 2.936 2.999 3.066 3.136 3.208

(2) 6.031 6.034 6.041 6.079 6.128 6.184 6.245 6.314 6.384 6.454 6.524 6.594 6.664 6.734 6.804

(3) 10.42 9.948 9.446 9.267 9.206 9.394 9.737 9.100 8.885 8.613 8.453 8.460 8.614 8.758 9.110

(4) 13.04 13.48 13.54 13.41 13.11 12.77 12.43 12.43 12.53 12.49 12.17 11.80 11.59 11.67 11.89

TABLE 8.1 : f1 (a = -6, a = -9)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.843 2.877 2.906 2.919 2.928 2.932 2.933 2.933 2.933 2.933 2.933 2.933 2.933 2.933 2.933

(2) 7.204 7.022 6.828 6.621 6.424 6.234 6.050 5.879 5.714 5.554 5.403 5.261 5.134 5.020 4.914

(3) 11.53 10.77 10.20 9.609 9.041 8.986 9.468 9.413 9.272 8.972 8.721 8.721 8.721 8.721 8.721

(4) 15.25 14.77 14.72 14.64 14.16 13.52 13.17 13.13 13.13 13.13 13.13 13.13 13.13 13.13 13.13

TABLE 8.1 : f1 (a = -6, a = -10)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 3.124 3.127 3.115 3.109 3.103 3.092 3.083 3.078 3.072 3.067 3.062 3.057 3.052 3.047 3.042

(2) 7.759 7.590 7.424 7.264 7.106 6.946 6.784 6.621 6.458 6.295 6.132 5.969 5.806 5.643 5.480

(3) 12.43 11.51 10.67 10.63 10.60 10.57 10.38 10.02 9.810 9.254 8.905 8.884 8.861 8.843 8.776

(4) 16.44 15.89 15.85 15.40 14.89 14.37 13.81 13.76 13.48 13.33 13.26 12.41 12.10 12.00 11.99

TABLE 8.1 : f1 (a = -7, a = -6)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.840 1.927 1.973 2.019 2.078 2.134 2.192 2.250 2.308 2.366 2.424 2.482 2.540 2.598 2.656

(2) 4.539 4.618 4.744 4.718 4.596 4.476 4.354 4.234 4.113 4.009 4.009 4.009 4.009 4.009 4.009

(3) 7.459 7.007 6.137 6.112 6.080 6.020 6.216 6.691 7.569 7.458 7.304 7.237 7.308 7.453 7.692

(4) 10.04 9.842 9.855 9.864 10.11 10.27 10.12 9.897 9.923 10.218 10.68 10.69 10.48 10.48 10.89

TABLE 8.1 : f1 (a = -7, a = -7)

NOTE: a = 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.101 2.119 2.157 2.211 2.299 2.454 2.481 2.504 2.491 2.454 2.399 2.331 2.257 2.179 2.101

(2) 5.150 5.207 5.207 5.103 4.945 4.792 4.636 4.479 4.321 4.164 4.009 3.846 3.683 3.520 3.357

(3) 8.305 8.170 7.886 7.712 7.718 7.842 7.987 8.054 7.987 7.845 7.718 7.712 7.886 8.100 8.395

(4) 11.28 10.90 10.87 11.04 11.21 11.12 10.88 10.75 10.68 11.12 11.21 11.06 10.87 10.90 11.28

TABLE 8.1 :  $f_1$  ( $\theta = -1.0$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.346 2.343 2.347 2.340 2.336 2.339 2.338 2.302 2.241 2.234 2.231 2.182 2.138 2.090 2.039

(2) 5.838 5.832 5.836 5.843 5.850 5.857 5.864 5.801 5.720 5.632 5.542 5.453 5.363 5.270 5.180

(3) 9.231 9.239 9.248 9.258 9.269 9.280 9.291 9.200 9.100 9.000 8.900 8.800 8.700 8.600 8.500

(4) 13.39 13.07 12.88 12.70 12.53 12.37 12.20 12.00 11.84 11.76 11.67 11.57 11.47 11.37 11.25

TABLE 8.1 :  $f_1$  ( $\theta = -0.8$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.431 1.459 1.570 1.644 1.720 1.783 1.862 1.922 1.988 2.012 2.033 2.053 2.073 2.093 2.113

(2) 3.532 3.642 3.721 3.757 3.792 3.827 3.862 3.897 3.932 3.967 4.002 4.037 4.072 4.107 4.142

(3) 5.813 5.815 5.816 5.816 5.816 5.816 5.816 5.816 5.816 5.816 5.816 5.816 5.816 5.816 5.816

(4) 7.989 7.922 7.931 8.009 8.109 8.218 8.346 8.484 8.632 8.789 8.956 9.133 9.319 9.517 9.715

TABLE 8.1 :  $f_1$  ( $\theta = -0.6$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.634 1.695 1.756 1.815 1.871 1.921 1.963 1.998 2.017 2.027 2.028 2.020 2.005 1.986 1.962

(2) 4.008 4.077 4.107 4.103 4.080 4.039 4.002 4.002 4.002 4.002 4.002 4.002 4.002 4.002 4.002

(3) 6.588 6.561 6.530 6.518 6.507 6.504 6.504 6.504 6.504 6.504 6.504 6.504 6.504 6.504 6.504

(4) 8.999 8.934 8.917 9.189 9.380 9.585 9.799 9.928 9.978 9.948 9.828 9.578 9.248 8.858 8.438

TABLE 8.1 :  $f_1$  ( $\theta = -0.4$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.812 1.858 1.900 1.939 1.973 2.000 2.021 2.035 2.041 2.041 2.035 2.023 2.006 1.984 1.962

(2) 4.437 4.465 4.465 4.465 4.465 4.465 4.465 4.465 4.465 4.465 4.465 4.465 4.465 4.465 4.465

(3) 7.281 7.246 7.211 7.215 7.212 7.216 7.225 7.236 7.245 7.253 7.259 7.263 7.268 7.273 7.278

(4) 10.03 10.00 10.09 10.13 10.23 10.24 10.24 10.24 10.24 10.24 10.24 10.24 10.24 10.24 10.24

TABLE 8.1 :  $f_1$  ( $\theta = -0.2$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.982 1.986 2.007 2.025 2.039 2.049 2.055 2.057 2.053 2.049 2.039 2.025 2.007 1.986 1.962

(2) 4.832 4.835 4.835 4.835 4.835 4.835 4.835 4.835 4.835 4.835 4.835 4.835 4.835 4.835 4.835

(3) 7.986 7.943 7.894 7.860 7.831 7.861 7.865 7.861 7.851 7.848 7.860 7.874 7.883 7.886

(4) 11.07 11.01 10.99 11.00 11.01 11.00 10.98 10.97 10.98 11.00 11.01 11.00 10.99 11.01 11.07

TABLE 8.1 :  $f_1$  ( $\theta = -0.0$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.084 2.083 2.083 2.081 2.079 2.075 2.070 2.063 2.053 2.041 2.026 2.008 1.986 1.962

(2) 5.212 5.204 5.187 5.156 5.113 5.061 5.005 4.932 4.908 4.875 4.855 4.866 4.845 4.844 4.835

(3) 8.697 8.635 8.577 8.430 8.348 8.204 8.092 8.021 8.025 8.021 8.021 8.021 8.021 8.021 8.021

(4) 12.11 11.96 11.81 11.79 11.78 11.75 11.66 11.54 11.46 11.44 11.43 11.39 11.30 11.21 11.16

TABLE 8.1 :  $f_1$  ( $\theta = 0.0$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.188 1.919 1.813 1.680 1.548 1.415 1.279 1.136 1.000 0.863 0.720 0.577 0.434 0.291 0.148

(2) 3.119 3.179 3.234 3.288 3.340 3.411 3.471 3.539 3.593 3.675 3.687 3.687 3.687 3.687 3.687

(3) 5.236 5.265 5.285 5.310 5.332 5.352 5.366 5.366 5.366 5.366 5.366 5.366 5.366 5.366 5.366

(4) 7.192 7.202 7.203 7.203 7.203 7.203 7.203 7.203 7.203 7.203 7.203 7.203 7.203 7.203 7.203

TABLE 8.1 :  $f_1$  ( $\theta = 1.0$ ,  $a = -7$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.471 1.525 1.580 1.634 1.688 1.741 1.791 1.804 1.810 1.804 1.804 1.804 1.804 1.804 1.804

(2) 3.608 3.602 3.596 3.587 3.577 3.567 3.557 3.547 3.537 3.527 3.517 3.507 3.497 3.487 3.477

(3) 5.919 5.937 5.943 5.951 5.959 5.967 5.975 5.983 5.991 5.999 6.007 6.015 6.023 6.031 6.039

(4) 8.136 8.137 8.137 8.137 8.137 8.137 8.137 8.137 8.137 8.137 8.137 8.137 8.137 8.137 8.137

TABLE 8.1 :  $f_1$  ( $\theta = 1.0$ ,  $a = -8$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.631 1.632 1.711 1.716 1.774 1.771 1.803 1.825 1.842 1.851 1.863 1.863 1.863 1.863 1.863

(2) 3.994 4.004 4.031 4.034 4.038 4.038 4.038 4.038 4.038 4.038 4.038 4.038 4.038 4.038 4.038

(3) 6.566 6.569 6.590 6.614 6.636 6.658 6.680 6.702 6.724 6.746 6.768 6.790 6.812 6.834 6.856

(4) 9.082 9.130 9.238 9.423 9.533 9.598 9.704 9.810 9.916 10.022 10.128 10.234 10.340 10.446 10.552

TABLE 8.1 :  $f_1$  ( $\theta = 1.0$ ,  $a = -9$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.746 1.768 1.807 1.823 1.837 1.849 1.858 1.863 1.867 1.871 1.874 1.874 1.874 1.874 1.874

(2) 4.351 4.359 4.368 4.384 4.397 4.407 4.417 4.427 4.437 4.447 4.457 4.467 4.477 4.487 4.497

(3) 7.208 7.211 7.235 7.248 7.264 7.278 7.292 7.306 7.320 7.334 7.348 7.362 7.376 7.390 7.404

(4) 10.04 10.09 10.17 10.25 10.29 10.31 10.38 10.43 10.48 10.53 10.58 10.63 10.68 10.73 10.78

TABLE 8.1 :  $f_1$  ( $\theta = 1.0$ ,  $a = -6$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.303 1.320 1.314 1.319 1.322 1.322 1.322 1.322 1.322 1.322 1.322 1.322 1.322 1.322 1.322

(2) 4.217 4.220 4.218 4.216 4.214 4.212 4.210 4.208 4.206 4.204 4.202 4.200 4.198 4.196 4.194

(3) 7.303 7.307 7.303 7.302 7.302 7.302 7.302 7.302 7.302 7.302 7.302 7.302 7.302 7.302 7.302

(4) 10.33 10.41 10.41 11.11 11.43 11.29 10.81 10.37 10.81 11.29 11.43 11.11 10.71 10.41 10.33

TABLE 8.1 :  $f_1$  ( $\theta = 1.0$ ,  $a = -5$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.272 1.311 1.301 1.300 1.300 1.300 1.300 1.300 1.300 1.300 1.300 1.300 1.300 1.300 1.300

(2) 3.723 3.720 3.727 3.725 3.727 3.725 3.727 3.725 3.727 3.725 3.727 3.725 3.727 3.725 3.727

(3) 6.438 6.439 6.436 6.433 6.430 6.427 6.424 6.421 6.418 6.415 6.412 6.409 6.406 6.403 6.400

(4) 9.133 9.275 9.432 9.781 10.131 10.15 9.922 9.955 10.31 10.70 10.71 10.48 10.37 10.26

TABLE 8.1 :  $f_1$  ( $\theta = 1.0$ ,  $a = -4$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.231 1.203 1.192 1.187 1.187 1.187 1.187 1.187 1.187 1.187 1.187 1.187 1.187 1.187 1.187

(2) 3.340 3.352 3.368 3.388 3.412 3.448 3.491 3.556 3.634 3.703 3.859 3.987 4.107 4.184 4.233

(3) 5.740 5.787 5.864 5.987 6.168 6.396 6.661 6.919 7.081 7.103 7.027 6.945 6.946 7.063 7.232

(4) 8.158 8.284 8.500 8.794 9.123 9.391 9.641 9.799 9.799 9.599 10.01 10.24 10.17 10.48 10.37 10.15

TABLE 8.1 :  $f_1$  ( $\theta = 1.0$ ,  $a = -3$ )

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.181 1.168 1.157 1.149 1.144 1.141 1.141 1.141 1.141 1.141 1.141 1.141 1.141 1.141 1.141

(2) 3.048 3.058 3.076 3.105 3.148 3.201 3.272 3.362 3.474 3.607 3.760 3.932 4.071 4.179 4.239

(3) 5.184 5.243 5.338 5.476 5.660 5.889 6.151 6.417 6.626 6.713 6.702 6.684 6.747 6.939 7.204

(4) 7.357 7.501 7.724 8.010 8.333 8.630 8.788 8.781 8.799 9.028 9.442 9.785 9.810 9.817 9.848

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.175	1.178	1.176	1.178	1.179	1.178	1.178	1.178	1.178	1.178	1.178	1.178	1.178	1.178	1.178
(2)	2.820	2.827	2.824	2.824	2.824	2.824	2.824	2.824	2.824	2.824	2.824	2.824	2.824	2.824	2.824
(3)	4.319	4.324	4.320	4.320	4.320	4.320	4.320	4.320	4.320	4.320	4.320	4.320	4.320	4.320	4.320
(4)	6.832	6.842	6.840	6.840	6.840	6.840	6.840	6.840	6.840	6.840	6.840	6.840	6.840	6.840	6.840

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.170	1.174	1.172	1.172	1.172	1.172	1.172	1.172	1.172	1.172	1.172	1.172	1.172	1.172	1.172
(2)	4.315	4.320	4.317	4.317	4.317	4.317	4.317	4.317	4.317	4.317	4.317	4.317	4.317	4.317	4.317
(3)	6.828	6.838	6.836	6.836	6.836	6.836	6.836	6.836	6.836	6.836	6.836	6.836	6.836	6.836	6.836
(4)	11.97	11.98	11.97	11.97	11.97	11.97	11.97	11.97	11.97	11.97	11.97	11.97	11.97	11.97	11.97

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.484	1.484	1.481	1.477	1.477	1.477	1.477	1.477	1.477	1.477	1.477	1.477	1.477	1.477	1.477
(2)	4.342	4.347	4.343	4.342	4.342	4.342	4.342	4.342	4.342	4.342	4.342	4.342	4.342	4.342	4.342
(3)	6.839	6.847	6.843	6.843	6.843	6.843	6.843	6.843	6.843	6.843	6.843	6.843	6.843	6.843	6.843
(4)	10.61	10.63	10.62	10.61	10.61	10.61	10.61	10.61	10.61	10.61	10.61	10.61	10.61	10.61	10.61

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.476	1.476	1.473	1.469	1.469	1.469	1.469	1.469	1.469	1.469	1.469	1.469	1.469	1.469	1.469
(2)	4.340	4.345	4.341	4.340	4.340	4.340	4.340	4.340	4.340	4.340	4.340	4.340	4.340	4.340	4.340
(3)	6.837	6.845	6.841	6.841	6.841	6.841	6.841	6.841	6.841	6.841	6.841	6.841	6.841	6.841	6.841
(4)	10.60	10.62	10.61	10.60	10.60	10.60	10.60	10.60	10.60	10.60	10.60	10.60	10.60	10.60	10.60

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.376	1.372	1.370	1.368	1.368	1.368	1.368	1.368	1.368	1.368	1.368	1.368	1.368	1.368	1.368
(2)	4.336	4.341	4.337	4.336	4.336	4.336	4.336	4.336	4.336	4.336	4.336	4.336	4.336	4.336	4.336
(3)	6.835	6.843	6.839	6.839	6.839	6.839	6.839	6.839	6.839	6.839	6.839	6.839	6.839	6.839	6.839
(4)	10.59	10.61	10.60	10.59	10.59	10.59	10.59	10.59	10.59	10.59	10.59	10.59	10.59	10.59	10.59

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.374	1.371	1.369	1.367	1.367	1.367	1.367	1.367	1.367	1.367	1.367	1.367	1.367	1.367	1.367
(2)	4.335	4.340	4.336	4.335	4.335	4.335	4.335	4.335	4.335	4.335	4.335	4.335	4.335	4.335	4.335
(3)	6.834	6.842	6.838	6.838	6.838	6.838	6.838	6.838	6.838	6.838	6.838	6.838	6.838	6.838	6.838
(4)	10.58	10.60	10.59	10.58	10.58	10.58	10.58	10.58	10.58	10.58	10.58	10.58	10.58	10.58	10.58

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.372	1.369	1.367	1.365	1.365	1.365	1.365	1.365	1.365	1.365	1.365	1.365	1.365	1.365	1.365
(2)	4.334	4.339	4.335	4.334	4.334	4.334	4.334	4.334	4.334	4.334	4.334	4.334	4.334	4.334	4.334
(3)	6.833	6.841	6.837	6.837	6.837	6.837	6.837	6.837	6.837	6.837	6.837	6.837	6.837	6.837	6.837
(4)	10.57	10.59	10.58	10.57	10.57	10.57	10.57	10.57	10.57	10.57	10.57	10.57	10.57	10.57	10.57

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.696	1.641	1.601	1.571	1.553	1.539	1.531	1.527	1.526	1.526	1.526	1.526	1.526	1.526	1.526
(2)	4.800	4.961	4.938	4.888	4.821	4.769	4.690	4.622	4.577	4.544	4.519	4.500	4.483	4.466	4.452
(3)	6.547	6.822	6.800	6.789	6.659	6.643	6.633	6.629	6.629	6.629	6.629	6.629	6.629	6.629	6.629
(4)	12.04	11.97	12.04	12.16	12.16	12.16	12.16	12.16	12.16	12.16	12.16	12.16	12.16	12.16	12.16

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.682	1.604	1.576	1.556	1.541	1.531	1.526	1.524	1.524	1.524	1.524	1.524	1.524	1.524	1.524
(2)	4.431	4.661	4.624	4.624	4.624	4.624	4.624	4.624	4.624	4.624	4.624	4.624	4.624	4.624	4.624
(3)	7.631	7.666	7.664	7.664	7.664	7.664	7.664	7.664	7.664	7.664	7.664	7.664	7.664	7.664	7.664
(4)	10.79	10.82	10.81	10.81	10.81	10.81	10.81	10.81	10.81	10.81	10.81	10.81	10.81	10.81	10.81

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.575	1.537	1.542	1.512	1.514	1.510	1.518	1.519	1.522	1.529	1.540	1.555	1.576	1.604	1.642
(2)	4.063	4.071	4.087	4.110	4.139	4.178	4.215	4.262	4.312	4.362	4.408	4.437	4.452	4.458	4.449
(3)	6.897	6.943	7.012	7.105	7.219	7.428	7.676	7.983	8.467	8.959	9.400	9.759	9.943	9.811	9.611
(4)	9.747	9.894	9.998	10.115	10.225	10.277	10.225	10.225	10.225	10.225	10.225	10.225	10.225	10.225	10.225

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.500	1.500	1.501	1.501	1.502	1.504	1.507	1.512	1.518	1.526	1.538	1.554	1.575	1.604	1.642
(2)	3.758	3.759	3.759	3.764	3.767	3.769	3.769	3.769	3.769	3.769	3.769	3.769	3.769	3.769	3.769
(3)	6.308	6.358	6.444	6.567	6.719	6.899	7.098	7.402	7.804	8.210	8.624	9.044	9.468	9.892	10.316
(4)	8.883	8.919	8.908	8.907	8.907	8.907	8.907	8.907	8.907	8.907	8.907	8.907	8.907	8.907	8.907

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.494	1.675	1.821	1.982	1.951	1.735	1.735	1.712	1.712	1.712	1.712	1.712	1.712	1.712	1.712
(2)	6.343	6.269	6.107	5.882	5.639	5.411	5.203	5.043	4.908	4.801	4.719	4.658	4.615	4.586	4.572
(3)	10.81	10.41	10.12	10.03	10.03	10.03	10.03	10.03	10.03	10.03	10.03	10.03	10.03	10.03	10.03
(4)	14.98	14.73	14.76	14.67	14.67	14.67	14.67	14.67	14.67	14.67	14.67	14.67	14.67	14.67	14.67

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.908	1.866	1.801	1.769	1.746	1.729	1.719	1.712	1.710	1.710	1.710	1.710	1.710	1.710	1.710
(2)	5.377	5.569	5.523	5.536	5.537	5.537	5.537	5.537	5.537	5.537	5.537	5.537	5.537	5.537	5.537
(3)	9.318	9.416	9.230	9.113	9.057	9.112	9.114	9.114	9.114	9.114	9.114	9.114	9.114	9.114	9.114
(4)	13.79	13.16	13.12	13.15	13.03	12.72	12.39	12.23	12.24	12.24	12.24	12.24	12.24	12.24	12.24

TABLE B.1 :  $\beta_1$  ( $1/\alpha = 0.5, 1/\alpha = 1.0$ )

MODE:

$\alpha$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	1.447	1.604	1.773	1.750	1.713	1.710	1.713	1.708	1.708	1.710	1.715	1.723	1.735	1.751	1.772
(2)	5.003	5													

TABLE B.1 :  $f_1 (1/4 - 9, 1/2 - 9)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 1.772 1.751 1.731 1.714 1.701 1.704 1.704 1.714 1.723 1.731 1.741 1.751 1.761 1.772

(2) 4.569 4.574 4.584 4.596 4.602 4.607 4.611 4.611 4.607 4.602 4.596 4.584 4.574 4.569

(3) 7.764 7.764 7.764 7.764 7.764 7.764 7.764 7.764 7.764 7.764 7.764 7.764 7.764 7.764

(4) 10.91 10.94 10.97 11.00 11.02 11.01 10.99 10.98 10.99 11.01 11.00 10.97 10.94 10.91

TABLE B.1 :  $f_1 (1/4 - 9, 1/2 - 1.0)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 1.688 1.688 1.688 1.689 1.690 1.692 1.695 1.699 1.701 1.712 1.722 1.735 1.751 1.772

(2) 4.293 4.231 4.240 4.257 4.282 4.315 4.355 4.401 4.459 4.531 4.628 4.752 4.903 5.086 5.307

(3) 7.065 7.118 7.174 7.230 7.313 7.406 7.504 7.612 7.731 7.861 8.003 8.156 8.321 8.497 8.722

(4) 9.954 10.04 10.15 10.25 10.29 10.30 10.32 10.39 10.49 10.58 10.64 10.61 10.45 10.24 10.81

TABLE B.1 :  $f_1 (1/4 - 1.0, 1/2 - 1.0)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 2.171 2.093 2.022 1.979 1.947 1.924 1.908 1.896 1.888 1.881 1.879 1.877 1.876 1.875 1.875

(2) 7.018 6.961 6.931 6.911 6.900 6.900 6.911 6.928 6.958 7.003 7.061 7.131 7.214 7.311 7.429

(3) 11.89 11.99 12.08 12.16 12.23 12.29 12.33 12.34 12.33 12.33 12.33 12.33 12.33 12.33 12.33

(4) 16.30 15.91 15.68 15.63 15.60 15.59 15.59 15.59 15.59 15.59 15.59 15.59 15.59 15.59 15.59

TABLE B.1 :  $f_1 (1/4 - 1.0, 1/2 - 1.0)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 2.170 2.093 2.022 1.979 1.947 1.924 1.908 1.896 1.888 1.881 1.879 1.877 1.876 1.875 1.875

(2) 6.191 6.168 6.088 5.943 5.754 5.549 5.355 5.183 5.038 4.923 4.839 4.773 4.732 4.709 4.699

(3) 10.18 10.29 9.967 9.725 9.463 9.206 8.953 8.711 8.498 8.317 8.161 8.113 8.073 8.064

(4) 14.68 14.23 14.11 14.09 13.66 13.40 13.08 12.79 12.46 12.09 11.67 11.25 11.23 11.23

TABLE B.1 :  $f_1 (1/4 - 1.0, 1/2 - 1.0)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 2.052 2.005 1.969 1.943 1.923 1.908 1.897 1.889 1.884 1.880 1.878 1.876 1.876 1.875 1.875

(2) 5.357 5.355 5.334 5.341 5.395 5.429 5.466 5.506 5.549 5.594 5.641 5.689 5.738 5.784 5.831

(3) 9.477 9.308 9.187 9.018 8.813 8.600 8.407 8.240 8.088 7.948 7.826 7.720 7.636 7.562 7.506

(4) 13.25 12.98 12.82 12.80 12.76 12.59 12.34 12.10 12.02 12.01 11.94 11.76 11.51 11.27 11.10

TABLE B.1 :  $f_1 (1/4 - 1.0, 1/2 - 1.0)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 1.969 1.946 1.927 1.913 1.902 1.894 1.888 1.883 1.880 1.878 1.877 1.876 1.876 1.875 1.875

(2) 5.074 5.073 5.070 5.057 5.031 4.992 4.943 4.890 4.839 4.784 4.732 4.709 4.711 4.701 4.686

(3) 8.578 8.552 8.490 8.412 8.347 8.304 8.264 8.232 8.209 8.188 8.168 8.158 8.157 8.157 8.157

(4) 12.01 11.93 11.83 11.79 11.79 11.76 11.66 11.54 11.46 11.44 11.44 11.39 11.28 11.16 11.06

TABLE B.2 :  $f_1 (1/4 - 1.0, 1/2 - 1.0)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 4.447 4.398 4.370 4.364 4.364 4.343 4.311 4.308 4.320 4.340 4.362 4.375 4.381 4.398 4.427

(2) 7.843 7.758 7.704 7.691 7.714 7.758 7.819 7.894 8.000 8.137 8.303 8.497 8.726 9.000 9.316

(3) 10.42 10.09 9.808 10.16 10.50 10.61 10.45 10.25 9.893 9.778 10.07 10.52 10.62 10.35 10.00 9.670

(4) 13.27 13.09 13.48 13.37 13.37 13.32 13.14 13.66 13.71 13.23 13.23 13.27 13.27 13.78 13.56 13.13

TABLE B.2 :  $f_1 (1/4 - 6, 1/2 - 7)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 5.052 5.146 5.155 5.085 4.832 4.389 4.348 4.132 4.000 3.894 3.810 3.804 3.811 3.841 3.878

(2) 8.014 8.480 8.936 9.332 9.738 9.844 9.785 9.525 9.068 8.519 7.899 7.161 6.384 5.494 4.503

(3) 11.99 11.29 11.23 11.45 11.39 11.37 10.93 10.45 10.47 10.93 11.09 10.92 10.38 10.26 10.06

(4) 14.97 14.93 15.23 15.07 14.45 14.30 14.40 14.34 14.11 13.86 13.80 14.15 14.17 13.79 13.18

TABLE B.2 :  $f_1 (1/4 - 6, 1/2 - 8)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 5.388 5.612 5.541 5.360 5.108 4.800 4.597 4.393 4.231 4.109 4.021 3.964 3.933 3.971 3.970

(2) 9.669 9.700 9.641 9.313 8.829 8.287 8.068 8.198 8.205 7.953 7.619 7.438 7.242 7.117 7.059

(3) 13.15 12.50 12.44 12.58 12.48 12.07 11.61 11.37 11.50 11.51 11.42 11.49 11.09 10.45 10.32 10.29

(4) 16.49 16.67 16.74 16.22 15.63 15.58 15.73 15.49 14.93 14.34 14.35 14.68 14.47 13.96 13.63

TABLE B.2 :  $f_1 (1/4 - 6, 1/2 - 9)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 6.098 6.014 5.905 5.636 5.300 5.008 4.834 4.607 4.421 4.271 4.155 4.069 4.007 3.967 3.943

(2) 10.35 9.929 9.798 9.598 9.016 8.283 8.017 8.058 8.247 8.543 8.697 8.438 7.268 7.268 7.233

(3) 14.31 13.60 13.53 13.34 13.29 12.75 12.25 11.99 11.99 11.99 11.99 11.99 11.48 11.09 10.73 10.45

(4) 18.37 18.20 18.10 17.33 16.74 16.70 16.65 16.34 15.32 15.15 15.12 15.09 14.75 14.28 13.61

TABLE B.2 :  $f_1 (1/4 - 6, 1/2 - 1.0)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 6.316 6.324 6.265 5.996 5.668 5.336 5.039 4.762 4.564 4.389 4.247 4.136 4.053 3.994 3.976

(2) 11.36 10.51 9.867 9.475 9.328 8.799 8.293 8.199 8.062 8.316 8.566 8.484 7.377 7.361 7.269

(3) 15.42 14.66 14.55 14.45 14.04 13.35 12.78 12.51 12.47 12.41 12.15 11.71 11.29 10.85 10.36

(4) 19.07 19.78 19.37 18.33 17.76 17.70 17.44 16.72 16.02 15.65 15.41 15.44 15.04 14.58 13.98

TABLE B.2 :  $f_1 (1/4 - 7, 1/2 - 6)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 3.812 3.944 4.050 4.097 4.066 3.866 3.611 3.403 3.259 3.159 3.107 3.102 3.102 3.102 3.102

(2) 6.743 6.760 6.600 6.348 6.212 6.244 6.108 6.017 6.039 6.007 6.072 6.144 6.201 6.258 6.316

(3) 9.555 9.116 8.898 9.016 9.296 9.553 9.710 9.740 9.704 9.316 8.692 10.002 10.02 9.812 9.658

(4) 11.86 11.59 11.81 12.18 12.27 11.98 11.93 12.30 12.32 12.67 12.78 12.45 13.00 13.19 12.95

TABLE B.2 :  $f_1 (1/4 - 7, 1/2 - 7)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 4.322 4.423 4.464 4.439 4.368 4.218 4.019 3.954 3.897 3.791 3.739 3.718 3.764 3.827 3.873

(2) 7.607 7.304 7.243 7.011 6.917 6.939 7.004 7.220 7.238 7.235 7.105 6.981 6.843 6.817 6.847

(3) 10.58 10.19 10.06 10.18 10.37 10.42 10.37 10.04 9.979 10.14 10.37 10.44 10.29 10.09 9.948

(4) 13.36 13.21 13.45 13.57 13.36 13.12 13.23 13.52 13.55 13.28 13.12 13.31 13.58 13.47 13.22

TABLE B.2 :  $f_1 (1/4 - 7, 1/2 - 8)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE: (1) 4.195 4.314 4.421 4.390 4.630 4.485 4.319 4.208 4.100 4.017 3.960 3.926 3.911 3.910 3.916

(2) 8.400 8.205 7.907 7.648 7.557 7.537 7.616 7.617 7.657 7.555 7.499 7.258 7.132 7.056 7.011

(3) 11.70 11.28 11.14 11.20 11.26 11.17 10.94 10.73 10.67 10.67 10.83 10.77 10.58 10.14 10.21

(4) 14.49 14.73 14.46 14.78 14.45 1.426 14.34 1.442 1.423 11.93 13.01 13.91 13.91 13.96 13.76 13.49

TABLE 8.2:  $f_1$  ( $\theta = -3, \alpha = -6$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 5.866 3.072 3.182 3.274 3.353 3.421 3.478 3.526 3.568 3.603 3.633 3.659 3.681 3.702

(2) 5.235 3.340 3.541 3.700 3.823 3.915 3.989 4.053 4.109 4.158 4.202 4.242 4.278 4.310 4.338

(3) 7.441 7.414 7.387 7.360 7.333 7.306 7.279 7.252 7.225 7.198 7.171 7.144 7.117 7.090 7.063

(4) 9.518 9.487 9.459 9.431 9.403 9.375 9.347 9.319 9.291 9.263 9.235 9.207 9.179 9.151 9.123

TABLE 8.2:  $f_1$  ( $\theta = -5, \alpha = -7$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 3.371 3.446 3.498 3.537 3.571 3.604 3.631 3.653 3.671 3.685 3.695 3.702 3.708 3.713 3.717

(2) 5.944 5.926 5.918 5.910 5.902 5.894 5.886 5.878 5.870 5.862 5.854 5.846 5.838 5.830 5.822

(3) 8.398 8.380 8.372 8.364 8.356 8.348 8.340 8.332 8.324 8.316 8.308 8.300 8.292 8.284 8.276

(4) 10.60 10.61 11.02 11.24 11.38 11.52 11.67 11.81 11.96 12.10 12.25 12.40 12.56 12.74 12.77

TABLE 8.2:  $f_1$  ( $\theta = -5, \alpha = -8$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 3.372 3.372 3.372 3.372 3.372 3.372 3.372 3.372 3.372 3.372 3.372 3.372 3.372 3.372 3.372

(2) 6.383 6.384 6.377 6.369 6.359 6.348 6.337 6.326 6.315 6.304 6.293 6.282 6.271 6.260 6.249

(3) 9.346 9.301 9.327 9.415 9.504 9.592 9.681 9.770 9.859 9.948 10.037 10.126 10.215 10.304 10.393

(4) 12.10 12.11 12.25 12.34 12.34 12.34 12.34 12.34 12.34 12.34 12.34 12.34 12.34 12.34 12.34

TABLE 8.2:  $f_1$  ( $\theta = -5, \alpha = -9$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 4.038 4.070 4.068 4.058 4.046 4.032 4.018 4.004 3.990 3.976 3.962 3.948 3.934 3.920 3.906

(2) 7.207 7.175 7.131 7.073 7.000 6.915 6.815 6.700 6.573 6.435 6.287 6.129 5.963 5.790 5.621

(3) 10.30 10.24 10.21 10.21 10.21 10.21 10.21 10.21 10.21 10.21 10.21 10.21 10.21 10.21 10.21

(4) 13.39 13.35 13.34 13.37 13.35 13.33 13.34 13.37 13.37 13.35 13.33 13.35 13.37 13.37 13.37

TABLE 8.2:  $f_1$  ( $\theta = -5, \alpha = -10$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 4.362 4.358 4.349 4.339 4.328 4.317 4.305 4.293 4.281 4.269 4.257 4.245 4.233 4.221 4.209

(2) 7.634 7.591 7.516 7.422 7.312 7.187 7.048 6.895 6.728 6.546 6.353 6.151 5.941 5.717 5.484

(3) 11.76 11.61 11.00 10.93 10.91 10.91 10.91 10.91 10.91 10.91 10.91 10.91 10.91 10.91 10.91

(4) 16.63 16.46 16.40 16.40 16.32 16.16 16.08 16.08 16.04 16.04 16.04 16.04 16.04 16.04 16.04

TABLE 8.2:  $f_1$  ( $\theta = -1.0, \alpha = -6$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 2.669 2.765 2.817 2.908 2.964 2.937 2.859 2.802 2.879 3.020 3.095 3.210 3.364 3.538 3.738

(2) 4.732 4.817 4.814 4.846 4.877 4.970 5.139 5.343 5.578 5.777 5.880 5.889 5.811 6.007 6.282

(3) 6.711 6.729 6.758 6.890 7.120 7.401 7.853 7.774 7.784 7.784 7.860 8.140 8.598 8.964 9.053 9.085

(4) 8.616 8.633 8.649 9.159 9.481 9.637 9.674 9.622 10.23 10.72 10.93 10.96 11.32 11.99 12.39

TABLE 8.2:  $f_1$  ( $\theta = -1.0, \alpha = -7$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 3.034 3.103 3.151 3.177 3.186 3.187 3.190 3.204 3.237 3.294 3.376 3.483 3.608 3.732 3.815

(2) 5.314 5.384 5.387 5.406 5.473 5.594 5.754 5.928 6.073 6.132 6.170 6.175 6.214 6.270 6.626

(3) 7.483 7.584 7.673 7.844 8.063 8.341 8.519 8.519 8.519 8.519 8.519 8.519 8.519 8.519 8.519

(4) 9.790 9.892 10.12 10.39 10.55 10.57 10.66 10.93 11.31 11.51 11.52 11.69 12.13 12.44 12.47

TABLE 8.2:  $f_1$  ( $\theta = -7, \alpha = -9$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 5.211 5.207 5.159 5.061 4.921 4.737 4.591 4.437 4.304 4.192 4.104 4.036 3.989 3.957 3.939

(2) 8.170 8.098 8.157 8.265 8.402 8.562 8.741 8.931 9.131 9.341 9.561 9.791 10.031 10.281 10.541

(3) 12.82 12.31 12.11 12.10 12.09 11.90 11.59 11.33 11.21 11.21 11.09 10.84 10.61 10.41 10.40

(4) 16.36 16.13 16.15 15.93 15.49 15.25 15.26 15.19 14.87 14.51 14.35 14.36 14.30 14.04 13.72

TABLE 8.2:  $f_1$  ( $\theta = -7, \alpha = -10$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 5.598 5.598 5.498 5.378 5.211 5.018 4.816 4.619 4.463 4.320 4.202 4.108 4.036 3.986 3.953

(2) 9.937 9.601 9.451 9.370 9.366 9.487 9.738 10.111 10.611 11.241 11.991 12.841 13.701 14.581 15.481

(3) 13.971 13.221 12.99 12.97 12.89 12.58 12.15 11.81 11.63 11.63 11.59 11.40 11.10 10.79 10.51

(4) 17.73 17.46 17.42 17.61 16.40 16.16 16.21 15.90 15.63 16.98 16.79 16.77 16.63 16.28 13.89

TABLE 8.2:  $f_1$  ( $\theta = -8, \alpha = -6$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 3.435 3.552 3.613 3.635 3.650 3.637 3.600 3.547 3.482 3.411 3.340 3.280 3.231 3.193 3.165

(2) 5.937 6.001 6.015 6.030 6.046 6.062 6.078 6.094 6.110 6.126 6.142 6.158 6.174 6.190 6.206

(3) 8.378 8.214 8.095 8.177 8.208 8.174 8.070 8.174 8.275 8.381 8.491 8.601 8.711 8.821 8.931

(4) 10.60 10.45 10.62 10.94 11.18 11.13 11.06 11.30 11.74 11.99 11.90 11.90 11.90 12.34 12.76 12.71

TABLE 8.2:  $f_1$  ( $\theta = -8, \alpha = -7$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 3.792 3.875 3.924 3.932 3.900 3.822 3.718 3.574 3.471 3.452 3.459 3.460 3.443 3.405 3.364

(2) 6.576 6.638 6.630 6.641 6.621 6.548 6.448 6.308 6.208 6.140 6.109 6.108 6.108 6.108 6.112

(3) 9.306 9.218 9.153 9.251 9.421 9.528 9.632 9.732 9.831 9.931 10.031 10.131 10.231 10.331 10.431

(4) 11.98 11.91 12.06 12.27 12.39 12.16 12.14 12.10 12.70 12.66 12.67 12.71 13.02 13.10 12.99

TABLE 8.2:  $f_1$  ( $\theta = -8, \alpha = -8$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 4.198 4.219 4.268 4.273 4.273 4.257 4.208 4.135 4.035 3.900 3.822 3.868 3.877 3.872 3.909

(2) 7.197 7.184 7.198 7.273 7.407 7.607 7.873 8.207 8.601 9.051 9.541 10.071 10.641 11.251 11.901

(3) 10.42 10.24 10.16 10.20 10.27 10.29 10.22 10.16 10.11 10.18 10.27 10.31 10.25 10.16 10.11

(4) 13.30 13.30 13.38 13.44 13.35 13.25 13.30 13.32 13.43 13.32 13.26 13.34 13.44 13.41 13.29

TABLE 8.2:  $f_1$  ( $\theta = -8, \alpha = -9$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 4.573 4.571 4.553 4.516 4.455 4.359 4.278 4.218 4.143 4.078 4.027 3.988 3.960 3.943 3.933

(2) 8.092 8.078 8.073 8.073 8.073 8.073 8.073 8.073 8.073 8.073 8.073 8.073 8.073 8.073 8.073

(3) 11.46 11.23 11.07 11.04 11.04 11.04 11.04 11.04 11.04 11.04 11.04 11.04 11.04 11.04 11.04

(4) 15.73 15.38 15.37 15.35 15.40 15.23 15.18 15.19 15.12 15.95 15.81 15.79 15.80 15.72 15.56

TABLE 8.2:  $f_1$  ( $\theta = -8, \alpha = -10$ )

u 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 4.703 4.692 4.684 4.645 4.541 4.446 4.357 4.275 4.217 4.178 4.148 4.128 4.111 4.094 4.077

(2) 8.173 8.059 8.027 8.013 8.013 8.013 8.013 8.013 8.013 8.013 8.013 8.013 8.013 8.013 8.013

(3) 12.48 12.14 12.14 12.11 12.11 12.11 12.11 12.11 12.11 12.11 12.11 12.11 12.11 12.11 12.11

(4) 16.08 15.77 15.71 15.65 15.65 15.65 15.65 15.65 15.65 15.65 15.65 15.65 15.65 15.65 15.65

TABLE 6.2 :  $f_1$  ( $1/a = 1.0, \sigma = 0$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.539	3.337	3.132	2.928	2.724	2.520	2.316	2.112	1.908	1.704	1.500	1.296	1.092	0.888
(2)	5.913	5.916	5.968	6.058	6.183	6.341	6.533	6.768	7.045	7.364	7.725	8.127	8.570	9.054	9.578
(3)	8.452	8.491	8.579	8.727	8.933	9.198	9.534	9.942	10.423	10.987	11.635	12.368	13.187	14.092	15.082
(4)	10.938	11.32	11.65	11.94	12.19	12.41	12.59	12.73	12.83	12.90	12.94	12.96	12.96	12.95	12.92

TABLE 6.2 :  $f_1$  ( $1/a = 1.0, \sigma = 0$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.633	3.466	3.321	3.197	3.093	2.999	2.914	2.837	2.767	2.702	2.641	2.583	2.528	2.474
(2)	6.300	6.301	6.312	6.345	6.397	6.466	6.552	6.656	6.778	6.918	7.075	7.248	7.436	7.638	7.854
(3)	9.130	9.162	9.233	9.344	9.498	9.698	9.945	10.239	10.581	10.972	11.412	11.901	12.438	13.025	13.662
(4)	12.18	12.28	12.37	12.45	12.52	12.57	12.60	12.62	12.63	12.64	12.64	12.64	12.64	12.64	12.64

TABLE 6.2 :  $f_1$  ( $1/a = 1.0, 1/a = 0$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.453	3.292	3.149	3.024	2.916	2.822	2.741	2.672	2.614	2.565	2.516	2.467	2.418	2.369
(2)	5.784	5.782	5.774	5.770	5.769	5.772	5.777	5.783	5.790	5.800	5.812	5.826	5.841	5.857	5.873
(3)	8.458	8.511	8.603	8.737	8.914	9.134	9.398	9.707	10.062	10.464	10.914	11.412	11.958	12.552	13.196
(4)	11.20	11.43	11.61	11.74	11.83	11.89	11.93	11.96	11.98	12.00	12.01	12.02	12.02	12.02	12.02

TABLE 6.2 :  $f_1$  ( $1/a = 1.0, 1/a = 0$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.453	3.292	3.149	3.024	2.916	2.822	2.741	2.672	2.614	2.565	2.516	2.467	2.418	2.369
(2)	5.784	5.782	5.774	5.770	5.769	5.772	5.777	5.783	5.790	5.800	5.812	5.826	5.841	5.857	5.873
(3)	8.458	8.511	8.603	8.737	8.914	9.134	9.398	9.707	10.062	10.464	10.914	11.412	11.958	12.552	13.196
(4)	11.20	11.43	11.61	11.74	11.83	11.89	11.93	11.96	11.98	12.00	12.01	12.02	12.02	12.02	12.02

TABLE 6.2 :  $f_1$  ( $1/a = 1.0, 1/a = 0$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.453	3.292	3.149	3.024	2.916	2.822	2.741	2.672	2.614	2.565	2.516	2.467	2.418	2.369
(2)	5.784	5.782	5.774	5.770	5.769	5.772	5.777	5.783	5.790	5.800	5.812	5.826	5.841	5.857	5.873
(3)	8.458	8.511	8.603	8.737	8.914	9.134	9.398	9.707	10.062	10.464	10.914	11.412	11.958	12.552	13.196
(4)	11.20	11.43	11.61	11.74	11.83	11.89	11.93	11.96	11.98	12.00	12.01	12.02	12.02	12.02	12.02

TABLE 6.2 :  $f_1$  ( $1/a = 1.0, 1/a = 0$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.453	3.292	3.149	3.024	2.916	2.822	2.741	2.672	2.614	2.565	2.516	2.467	2.418	2.369
(2)	5.784	5.782	5.774	5.770	5.769	5.772	5.777	5.783	5.790	5.800	5.812	5.826	5.841	5.857	5.873
(3)	8.458	8.511	8.603	8.737	8.914	9.134	9.398	9.707	10.062	10.464	10.914	11.412	11.958	12.552	13.196
(4)	11.20	11.43	11.61	11.74	11.83	11.89	11.93	11.96	11.98	12.00	12.01	12.02	12.02	12.02	12.02

TABLE 6.2 :  $f_1$  ( $1/a = 1.0, 1/a = 0$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.453	3.292	3.149	3.024	2.916	2.822	2.741	2.672	2.614	2.565	2.516	2.467	2.418	2.369
(2)	5.784	5.782	5.774	5.770	5.769	5.772	5.777	5.783	5.790	5.800	5.812	5.826	5.841	5.857	5.873
(3)	8.458	8.511	8.603	8.737	8.914	9.134	9.398	9.707	10.062	10.464	10.914	11.412	11.958	12.552	13.196
(4)	11.20	11.43	11.61	11.74	11.83	11.89	11.93	11.96	11.98	12.00	12.01	12.02	12.02	12.02	12.02

TABLE 6.2 :  $f_1$  ( $1/a = 7, 1/a = 6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	
MODE:	(1)	4.039	4.033	4.004	3.948	3.878	3.804	3.739	3.689	3.659	3.649	3.652	3.665	3.746	3.806	3.863
(2)	7.429	7.486	7.576	7.767	7.963	8.163	8.367	8.575	8.787	8.999	9.213	9.427	9.641	9.854	10.067	10.280
(3)	11.09	11.03	11.20	11.47	11.84	12.31	12.88	13.55	14.32	15.19	16.16	17.23	18.40	19.67	21.04	22.51
(4)	14.70	14.94	15.27	15.71	16.24	16.96	17.84	18.96	20.41	22.29	24.61	27.38	30.61	34.30	38.45	43.06

TABLE 6.2 :  $f_1$  ( $1/a = 7, 1/a = 6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	
MODE:	(1)	3.963	3.971	3.956	3.922	3.862	3.785	3.701	3.618	3.536	3.454	3.372	3.290	3.208	3.126	3.044
(2)	6.217	6.697	6.678	6.689	6.729	6.817	6.968	7.187	7.475	7.843	8.292	8.822	9.434	10.127	10.902	11.759
(3)	9.819	9.836	9.937	10.17	10.59	11.28	12.16	13.24	14.62	16.40	18.69	21.49	24.80	28.61	32.92	37.73
(4)	12.97	13.15	13.44	13.84	14.46	15.34	16.51	18.00	19.94	22.36	25.38	29.04	33.37	38.39	44.07	50.45

TABLE 6.2 :  $f_1$  ( $1/a = 7, 1/a = 6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	
MODE:	(1)	3.218	3.223	3.235	3.249	3.266	3.285	3.310	3.342	3.384	3.439	3.508	3.591	3.682	3.773	3.850
(2)	5.972	6.072	6.004	6.132	6.235	6.312	6.377	6.436	6.489	6.537	6.581	6.621	6.658	6.692	6.716	6.736
(3)	8.785	8.865	8.909	9.012	9.128	9.258	9.391	9.528	9.672	9.822	9.979	10.142	10.311	10.485	10.664	10.847
(4)	11.61	11.90	12.07	12.29	12.51	12.73	12.95	13.17	13.39	13.61	13.83	14.05	14.27	14.49	14.71	14.93

TABLE 6.2 :  $f_1$  ( $1/a = 7, 1/a = 6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	
MODE:	(1)	2.958	2.958	2.969	2.976	2.981	2.984	2.987	2.990	2.993	2.996	2.999	3.002	3.005	3.008	3.011
(2)	5.820	5.845	5.830	5.838	5.848	5.860	5.873	5.886	5.899	5.912	5.925	5.938	5.951	5.964	5.977	5.990
(3)	7.918	8.043	8.218	8.439	8.671	8.913	9.165	9.427	9.699	9.981	10.272	10.572	10.880	11.196	11.520	11.853
(4)	10.47	10.70	10.98	11.34	11.76	12.21	12.70	13.22	13.77	14.34	14.93	15.54	16.17	16.82	17.48	18.14

TABLE 6.2 :  $f_1$  ( $1/a = 7, 1/a = 6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	
MODE:	(1)	2.750	2.754	2.762	2.776	2.800	2.836	2.885	2.950	3.034	3.137	3.261	3.404	3.558	3.705	3.823
(2)	4.965	5.004	5.075	5.183	5.329	5.508	5.709	5.911	6.074	6.164	6.186	6.160	6.106	6.177	6.259	6.329
(3)	7.213	7.347	7.518	7.729	8.038	8.260	8.371	8.386	8.282	8.102	7.851	7.501	7.084	6.649	6.214	5.819
(4)	9.331	9.773	10.08	10.38	10.56	10.59	10.67	10.59	10.47	10.25	10.03	10.01	10.01	10.01	10.01	10.01

TABLE 6.2 :  $f_1$  ( $1/a = 4, 1/a = 6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	
MODE:	(1)	4.812	4.804	4.802	4.801	4.791	4.771	4.741	4.691	4.621	4.531	4.421	4.291	4.141	3.991	3.841
(2)	6.692	6.815	6.937	7.059	7.181	7.303	7.425	7.547	7.669	7.791	7.913	8.035	8.157	8.279	8.401	8.523
(3)	12.55	12.37	12.45	12.59	12.52	12.30	12.10	11.63	11.39	11.40	11.53	11.48	11.20	10.84	10.52	10.29
(4)	16.56	16.69	16.78	16.75	16.64	16.58	16.37	15.49	14.93	14.34	14.35	14.68	14.47	14.04	13.63	13.22

TABLE 6.2 :  $f_1$  ( $1/a = 4, 1/a = 6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	4.071	4.078	4.078	4.073	4.059	4.039	4.013	3.970	3.908	3.82				

TABLE 8.2 :  $f_1$  ( $1/A - 9, 1/A - 1.0$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.335	3.337	3.342	3.350	3.364	3.384	3.410	3.444	3.493	3.572	3.721	3.916	4.186	4.587
(2)	6.372	6.395	6.438	6.496	6.568	6.653	6.766	6.916	7.104	7.332	7.612	8.047	8.642	9.413	10.487
(3)	8.238	8.306	8.407	8.557	8.751	9.000	9.323	9.705	10.146	10.656	11.246	11.927	12.713	13.613	14.648
(4)	12.133	12.226	12.337	12.470	12.638	12.845	13.093	13.393	13.747	14.158	14.639	15.194	15.839	16.580	17.428

TABLE 8.2 :  $f_1$  ( $1/A - 1.0, 1/A - .8$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	5.259	5.332	5.440	5.579	5.750	5.965	6.228	6.544	6.928	7.393	7.953	8.623	9.423	10.378
(2)	10.34	10.25	9.983	9.653	9.277	8.854	8.384	7.867	7.314	6.735	6.134	5.514	4.887	4.257	3.627
(3)	15.19	14.67	14.38	14.51	14.67	14.87	15.10	15.36	15.66	16.00	16.38	16.80	17.26	17.76	18.30
(4)	19.89	19.81	19.40	18.34	17.76	17.70	17.43	16.72	16.02	15.65	15.61	15.46	15.02	14.48	13.98

TABLE 8.2 :  $f_1$  ( $1/A - 1.0, 1/A - .7$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.083	3.083	3.081	3.075	3.066	3.054	3.038	3.018	2.994	2.966	2.934	2.898	2.858	2.814
(2)	9.318	9.307	9.292	9.273	9.250	9.223	9.193	9.159	9.121	9.079	9.033	8.983	8.930	8.874	8.816
(3)	13.66	13.21	13.01	12.99	12.91	12.85	12.81	12.79	12.78	12.78	12.79	12.81	12.84	12.87	12.91
(4)	17.69	17.48	17.44	17.42	17.43	17.46	17.51	17.57	17.64	17.72	17.81	17.91	18.02	18.14	18.27

TABLE 8.2 :  $f_1$  ( $1/A - 1.0, 1/A - .6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	4.594	4.591	4.589	4.579	4.564	4.544	4.519	4.488	4.452	4.412	4.370	4.327	4.283	4.238
(2)	8.509	8.482	8.464	8.437	8.402	8.359	8.308	8.250	8.187	8.121	8.052	7.980	7.907	7.833	7.759
(3)	12.32	12.07	11.88	11.82	11.81	11.73	11.53	11.28	11.03	11.01	11.02	10.98	10.84	10.64	10.44
(4)	16.01	15.77	15.74	15.66	15.35	15.05	14.95	14.93	14.77	14.47	14.24	14.17	14.15	14.01	13.76

TABLE 8.2 :  $f_1$  ( $1/A - 1.0, 1/A - .5$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	4.223	4.218	4.216	4.216	4.207	4.191	4.166	4.134	4.098	4.057	4.014	3.970	3.920	3.877
(2)	7.708	7.694	7.672	7.647	7.614	7.574	7.529	7.479	7.426	7.372	7.317	7.251	7.184	7.124	7.064
(3)	11.17	11.09	10.99	10.93	10.92	10.91	10.86	10.76	10.65	10.58	10.57	10.56	10.53	10.44	10.34
(4)	14.58	14.46	14.41	14.40	14.13	14.10	14.08	14.06	14.03	13.93	13.80	13.71	13.71	13.58	13.58

TABLE 8.2 :  $f_1$  ( $1/A - .6, 1/A - .5$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.060	3.060	3.059	3.057	3.052	3.043	3.032	3.018	2.999	2.975	2.947	2.914	2.877	2.837
(2)	9.932	9.819	9.861	9.967	10.146	10.406	10.752	11.184	11.702	12.314	13.028	13.854	14.792	15.842	17.013
(3)	8.945	9.126	9.425	9.738	10.167	10.738	11.457	12.338	13.393	14.648	16.118	17.824	19.787	22.039	24.604
(4)	12.728	12.67	12.59	12.72	12.819	12.848	12.848	12.848	12.848	12.848	12.848	12.848	12.848	12.848	12.848

TABLE 8.2 :  $f_1$  ( $1/A - .6, 1/A - .4$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.546	3.487	3.392	3.241	3.144	3.047	3.010	2.974	2.937	2.901	2.874	2.857	2.840	2.823
(2)	6.827	6.705	6.482	6.168	5.784	5.352	4.888	4.402	3.907	3.412	2.927	2.452	2.007	1.602	1.237
(3)	10.34	10.45	10.68	10.83	10.65	10.22	9.491	8.444	7.111	5.611	4.011	2.411	1.011	0.011	0.011
(4)	14.133	14.43	14.41	14.32	14.08	13.68	13.19	12.62	12.00	11.34	10.64	9.90	9.13	8.34	7.54

TABLE 8.2 :  $f_1$  ( $1/A - .8, 1/A - .6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.417	3.401	3.401	3.409	3.421	3.438	3.461	3.490	3.524	3.564	3.611	3.664	3.723	3.788
(2)	6.339	6.316	6.298	6.278	6.256	6.232	6.207	6.180	6.151	6.120	6.088	6.054	6.019	5.983	5.946
(3)	10.00	10.02	10.09	10.23	10.31	10.24	10.15	10.12	10.10	10.22	10.30	10.25	10.16	10.11	10.11
(4)	13.14	13.23	13.39	13.56	13.73	13.76	13.70	13.43	13.13	12.78	12.38	11.94	11.46	10.94	10.39

TABLE 8.2 :  $f_1$  ( $1/A - .8, 1/A - .5$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.193	3.190	3.189	3.196	3.214	3.242	3.281	3.330	3.390	3.461	3.544	3.639	3.746	3.864
(2)	6.168	6.221	6.278	6.337	6.401	6.471	6.548	6.633	6.726	6.828	6.939	7.058	7.185	7.319	7.460
(3)	9.032	9.121	9.259	9.409	9.580	9.774	9.991	10.231	10.496	10.787	11.106	11.454	11.833	12.244	12.688
(4)	11.81	12.09	12.24	12.35	12.38	12.46	12.60	12.70	12.69	12.72	12.67	12.67	12.67	12.67	12.67

TABLE 8.2 :  $f_1$  ( $1/A - .8, 1/A - .4$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.133	3.146	3.151	3.166	3.187	3.218	3.263	3.323	3.399	3.493	3.604	3.733	3.881	4.049
(2)	5.571	5.705	5.798	5.852	5.904	5.954	6.004	6.054	6.104	6.154	6.204	6.254	6.304	6.354	6.404
(3)	8.235	8.162	8.151	8.201	8.269	8.354	8.456	8.574	8.707	8.854	9.014	9.184	9.364	9.554	9.754
(4)	10.95	11.06	11.29	11.56	11.89	12.28	12.73	13.24	13.81	14.44	15.14	15.91	16.74	17.64	18.61

TABLE 8.2 :  $f_1$  ( $1/A - .8, 1/A - .3$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.187	3.171	3.169	3.176	3.194	3.221	3.260	3.310	3.371	3.444	3.529	3.626	3.734	3.854
(2)	5.716	5.608	5.507	5.404	5.300	5.204	5.116	5.034	4.958	4.887	4.821	4.760	4.704	4.653	4.607
(3)	13.91	13.58	13.56	13.52	13.32	12.79	12.25	11.70	11.15	10.60	10.05	9.50	8.95	8.40	7.85
(4)	18.29	18.29	18.13	17.71	16.72	15.08	13.76	12.76	12.00	11.49	11.10	10.80	10.60	10.40	10.30

TABLE 8.2 :  $f_1$  ( $1/A - .8, 1/A - .2$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	4.578	4.587	4.578	4.571	4.568	4.564	4.561	4.558	4.555	4.552	4.549	4.546	4.543	4.540
(2)	8.596	8.485	8.384	8.283	8.182	8.081	7.980	7.879	7.778	7.677	7.576	7.475	7.374	7.273	7.172
(3)	12.45	12.21	12.11	12.11	12.11	12.11	12.11	12.11	12.11	12.11	12.11	12.11	12.11	12.11	12.11
(4)	16.25	16.15	16.17	16.15	16.15	16.15	16.15	16.15	16.15	16.15	16.15	16.15	16.15	16.15	16.15

TABLE 8.2 :  $f_1$  ( $1/A - .9, 1/A - .8$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	4.116	4.117	4.143	4.154	4.176	4.198	4.220	4.242	4.264	4.286	4.308	4.330	4.352	4.374
(2)	7.978	7.945	7.918	7.900	7.882	7.864	7.846	7.828	7.810	7.792	7.774	7.756	7.738	7.720	7.702
(3)	11.18	11.20	11.04	11.06	11.03	11.02	11.00	10.99	10.97	10.96	10.94	10.93	10.91	10.89	10.87
(4)	14.24	14.37	14.57	14.83	15.23	15.76	16.44	17.28	18.28	19.44	20.78	22.31	24.04	25.97	28.11

TABLE 8.2 :  $f_1$  ( $1/A - .9, 1/A - .7$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:	(1)	3.601	3.609	3.616	3.623	3.630	3.637	3.644	3.651	3.658	3.665	3.672	3.679	3.686	3.693
(2)	6.992	6.910	6.986	7.058	7.131	7.204	7.277	7.350	7.423	7.496	7.569	7.642	7.715	7.788	7.861
(3)	10.12	10.14	10.17	10											

TABLE 8.3 :  $f_1$  ( $\theta = -7, \alpha = 1.0$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -8, \alpha = .6$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -8, \alpha = .7$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -8, \alpha = .8$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -8, \alpha = .9$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -9, \alpha = .6$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -6, \alpha = .8$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -6, \alpha = .9$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -6, \alpha = 1.0$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -7, \alpha = .7$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -7, \alpha = .8$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\theta = -7, \alpha = .9$ )

Table with 4 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85. Rows (1) to (6) containing numerical values.

TABLE 8.3 :  $f_1$  ( $\rho = .9, \alpha = .7$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	2.447	2.453	2.463	2.478	2.501	2.531	2.570	2.619	2.677	2.745	2.821	2.901	2.979	3.047	3.098
(2)	4.911	4.921	4.938	4.961	5.001	5.051	5.119	5.201	5.297	5.407	5.536	5.685	5.774	5.847	5.901
(3)	7.459	7.459	7.464	7.474	7.490	7.520	7.569	7.637	7.721	7.821	7.936	8.066	8.211	8.367	8.501
(4)	10.002	10.025	10.047	10.056	10.055	10.050	10.041	10.029	10.015	10.000	9.984	9.966	9.945	9.921	9.894

TABLE 8.3 :  $f_1$  ( $\rho = .9, \alpha = .8$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	2.794	2.797	2.802	2.811	2.824	2.842	2.865	2.894	2.927	2.964	3.003	3.044	3.086	3.130	3.175
(2)	5.674	5.674	5.683	5.701	5.729	5.767	5.814	5.870	5.935	6.009	6.091	6.181	6.279	6.384	6.494
(3)	8.447	8.437	8.443	8.449	8.456	8.464	8.472	8.480	8.487	8.493	8.498	8.503	8.508	8.511	8.513
(4)	11.313	11.317	11.316	11.316	11.314	11.311	11.307	11.302	11.295	11.286	11.274	11.258	11.237	11.211	11.181

TABLE 8.3 :  $f_1$  ( $\rho = .9, \alpha = .9$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	3.119	3.116	3.112	3.108	3.104	3.101	3.098	3.095	3.092	3.089	3.086	3.083	3.080	3.077	3.074
(2)	6.270	6.263	6.261	6.266	6.277	6.290	6.301	6.309	6.316	6.322	6.328	6.334	6.340	6.345	6.349
(3)	9.403	9.407	9.425	9.443	9.466	9.492	9.520	9.554	9.593	9.637	9.686	9.739	9.795	9.853	9.913
(4)	12.532	12.537	12.557	12.575	12.591	12.604	12.613	12.619	12.622	12.623	12.623	12.622	12.620	12.617	12.613

TABLE 8.3 :  $f_1$  ( $\rho = 1.0, \alpha = 1.0$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	1.451	1.458	1.450	1.455	1.455	1.452	1.447	1.442	1.437	1.432	1.427	1.422	1.417	1.412	1.407
(2)	6.914	6.917	6.914	6.914	6.914	6.914	6.914	6.914	6.914	6.914	6.914	6.914	6.914	6.914	6.914
(3)	10.109	10.119	10.114	10.112	10.108	10.103	10.098	10.093	10.088	10.083	10.078	10.073	10.068	10.063	10.058
(4)	13.048	13.047	13.047	13.046	13.045	13.044	13.043	13.042	13.041	13.040	13.039	13.038	13.037	13.036	13.035

TABLE 8.3 :  $f_1$  ( $\rho = 1.0, \alpha = .8$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	1.8915	1.905	1.923	1.950	1.985	2.011	2.048	2.100	2.216	2.431	2.618	2.766	2.814	2.834	2.834
(2)	3.872	3.903	3.956	4.031	4.139	4.280	4.454	4.674	4.947	5.271	5.644	6.066	6.536	7.051	7.611
(3)	5.878	5.976	6.121	6.319	6.570	6.874	7.234	7.654	8.134	8.674	9.274	9.934	10.654	11.434	12.254
(4)	7.891	8.145	8.455	8.831	9.271	9.781	10.361	11.011	11.731	12.531	13.401	14.341	15.341	16.391	17.491

TABLE 8.3 :  $f_1$  ( $\rho = 1.0, \alpha = .7$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	2.704	2.720	2.748	2.791	2.849	2.924	2.999	3.074	3.149	3.224	3.299	3.374	3.449	3.524	3.599
(2)	4.401	4.413	4.429	4.450	4.466	4.478	4.486	4.491	4.494	4.496	4.497	4.498	4.499	4.500	4.501
(3)	6.101	6.109	6.114	6.117	6.119	6.120	6.121	6.122	6.123	6.124	6.125	6.126	6.127	6.128	6.129
(4)	7.801	7.807	7.811	7.814	7.816	7.818	7.819	7.820	7.821	7.822	7.823	7.824	7.825	7.826	7.827

TABLE 8.3 :  $f_1$  ( $\rho = 1.0, \alpha = .6$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	2.531	2.537	2.548	2.571	2.601	2.639	2.684	2.737	2.797	2.861	2.929	2.999	3.064	3.088	3.116
(2)	5.002	5.015	5.030	5.051	5.079	5.114	5.156	5.204	5.257	5.314	5.374	5.437	5.501	5.571	5.631
(3)	7.495	7.515	7.547	7.591	7.646	7.714	7.794	7.886	7.990	8.106	8.234	8.374	8.524	8.684	8.854
(4)	10.145	10.167	10.170	10.172	10.172	10.172	10.172	10.172	10.172	10.172	10.172	10.172	10.172	10.172	10.172

TABLE 8.3 :  $f_1$  ( $\rho = 1.0, \alpha = .9$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	2.033	2.038	2.050	2.064	2.081	2.106	2.138	2.174	2.214	2.258	2.306	2.358	2.414	2.474	2.538
(2)	4.061	4.071	4.088	4.110	4.138	4.171	4.209	4.251	4.297	4.347	4.401	4.459	4.521	4.587	4.657
(3)	6.091	6.101	6.118	6.140	6.167	6.200	6.238	6.280	6.326	6.376	6.429	6.486	6.547	6.611	6.679
(4)	8.121	8.131	8.148	8.170	8.197	8.230	8.268	8.310	8.356	8.406	8.459	8.516	8.577	8.641	8.709

TABLE 8.4 :  $f_2$  ( $\rho = .6, \alpha = .6$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	5.337	5.468	5.519	5.584	5.673	5.774	5.894	6.031	6.184	6.354	6.538	6.734	6.941	7.158	7.394
(2)	8.443	7.390	7.811	7.556	7.622	7.672	7.708	7.731	7.743	7.746	7.747	7.747	7.747	7.747	7.747
(3)	11.445	10.75	10.76	11.10	11.39	11.54	11.68	11.81	11.93	12.05	12.16	12.26	12.35	12.44	12.53
(4)	13.94	13.93	14.39	14.50	14.50	14.50	14.50	14.50	14.50	14.50	14.50	14.50	14.50	14.50	14.50

TABLE 8.4 :  $f_2$  ( $\rho = .6, \alpha = .7$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	6.052	6.107	6.001	5.728	5.394	5.088	4.843	4.651	4.516	4.416	4.337	4.268	4.209	4.150	4.091
(2)	9.872	9.164	8.637	8.461	8.552	8.741	8.964	9.211	9.481	9.774	10.091	10.428	10.784	11.151	11.528
(3)	12.69	12.17	12.22	12.47	12.45	12.60	12.65	12.70	12.75	12.80	12.85	12.90	12.95	13.00	13.05
(4)	15.87	15.94	16.18	16.35	16.42	16.48	16.54	16.60	16.66	16.72	16.78	16.84	16.90	16.96	17.02

TABLE 8.4 :  $f_2$  ( $\rho = .6, \alpha = .8$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	6.687	6.644	6.438	6.095	5.732	5.342	5.133	4.952	4.803	4.685	4.618	4.584	4.551	4.517	4.480
(2)	10.61	9.970	9.456	9.291	9.541	9.935	10.471	11.151	11.874	12.641	13.451	14.301	15.191	16.111	17.061
(3)	13.98	13.48	13.54	13.61	13.68	13.75	13.82	13.89	13.96	14.03	14.10	14.17	14.24	14.31	14.38
(4)	17.68	17.76	17.69	16.98	16.39	16.73	16.68	16.63	16.58	16.53	16.48	16.43	16.38	16.33	16.28

TABLE 8.4 :  $f_2$  ( $\rho = .6, \alpha = .9$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	7.485	7.143	6.859	6.466	6.064	5.732	5.421	5.190	5.011	4.874	4.771	4.700	4.650	4.619	4.604
(2)	11.53	10.73	10.22	10.02	10.03	10.05	10.06	10.07	10.08	10.09	10.10	10.11	10.12	10.13	10.14
(3)	15.25	14.73	14.72	14.64	14.54	14.43	14.33	14.23	14.13	14.03	13.93	13.83	13.73	13.63	13.53
(4)	19.44	19.44	19.07	18.19	17.45	17.69	17.89	18.04	18.14	18.24	18.34	18.44	18.54	18.64	18.74

TABLE 8.4 :  $f_2$  ( $\rho = .6, \alpha = 1.0$ )

$\rho$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
MODE:															
(1)	7.816	7.635	7.283	6.824	6.367	5.909	5.641	5.379	5.175	5.029	4.906	4.827	4.774	4.744	4.735
(2)	12.43	11.52	10.89	10.66	10.64	10.59	10.58	10.58	10.						

TABLE B.4 :  $f_1(t; -7, 1, -7)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	5.173	5.261	5.325	5.375	5.415	5.442	5.459	5.468	5.470	5.465	5.455	5.440	5.421	5.398	5.374
(2)	8.301	8.188	8.087	7.994	7.909	7.831	7.760	7.698	7.644	7.597	7.556	7.520	7.488	7.459	7.434
(3)	11.28	10.90	10.47	10.00	9.59	9.24	8.94	8.68	8.46	8.27	8.10	7.95	7.82	7.70	7.60
(4)	14.07	14.01	14.12	14.01	13.92	14.17	14.37	14.17	13.92	14.04	14.35	14.35	14.30	14.00	13.78

TABLE B.4 :  $f_1(t; -7, 1, -8)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	5.164	5.192	5.206	5.215	5.219	5.220	5.220	5.219	5.216	5.212	5.206	5.198	5.187	5.173	5.157
(2)	8.258	8.072	7.834	7.558	7.253	6.928	6.593	6.258	5.923	5.588	5.253	4.918	4.583	4.248	3.913
(3)	12.40	12.04	11.63	11.17	10.67	10.13	9.56	8.97	8.36	7.73	7.09	6.45	5.81	5.17	4.54
(4)	15.73	15.65	15.75	15.73	15.17	15.30	15.23	14.91	14.71	14.80	14.68	14.64	14.33	14.08	

TABLE B.4 :  $f_1(t; -7, 1, -9)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	6.248	6.208	6.092	5.904	5.639	5.323	4.978	4.623	4.268	3.913	3.558	3.203	2.848	2.493	2.138
(2)	10.11	9.752	9.179	8.324	7.204	5.984	4.698	3.372	2.046	7.811	6.485	5.159	3.833	2.507	1.181
(3)	13.59	13.22	13.10	13.01	12.70	12.18	11.46	10.46	9.22	7.81	6.25	4.56	2.75	0.84	-1.18
(4)	17.17	17.14	17.13	16.77	16.33	16.25	16.26	16.01	15.60	15.18	15.39	15.32	15.01	14.62	14.39

TABLE B.4 :  $f_1(t; -7, 1, -10)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	6.710	6.600	6.311	6.282	6.005	5.718	5.429	5.132	4.835	4.538	4.241	3.944	3.647	3.350	3.053
(2)	10.97	10.51	10.02	9.727	9.422	9.117	8.812	8.507	8.202	7.897	7.592	7.287	6.982	6.677	6.372
(3)	14.84	14.24	14.09	14.01	13.85	13.59	13.23	12.77	12.21	11.55	10.79	9.93	8.97	7.91	6.75
(4)	18.75	18.58	18.46	18.07	17.14	17.24	17.14	16.69	16.18	15.53	15.90	15.73	15.31	14.84	14.45

TABLE B.4 :  $f_1(t; -8, 1, -6)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	4.093	4.131	4.216	4.310	4.404	4.498	4.592	4.686	4.780	4.874	4.968	5.062	5.156	5.250	5.344
(2)	6.511	6.355	6.156	6.184	6.210	6.236	6.262	6.288	6.314	6.340	6.366	6.392	6.418	6.444	6.470
(3)	8.918	8.753	8.600	8.519	8.419	8.319	8.219	8.119	8.019	7.919	7.819	7.719	7.619	7.519	7.419
(4)	11.14	11.05	11.09	11.63	11.76	11.67	11.72	12.11	12.32	12.35	12.45	12.75	13.30	13.54	13.71

TABLE B.4 :  $f_1(t; -8, 1, -7)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	4.358	4.621	4.843	5.020	5.154	5.248	5.302	5.326	5.350	5.374	5.398	5.422	5.446	5.470	5.494
(2)	7.109	7.106	7.173	7.165	7.210	7.219	7.216	7.215	7.215	7.215	7.215	7.215	7.215	7.215	7.215
(3)	10.06	9.866	9.561	9.011	8.244	7.284	6.164	4.944	3.664	2.284	0.864	-0.576	-1.956	-3.276	-4.536
(4)	12.84	12.62	12.83	12.99	12.91	12.96	13.04	13.32	13.39	13.28	13.33	13.64	13.81	13.70	13.61

TABLE B.4 :  $f_1(t; -8, 1, -8)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	5.011	5.056	5.144	5.248	5.368	5.494	5.626	5.764	5.908	6.058	6.214	6.376	6.544	6.718	6.898
(2)	8.158	8.067	7.906	7.620	7.187	6.617	5.920	5.107	4.187	3.167	2.047	0.827	-0.403	-1.523	-2.543
(3)	11.18	10.95	10.55	10.08	9.54	8.91	8.107	7.107	5.907	4.507	2.907	1.107	-0.793	-2.493	-4.093
(4)	14.14	14.10	14.19	14.21	14.09	14.04	14.15	14.23	14.15	14.05	14.10	14.23	14.21	14.07	13.96

TABLE B.4 :  $f_1(t; -8, 1, -9)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	5.416	5.462	5.621	5.815	6.040	6.294	6.576	6.886	7.224	7.590	8.000	8.454	8.962	9.524	10.150
(2)	8.395	8.195	7.855	7.385	6.795	6.095	5.285	4.365	3.345	2.225	1.005	-0.215	-1.435	-2.855	-4.275
(3)	12.29	12.05	11.93	11.92	11.92	11.84	11.69	11.57	11.53	11.53	11.53	11.41	11.33	11.09	10.97
(4)	15.41	15.45	15.46	15.40	15.20	15.07	15.07	15.05	14.89	14.72	14.67	14.69	14.61	14.41	14.21

TABLE B.4 :  $f_1(t; -9, 1, -10)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	5.902	5.974	6.185	6.435	6.715	7.025	7.375	7.775	8.225	8.725	9.275	9.875	10.525	11.225	11.975
(2)	9.328	9.128	8.785	8.305	7.695	6.965	6.035	4.915	3.605	2.105	0.405	-1.495	-3.195	-4.695	-6.095
(3)	13.39	13.02	12.61	12.19	11.75	11.28	10.78	10.25	9.60	8.85	8.00	7.05	6.00	4.85	3.60
(4)	16.87	16.73	16.71	16.54	16.18	15.56	15.93	15.81	15.53	15.25	15.16	15.14	14.97	14.67	14.38

TABLE B.4 :  $f_1(t; -9, 1, -16)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	5.581	5.674	5.877	6.193	6.633	7.203	7.913	8.763	9.753	10.883	12.153	13.573	15.153	16.893	18.793
(2)	8.612	8.283	7.848	7.218	6.408	5.438	4.318	3.058	1.708	0.268	-1.212	-2.732	-4.252	-5.772	-7.292
(3)	12.79	12.42	12.01	11.57	11.10	10.60	10.07	9.52	8.95	8.37	7.78	7.18	6.57	5.95	5.33
(4)	16.87	16.73	16.71	16.54	16.18	15.56	15.93	15.81	15.53	15.25	15.16	15.14	14.97	14.67	14.38

TABLE B.4 :  $f_1(t; -9, 1, -17)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	6.042	6.144	6.447	6.913	7.523	8.273	9.163	10.193	11.373	12.703	14.183	15.823	17.623	19.583	21.703
(2)	9.366	8.938	8.318	7.508	6.438	5.138	3.718	2.198	0.578	-1.142	-2.762	-4.382	-6.002	-7.622	-9.242
(3)	13.40	13.03	12.62	12.18	11.71	11.21	10.68	10.13	9.56	8.97	8.37	7.76	7.14	6.51	5.88
(4)	17.40	17.26	17.26	17.09	16.64	16.03	15.27	14.37	13.33	12.18	10.93	9.58	8.13	6.58	4.93

TABLE B.4 :  $f_1(t; -9, 1, -18)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	6.474	6.586	6.966	7.526	8.276	9.226	10.376	11.726	13.276	15.026	16.976	19.226	21.776	24.626	27.776
(2)	9.798	9.270	8.550	7.540	6.280	4.800	3.120	1.280	-0.680	-2.440	-4.100	-5.660	-7.120	-8.480	-9.740
(3)	13.80	13.43	13.02	12.58	12.11	11.61	11.08	10.53	9.96	9.37	8.77	8.16	7.54	6.91	6.28
(4)	17.79	17.65	17.68	17.51	17.15	16.54	15.78	14.88	13.84	12.69	11.44	10.09	8.64	7.09	5.44

TABLE B.4 :  $f_1(t; -9, 1, -19)$

$w$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	6.872	7.004	7.484	8.144	8.994	10.044	11.294	12.744	14.394	16.244	18.294	20.544	23.094	25.944	29.094
(2)	10.216	9.588	8.768	7.658	6.298	4.718	2.938	1.018	-1.062	-2.822	-4.382	-			



TABLE 8.5 :  $f_1$  ( $\alpha = .8, \alpha = .6$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.445	1.744	1.855	1.952	2.027	2.086	2.131	2.172	2.210	2.245	2.277	2.307	2.334	2.359
(2)	4.119	4.197	4.246	4.286	4.326	4.358	4.390	4.422	4.453	4.485	4.516	4.547	4.578	4.609
(3)	8.231	8.108	7.975	7.831	7.676	7.512	7.340	7.162	6.980	6.803	6.631	6.465	6.305	6.151
(4)	6.530	10.23	10.26	10.155	10.017	9.854	9.676	9.494	9.309	9.122	8.935	8.748	8.561	8.374

TABLE 8.5 :  $f_1$  ( $\alpha = .8, \alpha = .7$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.910	1.921	2.146	2.279	2.408	2.534	2.653	2.767	2.876	2.981	3.082	3.179	3.272	3.361
(2)	4.794	5.093	5.376	5.644	5.898	6.139	6.368	6.585	6.791	6.986	7.171	7.346	7.511	7.666
(3)	6.097	5.435	5.000	4.579	4.170	3.773	3.389	3.028	2.691	2.378	2.089	1.824	1.583	1.366
(4)	11.21	11.46	12.16	12.53	13.00	13.31	13.57	13.78	13.95	14.09	14.21	14.31	14.40	14.48

TABLE 8.5 :  $f_1$  ( $\alpha = .8, \alpha = .8$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	2.435	2.516	2.586	2.653	2.717	2.778	2.835	2.888	2.937	2.982	3.024	3.062	3.097	3.130
(2)	5.487	5.782	6.079	6.367	6.649	6.925	7.196	7.462	7.723	7.979	8.231	8.479	8.724	8.966
(3)	9.008	8.623	8.248	7.884	7.531	7.191	6.865	6.554	6.258	5.976	5.708	5.455	5.216	5.000
(4)	12.74	13.45	14.12	14.67	15.13	15.51	15.83	16.11	16.35	16.56	16.74	16.89	17.02	17.14

TABLE 8.5 :  $f_1$  ( $\alpha = .8, \alpha = .9$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	2.857	2.970	3.063	3.148	3.224	3.291	3.349	3.400	3.445	3.485	3.521	3.553	3.582	3.608
(2)	6.661	6.937	7.199	7.447	7.683	7.907	8.119	8.319	8.507	8.684	8.849	9.004	9.149	9.285
(3)	11.18	11.26	11.30	11.31	11.31	11.31	11.31	11.31	11.31	11.31	11.31	11.31	11.31	11.31
(4)	15.69	16.45	16.44	16.43	16.42	16.41	16.40	16.39	16.38	16.37	16.36	16.35	16.34	16.33

TABLE 8.5 :  $f_1$  ( $\alpha = .9, \alpha = .6$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.442	1.550	1.649	1.734	1.801	1.854	1.903	1.948	1.989	2.026	2.060	2.091	2.119	2.145
(2)	3.661	3.691	3.719	3.747	3.775	3.802	3.828	3.854	3.879	3.904	3.928	3.952	3.975	3.998
(3)	6.127	6.409	6.712	7.037	7.384	7.752	8.142	8.554	8.988	9.444	9.922	10.422	10.944	11.488
(4)	6.578	6.955	7.331	7.707	8.083	8.459	8.835	9.211	9.587	9.963	10.339	10.715	11.091	11.467

TABLE 8.5 :  $f_1$  ( $\alpha = .9, \alpha = .7$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.698	1.797	1.897	1.997	2.097	2.197	2.297	2.397	2.497	2.597	2.697	2.797	2.897	2.997
(2)	4.279	4.501	4.723	4.945	5.167	5.389	5.611	5.833	6.055	6.277	6.499	6.721	6.943	7.165
(3)	7.118	7.315	7.512	7.709	7.906	8.103	8.300	8.497	8.694	8.891	9.088	9.285	9.482	9.679
(4)	9.964	10.339	10.715	11.091	11.467	11.843	12.219	12.595	12.971	13.347	13.723	14.099	14.475	14.851

TABLE 8.5 :  $f_1$  ( $\alpha = .9, \alpha = .8$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	2.011	2.101	2.191	2.281	2.371	2.461	2.551	2.641	2.731	2.821	2.911	3.001	3.091	3.181
(2)	4.872	5.084	5.296	5.508	5.720	5.932	6.144	6.356	6.568	6.780	6.992	7.204	7.416	7.628
(3)	7.809	8.011	8.213	8.415	8.617	8.819	9.021	9.223	9.425	9.627	9.829	10.031	10.233	10.435
(4)	10.339	10.715	11.091	11.467	11.843	12.219	12.595	12.971	13.347	13.723	14.099	14.475	14.851	15.227

TABLE 8.5 :  $f_1$  ( $\alpha = .9, \alpha = .8$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.977	2.033	2.132	2.238	2.338	2.438	2.538	2.638	2.738	2.838	2.938	3.038	3.138	3.238
(2)	4.879	5.099	5.404	5.749	6.141	6.583	7.078	7.626	8.231	8.900	9.639	10.449	11.332	12.289
(3)	8.066	8.543	9.035	9.535	10.135	10.845	11.665	12.505	13.365	14.245	15.145	16.065	16.995	17.935
(4)	11.33	11.96	12.64	13.36	14.12	14.92	15.76	16.64	17.56	18.52	19.52	20.56	21.64	22.76

TABLE 8.5 :  $f_1$  ( $\alpha = .9, \alpha = .9$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	2.146	2.236	2.339	2.437	2.537	2.637	2.737	2.837	2.937	3.037	3.137	3.237	3.337	3.437
(2)	5.384	5.673	5.958	6.245	6.534	6.824	7.116	7.411	7.709	8.011	8.316	8.624	8.934	9.246
(3)	9.028	9.519	10.007	10.494	11.081	11.668	12.255	12.842	13.429	14.016	14.603	15.190	15.777	16.364
(4)	12.46	13.34	14.01	14.78	15.62	16.46	17.31	18.16	19.01	19.86	20.71	21.56	22.41	23.26

TABLE 8.5 :  $f_1$  ( $\alpha = .9, \alpha = 1.0$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	2.355	2.462	2.584	2.720	2.875	3.051	3.253	3.481	3.736	4.019	4.330	4.668	5.034	5.427
(2)	5.921	6.221	6.562	6.949	7.384	7.867	8.399	8.981	9.614	10.298	11.034	11.824	12.668	13.567
(3)	9.843	10.44	11.04	11.61	12.16	12.69	13.21	13.73	14.25	14.77	15.29	15.81	16.33	16.85
(4)	13.96	14.68	15.27	15.86	16.45	17.04	17.63	18.22	18.81	19.40	20.00	20.59	21.18	21.77

TABLE 8.5 :  $f_1$  ( $\alpha = 1.0, \alpha = .6$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.318	1.385	1.448	1.506	1.562	1.617	1.672	1.727	1.782	1.837	1.892	1.947	2.002	2.057
(2)	3.795	3.893	3.971	4.042	4.108	4.174	4.240	4.306	4.372	4.438	4.504	4.570	4.636	4.702
(3)	5.314	5.446	5.621	5.841	6.106	6.417	6.774	7.178	7.629	8.128	8.675	9.270	9.910	10.596
(4)	7.720	8.185	8.705	9.295	9.958	10.712	11.560	12.513	13.573	14.741	16.019	17.407	18.906	20.516

TABLE 8.5 :  $f_1$  ( $\alpha = 1.0, \alpha = .7$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.518	1.617	1.717	1.830	1.959	2.108	2.281	2.486	2.727	3.011	3.405	3.917	4.679	5.644
(2)	3.877	4.053	4.302	4.588	4.935	5.352	5.851	6.445	7.137	7.941	8.871	10.044	11.471	13.164
(3)	6.408	6.781	7.204	7.681	8.223	8.835	9.519	10.278	11.124	12.058	13.081	14.194	15.397	16.690
(4)	8.970	9.496	10.08	10.72	11.40	12.13	12.91	13.74	14.61	15.53	16.50	17.52	18.59	19.72

TABLE 8.5 :  $f_1$  ( $\alpha = 1.0, \alpha = .8$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.724	1.820	1.917	2.018	2.125	2.235	2.350	2.481	2.629	2.795	2.979	3.181	3.411	3.669
(2)	4.346	4.598	4.864	5.175	5.531	5.935	6.397	6.927	7.537	8.239	9.034	9.924	10.910	12.003
(3)	7.278	7.690	8.151	8.671	9.253	9.897	10.615	11.419	12.311	13.293	14.366	15.531	16.788	18.145
(4)	10.39	10.77	11.41	12.02	12.76	13.54	14.37	15.25	16.18	17.16	18.19	19.27	20.40	21.58

TABLE 8.5 :  $f_1$  ( $\alpha = 1.0, \alpha = .9$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.932	2.031	2.141	2.265	2.406	2.566	2.748	2.968	3.232	3.545	3.918	4.362	4.877	5.464
(2)	4.847	5.108	5.399	5.731	6.110	6.538	7.017	7.549	8.137	8.785	9.495	10.268	11.107	12.016
(3)	8.116	8.520	8.989	9.517	10.115	10.793	11.561	12.421	13.385	14.456	15.635	16.924	18.324	19.836
(4)	11.39	12.07	12.69	13.31	14.04	14.88	15.83	16.89	18.06	19.34	20.73	22.24	23.87	25.62

TABLE 8.5 :  $f_1$  ( $\alpha = .8, \alpha = .6$ )

NOTE:  $\alpha = 0.15$  0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1)	1.977	2.033	2.132	2.238	2.338	2.438	2.538	2.638	2.738	2.838	2.938	3.038	3.138	3.238
(2)	4.879	5.099	5.404	5.749	6.141	6.583	7.078	7.626	8.231	8.900	9.639	10.449	11.332	12.289
(3)	8.066	8.543	9.035	9.535	10.135	10.845	11.665	12.505	13.365	14.245	15.145	16.065	16.995	17.935
(4)	11.33	11.96	12.64	13.36	14.12	14.92	15.76	16.64	17.56	18.52	19.52	20.56	21.64	22.76

TABLE 8.5 :  $f_1 (1/a - 6, 1/a - 6)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.784	1.819	1.852	1.881	1.906	1.927	1.944	1.958	1.969	1.977	1.982	1.985	1.987	1.988	1.989
(1)	1.784	1.819	1.852	1.881	1.906	1.927	1.944	1.958	1.969	1.977	1.982	1.985	1.987	1.988	1.989
(2)	4.315	4.724	5.173	5.614	6.020	6.394	6.728	7.020	7.273	7.491	7.671	7.818	7.936	8.021	8.083
(3)	2.634	2.855	3.073	3.284	3.486	3.679	3.856	4.018	4.167	4.304	4.430	4.547	4.655	4.754	4.844
(4)	10.81	11.41	12.09	12.80	13.50	14.20	14.90	15.59	16.27	16.94	17.60	18.25	18.89	19.52	20.14

TABLE 8.5 :  $f_1 (1/a - 7, 1/a - 7)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.416	1.481	1.531	1.573	1.609	1.640	1.667	1.691	1.712	1.730	1.746	1.760	1.772	1.782	1.790
(1)	1.416	1.481	1.531	1.573	1.609	1.640	1.667	1.691	1.712	1.730	1.746	1.760	1.772	1.782	1.790
(2)	4.088	4.766	5.435	6.094	6.745	7.389	8.028	8.663	9.294	9.922	10.548	11.172	11.794	12.414	13.033
(3)	6.890	7.332	7.691	7.978	8.198	8.451	8.728	9.029	9.354	9.703	10.076	10.474	10.897	11.345	11.818
(4)	9.998	10.723	11.500	12.320	13.183	14.090	15.042	16.041	17.088	18.185	19.333	20.533	21.786	23.094	24.458

TABLE 8.5 :  $f_1 (1/a - 7, 1/a - 1.0)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.482	1.551	1.608	1.658	1.703	1.743	1.779	1.811	1.839	1.863	1.884	1.902	1.918	1.932	1.944
(1)	1.482	1.551	1.608	1.658	1.703	1.743	1.779	1.811	1.839	1.863	1.884	1.902	1.918	1.932	1.944
(2)	3.730	4.270	4.784	5.273	5.739	6.184	6.610	7.018	7.409	7.784	8.144	8.491	8.826	9.149	9.461
(3)	6.786	6.806	6.984	7.223	7.525	7.891	8.323	8.822	9.389	9.925	10.530	11.205	11.950	12.765	13.650
(4)	6.789	7.284	7.834	8.439	9.099	9.814	10.584	11.410	12.292	13.231	14.228	15.284	16.399	17.574	18.809

TABLE 8.5 :  $f_1 (1/a - 8, 1/a - 8)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.367	1.431	1.482	1.529	1.571	1.609	1.643	1.673	1.700	1.724	1.745	1.763	1.779	1.793	1.805
(1)	1.367	1.431	1.482	1.529	1.571	1.609	1.643	1.673	1.700	1.724	1.745	1.763	1.779	1.793	1.805
(2)	6.582	6.828	7.084	7.351	7.628	7.916	8.214	8.523	8.843	9.174	9.517	9.872	10.239	10.618	11.009
(3)	11.35	11.98	12.69	13.43	14.20	15.00	15.83	16.69	17.58	18.49	19.43	20.40	21.40	22.43	23.49
(4)	16.20	17.11	17.73	18.43	19.20	20.04	20.94	21.89	22.89	23.94	25.04	26.19	27.39	28.64	29.94

TABLE 8.5 :  $f_1 (1/a - 8, 1/a - 7)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	2.194	2.237	2.282	2.324	2.363	2.399	2.432	2.462	2.489	2.513	2.536	2.557	2.576	2.593	2.608
(1)	2.194	2.237	2.282	2.324	2.363	2.399	2.432	2.462	2.489	2.513	2.536	2.557	2.576	2.593	2.608
(2)	5.758	6.024	6.304	6.599	6.908	7.231	7.568	7.919	8.284	8.663	9.056	9.464	9.887	10.325	10.778
(3)	9.837	10.38	10.99	11.66	12.38	13.15	13.98	14.86	15.79	16.76	17.78	18.84	19.94	21.08	22.26
(4)	13.99	14.77	15.57	16.43	17.34	18.30	19.32	20.39	21.51	22.68	23.90	25.17	26.49	27.86	29.28

TABLE 8.5 :  $f_1 (1/a - 8, 1/a - 6)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	2.016	2.078	2.131	2.175	2.213	2.247	2.278	2.306	2.332	2.356	2.378	2.398	2.416	2.432	2.446
(1)	2.016	2.078	2.131	2.175	2.213	2.247	2.278	2.306	2.332	2.356	2.378	2.398	2.416	2.432	2.446
(2)	5.160	5.483	5.831	6.194	6.573	6.968	7.380	7.809	8.255	8.718	9.197	9.692	10.204	10.733	11.280
(3)	6.767	6.783	6.733	6.633	6.493	6.323	6.133	5.923	5.693	5.443	5.183	4.923	4.663	4.403	4.143
(4)	12.36	13.03	13.77	14.57	15.43	16.34	17.31	18.34	19.43	20.58	21.79	23.05	24.37	25.74	27.16

TABLE 8.5 :  $f_1 (1/a - 8, 1/a - 5)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.447	1.521	1.581	1.628	1.671	1.710	1.745	1.777	1.806	1.832	1.856	1.878	1.898	1.916	1.932
(1)	1.447	1.521	1.581	1.628	1.671	1.710	1.745	1.777	1.806	1.832	1.856	1.878	1.898	1.916	1.932
(2)	4.672	5.160	5.684	6.244	6.841	7.474	8.143	8.848	9.590	10.370	11.188	12.044	12.938	13.870	14.840
(3)	7.074	6.286	5.361	4.399	3.417	2.433	1.466	0.533	-0.354	-1.083	-1.724	-2.277	-2.742	-3.118	-3.405
(4)	11.08	11.68	12.36	13.12	13.95	14.84	15.79	16.80	17.87	19.00	20.18	21.42	22.71	24.05	25.44

TABLE 8.5 :  $f_1 (1/a - 6, 1/a - 6)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.779	1.786	1.811	1.831	1.847	1.860	1.870	1.878	1.884	1.889	1.893	1.896	1.898	1.900	1.901
(1)	1.779	1.786	1.811	1.831	1.847	1.860	1.870	1.878	1.884	1.889	1.893	1.896	1.898	1.900	1.901
(2)	4.301	5.145	5.936	6.667	7.338	7.951	8.506	9.004	9.445	9.829	10.156	10.427	10.643	10.814	10.950
(3)	6.317	6.989	7.566	8.048	8.436	8.731	8.943	9.073	9.122	9.100	9.014	8.864	8.742	8.648	8.579
(4)	12.15	12.87	13.67	14.54	15.46	16.43	17.44	18.50	19.61	20.77	21.98	23.24	24.55	25.91	27.32

TABLE 8.5 :  $f_1 (1/a - 6, 1/a - 7)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.645	1.677	1.719	1.752	1.778	1.800	1.818	1.833	1.846	1.857	1.866	1.873	1.879	1.884	1.888
(1)	1.645	1.677	1.719	1.752	1.778	1.800	1.818	1.833	1.846	1.857	1.866	1.873	1.879	1.884	1.888
(2)	4.319	4.919	5.482	6.009	6.500	6.956	7.378	7.766	8.121	8.454	8.766	9.057	9.328	9.580	9.814
(3)	7.219	7.792	8.280	8.692	9.027	9.285	9.466	9.571	9.603	9.564	9.455	9.277	9.031	8.718	8.349
(4)	15.50	16.10	16.78	17.54	18.37	19.26	20.20	21.19	22.23	23.32	24.46	25.65	26.89	28.17	29.50

TABLE 8.5 :  $f_1 (1/a - 6, 1/a - 8)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.512	1.559	1.613	1.664	1.712	1.757	1.799	1.838	1.874	1.907	1.937	1.964	1.988	2.010	2.029
(1)	1.512	1.559	1.613	1.664	1.712	1.757	1.799	1.838	1.874	1.907	1.937	1.964	1.988	2.010	2.029
(2)	3.870	4.049	4.253	4.483	4.739	5.021	5.329	5.664	6.027	6.419	6.840	7.290	7.768	8.274	8.808
(3)	6.581	6.806	7.011	7.204	7.383	7.545	7.690	7.818	7.931	8.029	8.113	8.183	8.240	8.284	8.316
(4)	9.270	9.706	10.138	10.561	10.974	11.378	11.773	12.159	12.537	12.910	13.278	13.641	13.999	14.353	14.703

TABLE 8.5 :  $f_1 (1/a - 6, 1/a - 5)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.345	1.441	1.501	1.531	1.551	1.565	1.575	1.582	1.587	1.591	1.594	1.596	1.598	1.599	1.600
(1)	1.345	1.441	1.501	1.531	1.551	1.565	1.575	1.582	1.587	1.591	1.594	1.596	1.598	1.599	1.600
(2)	3.576	4.076	4.620	5.208	5.841	6.518	7.240	8.007	8.819	9.676	10.578	11.525	12.517	13.554	14.636
(3)	6.171	6.300	6.356	6.338	6.256	6.111	5.914	5.674	5.391	5.064	4.702	4.315	3.903	3.466	3.004
(4)	9.270	9.706	10.138	10.561	10.974	11.378	11.773	12.159	12.537	12.910	13.278	13.641	13.999	14.353	14.703

TABLE 8.5 :  $f_1 (1/a - 6, 1/a - 1.0)$

$a$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
NOTE:	1.271	1.330	1.385	1.433	1.475	1.512	1.544	1.572	1.596	1.616	1.633	1.647	1.658	1.666	1.672
(1)	1.271	1.330	1.385	1.433	1.475	1.512	1.544	1.572	1.596	1.616	1.633	1.647	1.658	1.666	1.672
(2)	3.147	3.500	3.856	4.214	4.573	4.934	5.297								

TABLE 8.5 :  $\beta_1$  ( $1/\theta - 1.0, 1/\theta - .7$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 1.894 1.771 1.660 1.559 1.460 1.372 1.294 1.226 1.158 1.090 1.022 0.954 0.886 0.818 0.750

(2) 4.281 4.478 4.701 4.945 5.208 5.481 5.764 6.057 6.360 6.683 7.026 7.389 7.772 8.175 8.598

(3) 7.100 7.511 7.979 8.474 8.999 9.513 10.057 10.631 11.235 11.869 12.533 13.227 13.951 14.705 15.489

(4) 10.000 10.611 11.222 11.866 12.544 13.256 13.999 14.774 15.581 16.421 17.287 18.179 19.097 19.941 20.810

TABLE 8.5 :  $\beta_1$  ( $1/\theta - 1.0, 1/\theta - .8$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 2.519 2.598 2.689 2.784 2.893 3.007 3.126 3.252 3.384 3.521 3.663 3.810 3.961 4.117 4.279

(2) 6.409 6.746 7.099 7.500 7.919 8.378 8.857 9.360 9.917 10.528 11.195 11.919 12.699 13.537 14.435

(3) 10.753 11.49 12.32 13.22 14.18 15.18 16.23 17.33 18.48 19.69 20.95 22.27 23.64 25.06 26.53

(4) 15.43 16.22 17.07 17.97 18.93 19.95 21.03 22.17 23.37 24.62 25.92 27.27 28.67 30.12 31.61

TABLE 8.5 :  $\beta_1$  ( $1/\theta - 1.0, 1/\theta - .9$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 2.209 2.401 2.509 2.622 2.744 2.872 3.007 3.148 3.294 3.445 3.601 3.762 3.928 4.100 4.278

(2) 5.860 6.121 6.457 6.822 7.219 7.648 8.107 8.607 9.149 9.735 10.358 11.019 11.719 12.457 13.233

(3) 9.641 10.35 11.11 11.92 12.78 13.68 14.61 15.57 16.57 17.61 18.69 19.82 20.99 22.20 23.45

(4) 13.85 14.57 15.33 16.14 17.00 17.91 18.87 19.84 20.92 22.03 23.18 24.36 25.57 26.82 28.11

TABLE 8.6 :  $\beta_1$  ( $q - .6, q - .7$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 4.594 4.870 5.181 5.519 5.900 6.402 6.970 7.537 8.122 8.726 9.358 10.019 10.711 11.434 12.189

(2) 8.270 8.767 9.314 9.914 10.560 11.253 11.994 12.782 13.618 14.504 15.441 16.429 17.467 18.556 19.696

(3) 11.85 12.66 13.41 14.17 14.97 15.78 16.62 17.50 18.43 19.41 20.44 21.52 22.65 23.83 25.06

(4) 15.62 16.51 17.41 18.32 19.25 20.20 21.18 22.21 23.28 24.39 25.53 26.70 27.92 29.18 30.48

TABLE 8.6 :  $\beta_1$  ( $q - .6, q - .8$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 5.335 5.646 5.996 6.391 6.838 7.339 7.887 8.494 9.162 9.892 10.684 11.539 12.457 13.439 14.485

(2) 9.606 10.17 10.78 11.40 12.05 12.73 13.46 14.24 15.07 15.94 16.86 17.83 18.85 19.92 21.04

(3) 13.68 14.46 15.18 15.97 16.82 17.72 18.67 19.66 20.70 21.79 22.92 24.09 25.31 26.58 27.91

(4) 18.13 18.86 19.64 20.47 21.34 22.26 23.22 24.23 25.28 26.37 27.50 28.68 30.00 31.37 32.79

TABLE 8.6 :  $\beta_1$  ( $q - .6, q - .9$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 6.037 6.393 6.773 7.197 7.663 8.178 8.747 9.374 10.060 10.809 11.623 12.501 13.444 14.453 15.529

(2) 10.91 11.52 12.16 12.83 13.53 14.27 15.04 15.85 16.71 17.62 18.58 19.58 20.63 21.74 22.90

(3) 15.78 16.35 16.95 17.61 18.33 19.08 19.87 20.71 21.61 22.56 23.54 24.57 25.64 26.76 27.93

(4) 20.57 20.84 21.27 21.85 22.47 23.14 23.86 24.62 25.44 26.31 27.22 28.17 29.16 30.20 31.29

TABLE 8.6 :  $\beta_1$  ( $q - .6, q - .9$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 6.734 7.109 7.507 7.948 8.433 8.971 9.561 10.253 11.057 11.975 12.999 14.139 15.397 16.775 18.273

(2) 12.18 12.61 13.07 13.57 14.10 14.67 15.28 15.93 16.63 17.38 18.17 19.01 19.89 20.81 21.77

(3) 17.59 18.26 18.93 19.64 20.39 21.18 22.01 22.88 23.79 24.74 25.73 26.76 27.83 28.94 30.09

(4) 23.09 23.99 24.92 25.89 26.90 27.95 29.04 30.17 31.34 32.54 33.78 35.06 36.39 37.76 39.18

TABLE 8.5 :  $\beta_1$  ( $1/\theta - .8, 1/\theta - 1.0$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 1.694 1.771 1.860 1.959 2.070 2.197 2.344 2.516 2.719 2.943 3.184 3.441 3.714 4.001 4.303

(2) 4.281 4.478 4.701 4.945 5.208 5.481 5.764 6.057 6.360 6.683 7.026 7.389 7.772 8.175 8.598

(3) 7.100 7.511 7.979 8.474 8.999 9.513 10.057 10.631 11.235 11.869 12.533 13.227 13.951 14.705 15.489

(4) 10.000 10.611 11.222 11.866 12.544 13.256 13.999 14.774 15.581 16.421 17.287 18.179 19.097 19.941 20.810

TABLE 8.5 :  $\beta_1$  ( $1/\theta - .8, 1/\theta - .6$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 2.647 2.679 2.717 2.776 2.856 2.959 3.086 3.243 3.431 3.650 3.907 4.184 4.474 4.784 5.112

(2) 7.359 7.714 8.106 8.531 8.999 9.513 10.074 10.683 11.341 12.051 12.813 13.687 14.584 15.504 16.447

(3) 12.76 13.44 14.23 15.06 15.94 16.87 17.84 18.85 19.90 20.99 22.12 23.29 24.51 25.78 27.10

(4) 18.71 19.17 19.68 20.24 20.84 21.48 22.16 22.88 23.64 24.44 25.28 26.16 27.09 28.06 29.07

TABLE 8.5 :  $\beta_1$  ( $1/\theta - .8, 1/\theta - .7$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 2.448 2.516 2.579 2.657 2.753 2.868 3.003 3.173 3.372 3.600 3.866 4.160 4.481 4.828 5.192

(2) 6.478 6.776 7.137 7.563 8.055 8.599 9.205 9.876 10.613 11.417 12.287 13.224 14.228 15.299 16.437

(3) 11.00 11.60 12.24 12.93 13.67 14.45 15.28 16.15 17.07 18.03 19.03 20.07 21.15 22.28 23.45

(4) 15.71 16.58 17.45 18.32 19.20 20.09 21.00 21.94 22.91 23.91 24.94 25.99 27.07 28.18 29.32

TABLE 8.5 :  $\beta_1$  ( $1/\theta - .9, 1/\theta - .8$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 2.248 2.316 2.420 2.554 2.718 2.912 3.137 3.392 3.678 4.006 4.375 4.784 5.234 5.726 6.261

(2) 5.804 6.071 6.391 6.765 7.187 7.667 8.206 8.806 9.476 10.217 11.031 11.920 12.885 13.926 15.044

(3) 9.809 10.30 10.83 11.39 12.00 12.65 13.34 14.08 14.86 15.69 16.56 17.47 18.42 19.41 20.44

(4) 13.99 14.64 15.33 16.05 16.81 17.61 18.44 19.31 20.22 21.16 22.14 23.16 24.21 25.29 26.41

TABLE 8.5 :  $\beta_1$  ( $1/\theta - .9, 1/\theta - .9$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 2.074 2.161 2.274 2.419 2.607 2.836 3.104 3.413 3.762 4.152 4.584 5.051 5.554 6.096 6.674

(2) 5.534 5.870 6.244 6.656 7.107 7.607 8.156 8.754 9.403 10.103 10.854 11.657 12.514 13.427 14.396

(3) 8.659 9.109 9.606 10.143 10.729 11.366 12.054 12.754 13.504 14.306 15.159 16.064 17.024 18.039 19.109

(4) 12.44 13.13 13.88 14.68 15.52 16.41 17.34 18.31 19.34 20.42 21.55 22.73 23.96 25.24 26.57

TABLE 8.5 :  $\beta_1$  ( $1/\theta - .9, 1/\theta - 1.0$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 1.906 1.984 2.074 2.174 2.297 2.452 2.640 2.864 3.124 3.421 3.756 4.127 4.534 4.978 5.456

(2) 4.704 5.079 5.491 5.942 6.434 6.967 7.544 8.167 8.837 9.556 10.324 11.143 12.004 12.917 13.882

(3) 8.035 8.641 9.290 9.982 10.718 11.500 12.331 13.213 14.146 15.130 16.165 17.251 18.388 19.576 20.814

(4) 11.21 11.92 12.59 13.32 14.10 14.94 15.83 16.77 17.76 18.80 19.89 21.03 22.22 23.46 24.74

TABLE 8.5 :  $\beta_1$  ( $1/\theta - 1.0, 1/\theta - .6$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

NOTE:

(1) 2.518 2.719 3.018 3.413 3.906 4.501 5.201 6.006 6.926 7.971 9.151 10.474 11.951 13.584 15.374

(2) 4.174 4.568 5.064 5.663 6.374 7.207 8.174 9.287 10.547 11.964 13.541 15.284 17.104 19.014 21.014

(3) 14.17 14.94 15.71 16.51 17.34 18.18 19.04 19.93 20.85 21.81 22.81 23.84 24.91 26.03 27.19

(4) 20.21 21.08 21.93 22.84 23.78 24.74 25.73 26.75 27.79 28.85 29.94 31.06 32.21 33.38 34.59

TABLE 8.6 :  $F_1$  ( $\alpha = 0.5, \alpha = -7$ )

NOTE: (1) 4.001 4.215 4.497 4.795 5.115 5.452 5.810 6.174 6.587 6.974 7.348 7.694 8.011 8.299 8.558 8.781 8.969 9.122 9.250 9.351 9.428 9.484 9.512 9.522 9.500 (2) 7.205 7.637 8.100 8.629 9.194 9.754 10.302 10.829 11.328 11.790 12.215 12.604 12.958 13.278 13.564 13.818 14.044 14.233 14.386 14.504 14.536 14.581 14.638 14.706 14.784 14.871 14.968 15.074 15.190 15.316 15.452 15.598 15.754 15.920 16.096 16.282 16.478 16.684 16.900 17.126 17.362 17.608 17.864 18.130 18.406 18.692 18.988 19.294 19.610 19.936 20.272 20.618 20.974 21.340 21.716 22.102 22.498 22.904 23.320 23.746 24.182 24.628 25.084 25.550 26.026 26.512 27.008 27.514 28.030 28.556 29.092 29.638 30.194 30.760 31.336 31.922 32.518 33.124 33.740 34.366 35.002 35.648 36.304 36.970 37.646 38.332 39.028 39.734 40.450 41.176 41.912 42.658 43.414 44.180 44.956 45.742 46.538 47.344 48.160 48.986 49.822 50.668 51.524 52.390 53.266 54.152 55.048 55.954 56.870 57.796 58.732 59.678 60.634 61.600 62.576 63.562 64.558 65.564 66.580 67.606 68.642 69.688 70.744 71.810 72.886 73.972 75.068 76.174 77.290 78.416 79.552 80.698 81.854 83.020 84.196 85.382 86.578 87.784 88.990 90.206 91.432 92.668 93.914 95.170 96.436 97.712 99.008 100.314 101.630 102.956 104.292 105.638 106.994 108.360 109.736 111.122 112.518 113.924 115.340 116.766 118.202 119.648 121.104 122.570 124.046 125.532 127.028 128.534 130.050 131.576 133.112 134.658 136.214 137.780 139.356 140.942 142.538 144.144 145.760 147.386 149.022 150.668 152.324 153.990 155.666 157.352 159.048 160.754 162.470 164.196 165.932 167.678 169.434 171.200 172.976 174.762 176.558 178.364 180.180 182.006 183.842 185.688 187.544 189.410 191.286 193.172 195.068 196.974 198.890 200.816 202.752 204.698 206.654 208.620 210.596 212.582 214.578 216.584 218.590 220.606 222.632 224.668 226.714 228.770 230.836 232.912 234.998 237.094 239.200 241.316 243.442 245.578 247.724 249.880 252.046 254.222 256.408 258.604 260.810 263.026 265.252 267.488 269.734 271.990 274.256 276.532 278.818 281.114 283.420 285.736 288.062 290.398 292.744 295.090 297.446 299.812 302.188 304.574 306.970 309.376 311.792 314.218 316.654 319.100 321.556 324.022 326.498 328.984 331.480 333.986 336.502 339.028 341.564 344.110 346.666 349.232 351.808 354.394 356.990 359.596 362.212 364.838 367.474 370.120 372.776 375.442 378.118 380.804 383.500 386.206 388.922 391.648 394.384 397.130 399.886 402.652 405.428 408.214 411.010 413.816 416.632 419.458 422.294 425.140 427.996 430.862 433.738 436.624 439.520 442.426 445.342 448.268 451.204 454.150 457.106 460.072 463.048 466.034 469.030 472.036 475.052 478.078 481.114 484.160 487.216 490.282 493.358 496.444 499.546 502.658 505.780 508.912 512.054 515.206 518.368 521.540 524.724 527.918 531.122 534.336 537.560 540.794 544.038 547.292 550.556 553.828 557.110 560.402 563.704 567.016 570.338 573.670 577.012 580.364 583.726 587.098 590.480 593.872 597.274 600.686 604.108 607.540 610.982 614.434 617.898 621.372 624.856 628.350 631.854 635.368 638.892 642.426 645.970 649.524 653.088 656.652 660.226 663.800 667.384 670.968 674.552 678.146 681.740 685.344 688.948 692.552 696.156 699.760 703.364 706.968 710.572 714.176 717.780 721.384 724.988 728.592 732.196 735.792 739.396 742.992 746.596 750.192 753.796 757.392 760.996 764.592 768.196 771.796 775.392 778.996 782.596 786.192 789.796 793.396 796.996 800.596 804.196 807.796 811.396 814.996 818.596 822.196 825.796 829.396 832.996 836.596 840.196 843.796 847.396 850.996 854.596 858.196 861.796 865.396 868.996 872.596 876.196 879.796 883.396 886.996 890.596 894.196 897.796 901.396 904.996 908.596 912.196 915.796 919.396 922.996 926.596 930.196 933.796 937.396 940.996 944.596 948.196 951.796 955.396 958.996 962.596 966.196 969.796 973.396 976.996 980.596 984.196 987.796 991.396 994.996 998.596 1002.196 1005.796 1009.396 1012.996 1016.596 1020.196 1023.796 1027.396 1030.996 1034.596 1038.196 1041.796 1045.396 1048.996 1052.596 1056.196 1059.796 1063.396 1066.996 1070.596 1074.196 1077.796 1081.396 1084.996 1088.596 1092.196 1095.796 1099.396 1102.996 1106.596 1110.196 1113.796 1117.396 1120.996 1124.596 1128.196 1131.796 1135.396 1138.996 1142.596 1146.196 1149.796 1153.396 1156.996 1160.596 1164.196 1167.796 1171.396 1174.996 1178.596 1182.196 1185.796 1189.396 1192.996 1196.596 1200.196 1203.796 1207.396 1210.996 1214.596 1218.196 1221.796 1225.396 1228.996 1232.596 1236.196 1239.796 1243.396 1246.996 1250.596 1254.196 1257.796 1261.396 1264.996 1268.596 1272.196 1275.796 1279.396 1282.996 1286.596 1290.196 1293.796 1297.396 1300.996 1304.596 1308.196 1311.796 1315.396 1318.996 1322.596 1326.196 1329.796 1333.396 1336.996 1340.596 1344.196 1347.796 1351.396 1354.996 1358.596 1362.196 1365.796 1369.396 1372.996 1376.596 1380.196 1383.796 1387.396 1390.996 1394.596 1398.196 1401.796 1405.396 1408.996 1412.596 1416.196 1419.796 1423.396 1426.996 1430.596 1434.196 1437.796 1441.396 1444.996 1448.596 1452.196 1455.796 1459.396 1462.996 1466.596 1470.196 1473.796 1477.396 1480.996 1484.596 1488.196 1491.796 1495.396 1498.996 1502.596 1506.196 1509.796 1513.396 1516.996 1520.596 1524.196 1527.796 1531.396 1534.996 1538.596 1542.196 1545.796 1549.396 1552.996 1556.596 1560.196 1563.796 1567.396 1570.996 1574.596 1578.196 1581.796 1585.396 1588.996 1592.596 1596.196 1599.796 1603.396 1606.996 1610.596 1614.196 1617.796 1621.396 1624.996 1628.596 1632.196 1635.796 1639.396 1642.996 1646.596 1650.196 1653.796 1657.396 1660.996 1664.596 1668.196 1671.796 1675.396 1678.996 1682.596 1686.196 1689.796 1693.396 1696.996 1700.596 1704.196 1707.796 1711.396 1714.996 1718.596 1722.196 1725.796 1729.396 1732.996 1736.596 1740.196 1743.796 1747.396 1750.996 1754.596 1758.196 1761.796 1765.396 1768.996 1772.596 1776.196 1779.796 1783.396 1786.996 1790.596 1794.196 1797.796 1801.396 1804.996 1808.596 1812.196 1815.796 1819.396 1822.996 1826.596 1830.196 1833.796 1837.396 1840.996 1844.596 1848.196 1851.796 1855.396 1858.996 1862.596 1866.196 1869.796 1873.396 1876.996 1880.596 1884.196 1887.796 1891.396 1894.996 1898.596 1902.196 1905.796 1909.396 1912.996 1916.596 1920.196 1923.796 1927.396 1930.996 1934.596 1938.196 1941.796 1945.396 1948.996 1952.596 1956.196 1959.796 1963.396 1966.996 1970.596 1974.196 1977.796 1981.396 1984.996 1988.596 1992.196 1995.796 1999.396 2002.996 2006.596 2010.196 2013.796 2017.396 2020.996 2024.596 2028.196 2031.796 2035.396 2038.996 2042.596 2046.196 2049.796 2053.396 2056.996 2060.596 2064.196 2067.796 2071.396 2074.996 2078.596 2082.196 2085.796 2089.396 2092.996 2096.596 2100.196 2103.796 2107.396 2110.996 2114.596 2118.196 2121.796 2125.396 2128.996 2132.596 2136.196 2139.796 2143.396 2146.996 2150.596 2154.196 2157.796 2161.396 2164.996 2168.596 2172.196 2175.796 2179.396 2182.996 2186.596 2190.196 2193.796 2197.396 2200.996 2204.596 2208.196 2211.796 2215.396 2218.996 2222.596 2226.196 2229.796 2233.396 2236.996 2240.596 2244.196 2247.796 2251.396 2254.996 2258.596 2262.196 2265.796 2269.396 2272.996 2276.596 2280.196 2283.796 2287.396 2290.996 2294.596 2298.196 2301.796 2305.396 2308.996 2312.596 2316.196 2319.796 2323.396 2326.996 2330.596 2334.196 2337.796 2341.396 2344.996 2348.596 2352.196 2355.796 2359.396 2362.996 2366.596 2370.196 2373.796 2377.396 2380.996 2384.596 2388.196 2391.796 2395.396 2398.996 2402.596 2406.196 2409.796 2413.396 2416.996 2420.596 2424.196 2427.796 2431.396 2434.996 2438.596 2442.196 2445.796 2449.396 2452.996 2456.596 2460.196 2463.796 2467.396 2470.996 2474.596 2478.196 2481.796 2485.396 2488.996 2492.596 2496.196 2499.796 2503.396 2506.996 2510.596 2514.196 2517.796 2521.396 2524.996 2528.596 2532.196 2535.796 2539.396 2542.996 2546.596 2550.196 2553.796 2557.396 2560.996 2564.596 2568.196 2571.796 2575.396 2578.996 2582.596 2586.196 2589.796 2593.396 2596.996 2600.596 2604.196 2607.796 2611.396 2614.996 2618.596 2622.196 2625.796 2629.396 2632.996 2636.596 2640.196 2643.796 2647.396 2650.996 2654.596 2658.196 2661.796 2665.396 2668.996 2672.596 2676.196 2679.796 2683.396 2686.996 2690.596 2694.196 2697.796 2701.396 2704.996 2708.596 2712.196 2715.796 2719.396 2722.996 2726.596 2730.196 2733.796 2737.396 2740.996 2744.596 2748.196 2751.796 2755.396 2758.996 2762.596 2766.196 2769.796 2773.396 2776.996 2780.596 2784.196 2787.796 2791.396 2794.996 2798.596 2802.196 2805.796 2809.396 2812.996 2816.596 2820.196 2823.796 2827.396 2830.996 2834.596 2838.196 2841.796 2845.396 2848.996 2852.596 2856.196 2859.796 2863.396 2866.996 2870.596 2874.196 2877.796 2881.396 2884.996 2888.596 2892.196 2895.796 2899.396 2902.996 2906.596 2910.196 2913.796 2917.396 2920.996 2924.596 2928.196 2931.796 2935.396 2938.996 2942.596 2946.196 2949.796 2953.396 2956.996 2960.596 2964.196 2967.796 2971.396 2974.996 2978.596 2982.196 2985.796 2989.396 2992.996 2996.596 3000.196 3003.796 3007.396 3010.996 3014.596 3018.196 3021.796 3025.396 3028.996 3032.596 3036.196 3039.796 3043.396 3046.996 3050.596 3054.196 3057.796 3061.396 3064.996 3068.596 3072.196 3075.796 3079.396 3082.996 3086.596 3090.196 3093.796 3097.396 3100.996 3104.596 3108.196 3111.796 3115.396 3118.996 3122.596 3126.196 3129.796 3133.396 3136.996 3140.596 3144.196 3147.796 3151.396 3154.996 3158.596 3162.196 3165.796 3169.396 3172.996 3176.596 3180.196 3183.796 3187.396 3190.996 3194.596 3198.196 3201.796 3205.396 3208.996 3212.596 3216.196 3219.796 3223.396 3226.996 3230.596 3234.196 3237.796 3241.396 3244.996 3248.596 3252.196 3255.796 3259.396 3262.996 3266.596 3270.196 3273.796 3277.396 3280.996 3284.596 3288.196 3291.796 3295.396 3298.996 3302.596 3306.196 3309.796 3313.396 3316.996 3320.596 3324.196 3327.796 3331.396 3334.996 3338.596 3342.196 3345.796 3349.396 3352.996 3356.596 3360.196 3363.796 3367.396 3370.996 3374.596 3378.196 3381.796 3385.396 3388.996 3392.596 3396.196 3399.796 3403.396 3406.996 3410.596 3414.196 3417.796 3421.396 3424.996 3428.596 3432.196 3435.796 3439.396 3442.996 3446.596 3450.196 3453.796 3457.396 3460.996 3464.596 3468.196 3471.796 3475.396 3478.996 3482.596 3486.196 3489.796 3493.396 3496.996 3500.596 3504.196 3507.796 3511.396 3514.996 3518.596 3522.196 3525.796 3529.396 3532.996 3536.596 3540.196 3543.796 3547.396 3550.996 3554.596 3558.196 3561.796 3565.396 3568.996 3572.596 3576.196 3579.796 3583.396 3586.996 3590.596 3594.196 3597.796 3601.396 3604.996 3608.596 3612.196 3615.796 3619.396 3622.996 3626.596 3630.196 3633.796 3637.396 3640.996 3644.596 3648.196 3651.796 3655.396 3658.996 3662.596 3666.196 3669.796 3673.396 3676.996 3680.596 3684.196 3687.796 3691.396 3694.996 3698.596 3702.196 3705.796 3709.396 3712.996 3716.596 3720.196 3723.796 3727.396 3730.996 3734.596 3738.196 3741.796 3745.396 3748.996 3752.596 3756.196 3759.796 3763.396 3766.996 3770.596 3774.196 3777.796 3781.396 3784.996 3788.596 3792.196 3795.796 3799.396 3802.996 3806.596 3810.196 3813.796 3817.396 3820.996 3824.596 3828.196 3831.796 3835.396 3838.996 3842.596 3846.196 3849.796 3853.396 3856.996 3860.596 3864.196 3867.796 3871.396 3874.996 3878.596 3882.196 3885.796 3889.396 3892.996 3896.596 3900.196 3903.796 3907.396 3910.996 3914.596 3918.196 3921.796 3925.396 3928.996 3932.596 3936.196 3939.796 3943.396 3946.996 3950.596 3954.196 3957.796 3961.396 3964.996 3968.596 3972.196 3975.796 3979.396 3982.996 3986.596 3990.196 3993.796 3997.396 4000.996 4004.596 4008.196 4011.796 4015.396 4018.996 4022.596 4026.196 4029.796 4033.396 4036.996 4040.596 4044.196 4047.796 4051.396 4054.996 4058.596 4062.196 4065.796 4069.396 4072.996 4076.596 4080.196 4083.796 4087.396 4090.996 4094.596 4098.196 4101.796 4105.396 4108.996 4112.596 4116.196 4119.796 4123.396 4126.996 4130.596 4134.196 4137.796 4141.396 4144.996 4148.596 4152.196 4155.796 4159.396 4162.996 4166.596 4170.196 4173.796 4177.396 4180.996 4184.596 4188.196 4191.796 4195.396 4198.996 4202.596 4206.196 4209.796 4213.396 4216.996 4220.596 4224.196 4227.796 4231.396 4234.996 4238.596 4242.196 4245.796 4249.396 4252.996 4256.596 4260.196 4263.796 4267.396 4270.996 4274.596 4278.196 4281.796 4285.396 4288.996 4292.596 4296.196 4299.796 4303.396 4306.996 4310.596 4314.196 4317.796 4321.396 4324.996 4328.596 4332.196 4335.796 4339.396 4342.996 4346.596 4350.196 4353.796 4357.396 4360.996 4364.596 4368.196 4371.796 4375.396 4378.996 4382.596 4386.196 4389



TABLE 8.6 : f<sub>1</sub> (1/a - 7, 1/a - 9)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.6 : f<sub>1</sub> (1/a - 7, 1/a - 10)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.6 : f<sub>1</sub> (1/a - 9, 1/a - 7)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.6 : f<sub>1</sub> (1/a - 8, 1/a - 6)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.6 : f<sub>1</sub> (1/a - 9, 1/a - 9)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.6 : f<sub>1</sub> (1/a - 8, 1/a - 8)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.6 : f<sub>1</sub> (1/a - 1.0, 1/a - 6)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.6 : f<sub>1</sub> (1/a - 7, 1/a - 10)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.6 : f<sub>1</sub> (1/a - 9, 1/a - 7)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.6 : f<sub>1</sub> (1/a - 8, 1/a - 10)

Table with 6 columns: a, f1, f2, f3, f4, f5. Rows include values for a from 0.15 to 27.29.

TABLE 8.7 :  $F_1$  ( $\theta = -7, \alpha = .6$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 5.930 4.184 4.427 4.725 5.083 5.466 5.890 6.351 6.743 7.138 7.538 7.931 8.355 8.785 9.180 9.579

(2) 7.076 7.596 7.993 8.468 8.776 9.011 7.282 6.906 7.439 8.232 9.101 8.916 8.461 7.980 7.560

(3) 10.22 10.82 11.39 10.69 9.687 10.008 10.187 11.48 10.63 10.12 11.04 12.34 11.63 11.63 11.20

(4) 13.36 14.06 12.96 12.54 13.37 14.3 15.25 14.05 12.97 13.40 14.79 14.56 13.64 13.31 15.48 14.45

TABLE 8.7 :  $F_1$  ( $\theta = -7, \alpha = .7$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 4.588 4.816 5.110 5.438 5.804 6.191 6.488 6.783 6.855 5.439 5.076 4.766 4.504 4.286 4.111

(2) 6.209 6.816 7.119 7.601 8.144 8.236 7.885 7.497 7.204 6.583 6.014 6.545 6.201 5.651

(3) 11.68 12.49 12.57 11.14 10.92 11.41 12.47 12.57 11.66 10.92 11.13 12.42 12.57 11.91 11.28

(4) 15.49 15.71 13.56 14.49 15.43 15.31 14.37 14.15 15.34 15.71 14.71 13.96 15.14 15.21 14.92

TABLE 8.7 :  $F_1$  ( $\theta = -7, \alpha = .8$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 5.166 5.444 5.756 6.102 6.474 6.807 6.834 6.447 5.984 5.202 4.493 4.637 4.413 4.233

(2) 9.309 9.807 10.30 10.38 9.450 8.607 8.314 8.740 9.437 10.01 9.739 9.100 8.659 8.181 7.781

(3) 13.45 14.03 13.17 11.94 12.27 11.05 11.41 12.84 11.90 11.47 12.78 13.17 12.69 12.01 11.39

(4) 17.52 16.57 15.58 16.38 17.20 16.14 15.06 15.80 16.76 15.94 14.96 14.79 16.26 15.67 15.01

TABLE 8.7 :  $F_1$  ( $\theta = -7, \alpha = .9$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 5.746 6.036 6.360 6.714 7.089 7.291 7.088 6.611 6.129 5.702 5.335 5.027 4.754 4.526 4.331

(2) 10.37 10.88 11.31 10.48 9.785 9.053 9.040 9.556 10.21 10.38 9.804 9.279 8.754 8.255 7.692

(3) 14.98 15.34 13.75 13.01 13.57 14.33 14.16 13.09 12.24 12.28 13.25 13.46 12.60 12.11 11.49

(4) 19.41 17.47 17.28 16.37 18.70 18.50 16.30 17.33 17.30 16.16 15.29 15.49 16.67 15.93 15.11

TABLE 8.7 :  $F_2$  ( $\theta = -7, \alpha = 1.0$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 6.278 6.593 6.924 7.278 7.601 7.681 7.212 6.781 6.279 5.840 5.463 5.140 4.860 4.619 4.408

(2) 11.39 11.91 12.18 11.29 10.15 9.558 9.223 10.79 10.81 10.62 10.07 9.422 8.878 8.402 7.988

(3) 16.46 16.36 16.41 14.13 14.00 15.33 14.49 13.36 12.68 13.11 13.94 13.65 12.92 12.27 11.59

(4) 21.07 18.62 18.98 19.78 18.73 17.26 17.68 18.48 17.58 16.40 15.83 16.46 16.89 16.05 15.22

TABLE 8.7 :  $F_1$  ( $\theta = -8, \alpha = .6$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 3.439 3.643 3.874 4.135 4.433 4.774 5.159 5.632 5.803 4.919 4.650 4.450 4.381 4.159 3.979

(2) 6.192 6.560 6.932 7.430 7.892 7.854 7.194 6.654 6.658 7.265 8.147 8.795 8.434 7.972 7.558

(3) 8.945 9.472 10.04 10.32 9.326 8.866 8.575 10.43 11.15 10.59 9.909 9.892 11.38 11.78 11.19

(4) 11.70 12.38 12.57 11.29 11.25 12.63 13.49 12.75 12.07 13.10 14.29 13.58 12.88 14.30 14.81

TABLE 8.7 :  $F_1$  ( $\theta = -8, \alpha = .7$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 3.989 4.216 4.472 4.761 5.072 5.451 5.822 6.013 5.702 5.415 5.094 4.761 4.509 4.285 4.111

(2) 7.184 7.594 8.069 8.521 8.244 8.133 7.468 7.176 7.562 8.270 9.052 9.497 8.976 8.526 8.064

(3) 10.38 10.98 11.48 10.86 9.588 10.25 11.01 11.82 11.55 10.32 11.12 12.79 11.88 11.28 11.28

(4) 13.57 14.22 13.23 12.80 13.58 14.49 14.16 13.15 13.60 14.87 14.62 13.77 13.69 13.43 14.90

TABLE 8.6 :  $F_1$  ( $1/\theta = 1.0, 1/\alpha = .8$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 5.170 5.603 5.891 6.105 6.518 7.022 7.689 7.854 7.261 7.235 6.315 6.433 6.071 5.743 5.407

(2) 8.899 10.10 10.87 11.47 11.91 11.09 10.15 9.516 9.232 10.45 11.02 10.60 10.11 9.554 9.055

(3) 12.44 13.52 14.20 14.18 12.94 12.63 13.35 14.04 13.56 12.70 12.19 12.87 13.71 13.24 12.59

(4) 18.81 19.63 19.17 17.52 18.56 18.65 17.13 16.70 17.67 17.72 16.62 15.81 16.73 17.13 16.32

TABLE 8.6 :  $F_1$  ( $1/\theta = 1.0, 1/\alpha = .9$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 4.884 5.301 5.763 5.566 6.016 6.417 6.960 7.260 7.413 7.127 6.722 6.311 5.928 5.465 5.189

(2) 8.842 9.293 9.808 10.37 10.88 10.68 9.863 9.221 9.119 9.679 10.43 10.45 9.968 9.447 8.973

(3) 12.44 13.52 14.20 14.18 12.94 12.63 13.35 14.04 13.56 12.70 12.19 12.87 13.71 13.24 12.59

(4) 18.86 17.69 17.36 16.05 16.49 17.53 16.71 15.71 16.36 17.19 16.40 15.32 15.72 16.84 16.20

TABLE 8.7 :  $F_1$  ( $\theta = -6, \alpha = .6$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 4.386 4.859 5.166 5.710 5.899 6.319 6.761 7.024 6.517 6.398 5.892 5.454 5.082 4.768 4.411

(2) 8.355 8.814 9.273 9.773 9.078 8.108 7.457 8.657 8.589 8.474 8.607 9.032 8.475 7.964 7.561

(3) 11.92 12.60 13.57 13.93 13.00 11.68 12.41 12.57 11.57 10.78 11.16 11.63 12.63 12.59 11.46 11.20

(4) 15.57 15.71 13.82 14.53 15.34 15.71 14.22 14.16 13.48 13.21 14.63 13.83 13.54 15.71 14.86

TABLE 8.7 :  $F_1$  ( $\theta = -6, \alpha = .7$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 5.318 5.621 5.960 6.317 6.743 7.024 6.517 6.398 5.892 5.454 5.082 4.768 4.411 4.233

(2) 9.216 10.11 10.44 10.47 9.315 8.492 8.404 8.398 9.281 10.23 9.723 9.209 8.536 8.025 7.466

(3) 13.87 14.42 13.03 11.98 12.57 13.45 13.81 12.76 11.78 11.55 12.67 13.31 12.45 11.93 11.29

(4) 18.00 16.41 15.87 14.62 14.56 16.01 15.15 14.23 16.98 15.87 14.83 15.07 16.53 15.26 14.93

TABLE 8.7 :  $F_1$  ( $\theta = -6, \alpha = .8$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 6.026 6.191 6.210 7.099 7.466 7.530 7.097 6.525 6.013 5.576 5.207 4.895 4.632 4.413 4.233

(2) 10.66 11.41 11.27 10.86 9.641 8.973 9.363 10.03 10.68 10.48 9.878 9.207 8.658 8.184 7.782

(3) 15.65 15.71 11.64 11.41 14.37 14.59 14.15 12.94 12.23 12.33 11.79 11.50 12.74 12.02 11.39

(4) 20.12 17.54 17.56 16.94 18.24 16.35 16.89 18.00 17.10 16.06 15.35 16.36 16.76 15.87 15.02

TABLE 8.7 :  $F_1$  ( $\theta = -6, \alpha = .9$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 7.356 7.686 8.054 8.407 8.576 8.104 7.444 6.817 6.297 5.848 5.466 5.161 4.843 4.519 4.209

(2) 12.57 12.83 12.57 11.19 10.08 9.817 10.26 10.82 11.18 10.43 9.945 9.319 8.770 8.297 7.893

(3) 17.19 16.48 14.57 14.85 15.64 15.71 14.42 11.79 12.99 13.81 14.31 13.63 12.84 12.92 11.50

(4) 21.62 19.22 19.89 20.58 18.68 17.67 18.60 18.45 17.55 16.33 16.33 17.48 16.89 15.97 15.12

TABLE 8.7 :  $F_1$  ( $\theta = -6, \alpha = 1.0$ )

$\nu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

WDFE:

(1) 7.356 7.686 8.054 8.407 8.576 8.104 7.444 6.817 6.297 5.848 5.466 5.161 4.843 4.519 4.209

(2) 13.37 13.25 13.14 11.57 10.59 10.316 11.62 11.62 11.47 10.78 10.07 9.436 8.803 8.209 7.809

(3) 19.01 17.16 15.71 15.21 16.90 16.13 14.70 13.32 13.86 14.70 14.57 13.16 12.95 12.33 11.60

(4) 22.48 21.01 21.90 21.45 19.14 19.11 20.01 19.21 17.76 16.79 17.39 17.88 17.01 16.07 15.27

TABLE B.7 :  $f_1$  ( $c = -9, a = -1.0$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	4.899	5.131	5.396	5.696	6.029	6.371	6.819	7.342	7.938	8.611	9.342	10.115	10.928	11.778	12.663
(2)	8.022	9.136	9.786	10.127	10.311	10.383	10.375	10.285	10.122	9.898	9.618	9.286	8.908	8.489	8.036
(3)	12.886	13.445	13.329	12.111	11.884	12.444	13.116	13.000	12.166	11.511	11.469	12.689	12.728	12.120	11.529
(4)	16.893	16.893	15.300	15.373	16.357	16.356	15.515	15.137	16.200	16.100	15.516	16.357	15.527	15.591	15.320

TABLE B.7 :  $f_1$  ( $c = -1.0, a = -0.9$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	2.751	2.815	3.079	3.308	3.588	3.874	4.165	4.453	4.707	5.097	5.604	6.311	7.078	7.943	8.918
(2)	4.934	5.248	5.580	5.915	6.246	6.570	6.893	7.207	7.507	7.893	8.261	8.741	9.341	10.063	10.907
(3)	7.156	7.482	7.822	8.168	8.520	8.878	9.242	9.612	9.987	10.367	10.751	11.139	11.531	11.927	12.327
(4)	9.339	9.911	10.500	10.539	9.760	10.118	11.011	11.941	12.915	13.934	14.998	16.107	17.261	18.461	19.703

TABLE B.7 :  $f_1$  ( $c = -1.0, a = -0.7$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	3.191	3.273	3.578	3.810	4.075	4.378	4.722	5.091	5.387	5.794	6.225	6.766	7.411	8.161	9.018
(2)	5.747	6.077	6.448	6.860	7.297	7.758	8.242	8.749	9.278	9.829	10.393	10.971	11.563	12.169	12.797
(3)	8.305	8.781	9.300	9.755	10.239	10.743	11.267	11.811	12.375	12.958	13.560	14.181	14.821	15.480	16.157
(4)	10.861	11.471	12.126	12.800	13.493	14.205	14.936	15.686	16.454	17.241	18.047	18.875	19.727	20.603	21.504

TABLE B.7 :  $f_1$  ( $c = -1.0, a = -0.5$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	3.616	3.812	4.072	4.281	4.584	4.884	5.218	5.582	5.886	6.219	6.582	6.976	7.401	7.856	8.341
(2)	6.318	6.827	7.278	7.715	8.111	8.511	8.911	9.311	9.711	10.111	10.511	10.911	11.311	11.711	12.111
(3)	8.423	8.942	9.461	9.980	10.500	11.020	11.540	12.060	12.580	13.100	13.620	14.140	14.660	15.180	15.700
(4)	11.223	12.123	13.023	13.923	14.823	15.723	16.623	17.523	18.423	19.323	20.223	21.123	22.023	22.923	23.823

TABLE B.7 :  $f_1$  ( $c = -1.0, a = -0.3$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	4.023	4.227	4.518	4.720	5.012	5.304	5.695	6.086	6.477	6.868	7.259	7.650	8.041	8.432	8.823
(2)	7.284	7.687	8.022	8.315	8.607	8.899	9.191	9.483	9.775	10.067	10.359	10.651	10.943	11.235	11.527
(3)	10.322	11.027	11.551	11.786	12.021	12.256	12.491	12.726	12.961	13.196	13.431	13.666	13.901	14.136	14.371
(4)	13.277	14.379	15.180	15.712	16.179	16.579	16.979	17.379	17.779	18.179	18.579	18.979	19.379	19.779	20.179

TABLE B.8 :  $f_1$  ( $c = -0.5, a = -0.9$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	5.534	5.867	6.242	6.657	7.100	7.588	8.115	8.682	9.289	9.936	10.624	11.352	12.120	12.928	13.776
(2)	9.188	9.740	10.316	10.914	11.534	12.176	12.841	13.529	14.242	14.980	15.743	16.531	17.344	18.181	19.043
(3)	12.866	13.693	14.538	15.392	16.254	17.124	18.002	18.888	19.781	20.681	21.584	22.496	23.417	24.347	25.286
(4)	16.544	17.456	18.369	19.281	20.194	21.107	22.020	22.933	23.846	24.759	25.672	26.585	27.498	28.411	29.324

TABLE B.8 :  $f_1$  ( $c = -0.5, a = -0.7$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	6.427	6.802	7.223	7.695	8.215	8.784	9.403	10.072	10.791	11.560	12.379	13.248	14.167	15.136	16.155
(2)	10.427	11.129	11.966	12.845	13.764	14.714	15.694	16.704	17.744	18.814	19.914	21.044	22.204	23.394	24.624
(3)	14.946	15.727	16.596	17.554	18.502	19.440	20.378	21.316	22.254	23.192	24.130	25.068	26.006	26.944	27.882
(4)	19.129	20.228	21.327	22.426	23.525	24.624	25.723	26.822	27.921	29.020	30.119	31.218	32.317	33.416	34.515

TABLE B.7 :  $f_1$  ( $c = -0.5, a = -0.8$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	4.520	4.784	5.039	5.316	5.619	5.958	6.334	6.748	7.201	7.694	8.227	8.800	9.413	10.066	10.759
(2)	8.127	8.580	9.066	9.576	10.121	10.701	11.316	11.966	12.651	13.366	14.111	14.886	15.691	16.526	17.391
(3)	11.777	12.378	12.979	13.580	14.181	14.782	15.383	15.984	16.585	17.186	17.787	18.388	18.989	19.590	20.191
(4)	15.328	16.129	16.930	17.731	18.532	19.333	20.134	20.935	21.736	22.537	23.338	24.139	24.940	25.741	26.542

TABLE B.7 :  $f_1$  ( $c = -0.5, a = -0.9$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	3.078	3.283	3.569	3.989	4.527	5.092	5.695	6.337	7.018	7.739	8.499	9.299	10.139	11.019	11.939
(2)	5.078	5.544	6.002	6.458	6.914	7.370	7.826	8.282	8.738	9.194	9.650	10.106	10.562	11.018	11.474
(3)	7.113	7.713	8.313	8.913	9.513	10.113	10.713	11.313	11.913	12.513	13.113	13.713	14.313	14.913	15.513
(4)	9.148	10.048	10.948	11.848	12.748	13.648	14.548	15.448	16.348	17.248	18.148	19.048	19.948	20.848	21.748

TABLE B.7 :  $f_1$  ( $c = -0.5, a = -1.0$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	3.511	3.771	4.066	4.393	4.762	5.174	5.621	6.104	6.618	7.163	7.739	8.346	8.984	9.653	10.353
(2)	5.977	6.446	6.922	7.404	7.891	8.393	8.911	9.444	9.992	10.555	11.133	11.726	12.334	12.957	13.595
(3)	8.453	9.022	9.600	10.188	10.786	11.394	12.012	12.640	13.278	13.926	14.584	15.252	15.930	16.618	17.316
(4)	10.929	11.627	12.345	13.083	13.841	14.609	15.397	16.205	17.033	17.881	18.749	19.637	20.545	21.473	22.421

TABLE B.7 :  $f_1$  ( $c = -0.5, a = -0.6$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	3.057	3.219	3.463	3.781	4.174	4.643	5.188	5.811	6.514	7.297	8.170	9.134	10.188	11.332	12.566
(2)	5.153	5.511	5.879	6.257	6.655	7.074	7.524	7.995	8.487	8.999	9.532	10.086	10.660	11.254	11.868
(3)	7.251	7.719	8.197	8.685	9.194	9.724	10.275	10.848	11.442	12.057	12.692	13.347	14.022	14.716	15.430
(4)	9.349	10.027	10.725	11.443	12.181	12.939	13.717	14.515	15.333	16.171	17.029	17.907	18.805	19.723	20.661

TABLE B.7 :  $f_1$  ( $c = -0.5, a = -0.3$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	3.544	3.748	3.976	4.233	4.526	4.854	5.226	5.633	6.076	6.550	7.055	7.591	8.158	8.756	9.385
(2)	6.184	6.592	7.041	7.534	8.072	8.655	9.284	9.958	10.677	11.441	12.250	13.104	14.003	14.947	15.935
(3)	8.727	9.352	10.027	10.752	11.527	12.352	13.227	14.152	15.127	16.152	17.227	18.352	19.527	20.752	22.027
(4)	11.270	12.027	12.827	13.672	14.562	15.497	16.477	17.502	18.572	19.687	20.847	22.052	23.302	24.597	25.937

TABLE B.7 :  $f_1$  ( $c = -0.5, a = -0.1$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	4.018	4.218	4.480	4.755	5.066	5.411	5.791	6.206	6.657	7.144	7.665	8.220	8.810	9.435	10.096
(2)	7.242	7.639	8.077	8.558	9.072	9.621	10.206	10.827	11.484	12.177	12.906	13.671	14.472	15.309	16.180
(3)	10.471	11.013	11.531	12.136	12.729	13.411	14.082	14.843	15.694	16.535	17.466	18.387	19.398	20.409	21.420
(4)	13.692	14.311	14.920	15.569	16.258	16.987	17.756	18.565	19.414	20.303	21.232	22.191	23.180	24.209	25.278

TABLE B.7 :  $f_1$  ( $c = -0.5, a = -0.9$ )

MODE:	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
-------	------	------	------	------	------	------	------	------	------	------	------	------	------

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -8$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -9$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -10$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -11$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -12$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -13$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -14$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -8$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -9$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -10$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -11$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -12$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -13$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.

TABLE 8.8 :  $f_1$  ( $\xi = -6, a = -14$ )

Table with 6 columns: u, 0.15, 0.20, 0.25, 0.30, 0.35. Rows (1) to (6) containing numerical data.



TABLE 8.9 :  $f_1$  ( $\theta = -7$ ,  $\alpha = -7$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	2.176	2.299	2.437	2.594	2.771	2.975	3.211	3.483	3.795	4.122	4.507	4.930	5.382	5.862	6.368
(1)	2.176	2.299	2.437	2.594	2.771	2.975	3.211	3.483	3.795	4.122	4.507	4.930	5.382	5.862	6.368
(2)	6.179	6.281	6.310	6.298	6.252	6.182	6.090	5.978	5.848	5.703	5.547	5.384	5.218	5.054	4.895
(3)	9.123	9.038	8.877	8.644	8.343	8.000	7.631	7.251	6.874	6.506	6.151	5.814	5.498	5.207	4.947
(4)	12.777	12.427	12.037	11.621	11.195	10.767	10.339	9.913	9.491	9.074	8.663	8.258	7.859	7.467	7.083

TABLE 8.9 :  $f_1$  ( $\theta = -7$ ,  $\alpha = -8$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	2.466	2.591	2.730	2.893	3.080	3.299	3.558	3.867	4.225	4.633	5.090	5.596	6.152	6.758	7.414
(1)	2.466	2.591	2.730	2.893	3.080	3.299	3.558	3.867	4.225	4.633	5.090	5.596	6.152	6.758	7.414
(2)	6.179	6.281	6.310	6.298	6.252	6.182	6.090	5.978	5.848	5.703	5.547	5.384	5.218	5.054	4.895
(3)	10.339	10.189	10.012	9.819	9.614	9.399	9.177	8.951	8.723	8.494	8.264	8.034	7.804	7.575	7.347
(4)	14.477	14.147	13.787	13.401	13.000	12.595	12.187	11.777	11.365	10.952	10.539	10.127	9.715	9.304	8.893

TABLE 8.9 :  $f_1$  ( $\theta = -7$ ,  $\alpha = -9$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	2.716	2.841	2.980	3.143	3.330	3.551	3.814	4.118	4.464	4.851	5.278	5.745	6.252	6.800	7.388
(1)	2.716	2.841	2.980	3.143	3.330	3.551	3.814	4.118	4.464	4.851	5.278	5.745	6.252	6.800	7.388
(2)	6.179	6.281	6.310	6.298	6.252	6.182	6.090	5.978	5.848	5.703	5.547	5.384	5.218	5.054	4.895
(3)	11.531	11.308	11.077	10.840	10.599	10.357	10.115	9.874	9.634	9.395	9.157	8.920	8.684	8.449	8.215
(4)	15.777	15.477	15.147	14.801	14.450	14.105	13.766	13.434	13.109	12.792	12.483	12.182	11.889	11.603	11.317

TABLE 8.9 :  $f_1$  ( $\theta = -7$ ,  $\alpha = -10$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	2.998	3.123	3.262	3.425	3.612	3.833	4.096	4.400	4.746	5.133	5.560	6.027	6.534	7.081	7.668
(1)	2.998	3.123	3.262	3.425	3.612	3.833	4.096	4.400	4.746	5.133	5.560	6.027	6.534	7.081	7.668
(2)	6.179	6.281	6.310	6.298	6.252	6.182	6.090	5.978	5.848	5.703	5.547	5.384	5.218	5.054	4.895
(3)	12.677	12.407	12.107	11.791	11.461	11.129	10.797	10.466	10.136	9.807	9.480	9.155	8.832	8.511	8.192
(4)	17.179	16.919	16.639	16.359	16.079	15.799	15.519	15.239	14.959	14.679	14.399	14.119	13.839	13.559	13.279

TABLE 8.9 :  $f_1$  ( $\theta = -4$ ,  $\alpha = -6$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	1.612	1.719	1.826	1.933	2.040	2.147	2.254	2.361	2.468	2.575	2.682	2.789	2.896	3.003	3.110
(1)	1.612	1.719	1.826	1.933	2.040	2.147	2.254	2.361	2.468	2.575	2.682	2.789	2.896	3.003	3.110
(2)	4.112	4.196	4.281	4.365	4.450	4.534	4.618	4.702	4.786	4.870	4.954	5.038	5.122	5.206	5.290
(3)	6.841	6.739	6.645	6.551	6.457	6.363	6.269	6.175	6.081	5.987	5.893	5.799	5.705	5.611	5.517
(4)	9.642	9.500	9.358	9.216	9.074	8.932	8.790	8.648	8.506	8.364	8.222	8.080	7.938	7.796	7.654

TABLE 8.9 :  $f_1$  ( $\theta = -4$ ,  $\alpha = -7$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	1.794	1.912	2.040	2.187	2.354	2.541	2.758	3.005	3.282	3.589	3.926	4.293	4.690	5.117	5.574
(1)	1.794	1.912	2.040	2.187	2.354	2.541	2.758	3.005	3.282	3.589	3.926	4.293	4.690	5.117	5.574
(2)	4.845	4.939	5.033	5.127	5.221	5.315	5.409	5.503	5.597	5.691	5.785	5.879	5.973	6.067	6.161
(3)	7.884	7.780	7.676	7.572	7.468	7.364	7.260	7.156	7.052	6.948	6.844	6.740	6.636	6.532	6.428
(4)	11.149	11.019	10.889	10.759	10.629	10.499	10.369	10.239	10.109	9.979	9.849	9.719	9.589	9.459	9.329

TABLE 8.9 :  $f_1$  ( $\theta = -6$ ,  $\alpha = -8$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	2.156	2.279	2.417	2.580	2.767	2.988	3.243	3.532	3.856	4.214	4.616	5.062	5.553	6.089	6.669
(1)	2.156	2.279	2.417	2.580	2.767	2.988	3.243	3.532	3.856	4.214	4.616	5.062	5.553	6.089	6.669
(2)	5.004	5.098	5.192	5.286	5.380	5.474	5.568	5.662	5.756	5.850	5.944	6.038	6.132	6.226	6.320
(3)	8.035	7.941	7.847	7.753	7.659	7.565	7.471	7.377	7.283	7.189	7.095	7.001	6.907	6.813	6.719
(4)	12.248	12.109	11.970	11.831	11.692	11.553	11.414	11.275	11.136	11.000	10.864	10.729	10.594	10.459	10.324

TABLE 8.9 :  $f_1$  ( $\theta = -4$ ,  $\alpha = -9$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	2.394	2.509	2.637	2.781	2.943	3.126	3.335	3.576	3.853	4.167	4.527	4.934	5.388	5.890	6.439
(1)	2.394	2.509	2.637	2.781	2.943	3.126	3.335	3.576	3.853	4.167	4.527	4.934	5.388	5.890	6.439
(2)	6.017	6.135	6.253	6.371	6.489	6.607	6.725	6.843	6.961	7.079	7.197	7.315	7.433	7.551	7.669
(3)	10.009	10.041	10.073	10.105	10.137	10.169	10.201	10.233	10.265	10.297	10.329	10.361	10.393	10.425	10.457
(4)	14.148	14.047	13.946	13.845	13.744	13.643	13.542	13.441	13.340	13.239	13.138	13.037	12.936	12.835	12.734

TABLE 8.9 :  $f_1$  ( $\theta = -8$ ,  $\alpha = -10$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	2.613	2.735	2.868	2.980	3.122	3.287	3.476	3.690	3.939	4.224	4.546	4.915	5.332	5.798	6.314
(1)	2.613	2.735	2.868	2.980	3.122	3.287	3.476	3.690	3.939	4.224	4.546	4.915	5.332	5.798	6.314
(2)	6.000	6.119	6.218	6.300	6.375	6.451	6.527	6.603	6.679	6.755	6.831	6.907	6.983	7.059	7.135
(3)	11.110	11.063	11.016	10.969	10.922	10.875	10.828	10.781	10.734	10.687	10.640	10.593	10.546	10.499	10.452
(4)	15.356	15.248	15.140	15.032	14.924	14.816	14.708	14.600	14.492	14.384	14.276	14.168	14.060	13.952	13.844

TABLE 8.9 :  $f_1$  ( $\theta = -9$ ,  $\alpha = -9$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	1.460	1.546	1.644	1.754	1.881	2.027	2.198	2.404	2.644	2.919	3.228	3.571	3.948	4.360	4.807
(1)	1.460	1.546	1.644	1.754	1.881	2.027	2.198	2.404	2.644	2.919	3.228	3.571	3.948	4.360	4.807
(2)	3.655	3.672	3.689	3.706	3.723	3.740	3.757	3.774	3.791	3.808	3.825	3.842	3.859	3.876	3.893
(3)	6.116	6.100	6.084	6.068	6.052	6.036	6.020	6.004	5.988	5.972	5.956	5.940	5.924	5.908	5.892
(4)	8.563	8.500	8.437	8.374	8.311	8.248	8.185	8.122	8.059	7.996	7.933	7.870	7.807	7.744	7.681

TABLE 8.9 :  $f_1$  ( $\theta = -9$ ,  $\alpha = -7$ )

$\mu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
WOM:	1.692	1.788	1.894	2.017	2.156	2.315	2.500	2.718	2.972	3.273	3.612	3.999	4.436		



TABLE 8.9 :  $f_1$  ( $1/\delta - 8, 1/\delta - 4$ )

MODEL

$\nu$	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
(1)	2.113	2.189	2.267	2.347	2.431	2.521	2.617	2.719	2.827	2.942	3.064	3.193	3.330	3.474	3.625
(2)	2.163	2.241	2.321	2.403	2.490	2.583	2.682	2.787	2.898	3.015	3.139	3.270	3.408	3.553	3.705
(3)	2.214	2.293	2.374	2.458	2.546	2.639	2.737	2.841	2.950	3.064	3.184	3.310	3.442	3.581	3.726
(4)	2.266	2.346	2.428	2.513	2.602	2.695	2.793	2.896	3.003	3.115	3.232	3.354	3.482	3.617	3.758
(5)	2.319	2.400	2.483	2.569	2.658	2.751	2.848	2.949	3.054	3.163	3.276	3.393	3.515	3.642	3.775
(6)	2.374	2.456	2.540	2.626	2.715	2.807	2.902	2.999	3.099	3.202	3.308	3.416	3.527	3.642	3.760
(7)	2.430	2.513	2.598	2.685	2.774	2.866	2.961	3.058	3.157	3.258	3.361	3.466	3.573	3.683	3.795
(8)	2.487	2.571	2.657	2.744	2.833	2.924	3.017	3.112	3.209	3.307	3.406	3.507	3.610	3.715	3.821
(9)	2.545	2.629	2.715	2.802	2.891	2.981	3.072	3.165	3.259	3.354	3.450	3.548	3.647	3.748	3.851
(10)	2.603	2.688	2.774	2.861	2.950	3.040	3.131	3.223	3.315	3.408	3.502	3.597	3.693	3.790	3.888
(11)	2.662	2.747	2.833	2.920	3.008	3.097	3.187	3.278	3.369	3.461	3.553	3.646	3.740	3.834	3.929
(12)	2.721	2.806	2.892	2.979	3.067	3.155	3.244	3.334	3.424	3.514	3.604	3.695	3.787	3.880	3.973
(13)	2.780	2.865	2.951	3.037	3.124	3.211	3.298	3.386	3.474	3.562	3.650	3.738	3.827	3.917	4.007
(14)	2.840	2.925	3.010	3.095	3.181	3.267	3.353	3.439	3.525	3.611	3.697	3.783	3.870	3.957	4.044
(15)	2.900	2.984	3.068	3.152	3.237	3.321	3.405	3.489	3.573	3.657	3.741	3.825	3.910	3.994	4.078
(16)	2.960	3.043	3.126	3.208	3.291	3.373	3.455	3.537	3.618	3.699	3.780	3.861	3.942	4.023	4.103
(17)	3.020	3.102	3.184	3.265	3.346	3.426	3.506	3.585	3.664	3.743	3.822	3.900	3.978	4.056	4.133
(18)	3.080	3.161	3.241	3.320	3.398	3.476	3.553	3.630	3.706	3.782	3.857	3.932	4.006	4.080	4.153
(19)	3.140	3.219	3.297	3.374	3.450	3.525	3.600	3.674	3.747	3.819	3.891	3.962	4.033	4.103	4.172
(20)	3.200	3.278	3.354	3.429	3.503	3.576	3.648	3.719	3.789	3.858	3.926	4.000	4.070	4.138	4.205
(21)	3.260	3.337	3.411	3.484	3.556	3.627	3.696	3.764	3.830	3.895	3.958	4.020	4.080	4.138	4.194
(22)	3.320	3.396	3.468	3.539	3.609	3.677	3.744	3.809	3.873	3.935	3.996	4.056	4.114	4.170	4.225
(23)	3.380	3.455	3.525	3.594	3.662	3.728	3.792	3.854	3.915	3.974	4.032	4.089	4.144	4.198	4.251
(24)	3.440	3.514	3.583	3.650	3.716	3.780	3.842	3.902	3.960	4.017	4.073	4.128	4.181	4.233	4.284
(25)	3.500	3.573	3.641	3.707	3.771	3.833	3.893	3.951	4.007	4.062	4.116	4.168	4.219	4.268	4.316
(26)	3.560	3.631	3.698	3.762	3.824	3.884	3.942	3.998	4.053	4.106	4.158	4.208	4.256	4.303	4.348
(27)	3.620	3.689	3.755	3.818	3.879	3.938	3.995	4.050	4.104	4.156	4.207	4.256	4.303	4.348	4.392
(28)	3.680	3.748	3.813	3.875	3.935	3.993	4.049	4.103	4.155	4.206	4.255	4.302	4.348	4.392	4.434
(29)	3.740	3.806	3.869	3.929	3.987	4.043	4.097	4.149	4.200	4.249	4.296	4.342	4.387	4.430	4.471
(30)	3.800	3.864	3.925	3.983	4.039	4.093	4.145	4.195	4.244	4.291	4.337	4.381	4.424	4.465	4.505
(31)	3.860	3.922	3.981	4.038	4.093	4.146	4.197	4.246	4.293	4.338	4.382	4.424	4.465	4.505	4.544
(32)	3.920	3.979	4.036	4.091	4.144	4.195	4.244	4.291	4.336	4.379	4.421	4.461	4.500	4.538	4.575
(33)	3.980	4.037	4.092	4.145	4.196	4.245	4.292	4.337	4.380	4.421	4.461	4.500	4.537	4.573	4.608
(34)	4.040	4.096	4.149	4.200	4.249	4.296	4.341	4.384	4.425	4.464	4.502	4.539	4.574	4.608	4.641
(35)	4.100	4.154	4.206	4.256	4.303	4.348	4.391	4.432	4.471	4.508	4.544	4.579	4.612	4.644	4.676
(36)	4.160	4.212	4.262	4.310	4.356	4.399	4.440	4.478	4.514	4.549	4.583	4.616	4.648	4.679	4.709
(37)	4.220	4.270	4.318	4.364	4.408	4.450	4.489	4.526	4.561	4.594	4.626	4.657	4.687	4.716	4.744
(38)	4.280	4.328	4.374	4.418	4.460	4.500	4.538	4.574	4.608	4.640	4.671	4.701	4.729	4.756	4.782
(39)	4.340	4.386	4.430	4.472	4.512	4.549	4.584	4.617	4.649	4.679	4.708	4.735	4.761	4.786	4.810
(40)	4.400	4.444	4.486	4.527	4.566	4.602	4.636	4.668	4.698	4.727	4.754	4.779	4.803	4.826	4.848
(41)	4.460	4.502	4.542	4.581	4.618	4.653	4.686	4.717	4.746	4.773	4.798	4.821	4.843	4.864	4.884
(42)	4.520	4.560	4.598	4.635	4.670	4.703	4.734	4.763	4.790	4.815	4.839	4.861	4.882	4.902	4.920
(43)	4.580	4.618	4.654	4.689	4.721	4.751	4.779	4.805	4.829	4.851	4.872	4.892	4.911	4.929	4.945
(44)	4.640	4.676	4.710	4.743	4.774	4.803	4.830	4.855	4.878	4.899	4.918	4.936	4.952	4.967	4.980
(45)	4.700	4.734	4.766	4.797	4.826	4.853	4.878	4.901	4.922	4.942	4.960	4.977	4.992	5.005	5.017
(46)	4.760	4.792	4.822	4.851	4.878	4.903	4.926	4.947	4.966	4.983	5.000	5.015	5.029	5.041	5.052
(47)	4.820	4.850	4.878	4.904	4.928	4.949	4.968	4.984	5.000	5.014	5.027	5.039	5.050	5.060	5.069
(48)	4.880	4.908	4.934	4.958	4.979	4.998	5.015	5.030	5.044	5.056	5.067	5.077	5.086	5.094	5.101
(49)	4.940	4.966	4.990	5.012	5.032	5.050	5.066	5.080	5.092	5.103	5.113	5.121	5.128	5.134	5.139
(50)	5.000	5.024	5.046	5.066	5.083	5.098	5.112	5.124	5.134	5.143	5.151	5.158	5.164	5.169	5.173
(51)	5.060	5.082	5.102	5.119	5.134	5.147	5.158	5.167	5.174	5.180	5.185	5.189	5.192	5.194	5.196
(52)	5.120	5.140	5.158	5.174	5.188	5.199	5.208	5.215	5.220	5.224	5.227	5.229	5.230	5.230	5.230
(53)	5.180	5.198	5.214	5.228	5.239	5.247	5.253	5.257	5.259	5.260	5.260	5.260	5.260	5.260	5.260
(54)	5.240	5.256	5.270	5.281	5.289	5.294	5.297	5.298	5.298	5.298	5.297	5.296	5.294	5.291	5.287
(55)	5.300	5.314	5.326	5.335	5.341	5.344	5.345	5.345	5.344	5.342	5.340	5.337	5.333	5.328	5.321
(56)	5.360	5.372	5.382	5.389	5.393	5.394	5.393	5.391	5.388	5.384	5.379	5.373	5.366	5.358	5.348
(57)	5.420	5.430	5.438	5.443	5.445	5.444	5.441	5.436	5.429	5.421	5.411	5.400	5.387	5.372	5.355
(58)	5.480	5.488	5.494	5.497	5.498	5.496	5.492	5.486	5.478	5.467	5.453	5.437	5.419	5.398	5.373
(59)	5.540	5.546	5.549	5.549	5.547	5.543	5.537	5.528	5.516	5.499	5.478	5.454	5.426	5.394	5.358
(60)	5.600	5.604	5.605	5.604	5.601	5.595	5.585	5.571	5.553	5.530	5.503	5.472	5.437	5.398	5.354
(61)	5.660	5.662	5.661	5.658	5.652	5.642	5.628	5.609	5.585	5.555	5.519	5.476	5.426	5.372	5.314
(62)	5.720	5.720	5.718	5.713	5.704	5.690	5.669	5.642	5.608	5.567	5.518	5.461	5.396	5.328	5.256
(63)	5.780	5.778	5.773	5.763	5.748	5.718	5.679	5.630	5.571	5.503	5.426	5.349	5.262	5.165	5.057
(64)	5.840	5.836	5.828	5.815	5.797	5.763	5.714	5.654	5.583	5.501	5.414	5.317	5.210	5.092	4.963
(65)	5.900	5.894	5.883	5.867	5.836	5.789	5.728	5.656	5.572	5.475	5.368	5.251	5.124	4.985	4.844
(66)	5.960	5.952	5.938	5.919	5.884	5.828	5.756	5.671	5.573	5.465	5.347	5.220	5.082	4.933	4.781
(67)	6.020	6.010	6.000	5.984	5.952	5.897	5.815	5.718	5.609	5.490	5.361	5.224	5.075	4.915	4.752
(68)	6.080	6.068	6.055	6.034	6.000	5.935	5.843	5.735	5.615	5.485	5.347	5.200	5.041	4.870	4.696
(69)	6.140	6.126	6.111	6.088	6.052	5.977	5.875	5.757	5.625	5.485	5.337	5.180	5.011	4.829	4.644
(70)	6.200	6.184	6.167	6.141	6.103	6.018	5.897	5.769	5.627	5.477	5.319	5.151	4.972	4.779	4.582
(71)	6.260	6.242	6.223	6.194	6.154	6.059	5.928	5.789	5.637	5.477	5.309	5.131	4.942	4.737	4.529
(72)	6.320	6.299	6.278	6.246	6.194	6.089	5.948	5.799	5.637	5.467	5.289	5.101	4.895	4.678	4.459
(73)	6.380	6.357	6.334	6.299	6.245	6.130	5.978	5.820	5.647	5.467	5.279	5.081	4.864	4.636	4.405
(74)	6.440	6.414	6.388	6.350	6.294	6.169	6.007	5.840	5.657	5.467	5.269	5.061	4.833	4.594	4.351
(75)	6.500	6.471	6.442	6.399	6.332	6.197	6.025	5.857	5.664	5.463	5.255	5.037	4.798	4.548	4.293
(76)	6.560														

TABLE 8.10 : $f_1$ ( $\alpha = -8, \alpha = -6$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.377 1.376 1.135 1.173 1.171 1.169 1.168 1.168 1.169 1.173 1.179 1.190 1.206 1.229 1.264
(2)	3.523 3.523 3.538 3.573 3.643 3.724 3.842 3.987 4.150 4.315 4.451 4.528 4.537 4.501 4.447
TABLE 8.10 : $f_2$ ( $\alpha = -8, \alpha = -7$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.373 1.372 1.370 1.367 1.363 1.360 1.356 1.353 1.352 1.351 1.354 1.359 1.368 1.383 1.406
(2)	4.104 4.094 4.095 4.114 4.136 4.220 4.305 4.404 4.506 4.595 4.653 4.670 4.651 4.611 4.565
TABLE 8.10 : $f_3$ ( $\alpha = -8, \alpha = -8$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.569 1.567 1.563 1.559 1.553 1.546 1.539 1.531 1.524 1.517 1.511 1.508 1.506 1.508 1.514
(2)	4.678 4.650 4.626 4.612 4.615 4.635 4.670 4.714 4.759 4.794 4.811 4.806 4.780 4.741 4.693
TABLE 8.10 : $f_4$ ( $\alpha = -8, \alpha = -9$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.765 1.761 1.756 1.748 1.738 1.727 1.713 1.698 1.682 1.666 1.649 1.633 1.618 1.604 1.592
(2)	5.243 5.185 5.118 5.053 5.002 4.967 4.951 4.949 4.957 4.965 4.964 4.954 4.923 4.882 4.830
TABLE 8.10 : $f_5$ ( $\alpha = -8, \alpha = -1.0$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.950 1.954 1.946 1.934 1.919 1.900 1.877 1.852 1.824 1.795 1.765 1.735 1.705 1.676 1.647
(2)	5.796 5.691 5.562 5.431 5.317 5.231 5.175 5.145 5.135 5.134 5.125 5.113 5.077 5.020 4.940
TABLE 8.10 : $f_6$ ( $\alpha = -9, \alpha = -6$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.048 1.048 1.050 1.052 1.055 1.059 1.064 1.071 1.081 1.093 1.108 1.128 1.154 1.187 1.233
(2)	3.158 3.181 3.221 3.280 3.350 3.433 3.529 3.738 3.902 4.068 4.211 4.304 4.319 4.315 4.317
TABLE 8.10 : $f_7$ ( $\alpha = -9, \alpha = -7$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.222 1.223 1.224 1.225 1.228 1.231 1.236 1.242 1.250 1.261 1.274 1.290 1.311 1.338 1.372
(2)	3.679 3.699 3.735 3.786 3.856 3.943 4.045 4.156 4.265 4.359 4.423 4.454 4.457 4.445 4.436
TABLE 8.10 : $f_8$ ( $\alpha = -9, \alpha = -8$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.398 1.397 1.397 1.398 1.399 1.401 1.403 1.407 1.411 1.417 1.424 1.434 1.446 1.461 1.480
(2)	4.195 4.207 4.228 4.260 4.303 4.356 4.414 4.472 4.542 4.564 4.584 4.590 4.585 4.576 4.572
TABLE 8.10 : $f_9$ ( $\alpha = -9, \alpha = -9$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.570 1.570 1.569 1.568 1.567 1.565 1.564 1.562 1.560 1.559 1.557 1.558 1.556 1.556 1.556
(2)	4.705 4.695 4.693 4.690 4.695 4.703 4.713 4.723 4.731 4.735 4.734 4.728 4.719 4.709
TABLE 8.10 : $f_{10}$ ( $\alpha = -9, \alpha = -1.0$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.744 1.742 1.740 1.738 1.736 1.734 1.724 1.716 1.706 1.695 1.683 1.670 1.656 1.642 1.628 1.613
(2)	5.203 5.169 5.121 5.067 5.019 4.967 4.931 4.908 4.897 4.893 4.891 4.881 4.862 4.833

TABLE 8.10 : $f_1$ ( $\alpha = -6, \alpha = -6$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.559 1.544 1.523 1.495 1.464 1.431 1.398 1.367 1.340 1.316 1.298 1.286 1.281 1.287 1.306
(2)	4.437 4.360 4.270 4.244 4.203 4.198 4.531 4.720 4.923 5.091 5.159 5.107 4.977 4.818 4.662
TABLE 8.10 : $f_2$ ( $\alpha = -6, \alpha = -7$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.818 1.821 1.774 1.741 1.703 1.662 1.621 1.581 1.545 1.513 1.486 1.465 1.451 1.445 1.450
(2)	5.230 5.049 4.915 4.861 4.826 4.819 5.026 5.156 5.274 5.318 5.317 5.219 5.016 4.921 4.774
TABLE 8.10 : $f_3$ ( $\alpha = -6, \alpha = -8$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 2.077 2.056 2.025 1.984 1.937 1.884 1.834 1.783 1.736 1.692 1.654 1.620 1.593 1.572 1.558
(2)	5.949 5.705 5.508 5.386 5.318 5.300 5.402 5.469 5.520 5.522 5.459 5.342 5.196 5.042 4.987
TABLE 8.10 : $f_4$ ( $\alpha = -6, \alpha = -9$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 2.326 2.311 2.273 2.223 2.165 2.101 2.035 1.970 1.908 1.850 1.797 1.749 1.706 1.669 1.636
(2)	6.645 6.316 6.032 5.837 5.728 5.687 5.691 5.713 5.723 5.692 5.609 5.482 5.329 5.169 5.017
TABLE 8.10 : $f_5$ ( $\alpha = -6, \alpha = -1.0$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 2.584 2.554 2.517 2.466 2.403 2.332 2.270 2.208 2.159 2.105 2.052 1.995 1.916 1.852 1.793 1.740 1.691
(2)	7.312 6.867 6.481 6.207 6.040 5.957 5.930 5.928 5.917 5.870 5.772 5.631 5.464 5.290 5.121
TABLE 8.10 : $f_6$ ( $\alpha = -7, \alpha = -6$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.432 1.377 1.310 1.320 1.328 1.325 1.282 1.270 1.258 1.250 1.244 1.244 1.249 1.263 1.288
(2)	3.967 3.919 3.869 3.892 3.934 4.016 4.138 4.294 4.471 4.463 4.766 4.803 4.760 4.670 4.566
TABLE 8.10 : $f_7$ ( $\alpha = -7, \alpha = -7$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.566 1.550 1.550 1.538 1.512 1.525 1.487 1.470 1.453 1.438 1.426 1.418 1.416 1.419 1.432
(2)	4.512 4.548 4.491 4.466 4.478 4.529 4.612 4.717 4.825 4.913 4.952 4.933 4.869 4.776 4.682
TABLE 8.10 : $f_8$ ( $\alpha = -7, \alpha = -8$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 1.769 1.751 1.749 1.753 1.713 1.710 1.666 1.650 1.635 1.611 1.590 1.571 1.556 1.545 1.560
(2)	5.201 5.155 5.059 4.982 4.947 4.977 5.025 5.075 5.108 5.107 5.066 4.994 4.903 4.810
TABLE 8.10 : $f_9$ ( $\alpha = -7, \alpha = -9$ )	
$\mu$	0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85
MODE:	(1) 2.012 2.022 2.021 2.014 2.013 2.017 2.050 2.060 1.990 1.899 1.849 1.801 1.756 1.713 1.673
(2)	6.590 6.418 6.017 5.801 5.641 5.518 5.484 5.465 5.463 5.458 5.431 5.371 5.280 5.168 5.046

TABLE 8.10 :  $f_1 (1/a - 7, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.566 1.500 1.550 1.538 1.522 1.505 1.487 1.470 1.453 1.438 1.426 1.418 1.410 1.419 1.432

(2) 4.818 4.546 4.491 4.466 4.478 4.529 4.612 4.717 4.825 4.913 4.952 4.933 4.868 4.776 4.622

TABLE 8.10 :  $f_1 (1/a - 7, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.373 1.372 1.370 1.367 1.363 1.360 1.356 1.353 1.351 1.354 1.359 1.368 1.383 1.406

(2) 4.104 4.084 4.095 4.114 4.156 4.220 4.305 4.404 4.506 4.595 4.653 4.670 4.651 4.611 4.565

TABLE 8.10 :  $f_1 (1/a - 7, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.222 1.223 1.224 1.225 1.228 1.231 1.236 1.242 1.250 1.263 1.274 1.280 1.311 1.318 1.372

(2) 3.679 3.699 3.735 3.786 3.856 3.943 4.045 4.156 4.265 4.359 4.423 4.454 4.457 4.445 4.436

TABLE 8.10 :  $f_1 (1/a - 7, 1/a - 10)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.101 1.102 1.105 1.109 1.114 1.121 1.130 1.141 1.155 1.172 1.192 1.217 1.248 1.285 1.331

(2) 3.327 3.362 3.415 3.486 3.576 3.683 3.804 3.934 4.062 4.171 4.247 4.295 4.296 4.297 4.208

TABLE 8.10 :  $f_1 (1/a - 8, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.077 2.056 2.025 1.984 1.937 1.886 1.835 1.783 1.736 1.692 1.654 1.620 1.593 1.572 1.558

(2) 5.949 5.705 5.508 5.368 5.285 5.250 5.269 5.292 5.322 5.359 5.342 5.196 5.042 4.897

TABLE 8.10 :  $f_1 (1/a - 8, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.789 1.781 1.769 1.751 1.733 1.710 1.686 1.660 1.635 1.611 1.590 1.571 1.556 1.545 1.540

(2) 5.261 5.155 5.055 4.962 4.947 4.947 4.977 5.025 5.075 5.108 5.107 5.068 4.994 4.903 4.810

TABLE 8.10 :  $f_1 (1/a - 8, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.649 1.667 1.693 1.729 1.773 1.823 1.879 1.939 1.999 2.063 2.129 2.197 2.267 2.337 2.406

(2) 4.878 4.650 4.626 4.612 4.615 4.635 4.670 4.714 4.759 4.794 4.811 4.806 4.780 4.741 4.698

TABLE 8.10 :  $f_1 (1/a - 8, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.356 1.397 1.397 1.398 1.399 1.401 1.403 1.407 1.411 1.417 1.424 1.434 1.446 1.461 1.480

(2) 4.195 4.207 4.228 4.260 4.303 4.356 4.414 4.472 4.524 4.562 4.584 4.590 4.565 4.576 4.572

TABLE 8.10 :  $f_1 (1/a - 8, 1/a - 10)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.258 1.259 1.261 1.265 1.269 1.275 1.283 1.293 1.304 1.318 1.335 1.355 1.378 1.406 1.437

(2) 3.755 3.826 3.872 3.932 4.006 4.089 4.177 4.261 4.332 4.381 4.411 4.421 4.421 4.425 4.444

TABLE 8.10 :  $f_1 (1/a - 9, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.316 2.311 2.273 2.223 2.165 2.101 2.035 1.970 1.908 1.850 1.797 1.749 1.708 1.669 1.636

(2) 6.645 6.316 6.032 5.837 5.728 5.689 5.691 5.713 5.723 5.692 5.609 5.482 5.329 5.169 5.017

TABLE 8.10 :  $f_1 (1/a - 10, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.855 2.899 2.942 2.984 3.014 3.037 3.052 3.059 3.060 3.059 3.057 3.053 3.048 3.043 3.039

(2) 4.383 4.340 4.300 4.276 4.260 4.250 4.242 4.236 4.232 4.229 4.227 4.226 4.226 4.226 4.226

TABLE 8.10 :  $f_1 (1/a - 10, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.101 1.102 1.105 1.109 1.114 1.121 1.130 1.141 1.155 1.172 1.192 1.217 1.248 1.285 1.331

(2) 3.327 3.362 3.415 3.486 3.576 3.683 3.804 3.934 4.062 4.171 4.247 4.295 4.296 4.297 4.208

TABLE 8.10 :  $f_1 (1/a - 10, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.258 1.259 1.261 1.265 1.269 1.275 1.283 1.293 1.304 1.318 1.335 1.355 1.378 1.406 1.437

(2) 3.755 3.826 3.872 3.932 4.006 4.089 4.177 4.261 4.332 4.381 4.411 4.421 4.421 4.425 4.444

TABLE 8.10 :  $f_1 (1/a - 10, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.044 1.045 1.047 1.051 1.056 1.063 1.071 1.080 1.090 1.101 1.112 1.124 1.137 1.151 1.166

(2) 3.127 3.162 3.215 3.286 3.376 3.483 3.604 3.734 3.862 3.971 4.047 4.095 4.096 4.097 4.008

TABLE 8.10 :  $f_1 (1/a - 10, 1/a - 10)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.174 1.174 1.175 1.177 1.180 1.184 1.190 1.197 1.206 1.216 1.226 1.236 1.246 1.256 1.266

(2) 4.477 4.360 4.270 4.244 4.233 4.232 4.232 4.232 4.232 4.232 4.232 4.232 4.232 4.232 4.232

TABLE 8.10 :  $f_1 (1/a - 10, 1/a - 11)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.327 1.327 1.328 1.330 1.333 1.337 1.343 1.350 1.358 1.366 1.374 1.382 1.390 1.398 1.406

(2) 3.967 3.913 3.869 3.832 3.804 3.784 3.771 3.764 3.761 3.760 3.760 3.760 3.760 3.760 3.760

TABLE 8.10 :  $f_1 (1/a - 11, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.077 1.077 1.078 1.080 1.083 1.087 1.092 1.097 1.103 1.109 1.116 1.123 1.130 1.137 1.144

(2) 3.523 3.523 3.523 3.523 3.523 3.523 3.523 3.523 3.523 3.523 3.523 3.523 3.523 3.523 3.523

TABLE 8.10 :  $f_1 (1/a - 11, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048

(2) 3.126 3.161 3.221 3.280 3.340 3.403 3.469 3.538 3.609 3.684 3.762 3.844 3.930 4.021 4.114

TABLE 8.10 :  $f_1 (1/a - 11, 1/a - 10)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048

(2) 4.873 4.910 4.947 4.984 5.021 5.058 5.095 5.132 5.169 5.206 5.243 5.280 5.317 5.354 5.391

TABLE 8.10 :  $f_1 (1/a - 11, 1/a - 11)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048 1.048

(2) 5.229 5.049 4.915 4.853 4.856 4.919 5.026 5.156 5.274 5.388 5.495 5.597 5.695 5.789 5.874

TABLE 8.10 :  $F_1 (1/a - 3, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.012 2.022 1.987 1.965 1.939 1.908 1.878 1.838 1.802 1.766 1.731 1.699 1.669 1.642 1.618

(2) 5.820 5.732 5.567 5.429 5.332 5.257 5.202 5.162 5.135 5.115 5.100 5.098 5.098 5.098 5.098

TABLE 8.10 :  $F_1 (1/a - 9, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.765 1.761 1.758 1.748 1.738 1.727 1.713 1.698 1.682 1.668 1.654 1.633 1.618 1.604 1.592

(2) 5.243 5.165 5.108 5.053 5.022 4.997 4.981 4.969 4.957 4.954 4.954 4.954 4.954 4.954 4.954

TABLE 8.10 :  $F_1 (1/a - 9, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.570 1.579 1.589 1.598 1.607 1.615 1.622 1.628 1.632 1.635 1.637 1.638 1.638 1.638 1.638

(2) 4.765 4.835 4.923 5.029 5.154 5.297 5.454 5.619 5.789 5.959 6.131 6.294 6.448 6.588 6.708

TABLE 8.10 :  $F_1 (1/a - 9, 1/a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.414 1.415 1.417 1.419 1.422 1.426 1.431 1.437 1.444 1.452 1.462 1.473 1.486 1.500 1.515

(2) 4.258 4.272 4.307 4.364 4.437 4.527 4.634 4.758 4.899 5.054 5.221 5.397 5.572 5.744 5.904

TABLE 8.10 :  $F_1 (1/a - 1.0, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.594 2.584 2.577 2.572 2.569 2.567 2.566 2.566 2.566 2.566 2.566 2.566 2.566 2.566 2.566

(2) 7.312 6.847 6.481 6.207 6.010 5.870 5.769 5.687 5.619 5.561 5.509 5.461 5.417 5.375 5.334

TABLE 8.10 :  $F_1 (1/a - 1.0, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.234 2.222 2.209 2.194 2.178 2.161 2.143 2.124 2.104 2.082 2.059 2.034 2.008 1.981 1.953

(2) 6.502 6.268 6.017 5.801 5.611 5.438 5.281 5.138 5.006 4.884 4.771 4.667 4.571 4.481 4.396

TABLE 8.10 :  $F_1 (1/a - 1.0, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.960 1.954 1.948 1.943 1.939 1.935 1.931 1.928 1.925 1.922 1.920 1.918 1.916 1.915 1.914

(2) 5.796 5.671 5.523 5.431 5.372 5.331 5.295 5.263 5.234 5.207 5.182 5.158 5.135 5.113 5.092

TABLE 8.10 :  $F_1 (1/a - 1.0, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.744 1.742 1.740 1.738 1.736 1.734 1.732 1.730 1.728 1.726 1.724 1.722 1.720 1.718 1.716

(2) 5.265 5.169 5.121 5.087 5.062 5.043 5.028 5.014 5.000 4.987 4.974 4.961 4.949 4.937 4.925

TABLE 8.11 :  $F_1 (a - 6, a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 3.407 3.471 3.517 3.550 3.568 3.573 3.575 3.576 3.576 3.576 3.576 3.576 3.576 3.576 3.576

(2) 7.705 7.551 7.485 7.427 7.372 7.318 7.265 7.214 7.162 7.111 7.060 7.010 6.960 6.910 6.860

TABLE 8.11 :  $F_1 (a - 6, a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 3.693 3.719 3.729 3.734 3.734 3.734 3.734 3.734 3.734 3.734 3.734 3.734 3.734 3.734 3.734

(2) 8.395 8.128 7.883 7.654 7.441 7.244 7.063 6.896 6.743 6.594 6.451 6.314 6.182 6.054 5.930

TABLE 8.11 :  $F_1 (a - 6, a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 3.939 3.932 3.913 3.896 3.874 3.851 3.828 3.804 3.779 3.753 3.726 3.698 3.669 3.639 3.609

(2) 9.027 8.697 8.365 8.032 7.699 7.366 7.033 6.700 6.367 6.034 5.701 5.368 5.035 4.702 4.369

TABLE 8.11 :  $F_1 (a - 7, a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.312 2.413 2.512 2.606 2.686 2.744 2.788 2.752 2.709 2.642 2.564 2.484 2.407 2.332 2.262

(2) 5.296 5.470 5.724 6.058 6.472 6.964 7.532 8.176 8.894 9.684 10.538 11.454 12.424 13.444 14.504

TABLE 8.11 :  $F_1 (a - 7, a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.635 2.720 2.795 2.856 2.896 2.909 2.884 2.851 2.808 2.714 2.634 2.556 2.483 2.419 2.366

(2) 5.998 6.033 5.942 5.753 5.559 5.409 5.287 5.188 5.108 5.043 5.000 5.000 5.000 5.000 5.000

TABLE 8.11 :  $F_1 (a - 7, a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.920 2.977 3.021 3.048 3.058 3.060 3.062 3.066 3.070 3.074 3.078 3.082 3.086 3.090 3.094

(2) 6.631 6.588 6.434 6.215 6.035 5.882 5.770 5.733 5.765 5.837 5.931 6.041 6.164 6.298 6.432

TABLE 8.11 :  $F_1 (a - 7, a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 3.166 3.191 3.206 3.207 3.192 3.161 3.113 3.050 2.976 2.895 2.812 2.730 2.651 2.578 2.512

(2) 7.222 7.128 6.931 6.676 6.429 6.235 6.110 6.048 6.034 6.046 6.057 6.072 6.072 6.072 6.072

TABLE 8.11 :  $F_1 (a - 7, a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 3.218 3.375 3.567 3.782 3.976 4.141 4.287 4.413 4.508 4.562 4.584 4.574 4.541 4.484 4.414

(2) 7.801 7.670 7.423 7.105 6.796 6.518 6.378 6.322 6.243 6.237 6.232 6.201 6.128 6.017 5.884

TABLE 8.11 :  $F_1 (a - 8, a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.023 2.111 2.199 2.282 2.356 2.415 2.453 2.465 2.454 2.426 2.381 2.326 2.266 2.205 2.144

(2) 4.838 4.755 4.605 4.470 4.361 4.284 4.228 4.184 4.151 4.128 4.114 4.101 4.088 4.075 4.062

TABLE 8.11 :  $F_1 (a - 8, a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.306 2.380 2.447 2.503 2.546 2.569 2.574 2.562 2.535 2.499 2.457 2.414 2.373 2.337 2.300

(2) 5.354 5.307 5.286 5.207 5.113 5.046 5.028 5.064 5.145 5.253 5.357 5.425 5.451 5.412 5.362

TABLE 8.11 :  $F_1 (a - 8, a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.306 2.380 2.447 2.503 2.546 2.569 2.574 2.562 2.535 2.499 2.457 2.414 2.373 2.337 2.300

(2) 5.354 5.307 5.286 5.207 5.113 5.046 5.028 5.064 5.145 5.253 5.357 5.425 5.451 5.412 5.362

TABLE 8.11 :  $F_1 (a - 8, a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.306 2.380 2.447 2.503 2.546 2.569 2.574 2.562 2.535 2.499 2.457 2.414 2.373 2.337 2.300

(2) 5.354 5.307 5.286 5.207 5.113 5.046 5.028 5.064 5.145 5.253 5.357 5.425 5.451 5.412 5.362

TABLE 8.11 :  $F_1$  ( $\alpha = 1.0, \alpha = 0$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.045 2.085 2.118 2.144 2.162 2.173 2.179 2.181 2.181 2.181 2.181 2.182 2.182 2.182 2.182  
 (2) 4.659 4.679 4.682 4.685 4.704 4.716 4.715 4.715 4.715 4.715 4.715 4.715 4.715 4.715 4.715

TABLE 8.11 :  $F_1$  ( $\alpha = 1.0, \alpha = 9$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.217 2.236 2.250 2.259 2.265 2.268 2.269 2.269 2.270 2.271 2.271 2.271 2.271 2.271 2.271  
 (2) 5.081 5.086 5.086 5.093 5.112 5.144 5.187 5.235 5.314 5.335 5.342 5.343 5.343 5.343 5.343

TABLE 8.11 :  $F_1$  ( $1/2 = 0.5, 1/2 = 0.6$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.895 1.889 1.891 1.894 1.893 1.889 1.888 1.888 1.885 1.885 1.884 1.884 1.884 1.884 1.884  
 (2) 5.025 4.977 4.905 4.845 4.830 4.874 4.984 5.160 5.394 5.658 5.883 5.966 5.889 5.725 5.539

TABLE 8.11 :  $F_1$  ( $1/2 = 0.5, 1/2 = 0.7$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.726 1.713 1.710 1.713 1.718 1.723 1.728 1.731 1.734 1.738 1.745 1.755 1.773 1.803 1.850  
 (2) 4.411 4.418 4.416 4.419 4.442 4.469 4.539 4.747 4.943 5.175 5.407 5.569 5.605 5.536 5.419

TABLE 8.11 :  $F_1$  ( $1/2 = 0.6, 1/2 = 0.8$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.604 1.590 1.585 1.585 1.589 1.595 1.604 1.615 1.627 1.642 1.660 1.683 1.714 1.758 1.817  
 (2) 3.844 3.964 3.991 4.029 4.084 4.164 4.276 4.424 4.611 4.828 5.054 5.240 5.333 5.331 5.276

TABLE 8.11 :  $F_1$  ( $1/2 = 0.6, 1/2 = 0.9$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.506 1.497 1.493 1.493 1.495 1.501 1.509 1.521 1.536 1.555 1.579 1.610 1.650 1.703 1.775  
 (2) 3.586 3.604 3.637 3.687 3.759 3.865 3.981 4.140 4.332 4.554 4.785 4.985 5.106 5.140 5.125

TABLE 8.11 :  $F_1$  ( $1/2 = 0.6, 1/2 = 1.0$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.419 1.419 1.420 1.422 1.428 1.430 1.437 1.448 1.463 1.482 1.508 1.540 1.587 1.647 1.728  
 (2) 3.305 3.319 3.348 3.398 3.472 3.575 3.708 3.895 4.078 4.312 4.560 4.782 4.934 4.977 4.963

TABLE 8.11 :  $F_1$  ( $1/2 = 0.7, 1/2 = 0.6$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.211 2.204 2.205 2.208 2.206 2.198 2.184 2.165 2.142 2.120 2.100 2.085 2.077 2.081 2.100  
 (2) 5.858 5.791 5.685 5.586 5.531 5.536 5.607 5.734 5.898 5.056 6.144 6.115 5.990 5.819 5.641

TABLE 8.11 :  $F_1$  ( $1/2 = 0.7, 1/2 = 0.7$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.013 1.998 1.995 1.998 2.003 2.007 2.010 2.012 2.012 2.011 2.012 2.015 2.025 2.043 2.074  
 (2) 5.144 5.145 5.131 5.112 5.109 5.130 5.200 5.304 5.439 5.585 5.704 5.755 5.726 5.637 5.525

TABLE 8.11 :  $F_1$  ( $1/2 = 0.7, 1/2 = 0.8$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.871 1.855 1.849 1.849 1.853 1.859 1.868 1.878 1.889 1.901 1.917 1.936 1.961 1.994 2.040  
 (2) 4.800 4.619 4.643 4.672 4.714 4.776 4.863 4.975 5.108 5.249 5.373 5.451 5.469 5.439 5.385

TABLE 8.11 :  $F_1$  ( $\alpha = 0.8, \alpha = 0$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.558 2.626 2.666 2.674 2.669 2.678 2.655 2.622 2.584 2.542 2.500 2.459 2.422 2.391  
 (2) 5.213 5.811 5.752 5.657 5.560 5.488 5.451 5.452 5.481 5.525 5.565 5.585 5.577 5.543 5.497

TABLE 8.11 :  $F_1$  ( $\alpha = 0.8, \alpha = 9$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.771 2.794 2.809 2.816 2.814 2.803 2.783 2.755 2.720 2.680 2.638 2.590 2.545 2.500 2.459  
 (2) 6.337 6.354 6.324 6.274 6.210 5.990 5.882 5.801 5.752 5.732 5.732 5.740 5.742 5.726 5.689 5.636

TABLE 8.11 :  $F_1$  ( $\alpha = 0.8, \alpha = 1.0$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.956 2.935 2.932 2.946 2.935 2.919 2.896 2.865 2.826 2.781 2.731 2.677 2.622 2.567 2.512  
 (2) 6.851 6.830 6.808 6.846 6.875 6.911 6.959 7.007 7.035 7.015 6.912 6.928 6.887 6.808 6.766

TABLE 8.11 :  $F_1$  ( $\alpha = 0.9, \alpha = 0.6$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.760 1.877 1.955 2.029 2.097 2.154 2.195 2.218 2.225 2.219 2.205 2.187 2.169 2.156 2.152  
 (2) 4.128 4.232 4.284 4.379 4.477 4.584 4.694 4.814 4.944 5.084 5.234 5.394 5.564 5.744 5.929

TABLE 8.11 :  $F_1$  ( $\alpha = 0.9, \alpha = 0.7$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.050 2.116 2.176 2.232 2.277 2.305 2.311 2.305 2.282 2.276 2.260 2.246 2.238 2.238  
 (2) 4.673 4.730 4.738 4.714 4.665 4.679 4.710 4.781 4.965 5.066 5.119 5.197 5.230 5.228 5.215

TABLE 8.11 :  $F_1$  ( $\alpha = 0.9, \alpha = 0.8$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.272 2.316 2.353 2.380 2.398 2.426 2.452 2.470 2.478 2.474 2.459 2.445 2.433 2.424 2.420  
 (2) 5.172 5.187 5.170 5.140 5.117 5.114 5.135 5.178 5.234 5.291 5.336 5.361 5.366 5.359 5.352

TABLE 8.11 :  $F_1$  ( $\alpha = 0.9, \alpha = 0.9$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.463 2.484 2.493 2.500 2.511 2.510 2.504 2.494 2.482 2.467 2.452 2.436 2.420 2.404 2.390  
 (2) 5.641 5.634 5.628 5.675 5.741 5.815 5.896 5.993 6.103 6.224 6.350 6.484 6.624 6.768 6.916

TABLE 8.11 :  $F_1$  ( $\alpha = 0.9, \alpha = 1.0$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.678 2.677 2.676 2.674 2.671 2.616 2.608 2.597 2.583 2.566 2.546 2.524 2.499 2.473 2.446  
 (2) 6.102 6.096 6.053 6.000 5.933 5.851 5.793 5.739 5.702 5.682 5.676 5.675 5.672 5.659 5.632

TABLE 8.11 :  $F_1$  ( $\alpha = 1.0, \alpha = 0.6$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.610 1.659 1.759 1.827 1.889 1.942 1.983 2.010 2.026 2.033 2.035 2.037 2.046 2.066  
 (2) 3.710 3.812 3.876 3.900 3.922 3.911 3.953 4.041 4.182 4.371 4.588 4.783 4.911 4.957 4.961

TABLE 8.11 :  $F_1$  ( $\alpha = 1.0, \alpha = 0.7$ )  
 MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.845 1.924 1.958 2.005 2.043 2.071 2.089 2.098 2.102 2.103 2.103 2.105 2.113 2.127 2.153  
 (2) 4.527 4.583 4.625 4.687 4.764 4.856 4.961 5.084 5.224 5.376 5.542 5.726 5.922 6.133 6.366

TABLE B.11 :  $f_1 (1/a - 7, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.559 2.245 2.238 2.236 2.236 2.240 2.244 2.250 2.257 2.264 2.272 2.280 2.289 2.300 2.312  
(2) 5.372 5.385 5.396 5.410 5.424 5.439 5.454 5.472 5.491 5.508 5.520 5.528 5.525 5.516 5.503

TABLE B.11 :  $f_1 (1/a - 9, 1/a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.129 2.129 2.129 2.130 2.132 2.135 2.139 2.145 2.153 2.164 2.177 2.194 2.213 2.236 2.263  
(2) 4.952 4.960 4.978 5.007 5.048 5.100 5.159 5.220 5.274 5.314 5.338 5.348 5.349 5.351 5.352

TABLE B.11 :  $f_1 (1/a - 1.0, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.156 3.146 3.145 3.142 3.128 3.098 3.052 2.993 2.924 2.850 2.774 2.701 2.631 2.566 2.507  
(2) 8.324 8.116 7.772 7.404 7.102 6.895 6.760 6.737 6.731 6.721 6.665 6.546 6.378 6.184 5.986

TABLE B.11 :  $f_1 (1/a - 1.0, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.805 2.853 2.846 2.845 2.844 2.838 2.823 2.799 2.766 2.725 2.679 2.630 2.580 2.531 2.484  
(2) 7.333 7.262 7.115 6.975 6.866 6.801 6.809 6.824 6.864 6.920 6.995 7.043 7.028 6.932 6.847 8.772

TABLE B.11 :  $f_1 (1/a - 1.0, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.673 2.649 2.638 2.634 2.633 2.630 2.623 2.611 2.594 2.571 2.544 2.515 2.484 2.452  
(2) 6.555 6.543 6.495 6.403 6.248 6.162 6.055 5.995 5.943 5.928 5.926 5.922 5.898 5.847 5.772

TABLE B.11 :  $f_1 (1/a - 1.0, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.509 2.484 2.468 2.462 2.460 2.460 2.460 2.479 2.476 2.471 2.463 2.453 2.441 2.427 2.412  
(2) 5.963 5.962 5.952 5.925 5.862 5.800 5.777 5.733 5.703 5.687 5.682 5.682 5.678 5.664 5.636

TABLE B.12 :  $f_1 (a - 6, a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.762 2.925 3.110 3.319 3.559 3.835 4.153 4.517 4.889 5.027 4.808 4.521 4.252 4.015 3.814  
(2) 6.421 6.801 7.227 7.698 8.129 8.554 7.718 6.493 6.055 6.059 6.052 5.811 5.504 5.262 5.043

TABLE B.12 :  $f_1 (a - 6, a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.202 3.384 3.568 3.819 4.021 4.378 4.711 5.058 5.271 5.149 4.865 4.574 4.310 4.082 3.893  
(2) 7.448 7.871 8.334 8.794 9.261 9.723 8.818 7.283 6.718 6.490 6.816 7.523 8.320 9.200 7.500 7.493

TABLE B.12 :  $f_1 (a - 6, a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.627 3.821 4.037 4.281 4.554 4.858 5.178 5.444 5.469 5.236 4.934 4.644 4.306 4.166 3.988  
(2) 8.442 8.893 9.365 9.801 10.146 10.245 9.732 7.055 6.520 6.219 6.641 8.378 7.959 7.560

TABLE B.12 :  $f_1 (a - 6, a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 4.031 4.228 4.450 4.697 4.974 5.268 5.552 5.712 5.607 5.328 5.018 4.729 4.476 4.282 4.099  
(2) 9.398 9.862 10.300 10.311 9.418 8.486 7.795 7.459 7.606 8.117 8.708 8.811 8.456 8.032 7.641

TABLE B.11 :  $f_1 (1/a - 7, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 1.757 1.747 1.742 1.741 1.744 1.749 1.758 1.769 1.784 1.802 1.825 1.854 1.890 1.936 1.996  
(2) 4.183 4.201 4.214 4.226 4.249 4.288 4.350 4.464 4.635 4.869 5.123 5.214 5.252 5.251 5.235

TABLE B.11 :  $f_1 (1/a - 7, 1/a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 1.656 1.656 1.657 1.659 1.662 1.667 1.674 1.685 1.700 1.719 1.745 1.778 1.821 1.876 1.947  
(2) 3.654 3.653 3.659 3.659 3.654 3.654 3.654 3.654 3.654 3.654 3.654 3.654 3.654 3.654 3.654

TABLE B.11 :  $f_1 (1/a - 8, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.577 2.518 2.520 2.521 2.517 2.505 2.484 2.455 2.422 2.387 2.354 2.323 2.289 2.260 2.237  
(2) 6.687 6.591 6.434 6.272 6.152 6.096 6.104 6.155 6.251 6.321 6.327 6.249 6.107 5.933 5.758

TABLE B.11 :  $f_1 (1/a - 8, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.351 2.383 2.379 2.382 2.382 2.389 2.389 2.389 2.389 2.389 2.389 2.389 2.389 2.389 2.389  
(2) 5.874 5.865 5.825 5.769 5.716 5.668 5.693 5.731 5.794 5.862 5.908 5.906 5.852 5.759 5.649

TABLE B.11 :  $f_1 (1/a - 8, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.138 2.122 2.112 2.112 2.115 2.121 2.128 2.135 2.142 2.149 2.157 2.167 2.179 2.195 2.218  
(2) 5.264 5.263 5.221 5.200 5.202 5.203 5.203 5.203 5.203 5.203 5.203 5.203 5.203 5.203 5.203

TABLE B.11 :  $f_1 (1/a - 8, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.507 1.995 1.990 1.989 1.991 1.995 2.073 2.073 2.073 2.073 2.073 2.073 2.073 2.073 2.073  
(2) 4.778 4.794 4.822 4.859 4.929 4.971 5.046 5.129 5.214 5.290 5.346 5.377 5.384 5.376 5.365

TABLE B.11 :  $f_1 (1/a - 8, 1/a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 1.672 1.672 1.673 1.675 1.678 1.678 1.678 1.678 1.678 1.678 1.678 1.678 1.678 1.678 1.678  
(2) 4.424 4.417 4.443 4.468 4.493 4.519 4.543 4.568 4.593 4.619 4.643 4.668 4.693 4.718 4.743

TABLE B.11 :  $f_1 (1/a - 9, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.642 2.632 2.633 2.633 2.625 2.625 2.625 2.625 2.625 2.625 2.625 2.625 2.625 2.625 2.625  
(2) 7.510 7.365 7.135 6.882 6.677 6.545 6.466 6.464 6.512 6.528 6.493 6.392 6.238 6.059 5.878

TABLE B.11 :  $f_1 (1/a - 9, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.568 2.568 2.563 2.564 2.566 2.566 2.566 2.566 2.566 2.566 2.566 2.566 2.566 2.566 2.566  
(2) 6.001 6.001 6.001 6.001 6.001 6.001 6.001 6.001 6.001 6.001 6.001 6.001 6.001 6.001 6.001

TABLE B.11 :  $f_1 (1/a - 9, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.405 2.384 2.375 2.376 2.379 2.383 2.385 2.384 2.382 2.377 2.371 2.364 2.358 2.353  
(2) 5.906 5.812 5.901 5.872 5.829 5.788 5.753 5.737 5.716 5.749 5.759 5.761 5.743 5.703 5.607

TABLE 8.12 :  $f_1 (s = -8, a = -1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.107 3.453 3.619 3.808 4.024 4.268 4.540 4.824 5.061 5.129 4.998 4.781 4.558 4.356 4.165  
(2) 7.741 8.118 8.531 8.925 9.022 8.478 7.792 7.239 6.917 6.958 7.388 8.002 8.273 8.049 7.713

TABLE 8.12 :  $f_1 (s = -9, a = -6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 1.841 1.950 2.073 2.213 2.373 2.558 2.775 3.030 3.335 3.697 4.089 4.500 4.189 3.995 3.808  
(2) 4.381 4.539 4.822 5.147 5.515 5.926 6.319 6.783 6.868 5.477 5.218 5.355 6.101 7.194 7.710

TABLE 8.12 :  $f_1 (s = -9, a = -7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.135 2.256 2.392 2.547 2.723 2.925 3.150 3.435 3.753 4.105 4.394 4.417 4.257 4.064 3.888  
(2) 4.866 5.250 5.569 5.928 6.325 6.724 6.862 6.490 6.043 6.697 5.592 5.975 6.805 7.560 7.615

TABLE 8.12 :  $f_1 (s = -9, a = -8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.418 2.547 2.692 2.855 3.041 3.255 3.500 3.782 4.098 4.405 4.582 4.517 4.340 4.151 3.984  
(2) 5.629 5.937 6.281 6.663 7.055 7.364 7.182 6.699 6.259 5.977 6.021 6.523 7.321 7.777 7.512

TABLE 8.12 :  $f_1 (s = -9, a = -9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.687 2.819 2.968 3.135 3.326 3.545 3.795 4.078 4.382 4.645 4.733 4.623 4.438 4.249 4.066  
(2) 6.268 6.593 6.956 7.353 7.734 7.854 7.469 6.921 6.495 6.275 6.423 6.975 7.679 7.854 7.601

TABLE 8.12 :  $f_1 (s = -9, a = -1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.939 3.070 3.218 3.386 3.579 3.801 4.054 4.337 4.628 4.849 4.878 4.738 4.453 4.351 4.183  
(2) 6.682 7.220 7.601 8.007 8.342 8.247 7.698 7.140 6.723 6.554 6.774 7.350 7.947 7.976 7.698

TABLE 8.12 :  $f_1 (s = -1.0, a = -6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 1.657 1.755 1.866 1.992 2.136 2.303 2.498 2.729 3.005 3.339 3.732 4.092 4.135 3.900 3.803  
(2) 3.853 4.022 4.340 4.633 4.967 5.346 5.759 6.041 5.812 5.441 5.140 5.064 5.574 6.637 7.298

TABLE 8.12 :  $f_1 (s = -1.0, a = -7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 1.921 2.031 2.153 2.292 2.451 2.633 2.846 3.095 3.393 3.730 4.085 4.284 4.216 4.051 3.884  
(2) 4.469 4.725 5.013 5.339 5.704 6.105 6.444 6.374 5.993 5.632 5.412 5.562 6.228 7.194 7.387

TABLE 8.12 :  $f_1 (s = -1.0, a = -8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.176 2.292 2.423 2.570 2.738 2.930 3.153 3.412 3.711 4.041 4.330 4.417 4.307 4.140 3.981  
(2) 5.067 5.344 5.655 6.005 6.389 6.769 6.923 6.616 6.198 5.864 5.738 6.028 6.752 7.495 7.475

TABLE 8.12 :  $f_1 (s = -1.0, a = -9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.418 2.537 2.671 2.822 2.995 3.193 3.421 3.665 3.944 4.295 4.523 4.542 4.410 4.241 4.084  
(2) 5.842 5.935 6.266 6.695 7.027 7.349 7.278 6.868 6.414 6.107 6.059 6.433 7.158 7.690 7.572

TABLE 8.12 :  $f_1 (s = -6, a = -1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 4.459 4.624 4.823 5.059 5.338 5.618 5.855 5.919 5.735 5.431 5.115 4.827 4.575 4.361 4.186  
(2) 10.31 10.78 11.14 10.77 9.692 8.748 8.102 7.868 8.095 8.603 9.039 9.441 8.543 8.116 7.729

TABLE 8.12 :  $f_1 (s = -7, a = -6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.367 2.508 2.662 2.845 3.051 3.288 3.564 3.887 4.257 4.617 4.694 4.488 4.241 4.011 3.812  
(2) 5.524 6.090 6.198 6.610 7.058 7.401 7.026 6.445 5.953 5.654 5.847 6.577 7.575 7.768 7.410

TABLE 8.12 :  $f_1 (s = -7, a = -7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.745 2.901 3.076 3.274 3.499 3.758 4.051 4.386 4.724 4.909 4.789 4.546 4.300 4.078 3.892  
(2) 6.304 6.748 7.154 7.594 7.824 7.210 6.621 6.203 6.132 6.584 7.381 8.039 7.854 7.482

TABLE 8.12 :  $f_1 (s = -7, a = -8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.123 3.275 3.441 3.670 3.928 4.176 4.476 4.792 5.047 5.068 4.873 4.620 4.377 4.163 3.988  
(2) 7.237 7.639 8.058 8.488 8.642 8.115 7.415 6.858 6.553 6.681 7.233 7.965 8.220 7.922 7.551

TABLE 8.12 :  $f_1 (s = -7, a = -9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.455 3.635 3.815 4.023 4.271 4.541 4.833 5.114 5.274 5.195 4.966 4.709 4.469 4.260 4.089  
(2) 8.058 8.457 8.924 9.271 9.110 8.364 7.651 7.138 6.944 7.169 7.763 8.340 8.340 8.001 7.633

TABLE 8.12 :  $f_1 (s = -7, a = -1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.779 3.945 4.134 4.352 4.591 4.859 5.138 5.379 5.460 5.32 5.071 4.809 4.568 4.359 4.185  
(2) 8.624 9.266 9.637 9.945 9.457 8.618 7.902 7.427 7.31 7.628 8.167 8.593 8.432 8.091 7.723

TABLE 8.12 :  $f_1 (s = -8, a = -6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.071 2.194 2.332 2.493 2.670 2.870 3.121 3.407 3.744 4.127 4.446 4.424 4.221 4.053 3.890  
(2) 4.816 5.102 5.425 5.789 6.195 6.626 6.812 6.390 5.968 5.529 5.405 6.051 6.775 7.613 7.409

TABLE 8.12 :  $f_1 (s = -8, a = -7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.462 2.538 2.631 2.865 3.053 3.291 3.552 3.856 4.186 4.522 4.641 4.459 4.281 4.072 3.890  
(2) 5.566 5.926 6.264 6.662 7.084 7.366 7.039 6.560 6.100 5.824 5.954 6.580 7.468 7.740 7.464

TABLE 8.12 :  $f_1 (s = -8, a = -8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.720 2.866 3.028 3.212 3.421 3.659 3.931 4.235 4.551 4.778 4.769 4.591 4.362 4.158 3.986  
(2) 6.333 6.678 7.043 7.476 7.856 7.854 7.324 6.768 6.353 6.207 6.505 7.180 7.868 7.854 7.535

TABLE 8.12 :  $f_1 (s = -8, a = -9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.023 3.172 3.338 3.527 3.740 3.983 4.246 4.552 4.828 4.965 4.878 4.676 4.456 4.256 4.087  
(2) 7.051 7.414 7.815 8.227 8.496 8.187 7.557 7.003 6.637 6.600 6.984 7.663 8.100 7.948 7.660

TABLE 8.12 :  $\epsilon_1 (1/a - 6, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 3.019 3.221 3.377 3.570 3.805 4.088 4.427 4.822 5.236 5.647 6.031 6.378 6.651 6.895 7.098  
(2) 7.675 8.074 8.544 9.072 9.668 10.318 10.991 11.732 12.527 13.363 14.229 15.116 16.014 16.924 17.838

TABLE 8.12 :  $\epsilon_1 (1/a - 8, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.744 2.822 2.927 3.060 3.224 3.423 3.663 3.951 4.293 4.677 5.099 5.562 6.047 6.554 7.084  
(2) 6.714 7.044 7.438 7.887 8.387 8.938 9.542 10.201 10.918 11.695 12.534 13.437 14.406 15.444 16.544

TABLE 8.12 :  $\epsilon_1 (1/a - 8, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.510 2.593 2.696 2.820 2.969 3.148 3.362 3.618 3.923 4.274 4.673 5.112 5.589 6.096 6.634  
(2) 5.937 6.263 6.623 7.016 7.451 7.957 8.537 9.194 9.839 10.574 11.391 12.284 13.256 14.308 15.444

TABLE 8.12 :  $\epsilon_1 (1/a - 8, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.302 2.392 2.496 2.619 2.762 2.930 3.130 3.367 3.648 3.976 4.321 4.694 5.103 5.548 6.029  
(2) 5.427 5.689 5.996 6.349 6.745 7.152 7.579 8.037 8.536 9.076 9.657 10.280 10.945 11.654 12.409

TABLE 8.12 :  $\epsilon_1 (1/a - 8, 1/a - 10)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.116 2.210 2.317 2.439 2.580 2.743 2.935 3.161 3.429 3.742 4.083 4.457 4.864 5.304 5.777  
(2) 4.956 5.203 5.487 5.813 6.181 6.575 6.996 7.476 7.996 8.556 9.157 9.799 10.484 11.214 12.000

TABLE 8.12 :  $\epsilon_1 (1/a - 9, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 3.397 3.469 3.624 3.798 4.014 4.274 4.586 4.949 5.343 5.768 6.226 6.721 7.247 7.804 8.394  
(2) 6.632 6.916 7.239 7.603 8.006 8.449 8.934 9.461 10.031 10.644 11.301 12.004 12.754 13.548 14.388

TABLE 8.12 :  $\epsilon_1 (1/a - 9, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 3.087 3.175 3.293 3.441 3.624 3.846 4.110 4.422 4.775 5.115 5.522 5.917 6.399 6.968 7.534  
(2) 7.552 7.920 8.353 8.833 9.253 9.765 10.319 10.916 11.556 12.239 12.966 13.738 14.556 15.421 16.334

TABLE 8.12 :  $\epsilon_1 (1/a - 9, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.824 2.918 3.033 3.172 3.339 3.538 3.775 4.053 4.373 4.706 5.064 5.454 5.874 6.324 6.804  
(2) 6.745 7.066 7.443 7.868 8.339 8.854 9.416 10.024 10.680 11.384 12.138 12.944 13.804 14.718 15.688

TABLE 8.12 :  $\epsilon_1 (1/a - 9, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.590 2.691 2.800 2.946 3.106 3.294 3.516 3.775 4.075 4.397 4.754 5.144 5.568 6.028 6.524  
(2) 6.105 6.399 6.739 7.126 7.556 7.984 8.464 8.994 9.574 10.204 10.884 11.614 12.394 13.224 14.104

TABLE 8.12 :  $\epsilon_1 (1/a - 9, 1/a - 10)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.381 2.466 2.607 2.744 2.901 3.084 3.298 3.547 3.835 4.154 4.442 4.781 5.151 5.564 6.014  
(2) 5.575 5.852 6.170 6.529 6.921 7.364 7.854 8.394 8.984 9.624 10.314 11.054 11.844 12.684 13.574

TABLE 8.12 :  $\epsilon_1 (1/a - 6, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.264 2.326 2.417 2.534 2.680 2.858 3.076 3.341 3.657 4.019 4.546 5.178 5.916 6.764 7.732  
(2) 7.577 8.059 8.642 9.324 10.114 11.024 12.064 13.244 14.574 16.064 17.724 19.574 21.624 23.974 26.624

TABLE 8.12 :  $\epsilon_1 (1/a - 6, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.058 2.117 2.198 2.296 2.419 2.570 2.754 2.979 3.256 3.589 4.020 4.488 5.004 5.564 6.168  
(2) 5.038 5.269 5.568 5.940 6.390 6.916 7.520 8.212 8.992 9.872 10.852 11.932 13.112 14.402 15.802

TABLE 8.12 :  $\epsilon_1 (1/a - 6, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.893 1.945 2.022 2.116 2.220 2.343 2.527 2.725 2.970 3.272 3.646 4.098 4.598 5.144 5.734  
(2) 4.498 4.714 4.974 5.279 5.636 6.048 6.522 7.068 7.686 8.354 9.084 9.874 10.724 11.634 12.604

TABLE 8.12 :  $\epsilon_1 (1/a - 6, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.726 1.774 1.872 1.965 2.072 2.200 2.351 2.534 2.757 3.032 3.373 3.785 4.269 4.814 5.424  
(2) 4.071 4.269 4.501 4.773 5.092 5.457 5.873 6.343 6.864 7.444 8.084 8.794 9.574 10.424 11.344

TABLE 8.12 :  $\epsilon_1 (1/a - 6, 1/a - 10)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.587 1.629 1.718 1.820 1.935 2.059 2.204 2.377 2.589 2.844 3.163 3.552 3.983 4.454 4.964  
(2) 3.717 3.873 4.118 4.367 4.657 4.992 5.375 5.784 6.220 6.704 7.234 7.814 8.444 9.124 9.854

TABLE 8.12 :  $\epsilon_1 (1/a - 7, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.642 2.714 2.819 2.945 3.125 3.333 3.584 3.889 4.256 4.669 5.117 5.618 6.166 6.764 7.414  
(2) 6.716 6.958 7.246 7.584 8.004 8.564 9.174 9.834 10.544 11.314 12.144 13.034 14.084 15.204 16.384

TABLE 8.12 :  $\epsilon_1 (1/a - 7, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.451 2.469 2.551 2.670 2.822 2.997 3.210 3.469 3.783 4.160 4.582 4.986 5.424 5.894 6.404  
(2) 5.875 6.105 6.373 6.679 7.079 7.514 8.011 8.564 9.164 9.814 10.514 11.264 12.014 12.864 13.714

TABLE 8.12 :  $\epsilon_1 (1/a - 7, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.214 2.253 2.345 2.464 2.592 2.740 2.918 3.136 3.394 3.694 4.036 4.418 4.844 5.314 5.824  
(2) 4.749 4.979 5.249 5.563 5.915 6.310 6.733 7.194 7.704 8.264 8.874 9.534 10.244 11.004 11.814

TABLE 8.12 :  $\epsilon_1 (1/a - 7, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 2.014 2.053 2.164 2.292 2.417 2.558 2.741 2.953 3.207 3.516 3.879 4.290 4.744 5.244 5.784  
(2) 4.449 4.679 4.949 5.263 5.615 6.010 6.454 6.944 7.484 8.074 8.714 9.404 10.144 10.944 11.804

TABLE 8.12 :  $\epsilon_1 (1/a - 7, 1/a - 10)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

(1) 1.852 1.914 2.027 2.134 2.258 2.401 2.570 2.770 3.011 3.302 3.647 4.018 4.418 4.844 5.294  
(2) 4.336 4.553 4.803 5.092 5.424 5.799 6.197 6.681 7.204 7.764 8.364 8.994 9.674 10.404 11.184

TABLE 8.12 :  $f_1 (1/a - 1.0, 1/a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.754 3.871 4.635 5.210 5.645 5.933 6.114 6.208 6.228 6.162 6.005 5.764 5.461 5.045 4.524  
 (2) 9.569 10.60 10.63 11.09 10.61 9.513 8.576 7.857 7.412 7.162 6.955 6.834 6.864 6.979 6.151

TABLE 8.12 :  $f_1 (1/a - 1.0, 1/a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 3.435 3.573 3.659 3.722 3.764 3.785 3.788 3.772 3.738 3.685 3.608 3.512 3.400 3.272 3.132  
 (2) 8.393 8.753 9.253 9.737 9.922 9.216 8.301 7.693 7.207 7.048 7.104 7.304 7.681 8.158 8.407 8.642

TABLE 8.12 :  $f_1 (1/a - 1.0, 1/a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 3.138 3.242 3.293 3.324 3.328 3.326 3.316 3.296 3.268 3.226 3.174 3.114 3.044 2.968 2.882 2.788  
 (2) 7.654 7.647 7.256 6.697 6.029 5.314 4.699 4.182 3.762 3.452 3.228 3.082 2.996 2.968 2.996 2.976

TABLE 8.12 :  $f_1 (1/a - 1.0, 1/a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.877 2.933 2.972 2.997 3.010 3.013 3.007 3.000 2.984 2.958 2.924 2.882 2.834 2.782 2.726  
 (2) 6.283 6.127 5.779 5.269 4.657 4.045 3.433 2.821 2.209 1.597 1.000 0.412 0.000 0.000 0.000 0.000

TABLE 8.13 :  $f_1 (1/a - 6, a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.767 2.933 3.127 3.314 3.478 3.614 3.712 3.782 3.825 3.842 3.835 3.805 3.756 3.692 3.618 3.534  
 (2) 6.432 6.819 7.225 7.647 8.098 8.671 9.274 9.914 10.591 11.316 12.083 12.943 13.940 15.080 16.370 17.804

TABLE 8.13 :  $f_1 (1/a - 6, a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 3.213 3.420 3.610 3.780 3.924 4.034 4.108 4.148 4.162 4.152 4.128 4.092 4.044 3.984 3.914 3.834  
 (2) 7.471 7.679 7.925 8.219 8.559 8.949 9.391 9.898 10.481 11.149 11.904 12.754 13.704 14.764 15.944 17.264

TABLE 8.13 :  $f_1 (1/a - 6, a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 3.647 3.849 4.035 4.201 4.342 4.453 4.528 4.574 4.598 4.598 4.574 4.534 4.480 4.418 4.352 4.282  
 (2) 8.464 8.758 9.143 9.614 10.174 10.834 11.604 12.494 13.514 14.684 15.934 17.284 18.744 20.324 22.044 23.924

TABLE 8.13 :  $f_1 (1/a - 6, a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 4.224 4.474 4.716 4.934 5.124 5.284 5.414 5.504 5.554 5.574 5.564 5.534 5.484 5.424 5.354 5.284  
 (2) 9.424 9.854 10.374 10.994 11.714 12.544 13.494 14.574 15.784 17.144 18.664 20.364 22.264 24.384 26.764 29.444

TABLE 8.13 :  $f_1 (1/a - 6, a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 4.454 4.674 4.874 5.054 5.214 5.354 5.474 5.574 5.654 5.714 5.754 5.774 5.784 5.784 5.774 5.754  
 (2) 10.41 10.63 11.48 11.91 12.42 12.47 12.51 12.51 12.51 12.51 12.51 12.51 12.51 12.51 12.51 12.51

TABLE 8.13 :  $f_1 (1/a - 7, a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.322 2.514 2.675 2.807 2.897 2.954 2.994 3.024 3.044 3.054 3.054 3.054 3.054 3.054 3.054 3.054  
 (2) 5.513 5.645 5.645 5.645 5.645 5.645 5.645 5.645 5.645 5.645 5.645 5.645 5.645 5.645 5.645 5.645

TABLE 8.13 :  $f_1 (1/a - 7, a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.754 3.914 4.694 5.332 5.801 6.114 6.288 6.368 6.408 6.408 6.368 6.288 6.114 5.801 5.332 4.694  
 (2) 6.408 6.778 7.198 7.670 8.193 8.762 9.386 10.057 10.781 11.554 12.386 13.274 14.214 15.204 16.244 17.334

TABLE 8.13 :  $f_1 (1/a - 7, a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 3.126 3.299 3.493 3.713 3.953 4.219 4.500 4.800 5.126 5.478 5.854 6.254 6.684 7.144 7.634 8.154  
 (2) 7.272 7.680 8.136 8.644 9.204 9.814 10.484 11.214 12.004 12.854 13.764 14.734 15.764 16.854 18.004 19.214

TABLE 8.13 :  $f_1 (1/a - 7, a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 3.483 3.664 3.866 4.094 4.354 4.651 4.989 5.367 5.786 6.246 6.746 7.286 7.866 8.486 9.146 9.846  
 (2) 8.113 8.546 9.028 9.548 10.101 10.697 11.334 12.014 12.734 13.494 14.294 15.134 16.014 16.944 17.914 18.924

TABLE 8.13 :  $f_1 (1/a - 7, a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 3.822 4.004 4.209 4.441 4.705 5.056 5.445 5.871 6.344 6.854 7.404 8.004 8.644 9.324 10.044 10.804  
 (2) 8.923 9.375 9.877 10.420 11.004 11.628 12.294 13.004 13.754 14.544 15.374 16.244 17.154 18.104 19.094 20.124

TABLE 8.13 :  $f_1 (1/a - 8, a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.075 2.200 2.340 2.500 2.684 2.896 3.146 3.442 3.798 4.230 4.744 5.205 5.705 6.244 6.824 7.444  
 (2) 4.824 5.114 5.442 5.814 6.239 6.735 7.299 7.939 8.654 9.454 10.344 11.324 12.404 13.584 14.864 16.244

TABLE 8.13 :  $f_1 (1/a - 8, a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.410 2.550 2.707 2.886 3.090 3.326 3.601 3.925 4.308 4.756 5.218 5.718 6.254 6.834 7.454 8.114  
 (2) 5.609 5.931 6.299 6.716 7.186 7.701 8.269 8.891 9.574 10.314 11.114 12.034 13.084 14.264 15.584 17.044

TABLE 8.13 :  $f_1 (1/a - 8, a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 2.715 2.886 3.087 3.249 3.468 3.720 4.012 4.351 4.745 5.175 5.624 6.104 6.624 7.184 7.784 8.424  
 (2) 6.369 6.721 7.122 7.573 8.069 8.596 9.164 9.774 10.424 11.114 11.844 12.614 13.424 14.274 15.164 16.094

TABLE 8.13 :  $f_1 (1/a - 8, a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 3.047 3.206 3.383 3.583 3.811 4.073 4.374 4.721 5.111 5.503 5.934 6.414 6.934 7.494 8.094 8.734  
 (2) 7.099 7.450 7.907 8.382 8.884 9.414 10.004 10.654 11.374 12.164 13.004 13.904 14.864 15.884 16.964 18.104

TABLE 8.13 :  $f_1 (1/a - 8, a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 3.344 3.504 3.683 3.887 4.119 4.386 4.693 5.043 5.425 5.774 6.192 6.654 7.154 7.694 8.274 8.894  
 (2) 7.808 8.207 8.655 9.150 9.637 10.164 10.734 11.344 12.004 12.714 13.474 14.264 15.094 15.964 16.874 17.824

TABLE 8.13 :  $f_1 (1/a - 9, a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
 (1) 1.845 1.955 2.080 2.222 2.386 2.575 2.786 3.020 3.278 3.564 3.884 4.244 4.644 5.084 5.564 6.084  
 (2) 4.288 4.546 4.837 5.169 5.546 5.984 6.483 7.044 7.664 8.344 9.084 9.864 10.704 11.614 12.594 13.644

TABLE 8.13 :  $f_1 (1/a - 6, 1/a - 8)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.921 1.991 2.075 2.174 2.291 2.431 2.599 2.803 3.067 3.374 3.764 4.273 4.866 5.301 5.743

(2) 4.548 4.768 5.028 5.336 5.697 6.120 6.615 7.174 7.641 7.478 7.048 6.660 6.435 6.899 8.092

TABLE 8.13 :  $f_1 (1/a - 6, 1/a - 9)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.753 1.828 1.913 2.012 2.126 2.260 2.419 2.609 2.840 3.128 3.491 3.957 4.537 5.050 5.113

(2) 4.112 4.315 4.552 4.869 5.262 5.748 6.406 7.203 8.099 8.546 8.307 6.900 7.638

TABLE 8.13 :  $f_1 (1/a - 6, 1/a - 10)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.605 1.682 1.768 1.866 1.979 2.109 2.262 2.444 2.664 2.935 3.276 3.713 4.265 4.820 4.991

(2) 3.748 3.941 4.163 4.418 4.714 5.060 5.468 5.939 6.484 6.888 6.723 6.417 6.180 6.278 7.272

TABLE 8.13 :  $f_1 (1/a - 7, 1/a - 6)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.717 2.790 2.893 3.030 3.199 3.405 3.657 3.965 4.343 4.811 5.379 5.935 6.588 7.311 5.449

(2) 6.777 7.123 7.539 8.025 8.587 9.216 9.969 10.872 11.969 13.287 14.872 16.779 19.013 9.018

TABLE 8.13 :  $f_1 (1/a - 7, 1/a - 7)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.462 2.537 2.634 2.754 2.900 3.078 3.294 3.557 3.881 4.283 4.780 5.350 5.702 5.597 5.366

(2) 5.940 6.228 6.576 6.985 7.463 8.010 8.591 9.312 10.209 11.317 12.688 14.377 16.459 19.013 8.825

TABLE 8.13 :  $f_1 (1/a - 7, 1/a - 8)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.241 2.323 2.421 2.536 2.673 2.836 3.031 3.268 3.557 3.915 4.360 4.892 5.365 5.430 5.262

(2) 5.306 5.362 5.665 6.222 6.938 7.820 8.960 10.411 12.566 15.466 19.279 24.169 29.226 33.579 18.559

TABLE 8.13 :  $f_1 (1/a - 7, 1/a - 9)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.045 2.132 2.232 2.347 2.481 2.637 2.823 3.052 3.330 3.640 4.049 4.548 5.055 5.251 5.146

(2) 4.797 5.034 5.311 5.632 6.005 6.438 6.933 7.468 8.141 9.045 10.205 11.679 13.508 15.737 8.261

TABLE 8.13 :  $f_1 (1/a - 7, 1/a - 10)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.873 1.962 2.063 2.177 2.308 2.460 2.638 2.850 3.105 3.417 3.804 4.280 4.799 5.022 5.031

(2) 4.373 4.598 4.856 5.153 5.497 5.895 6.353 6.857 7.256 7.162 6.815 6.503 6.395 6.895 7.977

TABLE 8.13 :  $f_1 (1/a - 8, 1/a - 6)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 3.105 3.189 3.308 3.453 3.655 3.890 4.176 4.523 4.946 5.453 6.009 6.244 6.039 5.740 5.450

(2) 7.744 8.139 8.611 9.159 9.778 10.36 10.10 9.253 6.495 7.883 7.452 7.708 7.004 9.370 9.040

TABLE 8.13 :  $f_1 (1/a - 8, 1/a - 7)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.814 2.902 3.010 3.147 3.314 3.517 3.762 4.060 4.423 4.864 5.375 5.812 6.208 6.621 5.370

(2) 6.788 7.117 7.512 7.975 8.506 9.084 9.467 9.027 8.354 7.768 7.318 7.251 7.908 8.896 8.893

TABLE 8.13 :  $f_1 (a - 9, a - 7)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.142 2.266 2.409 2.565 2.747 2.957 3.202 3.491 3.826 4.250 4.728 5.148 5.189 4.997 4.781

(2) 4.981 5.272 5.609 5.971 6.334 6.871 7.377 7.658 7.321 6.856 6.488 6.415 7.033 8.122 8.304

TABLE 8.13 :  $f_1 (a - 9, a - 8)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.431 2.566 2.717 2.880 3.063 3.267 3.508 3.823 4.233 4.652 5.091 5.355 5.285 5.001 4.872

(2) 5.556 5.974 6.332 6.736 7.191 7.684 8.108 8.009 7.574 7.063 6.769 6.893 7.638 8.459 8.386

TABLE 8.13 :  $f_1 (a - 9, a - 9)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.709 2.849 3.027 3.185 3.383 3.622 3.892 4.207 4.573 4.984 5.367 5.514 5.390 5.179 4.973

(2) 6.311 6.649 7.032 7.462 7.938 8.470 8.650 8.276 7.737 7.291 7.074 7.336 8.102 8.658 8.477

TABLE 8.13 :  $f_1 (a - 9, a - 10)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.732 3.114 3.274 3.456 3.653 3.902 4.178 4.500 4.870 5.270 5.598 5.662 5.503 5.283 5.073

(2) 6.341 7.297 7.720 8.154 8.646 9.084 9.061 8.517 7.945 7.512 7.363 7.320 8.467 8.817 8.576

TABLE 8.13 :  $f_1 (a - 10, a - 6)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.440 1.760 1.872 2.000 2.147 2.317 2.517 2.754 3.041 3.393 3.832 4.373 4.874 4.890 4.697

(2) 3.419 4.092 4.354 4.832 5.394 5.393 5.846 6.354 6.801 6.628 6.238 5.919 5.918 6.818 8.050

TABLE 8.13 :  $f_1 (a - 10, a - 7)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 1.828 2.049 2.166 2.359 2.472 2.661 2.862 3.143 3.456 3.835 4.292 4.792 5.064 4.965 4.773

(2) 4.463 4.746 5.040 5.375 5.759 6.195 6.685 7.156 7.792 8.606 9.624 10.823 11.503 11.539 8.214

TABLE 8.13 :  $f_1 (a - 10, a - 8)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.193 2.379 2.465 2.598 2.775 2.977 3.212 3.489 3.818 4.209 4.659 5.076 5.194 5.056 4.866

(2) 5.091 5.377 5.700 6.065 6.469 6.945 7.432 7.713 7.439 6.652 6.554 7.045 6.026 6.222

TABLE 8.13 :  $f_1 (a - 10, a - 9)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.438 2.555 2.766 2.867 3.050 3.260 3.505 3.792 4.130 4.526 4.958 5.291 5.319 5.159 4.968

(2) 5.650 5.995 6.330 6.721 7.162 7.644 8.280 8.935 9.666 10.483 11.394 12.408 12.496 12.344 8.428

TABLE 8.13 :  $f_1 (1/a - 6, 1/a - 6)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.319 2.392 2.481 2.598 2.742 2.970 3.136 3.402 3.731 4.103 4.664 5.307 5.810 5.710 5.446

(2) 5.869 6.108 6.464 6.884 7.375 7.944 8.581 9.361 10.415 11.785 13.489 16.329 18.102 8.950

TABLE 8.13 :  $f_1 (1/a - 6, 1/a - 7)$

$\mu$  0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85

MODE:

(1) 2.710 2.778 2.878 2.961 3.086 3.246 3.439 3.686 4.006 4.414 4.903 5.392 5.332 5.335 4.957

(2) 5.892 5.339 5.638 5.991 6.406 6.892 7.453 8.043 8.158 7.666 7.184 6.757 6.599 7.301 8.289

TABLE B.13 :  $F_1 (1/a - 1.0, 1/a - .8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.203 3.310 3.458 3.622 3.816 4.046 4.313 4.643 5.022 5.445 5.915 6.439 7.014 7.641 8.324  
(2) 7.580 7.942 8.367 8.754 9.204 9.704 10.244 10.824 11.444 12.104 12.804 13.544 14.324 15.144 16.004

TABLE B.13 :  $F_1 (1/a - 1.0, 1/a - .9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.921 3.046 3.189 3.353 3.542 3.763 4.022 4.327 4.685 5.092 5.548 6.054 6.611 7.221 7.886  
(2) 6.853 7.190 7.579 8.024 8.517 9.059 9.649 10.239 10.929 11.719 12.509 13.400 14.393 15.488 16.685

TABLE B.14 :  $F_1 (a - 6, a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 1.817 1.873 1.890 1.857 1.788 1.705 1.623 1.549 1.468 1.435 1.356 1.359 1.355 1.356 1.377  
(2) 4.709 4.823 4.903 4.951 4.914 4.835 4.755 4.673 4.591 4.509 4.427 4.345 4.263 4.181 4.100

TABLE B.14 :  $F_1 (a - 6, a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.119 2.166 2.204 2.165 2.082 1.983 1.885 1.797 1.721 1.659 1.611 1.576 1.554 1.547 1.559  
(2) 5.481 5.698 5.944 6.212 6.502 6.814 7.149 7.514 7.909 8.334 8.789 9.274 9.789 10.334 10.919

TABLE B.14 :  $F_1 (a - 6, a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.422 2.498 2.518 2.470 2.373 2.256 2.140 2.026 1.946 1.872 1.812 1.765 1.733 1.714 1.712  
(2) 6.242 6.549 6.901 7.314 7.789 8.274 8.771 9.280 9.801 10.334 10.879 11.439 12.014 12.604 13.209

TABLE B.14 :  $F_1 (a - 6, a - 9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.725 2.809 2.830 2.773 2.659 2.521 2.385 2.263 2.157 2.068 1.994 1.934 1.888 1.854 1.834  
(2) 6.586 6.965 7.352 7.747 8.151 8.574 9.014 9.471 9.944 10.434 10.944 11.474 12.024 12.594 13.184

TABLE B.14 :  $F_1 (a - 6, a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.028 3.121 3.142 3.072 2.937 2.775 2.617 2.475 2.351 2.245 2.155 2.080 2.018 1.977 1.928  
(2) 7.705 8.136 8.581 9.041 9.514 10.001 10.501 11.014 11.541 12.084 12.644 13.214 13.794 14.384 14.984

TABLE B.14 :  $F_1 (a - 7, a - 6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 1.569 1.640 1.692 1.713 1.697 1.652 1.594 1.533 1.478 1.431 1.394 1.368 1.355 1.356 1.377  
(2) 4.332 4.616 4.933 5.284 5.661 6.064 6.494 6.949 7.429 7.934 8.464 9.014 9.584 10.174 10.784

TABLE B.14 :  $F_1 (a - 7, a - 7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 1.831 1.913 1.974 1.978 1.923 1.852 1.779 1.712 1.655 1.609 1.574 1.553 1.547 1.547 1.559  
(2) 5.048 5.465 5.914 6.394 6.904 7.444 7.994 8.564 9.154 9.764 10.394 11.044 11.714 12.404 13.114

TABLE B.14 :  $F_1 (a - 7, a - 8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.092 2.186 2.255 2.281 2.256 2.190 2.105 2.018 1.937 1.867 1.809 1.764 1.732 1.714 1.712  
(2) 5.760 6.234 6.744 7.284 7.854 8.454 9.084 9.744 10.434 11.154 11.904 12.684 13.494 14.334 15.194

TABLE B.13 :  $F_1 (1/a - .8, 1/a - .8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.561 2.655 2.766 2.898 3.054 3.240 3.462 3.730 4.055 4.451 4.920 5.397 5.959 6.478 7.070  
(2) 6.064 6.536 7.011 7.512 8.036 8.604 9.214 9.864 10.554 11.284 12.014 12.744 13.474 14.204 14.934

TABLE B.13 :  $F_1 (1/a - .8, 1/a - .9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.317 2.437 2.551 2.683 2.835 3.013 3.223 3.473 3.775 4.142 4.588 5.059 5.564 6.111 6.694  
(2) 5.482 5.953 6.468 7.024 7.624 8.264 8.944 9.664 10.424 11.224 12.064 12.944 13.864 14.824 15.824

TABLE B.13 :  $F_1 (1/a - .8, 1/a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.140 2.242 2.358 2.488 2.638 2.811 3.014 3.255 3.543 3.882 4.274 4.709 5.157 5.611 6.082  
(2) 4.998 5.255 5.545 5.867 6.226 6.620 7.059 7.544 8.074 8.644 9.254 9.904 10.594 11.324 12.084

TABLE B.13 :  $F_1 (1/a - .9, 1/a - .6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.493 3.568 3.722 3.895 4.110 4.373 4.691 5.075 5.532 6.046 6.438 6.958 7.514 8.114 8.754  
(2) 8.719 9.154 9.679 10.281 10.971 11.741 12.591 13.521 14.531 15.621 16.791 18.041 19.371 20.781 22.271

TABLE B.13 :  $F_1 (1/a - .9, 1/a - .7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.165 3.262 3.366 3.540 3.728 3.954 4.227 4.557 4.952 5.413 5.913 6.457 7.044 7.674 8.344  
(2) 7.636 8.025 8.446 8.917 9.435 10.004 10.624 11.294 12.014 12.784 13.604 14.474 15.394 16.364 17.384

TABLE B.13 :  $F_1 (1/a - .9, 1/a - .8)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.681 2.742 2.812 2.960 3.126 3.312 3.522 3.752 4.009 4.284 4.584 4.914 5.274 5.664 6.074  
(2) 6.827 7.149 7.515 7.924 8.374 8.864 9.394 9.964 10.574 11.224 11.914 12.644 13.414 14.224 15.074

TABLE B.13 :  $F_1 (1/a - .9, 1/a - .9)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.629 2.742 2.870 3.018 3.189 3.388 3.623 3.892 4.199 4.545 4.935 5.371 5.849 6.364 6.914  
(2) 6.168 6.472 6.824 7.219 7.652 8.124 8.634 9.184 9.774 10.404 11.074 11.784 12.534 13.324 14.154

TABLE B.13 :  $F_1 (1/a - .9, 1/a - 1.0)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 2.629 2.742 2.870 2.998 3.146 3.316 3.508 3.724 3.964 4.234 4.544 4.894 5.284 5.714 6.184  
(2) 5.828 6.142 6.494 6.884 7.314 7.784 8.294 8.844 9.434 10.064 10.744 11.464 12.224 13.024 13.864

TABLE B.13 :  $F_1 (1/a - 1.0, 1/a - .6)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.883 3.986 4.135 4.324 4.546 4.804 5.104 5.444 5.824 6.244 6.704 7.204 7.744 8.324 8.944  
(2) 9.078 9.614 10.194 10.824 11.504 12.244 13.044 13.964 14.914 15.904 16.944 18.044 19.194 20.404 21.674

TABLE B.13 :  $F_1 (1/a - 1.0, 1/a - .7)$

MODE: 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
(1) 3.517 3.625 3.762 3.933 4.140 4.390 4.690 5.047 5.464 5.912 6.424 6.974 7.564 8.194 8.864  
(2) 8.484 8.992 9.542 10.144 10.804 11.524 12.304 13.144 14.044 15.004 16.024 17.104 18.244 19.444 20.764

TABLE 8.14 :  $f_1$  ( $\alpha = 9, \alpha = 9$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.848 1.945 2.047 2.140 2.209 2.239 2.266 2.178 2.114 2.047 1.984 1.930 1.886 1.854 1.834  
(2) 5.360 5.303 4.971 4.536 4.332 4.218 4.255 4.425 4.705 5.064 5.411 5.522 5.377 5.157 4.916

TABLE 8.14 :  $f_1$  ( $\alpha = 9, \alpha = 1.0$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 2.046 2.161 2.275 2.377 2.452 2.480 2.455 2.392 2.310 2.226 2.146 2.076 2.016 1.967 1.928  
(2) 5.591 5.859 6.431 6.972 6.658 6.512 6.539 6.709 6.992 7.339 7.628 7.658 7.482 7.255 7.042

TABLE 8.14 :  $f_1$  ( $\alpha = 1.0, \alpha = 6$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.106 1.171 1.238 1.305 1.365 1.412 1.437 1.439 1.425 1.403 1.380 1.361 1.351 1.354 1.376  
(2) 3.293 3.318 2.933 3.101 2.959 2.862 2.891 2.994 3.192 3.409 3.694 4.118 4.900 4.892 4.690

TABLE 8.14 :  $f_1$  ( $\alpha = 1.0, \alpha = 7$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.291 1.366 1.444 1.522 1.593 1.646 1.674 1.674 1.654 1.624 1.593 1.567 1.550 1.546 1.559  
(2) 3.759 3.868 3.781 3.567 3.408 3.306 3.305 3.411 3.623 3.940 4.360 4.837 5.079 4.951 4.759

TABLE 8.14 :  $f_1$  ( $\alpha = 1.0, \alpha = 8$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.475 1.561 1.651 1.740 1.820 1.880 1.908 1.904 1.876 1.836 1.794 1.757 1.729 1.713 1.712  
(2) 4.341 4.414 4.295 4.048 3.821 3.688 3.670 3.774 3.993 4.317 4.728 5.113 5.198 5.043 4.845

TABLE 8.14 :  $f_1$  ( $\alpha = 1.0, \alpha = 9$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.659 1.756 1.857 1.957 2.047 2.112 2.139 2.127 2.087 2.033 1.978 1.927 1.885 1.853 1.834  
(2) 4.883 4.954 4.789 4.471 4.169 4.019 3.983 4.082 4.303 4.630 5.019 5.314 5.312 5.139 4.941

TABLE 8.14 :  $f_1$  ( $1/\alpha = 6, 1/\alpha = 6$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.847 1.962 2.090 2.214 2.306 2.374 2.399 2.313 2.197 2.045 1.864 1.650 1.514 1.400 1.318  
(2) 5.529 5.838 6.082 5.882 5.547 4.995 4.786 4.944 5.350 5.825 6.285 6.314 5.792 5.269 5.405

TABLE 8.14 :  $f_1$  ( $1/\alpha = 6, 1/\alpha = 7$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.584 1.682 1.792 1.915 2.054 2.208 2.372 2.531 2.638 2.638 2.555 2.442 2.327 2.219 2.121  
(2) 4.739 5.007 5.257 5.308 4.923 4.455 4.076 4.018 4.375 4.857 5.265 4.729 4.329 4.501 5.318

TABLE 8.14 :  $f_1$  ( $1/\alpha = 6, 1/\alpha = 8$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.868 1.471 1.568 1.676 1.797 1.932 2.079 2.227 2.350 2.407 2.385 2.316 2.231 2.147 2.069  
(2) 4.147 4.303 4.612 4.753 4.608 4.262 3.936 3.696 3.578 3.632 3.696 4.325 4.920 5.279 5.207

TABLE 8.14 :  $f_1$  ( $1/\alpha = 6, 1/\alpha = 9$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.232 1.308 1.394 1.490 1.598 1.718 1.849 1.984 2.107 2.187 2.205 2.173 2.119 2.060 2.003  
(2) 3.686 3.698 4.105 4.264 4.247 4.026 3.785 3.557 3.443 3.443 3.663 4.037 4.569 5.047 5.082

TABLE 8.14 :  $f_1$  ( $\alpha = 7, \alpha = 9$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 2.354 2.459 2.573 2.684 2.751 2.819 2.845 2.748 2.663 2.583 2.512 2.447 2.393 2.349 2.314  
(2) 6.463 6.897 7.316 7.697 7.967 8.162 8.174 8.081 7.893 7.625 7.333 6.945 6.578 6.178 5.932

TABLE 8.14 :  $f_1$  ( $\alpha = 7, \alpha = 1.0$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 2.615 2.733 2.817 2.865 2.891 2.804 2.681 2.531 2.362 2.183 1.999 1.817 1.643 1.487 1.358  
(2) 7.155 6.463 5.793 5.396 5.256 5.320 5.528 5.824 6.178 6.500 6.789 6.989 6.989 6.724 6.406

TABLE 8.14 :  $f_1$  ( $\alpha = 8, \alpha = 6$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.378 1.450 1.516 1.565 1.589 1.583 1.553 1.510 1.466 1.424 1.391 1.367 1.354 1.355 1.377  
(2) 3.940 3.622 3.542 3.378 3.223 3.236 3.356 3.573 3.681 4.285 4.784 5.233 5.193 4.950 4.704

TABLE 8.14 :  $f_1$  ( $\alpha = 8, \alpha = 7$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.668 1.632 1.723 1.826 1.933 2.041 2.152 2.265 2.381 2.497 2.614 2.731 2.847 2.964 3.081  
(2) 4.595 4.423 4.103 3.841 3.769 3.712 3.819 4.072 4.402 4.819 5.263 5.435 5.257 5.007 4.771

TABLE 8.14 :  $f_1$  ( $\alpha = 8, \alpha = 8$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.808 1.933 2.071 2.215 2.362 2.512 2.665 2.819 2.974 3.131 3.287 3.444 3.601 3.758 3.915  
(2) 5.248 5.031 4.640 4.370 4.132 4.141 4.268 4.509 4.844 5.241 5.560 5.550 5.331 5.082 4.855

TABLE 8.14 :  $f_1$  ( $\alpha = 8, \alpha = 9$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 2.068 2.175 2.273 2.346 2.376 2.296 2.217 2.134 2.057 1.989 1.932 1.887 1.854 1.834  
(2) 5.897 5.622 5.142 4.755 4.547 4.517 4.639 4.880 5.210 5.563 5.758 5.655 5.418 5.170 4.950

TABLE 8.14 :  $f_1$  ( $\alpha = 9, \alpha = 1.0$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 2.237 2.377 2.575 2.834 3.064 3.288 3.490 3.639 3.725 3.750 3.698 3.598 3.457 3.287 3.117  
(2) 6.543 6.168 5.598 5.137 4.866 4.818 4.954 5.195 5.515 5.822 5.922 5.765 5.516 5.265 5.044

TABLE 8.14 :  $f_1$  ( $\alpha = 9, \alpha = 6$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.228 1.255 1.315 1.408 1.476 1.571 1.670 1.773 1.881 1.993 2.107 2.225 2.349 2.477 2.606  
(2) 3.576 3.465 3.402 3.205 3.026 3.026 3.064 3.241 3.491 3.819 4.254 4.829 5.092 4.928 4.698

TABLE 8.14 :  $f_1$  ( $\alpha = 9, \alpha = 7$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.432 1.532 1.665 1.821 1.999 2.198 2.416 2.644 2.881 3.126 3.371 3.616 3.861 4.106 4.351  
(2) 4.172 4.152 3.918 3.702 3.530 3.470 3.526 3.693 3.963 4.331 4.784 5.181 5.192 4.899 4.766

TABLE 8.14 :  $f_1$  ( $\alpha = 9, \alpha = 8$ )

$\mu$ : 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85  
MODE: (1) 1.637 1.728 1.820 1.946 2.091 2.248 2.416 2.594 2.781 2.976 3.171 3.366 3.561 3.756 3.951  
(2) 4.756 4.713 4.474 4.169 3.955 3.870 3.918 4.089 4.366 4.736 5.144 5.376 5.280 5.066 4.851

TABLE B.14 :  $f_1$  ( $1/a - 8, 1/a - 1.0$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 7.020 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 9, 1/a - 6$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 7.020 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 9, 1/a - 7$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 7.020 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 9, 1/a - 8$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 7.020 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 9, 1/a - 9$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 7.020 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 1.0, 1/a - 6$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 7.020 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 1.0, 1/a - 7$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 7.020 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 1.0, 1/a - 8$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 7.020 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 6, 1/a - 1.0$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 6.434 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 7, 1/a - 6$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 6.434 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 7, 1/a - 7$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 6.434 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 7, 1/a - 8$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 6.434 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 7, 1/a - 9$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 6.434 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 8, 1/a - 6$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 6.434 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 8, 1/a - 7$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 6.434 and MODE values for (1) and (2).

TABLE B.14 :  $f_1$  ( $1/a - 8, 1/a - 8$ )

Table with 2 columns: u and MODE. Rows include values for u from 0.15 to 6.434 and MODE values for (1) and (2).

TABLE 8.15 :  $f_1 (V/A - 7, 1/2 - 8)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 3.748 3.228 2.819 2.493 2.241 2.019 1.824 1.653 1.504 1.373 1.257 1.152 1.061 0.981 0.911  
 (2) 5.958 5.351 4.849 4.447 4.107 3.820 3.565 3.334 3.124 2.934 2.762 2.608 2.468 2.341 2.225  
 (3) 8.699 8.007 7.418 6.939 6.566 6.260 5.997 5.759 5.544 5.351 5.180 5.028 4.891 4.767 4.653  
 (4) 11.46 11.49 11.58 11.72 11.94 12.19 12.46 12.66 12.77 12.77 12.74 12.70 12.69 12.70 12.70 12.70

TABLE 8.15 :  $f_1 (V/A - 7, 1/2 - 3)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 2.950 2.910 2.845 2.766 2.674 2.569 2.454 2.333 2.208 2.081 1.953 1.824 1.693 1.561 1.429  
 (2) 5.408 5.414 5.434 5.467 5.516 5.581 5.663 5.769 5.895 6.043 6.213 6.403 6.615 6.848 7.106  
 (3) 7.863 7.923 7.992 8.074 8.161 8.264 8.384 8.516 8.662 8.822 9.007 9.217 9.453 9.714 10.000  
 (4) 10.34 10.44 10.58 10.76 11.05 11.37 11.81 12.25 12.61 12.85 13.13 13.16 13.16 13.16 13.16

TABLE 8.15 :  $f_1 (V/A - 7, 1/2 - 1.5)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 2.251 2.254 2.249 2.238 2.222 2.201 2.176 2.145 2.110 2.072 2.031 1.985 1.934 1.878 1.817  
 (2) 4.463 4.479 4.503 4.535 4.576 4.631 4.701 4.787 4.887 5.001 5.129 5.271 5.428 5.600 5.787  
 (3) 7.187 7.240 7.305 7.377 7.457 7.551 7.660 7.784 7.924 8.081 8.254 8.444 8.659 8.907 9.184  
 (4) 9.433 9.445 9.459 9.473 9.488 9.504 9.521 9.539 9.560 9.583 9.611 9.641 9.673 9.707 9.743

TABLE 8.15 :  $f_1 (V/A - 8, 1/2 - 4)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 4.622 4.461 4.317 4.183 4.058 3.943 3.836 3.736 3.643 3.557 3.476 3.400 3.328 3.261 3.200  
 (2) 8.658 8.521 8.397 8.282 8.175 8.075 7.981 7.892 7.807 7.726 7.649 7.577 7.510 7.448 7.391  
 (3) 12.59 12.26 11.94 11.69 11.56 11.46 11.39 11.33 11.28 11.24 11.21 11.19 11.17 11.15 11.13  
 (4) 16.44 16.02 15.75 15.67 15.66 15.66 15.66 15.66 15.66 15.66 15.66 15.66 15.66 15.66 15.66

TABLE 8.15 :  $f_1 (V/A - 8, 1/2 - 7)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 3.712 3.687 3.671 3.661 3.654 3.648 3.643 3.639 3.635 3.631 3.627 3.623 3.619 3.615 3.611  
 (2) 7.608 7.584 7.562 7.547 7.533 7.520 7.509 7.500 7.492 7.484 7.477 7.471 7.465 7.460 7.457  
 (3) 11.12 10.99 10.86 10.75 10.66 10.66 10.66 10.66 10.66 10.66 10.66 10.66 10.66 10.66 10.66  
 (4) 14.59 14.41 14.29 14.25 14.23 14.21 14.20 14.20 14.20 14.20 14.20 14.20 14.20 14.20 14.20

TABLE 8.15 :  $f_2 (V/A - 8, 1/2 - 3)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 3.712 3.687 3.671 3.661 3.654 3.648 3.643 3.639 3.635 3.631 3.627 3.623 3.619 3.615 3.611  
 (2) 6.865 6.799 6.746 6.705 6.676 6.656 6.641 6.628 6.616 6.606 6.598 6.591 6.584 6.578 6.573  
 (3) 9.925 9.829 9.757 9.698 9.654 9.623 9.599 9.579 9.561 9.545 9.531 9.519 9.509 9.499 9.489  
 (4) 13.05 13.04 13.05 13.10 13.19 13.31 13.42 13.49 13.49 13.44 13.36 13.28 13.22 13.21 13.23

TABLE 8.15 :  $f_2 (V/A - 8, 1/2 - 1.5)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 3.405 3.393 3.386 3.384 3.383 3.382 3.382 3.382 3.382 3.382 3.382 3.382 3.382 3.382 3.382  
 (2) 6.176 6.174 6.174 6.174 6.174 6.174 6.174 6.174 6.174 6.174 6.174 6.174 6.174 6.174 6.174  
 (3) 8.974 8.974 8.974 8.974 8.974 8.974 8.974 8.974 8.974 8.974 8.974 8.974 8.974 8.974 8.974  
 (4) 11.779 11.86 11.86 12.10 12.26 12.42 12.57 12.72 12.77 12.74 12.74 12.74 12.74 12.74 12.74

TABLE 8.15 :  $f_1 (V/A - 6, 1/2 - 4)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 3.423 3.424 3.395 3.360 3.319 3.273 3.228 3.180 3.140 3.108 3.087 3.073 3.068 3.064 3.061  
 (2) 6.305 6.411 6.495 6.579 6.678 6.778 6.884 6.995 7.118 7.244 7.373 7.505 7.639 7.774 7.911  
 (3) 9.489 9.720 9.961 10.209 10.463 10.722 10.988 11.260 11.538 11.818 12.099 12.379 12.655 12.926 13.191  
 (4) 12.46 12.49 12.27 12.47 12.76 13.21 13.69 13.68 13.68 13.36 13.13 12.86 12.67 12.47 12.27

TABLE 8.15 :  $f_1 (V/A - 6, 1/2 - 7)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 3.071 3.046 3.023 3.019 3.007 2.995 2.983 2.973 2.966 2.965 2.970 2.984 3.017 3.067 3.140  
 (2) 5.714 5.689 5.657 5.629 5.605 5.600 5.609 5.629 5.668 5.728 5.809 5.914 6.051 6.214 6.394  
 (3) 8.468 8.432 8.392 8.344 8.292 8.248 8.208 8.178 8.154 8.144 8.144 8.144 8.144 8.144 8.144  
 (4) 11.02 11.00 11.05 11.21 11.48 11.85 12.29 12.68 12.87 12.83 12.72 12.61 12.50 12.45 12.40

TABLE 8.15 :  $f_1 (V/A - 6, 1/2 - 8)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 2.426 2.467 2.516 2.573 2.638 2.713 2.798 2.897 2.997 3.097 3.197 3.297 3.397 3.497 3.597  
 (2) 5.105 5.107 5.114 5.131 5.160 5.205 5.269 5.356 5.470 5.617 5.799 6.018 6.266 6.549 6.868  
 (3) 7.465 7.484 7.519 7.577 7.677 7.827 8.041 8.324 8.684 9.134 9.680 10.319 10.959 11.699 12.449  
 (4) 9.639 9.501 10.02 10.22 10.49 10.84 11.24 11.64 12.06 12.58 13.17 13.77 14.48 15.37 16.34

TABLE 8.15 :  $f_1 (V/A - 6, 1/2 - 1.5)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 2.554 2.546 2.543 2.548 2.551 2.561 2.576 2.597 2.623 2.657 2.700 2.754 2.824 2.913 3.013  
 (2) 4.637 4.645 4.667 4.709 4.764 4.827 4.901 5.000 5.130 5.275 5.438 5.620 5.826 6.056 6.307  
 (3) 6.746 6.789 6.853 6.974 7.127 7.316 7.576 7.878 8.226 8.584 9.033 9.519 10.059 10.669 11.359  
 (4) 8.939 8.979 9.137 9.361 9.653 10.01 10.42 10.83 11.27 11.77 12.37 13.07 13.87 14.77 15.76

TABLE 8.15 :  $f_1 (V/A - 6, 1/2 - 7)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 2.359 2.372 2.387 2.402 2.422 2.448 2.480 2.521 2.572 2.636 2.717 2.821 2.949 3.097 3.260  
 (2) 4.254 4.372 4.501 4.645 4.804 4.988 5.192 5.424 5.684 6.031 6.401 6.801 7.231 7.691 8.181  
 (3) 6.165 6.317 6.501 6.723 7.001 7.341 7.759 8.264 8.864 9.564 10.364 11.264 12.264 13.364 14.564  
 (4) 8.056 8.207 8.378 8.613 8.920 9.289 9.709 10.115 10.511 10.79 10.94 11.08 11.23 11.47 11.68

TABLE 8.15 :  $f_2 (V/A - 6, 1/2 - 4)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 4.027 3.993 3.958 3.914 3.864 3.802 3.740 3.679 3.623 3.574 3.535 3.509 3.488 3.467 3.446  
 (2) 7.584 7.463 7.311 7.152 7.007 6.864 6.726 6.594 6.472 6.359 6.259 6.174 6.103 6.043 6.009  
 (3) 11.56 10.81 10.65 10.47 10.43 10.49 10.65 10.89 11.13 11.29 11.28 11.10 10.94 10.80 10.66  
 (4) 14.47 14.21 14.09 14.16 14.39 14.70 14.93 14.90 14.61 14.28 13.97 13.65 13.36 13.18 13.16

TABLE 8.15 :  $f_2 (V/A - 6, 1/2 - 7)$

$\mu$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: (1) 3.582 3.534 3.515 3.518 3.521 3.527 3.534 3.541 3.548 3.554 3.561 3.568 3.574 3.581 3.588  
 (2) 6.462 6.426 6.379 6.330 6.284 6.240 6.201 6.167 6.136 6.108 6.083 6.061 6.043 6.029 6.017  
 (3) 9.748 9.676 9.614 9.565 9.526 9.488 9.452 9.419 9.389 9.363 9.341 9.323 9.307 9.291 9.277  
 (4) 12.82 12.74 12.73 12.82 13.01 13.26 13.52 13.68 13.68 13.53 13.36 13.20 13.06 13.06 13.06

TABLE 6.15 :  $f_1$  ( $1/a = 1.0, 1/a = .7$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 5.119 5.656 5.972 6.183 6.329 6.453 6.562 6.657 6.736 6.801 6.861 6.916 6.966 7.012 7.053 7.091 7.127 (2) 9.482 9.365 9.184 8.976 8.738 8.493 8.271 8.059 7.865 7.693 7.537 7.407 7.302 7.229 7.189 (3) 12.78 13.47 13.08 12.71 12.39 12.11 11.82 11.60 11.74 11.70 11.62 11.49 11.31 11.03 10.84 (4) 17.95 17.42 16.93 16.55 16.22 16.15 16.20 15.72 15.35 14.98 14.58 14.23 13.92 13.67

TABLE 6.15 :  $f_1$  ( $1/a = 1.0, 1/a = .8$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 4.253 4.471 4.574 4.549 4.523 4.492 4.467 4.435 4.405 4.371 4.329 4.285 4.243 4.191 4.141 4.093 (2) 6.465 6.432 6.390 6.363 6.345 6.331 6.321 6.312 6.304 6.297 6.291 6.287 6.283 6.281 6.279 6.278 (3) 12.33 12.33 12.33 12.33 12.33 12.33 12.33 12.33 12.33 12.33 12.33 12.33 12.33 12.33 12.33 12.33 (4) 16.13 15.86 15.60 15.34 15.15 15.04 14.94 14.84 14.83 14.83 14.83 14.83 14.83 14.83 14.83 14.83

TABLE 6.15 :  $f_1$  ( $1/a = 1.0, 1/a = .9$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 4.253 4.234 4.219 4.206 4.195 4.183 4.169 4.154 4.137 4.119 4.097 4.074 4.051 4.029 4.008 (2) 2.203 2.660 2.654 2.647 2.641 2.639 2.639 2.638 2.638 2.637 2.637 2.637 2.637 2.637 2.637 (3) 11.16 11.12 11.06 10.98 10.89 10.79 10.67 10.53 10.42 10.30 10.19 10.08 9.97 9.86 9.75 9.64 (4) 14.61 14.51 14.42 14.39 14.20 14.12 14.00 14.05 14.04 13.93 13.91 13.80 13.69 13.57 13.42

TABLE 6.16 :  $f_2$  ( $a = .6, a = .8$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 5.578 5.858 6.130 6.453 7.137 7.658 8.145 8.698 9.316 10.28 8.772 9.171 8.637 8.167 7.781 (2) 3.179 3.278 3.335 3.350 3.359 3.362 3.362 3.362 3.362 3.362 3.362 3.362 3.362 3.362 3.362 (3) 12.85 13.42 14.48 15.44 16.41 16.93 17.45 17.95 18.45 18.92 19.38 19.81 20.22 20.60 20.96 (4) 16.53 17.51 18.60 19.65 18.45 18.07 17.65 16.46 17.20 19.09 19.39 18.25 16.17 17.75 18.63

TABLE 6.16 :  $f_2$  ( $a = .6, a = .7$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 6.475 6.784 7.196 7.667 8.200 8.810 9.504 10.26 10.81 10.54 9.975 9.376 8.799 8.351 7.963 (2) 10.65 11.27 11.96 12.73 13.58 14.37 15.13 15.79 16.42 17.02 17.57 18.07 18.53 18.96 19.37 (3) 14.92 15.78 16.73 17.72 18.77 19.81 20.91 22.05 23.22 24.42 25.66 26.94 28.26 29.61 30.99 (4) 19.18 20.28 21.42 22.61 23.87 25.19 26.56 27.94 29.34 30.75 32.19 33.64 35.10 36.56 38.03

TABLE 6.16 :  $f_2$  ( $a = .6, a = .6$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 7.274 7.669 8.112 8.611 9.175 9.809 10.50 11.14 11.78 10.75 10.11 9.513 8.994 8.563 (2) 12.08 12.75 13.48 14.29 15.12 15.47 15.48 15.58 15.65 15.69 15.72 15.73 15.73 15.73 15.73 (3) 16.93 17.65 18.55 19.67 20.91 22.27 23.72 25.26 26.87 28.53 30.24 31.99 33.83 35.66 37.49 (4) 21.77 22.93 24.37 26.08 27.97 29.94 31.97 34.14 36.42 38.80 41.28 43.85 46.51 49.26 52.09

TABLE 6.16 :  $f_2$  ( $a = .6, a = .5$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 8.094 8.502 9.060 9.673 10.35 11.08 11.81 12.58 13.39 11.28 11.59 10.91 10.31 9.718 9.199 8.758 8.394 (2) 13.45 14.15 14.91 15.73 16.42 16.42 16.42 16.42 16.42 16.42 16.42 16.42 16.42 16.42 16.42 (3) 18.86 19.83 20.81 21.89 22.97 24.05 25.13 26.21 27.29 28.37 29.45 30.53 31.61 32.69 33.77 34.85 (4) 24.28 25.44 26.67 28.07 29.57 31.16 32.84 34.61 36.47 38.34 40.21 42.08 43.95 45.82 47.69

TABLE 6.15 :  $f_1$  ( $1/a = .8, 1/a = 1.0$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 3.143 3.146 3.151 3.158 3.169 3.182 3.200 3.222 3.250 3.283 3.324 3.373 3.429 3.489 3.581 (2) 5.167 5.163 5.209 5.216 5.240 5.270 5.317 5.367 5.420 5.476 5.534 5.594 5.656 5.720 5.784 (3) 6.203 6.250 6.326 6.430 6.566 6.730 6.914 7.103 7.277 7.437 7.516 7.581 7.626 7.660 7.692 (4) 10.78 10.86 11.00 11.20 11.42 11.65 11.85 12.00 12.09 12.14 12.20 12.28 12.34 12.40 12.46 12.67 12.95

TABLE 6.15 :  $f_1$  ( $1/a = .9, 1/a = .8$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 5.175 5.127 5.073 5.003 4.915 4.812 4.702 4.589 4.478 4.374 4.278 4.193 4.118 4.055 4.004 (2) 9.723 9.518 9.243 8.919 8.548 8.376 8.131 7.912 7.721 7.558 7.423 7.314 7.231 7.170 7.126 (3) 14.10 13.62 13.16 12.77 12.46 12.20 12.09 12.13 11.33 11.33 11.32 11.28 11.16 11.03 10.84 10.64 (4) 18.32 17.68 17.21 16.99 16.90 16.79 16.76 16.76 16.76 16.76 16.76 16.76 16.76 16.76 16.76 16.76

TABLE 6.15 :  $f_1$  ( $1/a = .9, 1/a = .7$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 4.603 4.568 4.534 4.522 4.468 4.431 4.371 4.316 4.258 4.200 4.144 4.091 4.044 4.003 3.969 (2) 8.948 8.669 8.357 8.017 7.655 7.311 7.005 6.729 6.483 6.266 6.077 5.912 5.769 5.642 5.534 5.441 (3) 12.46 12.26 12.03 11.80 11.61 11.40 11.39 11.33 11.33 11.32 11.28 11.16 11.03 10.84 10.64 (4) 16.31 15.98 15.69 15.46 15.38 15.38 15.38 15.38 15.38 15.38 15.38 15.38 15.38 15.38 15.38 15.38

TABLE 6.15 :  $f_1$  ( $1/a = .9, 1/a = .6$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 4.175 4.145 4.124 4.108 4.054 4.019 4.002 4.004 4.024 4.054 4.094 4.144 4.194 4.254 4.314 (2) 7.647 7.617 7.587 7.537 7.483 7.425 7.366 7.309 7.256 7.210 7.170 7.139 7.114 7.096 7.084 (3) 11.14 11.09 11.00 10.91 10.83 10.77 10.70 10.70 10.70 10.70 10.70 10.70 10.69 10.62 10.54 10.44 (4) 14.42 14.41 14.39 14.30 14.25 14.23 14.23 14.23 14.23 14.23 14.23 14.23 14.23 14.23 14.23 14.23

TABLE 6.15 :  $f_1$  ( $1/a = .9, 1/a = .5$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 3.829 3.814 3.804 3.798 3.795 3.794 3.795 3.798 3.801 3.807 3.813 3.821 3.831 3.843 3.858 (2) 6.942 6.934 6.933 6.935 6.939 6.946 6.954 6.964 6.975 6.989 7.003 7.017 7.030 7.042 7.051 (3) 10.07 10.08 10.08 10.09 10.11 10.13 10.15 10.16 10.16 10.21 10.23 10.24 10.25 10.24 10.23 10.21 (4) 13.22 13.23 13.24 13.27 13.30 13.33 13.36 13.39 13.39 13.38 13.36 13.34 13.32 13.32 13.32

TABLE 6.15 :  $f_1$  ( $1/a = .9, 1/a = 1.0$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 3.535 3.537 3.540 3.545 3.552 3.551 3.572 3.588 3.603 3.623 3.647 3.675 3.707 3.742 3.782 (2) 6.359 6.360 6.367 6.422 6.456 6.498 6.550 6.610 6.675 6.744 6.815 6.894 6.932 6.977 7.031 (3) 9.212 9.243 9.291 9.356 9.437 9.530 9.635 9.753 9.889 9.948 9.979 9.990 9.970 9.900 9.794 (4) 12.07 12.13 12.22 12.34 12.46 12.57 12.65 12.70 12.75 12.79 12.79 12.79 12.85 12.95 13.07 13.19

TABLE 6.15 :  $f_1$  ( $1/a = 1.0, 1/a = .8$ )

$\mu = 0.075, 0.100, 0.125, 0.150, 0.175, 0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425$

MODE: (1) 5.748 5.690 5.622 5.531 5.416 5.280 5.133 4.982 4.833 4.682 4.560 4.459 4.378 4.289 4.190 (2) 10.78 10.50 10.13 9.717 9.293 8.961 8.632 8.305 8.000 7.638 7.241 6.837 6.419 6.000 5.582 5.175 (3) 15.55 14.89 14.25 13.70 13.15 12.94 12.77 12.68 12.62 12.62 12.62 12.62 12.62 12.62 12.62 12.62 (4) 20.09 19.19 18.49 18.08 17.91 17.82 17.76 17.71 17.65 17.61 17.54 17.54 17.54 17.54 17.54

TABLE 6.16 :  $f_1$  ( $\delta = -0.5, \alpha = -7$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 4.801 5.068 5.339 5.611 5.882 6.153 6.424 6.695 6.966 7.237 7.508 7.779 8.050 8.321 8.592

(1) 7.920 8.451 8.989 9.526 10.062 10.597 11.132 11.667 12.202 12.737 13.272 13.807 14.342 14.877 15.412

(2) 11.119 11.84 12.56 13.28 14.01 14.73 15.45 16.17 16.89 17.61 18.33 19.05 19.77 20.49 21.21

(3) 14.318 15.14 15.96 16.78 17.60 18.42 19.24 20.06 20.88 21.70 22.52 23.34 24.16 24.98 25.80

(4) 17.517 18.44 19.36 20.28 21.19 22.11 23.03 23.95 24.87 25.79 26.71 27.63 28.55 29.47 30.39

TABLE 6.16 :  $f_1$  ( $\delta = -0.5, \alpha = -8$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 4.655 5.152 5.649 6.146 6.643 7.140 7.637 8.134 8.631 9.128 9.625 10.122 10.619 11.116 11.613

(1) 6.003 6.581 7.159 7.737 8.315 8.893 9.471 10.049 10.627 11.205 11.783 12.361 12.939 13.517 14.095

(2) 12.270 13.140 14.010 14.880 15.750 16.620 17.490 18.360 19.230 20.100 20.970 21.840 22.710 23.580 24.450

(3) 16.333 17.24 18.15 19.06 19.97 20.88 21.79 22.70 23.61 24.52 25.43 26.34 27.25 28.16 29.07

(4) 20.397 21.44 22.49 23.54 24.59 25.64 26.69 27.74 28.79 29.84 30.89 31.94 33.00 34.05 35.10

TABLE 6.16 :  $f_1$  ( $\delta = -0.5, \alpha = -7$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 4.600 4.977 5.354 5.731 6.108 6.485 6.862 7.239 7.616 7.993 8.370 8.747 9.124 9.501 9.878

(1) 10.09 10.62 11.15 11.68 12.21 12.74 13.27 13.80 14.33 14.86 15.39 15.92 16.45 16.98 17.51

(2) 14.15 14.89 15.72 16.60 17.48 18.36 19.24 20.12 21.00 21.88 22.76 23.64 24.52 25.40 26.28

(3) 18.21 19.17 20.19 21.20 22.21 23.22 24.23 25.24 26.25 27.26 28.27 29.28 30.29 31.30 32.31

(4) 22.27 23.34 24.41 25.48 26.55 27.62 28.69 29.76 30.83 31.90 32.97 34.04 35.11 36.18 37.25

TABLE 6.16 :  $f_1$  ( $\delta = -0.5, \alpha = -1.0$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 6.650 6.957 7.262 7.567 7.872 8.177 8.482 8.787 9.092 9.397 9.702 10.007 10.312 10.617 10.922

(1) 11.08 11.61 12.22 12.80 13.45 14.06 14.67 15.28 15.89 16.50 17.11 17.72 18.33 18.94 19.55

(2) 15.55 16.32 17.19 18.06 18.93 19.80 20.67 21.54 22.41 23.28 24.15 25.02 25.89 26.76 27.63

(3) 20.03 21.04 22.06 23.07 24.08 25.09 26.10 27.11 28.12 29.13 30.14 31.15 32.16 33.17 34.18

(4) 24.51 25.63 26.75 27.86 28.97 30.08 31.19 32.30 33.41 34.52 35.63 36.74 37.85 38.96 40.07

TABLE 6.16 :  $f_1$  ( $\delta = -0.5, \alpha = -6$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 3.686 3.906 4.126 4.346 4.566 4.786 5.006 5.226 5.446 5.666 5.886 6.106 6.326 6.546 6.766

(1) 4.277 4.789 5.112 5.470 5.828 6.242 6.592 6.913 7.242 7.571 7.900 8.229 8.558 8.887 9.216

(2) 6.120 6.485 6.850 7.215 7.580 7.945 8.310 8.675 9.040 9.405 9.770 10.135 10.500 10.865 11.230

(3) 8.569 9.081 9.593 10.105 10.617 11.129 11.641 12.153 12.665 13.177 13.689 14.201 14.713 15.225 15.737

(4) 11.02 11.68 12.42 13.26 14.19 15.06 15.90 16.74 17.58 18.42 19.26 20.10 20.94 21.78 22.62

TABLE 6.16 :  $f_1$  ( $\delta = -0.5, \alpha = -7$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 4.277 4.789 5.112 5.470 5.828 6.242 6.592 6.913 7.242 7.571 7.900 8.229 8.558 8.887 9.216

(1) 4.849 5.113 5.409 5.743 6.124 6.541 6.997 7.454 7.911 8.368 8.825 9.282 9.739 10.196 10.653

(2) 7.102 7.512 7.973 8.496 9.082 9.722 10.34 10.95 11.56 12.17 12.78 13.39 14.00 14.61 15.22

(3) 9.946 10.52 11.17 11.90 12.72 13.59 14.52 15.49 16.46 17.43 18.40 19.37 20.34 21.31 22.28

(4) 12.79 13.51 14.36 15.28 16.27 17.24 18.21 19.18 20.15 21.12 22.09 23.06 24.03 25.00 25.97

TABLE 6.16 :  $f_1$  ( $\delta = -0.5, \alpha = -8$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 4.849 5.113 5.409 5.743 6.124 6.541 6.997 7.454 7.911 8.368 8.825 9.282 9.739 10.196 10.653

(1) 6.056 6.499 6.979 7.493 8.041 8.616 9.218 9.847 10.503 11.187 11.898 12.627 13.385 14.171 14.985

(2) 11.759 12.61 13.40 14.20 15.00 15.80 16.60 17.40 18.20 19.00 19.80 20.60 21.40 22.20 23.00

(3) 14.52 15.33 16.22 17.19 18.06 18.93 19.80 20.67 21.54 22.41 23.28 24.15 25.02 25.89 26.76

(4) 17.29 18.14 19.01 19.88 20.75 21.62 22.49 23.36 24.23 25.10 25.97 26.84 27.71 28.58 29.45

TABLE 6.16 :  $f_1$  ( $\delta = -6, \alpha = -1.0$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 6.627 6.925 7.223 7.521 7.819 8.117 8.415 8.713 9.011 9.309 9.607 9.905 10.203 10.501 10.800

(1) 14.27 15.14 16.01 16.88 17.75 18.62 19.49 20.36 21.23 22.10 22.97 23.84 24.71 25.58 26.45

(2) 20.72 21.70 22.65 23.61 24.57 25.53 26.49 27.45 28.41 29.37 30.33 31.29 32.25 33.21 34.17

(3) 26.67 27.78 28.89 29.99 31.09 32.19 33.29 34.39 35.49 36.59 37.69 38.79 39.89 40.99 42.09

(4) 32.62 33.84 35.06 36.28 37.50 38.72 39.94 41.16 42.38 43.60 44.82 46.04 47.26 48.48 49.70

TABLE 6.16 :  $f_1$  ( $\delta = -7, \alpha = -6$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 4.719 5.021 5.323 5.625 5.927 6.229 6.531 6.833 7.135 7.437 7.739 8.041 8.343 8.645 8.947

(1) 7.492 8.098 8.704 9.310 9.916 10.522 11.128 11.734 12.340 12.946 13.552 14.158 14.764 15.370 15.976

(2) 11.22 12.03 12.84 13.65 14.46 15.27 16.08 16.89 17.70 18.51 19.32 20.13 20.94 21.75 22.56

(3) 14.95 15.86 16.67 17.48 18.29 19.10 19.91 20.72 21.53 22.34 23.15 23.96 24.77 25.58 26.39

(4) 18.68 19.69 20.50 21.31 22.12 22.93 23.74 24.55 25.36 26.17 26.98 27.79 28.60 29.41 30.22

TABLE 6.16 :  $f_1$  ( $\delta = -7, \alpha = -7$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 5.469 5.815 6.162 6.508 6.854 7.200 7.546 7.892 8.238 8.584 8.930 9.276 9.622 9.968 10.314

(1) 9.326 9.958 10.590 11.222 11.854 12.486 13.118 13.750 14.382 15.014 15.646 16.278 16.910 17.542 18.174

(2) 13.22 14.03 14.84 15.65 16.46 17.27 18.08 18.89 19.70 20.51 21.32 22.13 22.94 23.75 24.56

(3) 16.64 17.59 18.41 19.22 20.03 20.84 21.65 22.46 23.27 24.08 24.89 25.70 26.51 27.32 28.13

(4) 20.06 21.14 22.22 23.30 24.38 25.46 26.54 27.62 28.70 29.78 30.86 31.94 33.02 34.10 35.18

TABLE 6.16 :  $f_1$  ( $\delta = -7, \alpha = -8$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 6.234 6.574 6.914 7.254 7.594 7.934 8.274 8.614 8.954 9.294 9.634 9.974 10.314 10.654 11.000

(1) 7.422 7.959 8.496 9.033 9.570 10.107 10.644 11.181 11.718 12.255 12.792 13.329 13.866 14.403 14.940

(2) 11.41 12.13 12.85 13.57 14.29 15.01 15.73 16.45 17.17 17.89 18.61 19.33 20.05 20.77 21.49

(3) 14.51 15.31 16.13 16.95 17.77 18.59 19.41 20.23 21.05 21.87 22.69 23.51 24.33 25.15 25.97

(4) 18.64 19.68 20.72 21.76 22.80 23.84 24.88 25.92 26.96 28.00 29.04 30.08 31.12 32.16 33.20

TABLE 6.16 :  $f_1$  ( $\delta = -7, \alpha = -9$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 6.939 7.280 7.621 7.962 8.303 8.644 8.985 9.326 9.667 10.008 10.349 10.690 11.031 11.372 11.713

(1) 11.54 12.33 13.12 13.91 14.70 15.49 16.28 17.07 17.86 18.65 19.44 20.23 21.02 21.81 22.60

(2) 16.17 17.11 18.05 18.99 19.93 20.87 21.81 22.75 23.69 24.63 25.57 26.51 27.45 28.39 29.33

(3) 20.81 21.92 23.03 24.14 25.25 26.36 27.47 28.58 29.69 30.80 31.91 33.02 34.13 35.24 36.35

(4) 25.45 26.67 27.89 29.10 30.31 31.52 32.73 33.94 35.15 36.36 37.57 38.78 39.99 41.20 42.41

TABLE 6.16 :  $f_1$  ( $\delta = -7, \alpha = -1.0$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 7.627 7.970 8.313 8.656 8.999 9.342 9.685 10.028 10.371 10.714 11.057 11.400 11.743 12.086 12.429

(1) 12.66 13.67 14.68 15.69 16.70 17.71 18.72 19.73 20.74 21.75 22.76 23.77 24.78 25.79 26.80

(2) 17.37 18.64 19.91 21.18 22.45 23.72 24.99 26.26 27.53 28.80 30.07 31.34 32.61 33.88 35.15

(3) 22.09 23.59 25.09 26.59 28.09 29.59 31.09 32.59 34.09 35.59 37.09 38.59 40.09 41.59 43.09

(4) 26.82 28.44 30.06 31.68 33.30 34.92 36.54 38.16 39.78 41.40 43.02 44.64 46.26 47.88 49.50

TABLE 6.16 :  $f_1$  ( $\delta = -8, \alpha = -6$ )

W: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MOE: 8.146 8.574 9.002 9.430 9.858 10.286 10.714 11.142 11.570 11.998 12.426 12.854 13.282 13.710 14.138

(1) 13.44 14.37 15.30 16.23 17.16 18.09 19.02 19.95 20.88 21.81 22.74 23.67 24.60 25.53 26.46

(2) 18.45 19.59 20.72 21.85 22.98 24.11 25.24 26.37 27.50 28.63 29.76 30.89 32.02 33.15 34.28

(3) 23.46 24.79 26.12 27.45 28.78 30.11 31.44 32.77 34.10 35.43 36.76 38.09 39.42 40.75 42.08

(4) 28.47 29.90 31.33 32.76 34.19 35.62 37.05 38.48 39.91 41.34 42.77 44.20 45.63 47.06 48.49

TABLE B.16 :  $f_1(1/a - 6, 1/a - 7)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.168 4.791 4.453 4.656 4.966 5.210 5.579 6.032 6.590 7.284 8.151 9.165 9.236 9.441 9.003

(2) 7.104 7.605 7.279 8.231 8.709 10.137 11.066 12.005 12.308 11.728 11.27 11.34 11.35 10.85

(3) 10.13 10.62 11.22 11.93 12.76 13.71 14.73 14.99 14.30 13.97 13.11 13.95 13.67 13.15 13.44 13.06 12.59 13.35 15.46

(4) 13.19 13.89 14.72 15.67 16.75 17.76 17.06 16.63 16.63 15.71 16.19 16.16 16.45 17.60 17.20

TABLE B.16 :  $f_1(1/a - 6, 1/a - 8)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.893 3.796 4.097 4.289 4.519 4.795 5.178 5.559 6.094 6.538 7.408 8.347 9.193 9.231 8.664

(2) 6.397 6.672 7.005 7.401 7.871 8.429 9.091 9.895 10.78 11.58 11.46 10.97 10.82 11.05 10.70

(3) 9.686 9.402 10.00 10.61 11.23 12.17 13.11 13.95 13.67 13.15 13.44 13.06 12.59 13.35 15.46

(4) 11.72 12.33 13.04 13.67 14.28 15.05 15.32 15.55 15.78 15.30 14.99 14.45 17.95 17.37 17.20

TABLE B.16 :  $f_1(1/a - 6, 1/a - 9)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.419 3.621 3.786 3.978 4.198 4.459 4.770 5.143 5.598 6.100 6.893 7.733 8.644 8.937 8.697

(2) 5.813 6.078 6.395 6.752 7.178 7.680 8.275 9.061 9.898 10.69 11.04 10.70 10.47 10.72 10.52

(3) 8.183 8.593 9.053 9.609 10.24 10.99 11.84 12.75 13.11 12.66 12.71 12.73 12.32 12.70 14.77

(4) 10.57 11.11 11.75 12.48 13.33 14.28 15.15 14.92 14.73 14.84 14.37 15.17 17.04 17.11 16.78

TABLE B.16 :  $f_1(1/a - 6, 1/a - 10)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.182 3.319 3.506 3.694 3.914 4.169 4.465 4.800 5.250 5.778 6.437 7.258 8.176 8.653 8.515

(2) 5.317 5.516 5.813 6.215 6.617 7.076 7.624 8.273 9.041 9.901 10.53 10.40 10.18 10.40 10.33

(3) 7.465 7.844 8.281 8.765 9.371 10.05 10.84 11.72 12.40 12.23 12.10 12.34 12.05 12.23 14.13

(4) 9.622 10.13 10.71 11.38 12.15 13.03 13.94 14.28 13.94 14.23 13.93 14.25 16.01 16.79 16.46

TABLE B.16 :  $f_1(1/a - 7, 1/a - 6)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 5.359 5.505 5.714 5.968 6.324 6.738 7.241 7.853 8.599 9.456 10.45 10.66 10.17 9.617 9.107

(2) 9.354 9.771 10.29 10.92 11.66 12.52 13.51 14.36 13.86 12.92 12.24 12.44 12.21 11.59 10.97

(3) 13.49 14.19 15.02 15.99 17.08 18.04 17.09 15.91 16.16 15.43 14.46 13.98 15.40 17.02 16.38

(4) 17.69 18.67 19.80 21.05 21.54 19.70 19.63 18.64 17.62 18.84 20.62 19.92 18.61 18.69 18.20

TABLE B.16 :  $f_1(1/a - 7, 1/a - 7)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.863 5.007 5.195 5.432 5.722 6.075 6.504 7.026 7.664 8.441 9.346 10.06 9.918 9.467 9.007

(2) 8.268 8.639 9.074 9.599 10.22 10.96 11.81 12.74 13.23 12.63 11.94 11.71 11.66 11.41 10.86

(3) 11.82 12.39 13.09 13.90 14.84 15.86 16.33 15.36 14.84 14.96 14.21 13.56 14.05 16.01 16.17

(4) 16.39 16.20 17.15 18.23 19.32 18.94 16.97 16.66 18.48 19.48 18.59 17.97 17.97

TABLE B.16 :  $f_1(1/a - 7, 1/a - 8)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.437 4.592 4.779 5.054 5.372 5.722 6.075 6.504 7.026 7.664 8.441 9.346 10.06 9.918 9.467 9.007

(2) 7.463 7.784 8.171 8.632 9.177 9.819 10.57 11.42 12.22 12.22 11.45 11.28 11.42 11.19 10.72

(3) 10.55 11.06 11.67 12.37 13.19 14.11 14.98 14.82 14.10 14.17 13.68 13.27 13.29 14.93 15.86

(4) 13.68 14.38 15.20 16.15 17.18 18.05 18.99 16.90 16.50 15.66 16.37 18.46 18.30 17.60 17.66

TABLE B.16 :  $f_1(a - 9, a - 9)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.396 3.669 3.915 4.200 4.516 4.868 5.262 5.694 6.171 6.714 7.324 8.014 8.789 9.639 10.56

(2) 8.232 8.437 8.561 10.55 11.22 11.96 12.68 12.94 12.44 11.91 11.76 11.64 11.19 10.69 10.24

(3) 12.58 13.24 13.99 14.81 15.68 16.59 16.94 15.71 15.34 14.63 14.33 13.69 12.79 12.78 15.45

(4) 16.19 17.05 18.00 18.95 19.05 18.11 17.99 17.19 17.01 16.14 15.25 14.75 14.11 17.96 17.90

TABLE B.16 :  $f_1(a - 9, a - 10)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.111 3.184 3.492 3.684 3.949 4.281 4.681 5.161 5.694 6.281 6.931 7.651 8.441 9.314 10.26 11.22

(2) 5.954 6.137 6.429 6.789 7.219 7.719 8.289 8.939 9.689 10.54 11.51 12.42 12.69 12.42 12.42

(3) 13.82 14.37 15.30 16.16 16.96 16.95 16.04 15.71 15.34 14.63 14.33 13.69 12.79 12.78 15.45

(4) 17.81 18.72 19.71 20.57 20.00 19.21 18.80 17.68 16.14 15.38 15.09 14.11 14.54 15.23 17.90

TABLE B.16 :  $f_1(a - 10, a - 6)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.317 3.315 3.718 3.932 4.285 4.620 5.014 5.463 5.966 6.522 7.159 7.859 8.618 9.440 10.32

(2) 5.508 5.637 6.208 6.835 7.514 8.273 9.098 9.933 10.82 11.77 12.77 13.84 14.91 16.07 17.31

(3) 7.732 8.173 8.694 9.295 9.951 10.73 11.60 12.50 13.44 14.41 15.49 16.69 17.91 19.15 21.40

(4) 9.916 10.51 11.18 11.94 12.79 13.73 14.73 15.80 16.97 18.24 19.61 21.07 22.54 24.02 26.49

TABLE B.16 :  $f_1(a - 10, a - 7)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.889 4.070 4.319 4.601 4.973 5.294 5.727 6.237 6.843 7.561 8.351 9.242 10.24 11.35 12.57

(2) 6.337 6.719 7.178 7.649 8.185 8.803 9.512 10.30 11.18 12.15 13.19 14.37 15.64 17.01 18.47

(3) 8.773 9.419 10.05 10.71 11.47 12.29 13.09 13.95 14.93 16.01 17.17 18.54 20.01 21.47 23.94

(4) 11.51 12.18 12.93 13.77 14.69 15.63 16.61 17.68 18.84 20.15 21.54 23.01 24.46 26.91 30.38

TABLE B.16 :  $f_1(a - 10, a - 8)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.324 4.627 4.918 5.199 5.572 5.977 6.415 6.970 7.566 8.240 8.922 9.741 10.68 11.74 12.91

(2) 7.750 7.649 8.218 8.628 9.195 9.844 10.58 11.33 12.16 13.07 14.09 15.27 16.54 17.91 19.37

(3) 10.16 10.77 11.52 12.07 12.87 13.70 14.58 15.46 16.44 17.51 18.67 19.94 21.31 22.77 25.24

(4) 13.07 13.80 14.61 15.52 16.42 17.39 18.41 19.54 20.77 22.09 23.51 25.04 26.67 28.39 31.16

TABLE B.16 :  $f_1(a - 10, a - 9)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.555 4.827 5.107 5.487 5.948 6.457 6.994 7.675 8.410 9.203 10.06 10.99 11.97 13.06 14.26

(2) 8.075 8.494 8.967 9.524 10.11 10.80 11.55 12.39 13.28 14.31 15.41 16.58 17.84 19.21 20.67

(3) 11.39 12.07 12.68 13.36 14.19 14.96 15.80 16.70 17.74 18.91 20.21 21.56 23.03 24.51 26.00

(4) 14.57 15.35 16.23 17.17 17.92 18.81 19.74 20.71 21.80 22.99 24.28 25.67 27.16 28.74 30.33

TABLE B.16 :  $f_1(a - 6, 1/a - 6)$

$\mu$  0.015 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.316 4.316 4.518 4.725 4.975 5.275 5.625 6.025 6.475 6.975 7.525 8.125 8.775 9.475 10.225

(2) 8.017 8.316 8.614 8.912 9.210 9.508 9.806 10.104 10.402 10.700 11.098 11.596 12.194 12.892 13.690

(3) 11.57 12.17 12.68 13.28 13.88 14.48 15.08 15.68 16.28 16.88 17.48 18.08 18.68 19.28 19.88

(4) 15.17 16.01 16.99 18.11 19.23 20.35 21.47 22.60 23.72 24.84 25.96 27.08 28.20 29.32 30.44

TABLE 8.16:  $\epsilon_1$  ( $1/\epsilon - 9, 1/\epsilon - 6$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	6.889	7.077	7.366	7.693	8.124	8.649	9.276	10.02	10.86	11.57	11.43	10.82	10.20	9.422	9.104
(2)	12.02	12.56	13.22	14.01	14.92	15.89	16.99	18.24	19.64	21.17	23.44	23.61	21.29	11.60	10.34
(3)	17.34	18.23	19.27	20.41	20.99	21.63	22.81	24.63	26.81	29.15	31.35	35.35	36.65	17.52	17.29
(4)	22.74	23.56	25.49	25.16	22.90	22.65	23.79	20.09	21.51	22.61	21.41	20.15	19.74	19.21	18.23

TABLE 8.16:  $\epsilon_2$  ( $1/\epsilon - 9, 1/\epsilon - 7$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	6.272	6.437	6.679	6.982	7.352	7.809	8.337	8.977	9.721	10.47	10.65	10.52	10.60	9.491	9.117
(2)	10.66	11.10	11.66	12.32	13.09	13.96	14.76	14.67	13.77	12.93	12.76	12.63	12.67	11.45	10.87
(3)	15.19	15.92	16.60	17.19	18.75	18.52	17.18	16.29	16.38	15.33	14.73	15.76	16.63	16.37	16.23
(4)	19.78	20.80	21.56	22.95	21.71	20.63	20.17	18.67	19.20	20.21	20.55	19.85	19.66	18.68	18.64

TABLE 8.16:  $\epsilon_3$  ( $1/\epsilon - 9, 1/\epsilon - 8$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	5.274	5.504	6.144	6.432	6.774	7.162	7.668	8.241	8.913	9.468	10.21	10.16	9.769	8.358	8.803
(2)	9.505	10.01	10.50	11.09	11.77	12.54	13.35	13.81	13.13	12.42	12.24	12.20	11.81	11.26	10.73
(3)	13.57	14.21	14.98	15.65	16.78	17.34	16.53	15.79	15.11	15.02	14.33	14.43	15.75	16.55	16.72
(4)	17.58	18.47	19.48	20.53	20.64	19.40	19.23	18.23	17.18	16.99	20.03	19.50	19.66	18.47	17.65

TABLE 8.16:  $\epsilon_4$  ( $1/\epsilon - 9, 1/\epsilon - 9$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	5.219	5.432	5.678	5.963	6.294	6.662	7.132	7.678	8.305	9.011	9.643	9.718	9.505	8.119	8.716
(2)	8.770	9.177	9.579	10.12	10.74	11.45	12.22	12.88	12.82	12.24	11.82	11.78	11.53	11.06	10.57
(3)	12.27	12.87	13.56	14.36	15.22	16.00	16.64	15.07	14.94	14.59	13.94	13.65	14.91	16.69	16.79
(4)	15.85	16.55	17.57	18.56	19.25	18.45	18.07	17.61	16.91	17.51	18.97	19.06	18.32	16.66	17.43

TABLE 8.16:  $\epsilon_5$  ( $1/\epsilon - 9, 1/\epsilon - 1.0$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	4.788	5.009	5.259	5.543	5.863	6.246	6.667	7.124	7.604	8.100	8.499	8.172	8.465	6.910	8.451
(2)	7.975	8.363	8.807	9.315	9.900	10.571	11.31	12.03	12.78	11.81	11.45	11.60	11.27	10.45	10.41
(3)	11.20	11.76	12.41	13.15	13.97	14.79	15.69	14.48	14.21	14.12	13.59	13.37	14.28	17.44	15.56
(4)	14.43	15.10	16.03	16.97	17.83	17.63	17.03	16.95	16.27	16.45	17.78	16.59	16.00	16.00	17.47

TABLE 8.16:  $\epsilon_6$  ( $1/\epsilon - 1.0, 1/\epsilon - 9$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	7.415	7.863	8.161	8.545	8.920	9.504	10.27	11.05	11.83	12.06	11.52	10.64	10.20	9.423	9.108
(2)	11.84	12.34	12.95	13.67	14.50	15.30	16.68	15.36	14.31	14.02	13.79	13.06	12.15	11.60	10.98
(3)	19.22	20.24	21.27	22.45	24.00	24.63	19.27	18.32	16.96	16.03	16.46	17.98	18.20	17.31	16.40
(4)	25.25	26.58	27.78	29.67	31.51	33.05	21.29	21.08	23.47	22.92	21.51	20.40	20.17	19.23	18.23

TABLE 8.16:  $\epsilon_7$  ( $1/\epsilon - 1.0, 1/\epsilon - 7$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	6.947	7.152	7.420	7.755	8.164	8.655	9.239	9.918	10.645	11.21	11.10	10.58	10.01	8.426	8.910
(2)	11.84	12.34	12.95	13.67	14.50	15.30	16.68	15.36	14.31	14.02	13.79	13.06	12.15	11.60	10.98
(3)	16.81	17.69	18.63	19.66	20.21	18.68	17.82	17.59	16.57	15.60	15.31	16.49	17.54	17.05	16.24
(4)	21.97	23.09	24.28	24.38	22.44	22.04	20.51	19.481	20.96	22.04	21.15	20.01	19.54	18.58	18.07

TABLE 8.16:  $\epsilon_1$  ( $1/\epsilon - 7, 1/\epsilon - 5$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	4.059	4.225	4.417	4.639	4.897	5.202	5.582	5.994	6.517	7.153	7.821	8.353	9.209	9.646	8.714
(2)	6.207	6.505	6.681	7.249	7.711	8.248	8.877	9.607	10.41	11.05	10.98	10.64	10.70	10.95	10.55
(3)	9.546	10.01	10.56	11.19	11.93	12.77	13.65	14.10	13.59	13.38	13.45	12.58	12.82	14.10	15.50
(4)	12.31	12.96	13.70	14.54	15.49	16.39	16.28	15.80	15.89	15.30	15.58	17.18	17.90	17.31	17.31

TABLE 8.16:  $\epsilon_2$  ( $1/\epsilon - 7, 1/\epsilon - 1.0$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	3.724	3.894	4.099	4.311	4.566	4.861	5.208	5.620	6.115	6.717	7.445	8.267	8.884	8.818	6.542
(2)	6.124	6.291	6.530	6.840	7.225	7.695	8.264	8.949	9.765	10.67	11.18	10.78	10.19	9.671	9.107
(3)	10.59	11.17	11.76	12.47	13.30	14.25	15.19	15.05	13.99	13.69	12.84	12.88	12.27	11.60	10.98
(4)	15.42	16.21	17.16	18.23	19.33	19.97	17.38	17.12	16.68	15.55	14.71	15.71	17.03	17.23	16.39

TABLE 8.16:  $\epsilon_3$  ( $1/\epsilon - 8, 1/\epsilon - 6$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	6.124	6.291	6.530	6.840	7.225	7.695	8.264	8.949	9.765	10.67	11.18	10.78	10.19	9.671	9.107
(2)	10.59	11.17	11.76	12.47	13.30	14.25	15.19	15.05	13.99	13.69	12.84	12.88	12.27	11.60	10.98
(3)	15.42	16.21	17.16	18.23	19.33	19.97	17.38	17.12	16.68	15.55	14.71	15.71	17.03	17.23	16.39
(4)	20.22	21.32	22.58	23.72	22.09	21.29	20.52	18.99	19.40	21.15	21.27	20.02	19.11	19.11	18.22

TABLE 8.16:  $\epsilon_4$  ( $1/\epsilon - 8, 1/\epsilon - 7$ )

MODE: 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

(1)	5.558	5.722	5.937	6.207	6.538	6.939	7.424	8.010	8.713	9.531	10.30	10.40	9.975	9.477	8.099

TABLE 8.16 :  $\epsilon_1 (1/a - 1.0, 1/a - 8)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 6.328 6.560 6.827 7.145 7.524 7.972 8.500 9.115 9.800 10.43 11.07 11.72 12.37 13.02 13.68

(2) 11.22 11.66 12.30 13.04 13.84 14.50 15.21 15.83 16.41 17.00 17.59 18.17 18.74 19.31 19.88

(3) 13.64 14.30 15.06 15.91 16.77 17.69 18.67 19.71 20.80 21.94 23.13 24.41 25.79 27.26 28.82

(4) 19.53 20.50 21.58 22.71 23.90 25.19 26.56 28.02 29.57 31.21 32.93 34.73 36.60 38.54 40.54

TABLE 8.16 :  $\epsilon_1 (1/a - 1.0, 1/a - 9)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 5.796 6.036 6.309 6.625 6.992 7.419 7.919 8.499 9.153 9.804 10.45 11.09 11.72 12.35 12.98

(2) 12.68 13.33 14.07 14.91 15.84 16.87 17.99 19.21 20.52 21.93 23.43 25.01 26.67 28.41 30.22

(3) 13.64 14.30 15.06 15.91 16.77 17.69 18.67 19.71 20.80 21.94 23.13 24.41 25.79 27.26 28.82

(4) 17.61 18.49 19.48 20.43 21.42 22.45 23.52 24.64 25.80 27.09 28.41 29.77 31.18 32.63 34.12

TABLE 8.17 :  $\epsilon_1 (1/a - 5, a - 6)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 4.540 4.687 4.711 4.735 4.741 4.739 4.729 4.713 4.688 4.653 4.608 4.554 4.491 4.419 4.337

(2) 7.482 7.621 7.717 7.779 7.809 7.813 7.801 7.774 7.733 7.678 7.608 7.523 7.425 7.316 7.197

(3) 10.66 10.33 10.61 11.19 11.94 12.84 13.87 14.74 15.25 15.23 15.20 15.19 15.19 15.19 15.19

(4) 13.47 13.84 14.59 15.54 16.62 17.72 17.41 15.94 14.65 13.70 13.76 15.40 17.62 17.59 16.71

TABLE 8.17 :  $\epsilon_1 (1/a - 6, a - 7)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 5.285 5.451 5.475 5.510 5.679 4.727 4.451 4.212 4.012 3.846 3.711 3.605 3.526 3.474 3.422

(2) 8.174 8.489 8.422 8.331 8.577 8.055 8.702 10.50 11.29 12.08 11.90 12.08 11.90 12.08 11.90

(3) 12.38 11.96 12.27 12.91 13.75 14.70 15.61 15.51 14.47 13.41 12.48 11.69 10.95 10.07 9.919

(4) 15.63 16.03 16.68 17.82 19.02 19.27 17.76 16.23 15.05 14.56 15.45 17.29 21.67 24.57 16.60

TABLE 8.17 :  $\epsilon_1 (1/a - 6, a - 8)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 6.019 6.203 6.222 6.222 6.222 6.222 6.222 6.222 6.222 6.222 6.222 6.222 6.222 6.222 6.222

(2) 10.06 10.64 9.511 9.371 9.516 10.11 10.78 11.57 12.32 12.54 12.07 11.41 10.78 10.21 9.740

(3) 14.06 13.53 13.84 14.54 15.41 16.32 16.72 15.87 14.70 13.64 12.31 11.92 11.26 10.69 10.23

(4) 17.72 18.15 19.07 20.14 20.93 19.91 18.13 16.55 15.68 15.74 16.99 18.57 18.66 17.81 16.53

TABLE 8.17 :  $\epsilon_1 (1/a - 6, a - 9)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 6.740 6.935 6.946 6.946 6.946 6.946 6.946 6.946 6.946 6.946 6.946 6.946 6.946 6.946 6.946

(2) 11.70 11.19 10.53 10.32 10.55 11.05 11.72 12.44 12.96 12.63 12.24 11.57 10.94 10.39 9.932

(3) 15.67 15.01 15.32 16.04 16.91 17.63 18.36 19.10 19.97 19.63 19.27 18.62 18.07 17.52 17.04

(4) 19.73 20.17 21.12 22.13 22.17 20.41 18.53 17.17 16.48 16.93 18.23 19.27 18.65 17.96 17.08

TABLE 8.17 :  $\epsilon_1 (1/a - 6, a - 10)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 7.442 7.644 7.641 7.632 7.627 7.627 7.627 7.627 7.627 7.627 7.627 7.627 7.627 7.627 7.627

(2) 12.92 12.27 11.45 11.37 11.47 12.51 13.14 13.43 13.09 12.44 11.75 11.13 10.58 10.12

(3) 17.21 16.38 16.69 17.42 18.24 18.64 17.65 16.58 15.28 14.16 13.22 12.41 11.72 11.12 10.60

(4) 21.62 22.09 23.05 23.06 23.01 20.90 19.05 17.75 17.32 17.97 19.18 19.69 19.04 18.32 17.25

TABLE 8.17 :  $\epsilon_1 (1/a - 7, a - 8)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 3.918 4.077 4.279 4.726 4.770 4.003 3.616 3.640 3.487 3.199 3.257 3.183 3.180 3.109 3.172

(2) 6.926 6.961 6.743 6.560 6.608 6.887 7.344 7.957 8.750 9.477 10.268 10.94 10.49 9.542 9.424

(3) 9.519 9.290 9.231 9.681 10.23 11.05 12.97 13.68 13.13 12.27 11.48 10.79 10.71 9.727

(4) 12.02 12.03 12.57 13.35 14.29 15.36 16.32 15.77 14.58 13.52 12.59 11.59 11.59 10.58 10.47

TABLE 8.17 :  $\epsilon_1 (1/a - 7, a - 7)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 4.101 4.165 4.914 4.946 4.834 4.630 4.203 4.183 4.020 3.852 3.709 3.624 3.528 3.473 3.452

(2) 8.882 8.692 8.703 8.753 8.750 8.964 9.403 9.667 9.481 9.279 9.138 9.114 9.12 9.053 9.063

(3) 11.20 10.76 10.74 11.17 11.84 12.68 13.62 14.44 14.22 13.74 12.46 11.68 11.68 10.79 10.71

(4) 13.35 13.31 14.55 15.41 16.43 17.44 17.38 16.10 14.68 13.35 12.68 12.68 12.68 11.79 11.79

TABLE 8.17 :  $\epsilon_1 (1/a - 7, a - 6)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 5.194 5.421 5.584 5.613 5.473 5.226 4.952 4.674 4.415 4.209 4.003 3.944 3.852 3.765 3.725

(2) 9.183 9.108 8.840 8.535 8.577 8.630 9.300 10.03 10.63 11.57 11.36 10.74 10.74 10.29 9.736

(3) 12.74 12.18 12.11 12.57 13.79 14.16 15.55 15.29 14.55 13.55 12.69 11.92 11.25 10.49 10.23

(4) 18.82 15.76 16.43 17.36 18.38 18.32 17.68 16.47 15.31 14.47 15.09 16.59 17.59 17.69 16.90

TABLE 8.17 :  $\epsilon_1 (1/a - 7, a - 5)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 5.815 6.005 6.238 6.254 6.075 5.762 5.455 5.148 4.876 4.639 4.435 4.301 4.194 4.114 4.032

(2) 10.28 10.27 9.603 9.336 9.335 9.645 10.13 10.85 11.59 12.59 11.56 11.49 11.91 12.38 9.959

(3) 14.24 13.51 13.28 13.88 14.82 15.47 16.32 15.83 14.68 13.85 12.95 12.17 11.59 10.92 10.44

(4) 17.61 17.50 18.21 19.17 20.06 19.63 18.34 16.90 15.84 15.47 16.19 17.64 18.41 17.96 17.05

TABLE 8.17 :  $\epsilon_1 (1/a - 7, a - 4)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 6.620 6.679 6.858 6.865 6.643 6.290 5.966 5.546 5.228 4.949 4.708 4.492 4.319 4.183 4.059

(2) 11.36 11.35 10.70 10.17 10.68 10.38 10.68 11.54 12.19 12.59 12.22 11.68 11.00 10.59 10.12

(3) 15.70 14.74 14.57 15.08 15.84 16.63 16.96 16.26 15.17 14.13 13.21 12.41 11.79 11.19 10.69

(4) 19.30 19.14 19.90 20.66 21.47 20.50 18.63 17.36 16.40 16.25 17.13 18.42 18.71 18.04 17.23

TABLE 8.17 :  $\epsilon_1 (1/a - 8, a - 6)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 3.139 3.418 3.705 3.965 3.940 3.876 3.750 3.606 3.469 3.350 3.251 3.179 3.129 3.123 3.122

(2) 6.129 6.308 6.386 6.330 6.061 6.195 6.574 7.022 7.681 8.515 9.153 9.444 10.38 9.94 9.416

(3) 8.673 8.500 8.428 8.478 9.066 9.703 10.49 11.43 12.44 12.64 12.20 11.46 10.79 10.21 9.726

(4) 10.56 10.77 11.10 11.73 12.53 13.48 14.54 15.23 14.45 13.44 12.63 12.42 13.65 16.34 16.59

TABLE 8.17 :  $\epsilon_1 (1/a - 8, a - 7)$

$\epsilon_1$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

NOTE: (1) 4.003 4.209 4.399 4.535 4.569 4.485 4.289 4.151 3.981 3.831 3.704 3.602 3.574 3.473 3.452

(2) 7.134 7.336 7.596 7.879 7.810 7.485 6.973 6.713 6.576 6.48 10.85 10.52 10.52 10.52 9.516

(3) 10.09 9.931 9.715 9.82 10.44 11.14 12.00 12.94 13.59 13.19 12.41 11.66 11.00 10.54 9.918

(4) 12.74 12.47 12.83 13.54 14.43 15.44 16.30 15.87 14.76 13.79 13.21 13.68 15.49 20.24 16.71

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	5.017	5.206	5.516	5.700	5.884	5.976	5.709	5.456	5.187	4.911	4.720	4.476	4.318	4.167	4.029
(2)	8.989	9.319	9.419	9.278	8.993	8.591	8.077	7.468	6.782	6.054	5.324	4.604	3.914	3.274	2.682
(3)	12.64	12.80	12.22	12.10	12.64	13.24	14.76	16.75	18.49	19.59	19.59	18.15	15.29	11.71	8.12
(4)	16.30	15.65	15.73	16.44	17.35	18.19	18.10	16.99	15.84	14.58	13.29	11.93	10.53	9.00	7.35

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	0.675	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	2.759	2.918	3.056	3.254	3.403	3.505	3.531	3.489	3.479	3.319	3.237	3.171	3.126	3.107	3.122
(3)	4.547	5.193	5.384	5.457	5.411	5.283	5.074	4.782	4.435	4.048	3.634	3.204	2.774	2.352	1.937
(4)	7.074	7.314	7.309	7.264	7.450	7.867	8.456	9.207	10.11	11.15	11.77	11.36	10.75	10.29	9.793
(5)	9.166	9.195	9.126	9.512	10.009	10.84	11.73	12.76	13.62	14.26	14.66	14.82	14.81	14.63	14.10

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	3.211	3.393	3.585	3.777	3.947	4.093	4.212	4.291	4.315	4.278	4.188	4.044	3.851	3.621	3.352
(2)	5.728	6.037	6.255	6.384	6.425	6.384	6.266	6.072	5.779	5.393	4.927	4.404	3.849	3.279	2.697
(3)	6.258	6.503	6.466	6.316	6.570	7.033	7.615	8.305	9.100	10.00	11.00	12.12	13.36	14.70	16.14
(4)	10.67	10.66	10.00	10.98	11.63	12.45	13.41	14.36	15.31	16.35	17.45	18.60	19.80	21.00	22.14

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	3.457	3.809	4.072	4.285	4.472	4.624	4.708	4.708	4.624	4.378	4.227	4.063	3.896	3.610	3.284
(3)	6.559	6.268	7.106	7.159	7.077	6.916	6.593	6.204	5.754	5.254	4.714	4.139	3.529	2.890	2.229
(4)	9.408	9.666	9.591	9.445	9.688	10.311	10.81	11.265	11.552	11.684	11.64	11.43	11.13	10.68	10.22
(5)	12.15	12.07	11.55	12.36	13.07	13.67	14.50	15.43	16.45	17.58	18.77	19.99	21.22	22.45	23.68

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	4.054	4.313	4.584	4.773	4.974	5.097	5.052	4.932	4.738	4.459	4.118	3.754	3.374	2.991	2.607
(2)	7.346	7.461	7.322	7.022	7.022	7.581	7.686	7.914	8.447	9.103	9.978	10.61	10.63	10.30	9.927
(3)	10.54	10.80	10.61	10.36	10.55	11.30	12.07	12.83	13.40	13.40	12.80	12.12	11.48	10.92	10.44
(4)	13.60	13.41	13.21	13.66	14.42	15.34	16.18	16.17	15.29	14.38	13.75	13.28	12.98	12.69	12.44

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	4.194	4.370	4.585	4.802	5.144	5.492	5.874	6.206	6.503	6.844	5.517	5.174	4.867	4.593	4.318
(2)	7.239	7.132	6.992	6.805	6.414	6.067	6.157	6.654	7.276	8.079	9.119	10.42	11.61	11.37	10.84
(3)	11.34	11.54	12.51	11.54	10.84	10.56	11.76	12.80	14.00	14.77	14.10	13.21	12.39	11.67	11.62
(4)	14.96	15.65	14.59	13.97	14.80	15.90	17.16	17.68	16.76	15.51	14.45	13.66	13.00	12.26	11.57

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	3.709	3.873	4.065	4.290	4.550	4.849	5.183	5.516	5.709	5.599	5.390	5.040	4.763	4.520	4.298
(3)	6.755	7.095	7.484	7.912	8.314	8.650	8.917	7.371	7.207	7.566	8.345	9.432	10.66	11.03	10.67
(4)	9.836	10.35	10.68	11.06	10.59	9.893	10.35	11.19	12.25	13.38	13.66	13.00	12.26	11.57	10.95
(5)	12.91	13.57	13.63	12.62	12.94	13.82	14.92	16.00	16.26	15.27	14.28	13.47	13.16	12.46	11.78

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	4.559	4.788	4.998	5.146	5.178	5.067	4.874	4.655	4.447	4.260	4.098	3.962	3.851	3.765	3.704
(2)	8.137	8.345	8.271	7.988	7.828	7.941	8.207	8.669	8.992	10.42	11.09	10.67	10.18	9.730	9.200
(3)	11.49	11.25	10.55	11.16	11.72	12.47	13.34	14.14	14.19	13.48	12.65	11.90	11.25	10.69	10.22
(4)	14.47	14.10	14.49	15.26	16.20	17.15	17.34	16.30	15.15	14.27	14.01	14.92	16.48	17.44	18.85

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	5.104	5.351	5.578	5.735	5.795	5.614	5.375	5.110	4.857	4.630	4.431	4.260	4.113	3.991	3.892
(3)	9.102	9.329	9.202	8.822	8.580	8.675	9.041	9.610	10.32	11.07	11.89	12.71	13.54	14.35	15.16
(4)	12.67	12.54	12.10	12.31	12.89	13.67	14.51	15.03	14.62	13.77	12.92	12.16	11.49	10.92	10.44
(5)	16.14	15.63	16.05	16.80	17.82	18.54	18.00	16.73	15.58	14.82	14.65	15.96	17.47	17.69	17.02

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	5.635	5.897	6.135	6.297	6.301	6.119	5.827	5.511	5.211	4.942	4.705	4.498	4.318	4.163	4.079
(2)	10.06	10.26	10.08	9.573	9.256	9.310	9.674	10.25	10.84	11.63	11.54	11.05	10.56	10.12	9.70
(3)	14.21	13.69	13.15	13.36	13.98	14.77	15.53	15.71	15.01	14.07	13.19	12.40	11.72	11.12	10.60
(4)	17.74	17.07	17.53	18.39	19.32	19.63	18.55	17.16	16.03	15.38	15.61	16.81	18.02	17.90	17.20

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	3.062	3.233	3.405	3.563	3.674	3.706	3.656	3.557	3.444	3.317	3.176	3.028	2.874	2.718	2.562
(3)	5.479	5.710	5.892	5.786	5.692	5.717	5.928	6.318	6.805	7.408	8.040	8.622	9.177	9.653	9.994
(4)	7.833	7.833	7.931	7.824	8.142	8.633	9.357	10.39	11.18	12.12	14.88	11.42	10.71	10.20	9.725
(5)	10.02	9.866	9.997	10.48	11.17	12.01	13.00	14.03	14.21	13.38	12.15	11.59	12.60	14.52	16.44

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	4.075	4.100	4.125	4.150	4.175	4.200	4.225	4.250	4.275	4.300	4.325	4.350	4.375	4.400	4.425
(2)	3.544	3.705	3.857	4.136	4.261	4.233	4.097	3.954	3.817	3.697	3.598	3.523	3.473	3.442	3.422
(3)	6.319	6.645	6.775	6.698	6.561	6.503	6.722	7.203	7.805	8.513	10.313	10.38	10.24	10.16	10.14
(4)	9.108	9.203	9.071	9.022	9.376	9.961	10.71	11.61	12.53	12.87	12.37	11.63	10.90	10.01	9.977
(5)	11.66	11.44	11.55	12.10	12.87	13.79	14.79	15.33	14.63	13.69	12.95	12.80	14.06	15.11	16.61

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	4.075	4.100	4.125	4.150	4.175	4.200	4.225	4.250	4.275	4.300	4.325	4.350	4.375	4.400	4.425
(2)	4.065	4.216	4.455	4.693	4.828	4.852	4.759	4.598	4.419	4.246	4.092	3.960	3.850	3.765	3.704
(3)	7.255	7.598	7.689	7.567	7.358	7.332	7.505	7.982	8.607	9.390	10.24	10.73	10.56	10.18	9.917
(4)	10.37	10.45	10.17	10.14	10.52	11.15	11.94	12.83	13.49	13.28	12.59	11.48	11.24	10.69	10.22
(5)	13.27	12.94	13.03	13.63	14.46	15.42	16.24	16.01	15.02	14.09	13.50	13.77	15.27	16.92	17.77

TABLE 6.17 :  $f_1, f_2, f_3, f_4, f_5$

MODE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.545	4.780	5.015	5.229	5.370	5.381	5.286	5.053	4.831	4.618	4.426	4.257	4.113	3.991	3.892
(3)	8.137	8.450	8.573	8.381	8.097	8.010	8.209	8.652	9.285	10.05	10.78	11.04	10.77	10.31	9.917
(4)	11.62	11.65	11.24	11.17	11.57	12.23	13.05	13.86	14.16	13.64	12.87	12.14	11.49	10.92	10.44
(5)	14.84	14.35	14.43	15.07	15.95	16.88	17.30	16.52	15.43	14.54	14.11	14.66	16.20	17.32	18.95

TABLE 8.17 :  $f_1 (1/a - 7, 1/a - 1.0)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 3.169 3.327 3.529 3.739 3.910 4.128 4.318 4.469 4.604 4.791 4.795 4.627 4.468 4.377 4.153 4.028

TABLE 8.17 :  $f_1 (1/a - 8, 1/a - 1.0)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 5.350 5.870 6.608 7.619 8.771 10.077 11.571 13.281 15.130 17.064 19.024 21.044 23.064 25.064 27.064 29.064

TABLE 8.17 :  $f_1 (1/a - 9, 1/a - 1.0)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 4.984 5.358 5.805 6.418 7.071 7.771 8.511 9.281 10.077 10.877 11.677 12.477 13.277 14.077 14.877 15.677

TABLE 8.17 :  $f_1 (1/a - 10, 1/a - 1.0)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 4.420 4.828 5.311 5.878 6.428 7.061 7.768 8.541 9.378 10.268 11.211 12.211 13.261 14.361 15.511 16.711

TABLE 8.17 :  $f_1 (1/a - 11, 1/a - 1.0)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 3.655 4.187 4.829 5.572 6.418 7.361 8.404 9.547 10.790 12.133 13.576 15.119 16.762 18.505 20.348 22.191

TABLE 8.17 :  $f_1 (1/a - 12, 1/a - 1.0)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 3.169 3.750 4.433 5.216 6.099 7.082 8.165 9.348 10.631 12.014 13.497 15.080 16.763 18.546 20.429 22.412

TABLE 8.17 :  $f_1 (1/a - 13, 1/a - 1.0)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.828 3.453 4.187 5.020 5.953 6.986 8.119 9.352 10.685 12.118 13.651 15.284 17.017 18.850 20.783 22.816

TABLE 8.17 :  $f_1 (1/a - 5, 1/a - 8)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 3.117 3.476 3.857 4.263 4.694 5.149 5.628 6.131 6.666 7.224 7.804 8.408 9.034 9.682 10.352 11.044

TABLE 8.17 :  $f_1 (1/a - 6, 1/a - 8)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.990 3.444 3.917 4.408 4.924 5.464 6.031 6.624 7.241 7.884 8.551 9.244 9.966 10.716 11.494 12.299

TABLE 8.17 :  $f_1 (1/a - 7, 1/a - 8)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.716 3.263 3.828 4.411 5.011 5.628 6.261 6.919 7.592 8.281 8.986 9.708 10.444 11.194 11.956 12.729

TABLE 8.17 :  $f_1 (1/a - 8, 1/a - 8)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.482 3.116 3.764 4.434 5.124 5.834 6.564 7.314 8.084 8.884 9.704 10.544 11.414 12.314 13.244 14.194

TABLE 8.17 :  $f_1 (1/a - 9, 1/a - 8)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.282 2.982 3.692 4.422 5.172 5.942 6.732 7.542 8.372 9.222 10.092 11.002 11.942 12.912 13.912 14.932

TABLE 8.17 :  $f_1 (1/a - 10, 1/a - 8)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.112 2.772 3.442 4.132 4.842 5.572 6.322 7.092 7.892 8.712 9.552 10.412 11.292 12.192 13.112 14.052

TABLE 8.17 :  $f_1 (1/a - 11, 1/a - 8)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 1.972 2.592 3.222 3.872 4.542 5.232 5.942 6.672 7.422 8.192 8.992 9.812 10.652 11.512 12.392 13.292

TABLE 8.17 :  $f_1 (1/a - 5, 1/a - 9)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 3.117 3.476 3.857 4.263 4.694 5.149 5.628 6.131 6.666 7.224 7.804 8.408 9.034 9.682 10.352 11.044

TABLE 8.17 :  $f_1 (1/a - 6, 1/a - 9)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.990 3.444 3.917 4.408 4.924 5.464 6.031 6.624 7.241 7.884 8.551 9.244 9.966 10.716 11.494 12.299

TABLE 8.17 :  $f_1 (1/a - 7, 1/a - 9)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.716 3.263 3.828 4.411 5.011 5.628 6.261 6.919 7.592 8.281 8.986 9.708 10.444 11.194 11.956 12.729

TABLE 8.17 :  $f_1 (1/a - 8, 1/a - 9)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.482 3.116 3.764 4.434 5.124 5.834 6.564 7.314 8.084 8.884 9.704 10.544 11.414 12.314 13.244 14.194

TABLE 8.17 :  $f_1 (1/a - 9, 1/a - 9)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.282 2.982 3.692 4.422 5.172 5.942 6.732 7.542 8.372 9.222 10.092 11.002 11.942 12.912 13.912 14.932

TABLE 8.17 :  $f_1 (1/a - 10, 1/a - 9)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 2.112 2.772 3.442 4.132 4.842 5.572 6.322 7.092 7.892 8.712 9.552 10.412 11.292 12.192 13.112 14.052

TABLE 8.17 :  $f_1 (1/a - 11, 1/a - 9)$

0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425
(1) 1.972 2.592 3.222 3.872 4.542 5.232 5.942 6.672 7.422 8.192 8.992 9.812 10.652 11.512 12.392 13.292

TABLE 8.17:  $f_1$  ( $1/a - 9, 1/a - 7$ )

MODE:  $\nu$

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.976	5.218	5.474	5.730	5.991	6.257	6.527	6.801	7.079	7.361	7.647	7.937	8.231	8.529	8.833
(3)	12.86	13.07	13.29	13.51	13.73	13.95	14.17	14.39	14.61	14.83	15.05	15.27	15.49	15.71	15.93
(4)	16.57	16.79	17.01	17.23	17.45	17.67	17.89	18.11	18.33	18.55	18.77	18.99	19.21	19.43	19.65

TABLE 8.17:  $f_1$  ( $1/a - 9, 1/a - 7$ )

MODE:  $\nu$

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	5.519	5.776	6.036	6.300	6.568	6.840	7.116	7.396	7.680	7.968	8.260	8.556	8.855	9.157	9.462
(3)	10.11	10.35	10.59	10.83	11.07	11.31	11.55	11.79	12.03	12.27	12.51	12.75	12.99	13.23	13.47
(4)	14.33	14.58	14.83	15.08	15.33	15.58	15.83	16.08	16.33	16.58	16.83	17.08	17.33	17.58	17.83

TABLE 8.18:  $f_1$  ( $a - 6, a - 8$ )

MODE:  $\nu$

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.538	4.681	4.765	4.870	4.937	5.025	5.085	5.157	5.231	5.309	5.394	5.475	5.554	5.631	5.707
(3)	7.816	7.614	7.270	7.077	7.434	7.863	8.444	9.160	9.956	10.29	9.273	9.173	8.634	8.109	7.784
(4)	10.65	10.32	10.59	11.16	11.90	12.76	13.44	14.26	15.11	15.66	11.70	11.55	11.33	10.67	10.08
(5)	13.46	13.82	14.57	15.49	16.44	15.94	14.76	15.13	14.44	13.49	13.56	15.26	16.58	16.89	15.96

TABLE 8.18:  $f_1$  ( $a - 6, a - 8$ )

MODE:  $\nu$

(1)	4.481	4.724	4.944	5.279	5.655	6.211	6.866	7.780	8.467	9.164	8.889	6.643	4.924	4.228	3.727
(2)	8.577	9.351	9.776	9.789	9.373	8.732	8.520	8.717	9.249	10.01	10.67	11.46	11.27	10.96	10.51
(3)	17.54	17.36	17.99	18.11	17.26	16.80	16.55	16.56	15.09	14.51	13.69	12.86	12.11	11.45	10.86
(4)	17.45	17.34	17.58	16.11	17.23	18.05	18.54	17.63	16.39	15.34	14.64	14.76	16.13	17.67	17.84

TABLE 8.18:  $f_1$  ( $a - 6, a - 7$ )

MODE:  $\nu$

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	5.276	5.440	5.463	5.500	5.520	5.570	5.616	5.670	5.726	5.784	5.843	5.903	5.964	6.026	6.089
(3)	9.161	8.838	8.408	8.307	8.542	9.002	9.618	10.33	10.85	10.55	9.930	9.300	8.693	8.354	7.967
(4)	12.36	11.94	12.23	12.66	13.65	14.41	14.14	13.11	12.49	12.64	12.70	11.48	10.80	10.25	9.770
(5)	15.61	16.00	16.82	17.77	17.99	16.32	16.22	15.06	14.78	14.24	15.23	16.68	16.89	15.96	15.15

TABLE 8.18:  $f_1$  ( $a - 6, a - 7$ )

MODE:  $\nu$

(1)	4.019	4.284	4.516	4.757	4.977	5.199	5.390	5.254	5.095	4.901	4.683	4.409	4.315	4.162	4.029
(2)	7.318	7.073	7.918	8.168	8.038	7.259	7.624	7.270	6.795	6.453	6.043	5.649	5.279	4.946	4.649
(3)	10.53	10.72	10.92	10.53	10.48	9.70	11.58	12.42	13.24	13.01	12.34	11.70	11.11	10.60	10.10
(4)	13.628	13.74	13.76	13.51	14.21	14.11	16.02	16.39	15.55	14.64	13.52	13.71	14.58	16.36	16.95

TABLE 8.18:  $f_1$  ( $a - 6, a - 8$ )

MODE:  $\nu$

(1)	6.003	6.182	6.201	6.004	5.672	5.316	4.970	4.706	4.464	4.261	4.092	3.952	3.840	3.753	3.694
(2)	10.43	10.02	9.489	9.329	9.554	10.02	10.63	11.22	11.30	10.76	10.17	9.518	8.899	8.558	8.198
(3)	14.03	13.49	13.78	14.44	15.20	15.50	14.57	13.89	13.28	13.09	12.40	11.67	11.01	10.45	9.981
(4)	17.69	16.09	16.95	19.71	18.74	17.50	17.34	16.27	15.30	15.43	16.70	17.63	17.00	16.12	15.32

TABLE 8.18:  $f_1$  ( $a - 6, a - 8$ )

MODE:  $\nu$

(1)	7.400	7.522	7.589	7.310	6.655	6.375	5.979	5.540	5.220	4.908	4.686	4.467	4.309	4.155	4.024
(2)	12.68	12.22	11.40	11.08	11.24	11.65	12.13	12.31	11.90	11.21	10.54	9.932	9.454	8.952	8.572
(3)	17.15	16.30	16.55	17.17	17.54	18.73	19.51	18.75	18.35	17.65	17.04	16.69	16.12	15.64	15.35
(4)	21.54	21.94	22.69	22.14	20.34	19.63	18.40	17.28	16.86	17.59	18.50	18.26	17.38	16.48	15.69

TABLE 8.18:  $f_1$  ( $a - 6, a - 9$ )

MODE:  $\nu$

(1)	6.713	6.903	6.911	6.676	6.288	5.870	5.488	5.155	4.830	4.628	4.422	4.248	4.107	3.981	3.884
(2)	11.66	11.15	10.48	10.26	10.45	10.90	11.46	11.64	11.61	10.89	10.32	9.724	9.204	8.762	8.397
(3)	15.64	14.95	15.22	15.87	16.49	16.18	14.18	13.38	13.62	13.68	11.68	11.22	10.64	10.18	9.811
(4)	19.67	20.07	20.91	21.12	19.49	18.68	17.97	16.73	16.05	16.59	17.77	17.99	17.18	16.30	15.51

TABLE 8.18:  $f_1$  ( $a - 6, a - 9$ )

MODE:  $\nu$

(1)	7.400	7.522	7.589	7.310	6.655	6.375	5.979	5.540	5.220	4.908	4.686	4.467	4.309	4.155	4.024
(2)	12.68	12.22	11.40	11.08	11.24	11.65	12.13	12.31	11.90	11.21	10.54	9.932	9.454	8.952	8.572
(3)	17.15	16.30	16.55	17.17	17.54	18.73	19.51	18.75	18.35	17.65	17.04	16.69	16.12	15.64	15.35
(4)	21.54	21.94	22.69	22.14	20.34	19.63	18.40	17.28	16.86	17.59	18.50	18.26	17.38	16.48	15.69

TABLE 8.18:  $f_1$  ( $a - 6, a - 10$ )

MODE:  $\nu$

(1)	7.400	7.522	7.589	7.310	6.655	6.375	5.979	5.540	5.220	4.908	4.686	4.467	4.309	4.155	4.024
(2)	12.68	12.22	11.40	11.08	11.24	11.65	12.13	12.31	11.90	11.21	10.54	9.932	9.454	8.952	8.572
(3)	17.15	16.30	16.55	17.17	17.54	18.73	19.51	18.75	18.35	17.65	17.04	16.69	16.12	15.64	15.35
(4)	21.54	21.94	22.69	22.14	20.34	19.63	18.40	17.28	16.86	17.59	18.50	18.26	17.38	16.48	15.69

TABLE 8.18:  $f_1$  ( $a - 6, a - 10$ )

MODE:  $\nu$

(1)	8.114	8.092	8.223	8.254	8.108	8.013	8.035	8.488	8.354	8.251	8.174	8.122	8.100	8.112	8.112
(2)	6.920	6.925	6.726	6.551	6.595	6.866	7.315	7.813	8.469	9.437	9.608	9.131	8.619	8.164	7.782
(3)	9.622	9.201	9.278	9.626	10.28	11.00	11.84	12.35	11.60	11.84	12.35	11.60	11.84	12.35	11.60
(4)	12.01	12.01	12.53	13.32	14.22	15.03	14.31	13.82	14.04	13.39	12.83	13.23	13.34	13.34	13.34

TABLE 8.18:  $f_1$  ( $a - 7, a - 8$ )

MODE:  $\nu$

(1)	3.814	4.092	4.223	4.254	4.108	4.013	4.035	4.488	4.354	4.251	4.174	4.122	4.100	4.112	4.112
(2)	6.920	6.925	6.726	6.551	6.595	6.866	7.315	7.813	8.469	9.437	9.608	9.131	8.619	8.164	7.782
(3)	9.622	9.201	9.278	9.626	10.28	11.00	11.84	12.35	11.60	11.84	12.35	11.60	11.84	12.35	11.60
(4)	12.01	12.01	12.53	13.32	14.22	15.03	14.31	13.82	14.04	13.39	12.83	13.23	13.34	13.34	13.34

TABLE 8.18:  $f_1$  ( $a - 7, a - 8$ )

MODE:  $\nu$

(1)	3.814	4.092	4.223	4.254	4.108	4.013	4.035	4.488	4.354	4.251	4.174	4.122	4.100	4.112	4.112
(2)	6.920	6.925	6.726	6.551	6.595	6.866	7.315	7.813	8.469	9.437	9.608	9.131	8.619	8.164	7.782
(3)	9.622	9.201	9.278	9.626	10.28	11.00	11.84	12.35	11.60	11.84	12.35	11.60	11.84	12.35	11.60
(4)	12.01	12.01	12.53	13.32	14.22	15.03	14.31	13.82	14.04	13.39	12.83	13.23	13.34	13.34	13.34

TABLE 8.18 :  $f_1$  ( $\phi = -8, \lambda = -9$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 5.033 5.323 5.545 5.698 5.718 5.554 5.351 5.092 4.819 4.533 4.245 3.961 3.681 3.401 3.124

(2) 9.659 9.250 8.765 8.285 7.819 7.359 6.909 6.478 6.066 5.673 5.308 4.967 4.649 4.354 4.084

(3) 12.63 12.48 12.05 12.23 12.27 13.44 13.66 13.41 12.70 12.34 12.22 11.75 11.10 10.64 10.18

(4) 16.11 15.68 15.56 16.71 17.37 16.85 15.96 15.79 15.13 14.41 14.47 15.61 16.58 16.18 15.48

TABLE 8.18 :  $f_1$  ( $\phi = -8, \lambda = -1.0$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 5.622 5.853 6.02 6.241 6.251 6.090 5.797 5.487 5.131 4.714 4.249 3.734 3.169 2.554 1.884

(2) 10.01 10.23 10.73 9.525 9.190 8.213 6.926 5.632 4.337 3.042 1.747 0.452 0.857 1.312 1.776

(3) 14.16 13.64 13.08 13.26 13.21 13.81 14.43 14.55 13.66 13.14 12.81 12.54 11.99 11.39 10.63 10.35

(4) 17.69 16.99 17.41 18.16 18.52 17.54 16.78 16.42 15.56 14.53 13.23 12.37 11.64 10.93 10.33 9.55

TABLE 8.18 :  $f_1$  ( $\phi = -9, \lambda = -8$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.620 3.273 3.400 3.556 3.668 3.698 3.645 3.551 3.418 3.240 3.040 2.812 2.562 2.282 1.971

(2) 5.474 5.204 5.824 5.275 5.683 5.704 5.506 5.200 4.833 4.537 4.204 3.842 3.443 3.012 2.576

(3) 7.816 7.315 7.762 7.811 8.127 8.483 8.313 8.111 10.06 10.65 10.16 10.23 10.47 10.03 9.563

(4) 10.02 9.869 9.362 10.46 11.14 11.59 12.81 12.80 12.06 12.56 12.54 12.30 11.75 12.41 14.61 14.31

TABLE 8.18 :  $f_1$  ( $\phi = -9, \lambda = -7$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.559 3.251 3.315 4.122 4.216 4.271 4.231 4.056 3.903 3.667 3.669 3.568 3.412 3.461 3.440

(2) 5.368 5.20 6.258 6.635 6.543 6.539 6.745 7.149 7.772 8.498 9.077 9.104 8.732 8.337 7.890

(3) 8.092 8.180 8.984 8.997 9.319 9.354 10.65 11.37 11.57 11.65 10.70 10.82 10.69 10.21 9.723

(4) 11.64 11.42 11.53 12.06 12.80 13.63 14.64 13.37 13.21 13.26 12.66 12.57 13.83 15.42 15.07

TABLE 8.18 :  $f_1$  ( $\phi = -9, \lambda = -6$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 6.048 4.251 4.375 4.688 4.822 4.828 4.739 4.581 4.403 4.232 4.078 3.946 3.837 3.752 3.684

(2) 7.297 7.533 7.653 7.543 7.319 7.257 7.426 7.695 8.487 9.115 9.513 9.334 8.935 8.526 8.182

(3) 10.35 10.43 10.10 10.46 11.00 11.69 11.76 12.29 12.03 11.47 11.23 11.32 10.95 10.47 9.972

(4) 13.24 12.91 12.59 13.56 14.33 15.04 14.76 14.10 14.15 13.72 13.14 13.45 14.83 15.77 15.28

TABLE 8.18 :  $f_1$  ( $\phi = -9, \lambda = -5$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.526 4.734 4.933 5.190 5.330 5.315 5.227 5.030 4.811 4.600 4.410 4.243 4.100 4.050 3.954

(2) 8.107 8.412 8.534 8.346 8.057 7.553 7.124 6.578 6.044 5.647 5.454 5.566 5.149 4.744 4.392

(3) 11.58 11.62 11.20 11.31 11.48 12.09 12.75 12.57 12.45 11.93 11.79 11.61 11.14 10.63 10.17

(4) 14.80 14.30 14.36 14.87 15.73 16.12 15.41 14.93 14.78 14.13 13.71 14.32 15.71 16.64 15.45

TABLE 8.18 :  $f_1$  ( $\phi = -9, \lambda = -1.0$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.389 5.285 5.467 6.265 6.628 6.829 6.673 6.428 6.145 5.813 5.437 5.075 4.734 4.414 4.124

(2) 8.944 9.265 9.365 9.092 8.688 8.320 8.030 7.670 7.249 6.784 6.419 6.044 5.666 5.149 4.744 4.392

(3) 12.79 12.75 12.17 12.02 12.41 13.64 13.61 13.52 12.47 12.35 12.47 12.35 12.20 11.80 11.35 10.82 10.35

(4) 16.31 15.59 15.64 16.29 17.02 18.97 16.05 15.71 15.78 14.55 14.28 15.07 16.29 16.28 15.65

TABLE 8.18 :  $f_1$  ( $\phi = -7, \lambda = -7$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.533 4.735 4.922 4.933 4.833 4.633 4.434 4.234 4.179 3.991 3.831 3.688 3.593 3.514 3.445 3.400

(2) 8.650 8.076 7.800 7.555 7.284 6.944 6.571 6.204 5.823 5.454 5.094 4.749 4.419 4.100 3.824 3.566

(3) 11.19 10.75 10.71 11.19 11.70 12.58 13.22 12.86 12.06 11.72 11.60 11.41 10.80 10.24 9.748

(4) 13.94 13.90 14.50 15.36 16.21 16.04 14.98 15.00 14.55 13.70 13.59 14.95 16.38 15.90 15.13

TABLE 8.18 :  $f_1$  ( $\phi = -7, \lambda = -6$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 5.180 5.402 5.352 5.590 5.453 5.209 4.937 4.600 4.462 4.255 4.089 3.951 3.839 3.751 3.684

(2) 9.171 8.174 8.016 8.614 8.481 8.242 7.847 7.468 7.043 6.603 6.166 5.734 5.308 4.887 4.471

(3) 12.72 12.15 12.07 12.59 13.10 13.49 14.05 13.27 12.56 12.44 12.22 11.62 11.00 10.44 9.879

(4) 15.80 15.71 16.35 17.20 17.71 16.81 15.99 15.65 14.95 14.39 14.80 16.22 16.76 16.07 15.31

TABLE 8.18 :  $f_1$  ( $\phi = -7, \lambda = -5$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 5.391 6.029 6.197 6.218 6.049 5.757 5.435 5.130 4.889 4.662 4.420 4.247 4.101 3.981 3.884

(2) 10.25 10.23 9.771 9.350 9.284 9.546 10.02 10.57 10.77 10.25 9.485 8.193 6.758 5.396

(3) 14.21 13.47 13.32 13.77 14.43 14.93 14.61 13.72 13.12 12.95 12.49 11.83 11.20 10.65 10.18

(4) 17.57 17.42 18.09 18.87 18.71 17.44 17.02 16.38 15.42 15.06 15.66 17.06 17.01 16.28 15.50

TABLE 8.18 :  $f_1$  ( $\phi = -7, \lambda = -1.0$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 6.383 6.639 6.803 6.813 6.606 6.259 5.889 5.525 5.208 4.932 4.693 4.487 4.308 4.155 4.024

(2) 11.31 11.25 10.65 10.11 9.895 10.22 10.67 11.16 11.27 11.04 10.47 9.908 9.395 8.849 8.371

(3) 15.64 14.68 14.48 14.93 15.56 15.84 15.11 14.17 13.64 13.34 12.74 12.05 11.41 10.84 10.35

(4) 19.24 19.04 19.71 20.33 19.51 18.27 17.82 16.85 15.93 15.83 16.74 17.41 17.24 16.45 15.68

TABLE 8.18 :  $f_1$  ( $\phi = -6, \lambda = -6$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.436 3.615 3.790 3.899 3.833 3.659 3.744 3.601 3.464 3.345 3.247 3.172 3.121 3.059 3.112

(2) 6.124 6.302 6.281 6.122 6.050 6.178 6.501 6.988 7.678 8.406 9.129 9.043 8.594 8.156 7.760

(3) 8.655 8.533 8.255 8.156 8.042 8.648 10.43 11.25 11.45 10.53 10.44 10.47 10.05 9.447

(4) 10.95 10.76 11.68 11.70 12.49 13.38 13.68 13.06 12.93 13.12 12.42 12.17 13.70 15.47 14.94

TABLE 8.18 :  $f_1$  ( $\phi = -6, \lambda = -5$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 3.997 4.200 4.387 4.521 4.555 4.473 4.318 4.141 3.972 3.821 3.694 3.591 3.513 3.461 3.440

(2) 7.123 7.312 7.252 7.070 6.958 7.078 7.419 7.937 8.596 9.325 9.977 9.229 8.707 8.242 7.893

(3) 10.08 9.916 9.695 9.591 10.39 11.07 11.64 12.33 13.01 13.45 13.25 13.20 12.84 12.53 12.16

(4) 12.72 12.45 12.80 13.49 14.33 15.05 14.53 13.68 14.09 13.50 13.80 13.44 13.18 13.77 15.11

TABLE 8.18 :  $f_1$  ( $\phi = -6, \lambda = -1.0$ )

$w$  0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

(1) 4.547 4.770 4.976 5.122 5.152 5.047 4.856 4.640 4.432 4.246 4.084 3.940 3.838 3.753 3.684

(2) 8.107 8.320 8.246 7.952 7.793 7.889 8.231 8.753 9.302 9.953 9.851 8.431 6.566 5.507 5.194

(3) 11.47 11.23 10.92 11.11 11.63 12.33 12.98 12.93 12.26 11.60 11.82 11.51 10.96 10.43 9.976

(4) 14.44 14.06 14.43 15.16 15.97 16.11 15.18 14.33 14.67 13.92 13.65 14.64 16.68 15.98 15.29

TABLE 8.18 :  $f_1$  ( $1/a = 6, 1/a = 9$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 2.865 3.112 3.278 3.466 3.679 3.921 4.190 4.475 4.779 5.093 5.426 5.779 6.146 6.527 6.924
- (2) 5.360 5.635 5.916 6.208 6.513 6.834 7.176 7.549 7.954 8.434 8.931 9.457 9.924 10.434 10.984
- (3) 7.762 8.167 8.598 9.058 9.548 10.068 10.618 11.200 11.814 12.462 13.137 13.842 14.578 15.346 16.146
- (4) 10.17 10.68 11.22 11.78 12.36 12.96 13.58 14.24 14.92 15.64 16.38 17.14 17.94 18.78 19.64

TABLE 8.18 :  $f_1$  ( $1/a = 6, 1/a = 1.0$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 2.700 2.842 3.000 3.179 3.380 3.608 3.858 4.120 4.384 4.514 4.511 4.419 4.284 4.145 4.022
- (2) 4.865 5.127 5.415 5.730 6.056 6.412 6.798 7.208 7.643 8.104 8.599 9.129 9.700 10.314 10.964
- (3) 7.039 7.412 7.808 8.177 8.535 8.911 9.333 9.799 10.300 10.836 11.408 12.018 12.664 13.346 14.064
- (4) 9.209 9.680 10.111 10.518 9.989 10.27 10.53 11.76 12.45 12.23 12.10 12.29 11.38 12.19 14.13

TABLE 8.18 :  $f_1$  ( $1/a = 7, 1/a = 6$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 4.815 5.016 5.265 5.560 5.899 6.301 6.645 6.922 7.151 7.234 7.176 6.967 6.631 6.203 5.708
- (2) 8.952 9.408 9.912 10.351 10.733 11.073 11.368 11.520 11.546 11.448 11.266 11.017 10.717 10.371 10.007
- (3) 13.115 13.600 13.779 12.32 11.94 12.60 13.53 14.37 15.06 15.92 16.74 17.54 18.33 19.11 19.88
- (4) 17.34 17.37 15.57 15.08 17.11 18.05 17.09 15.91 16.10 15.23 14.38 13.92 15.39 17.02 18.38

TABLE 8.18 :  $f_1$  ( $1/a = 7, 1/a = 7$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 4.357 4.448 4.603 4.815 5.064 5.323 5.600 6.008 6.345 6.623 6.847 6.918 6.788 6.520 6.297
- (2) 7.810 8.195 8.624 9.003 9.322 9.603 9.843 9.954 9.952 9.790 10.007 9.919 9.467 9.007
- (3) 11.29 11.95 12.34 11.67 11.35 11.15 11.07 12.76 13.24 12.64 11.94 11.70 11.47 11.23 11.078
- (4) 14.98 15.46 14.41 14.14 14.51 15.68 16.34 15.36 14.84 14.86 14.11 13.48 14.03 16.01 16.17

TABLE 8.18 :  $f_1$  ( $1/a = 7, 1/a = 8$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 3.857 4.001 4.200 4.428 4.688 4.959 5.257 5.584 5.925 6.284 6.659 7.053 7.467 7.895 8.338
- (2) 6.943 7.285 7.662 8.066 8.501 8.970 9.475 9.999 10.540 11.099 11.676 12.270 12.881 13.500 14.128
- (3) 10.06 10.57 11.00 10.86 10.24 10.17 10.70 11.48 12.24 12.22 11.46 11.28 11.28 11.38 11.12 10.64
- (4) 13.21 13.75 13.43 12.86 13.33 14.16 15.00 14.82 14.10 14.14 13.79 13.19 13.28 14.93 15.86

TABLE 8.18 :  $f_1$  ( $1/a = 7, 1/a = 9$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 3.480 3.629 3.820 4.033 4.271 4.530 4.795 5.020 5.216 5.448 5.704 6.000 6.329 6.684 7.064
- (2) 6.249 6.564 6.908 7.259 7.587 7.883 7.887 7.667 7.460 7.180 6.840 6.451 6.012 5.529 5.009 4.715
- (3) 9.044 9.469 9.899 9.998 9.413 8.462 8.030 10.50 11.28 11.68 11.23 10.95 10.98 10.68 10.48
- (4) 11.81 12.34 12.38 11.81 12.15 12.85 13.69 14.11 13.59 13.38 13.38 12.90 12.77 14.09 15.50

TABLE 8.18 :  $f_1$  ( $1/a = 7, 1/a = 1.0$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 3.149 3.313 3.495 3.699 3.923 4.165 4.412 4.628 4.746 4.725 4.606 4.452 4.295 4.151 4.023
- (2) 5.678 5.972 6.291 6.617 6.984 7.398 7.827 8.281 8.748 9.222 9.700 10.188 10.684 11.192 11.714
- (3) 8.258 8.612 8.999 9.387 9.823 10.312 10.859 11.429 12.024 12.638 13.272 13.924 14.594 15.284 15.992
- (4) 10.72 11.19 11.37 11.09 11.22 11.81 12.60 13.27 13.10 12.78 12.91 12.59 12.39 13.44 15.11

TABLE 8.18 :  $f_1$  ( $a = 1.0, a = 6$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 2.755 2.814 3.081 3.267 3.497 3.784 4.129 4.513 4.938 5.399 5.894 6.424 6.989 7.587 8.216
- (2) 4.540 5.184 5.316 5.469 5.603 5.723 5.788 5.799 5.741 5.613 5.421 5.178 4.875 4.514 4.121
- (3) 7.268 7.374 7.253 7.434 7.863 8.402 8.944 9.495 10.043 10.509 10.976 11.451 11.922 12.392 12.852
- (4) 9.959 9.183 9.164 9.494 10.07 10.79 11.64 12.32 13.13 13.06 12.55 12.79 12.51 12.07 12.74 14.67 15.00

TABLE 8.18 :  $f_1$  ( $a = 1.0, a = 7$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 3.274 3.335 3.575 3.764 3.932 4.044 4.067 4.098 3.994 3.797 3.677 3.583 3.510 3.460 3.440
- (2) 5.745 6.274 6.240 6.379 6.279 6.164 6.026 6.462 7.027 7.671 8.418 9.267 9.674 10.313 11.975
- (3) 8.248 8.497 8.450 8.335 8.139 8.987 9.815 10.36 11.01 10.88 10.43 10.44 10.57 10.18 9.753
- (4) 10.65 10.64 10.38 10.94 11.57 12.36 13.13 13.06 12.55 12.79 12.51 12.07 12.74 14.67 15.00

TABLE 8.18 :  $f_1$  ( $a = 1.0, a = 8$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 3.447 3.845 4.053 4.261 4.445 4.565 4.580 4.458 4.322 4.212 4.068 3.942 3.835 3.752 3.694
- (2) 6.545 7.846 7.950 7.134 7.001 6.893 6.946 7.234 7.720 8.353 9.093 9.163 8.697 8.572 8.188
- (3) 9.394 9.641 9.545 9.391 9.557 10.003 10.609 11.39 11.68 11.28 10.91 10.94 10.63 10.40 9.967
- (4) 12.12 12.65 11.91 12.37 12.90 13.76 14.29 13.67 13.41 13.43 12.82 12.71 13.79 15.34 15.20

TABLE 8.18 :  $f_1$  ( $a = 1.0, a = 9$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 4.077 4.252 4.513 4.726 4.933 5.055 5.069 4.915 4.700 4.401 4.229 4.098 3.960 3.804 3.694
- (2) 7.318 7.645 7.891 8.015 8.056 7.926 7.517 7.441 7.018 6.702 6.298 6.415 6.431 6.109 5.732 5.389
- (3) 12.52 10.16 10.53 10.31 10.46 11.39 11.65 12.24 12.19 11.68 11.38 11.38 11.07 10.61 10.17
- (4) 13.57 13.37 13.15 13.57 14.29 15.01 14.58 14.32 14.21 13.91 13.38 13.41 14.64 15.75 15.41

TABLE 8.18 :  $f_1$  ( $1/a = 8, 1/a = 6$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 4.152 4.322 4.517 4.716 4.900 5.073 5.012 4.812 4.585 4.276 4.016 3.713 3.456 3.192 2.948
- (2) 7.619 8.078 8.500 8.918 9.301 9.618 9.448 7.514 7.677 7.208 6.737 6.139 5.519 4.909 4.506
- (3) 11.29 11.90 12.63 11.92 10.82 10.32 11.20 12.69 13.44 12.61 12.00 11.59 11.34 11.07 10.61 10.17
- (4) 14.91 15.61 14.58 13.53 14.75 15.81 16.37 15.49 14.72 15.06 14.29 13.42 13.61 15.32 16.34

TABLE 8.18 :  $f_1$  ( $1/a = 8, 1/a = 7$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 3.450 3.614 3.802 4.023 4.281 4.590 4.916 5.244 5.584 5.928 6.275 6.631 6.995 7.369 7.754
- (2) 6.668 7.048 7.423 7.655 8.264 8.323 7.650 7.313 7.165 7.401 6.228 5.194 4.740 4.427 4.003
- (3) 9.280 12.78 10.23 11.03 12.77 9.853 10.29 11.10 12.04 12.38 11.78 11.22 11.32 11.27 10.77
- (4) 12.80 13.52 13.69 12.59 12.69 13.75 14.75 14.59 14.72 15.06 14.29 13.42 13.61 15.32 16.02

TABLE 8.18 :  $f_1$  ( $1/a = 8, 1/a = 8$ )

$\omega$ : 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425

MODE:

- (1) 3.181 3.431 3.654 3.865 4.016 4.300 4.566 4.728 4.815 4.720 4.575 4.385 4.231 4.119 4.028
- (2) 5.355 6.474 6.358 6.974 7.358 7.690 7.425 6.658 6.668 6.965 7.548 8.409 9.207 9.213 8.865
- (3) 8.659 6.699 6.577 6.943 6.666 6.216 6.346 6.469 6.669 6.042 6.159 6.146 6.679 6.682 6.699 6.662
- (4) 11.36 11.93 12.32 11.76 11.42 12.25 12.14 13.56 13.67 13.15 13.39 13.97 14.52 13.24 13.46

TABLE 8.18 :  $\theta_1$  ( $1/a - 9, 1/a - 8$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.937	5.134	5.337	5.541	5.750	5.966	6.190	6.422	6.662	6.911	7.168	7.434	7.709	7.994	8.288
(3)	8.907	9.302	9.698	10.095	10.493	10.893	11.294	11.697	12.102	12.509	12.918	13.329	13.742	14.157	14.574
(4)	12.629	13.228	13.826	14.424	15.022	15.620	16.218	16.816	17.414	18.012	18.610	19.208	19.806	20.404	21.002
(5)	16.777	17.429	18.081	18.733	19.385	20.037	20.689	21.341	21.993	22.645	23.297	23.949	24.601	25.253	25.905

TABLE 8.18 :  $\theta_1$  ( $1/a - 9, 1/a - 8$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.445	4.657	4.869	5.081	5.293	5.505	5.717	5.929	6.141	6.353	6.565	6.777	6.989	7.201	7.413
(3)	8.017	8.353	8.689	9.025	9.361	9.697	10.033	10.369	10.705	11.041	11.377	11.713	12.049	12.385	12.721
(4)	11.577	12.063	12.549	13.035	13.521	14.007	14.493	14.979	15.465	15.951	16.437	16.923	17.409	17.895	18.381
(5)	15.044	15.684	16.324	16.964	17.604	18.244	18.884	19.524	20.164	20.804	21.444	22.084	22.724	23.364	24.004

TABLE 8.18 :  $\theta_1$  ( $1/a - 9, 1/a - 8$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.046	4.258	4.470	4.682	4.894	5.106	5.318	5.530	5.742	5.954	6.166	6.378	6.590	6.802	7.014
(3)	7.281	7.627	7.973	8.319	8.665	9.011	9.357	9.703	10.049	10.395	10.741	11.087	11.433	11.779	12.125
(4)	10.497	10.887	11.277	11.667	12.057	12.447	12.837	13.227	13.617	14.007	14.397	14.787	15.177	15.567	15.957
(5)	13.633	14.081	14.529	14.977	15.425	15.873	16.321	16.769	17.217	17.665	18.113	18.561	19.009	19.457	19.905

TABLE 8.18 :  $\theta_1$  ( $1/a - 1.0, 1/a - 8$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	6.871	7.145	7.419	7.693	7.967	8.241	8.515	8.789	9.063	9.337	9.611	9.885	10.159	10.433	10.707
(3)	12.74	13.233	13.726	14.219	14.712	15.205	15.698	16.191	16.684	17.177	17.670	18.163	18.656	19.149	19.642
(4)	18.53	19.142	19.751	20.360	20.969	21.578	22.187	22.796	23.405	24.014	24.623	25.232	25.841	26.450	27.059
(5)	22.84	23.668	24.492	25.316	26.140	26.964	27.788	28.612	29.436	30.260	31.084	31.908	32.732	33.556	34.380

TABLE 8.18 :  $\theta_1$  ( $1/a - 1.0, 1/a - 7$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	6.068	6.334	6.600	6.866	7.132	7.398	7.664	7.930	8.196	8.462	8.728	8.994	9.260	9.526	9.792
(3)	11.11	11.55	12.00	12.44	12.89	13.33	13.78	14.22	14.67	15.11	15.56	16.00	16.45	16.89	17.34
(4)	16.09	16.53	17.00	17.44	17.89	18.33	18.78	19.22	19.67	20.11	20.56	21.00	21.45	21.89	22.34
(5)	20.58	21.02	21.47	21.91	22.36	22.80	23.25	23.69	24.14	24.58	25.03	25.47	25.92	26.36	26.81

TABLE 8.18 :  $\theta_1$  ( $1/a - 1.0, 1/a - 6$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	5.469	5.656	5.843	6.030	6.217	6.404	6.591	6.778	6.965	7.152	7.339	7.526	7.713	7.900	8.087
(3)	9.478	10.027	10.576	11.125	11.674	12.223	12.772	13.321	13.870	14.419	14.968	15.517	16.066	16.615	17.164
(4)	14.25	14.737	15.220	15.703	16.186	16.669	17.152	17.635	18.118	18.601	19.084	19.567	20.050	20.533	21.016
(5)	18.35	18.877	19.400	19.923	20.446	20.969	21.492	22.015	22.538	23.061	23.584	24.107	24.630	25.153	25.676

TABLE 8.18 :  $\theta_1$  ( $1/a - 1.0, 1/a - 5$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.869	5.056	5.243	5.430	5.617	5.804	5.991	6.178	6.365	6.552	6.739	6.926	7.113	7.300	7.487
(3)	8.699	9.258	9.817	10.376	10.935	11.494	12.053	12.612	13.171	13.730	14.289	14.848	15.407	15.966	16.525
(4)	12.79	13.301	13.803	14.305	14.807	15.309	15.811	16.313	16.815	17.317	17.819	18.321	18.823	19.325	19.827
(5)	16.50	17.044	17.588	18.132	18.676	19.220	19.764	20.308	20.852	21.396	21.940	22.484	23.028	23.572	24.116

TABLE 8.18 :  $\theta_1$  ( $1/a - 8, 1/a - 6$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.785	5.000	5.215	5.430	5.645	5.860	6.075	6.290	6.505	6.720	6.935	7.150	7.365	7.580	7.795
(3)	8.502	8.928	9.354	9.780	10.206	10.632	11.058	11.484	11.910	12.336	12.762	13.188	13.614	14.040	14.466
(4)	12.15	12.53	12.91	13.29	13.67	14.05	14.43	14.81	15.19	15.57	15.95	16.33	16.71	17.09	17.47
(5)	15.67	16.03	16.39	16.75	17.11	17.47	17.83	18.19	18.55	18.91	19.27	19.63	19.99	20.35	20.71

TABLE 8.18 :  $\theta_1$  ( $1/a - 8, 1/a - 7$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.372	4.587	4.802	5.017	5.232	5.447	5.662	5.877	6.092	6.307	6.522	6.737	6.952	7.167	7.382
(3)	7.928	8.353	8.778	9.203	9.628	10.053	10.478	10.903	11.328	11.753	12.178	12.603	13.028	13.453	13.878
(4)	11.49	11.96	12.43	12.90	13.37	13.84	14.31	14.78	15.25	15.72	16.19	16.66	17.13	17.60	18.07
(5)	15.04	15.52	16.00	16.48	16.96	17.44	17.92	18.40	18.88	19.36	19.84	20.32	20.80	21.28	21.76

TABLE 8.18 :  $\theta_1$  ( $1/a - 8, 1/a - 8$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	4.372	4.587	4.802	5.017	5.232	5.447	5.662	5.877	6.092	6.307	6.522	6.737	6.952	7.167	7.382
(3)	7.928	8.353	8.778	9.203	9.628	10.053	10.478	10.903	11.328	11.753	12.178	12.603	13.028	13.453	13.878
(4)	11.49	11.96	12.43	12.90	13.37	13.84	14.31	14.78	15.25	15.72	16.19	16.66	17.13	17.60	18.07
(5)	15.04	15.52	16.00	16.48	16.96	17.44	17.92	18.40	18.88	19.36	19.84	20.32	20.80	21.28	21.76

TABLE 8.18 :  $\theta_1$  ( $1/a - 8, 1/a - 9$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	3.953	4.164	4.375	4.586	4.797	5.008	5.219	5.430	5.641	5.852	6.063	6.274	6.485	6.696	6.907
(3)	7.135	7.487	7.839	8.191	8.543	8.895	9.247	9.599	9.951	10.303	10.655	11.007	11.359	11.711	12.063
(4)	10.31	10.77	11.24	11.70	12.16	12.62	13.08	13.54	14.00	14.46	14.92	15.38	15.84	16.30	16.76
(5)	13.47	13.93	14.39	14.85	15.31	15.77	16.23	16.69	17.15	17.61	18.07	18.53	18.99	19.45	19.91

TABLE 8.18 :  $\theta_1$  ( $1/a - 8, 1/a - 10$ )

NOTE:

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	3.538	3.754	3.970	4.186	4.402	4.618	4.834	5.050	5.266	5.482	5.698	5.914	6.130	6.346	6.562
(3)	6.481	6.875	7.269	7.663	8.057	8.451	8.845	9.239	9.633	10.027	10.421	10.815	11.209	11.603	12.000
(4)	9.324	9.774	10.224	10.674	11.124	11.574	12.024	12.474	12.924	13.374	13.824	14.274	14.724	15.174	15.624
(5)	12.170	12.56	12.95	13.34	13.73	14.12	14.51	14.90	15.29	15.68	16.07	16.46	16.85	17.24	17.63

TABLE 8.18 :  $\theta_1$  ( $1/a - 9, 1/a - 6$ )

NOTE:

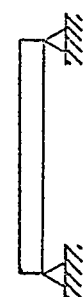

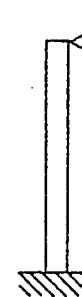
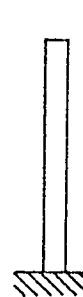
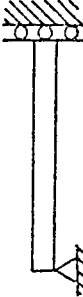

(1)	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275	0.300	0.325	0.350	0.375	0.400	0.425
(2)	6.167	6.433	6.699	6.965	7.231	7.497	7.763	8.029	8.295	8.561	8.827	9.093	9.359	9.625	9.891
(3)	11.25	11.75	12.25	12.75	13.25	13.75	14.25	14.75	15.25	15.75	16.25	16.75	17.25	17.75	18.25
(4)	16.40	16.87	17.34	17.81	18.28	18.75	19.22	19.69	20.16	20.63	21.10	21.57	22.04	22.51	22.98
(5)	21.63	22.09	22.56	23.03	23.50	23.97	24.44	24.91	25.38	25.85	26.32	26.79	27.26	27.73	28.20

TABLE 8.18 :  $\theta_1$  ( $1/a - 9, 1$

APPENDIXES

- A. Eigenvalues for Single-Span Uniform Beams
- B. Eigenvalues for Double-Span Uniform Beams
- C. Eigenvalues for Triple-Span Uniform Beams
- D. Computer Program

APPENDIX A. EIGENVALUES FOR SINGLE-SPAN UNIFORM BEAMS.

Case	Type	Mode	1st	2nd	3rd	4th	nth
1.	S-S		3.1416	6.2832	9.4248	12.5664	$\lambda_n = n\pi$
2.	C-C		4.7300	7.8532	10.9956	14.1372	For n large, $\lambda_n \approx (2n + 1) \pi/2$
3.	C-S		3.9266	7.0686	10.2102	13.3518	For n large, $\lambda_n \approx (4n + 1) \pi/4$
4.	C-F		1.8751	4.6941	7.8548	10.9955	For n large, $\lambda_n \approx (2n - 1) \pi/2$
5.	S-G		1.5708	4.7124	7.8540	10.9956	$\lambda_n = (2n - 1) \pi/2$
6.	C-G		2.3650	5.4978	8.6394	11.7810	For n large, $\lambda_n \approx (4n - 1) \pi/4$

The circular frequency is

$$\omega_n = \frac{\lambda_n^2}{L^2} \sqrt{\frac{EI}{\rho A}}$$

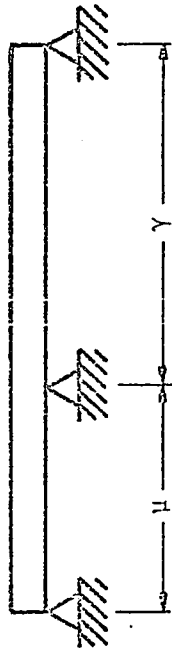
where EI = bending stiffness

$\rho A$  = mass per unit length

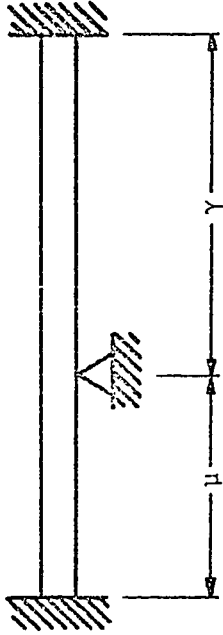
L = length of the beam

APPENDIX B. EIGENVALUES FOR DOUBLE-SPAN UNIFORM BEAMS.

(1) SSS Beam



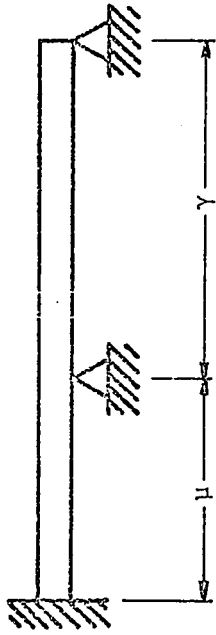
$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.150	4.4091	7.9860	11.5835	15.1779
0.200	4.6183	8.3915	12.1617	15.7080
0.250	4.8579	8.8399	12.5664	14.5129
0.300	5.1318	9.2769	11.7804	14.2845
0.350	5.4420	9.3721	11.0079	15.0596
0.400	5.7826	8.7679	11.3129	15.7080
0.450	6.1145	8.1313	12.0375	14.8628
0.500	6.2832	7.8532	12.5664	14.1372
0.550	6.1145	8.1313	12.0375	14.8628
0.600	5.7826	8.7679	11.3129	15.7080
0.650	5.4420	9.3721	11.0079	15.0596
0.700	5.1318	9.2769	11.7804	14.2845
0.750	4.8579	8.8399	12.5664	14.5129
0.800	4.6183	8.3915	12.1617	15.7080
0.850	4.4091	7.9860	11.5835	15.1779



(2) CSC Beam

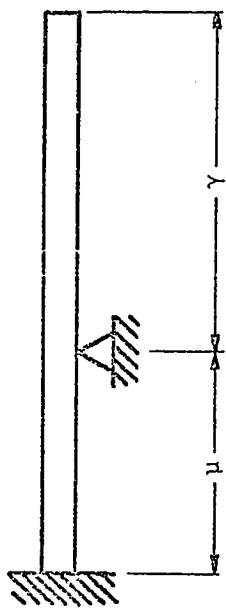
$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.150	5.3686	8.9479	12.5641	16.1847
0.200	5.6399	9.4215	13.2354	17.0022
0.250	5.9489	9.9545	13.9186	16.9305
0.300	6.3015	10.5280	13.9338	15.7260
0.350	6.7025	10.9794	12.8054	16.3585
0.400	7.1494	10.6107	12.7332	17.2339
0.450	7.6035	9.8381	13.5074	16.5519
0.500	7.8532	9.4501	14.1372	15.7064
0.550	7.6035	9.8381	13.5074	16.5519
0.600	7.1494	10.6107	12.7332	17.2339
0.650	6.7025	10.9794	12.8054	16.3585
0.700	6.3015	10.5280	13.9338	15.7260
0.750	5.9489	9.9545	13.9186	16.9305
0.800	5.6399	9.4215	13.2354	17.0022
0.850	5.3686	8.9479	12.5641	16.1847

(3) CSS Beam



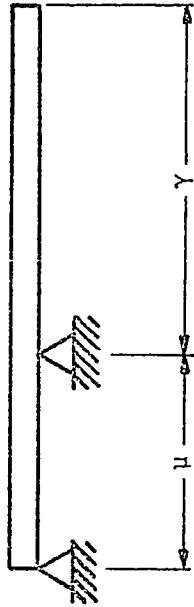
$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.150	4.4523	8.0470	11.6593	15.2797
0.200	4.6739	8.4595	12.2832	16.0717
0.250	4.9260	8.9431	12.9484	16.4162
0.300	5.2141	9.4785	13.3430	15.0778
0.350	5.5445	10.0079	12.4575	15.2686
0.400	5.9227	10.1680	11.7988	16.1768
0.450	6.3478	9.5888	12.2635	16.2622
0.500	6.7865	8.9266	13.0908	15.1832
0.550	7.0629	8.6586	13.2190	15.1526
0.600	6.9204	9.0529	12.4682	16.2960
0.650	6.5699	9.8321	11.8463	16.1516
0.700	6.2055	10.1984	12.1248	15.2855
0.750	5.8723	9.8236	13.2612	15.0124
0.800	5.5775	9.3370	13.0892	16.4345
0.850	5.3196	8.8334	12.4836	16.0716

(4) CSF Beam



$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.150	2.1178	5.3288	8.9496	12.5640
0.200	2.2160	5.5989	9.4232	13.2353
0.250	2.3255	5.9069	9.9561	13.9185
0.300	2.4484	6.2585	10.5294	13.9338
0.350	2.5874	6.6595	10.9797	12.9061
0.400	2.7461	7.1084	10.6081	12.7352
0.450	2.9290	7.5715	9.8283	13.5092
0.500	3.1416	7.8532	9.4248	14.1372
0.550	3.3907	7.6404	9.7821	13.5043
0.600	3.6830	7.2229	10.5636	12.7183
0.650	4.0196	6.8491	10.9975	12.7440
0.700	4.3737	6.6232	10.5721	13.8914
0.750	4.6396	6.7029	10.0593	13.9460
0.800	4.6826	7.2359	9.7364	13.3142
0.850	4.5486	7.8110	10.1394	12.8757

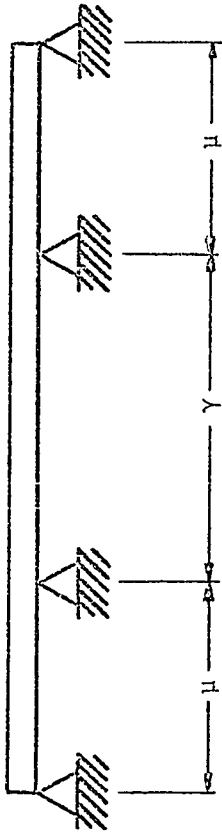
(5) SSF Beam



$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.150	2.0919	5.2808	8.8850	12.4835
0.200	2.1801	5.5381	9.3385	13.0891
0.250	2.2788	5.8326	9.8249	13.2613
0.300	2.3897	6.1662	10.1988	12.1253
0.350	2.5154	6.5334	9.8294	11.8482
0.400	2.6586	6.8941	9.0429	12.4701
0.450	2.8229	7.0673	8.6252	13.2201
0.500	3.0118	6.8262	8.8745	13.0891
0.550	3.2276	6.4247	9.5180	12.2565
0.600	3.4670	6.0590	10.1495	11.7629
0.650	3.7076	5.8402	10.0367	12.3931
0.700	3.8854	5.8352	9.5445	13.3334
0.750	3.9205	6.1599	9.1081	12.9865
0.800	3.8235	6.7405	8.9577	12.3888
0.850	3.6623	7.0576	9.6048	12.0925

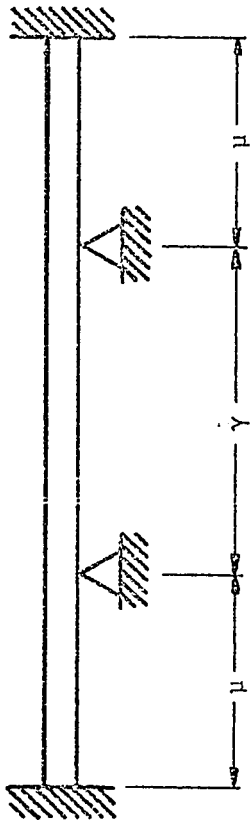
APPENDIX C. EIGENVALUES FOR THREE-SPAN UNIFORM BEAMS.

(1) SSSS Beam



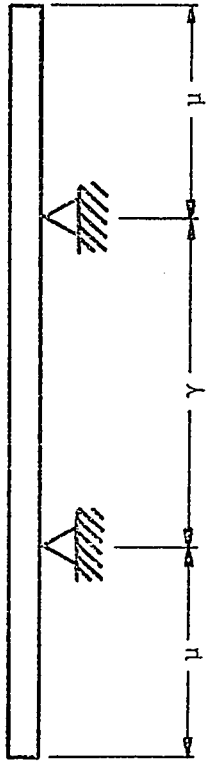
$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.075	5.2505	8.8182	12.3877	15.9720
0.100	5.5255	9.2366	13.0015	16.7831
0.125	5.7520	9.7159	13.6996	17.6798
0.150	6.0263	10.2536	14.4737	18.5539
0.175	6.4458	10.8340	15.2585	18.7442
0.200	6.8494	11.5552	15.7030	17.5357
0.225	7.3163	12.2290	15.1516	16.2626
0.250	7.8532	12.8664	14.61372	15.7064
0.275	8.4509	12.2290	13.2406	15.2526
0.300	9.0355	11.5552	12.6672	17.5357
0.325	9.4000	10.8840	12.5954	18.7442
0.350	9.3432	10.2536	13.5519	18.5539
0.375	9.0414	9.7159	14.0742	17.6798
0.400	8.6881	9.2366	15.7030	16.7831
0.425	8.3635	8.8182	15.3520	15.9720

(2) CSSC Beam

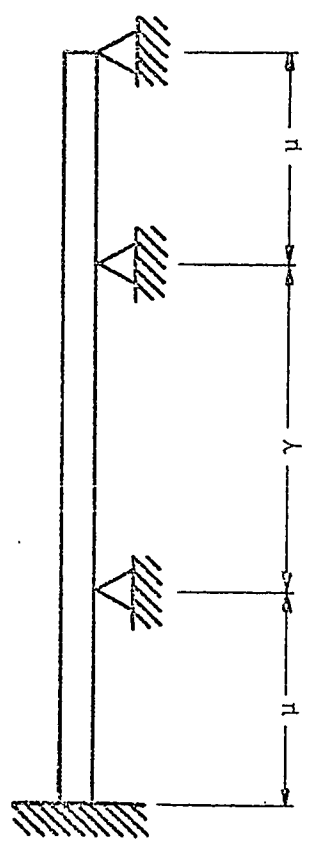


$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.075	5.3504	8.5046	12.4940	16.0240
0.100	5.6060	9.3479	13.1355	16.9389
0.125	5.8938	9.8520	13.8665	17.8961
0.150	6.2204	10.4283	14.6973	18.9570
0.175	6.5941	11.0890	15.6279	20.0159
0.200	7.0256	11.8453	16.6604	20.3359
0.225	7.5253	12.6956	17.2677	19.1376
0.250	8.1181	13.5729	16.7893	17.8533
0.275	8.8111	14.4258	15.7537	17.3171
0.300	9.6092	13.8408	14.8214	18.1058
0.325	10.4754	13.1358	14.2408	17.6642
0.350	10.9601	12.4109	14.4380	20.3969
0.375	10.8923	11.7446	15.7443	19.6471
0.400	10.5339	11.1551	17.1666	16.6739
0.425	10.1383	10.6392	17.0774	17.7669

(3) FSSF Beam



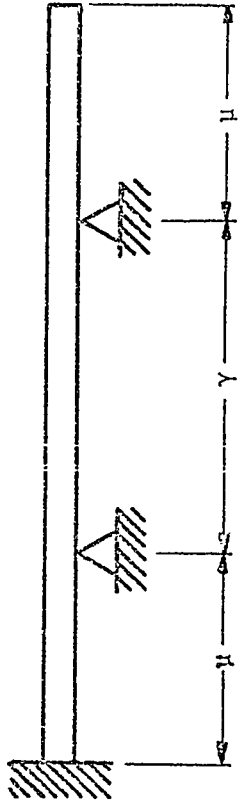
$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.075	3.6877	7.3246	10.8477	14.1352
0.100	3.9020	7.6470	10.9900	13.4809
0.125	4.1262	7.8411	10.5065	12.3198
0.150	4.3481	7.7707	9.6992	11.6704
0.175	4.5451	7.4152	9.0089	11.6803
0.200	4.6831	6.9340	8.5917	12.1379
0.225	4.7300	6.4553	8.4840	12.8493
0.250	4.6819	6.0237	8.6779	13.6524
0.275	4.5684	5.6459	9.1337	14.1346
0.300	4.4264	5.3173	9.7937	13.7982
0.325	4.2817	5.0307	10.5320	13.0669
0.350	4.1477	4.7794	10.9792	12.3325
0.375	4.0306	4.5575	10.9595	11.6651
0.400	3.9328	4.3603	10.4906	11.0762
0.425	3.8551	4.1838	10.0764	10.5616



(4) CSSS Beam

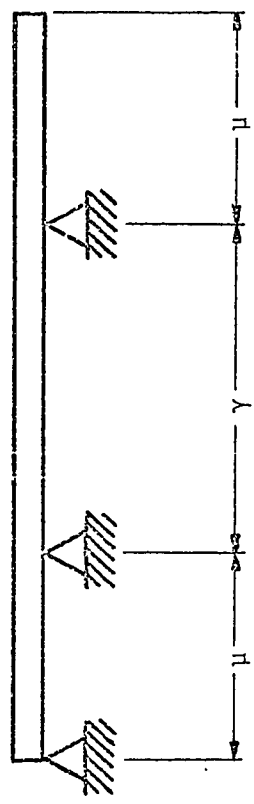
$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.075	5.3203	8.8613	12.4408	16.0329
0.100	5.5655	9.2921	13.0683	16.8607
0.125	5.8426	9.7837	13.7827	17.7870
0.150	6.1580	10.3455	14.5843	18.7466
0.175	6.5195	10.9856	15.4359	19.7171
0.200	6.9368	11.7023	16.0581	18.2583
0.225	7.4212	12.4457	15.6135	18.0541
0.250	7.9833	12.9243	14.9702	17.4035
0.275	8.6239	12.6277	14.8968	16.9433
0.300	9.2927	12.0672	14.3965	17.8287
0.325	9.7484	11.7870	13.8583	19.1017
0.350	9.6772	11.7294	14.0768	18.9096
0.375	9.3206	11.3648	15.3338	18.2042
0.400	8.9254	10.8781	16.0756	18.0254
0.425	8.5646	10.4133	15.6154	17.4656

(5) CSSF Beam



$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.075	4.5197	8.1151	11.6488	15.0290
0.100	4.7515	8.4726	11.8996	14.6575
0.125	4.9960	8.7283	11.5557	14.3741
0.150	5.2392	8.6927	11.2108	14.2825
0.175	5.4508	8.3771	11.4190	15.7102
0.200	5.5717	8.1154	11.9959	16.6441
0.225	5.5421	8.1261	12.7742	17.2720
0.250	5.3777	8.4364	13.6132	16.7524
0.275	5.1512	8.5837	14.1303	15.7070
0.300	4.9155	9.7048	13.8131	14.7762
0.325	4.6939	10.4849	13.1017	14.3016
0.350	4.4935	10.9700	12.3714	14.4165
0.375	4.3166	10.8750	11.7058	15.7378
0.400	4.1622	10.5054	11.1176	17.1556
0.425	4.0291	10.1043	10.6030	17.0763

(6) SSSF Beam



$\mu$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
0.075	4.4931	8.0745	11.5992	14.9760
0.150	4.7159	8.4218	11.8469	14.6057
0.225	4.9538	8.6745	11.5146	14.3921
0.300	5.1861	8.6464	11.1495	14.7735
0.375	5.3934	8.3349	11.3259	15.5138
0.450	5.5175	8.7615	11.8659	16.0926
0.525	5.4988	8.7451	12.5241	15.6248
0.600	5.3458	8.3167	12.9575	14.9808
0.675	5.1269	8.8134	12.6395	14.8685
0.750	4.8968	9.3378	12.0769	14.2234
0.825	4.6776	9.7994	11.7949	13.7866
0.900	4.4802	9.6912	11.6999	14.0444
0.975	4.3059	9.3267	11.3061	15.3264
1.050	4.1541	8.9268	10.8106	16.0745
1.125	4.0236	8.5665	10.3429	15.6151

```

0001 IMPLICIT REAL*8(A-H,O-Z)
0002 DIMENSION A(4,4), K(29,4), PMU(29)
0003 DATA DOT/4H...../
0004 WRITE (3,1)
0005 1 FORMAT (1H1///17X, 3HPHI, 10X, 5HALPHA, 10X, 3HPMU, 11X, 5HROOT1,
1 12X, 5HROOT2, 12X, 5HROOT3, 12X, 5HROOT4//)
0006 READ (1,2) (PMU(M), M = 1, 29)
0007 2 FORMAT (5X, F10.5)
0008 DO 900 K = 6, 10
0009 PHI = K/10.0
0010 DO 300 J = 6, 10
0011 ALPHA = J/10.0
0012 DO 200 M = 1, 29
0013 B = 1.0
0014 DO 13 N = 1, 4
0015 CR = 0.0
0016 CRT = 0.0
0017 4 CALL ARRAY(B, M, N, PMU, PHI, ALPHA, A)
0018 FB = DET(A)
0019 5 IF (CRT .GT. 0.0) GO TO 10
0020 IF (CR .GT. 0.0) GO TO 7
0021 FBS = FB
0022 CR = 1.0
0023 6 B = B + 0.5
0024 GO TO 4
0025 7 IF ((FBS*FB) .LT. 0.0) GO TO 8
0026 FBS = FB
0027 GO TO 6
0028 8 BI = B
0029 FBI = FB
0030 BO = B - 0.5
0031 FBO = FBS
0032 BS = B
0033 B = B0 + (BI - B0)*DABS(FBO)/DABS(FB0) + FBI
0034 I = (DABS(B - BS) .LT. 1.0D-6) GO TO 12
0035 CRT = 1.0
0036 GO TO 4
0037 10 IF ((FB*FBO) .LT. 0.0) GO TO 11
0038 FBO = FB
0039 BO = B
0040 GO TO 9
0041 FBI = FB
0042 BI = B
0043 GO TO 9
0044 R(M, N) = B
0045 B = B + 0.005
0046 13 CONTINUE
0047 200 CONTINUE
0048 DO 160 M = 1, 27, 2
0049 DO 150 N = 1, 4
0050 RINT = (R(M, N) + R(M+2, N))/2.0
0051 ERR = ((R(M+1, N) - RINT)/R(M+1, N)) * 200.0
0052 IF (DABS(ERR) - 1.0) 140, 140, 145
0053 140 R(M+1, N) = DOT
0054 GO TO 150
0055 145 R(M+1, N) = ERR
0056 150 CONTINUE
0057 160 CONTINUE

```

1 60 1

```

0058 DD 180 M = 1, 29
0059 WRITE (3, 170) PHI, ALPHA, PMU(M), (R(M, N), N = 1, 4)
0060 170 FORMAT (17X, F3.1, 11X, F3.1, 9X, F5.3, 4(10X, F7.4))
0061 180 CONTINUE
0062 WRITE (3, 250)
0063 250 FORMAT (1H //)
0064 800 CONTINUE
0065 900 CONTINUE
0066 STOP
0067 END

```

```

0001 IMPLICIT REAL*8(A-H,C-Z)
0002 DIMENSION A(4, 4), R(25, 4), PMU(29)
0003 DATA DOT/4H,.,.,./
0004 WRITE (3, 1)
0005
0006 1 FORMAT (1H1//17X, 3HPMU, 1CX, 3PHI, 10X, SHALPHA, 11X, SHROOT1,
0007 12X, SHROOT2, 12X, SHROOT3, 12X, SHROOT4//)
0008 2 FORMAT (5X, F10.5)
0009 DO 400 M = 1, 15
0010 DO 100 J = 6, 10
0011 PHI = J/10.0
0012 DO 200 MP = 60, 100, 5
0013 ALPHA = MP/100.000
0014 MM = (MP - 60)/5 + 1
0015 B = 1.0
0016 DO 13 N = 1, 4
0017 CR = 0.0
0018 3 CRT = 0.0
0019 4 CALL ARRAY(B, M, N, PMU, PHI, ALPHA, A)
0020 FB = DET(A)
0021 5 IF (CRT .GT. 0.0) GC TC 10
0022 IF (CR .GT. 0.0) GO TO 7
0023 FBS = FB
0024 CR = 1.0
0025 6 B = B + 0.5
0026 GO TO 4
0027 7 IF ((FBS*FB) .LT. 0.0) GO TO 8
0028 FBS = FR
0029 GO TO 6
0030 RI = 5
0031 FH1 = FB
0032 B0 = B - 0.5
0033 FB0 = FBS
0034 BS = B
0035 B = B0 + (B1 - B0)*CABS(FB0)/DAES(FB0 - FB1)
0036 IF (DAES(B - BS) .LT. 1.0D-6) GO TO 12
0037 CRT = 1.0
0038 GO TC 4
0039 FB0 = FB
0040 10 IF ((FB*FB0) .LT. 0.0) GO TO 11
0041 B0 = B
0042 GO TO 2
0043 FB1 = FB
0044 B1 = B
0045 GO TO 9
0046 12 R(MM, N) = B
0047 B = B + 0.005
0048 13 CONTINUE
0049 200 CONTINUE
0050 DO 160 MM = 1, 7, 2
0051 DO 150 N = 1, 4
0052 RINT = (R(MM, N) + R(MM+2, N))/2.000
0053 ERR = ((R(MM+1, N)-RINT)/R(MM+1, N))* 200.000
0054 IF (DAES(ERR) - 1.0) 140, 140, 145
0055 140 P(MM+1, N) = DCT
0056 GO TO 150
0057 145 R(MM+1, N) = ERR
0058 150 CONTINUE

```

```
0058  
0059  
0060  
0061  
0062  
0063  
0064  
0065  
0066  
0067  
0068  
0069  
  
160 CONTINUE  
DO 180 MM = 1, 9  
  ALPHA = (MM + 11)/20.0D0  
  WRITE (3, 170) PMU(N), PHI, ALPHA, (R(MM, N), N = 1, 4)  
170 FORMAT (15X, F5.3, 11X, F3.1, 10X, F4.2, 4(10X, F7.4))  
180 CONTINUE  
  WRITE (3, 250)  
250 FORMAT (1H //)  
100 CONTINUE  
400 CONTINUE  
  STOP  
  END
```

```

0001 IMPLICIT REAL*8(A-H,C-Z)
0002 DIMENSION A(4, 4), R(29, 4), PMU(29)
0003 DATA DOT/4H,....//
0004 WRITE (3, 1)
0005
0006 1 FORMAT (IH1///17X, 3HPMU, 10X, SHALPHA, 10X, 3PHI, 11X, 5HRCOT1,
0007 12X, 5HROOT2, 12X, 5HROOT3, 12X, 5HROOT4//)
0008 2 FORMAT (5X, F10.5)
0009 DO 400 M = 1, 15
0010 DO 100 J = 6, 10
0011 ALPHA = J/10.0
0012 DO 200 MP = 60, 100, 5
0013 PHI = MP/100.000
0014 MM = (MP - 50)/5 + 1
0015 B = 1.0
0016 DO 13 N = 1, 4
0017 CR = 0.0
0018 3 CRT = 0.0
0019 4 CALL ARRAY(B, M, N, PMU, PHI, ALPHA, A)
0020 FB = DET(A)
0021 5 IF (CRT .GT. 0.0) GC TC 10
0022 FUS = FB
0023 CR = 1.0
0024 6 B = B + 0.5
0025 GO TO 4
0026 7 IF ((FBS*FB) .LT. 0.0) GO TO 8
0027 FBS = FB
0028 GO TO 6
0029 8 B1 = B
0030 F31 = FB
0031 B0 = B - 0.5
0032 F30 = FRS
0033 9 BS = B
0034 B = B0 + (B1 - B0)*DABS(FE0)/DABS(FE0 - FB1)
0035 IF (DABS(B - BS) .LT. 1.0D-6) GO TO 12
0036 CRT = 1.0
0037 GO TO 4
0038 10 IF ((FB*F30) .LT. 0.0) GO TO 11
0039 F30 = FB
0040 B0 = B
0041 GO TO 9
0042 11 FB1 = FB
0043 B1 = B
0044 GO TO 9
0045 12 R(MM, N) = B
0046 E = B + 0.005
0047 13 CONTINUE
0048 200 CONTINUE
0049 DO 160 MM = 1, 7, 2
0050 DO 150 N = 1, 4
0051 RINT = (R(MM, N) + P(MM+2, N))/2.0D0
0052 ERR = ((R(MM+1, N) - RINT)/R(MM+1, N))* 200.0D0
0053 IF (DABS(ERR) - 1.0) 140, 140, 145
0054 140 R(MM+1, N) = DCT
0055 GO TO 150
0056 145 R(MM+1, N) = ERR
0057 150 CONTINUE

```

```
0058  
0059  
0060  
0061  
0062  
0063  
0064  
0065  
0066  
0067  
0068  
0069  
160 CONTINUE  
DO 180 MM = 1, 9  
  PHI = (MM + 11)/20,0D0  
  WRITE (3, 170) PMU(M), ALPHA, PHI, (R(MM, N), N = 1, 4)  
170 FORMAT (15X, F5.3, 10X, F3.1, 11X, F4.2, 4(10X, F7.4)/)  
180 CONTINUE  
  WRITE (3, 250)  
250 FORMAT (1H //)  
100 CCNTINUE  
400 CONTINUE  
STOP  
END
```

```

0001 IMPLICIT REAL*8(A-H,O-Z)
0002 DIMENSION A(4,4),R(4,15),ROOT(15),PMU(15)
0003 CALL RFPREAD
0004 READ (1,1) (PMU(M), M = 1, 15)
0005 1 FFORMAT (5X, F10.5)
0006 NTABLE = 1
0007 DO 900 K = 6, 10
0008   DK = K
0009   PHI = 10.000/DK
0010   PHII = DK/10.000
0011   DO 800 J = 6, 10
0012     DJ = J
0013     ALPHA = 10.000/DJ
0014     ALPHAI = DJ/10.000
0015     DO 200 M = 1, 15
0016       B = 1.0
0017       DU 13 N = 1, 4
0018       3 CR = 0.0
0019       CRT = 0.0
0020       4 CALL ARRAY(B, M, N, PMU, PHI, ALPHA, A)
0021       FB = DET(A)
0022       5 IF (CRT .GT. 0.0) GO TO 10
0023       IF (CR .GT. 0.0) GO TO 7
0024       FBS = FB
0025       CR = 1.0
0026       6 B = B + 0.5
0027       GO TO 4
0028       7 IF ((FBS*FB) .LT. 0.0) GO TO 8
0029       FBS = FB
0030       GO TO 6
0031       8 BI = B
0032       FBI = FB
0033       B0 = B - 0.5
0034       FBS0 = FBS
0035       9 BS = B
0036       B = B0 + (BI - B0)*DABS(FB0)/DABS(FB0 - FBI)
0037       IF (DABS(B - BS) .LT. 1.00-6) GO TO 12
0038       CRT = 1.0
0039       GO TO 4
0040       10 IF ((FB*FB0) .LT. 0.0) GO TO 11
0041       FB0 = FB
0042       B0 = B
0043       GO TO 9
0044       11 FBI = FB
0045       BI = B
0046       GO TO 9
0047       12 R(N, M) = B
0048       B = B + 0.005
0049       13 CONTINUE
0050       200 CONTINUE
0051       IF (NTABLE - (NTABLE/2)*2) 210, 230, 210
0052       210 WRITE (3, 220)
0053       220 FFORMAT (1H1/)
0054       GO TO 300
0055       230 WRITL (3, 240)
0056       240 FFORMAT (1H0////)
0057       300 WRITE (3, 350) NTABLE, PHII, ALPHA, PMU(M), M = 1, 15)
0058       350 FFORMAT (61X, 'TABLE XX.', I2//49X, '1/PHI = ', F3.1, 11X, '1/ALPH

```

IA = 1, F3.1//8X, 'PMU:', 15(3X, F5.3)/2X, 'MODE:')

DO 400 N = 1, 4

DO 370 M = 1, 15

Y = R(N, M)

IF (Y .GT. 0.999) GO TO 73

WRITE (99, 72) Y

72 FORMAT (F8.4)

GO TO 370

IF (Y .GT. 9.999) GO TO 75

WRITE (99, 74) Y

74 FORMAT (F8.3)

GO TO 370

IF (Y .GT. 99.999) GO TO 77

WRITE (99, 76) Y

76 FORMAT (F8.2)

GO TO 370

IF (Y .GT. 999.999) GO TO 79

WRITE (99, 78) Y

78 FORMAT (F8.1)

GO TO 370

IY = R(N, M)

WRITE (99, 80) IY

80 FORMAT (I8)

370 READ (99, 380) ROOT(M)

380 FORMAT (A8)

WRITE (3, 390) N, ROOT

390 FORMAT (I10, 2X, I2, 7X, 15(A8))

400 CONTINUE

NTABLE = NTABLE + 1

800 CONTINUE

900 CONTINUE

STOP

END

```

0001 DOUBLE PRECISION FUNCTION DET(A)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 DIMENSION A(4, 4)

```

```

C
0004 D1 = A(1,1) * (A(2,2)*(A(3,3)*A(4,4) - A(3,4)*A(4,3))
1 -A(2,3)*(A(3,2)*A(4,4) - A(3,4)*A(4,2))
2 +A(2,4)*(A(3,2)*A(4,3) - A(3,3)*A(4,2)))
0005 D2 = A(1,2) * (A(2,1)*(A(3,3)*A(4,4) - A(3,4)*A(4,3))
3 -A(2,3)*(A(3,1)*A(4,4) - A(3,4)*A(4,1))
4 +A(2,4)*(A(3,1)*A(4,3) - A(3,3)*A(4,1)))
0006 D3 = A(1,3) * (A(2,1)*(A(3,2)*A(4,4) - A(3,4)*A(4,2))
5 -A(2,2)*(A(3,1)*A(4,4) - A(3,4)*A(4,1))
6 +A(2,4)*(A(3,1)*A(4,2) - A(3,2)*A(4,1)))
0007 D4 = A(1,4) * (A(2,1)*(A(3,2)*A(4,3) - A(3,3)*A(4,2))
7 -A(2,2)*(A(3,1)*A(4,3) - A(3,3)*A(4,1))
8 +A(2,3)*(A(3,1)*A(4,2) - A(3,2)*A(4,1)))
C

```

```

0008 DET = D1 - D2 + D3 - D4
0009 RETURN
0010 END

```

```

0001 DOUBLE PRECISION FUNCTION DET(A)
0002 IMPLICIT REAL*8(A-H,C-Z)
0003 DIMENSION Z(3, 8), IPVCT(8), INDEX(8, 2), PIVCT(8)
0004 EQUIVALENCE (IROW, JRCW), (ICOL, JCOL)

```

C

```

0005 N = 5
0006 DLT = 1.0
0007 DO 1 J = 1, N
0008 IPVCT(J) = 0.0
0009 CONTINUE
0010 DO 9 I = 1, N
0011 T = 0.0
0012 DO 4 J = 1, N
0013 IF (IPVCT(J) .EQ. 1) GO TO 4
0014 DO 3 K = 1, N
0015 IF (IPVCT(K) - 1) 2, 3, 10
0016 IF (DABS(T) .GE. DABS(A(J, K))) GO TO 3
0017 IROW = J
0018 ICOL = K
0019 T = A(J, K)
0020 CONTINUE
0021 CONTINUE
0022 IPVCT(ICOL) = IPVCT(ICOL) + 1
0023 IF (IROW .EQ. ICOL) GO TO 6
0024 DET = -DET
0025 DO 5 L = 1, N
0026 T = A(IROW, L)
0027 A(IROW, L) = A(ICOL, L)
0028 A(ICOL, L) = T
0029 CONTINUE
0030 INDEX(I, 1) = IROW
0031 INDEX(I, 2) = ICOL
0032 PIVOT(I) = A(ICOL, ICOL)
0033 DET = DET * PIVOT(I)
0034 A(ICOL, ICOL) = 1.0
0035 DO 7 L = 1, N
0036 A(ICOL, L) = A(ICOL, L)/PIVOT(I)
0037 CONTINUE
0038 DO 9 LI = 1, N
0039 IF (LI .EQ. ICOL) GO TO 9
0040 T = A(LI, ICOL)
0041 A(LI, ICOL) = 0.0
0042 DO 8 L = 1, N
0043 A(LI, L) = A(LI, L) - A(ICOL, L) * T
0044 CONTINUE
0045 CONTINUE

```

C

```

0046 DO 10 RETURN
0047 END

```

CCC1      SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)  
 CCC2      IMPLICIT REAL\*8(A-H,O-Z)  
 CCC3      DIMENSION A(4, 4), FMU(29)

C

CCC4      BP = BETA1 \* PMU(M)  
 CCC5      SINBP = DSIN(BP)  
 CCC6      COSBP = DCOS(BP)  
 CCC7      SINHBP = DSINH(BP)  
 CCC8      COSHBP = DCOSH(BP)  
 CCC9      ALP = PHI/ALPHA  
 CCC10      GAMMA = 1.0 - PMU(M)  
 CCC11      ABG = ALP \* BETA1 \* GAMMA  
 CCC12      SINABG = DSIN(ABG)  
 CCC13      COSABG = DCOS(ABG)  
 CCC14      SIHABG = DSINH(ABG)  
 CCC15      COHABG = DCOSH(ABG)  
 CCC16      ALPHA4 = ALPHA\*\*4  
 CCC17      PSQA = ALPHA4 \* (ALP\*\*2)  
 CCC18      PCUA = ALPHA4 \* (ALP\*\*3)

C

CCC19      A(1, 1) = SINBP - SINHBP  
 CCC20      A(1, 2) = COSBP - COSHBP  
 CCC21      A(1, 3) = -SINABG - SIHABG  
 CCC22      A(1, 4) = -COSABG - COHABG  
 CCC23      A(2, 1) = COSBP - COSHP  
 CCC24      A(2, 2) = -SINBP - SINHP  
 CCC25      A(2, 3) = ALP \* (COSABG + COHABG)  
 CCC26      A(2, 4) = ALP \* (-SINABG + SIHABG)

C

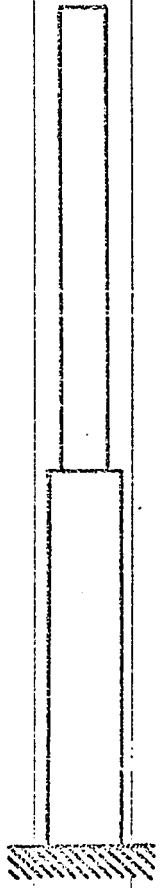
CCC27      A(3, 1) = -SINBP - SINHBP  
 CCC28      A(3, 2) = -COSBP - COSHBP  
 CCC29      A(3, 3) = PSQA \* (SINABG - SIHABG)  
 CCC30      A(3, 4) = PSQA \* (COSABG - COHABG)

C

CCC31      A(4, 1) = -COSBP - COSHBP  
 CCC32      A(4, 2) = SINBP - SINHBP  
 CCC33      A(4, 3) = PCUA \* (-COSABG + COHABG)  
 CCC34      A(4, 4) = PCUA \* (SINABG + SIHABG)

C

CCC35      RETURN  
 CCC36      END



CC01 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)  
 CC02 IMPLICIT REAL\*8(A-H,O-Z)  
 CC03 DIMENSION A(4, 4), PMU(29)

C

CC04 BP = BETA1 \* PMU(M)  
 CC05 SINBP = DSIN(BP)  
 CC06 COSBP = DCOS(BP)  
 CC07 SINHBP = DSINH(BP)  
 CC08 COSHBP = DCOSH(BP)  
 CC09 ALP = PHI/ALPHA

CC10 GAMMA = 1.0 - PMU(M)  
 CC11 ABG = ALP \* BETA1 \* GAMMA  
 CC12 SINABG = DSIN(ABG)  
 CC13 COSABG = DCOS(ABG)  
 CC14 SIHABG = DSINH(ABG)  
 CC15 COHABG = DCOSH(ABG)  
 CC16 ALPHA4 = ALPHA\*\*4  
 CC17 PSQA = ALPHA4 \* (ALP\*\*2)  
 CC18 PCUA = ALPHA4 \* (ALP\*\*3)

C

CC19 A(1, 1) = SINBP - SINHBP  
 CC20 A(1, 2) = COSBP - COSHBP  
 CC21 A(1, 3) = -SINABG  
 CC22 A(1, 4) = -SIHABG

C

CC23 A(2, 1) = COSBP - COSHBP  
 CC24 A(2, 2) = -SINBP - SINHBP  
 CC25 A(2, 3) = ALP \* COSABG  
 CC26 A(2, 4) = ALP \* COHABG

C

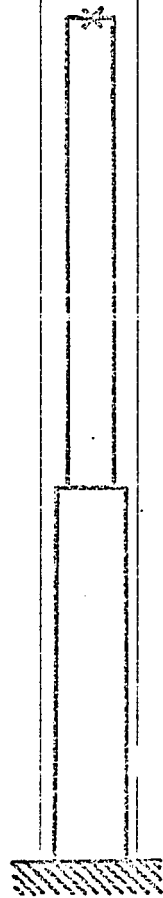
CC27 A(3, 1) = -SINBP - SINHBP  
 CC28 A(3, 2) = -COSBP - COSHBP  
 CC29 A(3, 3) = PSQA \* SINABG  
 CC30 A(3, 4) = -PSQA \* SIHABG

C

CC31 A(4, 1) = -COSBP - COSHBP  
 CC32 A(4, 2) = SINBP - SINHBP  
 CC33 A(4, 3) = -PCUA \* COSABG  
 CC34 A(4, 4) = PCUA \* COHABG

C

CC35 RETURN  
 CC36 END



```

0001 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 DIMENSION A(4, 4), PMU(29)

```

C

```

0004 BP = BETA1 * PMU(M)
0005 SINBP = DSIN(BP)
0006 COSBP = DCOS(BP)
0007 SINHP = DSINH(BP)
0008 COSHP = DCOSH(BP)
0009 ALP = PHIZ/ALPHA
0010 GAMMA = 1.0 - FMU(M)
0011 ABG = ALP * BETA1 * GAMMA
0012 SINABG = DSIN(ABG)
0013 COSABG = DCOS(ABG)
0014 SIHABG = DSINH(ABG)
0015 COHABG = DCOSH(ABG)
0016 ALPHA4 = ALPHA**4
0017 PSQA = ALPHA4 * (ALP**2)
0018 PCUA = ALPHA4 * (ALP**3)

```

C

```

0019 A(1, 1) = SINBP
0020 A(1, 2) = SINHP
0021 A(1, 3) = -SINABG
0022 A(1, 4) = -SIHABG

```

C

```

0023 A(2, 1) = COSBP
0024 A(2, 2) = CCSHP
0025 A(2, 3) = ALP * COSABG
0026 A(2, 4) = ALP * COHABG

```

C

```

0027 A(3, 1) = -SINBP
0028 A(3, 2) = SINHP
0029 A(3, 3) = PSQA * SINABG
0030 A(3, 4) = -PSQA * SIHABG

```

C

```

0031 A(4, 1) = -COSBP
0032 A(4, 2) = CCSHP
0033 A(4, 3) = -PCUA * COSABG
0034 A(4, 4) = PCUA * COHABG

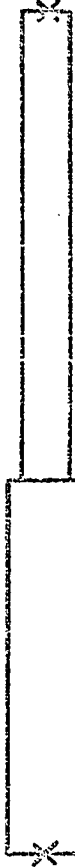
```

C

```

0035 RETURN
0036 END

```



0001 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)  
 0002 IMPLICIT REAL\*8(A-H, O-Z)  
 0003 DIMENSION A(4, 4), PMU(29)

C

0004 BP = BETA1 \* PMU(M)  
 0005 SINBP = DSIN(BP)  
 0006 COSBP = DCOS(BP)  
 0007 SINHRP = DSINH(BP)  
 0008 COSHRP = DCOSH(BP)  
 0009 ALP = PHI/ALPHA  
 0010 GAMMA = 1.0 - PMU(M)  
 0011 ABG = ALP \* BETA1 \* GAMMA  
 0012 SINABG = DSIN(ABG)  
 0013 COSABG = DCOS(ABG)  
 0014 SIHABG = DSINH(ABG)  
 0015 COHABG = DCOSH(ABG)  
 0016 ALPHA4 = ALPHA\*\*4  
 0017 PSQA = ALPHA4 \* (ALP\*\*2)  
 0018 PCUA = ALPHA4 \* (ALP\*\*3)

C

0019 A(1, 1) = SINBP - SINHRP  
 0020 A(1, 2) = COSBP - COSHRP  
 0021 A(1, 3) = -SINABG + SIHABG  
 0022 A(1, 4) = -COSABG + COHABG

C

0023 A(2, 1) = COSBP - COSHRP  
 0024 A(2, 2) = -SINBP - SINHRP  
 0025 A(2, 3) = ALP \* (COSABG - COHABG)  
 0026 A(2, 4) = ALP \* (-SINABG - SIHABG)

C

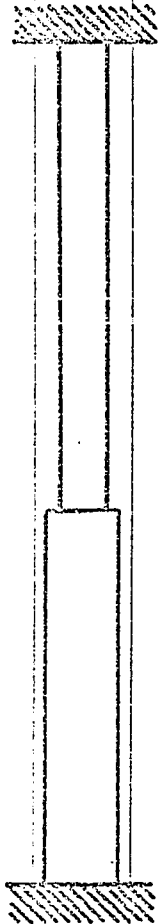
0027 A(3, 1) = -SINBP - SINHRP  
 0028 A(3, 2) = -COSBP - COSHRP  
 0029 A(3, 3) = PSQA \* (SINABG + SIHABG)  
 0030 A(3, 4) = PSQA \* (COSABG + COHABG)

C

0031 A(4, 1) = -COSBP - COSHRP  
 0032 A(4, 2) = SINBP - SINHRP  
 0033 A(4, 3) = PCUA \* (-COSABG - COHABG)  
 0034 A(4, 4) = PCUA \* (SINABG - SIHABG)

C

0035 RETURN  
 0036 END



```

SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(4, 4), PMU(29)

```

C

```

BP = BETA1 * PMU(M)
SINBP = DSIN(BP)
COSBP = DCOS(BP)
SINHBP = DSINH(BP)
COSHP = DCOSH(BP)
ALP = PHI/ALPHA

```

```

GAMMA = 1.0 - PMU(M)
ABG = ALP * BETA1 * GAMMA
SINABG = DSIN(ABG)
COSABG = DCOS(ABG)
SINHABG = DSINH(ABG)
COSHABG = DCOSH(ABG)
ALPHA4 = ALPHA**4
PSQA = ALPHA/4 * (ALP**2)

```

C

```

A(1, 1) = SINBP - SINHBP
A(1, 2) = COSBP - COSHP
A(1, 3) = 0.0
A(1, 4) = 0.0

```

C

```

A(2, 1) = 0.0
A(2, 2) = 0.0

```

```

A(2, 3) = SINABG + SINHABG
A(2, 4) = COSABG + COSHABG

```

C

```

A(3, 1) = COSBP - COSHP
A(3, 2) = -SINBP - SINHBP
A(3, 3) = ALP * (COSABG + COSHABG)
A(3, 4) = ALP * (-SINABG + SINHABG)

```

C

```

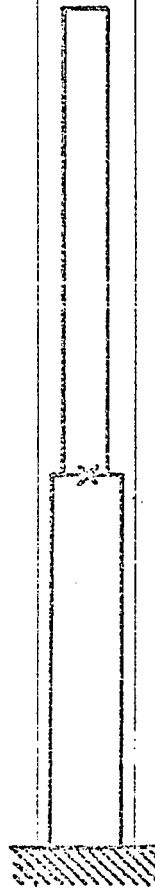
A(4, 1) = -SINBP - SINHBP
A(4, 2) = -COSBP - COSHP
A(4, 3) = PSQA * (SINABG - SINHABG)
A(4, 4) = PSQA * (COSABG - COSHABG)

```

```

RETURN
END

```



SUBROUTINE ARRAY(BLTAI, M, N, PMU, PHI, ALPHA, A)  
 IMPLICIT REAL\*8(A-H,O-Z)  
 DIMENSION A(4, 4), PMU(29)

C

BP = BETA1 \* PMU(M)  
 SINBP = DSIN(BP)  
 COSBP = DCOS(BP)  
 SINHBP = DSINH(BP)  
 COSHBP = DCOSH(BP)  
 ALP = PHI/ALPHA  
 GAMMA = 1.0 - PMU(K)  
 ABG = ALP \* BETA1 \* GAMMA  
 SINABG = DSIN(ABG)  
 COSABG = DCOS(ABG)  
 SINHABG = DSINH(ABG)  
 COHABG = DCOSH(ABG)  
 ALPHA4 = ALPHA\*\*4  
 PSQA = ALPHA4 \* (ALP\*\*2)

C

A(1, 1) = SINBP - SINHBP  
 A(1, 2) = COSBP - COSHBP  
 A(1, 3) = 0.0  
 A(1, 4) = 0.0

C

A(2, 1) = 0.0  
 A(2, 2) = 0.0  
 A(2, 3) = SINABG  
 A(2, 4) = SINHABG

C

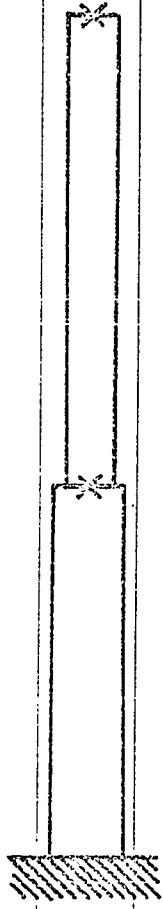
A(3, 1) = COSBP - COSHBP  
 A(3, 2) = -SINBP - SINHBP  
 A(3, 3) = ALP \* COSABG  
 A(3, 4) = ALP \* COHABG

C

A(4, 1) = -SINBP - SINHBP  
 A(4, 2) = -COSBP - COSHBP  
 A(4, 3) = PSQA \* SINABG  
 A(4, 4) = -PSQA \* SINHABG

C

RETURN  
 END



```

0001 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)
0002 I=1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100
0003 DIMENSION A(4, 4), PMU(29)

```

```

C
0004 BP = BETA1 * PMU(M)
0005 SINBP = DSIN(BP)
0006 COSBP = DCOS(BP)
0007 SINHP = DSINH(BP)
0008 COSHP = DCOSH(BP)
0009 ALP = PHI/ALPHA
0010 GAMMA = 1.0 - PMU(M)
0011 ABS = ALP * BETA1 * GAMMA
0012 SINABS = DSIN(ABS)
0013 COSABS = DCOS(ABS)
0014 SIHABS = DSINH(ABS)
0015 COHABS = DCOSH(ABS)
0016 ALPHA4 = ALPHA**4
0017 PSQA = ALPHA4 * (ALP**2)

```

```

C
0018 A(1, 1) = SINBP
0019 A(1, 2) = SINHP
0020 A(1, 3) = C.0
0021 A(1, 4) = C.0

```

```

C
0022 A(2, 1) = C.0
0023 A(2, 2) = C.0
0024 A(2, 3) = SINABS
0025 A(2, 4) = SIHABS

```

```

C
0026 A(3, 1) = COSBP
0027 A(3, 2) = COSHP
0028 A(3, 3) = ALP * COSABS
0029 A(3, 4) = ALP * COHABS

```

```

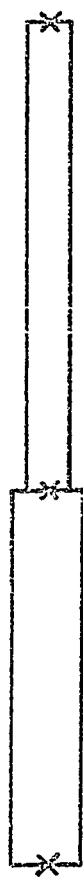
C
0030 A(4, 1) = -SINBP
0031 A(4, 2) = SINHP
0032 A(4, 3) = PSQA * SINABS
0033 A(4, 4) = -PSQA * SIHABS

```

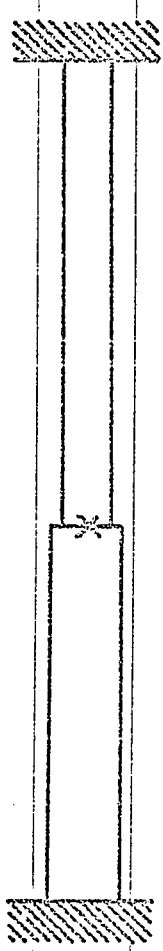
```

C
0034 RETURN
0035 END

```



```
0001 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 DIMENSION A(4, 4), PMU(29)
C
0004 BP = BETA1 * PMU(M)
0005 SINBP = DSIN(BP)
0006 COSBP = DCOS(BP)
0007 SINHP = DSINH(BP)
0008 COSHP = DCOSH(BP)
0009 ALP = PHI/ALPHA
0010 GAMMA = 1.0 - PMU(M)
0011 ABG = ALP * BETA1 * GAMMA
0012 SINAUG = DSIN(ABG)
0013 COSABG = DCOS(ABG)
0014 SIHAUG = DSINH(ABG)
0015 COHABG = DCOSH(ABG)
0016 ALPHA4 = ALPHA**4
0017 PSQA = ALPHA4 * (ALP**2)
C
0018 A(1, 1) = SINBP - SINHP
0019 A(1, 2) = COSBP - COSHP
0020 A(1, 3) = C.0
0021 A(1, 4) = C.0
C
0022 A(2, 1) = C.0
0023 A(2, 2) = C.0
0024 A(2, 3) = SINABG - SIHABG
0025 A(2, 4) = COSABG - COHABG
C
0026 A(3, 1) = COSBP - CCOSHP
0027 A(3, 2) = -SINBP - SINHP
0028 A(3, 3) = ALP * (COSABG - COHABG)
0029 A(3, 4) = ALP * (-SINABG - SIHABG)
C
0030 A(4, 1) = -SINBP - SINHP
0031 A(4, 2) = -COSBP - COSHP
0032 A(4, 3) = PSQA * (SINABG + SIHABG)
0033 A(4, 4) = PSQA * (COSABG + COHABG)
C
0034 RETURN
0035 END
```



CCCC SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)  
 CCCC J APPLICATION REAL\*8(A-H, O-Z)  
 CCCC DIMENSION A(4, 4), PMU(29)

C

CCCC BP = BETA1 \* PMU(M)  
 CCCC SINBP = DSIN(BP)  
 CCCC COSBP = DCOS(BP)  
 CCCC SINHBP = DSINH(BP)  
 CCCC COSHBP = DCOSH(BP)  
 CCCC ALP = PHI/ALPHA  
 CCCC GAMMA = 1.0 - PMU(M)  
 CCCC AUG = ALP \* BETA1 \* GAMMA  
 CCCC SINABG = DSIN(AUG)  
 CCCC COSABG = DCOS(AUG)  
 CCCC SINABG = DSINH(AUG)  
 CCCC COHABG = DCOSH(AUG)  
 CCCC ALPHA4 = ALPHA\*\*4  
 CCCC P50A = ALPHA4 \* (ALP\*\*2)

C

CCCC A(1, 1) = SINBP  
 CCCC A(1, 2) = SINHBP  
 CCCC A(1, 3) = 0.0  
 CCCC A(1, 4) = 0.0

C

CCCC A(2, 1) = 0.0  
 CCCC A(2, 2) = 0.0  
 CCCC A(2, 3) = SINABG + SINABG  
 CCCC A(2, 4) = CCSABG + COHABG

C

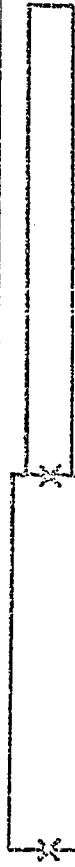
CCCC A(3, 1) = COSBP  
 CCCC A(3, 2) = COSHBP  
 CCCC A(3, 3) = ALP \* (COSABG + COHABG)  
 CCCC A(3, 4) = ALP \* (-SINABG + SINABG)

C

CCCC A(4, 1) = -SINBP  
 CCCC A(4, 2) = SINHBP  
 CCCC A(4, 3) = P50A \* (SINABG - SINABG)  
 CCCC A(4, 4) = P50A \* (COSABG - COHABG)

C

CCCC RETURN  
 CCCC END



CC01 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)  
 CC02 IMPLICIT REAL\*8(A-H,O-Z)  
 CC03 DIMENSION A(4, 4), PMU(29)

C

CC04 BP = BETA1 \* PMU(M)  
 CC05 SINBP = DSIN(BP)  
 CC06 CCSBP = DCCS(BP)  
 CC07 SINHBP = DSINH(BP)  
 CC08 CCSHBP = DCCSH(BP)  
 CC09 ALP = PHI/ALPHA  
 CC10 GAMMA = 1.0 - PMU(M)  
 CC11 ABG = ALP \* BETA1 \* GAMMA  
 CC12 SINABG = DSIN(ABG)  
 CC13 CCSABG = DCCS(ABG)  
 CC14 SIHABG = DSINH(ABG)  
 CC15 COHABG = DCCSH(ABG)  
 CC16 ALPHA4 = ALPHA\*\*4  
 CC17 PSQA = ALPHA4 \* (ALP\*\*2)  
 CC18 PCUA = ALPHA4 \* (ALP\*\*3)

C

CC19 N = 1, 3 SYMMETRIC MODES

1

CC20 A(1, 1) = SINBP  
 CC21 A(1, 2) = SINHBP  
 CC22 A(1, 3) = -CCSABG  
 CC23 A(1, 4) = -COHABG

C

CC24 A(2, 1) = COSBP  
 CC25 A(2, 2) = COSHBP  
 CC26 A(2, 3) = -ALP \* SINABG  
 CC27 A(2, 4) = ALP \* SIHABG

C

CC28 A(3, 1) = -SINBP  
 CC29 A(3, 2) = SINHBP  
 CC30 A(3, 3) = PSQA \* COSABG  
 CC31 A(3, 4) = -PSQA \* COHABG

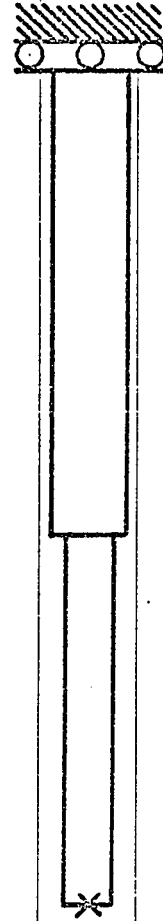
C

CC32 A(4, 1) = -COSBP  
 CC33 A(4, 2) = COSHBP  
 CC34 A(4, 3) = PCUA \* SINABG  
 CC35 A(4, 4) = PCUA \* SIHABG

C

CC36 RETURN  
 CC37 END

180



0001 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)  
 0002 IMPLICIT REAL\*8(A-H,O-Z)  
 0003 DIMENSION A(4, 4), PMU(29)

C  
 0004 DP = BETA1 \* PMU(M)  
 0005 SINBP = DSIN(BP)  
 0006 COSBP = DCOS(BP)  
 0007 SINHBP = DSINH(BP)  
 0008 COSHBP = DCOSH(BP)  
 0009 ALP = PHI/ALPHA  
 0010 GAMMA = 1.0 - PMU(M)  
 0011 ABG = ALP \* BETA1 \* GAMMA  
 0012 SINABG = DSIN(ABG)  
 0013 COSABG = DCOS(ABG)  
 0014 SIHABG = DSINH(ABG)  
 0015 COHABG = DCOSH(ABG)  
 0016 ALPHA4 = ALPHA\*\*4  
 0017 PSQA = ALPHA4 \* (ALP\*\*2)  
 0018 PCUA = ALPHA4 \* (ALP\*\*3)

C  
 C N = 1, 3 SYMMETRIC MODES  
 0019 1 A(1, 1) = SINBP - SINHBP  
 0020 A(1, 2) = COSBP - COSHBP  
 0021 A(1, 3) = -COSABG  
 0022 A(1, 4) = -COHABG

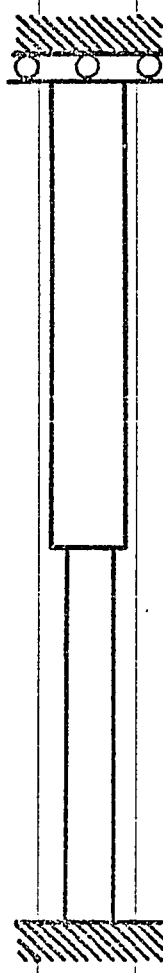
C  
 0023 A(2, 1) = COSBP - CCSHBP  
 0024 A(2, 2) = -SINBP - SINHBP  
 0025 A(2, 3) = -ALP \* SINABG  
 0026 A(2, 4) = ALP \* SIHABG

C  
 0027 A(3, 1) = -SINBP - SINHBP  
 0028 A(3, 2) = -COSBP - CCSHBP  
 0029 A(3, 3) = PSQA \* COSABG  
 0030 A(3, 4) = -PSQA \* COHABG

C  
 0031 A(4, 1) = -COSBP - COSHBP  
 0032 A(4, 2) = SINBP - SINHBP  
 0033 A(4, 3) = PCUA \* SINABG  
 0034 A(4, 4) = PCUA \* SIHABG

C  
 0035 RETURN  
 0036 END

181



```

C001 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)
C002 IMPLICIT REAL*8(A-H, O-Z)
C003 DIMENSION A(4, 4), PMU(29)

```

C

```

C004 DP = BETA1 * PMU(M)
C005 SINBP = DSIN(BP)
C006 COSBP = DCOS(BP)
C007 SINHP = DSINH(BP)
C008 COSHP = DCOSH(BP)
C009 ALP = PHI/ALPHA
C010 GAMMA = 1./D**2 - PMU(N)
C011 ABG = ALP * BETA1 * GAMMA
C012 SINABG = DSIN(ABG)
C013 COSABG = DCOS(ABG)
C014 SINABG = DSINH(ABG)
C015 COSHABG = DCOSH(ABG)
C016 ALPHA4 = ALPHA**4
C017 -PSOA = ALPHA4 * (ALP**2)

```

C

```

C N = 1, 3 SYMMETRIC MODES
C 1 A(1, 1) = SINBP
  A(1, 2) = SINHP

```

C

```

  A(1, 3) = P.C
  A(1, 4) = C.C

```

C

```

  A(2, 1) = C.C
  A(2, 2) = P.C
  A(2, 3) = COSABG
  A(2, 4) = COSHABG

```

C

```

  A(3, 1) = COSBP
  A(3, 2) = COSHP
  A(3, 3) = -ALP * SINABG
  A(3, 4) = ALP * SINHP

```

C

```

  A(4, 1) = -SINBP
  A(4, 2) = SINHP
  A(4, 3) = PSOA * COSABG
  A(4, 4) = -PSOA * COSHABG

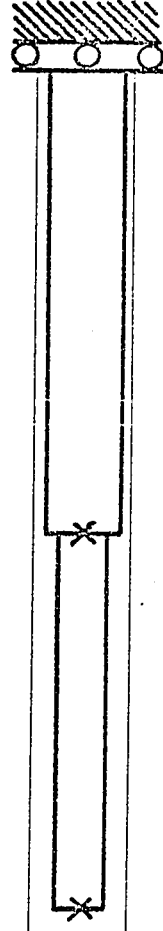
```

C

```

RETURN
END

```



```

CC01 SUBROUTINE AFRAY(BETA1, M, N, PMU, PHI, ALPHA, A)
CC02 IMPLICIT REAL*8(A-H,O-Z)
CC03 DIMENSION A(4, 4), PMU(29)
C
CC04 EF = BETA1 * PMU(M)
CC05 SINBP = DSIN(BP)
CC06 CCSEP = CCCS(EF)
CC07 SINHP = DSINH(BP)
CC08 CCSHEP = DCCSH(BP)
CC09 ALP = PHI/ALPHA
CC10 GAMMA = 1.0DC - PMU(M)
CC11 AEG = ALP * BETA1 * GAMMA
CC12 SINABG = DSIN(ABG)
CC13 CCSABG = DCCS(ABG)
CC14 SIHABG = DSINH(ABG)
CC15 CCHABG = DCCSH(ABG)
CC16 ALPHA4 = ALPHA**4
CC17 PSQA = ALPHA4 * (ALP**2)

```

```

C
C
C N = 1, 3 SYMMETRIC MCDES
CC18 1 A(1, 1) = SINBP - SINHP
CC19 A(1, 2) = CCSBP - CCSHP
CC20 A(1, 3) = C.C
CC21 A(1, 4) = 0.0
C
CC22 A(2, 1) = C.0
CC23 A(2, 2) = 0.0
CC24 A(2, 3) = CCSABG
CC25 A(2, 4) = CCHABG

```

```

C
CC26 A(3, 1) = CCSEP - CCSHEP
CC27 A(3, 2) = -SINBP - SINHP
CC28 A(3, 3) = -ALP * SINABG
CC29 A(3, 4) = ALP * SIHABG

```

```

C
CC30 A(4, 1) = -SINBP - SINHP
CC31 A(4, 2) = -CCSBP - CCSHP
CC32 A(4, 3) = PSQA * COSABG
CC33 A(4, 4) = -FSGA * CCHABG

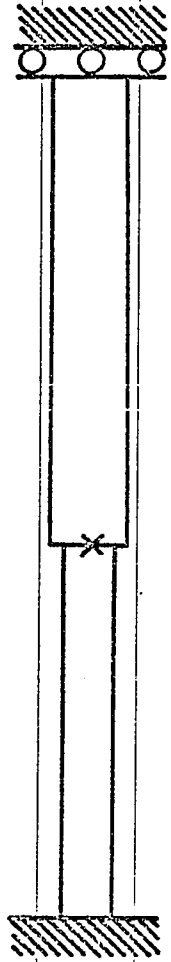
```

```

CC34 RETURN
CC35 END

```

- 183 -



0001 SUBROUTINE ARRAY(BETA, M, N, PHU, PHI, ALPHA, A)

0002 IMPLICIT REAL\*8(A-H, O-Z)

0003 DIMENSION A(4, 4), PHU(20)

0004 PP = BETA / PHU(M)

0005 SINPP = DSIN(PP)

0006 COSPP = DCOS(PP)

0007 SINHP = DSINH(PP)

0008 COSHP = DCOSH(PP)

0009 ALP = PHI / ALPHA

0010 GAMMA = 1. / (1. - PHU(M))

0011 ASG = ALP \* BETA \* GAMMA

0012 SINABG = DSIN(ASG)

0013 COSABG = DCOS(ASG)

0014 SINHABG = DSINH(ASG)

0015 CUSHABG = DCOSH(ASG)

0016 ALP4 = ALPHA\*\*4

0017 PSQA = ALPHA/4 \* (ALP\*\*2)

0018 N = 1, 3 SYMLTRIC MODES

0019 A(1, 1) = SINPP + SINHP

0020 A(1, 2) = COSPP + COSHP

0021 A(1, 3) = 0.0

0022 A(1, 4) = 0.0

0023 A(2, 1) = 0.0

0024 A(2, 2) = 0.0

0025 A(2, 3) = COSABG

0026 A(2, 4) = CUSHABG

0027 A(3, 1) = COSPP + COSHP

0028 A(3, 2) = -SINPP + SINHP

0029 A(3, 3) = -ALP \* SINABG

0030 A(3, 4) = ALP \* SINHABG

0031 A(4, 1) = -SINPP + SINHP

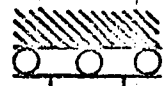
0032 A(4, 2) = -COSPP + COSHP

0033 A(4, 3) = PSQA \* COSABG

0034 A(4, 4) = -PSQA \* CUSHABG

0035 RETURN

0036 END



0001 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)  
 0002 IMPLICIT REAL\*8(A-H,O-Z)  
 0003 DIMENSION A(8, 8), PMU(29)

C

0004 BP = BETA1 \* PMU(M)  
 0005 SINGP = DSIN(BP)  
 0006 COSBP = DCOS(BP)  
 0007 SINHP = DSINH(BP)  
 0008 COSHP = DCOSH(BP)  
 0009 ALP = PHI/ALPHA  
 0010 GAMMA = 1.0 - 2.0 \* PMU(M)  
 0011 ABG = ALP \* BETA1 \* GAMMA  
 0012 SINABG = DSIN(ABG)  
 0013 COSABG = DCOS(ABG)  
 0014 SIHABG = DSINH(ABG)  
 0015 COHABG = DCOSH(ABG)  
 0016 ALPHA4 = ALPHA\*\*4  
 0017 PSOA = ALPHA4 \* (ALP\*\*2)  
 0018 PCUA = ALPHA4 \* (ALP\*\*3)

C

0019 DO 1 I = 1, 8  
 0020 DO 1 J = 1, 8  
 0021 A(I, J) = 0.0  
 0022 1 CONTINUE

C

0023 A(1, 1) = SINGP - SINHP  
 0024 A(1, 2) = COSBP - COSHP  
 0025 A(1, 4) = -1.0  
 0026 A(1, 6) = -1.0

C

0027 A(2, 1) = COSBP - COSHP  
 0028 A(2, 2) = -SINGP - SINHP  
 0029 A(2, 3) = -ALP  
 0030 A(2, 5) = -ALP

C

0031 A(3, 1) = -SINGP - SINHP  
 0032 A(3, 2) = -COSBP - COSHP  
 0033 A(3, 4) = PSOA  
 0034 A(3, 6) = -PSOA

C

0035 A(4, 1) = -COSBP - COSHP  
 0036 A(4, 2) = SINGP - SINHP  
 0037 A(4, 3) = PCUA  
 0038 A(4, 5) = -PCUA

C

0039 A(5, 3) = SINABG  
 0040 A(5, 4) = COSABG  
 0041 A(5, 5) = SIHABG  
 0042 A(5, 6) = COHABG  
 0043 A(5, 7) = -SINRP  
 0044 A(5, 8) = -SINHBP

C

0045 A(6, 3) = ALP \* COSABG  
 0046 A(6, 4) = -ALP \* SINABG  
 0047 A(6, 5) = ALP \* COHABG  
 0048 A(6, 6) = ALP \* SIHABG  
 0049 A(6, 7) = COSDP  
 0050 A(6, 8) = COSHP

C

0051 A(7, 3) = -FSOA \* SINABG  
 0052 A(7, 4) = -PSOA \* COSABG  
 0053 A(7, 5) = PSOA \* SIHABG  
 0054 A(7, 6) = PSGA \* COHABG  
 0055 A(7, 7) = SINRP  
 0056 A(7, 8) = -SINHBP

C

0057 A(8, 3) = -PCUA \* COSABG  
 0058 A(8, 4) = PCUA \* SINABG  
 0059 A(8, 5) = PCUA \* COHABG  
 0060 A(8, 6) = PCUA \* SIHABG  
 0061 A(8, 7) = -COSBP  
 0062 A(8, 8) = CUSHBP

C

0063 RETURN  
 0064 END



```

0001 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 DIMENSION A(8, 8), PMU(29)

```

```

C
0004      BP = BETA1 * PMU(M)
0005      SINBP = DSIN(BP)
0006      COSBP = DCOS(BP)
0007      SINHP = DSINH(BP)
0008      COSHP = DCOSH(BP)
0009      ALP = PHI/ALPHA
0010      GAMMA = 1.0 - 2.0 * PMU(M)
0011      ABG = ALP * BETA1 * GAMMA
0012      SINABG = DSIN(ABG)
0013      COSABG = DCOS(ABG)
0014      SIHABG = DSINH(ABG)
0015      COHABG = DCOSH(ABG)
0016      ALPHA4 = ALPHA**4
0017      PSQA = ALPHA4 * (ALP**2)

```

```

C
0018      DO 1 I = 1, 5
0019      DO 1 J = 1, 5
0020      A(I, J) = 0.0

```

1 CONTINUE

```

C
0022      A(1, 1) = (2.0 - 2.0 * COSBP * COSHP) / (COSBP - COSHP)
0023      A(1, 2) = -ALP
0024      A(1, 4) = -ALP
C
0025      A(2, 1) = (SINBP * COSHP - COSBP * SINHP) / (COSBP - COSHP)
0026      A(2, 3) = PSQA

```

```

C
0027      A(3, 2) = SINABG
0028      A(3, 3) = COSABG - COHABG
0029      A(3, 4) = SIHABG

```

```

C
0030      A(4, 2) = ALP * COSABG
0031      A(4, 3) = -ALP * (SINABG + SIHABG)
0032      A(4, 4) = ALP * COHABG

```

```

C
0033      A(4, 5) = (COSBP * SINHP - SINBP * COSHP) / SINHP

```

```

C
0034      A(5, 2) = -PSQA * SINABG
0035      A(5, 3) = -PSQA * (COSABG + COHABG)
0036      A(5, 4) = PSQA * SIHABG
0037      A(5, 5) = 2.0 * SINBP

```

```

C
0038      RETURN
0039      END

```



```

0001 SUBROUTINE ARRAY(BETA1, M, N, PMU, PHI, ALPHA, A)
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 DIMENSION A(8, 8), PMU(29)

```

C

```

0004 BP = BETA1 * PMU(M)
0005 SINBP = DSIN(BP)
0006 COSBP = DCOS(BP)
0007 SINHP = DSINH(BP)
0008 COSHP = DCOSH(BP)
0009 ALP = PHI/ALPHA
0010 GAMMA = 1.0 - 2.0 * PMU(M)
0011 ABG = ALP * BETA1 * GAMMA
0012 SINABG = DSIN(ABG)
0013 COSABG = DCOS(ABG)
0014 SIHABG = DSINH(ABG)
0015 COHABG = DCOSH(ABG)
0016 ALPHA4 = ALPHA**4
0017 PSQA = ALPHA/4 * (ALP**2)

```

C

```

0018 DO 1 I = 1, 5
0019 DC 1 J = 1, 5
0020 A(I, J) = 0.0
0021 I CONTINUE

```

C

```

0022 A(1, 1) = (2.0 - 2.0 * COSBP * COSHP) / (COSBP - CCOSHP)
0023 A(1, 2) = -ALP
0024 A(1, 4) = -ALP

```

C

```

0025 A(2, 1) = (SINBP * COSHP - COSBP * SINHP) / (COSEP - COSHP)
0026 A(2, 3) = PSQA

```

C

```

0027 A(3, 2) = SINABG
0028 A(3, 3) = CCSABG - CCHAEG
0029 A(3, 4) = SIHABG

```

C

```

0030 A(4, 2) = ALP * CCSABG
0031 A(4, 3) = -ALP * (SINABG + SIHABG)
0032 A(4, 4) = ALP * CCHAEG

```

C

```

0033 A(4, 5) = (2.0 + 2.0 * COSEP * COSHP) / (COSEP + COSHP)

```

C

```

0034 A(5, 2) = -PSQA * SINABG
0035 A(5, 3) = -PSQA * (CCSABG + COHABG)
0036 A(5, 4) = PSQA * SIHABG
0037 A(5, 5) = 2.0 * (SINBP * COSHP - COSBP * SINHP) / (COSBP + COSHP)

```

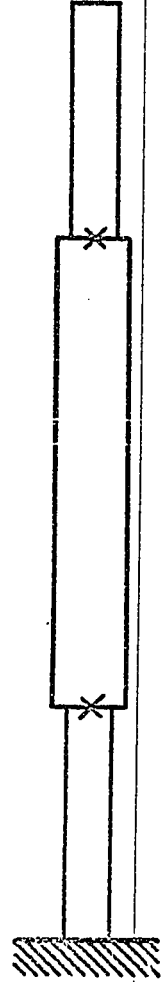
C

```

0038 RETURN
0039 END

```

188



0001 SUBROUTINE AKRAY(BETA1, M, N, PMU, PHI, ALPHA, A)  
 0002 IMPLICIT REAL\*8(A-H, O-Z)  
 0003 DIMENSION A(8, 8), PMU(29)

C

0004 BP = BETA1 \* PMU(M)  
 0005 SINBP = DSIN(BP)  
 0006 COSBP = DCOS(BP)  
 0007 SINHP = DSINH(BP)  
 0008 COSHP = DCOSH(BP)  
 0009 ALP = PHI/ALPHA  
 0010 GAMMA = 1.0 - 2.0 \* PMU(M)  
 0011 ABG = ALP \* BETA1 \* GAMMA  
 0012 SINABG = DSIN(ABG)  
 0013 COSABG = DCOS(ABG)  
 0014 SIHABG = DSINH(ABG)  
 0015 COHABG = DCOSH(ABG)  
 0016 ALPHA4 = ALPHA\*\*4  
 0017 PSQA = ALPHA4 \* (ALP\*\*2)

C

0018 DO 1 I = 1, 5  
 0019 DO 1 J = 1, 5  
 0020 A(I, J) = 0.0

0021 1 CONTINUE

C

0022 A(1, 1) = (COSBP\*SINHBP - SINBP\*COSHBP)/SINHBP  
 0023 A(1, 2) = -ALP  
 0024 A(1, 4) = -ALP

C

0025 A(2, 1) = -SINBP  
 0026 A(2, 3) = PSQA

C

0027 A(3, 2) = SINABG  
 0028 A(3, 3) = COSABG - COHABG  
 0029 A(3, 4) = SIHABG

C

0030 A(4, 2) = ALP \* COSABG  
 0031 A(4, 3) = -ALP \* (SINABG + SIHABG)  
 0032 A(4, 4) = ALP \* COHABG

C

0033 A(4, 5) = (2.0 + 2.0 \* COSBP \* COSHP) / (COSEP + COSHBP)

C

0034 A(5, 2) = -PSQA \* SINABG  
 0035 A(5, 3) = -PSQA \* (COSABG + COHABG)  
 0036 A(5, 4) = PSQA \* SIHABG  
 0037 A(5, 5) = 2.0 \* (SINBP \* COSHBP - COSBP \* SINHBP) / (COSEP + COSHBP)

C

0038 RETURN  
 0039 END

189



BIBLIOGRAPHY

A. General References

1. Flügge, W. (ed.). Handbook of Engineering Mechanics. New York: McGraw-Hill Book Co., 1962.
2. Meirovitch, L. Analytical Methods in Vibrations. New York: Macmillan Co., 1967.
3. Timoshenko, S. Vibration Problems in Engineering. Princeton: D. van Nostrand Co., Inc., 1955.
4. Volterra, E., and Zachmanoglou, E.C. Dynamics of Vibration. Columbus: Charles E. Merrill Books, Inc., 1965.

B. Collected Papers

5. Ayre, R.S., and Jacobsen, L.S. "Natural Frequencies of Continuous Beams of Uniform Span Length", Journal of Applied Mechanics, vol. 17 (1950), pp. 391-395.
6. Gorman, D.J. "Free Lateral Analysis of Double Span Uniform Beams", To be published in International Journal of Mechanical Sciences.
7. Inglis, C.E. "Natural Frequencies and Modes of Vibration in Beams of Non-Uniform mass and Section", Proceedings of the Institution of Naval Architects, March 1929, pp. 145-166.

C. Computer Methods.

8. Kuo, S.S. Computer Applications of Numerical Methods. Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1972.
9. McCracken, D.D. FØRTRAN with Engineering Applications. New York: John Wiley & Sons, Inc., 1967.